DESIGN RESEARCH ON MATHEMATICS EDUCATION: DEVELOPING STUDENTS' RELATIVE THINKING IN SOLVING PROPORTIONAL PROBLEMS

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Educaion (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By
WISNU SIWI SATITI
NIM 06122802003

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
JUNE 2014

APPROVAL PAGE

Research Title : Design Research on Mathematics Education: Developing Students' Relative Thinking in Solving Proportional Problems

Student Name : Wisnu Siwi Satiti

Student Number : 06122802003

Study Program : Magister of Mathematics Education/ International Master Program on Mathematics Education (IMPoME)

Approved by:

Prof. Dr. Zulkardi, M.I.Komp., M.Sc. Supervisor I	Dr. Yusuf Hartono, M. Sc. Supervisor II
Head of Mathematics Education Department	Dean of Faculty of Teacher Training and Education
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. NIP 196104201986031002	Sofendi, M.A., Ph.D. NIP 196009071987031002

Date of Approval: June , 2014

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NIM 06122802003

Approved by Examination Committee	Signature
1. Prof. Dr. Zulkardi, M.I.Komp., M.Sc. NIP 196104201986031002	------------------------
2. Dr. Yusuf Hartono, M.Sc. NIP 196411161990031002	------------------------
3. Dr. Ratu Ilma Indra Putri, M.Si. NIP 196908141993022001	------------------------
4. Dr. Darmawijoyo NIP 196508281991031003	------------------------
5. Dr. Somakim, M.Pd. NIP 196304061991031003	------------------------

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
JUNE 2014

STATEMENT PAGE

I hereby:

Name	: Wisnu Siwi Satiti
Place of birth	: Trenggalek
Date of birth	: August 06, 1988
Academic Major	: Mathematics Education

State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

Palembang,   July 2014
The one with the statement

Wisnu Siwi Satiti
NIM 06122802003

DEDICATION PAGE

I dedicated this thesis with great love to my mother- my first teacher.

She teaches me to dream big and work hard, to make a better future.

For her patience, never ending support and for her love (that never been spoken to me). Thank you Mama, the one who always understand.

Also, this thesis is dedicated to my great-childhood family: my father, my brother, my grandparents, my uncles, my aunts and my cousins for their great supports for my study.

Thank you.

ABSTRACT

Concept of proportions is not only fundamental to educational topics, but it also essential for everyday competences. However, proportional reasoning is difficult for many students, even it is not easy for adults. Students often find it hard to determine an appropriate reasoning to be used in a certain situation. One common error that many students should encounter is the misuse of absolute comparison in situation requiring relative comparison. Many students compare the situations in partial way without recognizing relative relationship between data. They tend to compare the absolute value instead of considering the proportional relationship of data. Thus, we need a learning design that can support the development of students' relative thinking in solving problems on proportions. Moreover, the present study is aimed to contribute to developing a local instruction theory about the domain as well as to identify how to support students developing relative thinking in solving problems on proportions. Consequently, design research is chosen as an approach to achieve the research aims. In addition, we designed the teaching-learning activities based on the tenets of *Pendidikan Matematika Realistik Indonesia* (PMRI) that was adapted from Realistic Mathematics Education (RME). Thirty nine students and one teacher of 5^{th} grade elementary school in Palembang, Indonesia, were involved in this study. The analysis of data collection of the learning process in the classroom show that the learning designed involving contextual-comparison problems may promote students' relative thinking in solving problems on proportion.

Keywords: Relative thinking, Proportional reasoning, Comparison tasks, RME, Design research

ABSTRAK

Konsep proporsi tidak hanya fundamental bagi materi pelajaran di sekolah, tetapi topik ini juga esensial di dalam kemampuan hidup sehari-hari. Akan tetapi, banyak siswa mengalami kesulitan di dalam bernalar sesuai dengan konsep proporsi, bahkan konsep ini juga tidak mudah bagi orang dewasa sekalipun. Para siswa seringkali mengalami kesulitan dalam menentukan cara penyelesaian yang sesuai untuk masalah tertentu. Salah satu kesalahan yang biasa dilakukan oleh siswa adalah mereka sering menerapkan perbandingan absolute padahal seharusna mereka menyelesaikan masalah dengan menggunakan perbandingan relatif. Terlebih lagi banyak siswa yang hanya menggunakan data atau informasi dari soal secara parsial tanpa memperhatikan keterkaitan antara data atau informasi. Oleh karena itu, diperlukan desain pembelajaran yang mampu mendukung berkembangnya kemampuan bernalar relatif pada siswa untuk menyelesaikan masalah yang melibatkan konsep proporsi. Selain itu, studi ini juga bertujuan untuk memberikan kontribusi terhadap pengembangan *local instruction theory* pada domain topik proporsi dan mendukung berkembangnya kemampuan bernalar relatif pada siswa untuk menyelesaikan masalah yang melibatkan konsep proporsi. Oleh karena itu, kami memilih design research sebagai pendekatan di dalam melaksanakan penelitian guna mencapai tujuan penelitian. Selain itu, di dalam membuat aktivitas belajar mengajar, kami menerapkan karaketristik dari Pendidikan Matematika Realistik Indonesia (PMRI). Penelitian ini mengikutsertakan 39 siswa kelas 5 dan seorang guru dari kelas yang bersangkutan di sekolah dasar di kota Palembang, Indonesia. Berdasarkan analisis terhadap data yang diperoleh dari kegiatan belajar mengajar, kami menyimpulkan bahwa desain pembelajaran yang terintegrasikan ke dalam permasalahan kontekstual dan permasalahan perbandingan (dengan menggunakan konsep proporsi) mampu meningkatkan kemampuan bernalar relatif siswa di dalam menyelesaikan masalah tentang proporsi.

Kata kunci: Bernalar relatif, Penalaran konsep proporsi, Masalah perbandingan, PMRI, *Design research*

SUMMARY

DESIGN RESEARCH ON MATHEMATICS EDUCATION: DEVELOPING STUDENTS' RELATIVE THINKING IN SOLVING PROPORTIONAL PROBLEMS
Scientific Paper in the form of Thesis, June 2014

Wisnu Siwi Satiti; Supervised by Prof. Dr. Zulkardi, M.I Komp., M.Sc. and Dr. Yusuf Hartono

xxi + 205 Pages, 9 tables, 58 Figures, 12 Attachments

Proportional reasoning is not only fundamental for mathematics topic (Ben-Chaims, *et al*. 1998) but, more importantly, it also essential for everyday application of numeracy (Hilton, *et al*. 2013). Bright, *et al*. (2003) explained that proportional reasoning includes the use of ratios in comparison quantities and it needs relational thinking. Despite of the importance of proportional reasoning in educational field and everyday life competence, Hilton, *et al.* (2013) found out that students often experience difficulty to recognize an appropriate reasoning to be used in a particular situation, one of them is the incorrectly used of absolute comparison in situation requiring relative comparison. The above finding is aligned to Van De Walle's, *et al*. (2010) findings that difficulties in identifying multiplicative or relative relationship is one of students' common proportional reasoning difficulties. Meanwhile, Lamon (2006) claimed that an ability to analyse situation in absolute and relative perspective is one of the most important types of thinking required for proportional reasoning.

In order to solve comparison problems, a student should compare two values of the intensive variable computed from the data (Karplus, *et al*., 1983), which is in line with what was stated by Sumarto *et al*. (2014). Therefore, in solving comparison problems, a student should consider proportional relationship between numbers or values instead of comparing the absolute value. For that reason, within a design research, we developed a learning trajectory on proportions involving comparison tasks, which may support students to understand different interpretations of proportional situations. Therefore, it may be beneficial to provide comparison problems that bring out different interpretations, and that helps students to understand that relative interpretation by employing concepts of proportions is the most appropriate.

This study consists of two cycles. First cycle is a pilot teaching experiment in which instructional design is tested with 9 students of 5th grade. The implementation of first cycle is aimed to know students' preliminary knowledge and to test the initial instructional design (students' worksheets, hypothetical learning trajectory, and teacher guide). The result of first cycle is analysed in

order to determine which parts or which contents of the design that are needed to be improved before being implemented into real teaching experiment (second cycle). In the first cycle, the class was taught by the researcher itself.

The second cycle is a real teaching experiment conducted in an actual teaching and learning environment, which was class 5F of SD YSP Pusri Palembang, Indonesia. The second cycle is carried out to collect data to be used in answering the proposed research questions. The experimental classroom is different from the class of pilot experiment. Moreover, the class of teaching experiment is taught by an actual teacher of it and involving all of students of that class.

Learning process in both cycles take 2x35 minutes each. There is a learning series consisting four learning activities, which is used in different lesson. The learning activities were designed based on the principles of *Pendidikan Matematika Realistik Indonesia (PMRI)* that is adapted from Realistic Mathematics Education (RME). As characteristic of PMRI, the learning activities include contextual problems, emphasizing on interactivity of students, the use of students' contributions in developing learning process and the intertwining between topic of proportions with another mathematics topic and daily situations.

According to result of the first cycle, the researcher improvs the details of the material, such as the chosen number for the problems, the words chosen for scaffolding and providing follow up and probing questions. Besides that, it is provided preliminary activities involving familiar situation for analogy (it has been adjusted in the HLT in chapter 4), the change of context for learning activity 2, and the switch of the mathematics activity for learning activity 3 and 4. After being refined, the learning series is as follows.

Learning activity 1 is about density-comparison task. Different sense of density emerged from a certain situations will lead somebody to give different interpretation. For instance, when a student is asked to determine which room is more crowded if there are two rooms in the same size occupied by different number of children, the student will determine room with more children is more crowded. But, when the student is asked to determine which room is more crowded if there are two rooms in different size occupied by different number of children, they may interpret the situation in different way instead of comparing the room size or the number of children occupying the rooms. Therefore, density-comparison task provides a helpful starting point for learning the proposed topic.

Learning activity 2 is about grasping the idea of part-to-whole (ratio) relationship in continuous quantity. It is easier for people to grasp the idea of part-whole relationship in continuous quantity. Therefore, it gives a helpful starting point to understand proportional relationship between numbers in term of part-to-whole. In learning activity 2 there is a context about three roads which are being asphalted, each each road has different amount of the asphalted part and the three roads have different length. The students are asked to determine which road that its asphalting project is mostly done. Learning activity 3 is about how to use given result of Dart games played by four children, which each child has different chance of shooting and score, in order to determine the most skilful player. The goals of learning activity 3 are supporting students to interpret the situation in relative perspective by grasping proportional relationship of discrete quantities and the students may use fraction to solve the given problem. Learning activity 4 manifests students' knowledge and experiences acquired at previous lesson to interpret the given survey data on students' interest, whether the students may compare the data partially without considering the relation among data (absolute thinking) or they may use the concept of proportionality.

In accordance with the abovementioned background of the study, the aims of this study is facilitating the development of 5th grade students' relative thinking in solving problems on proportions as well to contribute to the development of local instruction theory. Hence, researcher proposed a main research question as: "*How can we support students in developing relative thinking in solving problems on proportions?*"

To address the main research question, firstly the researcher answer the sub research questions according to findings of the second cycle as follows:

"*How do the 5th grade students use their initial understanding to solve proportional-comparison problems?*"

According to data analysis, we conclude that students may have different level of initial proportional reasoning ability. Most students used data in partial way and they compare the absolute value from the situations. However, several students use relative comparison by employing concept of proportionality in a simple way, by determining the number of things per one unit of measurement as elaborated by Sumarto *et al*. (2014) in their study. Furthermore, the comparison tasks help students to understand there is a set of quantities in a proportional situation that altogether affect the comparison. Moreover, the students also grasp the idea to use proportions and determine which proportion gives the largest fraction to solve comparison problems involving part-to-whole relationship.

"*How can proportional-comparison problems promote students' relative thinking?*"

Sumarto *et al*. (2014) stated that all information in comparison problems altogether influences the comparison, so we should not use the information in partial way. They added that in solving comparison tasks, ones should compute a set of numbers representing each situation, and ones have to determine which proportion will give a good comparison. The above statements are aligned with Karplus's, *et al*. (1983) ideas. All of those ideas about solving comparison tasks are aligned to the idea of relative thinking.

Based on data analysis on the learning process, we conclude that proportional-comparison tasks may promote students' relative thinking. At the first time, the proposed comparison problems lead students to come to different interpretation (absolute way or the use of relative comparison), and that relative interpretation by employing the concept of proportionality is the most appropriate. Furthermore, comparison tasks including proportions in term of part-to-whole relationship facilitate students to see the relative relationship of part and the whole.

"*How can the bar model as a visualization of proportional situations support students in developing relative thinking?*"

Based on the findings in this study, the students tended to works with numbers only. Model that is emerged from the situation seems to be visualization only and the use of it may not support the development of students' relative thinking in some extent.

Based on the answer to the sub research questions, we formulate the answer for the main research question as follow:

Boyer and Levine (2012) agreed that students' formal mathematics understanding about proportion can be fostered by using instructional activities that is built on students' intuitive understanding. Based on data analysis, in general, students have the sense of relative thinking by employing concept of proportionality in solving problems on comparison. Students' initial recognizing of relative comparison is represented on the use of different strategies in solving the task. Moreover, in comparing density the students may understand that they should not just compare the number of population or the size of the space, but they should consider the relationship of the number of population and the size of space. For that reason, we conclude that utilizing the initial ability and intuitive understanding may support students in developing relative thinking as well supporting them in solving problems on proportions. Therefore, in general, we conclude that comparison problems and empowering students' initial understanding together with teacher supports and the appropriate set up of the learning process may support students in developing relative thinking in solving problems on proportions.

Keywords : Relative thinking, Proportional reasoning, Comparison tasks, RME, Design research

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RINGKASAN

DESIGN RESEARCH ON MATHEMATICS EDUCATION: DEVELOPING STUDENTS' RELATIVETHINKING IN SOLVING PROPORTIONAL PROBLEMS
Karya tulis ilmiah berupa Tesis, Juni 2014

Wisnu Siwi Satiti; Dibimbing oleh Prof. Dr. Zulkardi, M.I Komp., M.Sc. dan Dr. Yusuf Hartono
xxi + 205 halaman, 9 tabel, 58 gambar, 12 lampiran

Proportional reasoning (kemampuan bernalar menggunakan konsep proporsi) tidak hanya fundamental untuk topic matematika (Ben-Chaims, *et al*. 1998), akan tetapi kemampuan ini esensial bagi kemampuan berhitung di dalam kehidupan sehari-hari (Hilton, *et al*. 2013). Bright, *et al*. (2003) menjelaskan bahwa *proportional reasoning* melibatkan penggunaan rasio untuk membandingkan kuantitas dan hal ini melibatkan relational thinking.. Meskipun *proportional reasoning* penting bagi topic pelajaran di sekolah dan kemampuan hidup sehari-hari, Hilton, *et al.* (2013) menemukan bahwa para siswa sering mengalami kesulitan untuk mengidentifikasi penalaran yang tepat digunakan untuk masalah tertentu, salah satu diantaranya yaitu siswa menggunakna pola pikir absolute untuk menyelesaikan masalah yang menuntut pola pikir relatif. Penemuan ini sesuai dengan penemuan Van De Walle's, *et al*. (2010) yang mengindikasikan bahwa kemampuan untuk menganalisis keterkaitan yang bersifat relatif antar data merupakan kesulitan yang umum dihadapi siswa. Padahal Lamon's (2006) menekankan bahwa kemampuan menganalisis situasi dari sudut pandang absolute dan relatif adalah salah kemampuan berpikir yang sangat penting dan dibutuhkan untuk *proportional reasoning*.

Untuk menyelesaikan masalah perbandingan, seseorang harus membandingkan dua nilai yang dihitung dari suatu data (Karplus, *et al*., 1983), yang mana hal ini sesuai dengan apa yang dinyatakan oleh Sumarto *et al*. (2014). Oleh karena itu, untuk menyelesaikan masalah proporsi, seorang siswa harus memperhatikan hibingan proporsional antar bilangan dari suatu situasi daripada sekedar membandingan nilai absolute. Untuk itu, dengan design research, kami mengembangkan suatu learning trajectory pada materi proporsi dengan menggunakan permasalahan perbandingan (*comparison tasks*), yang diharapkan mampu membantu siswa dalam memahami perbedaan interpretasi terhadap situasi proporsional. Oleh karena itu, hal ini mungkin cukup membantu jika siswa bekerja ada masalah perbandingan yang memunculkan perbedaan interpretasi dan membantu siswa untuk memahami bahwa interpretasi dengan menggunakan sudut pandang relatif dengan menerapkan konsep proporsi merupakan cara penyelesaian masalah yang paling sesuai.

Studi ini terdiri dari dua siklus. Siklus pertama melibatkan kelas percobaan yang terdiri dari 9 siswa kelas 5 sekolah dasar yang mana peneliti melakukan test terhadap desain pembelajaran awal. Pelaksanaan dari siklus pertama ditujukan untuk

mengetahui sejauh mana kemampuan awal siswa pada topic yang dipelajari. Selain itu, siklus pertama juga bertujuan untuk mencoba desain pembelajaran awal (LKS, hypothetical learning trajectory, panduan guru). Hasil dari sklus pertama digunakan untuk menentukan bagian mana dari desain yang harus diubah atau diperbaiki sebelum desain diterapkan pada experiment yang sesungguhnya di siklus 2. Di sklus pertama, kelas eksperiment diajar oleh peneliti.

Siklus kedua merupakan experiment yang sesungguhya yang dilaksanakan di kelas, yaitu kelas 5F SD YSP Pusri Palembang, Indonesia. pelaksaan siklus kedua ditujukan untuk mengumpulkan data guna menjawab rumusan masalah.kelas experiment berbeda dengan kelas percobaan. Kelas experiment melibatkan seluruh siswa dan diajar oleh guru asli dari kelas tersebut.

Proses pembelajaran di kedua siklus dilaksakan selama 2x35 menit untuk setiap pertemuan. Aktivitas belajar di suatu pertemuan berbeda dengan aktivitas pembelajaran di pertemuan yang lain. Aktivitas pembelajaran dibuat dengan menerapkan konsep *Pendidikan Matematika Realistik Indonesia (PMRI),* yaitu menggunakan masalah kontekstual, menekan kan pada interaksi siswa, menggunakan kontribusi siswa dan adanya keterkaitan antara topic utama yang dipelajari dengan topic matematika lain atau adanya keterkaina dengan situasi dari kehidupan sehari-hari.

Berdasarkan temuan di siklus pertama, peneliti melakukan beberapa perbaikan pada detail dari bahan ajar, diantaranya pemilihan angka pada soal, pilihan kata untuk pertanyan eksplorasi, penggunaan pertanyaan *follow up* . Selain itu, di siklus kedua, diberikan kegiatan perliminari sebelum pembelajaran inti yang melibatkan situasi yang familiar bagi siswa sebagai analogy untuk mempermudah siswa memasuki permasalahn utama. Selain itu, konteks pada kegiatan pembelajaran 2 diubah dan pertukana antara aktivitas pada kegiatan pembelajaran 3 dan 4. Berikut ini adalah kegiatan pembelajaran setelah dilakukan perbaikan dan penyesuaian.

Kegiatan pembelajaran 1 yaitu tentang membandingkan tingkat penuh atau sesak (kepadatan). Perbedaan persepsi tentang kepadatan akan mengarahkan siswa untuk menginterpretasi kepadatan secara berbeda. Contohnya, jika siswa diminta menentukan ruangan mana yang lebih penuh/sesak jika ada dua ruangan berukuran sama dan salah satu ruangan ditempati oleh lebih banyak anak, maka otomatis siswa tersebut akan menentukan bahwa ruangan yang ditempati lebih banyak anak adalah yang lebih penuh atau sesak. Hal ini menunjukkan bahwa siswa tersebut memperhatikan banyak absolute dari orang. Akan tetapi, siswa mungkin akan memberikan interpretasi berbeda jika mereka diminta untuk menentukan ruangan mana yang lebih penuh/sesak jika ada dua ruangan berbeda ukuran dan banyak anak yang menempati ruangan tersebut juga berbeda. Para siswa mungkin tidak sekedar membandingkan banyak anak yang menempati ruangan atau membandingkan ukuran ruangan. Oleh karena itu, maalah perbandingan yang melibatkan perbandingan penuh/sesak (kepadatan) memberikan awalan yang cukup membantu siswa untuk mempelajari topic utama.

Kegiatan pembelajaran 2 melibatkan konsep keterkaitan antara bagian dan keseluruhan yang dapat dinyatakan sebagai rasio, dan rasio ini merupakan rasio dari besaran kontinyu. Pada umumnya, seseorang mudah memahami adanya keterkaitan dalam bentuk bagian dari keseluruhan pada besaran kontinyu. Oleh karena itu, hal ini memberikan awalan yang bagus untuk membantu siswa memahami keterkaitan antara bagian dengan keseluruhan pada sebaarang besaran, termasuk besarn diskrit. Kegiatan pembelajaran 3 adalah tentang bagaimana siswa menggunakan hasil dari permaina Dart untuk menentukan pemain paling mahir. Ada 4 anak bermain Dart, setiap anak memilik total kesempatan menembak yang berbeda dan mereka mencetak skor yang berbeda pula. Kegatan pembelajaran 3 ditujukan untuk membantu siswa dalam menginterpretasi situasi dalam sudut pandang relatif dengan memahami keterkaitan proportional pada besaran diskret dan membantu siswa untuk memahami bahwa mereka dapat menyelesaikan masalah perbandingan dengan menggunakan pecahan sebagai alat. Kegiatan pembelajaran 4 merupakan manifestasi dari pengetahuan adan pengalaman yang diperoleh siswa dari ketiga pembelajaran sebelumnya. Di kegiatan 4 siswa akan menggunakan data dari suatu survey, apakah siswa membandingkan data secara parsial dan absolute, ataukah soswa akan menggunakan konsep proporsi di dalam membandingkan data.

Untuk mencapai tujuan dari studi yang dilaksanakan, kami mengajukan rumusan masalah utama sebagai berikut: *How can we support students in developing relatif thinking in solving problems on proportions?* (bagaimana kami dapat memberikan dukungan bagi siswa di dalam mengembangan kemampuan bernalar relatif untuk menyelesaikan masalah proporsi). Selain tiu dirumuskan pula tiga sub rumusan masalah sebagai berikut:

"*How do the 5th grade students use their initial understanding to solve proportional-comparison problems?* (bagaimana siswa kelas 5 SD menggunakan kemampuan awal mereka untuk menyelesaikan masalah perbandingan pada topic proporsi)".

Berdasarkan data yang diperoleh, kami menyimpulkan bahwa dimungkinkan siswa memiliki kemampuan awal yang berbeda. Sebagian besar siswa menggunakan data pada soal secara parsial dan membandingkan data secara absolute. Akan tetapi ada juga siswa yang membandingkan daa secara relatif dengan menggunakan konsep proporsi secara sederhana dengan menggunakan metode satuan (menentukan banyaknya sutau obyek persatuan ukuran). Selain itu, masalah perbandingan membantu siswa untuk memahami bahwa ada satu grup nilai dari suatu situasi yang mana nilai-nilai tersebut merupakan satu kesatuan yang secara bersama-sama mempengaruhi perbandingan. Selanjutnya siswa memahami bahwa di dalam membandingkan situasi mereka dapat menggunakan konsep proposri dengan menentukan proporsi dari situasi mana yang memberikan pecahan terbesar (dalam hal ini proporsi direpresentasikan ke dalam bentuk pecahan).

"*How can proportional-comparison problems promote students' relatif thinking?*(bagaimana masalah perbandingan pada topic proporsi mendorong siswa untuk bernalar relatif)".

Sumarto *et al*. (2014) menyatakan bahwa semua informasi pada masalah perbandingan bersama-sama mempengaruhi perbandingan tersebut, sehingga tidak seharusnya kita menggunakan data secara parsial. Mereka juga menambahkan bahwa untuk menyelesaikan masalah proporsi, sesorang harus menghitung satu grup dari bilangan yang merepresentasikan masing-masing situasi. Hal di atas sesuai dengan ide yang disampaikan oleh Karplus, *et al*. (1983). Dengan demikian, penalaran relatif berlaku untuk emnyelesaikan masalah perbandingan.

Berdasarkan analisis data, kami menyimpulkan bahwa masalah perbandingan meningkatkan kemampuan siswa dalam berpikur relatif. Pada awalnya mungkin akan muncul perbedaan interpretasi dari siswa terhadap situasi yang dihadapi (interpretasi secara absolute atau interpretasi seara relatif) akan tetapi siswa akan memahami bahwa interpretasi relatif dengan menggunakan konsep propors adalah yang sesuai. Selain itu, masalah perbandingan yang melibatkan konsep bagian terhadap keseluruhan memfasilitasi siswa untuk melihat keterkaitan relatif antara bagian dengan keseluruhan.

"*How can the bar model as a visualization of proportional situations support students in developing relatif thinking?* (bagaimana model batang sebagai visualisasi dari situasi proporsional membantu siswa mengembangkan pola pikir relatif)".

Berdasarkan temuan dari kegiatan penelitian para siswa cenderung bekerja hanya menggunakan angka. Model yang muncul dari konteks sekedar menjadi visualisasi tetapi kurang membantu sisa untuk mengembangkan pola pikir relatif dalam batas tertentu.

Boyer dan Levine (2012) setuju bahwa pemahaman matematika formal siswa tentang proporsi dapat ditingkatakn dengan menggunakan aktivitas pembelajaran yang dibangun dari pemahaman intuitif siwa. Berdasarkan analisis data, secara umum, para siswa memiliki sense pola pikir relatif dengan menggunakan konsep proporsi. Kemampuan awal dalam mengenali adanya perbandingan secara relatif dapat kita ketahui ketika siswa menggunakan strategi yang berbeda dalam menyelesaikan masalah. Terutama ketika siswa harus membandingkan tempat mana yang lebih penuh atau sesak, pada umumnya siswa memiliki pemahaman intuitif bahwa mereka tidah hanya sekedar membandingkan banyak populasi yang menempati suatu tempat atau membandingkan besar kecilya sutau tempat akan tetapi mereka harus memperhatikan pula keterkaitan antara banyak populasi dengan ukuran daerah yang ditempati. Oleh karena itu, kami menyimpulkan bahwa menggunakan kemampuan awal dan pemahaman intuitif siswa dapat membantu siswa dalam mengembangkan pola pikir relatif. Dengan demikian, secara umum, kami simpulkan bahwa masalah perbandingan dan memberdayakan pemahaman awal siswa bersama dengan dukungan dari guru dan desain pembelajaran yang sesuai akan membantu siswa dalam mengembangkan pola pikir relatif dalam menyelesaikan masalah proporsi.

Kata kunci : Pola pikir relatif, Bernalar konsep proporsi, Soal perbandingan, PMRI, *Design research*
Kepustakaan : 31 (1980-2014)

PREFACE

First of all I would like to express my deepest thanks to my Lord, Allah SWT, and thank you for all your blessing to me so I could finish writing this thesis.

This thesis is written to fulfill the requirements for the degree of Master of Science (M.Sc) in International Master Program on Mathematics Education (IMPoME), conducted by Sriwijaya University, Palembang-Indonesia in collaboration with Utrecht University, Utrecht-The Netherlands. I hope the results of this thesis might contribute to the development of mathematics education, especially the implementation of *Pendidikan Matematika Realistik Indonesia* (PMRI) in Indonesia.

Teachers take a great part in this study. Therefore I'd like to say my deepest gratitude and thank you to a teacher, my very first teacher, my Mother. Thank you for all those lessons, and thank you for never ending support and belief to me. I'd also want to say thank you to my father and my brother, thank you for the great support all this time.

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Palembang, July 2014

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A. CHAPTER 1
INTRODUCTION

1.1.Background

The concept of proportions is fundamental to many topics, not only mathematics, but also physics, chemistry, geography, economics, statistic and technological studies, such as: calculation in engineering, mechanics, robotics, computer science and others (Ben-Chaim, *et al.*, 2012). In line with that, Lamon (2007) claimed that among all of topics in mathematics curriculum; fraction, ratios and proportion are the most challenging and the most essential to success in higher mathematics and science education. However, the concept of proportions is not only useful, but it is also difficult to be mastered (Tourniaire and Pulos, 1985). In mathematics classroom, students experience varying difficulties with different proportional problems (Hilton**,** *et al.*, 2013). Moreover, approximately more than half of population of adults have difficulties in employing appropriate proportional reasoning (Lamon, 2005).

In general, there are two types of problems on proportions, missing value problems and comparison problems (Karplus, *et al.*, 1983; Tournaire and Pulos, 1985; Silvestre and Ponte, 2012). It has been examined in a study conducted by Sumarto, van Galen, Zulkardi and Darmawijoyo (2014) which indicated that comparison problems are more difficult than missing value problems. Missing value problems tend to lend students to algorithmic approach and do not necessarily provide students to apply proportional reasoning (Lesh, *et al.*, 1988). Therefore, it is important to provide a wide range of proportional problem in order to give more opportunities for students to develop proportional reasoning.

Noelting (1980) and Hilton *et al*. (2013) conducted studies to investigate students' proportional reasoning involving comparison problems. Noelting (1980) proposed mixture problem, meanwhile one of problem types used by Hilton *et al*. (2013) is relative thinking associated problems. In Noelting's mixture problems (1980), students were asked to determine which mixture of orange extract and water having a stronger orange taste. Most students didn't discern the appropriate interpretation. They compared the given data in partial way, which was by comparing the amount of orange extract or water only. Hilton *et al*. (2013) also discovered students' difficulties to elicit an appropriate proportional reasoning. In Hilton's *et al*. (2013) study, they also found that students who gave correct responses for the most problems (eight out of twelve problems) were less than 30% and there was less than 20% of total students who solved the relative thinking associated problems correctly.

One of common difficulties faced by students in employing proportional reasoning, which is also found in the studies by Noelting's (1980) and Hilton's *et al*. (2013), is the incorrectly used of absolute comparison in situation requiring relative comparison. In both studies, the problems required students to use relative thinking instead of using the given data or situation in partial way. As stated by Lamon (2006), there are two types of perspectives, absolute and relative perspective, which are important for proportional reasoning.

Other studies (Lamon, 1993; Silvestre and da Ponte, 2012) also point out students' difficulties to recognize relative perspective in proportional problems. This findings support van de Walle's, *et al.,* (2010) conclusion that an inability to identify a relative relationship in proportional situations is one of common errors made by students.

All those studies mentioned above contribute to the development of a theory about the students' relative thinking in solving proportional problems. However, these studies deal more with the theory and contribute less to the practice of teaching. Therefore, it is needed to do a study to develop instructional activities which can be done in the classroom, and can contribute to a local instruction theory. Hence, in this study we will develop a learning trajectory and instructional activities in a design research.

In this study, we will design contexts and instructional activities which support students to elicit their relative thinking in solving proportional problems. Therefore, the instructional activities are aimed to help students grasping a correct perspective to be used in solving problems on proportions, that is interpreting the situations in relative way and the use of concept of proportionality. Moreover, Realistic Mathematics Education (RME) is used as a theoretical framework in designing the instructional activities. Due of that, the aim of this study is to contribute to developing a local instruction theory that supports students to elicit their relative thinking in solving problems on proportions as well as supporting the development of students' proportional reasoning.

1.2. Research Aims

According to the background of the study, the purposes of this study are:

1. To develop a learning instruction that not only support the development of 5th grade students' relative thinking in solving problems on proportions, but also helps the students to develop proportional reasoning.
2. To contribute to the development of local instruction theory in proportions.

1.3. Research Questions

Regarding to the aforementioned background of this study, the researcher proposes the main research question as follows:

How can we support students in developing relative thinking in solving problems on proportions?

To address the main research question, the researcher proposes sub research questions in this study as follows:

1. *How do the 5th grade students use their initial understanding to solve proportional-comparison problems?*
2. *How can proportional-comparison problems promote students' relative thinking?*
3. *How can the bar model as a visualization of proportional situations support students in developing relative thinking?*

CHAPTER 2
THEORETICAL FRAMEWORK

2.1.Proportions

Proportions are common in daily life. People use proportions in many familiar situations, for example in enlarging and reducing photos and copies, price comparisons (van Galen, *et al*., 2008), density, speed and ingredients of a recipe (Karplus, *et al*., 1983). Moreover, a sense of proportions has been developed since an early age, for instance a kindergarten child knows that an adult needs fewer steps for the same distance than a five-year-old girl (Sumarto, 2013).

In school, proportions are used in numerous subject matters. Proportions are involved in mathematics lessons so frequently (van Galen *et al.*, 2008). Proportions are widely used to teach many school subjects such as sciences, geography, economics and statistics, and technological studies, though that is not always explicitly stated (Ben-Chaim, *et al.*, 2012). Furthermore, Lesh, *et al.*, (1988) and Lemon (2007) affirmed that proportions are cornerstone and important to success in higher-level areas of mathematics and science.

Hornby in Oxford Advanced Learner's Dictionary of Current English (1989), defined a proportion as a correct relation in size and degree, between one thing and another or between a part and a whole. Proportions are based on the concept of ratios (van Galen *et al*., 2008). According to Euclid in Grattan-Guinness (1996), a proportion is a comparison between two pairs of ratios. In line with that, Langrall and Swafford (2000) explained that a proportion is a statement in which two ratios are equal in the sense that both express the same relationship. Therefore, a proportion is a statement of equality of two ratios, i.e., $\frac{a}{b} = \frac{c}{d}$ (Tourniaire and Pulos, 1985)

Generally, there are two types of problems on proportion, missing value problems and comparison problems (Karplus, *et al*., 1983; Tournaire and Pulos, 1985; Silvestre and da Ponte, 2012). Tournaire and Pulos (1985) explained that a missing value problem is usually presented with three numbers, ***a*, *b*,** and ***c*,** and the task is to find the unknown $\boldsymbol{x}$ such that $\frac{a}{b} = \frac{c}{x}$. For example, how many cups of flour do we need to make a sticky mess with 6 cups of sugar, if we need 10 cups of flour to make a sticky mess with 4 cups of sugar (Hilton**,** *et al*., 2013). Meanwhile, in comparison problems, a student should compare two values of the intensive variable computed from the data (Karplus, *et al*., 1983).

2.2.Students' difficulties on proportional reasoning

Tourniaire and Pulos (1985) justified that concept of proportion is both useful and difficult to be mastered. Lamon (2005) revealed that more than half of adult populations are not able to use correct proportional reasoning. Moreover, Hilton, *et al*. (2013) found out that students experience varying degrees of difficulties with different proportional problems in mathematics classrooms. The variety of students' difficulties with different type of problems can happen because different types of proportional problems elicit different form of reasoning (Langrell& Swafford, 2000). In this cases, many students get difficulties in determining the appropriate reasoning for particular situation.

A previous study by Sumarto, *et al*., (2014) did investigation on students' proportional reasoning involving relative comparison problems. According to Sumarto, *et al*., (2014), comparison problems are believed to be more difficult than missing value problems. In Sumarto's, *et al*., (2014) study, the students were asked to analyse situations and determine which interpretation, relative or absolute, that is appropriate to be used in solving the comparison problems. Based on result of the study, Sumarto, *et al.*, (2014) concluded that in order to solve comparison problems, one needs to think about the relationship of data relative to other data. They also argued that instead of comparing the absolute values, ones should compute the set number for each situation in order to determine the correct response for solving the proposed problems.

There was a study by Hilton, *et al*. (2013) which focused on developing instruments to assess middle-year students' proportional reasoning. Result of this study showed that students who gave correct responses for the most problems (eight out of twelve problems) were less than 30%. Hilton, *et al*. (2013) reported that their findings aligned with previous studies, which figured out that concept of proportion is difficult for many students. One type of proportional problems which was used in this study was relative thinking associated problems.

This problem presented an end-of-term activity voted by twenty two year-5 and thirteen year-6 students. The result of the vote showed that eight students of year 5 chose the beach, fourteen students of year 5 chose the movies, seven students of year 6 chose the beach, and six students of year 6 chose the movies. The students were asked to determine whether this statement was true or not: going to the beach is relatively more popular choice with the year 6 students than the year 5 students. In this case, the students should compare the given data and identify which activity was more popular. Hilton, *et al*. (2013) explained that this problem required students to use relative comparison. However, the result showed that there were less than 10% of students in year 5 and 7, less than 20% of students in year 6 and 8, and 21.2% of students in year 9 who gave correct response.

There is another comparison problem which is widely known, namely Noelting's Orange Mixture (1980) in which students were asked to compare which mixture of orange extract and water had a stronger orange taste. Most students compared the given data in partial way, i.e. by comparing the amount of orange extract or water only. For instance, when the students had to decide which mixture; A (5 glasses of orange extract and 2 glasses of water) or B (7 glasses of orange extract and 3 glasses of water) had a stronger orange taste; some of them chose that mixture B had a stronger orange taste because there was more orange extract, and other students answered that the orange taste was the same because both mixtures had less amount of water than the amount of orange extract. The students didn't think in terms of a relative relationship between the parts (the amount of orange extract or water) and the whole. As the result, many students came to incorrect solution.

Based on studies conducted by Noelting (1980), Hilton, *et al*. (2013) and Sumarto et al. (2014), it is clear that instead of using absolute comparison, the students should utilize relative perspective in order to figure out correct proportional solution. Relative perspective means relative thinking which is important for proportional reasoning (Lamon, 2006). However, Noelting's (1980) and Hilton's, *et al*. (2013) findings showed it seems difficult for students to elicit relative thinking in solving problems on proportions. Lamon (1993) also found similar result that comparing the growth in relative perspective was the most challenging problem in her study. Silvestre & da Ponte (2012) reinforced that students are likely never have thought deeply about the situation, which they tend to see the problems in absolute perspective by focusing their intention on data in partial way. All of above findings support van de Walle's, *et al*., (2010) conclusion that one of common error in solving proportional problems is an inability to identify relative relationship among numbers or data in proportional situations.

In conclusion, students need to utilize an appropriate interpretation in order to determine the right strategy in solving the problem. Meanwhile, a proportional relationship may indicate relative perspective which become one stumbling blocks for students in solving problems on proportions (Noelting, 1980; Lamon, 1993; Van de Walle, *et al*.,2010; Hilton, *et al*.,2013). Due of that, it is necessary to design an innovative learning trajectory that supports students to elicit relevant proportional perspective which helps them to employ the appropriate proportional method in solving proportional problems. Furthermore, in solving particular comparison problems, it is necessary for students to use relative comparison. Therefore it is needed to design a learning trajectory that supports students to elicit their relative thinking in solving problems on proportions.

2.3.Relative Thinking

Lamon (2006) explained that to reason about problems in proportions, one should differentiate absolute situations and relative situations in order to decide appropriate perspective and reasoning. If someone thinks in a relative way, he/she relates the actual data, number or situation to the other data, for example: if a child analyzes growth of a tree in relative terms, he/she will consider the growth to the initial height, whereas if the child considers the change in an absolute way, he/she will count the difference of the present height and the initial height (the actual growth) only (Lamon, 2006). The notion of relative thinking is in accordance with how people should solve a comparison problem (Sumarto *et al.*, 2014), that is in order to solve comparison problems, one needs to think the relationship between numbers (data) and compute the set numbers for each situation in order to determine which proportion is the correct solution for the problem.

However, relative thinking involves more abstraction than absolute thinking (Lamon, 2006). Meanwhile, this study is aimed at students of the fifth grade who are in the age of 10-11 years. According to Piaget's theory of cognitive development in Wadsworth (1979), a child between the ages of 7 and 11 is in a period of concrete operation. In this period, a child can use logical operations to solve problems involving concrete objects, even though he/she is no longer dominated by perception (Wadsworth, 1979).

In regard to the characteristic of relative thinking and the cognitive phase of aimed students in this study, it is beneficial to use situation which is real and familiar to the students. This situation should support the students grasping the abstraction. In line with that, Lamon (2006) proposes a context which promotes students to grasp the abstraction which is not a perception as much as it is a conception, that is context of density. When ones are asked to determine which room that is more crowded if there are two rooms in the same size occupied by different number of children, ones will determine that room with more children is more crowded. In this case, ones compare the absolute number of children occupying the room. However, when ones are asked to determine which room that is more crowded if there are two rooms in different size occupied by different number of children, they may interpret the situation in different way instead of comparing the room size or the number of children occupying the rooms. Due the different interpretation that might be come up when people compare density, it gives opportunity to use density-comparison task as starting point to help students to understand the different interpretation in the proportional situation, and that proportional interpretation is most appropriate. This is in line with Karplus, *et al*., (1983) who assert that people employ proportional reasoning in context of density.

Moreover, Lamon (2006) also emphasized on the significance of relative thinking in fraction instruction, such as the need to compare fractions relative to the same unit, the meaning of factional numbers (for instance the notion of crowdedness), and the relationship between equivalent fractions ($\frac{3}{5}$ and $\frac{12}{20}$). This indicates that there is a relative relationship in fractional symbolization.

Hence, in this study, we design a learning sequence which employs familiar context and intertwine the relative reasoning with fractions that support the students to elicit relative thinking in solving proportional problems. The use of context and the intertwining among mathematical concept are in accordance with Realistic Mathematics Education (RME). So, the learning sequence is designed based on Realistic Mathematics Education.

2.4.Emergent perspective

Emergent perspective is a framework which can be used as guideline to interpret classroom activities (Cobb & Yackel, 1996). The frameworks can be seen as a reaction toward the subject of attempting in order to understand mathematics learning activities as it happens in the social context of the classroom (Gravemeijer & Cobb, 2006). The framework of learning proportion in this study is students' responses during the learning process in the classroom. And it is supposed that by participating in the lesson the students will be able to develop their knowledge and understanding into more advance level.

While the students are in the class discussion or even while they are in the group discussion, the students will build a negotiation, which doesn't only involving teacher and students, but there will also be negotiations among students. In this learning framework, the students negotiate by explaining their reasoning and justify solution, designate agreement and disagreement, try to make sense of the explanation given by others and pose some possible alternatives (Gravemeijer & Cobb, 2006). Therefore, it is expected the students will participate actively in accordance with the aimed framework of the learning activity.

2.5.(Indonesia) Realistic Mathematics Education Aspect in This Study

As stated at previous explanation, the learning series is designed based on Realistic Mathematics Education (RME). In Indonesian education, there is an adaptation of RME that is *Pendidikan Mathematika Realistik Indonesia*/PMRI (Sembiring, *et al.*, 2010).

In a study conducted by Bakker (2004), it was pointed out that based on Treffers (1987), there are five tenets of RME. The further explanation about how does we utilize those tenets in designing learning sequences to promote students' relative thinking is in the following part.

2.5.2. The use of context

Lamon (2006) explained that our language doesn't supply particular words to ask relative question or absolute question, and the same words may have different meaning in different context or situation. In regard to this fact and the cognitive characteristic of children at grade five (10-11 years-old), it is useful to use contextual problems to be solved in students' activities. Moreover, in order to help students to grasp the abstraction of relative thinking, Lamon (2006) proposed a context that is not only concrete for students, but also promotes students to compare the conception. The context she proposes is context of density (crowdedness).

Therefore, context of density (the crowdedness of chicken boxes) is used as starting point in the learning sequence. This context gives students different feeling of crowdedness and then it asks students to compare the crowdedness which lead them to see the proportional situation in relative perspective. Furthermore, the following learning activities also employ contextual problems, such as a context of favorite extracurricular at school, in which the students should use the given data to derive a decision. The use of context also helps students to grasp the idea that they often use mathematical concepts to solve familiar problems in daily life. Therefore, the use of context will make the mathematics learning becomes meaningful for students.

2.5.2. The use of models

Van Galen *et al.* (2008) emphasized that models are close to context situation, which the use of models is developed from model as representation of situation into models for reasoning. Besides that, models are concrete for students. It is easier for students to work and reason in something which is concrete for them. And then, because the models represent the initial context situations, it is easy for students to relate their works and reasoning to the initial problem by using the models.

The context of space is close to bar model. The bar gives geometrical representation of the space situation. Therefore it is easy for students to represent the situation into a bar. Besides that, the bar offers scrap paper for calculation process (van Galen and van Eerde, 2013), such as the students can determine and mark out how many part of space which is occupied by shading the bar.

The use of bar in this design is developed from representing the situation into using the bar as mathematics tool to grasp the relative relationship among data in the proportional problems. Due to the use of bar in the beginning of learning sequence, it will give students an understanding that they also can represent context in the next activities into bars. The bar also can be used as concrete representation of part-whole relationship which is the main proportional relationship in this design. Van Galen *et al.* (2008) explained that bar model can be used as concrete representation of fractional symbolization. Therefore, the use of bar as model of the situation is beneficial because the bar can be used as representation of the situation, the bar also represents the proportional relationship among data, the bar is a concrete representation of fractional symbolization (ratio) and the bar can supports students to reason proportionally.

2.5.3. The use of students' contribution

In a learning process, teacher should not be the one who judges the acceptable answer. It is important to build a learning activity which gives students wide opportunity to participate and contribute actively because the development of individual's reasoning can't be separated from his/her participation in sharing mathematical meaning (Cobb & Yackel, 1996). Therefore, the students should be encouraged to give contribution in the learning process.

Due of that, the tasks are designed to encourage students to contributes their ideas or strategies in the learning process. This is done by designing open questions (in the task and whole-class discussion) that encourage students to deliver their ideas and reasoning. Furthermore, this design is aimed to support teacher in orchestrating whole-class discussion. Therefore, this design is equipped with teacher guide to help teacher in conducting learning activities that encourage students to contribute their ideas.

