DEVELOPING THE NOTION OF SYMMETRY

THROUGH BATIK EXPLORATION

Master Thesis

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STATE UNIVERSITY OF SURABAYA

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DEVELOPING THE NOTION OF SYMMETRY THROUGH BATIK EXPLORATION

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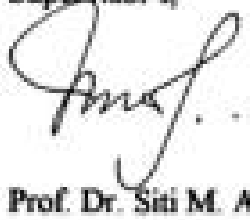

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DEDICATION

This thesis is dedicated to Ibu Dartiyah, my mom, my ibuk and my number one supporter since day one. Salute.

ABSTRACT

Wanita, C.T. 2014. *Developing the Notion of Symmetry through Batik Exploration.* Thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University. Supervisors: (I) Prof. Dr. Siti M. Amin, M.Pd. and (II) Dr. Abadi, M.Sc.

***Keywords**:* Symmetry, RME, design research, batik

The concept of symmetry is essential not only in geometry but also in human life. Therefore, students need to have a good basic understanding of the concept of symmetry. However, many studies found that students have difficulties in understanding it. By considering those difficulties, there is a need of developing a local instructional theory which can support students to have a better understanding of the concept of symmetry particularly line and rotational symmetry. Hence, this study aims at contributing a local instruction theory that can promote students' understanding of the concept of symmetry by exploring the characteristics of Batik, Indonesian traditional patterns. This study used Batik as the context because the patterns are not only familiar for students but also contain the concept of symmetry. This study used design research as the research approach and consisted of three cycles of teaching experiments which designed by implementing Realistic Mathematics Education (RME). The teaching experiments were conducted by orienting the designed hypothetical learning trajectory. Fifth-grade students of Laboratory Elementary School of Surabaya were involved in this study. Then, the data were collected from recording the teaching experiments, students' written works, and students' interview. The collected data were analyzed by confronting the hypothetical learning trajectory to the actual learning trajectory. The analysis result shows supporting evidence that exploring Batik pattern could make the students emerge with the concept of symmetry. Hence, the designed local instructional theory could be used to support students to have a better understanding of the concept of symmetry.

ABSTRAK

Wanita, C.T. 2014. *Developing the Notion of Symmetry through Batik Exploration.* Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Siti M. Amin, M.Pd. dan (II) Dr. Abadi, M.Sc.

***Kata Kunci**:* Simetri, RME, design research, batik

Konsep simetri tidak hanya berguna pada pembelajaran materi geometri tetapi juga pada kehidupan manusia. Oleh karena itu, siswa perlu mempunyai pemahaman dasar yang baik mengenai konsep simetri. Akan tetapi, banyak penelitian menunjukkan bahwa siswa mengalami kesulitan dalam memahami konsep simetri. Menyadari kesulitan siswa tersebut, perlu adanya pengembangan *Local Instruction Theory* (LIT) yang dapat mendukung siswa untuk mempunyai pemahaman yang lebih baik mengenai konsep simetri khususnya konsep simetri lipat dan simetri putar. Oleh karena itu, tujuan dari studi ini adalah berkontribusi *Local Instruction Theory* (LIT) yang dapat mendukung pemahaman siswa tentang konsep simetri dengan mengeksplorasi karakteristik batik, kain tradisional Indonesia. Studi ini menggunakan batik sebagai konteks pembelajaran bukan hanya karena batik sudah dikenal oleh siswa tetapi juga mengandung konsep simetri. Studi ini menggunakan *design research* sebagai pendekatan penelitian yang terdiri dari tiga siklus eksperimen pembelajaran. Pembelajaran tersebut didesain dengan mengimplementasi *Realistic Mathematics Education* (RME) dan terorientasi pada *Hypothetical Learning Trajectory* (HLT) yang sudah didesain sebelumnya. Studi ini melibatkan siswa kelas 5 SD Laboratorium UNESA Surabaya. Kemudian, data penelitian dikumpulkan dengan merekam eksperimen pembelajaran, mengumpulkan pekerjaan siswa dan wawancara siswa. Data yang sudah dikumpulkan kemudian dianalisa dengan membandingkan HLT dengan *actual learning trajectory*. Hasil analisa menunjukkan bukti bahwa pengeksplorasian motif batik dapat membantu siswa untuk belajar mengenai konsep simetri. Oleh karena itu, LIT yang sudah didesain di studi ini dapat digunakan untuk mendukung siswa untuk mempunyai pemahaman konsep simetri yang lebih baik.

PREFACE

I would like to express my deepest gratitude to Allah SWT for all the blessings. Besides, this thesis was possibly finished with the supports, guidance, and effort from many people.

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Last, I would like to express my greatest gratitude to my family in Jogjakarta for being my number one supporters and the best family anyone could have.

I hope this thesis can contribute to the improvement of mathematics education. Any critics or suggestions are welcomed to the improvement of this thesis.

Cici T. Wanita

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CHAPTER I
INTRODUCTION

A. Research Background

Symmetry is not only a part of geometrical concepts but also a part of human life. It has been used in countless applications such as culture, art, architecture, mechanic or science and mathematics (Yan, et al., 2003). Moreover, related to mathematics, *Principles and Standards for School Mathematics* (NCTM (2001), cited in Panaoura, et al., 2009) emphasizes symmetry as an important geometric concept. It is supported by Villiers (2011) and Marchis (2009) who stated that the concept of symmetry is essential in learning geometry as it is useful to be applied in problem solving and proving theorems and results. Therefore, Knuchel (2004) argued that it is very crucial for students in elementary school to have a good basic understanding of the concept of symmetry so that they can realize how symmetry is applied in their life. Unfortunately, not all students are aware of this (Knuchel, 2004). They even have difficulties in understanding the concept of symmetry. For example, Fierro (2013) and Roberts (2008) found that students often misunderstanding that diagonal of two-dimensional shapes always become their line symmetry, hence they think that a parallelogram has line symmetry. The students tend to assume line symmetry as a line which makes the shape becomes two congruent parts without considering the requirement that the two parts should be mirror images of each other (Leikin, et al., 2000a).

Consequently, the students get mixed up between rotational symmetry and the line symmetry (Panaoura, et al., 2009).

By considering students' difficulties in understanding the concept of symmetry, several studies have been done. Nevertheless, most of the studies (Gibbon, 2001; Hoyles & Healy, 1997; Knuchel, 2004; Mackrell, 2002; Seidel, 1998) have a tendency to use dynamic geometry software as a medium of learning the concept of symmetry rather than utilize the application of symmetry in students' daily life. However, using dynamic geometry software requires sufficient computer facilities and teachers' ability in using the software which will be rather difficult to be obtained in Indonesia since not every school provides computer facilities to the students and not every mathematics teacher has an ability to use the geometry software (Laksmiwati and Mahmudi, 2012). Therefore, besides using dynamic geometry software, exploring application of symmetry in students' daily life can be considered as an alternative strategy to learn symmetry in Indonesia. In fact, everyday application problems can be used as a milestone for students to start learning the mathematical concept (Heuvel-Panhuizen, 2003). An application of symmetry that can be viewed as a meaningful context as a starting point to learn it is batik, Indonesian traditional patterns. The patterns are rich resources to teach symmetry as the process of making it involves the concept of symmetry (Haake, 1989; Hariadi, et al., 2010). The use of batik is also supported by Yusuf & Yullys (2011) who stated that teaching mathematics by

combining mathematics and culture can be an innovation of educational practice mathematics. However, no study of utilizing batik as a medium for learning the concept of symmetry has been conducted in Indonesia.

Therefore, this research has the aim to contribute a local instruction theory that can promote students' understanding of the concept of symmetry by exploring the characteristics of batik, Indonesian traditional patterns.

B. Research Question

According to the aforementioned research background, the research question of this study is ***how can batik, Indonesian traditional patterns, promote students' understanding of the concept of symmetry?***

C. Research Aim

The aim of this study is to contribute a local instruction theory that can promote students' understanding of the concept of symmetry by exploring the characteristics of batik, Indonesian traditional patterns. In order to achieve the research aim, the researcher uses design research as the research approach and implements RME as the main theory in designing the instructional activities.

D. Definition of Key Terms

In order to avoid misinterpretation from the readers, several key terms which are used in this study will be defined as follow.

1. Symmetry (Line symmetry and rotational symmetry)

"*Symmetry is not a number or a shape, but a special kind of transformation – a way to move an object. If the object looks the same after being transformed, then the transformation concerned is a symmetry*" (Stewart, 2007, p. 12). By considering the notion of symmetry from Stewart above, it can be stated that symmetry is a transformation which makes an object stays the same. As this study only focuses on line and rotational symmetry, then both terms will be defined as follow. Line symmetry can be defined as a symmetry which is specified by its reflection line (axes of symmetry). A reflection line is a line that divides the object into two parts such that each part is a reflection or a mirror image of the other part (Harris, 2000). If an object has a reflection line, then it has line symmetry. Meanwhile, rotational symmetry can be defined as a symmetry which is specified by its centre point and its (counter clockwise) angle. An object has rotational symmetry if it can be rotated around a fixed point (the centre of rotation) before full rotation (less than 360^{o}) such that the orientation of the object remains the same as before the rotation (Harris, 2000).

2. Understanding of the concept of symmetry

In order to achieve understanding of the concept of symmetry, students need to develop their ability in implementing the notion of symmetry in proper context or new situations, explaining their ideas and developing them by giving relevant examples (Gardner 1991;

Gardner 1993; Sierpinska 1994, cited in Shaffer (1997)). Therefore, the designed activities are intended to support the students to achieve the understanding of the concept of symmetry. By considering this intention, the designed activities have these learning aims: (1) The students are able to identify the symmetric objects; (2) The students know the characteristics of line symmetry; (3) The students are able to differentiate the diagonal and the axes of symmetry of the objects; (4) The students are able to determine the characteristics of line symmetry in two-dimensional shapes; (5) The students know the characteristics of rotational symmetry; (6) The students are able to determine the characteristics of rotational symmetry; (7) The students are able to determine the characteristics of rotational symmetry in two-dimensional shapes; (8) The students are able to make the asymmetric pattern into the symmetric ones; (9) The students are able to complete the symmetric patterns; (10) The students are able to draw the symmetric patterns.

3. Characteristic of batik

The characteristic of batik patterns commonly shows self-similarity or self-affine (Hariadi, 2010). Hariadi also stated these characteristics mean that geometric details exist in various scales. Symmetry is a geometrical concept which can be found in batik patterns. As this study only focuses on supporting the students to discover the notion of line and rotational symmetry, this study only

employs the characteristics of batik patterns which show self-similarity in the same scale.

4. Local Instruction Theory (LIT)

"*A local instruction theory describes goals, envisioned learning route(s), and instructional activities or plans of action based on underlying assumption about teaching and learning*." (Nickerson and Whitacre, 2010, p. 233). Based on this definition of local instruction theory, the local instruction theory in this study describes the learning goals and the envisioned learning sequences and rationale for learning symmetry.

E. Significance of the research

This study expects two significances to be attained. The first significance is to contribute a local instruction theory that can promote students' understanding of the concept of symmetry. Then, the second significance is to provide mathematics teacher an insight of the instructional activities for supporting students' understanding of symmetry through batik exploration.

CHAPTER II
THEORETICAL FRAMEWORK

This theoretical framework is provided to address the structure of thinking for designing the instructional activities of this study. This literature review is useful as it can give insight how the basic concept of symmetry should be taught in elementary level by considering students' development of understanding it. In this study, Indonesian traditional patterns named batik will be employed as a medium of learning for students to discover the concept of symmetry which is embedded in the patterns. Therefore, literature about symmetry in batik patterns is required to explain how the concept of symmetry could embed in the patterns. Since this study will use batik, an example of the application of symmetry in contextual situations, as the medium of learning, the domain-specific instruction theory of Realistic Mathematics Education (RME) seems to be the proper theory as the base for designing the instructional activities. Moreover, this theory has been adapted to the Indonesian context which is known as *Pendidikan Matematika Realistik Indonesia* (PMRI). As the research will be conducted in Indonesia, an overview about the concept of symmetry for the elementary level in the Indonesian curriculum is also provided to give an insight of the mathematical goals of symmetry which Indonesian students should achieve.

A. The concept of symmetry in teaching and learning mathematics

1. The concept of symmetry

Understanding symmetry will help us to understand the world (Sautoy, 2009). It is possible to happen because symmetry is everywhere (Avital, 1996). Symmetry can be identified throughout nature, human products (i.e furniture, buildings) and also in chemistry, biology and art (Tapp, 2012 ; Marchis, 2009 ; Knuchel, 2004). Hann, (2013) defined symmetry as balance of physical form that can be identified in an image or an object with two equal parts in which each of them has the same size, shape and content, and one part is a reflection of the other (as if in a mirror). Moreover, symmetry of an object in a plane also implies that the object will stay the same after the plane moves or is repositioned (Tapp, 2012). It is supported by Rosen (2009, p. 4) who stated "*symmetry is immunity to a possible change*". Furthermore, there are three main types of symmetry which are described as follow.

a. Reflection symmetry

   Symmetry which is specified by its reflection line. A reflection line is a line that divides the object into two parts such that each part is a reflection or a mirror image of the other part (Harris, 2000). If an object has a reflection line, then it has line symmetry.

b. Rotational symmetry

   Symmetry which is specified by its centre point and its (counter clockwise) angle. An object has rotational symmetry if it can be

rotated around a fixed point (the centre of rotation) before full rotation (less than 360^o) such that the orientation of the object remains the same as before the rotation (Harris, 2000). The number of such orientations in which the object remain the same is called as the order of rotations of the object.

c. Translational symmetry

Symmetry which is specified by the length and direction of a single arrow (Tapp, 2012). In other words, a translation can be defined as a motion in which every point is moved by the same distance and same direction. Therefore, an object has a translational symmetry if it fits into the initial object when it is translated a given length at a given direction.

2. Students' understanding and misunderstanding of symmetry

Symmetry plays a fundamental role in mathematics (Knuchel, 2004). It is believed that understanding the concept of symmetry can be useful to understand other mathematical concepts such as algebra, geometry, probability or calculus. Furthermore, symmetry can be considered as an useful problem-solving tool since it can simplify the solution (Leikin, et al., 2000b). By considering the importance of the concept of symmetry, it is very crucial for the students to have a good understanding of it. Hoyles and Healy (1997) stated that in the process of understanding the concept of symmetry, the students use their prior knowledge. According to Harris (2000), many students in elementary school have symmetry sense with which they can identify whether an

object has line symmetry by looking at the balance of the object. A study of Tuckey (2005) also revealed the fact that the students have a subconscious awareness of symmetry.

However, the students also have difficulties in understanding the basic concept of symmetry. Several different studies (Fierro (2013); Roberts (2008); Harris (2000)) found students' misunderstanding that the diagonal of two-dimensional shapes always refers to the existence of line symmetry. Hence, they think that a parallelogram has line symmetry (Bagirova, 2012). In addition, the students think the line symmetry as a line which makes the shapes become two congruent parts without considering the requirement that the two parts should be mirror images of each other (Leikin, 2000a). As a result, the students get mixed up between rotational symmetry and line symmetry (Panaoura, et al., 2009).

B. Symmetry in batik patterns

In Indonesia, batik means drawing with wax. Precisely, it involves a dyeing process which starts by applying melted wax on a cloth with a special pen called "*canting*". After dyeing and doing fixation, the wax is removed by boiling. Repetitions of these steps lead to various patterns (Haake, 1989). Haake also stated an analysis of symmetry in Batik has been conducted on a representative sample of 110 repeating batik designs (fifty six from coastal regions and fifty four from Central Java). The designs were classified by reference to their symmetry characteristics. In total 505 traditional designs were examined, and 110 of these exhibited

regularity since they show repeating elements clearly. The analysis revealed the fact that 90% of the patterns showed four-direction reflection symmetry (Hann, 2013). Moreover, according to Haake (1989) symmetry which is embedded in the patterns shows Javanese philosophy as follows,

1. Translation, refers to meditation.
2. Rotation, refers to beliefs including religion
3. Mirror line, refers to coexistence. In this a case mirror line refers to line symmetry.

By considering those facts, batik patterns are considered as rich resources which can be exploited by the students as a medium to learn about the concept of symmetry.

C. Realistic Mathematics Education (RME)

"*Real in students' minds*" is the keyword of this theory. It refers to the contextual situation which will be used in the instructional activities. In this study, Batik patterns are set as the contextual problems for the students to learn the concept of symmetry. As the batik patterns are familiar for Indonesian students, it is in line with Freudenthal's notion that mathematics should be linked to reality through contextual problems. The instructional activities of this study are designed by following the five tenets of Realistic Mathematics Education (RME) by Treffers (1987) which are described as follow.

1. *The use of context*

   Contextual problems are used to give meaning to the mathematical learning and become the milestone for students to build the

mathematical concepts. Therefore, the instructional activities of this study begin with exploring the characteristics of batik patterns. Firstly, the teacher will present the problem of "*Javanese Batik Gallery"* in which students need to sort the batik fabrics into two types based on the regularity of the patterns. Then, the students will observe the details of the design of the batik patterns to discover the basic notion of line and rotational symmetry. The patterns are not only chosen because Indonesian students are already familiar with the batik patterns, but also because the process of making them employs the concept of symmetry.

2. *Using models and symbol for progressive mathematization*

   Models and symbols are used to promote the mathematical progress, from the informal concepts which students discovered from the context to the formal mathematical concepts. In this study, students will begin to discover the notion of regularity by exploring the details of the design of the batik patterns. After getting the notion of regularity in the batik exploration's activity, the students will have an activity named "*Batik Investigation*". In this activity, the students will get twelve regular patterns printed in a mirror, transparent cards and a pin. They are expected to utilize the mirror to discover the basic notion of line symmetry and the transparent batik cards and the pin to discover the notion of rotational symmetry. Therefore, this activity is

intended to become the bridge for bringing the concept of regularity into the concept of symmetry.

3. *Using students' own construction*

   "I hear and I forget. I see and I remember. I do and I understand" (Confucius). The passage is in line with the tenet of RME because students are considered as active learners in which they will learn better if they construct the mathematical concepts by themselves instead of just receiving them as "ready-made mathematics" (Heuvel-Panhuizen, 2000). Therefore, the teacher will give the students the opportunity to understand the mathematical concept by letting them to do the mathematical activities by themselves and try their own strategies. In this study, students have many opportunities to do the exploration and investigation in batik patterns and design their own batik patterns as well.

4. *Interactivity*

   Having mathematical interaction with others can be fruitful for students to gain more insights into the concept and to deepen their own thoughts. In this study, group and class discussion in each activity can be considered as an opportunity for the students to interact with the others by sharing ideas, comparing strategies, and reflecting about the mathematical concept.

5. *Intertwinement*

The instructional activities are not merely meant to teach the intended mathematical concept, but also to connect the learning to other domains. As symmetry is a fundamental part of geometry, understanding the concept of geometry will enhance students' sense of geometry.

D. The Concept of Symmetry in the Indonesian curriculum

The concept of symmetry is taught in the fourth and fifth grade in primary school. In primary school, the students learn about line symmetry and rotational symmetry. In the fourth grade, students start to learn line symmetry from nature such as animals or plants. In the fifth grade, the students learn further about line symmetry and rotational symmetry. The following table describes how the concept of symmetry is integrated in the Indonesian curriculum.

Table 2.1 The Concept of Symmetry for Primary School Grade Five in the Second Semester in the Indonesian Curriculum (BSNP, 2006).

The second semester of the fifth grade	
Geometry and Measurement	
6. Understanding the properties of three- dimensional shapes and the relation among two-dimensional shapes	6.4 Investigating the characteristics of similarity and symmetry

As described in the table 2.1, the basic competence of the concept of symmetry in the fifth grade of the Indonesian curriculum is investigating the characteristics of similarity and symmetry. However, this study just focuses on investigating the characteristics of symmetry, particularly line and rotational symmetry.

CHAPTER III
METHODOLOGY

A. Research approach

The aim of the present study is to contribute a local instruction theory that can promote students' understanding of the concept of symmetry by exploring the characteristics of batik, Indonesian traditional patterns. The local instruction theory is also intended to improve the teaching and learning of symmetry in Indonesia. Hence, this study focuses on answering the research question: "*How can batik, Indonesian traditional patterns, promote students' understanding of the concept of symmetry*?". Consequently, the researcher is required to make an innovation in designing instructional sequences in which students explore the characteristics of batik and research about how the design can supports students to understand the concept of symmetry. By considering the consequences, design research is the suitable research approach for this study for several reasons. First, the main purpose of design research is in line with the goal of this study which aims at educational innovation for improving educational practices. Second, design research perceives designing instructional sequences as the essential part of the research and it aims at developing theories of how the design supports the learning of students. Hence, this methodology allows the researchers to focus on students' understanding and its process in the educational setting activities so that they can study both aspects as integrated and meaningful phenomena (Akker et al., 2006). In addition, one main aspect of design research is the

adjustment of the learning trajectory throughout the research (Drijvers, 2004). In other words, the researcher can revise and improve the conjectures on the learning trajectory and the design after conducting the teaching experiment. It is in line with this study as the initial Hypothetical Learning Trajectory (HLT) and the instructional sequences of this study still need to be developed in order to contribute a local instruction theories theory that can promote students' understanding of the concept of symmetry.

The design experiment of this study consists of three cycles. It is in line with the characteristics of design research which is cyclic nature (Bakker and Van Eerde, 2013). By conducting the study in three cycles, the local instruction is expected to be more robust as the researcher can test the conjectures, generate or obtain alternative conjectures in learning. Furthermore, the design experiment is conducted in three phases which are (1) Preparing for the experiment. This is concerned with formulating local instruction theories which can be adapted throughout the experiment. The researcher starts with studying literature which is relevant to the concept of symmetry to get more insight in designing the instructional sequence and the HLT; (2) The design experiment. This aims at testing the instruction theories which were already designed in the first phase and at developing an understanding of how it can support students' understanding. The researcher starts to collect the data, such as by observing the target group during the lesson; (3) The retrospective analysis. This aims at supporting the researcher to revise and improve the local instruction theories. This phase can be done

by using the initial HLT to analyze the collected data so that the researcher knows whether they match or contradict and discuss the reason behind it.

B. Data Collection

1. Preparation phase

In this study, the participants of the study are fifth-grade students of elementary school in Surabaya namely SD Laboratorium UNESA and their mathematics teacher. This phase aims at obtaining relevant information about the participants of the study which can be used as an insight to conduct the design experiment such as: (1) the socio-mathematical norms, information that helps the researcher to arrange and manage the instructional sequence works as already designed, (2) the prior knowledge of students toward the concept of symmetry, information that helps the researcher to improve HLT and become the starting point of the instructional sequence. Mainly, the data of this phase are collected with three methods, classroom observation, a semi-structured interview with the teacher and a written test

Firstly, the researcher observes the mathematics classroom of the participants. It aims at gathering information about the social and socio-mathematical norms so that the researcher gets insights into how students learn, the mathematical interaction among the students and between students and teacher, and how the teacher conducts the learning process. To make sure that the observation obtains the intended information, it is guided by the observation scheme (*see appendix 3*).

Secondly, the semi-structured interview with the teacher is conducted after doing classroom observation. This interview is important to do as the researcher can ask about any aspects relevant to the students or the learning process which has already been observed. For example, the researcher can ask why the teacher asked the students to do the task individually instead of doing a group discussion. This information help the researcher to get an overview of the teacher's belief of teaching mathematics so that the researcher can think about the following steps that should be done to support the teacher to conduct the learning process as much as possible in the intended way. This phase also allows the researcher to investigate the level of students' understanding. This information is essential to determine the focus group in the teaching experiment of the second cycle. The interview are recorded and guided by the interview rubric (*see appendix 2*) to make sure all the important information is covered.

2. Preliminary teaching experiment (first cycle)

Design research This study consists of three cycles. The first cycle involves an experiment with a small group consisting of five fifth-grade students. These students are different from the target group of participants who participate in the next cycles. As this cycle is meant as a pilot study, the researcher only works with a small group of students so that it is easier for the researcher to investigate and analyze what students think and do during the experiment. In this cycle, the researcher conducts the lessons so

that students' thinking in solving the problems on the design can be explored and investigated thoroughly. It is done by giving follow-up questions for every students' answers or reactions toward the design. Therefore, the data from this preliminary teaching experiment can be used to test the learning conjecture and to know whether the design is suitable with students' prior knowledge so that the researcher can revise and improve the initial design. The result of the design revision are implemented in the next cycle. The data of this cycle will be collected by doing class observations which is recorded by video, making field notes and collecting students' written works.

3. **Teaching experiment (second and third cycles)**

The second cycle involves the target group of participants which are fifth-grade students in class C and Ibu Mardiati, the mathematics teacher, who conducts the lessons. Meanwhile, the third cycle involves fifth-grade students in class B and Ibu Mardiati. However, the researcher also has a focus group among the participants. This focus group is meant to be the group of students whose activities and written works are investigated and analyzed thoroughly to support the researcher in answering the research question. This focus group consists of students who have an intermediate level of understanding based on the teacher's interview. The students with an intermediate level of understanding are chosen because it can represent the level of understanding of fifth-grade students in general. The data of this cycle are collected by observing the lessons which are recorded by

video, making field notes and collecting students' written works. However, as the class consists of ± 23 students, the learning activities are recorded by two cameras, a static and a dynamic one. While the whole learning process is recorded by the static camera, the dynamic camera can focus on some interesting mathematical discussion among the students particularly in the focus group. Those interesting moments are transcribed and used as an evidence of what happened in the learning activities.

4. Pre and post-test

The pre-test and post-test are conducted in the three cycles in order to get an insight about students' prior knowledge before participating in the teaching experiment and students' recent knowledge after participating the teaching experiments. The problems in the pre-test and post-test are similar and referring to the learning goals of the designed activities. The pre-test and post-test are provided in the appendix 4 and 5.

The pre-test of the first cycle are held before the researcher conducts the lessons. Then, the pre-test of the second cycle are held after finishing the first cycle and before conducting the teaching experiment on the second cycle. Meanwhile, the pre-test on the third cycle are held before conducting the teaching experiment on the third cycle. All students who are involved in the three cycles do the pre-test but with different purposes. The pre-test in the first cycle is intended to determine students' prior knowledge of the concept of symmetry and to check whether the problems are feasible for fifth-grade students. The result of this pre-test is used to

revise and improve the initial pre-test so that the revised pre-test can be implemented in the next cycle and result more proper data. The result of the revised pre-test gives more general insight into students' prior knowledge so that the result can be used to revise and improve the initial design. In addition, it can be adjusted to the HLT about students' prior knowledge.

The post-test are held after finishing each cycle. All students who are involved in the three cycles do the post-test. The post-test aims at knowing how students develop their understanding of the concept of symmetry throughout the implementation of the design of instructional sequence. Furthermore, there is an additional interview for the students to clarify their thinking in doing the post-test.

The problems on the pre-test and post-test focus on the concept of line symmetry and rotational symmetry (*see appendix 4 and 5*). The problems in the both tests are similar. They are designed to not only make the students solve the problems about symmetry but also show their reasoning.

5. Validity and reliability

Validity and reliability are the main requirements of qualified scientific research. Therefore, this study tries to meet both aspects in collecting data by following these principles:

a) Data triangulation

This study collects the data from several different perspectives which are students' written works, interviews and video recordings of the classroom activities including the field notes. Therefore, the collected data enrich the understanding of the researcher about students' thinking and consequently contribute to the internal validity of the study.

b) Video registration and traceability

The video registration are used to increase the internal reliability of the study as video can show what really happened in the lesson. The traceability refers to the external reliability in which the researcher describes the learning process and the designing procedure in detail so that it will be easier to track or reconstruct the study.

C. Data Analysis

1. Classroom observation and teacher's interview

This study analyzes the classroom observation and the teacher's interview which have been conducted in the beginning of the study. It aims at giving insight for the researcher to improve the initial HLT.

2. Pre-test

This study analyzes the result of the pre-test from the three cycles in qualitative way. The researcher analyzes the students' written work qualitatively by looking at their reasoning in solving the problems. The result of analyzing the pre-test will be used to adjust the HLT.

3. Preliminary teaching experiment (first cycle)

This study analyzes the result of the first cycle by selecting some interesting fragments from the recorded video which show how the students explain their strategies in solving the problem on the learning sequence or when the learning conjectures do not occur. Then, the researcher describes how it can happen by grounding from the transcript of the fragments. In the analyzing process, the researcher uses the HLT to compare the conjectured learning with the actual learning of what students do during the lesson. Last, the analysis result is utilized to revise and improve the HLT during the lesson.

Moreover, the students' written works can be used as the visualization of students' thinking. Hence, the transcript of the interesting fragments and students' work can support each other in describing students' understanding of the concept of symmetry. The analysis result of this first cycle is used to revise and improve the HLT for the next cycle.

4. Teaching experiment (second and third cycle)

The researcher analyzes the result of the teaching experiment in the second and third cycle by selecting particular data which can be used to answer the research question. First, the researcher selects and transcribes the interesting fragments from the video of the learning process. For example, the fragment that shows how exploring batik patterns can help students to get the notion of line and rotational symmetry and develop their understanding of the concepts. These chosen segments can be supported

by the relevant field notes of the learning process. Second, the researcher analyzes students' written works in qualitative way to get insight into the development of students' understanding throughout the design experiment. In the process of analyzing, the researcher compares the students' written works with the learning conjecture in the HLT. If the strategies that are used by students contradict or different from the learning conjecture in the HLT, then the HLT is revised. Last, the interview result of the students is transcribed in order to strengthen the understanding of students' thinking. In the end, the analysis of each component data is triangulated to increase the validity of the research by clarifying the students' understandings of the concept of symmetry from their own perspective.

5. Post-test

The post-test is analyzed in qualitative way. It is similar with what has been discussed before in analyzing the result of the pre-test. Both of them are analyzed in the same way because we want to get an insight about the development of students' understandings of the concept of symmetry by exploring the characteristic of batik patterns.

6. Validity and reliability

This study will try to contribute to validity and reliability in analyzing data by following these principles:

a) Data triangulation

As the collected data are various and analyzed with various methods, all of the analysis result from each data is triangulated to get a detailed

and precise description of how batik, Indonesian traditional patterns can promote students' understanding of the concept of symmetry

b) Traceability and Inter-subjectivity

The traceability refers to the external reliability which means that others can understand the process of data collection and analysis. Therefore, the researcher describes the learning process and the designing procedure in details so that it is easier to track or reconstruct the study. Moreover, it increases the transparency of the study. Then, inter-subjectivity refers to internal reliability in which the researcher consults the data analysis with others. In this study, the researcher discusses the interpretation of the collected data with colleagues or supervisors to maximize the level of objectivity.

CHAPTER IV
HYPOTHETICAL LEARNING TRAJECTORY

"*A goal without a plan is just a wish*" (Antoine de Saint-Exupery). The passage proposes an act of planning for achieving any goals. Related to the teaching and learning process that are conducted in this study, a plan of structuring the process are needed to achieve the intended learning goals. Therefore, there is a need to hypothesize what the students will do and think when they participate in the designed instructional activities and plan the follow-up actions. By considering this need, Simon (1995, cited in Bakker & Van Eerde, 2006) emerged with the notion of hypothetical learning trajectory (HLT). HLT describes the conjectures of the learning processes regarding to the designed instructional activities (Simon (1995) as cited in Van Nes, 2009). According to Gravemeijer (2004, cited in Van Nes, 2009), HLT consists of mathematical learning goals, a plan of the instructional activities and conjectures of students' thinking and learning during the learning activities including the teacher's reactions. In this chapter, the researcher describes the HLT which is used as a guideline for teacher to conduct the teaching experiment. The researcher also provides the starting point of each activity in which the students use it to support them in achieving the learning goals.

The researcher uses the basic competence of the concept of symmetry in the fifth grade of Indonesian curriculum as the base to formulate the learning goals of the learning activities. The basic competence is *investigating the characteristics of*

symmetry. In order to support the students to achieve the basic competence, the researcher designs three meeting with the learning goals as described in table 4.1.

Table 4.1. The learning goal of the three meetings

Meeting	General learning goal	Sub-learning goals
1	The students are able to discover the notion of line symmetry by exploring the characteristic of batik patterns	The students are able to differentiate the patterns which have regularity (line symmetry) and the patterns that have no regularity (no line symmetry).
		The students are able to deduce the characteristics of line symmetry from the regular batik patterns by using a mirror.
		The students are able to differentiate between diagonal and the axes of symmetry on the batik patterns
2	The students are able to discover the notion of rotational symmetry by exploring the characteristic of batik patterns	The students are able to differentiate the patterns which have regularity (rotational symmetry) and the patterns that have no regularity
		The students are able to deduce the characteristics of rotational symmetry from the regular batik patterns by using a pin and the transparent batik cards
		The students are able to determine the characteristics of rotational symmetry (the order of rotation, the angle of rotation and the point of rotation)
3	The students are able to apply their understanding of line and rotational symmetry.	The students are able to make the asymmetric batik pattern into the symmetric ones
		The students are able to complete the symmetric pattern by considering the given axes of symmetry
		The students are able to create their own batik design by using provided batik units and considering the required angle of rotation

This HLT has been improved by considering the result of the classroom observations, the teacher's interview and the pre-test result. Therefore, the overview of those three aspects are described before describing the HLT.

A. Overview of the Classroom Observations and the Teacher's Interview

The classroom observations and the teacher's interview are conducted by following the schemes that can be seen in the appendix 2 and 3. Therefore, the overview is described based on the main points of the schemes as follow.

1. The classroom observations

   First, the classroom environments of three classes (5B, 5C and 5D) are similar. The teacher begins the lesson by posing some questions which lead the students to derive the idea of the mathematical concept (proportion). The teacher does not explain the concept that is going to be taught directly. After the students get the basic idea of the concept, the teacher gives several word problems which related to students' surroundings such as the proportion of the class equipments. The teacher gives the word problems orally without using any specific books as the teaching guidance or student worksheet.

   Second, the students' activity during the teaching-learning process is started with listening to teacher's explanation, answering the questions from the teacher and doing exercise individually. Most of the students are quite active in the lesson as they always answer the questions and respond to what the teacher says. The classes have a social norm in which the students should listen carefully if the teacher talks or the other students present their idea.

2. The teacher's interview

First, the teacher designs the structure of the learning process by considering the curriculum and the students' characteristics. For example, since most of students are quite talkative, the teacher realizes that she cannot only explain the concept. Therefore, she tries to involve the students during the learning process by posing some questions which engage students to think and obtain the mathematical concept behind the questions. However, the teacher also admits that she has a problem in managing the class because the students tend to do other things during the learning process. Then, in terms of the concept of symmetry, the teacher says that most of the students have difficulties in determining the number of line symmetry and rotational symmetry. She states that the difficulties may happen because both concepts require the students to imagine the objects. Related to the prior knowledge of the students toward the concept of symmetry, the teacher says that the students already learned about the basic concept of line symmetry in fourth grade. They learned to identify symmetric objects and draw the axes of symmetry.

Second, regarding the teacher's background, the teacher has been teaching for almost 20 years. She has an educational background of mathematics education. In addition, she ever involved in the Pendidikan Matematika Realistik Indonesia (PMRI) project and followed the PMRI training and workshops for mathematics teachers. Therefore, she thinks that the students should have an opportunity to be involved in the learning

process. Then, in order to attract the students to get involved in the lesson, the teacher tries to use learning materials which are familiar for students.

By considering the result of the classroom observations and the teacher's interview, the HLT and teacher guide are adjusted as follow.

a. As the teacher already had the same perspective of RME in which the students should be situated to learn the concept by themselves, the main role of the teacher is facilitating and guiding the students to derive the concept of symmetry by undertaking the designed activities. In order to support the teacher to facilitate and guide the students in the intended way, the possibility of students' strategies and the suggestion for the teacher to react toward them in the HLT are described in details.
b. As the students are quite talkative and tend to do other activities if they feel difficult in understanding the lesson, the student worksheets and the hands-on activities seem appropriate to be implemented in the classes because it can keep them busy. However, the implementation should be supported with the clear instruction on the worksheet so that the students do not face any difficulties in doing the activities.
c. The students of both classes have different range of academic achievements. Therefore, the teacher will form groups of students based on their academic achievement so that in a group consists of students with different levels of academic achievement. It is intended to make the students with higher achievement help the students with lower achievement to do the designed activities.

B. Overview of the Pre-test Result

The pre-test result shows that the students can identify the symmetrical objects. However, they tend to assume symmetric objects as the objects which consists of two identical parts without considering that the two identical parts should become each other's mirror images. Consequently, most of the students think a parallelogram as the symmetric object as it can be divided into two identical parts. Moreover, they assume the diagonal of a parallelogram as its axes of symmetry. Related to the concept of rotational symmetry, the students have difficulties in determining the characteristics as they have not learned about it before. For the details description of the pre-test result can be read in chapter V. This pre-test result gives an insight of how the students will respond toward the given activities in the learning sequence. Therefore, this result is used by the researcher to provide the possibilities of students' strategies in undertaking the designed activities.

C. The Hypothetical Learning Trajectory (HLT)

1. Meeting 1 – Javanese Batik Gallery (Line Symmetry)

a. The starting point

The starting point of this first activity is based on the students' prior knowledge of symmetry which already taught in fourth grade. However, it is also supported from the written works of the pre-test. In the fourth grade, the students already learned the following knowledge and skills of symmetry,

1) The students are able to identify the symmetric objects in daily life

2) The students are able to determine the symmetric shapes
3) The students define line symmetry as a line that determines whether the objects are symmetric.

Then, the pre-test result shows that the students think symmetric objects as the objects which consist of two identical parts without considering that the two parts should become each other's mirror images.

