PROMOTING STUDENTS' UNDERSTANDING

OF THE ADDITION OF FRACTIONS

Master Thesis

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STATE UNIVERSITY OF SURABAYA
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MASTER THESIS

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DEDICATION

This thesis is dedicated to my beloved family

Thanks for all your support and attention ☺

ABSTRACT

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***Keywords**:* Addition of fractions, RME, design research, paper strips, bar model, the estimation of the sum of two fractions

Many researchers have documented that students consider fractions as a difficult topic because there are many rules in operating fractions. In Indonesia, many teachers place more emphasize on the algorithms instead of understanding of the concept. Thus, students tend to memorize the algorithms without understanding the reasoning behinds it. Consequently, students often made mistakes in applying the algorithms, such as doing the procedure 'top+top over bottom+bottom' in solving the addition of fractions problems. Therefore, there is a need to develop instructional activities that support students' understanding of the addition of fractions. This study used design research approach and applied the theory of Realistic Mathematics Education (RME), which suggests that students have to construct their understanding actively by exploring contexts and models.

The focuses of this study were the use of paper strips and bar model, and the idea of estimation of the sum of two fractions to lead students to avoid the incorrect procedure 'top+top over bottom+bottom'. This study was conducted in the third grade of SD Laboratorium Unesa in three cycles, which each cycle consisted of three meetings. The result of this study revealed that the students began to develop their fraction sense after doing the fair sharing activity and producing fractions strips by using paper strips. Moreover, they were able to grasp the idea of the equivalent fractions and common denominator after exploring the fractions strips. Afterwards, the students started to know how to add fractions with either the same or the different denominators in the bar. In addition, after learning the comparison and the estimation of the sum of two fractions by using benchmarks, the students began to be aware that the output of the procedure 'top+top over bottom+bottom' in adding fractions is not reasonable.

In conclusion, the result of this study provides evidence that the use of paper strips and bar model can support students in understanding the addition of fractions. Moreover, the estimation of the sum of two fractions leads students to be aware that the procedure 'top+top over bottom+bottom' in adding fractions is incorrect.

ABSTRAK

Lestiana, H.T. 2014. *Promoting Students' Understandng of the Addition of Fractions.* Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Mega Teguh Budiarto, M.Pd. dan (II) Dr. Abadi, M.Sc.

Kata Kunci: Penjumlahan pecahan, RME, design research, strip kertas, model bar, estimasi hasil jumlah dua pecahan.

Banyak penelitian yang telah mendokumentasikan bahwa banyak siswa yang menganggap pecahan sebagai topik yang sulit. Di Indonesia, banyak guru yang lebih menekankan pada hafalan rumus daripada pemahaman konsep. Oleh karena itu, siswa cenderung menghafal rumus tanpa memahami bagaimana rumus-rumus tersebut berlaku. Akibatnya, siswa sering melakukan kesalahan seperti menerapkan rumus 'atas + atas per bawah + bawah' saat menyelesaikan soal penjumlahan pecahan. Oleh karena itu, perlu dikembangkan kegiatan pembelajaran yang mendukung pemahaman siswa terhadap konsep penjumlahan pecahan. Penelitian ini menggunakan pendekatan *design research* dan menerapkan teori *Realistic Mathematics Education* (RME), yang menekankan pembangunan konsep secara aktif dengan melakukan kegiatan eksplorasi konteks dan model.

Fokus dari penelitian ini adalah penggunaan strip kertas dan model bar, dan konsep estimasi hasil jumlah dua pecahan untuk membantu siswa memahami konsep penjumlahan pecahan dan menghindari rumus 'atas + atas per bawah + bawah'. Penelitian ini dilakukan di kelas III SD Laboratorium Unesa dalam tiga siklus, yang masing-masing siklus terdiri dari tiga pertemuan. Hasil penelitian menunjukkan bahwa siswa mulai memahami konsep pecahan setelah melakukan kegiatan *fair sharing* dan membuat strip pecahan dengan menggunakan strip kertas. Selain itu, mereka mulai memahami pecahan yang senilah dan konsep menyamakan penyebut setelah mengeksplorasi strip pecahan. Siswa juga mulai tahu bagaimana menambahkan pecahan baik yang berpenyebut yang sama atau yang berpenyebut berbeda dengan menggunakan model bar. Setelah mempelajari perbandingan pecahan dan estimasi jumlah dua pecahan, siswa mulai memahami bahwa rumus 'atas + atas per bawah + bawah' adalah rumus yang salah.

Dari hasil tersebut bisa disimpulkan bahwa penggunaan strip kertas dan model bar dapat membantu siswa dalam memahami penjumlahan pecahan. Selain itu, konsep estimasi penjumlahan dua pecahan membantu siswa memahami bahwa rumus 'atas + atas per bawah + bawah' adalah salah.

PREFACE

Thanks to Allah for His blessing so that I can finish this thesis. Special thanks to all people that have given great support and guidance during the process of completing this thesis. Those thanks are to:

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Finally, I hope this thesis can contribute in supporting and improving the quality of Indonesian education. Any supportive and constructive critics and ideas will be warmly accepted.

Surabaya, July 2014

Herani Tri Lestiana

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CHAPTER I
INTRODUCTION

A. Research Background

Many studies have documented that the topic of fractions is a difficult topic in mathematics (e.g. Charalambous & Pitta-Pantazi, 2007; Hasemann, 1981; Streefland, 1991). Many students find it difficult to learn fractions because there are many complicated rules in the operation of fractions compare to those in the natural numbers. According to Howard (1991) and Young-Loveridge (2007), a common mistake by students in solving an addition of fractions is the procedure 'top + top over bottom + bottom'. They argued that students do this incorrect procedure since they think a fraction as two different whole numbers. Howard (1991) also stated that students lack the understanding of fractions. Students tend to memorize the rules of the operation of fractions instead of understanding the reasoning behinds the rules. According to Cramer et al. (2008), estimation skill is helpful to lead students to be aware that the procedure 'top+top over bottom+bottom' is incorrect.

Another factor that contributes to this difficulty is that fractions have many interpretations, which are a fraction as a part-whole relation, a measure, a ratio, a quotient, and an operator (Charalambous & Pitta-Pantazi, 2007; Young-Loveridge, 2007). Charalambous et al. (2010) stated that focusing merely on one interpretation of fractions is a factor that can impede students' learning of fractions.

In addition, some studies have shown that a conventional instruction on fractions, that provides a set of algorithms, does not promote a meaningful learning for most students (Lamon, 2001). As has been stated by Freudenthal (1991), that mathematics is a human activity, so in learning mathematics, students should actively experience and construct their understanding. Regarding to this, Cramer et al. (2008) revealed that the role of representations, such as models and contexts, is very important since it can promote students' understanding of a concept and the relationship between concepts. They also argued that a model can promote students' arguments and reasoning about problems related to fractions so that students can avoid the incorrect procedure. Moreover, they suggested that models can support students in constructing the mental representations of the concept being taught. In the case of fractions, some researchers argued that a fraction circle is the best model to use in teaching fractions (e.g. Cramer et al., 2008) while other researchers suggested using a bar model or fraction strips (e.g. van Galen et al., 2008).

However, in Indonesia, many teachers still use a conventional method of teaching mathematics and place more emphasis on the rules and algorithms than on the students' understanding of the concept (Ullya et al., 2010; Sembiring et al., 2008). Moreover, in the topic of addition of fractions, Ullya et al. (2010) argued that the teachers do not use models and do not relate the topic to a real world to create a meaningful teaching and learning. Consequently, students know how to solve the problems of addition of fractions with a formal algorithm, but they cannot reason why such algorithms work (Kamii & Dominick, and Lappan & Bouck in Young-Loveridge, 2007).

To deal with this issue, this study is conducted to promote a meaningful teaching and learning in the addition of fractions. This study attempts to support students' understanding of the addition of fractions and lead students to avoid the incorrect procedure 'top + top over bottom + bottom by integrating the use of models and contexts in lessons. Moreover, this study also combines the interpretation of fractions as a quotient, a part-whole relationship, and a measure in the contexts to support students' understanding of the concept of fractions.

B. Research Questions

Based on the description of the background of this study above, the researcher formulates a research question as follows: *"How can instructional activities in this study support students' understanding of the addition of fractions?"*. To be more specific, this study seeks to answer these following research sub questions.

1. *How do paper strips and bar model promote students' understanding of the addition of fractions?*
2. *How does the estimation skill lead students to avoid the incorrect procedure 'top+top over bottom+bottom' in solving the addition of fractions problems?*

C. Research Aim

To deal with the issues elaborated in the background, there is a need to reform the teaching and learning of fractions in Indonesia. Inspired by the theory, tenets, and design heuristics of RME, the integration of contexts and models in the teaching and learning of fractions is considered fruitful to promote students meaningful learning. Therefore, the aim of this study is to contribute to the Local Instruction Theory (LIT) on the topic of the addition of fractions.

D. Definition of Key Terms

1. Addition of Fractions

A fraction has five different interpretations, those are a fraction as a part-whole relationship, as a measure, as a ratio, as an operation, and as a quotient. A fraction as a part-whole relation means a fraction is a part of a whole object that is divided into equal parts. A fraction as a measure means a fraction can be used to measure distance, length, or height, from the origin. As a ratio, a fraction is defined as a comparison between two quantities. A fraction also can be an operation, when a fraction acts as a function towards some numbers or objects. In a situation in which some quantities are divided or shared among some people, a fraction functions as a quotient.

Addition of fractions is one of the operations of fractions, that include the addition of two fractions with the same denominators and the addition of two fractions with different denominators.

2. Understanding of the Addition of Fractions

According to Skemp (in Kastberg, 2002), understanding means knowing what to do and knowing the reason why doing it. Moreover, NCTM (2000) also states that the ability of using representations or models of a concept is a sign of understanding. Therefore, in this study, understanding of the concept of the addition of fractions means that students are aware of how to use models to grasp the idea of common denominator in adding fractions, and know the reasoning of what they do with the models. Moreover, they know that they cannot apply the procedure 'top+top over bottom+bottom' in adding fractions.

3. Models

"Models are representations of relationships that can be used as tools to solve problems" (Fosnot and Dolk, 2002, p.90). Models can function as representations of situation or problems and to explore relationships between concepts and between numbers. Paper strips and the bar model are examples of models that can be used in teaching fractions.

4. Estimation Skill

Estimation is a process of finding a result that is close enough to the exact result without applying any complicated computation. Estimating the sum of two fractions means estimating the exact result by considering the benchmarks. For instance, by considering a half as the benchmark, the result of $\frac{2}{3} + \frac{1}{4}$ must be more than a half because $\frac{2}{3}$ is more than a half.

E. Significance of the Research

As has been described in the background, in learning the addition of fractions, students tend to memorize procedure rather than understanding the concept of the operation and the reasoning behind the procedure. As the result, students often make mistakes in solving problems about the addition of fractions and in applying the procedure. Therefore, this study attempts to provide an empirically grounded local instruction theory on the topic of the addition of fractions by utilizing models. The grounded instruction theories provide the description of how to support students' understanding of the addition of fractions. Moreover, this study gives an insight for the researcher about how to design the instructional activities by concerning some aspects, such as the theory about the learning process, the conjectures of students' thinking, and how teachers should react on students' thinking.

CHAPTER II
THEORETICAL FRAMEWORK

In this chapter, the researcher will elaborate the theoretical framework that underlies this study. A review of some literature about teaching and learning of the addition of fractions, either in Indonesia or in other countries, is presented as a framework for designing the learning material. Moreover, the theory about *Realistic Mathematics Education* (RME) is also described in this chapter as the grounding theory to investigate how students learn the addition of fractions through the principles of RME such as the use of contexts and models.

A. Issues on the Teaching and Learning of Fractions

Fractions are representations of part-whole relationships. Many researchers have reported that the topic of fractions is a complex and problematic topic in mathematics (Hasemann, 1981; Streefland, 1991; Cramer et al., 2002; van Galen et al., 2008; etc.). Those researchers also documented that in teaching fractions, commonly, teachers introduce fractions by illustrating it as a part-whole relation, giving some examples, and then presenting a set of rules to operate fractions. Teachers explain in a more conventional way, follow the textbook, and provide the formal definition and algorithms to students. Consequently, students know how to apply these rules yet they do not grasp the reasoning behind these rules. This will lead students to make mistakes in solving problems since they cannot ensure whether their solutions are reasonable or not. Moreover, many studies have revealed that this approach, in which the teacher places more emphasis on mechanistic teaching than on students' understanding, leads students to consider that the topic of fractions is a complicated field (Hasemann, 1981; Streefland, 1991; Cramer et al., 2002; Reys, 2009).

In addition, a multifaceted interpretation of fractions is another matter that causes students' difficulties in understanding fractions (Lamon, 2001; Charalambous & Pitta-Pantazi, 2007; Pantziara & Philippou, 2012). Fractions have five interpretations, which are a fraction as a part-whole relationship, as a measure, as a ratio, as a quotient, and as an operation. Charalambous et al. (2010) reported that many teachers and textbooks focus only on one interpretation, a fraction as a part-whole relation, and disregard the other interpretations. Therefore, students find it confusing when they deal with situations that comprise different interpretations of fractions. Besides, Charalambous et al. (2010) also argued that if teachers concentrate simply on one interpretation of fractions, it would disrupt the students' learning of fractions. In the next section below, we elucidate the five interpretations of fractions and issues on the addition of fractions.

1. Five Interpretations of Fractions

According to Behr et al. (in Charalambous & Pitta-Pantazi, 2007), there are five interpretations of fractions, which are a fraction as a part-whole relationship, as a measure, as a ratio, as an operation, and as a quotient. They also proposed a theoretical model that connects each interpretation to some operations of fractions.

A fraction as a part-whole relation means a fraction is a part of a whole object that is divided into equal parts. Based on Behr et al.'s theoretical model (in Charalambous & Pitta-Pantazi, 2007), the interpretation of fractions as a part-whole relation is a basic to build the understanding of the other interpretations. A fraction as a measure means a fraction can be used to measure distance, length, or height, from the origin. This interpretation is assumed helpful in constructing the understanding of the addition of fractions. As a ratio, a fraction is defined as a comparison between two quantities. This interpretation is regarded as important to develop the understanding of equivalence of fractions. A fraction also can be an operation, when a fraction acts as a function towards some numbers or objects. A fraction as an operation is deemed fruitful in supporting students' understanding of multiplication of fractions. In the situation of fair-sharing, in which some quantities are divided or shared among some people, a fraction functions as a quotient. A fraction as a quotient, together with the other interpretations, is essential in developing problem solving skills on fractions. The picture below represents the theoretical model proposed by Behr et al. (in Charalambous & Pitta-Pantazi, 2007).

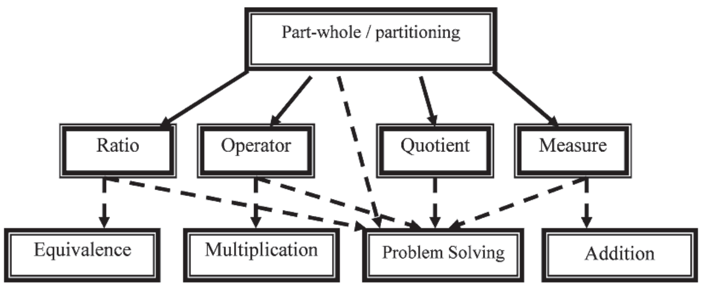


Figure 2.1 The Theoretical Model Connecting The Five Interpretations of Fractions to Some Operations of Fractions and The Problem Solving Skill.

2. Addition of Fractions

Addition of fractions is an operation of fractions in which students regard as difficult. Commonly, teachers provide a formal algorithm, which is students are required to find the common denominator by using the idea of *Least Common Multiple* (LCM) to solve the problem of the addition of fractions. Students might be able to use the algorithm, yet the students cannot reason why such an algorithm works (Kerslake, in Pantziara & Philippou, 2012). The students do not have a reasonable idea to evaluate their solutions. Consequently, it could cause misconceptions and lead them to do an incorrect procedure in solving the problems of the addition of fractions.

Bell et al. (in Amato, 2005) argued that some students' misconceptions about fractions occur because a new concept is not strongly linked to the previous concepts. Numerous studies have documented that a common mistake in adding two fractions is adding across the numerators and the denominators, or 'top+top over bottom+bottom' procedure (Howard, 1991; Young-Loveridge, 2007). For instance, students argue that the result of $\frac{2}{3}+\frac{1}{4}$ is $\frac{3}{7}$. Students who carry out that procedure might consider a fraction as two different whole numbers so they employ the same procedure as they do with whole numbers. In this case, students do not notice that a fraction is a part of the number system.

Some researchers agreed that teachers should introduce fractions as a part-whole relation since a part-whole model of fractions can represent the concepts and operations of fractions (Amato, 2005; Reys et al., 2009). This argument is in line with the theoretical model proposed by Behr and his associates (in Charalambous & Pitta-Pantazi, 2007), which argued that the interpretation of a fraction as a part-whole relation could support the understanding of other interpretations. Besides a part-whole relation, Behr and his associates also proposed that interpretation of fractions as a measure can promote students' understanding of the addition of fractions. Therefore, many researchers suggest not to focus on merely one interpretation to achieve better understanding of the concept of the addition of fractions (Lamon, 2001; Charalambous et al. 2010; Pantziara & Philippou, 2012). Van Galen et al. (2008) suggested that fractions can be represented as sharing and measuring situations. With respect to this suggestion, some researchers have integrated more these two interpretations of fractions in their studies such as Subramanian & Verma (2009). However, to our knowledge, there are no studies that integrate three interpretations, which are a fraction as a part-whole relation, a quotient, and a measure, in any case not in Indonesia. Therefore, the researcher integrates these interpretations in this study to be conducted in Indonesia.

Besides the consideration of the interpretations of fractions, teachers also need to take into account the knowledge that can support students' understanding of the addition of fractions. Some studies revealed that the concept of equivalence is helpful in constructing the understanding of a fraction as a single number and the understanding of the addition of fractions (Charalambous & Pitta-Pantazi, 2007; Cramer et al., 2002; Reys et al., 2009; Pantziara & Philippou, 2012). The concept of equivalence is useful in finding the common denominator in the addition of fractions with unlike denominators.

Moreover, an insight about comparing fractions and estimating the sum of two fractions by using benchmarks can be fruitful for students to promote their understanding of the addition of fractions (Reys et al., 2009; Cramer et al., 2008). When students are able to compare fractions by using benchmarks, it wil help them to find the reasonable estimation of the sum of two fractions (Johanning, 2011). Then, the estimation skill can encourage students' reasoning in examining whether their solutions of the addition of fractions problems are reasonable or not. Thus, it also will prevent students from doing an incorrect procedure 'top+top over bottom+bottom'. For instance, students will notice the result of $\frac{2}{3} + \frac{1}{4}$ can never be $\frac{3}{7}$ since they know that $\frac{2}{3}$ is more than $\frac{1}{2}$ so the result must be more than $\frac{1}{2}$.

3. Understanding of the Addition of Fractions

According to Skemp (in Kastberg, 2002), understanding means knowing what to do and knowing the reasoning why doing it. Moreover, NCTM (2000) also states that the ability of using representations or models of a concept is a sign of understanding. Therefore, in this study, the researcher identifies students' understanding of the concept of addition of fractions as below.

a. Students show their understanding in adding fractions when they are aware of the idea of common denominator in adding fractions by using models (paper strips and ar model), and know the reasoning behinds it.

b. Students notices that they the procedure 'top+top over bottom+bottom' in adding fractions is incorrect.

B. The Teaching and Learning of Fractions in Indonesia

Start from 2013, the curriculum used in the first and in the fourth grade of elementary is a new Indonesian curriculum, while the second, third, fifth, and sixth grade still the Indonesian curriculum of 2006.

The concept of fractions is firstly introduced in the third grade. In this grade, a teacher introduces the concepts and the representation of fractions by using pictures. Then, the teacher comes to the comparison between simple fractions, which is a comparison between a unit fraction and a unit fraction, and the comparison between fractions with the same denominators. Below are the description of the core and basic competence of the topic of fractions in the third grade (Badan Standar Nasional Pendidikan, 2006).

Table 2.1
The Core Competence and Basic Competence of The Topic of Fractions in the Third Grade of Indonesian Curriculum of 2006

Core Competence	Basic competence
Understanding fractions and using fractions to solve problems.	3.1 Understanding simple fractions 3.2 Comparing simple fractions 3.3 Solving problems including simple fractions

In the first semester of the following grade (fourth grade), the concept of fractions is re-explained and the operations of fractions are introduced. In this grade, the teacher should elucidates the concept of the addition of fractions with like and unlike denominators by using concrete models and pictures, as stated in the Basic Competence of the new Indonesian curriculum below (Kementrian Pendidikan dan Kebudayaan, 2013).

Table 2.2
The Core Competence and Basic Competence of The Addition of Fractions in the Fourth Grade of Indonesian Curriculum of 2013

Core Competence	Basic competence
Understanding the factual knowledge by observing (listening, looking, and reading) and asking based on a curiosity about themselves, other creatures and their activities, and objects they find at home, school, and playground.	Understanding the equivalence of fractions and the operations of fractions by using concrete models/pictures.

After the students of the third grade learned about the initial concept of fractions, they will learn about the concept of the addition of fractions. Thus, as a preparation in understanding the addition of fractions in the fourth grade and due to the limitation of time, this study will be conducted in the third grade of an elementary school in Indonesia, which is after the students learned the concept of fractions as stated in the curriculum of the third grade.

Many teachers in Indonesia still employ a conventional approach in the teaching and learning process. They focus and stress more on the rules and formal procedure than on students' understanding of the concept (Ullya et al., 2010; Sembiring et al., 2008). More specifically, in the topic of the addition of fractions, the teachers do not utilize models and do not relate it to the students' life (Ullya et al., 2010). The teachers do not explore their surroundings as a source in the learning process. Commonly, in the learning process the teachers start with a definition, followed by some properties and rules, and then provide some examples. The teachers do not engage students in the learning process and do not encourage them to be active in constructing a new knowledge. This situation is far away from a meaningful teaching and learning process.

Lamon (2001) argued that a mechanistic approach does not support students' meaningful learning. Moreover, according to Haji and Jailani (in Sembiring et al., 2008), students' difficulties in learning and understanding the concept of mathematics is a bad impact of this approach. Armanto (in Sembiring et al., 2008) also revealed that this teaching style can lead students to some misconceptions. Consequently, it can impede students' learning.

As stated in the core competence of the new Indonesian curriculum, the learning process should integrate students' surroundings and activities in the learning process. The teachers need to connect to the students' world in the teaching and learning process. Moreover, to create a meaningful learning, the teaching style should shift from 'teacher-centered' to 'students-centered'. The students have to be active in constructing their own knowledge facilitated by the teacher. It is in line with Freudenthal's idea about *Realistic Mathematics Education* (RME) approach, that teachers should relate the mathematics to the students' real world and utilize their surroundings as a tool to help students learn and understand (Gravemeijer, 2004a). Therefore, the researcher incorporates the RME theory as the ground theory in designing the teaching and learning materials for this study. The thorough description about RME will be elucidated in the next section below.

C. Realistic Mathematics Education (RME)

Constructing students' knowledge is the core of teaching mathematics. As has been outlined in the previous section, conventional teaching and learning can hamper students' learning. Many researchers, such as Charalambous & Pitta-

Pantazi (2007), suggested that teachers should emphasize students' conceptual understanding more instead of giving students abundant rules and algorithms. Teachers need to reform the conventional approach, in which mathematics is considered a ready-made knowledge, into an approach that can offer meaningful learning for students. In a meaningful learning process, students do not simply memorize and apply rules and algorithms, yet they need to actively construct their own knowledge by exploring connections between the mathematics and their surroundings. Freudenthal (1991) also has claimed that *mathematizing* is the heart of learning mathematics. *Mathematizing* is a process of building knowledge, in which students "explore situations mathematically, they are noticing and exploring relationships, putting forth explanations and conjectures, and trying to convince one another of their thinking" (Fosnot & Dolk, 2002, p. 9).

Furthermore, Freudenthal has stated that mathematics teaching and learning should be connected to reality (Gravemeijer, 2004a). To stay connected to the students' real world, the role of context as a means to support students in building relationships between a concept and the reality is very important (Fosnot & Dolk, 2002). 'Real world' means real or imaginable for students. Thus, the context does not merely relate to the students' daily life, but it also can relate to stories or fantasies. A rich context, which provides a space for students to explore many ideas, strategies, and solutions, can facilitate a meaningful learning. When students are exploring the contexts, the presence of a model is necessary. Models, such as paper strips and bar model, are tools to explore relationships among numbers and to solve problems. Both, contexts and models are essential elements for promoting students' *mathematizing*. It is also claimed by Cramer et al. (2008) that representations, such as models and contexts, can boost students in exploring and understanding concepts and relationships among numbers.

All facts described above are in line with the five tenets of RME. The RME theory suggests that mathematics teaching and learning should start and stay in reality and support students' meaningful learning (Gravemeijer, 2004a). A teacher needs to connect the material to a context that is real or imaginable for students. Besides the use of contexts, the use of models is also very useful for students as a tool to explore mathematical ideas and to solve problems. The contexts and models function to bridge students' preliminary and informal knowledge to a more formal knowledge. Thus, students can construct the knowledge meaningfully. The following section is the outline of the five tenets of RME and its relation to this study.

1. Five Tenets of RME

We ground the design of the learning activities in this study on the five tenets of RME as proposed by Treffers (1987). The five tenets of RME will be described below.

a. Phenomenological exploration by means of contexts

Teachers should not explain everything and should not be the center in the mathematics teaching and learning process. Instead, students have to be active in constructing and exploring concepts. Therefore, learning activities should start from an informal situation, in which teachers provide a meaningful context and encourage students to explore many ideas, strategies, and relationships in the context. In this study, the researcher uses a sequence of fair sharing and measure activities as the bridge to grasp the idea of equivalence of fractions, comparing and ordering fractions, the common denominator, and the addition of fractions.

b. Bridging by vertical instruments

Teachers should facilitate students' shift from informal knowledge to formal knowledge. As has been described above, models can be used as a helpful tool for students to explore relationships. In this study, the researcher uses paper strips and a bar as the models. As has been suggested by van Galen et al. (2008), paper strips and bar model can be conceptual models that help students construct the concept and support their reasoning of fractions. In this study, students will utilize measuring strips to explore the equivalence of fractions. Then, the teacher supports students to shift from measuring strips to the bar model to grasp the concept of the addition of fractions. By exploring the measuring strips and the bar model, the students will have a mental image for fractions and have a fractions sense. The mental image for fractions and the fractions sense is very useful in developing the understanding of the idea of the equivalence of fractions, comparing and ordering fractions, and the common denominator.

c. Students' own constructions and productions

To create a meaningful learning, students should be encouraged to participate actively in the learning process. It is in line with Freudenthal's notion that considers mathematics as a human activity (Gravemeijer, 2004a).

In this study, students will have more sense and understanding about what they learn by exploring and doing. When students do a fair sharing activity and create their own measuring strips, they will learn to partition and explore the equivalence of fractions.

d. Interactive instruction

Teachers have to establish a classroom culture, which offers a space for students to participate in interaction with the teacher and the other students. For instance, in this study, the teacher organizes a class discussion in each meeting, in which students learn to share their ideas and strategies in solving problems and learn to accept the opinions of others. In this discussion, the teacher facilitates and supports students to communicate their ideas and give comments to other ideas. By doing so, students will get more insight from different thoughts and reasoning.

e. Intertwining of learning strands

Concepts within mathematics correlate with each other, thus teachers can intertwine some concepts in an activity. For instance, in this design, when students explore fractions with the measuring strips, they will also learn about the concept of measurement.

2. Emergent Modeling

Besides the five tenets described above, RME design heuristics also support the design that aims at meaningful learning. The second tenet of RME is in line with the *emergent modeling* design heuristic, in which models are considered as a mean to help students to reason and shift from informal knowledge to formal knowledge (Gravemeijer, 2004a). The models emerge as students grapple with a context. As an example, in this design, the researcher creates a context that aims at the emergence of measuring strips and the bar model.

As has been described before, models build a bridge between an informal situation and a more formal knowledge. In the process of modeling, students start from a *model of* informal mathematical activities (context) to a *model for* a more formal mathematical reasoning. Models do not come from a formal mathematical knowledge. Instead, models derive from contexts that embody mathematical ideas. Gravemeijer (2004a) described the process of modeling through various levels below.

a. Situational level

   The situational level involves activities in the task setting. In this activity, students' understanding about how to act in the problem setting will influence their interpretations and solutions of the problem.

b. Referential level

   This level comprises activities that encourage students to come up with *models of* the problem setting in the learning activities.

c. General level

   In the general level, students shift from *models of* the situations to *models for* more formal mathematical reasoning that are independent from situation-specific imagery.

d. Formal level

In this level, students are independent from the support of models. Students start to think and reason in formal mathematics.

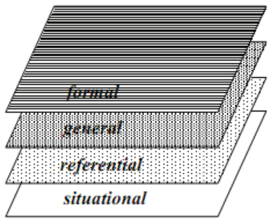


Figure 2.2. Four Levels of The Process of Emergent Modeling

The end goal of this study is the general level. In the beginning, the researcher designs the fair sharing and the measuring context that can lead students to use measuring strips as the *model of* the contexts. In the teaching and learning process, firstly, the teacher provides the contexts that reveal the need of measuring strips as the representation of the context. Then, the teacher facilitates students to shift from *the model of* the situations (measuring strips) to *model for* more formal mathematical reasoning (paper strips and bar model). In this phase, the teacher guides and supports students' understanding of the concept the addition of fractions by using paper strips and bar model.

CHAPTER III
METHODOLOGY

A. Research Approach

As has been mentioned in the first and second chapter, the purpose of this study is to support students' understanding of the addition of fractions by integrating the use of contexts and models. To reach this goal, the researcher needs to design and develop instructional activities about the addition of fractions. Besides, the researcher also needs to execute and examine the activities to find out how the activities support students' understanding of the addition of fractions. By designing and examining the instructional activities, the researcher intends to contribute to an innovation and an improvement of the teaching and learning of the addition of fractions. Therefore, the researcher employs a design research approach in this study.

Design research is an approach that can "bridge the gap between educational practice and theory" (Bakker and van Eerde, 2013, p. 2). Design research integrates a design and a research. Contributing an innovation in the field of education by designing instructional activities is a crucial part of design research. Besides, a research about how the design can promote students' learning is also essential to figure out how the design works or why the design does not work. Therefore, in the design research, the theory and the practice are intertwined to develop theories about the learning process and the activities or tools that can support the students' learning process.

There are three phases in the design research (Gravemeijer & Cobb, 2006), which are preparing for the experiment, experimenting in the classroom, and conducting retrospective.

1. Preparing for the Experiment

In this first phase, the researcher designs instructional activities and elaborates the students' conjectures toward the activities that will be examined and refined during the classroom experiment. The sequence of learning activities and the conjectures of students' thinking are included in the *Hypothetical Learning Trajectory* (HLT). An elaborated HLT comprises the learning goals, students' preliminary knowledge, the conjectures of students' thinking, and how teachers deal with the students' thinking (Bakker & van Eerde, 2013). In this phase, the HLT is helpful to organize the instructional activities that are formulated based on the grounded theories.

In addition, the researcher also develops learning materials (Worksheets), teacher guide, and pre- and post-test problems. Pre-test problems are used to investigate students' initial understanding of the addition of fractions, and post-test problems are used to figure out students' understanding after participating in the learning activities. The post-test contains the same problems as the pre-test so that the researcher can see students' progress from their answers. The pre- and post-test problems, the learning materials (Worksheets), and the teacher guide can be found in Appendix 3, 4, and 5 respectively.

To design the initial concept of HLT, the researcher refers to some theories that underlie the design and the researcher's experience. Therefore, firstly the researcher reads some references that relate to the teaching and learning of the topic of the addition of fractions. Then, based on the insights the researcher gets from reading the literatures, the researcher starts to design the instructional activities and discusses it with a supervisor who is experienced in designing learning materials. After the researcher makes the draft of the HLT, the researcher conducts a class observation and interview with the teacher to figure out the situation of the classroom, the characteristic of students, and the students' initial understanding of the topic of addition of fractions. Then, the researcher give pre-test to the students to identify students' initial understanding of the concept of fractions and the addition of fractions. Those information gotten from the observation, the interview, and the pre-test are used to adjust the instructional activities and the students' starting points in the HLT. Then, the HLT is ready to be examined in the teaching experiment.

2. Experimenting in the Classroom

After the researcher elaborates the HLT, the HLT is implemented in the actual classroom experiment. In this phase, this HLT functions as a guidance for the researcher and the teacher in conducting the teaching experiment. During this teaching experiment, the researcher collects some data to address the research questions, such as video recordings of the lessons, students' work, interview, and field notes.

There are three cycles in the teaching experiment of this study. Design research involves infinite iterative cyclic process. However, commonly, there are two or three cycle in the study of master students due to the limitation of time. The application of HLT in different settings may result differently. However, patterns of students' learning across different teaching experiments may occur. Those patterns and the insight of how the activities promote students' learning can contribute a more general instruction theory (Bakker & van Eerde, 2013). In this study, the researcher use three cycles in this study in order to contributeto a stronger empirically grounded local instruction theory.

The first cycle is a pilot study in which the researcher examines the initial HLT. In this study, the researcher conducts the pilot study in a small group of 5-6 students because the researcher wants to focus more and zoom in on students' thinking. The aim of the pilot study is to try out and to investigate how the initial HLT works. The result of the pilot study is used to refine and improve the content and the activities in the initial HLT. In the second cycle, the improved HLT is applied in a real classroom setting. Thereafter, the researcher improves the HLT based on the result of the teaching experiment of the second cycle. Then, in the last cycle, the improved HLT based on the second cycle is carried out in another classroom. The result of the third cycle is analyzed to answer the research questions.

The last improved HLT will contribute to the development of the *local instruction theory* on the topic of the addition of fractions. "Local instruction theories are the product of design research within which prototypical instructional sequences are developed in a cumulative process of designing and revising instructional activities" (Gravemeijer, 2004a, p. 9). In this study, the local instruction theory comprises both theories about the learning process of the addition of fractions and theories about the activities and tools to promote the learning process. The formulation of the local instruction theory is through the cyclic process is shown as the figure below.

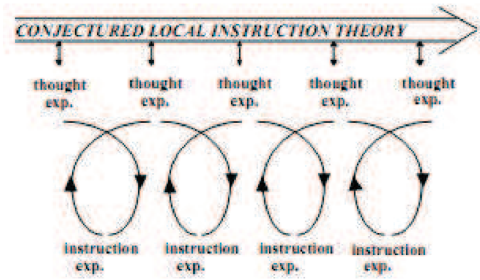


Figure 3.1. The Cyclic Process in Design Research (Gravemeijer, 2004a)

3. Retrospective Analysis

The data obtained from the teaching experiment is analyzed by referring to the HLT. In this phase, the researcher compares students' actual learning with the conjectures in the HLT. The result of the analysis describes not only how the design works, but also how and why the design does not work and the obstacles during the implementation of the HLT. This result will be used to improve the existing HLT.

B. Data Collection

1. Participants

This study is conducted in SD Laboratorium Unesa Surabaya, with a teacher and students of the third grade are involved in this study. In the first cycle, the participants are five students of 3A, namely Tya, Nabil, Samuel, Nanda, and Diva. Then, in the second cycle, the participants are the students of 3B. In the last cycle, the researcher takes the students of 3A, exclude the five students that have participated in the first cycle, as the participants.

2. Preparation Phase

Before carrying out the teaching experiment, the researcher conducts a classroom observation and interview with the teacher. Firstly, the researcher conducts the observation The purpose of the observation is to figure out the classroom setting, students' characteristics and how they interact with the teacher and other students, the classroom norms, how the teacher manage the class, and how the teacher explains a subject. Then, in the interview with the teacher, the researcher asks more information about the students' preliminary knowledge, the students' understanding and their difficulties about fractions, the students' characteristics, how the teacher manages the class, and how the teacher explains the addition of fractions. The researcher also asks about something interesting in the classroom observation, such as how the teacher manages the class, and the classroom norms. The elaborated observation and interview scheme can be seen in appendix 1 and 2. To collect the data, the researcher makes a video registration and field notes during the interview and the classroom observation.

3. Preliminary Teaching Experiment (First cycle)

Before the mathematical activities of the HLT are implemented in the first cycle, the students get a pre-test. After the students have completed the pre-test, the researcher conducts the instructional activities on the topic of the addition of fractions as described in the HLT, in which the researcher becomes the teacher. In this cycle, the researcher conducts the learning activities in a small group of 6-7 students. During the lessons, the researcher makes a video registration of the lessons with the help of the researcher's colleagues. The researcher also makes notes about important and interesting issues regarding how the learning activity works, students' responses, and how the teacher deal with it. Moreover, the researcher collects the students' work on the worksheet. Those data will be helpful for improving the HLT for the next cycles.

Thereafter, in the end of the last activity, the students get a post-test. After the students have done the post-test, the researcher interviews some students to get more information about their thinking and understanding of the addition of fractions. The problems of the pre-test and post-test can be found in appendix 3.

4. Teaching Experiment (Second and Third cycle)

In the second and the third cycle, the improved HLT is implemented in a real classroom setting. Before the teaching and learning process is conducted, the students get a pre-test. After the students have done the pre-test, they participate in the learning activities, in which the regular teacher of this class gives the lessons.

While the teacher conducts the activities, the researcher makes a video registration. The researcher records the activities of the whole class and records a small group discussion as the focus group to zoom in on students' thinking and reasoning. The researcher also makes field notes during the lessons that contain important and interesting issues regarding how the learning activity works, students' responses, and how the teacher deal with it. Moreover, the researcher also collects students' written work on the worksheets. In addition, after each lesson, the researcher consults with the teacher about the teacher's experience in conducting the lessons, such as what is missing in the lessons, what are the weaknesses, what is good about the lessons, and the teachers' difficulties. From this sharing, the researcher gets an insight into how the lessons work from the perspective of the teacher. This information is useful as a consideration in improving the HLT.

After participating in the teaching experiment, students get a post-test. This post-test is used to investigate the effect of the learning process and to figure out the extent to which students understand about the addition of fractions after joining the lessons. Besides conducting the post-test, the researcher also interviews some students to know more about students' thinking and reasoning. The researcher records the interview and makes field notes during the interview. The pre-test and post-test problems can be seen in appendix 3.

5. Validity and Reliability of Data Collection

Validity means to measure what is intended to measure. The various data used in this study such as video, written work, and field notes contribute to the internal validity of the study. The data the researcher gets from each method can be cross-checked with the other data so the researcher gets the data and the results from various perspectives. Moreover, these data can support each other. For instance, the interview will supplement the information of the observation, and conversely, the observation data can support the interpretation of interview results. Hence, the more data used, the more accurate the study will be.

Consequently, it contributes to the validity of the study. Moreover, collecting the data by using a video recorder can improve the reliability of the study since it minimizes the researcher's subjectivity and interference in the video data.

C. Data Analysis

1. Pre-test

The students' work in the pre-test is analyzed to figure out the students' preliminary knowledge about the addition of fractions. Their strategies and scratch in their written work reveal their initial understanding of the addition of fractions, the misconceptions they encounter, and how they solve the problems. The result of pre-test is used to adjust the students' starting points in the HLT. Moreover, the result of the pre-test of the each cycle is also used to find out whether the problems were understandable for students or not in order to improve the pre-test problems for the next cycle. The result of the pre-test is compared to the result of the post-test to investigate students' progress after participating in the learning activities.

2. Preliminary Teaching Experiment (First cycle)

The data collected in the first cycle are video registrations of the lessons, students' work, field notes, and interviews with students. Firstly, the researcher watches the videos and selected interesting fragments. The researcher chooses not only fragments showing students' understanding, but also fragments indicating that students struggle with a problem. Then, these fragments are transcribed and analyzed by referring to the HLT. The researcher compares the students' actual learning to the conjectures in the HLT. From this fragment, the researcher figures out which conjectures work well or do not work, and how and why the lessons work or do not work. Besides, the students' work, field notes, and interviews are used to clarify and supplement the findings of the video fragments. The findings of this analysis were a basis to improve and refine the learning activities and the conjectures of students' thinking in the initial HLT.

3. Teaching Experiment (Second and Third cycle)

As in the first cycle, the video of the class activities and the focus group of the second and third cycle is analyzed by firstly choosing the interesting fragments. The fragments are not only the fragments that show students' understanding, but also fragments that show how students grapple in solving the problems during the lessons. The chosen fragments reveal students' thinking and reasoning about a problem or an issue. Then, these fragments are transcribed and then are analyzed by comparing what really happens in the classroom to the conjectures in the HLT. The researcher analyzes why and how the conjectures work or do not work. Thereafter, the researcher also analyzes the students' written work, the field notes, and interview data to crosscheck and supplement the result of the video data. The students' work, field notes, and interviews can support and give more information to the result of the analysis of the fragments. From the analysis, the researcher figures out students' understanding and how and why the activities work or do not work. The result of the second cycle is used to improve the HLT for the next cycle, while the result of the third cycle is used to address the research questions, to derive the conclusion, and to contribute to the empirically grounded LIT on the topic of the addition of fractions.

4. Post test

The students' work of the post test is analyzed to investigate their understanding after participating in the lessons. Then, the results are compared to the results of the pre test to find out students' progress.

5. Validity and Reliability of Data Analysis

a. Validity

Internal validity is related to the quality of the data collection and the reasoning in drawing the conclusions. The use of various data such as students' written work, video recordings, interview, and field notes contribute the internal validity of this study. The findings that are drawn from the video recording can be supported and supplemented by other data. Thus, the more data are analyzed, the more accurate the findings will be. Moreover, comparing the analysis to the conjectures in the HLT is an attempt to keep focus on what is intended to measure. Hence, this can increase the internal validity of the analysis.

External validity means the generalizability of the findings. Framing issues as examples of something more general is a way to be able to generalize the findings of specific contexts to other contexts (Bakker & van Eerde, 2013). To improve the external validity, the researcher elaborates the students' activities and frames important episodes. Moreover, the researcher also makes a thorough description of what happens in the classroom and factors that might influence the learning activities. By doing so, the readers or other researchers can replicate and adjust the results of this study (instruction theory, HLT, educational activities) to their local setting.

In addition, the implementation of the instructional design in a real classroom also will strengthen the ecological validity of the study.

b. Reliability

Reliability means the independence of the researcher. During the analysis, the researcher discusses with the teacher and colleagues about the analysis and the interpretation of the fragments (peer examination). This will minimize the subjectivity of the researcher and thus increase the internal reliability of the study.

External reliability is related to the trackability of the study, which means the readers must be able to track the whole learning activities of this study and to reconstruct this study (Bakker & van Eerde, 2013). The description about the theories underlie the design, how the study has been carried out, the learning process, the failures and successes, and how the conclusions are derived must be clearly documented. Thus, a thorough and transparent description of the data collection, the learning processes, and the data analysis presented in this study will contribute to the external reliability.

CHAPTER IV
HYPOTHETICAL LEARNING TRAJECTORY

As has been described in Chapter III, Hypothetical Learning Trajectory (HLT) consists of three parts, which are the learning goal, the learning activities, and the conjectures of students thinking during the learning activities. Moreover, an elaborated HLT contains students' starting points that inform students' initial understanding of a concept, and the teacher's reaction toward all possibilities of students' thinking in order to support their understanding.

According to Bakker & van Eerde (2013), the intention of developing HLT is to offer empirically grounded results so that other researchers and teachers can adapt it in their learning ecologies. The implementation of an HLT in different situations might result different findings, yet patterns can be found in those different teaching experiments. Those patterns will show how to support students' learning with particular instructional activities. Thus, it will contribute to the local instruction theory of a particular domain.

A. The Process of Making HLT

1. Indicators of Each Meeting

As has been described in Table 2.2, the basic competence of the addition of fractions is "Understanding the equivalence of fractions and the operations of fractions by using concrete models/pictures" (Kementrian Pendidikan dan Kebudayaan, 2013). Thus, the researcher defines and maps the indicators on the topic of fractions as below:

a. Indicators of the first meeting

   1) Understanding the concept of fractions (partitioning and the notation of fractions)

   2) Finding the equivalence of fractions.

b. Indicators of the second meeting

   1) Comparing fractions.