2.5.4. Interactivity

Van den Heuvel-Panhuizen (2001) highlighted that mathematics learning is a social process. Therefore, the tasks in this learning series are designed as working group activities. Besides that, during the working group, the teacher also interacts with the students in some extent. Furthermore, the establishing of whole-class discussion give wider opportunity for teacher and students (and among the students themselves) to interact each other. In the whole-class discussion, the student interacts each other by giving explanation, asking question, criticizing, arguing and supporting other reasoning in order to gain the correct understanding.

2.5.5. Intertwining

Proportion can be described as fractions, percentages, and decimals (van Galen *et al.*, 2008). Besides that, fractional numbers might imply a relative relationship, such as part-whole relationship (Lamon, 2006). It indicates that relative relationship of proportional situation can be represented as fractions. Furthermore, proportional reasoning involves the use of ratios in comparing quantities (Bright, *et al*., 2003), and ratios can be represented as fraction, i.e., $\frac{a}{b}$ (Tourniaire & Pulos, 1985).

Due of that, fractions can be used as mathematics tools in solving problems in proportions. Furthermore, the use of fractions as mathematics tools to compare the proportion situation can reinforces students' sense of number position concept. Hence, it is clear that there is a relation between proportions and fractions and we can foster students' number position concept by employing it in solving proportional problems.

2.6. Proportion in the Indonesian curriculum

Zulkardi (2002), found out that most Indonesian mathematics textbooks contain set of formal rules which lack of application of the mathematics concept that can make the concept becomes real for students. That is similar to how concept of proportion is usually taught at class, which is started by giving a formal procedure in solving proportional problems (e.g. Soenarjo, 2007; Sumanto, Kusumawati, & Aksin, 2008). Proportion usually is taught by asking students to solve problems by utilizing representations, equalities between ratios, and/or the use of linear functions. However, by conducting this kind of teaching and learning, most students know proportion as a fixed rule to be used to solve problems. As the result, many students fail to develop an understanding about the concept and they may not be able to elicit the proper proportional reasoning.

In the Indonesian curriculum, proportion is formally taught in grade 5 until grade 8. However, the sense of proportion itself is learned before grade 5. Sumarto (2013) explained that 4th grades students have learnt about "solving problems which involve fractions" which involves patterns and relationships that are also related to proportional thinking. She also added that the topic of fractions can be expanded into a simple proportional problem. Therefore, it provides an opportunity to employ proportional problems in a more advance level than Sumarto (2013) did in grade 4. Hence, this study will be conducted with 5th graders and their teacher in Indonesia.

The basic competence used in this study is using fraction in solving proportional problems. This competence emphasizes on the relationship between fractions and proportion. Moreover, the previous basic competence is about mathematics operation by using fractions.

Table 2.1 Proportion for elementary school in the Indonesian curriculum (Depdiknas, 2006)

Standard Competence	**Basic Competence**
Numbers	
3. Using fractions to solve problems	5.1 Converting fractions into decimals and vice versa.
	5.2 Adding and subtracting fractions and decimals.
	5.3 Multiplying and dividing fractions and decimals.
	5.4 Using fractions in solving proportional problems and scale

2.7.Research aim and research questions

The aim of the present study is to develop a learning instruction that does not only support the development of 5th grade students' relative thinking in solving problems on proportions, but it also helps the students to develop proportional reasoning. Moreover, the study is aimed to contribute to developing a local instruction theory that supports students to elicit their relative thinking in solving problems on proportions. To reach this goal, the researcher proposes the following main research question: *How can we support students in developing relative thinking in solving problems on proportions?* There are three sub research questions in this study: *(i) how do the 5th grade students use their initial understanding to solve proportional-comparison problems? (ii) how can proportional-comparison problems promote students' relative thinking? (iii) how can the bar model as a visualization of proportional situations support students in developing relative thinking?*

CHAPTER 3
METHODOLOGY

3.1.Research approach

The aim of the present study is to contribute to developing a local instructional theory and for educational innovation in developing students' relative thinking in solving proportional problems. Due of that, we designed a learning series that is elaborated into hypothetical learning trajectory (HLT). The design is embodied into students' material which is aimed to help teacher in supporting students to elicit relative thinking in solving problems on proportions.

Literature about proportion and relative thinking is studied to identify the basic concepts required in order to correctly interpret problems on proportions. The theory studied is used as framework in developing a learning trajectory and an instructional design. The learning trajectory is tested in a real learning activity and the result is used to contribute in developing local instruction theory. Therefore, it is needed to apply a research approach that could mediate the theoretical side and the practical one. According to Bakker & van Eerde as cited in Sumarto (2013), design research is an approach that can bridge the practical side and the theoretical side. Therefore design research is employed in this study. Moreover, Bakker (2004) asserted that design research is evaluated against the metrics of innovation and usefulness, and that strength comes from its explanatory power and grounding in experience and results in products that are useful in educational practice because they have been developed in practice.

There are three phases in design research (Gravemeijer and Cobb, 2006). The three phases of design research in this study will be described in the following section.

3.1.1.Preparing for the experiment

In this phase, the main idea of the design is formulated which is referred to theories that has been studied. Therefore, relevant literature had been studied before designing the hypothetical learning trajectory (HLT).

a. Studying literature

This study is started by studying literature about proportion, proportional reasoning, the needs to reason relatively in solving proportional problems, and realistic mathematics education as framework in designing learning trajectory. Furthermore, design research is studied as the research method in this study.

b. Designing a hypothetical learning trajectory (HLT)

The hypothetical learning trajectory (HLT) contains several aspects that are important for designing the instructional activities, such as mathematics goals, students' starting point, mathematics activities, conjectures of students' thinking and strategies, and suggestions for the teacher regarding students' action toward certain mathematical activities. The HLT is flexible, which means it can be modified during the teaching experiment. Therefore this research is conducted in cycles that enable the researcher to do revisions and to implement the HLT in the next cycles.

3.1.2.The design experiment

In this phase, the instructional design is conducted and tested in a classroom experiment. There are four lessons in this design that are carried out in a fifth grade elementary school. The experiment is started by conducting a small scale pilot experiment as first cycle before implementing the design in a classroom experiment with a whole class.

a. First cycle (pilot teaching experiment)

The first cycle is a pilot teaching experiment. In the first cycle, the design is tested with a small group of students in which the researcher is the teacher. The implementation of the first cycle is aimed to know students' preliminary knowledge and to test the initial instructional design (students' worksheets, hypothetical learning trajectory, and teacher guide). The result of the first cycle will be analysed to determine which parts or which contents of the design that are needed to be improved before being used in the real teaching experiment (second cycle). Based on the result of learning process in the first cycle, the researcher does improvement on the details of hypothetical learning trajectory, teacher guide and the details of the mathematics activities. The revised instructional materials then are implemented at the real teaching experiment

b. Second cycle (teaching experiment)

The aim of the second cycle is collecting data in order to answer the proposed research questions. The second cycle is conducted in a class of 5th grade elementary school. However, the class of teaching experiment is different from the class of pilot experiment. The class of teaching experiment is taught by a regular teacher and it involved all of students from that class. There are four lessons in the second cycle for 2x35 minutes per lesson. Before doing the second cycle, the researcher discuss the learning materials with the teacher of the experimental class.

3.1.3.Retrospective analysis

In doing the analysis phase, the HLT is employed as a guideline (Bakker, 2004) and reference in analysing and interpreting the data collection. Retrospective analysis is aimed to answer the proposed research questions and to draw conclusions. The description about the analysis is elaborated in the part of *Data analysis*.

3.2.Data collection

In this study, some data are collected to answer the proposed research questions. The data are collected by using different methods and are used for different purposes. The further explanation is in the following part.

3.2.1.Preparation phase

Before conducting the second cycle, it is necessary to obtain some information as a starting point for the study. Therefore, the researcher conduct an interview with the teacher of the experimental class, classroom observations (in the experimental class) and a pre-test. The elaboration of pre-test will be presented in separate section.

a. Interview with the teacher

Interview with the teacher is needed to get insight about social norms, socio-mathematical norms, how the teacher usually teaches the topic, students' range of ability and classroom management (how the teacher usually designs the learning environment in the classroom). The interview is conducted as a semi-structured interview. The interview is recorded. Besides that, during the interview, the researcher makes field notes.

b. Classroom observation

Classroom observation is important, because by doing this observation, the researcher knew real situation of the class, the classroom culture and the socio-mathematics norms, the practical thing relates to how the class is used to be set, and how the teacher conducted the teaching-learning activity. Moreover, it is conducted to get insight about students' behaviour in the classroom. Similar to the interview, the researcher will make both record and field notes of the information about the situation of the class.

3.2.2.First cycle (pilot teaching experiment)

The first cycle or it is also called pilot experiment is conducted in order to try out the design of activities/HLT in a small group of students, consisting of nine students of year 5 (10-11 years old). The chosen students in the first cycle are not from the experimental class. It is done to assure the validity of data which are collected from the teaching experiment (second cycle). If the participants of the pilot experiment also become participants in the real teaching experiment, we couldn't ensure whether the learning process and learning outcome show the students' real ability or not due to their experiences in the pilot experiment. Which means that we couldn't guarantee whether the HLT measures what the researcher wants to measure from those participants or not.

In choosing students as participants, the researcher asks for teacher's suggestion, because teacher is the one who know the students' ability best. The researcher choose nine students that represent high achievement (2 students), five middle ones, and two students who represent the low achievement. The aim of this composition is to create a heterogenic group of learning. However, the researcher don’t choose students who are extremely clever or weak.

In the first cycle, the researcher itself is the teacher. All the learning activities and interviews with the students are recorded by using video recorder and camera. The interview is conducted to figure out students’ understanding about the topic. Moreover, the researcher collects the students’ written works.

The result from the first teaching experiment is evaluated in order to determine the HLT needs to be revised/ modified or not. This is why it is important to conduct the pilot experiment before conducting the real teaching experiment.

3.2.3.Second cycle (teaching experiment)

In the second cycle, the revised HLT is implemented in the experimental class. The second cycle is aimed to gather data in order to answer the proposed research questions, to improve the HLT and to develop an understanding of how the instructional design worked. The participants of the second cycle are all of students of the experiment class who are different from the students of the first cycle. In the second cycle, the class is taught by their own teacher (homeroom or mathematics teacher, in the Indonesian system, for elementary school usually there are homeroom teachers who teach all subjects to that class). While the teacher is teaching the class, the researcher observes the learning activities and makes field notes. The whole teaching and learning activities are recorded by using a video recorder. Besides that, the students’ written works are collected and used as data.

In the second cycle, a focus group is selected and closely observed. Moreover, their discussion is video recorded. The focus group is created due to the needs to get more detailed information about the learning process, i.e. the development of students’ relative thinking in solving problems on proportions. Moreover, in an Indonesian classroom, a class usually consists of about forty students. It is difficult to get details from each participant if the class is big.

3.2.4.Pre-test and post test

a. Pre-test

The pre-test is conducted twice, in the beginning of the first and second cycle. The pre-test (in both teaching experiments) is a written test. The test items are comparison problems. The test is aimed to get insight students' preliminary understanding about proportion and to diagnose students' proportional thinking, how the students interpret the proportional situation, and students' initial ability in solving problems on proportions. The researcher uses the information obtained from the pre-test as one of frameworks in building up the learning activity.

The participants of the first pre-test are nine students from the pilot class. Pre-test in the first cycle is aimed to know whether the questions of test items could be understood by the students or not, to know whether the problems are feasible for 5th graders or not, to know how students interpret the problems and to determine students' preliminary knowledge. Moreover, a mini interview is conducted to determine why the students employ such kinds of ideas or strategies, which this information is used to improve the HLT.

The second pre-test is conducted in the beginning of the second cycle, before the teaching experiment. The participants are all students from the experimental class. The teacher gives the test and the students work individually in specific amount of time. The aim of the second pre-test is getting insight about students' preliminary knowledge on proportions, and how they solved the problems and students' initial ability of proportional reasoning. The information that is obtained from the second pre-test is used to modify or adjust the HLT and the instructional activities.

b. Post-test

Post-test is conducted twice, at the end of the pilot experiment and at the end of teaching experiment. The purpose of the first post-test is to identify whether the test items are understandable or not. Moreover, the post-test is aimed to know about how the learning activities support the students to develop relative thinking and to identify how the students solve the problems on proportions after learning the topic.

The purposes of post-test in the second cycle are similar. The second post-test is carried out in order to get impression about how the learning activities support the development of the students' relative thinking as one of important type of proportional reasoning. The post-test is also aimed to examine how the students solve problems on proportions after they learn the topic.

The items used in the pre-test and post-test have same competencies but they are in different level of difficulties. Besides that, the researcher uses different numbers and presents different context of problems for pre-test and post-test. Students worked individually on both pre-test and post-test. The pre and post-test activities are not recorded. The data collected from pre-test and post-test are students' written works.

3.2.5.Validity and reliability

Validity is a term used to indicate whether the researcher really measures what he/she wants to measure or not, whereas, reliability designates the independence of researcher. There are three types of validity; internal, external and ecological validity. The internal validity is about the quality and credibility of data collection that leads to justifiable conclusions. In this study, several methods to collect data are used, namely interview, observation, pre-test and post-test, collecting students' written works and making field notes. Therefore, it is possible to do triangulation data which contributes to improve the internal validity of the data collection. Moreover, the teaching experiment is conducted in a real classroom with its real situation. So, it contributes to the ecological validity due to the data are obtained from the real environment in which the instructional design is applied.

3.3.Data analysis

3.3.1.Pre-test

Pre-test will be held in the both first and second cycles. The aim of analysis of pre-test' result in the first teaching experiment is to diagnose students' preliminary knowledge, to examine formulation of the items, i.e.: the formulation of the questions, to test the feasibility of problems, and to get insight on how the students interpret the problems. The students' answers are examined to determine the students' preliminary knowledge. By examining students' works, we also could verify whether the answers are in line with what is asked for. Moreover, it is conducted mini interview, which give more insight on what the students have already known about the subject and students' understanding and interpretation of the problems.

The pre-test in the second teaching experiment is aimed to identify students' proportional reasoning, how the students interpret the situation, and students' initial ability in solving problems on proportions. The researcher uses the information obtained from the pre-test as one of frameworks in building up the learning activity. The students' written works are examined carefully, how far they understand the problems, what strategies they use, what kind of misunderstand and mistakes that the students do. Therefore, through this analysis, it is expected to reveal what the students have already known about the concept of proportion and relative strategy in solving proportional problems. The students' preliminary knowledge gives an important impact toward the HLT because the HLT should be appropriate to students' preliminary knowledge.

3.3.2.First cycle (pilot teaching experiment)

Data collected in the first cycle are observation recorded by video, the students' written works, and information from mini interview. Data analyses are started by watching the registered video and choosing fragments which are important references to improve and modify the HLT. The selected fragments then is transcribed and analysed. The analysis focuses on students' thinking and strategies. These data, which are collected from the different kind of resources, are triangulated. The data triangulation is done by looking at students' work and it is checked with the students' response during the interview and/or class discussion. The data collections are compared against the HLT in order to see whether the prediction of students' actions occurred as predicted or not. It is also done to know if there are any students' actions that unexpectedly occurred in the learning process. Furthermore, this analysis is done to know whether the students' activities support the development of students' relative thinking in solving problems on proportions or not. All of the findings are used to improve the initial HLT before it is going to be applied in the teaching experiment (second cycle).

3.3.3.Second cycle (teaching experiment)

Data analysis of second teaching experiment is similar to the first teaching experiment. The collected data from the second teaching experiment are observation of the whole class activities and the focus group that are recorded by video, students' written works, and field notes. Data analyses are started by watching the registered video and choosing fragments which gave important data about the learning process and the fragments which contained essential evidences to answer the proposed research questions. The analysis is done by examining thoroughly all of the data collected. And then, the data are triangulated by looking at students' work which it is checked with the students' response during the interview and/or class discussion. The data collections are compared against the HLT (which has been revised) to determine whether the learning activities supports the development of students' relative thinking in solving proportional problems or not and whether it helps the students to reach the learning goals. All of the result and findings of the analysis are used to answer the proposed research questions, derive conclusions and adjust the HLT.

3.3.4.Post-test

Post-test is conducted at both first and second teaching experiments. The result of post-test from the first teaching experiment is analysed to know whether the problems are understandable or not and to know the students' understanding about the concept. Moreover, the findings from the analysis are used to improve the formulation of the test items.

On the other hand, the result of the post-test in the second teaching experiment is analysed by examining the students' written works thoroughly and it is used to get impression about how the learning activities support the development of the students' relative thinking as one of important type of proportional reasoning. In examining students' written works, the researcher looks at students' strategy and reasoning in solving the problems.

3.3.5.Validity and reliability

In analysing data collection, the researcher employs the (HLT) as a guideline. The data collections are tested against the HLT. As explained in the above section that theories are used as framework in designing the HLT. Therefore, HLT is a theoretical guideline in analysing the data. It contributes to the internal validity of the study. After examining thoroughly all of data collected, the data are triangulated by looking at students' work that it is checked with the students' response during the interview and/or class discussion. The data triangulation also increases the internal validity. Moreover, the analysis is reported clearly so the readers can adjust the HLT to their own teaching, which it improves external validity of this study.

In analysing the data, the researcher discusses and does a cross interpretation with supervisors. It means that more than one examiner analyse/examine the same data. The cross examination contributes to inter-subjectivity. Therefore the result of data analysis is independent from the researcher's subjectivity and it improves the reliability of this study. Furthermore, in this study, process of data collection is described clearly; i.e: what kind of data is collected, and what tools are used to collect the data. Therefore, it is possible for readers to trace the data (track-ability) which it improves the reliability of this study.

CHAPTER 4
HYPOTETHICAL LEARNING TRAJECTORY

The instructional activity is elaborated into a hypothetical learning trajectory (HLT). In this chapter, the researcher would like to elaborate the HLT which is designed to develop relative thinking in solving proportional problems for fifth grade (10-11 years old students). A HLT consists of learning goals, starting points, the description of mathematics activities, conjectures of students' thinking and strategies, and suggestions for teacher regarding to what and how the teacher should react to students' particular responses. Therefore, the HLT offers key aspects of the lesson plan. Moreover, the HLT is also a tool to do data analysis. HLT is a guideline to analyse data obtained from teaching experiment, which is used to answer the proposed research questions.

In this study, there are four mathematics activities. Each activity is given in different lesson. The first learning activity is a starting point to lead the students to understand different interpretations on proportional situation. Due to that aim, the context used for the first mathematics activity is context of density. People usually have intuitive understanding to interpret density in relative perspective, that is by considering the relationship of the size of space and the number of population occupying it.

The second, third and fourth learning activity are aimed to support the development of 5th graders' relative thinking in solving problems on proportions by employing comparison of part-to-whole (ratio of part and whole). In the second activity, the students work on determining which road project that is mostly done. In the third activity, the students will determine the best player of Dart games, and for the last activity, there will be data survey on students' interest and the students have to derive decision by using the given data. The HLT of each activity will be elaborated in the following part.

4.1. Learning activity 1: Comparing density (crowdedness)

4.1.1 Learning goals

a. Students are able to interpret proportional situation in relative perspective.
b. Students are able to understand the notion of ratio in problems of proportions.
c. Students are able to use relative comparison by using concept of proportionality to compare density.

4.1.2 Starting points

a. Students are able to do numbers operation, i.e. addition, subtraction, multiplication and division.
b. Students are able to do operation involving fraction and/or decimals.
c. Students are able to order fractions.
d. Students are able to do converting among units of area.

4.1.3 Mathematics activities

The first meeting is particularly used as starting point to help students to understand different interpretation in proportional situation is possible and that the proportional interpretation is the most appropriate. The learning activity 1 is about comparison-density task. When ones are asked to determine which room that is more crowded if there are two rooms in the same size occupied by different number of children, ones will determine room occupied by more children is more crowded. In this case, ones compare the absolute number of children occupying the room. However, when ones are asked to determine which room that is more crowded if there are two rooms in different size occupied by different number of children, they may interpret the situation in different way instead of comparing the room size or the number of children occupying the rooms. Due to different interpretation that might come up when people compare density, it gives opportunity to use density-comparison task as starting point to help students to understand different interpretations of proportional situation are possible, and that proportional interpretation is most appropriate. It aligns with Lamon (2006). Thus, the context of density might generate students' relative thinking.

The comparison-density tasks contain four situations of chickens' boxes. Each box has different size and there are different numbers of chickens in it. The students should compare the density of the boxes and make an order of the boxes from the most crowded to the least crowded. It is expected that there will be students who consider the relation of size of boxes and the number of chickens (relative perspective) instead of comparing the absolute number of population (chickens) or comparing the absolute size of the boxes (absolute perspective) only. Moreover, it is supposed that the students will realize the idea of proportionality of the problems.

a. Preliminary activities

Before the class work on the main mathematics activities, the teacher provides preliminary activity. There is a mind experiment in the preliminary activity in which the teacher will ask the students to compare density of several familiar situations. The mind experiment is aimed to help students recognizing the relative perspective of proportional situations. The teacher asks the students to compare the density of spaces, i.e.:

- Comparing the density of two rooms which are equal in size but there are different number of people in it.
- Comparing the density of two rooms which are different in size but there are same number of people in it.
- Comparing the density of two rooms which are different in size and there are different number of people in it.

b. Main activities

Working in groups (15 minutes)

After giving an overview of the context, the teacher gives worksheets to each group of students. Each group consists of two students. In the main activity, the task of students is determine which chickens' box is the most crowded. After that, the students are asked to put the chickens' boxes in an order from the most crowded to the least crowded. Each box has different size and different number of chicken.

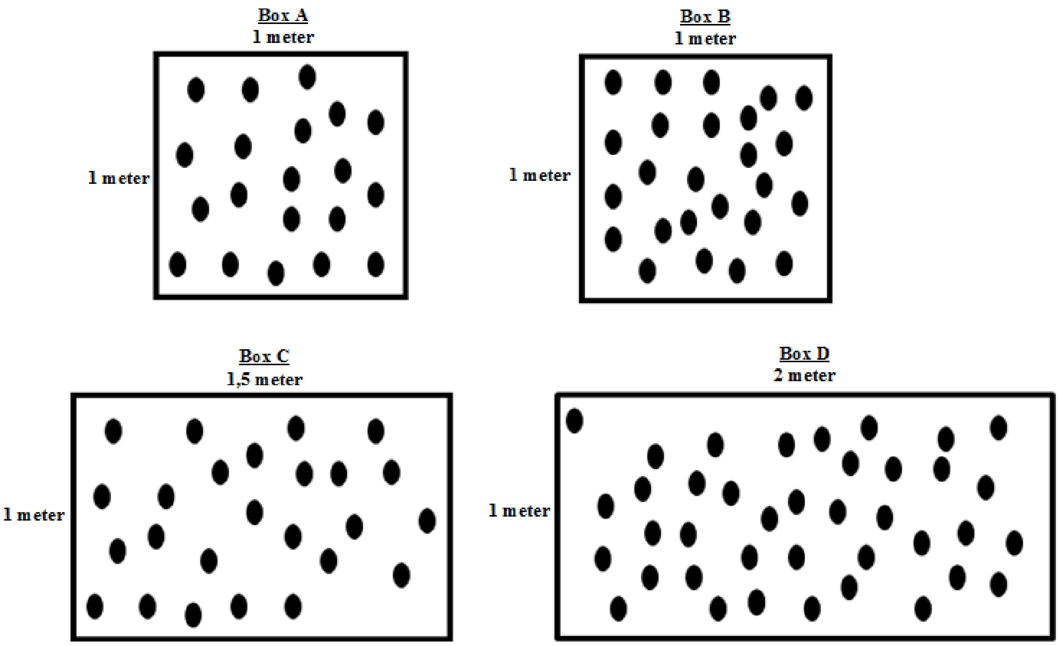


Figure 4.1 Chicken Boxes

c. Class discussion and deriving conclusion (35-40 minutes)

The teacher asks the students (in groups) to present their works in front of class.

4.1.4 Prediction of students' responses and teacher's actions

In comparing the population density, the students might come up with different ideas.

a. Several students might make some guesses. They might illustrate, what if the chickens move around and they might guess in which box the chickens have the furthest distance each other.

   Teacher's actions

   The teacher can ask students to convince others that their strategy might derive the right solution. Due of that, the students will be encouraged to use different strategy. The teacher might ask to other students who have different strategy.

b. Several students might answer that box D is the most crowded (it has the most number of chicken.

<u>Teacher's actions</u>

The teacher can provide this analogue:

There are 10 passengers in each minibus and bus. Which one is more crowded?

It is expected that the students will compare the population density in relative way, which they consider the relation of the number of objects and the size of space, instead of comparing the absolute number of objects.

c. Several students might determine that box B is the most crowded because it is small box with many chicken in it. However, in this case, the students compare the box B with box A only, because the both boxes are in same size.

<u>Teacher's actions</u>

It shows that the students are still thinking in absolute way. They just compare the number of chicken of boxes (space) that have same size. Then, the teacher can ask following questions,

What about box C and D? Don't you compare all boxes? Box D has the most number of chickens.

Then, the teacher can ask for different opinion. Moreover, the teacher can relate to following problems in which the students are asked to put the boxes in an order.

d. Several students might compare all those boxes, but they determine that box B is the most crowded, because it is in a same size with box A, but box B has more chicken. Moreover, box B has the same number of chicken with box C, but box C is bigger.

<u>Teacher's actions</u>

The teacher then can ask which is more crowded, box B or box D (box D has the most number of chickens). Several students might determine that box B is more crowded because box B is smaller. Then, the teacher can ask for the order of boxes, from the most crowded, to the least crowded. Therefore, the students will compare the density for each box.

e. It is expected that there will be students who use relative comparison by considering the relationship of the box size and the number of chickens in the box. The relative comparison can be implemented in term of proportion, which is students compare the situation in the following proportional way:
   - The students make the size of four boxes become the same, which it also change the number of chickens in proportional way. Therefore, the students compare the number of chickens occupying the boxes in the same size.
   - The students make the number of chickens of four boxes become the same, which it also change the size of boxes in proportional way. Therefore, the students compare the size of boxes, which the boxes are occupied with the same number of chickens.
   - It is also expected there will be students who look for the area that is occupied by each chicken.

4.2 Learning activity 2: Road Asphalting Project

4.2.1 Learning goals

a. Students are able to determine part-whole relationship in a proportional situation.

b. Students are able to interpret ratio (part-whole relationship) into fractions ($\frac{part}{whole}$).

c. Students are able to employ relative comparison by using comparison of part to whole to solve the problems involving continuous quantities.

4.2.2 Starting points

a. Students have already had understanding that different interpretation of proportional situation is possible, and that the proportional interpretation is most appropriate.

b. Students have already had experience in applying relative comparison (compare the students in proportional way) instead of absolute comparison.

c. Students are able to do operation involving fraction and/or decimals.

d. Students are able to make an order of fractions.

4.2.3 Mathematics activities

Mathematics activities in the second lesson are aimed to help student grasping the idea of part-whole relationship in a ratio. It is easier for students to grasp the idea of part-whole relationship in a continuous quantity. Due of that, the students will work on a context that involves continuous quantity. The second learning activity uses context of a road project. The students will work on problem about a project of making three new roads, road A, B and C. The current activity of the road project is asphalting activity. The three roads are being asphalted but it hasn't finished yet. Each road has different amount of the asphalted section.

In the learning activities, the students are asked to determine asphalting project in which road that is mostly done. It is expected that there will be students who use relative comparison by considering the part-whole relationship of the asphalted part and the total length of the road instead of comparing the absolute length of the asphalted part.

a. Preliminary activities

Before students work on the main activities, the teacher will give a short preliminary activity. In this activity the teacher will ask questions related to part-whole relationship in continuous quantity, i.e.: *The teacher has a chocolate bar. She gives out the chocolate to 4 students, which each of them has the same amount of chocolate. How much chocolate does each student get?*

b. Main activity

Working in groups (15 minutes)

There are three new roads that have been made. The current activity in the road project is asphalting. The progress on the road-asphalting is as follows:

Road A; total length: 5 km, the amount of asphalted section: 2 km

Road B; total length: 2 km, the amount of asphalted section: 1 km

Road C; total length: 8 km, the amount of asphalted section: 3 km

The chief of the project will make a report. In order to give clear overview about the progress of the project, the chief aims to make a visualization of the progress of the project. The roads are visualized into bars. The students are asked to help the chief by determining and shading the asphalting section.

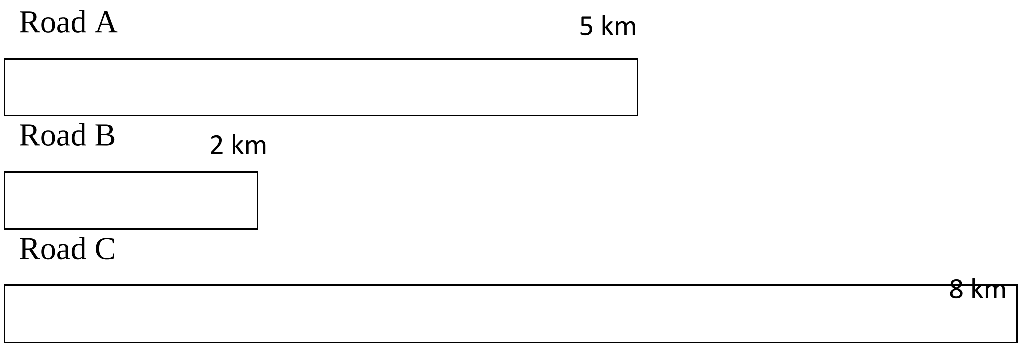


Figure 4.2 The bars of the roads

After visualizing the progress of the road project, the students are asked to determine which road project (A, B or C) is mostly done. And then, they have to make an order of the road based on the progress of the asphalting activity.

c. Class discussion and deriving conclusion (35-40 minutes)

The teacher asks the students (in groups) to present their works in front of class.

4.2.4 Prediction of students' responses and teacher's actions

a. In interpreting the asphalted section by shading the bar A, B and C, several students might not do it in proportional way. The teacher might ask for different strategy in shading the bars.
b. In determining the asphalting poject in which road that is mostly done, several students might compare the absolute length of the asphalted section (part) of the roads. Therefore, they might determine that road C is mostly done, because it has the longest asphalted section.

   Road C; asphalted section: 3 km

   Road A; asphalted section: 2 km

   Road B; asphalted section: 1 km

   (3>2>1)

   The students might not consider the relationship of asphalted section (part) and the total length (whole).

   <u>Teacher's actions</u>

   The teacher can ask to other students for different solution. It is expected that there will be students who consider the part-whole relationship.

c. Several students might compare the absolute non asphalted section, so they determined that road that has the shortest non asphalted section is the mostly done. It indicates that the students still think in absolute way.

   <u>Teacher's actions</u>

   The teacher can ask to other students for different solution. It is expected that there will be students who see the situations in relative perspective.

d. Several students might consider the relationship of asphalted section (part) and the total length (whole). It is expected that the students are able to represent the relationship of part out of whole in fraction.

   However, there might be several students who have difficulties in interpreting the part-whole into fractions ($\frac{part}{whole}$).

   <u>Teacher's actions</u>

   The teacher could use road B as a milestone, because it is easy for students to recognize that a half of road B has been asphalted. It is clear that a half of road B means 1 km out of 2 km. Therefore, by giving probing questions, it is expected the students will be able to figure out the part-whole relationship of each situation (road) and they will be able to interpret the relationship into fractions.

4.3 Learning activity 3: Dart games!

4.3.1 Learning goals

a. Students are able to interpret proportional situation in relative perspective.
b. Students are able to determine part whole relationship in discrete quantities.
c. Students are able to use the fractions to solve proportional problems.

4.3.2 Starting points

In the previous lessons,

a. The students have already had experiences in employing relative perspective.
b. The students have already had experiences in determining part whole relationship in continuous quantity.
c. Students have already had experiences in interpreting part whole relationship as fractions ($\frac{part}{whole}$).

d. The students have already had experiences employing the concept of proportionality, in term of part out of whole, instead of comparing the absolute value in solving the problems.
e. Students are able to put fractions in an order.

4.3.3 Mathematics activities

In the fourth lesson, it is provided result of Dart games played by four children. Each child has different chance of shooting and score. The students are asked to determine the most skilful)* player by employing the result. Several students may solve the problems by comparing the absolute score only. Several students might compare the number of failed shoots. However, the teacher can ask the students, whether their strategies are convincing, fair and appropriate or not. The students might consider different strategy and reasoning. Therefore, the problems are aimed to help students to understand different interpretation of situations.

a. Main activity

Working in groups (15 minutes)

This is the result of Dart games by four children: Gagah, Bayu, Rio and Fadli.

Gagah : ●●●●● ●●●●○ ○○○○○ ○○○○○
Bayu : ●○●○○ ●○●●○
Rio : ○●○●○ ○●○●○ ●○○●● ●○○○● ●●○○○
Fadli : ●●●●○ ○○○○○

The students are asked to determine the most skilful Dart player. And then, they should put the four players in order, from the most skilful to the less skilful.

b. Class discussion and deriving conclusion (35-40 minutes)

The teacher asks the students (in groups) to present their works in front of class.

)* students who have the most accuracy in shooting

4.3.4 Prediction of students' responses and teacher's actions

The teacher might make a table that can be used to organize the data

Name	Score	Total chance
Bayu	5	10
Gagah	9	20
Fadli	4	10
Rio	11	25

a. Several students might use absolute comparison.

1) Several students might consider the given data in partial way, by comparing the absolute score (black dots) only instead of considering the relation between data (the relationship of score and the total chance of shootings). Therefore, they might determine that player who made the most score (black dots) is the best player (that is Rio)

   Teacher's actions

   The teacher can ask the students to convince others that their strategy is fair for all players, due to each player had different chance of shootings. The teacher then can ask for different strategy.

   It is expected that there will be students who compare the situations in proportional way.

2) Several students may reason that player who made the least failed shoots is the most skilful (Bayu).

   Teacher's actions

   Then, the teacher can ask: *What about Rio? He made the most failed shoots, but he also made the most scores, 12, among all of children. Moreover, Rio made scored twice than Bayu did.*

3) Several students might determine Rio as the most skilful, because he made the most shooting.

   If the students still use absolute comparison, the teacher can provide this simple analogue:

   Maudi : ●○

   Soraya : ●●●●○ ○○○○○

   Which one is better in playing Dart?

b. To determine the order of the Dart player (**question no 2**), the students should compare all of the situations. Several students might realize the relative relationship of the score and the total chance of shootings, and they may use relative comparison.

   The students might represent the relationship in different mathematics tools:

   1) *The use of ratio*

      Several students might grasp the idea of part-whole relationship in term of ratio (the ratio of score and the total chance of shooting).

      Teacher's actions

      Due of that, the teacher can ask for different ways in solving the problems, which can be used to lead the students to use fractions, decimals or percentages in solving the problems.

   2) *The use of fraction (percentages/decimals)*

      Several students might realize that the proportional situations can be represented as fractions. Then, they need to compare the fractions in order to determine the most skilful player. However, the students might have constraint in determining common denominator, because the denominator of the fractions are 10,20, and 25.

      Teacher's actions

      The teacher can ask the class, who has different strategy in ordering the fractions. It is done to lead the students to think about different mathematics tools, i.e. the use of fractions, percentages or decimals.

c. Several students may do comparison gradually. They might start comparing the situations of players who have same total chance of shooting (Bayu and Fadli, 10 total chances). After that, the students might use the result of the previous comparison (Bayu and Fadli) to be compared with Gagah (who has 20 total chances), and Rio (total shoots is 25) might be the last comparison.

   Teacher's action

   1) Comparing the situation involving 100 as the common denominator can be used as a milestone to emerge the use of percentages.

2) Teacher also could lead the students to discuss the order of the situations. Comparison that involves 20 total shoots and 25 total shoots gives big chance (and easy way) to emerge the use of percentages.

If it is clear for everybody that the comparison can be done by using fractions and/or percentage, the teacher might ask the students about which strategy or tool that suits them. Each student might have different opinion about this.

4.4 Learning activity 4: Survey on students' interest

4.4.1 Learning goals

a. Students understand that different interpretations, absolute and relative, are possible in proportional situation.
b. Students are able to use the concept of proportionality instead of using absolute comparison in solving comparison tasks.

4.4.2 Starting points

a. The students have already had experiences in employing relative perspective.
b. The students have already had experiences in determining part whole relationship in continuous quantity (lesson 2) and discrete quantity (lesson 3).
c. The students have already had experiences in interpreting part whole relationship into fractions ($\frac{\boldsymbol{part}}{\boldsymbol{whole}}$).
d. The students have already had experiences employing the concept of proportionality, in term of part out of whole, instead of comparing the absolute value in solving proportional-comparison problems.
e. The students are able to put fractions in an order.

4.4.3 Mathematics activities

In the learning activity, the student will derive a decision by using survey data on children's interest. In using the data, several students might compare the data partially without considering the relation among data. It is expected that different solution, absolute value and the use of proportionality, will be emerged. Therefore, the class can discuss which strategy is the appropriate one in solving the problems.

a. **Main activity**

Working in groups (20 minutes)

A survey at ***Harapan Bangsa Elementary school*** *obtained data of students' interest on extracurricular activities as follows:*

1) *Basketball for class 5D:*

Girls

Girls who like to do basketball activity	Total girl students
7	15

Boys

Boys who like to do basketball activity	Total boy students
5	10

The students are asked to determine who is more interested on **Basketball** extracurricular, boys or girls. They are also asked to explain and give justification for their answers.

2) *Most of members of Silat and Pramuka ask to have twice a week activities. This is the information of students' preference in scheduling.*

Silat

Students who prefer twice a week	Total member
20	30

Pramuka

Students who prefer twice a week	Total member
30	50

But, because of the schedule of school activities, there is only one more extracurricular that can be scheduled twice a week.

In this situation, the students should determine which extracurricular (**Silat** or **Pramuka**) that will be scheduled twice a week. They are also asked to explain an give justification for their answers.

b. **Class discussion and deriving conclusion (35-40 minutes)**

The teacher asks the students (in groups) to present their works in front of class.

4.4.4 Prediction of students' responses and teacher's actions

There are predictions of students' strategies and thinking:

a. **Survey on basketball interest**

1) Several students might use absolute comparison, and they use the given data in partial way (comparing the absolute number only instead of considering the relation between numbers).
    a) The students might compare the absolute number of girls who like to do basketball and boys who like to do basketball and determine that girls are more interested on basketball than boys, because there are more girls (7) who like to do basketball than boys (5) who like to do it.
    b) Some students might compare the total number of boys and girls and determine that girls are more interested on basketball than boys, because the total number of girls (15) is more than the total number of boys (10).

    Teacher's action

    The teacher can ask for different answers. It is expected that there will be students who recognize the proportion of boys/girls who like to do basketball and the total number of boys/girls.

    Several students might compare the number of boys and girls who don't like to do basketball. There are 5 boys who don't like to do basketball. There are 8 girls who don't like to do basketball. Thus, they might determine that boys are more interested on basketball because there is less number of boys who don't like to do basketball than girls.

    Teacher's action

    Teacher could confront the students' idea by posing this statement:

    It's true that there are less boys who don't like to do basketball than girls. But, there are more girls who like to do basketball than boys.

Besides that, the teacher can confront students' answer with this information:

The boys who like to do basketball equals with the boys who don't like to do it (5-5).

It is expected that there will be students who realize the concept of proportion in the problems.

If it is still difficult for the students to grasp the relation of part (the number of boys/girls who like to do basketball activity) and the whole (the total number of boys/girls) and they still use absolute comparison instead of using the concept of proportionality, the teacher could ask the proportion of boys/girls who like to do basketball out of the total boys/girls. It is expected that there will be students who come up with a half (½) for boys (5 out of 10). A half can be use as milestone in supporting the students to use proportion in solving the problem.

b. **Survey on schedule preference**

In the first activity (survey on basketball interest), the students have had an experience in solving the problems by employing the concept of proportion. It is expected that the students consider the relationship among numbers (part-to-whole relationship) instead of using absolute comparison (comparing part and part, or comparing whole and whole). Moreover, it is supposed that the students will interpret the situation in proportional way. The students might make the total number of Silat members (whole) equals with Pramuka's (whole), which it will change the number of part (member of Silat/Pramuka who choose for twice a week activities) in proportional way. After that, they student can compare the number of Silat member and Pramuka member who choose for twice a week activities.

CHAPTER 5
RETROSPECTIVE ANALYSIS

In chapter 4, a hypothetical learning trajectory (HLT) was elaborated. There are four lessons in a learning series. Each lesson contains one mathematics activity. The learning series is designed based on some theoretical frameworks about developing relative thinking as one of the important required thinking for proportional reasoning. The mathematics activities are designed as a continuous learning process. What students have learnt in previous activity is starting point to learn the following mathematics activity.

The present study involves two cycles, first cycle and second cycle. The first cycle is a pilot teaching experiment that was conducted in grade 5, SDN 1 Palembang, Indonesia. Due to the policy of SDN 1 Palembang, the researcher could conduct the first cycle only. For that reason, the researcher continued the study at a different school; it was SD YSP Pusri Palembang, Indonesia. Because of the lack of time, the researcher couldn't start the study at SD YSP Pusri from a very beginning. At SD YSP Pusri Palembang, the researcher conducted the second cycle only, involving the class observation and teacher interview. In total, the study was conducted from 17 of February 2014 until 12 of April 2014.

The first cycle involved 9 fifth graders. The pilot class was taught by the researcher itself. The first cycle was aimed to determine students' preliminary knowledge and to try out the initial HLT. The try out was done in order to figure out whether the prediction of students' strategies and thinking that was created in the initial HLT occurred or not. In the first cycle, the initial HLT was tested. Based on the result of the first cycle, the researcher did several improvements in the details of the HLT and teacher guide.

After conducting the first cycle, the revised HLT and teacher guide were implemented into second cycle which is a real teaching experiment. The second cycle was conducted in a real classroom environment and it was taught by an actual teacher of the class. The teaching experiment involved 30 fifth graders. Each lesson in the both cycles was conducted in 2×35 minutes. Before following the main lesson, the students took a pre-test. At the end of the learning series, the students did a post-test.

5.1. Design experiment

In general, the mathematical activities and tasks in the instructional design are aimed to help students to understand different perspectives in comparison problems. The students may see the situation in absolute way, so they may use absolute comparison. However, the proportional interpretation is the most appropriate, in which the students use relative comparison by employing concept of proportionality. It doesn't mean that one perspective is the right one and the other is wrong. But, both perspectives are essential to foster students' proportional reasoning ability.

5.1.1. First cycle (pilot teaching experiment)

The first cycle was carried out at SDN 1 Palembang. It involved 9 students of class 5C. They were Gagah, Bayu, Rio, Fadli, Soraya, Maudi, Nina, Nisa and Fadiyah. The students were chosen by considering their achievement at studying. Gagah, Maudi and Rio were high achiever students. Fadli, Soraya, Nina and Nisa had about the same ability as middle achiever. Bayu and Fadiyah were considered as low achiever. However, they weren't not extremely clever or extremely low achiever.

The first cycle was initiated by conducting a pre-test and students' interviews. A week later, the pilot teaching experiment was started. The teacher in pilot teaching experiment was the researcher itself. There were four learning activities for four meetings (lessons). Each meeting took 70 minutes. The learning activities were done in groups. For every meeting, the groups were not always the same, because some students were absent in certain days. Due of that, the number of students in each group could be different for any lesson. In particular day, each group consisted of two students. At the other days, there were three students in a group.

Learning activity 1 was about comparing population density (crowdedness). The learning activity 1 was particularly used as starting point to help students to see that there were different ways in interpreting situations involved comparison on proportion. However, by working on the problems, it was supposed that the students would understand that proportional interpretation is the most appropriate. Therefore, the students might use relative comparison by using the concept of proportionality instead of using absolute comparison. Furthermore, the activities were designed for supporting students to understand the notion of ratio ($a\!:b$ or $\frac{a}{b}$) in proportional problems. In this activity the students were asked to solve comparison tasks involving four situations of chickens' boxes. Each box had different size and there were different numbers of chickens in it. The students should compare the density of the boxes and put the boxes in order from the most crowded up to the least crowded chicken box.

Leaning activity 2 was about comparing part-to-whole in continuous quantity. Part-to-whole comparison is one of problem types used in this study. The context used was about vegetables plots. Each plot had different area. There were several vegetables in the plots, one of them was spinach. There was different amount of spinach part in the plots. The task of the students was determining which plots having more part of spinach. Recognizing part-to-whole relationship in continuous quantity is easier than in discrete quantity. Therefore, the use of continuous quantity is milestone for helping students in determining part out of the whole in general situation (for instance situation including part out of whole in discrete quantity).

Learning activity 3 is about using survey data on students' interest in order to derive a decision. The problem was intended to support students' to understand about relative comparison. The students would use relative comparison by employing proportions representing part-whole relationship. The survey data showed information about boys' and girls' interest on extracurricular activities. The total number of girls was different to the total number of boys. Some children from girls group liked to do a certain extracurricular and some children from boys group liked to do as well. In this task, by using the given data, students were asked to decide which group, girls or boys, who were more interested doing the extracurricular activities**.**

The learning activity 4 was about Dart games played by four children. Each child had different chance of shooting and each of them made different number of score. The students were asked to determine the most skilful player by employing the given result. The students would see the situation in different perspective and that the proportional interpretation was the most appropriate. In solving the problems, several students might use absolute comparison (comparing the absolute score only). But, relative comparison by considering the proportional relationship of part (the score) and whole (total chance of shooting) was the most appropriate. The last activity manifested of all knowledge and experiences the students had already acquired from the three previous learning activities.

The last activity of the learning series was a post-test. The purposes of the post-test in the first cycle were to get impression about how the learning activities support the students to develop relative thinking and to determine how the students solved the problems on proportions after learning the topic.

