SUPPORTING 8TH GRADE STUDENTS' UNDERSTANDING OF THE AREA MEASUREMENT OF A CIRCLE

A THESIS

Submitted in Partial Fulfillment Requirements for the Degree of
Master of Science (M.Sc.)
In
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In collaboration between Sriwijaya University and Utrecht University)

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FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
JUNE 2014

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JUNE 2014

STATEMENT PAGE

I hereby:

Name : Sri Rejeki

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Date of birth : May 15, 1987

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State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

Palembang, June 2014
The one with the statement

Sri Rejeki
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ABSTRACT

Many studies revealed that most students were confuse between the concepts of area and perimeter of plane figures including the area and perimeter of a circle. The main reason is because the school practices make students focus on memorizing formulas instead of understanding concepts. Therefore, in this study, a set of learning activities about the area of a circle were designed based on Realistic Mathematics Education (RME) approach by using grid paper and applying reshaping strategy as the models. It aims at supporting 8^{th} graders to gradually understand the concept of the area of a circle. Consequently, design research was used as the research approach to achieve the aim. Thirty-eight 8^{th} graders of a Junior High School in Palembang, Indonesia were involved in this study. The results indicate that by using realistic problems, grid paper could stimulate students reasoning and understanding of the concept of area of a circle and the way to measure it informally. Moreover, the reshaping strategy of the sectors of circles which is grounded from the concept of area conservation support students understanding on the way to measure the area of a circle and how the formulas of the area of plane figures relate each other. Besides that, the whole activities gave important role to support students in distinguishing the area and perimeter of a circle.

Keywords: Area, plane figures, circle, design research, realistic mathematics education

ABSTRAK

Penelitian-penelitian terdahulu memaparkan bahwa siswa mengalami kesulitan dalam membedakan konsep luas dan keliling bangun datar termasuk luas dan keliling lingkaran. Penyebab utamanya adalah karena pembelajaran di sekolah yang membuat siswa lebih terfokus untuk menghafal rumus daripada memahami konsep. Oleh karena itu, satu rangkaian aktivitas pembelajaran pada materi luas lingkaran didesain berdasarkan prinsip pendekatan *Realistic Mathematics Education (RME)* dengan menggunakan kertas berpetak dan strategi *reshaping* sebagai model. Penelitian ini bertujuan untuk membantu siswa kelas 8 Sekolah Menengah Pertama (SMP) untuk secara bertahap memahami konsep luas lingkaran. Oleh karena itu, *design research* diterapkan sebagai pendekatan penelitian untuk mencapai tujuan penelitian. Sebanyak tiga puluh delapan siswa kelas 8 SMP di Palembang, Indonesia terlibat dalam penelitian ini. Hasil penelitian menunjukkan bahwa dengan menggunakan permasalahan realistik, kertas berpetak dapat memicu penalaran dan pemahaman siswa tentang luas lingkaran dan bagaimana untuk menentukan luas lingkaran dengan cara *informal*. Kemudian, strategi *reshaping* pada juring-juring lingkaran yang didasarkan pada teori konservasi luas dapat membantu pemahaman siswa tentang bagaimana menemukan rumus luas lingkaran dan bagaimana rumus luas bangun datar berkaitan satu dengan yang lainnya. Disamping itu, secara keseluruhan, aktivitas yang telah didesain berperan penting pada pemahaman siswa dalam membedakan luas dan keliling lingkaran.

Keywords: luas, bangun datar, lingkaran, *design research*, *realistic mathematics education*

SUMMARY

Sri Rejeki. Supporting 8th Grade Students' Understanding of the Area Measurement of A Circle

In Indonesia, the topic of the area of a circle is taught since students in the 6th grade of elementary school. And, it is continued when students in the 8th grade of junior high school. However, the school practices make students focus on memorizing formula instead of understanding concepts. Therefore, it cause a confusion between the concept of area and perimeter (Kidman, 1999; Oldham et al, 1999). For example students had counted perimeters to answer the questions about areas (Fauzan, 2002; Abdussakir and Achadiyah, 2009). In other words, students applied the formula of area and perimeter incorrectly. Therefore, in this study, we designed a set of learning activities based on Pendidikan Matematika Realistik Indonesia (PMRI) approach involving contexts which aims to bring students to the correct orientation of area. PMRI is the Indonesian version of Realistic Mathematics Education (RME). Furthermore, grid paper is used as a model to support students understanding of the area of a circle and the way to measure it by estimation. In addition, reshaping the sectors of a circle into a rectangle and other plane figures help students to derive the formula to measure the area of a circle.

Design research is chosen as the research approach in this study. The purpose of this type of research is to develop theories about both the process of learning and the means that are designed to support the learning (Gravemeijer and Cobb, 2006). And, it fits the aims of this study which are to develop theories about the process of learning on the topic of the area of a circle and about a set of activities which divided into six lessons which were designed to support the learning on the topic. Hence, what have been done in this study was developing a local instruction theory on the topic of the area measurement of a circle. Design research consists of three phases namely preparing for the experiment, design

experiment, and retrospective analysis (Gravemeijer and Cobb, 2006). This study was conducted based on those three phases mentioned.

Based on the finding of this study, it can be concluded that by designing realistic problems involving realistic contexts, it bring the students to the correct orientation of area. Moreover, the learning sequence which is involving grids as the representation of units of area measurement bring the students to gradually understand the concept of the area measurement of a circle which is related to counting or approximating the number of units covering a shape. In terms of the application of reshaping strategy, designing comparing activities involving irregular shapes bring students to apply reshaping strategy which can be a starting point into a discussion of the concept of area conservation. Moreover, the reshaping strategy of the sectors of circles which is grounded from the concept of area conservation support students understanding on the way to measure the area of a circle and how the formulas of the area of plane figures relate each other. Besides that, the whole activities gave important role to help students in distinguishing the area and perimeter of a circle.

RINGKASAN

Sri Rejeki. Supporting 8th Grade Students' Understanding of the Area Measurement of A Circle

Di Indonesia, materi luas lingkaran diajarkan di kelas 6 Sekolah Dasar (SD) dan diajarkan kembali ketika siswa duduk di kelas 8 Sekolah Menengah Pertama (SMP). Akan tetapi, pembelajaran matematika di sekolah membuat siswa fokus pada menghafal rumus, bukan memahami konsep. Hal ini menyebabkan siswa mengalami kesulitan untuk membedakan antara konsep luas dan keliling (Kidman, 1999; Oldham et al, 1999). Sebagai contoh, siswa menggunakan rumus keliling untuk menjawab pertanyaan tentang luas (Fauzan, 2002; Abdussakir dan Achadiyah, 2009). Oleh karena itu, dalam penelitian ini, kami merancang serangkaian kegiatan pembelajaran dengan pendekatan Pembelajaran Matematika Realistik Indonesia (PMRI) dengan menggunakan konteks yang bertujuan untuk membawa siswa pada pemahaman yang tepat tentang luas. PMRI adalah versi Indonesia dari pendekatan *Realistic Mathematics Education (RME)*. Selain itu, dalam penelitian ini, kertas berpetak digunakan sebagai model untuk membantu pemahaman siswa tentang konsep luas lingkaran dan cara untuk menentukan luas dengan estimasi. Selain itu, *reshaping* juring-juring lingkaran menjadi persegi panjang dan bangun-bangun datar lainnya dapat membantu pemahaman siswa tebtang bagaimana memperoleh rumus untuk mengukur luas lingkaran.

Design research dipilih sebagai pendekatan penelitian dalam penelitian ini. Tujuan dari jenis penelitian ini adalah untuk mengembangkan teori-teori tentang proses pembelajaran dan perangkat pembelajaran yang dirancang untuk mendukung proses belajar siswa (Gravemeijer dan Cobb, 2006). Hal ini sesuai dengan tujuan penelitian ini yaitu untuk mengembangkan teori tentang proses belajar pada materi pengukuran luas lingkaran dan tentang serangkaian kegiatan yang terbagi dalam enam pertemuan yang dirancang untuk mendukung pembelajaran pada materi tersebut. Oleh karena itu, apa yang telah dilakukan dalam penelitian ini adalah mengembangkan suatu *local instructional theory* pada

materi pengukuran luas lingkaran. Desain penelitian terdiri dari tiga tahap yaitu *preparing the experiment*, *design experiment*, dan *retrospective analysis* (Gravemeijer dan Cobb, 2006). Penelitian ini dilakukan berdasarkan tiga fase tersebut.

Berdasarkan temuan penelitian ini, dapat disimpulkan bahwa dengan merancang masalah-masalah realistik yang melibatkan konteks realistik, dapat membawa siswa pada pemahaman yang benar tentang luas. Selain itu, serangkaian pembelajaran yang melibatkan petak persegi sebagai representasi satuan pengukuran luas membawa siswa untuk secara bertahap memahami konsep pengukuran luas lingkaran yang berkaitan dengan menghitung atau memperkirakan jumlah satuan luas yang menutupi permukaan suatu bidang datar. Dalam hal penerapan strategi *reshaping*, dengan kegiatan merancang membandingkan yang melibatkan bentuk bangun datar yang tidak teratur membawa siswa untuk menerapkan strategi *reshaping* yang dapat menjadi titik awal dalam diskusi tentang konsep konservasi kawasan. Selain itu, strategi reshaping juring-juring lingkaran yang didasarkan pada konsep konservasi luas membantu pemahaman siswa tentang cara menemukan rumus luas lingkaran dan bagaimana rumus luas bangun datar berhubungan satu sama lain. Selain itu, secara keseluruhan, aktivitas-aktivitas yang telah didesain memberikan peran penting dalam membantu siswa untuk membedakan luas dan keliling lingkaran.

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I would also like to say thank you to the readers of this thesis. I consciously understand that this thesis is far away from being perfect. Thus, any supportive feedbacks and constructive ideas will be gladly accepted.

CURRICULUM VITAE

Palembang, June 2014

Sri Rejeki was born on May 15, 1987 in Grobogan, Central Java, Indonesia. She took her first formal education at Elementary School SD 1 Sedayu, Grobogan and graduated in 1999. She continued her study at Junior High School SMP N 1 Purwodadi, Grobogan and graduated in 2002. After that, she completed her secondary school at Senior High School SMA N 1 Purwodadi, Grobogan and graduated in 2005. In the same year, she started to study mathematics education at Mathematics Education Department, Faculty of Education and Teacher Training, Muhammadiyah University of Surakarta and awarded a bachelor degree sarjana pendidikan, S.Pd in 2009. During academic year 2010-2012, she was devoted to be a junior lecturer in the Mathematics Education Department, Muhammadiyah University of Surakarta. In the mid of 2012, she enrolled the International Master Program on Mathematics Education in collaboration between Sriwijaya University, Indonesia and Freudenthal Institute for Science and Mathematics Education, Utrecht University, Indonesia.

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CHAPTER I
INTRODUCTION

Area measurement is an important topic in mathematics (Kordaki and Potari, 1998). Moreover, according to Cavanagh (2008), the topic is an important part in middle-school curriculum because of two main reasons. Firstly, it is because of the wide variety of everyday application of area concepts in activities such as painting, gardening, tiling, and any tasks involving covering a two dimensional surface. Secondly, it is because area concepts are often used to introduce many other mathematical ideas. Moreover, Battista (2003) stated that area measurement is also the foundation to understand volume measurement in solid geometry.

However, both students and teachers experience difficulties with this topic. The main difficulty which is well documented in the literature is a confusion between the concepts of area and perimeter (Kidman, 1999; Oldham et al, 1999). For example in Indonesia, several studies found that some students had counted perimeters to answer the questions about areas (Fauzan, 2002; Abdussakir and Achadiyah, 2009). In other words, the students applied the formula of area and perimeter incorrectly. According to Baturo and Nason (1996), former studies show that the reasons behind students' difficulties are because the formal cultural practice for measuring area is based on the array notion of multiplication, and because of the learning experiences provided in school.

Related to these difficulties, there have been some studies which aimed at supporting students' understanding of the concepts of area (for example Winarti, 2011, and Fauzan, 2002). Yet, these studies were talking about the area of plane figures in general. And, there is not yet a particular investigation about the area of a circle. Moreover, based on an analysis of Indonesian matematics textbooks for the 8^{th} grade students of secondary school (for example Agus, N. A., 2007; Nuharini, D. and Wahyuni, T., 2008; Nugroho, H. and Meisaroh, L., 2009 Dris, J. and Tasari, 2011; Marsigit, 2011), those books have provided various contexts on

the topic of the area of a circle. However, in these cases, the role of contexts is only as a story for the introduction of the shape of a circle. Afterwards, the lesson directly jumps to the formal level. Hence, an innovation is needed to promote students' learning, particularly in the topic of the area measurement of a circle.

The innovation needed is not only about an implementation of a new way of teaching and learning mathematics, but also a new way of thinking about the purpose and practices of school mathematics. In this case, Pendidikan Matematika Realistik Indonesia (PMRI) or the Indonesian version of Realistic Mathematics Education (RME) is an approach which provides both of a new way of teaching and learning mathematics and a new way of thinking about the purpose and practices of school mathematics (Sembiring, Hadi, and Dolk, 2008).

Therefore, in this study, we designed a set of learning activities based on PMRI approach involving contexts which aims to bring students to the correct orientation of area. Furthermore, grid paper is used as a model to support students understanding of the area of a circle and the way to measure it by estimation. The grids are the representation of the units of area. And, according to Oldham et al. (1999) the area can be determined by counting or approximating the number of units covering a shape. In addition, reshaping the sectors of a circle into a rectangle and other plane figures help students to derive the formula to measure the area of a circle. It is related to the concept of area conservation and the area of the new shape is the sum of the original area (Oldham et al., 1999).

In fact, most Indonesian mathematics textbooks for 8^{th} grade students of secondary school have provided the strategy of cutting and reshaping the sectors of a circle into a rectangle to derive the formula to determine the area of a circle (for example Agus, N. A., 2007; Nuharini, D. and Wahyuni, T., 2008; Nugroho, H. and Meisaroh, L., 2009 Dris, J. and Tasari, 2011; Marsigit, 2011). However, what the students asked to do is just copying the steps told in the books with a ready used circle and a known shape of the rectangle. Therefore, we need to design less guided activities of reshaping the sectors of a circle. Moreover, to minimalize a big jump on students' thinking, the activities of reshaping the sectors of a circle might firstly into a parallelogram, then into a rectangle. In addition, an

activity of reshaping the sectors of a circle into other plane figures is important to strengthen students' understanding of the area measurement of a circle.

In terms of using grid paper in the area measurement of a circle, it is only a few mathematics textbook in Indonesia which provided this strategy (for example Nugroho, H. and Meisaroh, L., ; Marsigit, 2011). However, the grid paper is only used as an attribute to introduce square units. In other words, it is not really used as a tool to estimate the area of a circle. Thus, we need to focus more on the use of grid paper to support students' understanding of the area of a circle and as a tool to estimate the area of a circle.

Considering the issues about students' confusion of the area of a circle and the need of an innovation to promote students' learning about area particularly the area of a circle, this study aims to contribute to a local instructional theory which can support the development of students' understanding of the area measurement of a circle. Hence, the research question in this study is:

"*How can we support 8^{th} grade students' understanding of the area measurement of a circle?"*

CHAPTER II
THEORETICAL FRAMEWORK

2.1. The Concepts of Area

Area measurement is an important topic in mathematics, particularly in plane geometry. The topic is also the foundation for understanding volume measurement in solid geometry (Battista, 2003). Besides that, the topic also help to understand other topics in mathematics such as algebra and calculus.

The notion of area needs to be considered from two perspectives, namely a static perspective and a dynamic perspective (Baturo & Nason, 1996). The static perspective equates area with the amount of region (surface) which is enclosed within a boundary and the notion that this amount of region can be quantified. The dynamic perspective focuses on the relationship between the boundary of a shape and the amount of surface which it encloses, so that, as the boundary approaches a line, the area approaches zero. However, in the study of Baturo and Nason (1996), all students indicated that they had a static perspective of the notion of area. There were no students who came up with the dynamic perspective.

There are some basic aspects of area. Van der Valk & Broekman (1997) identified the following aspects:

a. The general idea of area

   Area is the closed contour of a two-dimensional shape.

b. Isometric transformation

   The area of a shape does not change when the shape is moved or canted, two shapes have the same area if surfaces fit, the area of a flat shape does not change when its surface is curved, and the area of a curved shape does not change when its surface is flattened.

c. Conservation of area (division and addition)

   The area of a shape equals the sum of the area of the parts, two shapes form a new shape when a part of the boundaries are put together and the area of the new shape is the sum of the original area.

d. Calculation

Involves the choice of a unit of area, can be determined by counting or approximating the number of units covering a shape, is dependent on the length, width, and configuration, can be calculated using a rule or formula, and is determined by the sum of small parts.

e. The area of spatial shapes

The area of spatial shapes can be specified or approximated by unfolding onto a flat plane and determining the area of the resulting net.

This study focuses on the (a), (c), and (d) aspects. We firstly focus on students' general idea of area and the area of a circle, continued by the calculation aspect involving units of area; determining area by counting or approximating the number of units covering a shape; dependent on the length, width, and configuration; calculating using a rule or formula; and determining area by the sum of small parts considering the aspect of conservation of area.

It is very important that students sufficiently understand those aspects in order to prevent them from experiencing conceptual difficulties. Therefore, school practices need to teach a more meaningful orientation of the concept of area (Kordaki & Potari, 1998). This attempt can be attained by connecting the concept to some examples of real life contexts. Common uses of the concept of area can be found in the activities of paving a courtyard with square paving stones, covering a floor with carpet, or covering a wall with wallpaper (Oldham, et al., 1999).

2.2. Students' and teachers' understanding of Area and Students' Difficulties

1. Students' and teachers' understanding of Area

The study of Kordaki and Potari (1998) has demonstrated the complexity of the concept of area and its measurement. The concept is not only a mathematical concept taught in school but also has something to do with students' personal experiences in their life. The findings of Kordaki and Potari's study also imply that school practices need to teach a more integrated and cultural orientation of the concept of area and place less

stress on mere calculation by formulas. Moreover, the study showed that students tend to use formulas in measuring area of plane figures, particularly for a regular shape. That is why it is necessary to give students an irregular shape and ask them to measure it without allowing them to use the area formulas and other conventional measurement tools.

Research in the field of mathematics education often reveals students' poor understanding in the topic of area measurement (Zacharos, 2006). In addition, this poor understanding is not only experienced by students but also by prospective teachers (Baturo & Nason, 1996). The study of Baturo and Nason found that the knowledge of perspective teachers about the nature and discourse of mathematics and about mathematics in culture and society was similarly alarming. It seemed to be based on assumptions such as: mathematics is mainly an arbitrary collection of facts and rules, most mathematical ideas have little or no relationship to real objects and therefore can only be represented symbolically, and the primary purpose of learning area measurement was the utilitarian one of being able to calculate areas of regular shapes.

2. Students' difficulties of area measurement

Many difficulties in the domain of area measurement have been described in the literature. Kamii (1996), in a study with 177 children in grades 4, 5, 7, and 9, questioned which students' difficulties in using area formula were due to their confusion between area and perimeter and which were due to their un-dimensional linear thinking. Oldham et al. (1999, p.29) also identified that there are four main difficulties regarding area measurement which are related to the aspects of area.

a. The first main difficulty has to do with measuring area. Students can measure the area of a plane figure by covering it with square units.
b. The second main difficulty is distinguishing between area and surface. In order to make the distinction, it is necessary to understand that the area can be expressed in a number which is connected to units and that the number does not change when the surface is changed.

c. The third main difficulty is distinguishing between area and perimeter. Some students may recognize area as perimeter and perimeter as area. Moreover, some others may suppose that shapes with the same areas also have the same perimeters and vice versa.
d. The fourth difficulty concerns the surface area of spatial shapes, such as cubes. Students may think a cube does not have an area, because they do not understand the addition aspect that is needed for summing the area of the sides.

Moreover, there were also studies which indicated that pupils in the middle years (year 5 to 8) get confused by the concepts of area and perimeter even though they may give correct answers to standard assessment questions requiring the use of formulas (Kidman & Cooper, 1997).

3. Promoting the learning of area of circles

In Indonesia, some previous studies have been conducted concerning area and perimeter. Fauzan (2002) emphasized the necessity of learning area and perimeter in a sequence. Learning this separately might cause students' confusion. Moreover it might also lead to the difficulty of distinguishing area and perimeter. Related to this confusion, the study of Winarti (2011) has showed that exploring the relations between area and perimeter can help students' learning in the domain of area and perimeter.

The study of Rahmi (2011) was focused on developing units of area measurement. The measurement process with the units improved when the students have had experiences with a covering activity by using units. The covering activity helps students to mentally partition the region into units. Through these activities the students can learn to use the unit to measure the area of two dimensional shapes, either regular or irregular shapes.

However, much attention in these studies has been devoted to area and perimeter only in regular polygons. Little is known about the circle as one of the plane figures. Hence, there is a need to promote a sequence of

learning activities which focuses on the area and the circumference of circles.

2.3. Applying Realistic Mathematics Education in Teaching Area of Circles

In this study, applying RME is in teaching the concepts of the area and the circumference of a circle is expected to be the solution of the need to teach a more meaningful orientation of those two concepts. This is because RME emphasizes both horizontal and vertical mathematizing. Horizontal mathematization means transforming a problem field into a mathematical problem and vertical mathematization means processing within the mathematical system (Gravemeijer, 1994). According to Treffers (1987), in horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. In this study, students will identify contextual problems of the size of some circular objects, formulate and visualize the problems in different ways, and discover relations between the objects and the mathematical concepts behind them. On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. In this study, students will use different models to estimate the area of shapes, combine and integrate the models, represent the relation with the formula of the area of a circle and other shapes of plane figures, and prove regularities in the ratio of the circumference and the diameter of a circle.

1. Learning activities based on the five tenets of RME

The process of designing a sequence of instructional activities in this study starts with experience-based activities. This is based on the five tenets for realistic mathematics education defined by Treffers (1987, p.248).

a. *Phenomenological exploration*

In the first instructional phase of a relatively new subject concrete contexts are needed for the mathematical activities. The aim is to get more insight into intuitive notions for concepts formation. This study

will use several contexts involving area measurement in daily life such as covering a floor with paving blocks, covering the bottom of a swimming pool with tiles, and covering a garden with grass.

b. *Using models and symbols for progressive mathematization*

From the very start, in elementary problem situations, a variety of vertical instruments such as models, schemas, diagrams, and symbols are offered, explored and developed as a bridge from the informal to the formal level. In this study, to support students' understanding of area measurement, grid paper can be used as a model to estimate the areas of regular and irregular plane figures. Moreover, it also involves a reshaping strategy which is grounded to the concept of area conservation.

c. *Using students' own construction*

Students are free to use their own strategies to solve problems as a starting point for the learning. Hence, these could lead to the emergence of various ways of solving a problem which can be used by the teacher to promote the learning process. The different ideas of strategies used by students are then used by the teacher to draw conclusions about the concept of area, the area measurement, and particularly the formula of the area of a circle.

d. *Interactivity*

The learning process is part of interactive instruction where individual work is combined with consulting fellow students, group discussion, collective work reviews, the presentation of one's own productions, the evaluation of various constructions on various levels and explanations by the teacher. In other words, the learning process of students is not only an individual process, but also a social process. In this study, students will work in a small group and discuss with their fellow students to solve the problem. Afterwards, the teacher will conduct a whole class discussion where the students can interact and contribute their ideas and their findings.

e. *Intertwinement*

What students do in the activity is not only about the topic of the area and the circumference of a circle itself. However, it relates to other domains. In this case, number sense and arithmetic operations play important roles in how they understand the area and the circumference of a circle and in how they discover its formulas.

2. Hypothetical learning trajectory on the domain of area and circumference of circles

Van den Heuvel-Panhuizen et al. (2008) distinguishes three phases in learning a (new) geometric concept or property. The phases are experiencing, explaining, and connecting. These phases offer a direction in outlining the teaching-learning trajectory of geometric concepts and the accompanying didactic approach. To indicate what should be understood in each of these phases, in this study these are explained as follows.

a. Experiencing

The basic assumption is that the start of the teaching-learning process should always take place in a natural way and that, from there on, insight develops to a higher and higher level (Van den Heuvel-Panhuizen et al., 2008). In this study, the activities of comparing objects; designing and reshaping a garden; and tiling a garden can be visual activities which seem natural for students.

b. Explaining

Teaching geometry in RME is required to bring the students to the desired insight, by making a model. In this study, grid paper can be a model which firstly can be used by students to estimate the area of a circle. Afterwards, the reshaping strategy which means reshaping a circle into other plane figures such as a parallelogram, a rectangle, a triangle, and a trapezoid can also be a model for students' experiencing phase and can also bring them to the explaining phase.

c. Connecting

The connecting phase means that the learned subject is connected to other concepts and phenomena, which should lead to a deepening of insight (Van den Heuvel-Panhuizen et al., 2008). In this study, one can think of the importance of an understanding about the area and a circumference of a circle to a daily life application. In this learning, students can estimate the budget needed for covering a circular garden with grass.

2.4. Area and circumference of circles for secondary school in Indonesia

1. Area of circles in Indonesian curriculum for secondary school

In Indonesia, the topic of circles is firstly introduced in the first semester of the 6th grade in elementary school. However, according to the Indonesian curriculum for elementary school (2006), the topic is only mentioned in one basic competence. Moreover, it only pays attention to the activity of calculating the area of a circle as described in this following table.

Standard Competence	Basic competence
Geometry and Measurement	
3. Calculate the area of a simple polygon, area of a circle and the volume of a triangular prism	3.1 Calculate the area of a polygon which is a combination of two simple plane figures 3.2 Calculate the area of circles 3.3 Calculate the volume of a triangular prism and a cylinder

Table 2.1 Circle for elementary school in Indonesian curriculum

Later on, in secondary school, the topic of circle is taught in the second semester of 8th grade students. Then, it is discussed more deeply in one standard competence (BNSP, 2006) as is described in table 2.2.

Standard Competence	Basic competence

Geometry and Measurement	
4. Determine the elements, the properties, and the size of a circle	4.1 Determine the properties and the elements of a circle 4.2 Calculate the circumference and the area of a circle 4.3 Use the relationship among central angles, arc length, and the area of sectors in problem solving 4.4 Calculate the length of common tangents of two circles 4.5 Draw the inner and the outer circle of a triangle

Table 2.2 Circle in secondary school in Indonesian curriculum

This study will be conducted in the 8th grade of secondary school and will only focus on the first and the second basic competence.

2. Curriculum implementation in the domain of area of circles for secondary school

As is stated in the Indonesian curriculum (2006, p.147), mathematics learning should start by introducing contextual problems. Hence, by using contextual problems, students can be gradually guided to understand the mathematics concept. Many mathematics textbooks for the 8thgrade of secondary school have provided various contexts on the topic of circles (for example Agus, N. A., 2007; Nuharini, D. and Wahyuni, T., 2008; Nugroho, H. and Meisaroh, L., 2009 Dris, J. and Tasari, 2011; Marsigit, 2011). However, in these cases, the role of contexts is only as a story for the introduction of the shape of a circle. Afterwards, the lesson directly jumps to the formal level. There is insufficient attention for students' reasoning in contexts to support students to develop their understanding.

Those books have also provided the strategy of cutting and reshaping the sectors of a circle into a rectangle to derive the formula to determine the area of a circle (for example Agus, N. A., 2007; Nuharini, D. and Wahyuni, T., 2008; Nugroho, H. and Meisaroh, L., 2009 Dris, J. and Tasari, 2011; Marsigit, 2011). However, what the students asked to do is just copying the steps told in the books with a ready used circle and a

known shape of the rectangle. In terms of using grid paper in the area measurement of a circle, it is only a few mathematics textbook in Indonesia which provided this strategy (for example Nugroho, H. and Meisaroh, L., ; Marsigit, 2011). However, the grid paper is only used as an attribute to introduce square units. In other words, it is not really used as a tool to estimate the area of a circle.

In addition, some small scale studies in Indonesia regarding the topic of circles have identified that teachers still tend to teach in a traditional way (Abdussakir, & Aschadiyah, 2009; Kurniawati, 2012; Halawa, 2009). The lesson is usually started by giving students the formula, applying the formula to solve some problems as examples and giving students some exercises to test their understanding about the topic. Hence, students focus more on applying the formula instead of understanding.

2.5. Research Question

Based on the above analysis, the aim of this research is to contribute to a local instructional theory which can support the development of students' understanding of the area measurement of a circle. Therefore, the research question in this research is "*How can we support 8th grade students' understanding of the area measurement of a circle?"* This general research question is elaborated into two sub-research questions:

1. *How can grid paper support 8th grade students' understanding of the area measurement of a circle?*
2. *How can a reshaping strategy support 8th grade students' understanding of the area measurement of a circle?*

CHAPTER III
METHODOLOGY

3.1. Research Approach

The aim of this study is to improve the teaching and learning in the topics of the area measurement of a circle. Therefore, in this study, a new learning trajectory about the area measurement of a circle was designed and tested. Moreover, a theory about those topics was also developed. This implies both the activities of designing a local instructional theory on the topic of the area measurement of a circle as a means to support the development of students' understanding and the activities of researching on how the means can support the development of students' understanding. Hence, design research is a research approach which fits with this study because according to Gravemeijer and Cobb(2006) the purpose of design research or sometimes called design experiment is to develop theories about both the process of learning and the means that are designed to support the learning.

As Gravemeijer and Cobb (2006) described, there are three phases of conducting a design experiment which are 1) preparing for the experiment, 2) design experiment, and 3) retrospective analysis. Each of those three phases will be elaborated in these following paragraphs,

1. **Preparing for The Experiment**

   From a design perspective, the goal of the preliminary phase of a design research experiment is to formulate a local instructional theory that can be elaborated and refined while conducting the intended design experiment from a research perspective; a crucial issue is that of clarifying its theoretical intent (Gravemeijer and Cobb, 2006). Accordingly, there are three important activities in this phase. The first activity is about clarifying the mathematical learning goals or the instructional end points and the instructional starting points. This is a crucial activity in order to develop a conjectured local instructional theory. In this part, researching existing studies and literature related to the topic research can be useful. The

second activity is discussing the conjectured local instruction theory which consists of encompassing both the instructional activities and the conjecture of the learning process. The last activity is elaborating on the theoretical intent of an experiment. Regarding to the topic research, in this study, firstly, the researcher will look at existing studies and literature related to the topic research which are area measurement and circle. Later on, based on the existing studies and the literature, the conjectures and the theoretical intent of the experiment about the topic of area measurement of a circle is elaborated.

2. Design Experiment

When the preparation has been done, the learning goals and the instructional starting points have been defined, and a conjectured local instructional theory has been formulated, the design experiment can be started. The experiment is a cyclic process of redesigning and testing instructional activities and other aspects of the design (Gravemeijer and Cobb, 2006). In this study, we did two cycles of teaching experiment. This is because in the first cycle, which is called a preliminary teaching experiment, we need to collect data for refining the initial HLT and in the second cycle, which is called the teaching experiment, we need to collect data for answering the research question. In the preliminary teaching experiment, the initial conjectured local instructional theory in the topic of the area measurement of a circle is tried out in a teaching and learning process of a small group of 8^{th} grade students which consists of six students. In the pilot experiment, the learning processes are observed and the students' pre-knowledge is investigated. Moreover, the data which are collected from the pilot experiment are used to support the refining of the initial conjectured local instructional theory. The refined conjectures are then used in conducting the second cycle of this experiment which is the teaching experiment in a real classroom of 8^{th} grade students. The teaching experiment aims at collecting data to answer the research question. The teacher and the researcher have a discussion before each activity. The

design research emphasizes the possibility of modifying the ideas and conjectures during the teaching experiment, based on the interpretation of students' learning and reasoning in the classroom.

3. **Retrospective analysis**

When all the data have been collected, the next phase is about analyzing the data to extract the useful information and to develop conclusions about how the local instructional theory on the topic of the area and the circumference of a circle works in this experiment. Gravemeijer and Cobb (2006) claimed that the main aim of retrospective analysis is to contribute to the development of a local instructional theory. In this analysis, the Hypothetical Learning Trajectory (HLT) is used as a guideline and a reference to analyze the entire data set during the teaching experiment. Each conjecture is compared with the actual teaching and learning process in the classroom. The analysis not only focuses on the parts which are in line the conjectures, but also on the parts which contradict the conjectures. The conclusions of the analysis help to answer the research questions.

3.2. Data collection

1. **Preliminary Teaching Experiment (First Cycle)**

In this study the preliminary teaching experiment which is also called the first teaching experiment is a pilot experiment. This teaching experiment is a teaching cycle in a small group of 8^{th} grade students who are in the middle level of mathematics achievement. The small group consists of students who will not be involved in the second cycle of the teaching experiment. The aims of this cycle are to try out the initial HLT and to investigate students' thinking and reasoning about a set of problems which has been designed in the HLT. The data that are collected are students' written work, video registrations of all the lessons, and field notes about important and interesting observations during the teaching and learning. In this cycle, the researcher is also the teacher. Hence, the video registration is done by the colleague of the researcher. The data collected

in this first phase will be analyzed and used to refine and improve the initial HLT.

2. **Teaching Experiment (Second Cycle)**

   The new HLT which results from the improvement of the initial HLT is used as a guideline to conduct the teaching experiment. The second cycle is an actual teaching experiment involving observations of whole class discussions and specifically observations of a focus group. The focus group is chosen based on the results of the pre-test and the interview with the teacher. The choice of the focus group students not only depends on the students' level of understanding, but also the students' activeness in group discussion. The following data are collected: students' written works, video registrations which focus on the whole lesson and on the focus group's discussion and field notes of the important and interesting observations. Two cameras are used to make the video registrations. One focuses on the focus group and the other one captures all activities of the whole lesson. In this cycle, the role of the researcher is as an observer and the teacher of the lesson is the real teacher of mathematics in the classroom. The data collected in this second phase will be analyzed to answer the research questions.

3. **Pre-test and Post-test**

   Before the teaching experiment for both the first cycle and the second cycle, the researcher conducts a pre-test involving all students in the pilot experiment and later also in the second teaching experiment. The pre-test aims at investigating students' starting points of the research topic. Moreover, it functions as a consideration to determine the students who will be in the focus group. The data collected from the pre-test are both students' answer sheets and students' interview about their reasoning by making video registrations.

   Moreover, at the end of the all the lessons, the researcher conducts a post-test for all students in both the pilot experiment and the teaching experiment. The post-test aims at assessing students' understanding about

the concept and the measurement of the area and the perimeter of a circle. The reason why we include perimeter is because it is needed to understand the concept of the area of a circle using reshaping strategy. The data collected from the post-test are both students' answer sheets and students' interviews about their reasoning in solving the problem given in the post-test. This interview is recorded as a video registration.

In this experiment, we made the same problems for the pre-test and the post-test, in order to make the results comparable to be analyzed. The pretest consists of four questions which are about comparing the area of two shapes with and without grids, determining the area of a circle with grids inside, and determining the perimeter of some plane figures. However, disregarding the results of the pretest and posttest, the pre-test focuses on investigating the starting points of the students and the post-test focuses on the understanding of the students which is compared to the pretest.

4. Validity and Reliability

In a research it is important to consider the aspects of validity and reliability of data collection. It not only strengthens the quality of the research but also determines the impact of the research results. In short, validity is related to whether a research measures what the researcher wants to measure and reliability refers to the independence of the researcher. During each of the iterations of a design study, various individuals may participate and various methods of data collection must be carefully chosen and applied; there are applications of triangulation (McKenney, Nieveen, and Akker, 2006). Hence, in this research, we use different types of methods of collecting data namely students' written works, video registrations, interviews, and field notes. Those different types of methods can be used for triangulation which contributes to the internal validity of the research. Besides that, the experiment which is carried out in a real classroom can also enhance the ecological validity.

Moreover, the data collection using video registration can improve the internal reliability of the research

3.3. Data Analysis

1. **Pre-test**

   The results of the pre-test are expected to describe students' understanding and students' misconceptions regarding their formal and informal knowledge about the concept and the measurement of the area of a circle. Moreover, the results are analyzed in order to investigate the students' starting points of the concept and the measurement of the area of a circle. The result of the analysis of the pre-test regarding the students' starting points and the students' level of understanding are used to refine the initial HLT.

2. **Preliminary Teaching Experiment (First Cycle)**

   The data collected in the preliminary teaching experiment, namely students' written worksheets, video registrations, and field notes from all activities are analyzed to describe the learning process of the students. The analysis is done by testing the conjectures in the initial HLT which means by comparing the conjectures about students' acts and responses in the learning processes with the actual learning. The focus of this analysis is not only on students' responses which are in line with the conjectures, but also on students' responses which contradict with the conjectures. Moreover, it also describes how the initial HLT contributes to students' learning of the concept and the measurement of the area and the perimeter of a circle. The results of the analysis of the first cycle are then used to refine and improve the initial HLT which will be implemented in the second cycle of the teaching experiment.

3. **Teaching Experiment (Second Cycle)**

   The data collected in the teaching experiment, namely students' written worksheets, video registrations of the whole class and the focus group, and field notes, are analyzed to obtain insights into the students' learning

processes. Therefore, there are several interesting and relevant fragments of the video which will be chosen to transcribe in order to describe students' thinking and reasoning. The interesting fragments are the fragments which show students' responses which are in line with or contradict the expectation in the HLT. Moreover, the fragments should be relevant to the research questions and can be grounded to answer the research questions. The selected fragments, together with the students' written work and considering the researcher's field notes, are compared with the conjectures in the new HLT from the first cycle. The results of the analysis of the second cycle are then used to answer the research questions, to draw the conclusion, and to redesign and improve the HLT.

4. Post-test

The results of the post-test are expected to describe the development of students' learning and understanding of the concept and the measurement of the area and the perimeter of a circle. Moreover, it is also expected to show students' strategies in solving the problems about the topic. The analysis is conducted by comparing the results of students' answers in this post-test with the results of students' answers in the pre-test. The results of this analysis also contribute to enhance the analysis of the teaching experiment and as a consideration in drawing the conclusion.

5. Validity and Reliability

Considering the aspects of validity and reliability is not only important in data collection, but also in data analysis. In data analysis, the validity is related to both internal and external validity and the reliability is also related to internal and external reliability. The internal validity is associated with the use of different sources which are analyzed using the method of triangulation. During the analysis, the researcher tests and analyzes the conjectures based on the students' written works, the video registrations, and the field notes. The external validity is related to the generalizability of the conclusion and the results of the HLT which can be adjusted by other researchers to their own local setting.

The internal reliability is enhanced by a discussion with others to interpret the data and to draw conclusions (inter subjectivity). Moreover, the external reliability refers to track ability. The teaching experiments and data analysis are guided by the conjectures in the HLT. The researcher describes this process in a systematic way so it can be followed by other researchers. Moreover, it should also be possible for other researcher to adapt the learning process and draw the same conclusion through the cycles of teaching experiments and data analysis.

CHAPTER IV
HYPOTHETICAL LEARNING TRAJECTORIES

Hypothetical Learning Trajectory (HLT) is a term for the teacher's prediction for the path which might proceed in the learning process (Simon, 1995). Moreover, Simon described that HLT consists of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process which is a prediction on how the students' thinking and understanding will evolve in the context of the learning activities. The HLT is developed in the first phase of design research which is preparing for the experiment.