   2) Estimating the sum of two fractions by using benchmarks.

c. Indicators of the third meeting

   1) Knowing that the procedure ‘top+top over bottom+bottom’ is incorrect.

   2) Finding a common denominator by utilizing paper strips.

   3) Adding fractions by using paper strips and bar model.

2. The Result of the Classroom Observation and the Interview with the Teacher

After the researcher makes the initial concept of HLT by referring to those indicators, and the grounded theories and the researcher’s experience, the researcher conducts a classroom observation to get information about the characteristics and the ability of the students, the social norms in the class, and the teaching style of the teacher. That information is used to adjust the instructional activities in the initial HLT. After being adjusted, the HLT are ready to be implemented in the teaching experiments.

Based on the classroom observation and the interview with the teacher, the researcher gets information as bellow.

a. The characteristics and the ability of the students

According to the teacher, students in the class 3A and 3B have the same characteristics. They are talkative students. Some students are active in asking during the lessons, while other students are a little bit quieter. In addition, the ability and the achievement of the students are heterogeneous. In each class, there are high achievers, middle achievers, and lower achievers.

On the topic of fractions, according to the teacher, the students have learned about the initial concept of fractions, which are the concept and the representation of fractions by using pictures. Moreover, they have learned the comparison between a unit fraction and another unit fraction, the comparison between fractions with the same denominators, and the addition of fractions with the same denominator. In addition, few students have known about the formal procedure (finding the Least Common Multiple of the denominators) in adding fractions with different denominators from their private course. However, according to the teacher, students who are able to use the formal algorithm usually find difficulties in understanding the reasoning behind the algorithm and in representing the algorithm in a model or pictures.

b. Teaching style and social norms in the class

In teaching the topic of fractions, the teacher usually follows the order of the book. To represent fractions, the teacher uses pictures as the model and uses the concept of fractions as a part-whole relationship. In addition, in teaching the addition of fractions, the teacher usually explains to the students that to add fractions, the denominators of both fractions have to be equal. The teacher does not let the students to explore the reasoning behind that concept.

In the regular lessons, the teacher not only explains a topic, but she also engages students to be active in the classroom by asking questions to the students. Sometimes, the teacher also divides the students into groups and asks them to discuss a problem with their group. In grouping the students, the teacher groups them such that there are high, middle, and lower achievers in the group.

Regarding the social norms, there are some rules or habits during the lessons. First, about the way the teacher points a student to answer a question. After the teacher poses a question to the students, the teacher will point a student who raises his/her hand without asking the other students who do not raise their hands. Consequently, other students will feel safe and have no responsibility to answer the question. Second, the students are not accustomed to have a group presentation and listen to other students. They seem busy with their own work so that they do not pay attention to their friends' argument. Third, the teacher usually gives a reward for students who are able to answer the question. The reward can be a point or stationery. According to the teacher, this reward can motivate students to be active in the classroom.

Based on the information the researcher gets from the interview and the classroom observation, there is a need to adjust the HLT and the teacher guide, such as:

a. Students can work in groups and the teacher should guide the groups and engage them to be active.
b. The teacher should not value students' answer. If there are differences in students' answer, the teacher should engage students to discuss it and give their argument.
c. The teacher gives some times to students to think before the teacher points the students. Then, the teacher needs to engage all students to be active by pointing not only students who raise their hands, but also students who seem quiet and shy.
d. The teacher will give rewards for students who are active and brave in giving their argument in order to engage students to be active.

3. The Result of Pre-test

The result of pre-test also contributes to the process of making HLT. The result of pre-test informs the researcher about the students' initial understanding of the concept of fractions and the addition of fractions. That information is used to adjust the HLT, in the part of students' starting points, the conjectures of students' thinking, and how the teacher should react to their thinking. The complete description of the result of pre-test can be seen in the Chapter V.

Based on the result of the pre-test and the interview result described in the previous section, the researcher summarizes the students' initial understanding of the concept of fractions and the addition of fractions as below.

a. Students are able to represent fractions in the form of pictures, and are able to label the fractions of given pictures.
b. Most of students do not know what equivalent fractions means, and do not know how to represent it in the form of bars.
c. Some students can compare fractions by using cross multiplication procedure. Other students try to compare fractions by using pictures, but they do not know about the idea of the unit of fractions, that to compare fractions, the unit has to have equal size and shape.
d. Students are not used to estimating the sum of two fractions.
e. Most of students are able to add fractions with the same denominator, yet they cannot add fractions with different denominators.

B. The HLT of This Study

The picture below is the learning line of the design of this study. This learning line shows the overview of the learning activities to support students' understanding of the addition of fractions. The detailed description of the learning activities; such as the learning goals, the students' starting points, the description of the activities, the conjectures of students' thinking, and the teacher's reaction; are elaborated in the HLT in the following sections.

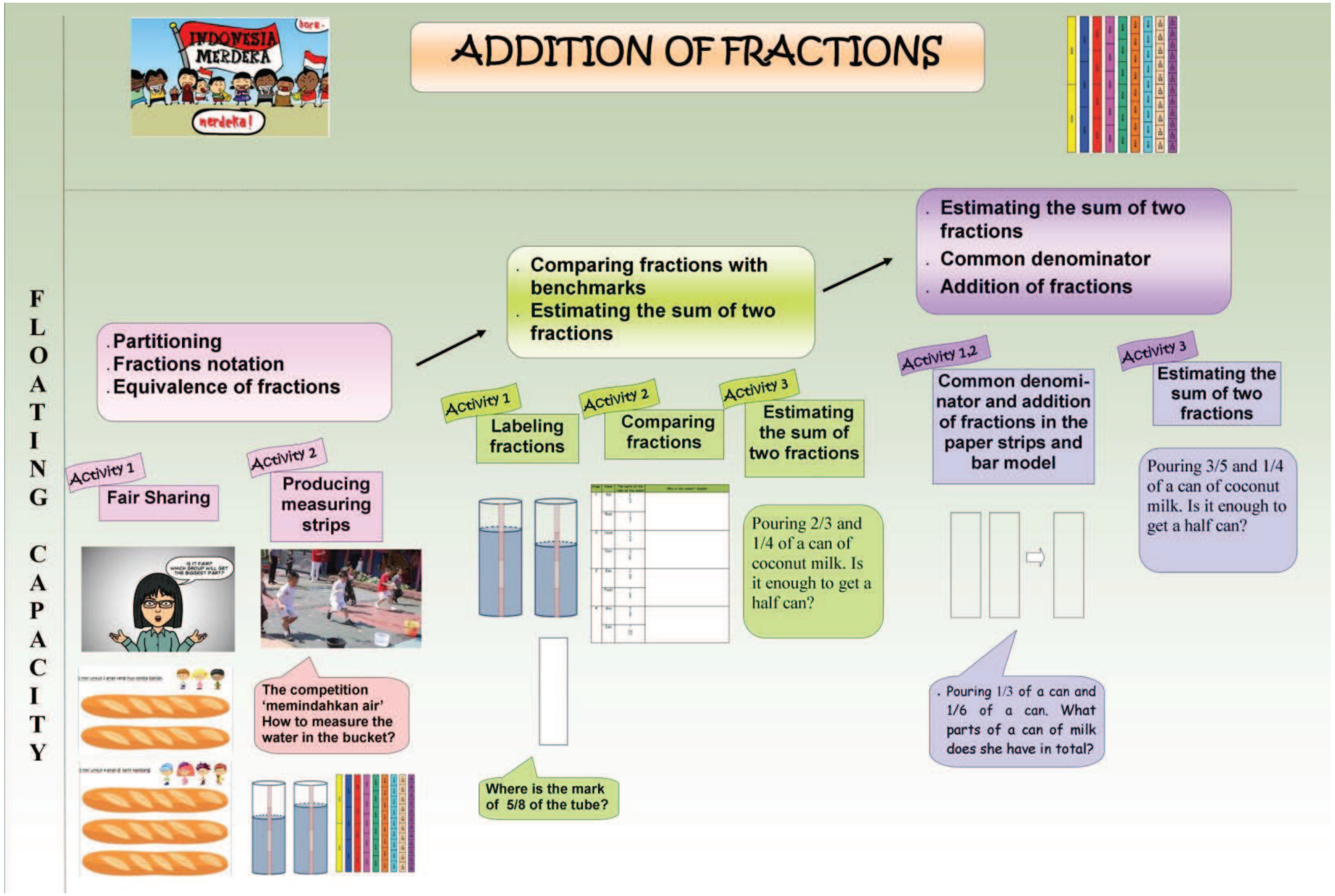


Figure 4.1. The Learning Line of The Addition of Fractions

1. **Meeting 1**

a. Students' starting points

1) Students are able to represent fractions in the form of pictures, and are able to label the fractions of given pictures.
2) Students are able to compare fractions with the procedure cross multiplication.
3) Students are able to add fractions with the same denominator.

b. Goal

1) Students are able to partition into equal parts.
2) Students can use the notation of fractions.
3) Students understand the idea of equivalent fractions.

c. Description of the activities

1) Activity 1: Fair sharing

Fair sharing is a rich activity since there are many ideas including in this activity. When students share some pieces of bread to a number of children, they learn how to divide the bread fairly. Students also learn about the notation of fractions when they are asked what parts of a piece of bread that each child gets. Moreover, there is the idea of the equivalence of fractions when students label the parts of bread that each child gets by using fractions notation. For instance, when students share 3 pieces of bread for 4 children, some students may think that each child gets $\frac{1}{2}$ and $\frac{1}{4}$ of a piece of bread, while other students may argue that each child gets $\frac{3}{4}$ of a piece of bread. From these differences of students' opinion, the teacher can raise the idea of equivalence of fractions. In this activity, the focus is on the idea of partitioning and the fractions notation. This notion will be useful for students in using the paper strips and bar model, in the case of partitioning the paper strips and bar model into equal parts and represent fractions on it.

In the beginning of the activity, the teacher tells this story:

In the celebration of the Independence Day, Mrs. Doni prepares some snacks for the children. After some children participate in some competitions, they get some bread to be eaten together. For children who participate in 'lomba bakiak', Mrs. Doni allocates 2 pieces of bread for 3 children, and for children who participate in 'lomba tarik tambang' 3 pieces of bread for 4 children. However, while she thinks that each child in a group will get the same parts of the bread, her friend argues that it is not fair because children in the group of 'lomba tarik tambang' will get different parts from children in the group of 'lomba bakiak'. Then, Mrs. Doni tries to figure it out. Does each child in each group get the same share of the bread? What parts of bread does each child in each group get?

2 pieces of bread for 3 children

3 pieces of bread for 4 children

Firstly, the teacher engages the students to help Mrs. Doni figuring out the questions (*Does each child in each group get the same share of the bread?)* in pairs. The possibilities of the students' answer:

a) Some students might argue that it is fair because the number of bread is one less than the number of children.
b) Some other students might think that children in the group '*tarik tambang*' will get a bigger share since they have more number of bread than in the group '*bakiak*'.
c) The other students might think that group '*bakiak'* will get a bigger share since the bread are divided into smaller number of children than in the group '*tarik tambang*'.

In this case, the teacher needs to encourage the students to consider both the number of bread and the number of children. Then, the teacher provides the students with activity 1 of worksheet 1 and asks them to work in a group of 2-3 students. In this worksheet, the students are asked to ensure and show in which group the children will a get bigger share of bread by dividing the representation of bread in the worksheet.

Conjecture of students' strategy and the teacher's reactions

a) Students might divide the representation of bread into equal parts as the number of children in each group. Then, they compare the parts of cake that each child in each group gets, such as:

3 pieces of bread for 4 children

The teacher’s reaction:

The teacher engages the students to consider both, the number of bread and the number of children. For example, the students might argue that 2 pieces of bread for 3 children is bigger because each part of each bread is bigger than in the situation 3 pieces of bread for 4 children. Then, the teacher can encourage them to think that even though each part in the situation 3 pieces of bread for 4 children is smaller, but there are more number of parts, which is 3 parts. Moreover, the teacher can engage the students to represent the students’ result in one picture, as below:

Parts of bread each child gets in the group ‘*bakiak’* 

Parts of bread each child gets in the group ‘*tarik tambang’* 

Then, the teacher asks the students which group gets bigger share of bread. In this case, the teacher can show students that 2 pieces of bread for 3 children is the same as two parts of three parts in a bread.

b) Students firstly halve the bread, and then they halve it again or divide it into equal parts as many as the number of children in each group. Then, they compare the parts of bread that each child in each group gets. They will get $\frac{1}{2}$ and $\frac{1}{6}$ for the group ‘*bakiak’*, and $\frac{1}{2}$ and $\frac{1}{4}$ for the group ‘*tarik tambang’*.

3 *pieces of bread for 4 children*

The teacher's reaction:

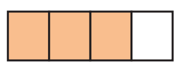
The teacher can encourage them to notice about the partition, that the more partition, the smaller part they get. The teacher also can ask them how to name the parts into fractions (the notation of fractions).

c) Students might solve the problems by directly dividing the number of cakes by the number of children and then compare it by considering the benchmark. For instance, they argue $\frac{3}{4}$ is greater than $\frac{2}{3}$ since $\frac{3}{4}$ needs $\frac{1}{4}$ to be a whole, and $\frac{2}{3}$ needs $\frac{1}{3}$ to be a whole. They might argue that $\frac{1}{4}$ is smaller than $\frac{1}{3}$, so they conclude that $\frac{3}{4}$ is greater than $\frac{2}{3}$. They also might solve it in formal ways, which they look for the common denominator for each fraction. Moreover, they might apply the cross multiplication procedure to compare the fractions.

The teacher's reaction:

The teacher can encourage the students to represent and explain their reasoning in a picture/bar. The teacher needs to encourage students to use pictures instead of algorithms in comparing fractions in order to build their fraction sense.

Class discussion

The teacher stresses on how to name the parts (by using the notation of fractions). If some students do not use fractions notation, the teacher introduces the notation of fractions as a part-whole relation to students by giving simple examples such as showing the following 

picture and asking '*What parts is the shaded area?'*

In determining what parts of the bread that each child in each group gets, students might get different answers. For instance, some students argue that each child in the group of *'lomba tarik tambang'* will get $\frac{3}{4}$ of a piece of bread while other students might get $\frac{1}{2}+\frac{1}{4}$ of a piece of bread. Then, the teacher can raise this issue to encourage students to think why the result can be different. Moreover, the teacher also has to point on the relation between two, four, and eight partitions. This knowledge is very important as an initial knowledge to learn the equivalence of fractions. In the next activity, students will learn more about the equivalence of fractions.

2) Activity 2: Producing Measuring strips

Learning by experiencing can support students to construct knowledge in their mind. In this activity, students will experience making the measuring strips with various numbers of partitions. From those measuring strips, students can see the relationships among partitions that can lead them to understand the relationship among fractions and the equivalence of fractions. Therefore, it is expected that the concept will embed in their mind. After students learn the reasoning of equivalent fractions by exploring the measuring strips, they can translate their reasoning in the bar model.

Firstly, the teacher tells how the previous story continues:

For the celebration of Independence Day, Mr. Doni and his friends have a task to arrange some creative competitions for children. His friends propose

a competition, namely 'lomba memindahkan air', which children need to move water with a plate from one bucket to another bucket. The children have to carry the water as much as possible within a given time. However, Mr. Doni wonders how they can know how much water that participants have filled in the bucket to determine the winner. What should they do?

For this problem, the students may come up with the idea of measuring the weight or the height of the water. Then, the teacher can ask '*how about if the weight of the buckets is different?*'. Afterwards, the teacher stresses that there are no scale and ruler. In order to lead them to the idea of measuring strips, the teacher shows the tube filled by around a half of the tube of water and asks them '*what parts of the tube is the water? How can we measure it?'*. The teacher demonstrates how to use paper strips as measuring strips by saying '*we can use this strip as a scale. This strip represents the tube. So, what parts of the tube are filled with this water?*'

Then, the teacher provides a tube filled by any scale of water, such as a half, one third, two third, one fourth, and three fourth of the tube, so that each group will make different measuring strip. The teacher engages the students to make two measuring strips with different number of partitions that can show what parts of the tube that is filled by water.

The possibilities of students' strategy in making the measuring strips

a) Divide the paper by estimating, without measuring.

The teacher's reaction:

The teacher can pose a question '*how can you sure that each part has the same length?'*

b) In folding the paper into 4 and 8 partitions, they may fold the paper into two several times. In partitioning the paper into three equal parts, the students may measure the length and then divided into three parts, or may use trial and error. Then, to make the 6 and 12 partitions, students might fold the 3-partitioned paper into two.

The teacher's reaction:

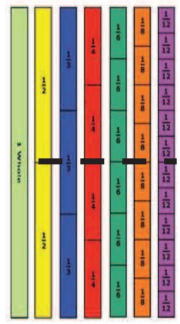
If the students do this way, the teacher can encourage students to name the fractions of each partition and notice the relation, for example between the eighth and fourth.

c) Measure the paper strips with a ruler and then divide the length of the paper strip into equal lengths.

The teacher's reaction:

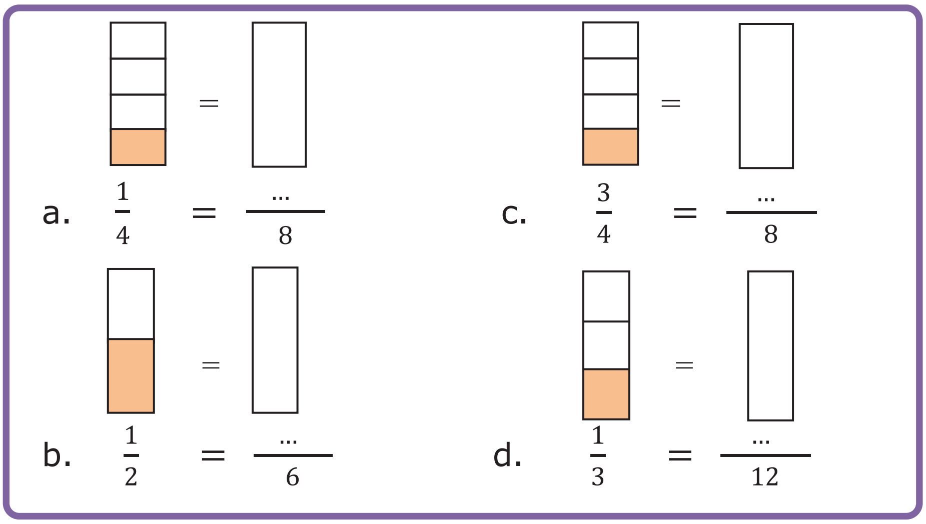
If students use this strategy, the teacher may pose '*what do you notice about the length of each part in eight partition paper in the four partition paper?*' Then, the teacher can encourage students to name the fractions of each partition and notice the relation between the length of each partition, for example the length between each part of the eighth and fourth.

Class Discussion



In the class discussion, students will discuss about how they make the measuring strips. The teacher has to stress on the notation of fractions in the measuring strips. Then, the teacher puts students' measuring strips together and encourages students to notice the extension of lines in the measuring strips to find the equivalent fractions as the black dot lines in the figure beside. Thereafter, the teacher supports the students to represent the equivalent fractions in the form of bars. In this discussion, the teacher needs to strengthen students' understanding of the equivalence of fractions by encouraging them to notice the pattern in finding the equivalence of fractions, that the denominator and the numerator have to be the same multiple of the initial fraction.

Then, the teacher gives some problems in which the students need to find the equivalence of fractions by using a bar individually (Activity 2 Worksheet 2).



The possibility of students' strategies in finding the equivalent fraction

a) Students partition the bar so that it has the same magnitude with the given bars.
b) Students find a pattern, that the denominator and the numerator have to have the same multiple as the initial fraction.

2. Meeting 2

a. Students' starting points

1) Students have learned about partitioning, the notation of fractions, and the equivalence of fractions in the first meeting.
2) Students can compare fractions by using cross multiplication procedure.

b. Goals

1) Students can compare fractions.
2) Students are able to estimate the sum of two fractions by using benchmarks.

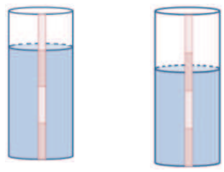
c. Description of the activities

1) Activity 1: What parts of the tube is it?

In this activity, students are asked to label the fractions of given fractions and to represent given fractions in the bars. The intention of this activity is to strengthen students' insight of partitioning and of how to represent fractions in the forms of bars. Moreover, after doing this activity, it is expected students use bars or their mental image of fractions to compare fractions.

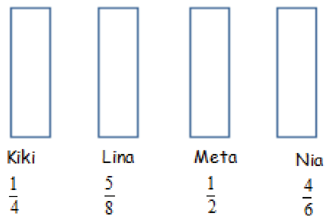
<u>First problem</u>

To determine which child has more water, Mr. Doni and his friends need to pour the water in each bucket in the tube and record what part of the tube that has been fulfilled. Now, we are going to help Mr. Doni and his friends to record what parts of the tube that each child has fulfilled. Rudi and Zacky participate in this competition. They are waiting for the announcement of the winner. The picture in the worksheet is the water from each of their buckets. What parts of the tube has their water fulfilled respectively?



<u>Second problem</u>

Can you show where the mark of the water?

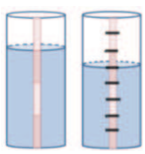


The possibilities of students' strategies in solving the first problem:

a) Students measure the height of the water by ruler and then convert it to fractions.

b) Students use paper strips and fold it such that it fits the height of the water.

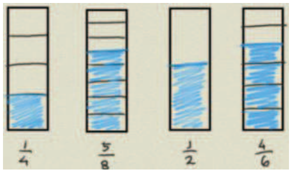
c) Students draw other lines that indicate eighth, sixteenth, third, sixth, etc., such that it fits the height of water.



The possibilities of students' strategies in solving the second problem:

a) Students measure the height of the tube by ruler and then find the corresponding height of the given fractions.

b) Students draw lines that indicate eighth, sixteenth, third, sixth, etc., such that it fits with the given fractions, as the figure below.



The teacher's reactions toward the students' strategy:

a) If the students measure the height of the water by using a ruler and then convert it to fractions, the teacher can ask how they convert it into fractions.

b) If the students make measuring strips by folding the paper strips, the teacher supports them and asks them to present how they did it in pictures.

c) If the students draw other lines that indicate eighth, sixteenth, third, sixth, etc., such that it fits the height of water, the teacher can ask them how to name the fractions.

d) If there are students who cannot name and label the fractions, the teacher can guide the students and remind them about the measuring strips.

2) Activity 2: Who will be the winner?

In this activity, students are engaged to compare fractions. The idea of comparing fractions is useful for students to grasp the idea of estimation of the sum of two fractions by using benchmarks. For instance, once students can compare and know that $\frac{2}{3}$ is more than a half, they will get that the sum of $\frac{2}{3}$ and $\frac{1}{4}$ is more than a half.

The problem in this activity (Activity 2 Worksheet 2) is:

After recording the water of three participants, the jury comes up with this result. Help the jury to determine the winner of each stage.

Stage	***Name***	***The parts of the tube of the water***	***Winner***
1	*Adi*	$\frac{1}{3}$	*?*
	Budi	$\frac{1}{5}$	
2	*Ucok*	$\frac{3}{4}$	*?*
	Doni	$\frac{3}{8}$	
3	*Edo*	$\frac{5}{6}$	*?*
	Fadil	$\frac{7}{8}$	

Conjectures of students' thinking in determining the winner

a) Students draw a bar (vertically or horizontally) and partition it as the fractions in the problem, and then compare or order it. If the students are going to use bar to solve the problem, the teacher can provide some bars in a paper such that it will result more precise picture.

The teacher's reaction:

The teacher can support their reasoning, and encourage them to consider the benchmark. For instance, in comparing $\frac{5}{6}$ and $\frac{7}{8}$, the teacher can ask

'*Which fraction is the nearest to be a whole?*'. Then, the teacher can pose more problems such as '*without drawing, which one is bigger,* $\frac{3}{8}$ *or* $\frac{4}{7}$*?*' In this case, the teacher encourages students to use a half as the benchmark.

b) Students might use the idea of a common denominator to compare or order the fractions.

<u>The teacher's reaction:</u>

The teacher asks their reasoning why they solve that way and asks them to represent their solution in a bar. For instance, by questioning '*why do you think that* $\frac{3}{4}$ *is greater than* $\frac{3}{8}$*? Can you draw that position in the bar?*'

c) Students might reason by using benchmarks and without drawing, for example, they know $\frac{7}{8}$ is greater than $\frac{5}{6}$ since $\frac{7}{8}$ needs $\frac{1}{8}$ to be a whole, and $\frac{5}{6}$ needs $\frac{1}{6}$ to be a whole. They know that $\frac{1}{8}$ is smaller than $\frac{1}{6}$, so they conclude that $\frac{7}{8}$ is greater than $\frac{5}{6}$.

<u>The teacher's reaction:</u>

The teacher can support their reasoning and pose more problems, such as '*which one is bigger,* $\frac{8}{9}$ *or* $\frac{8}{10}$*?* $\frac{6}{7}$ *or* $\frac{7}{8}$*?* $\frac{2}{5}$ *or* $\frac{3}{7}$*?*'.

d) Students apply the procedure cross multiplication to compare the fractions.

<u>The teacher's reaction:</u>

The teacher needs to engage students to use their fraction sense or pictures to compare fractions, for example by asking '*If you get* $\frac{1}{4}$ *of a cake and your brother gets* $\frac{1}{3}$ *of a cake, which one does get bigger parts of a cake?*'

Class discussion

In this discussion, the teacher points how to represent fractions in a bar so that students have a mental image for fractions and know the relative size of fractions in a bar. This mental image for fractions is very important for students in comparing fractions with benchmarks.

In comparing the fractions, some of the students might use a bar to represent the fractions, and the others might reason formally by using common denominators in solving the problems. In this discussion, the teacher encourages and supports the students to reason by using benchmarks, such as $\frac{1}{4}$ and $\frac{1}{2}$, to compare fractions. The knowledge about comparing fractions with benchmark will be useful for them to estimate the sum of two fractions in the next meeting.

3) Activity 3: Estimating the sum of two fractions

In this activity, firstly students are asked to determine fractions that are more than a half. This exercise is aimed at familiarizing the students to the relative size of fractions compared to a half as the benchmark and at helping students to use their fraction sense in estimating the sum of two fractions. For instance, when the students know that $\frac{2}{3}$ is more than a half, it is expected that they know that if $\frac{2}{3}$ is added with any fractions, the result must be more than a half.

The estimation skill is very useful for students to lead them to be aware that they cannot apply the procedure ‘top+top over bottom+bottom’ in adding fractions. As an example, when they know that the result of $\frac{2}{3} + \frac{1}{4}$ is more than a half, so they will not answer $\frac{3}{7}$ as the result because $\frac{3}{7}$ is less than a half.

The problems in this activity (Activity 3 Worksheet 2):

Circle the fractions which are more than $\frac{1}{2}$, and explain your strategy

$\frac{2}{3}$	$\frac{3}{7}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{4}{9}$	$\frac{3}{4}$
$\frac{3}{10}$	$\frac{5}{8}$	$\frac{5}{12}$	$\frac{4}{7}$	$\frac{5}{6}$	$\frac{1}{4}$

Answer these questions and explain your strategy!

1. $\frac{1}{5} + \frac{3}{4}$
   a. Is the result more or less than $\frac{1}{2}$? ……………………………
   b. Is the result more or less than 1? ……………………………

2. $\frac{3}{5} + \frac{1}{3}$
   a. Is the result more or less than $\frac{1}{2}$? …………………………
   b. Is the result more or less than 1? …………………………

After the students discuss about how to estimate the sum of two fractions, the teacher provides the application of the estimation skill in the word problem as below.

Mrs. Dewi will participate in the cooking competition. At home, she prepares the ingredients such as milk and coconut milk. She has two cans of milk which $\frac{2}{3}$ *and* $\frac{1}{4}$ *parts of it is filled respectively. If she pours the milk in the two cans into an empty can, is the total more or less than a half of can?*

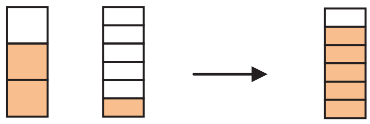
Conjectures of students' thinking in determining which fractions are more than a half:

a) Students might use bar to figure out the problems.

b) Students might reason by using the equivalence of fractions. For example, they argue that $\frac{5}{6}$ is more than a half because a half in sixth is $\frac{3}{6}$, so $\frac{5}{6}$ is more than a half.

Conjectures of students' thinking in estimating the sum of two fractions:

a) Students might use bar to figure out the problems, for example:



So, the result is more than a half.

<u>The teacher's reaction:</u>

The teacher encourages students to use benchmarks in solving the problem, which are asking them to compare the fractions, $\frac{2}{3}$ and $\frac{1}{4}$, to a half.

b) Students might reason by using benchmarks. For example, they argue that the result of $\frac{2}{3} + \frac{1}{6}$ must be more than a half because $\frac{2}{3}$ is more than a half.

<u>The teacher's reaction:</u>

The teacher supports their reasoning and asks follow up questions, for instance '*If Mr. Doni pours $\frac{3}{5}$ of a can of milk and $\frac{1}{3}$ of a can of milk, is it enough to get a half can?*'

c) Students find the exact result by finding the common denominator and then see whether the result is less or more than a half.

<u>The teacher's reaction:</u>

The teacher asks the students to explain their reasoning and represent it in a bar. Then, the teacher encourages them to use benchmarks in solving the problem, which are asking them to compare the fractions, $\frac{2}{3}$ and $\frac{1}{4}$, to a half.

Class discussion

To support students in determining which fractions are more than a half, the teacher may show the measuring strips they made in the previous meeting in order to encourage the students' mental image of the size of fractions. Moreover, the teacher also may encourage the students to use the idea of equivalent fractions to determine a half of the denominator.

In estimating, the teacher needs to support the students to use benchmarks, such as a half and one. When the students are able to estimate the sum of two fractions by using a benchmark, they will realize that they cannot do 'top+top over bottom+bottom'. To put more emphasize on it, the teacher can show the measuring strips and show that the procedure 'top+top over bottom+bottom' is incorrect.

3. Meeting 3

a. Students' starting points

In the previous meetings, students have learned about the notation of fractions, representing the size of fractions in a bar, equivalence of fractions by using measuring strips and a bar, and comparing and ordering fractions. Moreover, most of students know how to add fractions with the same denominator.

b. Goal

1) Students know that the procedure 'top+top over bottom+bottom' is incorrect.
2) Students are able to find common denominator by using paper strips and bar model.
3) Students are able to add fractions by using bar model.

c. Description of the activities

1) Activity 1: Adding Fractions by using paper strips

In this activity, students will experience the measuring activity. They measure what parts of tube filled with water before and after being poured by using paper strips. The measuring activity will support students' reasoning of the result of the addition of fractions. In the measuring activity, students learn how to make smaller measurements by using paper strips. The need of smaller measurement leads students to the concept of the equivalence of fractions. Then, the measuring activity and the concept of the equivalence of fractions support students to understand the addition of fractions. For example, when they measure the result of pouring $\frac{1}{2}$ and $\frac{1}{3}$ of a tube of water, they cannot use either the two- or three-partitioned paper strips. They need paper strips with smaller measurements (smaller partitions) that can represent the common denominator of the result.

Moreover, by exploring how to add fractions by using paper strips, it is expected that students grasp the reasoning behind the idea of common denominator by noticing the relation between the two fractions as in the idea of the equivalent fractions.

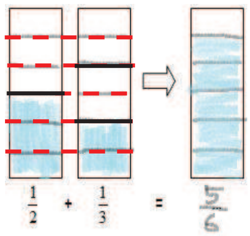
In the beginning of the activity, the teacher tells a new story that is still related to the previous story:

Instead of many competitions for children, there are also competitions for adults such as cooking competition. This year, the theme of the cooking competition is making pudding as creative as possible. Mrs. Dewi will participate in this competition. At home, she prepares the ingredients such as sugar, milk, coconut milk, etc. She needs a half can of milk. She remembered that she had left over two cans of milk. If Mrs. Dewi pours two cans of coconut milk that contain $\frac{1}{2}$ *and* $\frac{1}{4}$ *of a can respectively, what parts of can will be filled?*

In this activity, firstly the teacher demonstrates pouring two tubes of water that $\frac{1}{2}$ and $\frac{1}{4}$ parts of it filled with water respectively. Then, the teacher engages the students to guess what parts of the tube filled with water in total. Thereafter, the teacher engages the students to represent the process of adding $\frac{1}{2}$ and $\frac{1}{4}$ in the paper strips. After that, the teacher provides two tubes that $\frac{1}{2}$ and $\frac{1}{3}$ parts of it filled with water to each group of 2-3 students. The students experience in pouring the water and measure the water filling the tube after being poured by using paper strips. Then, the students explore why they can find such result by using the paper strips.

Conjectures of students' thinking in adding fractions in the paper strips:

a) Students partition the two paper strips representing the fractions being added into a number of parts that fit to both fractions. They might use the idea of common extension lines as they learned in the previous meeting to find the common number of partitions.



The teacher's reaction:

The teacher supports their understanding and encourages their reasoning, for instance by asking '*'Why did you divide the bar into eight (or twelve, etc.)?'* The teacher also can ask their reasoning how to find the exact result of the first problem.

b) Students might solve it in formal ways, which students add the fractions by finding the common denominator, and then representing the result in the paper strips

The teacher's reaction:

The teacher encourages students to represent and explain their reasoning in the form of bars.

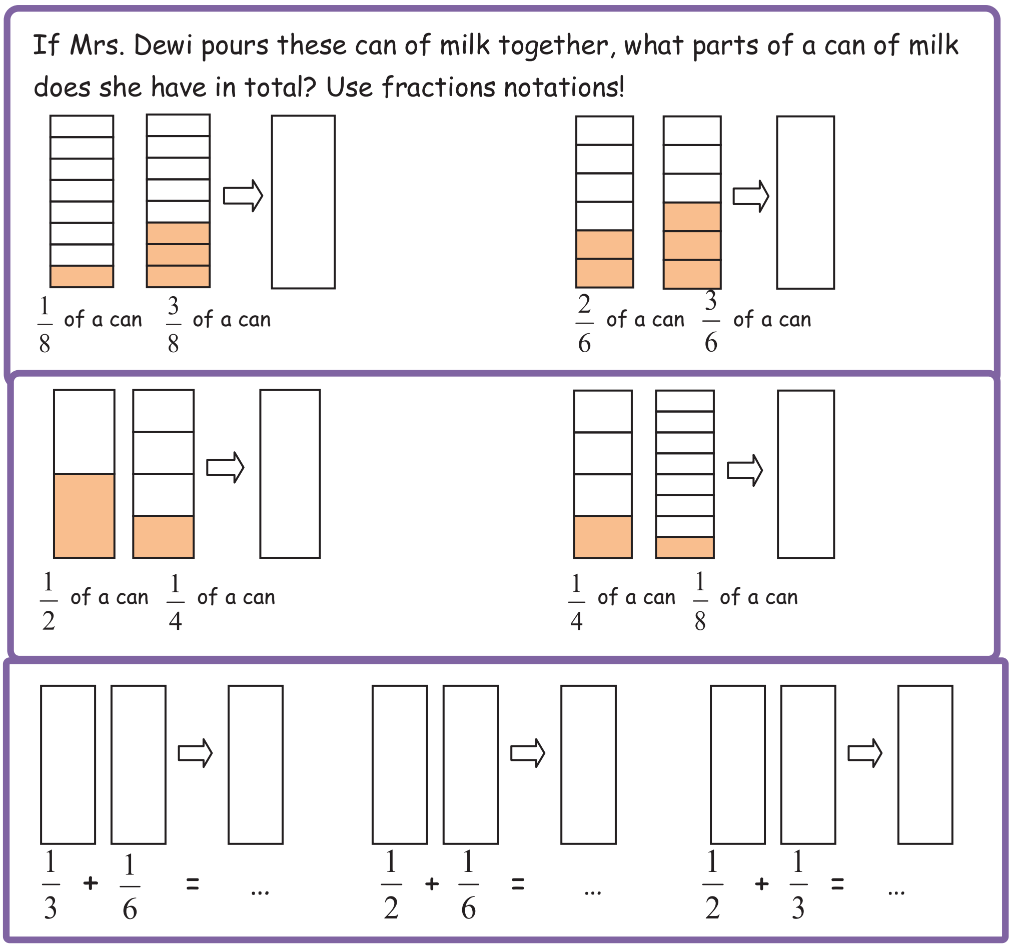
Class Discussion

In the discussion, the teacher engages the students to notice how to find the common number of partitions representing the common denominator and the reasoning of common denominator.

2) Activity 2: Adding Fractions by using bar model

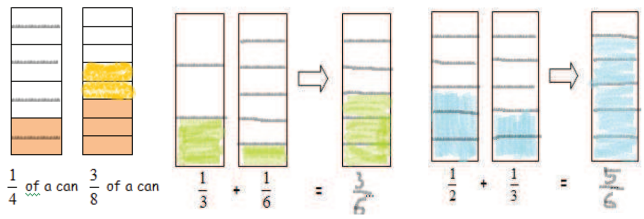
After students learn the idea of common denominator by using paper strips, the students are engaged to translate it into the bar model. It is expected that the students are able to translate what they notice in the paper strips in adding fractions into the bar model. These problems are also aimed at strengthening their understanding in adding fractions by using bars. By leaving the bar unpartitioned, the teacher can see the extent to which they understand the idea of common denominator and solve the problems in the bars.

The problems in this activity (Activity 2 of Worksheet 3) are:



Conjectures of students' thinking:

a) Students partition the bars into a number of parts that fit to both fractions. The find the common number of partitions by extending the lines of the bars as they do in the paper strips.



<u>The teacher's reaction:</u>

The teacher supports their understanding and encourages their reasoning, for instance by asking '*'Why did you divide the bar into eight (or twelve, etc.)?'* The teacher also can ask their reasoning how to find the exact result of the first problem.

b) Students might solve it in formal ways, which students add the fractions by finding the common denominator.

<u>The teacher's reaction:</u>

The teacher encourages students to represent and explain their reasoning in a bar.

3) Activity 3: Reviewing the estimation of the sum of two fractions

In last activity, the students review the estimation problem as in the second meeting. The intention of this activity is to strengthen students' understanding of the estimation of the sum of two fractions and to emphasize that the output of procedure 'top+top over bottom+bottom' is not reasonable.

As an example, when they know that the result of $\frac{2}{3} + \frac{1}{4}$ is more than a half, they will not answer $\frac{3}{7}$ as the result because $\frac{3}{7}$ is less than a half. The conjectures of students' answer are the same with the conjectures in the second meeting.

In the class discussion, the teacher should stimulate and strengthen students' reasoning in estimating the sum of two fractions by using a half as the benchmark. Moreover, the teacher also points that the procedure 'top+top and bottom+bottom' is not reasonable. To put more emphasize on it, the teacher can show the measuring strips and show that the procedure 'top+top over bottom+bottom' is incorrect.

CHAPTER V
RETROSPECTIVE ANALYSIS

In the retrospective analysis, the researcher contrasts the conjectures of students' learning in the Hypothetical Learning Trajectory (HLT) to the actual students' learning. Thereafter, the researcher uses the result of this analysis to reformulate and develop the existing HLT.

In the following sections, the researcher elaborates the retrospective analysis of the teaching experiment during the three cycles. At the end of the analysis of each meeting, the researcher describes the summary of the remarks of the activities and worksheets that were used to revise and improve the HLT and the worksheets for the next cycle. Later, the result of the analysis of the third cycle is used to answer the research questions.

This study is conducted in around a month, from 25 February 2014 untill 22 March 2014. There are three cycles of teaching experiments. Each cycle is conducted in three meetings. In the first cycle, five students participate in the lessons, namely Tya, Samuel, Diva, Nabil, Nanda. The participants of the second and the third cycle are the students of class 3B and 3A respectively.

In the second and third cycle, before conducting the lessons and during the lessons, the teacher and the researcher have a discussion about the teacher guide, those are about the learning activities, students's strategies, the teacher's role, and what to be stressed in the learning process. Before and after the teaching experiment, the students get a pre-test and post-test respectively. The problems of the pre-test and post-test, and the worksheets used in the teaching experiment can be found in appendix 3 and 4 respectively.

A. Pre-test

The result of the pre-test in the three cycles is similar. Perhaps, it is because the students have the similar heterogeneous of students' ability and because they get the same materials in the curriculum. The following description provides the summary of the result of the pre-tests in the three cycles.

1. The concept of fractions

All students in this class know how to represent fractions in pictures and how to label the fractions of given pictures. They seem to understand the concept of partitioning, that they need to partition the picture into equal parts to represent fractions. Moreover, they understand that the denominator corresponds to the number of partitions and the numerator represents the number of shaded parts. Most of the students have a similar solution for the first and second problem, as the figure below.

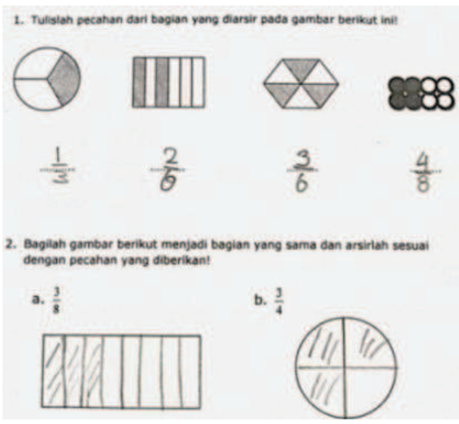
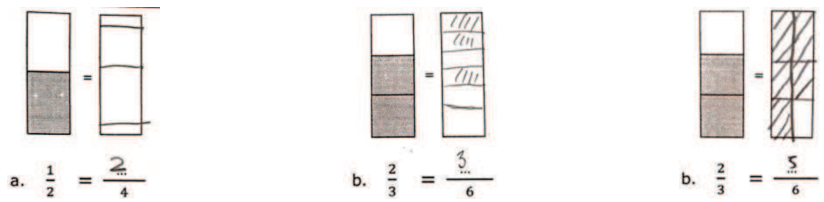


Figure 5.1. The Example of Students' Solution in the First Two Problems of Pre-Test

2. Equivalence of fractions

Only few students can find the equivalence of the given fractions, whereas most of the students cannot figure it out. The figures below are the examples of students' answer.



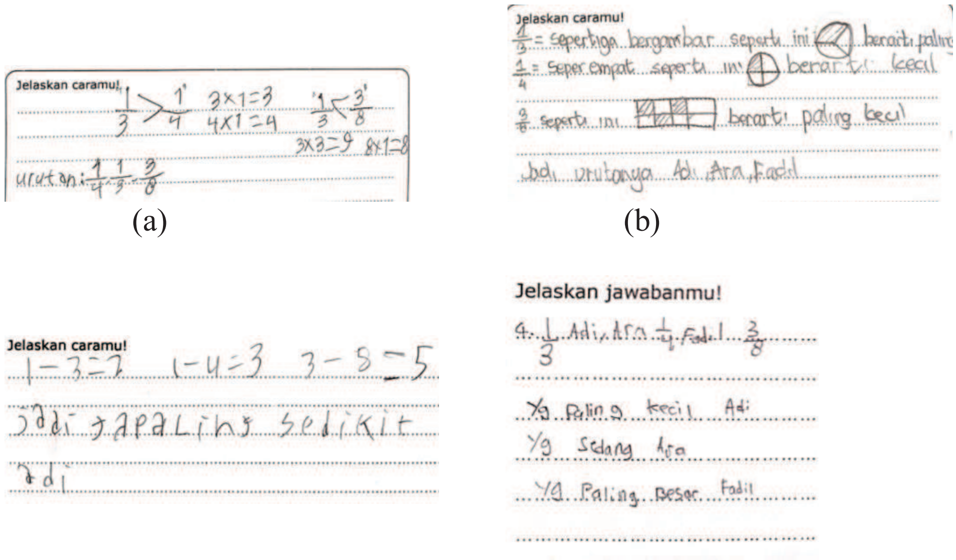
(a) (b) (c)

Figure 5.2. The Examples of Students' Work in Finding Equivalent Fractions

In Figure 5.2(a), the student is able to find the equivalent fractions of a half. However, he does not partition the bar into equal parts. It seems that he has not grasped the idea of partitioning yet, that each partition has to be equal in size. From Figure 5.2(b) and 5.2(c), it can be seen that the students seem have no sense to find the equivalent fractions of the given pictures. Possibly, it is because they do not know what the meaning of 'equivalent fractions' is.

