# DEVELOPING GRADE 5 STUDENTS'UNDERSTANDING OF MULTIPLICATION OF TWO FRACTIONS

**Master Thesis**

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# DEVELOPING GRADE 5 STUDENTS' UNDERSTANDING OF MULTIPLICATION OF TWO FRACTIONS

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**APPROVAL OF SUPERVISORS**

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***Subhanallah***

**This master"piece" is dedicated to my lovely family and them, the people who believe in their dreams.**

**You only need to work hard and do your best,**

**then let HE Finishes in HIS Way.**

**ABSTRACT**

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***Keywords**:* Multiplication of two fractions, PMRI, design based research

Multiplication of two fractions is an important part of learning about fractions. Therefore, instead of only knowing and applying the procedure to solve the problems students also need to have a deep understanding about this topic. However, students still have difficulties in the understanding about multiplication of two fractions. Many factors contribute to this difficulties such as the multifaceted interpretation of fractions; the influence of the notion about whole numbers; the shifting between the term "part of" into term "times". Hence, this study will design an instructional learning activity to teach students in grade 5 of elementary school where some ideas such as partitioning, taking a part of a part of a whole unit within contexts and using array model are concerned to support the students to develop their understanding about multiplication of two fractions.

Design research method is used as an approach in this study. The sequence of five lessons is design grounded by the *Pendidikan Matematika Realistik Indonesia (PMRI)* approach. The Hypothetical learning trajectory (HLT) becomes the base for research and learning instruments. Two cyclic of studies is conducted. The participants are 30 students of grade 5C of SD Al Hikmah Surabaya along with their mathematics teacher. The data are collected through video registration, students' written works and interviews. The retrospective analysis is conducted by confronting the HLT with the actual learning process of the students.

This study shows that these learning sequences could support students to develop their understanding of multiplication of two fractions. By providing students with the bar and the array models within contexts promotes them to recognize about the idea of partitioning. The students started to use the models in their reasoning about the taking a part of a part of a whole problem. Further, by experiencing the partitioning in the taking a part of a part of a whole problem activity, students recognized the idea of multiplying a fraction with another fraction. Moreover, it also reveals that the students could use the array models in solving the multiplication of two fractions problems.

# ABSTRAK

Rifandi, R. 2014. *Developing Grade 5 Students' Understanding of Multiplication of Two Fractions*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Mega T. Budiarto, M.Pd. dan (II) Dr. Agung Lukito, M.S.

***Kata Kunci**:* Perkalian dua pecahan, PMRI, design research,

Perkalian dua pecahan merupakan salah satu topik penting pada pembelajaran tentang pecahan. Oleh karena itu, siswa tidak hanya dituntut untuk tahu dan bisa menggunakan prosedur menyelesaikan perkalian dua pecahan tetapi juga diperlukan adanya sebuah pemahaman yang mendalam tentang topik tersebut. Namun, siswa masih memiliki kesulitan dalam membangun pemahaman mereka tentang topik perkalian dua pecahan. Banyak faktor yang memengaruhi munculnya kesulitan siswa dalam memahami perkalian dua pecahan. Faktor tersebut adalah pecahan memiliki pengertian yang beragam; adanya pengaruh pemahaman siswa tentang bilangan bulat; dan perubahan pemahaman istilah "bagian dari" menjadi istilah "kali". Dalam penelitian ini dirancang aktivitas pembelajaran untuk siswa kelas 5 sekolah dasar dengan menggunakan ide *partitioning*, mengambil bagian dari bagian dari keseluruhan dalam konteks, dan penggunaan model *array* untuk membantu siswa mengembangkan pemahaman mereka tentang perkalian dua pecahan.

Penelitian ini menggunakan pendekatan *design based research*. Susunan lima pertemuan dirancang berdasarkan pendekatan Pendidikan Matematika Realistik Indonesia (PMRI). *Hypothetical learning trajectory* (HLT) dijadikan sebagai dasar dalam melakukan penelitian dan menyusun instrumen pembelajaran. Penelitian ini terdiri dari dua siklus. Subjek penelitian ini adalah 30 orang siswa Kelas 5C SD Al Hikmah Surabaya beserta guru matematika kelas tersebut. Retrospektif analisis dilakukan dengan membandingkan HLT dengan proses pembelajaran aktual yang terjadi di kelas.

Penelitian ini menunjukkan bahwa disain pembelajaran yang telah dirancang dapat membantu siswa untuk mengembangkan pemahaman mereka tentang perkalian dua pecahan. Dengan memfasilitasi siswa menggunakan model *bar* dan model *array* dalam konteks membantu mereka untuk memunculkan ide tentang *partitioning*. Siswa mulai menggunakan model tersebut dalam menjelaskan proses mengambil bagian dari bagian dari keseluruhan. Selanjutnya dengan melakukan aktivitas *partitioning* dalam proses mengambil bagian dari bagian dari keseluruhan, siswa dapat menangkap ide tentang mengalikan sebuah pecahan dengan pecahan lainnya. Penelitian ini juga menunjukkan bahwa siswa mampu menggunakan model *array* dalam menyelesaikan perkalian dua pecahan.

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**Ronal Rifandi**

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# CHAPTER I

# INTRODUCTION

## A. Research Background

Learning about fractions is important for students because the understandings of fractions become a basic foundation to learn about proportions, decimal numbers and percentages which are very useful in the daily life (van Galen, et al, 2008). However, fractions itself is a complicated topic (Streefland, 2008).

Lamon (1999) stated in his study that one will have more complicated situation regarding the understanding of the symbols at the beginning of their moves from the whole number into fractions. The notion about operations in whole numbers influences students when they deal with fractions problems and this will make them confused. For instance, in the case of the multiplication of fractions, initially most students see the multiplication of fractions as repeated addition (van Galen, et.al, 2008). However, not all cases in the multiplication of fractions can be seen from that point of view. Instead of seeing as repeated addition, multiplication of a fraction with another fraction needs an understanding about a fraction as an operator. This in line with the result of the study of Charalambous and Pitta-Pantazi (2005) about fractions' sub constructs. Moreover, also based on their prior knowledge students assume that a multiplication always results a bigger number (cf. Bell et al., 1981). But, this does not hold for multiplication of fractions which also can produce a smaller number.

It is hard for students to make a shift from the term “of “to the term “times” which is symbolized by “×” within the multiplication of fractions (Streefland, 2008; van Galen, et. al, 2008). Students of the fifth grade will struggle to understand that the term “of” on fractions multiplication is the same with the term “times” and can be symbolized by “×”. In the traditional way of teaching a teacher might just say that the term “of” is the same with the term “times’ and the symbol “×”. Students can use it, but do not get the understanding about what exactly it is.

Regarding to the calculation of multiplying a fraction with another fraction, many students also have common mistakes, for example, if they multiply two fractions with the same denominator, then they just multiply the numerator and keep the denominator the same, $\frac{a}{b} \times \frac{c}{b} = \frac{ac}{b}$ (Wittmann, 2013). In other case, if the denominator is not the same they tend to find the least common multiple (LCM) of it and then multiply the numerator, e.g. $\frac{2}{3} \times \frac{1}{6} = \frac{4 \times 1}{6}$ as they do with the addition of fractions. Those common mistakes indicate that some students do not understand the use of the algorithm and the meaning behind the operations.

Many studies already done in Europe and in the United States about the use of context and models to support students’ understanding about multiplication of fractions. Fosnot and Dolk (2002) stated that models can help students to represent their computation strategies and use it as a tool to think with. Graeber and Tanenhaus (1993) argued that a model, in this case the area model, can help students to make sense that a multiplication can produce a smaller number. Taber (2007) in her study uses "Alice in wonderland context" to teach multiplication and she said that the rich context can be used by the students to explore and deepen their understanding. However, we still do not know how context and models support students' understanding about the multiplication of fractions topic, especially multiplication of a fraction with another fraction, in Indonesia.

Furthermore, in the teaching of mathematics, including the multiplication of fractions, many teachers in Indonesia still tend to use the abstract level and only focus on the algorithm (Sukayati & Marfuah, 2009). However, it is important to take and support the informal knowledge of the students within a meaningful learning activity as a starting point to build students understanding about the concept (Mack, 1990). Mack (2000) stated that the important informal knowledge of the students in understanding the multiplication of a fraction with another fraction is partitioning, particularly the notion about taking a part of a part of a whole.

## B. Research Question

Based on the explanation in the background above, the general research question in this study is:

*"How can models support 5$^{th}$ grade students' understanding of multiplication of a fraction with another fraction?"*

We derive two sub research questions from the general research question above as follows:

a. *How can models support students' understanding of taking a part of a part of a whole?*
b. *How can taking a part of a part of a whole activity using an array model support students' understanding of multiplication of two fractions?*

## C. Research Aim

Therefore, the purpose of this study is to contribute to the development of a local instruction theory in supporting students' understanding of multiplication of a fraction with another fraction. In the design of the activities, firstly, we support students to use their informal knowledge about partitioning and then provide them with a context which embed emergent model and use it to understand and to reason about the multiplication of a fraction with another fraction.

## D. Definition of Key Terms

1. Developing students' understanding

   The developing is defined as a progress of doing something. Meanwhile, the understanding is defined as an ability of making connection of new knowledge to be fitted with the existing schema that students have. Thus, developing students' understanding means as a progress of connecting the new knowledge to be fitted with the existing schema that students have.

2. Fraction

   Fraction is defined as any number that can be expressed as $\frac{a}{b}$ where *a* and *b* are integers, *a* is not multiple of *b*, and *b* is not equal to zero (Borowski &Borwein, 2002). However, in this present study we only focus in the proper fraction where the absolute value of the numerator is less than the absolute value of the denominator.

3. Multiplication of a fraction with another fraction

   Multiplication of a fraction with another fraction in this present study is based on the interpretation of the activity of taking a part of a part of a whole unit.

4. Taking a part of a part of a whole

   Taking a part of a part of a whole is defined as dividing a unit to produce a new quantity then finding a part of that new quantity.

5. Model

   Model is the representation of a situation or problem. The use of the model is to support students to bridge informal and formal mathematics and reach generalization and abstraction.

6. Array

   The array is a rectangular arrangement of unit in rows and columns.

7. The local instruction theory (LIT)

   The local instruction theory (LIT) in this study is developed based on the elaboration and the reflection of the designed HLT which is confronted to the actual learning process of the students. The LIT consists of conjectures about a possible learning process and possible tools of supporting that learning process. (Gravemeijer and Cobb, 2006).

**E. Significance of the Research**

We have two significance of conducting the present study. First, to give an insight for the mathematics teacher of developing students' understanding of multiplication of two fractions so that mathematics teacher can elaborate it in the learning activity in the classroom. The second significance is to give grounded instructional theory in understanding of multiplication of two fractions through the use of the model.

# CHAPTER II

# THEORETICAL FRAMEWORK

In this chapter we will provide the background framework that is used in this study. We will explain about fractions, multiplication of fractions, RME and teaching multiplication of fractions in the Indonesian curriculum.

## A. Fractions

Borowski and Borwein (2002) stated that fraction is any number that can be expressed as $\frac{a}{b}$ where *a* and *b* are integers, *a* is not multiple of *b*, and *b* is not equal to zero. Pupils start to have ideas about fractions in many situations in their daily life earlier than when it is taught in the classroom (Smith, 2002). An example of the situation is when they share candies or a cake with their friends. They informally build their own perception of fractions. Then students bring this initial knowledge to the classroom and they find something that is called "fractions". However, the topic of fraction indeed is very complicated. There is more than one interpretation of fractions and each of them has its own characteristic.

The first expert who distinguished the five constructs of fractions was Kieren (1976, in Charalambous and Pitta-Pantazi, 2007). He identified fractions as part-whole relations, ratio, quotient, measure and operator. These five constructs are also used and elaborated more in many other studies (Pantziara and Philippou, 2012; Behr et al., 1992). In a recent study, Smith (2002) refers to fractions as a quotient, divided quantity, ratio and proportion.

About thirty years ago, Freudenthal (1983) also explained about the multiple interpretations of fractions. He elucidated three interpretations of fractions in his book *Didactical phenomenology of mathematical structures*. First, he explained about fraction as fracture. Within this interpretation, fractions appear as a result of dividing a whole unit in some parts. The whole unit here can be definite and indefinite (Freudenthal, 1983). Second, he described fractions that can be used in comparison, especially in indirect comparison where a third object is used to mediate between the two objects to be measured. Third, he stated fractions that can be used in an operator.

According to the explanation above, it can be concluded that fractions have multifaceted interpretations. This condition makes students struggle in developing their understanding about fractions (Kieren 1993; Pantziara and Philippou, 2012). Therefore, we need to design an instructional theory which can be applied in mathematics classrooms in order to help students (and teachers) develop the understanding of fractions. To make it specific, in this present study, we only focus on developing students' understanding of multiplication of two fractions. We underpin this study by using notion about part-whole, or fractions as fractures (as in Freudenthal’s view), and the construct of fraction as an operator.

## B. Developing Students' Understanding of Multiplication of Two Fractions

As we described in the previous section in this chapter, the present study focuses on developing students' understanding of multiplication of two fractions. First, the researcher will describe definitions about understanding. Skemp (1987) argued about the idea of schema. He stated that "To understand something means to assimilate it into an appropriate schema." The term schema refers to the conceptual structures which relate to either the complex conceptual mathematical structures of mathematics or the simple structures which coordinate sensory motor activity (Skemp, 1987). Another expert that suggested a definition of understanding is Nickerson. He stated that understanding is the ability to build connections between one conceptual domain and another (Nickerson, 1985).

Based on the explanation above, we defined that understanding is the ability of making connection of new knowledge to be fitted with the existing schema that students have. Since the word developing means a progress of doing something, therefore in this present study, we defined developing students' understanding of multiplication of two fractions as the progress of connecting the new knowledge to be fitted with the existing schema in order to build the concept of multiplication of two fractions. The new knowledge in this study is the concept of multiplication of two fractions. Meanwhile, the existing schema can be derived from students' informal knowledge about partitioning.

Students learn to know fractions at home before they learn it at school, as mentioned before, which means that they already have initial knowledge about fractions itself. Previous studies which focus on developing students' understanding have documented that the informal knowledge of students should be an important consideration (Carpenter et al., 1989; Mack, 1990). Brown (1993) stated that students tend to use their informal knowledge of fractions to form a meaningful understanding of the algorithms. It also implies that in developing students’ understanding of multiplication of two fractions, teachers have to connect to the informal knowledge of the students as the starting point in the learning activity. In addition to this, Mack (1998) stated that it is important to build students’ informal knowledge before teaching them about the formal form of the multiplication of fractions. She argued that students need to know about two things when they learn the multiplication of fractions, those are equal-sized parts and equal representation of a fraction. In the understanding of equal-sized parts, the students have to know that the size of the part is based on the size of the unit. In the equal representation, students should know that every fraction has many other forms of fractions with the same value.

The informal knowledge that is important to support students in understanding the multiplication of two fractions is partitioning. Behr and Post (1992) defined partitioning as “dividing a region into equal parts or of separating a set of discrete objects into equivalent subsets”. They stated that the idea of partitioning is a fundamental point in order to grasp the knowledge about fractions. Moreover, supporting students to build their informal knowledge on partitioning may lead to the development of the understanding about multiplication of two fractions (Mack, 2000). Usually children can partition objects mentally in their daily activity or in their mind, but it is hard for them to represent it on paper (Smith, 2002). Therefore, Behr and Post (1992) recommended the teacher to ensure that students can experience the partitioning of various objects in the mathematics classroom.

The idea of partitioning will relate to the part-whole relation of fractions. When students can do partitioning properly, it will help them to quickly work with the collection of parts of the partitioning result (Smith, 2002). It is important for students to identify the construction of a fraction when the whole unit is known and also the other way around, determine the whole when the parts are known (Pantziara and Philippou, 2012). It will help them to deal with the activity of taking "a part of a part of a whole" which we define as dividing a unit to produce a new quantity then finding a part of that new quantity. The taking a part of a part of a whole unit has a connection with the interpretation of a fraction as an operator and further to help them in reasoning about multiplication of a fraction with another fraction.

Furthermore, in the multiplication of a fraction with another fraction, students need to extend their notion into interpreting fractions as an operator (Streefland, 1991, 1993). They also need to move from the understanding of multiplication as a repeated addition into seeing a fraction as a factor in multiplication (van Galen et al., 2008).

Based on the review of the studies above, we can conclude that the informal knowledge of students, that is partitioning, is important as a basis to understand the multiplication of two fractions. Moreover, knowing the part-whole relation and fractions as an operator are essential in order to get a formal knowledge of multiplication of two fractions. From the literature review we also found out that students struggle to understand the concept of multiplication of two fractions due to the multifaceted interpretation of fractions. It may get mixed up in students' mind. A difficulty that we already listed in the introduction is the distraction of natural number notions. Students may still think that multiplication always makes something bigger and interpret multiplication only as repeated addition. In addition, students also have difficulty to shift from the use of the term “out of” to the term “times” which is symbolized by “×”. Therefore, we need to embody the theory about supporting students’ understanding in a learning instruction so that teachers can apply it in the classroom.

## C. The Use of Realistic Mathematics Education (RME)

### 1. Mathematizing

Freudenthal (1968, in Keijzer, 2003) argued about learning mathematics as “mathematising”. He stated that mathematising is “watching the world from a mathematical perspective to thus make it more mathematical”. Gravemeijer (1994) proposed that “Learning mathematics means doing mathematics, of which solving everyday life problems is an essential part”. Treffers (1987, as cited in Keijzer, 2003) stated that in teaching mathematics a teacher needs to consider the initial knowledge of students and relate it to the realistic contexts as a base for the learning activity. The realistic contexts which consist of meaningful problems give students a chance to build their understanding of the mathematics (Greeno, Collin & Resnick, 1996, as cited in Keijzer, 2003). Furthermore, in the Realistic Mathematics Education (RME) approach, the mathematisation of meaningful problems becomes a tool for students to construct formal notions about the concept (Van den Heuvel-Panhuizen, 1996).

Freudenthal (1991) suggested that mathematics teaching and learning should be a process of "reinvention" by the students instead of just transferring the knowledge from the teacher to the students. The role of the teacher is to support students in this learning process. Later, Keijzer (2003) proposed that "When discussing this mathematising processes, we actually discuss the process of modeling, symbolizing, generalizing, formalizing, and abstracting". He said that these kinds of activities reflect the journey of the students in reaching the formal and abstract structures of the mathematical concepts. They experience every part of the activities by themselves which leads them to meaningful learning.

According to the explanation above, the present study aims at designing sequence of lessons by using the point of view of mathematics as mathematizing. The learning process starts with a rich context that allows students to experience some activities that lead to meaningful learning. In addition, regarding the process of mathematizing in Keijzer's view, the present study only addresses three of the activities, modeling, symbolizing and generalizing. We only use these three aspects of mathematizing because in this study we only focus on the informal parts and the journey of the students before they continue with the formal and the abstract notion of multiplication of two fractions.

**2. Five Tenets of RME**

The instructional design which starts from building the informal knowledge of the students in this study relies on the five tenets of RME. Treffers (1987, in Bakker, 2004) described the five tenets as follows:

a. Phenomenological exploration

It is important to use a meaningful context that can be explored as the foundation of the concept formation. Instead of starting to explain the abstract and procedural knowledge about multiplication of two fractions, in the present study, students are engaged with the context about a scout club which holds a hiking event. The students will explore how to divide the trail equally. In addition, students will also work on "time for exercises" and "sharing a chocolate block and dividing a *martabak telur* (Indonesia traditional food)" within a context about the preparation of the next hiking event. Some of the context we provide in the form of attractive comics so that the students will enjoy reading and then solve the problems.

b. Using models and symbols for progressive mathematization

The use of models is supporting students to bridge their informal knowledge and the formal mathematics and reach generalization and abstraction. First, students can use models as a representation of the condition and also the strategy they use to solve the problem. In the present study, there are two models that are expected to appear, those are bar model and array models.

c. Using students' own construction and productions

For conducting a meaningful learning, it is essential to use students' own productions. In the present study, students will construct their own array to solve the problem of the multiplication of two fractions. After the array model is elicited within the activity of sharing chocolate block in which the teacher provides the representation of the chocolate block on a grid, in the next problems the students try to make their own array. Further, as homework the teacher asks the students to draw the array with a different size to solve the problem of sharing chocolate.

d. Interactivity

In order to develop students' understanding of mathematics, they can learn from each other by sharing their ideas and the reasoning. In the present study, there are three kinds of discussions that will be conducted. First, in some activities in this study, students will work in a small group consisting of three or four students. They will discuss and solve the problem on the worksheet. Second, a whole class discussion is conducted as a follow-up of group work. In the whole class discussion students will share their ideas and answers about the given problem. And the third kind of discussion to hold is a “math congress”. In this discussion the teacher will support students to reflect on what they have done and what notions they already get.

e. Intertwinement

It is important to consider the relation of the specific topic in the design to other domains. In the RME approach this means that the theory and practice of a topic are not taught separately, but they have a wide relation to other topics and can support each other in the teaching and learning process. In this study the multiplication of two fractions has a relationship with proportion and also with the arithmetic operation of natural numbers.

**3. Emergent Modeling**

We zoom in to the second tenets of RME. It is important because in this study we want to know more about the use of models in supporting students' understanding of fractions. Gravemeijer (1994) stated that basically models are used as a concrete starting point for developing a formal notion of a concept in mathematics. Figure 2.1 shows the four levels of models that Gravemeijer (1994) proposes.

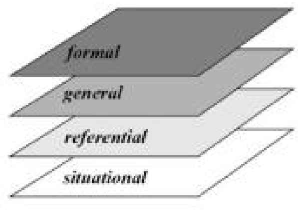



*Figure 2.1* Four levels of emergent modeling (Gravemeijer, 1994)

Gravemeijer described those levels as follows:

a. The level of situations, where domain specific, situational knowledge and strategies are used within the context of the situation (mainly out of school situations);
b. A referential level, where models and strategies refer to the situation which is sketched in the problem (mostly posed in a school setting);
c. A general level, where a mathematical focus on strategies dominates the reference to the context;
d. The level of formal arithmetic, where one works with conventional procedures and notations.

(Gravemeijer, 1994, p. 101)

In addition, the modeling in the referential level uses models as a representation of the activity in the instruction and in the general level the models become "models for" which are used in problem solving. This is independent of the situation (Gravemeijer, 1994). Furthermore, Gravemeijer (2000, in Fosnot and Dolk, 2002) stated that "The shift from *model of* to *model for* concurs with a shift in students' thinking, from thinking about the modeled context situation, to a focus on a mathematical relations" (p.74). It means that in the learning a teacher should provide the students with a context that can be modeled by the students, after which students, guided by the teacher will move to use the models as a tool for problem solving in a more general situation.

In this study, we only focused on using the initial knowledge of the students about partitioning as the starting point to understand about the multiplication of two fractions, therefore the levels of emergent model in this study only reach until a general level. The bar model is used to represent the result of a partitioning activity and used as a means in solving multiplication of a fraction with a whole number. The array models are used to support students to develop their understanding of multiplication of two fractions and later on use it in problem solving.

Therefore, we design a sequence of lessons to overcome students' difficulties in multiplication of two fractions grounded with the RME approach. The focus is on using models to support students' development in their understanding. In general, we prepare a learning-teaching trajectory as follows:

a. Address the initial and informal knowledge of the students.

   It is important to take the initial and informal knowledge of the students as a starting point. We have to activate the pre-knowledge of the students and build on that for further learning. In the present study we will conduct an activity of partitioning and notating the fractions as the result of the partition. By doing this activity students will experience by themselves how to make equal sized parts and relate it to the whole unit.

b. Modeling and symbolizing within an activity of taking a part of a part of a whole activity.

A rich and meaningful context will support students in the emergent model. In the present study, a context of sharing a chocolate block and a *martabak telur* can promote the use of the array model. First, they will work on an array that is provided in the form of a chocolate block. Further, the students will draw their own array to solve the problem in the context of dividing a *martabak telur* and they will make their own production regarding the array with a different size.

c. Generalizing.

In the next part, we leave the context out and support students to experience how to solve the problem of a multiplication of two fractions. In this activity the whole unit is not mentioned anymore. Moreover, in a math congress and a mini-lesson in the last lesson, guided by the teacher, students will make a shift from the term “of” to “times” and to the symbol “×”.

## D. Teaching Multiplication of Two Fractions in the Indonesian Curriculum

In Indonesia students start to learn the operations with fractions in grade four. The multiplication of fractions is taught in grade five in the second semester as can be seen in Table 2.1 (BSNP, 2006).

*Table 2.1* Fractions in the second semester of grade 5 in the Indonesian curriculum

| Standard Competence | Basic Competence |
|---|---|
| 5. To use fractions in problem solving | 5.1 Changing fractions to percentages and decimals and vice versa<br><br>5.2 Addition and subtraction of various forms of fractions<br><br>5.3 Multiplication and division of various forms of fractions<br><br>5.4 Using fractions to solve problems involving comparison and scale |

As we mention in the previous part in this chapter that we want to use the informal knowledge of the students to support students in developing their understanding of multiplication of two fractions. Mack (1998) stated that before learning about multiplication of two fractions students should know about equal size part and equal representation of fraction. Therefore, the preliminary knowledge that students should have is about the meaning of fractions in term of part- whole relationship and the equivalence of fractions. As we can see in our curriculum in Indonesia, our students already learned these topics in grade 4 (BSNP, 2006). Based on this explanation, we assume that the students in grade 5 already have the intended preliminary knowledge before they start learning about multiplication of two fractions.

## E. Local Instruction Theory (LIT)

Gravemeijer (2006) stated that a local instruction theory consists of conjectures about a possible learning process along with the possible tools to support that learning process. In specific, he argued about the outline of an LIT which consists of four points as follows (Gravemeijer, 2004; Doorman, 2005).

1. Tools.

   This part is where we put the means which is function as the foundation of the reinvention process that we support students to have (Gravemeijer 2004). Moreover, Doorman (2005) stated in a more simple term that the label "tools" refers to a physical representation.

2. Imagery.

   This part refers to a history or a record about students' prior experience which we assume that they already have.

3. Activity.

   This part contains the description of the opportunity for the students to learn on how to use the mathematical tools in order to establish the meaning.

4. Potential mathematical discourse topic.

   This part contains the potential concept that could be discussed based on several students' strategy on solving the task in the activity.

In supporting students learning, the LIT will become a source for the teacher to use the idea in the LIT by choosing the instructional activities and design the conjecture of students learning for their own students.

# CHAPTER III

# METHODOLOGY

In this chapter we describe the methodology that is used in this study in order to reach the aim and answer the research question. There are three key elements related to this method; research approach, data collection and data analysis.

## A. Research Approach

In general, the aim of this study is to contribute to the development of a local instructional theory in supporting students' understanding of multiplication of a fraction with another fraction. In this study we want to know how a model can support the development of students' understanding in this topic. In order to reach the aim and answer the research question, we make a design of learning sequences equipped with teaching and learning materials.

Based on the explanation above, the suitable approach for this design is a design based research (DBR) since we have an innovative goal that is to improve the teaching and learning of fractions. We use this approach because we have considered the characteristics of DBR. Cobb et al. (2003, in Bakker and van Eerde, in press) identified five characteristics of DBR, of which in this study we pointed out two. First, regarding the aim of the design based research, they stated that DBR not only develops theories about learning, but also the instruments which are designed to support the learning. In addition, Bakker and van Eerde (in press) stated that "DBR typically has an explanatory and advisory aim, namely to give theoretical insight into how particular ways of teaching and learning can be promoted" (p. 4). It means that the focus of this study is to understand the process of teaching and learning grounded by the theory so that it can support students' development. Further, in this study, we combine both the developing of the theoretical framework and also the development of local instruction theories (Gravemeijer and Cobb, 2006).

The second reason for choosing DBR is the characteristic that "DBR has prospective and reflective components that need not to be separated by a teaching experiment" (Cobb, et al., 2003, in Bakker and van Eerde, in press). The prospective component that we make in the form of the conjectures of the learning activities is confronted with the real fact in the learning activities that we observe. The reflective component is when we evaluate and make a reflection after we have collected and analyzed the data. It means that we can make a revision of our prospective components of the next lesson due to the evaluation and reflection of the previous lesson.

Furthermore, in the present study, we follow the three phases of the design based research. They are preparing for the experiment, experimenting in the classroom, and conducting a retrospective analysis (Gravemeijer and Cobb, 2006).

**1. Preparing for the Experiment**

The main goal of the preparing phase of DBR is to formulate a local instructional theory that can be elaborated and refined and also clarify its theoretical intent (Gravemeijer and Cobb, 2006). To reach these goals, in this study first we read and study the literature which is related to the topic of the multiplication of fractions. We do not only look at the journal articles on how other researchers conduct studies of the same topic and their results but also in the textbook to get insight into the topic. Moreover, we also read literature about the realistic mathematics education (RME) approach, and design based research because we use RME as the domain-specific instruction theory and DBR as the methods in this study as we already mentioned in the previous parts.

Furthermore, we design a learning sequence focusing on developing students' understanding of the multiplication of two fractions. We design a learning line and elaborate several activities with specific goals for each activity. In designing, we do a thought experiment in order to make a conjecture of what will happen in the real learning process of the students and then we include conjectures of how the teacher could interact with the students. All of this together is called a hypothetical learning trajectory (HLT). This HLT is used as a guideline for the teacher in the teaching experiment phase and also as the tools for the researcher in the retrospective analysis. During this preparation phase, we also discuss and consult some experts on the HLT to improve the design.

## 2. Experimenting in the Classroom

The main goal of the second phase is to test and improve the conjecture that is already made in the preparation phase and to develop the understanding of how it works (Gravemeijer and Cobb, 2006). In this phase the component that plays an important role is the HLT. We use the HLT as a guideline for both teacher and researcher (Bakker and van Eerde, in press). This means that the researcher uses the HLT in observing the teaching experiment process. Meanwhile, for the teacher, who conducts the lesson, the HLT is a guide for the teaching. Furthermore, in this study, the experimenting in the classroom phase is conducted in two cycles. The first cycle is the preliminary teaching experiment, aimed to see how the design can work and to evaluate and improve it for the next cycle. This pilot is conducted by the researcher as the teacher with a small group consisting of 5 students. The second cycle is the real teaching experiment in the classroom by the regular teacher of the students.

## 3. Retrospective Analysis

The third phase is the retrospective analysis. Gravemeijer and Cobb (2006) stated that the purpose of this phase depends on the theoretical intent of the design based research. Particularly in the present study, our intention is to contribute to the development of a local instruction theory in supporting students’ understanding of multiplication of a fraction with another fraction. The role of the HLT in this phase is to be the guideline for the researcher in determining the focus of the analysis (Bakker and van Eerde, in press). We confront the factual learning that took place in the classroom with our conjectures in the HLT. We analyze and describe not only the factors that indicate a successful learning, but also parts in which the conjectured learning did not take place. Based on this analysis, we can derive the conclusions of the study and answer the research question.

## B. Data Collection

### 1. Preparation Phase

In the present study, the participants are taken from the fifth grade students of SD Al Hikmah Surabaya and their mathematics teacher. We collect the data trough the interview with the teacher and the observation in the classroom. In the interview we collect data about classroom management, classroom norms (both social and socio-mathematical norms), teachers' beliefs and students' achievement and thinking process. We use these data as the base on constructing the HLT.

For the interview and the classroom observation, we make a scheme as a guideline as can be seen in the appendices A and B. For the interview with the teacher we make an audio registration and we make field notes during the interview and the observation as additional data. The data we get from this preparation phase are important in order to give a clear insight in the learning environment of the classroom and its nature. These data are used to prepare and to discuss the experiment with the teacher about the points that we need to agree on and improve.

2. **Preliminary Teaching Experiment (cycle 1)**

The preliminary teaching experiment functions as the pilot study of the design. In this pilot study the researcher plays the role of the teacher. And the study is conducted by the researcher involving a group of five students. The students are taken from class 5C of SD Al-Hikmah Surabaya. There are at least two considerations in choosing the students for the pilot study. First, the students in this pilot study are different from the students in our real experiment (the second teaching experiment). Second, the students have an average level of knowledge so that we expect they can follow the lesson and not dominate the lesson too much. The main purpose of this pilot study is to test the conjecture and also the worksheet that we have already prepared for the lesson. We collect data about students' thinking and also their learning process. All the lessons are recorded in a video registration supported with the students' written work. We have a small discussion with the students about the activities and their thinking after each lesson.

3. **The Teaching Experiment (cycle 2)**

The participants in this teaching experiment are 25 students in class 5C of SD Al-Hikmah Surabaya and their mathematics teacher, Ustadz Anshar. The cycle 2 is the real teaching experiment where we use the revised HLT as the guideline. The mathematics teacher conducts the lessons and the researcher acts as an observer who makes the video registration, the field notes and collect students' written work. From the second teaching experiment the researcher collect data about students' thinking and learning and data about how the design can help students to develop their understanding of multiplication of a fraction with another fraction. Moreover, we also observe the way the teacher implements the design. In practice, we use two video recorders, one for the static camera which focuses on the focus group of the study and the other as the dynamic camera which can move around to record the classroom activities.

**4. Pre-test and Post-test**

- Pre-test

  We conduct the pre-test to get data about the students' initial knowledge of the topic of multiplication of two fractions. The participants are the students in the teaching experiment class. This pre-test is held before the teaching experiment. The items we use in the test are designed to address students' ways or strategies in solving the problems. We include some problems that later will be elaborated in the lesson. We also consider the level of the difficulties of the items, easy, difficult and very difficult problems. The items can be seen in Appendix D and G.

- Post-test

  In order to check students' achievements after the learning process we conduct a post-test at the end of the learning sequence. Based on this result, we can see whether students have learned from the conducted lessons. The items we use in this post-test are similar to the problems in the pre-test. We only modify the context of the problems and the numbers. The problems for the post-test can be seen in appendix E and H.

In addition, the pre-test and post-test are conducted in both cycles. The reason for conducting it in the pilot phase is to ensure whether the items of the problems are understandable or not so that we can revise it for the next cycle. Furthermore, to check the validity of the items in these tests we consult it with several experts from Freudenthal Institute, PMRI team and our supervisors from State University of Surabaya.

**5. Validity and Reliability**

As mentioned before, this study is a design based research which has a specific view about the validity and reliability. In brief, validity refers to whether we measure what we want to measure and reliability refers to the independence of the researchers. To contribute to the validity of the data collection, we utilize data triangulation by collecting data with different methods such as observations and students' written work. Furthermore, to improve the reliability of the data collection, this study uses video registration as the main methods in collecting the data to ensure that it is independent from the researcher.

## C. Data Analysis

### 1. Pre-test

The analysis of the pre-test results is aimed to obtain information about students' current knowledge about the multiplication of two fractions. We analyze it qualitatively by using the goal for each item as the guideline. We zoom into each student's work to see the way students solve the problems and the strategies they use. This information will be used in determining how these results connect to the starting point of the lessons. The result also can be used as the consideration in arranging the small group discussions.

### 2. Preliminary Teaching Experiment (cycle 1)

The main focus in analyzing the data collected in the preliminary teaching experiment is to test the conjectured learning process of the students. We watch all the video registrations and zoom in into the interesting fragments. An interesting fragment is a fragment in which we can see the students' way of thinking in reaching the learning goals, in which students show their understanding of the topic. Moreover, it is also interesting to see and analyze a fragment that shows students struggling and that shows their efforts to find a strategy to solve the problems. We make transcripts of the interesting fragments. Further, we analyze the transcripts by comparing it with the HLT.

Moreover, in this analysis, we watch the video fragment first and then come back later to look in depth for what really happened. In addition, the analysis of students’ written work gives supporting data about students’ thinking and achievements.

The data collected from the first teaching experiment is analyzed trough a retrospective analysis. In this analysis, we compare the conjectured learning process in the HLT with the observed learning process. We can derive a conclusion whether the design of the learning sequence that we made can support the students’ understanding. Moreover, based on the discussions or small interviews with the students we can analyze what their difficulties are in following the lesson and also their comments about the worksheets we used, for example whether they understand the language and the instruction in the worksheet. All of these data are used in revising and improving the initial HLT.

**3. Teaching experiment (cycle 2)**

The goal is to investigate the students' learning process while they are on their way to develop the understanding of multiplication of two fractions. We analyze the whole class and the focus group video and then make a transcript of the interesting fragments of the video. To us, a fragment is not only interesting when we see students show their understanding but also when we see that our HLT is not working well or students do not understand. We look back into the fragments and the transcripts for two or three times, focusing on these interesting parts to see and interpret why this happens and also to interpret students' thinking. We support the analysis by looking at students' written work, including their scratch paper. From this analysis we derive a conclusion and answer the research questions.

**4. Post-test**

We analyzed the result of the post-test in qualitative ways. The purpose of this analysis is to investigate whether students have already reached the learning goals for the lessons. We analyze the way students answer the questions and the strategy they use. The result of this analysis can support us as additional information in drawing conclusions on the teaching experiments and to answer the research question of this study.

**5. Validity and Reliability**

- Validity

  In the analysis we pay attention to the internal and the external validity. In terms of internal validity, we triangulate the analysis of the video registration with the information we have in the field notes, moreover we also triangulate it with the analysis of students' written work. By doing this triangulation we contribute to a valid analysis. Furthermore, we contribute to the external validity which is interpreted as the generalizability of the results by providing this analysis in such a way that others can use it in other experiments.

- Reliability

Regarding the reliability we are concerned with both internal and external reliability. Bakker and Van Eerde (in press) stated that the internal reliability refers to how independent the researcher is toward the data analysis. Moreover, to improve the internal reliability, we discuss with others about the interpretation of the data and in making the conclusions.

Further, in terms of external reliability the research deals with the track ability so that others can follow the description of the analysis properly. To contribute to the external reliability, we document the research properly and describe how the analysis is conducted and how we derive the conclusion.

# CHAPTER IV

# HYPOTHETICAL LEARNING TRAJECTORY

In this chapter, we elaborate a hypothetical learning trajectory of learning multiplication of a fraction with another fraction by focusing on the use of the model to support students to develop their understanding of the topic. Simon (1995) stated that there are three main components in the hypothetical learning trajectory: the learning goal, the learning activities and the hypothetical learning process. In this study we include a brief description about the starting point of the students where we explain about the pre knowledge that we expect students already have before starting the learning activity in this design. Moreover, for the hypothetical learning process, we not only describe the conjecture of students' thinking, but also provide teacher reactions to the conjecture. This teacher reaction will be used as a help for the teacher in the teaching experiment phase and it is elaborated more in the teacher guide for each lesson.

As we already stated that the goal of this study is to support students' understanding of multiplication of two fractions the HLT in this chapter involve five lessons with its specific learning goals as can be seen in Table 4.1. In the table, we also give a brief overview about the description of the activity for each lesson. We use the data we have in the preparation phase as the base on designing this HLT.

*Table 4.1* Overview of the learning design

| Lesson | Learning Goals | Brief description |
|---|---|---|
| 1 | a. Students are able to do partitioning properly.<br>b. Students are able to label the result of the partitioning activity.<br>c. Students are able to get the idea of part-whole relationship. | Students work in a small group. They will determine the position of 6 flags and 4 game post along a hiking trail. And then put the label of each part in a bar.<br><br>Teacher extent the context of the activity 1. Students will determine the distance between the starting line with the first game post. |
| 2 | a. Students are able to indicate the partitioning activity within an array model.<br>b. Students are able to take a part of a part of a whole in a context.<br>c. Students are able to use the array model to solve the taking a part of a part of a whole within a context. | Students will work on 5 problems about sharing chocolate block. The story is presented in three comics.<br><br>The array model is introduced by the teacher in the form of chocolate block.<br><br>Students will try to solve two more problems about sharing chocolate block, but in the different size of chocolate blocks. |
| 3 | a. Students are able to take a part of a part of a whole within a context and without a context.<br>b. Students are able to construct their own array and use it in solving the taking a part of a part of a whole problems.<br>c. Students are able to choose an appropriate dimension of the array in solving the taking a part of a part of a whole problems.<br>d. Students are able to solve the taking a part of a part problems involving unit and non unit fractions. | First students discuss the homework about constructing an array with smaller size as another solution of the chocolate block problem in Lesson 2.<br><br>Students will work in Sharing *martabak telur* Problem, in which they start to construct their own array.<br><br>Students will discuss about an appropriate dimension of the array that can be a help for solving the problems.<br><br>Students will work on four bare |

| Lesson | Learning Goals | Brief description |
|---|---|---|
| | | problems about taking a part of a part in which the whole unit is not explicitly stated in the problems. |
| 4 | a. Students are able to make a shift from the word “of” into the symbol “x” in multiplication of a fraction by another fraction | **Math Congress**<br><br>Students will be reminded about the story and the context about taking a part of a part of a whole from the previous lessons.<br><br>The students will be invited to make a list of the solution of the taking a part of a part of a whole problems.<br><br>The students will recognize about the interpretation of taking a part of a part as a multiplication of two fractions.<br><br>Students will understand the use of an array model in multiplicative reasoning |
| 5 | a. Students are able to choose an appropriate array to help them in solving the multiplication of two fractions problems.<br>b. Students are able to determine the fraction notation of the result of the multiplication of two fractions based on the given array.<br>c. Students are able to determine the problem when the shaded array is given. | **Card Games**<br><br>Students will try to choose an appropriate array for each multiplication of two fractions on the card.<br><br>Students should find the problem for a shaded-array that is given on the card.<br><br>Students should interpret the result in fraction notation |

## A. Lesson 1: Partitioning

### 1. Learning goals

a. Students are able to do partitioning properly.

b. Students are able to label the result of the partitioning activity.

c. Students are able to get the idea of part-whole relationship**.**

### 2. Starting point

Students in fifth grade already learned about producing fractions, addition and subtraction of fractions, and equivalence of fractions.

**3. Materials:** Worksheet 1, ribbons, and markers**.**

### 4. Description of the activity, students' conjectures and teacher's actions

To start with, the teacher introduces a context about a scout club. As this context is familiar with the students the teacher can ask the students do they ever joint a scout activity or not. Let students mention what kind activities of a scout club usually hold. Then, the teacher brings the story about a scout club that will organize a hiking activity at the end of this month. This story is also included in Worksheet 1.

**Activity 1 – Locating the flags and the game posts**

In this activity, students are divided into a group of three or four. There will be a worksheet for every student with the context, the drawing of the hiking trail, and there are problems relate to the context. In practice, the teacher will give the problems one by one to the students and he/ she states that at the end of every problem solved there will be a whole class discussion.

The context and the problem of this activity is the following:

*A scout group plans to have a hiking activity at the end of this month. The length of the hiking trail is 6 km. The committee arranges several games during the out bond in 4 posts which are located at equal distance to each other along the hiking trail. The last post is at the finish line. Moreover, they want to put some flags along the trail as a sign for place to take a rest. They place a flag in each kilometer of the trail, and the last flag is in the finish line. You can see the trail in figure (in page 1), it is indicated by a red arrow.*

**Problem 1**

*You are a member of the committee of the hiking activity and your assignment is to think about how to locate the flags and the game posts. And draw on the picture of the trail the position of each flag and post! (Hints: You can use a ribbon to help you)*

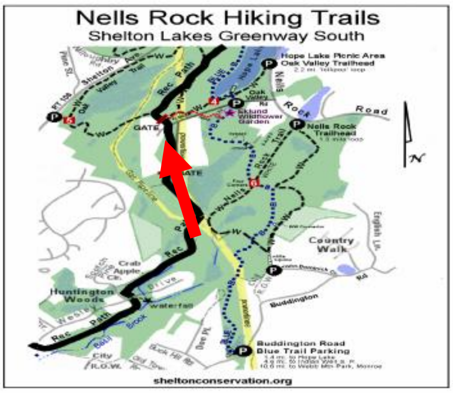



*Figure 4.1* The hiking trail

After students finish, the teacher leads a classroom discussion which focuses on students' strategy of solving the given problem.

**Conjecture of students' strategy**

The strategy that will be used by the students may vary. Sample strategies:

- Students only use their estimation on the figure and mark the position of the flags and the posts.
- Students use the ribbon to get the length of the trail in the figure. Then, they strengthen the ribbon. To find the position of the flags they fold the ribbon two times, it produces four equal parts, and they mark the folding lines. Furthermore, students draw a representation of the ribbon on the paper as can be seen in Figure 4.2.

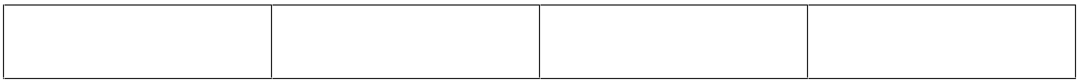


*Figure 4.2* Representation of the folded ribbon into 4 equal parts

After that, they put the ribbon again along the figure of the trail and mark the position of the posts. They also use the ribbon to help them in determining the position of the flags. Because it is not as easy as making the 4 equal parts of the ribbon, the students fold the ribbon randomly and by using the trial and error strategy they will get 6 equal parts of the ribbon as can be seen in Figure 4.3.

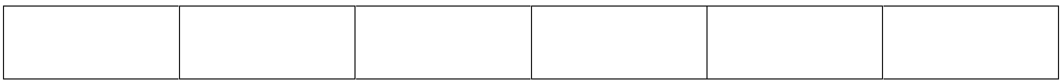


*Figure 4.3* Representation of the folded ribbon into 6 equal parts

- Students use the folded-ribbon to estimate the position of the flags and the game posts by overlapping the ribbon on the trail in the trail figure.

**Teacher reactions and discussion**

When the students only use their estimation, the teacher can give the following questions to be discussed.

- How can you do the estimation?
- Do you satisfy that your estimation is correct?
- What would you do to make it more precise?

Students might think that they need a strategy which assures the partition is in equal size.

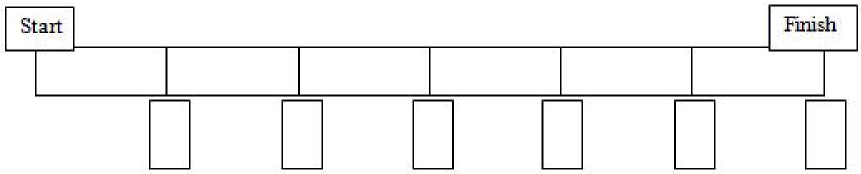
The teacher can encourage students who use the ribbon to share their strategies to the whole class and let the others react to it. Then the teacher can ask, *"Why don't you also try in your group and use the given ribbon to help you!"*

Furthermore, after students finish, the follow up activity involving the partitioning, is labeling the fractions. The aim is to make students understand the result of the partitioning activity. The teacher can use the result of the partitioning in the previous activity. The teacher invites students to think about the representation of the ribbon they fold (Figure 4.2 and 4.3). The teacher can start the discussion by posing questions about *"How many parts you get from the folding?"*, *"Can you think about the label of each position of the flag?"* Then, teacher shares the second problem of worksheet 1.

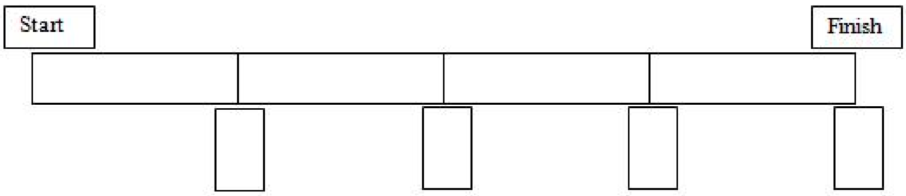
**Activity 2 – Making fractional notation of the result of the partitioning**

**Problem 2**
*Suppose the following bar is the representation of the trail included the flags and the posts you have put. Determine in what part of the trail the location of each of the flags and the post!*

2.a *Representation of the location of the flags*




2.b *Representation of the location of the game posts*




**Conjectures of students' solutions**

- Students just give the notation by using natural numbers which indicate the first flag, the second flag, the third flag and so on. The same strategy for the label of the game posts.
- Students give the notation only by using the unit fractions. They have not come up yet by using non-unit fraction. So they notate each part by $\frac{1}{6}$, because they get six equal parts for the flag's location and $\frac{1}{4}$ for the location of each game posts because they have four equal parts.

- Students use non-unit fractions to notate each part of the partition of the trail. They use $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}$ and so on for the location of the flags and $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ and $\frac{4}{4}$ for labeling the location of the game posts.

**Teacher reactions and discussion**

For the students who only give the notation in natural numbers teacher could ask students to think about the label respect to the whole unit and invites them to determine the label in term of fractions. When the students only use the unit fractions, the teacher can invite them to regard the folding line by asking *"What fraction should be put in the intended line with respect to the whole unit?"* The students also need to know that the size of the parts depends on the size of the whole unit.

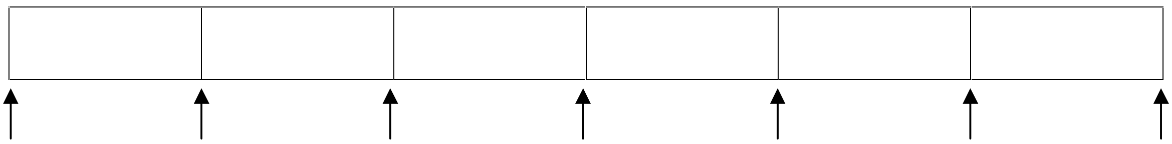


*Figure 4.4* Asking the students about the label of each folding line.

The idea of the first and the second problem in this lesson is to assure that students can do a partitioning and they know what fractions is produced by the partitioning. In the discussion, the teacher focuses on exploring how students do the partition and let them recognize the producing fractions.

Furthermore, the activity is continued by determining the distance between the starting point of the trail and the first game post. This problem is in the problem 3 of worksheet 1 which is given to the students after finishing the discussion of the second problem.

**Activity 3 – Determining the distance between the starting line and the first post**

**Problem 3**

> *In practice for locating the flags and the posts, the committee will ride a motorcycle to measure the distance. In what distance from the starting line should he put the first game post? (The total length of the trail is 6 km).*

**Conjectures of students' strategy**

- The students will determine the length of the middle point by splitting the total length of the trail, 6 km, and then halve it to get the distance of the first post location to the starting point.
- The students try to divide the total length of the trail with four because they know that the location is in the one fourth of the trail.

**Teacher reactions and discussion**

Regarding to the conjectures of students' answer to problem 3, the teacher invites the students to reflect on their strategy. Ask the students to explain and discuss the relation of their answer in problem 3 with the answer to problem 2. For students who use halving strategy, invite them to express their strategy in the fraction notation. The intention is to make the expression $\frac{1}{2}$ of $\frac{1}{2}$ of 6 km appear. Moreover, perhaps there is a student who recognizes that determining the distance of the first post with the starting point is the same by taking one fourth of 6 km. Then, the teacher can lead the students to compare the results. The students will notice that $\frac{1}{2}$ of $\frac{1}{2}$ of 6 km have the same result with $\frac{1}{4}$ of 6 km.

If there are no students come up with this idea, the teacher can ask students about the position of the first post regarding the whole trail. He or she can pose this question, "*Do you remember in what part of the trail is the location of the first post?*" Then, the teacher can show the ribbon representation again to the students and invite them to see the ribbon representation as a bar model which can they use to solve the problem 3. The teacher supports the students to represent their strategy in solving the problem 3 on the bar as can be seen in Figure 4.5.

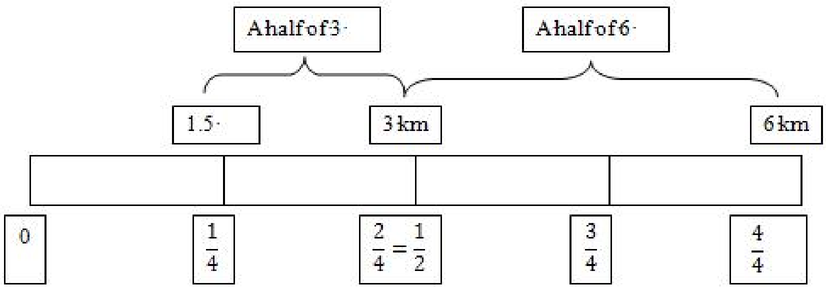



*Figure 4.5* The ribbon represented as a bar model in solving the problem 3

By solving the problem 3, the students will get the notion of taking a part of a whole and also start to take a part of a part of a whole. It will be useful for the next lesson.

## B. Lesson 2: Taking a part of a part of a whole

### 1. Learning goals

a. Students are able to indicate the partitioning activity within an array model.

b. Students are able to take a part of a part of a whole in a context.

c. Students are able to use the array model to solve the taking a part of a part of a whole within a context.

### 2. Starting point

Students already learned about how to do a partitioning properly and give labels to the result of the partitioning activity in fraction notation. They also already introduced to taking a part of a whole activity

### 3. Materials: worksheet 3, grid papers, markers.

### 4. Description of the activity, conjectures of students' thinking, and teacher's reactions.

In this lesson the context still relates to the hiking event. The teacher will provide a story about the Hafidz doing exercise to prepare his health for the next hiking event. The story is presented in three simple comic along with the questions. The Comics is can be seen in worksheet 3.

In this lesson the teacher starts to introduce the array model in the form of chocolate block. The students will informally experience multiplication of two fractions, but still in the taking a part of a part of a whole unit activity. This lesson is also as an extended activity of the "taking a part of a whole" that is already done in the lesson 1. Moreover, there are seven problems in the worksheet. Five of them are related to the story, and the others are bare problems about taking a part of a part of an amount.

Students will work on a small group consist of three or four students. First they will work on problem 1 until 5 of the worksheet 2 followed by a whole class discussion about the students' strategies. After getting the notion of using the chocolate blocks as a help in solving the problems, teacher asks students to solve the next problems and check whether they can use the model properly. The following are the story in the context and also the problems, conjectures of students and teacher reactions.

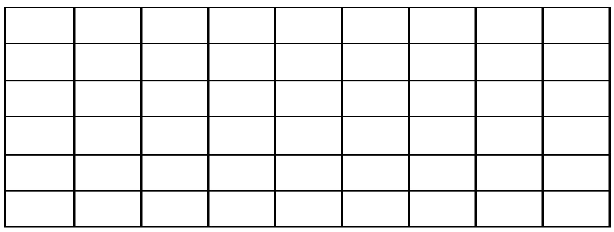
**Activity 1 – Sharing chocolate block**

**The Story:**

*To prepare for the next hiking event, Hafidz plan to have one hour exercises every week. This morning, he tells his father that he will jog with his friends. His father gives a chocolate block and said that Hafidz should share the chocolate to Aufan and his brother Siraj.*

**Problem 1**

*Can you help Hafidz to determine the parts of the chocolate for Aufa, Siraj and Hafidz?*
*Suppose that the following grid is the chocolate block that is given by Hafidz's father. Indicate by shading the part Aufa, Siraj and Hafidz will get!*

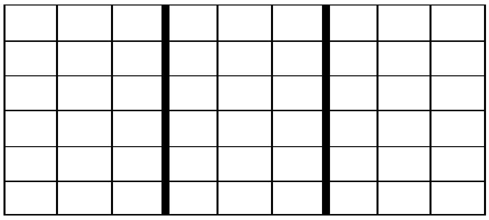


**Problem 2**

> *What part of the chocolate block is for Hafidz? Write your answer in fraction notation!*

**Conjectures of students' strategy of problem 1 and 2**

- The students divide the chocolate block diagonally or randomly.
- Students divide the chocolate block into three equal parts. First, they count the number of columns and divide it by three. And they make a line in every three columns. Each big part is for one person.



- Students might also divide the block horizontally by using the same strategy.
- To determine the part of Hafidz, students only look at the big part without counting the small parts. They get $\frac{1}{3}$ of the chocolate block as the answer.
- Students count the small parts, but not refer the whole unit of the chocolate block. They get 18 as the answer.
- Students count the small parts and relate it to the whole unit. They get a fraction form $\frac{18}{54}$ as the answer.