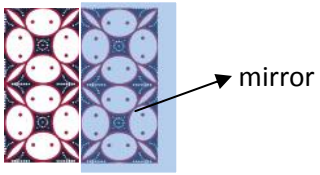
b. The mathematical learning aim

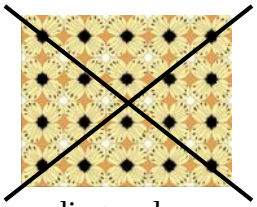
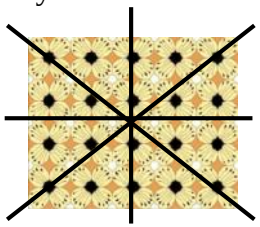
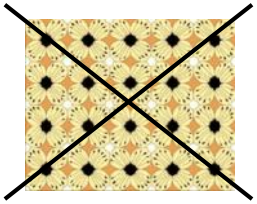
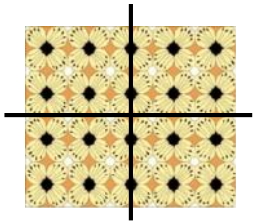
The first activity aims at supporting the students to discover the notion of line symmetry by exploring the characteristic of batik patterns. The aim is specified into these following sub-learning aims,

1) The students are able to differentiate the patterns which have regularity (line symmetry) and the patterns that have no regularity.
2) The students are able to deduce the characteristics of line symmetry from the regular batik patterns by using a mirror.
3) The students are able to differentiate between diagonal and the axes of symmetry on the batik patterns.

In order to achieve the learning aim, the researcher presents table 4.2 for giving an overview of the main activity and the details information in the following section.

Table 4.2 An overview of the main first meeting and the hypotheses of learning process

Main Activity	Mathematical Learning Goal	Conjectured of Students' Thinking	Teacher's reaction
Sorting the batik fabrics based on the regularity of the patterns	The students are able to differentiate the patterns which have regularity (line symmetry) and the patterns that have no regularity	▪ The students sort the patterns based on the similarity in motif Example: living creature motif Room 1 consists of Batik with living creature's motif (B, C, F, K) Room 2 consists of Batik with non living creature's motif (A, D, E, G, H, I, J, L)	Give several suggestions as follows, "*try to observe the patterns again and imagine how will you draw the patterns, relate it with the regularity of the patterns*?" Guide the students to notice that the regularity among the patterns is related to the details of the motif.
		The expected reaction ▪ The students sort the fabrics based on the regularity of the patterns in which whether the patterns present line symmetry or not Example: Room 1 : Batik A, D, G, H, I, J, L Room 2 : Batik B, C, E, F, K	Ask the students about the regularity that they mean, "*Why do you determine batik A as the pattern which has regularity*?" "*What kind of regularity that you mean*?" Guide the students to be aware that the regularity refers to the basic notion of line symmetry.
Discovering the characteristics of line symmetry from exploring the regular batik patterns by using a mirror	The students are able to deduce the characteristics of line symmetry from exploring the regular batik patterns by using a mirror.	The expected reaction: The students put the mirror in the middle of the pattern along the axes of symmetry and are aware that mirror reflection shows the same pattern as the pattern in the back of the mirror.  	Ask the students to do further exploration with a mirror to the regular patterns and guide them to determine the mirror positions of each pattern. "*Look at the mirror positions that you have already determined, what do you see from the mirror position*?"

Determining the diagonals and the axes of symmetry of the patterns	The students are able to differentiate between diagonal and the axes of symmetry of the batik patterns.	The students cannot draw the diagonals and the axes of symmetry correctly  diagonal  the axes of symmetry	The teacher can review the definition of diagonals and the axes of symmetry by asking some relevant questions, "*What do you know about diagonal*?" "*What do you know about the axes of symmetry*?" "*Do your axes of symmetry fulfil the definition of the axes of symmetry?*"
		The expected reaction: The students can draw the diagonals and the axes of symmetry correctly  diagonal  the axes of symmetry	The teacher can ask the further question such as, "*What is the difference between diagonal and the axes of symmetry*?" "*Is the diagonal of a shape always become its axes of symmetry*?" Guide the students to be aware that the diagonal is not always becomes the axes of symmetry. It just holds for particular objects.

c. The instructional activities

There are three main activities in this meeting. First, sorting batik fabrics based on the regularity of the pattern. It is designed to engage the students to use their sense of line symmetry as the provided fabrics consist of two types, the patterns with line symmetry (regular patterns) and the pattern with no line symmetry. Second, exploring the regular patterns by using a mirror. It is designed to support the students to discover the notion of line symmetry and view it as the symmetry that consists two identical parts in which both parts become each other's mirror images. Third, determining the diagonals and the axes of symmetry of the patterns. It is designed to support the students to be aware of the difference between diagonal and the axes of symmetry.

The learning sequence of the meeting is described as follow.

1) Introducing the context of Javanese batik gallery

This first activity uses the context of Javanese batik gallery in which the gallery will held an exhibition. As the gallery has only two rooms, the staffs need to sort the batik fabrics based on the regularity of the patterns. The teacher should ensure that all students understand the problem and exactly know what they should do. It can be done by asking several students to paraphrase the problem and asking the other students whether they agree with the statement. Example:

- *Could you explain the problem in your own words*?
- *Do you agree with your friend's statement? Why do you think so*?

2) Doing the worksheet

   After discussing what the context is about, the students will get oriented to do the worksheet in the group consisting of three to four students.

3) Classroom discussion

   Groups of students who have different answers in solving the problem on the worksheet will have an opportunity to present their answers. Then, the other students will have a chance to give comments or state their opinions whether they agree or disagree with the presentation. The teacher will lead the discussion so that all the groups have a chance to state their answers and keep the discussion focusing on the problem. In the end of the discussion, the teacher reviews the answers of the regular patterns, the definition of line symmetry and the difference between diagonal and the axes of symmetry. There are two important points of this lesson. First point is the notion of line symmetry in which it is not only about two identical parts but also both parts should become each other's mirror images. Second point is about the differences between the diagonal and the axes of symmetry.

4) Closing activity

   The teacher asks the students to reflect the lesson such as by asking these following questions

   - *What does line symmetry mean?*
   - *What are the differences between the diagonal and the axes of symmetry?*

d. The conjectures of students' thinking and learning

The conjectures of students' thinking and learning will be described based on the three tasks on the worksheet.

1) The first task

This task asks the students to fill the table in figure 4.1 with their sorting result.

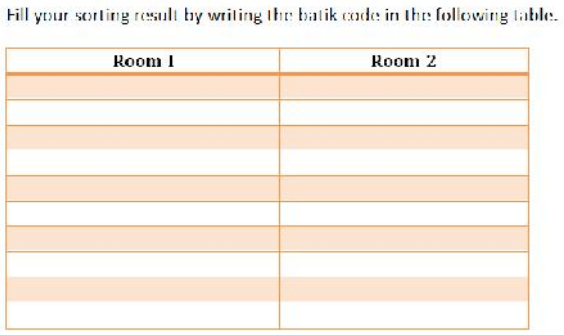


Figure 4.1. The figure of table to fill the sorting result

In line with table 4.1, the following are the possibilities of students' sorting result.

- The students sort the patterns based on the colour

  Room 1 : A, B, C, F, H, I, L

  Room 2 : D, E, G, J, K

  The students may answer that they sort the patterns by looking up the colour and they see that most of the patterns are brown, so that they sort the patterns by differentiating brown patterns and non-brown patterns.

- The students sort the patterns based on the similarity in motif (living creature motif)

  Room 1 : B, C, F, K

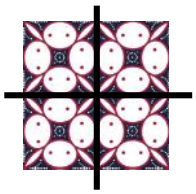
Room 2 : A, D, E, G, H, I, J, L

The students might answer that they sort the patterns by looking up the motif of the patterns. They see that there are several patterns which have motif of flowers or animals.

- The students sort the patterns based on the way of designing the motif

  Room 1: A, D, G, H, I, J, L
  Room 2: B, C, E, F, K

  The students might answer that they sort the patterns by looking up the way of designing the motif of the patterns. They see that there are several patterns which need to be equal in size among each other. Hence, it needs a line to draw as the following figure,



  Then, the following figure is the pattern that can be directly drawn without making a line.

  Therefore, they will answer that they sort the patterns based on the way of designing the motif.

- The students sort the patterns based on the regularity of the patterns whether they consist of the same motif

  Room 1 : Batik A, D, G, H, I, J, L

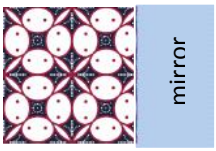
  Room 2 : Batik B, C, E, F, K

The students might answer that they sort the patterns by observing the motif of the patterns. They see that there are several patterns which consist of same motif, meanwhile other patterns are unique.

2) The second task

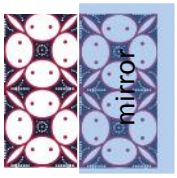
This task asks the students to explore the regular batik patterns by using a mirror in order to discover the notion of line symmetry. These are the possibilities of what students will do in using a mirror.

- The students put the mirror in the edge of the batik patterns



Then, the students are aware that the reflection on the mirror is the same with the whole pattern.

- The students put the mirror in the middle of the patterns along the axes of symmetry



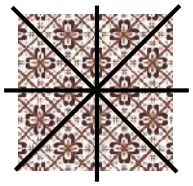
Then, the students are aware that the mirror reflection shows the same pattern as the pattern in the back of the mirror

3) The third task

This task asks the students to determine the diagonals and the axes of symmetry of the batik patterns. The following points are the possibilities of students' answers.

- The students draw the axes of symmetry and diagonals of the pattern correctly

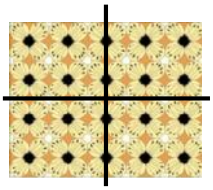
Pattern A



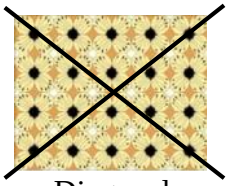
The axes of symmetry

Diagonal

Pattern B

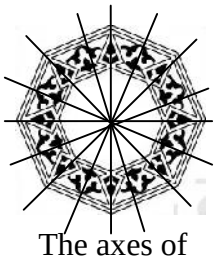


The axes of symmetry

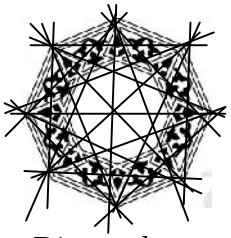


Diagonal

Pattern C



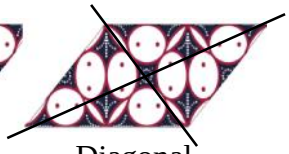
The axes of symmetry



Diagonal

Pattern D

No axes of symmetry



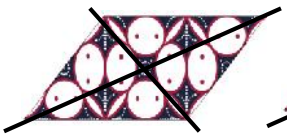
Diagonal

- The students think the diagonal always become the axes of symmetry of the pattern or vice versa.

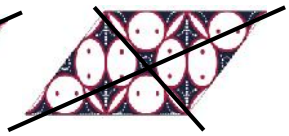
Example:

The axes of symmetry

Diagonal



The axes of symmetry



Diagonal

e. The teacher's reaction

By considering the possibilities of students' answer, the teacher can do these follow-up actions.

1) The first task

- The students sort the fabrics based on the colour

The teacher shows the batik patterns which are printed in black and white colour and asks them to sort the patterns. It aims at making the students realize that their way of sorting by looking up the colours is not general enough. The students should observe the motif of the patterns instead of their colour.

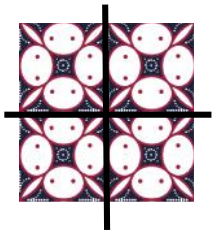
- The students sort the fabrics based on the similarity in the motif (living creatures motif)

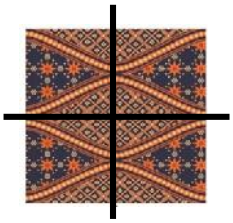
  The teacher takes one pattern with motif of flower like the following  figure. Then, the teacher asks the students to observe the motif more thoroughly. Then, the teacher can ask a follow-up question "*Imagine how you will draw the patterns, do you find any same motif inside the pattern*?" Teacher guides the students to notice that the patterns consist of the same motif (regular).

- The students sort the fabrics based on the way of designing the patterns

  The teacher asks the students to do further exploration to the patterns such as by asking "*what do you mean by making line, where will you draw the lines?*". The teacher also can ask the students to draw the line of each pattern that they assume as the patterns that need lines to draw. Then, the teacher can guide the students to notice that the lines refer to the lines which divide the patterns into same parts (the axes of symmetry).

For example,





- The students sort the fabrics based on the patterns whether they consist of the same motif

  The teacher asks the students about the regularity that the students mean,

  "*Why do you determine batik A as the pattern which has regularity*?"

  "*What kind of regularity that you mean*?"

  Guide the students to be aware that the regularity refers to the line symmetry in which the pattern consist of the same parts.

2) The second task

- The students put the mirror in the edge of the regular batik pattern then they are aware that the mirror reflection shows the same pattern as the whole pattern.
- The teacher can ask the students to put the mirror in the edge of the batik patterns which have no regularity. Then, the teacher asks about what the students see in the mirror. The students may answer that the mirror reflection shows the same pattern as the whole pattern. Then, the teacher poses the following question,

  "*If you think so, then there is no difference between Batik patterns which have regularity and no regularity*?"

"*What do you think, do you need to re-position the mirror in order to differentiate the regular and the irregular batik patterns?*"

The follow-up questions are intended to lead the students to position their mirror in the centre of the pattern along the axes of symmetry (vertically, horizontally and diagonally).

- The students put the mirror in the middle of the patterns along the axes of symmetry. The teacher can ask the students to do further exploration with a mirror to the regular patterns and guide them to determine the mirror positions of each pattern. It also can be followed up with questions such as

  "*Look at the mirror positions that you have already determined, what do you see from the mirror position*?"

  "*What do you usually name the mirror position*?"

  The follow-up questions are intended to lead the students to relate the mirror position with the axes of symmetry.

3) The third task

- The students draw the axes of symmetry and diagonals of the pattern correctly

  The teacher can give some following questions to make sure that the students understand the difference between the axes of symmetry and diagonals

  "*So, what is the difference between the axes of symmetry and diagonals*?"

"*Do the axes of symmetry always become the diagonal of the shape*?"

"*Do the diagonals always become the axes of symmetry of the shape*?"

- The students assume that the diagonal always become the axes of symmetry of the pattern

  The teacher can show batik pattern B or D and ask the students to draw the diagonal of each pattern. Then, ask the students to observe whether the diagonal divide the pattern into two same parts and the both parts become each other's mirror images. If the students still feel difficult in understanding that the diagonal is not the axes of symmetry, then the teacher can use a mirror to make them realize that the patterns are not reflecting each other. It is intended to make the students see and realize that the diagonal of the pattern is not always its axes of symmetry.

2. Meeting 2 – Javanese Batik Gallery (Rotational Symmetry)

a. The starting point

   To start the second activity, the students should understand the notion of regularity in which it refers to the patterns that consist of the same motif.

b. The mathematical learning aim

   The second activity aims at supporting the students to discover the notion of rotational symmetry by exploring the characteristic of batik patterns. The aim is specified into these following sub-learning aims,

   1) The students are able to differentiate the patterns which have regularity (rotational symmetry) and the patterns that have no regularity.

2) The students are able to deduce the characteristics of rotational symmetry from the regular batik patterns by using a pin and the transparent batik cards.
3) The students are able to determine the characteristics of rotational symmetry (the order of rotation, the angle of rotation and the point of rotation)

In order to achieve the learning goal, the researcher presents table 4.3 for giving an overview of the main activity and the hypotheses of learning process and the details information in the following section.

Table 4.3 An overview of the main second meeting and the hypotheses of learning process

Activity	**Mathematical Learning Goal**	**Conjectured of Students' Strategies**	**Teacher's reaction**
Sorting the Batik patterns into two types based on the regularity.	The students are able to differentiate the patterns which have regularity (rotational symmetry) and the patterns that have no regularity.	The expected strategy: The students sort the patterns based on the regularity of the patterns in which whether the patterns consist of repeating motif and the patterns that are unique. Example: Room 1 : Batik A, E, F, G, H, K, L Room 2 : Batik B,C,D,I,J	Ask the students about the regularity that they mean, "*Why do you determine batik F as the pattern which have regularity*?" "*What kind of regularity that you mean*?" Guide the students to be aware that the regularity refers to the same motif.

Activity	Mathematical Learning Goal	Conjectured of Students' Strategies	Teacher's reaction
Discovering the characteristics of rotational symmetry from the regular batik patterns by using a pin and transparent batik cards	The students are able to deduce the characteristics of rotational symmetry from the regular batik patterns by using a pin and the transparent batik cards.	The expected strategy: The students put the transparent batik cards above the corresponding patterns and position the pin in the centre of the card. Then, they turn around the transparent batik card and count how many times the pattern fit into itself in one round angle (360^o). Example: 	The teacher asks the students to observe the pattern and determine whether the pattern fit into itself as follows, "*What did happen to the pattern after you turn around*?" "*How about the position of the initial pattern and after you turn it around, do they have the same position*?"
Determining the characteristics of rotational symmetry from the provided patterns	The students are able to determine the characteristics of rotational symmetry (the order of rotation, the angle of rotation and the point of rotation)	The expected answer: The students can determine the characteristics of rotational symmetry properly such as - the order of rotation depends on how many the pattern fit into itself in one round angle - the angle of rotation can be determined by dividing 360^o with the order of rotation - the point of rotation is the centre point of the pattern which can be determined from the intersection of the diagonals or the axes of symmetry	The teacher can guide the students to have a further investigation about rotational symmetry such as by asking "*Does the pattern which has rotational symmetry always have line symmetry*?"

c. The instructional activities

There are three main activities in this meeting. First, sorting the batik fabrics based on the regularity of the patterns. It is designed to orient the students to the notion of rotational symmetry as the fabrics consist of two types, the patterns with rotational symmetry (regular patterns) and the patterns with no line symmetry. Second, exploring the regular patterns by using a pin and transparent batik cards. It is designed to engage the students to discover the basic notion of rotational symmetry. Third, determining the characteristics of rotational symmetry of the provided patterns. It is designed to develop students' basic notion of rotational symmetry which they discovered in the previous activity.

The learning sequence of the meeting is described as follow.

1) The initial activity

The teacher starts the lesson by showing the regular batik patterns which already discussed in the first meeting and asking the students about regularity in the pattern. It aims at reviewing the students' understanding of regularity so that it can help them to do the intended task. In addition, the size of one full angle, right angle and other special angle should be reviewed. It aims at supporting the students to do the worksheet and notice the relation between the order of rotation and the angle of rotation.

2) Doing the worksheet

The students get oriented to do the worksheet in a group which consists of three to four students. It is intended to make the students discuss and share their ideas so that they will obtain more ideas and do the task easier than doing individually.

3) Classroom discussion

Several groups of students who have different answers and strategies will have an opportunity to present their answers. Then, the other students will have a chance to give comments or state their opinions whether they agree or disagree with the presentation. The main point of the discussion is the characteristics of the regular batik patterns which lead the students to acknowledge the notion of rotational symmetry. Then, the students will have an opportunity to define the meaning of rotational symmetry individually. After five minutes, the students and the teacher discuss the meaning of rotational symmetry and its characteristics.

4) Closing activity

The teacher asks the students to reflect the lesson such as by asking the following questions,

- *What do we have learned today?*
- *Can you define rotational symmetry in your own words?*

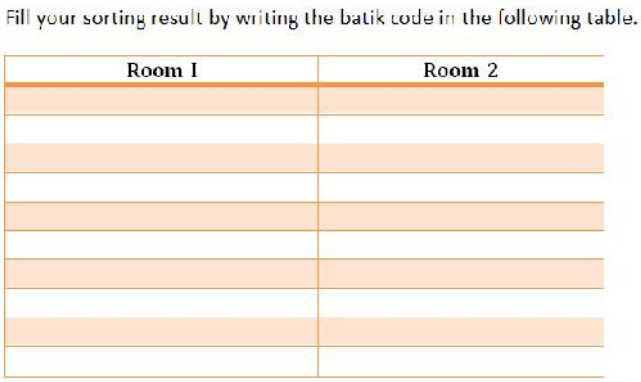
Besides, the teacher can give a regular batik pattern and ask the students whether it has a rotational symmetry.

d. The conjectures of students' thinking and learning

The conjectures of students' thinking and learning will be described based on the three tasks on the worksheet.

1) The first task

This task asks the students to fill the table (figure 4.2) with their sorting result.


Fill your sorting result by writing the batik code in the following table.

Room I	Room 2

Figure 4.2. The figure of table to fill the sorting result

In line with table 4.2, the following are the possibilities of students' sorting result.

- The students sort the batik fabrics based on the colour, the motif or other characteristics instead of the regularity of the patterns

  Room 1 (blue batik patterns) : B, C, G, H, J

  Room 2 (brown and black batik patterns) : A, D, E, F, I, K, L

  The students may answer that they sort the patterns by looking up the colour and they see that most of the patterns are blue and brown, so that they sort the patterns by differentiating blue patterns and brown & black patterns.

- The students sort the patterns based on the regularity of the patterns in which whether the patterns consist of the same motif or the pattern is unique.

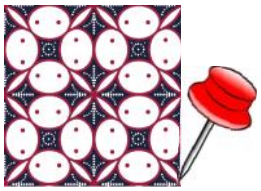
  Room 1 : Batik A, E, F, G, H, K, L

  Room 2 : Batik B, C, D, I, J

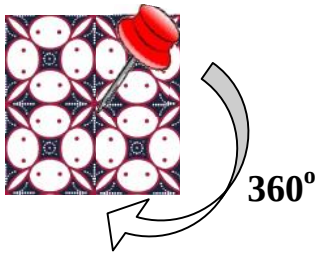
  The students may answer that they sort the patterns by looking up whether the pattern consist of the same patterns or the unique pattern. It may happen because they already experienced the similar activity in the first meeting.

2) The second task

a) The students put the transparent batik cards above the corresponding patterns and position the pin in the vertices of the cards and rotate it as follows



b) The students put the transparent batik cards above the corresponding patterns, position the pin in the centre of the card, and rotate it. However, they rotate the transparent batik cards for 360^o in every rotation. As the result, they think that the pattern can fit into itself for many times.



c) The students put the transparent batik cards above the corresponding patterns and position the pin in the centre of the card. Then, they turn around the transparent batik card and count how many times the pattern fit into itself in one round angle (360^o)

3) The third task

a) The students cannot determine the characteristics of rotational symmetry properly. Example:

- the order of rotation is the pattern fit into itself

- the angle of rotation is the degree of the rotation angle

The students have no idea in estimating the angle of rotation.

The students estimate the angle of rotation by seeing the movement of the pattern from the first position until the pattern fits into itself again.

- the point of rotation is the centre point of the pattern

b) The students can determine the characteristics of rotational symmetry properly such as,

- the order of rotation depends on how many the pattern fit into itself in one round angle
- the angle of rotation can be determined by dividing 360^o with the order of rotation
- the point of rotation is the centre point of the pattern which can be determined from the intersection of the diagonals or the axes of symmetry

e. The teacher's reaction

These are the description of teacher's reaction toward the conjectures of what students do in doing the given tasks

1) The first task

a) The students do not sort the batik patterns based on their regularity

If the students sort the patterns by their colour, then the teacher can ask the following questions:

"*How if the patterns are not printed in colour, how will you sort them*?"

If the students realize that their sorting strategy is not general enough, then the teacher can suggest the students to observe the details of the motif. It leads the students to notice about several patterns which consist of the same motif.

If the students sort the patterns based on the details of the motif or other characteristics, then the teacher can suggest them to review the regularity that they already discussed in the first meeting. It is intended to make the students understand that the regularity always refer to the same patterns.

b) The students sort the patterns based on the regularity of the patterns in which whether the patterns consist of the same motif or the pattern is unique.

The teacher asks the students about the regularity that they mean,

"*In this task, what do you mean by regularity*?"

This question is intended to know how the students sort the patterns based on their regularity.

The teacher also can give follow-up questions as follow,

"*Instead of the repeating patterns, what do you notice from the regular pattern*?"

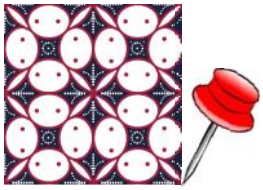
The students may answer that the regular patterns have line symmetry. Then, the teacher can refer to the pattern G and ask the following question,

"*Look at pattern G, it consists of the same pattern but does it have line symmetry*?"

This question aims at guiding the students to do the next task which is discovering the notion of rotational symmetry.

2) The second task

a) The students put the pin in the vertices of the cards as follows



The teacher asks follow-up question as follows,

"*What does happen to the pattern after you turn it around*?"

"*Do you see any differences between the initial pattern and the pattern after you turn it around*?"

"*If I want to turn the pattern around and make the pattern stays still in that position, where should I put the pin*?"

b) The students put the pin in the centre of the card, but they turn around the card for 360^o in every rotation. As the result, they think that the patterns fit into themselves for many times.

The teacher can tell the students that they should figure out whether the

pattern fit into itself for angle of rotation less than 360^o. The teacher can suggest the students to start rotating the pattern from the red mark on the top left of the pattern until the mark gets back to the top left of the pattern.

"*In this case, you are just allowed to turn the pattern around for one rotation (360^o), then count how many times the pattern fit into itself*?"

c) The students put the pin in the centre of the cards as follows,

The teacher asks follow up questions such as.

"Do you see any differences between the motif on the initial pattern and the patterns after you turn it around?"

"*How about the position of the initial pattern and after you turn it around, do they have the same position*?"

3) The third task

a) The students cannot determine the characteristics of rotational symmetry properly

The teacher can use pattern A as the example. Related to the order of rotation, the teacher can ask the students how many times the pattern A fits into itself in 360^o. Then, the teacher can refer students' answer to the order of rotation. Related to the angle of rotation, the teacher can review the size of one round angle (360^o) and ask the students to count how many pattern A fits into itself. It is intended to make the students notice that the angle of rotation can be determined by dividing 360^o with the order of rotation

"*Doing the rotation is turning the pattern around under one full angle. Hence, if the pattern can fit into itself for 4 times in one round, then what can you conclude*? *Determine the angle of rotation."*

Related to the point of rotation, the teacher can ask the students to show how they determine the centre of the pattern F and how they can be so sure that it is the centre of the pattern.

"*Do you have any strategy to make sure that the point of rotation is exactly in the centre of the pattern*?"

The questions aims at making the students draw the diagonals or the axes of symmetry of the pattern.

b) The students can determine the characteristics of rotational symmetry properly

The teacher can ask them whether the pattern which have rotational symmetry always have line symmetry. Then ask them to give examples.

"*Then, for which cases the centre of rotation is in the intersection of line symmetry or diagonals*?"

It aims at making the students aware that having rotational symmetry does not always mean having line symmetry, for example: parallelogram

3. Meeting 3 – Symmetric Patterns

a. The starting point

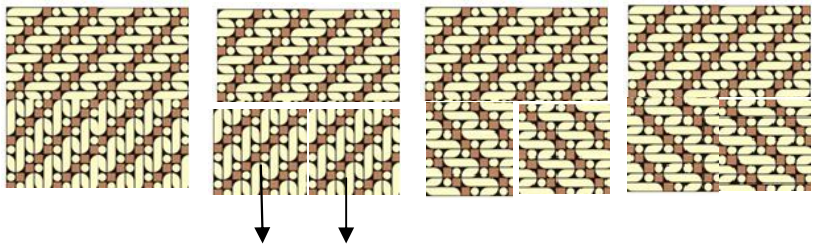
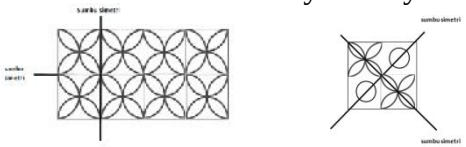
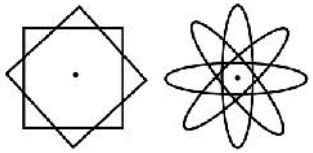
The students understand the concept of line symmetry in which the patterns can be divided into two parts and the two parts become each other's mirror image, and rotational symmetry in which the pattern can fit into itself under the angle of rotation.

b. The mathematical learning aim

The third activity aims at supporting the students to apply their understanding of line and rotational symmetry. The goal is specified into these following sub-learning aims,

1) The students are able to make the asymmetric batik pattern into the symmetric ones
2) The students are able to complete the symmetric pattern by considering the given axes of symmetry
3) The students are able to create their own batik design by using provided batik units and considering the required angle of rotation

In order to achieve the learning goal, the researcher presents table 4.4 for giving an overview of the main activity and the hypotheses of learning process and the details information in the following section.

Table 4.4. An overview of the main third meeting and the hypotheses of learning process			
Activity	**Mathematical Learning Goal**	**Conjectured of Students' Strategies**	**Teacher's reaction**
Restructuring asymmetric patterns into the symmetric ones	The students are able to make asymmetric patterns into the symmetric ones.	The students cut the pattern into several parts and paste them such that the patterns become symmetric. For example,  the pattern are being rotated and repositioned	The teacher asks the students about other strategies in making the patterns become symmetric.
Drawing the remaining pattern such that it becomes symmetric	The students are able to complete the symmetric pattern by considering the given axes of symmetry	The students complete the patterns by considering the unit pattern and the axes of symmetry as follows, 	The teacher asks the students to explain their reasoning and strategy in completing the pattern.
Designing the symmetric pattern by using the provided batik units	The students are able to create their own batik design by using provided batik units	The students draw their own pattern by using the provided batik units and considering the given angle of rotation 	The teacher asks the students to explain their strategies of creating the patterns and determine its symmetry

c. The instructional activities

There are three activities in this meeting. First, making the asymmetric pattern becomes the symmetric ones. It is designed to support the students to apply their understanding of line symmetry. Second, drawing the remaining part of the patterns such that they become symmetric. It is designed to support the students to apply their understanding of line symmetry as they have to draw the patterns by considering the axes of symmetry. Third, designing symmetric patterns by using given batik units. It is designed to support the students to apply their understanding of rotational symmetry as they have to draw the patterns by considering the angle of rotation and the point of rotation.

The learning sequence of the meeting is described as follow.

1) The initial activity

   In the beginning of the lesson, the definition and the characteristics of line and rotational symmetry should be reviewed. It aims at supporting the students to apply their understanding of both symmetry in making symmetric patterns.

2) Doing the worksheet

   The students do the worksheet individually. It is intended to see how each student applies his/her understanding of symmetry. While the students do the worksheet, the teacher will supervise how the students do the given tasks.

3) Classroom discussion

   The students will present their answers and strategies in creating symmetric patterns. During the discussion, the teacher should make sure that all students understand how to apply the characteristics of line and rotational symmetry in creating symmetric patterns. The most important point is that the line

symmetry always makes the pattern becomes two congruent parts in which the two parts become each other's mirror images and rotational symmetry always makes the pattern fits into itself.

4) Closing activity

In the end of the lesson, the students will reflect what they have been learned from the activity. The teacher can ask some following questions,

"*What do you have to consider in making symmetric patterns*?"

"*What is the role of the axes of symmetry in creating symmetric patterns*?"

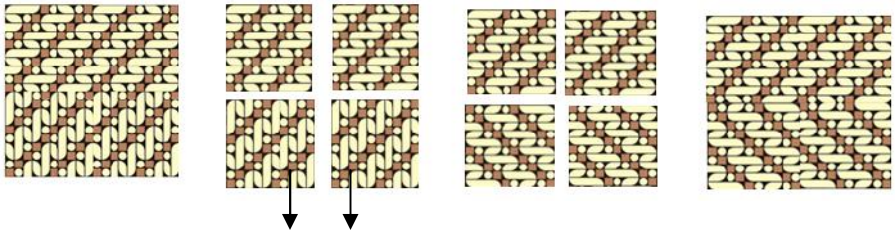
"*How do you draw the angle of rotation?*"

d. The conjecture of students' thinking and learning

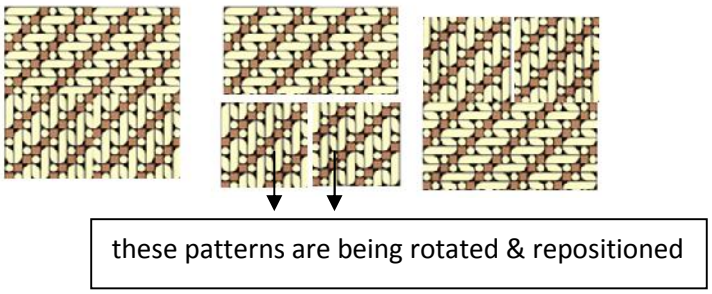
These are the possibilities of students' answers and strategies in doing the worksheet.

1) The first task

- The students cut the pattern into several parts and paste them such that the pattern becomes symmetric

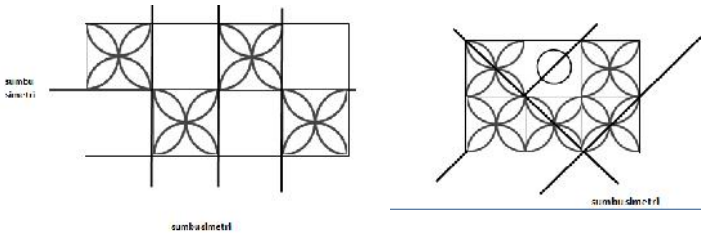


- The students cut the pattern into several parts, but they paste them in the wrong way so that the pattern does not become symmetrical. For example,

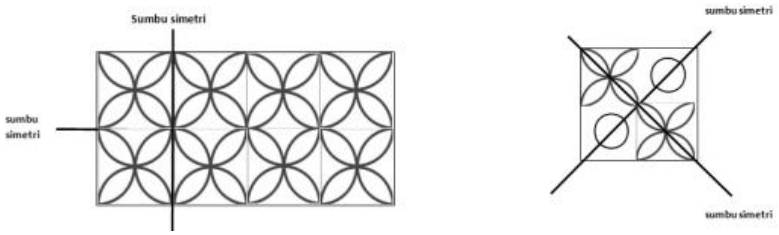


2) The second task

- The students complete the pattern with the same motif without considering the axes of symmetry as follow

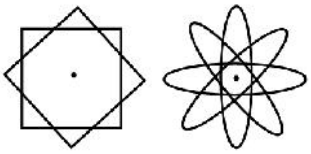


- The students complete the pattern with the same motif and consider the axes of symmetry such that the pattern becomes symmetrical as follow.



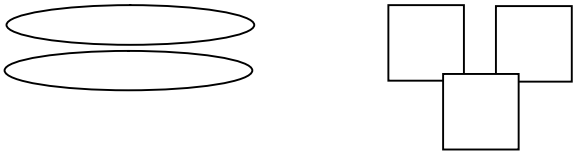
3) The third task

- The students draw their own pattern by using the provided batik units and considering the given angle of rotation



The students firstly draw the given angle in one full angle and its axes. Then, they imitate the provided unit by considering the given angle.

- The students draw their own pattern by using the provided batik units but without considering the given angle of rotation



e. The teacher's reaction

These are the description of the follow-up actions that teacher can do toward the conjectures of students' answers and strategies,

a) The first task

- The students cut the pattern into several parts and paste them such that the patterns become symmetric. The teacher can ask the students about other strategies in making the patterns become symmetric and their reasoning in arranging the patterns. For example:

  "*Is there any other way to arrange the pattern such that it becomes symmetric*?"

  "*Why do you arrange the pattern in that way, what do you consider*?"

- The students cut the pattern into several parts, but they paste them in the wrong way so that the pattern does not become symmetric.

  The teacher suggests the students to observe the pattern more thoroughly and review the definition of line symmetry. It is intended to make the students remember that there should be two identical parts in which both parts become each other mirror's image.

  "*What does line symmetry mean*?"

  "*What do you have to consider the make the pattern has line symmetry*?"

  It aims at encouraging the students to use the proper concept of line symmetry in arranging the patterns.

b) The second task

- The students complete the pattern with the same motif without considering the given axes of symmetry

The teacher asks the students to review the definition and the characteristics of the axes of symmetry. It aims at making the students realize that the half pattern should be identical and become each other mirror's image

"*What is the axes of symmetry*? *In order to make symmetric pattern, do you have to consider the given axes of symmetry?"*

- The students complete the pattern with the same motif and consider the given axes of symmetry such that the pattern becomes symmetric.

  The teacher asks the students to explain their reasoning and their procedure in completing the pattern in that way.

c) The third task

- The students draw their own pattern by using the provided batik units and considering the given angle of rotation

  The teacher asks the students to explain their way in designing the symmetric patterns and determine the number of line symmetry, the order of rotation, the angle of rotation and the centre rotation.

- The students draw their own pattern by using the provided batik units but without considering the given angle of rotation

  The teacher asks the students to review the definition rotational symmetry and its characteristics. The example of questions,

  "*What is the definition of rotational symmetry*?"

  "*Now, look at your pattern more thoroughly and see whether the definitions hold for your pattern*""*What is the angle of rotation*?"

This initial HLT is improved after the teaching experiment. The researcher analyzes whether the conjectures are in line with the actual learning process. It aims at increasing the accuracy of the conjectures of students' thinking and strategies under the designed instructional activities. Indeed, the HLT will not hold for any circumstance regarding some constraints such as the students, the class and the teacher. However, under the designed circumstance, the students will have the similar pattern in achieving the intended learning goals. Therefore, the final HLT can be used as the local instructional theory in learning line symmetry and rotational symmetry.

CHAPTER V

RETROSPECTIVE ANALYSIS

Gravemeijer & Cobb (2006, cited in Van den Akker, et.al (2006)) defined retrospective analysis as examining the whole data set collected during the teaching experiment which aims at supporting the researcher to revise and improve the local instruction theories and answering the research question. Therefore, this chapter focus on describing the analysis of the activities from the three cycles of the study. The research timeline can be seen in appendix 1.