3. Comparison of fractions

A Few students can solve the problem about ordering $\frac{1}{3}, \frac{1}{4}$, and $\frac{3}{8}$. Some of them compare each two fractions by using cross multiplication procedure (Figure 5.3(a)), while the other solve it by using common denominator. However, most of them cannot explain why the procedure works. In the procedure cross multiplication, the students multiply the denominator of one fraction with the numerator of the other fractions. As illustrated in the Figure 5.3(a), a fraction is bigger than another fraction if the multiplication result is bigger. On the other side, most of the students cannot compare fractions. Some of them try to draw pictures of corresponding fractions, yet the shape or the size of each picture is different. In this case, students still do not get about the idea of the unit of fractions, that to compare fractions, the unit has to have equal size and shape. Whereas, the other students just try any procedure they can (Figure 5.3(b) and Figure 5.3(c)).



(a) (b) (c) (d)

Figure 5.3. Students' Strategies in Comparing Fractions

In addition, there are some students who argue that the order of fractions from the greatest to the smallest is $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{3}{8}$ because they just look at the order of the fractions in the problem, as in the Figure 5.3(d) above. They argue this way: *since* $\frac{1}{3}$ *is the first written, so* $\frac{1}{3}$ *is the smallest fraction;* $\frac{1}{4}$ *is written in the middle, so* $\frac{1}{4}$ *is greater than* $\frac{1}{3}$ *and less than* $\frac{3}{8}$*;* $\frac{3}{8}$ *is the last written, so* $\frac{3}{8}$ *is the greatest fraction.*

4. Estimation of the sum of two fractions

In estimating the sum of two fractions, many students firstly add the fractions and then compare it to a half as the benchmark. Not many of them add the fractions by using the idea of common denominator, and the others do 'top+top over bottom+bottom'. In this case, students do not estimate the sum of the fractions by using their fraction sense or by using benchmarks. Moreover, the idea of the estimation to avoid the procedure 'top+top over bottom+bottom' does not appear since they add the fractions without estimating it. Moreover, they seem cannot compare it to a half. Below is the figure of the example of students' answer when they are asked to estimate the result of $\frac{2}{3}+\frac{1}{4}$.

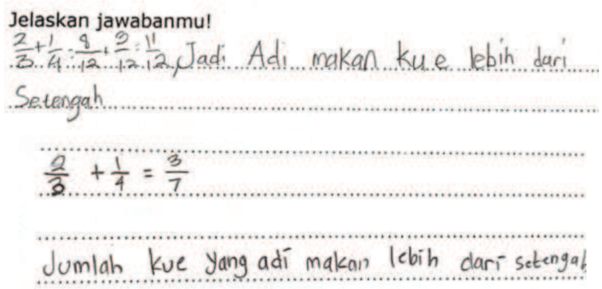
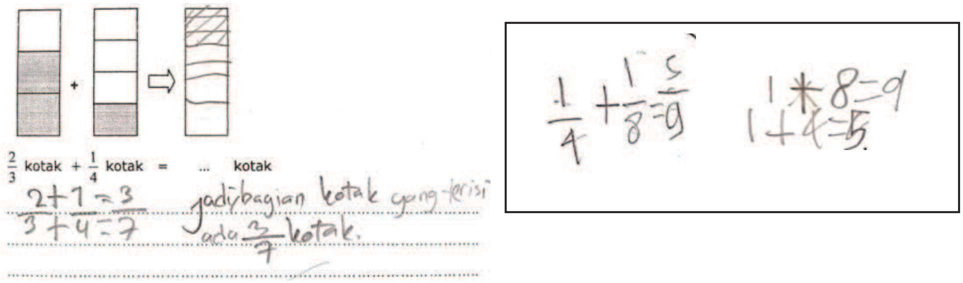


Figure 5.4. Examples of Students' Answer in Estimating the Result of $\frac{2}{3}+\frac{1}{4}$

5. Addition of fractions

In adding fractions of the same denominator, most of the students are able to solve it and represent it in the form of bars. However, almost all the students do not get the idea of how to add fractions with different denominators. As have been documented by many researchers, most of students apply the procedure 'top+top over bottom+bottom' in adding fractions, while other students apply a unique procedure as in the Figure 5.5(b).

In this unique procedure, the students use their knowledge about cross multiplication in comparing fractions. However, since the problem is about addition of fractions, they do cross addition. In this case, they still have no idea about the common denominator, especially when they add fractions with different denominators.



(a) (b)

Figure 5.5. Examples of Students' Solutions in Adding Fractions

Discussion of the Result of Pre-Test

Based on the curriculum, these students have learned about the concept of fractions, about how to compare unit fractions, and how to compare fractions with the same denominators. In line with what they have learned, the result of the pre-test above indicates that students are able to label the fractions of given pictures and represent fractions in a regional model. However, many students still do not get the idea of equivalent fractions and its representation in the bar.

In comparing fractions, only some students are able to compare fractions by using cross multiplication procedure and the common denominator. However, those students know the procedure without understanding the reasoning behind it. Moreover, some of the students are not aware of the concept of the unit in comparing fractions, in which they draw pictures with different sizes or different shapes.

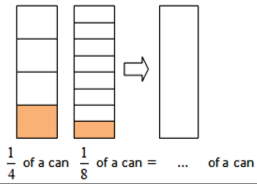
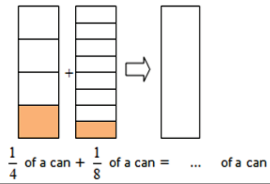
Furthermore, they do not know how to estimate the sum of two fractions by considering the benchmarks and how to add fractions with different denominators. In adding fractions, some students apply incorrect procedures, such as 'top+top over bottom+bottom' and cross addition.

Regarding the readability and the understandable of the problems in the first cycle, all the questions are clear enough for the students. However, in the problem number 6, the students do not know what to do with the bar, thus the researcher put the symbol '+' in the problem number 6 for the second cycle.

In the second cycle, some students do not know the meaning of the problem number 6 although there are already the symbol '+' between the two bars and between the fractions being added. Perhaps, it is because the students have not ever known yet how to use bars in adding fractions. Therefore, in the third cycle, the teacher gives more explanation about what to do with this problem. Moreover, some students are confused with the word '*total*' in the problem number 5 because some of them may not get used to hear the word. Thus, the researcher changes it into the word '*jumlah*' in the third cycle.

In the third cycle, the students seemed to understand all the questions, but still the teacher needs to guide the students in understanding the questions. The table below summarizes the improvement of pre-test problem during the three cycles.

Table 5.1 The Summary of the Changes Made in the Pre-test during the Three Cycles

Cycle	Improvement of Pre-test
Cycle 1	Problem number 6 (Addition of fractions in the bar) Put the symbol '+' in the problems so that students know what the meaning of the problems and what to do with the problems. The problem: *Mrs. Doni has* $\frac{1}{4}$ *and* $\frac{1}{8}$ *of a can of coconut milk. If she pours it together in one can, what part of the can will be filled?* Before the improvement  After the improvement 
Cycle 2	Problem number 5 (Estimating the sum of two fractions) Change the word '*total*' with the word '*jumlah*'. The problem before the improvement: *Sebelum berangkat sekolah, Adi memakan* $\frac{2}{3}$ *bagian kue. Sepulang sekolah, ia makan lagi* $\frac{1}{4}$ *bagian. Apakah total kue yang Adi makan lebih atau kurang dari setengah?* The problem after the improvement: *Sebelum berangkat sekolah, Adi memakan* $\frac{2}{3}$ *bagian kue. Sepulang sekolah, ia makan lagi* $\frac{1}{4}$ *bagian. Apakah jumlah kue yang Adi makan lebih atau kurang dari setengah?*

B. Meeting 1

At this meeting, students do two activities, those are fair sharing and producing measuring strips. The aims of the activities at this meeting are (1) to partition into equal parts, (2) to understand the notation of fractions, and (3) to understand the idea of equivalent fractions. At the beginning of the activities, the researcher, as the teacher, gives an introduction of the problem and then tells the fair sharing problem to the students. Then, the students work in groups of 2-3 students to solve the problems. After the students finish in solving the fair sharing problem, the teacher orchestrates a discussion. In the second activity, the teacher engages students to make their own measuring strips. Then, the students together with the teacher discuss about the equivalence of fractions by using the measuring strips they made. At the end of the lesson, the students are given a set of problems about the equivalence of fractions as the individual exercise.

1. First Activity

In this activity, the teacher gives a problem, in which students in group of 2-3 students have to determine which group gets bigger share of bread if group '*bakiak*' gets 2 pieces of bread for 3 children, and group '*tarik tambang'* gets 3 pieces of bread for 4 children. In dividing the bread, the researcher expects that students will either divide the bread as many as the number of children in each group, or firstly halve the bread and then divide the remaining parts as many as the number of children in each group. In comparing fractions, the researcher conjectures that students come up with fractions notation and then use their fraction sense to compare the fractions, use the idea of common denominator, or use their fraction sense.

a. First Cycle

Before dividing the picture of bread in the worksheet, the teacher engages students to think about which group gets bigger share. The fragment below shows the discussion between the teacher and the students in determining the group that will get bigger share.

The teacher : What do you think? Which group will get bigger parts? (*While showing the picture of the competitions and notes regarding the number of bread and the number of children in each group*).
Tya and Diva : This one (*pointing the group 'tarik tambang'*)
The teacher : Why?
Tya : I do not know (*smiling and getting confused)*
Samuel : I know, I know.
The teacher : Which group will get bigger share? (*Looking at Samuel*)
Samuel : The group '*tarik tambang'* because $\frac{2}{3}$ is more.. $\frac{3}{4}$ is greater than $\frac{2}{3}$.
The teacher : Why ($\frac{3}{4}$ is greater than $\frac{2}{3}$)? How did you get $\frac{3}{4}$? (*looking at all students)*
Students : Smile and look confused.

In the discussion above, Tya, Diva, and Sam think that children in the group '*tarik tambang'* get bigger share. However, they have different reasons. Tya and Diva argue that children in the group '*tarik tambang'* get bigger share but they do not know the reason, whereas Samuel already uses the fractions notation. He gets an idea that a fraction is a division. In this case, he uses the interpretation fractions as a quotient, which some quantities are divided or shared among some people. Moreover, he seems to use fraction sense in determining that $\frac{3}{4}$ is greater than $\frac{2}{3}$. However, he still does not know to explain it.

In dividing the picture of the bread in the worksheet, the conjectures stated in the HLT appear. Diva and Tya divide each piece of bread as many as the number of children, while Samuel and Nanda divide the bread by firstly halving each piece of bread and then divide the left over as many as the number of the children in each group. In comparing which group gets bigger share, Tya and Diva use ruler, while Nanda and Samuel can see obviously from their picture that children in the group '*tarik tambang*' get bigger share than children in the group *'bakiak'*. The figures below are the examples of their works in dividing the bread in the worksheet.

Nanda's and Samuel's work Tya's and Diva's work

Figure 5.6. Various Students' Answers in Dividing the Bread

In the discussion, by showing Nanda's and Samuel's work, the teacher engages the students to use fractions notation. It is easy for them to know that each child in the group '*tarik tambang'* gets a half and a quarter of a piece of bread. However, the students look confused in determining what parts of a piece of bread that each child in the group 'bakiak' gets. They get difficulties in determining a third of a half. In the first time, they think that each child in the group '*bakiak'* gets a half and a third of a piece of bread because the last half of the bread is divided into 3 parts. After the teacher reminds the students how a third of a piece of bread looks like, they are aware that each child in the group '*bakiak'* gets a half and a sixth of a piece of bread because the bread should be divided into equal parts.

Regarding the worksheet, students do not find any difficulties to understand the problem in the worksheet. However, it takes long time when students divide the pictures of bread by cutting and pasting the bread by using scissors and glue. Therefore, the researcher leaves out the hint to use scissors and glue on the worksheet.

b. Second Cycle

As have been expected in the HLT, most of the students divide the picture of each piece of the bread as many as the number of children in each group, including Krishna and his friends. Krishna, Mazta, and Satria try to measure the length of each piece of the bread and then divide it as many as the number of children in each group. After they finish dividing the bread, they try to figure out what parts of a piece of bread that each child in each group gets so that they can compare the fractions of both groups. The transcript below shows the students' struggle in finding the fractions of the parts of a piece of bread that each child gets in each group.

Krishna	: This one is three *(pointing the parts which each child gets in the group 'tarik tambang')*, overall there are four of three parts (*pointing the bread for the group 'tarik tambang'*)
Satria	: So, this one is three over what? (*pointing the fractions for the group 'tarik tambang')*
Mazta	: Eh Kris, how can it be $\frac{2}{6}$? It should be $\frac{2}{3}$ or two of three, isn't it? *(pointing 2 bread for 3 children in group 'bakiak')*
Krishna	: $\frac{2}{3}$?
Mazta	: Yes. It is $\frac{2}{3}$ or $\frac{2}{6}$?
Krishna	: There are six parts, aren't there? And every child gets 2 parts, so it is 2/6 *(pointing the parts of bread for each child in the group 'bakiak')*.
Mazta	: *(looks confused)*, over three or six? Oh, I don't know.

In the discussion between Mazta and Krishna above, Krishna looks the six parts of the bread as a whole. Since each child gets two parts, so Krishna thinks that each child gets two parts over six parts. Krishna's thought is correct when he thinks that the fraction is $\frac{2}{6}$ because each child gets two parts over six parts. However, he misinterprets what the whole is. In this problem, six parts should not be interpreted as a unit, but it should be two units because there are two different pieces of bread. Whereas Mazta, he considers the two pieces of bread as a unit and these two pieces of bread will be divided among 3 children. In this case, Mazta has an insight about fractions as a quotient, which some pieces of bread are shared among some children. Mazta are aware that a fraction is a division. The figure below presents Krishna's and his friends' work.

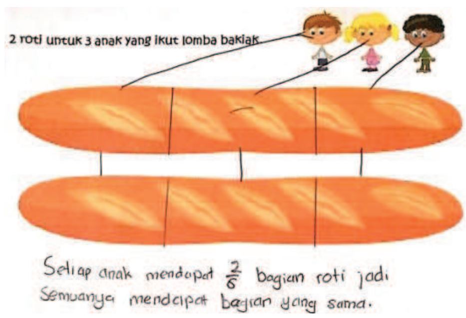
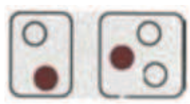


Figure 5.7. Krishna's and His friends' Solution of the First Activity

As has been documented by Howard (1991), students' misinterpretation of the concept of a whole or a unit can lead them to an incorrect concept of the operation of fractions. For instance, if the teacher asks what fraction each set is shaded in the figure beside, the students will answer $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If the teacher



asks students the fractions after they combine these two sets, the students might answer that the result is $\frac{2}{5}$. Therefore, the teacher needs to pay attention on the concept of a whole in teaching fractions. Moreover, the teacher needs to be careful in choosing a model in teaching fractions. To grasp the concept of the addition of fractions, the set model does not seem appropriate to be used since the students will not see the reasoning behind the concept of common denominator. The students may not be aware that in adding fractions, the unit must be the same (size or number), and thus it can lead students to do 'top+top over bottom+bottom' in adding fractions. Therefore, the researcher suggests that in teaching the addition of fractions, teachers should use models that can show apparently what the unit is, for instance bar model and fractions circle.

There are other groups' solutions in determining the fractions of each group besides the solution of Krishna's group. Emma's group argues that each child in the group '*bakiak'* gets $\frac{2}{3}$ parts of a piece of bread because each child gets two parts, which each part is $\frac{1}{3}$ of a piece of bread. They understand that the two pieces of bread are two different units, so they add $\frac{1}{3}$ of the first piece of bread and $\frac{1}{3}$ of the other piece of bread. Moreover, they also know how to add fractions with the same denominator. Below is the figure of the solution of Emma's group.

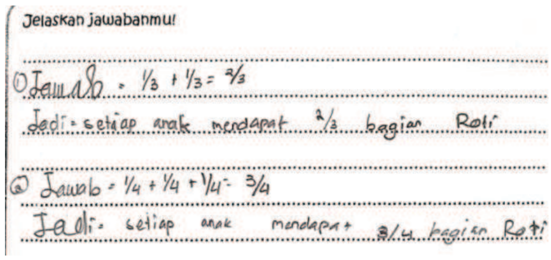
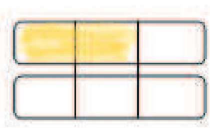


Figure 5.8. The Solution of Emma's Group of the Fair Sharing Problem

In determining which parts of bread is bigger, some students apply cross multiplication to compare the fractions. In the discussion, the teacher tries to engage students who compare the fractions by using cross multiplication and the idea of common denominator to compare the fractions by using pictures. By doing this, it is expected that students have a mental image of the size of fractions.

For the improvement of this activity in the next cycle, the researcher puts more questions in the teacher guide to support students in understanding the concept of the unit. For instance, in the problem 2 pieces of bread are shared among 3 children. If the students argue that each child will get $\frac{2}{6}$ parts of a piece of bread since there are 6 parts in total. In this case, the teacher can ask them:

'*look at the picture beside. Each child will get 2 parts, isn't it? So, what parts of* ***A PIECE OF BREAD*** *is it?'*



(The teacher puts an emphasize on 'a piece of bread' as a whole)

Moreover, the researcher also puts more questions to guide the students of how to compare fractions so that they do not use cross multiplication algorithm without knowing the reasoning. For example, '*without using the algorithm, you can use pictures to determine which one is bigger between* $\frac{2}{3}$ *and* $\frac{3}{4}$'.

Regarding the worksheet, based on the observation, the interview, and the field note, the researcher finds that some students do not understand the question of the first activity, that is '*Does each child in each group get the same share of bread?,* or '*Apakah anak-anak pada lomba bakiak dan tarik tambang akan mendapatkan bagian roti yang sama besar?'.* Some students answer it by '*Yes, because the bread is divided into equal parts'.* Students do not interpret the question as a hint to compare the parts of bread that each child in each group gets. Therefore, the researcher changes the question into '*Which group will get bigger share for each child?',* or *'Grup manakah yang akan mendapatkan bagian roti lebih besar?'*.

c. Third Cycle

In this cycle, all students partition the pieces of bread as many as the number of children. In determining which group gets bigger share of bread, as have been expected in the HLT, some students use the idea of common denominator, while other students, such as Dinda and her friends, use cross multiplication and use their fraction sense.

To find out students' reasoning, the researcher asks Dinda's group about their reasoning of their answer. Below is the transcript of the discussion between the researcher and the Dinda's group.

The researcher : Children in which group that get bigger share of bread?
Ersya : Tarik tambang.
The researcher : Why? (*asking to all members of the group)*
Ersya : Because the fractions is $\frac{3}{4}$.
The researcher : How did you know that $\frac{3}{4}$ is bigger (than $\frac{2}{3}$)?
Ersya was thinking, while Dinda was doing cross multiplication to find out which is bigger between $\frac{3}{4}$ and $\frac{2}{3}$.
Dinda : $\frac{2}{3}$, group 'bakiak' gets bigger share (*telling the researcher*)
The researcher : Group 'bakiak' gets bigger share? Bakiak or tarik tambang?
Dinda : I get group 'bakiak' that gets bigger share.
The researcher : You bakiak (*pointing at Dinda)*, and you tarik tambang *(pointing at Ersya)*, so?
Ersya : That's wrong, it's $\frac{2}{3}$, is it bigger (than $\frac{3}{4}$)? *(talking to Dinda)*
Dinda : Of course.
Ersya : $\frac{3}{4}$ is bigger
Dinda : it's my opinion.
(Dinda and Ersya was arguing each other)

In the discussion, Ersya and Dinda have different opinion about which group gets bigger share. Dinda seems to miscalculate when applying cross multiplication procedure so she argues that $\frac{2}{3}$ is bigger than $\frac{3}{4}$. On the other hand, Ersya may use their mental image of the size $\frac{2}{3}$ and $\frac{3}{4}$ so that she thinks that $\frac{3}{4}$ is bigger than $\frac{2}{3}$.

As has been revealed by Kamii and Dominick (1998), the description above is evidence that applying an algorithm without knowing the reasoning lead students to do a mistake. The students do not get used to use their fraction sense in solving the problem. Moreover, the students cannot review whether the result they get is reasonable or not.

Therefore, as in the previous cycle, in the class discussion, the teacher engages the students to not use the algorithm and accustoms the students to use pictures or their fraction sense. The teacher encourages the students to use pictures to represent fractions in order to build the mental image of fractions on students' mind. Then, the teacher engages students to use their fraction sense to compare fractions.

Conclusion

Based on the description of the above, it can be seen that students grasp the idea of partitioning into equal parts and fractions notation. Moreover, in this activity, some students already notice how to compare fractions, which is not the focus of the activity. Some students apply cross multiplication, which they do not know the reasoning behind it. Thus, the teacher engages students to use pictures in comparing fractions so that students can build their fraction sense.

2. Second Activity

The focus of this activity is the idea of equivalence of fractions. In this activity, the students produce their own measuring strips by using paper strips and then use it to discuss the idea of the equivalence of fractions. Then, the students are asked to solve three problems about the equivalence of fractions and its representation in the form of bars.

In folding the paper strips into some equal partitions, the researcher conjectures that students may estimate it, use a ruler, or halve the paper strips several times. While in solving the problems about the equivalence of fractions, the researcher expects that the students partition the bar by extending the lines of the given bar so that it has the same value as the given fraction. The students also may find the pattern of equivalent fractions, that the denominator and the numerator have to be the same multiple of the fraction in the problems.

a. First cycle

In this activity, Nanda, Sam, and Nabil get a tube which $\frac{1}{3}$ part of it filled with water, and Tya and Diva get a tube which $\frac{1}{4}$ part of it filled with water. The students are asked to make two measuring strips with different partitions. Nanda's group make measuring strips with 4 and 8 partitions, while Tya's group make measuring strips with 3 and 6 partitions. As have been conjectured in the HLT, the students use the idea of halving in folding the paper strips. After they finish making two different measuring strips, they put and arrange it together on the poster paper, as the figure below.

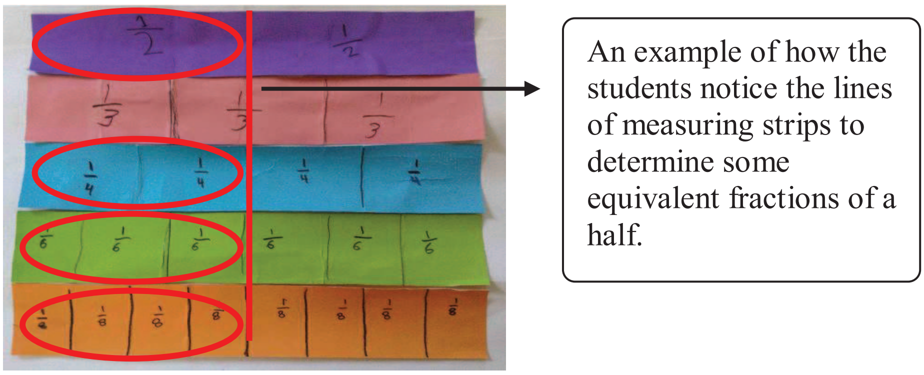


Figure 5.9. The Measuring Strips the Students Made and the Example of How They Find Some Equivalent Fractions of a Half

Afterwards, the teacher and the students discuss the equivalence of fractions by noticing the lines of each measuring strip they made as in the Figure 5.9. After engaging students to find equivalent fractions, the teacher engages the students to think about what they notice from the examples. Below is the transcript of the discussion.

The teacher	: Now look at the measuring strips. Can we represent a half with other fractions? (*showing the measuring strips that students made)*
Sam	: $\frac{2}{4}$ *(looking at the measuring strips)*
The teacher	: Is there other fractions? Look at the line (*pointing the line of a half*)
Students	: $\frac{3}{4}$, and $\frac{6}{8}$ *(looking at the measuring strips)*
The teacher	: What about $\frac{2}{3}$?
Diva	: $\frac{4}{6}$ *(looking at the line of* $\frac{2}{3}$ *in the measuring strips and extending it intothe sixth measuring strip)*
The teacher	: Let's write it down. Look at $\frac{3}{4}$ and $\frac{6}{8}$. Do you notice why 3 can be 6 and 4 can be 8? (*pointing the numerator and the denominator on* $\frac{2}{3}$ *and* $\frac{4}{6}$)
Tya	: I know I know.
The teacher	: Why? (*looking at Tya)*
Tya	: Because 3 times two is six, so the bottom must be multiplied by two too (*by pointing the number*).
The teacher	: What about this, how come a half equals to $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$? (*showing the fractions*)
Sam	: Because $\frac{4}{8}$ is 4 divided by 8… ehm, four.. is a half of eight.

As have been expected in the HLT, Tya and Samuel are able to notice the pattern of the equivalence of fractions after noticing the pattern of some equivalent fractions. Tya figures out that to get equivalent fractions, the numerator and denominator should have the same factors of multiple. For example, to get the equivalence of a fraction, if the numerator of the fraction is multiplied by 2, then the denominator also has to be multiplied by 2. Different from Tya, Samuel notices the relation between the numerator and the denominator. As can be seen in the fragment, he notices that 1 is a half of 2, so the numerators of the equivalent fractions is also a half of the denominators.

After the students get the idea of equivalence of fractions, the teacher asks them to find the equivalence of some fractions in the worksheets. The figure below shows the work of Sam.

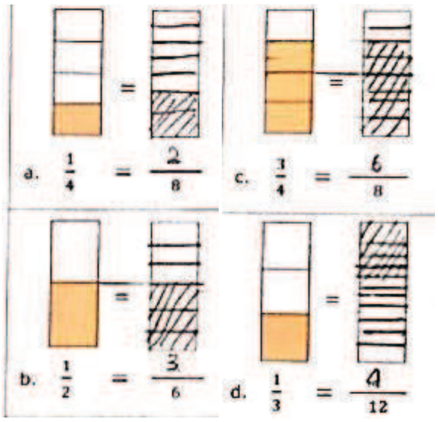


Figure 5.10. Sam's Work in Finding the Equivalence of Some Fractions

The figure above indicates that Sam starts to notice how to find the equivalent fractions by utilizing the common lines between two bars as he saw in the measuring strips before. Moreover, he begins to be aware that equivalent fractions have the same size of shaded parts.

After the students finish their work, the teacher discusses and asks them to name some equivalent fractions of $\frac{1}{5}$. The teacher poses the question to check students' understanding of equivalent fractions. Surprisingly, all students can answer it without looking at the measuring strips.

There is a note for this activity, that the teacher does not put more emphasize on how to represent equivalent fractions in the form of bars. Therefore, for the improvement of the second activity for the next cycle, the teacher needs to emphasize more on the idea of the equivalence of fractions and its representation in the form of bars. The representation of equivalent fractions in the form of bar will ease the students to get the idea of common denominator in adding fractions in the bars.

b. Second Cycle

As in the first cycle, most of the students in this cycle use the idea of halving in folding the paper. In the discussion, the teacher and the students discuss about the equivalent fractions by using measuring strips they make as in the Figure 5.11 below.

Figure 5.11. The Teacher and the Students Discuss about Equivalent Fractions

In solving the problems about the equivalent fractions, the conjectures in the HLT appear. Some students, who understand the pattern of the equivalence of fractions, draw the representation in the bars after they find the equivalent fractions. While some other students use the bar as a tool to find the equivalence of fractions, which is by extending the lines of the corresponding bars, partitioning into equal parts, and then shading the parts as many as the corresponding bars (Figure 5.12(d)).

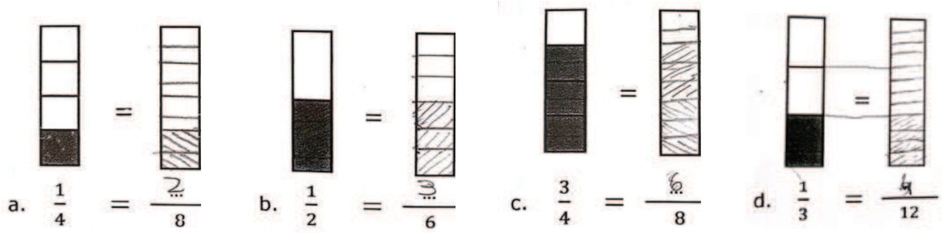


Figure 5.12. The Example of Students' Answer in Finding the Equivalent Fractions

In the Figure 5.12 above, it can be seen that students partition the bar as many as the denominator of the equivalent fractions. Then, they shade the parts so that it has the same size with the given bars.

There is a note in this cycle that the teacher does not point out the pattern of the equivalence of fractions. Thus, for the improvement of the next cycle, the researcher puts some questions in the teacher guide to support students to notice the patterns of equivalent fractions from some examples, for instance '*can you notice the relation between the numerators and the denominators between these equivalent fractions?'*.

c. Third Cycle

As in the previous cycle, the students do not find any difficulties in producing the measuring strips and all conjectures in the HLT in folding the paper strips occur. The following figure is the measuring strips they made.

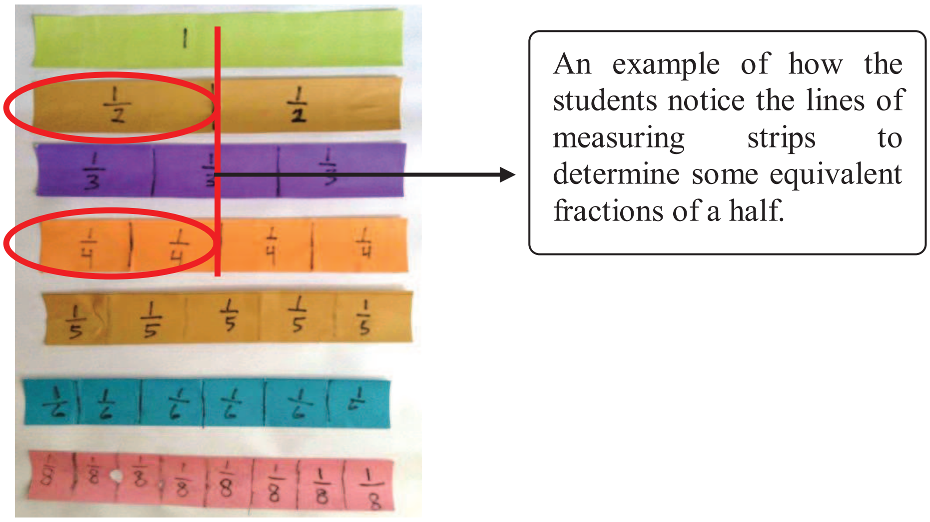


Figure 5.13. The Measuring Strips the Students Made and the Example of How They Find Some Equivalent Fractions of a Half

In the discussion about the equivalent fractions, the teacher engages the students to notice the lines of the fractions in the measuring strips as in the Figure 5.13 above. Then, the teacher gives the students a set of problems about the equivalent fractions.

As has been conjectured in the HLT, some students notice the pattern of equivalent fractions, so that they are able to solve the problems without using bars. Some other students partition the bar by extending the lines of given bars so that it has the same value as the given fractions. Their strategy is, firstly, they partition the bar by using the extension lines of the given partitioned-bar. Then, they shade the parts as many as the shaded parts in given bars. The figure below illustrates what has been explained above.

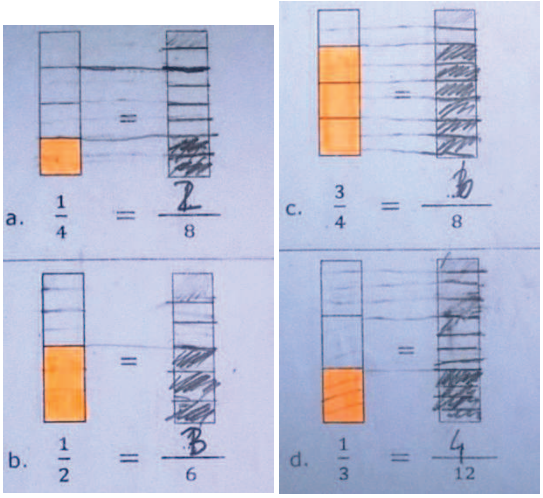


Figure 5.14. Examples of Students' Work in Finding the Equivalent Fractions

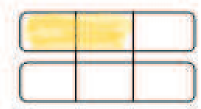
Conclusion

The description of the second activity above shows that students are able to partition the paper strips into equal parts. The fact that most of them use the idea of halving in folding the paper strips indicates that they notice the relation between, for example, 4 partitions and 8 partitions. In other words, they have an initial insight about the equivalence between fourth and eight. In addition, the students also can find the equivalence of fractions by noticing the lines in the measuring strips. Then, some students seem to notice the use of the extension lines that they observe in the measuring strips to determine the equivalent fractions in the bars. Moreover, some of them are aware of the pattern of equivalent fractions from some examples, that the numerator and the denominator of equivalent fractions are the multiple of each other.

The following table contains the summary of the refinement of the HLT and the worksheets of the first and second activity of meeting 1 during the three cycles.

Table 5.2. The Changes Made in the First and Second Activity of Meeting 1 during the Three Cycles

Cycle	Improvement of HLT and Teacher Guide	Improvement of Worksheet
Cycle 1	**First Activity** - Most of conjectures of students' thinking are in line with students' actual strategies. However, there is an addition of students' strategy in determining the parts of each child will get by using fractions notation, which is students thought that each child in the group '*bakiak'* gets $\frac{1}{2}$ and $\frac{1}{3}$ parts of a piece of bread instead of $\frac{1}{2}$ and $\frac{1}{6}$ parts of a piece of bread. In this case, the teacher can engage them to draw the representation of a third in the bar and compare it to the picture of a third of a half so that they realize that a third of a half is equal to a sixth. - When students argued that children in group '*tarik tambang'* get a bigger share than children in group '*bakiak'* because $\frac{3}{4}$ is bigger than $\frac{2}{3}$, the teacher needs to encourage students to explain why and how the students get $\frac{3}{4}$ and $\frac{2}{3}$, and how they know that $\frac{3}{4}$ is bigger than $\frac{2}{3}$. **Second Activity** - All students' actual strategies correspond to the conjectures in the HLT. - The teacher needs to put more emphasize the idea of equivalence of fractions in the bar. After experiencing finding the equivalent fractions in the measuring strips, the teacher should support students to translate the reasoning of equivalent fractions in the measuring strips to the bars.	**First Activity** Leave out the hints to use given tools (scissors) to divide the bread because it takes a long time when students cut and paste the picture of the pieces of bread in the worksheet. **Second Activity** Shorten the width of the bar. The purpose is to make students not divide the bar into 2 columns so that they can see the idea of equivalence of fractions in the bar.

Cycle	Improvement of HLT and Teacher Guide	Improvement of Worksheet
Cycle 2	**First Activity** - If there are students who misinterpret the concept of the whole (or the unit) of fractions, the teacher needs to put emphasize on it. The teacher can use pictures to explain it. For instance, in the problem 2 pieces of bread are shared among 3 children. There might be a student who argue that each child will get 2/6 parts of a piece of bread since there are 6 parts in total. In this case, the teacher can ask them:  '*look at the picture. Each child will get 2 parts, isn't it? So, what parts of* ***A PIECE OF BREAD*** *is it?'* (The teacher puts an emphasize on 'a piece of bread' as a whole). - The teacher needs to encourage students to use pictures instead of algorithms in comparing fractions in order to build their fraction sense. **Second Activity** - The teacher should engage students to notice the patterns of equivalent fractions from some examples.	**First Activity** The question of the fair sharing activity is changed from '*does each child in each group get the same share of bread?'* to '*Which group will get a bigger share for each child?'*. It is because some students answer the initial question by '*yes, because the bread is divided into equal parts'*. Students do not interpret the question as a hint to compare the parts of bread that each child in each group gets. **Second Activity** No changes

C. Meeting 2

There are three activities in this meeting. The aims of this meeting are (1) to compare fractions, and (2) to estimate the sum of two fractions with benchmarks. The first activity is aimed at getting the sense of fractions in a bar. Then, in the second activity, there is an idea about comparing or ordering fractions. In the last activity, the teacher engages the students to estimate the sum of two fractions.

In the description below, the researcher combines the analysis of the first and second activity because the aim of the first activity is to familiarize the students to use bars to represent fractions. The students do not find any difficulties in solving the first activity so there are no any discussions. Thus, the researcher combines the two activities to more focus on the second activity.

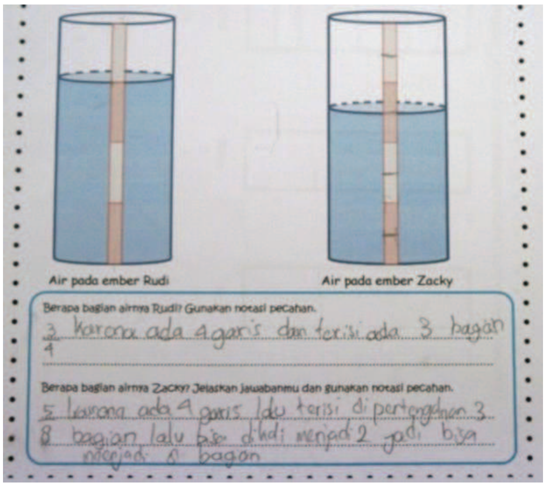
1. First and Second Activity

In these activities, the students work in group of 2-3 students as in the previous meeting. There are two problems in the first activity, in which the students need to determine the parts of the tube filled with water by using fractions notation, and to shade the area of corresponding fractions in the forms of bars. In the second activity, the focus is on the idea of comparing fractions. Then, in the second activity, the students are asked to compare the fractions to determine the winner of each elimination stage of the competition *'memindahkan air'* if the fractions representing the parts of the tube filled with water of each participant is given.

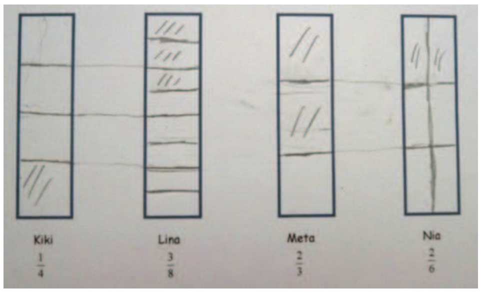
In the HLT for the first activity, the researcher expects that students use the idea of partitioning into equal parts to tackle the problems, which either to figure out the fractions of given pictures or to represent the given fractions in the bar. For the second activity, the expectation in the HLT are the students use bars, use the idea of common denominator, or use their fraction sense to compare the fractions.

a. First Cycle

During the first activity, students do not find difficulties. They are able to find the fractions of the corresponding tube filled with water. As in the first problem, students also are able to represent the corresponding fractions in the bars. Below is the figure of the example of students' work of the first activity.



(a)



(b)

Figure 5.15. The Example of Students' Answer in the Activity 1 of Worksheet 2

In the students' work above, as have been expected in the HLT, the students partition the bar into equal parts so that they can name the fractions notation of the given pictures (Figure 5.15(a)) and they can represent the given fractions in the bars (Figure 5.15(b)).

In the second activity, students have various strategies in comparing fractions as have been conjectured in the HLT. Nabil and Sam use pictures (bars) to solve the problem. Nabil says that to find the bigger fraction, he draws the pictures (bars) representing the two fractions and the bigger fraction is the fraction whose the largest shaded area. On the other hand, Diva and Nanda use an algorithm, in which they do cross multiplication to figure out which fraction is bigger. As can be seen in the Figure 5.16, in the procedure cross multiplication, the students multiply the denominator of one fraction with the numerator of the other fractions. The bigger fraction is the fraction whose bigger result of multiplication. The following figures are the examples of students' work that use bars and cross multiplication procedure.

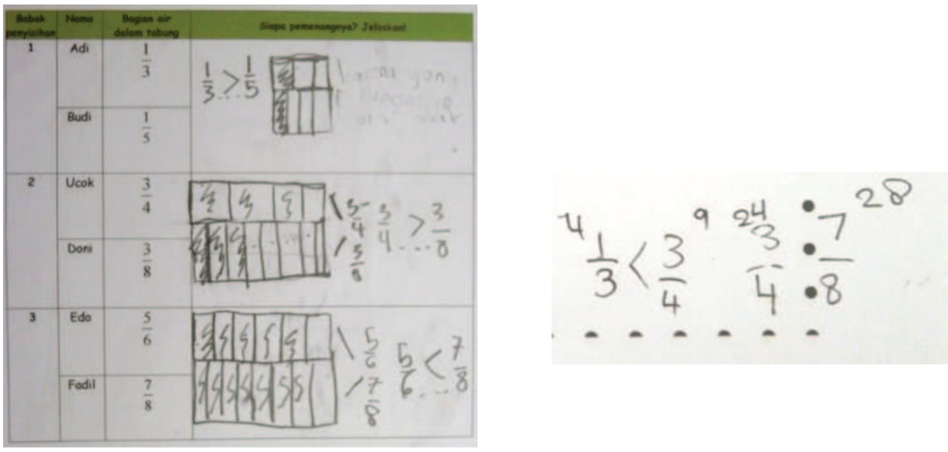


Figure 5.16. Students' Work in Comparing Two Fractions

Different from the other students, Tya does not use any pictures or algorithm. Tya just thinks and then knows which fraction is bigger. Below is the transcript of Tya's explanation.

The teacher	: *(said to all students)* you have to explain your strategy in the paper.
Tya	: I just have one reason in determining the winner.

The teacher : What is that?
Tya : I just know that.

Nabil and Samuel seem to get the idea about partitioning, that the bars and each partition on it has to be equal size to be able to compare two fractions. On the other hand, Nanda and Diva still apply an algorithm without knowing the reasoning of why and how the algorithm works to compare two fractions. Meanwhile, what Tya says in the conversation indicates that Tya uses her fraction sense. She knows which fraction is bigger without using any procedures. Perhaps, she uses benchmarks to compare the fractions or she has a mental image of relative size of the fractions.

Therefore, there are some notes to improve the HLT of the first and second activity of this meeting for the next cycles. First, the teacher needs to engage students more to use their fraction sense in comparing fractions, for example by imagining the relative size of the fractions or by using benchmarks. Second, the researcher puts more problems about comparing fractions in this activity. As can be seen in Table 5.3, the researcher puts a blank space to determine the winner of the final stage and to explain the students' strategy in ordering fractions to determine the winner of the final stage. The researcher also puts conjectures of students' strategies, such as using common denominator, using cross multiplication, and using bars, and using fraction sense.

b. Second Cycle

In this cycle, the students work in the group of 3-4 students. Similar with the first cycle, the students do not find any difficulties and solve the problems of the first activity as have been conjectured in the HLT. In the actual learning of the second activity, most of the groups compare the fractions by using cross multiplication, while the other solved it by using bars. The figures below are the examples of students' strategy.

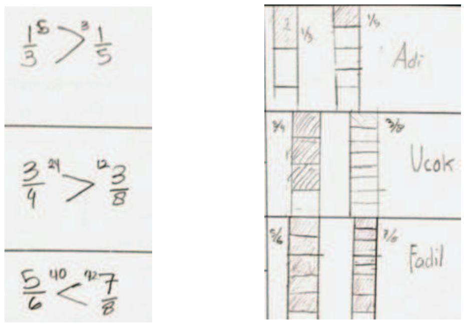


Figure 5.17. Examples of Students' Strategy in Comparing Fractions

Most of the students use cross multiplication procedure, yet they could not explain why they do such algorithm. This finding indicates that students still do not use their fraction sense, although it is very simple to compare $\frac{1}{3}$ and $\frac{1}{5}$, for example. In comparing $\frac{1}{3}$ and $\frac{1}{5}$, actually students do not need any procedure since they can imagine comparing one bread divided by three and one bread divided by five. Therefore, in the class discussion, the teacher engages the students to use their fraction sense or pictures to compare fractions. To get students used to use their fraction sense, the teacher poses a simple problem of comparison of fractions in a context, for instance, '*If you get $\frac{1}{4}$ of a cake and your brother gets $\frac{1}{3}$ of a cake, which one of you will get bigger parts of a cake?'*

In ordering fractions to determine the winner of the final stage, some students, such as Krishna's group, use the idea of common denominator to solve it. Below are the transcript of the discussion of Krishna's group in ordering $\frac{1}{3}, \frac{3}{4}, \frac{7}{8}$, and the figure of their work.

