FOSTERING STUDENTS' UNDERSTANDING ABOUT ANGLE AND ITS MAGNITUDE THROUGH REASONING ACTIVITIES

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program in Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:
BONI FASIUS HERY
NIM 06122802005

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
JUNE 2014

APPROVAL PAGE

Research Title	:	Fostering Students' Understanding about Angle and Its Magnitude through Reasoning Activities
Student Name	:	Boni Fasius Hery
Student Number	:	06122802005
Study Program	:	Mathematics Education

Approved by:

Prof. Dr. Zulkardi, M.I.Komp., M.Sc. Supervisor I	Dr. Darmawijoyo Supervisor II
Head of Mathematics Education Department, Sriwijaya University	Dean of Faculty of Teacher Training and Education, Sriwijaya University
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. NIP 19610420 198603 1 002	Sofendi, M.A., Ph.D. NIP 19600907 198703 1 002

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Approved by Examination Committee	Signature
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. **Sriwijaya University**	____________________
Dr. Darmowijoyo **Sriwijaya University**	____________________
Dr. Ratu Ilma Indra Putri, M.Si. **Sriwijaya University**	____________________
Dr. Yusuf Hartono, M.Sc. **Sriwijaya University**	____________________
Dr. Somakim, M.Pd. **Sriwijaya University**	____________________

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STATEMENT PAGE

I hereby:

Name : Boni Fasius Hery

Place of birth : Singkawang

Date of birth : August 8, 1987

Academic Major : Mathematics Education

State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
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This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

Palembang, June 2014
The one with the statement

Boni Fasius Hery
NIM 06122802005

ABSTRACT

The purpose of this study is to develop an innovative educational design to support seventh-grade students' learning about angle and its magnitude. This thesis reports on the outcomes of the three cycles of teaching experiments and their impact to the design and students' understanding toward the learning geometry. Angle situations that commonly encounter by students were selected as the contexts and Realistic Mathematics Education (RME) was employed as the design heuristic of the educational design. Design research was considered as the appropriate research approach to investigate how the design that consists of five lessons can help students to comprehend the important concepts of angles through reasoning activities. The data such as, the collection of students' written work, pre and post-test, interview with students, and video recording from the whole teaching experiments were analyzed using task-oriented method to continually improve the prediction power of the design. The results from the analysis suggest that the used of everyday-life angle situations in the teaching experiments could help the students to retrieve their prior-knowledge about angle, negate their misconceptions about angle and allow them to reinvent the relation between angles magnitudes in a parallel-transversal situation. It is shown how production tasks and reasoning activities supported the learning of important concepts of angles and its magnitude. In the teaching experiments, several students came to reason about the angle magnitude using informal measurement, overlapping and reshaping strategy.

KEY WORDS: *innovative educational design, realistic mathematics education, design research, angle, everyday life angle situations, reasoning activity*

ABSTRAK

Tujuan utama dari penelitian ini adalah untuk mengembangkan suatu desain pembelajaran inovatif guna mendukung siswa kelas tujuh dalam proses pembelajaran materi sudut dan ukurannya. Tesis ini melaporkan hasil dari tiga siklus pembelajaran serta pengaruhnya pada desain dan pemahaman siswa pada materi pembelajaran. Sudut dalam keseharian siswa digunakan sebagai konteks dan Realistic Mathematics Education (RME) dipilih sebagai acuan untuk mendesain pembelajaran. Design research dianggap sebagai pendekatan penelitian yang paling cocok untuk mengidentifikasi bagaimana desain yang dibuat dapat membantu siswa memahami konsep-konsep penting materi sudut melalui kegiatan bernalar. Data-data seperti hasil kerja siswa, pre-tes, post-tes, wawancara, dan rekaman video pembelajaran dianalisis dengan menggunakan metode 'task-oriented' guna secara berkelanjutan meningkatkan aspek prediktif dari desain. Hasil analisis menyarankan penggunaan konteks dari keseharian siswa dalam proses pembelajaran dapat membantu siswa mengingat kembali konsep sudut yang telah mereka pelajari, meluruskan kesalahan-kesalahan konsep mereka, dan membuat siswa menemukan kembali hubungan sudut-sudut bersesuaian. Telah ditunjukkan bagaimana kegiatan mencipta dan bernalar mendukung pemahaman siswa pada materi sudut dan ukurannya. Pada kegiatan pembelajaran dalam penelitian ini, beberapa siswa dapat menentukan ukuran sudut dengan menggunakan strategi pengukuran informal, strategi overlapping, dan strategi menyusun ulang.