5.1.2. Second cycle (teaching experiment)

Before starting the teaching experiment in the second cycle, the researcher interviewed the homeroom teacher (that also taught mathematics at the experimental class) and did a classroom observation. The observation and teacher's interview was done that we get insight about the real situation of the class, social norms, socio-mathematical norms, how the teacher usually teaches the topic, students' range of ability and how the teacher usually manage the learning process.

The second cycle involved one teacher and 30 students of fifth grade. It was initiated by carrying out a pre-test. There were just 25 students who did the pre-test because three students joined sport competition and the other two students were absent that day. On the following week after the pre-test, 15 students were interviewed.

In the first cycle, the pilot class was taught by the researcher itself. Meanwhile, the class experiment was taught by the actual teacher. There were several improvements and adjustments on the details of initial HLT and teacher guides, such as the word choices for questionings and scaffoldings. In general, mathematics activities carried out in the second cycle were the same as in the first cycle. However, we did change the context of learning activity 2 (vegetable plots) to the context of asphalting road project and we switched the learning activity 3 and 4. The reason on changing the context and the switch of mathematics activities would be elaborate in section of refined HLT.

Learning activity 2 in the second cycle was about road-asphalting project. The students worked on problem about a project of making three new roads, road A, B and C. The current activity of the road project was asphalting the roads. The three roads were being asphalted but it didn't finish yet. Each road had different amount of asphalted part. The students were asked to determine asphalting project in which road that was mostly done. It was expected that there would be students who used relative comparison by considering the part-to-whole relationship of the asphalted part and the total length of the road instead of comparing the absolute length of the asphalted part.

The order of learning activity 3 was switched with learning activity 4 in the second cycle. The switch was due to students' responses toward the two activities in the first cycle. The result of the first cycle shows that the situation of mathematics activity in the initial learning activity 4 (Dart games) was simpler than mathematics activity in the initial learning activity 3 (the use of survey data on students' interest). The number chosen in Dart games problem was smaller and its situation was less complex than the problem of survey on students' interest.

A post-test was conducted five days after the last lesson. The long interval between the last lesson and post-test happened due to the class preparation for the exam. Post-test was carried out to get impression about how the learning activities support the students to develop relative thinking as one of important type of proportional reasoning. The post-test was also aimed to know how the students solved problems on proportions after learning the topic.

5.2. Retrospective Analysis

5.2.1 First cycle (pilot teaching experiment)

a. Pre-test

There were three problems in the pre-test that were aimed to determine students' preliminary knowledge. The three problems were comparison problems that were used to identify whether the students compared the absolute value or they used the concept of proportionality in comparing the situation. The items on this pre-test were used in the second cycle as well.

- ***Problem 1***

  Problem 1 was a simple comparison problem. It was about two types of rice-package (package A and B) that contained different amount of rice and they had different price as well. For 2kg rice package A, the price is Rp. 20.000,00, and for 5kg rice package B the price is 45.000,00. The students were asked to determine which package (A or B) was the cheaper one.

  From nine students, there was only one student who couldn't give a correct answer. There were three types of students' solutions:

  Type 1

  Seven students looked for the unit price of rice package A (2kg rice-package), so they looked for price per kg and they came up with:

  $Rp. 20.000,00 : 2 = Rp. 10.000/kg.$

  After that, they looked for the price of package A and B if they made both of them into equal weight, which was 5kg. For package A; $Rp. 10.000,00 \times 5 = Rp. 50.000$, and 5kg package B was $Rp. 45.000$. The students then determined that package B was cheaper. The following students' work was one example of solution type 1.

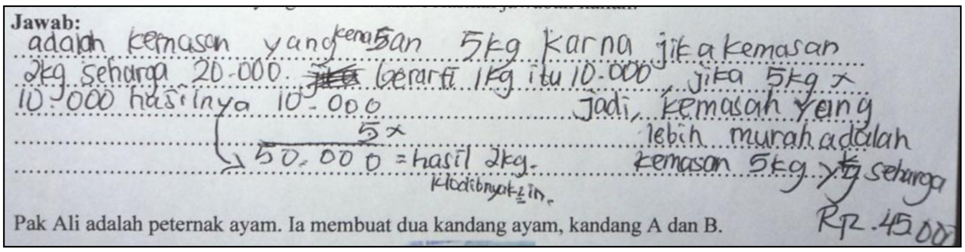


Figure 5.1 Nina's answer for pre-test problem 1

Type 2

One student looked for the unit price (price per one kg rice) of both packages, so she came up with

Package A; $Rp.20.000,00 : 2 = Rp.10.000/kg$

Package B; $Rp.45.000,00 : 5 = Rp.9.000/kg$

Due of that, the student determine that package B (5kg rice-package) was the cheaper one. The following student's work was of solution type 2.

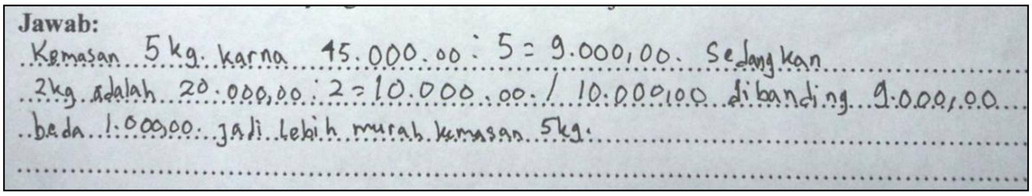

Jawab:
Kemasan 5kg. karna 45.000.00 : 5 = 9.000,00. Sedangkan
2kg adalah 20.000,00 : 2 = 10.000.00. / 10.000,00 dibanding 9.000,00
beda 1.000,00. jadi lebih murah kemasan 5kg.

Figure 5.2 Maudi's answer for pre-test problem 1

Type 3

One student compared the absolute price of two packages.

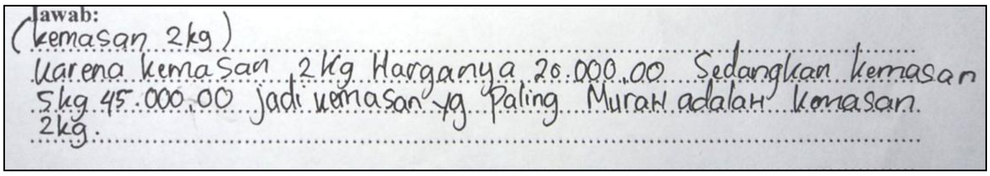

Jawab:
(kemasan 2kg)
karena kemasan 2kg Harganya 20.000,00 Sedangkan kemasan
5kg 45.000,00 jadi kemasan yg Paling MuraH adalaH kemasan
2kg.

Figure 5.3 Fadiah's answer for pre-test problem 1

Fadiah compared the absolute price of two packages. She came to a conclusion that lower price was cheaper. In this case, she didn't consider the relationship between the weight and the price of rice.

According to students' solutions, the solution type 1 and 2 showed that the students used relative comparison by considering the relationship between price and weight instead of comparing the absolute price. Solution type 1 showed that the students made the weight of both rice (A and B) became equal before they compared the price. Type 2 showed that the student looked for the unit price. However, based on type 3, it seemed that Fadiah didn't consider the idea of proportionality in comparison situations. She just compared the absolute value in the problem by noticing the price only. She didn't relate the amount of rice and its price. She concluded that the product which gave lower price was the cheaper one.

- ***Problem 2***

Problem 2 was a comparing density problem. It was about two chicken boxes, box A and box B, which had different size and contained different number of chicken. Chicken box A is $2m^2$ and there are 10 chickens in it. Chicken box B is $5m^2$ and there are 20 chickens in it. The students were asked to determine which box had more density (which box was more crowded).

From nine students, there were three students who employed relative comparison by considering the relationship between the number of chicken and the size of space instead of comparing the absolute value in the problem. There were three types of students' solutions:

Type 1

Three students looked for the number of chicken per $1m^2$, which means they used unit method (Sumarto, *et al*, 2014). These students came up with these solution: box A; 5 chickens in $1m^2$; box B; 4 chickens in $1m^2$

Therefore they concluded that box A was more crowded.

The following students' work was one example of answer type 1.

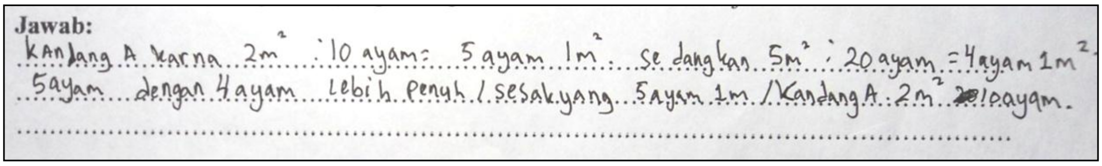

Jawab:
kandang A karna $2m^2$: 10 ayam = 5 ayam $1m^2$. sedangkan $5m^2$: 20 ayam = 4 ayam $1m^2$
5 ayam dengan 4 ayam lebih penuh / sesak yang 5 ayam 1m / Kandang A : $2m^2$ 10 ayam.

Figure 5.4 Maudi's answer for pre-test problem 2

Type 2

There were three students compared the absolute number of chickens in box A (10 chickens) and box B (absolute thinking). Because there were more chicken in box B (20 chicken), therefore they decided that box B was more crowded.

The following students' work was one example of answer type 2.

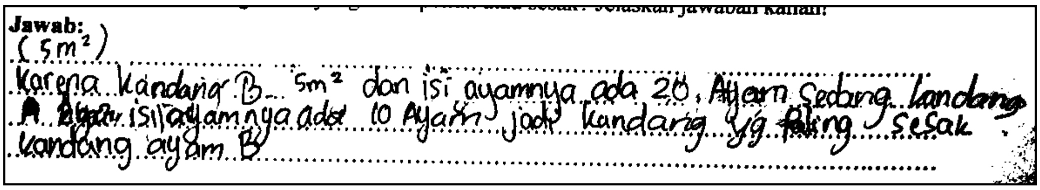
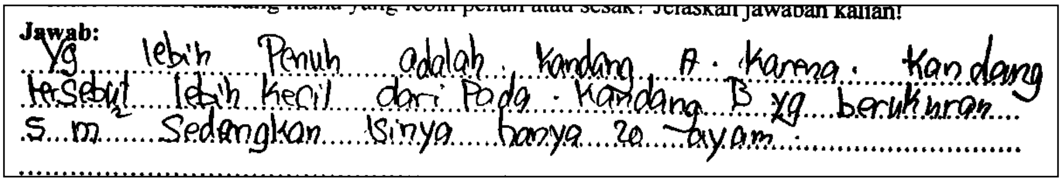

Jawab:
($5m^2$)
Karena kandang B $5m^2$ dan isi ayamnya ada 20, Ayam sedang kandang
A isi ayamnya ada 10 Ayam jadi kandang yg paling sesak
kandang ayam B

Figure 5.5 Fadiah's answer for pre-test problem 2

Type 3

Three students determined that box A was more crowded because box A ($2m^2$) was smaller than box B ($5m^2$). In this case these students compared the absolute size of the chicken boxes.

The following students' work was one example of answer type 3.


Jelaskan jawaban kalian!

Jawab: Yg lebih Penuh adalah kandang A karena kandang tersebut lebih kecil dari Pada kandang B yg berukuran 5 m Sedangkan isinya hanya 20 ayam.

Figure 5.6 Soraya's answer for pre-test problem 2

According to students' solutions, type 1 showed that students used relative comparison in the situations by seeing the relationship between the number of chicken and the size of box. Furthermore, the students used the concept of proportionality in term of unit method (Sumarto, *et al*, 2014). It was known that for $2m^2$ box A there were 10 chickens, therefore they determined the number of chicken for unit square (per $1m^2$). So they came up the solution that there were 5 chickens in box A. They did the similar computation for box B. For $5m^2$ box B there were 20 chickens, therefore they determined that for $1m^2$ box B there were 4 chickens in it. On the other way, based on students' answer type 2, the students determine which box was more crowded by comparing the absolute number of population in it. Moreover, from students' answer type 3, we could see that students compared the size of box in order to decide which box was more crowded.

- ***Problem 3***

Problem 3 was about determining for which class (class 5C or 5D) going to beach was more favourable. Total number of students in class 5C was 25 and 12 of them liked to go to beach. Total number of students in class 5D was 20 and 10 of them liked to go to beach. The students were asked to determine for which class (class 5C or 5D) going to beach was more favourable.

In solving problem 3, there were just two students who used relative comparison by considering the relationship between the number children who chose for going to beach (part) and the total children in each class (whole). There were three types of students' solutions:

Type 1

Two students recognized the relationship of the number of children who chose for going to beach (part) and the total children in each class (whole). Both students determined that there was a half (½) of children in class 5D who chose for going to beach.

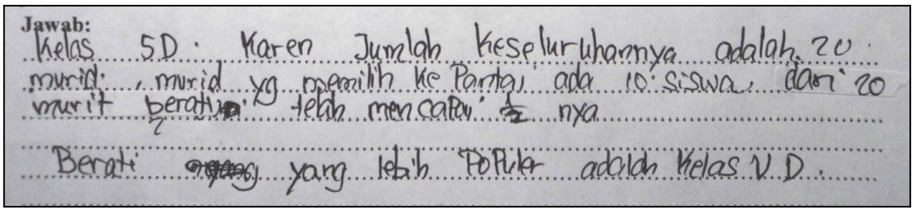

Jawab:
Kelas 5D : Karen Jumlah keseluruhannya adalah 20 murid, murid yg memilih ke Pantai ada 10 siswa, dan 20 murit beratian telah mencapai ½ nya
Berati yang lebih Populer adalah Kelas V D.

Figure 5.7 Soraya's answer for pre-test problem 3

In the interview, Soraya justified again that there was less than a half of class C students who chose for going to beach. Therefore she concluded that going to beach was more favourable for class 5D.

Type 2

There were five students who solved the problem by comparing the absolute number of children who chose for going to beach, and they concluded that going to beach was more popular for class 5C. The following students' work was one example of answer type 2

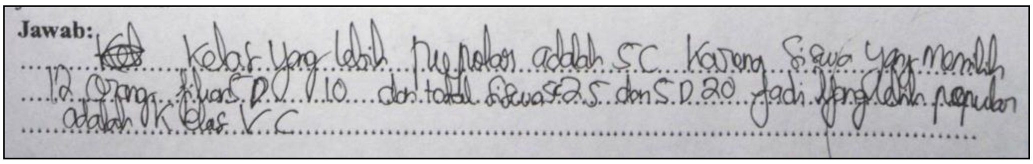

Jawab:
Kelas yang lebih populer adalah 5C karena siswa yang memilih 12 orang. Kelas 5D 10 dan total siswa 5C 25 dan 5D 20 jadi yang lebih populer adalah Kelas V.C

Figure 5.8 Bayu's answer for pre-test problem 3

Type 3

One student solved the problem by comparing the absolute number of total children from class 5C and 5D and she concluded that going to beach was more popular for class 5C.

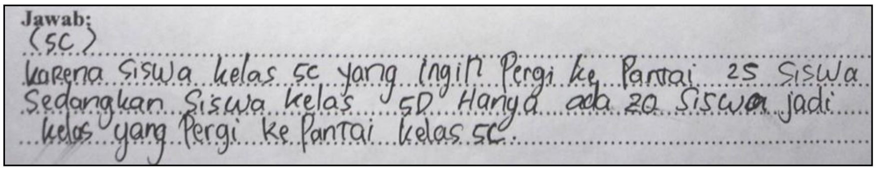

Jawab:
(5C)
Karena Siswa kelas 5c yang ingin Pergi ke Pantai 25 Siswa
Sedangkan Siswa kelas 5D Hanya ada 20 Siswa jadi
kelas yang Pergi ke Pantai kelas 5C.

Figure 5.9 Fadiah's answer for pre-test problem 3

According to students' solutions, type 1 showed that students interpreted the situation in relative way by looking upon the ratio of the number children who chose for going to beach (part) and the total children in each class (whole). It indicated that the students implemented concept of proportionality in solving the problem. However, the solution type 2 and 3 were evidence that most students used the data in partial way and they compared the absolute value instead of using proportions of relating the number children who chose for going to beach (part) and the total children in each class (whole).

- **Conclusion of pre-test**

Based on the analysis of the data collection, we may conclude that students have different level of initial proportional reasoning ability. Most students used the given data in partial way and they didn't recognize the relationship between the set of numbers for each situation. However, several students used relative comparison by employing concept of proportionality. It can be seen at students works in which they came up with the idea of proportionality in a simple way, for instance by determining unit amount of particular value, i.e.: students determine unit amount of rice (1kg), so they could compare the price only, and students determined unit size of boxes (1m^2), so they could compare the number of chicken only. The finding about unit amount and unit size in this study supports the findings of Sumarto, *et al* (2014) about unit method as one kind of students' strategy in comparing proportion. The other idea of proportionality is students compared the price of two different packages of rice in the same amount. Moreover, several students understood the idea of proportionality in term of part-to-whole and they compare the situation by determining which proportion that gave the largest fraction representing the part-to-whole.

After conducting the pre-test, the researcher did a short interview. According to the interview, the students explained that at the first time they didn't know what to do, because they were not used to solve this kind of problems. They were used to solve problems that were obvious for them, which they knew what kind of formulas and what kind of computation should be employed in order to solve the problems. Based on the interview, it appeared that the students got difficulties to understand the texts. For that reason, it is important to create simple texts and familiar context, so it will not take too much time for students to understand the problems.

b. Learning activity 1

Students' task in learning activity 1 is comparing situation of four chicken boxes. The students should determine which chicken box is the most crowded. Besides that, they are asked to put the boxes in an order, from the most crowded to the least crowded. Before students worked on the main activity, the teacher gave preliminary activity in which students did mind experiment as follows:

Which space is more crowded, A or B, both space are occupied by 10 objects, but B is twice larger than space A?

The following fragment shows students' answer and reasoning:

1	Maudi	:	Space A is more crowded, because it is smaller than B and both of them are occupied by the same number of object. So, B is more spacious.

In order to expand students' thinking, the teacher asked this follow up question:

2	Teacher	:	There are two desks, desk A and B. One student occupies desk A and two students occupy desk B. Which one is more crowded?
3	Nisa	:	Desk B is more crowded, because both desks equal in size but there are more students in desk B.

4 Teacher : So, in this case, what's your consideration in determining which desk is more crowded?
5 Maudi : The number of students.

Transcript 5.1

At line 1 and 5 in transcript 5.1, we could see that the student compared the situation in absolute way. The students didn't consider the relationship between the number of objects and the size of space. At line 1, the student compared the absolute size of space, and they explained that smaller space is more crowded. At line 4 and 5, the student compared the number of objects, and they came up with the idea that more students occupied a space, that made the space became more crowded.

But, at line 3, Nisa also noticed that since the two desks are equal in size, they could just compare the number of students. It indicates that Nisa realized size of space and the number of object altogether influence the density on space. Due of that, the teacher gave follow up questions:

1. Teacher : What if, there are desk C and desk D, which desk D is twice larger than desk A, and there are one student occupies desk C, two students occupy desk D.
Which desk is more crowded?
2. Soraya : Those are same.
Because desk D is twice larger than desk C, and there are two students occupy desk D. Therefore, desk D is divided for two students. That's way, desk C and D equal in density.

Transcript 5.2

Based on line 2 transcript 5.2, it is clear that the student, Soraya, considered the relationship between size of the space and number of objects. Soraya used relative comparison instead of comparing the absolute value. Moreover, Soraya computed the set of numbers (size of the space and the number of objects) in each desk C and D. By referring to her calculation, Soraya determined that desk C and D were equal in density. Based on student's discourse above, several students seemed already grasping the idea of comparing and seeing situations in relative perspective.

The main activity in the first lesson contained a problem about comparing density of four chicken boxes. The four chicken boxes were different in size. Each box contained different number of chicken.

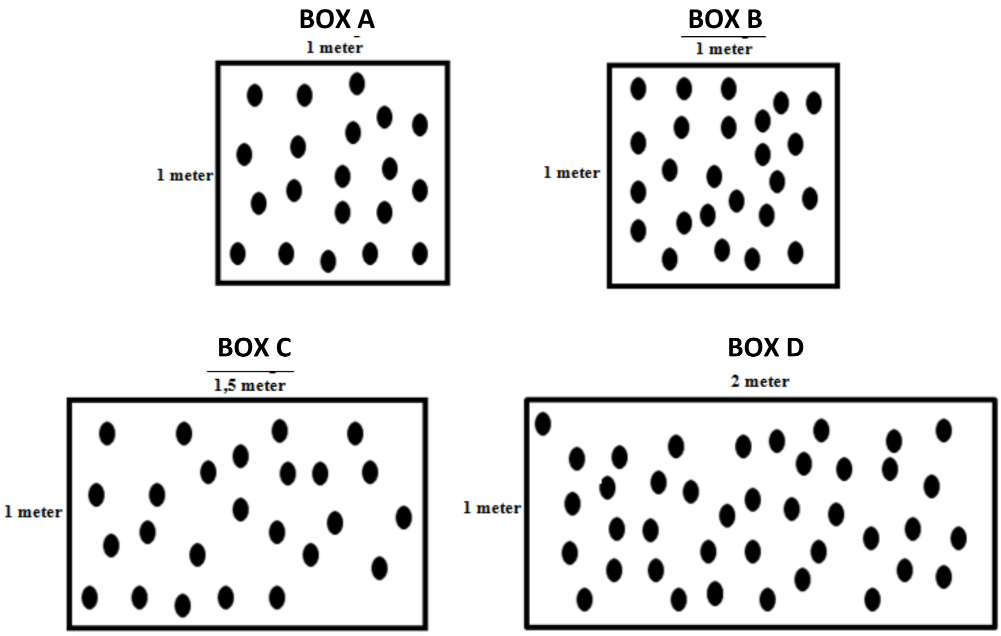


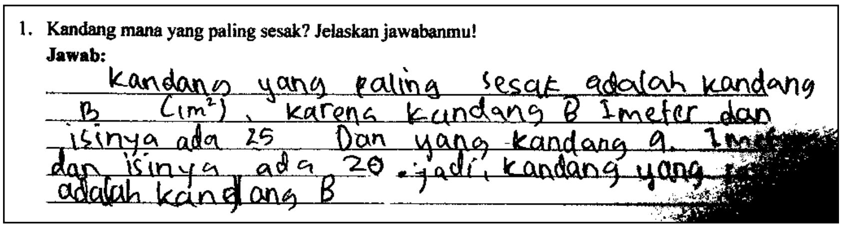
Figure 5.10 Chickens boxes

The students were asked to put the boxes in an order, from the most crowded box to the least crowded.

The preliminary activity has generated the idea of relative perspective in analyzing comparison situations. However, it was difficult for students bring upon the concept of proportionality in solving the proposed problem. The following figures are several students' answers:

Question

Which box is the most crowded?



1. Kandang mana yang paling sesak? Jelaskan jawabanmu!

Jawab:

kandang yang paling sesat adalah kandang B (1m²), karena kandang B 1meter dan isinya ada 25 Dan yang kandang A. 1m dan isinya ada 20. jadi, kandang yang adalah kandang B

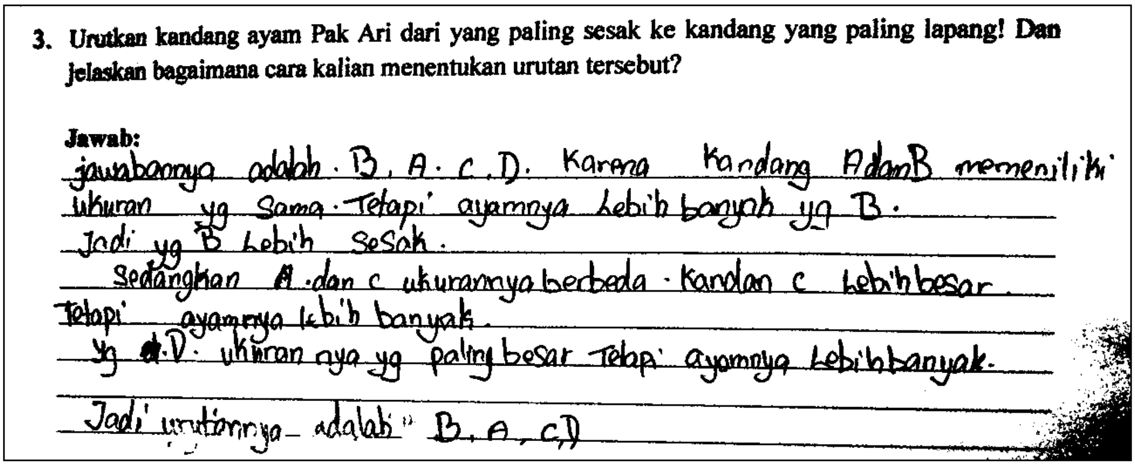
Figure 5.11 Nisa's and Bayu's works

Translation

Box B is the most crowded, because box B is $1m^2$ and it contains 25 chickens, and box A is $1m^2$ that contains 20.

Question

Put the chicken boxes of Pak Ari in an order, from the most crowded box to the least crowded!



3. Urutkan kandang ayam Pak Ari dari yang paling sesak ke kandang yang paling lapang! Dan jelaskan bagaimana cara kalian menentukan urutan tersebut?

Jawab:

jawabannya adalah . B, A . C, D. Karena kandang A dan B memeniliki ukuran yg Sama . Tetapi ayamnya lebih banyak yg B .
Jadi yg B lebih sesak .
Sedangkan A . dan c ukurannya berbeda . kandan c lebih besar . Tetapi ayamnya lebih banyak .
yg D . ukuran nya yg paling besar Tetap ayamnya lebih banyak .
Jadi urutannya adalah " B, A, c, D

Figure 5.12 Gagah's and Soraya's works

Translation

The order is box B, A, C and D. box A equals box B, but box B contains more chickens, so box B is more crowded. Box A and C are different in size, but box C is bigger than box A, and box B contains more chickens. Box D the biggest one and it contains most chickens.

Students' solution (figure 5.12) shows that the students got difficulty to compare density of boxes that were different in size and contained different number of objects (chickens). At the above students' solutions (figure 5.11 and figure 5.12), both groups compared the density of boxes that had equal size (box A and B). Because the size of box A equal box B, and box B contained more chicken, so the students concluded that box B was more crowded.

In comparing density in the boxes that were different in size and contained different number of objects (chickens), it wasn't clear how and why the students did come up with their solution. It indicates that students didn't understand how they should solve the problems. It implies they needs more support. The support can be scaffolding from teacher by giving strong follow up question. In order to make scaffolding and follow up questions became more effective, the teacher should use right words.

In density-comparison task the students have already had experiences about using the concept of proportionality instead of absolute value to solve proportional problems. That starting point is necessary for solving the next learning activity. In order to expand students' reasoning about proportional-comparison task, we designed different context for each learning activity. By experiencing proportionality in different contexts, it might foster students' proportional reasoning.

c. Learning activity 2

Leaning activity 2 was about comparing part-to-whole in continuous quantity. Part-to-whole comparison is one of problem types used in this study. Recognizing part-to-whole relationship in continuous quantity is easier than in discrete quantity. Therefore, the use of continuous quantity is milestone for helping students in determining proportions for a situation in term of part-to-whole.

There were two problems in this activity, which both of them used context of vegetables field. The problems were about a story of Pak Bakri who had vegetables plots as follows:

Figure 5.13 Vegetables plots

1) *There were two vegetables plots, A and B, in which Pak Bakri had read and green spinachs. Plot A was 2 m^2 and plot B was 5 m^2 as follows.*

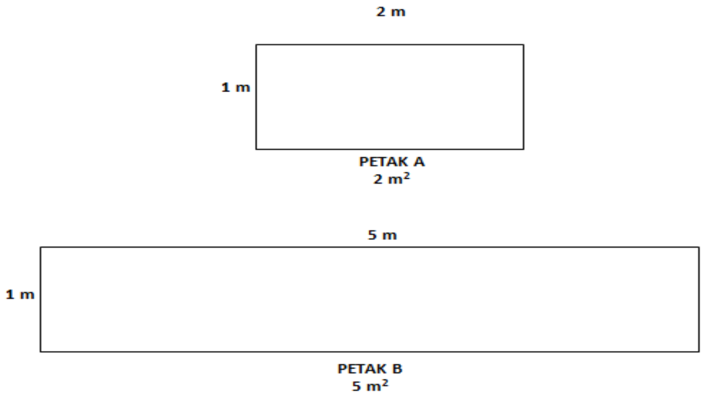


Figure 5.14 (a) The bars of the vegetables plots problem 1

Pak Bakri used 1m^2 plot A for red spinach and he also used 2m^2 of plot B for red spinaches.

In this activity, the students were asked to determine the part of red spinach in the both plots by shading bars that represented the plots. After that, they compared situation of the two plots and determined which statements below was true:

i. *Plot A has more part of red spinach than plot B.*
ii. *Plot B has more part of red spinach than plot A.*

2) *There were two vegetables plots, C and D, in which Pak Bakri had read and green cabbages. Plot C was 5m^2 and plot D was 3m^2 as follows.*

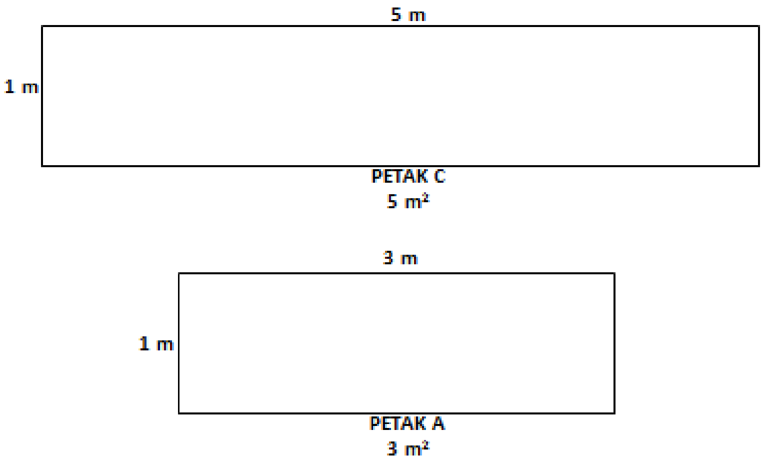


Figure 5.14 (b) The bars of the vegetables plots problem 2

Pak Bakri used 3m^2 plot C *for red cabbage and he aslo used 2m^2 of plot D for red cabbage*

In this activity, the students were asked to determine the part of red cabbage in the both plots by shading bars that represented the plots. After that, they compared situation of the two plots and determined which statements below was true:

iii. *Plot C has more part of red cabbage than plot D.*
iv. *Plot D has more part of red cabbage than plot C.*

The goal of learning activity 2 is to help students to understand that different interpretation of proportional situations is possible. Each pair of the above statements (*i* and *ii*; *iii* and *iv*) showed different interpretation of the situations. The problems might lead students to compare the absolute part of red spinach at plot A and B. In the other hand, it might lead students to realize the relationship between the red part and so they could compare the proportion of red spinach out of the whole part of the plots. Therefore, the problems aimed to support students in determining part-whole relationship. Furthermore, the learning activity was meant to facilitate students to understand that relative comparison by using comparison of part to whole was the most appropriate.

Problem 1 was similar to problem 2. It was aimed to use problem 1 as milestone to help students recognizing the notion of proportions in term of part-to-whole by using the idea a half, a half (plot A) and less than a half (plot B). A half is simple proportion and it is a familiar fraction for students. Therefore, the use of a half was a milestone for helping students to see the problem in proportional way. Problem 2 was used as follow up activity that was more complex because the proportions didn't include a half. and there were two situations that used the same number (plot B and plot C). The use of similar situations and the same numbers due to the focus of the study is on supporting students' proportional reasoning rather than on students' computation ability.

In the learning activity, students worked in peer group. While the students were working, the teacher went to the groups to observe the students' activities. The teacher came to the group of Soraya and Maudi. It seemed that the students were discussing how they should shade the vegetables plots. Transcript below showed the discussion of the students:

1.	Soraya	:	We should measure the length of bar A, and then we divide it into two
2.	Maudi	:	Oh ya. *The students measured the length of bar A by using ruler, and they divided the measurement by two.*
3.	Soraya	:	What about bar B? It is not 5cm [*bar B represented a vegetables plots that the actual length was 5m*]
4.	Maudi	:	We have to measure the length. *Both students measured the length of bar B by using ruler*,
5.	Teacher	:	How do you determine $2m^2$ in bar B?
6.	Soraya	:	It is difficult. We can't divide the number by 5 [*she meant that she couldn't divide the number that was the length of the bar by 5]*.
7.	Teacher	:	OK, but at first, can you tell me about your strategy in determining the part that you must shade?
8.	Maudi	:	We want to look for 1m and then we multiply by 2 because we want to get 2m.
9.	Teacher	:	What do you do to determine the 1m [*the researcher used the term that was used by students, that was 1m instead of $1m^2$*]?
10.	Maudi	:	We divide the bar into 5.

Transcript 5.3

Based on the students' discussion at transcript 5.3, we can see that the students made the shading in proportional way. Line 1 showed that the students understand that $1m^2$ was a half of $2m^2$ because they determined the $1m^2$ by dividing the $2m^2$ into two parts. Line 6 and 10 showed that the students understand that $2m^2$ out of $5m^2$ was two parts out of five equal parts. The students' explanation showed that they understand how to make partition in proportional way, which would lead the students to shade the bar in proportional way. The students also shaded the bars at problem 2 in proportional way. The figures below were Soraya's and Maudi's works on shading activities:

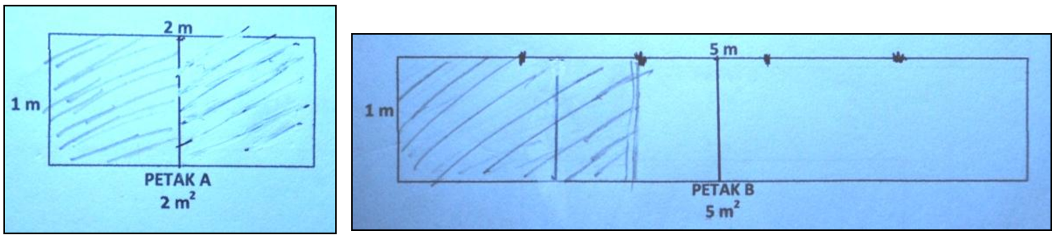


Figure 5.15 Soraya's and Maudi's works

From the above figures, it can be seen that in determining $1m^2$ out of $2m^2$, Soraya and Muadi divided the bar into two equal parts. They also divided bar B into five partitions and they took two partitions in order to get $2m^2$ out of $5m^2$. However, there were several students who did not shade the bar in proportional way, one of them was the work of Bayu and Gagah as follows.

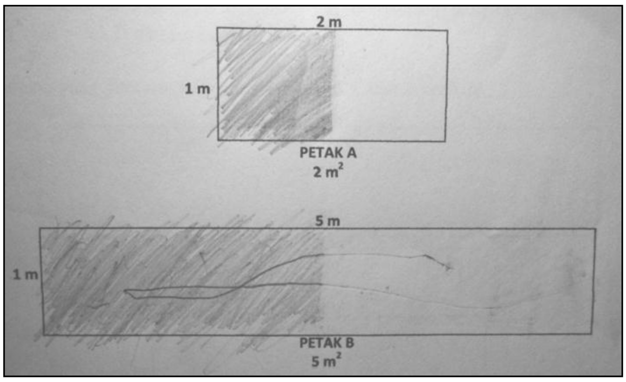


Figure 5.16 Bayu's and Gagah's works

Based on the above figure, it can be seen that in determining $1m^2$ out of $2m^2$, Bayu and Gagah divided the bar into two equal parts. But, they seemed doing the same thing in bar B. When Bayu and Gagah were asked to explain their way in shading the bars, they couldn't give a clear explanation.

The teacher asked the class whether there were similar way or different way in shading the bars. There was a group of Soraya and Maudi who presented their works in front of class. As we know from the group discussion, Soraya and Maudi already used the concept of proportionality in shading the bar. Soraya explained that they divided each bar into the number of equal parts. And then, they shaded the partition/s representing the part of red spinaches (problem 1) and red cabbages (problem 2).

In solving the comparison section, there were different interpretations emerged. Several students used absolute comparison, but there were other students who used relative comparison by employing proportion.

- **Question 1**

  Which statement is true?

  i. Plot A has more part of red spinach than plot B.
  ii. Plot B has more part of red spinach than plot A.

Comparing the absolute area of the vegetables plots

The figure below shows solution from the group of Gagah and Bayu who compare the absolute total area of the vegetables plot A and B.

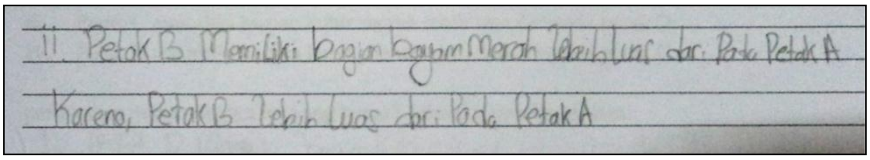

ii. Petak B Memiliki bagian bayam Merah lebih luas dari Pada Petak A
Karena, Petak B lebih luas dari Pada Petak A

Figure 5.17 Bayu's and Gagah's works

Translation

ii. Plot B has more part of red spinach than plot A, because plot B is larger than plot A.

Comparing the absolute red spinaches part

The figure below shows solution from the group of Nina and Nisa who compare the absolute red spinaches part.

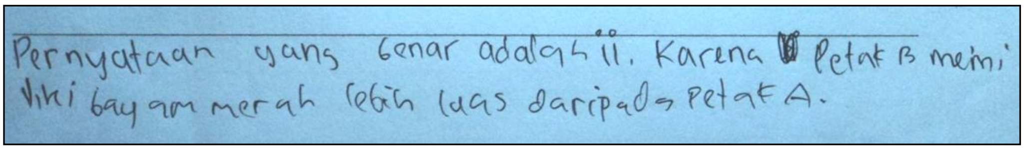

Pernyataan yang benar adalah ii. Karena Petak B memiliki bayam merah lebih luas daripada Petak A.

Figure 5.18 Nina's and Nisa's works

Translation

ii. *The true statement is ii, because plot B has more part of red spinach than plot A.*

According to the solution of Gagah and Bayu, it is obvious that the students used absolute comparison. They compared the absolute total area of the vegetables plots. Moreover, the group of Nina and Nisa also used absolute comparison of the absolute area for red spinaches. There was a different solution from the group of Rio and Fadli. The figure below was the works of Rio and Fadli

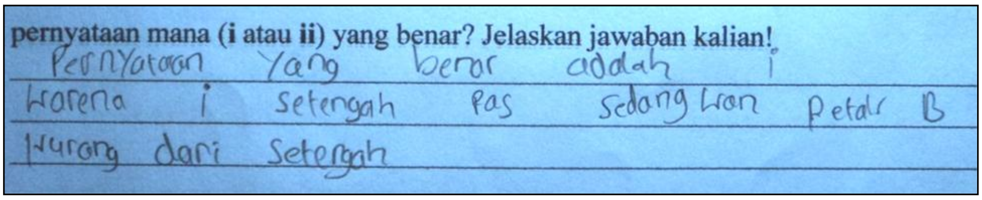

pernyataan mana (i atau ii) yang benar? Jelaskan jawaban kalian!
Pernyataan yang benar adalah i
karena i setengah pas sedangkan petak B
kurang dari setengah

Figure 5.19 Fadli's and Rio's works

Translation

The true statement is i (plot A has more part of red spinaches than plot B), because statement (i) is a half and plot B is less than a half.

According to students' written works, it doesn't clear what the students meant to say about the situation. Therefore, the teacher asked for clarification in the class discussion.

1.	Rio	:	The red spinaches part at plot A is a half and it is less than a half in plot B.
2.	Teacher	:	What do you mean by a half and less than a half? And why do you think in that way?
3.	Rio	:	The part of red spinaches at plot A is $1m^2$ that was a half of $2m^2$. And a half of $5m^2$ is 2½ m^2. And ... [*long pause*] $2m^2$ is less than 2½ isn't it? So ....[*he ended his explanation*]

Transcript 5.4

Line 1 at transcript 5.4 shows the student used relative comparison by using proportion representing the situations. Line 3 shows that the student understood the proportion denoted a part-to-whole relationship. The students already considered the relationship of numbers instead of comparing the absolute value. Moreover, the student understood that he had to determine which proportion gave the largest fraction (line 1 and 3).

- **Question 2**

  Which statement that is true?

  iii. *Plot C has more part of red cabbage than plot D.*
  iv. *Plot D has more part of red cabbage than plot C.*

Comparing the absolute red spinaches part

The figure below shows solution from the group of Nina and Nisa who compare the absolute red cabbages part.

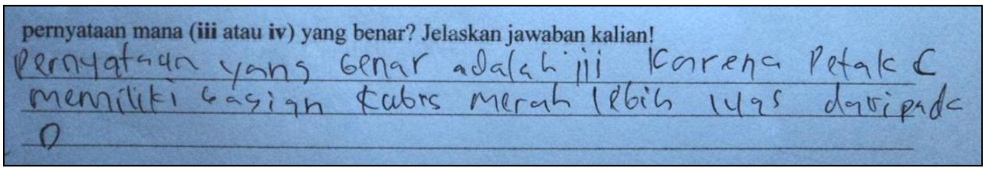

pernyataan mana (**iii** atau **iv**) yang benar? Jelaskan jawaban kalian!
Pernyataan yang benar adalah iii karena Petak C memiliki bagian kubis merah lebih luas daripada D

Figure 5.20 Nina's and Nisa's works

Translation

The true statement is iii because plot C has more part of red spinach than D.

There was no explanation for the answer, so the teacher asked for clarification from this group. Nisa explained that due to part of red cabbages in plots C was $3m^2$, which was larger than in plot D ($2m^2$).

The group of Rio and Fadli also interpret situation of question 2 in proportional way as follows:

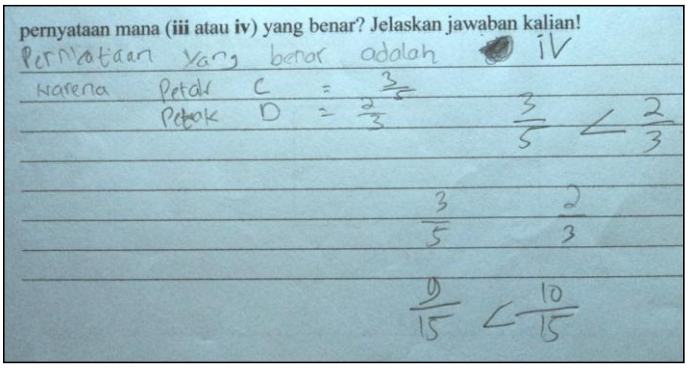

pernyataan mana (**iii** atau **iv**) yang benar? Jelaskan jawaban kalian!
Pernyataan yang benar adalah iV
karena Petak C = $\frac{3}{5}$
Petak D = $\frac{2}{3}$ $\frac{3}{5} < \frac{2}{3}$
$\frac{3}{5}$ $\frac{2}{3}$
$\frac{9}{15} < \frac{10}{15}$

Figure 5.21 Fadli's and Rio's works

Translation

Statement iv is true. Because: Plot C is $\frac{3}{5}$ *and plot D is* $\frac{2}{3}$

$\frac{3}{5} < \frac{2}{3}$ [there were several scratches]

$and\ finally\ \frac{9}{15} < \frac{10}{15}$

According to students' works and discussion in learning activity 2, we conclude that the problems may help students to see that there are different ways in interpreting the situation, and that proportional interpretation is the most appropriate. The use of continuous quantity may help students recognizing the relationship of numbers in term of part-whole relationship. Moreover, it may be easier for students to grasp the idea of proportions in continuous quantity. Therefore, the mathematics activity, including the context and the chosen numbers, may give a helpful starting point for students to understand relative comparison by employing part-to-whole relationship.

However, in using the concept of proportion, the students tended to works on numbers only. The students didn't use the bar to reason in proportional way anymore. It seems that bar model as visualization of proportional situations doesn't give significant support for students in solving the problems. Moreover, the students stated the unit measurement incorrectly. The context used unit measurement of area, but the students said it as unit measurement of length, for instance they students used 5m instead of $5m^2$ and so on. It indicates that the students tend to use something simpler. For that reason, the researcher changed context for the second cycle. The new context was road-asphalting project. The new context still involved continuous quantity including unit measurement of length.

d. Learning activity 3

The goal of learning activity 3 is to support students to solve proportional problem in a relative way. In the first and second learning activities, the students have already had experiences in using the concept of proportionality instead of absolute value to solve problems on proportions. Moreover, in the second activities, the students have determined part-to-whole relationship from a proportional situation. In the second activities, the context used involved continuous data (vegetables field), meanwhile in the third learning activities the students will work on discrete data.

In order to reach the proposed learning goals, the students worked on three mathematical activities about survey data on children's interests. The students were asked to compare situations by using the given data. In solving the problems, the students could use absolute thinking or they could think in proportional way by employing the relative relationship of a set of numbers.

The first problem was about children' (boys and girls) interest in traditional dance. The students were asked to determine who, boys or girls, were more interested in traditional dance.