In this study, we elaborate the HLT of six lessons in a learning sequence in the topic of the area measurement of a circle. The first two meetings and the first half of the third meeting are still about clarifying the students' understanding of area. The activities for clarifying the students' understanding of area is important as one of the students' difficulties regarding area measurement is distinguishing area and perimeter (for a review see Oldham et al, 1999). Started from the second half of the third meeting, the activities will focus on the area of a circle. However, there is also a meeting involving a set of activities about exploring the relation between the diameter and the circumference of a circle which is set up in the fifth meeting. To avoid the students' confusion about area and perimeter, it is important to involve perimeter in the learning activities of area. Moreover, the students' understanding about the pi value will be very useful to support the strategy for measuring the area of a circle by reshaping the sectors.

Area is the amount of region (surface) which is enclosed within a boundary (Baturo & Nason, 1996). Hence, the area of a circle means the amount of region which is enclosed within a circle. The learning activities in this study emphasized two aspects of area which are calculation and conservation of area. According to Van der Valk & Broekman (1997), calculation involves the choice of a unit of area, can be determined by counting or approximating the number of units covering a shape, is dependent on the length, width, and configuration, can be

calculated using a rule or formula, and is determined by the sum of small parts. Therefore, in this study we use grids as a set of similar units which can be a tool for students to estimate the area of a circle.

Moreover, regarding the conservation of area, the area of a shape equals the sum of the area of the parts, two shapes form a new shape when a part of the boundaries are put together and the area of the new shape is the sum of the original area (Van der Valk & Broekman, 1997). Hence, in this study, we use the idea of reshaping the sectors of a circle into other shapes of plane figures based on the concept of the conservation of area. The hypothetical learning trajectory of each meeting in this learning sequence will be elaborated as follows.

4.1. Lesson 1: Comparing and Ordering Shapes

1. Learning Goals

   The main goal

   Students are able to distinguish area and perimeter, as well as to explain the relation between those two concepts.

   The description of the goal

   - The students are able to put the islands in order.
   - Students are able to explain their strategies to compare and order irregular plane figures.
   - Students are able to explain the relation between the area and the perimeter of plane figures.

2. The starting points

   The concept of the area and the circumference of a circle is one of the topics which are taught in the eighth grade of secondary school in Indonesia. However, according to the Indonesian curriculum, the topics have been taught when the students in the sixth grade of elementary school. Moreover, the notion of the area and the perimeter of straight-sided plane figures have also been introduced in the fourth and fifth grade of elementary school and the seventh grade of secondary school. Hence, the starting points for the first lesson are:

- The students know attribute to be measured (area).
- The students have understood the way to compare two shapes.

3. Description of the activities and conjectures of students' thinking

   The mathematical idea in this meeting is using overlapping and reshaping strategies to compare shapes. This section begins by clarifying students' conception of area. There are two activities about comparing and ordering shapes. Students first use their own strategies to solve the problem. They can use papers, scissors, and markers as tools for comparing. Each of the activities is described as follows.

   1. Ordering the size of five biggest islands in Indonesia

      This activity is another activity of comparing. The students still work in their group and give them a large copy of Indonesian map. The problem is about determining which miniature of the five biggest islands in Indonesia will need more grass and which land will need less grass.

      Indonesian Archipelago

      A school will make a garden with the miniature of the five biggest islands in Indonesia namely Sumatera, Java, Kalimantan, Sulawesi, and Papua. If each of the land will be covered by grass, which land will need more grass than the other lands?

      Figure 4.1 Indonesian Archipelago

      Put the islands in order from the land which needs the most grass to the land which needs the least grass. Explain your way of working and reasoning.

   2. Designing two gardens with the same area

      This activity is involving open ended problem because students are asked to design two different shapes of gardens with the same area.

Designing Gardens

Mr. Joko buy a certain amount of grass to cover a part of the garden in his house. Help Mr. Joko to make the design of the land which will be covered by grass. Give Mr Joko two alternative designs.

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

Table 4.1 The Overview of The Activities in The First Meeting and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Ordering the size of five biggest islands in Indonesia.	The students are able to put the islands in order.	The students will determine the order only by considering the shapes of the islands which means that the wider the shapes the bigger the islands.	The teacher can ask the students to think about the shapes of Sulawesi islands. *"The shape is narrow, but how if we think the island as a whole?"*
		The students will cut the drawing of the islands and overlap them. Moreover, they will do cutting and pasting with the remaining space of the overlapping.	The teacher should make sure that the students know the reason why we can say that two shapes are in the same size and why we can say that two shapes are bigger or smaller.
Designing gardens	The students are able to justify that two shapes with the same area is not always have the same perimeter	The students will design two shape which are exactly the same because they think that it would be impossible if two different shapes will be covered by the same amount of grass.	The teacher should pose a question which can stimulate students' thingking. For example, *"How if we move some grass to other part of this land?"*

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Designing gardens		The students will design a rectangle as the first design and cut and move the part of the rectangle so that they get a new shape.	The teacher could ask the reason why the students choose a rectangle, not the other shape.
		The students will apply formula to calculate the area of two different plane figures with the same area.	The teacher should ask the students to explain their work by verbalizing explanation or by using figures.
Math congress: put the group works in a whole class discussion.	Students are able to explain their strategies to order and design shapes.	The students will share their work and their thinking about the idea of using overlapping strategy to compare two similar shapes.	The teacher can lead the students to understand that two shapes said to be in the same size if they fit each other with on gaps or overlaps.
		The students can combine the overlapping strategy with cutting and pasting strategies to compare two different shapes.	The teacher could guide the students to discuss and compare the different strategies they applied to solve the problem.
		The students will describe their answers which are expected that there will be various shapes designed by the students.	The teacher could guide the students to discuss the relations between are and perimeter.

4.2. Measuring Shapes through Tiling Activity

1. Learning goals

   The main goal

   Students understand the attribute to be measured and are able to measure objects using measurement units**.**

   The description of the goal

   – The students understand the attribute to be measured and are able to measure the area of an object using different size of measurement units.
   – The students are able to measure a more complicated object using some given measurement units with different size.
   – The students are able to explain their strategies to determine the number of measurement units needed to cover the L-floor and the bottom of the swimming pool.

2. The starting points

   – The students have understood the way to iterate a unit of area measurement properly.
   – The students have known about the way to find the area of a square.

3. Description of the activities and conjectures of students' thinking

   The mathematical idea in this meeting is unit iteration. This section begins by clarifying students' conception of area measurement using measurement units. There are two activities in this meeting. The idea of the first activity is recalling the students' knowledge about unit iteration in area measurement. Moreover, the second activity aims to recall both the students' knowledge about unit iteration in area measurement and the students' knowledge about the shape of a circle. Moreover, in the second activity, instead of iterate a unit to one another, they are expected to make grids based on the given grids.

   1. Covering an L-floor of a patio with tiles

      This activity is about covering an L-floor using two different sizes of tiles. The idea is recalling the students' knowledge about unit iteration in area measurement.

Covering a floor with tiles

Mr. Heri wants to cover the floor in his patio (terrace). There are two different size of square tiles which he can choose. Help Mr. Heri to know the number of tiles needed to cover the patio if he choose the small tile and the number of tiles needed to cover the patio if he choose the big tile. Draw the tiles on the design.

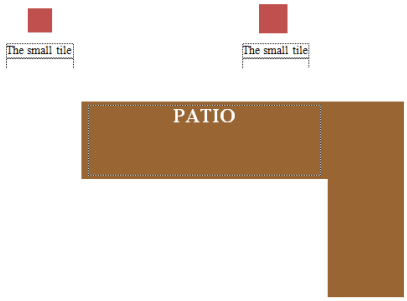


Figure 4.2 A patio design

There are three questions as the follow-up of that activity, as follows.

1. *How many small tiles do you need to cover the floor of the patio?*
2. *How many big tiles do you need to cover the floor of the patio?*
3. *Explain the differences of using the small tiles and using the big tiles.*

2. Covering the bottom of a swimming pool with tiles

   This activity is quite similar with the previous activity. The differences are about the shape of the floor which is a combination of a trapezoid and a semicircle and there are some given tiles on the floor. The intention is that the students will recall their knowledge about the shape of circle by providing the semicircle. Moreover, instead of iterate a unit to one another, they are expected to make grids based on the given grids. However, the teacher should not ask the students to make grids if in the activity they do not come up with the idea of making grids.

   Covering a swimming pool with tiles

   Mr. Heri will also make a swimming pool in his house. He has decided the shape of the swimming pool. Complete the work of tiling in the

design below. Estimate the number of square tiles needed to cover the bottom of the pool for the first design and the second design.

The first design

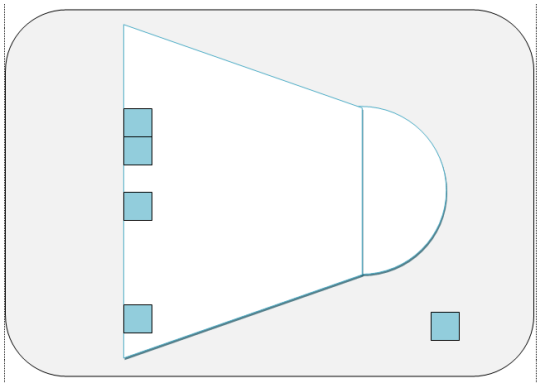


Figure 4.3 A design with small tiles

The design of the swimming pool with the small tiles

1. *Estimate the number of tiles needed to cover the bottom of the swimming pool?*
2. *Explain your way of working and reasoning.*

The second design

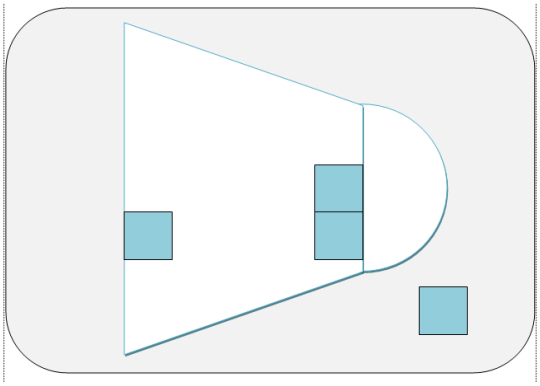


Figure 4.4 A design with big tiles

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

Table 4.2 The Overview of The Activities in The Second Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Covering an L-floor of a patio with tiles.	The students understand the attribute to be measured and are able to measure the area of an object using different size of measurement units.	The students will cut the tiles and make the drawing of the tiles on the patio one by one using the principle of unit iteration.	Ask the students about the difference of covering the patio using the small tiles and the big tiles.
		The students will use ruler to measure the length of the tile's sides and make the drawing of the tile on the patio using the ruler one by one using the principle of unit iteration.	Ask the students about another possible strategy of tiling besides drawing the tiles one by one.
		The students will use a ruler to measure the length of the tile sides and make the drawing of the tile on the patio based on the measurement (not one by one).	Ask the students about how to deal with the incomplete tiles resulted from their tiling activities by drawing the tiles using a ruler.
Covering the bottom of a swimming pool with tiles	The students are able to measure a more complicated object using some given measurement units with different size.	The students will continue to fulfill the bottom of the swimming pool with tiles by drawing the tile one by one.	Ask the students about another possible strategy of continuing the tiling process besides drawing the tiles one by one.
		The students will make grids through the given tiles inside the shape of the bottom of the swimming pool.	Ask the students to think about the similarities and the differences between tiling by drawing the tiles one by one and making grids.

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Math congress: put the group work in a whole class discussion	The students are able to explain their strategies to determine the number of measurement units needed to cover the L-floor and the bottom of the swimming pool.	The students will discuss about different strategies to determine the number of small tiles and big tiles needed to cover the L-floor of a patio and the bottom of the swimming pool.	Lead the students into a discussion about the strength and the weakness of the strategies used by the students.
		The students will discuss about the different number of tiles needed to cover the L-floor and the bottom of the swimming pool if they use the small tiles and if they use the big tiles.	Lead the students into a discussion that the number of tiles needed represents the area of the floor with the tiles as the measurement unit.

4.3. Measuring shapes using grid paper

1. Learning goals

   The main goal

   Students are able to measure shapes using a grid paper and by making their own grid.

   The description of the goal

   - The students are able to compare the size of two lakes using a grid paper.
   - Students are able to estimate the size of a shape by using the given grids and by making their own grid.
   - Students are able to explain their way of working and reasoning to estimate the grass needed to cover the circular garden.

2. The starting points
   - The students have understood about scaled figures.

- The students have had sense of sizes (smaller and bigger).
- The students have been able in doing arithmetic operation.

3. Description of the activities and conjectures of students thinking

   The mathematical idea in this meeting is using grid paper to measure area. This section begins by clarifying students' conception of area related to measurement units. Students try to compare two lakes on a grid paper and estimate the grass needed to cover the circular garden by looking at the relationship between the circular garden and the square land. The first activity aims to introduce the grids in area measurement and in the second activity the students are expected to see the relationship between the area of square and the area of circle which diameter is equals the side of the square land.

   1. Comparing two lakes on a grid paper

      The first activity is about comparing two lakes which are drawn with scale on a grid paper. The idea is that the students will have a sense of unit iteration as arranging a number of similar units to cover the attribute of the measured objects. Firstly, the teacher should ask the students to think of the problem individually. Then, the students can share their idea to solve the problem in group.

      Comparing two lakes

      Estimate which lake is bigger?

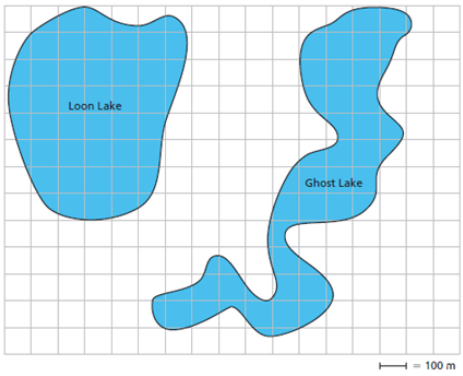


Figure 4.5 Figures of two lakes on a grid paper

There are three questions which the students have to answers related to this problem.

1. *Explain your way of comparing and reasoning.*
2. *How large is the Loon Lake and how large is the Ghost Lake?*
3. *How accurate is your estimation?*

2. Estimating the area of a circle using different size of grids

After the first activity which is about comparing the size of two irregular shapes on a grid paper. This activity is about estimating the grass needed to cover a circular garden. In this activity, the teacher should avoid mentioning the terms 'area' because to develop the students' understanding of area, the important thing is that the students get make sense what they call area is. Moreover, the teacher should explain explicitly that the drawing on the paper is a scaled design of a circular garden.

Covering a circular garden

Mr. Heri asks a garden designer to make a circular garden in his rectangular land. The circular garden should be covered by grass and the remaining land will be covered by paving blocks as the following figure.

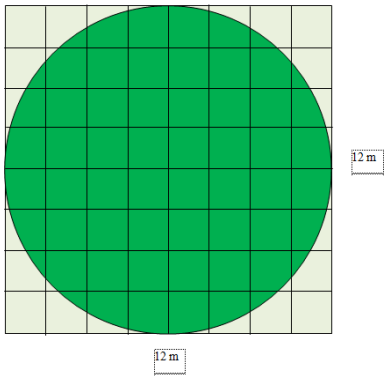


Figure 4.6 A design of a circular garden

There are two questions which the students have to answers related to the problem.

1. *How many meter square grass needed to cover the circular garden?*
2. *By making smaller size of paving, estimate how many meters square grass needed to cover the circular garden?*

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

Table 4.3 The Overview of The Activities in The Third Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Comparing two lakes on a grid paper	The students are able to compare the size of two lakes using a grid paper.	The students will disregard the grids and reshape the lake into two more similar and more regular shape.	Ask the students how to be sure that one of the lakes is bigger than the other one and ask them to consider the grids inside the boundary of the lakes.
		The students will count the grids inside the boundaries of each lake and compare the number of those grids.	Check if there are still students who compare the lakes based on the grids surrounded the lakes.
		The students will firstly count the complete grids and estimate the number of the incomplete grids.	Encourage the students to see the relation between unit iteration in tiling activities as they have done in the second lesson and the grid paper in terms of area measurement.
Estimating the area of a circle using different size of grids	Students are able to estimate the size of a shape by using the given grids and by making	The students will just say that the grass needed to cover the circle is three fourths of the area of the square land.	Ask the students to reason about their estimation and ask the students about the accuracy of their estimation.

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Estimating the area of a circle using different size of grids	Students are able to estimate the size of a shape by using the given grids and by making their own grid.	The students will estimate the area of a circle by considering the given grids. Hence, the area of the circle is more than $3r^2$ and less than $4r^2$.	Ask the students about the accuracy of their estimation and about their strategy to deal with the incomplete grids.
		The students might consider the bigger size of grids to make their work simpler. But, they might also make a smaller size of grid to get the more accurate estimation.	Ask the students to think about the advantages and the disadvantages of estimating the area inside the circle by considering the bigger size of grids and about the advantages and the disadvantages of estimating the area inside a circle by making the smaller size of grids.
Math congress: put the group work in a whole class discussion	Students are able to explain their way of working and reasoning to estimate the grass needed to cover the circular garden.	The students will discuss their findings of which might be various and discuss the accuracy of the findings.	Lead a discussion about the reason which might the students got the various results and the different strategies used by the students to which might cause the different results.
		The students will discuss about the advantages and the disadvantages of using the bigger grids and the smaller grids to estimate the area inside the circular garden.	Lead the students to the idea of the smaller grids used to estimate the area inside the circle, the more accurate their estimation.

4.4. Reshaping Circles

1. Learning goals

   The main goal

   Students can determine the estimation of the area of a circle using the sectors of the circle.

   The description of the goal

   – The students are able to reshape the four sectors of a circular garden into a shape resembles a parallelogram.
   – The students are able to reshape the 8, 12, and 16 sectors into three shapes resembles parallelograms
   – The students are able to reshape the parallelogram into a rectangle and are able to argue about the idea of determining the area of the rectangle.
   – The students are able to explain that the area of the rectangle formed from the sectors is $r \times {}^{1}\!/_{2}$ of circumference

2. The starting points
   – The students have understood the concept of conservation of area.
   – The students have known the way to find the area of a parallelogram.
3. Description of the activities and conjectures of students thinking

   The mathematical idea in this meeting is that the area of a circle equals the radius times half of its circumference. There is only one main activity in this meeting namely reshaping three circles which are cut into different numbers of sectors into a parallelogram. However, to make the students get the idea of reshaping the sectors of a circle into a parallelogram, there will be an activity of reshaping a circular garden into a shape resembles parallelogram which will be guided by the teacher.

   1. Reshaping a circular garden into a shape resembles parallelogram

      The first activity is about reshaping a circle into a shape resembles a parallelogram. Firstly, the students are asked about what strategies they used to estimate the grass needed to cover the circular garden in the previous activity. Then, the teacher can ask the students, *'Is there any*

other strategy to estimate?' When the students get stuck, the teacher can tell that some students (which is not in their class) use the different strategy as described in the first activity in their worksheet.

Reshaping a circular garden to estimate its area

To estimate the grass needed to cover the circular garden, Dewi's group cut the circle into four sectors and reshape the sectors into this following shape.

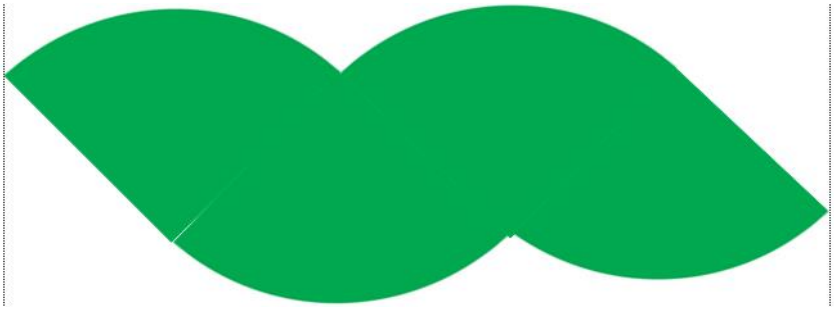


Figure 4.7 Reshaping four sectors of a circle

There are two questions which have to be answered by the students as follows.

1. *What kind of plane figure resembles the shape above?*
2. *Is the area of the shape equals the area of the circular garden?*

2. Reshaping three circles which are cut into different numbers of sectors into a parallelogram

   In this activity, the students will get three circles which each of them have been divided into different numbers of sectors, a scissor, glue, and a large paper poster. They will cut the sectors of the circle and try to arrange them in the same way like the shape which is shown by the teacher.

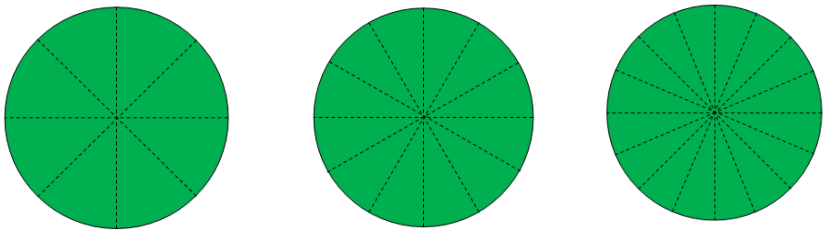


Figure 4.8 Circles which are divided into 8, 12, and 16 sectors

Cut the circle along the diameter so that there will be 8 sectors, 12 sectors, and 16 sectors, respectively as the figures above. Arrange the sectors like the shape in the first activity. Glue it in your poster.

1. *What kind of plane figure resembles the shapes which are formed from the sectors?*
2. *What will happen if we cut the circle into the more number of sectors?*

3. Reshaping the parallelograms into rectangles and determining the formula of its area

   The students have arranged the sectors into shapes resemble parallelograms. However, to connect the relation between the formula of a circle and a parallelogram might be quite difficult. Hence, in this activity, the students will reshape the parallelogram into a rectangle and make the relation between the way to find the area of the rectangle and the circle.

   There are two questions in this activity which are:

   1. *How can you reshape the shapes you formed into rectangles?*
   2. *What is your idea to find the area of the rectangles?*

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

Table 4.4 The Overview of The Activities in The Fourth Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Reshaping a circular garden into a shape resembles parallelogram	The students are able to reshape the four sectors of a circular garden into a shape resembles a parallelogram.	The students will reshape the sectors not into a shape shown in the worksheet because they do reshaping in a different way.	Guide the students to reshape the sectors in a way that can be formed as the shape in the worksheet.
		The students will reshape the sectors into a shape as shown in the worksheet. However they cannot still realize that the shape looks like a parallelogram.	Ask the students to imagine when the curve sides of the shape turn into straight sides. Moreover, the teacher can also ask the students to think about what will happen if the circle is cut into smaller sectors.
Reshaping three circles which are cut into different numbers of sectors into a parallelogram	The students are able to reshape the 8, 12, and 16 sectors into three shapes resembles parallelograms	The students will reshape the sectors not into shapes similar with the shape in the first activity because they still do reshaping in a different way.	Guide the students to reshape the sectors in a way that can be formed as the expected shapes.
		The students will reshape the sectors into shapes similar with the shape in the first activity. However they cannot still realize that the shape looks like a parallelogram.	Ask the students to pay attention to the changing which they can see from the shape of 4 sectors, 8, sectors, 12 sectors and 16 sectors.

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Reshaping three circles which are cut into different numbers of sectors into a parallelogram	The students are able to reshape the 8, 12, and 16 sectors into three shapes resembles parallelograms	The students will reshape the sectors into shapes similar with the shape in the first activity and realize that the shapes look like a parallelogram.	Ask the students to imagine about how the shapes will be when the sectors are getting smaller.
Reshaping the parallelograms into rectangles and determining the formula of its area	The students are able to reshape the parallelogram into a rectangle and are able to argue about the idea of determining the area of the rectangle.	The students will get stuck and confuse and do not have any idea to reshape the parallelogram into a rectangle.	Guide the students to get the idea of reshaping the parallelogram into a rectangle by seeing the shapes of both parallelogram and rectangle.
		The students will cut half of the sector on the left most part and put it on the right most part or the other way around.	Ask the students to think about the way to find the area of a rectangle.
Math congress: put the group work in a whole class discussion	The students are able to explain that the area of the rectangle formed from the sectors is $r \times 1/2$ of circumference	The students will discuss that the area of the circle equals the area of the rectangle. However, they determine the area of the rectangle as $p \times l$.	Guide the students to see the relation between the parts of the circle with the parts of the rectangle.
		The students will discuss that the area of the circle equals the area of the rectangle. Moreover, they determine the area of the rectangle as $r \times 1/2$ of circumference.	Make sure that all students understand that the area of the rectangle is $r \times 1/2$ of circumference by asking the students to show the parts of the circle in the rectangle.

4.5. Finding the relationship between the circumference and the diameter of circles

1. Learning goals

   The main goal

   The students can find the estimation value of pi and use it to formulate the formula of the circumference of a circle. Moreover, the students can use it to determine the area of a circle.

   The description of the goal

   - The students can determine the formula of the area of the rectangle the by considering the parts of the rectangle as the parts of the circle.
   - The students can estimate the length of the circumference of circles using its diameter.
   - The students are able to explain the relation between the diameter and the circumference of a circle.
   - The students are able to see the relations between circumference and diameter as a ratio which a little bit more than three times.

2. The starting points
   - The students have known the way to find the area of a rectangle.
   - The students have known the parts of a circle (the diameter and the circumference).
   - The students have been able in doing arithmetic.

3. Description of the activities and conjectures of students thinking

   The mathematical idea in this meeting is that the circumference of a cicle divided by its diameter yields the pi value. There are two main activities in this meeting which are making circles using a shoelace and a marker and investigating the relation between the circumference and the diameter of some circular objects. However, those two activities are begun by the first activity which is estimating the area of a rectangle which is formed from the sectors of a circle. This is because in the problem of the first activity, the students have to determine the circumference of a circle to find the area of the rectangle.

1. Estimating the area of a rectangle which is formed from the sectors of a circle

   This activity is connected to what the students have found in the previous meeting. They have to determine the area of a rectangle which is reshaped from the sectors of a circle.

   The rectangle of the circle

   In the previous lesson, we have reshaped the circle into this following form.

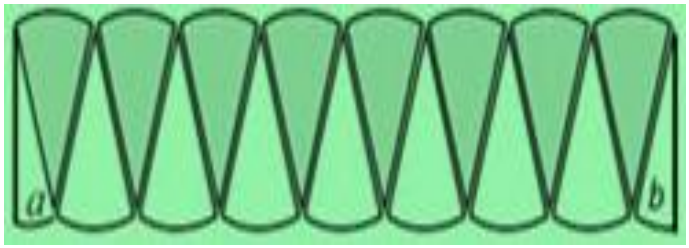


Figure 4.9 A circle which is reshaped into a rectangle

   1. *What measurement do you need to determine the area of the figure?*
   2. *Which measurement you know from the figure?*

2. Making circles using a shoelace and a marker

   In this activity, the students will use one of their shoelace and a marker to draw a circle. At first, the teacher will show the students the way to make the circle on the whiteboard. Afterwards, the students are invited into a short discussion about determining the length of the radius and diameter represented by the shoelace. When the students have really understood the way to draw the circles, the teachers let them to work with the problem in their group.

   Drawing circles using a pencil and a shoelace

   By using a pencil and a shoelace, draw five different sizes of circles on your paper. Measure the circumference of each circle using diameter (using the shoelace) and investigate the relations between the diameter and the circumference of the circles you draw on this following table.

The circle number-	The relation between the diameter and the circumference
1	
2	
3	
4	
5	

3. Investigating the relation between the circumference and the diameter of some circular objects

   The students will measure the diameter and the circumference of some circular objects. Based on the previous activity, the students should have known that the length of the circumference of a circle is slightly more than three times the length of its diameter. And, in this activity, they should also get the same idea.

   Comparing the diameter and the circumference of a circle

   There are some several objects which have circular shape in daily life as you have on your group.

Figure 4.10 Circular objects

   Trace along each of the objects above with a pencil on a piece of paper. Cut the drawing and measure the diameter and the circumference of the circle.

   Make a list of the diameter and the circumference of those circular objects.

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

Table 4.6 The Overview of The Activities in The Fifth Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Estimating the area of a rectangle which is formed from the sectors of a circle	The students can determine the formula of the area of the rectangle the by considering the parts of the rectangle as the parts of the circle.	The students will determine the area of the rectangle as $p \times l$ and disregard the relation between the parts of the rectangle with the parts of the circle.	Reminds the students about the last activity of the previous lesson, particularly about the relation between the parts of the circle and the parts of the rectangle.
		The students will remind about the last activity in the previous lesson and determine the area of the rectangle as $\mathrm{r} \times 1/2$ of circumference.	Ask the students about what part of the rectangle should be known so that they can find the area of the rectangle.
Making circles using a shoelace and a marker	The students can estimate the length of the circumference of circles using its diameter.	The students will draw five circles which have almost same size. Hence, they will conclude that the circumference is a little more than three times the diameter.	Ask the students whether they will get the same relation if they draw circles which significantly different size.
		The students will draw five circles which range from small to big circles. Hence, they will see that for all the circles, the circumference is a little more than three times the diameter.	The teacher should lead students to generalize that the circumference of a circle is a little more than three times the diameter.

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Investigating the relation between the circumference and the diameter of some circular objects	The students are able to explain the relation between the diameter and the circumference of a circle.	The students will only measure the circumference and the diameter of the circular objects without firstly drawing them on the paper.	The teacher can ask about how the students can be sure that they can measure the diameter of the circular objects by measuring it directly on the objects. For example the diameter of the armband.
		The students will draw the circular objects on a paper and measure the circumference and the diameter of the circles using paper ruler.	The teacher reminds the students about the properties of a circle especially which one is radius and diameter.
		The students will come up with the idea of the ratio of the circumference to the diameter is around 3.1.	The teacher can ask the students to share all their finding and lead them to draw a conclusion about the relation between the circumference and the diameter of circles.
		The students will come up with a different idea of the ratio which is looking the ratio from the opposite side (the diameter divided by the circumference)	The teacher can ask the students to try the other way around of thinking about the ratio which means the circumference divided by the diameter.

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Math congress: put the group work in a whole class discussion	The students are able to see the relations between circumference and diameter as a ratio which a little bit more than three times.	The students will discuss about two different ratios of the relation between the circumference and the diameter of circles which are the circumference divided by the diameter and the other way around.	The teacher should explain the students about the two different values and point out the ratio of the circumference to the diameter of circles as phi and tell them a little about the value of phi in the history of mathematics.
		The students will discuss about the formula of the area of the rectangle formed from the sectors of the circle and come up with $r \times {}^{1}\!/_{2} \times d \times \pi$ as the area of the rectangle.	The teacher can lead the students to draw a conclusion that the area of the circle equals the area of the rectangle which is $\pi \times r^2$.

4.6. Reshaping Circles into several plane figures

1. Learning goals

   The main goal

   Students can use grids and apply reshaping strategy to find the area of a circle. Moreover, they can also use those strategies to determine the area of a circle.

   The description of the goal

   – The students are able to reshape the sectors of the circle into several kinds of plane figures and formulate the area of the circle using the area of plane figures they formed.

   – The students can use grids and apply reshaping strategy to find the area of a circle.

- The students are able to argue about the similarities and the differences of estimating the area of a circle using grids and using reshaping strategy.

2. The starting points
   - The students have known about the concept of conservation of area.
   - The students have understood about how estimating area using grids work.
   - The students have understood about how estimating area by reshaping work.
3. Description of the activities and conjectures of students thinking

   The mathematical idea in this meeting is that the area of a circle equals pi times radius times radius. This section is kind of wrapping up activity. However, the first activity is still about reshaping. And, in the second activity, the students will estimate the amount of money needed to buy grass for covering a circular garden.

   1. Reshaping the sectors of a circle into several kinds of plane figures

      In this activity, the students will be given four circular papers which have been divided into 16 sectors. They should the sectors of the circles and reshape them into several kinds of plane figures.

      Circle and other plane figures

      In the previous lesson, we have reshaped the circle into this following form.

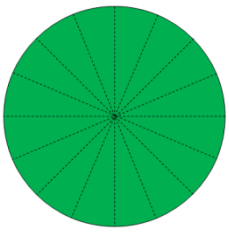


Figure 4.11 A circle divided into sixteen sectors

And reshape the sectors into this following shape.

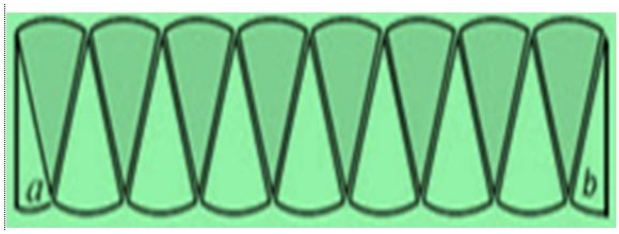


Figure 4.12 A rectangle formed from the sectors of a circle

Those sectors can also be arranged into other shapes. Rearrange the sectors into four different shapes of plane figures and glue them on your poster.

Determine the size of the circle using the shapes which are formed by the sectors.

The name of the new shapes	The area
1.	
2.	
3.	
4.	
5.	

What is the relationship between the area/size of the circle and the area of the new plane figure?

The students will cut the sectors, reshape the sectors of each circle into a plane figure, and glue them on a poster paper. The expectation is that the students will reshape the sectors into four different plane figures as follows.

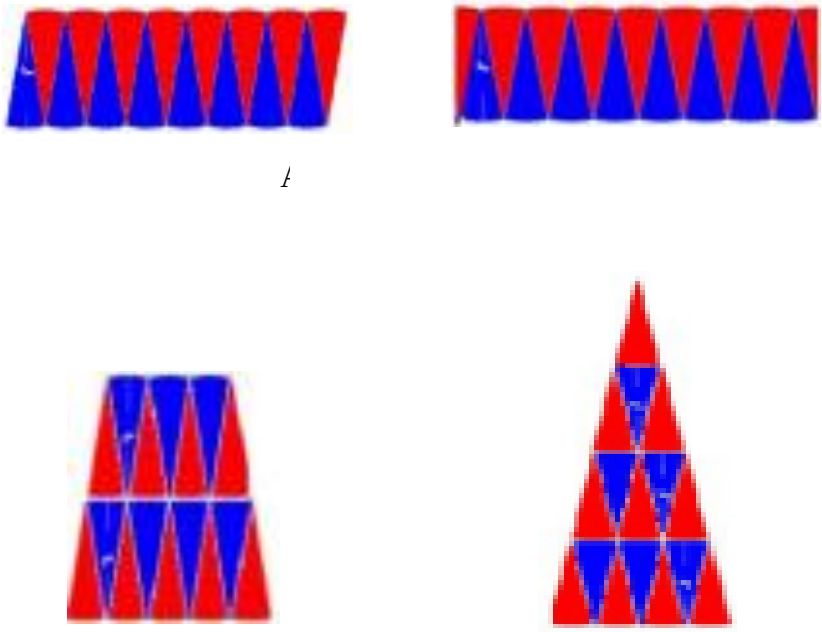


Figure 4.13 Four plane figures expected can be formed by students

However, the parallelogram and the rectangle can also be as these following figures.

Figure 4.3 Another shape of paralellogram and rectangle

It is not a must that they have to come up with all the shapes. The important thing is that they can form shapes of plane figures and try to find the area by considering the parts of the circle on the new shapes they formed.

2. Estimating the amount of money needed to buy grass for covering a circular garden

   In this activity the students will estimate the amount of money needed to buy grass for covering the circular garden. The idea is that the students will know the application of why they need to know the

amount of grass needed to cover the circular garden which means estimate the area inside a circle. Moreover, the students will apply the strategies which they have learned in the learning sequence. The result will also put on the poster paper of their group.

Covering a circular garden

Estimate the amount of money needed for the grass to cover the circular garden if 1 m^2 of grass is Rp. 150.000,00. Compare the results of using grids and reshaping strategy

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

Table 4.6 The Overview of The Activities in The Sixth Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Reshaping the sectors of a circle into several kinds of plane figures	The students are able to reshape the sectors of the circle into several kinds of plane figures and formulate the area of the circle using the area of plane figures they formed.	The students will only arrange the sectors into parallelogram and rectangle as they have done in the previous meetings.	Ask the students to try to reshape the sectors into other plane figures by remind them about the plane figures they have been learned in their 7^{th} grade.
		The students will try to reshape the sectors of the circle into several forms of plane figures including parallelogram, rectangle, triangle, and trapezoid.	Guide the students to investigate the relations between the formula of the plane figures they formed and the formula of the area of the circle by remind them about how the find the area of the rectangle in the previous meeting.

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Estimating the amount of money needed to buy grass for covering a circular garden	The students can use grids and apply reshaping strategy to find the area of a circle.	The students will directly use the formula of the area of a circle to estimate the area of the circle.	Ask the students to explain and reason how the formula works and why the formula can be applied to find the area inside the circle.
		The students will do both making grids first and reshaping the circle into some equal sectors to measure the circle.	The teacher can guide a discussion of the differences and the similarities of using grids and reshaping strategies to determine the area of a circle.
Math congress: put the group work in a whole class discussion	The students are able to argue about the similarities and the differences of estimating the area of a circle using grids and using reshaping strategy.	The students will still confuse to see the relation between the triangle and the trapezoid with the circle and find a difficulty to connect the way to find the area.	Guide the students to see the parts of the circle on both the triangle and the trapezoid. You can ask, for example, *'Which is the height of the triangle? What parts of the height on the circle?'*
		The students will discuss the different value of estimations of the amount of money needed to buy the grass. This difference is because the different estimation they get for the amount of grass needed to cover the circular land which is caused by the different strategies they used.	Lead the students to see the similarities of those strategies which lead us to get the value of a little bit more than three times r^2. Moreover, lead the students to discuss about which strategy they think is more helpful for them and the reason why it is more helpful.

CHAPTER V
RETROSPECTIVE ANALYSIS

In this chapter, we describe the retrospective analysis as the third phase of design research. However, firstly we describe the design experiment which is the second phase of design research. The design experiment consists of two cycles namely the preliminary teaching experiment (the first cycle) and the teaching experiment (the second cycle). In the first cycle, we tested the HLT which has been developed for the small group which consists of six students of 8th grade junior high school. However, firstly we conducted a pretest to know the starting points of students about the concept of the area of plane figures, particularly the area of a circle. Besides that, the remarks from the pretest are used as consideration to adjust the initial HLT into the first refined HLT or we can call it HLT1. Afterwards, we compare the HLT1 to the students' actual learning in the small group. We also analyze about how the learning activities which have been designed support students' understanding of the concept of the area of a circle. The last part of this cycle is conducting a post-test. All the findings and remarks from the first cycle are used to refine the HLT1 into the HLT2. The HLT2 are implemented in the second cycle. And, the implementation and the analysis of HLT2 is presented in the same order with the first cycle. In the conclusion part, we summarize the results of testing HLT to answer the two sub research questions in this study which is how grid paper and reshaping strategy support students' understanding of the area of a circle.

5.1. Design Experiment

5.1.1. Preliminary Teaching Experiment (The First Cycle)

This study was conducted at Sekolah Menengah Pertama Negeri (State Junior High School) 1 Palembang, with the subjects are students from the 8th grade. In the first cycle, 6 students were randomly selected from classroom VIII7. Those students are Dzaky, Attiyah, Alif, Siti, Belva, and Tasyila. The students are the representations of students from three different level. Dzaky

and Attiyah are from the high level, Alif and Siti are from the middle level, and Belva and Tasyila are from the low level.