KATA KUNCI: *desain pembelajaran inovatif, Realistic Mathematics Education, design research, sudut, sudut dalam kehidupan sehari-hari, kegiatan bernalar*

SUMMARY

In Indonesia, the concepts of angle and line are introduced simultaneously to the seventh graders. It is common for the teachers to begin the lesson by telling the definitions of angle and line to the students. Although it seems reasonable since the students have learnt about the definitions in primary school. They still need large amount of supports from their teacher in order to be mathematically mature to learn the further concepts in this subject matter. The further concept that students should learn after recalling the definitions is the concept of angle magnitude. Unfortunately, the teacher still uses the same approach to teach the concept of angle magnitude. The use of production tasks are rarely proposed compare with reproduction and comparison tasks. This makes the occurrence of students' misconceptions toward the subject matter is inevitable. Therefore, it raises the need to develop an innovative educational design that allows students to build the adequate knowledge about angle and its magnitude.

This study investigates on how a teaching and learning sequence that employs the selected angle situations can help students to understand the definitions of angle, comprehend the important concepts of angles, and grasp the sense of angle magnitude. Everyday-life angle situations were selected as the contexts and design research was selected as the research approach. An educational design that consists of five lessons was developed using Realistic Mathematics Education (RME) as the design heuristic. The design was applied in three cycles in SMPN 17 Palembang, where there were 52 seventh-grade students and their teacher involved. There were 6 students in the first cycle, 40 students in second cycle, and 6 students in the third cycle were involved for the advancement of the hypothetical learning trajectory.

The data such as, the collection of students' written work, pre and posttest, interview with students and teacher, and video recording from the whole teaching and learning process were analyzed using task-oriented method. Those data could help us as the educational designers to gain more understanding on how students

perceive this knowledge. The results from the analysis shows that the used of everyday-life angle situations in the teaching experiments could help the students to retrieve their prior-knowledge about angle, negate their misconceptions about angle and allow them to redefine the angle definitions. It is showed from the reasoning activities and production tasks enabled students to acquire the adequate knowledge about angle and its magnitude. The results of this study could help us as the educational designers to gain more understanding on how students perceive this knowledge.

RINGKASAN

Di Indonesia, konsep sudut dan garis diperkenalkan kepada siswa kelas VII. Biasanya guru mengawali pembelajaran dengan menyampaikan definisi sudut dan garis kepada siswa. Meskipun terlihat beralasan karena siswa telah mempelajarinya di sekolah dasar. Siswa masih membutuhkan banyak bantuan dari guru untuk memahami konsep sudut dan garis lebih lanjut. Konsep lanjutan yang harus dipelajari oleh siswa setelah memahami definisi adalah konsep ukuran sudut. Sayangnya, guru masih menggunakan pendekatan yang sama untuk menyampaikan konsep besaran sudut. Seringnya siswa hanya mengkonstruksi ulang tanpa disertai dengan kegiatan mencipta. Hal ini menyebabkan kesalahan konsep pada siswa tidak terelakkan. Oleh karena itu diperlukan suatu desain pembelajaran yang inovatif yang diharapkan mampu membangun pemahaman siswa tentang konsep sudut dan ukurannya.