Traditional dance

Table 5.1 Traditional dance for girls

Girls who like to do traditional dance	**Girls who don't like to do traditional dance**	**Total girl students**
100	150	250

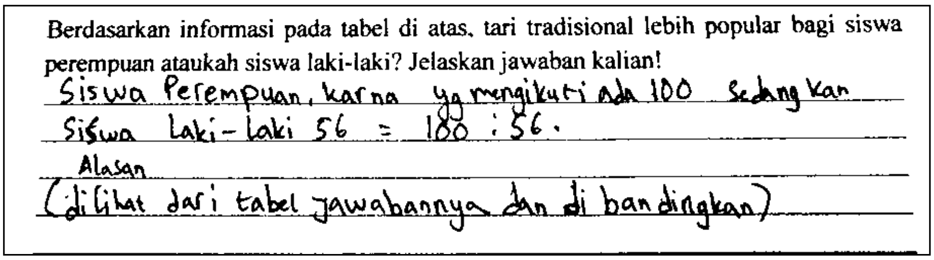
Table 5.2 Traditional dance for boys

Girls who like to do traditional dance	**Boys who don't like to do traditional dance**	**Total boys students**
56	94	150

In solving the first problem, all students stated that the number of girls who liked to do traditional dance was more than boys (100 > 56). Therefore, they concluded that girls were more interested on traditional dance than boys. The following students' works were example of the solutions.

Question 1

Determine, who (boys or girls) were more interested in traditional dance!


Berdasarkan informasi pada tabel di atas, tari tradisional lebih popular bagi siswa perempuan ataukah siswa laki-laki? Jelaskan jawaban kalian!

Siswa Perempuan, karna yg mengikuti ada 100 sedangkan siswa laki-laki 56 = 100 : 56.

Alasan

(dilihat dari tabel jawabannya dan di bandingkan)

Figure 5.22 Maudi's works for problem 1

Translation

Girls are more interested in traditional dance. Because there are 100 girls like to do traditional dance. There are 56 boys like to do traditional dance. So, 100:56.

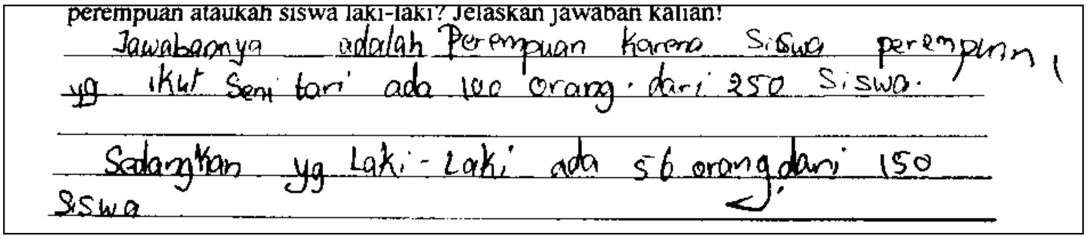

perempuan ataukah siswa laki-laki? Jelaskan jawaban kalian!
Jawabannya adalah Perempuan karena Siswa perempuan yg ikut Seni tari ada 100 orang. dari 250 Siswa.
Sedangkan yg Laki-Laki ada 56 orang dari 150 siswa

Figure 5.23. Soraya's works for problem 1

Translation

Girls are more interested in traditional dance. Because there are 100 girls out of 250 like to do traditional dance. There are 56 boys out of 100 like to do traditional dance.

The aim of giving the first problem, which the students could use both absolute and relative perspective, was to help students to understand comparison situation in different perspective. The idea was not that one perspective is wrong and the other was correct. Both perspectives were useful and it depended on the situations. Therefore, in the second problems, it was designed a problem that students should see the situation in different way instead of seeing the situation in absolute way.

The second problem was about children' (boys and girls) interest in *Basketball* extracurricular. The students were asked to determine who (boys or girls) were more interested in *Basketball* extracurricular.

***Basketball* extracurricular:**

Table 5.3 *Basketball* extracurricular for girls

Girls who like to do basketball activity	Girls who don't like to do basketball activity	Total girl students
100	250	250

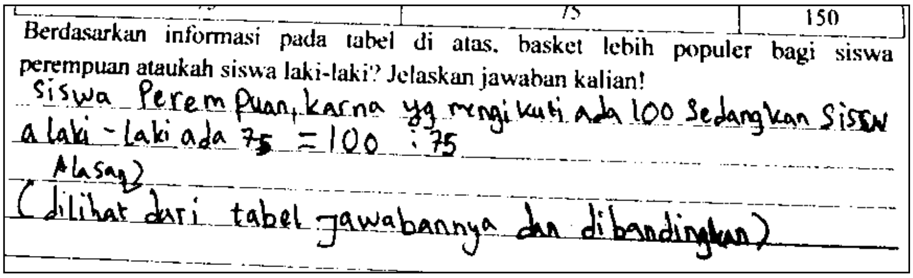
Table 5.4 *Basketball* extracurricular for boys

Boys who like to do basketball activity	Boys who don't like to do basketball activity	Total boys students
75	75	150

In the hypothetical learning trajectory (HLT), the researcher made two kinds of conjectures of students' responses. The students might use absolute comparison by comparing the absolute number of children (boys/girls) or they might employ the concept of proportion (part relative to the whole). Most students solved the problem in absolute way, one of them was Maudi's answer:

Question 2

Who (boys or girls) were more interested in Basketball extracurricular?


75 150

Berdasarkan informasi pada tabel di atas, basket lebih populer bagi siswa perempuan ataukah siswa laki-laki? Jelaskan jawaban kalian!

Siswa Perempuan, karna yg mengikuti ada 100 sedangkan siswa laki-laki ada 75 = 100 : 75

Alasan)

(dilihat dari tabel jawabannya dan dibandingkan)

Figure 5.24. Maudi's works for problem 2

Translation

Girls are more interested in basketball activity. Because there are 100 girls like to do it and there are just 75 boys like to do it. So, 100:75.

In a class discussion, the teacher asked students to elaborate their answers and the reasoning.

1.	Maudi	:	Because, 100 girls like to do it *[she meant that there were 100 girls who like to do basketball activity].*
2.	Nisa and Maudi	:	The total number of girls is 250
3.	Maudi	:	Nah, there are just 75 boys who like to do it *[she meant that there were 75 boys who like to do basket ball activity].*
4.	Teacher	:	There are 75 boys who like to do it. So in this case, what do you compare?
5.	Nisa	:	100 and 75

Transcript 5.5

Both class discussion and students' answer showed that students compared the absolute number of girls (100) and boys (75) who liked to do basketball activity.

One student, Soraya, gave a different answer. But it wasn't clear what she meant because she answered that there were just 150 boys.

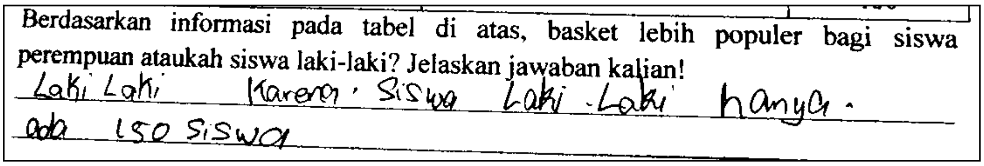

Berdasarkan informasi pada tabel di atas, basket lebih populer bagi siswa perempuan ataukah siswa laki-laki? Jelaskan jawaban kalian!
Laki Laki Karena siswa Laki-Laki hanya ada 150 siswa

Figure 5.25 Soraya's works for problem 2

In the class discussion, the teacher asked Soraya to clarify her answer.

6.	Soraya	:	The answer is boys
7,	Teacher	:	Why?
8,	Soraya	:	Because, total number of boys is less than total number of girls (150 < 250)
9,	Teacher	:	Wait, what do you meant by the total number of boys is less than the total number of girls?
10,	Soraya	:	Kan, like this, eee, usually, boys are more interested on playing basketball than girls. Just because the total number of girls is bigger than the total number of boys, but, the girls who like to do it is less than a half (*she meant that the number of girls who like to it is less than a half of the total number of girls*).
11.	Teacher	:	Wait, what did you say? It is less than a half?
12.	Teacher	:	There are 100 girls who like to do it, and you also consider the total number of girls. So, how do you compare the girls who like to do basketball and the total number of girls?
13.	Soraya	:	**100:250** (*she meant the ratio of girls who like to do basketball activity and the total number of girls*)
14.	Teacher	:	What about the boys?
15.	Soraya	:	**75:150** (*she meant the ratio of boys who like to do basketball activity and the total number of boys*)
16.	Teacher	:	Can you repeat your previous explanation?
17.	Soraya	:	Nah, 150 (*total number of boys*) is less a hundred than 250 (*total number of girls*), So, this is reasonable if there is less boys who like to do basket, because the total number of boys is less than girls.
18.	Teacher	:	So, your answer is that boys are more interested on basket than girls?
19.	 Soraya	:	*(Soraya nodded her head)* For the boys, it is a half Mam.
20.	Teacher	:	How do you come up with "a half" (1/2)
21.	Soraya	:	Because, 150 divided by 75 is 2, 2 times 75 is 150. So, it is a half (*she meant that 75 is a half of 150*)

Transcript 5.6

Soraya was able to describe the part-whole relationship in the set of numbers in each situation. She considered the relation between the numbers of children (boys/girls) who like to do basket (part) and the total number of boys/girls (whole). From line 8, 10 and 17 in the above fragment, it can be seen that she realized that the numbers of boys/girls who like to do basket and the total number of boys/girls altogether influenced the comparison situation.

Based on the discussion, other students came up with the use of fraction. Nina, one of the students stated that there was $\frac{1}{4}$ of girls who liked to do basketball. That was a wrong calculation. So, it seemed that the students understood that they had to find which proportion would give the largest fraction, but calculation appeared difficult for them. It also happened for the third problem. The students got difficulty in comparing the fractions.

Based on students' works, we conclude that the problem may help students to understand that different interpretations are possible, and that relative interpretation by employing the concept of proportionality in the most appropriate. Moreover, the problem may facilitate students to be able to determine proportions in term of part-to-whole for discrete quantity. Besides that, students' strategy and solution for the problems were consistent with the researcher's prediction on the HLT. Several students solved in absolute way and there was a student who employed proportionality in part-whole relationship. But, the calculation seemed difficult for students. Due of that, it was necessary to revise the numbers. It may be better to use numbers that are familiar for students.

e. Learning activity 4

Learning activity 4 is aimed to help students to understand different interpretation, absolute or relative, of proportional situations and to understand that proportional interpretation is the most appropriate. Besides that, the purpose of the activity is to support students to be able to use relative comparison by using concept of proportionality.

In learning activity 4, the students worked on context about Dart games. It was provided result of Dart games played by four children as follows:

Bayu : ●●●●● ●●●●● ●●●○○ ○○○○○

Fadli : ●●●●● ●●●●● ●○○○○ ○○○○○

Gagah : ○●●●○ ○●●●○ ●●●○● ●○○○● ●●●○○○

Rio : ●●●●● ●●●○○

Each child had different chance of shooting and score. The students were asked to determine the most skilful player put the four players in order, from the most skilful to the less skilful by employing the result.

All of students interpreted the situation in absolute way. All of students compared the absolute score of Bayu, Fadli, Gagah and Rio. The figure below shows solution of one student, that was Fadiah.

Question 1

Based on the above result, determine the most skilful Dart player?

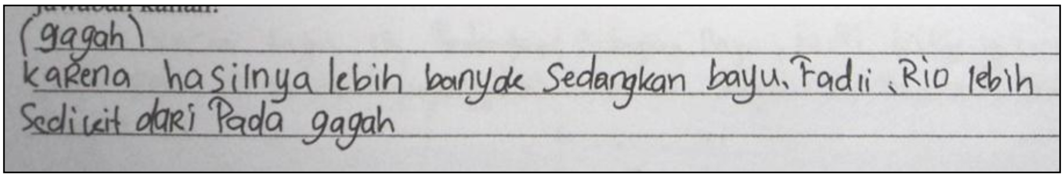


Figure 5.26 Fadiah's works for question 1

Translation

(Gagah)

Because he (Gagah) gets the most score, and Bayu, Fadli, Rio, each of them gets fewer score (than Gagah).

Question 2

Put the four players in order, from the most skilful to the less skilful!

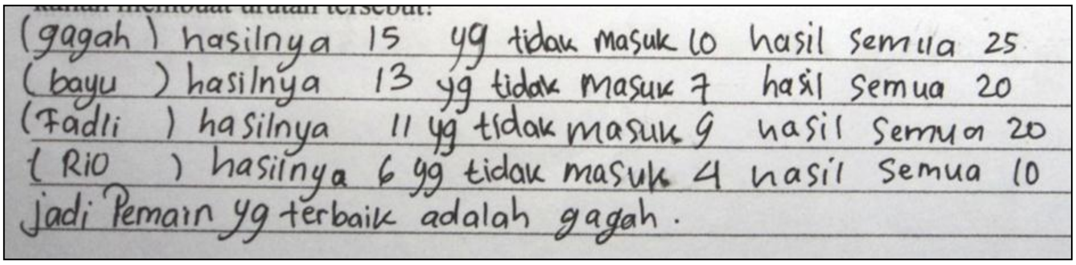

(gagah) hasilnya 15 yg tidak masuk 10 hasil semua 25
(bayu) hasilnya 13 yg tidak masuk 7 hasil semua 20
(Fadli) hasilnya 11 yg tidak masuk 9 hasil semua 20
(Rio) hasilnya 6 yg tidak masuk 4 hasil semua 10
jadi Pemain yg terbaik adalah gagah.

Figure 5.27 Fadiah's works for question 2

Translation

(Gagah), his score is 15, and the missed shootings is 10, the total shootings is 25.

(Bayu), his score is 13, and the missed shootings is 7, the total shootings is 20.

(Fadli), his score is 11, and the missed shootings is 9, the total shootings is 20.

(Rio), his score is 6, and the missed shootings is 4, the total shootings is 10.

The most skilful player is Gagah.

Fadiah' solutions in her written works were in line with her explanation as follows:

1.	Observer	:	Let me take a look at your answer! How do you know that Gagah is the most skilful player?
2.	Fadiah	:	Because it is more that the others [*she was pointing at the game result of Gagah]*
3.	Observer	:	What is more that others?
4.	Fadiah	:	The success shootings of Gagah [what *she meant is the score of Gagah*]
5.	Observer	:	The most success shootings. Who gets the most success shootings?
6.	Fadiah	:	Gagah
7.	Observer	:	How do you know? *And then the student started to count the number of success shootings of Gagah.*
8	Fadiah	:	Gagah gets 15.

Transcript 5.7

Based on Fadiah's works and her explanation in transcript 5.7, it it was clear that she interpret the situation in absolute way by comparing the absolute score of Bayu, Fadli, Gagah and Rio. The rest of class also determined the most skilful player and the order of the four players based on the obtained score.

Due to the students' solutions, the teacher provided a simpler situation as follows:

There are two children, Andi and Alex. They play Dart games.

Andi : ●○●○○

Alex : ●○●●○

1.	Teacher	:	Who is the most skilful player?
2.	Maudi	:	Alex, because he gets the most score.
3.	Teacher	:	OK [*And then, the teacher gave another situation*] What if the result is .. Edi : ●●●●● Eni : ●●●●● ●○●●○ Who is the most skilful player? [*the class was silent for a moment*]
4.	Gagah	:	Edi
5.	Teacher	:	Why do you think that Edi is more skilful than Eni? We can see that Eni gets 8 and Edi just gets 5. Edi's score is less that Eni sn't
6.	Gagah	:	Eh... , all of them are success.
7.	Teacher	:	What do you mean by that? What is success?
8	Gagah	:	I mean, al of Edi's shootings are success.
	Teacher	:	Yes, indeed. But, Eni gets the most score doesn't he? As what Maudi explained before that the child who got the most score was the most skilful player [the teacher confronted student' ideas] [*long pause from the class*]
9.	Teacher	:	Ok, what if, it is in this way, Jaya : ●○ Jaka : ●●●○ ○○○○ Which child is the most skilful?
10.	Gagah	:	Aaa, Jaya gets a half
11.	Teacher	:	What is a half Gagah? And why do you think in that way?
12.	Gagah	:	Because Jaya scores one, and there are two.
13.	Teacher	:	So, how do you get the "a half"?
14.	Gagah	:	Because Jaya get 1 and the total chance of shootings are 2 [The teacher wrote ½ next to Jaya's score]
15.	Soraya	:	Oh, this is similar to the survey data. So, Jaya is more skilful than Jaka. *[it seemed Soraya directly continued her explanation without waiting the teacher's instruction].* Eee, because 3 (Jaka's score) is less than a half of 8 (the total chance of shootings of Jaka), so.. Eh like this ... *[she paused her explanation for e few minutes due to interruption from another students].*

16.	Teacher	:	Students! Let Soraya continue her explanation first!
17.	Soraya	:	Like this Mam, because Jaya scored a half of the total chance and the score of Jaka is less than a half. So, the most skilfull player is Jaya.
18.	Gagah	:	Ah I see *[it seemed Gagah got the main point of Soraya's explanation]*.
19	Teacher	:	OK, that is Soraya's opinion. Any other opinion? Or, is anyone disagrees with Soraya? *[The class was silent again]* The situation of Jaya and Jaka is similar with the game result of Bayu, Fadli, Gagah and Rio isn't? Is there any different idea in solving the (initial) problems?

Transcript 5.8

At the first time, the teacher gave two situations of Andi and Alex, which both children had the same number of total chance of shootings. It was obvious that in order to compare the situation, we just need to compare the score. However, the aim of the learning activity was helping students to understand that different interpretations were possible. For that reason, the teacher proposed different situation, that was the game result of Edi and Eni.

Based on transcript 5.8, different interpretation was emerged after the teacher posed a simpler situation altogether with simpler number as an analogy. At line 4 and 6, it seemed Gagah changed his interpretation. He seemed considering the total chance of shootings instead of comparing the absolute number of score only. The teacher thought that it could be milestone to support students to interpret the given situation in different way. Therefore, the teacher posed other situation, that was the case of Jaya and Jaka.

In interpreting the game result of Jaya and Jaka, Gagah and Soraya were able to interpret it in proportional way (line 10, 12, 14, 15, 17). Moreover, Soraya was able to recognize the similarity of the ideas of the problem with the problem in learning activity 3 (survey data). Based on student' explanation at line 15 and 17, Soraya was able to recognize the part-whole relationship in the set of numbers in each situation. She considered the relation between the score (part) and the total chance of shootings (whole).

At the above discussion, Soraya already stated that due to Jaya's score was ahalf and Jaka's score was less than a half, then she concluded that Jaya was the most skilful. Furthermore, in the class discussion there was a student, Rio, who came up with the use of fraction. He also determined the fractions for each situation, they were $\frac{13}{20}, \frac{11}{20}, \frac{15}{25}, \frac{6}{10}$. It indicates the students understood that they had to find which proportion would give the largest fraction in order to figure out the most skilful player.

Due to students' experience in class discussion about determining the most skilful player, the students changed their strategy in solving the second question. The students compared the proportion for the situations and determining which proportion gave the largest fraction. However, calculating the fractions seemed too difficult for them, so it took too much time. Because of that, the researcher did revise the chosen number to be used in this problem that will be implemented in the second cycle. Furthermore, based on the class discussion, the use of half (as proportion) appeared helpful for students to grasp the idea of proportion in the situation. For that reason, the researcher also included the use of a half as one of proportion in this problem that would be implemented in the second cycle.

Generally, students' strategies in solving the problems in learning activity 4 are consistent with the prediction in the HLT. Besides that, students' written works were aligned with students' explanation in the discussion. At the first time, the students interpreted the situation in absolute way. By providing similar situation with simpler number, it seemed helpful to encourage student in grasping the main idea of the problem. Therefore, we conclude that the problem may promote students to understand that the different interpretations, absolute and relative, are possible in proportional situation, and that relative interpretation by using concept of proportionality is the most appropriate.

f. Post-test

In the end of learning series, the students did a post-test, which was aimed to get impression about how the learning activities support the students to develop relative thinking as one of important type of proportional reasoning. Besides that, the purposes of the post-test were to know how the students solved the problems on proportions after learning the topic.

There were four problems in the post-test of the first cycle including two problem types, density-comparison problems and part-to-whole comparison problems. The problems were different from pre-test items, but they were in the same competence. Post-test problems were in higher level, such as it included more difficult calculation than the pre-test problems. The post-test took 1×35 minutes.

- **Problem 1**

  Problem 1 was about comparing density of boxes for ducks, if there were two boxes, A and B. Box A was $3m^2$ containing 20 ducks. Box B was $4m^2$ containing 25 ducks. From 9 students who did the post-test, there two students who use absolute comparison by comparison the number of poultries and they conclude that box containing more poultries was more crowded. Seven students interpret the problem in different way by using relative comparison. They tried to determine the number of poultries per $1m^2$. But, four students made mistakes on calculating. It might be caused by the chosen number that resulted on non integer number after being computed.

- **Problem 2**

  Problem 2 was about part-to-whole comparison. The situation was about two children, Fadli and Fadlan who owned marbles. Fadli had 50 marbles and 25 out of them were milk marbles. Fadlan had 75 marbles and 37 out of them were milk marbles. The students were asked to determine who had most milk marbles.

  There were three students who employed absolute comparison. Two of them determined that Fadlan had most number of milk marbles because 37 is larger than 25. The other 5 students grasped the idea of part and the whole altogether affected the comparison. Therefore, these students figured out that a half of Fadli's marbles were milk marbles. Meanwhile Fadlan's milk marbles was less than a half out of the total marbles.

- **Problem 3**

Problem 3 was about determining which child was the most skilful in playing Dart. There were Dini and Sari who played Dart. Dini had 15 total chances of shootings and her score were 9. Sari had 6 total chance of shootings and her score were 4. There were 4 students who determined that Sari was more skilful due to she made the least mistakes (absolute comparison). One student compared the absolute number of score.

Two students tried to use proportions in term of part-to-whole and they understood that they had to determine which proportion would give the largest fraction. However, the calculating seemed too difficult for them and they couldn't determine the final result. But, it appeared that the two students understand the concepts of proportionality in this problem.

- **Problem 4**

Problem 4 was similar to problem 2, comparison problem involving part-to-whole relationship. But, problem 2 used less complex proportions than problem 4. The problem was about students' interest on mathematics subject. There were two classroom, 5C and 5D. There were 20 students of classroom 5C and 13 of them stated that mathematics was their favourite subject. For classroom 5D, there were 25 students in total, and 15 of them claimed that mathematics was their favourite subject. Based on this data, the students were asked to determine in which class mathematics was relatively favourable.

There were two students who solved the problem in proportional way (which these students were the same with students who used proportionality in solving problem 3). These two students looked for which proportion would give the largest fraction.

According to the post-test result, it can be seen that several students used relative comparison by employing the concept of proportionality. Some students also understood that instead of comparing the absolute value, they should compare proportions for each situation and determine which proportions would give the largest fraction. What the students did here were aligned to the idea of proportionality emerge in learning process.

g. Conclusion of the retrospective analysis on the first cycle

According to analysis of data collection at the first cycle, we conclude that the comparison problems may support students to understand that different interpretation, absolute and relative, are possible and that the proportional interpretation is the most appropriate. However, it is necessary to do several improvements toward the details if the learning activities. It seemed that students need more support from teacher. The instructional activities work in a learning situation that provides wide opportunity for students to deliver their opinion, reasoning and argumentation. Therefore, the discussion section takes essential role in the learning process. Moreover, the concept of relative and absolute in concept of proportion is rarely used as learning material to promote students' proportional reasoning.

The improvement on teacher's supports are about how does teacher propose questions, what kind of question does the teacher use to scaffold the students, how does teacher react to student's particular response and how does the teacher orchestra the class discussion, are fundamental in supporting his/her student. Therefore, the researcher did improve the teacher guide that later will be used in second cycle.

5.2.2 The Refined Hypothetical Learning Trajectory (HLT)

According to findings of the first cycle, the researcher did revise the initial instructional activities (HLT, students' mathematics activities and teacher guide). However, the improvement didn't bring a big change into it. The researcher improved the details of the material, such as the chosen number (see students' worksheets), the words chosen for scaffolding and the researcher also provided several examples of follow up and probing questions (see teacher guide appendix 5). Besides that, it was provided preliminary activities involving familiar situation for analogy (it has been adjusted in the HLT in chapter 4). The refined instructional activities are going to be implemented in teaching experiment in the second cycle. The improvement of the instructional activities will be elaborated in following section.

a. Learning activity 1

It was easy for students to compare density of two or more boxes that have same size. However, the students were struggling a lot to compare the density of boxes that were different in size and contained different number of objects (chickens). It indicates that students didn't understand how they should solve the problems. It implies that the students' needs more support. The support can be scaffolding from teacher of strong follow up question. For that reason, the researcher did improvement on scaffolding and follow up questions. Moreover, there will be preliminary activities in learning activity 1 in the second cycle.

b. Learning activity 2

The context used in learning activity 2 involving unit measurement of area, but the students always stated it as unit measurement of length. For instance the students stated $5m^2$ as 5m and so on. It indicates that students tended to use something simpler. For that reason, the researcher changed context for the second cycle. The new context was road-asphalting project. The new context still involved continuous quantity including unit measurement of length.

c. Learning activity 3

In the second cycle, the order of the learning activity 3 was switched with learning activity 4. The learning activity 4 in the first cycle was used as learning activity 3 in the second cycle, and the learning activity 3 in the first cycle was used as learning activity 4 in the second cycle. The switch was due to students' responses toward the two activities in the first cycle. The result of the first cycle shows that the situation used in the initial learning activity 4 (Dart games) was simpler and less complex than situation in the initial learning activity 3 (the use of survey data on students' interest). The number chosen in Dart games problem was also smaller than the number used in the problem of survey on students' interests. Moreover, the calculation appeared too difficult for students. Due of that, the researcher revised the chosen numbers with smaller number for the score of the Dart players.

d. Learning activity 4

In the learning activity, it seemed the students already understood that they needed to compare the proportion for the situations and determining which proportion gave the largest fraction. However, calculating the fractions seemed too difficult for them, so it took too much time. Because of that, the researcher did revise the chosen number to be used in this problem that will be implemented in the second cycle. Furthermore, based on the class discussion, the use of half (as proportion) appeared helpful for students to grasp the idea of proportion in the situation. For that reason, the researcher also included the use of a half as one of proportion in this problem that would be implemented in the second cycle.

5.2.3 Classroom observation and teacher interview

Classroom observation was conducted on experimental classroom, and an actual teacher of this class was interviewed. There are 30 students in a class, consisted of 11 girls and 19 boys. The class was arranged in parallel direction with individual desk for every student. The teacher arranged the class into groups, which each group consisted of different number of students for different subject study. There was no particular group works for mathematics lesson, because the mathematics assignment usually was an individual assignment. Therefore, the students usually didn’t have any working group for mathematics. However, there were working group assignment for other subjects, such as for natural sciences.

The researcher also asked to the teacher, which was better, having big group (for instance consist of 4-5 students) or having peer group. The teacher suggested having peer group, because she thought it would be more effective. A big group tended to make several students didn’t focus on their works, because some student might count on other students. In arranging groups, the teacher thought that ideally the group should be heterogenic. Due to the number of students from three different level of achievement was not proportional, so for some groups the teacher couldn’t arrange an ideal heterogenic group.

Regarding to the class discussion, it appeared the students just wrote down their answer on the board and recited it again. The teacher was the one who justified, which one was correct solution and she tended to explain directly about the correct works. This is also aligned with how the teaching and learning process were conducted. The teaching process was one way teaching, the students gave full attention while the teacher was explaining. The students usually had individual pen and paper task and then the teacher examined the students' works. The teacher also explained that the students could come to the teacher if they had difficulties in solving the problems. While the students were working on mathematics problems, the teacher was sitting in her desk.

The teacher informed that not many students who participate actively in the class discussion. She explained that it was not easy to encourage the students to participate actively. There were particular students who were eager to give their opinion, but most of students were not accustomed to speak up in the class. The teacher actually have already encouraged them to speak up, giving opinion or asking question if they had problems in understanding the topic. Based on the above explanation from the teacher, it seemed the students were passive enough. This is in line with researcher's finding from the observation activity, which the class was quite passive. In the learning process, the teacher explained the topic and the students would give full attention.

The actual major of the experimental classroom teacher wasn't mathematics, but it was biology. So it was first time for the teacher to know about PMRI. However, the teacher was willing to learn about PMRI.

5.2.4 Second cycle (teaching experiment)

Second cycle of the present study was conducted at different school, that was elementary school YSPP Pusri Palembang, Indonesia. The second cycle involved 30 students of class 5F and one teacher that was a home teacher of the experimental class. The teaching experiment consisted of four lessons. In each lesson, the students worked on a particular mathematics activity. There was one mathematics activity in each lesson as elaborated in chapter 4, hypothetical learning trajectory (HLT).

a. Pre-test

The second cycle was initiated with pre-test on proportional reasoning. The pre-test items used in the second cycle were the same as items used at pre-test in the first cycle. The pre-test was conducted to gain an insight about student's preliminary ability in solving problems on proportion. Moreover, the pre-test was aimed to identify students' initial understanding on proportional reasoning. There were 25 students who did the pre-test. Five students (out of 30) didn't do pre-test because 3 of them did a sport event and the 2 other students were absent. On the following week, the researcher carried out interview to 15 out of 25 students. There were several considerations in determining students as the interviewees, i.e.: the difference of students' strategy and the mathematics ideas on students' solutions.

- **Problem 1**

  Problem 1 was a simple comparison problem about determining which product was the cheaper one. There were two types of rice in packages; package A contained 2kg rice for Rp. 20.000,00 and package B contained 5kg rice for Rp. 45.000,00. The students were asked to determine which package of rice in the cheaper one.

  Due to the varied solution, the researcher classified students' solution on several types. The classification was done based on students' strategies and mathematics ideas on it. Based on students' works for problem 1, there were four solution types as follows:

  Relative comparison 1 (1RC1)

  Rice package B is cheaper than A.

  In solving this problem, students used relative comparison by considering the relationship of price relative to the amount of rice, in which the students determined the unit price of rice (the price per 1kg rice). If the students computed the price per 1kg rice correctly, they came up with

  Package A, Rp. 10.000,00/kg

  Package B, Rp. 9.000,00/kg

  Hence, they concluded that rice B was cheaper than rice A.

Relative comparison 2 (1RC2)

Rice package B is cheaper than A.

In solving this problem, students used relative comparison by considering the relationship of price relative to the amount of rice, in which the students made the amount of rice (package A and B) into the same weight. Several students made the amount of both packages of rice into 5kg or 10 kg. The change of the amount also changed the price in proportional way. At the end, students came up to the idea of proportionality in term of comparing the price of two different things in the same amount

Absolute comparison 1 (1AC1)

Rice package A is cheaper than B.

The students compared the absolute price of both packages and they came up with the idea that rice with smaller price (rice A, Rp. 20.000,00) was cheaper than rice B (45.000,00).

Absolute comparison 2 (1AC2)

Rice package A is cheaper than B.

The students compared the absolute amount of both packages and they came up with the idea that the lighter package (rice A, 2 kg) was cheaper than rice B (5kg).

The most common solution, given by 12 students, was 1AC1. There were 4 students giving solution 1RC1, 7 students gave solution 1RC2 and 2 students gave solution 1AC2. In general, there were 11 students used relative comparison and 14 students employed absolute comparison.

According to students' works on problem 1, most students solved the problem by comparing the absolute value of price, or comparing the absolute amount of rice (kg). The section below will present several example of students' works based on solution type.

Solution type: 1RC2

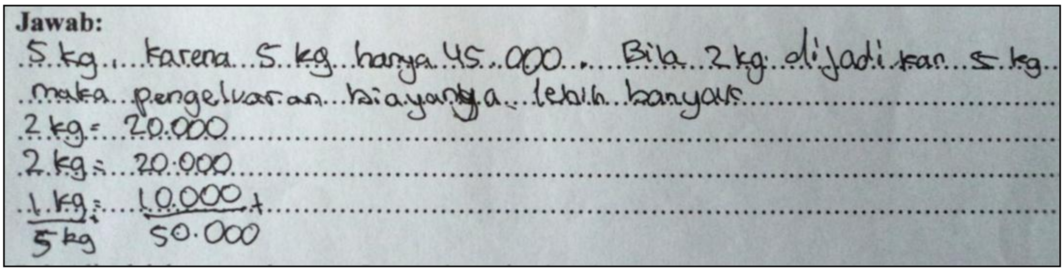

Jawab:
5 kg karena 5 kg hanya 45.000 Bila 2 kg dijadikan 5 kg
maka pengeluaran biayanya lebih banyak
2 kg = 20.000
2 kg = 20.000
1 kg = 10.000 +
5 kg 50.000

Figure 5.29 Bimo's works on pre-test problem 1

Translation

5kg, because 5kg rice costs Rp. 45.000,00. If we make the 2kg rice becomes 5kg rice, then it will cost more money.

2 kg = 20.000
2 kg = 20.000
1 kg = 10.000

---

5 kg = 50.000

According to Bimo's solution, it could be seen that the student came up with the idea of proportionality as follows:

- Because 2kg rice was Rp.20.000,00, he determined that 1kg rice was Rp.10.000,00.
- Bimo used repeated addition in proportional way in order to figure out the price of package A if he changed the original amount of package A (2kg) into 5kg.
- Bimo grasped the idea that he should compare the price of package A and B in the same amount.

There were other 6 students who gave this type of solution for problem 1. It indicates that several students have intuitive understanding of proportional reasoning. The students were also able to employ informal knowledge of proportional reasoning to solve the problem, even though they had never learnt the concept of proportionality yet.

Solution type: 1AC1

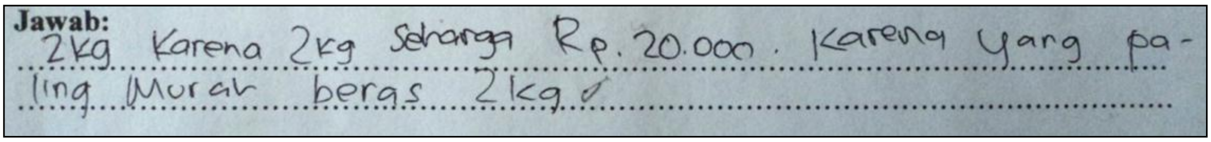

Jawab:
2kg Karena 2kg Seharga Rp. 20.000. Karena yang pa-
ling Murah beras 2kg

Figure 5.30 Yesta's works on pre-test problem 1

Translation

2kg is cheaper, because it is Rp. 20.000,00.

It wasn't clear about the student's reasoning. In order to get insight toward Yesta's reasoning, the researcher did clinical interview. In the clinical interview, the researcher gave follow up question about Yesta's solution. by presenting same context with different number. According to interview, Yesta thought that if we spent less money to buy the rice, then the rice must be the cheaper one. The researcher gave further clinical question by posing the same context with different number as follows:

There are two types of rice in packages; package C contained 1kg rice for Rp. 20.000,00 and package D contained 2kg rice for Rp. 30.000,00.

In solving the above problem, Yesta stood in the same perspective that rice which cost less money (package *C, Rp. 20.000,00*) was cheaper than package D (Rp. 30.000). Yesta couldn't realize that the amount of rice also affected the comparison. Therefore, the researcher built further discussion by giving another comparison situation:

There are two types of rice in packages; package E contained 1kg rice for Rp. 20.000,00 and package F contained 2kg rice for Rp. 20.000,00.

Yesta argued that package E as cheap as package F because both packages were same in price. Until at the end of the discussion, Yesta still thought that we should compare the price only in order to determine which one was cheaper. Therefore she concluded that if A cost less money than B, that A was cheaper than B. Yesta couldn't grasp the relationship of price and amount. The student also wasn't able to get the idea that price and amount of things affected the cost comparison.

Solution type: 1AC2

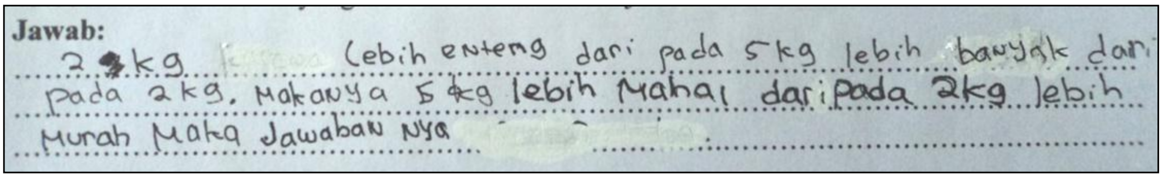

Jawab:
2 kg lebih enteng dari pada 5kg lebih banyak dari pada 2kg. Makanya 5kg lebih Mahal daripada 2kg lebih Murah Maka Jawaban nya

Figure 5.31 A.M Rafly's works on pre-test problem 1

Translation

2kg is lighter than 5kg, and 5kg is more than 2kg. Hence, 5kg rice is more expensive than 2kg.

According to the above solution, A.M Rafly thought that the lighter amount will cost cheaper. In order to get insight toward A.M Rafly's reasoning, the researcher did clinical interview. In the clinical interview, the researcher gave follow up question by presenting same context with different number. The researcher asked A.M Rafly to determine which package was cheaper:

There are two types of rice in packages; package C contained 1kg rice for Rp. 10.000,00 and package D contained 2kg rice for Rp. 18.000,00.

A.M Rafly gave different answer for this problem. After thinking for a few of minutes, he answered that package D was cheaper. The researcher then asked why he stated that package D was cheaper despite of its weight was heavier than package C. The student got difficulties to explain his reason. After a long silent, the researcher asked him about what was the consideration in determining which object was cheaper, whether we took a look at the amount only, or we should consider the relationship of amount relative to the price. The student explained that we should consider the amount and the price.

In solving problem in the follow up discussion, A.M Rafly gave different answer with his previous answer in the pre-test. However, A.M Rafly couldn't explain his reason. In seems that in the follow up question, the problem contained information of unit price. A.M Rafly didn't need to do computation in order to determine the unit price and he was able to see the price per 1kg rice directly. It indicates that less complex problem and the use of simpler number seems helping students to see the proportional idea (and other mathematics idea in general) of the problem easily. However, in order to support students to foster their proportional reasoning, it is need more complex problems. Therefore, the problem about simple comparison price is suitable as starting point to identify students' ability in proportional reasoning.

- **Problem 2**

Problem 2 was a comparing density problem. It was about two chicken boxes, box A and box B, which had different size and contained different number of chicken.

Mr Ali is a poultry farmer. He made two boxes of chicken, box A and box B.

Box A is $2m^2$ contains 10 chickens.

Box B is $5m^2$ contains 20 chickens.

The students were asked to determine which box had more density (which box was more crowded).

Due to the varied solution, the researcher classified students' solution on several types. The classification was done based on students' strategies and mathematics ideas on it. Based on students' works for problem 2, there were four types of solution as follows:

Relative comparison 1 (2RC1)

Box A is more crowded than box B

The students solve the problem by making the size of box A and B into the same. When students change the size of box A/B, then it will also change the number of chicken in proportional way. And finally, in order to determine which box is more crowded, the students can compare the number of chickens of Box A and box B are already same in size.

Relative comparison 2 (2RC2)

Box A is more crowded than box B

The students solve the problem by making the number of chicken in box A and B into the same. When students change the number of chicken of box A/B, then it will also change the size of each box in proportional way. And finally, in order to determine which box is more crowded, the students can compare the size of Box A and box B are already contain the same number of chicken.

Absolute comparison 1 (2AC1)

Students compare the absolute number of chicken in box A and box B

Box B is more crowded than box A, because box B contains more chicken than box A.

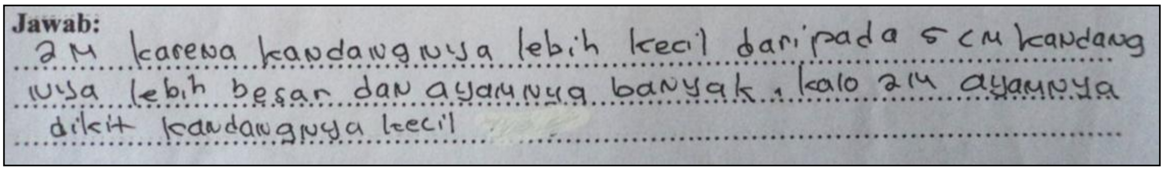
Absolute comparison 2 (2AC2)

Students compare the absolute size of box A and box B

Box A is more crowded than box B, because box A is smaller than box B ()

Most students, 19, solved the problem by using absolute comparison. Fourteen students determined that smaller box was more crowded than bigger box (2AC2). The student didn't think about the number of chicken in it. They were not able to grasp that the number of chicken also affected the density of the boxes.

Solution type: 2AC2



Jawab:
2 M karena kandangnya lebih kecil daripada 5 cM kandang nya lebih besar dan ayamnya banyak . kalo 2 M ayamnya dikit kandangnya kecil

Figure 5.32 A.M Rafly's works on pre-test problem 2

Translation

$2m^2$ box is more crowded, because it is smaller than $5m^2$ box (that $5m^2$) is bigger and contains more number of chicken. The $2m^2$ is smaller (than $5m^2$) and contains less chicken.

In order to get insight about students' reasoning, the researcher did clinical interview. In the clinical interview, A.M Rafly reasoned that we should consider the size of a space in order to determine the density of that space. The researcher gave follow up question as follows:

Let's think about a classroom and a football field. If there are 5 students in the classroom and 50 students in the football field, which place that is more crowded?

After thinking for a while, the student answered that no one of the place was more crowded than the others.

1.	A.M Rafly	:	Football field is large. And a classroom is quite large and there are just few students in it.
2.	Researcher	:	Therefore, in order to determine the density on space, what should we consider about?
3.	A.M Rafly	:	The size of space and the number of people

And then the researcher different situation:

Let's think about a minibus contains 10 students and a classroom that is occupied by 20 students, which one is more crowded?

The discourse below shows student's reasoning.

4.	A.M Rafly	:	The minibus is more crowded than the classroom because the minibus is smaller than the classroom
5.	Researcher	:	What if, there are two mini bus, minibus A and B. There are 10 students occupying minibus A and 5 students occupy minibus B. Which one is more crowded?
6.	A.M Rafly	:	Minibus A is more crowded than minibus B because there are more students occupy minibus A.
7.	Researcher	:	Why do you think that due to the more number of students who occupy minibus A that makes minibus A becomes more crowded?
8.	A.M Rafly	:	Because both minibus are the same (*he meant that both minibus are same in size*)

Transcript 5.9

At the above discourse, the researcher gave different situations, i.e.:

- two places in the same size and that are occupied by different number of people.
- two places in different size and that are occupied by different number of people.

At line 1 we can see that the student compared the conception of crowdedness in relative way, by considering the relationship of size of space (a football field and a classroom) and the number of people who occupy the space. At line 4, the student explained that because of minibus was smaller than classroom, then minibus was more crowded. However, when the students compare the density of two places which were same in size, the student determined than the place containing more people was more crowded.

According to the above discourses, we could see that each situation gave student different feeling about crowdedness. In certain situation, the student compared the size of places in order to figure out which place that was more crowded. Meanwhile, if the two places had same size (line 5, 6, and 8), the student compared the number of people in each place. It indicates that the different situation of density lead the student to use relative comparison. However, it was still difficult for student to discern proportional strategy in solving the proposed problem. Due of that, it is needed to expand the variety of context or situation to be used in order to foster students' proportional reasoning.

In solving problem 2, there were just 6 students who used relative comparison. There were two students who solved the problem by making the size of box A and B into the same and 4 students made the number of chicken in box A and B into the same.

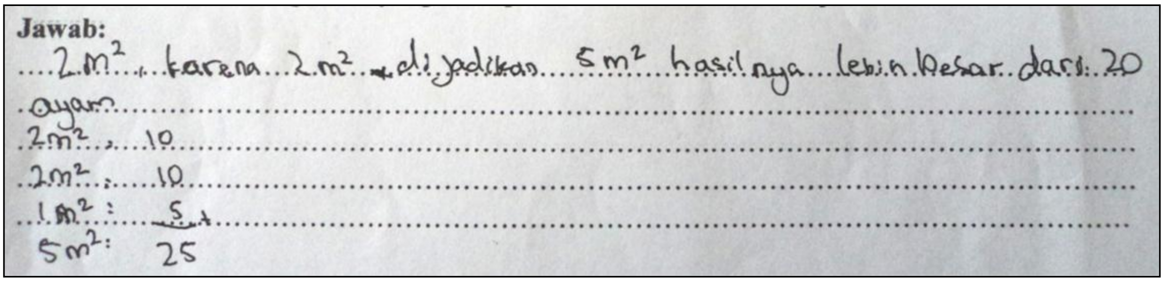


Figure 5.33 Bimo's works on pre-test problem 2

Translation

*the 2m^2 box (**is more crowded**), because if we make the 2m^2 box (**box A**) into 5m^2, then it (**box A**) will contain more than 20 chickens.*

2m^2 = 10
2m^2 = 10
1m^2 = 5
5m^2 = 25

According to Bimo's solution above, it could be seen that Bimo came up with the idea of proportionality as follows:

- Because $2m^2$ chicken-box can be occupied by 10 chickens, he determined that $1m^2$ can be occupied by 5 chickens.
- Bimo used repeated addition in proportional way in order to figure out the number of chicken for box A if he changed the original size ($2m^2$) into $5m^2$.
- Bimo grasped the idea that he should compare the number of chicken of box A and B if both chicken boxes were in the same size.

The students' works and discourse on discussion about problem 2 reveals that by presenting different feeling of density, it might lead students to employ relative comparison in order to figure out which space is more crowded. Moreover, several students have already employed the concept of proportionality in solving the problem. It indicates several students have intuitive understanding of proportional reasoning.

- **Problem 3**

Problem 3 was aimed to investigate how students used data in order to derive a conclusion. In using the data, the students might use the absolute data or they might consider the relationship among information in the data. Problem 3 was about determining for which class (class 5C or 5D) going to beach was more favourable as follows:

After doing the final exam, students of 5C and 5D will have vacation together. The teacher asked about the students' interest on going to beach. From 25 students of class 5C, 12 out of them chose for going to beach. Meanwhile, from 20 students of class 5D, 10 out of them chose for going to beach. For which class, 5C or 5D, going to beach is more popular? Explain your answer!

