DEVELOPING THE 5TH GRADE STUDENTS' UNDERSTANDING OF THE CONCEPT OF MEAN THROUGH MEASURING ACTIVITIES

Master Thesis

Said Fachry Assagaf

127785071

STATE UNIVERSITY OF SURABAYA

POSTGRADUATE PROGRAM

STUDY PROGRAM OF MATHEMATICS EDUCATION

2014

DEVELOPING THE 5TH GRADE SsTUDENTS' UNDERSTANDING OF THE CONCEPT OF MEAN THROUGH MEASURING ACTIVITIES

MASTER THESIS

A Thesis submitted to

Surabaya State University Postgraduate Program

as a Partial Fulfillment of the Requirement for the Degree of

Master of Science in Mathematics Education Program

Said Fachry Assagaf

NIM 127785071

SURABAYA STATE UNIVERSITY

POSTGRADUATE PROGRAM

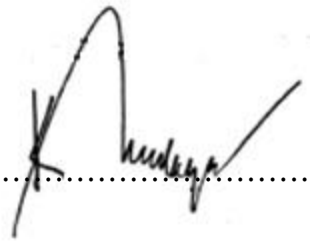
MATHEMATICS EDUCATION PROGRAM STUDY

2014

APPROVAL OF SUPERVISORS

Thesis by Said Fachry Assagaf, NIM 127785071, with the title *Developing the 5th Grade Students' Understanding of the Concept of Mean through Measuring Activities* has been qualified and approved to be tested.

Supervisor I, Date, July 8th 2014



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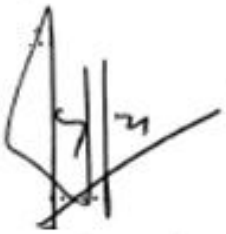

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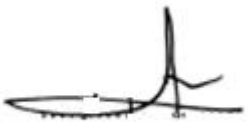


Dr. Agung Lukito, M.S
NIP 196201041991031002

APPROVAL

Thesis by Said Fachry Assagaf, NIM 127785071, had been defended in front of the Board of Examines on July 17th, 2014

Board of Examiners

Name	Signature	Position
Dr. Agung Lukito, M.S.	.....	Chairman/Member
Prof. I Ketut Budayasa, Ph.D.	.....	Member/Supervisor I
Dr. Tatag Eko Yuli Siswono, M.Pd.	.....	Member/Supervisor II
Prof. Dr. Siti Magfirotun Amin, M.Pd.	.....	Member
Prof. Dr. R.K. Sembiring	.....	Member

Acknowledged by
Director of Postgraduate Program

Prof. I Ketut Budayasa, Ph.D.
NIP 195712041994021001

DEDICATION

I dedicate this thesis to my mother

H. Daha, S.Pd., my father Y. S. Assagaf, SE.,

my brothers Said Hadly Assagaf, S.Sos., Said Asrul Adjmi Assagaf,

Said Chaerul Zany Assagaf, and my sister Syarifah Rahmi Inayyah Assagaf

for all support and pray.

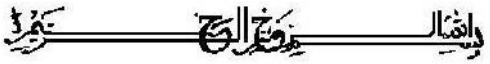
ABSTRACT

Assagaf, Said Fachry. 2014. *Developing the 5th Grade Students' Understanding of the Concept of Mean.* Thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University. Supervisors: (I) Prof. I Ketut Budayasa, Ph. D and (II) Dr. Tatag Yuli Eko Siswono, M.Pd.

***Keywords**:* The Concept of mean, Measuring activities, Realistic Mathematics Education (RME), Pendidikan Matematika Realistik Indonesia (PMRI), Design Research.

One of the first statistical measures that students encounter in school is the arithmetic mean, sometimes known as a mean or an average. Many studies have made an effort to promote students' understanding of the concept of mean. The present study also focused on developing students' understanding of the concept of mean based on Realistic Mathematics Education (RME) also known as Pendidikan Matematika Realistik Indonesia (PMRI). The goal is to contribute to a local instructional theory in learning the concept of mean. The research question of this study is how the measuring contexts can support students developing their understanding in learning the concept of mean. We used design research as the methodology to develop students' understanding in learning the concept of mean. We conducted two cycles of teaching experiments. The subjects were the 5th grade the students and one teacher in SD Inpres Galangan Kapal II Makassar. The first cycle consisted of 6 students, while the second cycle consisted of 29 students and one teacher. The data were collected through video-recordings of the teaching experiment, field notes, and students' written works. The data then analyzed by confronting the Hypothetical Learning Trajectory and the Actual Learning Trajectory. As the result of the analysis, six lessons on measuring were designed; (1) Average sentences; (2) repeated measurement contexts; (3) prediction on glider experiment; (4) prediction on glider experiment on the bar chart; (5) the students' heights; and (6) bookshelf context.

PREFACE



Assalamu Alaikum Wr.Wb.

The very first gratefulness is delivered to The Almighty, Allah SWT. The Greatest Creator and The Best Motivator for the blessing and mercy keeping me finish my thesis. Praying and greeting to beloved Prophet, Muhammad SAW who has shine the world with the light of Islam.

Although the author attempted to do the best, as a human being, I realize that there are still many shortcomings (the language, the contents, and the systematic) in writing this thesis. Suggestions and constructive criticism we hope for the best work in the future.

Furthermore, a lot of thanks to my beloved family; my father, Y.S.Assagaf,SE.; My mom Hj. Daha, S.Pd.;my brothers Said Hadly Assagaf, S.Sos., Said Asrul Adjmi Assagaf, Said Chairul Zany Assagaf, and my little sister, Syarifah Rahmi Inayyah Assagaf.

In addition, the highest appreciation also goes to :

1. Prof. I. Ketut Budayasa, Ph.D., Dr. Tatag Yuli Eko Siswono, M.Pd., as my supervisors in State University of Surabaya
2. Frans van Galen and Dolly van Eerde, as my supervisors in Utrecht University.
3. Dr. Agung Lukito. MS. and Maarten Dolk as the coordinator of the International Master Program on Mathematics Education (IMPoME) in State University of Surabaya and Utrecht University.

4. PMRI Center Board for the opportunity given to me as one of the grantees of the International Master Program on Mathematics Education.
5. All the lecturers and staff of Postgraduate, State University of Surabaya.
6. All the lesctures and Staff of Freudhental Institute, Utrecht University.
7. Dra. Mardawiyah, S.Pd., as a headmaster of SD Inpres Galangan Kapal II Makassar dan Agus, S.Pd., as the classroom teacher of grade 5 for the kindness during my research in the school.
8. All of the students in grade 5 SD Inpres Galangan Kapal II Makassar academic year 2013/2014 as the subject in my research.
9. Prof. Dr. H. Hamzah Upu, M.Ed. and Sabri, M.Sc. for the support and helps.
10. My friends of IMPOME batch 4; Ronal, Cici, Yoyo, Rani, Andrea, Wahid, Boni, Jeki, Siwi, Pipit, Dimas, Ahmad, (Alm) Jakia, Andika, Rafael, Rahmi and Lidya.
11. Indonesian community in the Netherlands.
12. My beloved friends from ICP Math 07.
13. Pak Usman, Pak Asdar, Pak Ali, Kak Aqil, Kak Ulla, Kak Agus, Kak Fajar, Kak Bustang, Ompes, Wilda, Wakkang, Siraj, Erwin and Rani.

Finally, I hope that God will always bless them along their life. Amin Ya Rabbal Alamin.

Surabaya, August 2014

Author,

Said Fachry Assagaf

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CHAPTER I

INTRODUCTION

A. Research Background

One of the first statistical measures that students encounter in school is the arithmetic mean, sometimes known as a mean or an average. Almost all countries introduce the mean starting from primary school. In Indonesia, for instance, the new curriculum 2013 mandates the schools to introduce the concept of mean from 5^{th} grade (the previous curriculum started from 6^{th}). It is important to understand the concept of mean because it is not only a mathematical school topic but it is also frequently used in everyday life, for example, the average of the velocity of a car, the average of the students' scores in a classroom, and the average of people's incomes in a country.

However, psychologists, educators, and statisticians all experience that many students, even in college do not understand many of the basic statistical concepts they have studied. Some studies also describe the difficulties regarding the concept of mean. For instance, Strauss & Bichler (1988) show the difficulties of students in the age of 8, 10, 12, and 14 years regarding the properties of the mean (e.g. the number representing the mean does not have to correspond with the physical reality). Gal et al. (1989) also described that students in grade 6 who have learnt the concept of mean still cannot employ the mean to compare two sets of data. In addition, Zazkis (2013) who investigated high school students (grade 12) states that most of the students

focused on calculating the mean and carried out the algorithm when the problem related to mean as the fixed total problems are given. Even in the college level, the students have a limited understanding of this concept (Hardiman et al., 1984).

Most students understand the mean as an “add-them-all-up-and-divide” algorithm (Zazkis, 2013). Moreover, many elementary and middle school mathematics textbooks have defined the mean as the way it is computed (Bremigan, 2003). It is also supported by the exercises and the examples elaborated which do not allow students to develop their understanding of the concept of mean. Most of them are procedural problems where the students only use the formula when the data are given. Unfortunately, in Indonesia, most teachers teach the concept of the mean in the traditional way, focusing on the computation but not the understanding of the concept of mean. They tend to follow the definition and the problems provided on the textbooks without elaborated more on developing students’ understanding of the concept.

There are many studies about the way the mean is introduced as an algorithmic procedure (e.g. Cai, 1998., Bremigan, 2013). This way leads students to focus on how to compute the mean, but not on the concept of mean, even though the application of the mean is more than an algorithm. Cai (1998) shows just a half of 250 six-grader students in his study were able to correctly apply the algorithm to solve a contextualized average problem even though the majority of the students knows the algorithm. The mean concept is

complicated by the different approaches (Mokros & Russel, 1995) and the properties (Strauss & Bichler, 1988). Therefore, we need to support students to learn not only how to calculate the mean but also to develop their understanding of the concept of the mean itself. However, hardly any Indonesian study with this concept of mean has been carried out neither in the theory nor practical studies.

Based on those issues, it is important to design meaningful contexts and activities in order to support the developing of students' understanding about the concept of the mean. Therefore, the present study concerned on developing students' understanding in learning the concept of mean.

B. Research Question

Based on the background above, the research question is formulated as follows "*how can measuring activities support 5th grade students developing their understanding in learning the concept of mean?"*. In supporting students, we design a series of learning sequences. The learning sequences consist of some activities on measuring which lead students developing their understanding of the concept of mean.

C. Objectives of Study

The research aim of this study is to contribute to the local instructional theory in developing students' understanding of the concept of mean. The contribution was an innovation in designing the series of learning sequences

(later we called HLT) that can support students developing their understanding of the concept of mean.

D. Definition of Some Key Terms

It is important to define the key terms involve in this study in order to avoid different or mis-interpretation for the terms. The following terms are

1. Measuring activities

   Measuring is to ascertain the extent, dimensions, quantity, capacity, etc by comparing with a standard unit. Measuring activities in this study refers to a series of learning activities (hands-on or non hands-on activities) related to measurement. Three types of measuring activities includes in this study are the height, the weight, and the distance measurement.

2. Understanding

   Understanding defined as ability to making connections between existed schemes or information and the new scheme or information.

3. Supporting

   Supporting defined as giving assistance or encouragement. In this study, supporting is not giving help for students all the time. It means to facilitate and encourage students in order to understand the concept.

4. Developing students' understanding

   Developing defined as a progress. Developing students' understanding means a progress connecting the existing knowledge and the elements of the network and the structures as a whole.

5. The concept of mean.

   The mean is one of the measures of central tendency together with median and mode. The mean is used to describe the data by taking one number as the center of the data by using the formula as add the data and divided it by the number of data.

6. Students' understanding of the concept of mean.

   Students' understanding of the concept of mean refers to the ability of students to make connection between the students' prior knowledge and the concept of mean itself as one of measures of central tendency. In this study, we elaborate the indicators of understanding of the concept of mean as follows:

   a. Distinguishing some interpretations of the word average in daily life
   b. Identifying the strategies to describe the data
   c. Using the diagram to represent the mean
   d. Know and apply the concept of the mean
   e. Compare two data sets

E. Significances of Study

Related to the objectives of the study above, the present study is expected to be able to contribute to the development of a local instructional theory in domain of the concept of mean. The contributions are the means to teach the concept of mean and also the description of the process of learning the concept of mean.

For the teacher, it is used as the information about how the measuring activities work in order to support students understanding of the concept. It is also expected to become references for other researchers who will conduct studies on relevant issues.

CHAPTER II

THEORETICAL FRAMEWORK

A. Measures of Central Tendency

The mean in this study refers to the arithmetic mean, one of the measures of central tendency in statistics together with the mode, median, and midrange. It is not an isolated topic. It is interrelated with the concepts of center and spread. Therefore, we also need to know other concepts that are related to the mean particularly the mode, median, and midrange. Moreover, this study uses those concepts as the basis in the instructional design. Further, the measure of central tendency will be discussed followed by a short description of the concept of mode, median, midrange and mean. And after that, the discussion will zoom into the concept of the mean and the studies related to promote students' understanding of the mean.

Measures of central tendency (or measures of center) are measures in descriptive statistics. The basic idea is to summarize the data distribution by giving one score as the representation of the data. Heiman (2011) described the measures of center as :

> … a number that is a summary that you can think of as indicating where on the variable most scores are located; or the score that everyone scored around; or the typical score; or the score that serves as the address for the distribution as a whole. (p.62).

For instance, data about the height of students in one classroom are given. Then, someone will say that the height of the students in the classroom

is around 120 cm. This is the idea of the measure of center, taking one height as the representative to describe the data distribution. The height indicates where the center of the distribution tends to be located.

Taking one number to summarize the data is not random. The trick is to compute and decide one correct number in such a way that the score can describe the distribution accurately. There are four methods that people commonly use to find the number: mode, median, midrange and mean.

The mode is used to find the number that occurs most frequently. For instance, the data of the height (cm) of five students are 130, 140, 140, 140, 120; then the mode is 140 cm because it occurs more often compared with other data. Regarding the frequency of the data, it is also possible that there are two or more data have the same frequency, for example: 130, 140, 130, 140, 120. In this case, there are two modes, 130 and 140. This distribution is called bimodal, and if there are three modes it is called trimodal and so on.

The median is the point that divides a distribution of scores into two parts that are equal in size (Spatz, 2008). To find the median, the data should be arranged first from the highest to the lowest or vice versa. For example, the data are 130, 140, 140, 140, 120. Firstly, the data should be ordered, 120, 130, 140, 140, 140. And then, the median is the middle point, 140 cm. However, there is a possibility that there is no middle point (if the number of data is even), for instance, 120, 130, 130, 140, 140, 140. Then, the median is half of the scores 130 and 140 which is 135. Thus, the median may or may not be an actual data.

The midrange is the halfway point between the maximum and minimum score. For instance, the data 110, 140, 140, 130, 120, have as the midrange 125 (add 110 and 140 and then divided the result by 2). The midrange is not as popular as the median and mode. Some people argue that it is not exactly accurate since it just considers two scores, maximum and minimum, instead of the whole data. However, this study uses the midrange as a simple way that students may come up with during the lesson about the mean.

The mean or arithmetic mean is also called the average. To find the mean, all of the data are added and after that divided by the number of data. For example, the data 110, 140, 140, 130, 120 have as the mean, 128 cm; 640 (the sum of all data) divided by 5 (the number of data). In the following, we will focus on the concept and related studies of the mean.

B. The Concept of Mean

The mean is the most common measure of central tendency that is used in many studies. It is because the mean includes every score and does not ignore any information in the data. In the school, most of the textbooks define the mean as the way it is computed, add the data and divide it by the number of data. However, it is not a simple mathematical entity.

The mean is not as simple as the algorithm. It is interrelated with the concepts of center and spread. The interrelation has been described by Strauss & Bichler (1988). The study investigated grade 4 through 8 students on the

properties of the mean. It proposed seven properties of the arithmetic mean. The properties are :

1. the average is located between the extreme values;
2. the sum of the deviations from the average is zero;
3. the average is influenced by values other than the average;
4. the average does not necessarily equal one of the values that was summed;
5. the average can be a fraction that has no counterpart in physical reality;
6. when one calculates the average, a value of zero, if it appears, must be taken into account;
7. the average value is representative of the values that were averaged.

These properties show clearly that the mean has a strong relation with other statistical measures. In addition, instead of all seven properties, the learning design of this study will focus on four properties (a, c, d, and g). The (b) property is quite difficult for primary school while the (e) and (f) properties are not appropriate to the context.

The complexity of the concept mean also is illustrated in a study by Mokross & Russel (1995). They found five predominant approaches used by students; (a) average as the mode, (b) average as an algorithm, (c) average as reasonable, (d) average as a midpoint, and (e) average as a mathematical point of balance. The students who used the first two approaches did not recognize the notion of representation, while the three other approaches were considered to imply the concept of mean as the representation of a data set.

This present study considers (a), (c), and (d) as the conjectures students may use during the lesson. Since the focus of the study was young students that have never been taught about the mean, (b) and (e) are excluded.

In addition, the interpretation of the mean is not easy. Konold & Pollatsek (2004) illustrates four interpretations of the mean : (a) data reduction, (b) fair share, (c) typical value, and (d) signal in noise. Table 2.1 is taken from Konold & Pollastek's article which provides the example context for the four interpretations. From the four interpretations, the main focuses of this study are the typical value and the signal in noise.

Table 2.1. The four interpretations of the mean

Interpretation/ Meaning	Example context
Data reduction	Ruth brought 5 pieces of candy, Yael brought 10 pieces, Nadav brought 20, and Ami brought 25. Can you tell me in one number how many pieces of candy each child brought? (From Strauss & Bichler, 1988)
Fair Share	Ruth brought 5 pieces of candy, Yael brought 10 pieces, Nadav brought 20, and Ami brought 25. The children who brought many gave some to those who brought few until everyone had the same number of candies. How many candies did each girl end up with? (Adapted from Strauss & Bichler, 1988)
Typical value	The numbers of comments made by eight students during a class period were 0, 5, 2, 22, 3, 2, 1, and 2. What was the typical number of comments made that day? (Adapted from Konold & Garfield, 1992)
Signal in noise	A small object was weighed on the same scale separately by nine students in a science class. The weights (in grams) recorded by each student were 6.2, 6.0, 6.0, 15.3, 6.1, 6.3, 6.2, 6.15, 6.2. What would you give as the best estimate of the actual weight of this object? (Adapted from Konold & Garfield, 1992)

Furthermore, an interesting article from Bakker & Gravemeijer (2006) provided an historical phenomenology of the mean. One way of using the

mean in ancient times is as estimation, which is quite similar to the interpretation of the mean as the signal in noise.

C. Developing Students' Understanding of the Concept of Mean

Nickerson (1985) defines the understanding as the ability to build a bridge as a connection between one conceptual domain and another. He also states that understanding always grows if ones know more about the subjects. Hiebert and Carpenter (Barmby, et al., 2007) specifically describe that the degree of understanding is determined by the number and the strength of the connection between the conceptual domains. In addition, Piere and Kieren (Meel, 2003) argue that understanding as a whole, dynamic, non-linear, and never-ending process. It is still develop. Meel (2003) describe the development of understanding as a process to connecting the representations to a structured and cohesive network. The connections require the recognition of the relationship between the concept and the elements inside the concept as a whole. Developing also means as a progress of doing something.

In particular, Meel (2003) provides a brief story of the development of the concept of understanding, one of them is conceptual understanding. Conceptual understanding refers as the comprehension of mathematical concepts, beliefs, and relations. NCTM describes ones may called has conceptual understanding when they provide evidence that they can :

1. recognize, label, and generate examples of concepts;

2. use and interrelate models, diagrams, manipulative, and varied representations of concepts;
3. identify and apply principles;
4. know and apply facts and definitions;
5. compare, contrast, and integrate related concept and principles;
6. recognize, interpret, and apply the signs, symbols, and terms used to represent concepts

Based on the theories above, understanding in this study is defined as making a connection between existed scheme or information and the new scheme or information. Therefore, students' understanding refers to the ability of students to make a connection between their prior knowledge and the new knowledge that have learned in the classroom. Moreover, developing means a progress of doing something. Thus, the developing of students' understanding refers to a progress of connecting the existing knowledge and the elements of the network and the structures as a whole.

This study focuses on the concept of mean. Thus, the developing of students' understanding of the concept of mean refers to the progress in making connection between the students' prior knowledge with the concept of mean itself as a measures of central tendency. In order to make the understanding visible, the present study provides indicators of understanding which refers to indicator of conceptual understanding from NCTM. The indicators as follows:

a. Distinguishing some interpretations of the word average in daily life

b. Identifying the strategies to describe the data

c. Using the diagram to represent the mean

d. Know and apply the concept of the mean

D. Measuring Activities

Based on the dictionary, measuring define as to ascertain the extent, dimensions, quantity, capacity, etc., of, especially by comparison with a standard (http://dictionary.reference.com/browse/measure). For instance, measuring the height of a student means to ascertain the height of the student by comparing in meter or centimeter (the length unit) using a length measurement tool such as a ruler or a measuring tape.

This present study focuses on measuring activities in order to support students developing their understanding of the concept of mean. Measuring activities here refers to a series of learning activities including hands-on and non hands on activities. The measuring is one of the strong activities or context used to introduce the concept of mean (Konold & Pollastek, 2001).

The activities in this study consist of measuring the length, the weight, and the distance. In the beginning, we focus on repeated measurement context on height and weight of an object. Repeated measurement refers to the situation on which multiple measurements obtain from each experimental unit (Davis, 2002). We also provided two hands-on activities which are the glider activity and the measuring height activity. The glider activity related to measure the distance of the glider from the person who throw the glider to the

place where the glide felt. Meanwhile, measuring height refers to measure their students' height by using measuring tape. These two hands-on activities lead the students to collect and play with their own data. By playing with their own data, it might motivate them to describe the data by using the idea measures of center such as the mean.

E. Studies on Promoting Students' Understanding of the Concept of Mean

Many studies focus on how the mean is taught in school. Some studies made an effort to promote students' understanding of the mean. Some models and contexts were tested in those studies. Hardiman et al. (1984) tested whether improving students' knowledge of balance rules through experience with a balance beam promotes their understanding of the mean. The results show that the students who were classified as non-balancers performed significantly better than the control group.

Zaskis (2013) explored the students' understanding of the statistical idea of the mean – inference from a fixed total. He investigated the way high school students solved three tasks related to the concept of mean as "inference from a fixed total". The idea of 'inference from a fixed total' means that even though the values of the data are different, as long as the total is the same, the average is also the same. He suggests that this reasoning should be an additional focus for the next study and instructional development since most of the participants still focused on the algorithm rather than on the notion of the mean as a fixed total.

Furthermore, Cortina (2002) also developed the instructional conjectures to promote students' understanding of the arithmetic mean as a ratio. Gal et al. (1990) argue that mastery of the proportional concept may be a prerequisite for learning the concept of "central tendency". The proportional concept is also shown in a comparing problem of Gal, et. al (1989). They use the context of comparing problems to investigate the development of statistical reasoning of elementary school children. This comparing problem is a good context to promote students' understanding of the statistical concept, because when students solve the problem, they have to consider the mean of the group, the size of the group, and the spread of the data in the group. Bremigan (2003) constructed two problems, the first is related to the comparing problems and the second is the "what if" problems based on the properties of the mean that Strauss and Bichler described. The aim of all these studies is to develop a meaningful understanding of the mean.

Promoting a meaningful understanding also means that the problems or the contexts should be meaningful for students. "*The function of the context is to describe such circumstances that give meaning to words, phrases, and sentences*" (Gilbert, 2007, p.960). One of the studies that used the context to promote the students' understanding of the mean was conducted by Lestariningsih, et al. (2012). They used a local fairy tale as the context. They investigated how the role of the context can support 6^{th} grade students to understand the mean. In the study, they used a bar chart to promote the understanding.

F. Realistic Mathematics Education (RME)

RME is a theory of mathematics education which believes that mathematics should be taught in a meaningful way to students. The idea of RME has largely been determined by Freudenthal's view of mathematics as a human activity. It must be related to the reality, close to the students' world, and relevant to the society (van den Heuvel-Panhuizen, 2001). The word 'realistic' means that the problems or contexts should be realistic for students. It does not necessarily mean that the students should be able to encounter the contexts in their lives. It could also be a problem that students can imagine as a real situation (Bakker, 2004).

In this study, we design a Hypothetical Learning Trajectory (HLT) based on the ideas of RME. The problems, contexts and activities in the HLT are developed in order to promote a meaningful understanding of the concept of the mean. The design is influenced by the following five tenets of RME (Treffers, 1987, cited in Bakker, 2004):

1. Phenomenological exploration

   The rich and meaningful contexts are explored to develop the basis for a meaningful understanding of the concept. The context in this study was measurement. One problem related to the repeated measurement and two problems about comparing two groups of data are used. The context of repeated measurement here is related to the error in measuring the height of one person which is a common problem that people regularly encounter in daily life. Meanwhile, comparing problems are rich contexts that allow

students to use many strategies to summarize, describe, and compare the data.

2. Using models and symbols for progressive mathematization

    The use of table and bar charts are considered as the model used in the design. The table acts as the way to organize and show the data in an effective way. Meanwhile, the bar chart supports the students' informal knowledge to find the mean with a visual representation of the formula on a more abstract level.

3. Using students' own construction and productions.

    It is assumed that using students' own data makes the learning meaningful and interesting for them. It allows them to collect their own data and compare them with other data. Therefore, using the students' own construction and productions is an important part of the instruction.

4. Interactivity

    Small and whole class discussions are included in the instruction. The discussions allow students to argue with and comment on each other. The different strategies students may use to summarize the data are the main part of the discussion.

5. Intertwinement

    It is important to consider the integration of the concept with other concepts within one domain or with other domains. In the instructional sequences, the students are not only learning about the mean but also other measures of central tendency such as mode, median, and midrange.

Moreover, the diagram and the table are also focused on the instruction. In addition, the measuring activity with the length measuring tool is also integrated into the lesson. Regarding the other domain, there is an experiment on that is adapted from a science lesson which can be considered an intertwinement with another domain. Nevertheless, the experiment focuses more on the data instead of the science aspect.