A. The Prior Knowledge of The Students in The Teaching Experiments

This study consists of three cycles of teaching experiments which are conducted in Laboratory Elementary School of Unesa. The preliminary teaching experiment involves five students of class 5D namely Andi, Eka, Sandi, Nico and Arka. The second cycle involves 23 fifth-grade students of class 5C. Then, the third cycle involves 23 fifth-grade students of class 5B. In general, the students have different levels of mathematics achievement. Most of the students are talkative and have difficulties to focus on doing the worksheets. Therefore, the teacher should supervise and encourage them intensely in order to make them focus on doing the worksheet. In the preliminary teaching experiment, the researcher plays the role as the teacher who conducts the lesson. However, Ibu Mardiati, the mathematics teacher, is the teacher who conducts the lesson in the second and third cycle. The prior knowledge of the students in the teaching experiment is obtained from the pre-test which consists of five problems of symmetry (the pre-test can be seen

in Appendix 4) and the small interview that has been conducted for clarifying the pre-test's answers. The analysis of the pre-test result and the follow-up interview corresponding to the students' prior knowledge in three cycles are described as follows.

1. The students can identify the symmetrical objects but assume a parallelogram as a symmetric object.

   The students are asked to determine the symmetric and asymmetric objects from the objects on figure 5.1.

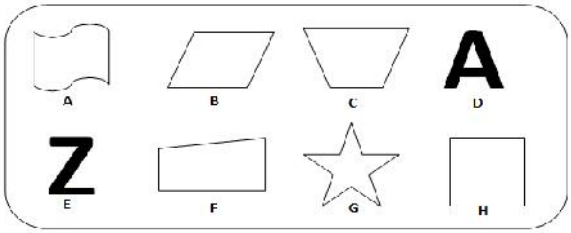


Figure 5.1. The Objects on The First Problem of Pre-test

In general, the students who involved in the three cycles of the teaching experiments can identify the symmetric objects that obviously look symmetric such as object C, D, G and H. However, most of them have difficulties to identify that object B (parallelogram) is asymmetric. Figure 5.2 is the example of students' answer.

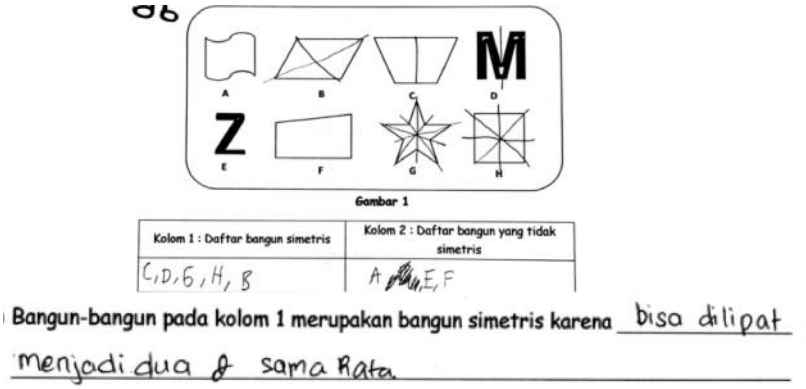


Figure 5.2. The Example of Students' Answers on The First Problem of Pre-test

They assume a parallelogram as a symmetric object. When they are asked to explain the reason, they say that a parallelogram can be divided into two same parts by drawing the diagonal. They assume the diagonal of the parallelogram as its axes of symmetry. These students' answers imply that the students think line symmetry as the symmetry which makes the object becomes two identical parts without considering that the two parts should become each other's mirror images.

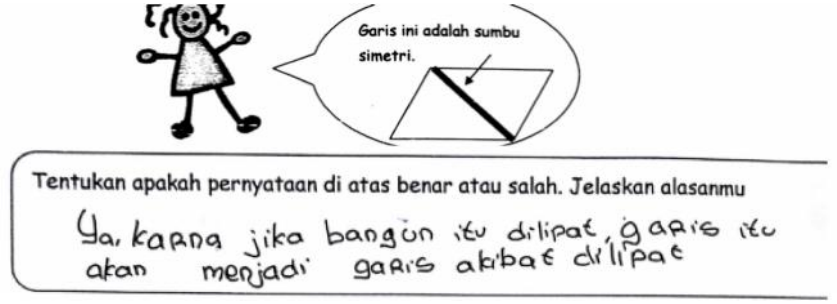
2. The students have a basic understanding of line symmetry

The students are asked to draw the axes of symmetry of the symmetric objects in figure 5.1 and give the reason of determining the object as the symmetric objects. As can be seen in figure 5.2, the students can draw the axes of symmetry of the symmetric objects C, D, G, and H correctly. However, since the students assume a parallelogram as a symmetric object, the students draw the diagonal of the parallelogram and assume it as its axes of symmetry. It may happen because the students only learned a basic concept of symmetry in fourth grade in which the symmetry of an object refers to object which can be folded along the line so that both parts matched perfectly (all edges matching). This basic understanding will be used in the learning sequence as a milestone for the students to develop their understanding that the symmetry is not only about two same parts but also the each other's mirror images.

Then, some of the students in the first cycle make a mistake by drawing the axes of symmetry of eight objects instead of drawing the symmetric objects that they already determined in the first problem. It may happen for two reasons that the students still have difficulties to understand what symmetric object is or the students do not understand the instruction. However, students' answers in the other problems show that they do understand about symmetry. Therefore, that mistake is possible to happen because of the unclear instruction. Therefore, the figure which contains eight objects will be omitted for the pre-test's second problem on the second and third cycle. The students will be asked to draw the axes of symmetry of the symmetric objects in the first figure of the first problem. Then, the instruction will be revised from "*Aku bisa menggambar sumbu simetri dari bangun-bangun simetris seperti berikut*" into "*Gambarlah sumbu simetri dari bangun-bangun simetris yang sudah kamu daftar di kolom 1 pada Gambar 1.*"

3. The students cannot differentiate the diagonal and the axes of symmetry of a parallelogram.

The students are asked to determine whether the diagonal of a parallelogram is its axes of symmetry and give their reasoning.



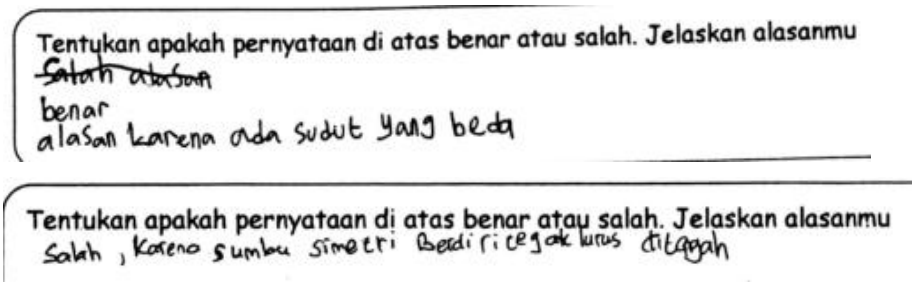


Figure 5.3. The Example of Students' Answers on The Third Problem of Pre-test

Most of the students think the diagonal of a parallelogram as its axes of symmetry. It can be seen in the first answer in figure 5.3. They think that if they fold the parallelogram along the diagonal, the both parts of the parallelogram will match perfectly (all edges matching). Meanwhile, the other students think that the diagonal of a parallelogram is not its axes of symmetry. However, as can be seen in figure 5.3, the students cannot give proper reasons. For example, the students think that the diagonal of a parallelogram is not its axes of symmetry because the line is not vertical. It implies that they do not have sufficient knowledge about the difference between the diagonal and the axes of symmetry.

4. The students are able to make asymmetric object becomes symmetric

The students are asked to determine whether the following given object in figure 5.4 is symmetric or not.

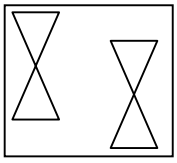


Figure 5.4. The Figure on The Fourth Problem of Pre-test

If the students think that the object is asymmetric, then they should make it becomes symmetric. The following figure 5.5 is the answers of most students. They are able to identify that the object is asymmetric and make it becomes symmetric by drawing the inside shapes in the symmetric position.

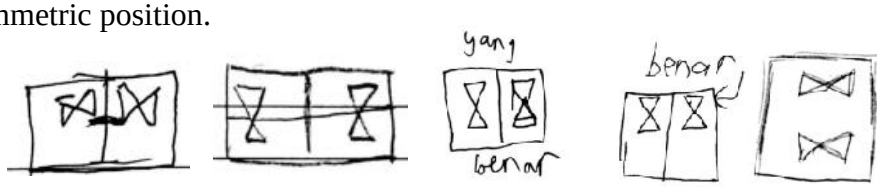


Figure 5.5. Students' Answers on The Fourth Problem of Pre-test

In general, the students have no difficulty in making an asymmetric object into the symmetric ones.

5. The students have difficulties in determining the number of rotational symmetry and its characteristics

The students are asked to determine the number of rotational symmetry, the angle of rotation and the centre of rotation of the object (parallelogram).

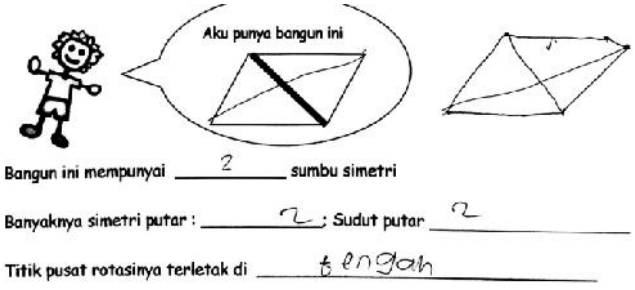


Figure 5.6. The Example of Students' Answers on The Fifth Problem of Pre-test

In the first cycle, the five students have difficulties in answering the problem as they have not learned about rotational symmetry yet. Then, as can be seen in figure 5.6, the form of the questions provoke the students

to assume that the object (parallelogram) has an axes of symmetry and rotational symmetry. Therefore, the question will be revised for the second and third cycle into "*Apakah bangun ini mempunyai sumbu simetri?;Apakah bangun ini mempunyai simetri putar? Jika bangun ini mempunyai simetri putar, maka jawablah pertanyaan berikut: Banyaknya simetri putar:____ .*

Based on the adjustment, the following figure 5.7 is the example of students' answers in the second and third cycle.

Figure 5.7. The Example of Students' Answers on The Fifth Problem of Pre-test

Even though the form of the questions is changed, but the students still cannot determine the characteristics of rotational symmetry of a parallelogram. When they are asked about their answer, they state that they only answer it randomly as they do not know the meaning of the order of rotation, the angle of rotation and the point of rotation.

In general, the questions on the pre-test can reveal sufficient information about the prior knowledge of students toward line symmetry and rotational symmetry.

B. The Teaching Experiment

The teaching experiment consists of three cycles in which each cycle has three meetings. The first cycle is conducted by involving five fifth grade students of class 5D. The second cycle involves the fifth grade students of class 5C. Then, the third cycle involves the fifth grade students of class 5B. The meetings will be described as the following description.

1. Meeting 1

The learning goal of this meeting is supporting the students to discover the notion of line symmetry. This meeting consists of four activities in which each of them has sub-learning goal. The activities are described as the following description.

a) First activity

This activity aims at supporting the students to be able to differentiate the patterns that have regularity (line symmetry) and the patterns that have no regularity. The activity starts with the context of Javanese batik gallery. The gallery has a package of various Javanese batik fabrics which consists of twelve batik patterns (figure 5.8). As the gallery only has two rooms for displaying the fabrics, the students need to sort the batik fabrics into two rooms based on the regularity of the patterns. Then, they must give their reasoning of sorting.

Figure 5.8. The Twelve Batik Patterns

Based on the HLT, the students are expected to sort the fabrics into two rooms based on the regularity of the patterns. Then, they reason that they sort the batik patterns into two rooms because there are two types of patterns which are regular and irregular patterns. It is expected that the students refer the regular patterns to the patterns which have line symmetry and vice versa. The following description is the analysis of the activity throughout the three cycles.

The first cycle

In the beginning of the activity, the students are oriented to the batik context and asked about their knowledge about batik. They respond it by relating batik with their uniform. They also recognize that batik has various kinds of patterns such as flowers or animals.

When the students sort the fabrics into two rooms based on the regularity of the patterns, they seem to use their sense of symmetry in determining the regular batik patterns. It can be deduced from the class observation in which the students use their finger and put it on the pattern as it is the axes of symmetry of the pattern. Then, when the researcher asks their reasoning, they state that the pattern can be considered as the regular ones if it can be divided into some identical parts. It implies that the students have a sense of symmetry in which they think that the regular patterns consist of identical parts. This students' strategy in determining the regular patterns by using their sense of symmetry has not stated in the HLT. Therefore, related to the

conjectures of students' strategies in the HLT, the HLT needs to be revised by adding the possibility of this students' strategy.

In general, the students do not have any difficulty in determining the regular patterns. However, there is a difference between the results of two groups in the class discussion, group 1 assumes H as an irregular batik pattern. Meanwhile, group 2 assumes it as a regular batik pattern. The following is the transcript of the class discussion.

Researcher : "Andi's group assumes H as the irregular pattern, but Sandi's group assumes it as a regular batik pattern. Why is it so?"
Sandi (group 1) : (*pointing batik pattern H*) "wait, don't you think that it will be the same if we fold it? "
Arka (group 1) : "It is symmetric"
Nico (group 2) : (*agreeing with Sandi's answer and talking to Adit*) "Yes Di, it will be the same, it's symmetric"
(*Other students start to realize that the natik pattern H is symmetric*)
Sandi (group 1) : "Oh yaa symmetric, I just realized it"
Nico (group 2) : "Yaa symmetric, why don't you say it from the beginning?
Researcher : "Why is it symmetric?"
Shandi (group 1) : (*pointing batik pattern H*) "It's symmetric because if we fold it, the pattern will become two same parts"

The students start to realize that batik pattern H is symmetric because if it is folded, then the two folded parts will have the same pattern. This awareness shows that the provided batik patterns can support the students to emerge with the basic concept of line symmetry. Nevertheless, when the students are asked to explain their reason of classifying the batik patterns into two rooms, they do not understand

how they should explain. As the result, they only write what they know about regularity in the pattern as the following figure 5.9.

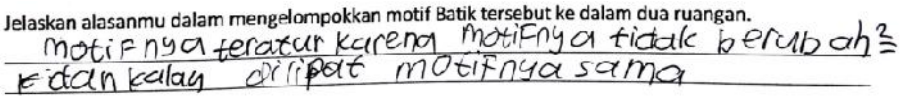


Figure 5.9. The Example of Students' Answers of The First Activity

Even though the answer on the figure 5.9 is relevant and shows the students' awareness about regularity, but this answer does not answer the question. Based on the HLT, they should answer that they classify the batik patterns into two rooms because there are two types of patterns which are regular and irregular patterns. Then, the regular patterns refer to the patterns which have line symmetry. It is possible to happen because of the unclear question in the worksheet. Therefore, the question needs to be revised for the next cycle. The question is revised from "*Jelaskan alasanmu dalam mengelompokkan motif Batik tersebut ke dalam dua ruangan*" into "*Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1 ; Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2*"

The second cycle

In general, the students have no difficulty in determining the batik patterns which have regularity. The following figure 5.10 is the example of most students' answers in determining the regular patterns.

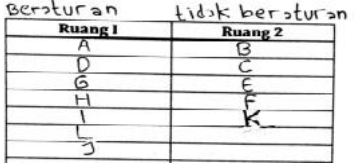


Figure 5.10. The Example of Students' Answers in Sorting the Patterns

However, as can be seen in figure 5.10, the students label the room 1 for the regular patterns and room 2 for the irregular patterns by themselves. Then, the classroom observation shows that several groups label the table reversely. Consequently, it takes time for the students to match the answers in the class discussion. Therefore, the table of the sorting results is revised by labelling the room 1 for the regular patterns for the next cycle as the following figure 5.11. It aims at making the students to have the same perception in using the rooms.

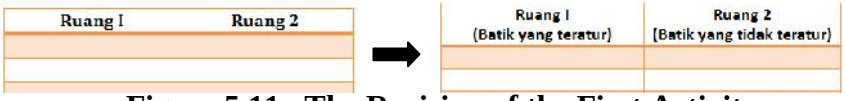
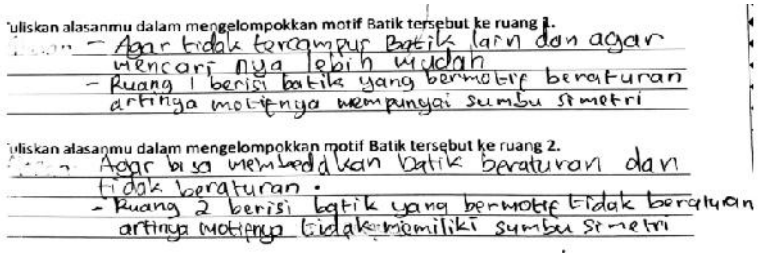


Figure 5.11. The Revision of the First Activity

Based on the adjustment of the first cycle, the question after sorting the patterns into two rooms is revised. It is intended to make the students give a proper mathematical reason of sorting the patterns into two rooms. However, when the students are asked to give their reasoning of sorting the patterns into two rooms, most of them answer that they do it to avoid the possibility of the regular patterns get mixed with the irregular patterns. In fact, it is not the expected answer. In the HLT, the students are expected to answer that they sort the patterns into two rooms because there are two types of patterns which are regular and irregular patterns. Then, the regular patterns refer to the patterns which have line symmetry. Therefore, the question should be revised again for the next cycle. The questions will be revised into "*Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur*." and "*Tuliskan*

perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur". It is expected that the students will give a proper mathematical reason of sorting the patterns into two rooms instead of giving technical reason.


Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1.
- Agar tidak tercampur Batik lain dan agar mencari nya lebih mudah
- Ruang 1 berisi batik yang bermotif beraturan artinya motifnya mempunyai sumbu simetri

Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2.
- Agar bisa membedakan batik beraturan dan tidak beraturan.
- Ruang 2 berisi batik yang bermotif tidak beraturan artinya motifnya tidak memiliki sumbu simetri

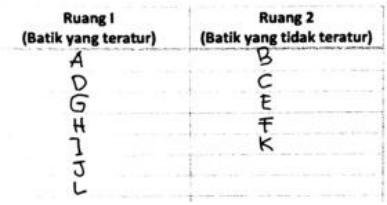
Figure 5.12. The Example of Students' Answers of Reasoning The First Problem

However, there is one group which answers the question by relating the patterns with the axes of symmetry (figure 5.12). They state that the regular patterns are the patterns which have axes of symmetry, meanwhile the irregular patterns are the patterns which have no axes of symmetry. This answer has not stated on the HLT yet because the concept of line symmetry is predicted to emerge in the second activity. The students may relate the regularity of the patterns with the axes of symmetry because they notice the symmetry of the patterns and the existence of the axes of symmetry on the patterns. Hence, this students' reasoning will be added into the conjectures of students' thinking and learning in the next HLT.

The third cycle

The following figure 5.13 is the example of students' answers in sorting the patterns based on the adjustment of the second cycle in

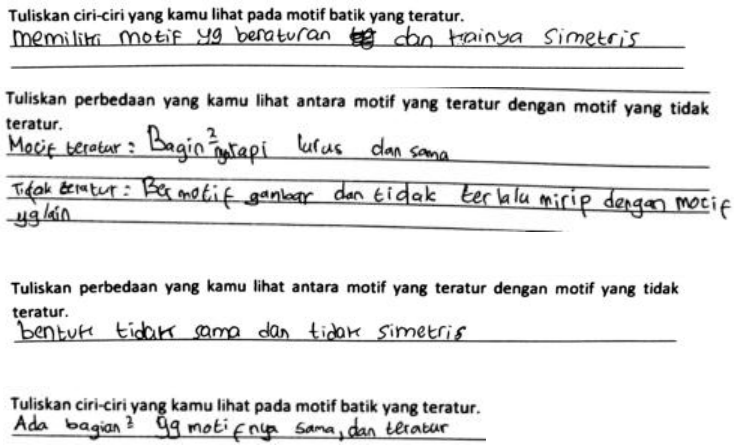
which the table is labelled. The class observation shows that labelling the rooms makes the class discussion works more effective.



Ruang I (Batik yang teratur)	Ruang 2 (Batik yang tidak teratur)
A	B
D	C
G	E
H	F
J	K
L	

Figure 5.13. The Example of Students' Answers in Sorting the Patterns

In line with the HLT, the students are able to determine the regular patterns. Then, based on the adjustment of the first meeting in the second cycle, the question after sorting the patterns into two rooms is revised. It is intended to make the students show their perception of regularity instead of reasoning about the rooms or the patterns.



Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.
memiliki motif yg beraturan dan lainya simetris

Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.
Motif teratur : Bagian 2 nya tapi lurus dan sama
Tidak teratur : Bermotif gambar dan tidak terlalu mirip dengan motif yg lain

Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.
bentuk tidak sama dan tidak simetris

Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.
Ada bagian 2 yg motifnya sama, dan teratur

Figure 5.14. The Example of Students ' Answers in Reasoning

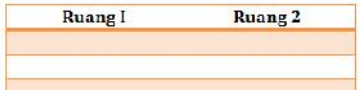
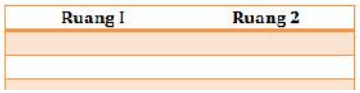
As can be seen in the figure 5.14, the revised questions can encourage the students to show their perspective toward regularity. They define the regular patterns as the patterns which consist of the same motif and vice versa. Meanwhile, one group emerges with the

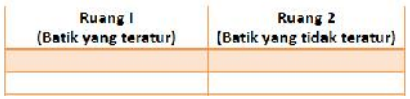
notion of line symmetry. The group states that regular patterns are the symmetric patterns. This conjecture in which the students emerge with the notion of symmetry in this activity is already in line with the revised HLT.

Conclusion

By considering the analysis result of the first activity in the three cycles, the students are able to determine the regular (patterns which have line symmetry) and irregular batik patterns. They determine the regular batik patterns by using their sense of symmetry. In addition, they can relate the regularity of the patterns with the basic concept of line symmetry by themselves. Therefore, it can be assumed that this activity supports the students to emerge with the notion of line symmetry. The following table 5.1 shows the refinement of the activity throughout the three cycles.

Table 5.1. The Refinement of the First Activity

	Student worksheet	HLT
The first cycle	The questions after sorting the patterns: "*Jelaskan alasanmu dalam mengelompokkan motif Batik tersebut ke dalam dua ruangan*" The result: The questions cannot make the students show their perception of regularity. Hence, the questions need to be revised.	The conjecture of students' strategy in determining the patterns which have regularity and no regularity: The students determine the regular patterns by using their sense of symmetry. For example: the students use their hand gesture to show the axes of symmetry in each pattern.
	Table of sorting result:  The result: The students have the same way in using the table because there are only two groups. Hence, the need of labelling the table has not appear.	
The second cycle	The questions after sorting the patterns: "*Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1"* *Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2*" The result: The students more focus on stating their perception about the rooms or the patterns instead of the concept of regularity. Hence, the questions need to be revised again to make the students give a proper reason.	The conjecture of students' answer in answering the questions after sorting the patterns: The students state that the regular patterns are the patterns which have axes of symmetry, meanwhile the patterns which have no regularity are the patterns which have no axes of symmetry
	Table of sorting result:  	

<table>
<tr><td></td><td><u>The result:</u>
The students have different ways in using the table. It takes time to discuss in the class discussion. Hence, the table needs to be labelled.</td><td></td></tr>
<tr><td rowspan="2">The third cycle</td><td><u>The questions after sorting the patterns:</u>
“Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.”
“Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.”
<u>The result:</u>
The students show their perception of regularity</td><td></td></tr>
<tr><td><u>Table of sorting result:</u>
 
<u>The result:</u>
The students have the same perception in using the table. Hence, it makes the class discussion works more effective</td><td></td></tr>
</table>

b) Second activity

This activity aims at supporting the students to be able to deduce the characteristics of line symmetry from the regular batik patterns by using a mirror. In this activity, each student gets a mirror to investigate the special characteristics of the regular patterns. Based on the HLT, the students are expected to discover the special characteristics of the regular patterns by putting the mirror in the middle of the patterns along the axes of symmetry. Then, they are aware that the mirror reflection shows the identical pattern as the pattern in the back of the mirror. The students are also expected to be aware that the mirror position refers to the axes of symmetry. In this case, the axes of symmetry refers to the line which divides the patterns into two identical parts and both of them become each other's mirror images. The following description is the analysis of the activity throughout the three cycles.

The first cycle

In line with the HLT, the students place the mirror in the middle of the patterns along the axes of symmetry. They are aware that the mirror reflection shows the identical pattern as the pattern in the back of the mirror. They also count the number of the mirror positions which refer to the axes of symmetry. However, the two groups start to argue when they have different number of the mirror positions of pattern D. Group 1 has four positions and group 2 has eight positions. Hence, they are asked to present how they place the mirror. Group 1 places the mirror in the middle of the pattern vertically, horizontally, right and left diagonally. Meanwhile, group 2 not only places the mirror like group 1 but also in the 4 edges of the pattern. Therefore, they have eight mirror positions as the result. Since this

situation is already predicted in the HLT, the researcher responds it by placing the mirror on the edge of the irregular pattern and asks the students whether the pattern look symmetric on the mirror. It is intended to make the students realize that the mirror positions on the edge of the pattern are not counted. As the students see that the reflection is identical with the pattern, then the researcher confronts it with the regular patterns. The students start to realize that if they place the mirror on the edge of the pattern, every pattern (regular and irregular) will look the same. However, not every student seems to understand why he or she cannot count the mirror position on the edge of the pattern. Hence, the teacher's reaction toward the students who put the mirror on the edge of the pattern needs to be revised. The teacher can ask the students to observe the pattern and find the mirror position which make the batik patterns do not change at all. If they put the mirror on the edge of the pattern, they are supposed to see that the pattern expands and different with the initial pattern. Then, it is expected that they will understand why they cannot place the mirror on the edge of the patterns

Then, in the whole group discussion, each group presents their result of counting the mirror position on the patterns. They also show how they place the mirror on the pattern. The following is the transcript of the discussion.

Researcher : "how about pattern J, how many mirror positions that you got?"

(*Eka and Nico present how they placed the mirror and count the number of positions*)

Nito : "Ehm...(*placing the mirror vertically*) one..".
(*placing the mirror horizontally*) "two...there are two positions"

Emir : (*placing the mirror vertically, horizontally and diagonally but he realizes that if he places the mirror diagonally, then the pattern will be different*)
"just two"

Researcher : (*talking to all students*) "Are you sure, two or four?"

All students : "two"

Researcher : “Yes two, how about L?”
(*every student place the mirror on the pattern L*)
Nito : (*placing the mirror vertically*) “one...”
(*placing the mirror horizontally*) “two”

(*placing the mirror diagonally*) “oh it is not. So, just two”
(*Other students agree to Nito’s answer*)
Researcher : “Yes two, now don’t you remember, what do you think about the mirror position, how should we call it?”
All students : “symmetry”
Emir : “line symmetry”

The transcript reveals the fact that the students can relate their prior knowledge of line symmetry which they learned in fourth grade with this hands-on activity. By observing how the students place the mirror in the pattern, they are aware that they should place the mirror in the middle of the pattern. Then, by considering the details of the pattern, they can determine the axes of symmetry of the pattern without using the mirror. In other words, the hands-on activity in which the students are asked to find the required mirror position in the batik pattern can support the students to deduce the characteristic of line symmetry (the axes of symmetry).

In general, this activity can support the students to be aware that line symmetry is not only about two identical parts, but they also consider that the both parts should become each other’s mirror images.

The second cycle

As already hypothesized in the HLT, the students put the mirror on the edge of the pattern. Then, based on the adjustment of the first cycle for this students’ strategy, the teacher responds it by asking the students to observe the pattern and find the mirror position which make the batik patterns do not change at all. The teacher asks the students to put the mirror on the edge of the pattern and see the

reflection in the mirror. The teacher makes the students notice that the pattern is expanded or changed. Then, the teacher puts the mirror in the position of the axes of symmetry and asks the students again to see the mirror reflection. By observing the difference of the mirror reflections, the students realize that the pattern stays the same if they put the mirror in the middle of the pattern along the axes of symmetry.

The following figure 5.15 is the example of students' answers after exploring the regular patterns by using a mirror

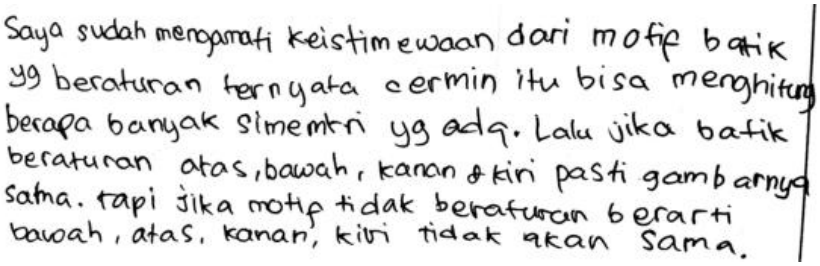

Saya sudah mengamati keistimewaan dari motif batik yg beraturan ternyata cermin itu bisa menghitung berapa banyak simentri yg ada. Lalu jika batik beraturan atas, bawah, kanan & kiri pasti gambarnya sama. tapi jika motif tidak beraturan berarti bawah, atas, kanan, kiri tidak akan sama.

Figure 5.15. The Example of Students' Answers in Discovering the Notion of Line Symmetry

The answer in figure 5.15 shows that exploring the patterns which have regularity with the mirror can support the students to notice that the mirror positions represent the axes of symmetry. The students state that they can count the number of the axes of symmetry by using the mirror. Moreover, there is an intriguing answer from the group of students who do not see any differences between the regular and irregular patterns. The figure 5.16 shows the students' answer.

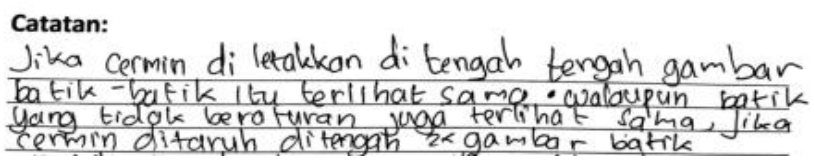

Catatan:
Jika cermin di letakkan di tengah tengah gambar batik - batik itu terlihat sama • walaupun batik yang tidak beraturan juga terlihat sama, jika cermin ditaruh di tengah 2 gambar batik

Figure 5.16. The Example of Students' Answers of The Second Activity

The answer in figure 5.16 in which the students do not see the differences between the regular and irregular patterns may happen because the students

compare the mirror reflection with the pattern in front of the mirror instead of the pattern in the back of the mirror. Therefore, they find it in regular and irregular patterns. Consequently, they do not notice any special characteristics in the regular patterns. By considering this possibility of students' thinking, there will be a refinement on the teacher guide to avoid this students' mistake. The teacher should ensure the students to observe the mirror reflection with the pattern in the back of the mirror.

The third cycle

The following figure 5.17 is the example of students' drawing of the mirror positions.

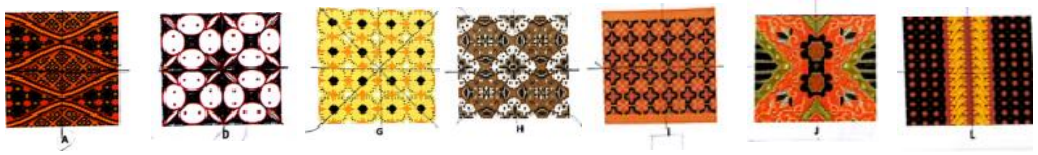


Figure 5.17. The Regular Batik Patterns with Their Mirror Positions

In line with the HLT, the students position the mirror properly by putting it along the axes of symmetry. They are also aware that those positions make the mirror reflection shows the identical pattern as the pattern in the back of the mirror. Interestingly, they only use the mirror for the first two regular patterns (A and D). They can directly deduce the mirror positions of the next regular patterns by observing the patterns. After determining the mirror positions of the regular patterns, the students are asked to state their investigation result. The following figure 5.18 is the example of students' finding after investigating the regular patterns by using a mirror.

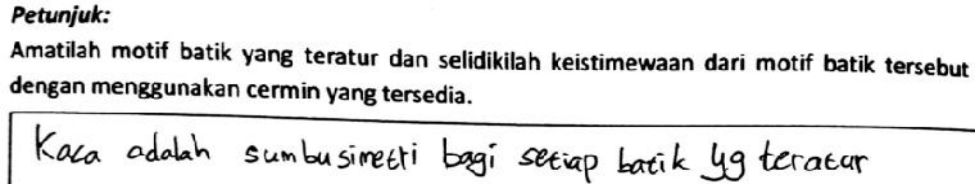


Figure 5.18. The Example of Students' Finding in Investigating the Patterns

The figure 5.18 shows how the students are able to deduce that the mirror position represents the axes of symmetry of the regular patterns. It implies that the activity of investigating the regular patterns by using a mirror can support the students to emerge with the notion of axes of symmetry.

Conclusions

By considering the analysis result of the second activity in the three cycles, the students are able to deduce the characteristics of line symmetry from the regular batik patterns by using a mirror. They position the mirror along the axes of symmetry and they are aware that the position makes the mirror reflects the same image with the initial pattern. They can deduce that the mirror positions refer to the axes of symmetry of the pattern. It seems that the students start to be aware that the symmetric object consists of two parts which reflect each other. The following table 5.2 shows the refinement of the activity throughout the three cycles.

Table 5.2. The Refinement of the Second Activity

	HLT	**Teacher Guide**
The first cycle	<u>The teacher's reaction toward the students who put the mirror on the edge of the pattern:</u> "The teacher asks the students to put the mirror on the edge of the irregular batik patterns. Then, the teacher asks about what the students see in the mirror. They will see that the mirror always reflects the same pattern if they put it on the edge. Then, it is expected that they	<u>The teacher's role during the activity</u> "The teacher situates the students to position the mirror along the axes of symmetry" <u>The result :</u> The students do not see any differences between the regular patterns and the

	will understand that they cannot place the mirror on the edge of the patterns" The result: Not every student seems to understand why he or she cannot count the mirror position on the edge of the pattern. Hence, the teacher's reaction needs to be revised.	patterns which have no regularity. Hence the teacher needs to ensure the students observe the mirror reflection with the pattern in the back of the mirror.
The second cycle	The teacher's reaction toward the students who put the mirror on the edge of the pattern: "The teacher makes the students to observe the pattern and find the mirror position which make the batik patterns do not change at all. If they put the mirror on the edge of the pattern, they are supposed to see that the pattern expands and different with the initial pattern. Then, it is expected that they will understand why they cannot place the mirror on the edge of the patterns." The result: The students seem to understand why they cannot count the mirror position on the edge of the pattern. Hence, there is no revision in the third cycle	The teacher's role during the activity "The teacher is not only situating the students to position the mirror along the axes of symmetry but also ensuring that the students observe the mirror reflection with the initial pattern" The result : The students aware the differences between the regular patterns and the patterns which have no regularity

c) The third activity

This activity aims at supporting the students to be able to differentiate between diagonal and the axes of symmetry of the batik patterns. In this activity, the students have four batik patterns (figure 5.19) and they must draw the diagonals and the axes of symmetry of each pattern. Then, there are two follow-up questions for students to determine whether the diagonal of two-dimensional shapes always refer to the axes of symmetry and vice versa. Based on the HLT, the students are expected to draw the diagonals and the axes of symmetry correctly. Then, they are

aware that the diagonal is not always the axes of symmetry of two-dimensional shapes and vice versa.

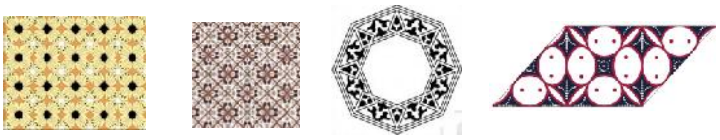


Figure 5.19. The four patterns in the third activity

The first cycle

In line with the expected students' answer in the HLT, the students are able to draw the diagonals and the axes of symmetry correctly. Figure 5.20 is the example of students' answers.

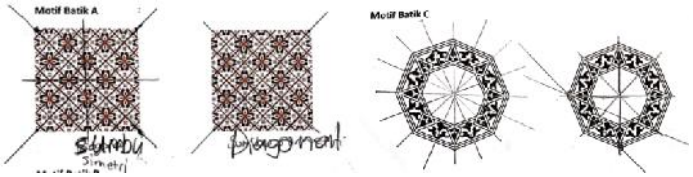


Figure 5.20. The Example of Students' Answers of The Third Activity

The students are able to draw the diagonals and the axes of symmetry of the given objects except the octagon. As can be seen in the figure 5.20, the students cannot draw the diagonals of the octagon completely. It may happen because the object is too complex for the students. It consists of many axes of symmetry and diagonal as well. Therefore, the object will be changed from octagon into pentagon (figure 5.21). This pentagon is chosen to replace the previous octagon because this object only has axes of symmetry and not so many diagonal as the octagon.

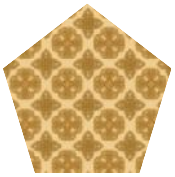


Figure 5.21. The Pentagon for the Third Activity on the Next Cycle

Furthermore, there is an interesting discussion in which the students show their understanding about the difference between the diagonal and the axes of symmetry. The following is the transcript of the discussion.

Researcher	: "*The axes of symmetry of 2D shapes is the diagonal of the shapes. What do you think, is the statement true*?"
Nico, Eka	: "*It is true*"
Sandi	: "*No, it is not. It does not work for all shapes*"
	"*Look at this* (*referring to batik pattern B*), *its axes of symmetry are not its diagonal*."
Andi	: "*It just work for a square*"

The transcript shows that the students are not only aware the difference between the axes of symmetry and diagonal, but also realize that the axes of symmetry is not always the diagonal of the shapes and vice versa. In addition, they give an example of shape which its axes of symmetry is also its diagonal.

The second cycle

Based on the adjustment of the first cycle, the students are able to differentiate the diagonals and the axes of symmetry of the given patterns as the expected answers in the HLT. The following figure 5.22 is the example of students' answers.