The preliminary teaching experiment was conducted in the school's library. There are 6 lessons which are designed in the HLT. The first two lessons are aimed to recall students' notion about area measurements and units in area measurement. Those lessons also give important basic knowledge for students related to the topic of the area of regular and irregular plane figures. The third lesson is about measuring the area of irregular plane figures on a grid paper and measuring the area of a circle by making grids. The fourth and the sixth lessons are about measuring the area of a circle by reshaping its sectors. And, in between the fourth and the sixth lessons, there is the fifth lesson which is about investigating the relations between the diameter and the circumference of a circle. This is because by using reshaping strategy to estimate the area of a circle, the students have to determine the circumference of the circle.

In this cycle, the researcher acted as both the teacher and the observer. The analysis on the preliminary teaching experiment will be focus on evaluating the instructional activities, particularly about what works and what does not work. Moreover, it is also discuss about the development of students' sense and understanding about the area of a circle after experiencing the learning sequence.

5.1.2. Teaching Experiment (The Second Cycle)

Before the implementation of the design on the second cycle,we did a classroom observation and a teacher inverview. The observation was conducted in the classroom of VIII8 at SMP N 1 Palembang, the classroom where the second cycle will be conducted. There were 32 students participated in this cycle. And, as what have been done in the first cycle, the second cycle is also started by conducting pretest and students' interviews. Among all student who were participating on the pretest, there were only six students who were interviewed on the following week after the test.

Compared with the activities in the first cycle, there were some changes on the activities of the lessons which were implemented in this cycle. Besides some revision of the worksheets, there are two pairs of consecutive lessons which were simplified and put together into one lesson which are the first lesson with the second lesson and the fifth lesson with the sixth lesson. Hence, in this cycle, there were only four lessons with 80 minutes of time allocation in each lesson. In the end of the learning sequence, there was a post-test which aimed at gaining impression on how the design support students understanding of the area of a circle by comparing the results with the pretest. The homeroom teacher acted as the teacher in this teaching experiment.

5.2. Retrospective Analysis

5.2.1. Preliminary Teaching Experiment (The First Cycle)

1. Pretest

The pretest was conducted to determine the students' preliminary knowledge and to know students' understanding about the topic would be discussed. The pretest consists of four questions which are about comparing the area of two irregular plane figures with and without grids, determining the area of a circle with grids inside, and determining the perimeter of some plane figures. The students had to solve those problems individually for about 30 minutes. Afterwards, we did an interview with each student. And, the interview give an over view that in general, the students still mainly recall and apply formulas to measure area and perimeter. Morever, some of them were still difficult to distinguish area and perimeter of a plane figure.

Problem 1: Comparing two shapes without grids inside

In this problem, the students are provided two pieces of lands with irregular shapes. And, they have to determine the land which need more grass to cover. The aim of this problem is to know students' sense about area which is represented by the amount of grass needed to cover the lands. Comparing

the amount of grass needed to cover the lands means comparing the area of those two lands.

Three students (Attiyah, Siti, and Syila) gave the right answer with the correct reasoning which is making relation between the amount of grass needed with the area of the lands. There are two students (Dzaky and Alif) who determined the amount of grass needed based on the perimeter of the lands. And, a student (Belva) gave incorrect answer with nonmathematical reasoning because she only said that A need more grass because it must be planted on all the surface of the land as shown on this following figure.

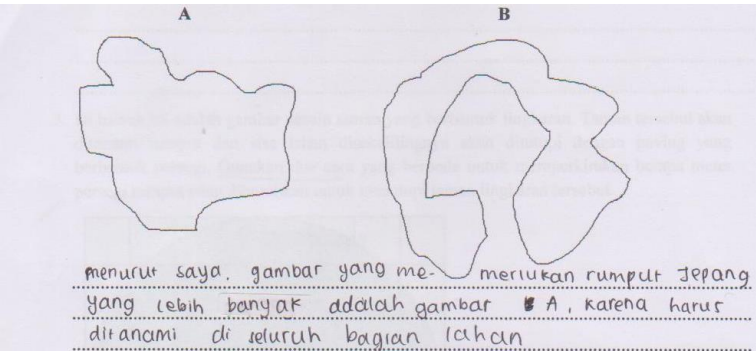


Figure 5.1 A student's written work on problem 1

Problem 2: Comparing two shapes with grids inside

The second problem is also about comparing two pieces of lands. However, there are paving blocks inside which are expected to make the students consider the grids as units of area in the process of comparing. The aim of this problem is that the students will have a sense of the notion unit area measurement and can consider the units to compare the size of those shapes.

There are three students (Dzaky, Alif, and Siti) who reasoned by considering the grids inside and derived the correct answers. Two students (Belva and Syila) pointed out that the regularity of those shapes influence the size of the shapes. However, there are two contradictory reasoning. Belva said that B is bigger because there are many curve on its side (more irregular than A) and Syila said that A is bigger because there is no curve on its side (more regular than B). Lastly, Attiyah give the correct answer that A needs

more grass because it is bigger than B without giving sufficient reasons on why A is bigger. The written work of Attiyah can be seen on this following figure.

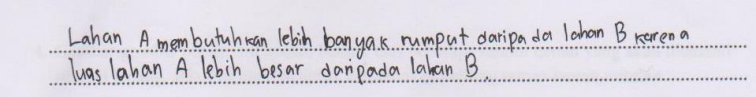


Figure 5.2 A student's written work on problem 2

Problem 3: Estimating the area of a circle with grids inside

This problem is a problem which is particularly about a circle itself. The students are asked to estimate the grass needed to cover the circular garden by using two possible ways. Grids are provided inside the square land in order to assist students in estimating the area of the circular garden.

Two students (Attiyah and Siti) solve the problem using two ways as asked on the sheet. However, Attiyah applied two different ways which all of them involving formulas. Siti use both formula and the grids inside to estimate the area as shown on figure 5.3. Two students (Dzaky and Alif) solve the problem using one way which is applying the formula of the area of a circle. Meanwhile, the other two students (Belva and Syila) did not have any idea to solve this problem on their sheet.

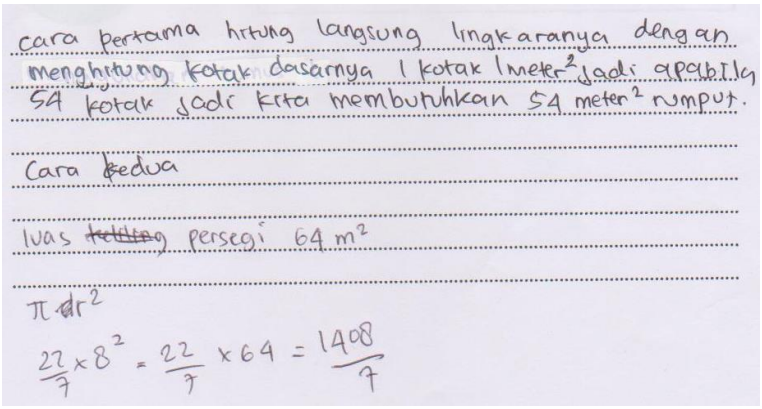


Figure 5.3 A student's written work on problem 3

In this problem, we did not put numbers as the length of the sides of the square land (Figure 5.4). And, based on the interview with the students, they were confuse because of that reasons. It also the reason why Belva and Syila

did not have any idea to solve the problem. Hence, we revised the problem by giving number as the length of the sides of the square land.

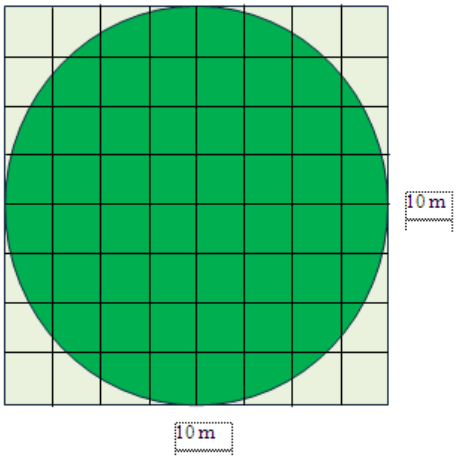


Figure 5.4 The revised version of problem 3

Problem 4: Determining the way to find the perimeter of plane figures

The last problem is about determining the perimeter of several plane figure which are a rectangle, a paralellogram, a trapezoid, a triangle, and a circle, respectively. The aim of this problem is that the students can reason how to determine the perimeter of plane figures with straight line sides and make the relation to find the circumference of a circle which is a plane figure with a curve side. Moreover, this problem give insight about students' understanding of the circumference of a circle as which is also the important topic related to the area of a circle.

Four students (Dzaky, Attiyah, Alif, and Siti) gave correct answers for all plane figures, but none of them have the idea how to find the circumference of a circle. And, two students (Belva and Syila) cannot give correct answer and just making up. Based on the interview with those two students. They could not explain the way to find the circumference of the circle because they forgot the formula which the used to only memorize it. The following figures are two example of students' written work on this topic.

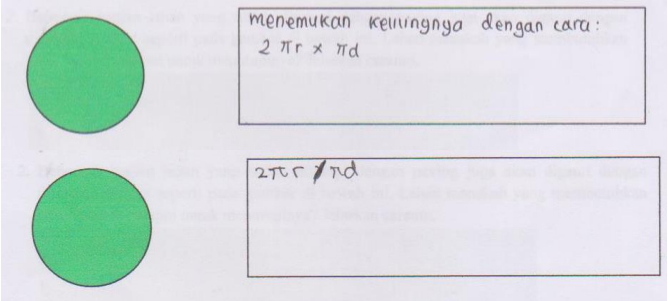


Figure 5.5 Two students' written works on problem 4

2. The teaching experiment of the first cycle

In this teaching experiment, we tested the initial HLT to a small group which consists of six 8th grade students of SMP N 1 Palembang. The researcher acted as the teacher in all the lessons in this cycle. Observation was conducted during the teaching experiment. The analysis about what happens in the teaching experiment are made by evaluating the conjectures and considering all remarks and finding in the teaching experiment. As a results, it is used to refine the initial HLT.

Lesson 1: Comparing and Ordering Shapes

In this lesson, the students are given a worksheet which consists of two activities (two problems). The first problem is about comparing and ordering the five biggest islands in Indonesia which are Sumatra, Kalimantan, Jawa, Sulawesi, and Irian Jaya. And the second problem is about designing two different shape of land with the same area. The students work in pairs in this activity. This following figure is the problem for the first activity.

Indonesian Archipelago

A school will make a garden with the miniature of the five biggest islands in Indonesia namely Sumatera, Java, Kalimantan, Sulawesi, and Papua. If each of the land will be covered by grass, which land will need more grass than the other lands?

Put the islands in order from the land which needs the most grass to the land which needs the least grass. Explain your way of working and reasoning.

Number	Land

Figure 5.6 The problem in activity 1-1

The problem is aimed to support students' understanding of the area of irregular plane figures. The students had to determine among the miniature of islands which needs more grass. The idea is that they will compare the area of those shapes which represent the amount of grass needed to cover the lands.

It was expected in the HLT that the students determined the order by considering the shape of the islands. Interestingly, none of the students came up with that idea. It was contradict with what most students did in the pretest when they determine the bigger land by consider the shapes of those lands. Moreover, there are also no students who determine the lands based on the perimeter of those shapes like some students did in the pretest. All students got the correct orientation of area. It might be because the questions is not determining the biggest land but determining the land which need more grass to cover. Hence, the students can reason that determining the amount of grass needed to cover the lands means determining the area of the lands.

However, the students applied different ways to determine the land which need more grass. As it was expected in the HLT, one group (two students) came up with the idea of overlapping the figures. The following transcript describe their reasoning.

1	Alif	: By **comparing** we can determine which miniature of
2		the islands which need more grass to cover. And,
3		Kalimantan is the miniature which need more grass
4		compared with the other islands.
5	Dzaky	: We know that by **comparing**, which is by **cutting the**
6		**figures and comparing them**. For example is what
7		we did with Kalimantan and Sumatra islands. So, the
8		area of Kalimantan is larger than Sumatra. (*He*
9		*overlap the figure of Kalimantan and Sumatra*
10		*islands*). In the same way, Sumatra is larger than Irian
11		Jaya, Irian Jaya is larger than Sulawesi, and Sulawesi
12		is larger than Java. Hence, Java is the smallest and
13		Kalimantan is the biggest.
14	Researcher	: So, the order is Kalimantan, Sumatra, Irian Jaya,
15		Sulawesi, and Java? *(Together with the teacher, the*
16		*students mentioned the order).* Therefore, we can
17		compare by overlap the figures, right?

Transcript 5.1 Comparing figures by overlapping strategy

The transcript described that the students get the idea of overlapping strategy in comparing the islands. At first, the students were difficult to explain their way. We can see that they just said "comparing" (in line 1 and line 5). However, later on, Dzaky could explain that they firstly did cutting the figures, then comparing them (line 5-6). Moreover, he demonstrate his explanation by overlapping the figures of Sumatra and Kalimantan (line 8-10). To deal with students' difficulty in describing their strategy in words like those two students did, it is important for teacher tell them the term of the strategy which might be new for them. In this activity, the teacher had tried to do that (line 16-17).

The rest of the students solve the problem by applying formulas. They determined the width and the length of the shapes and calculate the area of those shapes. The conversation with one group (Attiyah and Syila) when they share their work in the whole group is described as follows.

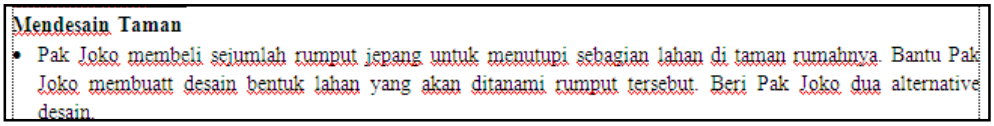
1	Attiyah	: We determine the length and the width of each figure
2		then calculate the area. For example, Sumatra, the
3		length is 1.2 centimeters and the width is 6
4		centimeters, hence the area is 7.2 centimeters. *(She*
5		*did the same way to determine the area of other*
6		*islands)*
7	Researcher	: Syila, could you please show us the length and the
8		width of Sumatra island.
9	Syila	: The length of Sumatra island is from this part to this
10		part *(pointing the longest part of the length of*
11		*Sumatra)* and the width of Sumatra island is from this
12		part to this part *(pointing the longest part of the width*
13		*of Sumatra).*
14	Researcher	: Do you all agree with the strategy which is applied by
15		Attiyah and Syila?
16	Siti	: No, because it is not a rectangle.
17	Alif	: The shape is abstract.

Transcript 5.2 Comparing figures using formulas

The transcript show how the students solve the problem by applying the formula of length times width like the formula to determine the area of a rectangle. It means that the students have misconception about the way to determine the area of those figures. However, the discussion might help the students to argue about others' answer and to reason about their answer. In this case, the teacher give opportunity to the students to expain their reasoning (line 9-13). Moreover, other students could justify their friend's reasoning (line 16-17).

Based on the fragments. In this problem, the students did not experience misconception about area and perimeter. The context gives clear orientation in what they have to do. Some students get the idea of overlapping strategy in comparing the islands. But, the majority applied the formula of length times width to determine the area of those figures. It might because of their mathematics learning experience which emphasize the use of formula to solve problems in area measurement. Hence, a discussion is needed to support their understanding in distinguising the situation when we can apply the length times width formula.

The second problem is about designing two different shapes of lands which need the same amount of grass to cover (see Figure 5.7). The idea is that the students will understand the concept of area conservation and know that it is possible that two shapes with the same area will have the different length of perimeter. It was expected in the HLT that the students will make a figure with the certain area and reshape the figure into different shape.


Mendesain Taman

- Pak Joko membeli sejumlah rumput jepang untuk menutupi sebagian lahan di taman rumahnya. Bantu Pak Joko membuatt desain bentuk lahan yang akan ditanami rumput tersebut. Beri Pak Joko dua alternative desain.

Figure 5.7 The problem in activity 1-2

However, none of the students come up with the idea of reshaping the figure. What students did is just play with formulas. They firstly make a rectangle with certain area and making calculation of another plane figure with the same area. Similar with the possible reason in the previous activity, the reason why the students just play with formula might because the students have got used to just applying formula and doing calculation in solving mathematics problem, particularly in area measurement. Here are an example of the student's worksheet of the activity.

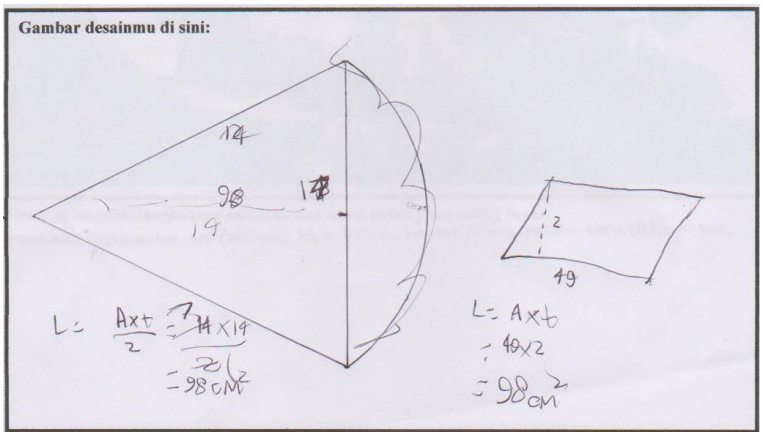


Figure 5.8 The example of students' written work on activity 1-2

Based on the students written work, the problem might not give essential support on students' understanding of the concept of area conservation, because what they did was just applying formulas. Hence, in the second cycle, this problem will not be used. Instead keeping use the activity, it will

be more necessary to discuss about the concept of area conservation and the relation of area and perimeter in the first activity.

Lesson 2: Measuring Shapes through Tiling Activity

The mathematical idea in this lesson is unit iteration. This section begins by clarifying students' understanding of the concept of area measurement using measurement units. There are two activities in this lesson and the students have to solve all the problems individually. The idea of the first activity is recalling the students' knowledge about unit iteration in area measurement and the second activity aims to recall both the students' knowledge about unit iteration in area measurement and the students' knowledge about the shape of a circle. Moreover, in the second activity, instead of iterate a unit to one another, they are expected to make grids based on the given grids.

The first activity is about tiling the floor of a patio with an L-shape with two different size of tiles. The problem can be seen on Figure 5.9. The idea is that the students will see that the smaller the tile they use to cover the floor, the more number of tiles needed. Hence, they will have a sense of units in area measurement.

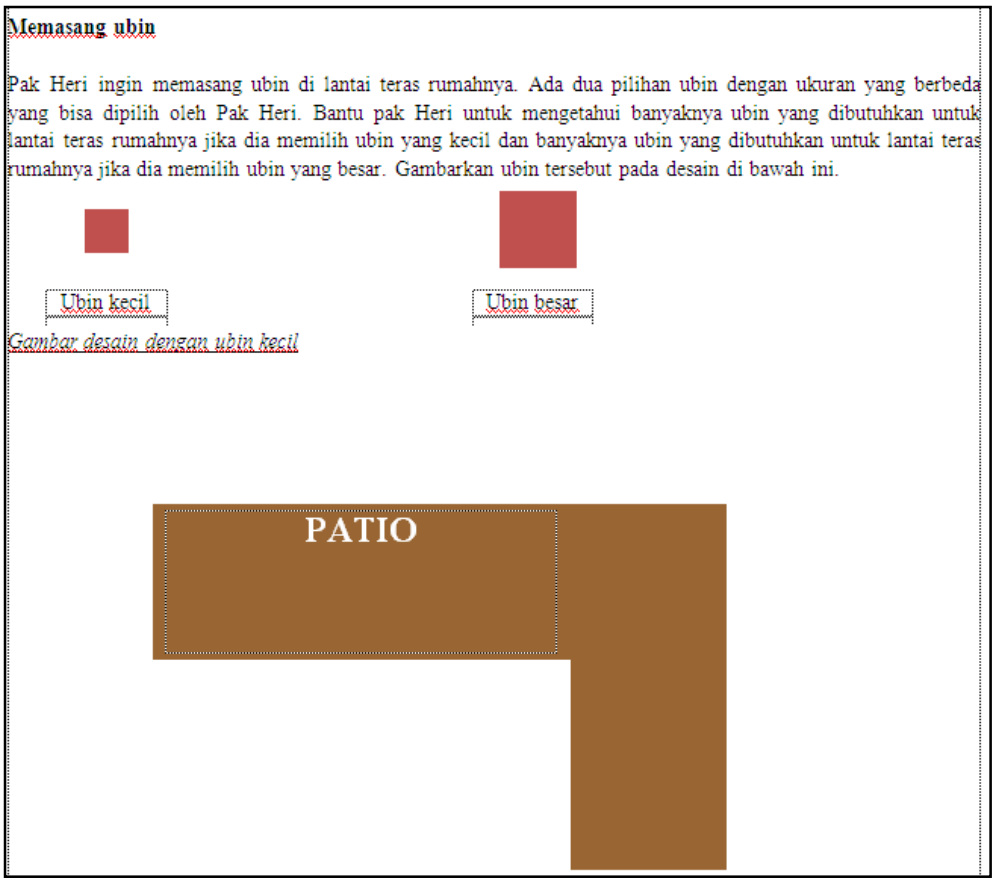


Figure 5.9 The problem in activity 2-1

As it was expected on the HLT, almost all students did cut the tiles and iterate those tiles on the patio. However, they have different way of tiling. Moreover, as it was also mentioned on the HLT, there is a student who did not cut the small tile and only use ruler to do the tiling process on the design. Instead of draw the tiles one by one, the student draw the tiles as a unity using the ruler. Interestingly, there is a student who cut the small tile and iterate the tile on the first floor design and did not cut the big tile for the second design. She used only the ruler to do the tiling process on the second design. It indicates that the students have known the computational knowledge related to area measurement. They just did multiplication on the way to find the total number of tiles needed to over the floor. The discussion on this activity is about the various answers of students in this problem. Here is the conversation of that fragment.

1	Alif	: The way to count is the same, but i count the
2		incomplete tile as one tile.
3	Siti	: I count the incomplete tile as half tile because it is not
4		fully one tile
5	Teacher	: So, Dzaky count an incomplete tile as two thirds of a
6		tile, Alif count an incomplete tile as one complete tile,
7		and Siti count an incomplete tile as half tile. Alif,
8		could you please tell us the reason why you count the
9		incomplete tile as one tile?
10	Alif	: More is better than less.

Transcript 5.3 Discussion on dealing with incomplete tiles

The transcript shows the discussion about this differences number of students' estimation and the reason behind it. The interesting part is that there is a student who has a significantly different number than the other (see line 1). And, the reason is that because in the estimation, he count the incomplete grids as one grid. Hence, the teacher should guide them to look back at the questions and emphasize that the students have to make estimation accurately.

The second problem is about continuing the tiling process of the bottom of a swimming pool which the shape is the combination of an isosceles trapezoid and a semicircle. Some tiles are provided on the swimming pool in order to make students think to do the tiling not one by one but as a unity. The problem can be seen in this following figure.

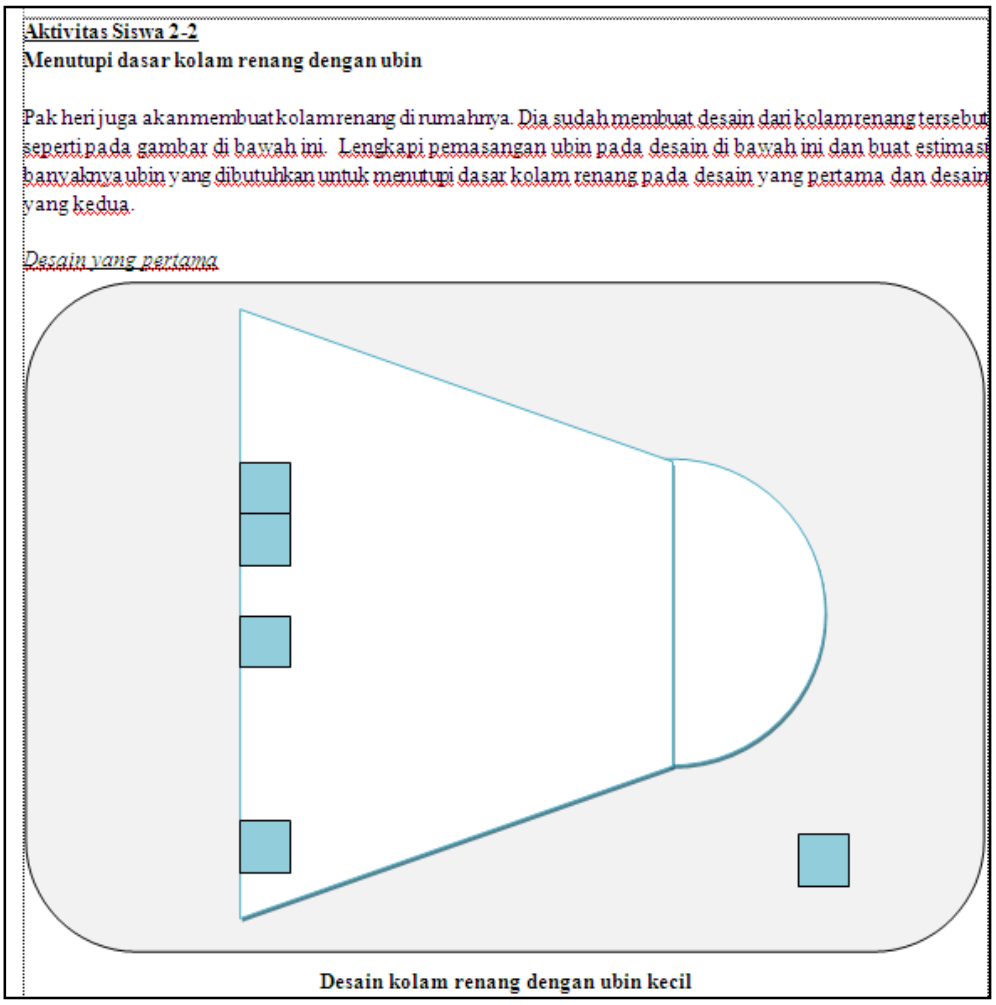


Figure 5.10 The problem in activity 2-2

As it was expected in the HLT, the students made the grids through the given tiles inside the shape of the bottom of the swimming pool. No one did continuing to fulfill the bottom of the swimming pool by drawing the tile one by one as mentioned as the first conjecture in the HLT. It might because all students have understood the way of tiling as the set of tiles which are put together, not always have to be put one by one.

The similar case happened as in the first activity, the discussion on this activity is about the various answers of students in this problem. The students discuss about this differences and the reason of the differences. The most important thing which have to be discussed is how to deal with the incomplete grids. Three students just did conventional calculation and add the incomplete grid. A student did estimation by justifying that the difference of

each row will be one. Hence, she can only add those number to know the total number of those tiles.

Interestingly, two students applied the formula to determine the area of trapezoid and the formula to determine the area of semicircle, then add the numbers they found. It might because they realize that what they have to found is the area of the combination of two plane figures with the tile as the unit. Hence, the can just apply formulas to calculate the area. The Figure 5.12 show the students work on that problem. The other important thing is that the students know the part to be measured and know that the more precise their way of measuring, the more accurate the number they get.

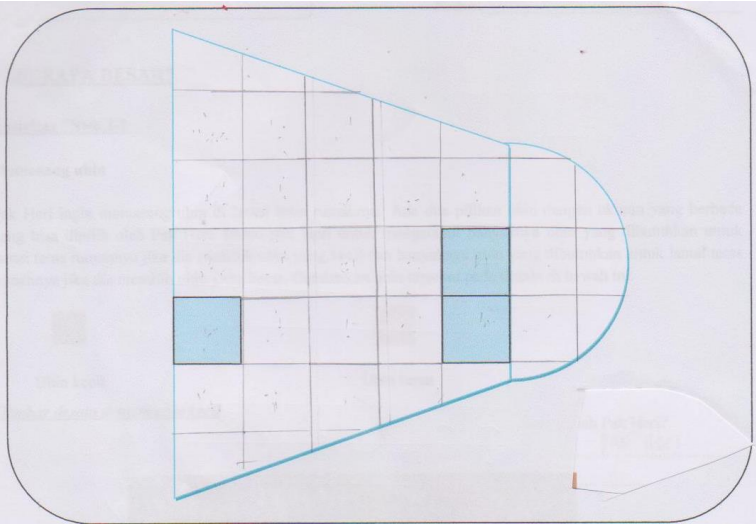


Figure 5.11 A Student's written work on activity 2-2

Based on the fragment and students' written work, since from the first activity, the students have understood the way of tiling as the set of tiles which are put together, not always have to be put one by one. Hence, in the refined HLT, the first activity was deleted and the second activity in this lesson will be put together.

Lesson 3: Measuring Shapes Using Grid Paper

The mathematical idea in this lesson is using grid paper to measure area. This section begins by clarifying students' conception of area related to measurement units. There are two activities in this meeting. Firstly, the students try to compare two lakes on a grid paper. Afterwards, for the second

activity, the students estimate the grass needed to cover a circular garden on a square land. The students worked individually in both meetings. The first activity aims to introduce the grids in area measurement and in the second activity the students are expected to see the relationship between the area of square and the area of circle which diameter equals the side of the square land.

For the first activity, the problem is about determining the bigger lake of two lakes which are Ghost Lake and Loon Lake (Figure 1). As it was expected on the HLT, to compare the size of those two lakes, the students tend to only count the grid and determining which lake cover more grids. None of them disregard the grids and reshape the lakes into two more similar shape as it was also mentioned on the HLT. It might because in the previous activity what the students have learned is about the units of area measurement involving square tiles. Hence, the students make an analogy of the grid as a square tile.

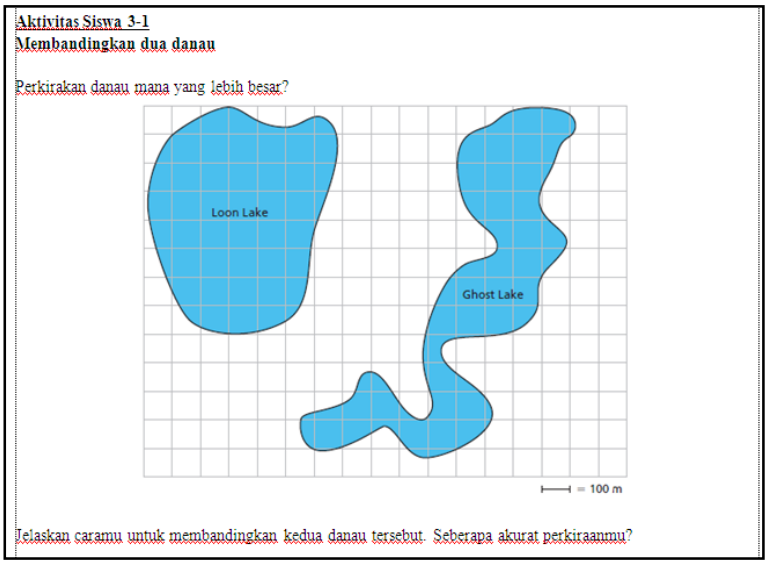


Figure 5.12 The problem in activity 3-1

The students' written works show that there are different ways of students in determining the number of grids. Most students just directly count the complete grids and add the incomple grids into one complete grids then add them all. Meanwhile, there two students who counted the grids by dividing the shapes into several parts which are they think more regular. Afterwards,

they did counting the grids of each shape and adding them all. The students' works can be seen on these following figures.

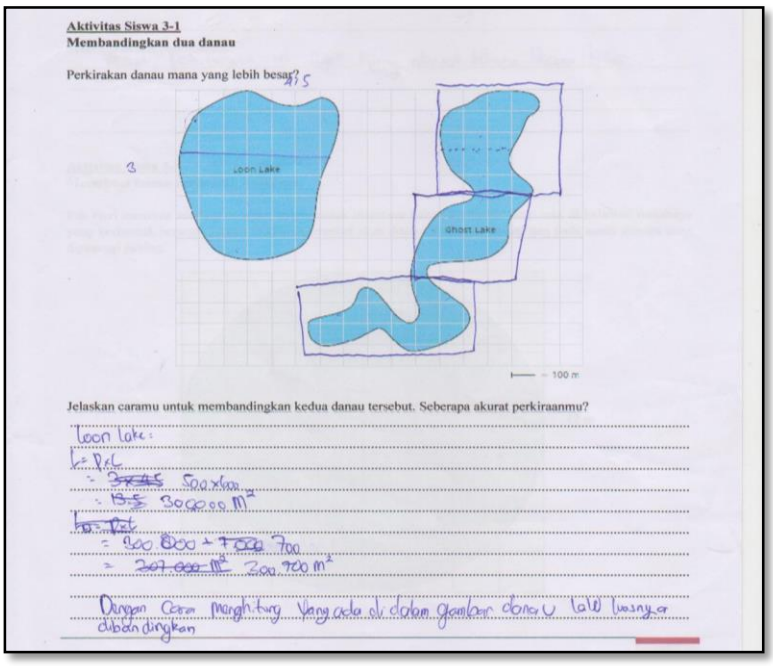


Figure 5.13 The example of a student's written work on problem 3-1

Moreover, a fragment of a short discussion related to the students' written work displayed above can be described as follows.

1	Researcher	: Which one is bigger?
2	The Boys	: **Loon Lake**
3	Researcher	: Loon like is bigger. How about the girls, do you have
4		the same answer?
5	The Girls	: Yes, **Loon Lake**
6	Researcher	: What is your reason?
7	The girls	: Because **we count the grids**. Loon Lake has 45 grids
8		and Ghost Lake 30 grids.
9	Researcher	: Alif's strategy is length times width, could you please
10		tell us which is the length and which is the width?
11		(pointing the Loon lake).
12	Alif	: The length is 45 and the width is 30. Oh no, we
13		change the length and 11the width become the actual
14		measurement which is **500 times 600** equals 300.000
15		m^2.

Transcript 5.4 Students' reasoning on problem 3-1

Based on the transcript (line 2 and line 5), all students agree that Loon lake is bigger than Ghost lake. Moreover, on line 7, the girls said that they applied strategy of counting the grids. Alif, one of the students with the reasoning

displayed on Figure 2 tried to explain their reasoning. They measured the area of a rectangle with 500 m wide and 600 m long (on line 14). The straight line on the figure of Loon lake represents the length of the rectangle. The students draw the rectangle to make them easier in estimating the number of grids on Loon lake. It indicates that they also reshape the figure into a regular figure and adding the rest of the grids outside the rectangle in their calculation.

For the second activity, the problem is about determining the amount of grass needed to cover a circular garden on a square land (Figure 15). The amount of grass will be in meter square and there are grids on the design as the representation of paving blocks covering the rest of the land which are not covered by grass. Surprisingly, almost all students directly apply the formulas of the area of a circle to determine the amount of grass needed to cover the circular garden. It might because when they looked at the diameter which are known as a number, the directly thought about applying the number on the formula.

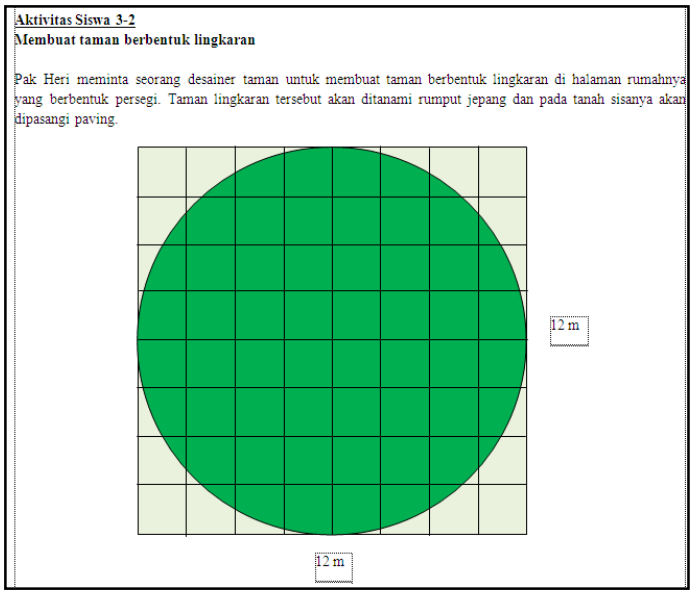


Figure 5.14 The problem in activity 3-2

In other words, the rest of the conjectures which have been mentioned on the HLT did not really revealed in the classroom. None of the students said that the grass needed to cover the circle is three fourths of the area of the square land. There were no students estimate the area of a circle by

considering the given grids which make the area of the circle is more than $3r^2$ and less than $4r^2$.

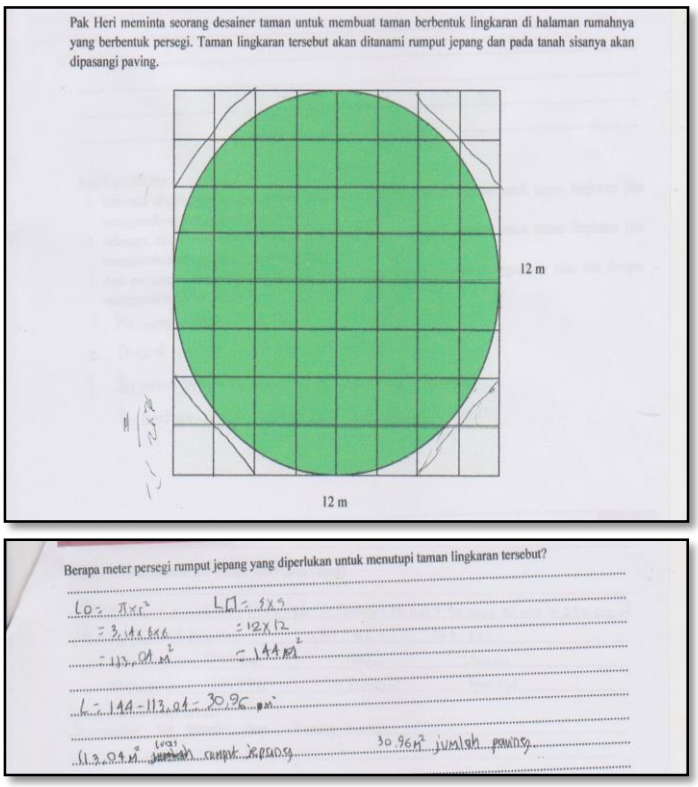


Figure 5.15 An example of students' written work on problem 3-2

An example of a student's written work (Figure 16) describes that in fact, the student tried to use the grids in estimating the area of the circular garden. The students tried to estimate the amount of grass needed to cover the circular garden by making four triangles in all four vertices of the square. However, in the conclusion of the problem, they directly applied formula to determine the area of a circle without mentioning about the estimation results.

Moreover, from the discussion, it revealed that the students mainly depend on the formula and thought that as long as they know the formula and they can determine the area of the circle by applying the formula, they do not need to apply other strategy.

There is no changing of the activities in this lesson. However, in the teacher guide, it should be explicitly mentioned that the teacher should really encourage the students to determine the area of the circular garden in the second activity by using their own way, not only by applying formulas.