G. Indonesian Curriculum

In the Indonesian curriculum, the concept of the mean is introduced in primary school. In primary school, the curriculum requires students to calculate the mean of a simple set of data. At the level of junior high school students are asked to deal with more complex data and variation of the use of the arithmetic mean. Moreover, at the senior high school level students are required to deal with the mean from the interval data or from the graph (Kemendiknas, 2013). The concept of mean is developed during these three levels of education. In the end, the students are expected to have a strong concept of the mean.

In 2013, the Indonesian curriculum has been changed. The new curriculum integrates some topics from different subject matters in one theme. Therefore, some topics including the mean are changed and shifted from one level to another. In the previous curriculum, the concept of mean was introduced in grade 6. The new curriculum 2013 divides the topic of the mean in two grades, grade 5 and 6. In grade 5, the students are expected to

understand the meaning of the mean, while grade 6 requires both understanding how to calculate the mean in a set of data and comparing the mean of two sets of data. These changes of the competences clearly show that the curriculum expects students to have a meaningful understanding of the concept of the mean instead of how to compute the mean.

Table 2.2. The concept of mean in the Indonesian curriculum.

The previous curriculum	The new curriculum 2013
Grade 6. Determining the arithmetic mean and the mode of the set of data.	Grade 5. Understanding the mean, median, and mode of set of data.
	Grade 6. • Understanding how to calculate the mean, median, and mode in a simple statistics. • Comparing the interpretation of mean, median, and mode of two different sets of data.

Based on the description above, the following conclusions can be drawn:

1. The concept of the mean is quite difficult to understand.
2. Many studies focus on how to promote a meaningful understanding by the students of the concept of the mean.
3. The shift in the Indonesian curriculum and the development of the concept of the mean at different levels of education indicate how important it is to introduce the mean in a meaningful way at an early stage.

However, so far, there has been little discussion and little studies in Indonesia on the concept of mean, neither in theory nor practice. Therefore, the aim of this study is to contribute to a local instructional theory for grade 5^{th} students in learning the concept of mean.

H. Hypothetical Learning Trajectory

Hypothetical Learning Trajectory (HLT) is a framework of learning activities. It consists of the learning goal, the learning activities, the predictions of students' thinking, and the teacher's reactions. This chapter describes the HLT of the topic mean. There are six lessons that designed to develop students' understanding of the concept. The activities and the contexts are constructed based on the theories, some result of previous studies and discussion with the experts. The HLT of the six lessons is elaborated as follows.

1. Starting Points

The mean is introduced in 5^{th} grade. Therefore, this is the first time students encountered the concept of mean. Some required knowledge before the lesson are needed to describe in order to support students to develop their understanding of the topic. The starting points of the lesson are as follows:

Students are able to:

a. understand the arithmetic operations on integer.
b. understand the arithmetic operations on decimal.
c. measure the length and the distance by using measuring tape or other length measurement tools.
d. make sentences and describe the meaning of a word in the sentence.

2. Lesson 1: Average sentences

Learning Goal

Students are expected to be able to distinguish some interpretation of the word “average” in daily life.

Materials

Worksheet.

The pictures of apples and its weights.

Mathematical Activities

1. ‘Average’ sentences

   Students are asked to write down two sentences they have ever used, seen, or heard in daily life with the word “average” and given the meaning of the word “average”. The teacher encourages students to write the different use of the meaning of the word “average”. After that, they discuss in their group about the meaning of “average” on their sentences. In the next session, there is a whole class discussion to hear and discuss the sentences from the students. The discussion stresses on what students know and their interpretation of the word “average”.

2. Apple Sentence.

   After the discussion on the students’ sentences, the teacher posts and discusses with the students a sentence of average as follows:

   “*The average weight of an apple in 1 kg apple is 0.25kg*”.

This sentence leads students to re-think about the meaning of the word average. The teacher shows the photo of 1 kg apples and some apples' weights from the 1 kg apples. All of the apples have different weights. Therefore, the average here does not mean that most of the apples have 0.25kg weight. The activity aims at developing students' understanding about the idea of average and to realize that the average is different with the mode.

Figure 2.1. The different weights of apples

3. Repeated measurements of height.

In this activity, the teacher tells a story of her friend – Anita's problem on her heights. Anita took five times measures of height in this month. In the beginning of the month, she was entering a fitness club. The fitness club required to measure the height and resulted 171.5 cm. Two days later, she was going to make a driving license at the police station. In the police station, the official was measuring her

height 170 cm. A week after, she checked up at the hospital. The nurse measured her height and got 171.3 cm. Yesterday, before taking a roller coaster, there was a measuring gate and resulted 172 cm. This week, she wants to apply for a job and fills the application form. In the form, there is information on height. However, she is confused. Now, she has four different measuring of her height. To make sure, she tried to measure herself in her house and resulted, 170.2 cm. She now has five different heights.

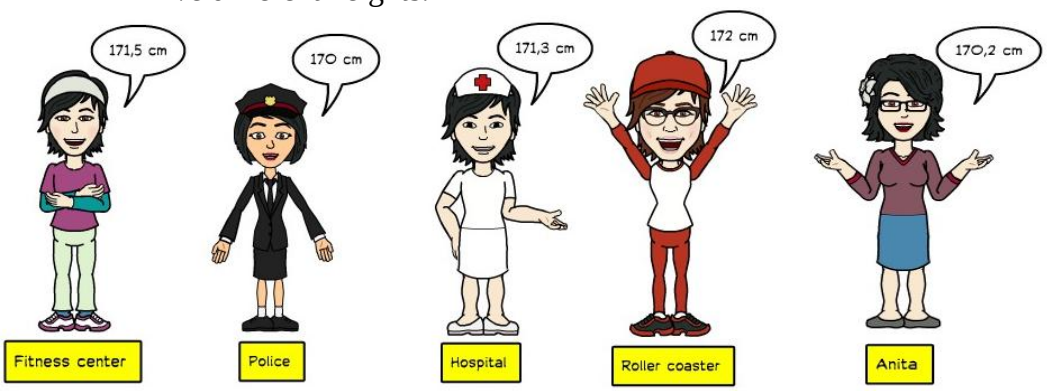


Figure 2.2. Anita's height

In this activity, students are asked to help the Anita to decide her height. The problem emphasizes on the chosen height by using the idea of measure of central tendency, such as mode, median, range, or average. However, this activity only focuses on the students' strategies, while introducing the name of the strategy (whether it is mode, median, range, or average) will be at the next meeting. Besides, before the worksheet is given, the teacher asks three or more students to measure the height of one student and make the students realize that the different measures of height might happen.

Prediction of students' responses

Regarding the sentences, the students may come in three kinds of sentences: (1) the sentences without a number (e.g. : average woman like a handsome man), (2) the sentence involve a number (e.g.: average height of students in a classroom is 150 cm), and (3) the unrelated sentences. There is also the possibility that one of the kind of the sentences does not appear during the lesson.

Concerning to the meaning of the word average, the students may answer that the average means the word such as 'most of' or 'many'. This answer indicates that the students aware of the idea of a mode. Besides, there is also the possibility that the student's answer indicates that she/he knows what the average (the mean) which is add all of the data and divided it by the number of the data. Similarly with the apple sentences, most of students may say "most of the apple weight is 100g". Since we assume that they will interpret the average as the mode.

In repeated measurement activity, the students may think randomly to choose the height. They may choose the number in the middle or the height, which are they thinking are more convincing such as the measurement from the hospital. However, there are also possibilities that they may use the median, midrange, or even mean.

Actions of the teacher

At the beginning, the teacher writes some sentences on the whiteboard. And then together with the students classify the sentences into

three parts regarding the classification above. Regarding the meaning, when the students' answer it by using the word such as 'most of' or 'many', the teacher should make the answer precise by giving follow up questions such as "what do you mean 'most of'?" "How much do you think 'most of' is?" or "Does the 'most of' mean more than 50%? Or less? Or equal?". Meanwhile, when they have the idea of median, midrange, or even mean, the teacher can bold their strategies in front of the class without telling them the name of those strategies.

During the "apple" sentence, the teacher should encourage students to realize that the average is different with the mode. Post a question such as "Do you think all most of the apples have 0.240kg of weight?"

In the second activity, the teacher encourages students to take all heights into account. The teacher asks students "how do you think we could do if we want to consider all of the heights?" Some students may give random strategies in order to take the heights into account. When the students grasp the idea of average to "add them up and divided" strategy, the teacher can introduce that is the idea of average. However, it does not matter if the students won't come up to the idea of the "Average". It will be stressed in the second meeting. Besides, some students may think about the midrange or the median. At this time, the teacher can stress these ideas. In the second activity, all these ideas will be introduced.

3. Lesson 2: Glider Experiment (Prediction)

Learning Goals

Students are expected to be able to identify the strategies to describe the data.

Materials

Worksheet	Papers
Measuring tape	Straw
Ruler	Glue
Scissor	

Mathematical Activities

Cylinder glider

At this meeting, the students have an experiment to make Hagravens' cylinder glider. This activity adapted by Ainley, J., Jarvis, T. and McKeon, F. (2011). Lawrence Hargrave (1850-1915) was an Australian aviation pioneer, inventor, explorer, mason and astronomer. One of his glider models is:

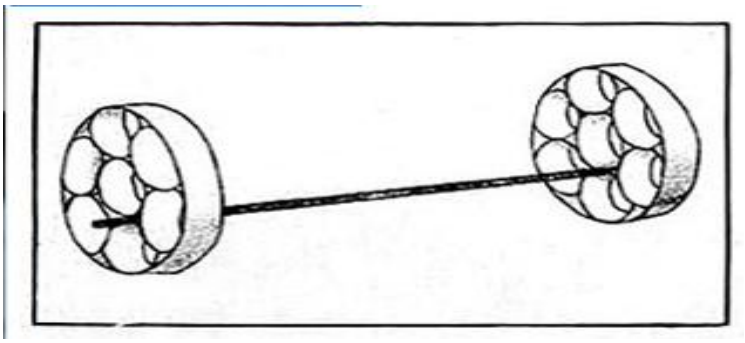


Figure 2.3. Hargravens' cylinder glider

The students in groups of 3 or 4 makes gliders from loops of paper attached to a drinking straw. The construction of the gliders allows (tail) loop to be moved along the straw. Look at the following picture (Figure 2.4). In this activity, the students will investigate the effect of the loops between the two gliders with the length of the flight. They will collect and compare the data of two gliders.

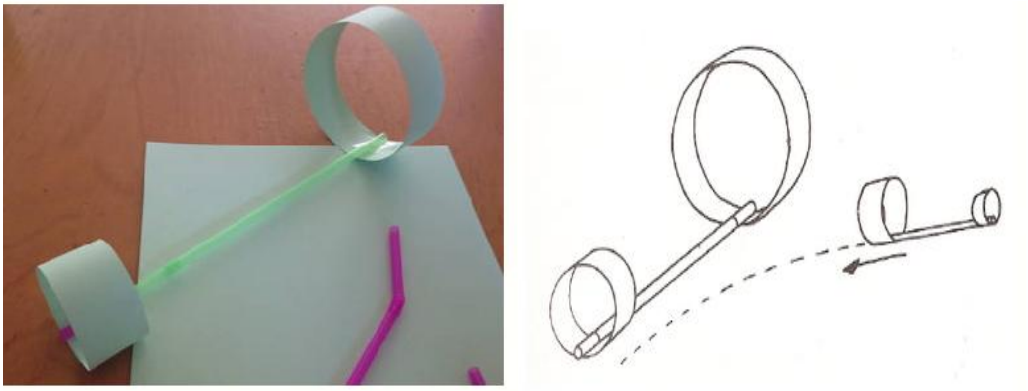


Figure 2.4. The glider and the way it is thrown

In the beginning, the teacher asks students to make some gliders. The teacher allows students to decide on the diameter as long as the two loops of the glider are different. After that, the students throw the gliders away and measure the length of the distance. The students will collect the data of one glider and write it down in the worksheet. It does not matter whether the students will produce different data. Instead of the trends, the activity focuses on the data (the numbers).

The students will work on the worksheet. There are three main questions in the worksheet: describing the glider characteristics, collecting the data, and predicting the distance of the glider. The first question expects the students to describe at least the color, the distance between the

loops, the circumference of the front and back loops of the glider. The second question is throwing the glider away and measure the distance from the student who throws the glider to the position where the glider stops. The third question is finding the glider distance by considering the five data. The distance is the prediction of what will be the distance of the next throw.

Two main discussions are held in this meeting: (1) how to measure the glider' distance and (2) the different strategies to decide the prediction of the distance of the glider. The first discussion purpose is to have an agreement in measure the distance of the glider. Some points to discuss as follows:

a. How to measure the distance?
    1) Is it from the person who throws away the glider to the nearest part of the glider? Or farther away? Or the middle?
    2) Is it when the glider hit the floor? Or until it stops?
b. How to see the measuring tape if the measurement is between two numbers? Do we need to boundary up? Or down?
c. How to throw the glider? Do we throw it with full energy or with a slowly?
d. Other points that students may found during the time practicing.

The second discussion focuses on the students' strategy to find the prediction. The discussion is expected to discuss the mean, median, mode,

and range strategy. After that, the teacher will introduce the name of those strategies, whether it is mean, median, mode, or range.

Prediction of students' responses

Related to predict the next throw, some strategies that students can come up, such as:

a. Median: taking one middle distance from the data,
b. Midrange: add the maximum and minimum distances and divided it by two,
c. Mode: the most appeared distance,
d. Maximum or minimum distance: taking the farthest or nearest distance.
e. Random: taking one distance not from the data or estimating without any procedure.

Actions of the teacher

If the students use the idea of the mode, median, midrange, or even mean. The teacher may ask them to present their idea in front of the class later on the whole class discussion. Particularly the median and midrange strategies, the teacher ask the students to present it together. In the whole class discussion, the teacher may discuss some points (Table 2.3).

Meanwhile, if the students use the strategy of maximum or minimum value, the teacher should stress that they have to consider other measures otherwise there is a distance which is far away from the prediction. The good prediction should be close enough to all the data that we have. Similarly with the random strategy, the teacher should ask the students

how they got the number. The teacher may ask questions such as: “how did you get the number? How do you estimate? Why did you choose this number?” Ask them to think about a reasonable procedure in determining the prediction. During the whole class discussion, the teacher may bring this idea to see other students’ opinion on these three strategies.

Table 2.3. Points to discuss

Mode	– If there is no same value, what did you do then? – If there are two values with appeared the same times what did you do? – The teacher can also relate the mode strategy to the sentences of mode in the first meeting.
Median and Range	– What do you think the difference between these two strategies? – What do you think the strong and weak points of these two strategies?
Mean	– How did you find the strategy? – The teacher emphasizes this strategy for the next meeting.

4. Lesson 3: Glider Experiment (Compensation Strategy on Bar Chart)

Learning Goals

Students are expected to be able to derive the formula of mean by using compensation strategy on the bar chart.

Materials

Worksheet

Glider

Ruler

Mathematical Activities

In this meeting, the teacher plays with his/her own glider. The teacher in the beginning of the lesson may show his/her glider for the

students and throw the glider once to show for students how the glider flies. After that, the teacher will show the second throw data as follow:

Table 2.4. The data of throwing the glider

Number of experiments	Glider 1
1	110 cm
2	100 cm

In this data, the teacher asks students to find the “typical distance” as the representation of the glider data. The bar chart is used in finding the typical distance (Look the worksheet number 1). The worksheet consists of three questions. The first question is to find the typical distance of the two data above. After that, the data and the typical distance interpret into a bar. In the second question, the students are given one more addition data, 60 cm. Again, the students are asked to directly interpret the data into the bar and find the typical height. This question emphasizes on the compensation strategy in the bar. The students are expected to realize the strategy to share the amount of value to another value in such a way that the value will result the same. The fourth data, 95 cm, is given as the third question in the worksheet. Interpreting the data into a bar and finding the typical distance from the bar also are the activities in this question. At this time, the students are expected to realize how to find the mean through the compensation strategy in the bar.

Every question is ended with a short discussion on how the students find the typical distance and interpret it into a bar. The discussion stressed at the idea of the average, which add the data and divided them by the

number of data through the compensation strategy on the bar. From the discussion of the first question, the teacher should start to emphasize these points:

a. The strategy between the median and midrange. These two strategies obtain the same answer.
b. The way students draw the bar. How the students put the number on the x-axis and how the length of the bar they drew.
c. The way students draw the typical distance on the bar.

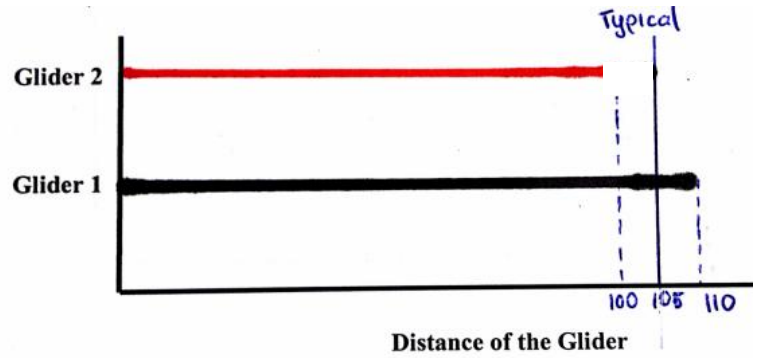
The further discussion also stresses on the points. The formula of mean is introduced implicitly through compensation strategy using the bar chart. Lastly, the students are expected to know how to find the average of the data.

Prediction of students' responses

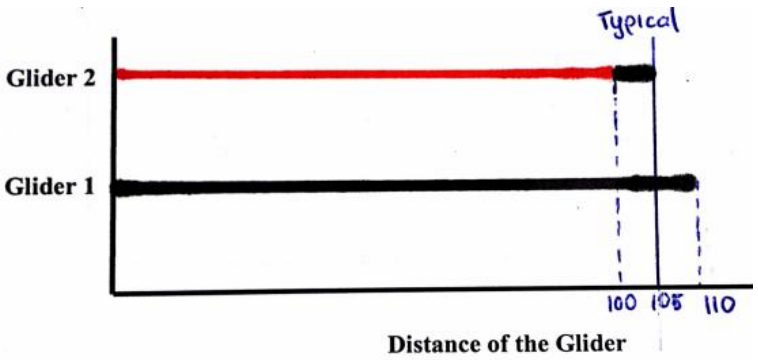
Regarding the strategy to get the typical distance, the students may use the median or midrange strategy. The median may take the middle number by dividing the first and the second data. Meanwhile, the midrange uses the maximum and minimum data and divided it by two. These two strategies obtain the same result in the first question, because there are only two data. The strategies do not obtain the same result for the second and the third questions.

In the bar, the students may directly draw the answer without realizing the compensation strategy (figure 2.5.a). They just draw the bar and the typical distance (figure 2.5.b). Similarly, when they ask to explain

their strategy based on the bar (Worksheet 1c, 2b, and 3b), the students may explain based on the median and midrange strategies instead of the compensation strategy.



a



b

Figure 2.5. (a) the bar without compensation strategy;

(b) the bar with compensation strategy

Actions of the teacher

In the typical distance question, particularly for the first question, the students are expected to use the median or midrange strategy. Therefore, when the students use the strategies, the teacher may ask them to present their result during the whole class discussion. It is different with the

second and the third strategies. When they still use the strategies, the teacher encourages the students to realize that the question in the worksheet aims at explaining their strategy based on the bar instead of the data. Since in the whole class discussion for the first question the teacher have already emphasized the compensation strategy, the teacher may post question for students to remember the strategy, “see the first question on how to find the typical distance based on the bar”.

Regarding the interpretation of the bar, the teacher can have a nice drawing of the bar with two colors of markers in the blackboard and starts to ask questions such as: “what do you think we can do to find 105 cm? What does make the 100 cm bar become 105cm? And what does make 110 cm bar become 105 cm?” The questions support students to realize the compensation strategy which is to give and take some amount of value in such a way that the bar obtains the result values.

Related to the question 1c, 2b, and 3b, the teacher may directly say to the students that the questions need to answer based on the bar instead of the strategy from the data. The teacher may stress from the first question on how to answer such kind of questions.

5. Lesson 4 : Panjat Pinang Context

Learning Goal

Students are expected to be able to compare two data sets.

Materials

Worksheet

Measuring tape

Mathematical Activities

'Panjat Pinang'

The context :

Figure 2.6. Panjat Pinang

The teacher met her friend from Netherland. She is also a teacher. They were talking about many things, such as their job as a teacher, their family, Indonesia and Netherlands cultures. One of the discussions was about Panjat Pinang. They imagined competing between Indonesian and Netherland students in Panjat Pinang. However, they see the difference of the height of the students between these two countries. Therefore, we need to make the Panjat Pinang contest fair.

In this meeting, the best way to have a fair play is to make the Pinang pole different. Therefore, we need to know the difference height between the two groups. In the beginning of the lesson, the teacher asks students to fill the Indonesian data with the male students in the class. After that, the students will find out how to compare this two data by taking one number as the representative of the data. The comparison may

use the idea of mean and median. The bar is still used in this activity to describe the two data sets. The discussion emphasizes on the strategy of students to find the different of these two datasets. The Netherland data is set to have different mean and median in order to engage students to argue about how well the mean and median can describe the data. The followings are the point to discuss:

a. What is the difference between the height of Indonesian and Dutch students based on the mean and the median?
b. What do you think about taking median or mean as the representative of the whole data? How well do mean and median represent the whole data?

Prediction of students' responses

Regarding the strategy to compare the data, some students may use the idea of maximum or minimum value, mean, mode, median, and midrange.

Actions of the teacher

a. If the students use the idea of mean and median, the teacher asks them for present their idea later in the whole class discussion.
b. If the students still use the idea of maximum or minimum value, the teacher may ask questions such as "why did you choose this number? Do you think it is a best way to describe the data? Do you think it represent all of the height?"

c. If the students use the idea of midrange, the teacher may posts question such as : why do you choose midrange? How many heights do you use to get the midrange? Is it all of the heights? You have to consider all of the heights of the students".

6. Lesson 5: Students' heights

Learning Goal

Students are expected to be able to demonstrate how to calculate the mean.

Materials

Worksheet

Measuring tape

Mathematical Activities

In this meeting, the teacher wants to compare the height of the students among the groups. The students just measure the height of their group and then find average to describe their group height. Besides, there is a question regarding an addition data. The discussion emphasizes on how students see the shift of the average because of one data. In addition, the students also still are asked to draw and interpret the data and the average into the bar. The students are expected to have varied ways regarding the question of the additional data. The discussion focuses on how the students the shift of the average if we add one data.

In the next, the teacher asks the following questions for students to discuss: Considering the height of the new data, what will happen to the average of the group?

a. If his height is more than the average?

b. If his height is less than the average?

c. If his height is as same as the average?.

Prediction of students' responses

Regarding to find the new mean from one additional value, some students may do the followings:

a. add the average and the new data, and then divided it by two.

b. add the average and the new data, and then divided it by the number of data plus 1.

c. add all the data again including the new data, and then divided by the number of data plus 1.

Actions of the teacher

Regarding the additional value problem, the teacher can ask all different strategy to explain their strategy in front of the class. From all possibilities, all of the students may agree on the strategy to add all of the data and the new data, and then divided it by the number of data plus one. However, the teacher may show others strategy as the comparison. The teacher also discusses the idea to use the average to combine the two data sets. The teacher may post a new problem such as: "if I have two data set, and I want to find the average of all two data together, may use the data and divided by 2?" Moreover, the bar chart may help to explain the shift of the average. The bar can show visual proof that the result when we use the strategy by adding the additional value with the initial average and

divided by two will obtain differently when we find the average as usual – add the data and divided it by the number of data. To make it clear, the teacher can use the formula.

7. Lesson 6: The Bookshelf Context

Learning Goal

Students are expected to be able to solve the problem by using the average.

Materials

Worksheet

Mathematical Activities

The context: Bookshelf

Figure 2.7. The bookshelf

In the classroom, the teacher has a plan to put a shelf on the wall. However, she does not know how high the best for the shelf. The teacher thinks that the shelf should be not too high and not too short. If it is too high, some students may not be able reach the book or some things on the shelf. If it is too short, some students may get hit by the shelf.

In this meeting, the teacher asks students to make a plan to determine the height of the shelf on the wall. The students are expected to be able to

use the idea of average to solve this problem. In the beginning, the teacher encourages the students to think what kind of information they need to carry out their plan. One of the issues is that they need the height of all students in the class.

In the previous meeting, the students have measured the height of their own group. Therefore, the teacher can ask every group to put their data on the blackboard or on the wall so all groups can use the data. The students can decide whether they want to use all data or they may pick certain samples. This is one of the ideas in this lesson in which students think to not use all of the data. This issue will be used in the whole class discussion.

During the discussion, the teacher asks some groups to present their plan in front of the class. In addition, the teacher points out the issue: “taking some students to decide on the number instead of all students. How? And what kind of students should we choose?” In the last session, the teacher may tell how the statisticians take some data as the sample instead of all the population. The teacher may use the context to know the income of Indonesian people.

Prediction of students’ responses

Regarding the way students making their own plan, the students' answers may as follow:

a. Taking the maximum and minimum height because if it is based on the highest student, the other students can use the chair. Meanwhile, if it is based on the shortest student, all people can reach the shelf.
b. Taking the midrange.
c. Taking a middle value by estimation.
d. Taking the mode, the mean, or the median.
e. Taking sample instead of all data and find the mean or median of the sample
f. Taking the mode, the mean, and the median, and add some cm in order to make the higher shelf.

Actions of the teacher

a. If the group uses to choose the tallest or the shortest students', the teacher can ask them whether it is too short or too high. It is just little students may reach the shelf if it is too high, meanwhile if it is too short most of the student will feel uncomfortable, why did you choose this number?
b. If the group is to just estimate the number, the teacher may drill the estimation. What kind of consideration they think to decide the number.
c. If the group uses midrange, the teacher may ask the generalization on every situation such as if there is one student is very high and one student is very short in a classroom, then can we still use the midrange?

d. The teacher allows the group who uses the idea of mean, median, and mode. In addition, if there is a group who use the sample, the teacher also should pay attention to this strategy. It can be a good point to discuss as fascinating planning.