Krishna : The winner of each elimination stage is $\frac{1}{3}, \frac{3}{4}, \frac{7}{8}$.

Acha : We have to order it?

Krishna : *(thinking, trying to find the common denominator of third, fourth, and eighth)* That can… oh, 16. 16..16..16 *(while writing 16 as the denominator of all fractions)* Oh..can 16 be divided by 3?

Acha : I do not know *(looks confused)*. Eh, Mazta, can 16 be divided by 3?

Mazta : No, it cannot. But it can be divided by 4.

Krishna : what number can be divided by … aah, I know… *(writing, finding the equivalent fractions of $\frac{1}{3}$)* 24 divided by 3? *(continue writing and make all fractions to twentyfourth, and get $\frac{8}{24}, \frac{18}{24}, \frac{21}{24}$)*

Acha : So, the winner is … Fadil? *(Fadil has the fraction of $\frac{21}{24}$)*

Krishna : Ya, the winner is.. Fadil.

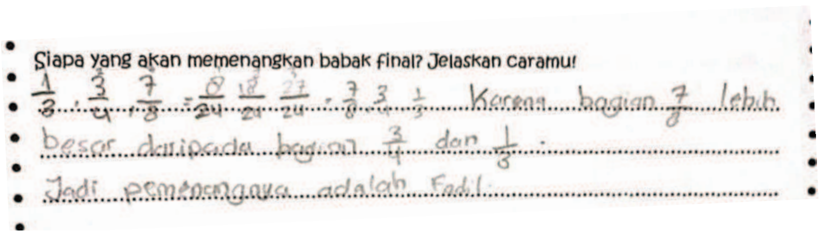


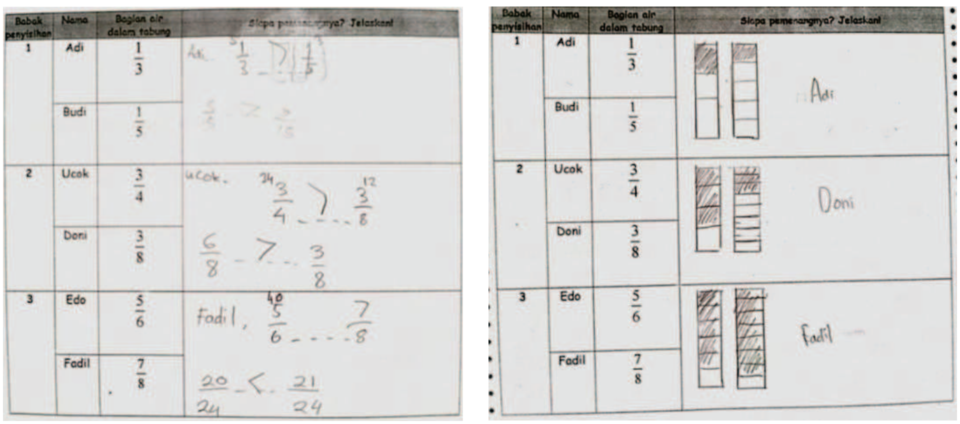
Figure 5.18. The Work of Krishna's Group in Ordering Fractions

From the transcript of the discussion and the work of Krishna's group, it can be seen that Krishna and his friends are able to find a common denominator. The fragment indicates that Krishna knows that to compare fractions, the denominator has to be the equal. However, it is not clear whether or not Krishna understands the reasoning of the algorithm he uses. Therefore, the teacher should encourage students to express their reasoning of doing the algorithm and engage students to use pictures or bars to understand the idea behind the algorithm.

For the improvement in the next cycle, the researcher leaves out a problem in the first activity in the worksheet, which was about representing the given fractions in the bar. The consideration is that most of students have learned about representing fractions in pictures as a part-whole relationship. Thus, most of them are already able to represent the given fractions in the bar. Moreover, the researcher wants to focus more on the second and third activity.

c. Third Cycle

As in the second cycle, the students also work in the group of 3-4 students in this cycle. In tackling the first activity, the students can solve it easily. In the second activity, as have been conjectured in the HLT, the students come up with various strategies, such as using bars, the idea of common denominator, and cross multiplication. The figures below are the examples of students' solution.



(a) (b)

Figure 5.19. Examples of Students' Answer in Comparing Fractions

Ersya and her friends solve the problem by using both, cross multiplication procedure and the idea of common denominator as in the Figure 5.19(a). To get the common denominator, Ersya firstly looks for a number that can be divided by the denominators of two fractions that will be compared. Then, she tries to find the equivalence of the fractions that will be compared by using the common denominator she finds. Ersya seems to notice that in comparing fractions, the denominator of the fractions should be the same. In line with the strategies in comparing fractions, the students also use the same strategy to order fractions.

Different from Ersya's group, Mitha's group starts using their fraction sense in comparing fractions. For example, in comparing $\frac{3}{4}$ and $\frac{3}{8}$, they argue that $\frac{3}{4}$ is bigger than $\frac{3}{8}$ because $\frac{3}{4}$ means 3 parts of 4 parts, while $\frac{3}{8}$ means 3 parts of 8 parts.

Conclusion

From the description of the result above, students uses the idea of partitioning into equal parts in the first activity. Moreover, the result of the second activity indicates that students are able to compare fractions with various strategies, such as by using bars, the idea of common denominator, the cross multiplication procedure, and by using their fraction sense. In using the bars, it can be seen that students begin to get used to use the bars and they are aware of the concept of the unit, that to compare fractions, the unit (bars) have to be equal size. Some of the students also begin to use their fraction sense in comparing fractions. In addition, after the class discussion, students who use the idea of common denominator and the cross multiplication algorithm start to get familiarize in using pictures (bars) to compare fractions. The use of pictures can help them build the mental image of fractions and the fraction sense.

2. Third Activity

In this activity, the students (in groups) discuss the estimation of the sum of two fractions by firstly determining the fractions that are more than a half. After determining which fractions are more than a half, the students estimate the sum of two fractions. Then, the teacher orchestrates a class discussion to support students in undersanding how to estimate the sum of two fractions.

In the HLT, the researcher conjectures that the students use bars or fraction sense to figure out the problem. Moreover, the researcher also assumes that the students firstly find the exact result of the addition of fractions and then compare it to the benchmarks.

a. First Cycle

In determining which fractions are more than a half, some of them use pictures to figure it out, while the other firstly find the half of the bottom number (denominator), and then check whether the numerator of the given fractions is more or less than a half of the denominator.

In estimating the sum of two fractions, students do not get the question. As has been conjectured in the HLT, most of them add the fractions and then compare the result to a half as the benchmark. Students still do not use their previous exercise about determining fractions which more than a half. In this case, what students know is that they have to add the fractions and then compare it to a half as the benchmark. Moreover, they do not know how to add fractions with different denominators yet. Thus, since their result of the addition of fractions is incorrect, their estimation is incorrect as well.

Therefore, the teacher guides the students that they do not need to add the fractions in the estimation, and engages them to consider a half as the benchmark. For example, in estimating the result of $\frac{1}{5}+\frac{3}{4}$, the teacher engages students to consider whether $\frac{3}{4}$ is less and more than a half. After the teacher gives the hint, the students begin to aware that the result must be more than a half because $\frac{3}{4}$ itself is more than a half.

Therefore, there is a note to improve this activity in the next cycles. The teacher needs to give more hints in estimating the sum of two fractions that lead them to use benchmarks. Therefore, as can be seen in Table 5.3, the researcher puts more hints to guide students in estimating the sum of two fractions. For instance, in estimating the result of $\frac{1}{5}+\frac{3}{4}$, the teacher can put a question '*Is* $\frac{3}{4}$ *more or less than a half?'*.

b. Second Cycle

In the beginning, the students do not know how to estimate fractions by using benchmarks. Then, noticing the hints in the problems and after the discussion with the teacher, some students seem to get the idea of how to estimate the sum of two fractions. It can be seen in the figure of the example of the students' answer below that he/she is able to use benchmarks to estimate the sum of $\frac{2}{3}$ and $\frac{1}{4}$.

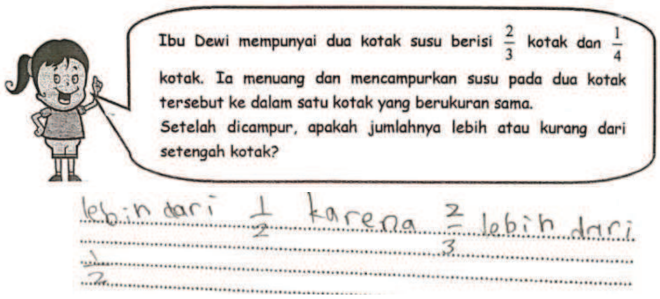


Figure 5.20. The Example of Students' Answer in Estimating the Sum of $\frac{2}{3}$ and $\frac{1}{4}$

The example of the students' work above indicates that some students start to know how to estimate the sum of two fractions by using benchmarks. As expected in the HLT, they use their fraction sense by looking at the fractions being added and compare it to the benchmark. For example, in estimating $\frac{2}{3}+\frac{1}{4}$, they look at $\frac{2}{3}$ and compare it to a half as the benchmark. After they know that $\frac{2}{3}$ is more than a half, they conclude that the result of the addition must be more than a half.

However, the idea of estimation to avoid the procedure 'top+top over bottom+bottom' still does not appear. Thus, the researcher also puts more probing questions in the teacher guide since the role of the teacher is very important to encourage students to notice that procedure 'top+top over bottom+bottom' is incorrect.

Regarding the worksheet, there is a note that can be used to improve the worksheet for the next cycle. The change made is reducing the number of fractions that have odd numbers as the denominator, including the fractions in the estimation problem, since some students cannot determine a half of odd numbers. Moreover, the researcher changes the estimation problem from

$\frac{2}{3}+\frac{1}{4}$ become $\frac{4}{6}+\frac{1}{4}$ in order to make students easier to find the exact result in the bars.

c. Third Cycle

After solving some problems that include hints how to estimate the sum of two fractions, the students are able to figure out how to estimate. In line with the conjectures in the HLT, they look at the size of the fractions being added and compare it to a half as the benchmark. For example, in estimating the result of $\frac{4}{6}+\frac{1}{4}$, the students are aware that $\frac{4}{6}$ is more than a half, so they argue that after being added, the result must be more than a half. Moreover, some of students estimate the result of $\frac{4}{6}+\frac{1}{4}$ by using bars as can be seen in the Figure 5.21 below.

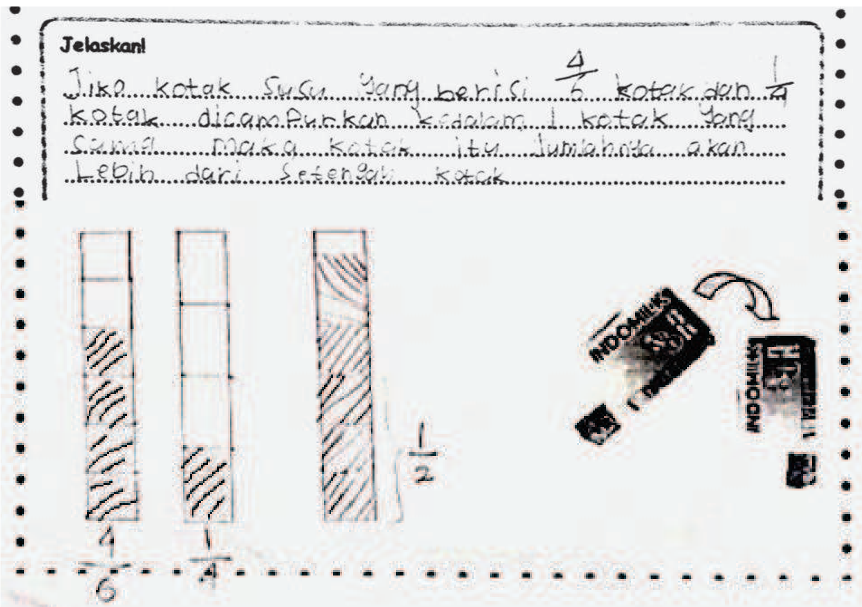


Figure 5.21. An Example of Students' Strategy in Estimating by Using Bars

After the students do an activity about estimating of the sum of two fractions, the teacher conducts a class discussion to show students that procedure 'top+top over bottom+bottom' is incorrect. Below is the transcript of the discussion.

The teacher	: Now, $\frac{3}{4}+\frac{1}{5}$ *(writing the problem in the whiteboard).* Yoga's answer is $\frac{4}{9}$. You add the top and the top, and add the bottom and the bottom, isn't it? (*pointing the numbers)*
The teacher	: Is $\frac{1}{5}$ more or less than a half?
Students	: Less than a half.
The teacher	: What about $\frac{3}{4}$?
Students	: More than a half.
The teacher	: So, $\frac{3}{4}$ is more than a half. If it is added by $\frac{1}{5}$, the result becomes more and more than a half, isn't it?
The teacher	: Now, is $\frac{4}{9}$ more or less than a half? *(pointing the number in the whiteboard)*
Students	: Less than
The teacher	: Whereas the result should be more than. So, can we apply top+top over bottom+bottom?
Students	: No.

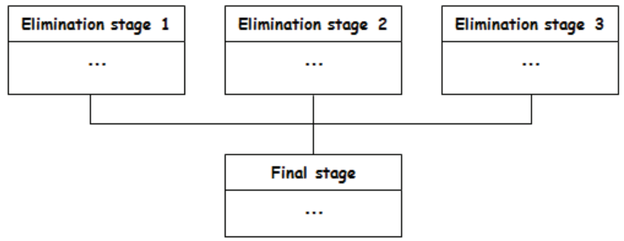
By using students' insight about estimation, the teacher engages the students to notice that the result of addition by using the procedure 'top+top over bottom+bottom' is not reasonable. The teacher shows an example that the sum of $\frac{3}{4}$ and $\frac{1}{5}$ should not be $\frac{4}{9}$ because the result must be more than a half. In the discussion above, it can be seen that the students begin to be aware that they cannot apply the procedure 'top+top over bottom+bottom' after noticing the estimation of the sum of $\frac{3}{4}$ and $\frac{1}{5}$.

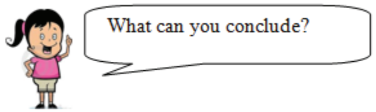
Conclusion

From the description above, it can be seen the students notice how to estimate the sum of two fractions after noticing the hints of the estimation problems in the worksheet. The students start to understand how to estimate the sum of two fractions by using their fraction sense, which they look at the size of the fractions being added. Moreover, after the discussion with the teacher, the students start to notice that the procedure 'top+top over bottom+bottom' is incorrect.

The following table provides a summary of the improvement of the HLT and worksheets of the meeting 3 during the three cycles.

Table 5.3. The Changes Made in the First, Second, and Third Activity of Meeting 2 during the Three Cycles

Cycle	Improvement of HLT and Teacher Guide	Improvement of Worksheet
Cycle 1	**First and Second Activity** - Most of conjectures of students' thinking are in line with students' actual strategies. - Add the conjectures of students' thinking in ordering fractions (to determine the winner of the final stage), for instance the students will use cross multiplication procedure, use common denominator, use pictures or use their fraction sense.	**First and Second Activity** Put a blank space to determine the winner of the final stage and to explain the students' strategy in determining the winner of the final stage. Diagram of the winner of each stage in the competition 'memindahkan air'  Who will be the winner of the final stage? Explain your strategy! ....

Cycle	Improvement of HLT and Teacher Guide	Improvement of Worksheet
Cycle 1	**Third Activity** The teacher needs to support students on how to estimate the sum of two fractions and to notice that the procedure 'top+top over bottom+bottom' is incorrect.	**Third Activity** - Add more hints to estimate the sum of two fractions, as below: 1. $\frac{1}{5}+\frac{3}{4}$ a. Is $\frac{1}{5}$ more or less than $\frac{1}{2}$? b. Is $\frac{3}{4}$ more or less than $\frac{1}{2}$?  c. Is the result more or less than $\frac{1}{2}$? .... d. Is the result more or less than 1? ....
Cycle 2	**First and Second Activity** - The teacher needs to engage students to use their fraction sense or pictures instead of cross multiplication to compare fractions. For example, by asking a problem *'If you get $\frac{1}{4}$ of a cake and your brother gets $\frac{1}{3}$ of a cake, can you imagine which one gets bigger parts of a cake?'* **Third Activity** - By utilizing the estimation skill, the teacher has to encourage students to notice that procedure 'top+top over bottom+bottom' is incorrect.	**First and Second Activity** Leave out the problems about representing the given fractions in the bar. **Third Activity** - Reducing the number of fractions that have odd numbers as the denominator since some students cannot determine a half of odd numbers. - Changing the estimation word problem from $\frac{2}{3}+\frac{1}{4}$ become $\frac{4}{6}+\frac{1}{4}$.

D. Meeting 3

The goals of the third meeting are (1) to estimate the sum of two fractions with benchmarks, (2) to grasp the idea of common denominator, and (3) to add fractions by using models. There are three activities in this meeting. In the first activity, the students experience in adding fractions by using paper strips, and the second activity was an individual exercise about adding fractions in the form of bars. The third activity is a review exercise about the estimation of the sum of two fractions.

In the description below, the researcher combine the analysis of the first and second activity because these two activities are related to each other. These two activities are about adding fractions by using models and are aimed at grasping the idea of common denominator in adding fractions. Firstly, the students explore the idea of common denominator by using paper strips, and then translate it into the form of bars in the second activity.

1. First and Second Activity

In the first activity, the students explore the idea of common denominator and experience in adding fractions by using paper strips. By exploring the idea of common denominator in the paper strips, it is expected that the students grasp the reasoning behind the idea of common denominator. In the HLT of the first activity, the researcher expects that the students extend the lines of each paper strips representing each fractions to find the common denominator. The researcher also conjectures that the students use the formal algorithm to find the common denominator.

In the second activity, the researcher poses a set of problems of the addition of fractions in the form of bars. The researcher expects the students to partition the bars into a number of parts that fit to both fractions as they do in the paper strips. The students also may use formal algorithm, which they use the idea of common denominator.

a. First Cycle

In this cycle, firstly the teacher demonstrates in pouring water from two tubes, whose a half and a quarter parts of it filled with water respectively. Then, the students practice by themselves in pouring water from two tubes, whose a half and a third of it filled with water respectively. After they measure the water after being poured, the students explore it by using measuring strips and then discuss it together with the teacher by noticing the measuring strips they make in the first meeting.

In the discussion, after the students get $\frac{5}{6}$ as the result of $\frac{1}{2}+\frac{1}{3}$, the teacher together with the students discuss why the result is $\frac{5}{6}$. Below is the transcript of their discussion.

The teacher	: Now, pay attention. $\frac{1}{2}$ and $\frac{1}{3}$, how can the result be $\frac{5}{6}$? *(showing the measuring strips)*
Tya	: Because a half and a third can be represented in sixths *(while looking at the measuring strips)*
The teacher	: Why do we choose sixths?
Nabil	: Because it’s the same.
Tya	: We can multiply it.

Corresponding to the conjectures in the HLT, the students explore the extension of lines in the paper strips and then start to notice the idea of common denominator. They use the idea of equivalence of fractions, which they try to find a denominator (or partitions in the bar) that can represent a half and a third. By saying '*because it's the same',* Nabil means that both fractions can be represented in sixths. Furthermore, Tya find that she also can multiply the denominator to get the common denominator.

In the second activity, the teacher gives a set of problems about the addition of fractions in the bars. The figure below is the example of students' work in solving the problem.

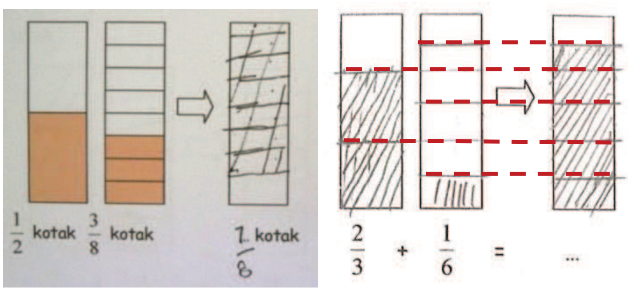


Figure 5.22. Students' Work in Adding Two Fractions by Using Bars

As expected in the HLT, the students are aware of the idea of common denominator. Students do not find any difficulties in adding fractions with the same denominator. In adding fractions with different denominators, they begin to notice that the partitions of the result bar should be able to represent the two fractions being added. It can be seen from their work in Figure 5.22 above, that they try to make the common extension lines between the three bars. Then, they partition the bar so that it can represent the two bars being added.

Regarding the activities and the worksheet, the researcher finds some remarks that can be used to improve the HLT for the next cycle. First, students do not need to experience in pouring the water because it is too messy and the water may spill. For the next cycle, the students just explore the result of $\frac{1}{3}+\frac{1}{6}$ and $\frac{1}{2}+\frac{1}{3}$ by using paper strips without experiencing pouring the water. Second, the symbol '+' on the worksheets should be introduced earlier since students do not know what to do with the bars. Third, the researcher omits some problem about the addition of fractions in the form of bars because the problem is too much for the students compared to the limitation of the time. The detail of the changes of the worksheet can be seen in Table 5.4.

b. Second Cycle

After the teacher represents why the result of $\frac{1}{2}+\frac{1}{4}$ can be $\frac{3}{4}$ by using paper strips, she asks the students to work in-group to explore and find the result of $\frac{1}{3}+\frac{1}{6}$ and $\frac{1}{2}+\frac{1}{3}$ by using paper strips. In the transcript below, Krishna and his friends try to figure out the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

… *(after make the paper strip of $\frac{1}{2}$ and $\frac{1}{3}$).*
Krishna : Now, we add it *(pointing the paper strips of $\frac{1}{2}$ and $\frac{1}{3}$)*
Emma : *(nodding)*
Krishna : *(thinking)* What is the result? *(asking the second student)* eh, what is the result of $\frac{1}{2}+\frac{1}{3}$?
Mazta : $\frac{1}{2}+\frac{1}{3}$? … $\frac{1}{2}+\frac{1}{3}$ is equal to $\frac{2}{5}$ *(thinking.)* What is being added?
Krishna : $\frac{1}{2}+\frac{1}{3}$
Mazta : $\frac{1}{2}+\frac{1}{3}$ is $\frac{2}{5}$
Krishna : Let me borrow a pencil. Where can I write?
Emma : *(giving a paper)*
Krishna : *(writing $\frac{1}{2}+\frac{1}{3}$ in the paper)*
Mazta : oh.. six! The denominator is six *(looking at Krishna's writing)*

Krishna : Six divided by 2 is 3 *(Finding the numerator of the equivalent of* $\frac{1}{2}$*)*
Mazta : So, its $\frac{5}{6}$
Krishna : *(still thinking)*, 6 divided by 3 is…*(Finding the numerator of the equivalent of* $\frac{1}{3}$*)*
Mazta : Bottom divided by bottom and then multiply it with the top..ya, that's right.
All : 2 *(pointing 6 divided by 3)* So, 5/6.

As has been stated in the HLT, in the transcript above Krishna and his friends do not use the paper strip as a tool to help them to find the common denominator (common number of partitions). Meanwhile, they use an algorithm, in which they try to find a number that can be divided by 2 and 3, and then make $\frac{1}{2}$ and $\frac{1}{3}$ into sixths by the idea of equivalence of fractions.

Different from Krishna's strategy, Nana's group uses the paper strips to help them to find the common denominators. They try to extend the lines from the two paper strips being added so that they get a common number of partitions for the three paper strips (two paper strips represent two fractions being added and another paper strip represented the result). The figure below is the work of Nana's group who uses the paper strips as a help to find out the result of $\frac{1}{3}+\frac{1}{6}$ and $\frac{1}{2}+\frac{1}{3}$.

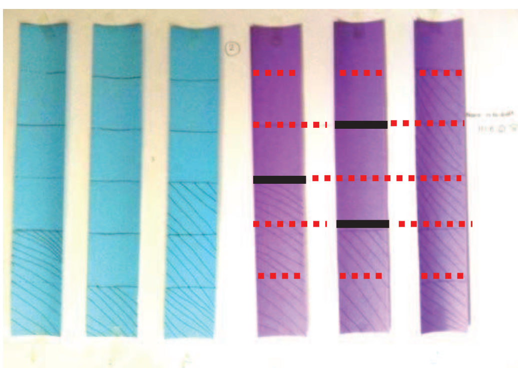


Figure 5.23. The Work of Nana's Group in Adding Fractions by Using Paper Strips

In solving the problems of the addition of fractions in the bars in the second activity, students come up with various strategies as have been conjectured in the HLT. In adding fractions with the same denominator, the students can solve it easily. While in adding fractions with different denominators, the students start to grasp the idea of common denominator, as can be seen in the Figure 5.24 below. They try to make the common extension lines and then partition the result bar so that it can represent the two fractions being added.

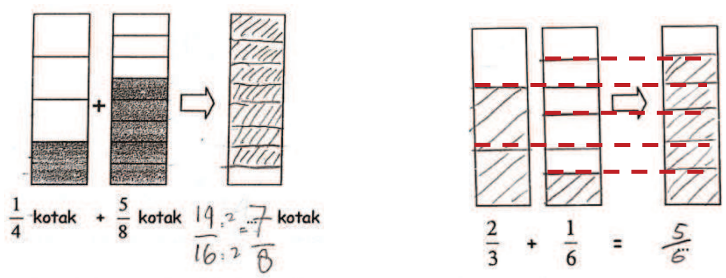


Figure 5.24. Examples of Students' Work in Adding Fractions by Using Bars

There are notes for the teacher for the improvement of the activity in the next cycle. The teacher needs to engage them to notice the relation between the common number of partitions and the denominator of two fractions being added.

c. Third Cycle

The strategies in adding fractions appear in this cycle are similar with those in the previous cycles. The transcript below shows the effort of Ersya's group in exploring the result of $\frac{1}{3} + \frac{1}{6}$ in the paper strips.

Shafa : If we add it, so it's ... *(showing around a half of the bar as the result by using her hand)*
Ersya : Just make dot lines *(showing her friends how to make the dot lines, by extending the lines of the other bars)*
Ajeng : How many parts is it?
Ersya : *(counting)* there are six parts
Ersya : Also make it into six parts (*pointing the result bar)*
Ersya : You can just extend the lines (from the other bar)
Ajeng : That's what I did
Shafa : Okay, if we add it it becomes a half.
Ersya : This is two *(pointing the shaded area of the six-partitioned bar of one third)*, and this is one *(pointing the bar of one sixth)*, so it becomes three partitions
Shafa : Yes it's right, it's a half
Ajeng : So, it's three? *(pointing the parts that would be shaded in the result bar)*
Ersya : Ya, it's three.

Ersya and her friends make the three and six-partitioned paper strips to represent $\frac{1}{3}$ and $\frac{1}{6}$. Then, they struggle to find out how many partitions that they should make in the resulting bar. They figure out the common number of partitions by extending the line of the six-partitioned bar to other bars. After they find that $\frac{1}{3}$ also can be represented as sixths, they partition the resulting bar into six partitions. Then, they count the parts that should be shaded in the result bar by adding the number of shaded parts of $\frac{1}{3}$ and $\frac{1}{6}$ in the six-partitioned bar, and they got three parts that should be shaded. In addition, from the beginning Shafa know that the result is a half. She looks at the bars representing $\frac{1}{3}$ and $\frac{1}{6}$, and then she seems imagining the total shaded area in the paper strips. Then, by using her hand she estimates that the sum is a half. Moreover, Shafa also shows their understanding of the equivalence of fractions when she argues that $\frac{3}{6}$ is equal to $\frac{1}{2}$.

In line with the work of Ersya's group, Aldy's group also has the same process in finding the result of $\frac{1}{2}+\frac{1}{3}$ and $\frac{1}{3}+\frac{1}{6}$. Figure 5.25 below presents the work of Ersya's group in solving $\frac{1}{3}+\frac{1}{6}$ and the work of Aldy's group in solving $\frac{1}{2}+\frac{1}{3}$ and $\frac{1}{3}+\frac{1}{6}$ in the paper strips.

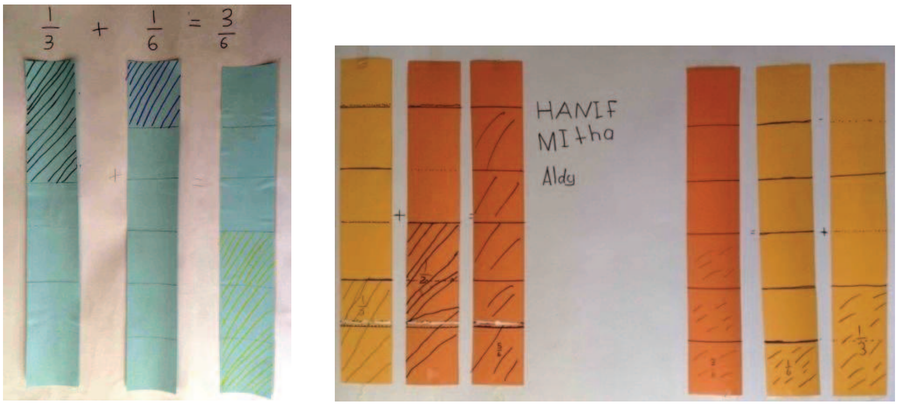


Figure 5.25. The Work of Ersya's Group (Left) and Aldy's Group (Right) in Adding Fractions by Using Paper Strips

In the third activity, the conjectures in the HLT appear, for example partitioning the bars into the same numbers of partitions. It can be seen in the figures below that the students try to extend the lines from the two bars being added to get the common number of partitions (the common denominator) as they do in the paper strips. Figure 5.26 shows how students struggle to add fractions by using bar model, while Figure 5.27 shows the example of students' solutions in adding fractions in the bar model.

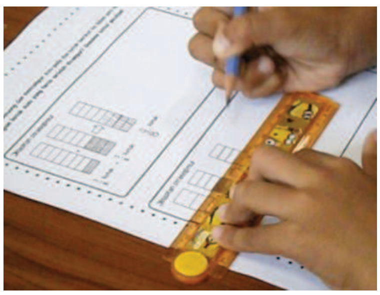


Figure 5.26. How Students struggle in Adding Fractions by Using Bars

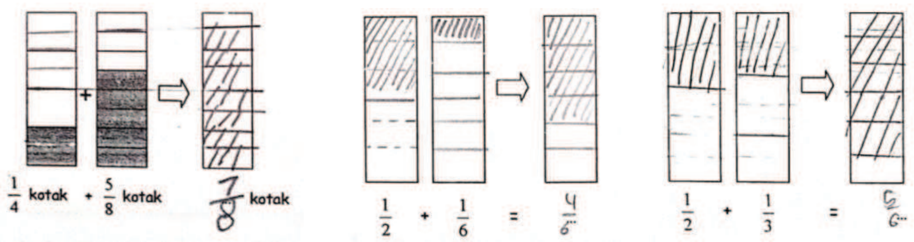


Figure 5.27. Examples of Students' Work in Adding Fractions by Using Bars

Conclusion

The description of the first and second activity above indicates that students begin to understand how to add fractions, especially fractions with different denominators. Students do not find any difficulties in solving the addition of fractions with the same denominator. In adding fractions with different denominators, they are aware that the two paper strips representing two fractions being added have to have the same number of partitions (as the common denominator). In other words, to add two fractions, the denominators of the fractions have to be equal. In addition, some students also are able to add fractions in the form of bars. They translate what they do in the paper strips into the bars.

2. Third Activity

In this activity, the students review the problem of estimation of the result of $\frac{2}{3}+\frac{1}{4}$ as in the second meeting. The researcher conjectures that students may represent the problem in the form of bars and notice that the result must be more than a half. The researcher also conjectures that students use the idea of common denominator or use a half as the benchmark.

a. First Cycle

As have been conjectured in the HLT, the students consider a half as the benchmark in the estimation. The transcript below shows the discussion between the teacher and the students to estimate the sum of $\frac{2}{3}$ and $\frac{1}{4}$.

The teacher : it's more than a half? (*emphasizing students' answer after telling the story of Mrs. Doni)*
Nabil and Sam : Because its more than a half.
The teacher : What is more than a half?
Nabil and sam : $\frac{2}{3}$.
The teacher : So, what should we do to estimate the sum of two fractions? *(asking the students)*
The students : Look at the fractions itself.

From this fragment, it can be seen that Nabil and Sam are able to find out that the sum of $\frac{2}{3}$ and $\frac{1}{4}$ is more than a half because they know that $\frac{2}{3}$ itself is more than a half. The fragment above indicates that the students understand how to estimate the sum of two fractions. They look at the fractions being added and then compare it to a half as the benchmark.

However, the idea that the estimation skill can lead students to be aware that the incorrect procedure 'top+top over bottom+bottom' does not appear yet. Therefore, to improve the activity in the next cycles, the teacher needs to emphasize and engage students to notice that they cannot do 'top+top and bottom+bottom' by showing the measuring strips and by using the estimation they have learned. Therefore, in the worksheet for the next cycles, besides the estimation problem, the researcher puts a question to ask students to find the exact result of $\frac{2}{3}+\frac{1}{4}$. Moreover, the researcher puts a question to encourage students to check whether or not their result is corresponding to their estimation and to be aware that the procedure ‘top+top over bottom+bottom’ is incorrect. The changes of the worksheet can be seen in Table 5.4.

Furthermore, the researcher puts the conjectures of students’ thinking in solving the improved problems. In finding the exact result of the addition, the researcher conjectures that students may use the bars or use the idea of common denominator without using the bar.

b. Second Cycle

In this activity, the students are asked to estimate the result of $\frac{2}{3}+\frac{1}{4}$, find the exact result of it, and then check it whether it is corresponding with their estimation. The figure below shows the example of students’ answer in solving the problems.

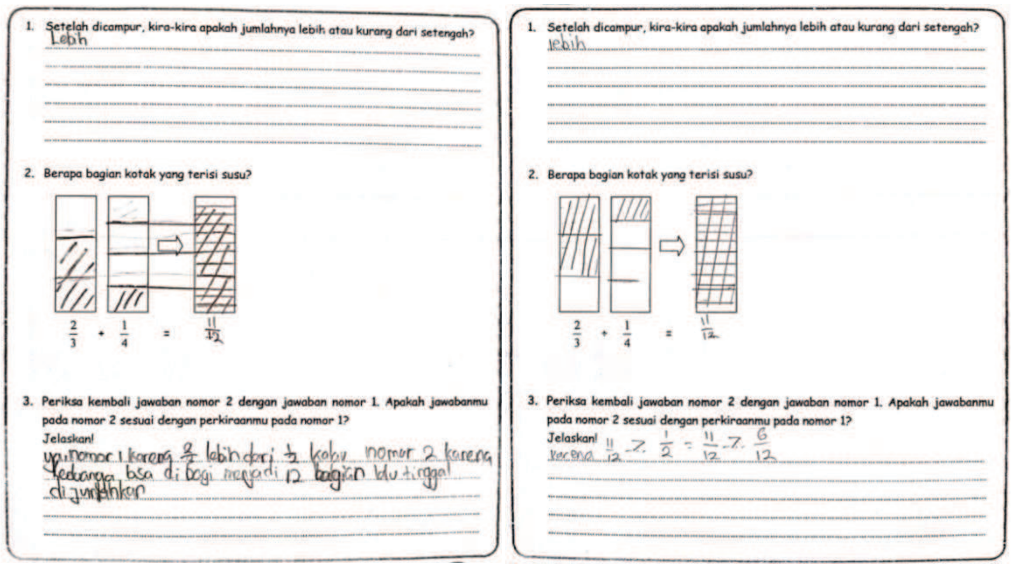


Figure 5.28. The Example of Students’ Work in Solving the Third Activity of Worksheet 3

In line with the conjectures in the HLT, most of students are able to estimate the sum of $\frac{2}{3}+\frac{1}{4}$ based on their previous experience in estimating in the second meeting. They are aware that $\frac{2}{3}$ is more than a half, so the result must be more than a half. In finding the exact result, the conjectures in the HLT appear, such as using the idea of common denominator or utilizing the bars. Figure 5.28 above also shows that the students can check whether their result and their estimation are in line.

However, based on the researcher's notes, many students cannot check their result with their estimation. For example, the students can estimate that the sum of $\frac{2}{3}+\frac{1}{4}$ is more than a half, but when they get $\frac{3}{7}$ as the result of the addition, they do not notice that the result is not corresponding with their estimation. Thus, the teacher encourages the students to check whether $\frac{3}{7}$ is more than a half or not. Then, the teacher asks and guides the students correct the exact result in the bars. After the discussion, the students begin to aware that they cannot apply the procedure 'top+top over bottom+bottom' in adding fractions.

Students might find difficulties to find the result $\frac{2}{3}+\frac{1}{4}$ because they find it difficult in finding the common number of partitions (common denominator) of $\frac{2}{3}+\frac{1}{4}$ in the bar. Therefore, for the improvement of the activity for the next cycle, the researcher change the estimation problem from $\frac{2}{3}+\frac{1}{4}$ become $\frac{4}{6}+\frac{1}{4}$ in order to make students easier to find the common number of partitions (common denominator) in the bars to find the exact result.

c. Third Cycle

In this cycle, some students show their ability in estimating and finding the exact result of $\frac{4}{6}+\frac{1}{4}$, as can be seen in the Figure 5.29 below.

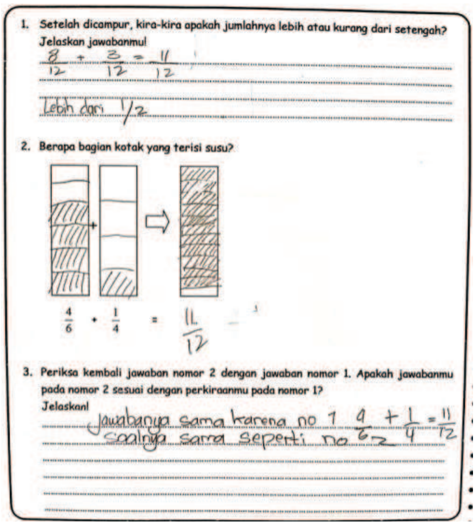


Figure 5.29. The Examples of Students' Work in Solving the Third Activity

As have been conjectured in the HLT, the students can estimate the sum of $\frac{4}{6}+\frac{1}{4}$. Most of students are able to estimate the result of $\frac{4}{6}+\frac{1}{4}$, either by using benchmarks or bars. In the picture above, the students estimate it by firstly find the exact result, and then compare it to a half as the benchmark. However, as in the previous cycle, some students still apply the procedure 'top+top over bottom+bottom' and get $\frac{5}{10}$ as the result of $\frac{4}{6}+\frac{1}{4}$. Moreover, many students still cannot relate between their estimation and their exact result.

It might because students do not get used to estimate in their mathematics class. They are used to use algorithms instead of the reasoning of estimation. Thus, it is rather difficult to engage students to use the estimation to check the reasonableness of their answer. Thus, for the next study, the teacher can extend the learning process to more focus on the estimation skill, for example by conducting a mini lesson that focuses on the estimation skill.

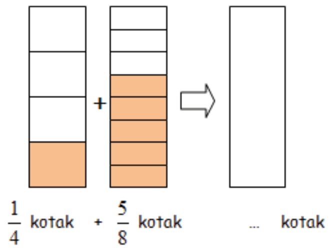
In the discussion, the teacher engages students to notice that the procedure 'top+top over bottom+bottom' by representing their result in the bars, and by showing their estimation. Moreover, the teacher guide the students to find the exact result of $\frac{4}{6}+\frac{1}{4}$ in the form of bars. In the end of the discussion, the students are aware that the procedure 'top+top over bottom+bottom' is incorrect because the output of that procedure is not reasonable.

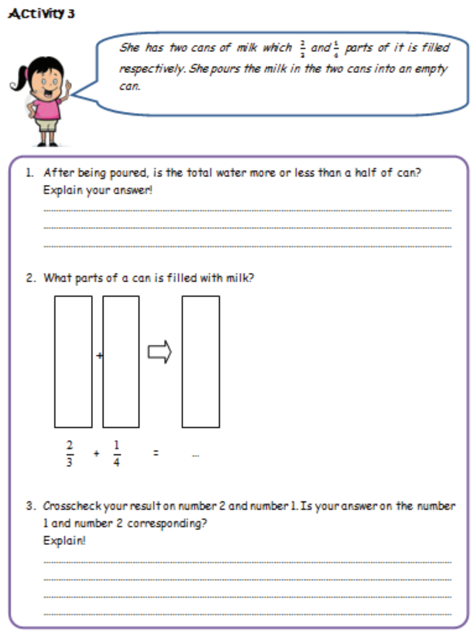
Conclusion

From the description above, it can be seen that students are able to estimate fractions by considering a half as the benchmark. Moreover, they start to be aware that the exact result and the estimation should be corresponding to each other. After the discussion, they also notice that the procedure 'top+top over bottom+bottom' is incorrect.

The table below presents the summary of the improvement of the activities in the third meeting during the three cycles.

Table 5.4. The Changes Made in the First, Second, and Third Activity of Meeting 3 during the Three Cycles

Cycle	Improvement of HLT and Teacher Guide	Improvement of Worksheet
Cycle 1	**First and Second Activity** - All conjectures of students' thinking are in line with students' actual strategies. - It is too messy if students measure the water before and after pouring the water from two tubes. Therefore, in the next cycle, it is better if the teacher demonstrates in pouring water from two tubes and the students would experience and investigate adding fractions ($\frac{1}{3}+\frac{1}{6}$ and $\frac{1}{2}+\frac{1}{3}$) by using paper strips. - The teacher needs to emphasize the reasoning of the common denominator by using paper strips or bars.	**First and Second Activity** • Put the symbol '+' between the two bars and between the fractions being added so that students know what to do with the bars  • Omit some problems about the addition of fractions in the form of bars. Before being changed: $\frac{1}{8}+\frac{3}{8}, \frac{2}{6}+\frac{3}{6}, \frac{1}{4}+\frac{1}{8}, \frac{2}{4}+\frac{3}{8}, \frac{2}{3}+\frac{1}{6}, \frac{1}{2}+\frac{1}{6}, \frac{1}{2}+\frac{1}{3}$ After being changed: $\frac{1}{8}+\frac{3}{8}, \frac{1}{4}+\frac{5}{8}, \frac{2}{3}+\frac{1}{6}, \frac{1}{2}+\frac{1}{6}, \frac{1}{2}+\frac{1}{3}$

Cycle	Improvement of HLT and Teacher Guide	Improvement of Worksheet
Cycle 1	**Third Activity** - The teacher should put more emphasize that students cannot do 'top+top over bottom+bottom' by showing the measuring strips and by using the estimation they have learned. - Add conjectures of students' thinking in solving the improved problem about reviewing the estimating problem. In finding the exact result of $\frac{2}{3}+\frac{1}{4}$, the researcher conjectured that students might use the bar or use the idea of common denominator without using the bar.	**Third Activity** This activity consists of: • The word problem about estimation of $\frac{2}{3}+\frac{1}{4}$. • A problem about adding fractions in the bar • A problem to review whether the result of the addition of fractions in the bar in line with the estimation or not in order to avoid the incorrect procedure 'top+top and bottom+bottom' 

Cycle	Improvement of HLT and Teacher Guide	Improvement of Worksheet
Cycle 2	**First and Second Activity** The teacher needs to support students' reasoning in adding fractions by using paper strips to grasp the relation between the common number of partitions and the denominator of two fractions being added.	**First and Second Activity** No changes
	Third Activity The teacher needs more time to carry out the third activity. Moreover, in this activity, the teacher should emphasize the use of estimation to avoid incorrect procedure 'top+top over bottom+bottom'.	**Third Activity** Change the problem from $\frac{2}{3}+\frac{1}{4}$ into $\frac{4}{6}+\frac{1}{4}$. The consideration is to make students easier to find the common number of partitions (common denominator) in the bars.

E. Post-test

In general, the varieties of students' strategies in the post-test of the three cycles are similar. The following description is the summary of the post-test in the three cycles.

1. The concept of fractions

As in the pre-test, students do not get any difficulties in solving the first two problems. They know how to label the fractions of corresponding pictures and know how to represent fractions in the pictures. From their answer, it can be seen that students comprehend the idea of partitioning, that they have to partition the pictures into equal parts to represent fractions. Moreover, they know that the denominator refers to the number of partitions and the numerator refers to the number of shaded areas. Below is the figure of the example of students' answer in solving the first problem.