Lesson 4: Reshaping Circles

The mathematical idea in this meeting is that the area of a circle equals the radius times half of its circumference. There is only one main activity in this meeting namely reshaping three circles which are cut into different numbers of sectors into a parallelogram. However, to make the students get the idea of reshaping the sectors of a circle into a parallelogram, there will be an activity of reshaping a circular garden into a shape resembles parallelogram which will be guided by the teacher. The students worked in a group of two.

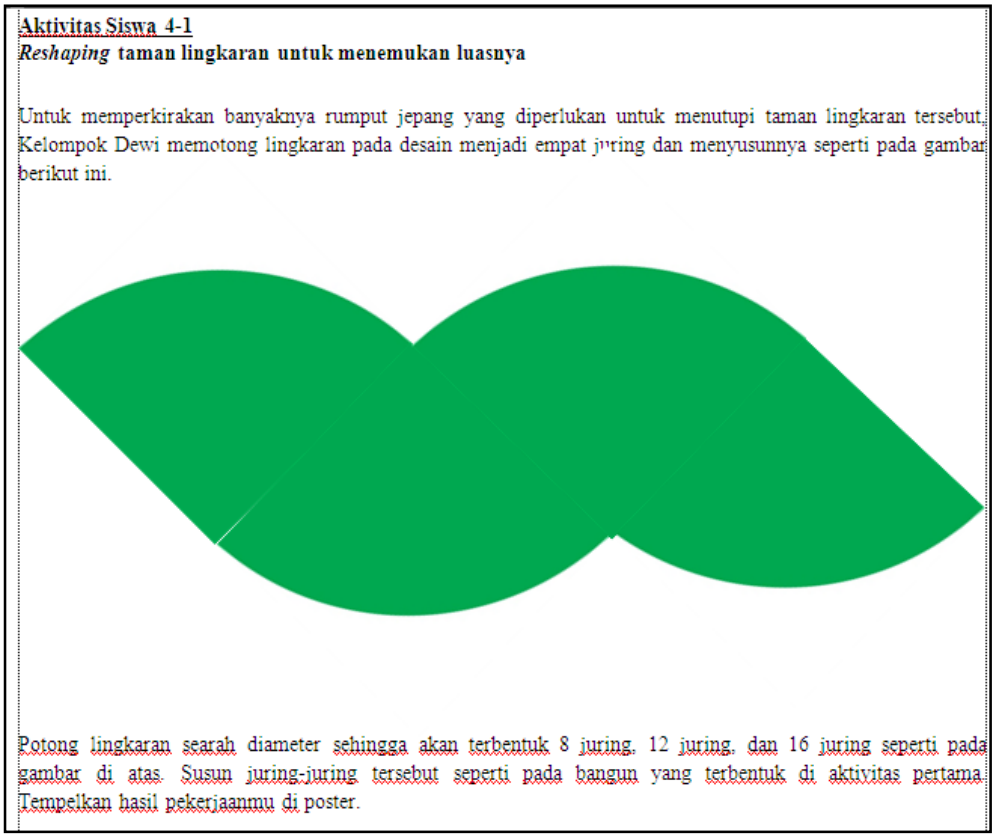


Figure 5.16 The Problem in activity 4

In the first activity of this meeting, the students are shown another way to estimate the grass needed to cover the circular garden which is by cutting the circle into four equal sectors and reshape it in certain way. There are no groups which recognize that the shape is resemble a parallelogram. It might because the arc are still too long and make the sides too curved. A group said that there is no plane figure resembles the shape. Two group decided to say that the shape resembles a circle, a rectangle, and a parallelogram. The

teacher should investigate students' reasoning by asking the properties of the shape so that they can say that the shape resembles the plane figures they mentioned. However, in this lesson, the teacher only asked why the shape resembles those plane figure without giving any clue. However for the second question, all students get the idea that the area of the shape are same as the area of the circular garden.

For the second problem all students were able to reshape the sectors of all three circle in the correct way. However, at the first time, all students thought that what they have to do is arranging the sectors into a shape shown in the first activity. Hence, the teacher must explain clearly that what students have to do is arranging the sectors in the same way as shown in the shape, not into the same shape.

After reshaping all three circles, all students can see that the shapes the formed resemble the shape of parallelogram. It was contradict with what that has been mentioned on the HLT. However, some students did not get the idea that the more number of sectors formed from the circle, the more resembling the shape with a parallelogram. In this part, the teacher did not also explore students' thinking about the reason why the more sectors will form the more resembling parallelogram.

For the last problem about changing the parallelogram into a rectangle. As it was expected on the HLT, only two students from one group who got the idea of halving one sector and put it on the right most or the left most of the shape. The rest of the students did not think that that shape can be changed into a rectangle. Hence, in this part, the teacher plays important role to remind students the relation between parallelogram and rectangle which they have been learned in the 7th grade. Here is an example of students' written work.

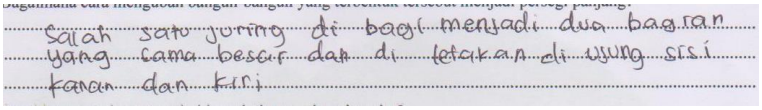


Figure 5.17 An example of students' written work on problem 4

Unfortunately the discussion about the way to determine the area of the rectangle could not be done in the end of this lesson due to the time

limitation. Therefore, it became a homework which would be discuss in the beginning of the following lesson.

Lesson 5: Finding the relationship between the circumference and the area of circles

The mathematical idea in this meeting is that the circumference of a circle divided by its diameter yields the pi value. There are two main activities in this meeting which are making circles using a shoelace and a marker and investigating the relation between the circumference and the diameter of some circular objects. However, those two activities are begun by the first activity which is estimating the area of a rectangle which is formed from the sectors of a circle. This is because in the problem of the first activity, the students have to determine the circumference of a circle to find the area of the rectangle.

Student Activity 5-2

Menggambar lingkaran dengan menggunakan pensil dan tali sepatu

Dengan menggunakan pensil dan tali sepatu, gambarlah lima buah lingkaran dengan ukuran yang berbeda. Ukur keliling setiap lingkaran dengan menggunakan ukuran diameter lingkaran tersebut (dengan menggunakan tali sepatu) dan tuliskan hubungan antara diameter dan keliling lingkaran pada tabel berikut ini.

Lingkaran nomor-	Hubungan antara diameter dan keliling
1	
2	
3	
4	
5	

Aktivitas Siswa 5-3

Membandingkan keliling dan diameter lingkaran

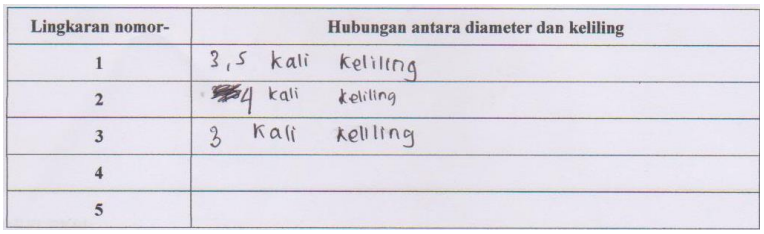
Terdapat banyak benda dalam kehidupan sehari-hari yang berbentuk lingkaran. Beberapa di antaranya adalah benda-benda di di bawah ini.

Buatlah lingkaran dari setiap gambar di atas di sebuah kertas dan ukurlah diameter dan keliling dari setiap lingkaran yang terbentuk.

Figure 5.18 The Problem of Activity 5

The first activity is just determining the way to determine the area of a rectangle which is formed from a circle which is divided into 16 sectors equally. Therefore, by the guidance from the teacher, the students did not find any difficulty. In the end, they determine the area of the rectangle as length times width or a half of the circumference of a circle times its radius.

In the second activity, some students were difficult to make a circle using a rope since they made the circle on a piece of A4 paper. Hence, for those who experienced difficulties, the teacher ask them to make the circles using a compass and measure the diameter and the circumference using the rope. In this activity, overall, the students conclude that the length of the circumference is about three times the length of its diameter.



Lingkaran nomor-	Hubungan antara diameter dan keliling
1	3,5 kali keliling
2	4 kali keliling
3	3 kali keliling
4	
5	

Figure 5.19 An example of students' written work on problem 5

In the third activity, the students were given some circular paper which were actually the drawing of some circular objects. They measured the diameter and the circumference and found the value of the circumference divided by the diameter. The interval of this finding is between 3 and 3.5. In this section, the explanation from the teacher plays essential role for students' understanding of the role of phi value in determining the circumference of a circle. The conclusion in this lesson is that, the circumference is around three times its diameter, and the exact value is 3.14 which is introduced by the teacher.

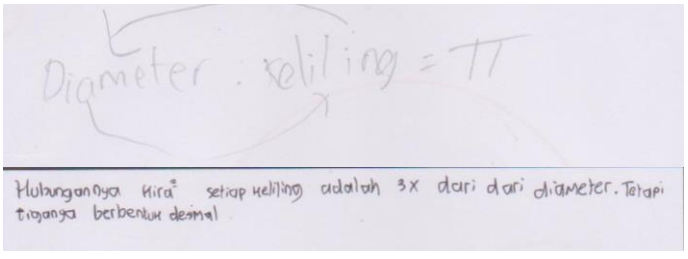
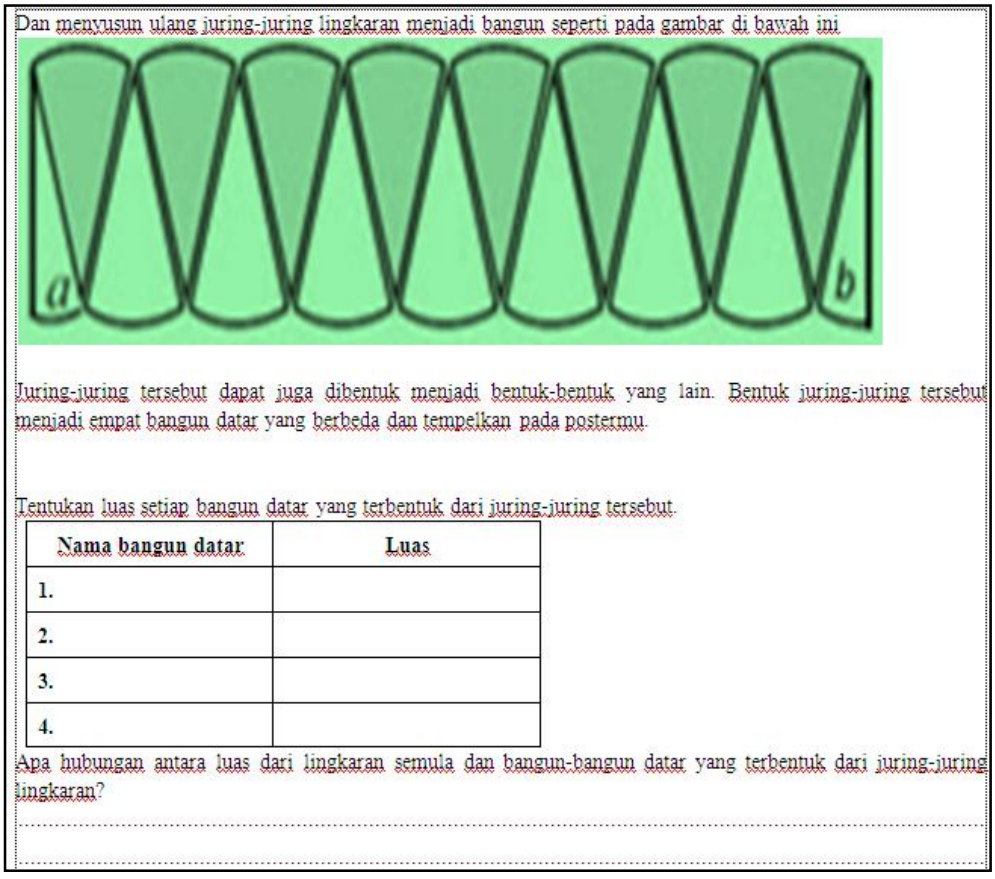


Figure 5.20 An example of students' written work on problem 5

Based on the activities in this lesson, the second activity will be deleted because of two reasons. Firstly, the students find difficulty in making the circle using a rope. Secondly, the activity is almost similar with the second activity. Hence, the second activity will be enough to support students' sense of the phi value as the ratio of the circumference and the diameter of a circle.

Lesson 6: Reshaping Circles into Several Plane Figures

The mathematical idea in this meeting is that the area of a circle equals pi times radius times radius. This section is kind of wrapping up activity. However, the first activity is still about reshaping. And, in the second activity, the students will estimate the amount of money needed to buy grass for covering a circular garden.


Dan menyusun ulang juring-juring lingkaran menjadi bangun seperti pada gambar di bawah ini

Juring-juring tersebut dapat juga dibentuk menjadi bentuk-bentuk yang lain. Bentuk juring-juring tersebut menjadi empat bangun datar yang berbeda dan tempelkan pada postermu.

Tentukan luas setiap bangun datar yang terbentuk dari juring-juring tersebut.

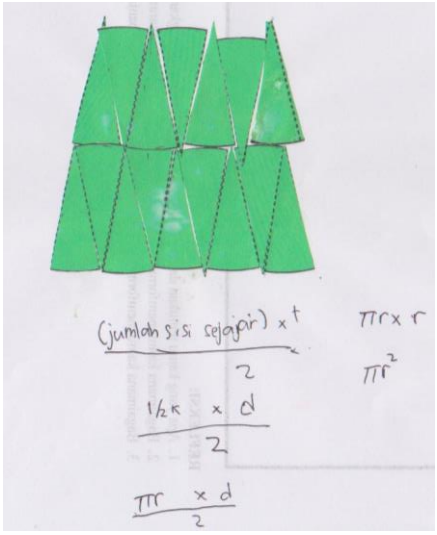
Nama bangun datar	Luas
1.	
2.	
3.	
4.	

Apa hubungan antara luas dari lingkaran semula dan bangun-bangun datar yang terbentuk dari juring-juring lingkaran?

Figure 5.21 The problem in activity 6-1

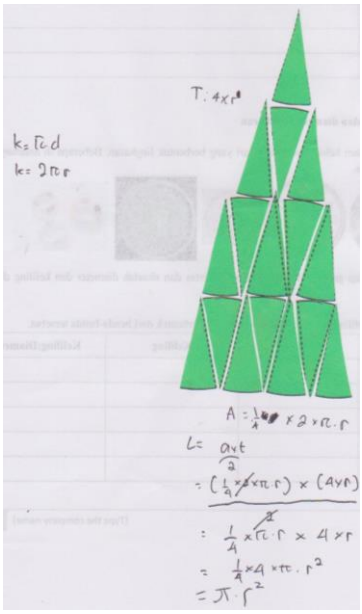
Due to the time limitation, each group of the students are asked to only form one shape of plane figure and investigate the formula to determine its area. As it was expected in the HLT, some students did not have any ideas to

form the sectors into plane figures different with what they have been formed in the previous meeting (parallelogram and rectangle). Hence, a group formed the sectors into a rectangle. The other two groups formed the sectors into a trapezoid and a triangle. However, they found it difficult to find the area of those figure. The students' work on trapezoid and triangle can be seen on these following transcripts.



A trapezoid

Figure 5.22 An example of students' written work on problem 6-1



A triangle

Figure 5.23 An example of students' written work on problem 6-1

The students' difficulty is on the arithmetic operation of the formula. Generally, by the teacher guidance, the students can see the relation between the part of the new figure and the part of the circle although doing the computation is a little bit complicated for the students. However, in the end, they can see that the formula of the area of the new shapes are same with the area of the circle which is πr^2.

Moreover, the students can conclude that the area of every plane figure which is formed from the sectors of a circle are same with the area of the circle. Interestingly, there is an answer which is not expected will come up in the classroom. Two students answer about the relation between the circle and the new plane figures in general, not focus on the area of those figures.

The second problem In this activity the students will estimate the amount of money needed to buy grass for covering the circular garden. The idea is that the students will know the application of why they need to know the amount of grass needed to cover the circular garden which means estimate the area inside a circle. Moreover, the students will apply the strategies which they have learned in the learning sequence. The result will also put on the poster paper of their group.

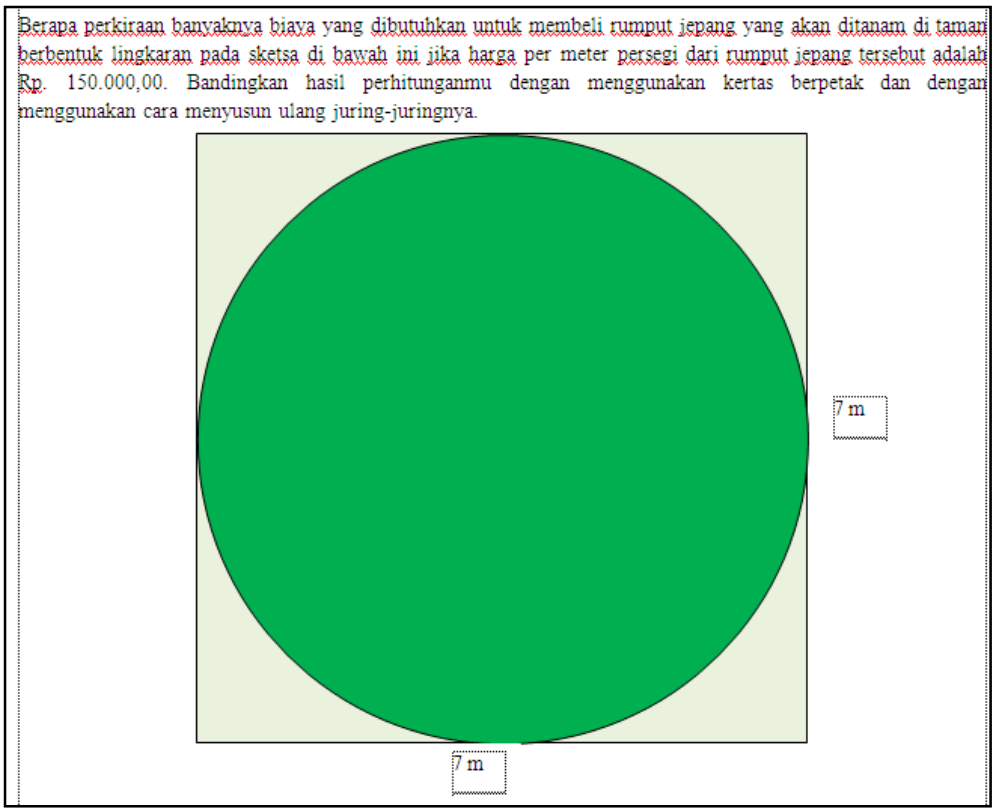


Figure 5.24 The problem in activity 6-2

However, due to the time limitation of the lesson, the teacher could not really explore students reasoning on this problem. The way students solve the problem could only be seen from the students' written work. An example of students' written work is described as follows.

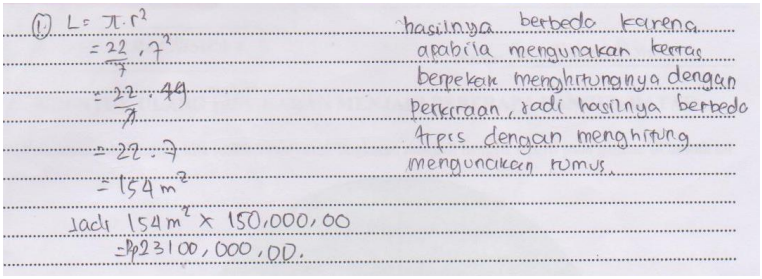


Figure 5.25 An example of students' written work on problem 6-2

The students know that they can solve the problem by making grids or applying the formula of the area of a circle. Yet, they just determine the value by applying the formula since, they think that it is more accurate. In the second cycle, we decided to leave this problem out because it would be better to spend more time in understanding for the activity 6-1.

3. Post-test

In the end of the learning sequence of the first cycle, a post-test was given for the students. There are only 5 students who did the post-test because one of the students had to join a school meeting. Same as the pretest, there are four questions in this postest. However, there were some changes on the questions.

There are no changes on the first, the second, and thefourth problems. A minor change is applied for the third problem. We put number as the length of the side of the square land because based on the students' interview and the written work in the pretest, the feel difficult when we do not put number on it. The other revison is on the command sentence used in the question. We change "explain your strategy" into "explain your reason". It is because based on the interview after the pretest, the old one was not understandable.

In general, most students solve the problems in a better way. In the first problem which is about comparing two lands without grids inside, they were able to give mathematical argumentation about the reason why one of them is bigger by explaining their strategy in comparing those two lands. Similar with the first problem, for the second problem, almost all students consider the grids to determine the bigger land.

4. Conclusion for the first cycle

Based on the observation and the students' work we may say that through the activities designed, grids can support students understanding of area and area measurement of plane figures, particularly irregular plane figures. However, to determine the area of a circle, the students were coming back to use the formal way which is applying formula πr^2. It might because the formula have been introduced since they were in the 6^{th} grade of elementary school. Moreover, by reshaping the sectors of a circle into other plane figures and determine the area of those plane figures, it helps students to understand the formula of the area of a circle come from and how the formula of the area of a circle related to the formula of the area of other plane figures. The idea is grounded from the concept of area conservation.

However, some changes should be done in order to improve the learning design. The details of the changing were described on section 5.2.

5. The refined of Hypothetical Learning Trajectory

Based on the findings of the first cycle and the interview with the teacher, the researcher refined the initial HLT. Related to the whole lessons, the reflection part was removed from the students' worksheet to the class discussion in the end of each lesson. Therefore, the teacher plays important role to guide the discussion. The revision of each lesson can be described as follows.

1. Lesson 1 and lesson 2

In the second cycle, the first and the second lessons are put together in one lesson and face 80 minutes length. This is because the second activity of the first lesson and the first activity of the second lessons are deleted.

The reason why the second activity of the first lesson is deleted is because in the activity the students only focus on applying formula and calculation. Hence, the problem might not be give essential support on students' understanding of the concept of area conservation as it was expected. Consequence, the teacher should really strengthen the students about the concept of area conservation and the relation of area and perimeter in the first activity.

The first activity of the second lesson is deleted because the students have understood the way of tiling as the set of tiles which are put together, not always have to be put one by one. Therefore, keeping both the first and the second activity will be kind of redundancy.

2. Lesson 3

There is no changing of the activities in this lesson. However, in the teacher guide, it should be explicitly mentioned that the teacher should really encourage the students to determine the area of the circular garden in the second activity by using their own way, not only by applying formulas.

Moreover, from the discussion, it revealed that the students mainly depend on the formula and thought that as long as they know the formula and they can determine the area of the circle by applying the formula, they do not need to apply other strategy. Therefore, asking students to prove their answer can be an idea to encourage them apply informal strategy.

3. Lesson 4

There is no significant changing in the fourth activity of the first cycle. The worksheets are exactly the same. For the last problem about changing the parallelogram into a rectangle. As it was expected on the HLT, only two students from one group who got the idea of halving one sector and put it on the right most or the left most of the shape. The rest of the students did not think that that shape can be changed into a rectangle. Hence, in this part, the teacher plays important role to remind students the relation between parallelogram and rectangle which they have been learned in the 7^{th} grade.

4. Lesson 5 and lesson 6

In the second cycle, the fifth and the sixth lessons are put together in one lesson and face 80 minutes length. This is because the first activity of the fourth lesson and the second activity of the sixth lessons were deleted.

Based on the activities in this lesson, the first activity of the fourth lesson were deleted because of two reasons. Firstly, the students find difficulty in making the circle using a rope. Secondly, the activity is almost similar with the second activity. Hence, the second activity will be enough to support students' sense of the phi value as the ratio of the circumference and the diameter of a circle.

Moreover, the second activity of the sixth lesson was deleted also because of two reasons. Firstly, due to the time limitation, it would be better to spend more time in understanding for the first activity. Secondly, the students have know the application of area in daily life from the problems and examples which they have learned in the learning sequence. In this part, the teacher plays important role.

5.2.2. Teaching Experiment (The Second Cycle)

1. Pre Test

The pretest was conducted to determine the students' preliminary knowledge and to know students' understanding about the topic would be discussed. Moreover, it is also used as consideration to choose the students for the focus group. The pretest consists of four questions which are about comparing the area of two shapes with and without grids, determining the area of a circle with grids inside, and determining the perimeter of some plane figures. The students had to solve those problems individually for about 30 minutes. Afterwards, we did an interview with six students which have quite different answers.

Problem 1: Comparing two shapes without grids inside

In this problem, the students are provided two pieces of lands with irregular shapes. And, they have to determine the land which need more grass to cover. The aim of this problem is to know students' sense about area which is represented by the amount of grass needed to cover the lands. Comparing the amount of grass needed to cover the lands means comparing the area of those two lands.

It was only a few students who gave the right answer with the correct reasoning which is making relation between the amount of grass needed with the area of the lands. Moreover, to compare the area, it is also only a few students who apply the idea of reshaping strategy. Most students determined the amount of grass needed based on the perimeter of the lands or the shapes of the lands. The rest of the students gave answer with non mathematical reasoning as shown on this following figure.

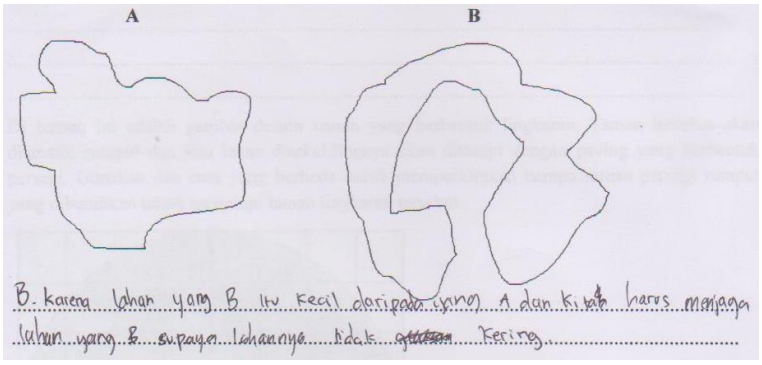


Figure 5.26 A student's written work on problem 1

In that written work, the student said, "B need more grass becauseB is smaller than A and we have to keep the land B not dry". The students' reasoning was out of mathematical reasoning because he think that the shape of B will make it tend to dry easily. Consequently, it needs more grass to cover. To prepare with students with this kind of reasoning, in the teaching experiment, the teacher should firstly check students' understanding of a problem and pose questions which can stimulate their reasoning.

However, in general, the students fulfilled the requirement to learn area. They could justify the part to be measured although only a few of them could relate with the notion of area. Moreover, the students also could made justification which land is bigger, even though they were not able to give any mathematical reasoning.

Problem 2: Comparing two shapes with grids inside

The second problem is also about comparing two pieces of lands. However, there are paving blocks inside which are expected to make the students consider the grids as units of area in the process of comparing. The aim of this problem is that the students have a sense of the notion unit area measurement and can consider the units to compare the size of those shapes.

Most students compared by considering the grids inside and derived the correct answers. Some students pointed out that the regularity of those shapes influence the size of the shapes. Whereas, some other looked at the length and the width of the lands to determine the bigger one. Moreover, there are still

some students who answer by giving non mathematical reasoning as shown in this following figure.

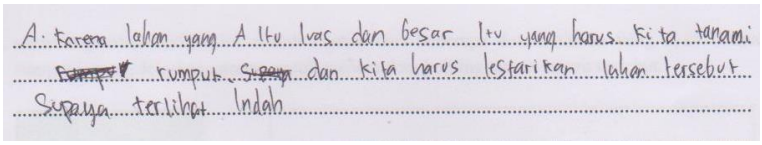


Figure 5.27 A student's written work on problem 2

On the written work, the student said, "A will need more grass because A is large and big. Moreover, we have to keep it looks beautiful". In this case, the student had have a correct understanding that the larger or bigger the land, the more grass needed to cover. However, this student gave other nonmathematical reasoning by mentioned that we have to keep it looks beautiful.

Based on the students' written work and the interview with the students, generally, most students have had sense on units of area measurement. It can be seen from the majority of students who did counted the grids to compare those lands. Yet, there were some students who determine the bigger lands based on the shapes of those islands.

Problem 3: Estimating the area of a circle with grids inside

This problem is a problem which is particularly about a circle itself. The students are asked to estimate the grass needed to cover the circular garden on a square land by using two possible ways. Grids are provided inside the square land in order to assist students in estimating the area of the circular garden. As mentioned in the refined HLT, we revised the problem by giving a value for the length of the sides of the square land.

Most students just applied one strategy to solve the problem. They did counted the grids to estimate the grass needed to cover the circular land. It indicates that they did not really have a sense of area related to the problem since they did not apply the formal way to determine the area of a circle. This following figure is an example of students' written works with the strategy.

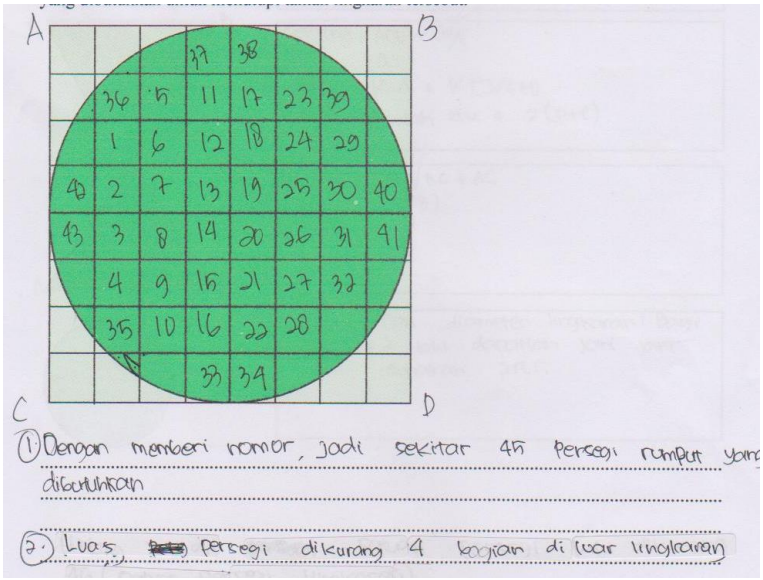


Figure 5.28 A student's written work on problem 3

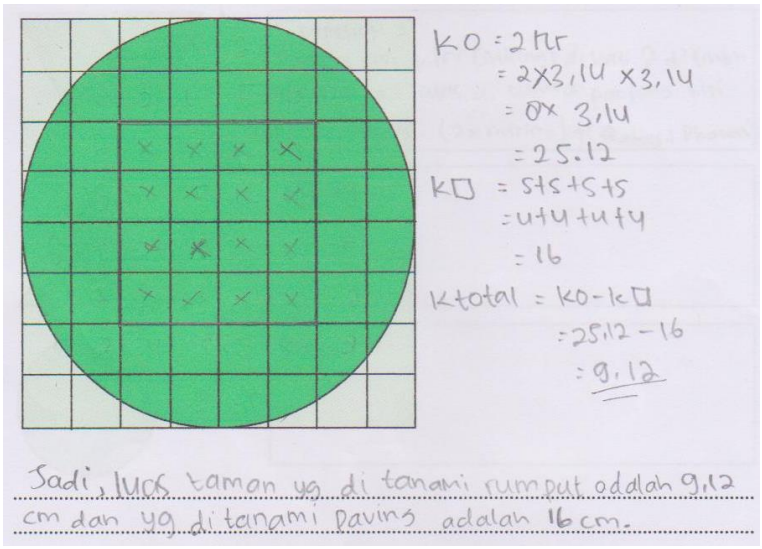


Figure 5.29 A student's written work on problem 3

Meanwhile, a few students applied formulas in solving the problem. The figure above shows an example of students' written work using the strategy. In fact, the students could show the green region inside the circle as the part which they have to determine which is. However, because there was a confusion about the area and perimeter, the student applied the formula to determine the perimeter of a circle.

In general, based on the students' written work and interview with the students, the students had known the part to be measured. Moreover, they

could consider the grids as units of area measurement. However, they still had a confusion about the terms of area and perimeter. It can be seen from the activity of applying the formula incorrectly.

Problem 4: Determining the way to find the perimeter of plane figures

The last problem is about determining the perimeter of several plane figure which are a rectangle, a paralellogram, a trapezoid, a triangle, and a circle, respectively. The aim of this problem is that the students can reason how to determine the perimeter of plane figures with straight line sides and make the relation to find the circumference of a circle which is a plane figure with a curve side. Moreover, this problem give insight about students' understanding of the circumfernce of a circle as which is also the important topic related to the area of a circle.

Most students directly wrote the formulas without giving any explanation about the way to find the perimeter. Therefore, we could not really said that giving correct answers means having understanding. It was only a few students who tried to reason and explain the way to determine the perimeter. The example of students written work can be seen on this following figures.

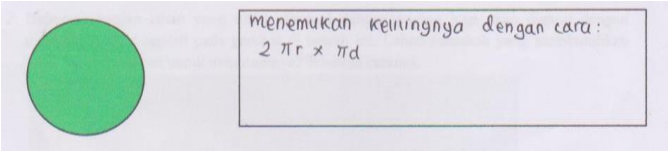


Figure 5.30 A student's written work on problem 4

Generally, the students' written works and the interview give an over view that in general, the students still mainly recall and apply formula to measure perimeter. Morever, some of them are still difficult to distinguish area and perimeter of a shape. As a result, the applied the formulas incorrectly.

2. The teaching experiment of the second cycle

In this teaching experiment, we used the refined HLT to a classroom of 8^{th} grade students of SMP N 1 Palembang which consists of 32 students. The homeroom teacher acted as the teacher in all the lessons in this cycle.

The researcher and a fellow researcher assisted the homeroom teacher during the learning process. Observation and video registration were conducted during the teaching experiment. The analysis about what happens in the teaching experiment are made by evaluating the conjectures and considering all remarks and finding in the teaching experiment. As a results, it is used to answer the research question.

Lesson 1: Comparing and Tiling

As described in the refined HLT, lesson 1 in this cycle is a squeezing of lesson 1 and lesson 2 in the first cycle. Therefore, the lesson titled "Comparing and Tiling". Before the students worked in group, the teacher gave an apperception related to the first activity in this lesson which is comparing the five biggest islands in Indonesia. What the teacher did was asked the students to compare the surfaces of two books with different size of surfaces and asked them to argue about the reason why it has different size. The apperception was planned in the HLT.

Activity 1-1

The learning activity was continued by working on the first worksheet in this lesson. The activity in the worksheet was about determining the miniature of the five biggest islands in Indonesia which need more grass to cover and put them in order. It means that the students have to compare and put the islands in order. The students worked on group of five or six and give them a large copy of Indonesian map. The reason why we choose the shapes of islands is because the shapes are irregular. Therefore, it might help us to know how students' understanding of area. In this case is about whether any confusion about perimeter and area.

Interestingly, the students did not experience misconception about area and perimeter as they did in the pretest. It might because the context give a clear orientation of area. Besides that, because they worked in group, the discussion in group might give them opportunity to learn from each other. Moreover, the teacher help students who still have confusion in

distinguishing area and perimeter by asking the students to show both area and perimeter of the islands.

As it was expected in the HLT, various strategies were used by the students. Some students were not able to give mathematical reasoning and did comparing just by looking at the figures of the islands, while some others, which are the students in the focus group, did cutting and overlapping to compare the islands. Two fragments of students' activities with those two strategies are described as follows.

(A conversation in a group)
Researcher : Kalimantan is bigger than the other islands. How do you know that?
Sstudent 1 : Because **we can easily see that**
Researcher : We just see that? Is it convincing? How can I believe?
Student 2 : This bigger than the other, **we can just see that**
Researcher : How if I say that Sumatra is bigger when I see that with my eyes?
Student 3 : **It's an agreement**
(A conversation in the focus group)
Student 1 : Firstly we cut all the islands
Student 2 : No, it's not all the islands
Student 3 : **The three islands were cut then divided into some parts then compared**
Researcher : How to compare it?
(The student overlap Sumatra and Kalimantan)
Researcher : When we **overlap** this one and that one, which one is bigger?
Student 3 : Kalimantan
Researcher : Why Kalimantan is bigger?
Student 4 : **Because we want it** (The student overlap Sumatra and Kalimantan then show the remaining parts although he said it in informal words)
Student 3 : Because there are still **remaining parts**
(The student shows the remaining parts of Kalimantan as a result of overlapping the figures of Sumatra and Kalimantan)

Transcript 5.5 Students reasoning in activity 1-1

The first part of the transcript describes the conversation in a group where students determine the bigger islands only by looking at the shapes. The students repeat twice about their way (line 4 and 6). Moreover, when

the researcher asked them to give a convincing reasoning, the students looked confuse to explain and said that it is an agreement in their group.

The second part of the transcript shows the conversation in the focus group. It was difficult for students in the group to explain what kind of strategy they applied. However, they explain that they cut the figures of the islands and demonstrate the way to compare it by overlap the figures of the islands (line 11-16). Afterwards, in line 17, the researcher introduce the terms “overlap” to the students in the follow up question about comparing Kalimantan and Sumatera islands. A student answer that Kalimantan is bigger because they want it. Yet, in the same time he demonstrate his explanation by overlap the figures of those two islands. Afterwards a students argued that it is because there are still remaining parts.

What students did in the fragment indicates that without knowing the name of the strategy they have applied, the students have applied the first step of reshaping strategy. It was because the cut the figures and then compare two figures by overlapping them each other. They could also argue about the reason why a figure is bigger than the other by pointing the remaining parts.

Surprisingly, there are some students who involved grids to solve the problem. A group did cutting the figures and putting them on a grid paper and another group did making grids on the figures. This was interesting because the use of grids or grid paper was expected to be used when they have learned about tiling in the second problem. The students works with this strategy is described as follows.

Figure 5.31 A student's written work in activity 1-1

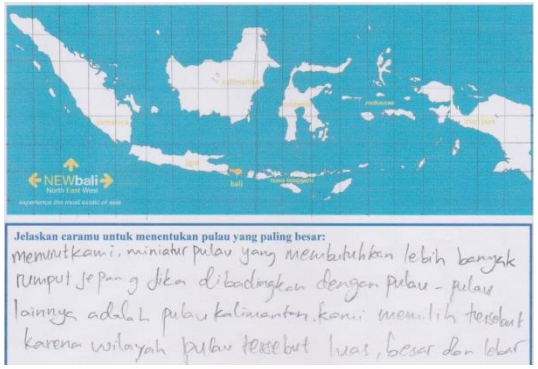


Figure 5.32 A student's written work in activity 1-1

The first figure shows the students work which was making a drawing of the figures of Irian Jaya and Kalimantan. Moreover, they made a rectangle cover each of the figures in order to help them counting the grids inside the figures of the islands. Therefore, we can conclude that they had really used the grids to support their comparing activity.

However, in the second figure, the students did making grids on the map. Based on the observation in the classroom. In the learning activity, they have tried to use the grids to compare the size of the islands. However, they experienced difficulties to count the grids because the islands have irregular shapes. It can be seen that in the written work, they just said that Kalimantan is the biggest island because it is big, large, and wide.

Activity 1-2

The second problem is about tiling the bottom of a swimming pool by using two different size of tiles. They goal is that the students are able to understand units of area measurement, particularly what can be justify from using two different size of area measurement. Moreover, the use of trapezoid and semicircle aims to stimulate the students' understanding about trapezoid and circle.