Penelitian ini menginvestigasi bagaimana kegiatan pembelajaran yang menggunakan konteks sudut dalam kehidupan sehari-hari dapat membantu siswa untuk memahami definisi sudut, memahami konsep-konsep penting tentang sudut dan memahami ukuran sudut. Design research dipilih sebagai pendekatan penelitian. Desain pembelajaran yang dikembangkan terdiri dari lima aktifitas pembelajaran menggunakan pendekatan RME (*Realistic Mathematics Education*). Desain pembelajaran ini diterapkan dalam tiga siklus di SMPN 17 Palembang yang melibatkan 52 siswa kelas VII beserta gurunya. Sebanyak 6 siswa terlibat dalam siklus pertama, 40 siswa pada siklus kedua dan 6 siswa lainnya pada siklus tiga untuk pemantapan *hypothetical learning trajectory*.

Data-data seperti hasil kerja siswa, pre-tes, post-tes, wawancara dan rekaman video dari seluruh kewgiatan pembelajaran dianalisis dengan menggunakan metode *task-oriented*. Data tersebut digunakan untuk memahami lebih dalam lagi bagaimana siswa memahami konsep yang diajarkan. Hasil dari analisis menunjukkan bahwa penggunaan konteks sudut dalam kehidupan sehari-hari pada kegiatan pembelajaran dapat membantu siswa untuk mengingat kembali konsep sudut sebelumnya, meluruskan kesalahan konsep, dan mendefinisikan

ulang definisi sudut. Kegiatan mencipta dan bernalar membantu siswa untuk menguasai konsep sudut dan ukurannya. Hasil dari penelitian ini dapat membantu para desainer pembelajaran untuk memahami lebih jauh lagi bagaimana siswa memahami konsep sudut dan ukuranya.

"I don't feel frightened by not knowing things.
By being lost in the mysterious universe without having any purpose.
Which is the way it really is, as far as I can tell."

Richard P. Feynman

For Cong Chi Cin and Emiliana

The most inspired parents in the entire universe.

PREFACE

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- SMPN 17 Palembang

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I am fully aware that I cannot mention all parties that have impacts to this study one by one, but for their kindness, I say thank you.

Palembang, June 2013

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CHAPTER 1
INTRODUCTION

In order to make students remember the definition and the concepts of the angle in a traditional mathematics classroom seems to be a fairly simple activity. For instance, the teacher displays several figures of regular polygons, claims the angle as the sub-figure of each polygon (the vertices), diagrammatically explains the definitions of angle and uses a protractor to make sense the magnitude of angle. There are so many ways to teach the students about the angle in a traditional mathematics classroom, however the idea is the same; start from an abstract domain and hope the students can apply this knowledge to any given situations. Unfortunately, the students interpreted this knowledge in so many different ways and a traditional teaching approach couldn't help us to gain a better understanding about how the students learn the concepts (Keiser, 2004; Mitchelmore and White, 2000; Devichi and Munier, 2013).

Keiser (2004) claimed that this approach allowed the concept to be introduced quickly but it robbed students' opportunities to experience angles that could help them to be more flexible on this area. Telling the definitions to the students is a typical approach in a traditional mathematics classroom, which Mitchelmore and White (2000) confirmed by stating that the definitions of angle are unlikely to help the young students. In addition to that, Devichi and Munier (2013) stated that production tasks are relevant to identify pupils' representations of the concept of angle. However, these tasks are rarely proposed in the traditional mathematics classroom, which is mainly based on reproduction and comparison tasks. In a reform mathematics classroom, the teacher does it in the reverse way; start from several concrete situations and guide the students progressively to make generalizations and abstractions of the situations.

Several studies have showed that many students still struggled in perceiving the concepts of angle (Munier and Merle, 2009; Devichi and Munier, 2013; Keiser, 2004; Mitchelmore, 1997). For example, Keiser (2004) in his study on comparing sixth-grade students' discourse to the history of the angle concept found many students were confuse about the angle concepts. For instance; the students thought that a sharper angle was the larger angle in turning contexts, some thought that the longer the rays the greater the measure of the angle was, others thought that the more space between the rays the larger the angle was, and some really struggled to adapt their concept image for angle so that it could include specifically the 0°, 180° and 360° angles. In addition to that, a study conducted by Mitchelmore and White (2000) revealed an interesting finding that even with a contextual classroom environment there is still a significant proportion of students who could not make the connection between the angles concepts.