CHAPTER III

RESEARCH METHOD

A. Research approach

The main goal of this study was to contribute to a local instructional theory for students in learning the concept of mean. To achieve this goal, an instructional sequence was designed as an innovation to improve mathematics education. The sequence was developed based on RME with a Hypothetical Learning Trajectory (HLT) and the instructional materials. The outcome of this study was the instructional means which can support students' understanding of the mean. The main result was to know how the instructional means can support the students' understanding. Therefore, a design research was chosen as the methodology in this study. It provided a methodology to understand and to improve the educational practices through an iterative process (van den Akker, et. al., in press).

The three phases of design research by Gravemeijer & Cobb (2013) are described as follows :

1. Preparing for the experiment

   In this phase, a local instructional theory was elaborated by reviewing related literature such as journals, articles, books, and existing designs. In this study, the review focused on the theory of mean as one of the measures of central tendency, on the theory of Realistic Mathematics Education, on the

Indonesian curriculum, and on the design research approach. Besides, the HLT was started to develop inspired by the literature review. The HLT consisted of three main components: the learning goal, the learning activities, and the hypothetical learning process – a prediction of students' thinking and teacher reactions (Simon, 1995). Activities and contexts were collected from the literature and discussions were held about the HLT with the lecturers who were experienced in designing for mathematics education. This HLT was dynamic and could be changed during the iterative process of the design research.

2. Teaching experiment

   This phase wass conducting the teaching experiment. The HLT and the material that have been designed in the previous phase were being tested and improved. This phase also aimed at understanding how the design works.

3. Retrospective analysis

   In this phase, the HLT was used as guidelines and references to analyze the data. The analysis was to compare the actual teaching and learning process with the hypothesis in the HLT. It not only meant looking for the actual observations that could confirm the hypothesis but also for observations that do not confirm it (van Eerde, 2013). The result of this phase was the conclusion that answers the research question.

B. Data Collection

1. Preparation phase

Two types of data were collected in this phase: classroom observation and an interview with the teacher. The aim of this phase was to get information about the students, the teacher, and the classroom management. The information was used to improve the initial HLT with the classroom observation.

a. Classroom observation

The purpose of this observation was to determine how the teaching and learning process takes place in the classroom setting. Video recordings and field notes were used to collect the data. The observations focused on the teacher's and the students' activities, the interaction between the teacher and the students and the interaction among the students, the classroom culture (including social norms and socio-mathematical norms), and the teaching and learning process.

b. Interview with the teacher

The interview was conducted after the classroom observation. The purpose of the interview was to figure out the teaching and learning process through the eyes of the teacher and also to clarify the classroom observation. A field notes were used during the interview.

2. The initial teaching experiment (Cycle 1)

This first teaching experiment is conducted as the pilot study and as the cycle 1 of the research. A small group with 6 students of the 5th grade in SD Inpres Galangan Kapal II Makassar was the participants of this first the teaching experiment. They were Ainun, Jenny, Ade, Fajri, Taufiq, and Zakaria. The six students were chosen based on their ability in mathematics, two students for every level of ability; low (Taufiq and Zakaria), middle (Ade and Jenny) and high (Ainun and Ade). They were recommended by the teacher. The aim was to test and to improve the HLT. In this phase, the researcher acted as the teacher. A video recording was used to collect the data of the teaching and learning process. In addition, the students' written works were also collected.

3. The second teaching experiment (Cycle 2)

The second teaching experiment was conducted in one class of 5th grade of SD Inpres Galangan Kapal II Makassar. The teacher implemented the improved HLT as the result of the first cycle. Instead of the researcher, the home room teacher acted as the teacher in this cycle. Video recording together with field notes were used to collect the data of the teaching and learning process. In addition, the students' written works were also collected as the data to investigate students' thinking during the teaching and learning process.

4. Pre-test and Post test

A pre-test and a post-test were conducted for all students in Cycle 1 and 2. The pre-test was given before the teaching experiment. The aim of the pre-test was to assess the prior knowledge of the students about the topic. Similarly, the purpose of the post-test was to assess students' knowledge after the teaching experiment phase. It was given at the end of the whole lesson.

The pre-test and posttest in this study consisted of three questions related to the concept of the mean. The questions were same for both tests. Two items concerned to the mean sentences and one item is about the comparing problem.

5. Validity and Reliability

The validity and reliability of data collection of a study determine the quality of it. In this study, various methods of data collection such as observation, field notes, document analysis and the audio or video recording were used. These various methods led to the triangulation of the data which contributes to the internal validity. In addition, the use of audio and video recordings improved the reliability. It showed what really happens during the teaching and learning process. Moreover, the selection of fragments of the video recordings enhanced the power of argumentation and reason for the inferences and assertions, because the arguments and reasons were built based on the reality.

C. Data Analysis

1. Pre-test

The pre-test analysis focused on the prior knowledge of the students. It described the students' informal knowledge, their misunderstanding, and their misconception of the concept of mean. The information wass used to describe the starting point in the HLT.

2. The initial teaching experiment (Cycle 1)

In the initial teaching experiment, the initial HLT was being tested. It showed how the initial HLT works. The video recording of the teaching and learning process were analyzed to reflect on the HLT. Besides, the students' written works also enriched the description of students' thinking and reasoning in solving the problems in HLT. The reflection led to an improvement of the HLT for the second teaching experiment.

3. The second teaching experiment (Cycle 2)

The video recording was transcribed and described to get an insight into the teaching and learning process. The field notes were used to identify the important episodes or fragments from the teaching and learning process. The selected episodes or fragments were transcribed to describe the group discussion and to interpret the students' thinking and reasoning. Besides, some interesting students' works also enriched the interpretation. The results of this analysis led to a revision of the HLT and to the conclusions that were needed to answer the research question.

4. Post-test

The post-test was analyzed to investigate the development of students in understanding the concept of mean. The analysis also included a comparison with the results of the pre-test. This result enriched the analysis of the teaching experiment in order to draw conclusions.

5. Validity and Reliability

The continuous activities for testing and improving the conjectures in the HLT based on the collected data enhance the internal validity of this study. Furthermore, the detailed descriptions of the participating students, the teaching experiment, the analysis, and the conclusion enhanced the external validity. These descriptions provided the opportunities for other researchers to adjust the situation to their own local setting. Besides, the detailed descriptions also enhanced the reliability that was related to trackability. Trackability means that the reader must be able to follow the teaching experiments and the analysis. It should provide the possibility to replicate the teaching and learning process.

In this study, the researcher discussed the results with colleagues, the lectures, and supervisors regarding the analysis and the conclusion of the study. The discussion increased the internal reliability regarding intersubjectivity. Intersubjectivity is concerned with the dialogue with other people about the interpretation of the data and the conclusion. In addition,

since the teaching experiment was conducted in the classroom as a real setting of the teaching and learning process, it contributes to the ecological validity.

CHAPTER IV

RETROSPECTIVE ANALYSIS

A. Data of Preparation Phase

Classroom observation and the teacher interview were conducted before the teaching experiment. In the beginning, we did the classroom observation continued with the teacher interview. The observation focuses on the teacher's and the students' activities, the interaction between the teacher and the students and the interaction among the students, the classroom culture (including social norms and socio-mathematical norms), and the teaching and learning process. Meanwhile, the purpose of the interview is to Figure out the teaching and learning process through the eyes of the teacher and also to clarify the classroom observation.

1. Classroom observation

In the observation, the teacher accustomed with direct instruction. The teacher started the lesson by explaining the topic and the formula for the students. In the beginning, the students listened and paid attention to the teacher. But, in the middle of the explanation, the teacher lost the students' attentions. Most of the students did not listen to the teacher. They chatted with their neighbors or did something else. The teacher then yelled to the students to get more attention. It succeeded but not long. The teacher also asked whether the students understand or not. Sometimes, he posed a question

for the students. The question was not too difficult for the students, because they just yelled and gave the right answer directly. The teacher lastly asked them to write and to carry out some exercises.

During exercises time, the teacher sometimes walked around to see what the students did. But, most of the time he led the students carried out the exercises by themselves. When the teacher did not pay attention to the students, some students walked around the class and chatted each other or did something else which did not relate to the exercises. It was only few students who are categorized as smart students focus on the exercises. The teacher again just yelled to handle the situation. In the last meeting, the teacher led the classroom discussion to check the answers of the exercises. He asked a student to write his answer on the whiteboard in front of the class and then judge whether the answer was right or not. If it was wrong, the teacher in some cases asked another student or he directly gave the right answer.

The teaching and learning process in the classroom was dominated by the teacher. The teacher explained the topic and the students listened. Even though the teacher posted a question for the students, it was just to take students' attention. The interaction was one way from the teacher to the students. During the exercises time, we observed that only few students who discuss about the exercises. However, the discussion was not meaningful because the smartest students always talked while others just listened and agreed without giving their reaction about the exercises.

Regarding the social norms, this classroom was very crowded. The teacher always yelled during the teaching and learning process. The students went silent for a while when they heard the teacher yelled, but no longer. Yelling was used not only by the teacher, but also by the students. When the teacher posted a question for the whole class, the students directly yelled the answer. We noticed that some students just yelled randomly. During the exercises time, the students walked around the classroom. Some students discussed with their neighborhood. However, the discussion was dominated by a smarter student.

The socio-mathemathical norms were also one of our concerns in the observation. It was shown clearly when the teacher and the students discussed the answer of the exercises. The classroom believed on one solution for one question. There is no chance for other different answers. The discussion also focused on the right answer, sometimes performed by the students and confirmed by the teacher. In addition, most of the solutions were about applying the formula and the number without reasoning why and how we used the formula. It may because the exercises did not challenge the students to perform their reasoning. Therefore, it was hard to find the sophisticated answers from the students.

2. Teacher Interview

Three main themes for this interview were about the background, the classroom management, and the students. The teacher's name is Mr. Agus. He

is a young teacher who has 5 years experience in teaching primary school. His background is a bachelor degree in PGSD (Teacher education for primary school). This year was the first time he deals with the fifth grade, he before taught grade 2, 3, and 4. Therefore, he never taught about statistics in primary school. He said that he has lack knowledge on statistics. Regarding the PMRI, he did not know what it is. The teacher understands the context as a story.

The teacher always used direct instructional as his method in teaching mathematics. He firstly explains the topic and provides examples and then asked the students to solve problems. He used worksheet and textbook as the sources in teaching mathematics.

Concerning about the role of discussion, the teacher stated that he did not use the group discussion often in his teaching and learning particularly in mathematics. Instead, the teacher sometimes posts a question during his explanation. This question sometimes helps the teacher to provide a discussion in a whole class. In addition, the students also did not familiar with the discussion.

Regarding the norm to give opinion and ask a question, the teacher provided it during the explanation. The teacher sometimes gives an opportunity in the middle or in the end of his explanation. However, the students usually did not use the opportunity to ask a question. They give a question during the exercises time when they have a difficulty to solve the exercises.

Furthermore, we also confirmed about the “yelling” norm. The teacher described that the students were very active. The active here refers to active which was not related to the teaching and learning process. Therefore, the teacher has to yelled in order to make the students silent and listen when he talks. Regarding, the different ability of the students, he said that he cannot do much to differentiate them. The best way is that sometimes he asked the smart students to teach their friends who did not understand.

3. Improvement on the HLT 1 to be HLT 2

Based on the observation and the interview results, we discussed the HLT with our supervisor in order to match the initial HLT with the classroom observation and the students in the school. Some changes and remarks as the follows:

a. The lesson 1 on the apple sentence problem was extended. We elaborated four conditions on apple weights, as follows:

   1) 200g, 200g, 200g, 100g, and 300g
   2) 200g , 200g , 140g , 230g , and 230g
   3) 200g, 130g, 260g, 180g, and 270g
   4) 170g, 130g, 250g, 300g, and 150g

   We tried to see whether the students directly can Figure out what the meaning of average in that sentence through the four conditions. It also aims at attacking the average as a mode as our conjecture that students may do in the HLT.

b. The Anita's problem was moved to the lesson 2. It is because the apple sentences were elaborated, the time allocation was not enough to include the Anita's problem on repeated measurement problem.
c. The comparing problem on the lesson 4 was deleted. We thought that the students will face difficulties to deal with those numbers of heights. Instead, we elaborated the comparing problem on the students' height problem. In the height problem, the students obtain the average of their group. From those group data, we can compare the data sets in order to introduce the comparing problem.
d. In the students' height problem, we removed the bar chart. Our intention was that to give the opportunity for the students to choose whether they use the bar chart or directly to the formula.
e. Other remarks on the teacher guide. We already described that the teacher and the students did not familiar with the discussion. Therefore, we provided more time for the discussion. We also stress the norm for discussion in the teacher guide especially for the first three lessons. We expected that the students can show a changing towards the discussion.

B. Analysis of the Initial Teaching Experiment (Cycle 1)

The initial teaching experiment was a pilot study. It aims at improving the hypothetical learning trajectory (HLT). This analysis focuses on how the teaching and learning process was going on and also testing the conjectures in

HLT. We described the pretest as the prior knowledge of the student, the teaching and learning process in order to get the insight about what is going on, and also the posttest to confirm the development of the learning process. We also compared the learning process with the conjectures in HLT. In the last, we provided remarks to revise the HLT for the second teaching experiment.

Six students were involved in this initial teaching experiment. They were Ade, Jenny, Ainun, Fajri, Taufiq, and Zakaria. They were grade 5 students in SD Inpres Galangan Kapal 2 Makassar. The students were chosen by the teacher. We asked the teacher to recommend the six students with different mathematical ability (low, medium, and high) from the classroom. In this teaching experiment, the researcher acted as the teacher.

1. Pretest

Pretest given before the teaching experiment was conducted. The aim is to get insight of students' prior knowledge about the concept of mean. The six students solved three questions in the first cycle; (1) making a sentence with the word "average" and the meaning, (2) interpreting the meaning of a sentence with the word "Average", and (3) comparing two group of data set. The first question asked students to write one sentence with the word average and describing the meaning of the word based on the sentence they had. In the second question, we provide a sentence about a criterion to pass the national exam which is the average score for three subjects (Mathematics, science, and Bahasa Indonesia) was 5.5. Three sub-questions elaborated; (1) write the

meaning of the word average in the sentence, (2) construct the score of three subjects in such a way one student pass the exam, (3) judge two students whether they pass the exam or not (the scores of two students were given). Meanwhile, the third question asked to find the different of two data sets.

As the result, four students can write the right sentences of the word average. All sentences were about the score of a student. However, they did not know what the meaning of average in their sentence is. They interpreted the average as the maximum score. In finding the different of two data sets, all students subtracted every single data in the data B from the data A. The results showed that some students can use the word average in the sentence. They may have heard or even used the word. However, they did know what the meaning of average is.

2. Lesson 1 : Average Sentences

The lesson 1 focuses on the ‘average’ sentences. The students investigated some sentences with the word “average”. In the beginning, the students were asked to write two sentences with the word “average” and then explain the meaning. In the next, one sentences “*The average weight of an apple in 1 kg apple is 200g*” were discussed to investigate the meaning of the word average in the sentence. Subsequently, four conditions of the weight of apples in the sentence were given:

a. 200g, 200g, 200g, 100g, and 300g

b. 200g , 200g , 140g , 230g , and 230g

c. 200g, 130g, 260g, 180g, and 270g

d. 170g, 130g, 250g, 300g, and 150g

The students were asked to interpret the meaning of the word average in the apple sentence based on the condition. The condition was given one by one and then there was a space in the worksheet where the students investigate the meaning of the word average based on the condition above.

The aim of this activity is that the students can distinguish some interpretations of the word “average” in daily life. The first problem which writing two sentences with the word average and the meaning allowed the students to show what they think of the word average. Our expectation is that the students can perform different kinds of sentences with the different meaning of the word average. The interpretations can be used as a starting point to describe the different meaning of the average and also to stress what kind of the word “average” we focuses on for the next meetings.

Meanwhile, the apple sentence and the conditions allowed them to think of the average as the arithmetic mean. It also aims at countering the word average as the mode. We expect that the students firstly recognize the average as the most (mode) as in the condition 1 and 2. In the next condition, the students encounter with the conditions where there is no apples with the same weights. The conditions provide a good opportunity for students to recognize the average as the arithmetic means.

In HLT, we conjecture our expectation is that the students may have different sentences and interpretation of the average. The students may come in three kinds of sentences: (1) the sentences without a number (e.g. : average woman like a handsome man), (2) the sentence involve a number (e.g.: average height of students in a classroom is 150 cm), and (3) the unrelated sentences. Regarding the meaning of the word average, the students may answer that the average means the word such as ‘most of’ or ‘many’. Similarly with the apple sentences, most of students may say “most of the apple weight is 200g” for condition 1 and 2. However, we conjecture that the students may have difficulties when they carry out the condition 3 and 4. Our high expectation is that the students may recognize the average as the arithmetic mean – add all of the data and divided it by the number of data.

In teaching experiment, the six students firstly had difficulties in writing two sentences. They took long time to produce a sentence individually. In the next, the teacher asked them to discuss with their neighbor; Taufiq and Ainun, Jeny and Zakaria, and Fajri and Ade. The discussion was not going as we expect. Instead explaining what they have done or sharing their ideas, all of the pairs were busy with their individually sentences. Even, some of them just copied their neighbor’ sentences without any discussion at all.

The way the students discussed with their neighbor indicates that they did not familiar with the discussion. This social norm is in line with the teacher statement in the interview that he rarely used a group discussion in the

teaching and learning process. Therefore, the students did not get use to group discussion.

Regarding the students' answers, they wrote similar sentences. Almost all sentences were about the average of their scores. For example, Jeni wrote "my score average is 8 of my report score" and Taufiq also wrote "my standard score average was 7". Only one sentence was about the average of height. Meanwhile, all of the students agreed that average was the maximum score. It was the same as what we saw in the pretest.

Based on the students' answers, we notice that the students described the average as a single value – maximum value. It clearly show in Ade and Fajri sentence; "*tinggi badan temanku rata-rata 53* (my friend's height average is 53). The word "*temanku* (my friend)" is single. This sentence indicates that Ade and Fajri possibly did not recognize the average works on two or more data. Unfortunately, the teacher did not follow up this case.

In the next, the teacher provided three sentences to bring the idea of average for them. Three sentences were (1) The average of woman like a handsome man; (2) My average score was 8; and (3) A road in front of my house is *flat / average* (in Bahasa Indonesia, average means *rata-rata* which can also mean as flat). As the conjecture in HLT, all students agreed that the average in the first two sentences means the most, many, and a half, while the third sentence was something different with the two sentences. When the

teacher asked about how much “the most, many”, the students described it as more than a half.

After the three sentences, the teacher gave the apple sentences. When the students read the apple sentences, they directly stated that the average as the most or many. However, when they face the four conditions of apples, they were confused and cannot explain it. The teacher next asked them to investigate why the average is still 200 gram even thought there was no 200 gram in the weight of apples for the condition 4. But, until the last minutes, they cannot realize the idea of average as the arithmetic mean. It shows that the students cannot directly go to the formula of the mean by providing several cases. However, they realize now that the average no longer means the most or many.

As the conclusion, the sentences’ problem cannot bring different kinds of average sentences with vary meaning of the word average as what we expected in HLT. The students recognized the average as one of the value / score as the maximum. The average as a mode (the most or many) appeared when the teacher provided three different sentences in order to emphasize the word average that used in daily life. Moreover, the apple sentence with the condition can attack the students opinion that the average as mode. However, they cannot go directly to investigate how to find the average through the apple sentences. Regarding the social norm of the students, we notice that they did not get use to discuss in a pairs or a group. The discussion for them means

to follow the smartest student among them. There was no sharing or explaining their opinion or answer during the discussion.

We summarize the conjectures and the students' actual reaction in the Table 4.1.

Table 4.1. Students' actual reaction on Lesson 1 of Cycle 1

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Writing two average sentences	Writing the different kinds of average sentences.	The students wrote similar sentences.
Describing the meaning of the word average	Describing the average as the mode ('most' or 'many') or the mean.	The students described the mean as maximum score (single value)
Describing the meaning of the word average in Apple sentence	Defining the average as the mode ('most' or 'many')	The students defined the average as the most or many.
Identifying the four conditions through the apple sentence	Realizing the idea of the arithmetic mean.	The students defined the average as the most or many.

3. Lesson 2 : Repeated Measurement Context

The sixth students solved one context of repeated measurement. The context was about a woman, namely Anita who had five times height measurements by different people in different places. All those five measurements obtained five different heights; fitness center resulted 171.5 cm, police station was 170 cm, hospital was 171.3 cm, roller coaster was 172 cm, and herself was 170.2 cm. The problem was to help Anita decided her height since she needs to fill out the form for getting a job.

The goal of the problem is that the students can identify one typical number through repeated measurement problem. The expectation is that the students can consider some or all measurements when they decided the height of Anita. They are expected to use one of the idea of measures of center whether median, mode, or even mean when they decided the typical number.

In the HLT, we include our expectation as one of the conjectures that the students might do. The students may consider the five measurements to decide the height of Anita by using the idea of measures of center (mean, median, and mode). However, we do not introduce the name of the measures of center in this activity. It will introduce in the glider activity. Besides, the students may also consider two, three, or four measurements instead of all five measurements. We also conjectured that the students may just randomly choose one measurement which is more convincing such as the Anita's result, 170.2 cm without considering other measurements. This conjecture is not what we expect for this context.

In the teaching experiment, the students firstly solved the problem individually and then worked together in a group to discuss one another answers. Ade, Ainun, and Jenny were together in one group, while Fajri, Taufiq, and Sakaria in another group. In Ade's group, Ainun and Jenny had the same answer, choosing Anita's measurement (170.2). Meanwhile Ade chose roller coaster measurement, 171.5 cm. In the group discussion, Ainun and Jenny just looked at Ade's answer. They showed disagreement face with

Ade's answer and suggested to choose Anita's measurement. Ade crossed out her answer and followed the majority answer. She did not give any reason to defend her answer. The next discussion in Ade's group was to construct a good sentence why they chose 170.2 cm. They discussed in lower voice and whispered. In Fajri's group, there was no discussion at all. The researcher as a teacher always engaged and asked them to discuss. But they just wrote and silent. Taufiq and Sakaria were just waiting for Fajri's answer since he is one of the smart students in their classroom.

It was not what we expected from the discussion. We expected that the students firstly showed their answer and their reason, and after that defend their answer. This showed the way the students discussed in a group. They were not discussed but they followed the majority answer or the smartest student's answer. In addition, we also found that the students tended to close their answer in such a way their neighbor cannot see their answer. Figure 4.1 shows how the students wrote the answer carefully. The way students discussed and closed their answers indicate that the students did not familiar to work in pairs or group.

Figure 4.1. The students closed their answer

In the classroom discussion, two groups read their answers. Ainun read the answer 170.2 cm. They argued that it was more convincing since Anita measured herself. When the teacher asked Fajri group's opinion, they just kept silent and looked down. And then the teacher moved to hear Fajri's group answer. Since there was no discussion, Fajri as the smartest student among them talked and answered 155.5 cm. He argued that if Anita wrote 170.2 cm, she will not accept in the job. The teacher tried to drill more; however, he cannot give any reasonable answer.

As in HLT, when the students chose one measurement as the answer, the teacher led students to consider about other five measurements. But, the

six students cannot give a reason considering other measurements. Instead, Ainun said that “some measurements were wrong and some were right”. The statement shows that Ainun did not recognize that different among the measurements was small. She also possibly did not realize that different measurement can happen in the measurement.

Therefore, we can conclude the Anita’s problem give opportunity for students to take one number from other measurements. However, they cannot consider other measurements to get the representative number. They chose one measurement that they believed it was true. It was because the context mentioned who took the measurements. In addition, we also should bold the social norms when the students discussed and closed their answer. The teaching and learning process should provide the learning environment where the students can build the ability to discuss and share their idea with their friends.

We summarize the conjectures and the students’ actual reaction in the Table 4.2.

Table 4.2. Students’ actual reaction on Lesson 2 of Cycle 1

Activity	**Conjectures of students’ reaction**	**Students’ actual reaction**
Deciding the Anita’s height from five times measurements.	Considering all or some measurements to decide the height of Anita. Choosing one measurement which is more convincing.	The students chose one measurement which is more convincing.

4. Lesson 3 : Glider Experiment (Prediction)

The six students had a hands-on experiment. They in a group made a glider from loops of paper attached to a drinking straw. One student thrown the glider away and then another student measured the distance of the glider. The distance measured from the student who thrown the glider to the place where glider landed. Every group did five times experiment to get five measurements of the distance of the glider. Lastly, the students were asked to discuss in a group to predict the next throw.

Figure 4.2. Students' glider

The intention of the glider activity is that the students can consider five times measurements to predict the next throw. We expected that they can use the idea of measures of center such as mode, median, midrange, or even mean. Previous meeting also have a similar intention that the students expected to consider other measurements and to use the idea of measures of center. However, this activity introduces the term of the strategy whether it is a mode, median, midrange or mean. Therefore, the goal of the activity is that the students can identify the strategies to describe the data.

The six students were grouped; Ade, Jenny, and Ainun in one group and Fajri and Taufiq in another group (Zakaria was absent). In the beginning, the students were made their own glider individually. They then tried to throw the glider away and decided which glider they used as a group glider. The students were enthusiastic to make their own glider and throw the glider away. They seemingly interested in making the glider (Figure 4.3).

Figure 4.3. The students made their own glider

In practicing, the teacher asked the Ade's group to show how to measure the distance of the glider. Jenny threw the glider and Ainun measured the distance. Jenny threw the glider to the Tables and the chairs places. Thus, it landed under the Table. Since it was difficult to measure under the Table, Ainun measured beside the Table (Figure 4.4. (a)). When the teacher asked whether they measured it correctly, Taufiq said that it should be gone directly to the glider. However, Ainun looks disagree with the statement. Therefore, the teacher asked Ainun to measure the distance to the glider and compared it with the previous measurement. Ainun then realize that the distance was different and agreed to Taufiq's statement that we should

measure the distance from the person who threw the glider directly to the glider itself. In addition, since the measurement tape consisted of two scale (cm and inch), Ainun read the inch. She did not realize that we used the cm scale. Therefore, the teacher brought this problem into a class discussion in order to get an agreement whether they used inch or cm.

(a) Before the discussion

(b) After the discussion

Figure 4.4. Ainun measured the glider

After trying out to throw the glider and to measure the distance of the glider, the teacher opened a discussion to discuss regarding how to throw the glider, how to measure, and rounding the measurements. Some agreements they obtained:

a. They throw the glider not too hard and not too slow.
b. They measured from the foot of the thrower to the nearest part of the glider.
c. Instead using decimal, they agreed to use the whole number.
d. They used the cm scale.