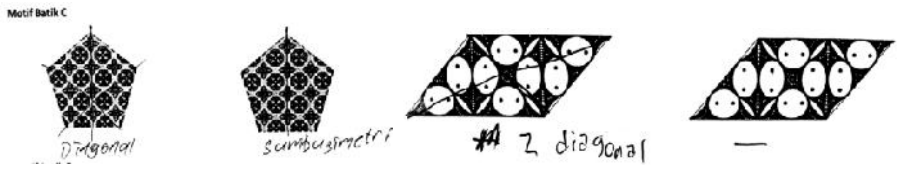


Figure 5.22. The Example of Students' Answers in the Third Activity

However, there are also students who get difficulty in determining the axes of symmetry of a parallelogram. They assume the diagonals as the axes of symmetry as well. Hence, the teacher points out this issue in the class discussion and asks a student to draw the diagonals of a parallelogram on the white board as shown in the figure 5.23.

Figure 5.23. The Student Draw The Diagonal of A Parallelogram

The following transcript is the discussion between the teacher and the students about the diagonal and the axes of symmetry of a parallelogram after the student draw the diagonal on the white board.

Teacher : "So, does a parallelogram have an axes of symmetry?"
The students : "No, it has not"
Teacher : "Why is it so?"
(*The students start stating their answers, hence the teacher raise her hand to ask the students to raise their hand if they want to answer the question*)
(*Several students raise their hands, the teacher appoint student A to answer the question*)
Student A : "Because it cannot be folded" (*saying unclear statement*)
(*The teacher seemed does not satisfy with the answer and appoint student B to state his answer*)
Student B : "Because if it is folded, it will not be the same, the sides will not be the same"
Teacher : "Yaa, good. So a parallelogram has diagonals but not.."
The students : "An axes of symmetry"

The teacher emphasizes the difference between the diagonal and the axes of symmetry by utilizing a parallelogram since the students often assume it as the shape which has an axes of symmetry. The students seem to have a better understanding after the class discussion. Therefore, this remark will be considered to revise the teacher guide for the third cycle. In general, the students in this cycle are able to be aware of the difference between diagonals and the axes of symmetry.

The third cycle

In the beginning of the activity, the students are asked to determine whether line A in the figure 5.24 is the diagonal or the axes of symmetry of the pattern.

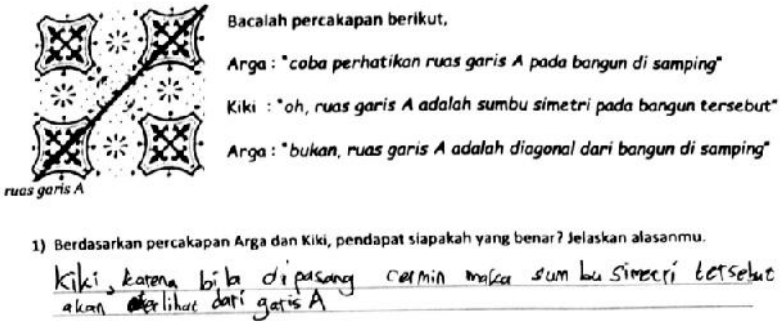


Figure 5.24. The Example of Students' Answers in Determining the Diagonal or the Axes of Symmetry

There are three kinds of students' answers in determining whether line A is the diagonal or the axes of symmetry. Some students state that line A is the axes of symmetry because it represents the mirror positions of the patterns as can be seen in figure 5.24. Meanwhile, other students assume line A as the diagonal because it connects two non-consecutive vertices. Then, one group of students identify it as the diagonal and the axes of symmetry. In general, most of the students can do the task as what has expected in the HLT. The students are aware that line A on figure 5.24 is the diagonal of the object and the axes of symmetry as well.

Conclusion

By considering the analysis result of the third activity in the three cycles, the students are able to differentiate the diagonals and the axes of symmetry. They are not only able to draw the diagonals and the axes of symmetry correctly, but also realize that the diagonal of the object is not always its axes of symmetry. It can be assumed that this activity supports the students to understand the differences between the diagonals and the axes of symmetry. The following table 5.3 shows the refinement of the activity throughout the three cycles.

Table 5.3. The Refinement of the Third Activity

	Student Worksheet	Teacher guide
The first cycle	Motif C  The result The students cannot draw the diagonals completely. Hence, the figure needs to be changed.	-
The second cycle	Motif C  The result The students can draw the diagonals completely. Hence, there is no revision for the third cycle.	The teacher asks the students about the difference between the diagonal & the axes of symmetry The result: The students assume the diagonals as the axes of symmetry as well. Hence, the teacher needs to emphasize the difference between them.
The third cycle	-	The teacher emphasizes the difference between the diagonal and the axes of symmetry by utilizing a parallelogram The result: The students have a better understanding of the difference between the diagonals and the axes of symmetry.

d) Fourth activity

This activity aims at supporting the students to implement their understanding of line symmetry that they already learned from exploring the batik patterns to the two-dimensional shapes. The students get a worksheet which consists of several two-dimensional shapes and alphabets, then determine whether the shapes are symmetric or not. If the shape is symmetric, then they should draw the axes of symmetry. Based on the HLT, the students are expected to determine the symmetric two-dimensional shapes and draw the axes of symmetry correctly.

The first cycle

In this cycle, the students do not get the worksheet to implement their understanding of line symmetry in two-dimensional shapes. The researcher only mentions the name of the shapes and asks the students to determine the number of the axes of symmetry. The class observation shows that the students are able to determine the number of the axes of symmetry but they do not have any opportunity to show how they draw the axes of symmetry. Therefore, in the next cycle, the students will get the worksheet so that the researcher can get sufficient data of students' understanding of line symmetry in two-dimensional shapes.

The second cycle

Due to the limitation of time, this activity is not done in the classroom. The teacher makes this activity as the homework for the students. As this activity is not done in the classroom, the collected data is not sufficient to determine whether the students achieve the learning goal. However, the students' written works show that the students are able to determine the axes of symmetry of the given two-dimensional shapes. The following figure 5.25 is the example of students' answers.

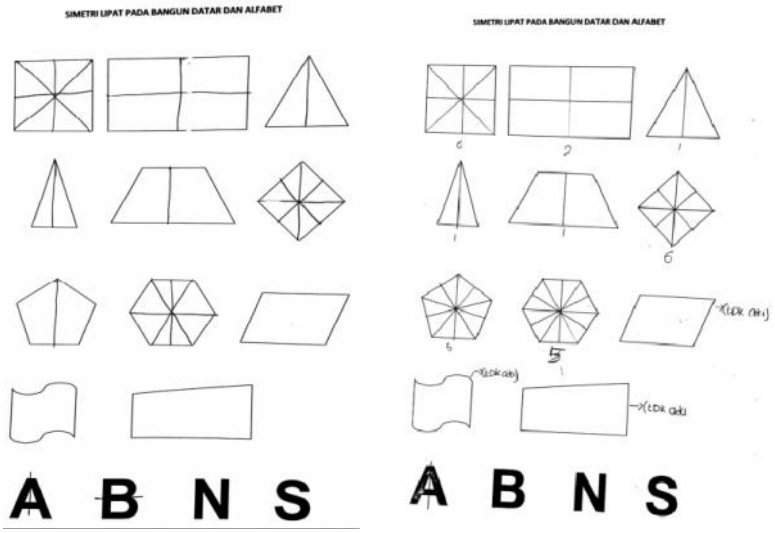


Figure 5.25. The Example of Students' Answers of The Fourth Activity

The sufficient data will be achieved if this activity is done in the classroom. Hence, the time management of the previous activities should be better so that the students can do this fourth activity in the classroom.

The third cycle

In line with the HLT, the students are able to determine the symmetric two-dimensional shapes and draw the axes of symmetry of each shape correctly. The following figure 5.26 is the example of students' answers.

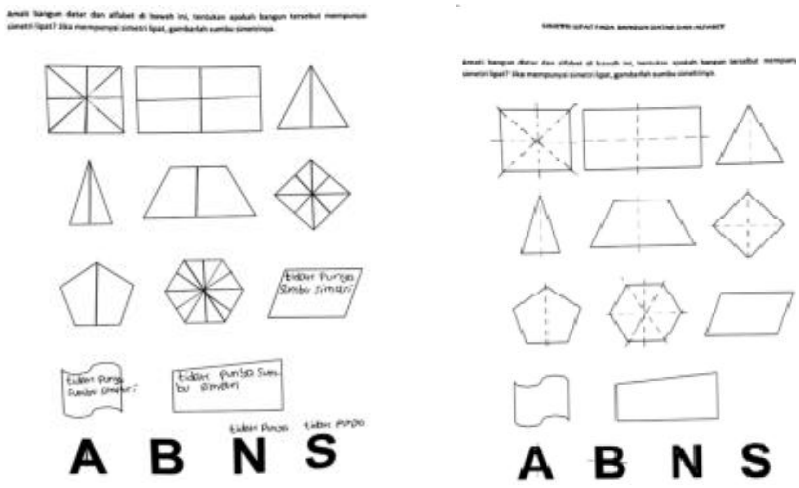


Figure 5.26. The Example of Students' Answers of The Fourth Activity

The class observation shows that the students can directly draw the axes of symmetry of the shapes without having any confusion in differentiating the number of axes symmetry in a square of batik pattern with the number of axes of symmetry in a blank square.

Conclusion

By considering the analysis result of the fourth activity in the three cycles, the students are able to implement their understanding of line symmetry to the two-dimensional shapes. They can determine the symmetric shapes and draw the axes

of symmetry. The following table 5.4 shows the refinement of the activity throughout the three cycles.

Table 5.4. The Refinement of the Fourth Activity

	Worksheet	**Teacher guide**
The first cycle	No worksheet The result : no students' written works. Hence, the students should get student worksheet	The teacher gives the task orally The result : no students' written works. Hence, the teacher needs to provide student worksheet
The second cycle	The students get the worksheet The result : there is students' written works. Hence, there is no revision in the third cycle	The activity is not done in the classroom The result : the teacher does not know how the students do the task. Hence, the activity should be done in the classroom in the third cycle
The third cycle		The activity is done in the classroom The result : sufficient data (students' written works & classroom observation)

2. Meeting 2

The learning goal of this meeting is supporting the students to discover the notion of rotational symmetry. This meeting consists of four activities in which each of them has sub-learning goal. The activities are described as follow.

a) First activity

This activity aims at supporting the students to be able to differentiate the patterns that have regularity (rotational symmetry) and the patterns that have no regularity. Similar with the first meeting, the activity starts with the context of Javanese batik gallery in which the gallery has a package of various Javanese batik fabrics which

consists of twelve batik patterns (figure 5.27). As the gallery only has two rooms for displaying the fabrics, the students need to sort the batik fabrics into two rooms based on the regularity of the patterns. Then, they must give their reasoning of sorting

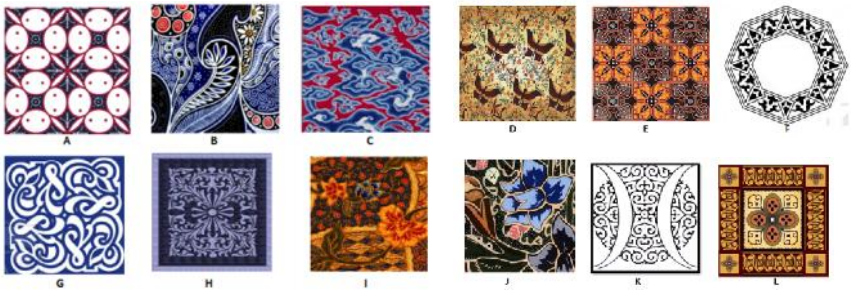


Figure 5.27. The Twelve Batik Patterns

Based on the HLT, the students are expected to sort the fabrics into two rooms based on the regularity of the patterns. Then, they reason that they sort the batik patterns into two rooms because there are two types of patterns which are regular patterns (refer to the patterns which have rotational symmetry) and irregular patterns. The following description is the analysis of the activity throughout the three cycles.

The first cycle

As the idea of this activity is similar with the first activity on meeting 1, the students can differentiate the regular and the irregular patterns easily. It is in line with the expected answer in the HLT. The following figure 5.28 is the answers of the students in sorting the patterns.

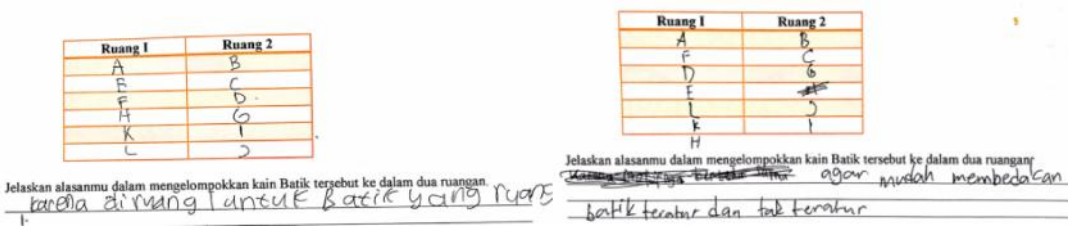


Figure 5.28. The Example of Students' Answers of The First Activity

As can be seen from the figure 5.28, pattern D becomes the different answer between two groups. It happens because the pattern consists of repeating motif but it is asymmetric. When discussing the pattern D in the class discussion, the students use their understanding of line symmetry. They try to fold the pattern and see whether the both parts become each other's images. In other words, they refer the regularity with their perspective of line symmetry.

However, the students have difficulty in giving their reason of sorting the fabrics into two rooms based on the regularity of the patterns. They tend to give technical reason instead of mathematical reason. For example, they sort the patterns into two rooms because it will be easier to distinguish the regular and the irregular patterns. Similar with the first activity in the first meeting, the question needs to be revised from "*Jelaskan alasanmu dalam mengelompokkan motif Batik tersebut ke dalam dua ruangan*" into "*Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1 ; Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2*"

The second cycle

The pattern G becomes the intriguing pattern among the students as the pattern has rotational symmetry but no line symmetry. The following transcript shows the discussion of the students about pattern G.

(Situation : Four students are discussing about the regularity in pattern G)

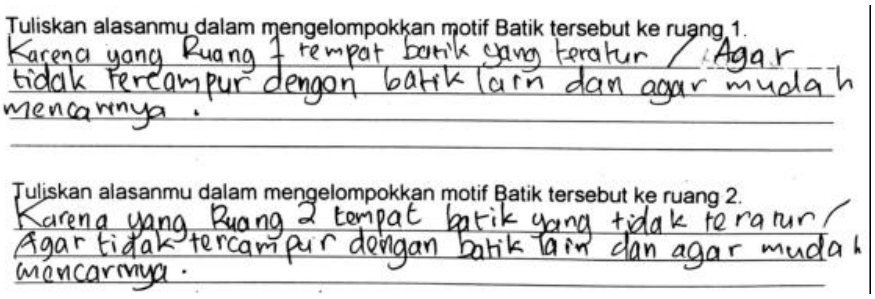
SF 1 : (*pointing her finger to pattern G*)
SF 3 : "No, it is not"
SF 1 : (*looking at SF 2, 3, 4*)
SF 4 : "It is not" (*pointing his finger and looking at SF 3*)

SF 1 : “But this has..this one” (*pointing her finger to the identical parts of pattern G*)

SF 3 : (*pointing his finger to the top and bottom part of the pattern*) “But, they are different”

When SF 1 points her finger to the identical parts of the pattern G, it can be meant that she thinks it is a regular pattern because it consists of the same pattern. However, SF 3 still assume pattern G as the irregular pattern because the parts are “different”. In this case, the term “different” implies that the parts are not reflecting each other as the other regular patterns. Therefore, he thinks that the pattern has no regularity. This pattern G can be a suitable pattern for introducing rotational symmetry to the students because it has rotational symmetry but not line symmetry. Therefore, this pattern will be pointed out in the class discussion of the next cycle.

Then, based on the adjustment of the first cycle, the question after sorting the patterns into two rooms is revised. It is intended to make the students give a proper mathematical reason of sorting the patterns into two rooms. The following figure 5.29 is the example of students’ answers in giving their reason of sorting the patterns.


Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1.
Karena yang Ruang 1 tempat batik yang teratur / Agar tidak tercampur dengan batik lain dan agar mudah mencarinya.

Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2.
Karena yang Ruang 2 tempat batik yang tidak teratur / Agar tidak tercampur dengan batik lain dan agar mudah mencarinya.

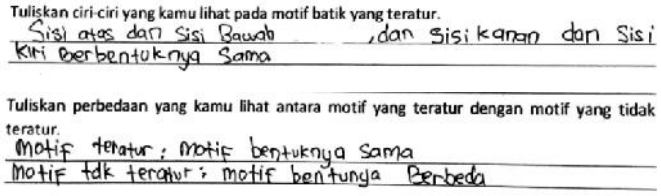
Figure 5.29. The Example of Students’ Answers in Reasoning

As can be seen in figure 5.29, the students' reason in sorting the patterns into two rooms because they do not want the regular batik patterns get mixed with the irregular patterns. It shows that the revised question still cannot make the students give a proper mathematical reason of sorting the patterns into two rooms. Hence, the questions need to be revised into "*Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur*." and "*Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur*.". It is expected that the students will give a proper mathematical reason of sorting the patterns into two rooms instead of giving technical reason.

The third cycle

In line with the HLT, the students are able to differentiate the patterns that have regularity (rotational symmetry) and the patterns that have no regularity. Related to pattern G, the students firstly assume it as a regular pattern because it consists of repeating patterns. When the teacher asks the students to determine whether the pattern G has line symmetry, they get confused about the pattern as it consists of repeating patterns but does not have line symmetry. Then, the teacher uses students' confusion to lead them to the next activity which is investigating the special characteristics of the regular patterns by using transparent batik cards and a pin. After doing the activity, the students realize that pattern G shows another kind of symmetry which is rotational symmetry.

Then, based on the adjustment of the second cycle, the question after sorting the patterns into two rooms is revised. It is intended to make the students give a proper mathematical reason of sorting the patterns into two rooms. The following figure 5.30 is the example of students' answers in giving their reason of sorting the patterns.


Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.
Sisi atas dan Sisi Bawah ,dan Sisi kanan dan Sisi kiri berbentuknya Sama

Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.
Motif teratur : motif bentuknya sama
Motif tdk teratur : motif bentunya Berbeda

Figure 5.30. The Example of Students' Answers in Reasoning

As can be seen in the figure 5.30, the students show their perspective of regularity. They refer the regularity with the concept of line symmetry. It is in line with the conjecture of students' answers in the HLT in which the students identify the regularity in the pattern by using their perspective of line symmetry.

Conclusion

By considering the analysis result of the first activity in the three cycles, the students are able to differentiate the batik patterns that have regularity (rotational symmetry) and the patterns that have no regularity. They determine the regular batik patterns by using their perspective of line symmetry. The following table 5.5 shows the refinement of the activity throughout the three cycles.

5.5. The Refinement of the First Activity

	Student worksheet	Teacher guide
The first cycle	The questions after sorting the patterns: "*Jelaskan alasanmu dalam mengelompokkan motif Batik tersebut ke dalam dua ruangan*" The result: The questions tend to give technical reason instead of mathematical reason. Hence, the questions need to be revised	-
The second cycle	The questions after sorting the patterns: "*Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1"* *Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2*" The result: The students do not give a proper mathematical reason of sorting the patterns. Hence, the question needs to be revised again.	Pattern G is not pointed out in the class discussion The result: The students get confused whether the pattern is regular or not because it does not have line symmetry but consists of repeating pattern. Hence, the pattern needs to be pointed out in the third cycle.
The third cycle	The questions after sorting the patterns: "*Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur*." "*Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur*." The result: The students show their perception of regularity	Pattern G is pointed out in the class discussion The result: The teacher can use students' confusion to lead them to the next activity which is investigating the speciality of the regular patterns by using transparent batik cards and a pin

b) Second activity

This activity aims at supporting the students to be able to deduce the characteristics of rotational symmetry from the regular batik

patterns by using a pin and the transparent batik cards. In this activity, the students only focus on investigating the regular patterns by using a pin and the transparent batik cards. Based on the HLT, the students are expected to discover the notion of rotational symmetry by doing an exploration of the regular batik pattern by using a pin and transparent batik cards. They are expected to place the transparent batik cards above the corresponding pattern and put the pin in the centre of the pattern. Then, they rotate the transparent batik card and aware that the pattern can fit into itself for several times depend on the order of the rotation.

The first cycle

In line with the HLT, the students put the transparent batik card above the corresponding pattern on the worksheet and put the pin in the centre of the pattern. However, they do not what to do next. Hence, the students are guided to do the rotation toward the pattern and they did it well. Similar with the first activity, the pattern G becomes an interesting discussion among the students because it only has rotational symmetry but not line symmetry. The following is the transcript of the discussion.

Nico	: "The pattern has rotational symmetry"
Andi	: "How about its line symmetry?"
Sandi	: "No, it does not have it"
Researcher	: "So, does the pattern G has a line symmetry*?*"
All students	: "Yes"
Researcher	: "Line symmetry*?*" (*questioning students' answer*)
Sandi	: "Eh wait, it does not have line symmetry"
Researcher	: "How about the rotational symmetry?"

Sandi : “It has rotational symmetry” (*while rotating the pattern G*)
Researcher : “How many the order of rotation? Arya, how many is it?”
Arya : “Four”

The discussion implies that pattern G can make the student revise their perspective toward the regularity. Firstly, the students always refer the regularity with the line symmetry. However, after discussing pattern G, they realize that the regularity can also be found in the patterns which have rotational symmetry. Moreover, the students can determine whether the pattern has rotational symmetry by utilizing a pin and transparent batik cards.

The second cycle

The class observation shows that the students are able to utilize the transparent batik cards and the pin properly. It seems that they have a sense of rotational symmetry in which they put the pin in the centre of the pattern and rotate the transparent batik card. In this activity, the students are supposed to investigate the regular patterns. However, there is one student investigates the irregular patterns. The following transcript shows how the student states his finding.

Teacher : “*Is that just a regular pattern which has a rotational symmetry*?”
Student : “*No, it is not, he patterns which have no regularity also have one rotational symmetry*”

He finds an interesting fact in which the irregular pattern can fit into itself like the regular patterns. He states that the irregular pattern can only fit into itself at once, meanwhile the regular pattern can fit into itself for several times. Then, the teacher responds it by confirming that the irregular pattern only fit into itself at once. However, related to rotational symmetry that is being discussed, the object can be defined as the object with rotational symmetry if the object can be rotated and fit into itself for more than once. Since this student's thinking has not stated in the HLT, it will be stated in the HLT for the next teaching experiment.

The third cycle

The students only use the transparent batik cards and the pin in the first two regular patterns. For the next regular patterns, the students can directly deduce the order of rotation by observing the patterns or count the sides of the object. For example, the students observe the details of pattern K and deduce that it has the order of rotation of two. Then, they count the sides of the pattern F to determine its order of rotation. The following figure 5.31 is the example of students' answer.

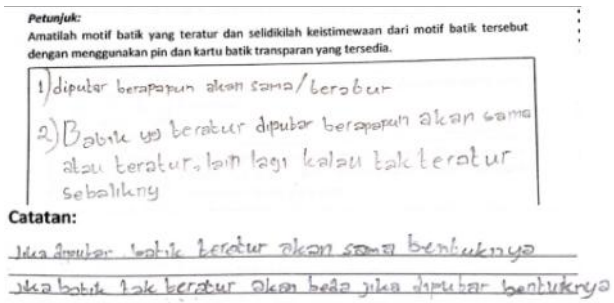


Figure 5.31. The Example of Students' Answers in Reasoning

As can be seen in the figure 5.31, the students can discover the special characteristics of the regular patterns by using the transparent batik cards and the pin. They are aware that rotating the regular pattern will make the pattern fit into itself for several times. Meanwhile, it does not hold for the irregular patterns.

Conclusion

By considering the analysis result of the second activity in the three cycles, the students are able to deduce the characteristics of rotational symmetry from exploring the regular batik patterns by using a pin and the transparent batik cards. They are aware that the regular pattern fit into itself for several times under the rotation. There is no refinement of the activity throughout the three cycles.

c) Third activity

This activity aims at supporting the students to be able to determine the characteristics of rotational symmetry (the order of rotation, the angle of rotation and the point of rotation). The students get five different Batik patterns and determine the characteristics of rotational symmetry of each pattern. In this activity, the students do not get transparent batik cards and the pin. They just rely on their ability to imagine the transition of the object if they do rotation. Based on the HLT, the students are expected to determine the characteristics of rotational symmetry properly.

The first cycle

The students determine the order of rotation of the pattern by rotating the worksheet. Rotating the worksheet seems to help the students to determine whether the object fit into itself. In general, they have no difficulty in determining the order of rotation. Then, related to the angle of rotation, the students have difficulty in determining the angle of rotation even though they have already knew the size of full rotation of angle and the order of rotation of the object. In line with the HLT, the students tend to multiply the angle of full rotation with the order of rotation instead of dividing them.

Last, related to the point of rotation, the pre-test's answers show that the students have an awareness that the point of rotation is always in the centre of the pattern. Therefore, when they are asked to determine the point of rotation of pattern A, they directly identify the centre point of the pattern as the point of rotation. However, for the pattern B which has no obvious centre, firstly the students just estimate the centre point and mark it. As already predicted in the HLT, the teacher responds it by asking how the students estimate the centre point. The students cannot answer it. Hence, they still need a guidance to determine the point of rotation properly. The following transcript is the discussion of determining the point of rotation.

(Situation : the researcher tried to guide the students in determining the point of rotation properly by using a rectangular paper)

Researcher	: (*referring to the rectangular paper*) “What shape is it?”
Nico	: “square”
Arka	: “rectangle”
Researcher	: “Yes, a rectangle. N*ow*, if I rotate it, how many is the order of rotation*?*”
Nico	: “Four”
Researcher	: “Two...two or four*?*”
Arka	: “Two”
Researcher	: “Now look at this. How should we determine the point of rotation”
Sandi	: “Rotate it”
Researcher	: “Now, if I choose this point as the point of rotation. Is it in the middle?” (*referring to the point which almost in the corner of the rectangle*). “Then, how should we determine the point of rotation*?*”
Sandi	: “We fold it and find its middle point”

The transcript shows that the students can emerge with the idea of making the axes of symmetry in determining the point of rotation by getting some guidance. They are aware that the axes of symmetry always divide the shape into two same parts. In other words, it is always located in the centre of the shape. Therefore, they can conclude that the point of rotation is the intersection of the axes of symmetry. This case shows the importance of the teacher’s role as the facilitator and guide for the students to give follow-up question which can support the students to emerge with the intended notion.

The second cycle

In general, the students are able to determine the characteristics of rotational symmetry of the given objects. However, most of the students have difficulty in determining the characteristics of rotational symmetry in the circle as the following figure 5.32.

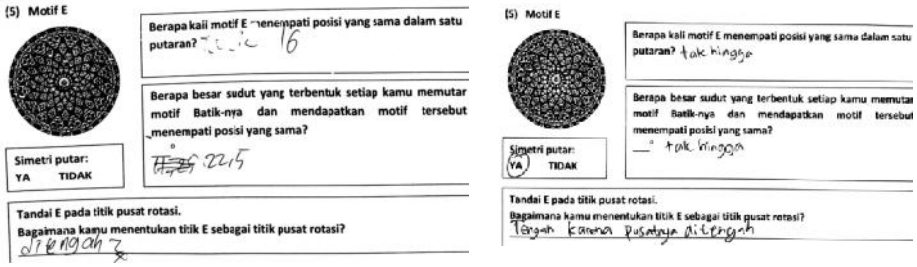


Figure 5.32. The Example of Students' Answers of The Third Problem

In line with the HLT, the students emerge with the answer that the order of rotation of the circle is 16 and the angle of rotation is 22.5^{o} (left answer in figure 5.32). It may occur because the students consider the details of the pattern. Meanwhile, the second answer states that the order of rotation and the angle of rotation are infinite. This answer may occur because the students do not consider the details of the pattern and assume it as blank circle. Unfortunately, these two different answers are not discussed in the class discussion. If it is discussed, then the teacher can use the difference to make the students notice that the order of rotation between circle with batik pattern and blank circle is different. It can also become the bridge for the students to transfer their understanding in determining the characteristics of rotational symmetry of batik patterns into two-dimensional shapes.

The third cycle

In line with the HLT, the students are able to determine the characteristics of rotational symmetry correctly. The following figure 5.33 is the example of students' answers in determining the characteristics of rotational symmetry.

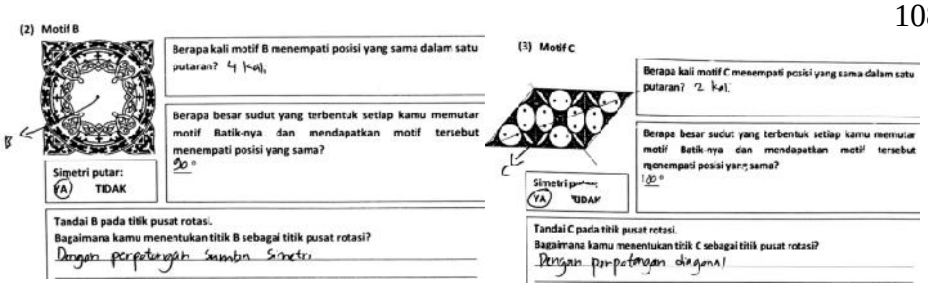


Figure 5.33. The Example of Students' Answers of The Third Problem

As can be seen in figure 5.33, the student can determine whether the object has rotational symmetry, the order of rotation and the angle of rotation correctly. The students can also determine the point of rotation precisely by drawing the axes of symmetry or the diagonals of the object. From this answer, it can be assumed that the students are able to decide whether they have to draw the diagonals or the axes of symmetry to determine the point of rotation.

Related to the characteristics of rotational symmetry in the circle, most of the students answer that the order of rotation of the circle is 16 and the angle of rotation is 22.5^{o}. There is no student who emerge with the answer of infinite as the order of rotation of the circle. Unfortunately, due to the limitation of time, the teacher does not get any opportunity to point out the issue in the class discussion.

Conclusion

By considering the analysis result of the third activity in the three cycles, the students are able to determine the characteristics of rotational symmetry (the order of rotation, the angle of rotation and the point of rotation) of the given objects correctly. There is no refinement of the activity throughout the three cycles.

d) Fourth activity

This activity aims at supporting the students to implement their understanding of rotational symmetry that they already learned from exploring the batik patterns to the two-dimensional shapes. The students get a worksheet which consists of several two-dimensional shapes and determine whether the shapes have rotational symmetry or not. If the shape has rotational symmetry, then they should determine its order of rotation. Then, there is a table in which the students must determine the order of rotation and the angle of rotation of regular two-dimensional shapes correctly. Based on the HLT, the students are expected to determine whether the shapes have rotational symmetry and the order of the rotation correctly. They are also expected to be aware that the angle of rotation can be determined by dividing the angle of full rotation with the order of rotation.

The first cycle

In this cycle, the students do not get the worksheet to implement their understanding of rotational symmetry in two-dimensional shapes. The researcher only emphasizes the relation between the order of rotation and the angle of rotation. The researcher mentions the name of the regular two-dimensional shapes and asks the students to determine the angle of rotation. In general, the students are able to determine the angle of rotation. Since there is no students' written works, the students will get the worksheet in the next cycle so that the researcher can get

sufficient data of students' understanding of line symmetry in two-dimensional shapes

The second cycle

Due to the limitation of time, this activity is not done in the classroom. The teacher makes this activity as the homework for the students. As this activity is not done in the classroom, the collected data is not sufficient to determine whether the students achieve the learning goal. The following figure 5.34 is the example of students' answers.

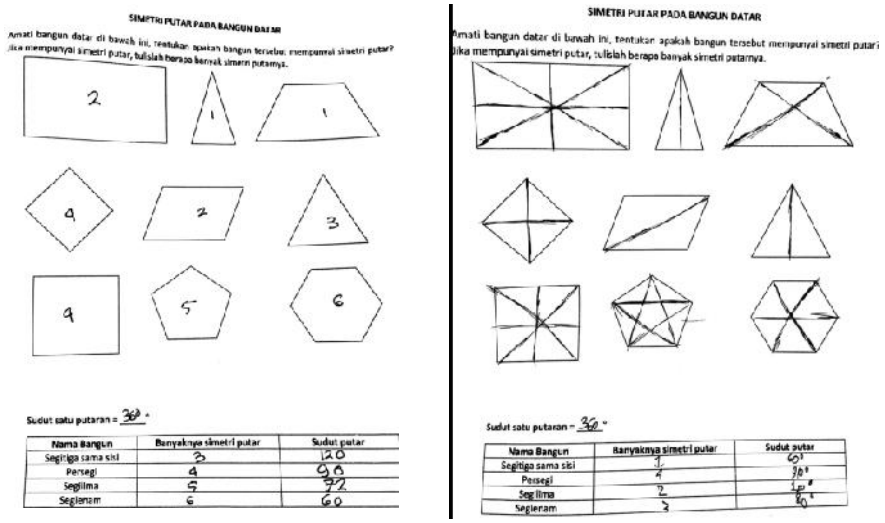


Figure 5.34. The Example of Students' Answers of The Fourth Problem

The left answer in figure 5.34 is the example of students' answer which are able to determine the rotational symmetry of the given shapes and the angle of rotation correctly. Meanwhile, the right answer in figure 5.34 shows how the students mix the concept of line symmetry with rotational symmetry. It may happen for two reasons, the students do not read the instruction thoroughly or the students cannot differentiate the concept of line symmetry with rotational symmetry.

Therefore, this activity should be conducted in classroom so that the teacher can supervise how the students do the task.

The third cycle

In line with the HLT, the students are able to determine the symmetric two-dimensional shapes and draw the axes of symmetry of each shape correctly. The following figure 5.35 is the example of students' answers.

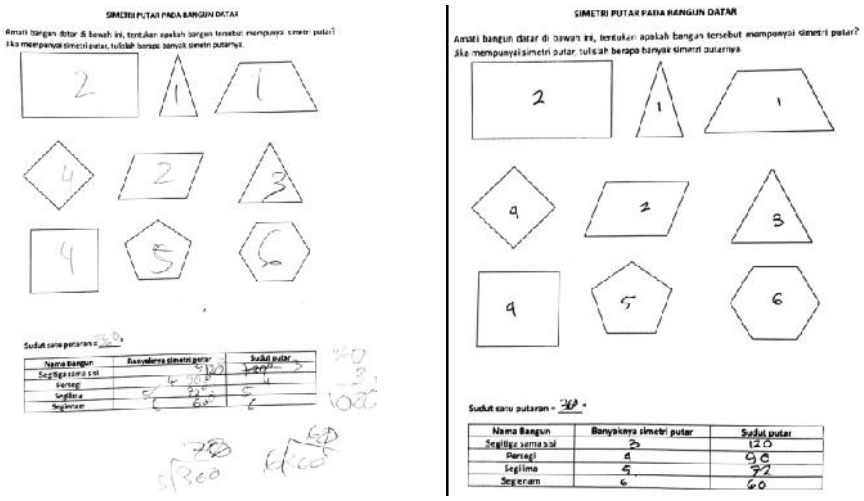


Figure 5.35. The Example of Students' Answers of The Fourth Problem

As can be seen in the figure 5.35 (left answer), the students aware that the angle of rotation can be determined by dividing the angle of full rotation with the order of rotation. They also seem to have no difficulty in determining the order of rotation of the given shapes.

Conclusion

By considering the analysis result of the fourth activity in the three cycles, the students are able to implement their understanding of

rotational symmetry to the two-dimensional shapes. They are aware of the relation between the order of rotation and the angle of rotation. The following table 5.6 shows the refinement of the activity throughout the three cycles.

Table 5.6. The Refinement of the Fourth Activity

	Worksheet	**Teacher guide**
The first cycle	No worksheet The result : no students' written works. Hence, the students should get student worksheet	The teacher gives the task orally The result : no students' written works, hence the students should get student worksheet
The second cycle	The students get the worksheet The result : there is students' written works. Hence, there is no refinement in the third cycle	The activity is not done in the classroom The result : the teacher does not know how the students do the task, hence the activity should be done in the classroom in the next xycle
The third cycle	-	The activity is done in the classroom The result : sufficient data (students' written works & classroom observation)

3. Meeting 3

The learning goal of this meeting is supporting the students to apply their understanding of line and rotational symmetry. It consists of three activities in which each of them has sub-learning goal. The activities are described as follows.

a) First activity

This activity aims at supporting the students to be able to make the asymmetric batik pattern into the symmetric ones. In this activity, the

students get an asymmetric batik pattern (left figure 5.36), then they should restructure the pattern into symmetric ones. Based on the HLT, the students are expected to cut the pattern into several parts and paste them such that the patterns become symmetric.

The first cycle

As already conjectured in the HLT, the students cut the patterns into several parts and arranged them together as can be seen in the right figure 5.36.

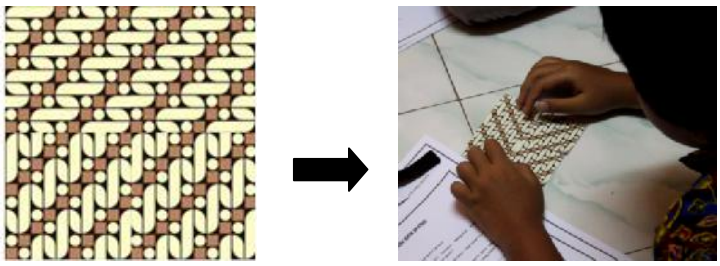


Figure 5.36. Students' Activity in Arranging the Asymmetric Pattern

It implies that the students are able to make the asymmetric pattern into symmetric ones. They can apply their understanding of line symmetry into this activity

The second cycle

The following figure 5.37 is the example of students' answers in restructuring the asymmetric pattern into symmetric ones.

Figure 5.37. The Example of Students' Answers of The First Problem

As can be seen in the figure 5.37, the students emerge with various solutions in making the asymmetric pattern into symmetric ones. However, one student has an answer as this following figure 5.38.