As it was expected in the HLT, the students can easily continue the tiling activity and determine the number of tiles needed to cover the bottom of the swimming pool. The students counted the complete tiles and combined the incomplete tiles into one tile, then added them up. This strategy was used by all group in this activity. However, some of them found difficulties on determining the incomplete grids, particularly on the semicircle.

Another conjecture which is applying three formula of the area of trapezoid and the area of semicircle was not appear in the activity. It might because the context make the students come up with only the informal way to determine the number of tiles needed. In other words, they did not realize the shape as a mathematical figures which can make them apply the formula of the area of a trapezoid and a circle. These two following figures show an example of students' written work on the problem involving both the big and the small tiles.

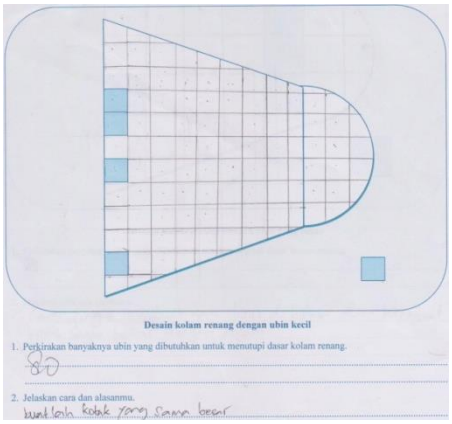


Figure 5.33 A student's written work in activity 1-2

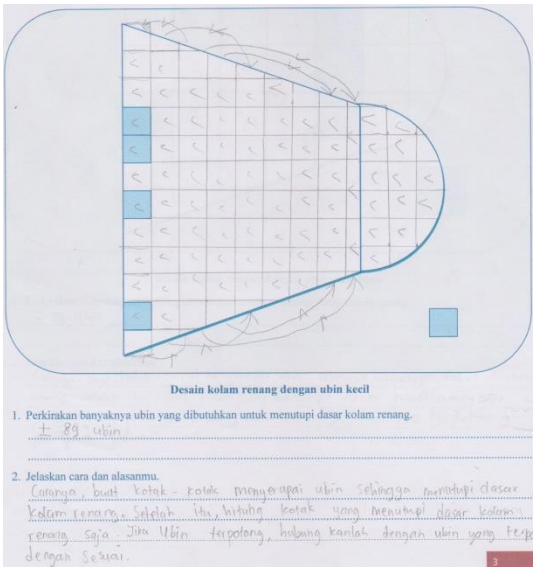


Figure 5.34 A student's written work in activity 1-2

Surprisingly, there is a group of students which did reshaping the trapezoid into a rectangle to simplify their work. It indicates that the students might relate what they did with the first problem which is cutting and pasting strategy could also be applied in this problem. The students' written work can be seen as follows.

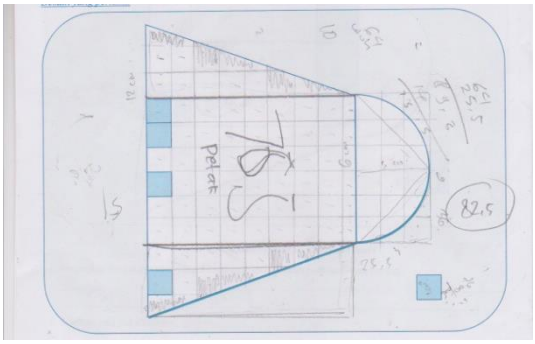


Figure 5.35 A student's written work in activity 1-2

In the classroom discussion at the end of the learning activities, firstly the teacher asked a group to share their work. For the first problem, the discussion was about different strategies which can be applied by the students to determine the miniature of the islands which need more grass The other groups which have different strategies also share and explain the strategy. The conclusion is that they could not compare only by looking at

the shapes. Yet, they can do both reshaping strategy and using grid paper to compare the size of the islands.

Afterwards, the students compare their results of the number of tiles needed to cover the bottom of the swimming pool. However, there was no discussion about the difference between tiling using the big and the small tiles which is related to the bigger the units of area measurement, the less number of the units. It was because the time limitation and the classroom management which not really well done.

Based on the fragments, the students' written works, and class discussion, there were no students who had confusion between area and perimeter in the comparing problem. Various strategy were used by students to compare the islands namely overlapping combining with reshaping and making grids. However, there are still some students who could not give mathematical reasoning. It is because they compared only by looking at the shapes of the islands.

In terms of the second problem. Most student just continued the tiling process and counted the complete tiles and added the incomplete tiles. None of students solved by using the formula of the area of trapezoid and circle. However, some students did cutting and pasting the trapezoid into a rectangle and counted the number of tiles by applying the formula of the area of a rectangle.

Lesson 2: Measuring Figures Using A Grid Paper

Activity 2-1

As aforementioned in the refined HLT, the second lesson is the third lesson in the first cycle. There are two activities in this lesson. The first lesson is about comparing two lakes on a grid paper and the second lesson is about estimating the amount of grass needed to cover a circular garden on a square land by making grids.

In the first activity, most students counted the grids to determine the larger lake. Like the students did in dealing with the incomplete tiles in the previous lesson, they combined the incomplete grids into one grids and

added them with the complete grids. An example of students written work using this strategy can be seen as follows.

A group of students applied different strategy which is reshaping strategy. The written work shows that the students cut the second figures and reshape it into more similar figure with the first one. Afterwards they compared those two figures and determine the larger lake by looking at the remaining parts.

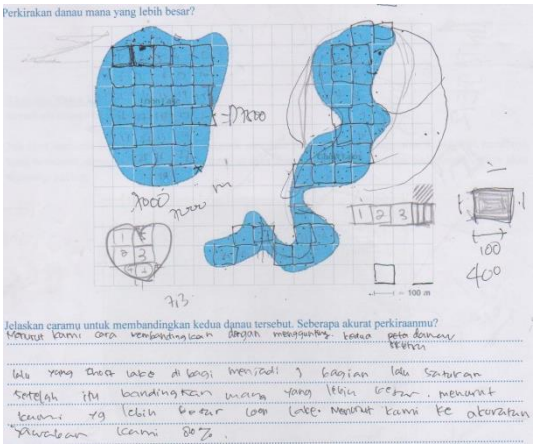


Figure 5.36 A student's written work in activity 2-1

Activity 2-2

In this activity, the students are asked to estimate the amount of grass needed to cover a circular garden on a square land. The expectation is that the students can imitate the strategy they applied in the first activity to solve the problem in this activity. And, as it was expected in the HLT,, most students estimate the grass needed to cover the area of the circular land by counting the complete grids and estimate the incomplete grids. The following fragment shows the activity of a group which applied the strategy.

1 Researcher : What is the length of the object you measured? (the
2 student measure the side of the square by using a
3 ruler)
4 Student 1 : **One point four (1.4)**
5 Researcher : Is that exactly 12 meters? That's in meter. Are you
6 sure? How if we use a ruler? Is that exactly 12?
7 Student 2 : No, so it will be twelve divided by 8
8 Researcher : Yes, so, how many grids are there?

9	Student 3	: **A grid is around 1.5 meter**
10	Researcher	: How many meter square is one grid?
11	Student 2	: 2.25 meter square
12	Researcher	: This is a square right? *(pointing the side of a grid)* So,
13		it's equal, hence it is also 1.5, how about the area?
14	Student 2	: 1.5 times 1.5
15	Student 1	: 2.25
16	Researcher	: Can we determine the area? Which part should we
17		find? The green or the white? Grass or what?
18	Student 3	: Grass which covered the circle
19	Researcher	: So, it would be the green one. Now we have known
20		that one square is 2.25 So how to get the estimation of
21		the area of that shape?
22	Student 1	: 2.25 times 32
23	Student 2	: 72

Transcript 5.6 Students reasoning in activity 2-2

The transcript shows that firstly the students used ruler to measure the length of the sides of the square. In other words, they disregarded the scale applied in the problem. Therefore, the teacher asked the students about the 12 meters as the length of the sides of the square. Hence, they see the grids as a scaled measurement (line 9). Afterwards, they can determine the area of a grid and then because there are 32 complete grids, they determine the area as 72. Moreover, they struggled to deal with the incomplete grids.

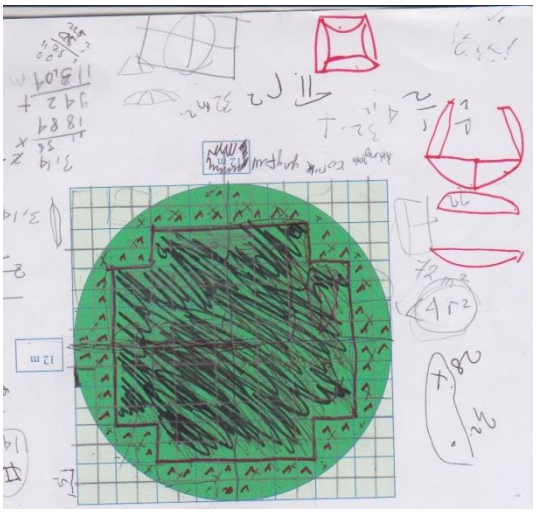


Figure 5.37 A student's written work in activity 2-2

To deal with the remaining parts, the students tried to reshape into a rectangle as shown in Figure 5.36. However, they realized that there is

empty space inside the rectangle while they have to only find the outer part. Moreover, the researcher asked their result when they applied formula to determine the area. Furthermore, the researcher asked the students to prove whether the calculation is true by counting the incomplete grids.

1 Researcher : So, how's the rest?
2 *(the students reshape the remaining part into a rectangle)*
3 Student 3 : It's possible, but the inside part is empty
4 Student 1 : And we just want to find the outer part
5 Researcher : You have calculated using formula right
6 Student 2 : Yes we did
7 Student 3 : The ticked part is 101.25

Transcript 5.7 A discussion in dealing with the incomplete grids

The students' written works show various way to determine the grass needed to cover the circular garden although the idea is the same which is counting the grids inside the circle. Interestingly, all worksheet show that students also applied formula to determine the area inside the circle.

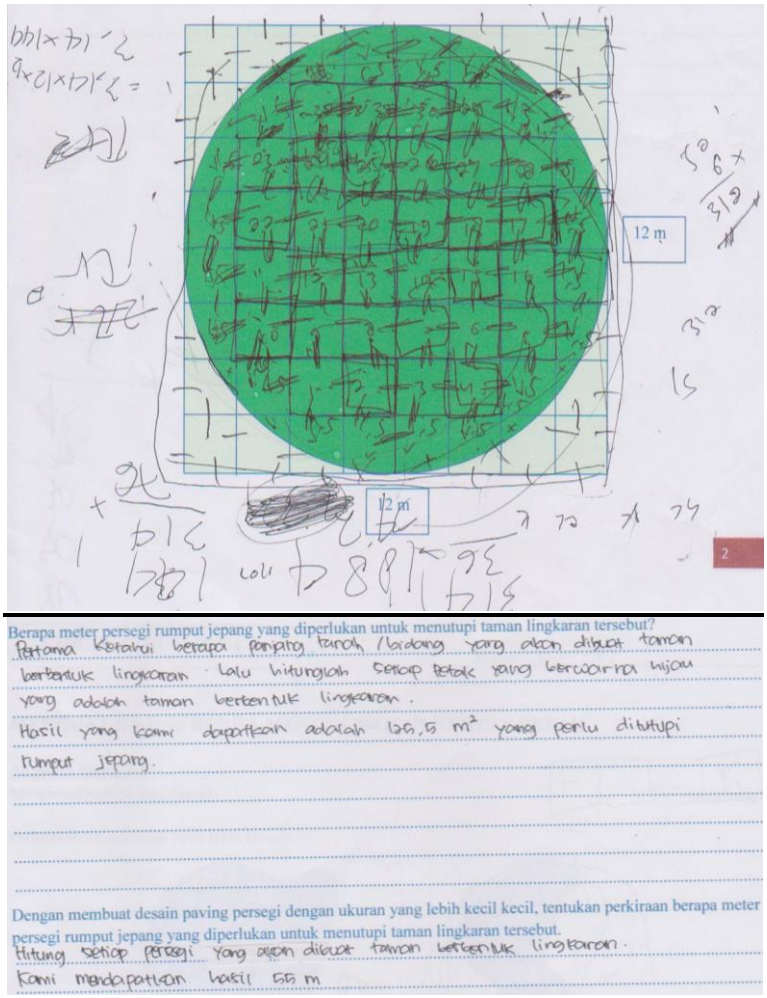


Figure 5.38 A student's written work in activity 2-2

The students put numbers on the complete square, we can see that they put number 1 to 32. Moreover, they group the incomplete grids into two types, the grids which are counted because the think that they are more than a half and the grids which are not counted because they think that they are less than a half. It can be seen from the written work, that the students wrote $3.14 \times 12 \times 12$. However, they seems not so sure about that and prefer to use the answer of counting grids.

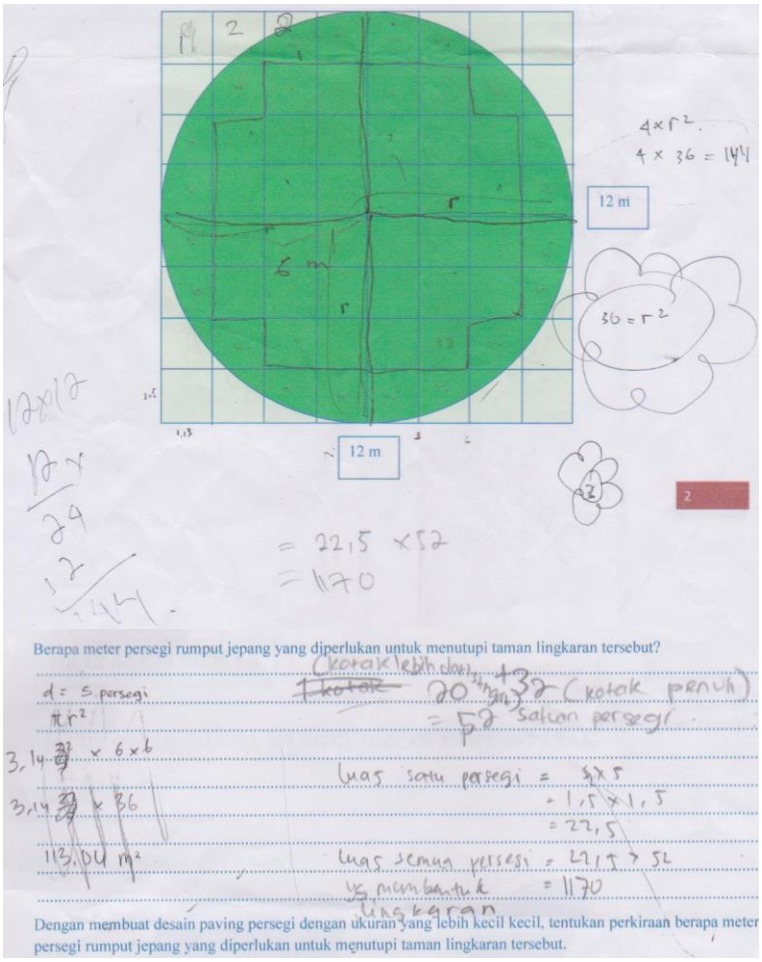


Figure 5.39 A student's written work in activity 2-2

The students did counting the complete grids by making new shape of those complete grids. The students might count the grids by multiplication of area because they did not put any numbers inside the grids. Moreover, they concluded that the complete grids are 32 and the rest is around 20, so that there will be 52 grids. The students determine the area of each grid which is $1.5 \times 1.5 = 22.5$. It can be seen that they did miscalculation which should be 2.25. Therefore, theyfound the area is 1170. In this case, they did not put measurement unit on their calculation.

Overall, the fragments and the students' written work show that the students used the grids to determine the grass needed to cover the circular land. Yet, they find it is difficult to deal with incomple grids. Interestingly, most of them also applied formula of the area of a circle. It indicates that the students seems not really sure about their calculation by using grids.

However, the students could make it into a discussion about the different value which they find when they counted the grids and when they applied formula.

Lesson 3: Reshaping Circle into A Rectangle

The mathematical idea in this activity is that the area of a circle equals the radius times half of its circumference. There is only one main activity in this meeting namely reshaping three circles which are cut into different numbers of sectors into a rectangle and determining the way to find its area. However, to make the students get the idea of reshaping the sectors of a circle into a rectangle, firstly there will be an activity of reshaping a circular garden into a shape resembles parallelogram which will be guided by the teacher. In the second problem, the students were given three circle which are divided into 8, 12, and 16 sectors, respectively. The students were asked to reshape the circle into a parallelogram, then into a rectangle. The students worked in the same group as they did in the previous meetings.

The students were able to reshape the sectors of all three circle in the correct way. However, although the teacher had explained that what they students had to do is arranging the shape at the first time, all students thought that what they have to do is arranging the sectors into a shape shown in the first activity. Hence, the teacher explain clearly that what students have to do is arranging the sectors in the same way as shown in the shape, not into the same shape. The following figures show some students works displayed on posters.

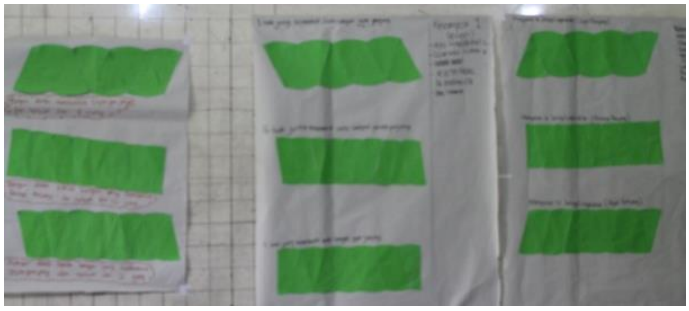


Figure 5.40 Students' work on posters

After reshaping all three circles, all students can see that the shapes the formed resemble the shape of parallelogram. It was contradict with what that has been mentioned on the HLT. However, some students did not get the idea that the more number of sectors formed from the circle, the more resembling the shape with a parallelogram. However, in this part, the teacher did not also explore students' thinking about the reason why the more sectors will form the more resembling parallelogram. The following conversation shows what happened in this part.

As it was expected in the HLT, the students could determine that the area of the rectangle is length times width. In this activity, they can clearly see that the width of the rectangle is the radius of the circle. However, it was difficult for some students to see the relation between the length of the rectangle and parts of the circle. Hence the teacher and the researcher gave them kinds of guidance as shown in the following transcript.

1 Researcher : How many sectors is this circle being cut?
2 Student 1 : Twelve
3 Together : One-two-three-four-five-six
4 *(the researcher is pointing the arc of the sectors of the circle which*
5 *are formed a rectangle)*
6 Researcher : How about this one? Which part of a circle is it?
7 *(the researcher is pointing the initial circle which has not been cut*
8 *into twelve sectors)*
9 Student 2 : **Circumference**
10 Together : One-two-three-four-five-six
11 *(pointing the arcs of six sectors of the initial circle)*
12 Student 2 : A half of its circumference

Transcript 5.8 Students reasoning in activity 3

The teacher asked the students to look at the initial circle and see that the length of all arcs of the twelve sectors is the circumference of the circle (line 6-9). Afterwards, the together with the researcher, the students pointed out the arcs of six sectors and a students realized that it is half of the circumference. This fragment indicates that students could see that the length of the rectangle equals half of the circumference of a circle. However, there were no depth discussion to check whether all students really understand this concept or it was only certain students.

In the end of the lesson, the teacher asked a group to share their work in front of class. There were two important conclusion in this lesson. Firstly, the students could conclude that the more number of sectors formed from the initial circle, the more similar the rectangle they formed. Secondly, the area of the rectangle can be determined by multiply the length and the width. The length is eight times the length of arcs and the width is the radius of the circle. In this case, the student who share their work in front of class did not state the length of the rectangle as half of circumference. Hence, the researcher posed a question to the class discussion as shown on Transcript 5.9.

Figure 5.41 A student share their group work in a class discussion

Researcher : Your friend said that the length of the rectangle is eight times the length of each arc, do you all agree whith his statement?
Students : Yes
(the students answered together, yet some students just mumbling because the other say yes)
Researcher : If you say yes, can you tell us how to determine the length of the arcs?
Student 1 : Mmm, firstly we find the length of an arc, and…
Teacher : How about the other? Who have another idea?
Student 2 : The length of the rectangle is half of its circumference miss, because the length consists of 8 arcs.

Transcript 5.9 A discussion on determining the length of the rectangle

Based on the fragment and the students' written work, we may conclude that some students can determine the area of the rectangle as radius times half of circumference. However, some students still confuse to determine the length of the rectangle and think that it is eight times the length of the

arcs without an idea to find the length of an arc. However, the discussion help them to understand about this concept.

Lesson 4: Reshaping Circles into Other Plane Figures

The mathematical idea in this activity is the concept of area conservation. In this case the area of a circle equals the area of other plane figures which are formed from its sectors. It is also about the relations between the formula of the area of a circle and the area of other plane figures. Moreover, it is also about deriving the formula of the area of a circle using the formula of the area of other plane figures.

However, in the beginning of the lesson, there was an activity of exploring the relation between the circumference and the diameter of a circle. The activity aims at supporting students understanding of phi value which is related to the activity of determining the length of the rectangle in the previous activity. It is important because what they found in the previous activity as the area of the rectangle formed from the sectors of a circle is the radius of a circle times half of its circumference. Therefore, phi value which is the ratio of the circumference and the diameter of a circle give the students clear orientation of the area of the rectangle which is πr^2.

Each student was asked to make a drawing of a circular object they bring from their home and measure the diameter and the circumference. Moreover, they had to determine the ratio of the circumference and the diameter of the drawing. Afterwards, the students put the result of their measuring activity on a list in their group.

The results of their calculation found that the ratio of the circumference and the diameter of a circle is slightly more than 3. An example is shown in Figure 4. In this part, the teacher introduced the ratio as phi value and related the value with the way to determine the length **of** the rectangle which is half of circumference. However, the activity of telling a little bit about the history of phi value which has been planned in the HLT was missing in this lesson.

Buat daftar diameter dan keliling dari setiap yangkaran yang terbentuk dari benda-benda tersebut.

	Nama Benda	Diameter	Keliling	Keliling:Diameter
1.	Botol Sabun	7 cm	23 cm	3,28 cm
2.	Botol Parfum	6,7 cm	20,9 cm	3,12 cm
3.	Gelas	7,7 cm	25,7 cm	3,33 cm
4.	Botol Minum	6,8 cm	21,3 cm	3,31 cm
5.	Botol Sabun	6,8 cm	21,3 cm	3,31 cm

Figure 4.42 A students' written work on the beginning of activity 4

After the students could determine the area of the rectangle as πr^2, the activity continued by giving the students four circular papers which have been divided into 16 sectors. They cut the sectors of the circles and reshaped them into several kinds of plane figures and put them on a poster paper. As it was expected in the HLT, the students could form into four different shapes of plane figures; a rectangle, parallelogram, a triangle, and a trapezoid. However, there were three different kinds of rectangle and two different kinds of parallelogram (Figure 5). A group came up with exactly the same idea mentioned in the HLT as shown on this following figure.

Figure 5.43 A students' written work on activity 4

Using algebraic operation, the students determine the area of the shapes and derive the area of circles. However, in the beginning, they experienced difficulties on relating the part of the plane figures and the part of the circle. The discussion in their group which observed by the researcher help them to solve this problem. This following transcript is an example of

students' discussion in this problem. In this case, they wanted to determine the area of the sectors which was formed into a triangle.

1	Student 1:	The area is length times altitude divided by two.
2		The length is **four radius** and the altitude is one-
3		two-three-… (the student suddenly realized that it
4		is not radius). It **is not radius but sector**.
5	Student 2:	**No, it is arc**. It is four times the arcs.
6	Student 1:	Yes, it is four arcs times four radius divided by
7		two.
8	Researcher:	What is **four arc**?
9	Student 1:	**A quarter of a circle**.
10	Researcher:	What part of a circle?
11	Student 3:	The circumference of a circle.
12	Student 1:	So, length times altitude divided by two is a
13		quarter of the circumference of a circle times four
14		radius divided by two.

Transcript 5.10 Students reasoning in activity 4

The transcript shows that the students have known how to determine the area of a triangle. However, in the beginning, they mentioned the arcs as radius, then as sectors. The discussion help them to understand that it was arcs, not a sector. Moreover, they also found difficulties on determining the length of the four arcs. Therefore the researcher posed a question about what part the four arcs in a circle is. And, in the end, the students realized that the length of four arcs equals a quarter of the circumference of a circle.

As fore-mentioned, the other students also made four different shapes of plane figures. The Figure 6 and Figure 7, respectively show two different kinds of parallelograms and two different kinds of rectangles formed by the students as displayed in their posters.

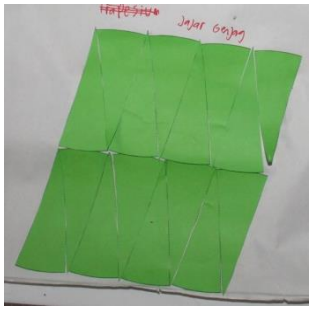
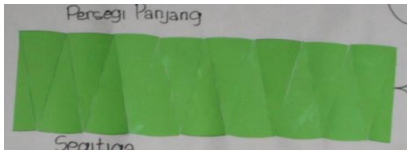



Figure 5.44 Two different kinds of parallelogram



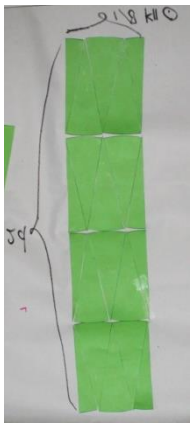
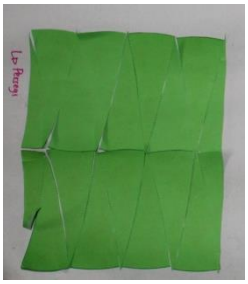



Figure 5.45 Three different kinds of rectangle

Moreover, after the students determine the area of the plane figures, the class discussion emphasized on students' explanation and argumentation about the reason why the area of all plane figures they formed equal πr^2, which means the area of a circle as shown in this following transcript.

1 Researcher : Why do the area of all the plane figures equals πr^2?
2 Students : Because each plane figure was formed from the sectors of a circle.

Transcript 5.11 Students' reasoning on the concept of area conservation

Based on the fragment, students' work on posters, and students' written work, by reshaping the sectors of a circle into several kinds of plane figures and determining the area, the students could see that the area of the new figures they formed is exactly the same with the area of the circle which is πr^2. They could also reason that it is because each figure consist

of the same part with the circle. Moreover, the various shape of parallelogram and rectangle could be a starting point of a discussion to enrich students' understanding of the concept or area conservation.

However, some students still find difficulties about the algebraic calculation in determining the area of the circle. The main reason was because algebraic operation. Yet, the support from the teacher and the researcher in each group and also the discussion in their group help them to overcome this problem.

3. Post Test

The post-test was conducted the day after the fourth lesson. There were 32 students participated in this test. Four problems were given to the students. The first three problems were exactly the same with the problem, but the last problem was different. It was because in this cycle, we want to know more about the students' understanding of area by reshaping strategy.

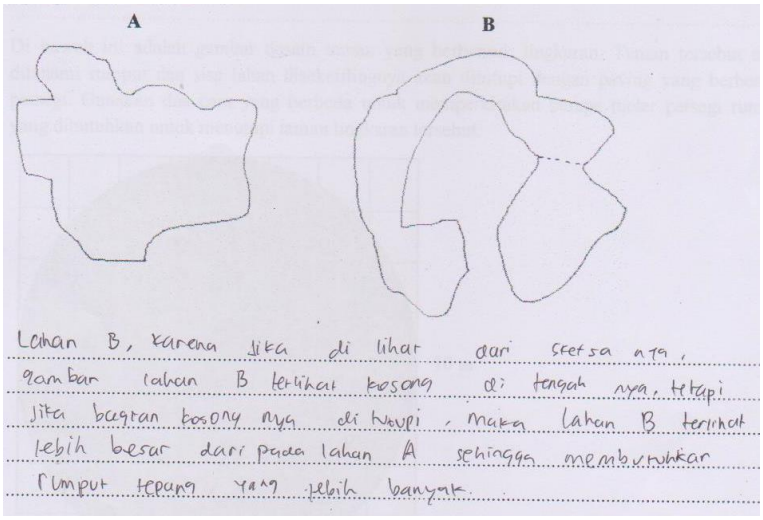
By comparing the result of the pretest and post-test we also want to know the development of students' preliminary knowledge and to know students' understanding about the topic discussed. The students had to solve those problems individually for about 30 minutes. Afterwards, we did an interview with five students from the focus group.

Problem 1: Comparing two shapes without grids inside

In this problem, the students are provided two pieces of lands with irregular shapes. And, they have to determine the land which need more grass to cover. The aim of this problem is to know students' sense about area which is represented by the amount of grass needed to cover the lands. Comparing the amount of grass needed to cover the lands means comparing the area of those two lands.

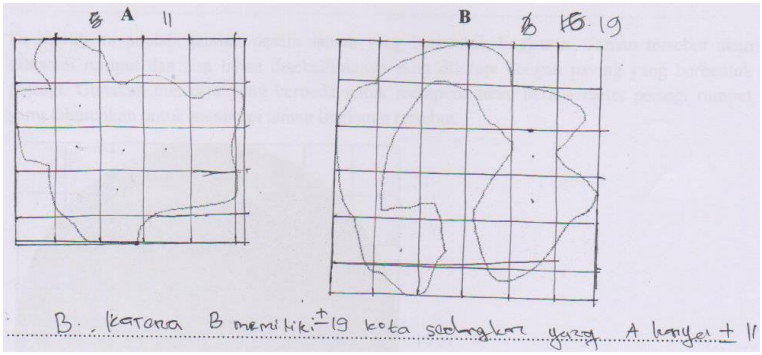
Almost all sudents could give good reasoning to solve this problem. Four students in the focus group came up with the idea of reshaping strategy, like the majority of students applied to solve this problem. A student in the focus group came up with the idea of making grids, like a few students did

to solve the problem. Yet, there were still some students who were not able to give mathematical reasoning or even did not have any ideas to solve the problem. The following figures are the examples of students' written work with the reshaping strategy, making grids, and giving non mathematical reasoning, respectively.



Solving by the idea of reshaping

Figure 5.46 A student's written work on problem 1



Soving by the idea of making grids

Figure 5.47 A student's written work on problem 1

Problem 2: Comparing two shapes with grids inside

The second problem is also about comparing two pieces of lands. However, there are paving blocks inside which are expected to make the students consider the grids as units of area in the process of comparing. The aim of this problem is that the students have a sense of the notion unit

area measurement and can consider the units to compare the size of those shapes.

Most students compared by counting the grids inside and derived the correct answers. Some students used reshaping strategy to compare those two shapes. And, some others only said that B need bigger pointed out that the regularity of those shapes influence the size of the shapes. Wheres, some other looked at the length and the width of the lands to determine the bigger one. Moreover, there are still some students who answer by giving non mathematical reasoning as shown in this following figure.

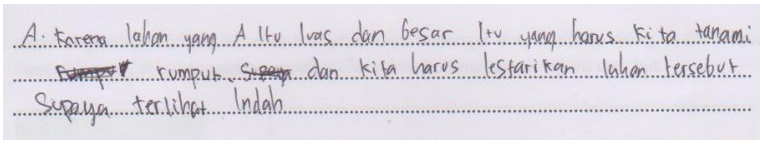


Figure 5.48 A student's written work on problem 2

On the written work, the student said, "A will need more grass because A is large and big. Moreover, we have to keep it looks beautiful". In this case, the student had have a correct understanding that the larger or bigger the land, the more grass needed to cover. However, this student gave other non mathematical reasoning by mentioned that we have to keep it looks beautiful.

Based on the students' written work and the interview with the students, generally, most students have had sense on units of area measurement. It can be seen from the majority of students who did counted the grids to compare those lands. Yet, there were some students who determine the bigger lands based on the shapes of those islands.

Problem 3: Estimating the area of a circle with grids inside

This problem is a problem which is particularly about a circle itself. The students are asked to estimate the grass needed to cover the circular garden on a square land by using two possible ways. Grids are provided inside the square land in order to assist students in estimating the area of the circular garden. As mentioned in the refined HLT, we revised the problem by giving a value for the length of the sides of the square land.

Most students just applied one strategy to solve the problem. They did counted the grids to estimate the grass needed to cover the circular land. It indicates that they did not really have a sense of area related to the problem since they did not apply the formal way to determine the area of a circle. This following figure is an example of students' written works with the strategy.

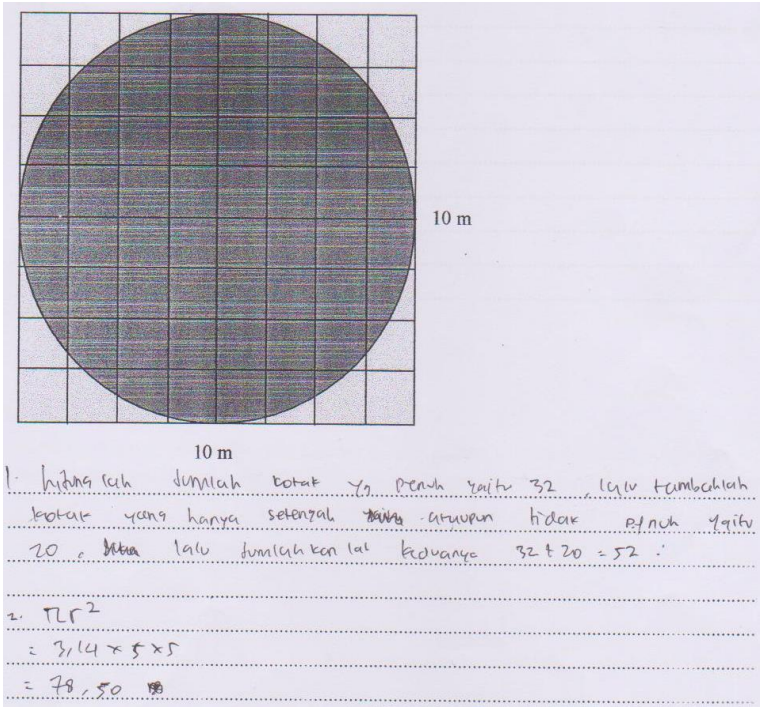


Figure 5.49 A student's written work on problem 3

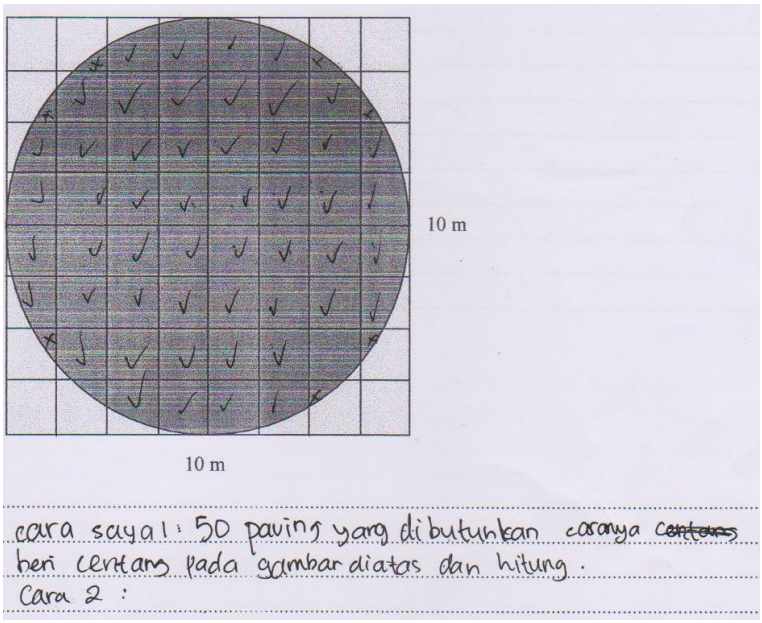


Figure 5.50 A student's written work on problem 3

Meanwhile, a few students applied formulas in solving the problem. The figure above shows an example of students' written work using the strategy. In fact, the students could show the green region inside the circle as the part which they have to determine which is. However, because there was a confusion about the area and perimeter, the student applied the formula to determine the perimeter of a circle.

In general, based on the students' written work and interview with the students, the students had known the part to be measured. Moreover, they could consider the grids as units of area measurement. However, they still had a confusion about the terms of area and perimeter. It can be seen from the activity of applying the formula incorrectly.

Problem 4: Determining the a rectangle formed from the sectors of a circle

The last problem is about determining the perimeter of several plane figure which are a rectangle, a paralellogram, a trapezoid, a triangle, and a circle, respectively. The aim of this problem is that the students can reason how to determine the perimeter of plane figures with straight line sides and make the relation to find the circumference of a circle which is a plane

figure with a curve side. Moreover, this problem give insight about students' understanding of the circumfernce of a circle as which is also the important topic related to the area of a circle.

Most students directly wrote the formulas without giving any explanation about the way to find the perimeter. Therefore, we could not really said that giving correct answers means having understanding. It was only a few students who tried to reason and explain the way to determine the perimeter. The example of students written work can be seen on this following figures.

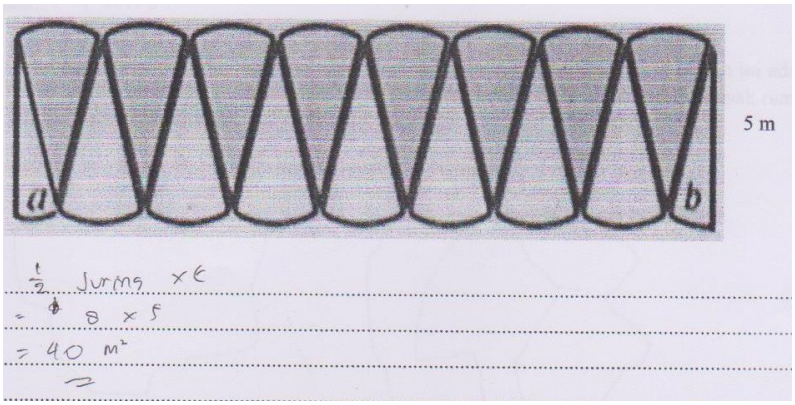


Figure 5.51 A student's written work on problem 4

Generally, the students' written works and the interview give an overview that most students were able to give mathematical reasoning in answering the problem of the post-test. It can be seen that they applied reshaping strategy and making grids as informal ways to compare and measure the area of irregular plane figures and a circle. It was only a few students who only applied the formula of the area of a circle. However, some of them are still difficult to distinguish area and perimeter of a shape.

6. Conclusion for the second cycle

Based on the observation and the students' work we may say that grids can support students understanding of area and area measurement of plane figures, particularly irregular plane figures and also a circle. However, to determine the area of a circle, the students also checked their calculation by using the formal way which is applying formula πr^2.It might because the

formula have been introduced since they were in the 6^{th} grade of elementary school.

Moreover, the reshaping strategy help students to understand the formula of the area of a circle come from and how the formula of the area of a circle related to the formula of the area of other plane figures. The idea is grounded from the concept of area conservation. Therefore, at the end, the students could conclude that the area of the planefigures formed from the sectors equals the area of the circle.