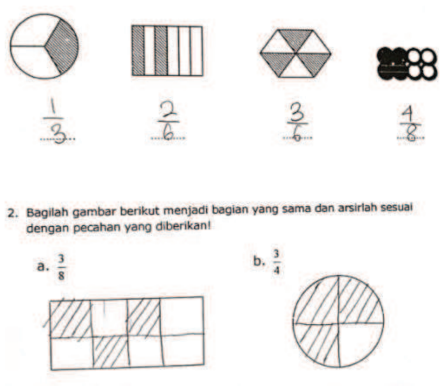


Figure 5.30. The Example of Students' Solution of the First and Second Problem of Post-Test

2. The equivalence of fractions

In the third problem, which students need to find the equivalence of fractions and its representation in the form of bars, the students are able to figure out the problem. Some of them try to extend the lines from the given bars to figure out the equivalence of the corresponding fractions as shown in the figure below. The others know the pattern of the equivalence fractions without utilizing the bars.

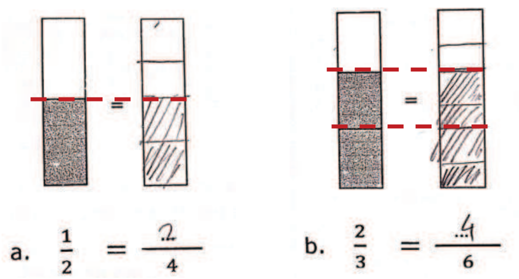


Figure 5.31. Students' Work in Finding the Equivalent Fractions

3. The comparison of fractions

In comparing fractions in the fourth problem, the students come up with different solutions in solving this problem, such as using cross multiplication, drawing the bar representing the fractions, and by using the common denominator. Most of the students represent the problems in the bars. There is an improvement in solving this problem, that students know how to draw the bars to compare fractions. They already understand that the bar should be in the same size and should be partitioned into equal parts. Figure 5.32 below presents the examples of students' answers in ordering $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{3}{8}$.

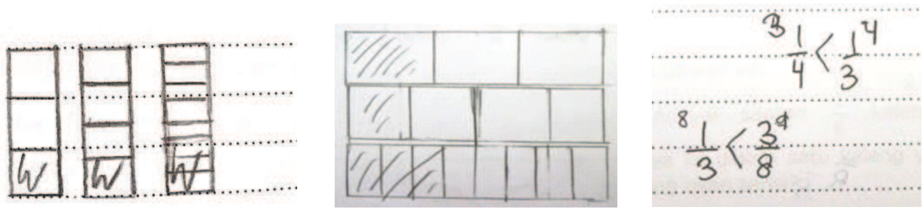


Figure 5.32. Examples of Students' Solution in Ordering Fractions

4. The estimation of the sum of two fractions

Most of the students start to consider a half as the benchmark in estimating. It can be seen from the examples of students' answer in the figure below that in estimating $\frac{2}{3}+\frac{1}{4}$, the students notice that $\frac{2}{3}$ itself is greater than a half, so the result must be more than a half. However, some students do not use their fraction sense and use benchmarks in solving the problem. They add the fractions by using the idea of common denominator and then compare it to a half. In this case, students start to understand how to add fractions with different denominator and how to find the common denominator. The figures below are the examples of students' answer in estimating $\frac{2}{3}+\frac{1}{4}$.

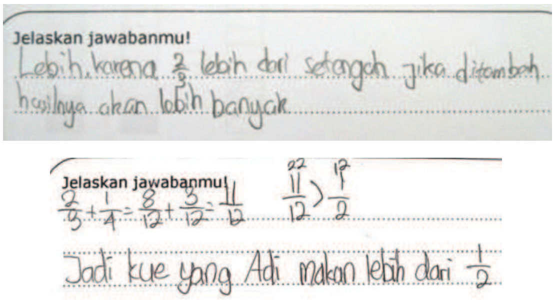


Figure 5.33. Examples of Students' Solution in Estimating $\frac{2}{3}+\frac{1}{4}$

5. Addition of fractions

In the last problem, students need to add fractions with either the same or the different denominators in the form of bars. The students do not find difficulties in solving the addition of fractions with the same denominators (Figure 5.34(a)). Moreover, the students show their understanding in adding fractions, which most of them use the idea common denominator of fractions. Some of them use the bars as a tool to find out the common denominator, which they try to make the common extension lines in the three bars (Figure 5.34(b)). The other students know how to find the common denominator and use the idea of equivalence of fractions (Figure 5.34(c), (d)).

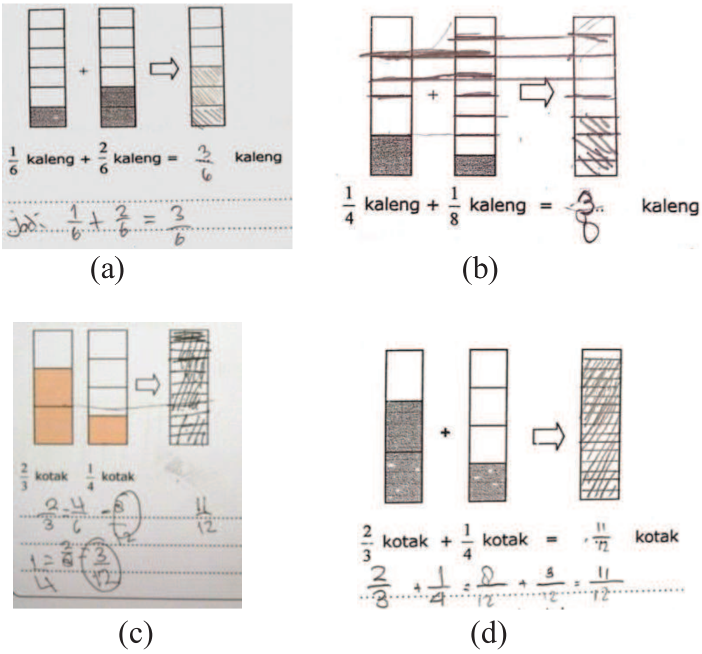


Figure 5.34. Examples of Students' Solution in Adding Fractions by using Bars

Conclusion

In general, there is an improvement of students' understanding after joining the lessons. They know how to compare fractions by using bars, which they start to use the equal size bars and partition it into equal parts. That indicates that students are aware the concept of the unit and the concept of partitioning. They also start to understand how to find the equivalence of fractions and its representation in the form of bars. Moreover, the result also indicates that some students are aware how to estimate the sum of two fractions by using benchmarks and begin to understand the idea of common denominator in adding fractions and its representation in the form of bars.

F. Validity and Realibility of the Analysis

As can be seen in the analysis above, the researcher uses various data in analyzing the data, those are students' written work, video recordings, and field notes. The findings obtained from the video are supported by students' work and field notes. Moreover, the researcher attempts to keep focus on what is intended to measure by always contrasting the result to the HLT. Thus, it contributes to the internal validity of the analysis.

Furthermore, the researcher frames some issues as examples of something more general in order to enable the generalization to other contexts, for instance the issue when students misinterpret the concept of a whole in determining a fraction (in the second cycle of activity 1 of meeting 1). In this issue, the researcher describes factors that might cause this issue and the anticipation of this issue. Those descriptions of some issues contribute to the external validity of the analysis. In addition, the implementation of the instructional design in the real classroom setting will strengthen the ecological validity of the study.

In regard to the reliability, there is a peer examination, in which the researcher discusses with the teacher and colleagues about the data analysis and the interpretation of the video fragments in order to minimize the subjectivity in the analysis. Thus, it increases the internal reliability of the study. Moreover, the researcher tries to describe thoroughly the theories underlying the design, how the study has been carried out, the learning process, the failures and successes, and how the conclusions are derived. Hence, it contributes to the external reliability of this study.

CHAPTER VI
CONCLUSION AND DISCUSSIONS

This chapter comprises the conclusion and the discussions based on the result elaborated in the previous cycle. In the conclusion, the researcher seeks to answer the research questions and includes the Local Instruction Theory (LIT). In the discussions, the weaknesses and the limitation of this study are presented. Moreover, the researcher also provides suggestions and recommendations for further studies.

A. Conclusion

The aim of this study is to investigate how teachers support students' understanding of the addition of fractions. To reach the goal, the researcher attempts to answer the research main question: *"How can instructional activities in this study support students' understanding of the addition of fractions?"*. In this study, the researcher integrates the use of paper strips and bar model. Moreover, the researcher pays attention on the use of estimation skill to lead students to avoid the procedure 'top+top over bottom+bottom' in adding fractions. Therefore, the researcher specifies the main research question into two research sub questions as follow:

1. *How do paper strips and bar model promote students' understanding of the addition of fractions?*
2. *How does the estimation skill lead students to avoid the incorrect procedure 'top+top over bottom+bottom' in solving the addition of fractions problems?*

In the next sections below, the researcher presents the description of the answer of those questions.

1. The Answer of the First Research Sub Question

The first question can be answered by summarizing the result of the first and third meeting, in which the students utilize paper strips and bar model in the learning process. The focus of the use of paper strips and bar model in the first meeting is the idea of equivalent fractions, while in the third meeting the focus is the idea of common denominator in adding fractions.

At the beginning of the first meeting, the students do a fair sharing activity that is aimed at grasping the idea of partitioning into equal parts and the notation of fractions. Then, the students construct their understanding about equivalent fractions by exploring the paper strips and then translate it into the form of bar model. As have been described in the result of the second activity of the first meeting, the students begin to notice the use of the common extension lines that they observe in the measuring strips with different number of partitions to determine the equivalent fractions in the form of bars. For example, as illustrated in the figure beside, the students extend the line representing a fourth in the measuring strip so that it coincides with the line of other measuring strips, such as two eighth and three twelfth.

In the third meeting, the students use the knowledge of the equivalence of fractions they have explored in the measuring strips and bar model to construct the idea of common denominator. To find the common denominator, the students use their insight of the common extension lines to get the common number of partitions (representing the common denominator) in the paper strips and in the bar model. The students do not find any difficulties in solving the addition of fractions with the same denominator. In adding fractions with different denominators, they are aware that the two paper strips or bars representing two fractions being added should have the same number of partitions (as the common denominator). In other words, they begin to notice that to add two fractions, the denominators of the fractions have to be equal.

From the result of this study, the researcher can deduce that the use of paper strips and bar model helps students to grasp the idea of the equivalence fractions and the common denominator, and the reasoning behinds it. Those ideas are useful to support students' understanding of the addition of fractions, either with the same or with different denominators.

2. The Answer of the Second Research Sub Question

The estimation of the sum of the two fractions is the focus of the second meeting and the last activity of the third meeting. Before the students learn about the estimation skill, firstly, they learn how to compare fractions. The result indicates that the students are able to compare fractions with various strategies, such as by using bars and by using their fraction sense.

The students use the knowledge of comparing fractions to estimate the sum of two fractions. The result of the estimation activity shows that the students are aware of how to estimate the sum of two fractions. They look at the size of the fractions being added, and then compare it to a half as the benchmark. For example, they know that the result of $\frac{2}{3}+\frac{1}{4}$ must be more than a half since $\frac{2}{3}$ itself is more than a half.

Furthermore, the students can check the exact result of the addition of fractions, such as $\frac{2}{3}+\frac{1}{4}$, with their estimation. When they do 'top+top over bottom+bottom' in adding fractions, they notice that the result is not corresponding to their estimation. At that time, the students begin to be aware that the procedure 'top+top over bottom+bottom' in incorrect. In conclusion, their estimation of the sum of two fractions can lead them to grasp that they cannot apply the procedure 'top+top over bottom+bottom' in adding fractions.

Based on the description of answers of the two research sub questions above, the researcher concludes that the instructional activities utilizing paper strip and bar model can support students in constructing the understanding of the addition of fractions. Moreover, the activities involving the estimation skill is also important to check whether the result of the addition is reasonable or not, and thus it leads students to be aware that the procedure 'top+top over bottom+bottom' in adding fractions is an incorrect procedure.

3. Local Instruction Theory (LIT)

The HLT that has been refined and improved during the three cycles forms an empirically grounded LIT. According to Gravemeijer (2004b), the overview of LIT comprises "the description of, and the rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topic" (p. 107). Moreover, according to Nickerson and Whitacre

(2010), "LIT describes goals, envisioned learning route(s), and instructional activities or plans of action based on underlying assumptions about teaching and learning" (p. 233). In the following description, the researcher provides the contribution of this study to the LIT on the topic the addition of fractions. The complete learning activities can be seen in the teacher guide on appendix 5.

a. Goals

1) Students are able to partition into equal parts.
2) Students can use fractions notation.
3) Students understand the idea of the equivalence of fractions.

Assumptions

Based on the teaching experiments, the researcher anticipates that when students are asked to share some pieces of bread to some children, students come up with partitioning the bread into equal parts as they have a common sense of how to share bread. The researcher also anticipates that students use halving method in dividing the bread. Moreover, the researcher predicts that some students have an insight of fractions as division, in which they argue that the number of pieces of bread shared represents the numerator and the number of children represents the denominator (van Galen et al., 2008; Fosnot & Dolk, 2002). Furthermore, the researcher predicts that some students use the addition of fractions in tackling the problem.

The use of using paper strips is important to achieve the first and second goal. In exploring paper strips, the researcher anticipates that students grasp the idea of partitioning and the relation between paper strips with different number of partitions that lead them to grasp the idea of the equivalence of fractions. Moreover, the researcher predicts that some students notice the formal pattern to find the equivalent of given fractions.

Envisioned Learning Route and Rationale

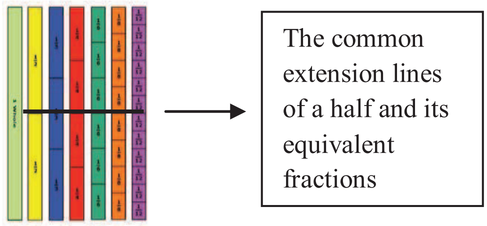
In order to support in achieving the goals, the context of fair sharing can be used as it is a rich context that comprises many ideas. When the students are asked to share some pieces of bread to a number of children, they learn how to divide the pieces of bread fairly. Moreover, the students can explore relationships between fractions that can lead them to the idea of the equivalence of fractions and the arithmetic with fractions (Fosnot & Dolk, 2002). For instance, when the students share 3 pieces of bread for 4 children, there are many possibilities of answers, such as each child gets $\frac{1}{2}$ and $\frac{1}{4}$ of a piece of bread, $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ of a piece of bread, and $\frac{3}{4}$ of a piece of bread. From these differences of students' opinion, the teacher can raise the idea of equivalence of fractions and the addition of fractions.

In addition, as suggested by the theory of RME, the use of contexts and models (paper strips and bar model) is important to support students to understand the idea of partitioning and the equivalence of fractions. According to van Galen et al. (2008), paper strips and bar model can be conceptual models that help students construct the concept and the reasoning of fractions. Producing and exploring the fractions strips made from the paper strips will familiarize the students to the image of the size of fractions, and thus it will help them build their fractions sense. Moreover, the students can explore the relationships among fractions and the reasoning of the equivalent of fractions in the fractions strips. Afterwards, the students take the idea of equivalent fractions into the bar model. Later, the insight of the equivalence of fractions is useful to grasp the idea of common denominator.

Instructional Activities

1) The teacher tells the context of fair sharing and asks the students to discuss it with their group.
2) In the discussion, the teacher anticipates of many strategies and ideas coming up in this problem. The teacher stresses on how to share fairly and name the parts (use the notation of fractions) of the bread. Then, the teacher needs to support the students to build their fractions sense to understand the reasoning of fractions, such as the equivalence of fractions and the comparison of fractions.
3) The teacher engages the students to the measuring activity. The students are asked to measure a tube whose particular parts of it filled with water and then to make measuring strips with different number of partitions. Then, the students put the measuring strips in the poster paper such that it forms a set of fraction strips.
4) The teacher has to point on the relation between various partitions of fractions strips to lead the students to grasp the initial idea of the equivalence of fractions. The teacher discusses the concept of the

equivalent of fractions by noticing the common extension lines in the measuring strips as in the figure below.



5) The teacher supports the students to translate the reasoning of equivalent fractions in the measuring strips to the bars. Then, the teacher engages the students to notice the patterns of equivalent fractions from some examples, that the denominator and the numerator have a common multiple of the denominator and the numerator of the initial fraction.

b. Goal

1) Students can compare fractions.
2) Students are able to estimate the sum of two fractions by using benchmarks.

Assumptions

Students of the third grade have learned about comparing a unit fraction with another unit fraction, and about comparing two fractions with the same denominators. Based on the researcher's observation and experience, teachers in some schools engage students to use algorithms to solve fractions problems. In comparing fractions, teachers introduce the cross multiplication and find the common denominator algorithm. Moreover, some teachers also engage students to use pictures to compare fractions. Thus, the researcher predicts that students use those strategies in comparing fractions. In order to reach the second goal, the teacher needs to make opportunities for students to build their fraction sense and use benchmarks to compare fractions. When students are able to compare fractions with benchmarks, they will find it easier to find reasonable estimation of the problems of the addition of fractions (Cramer, 2008).

Regarding students' estimation skill, the researcher anticipates that students add the fractions instead of estimating. It is because teachers usually do not put more emphasize on constructing students' fraction sense and do not familiarize students on the estimation skill. Since students have not learned about the addition of fractions with different denominators yet, they might add the fractions by using procedure 'top+top over bottom+bottom'. As the result, their estimation of the sum of two fractions is not reasonable.

Envisioned Learning Route and Rationale

To build the students ability in comparing fractions and estimating the sum of two fractions, the teacher needs to set a situation to engage the students to use their fraction sense. The use of bar model to compare fractions is useful to support them to construct their mental image of fractions. Moreover, the role of the teacher is important to encourage the students to use benchmarks to compare fractions. The idea of comparing fractions is useful for students to grasp the idea of estimation of the sum of two fractions by using benchmarks (Cramer et al., 2008). For instance, once the students can compare and notice that $\frac{2}{3}$ is more than a half, they will get that the sum of $\frac{2}{3}$ and $\frac{1}{4}$ is more than a half.

The estimation of the sum of two fractions by using benchmarks is a useful concept for students to support their understanding of the addition of fractions (Reys et al., 2009; Cramer et al., 2008). Students' estimation skill can encourage their reasoning to check whether their solution of the addition of fractions is reasonable or not. Thus, it will prevent the students from doing the procedure 'top+top over bottom+bottom'. For instance, students will notice the result of $\frac{2}{3}+\frac{1}{4}$ can never be $\frac{3}{7}$ since they know that $\frac{2}{3}$ is more than $\frac{1}{2}$ so the result must be more than $\frac{1}{2}$. In addition, in the learning process, the teacher has an important role to support the students to use benchmarks to compare fractions and to estimate the sum of two fractions.

Instructional Activities

1) The teacher asked the students to compare fractions in a context.
2) In the class discussion, the teacher supports the students to reason by using bars and benchmarks instead of by using algorithms to compare. The use of bars (pictures) will help the students to construct mental image and sense of fractions. Then, the teacher engages the students to use their fraction sense to use benchmarks in comparing fractions.
3) After the students learn about how to compare fractions, the teacher engages the students to estimate the sum of two fractions by using benchmarks.

4) In the class discussion, the teacher supports the students on how to estimate the sum of two fractions. The teacher also needs to support them to use benchmarks to estimate the sum of two fractions by comparing the added fractions with benchmarks such as a half and one. To put more emphasize on it, the teacher can show the fractions strips and help them to build their mental image of fractions and use the benchmarks. Moreover, the teacher also shows that the procedure 'top+top over bottom+bottom' is incorrect.

c. Goal

1) Students know that the procedure 'top+top over bottom+bottom' is incorrect.
2) Students are able to find common denominator by using paper strips and bar model.
3) Students are able to add fractions by using bar model.

Assumptions

Since students have learned about adding two fractions with the same denominator at glance, the researcher predicts that students are able to add two fractions with the same denominators. However, based on the researcher's experience and the result of many studies, in adding fractions with different denominators, students might apply the common mistake 'top+top over bottom+bottom'. It might because they consider a fraction as two different numbers, so they apply the algorithm of the addition of whole numbers (Howard, 1991; Young-Loveridge, 2007).

As the effect of the use of cross multiplication procedure in comparing fractions, the researcher also anticipates that some students will apply such algorithm to add fractions, namely the cross addition procedure. Students who apply the strategies 'top+top over bottom+bottom' and cross addition indicate that they do not understand about the idea of common denominator in adding fractions.

On the other hand, some students might know that to add fractions with different denominators, they have to find the common denominator by finding the Least Common Multiple (LCM) of the denominators.

After students learn about how to add fractions and how to estimate the sum of two fractions, the researcher anticipates that they are aware that the procedure 'top+top over bottom+bottom' is incorrect.

Envisioned Learning Route and Rationale

According to van Galen et al. (2008), a concrete context makes meaning to the concept of the addition of fractions. Measuring activity is an example of context in which students can observe the result of the addition of fractions and make reasoning towards it. Moreover, the role of models (paper strips and bar model) is important to model the situation and to explore the reasoning of adding fractions.

After experiencing in measuring the total water before and after being poured from two tubes, the students explore the reasoning of the result of the measuring activity in the paper strips. Here, the paper strips function as the model of the situation. The role of the teacher is essential to ensure the students know the relationships with fractions in the paper strips to grasp the idea of the common denominator. Then, the paper strips can function as the model for mathematical reasoning to solve other addition of fractions problems. Afterwards, the students shift from using the paper strips to a more abstract model, namely bar model.

In the learning process, it is important that the teacher should support the students to grasp the idea of common denominator by using paper strips and bar model. After the students learn how to add fractions, the teacher engages the students to use their estimation skill to show that the procedure 'top+top over bottom+bottom' is incorrect. Checking the result of the addition with their estimation will encourage the students to be aware that the outcome of the procedure 'top+top over bottom+bottom' is not reasonable For instance, when the students know that the result of $\frac{2}{3}+\frac{1}{4}$ must be more than $\frac{1}{2}$, they will notice that $\frac{3}{7}$ is not a reasonable answer.

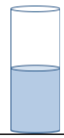
Instructional Activities

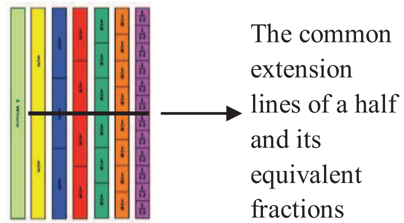
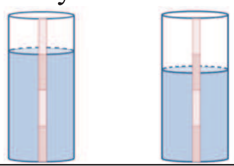
1) The teacher engages the students in the measuring activity, in which measure what parts of two tubes filled with water before after pouring the water into one tube.
2) The teacher represents the measuring situation in the paper strips and encourages the students to explore the result by using the paper strips. Then, the students are asked to solve other addition of fractions problem by using paper strips.

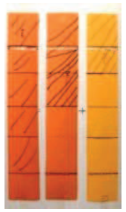
3) In the discussion, the teacher encourages the students to notice the reasoning of the common number of partitions representing the common denominator by using paper strips.
4) Moreover, the teacher needs to support the students' reasoning to grasp the relation between the common number of partitions and the pattern of common denominator of two fractions being added.
5) The teacher engages the students to show their reasoning in solving some problems about the addition of fractions in the form of bars.
6) The teacher strengthens the students' reasoning of the common number of partitions representing the common denominator in the form of bars. The teacher stimulates the students to notice that in adding two fractions, both bars have to have the same partitions. The teacher can use the fraction strips and remind them about the equivalence of fractions if some students do not understand at all how to solve it.
7) After the students are able to solve addition of fractions problems, the teacher reminds about the estimation of the sum of two fractions and points that the output of the procedure 'top+top over bottom+bottom' is not reasonable.

Based on the description above, the table below summarizes the LIT. As suggested by Gravemeijer (2004b), the table contains the tools used in the activities and its imagery, the activities, and the potential mathematical discourse topics.

Table 6.1. The Summary of the Local Instruction Theory on the Topic on the Addition of Fractions

Tools	Imagery	Activity	Potential Mathematical Discourse Topics
Representation of bread in the fair sharing activity (activity 1 worksheet 1) 	Signifying the situation of sharing fairly 2 pieces of bread to 3 children and 3 pieces of bread to 4 children.	Sharing 2 pieces of bread to 3 children and sharing 3 pieces of bread to 4 children. Then, determining which group get bigger share	- Partitioning into equal parts - The equivalence of fractions - Comparison of fractions - Addition of fractions
Tubes which $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}$ parts of it filled with water	The tubes filled with water signify the measuring situation. The tubes have the same size and are transparent. 	Measuring the water in the tube and then make the measuring strips with two different number of partitions	- Measuring activity - Partitioning into equal parts
Paper strips and poster paper	The number of partitions in the paper strips signifies what parts of the tubes filled with water in the measuring activity.	Producing measuring strips with different partitions and then putting together in the poster paper so that it forms measuring strips with various numbers of partitions.	- Partitioning into equal parts - Equivalent fractions

Tools	Imagery	Activity	Potential Mathematical Discourse Topics
Measuring strips (Partitioned paper strips)	The paper strips with different number of partitions are put together in the poster paper from the smallest number of partitions to the largest number of partitions.  The measuring strips signify the various measures of water filling the tubes.	Reasoning about the equivalent fractions by noticing the common extension lines in the measuring strips with different partitions 	- Mental image of fractions - Equivalent fractions
Bar model (activity 2 worksheet 1)	Bar model signifies the process of finding the equivalent fractions that is by extending the lines representing the fractions as in the measuring strips.	Reasoning about the equivalent fractions as they do in the measuring strips	- Representation of fractions in the bar model - Equivalent fractions
Representation of tube filled water (activity 1 worksheet 2) 	Signifying the representation of tubes filled with water in the measuring activity.	Labeling the fractions of given pictures	- Representation of fractions in the bar - Partitioning into equal parts

Tools	Imagery	Activity	Potential Mathematical Discourse Topics
Bar model	Signifying the situation in comparing fractions	Finding which fraction is bigger in the problem of comparison of fractions	Comparison of fractions
Activity 3 worksheet 2	Containing problems of comparing fractions to a half as the benchmark and hints how to estimate the sum of two fractions	Discussing how to estimate the sum of two fractions by using benchmarks	The estimation of the sum of two fractions
Two tubes which $\frac{1}{2}$ and $\frac{1}{4}$ parts of it filled with water and an empty tube	Signifying the measuring situation to explore the sum of two fractions. The tubes have the same size and are transparent.	Pouring the two tubes filled with water to an empty tube and then investigating the total parts of the tube filled with water	Addition of fractions
Paper strips	Signifying the process of adding fractions by pouring water from two tubes. Two paper strips represent the measures of water in the two tubes (the two fractions being added), and another paper strip represents the measure of the result. 	Exploring the idea of common denominator in adding fractions by extending the lines of each bar representing fractions being added to get the common number of partitions	The idea of common denominator in adding fractions

Tools	Imagery	Activity	Potential Mathematical Discourse Topics
Bar model (activity 2 worksheet 3)	Signifying the process of adding fractions	Reasoning about the common denominator as they do in the paper strips	Common denominator in adding fractions
Activity 3 worksheet 3	Containing problems to crosscheck the estimation and the exact result in order to prove that the procedure 'top+top over bottom+bottom' is incorrect.	Estimating the sum of two fractions, finding the exact result by using bars, and then checking whether or not the result is corresponding to the estimation.	The estimation of the sum of two fractions to lead students to avoid the procedure 'top+top over bottom+bottom'

B. Discussions

In this section, the researcher elaborates some weaknesses and limitations of this study. Moreover, the researcher also includes suggestions and the recommendations for further studies.

1. The Weakness Points of This Study

a. The teacher's role

The discussion between the researcher and the teacher about the learning activities and the teacher's role before the teaching experiments is not thorough. Although the researcher has a discussion with the teacher during the lessons, the teacher misses some points during the learning activities. For instance, the teacher does not ask the students' reasoning when they argue that the fraction of the situation three pieces of bread are shared among three children is $\frac{3}{4}$.

b. Students' discussion

Many students do not get accustomed to have a discussion and to state their opinion. When the researcher asks some students about their reasoning towards a problem, they are just silent because they are shy or afraid. In addition, in the class discussion, only a few students brave to speak. Thus, the expected students' participation in the discussion is not occurred.

c. Estimation Skill

In this study, the researcher want to focus on showing students that by using their estimation, they can notice that the procedure 'top+top over bottom+bottom' in adding fractions is incorrect. Thus, in this study, the researcher just uses a half as the benchmark in estimating. Moreover, in this study, students learn the estimation skill in a short time because the limitation of time. The researcher does not explore deeper on students' estimation skill.

2. Suggestions and Recommendations for Further Studies

Based on the result and the weaknesses of this study, the researcher provides suggestions and recommendations for further studies as follows.

a. Preparation before the teaching experiments

The researcher suggests that in the preparation before conducting the lessons, it is a need to discuss thoroughly with the teacher about the learning activities, the students' conjectures and how the teacher's responds toward it, and the teacher's role during the lessons. In addition, to deal with the students that are quiet, the researcher suggests that teachers create a situation to motivate the students to speak aloud, for example by pointing the students who are silent to express their opinion.

b. The use of context and models

Learning activities should start from an informal situation. Thus, contexts and models have an essential role to support students' meaningful learning. A rich context, such as the fair sharing problem, facilitates students to explore many ideas within the problem. Models, such as paper strips and bar model, are also important as tools to explore relationships among numbers and to solve problems. Papers strips and bar model are also useful to build the students' mental image of fractions and to develop their fraction sense. When students learn by exploring concepts within contexts and models, they actively construct the knowledge and thus the knowledge will embed in their mind.

c. The estimation skill

Besides supporting students in constructing their concept of the addition of fractions, teachers also have to pay attention to the common mistake in adding fractions, which is the procedure 'top+top over bottom+bottom'. Estimation skill is important to support students' reasoning in examining whether their result of the addition is reasonable or not. Thus, it also will prevent students from doing the procedure 'top+top over bottom+bottom'.

For further studies, the researcher suggests to explore and engage students more on their estimation skill and use various benchmarks. For instance, by conducting a mini lesson that focuses on the estimation skill. Moreover, to develop students' estimation skill, teachers need to encourage students' fraction sense. The use of models to visualize fractions can help students in developing their sense of the relative size of fractions. As suggested by Johanning (2011), the use of paper strips and number line will help students to see the relationship between a fraction and other fractions, and the relationship between a fraction and a whole. This will support them to build their sense of the size of fractions by using benchmarks.

When students have fraction sense, they will be able to find the reasonable estimation result of the addition of fractions. Hence, for further studies, it is important to pay more attention on the estimation skill of the addition of fractions.

d. The integration of other interpretations of fractions

There are three interpretations of fractions used in this study, those are a fraction as a part-whole relationship, as a measure, and as a quotient. The interpretation of a fraction as a quotient is included in the fair sharing activity, while the interpretation of a fraction as a measure is included in the measuring activity. During the learning activities, the teacher supports students' learning by involving the interpretation of a fraction as a part-whole relationship.

After students learn fractions involving the three interpretations, students start to have mental image and understanding of the concept of fractions. For instance, they begin to understand the idea of equivalent fractions and the common denominator in adding fractions. Thus, the use of various interpretations of fractions in the learning activities is helpful to support students' understanding of fractions.

For the next study, teachers can integrate other interpretation of fractions in the learning activities, such as a fraction as an operation. The example of the problem is *'Uncle Adi divides 20 candies. Dita gets a half of it and Uca gets a third of it. What fraction is the total candy that Uncle Adi gives to Dita and Uca?'*. In this case, teachers should support students the reasoning of the common denominator.

e. Limitation of time

Because the limitation of time, each cycle in this study consists of three meetings, in which there are two or three activities within each meeting. All activities in each meeting can be covered in around 90 minutes. However, the students seem to achieve too much material. Thus, for further studies, it will be better if the activities are re-arranged into four or five meetings. For example, the activity about estimation (the third activity of the second meeting and the third activity of the third meeting) can be combined as a meeting.

d. Formal algorithm

In this study, students learn the idea of common denominator in adding fractions by using paper strips and bar model. However, the students have not yet generalized the idea of the common denominator that they have learned in the paper strips and in the bar model into the formal algorithm. Thus, for further studies, teachers may encourage students to be aware of the pattern of the idea of common denominator so that they understand the reasoning behind the formal algorithm in adding fractions.

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Appendix 1

Interview Scheme

<u>Students' starting points</u>

- What do students know about fractions?
- What are students' difficulties in learning fractions?

<u>Classroom management</u>

- How is the interaction between the teacher and the students? Between the students and the students?
- Are there any rules in the class during the lessons?
- Do you usually let students work in small group or individually? If they work in small group, how many students are there in each group?
- How do you group the students? What is your consideration for grouping the students? Is it based on the score or performance level?

<u>Characteristic of students</u>

- How is students' ability or achievement?
- What do you think about the students? How active they are?

<u>The way the teacher explains the addition of fractions</u>

- How do you usually explain the addition of fractions?
- How do you engage students in the learning process?
- What books do you use?
- What tools do you use?
- How do you discuss a problem?

Appendix 2

Observation Scheme

<u>Practical Setting</u>

- How many students are there in the class?
- How is the classroom setting (the setting of desks and chairs)?

<u>The teaching and learning process</u>

- How does the teacher prepare the lesson?
- How does the teacher explain a topic? What teaching approach does the teacher use?
- What tools does the teacher use?
- How active are the students? Who is the most active student? Who is the least active students?
- How is the interaction between the teacher and the students? And between the students and the students?
- What does the teacher do when the students work?
- How does the teacher discuss a problem?
- Does the teacher judge the students' answer?
- How does the teacher point a student?
- Are there any rules (classroom norms)?
- How do the students work, in group or individually? If in group, how does the teacher group the students?
- How does the discussion take place in the classroom? How does the teacher lead the class discussion? And how are the students' responses?
- Do students get used to share and express their idea/opinion?

Appendix 3

Pre- dan Pos-tes

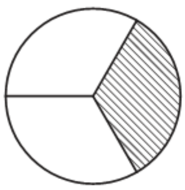
Nama : ………………………………………………….

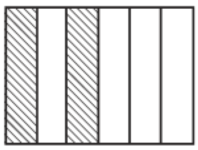
Kelas : ………………………………………………….

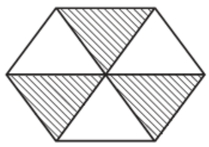
Tanggal : ………………………………………………….

Jawablah pertanyaan-pertanyaan berikut!

1. Tulislah pecahan dari bagian yang diarsir pada gambar berikut ini!



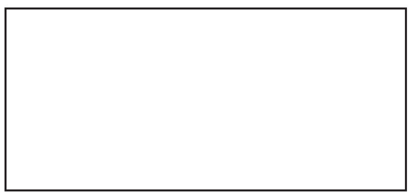




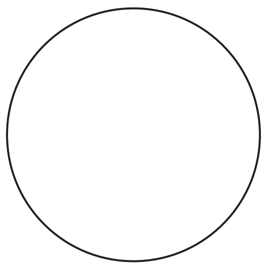
……….. ………. ………. ……….

2. Bagilah gambar berikut menjadi bagian yang sama dan arsirlah sesuai dengan pecahan yang diberikan!

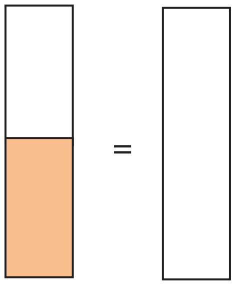
a. $\frac{3}{8}$



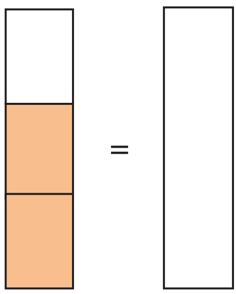
b. $\frac{3}{4}$



3. Bagilah strip pada sisi kanan dan arsirlah sehingga mempunyai nilai pecahan yang sama dengan strip pada sisi kiri, kemudian tuliskan pecahannya!

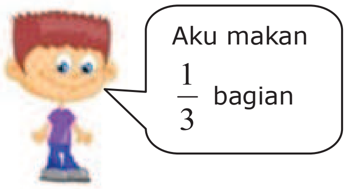


a. $\frac{1}{2} = \frac{\dots}{4}$



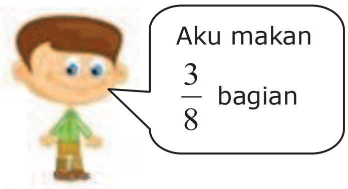
b. $\frac{2}{3} = \frac{\dots}{6}$

4. Adi, Ara, dan Fadil berbagi sebuah kue. 



Adi

Ara



Fadil

Urutkanlah anak-anak tersebut dari yang memakan bagian kue paling kecil sampai yang memakan bagian kue paling besar!

Jelaskan caramu!

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5. Sebelum berangkat sekolah, Adi memakan $\frac{2}{3}$ bagian kue. Sepulang sekolah, ia makan lagi $\frac{1}{4}$ bagian. Apakah jumlah kue yang Adi makan lebih atau kurang dari setengah?

Jelaskan jawabanmu!

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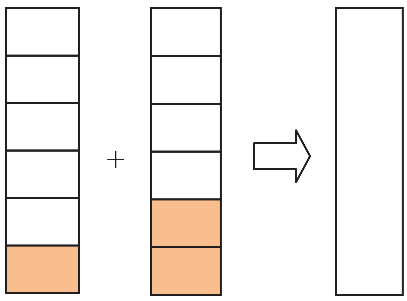
6. Selesaikanlah soal di bawah ini.

   a. Ibu Doni mempunyai santan $\frac{1}{6}$ kaleng dan $\frac{2}{6}$ kaleng. Jika ia mencampurkannya ke dalam satu kaleng yang berukuran sama, berapa bagian kaleng yang akan terisi?



$\frac{1}{6}$ kaleng + $\frac{2}{6}$ kaleng = ... kaleng

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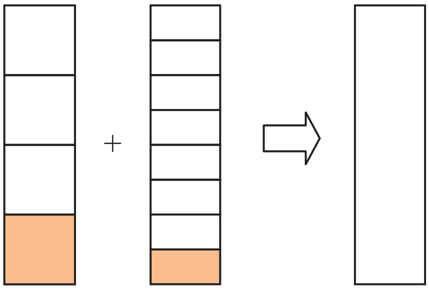
b. Ibu Doni mempunyai santan $\frac{1}{4}$ kaleng dan $\frac{1}{8}$ kaleng. Jika ia mencampurkannya ke dalam satu kaleng yang berukuran sama, berapa bagian kaleng yang akan terisi?



$\frac{1}{4}$ kaleng + $\frac{1}{8}$ kaleng = ... kaleng

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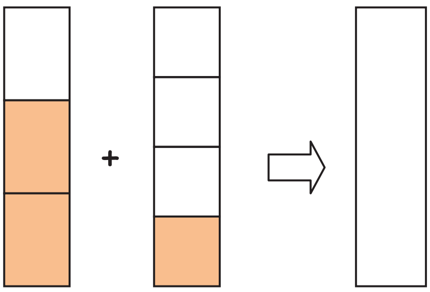
c. Ibu Doni juga mempunyai susu cair $\frac{2}{3}$ kotak dan $\frac{1}{4}$ kotak. Jika ia mencampurkannya ke dalam satu kotak yang berukuran sama, berapa bagian yang akan terisi?



$\frac{2}{3}$ kotak + $\frac{1}{4}$ kotak = ... kotak

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Pre and Post-test

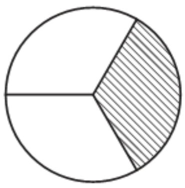
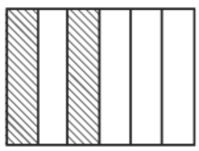
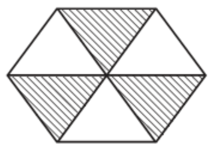
Name : ………………………………………………….

Class : ………………………………………………….

Date : ………………………………………………….

Fill in the blank!

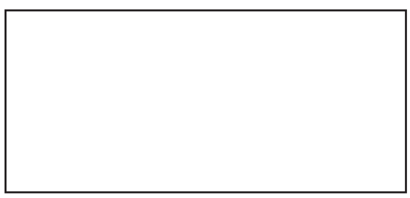
1. What fractions is the shaded area?

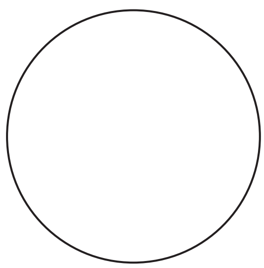

……….. ………. ……… ………

2. Shade the area as the corresponding fractions!

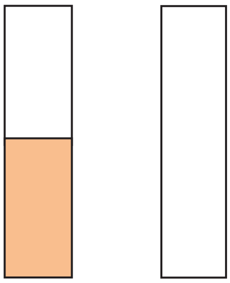
a. $\frac{3}{8}$



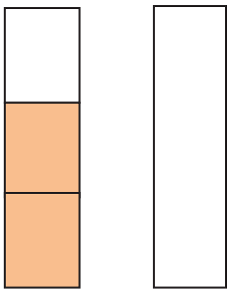
b. $\frac{3}{4}$



3. Partition the bar in the right side and shade the parts such that it has the same value as the bar in the left side.



a. $\frac{1}{2} = \frac{\dots}{4}$



b. $\frac{2}{3} = \frac{\dots}{6}$

4. Adi, Ara, and Fadil shared two similar cakes.

I eat $\frac{1}{3}$ parts of the bread

I eat $\frac{1}{4}$ parts of the bread

I eat $\frac{3}{8}$ parts of the bread

Order the children from who eats the smallest parts of bread to who eats the biggest parts of the bread!

Explain your strategy!

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5. Before going to school, Adi eats $\frac{2}{3}$ parts of bread. After back from the school, he eats $\frac{1}{4}$ more parts the bread. Is the total parts of bread that Adi eats more or less than a half of the bread?

Explain your answer!

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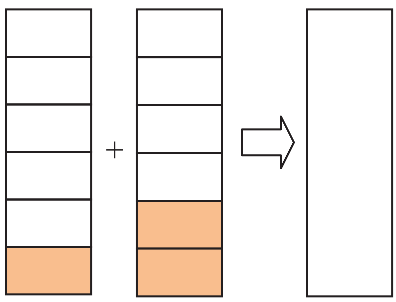
6. Solve the problems below.

   a. Mrs. Doni has $\frac{1}{6}$ and $\frac{2}{6}$ of a can of coconut milk. If she pours it together into one can, what part of the can will be filled?



$\frac{1}{6}$ of a can + $\frac{2}{6}$ of a can = ... of a can

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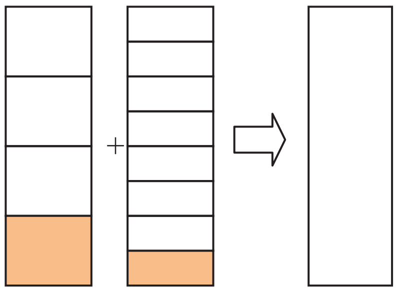
b. Mrs. Doni has $\frac{1}{4}$ and $\frac{1}{8}$ of a can of coconut milk. If she pours it together in one can, what part of the can will be filled?



$\frac{1}{4}$ of a can + $\frac{1}{8}$ of a can = ... of a can

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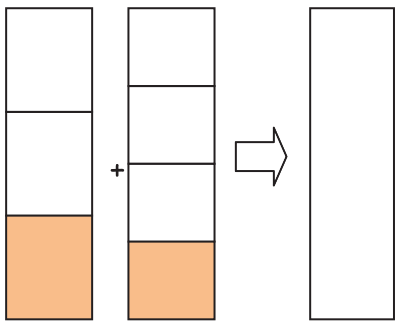
c. Mrs. Doni has $\frac{2}{3}$ and $\frac{1}{4}$ of a box of milk. If she pours it together in one box, what parts of the box will be filled? Is this more than a half can?



$\frac{2}{3}$ of a can + $\frac{1}{4}$ of a can = ... of a can

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Appendix 3

WORKSHEETS

Lembar Kerja Siswa 1

Nama : ....

Kelas : ....

Tanggal : ....