Due to the students' works, the researcher classified the solution into two types as follows:

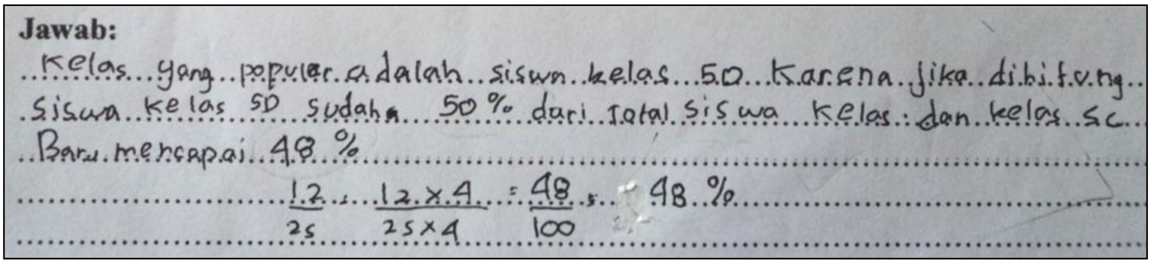
Relative comparison (3RC)

Going to beach is more popular for class 5D because a half of its students like to go to beach and there is less than a half of class 5C who like to go to beach.

Absolute comparison 1 (3AC)

Going to beach is more popular for class 5C because there are more students of class 5C (12) than class 5D (10) who like to go to beach.

There were just 4 students who used relative comparison by considering the relationship of the number of students who liked to go to beach and the total students for each class. These students determine which proportion that gave the largest fraction as shown by the works of Hanif (Hanif even interpreted the proportion into percentage):


Jawab:
Kelas yang populer adalah siswa kelas 5D karena jika dihitung siswa kelas 5D sudah 50% dari total siswa kelas dan kelas 5C baru mencapai 48 %

$\frac{12}{25} = \frac{12 \times 4}{25 \times 4} = \frac{48}{100} = 48\%$

Figure 5.34 Hanif's works on pre-test problem 3

Translation

[Going to beach] is more popular for class 5D. Because if we count it, it is already 50% of the total students of class 5D. And for class 5C, there is 48%.

This is the computation that Hanif made: $\frac{12}{25} = \frac{12\times4}{25\times4} = \frac{48}{100} = 48\%$

It appeared the student understood that he should compare the proportions (in term of part-to-whole relationship) and determined which proportion would result on the larger faction. The students also interpreted the situation into percentages.

One of example the absolute comparison was the works of Ishlah as follows:

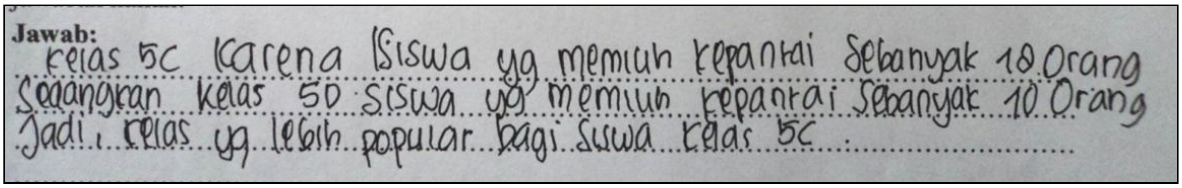

Jawab:
kelas 5C karena siswa yg memilih kepantai sebanyak 12 Orang
Sedangkan kelas 5D siswa yg memilih kepantai sebanyak 10 Orang
Jadi, kelas yg lebih popular bagi siswa kelas 5C

Figure 5.35 Ishlah's works on pre-test problem 3

Translation

[Going to beach] is more popular for class 5C, because there are 12 students choose for going to beach. Meanwhile, in class 5D, there are 10 students who choose for going to beach. Therefore, going to beach is more popular for class 5C

According to Ishlah above works, it is clear that the student compare the absolute number of students who like to go to beach.

Based on students' works, it can be seen that the problem lead the students to see the situation in different interpretations. Several students used absolute comparison. However, it seemed there were several students who understand that they had to consider the relationship between numbers (i.e.: the number of chickens and the size of chicken boxes). Moreover, some students realized that they should compare the proportion for the two situations and determined which proportion gave the largest fraction. It can be seen when the students compared between a half and less than a half.

Conclusion from pre-test and students' interview

Based on the students' works on pre-test and the information obtained from the interview, some students have already had sense on proportional reasoning. Moreover, several students have intuitive understanding on the concept of proportionality. Many students still use absolute thinking in analysing the situations. However, after getting follow up questions and experiencing different proportional situations, several students change their perspective on the situation. Therefore, we may conclude that the students have different level of initial understanding about proportion, which is in line with Sumarto *et al*. (2014) findings.

Besides that, Sumarto *et al*. (2014) pointed out that comparison problems are more difficult that missing value problems. As we have elaborated in the previous section that proportion is taught at the first time at grade five. The students might experience employing proportional reasoning at previous grade in informal way involving simpler proportional reasoning, such as solving missing value problems. With this consideration, it is important to build a discussion that allows the emergence of different reasoning and strategies. Furthermore, it is essential to pose deeper relative questions and posing different type of proportional situations.

b. Learning activity 1

The learning activity 1 in the second cycle was same as in first cycle, which was about density-comparison task. In the first cycle, it was found that grasping an idea of relative comparison on proportional situation was not easy for 5th grade students. It seemed the students needed more support to understand that different interpretations on proportional situations are possible. Therefore, in the beginning of the first lesson, it was given preliminary activities involving simple proportional situations that were close to students' daily life. Furthermore, the preliminary activity was aimed to bring the idea of proportionality in terms of relative comparison and to evoke students' enthusiastic to do the learning process.

First of all, the teacher explained that what the class would learn that day related to mathematics activities in the pre-test. Actually, it wasn't in the teacher guide. However, it was a good idea, because it reminded the students to pre-test activity, so it was supposed helping students to get into mathematics activities.

In the preliminary activities, the teacher posed mind experiment about comparing density. The first mind experiment was about two rooms, room A and room B, which occupied by different number of children. The rooms were same in size. The teacher asked the students to determine which room was more crowded. All students agreed that the room contained more children must be more crowded than the other one. However, there was no discussion in order to raise different interpretation about the situation. It seemed that the students compared the density of room A and room B based on the number of people occupying the rooms.

Therefore, it was not known yet whether the students recognized about factor size of the room or not.

In the next mind experiment, the teacher posed situation of two public transports, "***Angkot***" and "***Trans-Musi***" that are different in size (***Trans-Musi*** is bigger than ***angkot*** and ***Trans-Musi*** can load more passengers than ***angkot***). Each of the public transport was occupied by the same number of passengers (10 passengers). All of students agreed that ***Angkot*** must be more crowded than ***Trans-Musi***.

The teacher asked for the students' reasoning in order to get insight about students' understanding.

1. Teacher : Why do you think that Angkot is more crowded? Each of them (***angkot***, ***trans-musi***) contains 10 passengers doesn't it? [*as we know that in the previous discussion, the students determined that space occupied by more people was more crowded. In this case, there are same number of people in* ***angkot*** *and* ***trans-musi***].
2. Damar : Because angkot and trans-musi are different in size. Even though the number of passengers in both angkot and ***trans-musi*** are same, but ***trans-musi*** is bigger.
3. Teacher : So, what factors do influence density?
4. Students : The size and the number of passengers [*the students answered in choir*].

Transcript 5.10

At line 2 and 4 in transcript 5.10, we can see that the students started thinking about the relationship of size of space and the number of people occupying the space. It indicated that experiencing different feeling of density may help students to recognize different interpretation of the proportional situation.

In the main learning activity, the students worked in peer-group. The first lesson involved **30** students, so there were **15** peer-groups. Actually, before the class began, the teacher had already created groups and the students already sat with their own peer. The students worked on the mathematics activity for 20 minutes. The teacher explained a little bit about the context.

There is Pak Ali, a poultry farmer, who makes four chicken boxes, A, B, C and D. Each box is in different size to other boxes **(*box A = 1 m²*, *box B = 1 m²*, *box C =* $1\frac{1}{2}$ *m²*, *box D = 2 m²*)**. *Moreover, each box is occupied by different number of chicken. There is a regulation that for 1m² chicken box, the maximum number of chickens that can be loaded is 20.*

The students were asked to help Pak Ari examining which box was the most crowded and making an order of Pak Ari's chicken boxes from the most crowded to the least crowded. The figure below is the representation of Pak Ari's chicken boxes.

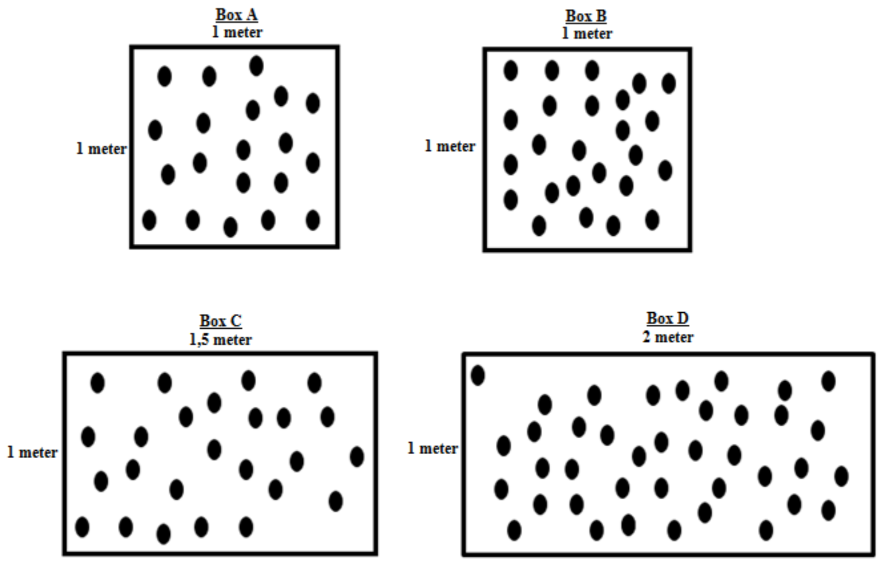


Figure 5.36 Chickens boxes

During the group work activity, the researcher was observing how each group worked. The researcher then came to the group of Hanif and Adam. The researcher asked about students' strategies and reasoning. At the first time, the researcher asked which box was the most crowded. Adam, one of the students, answered that box D was the most crowded because box D was the biggest one. And then, the researcher posed follow up questions as follows:

1.	Researcher	:	Why do you think that the biggest box is the most crowded?
2.	Adam	:	Because it can load the most number of chicken [*he is pointing to the box D*]
3.	Researcher	:	Ooo, it can load the most number of chickens. *[And then, the researcher continued asking about students' reasoning].*
4.	Researcher	:	You said that the most crowded is box D, because it is the biggest one, so it can load the most number of chickens.

After that, the researcher reminded the students about the situation of ***Angkot*** and ***Trans-musi***. The researcher used the situation of ***Angkot*** or ***Trans-musi*** as an analogy for the situation of chicken boxes due to students were familiar with ***Angkot*** or ***Trans-musi.***

	Researcher		Then, which one, ***Angkot*** or ***Trans-musi*** that can load more passengers?
4	Adam	:	Trans-musi.
5	Researcher	:	*(The researcher continued...)* What if there are 10 passengers in ***angkot*** and 10 passengers in ***Trans-musi***. Which one is more crowded**,** ***angkot*** or ***Trans-musi***?
6	Adam	:	Angkot [*Adam directly answered*]
7	Researcher	:	But, Transmusi can load more passengers! *[The researcher confronted the students' late answer with the students' previous answer that public transport which can load more passengers is more crowded (line 2)].*
8	Hanif	:	The size.
9	Adam	:	O ya, the size!
10	Hanif	:	1 meter, 1 meter, 1 and a half meter... [*Hannif was stating the size of the boxes while pointing at the boxes*].
11	Hanif	:	40 isn't it?
12	Researcher	:	Then, how do you solve the problems?
13			
	Hanif	:	If it (box B) is 2m^2, there will be 50 chickens [*Hanif was pointing at his computation of box B*]. *(The original size of box B is 1m^2 and it contains 25 chickens).*
14	Adam	:	Oh, that's true.. [*he was nodding for Hanif's statement*]
15	Researcher	:	He-eh, and then?
16	Hanif	:	If I make it (box B) into 2m^2, there will be 50 chickens [*he was computing*]. And for this one, 40. *[He was pointing at box A*, and that *he might mean that if he made box A into 2m^2, there would be 40 chickens].*

Transcript 5.11

According to the above discussion, we could see that at the first time, Adam used absolute comparison. As stated at line 2, Adam just considered the number of chicken occupying the box, so he concluded that box contained the most number of chicken must the most crowded. The researcher posed simpler situation as an analogy for the initial problems, in which the researcher used a context that was familiar for students (context of density on public transport), and the students began to consider the relationship of the number of population and the size of place.

It can be that seen that at line 4 and 6, Adam developed different recognition about the concept of density. At line 8, Hanif started thinking about the size of space. Moreover, it is obvious that at line 13, the student began employing the concept of proportionality in solving the problem. It seemed the given context helped students to see the situation in different interpretation. Moreover, the use of familiar context helped students to get more insight about the idea of problems. But, due to the lime limit, the discussion must be stopped and he students continued to finish their works.

After all groups finished their works, the teacher held a whole class discussion. The teacher pointed out several groups to present their works in front of class. In determining which group that presented their work in front of class, the teacher considered the variety of students' solutions and strategies.

The first group presenting their work in front of class used absolute comparison to solve the problem.

Question 1

Which box is the most crowded?

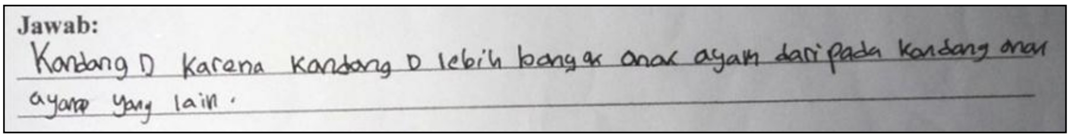

Jawab:
Kandang D karena kandang D lebih banyak anak ayam daripada kandang anak ayam yang lain.

Figure 5.37 Daffa and Habib's works on question 1 learning activity 1

Translation

Box D (is the most crowded), because box D contains the most number of chicken than the other boxes.

Question 2

Make an order of the boxes from the most crowded to the least crowded box!

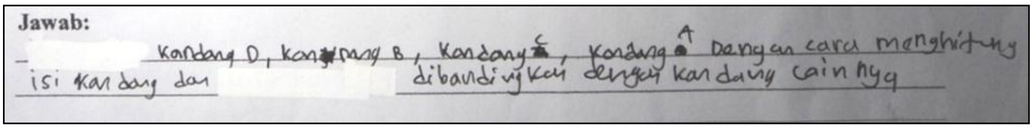


Figure 5.38 Daffa and Habib’s works on question 2 learning activity 1

Translation

(The order is) box D, B, C, A, by counting the number of chickens and comparing the number of chicken occupying the boxes.

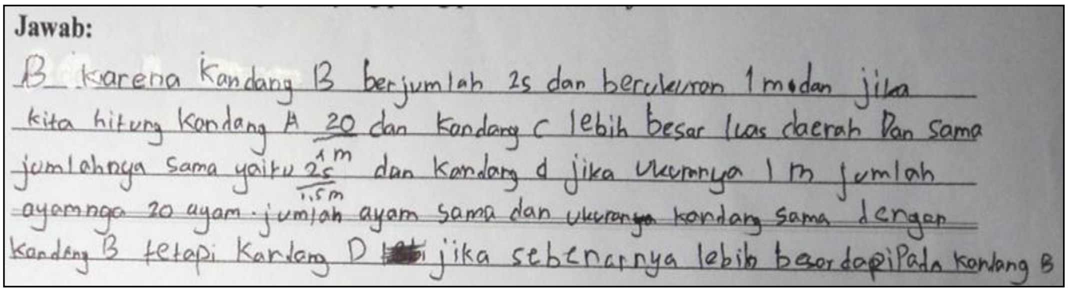
Based on students’ works above, it was clear that the students compared the absolute number of chicken in each box. The teacher asked the above group to give motivation for their solution. But, it was difficult for students to support their answer with reasoning or argumentation. Both students just smiled.

The students in this class were not used to explain or supporting their solution with reasoning. Due of that, the teacher changed her wording to ask for explanation. Instead of using: “*explain!; give explanation!; prove that your solution is the correct one!*”, the teacher asked the student to give comment toward their own works or other students’ works. Asking the students to give a comment was more effective in engaging them in reasoning and discussion. But, there were just few students who were eager to participate actively.

The teacher asked for comments from another groups about the works of Daffa and Habib. There was no comment raised on Daffa’s and Habib’s works. The passive response also happened when the teacher asked whether there were other students who agreed or disagreed. And finally, the teacher asked whether any different solution. That was a group of Hanif and Adam who proposed different solution for the problem.

Question 1

Which box is the most crowded?


Jawab:

B karena Kandang B berjumlah 25 dan berukuran 1 m dan jika kita hitung Kandang A 20 dan Kandang C lebih besar luas daerah Dan sama jumlahnya sama yaitu $\frac{25}{1,5 m}$ dan Kandang d jika ukurannya 1 m jumlah ayamnya 20 ayam. jumlah ayam sama dan ukurannya kandang sama dengan Kandang B tetapi Kandang D jika sebenarnya lebih besar dapi Pada kandang B

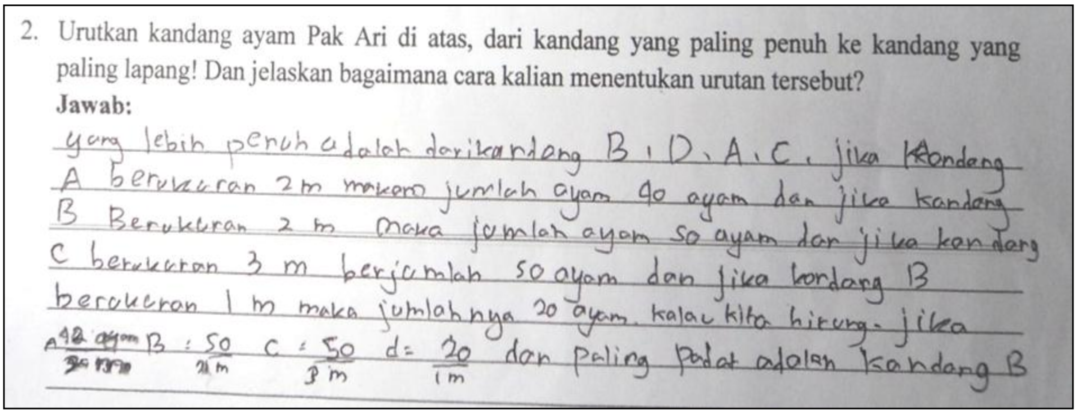
Figure 5.39 Hanif's and Adam's works on question 1 learning activity 1

Translation

The most crowded is box B. The number of chicken in box B is 25 and the size of box B is 1 m^2. If we count for box A=$\frac{20}{1\,m^2}$ and box C is bigger than box B and the number of chicken in box B and C is equal, that is $\frac{25}{1½\,m^2}$; And for box D, if its size is 1m, it can load 20 chickens, and box D has the same size with box B, but in fact, box D is bigger than box B.

Question 2

Make an order of the boxes from the most crowded to the least crowded box!


2. Urutkan kandang ayam Pak Ari di atas, dari kandang yang paling penuh ke kandang yang paling lapang! Dan jelaskan bagaimana cara kalian menentukan urutan tersebut?

Jawab:

yang lebih penuh adalah dari kandang B, D, A, C. jika Kandang A berukuran 2 m makam jumlah ayam 40 ayam dan jika kandang B Berukuran 2 m maka jumlah ayam 50 ayam dan jika kandang C berukuran 3 m berjumlah 50 ayam dan jika kandang B berukuran 1 m maka jumlahnya 20 ayam. kalau kita hitung. jika A $\frac{40 \text{ ayam}}{2 m}$ B : $\frac{50}{2 m}$ C : $\frac{50}{3 m}$ d= $\frac{20}{1 m}$ dan paling padat adalah kandang B

Figure 5.40 Hanif's and Adam's works on question 2 learning activity 1

Translation

*The order of boxes is B, D, A, C; If box A is 2 m^2 then it can load 40 chicken; If box **B** is 2 m^2, then it can load 50 chicken; If box C is 3 m^2, then it can load 50 chicken; If box B is 1 m^2, then it can load 20 chicken; (they might mean for box D) A= $\frac{40\ chickens}{2\,m^2}$; B= $\frac{50}{2\,m^2}$; C= $\frac{50}{3\,m^2}$; $D = \frac{40}{2\,m^2}$. The most crowded is box B.*

Based on Hanif's and Adam's works, it can be seen that the students employed relative comparison because they already considered the relationship between the number of chicken relative to the size of box instead of comparing the absolute number of chicken in box A, B, C and D.

Moreover, the students came up with the idea of proportionality that was they could figure out the density of the chicken boxes by comparing the number of chicken if the boxes were in the same size. In this problem, Hanif and Adam doubled the size of all boxes, except box D. It might be caused the biggest box was twice large than other boxes (except box C). Due to the doubling of box size, then the number of chicken was also being double. The students' strategy in solving the problem indicates that the students understood that the size of box and the number of chicken altogether affected the comparison.

There was other solution of Bimo's and Ajeng's works. Their solution was similar as Hanif's and Adam's. But, Bimo and Ajeng used different strategy.

Question 1

Which box is the most crowded?

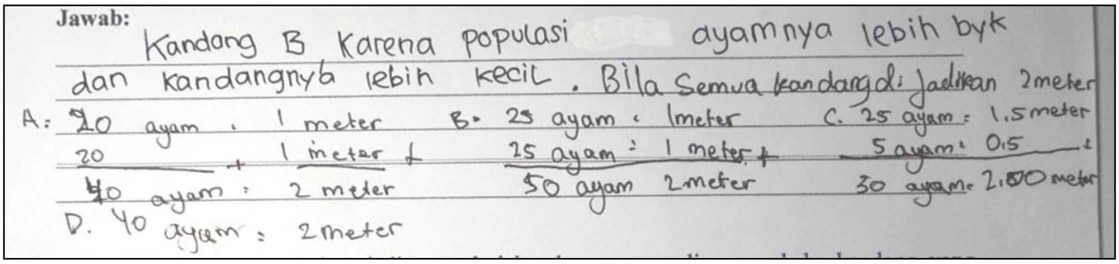

Jawab: Kandang B karena populasi ayamnya lebih byk dan kandangnya lebih kecil. Bila semua kandang di jadikan 2meter
A: 20 ayam : 1 meter B: 25 ayam : 1meter C. 25 ayam : 1,5meter
20 + 1 meter + 25 ayam : 1 meter + 5 ayam : 0,5
40 ayam : 2 meter 50 ayam 2meter 30 ayam : 2,00 meter
D. 40 ayam : 2meter

Figure 5.41 Bimo's and Ajeng's works on question 1 learning activity 1

Translation:

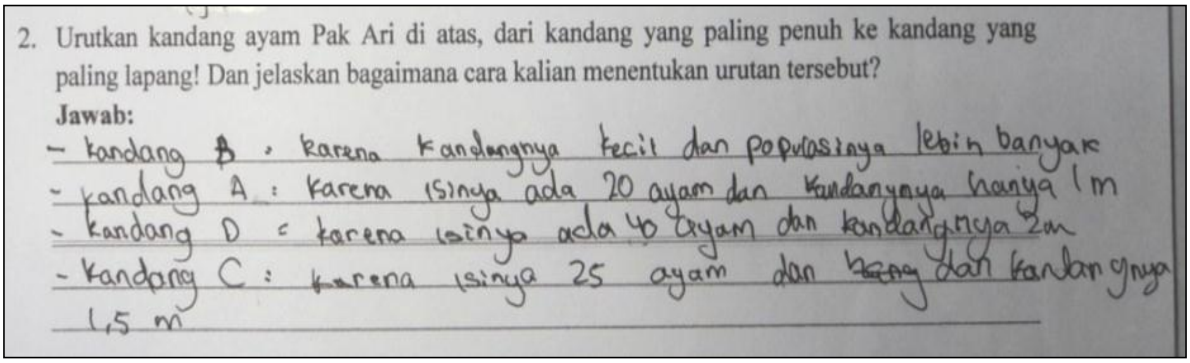
The most crowded is box B, because it has more chicken and the box is smaller.

A	20 chickens =	1 m	**B**	25 chickens =	1 m	**C**	25 chickens =	1 ½ m
	20 chickens +	1 m +		25 chickens +	1 m +		5 chickens +	½ m +
	40 chickens =	2 m		50 chickens =	2 m		30 chickens =	2 m

D contains 40 chickens for 2m² box.

Question 2

Make an order of the boxes from the most crowded to the least crowded box!



2. Urutkan kandang ayam Pak Ari di atas, dari kandang yang paling penuh ke kandang yang paling lapang! Dan jelaskan bagaimana cara kalian menentukan urutan tersebut?

Jawab:

- kandang B : karena kandangnya kecil dan populasinya lebih banyak
- kandang A : karena isinya ada 20 ayam dan kandangnya hanya 1 m
- kandang D : karena isinya ada 40 ayam dan kandangnya 2m
- kandang C : karena isinya 25 ayam dan ~~[illegible]~~ dan kandangnya 1,5 m

Figure 5.42 Bimo's and Ajeng's works on question 2 learning activity 1

Translation

The order of boxes is B, A, D, C

Box B; it is small and it has more number of population (chicken)

Box A; because there are 20 chicken in it and the box is only 1 m^2

Box D; because there are 40 chickens and the box is 2 m^2

Box C; because there are 25 chicken and the box is 1 ½ m^2.

In solving question 1, Bimo and Ajeng made the size of boxes into the same, which it also changed the number of chicken in each box in proportional way. Bimo and Ajeng used repeated addition strategy in proportional way, meanwhile Hanif and Adam use doubling.

Bimo and Ajeng made a little mistake at their computation for box C. It might happen due to the number was not easy to be calculated. At their works, Bimo and Ajeng aimed to make box C into $2m^2$. Therefore they wanted to add the initial 1½ m^2 box with ½ m^2. However, it seemed they got difficulty to determine the number of chicken at box C if the size of box was ½ m^2. It wasn't clear why did they determine that there were 5 chickens for ½ m^2 box C. There was no discussion about this matter.

Furthermore, in answering question 1, at the first place Bimo and Ajeng compared the density of boxes that were in the same size. It supported the findings of Sumarto, *et al*., (2014) that students might come up with the idea of proportionality in simple that we should compare the number of chicken of boxes that were in same size. This kind of proportional reasoning was also shown in students' way to solve the problem by making one value (the number of chickens or the size of boxes) into the same, and then the students would compare the other value. Therefore, the students neither just compared the absolute number of chickens in the four boxes nor compared the absolute size of the four boxes.

Based on students' works and the students' discussion on learning activity 1, we conclude that the comparing density tasks may help students to begin to understand that different interpretations of situation are possible, and that relative comparison by considering the relationship of information (data) would lead to the most appropriate solution. The students' different way in interpreting the situation and the variety of students' solutions are aligned to what is predicted in the HLT. The use of context of density lead students experiencing different feeling of crowdedness, and it support Lamon's (2006) statement. Besides that, students' familiarity to the context may lead them to get insight about the problem more easily, which is in line with Sumarto's, *et al*., (2014) finding. According to students' strategy in solving the problem, it seems students understand that they should make one value (the number of chickens or the size of boxes) be the same, and then the students could compare the other value. Sumarto's, *et al*., (2014) also found this kind of proportional reasoning in their study.

In the learning process, the teacher also already did support students to elicit proportional reasoning by posing open questions, explanatory questions and follow up questions. However, it seems that the teacher is still the one who justify whether a particular mathematical explanation is acceptable or not. After the last group of Bimo and Ajeng presented their works, the teacher directly justified that the solution of the two last groups were the correct one. Moreover, when the teacher found the correct solution of students, it seems that the teacher will directly determine the solution is the correct one instead of confronting the solution to encourage students to think aloud. In addition, most students are passive and it is difficult to engage them to participate actively in the class discussion. Therefore, it seemed that there was no real class agreement, and the teacher was the one who derive the conclusion of the learning activity.

In analysing the result, the researcher did examine what student did in their written work and what the students' explanation was in the discussion. The researcher found the data collected form students' discussion are in accordance with students' works.

c. Learning activity 2

The goal of the learning activity 2 is to help students to understand that different interpretation of proportional situations is possible. Moreover, the mathematics activity is aimed to support students in determining part-whole relationship in problems involving continuous quantities and the students may interpret the relationship into fractions. Besides that, the learning activity is designed to facilitate students in applying relative comparison by using comparison of part to whole to solve the problems.

The context used in the second mathematics activity for second cycle is different from the first cycle. In the second cycle, it was used a context of of asphalting road project. The students worked on problem about a project of making three new roads, road A, B and C. The current activity of the road project was asphalting the roads. The three roads were being asphalted but it didn't finish yet. Each road had different amount of asphalted part. The students were asked to determine asphalting project in which road that was mostly done. It was expected that there would be students who used relative comparison by considering the part-whole relationship of the asphalted part and the total length of the road instead of comparing the absolute length of the asphalted part.

The students were asked to help the chief making a report of the progress of the road project. In order to make a clear description of the project, the chief is aimed to make a visualization of the progress of the project. The roads are visualized into bars. The students are asked to determine the asphalted section by shading the bars.

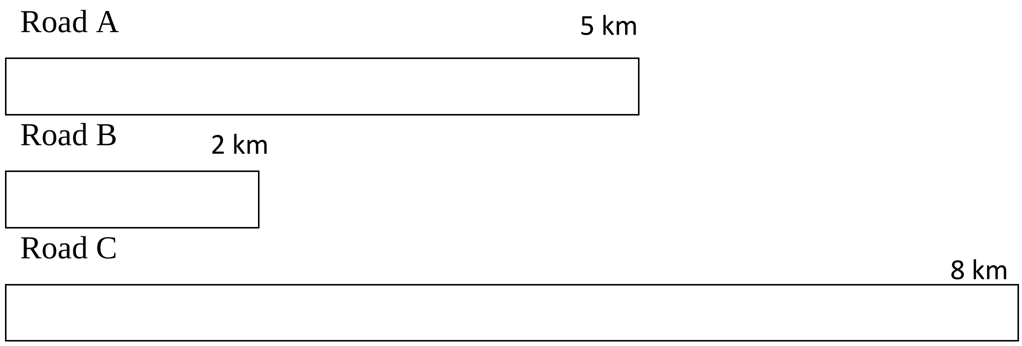


Figure 5.43 Bars representation of the roads

After visualizing the progress of the road-asphalting project, the students were asked to determine which road project (A, B or C) is mostly done. And then, they had to put the road in an order based on the progress of the project.

Before the students worked on the main activity, the teacher gave preliminary activities, which were aimed to bring the idea of proportionality in terms part-to-whole comparison and to evoke students' enthusiastic in doing the learning process. In the preliminary activities, the teacher provided a situation that the students might interpret in absolute or relative way. The teacher explained that she had a bamboo 3 m long. She would use the bamboo to put up Indonesian flag for celebrating Indonesian Independence Day August 17. Therefore she wanted to painted in red and while colour. She painted the 1 m in red. The teacher asked her students to determine how much part of the bamboo that was painted in red. The transcript below shows the class discussion.

1.	Hegel		1 m. Because you painted the 1 m of it.
2.	Teacher	:	OK, is there any comments or objection toward Hagel' answer? *[The class was silent. And then, the teacher continued:]* Is there any different idea?
3.	Damar	:	That is 1m, so a quarter ($\frac{1}{4}$) of it is red.
4.	Teacher	:	Why do think that the red part is a quarter ($\frac{1}{4}$) of the bamboo?
5.	Damar	:	Because there is 1 m, and there are 3 m. Eh, 3m is the total length isn't it? [*Damar clarified his answer*].
6.	Rafly	:	It is $\frac{1}{3}$!
7.	Damar	:	Oh ya, that is $\frac{1}{3}$ [*Damar nodded his head*]
8.	Teacher	:	What do you mean by $\frac{1}{3}$? And why do you think that way?

			As we know that the red part is 1m.
9.	Damar	:	Because the red part is 1m and the total length is 3m. So, the red part is $\frac{1}{3}$. yes, $\frac{1}{3}$ out of the total
10.	Teacher	:	OK, it is different answer from Damar. Damar answered that the red part is $\frac{1}{3}$ out of the total. What do you think about that? Is there anyone that wants to give comments?
11.	Rafly	:	I think it is $\frac{1}{3}$, because the red part is 1m and the total is 3m.
12.	Teacher	:	Rafly also think that it is $\frac{1}{3}$. Is there any other comments?

The class was silent. And then the teacher went directly to the main activity. The teacher and the researcher gave students' worksheet for each group.

Transcript 5.12

According to class discussion in transcript 5.12, it can be seen there was a student who interpret the situation in absolute way by considering the absolute length of the red part (line 1). There were also students who interpreted the situation in relative perspective by recognizing the relationship of part out of the whole (line 5, 6, 7, 8 and 11). However, it was not obvious yet whether the class (as a whole) began to understand the different interpretation of the situation because there were just some of them who gave responds toward teacher's question. The rest of the class was silent. However, it was supposed that the discussion in the preliminary activities could give an overview and insight about the concepts that the students were going to learn. After that, the students worked on the main mathematics activity in a peer group for 15-20 minutes. While the students were doing working group, the researcher observed the class activity.

Based on students' works, students' strategies in shading the bar can be distinguished into two, by using the concept of proportionality and not and non-proportional strategy.

Using the concept of proportionality

- Several groups divide the bars into the number of equal parts, 5 equal parts for bar A (road A), 2 equal parts for bar B (road B), and 8 equal parts for bar C (road C), and then they shaded the part that represented the asphalted section (2 parts for bar A, 1 part for bar B, 3 equal parts for bar C). In determining the part, most students did measure the actual length of the bars by using ruler.

The students divided the number that was the actual length of the bars by the number of parts of each bars (5 or 2 or 3, depended on the bar).

- Several groups shaded bar B first in order to obtain 1 km, because in order to determine 1 km of bar B, they just needed to divide the bar into two equal parts. After that, the students used the 1 km they obtained from bar B as a unit measurement.
- In determining 3km out of 8km in bar C, there was a group who moved bar A into bar C. As we know that bar a represented 5km road. Therefore, they could obtain 3km by subtracting 8km with 5km.

The figure below shows one example of students' works on shading the bar in proportional way.

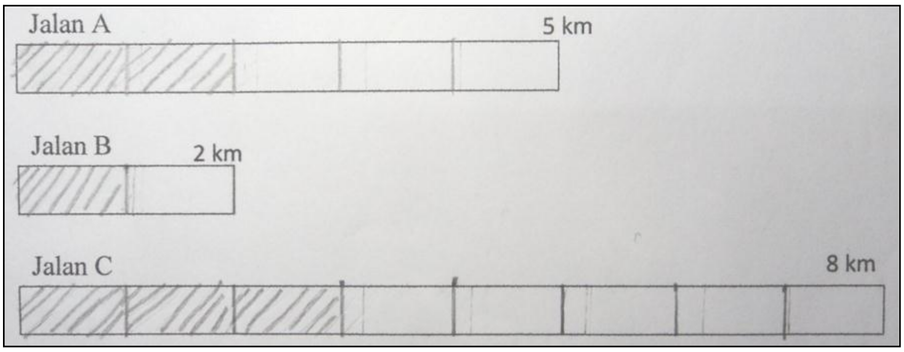


Figure 5.44 (a) Student's strategy on shading the bars

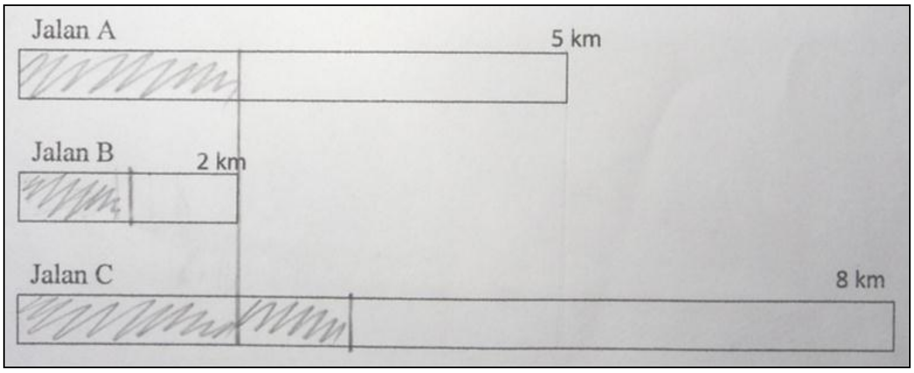


Figure 5.44 (b) Student's strategy on shading the bars

From figure 5.49.a, it can be seen that the students divided the bars into the number of equal parts. Meanwhile figure 5.49.b shows that the students utilized their partition on the preceding bars.

Using non proportionality concept

Several groups interpreted 1km as 1cm on the bar. However, actually 1cm didn't represent the 1 km of the road. Therefore, the shaded parts were not in the right proportion. One of groups who shaded the bar not in proportional way was the group of Conny and Farhan. When the teacher asked why they did in that way, they argued that 5km (the length of the road A) was represented by 5cm (the length of bar A). The teacher asked both students to prove their argument, but they failed to do it. And then, the teacher asked whether the students had different strategy in shading the bar accurately, but both students just smiled.

Students' strategies in shading the bar to represent the asphalted section of the road are in line with the conjectures in the HLT. It was predicted that several students might shade the bar in proportional way and there might be several students who didn't shade the bar in proportional way.

The researcher then stopped by in the focus group, the group of Hanif and Bimo. The focus group in the second lesson was different to focus group in the first lesson. It happened because one of students in the previous focus group (Hanif and Adam), that was Adam, was absent at that time. However, in the third and fourth (last) lesson, the focus group remained the same (the group of Hanif and Bimo). The following transcript showed the discussion of both students.

1.	Researcher		How do you determine the order of the road-asphalting activity from the mostly done to the least one?
2.	Bimo	:	The order is based on the length of the asphalted section of each road.
3.	Researcher	:	What is the order?
4.	Bimo	:	The order is C, A and B. Because the asphalted section of road C is 3km, and road A is 2km, road B is 1km [*in this case, he compared the absolute length of the asphalted section of road A, B, and C*].
5.	Researcher	:	So, what do you see to determine the order of the road project?
6.	Bimo	:	The length of the asphalted section of the road. 3km is the longest, and then 2km and 1km is the shortest.
7.	Researcher	:	Do you have another idea in comparing the situation? What do you think about our discussion about the painted bamboo before [*the researcher tried to remind the students toward the discussion on the preliminary activity*]
8.	Bimo & Hanif.	:	*Both students saw to each other. They read the problem again and analyzed.* *After few minutes, the researcher came back to that group.*

9.	Hanif	:	Three eight ($\frac{3}{8}$) [*Hanif said something to Bimo*].
10.	Bimo & Hanif.	:	Two fifth ($\frac{2}{5}$), a half ($\frac{1}{2}$), three eight ($\frac{3}{8}$). (*The students added the fractional representation on their works*)
11.	Researcher	:	What is three eight ($\frac{3}{8}$)? And how do you come up with the fractions? [*the researcher was pointing at students' works*].
12.	Bimo	:	Three eight ($\frac{3}{8}$) comes from 3km asphalted section out of the total length 8km.
13.	Researcher		And now, how do you use the fractions to solve the problem? Or, do you solve the problem by comparing this one [*the researcher was pointing at the absolute length of asphalted section of the three*]?
14.	Hanif		Which one should we use? [*Hanif asked to Bimo*] I think we should use this one [*while pointing at the fraction as proportional representation of the situations*]
15.	Researcher		So, which one do you use to solve the problem?
16.	Bimo		We solve the problem by comparing the fractions [*he was pointing at two fifth ($\frac{2}{5}$), a half ($\frac{1}{2}$), three eight ($\frac{3}{8}$)].*
17.	Researcher		Why do you compare the fractions? At the first time, you made the order by comparing the (absolute) length of the asphalted section didn't you?

The discussion of the group should be stopped due to the lack of the time. Hanif and Bimo then finished their work before the teacher orchestrated a class discussion

Transcript 5.13

Based on the discussion in transcript 5.13, we can see that at the first time, the group of Hanif and Bimo used absolute comparison. But, after the researcher reminded them toward the discussion of preliminary activity that involved a similar problem which was simpler that the initial problem, the students were encouraged to find a different idea. It seemed that simpler situation, altogether with the use of simpler number, might affect students' responses. The different interpretation in the problem, the use of absolute and relative comparison, is in line with what is predicted in the HLT.

Based on students' response at line 9, Hanif came up with the idea of proportionality in term of part-whole relationship. Moreover, Hanif directly represented the part-whole relationship into fractions. And finally, both students agreed to use fractions. It indicates that the students are able to determine part-whole relationship in a proportional situation. Line 12 shows us that the students understood the meaning of the fractions as representations of part out of the whole. As we can see at line 16, Bimo stated that they needed to compare the fraction in order to solve the problem. It shows that the students understood that they had to find which proportion would give the largest fraction.

In the class discussion, the teacher asked the group of Meutia and Ariq to present their works. This is the works of Meutia and Ariq.

Question 2

Make an order of the road project, from the mostly done to least one.

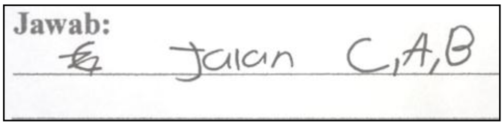

Jawab:
Jalan C, A, B

Figure 5.45 Meutia's and Ariq's works on question 2 learning activity 2

Translation

The order is C, A, B.

Question 3

How do you make the order of the project?

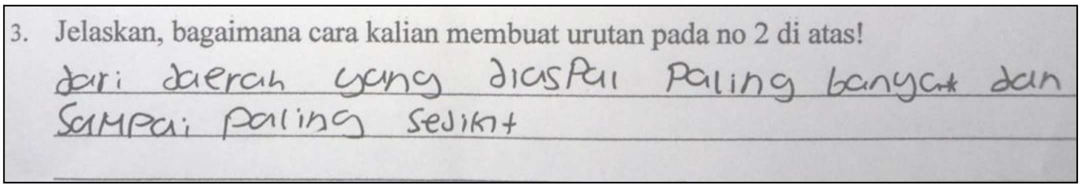

3. Jelaskan, bagaimana cara kalian membuat urutan pada no 2 di atas!
dari daerah yang diaspal paling banyak dan sampai paling sedikit

Figure 5.46 Meutia's and Ariq's works on question 3 learning activity 2

Translation

The order is from the road that has the longest asphalted section to the road that has the shortest asphalted section.

The teacher asked for the students' reasoning for their answer. However, Meutia and Ariq were just silent. The teacher asked whether there were other groups who had similar answered and wanted to explain their works.

And then, the teacher came to the group of Rafi and Aulia that also did absolute comparison as follows.

Question 3

How do you make the order of the project?

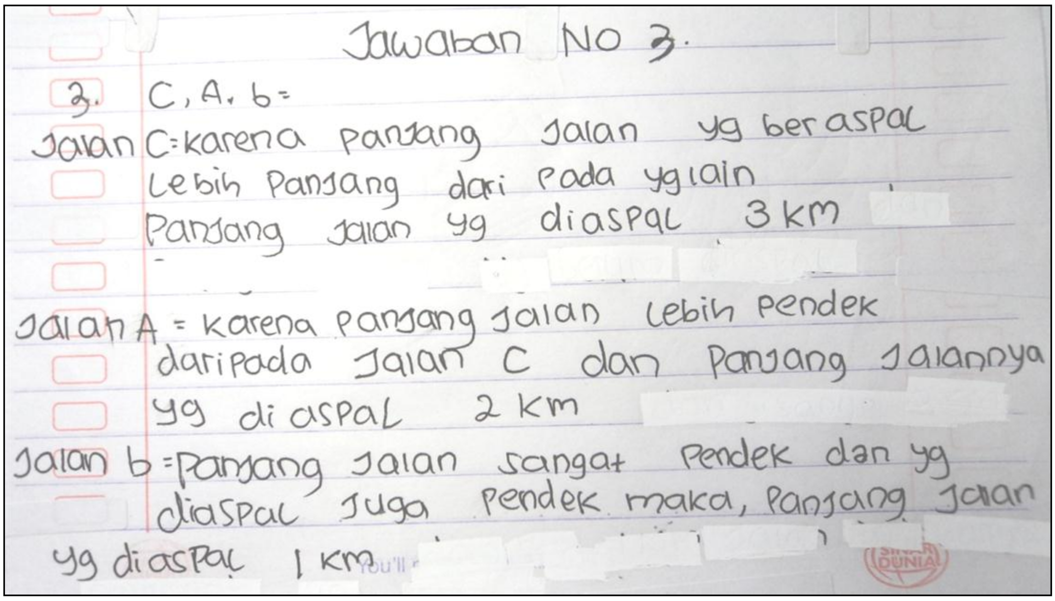


Figure 5.47 Rafi's and Aulia's works on question 3 learning activity 2

Translation

The order of the project is C, A, B

Road C, because it has the longest asphalted section, that is 3km.

Road A, because the asphalted section of road A is shorter than road C, that is 2km. Road B, because road B is the shortest and the asphalted section of it is also the shortest one, that is 1km.

According to the works of two groups above, it can be seen that the students used absolute comparison by comparing the absolute length of asphalted section. It indicates that the students interpreted the situation in absolute perspective. They didn't recognize the part-whole relationship in the situations.