**Teacher reactions and discussion**

The activity on lesson 3 focuses on how to build students' understanding about the relation on relation, taking a part of a part of a whole. By providing the problems in this lesson teacher could introduce the array model within the context. Moreover, students will get use of representing the situation into the model then use it as a tool for thinking and also to reason in solving the problem.

This lesson readdresses students' ability in doing partitioning activity. In the first lesson the partition is in a bar as the representation of the hiking trail, in this lesson the students do partitioning on a chocolate block. If there is a student divide the chocolate block diagonally or randomly encourage them to reflect whether they produce the equal parts or not. The students should understand that the partitioning activity should produce equal parts.

To determine the parts of Hafidz, when the students just answer in whole number, for example, they answer with 18 parts, the teacher asks the others to react about it. May be there is a student says about answering in fraction notation. Then, lead a discussion about the part-whole relation until the student understand how to produce a fraction in this activity. For example, by counting the total number of small parts in the chocolate block, then we take 18 smaller parts so it means $\frac{18}{54}$.

**Activity 2 – Time for reaching Aufa's house**

**Problem 3**

The next problems are:

> *In his planning, Hafidzh allocates a half of an hour exercise for jogging. He starts to jog from his house and usually he reach Aufa's house after a third of the jogging time. Can you determine how many minutes is that?*

**Conjectures of students' thinking of problem 3**

- Students do the calculation as follows. Time for exercise is 1 hour, it equals to 60 minutes. Time for jogging is a half of the exercise time. They divide it by 2 and get 30 minutes. Then they divide the 30 minutes by 3 and get 10 minutes as the answer.
- Students draw a clock to represent the situation. They focus on the minutes and they know that the whole circle (the clock) is 60 minutes. They shade a half of it and then dived the shaded area into three equal parts. And they can see that one part is equal to 10 minutes.

**Teacher reactions and discussion**

As a starting point to get the knowledge of taking a part of a part of an amount, teacher leads a discussion about the students' answer to problem 3 in worksheet 2. The teacher invites students to reflect to their answer, "*What is the initial time? And what is the different meaning of 30 minutes and the 60 minutes in the context?*" Students may answer that 60 minutes is the total time which is known from the information in problem 3. And the 30 minutes is the time for jogging. When they just divide the 60 minutes by 2 invite them to think about the other expression to say about it. Students should know that the 30 minutes is the result of taking a half of 60 minutes. Further, to get the time for reaching Aufa's house they need to take a third of 30 minutes. So, hopefully students can recognize that they take $\frac{1}{3}$ of $\frac{1}{2}$ of an hour. The result is 10 minutes.

**Activity 3 – Taking a part of a part of a chocolate block**

**Problem 4**

*When Hafidz arrive at Aufa's house he shares the chocolate block with Aufa and Siraj, but suddenly he remembers about her sister Nazifah who also likes chocolate. Then he just thinks to split his part and share it with Nazifah.*

*How about Nazifah's part, can you show it in the following grid? (Hint: Use the result in question 1. Draw line from the Hafidz's share).*

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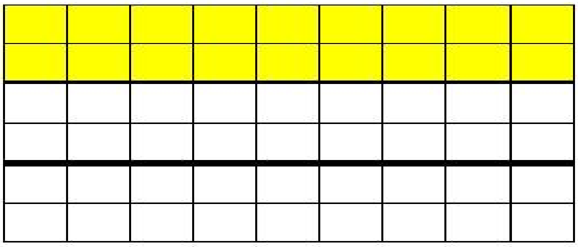
**Problem 5**

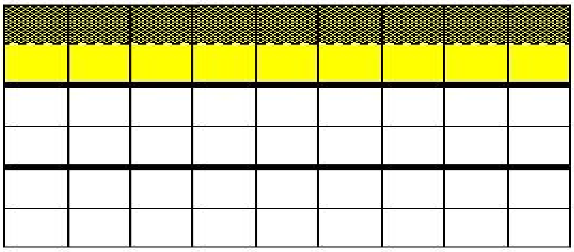
*What fractions of the chocolate block did Nazifah get?*

**Conjecture of students' strategy for problem 4**

- Students use the answer of the first problem as the starting point.



Students divide it horizontally into three parts then they shade the first two rows.



To determine the Nazifah's share, the students divide the shaded area into two parts equally and one part of those is the Nazifah's share.

- Students do the similar strategy, but in a different direction. Instead of dividing the block horizontally, they divide it vertically into three equal parts as the part for Hafidz and then split it up into two parts.

**Conjecture of students' strategy of problem 5**

It means to determine the fraction of Nazifah's share, the students can use the shaded area on the answer to problem 4.

- To determine the fraction of the Nazifah's share students counts the pieces of it and relate to the whole unit. The students conclude that Nazifah will get $\frac{9}{54}$ of the chocolate block.
- Students only look at the rows of the block and they conclude that Nazifah get $\frac{1}{6}$ of the chocolate block.

- Students may have a misunderstanding on determining the fractions. Instead of using the whole as a unit, they just consider the Hafidz parts as a unit. They come up with $\frac{9}{18}$ or $\frac{1}{2}$.of the chocolate block.

Furthermore, the teacher invites students to solve two bare problems in the array that already given by the teacher.

**Teacher reactions and discussion**

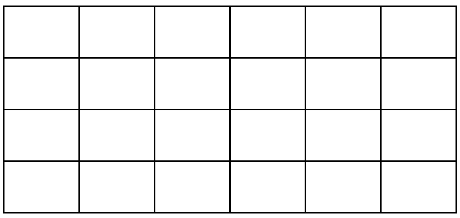
The discussion focuses on the strategy that is used by the students when deal with the given array. The teacher can explore the way students interpret the dividing and the shading activity of the chocolate block.

The teacher needs to overcome the misunderstanding of the students about the "whole" unit. The teacher can pose a question such as *"We get the Nazifah's share of chocolate block as the shaded part in the picture, but if we want to make a fraction of it, we only consider from the Hafidz part or the whole chocolate?"* The following question also can help students realize that the fraction should be something out of the whole unit. *"Based on the question "What fraction of **the block of chocolate** did Nazifah receive?" It means we refer to what?"* By emphasizing on "**the block of chocolate"** students can recognize that the fraction of Nazifah's share is referring to the whole of the chocolate. Moreover, it is also important that students have a clear understanding that the process they did in answering the problem 5 is that they take $\frac{1}{2}$ of $\frac{1}{3}$ of the chocolate block.

Regarding the different form of fractions that arise from students answer such as students with $\frac{9}{54}$ and $\frac{1}{6}$ as the answer to problem 5, teacher can invite students to think about the representation of each fraction on the figure. Perhaps there is a student that remembers about simplifying fraction or fraction equivalency. He or she will recognize that $\frac{9}{54}$ can be simplified become $\frac{1}{6}$. If there is no students come up with this idea, the teacher can invite them to look at the drawing. In the drawing they can see that the shaded area for Nazifah is the same. And the teacher can lead students to conclude that if we refer to the same unit (the chocolate block with the same size), the result will be the same. It means that $\frac{9}{54}$ $of\ the\ chocolate\ block$ is same with $\frac{1}{6}$ $of\ the\ chocolate\ block$ in case the whole chocolate block is the same.

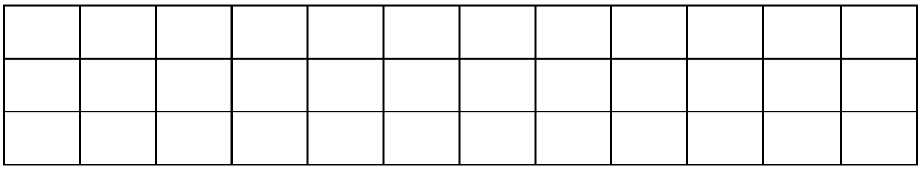
**Activity 4 – Working on problem 6 and 7 on worksheet 2**

*6. a. Determine* $\frac{2}{3}$ *of* $\frac{1}{2}$ *of the following chocolate block!*



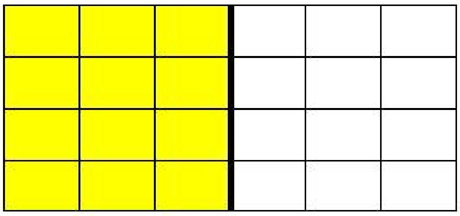
*b. What part of the chocolate block do you get?*

*7. a. Determine $\frac{1}{6}$ of $\frac{2}{3}$ of the following chocolate block!*

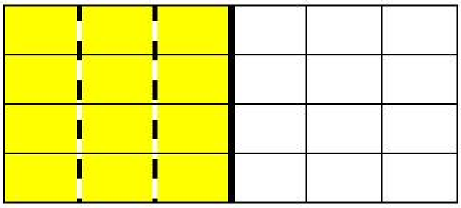


*b. What part do you get?*

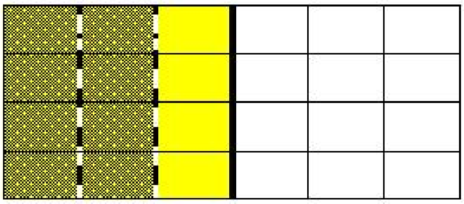
**Conjectures of students' answer to problem 6**



First they divide the block into two parts equally and the yellow shaded part in the left figure indicates the half of the chocolate block.



Then, students divide the yellow part into three equal parts



Shade two parts of the parts from the second step above as the figure beside. The answer of $\frac{2}{3}$ of $\frac{1}{2}$ of the whole chocolate block is the part that shaded twice.

There will be several forms of fractions that will be the answer to this problem. Some students will conclude that the answer is $\frac{8}{24}$, they count the small pieces that shaded twice and respect to the whole chocolate block. Other students will answer with $\frac{4}{12}$, because they count the small pieces in the group of two and respect to the whole block (in a group of two small pieces). Other possible answers that will appear are $\frac{2}{6}$ and $\frac{1}{3}$. There may be still some students that have misinterpretation and come up with $\frac{8}{12}$ or $\frac{2}{3}$ as the result.

**Conjecture of students' strategy of problem 7**

- Students use the same strategy as the answer of number 5. But the way they divide the block may be vary.
- Students will come up with different forms of fractions depend on the way they divide the block and the way they count the small pieces respect to the whole unit. Some possible answers are $\frac{4}{36}$, $\frac{2}{18}$, and $\frac{1}{9}$.
- The misunderstanding that may appear is the same with the problem before, about the part and the whole.

**Teacher reactions and discussion**

In the discussion, the teacher allows students to share their ideas in solving the problems, especially on how they shade the part of the chocolate block and how they interpret the result of the dividing and shading activity of the chocolate blocks. If there is a student who still has doubtfulness of the various fraction notations that come up with the answer, perhaps other students explain that they can reflect to the answer of the previous problems. Furthermore, the teacher can lead a discussion about choosing the simplest fraction notation as the answer. The teacher invites the students to think again about fraction equivalents as they already learned.

At the end of the lesson, teacher provides students with worksheet 3 as homework. The instructions are:

1. *Look back to the comic 1 story in worksheet 3. Can you show the chocolate block share for Aufa, Siraj and Hafidz with your own rectangle with grids inside?. You can try to solve it by constructing your own rectangle with smaller sizes. There will be more than one answer.*
2. *Determine what part you get for Aufa's share based on the drawing that you make! Write your answer in fraction notation!*

The teacher states that the answer of this homework will be discussed in lesson 3.

## C. Lesson 3: Sharing the *Martabak Telur*

### 1. Learning goals

a. Students are able to take a part of a part of a whole within a context and without a context.

b. Students are able to construct their own array and use it in solving the taking a part of a part of a whole problems.

c. Students are able to take a part of a part of a whole of non unit fractions.

### 2. Starting point

Students already learned about how to do a partitioning properly and give labels to the result of the partitioning activity in fraction notation. And they have already introduced to the use of an array model to help them in solving problems about taking a part of a part of a unit.

3. **Materials:** students' answers of worksheet 3, worksheet 4, grid paper, and markers

4. **Description of the activity, conjectures of students' thinking, and teacher reactions**

Teacher asks students to look back on the chocolate block they have in the activity in lesson 2. And then discuss the solution of the homework. The aim of the problem in the homework is to give chance for students to reflect and elaborate more the use of the array model. This activity also allows students to construct their own array in solving the problem. Using students own construction is useful to deepen their understanding.

**Preliminary activity – Discussing the homework**

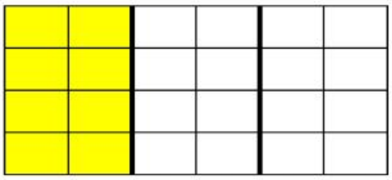
**The problems on worksheet 3 (homework)**

1. *Can you show the chocolate block share for Aufa, Siraj and Hafidz with your own rectangle with grids inside? You can try to solve it by constructing your own rectangle with smaller sizes. Then, indicate by shading the part for Nazifah which is a half of Hafidz's part. There will be more than one possible answer.*
2. *Determine what part you get for Nazifah's share based on the drawing that you make! Write your answer in fraction notation!*

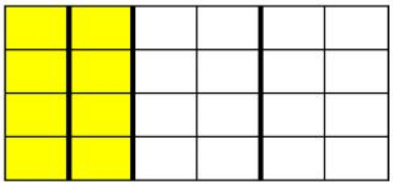
**Conjectures of students' solution of the homework**

- Students draw several sizes of chocolate block and then divide and shaded it as the strategy they already discussed in the activity 3. One of the examples is the following:

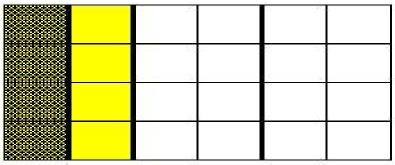
- 6 × 3 chocolate block.



They divide the block into three equal parts vertically. And shade the first three columns as the part for Hafidzh



They divide the shaded area into two equal parts vertically.



They shade the first columns of the yellow area. And that is the part for Nazifah

Students will interpret the result above in the form of fractions. There will be various answers such as $\frac{1}{6}$ , $\frac{4}{24}$, or $\frac{2}{12}$. It depends on how the students relate the shaded area with respect to the whole block.

**Teacher reactions and discussion of the homework**

In the discussion the teacher addresses whether the students really understand the idea of constructing an array model. Due to the different form of fraction they have as the answer to problem 2, the teacher can engage students to think about fraction equivalence which is already learned before they start to learn about fraction multiplication. During the discussion, the teacher support students to conclude that $\frac{4}{24}$, or $\frac{2}{12}$ can be simplified as $\frac{1}{6}$. Therefore, in the reflection, the teacher lets students recognize that the activity they have done is about taking $\frac{1}{2}$ of $\frac{1}{3}$ of a chocolate block with the simplest result is $\frac{1}{6}$ of the chocolate block.

The teacher invites students to compare the result they get to the answer to problem 5 (the same problem) in worksheet 2. The discussion is continued to the relation between the two answers. The students can see that the simplest fraction form of the solution of problem 5 on worksheet 2 which is also about taking $\frac{1}{2}$ of $\frac{1}{3}$ of a chocolate block is the same with the simplest solution of the problem in the homework although the size of the chocolate blocks are different. The teacher emphasizes this knowledge and leads the students realize that they can use the similar strategy in solving the similar problems.

**Activity 1 – Sharing *martabak telur***

After discussing the answer of the homework, the teacher gives the worksheet 4 and asks students to work in pair. There are two parts in the worksheet. Part A is about sharing *martabak telur,* an Indonesian traditional food and for part B is 4 bare problems which aims at addressing the students' ability to construct their own array in solving the problem. The problems are not only about unit fractions, but also non unit fractions. First, students will work in pair on the problems of part A followed by a class discussion after each problem. Next, the teacher asks students to work on part B and followed by a class discussion.

**Problem 1**

*The Hafidz's motherr makes a martabak telur for the desert at lunch. However, Hafidz went home lately after doing the exercise in the morning. They just found $\frac{1}{2}$ of the martabak telur telur in the kitchen. Hafidz eats $\frac{1}{4}$ of the left-martabak telur. What part is that if we compare to the whole martabak telur? (Hint: You can draw a picture to help you in solving this problem).*
*Write your answer in fraction notation!*

**Conjectures of students' strategy of problem 1**

- Students represent the whole *martabak telur* in a rectangle as a starting point. Then they divide it into two and shade one of it as the representation of the *martabak telur* left in the kitchen. Because they know that Hafidz eat $\frac{1}{4}$ of the remaining *martabak telur* so they divide the shaded area into four equal parts and indicate one part of it as the part that is eaten by Hafidz.

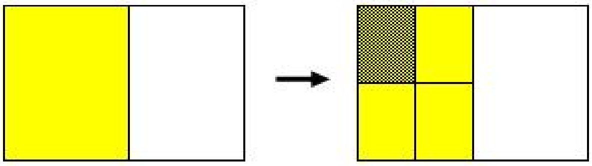


*Figure 4.6* Representation of a whole *martabak telur* in a rectangle

- To get the answer to the problem, students need to compare the Hafidz's part with the whole *martabak telur*. They get $\frac{1}{4}$ of $\frac{1}{2}$ of a whole *martabak telur*. The students may not get the final solution since they struggle on how to determine the fraction notation of Hafidz's part compare to the whole *martabak telur*. Some of the students may be just count the shaded area not the blank one, so they get $\frac{1}{4}$ as the answer, which is not correct.

- Students use the representation of the half of the *martabak telur* in a rectangle as a starting point and divide it into four as can be seen in figure 9. But they do not relate the parts with the whole cake as the unit. They cannot determine the whole unit and as an effect they cannot come up with a fraction notation for the Hafidz's part respect to the whole *martabak telur*.

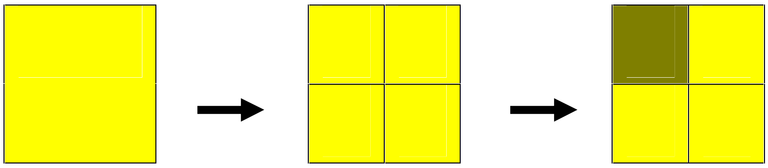


*Figure 4.7* Representation of a half of *martabak telur* as a starting point

**Teacher reactions and discussion**

If the conjecture of students who start with the representation of the whole *martabak telur* but they cannot determine the fraction notation of Hafidz's part happens, the teacher can invite the students to think about the Hafidz's part respect to the whole *martabak telur*. Teacher can ask the student that *"Can you think about how many times the small part (the Hafidz's part) fit into the whole martabak telur representation?"* Then, they can draw a dot line to help them as can be seen in Figure 4.8.

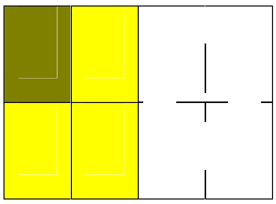


*Figure 4.8* Making dot lines inside the rectangle

Students may answer that the fraction notation of the Hafidz's part is $\frac{1}{7}$ which is not correct. The teacher can suggest students to think again by posing the question such as *"If we only arrange 7 small parts of the martabak telur then will they form a rectangle?"* Since the whole *martabak telur* is represented in a rectangle so there is one more small part needed. Therefore, the Hafidz's part must fit 8 times in the rectangle and one small part is equal to $\frac{1}{8}$ of the whole *martabak telur*.

In addition, if the second conjecture of students' answer to problem 1 happens, the teacher can ask the students about how to draw the whole *martabak telur* if we have a half of it. May be students will realize that they need a half more to complete the rectangle as the representation of the whole *martabak telur*. Furthermore, the discussion can be continued to determine the fraction notation for Hafidz's part which is already explained in teacher reaction of the first conjecture.

**Activity 2 – Choosing an appropriate array**

After finishing the class discussion about the first problem, the teacher asks the students to work on the second problem.

**Problem 2**

*Three students try to solve the problem 1 by drawing a rectangle on grid paper. As you can see in the following:*

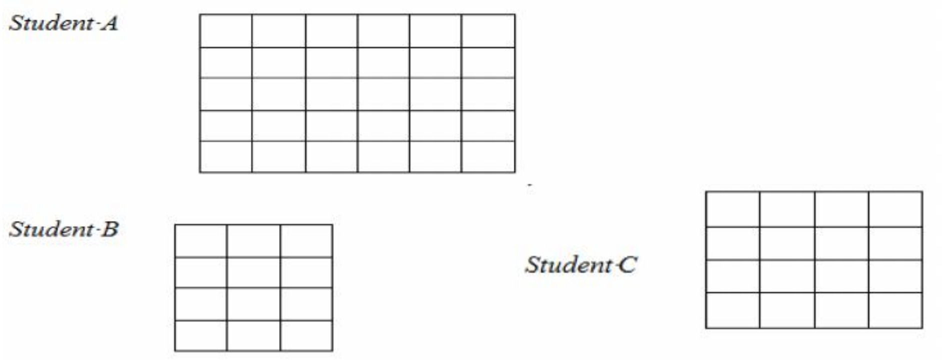



*Figure 4.9* The rectangle on a grid paper for solving the sharing *martabak telur*

*Which drawing do you prefer to solve the problem? Explain your answer!*

**Conjectures of students' strategy of problem 2**

- Students will try to apply the divide and the shade strategy for all samples. Perhaps, they will notice that for students A and B they cannot divide the array properly. The only one that might be as an easy help to solve the problem is the array of students C. It will come up with $\frac{1}{8}$ or $\frac{2}{16}$ as the result.

**Teacher reactions and discussion**

The problem 2 in worksheet 4 will lead students to think about the dimension of the array. We hope that they will recognize the idea of using an appropriate array size, which is depends on the number used in the problems. Moreover, the teacher also can invite students to compare the answer to this problem (problem 2) with the solution that has been discussed in problem 1. If they get confused because of the different drawing and the different fraction form, then the teacher can bring the idea of fraction equivalency to the students.

It also can be used to strengthen students' understanding that in taking a part of a part of a whole, we need to consider the result respects to the initial whole unit.

**Activity 3 – The exercise**

The next activity is working in pair on part B of worksheet 4.

**Problems in part B**

1. *Determine* $\frac{1}{4}$ *of* $\frac{1}{3}$ *!*
2. *Determine* $\frac{1}{4}$ *of* $\frac{2}{3}$ *!*
3. *Determine* $\frac{3}{4}$ *of* $\frac{1}{3}$ *!*
4. *Determine* $\frac{3}{4}$ *of* $\frac{2}{3}$ *!*

Students will work in pairs to solve these bare problems. The teacher states to the students that they can draw a rectangle on a grid paper for each problem to help them in finding the solution of the problems. These four problems are in the next level of understanding of the students where the whole unit is not explicitly stated.

**Conjectures of students' answers of part B**

- It is conjectured that maybe there is a student who will get confused because he or she cannot see what the whole unit in this problem is.
- For problem 1, the students will draw a rectangle on a grid paper with the dimension 3x4. They choose this size because they look at the denominator

of the fractions in the problems. Then they try to shade the part as they have done in part A of worksheet 4. They will come up with $\frac{1}{12}$ as the answer.

- The students use the similar strategy for the next problems. The students may struggle when deal with non unit fractions.

**Teacher reactions and discussion of part B**

When the students get confused of the problems because they are different with the previous problems, the teacher can invite them to think about what is the quantity in the problem. For example, in problem 1, the quantity is $\frac{1}{3}$ and it means that there is one third of a whole unit. The whole unit can be modeled with a rectangle, so first we need to divide the rectangle into three and take or shade 1 part of it. Further, the teacher lets students to revisit what they have done with the problems in part A; in this case they will divide the one third part into four and shade one of it.

Furthermore, when dealing with non unit fractions maybe there is a student that can solve it properly, then the teacher asks him or her to share their ideas. If it is not, the teacher may start the discussion on how to represent a $\frac{2}{3}$ in a rectangle, and if we want to take a $\frac{3}{4}$ of that $\frac{2}{3}$ we need to divide the $\frac{2}{3}$ part into four, and then take 3 parts of it.

In addition, to interpret the result of the drawing into a fraction notation, perhaps there are students who can come up with the fraction notation they relate the intended parts respect to the total number or small parts in the rectangle. If it is not, then the teacher invites them to reflect again on how to relate the part with the whole.

## D. Lesson 4: Math Congress

### 1. Learning goals

Students are able to make a shift from the word “of” into the symbol “x” in multiplication of a fraction with another fraction

### 2. Starting point

Students the use of an array model to help them in solving problems about taking a part of a part of a unit. They also already learned about how to take a part of a part of a whole and construct their own array to solve the problems

### 3. Materials: students’ work on the previous worksheet.

### 4. Description of the activity, conjectures of students’ thinking and teacher reactions

To start the math congress, the teacher remind the students about the context they already learned in the previous lessons, for example about the time that is used by Hafidz to reach Aufa’s house, the sharing chocolate block, sharing a *martabak telur* and the bare problems they already solved in the worksheet 3. Let the students look back at their solution of the taking a part of a part of a whole problems that they already solved in the previous lessons. The teacher gives instruction to the students to share the result of those problems in a complete sentence in the class discussion.

**Conjectures of students' thinking**

It is conjectured that the students will share the solution of the previous problems about the taking a part of a part problems as follows

$\frac{1}{3}$ of $\frac{1}{2}$ of 60 minutes = 10 minutes

$\frac{1}{6}$ of 60 minutes = 10 minutes

$\frac{1}{2}$ of $\frac{1}{3}$ of a chocolate block = $\frac{1}{6}$ of a chocolate block

$\frac{2}{3}$ of $\frac{1}{2}$ of a chocolate block = $\frac{2}{6}$ of a chocolate block

$\frac{1}{4}$ of $\frac{1}{2}$ of a *martabak telur* = $\frac{1}{8}$ of a *martabak telur*

$\frac{3}{4}$ of $\frac{2}{3}$ = $\frac{6}{12}$ = $\frac{1}{2}$

$\frac{2}{5}$ of $\frac{2}{3}$ = $\mathbf{\frac{4}{15}}$

**Teacher reactions and discussion**

In the discussion the teacher invites students to look at the relationship between the fractions in the list. The teacher will give the students time to think individually. Further, the teacher invites students to share their ideas.

We expect that there will be a student recognize about the relationship between the numerators of the fractions and also between the denominators of the fractions. The teacher will elaborate it until the students notice about the multiplication of two fractions. To lead students into that idea the teacher could make the new list from the previous one. The teacher invites the students to leave the whole unit of each part of the previous list as follows.

$\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

$\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{6}$

And so on.

We conjecture that the students will see clearly that the result of taking a part of a part can be determined by multiplying the numerator of the first fraction with the numerator of the second fraction over the multiplication of the denominator of the first fraction and the denominator of the second fraction in the problem.

Therefore, the teacher can ask the students to write the final result of the discussion in the whiteboard.

$\frac{1}{3}$ of $\frac{1}{2} \quad = \quad \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

$\frac{2}{3}$ of $\frac{1}{2} \quad = \quad \frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

And soon.

Trough the class discussion the teacher invites all of the students to closely see this idea and asks them whether they understand about it or not. When the students still struggle to grasp this idea the teacher can support them by investigating an example of the list. State that the students could think how they got the result of determining a part of a part or in another word determining the result of a fraction of a fraction. Hopefully, there will be one of the students realize that they could use a multiplication between the numerators and the multiplication of the denominators.

## E. Lesson 5: Card Games

### 1. Learning goals

a. Students are able to choose an appropriate array to help them in solving the multiplication of two fractions problems.

b. Students are able to determine the fraction notation of the result of the multiplication of two fractions based on the given array.

c. Students are able to determine the problem when the shaded array is given.

### 2. Starting point

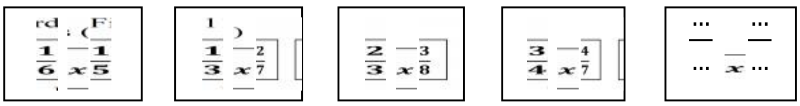
Students already learned about how to take a part of a part of a whole and construct their own array to solve the problems. They have also already made a shift from term “of” into term “times” in the taking a part of a part of a whole. The students already know the use of symbol “x” in multiplication of two fractions.

**3. Material:** Cards with problem, cards with the array, and cards for the result in fractional notation.

### 4. Description of the activity, students’ conjectures and teacher’s actions

After students have been introduced to the array model and try to construct their own array, in this lesson it is expected that the students get use of using array in solving the problems. This lesson is formatted in a card game. There will be three groups of cards: the problem cards (P cards), the array cards (A cards), and the solution cards (S cards). In each of the problem cards, there is a multiplication of two fractions problem, meanwhile in each of the array cards, there is an array that is correspondence with the problem. The S cards are blank where the students write the solution on it.

There are five P cards that should be solved by the students and one of them is a blank card (Figure 4.10). There are also five array cards, four of the arrays are correspondence with problem $P_1$ until $P_4$ but they are in different order *a*, *b*, *c* and *d*. The last array card (card *e*) contains an array that is already shaded as can be seen in Figure 14. This card is correspondence to the $P_5$. Moreover to write the final solution for each problem, there will be five solution cards (Figure 4.11)



$P_1$ $P_2$ $P_3$ $P_4$ $P_5$

*Figure 4.10* The Problem cards

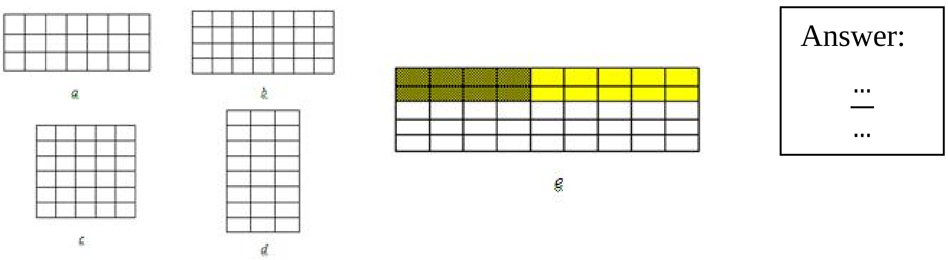



*Figure 4.11* The array cards (a-e) and an example of the S cards

**Activity 1 – Working on the first 4 sets of the cards**

The instruction of the game is the following:

a. Work in pairs
b. Find the appropriate array for the problem in P cards.
c. Indicate in the array by shading the solution of the problem.
d. Write the solution of the problem in a fraction notation with the S card.
e. The shaded- array in card *e* is corresponding to the $P_5$ card. Determine the problem which is represented by the shaded- array. (Hint: the dark yellow parts indicate the parts which are shaded twice).
f. Write the solution of the problem you get for $P_5$ card in the S card.

**Conjecture of students' strategy**

- To find the appropriate array for the problem in the first four problem cards, the students may only do a trial and error strategy.
- Some students may consider the denominator of the problem to determine the appropriate array dimension.
- The students indicate the solution in the array by shading the parts depends on the problem.
- To make the fraction notation of the solution in the array, the students will count the shaded parts and respect it to the total number of small parts in the array.

- For the last problem ($P_5$ and card *e*), the students will get confused because they don't know to take what part of what part of the array. Some of the students may answer with $\frac{8}{45}\ of\ \frac{10}{45}$ or $\frac{8}{18}\ of\ \frac{18}{45}$ which is not correct.
- For the last problem, the students answer with $\frac{4}{9}\ of\ \frac{2}{5}$ because they consider the dimension of the array and also the information in the instruction of the game about the parts that is shaded twice.

**Teacher reactions and discussion**

In the discussion, the teacher encourages the students who have consideration of choosing the array to share their idea instead of just trial and error. The students may explain that they just look at the pattern of the answer in the previous lesson. Perhaps, there is a student that recognized that when we want to find $\frac{1}{6}\ of\ \frac{1}{5}$ we need to divide a rectangle vertically into five and then divide it horizontally into six or vise versa. It produces an array with 5x6 as its dimension. They can continue to discuss about the way they shade the array to indicate the solution and then how to interpret the solution into a fraction notation.

**Activity 2 – Working on the last set of the cards**

Furthermore, for the last problem, the teacher invites the students with non correct answers to explain their strategy, why he or she can come up with that solution. Then, the students with the correct answer will react to this. They will remind the other about the given hints that the dark shaded parts are the part which are shaded twice. It means that first we shade the first two rows (the yellow parts). There are five rows in that rectangle so it means $\frac{2}{5}$. Next, the total number of columns is 9 and there are 4 columns that are shaded overlap with the yellow parts. The teacher asks *"What does it mean? What fraction is that?"* the intention of the question is to lead students to recognize that it means they take four parts (columns) over the nine parts (columns), $\frac{4}{9}$. Finally, the students will recognize that they have $\frac{2}{5}$ of the array and we take $\frac{4}{9}$ of it. The result is the parts that are shaded twice.

# CHAPTER V

# RETROSPECTIVE ANALYSIS

This chapter provides the data analysis of the two cycles of our study. At the beginning of the analysis, we describe some remarks we got from the classroom observation and the teacher interview, which we conducted before we started the first cycle.

The participants for the both cycles were taken from class 5C of SD Al Hikmah Surabaya. The reason of why we only took students from one class was based on the recommendation from the school. The vice principal of the school only gave one class of the three classes provided. It was because the topic that would be focused on this study would be being thought soon at that time. The vice principal said that it would be better if the other class still follow the school program and the teaching of multiplication of two fractions topic in class 5C was postponed until the researcher ready to conduct the lesson (the cycle 2) in that class.

For the first cycle we took 5 students from class 5C with the recommendation of the teacher. For the second cycle the participants were all of the students in Class 5C excluded the 5 students of the cycle 1 with the home run teacher as the teacher. The consideration to choose the five students for the cycle 1 was based on the characteristic which represented the characteristic of the students in the class (the class for the cycle 2).

We analyze the result of the two cycles grounded by the result of the HLT that we were tested in the teaching experiment phase. The main focus of the analysis is the learning process of the students, the mathematical ideas and students' development of understanding of the multiplication of two fractions.

## A. The Research Timeline

*Table 5.1* The research timeline.

| **Dates** | **Activities** | **Participants** |
|---|---|---|
| | Preparation and designing the initial HLT of the multiplication of two fractions. | Researcher and the Dutch supervisor. |
| | Supervising the design, preparing the learning materials and teacher support in Bahasa Indonesia | Researcher and the Indonesian supervisor |
| 13 February 2014 | Observing the classroom. | The students and the mathematics teacher of Class 5 C SD Al Hikmah Surabaya who would become the classroom and the teacher of cycle 2) |
| 13 February 2014 | Interviewing the teacher | The mathematics teacher of Class 5C SD Al Hikmah. |
| **Cycle 1** | | |
| 17 February 2014 | Pre-test before cycle 1 | 5 students in preliminary teaching experiment (cycle 1). The students are from SD AL-Hikmah Surabaya. |
| | | |
| 17 February 2014 | Lesson 1: The Hiking Trail | |
| 18 and 19 February 2014 | Lesson 2:Sharing the Chocolate Block | |
| 19 February 2014 | Lesson 3:Sharing The *Martabak telur* | |

| Dates | Activities | Participants |
|---|---|---|
| 20 February 2014 | Lesson 4: Math Congress | 5 students in preliminary teaching experiment (cycle 1). The students are from SD AL-Hikmah Surabaya. |
| 21 February 2014 | Lesson 5: Card Games | |
| 25 February 2014 | Post-test after cycle 1 | |
| **Cycle 2** | | |
| 21 February 2014 | Pre-test before cycle 2 | The students of the teaching experiment classroom (cycle 2) from SD AL-Hikmah Surabaya. |
| 24 February 2014 | Lesson 1: The Hiking Trail | The students and the teacher of the teaching experiment (cycle 2) from SD Al-Hikmah Surabaya and the focus group of the teaching experiment (cycle 2) from SD Al-Hikmah Surabaya. |
| 25 February 2014 | Lesson 2: Sharing the Chocolate Block | |
| 26 February 2014 | Lesson 3: Sharing The *Martabak telur* | |
| 27 February 2014 | Lesson 4: Math Congress and Card Games | |
| 5 March 2014 | Post-test after cycle 2 | |
| 7 March 2014 | Small interview for post-test clarification. | The focus group of the students in cycle 2 from SD Al-Hikmah Surabaya. |

## B. The Information We Got from the Observation and the Interview with the Teacher (the Classroom and the Teacher of the Cycle 2)

To collect the data in the classroom observation, we took a video and also make some notes meanwhile in the interview with the teacher we make a record and notes. As a guide in the observation and the interview, we use some items in Appendix A and B.

The teacher is Ustadz Anshar, he has 2.5 years experience as a mathematics teacher in junior high school and then he continued to work in the elementary school for around 3 years until now. He graduated from ITS (Institute of

Technology of *Sepuluh November*) and got the skill for educational teaching by following workshop, training and sharing with his college.

After conducting the classroom observation and the interview with the teacher we concluded the following five points. Firstly, in conducting the lesson, the teacher usually uses students centered approach. He usually gives a chance for students to think and try to find the knowledge through exploration. The teacher also gets used by asking students to share their strategy in front of the class and let the other react to it in a discussion. However, the teacher stated that he rarely used small group discussion. As an effect, based on our observation, the students only tend to solve the problem individually.

Secondly, we pointed out about the social and mathematical norms in the classroom. In the class discussion if someone talks in front of the class then the others should pay attention to it. There is time allocated for the other students to react to others statement and they can defend their own and the role of the teacher is as a facilitator. In addition, the teacher engages the students in the learning activity by giving and reducing point for each student and at the end, the students with the highest rank will get reward from the teacher.

Moreover, the teacher and the students quite get used of socio-mathematical norms in the classroom. For example, the teacher allows students to use various strategies to solve a mathematics problem as long as they can explain and reason about it. The correctness of the answers are not told by the teacher, but come up during the discussion. The teacher gives direct instruction if the students hardly found the right strategy or answer for a problem.

Thirdly, about PMRI, the teacher said that he already tried to use the PMRI approach in mathematics teaching and learning. However, he partly understands about the RME, in his opinion, it is more like students centered approach. In the implementation, based on our observation the teacher often uses the example that is close to students' life, for example, when thought about percentage the teacher show the students the loading bar on the screen, asked the students do they ever see things like that in the computer games or when uploading the phone cell battery.

Fourthly, about the character of the students in class 5C, the teacher said that the students have an active and dynamic characteristic. And the level knowledge of the students is in the middle and upper level. This point also supported by the data we collected based on the discussion with the vice principle of the school, she said that the classification of the students in SD Al Hikmah is based on the multiple intelligence test and for the students in class 5C, they have "*cerdas matematika* (mathematical intelligence)".

Lastly, the integration of the religious value in the teaching process, for examples, sometimes before answering the question, the students should recite a verse of the holy Quran which already an agreement together. Although it is not related to the mathematics, we have seen that almost all of the students' enthusiasms to do this activity. It implies that they are willing to learn mathematics in the classroom and get points because they allocate their time outside the school to memorize the Quran as the key point before answering a question in the classroom.

**Some adjustments to the design of this study**

Considering the findings in the classroom observation and the result of the interview with the teacher we make some adjustments about the design in our study as follows:

1. Ask students to think of their own first, then discuss it with his peers or his small group before bring it into the class discussion. It will help students to share their ideas and difficulty with their students. It also will make sure that all students get the opportunity to join the discussion and talk even not all will be brought to the whole class discussion.
2. The teacher and the students already have socio and socio-mathematical norms, then the teacher have to strengthen it so that it can support the implementation of the design.
3. The students quite competitive and the use of points to motivate students will increase their willingness in joining the activity in our design.
4. Since the students in this class have the characteristic of "*cerdas matematika*", then we find that our design suitable to be implemented in this class.

## C. The Result of the Cycle 1

In the first cycle we tested the five lessons we have designed. We conducted a pre-test before starting the cycle 1 and post-test after we conducted the cycle 1.

The participants in this cycle were five students from Class 5C SD Al-Hikmah Surabaya, they were in the middle of learning about Fractions topic. The students are Adrian, Abdul, Arfan, Izmi and Celvin (not the real names). The researcher took a role as a teacher in this cycle.

In the following we describe the result and the analysis of cycle 1 start with the pre-test analysis, analysis of cycle 1 results, analysis of post-test, the conclusion and the remarks about the learning design and the learning materials.

**1. The pre Knowledge of the Students in the Cycle 1 on Multiplication of Two Fractions**

The aim of conducting the pre-test was to collect data about students' initial knowledge about multiplication of two fractions. Moreover, we also wanted to test about the correctness of the items, especially about the language we used in the items. There were four problems where some of them have two or three sub questions (see appendix D). The following are the analysis of students' written works for each item.

In problem 1, the students were given a rectangle as the representation of a *mango tart*. They were asked to show in the figure if they take $\frac{1}{2}$ of $\frac{1}{3}$ of that cake. Two out of five students can indicate $\frac{1}{2}$ of $\frac{1}{3}$ by shading a rectangle correctly. One of them makes a drawing as can we call an array model. The other three students made a wrong step in taking a half of a third in the rectangle.

Based on students' solutions, it implies that students are familiar with drawing and shading a rectangle when deal with fractions, although some of them still struggling on how doing it properly.

In problem 2 the students were given a story as follows,

*The total number of students in grade 5 SD Tanah Air is 40. A half of them are male students and a quarter of the male students like playing football.*

- *How many male students like playing football?*
- *How many parts of the male students like playing football respect to the total number of students in grade 5 SD Tanah Air? Write your answer in fraction notation.*

Two students solved it correctly. Abdul determined the number of male students by taking $\frac{1}{2}$ of the number of the total students that is 20 students. Then, to determine the quarter of the male students he did a multiplication, $\frac{1}{4} \times 20 = 5$ students. To determine the part of the students who like playing football respect to the total number of students, Abdul just compares the number of male students over the total number of students. He wrote $\frac{5}{40} = \frac{1}{8}$.

Celvin determined the number of male students who like playing football by doing two division processes. First he divided 40 by 2 and got 20. Then, he divided 20 by 4 to get 5 as the answer. To answer the second part of the question, he drew an array. The way he drew is first he drew 4 small squares vertically and then drew small squares and counted it 1 till 10 horizontally. Furthermore, he completed the row and the column in the rectangle following the pattern he made. Lastly, he shaded five small squares which indicate the male students who like playing football. For the fractional notation he wrote $\frac{5}{40}$.

The other students could not answer the problem properly. They did not use fraction notation yet in solving the problem 2 a. And for problem 2 b they know about how to relate the part to the whole unit. However, as they have wrong solutions to the problem 2 a then the result of the 2b are also wrong.

Based on the students' answer to problem 2, it implies that only one student already knew how to multiply a fraction with a whole number. He used this strategy as a step in solving the problem. Moreover, most of the students, partly know about how to relate *part* and a *whole unit.* Although, it's still not correct since they did not determine a correct solution for the *part*.

In problem 3, when the array was given in the form of chocolate block and the instruction for problem 3a was to indicate by shading the part for Roni and Ridho if they share the chocolate block equally. All students can show the part of Ridho and Roni correctly because it is a bit easy for them since they only need to split the chocolate block into two.

The next question (3b) was to indicate by shading if Roni shares a third of his part with his sister Rosi. The same drawing of the representation of the chocolate block is also given in this problem. There are various ways of the students in solving the problem. But, in general, there are two kinds of interpretation of the given drawing in the 3b.

- The students interpreted that the chocolate block in problem 3b as the Roni's part as the result of the answer of 3a. Then, they try to indicate a third of it as the part for Rosi. Two students can show it correctly, they are Arfan and Adrian. But, Celvin instead of taking a third of the Roni's part, he shade 3 columns of the six columns in the drawing on 3b. It implies that Celvin cannot show a third in that array.
- The students interpreted that the chocolate block in problem 3b as the initial chocolate block. Abdul answer this problem correctly, he can show that the part of Rosi is only one column of the three columns of the Roni's part. And it means only one column over the total six columns of the chocolate block. The other student, Izmi, only divided the initial chocolate block by 3, each of it for Roni, Ridho and Rosi. This answer is not correct because he was not following the instruction in the story of the problem 3b.

Based on the explanation above, we can conclude that most of the students understand how to make a partition into equal size part. However, only two of them can show in the figure (the given array) if they take a part of a part of a whole unit. In addition, regarding the language used in the instructions, this result shows that students could have miss interpretation of the problem 3b. Some students think that the given drawing in this problem as the part of Roni not the initial chocolate block. Therefore, the language for this item should be revised.

For problem 4, when the students are asked to determine $\frac{1}{3} \times \frac{1}{2}$, only one student could give a correct solution, Abdul. However, the way he solved the problem was still influenced by the procedure in addition and subtraction of fractions. Abdul found the least common multiple (LCM) of the denominators first and it changed the numerators.

The other four students could not solve the problem properly. Some of them tried to find the LCM of the denominators, then multiplied the numerator and they kept the denominator the same as the LCM of the denominators.

Based on the description, we conclude the following points regarding to the students' pre-knowledge of multiplication of two fractions.

*Table 5.2* Overview of the students' pre-knowledge of multiplication of two fractions

| **The students can** | **The students cannot** |
|---|---|
| The students already get used in drawing and shading a rectangle when deal with fractions. | Some of them still struggling on how to represent a fraction in a figure or to interpret the rectangle they have drawn and shaded. |
| Some students already know how to relate part and the whole unit. | Most of the students cannot interpret the process of taking a part of a whole as the multiplication of a fraction with a whole number. |
| The students understand that in partitioning they should get equal size parts. | Most of the students cannot show the part of a part of a whole unit correctly. |

| **The students can** | **The students cannot** |
|---|---|
| Only one student can multiply a fraction with another fraction in an abstract way. However, he still influenced by the procedure in solving the addition of fraction. | Most students cannot solve multiplication of a fraction with another fraction. |

Based on the result and the analysis above, we conclude these points regarding the pre-test items.

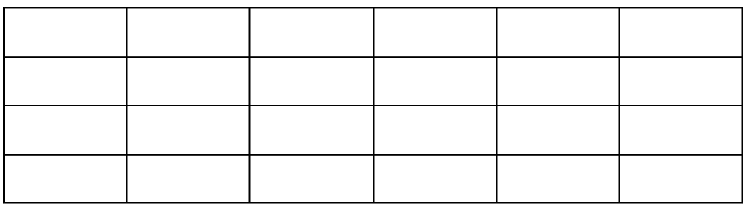
a. Problem 1, 2 and 4 are appropriate to be used in the pre-test of the second cycle.

b. We revised the problem 3 b because the language and the information is not good enough. It makes students miss understood about the representation of the chocolate block in the drawing.

**The revisions of the pre-test problems are:**

Problem 3b.

*However, Roni shares a third of his parts with his sister Rosi. Indicate the part of Rosi in the same chocolate block figure as the drawing below!*



c. We add one problem to number 3. The intension is to extend the difficulty of this test and to see whether the students know how to interpret the result of shading the array into the fractional notation.

Problem 3c

*In what part of the chocolate block will Rosi get if it respects to the whole chocolate block? Write your answer in a fractional notation!*

**2. Data and Analysis of the Students' Learning process in Cycle 1**

In this part we provide the retrospective analysis by confronting the initial HLT with the actual learning process of the students for each lesson. The initial HLT for this cycle refers to our description of the detail conjecture of students' strategy in the chapter 4.

**a. Lesson 1- Partitioning**

In this lesson, the students were asked to work in small groups. Celvin and Adrian in group 1 and Abdul, Arfan and Izmi in group 2. The context in this lesson is about a hiking trail. The context and the story about hiking activity in this lesson are familiar with the students. Based on this context, these students would work on three activities. The activities started with locating 6 flags and 4 game posts in equal distance along a hiking trail. This activity supports students to experience the partitioning activity by producing equal size parts. Furthermore, the students should notate the result of the partitioning by using fractional notation which would lead them into the notion of part-whole relationship. At last, in order to introduce students into the notion of taking a part of a whole, the students should determine the distance between the first game post and the starting line. The following are the analysis of students' activity in solving the problems.

**Activity 1- Locating the flags and the game posts**

The students seemingly understood that their first task was to locate the position of six flags and four game post along the given hiking trail figure in worksheet 1. The requirement is that the flags have equal distance to each other and so do the game posts. As an addition to the information on the problem, there is neither flag, nor game post in the starting line. As the tool in this activity, there were two pieces of ribbons for each student. The only hint that was given to the students was that they could use the ribbons as a mean to help.

The goal of this problem is to make the students recognize about the partitioning activity and the equal size parts as the result of the partition. We provided the students with the worksheet 1 and also two pieces of ribbons for each student. In our expectation in the HLT, the students would use the ribbon to help them to produce equal size parts by folding strategy and to mark the position of the flags and the game posts in the figure. Then, the students would overlap the ribbon to the trail figure.

In the teaching experiment we can see that at first, the students struggled on how to solve the problem. When the ribbons were given to the students, they look confused about the use of it. All of the students tried to overlap the trail figure with the ribbon. There were various students' strategies. Abdul and Izmi tried to represent the trail by combining the two ribbons so it looks like the same with the figure of the trail. Celvin, Adrian and Arfan tried to use only one ribbon first. However the students could not came up to the right solution. They kept exploring without getting into the idea of folding the ribbon.

*Figure 5.1* Students work in problem 1 Worksheet 1

After a while, to overcome the difficulty of the students the researcher gave hint that the students could use one ribbon for helping in determining the location of the flags and another one for helping in locating the game posts. In addition, the teacher reminded the students that each post has equal distance to the next one. Further, the teacher asked about how making the equal distance.

One student came up with the idea of folding the ribbon. Then, the other students tried to use the same strategy. The following is the transcript of the discussion about making four equal parts by using a ribbon.

**Transcript 1**

Adrian : So the ribbon is divided into four parts *(Folding the ribbon twice).*
The researcher : It is divided into two, then?
Adrian : Folded it again a halve of it
The researcher : It is folded again a halve of it
Adrian : It becomes four parts *(unfold the ribbon and show the four parts of it).*

The discussion, which is transcribed in Transcript 1, shows indication that the students recognized the way of producing equal sizes parts using the ribbon. The other students convinced that the folding produce four equal parts. Subsequently, all of the students used the folding strategy and they could determine the four equal parts properly as the position of the game posts.

The next discussion was about how to give a mark in the figure of the trail which indicated the location of each game post. Izmi explained that after folding the ribbon twice, instead of unfolding the ribbon, he just overlapped it into the figure and gave mark to indicate the first post. Then, he moved and overlapped the unfold-ribbon again by starting to put it from the mark of the first post and gave the next mark at the end of another side of the ribbon as the location for the second post and did it again for the location of the third and the last post. Indeed, the other students agreed with the strategy proposed by Izmi.

We could see that our students in this activity did not unfold the ribbon before they overlapped it to the figure as we assumed in our HLT. Hence, there will be refinement toward the students' conjecture in the HLT for the next cycle. Actually, it is not a big problem as the students still reach the idea of equal size parts. We were sure about this point because before the students used the strategy of Izmi in their work, they already discussed about making equal size parts (see Transcript 1). However, we still want to make the students come up with the expected strategy in which they overlap the ribbon into the hiking trail. Therefore, we decrease the size of the ribbon into the half of the initial ones. As the ribbon becomes smaller, the students still see the hiking trail while overlapping the ribbon. Hence, they are expected to overlap the ribbon into the hiking trail to locate the positions of the flags and the game posts.

Regarding the data analysis, we triangulate the colected data through video registration with students' written work. The Figure 5.2 is the example of students' written work on the problem 1. Look at the position of letter *P* in the figure.

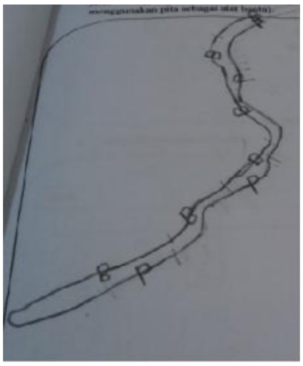


*Figure 5.2.* An example of students' solution on problem 1

To understand the figure, the letter *P* indicates the location of the game post and the letter *B* indicate the location of each flag. Just focus on the letter P since these are the first part that students did in the figure before they tried to locate the position of the *B*s. From Figure 5.2 we could see that the students could locate the position of the four *P*s in an equal distance as the result of the overlapped strategy they used. Based on this data, we could see that the students' written work and the observation analysis for the activity of locating the flags in this lesson are consistent.

Furthermore, the next part of the problem 1 was to locate the six flags along the figure of the trail. From the video registration, we can see that all of the students tried to fold the ribbon. However, they struggled to do it in a right way. Calvin, Izmi and Abdul tried to fold it three times, but Arfan reacted on it since it would not work because the result of it is eight equal parts. Arfan and Adrian got six equal parts of the ribbon. Arfan explained their strategy, first he folded the ribbon once, then by estimating, he divided the unfold ribbon into three equal parts and folded it again. Figure 5.3 shows the step after Arfan folded the ribbon once.

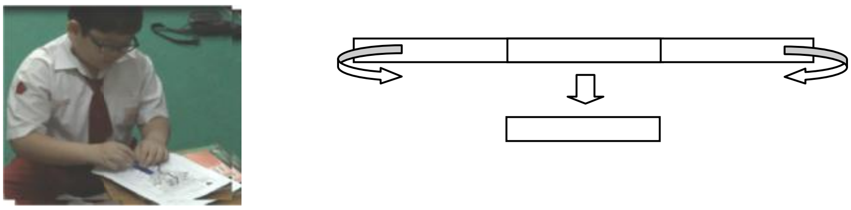


*Figure 5.3* The illustration of the step of making six equal parts of the once-folded ribbon

As we expected, the students recognized the idea of folding the ribbon to produce six equal parts which was more complicated than producing four equal parts. The use of estimation is one of the conjectures of students' strategy in our initial HLT. It implies that our students could explore the use of the given tool to help them in solving the problem.

At last in this activity, the students used the same strategy to give mark for the location of the six flags along the trail in the figure on worksheet 1 as they did for the locating the game posts.

To support this analysis, the student's written work on Figure 5.2 shows that the student located the letter *B*s which is the indication of the flags in an equal distance along the trail. This implies that the two data collection methods, the observation through a video registration and the students' written work, have a consistency. It is contribute to the validity of the analysis.

Based on the explanation above, we concluded that the students reached the goal of the activity of locating the game posts and flags along the hiking trail. It shows that they could do partitioning activity properly, although they struggled in the beginning of the activity. Moreover, they could get the notion about the result of partitioning activity is should be in equal size parts.

**Activity 2- Making fraction notation of the result of the partitioning**

The second activity was to solve the problem 2 of the worksheet 1. But before that the teacher invited the students to strengthen the ribbon and told them that they already provided with the picture of the strengthen ribbon on page 3 of worksheet 1.

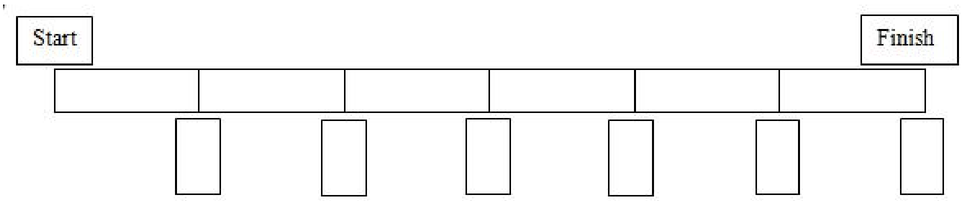



*Figure 5.4* The ribbon representation of the flags' location

The aim of this activity was to support students in developing their initial understanding about the part-whole relation. Moreover, we expected that the students would use fractions in an ordinal way to indicate the result of the partition.

In the beginning, when the students saw the figure, they got confused about what was it mean and what to do. But when the teacher gave the special tones in the instruction *"In what part of the whole trail is the location of the first flag?"* the students seemingly understood that they would work with fractions. As we expected in the HLT, all of the students started to use the fractions in the ordinal way. As we observed from the video registration which is transcribed in Transcript 2 below, the students discussed about their strategy in solving the problem.

**Transcript 2**

The researcher : Who can explain this to us?

Celvin : This is a trail. On this trail there are six posts, so this trail is divided by six. So it is $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}$. Divided by 6.

The researcher : Divided by 6?

Celvin : Yes, it is divided by 6.