Kegiatan 1

Pada perayaan 17 Agustus, Bu Doni menyiapkan makanan kecil untuk anak-anak. Setelah berpartisipasi dalam beberapa lomba, anak-anak akan mendapatkan roti untuk dimakan bersama-sama. Untuk anak-anak yang berpartisipasi dalam lomba bakiak, Bu Doni menyiapkan 2 roti untuk tiap 3 anak, dan untuk anak-anak yang berpartisipasi dalam lomba tarik tambang, Bu Doni menyiapkan 3 roti untuk tiap 4 anak.

→ 2 roti untuk 3 anak

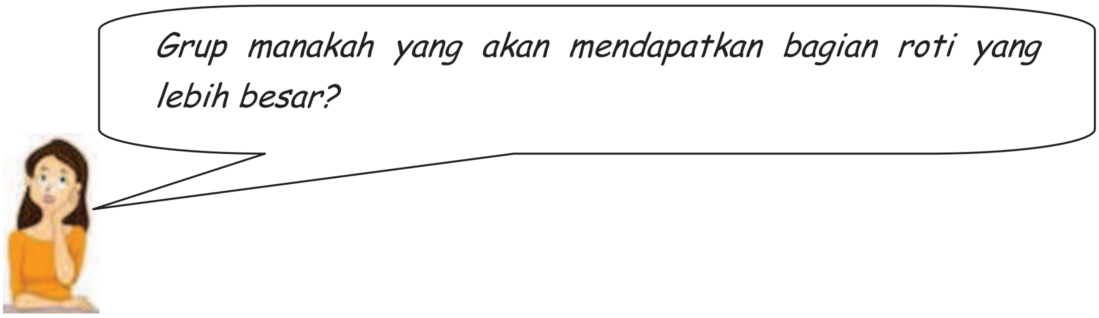
→ 3 roti untuk 4 anak

Grup manakah yang akan mendapatkan bagian roti yang lebih besar? Selidikilah jawabannya dengan melakukan kegiatan berikut!

Berikut adalah roti-roti yang akan dibagikan kepada anak-anak pada lomba bakiak dan tarik tambang. Bagilah roti-roti berikut sesuai jumlah anak pada masing-masing lomba dan tentukan grup mana yang akan mendapatkan bagian roti yang lebih besar.

2 roti untuk 3 anak yang ikut lomba bakiak.

3 roti untuk 4 anak di tarik tambang.



Jelaskan jawabanmu!

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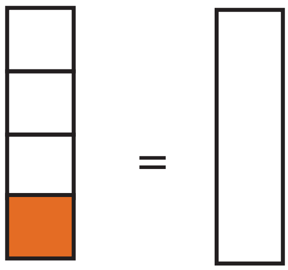
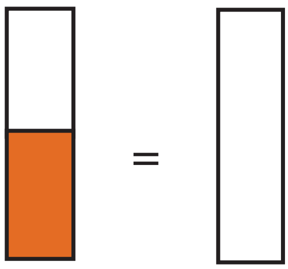
Kegiatan 2

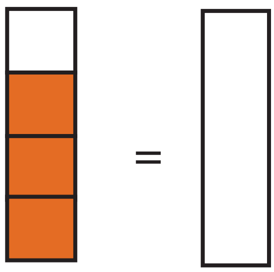
Bagilah strip pada sisi kanan dan arsirlah sehingga mempunyai nilai pecahan yang sama dengan strip pada sisi kiri, kemudian tuliskan pecahannya!



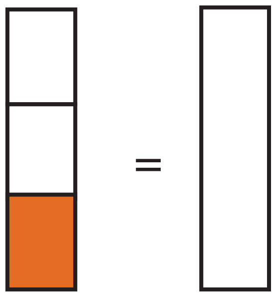
a. $\frac{1}{4} = \frac{\dots}{8}$



b. $\frac{1}{2} = \frac{\dots}{6}$



c. $\frac{3}{4} = \frac{\dots}{8}$



d. $\frac{1}{3} = \frac{\dots}{12}$

Lembar Kerja Siswa 2

Nama : ....

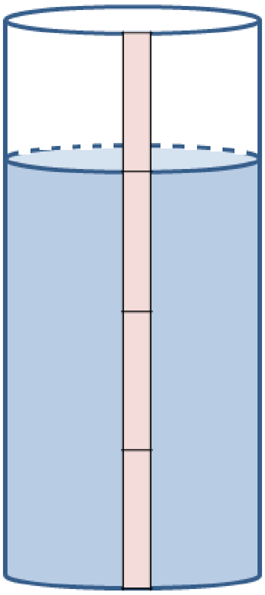
Kelas : ....

Tanggal : ....

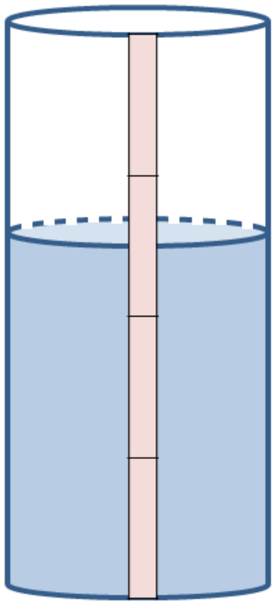
Kegiatan 1

Untuk menentukan siapa yang lebih banyak mengisi air ke dalam ember pada lomba memindahkan air ke ember, Pak Doni beserta panitia menuang air pada ember setiap peserta ke dalam tabung. Kemudian, mereka mengukurnya dengan kertas ukur dan mencatat berapa bagian tabung yang telah terisi dengan melihat tinggi air pada kertas ukur. Rudi dan Zacky baru saja mengikuti lomba itu dan sekarang sedang menunggu pengumuman.

Gambar berikut adalah gambar tabung yang berisi air yang telah mereka pindahkan ke ember. Berapa bagian tabung yang telah terisi air dari ember Rudi dan Zacky?



Air pada ember Rudi



Air pada ember Zacky

Berapa bagian air pada ember Rudi? Gunakan notasi pecahan.

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Berapa bagian air pada ember Zacky? Jelaskan jawabanmu dan gunakan notasi pecahan.

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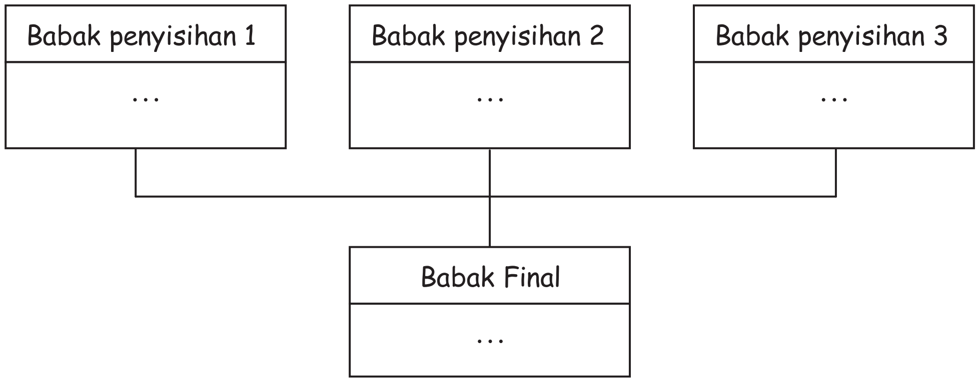
Kegiatan 2

Setelah mendata air dari setiap peserta pada babak penyisihan, panitia mendapatkan hasil sebagai berikut. Bantulah panitia untuk menentukan pemenang pada babak penyisihan pertama, kedua, dan ketiga berdasarkan tabel hasil lomba berikut ini. Kemudian, isilah nama pemenang pada tiap babak penyisihan pada bagan di bawah.

Babak penyisihan	Nama	Bagian air dalam tabung	Siapa pemenangnya? Jelaskan!
1	Adi	$\frac{1}{3}$	
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	
	Doni	$\frac{3}{8}$	
3	Edo	$\frac{5}{6}$	
	Fadil	$\frac{7}{8}$	

Bagan para pemenang dari tiap babak penyisihan lomba memindahkan air.



Siapa yang akan memenangkan babak final? Jelaskan caramu!

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Kegiatan 3

Lingkarilah pecahan yang lebih dari $\frac{1}{2}$ dan jelaskan!

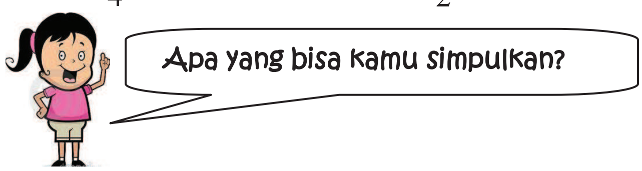
$\frac{3}{4}$	$\frac{4}{12}$	$\frac{2}{8}$	$\frac{5}{8}$
$\frac{3}{10}$	$\frac{4}{6}$	$\frac{1}{3}$	$\frac{9}{12}$

Jawablah pertanyaan berikut dan jelaskan!

1. $\frac{1}{5} + \frac{3}{4}$

   a. Apakah $\frac{1}{5}$ lebih atau kurang dari $\frac{1}{2}$? ....

   b. Apakah $\frac{3}{4}$ lebih atau kurang dari $\frac{1}{2}$? ....



   c. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari $\frac{1}{2}$? ....

   d. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari 1? ....

2. $\frac{3}{5} + \frac{1}{3}$

   a. Apakah $\frac{3}{5}$ lebih atau kurang dari $\frac{1}{2}$? ....

   b. Apakah $\frac{1}{3}$ lebih atau kurang dari $\frac{1}{2}$? ....

   c. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari $\frac{1}{2}$? ....

   d. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari 1? ....

Ibu Dewi akan berpartisipasi dalam lomba masak. Dirumah, ia menyiapkan bahan-bahan yang diperlukan seperti susu cair dan santan. Ia membutuhkan setengah kotak susu cair.

Ibu Dewi mempunyai dua kotak susu berisi $\frac{4}{6}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama.
Setelah dicampur, apakah jumlahnya lebih atau kurang dari setengah kotak?

Jelaskan!

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Lembar Kerja Siswa 3

Nama : ....

Kelas : ....

Tanggal : ....

Kegiatan 1

Gunakan strip kertas untuk menunjukkan hasil penjumlahan berikut!

1. $\frac{1}{3} + \frac{1}{6}$

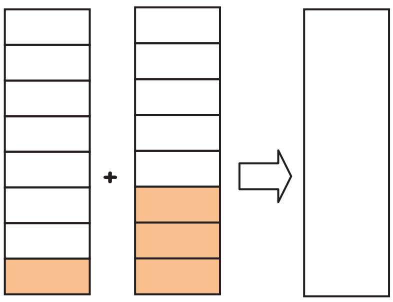
2. $\frac{1}{2} + \frac{1}{3}$

Tempelkan strip kertas kalian pada kertas yang disediakan dan jelaskan hasil penjumlahan yang kalian peroleh!

Kegiatan 2

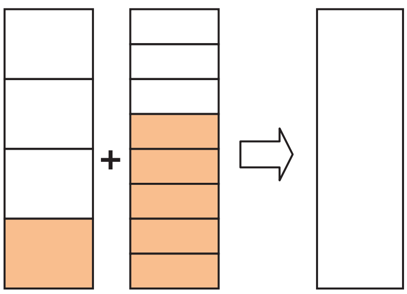
Jika Bu Dewi menuang dan mencampur susu pada dua kotak berikut ke dalam satu kotak, **berapa bagian kotak susu yang terisi setelah dicampur? Gunakan notasi pecahan!**

Jelaskan strategimu!



$\frac{1}{8}$ kotak + $\frac{3}{8}$ kotak ... kotak

Jelaskan strategimu!

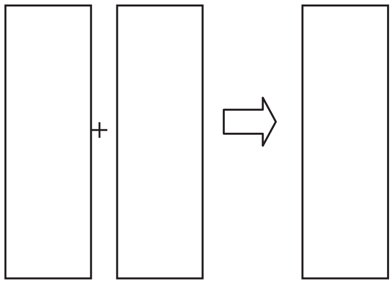


$\frac{1}{4}$ kotak + $\frac{5}{8}$ kotak ... kotak

Jika Bu Dewi menuang dan mencampur santan pada dua kotak berikut ke dalam satu kotak, **berapa bagian kotak santan yang terisi setelah dicampur?**

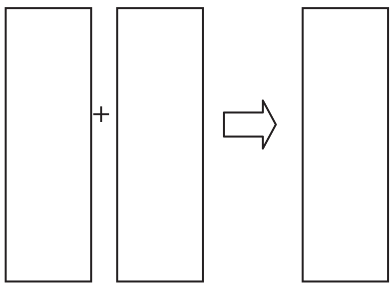
Petunjuk: Bagilah dan arsirlah strip berikut sesuai pecahan yang tertera di bawahnya.

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{2}{3}$ kotak dan $\frac{1}{6}$ kotak.



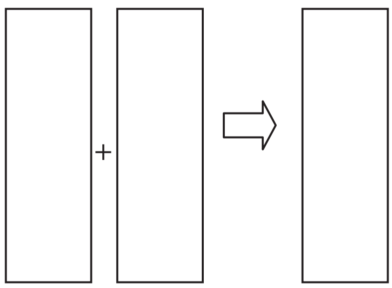
$\frac{2}{3}$ + $\frac{1}{6}$ = ...

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{6}$ kotak.



$\frac{1}{2}$ + $\frac{1}{6}$ = ...

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{3}$ kotak.



$\frac{1}{2}$ + $\frac{1}{3}$ = ...

Kegiatan 3

Ibu Dewi mempunyai dua kotak susu berisi $\frac{4}{6}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama.

1. Setelah dicampur, kira-kira apakah jumlahnya lebih atau kurang dari setengah?
   Jelaskan jawabanmu!
   ....
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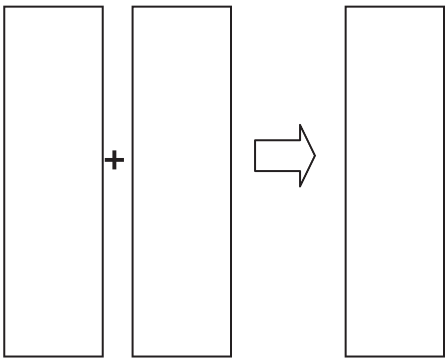
2. Berapa bagian kotak yang terisi susu?



$\frac{4}{6}$ + $\frac{1}{4}$ = ...

3. Periksa kembali jawaban nomor 2 dengan jawaban nomor 1. Apakah jawabanmu pada nomor 2 sesuai dengan perkiraanmu pada nomor 1?
   Jelaskan!
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Worksheet 1

Name : ………………………………………………

Class : ………………………………………………

Date : ………………………………………………

Activity 1

In the celebration of the Independence Day, Mrs. Doni prepares some snacks for the children. After some children participate in some competitions, they get some bread to be eaten together. For children who participate in 'lomba bakiak', Mrs. Doni allocates 2 bread for 3 children, and for children who participate in 'lomba tarik tambang' 3 bread for 4 children.

→ 2 bread 3 children

→ 3 bread for 4 children

Which group will get bigger parts for each child?
Investigate it by doing the following activity!

The pictures below represent bread that will be shared among children in the group 'bakiak' and 'tarik tambang'. Divide the bread to show what parts of bread that each child in each group gets.

2 bread for 3 children in the group 'bakiak'.

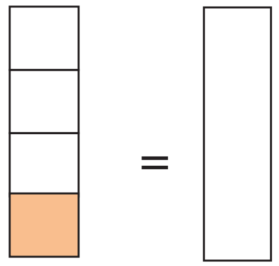
4 bread for 4 children in the group 'tarik tambang'.

Which group will get bigger parts for each child?

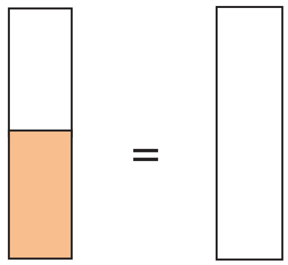
Explain your answer!

Activity 2

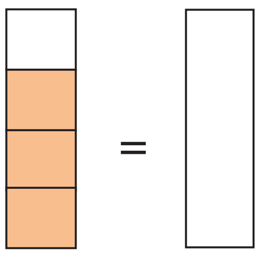
Partition the bar on the right side into equal parts and shade it such that it has the same value as the fractions on the bar on the left side.



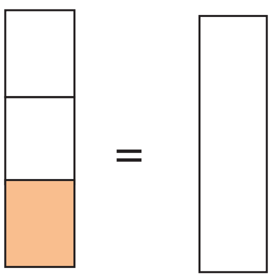
a. $\frac{1}{4} = \frac{\dots}{8}$



b. $\frac{1}{2} = \frac{\dots}{6}$



c. $\frac{3}{4} = \frac{\dots}{8}$



d. $\frac{1}{3} = \frac{\dots}{12}$

Worksheet 2

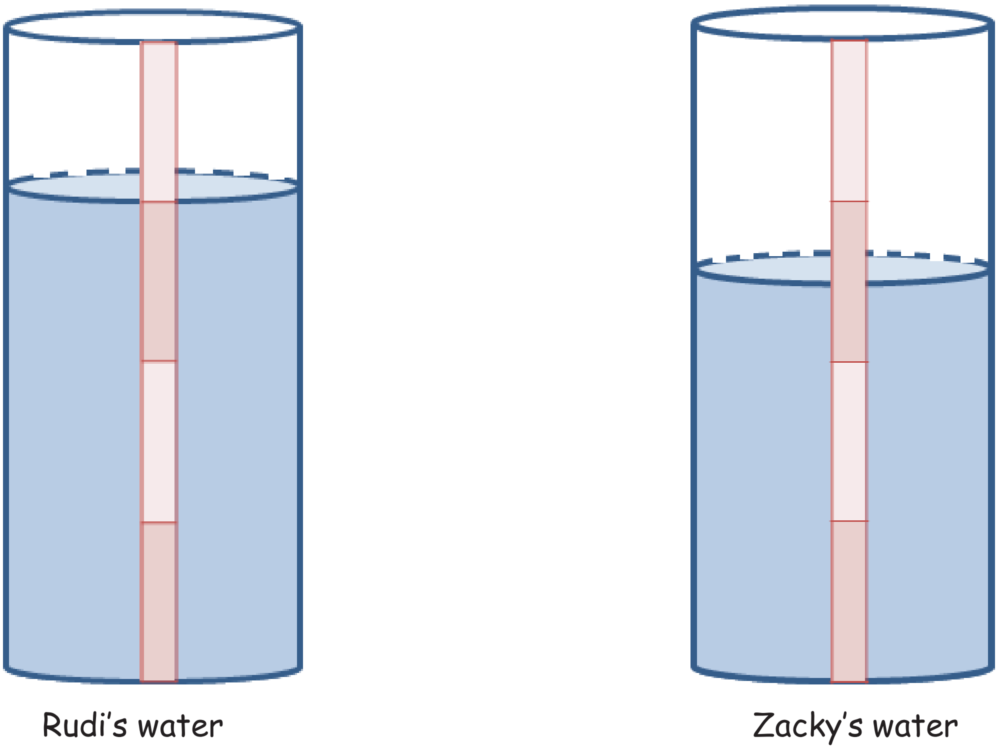
Name : ....

Class : ....

Date : ....

Activity 1

To determine which child has more water, Mr. Doni and his friends need to pour the water in each bucket in the tube and record what part of the tube that has been fulfilled. Now, we are going to help Mr. Doni and his friends to record what parts of the tube that each child has fulfilled. Rudi and Zacky participate in this competition. They are waiting for the announcement of the winner. The picture in the worksheet is the water from each of their buckets. What parts of the tube has their water fulfilled respectively?



What parts of the tube is Rudi's water? Use fractions notation!

....

....

What parts of the tube is Zacky's water? Explain and use fractions notation!

....

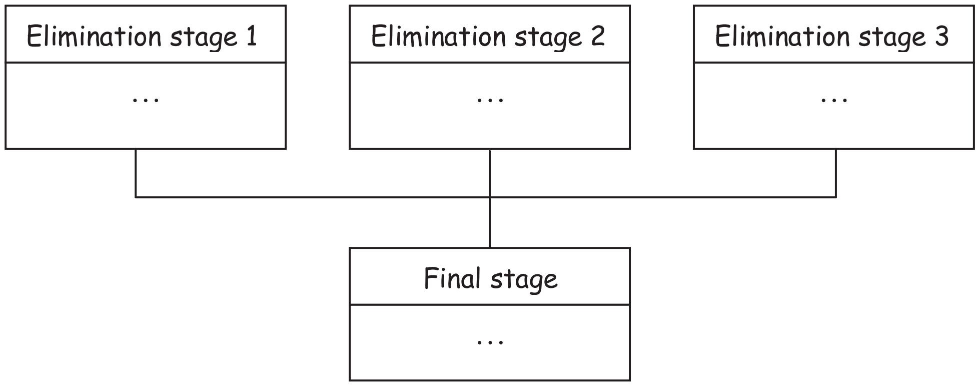
....

Activity 2

After recording the water of some stages, the jury comes up with the following result. Help the jury to determine the winner of each elimination stage.

Stage	Name	The parts of the tube of the water	Who is the winner? Explain!!
1	Adi	$\frac{1}{3}$	
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	
	Doni	$\frac{3}{8}$	
3	Edo	$\frac{5}{6}$	
	Fadil	$\frac{7}{8}$	

Diagram of the winner of each stage in the competition 'memindahkan air'



Who will be the winner of the final stage? Explain your strategy!

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Activity 3

Circle the fractions which are more than $\frac{1}{2}$ and explain your strategy!

$\frac{3}{4}$	$\frac{4}{12}$	$\frac{2}{8}$	$\frac{5}{8}$
$\frac{3}{10}$	$\frac{4}{6}$	$\frac{1}{3}$	$\frac{9}{12}$

Answer therse questions and explain your strategy!

1. $\frac{1}{5} + \frac{3}{4}$

   a. Is $\frac{1}{5}$ more or less than $\frac{1}{2}$?

   b. Is $\frac{3}{4}$ more or less than $\frac{1}{2}$?

What can you conclude?

   c. Is the result more or less than $\frac{1}{2}$? ....

   d. Is the result more or less than 1? ....

2. $\frac{3}{5} + \frac{1}{3}$

   a. Is $\frac{3}{5}$ more or less than $\frac{1}{2}$?

   b. Is $\frac{1}{3}$ more or less than $\frac{1}{2}$

What can you conclude?

   c. Is the result more or less than $\frac{1}{2}$? ....

   d. Is the result more or less than 1? ....

Mrs. Dewi will participate in the cooking competition. At home, she prepares the ingredients such as milk and coconut milk.

She has two cans of milk which $\frac{2}{3}$ *and* $\frac{1}{4}$ *parts of it is filled respectively. If she pours the milk in the two cans into an empty can, is the total more or less than a half of can?*

Explain!

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Worksheet 3

Name : ....

Class : ....

Date : ....

Activity 1

Use the paper strips to show the result of the addition below!

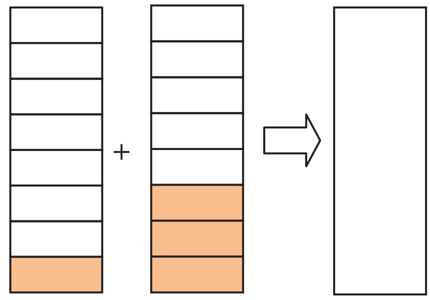
1. $\frac{1}{3} + \frac{1}{6}$

2. $\frac{1}{2} + \frac{1}{3}$

Put the paper strips on the provided poster paper and explain the result you got!

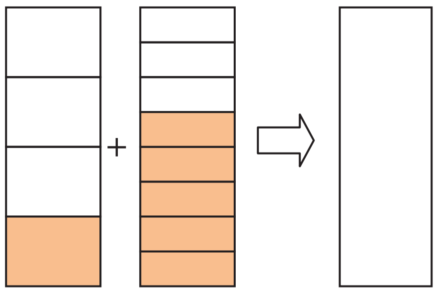
If Mrs. Dewi pours these cans of milk together, **what parts of a can of milk does she have in total? Use fractions notation!**

Explain your strategy!



$\frac{1}{8}$ of a can + $\frac{3}{8}$ of a can .... of a can

Explain your strategy!

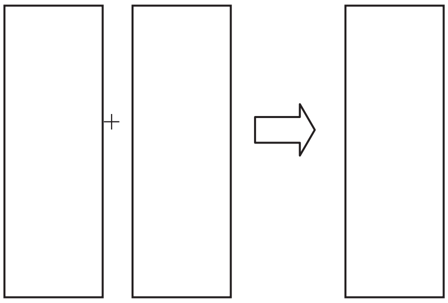


$\frac{1}{4}$ of a can + $\frac{5}{8}$ of a can ...... of a can

If Mrs. Dewi pours these cans of coconut milk together, **what parts of a can of milk does she have in total? Use fractions notation!**

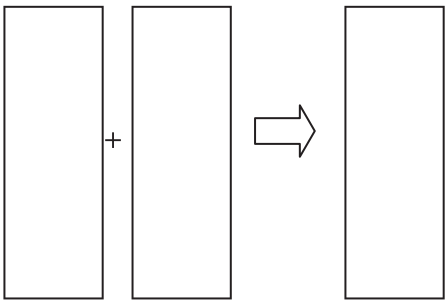
Hint: Partition and shade the bars below as the corresponding fractions.

Pouring $\frac{2}{3}$ of a can and $\frac{1}{6}$ of a can. What parts of a can of milk does she have in total?



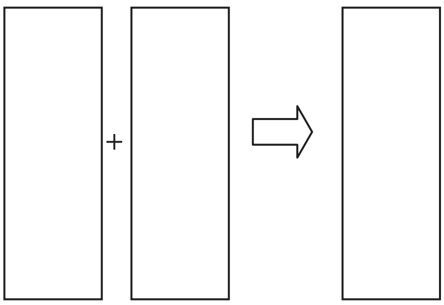
$\frac{2}{3}$ + $\frac{1}{6}$ = ...

Pouring $\frac{1}{2}$ of a can and $\frac{1}{6}$ of a can. What parts of a can of milk does she have in total?



$\frac{1}{2}$ + $\frac{1}{6}$ = ...

Pouring $\frac{1}{2}$ of a can and $\frac{1}{3}$ of a can. What parts of a can of milk does she have in total?



$\frac{1}{2}$ + $\frac{1}{3}$ = ...

Activity 3

She has two cans of milk which $\frac{2}{3}$ and $\frac{1}{4}$ parts of it is filled respectively. She pours the milk in the two cans into an empty can.

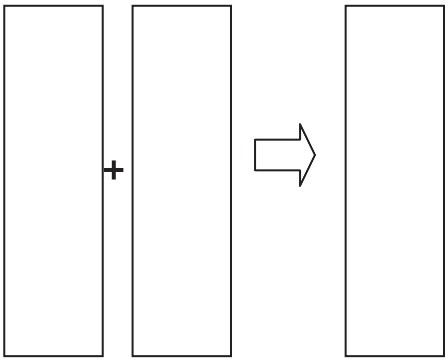
1. After being poured, is the total water more or less than a half of can? Explain your answer!

   ....

   ....

   ....

2. What parts of a can is filled with milk?



$$\frac{4}{6} + \frac{1}{4} = \ldots$$

3. Crosscheck your result on number 2 and number 1. Is your result on the number 1 corresponding to your estimation on the number 1? Explain!

   ....

   ....

   ....

   ....

Appendix 5

Teacher Guide

Meeting 1

Time allocation
90 minutes

Learning material
Student worksheet 1, tubes filled with water, paper strips, poster paper.

Learning Goals

- Students are able to partition into equal parts.
- Students understand the notation of fractions.
- Students understand the idea of equivalent fractions.

Description of the activity

Activity 1: Fair Sharing

Opening (5 minutes)

- The teacher asks students about various competition in the celebration of Independence Day. Then, the teacher tells the students about Mrs. Doni's story in preparing snacks for the children who participate in some competitions in the celebration of Independence Day as follows:

  In the celebration of the Independence Day, Mrs. Doni prepares some snacks for the children. After some children participate in some competitions, they get some bread to be eaten together. For children who participate in 'lomba bakiak', Mrs. Doni allocates 2 bread for 3 children, and for children who participate in 'lomba tarik tambang' 3 bread for 4 children. However, while she thinks that each child in a group will get the same parts of the bread, her friend argues that it is not fair because children in the group of 'lomba tarik tambang' will get different parts from children in the group of 'lomba bakiak'. Then, Mrs. Doni tries to figure it out. Does each child in each group get the same parts of the bread? What parts of bread does each child in each group get?

→ 2 bread for 3 children

→ 3 bread for 4 children

- The teacher shows the bread and the picture of the competitions such that students get the sense of the story. Then, the teacher engages students to help Mrs. Doni figuring out the questions (*which group will get bigger parts of bread?)*. Firstly, the teacher lets students to think in pairs about the questions.

Students might come up with different answers:

1. Some students might argue that it is fair because the number of bread is one less than the number of children.
   The teacher's reaction:
   The teacher asks students how to be sure by asking *'how do you know?'*
2. Students might think that children in the group '*bakiak'* get the biggest part since the bread are divided into smaller number of children than in the group '*tarik tambang*'.
   The teacher's reaction:
   The teacher can pose '*but the group of 'tarik tambang' has more number of bread, what do you think?'*
3. Students might think that children in the group '*tarik rambang'* get the biggest part since they have more number of bread than in the group '*bakiak*'.
   The teacher's reaction:
   The teacher can pose '*but the group of 'tarik tambang' has more number of children than the group of 'bakiak', what do you think?'*

Group discussion (10 minutes)

- After students have a glance idea about the solution of the problem, the teacher makes groups consisting of 2-3 students.
- The teacher provides Activity 1 of Worksheet 1 to each group, and asks them to show their strategy to ensure in which group the children will get the biggest part of the bread.
- During the discussion, the teacher encourages students to raise their arguments by posing questions such as '*how do you know?*', '*why/how did you do that?*', '*how did you divide the bread?'*

Students' strategies are:

1. Students divide the bread into equal parts as the number of children in each group. Then, they compare the parts of cake that each child in each group gets, for example:

3 bread for 4 children

   The teacher's reaction:
   The teacher engages students to consider both, the number of bread and the number of children.
   - If the students answer that the first group get bigger parts because each part of

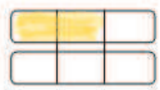
the bread is bigger than that of in the second group, the teacher can pose questions such as ‘*But the second group gets more number of parts, what do you think?’* If students become confuse or keep on their argument, the teacher can ask ‘*how can you be sure that the first group gets bigger parts?’*. Moreover, the teacher can enganges the students to represent the students’ result in one picture, as below:

Parts of bread each child gets in the group ‘*bakiak’* 

Parts of bread each child gets in the group ‘*tarik tambang’* 

Then, the teacher asks the students which group gets bigger parts of bread. In this case, the teacher can show students that 2 bread for 3 children is the same as two parts of three parts in a bread.

- If there are students who compare the fractions by applying cross multiplication, the teacher needs to encourage students to use pictures instead of algorithms in comparing fractions in order to build their fractions sense.
- If there are students who misinterpret the concept of the whole (or the unit) of fractions, the teacher needs to put emphasize on it. The teacher can use pictures to explain it. For instance, in the problem 2 bread are shared among 3 children. There might be a student who argue that each child will get 2/6 part of bread since there are 6 parts in total. In this case, the teacher can ask them: ‘*look at the picture. Each child will get 2 parts, isn’t it? So, what parts of **A BREAD** is it?’* (The teacher puts an emphasize on ‘a bread’ as a whole).



2. Students firstly halve the bread, and then they halve it again or divide it into equal parts as the number of children in each group. Then, they compare the parts of bread that each child in each group gets. They will get $\frac{1}{2}$ and $\frac{1}{6}$ for each child in the group ‘*bakiak’*, and $\frac{1}{2}$ and $\frac{1}{4}$ for each child the group ‘*tarik tambang’*.

3 bread for 4 children

<u>The teacher’s reaction:</u>

If students use this strategy, it is easier for them to find out which group gets bigger parts. Then, the teacher can encourage them to notice about the partition, that the more partition, the smaller part they get. The teacher also can ask them how to name the parts into fractions (the notation of fractions). If they will get $\frac{1}{2}$ and $\frac{1}{6}$ for the group ‘*bakiak’*, and $\frac{1}{2}$ and $\frac{1}{4}$ for the group ‘*tarik tambang’*, the teacher can ask students how to determine which one is bigger. If there are students who argue that children in the group '*bakiak*' get $\frac{1}{2}$ and $\frac{1}{3}$ because there are a half and a third of a half, the teacher can engage them to draw the representation of a third in the bar and compare it to the picture of a third of a half so that they realize that a third of a half is equal to a sixth. In this case, the teacher should stress on the idea of partitioning into equal parts.

3. Students might solve the problems by directly dividing the number of cakes by the number of children and then compare it by considering the benchmark. For instance, they argue $\frac{3}{4}$ is greater than $\frac{2}{3}$ since $\frac{3}{4}$ needs $\frac{1}{4}$ to be a whole, and $\frac{2}{3}$ needs $\frac{1}{3}$ to be a whole. They might argue that $\frac{1}{4}$ is smaller than $\frac{1}{3}$, so they conclude that $\frac{3}{4}$ is greater than $\frac{2}{3}$. They also might solve it in formal ways, which students look for the common denominator for each fraction.

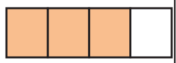
   <u>The teacher's reaction:</u>

   The teacher can encourage students to explain their reasoning and represent it in a picture/bar. The teacher can ask '*Why did you divide the number of bread by the number of children? How did you get* $\frac{3}{4}$ and $\frac{2}{3}$ *?*'

<u>Class discussion (around 10 minutes)</u>

After students finish solving the task, the teacher generates a class discussion and asks some groups that have different strategies to present their works. The teacher encourages the students to be active by asking questions or giving comments.

- In this discussion, the teacher stresses on how to name the parts (use the notation of fractions). If some students do not use fractions notation, the teacher introduces the notation of fractions as a part-whole relation to students by giving simple examples such as showing the following picture (or taking one of the students' pictures) and asking '*What fractions is the shaded area?*'



- In determining what parts of the bread that each child in each group gets, students might get different answers. For instance, some students argue that each child in the group of '*lomba bakiak*' will get ¾ of the bread while other students might get ½ + ¼ of the bread. Then, the teacher can raise this issue to encourage students to think why the result can be different. In this case, the teacher has to point on the relation between two, four, and eight partitions, for example by putting the 4-partitioned bread and the 8-partitioned bread together, and then asking '*What do you notice about the parts of these two bread?*'. This issue will lead students to the initial idea of the relation between partitions or fractions, such as ½ and ¼, and the equivalence of fractions.

- The teacher needs to engage students to consider what is the whole in naming the fractions of the parts of a whole.
- The teacher should engage students to use pictures in comparing the parts of bread in each group instead of using any algorithm in order to support their mental image of fractions.

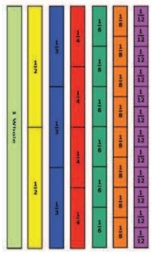
<u>Activity 2: Producing measuring strips</u>

Opening (around 10 minutes)

- The teacher asks students whether they know and ever participate in '*lomba memindahkan air ke dalam ember*', then he/she tells how the previous story about the celebration of the Independence Day continues:
  For the celebration of Independence Day, Mr. Doni and his friends have a task to arrange some creative competitions for children. His friends propose a competition namely 'lomba memindahkan air', which children need to move water with a plate from one bucket to another bucket. The children have to carry the water as much as possible within a given time. However, Mr. Doni wonders how they can know how much water that participants have filled in the bucket to determine the winner. What should they do?
- Students may come up with different ideas such as measuring the weight or the height of the water. Then, the teacher can ask '*how about if the weight of the buckets is different?*'. Afterwards, the teacher stresses that there are no scale and ruler. In order to lead them to the idea of measuring strips, the teacher shows the tube filled by around a half of the tube of water and asks them '*what parts of the tube is the water? How can we measure it?'*. The teacher demonstrates how to use paper strips as measuring strips by saying '*we can use this strip as a scale. This strip represents the tube. So, what parts of the tube is this water?*' The students might answer that the water fills a half of the tube. Then, the teacher asks '*How can you be sure that it is a half of the tube?'* to stimulate students that they can fold the paper strip into two to make a half.
- After telling the story, the teacher provides a tube filled by any scale of water, such as a half, one third, two third, one fourth, and three fourth of the tube, so that each group will make different measuring strips. The teacher also provides a two-partitioned paper strip to each group. Then, the teacher engages students to make two measuring strips with different number of partitions that can show what parts of the tube that is filled by water. By using the two-partitioned paper strip in measuring the water in their tube, the students might have an idea that they can fold the paper into four or eight partitions.

Group discussion (15 minutes)

- The teacher provides paper strips to each group and engages them to make two measuring strips with different number of partitions that can fit to measure the water in the tube.
- While the students work in their group, the teacher encourages them to raise their arguments in partition the paper strips by posing questions such as '*how do you know?*', '*why/how did you do that?*', '*how did you divide the paper strips?*'
- The teacher encourages students to notice the relation between two measuring strips with different number of partitions that they make.
- Each group puts the measuring strips together in the poster paper such that it forms a fraction strips as the figure below.

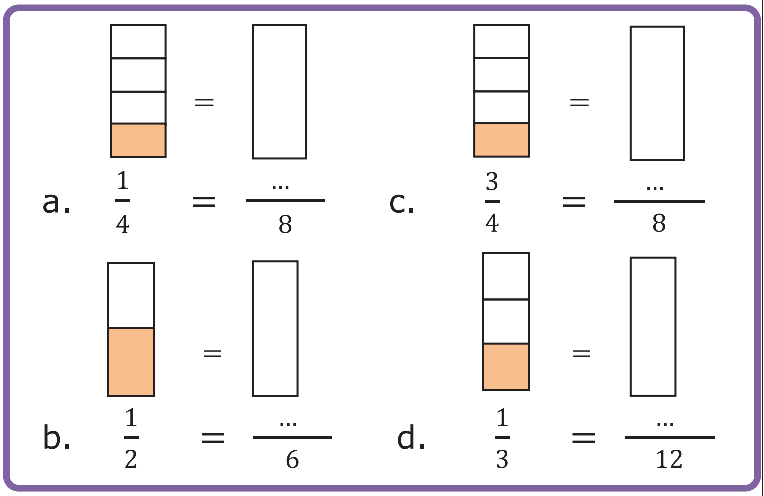


Class discussion (around 25 minutes)

After students finish solving the task, the teacher generates a class discussion and asks some groups who have different strategies to present their works. The teacher encourages the students to be active by asking questions or giving comments.

- In the class discussion, the teacher raises the issue of partitioning and the notation of fractions. The teacher also will stress on the equivalence of fractions by noticing the extention lines in the measuring strips they have made. For example, as can be seen in the figure beside, the teacher engages the students to notice the extention line of a half to determine the equivalent fractions of a half. For instance by asking '*from these measuring strips, what do you notice? if the water has filled* $\frac{1}{2}$ *of the tube, can we express with other fractions?*'.
- After experiencing finding the equivalent fractions in the measuring strips, the teacher should support students to translate the reasoning of equivalent fractions in the measuring strips to the bars.
- The teacher should engage students to notice the patterns of equivalent fractions from some examples. that the denominator and the numerator have to be the same multiple of the initial fraction, for example by questioning '*what did you notice about the pattern?*' To put more emphasize, the teacher can show them how the pattern works in the measuring strips that they have made in the previous meeting. The teacher also can pose more questions such as '*can we represent* $\frac{2}{5}$ *with another fraction? Why?*'.

Individual exercise (around 15 minutes)
After the students learn the idea of the equivalent fractions, the teacher engages students to solve the Activity 2 of Worksheet 1) individually. The problems are about finding the equivalence of fractions in the form of bars, as beside:



Students' strategies:

1. Students partition the bar so that it has the same value, for example:

2. Students find a pattern, that the denominator and the numerator have to be the same multiple of the initial fraction.

At the end of the lesson, the teacher, together with the students, sums up what they have learned, about the notation of fractions and the equivalence of fractions.

Didactical suggestions
The teacher may not value the students' answer, and give them some times to think before pointing them to share their ideas.

Meeting 2

Time allocation
90 minutes

Learning materials
Student worksheet 2.

Learning Goals

- Students can compare fractions.
- Students are able to estimate the sum of two fractions with benchmarks.

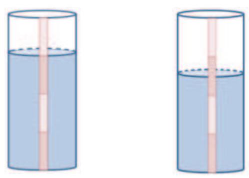
Description of the activity

Activity 1: What parts of the tube is it?

Opening (5 minutes)

- The teacher reminds students about the previous meeting and asks them a problem about the equivalence of fractions, such as '*Can we represent* $\frac{1}{2}$ *with other fractions?*' Then, he/she tells the story about how Mr. Doni and the committee measure the water in the tube to determine the winner, as follows (Activity 1 of Worksheet 2).

  To determine which child has more water, Mr. Doni and his friends need to pour the water in each bucket in the tube and record what part of the tube that has been fulfilled. Now, we are going to help Mr. Doni and his friends to record what parts of the tube that each child has fulfilled. Rudi and Zacky participate in this competition. They are waiting for the announcement of the winner. The picture in the worksheet is the water from each of their buckets. What parts of the tube has their water fulfilled respectively?

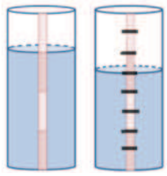


Group discussion (10 minutes)

- The teacher provides the worksheet 2 (Activity 1) to each group and and tells that after they solve the problem, some groups will present their solution in the class discussion. While the students discuss with their group, the teacher guides them and encourages them to raise their arguments by posing questions such as '*how do you know?*', '*why/how did you do that?*', '*how did you divide the paper strips?'*
- In this activity, the teacher stresses on the idea of partitioning and lead students to get the fractions sense and have a mental image for fractions in a bar.

Students' strategies:

1. Students measure the height of the water by ruler and then convert it to fractions.
2. Students use paper strips and fold it such that it fits the height of the water.
3. Students draw other lines that indicate eighth, sixteenth, third, sixth, etc., such that it fits the height of water.



<u>The teacher's reaction:</u>

1. If students measure the height of the water by using a ruler and then convert it to fractions, the teacher can ask how they convert it into fractions and encourage them to think whether there is another way to determine the parts of the water without using a ruler.
2. If students make measuring strips by folding the paper strips, the teacher supports them and asks them to present how they did it in pictures.
3. If students draw other lines that indicate eighth, sixteenth, third, sixth, etc., such that it fits the height of water, the teacher can ask them how to name the fractions.
4. If there are students who cannot name and label the fractions, the teacher can guide the students and remind them about the measuring strips.

Class discussion (around 5 minutes)

- The teacher generates a class discussion and asks some groups who have different strategies to present their works. The teacher encourages the students to be active by asking questions or giving comments.
- In the discussion, the teacher engages students to determine the exact measurement of the water by using the notation of fractions. The teacher stresses on the idea of partitioning and the equivalence. For example, after students are able to determine the fractions of the second tube, the teacher can ask them whether they also can use the same partition for the first tube.

<u>Activity 2: Who will be the winner?</u>

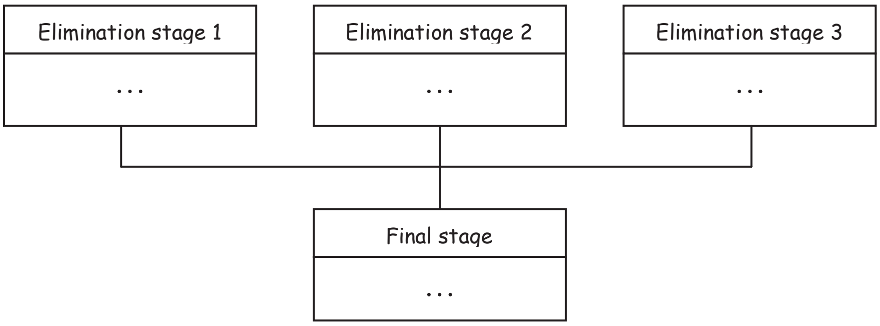
Opening (around 5 minutes)

- The teacher engages students to solve the following problem (Activity 2 of Worksheet 2):

After recording the water of all participants in each elimination stage, the jury comes up with this result. The winner of each elimination stage will participate in the final stage. Help the jury to determine the winner of each elimination stage. Who will be the winner of each elimination stage?