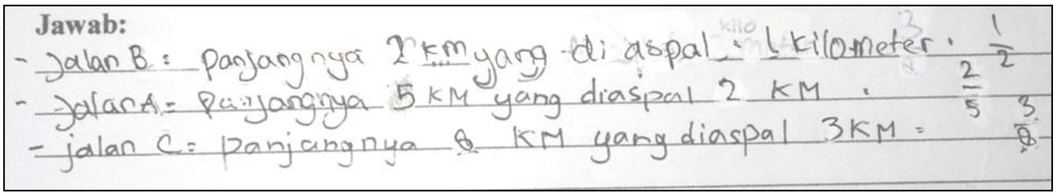
After that, the teacher asked for different solution. There was the group of Hanif and Bimo who presented their works.

1.	Bimo		The project that mostly done is the asphalting activity in road B. So, the order is B, A, C.
2.	Teacher	:	How do you come to that answer? OK, write down your answer! [*the teacher asked the group of Bimo and Hanif to write down their solution in the board*].

Transcript 5.14

Question 2

Make an order of the road project, from the mostly done to least one.



Jawab:
- Jalan B : panjangnya 2 KM yang di aspal : 1 kilometer · $\frac{1}{2}$
- Jalan A : panjangnya 5 KM yang diaspal 2 KM · $\frac{2}{5}$
- jalan C : panjangnya 8 KM yang diaspal 3 KM = $\frac{3}{8}$

Figure 5.48 Hanif's and Bimo's works on question 2 learning activity 2

Translation

The order is B, A, C

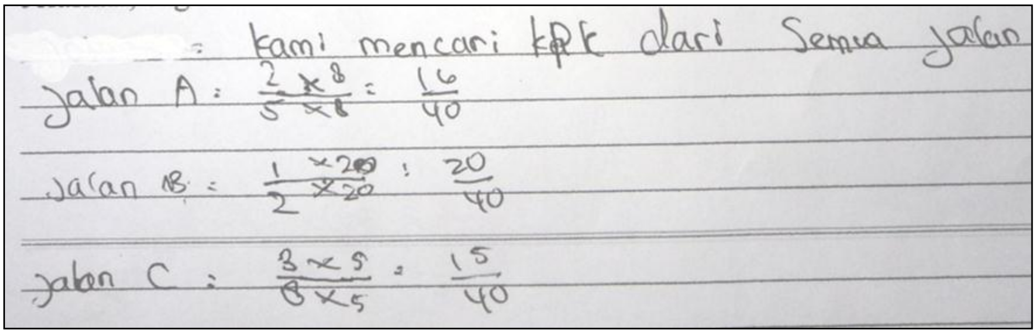
Road B : the total length is 2km, the asphalted section is 1km $(\frac{1}{2})$

Road A : the total length is 5km, the asphalted section is 2km $(\frac{2}{5})$

Road C : the total length is 8km, the asphalted section is 3km $(\frac{3}{8})$

Question 3

How do you make the order of the project?



: kami mencari kpk dari Semua jalan

Jalan A : $\frac{2 \times 8}{5 \times 8} = \frac{16}{40}$

Jalan B : $\frac{1 \times 20}{2 \times 20} : \frac{20}{40}$

Jalan C : $\frac{3 \times 5}{8 \times 5} = \frac{15}{40}$

Figure 5.49 Hanif's and Bimo's works on question 3 learning activity 2

Translation

We look for the least common multiple of the total length of all roads (A, B, and C).

According to the discussion in transcript 5.13 (line 9, 10, 12, 14 and 16) and the works of Hanif and Bimo, it is clear that they looked for which proportion that gave the biggest fraction. It indicates that the students used relative comparison by employing proportions. In Indonesian mathematics classroom, it is common that a teacher asks students to use the least common multiple to look for the common denominator.

After the group of Hanif and Bimo presented their works in front of class, instead of supporting the class to derive conclusion about which interpretation, absolute of relative (by using the concept of proportionality), was acceptable and the most appropriate, the teacher directly justified that the works of Hanif and Bimo was the correct one. The teacher explained that proportional interpretation by employing part-whole relationship was the appropriate one. The teacher stated they should use relative comparison by considering the relationship of the asphalted section (part) and the total length of the road (whole).

Based on students' works and the students' discussion on learning activity 2, we conclude that the problems may help students to see that there are two ways to interpret the situation, and that relative comparison by using the concept of proportionality (in term of part-to-whole relationship) is the most appropriate. It is easier for students the grasp the relationship of part-whole in continuous quantity then it will be helpful for students to determine part-to-whole relationship in the proportional situation. Moreover, it is easier for students to understand that part-to-whole relationship in continuous quantity can be represented into fractions ($\frac{part}{whole}$). It is a starting point for students to begin to understand that one kind of proportional relationship is part-to-whole and it will be helpful for student to determine part-to-whole relationship in discrete quantities.

However, in using the concept of proportion, the students tended to works on numbers only. The students didn't use the bar in proportional reasoning anymore. It seems that bar model as visualization of proportional situations doesn't give significant support for students in solving the problems.

All of students' strategies and thinking that are emerged in the learning process are in accordance with the prediction in the HLT. Students' written works are in line with students' verbal explanation (as we can see at students' written solutions and students' explanation in the transcript). In the learning process, the teacher also already supported students in order to come up with the appropriate reasoning by giving open questions and follow up questions. However, it seems that the teacher is still the one who justify whether a particular mathematical explanation is acceptable or not.

d. Learning activity 3

The goals of learning activity 3 are supporting students to interpret proportional situation involving discrete quantities in relative perspective and the students may use fraction as mathematics tools to solve the given problem. In the learning activity 2, the students have already had experience in determining proportion in term of part-whole relationship in continuous quantity. In this learning activity, the students were going to determine proportions in discrete quantity.

The context for learning activity 3 is Dart games. It was provided result of Dart games played by four children. Each child had different chance of shooting and each of them got different score. The students were asked to determine the most skilful player by employing the result.

Before the class worked on the main mathematics activities, the teacher gave a preliminary activity in order to raise students' enthusiasm toward the learning. In the activity, several students were asked to play Dart game. Two students, Rafi and Fajri, as the class representatives played the game. Each student had different chance of shooting and each of them got different scores. After the play, the class discussed about the result as follows:

1.	Teacher	:	Rafi made 2 shots and he scored 1. Fajri made 5 shots and he scored 1. Which student that is better *[more skilful]* in playing Dart?
2.	Students	:	*[Many students agree that Rafi is better than Fajri]*.
3.	Damar	:	A half Mam.
4.	Teacher	:	What is "a half" Damar?
5.	Damar	:	Kan Fajri has more chance of shooting.
6.	Teacher	:	Fajri has more chance of shootings, that is 5, and he scored 1.
7.	Damar	:	One (1). One fifth ($\frac{1}{5}$) [*he meant that* $\frac{1}{5}$ *was for Fajri*]
8.	Teacher	:	So, $\frac{1}{5}$ *[score for Fajri is]?*
9.	A student	:	Yes Mam.
10.	Teacher	:	But, both students made the same score 1 [*the teacher intended to ask students to clarify their answer, why didn't Fajri and Fajri get the same score due to both students, Fajri and Rafi scored 1]*.
11.	Damar	:	The students (*Rafi and Fajri*) have different number of chance *[of shooting]*.
12.	Teacher	:	So, the chance of shooting, Rafi made two shots *[two chance of*

			shooting] and he scored 1, so [*she intended to ask what was the score for Rafi*]?
13.	Students	:	A half ($\frac{1}{2}$).
14.	Teacher		A half ($\frac{1}{2}$) [*the teacher recited the students' answer*].

Transcript 5.15

According to the class discussion in the transcript 5.15, it can be seen that several students interpret the situation in relative way. There were students who determined that Rafi was better than Fajri in spite of their score were same. At line 5 and 11, we can see Damar gave an explanation that due to Rafi and Fajri had different chance of shooting, so they couldn't simply compare the absolute score of Rafi and Fajri. At line 3 and 7, Damar determined the proportion of score out of the total chance of shootings. Based on the discussion, it seemed that several students understood that the student who had proportion which gave largest fraction was the best (the most skilful).

The main mathematics activity will support students to learn more about the concept and the relative comparison of proportional situation. The following situation is the result of Dart games played by four children: Gagah, Bayu, Rio and Fadli.

Gagah : ●●●●● ●●●●○ ○○○○○ ○○○○○

Bayu : ●○●○○ ●○●●○

Rio : ○●○●○ ○●○●○ ●○○●● ●○○○● ●●○○○

Fadli : ●●●●○ ○○○○○

The students were asked to determine the most skilful Dart player. And then, they should put the four players in order, from the most skilful to the less skilful.

While the students were working in the group, the teacher observed the learning process. The group of Damar and Adam was the first group who finished the task.

1.	Researcher	:	What do you think about the total chance of shooting for the four children?
2.	Damar	:	The total chance of shooting for each child is different.
3.	Researcher	:	So, how do you compare the result of that game? Can you just compare the score of those children?
4.	Damar	:	No, I can't.
5.	Researcher	:	So, what do you do to solve the problem?

6.	Adam	:	Look at this![*he was pointing at the score of the four children*]. *[Adam intended to show his opinion to Damar].* I think we can directly compare the score of the four children.
		:	*[Damar shacked his head disagreed with Adam. And then, both students seemed discussing about how they should interpret the situations.]*
7.	Researcher	:	So, how do you solve the problem?
8.	Damar	:	Like this, by determining the common denominator of the fractions.
9.	Researcher	:	The common denominator of what? As we know that the students have different chance of shooting.
10.	Damar	:	The common denominator of 10, 20 and 25.
11.	Researcher	:	What do you mean by determining the common denominator of 10, 20 and 25?
12.	Damar	:	It means that we make the number into the same.

Transcript 5.16

According to the discussion in transcript 5.16, we can see that Damar justified his answer that we couldn't just compare the absolute score of Gagah, Bayu, Rio and Fadli because they had different total chance of shootings (line 2, 7 and 11). Damar considered the relative relationship of the score and the total chance of shootings. Damar's explanations in this discussion were aligned with his previous explanation in preliminary activities. Line 8, 10 and 12 showed that Damar realized that he should determine which proportion gave the largest fraction. In other words, he understood he should make the total chance of shooting became the same in order compare the score. This kind of proportional reasoning was also founded by Sumarto, et al., (2014) at their study. However, Adam, the other member of the group, seemed using absolute comparison. It indicates that the problem raised different interpretation.

The researcher also came to the group of Hanif and Bimo.

1.	Hanif	:	I count the number of score out of the total chance of shootings.
2.	Researcher	:	Oo, you count the number of score out of the total shots [*the researcher recited the students' answer*]. *The researcher continued....* What is the representation of the score in that figure?
3.	Both students	:	Black dots.
4.	Researcher	:	Give an example! Eee, count for Bayu!

Then, the students count the score of Bayu.

5. Hanif : Bayu made score five. From the total shots, that is ten (10).

Bimo also looks at the problem. And he added,

So $\frac{5}{10}$.

Bimo seemed writing something in his book.

6. Researcher : What do you mean by $\frac{5}{10}$?

7. Hanif and Bimo : Because he scored 5 out of 10 total chance.

*Bimo and Hanif represented the result of each child (*Gagah, Bayu, Rio and Fadli*) into fractions. They wrote the fraction and they did several computation. The students change the denominator into the same as follows:*

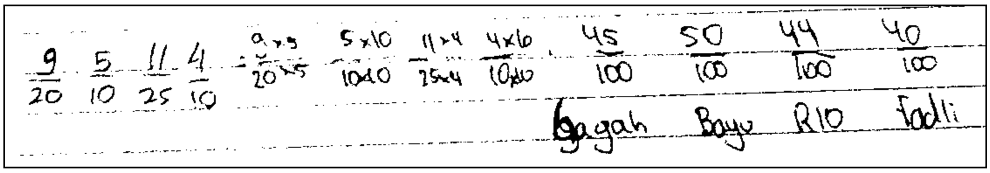


Figure 5.50 Hanif's and Bimo's works

Due of the students' task to compare four (4) situations of Dart games result, then the researcher asked how Bimo and Hanif used the fraction to solve the comparison problems.

8. Researcher : In this case, how do you compare? [*she meant that how the students used the fraction to compare the situations*].
9. Bimo : We compare the numerator.
10. Researcher : You compare the numerator.

Here, you made the denominator into the same (*the researcher was pointing into* $\frac{45}{100}$; $\frac{50}{100}$; $\frac{44}{100}$; $\frac{40}{100}$ *on the students' book*)

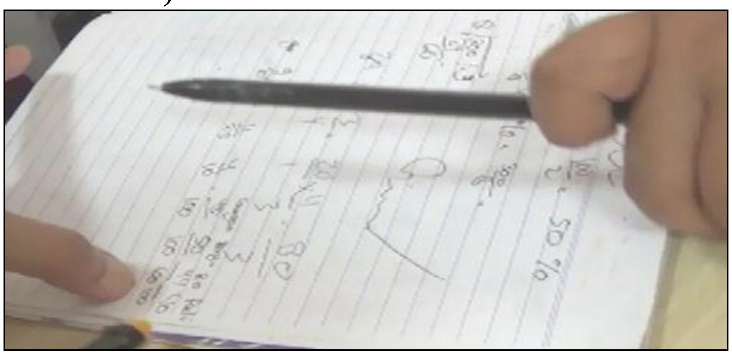


Figure 5.51 Hanif's and Bimo's works

Why don't you compare this one? (*she meant that why didn't the students directly compare the initial fractions, i.e.:* $\frac{9}{20}$; $\frac{5}{10}$; $\frac{11}{25}$; $\frac{4}{10}$)

11.	Bimo	:	Because, the denominator should be the same.
	Hanif	:	Because, the denominator are different.
12.	Researcher	:	Oo, because, the denominators are different. Why can't we compare the situation if the denominators of the fractions are different??
13.	Hanif	:	Because ... [*long pause*], Because the total chance for each child is different.
14.	Researcher	:	OK. Because the total chance for each child is different [*the researcher recited Hanif's answer*].

Transcript 5.17

Based on the discussion in transcript 5.17 the students interpret the situation in proportional way (line 1, 5 and 7). Moreover, based on students; explanation at line 7, it seemed the students understood that they had to figure out which proportion would give the largest fraction. Besides that, based on line 13, the student understood that in order to compare the situation, they should make the total chance into the same. In the fractions, the total chance was represented by the denominator($\frac{\boldsymbol{score}}{\boldsymbol{total\ chance\ of\ shootings}}$).

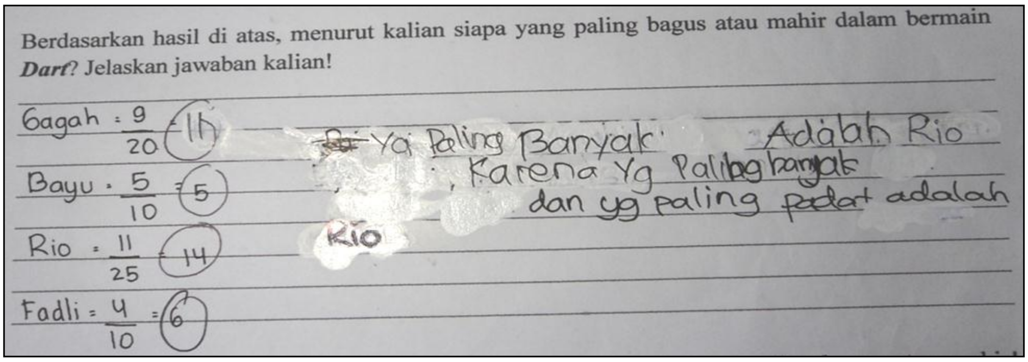
In the class discussion, there was the group of Farhan and Conny who presented their works. They determine Rio was the most skilful player.

1.	Teacher	:	Who is the most skilful player?
2.	Farhan	:	Rio
3.	Teacher	:	Why do you think that Rio is the most skilful?
4.	Conny	:	Because he got the most score, 11.

Transcript 5.18

The figure below shows Farhan’s and Conny’s works.

Question 1: *Who is the most skilful Dart player? Explain your answer!*



Berdasarkan hasil di atas, menurut kalian siapa yang paling bagus atau mahir dalam bermain ***Dart***? Jelaskan jawaban kalian!

Gagah = 9/20 = 11

Bayu = 5/10 = 5

Rio = 11/25 = 14

Fadli = 4/10 = 6

Yg Paling Banyak Adalah Rio , Karena Yg Paling banyak dan yg paling padat adalah Rio

Figure 5.52 Farhan’s and Conny’s work on learning activity 3

Translation

The explanation part: the most is Rio, because it is the most, and the one who had the most density is Rio.

Based on the works above, it appeared the students came up with the use of fractions. However, it seemed the students didn’t understand the meaning of the fractions. In this case, tFarhan Conny came up with fractions interpretation might be due to the use of fractions as mathematics tools to solve problem in previous lesson. Therefore, their conclusion was not aligned with the use of fractions. In their explanation, they stated that Rio was the most skilful player due to his most scores. It also was not clear why they wrote about density. It indicates these students didn’t understand about the problems, the relationship between numbers, and the relationship of what the information was and what was being asked. Moreover, the teacher didn’t explore what Farhan and Conny did, and she just considered the final answer of both students. After that, she brought directly the solution into the class. She asked for comments from other students. There was Hanif, who gave comments regarding the solution of the proposed problem.

1. Hanif : Rio’s score is less than a half of the total chance of shootings.
2. Teacher : So, how do you compare the situations?
3. Hanif : Rio is $\frac{\mathbf{11}}{\mathbf{25}}$
4. Teacher : What is a half of 25?
5. Hanif : 12 ½
6. Teacher : Ha, 12 ½ .
   So, how should we compare the result of the game? Can we compare the score only?

7.	Some students	:	No, we can't [*the students answered teacher's question in choir*].
8.	Damar	:	Because the total chance of shootings are different among the children.
9.	Teacher	:	So, what should we do in comparing the children from the most skilful player to the least skilful?
10.	Some students		We should compare the fraction.
11.	Teacher		OK, now the group of Rafi and Aulia, please present your works in front of class!

Transcript 5.19

The figure below shows the works of Rafi and Aulia.

Question 1

Who is the most skilful Dart player? Explain your answer!

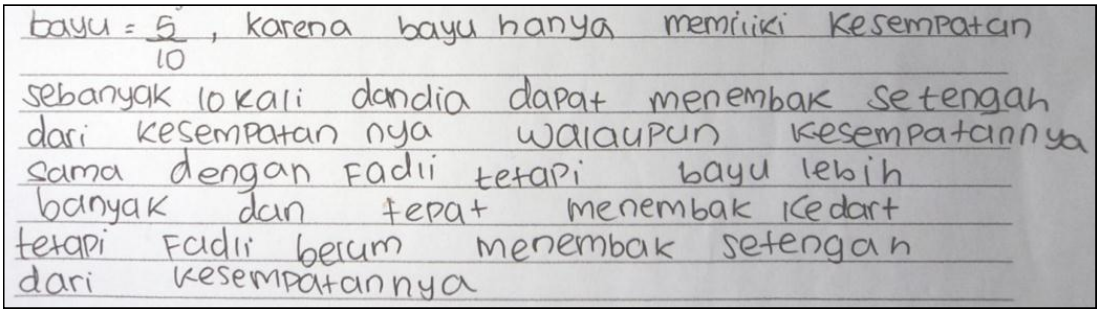

bayu = $\frac{5}{10}$, karena bayu hanya memiliki kesempatan sebanyak 10 kali dandia dapat menembak setengah dari kesempatan nya walaupun kesempatannya sama dengan Fadli tetapi bayu lebih banyak dan tepat menembak kedart tetapi Fadli belum menembak setengah dari kesempatannya

Figure 5.53 Rafi's and Aulia's work on question 1 learning activity 3

Translation

Bayu= $\frac{5}{10}$*, because Bayu has only 10 chances of shootings, and his half of total shootings are success even though his chance equals to Fadli's. But, Bayu has more score (than Fadli), and Fadli's score is less than a half out of his total chance of shootings.*

Question 2

Put the four players in an order, from the most skilful player! Explain how do you determine the order!

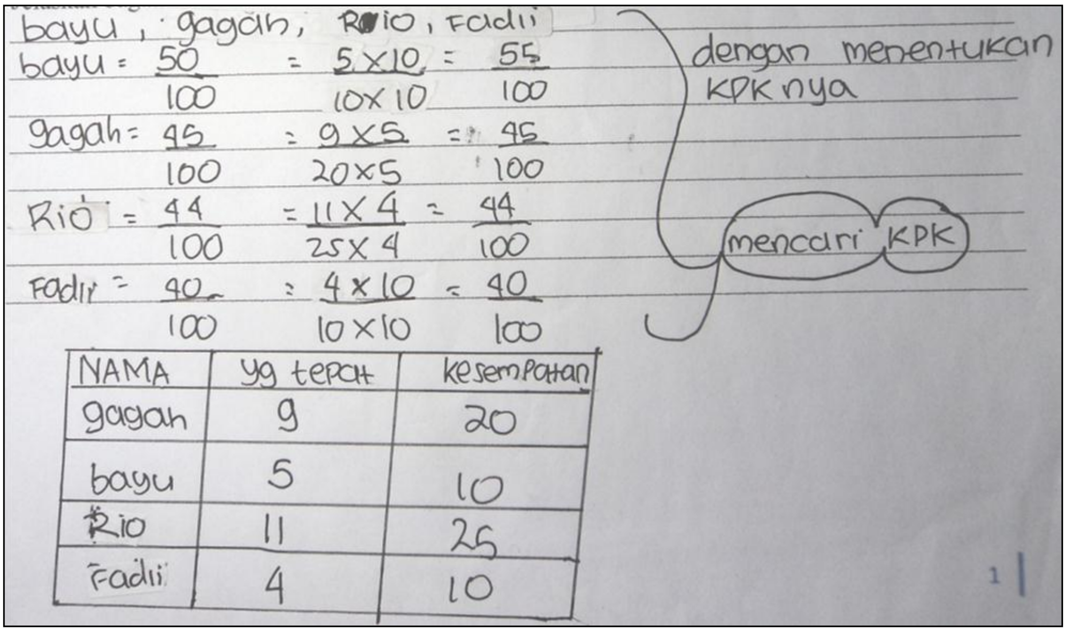


Figure 5.54 Rafi's and Aulia's work on question 2 learning activity 3

Translation

Bayu, Gagah, Rio, Fadli [the order]. *By determining the least common multiple* [the students used the last common multiple as denominator in order to compare the proportions of the situations].

There was no explanation from the group. Rafi just recited their works. The teacher directly justified that Rafi's and Aulia's works was correct. However, based on Rafi's and Aulia's works, it was clear that the students didn't used he information in partial way. According to Rafi's and Aulia's solution for question 1, it appeared they recognized the relationship among numbers, so they came up with part-to-whole relationship. It indicates that the students understood that part (score) and the whole (total chance of shootings) altogether influenced the comparison. And finally, the students solved the problems the students used proportion for each situation and they determined which proportion would give the largest fraction into the smallest fraction (question 2).

Based on the discussion in transcript 5.17, 5.18 and 5.19, the group's works of Farhan and Conny; Rafi and Aulia, it can be seen that the problem leads to different interpretation. There were several students who interpret the problem in relative way instead of comparing the absolute score of the children. The students were able to determine the relationship of score relative to the total chance of shootings. The use of fraction comparison indicates that the students understood that they had to find which proportion would give the largest fraction. And then they would make an order of the fractions as the order of the Dart player.

According to the students' discussion and students' works, students interpreted the situation in different perspective. Several students used absolute comparison (they compared the absolute score). Some students used relative comparison by considering the relationship of the score and the total chance of shootings, which was justified by Damar's statements (transcript 5.15 line 11 and transcript 5.16 line 2) and Hanif's statements (transcript 5.17 line 13) that due to the difference of total shootings, they couldn't simply compare the score of the children.

The strategies and thinking come out from students are in line with the prediction on the HLT. Students' written works are in accordance with students' verbal explanation (as we can see at students' written solutions and students' explanation in the transcript). The teacher also already supported students to elicit relative thinking in solving the problems by giving probing questions and follow up questions. However, it seems that the teacher is still the one who justify whether a particular mathematical explanation is acceptable or not.

e. Learning activity 4

Learning activity 4 manifested students' knowledge and experiences acquired in three previous learning activities. By working on the three previous learning activities, it was supposed that students understood the different interpretation in proportional-comparison tasks. The students might use absolute comparison, but through the discussion, that can be either group discussion or class discussion, it was supposed the students would understand that relative comparison by employing the concept of proportionality was the most appropriate.

The mathematics activity in the learning activity 4 was about how did the students interpret the given survey data on students' interest. There were two problems in the mathematics activity as follows:

A survey at ***Harapan Bangsa Elementary school*** *obtained data of students' interest on extracurricular activities as follows:*

1) *Basketball for class 5D:*

Table 5.5 Girls' interest on Basketball extracurricular

Girls who like to do basketball activity	Total girl students
7	15

Table 5.6 Boys' interest on Basketball extracurricular

Boys who like to do basketball activity	Total boy students
5	10

The students were asked to determine who was relatively more interested on **Basketball** extracurricular, boys or girls. They were also asked to either explain or give justification to their answers.

2) *Most of members of Silat and Pramuka ask to have twice a week activities. This is the information of students' preference in scheduling.*

Table 5.7 Silat members' preference on scheduling

Students who prefer twice a week	Total member
20	30

Table 5.8 Pramuka members' preference on scheduling

Students who prefer twice a week	Total member
30	50

But, because of the schedule of school activities, there is only one more extracurricular that can be scheduled twice a week.

The students were asked to determine which *extracurricular* that shoudl be scheduled twice a week by using the information of the above give data.

When the students were working the groups, both teacher and the researcher observed the learning process. Based on the observation and the students' written works, it was known that the students started to consider the relative comparison instead of comparing the absolute value, which also happen toward the focus group (Hanif and Bimo).

1.	Bimo	:	Because the number of boys who like to do basketball activity is a half out of the total, and it is less than a half of girls who like to do it.

Transcript 5.20

Bimo's above answer was in line with their written works. In this case, Hanif and Bimo directly reasoned in proportional way. Moreover, the group of Hanif and Bimo also determined the percentage of boys and girls who like to do basketball activities as follows:

Question 1

Who (boys or girls) were more interested in Basketball extracurricular?

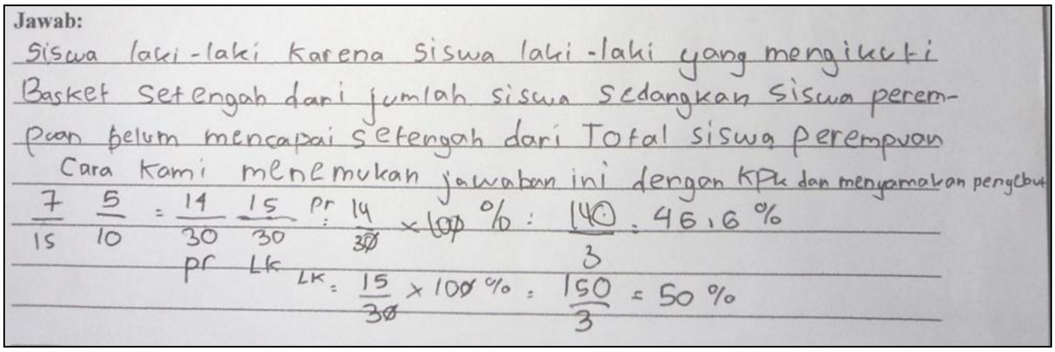

Jawab:
Siswa laki-laki karena siswa laki-laki yang mengikuti Basket setengah dari jumlah siswa sedangkan siswa perem-puan belum mencapai setengah dari Total siswa perempuan
Cara Kami menemukan jawaban ini dengan KPK dan menyamakan penyebut
$\frac{7}{15}$ $\frac{5}{10}$ = $\frac{14}{30}$ $\frac{15}{30}$ pr : $\frac{14}{30} \times 100\%$ = $\frac{140}{3}$ = 46,6 %
pr LK LK : $\frac{15}{30} \times 100\%$ = $\frac{150}{3}$ = 50 %

Figure 5.55 Hanif's and Bimo's works on problem 1 learning activity 4

Translation:

The boys because the boys who like to do basketball activity is a half out of the total boys, meanwhile the girls who like to do basketball activity is less than a half out of the total girls.

$\frac{7}{15}\ \ \frac{5}{10} = \frac{14}{30}\ \ \frac{15}{30},$

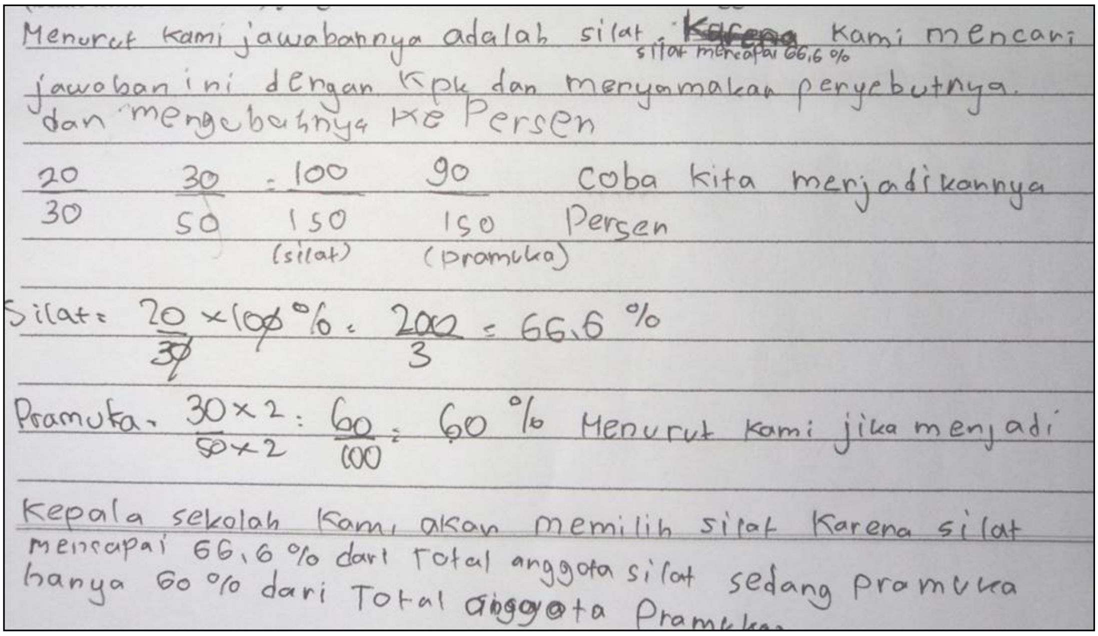
$\frac{14}{30} \times 100\% = \frac{140}{3} = 46{,}6\%$ [the percentage for the girls who like to do basketball activity].

$\frac{15}{30} \times 100\% = \frac{150}{3} = 50\%$ [the percentage for the boys who like to do basketball activity].

Furthermore, for the second problem, Hanif and Bimo also used relative comparison by determining the proportion of the students who voted for twice time activities for each extracurricular (*Silat* and *Pramuka*). And then, they found out which proportion that gave the largest fraction in this way:

Question 2

If you are the school principle, how do you determine which extracurricular (Silat or Pramuka) that will be scheduled twice a week? Explain your strategy!


Menurut kami jawabannya adalah silat. ~~Karena~~ silat mencapai 66,6 % Kami mencari jawaban ini dengan KPK dan menyamakan penyebutnya. dan mengubahnya ke Persen

$\frac{20}{30}$ $\frac{30}{50}$ = $\frac{100}{150}$ (silat) $\frac{90}{150}$ (pramuka) coba kita menjadikannya Persen

Silat = $\frac{20}{30} \times 100\% = \frac{200}{3} = 66{,}6\%$

Pramuka = $\frac{30 \times 2}{50 \times 2} = \frac{60}{100} = 60\%$ Menurut kami jika menjadi Kepala sekolah kami akan memilih silat Karena silat mencapai 66,6 % dari Total anggota silat sedang pramuka hanya 60 % dari Total anggota Pramuka

Figure 5.56 Hanif's and Bimo's works on problem 2 learning activity 4

Translation:

We think that Silat should be scheduled twice times a week. We solved the problem by determining the least common multiple [in order to determine the common denominator of fractions] *and we make the denominator into the same and we also look for the percentages* [that represents the situation].

$\frac{20}{30}$ $\frac{30}{50}$ = $\frac{100}{150}$*(Silat)* $\frac{90}{150}$ *(Pramuka)* *we tried to determine the percentages,* *Silat* $= \frac{20}{30} \times 100\% = \frac{200}{3} = 66.6\%$ [the students did miss-wrote the % symbol after $\frac{200}{3}$].

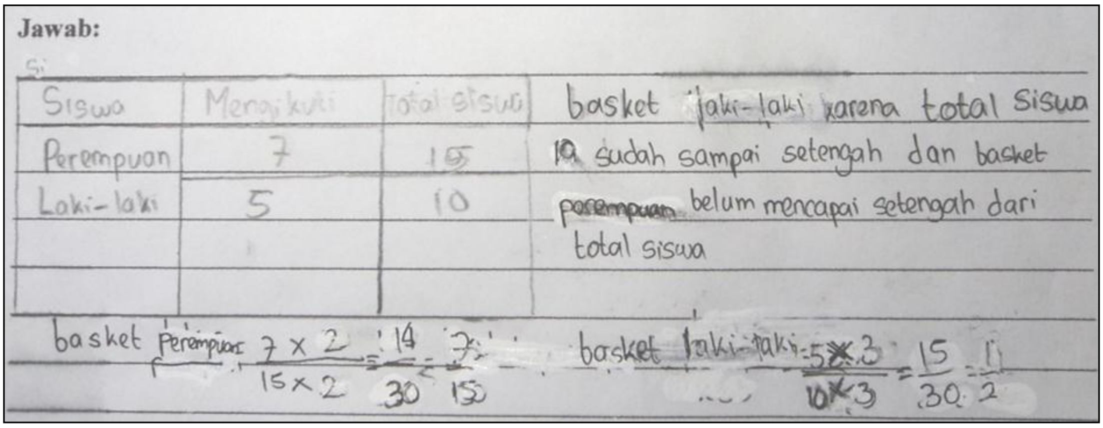
Pramuka $\frac{30}{50} \times 100\% = \frac{60}{100} = 60\%$

If we are the school principle, we will decide that Silat will be scheduled twice a weeks because 66.6% of its members [preferred for twice a week activities], *and there is 60% of Pramuka's members* [who preferred for twice a week activity].

In the class discussion, the class also agreed that they should interpret the situation in relative way by employing the concept of proportionality. There was the group of Rahma and Farel. Instead of explaining their answer, Rahma and Farel just read what they had done to solve the problem. This is their solution for the problem.

Question 1

Who (boys or girls) were more interested in Basketball extracurricular?


Jawab:

Siswa	Mengikuti	total siswa	
Perempuan	7	15	basket laki-laki karena total siswa 10 sudah sampai setengah dan basket
Laki-laki	5	10	perempuan belum mencapai setengah dari total siswa

basket perempuan $\frac{7 \times 2}{15 \times 2} = \frac{14}{30}$ $\frac{7}{15}$ basket laki-laki $\frac{5 \times 3}{10 \times 3} = \frac{15}{30} = \frac{1}{2}$

Figure 5.57 Rahma's and Farel's works on problem 1 learning activity 4

Translation:

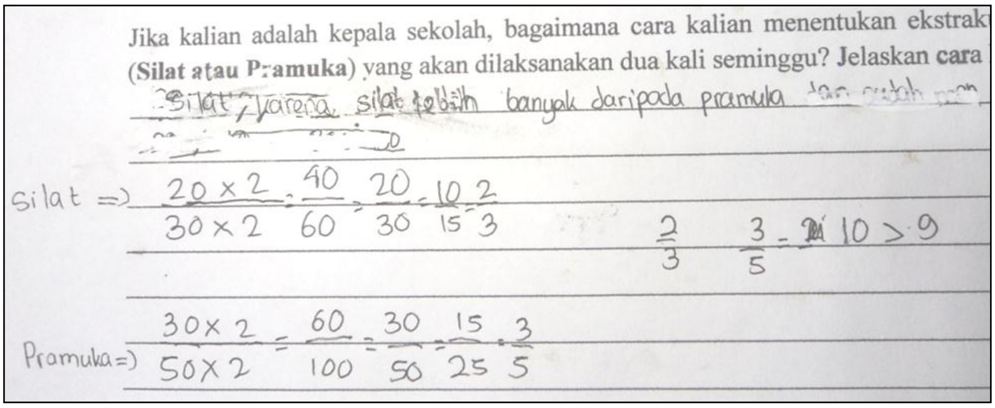
The total number of boys is 10, and a half of them like to do basketball activity, and for the girls, there is less than a half of them who like to do basketball activity.

For girls $\frac{7 \times 2}{15 \times 2} = \frac{14}{30}$

For boys $\frac{5 \times 3}{10 \times 3} = \frac{15}{30}$

Question 2

If you are the school principle, how do you determine which extracurricular (Silat or Pramuka) that will be scheduled twice a week? Explain your strategy!


Jika kalian adalah kepala sekolah, bagaimana cara kalian menentukan ekstrak
(**Silat atau Pramuka**) yang akan dilaksanakan dua kali seminggu? **Jelaskan cara**

Silat, karena silat lebih banyak daripada pramuka

Silat => $\frac{20\times 2}{30\times 2} = \frac{40}{60} = \frac{20}{30} = \frac{10}{15} = \frac{2}{3}$ $\frac{2}{3}$ $\frac{3}{5}$ = 10 > 9

Pramuka => $\frac{30\times 2}{50\times 2} = \frac{60}{100} = \frac{30}{50} = \frac{15}{25} = \frac{3}{5}$

Figure 5.58 Rahma's and Farel's works on problem 2 learning activity 4

Translation:

The solution of Rahma and Farel for problem 2 was Parmuka should be scheduled two times a week.

Rahma also justified again the solution of her group in the class discussion as follows:

1. Rahma : Parmuka should be scheduled two times a week, because it is larger than Silat [*she meant that the proportion of member of Pramuka who votes for twice a week activities gave a larger fraction than Silat*].

Transcript 5.21

In the class discussion, the teacher didn't ask for further reasoning. It might be caused by the obvious solution of students. Based on the students' verbal explanation and works, it is obvious that they didn't use absolute comparison. The students interpret the situation into proportions and then they determined which proportion that gave the largest fraction. Moreover, when the teacher asked whether there were comments of different solution, the class seemed agreed to the solution of Rahma and Farel. Due of that, the teacher directly justified the solution of Rahma and Farel was the correct one.

f. Post-test

At the end of learning series in the second cycle, the students did a post-test. The goal of the post-test was to get impression about how the learning activities support the students to develop relative thinking as one of important type of proportional reasoning. The post-test was also aimed to know how the students solved problems on proportions after learning the topic. There were four problems in the post-test. The post-test items of the second cycle were different to post-test items of the first cycle. The proposed competences of post-test in both cycles were same, but the chosen numbers were different. Moreover, post-test of the second cycle used two contexts that were different from the contexts used in post-test of the first cycle. It was aimed to expand the variety of contexts that were used to promote students relative thinking.

There were four problems in post-test of the second cycle including two problem types, density-comparison problems and part-to-whole comparison problems. The last problem was part-to-whole comparison involved growth problem. The students may analyse the growth in absolute way. Besides that, they may identify the growth in relative way and they evaluate the growth by comparing the proportions instead of comparing the absolute amount of the growth. The post-test took 1×35 minutes. There were 29 students who did the post-test because one of them was absent at that day.

- **Problem 1**

Problem 1 was about comparing density of boxes for ducks, if there were two boxes, A and B. Box A was $5m^2$ containing 100 ducks. Box B was $3m^2$ containing 75 ducks. Due to the varied solution, the researcher classified students' solutions into several types. The classification was done based on students' strategies and mathematics ideas on it. Based on students' works, there were four solution types as follows:

Absolute comparison 1

There were 6 students compared the absolute number of ducks in box A and B, and they concluded box A was more crowded than box B because there were more duck in box A (100) than in box B (75).

Absolute comparison 2

There were 3 students compared the absolute size of box A and B, and they concluded box B was more crowded than box A because box B was smaller than box A.

Relative comparison 1

There were 14 students used the concept of proportionality in a simple way by comparing the number of ducks of box A and box B in the same size. There were 3 students who changed the size of box B into $5m^2$, and 11 students made the two boxes into $15m^2$, which changed the number of the ducks (proportionally). After they got the boxes in the same size, the students compared the number of ducks. The students concluded that box B was more crowded because if box B was in the same size with box A, box B contained more number of ducks.

Relative comparison 2

There were 6 students interpreted the situation in proportional way by comparing ratio of box size with the number of duck ($\frac{the\ size\ of\ box}{the\ number\ of\ ducks}$), and they determined which ratio gave the biggest fraction.

- **Problem 2**

Problem 2 was about using the survey data in comparison situation. it was given these data:

Table 5.9 Students' interest on reading folk story

Class	Total students	Students who like to read folk story
5A	25	13
5B	20	11

The students were asked to determine reading folk story was relatively more popular in which class.

In analysing the survey data, the class interpret the data in different way. Eight students compared the situations by comparing the absolute number of students who liked to read folks story, and they claimed that reading folk story was more popular for class 5A. In the other hand, 21 students recognized that the number of students who liked to read folk story and total number of students in each class altogether influenced the comparison. These students employed part-to-whole relationship for each situation and they determined which proportion giving them the largest fraction in order to answer the question.

- **Problem 3**

There was a Dart games results played by two children, Sari and Dini. Each child had different chance of shooting and each of them got different score as follows:

Dini : ●●●●● ●●●●○ ○○○○○

Sari : ●●●●● ●○○○

Based on the above result, the students were asked to determine the true statement! And explain your reason!

a. *Dini as skilful as Sari in playing **Dart**.*
b. *Dini is more skilful than Sari in playing **Dart**.*
c. *Sari is more skilful than Dini in playing **Dart**.*

The students were asked to determine the most skilful player by employing the result. There were 3 students who compared the absolute score. The other students directly came up with the idea of part out of the whole, so they determine that statement B was true. The other students also understood that they had to determine which proportion gave the biggest fraction for the each information, so they figure out that statement was true.

- **Problem 4**

Problem 4 was quite different with the three other problems. It wasn't about part-to-whole, but how the students would recognize and determine the correct relationship between data (numbers). The problem was about the growth of population as follows:

At 2005, the population of rusa-sambar was 30, and the population of kijang was 20. At 2010, the population of rusa-sambar increased to 50, and the population of kijang increased to 40.

Based on the above information, which statement is true! And explain your reason!

a. *From 2005 to 2010, the population of rusa-sambar grew more than kijang.*
b. *From 2005 to 2010, the population of kijang grew more than rusa-sambar.*

c. *From 2005 to 2010, the population of rusa-sambar and kijang grew in the same amount.*

Most students interpreted the situation in absolute way. There were 18 students determined that statement C was true because population of both deer types grew in the same amount, that was 20. In this case, the students just saw the absolute difference for the number of population at 2005 and 2010. However, there were other students who analysed the situation in different way. There were 4 students who considered the relationship among numbers for each situation, and they recognized that the population of *kijang* at 2010 grew twice more than its population at 2005. In the other hand, the population of *rusa-sambar* at 2010 grew less than twice its initial number at 2005. For that reason, these students determined that statement B was true.

There were 7 students who tried to figure out the relationship among numbers for each situation, but they failed figuring out the appropriate relationship, so they came up with wrong proportions. These students compare the proportion in this way:

$Rusa\ sambar = \frac{30}{50}$

$Kijang = \frac{20}{40}$

After that, they determined which proportion gave the largest fraction.

Conclusion of post-test on the second cycle

Based on the students' answer on the posttest, we may conclude that the problems help students to interpret the situation in different way. Some still used absolute comparison, but there are many students understand that they have to use relative comparison. Some students understand that proportional interpretation is the most appropriate. In general, the students use the idea of proportionality in simple, way that is they should make one value into the same in order to compare another value. Moreover, many students tend to work directly using the numbers. Many students come directly on the proportions represented the situation and determine which proportion will give the largest fraction. They are not accustomed to give a little bit explanation about their idea, so it may not clear enough whether the students understand the problem and they really use the concept of proportionality, or they just take any numbers and compute them. Due of that, it seemed some of the students also still need more discussion about proportional interpretation, so the students don't just take and compute any number without knowing the relationship among numbers and why do they do in that way.

It may be the reason way there are just a few students who are able to give an appropriate solution for problem 4. Just taking any number and computing them will not lead the students to determine the appropriate solution, to solve problem 4, the students have to understand the problem, and they have to understand whether the problem needs absolute or relative solution. And when the students want to use relative comparison by employing proportions, they have to understand, which set of numbers that represent the proportion for the situation.

g. Conclusions of the retrospective analysis on the second cycle

According to analysis of the data collection from the second cycle, we could draw conclusions as follows:

Based on the obtained data from learning activity 1 until learning activity 4, we conclude that proportional-comparison tasks may promote students to see the proportional situation in different perspective. At the first time, the proposed comparison problems (learning activity 1-3) lead several students to interpret the situation in absolute way and few students used relative comparison. However, the difference interpretations that are emerged helps students to begin to understand that the different interpretations are possible, but relative interpretation by employing the concept of proportionality is the most appropriate.

For instance, when the students were asked to determine which one of two places in the same size and contain different number of objects is more crowded. In this case, the use of absolute comparison by comparing the absolute number of object is appropriate. In the other hand, to compare the density of two places in different size and contain different number of objects, the students couldn't compare the absolute value only. Therefore, the students may interpret in different way, and several students may consider the relationship of the size and the number of objects.