CHAPTER VI
CONCLUSION AND SUGGESTION

6.1. Conclusion

The aims of this study are to support 8th grade students' understanding of the area of a circle and to promote to the development of the local instruction theory on that topic. As mentioned in the first chapter of this report, there is a general research question in this study, which is *"How can we support 8th grade students understanding of the area measurement of a circle?"* To answer the general research question, two sub-research questions are elaborated:

3. *How can grid paper support students' understanding of the area measurement of a circle?*
4. *How can a reshaping strategy support students' understanding of the area measurement of a circle?*

To gain the aims and to answer the research question, we developed and tested a Hypothetical Learning Trajectory (HLT) as described in the previous chapters. And, based on the data analysis, we may answer the research question as follows.

Answer to the first sub-research question

"How can grid paper support students' understanding of the area measurement of a circle?"

Based on the results and the discussion of pre-test, teaching experiment, and post-test, there are two main things which we can conclude. Firstly, by designing realistic problems involving comparing activity, it could stimulate students to apply the strategy of making grids. Moreover, it also bring students to the correct orientation of area. Secondly, the learning sequence which is involving the use of grids as the representation of the units bring the students to gradually understand the concept of the area measurement of a circle which is related to counting or approximating the number of units covering a shape.

The first lesson about ordering the five biggest islands in Indonesia stimulated students to use grid paper and to make grids to compare and order the islands. Moreover, the tiling activity bring students to see the two different sizes of tiles as unit of area measurements and to measure area as the number of units covering a plane figures. The activity of comparing and measuring two lakes on a grid paper also strengthen students understanding on the use of grids in measuring area. Therefore, they could do estimating the area of a circle using grid paper. However, the students were sometimes coming back to the formal way to determine the area of a circle because it is difficult for them to estimate the area of a circle using the grid paper.

Answer to the second sub-research question

"How can reshaping strategy support students' understanding of the area measurement of a circle?"

Based on the results and the discussion of pre-test, teaching experiment, and post-test, there are two main things which we can conclude. Firstly, designing comparing activities involving irregular shapes bring students to apply reshaping strategy which can be a starting point into a discussion of the concept of area conservation. Secondly, the activities designed bring the students to gradually understand the concept of area measurement of a circle especially on how to derive the formula of the area of a circle and how the formula of the area of a circle and others plane figures relate each other.

The activity in the first lesson which is about ordering the five biggest islands in Indonesia could stimulate students to use the strategy of overlapping combined with cutting and pasting. This strategy can be seen as reshaping strategy which related to the concept of area conservation. Moreover, the activity of reshaping the sectors of a circle into a rectangle and determining the area bring the students to understand on how the to derive the formula of the area of a circle. Furthermore, it also makes a clear distinction between the area and the circumference of a circle. It can also be seen from their activity of determining the area of the plane figures which yielded the area of the circle that from the activity, the students could also conclude the

reason why the area of those plane figures equal the area of the circle is because the plane figures were formed from the sectors of the circle.

6.2. Reflection

The implementation of RME on the learning design

As the first tenet of RME, phenomenological exploration, we used concrete contexts as the starting point of the learning process. As a result, the students did not experience any misconception about area and perimeter as some students did in the pretest. The context gives clear orientation in what they have to do. It is in line with what Kordaki and Potari (1998) said that school practices need to teach a more meaningful orientation of the concept of area.

Together with the concrete context, we use models as the second tenet of RME. The models are grid paper and reshaping strategy. Through the learning sequence, by comparing activity and estimating area, the grid paper support students' understanding of the area of plane figures, particularly the area of a circle. Moreover, by reshaping the sectors of a circle into several kinds of plane figures and determining the areas, the students could understand the concept of area conservation and how the formula of plane figures relate each other.

In fact, the models mentioned before were emerged from the students' own construction in the first lesson of this learning sequence. Using students own construction is the third tenet of RME. The students could use any strategy to solve problems in the first lesson. Various strategies were came up by the students. There were mathematical strategies and non mathematical strategies. And, in the classroom discussion, the teacher guide them to draw the mathematical strategies which can be applied to solve the problem which are using grid paper and reshaping strategy.

Interactivity is the fourth tenet of RME. And, in this study, the group discussion and the classroom discussion during the lesson may enable the

interaction among the students and the teacher. This interaction lead the students to avoid misconception and develop their understanding.

Lastly, the contexts were used in this design also intertwined with other field, such as art geography, art, and daily life problem. Moreover, it is also related to other topic in mathematics such as number sense and arithmetic operations. Therefore, the students did not only learn about the topic of area itself.

The contribution to the local instructional theory of the area of a circle

Based on the analysis of several mathematics textbook for junior high school (for example Agus, N. A., 2007; Nuharini, D. and Wahyuni, T., 2008; Nugroho, H. and Meisaroh, L., 2009 Dris, J. and Tasari, 2011; Marsigit, 2011), the contexts provided in those books are only as decoration. It is because the role of contexts is only as a story for the introduction of the shape of a circle and the lesson directly jumps to the formal level. In other words, in this case, contexts do not play essential role and can be dropped without affecting the mathematics required (Oldham et al., 1999).

The Indonesian mathematics textbooks for 8th grade students of secondary school have provided the strategy of cutting and reshaping the sectors of a circle into a rectangle to derive the formula to determine the area of a circle (for example Agus, N. A., 2007; Nuharini, D. and Wahyuni, T., 2008; Nugroho, H. and Meisaroh, L., 2009 Dris, J. and Tasari, 2011; Marsigit, 2011). However, what the students asked to do is just copying the steps told in the books with a ready used circle and a known shape of the rectangle. Therefore, this study contribute less guided activities of reshaping the sectors of a circle. Moreover, to minimize a big jump on students' thinking, the activities of reshaping the sectors of a circle were designed by firstly reshaping the sectors of a circle into a parallelogram, then into a rectangle. In addition, an activity of reshaping the sectors of a circle into other plane figures was designed to strengthen students' understanding of the area measurement of a circle.

In terms of using grid paper in the area measurement of a circle, it is only a few mathematics textbook in Indonesia which provided this strategy (for example Nugroho, H. and Meisaroh, L., ; Marsigit, 2011). However, the grid paper is only used as an attribute to introduce square units. In other words, it is not really used as a tool to estimate the area of a circle. Thus, this study contribute activities which really focus on the use of grid paper to support students' understanding of the area of a circle and as a tool to estimate the area of a circle.

Furthermore, based on the previous study (Zulkardi, 2002), most of textbook that used in Indonesia contain mainly the set of rules and algorithm which is already formal and lack of applications which are needed by students in order to make the concept be real for them. The learning process only emphasized the procedures to solve the problems and there is insufficient attention for students' reasoning in order to develop their understanding.

Therefore, in this study, we proposed a learning trajectory and the designed activities which emphasized on students' understanding of the concept of the area of a circle. Together with the contextual problems, the grid paper and reshaping strategy, which used as the models in this learning, helped the students to gradually develop their understanding of the area of a circle. Moreover, the teacher guide, which was provided in this learning design, may help teachers to implement the HLT in their class. And, because of the contextual problems which used in this design were quite universal and familiar for students, we may say that this design could be implemented in other school in Indonesia.

6.3. Suggestions

Suggestion for the teacher

Applying a new social norm in a classroom is not easy. It takes time to introduce and make both students and teachers accustom to a new learning atmosphere. We can see from the teaching experiment which is explained in the previous chapter that some students tend to silent and did not actively participated in the classroom discussion, even when they have different answer or ideas in solving a problem. The main reason was because they felt shy and afraid of incorrect answer. Therefore, teachers should give more support and encourage their students not to shy and afraid of making mistake in order to change this norm.

Furthermore, most teachers tend not to be quite patient to wait for their students' ideas. Hence, they explain the answer or the strategies to solve problems directly to the students. It might also become one of the factors which make students become passive. They tend to wait for their teacher explanation and copy the procedures. Therefore, instead of telling the students the right answer or the right strategies, it is better to pose questions which can lead the students to gain an idea to solve problems. In this case, the class discussion might becomes more alive.

In terms of socio-mathematical norms, the implementation of this norms in a classroom is also not easy. A teacher should emphasize the normative understanding about what counts as a mathematical difference, a mathematical sophistication, and an acceptable mathematical explanation and justification in the classroom. According to Yackel & Cobb (1996), the teacher plays a crucial role in establishing both the mathematical quality of the classroom environment and norms for mathematical aspects of students' activity.

Based on the aforementioned statements, we may say that teachers have an important role to create a good learning situation in a classroom. Hence, it is important to always look back on themselves, upgrade their knowledge, and open minded into a change to improve the learning situation in their classroom. As a result, the implementation of RME approach will be more

effective when the teacher is able to lead the classroom situation by considering socio norms and socio-mathematical norms.

Recommendations for the future study

The grid paper and reshaping strategy may support students understanding of the area of a circle. However, the students are sometimes coming back to the formal way in determining the area of a circle. Therefore, this study can be done in the very beginning students learn about the area of a circle which is in the sixth grade of elementary school. Yet, we should consider the level of difficulties on the problems designed for the elementary school students.

Furthermore, through the learning activities designed in this study, the grid paper and reshaping strategy were also support students understanding of the area of irregular plane figures. Thus, a future study could also apply these strategies the topic to other plane figures such as parallelogram, rectangle, triangle, and else.

Moreover, the contexts which used in the problems on each activity were familiar for the students, especially garden and tiling context. However, it is necessary to expand the variation of the context so that the students do not limit the application of the area of a circle just in certain context. Thus, the other suggestion for the next study is give some more contexts as the enrichment material to the students.

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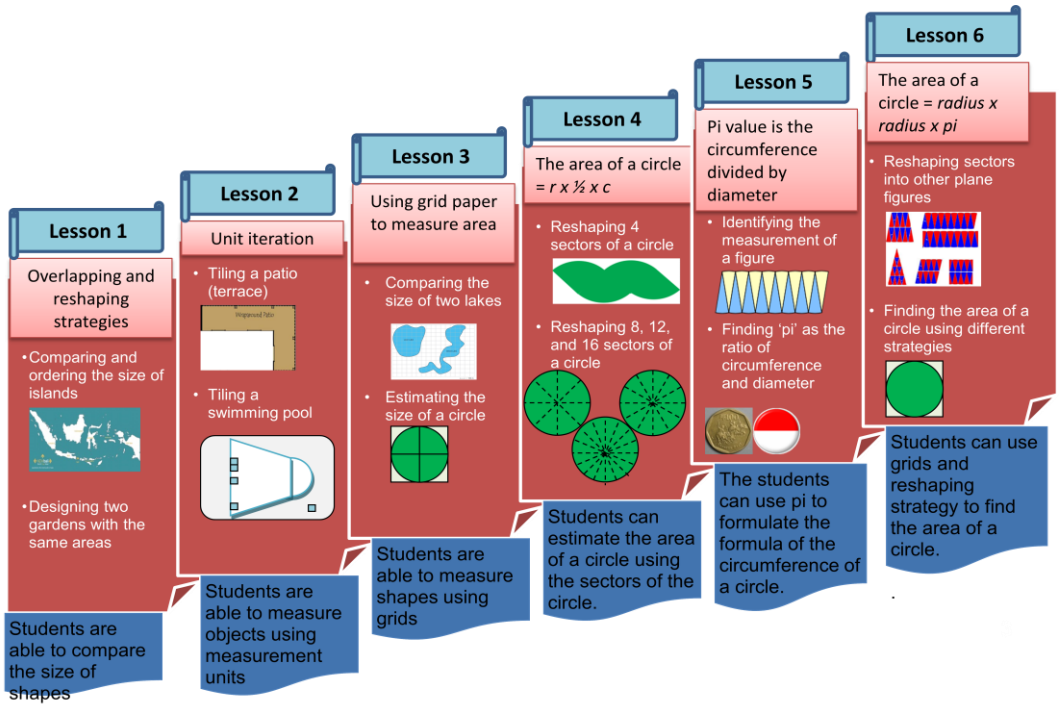
THE TIMELINE OF THE STUDY

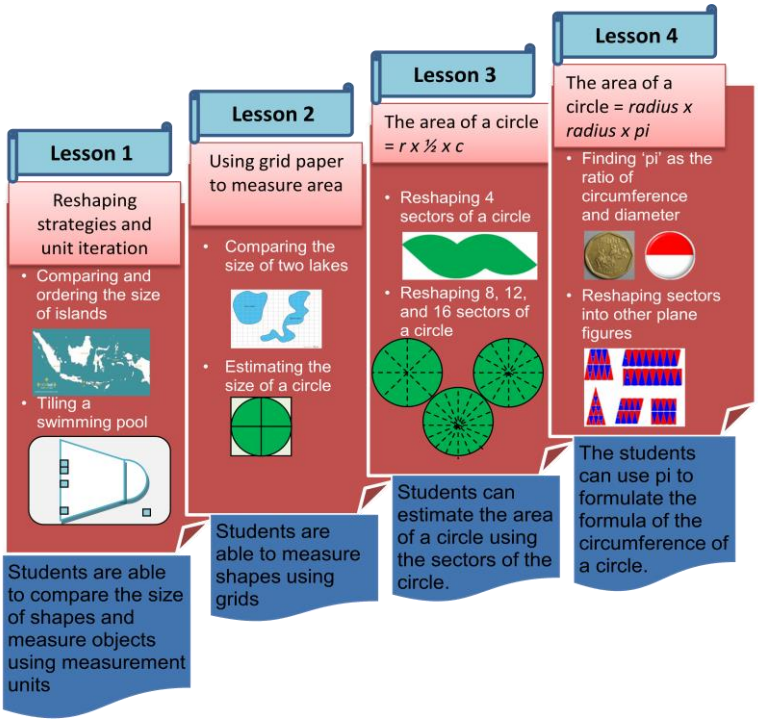
	Date	Description
Preparing for the Experiment		
Studying literature and designing the initial HLT	July 2013 – January 2014	Looking at the existing studies and literature about the topic of the area measurement of a circle and elaborating the conjectures and the theoretical intent of the experiment about the topic.
Discussing with the teacher	February 2014	Communicating the designed HLT.
Preliminary teaching experiment (The First Cycle)		
Observing the teaching and learning process in grade 8	February 2014	Investigating the social between the teacher and the students.
Testing the initial HLT	February 2014	Trying out the initial HLT
Refining the initial HLT	February 2014	Refining the initial HLT based on the preliminary teaching experiment and will be applied in the teaching experiment.
Teaching experiment (The Second Cycle)		
Pre-test	February 2014	Investigating the students' preliminary knowledge.
Lesson 1	April 2014	Comparing and Tiling
Lesson 2	April 2014	Measuring Figures Using A Grid Paper
Lesson 3	April 2014	Reshaping Circle into A Rectangle
Lesson 4	April 2014	Reshaping Circles into Other Plane Figures
Post-test	April 2014	Describing the development of students' understanding of the area measurement of a circle.
Retrospective Analysis		
Data Analysis	April – June 2014	Analyzing the data collected from the experiment.

OVERVIEW OF THE ACTIVITIES

The initial Activities

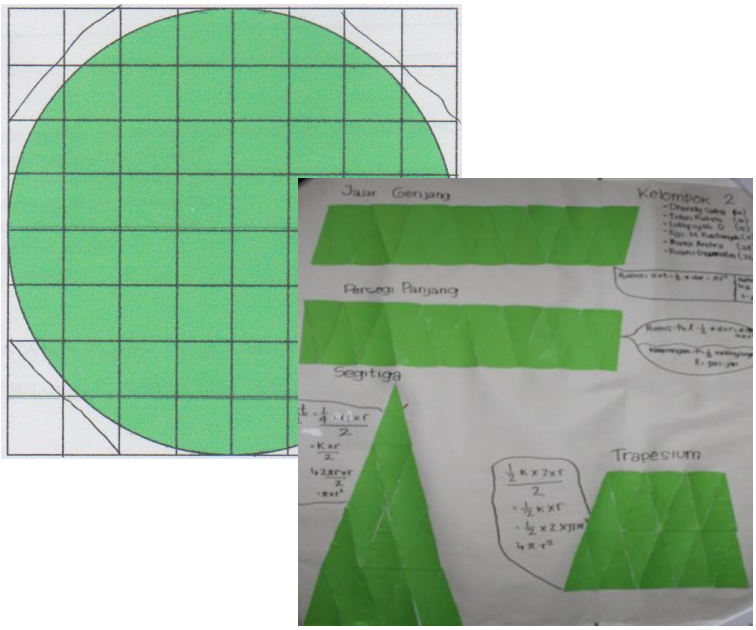


The refined Activities



THESIS BOOKLET

SUPPORTING 8TH GRADE STUDENTS' UNDERSTANDING OF THE AREA MEASUREMENT OF A CIRCLE



By:

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INTERNATIONAL MASTER PROGRAM ON MATHEMATICS EDUCATION

FACULTY OF TEACHER TRAINING AND EDUCATION

SRIWIJAYA UNIVERSITY

2014

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The Scheme of Teacher's Interview

1. **Teacher's experience**
   - Do you have an experience with teaching area and perimeter in secondary school? If you don't have it yet, how do you think you will teach the topic?
   - Do you have an experience with teaching area and circumference of a circle in 8^{th} grade of secondary school? If you don't have it yet, how do you think you will teach the topic?
2. **Teaching process**
   - What kinds of mathematics textbooks do you usually use to teach mathematics?
   - Besides mathematics textbooks, do you also use other sources to teach mathematics? If yes, what kind of sources do you use?
   - What do you think may be the difficulties in teaching this topic?
   - What do you think may be the difficulties for students in learning this topic?
   - Do you usually make a lesson plan for your teaching?
   - What kind of methods do you usually apply in your teaching?
   - Do you usually guide students to work in groups and to be involved in a small group discussion and a whole class discussion? If yes, how do you guide them?
   - How do you usually reply to correct or incorrect ideas students have?
   - Do you ever use realistic contexts to introduce a topic?
3. **Students' profile**
   - How old are the students in the 8^{th} grade?
   - What do students already know about area and perimeter?
   - What do students already know about area and circumference of a circle?
   - What is the level of students in the 8^{th} grade, particularly in mathematics?
   - Are there any criteria in grouping students in the mathematics classroom?

- Are students used to working in a small group in the mathematics classroom?
- Are students used to being involved in a discussion in their learning?
- Are students used to arguing and expressing their thinking?

4. PMRI

- What you know about PMRI?
- How long have you been teaching using the PMRI approach?
- Do you have any difficulties in teaching using this approach?
- What do you think about teaching mathematics, using the PMRI approach?

The Scheme for Classroom Observation

1. **Teacher's role**
   - How's the time management of the teacher?
   - How does the teacher response to an incorrect answer of the students?
   - How does the teacher response to a correct answer of the students?
   - Does the teacher discuss with the students about elegant, efficient, and sophisticated solutions?
   - How does the teacher deal with the different answers from the students?
   - Does the teacher use students' ideas to draw the conclusion?
   - Does the teacher give time to students individually to think?
   - Does the teacher encourage students to share their ideas in a small group discussion?
   - How does the teacher facilitate a small group discussion and a whole class discussion? Is she moving around or does she only focus on the groups which experience difficulties?
2. **Teaching and Learning Process**
   - How does the teacher open the lesson?
   - What kind of methods are used by the teacher? Does the teacher use realistic contexts in the teaching?
   - Does the teacher always follow the textbooks during the lesson?
   - Does the teacher use students' worksheets in the teaching and learning activities?
   - Does the teaching and learning invite students to work in a small group and invite students to be involved in a whole class discussion?
   - How do students work in a small group? Do they actively share their ideas? Or is it only dominated by a few students?
   - How do the students participate in the class discussion?
   - Are there any students who don't want to contribute in the lesson?
   - Are there any students who are noisy during the classroom activities?

- How does the teacher end the lesson?

3. **Students' activities**
    - How do students communicate with each other?
    - How do students contribute to the class discussion?
    - How do students explain their strategy?
    - How do students challenge other thinking and justify their own interpretation? Do different strategies come up in students' understanding?

Teacher Guide

Pertemuan Pertama

Aktivitas:

Mengurutkan Benda Berdasarkan Luasnya Dan Mengukur Luas Menggunakan Satuan Luas Yang Berbeda

Alokasi Waktu:

2 x 40 menit

Materi:

- Lembar Kerja Siswa 1
- Kertas
- Gunting
- Peta Indonesia
- Penggaris

Tujuan Pembelajaran:

Tujuan utama

Siswa dapat memahami luas daerah dari suatu benda dan dapat mengukur luas permukaan benda menggunakan satuan luas dalam kehidupan sehari-hari.

Deskripsi tujuan pembelajaran

- Siswa dapat mengurutkan lima pulau terbesar di Indonesia berdasarkan luasnya.
- Siswa dapat menjelaskan cara yang mereka gunakan untuk membandingkan dan mengurutkan benda datar.
- Siswa dapat mengukur luas benda dengan bentuk yang tidak sederhana menggunakan satuan luas dengan ukuran yang berbeda.

– Siswa dapat menjelaskan strategi mereka untuk menentukan banyaknya ubin yang diperlukan untuk menutupi dasar kolam renang yang berbentuk gabungan trapesium sama kaki dan setengah lingkaran.

Gambaran aktivitas:

***Pembukaan* (*15 menit*)**

Guru mengecek pemahaman siswa tentang istilah *"lebih besar dan lebih kecil"* dengan meminta siswa untuk menjelaskan maksud istilah tersebut dengan kalimat mereka sendiri. Akan tetapi, sebelum siswa menyampaikan pemahaman mereka, guru sebaiknya memberi kesempatan kepada siswa untuk berpikir terlebih dahulu. Selanjutnya, guru dapat membimbing siswa untuk mendiskusikan jawaban mereka. Dalam diskusi tersebut, guru dapat bertanya kepada beberapa siswa untuk memberikan pendapat tentang jawaban daris siswa lain. Contohnya, guru dapat bertanya, *'Menurut kamu bagaimana jawaban dari temanmu? Apa kamu setuju? Apa persamaan dan/atau perbedaan dari pengertian yang disampaikan oleh temanmu dan pengertianmu?'*. Dalam diskusi singkat ini, guru memberikan kesempatan siswa untuk berpikir tentang apa yang dimaksud dengan lebih kecil dan lebih besar tanpa membuat kesimpulan yang formal. Setelah diskusi singkat, guru mengelompokkan siswa menjadi kelompok-kelompok yang terdiri dari 5-6 siswa. Kemudian, pada setiap kelompok diberikan LKS yang terdiri dari dua aktivitas.

***Kegiatan inti* (*55 menit*)**

1. Mengurutkan lima pulau terbesar di Indonesia berdasarkan besarnya (20 menit)

   Aktivitas ini merupakan aktivitas membandingkan sekaligus mengurutkan lima pulau terbesar di Indonesia. Siswa akan bekerja dalam kelompok. Guru memberikan peta Indonesia dengan ukuran yang cukup besar dan tugas siswa adalah menentukan manakah dari pulau-pulau tersebut yang akan membutuhkan lebih banyak rumput. Permasalahan dan konjektur pemikiran siswa digambarkan sebagai berikut.

Kepulauan Indonesia

Sebuah sekolah akan membuat taman yang dilengkapi dengan miniatur lima pulau terbesar di Indonesia yaitu Sumatera, Jawa, Kalimantan, Sulawesi, dan Irian Jaya. Pertama-tama setiap tanah yang membentuk pulau ini akan ditanami rumput jepang. Miniatur pulau manakah yang akan membutuhkan lebih banyak rumput jepang jika dibandingkan dengan pulau-pulau yang lainnya?

Urutkan pulau-pulau tersebut dari yang memerlukan rumput paling banyak ke pulau yang memerlukan rumput paling sedikit. Jelaskan caramu mengurutkan dan alasannya.

Nomor	Pulau

Konjektur pemikiran siswa dalam kongres matematika dijabarkan dalam tabel berikut ini.

Konjektur Pemikiran Siswa	Respon Guru
Siswa akan mengurutkan hanya dengan mempertimbangkan bentuk dari pulau-pulau yang artinya semakin lebar permukaannya maka semakin besar pulau tersebut.	Guru dapat meminta siswa untuk memperhatikan bentuk pulau Sulawesi. *"Pulau Sulawesi berbentuk kecil memanjang, tapi apakah ia merupakan pulau yang terkecil? Bagaimana jika kita melihatnya sebagai satu kesatuan?"*
Siswa akan menggunakan penggaris untuk menghitung luas setiap pulau dan mengurutkannya berdasarkan hasil perhitungan yang mereka peroleh.	Guru dapat meminta siswa untuk menunjukkan unsur-unsur pada pulau tersebut misalnya, *"Coba tolong tunjukkan yang manakah yang merupakan panjang dan lebarnya?"* Kemudian guru dapat menunjukkan bahwa ketika kita mengukur panjang dan lebar dari benda tersebut dari bagian yang lain mereka akan menemukan hasil perhitungan yang berbeda.
Siswa akan menggunting gambar pulau dan menumpuk gambar satu dan yang lainnya. Selanjutnya, mereka akan memotong dan menempel gambar pada bagian sisa yang tidak tertutupi.	Guru dapat mendorong siswa untuk memberikan alasan bagaimana kita bisa mengatakan suatu benda lebih besar dari benda yang lain.

2. Memasang ubin untuk menutupi dasar kolam renang (15 menit)

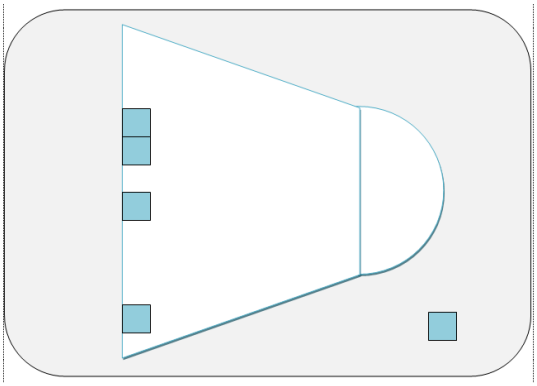
   Bentuk daerah yang akan dipasangi ubin merupakan gabungan dari bentuk bangun trapesium dan setengah lingkaran. Kemudian, terdapat beberapa ubin

yang sudah terpasang di dasar kolam renang tersebut. Tujuan dari dipergunakannya setengah lingkaran adalah agar siswa dapat mengingat pengetahuan mereka tentang bentuk lingkaran. Kemudian, siswa dapat membuat petak-petak berdasarkan ubin yang telah disediakan daripada membuat ubin satu-persatu melalui proses pengulangan. Akan tetapi, guru tidak boleh meminta siswa untuk membuat kotak-kotak jika itu tidak muncul dari ide mereka sendiri. Permasalahan dan konjektur pemikiran siswa dijabarkan sebagai berikut.

Menutupi dasar kolam renang dengan ubin

Pak heri juga akan membuat kolam renang di rumahnya. Dia sudah membuat desain dari kolam renang tersebut seperti pada gambar di bawah ini. Lengkapi pemasangan ubin pada desain di bawah ini dan buat estimasi banyaknya ubin yang dibutuhkan untuk menutupi dasar kolam renang pada desain yang pertama dan desain yang kedua.

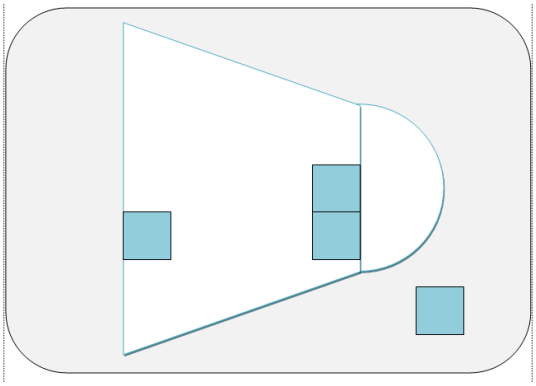
Desain pertama



Desain kolam renang dengan ubin kecil

3. *Perkirakan banyaknya ubin yang dibutuhkan untuk menutupi dasar kolam renang.*
4. *Jelaskan cara dan alasanmu.*

Desain kedua



Desain kolam renang dengan ubin besar

Konjektur pemikiran siswa dalam kongres matematika dijabarkan dalam tabel berikut ini.

Konjektur pemikiran siswa	Respon guru
Siswa akan meneruskan untuk memenuhi dasar kolam renang dengan cara menggambar ubin sati per satu.	Guru dapat bertanya kepada siswa tentang cara lain yang dapat digunakan untuk melanjutkan proses pengubinan selain dengan menggambarnya satu per satu.
Siswa akan membuat petak-petak berdasarkan ubin-ubin yang telah diberikan pada gambar.	Guru dapat meminta siswa untuk berpikir tentang persamaan dan perbedaan antara memasang ubin dengan menggambarnya satu persatu dan membuat petak-petak.

3. Kongres Matematika: mendiskusikan hasil pekerjaan kelompok dalam diskusi kelas (20 menit)

   Dalam diskusi, tidak mungkin meminta setiap kelompok untuk mempresentasikan jawabannya. Oleh karena itu, minta salah satu kelompok secara sukarela untuk mempresentasikan jawabannya. Kelompok yang tidak

mempresentasikan jawabannya dapat memberikan komentar dan pendapat berkaitan dengan pekerjaan kelompok yang tampil di depan kelas.

Konjektur pemikiran siswa dalam kongres matematika dijabarkan dalam tabel berikut ini.

Konjektur Pemikiran Siswa	**Respon Guru**
Siswa akan mendeskripsikan jawaban dan pemikiran mereka tentang penggunaan strategi menumpuk untuk membandingkan dua buah benda.	Guru membimbing siswa untuk memahami bahwa dua buah benda dikatakan memiliki ukuran yang sama jika keduanya saling menutup tanpa sisa.
Siswa juga dapat untuk mengkombinasikan penggunaan strategi overlapping dan cutting and pasting untuk membandingkan dua benda yang berbeda.	Guru dapat membimbing siswa untuk mendiskusikan cara-cara yang berbeda dan membandingkan keefektifan cara-cara tersebut.
Siswa akan mendiskusikan tentang berbagai macam strategi untuk menentukan banyaknya ubin kecil dan ubin besar yang dibutuhkan untuk menutupi lantai teras dan dasar kolam renang.	Guru dapat membimbing siswa untuk berdiskusi tentang kelebihan dan kelamahan dari cara yang mereka gunakan.
Siswa akan mendiskusikan tentang perbedaan banyaknya ubin yang dibutuhkan untuk menutupi lantai teras dan dasar kolam renang ketika mereka menggunakan ubin kecil dan ubin besar.	Guru dapat membimbing siswa untuk berdiskusi tentang banyaknya ubin yang dibutuhkan untuk menutupi lantai dan dasar kolam renang merepresentasikan luas kolam renang dengan ubin sebagai satuan luasnya

***Kegiatan Akhir* (*10 minutes*)**

Setelah diskusi, dengan menggunakan pertanyaan yang terdapat pada bagian refleksi, kita dapat mengecek pemahaman siswa dari keseluruhan pembelajaran pada pertemuan ini. Di sini dapat dilakukan dengan memberikan beberapa pertanyaan sebagai berikut.

1. *Strategi apa saja yang dapat digunakan untuk membandingkan besarnya suatu benda?*
2. *Bagaimana bisa dikatakan suatu benda lebih besar dibandingkan benda yang lain?*
3. *Seberapa akurat perkiraanmu terhadap banyaknya ubin yang diperlukan untuk dasar kolam renang?*

Guru dapat meminta beberapa siswa untuk menjawab pertanyaan-pertanyaan tersebut dan memberikan kesempatan kepada siswa lain untuk mengulangi dan memberikan jastifikasi atas jawaban temannya.

Peran Guru:

Selama siswa ekerja di dalam kelompok

- Guru membimbing siswa untuk berdiskusi tentang cara membandingkan besarnya dua pulau terlebih dahulu. Misalnya dapat diawali dengan menentukan pulau yang menurut mereka terbesar atau terkecil, dilanjutkan dengan membandingkan dengan pulau yang menurut mereka lebih besar atau lebih kecil dari pulau yang menurut mereka terbesar atau terkecil. Beberapa pertanyaan yang mungkin dapat diajukan oleh guru misalnya.

  Menurut perkiraan awalmu, bagaimana urutan pulau-pulau ini dari yang terbesar sampai yang terkecil?

  Bagaimana caramu membuktikan bahwa suatu pulau lebih besar daripada pulau yang lain?

  Guru juga harus mengingat bahwa kemungkinan siswa akan menghubungkan pertanyaan ini dengan pelajaran geografi dan menjawabnya sesuai dengan pengetahuan yang mereka dapat di pelajaran tersebut. Maka, untuk mengantisipasi, guru dapat memberikan pertanyaan.

Apakah benar urutan yang kita pelajari di pelajaran geografi? Mari kita berpikir bagaimana cara untuk membuktikannya.

Guru juga berperan penting untuk membimbing siswa yang mengalami kesulitan dalam aktivitas ini, misalnya dengan meminta siswa untuk memaksimalkan alat pembelajaran secara maksimal (peta dan gunting).

- Pada permasalahan kedua, guru dapat melihat teknik siswa dalam pengubinan, yaitu apakah masih menggunakan prinsip iterasi atau sudah membuat petak-petak berdasarkan kotak-kotak tang sudah diberikan pada gambar. Pada bagian ini, guru dapat mengajukan pertanyaan-pertanyaan sebagai pancingan, misalnya:

  Selain dengan menggambar ubin satu per satu, apakah ada cara lain untuk memperkirakan banyaknya ubin yang diperlukan?

  Bagaimana caramu memasang ubin pada bagian lantai yang berbentuk setengah lingkaran?

Selama diskusi kelas

- Guru meminta satu kelompok untuk mempresentasikan hasil kerja kelompok di depan kelas. Fokus diskusi ini adalah strategi dan penalaran siswa tentang bagaimana cara membandingkan dan mengurutkan benda datar. Guru harus menciptakan situasi yang membuat siswa-siswa kelompok lain menanggapi jawaban kelompok yang mempresentasikan jawaban di depan kelas. Selanjutnya guru dapat memberikan pertanyaan yang bertujuan memunculkan konsep luas pada diri siswa, misalnya.

  Apa yang kita bandingkan dari pulau-pulau tersebut?

  Bagian manakah yang disebut luas?

  Bagian manakah yang disebut keliling?

- Guru dapat memberikan pertanyaan yang bertujuan memunculkan konsep satuan pengukuran luas pada diri siswa, misalnya.

  Jika ubin yang digunakan semakin kecil, bagaimana dengan banyaknya ubin yang diperlukan untuk menutupi suatu daerah?

  Jika sisi ubin tersebut berukuran 1 m, berapa luas satu ubin, dan berapa luas lantai teras serta dasar kolam renang?

Pertemuan Kedua

Aktivitas:

Mengukur Luas Dengan Kertas Berpetak

Alokasi Waktu:

2 x 40 menit

Materi:

- Lembar Kerja Siswa 2
- Kertas
- Penggaris

Tujuan Pembelajaran:

Tujuan Utama

Siswa dapat mengukur luas suatu daerah dengan menggunakan kertas berpetak dan dengan membuat petak-petak persegi sendiri.

Deskripsi tujuan pembelajaran

- Siswa dapat membandingkan luas dua buah danau dengan menggunakan kertas petak.
- Siswa dapat memperkirakan luas suatu daerah dengan menggunakan kertas petak dan dengan membuat petak-petak persegi sendiri.
- Siswa dapat menjelaskan cara dan penalarannya untuk memperkirakan banyaknya rumput yang diperlukan untuk taman yang berbentuk lingkaran.

Gambaran Aktivitas:

***Kegiatan Awal* (*10 menit*)**

Guru dapat mengingatkan siswa tentang aktivitas yang sudah mereka lakukan pada pertemuan sebelumnya dan mendorong mereka untuk dapat berpendapat tentang hubungan dari cara yang digunakan untuk memperkirakan banyaknya ubin yang dibutuhkan pada aktivitas pertaman dan kedua. Guru juga dapat

bertanya kepada siswa, '*Apakah ada cara lain yang dapat digunakan untuk menemukan banyaknya ubin yang dibutuhkan selain dengan menggambar ubin satu per satu? Strategi apakah itu?'* setelah diskusi singkat, guru meminta siswa untuk duduk dalam kelompok seperti pada pertemuan sebelumnya. kemudian guru memberikan LKS kepada setiap kelompok dan memastikan bahwa setiap siswa akan berkontribusi dalam kerja kelompok tersebut.

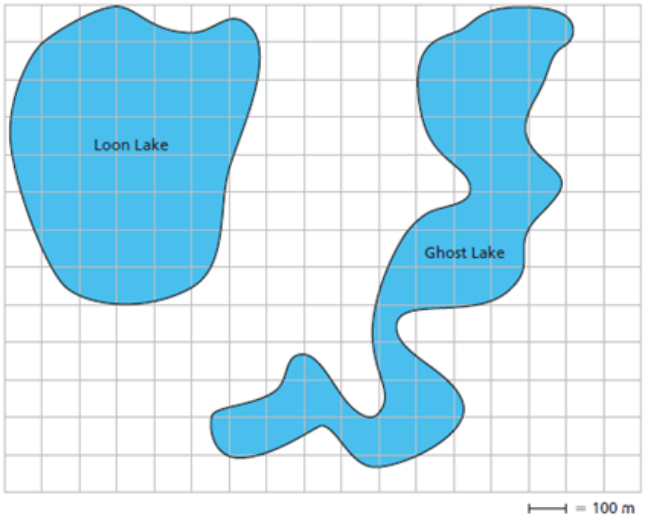
***Kegiatan Inti* (*60 menit*)**

3. Membandingkan dua danau pada kertas berpetak (15 menit)

   Aktivitas pertama adalah tentang membandingkan dua buah danau yang digambarkan dengan skala pada kertas berpetak. Ide pada permasalahan ini adalah siswa akan memahami prinsip iterasi sebagai penyusunan sejumlah satuan luas untuk menutupi suatu obyek yang aka diukur luasnya. Pertama-tama, guru meminta siswa untuk memikirkan masalah ini secara individu, kemudian mendiskusikan ide untuk memecahkan masalah di dalam kelompok. Berikut ini adalah masalah pada aktivitas pertama.

   Membandingkan dua danau

   Perkirakan danau mana yang lebih besar?



   Terdapat tiga pertanyaan yang harus dijawab oleh siswa pada permasalahan ini.

   4. *Jelaskan caramu untuk membandingkan kedua danau tersebut. Seberapa akurat perkiraanmu?*

5. *Berapa luas Danau Loon dan berapa luas Danau Ghost?*
6. *Seberapa akurat perkiraanmu?*

Pada tabel berikut ini, kami memaparkan konjektur pemikiran siswa.

Kojektur Pemikiran Siswa	**Respon Guru**
Siswa akan mengabaikan petak-petak dan membentuk ulang danau menjadi dua bentuk yang lebih mirip dan beraturan.	Guru bertanya kepada siswa tentang cara untuk membuktikan bahwa salah satu danau lebih besar daripada danau yang lain dan meminta siswa mempertimbangkan petak-petak di dalam batas-batas keliling danau.
Siswa akan menghitung petak yang penuh dan memperkirakan banyaknya petak yang tidak penuh.	Guru mendorong siswa untuk melihat hubungan antara iterasi satuan dalam proses memasang ubin dan kertas berpetak dengan pengukuran luas.