Stage	*Name*	*The parts of the tube of the water*	*Winner*
1	*Adi*	$\frac{1}{3}$	*?*
	Budi	$\frac{1}{5}$	
2	*Ucok*	$\frac{3}{4}$	*?*
	Doni	$\frac{3}{8}$	
3	*Edo*	$\frac{5}{6}$	*?*
	Fadil	$\frac{7}{8}$	

Then, the students are asked to fill the diagrams of winners in the competition '*memindahkan air'* below.



<u>Group discussion (15 minutes)</u>

- The teacher provides the Activity 2 of Worksheet 2 to each group. While the students discuss with their group, the teacher guides them and encourages them to raise their arguments by posing questions such as '*how do you know?*', '*why/how did you do that?*', '*how did you divide the paper strips?'*
- In this activity, the teacher stresses on comparing fractions by using bars or benchmarks instead of by using any algorithms

.

Students' strategies in comparing fractions

1. Students draw a bar (vertically or horizontally) and partition it as the fractions in the problem, and then compare it or order it. If students are going to use bar to solve the problem, the teacher can provide some bars in a paper such that it will result more precise picture.

The teacher's reaction:

The teacher can support their reasoning, and encourage them to consider the benchmark. For instance, in comparing $\frac{5}{6}$ and $\frac{7}{8}$, the teacher can ask '*Which fraction is the nearest to be a whole?'*. Then, the teacher can pose more problems such as '*without drawing, which one is bigger,* $\frac{3}{8}$ *or* $\frac{4}{7}$*?*' In this case, the teacher encourages students to use a half as the benchmark. In answering this question, students might draw bars to represent the fractions. They also might reason that $\frac{4}{7}$ is bigger because it has 4 parts of 7-partitioned bar while $\frac{3}{8}$ means it has 3 parts of 8-partitioned bar. They also might think that $\frac{4}{7}$ is bigger because it is more than a half.

2. Students might use the idea of a common denominator to compare and order fractions.
   The teacher's reaction:
   The teacher asks their reasoning why they solve that way and asks them to represent their solution in a bar. For instance, by questioning '*why do you think that* $\frac{3}{4}$ *is greater than* $\frac{3}{8}$*? Can you draw that situation in the bar?'*
3. Students might reason by using benchmarks and without drawing, for example, they know $\frac{7}{8}$ is greater than $\frac{5}{6}$ since $\frac{7}{8}$ needs $\frac{1}{8}$ to be a whole, and $\frac{5}{6}$ needs $\frac{1}{6}$ to be a whole. They know that $\frac{1}{8}$ is smaller than $\frac{1}{6}$, so they conclude that $\frac{7}{8}$ is greater than $\frac{5}{6}$
   The teacher's reaction:
   The teacher can support their reasoning and pose more problems, such as '*which one is bigger,* $\frac{8}{9}$ *or* $\frac{8}{10}$*?* $\frac{6}{7}$ *or* $\frac{7}{8}$*?* $\frac{2}{5}$ *or* $\frac{3}{7}$*?'*.
4. Students use cross multiplication to compare two fractions.
   The teacher's reaction:
   The teacher needs to engage students to use their fractions sense or pictures to compare fractions, for example by asking '*If you get* $\frac{1}{4}$ *of a cake and your brother gets* $\frac{1}{3}$ *of a cake, can you imagine which one does get bigger parts of a cake?'*

Class discussion (around 15 minutes)

- The teacher asks groups who have different strategies to present their works. The teacher encourages the students to be active by asking questions or giving comments.
- The teacher encourages and supports students to reason by using benchmarks, such as $\frac{1}{4}$ and $\frac{1}{2}$, to compare or order fractions. To use the benchmarks such as $\frac{1}{4}$ and $\frac{1}{2}$, the teacher can remind the students about the equivalence of fractions to determine whether the fractions more or less than the benchmark. Then, the teacher can pose more problems to strengthen their understanding in comparing fractions by using benchmarks, such as *which one is bigger, or* $\frac{8}{10}$*?* $\frac{6}{7}$ *or* $\frac{7}{8}$*?* $\frac{2}{5}$ *or* $\frac{3}{7}$*?'*.

- If there are students who cannot understand how to solve the problems, the teacher can engages them to use pictures (bars) to help them to build their mental image of fractions. The teacher also may show the series of measuring strips they have made in the previous problem, and then encourage them to represent the problems in the measuring strips.

<u>Activity 3: Estimating the sum of two fractions</u>

Group discussion (around 15 minutes)

- Firstly, the teacher asks the students to determine which fractions are more than a half. Then, the teacher enganges the students to discuss how to estimate the sum of two fractions in the Activity 3 Worksheet 2 as below.

Circle the fractions which are more than $\frac{1}{2}$, and explain your strategy

$\frac{3}{4}$ $\frac{4}{12}$ $\frac{2}{8}$ $\frac{5}{8}$ $\frac{3}{10}$ $\frac{4}{6}$ $\frac{1}{3}$ $\frac{9}{12}$

Answer therse questions and explain your strategy!

1. $\frac{1}{5}+\frac{3}{4}$

   a. Is $\frac{1}{5}$ more or less than $\frac{1}{2}$?

   b. Is $\frac{3}{4}$ more or less than $\frac{1}{2}$?

   What can you conclude?

   c. Is the result more or less than $\frac{1}{2}$? ……………………………………….

   d. Is the result more or less than 1? ……………………………………….

2. $\frac{3}{5}+\frac{1}{3}$

   a. Is $\frac{3}{5}$ more or less than $\frac{1}{2}$?

   b. Is $\frac{1}{3}$ more or less than $\frac{1}{2}$

   What can you conclude?

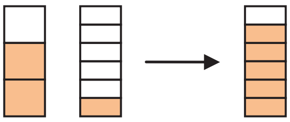
   c. Is the result more or less than $\frac{1}{2}$? ……………………………………….

Students' strategies in determining which fractions is more than a half:

1. Students might use bar to figure out the problems.
2. Students might reason by using the equivalence of fractions. For example, they argue that *or* $\frac{5}{6}$ is more than a half because a half in sixth is $\frac{3}{6}$, so $\frac{5}{6}$ is more than a half.

Students' strategies in estimating the sum of two fractions:

1. Students might use bar to figure out the problems, for example:

   So, the result is more than a half.

   The teacher's reaction:

   The teacher encourages students to use benchmarks in solving the problem, which are asking them to compare the fractions, $\frac{3}{4}$ and $\frac{1}{5}$, to a half.
2. Students might reason by using benchmarks. For example, they argue that the result of $\frac{3}{5}+\frac{1}{3}$ must be more than a half because $\frac{3}{5}$ is more than a half.

   The teacher's reaction:

   The teacher supports their reasoning and asks follow up questions, for instance '*If Mr. Doni pours* $\frac{2}{3}$ *of a can of milk and* $\frac{1}{6}$ *of a can of milk, is it enough to get a half can?'*
3. Students find the exact result by finding the common denominator and then see whether the result is less or more than a half.

   The teacher's reaction:

   The teacher asks the students to explain their reasoning and represent it in a bar. Then, the teacher encourages them to use benchmarks in solving the problem.

Class discussion (10 minutes)

- The teacher has to support students on how to estimate the sum of two fractions by comparing the added fractions with benchmarks such as a half and one. For instance, the teacher asks *'is* $\frac{2}{3}$ *more or less than a half? So, what can you conclude about the result? Should it be less or more than a half?'* When the students are able to estimate the sum of two fractions by using a benchmark, they will realize that they cannot do 'top+top over bottom+bottom'. To put more emphasize on it, the teacher can show the measuring strips and show that the procedure 'top+top over bottom+bottom' is incorrect.

Individual exercise (around 10 minutes)

After the students discuss about how to estimate the sum of two fractions, the teacher provides the application of the estimation skill in the word problem as below.

Mrs. Dewi has two cans of milk which $\frac{4}{6}$ *and* $\frac{1}{4}$ *parts of it is filled respectively. If she pours the milk in the two cans into an empty can, is the total more or less than a half of can?*

Didactical Suggestions:

The teacher may not value the students' answer, and give them some times to think before pointing them to share their ideas.

Meeting 3

Time allocation
90 minutes

Learning material
Student worksheet 3, tubes filled with water.

Learning Goals

- Students are able to estimate the sum of two fractions with benchmarks.
- Students grasp the idea of common denominator.
- Students are able to add fractions by using a bar model.

Description of the activity

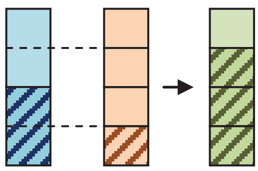
Activity 1: Adding fractions by using paper strips

Opening (around 10 minutes)

- Before going to the main activity, the teacher tells a new story that is still related to the celebration of the Independence Day.

  Instead of many competitions for children, there are also competitions for adults such as cooking competition. This year, the theme of the cooking competition is making pudding as creative as possible. Mrs. Dewi will participate in this competition. At home, she prepares the ingredients such as sugar, milk, coconut milk, etc. She needs a half can of milk. She remembered that she had left over two cans of milk. If Mrs. Dewi pours two cans of coconut milk that contain $\frac{1}{2}$ and $\frac{1}{4}$ of a can respectively, what parts of can will be filled?

- Then, the teacher shows them two (transparent) tubes contain $\frac{1}{2}$ and $\frac{1}{4}$ of a tube of water respectively. The teacher demonstrates in pouring these two tubes. The teacher asks one student to measure the water before and after being poured by using measuring strips. Thereafter, the teacher engages and asks students why the result can be $\frac{3}{4}$. Students might come up with different answers such as reasoning that $\frac{1}{2}$ is equal to $\frac{2}{4}$. Then, the teacher engages the students to represent the process of adding $\frac{1}{2}$ and $\frac{1}{4}$ in the paper strips as in the figure below.



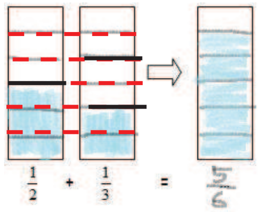
- The teacher provides students to work in their group to find the result of $\frac{1}{3}+\frac{1}{6}$ and $\frac{1}{2}+\frac{1}{3}$ by using paper strips.

<u>Group discussion (around 20 minutes)</u>

- While students work in their group, the teacher guides them by asking '*why do you think so?*' or '*how do you do it?*' to encourage students to give their arguments.
- The teacher engages students to notice the idea of common denominator by using paper strips.

Students' strategies in adding fractions in the paper strips:

1. Students partition the two paper strips representing the fractions being added into a number of parts that fit to both fractions. They might use the idea of common extension lines as they learned in the previous meeting to find the common number of partitions.

   <u>The teacher's reaction:</u>
   The teacher supports their understanding and encourages their reasoning, for instance by asking ' '*Why did you divide the bar into eight (or twelve, etc.)?*' The teacher also can ask their reasoning how to find the exact result of the first problem.
2. Students might solve it in formal ways, which students add the fractions by finding the common denominator, and then representing the result in the paper strips
   <u>The teacher's reaction:</u>
   The teacher encourages students to represent and explain their reasoning in the form of paper strips.

<u>Class discussion (around 10 minutes)</u>

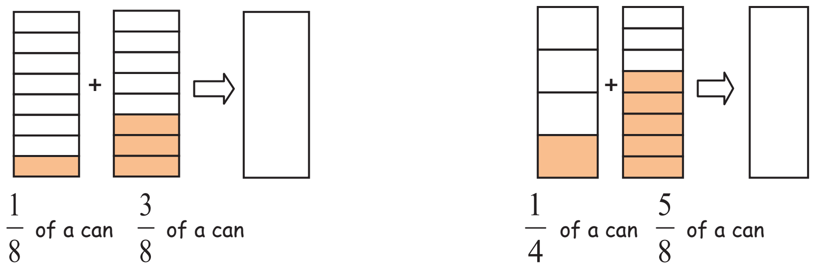
In the discussion, the teacher engages the students to notice the reasoning of the common number of partitions representing the common denominator by using paper strips. Moreover, the teacher needs to support students' reasoning to grasp the relation between the common number of partitions and the pattern of common denominator of two fractions being added.

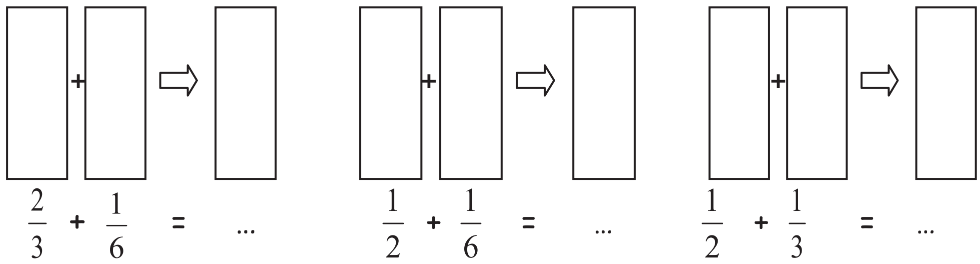
<u>Activity 2: Adding fractions by using bar model</u>

Individual exercise (around 20 minutes)

- The teacher poses the problems of the addition of fractions in the form of bars (Activity 2 Worksheet 3).

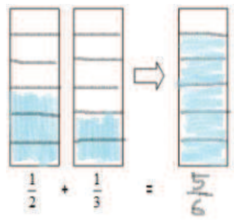
If Mrs. Dewi pours these can of milk together, what parts of a can of milk does she have in total? Use fraction notation!





Students' strategies:

1. Students partition the bars into a number of parts that fit to both fractions. The find the common number of partitions by extending the lines of the bars as they do in the paper strips.



   The teacher's reaction:
   The teacher supports their understanding and encourages their reasoning, for instance by asking '*'Why did you divide the bar into eight (or twelve, etc.)?'*
2. Students might solve it in formal ways, which students add the fractions by finding the common denominator.
   The teacher's reaction:
   The teacher encourages students to represent and explain their reasoning in the bars.

Activity 3: Reviewing the estimation of the sum of two fractions

Individual exercise (around 15 minutes)

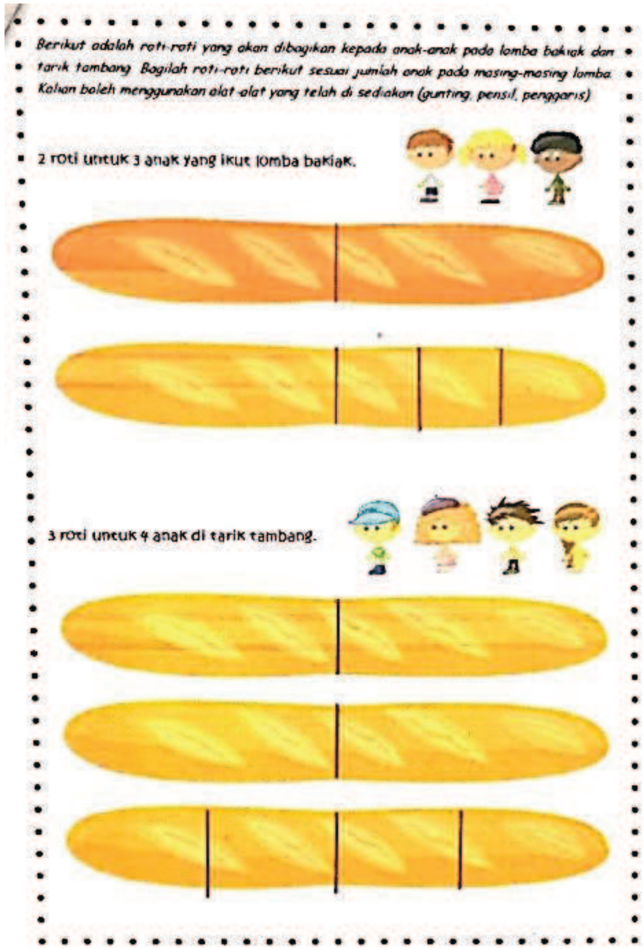
The teacher re-poses the estimation problem as in the second meeting, which is about estimating the sum of $\frac{4}{6}$ and $\frac{1}{4}$. Moreover, in the worksheet, there are a problem in which the students have to determine the exact result of $\frac{4}{6} + \frac{1}{4}$ in the for of bars, and then they

are asked to crosscheck the exact result and their estimation whether the exact result is corresponding with the estimation or not (Activity 3 of Worksheet 3).
Class discussion (around 15 minutes) • The teacher discuss the second and third activity. • In the problems of the second activity, the teacher encourages students' reasoning of the common number of partitions representing the common denominator in the form of bars. Moreover, the teacher needs to support students' reasoning to grasp the relation between the common number of partitions and pattern of the common denominator of two fractions being added. The teacher also needs to stimulate students to notice that in adding two fractions, both bars have to have the same partitions. If there are students who do not understand at all how to solve it, the teacher can use the measuring strips as the model of to represent the problems and remind them about the equivalence of fractions. • In the problems of the third activity, the teacher stimulates and strengthens students' reasoning in estimating the sum of two fractions by using a half as the benchmark. Moreover, the teacher also points that the procedure 'top+top over bottom+bottom' is not reasonable. The teacher can ask more problems about estimation, such as '*Is the result of* $\frac{3}{5}+\frac{1}{4}$ *more or less than a half? Is it possible if we do 3+1 per 5+4?*' If there are students who do not understand at all how to solve it, the teacher can use the measuring strips to show the exact result and to show that the procedure 'top+top over bottom+bottom' is not reasonable.

Appendix 6

Examples of Students' Work

Meeting 1, Activity 1
Cycle 1



Berikut adalah roti-roti yang akan dibagikan kepada anak-anak pada lomba bakiak dan tarik tambang. Bagilah roti-roti berikut sesuai jumlah anak pada masing-masing lomba. Kalian boleh menggunakan alat-alat yang telah di sediakan (gunting, pensil, penggaris)

2 roti untuk 3 anak yang ikut lomba bakiak.

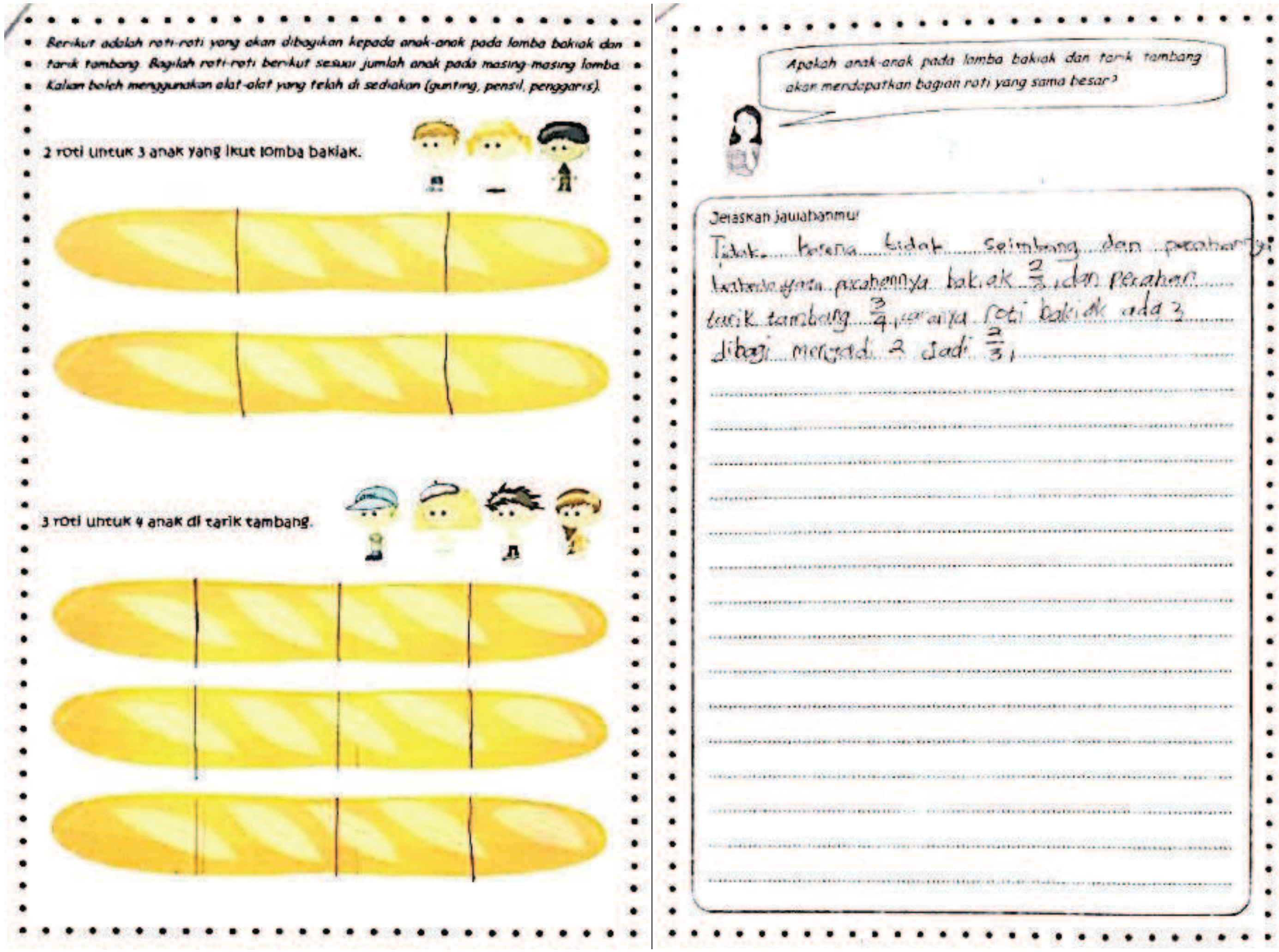
3 roti untuk 4 anak di tarik tambang.

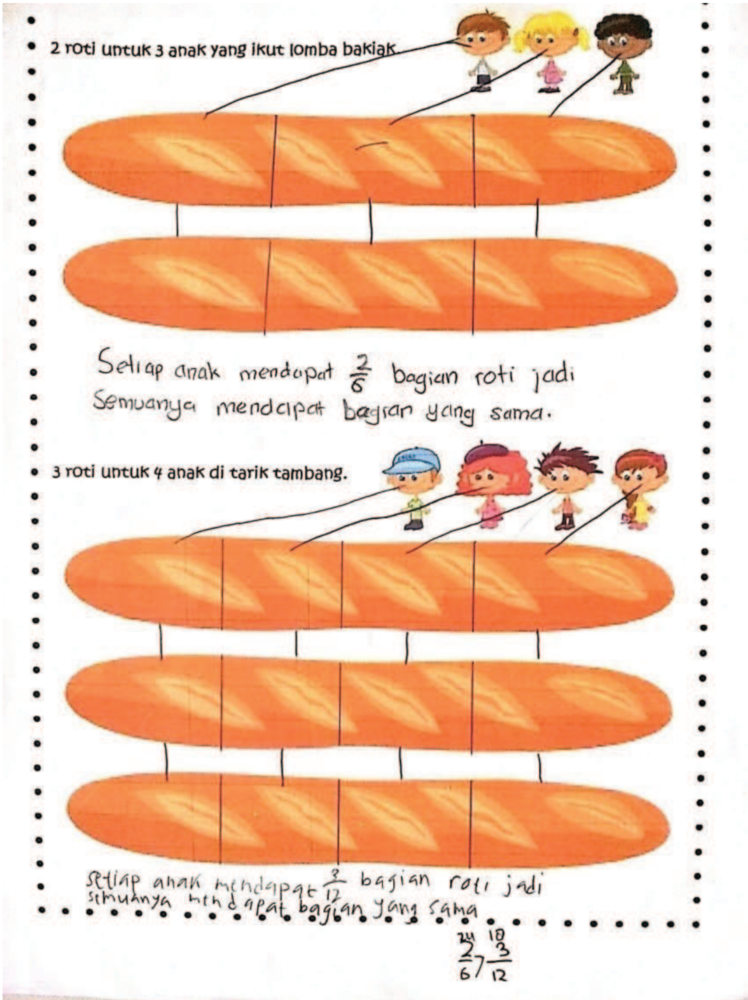
Apakah anak-anak pada lomba bakiak dan tarik tambang akan mendapatkan bagian roti yang sama besar?

Jelaskan jawabanmu!

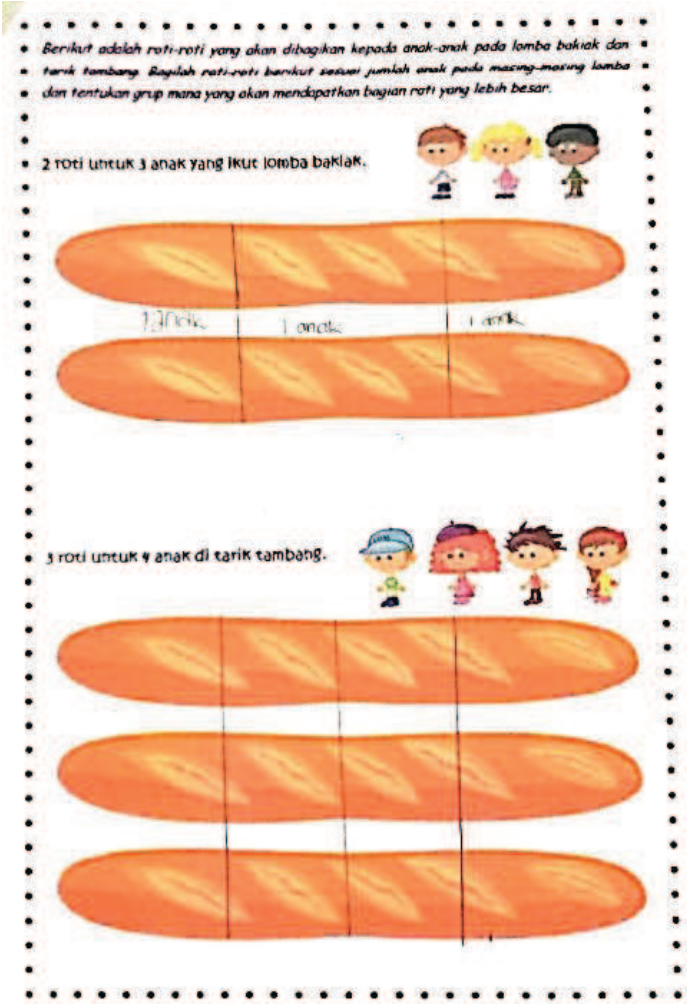
ya, karena jika roti ada 2 buah
maka bisa dibagi menjadi 3 bagian
lalu masih ada sisa 1 potong. jadi yang 1 potong
Bia dibagi menjadi 3 potong. juga kalau
3 roti dibagi untuk 4 anak akan sisa
1 potong lalu yang 1 potong tersebut
bisa dibagi menjadi 4 bagian

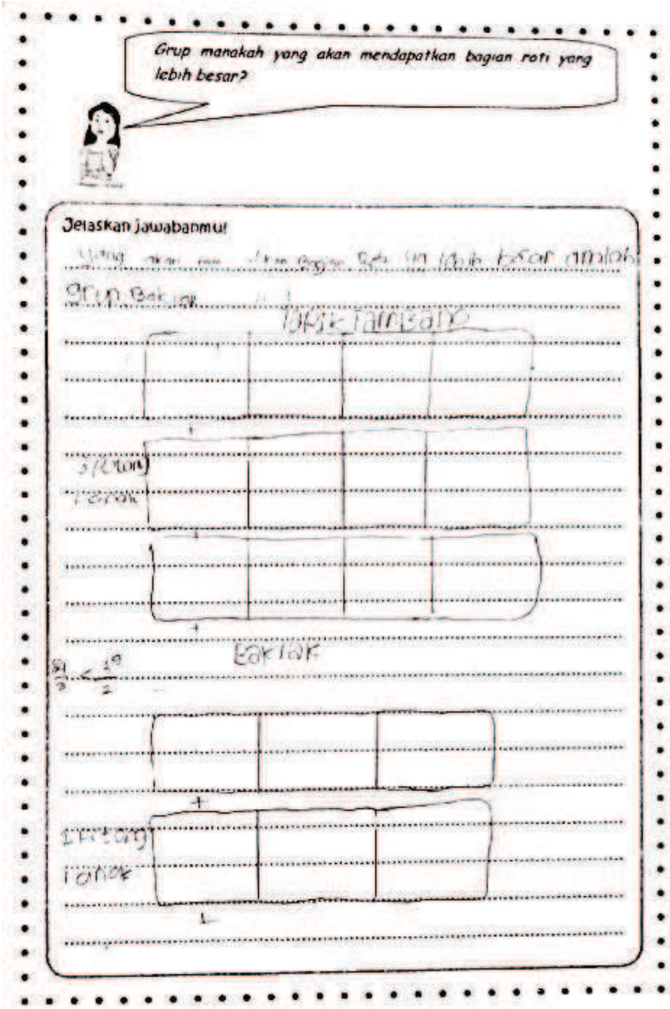
Meeting 1, Activity 1
Cycle 2




2 roti untuk 3 anak yang ikut lomba bakiak
Setiap anak mendapat 2/6 bagian roti jadi semuanya mendapat bagian yang sama.
3 roti untuk 4 anak di tarik tambang.
Setiap anak mendapat 3/12 bagian roti jadi semuanya mendapat bagian yang sama
24 18
2/6 > 3/12

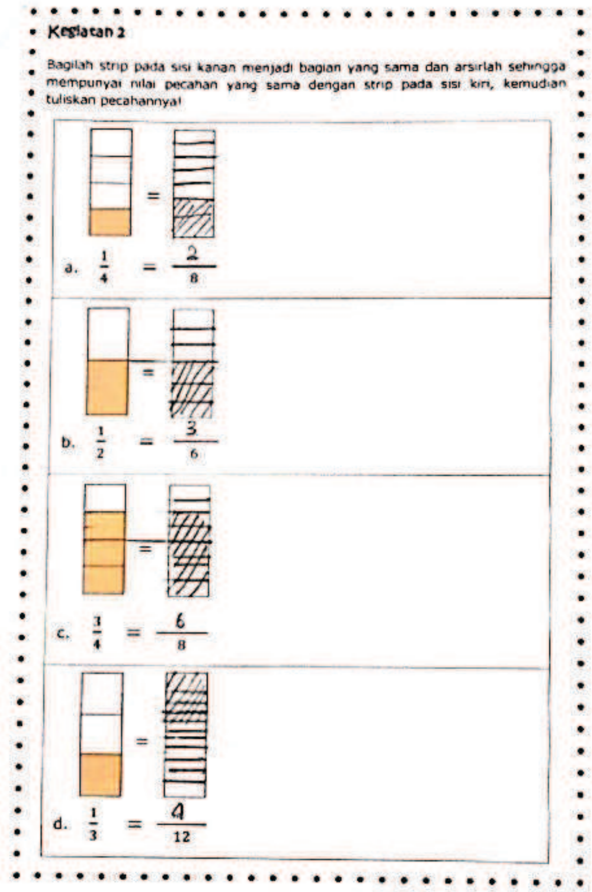
Meeting 1, Activity 1
Cycle 3

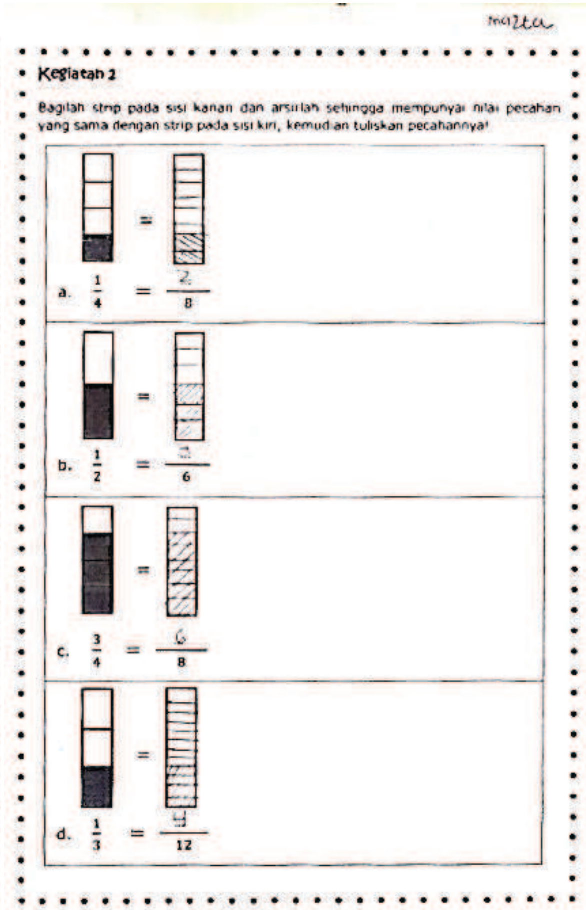


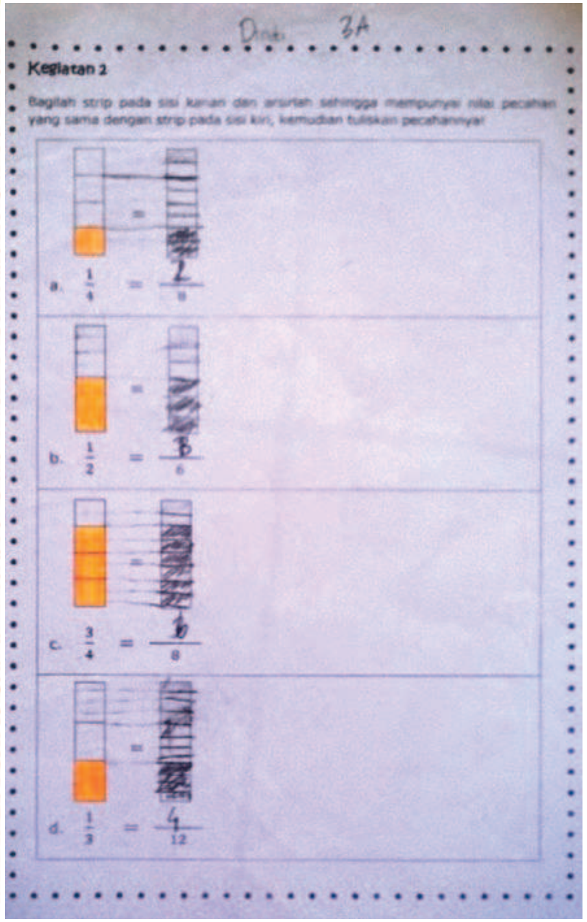


Meeting 1, Activity 2

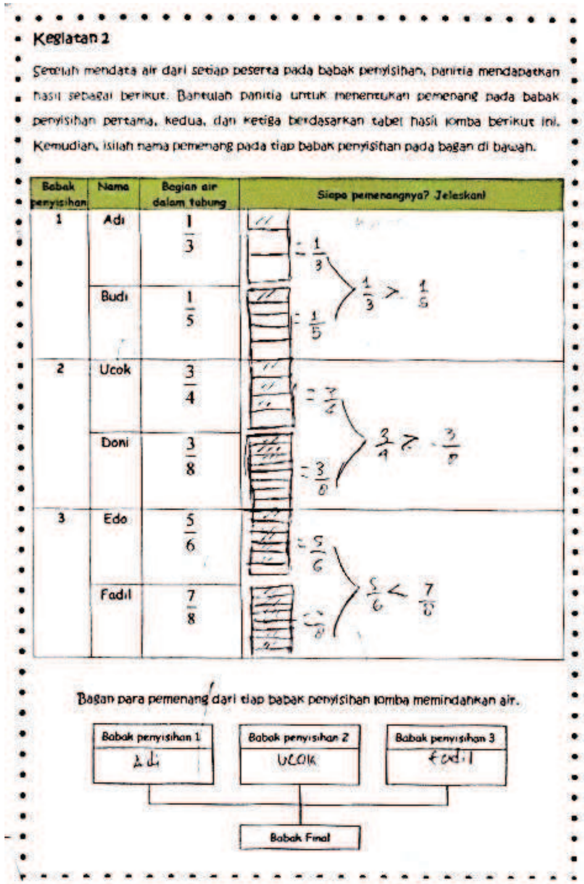
Cycle 1 **Cycle 2** **Cycle 3**







Meeting 2, Activity 2
Cycle 1



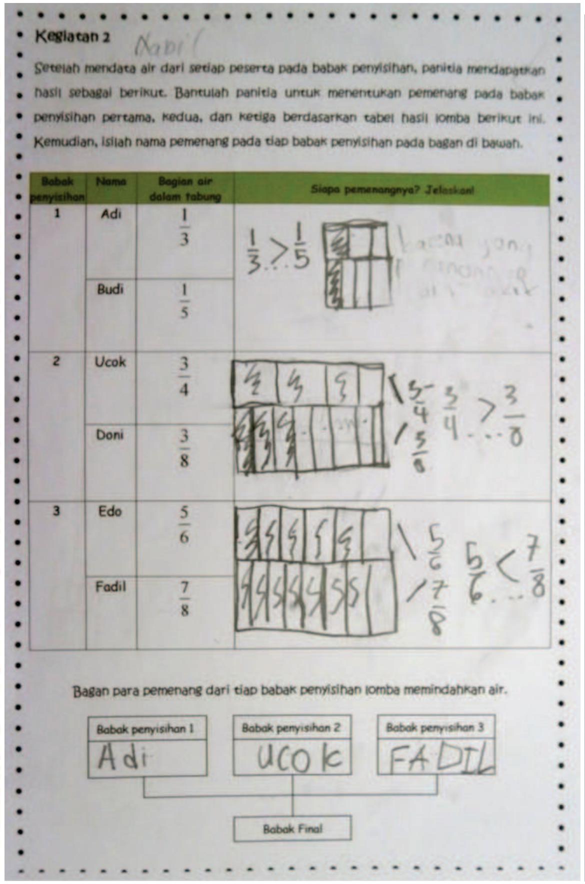
Kegiatan 2

Setelah mendata air dari setiap peserta pada babak penyisihan, panitia mendapatkan hasil sebagai berikut. Bantulah panitia untuk menentukan pemenang pada babak penyisihan pertama, kedua, dan ketiga berdasarkan tabel hasil lomba berikut ini. Kemudian, isilah nama pemenang pada tiap babak penyisihan pada bagan di bawah.

Babak penyisihan	Nama	Bagian air dalam tabung	Siapa pemenangnya? Jelaskan!
1	Adi	$\frac{1}{3}$	$\frac{1}{3} > \frac{1}{5}$
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	$\frac{3}{4} > \frac{3}{8}$
	Doni	$\frac{3}{8}$	
3	Edo	$\frac{5}{6}$	$\frac{5}{6} < \frac{7}{8}$
	Fadil	$\frac{7}{8}$	

Bagan para pemenang dari tiap babak penyisihan lomba memindahkan air.

Babak penyisihan 1: Adi
Babak penyisihan 2: UCOK
Babak penyisihan 3: Fadil
Babak Final



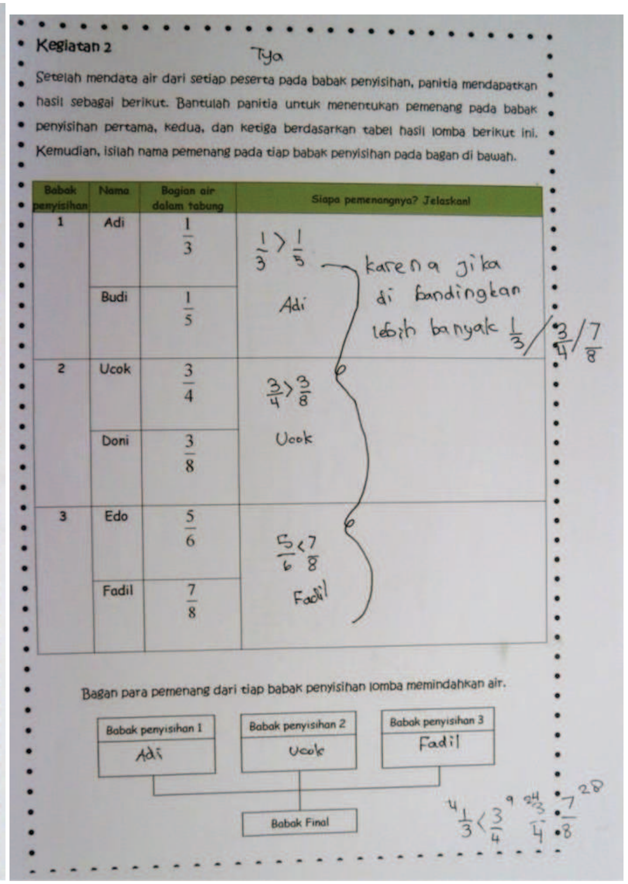
Kegiatan 2 Nabil

Setelah mendata air dari setiap peserta pada babak penyisihan, panitia mendapatkan hasil sebagai berikut. Bantulah panitia untuk menentukan pemenang pada babak penyisihan pertama, kedua, dan ketiga berdasarkan tabel hasil lomba berikut ini. Kemudian, isilah nama pemenang pada tiap babak penyisihan pada bagan di bawah.

Babak penyisihan	Nama	Bagian air dalam tabung	Siapa pemenangnya? Jelaskan!
1	Adi	$\frac{1}{3}$	$\frac{1}{3} > \frac{1}{5}$
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	$\frac{3}{4} > \frac{3}{8}$
	Doni	$\frac{3}{8}$	
3	Edo	$\frac{5}{6}$	$\frac{5}{6} < \frac{7}{8}$
	Fadil	$\frac{7}{8}$	

Bagan para pemenang dari tiap babak penyisihan lomba memindahkan air.

Babak penyisihan 1: Adi
Babak penyisihan 2: UCOK
Babak penyisihan 3: FADIL
Babak Final



Kegiatan 2 Tya

Setelah mendata air dari setiap peserta pada babak penyisihan, panitia mendapatkan hasil sebagai berikut. Bantulah panitia untuk menentukan pemenang pada babak penyisihan pertama, kedua, dan ketiga berdasarkan tabel hasil lomba berikut ini. Kemudian, isilah nama pemenang pada tiap babak penyisihan pada bagan di bawah.

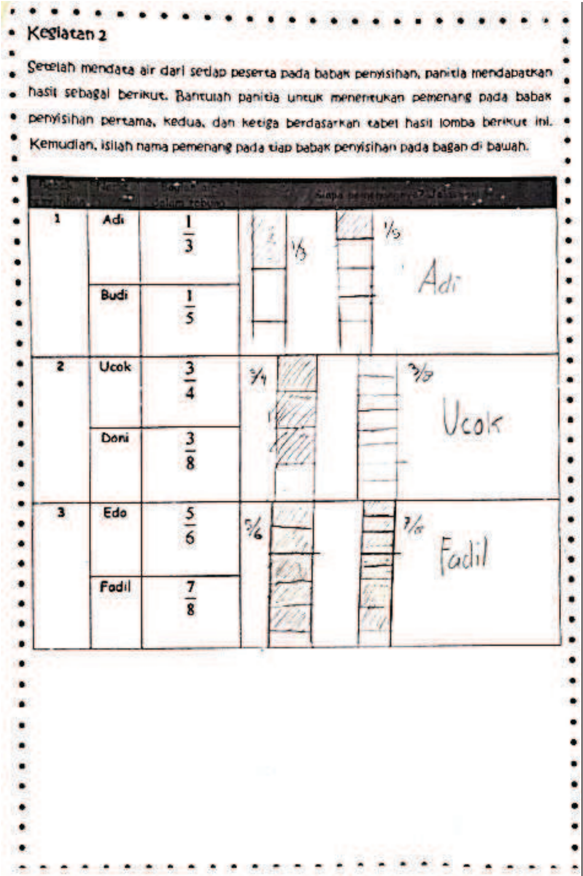
Babak penyisihan	Nama	Bagian air dalam tabung	Siapa pemenangnya? Jelaskan!
1	Adi	$\frac{1}{3}$	$\frac{1}{3} > \frac{1}{5}$ Adi — karena jika di bandingkan lebih banyak $\frac{1}{3}$ / $\frac{3}{4}$ / $\frac{7}{8}$
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	$\frac{3}{4} > \frac{3}{8}$ Ucok
	Doni	$\frac{3}{8}$	
3	Edo	$\frac{5}{6}$	$\frac{5}{6} < \frac{7}{8}$ Fadil
	Fadil	$\frac{7}{8}$	

Bagan para pemenang dari tiap babak penyisihan lomba memindahkan air.