The researcher : Is there any question about it? Do you understand the strategy? *(Look at the other students)*

The students : *(Nodding)*
The researcher : What is the meaning of the $\frac{3}{6}$?
Celvin : $\frac{3}{6}$ means that we reach the third post.
The researcher : it is about the post or flags?
Celvin : Eh, it is about flags. Already reach the third flags.
The researcher : How do you think Izmi, what does that mean?
Izmi : The third flag of the six flags.
The researcher : Adrian, you want to say something?
Adrian : A half way of it.

Based on the Transcript 2, we can see that the students could label the result of the partitioning with fractional notation. We also interpret that they knew about part and whole. Moreover, in terms of fractions, the students recognized that when we reach the third of the six flags, it means we already reach a halve way of the trail (Adrian said). Based on this statement, the teacher encouraged the students to think if there are any other fractions in the figure that could be simplified. The students noticed that $\frac{2}{6}$ is the same with $\frac{1}{3}$ and $\frac{4}{6}$ is the same with $\frac{2}{3}$ and $\frac{6}{6}$ equals to 1. They remembered about the equivalence fractions topic they already learned.

As a data triangulation, the explanation above is in line with what we found on the students' written work. The Figure 5.5 is one of the examples that our students used fractional notation in an ordinal way.

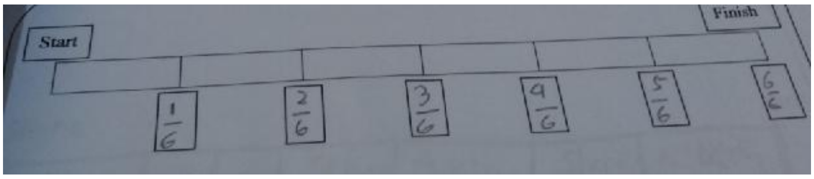



*Figure 5.5* Example of students' solution to problem 2a worksheet 1

Furthermore, the discussion continued to solve the problem 2b about the fractional notation of the location of the posts. The students used the similar explanation as they did to solve the problem 2a.

As the analysis of the activity of problem 2, we concluded that the students could make a fraction notation for the result of the partitioning activity. They also knew about the relation between part and whole unit and they could simplify fractions by looking at the figure and by remembering the procedure they learned.

Regarding the learning materials for the problem 2, it seemed that the students difficult to see the figure as the representation of the unfold ribbon for the location of flags and game posts. Therefore, we planned to add the flags and the post drawing in the folding line in the figure, so it would help the students to recognize the figure and related it to the story.

**Activity 3- Determining the distance between the starting line and the first post**

The last activity in this lesson was to determine the distance between the first post and the starting line. Our intention in this activity is to strengthen the students' initial notion about part-whole relation. We expected that the students would realize about taking a part of the whole length of the hiking trail. We also assumed that the students would use fractions in the operation when solving this problem.

The students understood the problem well. It is derived from the way the students explained about the problem in their own sentences. The researcher gave students time to do it individually then discuss it with their peers. However, not all students involved in the peer discussion, they tended to solve the problem by themselves. To overcome this condition, the researcher emphasized more than two times that the students could discuss with their peers. Then, we could see some students started to discuss the strategy they used.

After a while, the whole discussion began. There were three different strategies that had been discussed. In general, all of the students drew a bar to represent the whole hiking trail. First, we pointed out the strategy by Abdul. He divided the bar in six and got 1 km for each part. Then, he drew 4 lines and gave mark $P_1$ to $P_4$ to indicate the position of the posts. To determine the distance between the posts he said that he divided the length of the trail with four and got 1.5 km (see Figure 5.6).

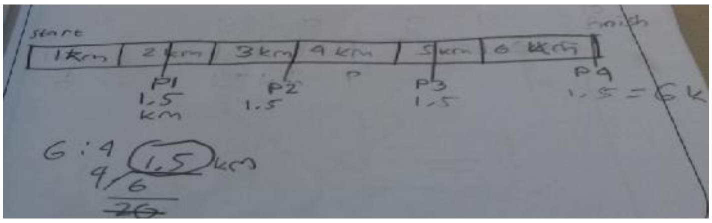



*Figure 5.6* The work of Abdul on Problem 3 worksheet 1

The second strategy was the strategy by Celvin and Adrian. Their strategies were a bit similar to the strategy of Abdul. But the way they got 1.5 km is different. First, they divided the 6 km by 4 and got 1 km with 2 km as the leftover. Then, they divided the leftover by 4 and got 0.5 km. Further, they added 1 km and 0.5 km and came up with 1.5 as the result (see Figure 5.7).

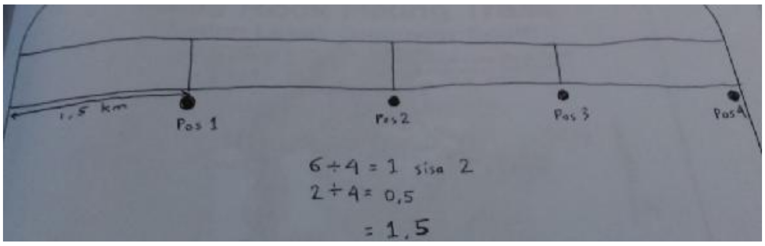



*Figure 5.7* The work of Adrian on problem 3 worksheet 1

The next strategy was the strategy by Arfan and Izmi. The Transcript 3 is the discussion when Arfan explained his strategy.

**Transcript 3**

Arfan : There are 4 posts and all of it is 6 km, a half of it there are 2 posts, it means 6 divide by 2, 3 km. Then, there are 2 posts in the 3 km, so 3 divide by 2 equals 1.5 *(see figure 5.8).*

Researcher : Is it clear what Arfan explained?

Students : Yes, it is clear.

To support this data collected, we provide Arfan's written work in the Figure 5.8 below. The figure shows that Arfan used jumping marks to indicate the strategy he used to find 1.5 as the solution of the problem. We could see that the data is in line with Arfan's explaination in the discussion (see Transcript 3). Therefore, it contributes to the validity of the analysis.

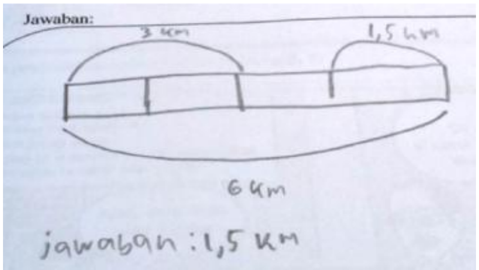



*Figure 5.8* The work of Arfan on problem 3 of worksheet 1

The discussion transcribed in the Transcript 3 shows that Arfan started to use the fraction, although he used the word which indicated the fraction $\frac{1}{2}$ unconsciously. We could see that the word a half is quite close to the students since they usually use it in the daily life. This finding shows that the choice we made about using the simple fraction in this problem helped students to use the word in the solution. Therefore, we could use it in the discussion in order to bridge the students to understand the part-whole relation.

The researcher brought the strategy of Arfan to be discussed more. The researcher transferred the work of Arfan on the whiteboard and emphasized in the language that was used by Arfan. The researcher said that Arfan take a half of the whole trail and get 3 km, and then take a half of 3 km to get 1.5 km as the distance between the first post and the starting line. The word a half is transferred into fractional notation $\frac{1}{2}$. Then the researcher invited the students to rewrite the step that had been done by Arfan in a complete sentence. The students recognized that Arfan took $\frac{1}{2}$ of $\frac{1}{2}$ of 6 km.

Furthermore, the researcher put the strategy used by Abdul, which was about dividing the 6 km by 4, to the whole class discussion. The researcher invited students to think about writing it in term of fractions, the students could not come up directly about the answer. To deal with that condition, the teacher said "*how about take $\frac{1}{4}$ of 6 km, is it the same with 6 divided by 4?*". As the result the students could grasp the idea and they recognized that to determine the location of the first post from the starting line they also could take $\frac{1}{4}$ of 6 km. In conclusion, they agreed that taking $\frac{1}{2}$ of $\frac{1}{2}$ of 6 km is the same with taking $\frac{1}{4}$ of 6 km. The discussion on this part supported our conjecture that by comparing the two strategies which appeared from the students supported them to notice about the idea of taking a part of a whole unit.

**Conclusion about lesson 1 analysis**

After conducting the analysis of the first lesson in cycle 1 we conclude these three points.

- The students could do a partitioning activity properly. And they recognized that result of the partitioning should be in the equal size.
- The students could give a fraction notation of the result of the partitioning activity. They recognized about the use of fraction when the teacher asked about *"In what part of the trail …?"*
- The students tended to use the division algorithm when they deal with the taking a part of a whole problems. They were not familiar with interpreting it by using fraction notation.

**b. Lesson 2 – Taking a part of a part of a whole**

There were four activities in this lesson in which the students will work either individually or in pairs on several problems provided in the worksheet 2. The array model was introduced in a form of a chocolate block in activity one. They would share the chocolate block for three children. then the students were introduced into the term taking a part of a part of a whole in the time for exercise context. Further, in activity 3 the students would apply the taking a part of a part of a whole on the array model. At the end, we provided students with exercise to solve the sharing chocolate block with different dimensions.

Actually, in the practice, the lesson 2 was conducted in two meetings with 35 minutes for each. It is because the time for learning mathematics in the school was settled in that way and the researcher just followed the schedule. However, in this analysis, we analyze the whole activity in lesson 2 for the both meetings.

In this lesson, the teacher continued the story about the hiking event. That was about a student named Hafidz doing exercise to prepare himself for the next hiking event.

**Activity 1 – Sharing chocolate block**

The introduction of problem 1 and 2 was given in the comic 1 (on worksheet 2). It was about the sharing of a chocolate block among three scout boys, Hafidz, Aufa and Siraj, who prepare their health for the hiking activity. The instruction in the problem 1 was to show in the given array (the drawing of the representation of a chocolate block) the part for each boy. Meanwhile, the problem 2 was asking about what part of the chocolate block that Hafidz get.

The intention of giving this problem to the students was to get started doing partitioning and interpret the result of the partitioning in a fraction notation. In this activity the students would start to work with the array model which was introduced in the form of a chocolate block. In solving these problems we expected that the students would divide the chocolate block figure vertically or horizontally and use counting strategy to determine the fractional notation of the intended part.

In the teaching experiment, all of the students solved these two problems correctly. As the conjectures for the problem 1, they could show the parts for Hafidz, Aufa and Siraj by dividing the given array into three equal parts vertically. Moreover, there were two different strategies that were used by the students in solving problem 2. Most of them count the total number of small pieces in the chocolate blocks and then count the small pieces of the Hafidz parts. They did it by multiplying the dimension of the block, the number of rows multiplied by the number of columns. Then the fractional notation for it is the number of small pieces of Hafidz over the total number of small pieces in the chocolate block (as can be seen in Figure 5.9). They came up with $\frac{18}{54}$. Furthermore, they simplified the fraction by dividing both the numerator and the denominator of that fraction by 18 and got $\frac{1}{3}$ as a final result.

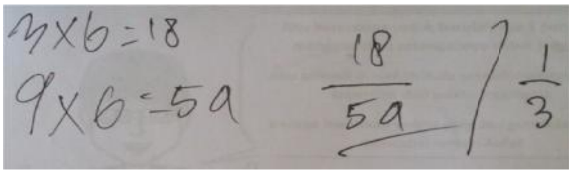



*Figure 5.9* Multiplying rows and columns strategy in determining the fraction notation of problem 1 and 2 in worksheet 2

Only one student who directly interpreted the drawing of Hafidz parts as $\frac{1}{3}$. When the researcher asked why he directly answered like that without counting the number of small pieces as the others did. The following Transcript 4 is the discussion about this idea.

**Transcript 4**

| | |
|---|---|
| Adrian | : It is divided by 3, Hafidz, Siraj … There are three children. |
| Researcher | : There are three children, then? |
| Adrian | : Hafidz gets one over three of it. |
| Researcher | : One over three of it, Aufa gets? |
| Adrian | : One over three of it. |
| Researcher | : Also gets one over three. It is also for Siraj? |
| Adrian | : Yes. |
| Researcher | : Why each of them got one over three? Because what does it says in the problem? |
| The students | : Divided into equal parts. |

As in Transcript 4, Adrian said that he knew the chocolate block was divided into three, so each part is the same as $\frac{1}{3}$. In our interpretation, it implies that this student could realize that the unit is 1 then this unit was divided into three equal parts. He could relate these two numbers in order to come up with the answer one over three. As the compliment of this explanation, we provide students' written work on the activity 1 in Figure 5.10.

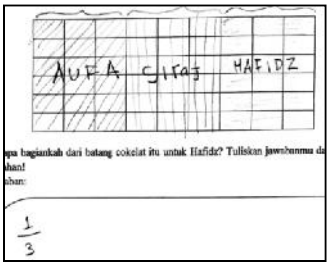



*Figure 5.10* Students' written work on activity 1

Further, in the discussion, the researcher asked the students to compare the solutions of problem 2, whether they agreed or not or did they understand the way each of them got the fractional notation. The result of this discussion shows about the agreement among the students where they concluded the two solutions were correct.

Based on the analysis above, we conclude that the students already knew how to make three equal parts and interpreted it into a fraction notation. They knew about taking a part of a whole unit. They also could relate the part and the whole unit in a form of a fraction. They also get used of simplifying the fraction in the final result.

**Activity 2 – Time for reaching Aufa's House**

In this activity, the students' task was to determine how many minutes needed by Hafidz to reach Aufa house if he get into Aufa's house after using a third of his jogging time. In the story in comic 2 it is stated that the time for exercise is one hour and a half of it is used for jogging by Hafidz. The intention of this activity is to let students discover the result of taking a third of a half of an hour. We proposed this context to engage students with an idea of part-part-whole which really close to their life since in our expectation, the context about times is quite simple for the students. We conjectured in the HLT that the students could interpret the term part of into mathematical operation.

At the beginning, the researcher did not really check students understanding about the problem. It led to a misinterpretation of the given problem to most of the students. There is only one student could follow the story and also the steps in solving the problem correctly, Abdul. In the following Transcript 5 we provided the idea of the student in solving the problem.

**Transcript 5**

| | |
|---|---|
| Abdul | : Hafidz planned to have an exercise in one hour. |
| The researcher | : Yes, then? |
| Abdul | : In a Sunday morning he uses a half of the total time for jogging. It is equal to 30 minutes. He starts to jog and need a third of the jogging time to reach Aufa's house. |
| The researcher | : What does it mean? |
| Abdul | : It is mean 30 minutes divided by 3. Hmm, it is 30 minutes multiplied by one over three. |

Based on the Transcript 5, we could see that the student understand the information of the problem. He represented the information in a mathematical operation in order to get the final result. As can we interpret from the transcript, he understand that multiplied the 30 minutes with $\frac{1}{3}$ is the same with dividing 30 minutes by 3.

As a compliment of the data collected from the video registration above we provide the students written work in Figure 5.11

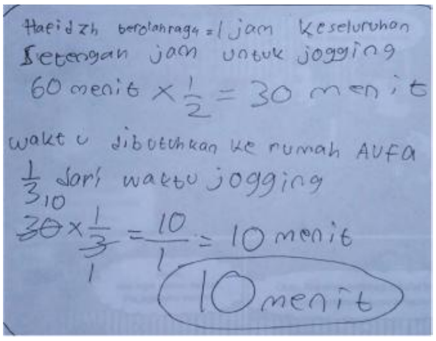



*Figure 5.11* Abdul's work on problem 3 of worksheet 2

In his written work (Figure 5.11), as we expected, Abdul addressed the use of multiplication with fractions. We can infer from his work that he started to realize about determining a result of taking a part of the whole through multiplication operation, although in his writing, we could see that he multiplied the time with the fraction instead of the other way around. It implies that he still needed to understand the real meaning of the multiplication he did. As a conclusion in term of validity of the analysis, we could see that the data of this students' written work is consistent with the data collected through the video registration.

Moreover, from the written works of the other students, we interpret that they did not carefully took the important information about the story in comic 2. They tried to find the time for reaching Aufa's house from the initial time for exercises not from the time for jogging. Adrian and Izmi divided 60 minutes by 3 and got 20 minutes as the answer as can be seen in Figure 5.12.

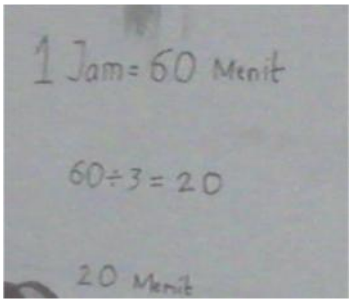



*Figure 5.12* Incorrect solution of the student on problem 3 of worksheet 2

Furthermore, to engage the students to recognize the part-part-whole relationship we conducted a class discussion. In the discussion the researcher elaborated the work of Abdul which was already written on the whiteboard. The teacher invited the students to rewrite the strategy that was used by Abdul in a complete sentence. The intention of asking such question was to make the students realized the taking a part of a part of a whole unit in the activity. However, at first none of the students could come up with this idea, so that the teacher helped them by introducing how to write the process in a sentence. The researcher reminded the students about the problem in the story. Then, the researcher, guided the students to write that the time for reaching the Aufa's house was $\frac{1}{3}$ $of$ $\frac{1}{2}$ $of$ $an$ $hour.$

The result of the discussion reveals that the students noticed and started to acknowledge the use of the term a part of a part of a whole unit in the solution of the problem. However, it was not coming up from the students, but by guidance from the teacher. The initial notion of a part of a part of a whole would be elaborated more in the next activity.

**Activity 3 – Taking a part of a part of a chocolate block**

In this activity the problems were related to the problem 1 and 2, about the sharing of the chocolate block. There was an extension of the story in which Hafidz wants to share his chocolate parts equally with his sister Nazifah. The students were asked to show the part of Nazifah on the given drawing, an array that represents the initial chocolate block. They also need to interpret the drawing into a fraction notation.

The aim of this activity was to allow students to experience the use of an array model to solve the taking a part of a part of a whole unit within a context. We tended to support them to be able of using the array to help them in solving the problem. In the initial HLT we expected that the students would divide the array as the information given in the problem. Moreover, to determine the fractional notation of the intended part the students would use either counting strategy by relating the intended part with the whole unit or try to think about how many times the intended part fit with the whole chocolate block.

In the teaching experiment, for problem 4, based on our observation, all of the students could show the part of Nazifah correctly. They split up the parts of Hafidz into two equal parts and shaded it. This data in line with what we found in the students written work as can be seen in Figure 5.13. It implies that the students could do the partitioning activity correctly within the array.


*Figure 5.13* Students' indicate the part for Nazifah

Furthermore, for problem 5 most students except Abdul answered by $\frac{9}{18}$ and then simplified it became $\frac{1}{2}$. Abdul answered by $\frac{1}{6}$. In the discussion, the students explained their strategies. It was similar to the strategy they used in solving the problem 3 and 4. As we conjectured in the initial HLT, the students counted the small pieces of the intended part, over the small pieces of the unit. The difference between the two answers for problem 5 was due to the unit that they refer to. The students who answer with $\frac{1}{2}$ said that they refer to the parts of Hafidz. The meaning of $\frac{1}{2}$ they wrote on the answer box was that the parts for Nazifah is $\frac{1}{2}$ of Hafidz's parts.

Meanwhile, Abdul said that to determine the fraction notation of Nazifah's parts he not only refer to the Hafidz's part but to the whole chocolate block. As can be seen in Figure 5.14, Abdul did not count the small pieces of the chocolate block. He started with the part for Hafidz was $\frac{1}{3}$ of the chocolate block, then the part for Nazifah is $\frac{1}{2}$ of that $\frac{1}{3}$ part. However, the way he got $\frac{1}{6}$ as the answer is not correct. We can see that Abdul used the strategy of solving the subtraction of fractions. He subtracted $\frac{1}{2}$ with $\frac{1}{3}$ and by using the procedure he got $\frac{1}{6}$ as the answer. To correct that strategy, the researcher asked all of the students to determine the fractional notation of Nazifah respect to the whole chocolate block.

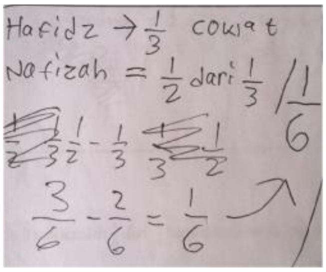



*Figure 5.14* Abdul's work on problem 5 of worksheet 2

Celvin tried to explain his solution in front of the class. He got $\frac{9}{45}$ as the solution. But Abdul had a question regarding the answer that was shown by Celvin. The discussion is transcribed in the Transcript 6.

**Transcript 6**

Celvin : This is the Hafidz's parts *(pointed at the number 9 in Figure 5.12)*, this is Aufa's and Siraj's *(pointed at number 36)*. We added up became 45. It means the parts of Hafidz, Aufa and Siraj are 45 in total. And this 9 is Nazifah's (pointed at the number 9 in the numerator of the result).

The researcher: Then?

Abdul : Why it is 45? Since initially the Hafidz's parts has not included the Nazifah's parts.

The researcher: Do you understand the question of Abdul, Celvin?

Celvin : No, I don't.

Abdul : I mean, you added up the part of Aufa, Siraj and the part of Hafidz excluded the Nazifah's part. Why don't you add it up with the initial part of Hafidz?

The researcher: The part that is not shared yet [with Nazifah].

Abdul : The part that is not shared yet [with Nazifah].
Celvin : Because [the information of] the problem said that the parts [of Hafidz] already cut before.
The researcher: What do you think Arfan?
Arfan : I think, it is from the whole part. Why don't you take the whole parts of Hafidz.
Abdul : Because it is asked about the parts of Nazifah respect to the whole [chocolate block]

Based on the Transcript 6, we can interpret that Celvin still got confused in determining the whole unit in the taking a part of a part of a whole unit. Instead of just taking the part of Hafidz after he split it up with Nazifah, Abdul and Arfan suggested that they need to consider the whole part of Hafidz before the splitting when counting the total number of small pieces of the chocolate block. It implies that Abdul and Arfan know how to relate the intended part (the part of Nazifah) with the whole unit. Together they corrected the answer and got $\frac{9}{54}$ and simplified it became $\frac{1}{6}$.

As the compliment for the analysis of the video registration above, we provide student's written work which show that he had a misinterpretation, that he is not relate the intended part with the whole chocolate block but only relate it to the remaining of the block.

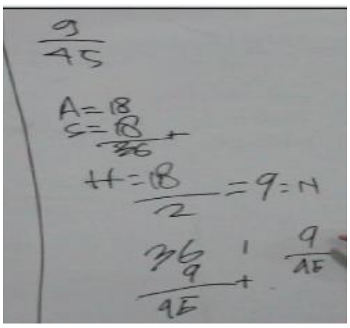


*Figure 5.15* Celvin explained his answer in front of the class

The student's written work in Figure 5.15 is consistent with what we found in the students' discussion which is transcribed in Transcript 6.

Furthermore, the researcher asked the students, whether the strategy that they already discussed, had the same meaning of the subtraction that was done by Abdul (Figure5.14) to find the fractional notation $\frac{1}{6}$. As the answer to problem 5. The students could recognize that it was not the same. They could not use it in solving the taking a part of a part of a whole problems.

As the description above, we noticed that none of the students thought about how many times the parts of Nazifah fit into the initial chocolate block. The conjecture about this strategy not appeared in the teaching experiment. In our interpretation, might be our students only tended to count the small pieces in the chocolate block figure and related it with the whole unit.

**Activity 4 - Working on problem 6 and 7 on worksheet 2**

The next activity was to solve two problems of taking a part of a part of a chocolate block. These problems were similar to the problem 4 and 5. There were 2 arrays as the representation of the chocolate blocks. The dimension of the array was different for each problem. For problem 6, the students needed to show in the drawing $\frac{2}{3}$ of $\frac{1}{2}$ of a chocolate block with dimension 4 × 6. Meanwhile, for problem 7, they need to determine $\frac{1}{6}$ of $\frac{2}{3}$ of a chocolate block with dimension 3 × 12. Then they also need to determine the fraction notation of each answer.

The objective of this activity was to give more chances for the students on practicing the use of array in solving the problem of taking a part of a part of a whole unit. Moreover, the researcher also wanted to know whether the students could use the strategy they already discussed when dealing with non unit fractions. In our expectation in the HLT, the students would use the similar strategy as they applied in the previous problem that is determining the fractional notation by relating the number of cells in the shaded part and the whole chocolate block. The difficulty that could appear was about the part and the whole unit, since this activity involved the non unit fractions.

For problem 6, the dimension of the chocolate block was 4 × 6 (4 rows and 6 columns). The following Transcript 7 is the discussion between the students to discuss about partitioning the given chocolate block.

**Transcript 7**

The researcher : Celvin, what do we have?
Celvin : a chocolate block
The researcher : Then, what should we do?
Celvin : This is a half of the chocolate block, it means divided into a half. *(put a line in the middle of the chocolate block vertically)*.
Researcher : Then the task is to determine 2 over 3 of the half right?
Celvin : First, we take a half [of the chocolate block].
The researcher : Yes, and then?
Celvin : Hmm
The researcher : Who can help Celvin?
Arfan : That is already a half then we should determine the 2 over 3 of the half and shade it. Further, try to find the total number of cells and the number of cells in the shaded part.

Based on Transcript 7, we could interpret that the students could show the intended part in the drawing. They took a half of the figure first and divided it into three parts then shaded two parts of it. This explanation implies that the students could determine the taking 2 over 3 of a half of a chocolate block properly. In addition, to support the data collected in the transcript above, we triangulate it with student's written work of this problem as can be seen in Figure 5.16.

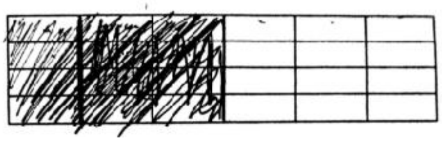


*Figure 5.16* Students' written work on problem 6 of worksheet 2

Furthermore, we provide the analysis of students' written work on determining the fractional notations of the parts. Most of the students answered with $\frac{8}{24}$ and they simplified it became $\frac{1}{3}$. However, they used various strategies. Celvin and Adrian calculated the small pieces in the intended parts over the total number of small pieces in the whole chocolate block (see Figure 5.17a). Arfan divided the total number of small pieces by 3 and got 8 then he related it with the whole chocolate block to get $\frac{8}{24}$ (see Figure 5.17b). Abdul wrote that he knew a half of the whole chocolate block was 12 small pieces and he multiplied it with $\frac{2}{3}$ to get the number of small pieces in the intended part (see Figure 5.17c).

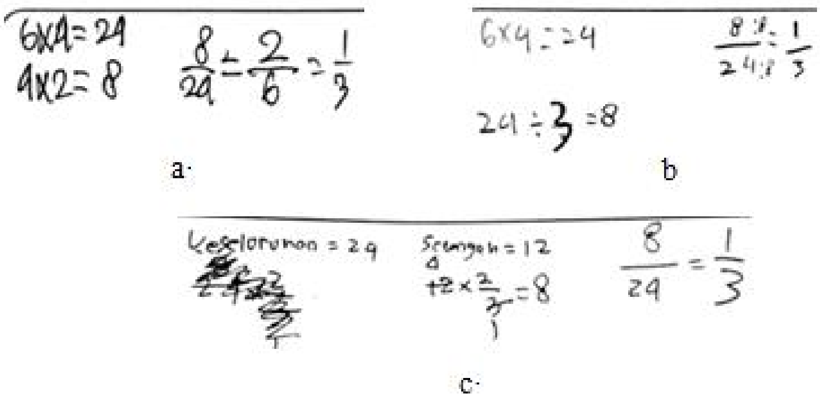



*Figure 5.17* Students' written works of problem 6c on worksheet 2

Based on the analysis above we could see that the data collected through different method are compliment to each other to support us in figuring students' learning process.

Similarly, for problem 7, most of the students could show the result of taking $\frac{1}{6}$ of $\frac{2}{3}$ of a chocolate block in the drawing. Furthermore, to determine the fractional notation of the intended part, Celvin and Adrian did as same as they did for the problem 6. They got $\frac{4}{36}$ as the result and simplified it became $\frac{1}{9}$ (see Figure 5.18a). Arfan and Izmi did three multiplications to find the fractional notation of it. First, they multiplied the dimension of the whole chocolate block to get the total number of small pieces in the block. Then they multiplied it with $\frac{2}{3}$ and got 24. Further, they multiplied the 24 with $\frac{1}{6}$ and got 4. They related the 4 with the total number of small pieces in the block and they came up with $\frac{4}{36}$ as the result (see Figure 5.18b). The last student, Abdul, divided the total number of small pieces by 3 and multiplied the result with 2 to get the result of taking $\frac{2}{3}$ of the chocolate block that was 24. Further, he multiplied the 24 with $\frac{1}{6}$ to find the final solution. He got $\frac{4}{36}$ and simplified it became $\frac{1}{9}$ (see Figure 5.18c).

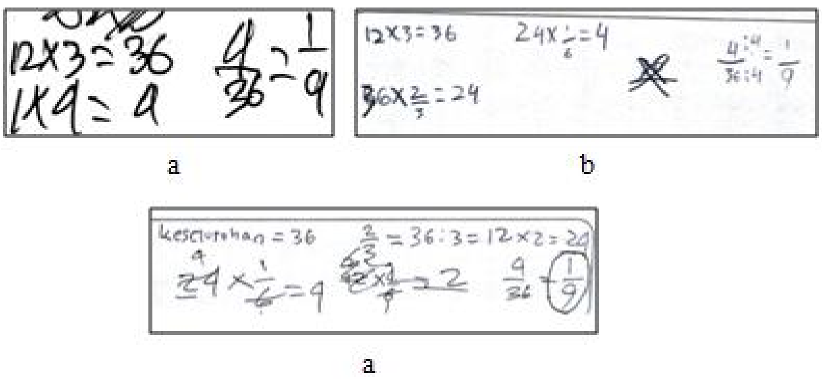



*Figure 5.18* Students' written work on problem 7 of worksheet 2

The description of students' strategies in solving the problems 6 and 7 above indicates that most of our students could use the array model to show when they produce the new quantity from the initial chocolate block and then take a part of the new quantity. They also could deal with non unit fractions and determined the intended part in the drawing. In addition, we notice that our prediction that the students would have difficulty when dealing with non unit fractions was not happen. It might be because the students already get the notion of doing partition within the array and interpret it into a fractional notation.

Furthermore, regarding the worksheet 3, which we planned as the homework of the lesson 2, was not used because of the limitation of the time and the researcher want to address the more important part of the lesson.

**Conclusion of the lesson 2 analysis**

Based on the analysis of the video registration and the students' written work of this lesson, we derived the following conclusions. First, the activity of determining the time for reaching Aufa's house within the time for exercise context along with the discussion facilitated by the teacher could support students to start working on the form of a part of a part of a whole unit using the fractions. Second, the students could do the partitioning activity within an array model properly for both unit fractions and non unit fractions. Moreover, in the activity of sharing chocolate block in this lesson, the students started to use the context and the array model to deal with the taking a part of a part of a whole problems. It shows that the context and the array model helps students to convince each other about the idea of part-whole relationship.

**c. Lesson 3- Sharing *martabak telur***

The lesson 3 was also conducted in two meetings because there was not enough time to conduct all of the content in this lesson. This condition happened because in the beginning of the lesson 3 we still discussed about the last part of the worksheet that was part of the lesson 2.

After students experienced the activity of partitioning on the given array to solve the taking a part of a part of a unit problems, in this lesson they started to construct their own array. Students would work on worksheet 4. This worksheet consists of three activities. The students would work on the problem of sharing *martabak telur* in which they would use their own array in solving the taking a part of a part of a whole problem. Furthermore, they would ask to choose one of the three array figure that suitable in solving the same problem of the *martabak telur* as in activity 1. Then at last, the students would work in 4 bare problems about determining a result of taking a fraction of a fraction, in condition that the whole unit was not mentioned anymore in the problems. The following are the analysis of these two parts.

**Activity 1 - Sharing the *martabak telur*.**

The context was about sharing a *martabak telur*. In the story, it is stated that the Hafidz's mother made a *martabak telur*. However, because Hafidz come home late, he just found a half of that *martabak telur*, then he eats a quarter of it. The students' task in the first problem was to determine what part of the *martabak telur* that was eaten by Hafidz respects to the initial *martabak telur*. The researcher gave hints that the students could make a drawing to help them.

The goal of this activity was to give more chances for the students in dealing with the taking a part of a part of a whole problem within a context. In this activity we did not provide the students with the drawing, our intention was that the students started to construct their own array model. In our expectation in the initial HLT there would be two strategies of the students.

First, they start to draw the whole *martabak telur* and the second they start to draw the *martabak telur* that available before Hafidz eat it.

To solve the problem 1, based on our observation of the video registration, all of the students started to make a drawing to help them in solving the problem (see Transcript 8).

**Transcript 8**

Adrian : *(Draw a rectangle and split it into 2 big parts and then split each of it into four equal parts)*.

The researcher : Can you explain your drawing?

Adrian : This part is already eaten *(indicate a half of the rectangle by his hand)*. Then, Hafidz eat a quarter of the leftover. So it means 1 over 4 of 1 over 2.

The student drew a square and split it up into two parts first and then divided it into four parts then shaded one small piece of the last partition. We can interpret that the student could construct his own array by showing a new quantity as the result of the first partition and take a part of the new quantity.

We triangulate the data collected in the Transcript 8 above by providing student’s written work on this problem as can be seen in Figure 5.19a. This two data shows a consistency in which we can see in his work, the student shade one small pieces of the array they have constructed.

As a compliment for the analysis above, we also pointed at different strategy used by other students as can be seen in Figure 5.19 b and c. In our interpretation, Izmi drew a small rectangle first and drew another seven small rectangle in order to complete the *martabak telur* as the initial *martabak telur*.

Abdul also drew a rectangle and then he split it up into two and then made it became an array with dimension $4 \times 8$.

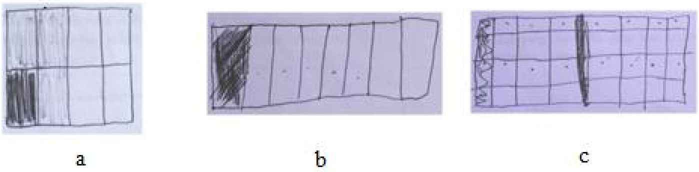


*Figure 5.19* Students' answers of problem 1 on part A of worksheet 4

Based on students' strategies above, we could see that all of our conjectures in the initial HLT happened. As a conclusion, it implies that the students grasped the idea of part-whole relationship well. They knew that they should represent the whole *martabak telur* in order to determine the fractional notation of the intended part of the problem.

Furthermore, to find the final solution of problem 1, there were two different strategies that were used by the students. First, they calculated the number of small pieces in the drawing by multiplying the dimension, they got $\frac{1}{8}$ as the answer (see Figure 5.20a). Second, they did two multiplication steps, at first they multiplied the total number of small pieces in the initial *martabak telur* with $\frac{1}{2}$ and then multiplied the result with $\frac{1}{4}$ to get the pieces that were eaten by Hafidz. Furthermore, they related the last result with the total number of the small pieces in the initial *martabak telur*, they came up with $\frac{1}{8}$ as the answer (see Figure 5.20b).

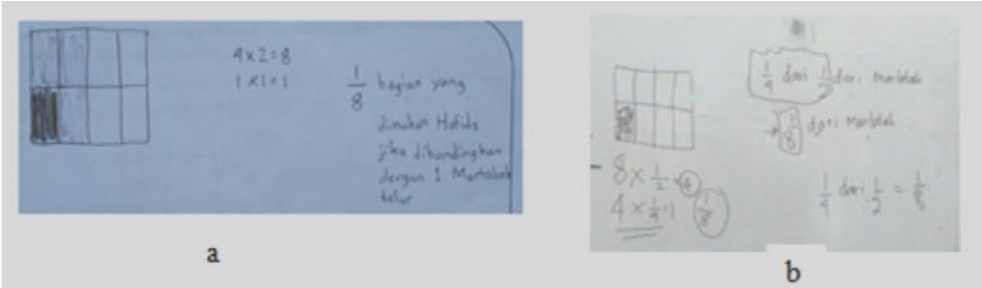



*Figure 5.20* Two strategies in solving problem 1 of part A on worksheet 2

The students who solved the problem with the second strategy above indicate that they started to work with fraction operation in solving this problem. Partially, they tried to relate the “taking a part of” into the multiplication involving fractions.

**Activity 2 - Choosing an appropriate array**

Furthermore, the next activity was to choose one of the three given arrays that already made by three pupils in the story, A, B and C with different dimensions to solve the problem 1. The dimensions of the arrays in the problem 2 of worksheet 4 were $5 \times 6$, $4 \times 3$, and $4 \times 4$ respectively.

The aim of this activity was to assure that the students recognized the idea of choosing an appropriate dimension of the array that they used to help them in solving the taking a part of a part of a whole problems. We expected that the students would do a trial and error strategy where they tried to apply the taking a quarter of a half of the *martabak telur* on the three arrays.

Based on students' written works and also based on the video registration, we can see that all of the students chose that the last figure was the easiest tool that could help them in solving the problem. However, they still struggled to explain the reason why they chose the last figure. They said that because the first figure has too many small pieces, then they researcher consulted their answer, "*if the first has too many small pieces, why they did not choose the second figure*?" The Transcript 9 is the discussion about the reason why they could not choose the second figure in the worksheet (figure B).

**Transcript 9**

The researcher: Why don't you choose the student B. If the reason is because too many small boxes, why don't you choose the student B since [his or her drawing] has the least number of small box in it. Why it is not?

Abdul : Hmm…

The researcher: Before, you said that because the student A's figure has too much small box, so then it mean student B as the answer right? [Why not?]

Adrian : Because the student B has least number of the small boxes in it.

Arfan : Because the quarter… em a half of the quarter cannot be shown in the figure.

Abdul : Yes, we cannot show a half of the quarter in the figure.

The researcher: Okay. Can you repeat it loudly Arfan?

Arfan : Because we cannot show a quarter of the half in the figure.

The Researcher: We cannot show in the figure.

Arfan : Yes.

Based on the Transcript 9 we can see that at first the students said that the second figure (figure of student B on the worksheet) has least number of small pieces. These answers imply that they still could not grasp the idea of using the suitable dimension of the array properly. However, one of them recognized a reason that they could not use the figure of student B because it could not show the quarter of a half of the *martabak telur*.

As a compliment to the transcript, we found in students' written work, there is student came up with the idea that the last figure was easy to divide to produce even number (see Figure 5.21).

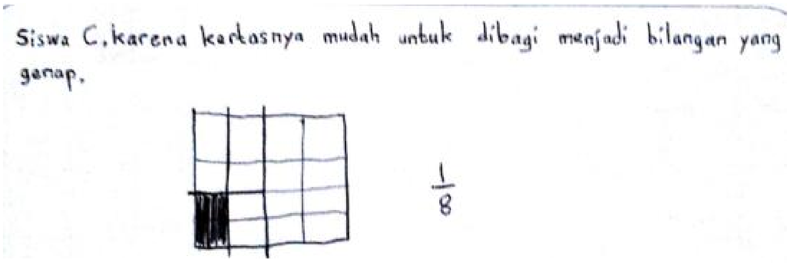



*Figure 5.21* Students written work on problem 2 of lesson 3

The opinion above is closer to the core idea of this activity. After the discussion as transcribed in the Transcript 9 the students could recognize that they could show the half of the *martabak telur* easily and so did for the quarter of the half of the *martabak telur* in the last figure (figure of student C on the worksheet). We could see that the data collected through different method support each other so that we could derive a complete figure of the learning process of the students.

Based on the analysis of the students' written works and the video registration about the problem 2 beforehand, we conclude that the students could grasp the idea of using the appropriate dimension of the array models that could be used as a tool in solving the taking a part of a part of a whole problem. All of the students could reason about the solution after the researcher lead a discussion addressing this idea.

**Activity 3 - The Exercise**

The part B of the worksheet 4 was given as homework for the students and it was being discussed in the next meeting. The objective of the four problems in this part was to allow students to have an experience of making their own array to solve the problem taking a part of a part. We expected students to represent their strategy on solving these problems by drawing their own array.

In the teaching experiment, based on the students' written work, we can see that all of the students seemingly understand how to build their own array in solving the problems. In the discussion the researcher asked the students to choose a problem to be discussed together. The students chose to discuss the problem 2 which was to determine $\frac{1}{4}\ of\ \frac{2}{3}$. For the other numbers the researcher only asked students to mention their answer and checked whether they had questions and difficulty or not. In Figure 5.22 there are some of students' solutions of problem 2 in part B of worksheet 4.

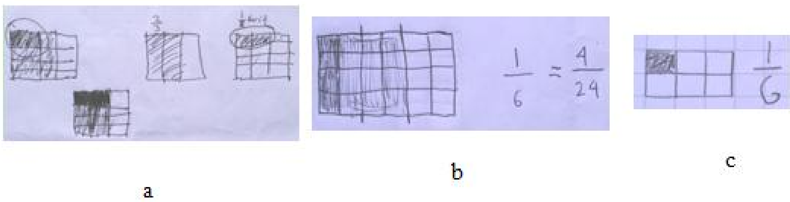



*Figure 5.22* Examples of Students' work on problem 2 of part B of worksheet 4

Based on Figure 5.22 we can interpret that the students came up with different size of arrays. However, they could determine the $\frac{2}{3}$ in the figure and then take $\frac{1}{4}$ of the $\frac{2}{3}$ they made. Further, they used the same strategy that they already used in solving the previous problem to get the fractional notation of the result. All of them agreed that the result of taking $\frac{1}{4}$ of $\frac{2}{3}$ was $\frac{1}{6}$. To deal with different drawing and fraction notation that came up from the students, the researcher invited them to share the ideas in the discussion. Based on the video registration we transcribe in Transcript 10 the fragment of Abdul explained his different drawing in front of the class. This fragment after another student shared his drawing which use 2x 3 array dimension to solve $\frac{1}{4}$ of $\frac{2}{3}$ (see Figure 5.23).

**Transcript 10**

Abdul : First we draw a rectangle. And then we divide it.
The researcher : We divided the rectangle into what parts?
Abdul : Hmm… divided by 3 (split the rectangle into 3 columns)
The researcher : Divided by 3, then ?
Abdul :Hmm … *(thinking).*
The researcher : You divided by 3 in order to get what part?
Abdul : To get the $\frac{2}{3}$.
The researcher : Then, which is the $\frac{1}{4}$ of the $\frac{2}{3}$?
Abdul : This one *(point at one cell the* $\frac{2}{3}$ *).*
The researcher : How can you get it?
Abdul : We divide it by 3 then split it into 2 *(indicate the row in the figure he made).*

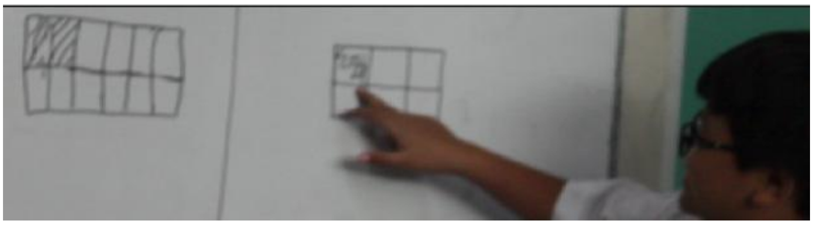


*Figure 5.23* Abdul explain his drawing in front of the class

The result of the discussion indicates that the students realized that they could use different sizes of the array as long as it fit with the fraction of the problems. When they came up with the different fractional notation based on their solutions, they knew that the fractions are equal. Further, the researcher invited the students to compare the drawing. Through this comparing activity the students recognized that the figures also showed the same parts. It led to a more convince proof since they could see directly on the drawing they made.

In term of validity of the analysis, we could see that the transcript is consistent with what we found in Figure 5.22 and 5.23 beforehand where the students use different array dimension for solving the same problem.

Furthermore, to strengthen students' understanding of using the array in solving the problems, the researcher gave one additional problem about taking a part of a part. The problem was to determine $\frac{2}{5}\ of\ \frac{3}{4}$. After a while the students can solve the problem by drawing their own array and interpret the result in a fraction notation. They used the similar strategy as they did in the previous problem. They got $\frac{6}{20}$ or $\frac{3}{10}$ as the solution.

As conclusion of the analysis of this activity we could derive that the students already started to get used of using an array model in helping them to solve the taking a part of a part problems. They used the array model to reason about the result of the partition and then determine the fractional notation based on the drawing (the array). However, none of the students got an idea of relating the fractions involving in the problems. We need to support students to recognize about the relation between the fractions in the taking a part of a part problem in order to start seeing the taking a part of a part as a multiplication of a fraction with another fraction.

**Conclusion of the lesson 3**

Generally, based on the analysis of the students' written works and the video registration of this lesson, we conclude the following points. First, the context about sharing *martabak telur* supported the students in constructing their own array. It helps when the students determine the part and the whole unit in their drawing. Second, trough the activity of choosing the right array figure in solving the *martabak telur* problem, the students recognized about the use of an appropriate array dimension to be used in solving the taking a part of a part of a whole problem. Third, the students could solve the taking a part of a part problem, although the initial whole unit was not mentioned explicitly. In addition, the students were able to represent the taking a part of a part in a suitable array model and determine the fractional notation of the result of the partition for both unit fractions and non unit fractions.

**d. Lesson 4- Math Congress**

In this lesson, the researcher would lead a discussion as a reflection of the main activity of the previous lessons. The researcher started the discussion by inviting the students to think about several taking "a part of a part of a whole unit" problems that they already discussed from the lesson 1 until the lesson 3.

The aim of this activity was to support students to see the relation between the taking a part of a part problems with the multiplication of two fractions. Moreover, we also wanted students to realize the strategy of solving the multiplication of two fractions problem. In our conjecture in the initial HLT, the students would remind about the solution of the taking a part of a part of a whole problem they already solved. Then, the student would recognize about the relation between the numerators and the denominators of the fractions in the problems and also between the denominators of the fractions in the solutions.

The Figure 5.24 shows the list of the solutions of some taking a part of a part of a whole unit that the students already solved.

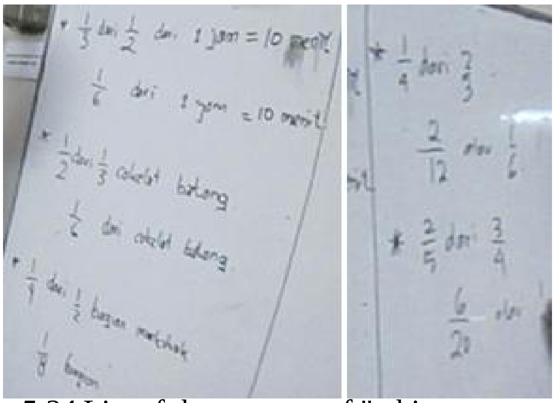


*Figure 5.24* List of the answers of "taking a part of a part of a whole problems

The researcher asked the students to look closely at the list on the whiteboard. Then the researcher asked about what ideas that came to the students' mind. The researcher also asked about what the students could tell about the list. The students needed more time to think before they reacted to these questions.

A student, Abdul, tried to share his idea about the list. The Transcript 11 is the discussion about it.

**Transcript 11**

| | |
|---|---|
| Abdul | : Hm… That.. What we call.. Um, what is the name of the number above [the per sign]? |
| The researcher | : What can we call the number above the sign "—"? |
| Abdul | : Denominator, eh numerator. Numerator times numerator, denominator times the denominator, it means something of a something. |
| The researcher | : Well, Do you understand what Abhi had explained to you? |
| Students | : Yes, we do. |
| The researcher | : Abdul, What is the reason of your idea? |
| Abdul | : Because we can see from the results. [For example] $\frac{1}{3}$ of $\frac{1}{2}$ is equal to $\frac{1}{6}$. Numerator times numerator, 1 times 1. [Then] the denominator times the denominator, 3 times 2. Then it is equal to $\frac{1}{6}$. |

From the Transcript 11, as we expected in the initial HLT, we can see that the student recognized from the list that to determine the result of the taking a part of a part problems which mean in mathematics taking a fraction of another fraction, we can multiply the numerators and also multiply the denominators.

Moreover, to let students grasp the idea of using the multiplication symbols, the researcher, extended the discussion to address this point. The researcher invited the students to simplify the list by not mentioning the whole unit, only listing the part of the parts and the results as can be seen in the following list.

$\frac{1}{3}$ of $\frac{1}{2}$ $= \frac{1}{6}$

$\frac{1}{4}$ of $\frac{1}{2}$ $= \frac{1}{8}$

$\frac{1}{4}$ of $\frac{2}{3}$ $= \frac{2}{12} = \frac{1}{6}$

$\frac{2}{5}$ of $\frac{3}{4}$ $= \frac{6}{20} = \frac{3}{10}$

Furthermore, in the discussion the researcher pointed out the word “times” that was used by Abdul in explaining his idea as can be seen in the Transcript 11. The researcher invited the students not only to use the term “times” between the multiplication of the numerators and the denominators, but also use it in the operation of as a fraction times another fraction. This discussion led students to get the notion that the activity of the taking a part of a part of a whole activity can be interpreted as a multiplication of a fraction with another fraction.

The weakness of the analysis of this lesson was that we only have one data sources that is the video registration. We only can observe what happen in the class and describe it as we already provided above. We cannot triangulate it with students’ written work since in this activity there was no task for the students that could be traced from their written work.

**Conclusion of lesson 4 analysis**

Based on the result of the discussion at this math congress we conclude that the students noticed about interpreting the taking a part of a part activity as the multiplying a fraction with another fraction. The discussion also led students to realize that they could make a shift from the term of or part of into the multiplication symbol “×”. In addition, the result of this analysis also reveals that through the class discussion by looking at the relationship between the fractions in the solution list of taking a part of a part activity, the students recognize about the strategy for solving the multiplication of two fractions problems.

**e. Lesson 5- Card Games**

Lesson 5 was the last lesson in the learning sequence in this learning design. The activities in this lesson were a card game about solving multiplication of two fractions problems using an array model.

In this lesson the students were divided into two small groups. The Abu Bakar group consisted of Abdul, Adrian and Celvin and The Umar Group consisted of Arfan and Izmi. There were two parts in this lesson. First, the students worked the 4 set cards and discussed the solution together. Second, they worked on the 1 set of cards in which they should determine the problem when the shaded array was given.

**Activity 1 - The first 4 set of the cards**

The researcher explained the instruction and gave time for the students to understand it or had any question about it. The instruction of the game can be seen in the lesson plan for lesson 5 (Appendix C) and the problems and the array cards can be seen in Figure 5.25.

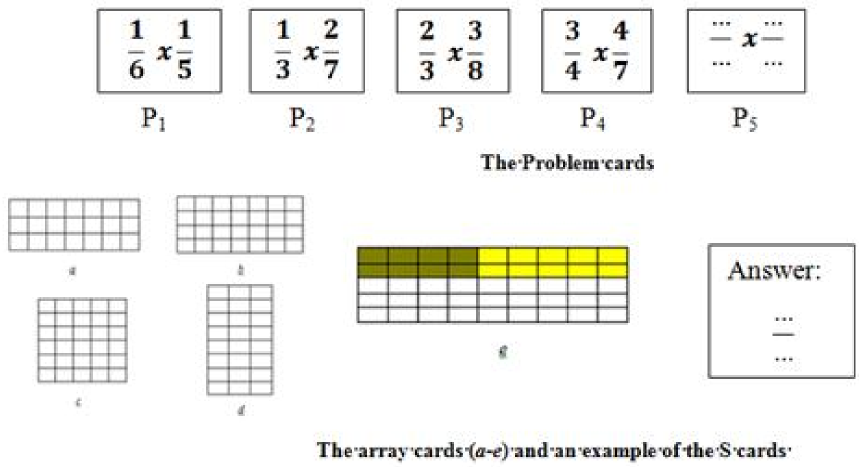



*Figure 5.25* A set of cards for the card games

The objective of this lesson was to assure that the students could choose an appropriate array to help them in solving the multiplication of two fractions problems. We also expected that the students could determine the fractional notation of the result of multiplication of two fractions on the array.

In the teaching experiment, the researcher shared the green cards first which contained the problems that should be solved by the students along with the yellow cards for writing the fraction notation of the answer. While the researcher prepared the blue cards, the cards with the arrays on it, the students started to look at the problems on the green cards.

From the observation, we can see that the Umar group started to figure out the answer of each problem on the green cards. They just solved it by thinking without making any scrap on the paper. It seemed that they started to use the strategy that already discussed in the lesson 4. They just multiplied the numerators and did the same ways to the denominators. Meanwhile, after looking at the fellow group, the Abu Bakar group also started to use the same strategy to determine the answer of the problems.

After the researcher shared the arrays within the blue cards the students discussed which arrays were appropriate to each problem. The Umar group could determine the pairs of the cards in a short time. In the discussion, the researcher invited the Umar group to share their strategies to the whole class. The Transcript 12 is a transcript of that discussion.

**Transcript 12**

*Situation* : *Umar group (Izmi and Arfan) shares their idea in the discussion. Izmi picks a blue card contains an array with dimension 3×8.*

Izmi : We already had the solution for the *problems (pointed at one of the yellow card)*. We try to find em the one with 8 in total. Then we count it, 1, 2, 3, 4, 5, 6, 7, 8 *(counting the number of rows of a figure on one of the blue cards).*

Arfan : Multiplied by 1, 2, 3 *(counting the columns in the array on that blue card).*

Izmi : Multiplied by 3, 8, 16.. ehm…

Arfan : [The number of the columns] multiplied by 8, [equals to] 24.

Izmi : Seven, the seven one *(looking for the other blue cards).*

Arfan : This is a wrong card [you picked up].

The researcher: Hm. The strategy is… emm, do you understand the strategy that was used by this group? I get their point. Can you [the other group] give comment to their explanation? Can you repeat it with your own words?

Celvin : We try to find the answer of this one first *(pointed at the problems in the green cards)* and then shade the array.

The researcher: But, which one that you shaded?

Celvin : This one *(pointed at the array)*. For example, this one is $\frac{1}{30}$ *(picked up a yellow card)*. We try to find the array with the number of small boxes is 30.

The researcher: Determine the number of the small boxes.

Celvin : This one is 1, 2,3 ,4 ,5 *(counting the number of columns in an array on the blue card)*. Then, 1, 2,3 ,4 ,5, 6 *(counting the number of rows in the array)*.

Arfan : 5 times 6

Celvin : 5 times 6 is 30, it means that one is for $\frac{1}{30}$.

*Figure 5.26* The Umar group work on the cards game

From the Transcript 12 we can interpret that Izmi (member of Umar group) still had difficulty explaining the strategy they used. He got the wrong result of multiplying 3 with 8 then his peer, Arfan, corrected it. But, since he got confused, he said that he needs to find the other array figure with contains seven rows. We do not know what he meant by saying that sentence. We did not explore it more in that discussion. However, when we observed their group work, we could conclude that they look at the number of fraction they had as the answer in the yellow cards and then count the total number of the cells in the array by multiplying the number of columns and the number of the rows (Figure 5.26).

In the next part of the Transcript 12, we can see that Celvin tried re-explain the strategy, he understood that he found the answer first and then tried to find an array figure that has the number of small pieces in it as much as the denominator of the result. He counted the number of rows and multiplied it with the number of the rows. Further, they just shaded the parts that were indicated by the numerator of the answer.

To contribute to the validity of the analysis, we provide students' written work on Figure 5.27.

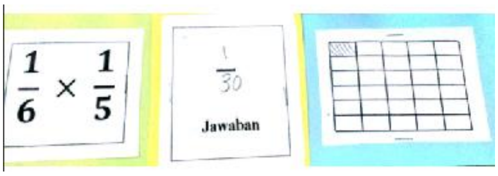



Figure 5.27 Students' written work on the card game

Based on Figure 5.27, we could see that the students determined the result of the problem and found the correspondence array. It is consistent with what we explain in the analysis of the video registration beforehand.