Figure 5.38. The Example of Students' Answers of The First Problem

The pattern in figure 5.38 is still an asymmetric pattern. However, if it is divided into two parts which are the left side and the right side, then each side will become a symmetric pattern. It seems that the students do not understand the instruction thoroughly and assume that the whole pattern does not have to be symmetric. Hence, the teacher needs to ensure that all students understand the instruction.

The third cycle

In line with the expected answer in the HLT, the students are able to restructure the given asymmetric pattern into symmetric ones. The students' answer are similar with the students' answers in the second cycle (figure 5.37).

Conclusion

By considering the analysis result of the first activity in the three cycles, the students are able to make the asymmetric batik pattern into the symmetric ones. They cut the pattern into several parts and paste

them such that the patterns become symmetric. There is no refinement of the activity throughout the three cycles

b) Second activity

This activity aims at supporting the students to be able to complete the pattern by considering the given axes of symmetry. In this activity, the students get two incomplete patterns. Then, they must draw the remaining patterns such that the whole patterns become symmetric. Based on the HLT, the students are expected to complete the pattern by considering the unit pattern and the axes of symmetry.

The first cycle

The students tend to imitate the pattern instead of drawing it by considering the axes of symmetry. The following figure 5.39 is the example of students' answers in completing the pattern.

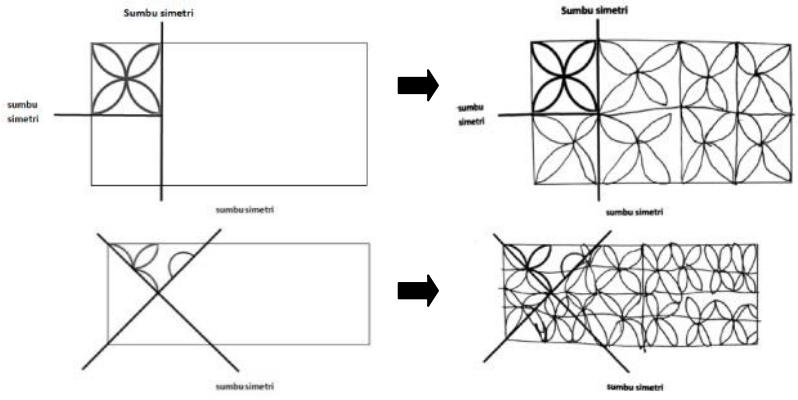


Figure 5.39. The Example of Students' Answer of Completing Patterns

As can be seen in the figure 5.39, the students complete the patterns without considering the axes of symmetry. In the first pattern, the students can complete the pattern easily. However, they tend to imitate the unit pattern. Hence, the teacher of the next teaching experiment

should emphasize the importance of the axes of symmetry in doing this task. The teacher can review the definition of line symmetry and its characteristics. Then, the second pattern seems too difficult for the students. Hence, the unit pattern will be revised as the following figure 5.40. The unit pattern will be different from the first pattern in order to minimize the students' possibility to draw the same pattern as the first ones.

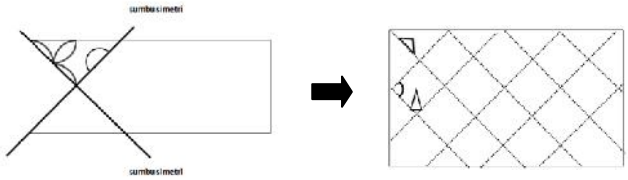


Figure 5.40. The Revision of The Second Task's Pattern

Then, to make the students easier and faster in doing the tasks on this second activity, all of the axes of symmetry to complete the pattern will be given directly. The students do not have to draw the required axes of symmetry.

The second cycle

Based on the adjustment of the first cycle, the following figure 5.41 is the example of students' answer in completing the patterns.

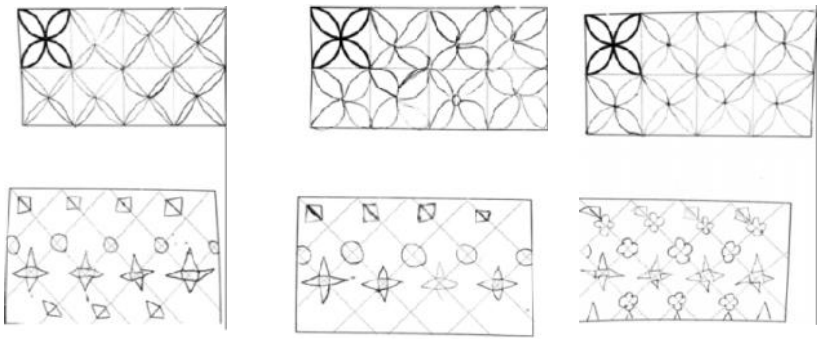


Figure 5.41. The Example of Students' Answers of the Second Problem

As can be seen in figure 5.41, the first pattern can be easily completed by the students as the unit pattern is not complex enough. However, it is still hard to analyze whether the students complete the pattern by considering the axes of symmetry or not. Therefore, the first unit pattern will be revised (figure 5.42). The students also seem to find the second pattern more difficult than the previous pattern in the first cycle. They cannot complete the pattern correctly even though the pattern consists of familiar shapes. Hence, the pattern of the second task will be revised as the following figure 5.42.

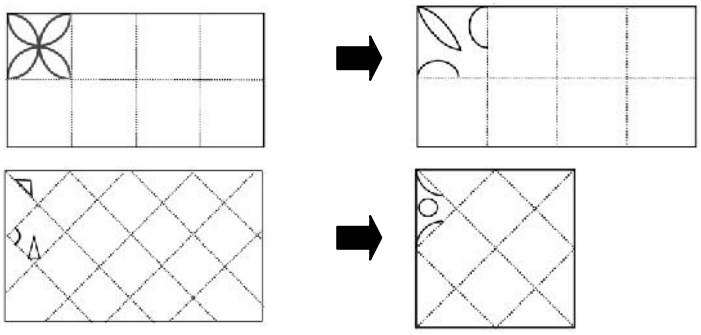


Figure 5.42. The Revision of The Unit Pattern

The third cycle

Based on the adjustment of the second cycle, the following figure 5.43 is the example of students' answers in completing the patterns.

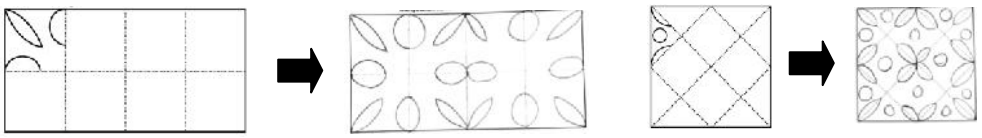


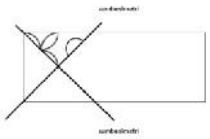
Figure 5.43. The Example of Students' Answers in Completing the Pattern

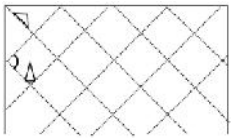
As can be seen in the figure 5.43, the students are able to complete the patterns by considering the given axes of symmetry. It is in line with the expected answer in the HLT. The chosen unit patterns can foster the students to draw the patterns by considering the axes of symmetry. It minimizes the possibility of the students to imitate the unit patterns.

Conclusion

By considering the analysis result of the second activity in the three cycles, the students are able to complete the pattern by considering the given axes of symmetry. The following table 5.7 shows the refinement of the activity throughout the three cycles.

Table 5.7. The Refinement of the Second Activity

	Student worksheet	**Teacher guide**
The first cycle	The pattern for the second task  The result: the students tend to imitate the pattern, hence the pattern needs to be revised	The teacher does not emphasize the importance of the axes of symmetry in doing the task. The result: the students cannot complete the pattern correctly. Hence, the teacher needs to emphasize the role of axes of symmetry.
The second cycle	The pattern of the first's task  The result: the students tend to imitate the pattern, hence the pattern needs to be revised again	The teacher emphasizes the importance of the axes of symmetry in doing the task by reviewing its characteristics. The result: the students still cannot complete the pattern correctly. Hence, the teacher needs to demonstrate how the axes of symmetry works in symmetric objects

	The pattern for the second task  The result: the students cannot complete the pattern correctly. hence the pattern needs to be revised again	
The third cycle	The pattern of the first's task  The result: the students complete the pattern by considering the given axes of symmetry	The teacher reviews the characteristics of the axes of symmetry by demonstrating the correct way of completing the patterns The result: the students can complete the patterns correctly.
	The pattern of the second's task  The result: the students complete the pattern by considering the given axes of symmetry	

c) Third activity

This activity aims at supporting the students to be able to create their own batik design by using provided batik units. In this activity, the students should create their own batik design by using the provided batik units and considering the given angle of rotation. Based on the HLT, the students are expected to draw their own pattern by using the provided batik units and considering the given angle of rotation.

The first cycle

This activity requires the students' ability in drawing the angle by using protractor. However, the classroom observation shows that the students have insufficient knowledge of drawing the angle. Hence, they are firstly guided to draw the angle in one full rotation and imitate the provided pattern unit by considering the angle. This following figure 5.44 is the example of students' answers in creating their batik patterns.

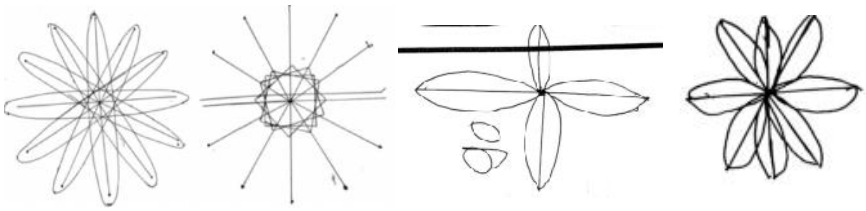


Figure 5.44. The Example of Students' Answer in Creating Patterns

The two left figures in figure 5.44 are the expected answer of this activity. It shows that the students are able to create their own pattern by considering the size of the angle and utilizing the provided unit pattern. Even though the students are told to make the point of rotation in any point, they tend to choose the middle point of the pattern as the point of rotation. It may happen because in the previous lesson they learned that the point of doing rotational is always in the centre. Meanwhile, the two right figures in figure 5.44 reveal the fact that the students do not read the instruction the worksheet thoroughly as they only draw the pattern by considering the required angle without using the provided unit pattern. Therefore, the teacher of the next teaching

experiment should make the students use the provided batik unit pattern.

The second cycle

The following figure 5.45 is the example of patterns which are created by the students.

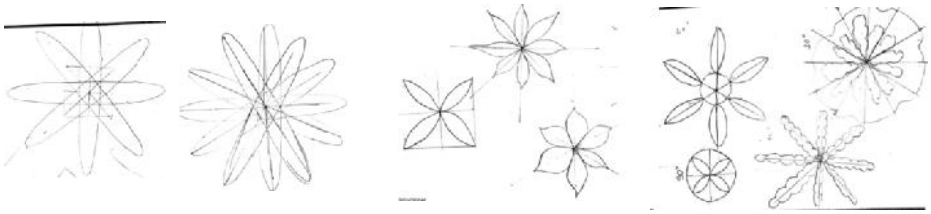


Figure 5.45. The Example of Students' Answer in Creating Patterns

The two left figures in figure 5.45 show the expected answers of the activity. In line with the HLT, the students use the given unit pattern and consider the given angle of rotation in creating their patterns. Meanwhile, the right parts of the figure 5.45 show the pattern which are created by the students without using the given unit pattern. However, if it is observed thoroughly, the patterns are created by considering the given angle of rotation. It seems that the students do not use the given unit patterns, but they create the pattern by considering the given angle. Even their patterns are symmetric, but the students are asked to use the given unit patterns. Therefore, the teacher must ensure that all the students use the given patterns in creating their own patterns.

The third cycle

The students tend to use the unit pattern which looks like oval instead of using the square ones. Hence, most of the students have the same result as the following figure 5.46.

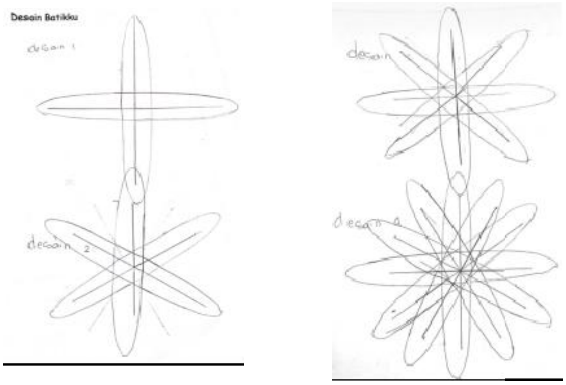


Figure 5.46. The Example of Students' Answer in Creating Patterns

In line with the expected answer in the HLT, the students are able to create their own pattern by using the provided unit pattern and considering the given angle of rotation.

Conclusion

By considering the analysis result of the third activity in the three cycles, the students are able to create the pattern by using the provided unit pattern and considering the given angle of rotation. There is no refinement of the activity throughout the three cycles.

C. The Recent Knowledge of The Students toward The Concept of Symmetry after The Teaching Experiment

The post-test is conducted after finishing the preliminary teaching experiment. It aims at knowing their recent knowledge of symmetry after getting involved in the teaching experiment instead of comparing their prior

knowledge with the recent ones. The following is the description of the students' recent knowledge based on the post-test's result in the three cycles.

1. The students think a parallelogram as an asymmetric object.

   The students are asked to determine the symmetric and asymmetric objects from the objects on figure 5.47. The following figure 5.47 is the example of students' answer.

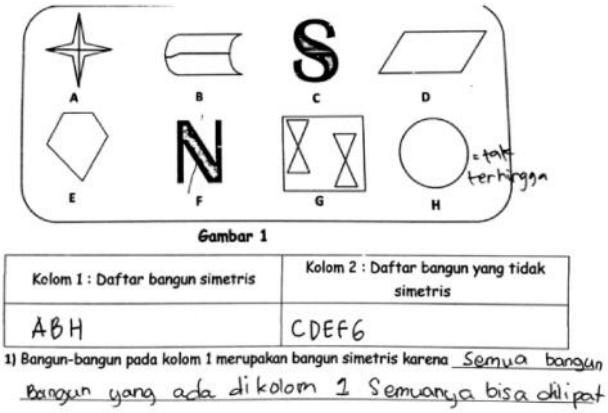


Figure 5.47. The Example of Students' Answer in Identifying Symmetric Objects

Most of the students in the three cycles are able to determine the symmetric objects in figure 5.47. Previously, the students often assume a parallelogram as a symmetric object, however their post-test's answers show that they change their perception and classify it as an asymmetric object. It implies that the students' understanding of the concept of line symmetry has developed. They do not perceive the line symmetry as the symmetry in which the object consists of two identical parts anymore. In other words, the teaching experiment can support the students to develop their understanding of the concept of symmetry.

2. The students are able to determine the axes of symmetry correctly

   The students are asked to draw the axes of symmetry of the symmetric objects and give the reason of determining the object as the symmetric objects. As can be seen in the figure 5.47, the students are able to draw the axes of symmetry of each shape. They even aware that a circle has infinite axes of symmetry. Then, they reason that the objects are symmetric because if it is folded, the two parts will match perfectly.

3. The students are aware of the difference between the axes of symmetry and diagonal

   The students are asked to determine whether the diagonal of the parallelogram is its axes of symmetry. The following figure 5.48 is the example of students' answers in giving their reasons.

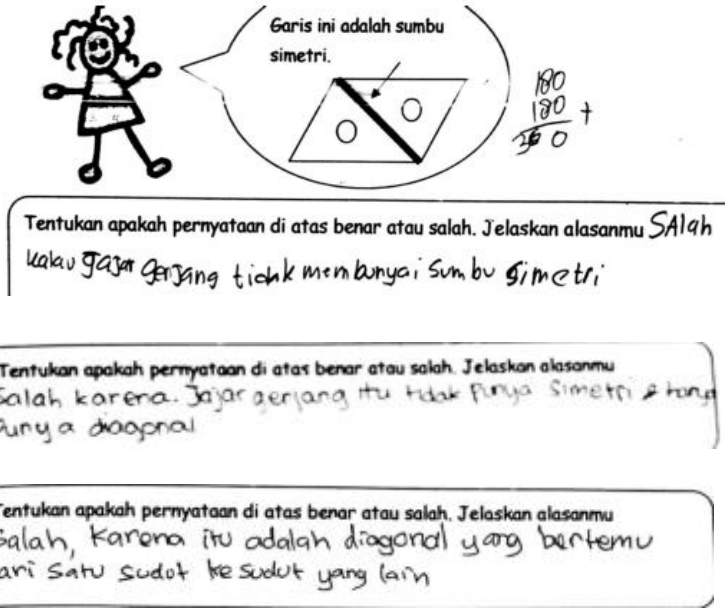


Figure 5.48. The Example of Students' Answer of The Post-test's Third Problem

As can be seen in the figure 5.48, the students are able to determine that the diagonal of a parallelogram is not its axes of symmetry. They are aware that a parallelogram has no axes of symmetry.

4. The students can make an asymmetric objects becomes symmetric.

The students are asked to determine whether the given object is symmetric or not. If the object is asymmetric, then they should make it becomes symmetric. The following figure 5.49 is the example of students' answers in making the asymmetric objects into the symmetric ones.

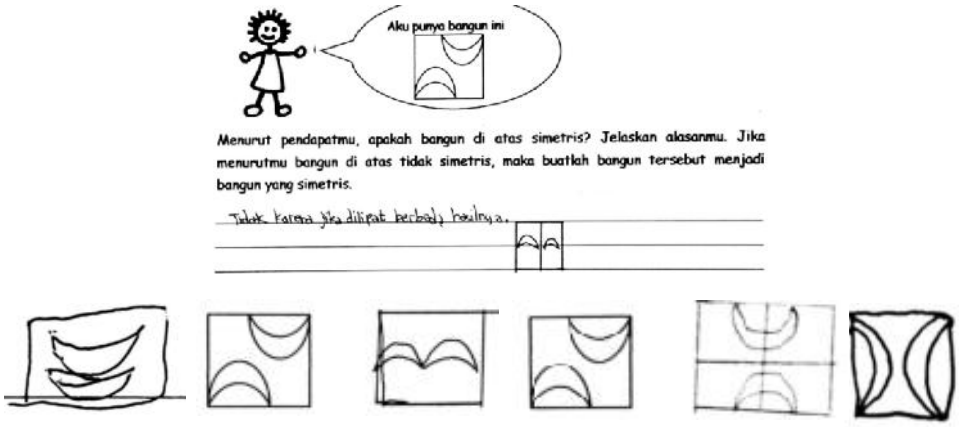


Figure 5.49. The Example of Students' Answer of The Post-test's Fourth Problem

The students in the three cycles seem to have no difficulty in rearranging the inside objects become symmetric.

5. The students are able to determine the basic characteristics of rotational symmetry

The students are asked to determine the characteristics of rotational symmetry of a parallelogram. The following figure 5.50 shows that the students are able to determine the order of rotation. The students can

determine that a parallelogram does not have axes of symmetry but the order rotation of two. They can also determine the angle of rotation which is 180^{o}. However, only several students can determine that the point of rotation of a parallelogram is in its intersection of the diagonals. When the students are interviewed, they state that they firstly made the diagonals because they want to ensure that they exactly choose the centre point of the parallelogram. Meanwhile, most of the students only state that the point of rotation of a parallelogram is in the centre of the shape. Those students' answers can be seen in the following figure 5.50.

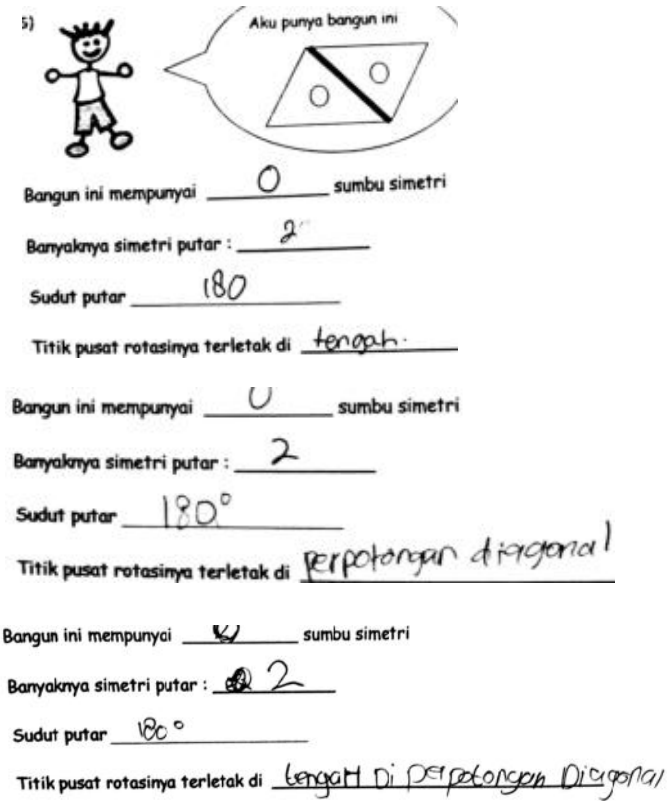


Figure 5.50. The Example of Students ' Answers of the Post-Test's Fifth Problem

D. Validity and Reliability

The validity and reliability of the study are obtained by doing data triangulation and having traceability and inter-subjectivity. First, related to the validity, the researcher triangulated the collected data in the process of analyzing. As the collected data are achieved by various method such as classroom observation, students' written works and students' interview, the researcher compared the data from one method to the other ones and analyze whether they are in line or not. Thereafter, the researcher discussed the possible reason of the data which are inline or not and resulted in theories.

Second, related to the traceability and inter-subjectivity of this study, the readers are provided with the details of the methodology of the study which can be found in chapter 3 and the process of learning during the teaching experiments in chapter 5. Hence, the reader can easily track or reconstruct the study. Besides, the researcher discussed the collected data during the data analysis with the supervisors and colleagues to minimize the level of subjectivity.

CHAPTER VI
CONCLUSION AND DISCUSSION

This chapter consists of two sections which are conclusion and discussion. The conclusion will summarize the answer of the research question and describe the local instruction theory in learning the concept of symmetry through batik exploration. The discussion will discuss the important issues during the study and the recommendation for further educational research in the concept of symmetry through batik exploration based on the remarks of this study.

A. Conclusion

The present study aims at contributing a local instruction theory that can promote students' understanding of the concept of symmetry by exploring the characteristics of batik. Hence, the research question of this study is: "*How can batik, Indonesian traditional patterns, promote students' understanding of the concept of symmetry?"*. The teaching experiment and the retrospective analysis show that the designed instructional activities can promote students' understanding of the concept of symmetry. Therefore, the research question is answered by summarizing the learning activities and considering the important remarks that have been learned from the study as follow.

1. *Sorting the batik fabrics based on the regularity of the patterns (line symmetry)*

   The students get twelve batik fabrics and they have to sort them based on the regularity of the patterns. When the students do the sorting

process, they are expected to sort them into two types, the regular and the irregular patterns. In determining the regular patterns, they are tested whether they use their sense of line symmetry or not. In general, the students show two strategies in sorting the fabrics based on their perspective of regularity. Some students refer regularity to the basic concept of line symmetry in which the right and left parts of the patterns should be identical. As a result, they can determine the regular patterns correctly. Then, the other students refer regularity to the repeating motif of the patterns. Hence, they include the patterns which consist of repeating motif but do not have line symmetry as the regular patterns. These different answers can be managed by having class discussion as the students who use their sense of line symmetry and determine the regular patterns correctly can share their perspective of regularity properly. In general, the students are able to sort the batik fabrics based on the regularity of the pattern correctly and refer the regularity to line symmetry. Hence, it can be concluded that this activity can be a starting point to support the students learn the basic notion of line symmetry by observing the batik patterns.

2. *Exploring the regular batik patterns by using a mirror*

After having the same answer of the regular patterns, the students explore the patterns by using a mirror in order to discover their special characteristics. Basically, the students have found one of the characteristic in the previous activity as they stated that regular patterns

consist of two identical parts. However, as already described in the theoretical background, the students often think that line symmetry is always about two identical parts without considering that the two parts should be each other's mirror image. Therefore, this activity is intended to revise their thought. When the students put the mirror in the centre of the patterns along the axes of symmetry, they see that the mirror reflection shows the similar pattern as the pattern on the back of the mirror. Consequently, they are aware that the two parts of the regular patterns are not only identical but also become each other's mirror images. In addition, the students notice that the mirror positions refer to the axes of symmetry. Therefore, it can be concluded that this activity can support the students to develop their understanding of line symmetry.

3. *Differentiating the diagonals and the axes of symmetry of the given patterns*

   The students are asked to draw the diagonals and the axes of symmetry of the given patterns and determine whether the diagonals always refer to the axes of symmetry. In this activity, the students are stimulated to be aware of the differences between the diagonals and the axes of symmetry by drawing them separately. Then, they will realize that the diagonal of the object does not always refer to its axes of symmetry. In this way, students' misunderstanding that they often refer the diagonal of the object as the axes of symmetry can be avoided.

Therefore, it can be concluded that drawing the diagonals and the axes of symmetry of the objects separately can support the students to be aware of the differences between them.

4. *Determining the characteristics of line symmetry of two-dimensional shapes*

   If the previous activities always involve the shapes with batik details, this activity only involves two-dimensional shapes without batik details. Therefore, this activity can be a test for the students whether they are able to implement their understanding of line symmetry which they have learned from exploring the batik patterns or not. In this activity, the students get several two-dimensional shapes and they need to determine whether the shapes have line symmetry and draw the axes of symmetry. The collected data shows that the students are able to determine whether the shapes have line symmetry and draw the axes of symmetry. Hence, it can be concluded that asking the students to determine the axes of symmetry of two-dimensional shapes can be a proper way to support the students to implement their understanding of line symmetry.

5. *Sorting the batik fabrics based on the regularity of the patterns (rotational symmetry)*

   This activity is intended to support the students to discover the notion of rotational symmetry. Similar with the previous activity, the students are asked to sort the given batik patterns based on the

regularity of the patterns. As they have learned the concept of line symmetry in the previous activities, they refer the regularity to line symmetry. Consequently, they get confused in classifying a particular pattern which has rotational symmetry but not line symmetry. In dealing with this issue, the teacher's guidance is needed to engage the students to investigate the particular pattern in the following activity and determine its regularity. This particular pattern also can be used to introduce the notion of rotational symmetry to the students. Hence, it can be concluded that such a discussion is needed to support the students to be able to sort the given batik fabrics based on the regularity of the patterns.

6. *Exploring the regular batik patterns by using a pin and transparent batik cards*

   The students are asked to discover the special characteristics of the regular patterns by using a pin and transparent batik cards. The special characteristics will be discovered if the students place the transparent batik card above the corresponding pattern and put the pin in the centre of the pattern. By rotating the transparent batik cards with the pin as the point of rotation, the students notice that the pattern can fit into itself for several times depend on the details of the pattern. Interestingly, the students are able to determine the number of the pattern fits into itself by observing the details of pattern without rotating the transparent batik

cards. It implies that the activity support the students to discover the notion of the order of rotation.

7. *Determining the characteristics of rotational symmetry of the given objects*

   The students get several objects and determine their characteristics of rotational symmetry such as the order of rotation, the point of rotation and the angle of rotation. As the students have not known those mathematical terms, they are guided with the relevant questions which can lead them to determine their properties. Even though the collected data show that the students are not able to determine the properties completely correct, but they show their basic understanding of rotational symmetry. For example, most of the students determine the point of rotation by estimating the centre of the pattern without drawing the intersection of the diagonals or the axes of symmetry. It implies that the students are aware that the point of rotation is always in the centre of the pattern. They just have not notice that it is on the intersection of the diagonals or the axes of symmetry. However, after being asked to determine the exact position of the point of rotation and ensure that it is in the centre of the pattern, they can deduce that they need to draw the axes of symmetry or the diagonals of the pattern. It implies that the students still need guidance from the teacher to develop their basic understanding of rotational symmetry.

8. *Determining the characteristics of rotational symmetry of two-dimensional shapes*

   Similar with the previous activity in determining the characteristics of line symmetry of two-dimensional shapes, this activity situates the students to implement their understanding of rotational symmetry to two-dimensional shapes. They get several two-dimensional shapes and determine the order of rotation of each shape. Then, they are guided to understand the relation between the angle of rotation and the order of rotation by filling the provided table. In general, the students are able to achieve the learning goal of the activity. Therefore, it can be concluded that asking the students to determine the characteristics of rotational symmetry of two-dimensional shapes can be a good way to implement their understanding of rotational symmetry.

9. *Making an asymmetric pattern becomes symmetric ones*

   The students get an asymmetric pattern and they have to make it becomes symmetric ones. The students directly cut the pattern into several parts and rearrange its structure in order to make it becomes symmetric. They emerge with various new symmetric structures. It implies that the students are able to make an asymmetric becomes symmetric ones.

10. *Completing the symmetric patterns*

    The students get a small part of the symmetric pattern and they have to complete the remaining part by considering the given axes of

symmetry. They are provided with two types of axes of symmetry. The first axes of symmetry are vertical and horizontal, then the second ones are oblique. For the vertical and horizontal axes of symmetry, the students tend to imitate the small part of the pattern instead of drawing it by considering the axes of symmetry. Then, for the oblique axes of symmetry, the students tend to treat it as vertical/horizontal axes of symmetry. In dealing with these students' mistakes, the teacher should ensure that the students understand the instruction and the meaning of considering the axes of symmetry in completing the pattern. It can be done by firstly demonstrating the proper way of completing the symmetric pattern by considering the axes of symmetry. Hence, it can be concluded that teacher's guidance is needed to support the students to complete the symmetric patterns properly.

11. *Drawing the symmetric patterns*

    In this activity, the students are asked to create their own symmetric patterns by using the unit batik pattern, a ruler and a protractor and considering the given angle of rotation. As the students have to consider the given angle in creating their symmetric patterns, they need to firstly draw the angle of rotation in one full angle. Hence, the teacher needs to ensure that the students have an ability to draw the angle by using a protractor because it plays an important role in drawing the required patterns. If the students are able to create the required symmetric pattern, then it means that they can implement their

understanding of rotational symmetry to this activity. The analysis result shows that the students are able to create the pattern by using the provided unit pattern and considering the given angle of rotation. Hence, it can be concluded that this activity supports the students to implement their understanding of rotational symmetry.

By considering the descriptions of the activities, it can be concluded that the learning activities in which the students explore the characteristics of batik patterns can promote students' understanding of the concept of symmetry. As the product of this study, the local instruction theory is presented in the following descriptions.

- Learning Goal 1 : Students Discover the Notion of Line Symmetry

  Assumptions

  The study of Marchis (2009) suggested sorting symmetric patterns as the effective way to teach line symmetry. It is also proven in this study. When the students are asked to sort the symmetric patterns, they emerge with the basic notion line symmetry in which the patterns consist of two identical parts. As this study uses RME as the domain-specific theory, the students are situated to *reinvent* the notion of line symmetry by using a mirror and regular patterns. However, it does not mean that the students reinvent the concept by themselves but it refers to *guided reinvention* (Freudenthal (1991), cited in Gravemeijer and Doorman (1999)). Hence, when the students put the mirror on the edge of the patterns, the teacher is allowed to guide them to put the mirror in

the centre of the patterns along the axes of symmetry. Note that some students are not aware of the notion of line symmetry when doing the activity. Therefore, the teacher must guide the students to see that the mirror reflection shows the half of the pattern which similar with the half pattern in the back of the mirror. In other words, if the half pattern is combined with its reflection then it will look the same as the original pattern.

Envisioned Learning Route and Rationale

Mathematizing *everyday-life* subject matter can be the starting point of learning (Freudenthal (1971, 1973, 1991), cited in Gravemeijer and Doorman (1999)). Therefore, this study employs batik, traditional Indonesia patterns, to be mathematized by students. The characteristic of batik patterns which exhibits the regularity lead the students to discover the notion of line symmetry. Based on Haake (1989), this study refers the regularity to repeating motif of the patterns. Then, the students are situated to investigate the regularity of the patterns by using mirror. Based on the teaching experiments of this study, a mirror can help the students to be aware that the symmetric pattern is not only about two identical parts but the two parts also become each other mirror's image. As a result, students' misunderstanding in which they assume the diagonals of an object as the axes of symmetry can be managed.

The instructional activities

In order to support the students to achieve the intended learning goal, the following are the sequence of the instructional activities.

Sorting the batik fabrics based on the regularity of the pattern	*Investigating the regular patterns by using a mirror*	*Relating the regular patterns with the notion of line symmetry*	*Drawing the diagonals and the axes of symmetry in order to understand the difference between them*	*Determining the characteristics of line symmetry to two dimensional shapes*

❖ Learning Goal 2 : Students Discover the Notion of Rotational Symmetry

Assumptions

As the students learn the concept of line symmetry in the previous activity, the students tend to do this activity with their perspective of line symmetry. Therefore, the pattern which has rotational symmetry but not line symmetry can be seen as an effective way to engage the students to figure out the notion of rotational symmetry. Then, when the students do the investigations, the students may do as the expected strategy but they are not aware of the notion of rotational symmetry. Hence, the teacher must anticipate this possibility by guiding the students to be aware that the regular pattern can fit into itself for more than once and under particular angle.

Envisioned Learning Route and Rationale

Rotational symmetry is a new concept for fifth-grade students. Hence, they need an interest and inclination to learn it (Bagirova, 2012). This

study shows that the students get interested to learn the concept of symmetry by asking them to investigate the regular batik patterns with transparent batik cards and a pin. The activity can challenge them to figure out the meaning behind the investigation. It is in line with Freudenthal's idea (Gravemeijer and Doorman ,1999) who emphasized *mathematics as an activity and not as a ready-made system*. The students are situated to reinvent the concept of rotational symmetry by placing the batik card above the corresponding pattern and putting the pin in the centre of the pattern.

The instructional activities

In order to support the students to achieve the intended learning goal, the following are the sequence of the instructional activities.

Sorting the batik fabrics based on the regularity of the pattern	*Investigating the regular patterns by using transparent batik cards and pin*	*Relating the regular patterns with the notion of rotational symmetry*	*Determining the basic notion of rotational symmetry*	*Determining the characteristics of rotational symmetry to two dimensional shapes*

❖ Learning Goal 3 : Students Apply their Understanding of Line and Rotational Symmetry

Assumptions

Based on the teaching experiment, the students are not aware with the oblique axes of symmetry. To anticipate this possibility, the teacher needs to emphasize the definition of axes of symmetry or demonstrate

how to draw the symmetric figures with the oblique axes of symmetry. Then, drawing the angle becomes the main obstacle for the students to creating the symmetric patterns which show rotational symmetry. Hence, the teacher needs to firstly ensure that the students have ability in drawing angle.

Envisioned Learning Route and Rationale

Based on Taxonomy Bloom, the higher stage of learning after understanding is application. Therefore, in this last activity, the students are asked to apply their understanding of symmetry by making the asymmetric pattern into the symmetric ones. Then, in order to develop students' understanding, this study uses the recommended instruction for learning symmetry in elementary levels by asking the students to complete some figures to become symmetric by considering the axes of symmetry (Aurelie and Valerie, 2012). Last, creating the symmetric patterns which show rotational symmetry becomes the last activity of the learning sequence of this study.

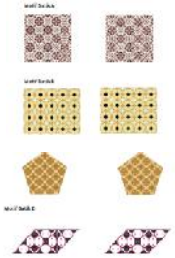
The instructional activities

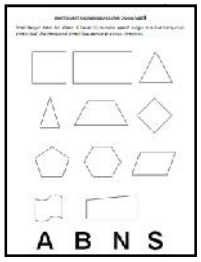
In order to support the students to achieve the intended learning goal, the following are the sequence of the instructional activities.

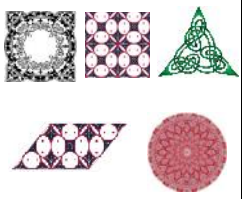
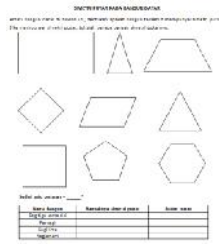
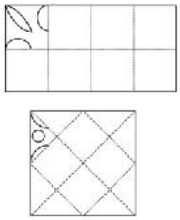
Making the asymmetric pattern into the symmetric ones	*The teacher ensures that the students know how the axes of symmetry works*	*Completing the symmetric patterns by considering the given axes of symmetry*	*The teacher ensures the students how to draw the angles*	*Drawing the patterns which have rotational symmetry by considering the given angle of rotation*

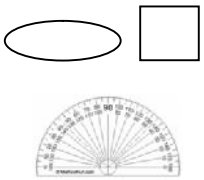
Then, the following table 6.1 gives an overview of the learning tools, the corresponding imagery, the mathematical activity and the topics of mathematical discourse (Gravemeijer, 2004).