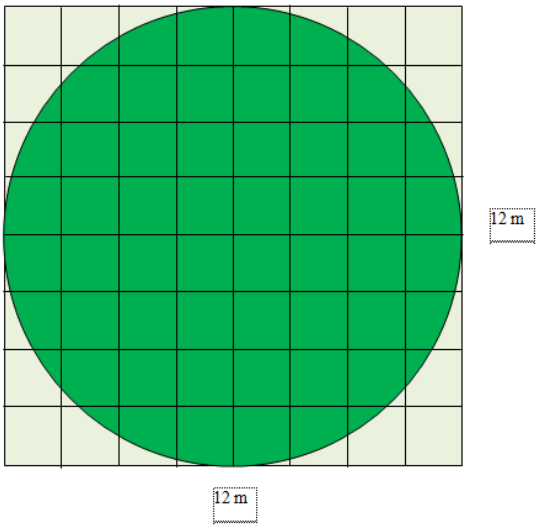
4. Memperkirakan luas lingkaran dengan menggunakan ukuran petak yang berbeda (25 menit)

   Setelah aktivitas pertama tentang membandingkan luas dari dua daerah yang tidak beraturan pada kertas berpetak, aktivitas ini adalah tentang memperkirakan banyaknya rumput yang diperlukan untuk menutupi taman yang berbentuk lingkaran. Pada aktivitas ini, guru hendaknya menghindari menggunakan istilah ‘luas’ karena untuk mengembangkan pemahaman siswa tentang luas, hal yang paling penting adalah siswa dapat memahami apa yang dimaksud denga luas. Kemudian, jelaskan dengan eksplisit bahwa gambar di dalam LKS adalah desain dengan skala dari sebuah taman yang berbentuk lingkaran. Permasalahan berikut ini adalah soal pada aktivitas ini.

   Membuat taman berbentuk lingkaran

   Pak Heri meminta seorang desainer taman untuk membuat taman berbentuk lingkaran di halaman rumahnya yang berbentuk persegi. Taman lingkaran

tersebut akan ditanami rumput jepang dan pada tanah sisanya akan dipasangi paving.



Terdapat dua pertanyaan yang harus dijawab oleh siswa berkaitan dengan masalah ini.

3. *Berapa meter persegi rumput jepang yang diperlukan untuk menutupi taman lingkaran tersebut?*
4. *Dengan membuat desain paving persegi dengan ukuran yang lebih kecil kecil, tentukan perkiraan berapa meter persegi rumput jepang yang diperlukan untuk menutupi taman lingkaran tersebut.*

Tabel berikut ini menjabarkan konjektur pemikiran siswa.

Konjektur Pemikiran Siswa	**Respon Guru**
Siswa akan mengatakan bahwa rumput yang dibutuhkan untuk taman tersebut adalah ¾ bagian dari luas lahan yang berbentuk persegi.	Guru meminta siswa untuk menjelaskan cara mereka memperkirakan dan akurasi dari hasil perkiraannya.
Siswa akan memperkirakan luas taman yang berbentuk lingkaran dengan mempertimbangkan petak-petak yang diberikan. Sehingga, luas lingkaran adalah lebih dari $3r^2$ dan kurang dari $4r^2$.	Guru bertanya kepada siswa tentang akurasi dari perkiraan mereka dan cara yang digunakan untuk menghitung petak yang tidak penuh

Konjektur Pemikiran Siswa	Respon Guru
Siswa mungkin akan menggunakan petak yang lebih besar untuk membuat pekerjaan mereka lebih mudah. Akan tetapi, mereka mungkin juga menggunakan petak yang lebih kecil untuk membuat perkiraan mereka lebih akurat.	Guru meminta siswa untuk berpikir tentang kelebihan dan kelemahan memperkirakan luas daerah di dalam lingkaran dengan menggunakan petak yang lebih besar dan dengan mengunakan petak yang lebih kecil.

5. Kongres Matematika: mendiskusikan hasil pekerjaan kelompok dalam diskusi kelas (25 menit)

   Diskusi ini akan memakan waktu yang cukup lama karena pada pertemuan ini terdapat aktivitas yang sudah fokus pada luas lingkaran. Guru meminta salah satu kelompok untuk menjelaskan jawabannya di depan kelas. Selama diskusi, guru dapat mendorong siswa dari kelompok yang tidak mempresentasikan jawabannya untuk berpartisipasi berpendapat di dalam diskusi. Misalnya dengan bertanya, *"Apakah semuanya mempanyai ide yang sama dalam memecahkan masalah ini? Apakah ada cara lain yang bisa digunakan untuk memperkirakan banyaknya rumput yang harus ditanam di taman yang berbentuk lingkaran tersebut? Untuk dapat mengetahui banyaknya rumput yang diperlukan untuk taman tersebut, apa yang harus kita cari?"*

   Tabel berikut ini menjabarkan konjektur permikiran siswa dan respon guru.

Konjektur Pemikiran Siswa	Respon Guru
Siswa akan mendiskusikan penemuan mereka yang mungkin akan terdiri dari beberapa jawaban yang berbeda dan mendiskusikan akurasi perhitungan mereka.	Guru membimbing diskusi tentang alasan kenapa siswa mendapatkan hasil yang berbeda-beda dan cara-cara yang berbeda untuk memperkirakan luas daerah tertentu.

Konjektur Pemikiran Siswa	Respon Guru
Siswa akan mendiskusikan kelebihan dan kekurangan menggunakan petak yang lebih besar dan petak yang lebih kecil dalam memperkirakan luas daerah di dalam lingkaran.	Guru membimbing siswa untuk menemukan ide bahwa semakin kecil petak yang digunakan untuk memperkirakan luas daerah di dalam lingkaran semakin akurat perkiraan mereka.

***Kegiatan Akhir* (*10 minutes*)**

Setelah diskusi, dengan menggunakan pertanyaan yang terdapat pada bagian refleksi, kita dapat mengecek pemahaman siswa dari keseluruhan pembelajaran pada pertemuan ini. Di sini dapat dilakukan dengan memberikan beberapa pertanyaan sebagai berikut.

1. *Seberapa akurat perkiraanmu tentang banyaknya rumput yang diperlukan untuk taman lingkaran jika menggunakan petak dengan ukuran besar?*
2. *Seberapa akurat perkiraanmu tentang banyaknya rumput yang diperlukan untuk taman lingkaran jika menggunakan paving dengan ukuran kecil?*
3. *Apa persamaan dan perbedaan mengukur luas suatu bidang dengan menggunakan ubin dan dengan menggunakan kertas petak?*

Guru dapat meminta beberapa siswa untuk menjawab pertanyaan-pertanyaan tersebut dan memberikan kesempatan kepada siswa lain untuk mengulangi dan memberikan jastifikasi atas jawaban temannya.

Peran Guru:

Selama siswa bekerja di dalam kelompok

- Guru harus membimbing siswa untuk berdiskusi tentang cara membandingkan besarnya dua danau tersebut terlebih dahulu. Misalnya dengan mendorong siswa untuk mengemukakan ide secara individu terlebih dahulu sebelum membuat keputusan kelompok.

  Apa ide dari setiap anggota kelompok untuk membandingkan besarnya kedua danau ini?

Mengapa kita bisa mengatakan bahwa danau yang satu lebih besar daripada danau yang lain?

- Guru juga dapat bertanya kepada siswa tentang fungsi dari petak-petak yang ada di dalam gambar danau tersebut ketika tidak satu pun siswa di dalam kelompok yang memunculkan ide tersebut.

  Menurut kamu, bisa tidak kita gunakan petak-petak ini untuk membandingkan besar kedua danau? Kalau tidak bisa, mengapa tidak bisa? Dan kalau bis,a bagaimana caranya?
- Guru mendorong siswa untuk menggunakan cara informal dalam menentukan banyaknya rumput yang diperlukan untuk menutupi taman lingkaran di dalam lahan yang berbentuk persegi, terutama jika siswa langsung menerapkan rumus luas lingkaran untuk memecahkan maasalah tersebut.

  Bagaimana kamu yakin bahwa perhitunganmu itu benar? Apakah ada cara lain selain menggunakan rumus? Bagaimana kalau kita bandingkan hasil perhitungan dari menggunakan rumus dan tidak menggunakan rumus?
- Guru membantu siswa untuk memperkirakan luas lingkaran dengan membimbing siswa untuk melihat hubungan antara luas persegi dan luas lingkaran di dalam persegi.

  Bagaimana cara mencari luas persegi? Kalau kita lihat lingkaran ini terletak di dalam sebuat persegi, apa hubungan luas lingkaran dan luas persegi?

Selama diskusi kelas

- Guru meminta satu kelompok untuk mempresentasikan hasil kerja kelompok di depan kelas dan memberi kesempatan kepada kelompok lain untuk memberikan komentar dalam diskusi. Fokus diskusi ini adalah strategi dan penalaran siswa tentang bagaimana cara membandingkan dan menentukan luas benda datar tidak beraturan dengan mempertimbangkan petak-petak yang ada didalam gambar.

  Tanpa menggunakan kertas berpetak tersebut, bagaimana kita bisa membandingkan luas kedua danau? Bagaimana jika menggunakan kertas berpetak?

- Guru dapat memberikan pertanyaan tentang hasil perkiraan dari setiap kelompok. Kemudian, dilanjutkan dengan diskusi yang bertujuan untuk memunculkan strategi-strategi estimasi yang digunakan oleh siswa untuk memperkirakan berapa meter persegi rumput yang diperlukan untuk menutupi taman lingkaran.

  Apa yang menyebabkan hasil-hasil perkiraan dari tiap kelompok menunjukkan angka yang berbeda-beda? Bagaimana cara setiap kelompok untuk memperkirakan luasnya? Adakah yang membuat petak-petak lebih kecil? Atau lebih besar?

Pertemuan Ketiga

Aktivitas:

Menemukan Luas Lingkaran Dengan Menggunakan Strategi *Reshaping* (Membentuk Ulang) Juring-juring Lingkaran

Alokasi Waktu:

2 x 40 menit

Materi:

- Lembar Kerja Siswa 3
- Kertas (kertas berbentuk lingkaran yang sudah dibagi menjadi beberapa buah juring)
- Kertas poster
- Gunting
- Lem

Tujuan Pembelajaran:

Tujuan utama

Siswa dapat memperkirakan luas lingkaran menggunakan juring-juring dari lingkaran tersebut.

Deskripsi tujuan pembelajaran

- Siswa dapat membentuk ulang empat juring lingkaran menjadi suatu bangun yang menyerupai jajar genjang.
- Siswa dapat membentuk ulang 8, 12, dan 16 juring menjadi tiga bangun yang menyerupai jajar genjang.
- Siswa dapat membentuk ulang jajar genjang menjadi sebuah persegi panjang dan dapat mengemukakan ide tentang cara menemukan luas persegi panjang tersebut.
- Siswa dapat menjelaskan bahwa luas persegi panjang yang dibentuk dari juring-juring tersebut adalah $r \times {}^{1}\!/_{2}$ keliling

Gambaran Aktivitas:

***Pembukaan* (*5 menit*)**

Pada pertemuan ini, guru langsung meminta siswa untuk duduk dalam kelompok yang sama seperti pada pertemuan-pertemuan sebelumnya dan membagikan lembar kerja siswa ke setiap kelompok. Tidak diperlukan pembukaan yang lama karena aktivitas pertama di pertemuan ini adalah tentang mengingat kembali apa yang sudah dipelajari oleh siswa pada pertemuan sebelumnya.

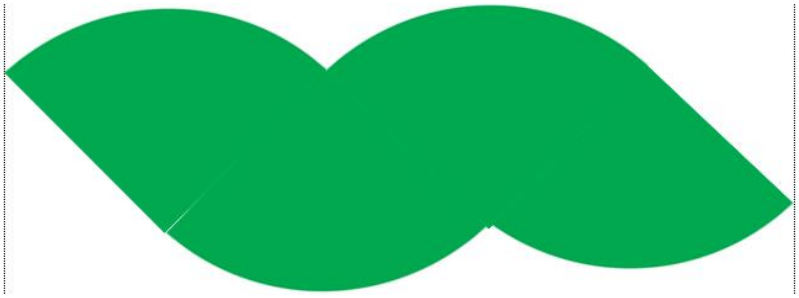
***Aktivitas Utama* (*65 menit*)**

4. Membentuk ulang taman lingkaran menjadi bentuk yang menyerupai jajar genjang (10 menit)

   Aktivitas pertama adalah tentang membentuk ulang sebuah lingkaran menjadi bentuk yang menyerupai jajar genjang. Pertama-tama guru bertanya pada siswa tentang strategi yang mereka gunakan untuk memperkirakan luas rumput yang diperlukan untuk menutupi taman lingkaran di pertemuan sebelumnya. kemudian, guru dapat bertanya, "Apakah ada strategi lain untuk memperkirakan rumput yang dibutuhkan?" ketika siswa tidak puny ide, guru dapat menceritakan tenang beberapa siswa (yang tidak di kelas tersebut) menggunakan strategi yang berbeda seperti yang digambarkan pada aktivitas pertama di dalam LKS.

***Reshaping* taman lingkaran untuk menemukan luasnya**

Untuk memperkirakan banyaknya rumput jepang yang diperlukan untuk menutupi taman lingkaran tersebut, Kelompok Dewi memotong lingkaran pada desain menjadi empat juring dan menyusunnya seperti pada gambar berikut ini.



Bangun yang terbentuk dari juring-juring lingkaran

Terdapat dua pertanyaan yang harus dijawab oleh siswa yaitu sebagai berikut.

3. Bangun datar apakah yang menyerupai bentuk di atas?

4. Apakah luas bangun di atas sama dengan luas taman lingkaran?

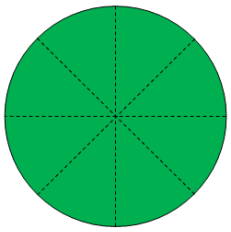
Tabel berikut ini menggambarkan konjektur pemikiran siswa.

Konjektur Pemikiran Siswa	Respon Guru
Siswa akan membentuk ulang juring-juring tidak menjadi bentuk seperti yang digambarkan di LKS karena mereka membentuk ulang dengan cara yang berbeda.	Guru membimbing siswa untuk membentuk ulang juring-juring dengan cara yang dapat membentuk bangun seperti yang digambarkan di LKS.
Siswa akan membentuk ulang juring-juring menjadi bentuk seperti yang digambarkan di LKS. Akan tetapi mereka belum dapat menganggap bahwa bangun tersebut menyerupai jajar genjang.	Guru meminta siswa untuk membayangkan ketika sisi lengkung dari bangun tersebut menjadi garis lurus. Kemudian, bisa juga dengan meminta siswa berpikir tentang apa yang akan terjadi jika lingkaran tersebut dipotong-potong menjadi juring-juring yang lebih kecil.

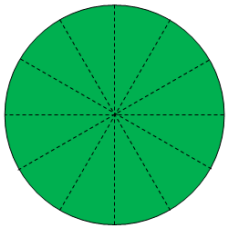
5. Membentuk ulang tiga lingkaran yang dipotong menjadi juring-juring yang berbeda banyaknya menjadi sebuah jajar genjang (25 menit)

   Di aktivitas ini, bagikan kepada setiap kelompok tiga buah lingkaran yang dibagi menjadi juring-juring yang banyaknya berbeda, sebuah gunting, lem, dan sebuah kertas poster. Siswa akan menggunting juring-juring lingkaran dan menyusunnya dengan cara yang sama sepert pada aktivitas pertama. Masalah pada LKS digambarkan sebagai berikut.

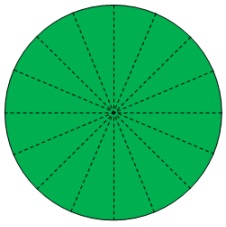
Menyusun ulang 8-juring lingkaran



Menyusun ulang 12-juring lingkaran



Menyusun ulang 16-juring lingkaran



Potong lingkaran searah diameter sehingga akan terbentuk 8 juring, 12 juring, dan 16 juring seperti pada gambar di atas. Susun juring-juring tersebut seperti pada bangun yang terbentuk di aktivitas pertama. Tempelkan hasil pekerjaanmu di poster.

3. *Bangun datar apa yang menyerupai bangun yang terbentuk dari juring-juring tersebut?*
4. *Bagaimana jika kita memotong lingkaran tersebut menjadi juring-juring yang lebih banyak?*

Tabel berikut ini menjabarkan konjektur pemikiran siswa.

Konjektur Pemikiran Siswa	Respon Guru
Siswa menyusun ulang juring-juring tidak menjadi bentuk yang menyerupai bangun di aktivitas pertama karena mereka masih melakukan reshaping dengan langkah yang berbeda.	Guru meminta siswa untuk memperhatikan perubahan yang dapat siswa lihat dari bentuk jajar genjang 4 juring, 8 juring, 12 juring, dan 16 juring.
Siswa akan membentuk ulang juring-juring menjadi bangun yang menyerupai bangun di aktivitas pertama dan menyadari bahwa bangun tersebut menyerupai jajar genjang.	Guru meminta siswa untuk membayangkan tentang bagaimana bentuk bangun tersebut jika juring-juringnya semakin kecil.

5. Membentuk ulang jajar genjang menjadi persegi panjang dan menentukan rumus luasnya (10 menit)

   Ketika siswa sudah menyusun juring-juring lingkaran menjadi bentuk yang menyerupai jajar genjang, kita bisa saja langsung mencari luas lingkaran dari bentuk tersebut. Akan tetapi, hal ini mungkin akan sedikit sulit untuk siswa mengkaitkan luas lingkaran dengan luas jajar genjang. Oleh karena itu, di aktivitas ini, siswa akan menyusun ulang jajar genjang tersebut menjadi bentuk persegi panjang dan menghubungkan cara menemukan luas persegi panjang dan lingkaran.

   Berikut ini dua pertanyaan yang harus dijawab siswa di aktivitas ini.

   3. *Bagaimana cara mengubah bangun-bangun yang terbentuk tersebut menjadi persegi panjang?*
   4. *Apa idemu untuk mengetahui luas dari persegi panjang itu?*

Dan tabel berikut menggambarkan konjektur pemikiran siswa.

Konjektur Pemikiran Siswa	Respon Guru
Siswa tidak dapat menemukan ide dan bingung bagaimana membentuk jajar genjang menjadi sebuah persegi panjang.	Guru membimbing siswa untuk mendapatkan ide tentang bagaimana cara membentuk ulang jajar genjang menjadi persegi panjang dengan melihat bentuk dari kedua bangun tersebut.
Siswa akan memotong setengah dari juring di bagian paling kiri dan meletakkannya di bagian paling kanan, atau sebaliknya.	Guru meminta siswa untuk berpikir tentang cara untuk menemukan luas persegi panjang.

6. *Kongres Matematika: mendiskusika perkerjaan kelompok di dalam forum diskusi (20 menit)*

   Mulai kongres matematika dengan aktivitas galeri berjalan. Minta siswa untuk menempelkan posternya di dinding ruang kelas dan setiap kelompok harus mengunjungi poster kelompok lain. Ketika mereka mengunjungi poster kelompok lain, mereka juga harus memberikan komentar, catatan, dan saran. Kemudian, di dalam diskusi, setiap kelompok harus memberikan respon berkaitan dengan komentar, cacatan, dan saran yang mereka dapat. Tabel berikut ini menggambarkan konjektur pemikiran siswa di dalam diskusi.

Konjektur Pemikiran Siswa	Respon Guru
Siswa akan berdiskusi bahwa luas lingkaran sama dengan luas persegi panjang. Akan tetapi, mereka menyatakan luas persegi panjang sebagai $p \times l$.	Guru membimbing siswa untuk melihat hubungan antara bagian-bagian lingkaran dengan bagian-bagian persegi panjang.

Konjektur Pemikiran Siswa	Respon Guru
Siswa akan berdiskusi bahwa luas lingkaran sama dengan luas persegi panjang. Kemudian, mereka akan menentukan bahwa luas persegi panjang tersebut adalah $r \times 1/2$ keliling.	Untuk memastikan bahwa siswa mengerti luas dari persegi panjang adalah $r \times 1/2$ keliling dengan meminta siswa untuk menunjukkan bagian-bagian dari lingkaran di persegi panjang tersebut.

***Kegiatan Akhir* (*10 menit*)**

Setelah diskusi, dengan menggunakan pertanyaan yang terdapat pada bagian refleksi, kita dapat mengecek pemahaman siswa dari keseluruhan pembelajaran pada pertemuan ini. Di sini dapat dilakukan dengan memberikan beberapa pertanyaan sebagai berikut.

1. *Seberapa akurat perkiraanmu tentang luas lingkaran jika kita menggunakan cara memotong lingkaran tersebut menjadi juring-juring yang sama besar dan menyusunnya menjadi bangun datar yang lain?*
2. *Apakah unsur yang belum diketahui dari bangun tersebut?*
3. *Bagaimana kamu memformulasikan rumus dari luas dari bangun yang baru?*

Guru dapat meminta beberapa siswa untuk menjawab pertanyaan-pertanyaan tersebut dan memberikan kesempatan kepada siswa lain untuk mengulangi dan memberikan jastifikasi atas jawaban temannya.

Peran Guru:

Selama siswa bekerja di dalam kelompok

- Guru harus membimbing siswa untuk menyusun juring-juring lingkaran sesuai dengan pola yang ditunjukkan pada LKS, tidak menyusun juring-juring tersebut menjadi bentuk yang sama persis seperti yang ditunjukkan pada LKS. *Apakah semua anggota kelompok sudah mengerti apa yang harus dilakukan dengan juring-juring ini? Dapatkah kalian menjelaskan kembali dengan*

kalimat kalian sendiri tentang pertanyaan dan perintah yang terdapat pada LKS?

- Guru membimbing siswa untuk melihat hubungan bahwa semakin semakin banyak juring yang terbentuk dari suatu lingkaran, maka bangun yang disusun-dari juring-juring tersebut akan semakin mirip dengan bangun jajar genjang. Kemudian, guru dapat meminta siswa untuk menjelaskan strateginya untuk mengubang bangun jajar genjang tersebut menjadi sebuah persegi panjang.

  Perhatikan bangun pertama, kedua, dan ketiga, apakah hubungan antara ketiganya? Jika bangun-bangun tersebut menyerupai bangun jajar genjang, bagaimana cara mengubah jajar genjang tersebut menjadi persegi panjang?
- Guru mengecek pemahaman siswa tentang hubungan unsur-unsur persegi panjang yang terbentuk dari juring-juring lingkaran dengan unsur-unsur lingkaran awal.

  Bagian dari lingkaran apakah lebar dan panjang daribangun persegi panjang ini? Jika kurang jelas, perhatikan juring-juringnya dan gambar lingkaran yang belim dipoting mrnjadi juring-juring.

Selama diskusi kelas

- Guru mempersilahkan salah satu kelompok untuk memberikan respon dari komentar- komentar dan saran saran yang diberikan oleh kelompok lain pada posternya.

  Apa tanggapan kalian dari komentar dan saran-saran yang telah diberikan oleh teman-teman kalian? Jika setuju jelaskan alasannya dan jika tidak setuju apa alasannya?
- Guru mengarahkan diskusi pada konsep konservasi luas pada lingkaran yang diubah menjadi jajar genjang, kemudian menjadi persegi panjang. Kemudian, diskusi membahas tentang strategi siswa dalam menentukan luas persegi panjang yang terbentuk dari juring-juring lingkaran tersebut.

  Bagaimana hubungan luas lingkaran, luas jajar genjang, dan luas persegi panjang yang telah kita bentuk? Bagaimana cara menemukan luas persegi panjang? Bagaimana dengan luas persegi panjang yang terbentuk dari juring-juring lingkaran tersebut?

Pertemuan Keempat

Aktivitas:

Mengukur keliling lingkaran dan membentuk bangun-bangun datar dari juring-juring lingkaran.

Alokasi Waktu:

2 x 40 menit

Materi:

- Lembar Kerja Siswa 4
- Kertas (hasil menjiplak benda yang berbentuk lingkaran)
- Kertas (kertas berbentuk lingkaran yang sudah dibagi menjadi 16 buah juring)
- Kertas Poster
- Gunting
- Lem
- Penggaris

Tujuan Pembelajaran:

- Siswa dapat menemukan perkiraan nilai pi dan menggunakannya untuk memformulasikan rumus keliling lingkaran. Kemudian, siswa dapat menggunakannya untuk menemukan luas lingkaran.
- Siswa strategi *reshaping* untuk menemukan luas lingkaran.

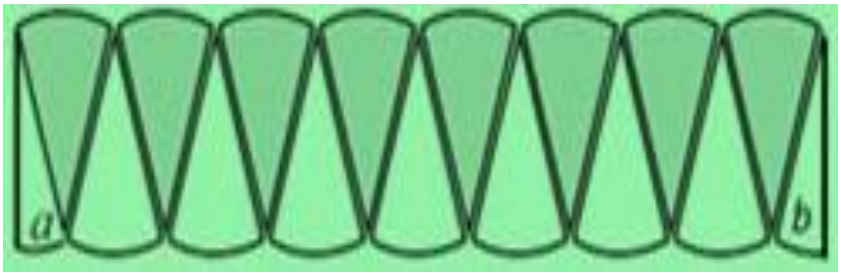
Gambaran Aktivitas:

***Kegiatan Awal* (*15 menit*)**

Aktivitas ini berhubungan dengan temuan siswa pada aktivitas di pertemuan sebelumnya bahwa luas persegi panjang yang terbentuk dari juring-juring lingkaran adalah $r \times {}^{1}\!/_{2}$ keliling.

Jajar genjang yang terbentuk dari lingkaran

Di pertemuan sebelumnya, kita sudah membentuk ulang lingkaran menjadi bangun di bawah ini.



1. *Unsur apa saja yang harus diketahui untuk menentukan luas dari bangun di atas?*
2. *Unsur apa yang sudah diketahui pada bangun di atas?*

Kemudian, aktivitas akan difokuskan dengan mencari hubungan antara keliling dan diameter lingkaran yang bertujuan untuk memahami nilai pi dalam perhitungan keliling lingkaran.

***Kegiatan Inti* (55 *menit*)**

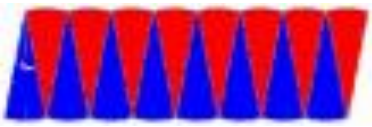
3. Dalam pembelajaran ini siswa akan menyelidiki hubungan antara keliling dan diameter dari benda-benda yang berbentuk lingkaran. Setiap kelompok akan memiliki 5-6 kertas yang berbentuk lingkaran yang merupakan hasil penjiplakan benda-benda yang berbentuk lingkaran tersebut. Siswa akan membuat daftar nama benda, keliling, diameter dan rasio keliling dan diameter. Siswa akan mendapatkan angka pendekatan phi yang merupakan rasio dari keliling terhadap diameter. Di sini, guru membimbing siswa untuk menyimpulkan bahwa keliling dibagi diameter adalah phi atau bisa dikatakan keliling adalah phi dikalikan dengan diameter.

Nama Benda	Diameter	Keliling	Keliling:Diameter

Kemudian, guru membimbing siswa untuk menyimpulkan bahwa berdasarkan hasil pada pertemuan sebelumnya bahwa luas lingkaran yang juring-juringna diubah ke dalam bentuk persegi panjang adalah jari-jari dikalikan setengah keliling lingkaran maka: Luas Lingkaran = $r \times \frac{1}{2} d = r \times \frac{1}{2} (\pi d) = r \times \pi \times r = \pi r^2$

4. Menyusun ulang juring-juring lingkaran menjadi beberapa bangun datar (20 menit)

   Guru memberikan kepada setiap kelompok pat kertas berbentuk lingkaran yang sudah dibagi menjadi 16 juring, sebuah gunting, kertas poster, dan lem. Siswa akan menggunting juring-juring tersebut, menyusun ulang juring-juring dari setiap lingkaran menjadi sebuah bangun datar dan mengelemnya pada sebuah kertas poster. Ekspektasinya adalah bahwa siswa akan menyusun ulang juring-juring tersebut menjadi empat buah bangun datar sebagai berikut.



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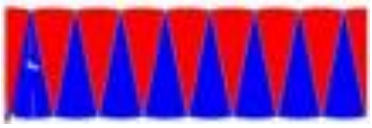

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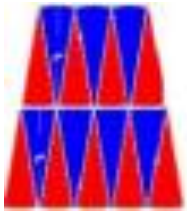
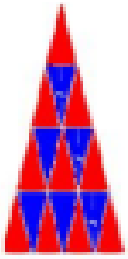
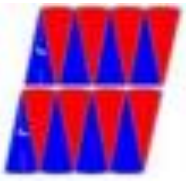




Akan tetapi, jajar genjang dan persegi panjang tersebut dapat juga berbentuk seperti pada gambar di bawah ini.



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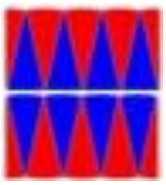

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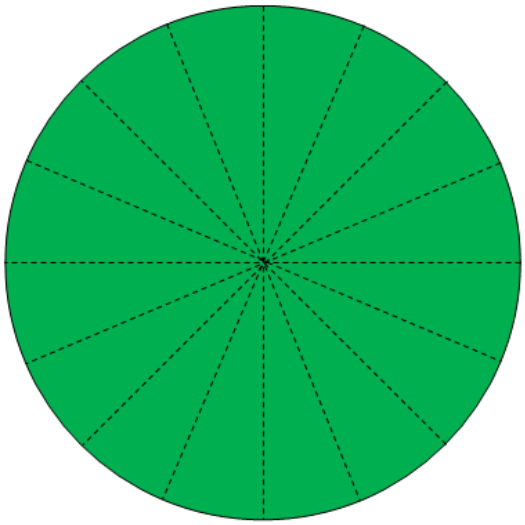
Guru mendorong siswa untuk menunjukkan kreativitasnya. Bukan menjadi suatu keharusan bahwa siswa harus dapat menemukan semua bentuk di atas. Yang terpenting adalah mereka dapat membentuk bangun-bangun datar dan mencoba menemukan luasnya dengan memperhatikan bagian-bagian lingkaran dan bagian-bagian bangun datar baru yang mereka bentuk.

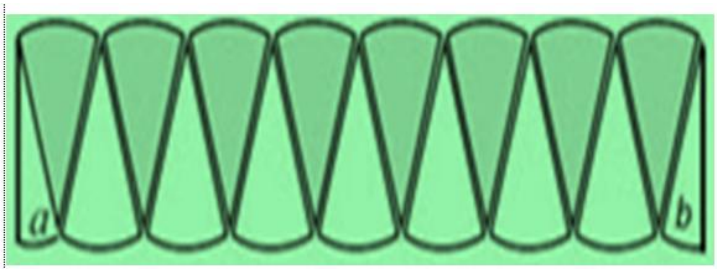
Permasalahan dalam aktivitas ini dan konjektur pemikiran siswa digambarkan sebagai berikut.

Lingkaran dan bangun datar lainnya

Di pertemuan sebelumnya, kita sudah membentuk lingkaran menjadi bangun seperti pada gambar di bawah ini.



Dan menyusun ulang juring-juring lingkaran menjadi bangun seperti pada gambar di bawah ini



Juring-juring tersebut dapat juga dibentuk menjadi bentuk-bentuk yang lain. Bentuk juring-juring tersebut menjadi empat bangun datar yang berbeda dan tempelkan pada postermu.

Tentukan luas setiap bangun datar yang terbentuk dari juring-juring tersebut.

Nama Bangun Datar	Luas
1.	
2.	
3.	
4.	

Apa hubungan antara luas dari lingkaran semula dan bangun-bangun datar yang terbentuk dari juring-juring lingkaran?

Konjektur pemikiran siswa	Respon Guru
Siswa hanya akan menyusun juring-juring tersebut menjadi jajar genjang dan persegi panjang seperti yang mereka lakukan pada pertemuan sebelumnya.	Guru meminta siswa untuk mencoba membentuk ulang juring-juring menjadi bangun datar lain dengan mengingatkan mereka tentang macam-macam bangun datar yang sudah dipelajari di kelas 7.
Siswa akan mencoba untuk menyusun ulang juring-juring lingkaran menjadi beberapa bentuk bangun datar termasuk jajar genjang, persegi panjang, segitiga, dan trapesium.	Guru membimbing siswa menginvestigasi hubungan antara rumus luas bangun datar yang mereka bentuk dan luas lingkaran dengan mengingatkan mereka tentang bagaimana menemukan luas persegi panjang pada pertemuan sebelumnya.

5. Memperkirakan banyaknya uang yang diperlukan untuk membeli rumput yang akan digunakan untuk menutupi sebuah taman berbentuk lingkaran (15 menit)
   Di aktivitas ini, siswa akan memperkirakan banyaknya uang yang diperlukan untuk membeli rumput yang akan digunakan untuk menutupi taman berbentuk lingkaran. Ide dari pembelajaran ini adalah siswa akan mengetahui pentingnya mencari luas daerah lingkaran dalam kehidupan sehari-hari. Kemudian, siswa

akan menggunakan cara yang mereka gunakan pada pertemuan-pertemuan sebelumnya. Hasil diskusi ini selanjutnya akan ditempelkan pada kertas poster. Permasalahan dan konjektur pemikiran siswa digambarkan sebagai berikut.

Menutupi taman berbentuk lingkaran

Berapa perkiraan banyaknya biaya yang dibutuhkan untuk membeli rumput jepang yang akan ditanam di taman berbentuk lingkaran pada sketsa di bawah ini jika harga per meter persegi dari rumput jepang tersebut adalah Rp. 150.000,00. Bandingkan hasil perhitunganmu dengan menggunakan kertas berpetak dan dengan menggunakan cara menyusun ulang juring-juringnya.

Konjektur Pemikiran Siswa	**Respon Guru**
Siswa akan langsung menggunakan rumus untuk menemukan luas lingkaran.	Guru dapat meminta siswa untuk menjelaskan bagaimana rumus tersebut bekerja dan mengapa rumus tersebut dapat digunakan untuk menghitung luas linkaran.
Sebagian siswa akan membuat petak-petak dan sebagian siswa akan menggunakan strategy *reshaping* untuk memperkrakan luas lingkaran tersebut.	Guru membimbing siswa untuk mendiskusikan persamaan dan perbedaan menggunakan petak-petak persegi dan menggunakan strategi *reshaping* untuk menentukan luas lingkaran.

6. Kongres Matematika: mendiskusikan hasil pekerjaan kelompok dalam diskusi kelas (20 minutes)

   Seperti di pertemuan keempat ketika siswa bekerja dengan poster, kongres matematika ini dimulai dengan aktivitas *walking galery*. Guru meminta siswa untuk menempelkan poster di dinding sekolah dan setiap kelompok diharuskan untuk mengunjungi poster kelompok lain. Setiap kali mengunjungi poster kelompok lain, siswa juga harus memberikan komentar, catatan, dan saran. Kemudian, pada waktu diskusi, setiap kelompok harus memberikan respon atas

komentar, catatan, dan saran yang mereka perleh dari kelompok lain. Tabel berikut ini menggambarkan konjektur pemikiran siswa dalam diskusi.

Konjektur Pemikiran Siswa	**Respon Guru**
Siswa masih bingung untuk melihat hubungan antara segitiga dan trapesium dengan lingkaran. Kemudian, siswa mengalami kesulitan untuk menemukan cara untuk menghitung luasnya.	Guru membimbing siswa untuk melihat bagian-bagian lingkaran pada segitiga dan trapesium. *"Bagian manakah yang merupakan tinggi segitiga? Bagian manakah yang merupakan tinggi segitiga?"*
Siswa akan mendiskusikan perbedaan hasil perkiraan uang yang dibutuhkan untuk membeli rumput jepang. Perbedaan ini disebabkan oleh perbedaan hasil perhitungan pada banyaknya rumput yang diperlukan yang merupakan representasi dari luas lingkaran.	Guru membimbing siswa untuk melihat persamaan dan perbedaan dari kedua strategi itu yang akan membawa siswa untuk menemukan bahwa luas lingkaran adalah tiga lebih sedikit dikalikan r^2. Kemudian, guru membimbing siswa untuk mendiskusikan strategi mana yang menurut mereka lebih membantu dan alasannya.

***Kegiatan Akhir* (*10 menit*)**

Setelah diskusi, dengan menggunakan pertanyaan yang terdapat pada bagian refleksi, kita dapat mengecek pemahaman siswa dari keseluruhan pembelajaran pada pertemuan ini. Di sini dapat dilakukan dengan memberikan beberapa pertanyaan sebagai berikut.

1. *Strategi apa saja yang dapat digunakan untuk memperkirakan luas di dalam lingkaran?*
2. *Apa kelebihan dan kekurangan dari strategi-strategi tersebut?*

Guru dapat meminta beberapa siswa untuk menjawab pertanyaan-pertanyaan tersebut dan memberikan kesempatan kepada siswa lain untuk mengulangi dan memberikan jastifikasi atas jawaban temannya.

Students' Worksheets

LEMBAR KERJA SISWA 1

Nama:_____

Aktivitas Siswa 1-1

Kepulauan Indonesia

Sebuah sekolah akan membuat taman yang dilengkapi dengan miniatur lima pulau terbesar di Indonesia yaitu Sumatera, Jawa, Kalimantan, Sulawesi, dan Papua. Pertama-tama setiap tanah yang membentuk pulau ini akan ditanami rumput jepang. Miniatur pulau manakah yang akan membutuhkan lebih banyak rumput jepang jika dibandingkan dengan pulau-pulau yang lainnya?

Jelaskan caramu untuk menentukan pulau yang paling besar:

Urutkan pulau-pulau tersebut dari yang memerlukan rumput paling banyak ke pulau yang memerlukan rumput paling sedikit. Jelaskan caramu mengurutkan.

Nomor	Pulau

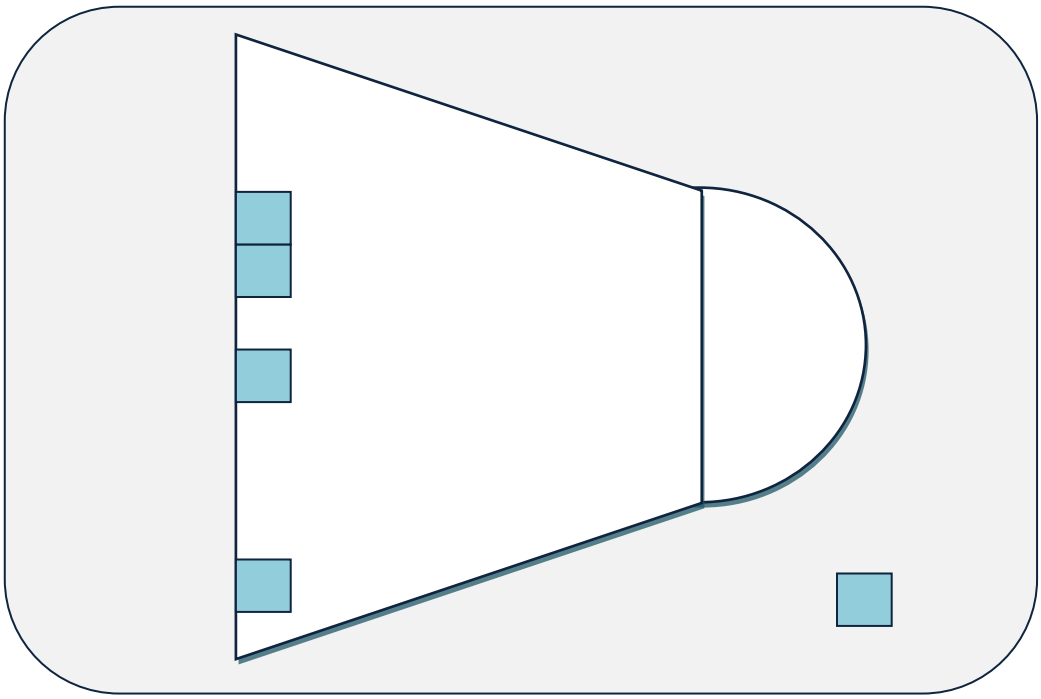
Jelaskan jawabanmu di sini:

Aktivitas Siswa 1-2

Menutupi dasar kolam renang dengan ubin

Pak Heri akan membuat kolam renang di rumahnya. Dia sudah membuat desain dari kolam renang tersebut seperti pada gambar di bawah ini. Lengkapi pemasangan ubin pada desain di bawah ini dan buat estimasi banyaknya ubin yang dibutuhkan untuk menutupi dasar kolam renang pada desain yang pertama dan desain yang kedua.