Based on the analysis above, we can conclude that the students already have ability to solve the multiplication of two fractions. They solved it by using the multiplication between the numerators and the multiplication of the denominators of the fractions in the problem. In choosing the suitable array figure for each problem, they look at the answer they wrote on the yellow cards, then tried to find an array which has the total number of small pieces in it as much as the number in the denominator. However, in shading the array, they did not show the first part they took from a whole unit. They directly shaded the small pieces regards to the number in the numerators of the answer. It only shows that they interpreted a fraction on an array not the taking a part of a part process.

**Activity 2 - Working on the last set of the cards**

The next activity was to work on the last set of cards. The difference between this activity and the previous one was there was no problem on the green card. Instead, the blue card consist a shaded array. The students' task in here was to determine the problem and also the final result based on the given shaded array as can be seen in Figure 5.28.

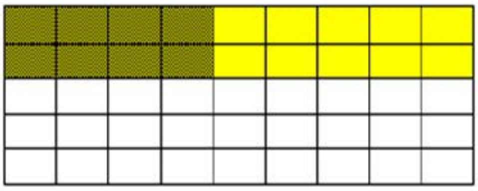


*Figure 5.28* The given shaded array in the card game

The teacher explained about the shaded array to the students. The light yellow parts were the part that shaded once and the dark yellow parts were the parts that shaded twice. Almost all of the students confused and need more time to understand the information. After a while, the Abu Bakar group, could interpret the shaded array after using trial and error strategy, they got $\frac{8}{18} \times \frac{2}{5}$ or $\frac{4}{9} \times \frac{2}{5}$. Meanwhile, Umar group struggled to determine the problem of the array. They took too much time explored and discussed what is the meaning of the shaded array. To overcome that condition the researcher gave hints to assume that at first the rectangle was blank and it was divided into some parts, then some parts of it were shaded once, it was indicated by the first two rows that was shaded with a light yellow color. Then, these parts were divided again into some equal parts and it was shaded to get the dark yellow parts.

The discussion between Izmi and Arfan was continued. It was transcribed in Transcript 13.

**Transcript 13**

| | |
|---|---|
| Izmi | : Assume that, it is a blank box. There is no this one. *(Pointed at the columns and the rows in the array)* |
| Arfan | : Then? |
| Izmi | : We draw these lines 1, 2, 3, 4, and 5 *(pointed at the rows in the array)* we shaded 2 parts. [It means] $\frac{2}{5}$. |

Based on Transcript 13, we can see that Izmi already got the idea about the light yellow part. He explained to his pair Arfan that first they divide the whole rectangle into 5 rows and shaded 2 rows of it that is the shaded parts. It seems that they could determine that at first they had the $\frac{2}{5}$.

However, Izmi and Arfan still struggled on how to interpret the fraction of the dark yellow parts. The researcher went to that group and supported them to interpret the shaded park. The researcher invited them to look at the initial yellow parts and then asked that the initial yellow parts were divided into what part. Finally the students recognized that they divided the yellow parts into nine equal parts and shaded again 4 parts of it. They came up with $\frac{4}{9} \times \frac{2}{5}$ as the final answer.

As the conclusion of this activity, we could see that this problem more complicated for the students. They need more time to explore and find what the figure mean. The shaded array we provided made students confuse, since there were three parts, the dark shaded, the light shaded and the un-shaded part. After the guidance from the researcher, the students could find the multiplication of the two fractions of the array figure, but it seemed that some of them still not convincing enough. They also did not get the sense of taking a part of a part of a whole unit in this activity.

**Conclusion of lesson 5 analysis**

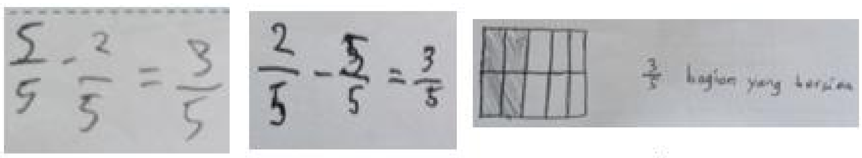
Based on the analysis of the lesson 5, we could conclude that the first part of this game card gave students experience to choose an appropriate array in solving the multiplication of two fractions problems. Although some of the students started using the strategy of solving the problems as they did in lesson 4 without looking at the array first.

Furthermore, the last part of the game card where the shaded array was given shows that most students had difficulties in understanding the array figure since the figure was complicated then they had before. However, based on the guidance from the researcher the students could determine the intended multiplication of two fractions as it was represented in the shaded array drawing. In addition, we could interpret that through this last problem the students could not see the taking a part of a part of a whole within the array drawing.

### 3. Analysis of the Result of Post-test after Conducting Cycle 1

The post-test was conducted in order to see students' understanding about multiplication of two fractions after the students involved in the five lessons of the cycle 1. There were seven problems in this test that should be finished by the students in 35 minutes. The following was the analysis of the students' written work along with the interview that we conducted after the post-test.

The problem 1 in the test was about sharing a *Bika Ambon*. The task 1a was to determine the leftover when Pak Gunawan and Bu Susi eat $\frac{2}{5}$ of the *Bika Ambon* cake. The task for 1b was to determine the part of each child of Pak Gunawan when they share the leftover of the cake equally for Andika and Audi. For problem 1a, there are two different strategies that were shown by the students. Abdul, Izmi and Arfan use a fraction subtraction to find the answer (Figure 5.29a), meanwhile Adrian and Celvin only drew and shaded an array to indicate the result of this problem (Figure 5.29b). Although the students who use the subtraction strategy have the same final answer, only one student did the subtraction correctly.

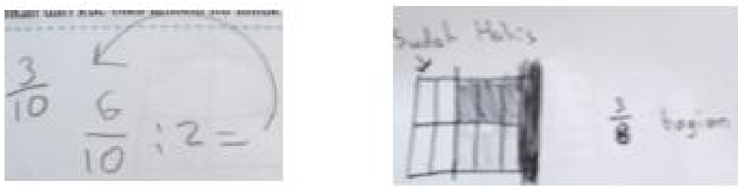


a b

*Figure 5.29* Students' written works of problem 1a

Based on Figure 5.29a, we interpret that the students know about the whole cake is equal to $\frac{5}{5}$. However, although the students who use the subtraction strategy have the same final answer, only one student did the subtraction correctly. He subtracted the $\frac{2}{5}$ from the $\frac{5}{5}$ and he got $\frac{3}{5}$. He did not use an array figure to help him. Moreover, based on Figure 5.29b we can interpret that Celvin and Adrian could represent the problem in an array and indicate the left over on it but Celvin did not interpret the result of the drawing into a fraction notation.

The students solved the problem 1b. And there are also two different strategies that were used by the students, drawing an array and using division. They got $\frac{3}{10}$ or indicate in the array the part for Audi and Andika. The Figure 5.30 is examples of the students' written work on problem 1b.




a b

*Figure 5.30* Students' written work of problem 1b on post test

As can be seen in Figure 5.30a, we interpret that at first the students tried to find an equal fraction of $\frac{3}{5}$, he got $\frac{6}{10}$. We assume that it is to make it easier for the student to divide by 2 because it is stated in the problem that the leftover is divided equally for Andika and Audi. This student came up with $\frac{3}{10}$

as the result. The other example of student's answer is on Figure 5.30b, we interpret that this student could represent the problem by drawing an array to show the sharing of the leftover of the cake into two equal parts. However, he misinterpreted the result of the partitioning, he came up with $\frac{3}{8}$ as the answer.

The problem 2 was about sharing a chocolate block. There were three sub tasks in this problem. The task 2a was to determine the part in a fraction notation if the chocolate block is shared among three children equally. The task 2b was to show the answer of the problem 2a into a figure. And the last task (2c) is the extended of the story of the problem when one of the children shared his part with his two brothers.

All of the students solved the problem 2a and 2b correctly. Most of them directly write that the parts of each child are equal to $\frac{1}{3}$. One of the students had a different fraction notation that was $\frac{3}{9}$. It is because he drew an array with dimension $3 \times 3$ then he represented that 1 child will get $\frac{3}{9}$ of the chocolate block. For the subtask 2c, only two students had the correct answer. They used an array to help them in solving the problem. The other two students only indicate the answer in the drawing and the last student misinterpreted the problem so he could not come up with the right answer. The Figure 5.31 is the examples of the students' written works on problem 2c.

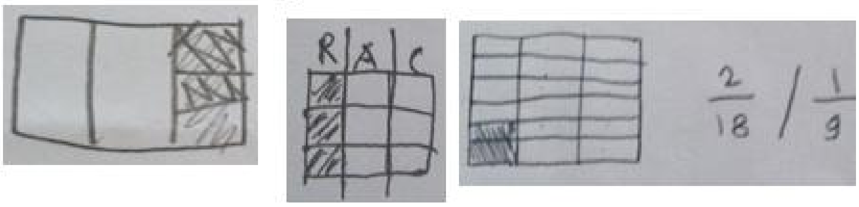



*Figure 5.31* Examples of students' written work on problem 2c in the post-test

Based on Figure 5.31 and the students’ written work, we can conclude that most of the students could use the array model to help them in solving the sharing the chocolate block problem. They could indicate the partitioning process properly. They know that they have to split the whole chocolate block into three equal parts and then they divide one part of it into three. However, not all of them drew the complete array and interpret the result in a fraction notation as can be seen in the Figure 5.31 above.

The third problem of this post-test is to determine $\frac{2}{9}$ of $\frac{3}{4}$. The two strategies, drawing an array and formal multiplication, also appear on the students written works as can be seen in Figure 5.32. Two students use an array and the other students used formal multiplication.

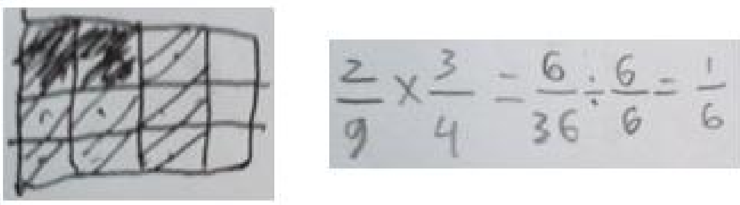



*Figure 5.32* Examples of students' written works on problem 3 of the post-test

Based on Figure 5.32, we can interpret that the students could use the array model properly and they also could interpret the result in the right fraction notations. Moreover, some of the students also could directly interpret the taking a part of a part as the multiplication of two fractions. And as can be seen in the figure, they could solve the multiplication of two fractions by using its procedure.

The last problem was a formal multiplication of two fraction problems. The students should determine the answer of $\frac{3}{7} \times \frac{2}{5}$. All of the students answered this problem correctly with $\frac{6}{35}$ as the final result. None of them used an array model in solving this problem. Instead, they directly found that the result is $\frac{6}{35}$. In our interpretation the students used the procedure of the multiplication of the numerators and multiplication of the denominators.

Based on the description of the analysis of the post-test, we conclude these findings:

a. Students could do a partitioning activity properly in the array.
b. The students could show that the partition should produce equal size parts.
c. The students could use an array model to help them in solving a taking a part of a part problems.
d. The students could solve multiplication of two fraction problems by doing the formal procedure.
e. Students use the array to help them finding the answer for taking a part of a part problem.

## 4. Summary of the Cycle 1 Analysis

In this part we summarize the remarks that we found during the cycle 1 implementation especially on the design of the learning activity and its learning materials. The first remark is about the figure of the hiking trail in the first page of the worksheet. We found that our students got confused to put the mark for the location of each post and flag, some of them did an unnecessary action by redrawing the figure on the answer box. It took too much time and we recommended it to be revised for the cycle 2.

The second remark is about the figure of the ribbon representation in activity 2 of lesson 1. We found that our students could not recognize the figure in problem 2 of the worksheet 1 as the representation of the hiking trail and the position of each flag and each game post. They got confused about what does the figure mean and what should they do. We should revise the materials so that it is clear for the students about the figure as the representation of the hiking trail in a ribbon.

The third remark is about the content in the lesson 2. We found that the time allocation in one meeting is not enough to conduct the lesson 2. Therefore, we need to rearrange the content so that it fit with the time and in the same time we still can reach the learning goal of the lesson.

The fourth remark is about the integration of the lesson 4 and lesson 5. We found that the math congress where the students made reflection on what they have learned could be done in about 35 minutes. It is suggested that it would be better if we combine the math congress with the lesson 5.

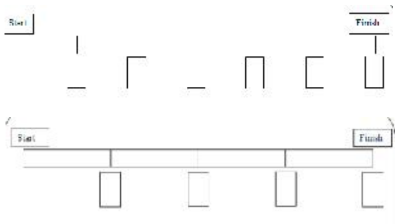
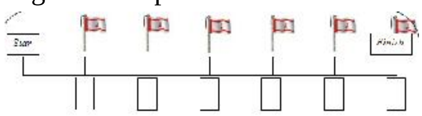
Furthermore, in lesson 5, we found that the last problem of the card games is not support our students in reaching the goal about experiencing the use of an array to solve the multiplication of two fractions problem. The last problem about determining the multiplication problem of the given shaded array made our students confused. They took too much time in this part and it is not the focus of this lesson. We recommended to omit this last problem.

## D. The Improvement of the HLT and the Learning Materials for the Cycle 2

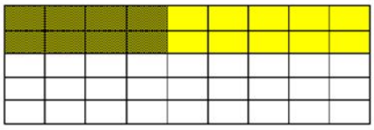
In general, based on the analysis of the first cycle we revised some activities in some lessons and some parts of the learning instruments. The five lessons in the initial HLT were revised to be four lessons in the cycle 2. We provide the general overview of the improvement in the Table 5.8 as follows.

*Table 5.3* The refinement of the HLT for the cycle 2

| **In the cycle 1** | **Refinement for the cycle 2** |
|---|---|
| **Lesson 1**<br>Three tasks about hiking trail to propose the initial understanding of the students about partitioning activity, labeling the result of the partition with fractional notation, and the idea of part-whole relationship.<br><br><u>The learning material</u><br>- Activity 1: The hiking trail figure in worksheet 1 is placed in first page and there is an answer box after the problem. | **Lesson 1**<br>We kept the context and the three tasks as the same as in the cycle 1.<br><br><u>The revision of the learning material</u><br>- Activity 1: The hiking trail figure is put after the problem and there is no answer box. The students directly put the marks for the location of flag and post in the |

| In the cycle 1 | Refinement for the cycle 2 |
|---|---|
| The size of the ribbon is bigger than the size of the hiking trail in the worksheet<br><br>- Activity 2:<br><br><br>The HLT<br>- Activity 1<br>There is no conjecture of students' strategy in which they overlap the unfold ribbon into the hiking trail. | given figure.<br>The sise of the ribbon is decereased so that it has similar size as the size of the hiking trail<br><br>- Activity 2: we add the visualization of the flags and the posts<br><br><br><br>The revision of the HLT<br>- Activity 1<br>There is an additional conjecture of students' strategy in which they overlap the unfold ribbon into the hiking trail |
| **Lesson 2**<br><br>There are four activities in this lesson; sharing chocolate block among three children, determining the time for reaching Aufa's house which is a third of the jogging time, taking a part of a part of the chocolate block and some exercise of taking a part of a part of a chocolate block involving non unit fractions.<br><br>The learning material<br>The initial worksheet 3 contains a task to solve the sharing chocolate block among three children by drawing their own array.<br><br>The HLT<br>- Activity 2: There is no discussion about introducing the context of time for | **Lesson 2**<br><br>We keep the activity of *sharing chocolate block*, taking a part of a part of a chocolate block and the *time for exercises*. However, for the last activity about several problems in determining a part of a part of several chocolate blocks with different dimension would not be discussed completely. We would ask students to do it at home as the homework.<br><br>The revision of the learning material:<br>The problem in the initial worksheet 3 about students solve the sharing chocolate block among three children by drawing their own array is not used.<br><br>The revision of the HLT<br>- Activity 2: The teacher holds a discussion to check whether the students understand |

| In the cycle 1 | Refinement for the cycle 2 |
|---|---|
| jogging. It lead to misinterpretation of the students.<br>- Activity 3: It is conjectured that to find the fractional notation, the students would think about how many times the intended part fit to the whole unit. | the instruction or not in introducing the context about time for jogging.<br>- Activity 3: When the conjecture in the cycle 1 not appear, the teacher should engage students to discuss about how many times the intended part fit with the whole unit. |
| **Lesson 3**<br><br>Three activities about sharing martabak telur where the students start constructing their own array, choosing an appropriate array and some exercise about taking a part of a part problems.<br><br><u>The HLT</u><br>- Activity 2: The students reason about the choosing an appropriate array dimension to solve the taking a part of a part of the *martabak telur* properly. | **Lesson 3**<br><br>We keep the context and the students' tasks in the three activities of this lesson the same as in the cycle 1.<br><br><u>The revision of the HLT</u><br>- Activity 2: If the conjecture in the cycle 1 not appear, the teacher engage students to do a trial and error strategy on the three available figures and make a reflection of the result. |
| **Lesson 4**<br><br>A math congress where the students make a reflection about the taking a part of a part of a whole activities they have done in the previous lesson.<br><br>Students discuss the idea about interpreting the taking a part of a part as the multiplication of a fraction with another fraction<br><br>**Lesson 5**<br><br>A card game where the students tried to solve the multiplication of two fractions and choose the appropriate array in helping them in solving the problems.<br><br><u>The learning material</u><br>- Activity 2: The last problem in the card game is about determining the | **Lesson 4**<br><br>Considering the time used for the lesson 4 and lesson 5 in the cycle 1 which only took 70 minutes for both, we decided to combine these two lessons. We arrange that there would be 2 activities in this lesson. The first activity is the math congress in order to support students to make a shift from the term part of into term times which is symbolized with "×". The second activity is the card game where we support students to experience the use of array model in solving the multiplication of two fractions problems.<br><br><u>The revision of the learning material</u><br>We omit the last problem in the card game. |

| In the cycle 1 | Refinement for the cycle 2 |
|---|---|
| multiplication of two fractions when there is a shaded array given. The shaded array figure is the following.  | |

## E. The Result of the Cycle 2

The participants in this cycle 2 were 25 students of Class 5C SD Al Hikmah Surabaya. The teacher was the mathematics teacher who conducted the four lessons we designed. In this cycle we focused on observing the learning process of a focus group consist of five students; Afdal, Khairul, Dani, Defri, and Didi. The consideration of choosing this focus group was based on the discussion with the teacher and also based on the similarity of their character with the students in our students in cycle 1 of this study.

### 1. Prior Knowledge of Students of the Cycle 2

We gave a pre-test for all of the students in class 5C before conducting the lessons in order to collect information about their prior knowledge in multiplication of two fractions topic. There were four problems in this pre-test. Some of the problems have two or three subquestions. In total there were seven items. The complete items of this pre-test can be seen in Appendix G.

The problem 1 was about sharing a tart cake. The task for the students was to indicate in the given rectangle $\frac{1}{2}$ of $\frac{1}{3}$ of the tart cake. Based on students' written works, there were various kinds of solutions came up from the students. In general almost all of the students tried to divide the rectangle in regular pattern become rows and column, only one of them divide it irregularly. However, only nine students have a correct drawing. These nine students could make a clear partition in the rectangle. In our interpretation the students divided the rectangle into three equal parts, then split one of the parts into two and they shaded it to indicate the intended part. This interpretation is supported by the explanation of some students in their work about this explanation. However, some other students who have the correct drawing provided calculation to find the fractional notation of the part by using subtraction between $\frac{1}{3}$ and $\frac{1}{2}$ and they could not get the right answer. Some students interpret that they show a half of the whole cake and also a third of the whole cake. Then, they add it up (4 students) or subtract it up (4 students) as the answer.

Two students draw a 2 × 3 array and shaded one row, they wrote that they determine the partition in the rectangle by finding the LCM of 2 and 3. But they did not give a proper explanation why they should shaded one row in the rectangle. Three other students could not give a correct drawing and they also did not give an explanation in their written work.

Based on the analysis of the problem 1 above, we can conclude that most of the students know that the partitioning should be in a regular way in order to produce equal parts. However, only some of them could take a half of a third of the rectangle it implies that only some of them started to use the model (the drawing) to solve the taking a part of a part problem. In addition, most of the students had difficulties in interpreting the result of the partition into a fraction notation.

In the problem 2, the information was that there are 40 pupils in class 5 SD Tanah Air. A half of them are male pupils and a quarter of the male pupils like playing football. The students' task in this problem was to determine the number of male pupils who like playing football and determine what part of the whole number of the pupils like playing football.

Fourteen students in our cycle 2 class have a correct answer for the first sub-question in the problem 2. Most of them get 5 pupils as the answer by doing two divisions, first they divide 40 by 2 and got 20 then they divided the 20 by 4. In addition, four of them started to use the fractions and surprisingly three of these four students use a multiplication of the fraction with the number of pupils to get the answer.

Furthermore, for the second sub-question in the problem 2, only 6 students could answer it correctly. They related the number of male pupils who like playing football with the total number of pupils. They got $\frac{5}{40}$, some of them simplified it became $\frac{1}{8}$. The other students could not answer this question, some of the students did not relate the answer to the total number of the pupils of class 5C SD Tanah Air. Some other students could not give the answer.

Based on the analysis of the students' written answers to the second problem we can conclude that most of the students in our cycle 2 still could not see the part-whole relation clearly. Although more than a half of the students could determine the result of taking a quarter of a half of 40 pupils, but they could not recognize that it is as a part of a part of a whole activity. The students still tended to do division between whole numbers. Only a few of them start to see this problem as taking a certain number of the total number in the whole unit.

The problem 3 in this pre-test was about sharing a chocolate block between two children, Ridho and Roni. Later, Roni share a third of his part for his sister Rosi. The students should answer three sub question in this problem. First, the students need to indicate the initial part of Roni and Ridho in a given array with dimension 4×6. Second, they need to show in the given array the parts for Rosi. Third, the students need to determine the fraction notation of Rosi's part.

Fifteen students could solve the first and the second sub questions in the problem 3 correctly. Based on the drawing on their written works we can see that they split the whole chocolate block (the array) into two equal parts vertically, three columns for Ridho and three columns for Roni. Then, they shaded one of the three columns of Roni as the chocolate part for Rosi. There is an interesting figure of one student in these fifteen students, to determine the parts of Rosi he did not shade a column of Roni's part, but he shaded four cells of it as can be seen in the Figure 5.33.

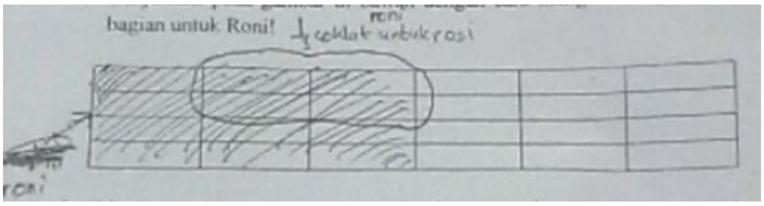



*Figure 5.33* Students' written work on problem 3b of the pre-test of the cycle 2

Based on Figure 5.33 we could see that the student knew about a third of the Roni parts equals to four cells of the block. He circled four cells of the Roni's part and Wrote $\frac{1}{3}$ to indicate these parts.

Furthermore, we identified two common mistakes of the students who had a wrong answer for subquestion 2 of problem 3. The examples can be seen in Figure 5.34.

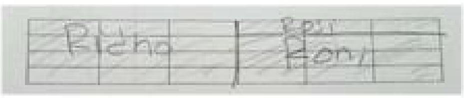


*Figure 5.34* Students' common mistake on solving problem 3b of the pre-test

From Figure 5.34 we can see two examples of wrong solution of the students. First, four of our students only shaded three cells of Roni's part. In our interpretation these students thought about interpreting a third of Roni's part as three small pieces (cells) of Roni's part. The remaining four students seemed that they misinterpreted to the given instruction. Instead of indicating the part of Rosi from Roni's part they shaded one column of the Ridho's part.

For the next sub question there are only nine students gave a correct answer. Four of them write $\frac{4}{24}$ as the part of Rosi respects to the whole chocolate block. In our interpretation they got this solution by relating the number of cells in the array for Rosi with the total number of cells in the array which represent the initial chocolate block. The other five students answer with $\frac{1}{6}$. There are two strategies the used to get this answer. First, the students used the same strategy as we explained beforehand and they got $\frac{4}{24}$, then they simplified this fraction became $\frac{1}{6}$. Second, the students made a drawing and indicate the Rosi's part in the drawing as can be seen in Figure 5.35.

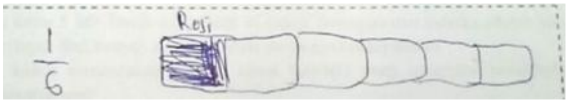
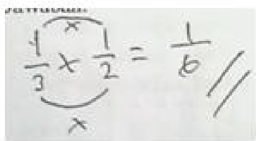
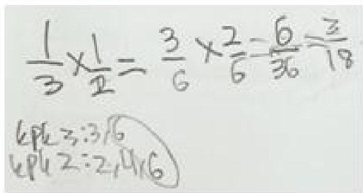



*Figure 5.35* Students' strategy on solving problem 3c of the pre-test in the cycle 2

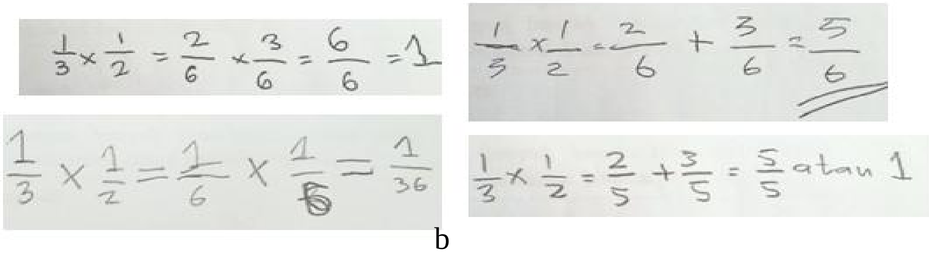
Based on Figure 5.35 we can see that the students drew a bar and divided it into six equal parts. Further, they shaded one part of the bar and wrote $\frac{1}{6}$ to indicate the intended part. In our interpretation these students tried to represent the chocolate block in the array into a bar to help them seeing the Rosi's part easily.

Based on the analysis of the students' written work of problem 3 in this pre-test we conclude that most of the students could do the partitioning in the given array properly. Their answers to sub-question 1 and 2 show that they could do the partitioning in the array in order to show the taking a third of a half of the chocolate block. However, only some of the students could interpret the result of taking a part of a part of a whole partition into a fraction notation. Moreover, only a few of them use the drawing to interpret the intended part into a fraction notation.

The last problem in this pre-test was to determine $\frac{1}{3}\times\frac{1}{2}$ . There are various answers from the students' they are $\frac{1}{6}$ (11 students), $\frac{3}{18}$ (1 student), $\frac{6}{6}$ or 1 (5 students), $\frac{5}{6}$ (1 student), $\frac{2}{3}$ (1 student), $\frac{1}{36}$ (1 student), $\frac{5}{5}$ or 1 (1 student) and the other two students did not give their answer. The Figure 5.36a is the examples of students’ correct answer and Figure 5.36b is the examples of students who did not give a correct solutions.

a




b

*Figure 5.36* Students' works on problem 4 of the pre-test in cycle 2

Based on the explanation in the previous paragraph and supported by Figure 5.36a we can see that almost a half of our students (11 students) could solve the multiplication of two fractions directly. In our interpretation these students know that they could do multiplication between the numerators and the denominators of the fractions (see Figure 5.36a). One of the students made an interesting strategy in solving this problem. He also multiplied the numerator of the first fraction with the numerator of the second fraction and multiplied the denominator of the both fractions, but first he changed the fractions in the problem with its corresponding fractions which have the same denominator that is $\frac{3}{6}$ and $\frac{2}{6}$. Then, he multiplied 3 with 2 and 6 with 6 to get $\frac{6}{36}$ and simplified it became $\frac{3}{18}$. To find the common denominator, this student determined the LCM of 3 and 2 as can be seen in the second part of Figure 5.36a.

Based on the examples of the incorrect answers of the students (see Figure 5.36b), we can see that most of them influenced by their notion in solving addition of fraction. They tried to find the common denominator of the fractions and then modified the initial fraction is as the procedure in solving the addition of fractions. Further, they only multiplied the numerators and got $\frac{6}{6}$ or a few of them add the numerators and came up with $\frac{5}{6}$ as the answer.

Therefore, as explained in the analysis of the students' strategies in solving the last problem in this pretest we can conclude that a half of our cycle 2 students could solve multiplication of two fractions problem using a formal procedure. However the others still think that they could use the algorithm of solving addition of fractions in this problem.

After looking at the analysis of the pre-test result, we could make a general conclusion as follows.

a. Some of our students in this cycle 2 could make a partition properly in the given figure.
b. Some of them could show the process of taking a part of a part of a whole unit in the given array, however, they still struggle on interpreting the result of taking a part of a part of a whole into a fraction notation.
c. The students' written works also show that some of the students have an idea of constructing their own array based on the context of the problem.
d. Moreover, in solving a multiplication of two fractions problem, only a half of them could solve it in a formal way, the others tended to use the algorithm of solving an addition of fractions and came up with incorrect

answers.

## 2. Data and Analysis of the Students' Learning Process in Cycle 2

The analysis of the cycle 2 was conducted by confronting the actual learning process of the students with the improved HLT. The HLT can be seen in the chapter IV and the improvement of the HLT can be seen in part D of this chapter. In the HLT we already described the goal of each lesson along with our expectation and conjectured of students' thinking, therefore in this part we directly analyze and just refer to the HLT. In addition the learning materials and the lesson plan for this cycle 2 can be seen in the Appendix I and J.

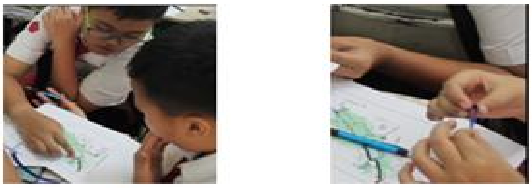
### a. Lesson 1- Partitioning

There are three activities in this lesson which is related to each other; determining the location of flags and posts along the hiking trail; notating the fraction of the position of each flag and post; and determining the distance between the starting line and the first post. Through this activities we expect the students to experience the partitioning activity and interpret the result by using fractions and get the notion of part-whole relationship. The problems in this lesson refer to the problems in the Worksheet 1 in the appendix J. As a starting point, the teacher introduced the context about a scout club to the students and invited them to share about what kind of activities a scout club usually had.

**Activity 1- Locating the flags and the game posts**

The students’ task in this activity was to determine the location of 6 flags in a hiking trail figure which the distance of each flag is 1 km. They also needed to put 4 game posts with equal distance along the trail. The total length of the trail is 6 km. The aim of this activity was to support students to be able to partitioning activity. We conjectured that the students would use the folding ribbon strategy to produce equal size parts and try to overlap it into the figure to give mark for its position.

Students seemingly understand about the context in this lesson. Based on the observation we can see that the students are familiar with the hiking trail as one of the activities of the scout club. The teacher started to introduce the problem of the activity 1 and asked students to work in their small group focusing on the first and the second problem in the worksheet 1. Each student was provided with 2 pieces of ribbons and the teacher said that they could use the ribbon to help them in solving the problem. Figure 5.37 shows students work on problem 1 of worksheet 1.



a b

*Figure 5.37* Students work on problem 1 of worksheet 1

In the teaching experiment, there were students in our focus group started to do estimation about the location of the flags (see Figure 5.37a). However, they were not convincing enough about whether the position of the flags has an equal distance to each other. One of them asked about the use of rulers, but his friends reacted that they only could use the given tools, ribbon. Based on the observation, similar with what our cycle 1's students did, in the middle of the group work there were two students had an idea of folding the ribbon (see Figure 5.37b). The following is the transcript of discussion about the folding strategy.

**Transcript 14**

| | |
|---|---|
| The teacher | : Could you make the four same distances in the ribbon? How will you do that? |
| Students | : (*the students do not seem understand with the teacher's question* |
| The teacher | : If there is a ribbon, then you have to divide it into four same parts, how will you do it? |
| Dani | : Oh ya we have to fold it.. (*he folds the ribbon into two same parts then folds it again into four same parts*) |
| Khairul | : (*he folds the ribbon into four same parts and marks the folding line in the ribbon*) |
| Defri | : Oh ya I get it |

Based on the Transcript 14, we could see that the students recognize the idea of folding the ribbons. It showed them that the result of the partitioning they did produce the equal size parts.

Furthermore, our students could not transfer the folding ribbon into the trail since the trail is not a straight line. We notice that not as in cycle 1 where the students got the idea of overlapped the ribbon into the drawing by themselves in this cycle 2 the teacher reminded the students about making an equal distance in the hiking trail figure and could use the overlapped strategy.

In addition, the width of the ribbon which is smaller than what we used in the cycle 1 makes the overlapping process easier for the students.

To support the description above, Figure 5.38 is the example of students' written works in determining the location of the post in the figure. It shows that the data collected through two different methods are consistent.

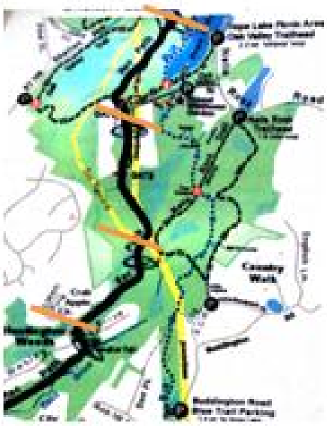


*Figure 5.38* Students' written work of problem 1 in lesson 1 of cycle 2

Furthermore, the activity continued to determine the position of the flags. The students used the similar strategy to produce six equal parts of the ribbon. But, the teacher did not give enough time for the students to explore it more. The teacher directly invited the students to make conclusion about the answer of the activity 1. It was about making four equal parts of the location of the posts and six equal parts of the location of the flags in the ribbon.

Based on the description of the first activity above, we can conclude that the students could do partitioning activity properly. They understood about the result of the partitioning activity should be of equal size. Some of them could come up with the idea of folding the given ribbon to produce the equal parts of the ribbon, but they struggled on how to transfer it into the hiking trail figure because it is not a straight line. To deal with it, they overlapped the ribbon following the path of the trail as the students in our cycle 1 did.

**Activity 2- Making fractional notation of the result of the partitioning**

In the next activity, the teacher invited the students to look at the ribbon representation in the answer of problem 1. The representation of the ribbon is two bars for the location of the posts and the location of the flags. These bars had already drawn on the whiteboard. The teacher asked students to work on problem 2 and they could discuss in their group. The aim of this activity was to support students to be able to label the result of the partitioning activity they did with a fractional notation. Our expectation is the students use fraction in an ordinal way to solve this problem.

In the teaching experiment, the teacher introduce the problem by stating that the students should find the answer regards to fraction notation by stating about what part of the bar regard to fractions term. It made the instruction of the task clear for the students. Moreover, based on the observation, our students did not confuse about the figure we provided in the worksheet 1 since we already added the figure of flags and game post into the bar so it is clear for the students that these are the representation of the trail in a bar model. The students got the idea of using appropriate fraction notation to indicate the position of each flag and posts in the bar representation. The Transcript 15 is the transcript of the discussion in our focus group.

**Transcript 15**

| | |
|---|---|
| Defri | : [For the problem 2a] there are 6 boxes, 1, 2, 3, 4, 5, 6. So it means $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}$ |

*(the other students agree to this idea, then they continued to the problem 2b about locating 4 game posts)*

| | |
|---|---|
| Dani | : 6 divided by 4, It is equal to? |
| Afdal | : 6 Divided by 4? *(Look a bit confuse)* |
| Defri | : 1.5 It means… |
| Dani | : 1 $\frac{5}{6}$ |
| Defri | : 1.5 over…? |
| Khairul | : Hmm, my answer is $\frac{1}{4}$ |
| Defri | : yeah, we can also write under this fraction |
| Dani | : Is it $\frac{1}{4}$? |
| Khairul | : Yes, it is. |
| Dani | : Hmm, because it is 6 km. Hmm the over here (pointed at the denominator) should be 6. |
| Khairul | : No, it should not. Since this is divided by 4 and the question is about what part of. |

Based on Transcript 15, we can see that most students understand that they need to give a label of each part regard to the total parts of the partition results. However, in our interpretation Dani got confused when he tried to answer the problem 2b which was about the location of the posts. He thought that the denominator of the fractions should be 6 not 4 because the length of the trail is 6 km. In our interpretation, Dani also had the same point of view with his answer to the problem 2a. He used 6 as the denominator because of the total length of the trail not because the bar was divided into 6 equal parts. To deal with this, in the discussion transcribed in Transcript 15 we can see that Khairul helped Dani by reminding him about the instruction in the problem. Khairul stated that the denominator regard to the whole number of parts of the partitioning results. It implies that Khairul already got the idea about relating the parts to the whole unit.

Furthermore, in the class discussion, the idea of using the length also appeared from another group. One of the students of the other groups explained in front of the class that the first position of the post is in 1.5 km, the second post is 3 km and so on. He explained that at first he divided the total length by 4 it means one part is equal to 1.5 km. However, the teacher reacted to this answer by asking the students to state about what he refers to, whether it is part of the trail or part of the length of the trail. Based on this discussion the students grasped the idea that there were two kinds of answers depended on the whole unit they refer to, whether a trail as a unit or the length of the trail.

To contribute to the validity of this analysis, we provide the students' written work on this problem in Figure 5.39 as follows.

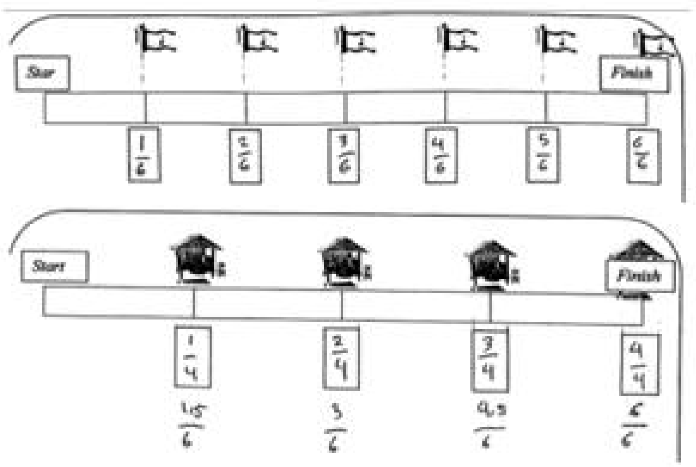



Figure 5.39 Students written work on activity 2 of lesson 1

The Figure 5.39 shows that the students wrote the fraction in an ordinal way for the location of the flags and post as we expected in the conjecture. Furthermore, they consider the whole unit when they determine the fraction in each box. These data show a consistency with what they discussed in Transcript 15.

Based on the description above, the activity 2 in this lesson promotes students' understanding about interpreting the result of partitioning in term of fractional notation, although the idea of using fractions was guided by the teacher, not came up from the students. Moreover the students started to see the relation between the parts and the whole unit which was represented in the bars.

**Activity 3- Determining the distance between the starting line and the first post**

Activity 3 was the last activity in this lesson. The students' task was to determine the distance between the starting line with the first game post. The teacher asked the students to work in their group and discussed the strategy they used before they shared it in front of the class in the whole class discussion. The aim of this activity was to support students to be able to get the idea of part-whole relationship. We expected students to start use fractions in their operation and grasp the initial idea about part-whole relationship

In the teaching experiment, our conjecture about the use of fractions notation was not appear yet in the solution of our focus group students. All of them started by drawing a bar to represent the hiking trail and then divide the bar into four equal parts as our students in cycle 1 did. The students did a division between the whole numbers. They divided the total length of the hiking trail by the number of the game posts. The students got that each part of the bar is equal to 1.5 km. Further, some of the students also determined the distance between the starting line with the 2nd post, 3rd post and the last post.

To support the data from the observation above we provide the example of students' written work of problem 3 in Figure 5.40 below.

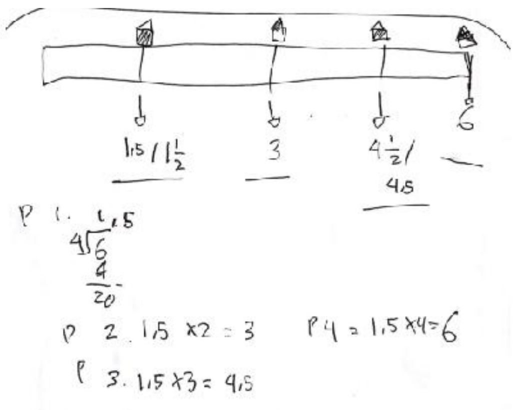


Figure 5.40 Student's written work on activity 3 of lesson 1

Based on the Figure 5.40 we could see that the students did a long division of 6 by 4 to get the distance of each part of the bar. Then they determine the distance of each post regarding the starting line. This implies that the students' written work in line whit what we described in the analysis of the video registration.

Since the students did not use the fractions on the calculation in the solution, the teacher invited them to think about using the fractions. The objective was to engage students to see the relation between the part and the whole in term of the fraction. The following Transcript 16 is about the class discussion of this problem.

**Transcript 16**

| | |
|---|---|
| The teacher | : It is $\frac{1}{4}$. For example, if we relate the answer with a fraction notation. How if we use the word a quarter. You use the one over four in your answer. How? |
| Students A | : 6 km times one over four. |
| The teacher | : Or in other words? Yes. It is one over four, then, since the total length is 6, we can conclude, one over four of 6 km? You can find the answer? 1.5 km. |
| The teacher | : If I asked you about this one until this one? *(Pointed at the starting line and the second post)* |
| Students | : 3 km |
| The teacher | : How can you get it? |
| Defri | : Yaa. I add 1.5 to 1.5 |
| The teacher | : The relation with fractions? |
| Students A | : $\frac{2}{4}$ of 6 km |
| The teacher | : $\frac{2}{4}$ of ? |
| Defri | : we add $\frac{1}{4}$ with $\frac{1}{4}$. |
| The teacher | : $\frac{2}{4}$ of ? |
| Students B | : 6 |
| The teacher | : $\frac{2}{4}$ of 6. What is the answer? It equals to3 km. Now, If it is like this one, It is 6, I split it up into 2, It means a half of it right, so a half of? |
| Students | : 6 |
| The teacher | : Of 6, what is the answer? |
| Students | : 3 |
| The teacher | : If I split it up again in 2, is it allowed or not? |
| Students | : Yes, it is. |
| The teacher | : So, if I split it up, is it allow or not? So it means a half of a half of 6. It could be or not? |

*(Some students said yes, the others said no)*

| | |
|---|---|
| The teacher | : To find this part, I split it up into two. At first a half of this one, a half of 6 then I split it up again. A half of a half of 6 |
| Students | : 1.5 |
| The teacher | : It could be or not? |

Students : Yes, it could be.

Based on the Transcript 16 we can interpret that there is a student came up with the idea of multiplication of a whole number with a fraction. However, the teacher not used the term times yet, instead he used the term part of. Moreover, to introduce the taking a part of a part of a unit activity, the teacher invited the students to think about how if they took a half of the trail first and then take a half of it again to find the distance between the starting line and the first post. The students could see that the result is the same with the answer of their divisional strategy that was 1.5 km. Further, at the end of the discussion, the students recognized that $\frac{1}{2}$ of $\frac{1}{2}$ of 6 km is equal to $\frac{1}{4}$ of 6 km.

Based on the analysis of the third activity above, we can conclude that initially most students did not use the fraction yet in solving the problems. They could not see the problem as taking a part of a whole unit. However, the teacher overcome this by inviting them to discuss about using fractions and introduced the taking a part of a part of a whole activity to the students.

**Conclusion of lesson 1**

Based on the description in the analysis of the lesson 1 of cycle 2, we conclude some remarks respect to the learning goal of this lesson. The activity of determining the location of the flags and the game posts along a hiking trail figure gave students experience of doing partitioning activity, they recognize that the result of the partitioning should be in an equal size parts. The notating the fraction activity could help students in producing fraction when they gave label into the result of the partitioning activity. Moreover, in the determining the distance of the first post and the starting line activity most students tend to use division and multiplication between whole numbers. They still had not started to see this activity as taking a part of a whole unit or using fractions in the multiplication process. To deal with that condition, the teacher introduced the taking a part of a whole unit in the class discussion. We interpret that the students could have an initial understanding about the part-whole relationship.

**b. Lesson 2- Taking a part of a part of a whole**

This lesson provides students with activities to start doing the partitioning within the array model (activity 1). The array model was introduced in a form of a chocolate block. We also started to engage students to the term part of a part of a whole within the context of time for reaching Aufa's house (activity 2). Further, we would apply the notion about the part-part-whole into the activity of partitioning a chocolate block (activity 3). At last, to strengthen students' ability on taking a part of a part of a whole, we provide them with some exercises (activity 4) The problems in this analysis refer to the problem of worksheet 2 as can be seen in Appendix J. The following are the analysis of each activity.

**Activity 1- Sharing chocolate block**

The activity 1 was working on problem 1 and 2 of worksheet 2. The teacher started with explaining the story about Hafidz preparing himself for the next hiking event. Hafidz planned to have an exercise once a week. He uses a half of the time for jogging. On a Sunday morning Hafidz jog to Aufa's house. His father gives a chocolate block for him and asks him to share it with Aufa and Siraj. The task was to show in the given array the parts for each child and determine the fractional notation of it.

The aim of the first problem in this activity was to let students indicate partitioning activity within an array model. The array model was introduced to the students in the form of the chocolate block figure. We expect students to produce three equal size parts in the chocolate block figure and relate the part with the whole unit in determining its fractional notation.

In the teaching experiment, based on students written works, we can see that all of the students in our focus group could show the parts correctly in the figure. As we conjectured in the HLT, similar with the students in cycle 1 did, in this cycle 2 the students divided the chocolate block into three equal parts vertically (see Figure 5.41). Furthermore, to interpret the result of the partitioning they did in the first problem there were two different strategies of the students. The Transcript 17 shows Defri argued his idea of solving the problem 2.

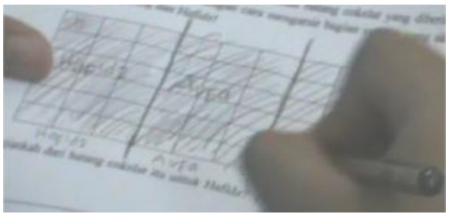


*Figure 5.41* Students work on problem 1 of worksheet 2

**Transcript 17**

| | |
|---|---|
| Dani | : Make it become a fraction notation? |
| Khairul | : 1, 2, 3, *(Counting the rows and the columns of Hafidz parts)*, 18 |
| Defri | : Yes, we can easily say that it is one over three. |
| Didi | : one over three? |
| Dani | : Yes, we can say it as one over three. |
| Defri | : Wait, wait, wait. 1, 2, 3, 4, 5, 6, 7, 8, 9 *(counting the number of columns in the drawing)*. 9 multiply with 1, 2, 3, 4, 5, 6 *(Counting the number of rows)*. 9 times 6 equals to 54. |
| Khairul | : We need to count it first, then simplify it. |

Based on Transcript 17, we can see that the students discussed about the fractional notation of the Hafidz's part. We can interpret that Defri understood if they divided the block into three equal parts, it means each of the parts equals to $\frac{1}{3}$. However, there was a student doubted about the solution, then to make it sure Khairul said that they could count it first then simplified it to get the final result. The Figure 5.42 is an example of student's work in this activity.

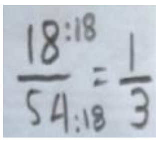



*Figure 5.42* Students' answer of problem 2 on worksheet 2

Based on Figure 5.42, we can see that the students did as it was conjectured in our HLT. They count the small pieces in Hafidz parts and in the whole chocolate block. They multiplied the number of the columns and the number of the rows. The final result was the number of small pieces in Hafidz's part over the total number of small pieces in the chocolate block. They got $\frac{18}{54}$ which is simplified became $\frac{1}{3}$. This answer is the same with the idea of Defri which directly said that the fraction is $\frac{1}{3}$ (see Transcript 17). This description implies that the students could realize about part and whole based on the context. They started to relate the part with the whole to determine the fractional notation of the intended part, as our students in cycle 1 did.

In term of validity of the analysis in this activity 1, we could see the consistency of the data collected through video registration which is transcribed in Transcript 17 with the data we provide from the students' written work in Figure 5.42. It shows that the students got $\frac{1}{3}$ as the final solution.

Based on the description of the activity 1 in lesson 2 we can conclude that this activity helped students start working with the array. The students could do the partitioning in the array and they could interpret the result in term of fraction notations. This result also reveals about students grasped the idea of the part-whole relationship, although they did not state it clearly in their written works.

**Activity 2- Time for reaching Aufa's house**

In this activity, students worked on problem 3 of the worksheet 2. The students' task was to determine the time that is used by Hafidz to reach Aufa's house when he arrived at Aufa's house after a third of his jogging time. The jogging time is a half of an hour. The goal of this activity was to introduce the part-part-whole relation to the students. We expected that trough this activity the students recognize about representing the taking a third of a half of an hour in a fractional notation.

In the teaching experiment, to overcome the misinterpretation that appears in the cycle 1, before doing the task in the small group, the teacher in this cycle 2 invited the students to discuss about the information in the story of the context. Transcript 18 shows the part of the discussion.

**Transcript 18**

| | |
|---|---|
| The teacher | : How long the time for jogging that is used? |
| Khairul | : 30 |
| Students | : Why it is 30? |
| The teacher | : Why it should be 30? |
| Khairul | : Because it is said that he use a half of the exercise time. |
| The teacher | : It means? |
| Khairul | : It means umm. He uses 30 minutes of the jogging time. |
| The teacher | : a half of 60, right? It is equal to 30 minutes. |

Based on Transcript 18, we can see that Khairul knew about the time for jogging because he took *separoh* (a half) of the exercise time. The teacher clarified Khairul's answer into a term a half of the exercise time. Because this was a part of the class discussion, it implies that all groups already had the same information that the time for jogging is 30 minutes. Further, the students started to work in their small group.

Based on the observation, the students in our focus group had a discussion about the time, there were students came up with 20 minutes as the time for reaching Aufa's house but he did not explain how could he get it. The other students in this group said that the time should be 10 minutes. The Figure 5.43 shows examples of students' written works on problem 3 of worksheet 2.

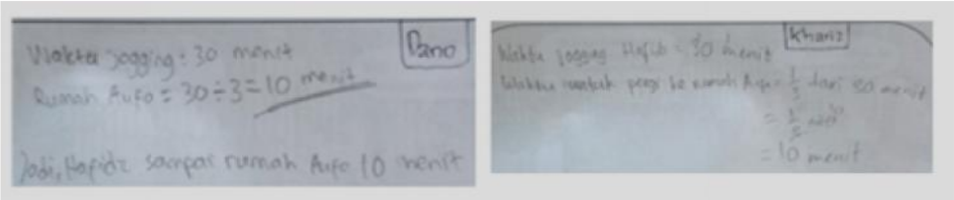



*Figure 5.43* Students' written work on problem 3 of worksheet 2

Based on Figure 5.43 we can see that the first student divided the time for jogging by 3 and got 10 minutes as the answer. We do not have data about why he directly divided it by 3. Meanwhile, the second student tried to take $\frac{1}{3}$ of the 30 minutes. He multiplied $\frac{1}{3}$ with 30 minutes to get 10 minutes. In our interpretation, it implies that both students know that they should take $\frac{1}{3}$ of the jogging time, however the strategy they used were different. In addition, Khairul started to relate the taking a part of a whole process as multiplication of a fraction with a whole number.

The conjecture in the HLT about students wrote the solution in the form of taking $\frac{1}{3}$ of $\frac{1}{2}$ of 60 minutes was not happening, then the teacher led a class discussion to address this point. He reminded the students to think about the relation between the time of reaching Aufa's house, time for jogging and the total time for the exercise. He also told that the students could relate it in term of fraction as they did in the previous problem. The key word that the teacher said to the students was about *"a part of a part of …"*. The figure 5.44 shows Defri explained in front of the class.

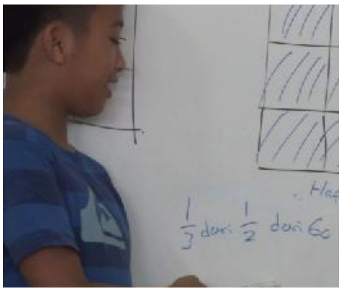


*Figure 5.44* Defri explain about the part of a part of a whole relation on problem 3 of worksheet 2

Based on the observation (supported by the Figure 5.44), we could see that the student recognize the idea of taking a part of a part of a whole unit. They understood that the process of finding the time of Hafidz to reach Aufa's house is the same as a taking $\frac{1}{3}$ of $\frac{1}{2}$ of the total time for exercising.

In conclusion, the activity 2 in lesson 2 shows that the students could start to understand about the taking a part of a part of a whole activity. Although at first they did not use fractions in their calculation on finding the time that is used for Hafidz to reach Aufa's house. Some of the students already started to interpret the taking a part of a whole unit as a multiplication of a fraction with a whole number. However, at that time, the teacher only tended to use the term part of something.

**Activity 3- Taking a part of a part of a chocolate block**

The next activity was about sharing the chocolate block. The teacher extended the story of Hafidz by inviting the students to read the comic 3 on the worksheet 2. The students' task was to determine the part of Nazifah when Hafidz share his chocolate parts with Nazifah equally. Problem 4 asked students to indicate the sharing in the given array. The array is the same representation of the chocolate block in the problem 1. Further, problem 5 asked students to determine the fraction notation of Nazifah's part.

The aim of this activity was to allow students to experience the use of an array model to solve the taking a part of a part of a whole unit within a context. Moreover, we expected that the students would interpret the result of the partition into a fractional notation by relating the intended part with the whole unit. We also want the students could think about how many times the intended part fit with the whole chocolate block.

In the teaching experiment, to answer the problem 4 all of the students in our focus group could divide and shaded the given array properly. They split the Hafidz's part into two equal parts horizontally (see Figure 5.45). Further the discussion happened to solve the problem 5 where they should interpret the shaded part into a fraction notation. We transcribed the discussion in the

Transcript 19 below. In addition, we provide Didi's answer to problem 5 in Figure 5.46 since he wrote about a part of a part of a whole.


*Figure 5.45* Students' answer of problem 4 on worksheet 2

**Transcript 19**

Afdal : Okay, wait. A half of… It is 18, we divided by 2 *(split the Hafidz part in the figure into two equal parts)*. 18 divided by 2 equals to 9.

Didi : How many parts of the whole chocolate block for Nazifah. So, it means we need to know this one (*the total number of small pieces)*.

Didi : Hmm, how many is that?, A half… These are three parts.

*(Defri, Dani, Khairul, count the number of small pieces of Hafidz and then the total number of small pieces in the whole chocolate block)*

Defri : *(Wrote* $\frac{9}{54} = \frac{1}{6}$*)*. $\frac{9}{54}$ equals to $\frac{1}{6}$. I already divided it [simplified it].

Based on Transcript 19 we can see that the strategy that was used by most of the students was determining the number of small pieces in the part of Nazifah and also in the whole chocolate block. Most of them multiplied the number of columns with the number of rows. The final solution was the part of Nazifah over the whole chocolate block. The conjecture in our HLT about students tried to think of how many times the Nazifah parts fit with the whole chocolate block did not happen. However, the teacher did not provide students with a support to grasp this idea.

To support the analysis above, we triangulate it with students' written work in Figure 5.46 which is the work of Didi. These data obtained through video registration and the students written work is consistent.

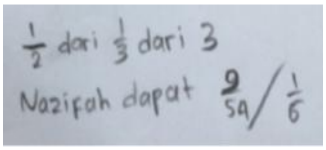



*Figure 5.46* Didi' written work on problem 5 of worksheet 2.