Babak penyisihan 1: Adi
Babak penyisihan 2: Ucok
Babak penyisihan 3: Fadil
Babak Final

$\frac{1}{3} < \frac{3}{4} \quad \frac{3}{4} \quad \frac{7}{8}$

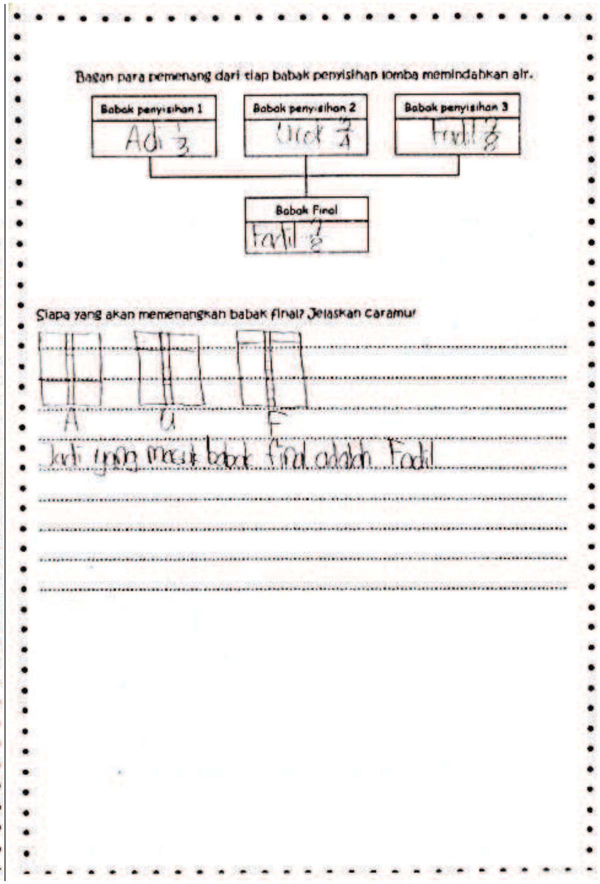
Meeting 2, Activity 2
Cycle 2



Kegiatan 2

Setelah mendata air dari setiap peserta pada babak penyisihan, panitia mendapatkan hasil sebagai berikut. Bantulah panitia untuk menentukan pemenang pada babak penyisihan pertama, kedua, dan ketiga berdasarkan tabel hasil lomba berikut ini. Kemudian, isilah nama pemenang pada tiap babak penyisihan pada bagan di bawah.

Babak penyisihan	Nama	Banyak air dalam tabung	Siapa pemenangnya? Jelaskan!
1	Adi	$\frac{1}{3}$	1/3 1/5 Adi
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	3/4 3/8 Ucok
	Doni	$\frac{3}{8}$	
3	Edo	$\frac{5}{6}$	5/6 7/8 Fadil
	Fadil	$\frac{7}{8}$	



Bagan para pemenang dari tiap babak penyisihan lomba memindahkan air.

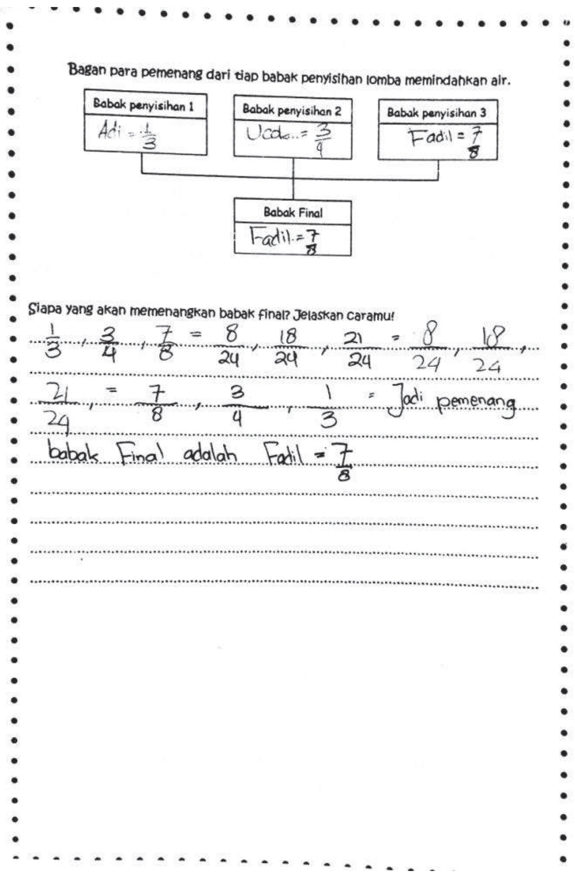
Babak penyisihan 1	Babak penyisihan 2	Babak penyisihan 3
Adi $\frac{1}{3}$	Ucok $\frac{3}{4}$	Fadil $\frac{7}{8}$

Babak Final
Fadil $\frac{7}{8}$

Siapa yang akan memenangkan babak final? Jelaskan caramu!

A U F

Jadi yang masuk babak final adalah Fadil



Bagan para pemenang dari tiap babak penyisihan lomba memindahkan air.

Babak penyisihan 1	Babak penyisihan 2	Babak penyisihan 3
Adi = $\frac{1}{3}$	Ucok = $\frac{3}{4}$	Fadil = $\frac{7}{8}$

Babak Final
Fadil = $\frac{7}{8}$

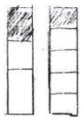
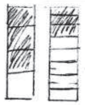
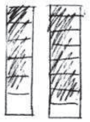
Siapa yang akan memenangkan babak final? Jelaskan caramu!

$\frac{1}{3}, \frac{3}{4}, \frac{7}{8} = \frac{8}{24}, \frac{18}{24}, \frac{21}{24} = \frac{8}{24}, \frac{18}{24}, \frac{21}{24} = \frac{7}{8}, \frac{3}{4}, \frac{1}{3}$ = Jadi pemenang babak Final adalah Fadil = $\frac{7}{8}$

Meeting 2, Activity 2
Cycle 3

Kegiatan 2

Setelah mendata air dari setiap peserta pada babak penyisihan, panitia mendapatkan hasil sebagai berikut. Bantulah panitia untuk menentukan pemenang pada babak penyisihan pertama, kedua, dan ketiga berdasarkan tabel hasil lomba berikut ini. Kemudian, isilah nama pemenang pada tiap babak penyisihan pada bagan di bawah.

Babak penyisihan	Nama	Bagian air dalam tabung	Siapa pemenangnya? Jelaskan!
1	Adi	$\frac{1}{3}$	 Adi
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	 Doni
	Doni	$\frac{3}{8}$	
3	Edo	$\frac{5}{6}$	 Fadil
	Fadil	$\frac{7}{8}$	

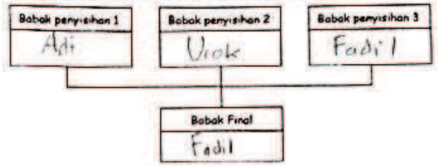
Ezra, Lia, ~~apa~~ afnan, yoga

Kegiatan 2

Setelah mendata air dari setiap peserta pada babak penyisihan, panitia mendapatkan hasil sebagai berikut. Bantulah panitia untuk menentukan pemenang pada babak penyisihan pertama, kedua, dan ketiga berdasarkan tabel hasil lomba berikut ini. Kemudian, isilah nama pemenang pada tiap babak penyisihan pada bagan di bawah.

Babak penyisihan	Nama	Bagian air dalam tabung	Siapa pemenangnya? Jelaskan!
1	Adi	$\frac{1}{3}$	Adi, $\frac{1}{3} > \frac{1}{5}$
	Budi	$\frac{1}{5}$	
2	Ucok	$\frac{3}{4}$	ucok. $\frac{3}{4} > \frac{3}{8}$
	Doni	$\frac{3}{8}$	$\frac{6}{8} > \frac{3}{8}$
3	Edo	$\frac{5}{6}$	Fadil, $\frac{5}{6} \dots \frac{7}{8}$
	Fadil	$\frac{7}{8}$	$\frac{20}{24} < \frac{21}{24}$

Bagan para pemenang dari tiap babak penyisihan lomba memindahkan air.



Siapa yang akan memenangkan babak final? Jelaskan caramu!

Yang akan memenangkan Babak final adalah Fadil

Caraku: Fadil paling banyak karena 7/8

$\frac{5}{6} < \frac{7}{8}$

Meeting 2, Activity 3
Cycle 1

Bahan Diskusi Pertemuan ke 2

Ungkaplah pecahan yang lebih dari $\frac{1}{2}$ dan jelaskan!

$\frac{2}{3}$ $\frac{3}{7}$ $\frac{2}{5}$ $\frac{1}{3}$ $\frac{4}{9}$ $\frac{3}{4}$

$\frac{3}{10}$ $\frac{5}{8}$ $\frac{5}{12}$ $\frac{4}{7}$ $\frac{5}{6}$ $\frac{1}{4}$

Jawablah pertanyaan berikut dan jelaskan!

1. $\frac{1}{5}+\frac{3}{4}$
   a. Apakah hasilnya lebih atau kurang dari $\frac{1}{2}$?
   b. Apakah hasilnya lebih atau kurang dari 1? kurang

2. $\frac{2}{3}+\frac{1}{6}$
   a. Apakah hasilnya lebih atau kurang dari $\frac{1}{2}$?
   b. Apakah hasilnya lebih atau kurang dari 1?

c. $\frac{3}{5}+\frac{1}{2}$
   a. Apakah hasilnya lebih atau kurang dari $\frac{1}{2}$?
   b. Apakah hasilnya lebih atau kurang dari 1?

Ibu Dewi akan berpartisipasi dalam lomba masak. Dirumah, ia menyiapkan bahan-bahan yang diperlukan seperti susu cair dan santan. Ia membutuhkan setengah kotak susu cair

Ibu Dewi mempunyai dua kotak susu berisi $\frac{2}{3}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama. Setelah dicampur, apakah totalnya lebih atau kurang dari setengah kotak?

Jelaskan!

karena lebih dari $\frac{1}{2}$

Meeting 2, Activity 3
Cycle 2

Bahan Diskusi Pertemuan ke 2 Nama : Ditarana

Lingkarilah pecahan yang lebih dari $\frac{1}{2}$ dan jelaskan!

($\frac{3}{4}$)	$\frac{4}{12}$	$\frac{2}{8}$	($\frac{5}{8}$)
$\frac{3}{10}$	($\frac{4}{6}$)	$\frac{1}{3}$	($\frac{9}{12}$)

Jawablah pertanyaan berikut dan jelaskan!

1. $\frac{1}{5}+\frac{3}{4} = \frac{4}{20} + \frac{12}{20} = \frac{16}{20}$
   a. Apakah $\frac{1}{5}$ lebih atau kurang dari $\frac{1}{2}$? Kurang
   b. Apakah $\frac{3}{4}$ lebih atau kurang dari $\frac{1}{2}$? Lebih

Apa yang bisa kamu simpulkan?

   c. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari $\frac{1}{2}$? Lebih
   d. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari 1? Kurang

2. $\frac{4}{6}+\frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$
   a. Apakah $\frac{4}{6}$ lebih atau kurang dari $\frac{1}{2}$? lebih
   b. Apakah $\frac{1}{4}$ lebih atau kurang dari $\frac{1}{2}$? kurang

Apa yang bisa kamu simpulkan?

   c. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari $\frac{1}{2}$? Lebih
   d. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari 1? Kurang

Kegiatan

Ibu Dewi akan berpartisipasi dalam lomba masak. Dirumah, ia menyiapkan bahan-bahan yang diperlukan seperti susu cair dan santan. Ia membutuhkan setengah kotak susu cair.

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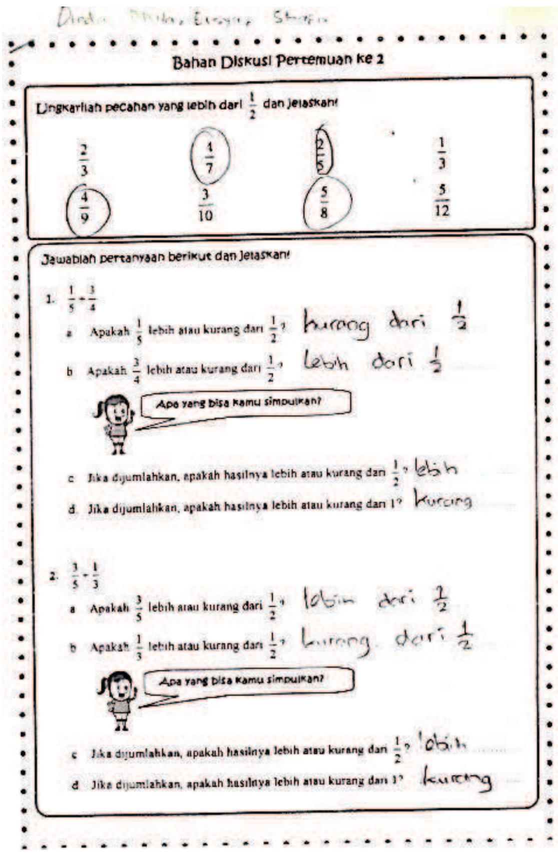
Setelah dicampur, apakah jumlahnya lebih atau kurang dari setengah kotak?

Jelaskan!

lebih dari $\frac{1}{2}$ karena $\frac{2}{3}$ lebih dari $\frac{1}{2}$

Meeting 2, Activity 3
Cycle 3



Bahan Diskusi Pertemuan ke 2

Lingkarilah pecahan yang lebih dari $\frac{1}{2}$ dan jelaskan!

$\frac{2}{3}$ $\frac{4}{7}$ $\frac{2}{5}$ $\frac{1}{3}$

$\frac{4}{9}$ $\frac{3}{10}$ $\frac{5}{8}$ $\frac{5}{12}$

Jawablah pertanyaan berikut dan jelaskan!

1. $\frac{1}{5} + \frac{3}{4}$
   - a. Apakah $\frac{1}{5}$ lebih atau kurang dari $\frac{1}{2}$? kurang dari $\frac{1}{2}$
   - b. Apakah $\frac{3}{4}$ lebih atau kurang dari $\frac{1}{2}$? lebih dari $\frac{1}{2}$

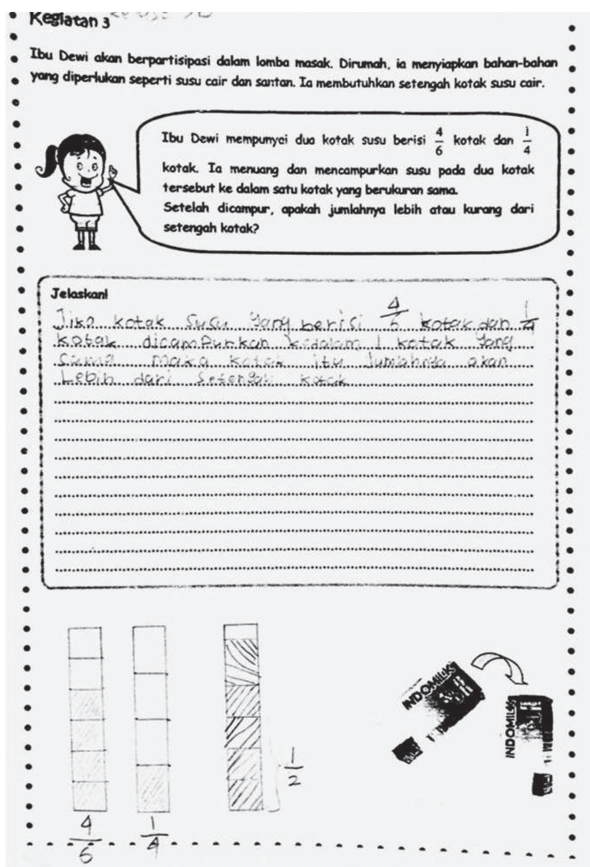
   Apa yang bisa kamu simpulkan?

   - c. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari $\frac{1}{2}$? lebih
   - d. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari 1? kurang

2. $\frac{3}{5} + \frac{1}{3}$
   - a. Apakah $\frac{3}{5}$ lebih atau kurang dari $\frac{1}{2}$? lebih dari $\frac{1}{2}$
   - b. Apakah $\frac{1}{3}$ lebih atau kurang dari $\frac{1}{2}$? kurang dari $\frac{1}{2}$

   Apa yang bisa kamu simpulkan?

   - c. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari $\frac{1}{2}$? lebih
   - d. Jika dijumlahkan, apakah hasilnya lebih atau kurang dari 1? kurang



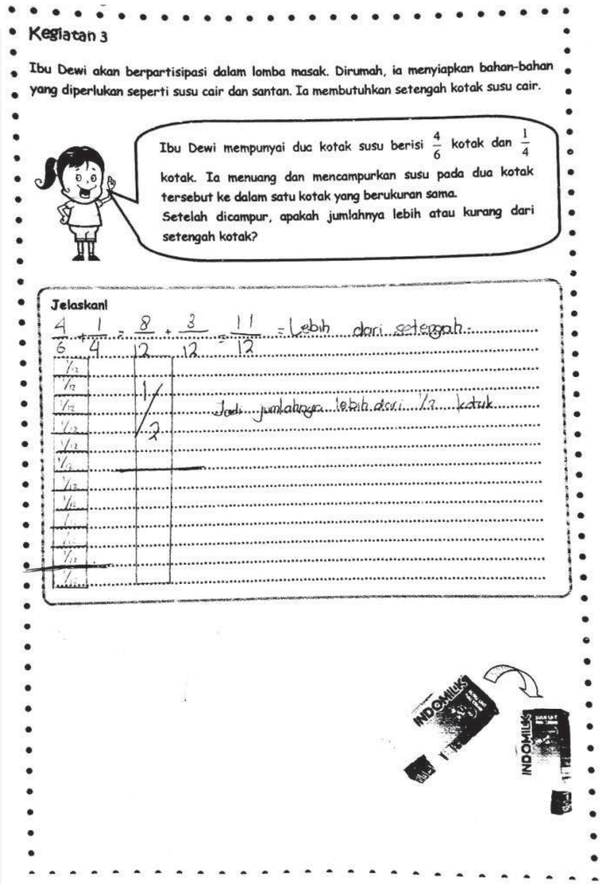
Kegiatan 3

Ibu Dewi akan berpartisipasi dalam lomba masak. Dirumah, ia menyiapkan bahan-bahan yang diperlukan seperti susu cair dan santan. Ia membutuhkan setengah kotak susu cair.

Ibu Dewi mempunyai dua kotak susu berisi $\frac{4}{6}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama. Setelah dicampur, apakah jumlahnya lebih atau kurang dari setengah kotak?

Jelaskan!

Jika kotak susu yang berisi $\frac{4}{5}$ kotak dan $\frac{1}{4}$ kotak dicampurkan kedalam 1 kotak yang sama maka kotak itu jumlahnya akan lebih dari setengah kotak



Kegiatan 3

Ibu Dewi akan berpartisipasi dalam lomba masak. Dirumah, ia menyiapkan bahan-bahan yang diperlukan seperti susu cair dan santan. Ia membutuhkan setengah kotak susu cair.

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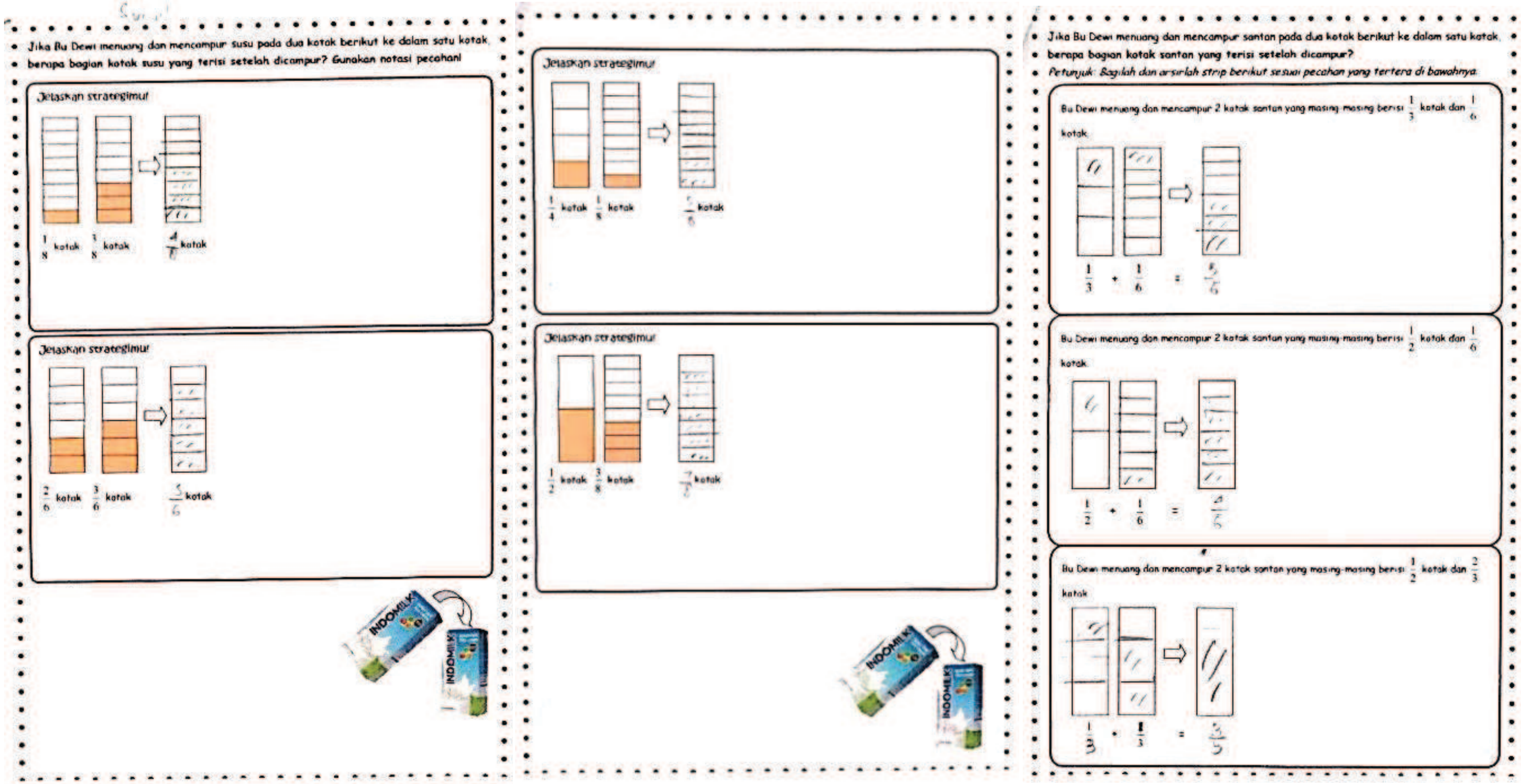
Jelaskan!

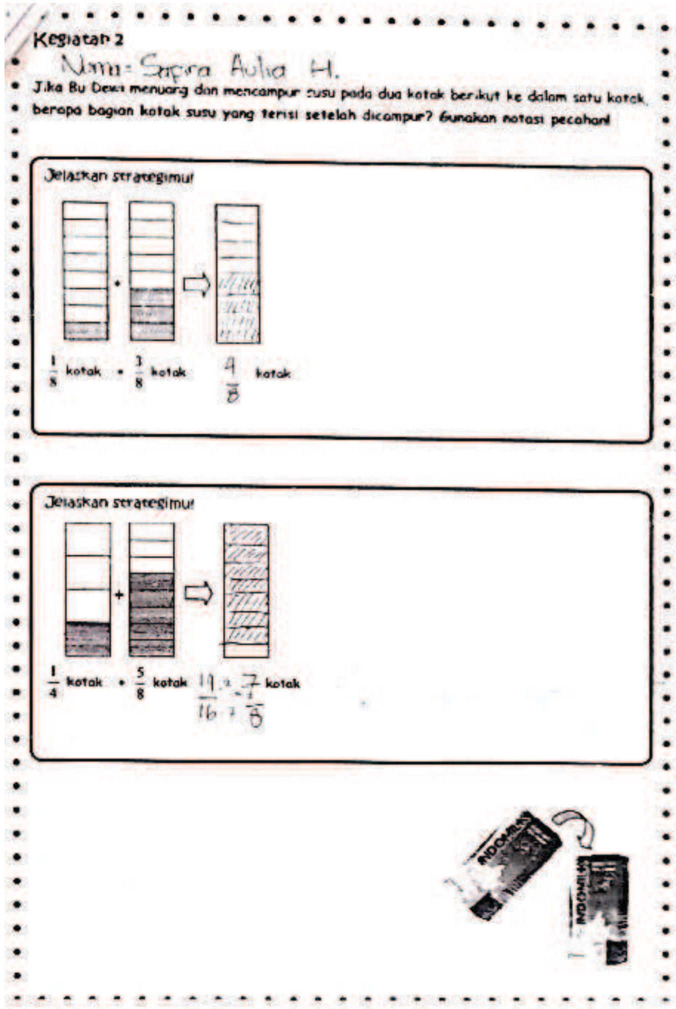
$\frac{4}{6} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$ = Lebih dari setengah.

Jadi jumlahnya lebih dari $\frac{1}{2}$ kotak

Meeting 3, Activity 2
Cycle 1



Meeting 3, Activity 2
Cycle 2



Kegiatan 2

Nama: Safira Aulia H.

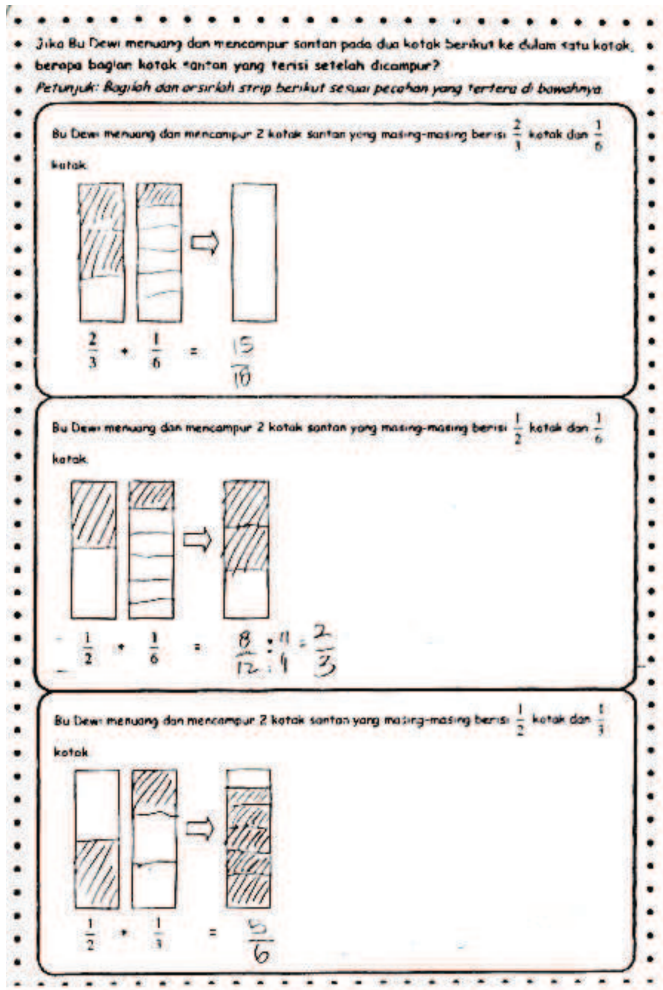
Jika Bu Dewi menuang dan mencampur susu pada dua kotak berikut ke dalam satu kotak, berapa bagian kotak susu yang terisi setelah dicampur? Gunakan notasi pecahan!

Jelaskan strategimu!

$\frac{1}{8}$ kotak + $\frac{3}{8}$ kotak $\frac{4}{8}$ kotak

Jelaskan strategimu!

$\frac{1}{4}$ kotak + $\frac{5}{8}$ kotak $\frac{11}{16} = \frac{7}{8}$ kotak



Jika Bu Dewi menuang dan mencampur santan pada dua kotak berikut ke dalam satu kotak, berapa bagian kotak santan yang terisi setelah dicampur?

Petunjuk: Bagilah dan arsirlah strip berikut sesuai pecahan yang tertera di bawahnya.

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{2}{3}$ kotak dan $\frac{1}{6}$ kotak

$\frac{2}{3} + \frac{1}{6} = \frac{15}{18}$

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{6}$ kotak

$\frac{1}{2} + \frac{1}{6} = \frac{8}{12} : \frac{4}{4} = \frac{2}{3}$

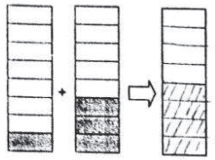
Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{3}$ kotak

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

Kegiatan 2 Krishna suryadi prabowo 3.B

Jika Bu Dewi menuang dan mencampur susu pada dua kotak berikut ke dalam satu kotak, berapa bagian kotak susu yang terisi setelah dicampur? Gunakan notasi pecahan!

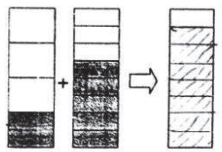
Jelaskan strategimu!



$\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

$\frac{1}{8}$ kotak + $\frac{3}{8}$ kotak $\frac{4}{8}$ kotak

Jelaskan strategimu!



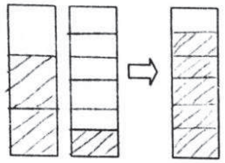
$\frac{1}{4} + \frac{5}{8} = \frac{2}{8} + \frac{5}{8} = \frac{7}{8}$

$\frac{1}{4}$ kotak + $\frac{5}{8}$ kotak $\frac{7}{8}$ kotak

Jika Bu Dewi menuang dan mencampur santan pada dua kotak berikut ke dalam satu kotak, berapa bagian kotak santan yang terisi setelah dicampur?

Petunjuk: Bagilah dan arsirlah strip berikut sesuai pecahan yang tertera di bawahnya.

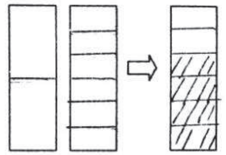
Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{2}{3}$ kotak dan $\frac{1}{6}$ kotak.



$\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

$\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$

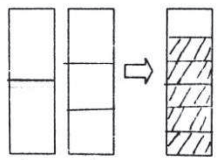
Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{6}$ kotak.



$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$

$\frac{1}{2} + \frac{1}{6} = \frac{4}{6}$

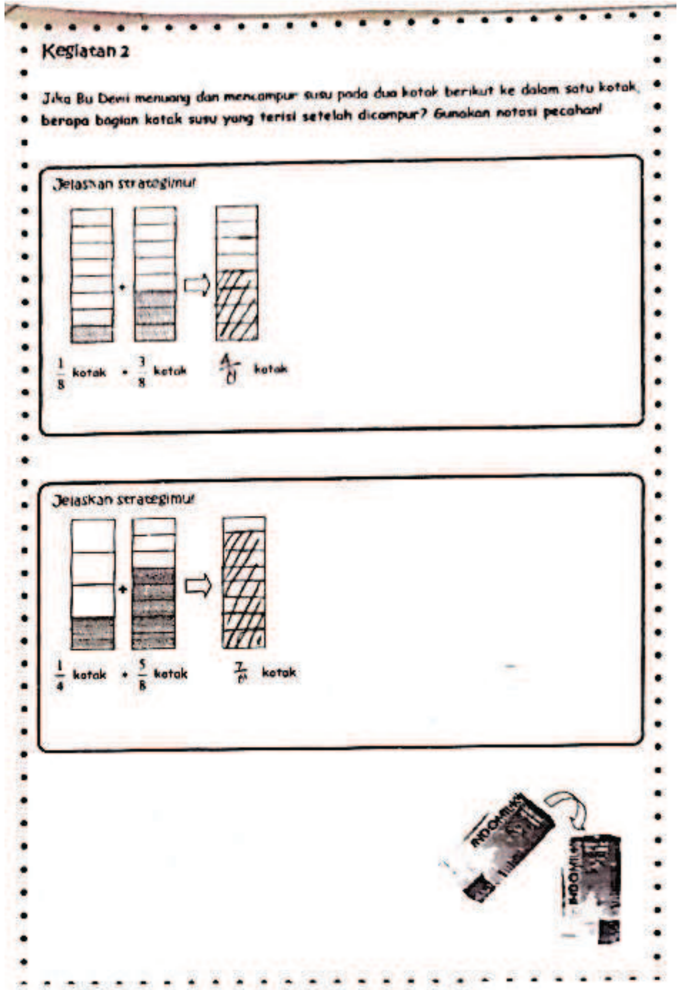
Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{3}$ kotak.



$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

Meeting 3, Activity 2
Cycle 3


Kegiatan 2

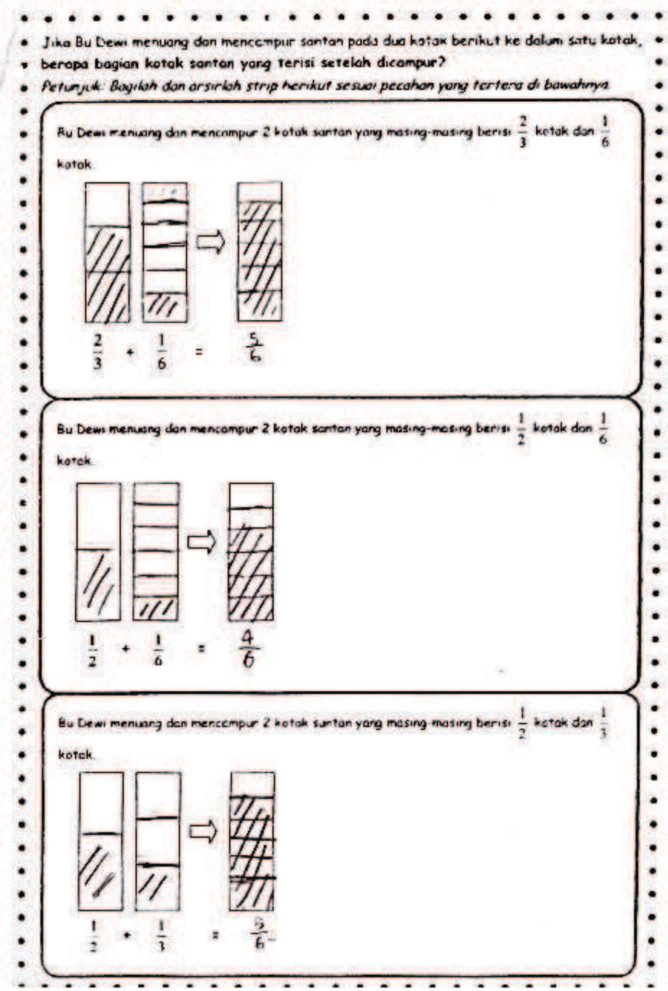
Jika Bu Dewi menuang dan mencampur susu pada dua kotak berikut ke dalam satu kotak, berapa bagian kotak susu yang terisi setelah dicampur? Gunakan notasi pecahan!

Jelaskan strategimu!

$\frac{1}{8}$ kotak + $\frac{3}{8}$ kotak $\frac{4}{8}$ kotak

Jelaskan strategimu!

$\frac{1}{4}$ kotak + $\frac{5}{8}$ kotak $\frac{7}{8}$ kotak


Jika Bu Dewi menuang dan mencampur santan pada dua kotak berikut ke dalam satu kotak, berapa bagian kotak santan yang terisi setelah dicampur?

Petunjuk: Bagilah dan arsirlah strip berikut sesuai pecahan yang tertera di bawahnya.

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{2}{3}$ kotak dan $\frac{1}{6}$ kotak.

$\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{6}$ kotak.

$\frac{1}{2} + \frac{1}{6} = \frac{4}{6}$

Bu Dewi menuang dan mencampur 2 kotak santan yang masing-masing berisi $\frac{1}{2}$ kotak dan $\frac{1}{3}$ kotak.

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

Meeting 3, Activity 3

Cycle 1

Kegiatan 3

Ibu Dewi akan berpartisipasi dalam lomba masak. Dirumah, ia menyiapkan bahan-bahan yang diperlukan seperti susu cair dan santan. Ia membutuhkan setengah kotak susu cair.

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Setelah dicampur, apakah jumlahnya lebih atau kurang dari setengah kotak?

Jelaskan!

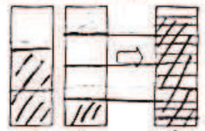
lebih karena $\frac{11}{12} > \frac{1}{2}$ $\frac{11}{12} > \frac{6}{12}$

Cycle 2

Ibu Dewi mempunyai dua kotak susu berisi $\frac{2}{3}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama.

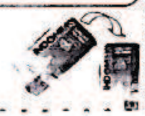
1. Setelah dicampur, kira-kira apakah jumlahnya lebih atau kurang dari setengah?
   lebih

2. Berapa bagian kotak yang terisi susu?

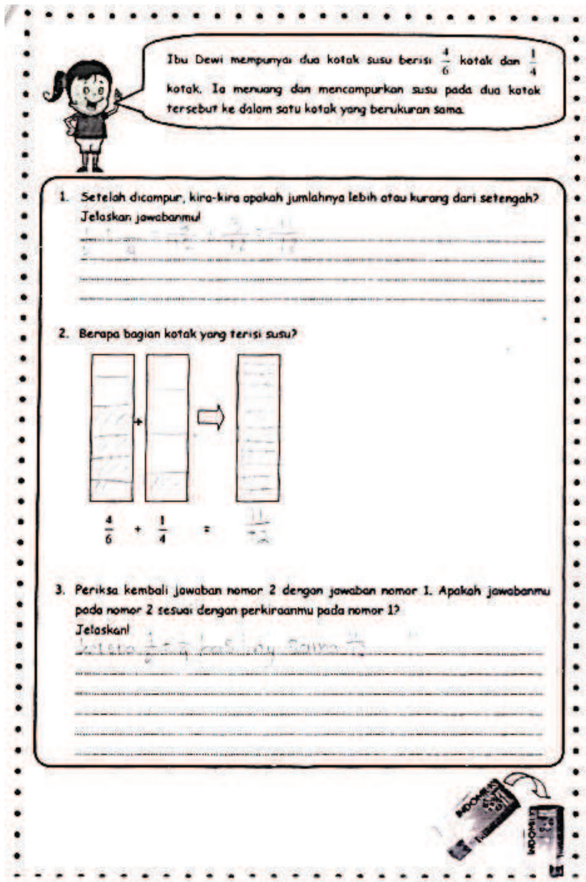


$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$

3. Periksa kembali jawaban nomor 2 dengan jawaban nomor 1. Apakah jawabanmu pada nomor 2 sesuai dengan perkiraanmu pada nomor 1?
   Jelaskan!
   ya nomor 1 karena $\frac{2}{3}$ lebih dari $\frac{1}{2}$ kotak nomor 2 karena kotaknya bisa di bagi menjadi 12 bagian lalu tinggal dijumlahkan



Cycle 3

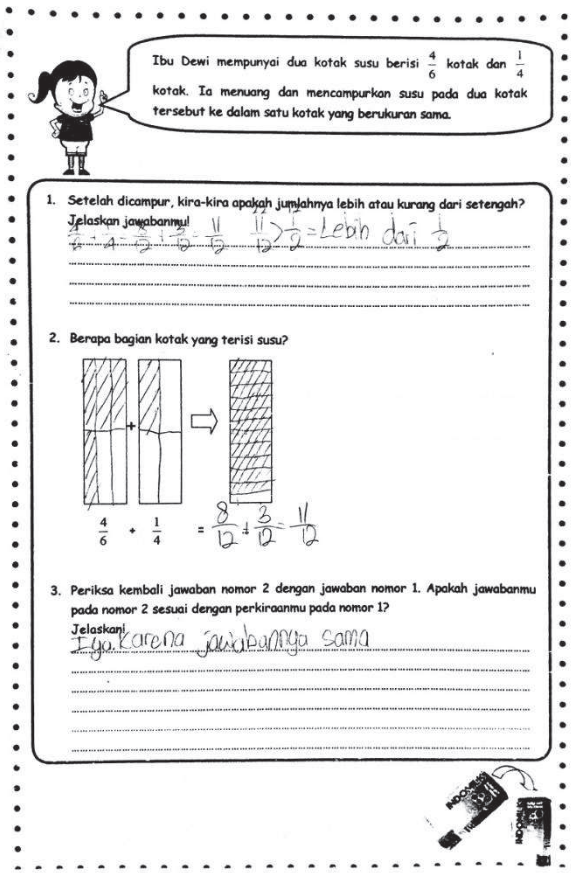


Ibu Dewi mempunyai dua kotak susu berisi $\frac{4}{6}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama.

1. Setelah dicampur, kira-kira apakah jumlahnya lebih atau kurang dari setengah? Jelaskan jawabanmu!
2. Berapa bagian kotak yang terisi susu?

$\frac{4}{6} + \frac{1}{4} =$

3. Periksa kembali jawaban nomor 2 dengan jawaban nomor 1. Apakah jawabanmu pada nomor 2 sesuai dengan perkiraanmu pada nomor 1? Jelaskan!

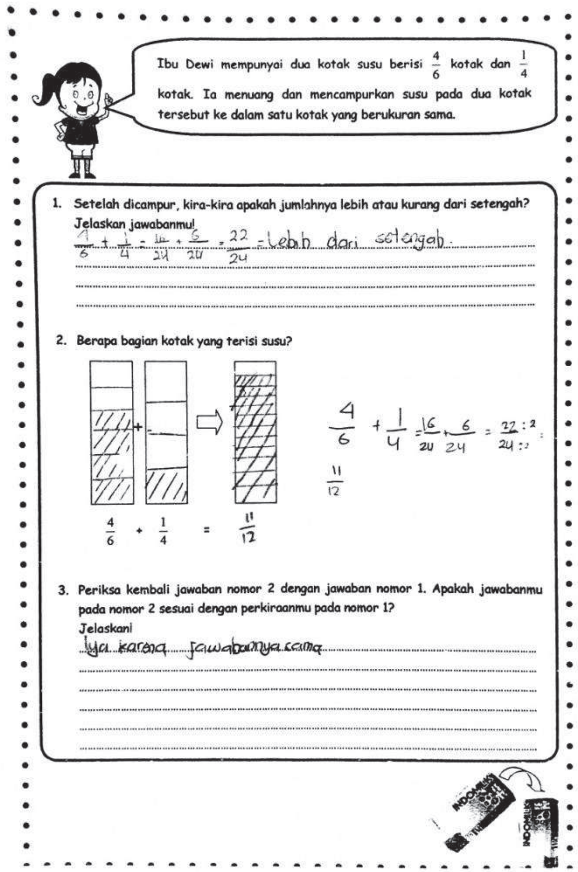


Ibu Dewi mempunyai dua kotak susu berisi $\frac{4}{6}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama.

1. Setelah dicampur, kira-kira apakah jumlahnya lebih atau kurang dari setengah? Jelaskan jawabanmu!
   $\frac{4}{6} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$; $\frac{11}{12} > \frac{1}{2}$ = Lebih dari $\frac{1}{2}$
2. Berapa bagian kotak yang terisi susu?

$\frac{4}{6} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

3. Periksa kembali jawaban nomor 2 dengan jawaban nomor 1. Apakah jawabanmu pada nomor 2 sesuai dengan perkiraanmu pada nomor 1? Jelaskan!
   Iya karena jawabannya sama



Ibu Dewi mempunyai dua kotak susu berisi $\frac{4}{6}$ kotak dan $\frac{1}{4}$ kotak. Ia menuang dan mencampurkan susu pada dua kotak tersebut ke dalam satu kotak yang berukuran sama.

1. Setelah dicampur, kira-kira apakah jumlahnya lebih atau kurang dari setengah? Jelaskan jawabanmu!
   $\frac{4}{6} + \frac{1}{4} = \frac{16}{24} + \frac{6}{24} = \frac{22}{24}$ = Lebih dari setengah.
2. Berapa bagian kotak yang terisi susu?

$\frac{4}{6} + \frac{1}{4} = \frac{16}{24} + \frac{6}{24} = \frac{22:2}{24:2} = \frac{11}{12}$

$\frac{4}{6} + \frac{1}{4} = \frac{11}{12}$

3. Periksa kembali jawaban nomor 2 dengan jawaban nomor 1. Apakah jawabanmu pada nomor 2 sesuai dengan perkiraanmu pada nomor 1? Jelaskan!
   Iya karena jawabannya sama