The concrete situations that were used by the researchers in those studies differ from each other. Mainly they are related to intersection, corner, bend, slope, turn, and rotation to put the angle concepts into a context. Those studies stressed their attention on how the elementary students perceived the definitions of angle relate to the angles situations that presented. However, further analysis on how students comprehended the concept of angle magnitude seems not enough, especially in the secondary level. In the secondary level, the students learn about the magnitude of angles by studying the proposition 29 in book 1 of Euclid's Element. They study this knowledge in rather formal way. Usually, the teacher display a straight line that falling across two parallel lines, claims that the alternate angles are equal to one another, tells the students all the possible consequences of this condition, and drills the students with problems. This less context approach tells us very little about students' understanding toward the knowledge.

There are several important findings that can justify the use of contexts in learning about the angle. However, some contexts may produce the intended outcomes but other may not, depend on many external factors. An example from Mitchelmore's study about children's informal knowledge of physical angle situations (1997) found that some specific features of each angle situation strongly hindered recognition of the common features which define the angle concept (e.g. in turns context, and size of small angles involves the fraction concept). Therefore, the finding suggested that we as the educational designers had to be very careful in selecting the contexts of angle in order to maintain the obviousness of the concepts.

Of course we cannot be absolutely sure about which angle situations that can be used to create the best learning environment for the students. However, we still can carefully chose and calibrate the angle situations that can provide the students with a meaningful learning environment and give them the opportunity to gain the intended knowledge. Devichi and Munier (2013) suggested that it would be interesting to analyze the link between the type of angle produced and the ability to change its size in countries where the right angle, the other angles, and the measurement of angles are introduced simultaneously. Indeed in Indonesia, these concepts are introduced simultaneously as it is clear from the national curriculum and the standard mathematical text books that have been used recently. However, Indonesia still lacks of studies that intensively focus on the effectiveness of an innovative educational design that employs the angle situations. In particular, the educational design that aimed to investigate students' comprehension about angle and its magnitude in the secondary school level.

The aims of this study are to investigate how a teaching and learning sequence that employs the selected angle situations can help students understand the definitions of angle, grasp the sense of angle magnitude, and comprehend the important concepts of angles. We are also interested in analyzing the aspects from the selected angle situations that have the positive impacts on the students, and we want to contribute to mathematics education literature by providing ideas in teaching and learning activities about angle and its magnitude in the secondary school level. Therefore, the research question of this study formulated as follows.

"*How can we support 7th graders to comprehend the magnitude of angles through reasoning activities?*"

CHAPTER 2
THEORETICAL BACKGROUND

This chapter highlights the framework of thinking that will be used in the process of designing a lesson sequence in order to understand how students perceive the angle and its magnitude. This chapter begins with a mathematical overview of angle concepts that is commonly used in the mathematics education domain and several related studies on this area. The purpose of reviewing the angle concepts is to emphasize the fact that the concepts have several interpretations depending on what aspect of angle we stress. This chapter continues to describe students' knowledge about the angle. It highlights aspects that we have already known from previous studies about numbers of difficulties encountered by students. We review the practical aspects of those studies in a classroom context in order to get some ideas for designing our lesson sequence.

We also explain how realistic mathematics education (RME) is used to ground the development of the design. The RME is needed in order to investigate and to explain how the learning activities in the lesson sequence help the students to comprehend the intended mathematical concepts. Since the study was conducted in Indonesia, this chapter provides a general overview of the concepts of angle in the Indonesian curriculum as well. At the end of this chapter, we also describe the research aim and research questions of this study.

2.1 Different conceptions of angles

According to Sbaragli and Santi (2011, p. 15), there are 8 definitions of angle based on the interpretation of Euclid and one definition from Hilbert. However, it is not favorable for this study to analyze the nine interpretations in order to investigate how seventh graders perceive the angle and its magnitude. Therefore, we use Schotten's classification of the definitions that concentrates mostly on three particular classes of definitions of this concept: angle as the portion of a plane included in between two rays in the plane which meet in a point, angle as the difference of direction between two rays, and angle as the amount of turn/rotation between two rays (Schotten, 1893, pp. 94–183; cited by Dimitric, 2012). In this part, we will discuss the three groups of definitions in general.