Didi, thought that the Nazifah's part is a half of Hafidz parts (see Transcript 19) and Hafidz had the parts as the result of dividing the whole chocolate block into three equal parts. Based on Figure 5.46 we interpret that Didi tried to determine the part for Nazifah by thinking of taking a part of a part of a whole unit. However, in solving the problem 5, the whole unit he referred was 3. It might be because he looked at the result of the partition of the whole chocolate block for Hafidz, Aufa, and Siraj.

Based on the analysis of the activity 3 on lesson 2, we conclude that the problem about sharing chocolate block could provide students with the experience of taking a part of a part of a whole unit in the given array. Further, in interpreting the result of this activity into a fraction notation, the students tended to count the cells in the array, as we explained beforehand, none of them determined the fraction only by looking at how many times the intended part fit with the whole chocolate block. Moreover, we could see that most of the students still think in term of part of a whole, only one of them started to think as a part of a part of a whole unit.

**Activity 4 – The exercise**

In our planning, there would be two problems in this activity, problem 6 and 7 on worksheet 2. The problems were about solving the taking a part of a part of a chocolate block involving non unit fractions. In problem 6, the students were given an array figure with 4 × 6 as the dimension and they should determine $\frac{2}{3}$ of $\frac{1}{2}$ on the array and find the fractional notation of it. The problem 7 is to show $\frac{1}{6}$ of $\frac{2}{3}$ of a chocolate block with 3 × 12 as its dimension.

The activity 4 in this lesson aimed to give students more experience about the use of an array model to solve the taking a part of a part of a whole problems. Moreover, we also wanted to support students about dealing with the non unit fractions. We expected that the students could show in the drawing the partitioning and they recognized the part-part-whole relationship properly.

In the teaching experiment, the teacher only asked students to work on problem 6, the last problem should be done at home as a homework for the students. The teacher asked students to reflect on what they did to solve the previous problem. Then, the students started to work on this problem.

The following is the transcript when the student explained how he determined $\frac{2}{3}$ of $\frac{1}{2}$ of the given chocolate block.

**Transcript 20**

Defri : $\frac{2}{3}$ of $\frac{1}{2}$ of the given chocolate is... (*reading his worksheet*)

The teacher : Which $\frac{1}{2}$ of the given chocolate do you mean? Show it.
Defri : (*pointing his finger into the mark of the half of the chocolate blocks*)

*(Defri divides the chocolate blocks into two same parts by using red marker and divides the half part into three same parts by using black market by using a black marker. Then, he determines the $\frac{2}{3}$ of $\frac{1}{2}$ of the given chocolate block by shading it with red and black markers as the following figure )*

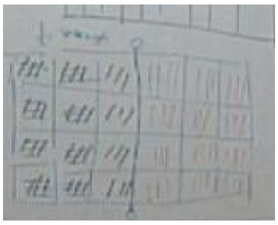
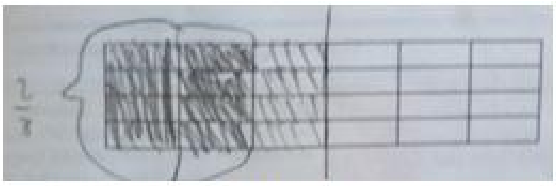


*Figure 5.47* Students' written works on problem 6

Defri : Then $\frac{2}{3}$ of $\frac{1}{2}$ of the given chocolate block is ..
(*pointing his finger into the black and red shaded parts*)

Based on the video registration, Defri determined the $\frac{2}{3}$ of $\frac{1}{2}$ of the given chocolate block by firstly determining the half part, then the two-third part of it. In line with the students' explanation in determining $\frac{2}{3}$ of $\frac{1}{2}$ of the given chocolate block, the Figure 5.48 is the example of students' written work in our focus group.



Problem-6

*Figure 5.48* Students' written works on problem 6 of worksheet 2

Based on Figure 5.48, we could see that to determine $\frac{2}{3}$ of $\frac{1}{2}$ of the given chocolate block, the students divided the array into two equal parts equally, then shade two columns of the first part to indicate the result. It implies that the students knew that they needed to determine the $\frac{1}{4}$ of the chocolate block first and further they determined the $\frac{2}{3}$ of the $\frac{1}{2}$. In the interpretation into a fraction, they still did the similar strategy as they did to solve problem 5 (activity 3). They counted the number of small pieces in the intended part regards to the total number of small pieces in the whole chocolate block.

Based on the description above it implies that the students could use the array in determining the taking a part of a part of a whole problems and interpret the result in a fraction notation by looking at the number of the cells in the intended part regard to the total cells in the array. In addition, this activity promotes students' understanding of using an array to solve the problem involved non unit fractions.

**Conclusion of lesson 2**

Based on the analysis of the activities on lesson 2 of the cycle 2, we conclude that the students already reached the goal of lesson 2. As we described in the analysis, the activity of sharing chocolate block for three children could be the warming up for the students to start use an array in partitioning and see the part-whole relation. The activity of determining the time of Hafidz reaching Aufa's house promotes students' initial understanding of taking a part of a whole unit. Some of the students already started to interpret the taking a part of a whole unit as a multiplication of a fraction with a whole number.

Most of the students difficult to make a move on their understanding from the taking part of a whole understanding into a part of a part of a whole unit. Therefore, in order to engage students to see the taking a part of a part of a whole unit clearly in their work, the discussion was held focusing on the term part of a part of a something. Moreover, the activity of sharing chocolate blocks in problem 4 till 7 could provide students with the experience of taking a part of a part of a whole unit in the given array for both unit fractions and non unit fractions. Furthermore, to determine the final answer of the taking a part of a part problem in term of a fraction notation, the students tended to count the cells in the array.

**c. Lesson 3- Sharing *martabak telur***

There were three activities in this lesson. In activity 1 the students will work on problem 1 of Part A on worksheet 3 about sharing *martabak telur* context. Activity 2 was the extension of the problem 1, the students' task was to choose an appropriate array for solving the sharing *martabak telur* problem. The last activity was working on part B of worksheet 3 in which students should solve some bare problems about taking a part of a part. The problems in this lesson we refer to the problems in worksheet 3 in Appendix J.

**Activity 1- Sharing *martabak telur***

The students' task in this activity was to determine what part of the whole *martabak telur* if Hafidz eat a quarter of a half of the *martabak telur*. This activity aimed to support students to be able in solving the taking a part of a part of a whole problem by constructing their own array. In the HLT we expected students to represent the *martabak telur* in a rectangle. They would start to think either from a representation of the whole martabak telur or from a representation of the part of the *martabak* that was eaten by Hafidz.

In the teaching experiment, we could observe that all of our focus group's students drew a rectangle first then they divided the rectangle in order to find the intended part. Different with the students in cycle 1, in this cycle 2 the conjecture about students start to represent the part of *martabak telur* first was not appear. However, it is not a big problem since the students could show the right partition of the rectangle. Furthermore, they discuss about how to determine the fractional notation of the result of the partition they did in the rectangle. The following Transcript 21 is about their discussion on interpreting the drawing into a fraction notation.

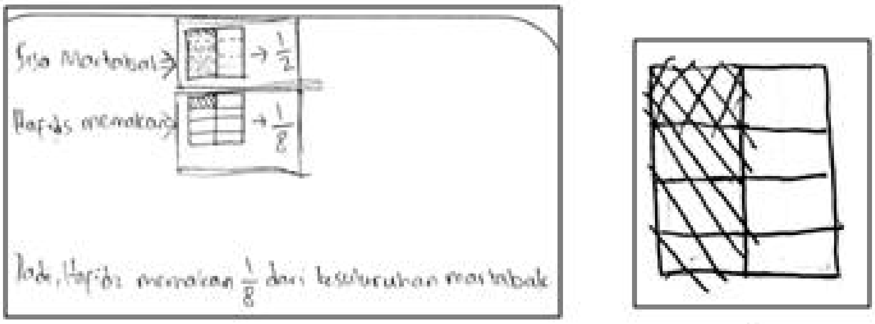
**Transcript 21**

| | |
|---|---|
| Defri | : All of us draw a rectangle. Then we divided it into four. |
| Khairul | : Hmm, we should halve it first.<br>*(Draw and divide the rectangle into two parts than divide the half part into four)* |
| Didi | : Into a fraction form? Wow! |
| Kharis | : A half of eh.. first one is a quarter of a half of. |
| Didi | : Of 1 |
| Khairul | : Hmm, we count the parts first *(counting the cells inside the rectangle)* one, two, three, four, five, six, seven, eight. |
| Afdal | : Yes, of eight. |

Didi : Not part of one?
Khairul : The simple way is $\frac{1}{4}$ times $\frac{1}{2}$ times 8

Based on Transcript 21 we could see that all of our focus group students using the representation of a whole *martabak* as a starting point. It is in line with one of the conjectured in our HLT. Khairul knew how to draw the correct array. He said that they should show the $\frac{1}{2}$ first in the figure. It implies that he understood that they needed to show the parts that was already eaten by Hafid's family, because it would be the new quantity. A quarter of the new quantity will be eaten by Hafidz later on. Furthermore, the students started to recognize that the process they represent in the array is the process of taking $\frac{1}{4}$ of $\frac{1}{2}$ of a *something*. They discussed about what the *something* refers to. Didi's idea was they should refer to 1. In our interpretation, Didi was thinking about the whole *martabak telur* as a unit, the one. But Khairul and Afdal stated that it should be eight since the small pieces in the array were 8 parts. This part of the discussion implies that although they came up with different number to represent the whole, the students already know that in a *taking a part of a part* problem they should refer to the initial unit.

As a compliment for the analysis above we provide the example of students written work on activity 1 of lesson 3 as can be seen in Figure 5.49 as follows.



a b

Figure 5.49 Student' written work on activity 1 of lesson 3

Based on students written work in Figure 5.49 we could see that it shows a consistency with what we observe from the video registration as transcribed in Transcript 21. The students represented the whole *martabak telur* in a rectangle and then do the partition to find the part that was eaten by Hafidz.

In addition, Khairul started to conclude that in solving the problem they could use multiplication operation. He said that it is the same with $\frac{1}{4}$ times $\frac{1}{2}$ times 8 (see Transcript 21). However, since he did not share it in the whole class discussion, we do not have data about how the teacher explored Khairul's idea.

**Activity 2- Choosing an appropriate array**

In this activity, the teacher explained that there were three students, A, B and C, tried to draw an array to help them in solving the sharing *martabak telur* problem (the problem in activity 1). Each of them drew an array with different dimension, 5 × 6, 4 × 3, and 4 × 4 respectively. The students' task was to choose which array would be an easy help for the students. The aim of this activity was to assure that the students recognized the idea of using an appropriate dimension of the array as the means in solving the taking a part of a part of a whole problems. We conjectured that the students would relate the number of rows and column in the array figure whether it could be a help in the partitioning of the *martabak telur* problem.

In the teaching experiment, we observed that the students choose the figure of students C as the answer. The following transcript is the discussion among the students in the focus group when they determine C as the proper array.

**Transcript 22**

*(Defri asks his friends' answers about the proper array to determine $\frac{1}{4}$ of $\frac{1}{2}$)*

Defri : (*ask his friends*) what is your answer?
Students : (*the other students answer C*)
Defri : okay, then the answer is C
(*all the students write C as the answer in their worksheet and the reason*)
Defri : It is C because the amount of the array is appropriate.
Denis : appropriate?
Defri : Yes, it is appropriate if we divide it

Based on the Transcript 22, we interpret the term appropriate that was being used by the students with the amount of array 4 x 4 which fit with the amount of array that they need to determine the value of $\frac{1}{4}$ of $\frac{1}{2}$ . Hence, the student can divide the half part of the array into four same parts.

In line with the description of video registration above, the Figure 5.50 is the example of students' written work on problem 2 of part A of worksheet 3.

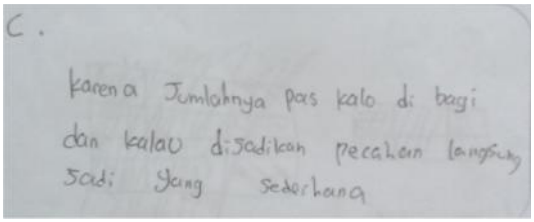



*Figure 5.50* Students' reason about choosing the figure of student C in problem 2 of par A of worksheet 3

Based on Figure 5.50 we can see that the student said about the reason they choose the figure C since the number is suitable to be divided and if they interpret it into a fraction notation it will be the most simple fraction. Based on this answer we could interpret that the students thought about the dividing of the array following the story in the sharing *martabak telur* context. Although we don't have data about what they mean by "the number" there, we interpret that they refer to number of rows and the number of columns in the array. Might be, the students thought that they could show the half and the quarter clearly in the figure of student C.

Based on the analysis of activity 2, we conclude that this activity could help students to recognize that they need to use an appropriate array to solve the taking a part of a part of a whole problem. The students reason by pointing about the suitableness of the figure to be divided to show the $\frac{1}{2}$ and the $\frac{1}{4}$ of the $\frac{1}{2}$.

**Activity 3 – The exercise**

In this activity the students' task was to solve four bare problems about taking a part of a part on part B of worksheet 3. The students deal with unit fractions and non unit fractions. They discussed in their small group about how to find the solution of the problems. The aim of this activity was to support students to have an experience of constructing their own array in solving the taking a part of a part problems for both unit and non unit fractions. in the conjecture we expected that the students would represent the taking a part of a part by doing a partitioning within they own array.

Based on the observation, as we conjectured, all of the students in our focus group started by drawing an array and discussed about how many part they should divide the rectangle first. For example the students discussed about how to solve $\frac{1}{4}\ of\ \frac{1}{3}$. The discussion is transcribed in the Transcript 23 below.

**Transcript 23**

| | |
|---|---|
| Defri | : This is one over three… *(show the rectangle that has been divided into three parts vertically)* |
| Denis | : [One over three] of? |
| Defri | : Hmm yaa… it means one over four of the one over three. |
| Denis | : O yes, you are right. Hmm, but how about the one over four. |
| Defri | : *(Divide the rectangle into four parts horizontally)* |

Based on Transcript 23, we could see that the students use the idea of indicating the second fraction first in the rectangle as the new quantity. Further, they divided the new quantity into four equal parts horizontally to determine the one over four of the new quantity. Finally, they got the intended part as the result of taking $\frac{1}{4}\ of\ \frac{1}{3}$. In addition to the data of video registration on Transcript 23, we provide Figure 5.51 which shows students' written work in solving the taking $\frac{1}{4}\ of\ \frac{1}{3}$ problem.

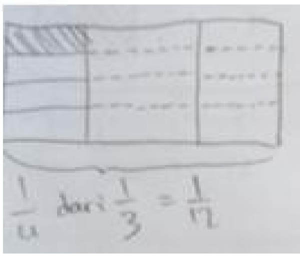


*Figure 5.51* Example students' written work on part B of worksheet 3

The students' written work in Figure 5.51 are consistent with the result of the discussion of the students in solving the problem of taking one over four of one over three. Through these data collected through different sources help us on making a complete figure on students' strategy in solving the problem.

Another example of students work in this activity is can be seen in Figure 5.52 where the students tried to solve $\frac{3}{4}\ of\ \frac{2}{3}$ .

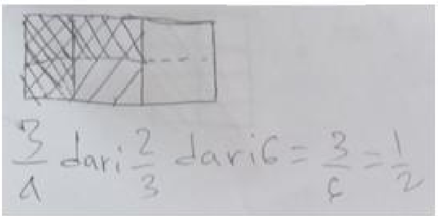


Figure 5.52 Students' written work on solving $\frac{3}{4}\ of\ \frac{2}{3}$

Based on Figure 5.52 to find the result in a fraction notation, we pointed out that none of the students tried to relate the numerator of the first fraction with the numerator of the second fraction and the denominator of the first fraction with the denominator of the second fraction in the problems. As same as in the previous activity, the students only tended to count the number of cells of the intended part regard to the total number of cells in the array.

Based on the explanation above, we can conclude that the activity on part B of worksheet 2 could help students to recognize that the array could help them in solving the taking a part of a part problem. However, the students still could not see the relation between the numerators and the denominators of the fractions in finding the final result of the problem of taking a part of a part activity.

**Conclusion of lesson 3**

Based on the analysis of the lesson 3 beforehand, we conclude the following remarks. The sharing *martabak telur* activity in lesson promotes the student's ability on constructing their own array in solving the taking a part of a part problems. They also recognized that they should use the appropriate dimension of the array to help them in solving the problems. Moreover, the analysis of the lesson 3 also shows evidence that the students could use the array models to help them to solve bare problems about taking a part of a part despite the whole unit was not mentioned in the problem. Through experiencing the activities in lesson 3, students recognized about how to construct an array model and use it as a tool to help them in finding the result of taking a part of a part problem. However, none of the students started to relate the numerators and the denominators of the fraction in the taking a part of a part problems in finding the solution. They just counted the cells in the array as they did in lesson 1 and 2.

**d. Lesson 4- Math congress and card games**

There were two activities in this lesson. The first activities was a math congress, which aimed to support students to make a shift in using the terms “times” in taking a part of a part problems. The second activity was a card games where the students would experience about using an array to solve multiplication of two fractions problems. The following is the analysis of each activity.

**Activity 1- Math congress**

At the beginning of this lesson the teacher asked the students to explain in brief about the main activity of the previous lessons. Then he invited the students to remind about the answer of some problems they already solved in the lesson 1, 2, and 3. He made a list on the whiteboard based on the students’ answer. The following Figure 5.53 is the list of the answer. The objective of this activity was to support students to be able to interpret the taking a part of a part as the multiplication of two fractions. We expected the students to grasp the initial notion of the procedure of solving multiplication of two fractions.

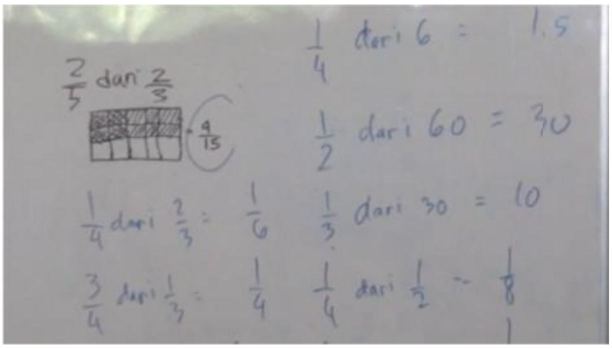



*Figure 5.53* List of the answer of taking a part of a part of a whole problems

The teacher asked the students to look at the list on the whiteboard. He gave students time to think about what kind of ideas came up to students' mind. At first, students still confuse what should they do or explain then the teacher retold the instruction to make it clear to the students.

One of the students came with the idea of multiplication. The teacher asked the student to share about his idea into the class discussion. The following Transcript 24 is the transcript about the student explained his idea in front of the class.

**Transcript 24**

The teacher : Who can make a conclusion based on the list?

Khairul : The conclusion is, for example $\frac{1}{2}$ of 60, it means we multiplied $\frac{1}{2}$ with 60.

The teacher : What is the result?

Khairul : It is 30.

The teacher : Okay, could you give another example?

Khairul : For example $\frac{2}{5}$ of $\frac{2}{3}$, it means $\frac{2}{5}$ times $\frac{2}{3}$. The result is $\frac{4}{15}$.

The teacher : Well, one more example from the list please!

Khairul : For example this one. $\frac{1}{4}$ of $\frac{2}{3}$ and it is equal to $\frac{2}{12}$ and we can simplify it become $\frac{1}{6}$.

Based on the Transcript 24, we can derive that the student could interpret the taking a part of a part problems as multiplication of a fraction with another fraction. In our interpretation, the student looked at the numerators and the denominators of the fraction in the list. To find the solution of the taking a part of a part problem, he could determine a new fraction where the numerator is the product of the multiplication between the numerators and the denominator of the new fraction is the result of multiplication of the denominator of the fractions in the problem. Based on the idea of this student, the teacher invited the other students to make a reflection whether they agree or not with this idea. As we observed from the class discussion, the students could grasp the idea of shifting the term part of into the term times which is symbolized with "×".

Based on the explanation above, we can conclude that the students recognized about the taking a part of a part problem could be seen as a multiplication of a fraction with another fractions. And to find the answer they could multiply the numerators and also multiplied the denominators. It implies that the students already made a move from using term part of into seeing this problem as a multiplication of two fractions.

**Activity 2 – Card games**

The next activity was the card games. The students played in pair in their small group. The teacher shared the cards to each group and explained the instruction of the game. The cards of the game can be seen in the Appendix J.

This activity aimed to assure that the students could choose an appropriate array in solving the multiplication of two fractions. We expected students would use their notion about the procedure of solving the multiplication of two fractions which already discussed in the activity 1 and represent it in an appropriate array figure.

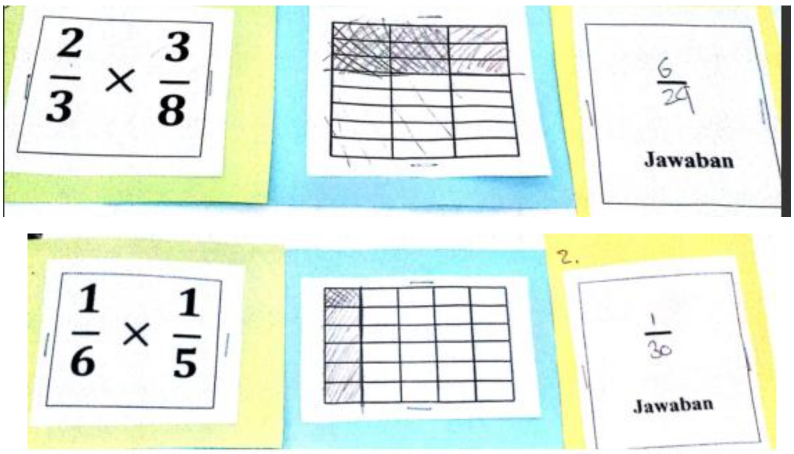
*Figure 5.54* Students work on the card game

The students started to discuss in their group to find the right set of cards. The following transcript is the discussion among the students in the focus group.

**Transcript 25**
(*The students choose and try to match the proper array with the fractions*)
Dani : (*he counts the number of the rows and columns of the array 3 x 8*)
it fits with this one
(*he refers the 3 x 8 array with the multiplication of* $\frac{1}{4}$ *of* $\frac{2}{3}$)
Dani : (*he transfer the value of* $\frac{1}{4}$ *of* $\frac{2}{3}$ *by shading the array 3 x 8*)
Defri : (*he counts the number of the rows and columns of the array 5 x 6*)
this one is for this one
(*he refers the 5 x 6 array with the multiplication of* $\frac{1}{6}$ *of* $\frac{1}{5}$)

Based on the Trancript 25, the students determined the pairs of the card by looking at the array first, They count the number of the rows and the columns of the array. Further, they tried to look at the denominator of the fractions in the problem cards. They looked for the denominators that the same as the dimension of the array. After that, they shaded the array and interpret the result in a fraction notation. Besides, there is another strategy which has shown by Khairul, Afdal and Didi. They tried to determine the answer of the problem cards first. Then they tried to find the appropriate array of the problems. In interpreting the result they used two strategies to assure their answers. First, they used multiplication between the numerators and between the denominators and simplified it. Second, they counted the small pieces in the intended part regards to the total number of small pieces in the whole array.

To support the data collected above, we provide students' written work of this activity as can be seen in Figure 5.55. It is in line with what we found in the video registration.




F*igure 5.55* Students' written work on the card games

Based on the analysis of the activity 2 of lesson 4, we can conclude that the students could use the array to solve the multiplication of two fractions problems. Some of them recognized that the dimension of the array has a relation with the denominators of the fractions in the array.

**Conclusion of lesson 4**

Based on the description of the analysis of lesson 4 above, we make the following conclusions. The math congress could be a time for the students to have a reflection about the activities they had already done in lesson 1, 2 and 3. The analysis of this lesson shows that the students could recognize that they can interpret the taking a part of a part activity as a multiplication of two fractions. Moreover the card game shows that the students could use the array in solving the multiplication of two fractions problems.

### 3. Analysis of the Result of Post-test after Conducting Cycle 2

We held a post-test after the four lessons in the second cycle was conducted. The aim of this post-test was to check students' achievements after the learning process. Based on this result, we can see whether students have learned from the conducted lessons. There were four problems in this post - test in which some of the problems consisted of two or three sub questions. The participant in this post-test was our students in cycle 2 of this study. (24 students), one student did not join this post test since he did not attend the class at that time. We conducted this post-test in 35 minutes. In the following we will describe the analysis of each item in the post-test based on the students' written works. After that, we provide general remarks about students' knowledge after the lessons.

The first problem was about sharing a *Bika Ambon* cake. There were two students' tasks in this problem. First, the students needed to determine the leftover of the *Bika Ambon* if $\frac{2}{5}$ of it were eaten by Bapak Gunawan and Ibu Susi. Second, the students needed to determine the part of two children who shared the leftover of the cake equally. Twenty students gave a correct answer for the first task of the problem 1 (1a) including four of our focus group students. However, there were two different strategies used by the students. The following Figure 5.56 shows the written works of Khairul (a) and Dani (b) as examples of each strategy used by the students.

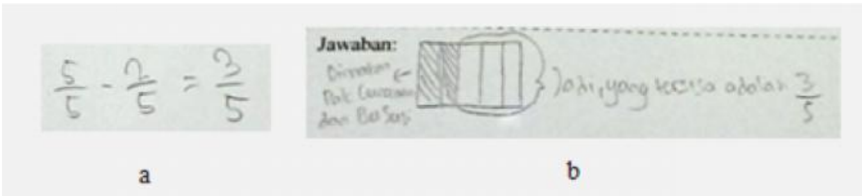
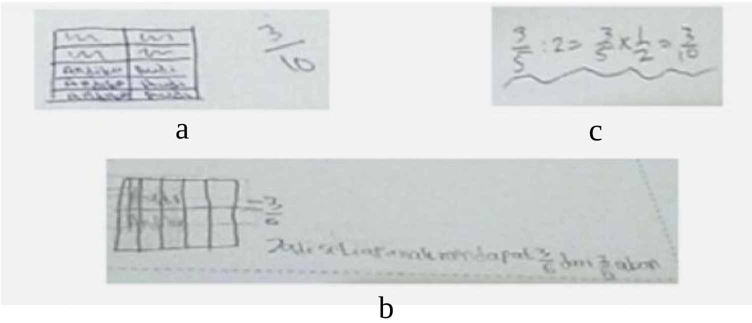



*Figure 5.56* Examples of students' written works of problem 1a in the post-test

Fourteen students used subtraction strategy to solve the problem. At first they knew that the whole *Bika Ambon* cake equal to $\frac{5}{5}$ then they subtracted $\frac{2}{5}$ of it and got $\frac{3}{5}$ as the answer. Six students drew a rectangle to represent the whole *Bika Ambon* and divided it into 5 equal parts, then shaded 2 parts of it to indicate the parts that were eaten.

Furthermore, for the answer of the second task of the problem 1 only eleven students have a correct answer included Defri, Dani and Kharul. The Figure 5.57 is the examples of correct answers of the students on problem 1b.



a c b

*Figure 5.57 C*orret answer of the students on problem 1b

of the post-test in cycle 2

Only six of the eleven students used an array to help them in finding the fractional notation. They came up with two different solutions, two of the six students got $\frac{3}{10}$ of the *Bika Ambon* as the answer (see Figure 5.57a) meanwhile the remaining four students got $\frac{3}{6}$ of the leftover of the *Bika Ambon* as the solution (see Figure 5.57b). Surprisingly, five students who have the correct answer used a division between $\frac{3}{5}$ and 2 to get $\frac{3}{10}$ (see Figure 5.57c). We do not have data about how it could happen. In our assumptions, it due to the time we conducted the post test which was a week after the last lesson. And during the week after the last lesson the students started to learn about division of fractions.

Based on the analysis of the students written work of the problem 1, we could conclude that most of the students knew that a whole *Bika Ambon* equal to 1 and to find the leftover they should subtract $\frac{2}{5}$ from the 1. However, only some of the students used a drawing instead of a formal subtraction to indicate the leftover. Moreover, almost a half of them could solve the second task in which we can interpret as taking a half of the $\frac{3}{5}$ of the *Bika Ambon*, but only some of them used an array to help them to indicate the parts. We also found that a few of the students could use the division of a fraction with a whole number in solving the problem.

The second problem in this post-test was about sharing a chocolate block among three children. There were three sub questions in this problem. First, the students should determine what part of the chocolate block for each child. Second, they should indicate in a drawing the parts of each child. Third, one of the three children wanted to share her parts with her brother Badu and Andi, the students needed to determine the chocolate part for Badu respect to the initial chocolate block.

Almost all of our students could solve the first and the second sub questions correctly. 19 students had $\frac{1}{3}$ as the answer of the first sub question and they could show the partition in a drawing as the solution of the second sub question (see Figure 5.58 a and b). Five students had $\frac{1}{2}$ as the solution along with the drawing as can be seen in the Figure 5.58c.

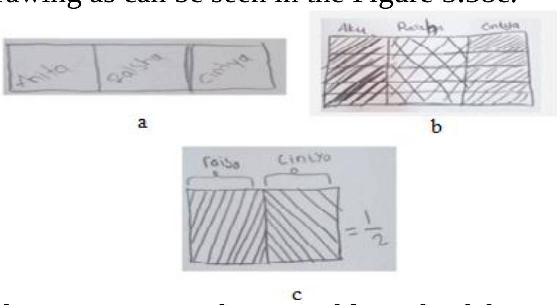
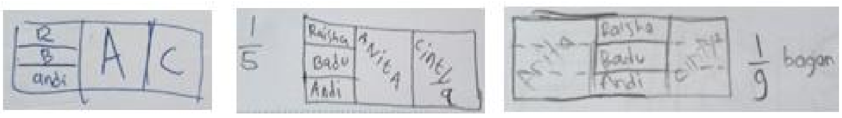


*Figure 5.58* Students' written works on problem 2b of the post-test in the cycle 2

As can be seen in Figure 5.58, to show the initial partition of the chocolate block there were two kinds of drawing came up with our students' written works. Some of the students drew a bar and then divided it into three parts (see Figure 5.58a). Meanwhile, some others drew an array and then shaded the three big parts of the array to indicate the parts for each child, Anita Raisya and Cintya (see Figure 5.58b). Although the students used an array, but they still could show in their drawing the part for Anita, Cyntia and Raisha by shading three equal parts of the array vertically.

Furthermore, we analyzed the students' written works which had $\frac{1}{2}$ as the solution. We can see from their works that they miss interpret the instruction of the problem. Instead of dividing the whole chocolate block into three they only divided it into two and showed that one part is for Raisha and another part is for Cyntia. It is not correct since the instruction was not to give the chocolate block only for Raisha and Cyntia but sharing the chocolate block among the three children, the owner Anita and her friends Raisha and Cyntia. Then the chocolate block should be divided by three.

For the last sub question in the problem 2, there were eleven students could show the correct solution in the drawing they made. The following figure 5.59 is the examples of three different drawings of our students.




a b c

*Figure 5.59* Student's written work on problem 2c of the post-test in the cycle 2

Based on Figure 5.59a we can see that the students had a correct drawing where they split the part of Raisha into three equal parts, but these students did not interpret the drawing into a fraction notation. In Figure 5.59b, we can see that the student also split the part of Raisha into three and he interpreted each part of the three parts as $\frac{1}{5}$. In our interpretation, this student only counted the parts in the drawing without considering the size of each part. The last figure (c) shows the complete solution of this problem. The students not only split the part of Raisha, but also dividing the part of Anita and Cyntia into three equal parts for each. It helps them in interpreting the part of Badu and Andi into a fraction notation. They got $\frac{1}{9}$ as the solution.

Based on the description above we can conclude that most of our students could do a partitioning of a whole into some parts. They also could interpret it into a fraction notation. It implies that the students recognized the idea of taking a part of a whole. However, when the problem was extended into splitting the part of the first partition into some parts, most of our students could show it in the drawing, but only some of them could interpret it into a fraction notation correctly. Not all of them got the idea of taking a part of a part of a whole within this problem.

The problem 3 was to determine $\frac{2}{9}$ of $\frac{3}{4}$. We provided examples of students' solution of this problem in Figure 5.60 below.

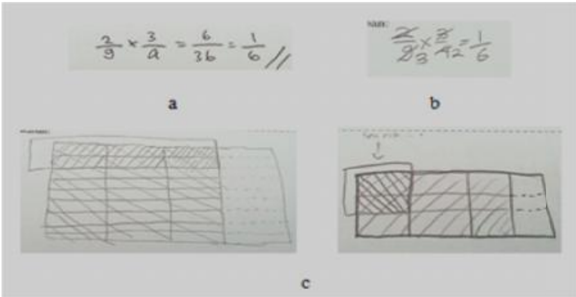


*Figure 5.60* Students' written works on problem 3
of the post-test in the cycle 2

Sixteen of our students could give a correct answer for this problem. Most of them directly interpret the problem into a multiplication of two fractions. So they tried to solve $\frac{2}{9} \times \frac{3}{4}$. Eight of them multiplied the numerator of the first fraction with the numerator of the second fraction. They also did the same for the denominators. These students came up with $\frac{6}{36}$ and they simplified it into $\frac{1}{6}$ as the result (see Figure 5.60a). One student used a cross-division strategy where he divided the numerator of the first fraction and the denominator of the second fraction by 2. He divided the numerator of the second fraction and the denominator of the first fraction by 3. Further, he did the multiplication and got $\frac{1}{6}$ as the answer of the problem (see Figure 5.60b).

Eleven students tried to represent the taking $\frac{2}{9}$ of $\frac{3}{4}$ in a drawing (an array model) as can be seen in figure 5.60c. However, only seven of them could show the correct drawing. Six of them used an array with 4×9 as the dimension. In our interpretation, first, these students divided a rectangle into 4 equal parts and shaded three parts of it. Further, they divided the 3 parts into nine and shaded 2 parts of the nine parts. They indicated that the parts that were shaded twice are the answer. However, only three of them interpret the drawing into fraction notation $\frac{6}{36}$ and simplified it became $\frac{1}{6}$. One Student (one of our focus group student) came up with an array with dimension 3×4 as can be seen in the Figure 5.60c but he also did not interpret it into a fraction notation. For the students who had a wrong drawing, it's because they made a wrong step in constructing their array, they represented the $\frac{2}{9}$ first instead of $\frac{3}{4}$.

Based on the analysis above, we can conclude that most of our cycle 2 students could solve the taking a part of a part problem. They interpreted this problem as multiplying two fractions. In solving the multiplication they multiplied the numerators of both fractions over the multiplication of both denominators. Moreover, we also found that some students used an array model to show the part-part-whole relationship and they could come up with a right figure as the result However, in interpreting into a fraction notation for the final result these students relate the number of cells the intended part over the total number of cells in the array.

The last problem in this post-test was to determine the result of $\frac{3}{7}\times \frac{2}{5}$. The aim of this problem was to investigate whether our students could solve a formal multiplication of two fractions. Based on students' written works we could see that almost all of our cycle 2 students have a correct answer for this problem. They came up with $\frac{6}{35}$ as the answer. One of the students solved with

$\frac{14}{15}$ and one student did not give his answer. Figure 5.61a is one of an example of students' correct answer and Figure 5.61b is the incorrect answer of the student on problem 4.

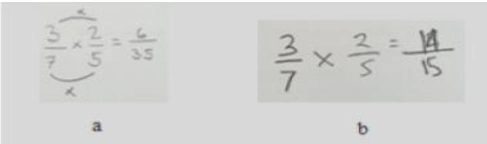


*Figure 5.61* Students' written works on problem 4 of the post-test

Based on Figure 5.61a we could see that these students used the same strategy as the strategy of multiplying two fractions in the previous problem as we describe beforehand. They multiplied between both the numerators and denominators of the fractions to get $\frac{6}{35}$. Meanwhile, one of our students did a wrong procedure (see Figure 5.61b). In our interpretation, to get the result, this student multiplied the denominator of the first fraction with the numerator of the second fraction over the result of multiplication of the numerator of the first fraction with the denominator of the second fraction. It seems that this student mixed up about multiplying two fractions with determining cross division of two fractions.

As our description and analysis of students' written works on problem 4, we can conclude that most of our students could solve the multiplication of two fraction problems by using the formal procedure of multiplying two fractions.

After looking at the analysis of the post-test result, we could make a general conclusion as follows:

a. The students knew that the whole unit equals to 1 and they found the leftover by using the subtraction strategy. Moreover, not more than a half of the students could solve the taking a part of a part task and we also found that not all of them used a drawing to indicate the process of taking a part of a whole unit on the problem of sharing a *Bika Ambon* cake.
b. The students could do a partitioning in the drawing properly and most of them knew that the result of the partition should be in the equal size.
c. Almost all of the students could grasp the idea of taking a part of a whole properly and they could interpret the result of this process in a fraction notation.
d. Not all of the students could recognize the idea of taking a part of a part of a whole unit when the context was extended. Although most of them could show the array the taking a part of a part of a whole process in, only some of them could interpret the result into a correct fraction notation.
e. Most of the students recognized that they can interpret the taking a fraction of another fraction same as multiplication of two fractions. Some of them could show the process in a drawing.
f. The students could solve the multiplication of two fractions in a formal procedure.

## 4. Summary of the cycle 2

To summarize the cycle 2, we provide the following Table 5.4 to give an overview of the four learning sequences and its remarks. The content of the table is based on the conjecture in the HLT and the actual learning process of the students.

*Table 5.4* Summary of cycle 2

| **Lesson** | **Activity** | **Remark** |
|---|---|---|
| Lesson1: Partitioning | **Activity 1.** Students determine the location of 6 flags and 4 game posts along a hiking trail in an equal distance. The tool that is used is two pieces of ribbons to help the students in doing the partitioning. | In locating the marks in the figure the students did not get the idea of overlapped the ribbons into the figure. It might be because there is no clear instruction for the students in the worksheet that they should put mark by using letter P for post and F for flags in the same hiking figure. |
| | **Activity 2.** Students interpret the result of the partitioning by using fractional notation. | The figure of flags and game post in the ribbon representation in the worksheet, help students to recognize that the bar is the representation of the hiking trail. In addition, it help them to easily come up with the udea of using fractional notation to relate the part and the whole unit. |
| | **Activity 3.** Determining the distance between the first game post and the starting line. | The idea of involving fraction in the operation appears on the class discussion and the students could recognize the part-whole relationship during this activity. |
| **Lesson 2**: Taking a part of a part of a whole | **Activity 1.** Sharing chocolate block among three children. | It shows that the students get the initial notion of doing partition within an array model and interpret it into a fractional notation by relating the part and the whole. |
| | **Activity 2.** Determining the time for reaching Aufa's house. | By discussing in the beginning of the lesson about the context that the time for exercise is one hour, a half of it is used |

| Lesson | Activity | Remark |
|---|---|---|
| | | for jogging and a third of the jogging time the boy arrive at Aufa's house, helps students to minimize their misinterpretation of the problem. |
| | **Activity 3.** Taking a part of a part of a chocolate block. | Students could clearly determine the partitioning within the array. They started using the array model to reason about part and whole in determining the fractional notation of the intended part. However, we notice that it would be better if the teacher also provide a discussion about how many times the intended part fit with the whole unit. It will be easier for the students to use this strategy in determining the fractional notation of the result of the partition. |
| | **Activity 4.** Taking a part of a part of a chocolate block involving non unit fractions. | Students could determine the new quantity by taking a part of the chocolate block figure and split the new quantity to find the last partition. They determine the fractional notation by looking at the number of the cells in the intended part over the total number of cells in the array. |
| Lesson 3: Sharing *Martabak Telur* | **Activity 1**. Sahing *martabak telur*. Which can be summarize as taking a quarter of a half of a *martabak telur*. | Students could construct their own array properly. They can interpret the result of the partition in a fractional notation. |
| | **Activity 2.** Choosing an appropriate array to solve the problem in activity 1. | Students recognized about the use of an appropriate array as a help in solving the taking a part of a part of a whole problem. |
| | **Activity 3.** Solving the taking a part of a part problem. | Students construct their own array in solving the taking a part of a part problem properly even the whole unit is not explicitly stated in the problem. |
| Lesson 4: Math Congress and Card games | Activity 1. Math congress where the students reflect on the partitioning activities they have done in the previous lessons. | Students recognize about interpreting the taking apart of a part problems into a multiplication of a fraction with another fraction. They also could make a shift into the use of symbol "×" and the procedure of multiplying the numerators |

| Lesson | Activity | Remark |
|---|---|---|
| | | and the denominators of the fraction (top-top and bottom-bottom) |
| | Activity 2.<br>The students match the multiplication problems in the problem cards with the appropriate array figure in the array cards then determine the product of the multiplication. | Students started to use the formal procedure in solving the multiplication of two fractions problems. They consider the denominators of the fraction in the problem card to match it with the appropriate array dimension on the array cards. |

## F. Validity and Reliability

In term of validity and reliability of the analysis as we stated in the chapter III, we are concerned with both internal and external validity and reliability. In contributing to the internal validity of the analysis, we use different sources of data, they are the video registration of the lessons and the students' written works. In conducting the analysis, these two sources of data support to each other. Moreover, to contribute to the external validity, we present the learning goal of each activity, the conjectures of students' strategies and the analysis of the actual learning process of the students in a clear ways so that others can follow it properly and they can adjust them to their local circumstances.

Furthermore, in term of internal reliability which is deal with how independent the researcher towards the data analysis, we discussed the data collection we have with our fellow master students and our supervisors. The main data in this study were collected trough video registrations which ensure that it is independent from the researcher. In addition, to contribute to the external reliability, which is known as trackability, the researcher documented the research in a clear way. We provide the timeline of the research, the learning materials and the lesson plans for each lesson. In analyzing the learning activities of the students, we provide the description based on the real activity we observe in the video registration and in the students' written works so that the reader could see from what data our interpretation are come from. We belief that by documenting the research in such way will make it clear for the reader how the research has been conducted and how we derive the conclusion.

# CHAPTER VI

# CONCLUSION AND DISCUSSION

This chapter consists of three parts; conclusion of the study, discussions, and recommendation. In the conclusion, we will provide the answer of the research questions and the local instruction theory on multiplication of two fractions. The important remarks will be discussed in the discussion part. Then, at the end of this chapter, we provide some recommendation for future study about supporting students' understanding of multiplication of two fractions.

## A. Conclusion

### 1. The Answer of the Research Questions.

The main research question in this study is *"How can models support students' understanding of multiplication of a fraction with another fraction?"*. We derive two research sub-questions from that main research question as follows:

*a. How can models support students' understanding of taking a part of a part of a whole?*

b. *How can taking a part of a part of a whole activity using an array model support students' understanding of multiplication of two fractions?*

**The answer of the first research sub-questions**

The activities in this study emphasized the use of informal knowledge of the students as a starting point in developing their understanding about multiplication of two fractions. One of the informal knowledge that is important in this topic is partitioning (Mack, 2000; Behr and Post, 1992). The idea of partitioning will lead students to recognize about part-whole relation and we extend it into the notion of part-part-whole relation.

In order to support students to build their understanding about those ideas, we provided them with contextual problems which the use of models emerged. The analysis of students' written works and the video registration of the lesson 1, 2 and 3 shows evidence that the use of models help students understand about part-whole relation and start to use the models in reasoning about taking a part of a part of a whole problems.

The activity of locating flags and game posts along a hiking trail promote students' initial knowledge about partitioning. The bar model which is the representation of the unfold ribbon that was used by the students in this activity helped the students see the result of the partition. The students recognize that the partitioning should produce equal size parts. Moreover, within the bar models the students could use fractions in an ordinal way to notate the position of the flags and the game post with respect to the whole hiking trail. They started to recognize the idea of part-whole relationship.

Furthermore, the idea of part-whole relations was extended into taking a part of a part of a whole unit. The activity of determining the time for reaching Aufa's house could help students start exploring the taking a part of a part of a whole problem. Within this context the students were invited to see the form of a part of a something where the something is also a part of the whole thing. Then the activity of sharing chocolate block in which the array model was introduced promote students understanding about taking a part of a part of a whole unit. The students used the array model to indicate the process of taking a part of a part of a whole unit and then started to reason about the fraction notation of the result. However, they still tend to count the small pieces (the cells) in the array to find the fraction notation. Only a few of them thought about determining the fraction notation by comparing the intended part with the whole part of the array.

Moreover, the students started constructing their own array in solving the taking a part of a part of a whole unit in the activity of sharing *martabak telur*. The students also recognized that in constructing the array they should consider the dimension of the array so that the array models could help them in solving the taking a part of a part of a whole problem.

Based on the description above, we can conclude that the activities involving the models could support students in developing their understanding of taking a part of a part of a whole unit. They started to use the array to reason about part-part-whole relation. The taking a part of a part of a whole understanding was used to bridge students in the developing an understanding of multiplication of two fractions.

**The answer of the second research sub-question**

We derive the answer of this sub-research question based on the analysis of students' written works and video registration of the lesson 3 and 4 in this study.

The idea of taking a part of a part of a whole unit was elaborated more in those lessons. The students used their notion about part-part-whole to start seeing the relationship between the two fractions in the problem. The activity of sharing chocolate block and sharing *martabak telur* engaged students to recognize that in solving the taking a part of a part of a whole problem, the first fraction which indicates the intended part is taken from the second fraction. The second fraction is the new quantity which is taken from the whole unit (the initial unit before the partition). Our result reveals that our students in this setting perform an ability to show the process of making the new quantity and taking a part of that new quantity in the array model. Although, some of the students still find difficulties to interpret the result of this taking a part of a part of a whole into a fraction notation. The common mistake that they made is that they did not refer or consider to the whole unit as the initial quantity.

Moreover, in order to support students to start thinking about the relationship between the two fractions in depth we provide some bare problems about determining a fraction of another fraction along with the array figures. In these problems the whole unit is not stated explicitly. The result shows that this activity could help students start seeing the relationship between the two fractions in the problem of taking a part of a part. However, some of the students still count the total number of cells in the array as the whole unit, then they used a counting strategy to relate the intended part with the whole unit to come up with the fraction notation.

Furthermore, to make a shift of the students from interpreting the term *part of* into a term *times*, which is symbolized by "×", the teacher engaged students to look at the list of the solution of the taking a part of a part problems that they already solved. The result of this activity shows that some of the students could recognize that they could interpret the taking a part of a part as the multiplication of a fraction with another fractions. Further, through a class discussion, this idea was elaborated more. The students recognized that the product of a fraction times another fraction can be determined by multiplying the numerator of the fractions over the multiplication product of the denominators of the fractions. The students grasped this idea after they discussed about the relationship between the numerators of the fraction and the denominators of the fractions in the problems on the list in the math congress activity.

In addition, the last activity of card games could give students experience to solve a multiplication of two fractions using an appropriate array. However, since the students already got the strategy of solving the multiplication of two fractions in the math congress activity, most of them tended to solve the multiplication by using the formal procedure first then determined the appropriate array by looking at the denominator of the fractions as the dimension of the array.

In conclusion, based on the answers we provided for both of the sub-research questions beforehand, we found that these learning sequences could support students to develop their understanding of multiplication of two fractions. By providing students with the bar and the array models within contexts promotes the recognition the idea of partitioning. The students started to use the models in their reasoning about the taking a part of a part of a whole problem. Further, by experiencing the partitioning in the taking a part of a part of a whole problem activity, students recognized the idea of multiplying a fraction with another fraction. Moreover, it also reveals that the students could use the array models in solving the multiplication of two fractions problems.

**2. The Local Instruction Theory of Multiplication of Two Fractions.**

This study aimed to contribute to the development of a local instruction theory in supporting students' understanding of multiplication of a fraction with another fraction. Gravemeijer (2004) use the term local instruction theories to refer to "the description of, and rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topics". In the Table 6.1 we provide the summary of tools and the contextual activities we proposed in the instructional design. The outline of the table was adapted from Gravemeijer et.al (2003) where they described about a learning trajectory for measurement and flexible arithmetic.

*Table 6.1*: Local instruction theory in developing students' understanding of multiplication of two fractions

| **Tool** | **Imagery** | **Activity** | **Potential Mathematical discourse topics** |
|---|---|---|---|
| Hiking trail figure and two pieces of ribbons | Signifies the initial idea about partitioning | Determining the location of 6 flags and 4 game post along the hiking | Partitioning, producing equal-size parts |

| **Tool** | **Imagery** | **Activity** | **Potential Mathematical discourse topics** |
|---|---|---|---|
| | | trail figure. | |
| Bar representation of the ribbons | Signifies the relation between the partitioning result and fraction notation of it | Notating the location of the flags and the game post by using fraction notation in the ribbon representation figure. | The use of fraction in notating the result of the partitioning |
| A context about measuring the distance of the first game post and the starting line of the hiking trail | Signifies the part-whole relation | Determine the distance between the first game post and the starting line of the hiking trail | Start using the understanding of the part-whole relation. |
| A story of time for reaching Aufa’s house within a comic | Signifies the acquisition of the initial form of taking a part of a part of a whole unit | Determine the time for reaching Aufa’s house when it was a third of a jogging time and the jogging time is a half of an hour. | Introducing to the initial understanding about part-part-whole relation |
| Chocolate block figure | Signifies the process of taking a part of a part of a whole unit within an array model | Sharing a given chocolate block figure among three children. Then splitting one of the parts into two. | Indicating the part-part-whole relation in the array model.<br><br>Interpreting the result of the taking a part of a part of a whole into a fraction respect to the initial whole unit. |
| Chocolate block with different dimensions | Signifies the use of array models in solving the taking a part of a part of a whole unit | Determine a part of a part of the given chocolate block figures. | Determine the result of the taking a part of a part of a whole unit by using a fraction notation. |
| *Martabak telur* | Signifies the | Solving the sharing | Constructing an array |

| **Tool** | **Imagery** | **Activity** | **Potential Mathematical discourse topics** |
|---|---|---|---|
| figure | acquisition of constructing an array models | of a *martabak telur* problem by constructing an array model | model to solve the taking a part of a part of a whole unit |
| Three different array figure relates to the sharing *martabak telur* problem | Signifies the acquisition of using an appropriate dimension of an array model in solving the problems | Choosing an appropriate array figure to solve the sharing martabak telur problem | Using an appropriate dimension of the array to solve the taking a part of a part of a whole problem. |
| A set of taking a part of a part problems | Signifies the acquisition of solving the taking a part of a part problems | Solving bare problems about taking a part of a part problem by constructing the array model for each problem. | To see the relation between the fractions in the taking a part of a part problems.<br><br>Start to recognize the relation without referring to the initial whole unit explicitly |
| List of problems and solutions of the taking a part of a part problems | Signifies the acquisition of the shift from the term part of into term times, which is symbolized by "×" | Discuss about the idea that emerge when looking at the relation of the fraction in the list of the solution of the taking a part of a part problem | Interpreting the taking a part of a part problem as the multiplication of two fractions. |
| Sets of cards (the multiplication of two fraction cards, the arrays figure cards, and the blank solution cards) | Signifies the use of array models in solving multiplication of two fractions | Choosing an appropriate array to solve the multiplication of two fractions problems, then find its solution using the array | Using array models in solving multiplication of two fractions. |

## B. Discussions

### 1. The Weaknesses of the Study

In this part we will describe about three points that we consider as the weaknesses in our study. The first weakness is about the time between conducting the cycle 1 and the cycle 2. We conducted the cycle 2 right after the week of conducting the cycle 1. This condition due to the academic calendar of the school, since the school need to continue to other topics which were the requirement for the students for joining the mid semester test so the vice principle of the school discussed together with the teacher and the researcher and concluded that the cycle 2 should be conducted in the week right after the cycle 1.

The limitation of time between the two cycles caused the limitation of our preparation in revising the materials and the design for the cycle 2. To overcome this condition the researcher directly drafted an analysis and notated the important remark of each lesson in the cycle 1 and discussed about the revision with the supervisors before conducting the cycle 2. In addition, it also affected the time for the teacher to prepare himself for the teaching. The researcher overcame this problem by having a discussion with the teacher before each lesson was conducted.

The second weakness is about the clarification of the students' thinking in the focus group. In some instances, we could not get a clear data about students' thinking when they discussed or worked at their small group. When the cycle 2 was conducted the researcher only took role as an observer. We chose not interfere the learning process since we wanted to keep the originality of student learning activity. Since in our design the only one person who could give questions to explore students' thinking or to clarify it was the teacher or it also could happen within the discussion among the children themselves.

To overcome this condition in the rest of the cycle 2, we discussed with the teacher and asked the teacher to explore students thinking even when they work in the small group discussion not only when the students present their work in the class discussion. However, in some cases, we saw that the teacher didn't really do it well. In our interpretation, it due to the large number of students so that it was hard for the teacher to only focus to our focus group students. Therefore, it would be better if the researcher conduct an interview with the students after the lesson to triangulate what we observe and what the students did in their written work.

The third weakness is about the validation report of the items in the pre-test and poet-test. We conducted the validation with the experts for the item in pre-test and post-test of our study, however we did not provide and report about the validation form. Hence, the reader could not check the feedback from the reviewer.

**2. Reflection on the Important Issue**

In this part we will describe about some important issues that we found during conducting the present study. In general, we conclude these issues in two points as follows.

**The role of the teacher**

In our chapter 2, we explain that we used the realistic mathematics education (RME) approach, which is in Indonesian version known as *Pendidikan Matematika Realsitik Indonesia* (PMRI), as a domain specific theory in designing the learning sequences and also the materials. In our study, the mathematics teacher who we collaborate with was already familiar with the PMRI approach since the school is one of the PMRI schools in Surabaya. We found that the teacher quite cooperative with the new idea in teaching and also he gave good effort on conducting the lesson based on the teacher guide we provided.

In the classroom, the teacher took a role as facilitator of the learning process of the students. The teacher did not instruct the students to the correct solution, but he tried to guide them naturally to find the good and appropriate solution together. It is important since in our design we expected that the students would come up with a different strategy and different kinds of solutions so that it made the discussion more dynamic. However, we also notice a minus point of the teacher when he conducted the lesson that is, he did not really look at the students' work when the students worked in their small group. Therefore, sometimes we found that some potential strategies (either correct or not correct) strategy by the students that could be elaborated more was not bringing in up into the class discussion.

**The classroom norms in the class 5c**

The participants of our study are the students of class 5C of SDI Al Hikmah Surabaya. There are 30 students in this class and all of them are male students. Since, in this school the boys and the girls of grade 4, 5 and 6 are separated. The regular sitting arrangement in this classroom consists of five rows and four columns and the students sit in pairs. However, in our study, we rearranged it became sitting in a small group where the students sit in a circle around the table.

As we stated beforehand that the school implement the PMRI approach in the mathematics classroom so that the learning are based on students centered. The students get used to share their ideas in the whole class discussion and they also gave attention when there is a student present his work in the discussion. They also show an ability of giving comment in the whole class discussion. We also noted that the students have communication skill but it is not in depth of the communication skill in mathematics. They still found difficulties in elaborate their mathematical ideas. When they present their work, mostly they just relied on what they have prepared such as their solution in the worksheet.

Although the students were involved well in the class discussion, we found that in the beginning of conducting the lesson in each cycle of our study, the students did not get used in discussing in pairs or in their small groups. They tended to find the solution individually. In order to engage students to work in pair or discuss with their small group the teacher stated clearly that they should discuss in their group before presenting in the class discussion. This clear instruction makes students tried to discuss, but we still see in some group there were students still work individually and there were also students that just copied their fellow group member solution.

Moreover, we noticed the way the teacher increase students' motivation in joining the learning activity and even prepare themselves at home before the lesson. The teacher used plus and minus point systems. There is a record on the teacher computers for each student. When the students were involved in the discussion, presented their work, or asking questions the teacher give an additional point for them. On the other hand, the teacher sometimes stated that he would reduce the point for the students who did not pay attention or make an unimportant noise during the lesson. At the end, there will be a reward for the students with the highest score. We found that this method is quite effective to motivate students in the learning process and also become an important tool to get students attention back when they started doing unimportant activities and making noises.

## C. Recommendation for Further Research

In conducting this present study and based on the descriptions of the weakness and the important issues beforehand, we propose the following points for the further research on the topic of multiplication of two fractions.