Table 6.1 Outline of the local instruction theory

Tool	**Imagery**	**Activity**	**Potential mathematical discourse topics**
A worksheet consists of twelve batik patterns 	The regular patterns signify the notion of line symmetry in real life	Sorting the batik fabrics based on the regularity of the pattern	Referring the concept of regularity to line symmetry
		Resulting in an interest of knowing the special characteristics of the regular batik patterns	
The regular batik patterns and a mirror 	The regular patterns signify the objects with line symmetry The mirror signifies the notion of line symmetry in which the two parts should be each other's mirror image	Exploring the regular patterns by using a mirror: placing the mirror in the centre of the pattern along the axes of symmetry	Recognizing that the two identical parts always become each other's mirror image. Referring the mirror position to the axes of symmetry.
		Resulting in awareness that the concept of line symmetry is not only about two identical parts but also the two parts should also become each other's mirror image.	
The shapes with batik details 	The shapes with batik details signify the two-dimensional shapes.	Drawing the diagonals and the axes of symmetry of the shapes with batik details separately	Being aware of the difference between the diagonals and the axes of symmetry
		Developing students' understanding of the differences between the diagonals and the axes of symmetry	

<table>
<tr><td rowspan="2">A worksheet consists of two dimensional shapes & alphabet
 
</td><td rowspan="2">The two-dimensional shapes signify the geometrical shapes</td><td>Determining the line symmetry of two-dimensional shapes and drawing the axes of symmetry</td><td>Knowing the application of line symmetry in two-dimensional shapes</td></tr>
<tr><td colspan="2">Promoting students' understanding of line symmetry in two-dimensional shapes</td></tr>
<tr><td rowspan="2">A worksheet consists of twelve batik patterns
  </td><td rowspan="2">The regular patterns signify the notion of rotational symmetry in real life</td><td>Sorting the batik fabrics based on the regularity of the pattern</td><td>Referring the concept of regularity to the basic characteristics of rotational symmetry</td></tr>
<tr><td colspan="2">Resulting in an interest of knowing the special characteristics of the regular batik patterns</td></tr>
<tr><td rowspan="2">The regular batik patterns, a pin and transparent batik cards
  </td><td rowspan="2">The regular batik patterns in the worksheet signify the objects with rotational symmetry
The transparent batik cards signify how the rotation works
The pin signify the point of rotation</td><td>Exploring the regular patterns by using a pin and transparent batik cards : placing the pin in the centre of the pattern and doing the rotation with the corresponding transparent batik cards</td><td>Recognizing that the transparent batik card can fit into the corresponding pattern on the worksheet for several times</td></tr>
<tr><td colspan="2">Resulting in an awareness that the concept of rotational symmetry is rotating an object with a centre point as the point of rotation and making the object fits into itself for several times</td></tr>
</table>

<table>
<tr><td rowspan="2">The shapes with batik details
  </td><td rowspan="2">The shapes with batik details signify the two-dimensional shapes.</td><td>Determining the characteristics of rotational symmetry</td><td>Knowing the basic characteristics of rotational symmetry</td></tr>
<tr><td colspan="2">Developing students' understanding of the basic notion of rotational symmetry</td></tr>
<tr><td rowspan="2">A worksheet consists of two dimensional shapes
  </td><td rowspan="2">The two-dimensional shapes signify the geometrical shapes
The table signifies the relation between the order of rotation and the angle of rotation</td><td>Determining the order of rotation of two-dimensional shapes and the relation between the order of rotation and the angle of rotation</td><td>Knowing the application of rotational symmetry in two-dimensional shapes</td></tr>
<tr><td colspan="2">Promoting students' understanding of rotational symmetry in two-dimensional shapes</td></tr>
<tr><td rowspan="2">An asymmetric batik pattern
  </td><td rowspan="2">The asymmetric batik patterns signify the asymmetric object in real life</td><td>Making the asymmetric pattern becomes symmetric ones : cutting the pattern into several parts and restructuring it into the symmetric ones</td><td>Applying the concept of line symmetry to make an asymmetric pattern becomes the symmetric ones</td></tr>
<tr><td colspan="2">Being able to apply the concept of line symmetry to make an asymmetric pattern becomes the symmetric ones</td></tr>
<tr><td rowspan="2">An incomplete pattern and the axes of symmetry
  </td><td rowspan="2">An incomplete pattern signifies how the concept of line symmetry can be used to complete the pattern</td><td>Drawing the remaining part of the patterns by considering the given axes of symmetry</td><td>The role of axes of symmetry in the concept of line symmetry</td></tr>
<tr><td colspan="2">Being able to draw the remaining part of the incomplete pattern by considering the given axes of symmetry</td></tr>
</table>

<table>
<tr><td rowspan="2">The transparent batik unit and a protractor
  </td><td rowspan="2">The transparent batik unit signify the smallest part of symmetric pattern
The protractor signify the tool to draw the angle of rotation</td><td>Drawing the symmetric patterns by using the batik units and considering the required angle of rotation</td><td>The role of angle of rotation</td></tr>
<tr><td colspan="2">Being able to create the symmetric patterns by using the batik unit and considering the required angle of rotation</td></tr>
</table>

B. Discussion

This section will discuss the important issues during the study which are related to the role of the theory, the methodology and the limitation of the research. There are also several recommendations for further educational research in the concept of symmetry through batik exploration based on the remarks of this study.

As this study uses Realistic Mathematics Education (RME) as the domain-specific instruction theory, there are several important issues related to the role of the theory. First, related to the use of RME in this study, this study uses batik patterns as the context of this study (cf. the first RME tenet of the use of context). The batik patterns can lead the students to be aware of the notion of line and rotational symmetry which are embedded on the patterns. The chosen problem in which the students should sort the batik fabrics based on the regularity of the patterns can stimulate the students to emerge with their symmetry sense. They can determine the regular batik patterns as the patterns which consist of two identical parts. Therefore, the

batik patterns should be chosen thoughtfully so that the students are able to differentiate the patterns with symmetry and no symmetry. Second, the students are situated to discover the notion of line and rotational symmetry by exploring the regular batik patterns (cf, the second RME tenet of students' own construction) and having discussion with other students (cf. the third RME tenet of interactivity). For example, when exploring the regular batik patterns by using a mirror, the students can develop their understanding of line symmetry in which it is not only about two identical parts but also two parts which become each other's mirror images. It can be achieved as long as the students are supported by the teacher's guidance. The teacher needs to supervise how the students do the exploration activities and ensure they work in the intended way. Then, related to the students' discussion, the teacher also needs to supervise and ensure the students discuss the mathematical issues of the activity. If the students seem to discuss other topics, then the teacher can pose relevant questions toward the activity and ask them to discuss it. It is also intended to guide the students to formalize their reasoning and develop their ideas to become more general.

Related to the methodology, design research is appropriate for the purpose of this study which aims at educational innovation for improving educational practices. The cyclic process of design research allows us to be aware of the strength and the weakness of the designed activities. Hence, we can improve the quality of the designed instructional activities based on what happened during the teaching experiment. Then, related to the

generality of the result of this study, this study contributes to the local instruction theory in learning the concept of symmetry. Nevertheless, this contribution is only based upon three teaching experiments in one school. Therefore, for other researchers who want to conduct similar study, this local instruction theory have to be adjusted to the condition of the local circumstances (Barab & Kirshner (2001), cited in Bakker (2004)). We also suggest to conduct more teaching experiments in order to obtain a robust instruction theory which can works in different educational practices.

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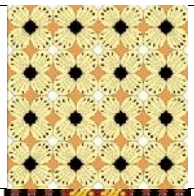
http://www.hansdoddema.com/batik-pattern-kawung-prabu/	
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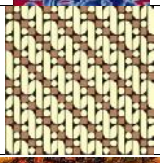
http://1.bp.blogspot.com/_VE5NvpKGb-U/SslPFDeJkdI/AAAAAAAADXY/xVdGOyx5_X0/s400/btk.jpg	
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APPENDIX 1

THE RESEARCH TIMELINE

The Research Timeline

Data Collection		Date	Participants
Classroom observations and teacher's interview		February 21, 2014	The students of class 5B, 5C and the mathematics teacher of Laboratory Elementary School of Unesa
Pre test and small interview for clarifying pre-test's answers		February 24, 2014	A small group of 5 students from class 5D of Laboratory Elementary School of Unesa and the researcher as the teacher
Cycle 1	Activity 1	February 25, 2014	
	Activity 2	February 26, 2014	
	Activity 3	February 27, 2014	
Post test and small interview for clarifying post-test's answers		February 28, 2014	
Pre test and small interview for clarifying pre-test's answers		March 3, 2014	The students of class 5C and the mathematics teacher of Laboratory Elementary School of Unesa
Cycle 2	Activity 1	March 5, 2014	
	Activity 2	March 7, 2014	
	Activity 3	March 10, 2014	
Post test and small interview for clarifying post-test's answers		March 11, 2014	
Pre test and small interview for clarifying pre-test's answers		March 12, 2014	The students of class 5B and the mathematics teacher of Laboratory Elementary School of Unesa
Cycle 3	Activity 1	March 13, 2014	
	Activity 2	March 14, 2014	
	Activity 3	March 18, 2014	
Post test and small interview for clarifying post-test's answers		March 20, 2014	

APPENDIX 2

The Scheme for Classroom Observation

- Classroom environment
  a) The role of the teacher, it concerns about how the teacher conduct the lesson.
     - How does the teacher begin the lesson?
     - Does the teacher state the learning goal of the lesson?
     - How does the teacher present the learning material?
  b) Learning materials that are used by the teacher and students during the learning process.
     - The books that are used by the teacher
     - Does the teacher use worksheet to support the teaching?
     - The others learning material
  c) Students' activity, it concerns about the activity of students during the learning process whether they are just listening to teacher's explanation, doing exercise individually, doing hands-on activities, discussing in a group or other activities.
     - What are the students doing during the lesson?
     - How does the students work individually, work in group?
  d) Interaction between the teacher and students

     It concerns about how the students present their questions or opinions and how the teacher reacts toward their questions or opinions.
     - How does the teacher encourage the students to get involved during the lesson?
     - How does the teacher ask question to the students?
     - How does the teacher react to students' answers, questions or opinions during the lesson?
- Mathematical discussion among students

  If there is a discussion during the learning process, then it will observe about what mathematical ideas that are discussed by the students and how they discuss their ideas.
  - How do the students present their ideas?
  - How do the other students react toward their friends' ideas?

APPENDIX 3

The Scheme for Teacher's Interview

- The structure of the learning process

  It discusses the process of designing the learning sequence of the lesson, whether the teachers just follow the Indonesian curriculum and its lesson plans or they modify the content and the sequence. If the teachers modify the learning sequence, then we can ask about their considerations of doing it.

  - How do you plan the teaching-learning process?
  - Do you design your own instructional activities? If you design your own instructional activities, then what are your considerations in designing?

- Background of the teacher

  It discusses about the teachers' experience in teaching mathematics and their perspective about mathematics education. We can also ask whether they know about PMRI.

  - How long have you become a mathematics teacher?
  - Would you like to share your perspective about teaching mathematics in Indonesia?
  - Have you ever heard about PMRI?
  - If yes, what do you think about PMRI compared to traditional mathematics teaching?

- Students' understanding and difficulties of the topic

  It discusses what students already knew about the concept of symmetry and what difficulties that students often deal during the process of learning symmetry.

  - Regarding the topic of symmetry, what are the knowledge and the skills that students already learned from grade 4?
  - Based on your experience, what are the main difficulties of students in learning symmetry?

- Classroom management

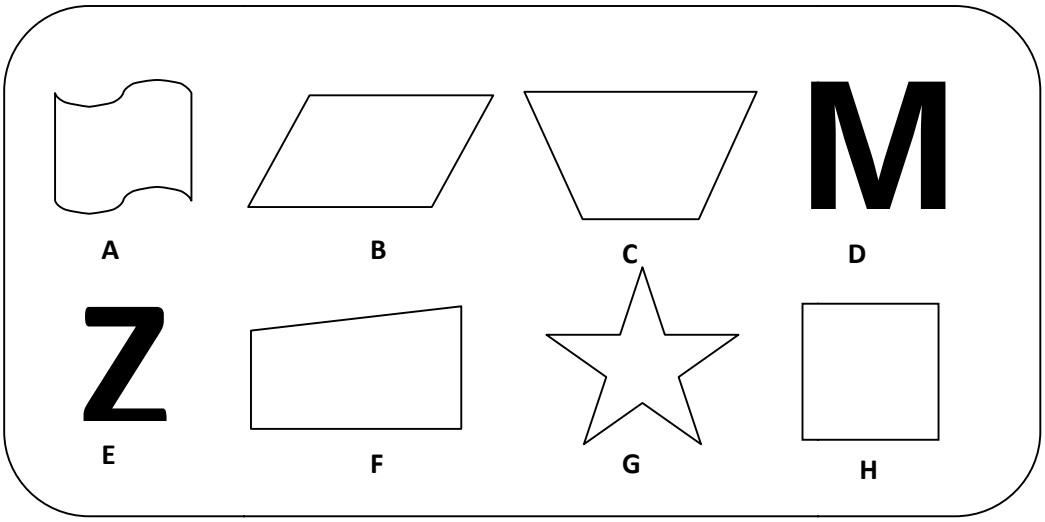
  It discusses about the strategies of the teacher to manage the students to get involved in the learning process.

  - How do you manage the mathematics class?
  - Do you have any difficulties in managing the class?
  - How do you keep the students to get involved in the learning process?
  - How do you make the passive students to get involved in the learning process?

Nama : _____

PRE-TEST

Aku punya kumpulan bangun berikut. Aku ingin memilih bangun yang simetris. Bantu aku ya untuk memilihnya dan menjawab pertanyaan-pertanyaan selanjutnya



Gambar 1

Kolom 1 : Daftar bangun simetris	Kolom 2 : Daftar bangun yang tidak simetris

1) Bangun-bangun pada kolom 1 merupakan bangun simetris karena _____

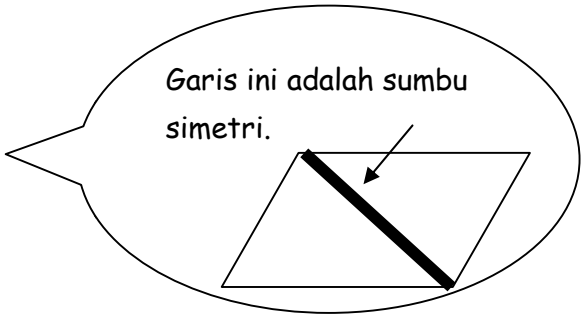
_____

_____

_____

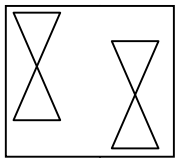
2) Gambarlah sumbu simetri dari bangun-bangun simetris yang sudah kamu daftar di kolom 1 pada *Gambar 1*.

3)



Tentukan apakah pernyataan di atas benar atau salah. Jelaskan alasanmu

4)



Menurut pendapatmu, apakah bangun di atas simetris? _____

Jelaskan alasanmu. _____

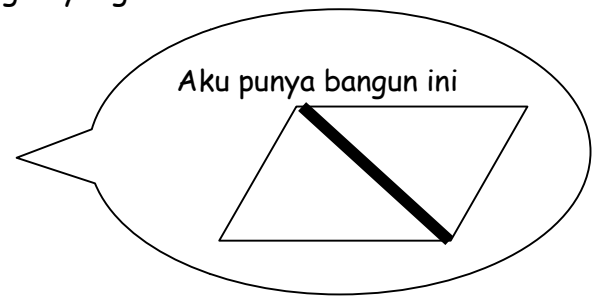
_____

_____

_____

Jika menurutmu bangun di atas tidak simetris, maka gambarlah bangun tersebut menjadi bangun yang simetris. Gambarlah di sebelah bangun tersebut.

5)



Apakah bangun ini mempunyai sumbu simetri?_____

Apakah bangun ini mempunyai simetri putar?_____

Jika bangun ini mempunyai simetri putar, maka jawablah pertanyaan berikut:

Banyaknya simetri putar : _____

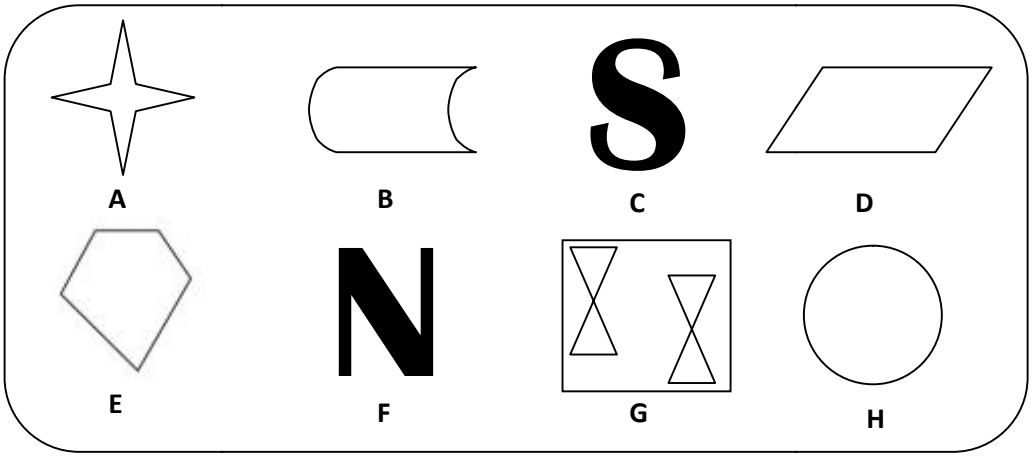
Sudut putarnya : _____^{o}

Titik pusat rotasinya terletak di _____

Name : _____

POST-TEST

Aku punya kumpulan bangun berikut. Aku ingin memilih bangun yang simetris. Bantu aku ya untuk memilihnya dan menjawab pertanyaan-pertanyaan selanjutnya



Gambar 1

Kolom 1 : Daftar bangun simetris	Kolom 2 : Daftar bangun yang tidak simetris

1) Bangun-bangun pada kolom 1 merupakan bangun simetris karena _____

_____

_____

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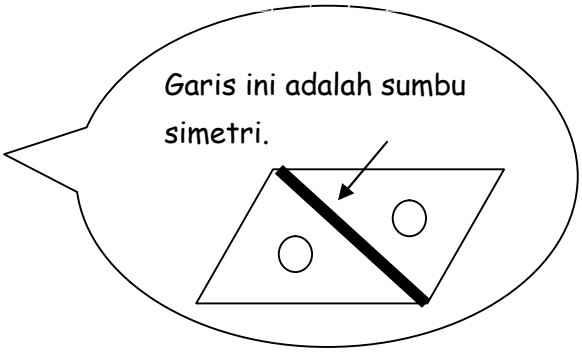
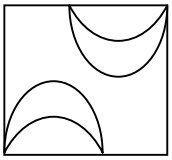
2) Buatlah sumbu simetri dari bangun-bangun simetris yang sudah kamu daftar di kolom 1 pada *Gambar 1*.

3)



Tentukan apakah pernyataan di atas benar atau salah. Jelaskan alasanmu

4)



Menurut pendapatmu, apakah bangun di atas simetris? _____

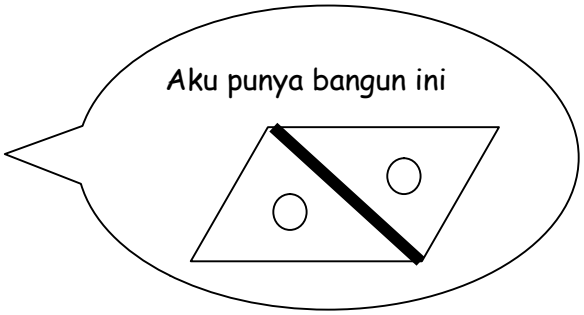
Jelaskan alasanmu. _____

_____

_____

Jika menurutmu bangun di atas tidak simetris, maka buatlah bangun tersebut menjadi bangun yang simetris.

5)



Apakah bangun ini mempunyai sumbu simetri?_____

Apakah bangun ini mempunyai simetri putar?_____

Jika bangun ini mempunyai simetri putar, maka jawablah pertanyaan berikut:

Banyaknya simetri putar : _____

Sudut putarnya : _____^{o}

Titik pusat rotasinya terletak di _____

Nama : _____

Kelompok : _____

Galeri Batik Jawa

Galeri batik di Jogjakarta baru saja menerima paket yang berisi berbagai macam kain batik untuk dipajang di pameran. Agar pamerannya tertata dengan bagus, pegawai galeri berencana untuk memajang kain batik tersebut berdasarkan keteraturan pada motifnya. Akan tetapi galeri tersebut hanya mempunyai dua ruangan (ruang 1 dan ruang 2) untuk memajang kain batik. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan pada motifnya?

Tuliskan hasil pengelompokkan motif Batik ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Ruang I (Batik yang teratur)	**Ruang 2 (Batik yang tidak teratur)**

Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.

_____

_____

_____

_____

Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.

_____

_____

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_____

LATIHAN 1

Petunjuk:

Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan menggunakan cermin yang tersedia.

Catatan:

_____
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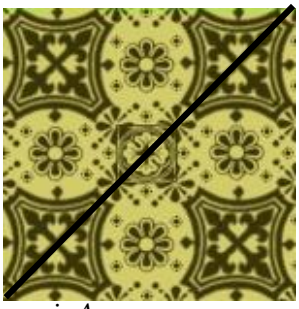
LATIHAN 2



ruas garis A

Bacalah percakapan berikut,

Arga : "*coba perhatikan ruas garis A pada bangun di samping*"

Kiki : "*oh, ruas garis A adalah sumbu simetri pada bangun tersebut*"

Arga : "*bukan, ruas garis A adalah diagonal dari bangun di samping*"

1) Berdasarkan percakapan Arga dan Kiki, pendapat siapakah yang benar? Jelaskan alasanmu.

_____
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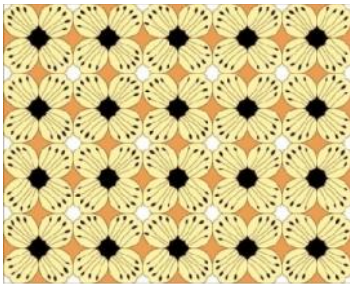
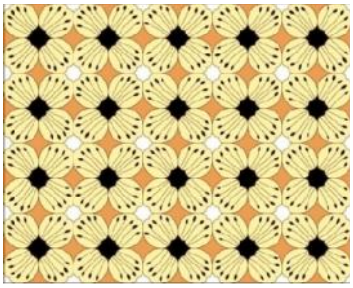
_____

2) Amatilah motif – motif batik berikut. Tentukan apakah motif tersebut mempunyai sumbu simetri. Jika motif tersebut mempunyai sumbu simetri, maka gambarlah sumbu simetri serta diagonal dari setiap motif berikut pada motif batik yang terpisah.

Motif Batik A

Motif Batik B





Motif Batik C

Motif Batik D

3) **Sumbu simetri dari suatu bangun datar merupakan diagonal bangun datar tersebut**. Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

_____
_____
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_____

4) **Diagonal dari suatu bangun datar merupakan sumbu simetri bangun datar tersebut**. Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

_____
_____
_____
_____

A

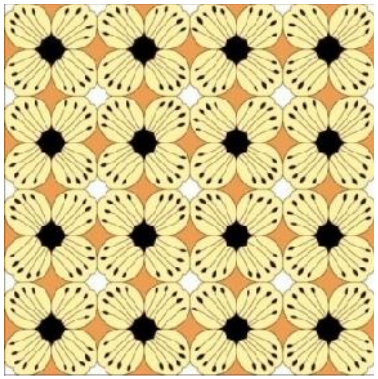
B

C

D

E

F



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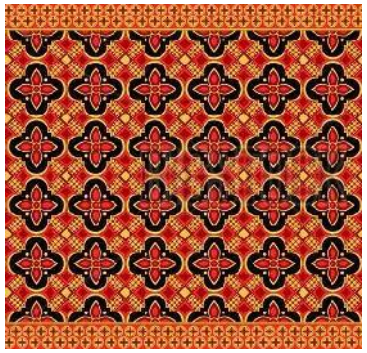


I

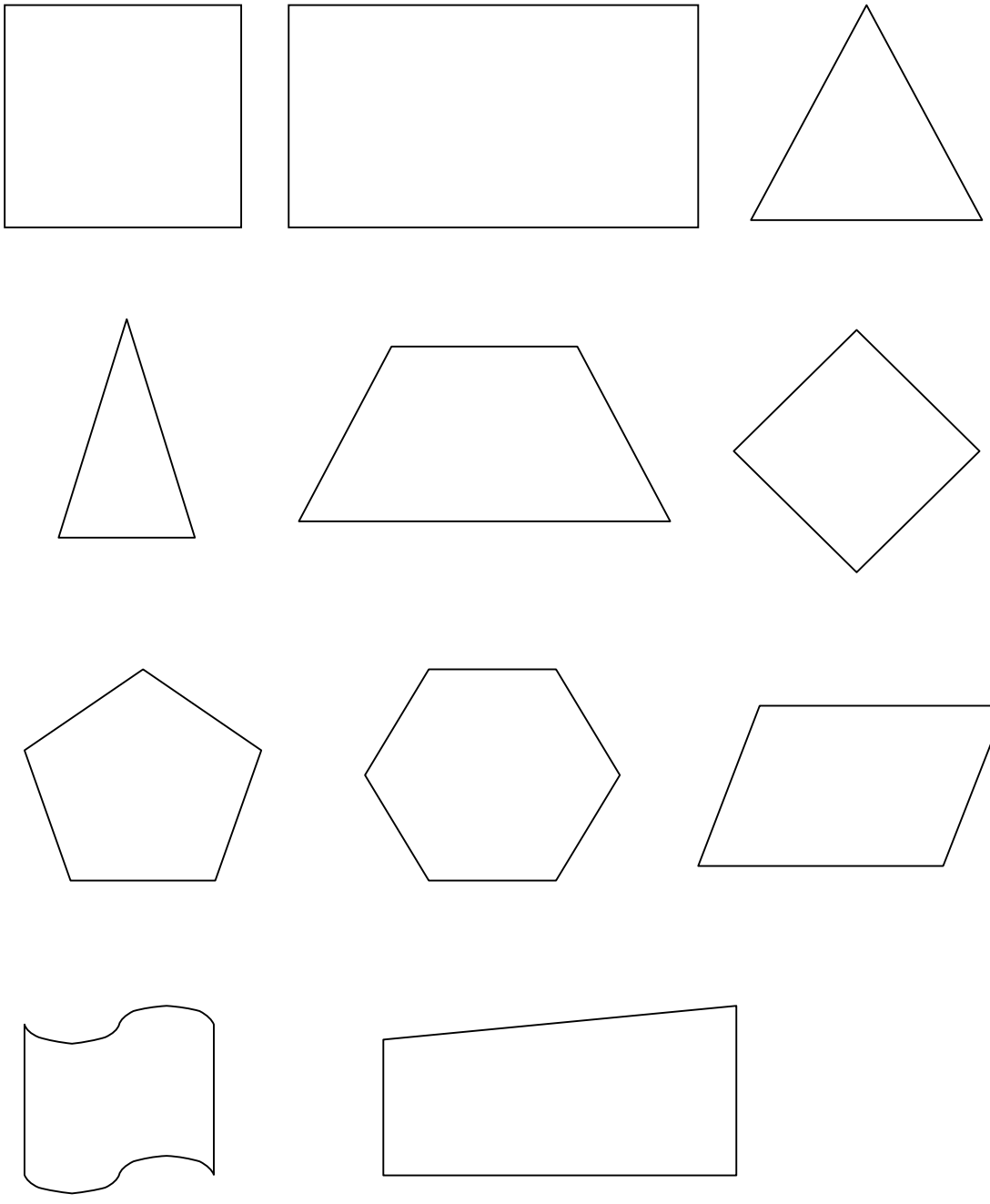
J

K

L

SIMETRI LIPAT PADA BANGUN DATAR DAN ALFABET

Amati bangun datar dan alfabet di bawah ini, tentukan apakah bangun tersebut mempunyai simetri lipat? Jika mempunyai simetri lipat, gambarlah sumbu simetrinya.



A B N S

Nama : _____

_____

Galeri Batik Jawa

Beberapa bulan berikutnya, galeri batik di Jogjakarta menerima paket lagi yang berisi berbagai macam kain batik untuk dipajang di pameran. Pegawai galeri tersebut berencana untuk memajang kain batik tersebut berdasarkan keteraturan motifnya seperti pameran pertamanya. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan motifnya?

Tuliskan hasil pengelompokkan motif Batik ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Ruang 1	Ruang 2

Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.

_____

_____

_____

_____

Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.

_____

_____

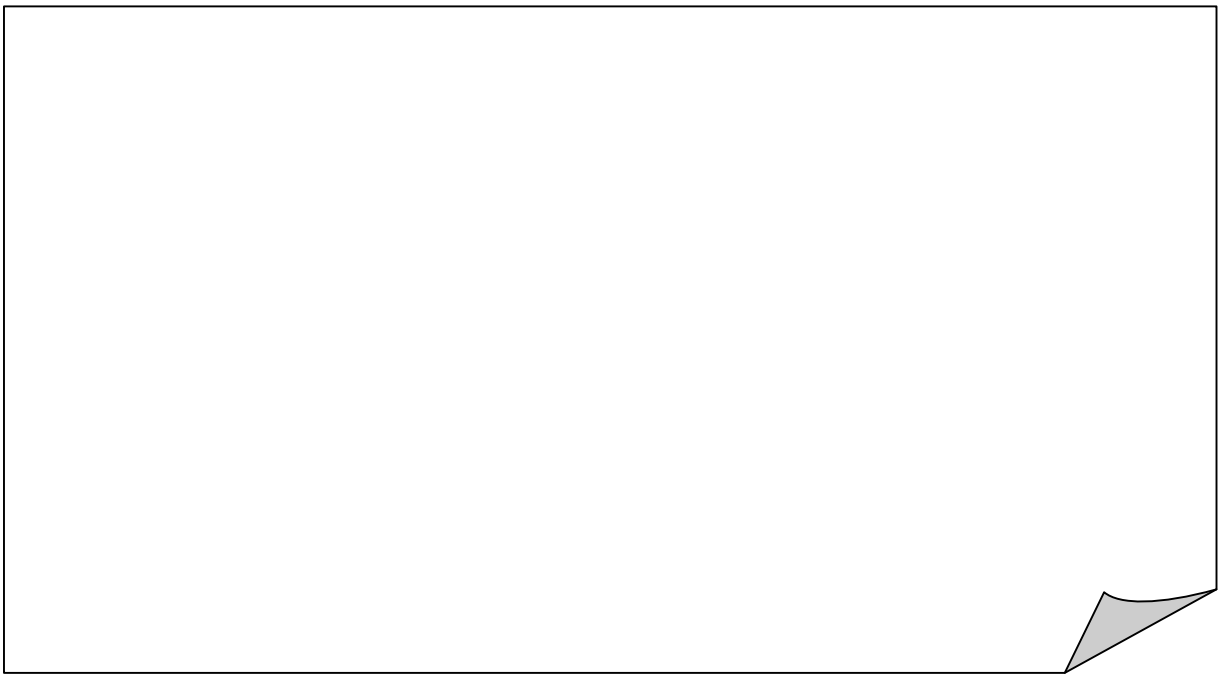
_____

_____

LATIHAN 1

Petunjuk:

Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan menggunakan pin dan kartu batik transparan yang tersedia.



Catatan:

_____
_____
_____
_____
_____
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_____
_____

LATIHAN 2

Petunjuk:

Amati kelima motif batik berikut. Tentukan apakah setiap motif mempunyai simeri putar. Jika motif tersebut mempunyai simetri putar, tandai titik yang kamu jadikan sebagai pusat rotasi.

(1) Motif A

Simetri putar:
YA TIDAK

Berapa kali motif A menempati posisi yang sama dalam satu putaran?

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?

____^{o}

Tandai A pada titik pusat rotasi.
Bagaimana kamu menentukan titik A sebagai titik pusat rotasi?

_____
_____
_____
_____

(2) Motif B

Simetri putar:
YA TIDAK

Berapa kali motif B menempati posisi yang sama dalam satu putaran?

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?

____^{o}

Tandai B pada titik pusat rotasi.
Bagaimana kamu menentukan titik B sebagai titik pusat rotasi?

_____
_____
_____
_____

(3) Motif C

Berapa kali motif C menempati posisi yang sama dalam satu putaran?

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?

____^{o}

Simetri putar:
YA TIDAK

Tandai C pada titik pusat rotasi.
Bagaimana kamu menentukan titik C sebagai titik pusat rotasi?

_____
_____
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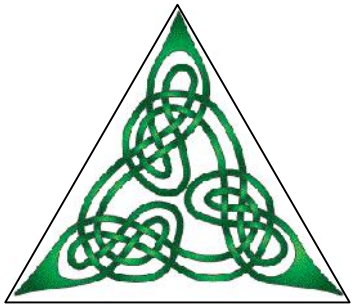
(4) Motif D



Berapa kali motif D menempati posisi yang sama dalam satu putaran?

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?

____^{o}

Simetri putar:
YA TIDAK

Tandai D pada titik pusat rotasi.
Bagaimana kamu menentukan titik D sebagai titik pusat rotasi?

_____
_____
_____
_____

(5) Motif E

Berapa kali motif E menempati posisi yang sama dalam satu putaran?

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?

____^{o}

Simetri putar:
YA TIDAK

Tandai E pada titik pusat rotasi.
Bagaimana kamu menentukan titik E sebagai titik pusat rotasi?

_____
_____
_____
_____

Catatan!

Hari ini saya belajar _____

_____

Banyaknya simetri putar adalah_____

_____

Titik pusat rotasi terletak di _____

_____

Hubungan antara banyaknya simetri putar dan besar sudut putaran adalah_____

_____

_____

A

B

C

D

E

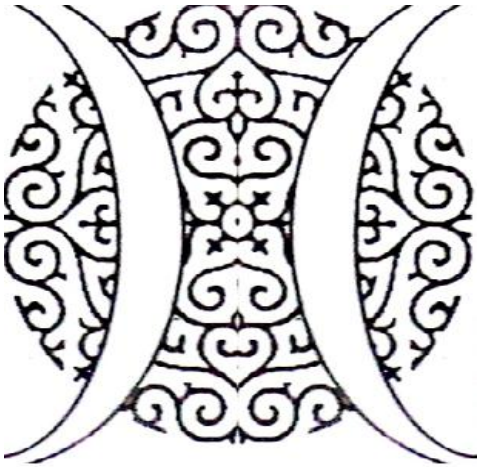
F

G

H

I

J

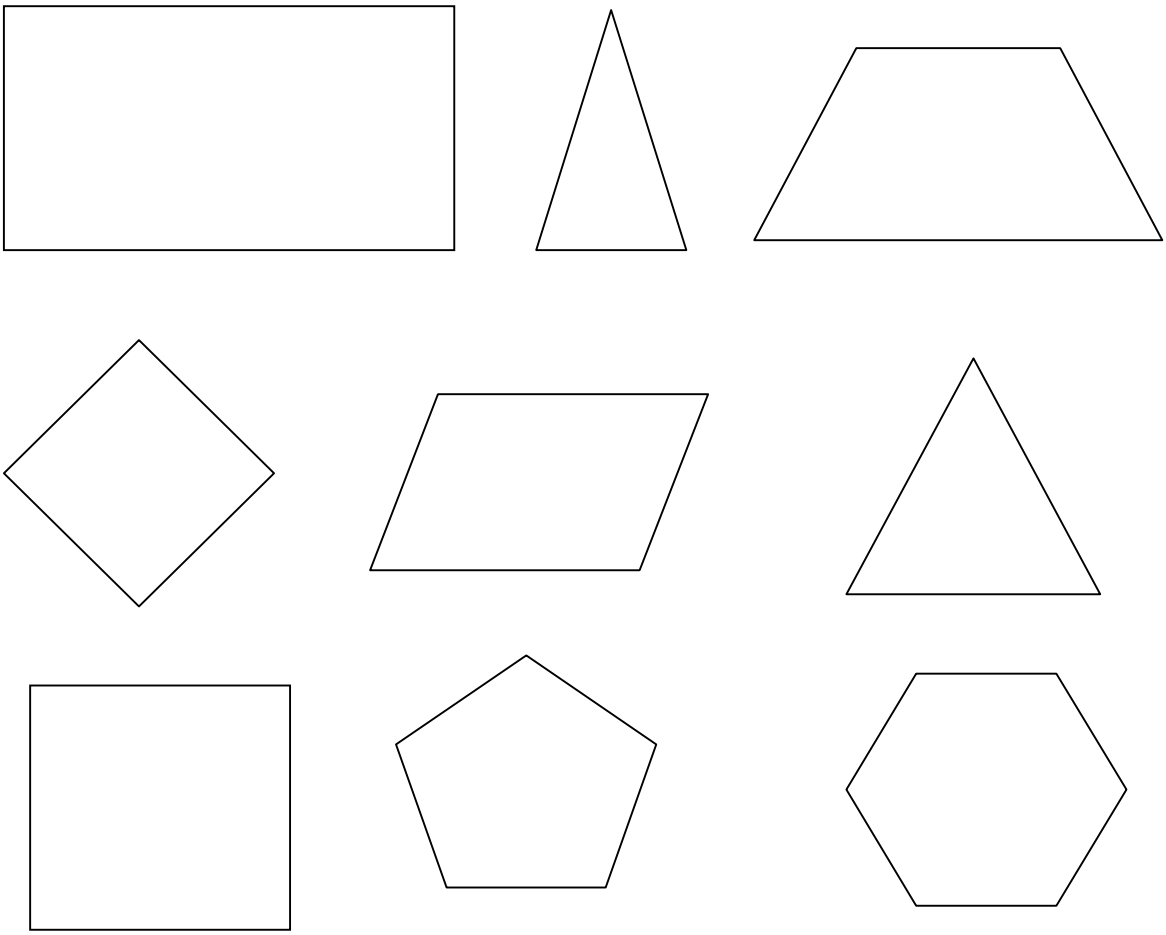


K

L

SIMETRI PUTAR PADA BANGUN DATAR

Amati bangun datar di bawah ini, tentukan apakah bangun tersebut mempunyai simetri putar? Jika mempunyai simetri putar, tulislah berapa banyak simetri putarnya.



Sudut satu putaran = _____^{o}

Nama Bangun	Banyaknya simetri putar	Sudut putar
Segitiga sama sisi		
Persegi		
Segilima		
Segienam		

Nama :

_____

PAMERAN BATIK SIMETRIS

Tantangan I

Perhatikan motif berikut.

Apakah motif tersebut mempunyai simetri lipat atau/dan simetri putar?