The discussion aims at having the same perception on how to do the experiment. The discussion also expected to build social norms to discuss and to take agreements regarding the activities. The six students showed a change attitude in discussion. All students tried to give their opinion and to answer the teacher questions. They seemingly feel free to say something compared to the previous activities.

In the next, the teacher allowed the two groups to do the five times experiment by throw their group's glider five times. One student threw the glider away and another student measured the distance and wrote their result in the worksheet. Ade's group obtained 187 cm, 238 cm, 252 cm, 251 cm, and 300 cm, while Fajri's group resulted 380 cm, 201 cm, 310 cm, 304 cm, and 250 cm, respectively.

After both group got the data, the teacher asked the students to predict the next throw. Ade's group predicted 238 cm and Fajri's was 250 cm. Both groups cannot provide the reason why they choose this number. However, from the both group prediction, we can see that they chose the middle data

instead taking the maximum or minimum data. They considered the data when they predicted but they did not realize it.

Figure 4.5. The students measured the distance

We summarize the conjectures and the students' actual reaction in the Table 4.3.

Table 4.3. Students' actual reaction on Lesson 3 of Cycle 1

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Predicting the next throw of the glider	Considering five times measurements of the glider. Using the idea of measure of center. Choosing one measurement that they believed in (maximum or minimum value Estimating the prediction.	The students chose one measurement in the middle (not the maximum or the minimum value)

5. Lesson 4 : Glider Experiment (Compensation strategy on bar chart)

Instead of playing with students' data, this activity played with the data from the teacher's glider. In the beginning, the teacher gave two data of glider measurements. And the students were asked to predict the next throwing and also interpret the way they predict into the bar. In the next question, the students were given the third data, and again asked to find the fourth throwing. Similarly, they have to interpret their prediction into the bar. Lastly, they were given the fourth throw data and ask for the next throw and also the bar. The four data were given in Table 4.4.

Table 4.4. The data of glider

Number of experiment	Glider
1	110 cm
2	100 cm
3	60 cm
4	95 cm

The aim of this activity was that the students can be able to derive the formula of mean by using compensation strategy on the bar. We expected that the students can use the bar to predict the next throwing. In the beginning, the students expected to predict the third throw by taking the median 105 cm. Therefore, the teacher can introduce the compensation strategy for the students through the bar. Lastly, we expected that they can figure out the fifth prediction by using the bar and also the formula.

The six students divided into two groups; Jenny, Ainun, and Zakaria in one group, while Ade, Taufiq, and Fajri in another group. The teacher firstly asked them to predict the third throw. Jenny's group answered 120 cm and Ade's group obtained 100 cm. When we asked why, they cannot provide a reason.

The teacher then provided the median strategy to predict the glider in the classroom discussion. The teacher posted a question to find the middle between 100 and 110. Then, Zakaria directly answered 105 cm. The teacher also asked students which strategy was more convincing. Most of them agreed that 105 cm since it was in the middle. After that, the teacher stress that they focused on this strategy in this meeting.

Furthermore, the two groups interpreted the data into a bar. Since this was the first time students encountered with a graph, the drawing was not a bar as what we expected (Figure 4.6). They drew the bar as the glider felt down to the floor. Since, it was not as we expected, the teacher introduced the bar to the students.

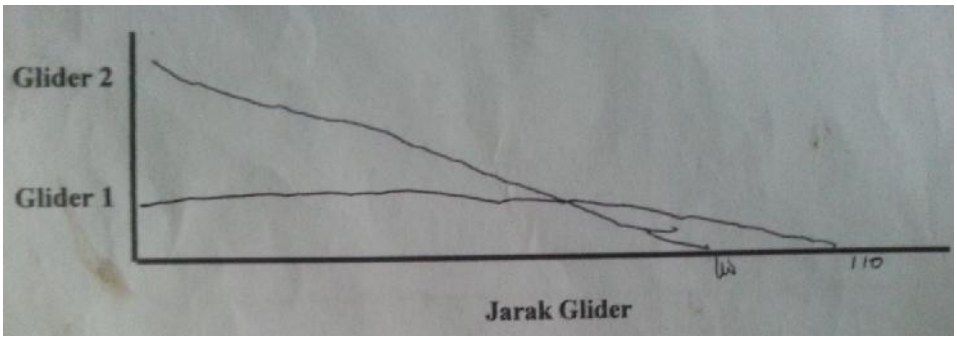
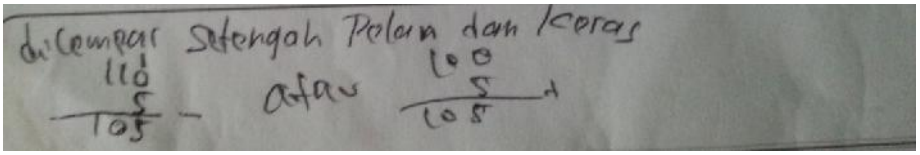
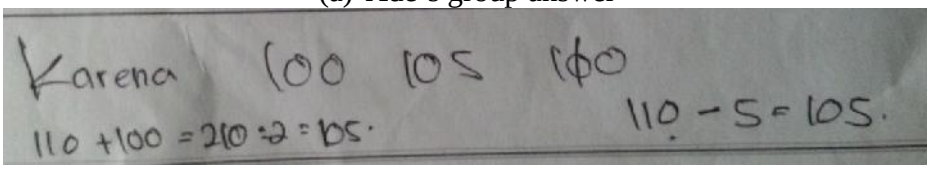


Figure 4.6. Jenny's group graph

After the teacher drew the bar with the data of two gliders, the teacher then asked the students to think how to find the 105 cm. Figure 4.7 shows the two group answers. Ade's group used the idea of compensation strategy by looking at the bar. They subtracted five from the 110 and added it into 100. Meanwhile, Jenny's group used the idea of median, added the two data and divided it by two. Since both groups did not draw their prediction on the bar, the teacher then introduced their algorithm into the bar.



(a) Ade's group answer



(b) Jenny's group answer

Figure 4.7. Students answer for the first problem.

In the next session, the data of the third throw was given to the students. The data were 110, 100, and 60. The students were asked to predict the fourth data by using the bar. Both groups tried to subtract and add the bar in order to get the same value of the bar. In Jenny's group discussion, they firstly took 20 from 110 and added it to 60. They obtained 80, 100, and 90. They then subtracted 10 by 100 and added it to 80. As the result they got 90 as the answer.

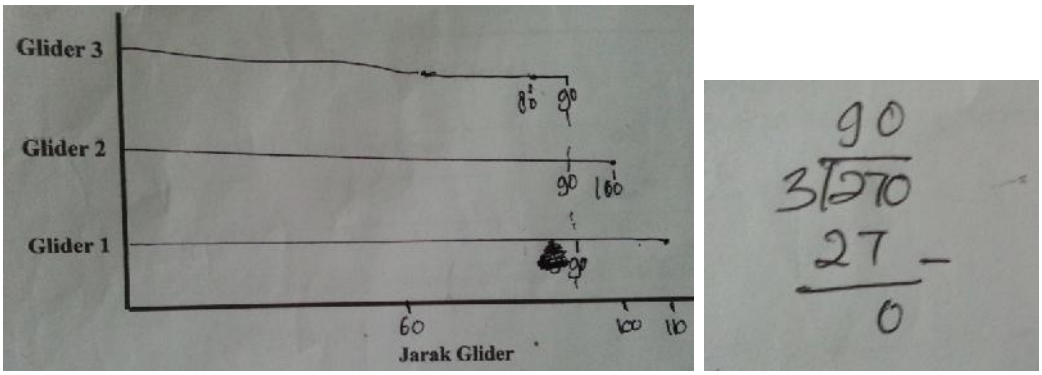


Figure 4.8. Jenny's group work on the third problem

Meanwhile, Ade's group obtained the different result, 100 cm. In the classroom discussion, the teacher asked them to show how they got the answer. Fajri then drew the bar on the whiteboard and explained their strategy. First, they subtracted 10 by 110 and put it into 60. As the result, they got 70 cm, 100 cm, and 100 cm. Second, they added 30 to 70 in order to get 100 cm. When the teacher asked where they get 30, they said that they want to get 100 cm. Ade's group seemingly did not understand yet that they should take apart from one bar to another bar in such a way they got the same result for all bars. It indicates that they did not understand yet how the compensation strategy works. Therefore, they cannot provide the formula for the second bar. The teacher then asked Jenny's group presented their bar in front of the class. Ainun drew the bar and explained their strategy to Fajri's group.

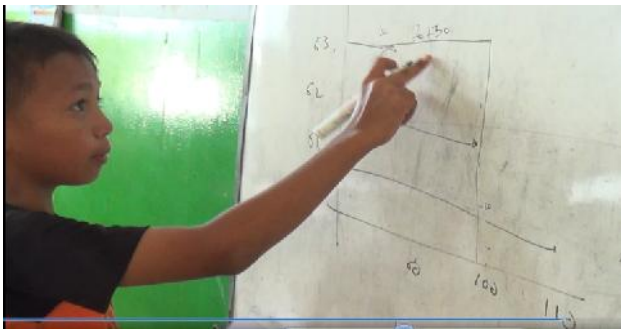


Figure 4.9. Fajri explained their group strategy on the bar

As the last problem, the teacher then told the fourth data which was 95 cm and gave time to group discussion. Both groups have a difficulty in drawing the bar. In Jenny's group, Ainun and Jenny tried to calculate the result directly. They added the fourth data and divided it by 2 and then 3 (Figure 4.10). However, they did not use it as the answer since the result was not an integer. Meanwhile, Ade's group still looked at the previous problem. They seemingly confused how to solve the last problem.

Figure 4.10. Ainun tried to divide the sum of the four data by two and three.

The teacher then discussed it together and back to explain the first and the second problem. We saw that the fourth data was hard for the students. The number 95 cm made the average was not an integer. And the students still had a difficulty to divide the number which obtained a decimal. Lastly, the

students then realized that they should add the data and divided it by four. The teacher then emphasized the formula of mean.

This activity invited the students to predict the next throw by using the compensation strategy on the bar. In the first and the second problem, the students can predict by using the idea of mean. However, it still needs more support from the teacher. In addition, the number involved should be easy for them since we expected the students to focus on the strategy instead of the number.

We summarize the conjectures and the students' actual reaction in the Table 4.5.

Table 4.5. Students' actual reaction on Lesson 4 of Cycle 1

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Predicting the third throw by using two data	Using the median or midrange strategy.	The students looked the pattern of the data The students chose one of the measurements.
Interpreting the prediction into the digram.	Figure 2.5	Figure 4.6
Predicting the fourth throw by using three data	Using the compensation strategy on the bar.	The students used the compensation strategy on the bar.
Interpreting the bar into the formula	Using the idea of mean	The students confused
Predicting the fourth throw by using three data	Using the compensation strategy on the bar.	The students used the compensation strategy on the bar.
Interpreting the bar into the formula	Using the formula of mean	The students used the formula of mean

6. Lesson 5 : Students' Heights

Two problems were given in this lesson. Firstly, the six students were asked to find their average of their group heights. In the beginning, they measured the height of every member in the group and write them down in the worksheet. Subsequently, they were asked to find their group average. Secondly, there was a problem about an additional data. One student, namely Toni with height 150 cm joined in their group. The students' task was finding the new average of their group now.

In previous meeting, they have learnt how to find the average by using the compensation strategy on the bar and also the formula. Therefore, this activity aims at demonstrating how to calculate the mean. The students were expected to show how they find the average of their group heights. The activity allowed them to use whether the compensation strategy or directly to the formula – add all of the data and divided it by the number of data.

Regarding the additional data problem, we hypothesized that the students may add the average and Toni's data and then divided it by two. They may also add the average and Toni's data and divided it by the number of students in their group plus one (Toni). However, it is not what we expected. Our expectation is that the students can realized that instead using the average, they should add the data and also the Toni's data, and divided it by the number of data plus one (Toni).

In teaching experiment, two groups were made; Ade, Ainun, and Jenny in one group, and Fajri, Taufiq, and Zakaria in another group. In the beginning, the two groups measured their member's heights. All students seemingly have no difficulties in measuring the height. They still used the agreement to use the whole number instead of decimal.

After they got their data, both groups tried to determine the average of their heights. As in HLT, both groups directly used the formula to find the average. They added all of the data and divided it by the number of member in their group. Both groups also used long division to divide the sum of the data and the number of data.

They did not use the compensation strategy anymore. It may because there was no instruction or place to draw the bar like in the previous meeting. Besides, it may also because in each group there was a member who can perform long division correctly so they did not have to worry to use the formula.

In Ade's group, we saw that Ainun and Jenny worked together to do the long division. Meanwhile, Fajri took all of the responsibility to find the average by using long division in another group. When the teacher asked Ade why she did not participate in calculating, she said that she still have difficulty to use the long division. Similarly, when the teacher asked Taufiq and Zakaria to also calculate the average, both said that they did not know how to divide the number.

During the classroom discussion, the teacher cannot compare the two groups' average. It was because the average for both group were equal, 117 cm. Therefore, the discussion on comparing two data sets did not conduct in this activity.

In the next session, the teacher moved to introduce the second problem. The teacher firstly discussed what the meaning of the problem and then asked them to discuss in their group to solve the problem. As the result, both groups added all of the data including Toni's height and divided it by four (the number of member was three plus Toni). In the group discussion, Fajri Taufiq, and Zakaria worked together to solve the problem, it was different from the first problem where Fajri did the calculation by himself. Similarly, Ade, Jenny, and Ainun also solved the exercises together; even they resulted incorrectly because they miscalculated.

As the conclusion, the students show that they can find the average. This is what we expected in the HLT. However, the students used the formula instead used the compensation strategy on the bar. Unfortunately, we cannot discuss on compare problem since the two averages resulted the same average. Regarding the additional data problem, the students showed the solution where they did not think about the average to find a new average from additional data. They still add all of the data including the additional data and divided it by the number of data.

We summarize the conjectures and the students' actual reaction in the Table 4.6.

Table 4.6. Students' actual reaction on Lesson 5 of Cycle 1

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Finding the mean of the group.	Used the formula of mean Used the compensation strategy on the bar	The students used the formula of the mean
Finding the new mean if one data was added	Added the average and the new data, and the divided it by two Add the average and the new data, and then divided it by the number of data plus one. Used the formula of mean	The students used the formula of mean

7. Activity 6 : Bookshelf context

A bookshelf context was a story to put a bookshelf on the wall. The students were determined the height of the bookshelf on the wall of the classroom. The shelf should be not too high and not too short. If it is too high, some students may not be able reach the book or some things on the shelf. If it is too short, some students may get hit by the shelf. Therefore, the students were asked to have a strategy to find the height of the shelf.

In previous meeting, the students have the data of the height of all students. The heights of all students and the average of all groups were given for all students. We provide the data. The students' task was to use the data in order to construct their own strategy to determine the height of the shelf.

The aim of this activity was that the students can solve the problem by using the average. We expected that the students can find a sophisticated strategy which involved the average to determine the height of the shelf. For example, the students may take some students (taking sample) and find the average of the height of those students instead of finding the average of all students in the classroom.

In HLT, we conjectured that the students may use the idea of measures of center such as mean, median, mode, or midrange. They also may just take randomly or estimate the height. The worst thing was that the students possibly just take a maximum or a minimum or a value that they think it was good without considering all of the data.

In the teaching experiment, the teacher firstly divided the six students into two groups. Ade, Jenny, and Ainun were in one group, while Fajri, Taufiq, and Zakaria were in another group. The teacher then introduced the story to the students and then allowed them to ask questions regarding the context. After the context was clear, the two groups discussed to solve the problem.

In the group discussion, we saw that the two groups' strategies were the same. They simply added all of the heights of the six students, and divided it by the number of data, six. There was no long discussion, since all the students in both groups thought the same way. We just saw that they struggling in divided the sum by six.

As the conclusion, the students through this activity demonstrated that they can use the average formula. They achieve the goal of the activities. However, we did not find any sophisticated solution from the students. It was possibly because this activity essentially worked for the big classroom. Since, the students can deal with many data of heights of all students in the classroom. Therefore, the needs of sampling, or simplify the calculation was necessary.

We summarize the conjectures and the students' actual reaction in the Table 4.7.

Table 4.7. Students' actual reaction on Lesson 6 of Cycle 1

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Deciding the height of the bookshelf on the wall. (not too high and not too low)	Taking the maximum or minimum height of student Used the midrange, mean, mode, or median Used the sample Estimating the height	The students used the formula of mean.

8. Posttest

The posttest was conducted in the end of the cycle 1. The questions in the posttest were the same as the questions in the pretest. However, it was not intended to compare between the pretest. It aims is at clarifying the students'

knowledge development that had been observed and analyze in teaching experiment phase.

As the result, the students did not know how to interpret the word average in the sentences. They still thought that it was a maximum value. The average sentence was the first activity in teaching experiment. In this activity, the teacher did not stress on the real meaning of average sentences. In the analysis of the first activities, our conjecture that the students may have varied sentences did not work. Therefore, the first activity did not work as what we expected. It may the reason why the students cannot provide a correctly meaning of the word average in the sentences.

Regarding the second questions, all students correctly constructed the score of three subjects so one student was called pass the exam. Some students realized that they have to put the score which is the same or even more than the average standard. It was sure that the average score will be more than the average standard. For example, Taufiq wrote the score 8, 9, and 10 and Ade wrote 5.5, 5.6, and 5.5. In activity five, we have mentioned that we cannot conduct a comparing two data sets discussion since the two groups result the same average. Therefore, the students did not answer it correctly for the third problem in the posttest.

9. Discussion

Based on the analysis for the six activities, we obtained remarks to revise the HLT for the teaching experiment in the big classroom setting. Some activities were extended and also reduced. The conjectures that students provided in the initial teaching experiment were elaborated in the next HLT. Some remarks as follows:

a. In the lesson 1, we reduced the number of sentences that students have to write become one sentence instead of two. It was because most students wrote the similar sentences for the both sentences. In addition, some students also show that they had difficulty in writing the sentences.

b. The lesson 1 focused on the sentences that the teacher provided in order to bring the students opinion about the average as a mode. We excluded the four conditions of apples weights since the students cannot provide the meaning of the sentences through the four conditions as we expected. We used the apple sentence as one of the third sentences to make the students realized that the average we mean here was not a mode. Therefore, after hearing some examples of the students' sentences, the teacher can directly posted three sentences; (1) the average women like a handsome man; (2) the average height of the fifth grade students was 117 cm; (3) the average weight of an apple in 1 kg apples was 0.240 kg. In the third sentence, the teacher provided five measurements of apples' weights

(Figure 2.1) and then discussed with the students whether they still thought that the average was a mode.

c. In Anita's problem of heights, we changed the fitness center's measurement from 171.5 cm became 171.2 cm. We realized that if we did not change the number, the students might obtain a difficult number when they added the five measurements which was 855.8 cm and divided it by five. Therefore, we changed the sum to 855.5 cm.

d. In the second problem after Anita's height problem, we added one context related to the repeated measurement. We saw that the students did not consider the five measurements when they asked to decide the Anita's height. They tended to chose one number that they thought was convincing. They focused on the person who took the measurement. Therefore, we added one context about repeated measurement on the weight of an object. This context did not describe the person who took the measurement. The story was that to decide the weight of a small object from the ten measurements that was taken from ten students in one classroom. In addition, we also added the outlier data in that problem to make more complicated for students.

e. In the lesson 4 on the glider experiment on bar chart, we changed the number. The previous number was difficult to find the average which was 91.25 cm. Table 4.8 showed the new data with the average 110 cm. Besides, we also exchanged the first and the second data to invite students

to predict 120 cm since it was getting far away from 100 cm, 110cm, and then 120 cm. We also expected that the numbers could make students easy to realize the compensation strategy on the bar. They focused on the strategy instead of the number involved.

Table 4.8. The revision of glider data.

Number of experiment	Glider
1	100 cm
2	110 cm
3	90 cm
4	140 cm

C. Analysis of the second teaching experiment (Cycle 2)

The second teaching experiment was conducted in the 5th grade of SD Inpres Galangan Kapal 2 (elementary school) in Makassar. There were six meeting with 29 students and one teacher involved in this cycle. The teacher was their home room teacher, Pak Agus. The teacher implemented the improved HLT as the result of the cycle 1. In this cycle, the teacher and the researcher decided to choose one group as the focus group by considering their ability in mathematics and also in discussion.

1. Pretest

The pretest was given before the second experiment was conducted. It aims at getting insight of students' prior knowledge about the concept of mean. Three questions provided in the pretest; (1) making a sentence with the word "average" and the meaning, (2) interpreting the meaning of a sentence

with the word “Average”, and (3) comparing two group of data set. The questions were same with the pretest and posttest in the initial teaching experiment. They were given to all students in the classroom.

As the result, most of the students wrote correct sentences regarding the word average. Most of the students wrote the sentences regarding the average of the students’ score, some others related to the average of a price of something. It indicates that the students familiar with the word. However, they cannot provide a correct meaning of the word average. Some students who wrote the score sentences defined the average as the maximum point, some others just randomly described the word based on the sentences. Similarly, there were no students wrote a correct answer for the second question, even the question still related to the students score which most of the students familiar with. From the two questions, we can conclude that the students did not know what the average is.

Regarding comparing problem, all students also did not success in solving the problem. Some just subtracted every single data in class A by the data in class B. It implies that they also did not know how to compare two data sets.

2. Lesson 1 : Average Sentences

Two main problems involved in this activity. Firstly, the students wrote a sentence using the word “average” and described the meaning of the word in their sentence. The classroom discussion followed after the students discuss

with their neighbor about their own sentence. The second problem was investigating three sentences with the word average. The sentences were:

a. *Rata-rata wanita menyukai pria yang tampan* (The average woman like a handsome man)
b. *Rata-rata tinggi siswa kelas 5 adalah 171 cm* (The average height of grade 5 students is 171 cm)
c. *Rata-rata berat sebuah apel dalam satu kilogram apel adalah 0.240 kg* (The average of an apple in one kilogram apples is 0.240)

The aim of this activity was to distinguish some interpretations of the word average in daily life. Through the first and the second sentences, we expected that the students described the word average as the mode. However, in the third problem, we provided pictures of five apples weight measurements. The measurements were different. Therefore, we expected that the students confused and curious what the average is. Lastly, we emphasized to the students that we were going to deal with this kind of average and they were going to know what the meaning of the average in that sentence through several lessons.

In making the sentences, some students wrote the sentences regarding students' score, for example the average of Andi's score was 5.5. This sentence similar to what we found in the initial teaching experiment. Some showed different kinds of sentence such as the average price of clothes. Regarding the meaning, some of them stated that the average as the maximum

value. Almost all students who used score sentences described the mean as the maximum score. Meanwhile, others tried to fix the meaning with the sentences. They use a word or a description that can describe or change the word average in their sentence. However, all of them failed to describe correctly about the mean.

Interestingly, we found some characteristics of average sentences from students' written works. The sentences did not write well based on Bahasa Indonesia's rule. However, some sentences indicates that the sentences belong to the following categorizations :

a. Average as the exact number

Figure 4.11 shows that the student defined an average (*rata-rata*) as an exact price of two fruits, *lenkeng* and *rambutan*. Some students did the same thing. They wrote one object followed by a number as the average. They saw the average as one value from a single object.

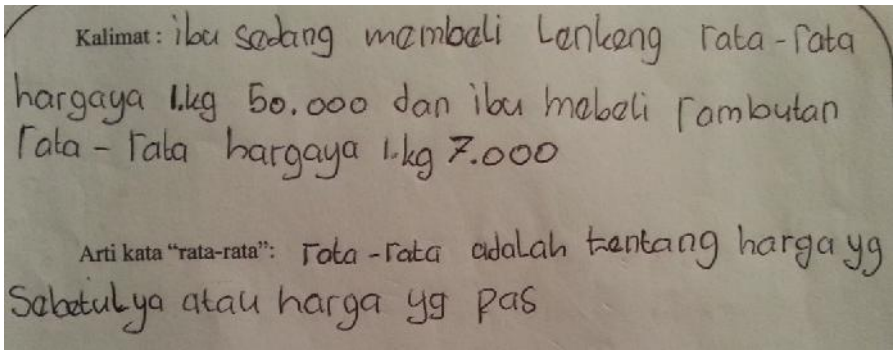


Figure 4.11 Average as the exact number

b. Average as the arithmetic mean

Most students wrote the score sentences. It implies that most of them familiar with the word average in scoring. For instance, the average of Andi's scores was 5.5 (Figure 4.12). The sentence was correct and related to the arithmetic mean. However, most of them defined the average as the maximum score rather than as the arithmetic mean.

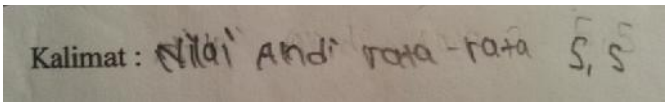


Figure 4.12 Average as the arithmetic mean

c. Average as the mode

Figure 4.13 shows a sentence: "*My mom bought (a) cloth(es) in the market with the average price IDR 24000"*. In Bahasa Indonesia, we most of the time are hard to distinguish singular or plural words of an object. It was a big problem to categorize the sentence in the Figure 4.13. The word "cloth" (*baju)* can be a singular or plural word. If it was a singular, it might be categorized as the sentence with average as the exact number. Meanwhile, if it was a plural, we considered it as the mode sentence. However, we decided to put this sentence as the mode sentence.

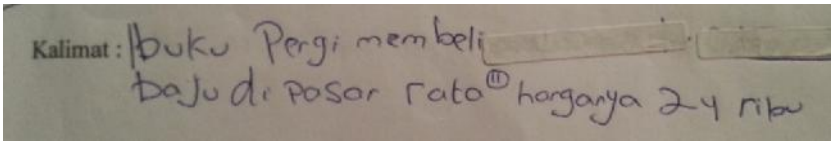


Figure 4.13 Average as the mode

d. Average as the fair sharing

One student provided a sentence as a fair sharing. Figure 4.14 shows a sentence : *a mother bought a cake and gave it to their children ½ as the average*. The student defined the mean as *for each/ shared*.