The first recommendation is about the time between the first and the second cycles. We suggest that the researcher should give enough time to elaborate more and analyze the result of the cycle 1. It also will give time for the researcher to have a depth discussion with the teacher about the teacher guide for each lesson before conducting the cycle 2. In addition, when there is enough time before the cycle 2, the teacher and the researcher also can discuss about how to improve the materials and set the socio and socio-mathematical norms in the cycle 2 class.

The second recommendation is about integrating the design which also studies about students' communication skills. We found that this is an important issue for the researcher and the teacher when we want to develop students' understanding, since the understanding is inside the student's mind and we cannot see the students' mind. The only data we have is the students' work and what students say. Therefore, we need to develop students' communication skill in order to get the inside of students' minds.

The last recommendation is about considering the new curriculum in Indonesia. In the present study we still design the learning sequences grounded by the old curriculum where the multiplication of two fractions is a topic in mathematic subject in grade 5. Meanwhile, the new curriculum use thematic based learning. In the thematic based learning, the mathematical topic will be integrated with other subject topic. Therefore, in designing the learning sequences for developing students' understanding of multiplication of two fractions should smoothly integrated with the other topic in the specific theme.

**Appendix A** The general schemes for Interview with the teacher

1. Classroom management
   - What is the teaching method that is usually used by the teacher?
   - How does a teacher makes small groups of the students
   - How does the teacher usually lead the discussion
   - How does the teacher engage students in the learning activity
2. Classroom norms
   - What are the social norms in the classroom and how does the teacher deal with it?
   - What is the socio-mathematical norms and how does the teacher deal with it in the classroom
3. Teachers' beliefs
   - What are the teacher's understanding of and experience about PMRI
   - How does the teacher usually teach the topic of multiplication of two fractions.
4. Students' achievement
   - What is the preliminary knowledge of the students?
   - How do students' levels of knowledge differ
   - What are the students' understanding and difficulties in learning mathematics?

**Appendix B** The general scheme of the classroom observations

1. The role of the teacher
   - How does the teacher start the lessons
   - How does the teacher engage the students' attention
   - How does the teacher deal with the students’ reaction which is not in the HLT
   - How does the teacher lead a discussion
   - How does the teacher use students’ answers or students’ own constructions in the learning process
   - How does the teacher elaborate students’ answer
   - How does the teacher scaffold students
   - How does the teacher deal with the different levels of the students
   - How does the teacher end the learning session
2. The role of students
   - Are students willing to join in on the learning activity
   - Students’ interactivity in the whole class discussion and also in the focus group discussion
   - How do students explain their strategy
   - How do students react to their friend's argument
   - What students do, write, read or say
   - How do students deal with their difficulties during the learning activity

3. The classroom
   - How is the seating arrangement of the classroom

4. About the PMRI approach
   - How does the teacher implement the PMRI approach in the classroom activity

**Appendix C** Teacher Guide of the cycle 1

# Teacher Guide

**Subject : Mathematics**
**Class/ Semester : V**
**Semester : 2**
**Time Allocation : 2 x 35 minutes**
**Lesson : 1**

**A. Learning Objectives**

1. Students are able to do partitioning properly.
2. Students are able to label the result of the partitioning activity.
3. Students are able to multiply a fraction with a whole number in a context.

**B. Starting point :**

Students in fifth grade already learned about producing fractions, addition and subtraction of fractions, and equivalence of fractions.

**C. Learning Methods :** Hands on activity, working on worksheet, class discussion.

**D. Learning approach :** PMRI

**E. Learning materials :** Worksheet 1, ribbon, markers.

**F. Learning Activities**

**1. Orientation (5 minutes)**

- The teacher introduces a story about a scout club. Ask the students whether they ever join a scout club activity or not. Let students mention what kind of activities of a scout club usually hold.
- Introducing the context :
  *A scout group plans to have a hiking activity at the end of this month. The length of the hiking trail is 6 km. The committee arranges several games during the out bond in 4 posts which are located at equal distance to each other along the hiking trail. The last post is at the finish line. Moreover, they want to put some flags along the trail as a sign for place to take a rest. They place a flag in each kilometer of the trail, and the last flag is in the finish line.*
- Ask the students to make group consist of three of four students. Then the teacher gives the problem one of the worksheet 1 for each of them. The teacher also provides some ribbons and markers for each group.

**2. Working on Worksheet 1: Problem 1 (10 minutes)**

- The teacher tells the students to focus on problem 1. The instruction of the problem for the students is to act as the committee of the hiking activity and think about how to locate the flags and the game posts. (The picture of the hiking trail is provided in worksheet 1).
- While the students work in small groups the teacher walk around and help the group if they have difficulties regarding the understanding of the problems.
- The teacher also make note about the strategy that is used by the students to make decision on which group that should explain their strategy first in the class discussion.
- When there is a group which took much time on thinking about the strategy, the teacher can give hints that they can use the ribbon to help them.

<u>Some of the possible answers by the students</u>

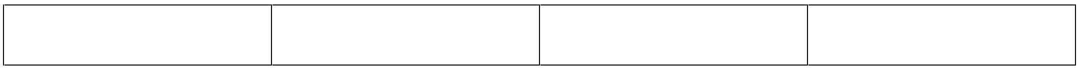
- Students only use their estimation on the figure and mark the position of the flags and the posts.
- Students use the ribbon to get the length of the trail in the figure. Then, they strengthen the ribbon.
- To find the position for the game posts they fold the ribbon two times.
- To find the position for the flags the students fold the ribbon randomly and by using the trial and error strategy they will get 6 equal parts of the ribbon
- Students use the folded-ribbon to estimate the position of the flags and the game posts by overlapping the ribbon on the trail in the trail figure.

**3. Class discussion on problem 1 (15 minutes)**

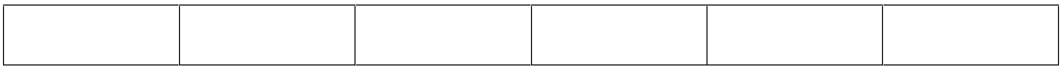
- When there is student who only uses estimation strategy, ask them to share their work and let the other comment about it.
- The discussion could be about:
  - *How can you do the estimation?*
  - *Do you satisfy that your estimation is correct?*
  - *What would we do to make it more precise?*

Students might think that they need a strategy which assures the partition is in equal size.

- Teacher can encourage students who use the ribbon to share their strategy to the whole class and let the others react to it.
- Then teacher can ask, "*Why don't you also try in your group and use the given ribbon to help you!*"
- Close this activity by inviting the students to represent the ribbon into a bar along with the folding line.



Representation of the folded ribbon into 4 equal parts



Representation of the folded ribbon into 6 equal parts

**4. Working on worksheet 1 : Problem 2. Labeling the fractions (15 minutes)**

- The teacher gives the problem 2 of the worksheet to the students.
- The students will work again in their group.
- Orient students that they need to determine the fraction notation for each position of the flags and the posts.
- While the students work in groups, the teacher walks around and support the students to understand the instruction.

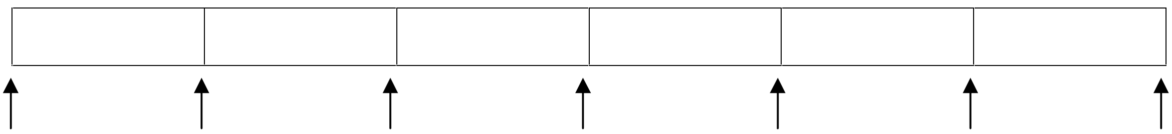
<u>Some of the possible answers by the students</u>

- Students only give the notation by using natural numbers which indicate the first flag, the second flag, the third flag and so on. The same strategy for the label of the game posts.
- Students give the notation only by using the unit fractions. They notate each part by $\frac{1}{6}$ for the flags location and $\frac{1}{4}$ for the location of each game posts.
- Students use non-unit fractions. They use $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$ and so on for the location of the flags and $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$ for labeling the location of the game posts.

**5. Class discussion of problem 2 (15 minutes)**

- Ask one of the groups to present their works and let the other react to it.
- Let the other react and give feedback to them.

- For the students who only give the notation in natural numbers teacher could ask students to think about the label respect to the whole unit and invites them to determine the label in term of fractions.
- When the students only use the unit fractions, the teacher can invite them to look at the folding line in the bar (the ribbon representation)
- The teacher draws the representation of the ribbon in the white board. And then discuss with the students. *“What fraction should be put in the intended line respect to the whole unit?”*



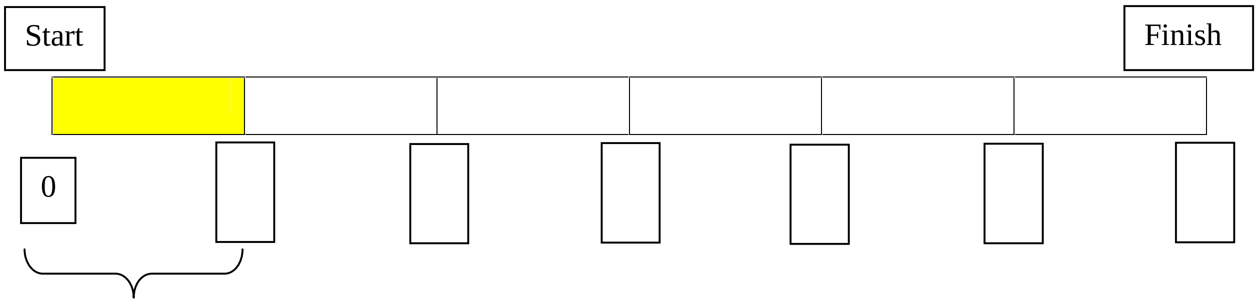
**Asking about the label of each folding line.**

- Let students state the position of the flags in ordinal number. May be there is students said that the first flag is in the beginning of the bar. Let other students react on it.

| | |
|---|---|
| Teacher | : Is it true that the first flags is in the beginning of the bar? |
| Students | : (confuse) |
| Teacher | : What is the information in the beginning of the story? Do you remember? |
| Student | : Oh yaa… The beginning of the bar represents the starting line and there is neither flag nor game post there. |
| Teacher | : Yes, that is right. What number should we put in that point? (the teacher pointed out to the beginning of the bar). |
| Student | : Zero. |

- To start discussing about labeling the position by using fraction notation, lead students recognize the result of the partition they made.
- Ask them about the part and the whole. The teacher can use the representation of the answer on the worksheet.

Ribbon representation of the position of the flags:




Ask the students about this part respect to the whole bar. Students may said that there are 6 parts in the bar and the yellow part is 1 of them, so it means *one over six, or* $\frac{1}{6}$.

- May be there is students remember about number line in whole number, the teacher can use it to encourage students to see it as a number line but in the form of a bar.
- Then discuss about how if we shade two parts of it. Students may see that they take two parts of the whole six parts in the bar. And it is $\frac{2}{6}$ then the students can see the pattern and continue until the position of the last flags.
- Let the students reflect on this discussion and look back on their work.
- Give the students time to reflect and revise their work for both the labeling fraction for the flags and the posts.

**6. Working on worksheet 1: Problem 3 (10 minutes)**

- Furthermore, the activity is continued by determining the distance between the starting point of the trail and the first game post. In the worksheet it is stated that the length of the trail is 6 km.
- Students will work in their small group and teacher walk around to see the students work.

<u>Some of the possible answers by the students</u>

- The students will determine the length of the middle point by splitting the total length of the trail, 6 km, and then halve it to get the distance of the first post location to the starting point.
- The students try to divide the total length of the trail with four but may be they will struggle to find the answer.

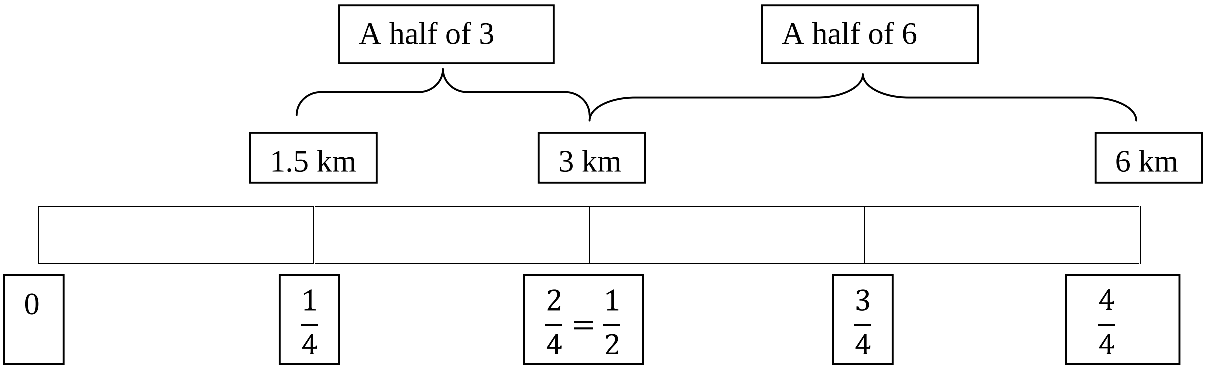
**7. Class discussion of problem 3 (10 minutes)**

- The teacher invites the students to reflect on their strategy. Ask the students to explain and discuss the relation of their answer in problem 3 with the answer of problem 2.
- For students who use halving strategy, invite them to express their strategy in the fraction notation.

| | |
|---|---|
| Teacher | : What did you do first? |
| Student A | : I divide the whole length of the trail with 2. |
| Teacher | : what does it mean? |
| Student A | : (Confuse) |
| Teacher | : Can you express the process you did in other word? |
| Student A | : I took a half of 6 km to get the middle. |
| Teacher | : Yes, good. So first you take $\frac{1}{2}$ of 6 km. Then? |
| Student A | : Then I halve it again to get the final solution. |
| Student B | : It means you take $\frac{1}{2}$ of 3km. and we get one and a half km. |
| Teacher | : Yes you are right. |
| Teacher | : So, can somebody conclude I a complete sentence about the strategy of student A? |
| Student C | : He took $\frac{1}{2}$ of $\frac{1}{2}$ of 6 km |

- Perhaps there is a student who directly divides 6 km by four to determine the distance of the first post with the starting point. Let students compare the solution.
- If there is no students come up with this idea, the teacher can ask students about the position of the first post regarding to the whole trail. He or she can pose this question, "*Do you remember in what part of the trail is the location of the first post?*" Then, the teacher can show the ribbon representation again to the students and invite them to see the ribbon representation as a bar model which can they use to solve the problem 3. The teacher supports the students

to represent their strategy in solving the problem 3 on the bar as can be seen in the figure.




- Furthermore, invite students to compare the result. The student may recognize that find the distance between the first post and the starting point is the same of taking $\frac{1}{4}$ of 6 km.
- The result is the same with the students who uses $\frac{1}{2}$ of $\frac{1}{2}$ of 6 km.

**8. Closing (5 minutes)**

- Lead students to make a conclusion in their group and invite them to reflect on what they already learned during the lesson.
- Focus on the experience of partitioning activity, making fraction notation and also taking a part of a whole.

# Teacher Guide

**Subject** **: Mathematics**
**Class/ Semester** **: V**
**Semester** **: 2**
**Time Allocation** **: 2 x 35 minutes**
**Lesson** **: 2**

**A. Learning Objectives**

- Students are able to take a part of a part of a whole in a context.
- Students are able to use the array model to solve taking a part of a part of a whole within a context

**B. Starting point** **:**

Students already learned about how to do a partitioning properly and give label to the result of the partitioning activity in fraction notation. They also already introduced to taking a part of a whole activity.

**C. Learning Methods** **:** working on worksheet, small group discussion and class discussion.

**D. Learning approach** **:** PMRI

**E. Learning materials** **:** Worksheet 2, Worksheet 3, grid papers, markers.

**F. Learning Activities**

**a. Orientation (5 minutes)**

- Remind the students about hiking trail activity. Extent the story that Hafidz want to join the next hiking event, so that he prepare his self with exercises. *Hafidz plan to have one hour exercises every week. This morning, he tells his father that he will jog with his friends. His father give a chocolate block and said that Hafidz should share the chocolate to Aufan and his brother Siraj.*
- The teacher groups the students on three or four and shares the worksheet 2.

**b. Working in worksheet 2: problem 1, 2 and 3 (10 minutes)**

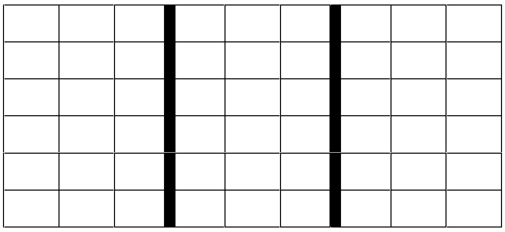
- The context in these problems is represented in Comic 1 and 2. Ask the students to read the comic first and take the important information of it.
- Ask some students to tell about the problems to see whether they understand or not. Problem 1 and 2 is related.
  In problem 1, there is a representation of the chocolate block and the students need to indicate the part for Hafidz, Aufa and Siraj. Next in problem 2, they need to determine what part is that and write the answer in fraction notation.

Meanwhile, the problem 3 is about the time that is used to reach Aufa house. Hafidz have an hour exercise, he uses a half of it for a jog. And when he jogs in this morning he uses a third of the jogging time to reach Aufa's house. The students need to determine how many minutes is the time of Hafidz to reach Aufa's house?

- After the students understand about the problem, ask them to think about their own idea to solve the problem for 1 or two minutes for each problem. Then, ask them to discuss and start to write the solution.
- While students work in their group, the teacher walks around to see the students' work

<u>Some of the possible strategies by the students for problem 1 and 2</u>

- The students divide the given chocolate block diagonally or randomly.
- Students divide the chocolate block into three equal parts. First, they count the number of columns and divide it by three. And they make a line in every three columns. Each big part is for one person.



- Students might also divide the block horizontally by using the same strategy.
- To determine the part of Hafidz, students only look at the big part without counting the small parts. They get $\frac{1}{3}$ of the chocolate block as the answer.
- Students count the small parts but not refer the whole unit of the chocolate block. They get 18 as the answer.
- Students count the small parts and relate it to the whole unit. They get a fraction form $\frac{18}{54}$ as the answer.

<u>Some of the possible strategies by the students for problem 3</u>

- Students do the calculation as follows.
  Time for exercise is 1 hour, it equals to 60 minutes. They divide it by 2 and get 30 minutes. Then they divide the 30 minutes by 3 and get 10 minutes as the answer.

- Students draw a clock to represent the situation. They focus on the minutes and they know that the whole circle (the clock) is 60 minutes. They shade a half of it and then dived the shaded area into three equal parts. And they can see that one part is equal to 10 minutes.

**c. Class discussion of problem 1, 2 and 3 (15 Minutes)**

- First focus on problem 1 and 2.
- If the conjecture of students divides the given chocolate block representation diagonally or randomly, let them think about the result of the partition, *is it produce equal parts or not?*
- Encourage students to react at this point until we assure that they get the notion that the partitioning activity should produce equal parts.
- To determine the parts of Hafidz, when the students just answer in whole number for example, they answer with 18 parts, the teacher asks the others to react about it. May be there is a student says about answering in fraction notation.
- Then, lead a discussion about the part-whole relation until the students understand how to produce fraction in this activity.
- For example, by counting the total number of small parts in the chocolate block, then we take 18 small parts of it. Ask the students about the meaning of it.
- The students may answer that it means we have $\frac{18}{54}$. Of the chocolate block.
- Let student who have $\frac{1}{3}$ as the answer to react to the previous answer. Then the class can discuss about the equivalence of fractions.
- Next, invite students to discuss about the answer of problem 3.
- Teacher invites students to reflect to their answer,

  Teacher : What is the initial time?
  Student A : An hour.
  Teacher : Yes you are right, how many minutes is that?
  Student B : 60 minutes.
  Teacher : Okay. Then, how many minutes that is used for jogging?
  Student A : 30 minutes.
  Teacher : how can you get 30 minutes?
  Student B : I divide the 60 minutes by 2.
  Student A : I take a half of 60 minutes.

Teacher : Yes, you are right. The result of 60 divided by 2 is 30 minutes and B said that she also get 30 minutes but she said that se take a half of 60 minutes.

Student B : So in other words we can express that we take $\frac{1}{2}$ of 60 minutes. And 30 minutes is the time for jogging.

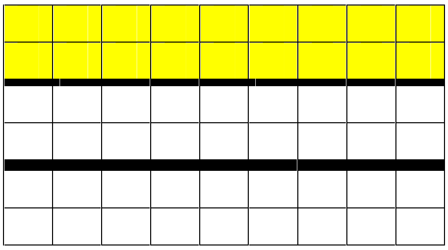
- Students should know that the 30 minutes is the result of taking a half of 60 minutes
- Continue the discussion until the student gets the final answer of the question that Hafidz reach Aufa house in ten minutes which is a third of 30 minutes.
- As the result of this discussion, assure students to recognize that they take $\frac{1}{3}$ of $\frac{1}{2}$ of an hour.

**d. Working on worksheet 2: problem 4 and 5 (5 minutes)**

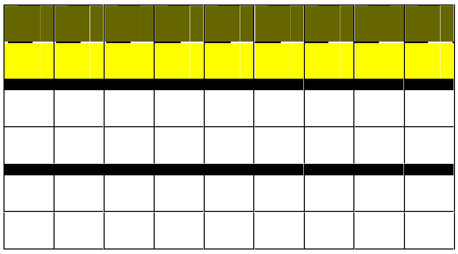
- The next problem is about sharing the chocolate parts of Hafidz to her sister Nazifah. This problem related to problem 1 and 2 in this worksheet.
  In problem 4, the context remembers about Hafidz that his sister Nazifah also like chocolate and he plans to share his chocolate parts with Nazifa equally. Then in Problem 5, the students need to write the part of Nazifah in fraction notation.
- Give time for students to understand the problem and asks one or two of them to explain what the problem is in their own sentence.
- Let the students start to work in their small group.
- The teacher walks around to see the students' work.
- If there are students that still don't know what to do, support them that they can start from the drawing of the answer of problem 1.

<u>Some of the possible strategies by the students for problem 4</u>

- Students use the answer of the first problem as the starting point.



Students divide it horizontally into three parts then they shade the first two rows.



To determine the Nazifah's share, the students divide the shaded area into two parts equally and one part of those is the Nazifah's share.

- Students do the similar strategy, but in a different direction. Instead of dividing the block horizontally, they divide it vertically into three equal parts as the part for Hafidz and then split it up into two parts

Some of the possible strategies by the students for problem 5

- To determine the fraction of the Nazifah's share students counts the pieces of it and relate with the whole unit. They get $\frac{9}{54}$ of the chocolate block as the answer.
- Students only look at the rows of the block and they conclude that Nazifah get $\frac{1}{6}$ of the chocolate block.
- Students may have a misunderstanding on determining the fractions. Instead of using the whole as a unit, they just consider the Hafidz parts as a unit. They come up with $\frac{9}{18}$ or $\frac{1}{2}$.of the chocolate block.

**e. Class discussion of problem 4 and 5 (10 minutes)**

- The focus of the discussion is the strategy that is used by the students when deal with the given array. The teacher can explore the way students interpret the dividing and the shading activity of the chocolate block.
- To deal with the misunderstanding of the students about the "whole" unit. The teacher can pose a question such as *"We get the Nazifah's share of chocolate block as the shaded part in the picture, but if we want to make a fraction of it, we only consider from the Hafidz part or the whole Chocolate?"*
- The following question also can help students realize that the fraction should be something out of the whole unit. *"Based on the question "What fraction of* ***the block of chocolate*** *did Nazifah receive?" It means we refer to what?"*
- By emphasizing on "**the block of chocolate"** students can recognize that the fraction of Nazifah's share is referring to the whole of the chocolate.

- Let students reflect on what they did. Perhaps one student will say that the process they did in answering the problem 5 is they take $\frac{1}{2}$ of $\frac{1}{3}$ of the chocolate block.
- Regarding the different form of a fraction that arise from students answer such as students with $\frac{9}{54}$ and $\frac{1}{6}$ as the answer for problem 5, the teacher can invite students to think about the representation of each fraction on the figure.
- Perhaps there is a student that remembers about simplifying fraction or fraction equivalency. He or she will recognize that $\frac{9}{54}$ can be simplified become $\frac{1}{6}$.
- If there is no students come up with this idea, the teacher can invite them to look at the drawing. In the drawing they can see that the shaded area for Nazifah is the same.
- And the teacher can lead students to conclude that if we refer to the same unit (the chocolate block with the same size), the result will be the same.
- It means that $\frac{9}{54}$ *of the chocolate block* is same with $\frac{1}{6}$ *of the chocolate block* in case the whole chocolate block is the same.

**f. Working on worksheet 2: problem 6 and 7 (10 minutes)**

- Furthermore, the teacher invites the students to work on problem 6 and 7.
- In problem 6 there is an array with dimension 4x6 as the representation of a chocolate block. The students should determine $\frac{2}{3}$ of $\frac{1}{2}$ of that chocolate block!
- Problem 7 is similar to problem 6. The dimension of the array is 3x12 and the students should determine $\frac{1}{6}$ *of* $\frac{2}{3}$ of that chocolate block.

<u>Some of the possible strategies by the students for problem 6 and 7</u>

- The students will use divide and shade strategy to answer these problems.
- Students will come up with different forms of fractions depend on the way they divide the block and the way they count the small pieces respect to the whole unit.

**g. Class discussion of problem 6 and 7 (10 minutes)**

- In the discussion, the teacher allows students to share their ideas in solving the problems, especially on how they shade the part of the chocolate block and how they interpret the result of the dividing and shading activity of the chocolate blocks.
- If there is a student who still have doubtfulness of the various fraction notations that come up with the answer, perhaps other students explain that they can reflect to the answer of the previous problems.
- Furthermore, the teacher can lead a discussion about choosing the simplest fraction notation as the answer. The teacher invites the students to think again about fraction equivalency as they already learned.

**h. Closing (5 minutes)**

- At the end of the lesson, support the students to make conclusion about the activity they have done in this lesson. Especially about the activity of taking a part of a part of a whole within the sharing chocolate context.
- Students may understand the use or an array model to help them in solving the taking a part of a part of a whole problems.
- Furthermore, the teacher gives worksheet 3 as homework for the students. The problem in the worksheet is:

    1. *Look back to the comic 1 story in worksheet 3. Can you show the chocolate block share for Aufa, Siraj and Hafidz with your own rectangle with grids inside?. You can try to solve it by constructing your own rectangle with smaller sizes. There will be more than one answer.*

    2. *Determine what part you get for Aufa's share based on the drawing that you make! Write your answer in fraction notation!*

- The teacher states that the answer of this homework will be discussed in lesson 3

# Teacher Guide

**Subject** **: Mathematics**
**Class/ Semester** **: V**
**Semester** **: 2**
**Time Allocation** **: 2 x 35 minutes**
**Lesson** **: 3**

**A. Learning Objectives**

- Students are able to take a part of a part of a whole within a context and without a context.
- Students are able to construct their own array and use it in solving the taking a part of a part of a whole problems.
- Students are able to take a part of a part of a whole of non unit fractions.

**B. Starting point :**

Students already learned about how to do a partitioning properly and give label to the result of the partitioning activity in fraction notation. And they already introduced to the use of an array model to help them in solving problems about taking a part of a part of a unit.

**C. Learning Methods :** working on worksheet, small group discussion and class discussion.

**D. Learning approach :** PMRI

**E. Learning materials :** Worksheet 3, Worksheet 4, grid papers, markers.

**F. Learning Activities**

**a. Orientation (5 minutes)**

**i.** Teacher asks students to look back on the chocolate block they have in the activity in lesson 2. And then discuss the solution of the homework

**ii.** Ask the students to tell about the story and the problems

**b. Class discussion of the homework (worksheet 3) (5 minutes)**

- To start the discussion the teacher invites a student to share his or her answer for the homework. Ask also about the strategy he or she did.
- Lets other students react to the student and compare with themselves.

<u>Some of the possible strategies by the students for problem in the homework</u>

- Students draw several sizes of chocolate block and then divide and shaded it as the strategy they already discussed on the activity 3
- There will be various answers such as $\frac{1}{6}$ , $\frac{4}{24}$, or $\frac{2}{12}$. It depends on how the students relate the shaded area respect to the whole block.

<u>The discussion</u>

- In the discussion the teacher address whether the students really understand the idea of constructing an array model.
- Due to the different form of fraction they have as the answer of problem 2, perhaps there is student remember about fraction equivalency that they already lean.
- During the discussion, the teacher support students to conclude that $\frac{4}{24}$, or $\frac{2}{12}$ can be simplified as $\frac{1}{6}$.
- Therefore, in the reflection, teacher let students recognize that the activity they have done is about taking $\frac{1}{2}$ of $\frac{1}{3}$ of a chocolate block with the simplest result is $\frac{1}{6}$ of the chocolate block.
- The teacher invites students to compare the result they get with the answer of problem 5 (the same problem) in worksheet 2. The discussion is continued to the relation between the two answers.
- The students can see that the simplest fraction form of the solution of problem 5 on worksheet 2 which is also about taking $\frac{1}{2}$ of $\frac{1}{3}$ of a chocolate block is the same with the simplest solution of the problem in the homework although the size of the chocolate blocks are different.
- The teacher emphasizes this knowledge and leads the students realize that they can use the similar strategy in solving the similar problems.

**c. Working on worksheet 4: part A, problem 1 (5 minutes)**

- After discussing the homework, the teacher introduces the new context and gives the part A of worksheet 4.

*Hafidz's mother makes a Martabak Telur for the desert at lunch. However, Hafidz went home lately after doing the exercise in the morning. They just found $\frac{1}{2}$ of the martabak telur in the kitchen.*

**A whole martabak telur**

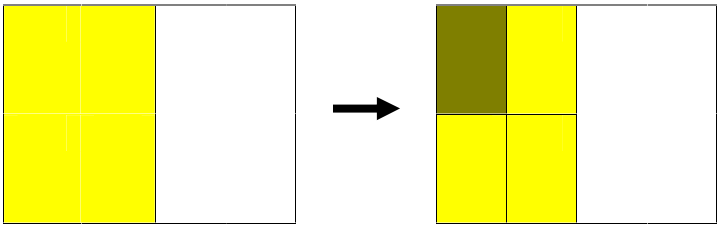
*Hafidz eats $\frac{1}{4}$ of the left-martabak. What part is that if we compare to the whole martabak? (Hint: You can draw a picture to help you in solving this problem)*

*Write your answer in fraction notation!*

- The teacher ask the students to think individually first and then let them discuss in pair.

Some of the possible strategies by the students for problem 1

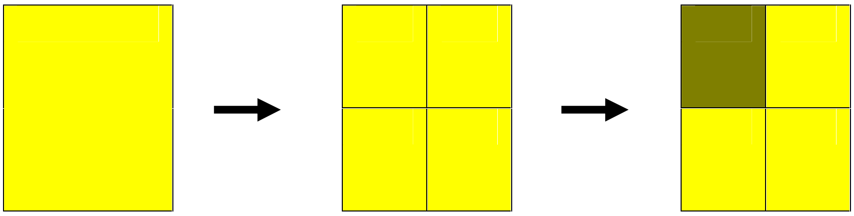
- Students represent the whole *martabak telur* in a rectangle as a starting point. Then they divide it into two and shade one of it. Then, they divide the shaded area into four equal parts and indicate one part of it as the part that is ate by Hafidz.



**Representation of a whole martabak in a rectangle**

To get the answer to the problem, students need to compare the Hafidz's part with the whole *martabak telur*. They get $\frac{1}{4}$ of $\frac{1}{2}$ of a whole *martabak telur*. The students may not get the final solution since they struggle on how to determine the fraction notation of Hafidz's part compare to the whole martabak telur. Some of the students may be just count the shaded area not the blank one, so they get $\frac{1}{4}$ as the answer, which is not correct.

- Students use the representation of the half of the *martabak telur* in a rectangle as a starting point and divide it into four. But they do not relate the parts with the whole cake as the unit.

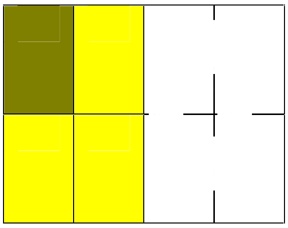


**Representation of a half of martabak telur as a starting point**

They cannot determine the whole unit and as an effect they cannot come up with a fraction notation for the Hafidz's part respect to the whole martabak telur.

**d. Class discussion of problem 1 (10 minutes)**

- If the conjecture of students who start with the representation of the whole martabak but they cannot determine the fraction notation of Hafidz's part happens, the teacher can invite the students to think about the Hafidz's part respect to the whole martabak. Teacher can ask the student that *"Can you think about how many times the small part (the Hafidz's part) fit into the whole martabak representation?"* Then, they can draw dot line to help them.



**Making dot lines inside the rectangle**

- Students may answer that the fraction notation of the Hafidz's part is $\frac{1}{7}$ which is not correct.
- Teacher can suggest students to think again by posing the question such as *"If we only arrange 7 small parts of the martabak then will they form a rectangle?"* Since the whole martabak is represented in a rectangle so there is

one more small part needed. Therefore, the Hafidz's part must fit 8 times in the rectangle and one small part is equal to $\frac{1}{8}$ of the whole martabak.

- In addition, if the second conjecture of students' answer of problem 1 happens, the teacher can ask the students about how to draw the whole martabak if we have a half of it. May be students will realize that they need a half more to complete the rectangle as the representation of the whole martabak. Furthermore, the discussion can be continued to determine the fraction notation for Hafidz's part which is already explained in teacher reaction of the first conjecture.

**e. Working on worksheet 4: part A, problem 2 (5 minutes)**

- After finishing the class discussion about the first problem, the teacher asks the students to work on the second problem.
- The teacher extent the story of problem 1. In problem 2 the teacher says that three students try to solve the problem 1 by drawing a rectangle on grid. The three rectangle have different dimension as can be seen in worksheet 4. The instruction is to let the student think about which drawing will be an easy help to solve the sharing martabak problem.
- The students will work in pair.

**f. Class discussion of problem 2 (10 minutes)**

- To start the discussion invites some students to share their answers.
- Let the other students react on it and compare to their own answers.
- Encourage the students to discuss about the strategy in choosing the drawing.
- The students may recognize the idea of using an appropriate array size which is depends on the number used in the problems.
- Moreover, teacher also can invite students to compare the answer of this problem (problem 2) with the solution that has been discussed in problem 1.
- If they get confused because of the different drawing and the different fraction form, then the teacher can bring the idea of fraction equivalence again to the students.
- It also can be used to strengthen students' understanding that in taking a part of a part of a whole, we need to consider the result respects to the initial whole unit.

**g. Working on worksheet 4: part B (15 minutes)**

- Furthermore, the teacher shares the part B of the worksheet.

- Give students time to work on the four bare problems given in this part.

**Problems in part B**

1. *Determine $\frac{1}{4}$ of $\frac{1}{3}$ !*
2. *Determine $\frac{1}{4}$ of $\frac{2}{3}$ !*
3. *Determine $\frac{3}{4}$ of $\frac{1}{3}$ !*
4. *Determine $\frac{3}{4}$ of $\frac{2}{3}$ !*

<u>Some of the possible answer by the students for problem 1</u>

- It is conjectured that may be there is a student who will get confuse because he or she cannot see what the whole unit in this problem is.
- For problem 1, the students will draw a rectangle on a grid paper with the dimension 3x4. They choose this size because they look at the denominator of the fractions in the problems. Then they try to shade the part as they have done in part A of worksheet 4. They will come up with $\frac{1}{12}$ as the answer.
- The students use the similar strategy for the next problems. The students may struggle when deal with non unit fractions

**h. Class discussion of part B (10 minutes)**

- When the students get confuse of the problems because they are different with the previous problems, the teacher can invite them to think about what is the quantity in the problem.
- For example in problem 1, the quantity is $\frac{1}{3}$ and it means that there is one third of a whole unit. The whole unit can be modeled with a rectangle, so first we need to divide the rectangle into three and take or shade 1 part of it.
- Further, teacher lets students to revisit what they have done with the problems in part A; in this case they will divide the one third parts into four and shade one of it.
- When dealing with non unit fractions the teacher may start the discussion on how to represent a $\frac{2}{3}$ in a rectangle, and if we want to take a $\frac{3}{4}$ of that $\frac{2}{3}$ we need to divide the $\frac{2}{3}$ part into four, and then take 3 parts of it.

- In addition, to interpret the result of the drawing into a fraction notation, perhaps there are students who can come up with the fraction notation they relate the intended parts respects to the total number or small parts in the rectangle. If it is not, then the teacher invites them to reflect again on how to relate the part with the whole

**i. Closing (5 minutes)**

- Support students to make a conclusion about what they have done in this lesson.
- Check at a glance about the main points in this lesson, whether the students have already gotten or not.
- Ask them to make a reflection and put it on their notebook.

# Teacher Guide

**Subject : Mathematics**
**Class/ Semester : V**
**Semester : 2**
**Time Allocation : 2 x 35 minutes**
**Lesson : 4**

A. **Learning Objectives**
   - Students are able to make a shift from the word “of” into the symbol “x” in multiplication of a fraction by another fraction
   - Students will understand the use of an array model in multiplicative reasoning.

B. **Starting point :**
   Students the use of an array model to help them in solving problems about taking a part of a part of a unit. They also already learned about how to take a part of a part of a whole and construct their own array to solve the problems.

C. **Learning Methods :** class discussion.

D. **Learning approach :** PMRI

E. **Learning materials :** Students’ work on Worksheet 2 and 4

F. **Learning Activities**

   a. **Orientation (10 minutes)**
   - Ask the students to prepare their work on worksheet 1, 2 and 4.
   - Remind students about the context in the activity of the previous lesson and also about several problems they already solved.

   b. **Class discussion (50 minutes)**

   - The teacher reminds the students about the context they already learned in the previous lessons, for example about the time that is used by Hafidz to reach Aufa’s house, the sharing chocolate block, sharing a *martabak telur* and the bare problems they already solved in the worksheet 3.
   - Let the students look back at their solution of the taking a part of a part of a whole problems that they already solved in the previous lessons.
   - The teacher gives instruction to the students to share the result of those problems in a complete sentence in the class discussion.

- It is conjectured that the students will share the solution of the previous problems about the taking a part of a part problems as follows

  $\frac{1}{3}$ of $\frac{1}{2}$ of 60 minutes = 10 minutes

  $\frac{1}{6}$ of 60 minutes = 10 minutes

  $\frac{1}{2}$ of $\frac{1}{3}$ of a chocolate block = $\frac{1}{6}$ of a chocolate block

  $\frac{2}{3}$ of $\frac{1}{2}$ of a chocolate block = $\frac{2}{6}$ of a chocolate block

  $\frac{1}{4}$ of $\frac{1}{2}$ of a *martabak telur* = $\frac{1}{8}$ of a *martabak telur*

  $\frac{3}{4}$ of $\frac{2}{3}$ $= \frac{6}{12}$ $= \frac{1}{2}$

  $\frac{2}{5}$ of $\frac{2}{3}$ $= \mathbf{\frac{4}{15}}$
- In the discussion the teacher invites students to look at the relationship between the fractions in the list. The teacher will give the students time to think individually. Further, the teacher invites students to share their ideas.
- We expect that there will be a student recognize about the relationship between the numerators of the fractions and also between the denominators of the fractions.
- The teacher will elaborate it until the students notice about the multiplication of two fractions.
- To lead students into that idea the teacher could make the new list from the previous one.
- The teacher invites the students to leave the whole unit of each part of the previous list as follows.

  $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

  $\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{6}$

  And so on.
- We conjecture that the students will see clearly that the result of taking a part of a part can be determined by multiplying the numerator of the first fraction with the numerator of the second fraction over the multiplication of the denominator of the first fraction and the denominator of the second fraction in the problem.
- Therefore, the teacher can ask the students to write the final result of the discussion in the whiteboard.

$$\frac{1}{3} \text{ of } \frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

$$\frac{2}{3} \text{ of } \frac{1}{2} = \frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$$

And soon.

- Trough the class discussion the teacher invites all of the students to closely see this idea and asks them whether they understand about it or not.
- When the students still struggle to grasp this idea the teacher can support them by investigating an example of the list.
- State that the students could think how they got the result of determining a part of a part or in another word determining the result of a fraction of a fraction.
- We expect that there will be one of the students realize that they could use a multiplication between the numerators and the multiplication of the denominators.

**c. Closing (10 minutes)**

- Support students to make a conclusion of the discussions.
- Ask them to make a note and put it on their notebook.

# Teacher Guide

**Subject : Mathematics**
**Class/ Semester : V**
**Semester : 2**
**Time Allocation : 2 x 35 minutes**
**Lesson : 5**

**A. Learning Objectives**

- Students are able to choose an appropriate array to help them in solving the multiplication of two fractions problems.
- Students are able to determine the fraction notation of the result of the multiplication of two fractions based on the given array.
- Students are able to determine the problem when the shaded array is given.

**B. Starting point :**

Students already learned about how to take a part of a part of a whole and construct their own array to solve the problems. They also already make a shift from term "of" into term "times" in the taking a part of a part of a whole. The students already know the use of symbol "x" in multiplication of two fractions.

**C. Learning Methods :** card game, class discussion.

**D. Learning approach :** PMRI

**E. Learning materials :** Cards with problem, cards with the array, and cards for the result in fraction notation

**F. Learning Activities**

**a. Orientation (5 minutes)**

- Let the students reflect on the previous lesson.
- Ask about what kind of big ideas they already get.
- Tell the students that in this lesson they will work on card games.

**b. Giving instruction (10 minutes)**

- Explain the rule of the game to the students. and then ask wether they understand or not.

The instruction of this game is the following:

- Work in pair
- Find the appropriate array for the problem in P cards.
- Indicate in the array by shading the solution of the problem.

- Write the solution of the problem in a fraction notation on the S card.
- The shaded- array in card *e* is correspondence to the $P_5$ card. Determine the problem which is represented by the shaded- array. (Hint: the dark yellow parts indicate the parts which are shaded twice).
- Write the solution of the problem you get for $P_5$ card in the S card.

**c. Playing the cards game (30 minutes)**

- After the students understand about the instruction. Give the students time to finish the game.
- While the students work on the cards, the teacher walks around to see students' work.
- The problems in this game are the following.

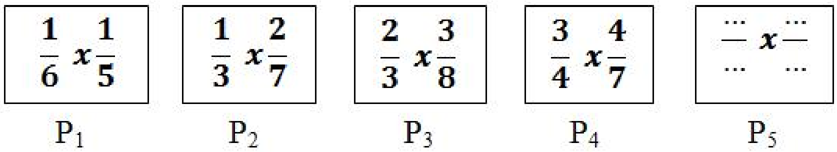



**Figure 11.** **The Problem cards**

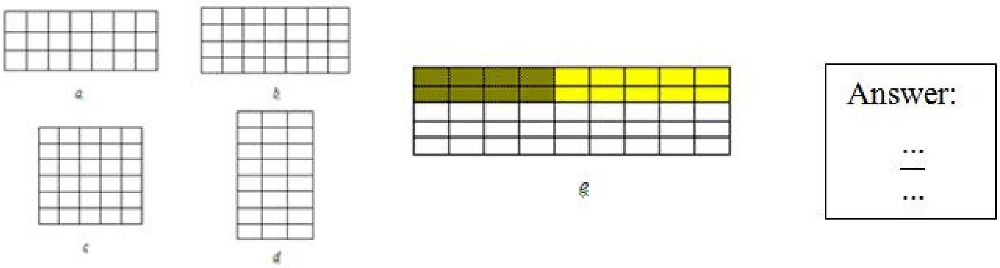



**Figure 12.** **The array cards (*a*-*e*) and an example of the S cards**

Information for the students, that on the *e* card, the dark shaded parts are the part which are shaded twice.

<u>Some of possible strategies by the students</u>

- To find the appropriate array for the problem in the first four problem cards, the students may only do a trial and error strategy.
- Some students may consider the denominator of the problem to determine the appropriate array dimension.

- The students indicate the solution in the array by shading the parts depends on the problem.
- To make the fraction notation of the solution in the array, the students will count the shaded parts and respect it to the total number of small parts in the array.
- For the last problem ($P_5$ and card *e*), the students will get confuse because they don't know to take what part of what part in the array. Some of the students may answer with $\frac{8}{45}\ of\ \frac{10}{45}$ or $\frac{8}{18}\ of\ \frac{18}{45}$ which is not correct.

  For the last problem, the students answer with $\frac{4}{9}\ of\ \frac{2}{5}$ because they consider the dimension of the array and also the information in the instruction of the game about the parts that is shaded twice.

**d. Class discussion (20 minutes)**

- In the discussion, teacher encourages the students who have consideration of choosing the array to share their idea instead of just trial and error.
- The students may explain that they just look at the pattern of the answer in the previous lesson.
- Perhaps, there is a student that recognized that when we want to find $\frac{1}{6}\ of\ \frac{1}{5}$ we need to divide a rectangle vertically into five and then divide it horizontally into six or vise versa. It produces an array with 5x6 as its dimension.
- They can continue to discuss about the way they shade the array to indicate the solution and then how to interpret the solution into a fraction notation.
- Furthermore, for the last problem, the teacher invites the students with non correct answer to explain their strategy, why he or she can come up with that solution. Let other students react to this explanation.
- The students may use the information given at the beginning of the game that the dark shaded parts in the array are the parts which are shaded twice.
- It means that first we shade the first two rows (the yellow parts). There are five rows in that rectangle so it means $\frac{2}{5}$. Next, the total number of columns is 9 and there are 4 columns that are shaded overlap with the yellow parts.
- Teacher asks "*What does it mean? What fraction is that?*" the intention of the question is to lead students to recognize that it means they take four parts (columns) over the nine parts (columns), $\frac{4}{9}$. Finally, the students will recognize that they have $\frac{2}{5}$ of the array and we take $\frac{4}{9}$ of it.

**e. Closing (5 minutes)**

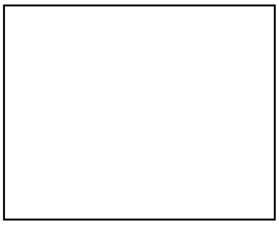
- Support students to make a reflection of the activity in this lesson.
- Ask them to write about the big ideas that is used in this lesson and put it their notebook.

**Appendix D** Pre-test of the cycle 1

**Nama :** **Tanggal:**

1. Ibu membuat sebuah kue *tart* rasa Mangga. Misalkan, gambar persegi di bawah ini adalah gambar kue rasa mangga itu. Dapatkah kamu menunjukkan pada gambar itu jika kamu mengambil $\frac{1}{2}$ dari $\frac{1}{3}$ dari kue tersebut? Jelaskan caramu!

Jawaban:



2. Jumlas siswa SD kelas 5 SD Tanah Air adalah 40 orang. Setengah dari mereka adalah laki-laki dan seperempat dari jumlah siswa laki-laki itu menyukai sepakbola.
   a. Dapatkah kamu menentukan jumlah siswa laki-laki yang menyukai sepakbola? Jelaskan jawabanmu!

Jawaban:

   b. Berapa bagiankah jumlah siswa laki-laki yang menyukai olahraga sepakbola jika dihubungkan dengan jumlah keseluruhan siswa kelas 5 SD Tanah Air tersebut? Tuliskan jawabanmu dalam bentuk pecahan!

Answer:

3. Ridho memiliki sebuah cokelat batang seperti yang terlihat pada gambar di bawah ini. Dia membagi cokelat batang itu dengan Roni sama banyak.
   a. Tunjukkan pada gambar dengan mengarsir bagian untuk Ridho dan bagian untuk Roni!

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   b. Namun, Roni memberikan sepertiga dari bagiannya kepada adik perempuannya yang bernama Rosi. Tunjukkan pada gambar dibawah ini bagian untuk Rosi!

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4. Tentukanlah $\frac{1}{3} \times \frac{1}{2}$ !

Jawaban:

**Appendix E** Post-test of the cycle 1

**Nama:** **Tanggal:**

1. Pak Gunawan bring a bika ambon home. The bika ambon is given by his friend from Medan. He and Bu Susi eat $\frac{2}{5}$ of eat.

   a. What part left?

   Answer:

   b. His children Andika and Audi share the leftover equally. What part of the bika ambon is that for each of them?

   Answer:

**2.** Anita has a big chocolate bar. She wants to share this chocolate bar with Raisha and Cintya.

   a. What part of the chocolate bar is for each of them?

   Answer:

   b. Can you show it in a figure of a chocolate bar?

   Answer:

c. Raisha take her parts home, and then she shares her part equally with her brothers Badu and Andi. Can you determine what part is for Badu respect to the initial chocolate bar?

Answer:

**3.** Find $\frac{2}{9}$ of $\frac{3}{4}$!

Answer:

**4.** Determine $\frac{3}{7}$ $x$ $\frac{2}{5}$ !

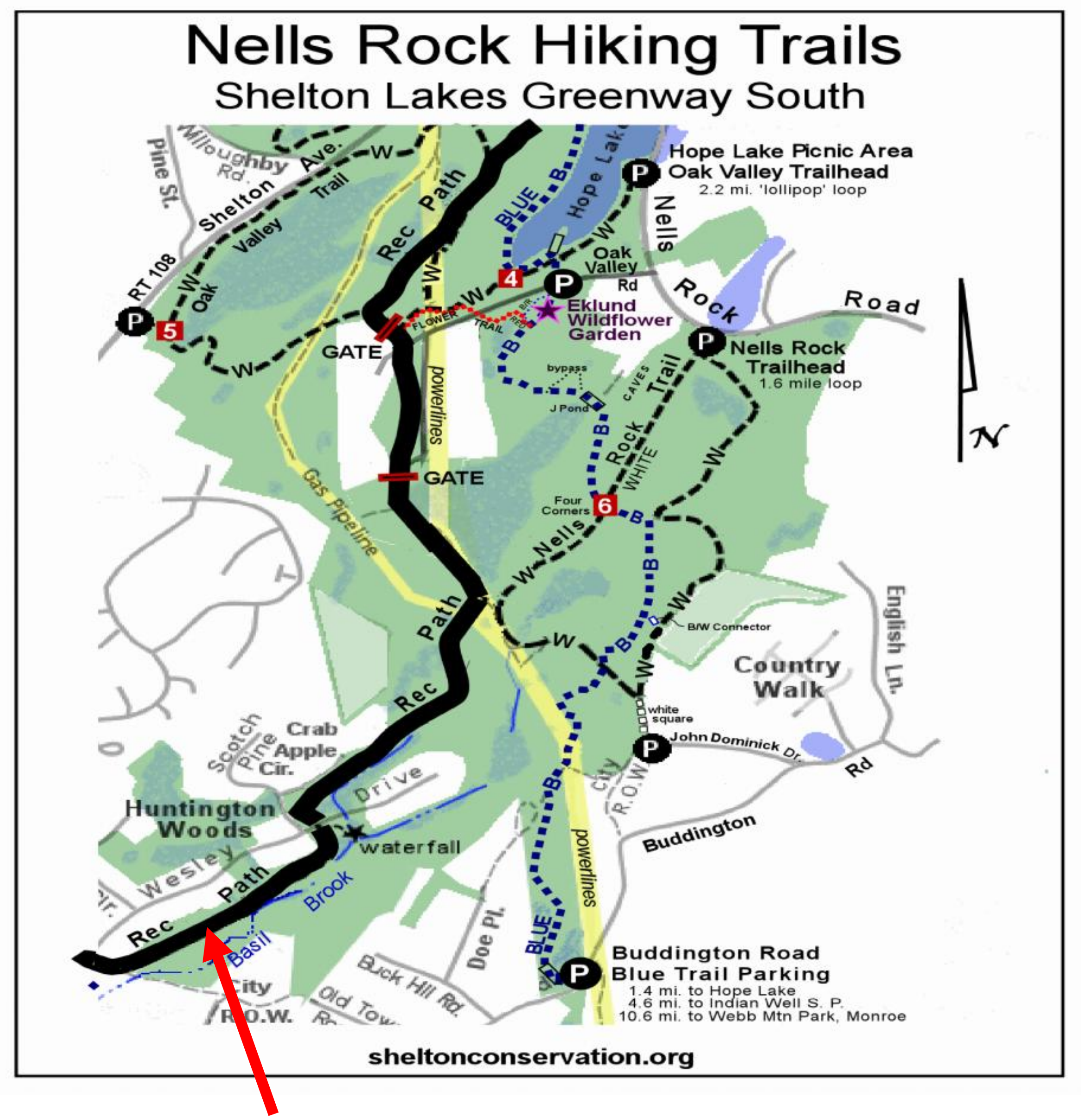
Answer:

**Appendix F** Worksheet of the cycle 1

**Lembar Kerja Siswa 1**

Nama : …………………………….

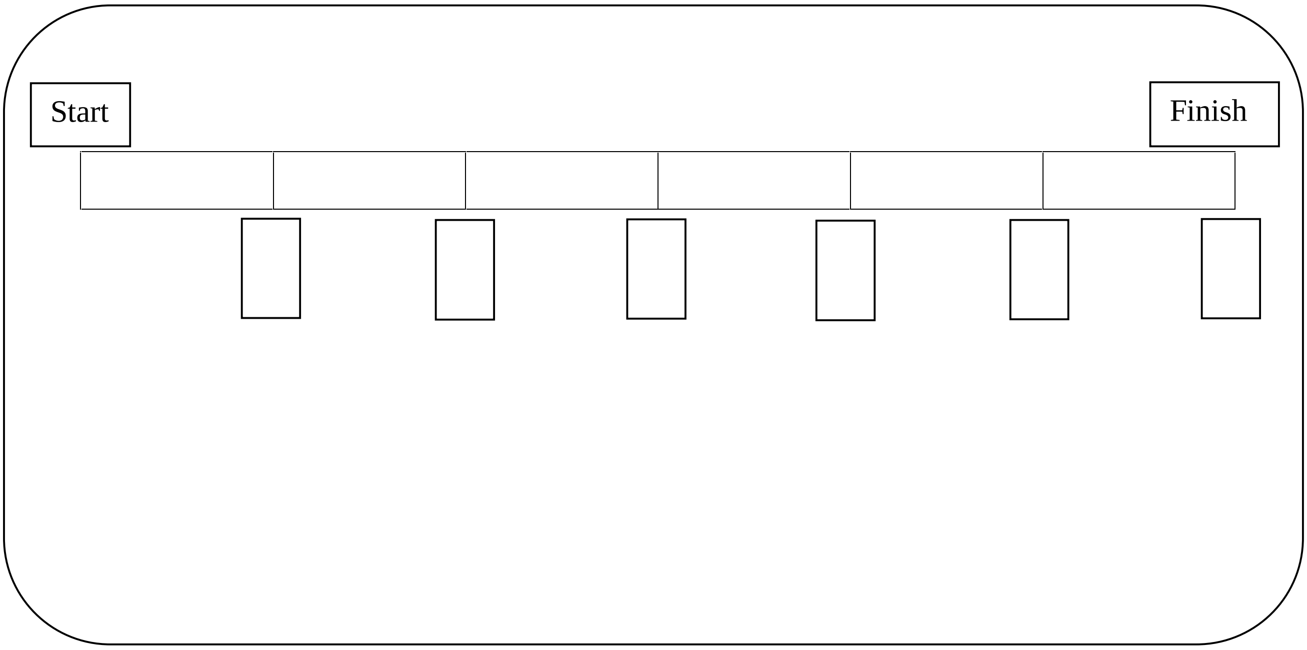
Kelompok : ……………………………. Tanggal: ………………...