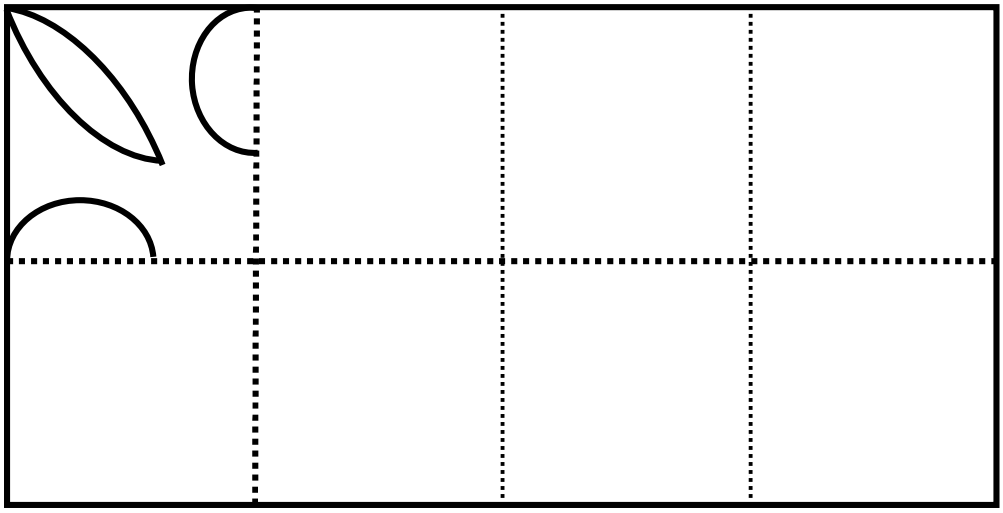
Jika motif Batik tersebut tidak simetris, maka buatlah menjadi simetris

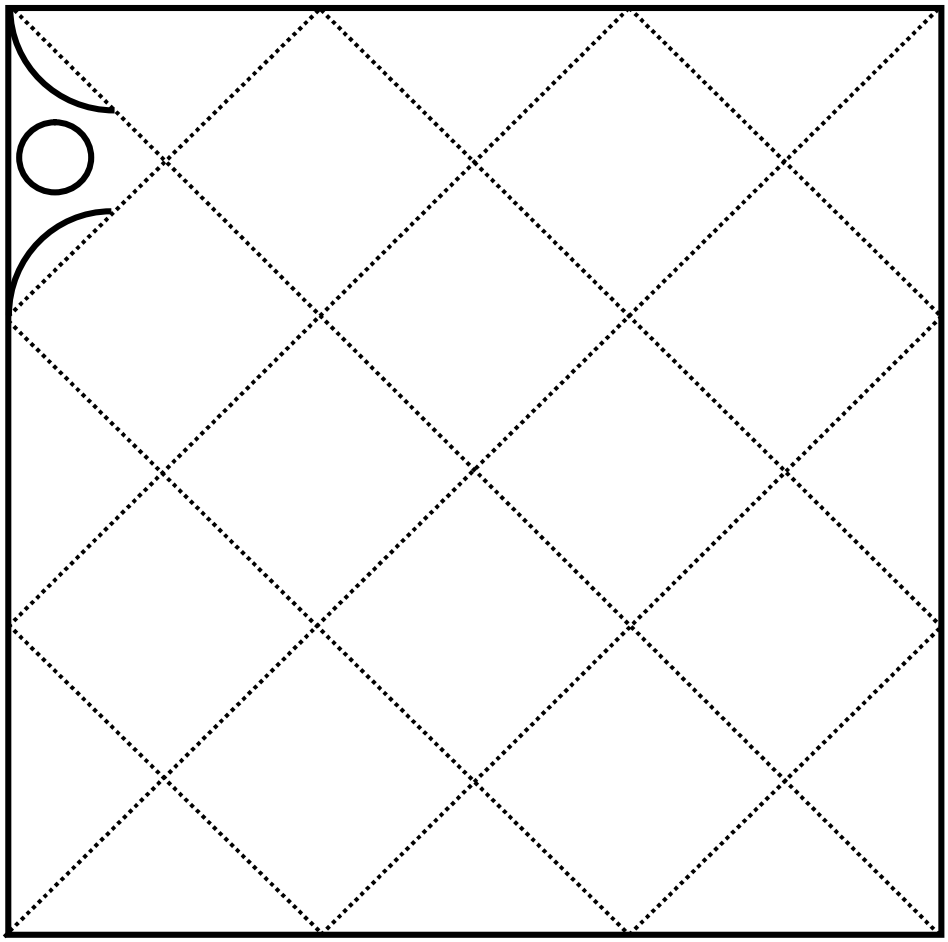
Kamu boleh memotong motif Batik yang tersedia dan menempelkannya di halaman yang sudah tersedia.

Tantangan 2

Lengkapilah motif batik berikut sehingga bisa menjadi batik yang simetris. Perhatikan sumbu simetrinya. Sumbu simetri digambarkan dengan garis putus-putus

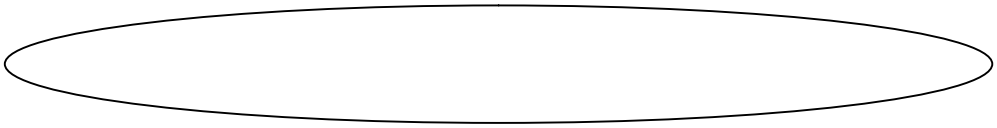
garis ..... menyatakan sumbu simetri



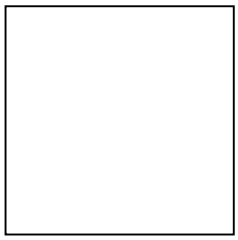


Tantangan 3

Dengan menggunakan unit Batik berikut,



atau



Buatlah motif Batik yang mempunyai simetri lipat dan simetri putar sesuai dengan sudut putar yang sudah ditentukan sebagai berikut,

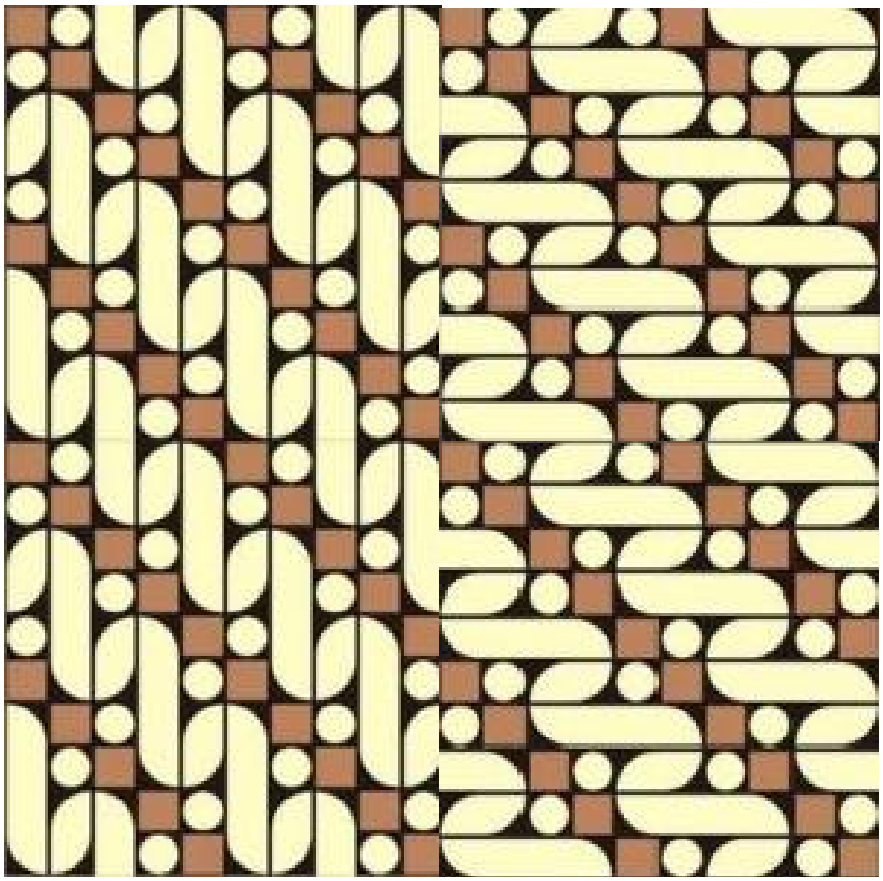
- Desain 1 : sudut putar 90°
- Desain 2 : sudut putar 60°
- Desain 3 : sudut putar 45°
- Desain 4 : sudut putar 30°

Selamat mendesain Batik-mu!

Petunjuk:

a) Tentukan titik pusat rotasi
b) Tentukan banyaknya simetri putar pada setiap desain Batikmu.
c) Jelaskan bagaimana caramu mendesain motif Batik dari unit Batik yang tersedia.

Desain Batikku



Teacher Guide of Learning Symmetry

Target group : fifth-grade students

The general learning goal : investigating the characteristics of line symmetry and rotational symmetry

Activity 1 - Javanese Batik Gallery (Line Symmetry)

❖ **Overview**

Students will explore the characteristics of line symmetry which are embedded in the batik patterns in informal way by employing their symmetrical sense. They will observe the various batik fabrics and try to sort the fabrics based on the regularity of the patterns. The students are expected to figure out that several patterns have regularity in which they consist of repeating motif. After finding the regular patterns, the students will investigate them by using a mirror in order to discover the notion of line symmetry and define its meaning. By using their understanding of line symmetry, they will differentiate the axes of symmetry and the diagonal of the patterns.

❖ **About the Mathematics**

The first activity of this meeting focuses on exploring the characteristics of line symmetry which are embedded in the regular Batik patterns. In the beginning, the characteristics of line symmetry refer to the regularity (repeating patterns) that can be found among the patterns. Second, the students employ the regular patterns to discover the notion of line symmetry. For the patterns which have line symmetry, the students will see two identical patterns by putting the mirror along the axes of symmetry of the patterns. The mirror reflection is identical with the pattern which is covered by the mirror. Last, the students will draw the axes of symmetry and diagonal of the given patterns to make them aware that not all diagonals are the axes of symmetry.

Line symmetry is a symmetry which is specified by its axes of symmetry.

An axes of symmetry is a line that divides the object into two parts such that the two parts become each other's image.

Diagonal is a line which connects two non-consecutive vertices.

❖ **Mathematical Goal**

The students are able to discover the notion of line symmetry by exploring the characteristic of batik patterns. This goal can be elaborated into the following sub-learning goals.

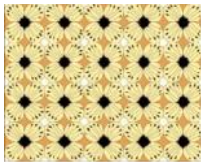
- The students are able to differentiate the patterns which have regularity (line symmetry) and the patterns that have no regularity.
- The students are able to deduce the characteristics of line symmetry from the regular batik patterns by using a mirror.
- The students are able to differentiate diagonal and line symmetry on the batik patterns.
- The students are able to implement their understanding of line symmetry into two-dimensional shapes

❖ **Learning Materials**

- Student worksheet
- A pencil, a pen
- A mirror

❖ **Learning Activities**

1. Ask the students about the famous cultural heritage of Indonesia. If the students do not mention batik patterns, then engage them to mention it by asking the following question "*How about the traditional cloth that people usually wear*?"
2. Show some pictures of batik patterns in Java, and explain the meaning of several patterns such as Batik Parang represents braveness and leadership and Batik Kawung represents simplicity.


Kawung


Parang

3. Tell the students about the story of a batik gallery that wants to held a batik exhibition. Pose a problem that is being faced by the gallery staff in which they only has two rooms to display the various batik fabrics. Consequently, they should sort the fabrics into two types based on the regularity of the patterns. Show the printed Batik patterns and ask the students to observe the patterns to figure out the whether the patterns are regular. Then, make sure that all students understand the problem and exactly know what they should do. It can be done by asking several students to paraphrase the problem and asking the other students whether they agree with the statement. The example of questions,
    - *Could you explain the problem in your own words*?
    - *Do you agree with your friend's statement? Why do you think so*?

4. Situate the students to do the worksheet in a group consists of three to four students. For the first discussion, the students only have to do the first task in which they need to sort the twelve batik fabrics based on the regularity of the patterns. (10 minutes)

   Teacher note:

   While the students discuss the problem in their group, walk around the class to supervise how the discussion is going and ensure that all students get involved in solving the problems. In addition, if there is a group of students who have no idea in solving the problem, then you can support them by posing the following question.

   "*Try to observe the pattern in details, do you see something regular in the pattern? Which are the regular parts?* "

   Moreover, it is important to note the pairs of students who have different answers and strategies in solving the problems and figure out the concept of regularity of the patterns so that their findings can be discussed in the class discussion.
5. After 10 minutes, conduct the class discussion to talk about the regular patterns. Ask the groups which have different answers to state their answer of the regular patterns and the way they determine the regularity of the patterns.
6. After having the same answer of the regular patterns, asks the students to do the second task in which they have to discover the special characteristics of the regular patterns by using a mirror.

   Teacher note:

   In exploring the regular patterns by using a mirror, the students may put the mirror in the edge of the patterns and do not see any special characteristics of the regular patterns. Therefore, ensure that the students do the proper exploration by putting the mirror in the centre of the patterns along the axes of symmetry and discovering the special characteristics that the mirror reflection shows the same pattern as the pattern in the back of the mirror.
7. Ask the students to present their findings about the special characteristics of the regular patterns and engage them to refer their findings to the notion of line symmetry. The teacher can pose these following questions,

   "*Where do you put the mirror*? *What do you see from the mirror reflection?"*

   "What do you notice by putting the mirror in the centre of the pattern?"

   If the students do not come up with the notion of line symmetry, then review the concept of line symmetry which the students already learned in fourth grade by asking the following questions.

"*Do you still remember about line symmetry? What do you think about the regular patterns, do they have line symmetry?*"

If the students are aware that the regular patterns have line symmetry, then lead them to the special characteristics which they already found in the previous task. It aims at making the students know that the line symmetry is about two identical parts which become each other's mirror images.

Then, refer the mirror positions to the axes of symmetry of the patterns by asking the following questions,

"*What do you notice about the mirror position? Does it refer to particular line?*"

8. Lead the students to define line symmetry and the axes of symmetry by themselves based on the class discussion. It can be done by posing these following questions,

   "*If the regular patterns signify the concept of line symmetry, then what is line symmetry? Define with your own words*"

   "*How about the axes of symmetry? What is the special characteristics of it*?"

9. Review the definition of diagonals and the axes of symmetry to support the students to do the third task by asking the following questions

   "*In two-dimensional shapes, there is a line namely diagonal, what kind of line is it?"*

   "*Can you state your own definition of the axes of symmetry*?"

10. Ask the students to do the third task in the group.

    Teacher note:

    There will be students who cannot draw the diagonals and the axes of symmetry properly. Let the students have different answers and discuss about the differences in the class discussion.

11. Ask some students who have different answers and discuss their drawing of the diagonals and the axes of symmetry. Ensure students' understanding about the difference between them by asking some following questions,

    "*Does the diagonal of the object always becomes its axes of symmetry? Give some examples*"

    "*What are the differences between the diagonal and the axes of symmetry?*"

12. After the students show their understanding about the differences between the diagonals and the axes of symmetry, ask the students to do the fourth task individually.

13. When the students do the fourth task, walk around the class to see whether the students have difficulty in implementing their understanding of line symmetry to two-dimensional shapes.
14. Discuss the number of line symmetry and how the students draw the axes of symmetry in two-dimensional shapes.
15. In the closing activity, ask the students to reflect the lesson such as by asking the following questions,

    "*What do we have learned today*?"

    "*Can you define line symmetry in your own words*?"

Activity 2 - Javanese Batik Gallery (Rotational Symmetry)

❖ **Overview**

Similar with the first activity, the students will explore the characteristics of rotational symmetry which are embedded in the batik patterns in informal way by their understanding of symmetry. They will observe the various batik fabrics and try to sort the fabrics based on the regularity of the patterns. The students are expected to figure out that there is a pattern which consists of repeating motif but not having line symmetry. The pattern will be used to introduce the notion of line symmetry. After finding the regular patterns, the students will investigate them by using transparent batik cards and a pin in order to discover the notion of rotational symmetry and define its meaning. They are expected to put the transparent batik cards above the corresponding patterns and the pin in the centre of the patterns. Then, the students will be aware that the regular pattern can fit into itself for more than once.

❖ **About the Mathematics**

The first activity of this meeting focuses on determining the regular patterns. Then, the students employ the regular patterns to discover the notion of rotational symmetry. For the patterns which have rotational symmetry, the students can rotate the pattern and figure out that it can fit into itself for more than once.

Rotational symmetry is a symmetry which is specified by its centre point and angle of rotation.

The order of rotation is the number of times the figure can fit into itself

The angle of rotation is the size of angle which can make the pattern fit into itself

The point of rotation is the centre point of the pattern which can be determined from the intersection of the diagonals or the axes of symmetry

❖ **Mathematical Goal**

The students are able to discover the notion of rotational symmetry by exploring the characteristic of batik patterns. This goal can be elaborated into the following sub-learning goals.

- The students are able to differentiate the patterns which have regularity (rotational symmetry) and the patterns that have no regularity.
- The students are able to deduce the characteristics of rotational symmetry from the regular batik patterns by using a pin and the transparent batik cards.
- The students are able to determine the characteristics of rotational symmetry (the order of rotation, the angle of rotation and the point of rotation)
- The students are able to implement their understanding of rotational symmetry into two-dimensional shapes

❖ **Learning Materials**

- Student worksheet
- A pencil, a pen
- Transparent batik cards
- A pin

❖ **Learning Activities**

1. Start the lesson by showing the regular batik patterns which already discussed in the first meeting and asking the students about regularity in the pattern.


"Are the patterns regular?"

"What do you mean by regular patterns?"

2. Ask the students to help the gallery staff again to sort the batik fabrics based on the regularity of the patterns. Show the printed Batik patterns and ask the students to observe the patterns to figure out the whether the patterns are regular. Then, make sure that all students understand the problem and exactly know what they should do. It can be done by asking several students to paraphrase the problem and asking the other students whether they agree with the statement. The example of questions,
    - *Could you explain the problem in your own words*?
    - *Do you agree with your friend's statement? Why do you think so*?

3. Situate the students to do the worksheet in a group consists of three to four students. For the first discussion, the students only have to do the first task in which they need to sort the twelve batik fabrics based on the regularity of the patterns. (10 minutes)

   Teacher note:

   While the students discuss the problem in their group, walk around the class to supervise how the discussion is going and ensure that all students get involved in solving the problems. In addition, if there is a group of students who have no idea in solving the problem, then you can support them by posing the following question.

   "*Try to observe the pattern in details like what has been discussed previously? Which are the regular parts?* "

   Moreover, it is important to note the pairs of students who have different answers and strategies in solving the problems and figure out the concept of regularity of the patterns so that their findings can be discussed in the class discussion.
4. After 10 minutes, conduct the class discussion to talk about the regular patterns. Ask the groups which have different answers to state their answer of the regular patterns and the way they determine the regularity of the patterns.
5. After having the same answer of the regular patterns, asks the students to do the second task in which they have to discover the special characteristics of the regular patterns by using transparent batik cards and a pin.

   Teacher note:

   In exploring the regular patterns by using transparent batik cards and a pin, the students may do the intended way by putting the transparent batik cards above the corresponding patterns and the pin in the centre of the patterns. However, they are not aware that the regular pattern can fit into itself for more than once. Therefore, guide the students to be aware of the notion of rotational symmetry by asking the following questions,

   "*What do you see after rotating the pattern?*"

   "*Don't you see that the pattern can fit into itself?*"

   "*Try to count the number of times the pattern can fit into itself?*"
6. Ask the students to present their findings about the special characteristics of the regular patterns and engage them to refer their findings to the notion of rotational symmetry. The teacher can pose these following questions,

   "*Where do you put the pin*? *Why do you put in there?"*

   "What do you notice by rotating the pattern?"

If the students do not come up with the notion of rotational symmetry, then the teacher can show the pattern which consist of repeating motif but not having line symmetry and ask the following questions.

"*Look at the pattern? Is it regular? Does it have line symmetry*"

"*Then, what kind of symmetry is it?*"

"*Now, try to put the corresponding transparent batik cards above it and the pin in the centre of the pattern, then rotate it?*"

The students will be aware that if they rotate the regular pattern, then the pattern can fit into itself. Relate their awareness with the notion of rotational symmetry.

7. Lead the students to define rotational symmetry by themselves based on the class discussion. It can be done by posing these following questions,

   "*If the regular patterns signify the concept of rotational symmetry, then what is rotational symmetry? Define with your own words*"

8. Review the size of special angle and full angle by asking the following questions

   "*Do you still remember about special angles*? *Can you mention them*?"

   "*How about the size of full angle?*"

9. Ask the students to do the third task in the group.

   Teacher note:

   There will be students who cannot determine the angle of rotation and the point of rotation. Let them to have different answers so that it can be discussed in the class discussion

10. Ask some students who have different answers and discuss their answers. Ensure students' understanding about the characteristics of rotational symmetry.

    Teacher note:

    If the students still have difficulty in determining the characteristics of line symmetry, then the teacher can use pattern A as the example. Related to the order of rotation, the teacher can ask the students how many times the pattern A fits into itself in 360^o. Then, the teacher can refer students' answer to the order of rotation

    Related to the angle of rotation, the teacher can firstly review the size of one round angle (360^o) and ask the students to count how many pattern A fits into itself. It is intended to make the students notice that the angle of rotation can be determined by dividing 360^o with the order of rotation

"*Doing the rotation is turning the pattern around under one full angle. Hence, if the pattern can fit into itself for 4 times in one round, then what can you conclude*? *Determine the angle of rotation."*

Related to the point of rotation, the teacher can ask the students to show how they determine the centre of the pattern F and how they can be so sure that it is the centre of the pattern.

"*Do you have any strategy to make sure that the point of rotation is exactly in the centre of the pattern*?"

11. After the students show their understanding about the characteristics of rotational symmetry, ask the students to do the fourth task individually.

    When the students do the fourth task, walk around the class to see whether the students have difficulty in implementing their understanding of line symmetry to two-dimensional shapes.
12. Discuss the order of rotation, the angle of rotation in two-dimensional shapes and the relation between them.
13. In the closing activity, ask the students to reflect the lesson such as by asking the following questions,

    "*What do we have learned today*?"

    "*Can you define rotational symmetry in your own words*?"

    "*How do you determine the point of rotation?*"

    "*How do you determine the angle of rotation?*"

    "*Is there any relation between the angle of rotation and the order of rotation?*"

Activity 3 – Symmetric Patterns

❖ **Overview**

Students will implement their understanding of line and rotational symmetry by making the asymmetric patterns into the symmetric ones, completing the symmetric pattern by considering the given axes of symmetry and drawing the symmetric patterns which show rotational symmetry.

❖ **About the Mathematics**

The way of restructuring the asymmetric pattern into the symmetric ones can be various because it depends on the way they determine the axes of symmetry. Then, for completing

the symmetric patterns, the students must know how the axes of symmetry works in symmetric patterns. There are three types of axes of symmetry in the worksheet (horizontal, vertical, and oblique). Last, the students need ability to draw angle in order to draw the symmetric patterns by considering the required angle.

❖ **Mathematical Goal**

The students are able to apply their understanding of line and rotational symmetry. This goal can be elaborated into the following sub-learning goals.

- The students are able to make the asymmetric batik pattern into the symmetric ones
- The students are able to complete the symmetric pattern by considering the given axes of symmetry
- The students are able to create their own batik design by using provided batik units and considering the required angle of rotation

❖ **Learning Materials**

- Student worksheet
- A pencil, a pen, a scissors
- A ruler, a protractor

❖ **Learning Activities**

1. Review the definition of line and rotational symmetry in order to help the students do the tasks. It can be done by posing the following questions
   "*Can you state the definition of line symmetry and rotational symmetry?*"
   "*What is the axes of symmetry*?"
2. Ensure the students understand the notion of line and rotational symmetry.
3. Situate the students to do the worksheet individually because this activity is intended to see whether the students can apply their understanding of line and rotational symmetry.
   Teacher note:
   While the students do the tasks, walk around the class to see how the students make the symmetric patterns, complete the symmetric pattern and draw the symmetric patterns. If the students can do the task properly, then the teacher can ask several questions in order to strengthen their understanding such as,
   "*Is there any other way to arrange the pattern such that it still become symmetric?*"
   "*Why do you arrange the pattern in that way, what do you consider*?"

However, if the students cannot do the task properly, then the teacher can do the following action.

The first task

If the students cut the pattern into several parts, but they paste them in the wrong way so that the pattern does not become symmetric, then suggests the students to observe the pattern more thoroughly and review the definition of line symmetry

"*What does line symmetry mean*?"

"*What do you have to consider the make the pattern has line symmetry*?"

The second task

If the students complete the pattern with the same motif without considering the given axes of symmetry, then ask them to review the definition and the characteristics of the axes of symmetry.

"*What is the axes of symmetry*? *In order to make symmetric pattern, do you have to consider the given axes of symmetry?"*

The third task

If the students draw their own pattern by using the provided batik units but without considering the given angle of rotation, then ask them to review the definition rotational symmetry and its characteristics. The example of questions,

"*What is the definition of rotational symmetry*?"

"*Now, look at your pattern more thoroughly and see whether the definitions hold for your pattern*"

"*What is the angle of rotation*?"

If the students get difficulty in drawing the angle of rotation, demonstrate the way of drawing the angles properly.

4. Ask several students who have different result of symmetric patterns to present their works and explain how they design the patterns.
5. In the closing activity, ask the students about the use of the notion of symmetry in their activity.

   Example of questions:

   "*How is the application of line or rotational symmetry in designing pattern?*"

THE FIRST MEETING

ACTIVITY 1

Cycle 1

Galeri Batik Jawa

Galeri batik di Jogjakarta baru saja menerima paket yang berisi berbagai macam kain batik untuk dipajang di pameran. Agar pamerannya tertata dengan bagus, pegawai galeri berencana untuk memajang kain batik tersebut berdasarkan keteraturan pada motifnya. Akan tetapi galeri tersebut hanya mempunyai dua ruangan (ruang 1 dan ruang 2) untuk memajang kain batik. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan pada motifnya?

LATIHAN 1

Tuliskan hasil pengelompokkan motif Batik ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Ruang I	Ruang 2
A	B
D	C
G	F
H	K
I	
J	
L	

Jelaskan alasanmu dalam mengelompokkan motif Batik tersebut ke dalam dua ruangan.

motifnya teratur karena motifnya tidak berubah $\frac{1}{2}$ ke dan kalau dilipat motifnya sama

Cycle 2

Galeri Batik Jawa

Galeri batik di Jogjakarta baru saja menerima paket yang berisi berbagai macam kain batik untuk dipajang di pameran. Agar pamerannya tertata dengan bagus, pegawai galeri berencana untuk memajang kain batik tersebut berdasarkan keteraturan pada motifnya. Akan tetapi galeri tersebut hanya mempunyai dua ruangan (ruang 1 dan ruang 2) untuk memajang kain batik. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan pada motifnya?

Tuliskan hasil pengelompokkan motif Batik ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Batik beraturan Batik tidak beraturan

Ruang 1	Ruang 2
A	B
D	C
G	E
H	F
I	K
J	

Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1.

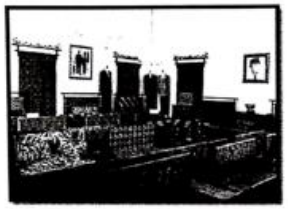
Agar tidak tercampur dengan batik lain dan mengenalnya lebih gampang.

Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2.

Agar bisa membedakan batik beraturan dan tidak beraturan

Cycle 3

Galeri Batik Jawa



Galeri batik di Jogjakarta baru saja menerima paket yang berisi berbagai macam kain batik untuk dipajang di pameran. Agar pamerannya tertata dengan bagus, pegawai galeri berencana untuk memajang kain batik tersebut berdasarkan keteraturan pada motifnya. Akan tetapi galeri tersebut hanya mempunyai dua ruangan (ruang 1 dan ruang 2) untuk memajang kain batik. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan pada motifnya?

Tuliskan hasil pengelompokkan motif Batik ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Ruang I (Batik yang teratur)	Ruang 2 (Batik yang tidak teratur)
A	B
D	C
G	E
H	F
I	K
J	
L	

Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.

memiliki motif yg beraturan dan hainya simetris

Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.

bentuk tidak sama dan tidak simetris

ACTIVITY 2

Cycle 1

Petunjuk:
Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan menggunakan cermin yang tersedia.

Sumbu simetri adalah sumbu yang mencerminkan ~~AB~~ / membagi 2 dengan sama besar ~~dengan~~ dan saling mencerminkan

Cycle 2

LATIHAN 1

Petunjuk:
Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan menggunakan cermin yang tersedia.

Batik yg beraturan karan kiri dan atas bawah terlihat sama
Batik yg tidak beraturan karan kiri dan atas bawah tidak sama

Catatan:

- Jika cermin diletakkan di tengah ×× gambar batik, batik itu, terlihat sama walaupun batik yg tidak beraturan juga terlihat sama jika cermin diputar di tengah ×× gambar batik.
- Batik yg beraturan mempunyai sumbu simetri 4
- Batik yg tidak beraturan mempunyai sumbu simetri 4

Cycle 3

LATIHAN 1

Petunjuk:
Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan menggunakan cermin yang tersedia.

Kaca adalah sumbu simetri bagi setiap batik yg teratur

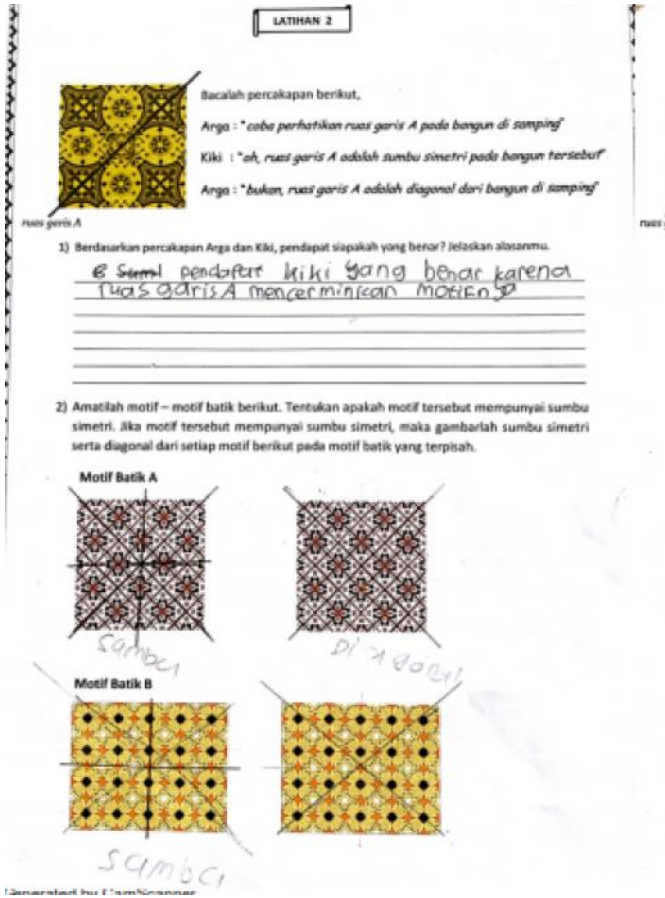
Catatan:

Sumbu simetri adalah garis yg bisa membelah benda menjadi 2

ACTIVITY 3

Part 1

Cycle 1



LATIHAN 2

ruas garis A

Bacalah percakapan berikut,

Arga : "coba perhatikan ruas garis A pada bangun di samping"

Kiki : "oh, ruas garis A adalah sumbu simetri pada bangun tersebut"

Arga : "bukan, ruas garis A adalah diagonal dari bangun di samping"

1) Berdasarkan percakapan Arga dan Kiki, pendapat siapakah yang benar? Jelaskan alasanmu.

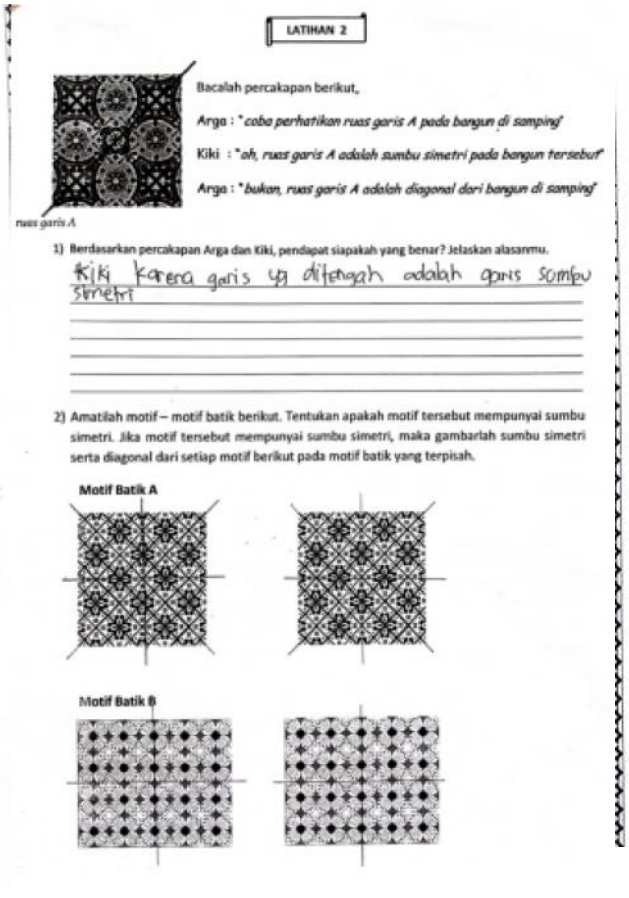
pendapat kiki yang benar karena ruas garis A mencerminkan motifnya

2) Amatilah motif – motif batik berikut. Tentukan apakah motif tersebut mempunyai sumbu simetri. Jika motif tersebut mempunyai sumbu simetri, maka gambarlah sumbu simetri serta diagonal dari setiap motif berikut pada motif batik yang terpisah.

Motif Batik A

Motif Batik B

Cycle 2



LATIHAN 2

ruas garis A

Bacalah percakapan berikut,

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Arga : "bukan, ruas garis A adalah diagonal dari bangun di samping"

1) Berdasarkan percakapan Arga dan Kiki, pendapat siapakah yang benar? Jelaskan alasanmu.

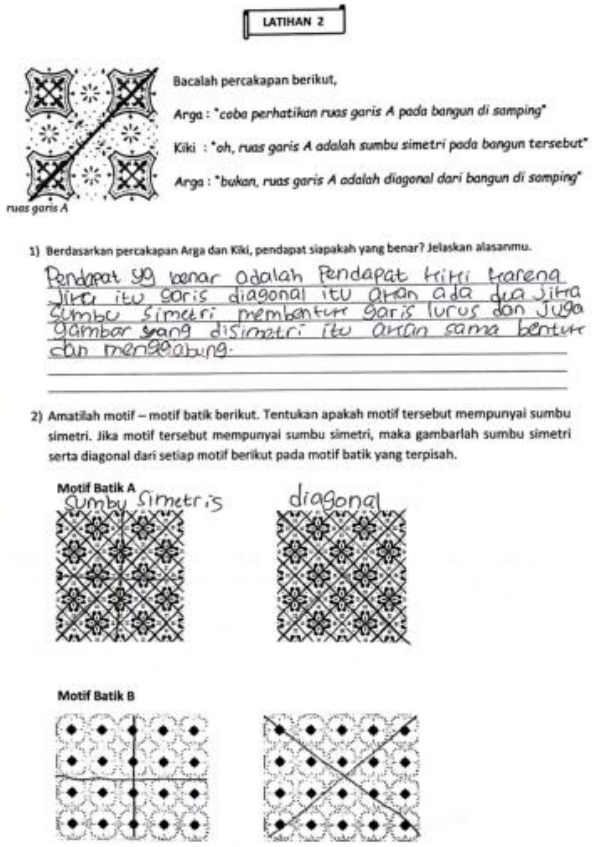
Kiki karena garis yg ditengah adalah garis sumbu simetri

2) Amatilah motif – motif batik berikut. Tentukan apakah motif tersebut mempunyai sumbu simetri. Jika motif tersebut mempunyai sumbu simetri, maka gambarlah sumbu simetri serta diagonal dari setiap motif berikut pada motif batik yang terpisah.

Motif Batik A

Motif Batik B

Cycle 3



LATIHAN 2

ruas garis A

Bacalah percakapan berikut,

Arga : "coba perhatikan ruas garis A pada bangun di samping"

Kiki : "oh, ruas garis A adalah sumbu simetri pada bangun tersebut"

Arga : "bukan, ruas garis A adalah diagonal dari bangun di samping"

1) Berdasarkan percakapan Arga dan Kiki, pendapat siapakah yang benar? Jelaskan alasanmu.

Pendapat yg benar adalah Pendapat Triki karena jika itu garis diagonal itu akan ada dua jika sumbu simetri membentuk garis lurus dan juga gambar yang disimetri itu akan sama bentuk dan menggabung.

2) Amatilah motif – motif batik berikut. Tentukan apakah motif tersebut mempunyai sumbu simetri. Jika motif tersebut mempunyai sumbu simetri, maka gambarlah sumbu simetri serta diagonal dari setiap motif berikut pada motif batik yang terpisah.

Motif Batik A

sumbu simetris

diagonal

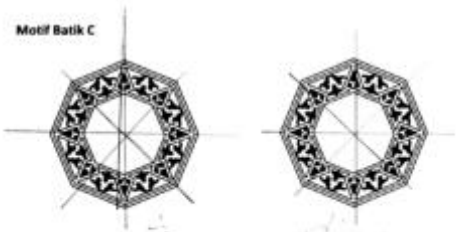
Motif Batik B

ACTIVITY 3

Part 2

Cycle 1

Motif Batik C



Motif Batik D

3) **Sumbu simetri dari suatu bangun datar merupakan diagonal bangun datar tersebut.**
Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

Salah, karena tidak selalu diagonal yang merupakan sumbu simetri dari sebuah bangun datar

4) **Diagonal dari suatu bangun datar merupakan sumbu simetri bangun datar tersebut.**
Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

Salah karena tidak selalu diagonal adalah sumbu simetri suatu bangun datar

Cycle 2

Motif Batik C

Motif Batik D

3) **Sumbu simetri dari suatu bangun datar merupakan diagonal bangun datar tersebut.**
Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

salah, belum tentu diagonal bangun tersebut

4) **Diagonal dari suatu bangun datar merupakan sumbu simetri bangun datar tersebut.**
Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

salah, belum tentu sumbu simetri karna belum tentu bisa dilipat. hanya pada bangun tertentu.