Moreover, the use of different density situation (in mind experiment, learning activity 1) leads students to have different feeling of crowdedness and it encourages them to recognize relative relationship of the size of a place with the number of objects occupying. In addition, people seem understand that in comparing density, we should consider the relationship of the size of place and the number of population. Therefore, the use of density-comparison task may be a beneficial starting point to promote students' relative thinking.

Comparison tasks including proportions in term of part-to-whole relationship (learning activity 2-4) may facilitate students to see the relative relationship of part and the whole. Instead of comparing part and part (absolute thinking), the situations help students to realize that they should compare the proportion representing part-to-whole and determine which proportion will give the largest fraction. Consequently, the students may use fractions as mathematics tools to solve the comparison problem

Besides that, it is easier for students to determine part out of the whole of continuous quantity (learning activity 2). It is also easier for students to understand that part out of the whole in continuous quantity can be represented into fractions ($\frac{part}{whole}$). It may help students to begin to understand that one of proportional relationship is part-to-whole. In addition, it may give a helpful starting point for student to determine part-to-whole in discrete quantities (learning activity 3 and 4).

However, in solving the task, the students tended to works with numbers only. This is aligned with the behaviour of Indonesian mathematics classroom that students are accustomed to work by using number only without giving any elaboration or reasoning. In the learning activities, the students don't use the bar as representation of the situation to reason proportionally. It seems that bar model as visualization of proportional situations doesn't give significant support for students in developing relative thinking. Therefore, we conclude that the bar model as a visualization of proportional situations may not support the development of students' relative thinking in some extent.

In the learning process, the teacher also already supported students in order to come up with the appropriate reasoning by giving open questions and follow up questions. However, most students were passive and they tended to give answers in choir. Furthermore, it seemed that the teacher was still the one who justify whether a particular mathematical explanation was acceptable or not.

CHAPTER 6
CONCLUSIONS AND SUGGESTIONS

6.1.Conclusions

The aims of the study are to develop a learning instruction that does not only support the development of 5th grade students' relative thinking in solving problems on proportions, but it also helps students to develop proportional reasoning. Besides that, the present study aims to contribute to the development of local instruction theory in proportions. A set of instructional activities are developed and implemented in teaching experiment. Throughout data analysis as elaborated in the previous chapter, the researcher attempts to answer the proposed research questions as follow.

6.1.1.Answer to the first sub research question

The proposed first sub research question in this study is *how do the 5th grade students use their initial understanding to solve proportional-comparison problems?*

The answer for the first sub research question can be derived from the data collection of pre-test in the beginning of the teaching experiment due to the aim of the pre-test are in order to impression on how the students may use preliminary ability in solving problems on proportion and identifying students' initial understanding on proportional reasoning. In the pre-test the students worked on three comparison problems.

According to data analysis, we may conclude that students have different level of initial proportional reasoning ability. Most students used the given data in partial way and they didn't recognize the relationship between the set of numbers (quantities) for each situation. However, several students used relative comparison by employing concept of proportionality. Based on students' works, several students employed an idea of proportionality in a simple way, for instance the students used unit one method (unit amount) by determining the number of things per one unit of measurement., i.e.: students determine unit amount of rice (1kg), so they could compare the price only, and students determined unit size of boxes ($1m^2$), so they could compare the number of chicken only.

The students understand that there is a set of quantities in a proportional situation, for instance quantity representing the number of chicken and the size of chicken box. The students solve the proportional-comparison problems by making the sizes of boxes (one quantity) into the same that the students can compare the number of chicken (the other quantity). Moreover, the students also solved the proportional-comparison problems by comparing proportion (in term of part-to-whole) of the situation and determine which proportion that gave the largest fraction.

In addition, we conclude that some 5^{th} grade students have initial understanding on proportional situations in term of interpreting the situations in relative way. Besides that, some students employ proportional reasoning. Therefore, the study on proposed topic by using the proposed problems is possible to be conducted toward 5^{th} grade students.

6.1.2.Answer to the second sub research question

The proposed second sub research question in this study is *how can proportional-comparison problems promote students' relative thinking?*

Learning activities in this study emphasized on supporting student in developing relative thinking that is important for proportional reasoning. The learning activities involve comparison tasks. As stated by Sumarto *et al*. (2014) that in comparison problems, all of information altogether influenced the comparison, which we should not use the information in partial way. In comparison problems, a student should compare two values of the intensive variable computed from the data (Karplus, *et al*., 1983). In line with Karplus, *et al*., (1983), Sumarto *et al*. (2014) determined that in solving comparison tasks, ones should compute a set of numbers representing each situation, and ones have to determine which proportion will give a good comparison. All of those ideas about solving comparison tasks are aligned to the idea of relative thinking.

Based on data analysis on the learning process from learning activity 1 until learning activity 4, we conclude that proportional-comparison tasks may promote students' relative thinking. At the first time, the proposed comparison problems (learning activity 1-3) lead students to come to different interpretation about them. There are students who interpreted the situation in absolute way and a few of them used relative comparison. However, the difference interpretations that are emerged helps students begin to understand that the different interpretations are possible, but relative interpretation by employing the concept of proportionality is the most appropriate solution.

For instance, in comparing density of two places in the same size containing different number of objects, the use of absolute comparison by comparing the absolute number of object is appropriate. In the other hand, in comparing density of two places in different size and contain different number of objects, ones should consider the relationship of the size and the number of objects. Moreover, the use of different density situation (in mind experiment learning activity 1) leads students to have different feeling of crowdedness and that encourage students to recognize relative relationship of the size of a place with the number of objects occupying. In addition, people appear to have an intuitive understanding that in comparing density, we should consider the relationship of the size of place and the number of population. Therefore, the use of density-comparison task is a beneficial starting point to promote students' relative thinking.

Furthermore comparison tasks including proportions in term of part-to-whole relationship (learning activity 2-4) facilitate students to see the relative relationship of part and the whole. Instead of comparing part and part (absolute thinking), the situations help students to understand that they should compare the proportion representing part-to-whole and determine which proportion will give the largest fraction.

In addition, it is easier for students to determine part out of the whole in continuous quantity. Besides that, it is also easier for students to understand part out of the whole in continuous quantity can be represented into fractions ($\frac{part}{whole}$). It is a starting point for students to begin to understand that one of proportional relationship in part-to-whole and that it is helpful for student to determine part-to-whole in discrete quantities.

6.1.3.Answer to the third sub research question

The proposed third sub research question in this study is *how can the bar model as a visualization of proportional situations support students in developing relative thinking?*

Van Galen *et al.* (2008) claimed that models are close to context situation, which the use of models is developed from model as representation of situation into models for reasoning. Besides that, models are real for students. It is easier for students to work and reason in something which is real for them. And then, because the models represent the initial context situations, it is easy for students to relate their works and reasoning by using models and the initial problems.

However, in solving the problems, the students tended to works with numbers only. The students didn’t use the bar as representation of the situation to reason proportionally. It seems that bar model as visualization of proportional situations doesn’t give significant support for students in developing relative thinking. Consequently, we conclude that the bar model as a visualization of proportional situations may not support the development of students’ relative thinking in some extent.

6.1.4.Answer to the main research question

The proposed main research question in this study is *how can we support students in developing relative thinking in solving problems on proportions?*

Boyer and Levine (2012) agreed that students’ formal mathematics understanding about proportion can be fostered by using instructional activities that is built on students’ intuitive understanding of proportional relationship. Based on data analysis, in general, students have the sense of relative thinking by employing concept of proportionality in comparison situation. The students’ initial recognizing of relative comparison is the use of simple proportional strategies in solving the comparison task. Moreover, the use of density-comparison tasks as starting point may generate students’ intuitive understanding that the relationship among numbers altogether influences the comparison, so we should not compare the data in partial way. For that reason, we conclude that utilizing the initial ability and intuitive understanding may support students in developing relative thinking as well supporting students in solving problems on proportions.

Karplus, *et al*. (1983) argued that in comparison problems, a student should compare two values of the intensive variable computed from the data. It indicates that to solve comparison problems, there is a relation between numbers in a set. Therefore, comparison problems may help students to understand that instead of comparing the absolute value, they should compare proportion representing the relationship of a set of number. For that reason, we conclude that comparison tasks may support students in developing relative thinking.

Furthermore, in order to support students to elicit relative thinking, the teacher may pose open questions, explanatory questions and follow up questions. Therefore, the teacher support takes an important role in the learning process.

6.2. Reflection

6.2.1 The implementation of PMRI on the learning activities

The learning activities employed in this study utilize contexts that are familiar for students. The use of context is the first characteristic of PMRI. The mathematics activities are set up as group activities. The group discussion and the class discussion give opportunities for students to develop interaction among students and between the teacher and the students. The interactions are aimed to support students to develop relative thinking as well as solving problems on proportions. Therefore, the discussion may encourage students to help each other and share ideas about the concept that is being learnt, so the students could give contribution in the mathematics learning. Moreover, the learning topics in this study is related to another topic in mathematics. Proportion is closed related to fraction and measurement. Therefore, the instructional activities intertwines with other topic of mathematics.

6.2.2 The contribution to the local instruction theory of proportions

In Indonesian educational system, the concepts of proportions are taught formally for the first time at grade 5 (Depdiknas, 2006). Zulkardi (2002) revealed that mathematics textbooks used in Indonesian classroom usually present the set of formal rules and algorithm that deals with formal way in solving mathematics problems. However, in spite of students' ability to employ formal procedures to solve problems on proportions, it doesn't guarantee that students understand the concept of proportionality (Sumarto, *et al.*, 2014).

In this study, researcher designs and utilizes a learning trajectory and instructional activities which emphasizes the insight of proportional reasoning. Lamon's (2006) claimed that an ability to analyse situation in absolute and relative perspective is one of the most important types of thinking required for proportional reasoning. For that reason, the instructional activities including comparison tasks are designed to help students to elicit relative thinking in solving problems on proportions. Besides that, the researcher also creates teacher guide to support teacher in implementing the learning materials and the HLT. In addition, due to the familiar contexts which are used in this design, we may say that this design could be implemented in other school in Indonesia.

6.3. Suggestions

6.3.1 Suggestion for teachers

Applying a new social norm in a classroom is not an easy task. Introducing a new learning behavior and making the students become accustomed to it will take time. As it was elaborated at chapter 5, there were just several (particular) students who participated actively in the learning process, so the students' interaction and the use of students' contribution is not optimal. Most students were shy to speak up and most of them were afraid to give explanation or arguments. They were afraid if they made any mistakes on it. Therefore, many students tended to wait for other students presenting the works.

Nevertheless, it does not mean that the social norm can't be changed. Teacher should never stop encouraging his/her students to participate actively in learning process and not being hesitated to contribute in the discussion. Due of that, it is supposed that socio-mathematics norms in the classroom may be changed.

Furthermore, most of the teacher tends to justify, by him/herself, which solution is the correct one. It may be one kind of factors that leads the passiveness of the students. Moreover the students tend to accept any teacher's explanation. It is better for teachers to guide students to find solution by themselves. The teacher can poses any questions which lead the students to grasp mathematics idea and strategies to solve the problem. By giving active guidance for students, the teacher also activates the class discussion.

Based on the abovementioned statements, we might say that a teacher takes an important role in building a good learning environment. Therefore, it is crucial for teachers to do some reflection on her/his way of teaching. Evaluation is one of important ways to enhance our quality. In addition, it will be helpful for teachers to know recent studies about education that will enhance the teachers' knowledge and ability.

6.3.2 Recommendation for future research

The previous studies pointed out that many students fail to identify the appropriate proportional relationship, which leads them fail in determining the appropriate proportional reasoning. In this study, it is used comparison problems involving the notion of density and part-to-whole relationship. The idea of creating this kind of problem is supporting students to analyse the correct interpretation of proportional relationship. However, it is necessary to expand the variation of the context and proportional relationship represented the situations so that the students do not limit the proportional reasoning just in certain situations.

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APENDIX 1

TEACHER INTERVIEW SCHEME

1. **The students**
   - How many students in class 5F? How many girls and boys?
   - In general, are the students typically active or passive?
   - Are the students accustomed to work in group?
   - How do the students build an interaction among others in doing working group?
   - Do the students do a working group together, or do some students count on the other students?
   - Are the students accustomed to work by using worksheets?
   - Do the students have difficulties in understanding text?
2. **The teaching and learning activities**
   - Does the teacher arrange group work for mathematics class? How many students in each group?
   - What are the teacher considerations in arranging groups?
   - Do the students usually have discussion in the learning process?
   - How does the teacher manage the discussion? For instance, how does the teacher set up class presentation and class discussion? How does the teacher usually do to encourage students to participate actively in the discussion? What kind of instructions or questions that the teacher usually gives to students to enhance their participation in the discussion.
   - How do the students response the discussion?
   - Are the students accustomed to give opinions or ideas in verbal way during the class discussion?
   - Is there any norm in the classroom regarding to the learning process? For instance what do the students usually do if they want to ask about mathematics topic or clarify their works?
   - What kind of books do the students use in mathematics classroom?
3. **Pendidikan Matematika Realistik Indonesia (PMRI)**
   - What does the teacher know about PMRI?

APENDIX 2

CLASSROOM OBSERVATION SCHEME

1. **The teaching and learning process**
   - How is the class organized? For instance, do the students sit in a group, individual desk, or in pair desk?
   - How is the interaction between the teacher and the students?
   - How is students' interest toward the learning activity?
   - How does the teacher usually teach mathematics topic?
   - Where is teacher's position during the teaching and learning process? Is the teacher always stand in one particular place, or is the teacher usually moving around the classroom?
   - What type of questions that the teacher usually poses to the class?
   - Does the teacher give the students time to think before they give responses during the learning process?
   - Does the teacher usually generate different thinking of students? How does the teacher usually generate the different of students' responses of thinking?
2. **The social norm in the classroom**
   - How is the rule in the class? What the students should do in order to give opinion or responses toward particular mathematics matter?
   - What the students usually do if they have difficulties in understanding a particular topic? Do they ask directly by coming to the teacher (while the teacher is in a free time), or their rise their hand for asking?
3. **The students' activities**
   - What the students usually do when the teacher giving explanation?
   - What is the characteristic of the class, whether the students participate actively or they prefer to wait and accept any information?

APENDIX 3

PRE-TEST PROBLEMS

Nama :
Kelas/No. Absen :
Tanggal :

1. Bu Siti menjual beras dalam kemasan.
   Kemasan A; berat 2kg, harga Rp 20.000,00.
   Kemasan B; berat 5kg, harga Rp. 45.000,00.
   Beras kemasan mana yang lebih murah? Jelaskan jawaban kalian!
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2. Pak Ali adalah peternak ayam. Ia membuat dua kandang ayam, kandang A dan B.
   Kandang A berukuran $2m^2$ dan berisi 10 ayam. Kandang B berukuran 5 m^2 dan berisi 20 ayam.
   Menurut kalian, kandang mana yang lebih penuh atau sesak? Jelaskan jawaban kalian!

   

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3. Setelah ujian semester, kealas 5C dan 5D akan berlibur bersama. Ibu guru menyarankan untuk pergi ke pantai. Tetapi, ibu guru juga menanyakan minat siswa untuk pergi ke pantai dan berikut ini informasi yang diperoleh:

	Siswa yang memilih ke pantai	**Total siswa**
Siswa kelas 5C	12	25
Siswa kelas 5D	10	20

Menurut kalian, pergi ke pantai lebih popular bagi siswa kelas 5C atau siswa kelas 5D? Jelaskan jawaban kalian!

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APENDIX 4

POST-TEST PROBLEMS

Nama :
Kelas/No. Absen :
Tanggal :

1. Pak Saiful adalah peternak itik. Ia membuat kandang itik sebagai berikut.

Dua dari kandang itik Pak Saiful, kandang A berukuran 5m^2 dan berisi 100 itik. Kandang B berukuran 3 m^2 dan berisi 75 itik. Menurut kalian, kandang mana yang lebih penuh atau sesak? Jelaskan jawaban kalian!

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2. Kepala SD Harapan Bangsa mengadakan survey “Buku Kesukaan ku” untuk mengetahui minat baca siswa. Dari kegiatan itu, diperoleh hasil sebagai berikut:

Kelas	Total siswa	Siswa yang suka membaca buku ***cerita daerah***
5A	25	13
5B	20	11

Menurut kalian, ***cerita daerah*** lebih popular di kelas mana? Jelaskan jawaban kalian!

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3. Dini, dan Sari suka bermain dart. Dari permainan tersebut, diperoleh hasil sebagai berikut:

Dini : ●●●●● ●●●●○ ○○○○○

Sari : ●●●●● ●○○○

Berdasarkan hasil di atas, dari tiga pernyataan di bawah ini, pernyataan mana (***a***, ***b***, atau ***c***) yang benar? Jelaskan, mengapa pernyatan itu benar!

a. Dini dan Sari sama-sama mahir dalam bermain ***Dart***.

b. Dini lebih mahir dalam bermain ***Dart*** daripada Sari.

c. Sari lebih mahir dalam bermain ***Dart*** daripada Dini

Jawab:

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4. Suatu taman konservasi di Sumatera Selatan memiliki dua jenis rusa, rusa-sambar dan kijang.


Rusa Sambar **Kijang**

Tahun 2005, populasi rusa-sambar adalah 30, dan populasi kijang adalah 20. Tahun 2010, populasi rusa-sambar meningkat menjadi 50, dan populasi kijang menjadi 40.

Berdasarkan informasi tersebut, dari tiga pernyataan di bawah ini, pernyataan mana (***a, b***, atau ***c***) yang benar? Dan jelaskan, mengapa pernyatan itu benar!

a. Dari tahun 2005 ke 2010, peningkatan populasi rusa-sambar lebih tinggi daripada kijang
b. Dari tahun 2005 ke 2010, peningkatan populasi kijang lebih tinggi daripada rusa-sambar.
c. Dari tahun 2005 ke 2010, peningkatan populasi rusa-sambar dan kijang adalah sama.

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APENDIX 5
TEACHER GUIDE
Panduan Guru

Pendahuluan

Pemikiran relative atau ***relative thinking*** merupakan salah satu cara bernalar yang penting di dalam ***proportional reasoning*** (cara bernalar dalam memecahkan masalah perbandingan/proporsi). Lawan dari ***relative thinking*** adalah ***absolute thinking***. Pada saat siswa berpikir secara relative, siswa akan melihat situasi secara relative dengan memperhatikan keterkaitan antar infomasi. Sedangkan jika siswa berpikir secara absolute, siswa cenderung untuk tidak mengindahkan keterkaitan antar informasi.

Salah tipe permasalahan pada topic proporsi adalah *comparison problems*. Karplus, *et al*., 1983 menjelaskan bahwa di dalam menyelesaikan *comparison problems*, seorang siswa harus membandingkan dua nilai dari variabel intensif yang dihitung dari suatu data. Hal ini sesuai dengan yang diungkapkan oleh Sumarto et al. (2014) bahwa dalam menyelesaikan *comparison problems* siswa harus menghitung setiap set bilangan dari situasi dan kemudian menentukan hasil perhitungan mana yang memberikan perbandingan terbaik. Hal ini menunjukkan bahwa untuk menyelesaikan *comparison problems*, siswa tidak bisa hanya membandingkan salah satu angka dari setiap set angka. Siswa harus mempertimbangkan keterkaitan antar angka dan siswa harus memahami bahwa keseluruhan informasi mempengarushi perbandingan yang ada. penjelasan tentang bagaimana seharusnya menyelesaikan *comparison problems* di atas sesuai dengan prinsip pola pikir relative (Lamon, 2006).

Salah satu contoh permasalah yang mana pada umumnya orang-orang seara intuitif akan memperhatikan keterkaitan antar angka adalah ketika diminta utuk membandingkan kepadatan populasi di dua daerah, misalnya daerah A dan B. Daerah A memiliki populasi 250.000 jiwa, sedangkan daerah B memiliki populasi 200.000 jiwa. Jika seseorang ditanya, daerah mana yang lebih padat populasinya, secara otomatis orang tersebut juga akan membandingkan banyak populasi dengan luas daerah yang ditempati oleh populasi. Dengan demikan, orang tersebut mengaitkan informasi banyak populasi dengan luas daerah yang ditempati. Pola pikir atau penalaran seperti ini disebut dengan pola pikir relative.

PERTEMUAN 1

Alokasi waktu

2×35 menit

Bahan

Lembar Kerja Siswa (LKS1)

Tujuan Pembelajaran

- Siswa mampu menginterpretasi situasi proporsional dalam sudut pandang relatif (*relative perspective*).
- Siswa memahami adanya rasio ($a\colon b$ atau $\frac{a}{b}$) dalam masalah perbandingan.
- Siswa mampu membandingkan kepadatan populasi dengan menggunakan perbandingan relative dengan menggunakan konsep proporsi.

Deskripsi Aktivitas Pembelajaran

1. Guru membuat kelompok yang terdiri atas 2 orang (peer group) siswa dan siswa diminta untuk berada di dalam kelompok yang sama setiap kali mereka melakukan kerja kelompok.
2. Apersepsi
    - Sebelum siswa bekerja pada kegiatan inti, guru mengajukan pertanyaan sebegai berikut:

      Siapa yang tahu, apa itu perbandingan?

      atau

      siapa yang bisa memberikan contoh perbandingan?

      Guru juga bisa mengangkat perbandingan dari situasi di dalam kelas, diantaranya:

      Siapa yang tahu, bagaimana perbandingan antara banyak siswa laki-laki dan perempuan di kelas kita?

      Siapa yang tahu, bagaimana perbandingan antara banyak bangku dengan banyak siswa di kelas kita?

      Dari kegiatan apersepsi di atas, diharapkan siswa mengenal apa itu perbandingan (rasio) dan bagaimana menuliskan bentuk perbandingan (rasio).
    - Untuk membangkitkan minat siswa terhadap pembelajaran dan sebagai starting point untuk memasuki materi inti, guru memberikan *mind experiment* (bagian dari kegiatan apersepsi) sebegai berikut:

      Mana yang lebih sesak/penuh, kelas kita berisi 10 siswa atau kelas kita berisi 30 siswa? Mengapa?

      Mana yang lebih penuh/sesak, angkot yang berisi 10 siswa atau kah trans musi yang berisi 10 siswa? Mengapa, padahal kan banyak nya siswa sama-sama 10 kan?

      Tempat mana yang lebih sesak, lapangan bulu tangkis yang ditempati 50 orang atau lapangan sepak bola Jaka-Baring yang ditempati oleh 200 orang?

Melalui *mind experiment* ini, diharapkan siswa dapat mendeskripsikan kepadatan populasi dalam sudut pandang relative (*relative thinking*), karena kepadatan populasi dipengaruhi oleh banyak orang dan ukuran tempat (tidak sekedar dipengaruhi oleh banyak orang atau ukuran tempat saja).

3. Kegiatan inti
   - Guru membagikan LKS 2 pada tiap kelompok.
   - Guru meminta masing-masing kelompok untuk menuliskan nama anggota kelompok mereka pada LKS yang disediakan. Siswa dapat menuliskan proses penghitungan pada lembar LKS ini.
   - Guru memberikan waktu kepada siswa untuk membaca dan memikirkan soal tersebut selama 2 menit sebelum mereka berdiskusi dan bekerjasama dalam kelompok. Waktu yang diberikan untuk menyelesaikan kerja kelmpok adalah 15 menit.
   - Setelah waktu kerja kelompok selesai, guru memimpin diskusi di kelas.

Peranan Guru

Pada saat siswa bekerja di dalam kelompok

1. Siswa diminta untuk membaca dan memahamimkonteks serta permasalahan di 2 menit pertama. Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut:

   Apakah kalian memahami informasi yag ada?
   Apakah kalian memahami apa yag dimaksud soal?
   Pertanyaan ini tentang apa?
   Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?

   Memastikan pemahaman siswa terhadap soal yang diberikan itu penting dilakukan karena seringkali hambatan siswa dalam menyelesaikan suatu soal bukan karena siswa tidak bisa menyelesaikan persoalan tersebut. Akan tetapi hal itu diarenakan kurangnya pemahaman siswa akan maksud dari soal yang diberikan.
2. Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas.

   Beberapa pertimbangan dalam memilih jawaban siswa:
   - Variasi jawaban, siswa yang membandingkan kepadatan populasi secara absolut (hanya memperhatikan banyak ayam/populasi saja) dan kelompok yang memperhatikan keterkaitan antara banyak populasi dengan ukuran tempat (berpikir secara relatif).
   - Variasi strategi dalam menjawab.
   - Jawaban atau penjelasan yang menarik.

<u>Pada saat diskusi kelas</u>

4. Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama.

   Bagi siswayang sudah mengetahui jawaban dari permasalahan, mereka diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.)
5. Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu.
6. Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***.
7. Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas.
8. Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan.
9. Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis
10. Guru dapat membuat tabel di papan tulis untuk mempermudah siswa melihat hubungan antara luas kandang dengan banyak anak ayam (*relative thinking*).

Nama Kandang	**Ukuran kandang**	**Banyak anak ayam**
A	$1m^2$	20
B	$1m^2$	25
C	$1\frac{1}{2}m^2$	25
D	$2m^2$	40

11. Dengan mengingatkan kembali tentang perbandingan pada minilesson, melalui diskusi di dalam kelas diharapkan siswa dapat menuliskan perbandingan antara ukuran kandang dengan banyak anak ayam dalam pernyataan matematika $a{:}\,b$ atau $\frac{a}{b}$. Dengan demikian, siswa akan memahami bahwa ada konsep rasio di dalam masalah perbandingan.
12. Jika siswa belum sepakat bahwa siswa harus membandingkan kepadatan populasi ayam secara relative (dengan memperhatikan banyak anak ayam dengan ukuran kandang), guru dapat menanyakan kembali tentang mana yang lebih penuh/sesak, angkot yang berisi 10 siswa atau transmusi yang berisi 10 siswa. Dengan demikian diharapkan bahwa siswa akan berpikir secara relative.

13. Melalui diskusi kelas, diharapkan akan ada siswa yang membandingkan tingkat kepadatan populasi kandang dengan mencari luas daerah per ayam.
14. Selain itu, mungkin akan ada siswa yang membandingkan tingkat kepadatan kandang sebagai banyak anak ayam yang menempati per 1 m^2 kandang (rate=$\frac{banyak\ \ obyek}{m^2}$).
15. Siswa juga dimungkinkan akan menyelesaikan masalah perbandingan kepadatan populaso ini dengan cara:
    - menyamakan ukuran kandang,
      jika ukuran kandang diubah, maka banyak ayam juga akan berubah secara proporsional. Karena ukuran kandang telah sama, selanjutnya siswa dapat melihat kandang mana yang berisi lebih banyak ayam. Semakin bayak ayam di dalam suatu kandang, maka kandang itu semakin penuh/sesak/padat.
    - menyamakan banyak ayam.
      jika banyak ayam diubah, maka ukuran kandang juga akan berubah secara proporsional. Karena banyak ayam telah sama, selanjutnya siswa dapat melihat kandang mana yang berukuran lebih sempat. Dikarenakan banyak ayam di dalam kandang telah dibuat sama, maka semakin sempit atau kecil ukuran kandang akan membuat kandang tersebut semakin penuh/sesak/padat.

Di akhir kegiatan pembelajaran

16. Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi
17. Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran
    Apa yang kita pelajari hari ini?
    Hal penting apa yang kita pelajari hari ini?
    Apa yang harus diperhatikan dalam menentukan tempat mana yang lebih penuh/sesak/padat.
18. Guru memotivasi siswa untuk lebih aktif dalam belajar dan berdiskusi.

PERTEMUAN 2

Alokasi waktu

2×35 menit

Bahan

Lembar Kerja Siswa (LKS2)

Tujuan Pembelajaran

- Siswa mampu menentukan keterkaitan antara bagian (***part***) relative terhadap keseluruhan (***whole***) pada suatu rasio.
- Siswa mampu mewujudkan ***part-whole relationship*** (rasio) ke dalam bentuk pecahan ($\frac{part}{whole}$).
- Siswa mampu menggunakan perbandingan relative dalam menyelesaikan masalah perbandingan dengan membandingakan ***part-whole.***

Deskripsi Aktivitas Pembelajaran

1. Guru meminta siswa untuk duduk dalam kelompok masing-masing seperti yang sudah dibagi pada pertemuan pertama.
2. Apersepsi
   - Guru memulai pembelajaran dengan mengajukan pertanyaan,
     Misalnya:
     - *Ibu memiliki satu batang cokelat, kemudian ibu belah menjadi dua bagian yang sama. Satu bagian ibu berikan kepada Dewi dan satu bagian yang lain ibu simpan sendiri. Berapa bagian cokelat yang ibu berikan kepada Dewi?*
     - *Ibu memiliki galah sepanjang 3 m. Ibu mengecat 1 m dari galah tersebut dengan warna merah. Berapa bagian dari galah yang Ibu cat merah?*

   Melalui pemberian pertanyaan-pertanyaan pada apersepsi ini, diharapkan siswa akan menyadari hubungan antara bagian dengan keseluruhan (*part-whole relationship*) pada besaran kontinu (*continue quantity*). Sehingga di dalam membandingkan, siswa tidak hanya membandingkan antar bagian dengan bagian (*part- part*), tetapi siswa juga mampu membandingkan situasi yang melibatkan *part-whole relationship*.

3. Kegiatan inti
   - Guru membagikan LKS 2 pada tiap kelompok.
   - Guru meminta masing-masing kelompok untuk menuliskan nama anggota kelompok mereka pada LKS yang disediakan. Siswa dapat menuliskan proses penghitungan pada lembar LKS ini.
   - Guru memberikan waktu kepada siswa untuk membaca dan memikirkan soal tersebut selama 2 menit sebelum mereka berdiskusi dan bekerjasama dalam kelompok. Waktu yang diberikan untuk menyelesaikan kerja kelmpok adalah 15 menit.
   - Setelah waktu kerja kelompok selesai, guru memimpin diskusi di kelas.

Peranan Guru

Pada saat siswa bekerja di dalam kelompok

1. Siswa diminta untuk membaca dan memahamim konteks serta permasalahan di 2 menit pertama. Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut:
   Apakah kalian memahami informasi yag ada?
   Apakah kalian memahami apa yag dimaksud soal?
   Pertanyaan ini tentang apa?
   Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?
2. Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas.
   Beberapa pertimbangan dalam memilih jawaban siswa:
   - Variasi strategi dalam membuat visualisasi dari bagian jalan yang telah diaspal.
   - Variasi strategi dalam membuat urutan (soal no 2 dan 3), apakah siswa menggunakan ***absolute*** atau ***relative thinking***.
   - Jawaban atau penjelasan yang menarik, terutama yang berkaitan dengan ***part-whole relationship*** pada perbandingan antara bagian beraspal dan keseluruhan jalan.

Pada saat diskusi kelas

4. Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama.
   Bagi siswayang sudah mengetahui jawaban dari permasalahan, mereka diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.
5. Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu.
6. Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***.
7. Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas.
8. Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan.

9. Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis
10. Guru dapat membuat tabel di papan tulis untuk mempermudah siswa melihat hubungan antara bagian beraspal (***part***) dengan total panjang jalan (***whole)***.

Jalan	**Bagian berasapal**	**Total panjang jalan**
A	2 km	5 km
B	1 km	2 km
C	3 km	8 km

11. Beberapa siswa mungkin akan mengurutkan jalan mulai dari jalan C,A dan B karena siswa membandingkan panjang absolute dari bagian yang telah diaspal saja (***absolute thinking***), tanpa memperhatikan perbandingan (rasio) bagian beraspal terhadap keseluruhan panjang jalan.
12. Selain itu, beberapa siswa mungkin membandingkan keterlaksanaan pengaspalan jalan dengan melihat panjang bagian yang belum beraspal (***absolute value*** dari bagian yang belu diaspal). Mereka mungkin akan menentukan bahwa jalan memiliki bagian belum diaspal terpendek sebagain jalan yang proyek pengerjaannya hampil selesai. Di dalam hal ini, siswa tidak memperhatikan keterkaitan antara *part* dan *whole*. Penalaran yang seperti ini disebut ***absolute thinking***.

    Berikut ini adalah contoh pertanyaan yang dapat digunakan untuk memunculkan ***relative thinking*** (***part*** dengan ***whole***):

    Berapa bagian yang diaspal?
13. Guru bisa membahas terlebih dahulu situasi jalan 2, yg mana total panjang adalah 2 km, diaspal 1 km. Siswa dapat dengan mudah memahami bahwa bagian yang diaspal pada jalan 2 adalah setengah (½).
14. Setelah siswa mampu memahami "bagian" dari jalan, guru dapat mengembalikan ke soal utama yang meminta siswa untuk membandingkan jalan berdasarkan bagian yang beraspal. Sehingga, diharapkan siswa akan menyadari bahwa selanjutnya mereka harus membandingkan antar pecahan yang menyatakan bagian (***part-whole relationship***) tersebut.

Di akhir kegiatan pembelajaran

15. Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi
16. Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran

    Apa yang kita pelajari hari ini?

    Hal penting apa yang kita pelajari hari ini?

    Apa yang harus diperhatikan dalam menentukan tempat mana yang lebih penuh/sesak/padat.
17. Guru memotivasi siswa untuk lebih aktif dalam belajar dan berdiskusi.

PERTEMUAN 3
Alokasi waktu
2×35 menit
Bahan
Lembar Kerja Siswa (LKS3)
Tujuan Pembelajaran

- Siswa mampu menyelesaikan perbandingan dengan memperhatikan hubungan antara bagian dan keseluruhan (***relative thinking*** dan ***part-whole relationship***).
- Siswa mampu mendeskripsikan adanya perbedaan dari total kesempatan menembak.
- Siswa mampu mewujudkan rasio yang berupa ***part-whole relationship*** ke dalam bentuk pecahan.
- Siswa mampu mengunakan pecahan untuk menyelesaikan masalah perbandingan

Deskripsi Aktivitas Pembelajaran

1. Guru meminta siswa untuk duduk dalam kelompok masing-masing seperti yang sudah dibagi pada pertemuan pertama.
2. Apersepsi
   Guru memulai pembelajaran dengan memaparkan konteks, dengan menunjukkan papan Dart.
   Misalnya:
   Siapa yang tahu, ini papan apa?
   Siapa yang pernah bermain menggunakan papan ini?
   Apa yang harus kita lakukan untuk mencetak skor dalam permainan ini?
   Karena dimungkinakan siswa akan memahami aturan permainan Dart secara berbeda-beda, guru dapat memberi batasan bahwa untuk mencetak skor, seseorang harus melempar tepat mengenai tengah papan.
3. Kegiatan inti
   - Guru membagikan LKS 3 pada tiap kelompok.
   - Guru meminta masing-masing kelompok untuk menuliskan nama anggota kelompok mereka pada LKS yang disediakan. Siswa dapat menuliskan proses penghitungan pada lembar LKS ini.
   - Guru memberikan waktu kepada siswa untuk membaca dan memikirkan soal tersebut selama 2 menit sebelum mereka berdiskusi dan bekerjasama dalam kelompok. Waktu yang diberikan untuk menyelesaikan kerja kelmpok adalah 15 menit.
   - Setelah waktu kerja kelompok selesai, guru memimpin diskusi di kelas.

Peranan Guru

Pada saat siswa bekerja di dalam kelompok

1. Siswa diminta untuk membaca dan memahamim konteks serta permasalahan di 2 menit pertama. Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut:
   Apakah kalian memahami informasi yag ada?
   Apakah kalian memahami apa yag dimaksud soal?
   Pertanyaan ini tentang apa?
   Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?
2. Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas.
   Beberapa pertimbangan dalam memilih jawaban siswa:
   - Variasi jawaban, penalaran dan strategi dalam menentukan anak mana yang memenangkan permainan (Bayu, Gagah, Fadi atau Rio).
   - Variasi strategi dalam membuat urutan (soal no 2), apakah siswa menggunakan ***absolute*** atau relative ***thinking***.
   - Jawaban atau penjelasan yang menarik, terutama yang berkaitan dengan ***part-whole relationship*** antara skor menembak dan total kesempatan menembak

Pada saat diskusi kelas

4. Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama.
   Bagi siswayang sudah mengetahui jawaban dari permasalahan, mereka diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.
5. Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu.
6. Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***.
7. Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas.
8. Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan.

9. Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis
10. Guru dapat membuat tabel di papan tulis untuk mempermudah siswa melihat hubungan antara skor (***part***) dengan total menembak (***whole***) → *relative thinking*.

Nama anak	**Skor**	**Total menembak**
Bayu	5	10
Gagah	9	20
Fadli	4	10
Rio	11	25

11. Jika siswa masih berpikir secara absolut, guru dapat memberikan metaphor situasi serupa dengan angka yang sederhana

    Ada dua siswa, Maudi dan Soraya. Mereka berdua bermain Dart dengan hasil sebagai berikut:

    Maudi : ○● *Soraya : ●●●●○ ○○○○○*

    Dapatkah kalian menentukan, siapa yang lebih mahir bermain Dart?

    Melalui diskusi kelas, diharapkan akan ada siswa yang berpendapat bahwa skor masuk Maudi adalah "**setengah**" dan skor masuk Soraya kurang dari "**setengah**".
12. Selanjutnya, guru dapat menanyakan, *apa yang kalian maksud dengan **"setengah"** disini?*

    Hal ini ditujukan agar siswa mengungkapkan bagaimana dia menemukan kata-kata **"setengah",** karena pada umumnya siswa hanya akan melihat banyak bulatan penuh/skor menembak (nilai absolut).
13. Selanjutnya, guru dapat mengembalikan ke soal utama.

    Bukankah soal ini serupa dengan soal Bayu, Gagah, Fadli dan Rio?
14. Pertanyaan guru pada saat pemberian metaphor di atas, sebenarnya serupa dengan pertanyaan guru pada konsep ***part-whole*** dengan konteks jalan. Sehingga, diharapkan ada siswa yang masih mengingat dan menemukan keterkaitan antara dua konteks ini.
15. Selanjutnya, guru mengarahkan siswa dalam berdiskusi. Guru juga dapat memberikan penguatan pada jawaban siswa. Guru juga dapat mengingatkan kembali pada apa yang telah siswa pelajari di pertemuan sebelumnya.

Di akhir kegiatan pembelajaran

16. Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi
17. Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran

    Apa yang kita pelajari hari ini?

    Hal penting apa yang kita pelajari hari ini?

    Apa yang harus diperhatikan dalam menentukan tempat mana yang lebih penuh/sesak/padat.
18. Guru memotivasi siswa untuk lebih aktif dalam belajar dan berdiskusi.

PERTEMUAN 4

Alokasi waktu

2×35 menit

Bahan

Lembar Kerja Siswa (LKS4)

Tujuan Pembelajaran

- Siswa memehami bahwa perbedaan interpreatsi, absolute dan relative, pada masalah perbandingan itu bisa.
- Siswa mampu menerapkan konsep perbandingan dalam menyelesaikan masalah perbandingan.

Deskripsi Aktivitas Pembelajaran

1. Guru meminta siswa untuk duduk dalam kelompok masing-masing seperti yang sudah dibagi pada pertemuan pertama.
2. Apersepsi
   - Guru memulai pembelajaran dengan memaparkan konteks. Konteks pada pertemuan empat ini adalah survey yang dilakukan di SD Harapan Bangsa. Dari survey ini diketahui tentang minat siswa terhadap kegiatan ekstra kurikuler. Guru juga menginformasikan tentang survey minat ekstrakurikuler basket di kelas 5F. Dari survey ini, kita dapat mengetahui minat siswa laki-laki dan perempuan terhadap ekstrakurikuler basket.
   - Guru juga dapat mengajukan beberapa pertanyaaan pembuka, diantaranya:
     Siapa yang suka main basket?
     Siapa yang enjadi anggota tim basket sekolah?
     Siapa yang suka pramuka?
     Siapa yang ikut kelas bela diri?
     Melalui pemberian apersepsi ini, diharapkan siswa akan tertarik dalam mengikuti pembelajaran, terutama karena tema yang dibahas hari ini adalah tentang kegiatan ekstrakurikuler di sekolah.
3. Kegiatan inti
   - Guru membagikan LKS 3 pada tiap kelompok.
   - Guru meminta masing-masing kelompok untuk menuliskan nama anggota kelompok mereka pada LKS yang disediakan. Siswa dapat menuliskan proses penghitungan pada lembar LKS ini.
   - Guru memberikan waktu kepada siswa untuk membaca dan memikirkan soal tersebut selama 2 menit sebelum mereka berdiskusi dan bekerjasama dalam kelompok. Waktu yang diberikan untuk menyelesaikan kerja kelmpok adalah 15 menit.
   - Setelah waktu kerja kelompok selesai, guru memimpin diskusi di kelas.

<u>Peranan Guru</u>

<u>Pada saat siswa bekerja di dalam kelompok</u>

4. Siswa diminta untuk membaca dan memahamim konteks serta permasalahan di 2 menit pertama. Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut:
   Apakah kalian memahami informasi yag ada?
   Apakah kalian memahami apa yag dimaksud soal?
   Pertanyaan ini tentang apa?
   Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?
5. Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas.
   Beberapa pertimbangan dalam memilih jawaban siswa:
   - Variasi jawaban dan penalaran dalam menggunakan data untuk menentukan apakah basket lebih populer bagi siswa perempuan atau siswa laki-laki.
   - Variasi strategi dalam menjawab pertanyaan, apakah siswa menggunakan ***absolute*** atau relative ***thinking***.
   - Jawaban atau penjelasan yang menarik, terutama yang berkaitan dengan ***part-whole relationship.***

<u>Pada saat diskusi kelas</u>

6. Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama.
   Bagi siswayang sudah mengetahui jawaban dari permasalahan, mereka diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.
7. Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu.
8. Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***.
9. Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas.
10. Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan.

11. Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis
12. Siswa telah memiliki pengalaman dalam menyelesaikan masalah perbandingan serupa, sehingga jia siswa merasa kesulitan, guru dapat mengingatkan kembali akan aktivtas di pertemuan sebelumnya.
13. Selain itu, hendaknya guru menulis kembali data pada tabel sehingga akan membantu siswa dalam menjawab soal dan menjelaskan penalaran.
14. Kegiatan dua dikerjakan setelah siswa selesai mendiskusikan kegiatan 1.
    Bentuk dukungan guru terhadap penalaran siswa dalam menyelesaikan masalah di kegiatan 2 adalah serupa dengan apa yang guru lakukan di kegiatan 1 dan juga serupa dengan apa bimbingan yang guru lakukan untuk membantu siswa menyelesaikan permasalahan pada pertemuan 3.

Di akhir kegiatan pembelajaran

15. Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi
16. Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran
    Apa yang kita pelajari hari ini?
    Hal penting apa yang kita pelajari hari ini?
    Apa yang harus diperhatikan dalam menentukan tempat mana yang lebih penuh/sesak/padat.
17. Guru memotivasi siswa untuk lebih aktif dalam belajar dan berdiskusi.

APPENDIX 6

Rencana Pelaksanaan Pembelajaran (RPP)

Sekolah	:	SD YSP Pusri Palembang
Mata Pelajaran	:	Matematika
Kelas/Semester	:	V/Genap
Pertemuan	:	1 (pertama)
Alokasi Waktu	:	2 x 35 menit
Standar Kompetensi	:	Menggunakan pecahan dalam pemecahan masalah
Kompetensi Dasar	:	Menggunakan pecahan dalam masalah perbandingan dan skala

A. Tujuan Pembelajaran

- Siswa mampu menginterpretasi situasi proporsional dalam sudut pandang relatif (*relative perspective*).
- Siswa memahami adanya rasio ($a\!:b$ atau $\frac{a}{b}$) dalam masalah perbandingan.
- Siswa mampu membandingkan kepadatan populasi dengan menggunakan perbandingan relative dengan menggunakan konsep proporsi.

B. Indikator

- Siswa mampu menjelaskan bahwa kepadatan suatu populasi tidak hanya ditentukan oleh banyak populasi yang menempati suatu daerah (***absolute thinking***), akan tetapi kepadatan populasi bersifat relative karena dipengaruhi oleh banyak populasi dan luas daerah yang ditempati (***relative thinking***).
- Siswa mampu mendeskripsikan hubungan antara ukuran tempat (kandang) dan banyak populasi (ayam) yang menempati kandang sebagai rasio $a\!:b$ atau $\frac{a}{b}$ (rate).
- Siswa mampu mengunakan perbandingan relative terhadap kepadatan populasi ayam pada suatu kandang dengan menggunakan konsep proporsi, antara lain:
  - menentukan luas kandang yang ditempati per anak ayam (rasio antara luas kandang dengan banyak anak ayam),
    dan/atau
  - menentukan banyak anak ayam per $1m^2$ (rate=banyak anak ayam/m^2)
  - menyamakan ukuran kandang,
    jika ukuran kandang diubah, maka banyak ayam juga akan berubah secara proporsional. Karena ukuran kandang telah sama, selanjutnya

siswa dapat melihat kandang mana yang berisi lebih banyak ayam. Semakin bayak ayam di dalam suatu kandang, maka kandang itu semakin penuh/sesak/padat.

– menyamakan banyak ayam.
jika banyak ayam diubah, maka ukuran kandang juga akan berubah secara proporsional. Karena banyak ayam telah sama, selanjutnya siswa dapat melihat kandang mana yang berukuran lebih sempat. Dikarenakan banyak ayam di dalam kandang telah dibuat sama, maka semakin sempit atau kecil ukuran kandang akan membuat kandang tersebut semakin penuh/sesak/padat.