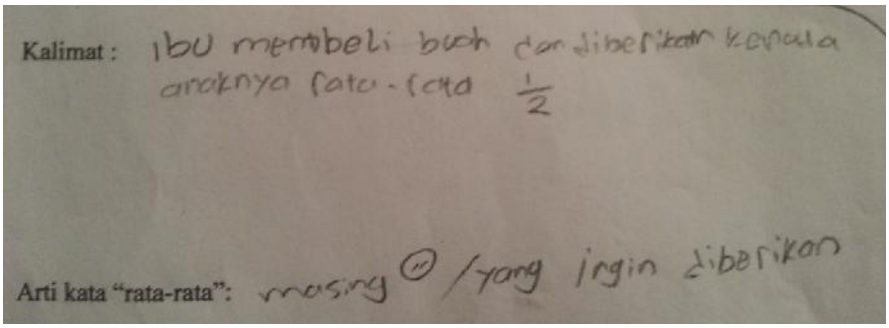


Figure 4.14 Average as the fair sharing

In HLT, we included the average as the arithmetic mean and the mode in our conjectures. We did not predict the first and the fourth sentences since they did not appear in the initial teaching experiment. Therefore, the teacher just focuses on the mode sentence and did not bring this interpretation in the classroom discussion.

After the classroom discussion on the students' sentences, the teacher provided the three kinds sentences for the students. The sentences were given for the students one by one respectively. Firstly, the teacher wrote the first sentence on the whiteboard and then asked for the meaning. After that, he continued to the second and the third sentence.

In the Transcript 4.1, the teacher asked the students to define the average of the first sentence in the whole class discussion. Two students

(Nabila and Azizah) discussed about the meaning of the average in the sentence after the teacher posted the question.

1	Azizah	:	(talking to Nabila) A part of a woman likes a handsome man.
2	Nabila	:	(whispering) A part of a woman likes a handsome man. (looking agree).
3	Teacher	:	Anybodies know about that? (talking to the whole class)
4	Nabila	:	Me, sir… (raising her hand)
5	Teacher	:	Yap, what is it?
6	Nabila	:	A part of
7	Teacher	:	Ok, a part of. Other ideas?
8	Fiqri	:	Almost all.
9	Teacher	:	Ok, almost all. Others?
10	Students	:	(Some said *a part of* and some others was *almost all*)

Based on the Transcript 4.1, Azizah and Nabila found the meaning of the word average by substituting the word average with another word into the sentence (line 1 and 2). They agreed to define the mean as *a part* (line 1 and 2). When the teacher asked the meaning to the class, Fiqri argued that it was *almost all* (line 8). All students agreed with these two meaning of the word average. It also happened in the second sentence. The students exchanged the word average with the word *a part* and *almost all* in the sentences.

When the teacher provided the third sentence and the pictures of apples weight measurements, the students seemingly confused to define the word average. It was our expectation that the students would get a conflict with their own meaning of the average in the first and the second sentences. It would be emphasized that the meaning of average as the mode was not our focus for the next meeting.

In fact, the teacher did not close the meeting as what we expected. He emphasized the meaning of average as *a part* and *almost all*. He did not pay attention on the last part of the teacher guide. Besides, he looked also confuse with the sentences. As in the interview, he said that he did not have a good understanding of the concept of mean.

Concerning to the norms, the students and teacher both did not familiar with the discussion. The teacher was seemingly hard to guide the whole class discussion. He tended to take a lead and provide the information for the students. He also did not familiar to follow up questions and giving an opportunity for students to express their idea. Besides, he also faced a difficulty to deal with different types of answered.

As the conclusion, the average activity allowed the students to have different kinds of the average sentences. The sentences can be used as a starting point to discuss on the different interpretations of the average. In the initial teaching experiment, we just found one interpretation which was the average as the arithmetic mean. Meanwhile, we found four interpretations in the second teaching experiment. It implies that the students was familiar with the word average, however, they did not know what the average is.

We summarize the conjectures and the students' actual reaction in the Table 4.9.

Table 4.9. Students' actual reaction on Lesson 1 of Cycle 2

Activity	Conjectures of students' reaction	Students' actual reaction
Writing one average sentence	Writing the different kinds of average sentences.	The students wrote different kinds of sentences.
Investigating three kinds of average sentences	Describing the average as the mode ('most' or 'many') or the mean.	The students described the mean as 'the most', 'many', 'a half' , 'almost all'

3. Lesson 2 : Repeated Measurement Contexts

The students carried out two main problems related repeated measurement contexts; Anita's and a small object measurement. Anita's measurement related to the repeated measurement on height. It was the same with the activity on the initial teaching experiment. The problem was about a woman, namely Anita was taking five times height measurements in a month (police station, hospital, roller coaster, fitness center, and herself). The result showed the five different measurements. The task was to help her decide her height from those five different measurements. Meanwhile, a small object measurement related to repeated measurement on weight. Similarly wit Anita's problem, the students have to decide what the weight of the object based on ten times measurements. The different was that Anita's problem provided the information who took the measurements; meanwhile a small object measurement did not specify the person. We also added an outlier problem (Table 4.10).

Table 4.10. The data of the weight of a small object

6 gram	46 gram	8.5 gram	7.5 gram	7 gram
6 gram	7 gram	9 gram	8 gram	6.4 gram

The aim of this activity was that the students can identify one typical number through repeated measurement problem. We expected that the students can take one number by considering others measurements. As in the initial teaching experiment, the students just chose one measurement that they believed in Anita's problem without considering other measurements. It was because the problem provided the person who took the measurement. Therefore, in the second teaching experiment, we added one context (an object measurement) which did not specify the person. In addition, we also included the outlier problem in the small object problem. We expected that the students realized about the outliers concept and did not take it into account.

In teaching experiment, the teacher firstly divided the class into several groups which consisted of four-five students in one group. He then told a story of Anita. After telling the story, he then gave time for students to discuss in their group deciding Anita's height.

As the result, some groups directly chose one measurement that they thought more convincing. In Nabila's group, they firstly wrote 171 cm. They took the measurement because it was the majority answer. They discussed in low voice while pointing the picture (Figure 4.15). After deciding 171 cm,

they did randomly calculation to decide the number. It was because they heard from another group used a calculation. Since they did not success in calculating, they then change their answer to 172 cm (Roller Coaster measurement).

(a) (b)

Figure 4.15. (a) Nabila's group chose one measurement that they believed in; (b) Nabila's group did some calculation

Similarly, we also found another group that chose one measurement; however, they did not choose the measurement randomly as they believed. During the group discussion, the researcher heard Abdul's group discussion. They eagerly discussed who between those five measurements was good to chose. Transcript 4.2 shows the discussion between the researcher and Abdul's group.

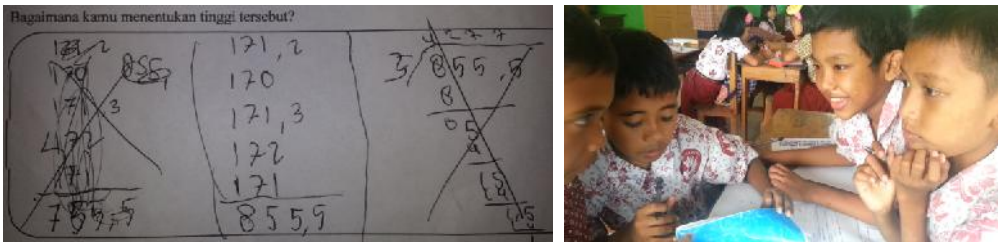
Transcript 4.2

1.	Researcher	:	Ok. Which one did you choose? (Asking Irwan)
2.	Irwan	:	This (Choosing roller coaster measurement 172 cm)
3.	Researcher	:	Why?
4.	Irwan	:	Because her height increased
5.	Researcher	:	What about you? (Asking Abdul)
6.	Abdul	:	This (Choosing hospital measurement 171.3 cm)
7.	Researcher	:	Why?

8. Abdul : Because the height was medium among other measurement
9. Researcher : How about you? (Asking Irham and Hamdani)
10. (Both agreed with Abduls' opinion)

Based on the Transcript 4.2, Irwan chose the roller coaster measurement since the height increased. He meant that the roller coaster measurement was the highest measurement among other measurements (line 4). Abdul also chose one measurement, hospital measurement because it was in the middle (line 8). These two answers chose one measurement as the answer. However, Irwan did not take into account other measurements like what Abdul did. Abdul looked at all the measurements and took the middle value from those measurements. Abdul used the idea of median, taking the middle value from those measurements instead randomly chose one number that more convincing. It was one of our expectations that the students can consider other measurements to decide Anita's height or even used the idea of median.

Another interesting idea also appeared from Fiqri's group. They did calculation to find the height of Anita. Figure 4.16 shows that they firstly add all of the data and the divided it by 2. However, since the result was not convincing they scratched on other paper and divided it by 5. When the researcher asked why they add the data and divided it by 5, Fiqri said that "in order to make all measurements equal". The students indeed used the idea of average as the arithmetic mean. Based on the Fiqri's statement, it indicates that he thought about the mean as fair sharing, making all measurements equal.



(a) (b)

Figure 4.16. (a) the students written work in the worksheet;
(b) Fiqri's group calculated the mean on the scratched paper

Lastly, the teacher asked Nabila's, Fiqri's, and Abdul's groups to present their answer. The representative of each group read their answer and the reason. Most of the reasons were about how they find the measurement rather then why. It shows the mathematical norms of the students in providing a reason of an answer. They usually more focuses on how they found the answer, rather than why. It also happened during the group discussion. The discussion focused on the way the students got the answer rather than the reason why they chose the answer. Unfortunately, the teacher did not ask why they chose the measurement, instead he more focused on confirmation questions such as "what do you mean?", "what did you add?", and "so you chose it because of ….".

The second problem was given after the classroom discussion. The teacher firstly told the story of the small object measurements. Then, he asked the students to discuss in group to decide the weight from those ten measurements.

In the middle of group discussion, the teacher posted a question to the whole class regarding the outlier. He asked whether they found some strange on the data. Fiqri then shouted that it was 46 gram. His reason was that because among the measurements there were no the same weight with 46 gram. It indicates that the student realize the outliers among the data. After that, the teacher asked the students to continue group discussion.

As the result, almost all groups answered 6 grams, while just a few including Nabila's group answered 7.5 grams. The reason why almost all students chose 6 grams was that because in their worksheet, it was written *a small object* as the context. Therefore, they took the small measurement as the answer. Meanwhile, Nabila's group who answered 7.5 grams stated that they chose the answer since 7.5 grams was more convincing as the maximum measurements among all measurements. This problem failed to bring the students to consider others measurements in order to decide the weight of that object.

As the conclusion, we saw differently between this second cycle and the initial cycle. In the initial cycle, there was no student considered other measurements to decide the Anita's height. They all chose the measurement that they believed in since we specify the person who took the measurements in the context. We then added a context where the students cannot choose one of the measurements for the second cycle. The context provided no information on who took the measurement. As the result, in the second cycle,

the students considered five measurements in Anita's problem which was opposite with the initial cycle. Moreover, the students also performed the idea of median and the mean. On the contrary, the second problem failed to bring students considering other measurements. However, the students recognized the outlier on the problem.

We summarize the conjectures and the students' actual reaction in the Table 4.11.

Table 4.11. Students' actual reaction on Lesson 2 of Cycle 2

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Deciding the Anita's height from five times measurements.	Considering all or some measurements to decide the height of Anita. Choosing one measurement which is more convincing.	The students chose one measurement which is more convincing. The students chose the middle value The students used the formula of mean
Deciding a small object weight from ten times measurements	Considering all or some measurements to decide the height of Anita. Choosing one measurement which is more convincing.	The students chose the minimum or maximum data.

4. Lesson 3 : Glider Experiment (Prediction)

This lesson was same as what we did in initial teaching experiment. The students firstly made their own glider. The students then throw the glider and measured the distance. The distance measured from the person who threw the glider to the place where the glider felt. The students threw the glider five times to get five measurements data. Lastly, they predicted the next throw. The intention of this activity was that the students can consider five measurements to predict the next throw.

The aim of this activity was that the students can identify the strategies to describe the data. We expected that the students use the idea of measures of center such as mode, median, midrange, or mean in predicting the next throw. Therefore, the teacher introduced the idea of measures of center.

In teaching experiment, the teacher started to divide the class into four-five members groups. He then introduced the glider and asked them to make their own glider. The students were excited and work together in making their group glider (Figure 4.17). After they finished, the teacher allowed them to throw the glider away to practice.

Figure 4.17. The students made the glider

In the next session, the teacher asked Nabila's group to demonstrate how to throw away and measured the glider and the other groups looked (Figure 4.18). The teacher discussed with the students how to measure the distance and how to throw the glider. In the discussion, the teacher dominantly talked without giving time for the students to have an agreement like what we expected in the HLT.

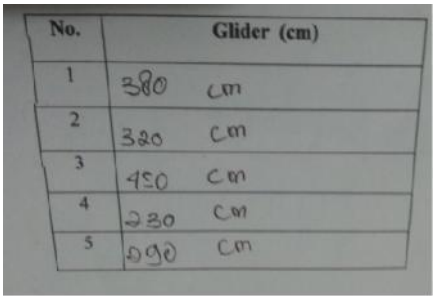
Figure 4.18. Demonstrating how to throw and measure the glider

The students then did their experiment. Some of the groups threw the glider in the classroom, while others did in the outside of the classroom. Nabila's group did the experiment in the classroom; Nabila as the thrower, Azizah and Febby as the measures, and Mutiara as the writer. In the

classroom, there were Tables and chairs. Nabila threw the glider directly to where chairs and Tables were. Figure 4.19 shows how they measured the distance. Instead measuring the glider through the floor, they measured it in the air. It may cause some inaccuracy in measuring; however, we focused on the data not the accuracy in measuring the distance. We also found an interesting way of students in measuring the distance.

Figure 4.19. Nabila's group measured the distance of the glider

After all groups got the data, every group back to their place discussed the prediction. In Nabila's group, they started to describe their glider on the worksheet and then discussed about the prediction. Figure 4.20 shows Nabila's group data.



No.	Glider (cm)
1	380 cm
2	320 cm
3	450 cm
4	230 cm
5	590 cm

Figure 4.20. Nabila's group data

Transcript 4.3 shows a discussion between Nabila and Azizah to decide the prediction. Febby and Mutiara paid attention to the discussion.

Transcript 4.3

1. Nabila : I think, it will be between this (pointing 230 cm) and this (pointing 320 cm).
   This (pointing 450) is not possible because it was an accident. (looking at her friend with smile)
   ……
2. Nabila : So,330 and 320. Do I write it down? (looking at her friend)
3. Aziah : (Nodding) write down “or”

In Transcript 4.3, the students predicted the next glider by considering all of the data. This is what we expected from the HLT that the students will consider all of the data in deciding the prediction even thought the prediction did not use the idea of measure of center. During the discussion, Febby followed the discussion without giving her opinion or even commenting other opinions. Meanwhile, Mutiara did not pay attention at all to the discussion. They both seemingly trusted Nabila since she is the smartest students in their group.

Nabila’s group answered two measurements, 330 cm and 320 cm. The teacher then came to see and asked why they answered two and then leaved. They then crossed the 330 cm and answered 320 cm. They reasoned that was because it was more convincing to them. The researcher then also came to their place and asked why they did not choose others measurement. They just smiled and cannot give their reason. The researcher then suggested

considering other five measurements. Transcript 4.4 shows their next strategy to find the prediction of the next glider.

Transcript 4.4

1.	Nabila :	Add then divided it by 5 (Scratching on paper)
2.	Azizah :	mm… (looking at her friend's writing)
3.	Nabila :	Subtract it … Zero … eight minus two .. hmm six .. six minus five is one … one minus two hmm it cannot … (Crossing the scratch paper)
4.		(Azizah, Febry, and Mutiara did not pay attention. They did something else)
	……	
5.	Nabila :	(She then added the data) What is 1670 divided by 5? (Asking Azizah).
6.	Azizah :	I don't know.
7.	Nabila :	Argggh.. I did not know division.
8.	Azizah :	That's your business
	….	
9.	Nabila :	What times five is getting 1670?
10.	Azizah :	(Give a did-not-know expression)
11.	Nabila :	Hmm… I am sure it can be divided..

Nabila tried to apply random formulas to decide the prediction. Interestingly, she used the idea of mean – add all of the data and divided it by the number of data (line 1). However, during execution, she changed her mind to subtract the data (line 3). Since she did not succeed, she tried another strategy. She then added the data and divided it by 5 (line 5). However, she found a difficulty to divide the number. This short discussion showed that the formula of mean did exist in Nabila's strategy. However, since there is no teacher around and also she faced difficulty with formula, they did not continue their thinking. Transcript 4.4 also shows the way the Nabila's group

discussed. The three students did not contribute their idea in solving the problem. Nabila did all of the calculation and the thinking.

Furthermore, there were interesting strategies to predict the glider using the idea of a mode. Figure 3 shows the data of a group of students and the reason why they chose the number. In the data, we saw that they got the same measurement which was 290 cm. Therefore, they decided to answer 290 cm. They wrote that they chose the number since it appeared two times. This was the idea of mode. However, when the teacher asked how if there was no the same result, they said that they will choose the maximum score (Figure 3).

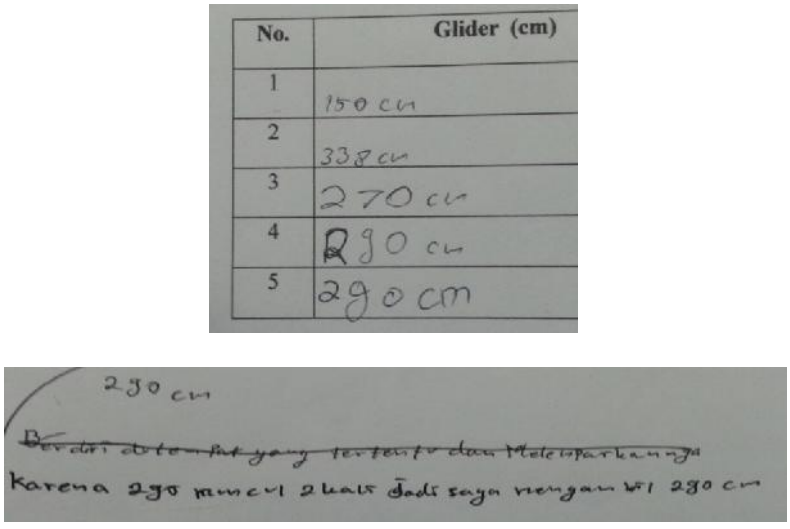


Figure 4.21. Students' strategy using mode

Besides, there was also a group that used the idea of mode but they used the boundary (Figure 4). In their data, they probably saw that 430 cm, 450 cm, and 490 cm. Therefore, they predicted that 400 cm was the next throw of the glider.

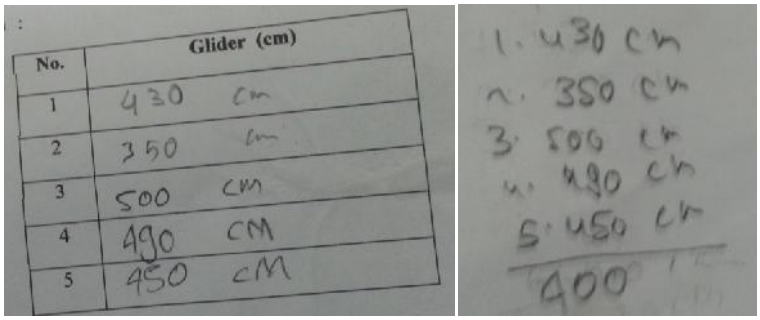


Figure 4.22. Students' strategy using mode and boundary

As the conclusion, the glider activity was rich context that allow students to have their own strategy. We saw that the students realized that they had to consider all of the data in order to predict the next throw. From the students' strategy, the idea of measures of central tendency such as the mode and the mean was existed thought it was not explicitly seen in their reason. In this situation, the role of the teacher to guide the students was needed.

We summarize the conjectures and the students' actual reaction in the Table 4.12.

Table 4.12. Students' actual reaction on Lesson 3 of Cycle 2

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Predicting the next throw of the glider	Considering five times measurements of the glider. Using the idea of measure of center. Choosing one measurement that they believed in (maximum or minimum value)	The students considered five times measurements of the glider. The students used the mode.

	Estimating the prediction. The students chose one measurement in the middle (not the maximum or the minimum value)	The students used the boundary mode.

5. Lesson 4 : Glider Experiment (Compensation Strategy on Bar Chart)

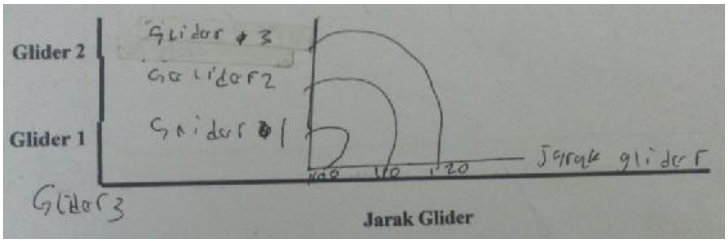
The glider activity continued in this activity. The students played with the teacher glider data. Four times measurements of the distance of the glider were given to the students one by one. Based on the data, they have to predict the next throw. In the beginning, two data were given and then the students predicted the third throw. The third data again was given and predicted the fourth, and so on until they predict the fifth throw with considering the four data. In every prediction, the students also have to interpret their answer in the bar. They used the bar in order to predict the next throw. This activity was the same with the activity in the initial teaching experiment. We only changed the data.

In the teaching experiment, the teacher firstly divided the class into four-five-members groups and introduced the problem to them. Two data were given to the students; 100 cm and 110 cm. The students predicted the third throw. They should provide a reason why they choose the number. Then, they interpreted the prediction into the bar.

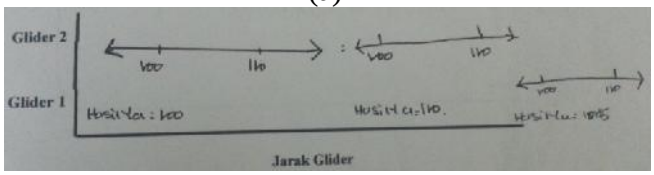
In Nabila's group, they predicted 105 cm. They chose the median of 100 and 110. This is what we expected that the students choose 105 cm.

However, they cannot provide the reason why they chose the number. Meanwhile, other group also predicted differently. Fiqri's group predicted 120 cm since they saw an increase pattern from 100 to 110 cm. Some just chose 100 cm or 110 cm.

Regarding the chart, we found two types of charts students used (Figure 4.23). In the initial teaching experiment, we found the chart as in Figure 4.23 (a). The students drew the bar as the way the glider felt on the floor. While, the Figure 4.23 (b) shows the students' bar predicting 105. However, we did not understand how the students used the bar to get 105 cm. Since it was the first time students encountered with the chart, we can see the different bar drawings.



(a)



(b)

Figure 4.23. Students' first drawing of the graph.

The teacher then conducted the classroom discussion. In the beginning, the teacher asked Nabila's group to present their prediction. The teacher

emphasized that they focused on this prediction to take in the middle rather than taking maximum or minimum value. In the next, the teacher drew the bar and show how the compensation bar works to get 105 cm.

In the next session, the teacher then introduced the third data, 90 cm. The students then asked to determine the fourth throw by using the bar. The transcript 4.5 shows two students, Fiqri and ibnu's discussion to understand the compensation strategy. Figure 4.23 shows their work.

The transcript showed that Ibnu explained how to draw the prediction of the next throwing using the bar. The discussion showed that he realized the compensation strategy on the bar chart by adding or subtracting some part of a bar to another part. And then they can Figure out the formula by adding all of the data and divide it by three.

Transcript 4.5

1	Fiqri :	Adding five .. (start writing)
2	Ibnu :	Adding ten, idiot (taking a pen from student A)
3	Fiqri :	Adding ten? … hmm… adding ten, here (taking a pen from student B)
4	Ibnu :	Adding ten's here (start writing) .. Finish … So, a hundred is the result (based on the bar)
5	Fiqri :	(Looking at the whiteboard) This is the first, right? (pointing at the picture)
6	Ibnu :	One hundred and ten minus ten… ten.. after that put the ten here (Pointing at 90 and explaining the drawing)
7	Fiqri :	(Writing the formula 90 + 100 and stop)
8	Ibnu :	Add it by 100 ..
9	Fiqri :	What is the divisor here?
10	Ibnu :	Add it by 100 …
11	Fiqri :	(Adding 100)
12	Ibnu :	What is the sum?
13	Fiqri :	(Scratching on his hand) Wait a minute I calculate it first.

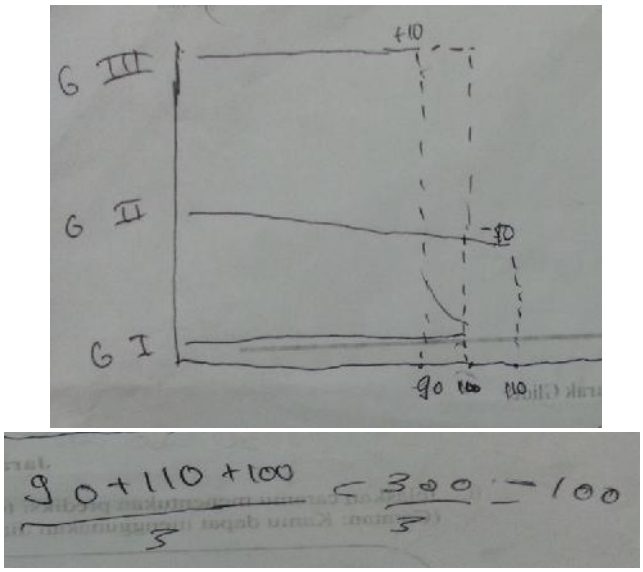


Figure 4.24. Students' strategy in predicting the fourth throw

Lastly, the teacher provided the fourth data, 140 cm. All of the groups succeed to draw the bar and got the result by using the bar. They also shows the formula that was add all of the four data and divided it by the number of data, four. The teacher then closed the meeting by introducing that they used the formula of the mean.

As the conclusion, the students realized the compensation strategy on the bar chart by adding or subtracting some part of a bar. And then they can Figure out the formula by adding all of the data and divide it by three. The students can interpret the compensation strategy into the formula. Thus, compensation strategy supports students to visualize the formula of mean.

We summarize the conjectures and the students' actual reaction in the Table 4.13.