2.1.1 Angle as the space in between two lines in the plane which meet in a point

Euclid's elements of geometry is one of the most influential texts in geometry that has ever written. It covers almost all important concepts in plane geometry that we still use today. The first description of the concepts of angle in this text is in Book I, definition 8-12:

> 8. And a plane angle is the inclination of the lines to one another, when two lines in a plane meet one another, and are not lying in a straight-line.
> 9. And when the lines containing the angle are straight then the angle is called rectilinear.
> 10. And when a straight-line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle, and the former straight-line is called a perpendicular to that upon which it stands.
> 11. An obtuse angle is one greater than a right-angle.
> 12. And an acute angle (is) one less than a right-angle.

One of the interesting properties of the angle in this book is the two lines are not lying in a straight-line. The logical consequences of this property are there will be no zero angles, straight angles, or any angles that are bigger than a straight angle. Lo, Gaddis, and Henderson (1996) reported that in several plane geometry texts in the Cornell library, the definition interpreted angle as the space between two lines. As Freudenthal (1973) explained, Euclid takes the liberty of adding angles beyond two or even four right angles; the result cannot be angles according to the original definitions. Although the students can immediately see the angles as the space in between two lines, but understanding the angles in this way can result in ambiguity when the arms of the same angles are of a different length. In addition to that, it may result incompleteness in students' understanding about the magnitude of angles.

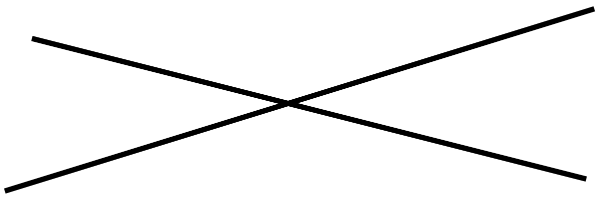


Figure 2.1. Diagrammatic interpretation of angle as the spaces between two lines.

2.1.2 Angle as the difference of direction between two lines

A well-known German mathematician, David Hilbert (1902, p. 8) defined the angle in his Foundation of Geometry as follows:

> Let α be any arbitrary plane and h, k any two distinct half-rays lying in α and emanating from the point O so as to form a part of two different straight lines. We call the system formed by these two half-rays h, k an angle and represented it by the symbol $\angle(h, k)$ or by $\angle(k, h)$.

This definition is clear and straightforward in defining angles that are less than $360°$. Defining the angle in this way may overcome students' perplexity that is caused by the length of the arms that form the angles that occur when we define the angle as the portion of a plane included in between two rays in the plane which meet in a point. However, it happen that the students don't realize the existence of reflex angle because they might focus solely on the angle that less than $180°$. We are fully aware that the definition has its own limitations in order to explain the angles that are larger than $180°$ and to make sense the existence of vertical angles.

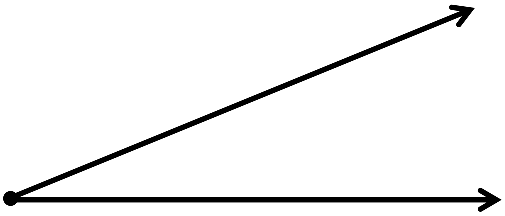


Figure 2.2. Diagrammatic interpretation of angle as the difference of direction between two lines.

2.1.3 Angle as the amount of turn between two lines

Angles have been defined as the amount of rotation necessary to bring one of its rays to the other ray without moving out of the plane (Kieran, 1986; cited by Clements and Burns). This definition fills the gap from the previous definitions of angle by allowing the students to be aware of the existence of a straight-angel and angles that are bigger than 180°. Presumably, introducing this dynamic angle situation may be too early for the students if they do not have sufficient experiences about the angle and its magnitude.