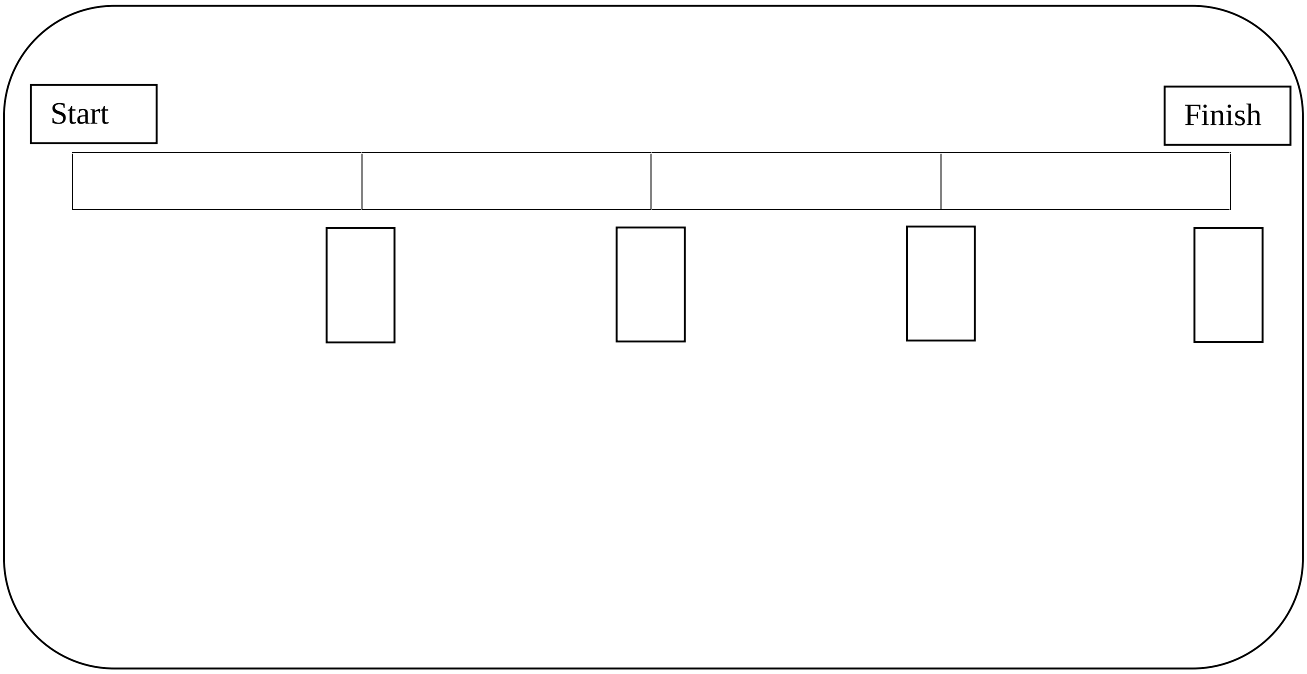
Sebuah kelompok pramuka berencana untuk mengadakan kegiatan *hiking* pada akhir bulan ini. Panjang jalur hiking adalah 6 km. Panitia menyiapkan beberapa permainan *out bond* di 4 pos yang terletak di jarak yang sama satu sama lain di sepanjang jalur *hiking*. Pos yang terakhir berada di garis finish. Selain itu, panitia akan menempatkan beberapa bendera di sepanjang jalur *hiking* sebagai tanda untuk tempat beristirahat. Mereka menempatkan 1 bendera disetiap 1 km dari jalur *hiking* tersebut, dan bendera terakhir ada di garis finish. Kamu dapat melihat jalur *hiking* pada gambar (di halaman 1), ditunjukkan oleh tanda panah pada gambar.

1. Kamu adalah anggota panitia kegiatan *hiking* dan tugas kamu adalah untuk berpikir tentang bagaimana menempatkan bendera dan pos permainan. Gambarkanlah pada peta jalur *hiking* di halaman 1 posisi tiap-tiap bendera dan pos! (Tips: Kamu bisa menggunakan pita sebagai alat bantu).

2. Misalkan gambar di bawah ini sebagai jalur *hiking* termasuk dengan lokasi dari bendera dan pos bermain yang telah kamu letakkan. Tentukanlah pada seberapa bagian dari jalur hiking itu posisi untuk setiap bendera dan pos bermain!
   a. Untuk lokasi bendera.




   b. Untuk lokasi pos bermain.



3. Di saat menempatkan bendera dan pos bermain di lokasi jalur *hiking*, panitia akan mengendarai motor untuk mengukur jarak. Pada jarak berapakah dari garis start, panitia harus menempatkan pos bermain pertama? (Panjang total jalur *hiking* tersebut adalah 6 km)

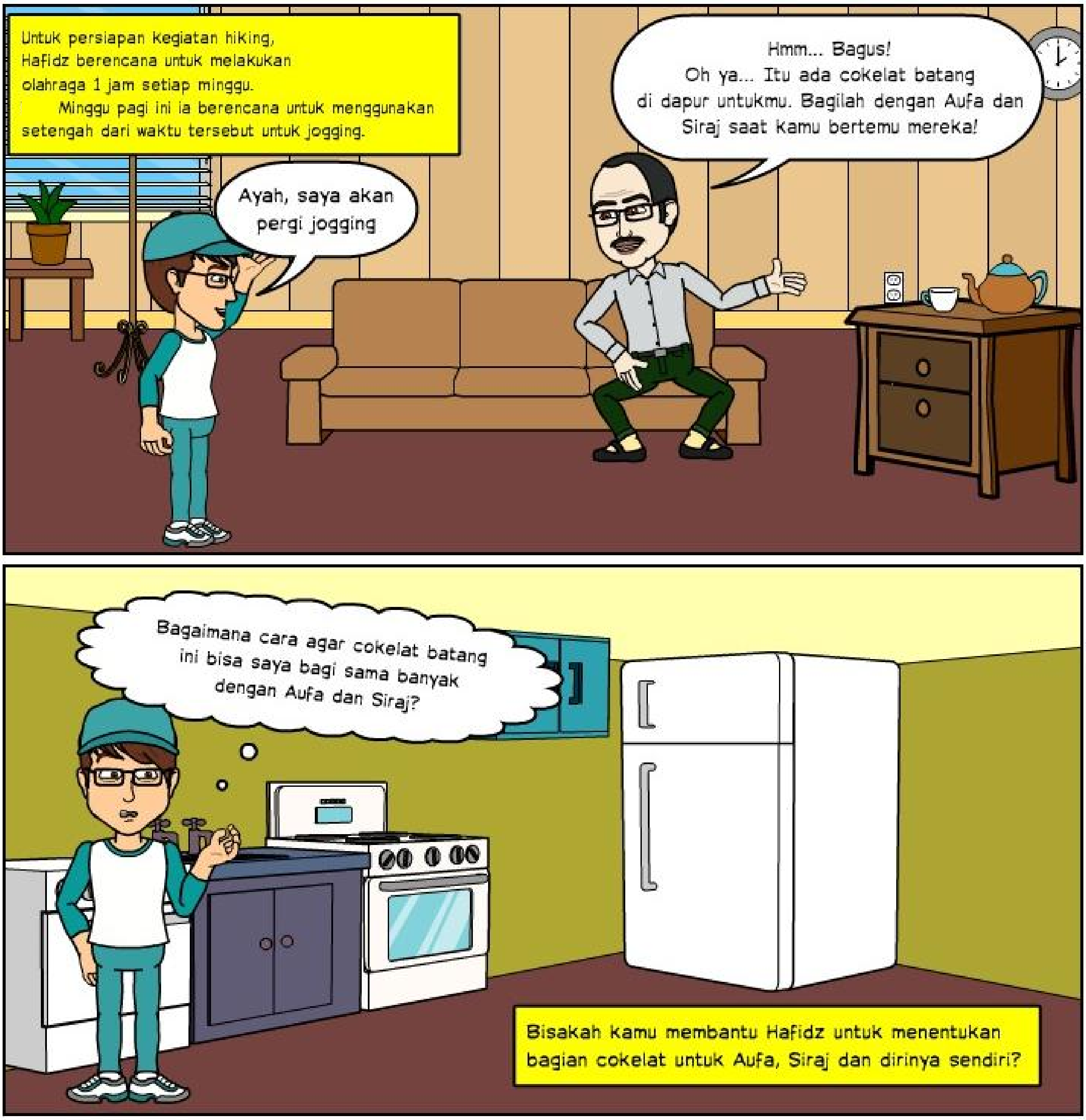
   Jawaban:

**Lembar Kerja Siswa 2**

Nama : …………………………….

Kelompok : ……………………………. Tanggal: …………………..

**Mari kita perhatikan cerita berikut! (Komik 1)**



1. Misalkan gambar *grid* di bawah ini adalah cokelat batang yang diberikan oleh ayah Hafidz. Tunjukkanlah dengan cara mengarsir bagian cokelat yang akan diperoleh oleh Aufa, Siraj dan Hafidz!

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2. Berapa bagiankah dari cokelat batang itu untuk Hafidz? Tuliskan jawabanmu dalam bentuk pecahan!

   Jawaban:

**Apakah kamu sudah selesai? Baiklah, mari kita kembali ke cerita tentang Hafidz!**

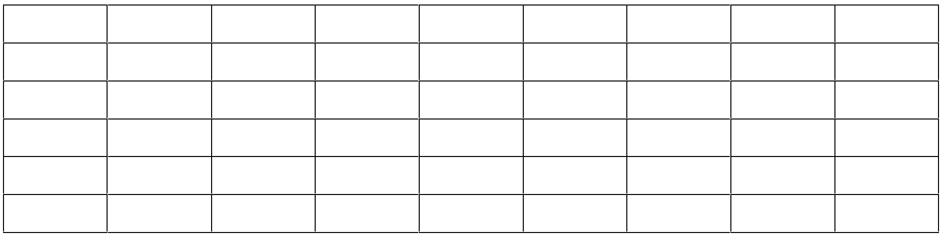
**(Komik 2)**

3. Kamu dapat menuliskan jawabanmu untuk pertanyaan Hafidz di kotak di bawah ini!
   Jawaban:

**Komik 3**


Oh ya teman-teman, Aku punya cokelat nih.
Kita bisa membaginya bersama.
Sip!! Terima kasih ya, Bro.
Aku sangat suka cokelat.
Setelah saling menyapa, Hafidz menawarkan cokelat kepada Aufa dan Siraj.
Tapi....
Hmmm,
Aku ingat adikku Nazifah.
Dia juga suka cokelat.
Okay, Aku akan membagi cokelat bagianku dan memberikannya kepada Nazifah.

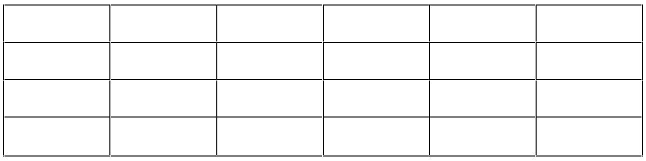
4. Bagaimana dengan bagian cokelat untuk Nazifah, dapatkah kamu menunjukkannya dalam gambar di bawah ini? (Tips: Gunakanlah jawaban dari pertanyaan no 1. Gambarlah garis pada bagian cokelat milik Hafidz).



5. Berapa bagian dari cokelat batang itu yang diperoleh Nazifah? Tuliskan jawabanmu dalam bentuk pecahan!

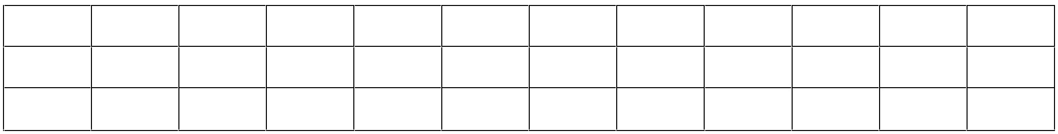
**Sekarang, misalkan gambar di bawah ini adalah sebuah cokelat batang. Gunakanlah gambar yang diberikan dengan cara mengarsir untuk menentukan jawaban dari soal-soal berikut!**

6. a. Tentukanlah $\frac{2}{3}$ dari $\frac{1}{2}$ bagian cokelat batang di bawah ini!



b. Berapa bagian dari cokelat batang yang kamu dapatkan? Tulis jawabanmu dalam bentuk pecahan

7. a. Tentukanlah $\frac{1}{6}$ dari $\frac{2}{3}$ bagian cokelat batang di bawah ini!



b. Berapa bagian yang kamu peroleh? Tuliskan jawabanmu dalam bentuk pecahan!

**Lembar Kerja Siswa 3 (PR)**

Nama : …………………………….

Kelompok : ……………………………. Tanggal: ………………….

1. Lihat kembali cerita Komik 1 pada LKS 2. Dapatkah kamu menunjukkan bagian cokelat batang untuk Aufa, Siraj dan Hafidz dengan mengambarkan persegipanjang dengan ukuran yang lebih kecil pada kertas berpetak di bawah ini! Akan ada lebih dari satu kemungkinan jawaban.

2. Tentukanlah dalam bentuk pecahan berapa bagian yang akan diterima oleh Nazifah berdasarkan gambar yang kamu buat!

**Lembar Kerja Siswa4**

Nama : …………………………….

Kelompok : ……………………………. Tanggal: …………………..

**Bagian A**

1. Ibu Hafidz membuat sebuah martabak telur untuk makan siang. Namun, Hafidz terlambat pulang setelah melakukan olahraga pagi. Ia hanya menemukan $\frac{1}{2}$ dari martabak telur itu di dapur.

   Hafidz memakan $\frac{1}{4}$ dari bagian yang ada itu. Berapa bagiankah itu jika kita bandingkan dengan keseluruhan martabak telur. (Kamu dapat membuat gambar untuk membantumu menyelesaikan soal ini)

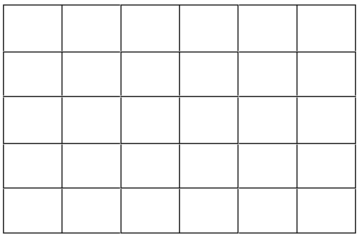
**A whole Martabak Telur**

   Tuliskan jawabanmu dalam bentuk pecahan!

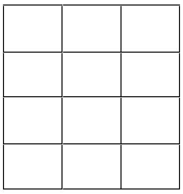
   Jawaban:

2. Tiga orang siswa mencoba menyelesaikan soal nomor 1 di atas dengan menggambar sebuah persegi panjang di kertas berpetak. Seperti yang dapat kamu lihat di bawah ini:

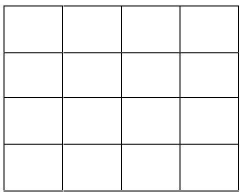
   Siswa A

   Siswa B

   Siswa C

   Gambar yang manakah yang kamu pilih untuk bisa membantumu menyelesaikan soal nomor 1 dengan mudah? Jelaskan jawabanmu!

   Jawaban:

**Bagian B**

**Nama:**

Tips:

Saya kira, saya bisa menggunakan persegipanjang untuk membantu saya menyelesaikan soal ini. ^_^

**Selesaikan soal-soal berikut ini!**

1. **Tentukanlah $\frac{1}{4}$ dari $\frac{1}{3}$ !**

2. **Tentukanlah $\frac{1}{4}$ dari $\frac{2}{3}$ !**

3. **Tentukanlah $\frac{3}{4}$ dari $\frac{1}{3}$ !**

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4. **Tentukanlah $\frac{3}{4}$ dari $\frac{2}{3}$ !**

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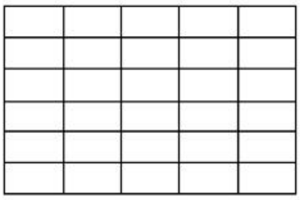
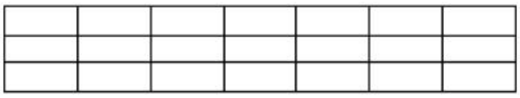
**Kartu untuk *Card Game***

1. $\frac{1}{6} \times \frac{1}{5}$



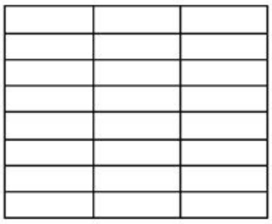
Jawaban: $\frac{\dots}{\dots}$

2. $\frac{1}{3} \times \frac{2}{7}$



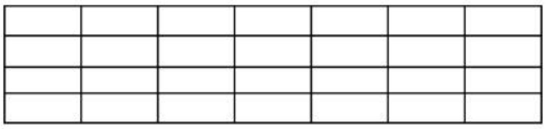
Jawaban: $\frac{\dots}{\dots}$

3. $\frac{2}{3} \times \frac{3}{8}$



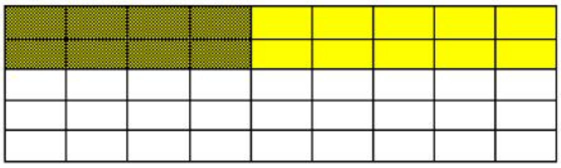
Jawaban: $\frac{\dots}{\dots}$

4. $\frac{3}{4} \times \frac{4}{7}$



Jawaban: $\frac{\dots}{\dots}$

5. $\frac{\dots}{\dots} \times \frac{\dots}{\dots}$



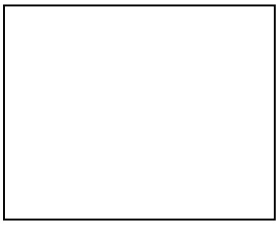
Jawaban: $\frac{\dots}{\dots}$

**Appendix G** Pre-test of the cycle 2

**Nama :** **Tanggal:**

1. Ibu membuat sebuah kue *tart* rasa mangga. Misalkan, gambar persegi di bawah ini adalah gambar kue rasa mangga itu. Dapatkah kamu menunjukkan pada gambar itu jika kamu mengambil $\frac{1}{2}$ dari $\frac{1}{3}$ dari kue tersebut? Jelaskan caramu!

Jawaban:



2. Banyak siswa kelas 5 SD Tanah Air adalah 40 orang. Setengah dari mereka adalah laki-laki dan seperempat dari banyak siswa laki-laki itu menyukai sepakbola.
   a. Dapatkah kamu menentukan banyak siswa laki-laki yang menyukai sepakbola? Jelaskan jawabanmu!

Jawaban:

   b. Berapa bagiankah banyak siswa laki-laki yang menyukai sepakbola jika dihubungkan dengan banyak keseluruhan siswa kelas 5 SD Tanah Air tersebut? Tuliskan jawabanmu dalam bentuk pecahan!

Answer:

3. Ridho memiliki sebatang cokelat seperti yang terlihat pada gambar di bawah ini. Dia membagi batang cokelat itu dengan Roni sama banyak.
   a. Tunjukkan pada gambar di bawah dengan cara mengarsir bagian untuk Ridho dan bagian untuk Roni!

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   b. Namun, Roni memberikan sepertiga bagiannya kepada adiknya yang bernama Rosi. Tunjukkanlah bagian untuk Rosi pada gambar batang cokelat di atas!

   c. Berapa bagiankah dari batang cokelat itu untuk Rosi jika dihubungkan dengan keseluruhan batang cokelat? Tuliskan jawabanmu dalam bentuk pecahan!

Jawaban:

4. Tentukanlah $\frac{1}{3} \times \frac{1}{2}$ !

Jawaban:

**Appendix H** Post-test of the cycle 2

**Nama:** **Tanggal:**

1. Pak Gunawan membawa sebuah bika ambon. Bika ambon itu adalah pemberian dari temannya yang baru datang dari kota Medan. Pak Gunawan dan Bu Susi memakan $\frac{2}{5}$ bagiannya.

a. Berapa bagiankah yang tersisa?

Jawaban:

b. Anak mereka, Andika dan Audi membagi kue yang masih ada itu sama banyak. Berapa bagiankah dari kue bika ambon itu untuk tiap-tiap anak?

Jawaban:

**2.** Anita mempunyai sebatang cokelat yang berukuran besar. Dia ingin membaginya dengan Raisha dan Cintya sama banyak.

a. Berapa bagiankah dari batang cokelat itu untuk tiap-tiap anak?

Jawaban:

b. Dapatkah kamu menunjukkannya ke dalam gambar sebatang cokelat?

Jawaban:

c. Raisha membawa bagiannya pulang dan membaginya dengan 2 orang adiknya, Badu dan Andi. Dapatkah kamu menentukan berapa bagiankah untuk Badu jika dihubungkan pada batang cokelat yang pertama?

Jawaban:

**3.** Tentukanlah $\frac{2}{9}$ dari $\frac{3}{4}$!

Jawaban:

**4.** Tentukanlah $\frac{3}{7} \times \frac{2}{5}$ !

Jawaban:

**Appendix I** Lesson Plan of the cycle 2

# Rencana Pelaksanaan Pembelajaran (RPP) 1

**Satuan Pendidikan : SD Al- Hikmah Surabaya**
**Mata Pelajaran : Matematika**
**Kelas : V**
**Semester : 2**
**Alokasi Waktu : 2 × 35 minutes**

**A. Standar Kompetensi**
Menggunakan pecahan dalam pemecahan masalah.

**B. Kompetensi Dasar**
Perkalian dan pembagian berbagai bentuk pecahan.

**C. Tujuan Pembelajaran**
1. Siswa dapat melakukan aktivitas *partitioning* dengan benar.
2. Siswa dapat menamai hasil dari aktivitas *partitioning* dengan benar.
3. Siswa dapat melakukan perkalian sebuah pecahan dengan bilangan bulat dalam sebuah konteks.

**D. Starting point :**
Siswa pada kelas 5 yang telah belajar tentang memproduksi pecahan, penjumlahan dan pengurangan pecahan, dan pecahan senilai.

**E. Metode Pembelajaran :** *Hands on activity*, mengerjakan LKS, diskusi kelas.
**F. Strategi Pembelajaran :** PMRI
**G. Alat dan Bahan :** LKS 1, pita, Spidol.
**H. Aktivitas Pembelajaran**

**1. Pendahuluan (5 menit)**

- Guru memberikan sebuah cerita tentang sebuah klub pramuka. Guru menanyakan kepada siswa apakah mereka pernah mengikuti kegiatan pramuka. Berikan kesempatan kepada siswa untuk menyebutkan kegiatan apa saja yang biasanya diadakan oleh klub pramuka.
- Memberikan konteks :
  *Sebuah kelompok pramuka berencana untuk mengadakan kegiatan hiking pada akhir bulan ini. Panjang jalur hiking adalah 6 km. Panitia menyiapkan beberapa permainan out bond di 4 pos yang terletak di jarak yang sama satu sama lain di sepanjang jalur hiking. Pos yang terakhir berada di garis finish.*

- Guru membagi siswa menjadi beberapa kelompok kecil, setiap kelompok terdiri dari 5 anak. Kemudian guru membagikan LKS 1 dan meminta siswa untuk focus pada soal yang pertama terlebih dahulu. Guru juga menyediakan beberapa buah pita dan spidol untuk tiap-tiap kelompok.

**2. Mengerjakan LKS 1: Soal 1 (10 menit)**

- Guru memberitahu siswa untuk focus pada soal yang pertama. Instruksi pada soal yang pertama tersebut adalah agar siswa berperan sebagai panitia dari kegiatan *hiking* dan pikirkan tentang cara untuk menempatkan bendera dan pos-pos permainan di sepanjang jalur *hiking.* (Gambar dari jalur *hiking* dapat dilihat pada LKS 1 halaman 1).
- Ketika siswa mengerjakan LKS 1 di dalam kelompoknya, guru berkeliling dan memberikan bantuan kepada kelompok yang memiliki kesulitan dalam memahami soal yang ada.
- Guru membuat catatan tentang strategy yang dipakai oleh siswa sebagai pertimbangan untuk memilih kelompok yang akan menampilkan hasil kelompoknya terlebih dahulu di depan kelas saat diskusi.
- Jika ada kelompok yang membutuhkan waktu yang terlalu lama untuk berpikir tentang strategi dalam menyelesaika soal, guru bisa memberikan tips, bahwa mereka bisa menggunakan pita yang disediakan untuk membantu mereka.

<u>Beberapa kemungkinan jawaban siswa</u>

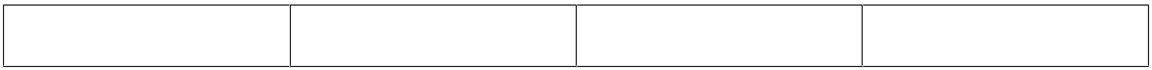
- Siswa hanya menggunakan perkiraan pada gambar dan menandai pada gambar posisi untuk tiap-tiap bendera dan posisi untuk pos-pos permainan pada jalur hiking pada gambar di halaman 1 LKS 1.
- Siswa menggunakan pita untuk mendapatkan representasi pajang dari jalur *hiking* pada gambar. Kemudian siswa meluruskan pita tersebut.
- Untuk menentukan letak dari pos-pos permainan, siswa melipat pita tersebut dua kali.
- Untuk menentukan posisi dari bendera-bendera, siswa melipat pita secara acak dan menggunakan strategi *trial and error.* Mereka memperoleh 6 bagian yang sama dari pita tersebut.
- Siswa menggunakan pita yang telah dilipat untuk memprediksi letak dari tiap-tiap bendera dan pos-pos permainan dengan menghimpitkan pita tersebut pada jalur *hiking* pada gambar di halaman 1 LKS 1.

**3. Diskusi Kelas untuk soal 1 (15 menit)**

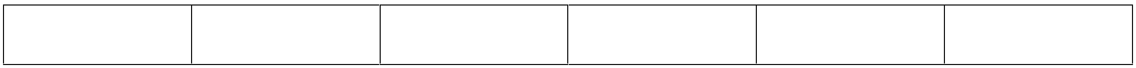
- Jika ada siswa yang hanya menggunakan perkiraan saja, maka mintalah siswa tersebut untuk men *share* hasil kerjanya dan mintalah pendapat dari siswa lainnya.
- Hal-hal yang akan didiskusikan:
  - *Bagaimana caramu melakukan estimasi/ perkiraan?*
  - *Apakah kamu puas dan yakin bahwa perkiraan mu itu sudah benar?*
  - *Apa yang dapat kamu lakukan untuk membuatnya menjadi lebih tepat?*

  Siswa mungkin berpikir bahwa mereka butuh strategi yang bisa meyakinkan bahwa *partisi* yang mereka lakukan sudah benar.

- Guru dapat menarik siswa yang menggunakan pita sebagai alat bantu untuk menjelaskan strategi mereka pada saat diskusi kelas dan berikan kesempatan kepada yang lain untuk memberikan tanggapan.
- Kemudian, guru dapat meminta siswa kelompok lainnya untuk menggunakan pita. Misalnya dengan bertanya sebagai berikut, *"Mengapa kamu tidak mencoba untuk menggunakan pita juga sebagai alat bantu di kelompokmu masing-masing?"*
- Tutuplah kegiatan ini dengan mengajak siswa untuk merepresentasikan pita menjadi sebuah diagram batang (horizontal) beserta dengan garis lipat nya.



Representasi dari pita yang telah dilipan menjadi 4 bagian yang sama



Representasi dari pita yang telah dilipan menjadi 6 bagian yang sama

**4. Mengerjakan LKS 1 : Soal 2. Menamai dengan notasi pecahan (15 menit)**

- Guru memberikan soal kedua yang ada di LKS 1 kepada siswa.
- Siswa kembali bekerja pada kelompoknya masing-masing.
- Berikan orientasi kepada siswa bahwa mereka harus menentukan notasi pecahan untuk tiap-tiap posisi dari bendera-bendera dan pos-pos permainan.
- Saat siswa bekerja pada kelompoknya, guru berkeliling dan memberikan bantuan kepada siswa untuk memahami instruksi yang diberikan.

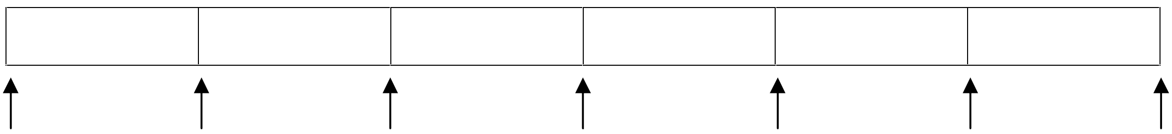
  <u>Beberapa kemungkinan jawaban dari siswa</u>

- Siswa hanya memberikan notasi dengan bilangan bulat yang mengindikasikan bendera pertama, bendera kedua, ketiga dan keempat. Mereka menggunakan strategi yang sama untuk memberikan label pada posisi pos-pos bermain.

- Siswa memberikan label untuk posisi-posisi bendera dan pos permainan hanya dengan menggunakan pecahan satuan. Mereka memberikan notasi untuk masing-masing bagian dengan $\frac{1}{6}$ untuk posisi tiap-tiap bendera dan $\frac{1}{4}$ untuk posisi tiap-tiap pos permainan.
- Siswa menggunakan pecahan non satuan. Mereka menggunakan pecahan $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}$ dan seterusnya untuk posisi bendera dan pecahan $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ dan $\frac{4}{4}$ untuk menandai posisi dari pos-pos permainan.

**5. Diskusi kelas pada soal 2 (15 menit)**

- Tunjuklah salah satu kelompok untuk mempresentasikan pekerjaannya.
- Beri kesempatan kepada kelompok lainnya untuk memberikan tanggapan dan pertanyaan.
- Untuk siswa yang hanya menggunakan bilangan asli sebagai label, maka guru dapat mengajak siswa tersebut untuk berpikir tentang label yang dihubungkan dengan keseluruhan bagian. Mintalah siswa untuk memberikan label dalam bentuk notasi pecahan.
- Jika siswa hanya menggunkan pecahan satuan, maka guru dapat mengajak siswa tersebut melihat kembali pada garis lipat pada bar (representasi dari pita).
- Guru menggambarkan representasi dari pita di papan tulis. Kemudian diskusikan bersama siswa. *"Pecahan apa yang dapat diletakkan pada garis lipat yang ditunjukkan oleh tanda panah dihubungkan dengan keseluruhan bagian pita?"*



**Menanyakan tentang label dari setiap garis lipat.**

- Arahkan siswa untuk memberikan label dalam bentuk bilangan ordinal.
- Mungkin ada siswa yang berpendapat bahwa bendera pertama berada pada bagian awal sebelah kiri dari batang horizontal tersebut. Beri kesempatan pada siswa lainnya untuk menanggapi pendapat ini.

Guru : Apakah benar bendera pertama terletak pada bagian awal dari batang horizontal tersebut?

Siswa : (Terlihat bingung)

Guru : Apa informasi yang ada pada bagian awal dari cerita tentang kegiatan hiking ini? Apakah kamu masih ingat?

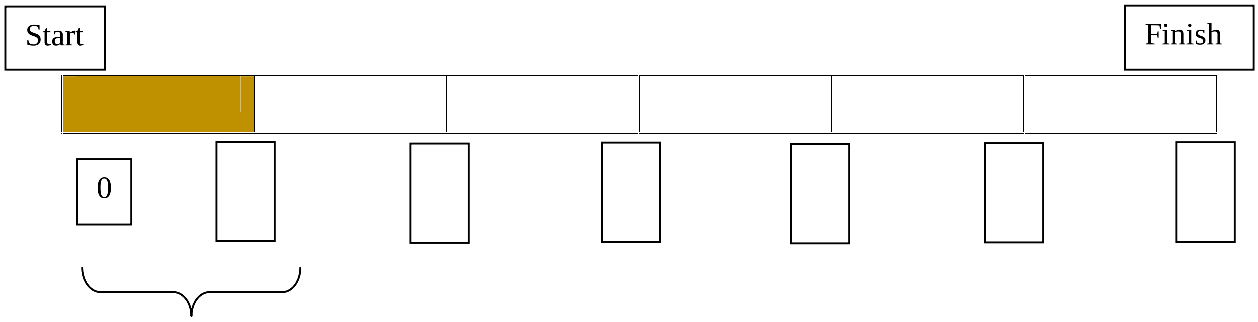
Siswa : Oh yaa… Bagian awal dari batang horizontal itu menunjukkan posisi garis start. Pada garis start tidak ada bendera dan juga pos permainan.

Guru : Ya, kamu benar. Jadi, angka berapa yang harus diletakkan pada titik tersebut? (Guru menunjuk area pada bagian awal batang horizontal).

Siswa : Nol.

- Untuk memulai diskusi tetang memberikan label dengan notasi pecahan, arahkan siswa untuk melihat kembali hasil partisi yang telah mereka buat.
- Tanyakan kepada siswa tentang bagian dan keseluruhan.Guru dapat menggunakan representasi dari jawaban siswa yang ada di LKS.

Gambar berikut ini merupakan representasi dari pita yang telah dilipat kemudian diregangkan untuk menentukan posisi dari bendera:




Tanyakan kepada siswa tentang bagian ini dihubungkan dengan keseluruhan batang horizontal.Siswa mungkin akan mengatakan bahwa ada 6 bagian pada batang horizontal tersebut dan bagian yang diarsir adalah salah satunya, jadi itu berarti 1 dari 6 bagian atau $\frac{1}{6}$.

- Mungkin ada siswa yang ingat tentang garis bilangan seperti yang mereka pelajari pada bilangan bulat. Guru dapat menggunakan pengetahuan ini untuk mengajak siswa melihat batang horizontal tersebut sebagai garis bilangan, hanya saja dalam bentuk yang berbeda
- Kemudian diskusikan bersama siswa tentang bagaimana jika kita mengarsir 2 bagian dari batang horizontal tersebut.Siswa dapat melihat bahwa mereka mengarsir 2 bagian dari 6 bagian yang ada, ini berarti kita telah mengarsir $\frac{2}{6}$. Kemudian, siswa dapat melihat polanya dan mereka melanjutkan sampai posisi untuk bendera yang terakhir.

- Berikan kesempatan kepada siswa untuk melakukan refleksi dari diskusi yang telah dilakukan dan juga pada pekerjaan mereka.
- Beri siswa waktu untuk merevisi pekerjaannya untuk posisi tiap-tiap bendera dan juga pos permainan.

**6. Mengerjakan LKS 1: Soal 3 (10 menit)**

- Selanjutnya, aktivitas dilanjutkan dengan mengerjakan soal ketiga. Instruksi pada soal ketiga yaitu untuk menentukan jarak antara garis start dan pos permainan pertama. Pada cerita di LKS 1 disebutkan bahwa panjang dari jalur *hiking* tersebut adalah 6 km.
- Siswa akan bekerja kembali di kelompok-kelompok kecil yang telah dibentuk.

<u>Beberapa kemungkinan jawaban dari siswa</u>

- Siswa menentukan panjang dari titik start ke titik tengah dari jalur *hiking* tersebut dengan membagi dua panjang total dan kemudian membagi duanya lagi untuk mendapatkan jarak dari pos pertama dan garis start.
- Siswa mencoba membagi total panjang jalur *hiking* dengan 4 karena jumlah pos permainan ada 4. Mungkin siswa akan mengalami kesulitan dlam mendapatkan hasilnya.

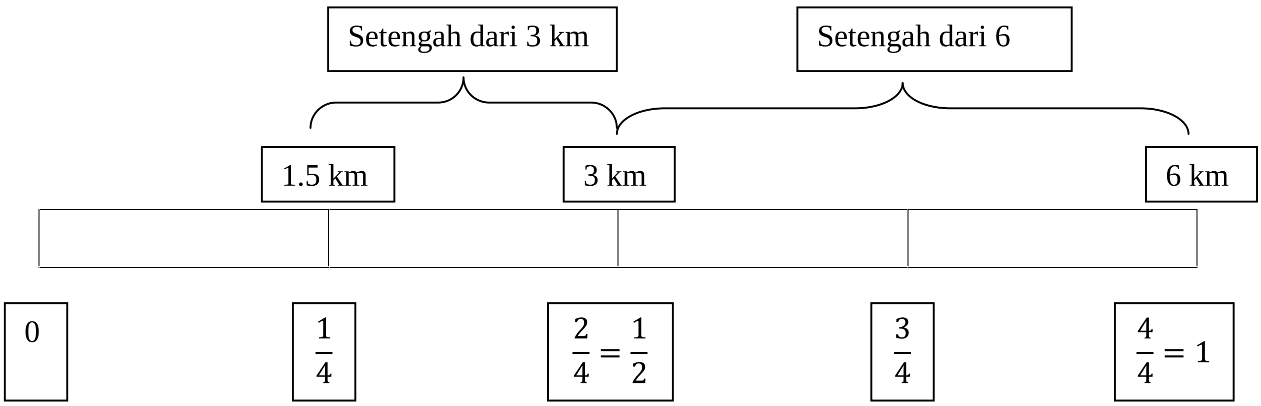
**7. Diskusi kelas untuk soal 3 (10 menit)**

- Guru mengajak siswa untuk melakukan refleksi pada strategi yang mereka gunakan. Suruh siswa untuk menjelaskan dan mendiskusikan hubungan dari jawaban mereka untuk soal 3 dengan jawaban pada soal 2.
- Untuk siswa yang menggunakan *halving strategy*, ajak mereka untuk mengekspresikan strategi mereka dalam notasi pecahan.

| | |
|---|---|
| Guru | : Apa yang pertama kamu lakukan? |
| Siswa A | : Saya membagi total panjang jalur *hiking* dengan 2. |
| Guru | : Apa maksudnya itu? |
| Siswa A | : (Bingung) |
| Guru | : Dapatkah kamu menjelaskan proses yang kamu lakukan dengan kalimat lain? |
| Siswa A | : Saya mengambil setengah dari 6 km untuk mendapatkan tengahnya. |
| Guru | : ya, bagus. Jadi pertama kamu mengambil $\frac{1}{2}$ dari 6 km. Kemudian? |
| Siswa A | : Kemudian saya bagi dua lagi untuk mendapatkan jawabanya. |
| Siswa B | : Itu berarti kamu mengambil $\frac{1}{2}$ dari 3km dan kita mendapatkan 1 setengah kilometer. |
| Guru | : Ya, kamu benar. Jadi, adakah yang bisa menyimpulkannya dengan lengkap strategi yang dilakukan oleh siswa A? |

Siswa C : Kita mengambil $\frac{1}{2}$ dari $\frac{1}{2}$ dari 6 km

- Mungkin ada siswa yang langsung membagi 6 km dengan 4 untuk menentukan jarak dari pos permainan pertama dengan garis start. Berikan kesempatan pada siswa untuk membandingkan jawabannya.
- Jika tidak ada siswa yang muncul dengan ide tersebut, guru dapat menanyakan tentang posisi dari pos permainan pertama dihubungkan dengan keseluruhan panjang jalur *hiking*. Guru dapat menanyakan sebagai berikut, "*Apakah kamu masih ingat, pada bagian yang mana dari jalur hiking itu posisi dari pos pertama?*" Kemudian, guru menunjukkan kembali representasi dari pita dan mengajak siswa untuk melihatnya sebagai sebagai *bar model* (model batang horizontal) yang dapat mereka gunakan untuk membantu menyelesaikan soal 3. Guru memberikan support kepada siswa untuk merepresentasikan strategi penyelesaian soal 3 pada bar model sebagai mana dapat dilihat pada gambar berikut ini.




Selanjutnya, ajak siswa untuk membandingkan hasil yang mereka peroleh. Siswa akan mendapatkan bahwa menentukan jarak antara garis start dengan pos permainan yang pertama sama dengan menentukan $\frac{1}{4}$ dari 6 km.

- Hasilnya sama dengan siswa yang menentukan $\frac{1}{2}$ dari $\frac{1}{2}$ dari 6 km.

**8. Penutup (5 menit)**

- Arahkan siswa untuk membuat kesimpulan di kelompoknya masing-masing dan ajak mereka untuk melakukan refleksi tentang apa yang telah mereka pelajari pada pembelajaran kali ini.
- Fokus pada pengalaman siswa dalam aktivitas *partitioning*, melabeli dengan notasi pecahan dan aktivitas mengambil sebuah bagian dari keseluruhan.

# Rencana Pelaksanaan Pembelajaran (RPP) 2

**Satuan Pendidikan : SD Al- Hikmah Surabaya**
**Mata Pelajaran : Matematika**
**Kelas : V**
**Semester : 2**
**Alokasi Waktu : 2 × 35 menit**

**A. Standar Kompetensi**
Menggunakan pecahan dalam pemecahan masalah.

**B. Kompetensi Dasar**
Perkalian dan pembagian berbagai bentuk pecahan.

**C. Tujuan Pembelajaran**
- Siswa dapat mengambil sebagian dari sebagian dari keseluruhan dalam konteks.
- Siswa dapat menggunakan model array untuk menyelesaikan masalah mengambil sebagian dari sebagian dari keseluruhan dalam sebuah konteks.

**D. *Starting point* :**
Siswa telah belajat tentang bagaimana cara melakukan *partitioning* dengan benar dan memberikan label dari hasil *partitioning* tersebut dalam bentuk notasi pecahan. Siswa telah diperkanalkan pada kegiatan mengambil sebagian dari keseluruhan dalam sebuah konteks.

**E. Metode Pembelajaran :** mengerjakan LKS, diskusi kelompok dan diskusi kelas.
**F. Strategi Pembelajaran :** PMRI
**G. Alat dan Bahan :** LKS 2, LKS 3, kertas berpetak, spidol.
**H. Aktivitas Pembelajaran :**

**a. Pendahuluan (5 menit)**
- Guru mengingatkan siswa tentang aktivitas yang berkaitan dengan jalur *hiking*. Kemudian guru meluaskan cerita tersebut dengan menambahkan informasi bahwa seorang anak pramuka bernama Hafidz ingin berpartisipasi dalam kegiatan hiking berikutnya. Hafiz melakukan aktivitas olahraga untuk mempersiapkan dirinya sebelum mengikuti kegiatan *hiking* tersebut.
  *Hafidz berencana untuk melakukan olahraga satu jam setiap minggunya. Minggu pagi ini ia mengatakan pada ayahnya bahwa ia ingin pergi jogging bersama temannya, Aufa dan Siraj. Ayah Hafidz memberikan sebuah cokelat batang dan mengatakan bahwa Hafidz harus membaginya dengan Aufa dan Siraj sama banyak.*
- Guru membagi siswa kedalam kelompok-kelompok kecil.

**b. Mengerjakan LKS 2: soal 1, 2 dan 3 (10 menit)**

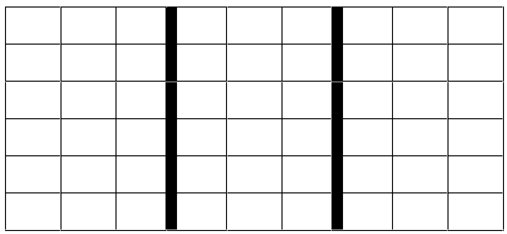
- Konteks dalam soal ini ditampilkan dalam bentuk cerita bergambar (komik) 1 dan 2. Guru meminta siswa untuk membaca komik 1 dan mengambil informasi penting yang ada didalamnya.
- Guru meminta beberapa siswa untuk menyampaikan kembali dengan bahasa mereka sendiri untuk mengetahui apakah siswa mengerti tentang cerita dan soal yang ada.
- Soa 1 dan soal 2 pada LKS 2 ini saling berkaitan.

  Pada soal 1 disajikan sebuah representasi dari cokelat batang yang dimaksud dalam cerita. Siswa harus menunjukkan bagian cokelat untuk Hafidz, Aufa dan Siraj. Pada soal 2, siswa harus menentukan berapa bagiankah untuk tiap-tiap anak dan menuliskannya dalam bentuk notasi pecahan.

  Sedangkan, soal 3 adalah tentang waktu yang digunakan oleh Hafidz untuk mencapai rumah Aufa. Hafidz memiliki waktu 1 jam untuk berolahraga, dia menggunakan separuhnya untuk *jogging.* Ketika Hafidz melakukan *jogging* Minggu pagi, dia menggunakan sepertiga dari waktu joggingnya itu untuk mencapai rumah Aufa. Siswa harus menentukan berapa menitkah waktu yang diperlukan Hafidz untuk mencapai rumah Aufa?
- Setelah siswa mengerti tentang soal yang diberikan, mintalah mereka untuk memikirkan ide untuk menyelesaikan soal tersebut dalam satu atau dua menit. Setelah itu mintalah siswa untuk mulai mengerjakan dan berdiskusi dengan teman disebelahnya.
- Ketika siswa sedang bekerja menyelesaikan soal dalam kelompok-kelompok kecilnya, guru berkeliling untuk memperhatikan kerja siswa.

<u>Beberapa kemungkinan strategi siswa untuk soal 1 dan 2</u>

- Siswa membagi gambar cokelat batang yang diberikan secara diagonal atau bahkan secara tidak beraturan.
- Siswa membagi gambar cokelat batang yang diberikan menjadi tiga bagian yang sama. Pertama mereka menghitung jumlah kolom dan membaginya menjadi tiga. Kemudian mereka membuat sebuah garis pada setiap tiga kolom. Satu bagian besar adalah untuk satu anak.



- Siswa juga mungkin membagi gambar cokelat batang itu secara horizontal dengan menggunakan strategi yang sama.

- Untuk menentukan bagian untuk Hafidz, siswa hanya melihat pada bagian besar yang sudah mereka buat tanpa menghitung bagian-bagian kecil yang ada didalamnya. Mereka memperoleh $\frac{1}{3}$ dari cokelat batang itu sebagai jawaban.
- Siswa menghitung bagian-bagian kecil dalam bagian besar itu, tetapi tidak menghubungkannya dengan jumlah total bagian kecil untuk sebuah cokelat batang yang utuh. Mereka mendapatkan 18 sebagai jawaban.
- Siswa menghitung bagian-bagian kecil yang ada di dalam bagian yang besar dan menghubungnya dengan jumlah total dari bagian kecil yang ada di satu cokelat batang yang utuh. Mereka memperoleh $\frac{18}{54}$ dari cokelat batang itu sebagai jawaban.

<u>Beberapa kemungkinan jawaban siswa untuk soal 3</u>

- Siswa melakukan perhitungan matematika sebagai berikut.
  Waktu untuk berolahraga adalah 1 jam sama dengan 60 menit. Siswa membaginya dengan 2 dan memperoleh 30 menit. Kemudian, siswa membagi 30 menit itu dengan 3 dan mendapatkan 10 menit sebagai jawaban.
- Siswa menggambar sebuah jam berbentuk lingkaran untuk merepresentasikan cerita pada soal. Mereka fokus pada penanda untuk menit. Mereka tahu bahwa satu lingkaran penuh itu adalah 60 menit. Mereka mengarsir separuhnya dan kemudian membagi bagian yang diarsir menjadi 3 bagian yang sama. Siswa dapat melihat satu bagian nya itu sama dengan 10 menit sebagai jawaban untuk soal 3

**c. Diskusi kelas untuk soal 1, 2 dan 3 (15 menit)**

- Sebagai awal diskusi, fokus terlebih dahulu pada soal 1 dan 2.
- Jika siswa membagi gambar cokelat batang secara diagonal ataupun acak, mintalah mereka melihat pada hasil pembagian yang mereka lakukan, *apakah itu menghasilkan bagian yang sama atau tidak?*
  Guru mengajak siswa lainnya untuk menanggapi pertanyaan tersebut sehingga guru bisa yakin bahwa siswa telah memahami bahwa *partitioning* yang dilakukan haruslah menghasilkan bagian yang sama.
- Untuk menentukan bagian untuk Hafidz, ketika siswa hanya menjawab dengan memberikan bilangan bulat misalnya dengan jawaban 18 bagian, maka guru meminta siswa lainnya untuk menanggapi. Mungkin akan ada siswa yang menyampaikan bahwa jawaban yang diberikan adalah dalam bentuk pecahan sesuai dengan yang diminta pada soal.
- Kemudian, guru mengarahkan diskusi pada pemahaman tentang hubungan antara *bagian dan keseluruhan (part-whole relation)* sampai siswa memahami bagaimana cara untuk menyajikan hasil *partitioning* dalam bentuk pecahan.

- Sebagai contoh adalah dengan cara menghitung jumlah total bagian-bagian kecil yang ada pada gambar cokelat batang tersebut, kemudian kita mengambil 18 bagian kecilnya. Mintalah siswa untuk menjelaskan maksudnya.
- Siswa mungkin akan menjawab bahwa itu berarti kita punya $\frac{18}{54}$ dari cokelat batang itu.

  Mintalah siswa yang menjawab dengan jawaban $\frac{1}{3}$ untuk menanggapi jawaban diatas. Kemudian guru bisa melanjutkan diskusi kelas tentang pecahan senilai.
- Selanjutnya, guru meminta siswa untuk mendiskusikan jawaban untuk soal 3.

- Guru mengajak siswa untuk melakuakan refleksi terhadap jawabannya.

  Guru : Berapa waktu awal atau waktu total yang digunakan untuk olahraga?

  Siswa A : Satu jam.

  Guru : Benar. Berapa menitkah itu?

  Siswa B : 60 menit.

  Guru : Baiklah. Kemudian, berapa menitkah yang digunakan untuk *jogging*?

  Siswa A : 30 menit.

  Guru : Bagaimana caramu mendapatkan 30 menit?

  Siswa B : Saya membagi 60 menit dengan 2.

  Siswa A : saya mengambil setengah dari 60 menit.

  Guru : Ya, benar. Hasil dari 60 menit dibagi dengan 2 adalah 30 menit dan B mengatakan bahwa ia juga memperoleh 30 menit tetapi ia mengatakan bahwa ia mengambil setengah dari 60 menit.

  Siswa B : Jadi dengan kata lain, kita dapat mengatakannya bahwa kita mengambil $\frac{1}{2}$ dari 60 menit. Dan 30 menit adalah waktu untuk *jogging*.

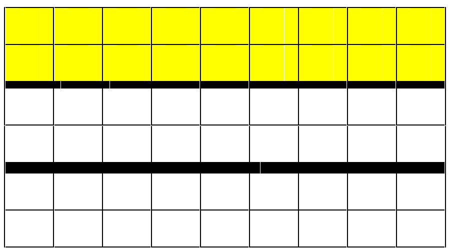
- Siswa harus mengetahui bahwa 30 menit adalah hasil dari mengambil setengah dari 60 menit
- Lanjutkan diskusi sampai siswa memperoleh hasil akhir bahwa Hafidz mencapai rumah Aufa setelah menggunakan sepertiga dari 30 menit yaitu 10 menit.
- Sebagai hasil dari diskusi ini, siswa dapat memahamibahwa mereka mengambil $\frac{1}{3}$ dari $\frac{1}{2}$ dari satu jam.

**d. Mengerjakan LKS 2: soal 4 dan 5 (5 menit)**

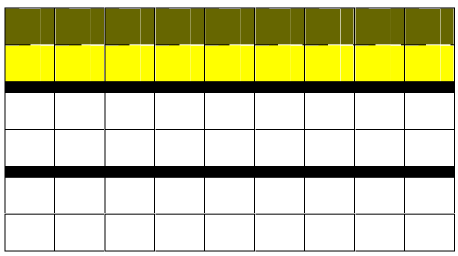
- Soal selanjutnya adalah tentang membagi bagian cokelat batang milik Hafidz untuk adiknya Nazifah. Soal ini berhubungan dengan soal 1 dan 2 pada LKS ini.
  Pada soal 4, konteks nya adalah bahwa Hafidz ingat bahwa adiknya yang bernama Nazifah juga menyukai cokelat dan ia berencana untuk membagi bagiannya untuk Nazifah sama banyak. Kemudian pada soal 5, siswa diminta untuk menuliskan bagian untuk Nazifah dalam notasi pecahan.
- Beri siswa waktu untuk memahami soal dan mintalah salah satu dari sisa untuk menyampaikan kembali permasalahan yang ada di soal dengan bahasanya sendiri untuk melihat apakah mereka sudah mengerti maksud dari soal atau belum.
- Mintalah siswa untuk bekerja pada kelompok kecilnya masing-masing.
- Guru berjalan berkeliling untuk melihat pekerjaan siswa.
- Jika ada siswa yang masih belum tahu tentang apa yang harus dilakukan, guru memberikan support bahwa mereka bisa memulai dengan melihat kembali gambar yang telah mereka arsir pada jawaban soal nomor 1.

<u>Beberapa kemungkinan strategi siswa untuk soal 4</u>

- Siswa menggunakan jawabannya pada soal 1 sebagai langkah awal untuk mnyelesaikan soal ini.



Siswa membagi gambar tersebut secara horizontal menjadi 3 bagian sama besar.



Untuk menentukan bagian untuk Nazifah, siswa membagi bagian yang sudah diarsir menjadi 2 sama banyak. Satu bagiannya adalah untuk Nazifah.

- Siswa melakukan strategi yang sama tapi dengan arah yang berbeda. Mereka membagi gambar cokelat batang itu secara vertical.

<u>Beberapa kemungkinan strategi siswa untuk soal 5</u>

- Untuk menentukan notasi pecahan untuk bagian Nazifah siswa menghitung bagian Nazifah dan menghubungkannya dengan keseluruhan cokelat batang itu. Mereka memperoleh $\frac{9}{54}$ bagian dari cokelat batang itusebagai jawaban.
- Siswa hanya melihat pada 2 baris dari cokelat batang itudan menyimpulkan bahwa bagian untuk Nazifah adalah $\frac{1}{6}$ dari cokelat batang tersebut. Jawaban ini dengan memperhatikan berapa kali bagian Nazifah itu memenuhi keseluruhan cokelat batang tersebut.
- Siswa mungkin menemukan *misunderstanding* dalam menentukan notasi pecahan untuk bagian Nazifah. Sebagian dari siswa mungkin hanya menghubungkan bagian Nazifah dengan bagian milik Hafidz bukan dengan keseluruhan Cokelat batang. Mereka mendapatkan jawaban yang tidak tepat, misalnya $\frac{9}{18}$ atau $\frac{1}{2}$.bagian dari cokelat batang.

**e. Diskusi kelas untuk soal 4 dan 5 (10 menit)**

- Fokus dari diskusi ini adalah tentang strategi yang digunakan oleh siswa ketika mereka menyelesaikan soal dengan model array yang telah diberikan. Guru dapat mengeksplorasi cara siswa menginterpretasikan hasil dari akivitas membagi dan mengarsir gambar cokelat batang yang diberikan.
- Untuk mengatasi *misunderstanding* siswa tentang "bagian" dan "keseluruhan unit" guru dapat mengajukan beberapa pertanyaan seperti, *"Kita mendapatkan bagian cokelat untuk Nazifah berupa bagian yang diarsir pada gambar, nah, jika kita ingin menentukan notasi pecahan untuk bagian Nazifah itu, apakah kita harus menghubungkannya pada bagian Hafidz saja atau pada keseluruhan bagian dari cokelat batang semula?"*
- Pertanyan berikut juga dapat membantu siswa untuk menyadari bahwa pecahan itu adalah sesuatu bagian dari keseluruhan bagian. *"Dari pertanyaan di soal " Berapa bagian* ***dari cokelat batang*** *yang diperoleh oleh Nazifah?" Ini berarti kita merujuk kepada apa?"*
- Dengan member penekanan pada "**dari cokelat batang"** siswa dapat menyadari bahwa notasi pecahan untuk bagian dari Nazifah yang ditanyakan adalah merujuk kepada keseluruhan bagian cokelat batang semula.
- Jika tidak muncul startegi untuk melihat berapa kali bagian Nazifah itu memenuhi keseluruhan batang cokelat, maka guru bisa mengajak siswa untuk melihat bagian yang diarsir sebagai satu kesatuan dan mintalah mereka untuk melihat bahwa bagian itu dapat memenuhi keseluruhuan batang cokelat dalam berapa kali *overlapping.*

- Mintalah siswa untuk melakukan refleksi terhadapa apa yang telah mereka kerjakan. Mungkin aka nada siswa yang mengatakan bahwa yang mereka lakukan dalam proses menjawab soal 5 adalah mengambi $\frac{1}{2}$ dari $\frac{1}{3}$ bagian dari cokelat batang.
- Untuk perbedaan jawaban pada notasi pecahan yang dihasilkan oleh siswa, misalnya antara siswa yang menjawab dengan $\frac{9}{54}$ dan siswa yang menjawab dengan $\frac{1}{6}$ maka guru dapat mengajak siswa untuk melihat kembali representasi gambar yang diarsir untuk kedua pecahan itu. Siswa dapat melihat bahwa besar daerah yang diarsir untuk kedua pecahan itu adalah sama.
- Mungkin siswa akan ingat tentang menyederhanakan bentuk pecahan atau pecahan senilai.
- Guru memimpin diskusi kelas agar siswa dapat menyimpulkan bahwa jika kita merujuk pada unit yang sama (cokelat batang yang berukuran sama) maka hasil yang diperoleh untuk kedua noasi pecahan yang berbeda pada contoh di atas adalah sama.
- Ini berartinahwa $\frac{9}{54}\ bagian\ dari\ cokelat\ batang$ adalah sama dengan $\frac{1}{6}\ bagian\ dari\ cokelat\ batang$ dengan catatan cokelat batang yang dimaksud adalah sama.