Desain yang pertama



5. Perkirakan banyaknya ubin yang dibutuhkan untuk menutupi dasar kolam renang.

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6. Jelaskan cara dan alasanmu.

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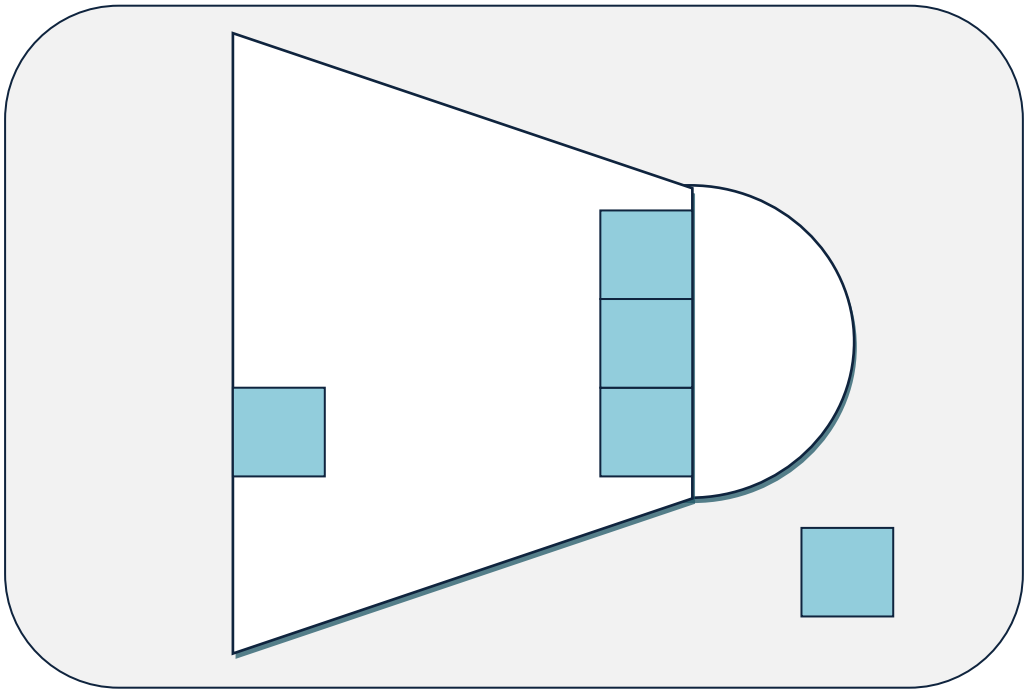
Desain kedua



Desain kolam renang dengan ubin yang besar

1. Perkirakan banyaknya ubin yang dibutuhkan untuk menutupi dasar kolam renang.

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2. Jelaskan cara dan alasanmu.

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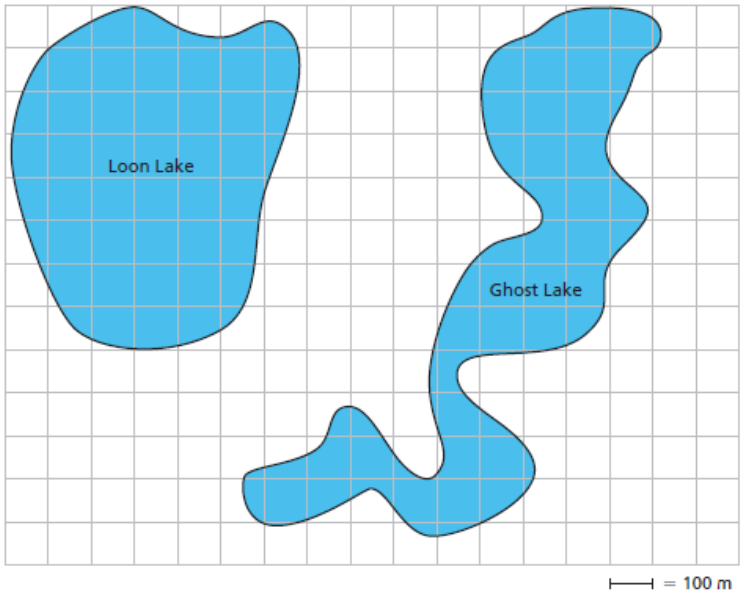
LEMBAR KERJA SISWA 2

Nama:_____

<u>Aktivitas Siswa 2-1</u>

Membandingkan dua danau

Perkirakan danau mana yang lebih besar?



Jelaskan caramu untuk membandingkan kedua danau tersebut. Seberapa akurat perkiraanmu?

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Berapa luas Danau Loon dan berapa luas Danau Ghost?

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Seberapa akurat perkiraanmu?

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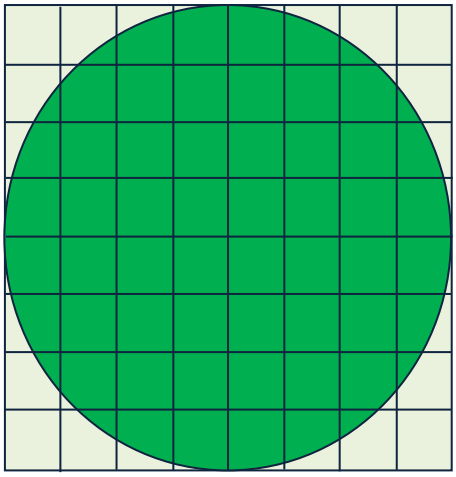
Aktivitas Siswa 2-2

Membuat taman berbentuk lingkaran

Pak Heri meminta seorang desainer taman untuk membuat taman berbentuk lingkaran di halaman rumahnya yang berbentuk persegi. Taman lingkaran tersebut akan ditanami rumput jepang dan pada tanah sisanya akan dipasangi paving.



Berapa meter persegi rumput jepang yang diperlukan untuk menutupi taman lingkaran tersebut?

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Dengan membuat desain paving persegi dengan ukuran yang lebih kecil kecil, tentukan perkiraan berapa meter persegi rumput jepang yang diperlukan untuk menutupi taman lingkaran tersebut.

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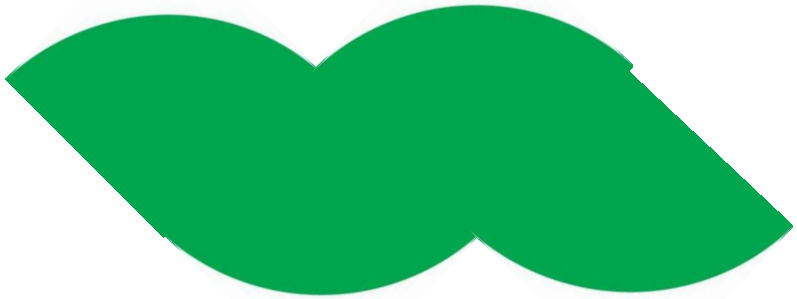
LEMBAR KERJA SISWA 3

Nama:_____

Aktivitas Siswa 4-1

***Reshaping* taman lingkaran untuk menemukan luasnya**

Untuk memperkirakan banyaknya rumput jepang yang diperlukan untuk menutupi taman lingkaran tersebut, Kelompok Dewi memotong lingkaran pada desain menjadi empat juring dan menyusunnya seperti pada gambar berikut ini.



Bangun yang terbentuk dari juring-juring lingkaran

Bangun datar apakah yang menyerupai bentuk di atas?

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Apakah luas bangun di atas sama dengan luas taman lingkaran?

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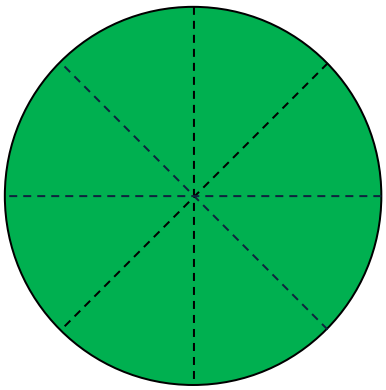
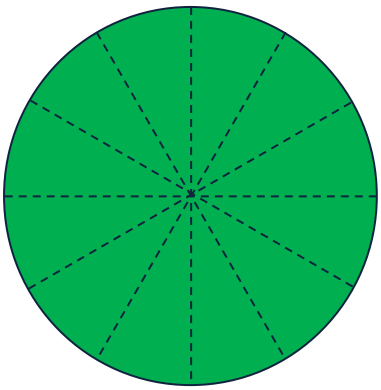
Aktivitas Siswa 4-2

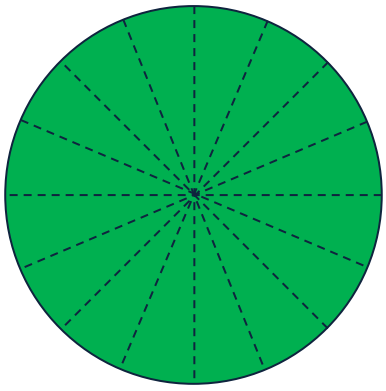
Menyusun ulang 8-juring lingkaran



Menyusun ulang 12-juring lingkaran



Menyusun ulang 16-juring lingkaran



Potong lingkaran searah diameter sehingga akan terbentuk 8 juring, 12 juring, dan 16 juring seperti pada gambar di atas. Susun juring-juring tersebut seperti pada bangun yang terbentuk di aktivitas pertama. Tempelkan hasil pekerjaanmu di poster.

Bangun datar apa yang menyerupai bangun yang terbentuk dari juring-juring tersebut?

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Bagaimana jika kita memotong lingkaran tersebut menjadi juring-juring yang lebih banyak?

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Bagaimana cara mengubah bangun-bangun yang terbentuk tersebut menjadi persegi panjang?

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Apa idemu untuk mengetahui luas dari persegi panjang itu?

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LEMBAR KERJA SISWA 4

Nama:_____

Aktivitas Siswa 5-1

Membandingkan keliling dan diameter lingkaran

Terdapat banyak benda dalam kehidupan sehari-hari yang berbentuk lingkaran. Beberapa di antaranya adalah benda-benda yang sudah kalian jiplak pada sebuah kertas.

Buat daftar diameter dan keliling dari setiap yangkaran yang terbentuk dari benda-benda tersebut.

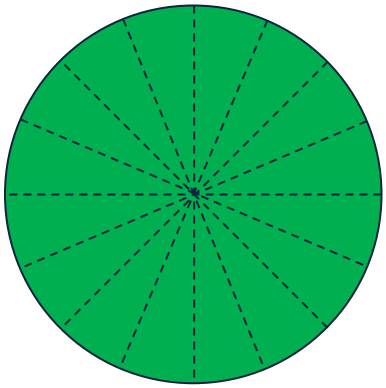
Nama Benda	Diameter	Keliling	Keliling:Diameter

Jelaskan hubungan antara diameter dan keliling.

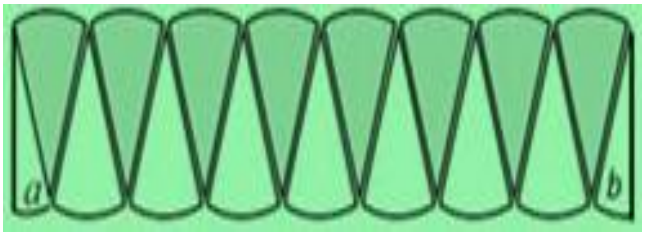
Aktivitas Siswa 4-1

Membentuk ulang lingkaran menjadi beberapa bangun datar

Di pertemuan sebelumnya, kita sudah membentuk lingkaran menjadi bangun seperti pada gambar di bawah ini.



Dan menyusun ulang juring-juring lingkaran menjadi bangun seperti pada gambar di bawah ini



Sehingga luasnya adalah *p x l = ½ x keliling lingkaran x r = ½ x (phi x d) x r = ½ x (phi x (2r)) x r = phi x* r^2

Juring-juring tersebut dapat juga dibentuk menjadi bentuk-bentuk yang lain. Bentuk juring-juring tersebut menjadi empat bangun datar yang berbeda dan tempelkan pada postermu.

Tentukan luas setiap bangun datar yang terbentuk dari juring juring tersebut.

Nama bangun datar	Luas
1.	
2.	
3.	
4.	

Apa hubungan antara luas dari lingkaran semula dan bangun-bangun datar yang terbentuk dari juring-juring lingkaran?

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LEARNING TRAJECTORIES

Lesson 1: Ordering and Tiling

4. Learning Goals

   The main goal

   Students are able to compare the size of shapes and put them in order.

   The description of the goal

   – The students can reason that their left shoe is as big as their right shoe.
   – The students are able to put their shoes in order based on its sizes.
   – The students are able to put the islands in order.
   – Students are able to explain their strategies to compare and order shapes.

5. The starting points

   The concept of the area and the circumference of a circle is one of the topics which are taught in the eighth grade of secondary school in Indonesia. However, according to the Indonesian curriculum, the topics have been taught when the students in the sixth grade of elementary school. Moreover, the notion of the area and the perimeter of straight-sided plane figures have also been introduced in the fourth and fifth grade of elementary school and the seventh grade of secondary school. Hence, the starting points for the first lesson are:

   – The students know attribute to be measured (area).
   – The students have understood the way to compare the sizes of two shapes.

6. Description of the activities and conjectures of students' thinking

   The mathematical idea in this meeting is using overlapping and reshaping strategies to compare shapes. This section begins by clarifying students' conception of area. There are three activities about comparing and ordering shapes. Students first use their own strategies to solve the problem. They can use papers, scissors, and markers as tools for comparing. Each of the activities is described as follows.

3. Ordering the size of five biggest islands in Indonesia

   This activity is another activity of comparing. The students will still work in their group and give them a large copy of Indonesian map. The problem is about determining which miniature of the five biggest islands in Indonesia will need more grass and which land will need less grass.

   Indonesian Archipelago

   A school will make a garden with the miniature of the five biggest islands in Indonesia namely Sumatera, Java, Kalimantan, Sulawesi, and Papua. If each of the land will be covered by grass, which land will need more grass than the other lands?

   Put the islands in order from the land which needs the most grass to the land which needs the least grass. Explain your way of working and reasoning.

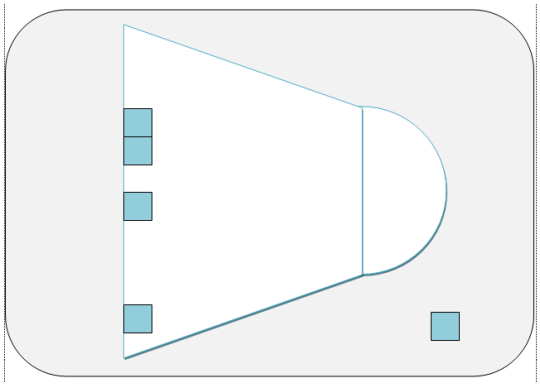
4. Covering the bottom of a swimming pool with tiles

   This activity is quite similar with the previous activity. The differences are about the shape of the floor which is a combination of a trapezoid and a semicircle and there are some given tiles on the floor. The intention is that the students will recall their knowledge about the shape of circle by providing the semicircle. Moreover, instead of iterate a unit to one another, they are expected to make grids based on the given grids. However, the teacher should not ask the students to make grids if in the activity they do not come up with the idea of making grids.

Covering a swimming pool with tiles

Mr. Heri will also make a swimming pool in his house. He has decided the shape of the swimming pool. Complete the work of tiling in the design below. Estimate the number of square tiles needed to cover the bottom of the pool for the first design and the second design.

The first design

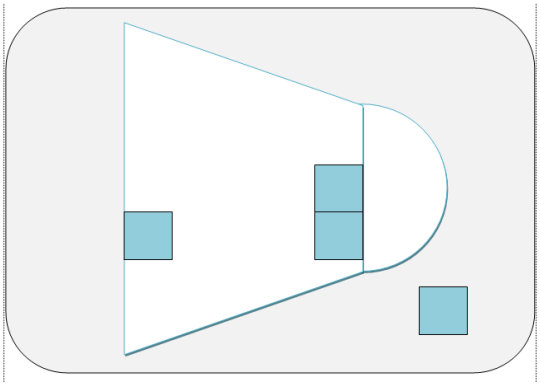


A design with small tiles

The design of the swimming pool with the small tiles

1. *Estimate the number of tiles needed to cover the bottom of the swimming pool?*
2. *Explain your way of working and reasoning.*

The second design



A design with big tiles

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

The Overview of The Activities in The First Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Ordering the size of five biggest islands in Indonesia.	The students are able to put the islands in order.	The students will determine the order only by considering the shapes of the islands which means that the wider the shapes the bigger the islands. The students will cut the drawing of the islands and overlap them. Moreover, they will do cutting and pasting with the remaining space of the overlapping.	The teacher can ask the students to think about the shapes of Sulawesi islands. *"The shape is narrow, but how if we think the island as a whole?"* The teacher should make sure that the students know the reason why we can say that two shapes are in the same size and why we can say that two shapes are bigger or smaller.
Covering the bottom of a swimming pool with tiles	The students are able to measure a more complicated object using some given measurement units with different size.	The students will continue to fulfill the bottom of the swimming pool with tiles by drawing the tile one by one. The students will make grids through the given tiles inside the shape of the bottom of the swimming pool.	Ask the students about another possible strategy of continuing the tiling process besides drawing the tiles one by one. Ask the students to think about the similarities and the differences between tiling by drawing the tiles one by one and by making grids.
Math congress: put the group works in a whole class	Students are able to explain their strategies to order and design shapes.	The students will share their work and their thinking about the idea of using overlapping strategy	The teacher can lead the students to understand that two shapes said to be in the same size if they

discussion.		to compare two similar shapes. The students can combine the overlapping strategy with cutting and pasting strategies to compare two different shapes. The students will describe their answers which are expected that there will be various shapes designed by the students.	fit each others with on gaps or overlaps. The teacher could guide the students to discuss and compare the different strategies they applied to solve the problem. The teacher could guide the students to discuss the relations between are and perimeter.

Lesson 2: Measuring shapes using grid paper

1. Learning goals

   The main goal

   Students are able to measure shapes using a grid paper and by making their own grid.

   The description of the goal

   – The students are able to compare the size of two lakes using a grid paper.
   – Students are able to estimate the size of a shape by using the given grids and by making their own grid.
   – Students are able to explain their way of working and reasoning to estimate the grass needed to cover the circular garden.

2. The starting points
   – The students have understood about scaled figures.
   – The students have had sense of sizes (smaller and bigger).
   – The students have been able in doing arithmetic operation.

3. Description of the activities and conjectures of students thinking

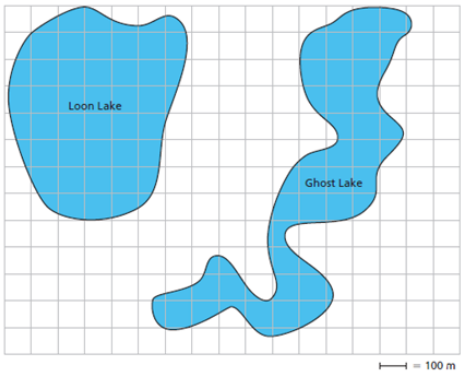
The mathematical idea in this meeting is using grid paper to measure area. This section begins by clarifying students' conception of area related to measurement units. Students try to compare two lakes on a grid paper and estimate the grass needed to cover the circular garden by looking at the relationship between the circular garden and the square land. The first activity aims to introduce the grids in area measurement and in the second activity the students are expected to see the relationship between the area of square and the area of circle which diameter is equals the side of the square land.

1. Comparing two lakes on a grid paper

   The first activity is about comparing two lakes which are drawn with scale on a grid paper. The idea is that the students will have a sense of unit iteration as arranging a number of similar units to cover the attribute of the measured objects. Firstly, the teacher should ask the students to think of the problem individually. Then, the students can share their idea to solve the problem in group.

   Comparing two lakes

   Estimate which lake is bigger?



Figures of two lakes on a grid paper

There are three questions which the students have to answers related to this problem.

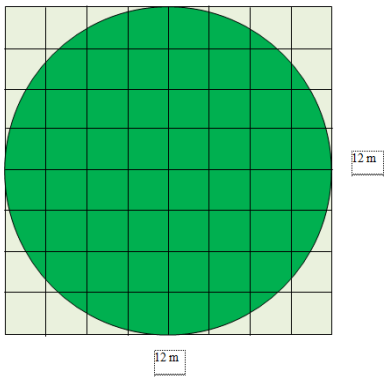
1. *Explain your way of comparing and reasoning.*
2. *How large is the Loon Lake and how large is the Ghost Lake?*
3. *How accurate is your estimation?*

2. Estimating the area of a circle using different size of grids

   After the first activity which is about comparing the size of two irregular shapes on a grid paper. This activity is about estimating the grass needed to cover a circular garden. In this activity, the teacher should avoid mentioning the terms 'area' because to develop the students' understanding of area, the important thing is that the students get make sense what they call area is. Moreover, the teacher should explain explicitly that the drawing on the paper is a scaled design of a circular garden.

   Covering a circular garden

   Mr. Heri asks a garden designer to make a circular garden in his rectangular land. The circular garden should be covered by grass and the remaining land will be covered by paving blocks as the following figure.



   There are two questions which the students have to answers related to the problem.

   1. *How many meter square grass needed to cover the circular garden?*
   2. *By making smaller size of paving, estimate how many meters square grass needed to cover the circular garden?*

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

The Overview of The Activities in The Second Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Comparing two lakes on a grid paper	The students are able to compare the size of two lakes using a grid paper.	The students will disregard the grids and reshape the lake into two more similar and more regular shape. The students will count the grids inside the boundaries of each lake and compare the number of those grids. The students will firstly count the complete grids and estimate the number of the incomplete grids.	Ask the students how to be sure that one of the lakes is bigger than the other one and ask them to consider the grids inside the boundary of the lakes. Check if there are still students who compare the lakes based on the grids surrounded the lakes. Encourage the students to see the relation between unit iteration in tiling activities as they have done in the second lesson and the grid paper in terms of area measurement.
Estimating the area of a circle using different size of grids	Students are able to estimate the size of a shape by using the given grids and by making their own grid.	The students will just say that the grass needed to cover the circle is three fourths of the area of the square land. The students will estimate the area of a circle by considering the given grids. Hence, the area of the circle is more than $3r^2$	Ask the students to reason about their estimation and ask the students about the accuracy of their estimation. Ask the students about the accuracy of their estimation and about their strategy to deal with the incomplete

		and less than $4r^2$. The students might consider the bigger size of grids to make their work simpler. But, they might also make a smaller size of grid to get the more accurate estimation.	grids. Ask the students to think about the advantages and the disadvantages of estimating the area inside the circle by considering the bigger size of grids and about the advantages and the disadvantages of estimating the area inside a circle by making the smaller size of grids.
Math congress: put the group work in a whole class discussion	Students are able to explain their way of working and reasoning to estimate the grass needed to cover the circular garden.	The students will discuss their findings of which might be various and discuss the accuracy of the findings. The students will discuss about the advantages and the disadvantages of using the bigger grids and the smaller grids to estimate the area inside the circular garden.	Lead a discussion about the reason which might the students got the various results and the different strategies used by the students to which might cause the different results. Lead the students to the idea of the smaller grids used to estimate the area inside the circle, the more accurate their estimation.

Lesson 3: Reshaping Circles

1. Learning goals

   The main goal

   Students can determine the estimation of the area of a circle using the sectors of the circle.

The description of the goal

- The students are able to reshape the four sectors of a circular garden into a shape resembles a parallelogram.
- The students are able to reshape the 8, 12, and 16 sectors into three shapes resembles parallelograms
- The students are able to reshape the parallelogram into a rectangle and are able to argue about the idea of determining the area of the rectangle.
- The students are able to explain that the area of the rectangle formed from the sectors is $r \times {}^{1}\!/_{2}$ of circumference

2. The starting points
   - The students have understood the concept of conservation of area.
   - The students have known the way to find the area of a parallelogram.
3. Description of the activities and conjectures of students thinking

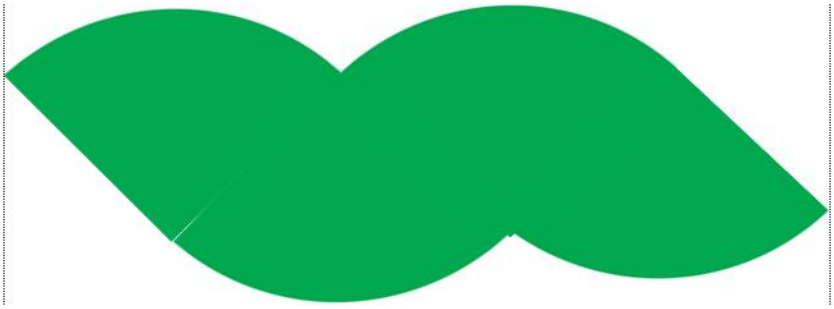
   The mathematical idea in this meeting is that the area of a circle equals the radius times half of its circumference. There is only one main activity in this meeting namely reshaping three circles which are cut into different numbers of sectors into a parallelogram. However, to make the students get the idea of reshaping the sectors of a circle into a parallelogram, there will be an activity of reshaping a circular garden into a shape resembles parallelogram which will be guided by the teacher.

   1. Reshaping a circular garden into a shape resembles parallelogram

      The first activity is about reshaping a circle into a shape resembles a parallelogram. Firstly, the students are asked about what strategies they used to estimate the grass needed to cover the circular garden in the previous activity. Then, the teacher can ask the students, *'Is there any other strategy to estimate?'* When the students get stuck, the teacher can tell that some students (which is not in their class) use the different strategy as described in the first activity in their worksheet.

Reshaping a circular garden to estimate its area

To estimate the grass needed to cover the circular garden, Dewi's group cut the circle into four sectors and reshape the sectors into this following shape.



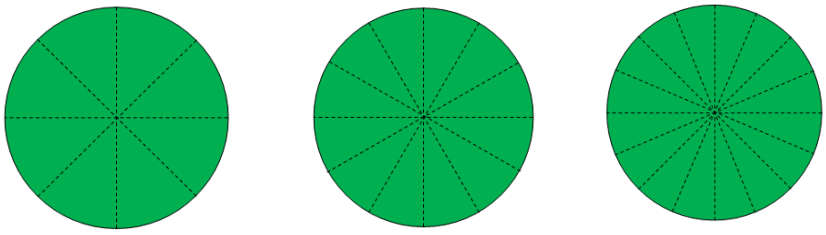
Reshaping four sectors of a circle

There are two questions which have to be answered by the students as follows.

1. *What kind of plane figure resembles the shape above?*
2. *Is the area of the shape equals the area of the circular garden?*

2. Reshaping three circles which are cut into different numbers of sectors into a parallelogram

In this activity, the students will get three circles which each of them have been divided into different numbers of sectors, a scissor, glue, and a large paper poster. They will cut the sectors of the circle and try to arrange them in the same way like the shape which is shown by the teacher.



Circles which are divided into 8, 12, and 16 sectors

Cut the circle along the diameter so that there will be 8 sectors, 12 sectors, and 16 sectors, respectively as the figures above. Arrange the sectors like the shape in the first activity. Glue it in your poster.

1. *What kind of plane figure resembles the shapes which are formed from the sectors?*
2. *What will happen if we cut the circle into the more number of sectors?*

3. Reshaping the parallelograms into rectangles and determining the formula of its area

   The students have arranged the sectors into shapes resemble parallelograms. However, to connect the relation between the formula of a circle and a parallelogram might be quite difficult. Hence, in this activity, the students will reshape the parallelogram into a rectangle and make the relation between the way to find the area of the rectangle and the circle.

   There are two questions in this activity which are:

   1. *How can you reshape the shapes you formed into rectangles?*
   2. *What is your idea to find the area of the rectangles?*

The overview of the activities in the first lesson and the conjectures of students thinking are described in the following table.

The Overview of The Activities in The Third Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Reshaping a circular garden into a shape resembles parallelogram	The students are able to reshape the four sectors of a circular garden into a shape resembles a parallelogram.	The students will reshape the sectors not into a shape shown in the worksheet because they do reshaping in a different way. The students will reshape the sectors into a shape as shown in the worksheet. However they cannot still realize	Guide the students to reshape the sectors in a way that can be formed as the shape in the worksheet. Ask the students to imagine when the curve sides of the shape turn into straight sides. Moreover, the teacher can also ask

		that the shape looks like a parallelogram.	the students to think about what will happen if the circle is cut into smaller sectors.
Reshaping three circles which are cut into different numbers of sectors into a parallelogram	The students are able to reshape the 8, 12, and 16 sectors into three shapes resembles parallelograms	The students will reshape the sectors not into shapes similar with the shape in the first activity because they still do reshaping in a different way. The students will reshape the sectors into shapes similar with the shape in the first activity. However they cannot still realize that the shape looks like a parallelogram. The students will reshape the sectors into shapes similar with the shape in the first activity and realize that the shapes look like a parallelogram.	Guide the students to reshape the sectors in a way that can be formed as the expected shapes. Ask the students to pay attention to the changing which they can see from the shape of 4 sectors, 8, sectors, 12 sectors and 16 sectors. Ask the students to imagine about how the shapes will be when the sectors are getting smaller.
Reshaping the parallelograms into rectangles and determining the formula of its area	The students are able to reshape the parallelogram into a rectangle and are able to argue about the idea of determining the area of the rectangle.	The students will get stuck and confuse and do not have any idea to reshape the parallelogram into a rectangle. The students will cut half of the sector on the left	Guide the students to get the idea of reshaping the parallelogram into a rectangle by seeing the shapes of both parallelogram and rectangle. Ask the students to think about the way to find the area of a

		most part and put it on the right most part or the other way around.	rectangle.
Math congress: put the group work in a whole class discussion	The students are able to explain that the area of the rectangle formed from the sectors is $r \times 1/2$ of circumference	The students will discuss that the area of the circle equals the area of the rectangle. However, they determine the area of the rectangle as $p \times l$. The students will discuss that the area of the circle equals the area of the rectangle. Moreover, they determine the area of the rectangle as $r \times 1/2$ of circumference.	Guide the students to see the relation between the parts of the circle with the parts of the rectangle. Make sure that all students understand that the area of the rectangle is $r \times 1/2$ of circumference by asking the students to show the parts of the circle in the rectangle.

Lesson 4: Reshaping Circles into several plane figures

1. Learning goals
   - The students are able to find the estimation of phi value and use it to formulate the formula of the circumference of a circle. Furthermore, the students are able to use it to formulate the area of a circle.
   - The students are able to use reshaping strategy to find the area of a circle
2. The starting points
   - The students' have known the concept of ratio.
   - The students have known about the concept of conservation of area.
   - The students have understood about how estimating area by reshaping.
3. Description of the activities and conjectures of students thinking

   The mathematical idea in this meeting is that the area of a circle equals pi times radius times radius. This section is kind of wrapping up activity.

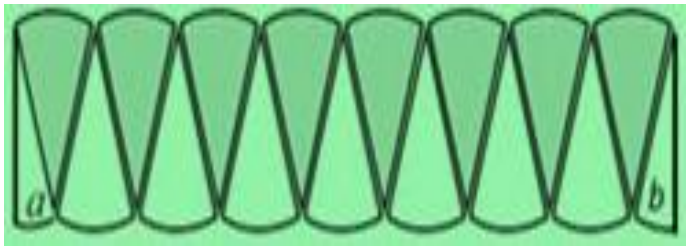
However, the first activity is still about reshaping. And, in the second activity, the students will estimate the amount of money needed to buy grass for covering a circular garden.

4. Estimating the area of a rectangle which is formed from the sectors of a circle

   This activity is connected to what the students have found in the previous meeting. They have to determine the area of a rectangle which is reshaped from the sectors of a circle.

   The rectangle of the circle

   In the previous lesson, we have reshaped the circle into this following form.



A circle which is reshaped into a rectangle

1. *What measurement do you need to determine the area of the figure?*
2. *Which measurement you know from the figure?*

There are some several objects which have circular shape in daily life as you have on your group.

Circular objects

Trace along each of the objects above with a pencil on a piece of paper. Cut the drawing and measure the diameter and the circumference of the circle.

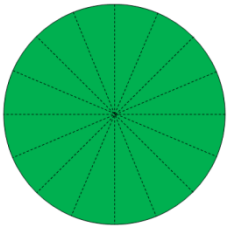
Make a list of the diameter and the circumference of those circular objects.

5. Reshaping the sectors of a circle into several kinds of plane figures

In this activity, the students will be given four circular papers which have been divided into 16 sectors. They should the sectors of the circles and reshape them into several kinds of plane figures.

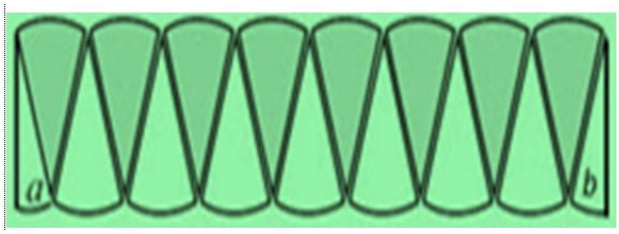
Circle and other plane figures

In the previous lesson, we have reshaped the circle into this following form.



A circle divided into sixteen sectors

And reshape the sectors into this following shape.



A rectangle formed from the sectors of a circle

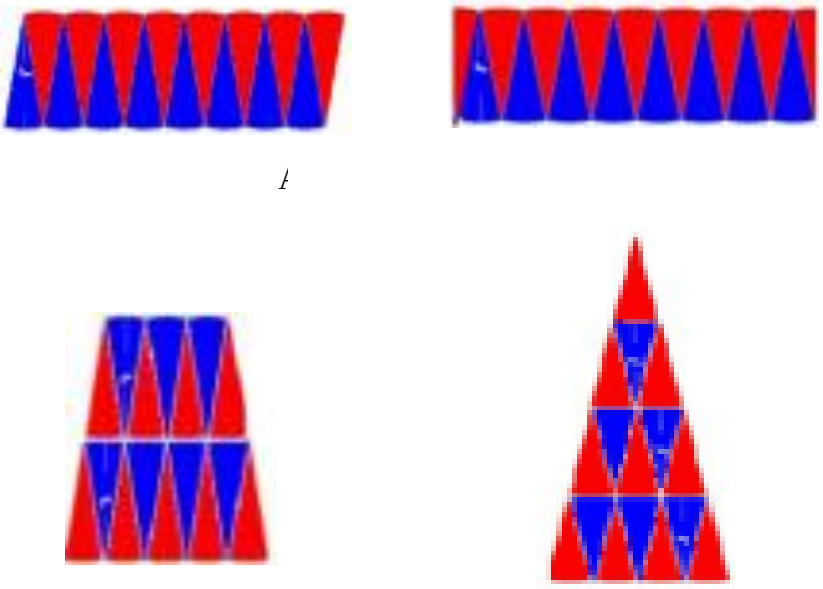
Those sectors can also be arranged into other shapes. Rearrange the sectors into four different shapes of plane figures and glue them on your poster.

Determine the size of the circle using the shapes which are formed by the sectors.

The name of the new shapes	**The area**
1.	
2.	
3.	
4.	
5.	

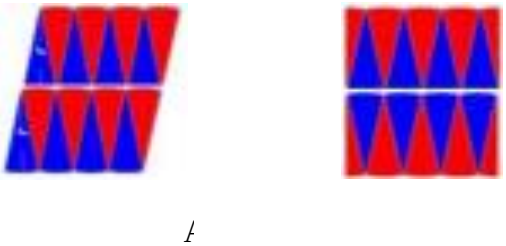
What is the relationship between the area/size of the circle and the area of the new plane figure?

The students will cut the sectors, reshape the sectors of each circle into a plane figure, and glue them on a poster paper. The expectation is that the students will reshape the sectors into four different plane figures as follows.



Four plane figures expected can be formed by students

However, the parallelogram and the rectangle can also be as these following figures.



Another shape of paralellogram and rectangle

It is not a must that they have to come up with all the shapes. The important thing is that they can form shapes of plane figures and try to find the area by considering the parts of the circle on the new shapes they formed.

The Overview of The Activities in The Fourth Lesson and Conjectures of Students' Thinking

Activity	Learning Goal	Conjectures of Students' Thinking	Actions of the Teacher
Estimating the area of a rectangle which is formed from the sectors of a circle	The students can determine the formula of the area of the rectangle the by considering the parts of the rectangle as the parts of the circle.	The students will determine the area of the rectangle as $p \times l$ and disregard the relation between the parts of the rectangle with the parts of the circle. The students will remind about the last activity in the previous lesson and determine the area of the rectangle as $\text{r} \times 1/2$ of circumference.	Reminds the students about the last activity of the previous lesson, particularly about the relation between the parts of the circle and the parts of the rectangle. Ask the students about what part of the rectangle should be known so that they can find the area of the rectangle.
Investigating the relation between the circumference and the diameter of some circular objects	The students are able to explain the relation between the diameter and the circumference of a circle.	The students will only measure the circumference and the diameter of the circular objects without firstly drawing them on the paper. The students will draw the circular objects on a paper and measure the circumference and the diameter of the circles using paper ruler.	The teacher can ask about how the students can be sure that they can measure the diameter of the circular objects by measuring it directly on the objects. For example the diameter of the armband. The teacher reminds the students about the properties of a circle especially which one is radius and diameter.

		The students will come up with the idea of the ratio of the circumference to the diameter is around 3.1. The students will come up with a different idea of the ratio which is looking the ratio from the opposite side.	The teacher can ask the students to share all their finding and lead them to draw a conclusion about the relation between the circumference and the diameter of circles. The teacher can ask the students to try the other way of thinking about the ratio which means the circumference divided by the diameter.
Reshaping the sectors of a circle into several kinds of plane figures	The students are able to reshape the sectors of the circle into several kinds of plane figures and formulate the area of the circle using the area of plane figures they formed.	The students will only arrange the sectors into parallelogram and rectangle as they have done in the previous meetings. The students will try to reshape the sectors of the circle into several forms of plane figures including parallelogram, rectangle, triangle, and trapezoid.	Ask the students to try to reshape the sectors into other plane figures by remind them about the plane figures they have been learned in their 7^{th} grade. Guide the students to investigate the relations between the formula of the plane figures they formed and the formula of the area of the circle by remind them about how the find the area of the rectangle in the previous meeting.