Table 4.13. Students' actual reaction on Lesson 4 of Cycle 2

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Predicting the third throw by using two data	Using the median or midrange strategy. Looking the pattern of the data Choosing one of the measurements.	The students used the median. The students looked pattern of the data. The students chose one of the measurements
Interpreting the prediction into the diagram.	Figure 2.5 or Figure 4.26	Figure 4.23
Predicting the fourth throw by using three data	Using the compensation strategy on the bar.	The students used the compensation strategy on the bar.
Interpreting the bar into the formula	Using the idea of mean	The students used the idea of mean
Predicting the fourth throw by using three data	Using the compensation strategy on the bar.	The students used the compensation strategy on the bar.
Interpreting the bar into the formula	Using the formula of mean	The students used the formula of mean

6. Activity 5 : Students' heights

This activity was similar with the initial teaching experiment. The students measured their teammates' heights and finding the average of their group. In addition, there was a problem when an additional data included in their group average. The aims were that the students can demonstrate how to

calculate the mean. We expected that the students find the average by using formula or using the compensation strategy on the bar.

In teaching experiment, the teacher firstly divided the class into four-five students in one group. Every group got a measuring tape and the worksheet. The students were given time to collect the data of their member heights. They then were asked to find their average height in the worksheet. The second problem was given after they got their own data. The classroom discussion was conducted to hear the students' answer of the additional data.

During collecting the data, the students were excited. Figure 4.25 shows how the students measured their friend height. Figure 4.25 (a) and (b) shows that the teacher helps students to measure the height of the students. The student stand in front of the whiteboard and then the teacher marked the height. Another student then measured the height from the floor to the mark. Meanwhile, Figure 4.25 (c) and (d) shows three students worked together collecting the data. The student divided the tasks; one as writer, one as the person who measuring the height, and one as the object of measuring. We can see also that this group measured their length with full of motivation. The enthusiasm can be seen in Figure 4.25 (d) where one student even opened his shoes while his friend measured his height.

(a) (b)

(c) (d)

Figure 4.25. The measuring height activities

All groups succeed in calculating the mean. They used the formula instead of using the compensation strategy. They directly added all of the data and divided it by the number of data. It indicates that they can demonstrate how to calculate the mean correctly.

After all groups collected their data, the teacher asked them to write the data including the sum and the average on the whiteboard (Figure 4.26). The classroom discussion was held after all the groups wrote their data on the whiteboard.

Figure 4.26. The representative of the group wrote their data including the sum and the average on the whiteboard

In the classroom discussion, we expected that the teacher discuss about comparing two group data of the students height. The teacher firstly posted a question about the tallest and the shortest group. The students directly answered that the tallest was the group who had the biggest average, meanwhile the shortest was the group who had the smallest average. When the teacher confirmed the tallest and the shortest groups, the classroom was getting crowded with students' voices. In the next, the teacher then directly stated that the difference between the tallest and the shortest group was just subtracted the averages. He did not pose the question to the students since the classroom was getting noisy and did not pay attention to the teacher. The teacher then yelled and was angry to the students.

After the class was silent, the teacher then introduced the second problem regarding the additional data. The teacher then gave the students time to discuss within their group. In the group discussion, the students in their group directly added the data and the additional data and then divided it by the

number of data plus one – an additional data. All the groups did the same way. It implies that they can use the formula correctly.

As the conclusion, the students can demonstrate how to find the mean even if there was an additional data involved. They used a formula rather than a bar to calculate the mean. Besides, we also pointed that collecting and playing their own data motivate the students to involve in the activity. Regarding the comparing two data sets, the discussion did not success because the classroom was getting messy and the students did not pay attention to the teacher.

We summarize the conjectures and the students' actual reaction in the Table 4.14.

Table 4.14. Students' actual reaction on Lesson 5 of Cycle 2

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Finding the mean of the group.	Used the formula of mean Used the compensation strategy on the bar	The students used the formula of the mean
Finding the new mean if one data was added	Added the average and the new data, and the divided it by two Add the average and the new data, and then divided it by the number of data plus one. Used the formula of mean	The students used the formula of mean

7. Activity 6 : The bookshelf context

The bookshelf context was a story to decide the height of a bookshelf on the wall. The height should be not too high or too low. This activity was exactly the same as in the initial teaching experiment. The aim of this activity was that the students can solve the problem by using the average. We expected that the students can show sophisticated answers by using the idea of average. For example, the students may take some students (taking sample) and find the average of the height of those students instead of finding the average of all students in the classroom.

In the beginning, the teacher told a story of the bookshelf. The teacher and students then needed all of the students' heights in the classroom. Since they had wrote all the data on the whiteboard in the previous meeting, the teacher did not need to ask each group to write again on the whiteboard (Figure 4.27). The teacher then asked to discuss in their group.

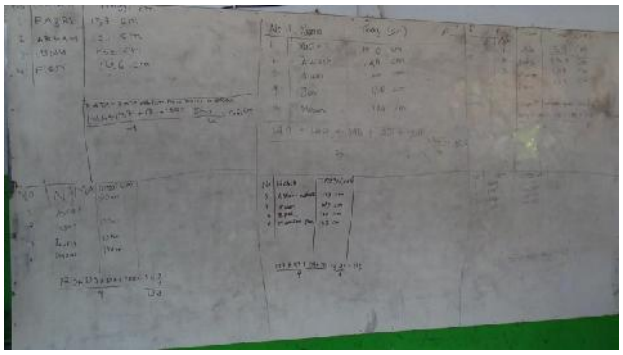


Figure 4.27 All students' height data on the whiteboard

Unfortunately, this activity did not success. All the groups answered incorrectly. They add all of the average from each group and divided it by the number of group. It was wrong since the number of students for every group was different. There were some group with four members and some others with five members. The main reason why the students did it was because the teacher in the middle of the group discussion emphasized to use the average of every group. He said that "we had the data from every group and also the average. You can use the average". He did not know that we cannot just add two averages to find the average total of two data sets if the size of the groups were unequal.

We summarize the conjectures and the students' actual reaction in the Table 4.15.

Table 4.15. Students' actual reaction on Lesson 6 of Cycle 2

Activity	**Conjectures of students' reaction**	**Students' actual reaction**
Deciding the height of the bookshelf on the wall. (not too high and not too low)	Taking the maximum or minimum height of student Used the midrange, mean, mode, or median Used the sample Estimating the height	The students used the formula of mean by adding the mean of all groups and divided it by the number of group without considering the size of the group.

8. Posttest

The posttest was conducted after the six meeting. The problems were the same in the pretest. It aimed at clarifying the students' knowledge

development that had been observed and analyzed in the teaching experiment phase.

As the result, most of the students wrote a correct sentence about the average. Most of them still wrote the sentence related to the average score, some others used the sentence that appeared in the first activity, including Nabila and Febri . However, some still used a maximum score as the meaning of the average. Interestingly, we also found a student that wrote the average was “the score obtained by adding and dividing based on the number of object”. There was also a student that used their answer to explain the average meaning (Figure 4.28). The answer was wrong but the idea of calculate the average exists.

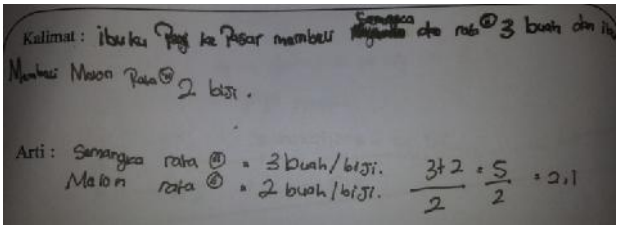
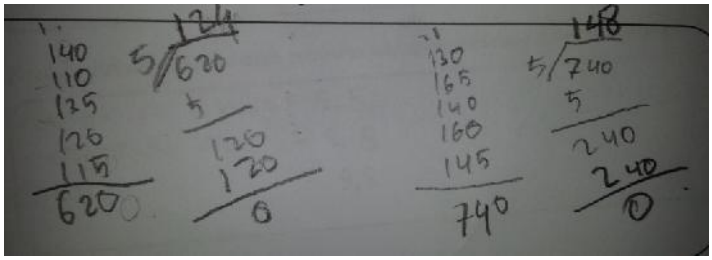


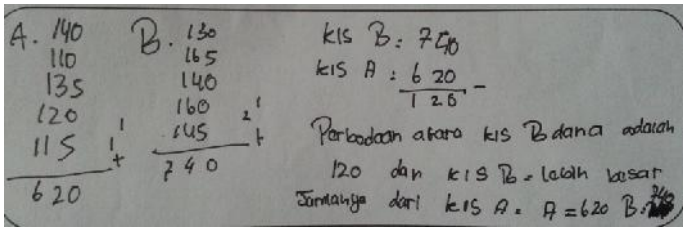
Figure 4.28. An example of students answer for the first question

Most of the students was correct to answer the second question regarding making a score in such a way one can pass the national exam. The students knew that they need to write the scores which were higher than the average for those three subjects. On the contrary, they did not use the average to answer the problem where they have to decide whether the two students passed the exam or not based on their scores.

In comparing the two data sets, we found different solutions. Febby, Azizah, and Mutiara subtracted every single data A by the data B and then divided it by five. Differently, Nabila added every single data A with the data B and then divided it by five. Only few students calculated the average for every group. However, they did not subtract the averages to get the different. They just compared the two averages (Figure 4.29 (a)). They possibly did not understand the question; to find the different. Besides, we also found the students who compared the two data sets by using the sum of the data (Figure 4.29 (b)).



(a)



(b)

Figure 4.29. Examples of students answer comparing two data sets

9. Discussion

The students before the lesson have ever been heard or seen the word average in their daily life. It was shown in the first activity. Most of the students were able to write the sentences regarding the average. However, they still did not know what the meaning of the average in their sentences.

In the teaching experiment, we found four kinds of averages sentences; (1) average as the exact number, (2) average as the arithmetic mean, (3) average as the mode, and (4) average as the fair sharing. The first sentence was incorrect sentence. Meanwhile the other three sentences were correct sentences. The average as arithmetic mean and as mode was mentioned by Mokross & Russel (1995), while average as the fair sharing described in Konnold & Pollastek (2004).

The meaning of average was not a single meaning. There were some interpretations and approaches to describe it. Konnold & Pollastek and Mokross & Russel provided some meaning of the averages. The students' sentences can also have a different interpretation of the word average. One of them was arithmetic mean. Therefore, the average sentences activity was a good starting point to describe the different meaning of the word average for the students. Teacher can emphasize on the idea of arithmetic mean from the sentences.

The repeated measurement context invited students to use the idea of measures of center to solve the problem. However, we should be aware of the

context. The contexts sometimes make students did not consider the others measurements to decide a representative measurement. In the teaching experiment, some students just chose one measurement that more convincing for them without considering others measurements. But, we also saw the students used the measures of center such as the arithmetic mean. However, they did not continue since they got a difficulty to divide the number. The role of teacher to aware of the students' strategy was needed during the solving problem process. The students sometimes have a sophisticated idea to solve the problem. But, when they found an obstacle, they will try to find a simple way to solve the problem and ignore their idea. The number involved also should be considered to design a problem.

In the glider experiment, we saw students used some ideas of measures of center to predict the next throw. By experiencing how to collect the data, the students seemingly understand how the data existed in their experiment. It brings them considered the data in order to predict the distance. Moreover, a hands-on activity on glider motivated students to get involved in the activity.

Compensation strategy on the bar chart helps the students to visualize the formula of the mean. Taking apart and give it to another bar makes them Figure out the formula of mean. However, since this was the first time students encountered with the bar chart, the teacher needs to introduce for the students how to use the bar chart with compensation strategy. In addition, Lestariningsih, et al. (2012) also used the compensation strategy on the bar

chart to promote students understanding of the concept of mean. Their result showed that the students could use the bar in order to find the mean. Bakker (2004) also used the compensation strategy to estimate the mean. As the result, the students easily reinvent the compensation strategy on the bar chart. However, since this was the first time students encountered with the bar chart, the teacher needs to introduce for the students how to use the bar chart with compensation strategy.

Collecting and playing with their own data motivated the students to get involved in this study. We also saw the students' excitement when the students measured their own height. They also could easily performed how to find the mean of their group' height. Moreover, they showed that they can solve the problem when an additional data involved.

As the last problem, both teacher and students did have misconception on finding the classroom average. The teacher asked the students to add all averages from every group and then divided it by the number of groups even thought the group size were different. Gal, et.al. (1989) also described this misconception on their study. They stated that many students in their study blindly added the data even when groups of unequal sizes.

CHAPTER V

CONCLUSION AND SUGGESTION

A. Conclusion

Based on the retrospective analysis, we can draw conclusions about how the measuring activities support students developing their understanding of the concept of mean, as follows:

1. The average sentences activity was a good starting point to describe the different meaning of the word average for the students. The grade 5 students could create several average sentences with different interpretation. It could be used by the teacher as the tools to clarify the meaning of the word average in daily life and also support the students to recognize the concept of mean.
2. The repeated measurement context invited students to use the idea of measures of center by considering others measurements to take one representative measurement. However, we should be aware of the context. The contexts sometimes make students did not consider the others measurements to decide a representative measurement.
3. The compensation strategy on the bar helped the students visualize the formula of the mean. Taking apart and give it to another bar made them figure out the formula of mean.

4. Collecting and playing with the students' data engaged the students to demonstrate how to calculate the mean. The students could easily calculate the mean by using the formula instead of using the bar.
5. The students and also the teacher were not aware of the size of the groups in calculating the mean from several groups' averages. They blindly added the data and divided it by the number of groups, even when groups were unequal size.

The aim of this research was to contribute to a local instructional theory in learning the concept of mean. We summarized the LIT in the table 5.1 as follows:

Table 5.1 LIT of the concept of mean

Tools	**Imagery**	**Activity**	**Concepts**
Average sentences	The daily life used of the word average signifies the different meaning of the word average.	Write the sentence with the word average and describe the meaning of the word. Investigating the average sentences.	Interpretations of the word average : Average as mode; average as the arithmetic mean; average as fair sharing;
Repeated measurement context	Signifies the measures of center through repeated measurement.	Take one measurements with considering others measurements	Average as typical number
Glider, Measuring tape	Signifies the mean, median, midrange, or mode	Throw the glider and measured the distance. Collect the data and predict the next throw by considering the data.	Measures of center such as mean, median, mode, and midrange
Bar chart	The median as one of the measures of	Interpret the data and prediction into the bar.	The arithmetic mean;

	center signifies the compensation strategy on the bar.		Compensation strategy on the bar;
Measuring tape	Demonstrating how to calculate the mean	Measuring the students heights and find the group average	Arithmetic mean
Students' height data	The formula of mean signifies the average of the whole class.	Discussing a strategy to put a bookshelf on the wall. It should be not too high or too low.	Sampling, Arithmetic mean on two or more data sets.

B. Weaknesses of study

During implementing the study, we faced some obstacles and limitation that cannot be avoided. We described them as the weaknesses of the study, as follows :

1. Limitation of the time

   We had limited time to conduct this study. In Indonesian curriculum, primary school students had final exams at June. It was the end of the academic year 2013/2014. Hence, the school did not give us enough time to conduct the study. Therefore, we stopped until the second cycle. Methodologically, the cycle should be continued until the HLT can accommodate the students' conjectures and also reach the goal for every lesson.

2. Social norms and Socio-mathematical norms of the students

   We had the students and the teacher who did not familiar with the discussion. The students did not know how to share their ideas and discussed

with their teammates and also in front of the class. Most of the time, we saw the smartest students always was dominant in group discussion. The other students sometimes followed the smartest students' opinion and kept silent even thought they have a good answer and reasoning. They also did not familiar to have different opinion or answers. Mathematics for them should result in one correct answer.

3. Students' learning style

This study did not focus on the learning style of the students. We designed the activities and contexts without considering the different learning style that students have.

4. Role of the teacher

The teacher in this study also did not familiar with the discussion. He usually uses a direct instructional method where the teacher as the center in teaching and learning process. In addition, he also lack on the RME and the statistics topic. We did not provide a teacher training for the teacher to deal with these problems. It was one of the weaknesses from this study. However, we tried to minimize it by discussed with the teacher thirty to sixty minutes before he implemented the lesson. The discussion was related to the implementation on the teacher guide and also the mathematical idea behind the lesson. Moreover, we also allowed the teacher to ask anything during the teaching and learning process.

C. Suggestion

We suggested for the teachers or the next researchers to give an orientation for students how to discuss in group or in whole classroom discussion. Give the students rule to discuss in the beginning of the lesson. Besides, the context and the story should trigger the discussion. In addition, the worksheet should have a clear instruction in order to make students wrote the intended answer.

It was better to have a good and clear teacher guide especially in how to post questions and deal with different answers from the students in the discussion. It is also good to have a special training for the teacher to introduce the RME and also the topic.

The arithmetic mean was not a simple mathematical concept. It has many interpretations and approaches as we mentioned in the theoretical chapter. The further researcher can use one or two of these interpretation and approaches as a starting point to design their own learning trajectory.

In this study, we focus on the measuring activity that we believed can support students developing their understanding of the concept. However, the concept also applied in vary domain of contexts and subjects. Therefore, we suggest for the further researchers to study on different contexts and application of the concept of mean. Furthermore, the mean also related to other concepts such as mode, median, distribution, midrange, and other statistical concepts. It was better to design a learning trajectory that can combine all of the concepts.

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APPENDIX

The Classroom Observation Scheme

1. **Practical Setting**
   - How many students in the classroom? (Boys and girls)
   - How does the arrangement of classroom setting? (the position of teacher and students' desks, the area of classroom, etc)
   - What is the typical height of the students in the classroom?
   - What tools are the teacher used during the teaching and learning process? (Textbook, worksheet, power point, etc).

2. **Teaching and learning process**
   - How does the teacher start the lesson?
   - How does the teacher organize the classroom?
   - How does the teacher teach mathematics in the classroom? (Explaining all the time, promoting a discussion, carrying out the exercises, etc).
   - How does the teacher involve students in the classroom activity?
   - Does the teacher give a chance for students to ask a question or to give an opinion?
   - How does the teacher deal with the students' question or opinion?
   - How does the teacher facilitate the different opinion or answer among the students?
   - Is there any discussion about students' thinking and reasoning? (if yes, how the teacher deals with the discussion?)
   - If the teacher uses tools such as textbook or worksheet, how does the tools' role during the teaching and learning process? (follow the textbooks, give the worksheet for students)
   - How does the teacher know whether the students understand the topic or not?
   - Where is the position of the teacher during the teaching and learning process? (in front of the class, moving around the class, etc)
   - How does the teacher end the lesson?

3. **Students' activities**
   - How do the students follow the teacher explanation?
   - How do the students work? (Individually, in pairs, or groups)
   - If there is a discussion, how do the students discuss with their friends? (just one or two students talk, sharing opinion, etc)
   - Do the students give their opinion or answer in the classroom?
   - Do the students ask a question if they do not understand?
   - Is there any irrelevant behavior of the students during the lesson? How does the teacher deal with it?
   - Are there any students who talkative and too silent? How does the teacher deal with it?
   - What is the general ability of the students?

The Teacher Interview Scheme

1. **The teacher background**
   - What is your educational background? What is the subject?
   - How long have you been teaching especially in primary school?
   - Have you ever taught statistics before?
   - What do you think about statistics topic in primary school?
   - Have you ever heard about PMRI? What do you think about it?
   - Do you have any experience to teach mathematics by using real context?

2. **Classroom Management**
   - What is the method of teaching do you always use in teaching mathematics?
   - What are the tools, media, or material you often use to teach mathematics?
   - What do you think about the discussion in the classroom?
   - Do you often involve the discussion in your teaching? How do you manage the discussion?
   - Do you have any specific rule in the class? (raising finger, reward or punishment)

3. **The students**
   - Do the students familiar with the group discussion? How do you group them?
   - Do the students get used to give their opinion and ask the questions during the lesson? How do you engage students to give their opinion or to ask a question when they do not understand?
   - Do you invite students to be more active in classroom activities? How?
   - Do you know the students who have high and low ability in mathematics? How do you deal with the different ability of students?
   - Do you know the students who are talkative and too silent in the class? How do you deal with them?

PRETEST & POSTTEST

Nama :
Kelas :

Jawablah pertanyaan berikut

1. Tulislah sebuah kalimat menggunakan kata “rata-rata” dan jelaskan apa arti kata tersebut dalam kalimat yang kamu buat.

   Kalimat :

   Arti :

2. Di dalam sebuah surat kabat tertulis:

   “*Salah satu prasyarat siswa lulus ujian nasional adalah rata rata mata pelajaran yang termasuk UN (Matematika, IPA, dan Bahasa Indonesia) adalah*”

   a. Jelaskan apa yang dimaksud dengan kata “rata-rata” dalam kalimat di atas !

   b. Buatlah nilai untuk ketiga mata pelajaran sehingga rata ratanya adalah 5.

c. Berikut adalah nilai UN dua siswa, Andi dan Rani :

Mata Pelajaran	**Andi**	**Rani**
Matematika	4	5
IPA	5	6
Bahasa Indonesia	5	4

Apakah Andi dan Rani lulus Ujian Nasional? Berikan alasanmu !

3. Dua data tinggi badan dua kelas A dan B.

Kelas A (cm)	**Kelas B (cm)**
140	130
110	165
135	140
120	160
115	145

Berapakah perbedaan selisih tinggi badan antara dua kelas di atas?

Bagaimana kamu menentukan selisihnya ?

Hypothetical Learning Trajectory of the Cycle 2

1. Starting Points

The mean is introduced in 5th grade. Therefore, this is the first time students encountered the concept of mean. Some required knowledge before the lesson are needed to describe in order to support students to develop their understanding of the topic. The starting points of the lesson are as follows:

Students are able to:

a. understand the arithmetic operations on integer.
b. understand the arithmetic operations on decimal.
c. measure the length and the distance by using measuring tape or other length measurement tools.
d. make sentences and describe the meaning of a word in the sentence.

2. Lesson 1: Average sentences

Learning Goal

Students are expected to be able to distinguish some interpretation of the word “average” in daily life.

Materials

Worksheet.

The pictures of apples and its weights.

Mathematical Activities

a. *‘Average’ sentences*

Students are asked to write down one sentence they have ever used, seen, or heard in daily life with the word “average” and given the meaning of the word “average”. The teacher encourages students to write the different use of the meaning of the word “average”. After that, they discuss with their neighbor about the meaning of “average” on their sentences. In the next session, there is a whole class discussion to hear and discuss the sentences from the students. The

discussion stresses on what students know and their interpretation of the word “average”.

b. *Three different sentences.*

After the discussion on the students’ sentences, the students discuss the three sentences;

(1) the average women like a handsome man;

(2) the average height of the fifth grade students was 117 cm;

(3) the average weight of an apple in 1 kg apples was 0.240 kg.

The sentences were given one by one. The students discuss on the meaning of the word average in the sentence.

The last sentence, apple sentence leads students to re-think about the meaning of the word average. The teacher shows the photo of 1 kg apples and some apples’ weights from the 1 kg apples. All of the apples have different weights. Therefore, the average here does not mean that most of the apples have 0.250kg weight. The activity aims at developing students’ understanding about the idea of average and to realize that the average is different with the mode.

The different weights of apples

Prediction of students' responses

Regarding the sentences, the students may come in different kinds of sentences. It shows the students prior knowledge of the word average. Concerning to the meaning of the word average, the students may answer that the average means the word such as 'most of' or 'many'. This answer indicates that the students aware of the idea of a mode. Besides, there is also the possibility that the student's answers indicate that she/he knows what the average (the mean) which is add all of the data and divided it by the number of the data. Similarly with the apple sentences, most of students may say "most of the apple weight is 250g". Since we assume that they will interpret the average as the mode.

Actions of the teacher

Regarding the meaning, when the students' answer it by using the word such as 'most of' or 'many', the teacher should make the answer precise by giving follow up questions such as "what do you mean 'most of'?" "How much do you think 'most of' is?" or "Does the 'most of' mean more than 50%? Or less? Or equal?". Meanwhile, when they have the idea of median, midrange, or even mean, the teacher can bold their strategies in front of the class without telling them the name of those strategies.

During the "apple" sentence, the teacher should encourage students to realize that the average is different with the mode. Post a question such as "Do you think all most of the apples have 0.240kg of weight?"

3. Lesson 2 : Repeated Measurement Contexts

Learning objectives

Students are expected to be able to identify one typical number through repeated measurement.

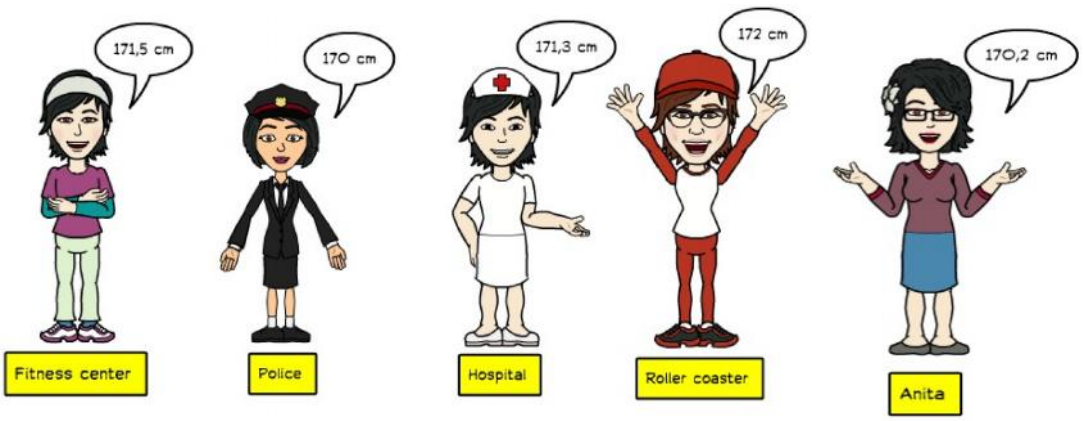
Materials

Students' worksheet.

Mathemathical Activities

1) Repeated measurements of height.

In this activity, the teacher tells a story of her friend – Anita's problem on her heights. Anita took five times measures of height in this month. In the beginning of the month, she was entering a fitness club. The fitness club required to measure the height and resulted 171.3 cm. Two days later, she was going to make a driving license at the police station. In the police station, the official was measuring her height 170 cm. A week after, she checked up at the hospital. The nurse measured her height and got 171.3 cm. Yesterday, before taking a roller coaster, there was a measuring gate and resulted 172 cm. This week, she wants to apply for a job and fills the application form. In the form, there is information on height. However, she is confused. Now, she has four different measuring of her height. To make sure, she tried to measure herself in her house and resulted, 170.2 cm. She now has five different heights.