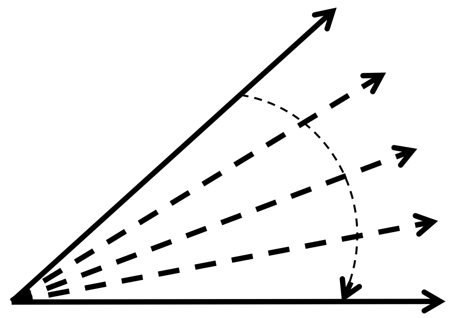


Figure 2.3. Diagrammatic interpretation of angle as the amount of turn between two lines.

Making a definition that can covers all the crucial aspects from the concept of angle is a difficult task due to all definitions have their own limitations in describing the concept by emphasizing one aspect more heavily than others (Keiser, 2004). The teacher may have one or more definitions at hand before s/he enters the classroom. It will be excellent if s/he knows the three definitions in order to anticipate students' reactions in the teaching and learning process.

2.2 Students' knowledge about angles

In this part of the chapter, we will identify four main difficulties encountered by the students in the process of knowledge acquisition of angle and its magnitude that we have already known from the previous studies.

2.2.1 Students' tendency to see the length of arms affects the angles magnitudes

It seems to be a global tendency of students' misunderstanding about the definition of angle that the students seem to associate the magnitude of an angle with the length of its arms (Mitchelmore and White, 1998; Munier and Merle, 2009; Keiser, 2004; Sbaragli and Santi, 2011). In this case, the students judge that the length of the arms of an angle affects the magnitude of the angle. Moreover, according to a study conducted by Munier and Merle (2009) this difficulty exists irrespective of the country, and appears to be relatively hard to overcome.

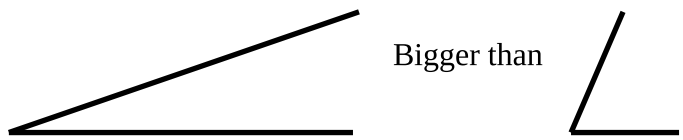


Figure 2.4. The length of arms affects the magnitude of angles.

2.2.2 Students' tendency to see sharper angles as the larger angles

In a study conducted by Keiser (2004) he highlighted the similarities between sixth-grade students' developing notions of angle and mathematicians' struggles to define the complex concept of angle. On the fourth day of his study, the teacher posted a story about a triangle that iteratively added new sides to become a 4-gon, a 5-gon, and so on. As was expected, the angles of each new shape increased in magnitude. The teacher then invited the students to a classroom discourse and found that some of the students were confused about the sharpness of the vertex and the magnitude of the angle. They claimed that, the sharper the vertex, the bigger the angle.

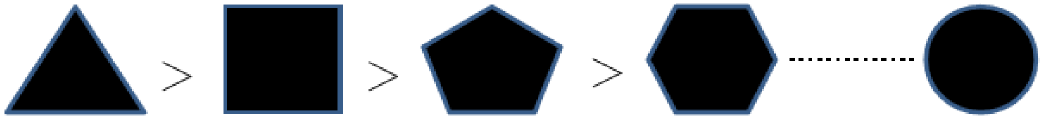


Figure 2.5. The sharper the vertex, the bigger the angle.

2.2.3 Students' difficulties in identifying a right-angle that does not have one horizontal arm

Some students showed a tendency only to recognize the right-angle in some special orientation, and often do not recognize the right-angle anymore if it is displayed in a different orientation. Several studies in France have shown this tendency. For example, some adults in France still struggled for identifying right-angles that did not have at least one horizontal arm (Browning et al., 2007, p. 286; cited by Devichi and Munier, 2013). Another interesting finding related to the right-angle is that when students, especially young ones, were asked to draw an angle they usually drew a right-angle (Baldy et al., 2005; cited by Devichi and Munier, 2013).