**f. Mengerjakan LKS 2: soal 6 dan 7 (10 menit)**

- Selanjutnya, gurumengajak siswa untuk mengerjakan soal 6 dan 7.
- Pada soal 6 disajikan sebuah model array dengan ukuran 4 × 6 sebagai representasi dari sebuah cokelat batang. Siswa harus menentukan $\frac{2}{3}$ dari $\frac{1}{2}$ bagian dari cokelat batang tersebut!
- Soal 7 merupakan soal yang mirip dengan soal 6. Namun, pada soal ini ukuran dari model arraynya adalah 3 × 12 dan siswa harus menentukan $\frac{1}{6}\ dari\ \frac{2}{3}$ bagian dari cokelat batang itu.

<u>Beberapa kemungkinan strategi siswa untuk soal 6 dan 7</u>

- Siswa akan menggunakan strategi membagi dan mengarsir untuk menjawab soal ini.
- Siswa akan memberikan jawaban dalam bentuk pecahan yang beragam tergantung cara mereka membagi cokelat batang itu dan cara mereka menghitung bagian-bagian kecil yang ada dihubungkan dengan keseluruhan bagian cokelat batang yang dimaksud.

**g. Diskusi kelas untuk soal 6 dan 7 (10 menit)**

- Pada sesi diskusi, guru meminta siswa untuk men *share* ide mereka dalam menyelesaikan soal-soal ini. Topik yang didiskusikan terutama pada strategi siswa dalam membagi, mengarsir dan menginterpretasikan hasil yang diperoleh dalam bentuk pecahan.

- Jika ada siswa yang masih punya keraguan tentang bermacam-macam jawaban yang muncul, mungkin siswa lainnya dapat menjelaskan bahwa mereka bisa merefleksi pada jawaban untuk soal sebelumnya.
- Selanjutnya, guru memimpin diskusi tentang memilih bentuk pecahan yang paling sederhana sebagai jawaban untuk soal ini. Guru meminta siswa untuk berpikir kembali tentang menyederhanakan pecahan atau pechan senilai yang telah pernah mereka pelajari.

**h. Penutup (5 menit)**

- Pada akhir pembelajaran, guru memberikan *support* kepada siswa untuk membuat kesimpulantentang aktivitas yang telah mereka lakukan. Terutama tentang aktivitas mengambil sebagian dari sebagian dari keseluruhan dalam konteks berbagi cokelat batang.
- Siswa bisa memahami penggunaanmodel array sebagai alat bantu dalam menyelesaikan persoalan yang berkaitan dengan mengambil sebagian dari sebagian dari sebuah unit.
- Selanjutnya guru memberikan LKS 3 sebagai tugas rumah siswa. Soal pada LKS 3 adalah sebagai berikut:

    1. *Lihat kembali cerita Komik 1 pada LKS 2. Dapatkah kamu menunjukkan bagian cokelat batang untuk Aufa, Siraj dan Hafidz dengan mengambarkan persegipanjang dengan ukuran yang lebih kecil pada kertas berpetak di bawah ini! Akan ada lebih dari satu kemungkinan.*

    2. *Tentukanlah dalam bentuk pecahan berapa bagian yang akan diterima oleh Nazifah berdasarkan gambar yang kamu buat!*

- Guru menyampaikan bahwa jawaban dari LKS 3 akan didiskusikan pada pertemuan berikutnya.

# Rencana Pelaksanaan Pembelajaran (RPP) 3

**Satuan Pendidikan : SD Al- Hikmah Surabaya**
**Mata Pelajaran : Matematika**
**Kelas : V**
**Semester : 2**
**Alokasi Waktu : 2 × 35 menit**

**A. Standar Kompetensi**

Menggunakan pecahan dalam pemecahan masalah.

**B. Kompetensi Dasar**

Perkalian dan pembagian berbagai bentuk pecahan.

**C. Tujuan Pembelajaran**

- Siswa mampu mengambil bagian dari bagian dari keseluruhan dalam konteks dan tanpa konteks.
- Siswa mampu membangun *array* mereka sendiri dan menggunakannya dalam memecahkan masalah mengambil bagian dari bagian dari keseluruhan.
- Siswa mampu mengambil bagian dari bagian dari keseluruhan untuk pecahan non satuan.

**D. *Starting point* :**

Siswa sudah belajar tentang bagaimana melakukan partisi dengan benar dan memberikan label terhadap hasil kegiatan partisi tersebut dalam bentuk notasi pecahan. Mereka sudah diperkenalkan dengan penggunaan model *array* untuk membantu mereka dalam memecahkan masalah tentang mengambil bagian dari bagian dari keseluruhan.

**E. Metode Pembelajaran :** Mengerjakan LKS, diskusi kelompok dan diskusi kelas.
**F. Strategi Pemebelajaran :** PMRI
**G. Alat dan Bahan :** LKS 2, LKS 3, Spidol berwarna hitam, biru dan merah..
**H. Aktivitas Pembelajaran**

**a. Pembukaan (5 menit)**

i. Guru meminta siswa untuk melihat kembali pekerjaan mereka tentang membagi batang cokelat yang telah mereka pelajari dalam aktivitas dalam pertemuan 2. Kemudian, guru mengajak siswa untuk mendiskusikan jawaban dari pekerjaan rumah.

**ii.** Mintalah siswa untuk menceritakan kendala yang dihadapi dalam menyelesaikan soal.

**b. Diskusi kelas untuk pekerjaan rumah (LKS 3) (5 menit)**

- Untuk memulai diskusi guru mengajak siswa untuk berbagi tentang jawaban mereka. Tanyakan juga tentang strategi yang mereka gunakan. Tunjuk satu atau dua orang siswa untuk menjelaskan caranya di depan kelas.
- Beri kesempatan pada siswa lainnya untuk menanggapi jawaban dari teman mereka dan minta mereka membandingkan dengan jawaban mereka sendiri.

<u>Beberapa kemungkinan strategi siswa untuk soal pada PR</u>

- Siswa menggambar beberapa ukuran batang cokelat dan menggunakan strategi membagi dan mengarsir untuk menyelesaikannya seperti yang sudah mereka pelajari di pertemuan 2.
- Akan ada jawaban yang beragam dari siswa seperti $\frac{1}{6}$ , $\frac{4}{24}$, atau $\frac{2}{12}$. Hal ini tergantung dengan bagaimana siswa menghubungkan bagian yang mereka arsir dengan keseluruhan bagian dari batang cokelat.

<u>Diskusi</u>

- Dalam diskusi guru membahas apakah siswa benar-benar mengerti tentang membangun model *array*.
- Untuk mengatasi keraguan siswa tentang jawaban yang beragam, mungkin ada siswa ingat tentang kesetaraan pecahan atau penyederhanaan pecahan yang telah mereka pelajari.
- Selama diskusi, guru memberikan *support* pada siswa untuk membuat kesimpulan bahwa $\frac{4}{24}$, atau $\frac{2}{12}$ dapat disederhanakan menjadi $\frac{1}{6}$.
- 1/2 dari 1/3 dari blok cokelat dengan hasil yang paling sederhana adalah 1/6 dari blok cokelat
- Dalam refleksi, guru membiarkan siswa menyadari dan menyimpulkan bahwa kegiatan yang mereka lakukan adalah tentang mengambil $\frac{1}{2}$ dari $\frac{1}{3}$ dari sebatang cokelatdengan jawaban yang paling sederhana adalah $\frac{1}{6}$ bagian dari batang cokelat itu.
- Guru mengajak siswa untuk membandingkan hasil yang mereka dapatkan dengan jawaban dari soal 5 (soal yang sama) dalam LKS 2. Diskusi dilanjutkan dengan hubungan antara dua jawaban.
- Para siswa dapat melihat bahwa bentuk pecahan yang paling sederhana dari solusi dari masalah 5 pada LKS 2 yang juga tentang mengambil $\frac{1}{2}$ dari $\frac{1}{3}$ bagian dari batang cokelat adalah sama dengan solusi paling sederhana dari soal dalam lembar

pekerjaan rumah (LKS 3) meskipun ukuran dari batang cokelat yang digunakan berbeda.

- Guru menekankan pengetahuan ini dan memimpin siswa menyadari bahwa mereka dapat menggunakan strategi yang sama dalam memecahkan soal yang serupa.

**c. Mengerjakan LKS 4: bagian A, soal 1 (5 menit)**

- Setelah membahas jawaban dari PR, guru memperkenalkan konteks baru dan membagikan bagian A dari LKS 4.

*Ibu Hafidz membuat sebuah martabak telur untuk makan siang. Namun, Hafidz terlambat pulang setelah melakukan olahraga pagi. Ia hanya menemukan* $\frac{1}{2}$ *dari martabak telur itu di dapur.*

**Sebuah martabak telur yang utuh**

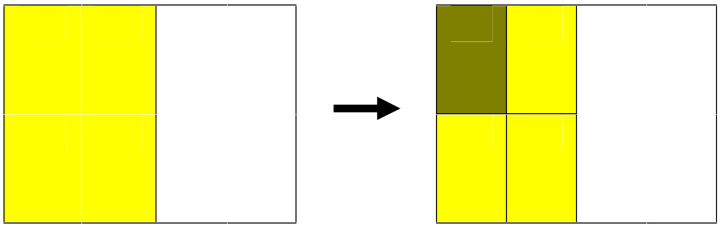
*Hafidz memakan* $\frac{1}{4}$ *dari bagian yang ada itu. Berapa bagiankah itu jika kita bandingkan dengan keseluruhan martabak telur. (Kamu dapat membuat gambar untuk membantumu menyelesaikan soal ini)*

*Tuliskan jawabanmu dalam bentuk notasi pecahan!*

- Guru meminta siswa untuk memikirkan strategi untuk menyelesaikan soal ini secara individu terlebih dahulu. Kemudian beri waktu kepada siswa untuk mendiskusikannya dengan teman disebelah mereka.

<u>Beberapa kemungkinan strategi siswa untuk soal 1</u>

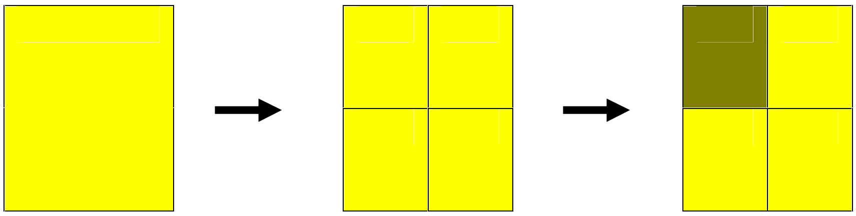
- Siswa menggambarkan keseluruhan martabak telur dalam bentuk sebuah persegi panjang sebagai langkah awal penyelesaian soal. Kemudian mereka membagi persegi panjang tersebut menjadi dua. Salah satu dari bagian itu mereka arsir. Kemudian, mereka membagi daerah yang diarsir menjadi empat bagian yang sama Dan menunjukkan satu bagian dari itu sebagai bagian yang makan oleh Hafidz



**Representasi sebuah martabak telur pada sebuah persegi panjang**

Untuk mendapatkan jawabannya, siswa harus membandingkan bagian yang dimakan oleh Hafidz dengan keseluruhan martabak telur. Mereka memperoleh $\frac{1}{4}$ dari $\frac{1}{2}$ bagian dari sebuah *martabak telur*. Siswa mungkin tidak mendapatkan solusi akhir karena mereka mengalami kesulitan tentang cara untuk menentukan notasi pecahan untuk bagian Hafidz jika dibandingkan dengan keseluruhan martabak Telur itu. Beberapa siswa mungkin hanya menghitung daerah yang diarsir dengan tidak memasukkan daerah yang bukan yang kosong, sehingga mereka mendapatkan $\frac{1}{4}$ bagian sebagai jawaban, dan jawaban ini yang tidak benar.

- Siswa menggunakan representasi dari setengah dari martabak telur berupa sebuah persegi panjang sebagai langkah awal penyelesaian soal. Siswa membaginya menjadi empat bagian. Tapi mereka tidak bisa menghubungkannya dengan bagian-bagian keseluruhan martabak telur tersebut.

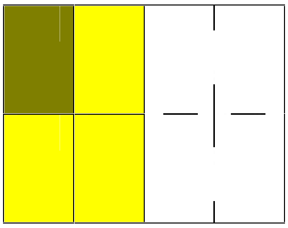


**Representasi separuh martabak telur sebagai langkah awal**

Mereka tidak bisa menentukan keseluruhan bagian martabak telur semula dan sebagai akibat mereka tidak bisa menentukan notasi pecahan yang benar untuk bagian yang dimakan oleh Hafidz jika dihubungkan dengan keseluruhan bagian martabak telur.

**d. Diskusi kelas untuk soal 1 (10 menit)**

- Jika dugaan tentang siswa yang mulai dengan representasi dari **keseluruhan bagian martabak** tetapi mereka tidak dapat menentukan notasi pecahan untuk bagian Hafidz terjadi, guru dapat mengajak siswa untuk berpikir tentang bagian yang dimakan oleh Hafidz dihubungkan dengan keseluruhan bagian martabak. Guru dapat menanyakan kepada siswa sebagai berikut "*Dapatkah kamu memikirkan tentang berapa kali bagian kecil (bagian Hafidz itu) masuk ke dalam representasi* *keseluruhan martabak?*" Kemudian, siswa dapat menarik garis putus-putus untuk membantu mereka



**Membuat garis putus-putus di dalam persegi panjang**

- Siswa mungkin menjawabdengan memberikan notasi pecahan $\frac{1}{7}$ untuk bagian yang dimakan Hafidz. Jawaban ini belum tepat.
- Guru dapat menyarankan siswa untuk berpikir lagi dengan mengajukan pertanyaan seperti berikut ini, "*Jika kita hanya mengatur 7 bagian kecil dari martabak kedalam persegi panjang itu, apakah itu cukup untuk memenuhi untuk persegi panjang tersebut dengan utuh?* " Karena seluruh martabak direpresentasikan dalam persegi panjang sehingga ada satu bagian kecil lagi yang diperlukan. Oleh karena itu, bagian Hafidz harus sesuai 8 kali dalam persegi panjang tersebut. Dan satu bagian kecil itu sama dengan $\frac{1}{8}$ dari keseluruhan bagian martabak telur.
- Selain itu, jika dugaan kedua dari jawaban siswa dari soal 1 terjadi, guru dapat menanyakan kepada siswa tentang cara menggambar keseluruhan bagian martabak jika kita memiliki setengah dari martabak itu sebagai awal. Mungkin siswa akan menyadari bahwa mereka perlu setengah bagian lagi untuk membuat sebuah persegi panjang sebagai representasi dari keseluruhan martabak yang dibuat oleh Ibu Hafidz. Selain itu, diskusi dapat dilanjutkan untuk menentukan notasi pecahan untuk bagian yang dimakan oleh Hafidz. Cara untuk menentukan notasi pecahan sama seperti cara yang sudah dijelaskan pada point sebelum ini.

**e. Mengerjakan LKS 4: bagian A, soal 2 (5 menit)**

- Setelah menyelesaikan diskusi kelas tentang soal pertama, guru meminta siswa untuk bekerja pada soal kedua.
- Pada soal 2 guru menceritakan bahwa ada tiga anak yang mencoba untuk memecahkan soal 1 dengan menggambar persegi panjang di kertas berpetak. Tiga persegi panjang itu memiliki dimensi ukuran yang berbeda seperti dapat dilihat dalam LKS 4. Instruksi yang diberikan kepada siswa adalah siswa harus berpikir tentang gambar yang paling tepan dan dapat menjadi alat bantu yang mudah untuk memecahkan masalah berbagi martabak tersebut.
- Siswa akan bekerja dengan teman disebelahnya.

**f. Diskusi kelas untuk soal 2 (10 menit)**

- Untuk memulai diskusi mintalah satu atau dua siswa untuk berbagi di depan kelas tentang jawaban mereka.
- Beri kesempatan kepada siswa lainnya untuk memberikan komentar tentang jawaban temannya tersebut.
- Dorong siswa untuk mendiskusikan tentang strategi dalam memilih gambar yang mana yang lebih pas, gambar A, B atau C.
- Siswa dapat mengenali ide menggunakan ukuran *array* yang tepat yang disesuaikan dengan angka yang ada pada soal.
- Selain itu, guru juga dapat meminta siswa untuk membandingkan jawaban dari soal ini (soal 2) dengan solusi yang telah dibahas dalam soal 1.
- Jika mereka memiliki kebingungan karena gambar yang berbeda dan juga bentuk notasi pecahan yang berbeda, maka guru dapat membawa ide pecahan senilai dan penyederhanaan pecahan.
- Hal ini juga dapat digunakan untuk memperkuat pemahaman siswa bahwa dalam mengambil bagian dari bagian dari keseluruhan, kita perlu mempertimbangkan hasil yang diperoleh ke keseluruhan bagian unit awal

**g. Mengerjakan LKS 4: bagian B (15 menit)**

- Selanjutnya, guru membagikan bagian B dari LKS 4.
- Guru memberikan waktu untuk bekerja untuk menyelesaikan 4 buah soal pada bagian B LKS 4.

**Soal pada bagian B**

1. *Tentukanlah* $\frac{1}{4}$ *dari* $\frac{1}{3}$ *!*
2. *Tentukanlah* $\frac{1}{4}$ *dari* $\frac{2}{3}$ *!*
3. *Tentukanlah* $\frac{3}{4}$ *dari* $\frac{1}{3}$ *!*
4. *Tentukanlah* $\frac{3}{4}$ *dari* $\frac{2}{3}$ *!*

<u>Beberapa kemungkinan jawaban siswa untuk soal 1</u>

- Mungkin ada siswa yang akan bingung karena tidak melihat adanya keseluruhan unit pada soal tersebut.

- Untuk soal 1, siswa akan menggambar persegi panjang dengan ukuran 3 × 4. Mereka memilih ukuran ini karena mereka melihat penyebut dari pecahan-pecahan dalam soal. Kemudian siswa mencoba untuk mengarsir bagian yang ingin ditunjukkan seperti yang telah mereka lakukan di bagian A dari LKS 4. Mereka akan memperoleh $\frac{1}{12}$ sebagai jawaban.
- Para siswa menggunakan strategi yang sama untuk soal-soal berikutnya. Para siswa mungkin memperoleh kesulitan ketika berhadapan dengan pecahan non satuan.

**h. Diskusi kelas untuk bagian B (10 menit)**

- Ketika siswa bingung karena soal pada bagian B berbeda dengan soal-soal sebelumnya, maka guru dapat mengajak siswa untuk memikirkan tentang kuantiti pada soal-soal ini.
- Misalnya dalam soal 1, kuantitas adalah $\frac{1}{3}$dan itu berarti bahwa ada sepertiga dari seluruh unit. Seluruh unit dapat dimodelkan dengan persegi panjang, jadi pertama kita perlu membagi persegi panjang menjadi tiga dan mengambil atau mengarsir 1 bagian dari itu.
- Selanjutnya, guru mengajak siswa untuk mengingat kembali apa yang sudah mereka lakukan pada bagian A, dalam soal 1 ini mereka akan membagi satu bagian dari ketiga bagian pertama tadi menjadi empat bagian yang sama. Setelah itu siswa mengarsir salah satu bagiannya sebagai bagian yang ditunjukkan untuk solusi dari soal 1.
- Ketika berhadapan dengan pecahan non satuan guru dapat memulai diskusi tentang bagaimana untuk merepresentasikan $\frac{2}{3}$ di persegi panjang, Dan jika kita ingin mengambil $\frac{3}{4}$ bagian dari yang $\frac{2}{3}$ bagian tersebut, kita perlu membagi $\frac{2}{3}$bagian itu menjadi empat bagian yang sama dan kemudian mengarsir 3 bagiannya. Untuk membuat prosesnya jelas, di papan tulis gunakan spidol yang berbeda warna untuk pembagian pertama dan pembagian kedua.
- Selain itu, untuk menginterpretasikan hasil gambar kedalam bentuk notasi pecahan, mungkin ada mahasiswa yang bisa muncul dengan notasi pecahan karena mereka menghubungkan bagian-bagian kecil yang dimaksudkan dengan jumlah total bagian-bagian kecil yang ada pada persegi panjang yang utuh. Jika siswa tidak bisa memberikan jawaban yang tepat, maka guru mengajak mereka untuk merefleksikan kembali tentang bagaimana menghubungkan *bagian dan keseluruhan* dalam topik pecahan.

**i. Penutup (5 menit)**

- Guru memfasilitasi siswa untuk membuat kesimpulan tentang apa yang telah mereka lakukan dalam pelajaran ini.
- Periksa secara garis besar tentang poin-poin utama dalam pelajaran ini apakah siswa sudah mendapatkan atau tidak.

# Rencana Pelaksanaan Pembelajaran (RPP) 4

**Satuan Pendidikan : SD Al- Hikmah Surabaya**
**Mata Pelajaran : Matematika**
**Kelas : V**
**Semester : 2**
**Alokasi Waktu : 2 × 35 menit**

**A. Stdanar Kompetensi**

Menggunakan pecahan dalam pemecahan masalah.

**B. Kompetensi Dasar**

Perkalian dan pembagian berbagai bentuk pecahan.

**C. Tujuan Pembelajaran**

- Siswa berbagi tentang strategi yang mereka gunakan dalam menyelesaikan persoalan mengambil bagian dari bagian dari keseluruhan dan mampu menyelesaikan persoalan perkalian pecahan dengan pecahan.
- Siswa dapat memahami dan menginterpretasikan istilah "bagian dari" menjadi operasi perkalian pecahan dengan pecahan dan menggunakan simbol "×".
- Siswa dapat menggunakan model *array* sebagai alat bantu dalam menyelesaikan dan memahami perkalian pecahan dengan pecahan.

**D. *Starting Point* :**

Siswa menggunakan model *array* untuk membantu mereka dalam menyelesaikan soal-soal tentang mengambil bagian dari bagian dari keseluruhan. Siswa juga telah mempelajari cara mengambil bagian dari bagian dari keseluruhan unit dan membuat array mereka sendiri.

**E. Metode Pembelajaran :** diskusi kelas.
**F. Strategi Pembelajaran :** PMRI
**G. Alat dan Bahan :** Jawaban siswa untuk beberpa soal di LKS 2 dan 4
**H. Aktivitas Pembelajaran**

**a. Pendahuluan (10 menit)**

- Kondisikan siswa untuk duduk di kelompoknya masing-masing.
- Ajaklah siswa mengingat kembali tentang pelajaran dalam tiga pertemuan sebelumnya. Hal ini dilakukan secara garis besar saja.

- Perkenalkan kepada siswa istilah model *array*, yaitu gambar persegi panjang yang terdiri dari bagian-bagian kecil yang sama besar, atau terdiri dari baris dan kolom. Model *array* inilah yang sebelumnya dipakai dalam menyelesaikan soal-soal tentang berbagi cokelat dan berbagi martabak telur.
- Sebagai pemanasan, berikan satu atau dua buah soal yang mirip dengan soal di LKS sebelumnya.

  Misalnya:

  **Tunjukkanlah dengan gambar $\frac{2}{5}$ dari $\frac{2}{3}$ !**

  **Kemudian, tentukanlah hasilnya dalam bentuk notasi pecahan!**

**b. Diskusi kelas bagian 1 (10 menit)**

- Mintalah satu atau dua orang siswa untuk menjelaskan strategi yang ia gunakan untuk menyelesaiakan soal tersebut.
- Beri kesempatan kepada siswa untuk saling menanggapi terhadap jawaban yang telah dijelaskan oleh siswa di depan kelas.

Kemungkinan strategi siswa untuk soal pemanasan:

- Siswa akan menggambarkan sebuah persegi panjang. Siswa membaginya menjadi 3 bagian dan mengarsir 2 bagiannya untuk merepresentasikan pecahan $\frac{2}{3}$. Kemudian, siswa membagi $\frac{2}{3}$ bagian tersebut menjadi 5 bagian yang sama dan kemudian mengarsir kembali 2 diantaranya. Bagian yang diarsir dua kali adalah solusi untuk soal ini.
- Siswa melihat penyebut dari soal yang diberikan yakni 5 dan 3, kemudian mereka membuat *array* yang berukuran 3 × 5. Setelah itu mereka menggunkan cara yang serupa untuk menyelesaikan soal ini, yaitu dengan mengarsir bagian-bagian yang diminta oleh soal.
- Siswa akan mendapatkan pecahan $\frac{4}{15}$ sebagai jawaban.

- Mungkin masih ada siswa yang bingung tentang cara membagi dan mengarsir sehingga mereka tidak bisa menyelesaikan soal ini dengan baik.

**Poin-poin diskusi bagian 1:**

- Guru meminta siswa yang masih bingung untuk menjelaskan bagian mana yang masih belum dimengerti. Mintalah mereka untuk menampilkan jawabannya, walau masih belum tepat.
- Mintalah siswa lainnya untuk memberikan tanggapan dan memberikan bantuan agar siswa tersebut memahami letak kekeliruannya dan bisa menyelesaikan soal ini dengan benar.
- Biarkan siswa mengambil kesimpulan bahwa mereka harus menentukan dulu bagian yang pertama yang diambil dari keseluruhan. Kemudian menentukan bagian kedua yang diambil dari bagian yg pertama tadi.

**3. Diskusi kelas bagian 2 (15 menit)**

- Selanjutnya, guru mengajak siswa untuk membuat list dari jawaban untuk beberapa soal yang sudah mereka kerjakan.
- Guru mengingatkan tentang soal waktu *jogging* Hafidz, soal tentang berbagi cokelat, soal tentang berbagi martabak telur, beberapa soal bagian B LKS 4 dan soal yang dibahas pada sesi diskusi bagian satu di atas.
- Guru bisa meminta siswa membantu menyebutkan jawaban-jawabannya dan guru menuliskannya di depan kelas misalnya sebagai berikut.
  - $\frac{1}{3}$ dari $\frac{1}{2}$ bagian dari 60 menit = 10 menit
    $\frac{1}{6}$ dari 60 menit = 10 menit
  - $\frac{1}{2}$ dari $\frac{1}{3}$ bagian dari batang cokelat = $\frac{1}{6}$ bagian dari batang cokelat
  - $\frac{2}{3}$ dari $\frac{1}{2}$ bagian dari batang cokelat = $\frac{2}{6}$ bagian dari batang cokelat
  - $\frac{1}{4}$ dari $\frac{1}{2}$ bagian dari martabak telur = $\frac{1}{8}$ bagian dari martabak telur

- $\frac{3}{4}$ dari $\frac{2}{3}$ $= \frac{6}{12}$ $= \frac{1}{2}$
- $\frac{2}{5}$ dari $\frac{2}{3}$ $= \mathbf{\frac{4}{15}}$

- Guru meminta siswa untuk melihat list tersebut secara seksama. Mintalah mereka secara individu untuk memikirkan ide apa yang bisa mereka ambil dari list tersebut.
- Diharapkan ada siswa yang menyadari hubungan-hubungan antara pecahan-pecahan tersebut.

Siswa dapat menyimpulkan bahwa

- $\frac{1}{3}$ dari $\frac{1}{2} = \frac{1}{6}$
- $\frac{2}{3}$ dari $\frac{1}{2} = \frac{2}{6}$
- Dan seterusnya.

- Mungkin ada siswa yang akan menghubungkan list tersebut pada operasi perkalian. Guru mengankat pendapat siswa tersebut ke diskusi kelas dan bersama-sama dengan siswa lainnya mendiskusikan bahwa aktivitas mengambil bagian dari bagian dari keseluruhan dalam bahasa matematika bisa diinterpretasikan sebagai operasi perkalian dari sebuah pecahan dengan pecahan.
- Sehingga,
  - $\frac{1}{3}$ dari $\frac{1}{2}$ $=$ $\frac{1}{3} \times \frac{1}{2}$ $= \frac{1}{6}$
  - $\frac{2}{3}$ dari $\frac{1}{2}$ $=$ $\frac{2}{3} \times \frac{1}{2}$ $= \frac{2}{6} = \frac{1}{3}$
  - Dan seterusnya

**c. Bermain Kartu (30 menit)**

- Sebagai bentuk latihan akan diadakan permainan mencocokkan kartu.

Peraturan dalam permainan kartu ini adalah:

- Kerjakanlah bersama teman disebelahmu (2 atau tiga orang)
- Ada tiga jenis kartu: biru, hijau dan kuning.
  Pada kartu hijau (1- 4) akan ada soal tentang perkalian pecahan
  Pada kartu biru (a-d) aka nada gambar array dengan ukuran yang berbeda-beda.
  Kartu kining adalah kartu untuk menuliskan jawabannya.
- Cocokkanlah masing-masing soal pada kartu hijau dengan gambar array yang disediakan pada kartu biru,
- Tunjukkan jawabanmu dengan cara mengarsir.
- Tuliskan jawabanmu pada kartu kuning.

- Beri kesempatan pada siswa untuk bereksplorasi untuk mencocokkan gambar *array* dan soal perkalian pecahannya. Kemudian menuliskan jawabannya di kartu kuning.
- Setelah waktu permainan selesai, guru meminta perwakilan beberapa kelompok untuk menuliskan kombinasi pasangan kartu dan jawaban yang diperoleh di papan tulis.
- Mintalah beberapa siswa untuk menjelaskan cara yang mereka gunakan. Jika ada kelompok yang lebih cepat selesai, maka tanyakan, mengapa mereka cepat selesai, apa strategi yang mereka gunkan atau langkah apa yang mereka lakukan terlebih dahulu.
- Mungkin ada siswa yang menyelesaikan soal perkalian pecahannya terlebih dahulu baru kemudian menentukan gambar *array* yang sesuai. Mungkin mereka melihat penyebut di kedua pecahan pada soal dan mencari array yang berukuran sama dengan perkalian dari penyebut tersebut.

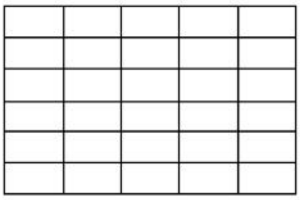
i. Tanyakan apakah ada siswa yang masih terkendala dalam penyelesaian soal perkalian pecahan dengan pecahan dan bantulah mereka dengan metode diskusi kelas bersama.

**d. Penutup (5 menit)**

- Suport siswa untuk menyimpulkan pembelajaran hari ini dengan bahasa mereka masing-masing.
- Minta mereka untuk memperhatikan poin-poin utama dalam pembelajaran.

**Kartu untuk *Card Game***

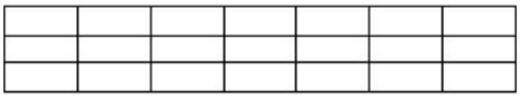
1. $\frac{1}{6} \times \frac{1}{5}$



Jawaban:

$\frac{\dots}{\dots}$

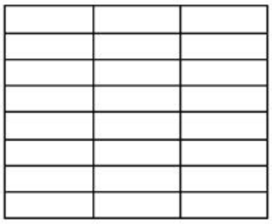
2. $\frac{1}{3} \times \frac{2}{7}$



Jawaban:

$\frac{\dots}{\dots}$

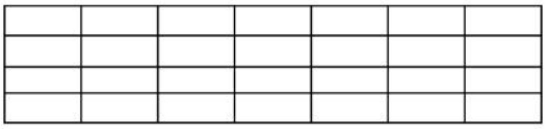
3. $\frac{2}{3} \times \frac{3}{8}$



Jawaban:

$\frac{\dots}{\dots}$

4. $\frac{3}{4} \times \frac{4}{7}$



Jawaban:

$\frac{\dots}{\dots}$

**Appendix J** Worksheet of the cycle 2

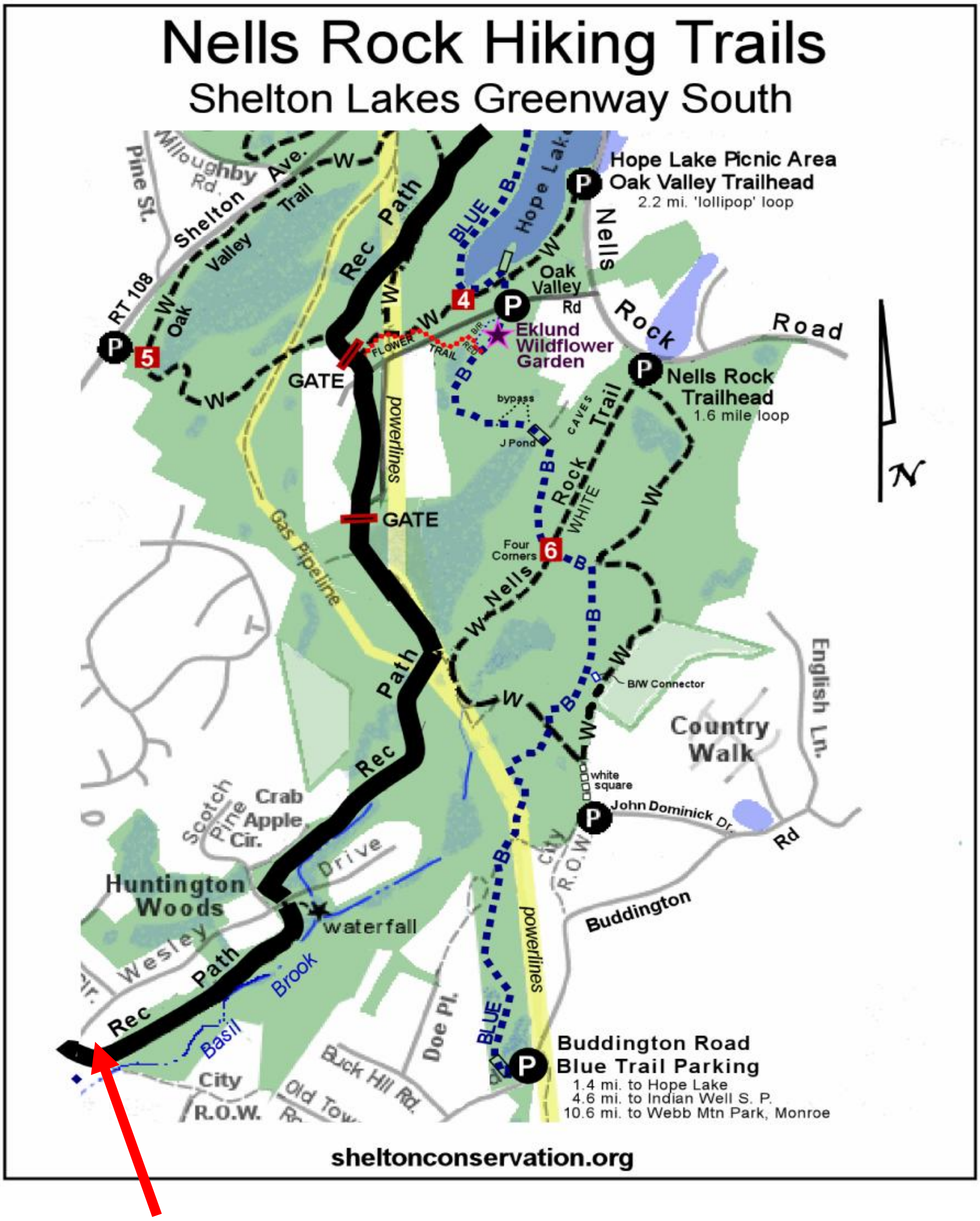
**Lembar Kerja Siswa 1**

Nama : …………………………….

Kelompok : ……………………………. Tanggal: ………………...

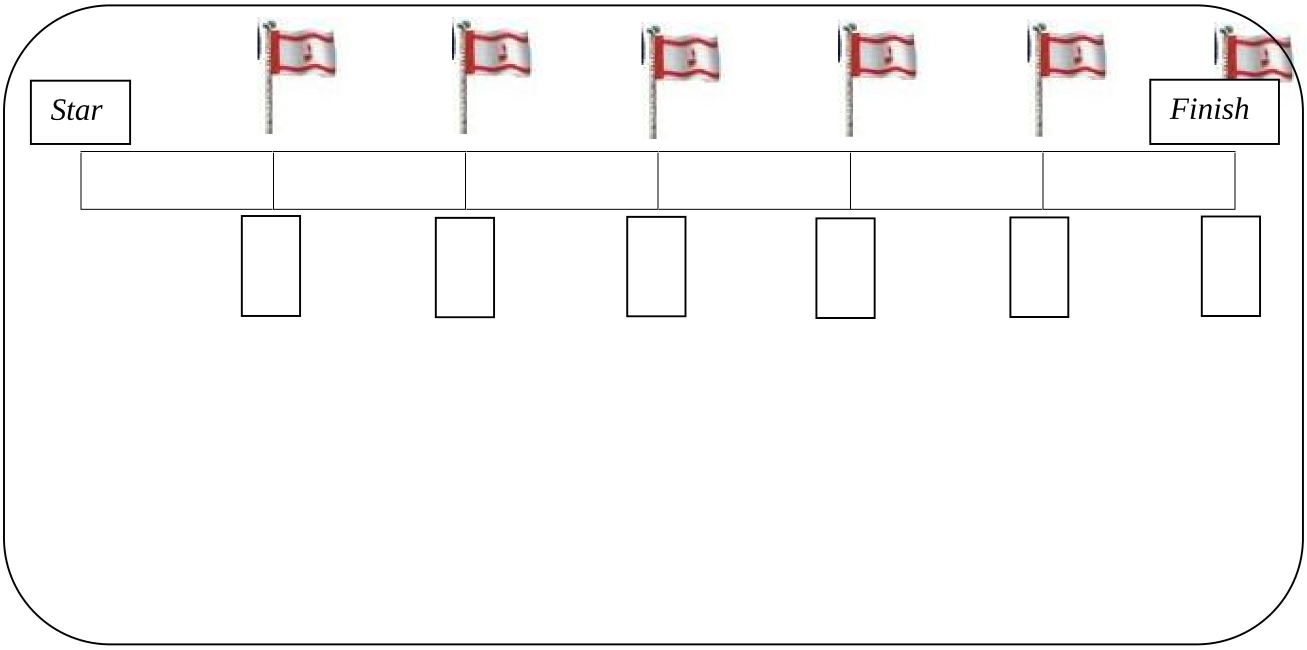
Sebuah kelompok pramuka berencana untuk mengadakan kegiatan *hiking* pada akhir bulan ini. Panjang jalur hiking adalah 6 km. Panitia menyiapkan beberapa permainan *outbond* di 4 pos yang terletak di jarak yang sama satu sama lain di sepanjang jalur *hiking*. Pos yang terakhir berada di garis finish. Selain itu, panitia akan menempatkan beberapa bendera di sepanjang jalur *hiking* sebagai tanda untuk tempat beristirahat. Mereka menempatkan 1 bendera disetiap 1 km dari jalur *hiking* tersebut, dan bendera terakhir ada di garis finish. Kamu dapat melihat jalur *hiking* pada gambar (di halaman 2) berupa garis tebal dan ditunjukkan oleh tanda panah.

1. Kamu adalah anggota panitia kegiatan *hiking* dan tugas kamu adalah untuk berpikir tentang bagaimana menempatkan bendera dan pos permainan. Gambarkanlah pada peta jalur *hiking* di halaman 2 posisi tiap-tiap bendera dan pos! (Tips: Kamu bisa menggunakan pita sebagai alat bantu).

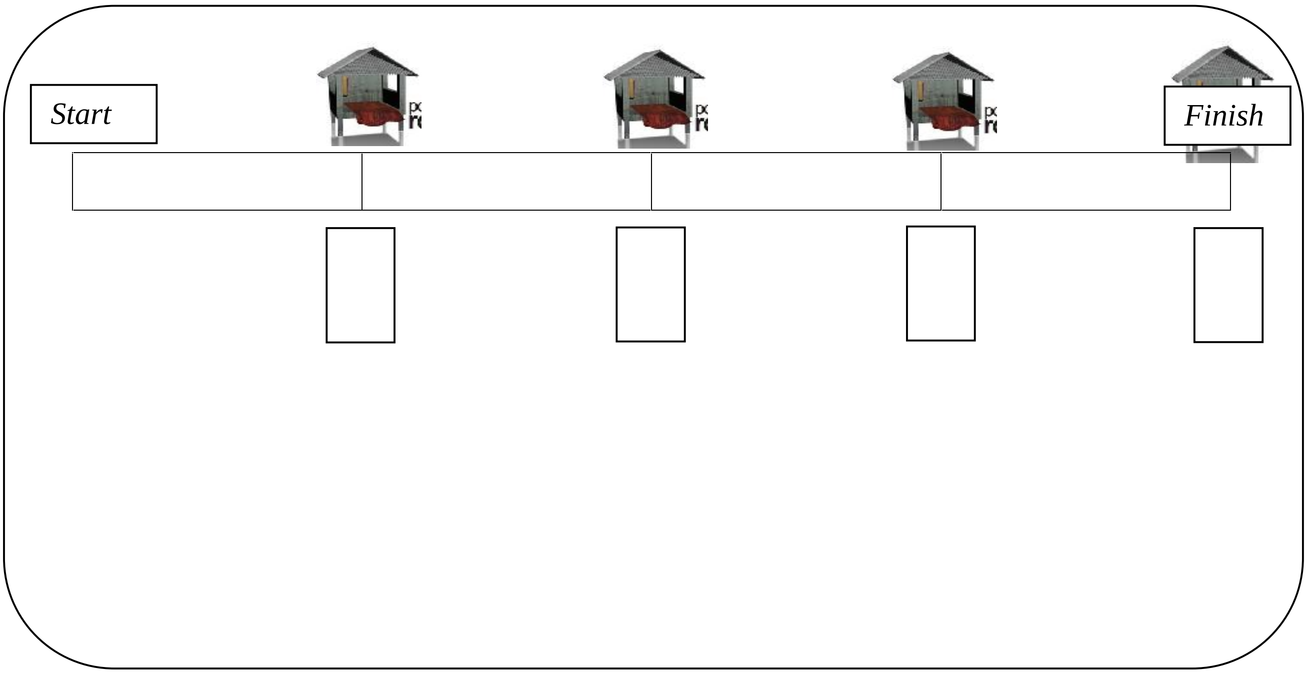

Nells Rock Hiking Trails
Shelton Lakes Greenway South
Hope Lake Picnic Area
Oak Valley Trailhead
2.2 mi. 'lollipop' loop
Nells Rock Road
Nells Rock Trailhead
1.6 mile loop
Eklund Wildflower Garden
Oak Valley Rd
Hope Lake
BLUE
Pine St.
Willoughby Rd.
Shelton Ave.
Valley Trail
Oak
RT 108
Rec Path
GATE
FLOWER TRAIL
powerlines
Gas Pipeline
bypass
J Pond
CAVES
Rock Trail
WHITE
Four Corners
Nells
B/W Connector
Country Walk
English Ln.
white square
John Dominick Dr.
Rd
City R.O.W.
Buddington
Crab Apple Cir.
Scotch Pine
Drive
Huntington Woods
waterfall
Wesley
Brook
Basil
City R.O.W.
Buck Hill Rd.
Old Town Rd.
Doe Pl.
Buddington Road Blue Trail Parking
1.4 mi. to Hope Lake
4.6 mi. to Indian Well S. P.
10.6 mi. to Webb Mtn Park, Monroe
N
sheltonconservation.org

2. Misalkan gambar di bawah ini sebagai jalur *hiking* termasuk dengan lokasi dari bendera dan pos bermain yang telah kamu letakkan. Tentukanlah pada seberapa bagian dari jalur hiking itu posisi untuk setiap bendera dan pos bermain!

   a. Untuk lokasi bendera.




   b. Untuk lokasi pos permainan.



3. Di saat menempatkan bendera dan pos permainan di lokasi jalur *hiking* yang berjarak 6 km, panitia akan mengendarai motor untuk mengukur jarak. Pada jarak berapakah dari garis *start*, panitia harus menempatkan pos permainan pertama?

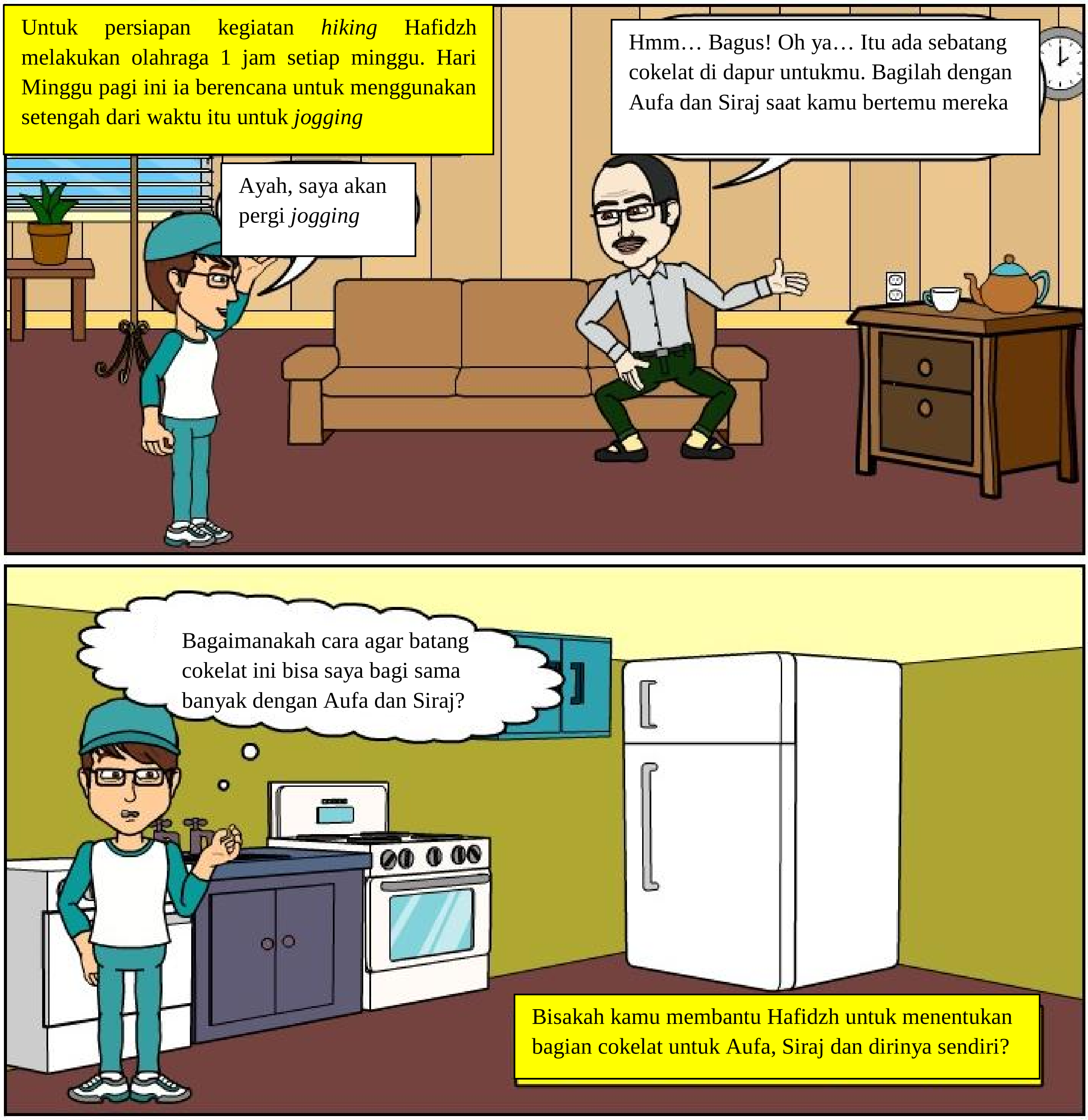
   Jawaban:

**Lembar Kerja Siswa 2**

Nama : …………………………….

Kelompok : ……………………………. Tanggal: …………………..

**Mari kita perhatikan cerita berikut! (Komik 1)**



1. Misalkan gambar petak-petak di bawah ini adalah batang cokelat yang diberikan oleh ayah Hafidz. Tunjukkanlah dengan cara mengarsir bagian cokelat yang akan diperoleh oleh Aufa, Siraj dan Hafidz!

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2. Berapa bagiankah dari batang cokelat itu untuk Hafidz? Tuliskan jawabanmu dalam bentuk pecahan!
   Jawaban:

**Apakah kamu sudah selesai? Baiklah, mari kita kembali ke cerita tentang Hafidz!**

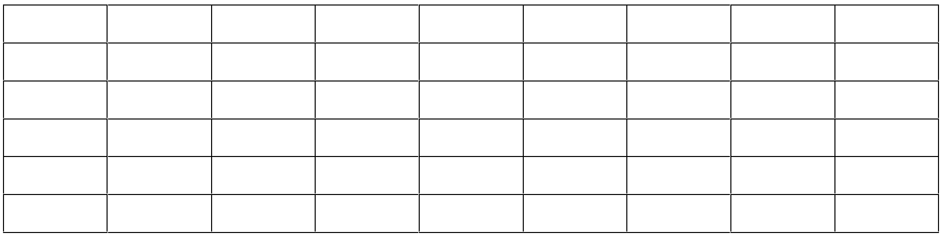
**(Komik 2)**

3. Kamu dapat menuliskan jawabanmu untuk pertanyaan Hafidz di bawah ini!
   Jawaban:

**Komik 3**


Oh ya teman-teman, aku punya cokelat nih. Kita bisa membaginya bersama.
Sip!! Terima kasih ya , Bro. Aku sangat suka cokelat.
Setelah saling menyapa, Hafidz menawarkan cokelat kepada Aufa dan Siraj.
Tapi…
Hmmm, Aku ingat adikku Nazifah. Dia juga suka cokelat.
Okay, Aku akan membagi cokelat bagianku dan memberikannya kepada Nazifah.

4. Bagaimana dengan bagian cokelat untuk Nazifah, dapatkah kamu menunjukkannya dalam gambar di bawah ini? (Tips: Gunakanlah jawaban dari pertanyaan no 1. Gambarlah garis pada bagian cokelat milik Hafidz).



5. Berapa bagian dari keseluruhan batang cokelat semula yang diperoleh Nazifah? Tuliskan jawabanmu dalam bentuk pecahan!

**Sekarang, misalkan gambar di bawah ini adalah sebuah batang cokelat. Gunakanlah gambar yang diberikan dengan cara mengarsir untuk menentukan jawaban dari soal-soal berikut!**

6. a. Tentukanlah $\frac{2}{3}$ dari $\frac{1}{2}$ bagian batang cokelat di bawah ini!

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|---|---|---|---|---|---|
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| | | | | | |

b. Berapa bagian dari batang cokelat yang kamu dapatkan? Tulis jawabanmu dalam bentuk pecahan

7. a. Tentukanlah $\frac{1}{6}$ dari $\frac{2}{3}$ bagian batang cokelat di bawah ini!

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| | | | | | | | | | | | |

b. Berapa bagian yang kamu peroleh? Tuliskan jawabanmu dalam bentuk pecahan!

**Lembar Kerja Siswa3**

Nama : …………………………….

Kelompok : ……………………………. Tanggal: …………………..

**Bagian A**

**A whole Martabak Telur**

1. Ibu Hafidz membuat sebuah martabak telur untuk makan siang. Namun, Hafidz terlambat pulang setelah melakukan olahraga pagi. Ia hanya menemukan $\frac{1}{2}$ dari martabak telur itu di dapur.

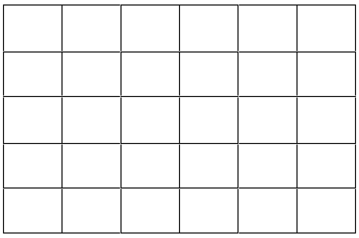
   Hafidz memakan $\frac{1}{4}$ dari bagian yang ada itu. Berapa bagiankah itu jika kita bandingkan dengan keseluruhan martabak telur. (Kamu dapat membuat gambar untuk membantumu menyelesaikan soal ini)

   Tuliskan jawabanmu dalam bentuk pecahan!

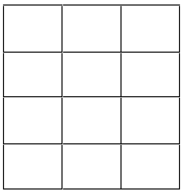
   Jawaban:

2. Tiga orang siswa mencoba menyelesaikan soal nomor 1 di atas dengan menggambar sebuah persegi panjang di kertas berpetak. Seperti yang dapat kamu lihat di bawah ini:

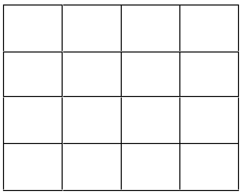
   Siswa A

   Siswa B

   Siswa C

   Gambar yang manakah yang kamu pilih untuk bisa membantumu menyelesaikan soal nomor 1 dengan mudah? Jelaskan jawabanmu!

   Jawaban:

**Bagian B** **Nama:**

Tips:

Saya kira, saya bisa menggunakan persegipanjang untuk membantu saya menyelesaikan soal ini. ^_^

**Selesaikan soal-soal berikut ini!**

1. **Tentukanlah $\frac{1}{4}$ dari $\frac{1}{3}$ !**
   **Tuliskan dalam bentuk pecahan!**

   **Jawaban:**

2. **Tentukanlah $\frac{1}{4}$ dari $\frac{2}{3}$ ! Tuliskan jawabanmu dalam bentuk notasi pecahan!**

   **Jawaban:**

**3. Tentukanlah $\frac{3}{4}$ dari $\frac{1}{3}$ ! Tuliskan jawabanmu dalam bentuk notasi pecahan!**

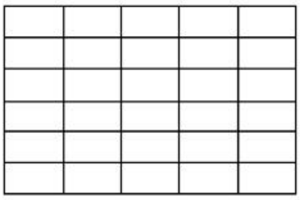
**Jawban:**

**4. Tentukanlah $\frac{3}{4}$ dari $\frac{2}{3}$ ! Tuliskan jawabanmu dalam bentuk notasi pecahan!**

**Jawaban:**

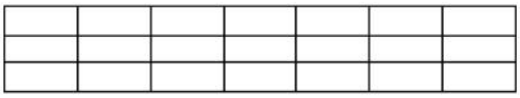
**Kartu untuk *Card Game***

1. $\frac{1}{6} \times \frac{1}{5}$



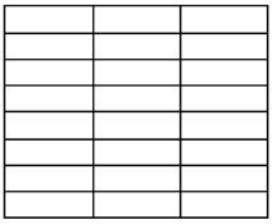
Jawaban: $\frac{\dots}{\dots}$

2. $\frac{1}{3} \times \frac{2}{7}$



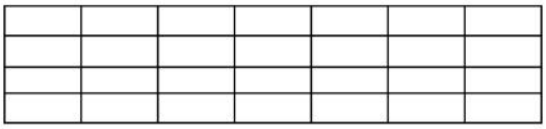
Jawaban: $\frac{\dots}{\dots}$

3. $\frac{2}{3} \times \frac{3}{8}$



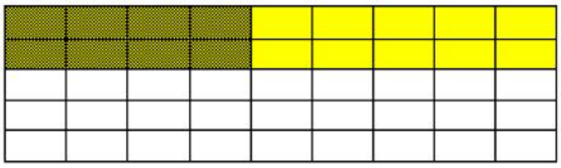
Jawaban: $\frac{\dots}{\dots}$

4. $\frac{3}{4} \times \frac{4}{7}$



Jawaban: $\frac{\dots}{\dots}$

5. $\frac{\dots}{\dots} \times \frac{\dots}{\dots}$



Jawaban: $\frac{\dots}{\dots}$