Anita's height

In this activity, students are asked to help the Anita to decide her height. The problem emphasizes on the chosen height by using the idea of measure of central tendency, such as mode, median, range, or average. However, this activity only focuses on the students' strategies, while introducing the name of the strategy (whether it is mode, median, range, or average) will be at the next meeting. Besides, before the worksheet is given, the teacher asks three or more students to measure the height of one student and make the students realize that the different measures of height might happen.

2) Repeated Measurements of weight

The story about the group students in science class measured the weight of an object and obtained ten data of measurement as follows :

6 gram	46 gram	8.5 gram	7.5 gram	7 gram
6 gram	7 gram	9 gram	8 gram	6.4 gram

The students discussed in their group how to decide the weight of the object. In the first problem, the students may not consider the five measurements when they asked to decide the Anita's height. They tended to chose one number that they thought was convincing. They focused on the person who took the measurement. Therefore, we added one context about repeated measurement on the weight of an object. This context did not describe the person who took the measurement. The story was that to decide the weight of a small object from the ten measurements that was taken from ten students in one classroom. In addition, we also added the outlier data in that problem to make more complicated for students.

<u>Prediction of students' responses</u>

In repeated measurement activity, the students may think randomly to choose the height. They may choose the number in the middle or the height, which are they thinking are more convincing such as the measurement from the hospital. However, in the second problems, the students cannot choose one measurement as in the first problem since it does not specify the person who took the measurements. The possibilities that they may use the median, midrange, or even mean.

Actions of the teacher

In this activity, the teacher encourages students to take all heights into account. The teacher asks students “how do you think we could do if we want to consider all of the heights?” Some students may give random strategies in order to take the heights into account. When the students grasp the idea of average to “add them up and divided” strategy, the teacher can introduce that is the idea of average. However, it does not matter if the students won’t come up to the idea of the “Average”. It will be stressed in the second meeting. Besides, some students may think about the midrange or the median. At this time, the teacher can stress these ideas. In the next lesson, all these ideas will be introduced.

4. Lesson 3: Glider Experiment (Prediction)

Learning Goals

Students are expected to be able to identify the strategies to describe the data.

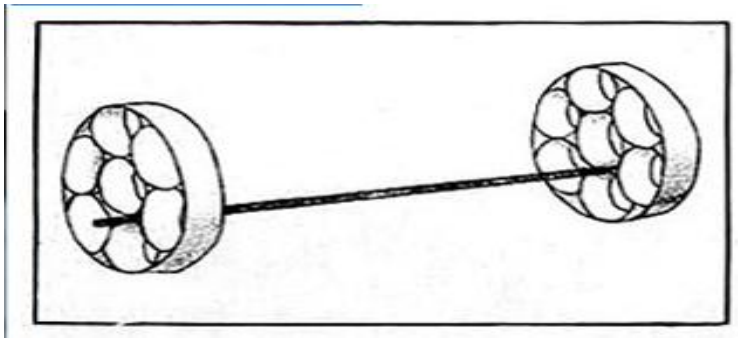
Materials

Worksheet	Papers
Measuring tape	Straw
Ruler	Glue
Scissor	

Mathematical Activities

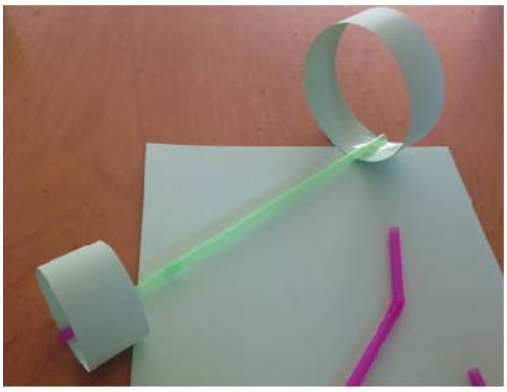
Cylinder glider

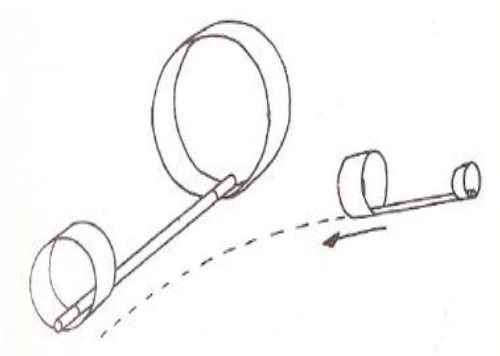
At this meeting, the students have an experiment to make Hagravens’ cylinder glider. This activity adapted by Ainley, J., Jarvis, T. and McKeon, F. (2011). Lawrence Hargrave (1850-1915) was an Australian aviation pioneer, inventor, explorer, mason and astronomer. One of his glider models is:



Hargravens' cylinder glider

The students in groups of 3 or 4 makes gliders from loops of paper attached to a drinking straw. The construction of the gliders allows (tail) loop to be moved along the straw. Look at the following picture. In this activity, the students will investigate the effect of the loops between the two gliders with the length of the flight. They will collect and compare the data of two gliders.





The glider and the way it is thrown

In the beginning, the teacher asks students to make some gliders. The teacher allows students to decide on the diameter as long as the two loops of the glider are different. After that, the students throw the gliders away and measure the length of the distance. The students will collect the data of one glider and write it down in the worksheet. It does not matter whether the students will produce different data. Instead of the trends, the activity focuses on the data (the numbers).

The students will work on the worksheet. There are three main questions in the worksheet: describing the glider characteristics, collecting

the data, and predicting the distance of the glider. The first question expects the students to describe at least the color, the distance between the loops, the circumference of the front and back loops of the glider. The second question is throwing the glider away and measure the distance from the student who throws the glider to the position where the glider stops. The third question is finding the glider distance by considering the five data. The distance is the prediction of what will be the distance of the next throw.

Two main discussions are held in this meeting: (1) how to measure the glider' distance and (2) the different strategies to decide the prediction of the distance of the glider. The first discussion purpose is to have an agreement in measure the distance of the glider. Some points to discuss as follows:

a. How to measure the distance?
    1) Is it from the person who throws away the glider to the nearest part of the glider? Or farther away? Or the middle?
    2) Is it when the glider hit the floor? Or until it stops?
b. How to see the measuring tape if the measurement is between two numbers? Do we need to boundary up? Or down?
c. How to throw the glider? Do we throw it with full energy or with a slowly?
d. Other points that students may found during the time practicing.

The second discussion focuses on the students' strategy to find the prediction. The discussion is expected to discuss the mean, median, mode, and range strategy. After that, the teacher will introduce the name of those strategies, whether it is mean, median, mode, or range.

<u>Prediction of students' responses</u>

Related to predict the next throw, some strategies that students can come up, such as:

a. Median: taking one middle distance from the data,

b. Midrange: add the maximum and minimum distances and divided it by two,
c. Mode: the most appeared distance,
d. Maximum or minimum distance: taking the farthest or nearest distance.
e. Random: taking one distance not from the data or estimating without any procedure.

Actions of the teacher

If the students use the idea of the mode, median, midrange, or even mean. The teacher may ask them to present their idea in front of the class later on the whole class discussion. Particularly the median and midrange strategies, the teacher ask the students to present it together. In the whole class discussion, the teacher may discuss some points, such as:

Some point to discuss when each strategy is presented:

Table 2.3. Points to discuss

Mode	– If there is no same value, what did you do then? – If there are two values with appeared the same times what did you do? – The teacher can also relate the mode strategy to the sentences of mode in the first meeting.
Median and Range	– What do you think the difference between these two strategies? – What do you think the strong and weak points of these two strategies?
Mean	– How did you find the strategy? – The teacher emphasizes this strategy for the next meeting.

Meanwhile, if the students use the strategy of maximum or minimum value, the teacher should stress that they have to consider other measures otherwise there is a distance which is far away from the prediction. The good prediction should be close enough to all the data that we have.

Similarly with the random strategy, the teacher should ask the students how they got the number. The teacher may ask questions such as: “how did you get the number? How do you estimate? Why did you choose this number?” Ask them to think about a reasonable procedure in determining the prediction. During the whole class discussion, the teacher may bring this idea to see other students’ opinion on these three strategies.

5. Lesson 4: Glider Experiment (Compensation Strategy on Bar Chart)

Learning Goals

Students are expected to be able to derive the formula of mean by using compensation strategy on the bar chart.

Materials

Worksheet

Glider

Ruler

Mathematical Activities

In this meeting, the teacher plays with his/her own glider. The teacher in the beginning of the lesson may show his/her glider for the students and throw the glider once to show for students how the glider flies. After that, the teacher will show the second throw data as follow:

The data of throwing the glider

Number of experiments	Glider 1
1	100 cm
2	110 cm

In this data, the teacher asks students to find the “typical distance” as the representation of the glider data. The bar chart is used in finding the typical distance (Look the worksheet number 1). The worksheet consists of three questions. The first question is to find the typical distance of the two data above. After that, the data and the typical distance interpret into a bar. In the second question, the students are given one more addition data, 90

cm. Again, the students are asked to directly interpret the data into the bar and find the typical height. This question emphasizes on the compensation strategy in the bar. The students are expected to realize the strategy to share the amount of value to another value in such a way that the value will result the same. The fourth data, 140 cm, is given as the third question in the worksheet. Interpreting the data into a bar and finding the typical distance from the bar also are the activities in this question. At this time, the students are expected to realize how to find the mean through the compensation strategy in the bar.

Every question is ended with a short discussion on how the students find the typical distance and interpret it into a bar. The discussion stressed at the idea of the average, which add the data and divided them by the number of data through the compensation strategy on the bar. From the discussion of the first question, the teacher should start to emphasize these points:

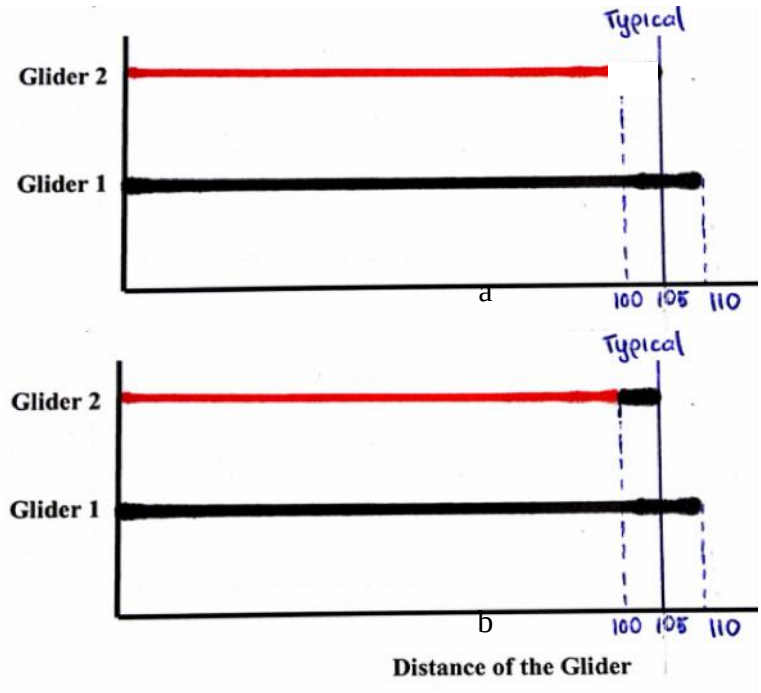
a. The strategy between the median and midrange. These two strategies obtain the same answer.
b. The way students draw the bar. How the students put the number on the x-axis and how the length of the bar they drew.
c. The way students draw the typical distance on the bar.

The further discussion also stresses on the points. The formula of mean is introduced implicitly through compensation strategy using the bar chart. Lastly, the students are expected to know how to find the average of the data.

Prediction of students' responses

Regarding the strategy to get the typical distance, the students may use the median or midrange strategy. The median may take the middle number by dividing the first and the second data. Meanwhile, the midrange uses the maximum and minimum data and divided it by two. These two strategies obtain the same result in the first question, because there are

only two data. The strategies do not obtain the same result for the second and the third questions.



(a) the bar without compensation strategy;

(b) the bar with compensation strategy

In the bar, the students may directly draw the answer without realizing the compensation strategy (figure a). They just draw the bar and the typical distance (figure b). Similarly, when they ask to explain their strategy based on the bar (worksheet 1c, 2b, and 3b), the students may explain based on the median and midrange strategies instead of the compensation strategy.

Actions of the teacher

In the typical distance question, particularly for the first question, the students are expected to use the median or midrange strategy. Therefore, when the students use the strategies, the teacher may ask them to present their result during the whole class discussion. It is different with the second and the third strategies. When they still use the strategies, the teacher encourages the students to realize that the question in the worksheet aims at explaining their strategy based on the bar instead of the

data. Since in the whole class discussion for the first question the teacher have already emphasized the compensation strategy, the teacher may post question for students to remember the strategy, “see the first question on how to find the typical distance based on the bar”.

Regarding the interpretation of the bar, the teacher can have a nice drawing of the bar with two colors of markers in the blackboard and starts to ask questions such as: “what do you think we can do to find 105 cm? What does make the 100 cm bar become 105cm? And what does make 110 cm bar become 105 cm?” The questions support students to realize the compensation strategy which is to give and take some amount of value in such a way that the bar obtains the result values.

Related to the question 1c, 2b, and 3b, the teacher may directly say to the students that the questions need to answer based on the bar instead of the strategy from the data. The teacher may stress from the first question on how to answer such kind of questions.

6. Lesson 5: Students’ heights

Learning Goal

Students are expected to be able to demonstrate how to calculate the mean.

Materials

Worksheet

Measuring tape

Mathematical Activities

In this meeting, the teacher wants to compare the height of the students among the groups. The students just measure the height of their group and then find average to describe their group height. Besides, there is a question regarding an addition data. The discussion emphasizes on how students see the shift of the average because of one data. In addition, the students also still are asked to draw and interpret the data and the average into the bar. The students are expected to have varied ways

regarding the question of the additional data. The discussion focuses on how the students the shift of the average if we add one data.

In the next, the teacher asks the following questions for students to discuss: Considering the height of the new data, what will happen to the average of the group?

a. If his height is more than the average?
b. If his height is less than the average?
c. If his height is as same as the average?.

Prediction of students' responses

Regarding to find the new mean from one additional value, some students may do the followings:

a. add the average and the new data, and then divided it by two.
b. add the average and the new data, and then divided it by the number of data plus 1.
c. add all the data again including the new data, and then divided by the number of data plus 1.

Actions of the teacher

Regarding the additional value problem, the teacher can ask all different strategy to explain their strategy in front of the class. From all possibilities, all of the students may agree on the strategy to add all of the data and the new data, and then divided it by the number of data plus one. However, the teacher may show others strategy as the comparison. The teacher also discusses the idea to use the average to combine the two data sets. The teacher may post a new problem such as: "if I have two data set, and I want to find the average of all two data together, may use the data and divided by 2?" Moreover, the bar chart may help to explain the shift of the average. The bar can show visual proof that the result when we use the strategy by adding the additional value with the initial average and divided by two will obtain differently when we find the average as usual – add the data and divided it by the number of data. To make it clear, the teacher can use the formula.

7. Lesson 6: The Bookshelf Context

Learning Goal

Students are expected to be able to solve the problem by using the average.

Materials

Worksheet

Mathematical Activities

The context: Bookshelf

Figure 2.7. The bookshelf

In the classroom, the teacher has a plan to put a shelf on the wall. However, she does not know how high the best for the shelf. The teacher thinks that the shelf should be not too high and not too short. If it is too high, some students may not be able reach the book or some things on the shelf. If it is too short, some students may get hit by the shelf.

In this meeting, the teacher asks students to make a plan to determine the height of the shelf on the wall. The students are expected to be able to use the idea of average to solve this problem. In the beginning, the teacher encourages the students to think what kind of information they need to carry out their plan. One of the issues is that they need the height of all students in the class.

In the previous meeting, the students have measured the height of their own group. Therefore, the teacher can ask every group to put their data on the blackboard or on the wall so all groups can use the data. The students can decide whether they want to use all data or they may pick certain samples. This is one of the ideas in this lesson in which students

think to not use all of the data. This issue will be used in the whole class discussion.

During the discussion, the teacher asks some groups to present their plan in front of the class. In addition, the teacher points out the issue: “taking some students to decide on the number instead of all students. How? And what kind of students should we choose?” In the last session, the teacher may tell how the statisticians take some data as the sample instead of all the population. The teacher may use the context to know the income of Indonesian people.

Prediction of students’ responses

Regarding the way students making their own plan, the students' answers may as follow:

a. Taking the maximum and minimum height because if it is based on the highest student, the other students can use the chair. Meanwhile, if it is based on the shortest student, all people can reach the shelf.
b. Taking the midrange.
c. Taking a middle value by estimation.
d. Taking the mode, the mean, or the median.
e. Taking sample instead of all data and find the mean or median of the sample
f. Taking the mode, the mean, and the median, and add some cm in order to make the higher shelf.

Actions of the teacher

a. If the group uses to choose the tallest or the shortest students’, the teacher can ask them whether it is too short or too high. It is just little students may reach the shelf if it is too high, meanwhile if it is too short most of the student will feel uncomfortable, why did you choose this number?
b. If the group is to just estimate the number, the teacher may drill the estimation. What kind of consideration they think to decide the number.

c. If the group uses midrange, the teacher may ask the generalization on every situation such as if there is one student is very high and one student is very short in a classroom, then can we still use the midrange?
d. The teacher allows the group who uses the idea of mean, median, and mode. In addition, if there is a group who use the sample, the teacher also should pay attention to this strategy. It can be a good point to discuss as fascinating planning.

TEACHER GUIDE

The 1st Meeting

Topic	:	Mean
Grade / Semester	:	5th / Even
Meeting	:	1
Time Allocation	:	2 × 35 minutes

Learning objectives

Students are expected to be able to distinguish some interpretation of the word "average" in daily life.

Materials

Students' worksheet.

Learning Activities

1. Orientation of the problem (5 minutes)
    - The teacher starts the lesson by asks students whether they have ever heard, seen, or read the word "average". The teacher may also ask some students who want to talk what type of the sentences they have found and where they have found that word. In this time, the teacher tries to encourage students to feel free to talk and give their own opinion. The teacher starts to establish the socio norm that the students can express their idea and tell their own story in the class.
2. Working on worksheet (5 minutes)
    - The teacher asks the students to write a sentence using the word "average" and then explain the meaning of the word in the sentences. The students work on the worksheet individually.
    - The teacher goes around the class to check which students have difficulties during the worksheet time. In this time, the teacher should be aware of the pair

that both students have difficulty in making the sentences, since in the next step is that the students will discuss in pair with their neighbor. The teacher may directly change the pairs.

3. Pair discussion (5 minutes)
   - The teacher asks the students to discuss with their neighbor. The discussion aims at explaining and understanding the students' sentences and the meaning of the word average to each other.
   - The teacher goes around the class to check some of the students' sentences. In this time, the teacher tried to find different types of the sentences..
4. Whole class discussion (30 minutes)
   - The teacher gives opportunity for some students with different kinds of sentences to read their sentences and describe the meaning.
   - The teacher then started to discuss a sentence "The average of woman likes a handsome man". The teacher gives the students time to think first for one minute. The teacher then gives the opportunity for students to explain the meaning of the word average. Some may answered "the mos" or "many".
   - The teacher then started to discuss the second sentence "The average height of grade five students is 171 cm". The teacher gives the students time to think first for one minute. The teacher then gives the opportunity for students to explain the meaning of the word average.
   - In this time, if the students' answer that the average means the word such as 'most of' or 'many', the teacher should make the answer precise by giving follow up questions such as "what do you mean 'most of'?" "How much do you think 'most of' is?" or "Does the 'most of' mean more than 50%? Or less? Or equal?".
   - If the student's answers indicate that she/he knows what average (the mean) is, the teacher could drill to find out how far the student understands the

concept. And then, the teacher bolds the idea from that students to the whole class.

5. The apple sentences. (25 minutes).
   - The teacher posts a sentence that "*The average weight of an apple in 1 kg apple is 0.240kg*" and asks the students to think individually about the meaning of the word average in the sentences in 1 minute. After that, they can talk to their neighbor.
   - The teacher gives opportunity for the students who want to explain their opinion. Some students may still argue that it means "many" or "most of".
   - The teacher shows the photo of 1 kg apples and some apples' weights from the 1 kg apples. All of the apples have different weights. Therefore, the average here does not mean that most of the apples have 240g weight. The activity aims at developing students' understanding about the idea of average and to realize that the average is different with the mode.
   - The teacher then emphasize that the next meeting focuses on this kinds of average.

TEACHER GUIDE

The 2st Meeting

Topic	:	Mean
Grade / Semester	:	5th / Even
Meeting	:	2
Time Allocation	:	2 × 35 minutes

Learning objectives

Students are expected to be able to identify one typical number through repeated measurement.

Materials

Students' worksheet.

Learning Activities

1. Anita's height problem (35 minutes)
    - The teacher tells a story of Anita:

      Anita took five times measures of height in this month. In the beginning of the month, she was entering a fitness club. The fitness club required to measure the height and resulted 171.5 cm. Two days later, she was going to make a driving license at the police station. In the police station, the official was measuring her height 170 cm. A week after, she checked up at the hospital. The nurse measured her height and got 171.3 cm. Yesterday, before taking a rollercoaster, there was a measuring gate and resulted 172 cm. This week, she wants to apply for a job and fills the application form. In the form, there is information of height. However, she confused. Now, she has four different measuring of her height. To make sure, she tried to measure herself in her house and resulted, 170.2 cm. She now has five different heights.

- The teacher asks to help the Anita to decide her height. The problem emphasizes on the chosen height by using the idea of measure of central tendency, such as mode, median, range, or average.
- The teacher asks the students to work on the worksheet in groups. The teacher goes around the class to check how students decide the Anita's problem.
- Some students may think randomly to choose the height. They may choose the number in the middle or the height which is they think are more convincing such as the measurement from the hospital. In this time, the teacher suggests the students to consider the five measurements. The teacher can pose questions such as : "so, do you think others measurements are wrong?" "How if we consider the five measurements"?
- The teacher tried to find the different reasoning from students to be discussed in the whole class discussion.
- The teacher then conducts the whole class discussion to hear and discuss the student's strategy on how they decided the anita's height.

2. A small object weight problem (35 minutes)
   - The teacher tells a story:

     In a science class, the students were asked to measure the weight of a small object. Ten students in one group measured the weight by using the scale. The following table shows the data

6 gram	46 gram	8.5 gram	7.5 gram	7 gram
6 gram	7 gram	9 gram	8 gram	6.4 gram

   - The teacher asks to help the group of students to decide its weight. The problem emphasizes on the chosen weight by using the idea of measure of central tendency, such as mode, median, range, or average.

- The teacher asks the students to work on the worksheet in groups. The teacher goes around the class to check how students decide the weigth's problem.
- Some students may think randomly to choose the height. They may choose the number in the middle or the height which is they think are more convincing such as the measurement from the hospital. In this time, the teacher suggests the students to consider the five measurements. The teacher can pose questions such as : "so, do you think others measurements are wrong?" "How if we consider the five measurements"?
- The teacher tried to find the different reasoning from students to be discussed in the whole class discussion.
- The teacher then conducts the whole class discussion to hear and discuss the student's strategy on how they decided the weight.
- In the beginning, the teacher discusses the outlier in the data. The teacher can pose a question such as :"is there any something wrong with the data?"
- In the next, the teacher heard the students' reasoning to answer the problem.
- Lastly, the teacher emphasizes the different strategy to find the typical number that appeasr during the discussion.

TEACHER GUIDE

The 3rd Meeting

Topic	:	Mean
Grade / Semester	:	5th / Even
Meeting	:	3
Time Allocation	:	2 × 35 minutes

Learning objectives

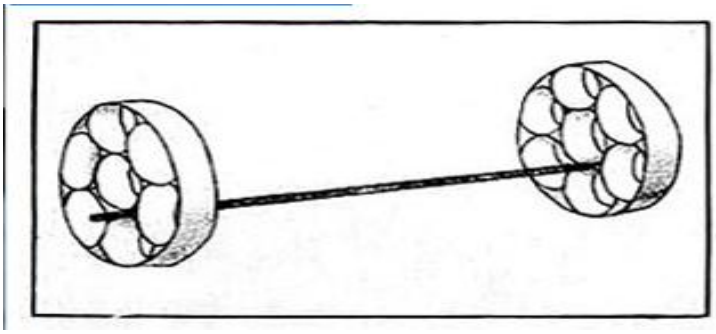
Students are expected to be able to identify the strategies to describe the data.

Materials

- Worksheet
- Measuring tape
- Ruler
- Scissor
- Papers
- Straw
- Glue

Learning Activities

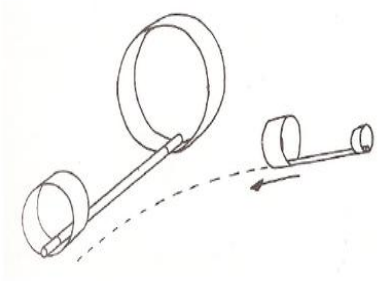
1. Orientation of the problem (10 minutes)
   - The teacher tells to the students that in this meeting the students will make Hagravens' cylinder glider. Lawrence Hargrave (1850-1915) was an Australian aviation pioneer, inventor, explorer, mason and astronomer. One of his glider models is:

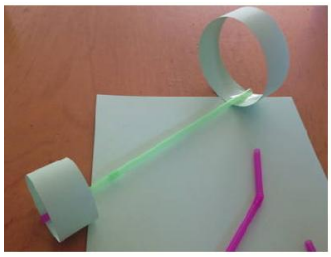


   - The teacher divides the class into 3-or-4 members groups.

- The teacher explains that later they will make their own glider and measure the distance of the glider. They will investigate how far the glider can fly and estimate the distance by looking at the data of throwing that we have.

2. Making a glider (10 minutes)
   - The teacher explains how to make a glider. They make a glider from loops of paper attached to a drinking straw. The construction of the gliders allows (tail) loop to be moved along the straw. The teacher can provide an example of the glider for the students





   - The teacher asks the students to make their own glider.
   - During this session, the teacher goes around the class to help the students. The students allow making two or three gliders. In this time, the teacher may ask the students to try out their own glider.