C. Materi Pembelajaran

Perbandingan adalah topik yang penting dalam Matematika. Kemampuan bernalar dalam perbandingan banyak dibutuhkan untuk mempelajari beragam materi Matematika, antara lain aljabar dan geometri. Rasio ($a{:}b$ atau $\frac{a}{b}$) merupakan dasar dari perbandingan karena berdasarkan pengertiannya, perbandingan merupakan persamaan dua rasio. Rasio juga disebut sebagai *quotient* atau perbandingan dari dua bilangan, besaran, kuantitas atau ekspresi, misalnya rasio antara sisi persegi dengan diagonal persegi adalah $1{:}\sqrt{2}$. Oleh karena itu, nilai suatu rasio tergantung dari nilai dua hal yang dibandingkan tersebut. Sehingga, nilai suatu rasio bersifat relatif.

Dengan demikian, dalam menyelesaikan masalah perbandingan dan melibatkan rasio, siswa diharapkan mampu mengembangkan penalaran relatif. Misalnya, di dalam membandingkan kepadatan populasi, beberapa siswa mungkin bernalar secara absolut dengan hanya membandingkan banyak populasi. Akan tetapi, siswa harus memperhatikan keterkaitan antara banyak populasi dengan luas daerah yang ditempati (*relative perspective*).

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	– Berdoa – Guru mengkondisikan kelas pada situasi belajar (misalnya siswa dikondisikan dalam kelompok belajar, jika siswa belum duduk secara berkelompok) – Guru menyampaikan kepada siswa bahwa hari ini mereka akan belajar tentang perbandingan (guru menulis judul materi di papan tulis)	3 menit

	<u>**Apersepsi**</u> ***Menanya*** – Guru memulai pembelajaran dengan mengajukan pertanyaan, ***Misalnya***: *Siapa yang tahu, apa itu perbandingan?* atau *siapa yang bisa memberikan contoh perbandingan?* ***Guru menanya da siswa mengamati (situasi dari konteks permasalahan)*** Guru juga bisa mengangkat perbandingan dari situasi di dalam kelas, diantaranya: *Siapa yang tahu, bagaimana perbandingan antara banyak siswa laki-laki dan perempuan di kelas kita?* *Siapa yang tahu, bagaimana perbandingan antara banyak bangku dengan banyak siswa di kelas kita?* Dari kegiatan apersepsi di atas, diharapkan siswa mengenal apa itu perbandingan (rasio) dan bagaimana menuliskan bentuk perbandingan (rasio).	
	<u>**Minilesson**</u> ***Guru menanya, siswa mengamati dan mengasosiasikan topic pembelajaran dengan pemahaman akan pengalaman dari kehidupan nyata*** <u>*Relative perspective* pada kepadatan-populasi (*population-density*)</u> – Guru memberikan *mind experiment* dengan cara memeberikan beberapa situasi yang mana siswa diminta untuk membandingkan kepadatan antar situasi tersebut, anatara lain: • *Mana yang lebih sesak/penuh, kelas kita berisi 10 siswa atau kelas kita berisi 30 siswa? Mengapa?* • *Mana yang lebih penuh/sesak, angkot yang berisi 10 siswa atau kah trans musi yang berisi 10 siswa? Mengapa, padahal kan banyak nya siswa sama-sama 10 kan?* • *Tempat mana yang lebih sesak, lapangan bulu tangkis yang ditempati 50 orang atau lapangan sepak bola Jaka-Baring yang ditempati oleh 200 orang?* Melalui minilesson ini, diharapkan siswa dapat mendeskripsikan kepadatan populasi dalam sudut pandang relative (*relative thinking*), karena kepadatan populasi dipengaruhi oleh banyak orang dan ukuran tempat (tidak sekedar dipengaruhi oleh banyak orang atau ukuran tempat saja).	5-7 menit

	<u>Setelah minilesson</u> – Guru menyampaikan kepada siswa bahwa untuk memperdalam pemahaman siswa, siswa akan bekerja dalam kelompok (2 siswa) dengan menggunakan lembar kerja siswa.	
Kegiatan Inti	***Guru menanya dan siswa mengamati situasi dari konteks permasalahan*** – Guru menyampaikan konteks permasalahan *Ada seorang peternak bernama Pak Ari membuat 4 kandang anak ayam.* – Guru menunjukkan gambar ayam dan kandang milik Pak Ari (terdapat di LKS). – Siswa mengamati situasi dari kandang ayam Pak Ari *Ada aturan dalam membuat kandang ayam, yaitu 1m2 kandang ayam sebaiknya diisi paling banak 20. Selain itu, kandang yang terlalu sesak/penuh tidak baik untuk perkembangan ayam.* *Saat ini, tugas kalian adalah membantu Pak Ari membandingkan kepadatan kandang ayam Pak Ari dan menentuka kandang mana yang layak atau kurang layak untuk ayam.* – Guru (dengan bantuan peneliti) membagikan lembar kerja siswa (LKS)	2-3 menit
	<u>Eksplorasi</u> – Siswa diminta mengerjakan LKS. – Siswa dipersilakan berdiskusi dalam kelompok saat bekerja. ***Menanya*** – Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut: *Apakah kalian memahami informasi yag ada?* *Apakah kalian memahami apa yag dimaksud soal?* *Pertanyaan ini tentang apa?* *Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?* Memastikan pemahaman siswa terhadap soal yang diberikan itu penting dilakukan karena seringkali hambatan siswa dalam menyelesaikan suatu soal bukan karena siswa tidak bisa menyelesaikan persoalan tersebut. Akan tetapi hal itu diarenakan	15 menit

	kurangnya pemahaman siswa akan maksud dari soal yang diberikan. – Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas. Beberapa pertimbangan dalam memilih jawaban siswa: o Variasi jawaban, siswa yang membandingkan kepadatan populasi secara absolut (hanya memperhatikan banyak ayam/populasi saja) dan kelompok yang memperhatikan keterkaitan antara banyak populasi dengan ukuran tempat (berpikir secara relatif). o Variasi strategi dalam menjawab. o Jawaban atau penjelasan yang menarik.	
	<u>Elaborasi</u> (Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama. Bagi siswayang sudah mengetahui jawaban dari permasalahan, mereka diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.) ***Mengkomunikasikan*** – Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu. – Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***. – Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas. ***Mengkomunikasikan dalam bentuk guru meminta klarifikasi siswa*** – Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang	37 menit

kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan.

- Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis
- Guru dapat membuat tabel di papan tulis untuk mempermudah siswa melihat hubungan antara luas kandang dengan banyak anak ayam (*relative thinking*).

Nama Kandang	Ukuran kandang	Banyak anak ayam
A	$1m^2$	20
B	$1m^2$	25
C	$1\frac{1}{2}m^2$	25
D	$2m^2$	40

Mengasosiasikan

- Dengan mengingatkan kembali tentang perbandingan pada minilesson, melalui diskusi di dalam kelas diharapkan siswa dapat menuliskan perbandingan antara ukuran kandang dengan banyak anak ayam dalam pernyataan matematika $a: b$ atau $\frac{a}{b}$. Dengan demikian, siswa akan memahami bahwa ada konsep rasio di dalam masalah perbandingan.

Menanya dan mengasosiasikan

- Jika siswa belum sepakat bahwa siswa harus membandingkan kepadatan populasi ayam secara relative (dengan memperhatikan banyak anak ayam dengan ukuran kandang), guru dapat menanyakan kembali tentang mana yang lebih penuh/sesak, angkot yang berisi 10 siswa atau transmusi yang berisi 10 siswa. Dengan demikian diharapkan bahwa siswa akan berpikir secara relative.

Mengkomunikasikan

- Melalui diskusi kelas, diharapkan akan ada siswa yang membandingkan tingkat kepadatan populasi kandang dengan mencari luas daerah per ayam.
- Selain itu, mungkin akan ada siswa yang membandingkan tingkat kepadatan kandang sebagai banyak anak ayam yang menempati per 1 m^2 kandang (rate=$\frac{banyak\ \ obyek}{m^2}$).

	Mengkomunikasikan – Siswa juga dimungkinkan akan menyelesaikan masalah perbandingan kepadatan populaso ini dengan cara: • menyamakan ukuran kandang, jika ukuran kandang diubah, maka banyak ayam juga akan berubah secara proporsional. Karena ukuran kandang telah sama, selanjutnya siswa dapat melihat kandang mana yang berisi lebih banyak ayam. Semakin bayak ayam di dalam suatu kandang, maka kandang itu semakin penuh/sesak/padat. • menyamakan banyak ayam. jika banyak ayam diubah, maka ukuran kandang juga akan berubah secara proporsional. Karena banyak ayam telah sama, selanjutnya siswa dapat melihat kandang mana yang berukuran lebih sempat. Dikarenakan banyak ayam di dalam kandang telah dibuat sama, maka semakin sempit atau kecil ukuran kandang akan membuat kandang tersebut semakin penuh/sesak/padat.	
Kegiatan akhir	**Konfirmasi** – Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi ***Menanya dan mengkomunikasikan*** – Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran *Apa yang kita pelajari hari ini?* *Hal penting apa yang kita pelajari hari ini?* *Apa yang harus diperhatikan dalam menentukan tempat mana yang lebih penuh/sesak/padat.* – Guru memotivasi siswa untuk lebih aktif dalam belajar dan berdiskusi.	5 menit

F. Media Pembelajaran
LKS (terlampir)

G. Penilaian
Tidak ada penilain khusus untuk pertemuan pertama karena masih memperkenalkan siswa akan konsep relativitas pada masalah perbandingan.

Palembang, 17 Maret 2014

Guru Kelas V F	Peneliti
R.A. Masturoh. Z, S.Pd	Wisnu Siwi Satiti, S.Pd NIM. 06122802003

Mengetahui
Kepala SD YSP Pusri Palembang

Hesti Pariza, S.Ag

Rencana Pelaksanaan Pembelajaran

(RPP)

Sekolah	:	SD YSP Pusri Palembang
Mata Pelajaran	:	Matematika
Kelas/Semester	:	V/Genap
Pertemuan	:	2 (kedua)
Alokasi Waktu	:	2 x 35 menit
Standar Kompetensi	:	Menggunakan pecahan dalam pemecahan masalah
Kompetensi Dasar	:	Menggunakan pecahan dalam masalah perbandingan dan skala

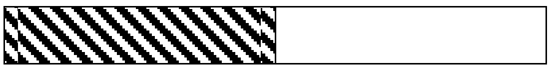
A. Tujuan Pembelajaran

- Siswa mampu menentukan keterkaitan antara bagian (***part***) relative terhadap keseluruhan (***whole***) pada suatu rasio.
- Siswa mampu mewujudkan ***part-whole relationship*** (rasio) ke dalam bentuk pecahan ($\frac{part}{whole}$).
- Siswa mampu menggunakan perbandingan relative dalam menyelesaikan masalah perbandingan dengan membandingakan ***part-whole.***

B. Indikator

- Siswa mampu menentukan rasio antara bagian dan keseluruhan.

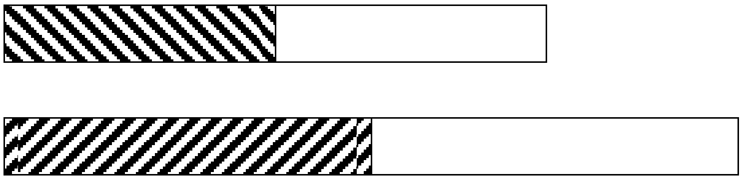
  Contoh:

  Rasio (perbandingan) antara bagian yang diarsir dengan keseluruhan adalah 1:2.
- Siswa mampu menjelaskan bahwa nilai dari bagian (***part***) bersifat relatif, tergantung dari keseluruhannya (***whole***).

  Contoh:

  Rasio antara bagian yang diarsir dengan keseluruhan pada kedua benda berikut adalah sama, yaitu 1:2. Akan tetapi, karena nilai keseluruhan dari kedua benda adalah berbeda, maka rasio keduanya (1:2) bersifat relatif meskipun memiliki nilai yang sama (setengah)



- Siswa mampu merepresentasikan rasio yang berupa ***part-whole relationship*** ke dalam bentuk pecahan.
- Siswa mampu membandingkan dua atau lebih situasi dengan menggunakan rasio antara bagian (***part***) dengan keseluruhannya (***whole***).

C. Materi Pembelajaran

- ***Part-whole relationship*** pada masalah perbandingan.
- Pecahan sebagai perwujudan dari ***part-whole relationship***

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	– Berdoa – Guru mengkondisikan kelas pada situasi belajar (misalnya siswa dikondisikan dalam kelompok belajar jika siswa belum duduk secara berkelompok) – Guru menyampaikan kepada siswa bahwa hari ini mereka akan belajar tentang perbandingan (guru menulis judul materi di papan tulis) **<u>Apersepsi</u>** ***Guru menanya dan siswa mengamati konteks dari permasalahan*** – Guru memulai pembelajaran dengan mengajukan pertanyaan, ***Misalnya***: • *Ibu memiliki satu batang cokelat, kemudian ibu belah menjadi dua bagian yang sama. Satu bagian ibu berikan kepada Dewi dan satu bagian yang lain ibu simpan sendiri. Berapa bagian cokelat yang ibu berikan kepada Dewi?* • *Ibu memiliki galah sepanjang 3 m. Ibu mengecat 1 m dari galah tersebut dengan warna merah. Berapa bagian dari galah yang Ibu cat merah?*	3 menit

	Melalui pemberian pertanyaan-pertanyaan pada apersepsi ini, diharapkan siswa akan menyadari hubungan antara bagian dengan keseluruhan (***part-whole relationship***) pada besaran kontinu (*continue quantity*). Sehingga di dalam membandingkan, siswa tidak hanya membandingkan antar bagian dengan bagian (***part- part***), tetapi siswa juga mampu membandingkan situasi yang melibatkan ***part-whole relationship***.	
Kegiatan Inti	***Guru menanya dan siswa mengamati konteks dari permasalahan*** – Guru menyampaikan konteks permasalahan *Ada proyek pengaspalan jalan di suatu kecamatan yang mana ketua pelaksana proyek adalah teman Ibu guru. Ketua pelaksana akan membuat laporan kemajuan proyek, untuk itu ia harus mendeskripsikan keterlaksanaan proyek pada laporan yang akan ia buat.* *Untuk memperjelas laporan, ketua pelaksana juga akan menggambarkan situasi jalan yang tengah diaspal sampai saat ini.* – Guru menunjukkan gambar aktivitas pengaspalan jalan (terlampir). Selanjutnya guru meminta siswa untuk membantu ketua pelaksana membuat laporan. – Guna membantu siswa menyusun laporan, ada beberapa aktivitas yang harus dilakukan oleh siswa. Untuk membantu siswa melaksanakan aktivitas tersebut, guru telah menyediakan LKS. – Guru (dengan bantuan peneliti) membagikan LKS	2 menit
	<u>Eksplorasi</u> – Siswa diminta mengerjakan LKS. – Siswa dipersilakan berdiskusi dalam kelompok saat bekerja. ***Menanya*** – Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut: *Apakah kalian memahami informasi yag ada?* *Apakah kalian memahami apa yag dimaksud soal?* *Pertanyaan ini tentang apa?* *Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?* Memastikan pemahaman siswa terhadap soal yang diberikan itu penting dilakukan karena seringkali hambatan siswa dalam menyelesaikan suatu soal bukan karena siswa tidak bisa menyelesaikan persoalan tersebut. Akan tetapi hal itu diarenakan kurangnya pemahaman siswa akan maksud dari soal yang diberikan. – Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai	15 menit

	menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas. Beberapa pertimbangan dalam memilih jawaban siswa: o Variasi strategi dalam membuat visualisasi dari bagian jalan yang telah diaspal. o Variasi strategi dalam membuat urutan (soal no 2 dan 3), apakah siswa menggunakan ***absolute*** atau ***relative thinking***. o Jawaban atau penjelasan yang menarik, terutama yang berkaitan dengan ***part-whole relationship*** pada perbandingan antara bagian beraspal dan keseluruhan jalan.	
	<u>Elaborasi</u> (Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama. Bagi mereka yang sudah mengetahui jawaban dari permasalahan, siswa diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.) ***Mengkomunikasikan*** – Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu. – Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***. – Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas. ***Menanya (dalam bentuk guru meminta klarifikasi)*** – Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan. – Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis. – Guru dapat membuat tabel di papan tulis untuk mempermudah siswa melihat hubungan antara bagian beraspal (***part***) dengan total panjang jalan (***whole)***.	40 - 45 menit

Jalan	Bagian berasapal	Total panjang jalan
A	2 km	5 km
B	1 km	2 km
C	3 km	8 km

Mengkomunikasikan

- Beberapa siswa mungkin akan mengurutkan jalan mulai dari jalan C,A dan B karena siswa membandingkan panjang absolute dari bagian yang telah diaspal saja (***absolute thinking***), tanpa memperhatikan perbandingan (rasio) bagian beraspal terhadap keseluruhan panjang jalan.
- Selain itu, beberapa siswa mungkin membandingkan keterlaksanaan pengaspalan jalan dengan melihat panjang bagian yang belum beraspal (***absolute value*** dari bagian yang belu diaspal). Mereka mungkin akan menentukan bahwa jalan memiliki bagian belum diaspal terpendek sebagain jalan yang proyek pengerjaannya hampil selesai. Di dalam hal ini, siswa tidak memperhatikan keterkaitan antara *part* dan *whole*. Penalaran yang seperti ini disebut ***absolute thinking***.
  Berikut ini adalah contoh pertanyaan yang dapat digunakan untuk memunculkan ***relative thinking*** (***part*** dengan ***whole***):
  Berapa bagian yang diaspal?
  Penekanan pada kata "bagian" (keterkaitan antara ***part*** dengan ***whole*)**
- Guru bisa membahas terlebih dahulu situasi jalan 2, yg mana total panjang adalah 2 km, diaspal 1 km. Siswa dapat dengan mudah memahami bahwa bagian yang diaspal pada jalan 2 adalah setengah (½).
- Guru bisa langsung memberi penekanan pada kata ***setengah***, dan menuliskan kan setengah dalam bentuk ½ .
- Bagaimana guru merepresentasikan suatu situasi ke dalam notasi matematika memiliki peran penting dalam proses pembelajaran. Hal ini dikarenakan, mungkin siswa dapat menjawab dan menjelaskan secara lisan tentang stengah (½) jalan yang telah diaspal. Akan tetapi, mungkin ada sebagain siswa yang tidak mengetahui bagaiaman menuliskan setengah ke dalam notasi matematika.
- Dengan menuliskan **setengah sebagai pecahan**, hal ini akan memberikan ide bagi siswa bahwa **bagian** (sebanyak ***a*** dari ***b***)dapat dinyatakan ke dalam bentuk pecahan ($\frac{a}{b}$).

Mengasosiasikan

- Setelah siswa mampu memahami "bagian" dari jalan, guru dapat mengembalikan ke soal utama yang meminta siswa untuk membandingkan jalan berdasarkan bagian yang beraspal. Sehingga, diharapkan siswa akan menyadari bahwa selanjutnya mereka harus membandingkan antar pecahan yang menyatakan bagian (***part-whole***

	relationship) tersebut.	
Kegiatan akhir	**Konfirmasi** – Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi ***Menanya dan mengkomunikasikan*** – Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran *Apa yang kita pelajari hari ini?* *Hal penting apa yang kita pelajari hari ini?* – Guru memotivasi siswa untuk lebih aktif	5-10 menit

F. Media Pembelajaran

LKS (terlampir)

G. Penilaian

Teknik : tes

Bentuk tes : tertulis

Instrument : LKS pertemuan 2(terlampir)

Rubrik penilaian

Kriteria	**Skor**
Siswa mampu memberikan jawaban benar disertai dengan penjelasan pendukung yang benar dan masuk akal.	10
Siswa mampu memberikan jawaban benar disertai dengan penjelasan pendukung yang benar, tetapi alasannya kurang lengkap.	8
Siswa mampu memberikan jawaban benar, tetapi penjelasan yang diberikan tidak mendukung diperolehnya jawaban.	5
Siswa mampu memberikan jawaban benar, tetapi tidak disertai dengan penjelasan pendukung atau argument.	3
Siswa tidak mampu memberikan jawaban yang benar.	2
Siswa tidak memberikan jawaban.	0

Kunci Jawaban.

1. Jalan yang keterlaksanaan proyek pengasapalannya paling banyak adalah jalan B. Karena, setengah dari keseluruhan jalan telah diaspal. Sedangkan jalan A dan C, belum ada setengah bagian yang diaspal.
   (Skor 10)
2. Urutan jalan berdasarkan bagian yang beraspal
   Jalan B ($\frac{1}{2}$ bagian), jalan A ($\frac{2}{5}$ bagian) dan jalan C ($\frac{3}{8}$bagian)
   Karena, $\frac{1}{2} = \frac{20}{40} > \frac{2}{5} = \frac{16}{40} > \frac{3}{8} = \frac{15}{40}$ (Skor 10)

Palembang, 18 Maret 2014

Guru Kelas V F	Peneliti
R.A. Masturoh. Z, S.Pd	Wisnu Siwi Satiti, S.Pd NIM. 06122802003

Mengetahui
Kepala SD YSP Pusri Palembang

Hesti Pariza, S.Ag

Rencana Pelaksanaan Pembelajaran
(RPP)

Sekolah	:	SD YSP Pusri Palembang
Mata Pelajaran	:	Matematika
Kelas/Semester	:	V/Genap
Pertemuan	:	3 (ketiga)
Alokasi Waktu	:	2 x 35 menit
Standar Kompetensi	:	Menggunakan pecahan dalam pemecahan masalah
Kompetensi Dasar	:	Menggunakan pecahan dalam masalah perbandingan dan skala

A. Tujuan Pembelajaran

a. Siswa mampu menginterpretasi masalah perbandingan dari sudut pandang relative (***relative thinking***).
b. Siswa mampu menentukan keterkaitan antara bagian (*part*) relative terhadap keseluruhan (*whole*) pada besaran diskret.
c. Siswa mampu menggunakan pecahan dan/atau decimal untuk menyelesaikan masalah perbandingan.

B. Indikator

- Siswa mampu menyelesaikan perbandingan dengan memperhatikan hubungan antara bagian dan keseluruhan (***relative thinking*** dan ***part-whole relationship***).
- Siswa mampu mendeskripsikan adanya perbedaan dari total kesempatan menembak.
- Siswa mampu mewujudkan rasio yang berupa ***part-whole relationship*** ke dalam bentuk pecahan.
- Siswa mampu mengunakan pecahan untuk menyelesaikan masalah perbandingan

C. Materi Pembelajaran

- Perbandingan yang melibatkan relative thinking
- Perbandingan yang melibatkan pecahan.
- Pecahan sebagai perwujudan dari ***part-whole relationship***.

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	– Berdoa – Guru mengkondisikan kelas pada situasi belajar (misalnya siswa dikondisikan dalam kelompok belajar jika siswa belum duduk secara berkelompok) – Guru menyampaikan kepada siswa bahwa hari ini mereka akan belajar tentang perbandingan (guru menulis judul materi di papan tulis) **<u>Apersepsi</u>** ***Guru menanya dan siswa mengamati konteks dari permasalahan*** – Guru memulai pembelajaran dengan memaparkan konteks, dengan menunjukkan papan Dart (disediakan oleh peneliti). ***Misalnya***: *Siapa yang tahu, ini papan apa?* *Siapa yang pernah bermain menggunakan papan ini?* *Apa yang harus kita lakukan untuk mencetak skor dalam permainan ini?* Karena dimungkinakan siswa akan memahami aturan permainan Dart secara berbeda-beda, guru dapat memberi batasan bahwa untuk mencetak skor, seseorang harus melempar tepat mengenai tengah papan. Selanjutnya, guru memaparkan konteks, *Ada empat siswa bermain Dart, Bayu, Gagah, Fadli dan Rio.* *Setiap dari mereka memiliki kesempatan menembak yang berbeda dan mereka membuat skor yang berbeda pula.* *Dari permainan ini, pemenang akan mendapatkan papan Dart baru.* *Tugas kalian adalah menentukan siapa yang memenangkan permainan Dart ini dan bagaimana cara kalian menentukan si pemenang.* Melalui pemberian apersepsi dan guru menunjukkan papan Dart yang asli, diharapkan siswa akan lebih antusias dalam mengikuti pembelajaran. Siswa akan merasa tertantang, karena sangat dimungkinkan banyak dari siswa pernah bermain Dart. Guru juga dapat mendemonstrasikan cara bermain Dart.	3-5 menit

Kegiatan Inti	– Guru (dengan bantuan peneliti) membagikan LKS	
	<u>Eksplorasi</u> – Siswa diminta mengerjakan LKS. – Siswa dipersilakan berdiskusi dalam kelompok saat bekerja. ***Menanya*** – Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut: *Apakah kalian memahami informasi yag ada?* *Apakah kalian memahami apa yag dimaksud soal?* *Pertanyaan ini tentang apa?* *Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?* – Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas. Beberapa pertimbangan dalam memilih jawaban siswa: o Variasi jawaban, penalaran dan strategi dalam menentukan anak mana yang memenangkan permainan (Bayu, Gagah, Fadi atau Rio). o Variasi strategi dalam membuat urutan (soal no 2), apakah siswa menggunakan ***absolute*** atau relative ***thinking***. o Jawaban atau penjelasan yang menarik, terutama yang berkaitan dengan ***part-whole relationship*** antara skor menembak dan total kesempatan menembak	15 menit
	<u>Elaborasi</u> (Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama. Bagi mereka yang sudah mengetahui jawaban dari permasalahan, siswa diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang	40-45 menit

berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.)

Mengkomunikasikan

– Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu.
– Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***.
– Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas.
– Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan.
– Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis
– Guru dapat membuat tabel di papan tulis untuk mempermudah siswa melihat hubungan antara skor (***part***) dengan total menembak (***whole***) → *relative thinking*.

Nama anak	Skor	Total menembak
Bayu	5	10
Gagah	9	20
Fadli	4	10
Rio	11	25

Mengasosiasikan

– Jika siswa masih berpikir secara absolut, guru dapat memberikan metaphor situasi serupa dengan angka yang sederhana
Ada dua siswa, Maudi dan Soraya. Mereka berdua bermain Dart dengan hasil sebagai berikut:
Maudi : ○●
Soraya : ●●●●○ ○○○○○
Dapatkah kalian menentukan, siapa yang lebih mahir bermain Dart?

Melalui diskusi kelas, diharapkan akan ada siswa yang berpendapat bahwa skor masuk Maudi adalah "**setengah**" dan skor masuk Soraya kurang dari "**setengah**".

	Menanya – Selanjutnya, guru dapat menanyakan, *apa yang kalian maksud dengan **"setengah"** disini?* Hal ini ditujukan agar siswa mengungkapkan bagaimana dia menemukan kata-kata **"setengah",** karena pada umumnya siswa hanya akan melihat banyak bulatan penuh/skor menembak (nilai absolut). ***Mengasosiasikan*** – Selanjutnya, guru dapat mengembalikan ke soal utama. *Bukankah soal ini serupa dengan soal Bayu, Gagah, Fadli dan Rio?* – Pertanyaan guru pada saat pemberian metaphor di atas, sebenarnya serupa dengan pertanyaan guru pada konsep ***part-whole*** dengan konteks jalan. Sehingga, diharapkan ada siswa yang masih mengingat dan menemukan keterkaitan antara dua konteks ini. – Selanjutnya, guru mengarahkan siswa dalam berdiskusi. Guru juga dapat memberikan penguatan pada jawaban siswa. Guru juga dapat mengingatkan kembali pada apa yang telah siswa pelajari di pertemuan sebelumnya.	
Kegiatan akhir	**Konfirmasi** – Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi ***Menanya dan mengkomunikasikan*** – Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran: *hal apa yang harus diperhatikan dalam menentukan pemain Dart yang aling mahir?* – Guru memotivasi siswa untuk lebih aktif dalam belajar dan berdiskusi.	5-10 menit

F. Media Pembelajaran

LKS (terlampir)
Gambar (terlampir)

G. Penilaian

Teknik : tes
Bentuk tes : tertulis
Instrument : LKS pertemuan 3 (terlampir)

Rubrik penilaian

Kriteria	Skor
Siswa mampu memberikan jawaban benar disertai dengan penjelasan pendukung yang benar dan masuk akal.	10
Siswa mampu memberikan jawaban benar disertai dengan penjelasan pendukung yang benar, tetapi alasannya kurang lengkap.	8
Siswa mampu memberikan jawaban benar, tetapi penjelasan yang diberikan tidak mendukung diperolehnya jawaban.	5
Siswa mampu memberikan jawaban benar, tetapi tidak disertai dengan penjelasan pendukung atau argument.	3
Siswa tidak mampu memberikan jawaban yang benar.	2
Siswa tidak memberikan jawaban.	0

Kunci jawaban

1. Bayu adalah yang paling mahir bermain Dart, karena skor yang ia peroleh adalah **setengah** (½) dari keseluruhan kesempatan menembak, yang mana siswa yang lain tidak ada yang mencapai skor **setengah** dari keseluruahn menembak.
   (Skor 10)
2. Urutan pemain:
   Bayu ($\frac{5}{10} = \frac{50}{100}$), Gagah ($\frac{9}{20} = \frac{45}{100}$), Rio ($\frac{11}{25} = \frac{44}{100}$), , Fadli ($\frac{4}{10} = \frac{40}{100}$)
   (Skor 10)

Palembang, 19 Maret 2014

Guru Kelas V F	Peneliti
R.A. Masturoh. Z, S.Pd	Wisnu Siwi Satiti, S.Pd NIM. 06122802003

Mengetahui
Kepala SD YSP Pusri Palembang

Hesti Pariza, S.Ag

Rencana Pelaksanaan Pembelajaran
(RPP)

Sekolah	:	SD YSP Pusri Palembang
Mata Pelajaran	:	Matematika
Kelas/Semester	:	V/Genap
Pertemuan	:	4 (keempat)
Alokasi Waktu	:	2 x 35 menit
Standar Kompetensi	:	Menggunakan pecahan dalam pemecahan masalah
Kompetensi Dasar	:	Menggunakan pecahan dalam masalah perbandingan dan skala

A. Tujuan Pembelajaran

a. Siswa memehami bahwa perbedaan interpreatsi, absolute dan relative, pada masalah perbandingan itu bisa.
b. Siswa mampu menerapkan konsep perbandingan dalam menyelesaikan masalah pernadingan.

B. Indikator

- Siswa mampu menyelesaikan masalah perbandingan dengan memperhatikan hubungan antar data atau informasi (sudut pandang relatif), bukan sekedar menggunakan informasi secara parsial.
- Siswa mampu menyelesaikan masalah perbandingan dengan menggunakan hubungan antara bagian dengan keseluruhan.
- Siswa mampu merepresentasikan hubungan antara bagian dengan keseluruhan ke dalam bentuk pecahan.
- Siswa dapat menggunakan pecahan dalam menyelesaikan masalah perbandingan.

C. Materi Pembelajaran

- Perbandingan yang melibatkan *relative thinking*
- Perbandingan yang melibatkan pecahan.
- Pecahan sebagai perwujudan dari ***part-whole relationship***

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	– Berdoa – Guru mengkondisikan kelas pada situasi belajar (misalnya siswa dikondisikan dalam kelompok belajar jika siswa belum duduk secara berkelompok) – Guru menyampaikan kepada siswa bahwa hari ini mereka akan belajar tentang perbandingan (guru menulis judul materi di papan tulis) **<u>Apersepsi</u>** ***Guru menanya dan siswa mengamati situasi dari konteks permasalahan*** – Guru memulai pembelajaran dengan memaparkan konteks. Konteks pada pertemuan empat ini adalah survey yang dilakukan di SD Harapan Bangsa. Dari survey ini diketahui tentang minat siswa terhadap kegiatan ekstra kurikuler. Guru juga menginformasikan tentang survey minat ekstrakurikuler basket di kelas 5F. Dari survey ini, kita dapat mengetahui minat siswa laki-laki dan perempuan terhadap ekstrakurikuler basket. – Guru juga dapat mengajukan beberapa pertanyaaan pembuka, diantaranya: *Siapa yang suka main basket?* *Siapa yang enjadi anggota tim basket sekolah?* *Siapa yang suka pramuka?* *Siapa yang ikut kelas bela diri?* Melalui pemberian apersepsi ini, diharapkan siswa akan tertarik dalam mengikuti pembelajaran, terutama karena tema yang dibahas hari ini adalah tentang kegiatan ekstrakurikuler di sekolah. – Selanjutnya guru menyampaikan: *Nah, sekarang kita akan mengetahui minat siswa SD Harapan bangsa terhadap kegiatan ekstrakurikuler.*	5 menit
Kegiatan Inti	– Guru (dengan bantuan peneliti) membagikan LKS Kegiatan 1	

	<u>**Eksplorasi**</u> **Kegiatan 1** – Siswa diminta mengerjakan LKS. – Siswa dipersilakan berdiskusi dalam kelompok saat bekerja. ***Menanya*** – Setelah 2 menit pertama, guru bertanya kepada siswa apakah siswa telah memahami informasi dan soal yang ada. Guru dapat menggunakan kalimat tanya berikut: *Apakah kalian memahami informasi yag ada?* *Apakah kalian memahami apa yag dimaksud soal?* *Pertanyaan ini tentang apa?* *Dapatkah kalian mengulang pertanyaan itu dengan bahasa kalian sendiri?* – Ketika siswa sedang bekerja di dalam kelompok, guru berkeliling untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru hendaknya juga melihat jawaban siswa dan mulai menentukan kelompok mana yang akan diminta mempresentasikan hasil kerja di depan kelas. Beberapa pertimbangan dalam memilih jawaban siswa: o Variasi jawaban dan penalaran dalam menggunakan data untuk menentukan apakah basket lebih populer bagi siswa perempuan atau siswa laki-laki. o Variasi strategi dalam menjawab pertanyaan, apakah siswa menggunakan ***absolute*** atau relative ***thinking***. o Jawaban atau penjelasan yang menarik, terutama yang berkaitan dengan ***part-whole relationship.***	5 - 10 menit
	<u>**Elaborasi**</u> (Di dalam diskusi kelas, penting untuk dibuat aturan yang mana siswa harus berpikir terlebih dahulu/***time thinking*** sebelum menjawab pertanyaan dari guru. ***Time thinking*** ini diharapkan dapat membantu siswa yang mungkin membutuhkan waktu berpikir lebih lama. Bagi mereka yang sudah mengetahui jawaban dari permasalahan, siswa diminta meletakkan ibu jari di depan mulut. Kamudian guru akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar di dalam proses diskusi, tidak hanya siswa yang aktif saja yang berkontribusi, akan tetapi semua siswa dapat berartisipasi aktif di dalam pembelajaran.)	10-15 menit

	Mengkomunikasikan – Diskusi di dalam kelas tentang beragam penalaran dan jawaban siswa serta alasan mengapa siswa berpendapat seperti itu. – Guru menekankan pada alasan dan penalaran siswa, sehingga guru hendaknya banyak menggunakan kata tanya: ***mengapa*** dan ***bagaimana***. – Guru meminta kelompok yang telah dipilih untuk mempresentsikan hasil kerja di depan kelas. – Sebelum guru memberikan konfirmasi apakah jawaban siswa benar atau salah, guru menanyakan kepada kelas, apakah ada yang kurang setuju atau mungkin ada pendapat dan strategi lain dalam menyelesaikan permasalahan. – Guru juga dapat mempertemukan pendapat dan alasan yang berbeda sehingga siswa dapat berpikir kritis. ***Mengasosiasikan*** – Siswa telah memiliki pengalaman dalam menyelesaikan masalah perbandingan serupa, sehingga jia siswa merasa kesulitan, guru dapat mengingatkan kembali akan aktivtas di pertemuan sebelumnya. ***Mengasosiasikan*** – Selain itu, hendaknya guru menulis kembali data pada tabel sehingga akan membantu siswa dalam menjawab soal dan menjelaskan penalaran.	
	Kegiatan 2 – Kegiatan dua dikerjakan setelah siswa selesai mendiskusikan kegiatan 1. – Bentuk dukungan guru terhadap penalaran siswa dalam menyelesaikan masalah di kegiatan 2 adalah serupa dengan apa yang guru lakukan di kegiatan 1 dan juga serupa dengan apa bimbingan yang guru lakukan untuk membantu siswa menyelesaikan permasalahan pada pertemuan 3.	Eksplorasi dan elaborasi 20 - 25 menit
Kegiatan akhir	**Konfirmasi** – Guru memberikan pujian untuk siswa yang berpartisipasi dalam diskusi ***Menanya dan mengkomunikasikan*** – Dengan bimbingan guru, siswa melakukan refleksi terhadap kegiatan pembelajaran *Apa yang kita pelajari hari ini?*	5-10 menit

	Hal penting apa yang kita pelajari hari ini? – Guru memotivasi siswa untuk lebih aktif	

F. Media Pembelajaran

LKS (terlampir)

G. Penilaian

Teknik : tes
Bentuk tes : tertulis
Instrument : LKS pertemuan 4 (terlampir)

Rubrik penilaian

Kriteria	Skor
Siswa mampu memberikan jawaban benar disertai dengan penjelasan pendukung yang benar dan masuk akal.	10
Siswa mampu memberikan jawaban benar disertai dengan penjelasan pendukung yang benar, tetapi alasannya kurang lengkap.	8
Siswa mampu memberikan jawaban benar, tetapi penjelasan yang diberikan tidak mendukung diperolehnya jawaban.	5
Siswa mampu memberikan jawaban benar, tetapi tidak disertai dengan penjelasan pendukung atau argument.	3
Siswa tidak mampu memberikan jawaban yang benar.	2
Siswa tidak memberikan jawaban.	0

Kunci Jawaban

1. Basket lebih populer bagi siswa laki-laki karena setengah dari total siswa laki-laki (5 siswa dari 10 siswa) tertarik untuk mengikuti basket. Sedangkan untuk siswa perempuan, hanya 7 dari 15 siswa yang tertarik mengikuti basket, yang mana ini belu setengah dari keseluruhannya.
   $\frac{5}{10} = \frac{15}{30} > \frac{7}{15} = \frac{14}{30}$.
   (Skor 10)
2. Ekstrakurikuler Silat ($\frac{20}{30}$) yang seharusnya dijadwalkan 2 kali seminggu,bukan ekstrakuler Pramuka, karena:
   anggota silat yang memilih dua kali kegiatan dalam seminggu=$\frac{20}{30}$; dan
   anggota pramuka yang memilih dua kali kegiatan dalam seminggu=$\frac{30}{50}$,
   yang mana $\frac{20}{30} = \frac{100}{150} > \frac{30}{50} = \frac{90}{150}$
   (Skor 10)

Palembang, 19 Maret 2014

Guru Kelas V F | Peneliti

R.A. Masturoh. Z, S.Pd | Wisnu Siwi Satiti, S.Pd
NIM. 06122802003

Mengetahui
Kepala SD YSP Pusri Palembang

Hesti Pariza, S.Ag

APENDIX 7

STUDENTS' MATERIALS

Activity 1 (Worksheets)-LKS1

Nama : ____________________ **Kelas** : ____________________

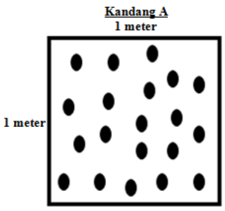
____________________ **Tanggal** : ____________________

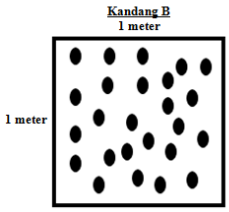
Baca pertanyaan dan informasi dengan teliti!

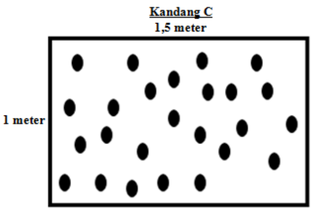
Pak Ari membuat empat kandang anak ayam, kandang A, B, C dan D.

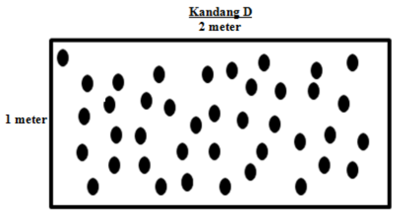
Setiap kandang memiliki ukuran berbeda **(kandang A = *1 m*^{2} , kandang B = *1 m*^{2}, kandang C = $1\frac{1}{2}$ *m*^{2}, *dan* kandang D = *2 m*^{2})**

Di bawah ini adalah gambar kandang anak ayam Pak Ari dan banyak anak ayam di dalamnya (bulatan hitam)









1. Menurut kalian, kandang mana yang paling penuh? Jelaskan jawaban kalian!
   Jawab:
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2. Urutkan kandang ayam Pak Ari di atas, dari kandang yang paling penuh ke kandang yang paling lapang! Dan jelaskan bagaimana cara kalian menentukan urutan tersebut?
   Jawab:
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Activity 2 (Worksheets)-LKS 2

Nama : ____________________ **Kelas** : ____________________

____________________ **Tanggal** : ____________________

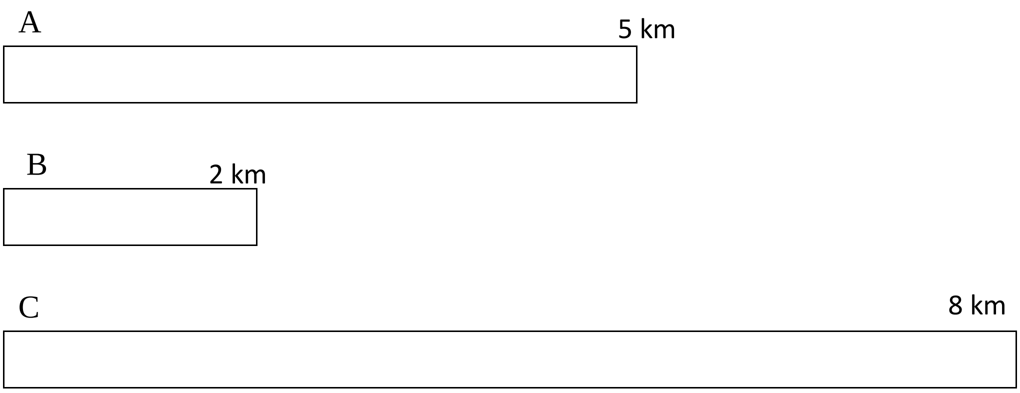
Baca pertanyaan dan informasi dengan teliti!

Dinas Bina Marga membuat tiga jalan baru di kecamatan Suka Maju, yaitu jalan A, B and C. Saat ini, ketiga jalan tengah diaspal.

Ketua pelaksana pengaspalan jalan akan membuat laporan. Untuk itu, ia akan mempersiapkan gambar keterlaksaan pengaspalan jalan.

Jalan A, total panjang 5 km, bagian yang telah diaspal 2 km
Jalan B, total panjang 2 km, bagian yang telah diaspal 1 km
Jalan C, total panjang 8 km, bagian yang telah diaspal 3 km.

1. Arsirlah bagian yang telah diaspal pada jalan A, B, dan C berikut!



2. Sebagai laporan, ketua pelaksana perlu mengurutkan keterlaksaan pengaspalan jalan A, B, C.

   Sekarang bantulah ketua pelaksana untuk mengurutkan ketiga jalan di atas mulai dari jalan yang memiliki bagian beraspal nya paling banyak!

   Jawab:

3. Jelaskan bagaimana cara kalian dalam membuat urutan tersebut!

Activity 3 (Worksheets)-LKS 3

Nama : ____________________ **Kelas** : ____________________
____________________ **Tanggal** : ____________________

Baca pertanyaan dan informasi dengan teliti!

Berikut ini adalah hasil permainan ***Dart*** yang dilakukan oleh Gagah, Bayu, Rio dan Fadli.
Gagah : ●●●●● ●●●●○ ○○○○○ ○○○○○
Bayu : ●○●○○ ●○●●○
Rio : ○●○●○ ○●○●○ ●○○●● ●○○○● ●●○○○
Fadli : ●●●●○ ○○○○○

a. Berdasarkan hasil di atas, menurut kalian siapa yang paling bagus atau mahir dalam bermain ***Dart***? Jelaskan jawaban kalian!
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b. Sekarang urutkan ke empat pemain di atas mulai dari pemain yang paling bagus atau mahir! Jelaskan bagaimana cara kalian membuat urutan tersebut!
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Activity 4 (Worksheets)-LKS 4

Nama : ____________________ **Kelas** : ____________________
____________________ **Tanggal** : ____________________

Baca pertanyaan dan informasi dengan teliti!

Kegiatan 1

Dari pengisian angket kegiatan ekstrakurikuler di SD *Harapan Bangsa*, diperoleh informasi tentang minat siswa terhadap kegiatan ekstrakurikuler.

1. Basket kelas 5D:

   Siswa perempuan

Siswa perempuan yang mengikuti basket	Total siswa
7	15

   Siswa laki-laki

Siswa laki-laki yang mengikuti basket	Total siswa
5	10

   Berdasarkan informasi pada tabel di atas, basket lebih populer bagi siswa perempuan ataukah siswa laki-laki? Jelaskan jawaban kalian!

   Jawab:

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Kegiatan 2

2. Sebagian besar anggota *Silat* and *Pramuka* meminta jadwal latihan dua kali seminggu.

   Berikut ini informasi permintaan siswa untuk jadwal latihan Silat dan Pramuka:

   Silat

Siswa yang memilih dua kali seminggu	Total anggota Silat
20	30

   Pramuka

Siswa yang memilih dua kali seminggu	Total anggota Pramuka
30	50

   Tetapi, karena jadwal kegiatan sekolah sudah padat, hanya satu ekstrakurikuler yang bisa diadakan dua kali seminggu.

   Jika kalian adalah kepala sekolah, bagaimana cara kalian menentukan ekstrakurikuler mana (**Silat atau Pramuka**) yang akan dilaksanakan dua kali seminggu? Jelaskan cara kalian!

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