Cycle 3

Motif Batik C

Motif Batik D

tidak mempunyai Sumbu Simetri

3) **Sumbu simetri dari suatu bangun datar merupakan diagonal bangun datar tersebut.**
Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

Salah, karena Sumbu simetri bukan diagonal

4) **Diagonal dari suatu bangun datar merupakan sumbu simetri bangun datar tersebut.**
Tentukan apakah pernyataan tersebut salah atau benar. Jelaskan alasanmu.

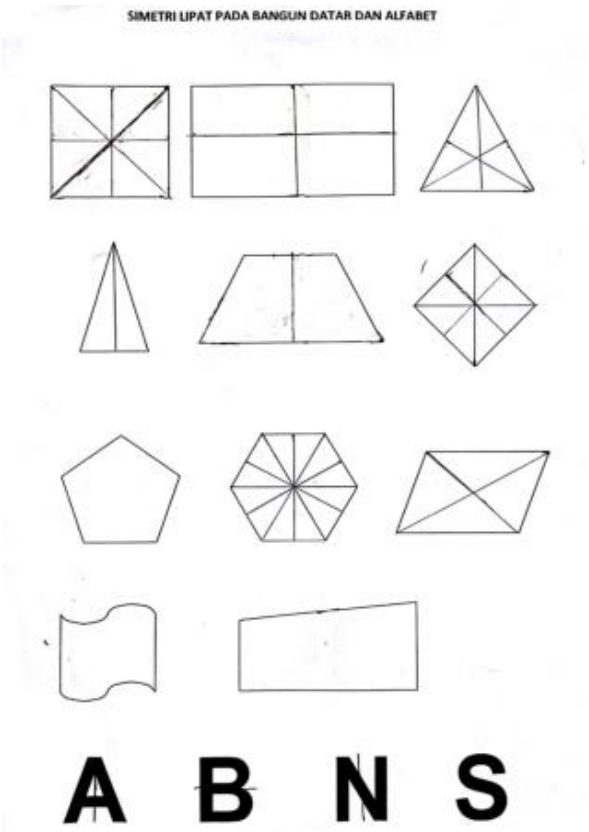
Jawaban itu bisa benar bisa juga salah karena bangun datar itu berbeda ada yang 2 sumbu simetri dan 2 diagonal

ACTIVITY 4

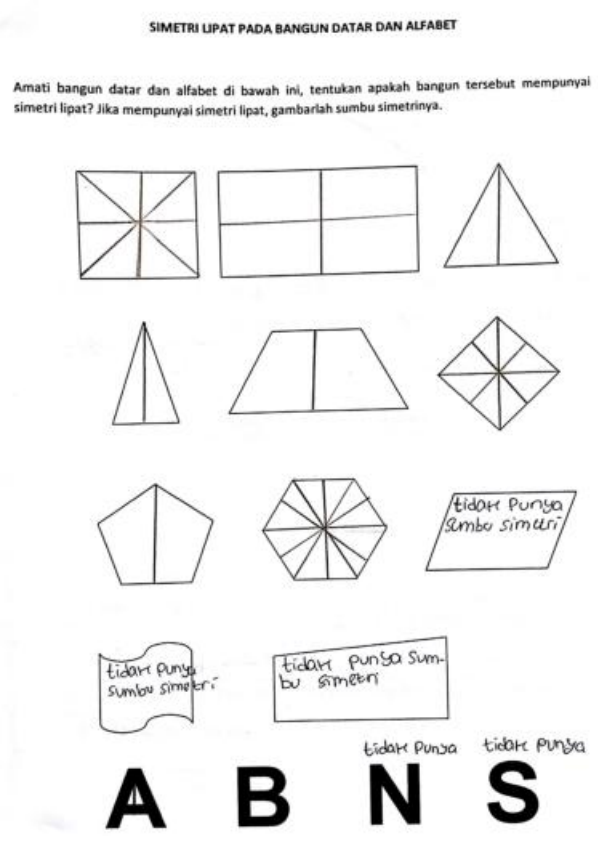
Cycle 1

No students' written works

Cycle 2



Cycle 3



THE SECOND MEETING

ACTIVITY 1

Cycle 1

Galeri Batik Jawa

Beberapa bulan berikutnya, galeri batik di Jogjakarta menerima paket lagi yang berisi berbagai macam kain batik untuk dipajang di pameran. Pegawai galeri tersebut berencana untuk memajang kain batik tersebut berdasarkan ketraturan motifnya seperti pameran pertamanya. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan motifnya?

Tuliskan hasil pengelompokkan kain Batik di atas ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Ruang 1	Ruang 2
A	B
F	C
D	G
E	
L	J
K	I

H

Jelaskan alasanmu dalam mengelompokkan kain Batik tersebut ke dalam dua ruangan

agar mudah membedakan batik teratur dan tak teratur

Cycle 2

Galeri Batik Jawa

Beberapa bulan berikutnya, galeri batik di Jogjakarta menerima paket lagi yang berisi berbagai macam kain batik untuk dipajang di pameran. Pegawai galeri tersebut berencana untuk memajang kain batik tersebut berdasarkan keteraturan motifnya seperti pameran pertamanya. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan motifnya?

Tuliskan hasil pengelompokkan motif Batik ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Ruang 1	Ruang 2
A	B
F	C
P	D
H	G
K	J
L	

Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 1.

karena motif batik tersebut termasuk yang teratur

Tuliskan alasanmu dalam mengelompokkan motif Batik tersebut ke ruang 2.

karena motif batik tersebut termasuk yang tidak teratur

Cycle 3

Galeri Batik Jawa

Beberapa bulan berikutnya, galeri batik di Jogjakarta menerima paket lagi yang berisi berbagai macam kain batik untuk dipajang di pameran. Pegawai galeri tersebut berencana untuk memajang kain batik tersebut berdasarkan keteraturan motifnya seperti pameran pertamanya. Jika kalian adalah pegawai galeri tersebut, bagaimana kalian akan mengelompokkan kain Batik tersebut ke dalam dua ruangan yang tersedia berdasarkan keteraturan motifnya?

Tuliskan hasil pengelompokkan motif Batik ke dalam dua ruangan dengan menulis kode Batik pada tabel berikut ini.

Ruang 1	Ruang 2
a	B
d	C
e	K
f	I
h	J
L	
g	

Tuliskan ciri-ciri yang kamu lihat pada motif batik yang teratur.

gg teratur:
- atas dan bawah sama
- kanan dan kiri sama

Tuliskan perbedaan yang kamu lihat antara motif yang teratur dengan motif yang tidak teratur.

gg teratur:
- atas dan bawah sama
- kanan dan kiri sama

tidak teratur:
- atas dan bawah beda
- kanan dan kiri beda

ACTIVITY 2

Cycle 1

Petunjuk:
Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan. menggunakan pin dan kartu transparant batik yang tersedia.

Simetri putar adalah simetri yang kalau gambarnya diputar bisa menutup bingkainya

Cycle 2

LATIHAN 1

Petunjuk:
Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan menggunakan pin dan kartu batik transparan yang tersedia.

Catatan:
Jika motif yang teratur, kalau diputar bisa tepat gambarnya

Cycle 3

LATIHAN 1

Petunjuk:
Amatilah motif batik yang teratur dan selidikilah keistimewaan dari motif batik tersebut dengan menggunakan pin dan kartu batik transparan yang tersedia.

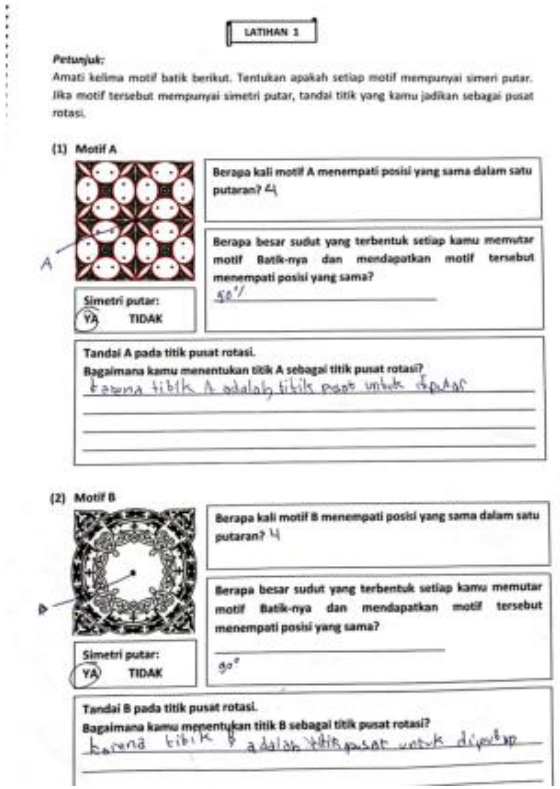
1. diputar berapapun, akan sama (teratur)
2. diputar berapapun, akan beda (tak teratur)

Catatan:
Jika diputar, batik teraturakan sama bentuknya. Jika batik tak teratur akan beda bentuknya.

ACTIVITY 3

Part 1

Cycle 1



LATIHAN 1

Petunjuk:
Amati kelima motif batik berikut. Tentukan apakah setiap motif mempunyai simeri putar. Jika motif tersebut mempunyai simetri putar, tandai titik yang kamu jadikan sebagai pusat rotasi.

(1) Motif A

A

Berapa kali motif A menempati posisi yang sama dalam satu putaran? 4

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
90°

Simetri putar:
(YA) TIDAK

Tandai A pada titik pusat rotasi.
Bagaimana kamu menentukan titik A sebagai titik pusat rotasi?
karena titik A adalah titik pusat untuk diputar

(2) Motif B

B

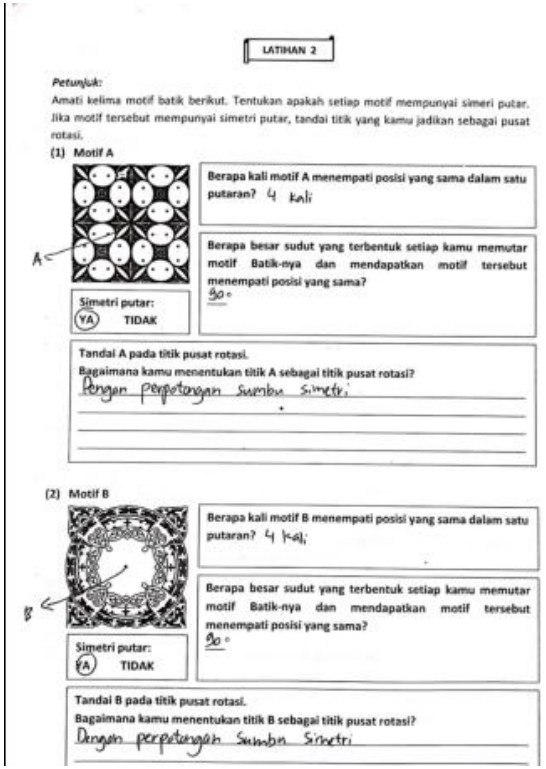
Berapa kali motif B menempati posisi yang sama dalam satu putaran? 4

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
90°

Simetri putar:
(YA) TIDAK

Tandai B pada titik pusat rotasi.
Bagaimana kamu menentukan titik B sebagai titik pusat rotasi?
karena titik B adalah titik pusat untuk diputar

Cycle 2



LATIHAN 2

Petunjuk:
Amati kelima motif batik berikut. Tentukan apakah setiap motif mempunyai simeri putar. Jika motif tersebut mempunyai simetri putar, tandai titik yang kamu jadikan sebagai pusat rotasi.

(1) Motif A

A

Berapa kali motif A menempati posisi yang sama dalam satu putaran? 4 kali

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
90°

Simetri putar:
(YA) TIDAK

Tandai A pada titik pusat rotasi.
Bagaimana kamu menentukan titik A sebagai titik pusat rotasi?
Dengan perpotongan sumbu simetri

(2) Motif B

B

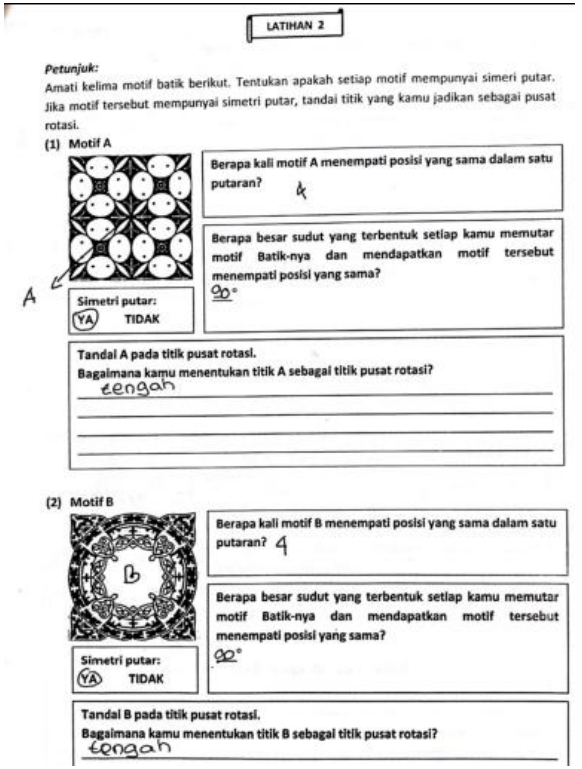
Berapa kali motif B menempati posisi yang sama dalam satu putaran? 4 kali

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
90°

Simetri putar:
(YA) TIDAK

Tandai B pada titik pusat rotasi.
Bagaimana kamu menentukan titik B sebagai titik pusat rotasi?
Dengan perpotongan sumbu simetri

Cycle 3



LATIHAN 2

Petunjuk:
Amati kelima motif batik berikut. Tentukan apakah setiap motif mempunyai simeri putar. Jika motif tersebut mempunyai simetri putar, tandai titik yang kamu jadikan sebagai pusat rotasi.

(1) Motif A

A

Berapa kali motif A menempati posisi yang sama dalam satu putaran? 4

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
90°

Simetri putar:
(YA) TIDAK

Tandai A pada titik pusat rotasi.
Bagaimana kamu menentukan titik A sebagai titik pusat rotasi?
tengah

(2) Motif B

B

Berapa kali motif B menempati posisi yang sama dalam satu putaran? 4

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
90°

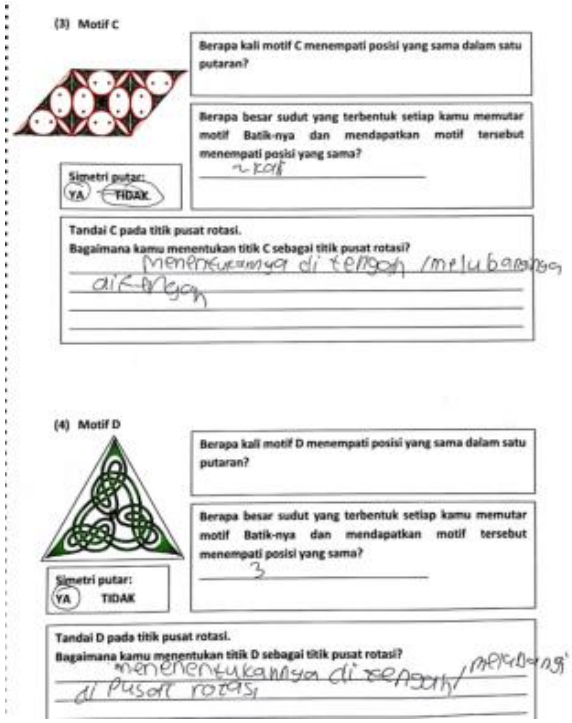
Simetri putar:
(YA) TIDAK

Tandai B pada titik pusat rotasi.
Bagaimana kamu menentukan titik B sebagai titik pusat rotasi?
tengah

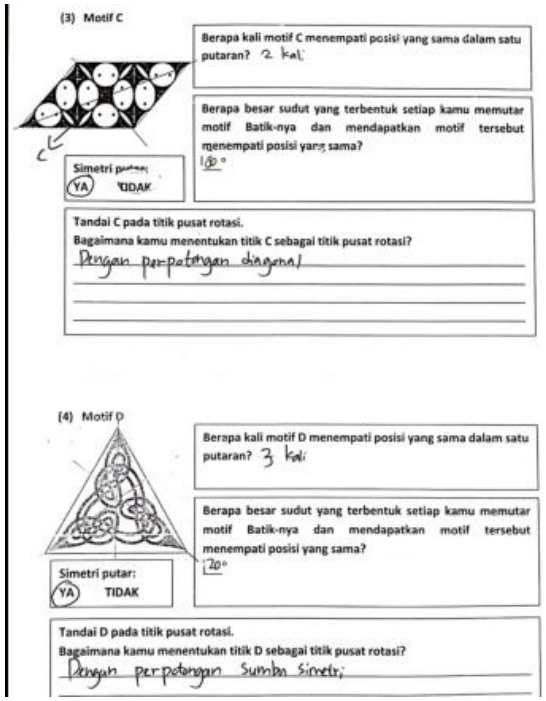
ACTIVITY 3

Part 2

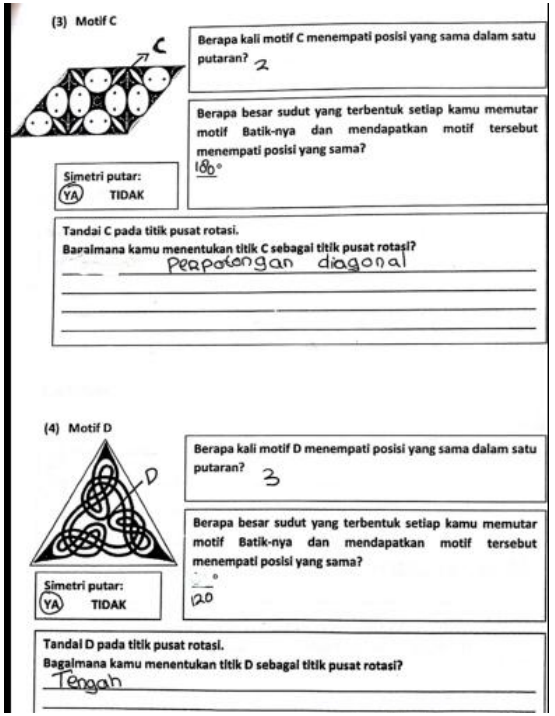
Cycle 1



Cycle 2



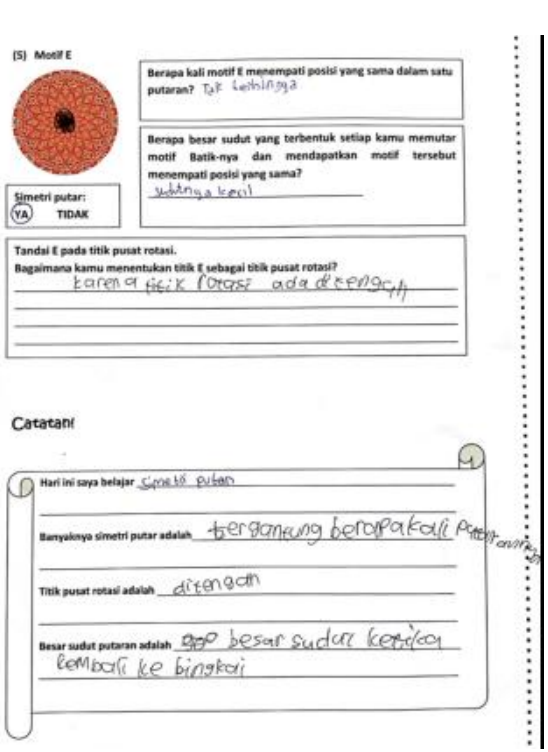
Cycle 3



ACTIVITY 3

Part 3

Cycle 1


(5) Motif E

Berapa kali motif E menempati posisi yang sama dalam satu putaran? Tak terhingga

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
sudutnya kecil

Simetri putar: (YA) TIDAK

Tandai E pada titik pusat rotasi.
Bagaimana kamu menentukan titik E sebagai titik pusat rotasi?
karena titik rotasi ada ditengah

Catatan!

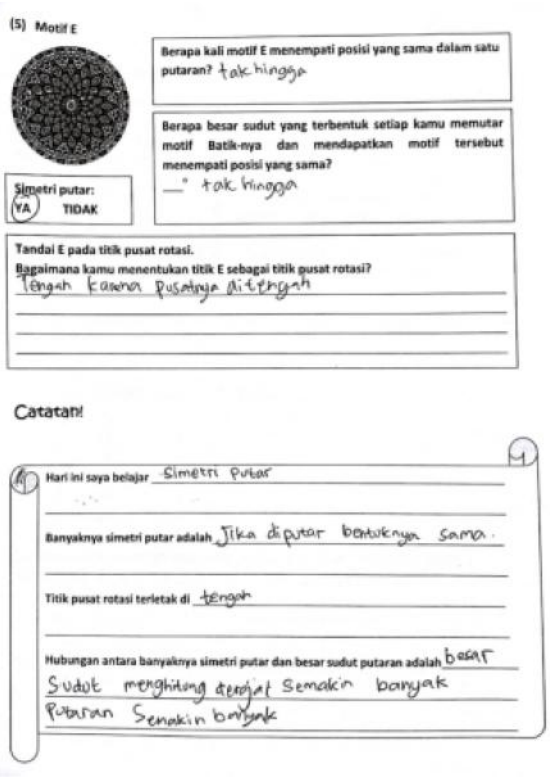
Hari ini saya belajar simetri putar

Banyaknya simetri putar adalah tergantung berapakali

Titik pusat rotasi adalah ditengah

Besar sudut putaran adalah ... besar sudut kembali ke bingkai

Cycle 2


(5) Motif E

Berapa kali motif E menempati posisi yang sama dalam satu putaran? tak hingga

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
... tak hingga

Simetri putar: (YA) TIDAK

Tandai E pada titik pusat rotasi.
Bagaimana kamu menentukan titik E sebagai titik pusat rotasi?
Tengah karena pusatnya ditengah

Catatan!

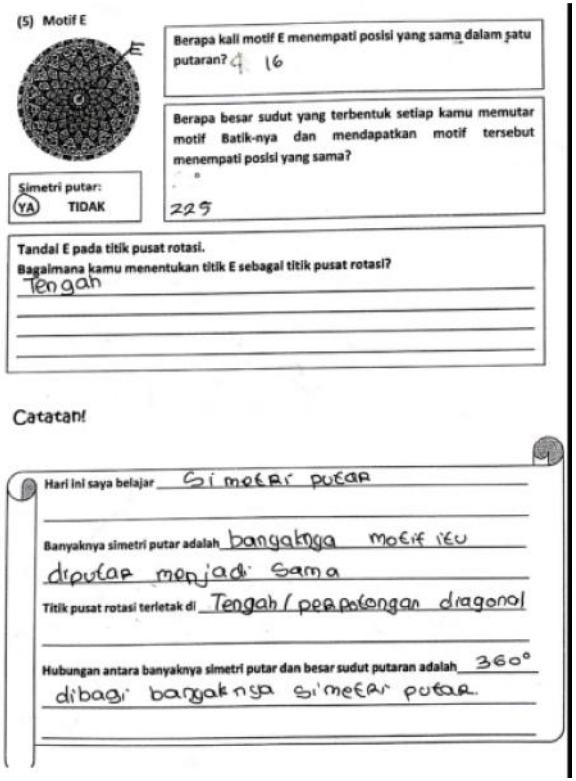
Hari ini saya belajar Simetri Putar

Banyaknya simetri putar adalah Jika diputar bentuknya sama.

Titik pusat rotasi terletak di tengah

Hubungan antara banyaknya simetri putar dan besar sudut putaran adalah besar sudut menghitung derajat semakin banyak putaran semakin banyak

Cycle 3


(5) Motif E

Berapa kali motif E menempati posisi yang sama dalam satu putaran? 16

Berapa besar sudut yang terbentuk setiap kamu memutar motif Batik-nya dan mendapatkan motif tersebut menempati posisi yang sama?
225

Simetri putar: (YA) TIDAK

Tandai E pada titik pusat rotasi.
Bagaimana kamu menentukan titik E sebagai titik pusat rotasi?
Tengah

Catatan!

Hari ini saya belajar Simetri putar

Banyaknya simetri putar adalah banyaknya motif itu dipotar menjadi sama

Titik pusat rotasi terletak di Tengah / perpotongan diagonal

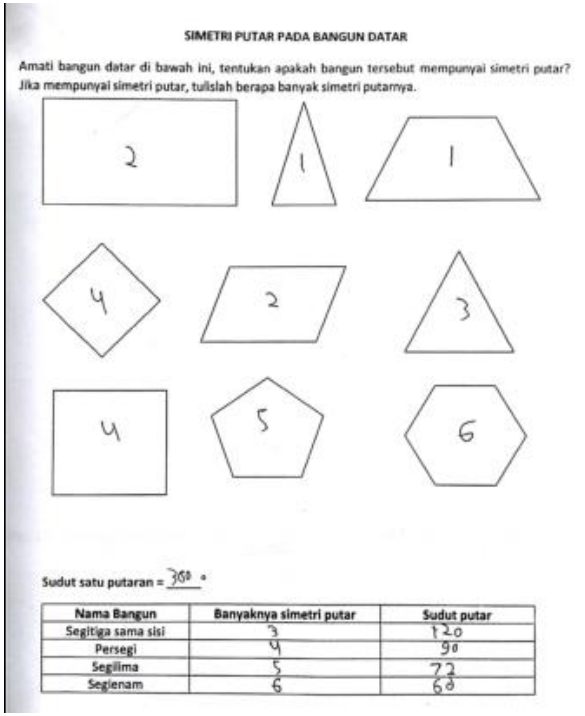
Hubungan antara banyaknya simetri putar dan besar sudut putaran adalah 360° dibagi banyaknya simetri putar.

ACTIVITY 4

Cycle 1

No students' written works

Cycle 2



SIMETRI PUTAR PADA BANGUN DATAR

Amati bangun datar di bawah ini, tentukan apakah bangun tersebut mempunyai simetri putar? Jika mempunyai simetri putar, tulislah berapa banyak simetri putarnya.

2 1 1

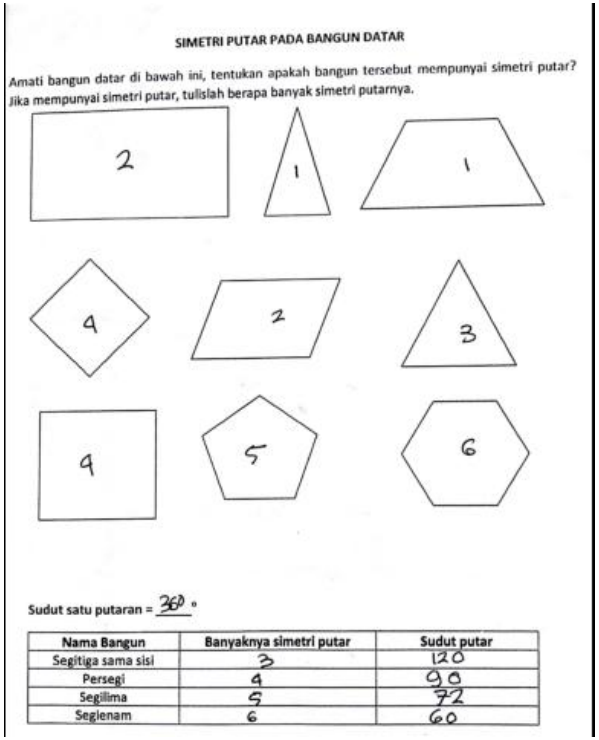
4 2 3

4 5 6

Sudut satu putaran = 360 °

Nama Bangun	Banyaknya simetri putar	Sudut putar
Segitiga sama sisi	3	120
Persegi	4	90
Segilima	5	72
Segienam	6	60

Cycle 3



SIMETRI PUTAR PADA BANGUN DATAR

Amati bangun datar di bawah ini, tentukan apakah bangun tersebut mempunyai simetri putar? Jika mempunyai simetri putar, tulislah berapa banyak simetri putarnya.

2 1 1

4 2 3

4 5 6

Sudut satu putaran = 360 °

Nama Bangun	Banyaknya simetri putar	Sudut putar
Segitiga sama sisi	3	120
Persegi	4	90
Segilima	5	72
Segienam	6	60

THE THIRD MEETING

ACTIVITY 1

Cycle 1

PAMERAN BATIK SIMETRIS

Tantangan I

Perhatikan motif berikut.

Apakah motif tersebut mempunyai simetri lipat atau/dan simetri putar?

Jika motif Batik tersebut tidak simetris, maka buatlah menjadi simetris

Kamu boleh memotong motif Batik yang tersedia dan menempelkannya di halaman yang sudah tersedia.

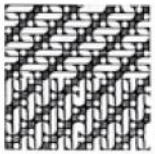
Cycle 2

PAMERAN BATIK SIMETRIS

Kelas = 5C

Tantangan I

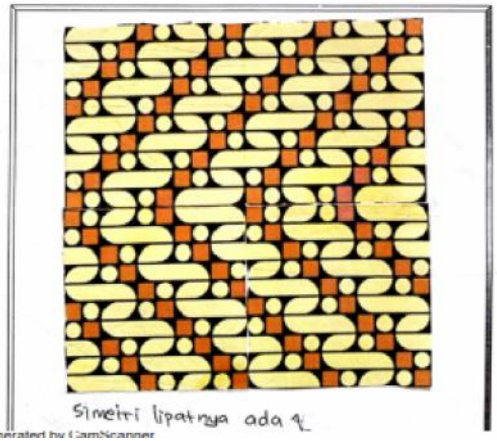
Perhatikan motif berikut.



Apakah motif tersebut mempunyai simetri lipat atau/dan simetri putar?

Jika motif Batik tersebut tidak simetris, maka buatlah menjadi simetris

Kamu boleh memotong motif Batik yang tersedia dan menempelkannya di halaman yang sudah tersedia.



Simetri lipatnya ada 4

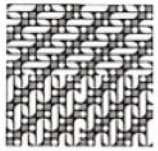
Cycle 3

PAMERAN BATIK SIMETRIS

Talitha khairiyah s.

Tantangan I

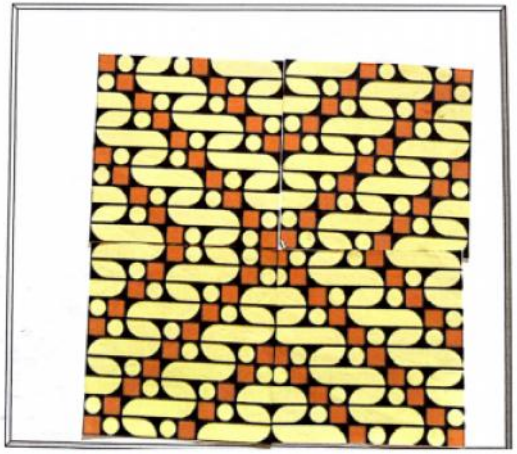
Perhatikan motif berikut.



Apakah motif tersebut mempunyai simetri lipat atau/dan simetri putar? Ya

Jika motif Batik tersebut tidak simetris, maka buatlah menjadi simetris

Kamu boleh memotong motif Batik yang tersedia dan menempelkannya di halaman yang sudah tersedia.

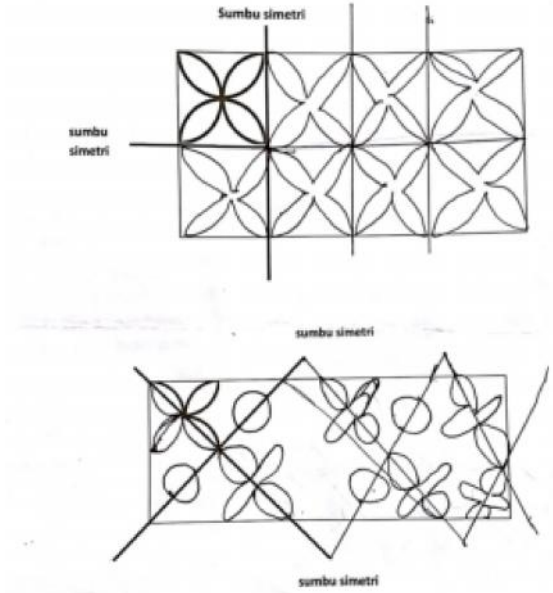


ACTIVITY 2

Cycle 1

Tantangan 2

Lengkapilah motif batik berikut sehingga bisa menjadi batik yang simetris. Perhatikan sumbu simetrinya.

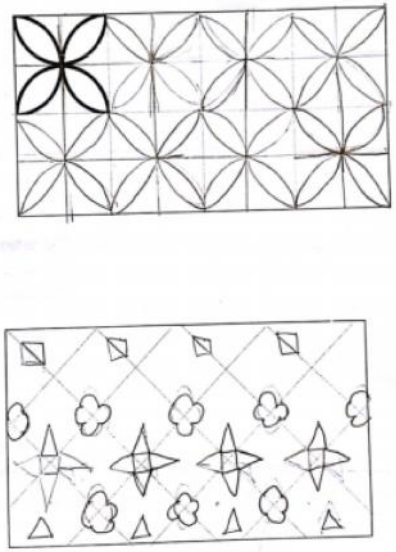


Cycle 2

Tantangan 2

Lengkapilah motif batik berikut sehingga bisa menjadi batik yang simetris. Perhatikan sumbu simetrinya. Sumbu simetri digambarkan dengan garis putus-putus

garis ..... menyatakan sumbu simetri



Cycle 3

Tantangan 2

Lengkapilah motif batik berikut sehingga bisa menjadi batik yang simetris. Perhatikan sumbu simetrinya. Sumbu simetri digambarkan dengan garis putus-putus

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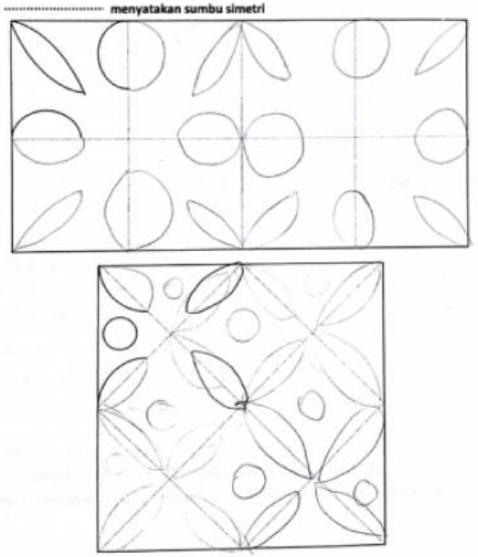
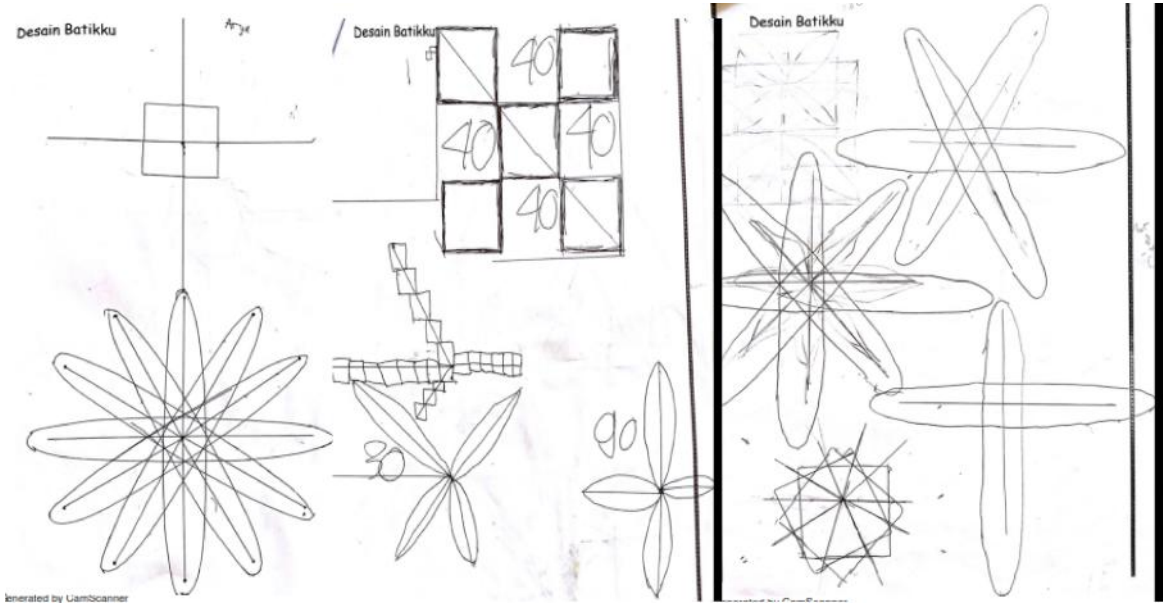
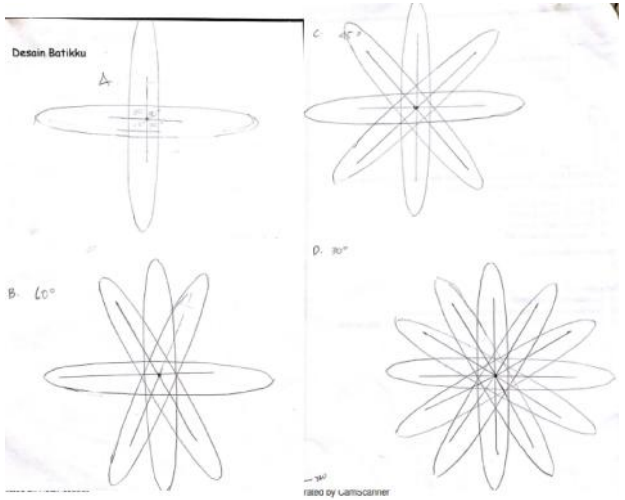


ACTIVITY 3

Cycle 1



Cycle 2



Cycle 3