3. Measuring the distance (Working on the worksheet). (20 minutes)
   - The teacher shares the worksheet for every group. And explain the worksheet.
   - The worksheet consists of three questions. The first question is the glider characteristics. The students are expected at least to describe the color, the distance between the loops, the circumference of the front and back loops. The second question is collecting the data. The students throw the glider away, and measure the distance from the student who throws the glider to the position where the glider stop. The third question is to predict. In this time, the teacher does not need to explain this question now.

- The teacher asks students to choose one glider as the best glider they made. The teacher, at the first, asks the students to describe their own glider. The next, they can throw the glider away and measure the distance for practicing about 5 minutes. The practice activity may lead students to face some difficulties to measure the distance.
- The teacher opens a discussion related to measure the glider. The followings are several points to discuss.
  a) How to measure the distance?
     3) Is it from the person who throws away the glider to the nearest part of the glider? Or the far away? Or the middle?
     4) Is it when the glider hit the floor? Or until it is stop?
  b) How to see the measuring tape if the measurement is between two numbers? Do we need to boundary up? Or down?
  c) How to throw the glider? Do we throw it with the full of energy or with a slowly?
  d) Other points that students may found during the practicing time.

  In the discussion, the teacher should make sure that they have the agreement of those points above. The teacher may ask a student to repeat the agreement and explain how they are going to collect the data.
- The teacher allows students to collect their own data by five times throwing. During the experiment time, the teacher goes around the class to make sure they throw and measure the distance well and follow the agreement.

4. Estimate the glider distance for the next throwing. (10 minutes)
   - After seeing the students find the five throwing data, the teacher explains the third question on the worksheet which is to find the glider distance by considering the five data. The distance is the prediction of what will be the distance of the next throwing.
   - The teachers give 5 minutes for students to discuss in their group.

- During the discussion, the teacher goes around the class to see how students' predict the distance.
- If the group' prediction related to the mode, median, mean, or midrange, then the teacher may asked the group to present their results in front of the class later on the discussion.
- If the group' prediction is taking one number randomly from the data (not the median), the teacher should drills how they take that number. The teacher may ask questions such us :

  Teacher : Why did you choose that number?

  Student : We just take one number (for example, they took the minimum value), which is 150 cm.

  Teacher : But, you see here (point the maximum value), it is too far from those number. It means that your prediction is not considering this number. What do you think the best prediction? It should be close enough to all data that we have. Could you think other number?
- If the group' prediction is only estimate the middle value (not the median) without any reasonable procedure, the teacher could ask them how can they find the number.

  Teacher : How did you get that number?

  Student : We estimate.

  Teacher : How do you estimate?

  Student : We estimate the middle value. We just take one number that we think it is in the middle.

  Teacher : Do you think it is in the middle? Could you find a way to explain how you estimate the middle value that you got?

5. Whole class discussion (20 minutes)
   - The teacher starts the whole class discussion by asking some groups with the different strategies present their strategy in front of the class. The teacher asks

the mode, the midrange and the median, and the mean strategy to present in order.

- The teacher gives opportunity for others group to give comments, opinions, doubts, or questions for the presented team. Some point to discuss when each strategy is presented:

Mode	– If there is no same value, what did you do then? – If there is two values with appeared the same times what did you do? – The teacher can also relate the mode strategy to the sentences of mode in the first meeting.
Median and Range	– What do you think the different between these two strategies? – What do you think the strong and weak points of these two strategies?
Mean	– How did you find the strategy? – The teacher emphasizes this strategy for the next meeting.

After each discussion, the teacher gives the name (mode, midrange, median, and mean) for every strategy that students' present.

TEACHER GUIDE

The 4th Meeting

Topic	:	Mean
Grade / Semester	:	5th / Even
Meeting	:	4
Time Allocation	:	2 × 35 minutes

Learning objectives

Students are expected to be able to determine the mean by using the bar chart.

Materials

- Worksheet
- Glider
- Ruler

Learning Activities

1. Orientation of the problem (10 minutes)
    - The teacher starts the lesson by showing her/his glider for the students and describing the characteristic of the glider. The teacher tells the students that this meeting is about the teacher's glider data. They will find out the prediction of the teacher's glider.
    - The teacher throws the gliders several times in front of the class and shows how well the glider flies. The teacher also shows how he/she measures the distance of the glider. Besides, the teacher also emphasizes that in this meeting we will works on the bar chart. The teacher can ask the students whether they know the bar chart and ask them to draw one example of the bar.
2. Working on the worksheet: Question 1. (10 minutes)
    - The teacher shares the worksheet for every group.

- The teacher asks them to focus on the first question in the worksheet. And then, the teacher explains the first question. In the first question, the teacher throws the glider two times and measures the distance. The students are asked to determine the typical distance of the glider. There is also the bar chart drawing in the worksheet where the students have to interpret the data into the bar.
- The teacher goes around the class to see what's going on in the discussion. Three points that the teacher should pay attention during the worksheet time; how students decide the number, how students interpret the number into the bar, and how they draw the typical distance on the bar. In addition, the teacher also tries to find two teams who use the idea of midrange and median.

3. Whole class discussion: Question 1. (10 minutes)
   - The teacher gives opportunity for the team who wants to share their answer and draw the bar in the blackboard. The presenting team explains their result to the whole class.
   - The teacher allows other teams to give comments, ideas, questions, or different answers of the first question. Besides, the teacher can also discuss about :
     - The strategy between the median and midrange. These two strategies obtain the same answer.
     - The way students draw the bar. How the students put the number on the x-axis and how the length of the bar they drew.
     - The way students draw the typical distance on the bar.
     - The way the students interprets the strategy of point (1a) into the bar. The students should realize the compensation strategy in the bar.
4. Whole class discussion: Orientation of the question 2. (10 minutes)

- The teacher orients students with the second question. The situation is that the teacher wants to throw the glider once again. In order to orient the students, the teacher encourages a discussion as follows :

  Teacher : What do you think the distance of the third throwing will be?

  Student : 105 cm

  Teacher : Why?

  Student : Because it is the result of the first question
- The teacher shares the result of the third throwing which obtains totally different prediction 90 cm. Then, the teacher asks questions to the students such as: what is happening? Do you think that our prediction is wrong? In this time the teacher listens to the students arguments. The teacher may give time one minute for students to think about the answer and then talk with their neighbor to react on the question.
- The teacher again posts a question such as :
  - What will happen to our prediction?
  - Should we change the prediction?
  - If we change the prediction, is it getting bigger? Or same? Or equal?

  In this time, the teacher listens to what students' opinion. If the students needs more time to think, give them time to think and talk to their neighbor.

5. Working on the worksheet: Question 2 (10 minutes)
   - The teacher encourages students to once again find the typical number including the third data. It is the time to work on the second question in the worksheet.
   - The teacher goes around the class to check students' work. The teacher should pay attention to the students answer.
   - If the students use the idea of midrange and median, the teacher asks them to look the worksheet carefully. The question 2 asks to find the typical height by using the bar chart.

6. Whole class discussion: Question 2 and the orientation of question 3. (10 minutes)
    - The teacher asks one team to describe their strategy to find the typical distance after the third throwing. The discussion points out on the compensation strategy that they take some parts of the bar to fill the other parts in such a way they will get the same result. The teacher leads the discussion in such a way they realized that the compensation strategy in this problem is adding all of the data and divided it by 3. The teacher may post a question such as “do you have any idea how to find the result without using the bar?” “what do you think we can do to find 105 cm? What does make the 100 cm bar become 105cm? And what does make 110 cm bar become 105 cm?”
    - The teacher has one more data of throwing the glider. The teacher post questions again such as: “We already know that the fourth throwing will change our prediction, however, what do you think the condition of the fourth throwing such that our prediction is not change?” The teacher gives one minute for students to think of the answer, and talk to their neighbor. And after that listen to the students answer.
7. Working on the worksheet: Question 3. (10 minutes)
    - The teacher asks students to work on the question 3.
    - The teacher goes around the class to see the students’ answers. The teacher tries to find the team who results the idea of mean which is add all of the data and divided it by the number of data. The teacher asks the team to present in front of the class later.
    - If there is a team still used the idea of median or midrange, the teacher may ask them to realize the compensation strategy on the second problem.
    - The teacher names the strategy. The teacher also introduce that the mean sometimes called average. This is one of the strategies with median, mode, and midrange that we can use to summarize the data.

The 5th Meeting

Topic	:	Mean
Grade / Semester	:	5th / Even
Meeting	:	5
Time Allocation	:	2 × 35 minutes

Learning objectives

Students are expected to be able to demonstrate how to calculate the mean.

Materials

- Worksheet
- Measuring tape

Learning Activities

1. Orientation of the problem (5 minutes)
    - The teacher starts to tell that this meeting he/she wants to know the height of all students in the class and compare the height of the students among the groups. They will work on the worksheet.
2. Measure the height of the group members (25 minutes)
    - The teacher shares the worksheet and gives them time to read the worksheet.
    - The teacher points one student to explain the worksheet. The worksheet has three main questions: the height of the groups, the bar with the average, and an additional data that makes the average moves. The teacher should make sure that the students know what they will do.
    - The teacher asks the students to start working in their group. The teacher goes around the classroom to see how the students measure the height of their

group. The teacher also pays attention to how the students answer the question number 3 and the way they interpret the data into the bar.

- The teacher tries to find the different ways in solving the problem of the new additional data and ask the group to present their result.

3. Whole class discussion : The answer of the worksheet (25 minutes)
   - The teacher gives the opportunity for the group who has different result to explain their answer in front of the class. The discussion emphasizes on how students see the shift of the average because of one data.
   - The teacher also discusses the idea to use the average to combine two data sets. The teacher may post a new problem such as: “if I have two data set, and I want to find the average of all two data together, may use the data and divided by 2?”
4. Whole class discussion : The new problem (15 minutes)
   - The teacher asks the following questions for students to discuss:
     Considering the height of the new data, what will happen with the average of the group?
     (a) If his height is more than the average?
     (b) If his height is less than the average?
     (c) If his height is as same as the average?.
   - The teacher asks them to think individually, and then talk to their neighbor about the answer.
   - The teacher conducts the whole class discussion to hear the students’ answer related to the questions.

TEACHER GUIDE

The 6th Meeting

Topic	:	Mean
Grade / Semester	:	5th / Even
Meeting	:	6
Time Allocation	:	2 × 35 minutes

Learning objectives

Students are expected to be able to solve the problem by using the idea of average.

Materials

- Worksheet

Learning Activities

1. Orientation of the problem (15 minutes)
    - The teacher starts to tell a story about the shelf on the wall.

      The context :

      In the classroom, the teacher has a plan to put a shelf on the wall. However, she does not know how high the best for the shelf. The teacher thinks that the shelf should be not too high and not too short. If it is too high, some students may not be able reach the book or any things on the shelf. If it is too short, some students may get hit by the shelf.
    - The teacher asks students to make a plan to determine the height of the shelf on the wall. The teacher gives time to the students to think what they need to know. Some students may answer that they need the height of all students in the class.
    - The teacher asks students to put their result on the previous meeting on the wall so other groups can see the result and use it to carrying out their plan.
2. Working on their plan (30 minutes)

e. The teacher gives the students 20 minutes to carry out their plan and make a poster.
f. The teacher goes around the class to hear every team plans. The teacher decides two or three planning among the groups who includes the median, mode, and mean idea in their plan.
g. If the students use to choose the tallest or the shortest students', the teacher can ask them whether it is too short or too high. It is just little student may reach the shelf if it is too high, meanwhile if it is too short most of the student will feel uncomfortable, why did you choose this number?
h. If the students use to just estimate the number, the teacher may drill the estimation. What kind of consideration they think to decide the number.

3. Whole class discussion (25 minutes)
   - The teacher asks the chosen groups to present their idea in front of the class.
   - The teacher gives opportunity for other students to give comments, doubts, or remarks on the presented team.
   - The teacher point out the issue: "taking some students to decide on the number instead of all students. How? And what kind of students should we choose?"
   - The teacher gives the students 2 minutes to think individually, and talk with their group about the possibilities.
   - The teacher conducts the discussion on the issue by giving opportunity to the group to share their idea.
   - In the last discussion, the teacher tells that some people do the issues often. The teacher can talk about how the statisticians take sample from the population. The teacher may give the example of determining average income of Indonesian people.

PERTEMUAN 1

Nama : …….
Kelas : ……..

Lembar Kerja Siswa

Tulislah sebuah kalimat dengan menggunakan kata “rata-rata” dan jelaskan arti kata tersebut di dalam kalimat yang telah kamu buat.

Kalimat :

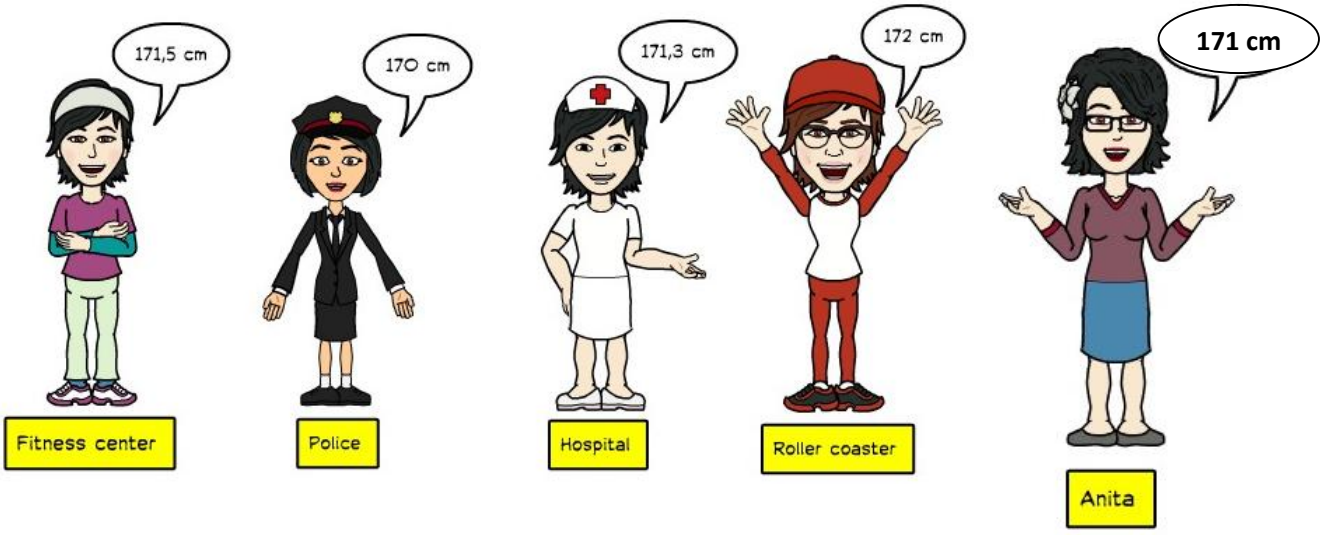
Arti kata “rata-rata”:

PERTEMUAN 2

Kelompok : ……..
Nama : …….
: …….
: ……..
Kelas : ……..

Lembar Kerja Siswa 1

Tinggi Anita



Menurutmu, berapakah tinggi yang sebaiknya Anita tuliskan?

Bagaimana kamu menentukan tinggi tersebut?

PERTEMUAN 2

Kelompok : ……..
Nama : …….
: …….
: ……..
Kelas : ……..

Lembar Kerja Siswa 2

Dalam sebuah kelas IPA, siswa diberikan tugas untuk mengukur berat sebuah benda kecil. Sebelas anak membuat sebuah kelompok dan mencoba mengukur berat benda tersebut. Berikut adalah hasil kesebelas anak.

6 gram	46 gram	8.5 gram	7.5 gram	7 gram
6 gram	7 gram	9 gram	8 gram	6.4 gram

Bantulah mereka untuk menemukan berat benda kecil tersebut.
Diskusikanlah dengan teman kelompokmu berapa berat benda kecil tersebut?
Jelaskan jawaban dan alasanmu di dalam kotak di bawah ini.

PERTEMUAN 3

Kelompok : ……..
Nama : …….
: …….
: ……..
Kelas : ……..

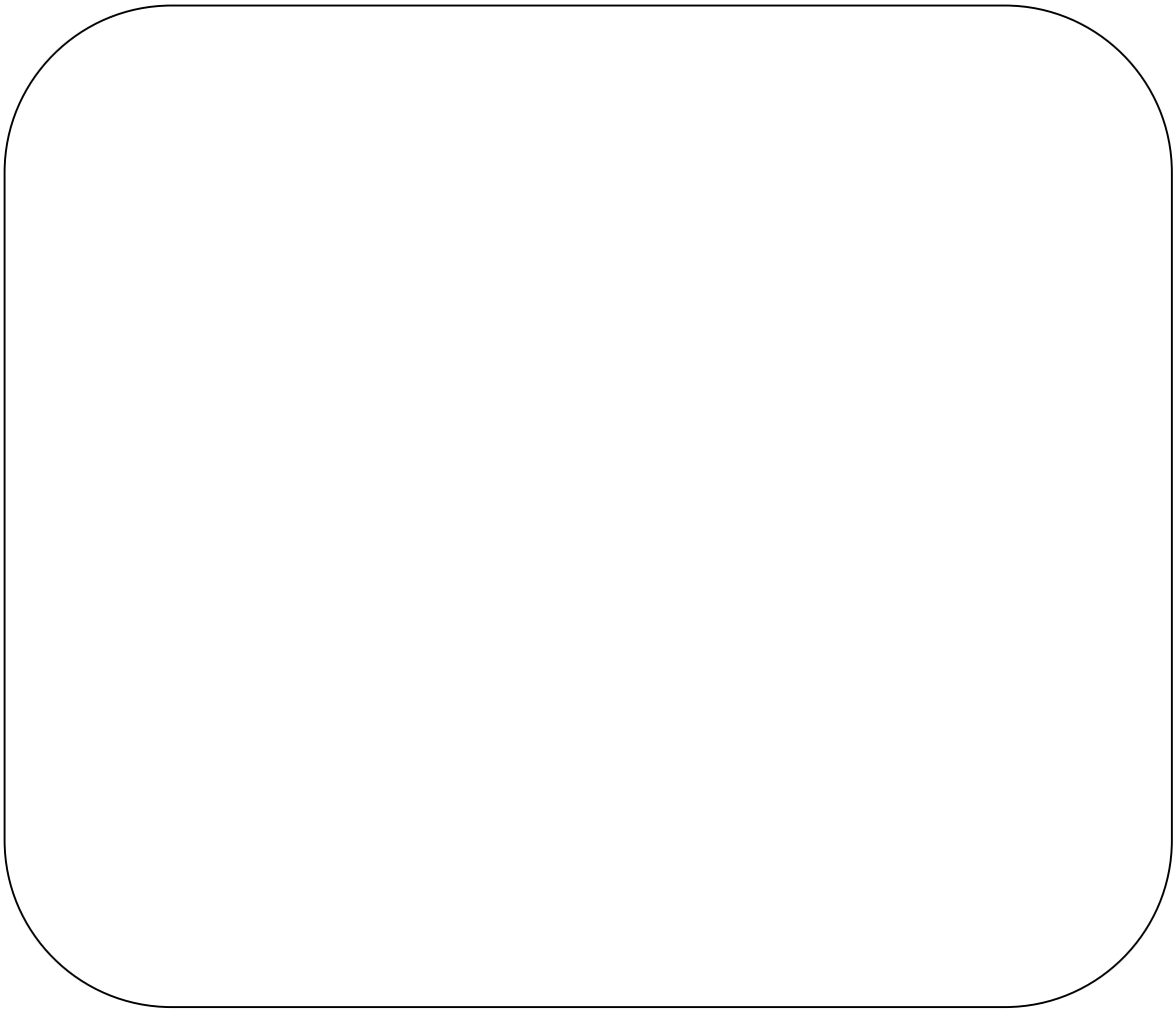
Lembar Kerja Siswa

1. Gambarkan ciri-ciri glidermu. (warna, jarak antar lup, keliling lup depan dan belakang, dan ciri lainnya yang kalian anggap perlu).

2. Masukkan datamu :

No.	Glider (cm)
1	
2	
3	
4	
5	

3. Berapakah prediksi jarak glidermu? Jelaskan strategimu untuk mendapatkan jarak tersebut !



PERTEMUAN 4

Kelompok : ……..
Nama : …….
: …….
: ……..
Kelas : ……..

Lembar Kerja Siswa

1. Data

Banyaknya Percobaan	Glider
1	100 cm
2	110 cm

a. Berapakah prediksi jarak glider berdasarkan dua data di atas? Jelaskan strategimu !

b. Gambarkan diagram batang data di atas serta tentukan prediksi jarak glidermu pada diagram tersebut.
(Catatan: Gunakan warna yang berbeda untuk menggambarkan diagram batangnya)

Glider 2

Glider 1

Jarak Glider

2. Percobaan ketiga menghasilkan 90 cm.
   a. Gambarkan diagram batangnya dan tentukan prediksi jarakglider berdasarkan ketiga data yang diperoleh.

Jarak Glider

   b. Jelaskan caramu menentukan prediksi tersebut menggunakan diagram batang di atas. (Catatan: Kamu dapat menggunakan diagram batang di atas)

3. Data keempat percobaan

Banyaknya percobaan	Glider
1	100 cm
2	110 cm
3	90 cm
4	140 cm

Gambarkan diagram batangnya dan tentukan prediksi jarak glider berdasarkan datadi atas. Jelaskan caramu menentukan jarak tersebut.

Jarak Glider

Caramu :

PERTEMUAN 5

Kelompok : ……..
Nama : …….
: …….
: ……..
Kelas : ……..

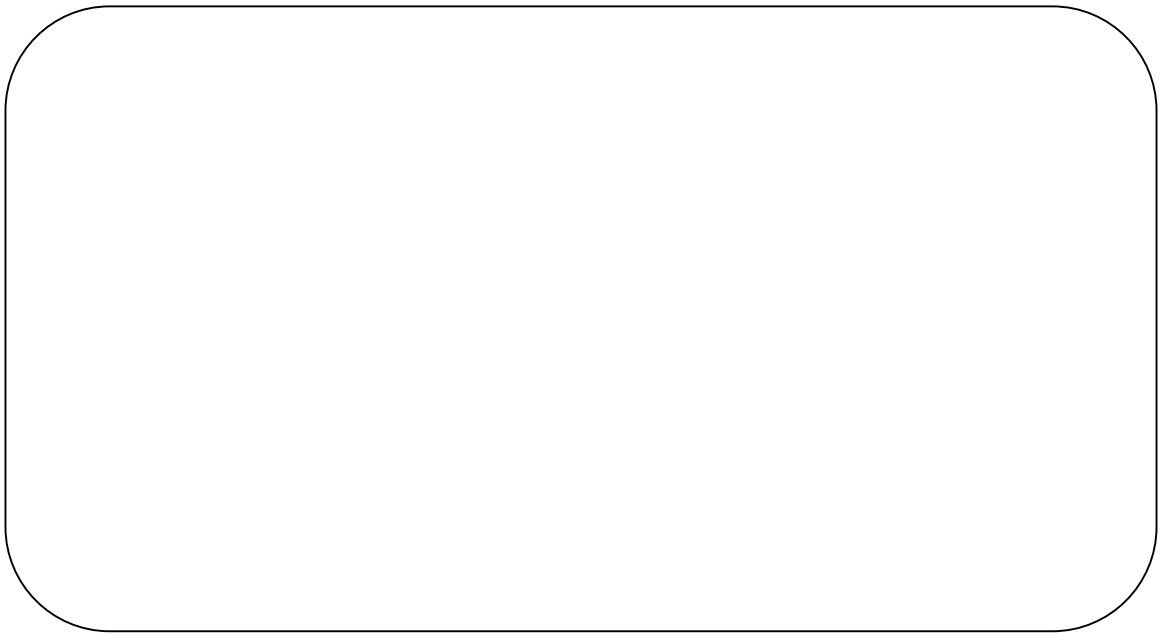
Lembar Kerja 1

1. Ukurlah tinggi teman kelompokmu

No.	Nama	Tinggi (cm)
1		
2		
3		
4		
5		

RATA RATA KELOMPOK KAMI ADALAH :

2. Jika seorang siswa bernama Toni dengan tinggi 150 cm bergabung di kelompokmu, berapa tinggi rata rata kelompokmu sekarang? Jelaskan cara menentukannya !



PERTEMUAN 6

Kelompok : ……..
Nama : …….
: …….
: ……..
Kelas : ……..

Lembar Kerja

Rak Buku

Di dalam kelas, guru ingin menempatkan rak buku di dinding. Akan tetapi, ia tidak tahu seberapa tinggi sebaiknya rak tersebut. Guru berencana rak itu tidak terlalu tinggi dan juga tidak terlalu rendah. Jika terlalu tinggi, beberapa siswa mungkin akan sulit menjangkau buku atau apapun yang berada pada rak. Sebaliknya, jika terlalu rendah, beberapa siswa mungkin akan terbentur rak atau akan terjadi kecelakaan dikarenakan rak tersebut.

Tugas kalian

- Bantulah guru menentukan tinggi rak buku tersebut di dinding. Diskusikan dalam kelompokmu tinggi dan cara menentukan tinggi rak buku tersebut.

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The Learning Line of cycle 1

Average Sentence

Making two sentences with the word average and explain the meaning

Investigating the meaning of the sentences : *The average weight of an apple in 1 kg apple is 200g*"

Repeated Measurement

Deciding the height of Anita

Glider Experiment (Prediction)

Throwing the glider five times and measuring the distance. Predict the next throw.

Glider Experiment (Compensation strategy on bar chart)

Predicting the next throw using the bar

The students' height

Measuring the height of the students and calculating the mean

The bookshelf context

Deciding the height of the bookshelf by using the mean

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The Learning Line of cycle 2

Average Sentence

Making one sentences with the word average and explain in the meaning

Investigating the meaning of the sentences : *The average weight of an apple in 1 kg apple is 0.240kg*"

Repeated Measurement

Deciding the height of Anita

Repeated Measurement

Deciding the weight of a small object

Glider Experiment (Prediction)

Throwing the glider five times and measuring the distance. Predict the next throw.

Glider Experiment (Compensation strategy on bar chart)

Predicting the next throw using the bar

The students' height

Measuring the height of the students and calculating the mean

The bookshelf context

Deciding the height of the bookshelf by using the mean