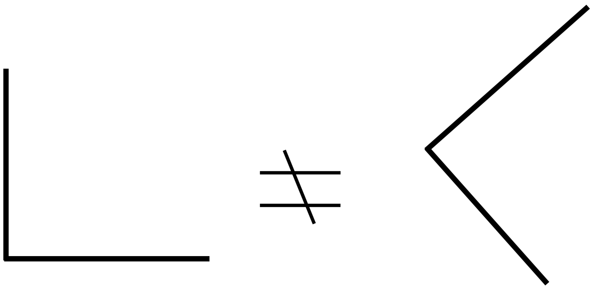


Figure 2.6. The right-angle that doesn't have a horizontal ray doesn't consider to be a right-angle.

2.2.4 Students' difficulties in perceiving 0°, 180°, 270°, 360°, or larger angles

Keiser (2004, p. 300) had shown that students still encountered difficulties when perceiving special angles such as 0°, 180°, 270°, 360°, or ones even larger. He claimed that it might be the result of the students' conception of the angles as the distance between two rays. This is not surprising, since the nature of the definition itself doesn't allow any angle that is greater than or equal to 180°.

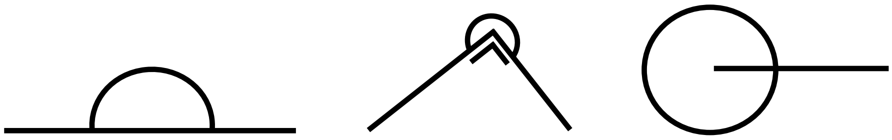


Figure 2.7. Special angles.

2.3 Promoting learning about angles

Several studies on this area utilized the power of contexts in making a meaningful learning environment to promote students' learning about the concept of angle. Mitchelmore and White (2000) for example utilized real world objects that were commonly associated with or have strong relations with the attribute of angles, such as: a wheel, door, scissors, fan, signpost, hill, junction, tile and wall. Their study revealed that there is a hierarchical relationship between students' recognition of angles and their grade level as is shown in figure 2.8. Furthermore, they claimed that the students' conception of angle develops from a physical angle domain and grows steadily to more abstract concepts of angle.

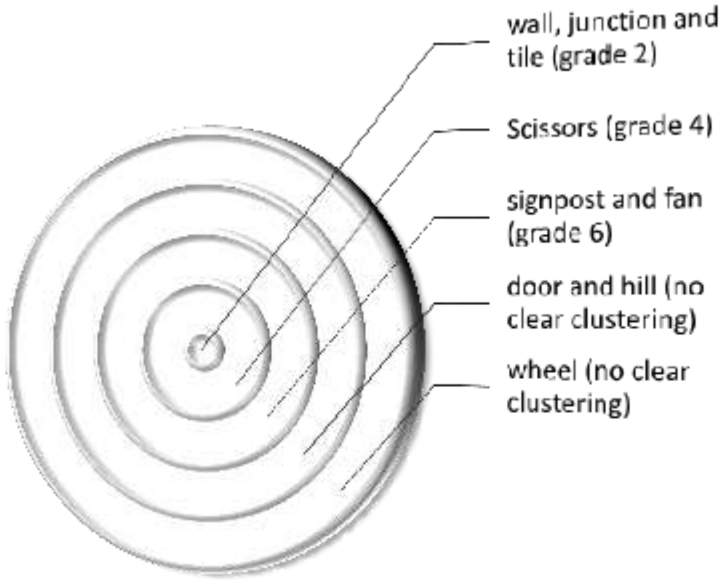


Figure 2.8. The hierarchical of students' recognition of angle.

Munier and Merle (2009, p. 1889-1891) investigated how their designed teaching sequence supported students' understanding about the concepts of angle. They employed three learning contexts; a mirror, compass, and visual field. In the mirror sequence, the students had to determine which of three objects would be lit up by a reflected ray from a mirror that was put in a 'random' orientation. In the compass sequence, the students had a map showing the position of a treasure and had to explain how to find it to two other children. This activity requires students to apply the triangulation principle using the azimuth that is displayed on the compass. In the visual field sequence, the students were told that a child was seated facing a screen, but they were not given a diagram. They had to state whether the hidden area would get bigger, smaller, or stay the same when the distance between the child and the screen changed, or when the screen was made wider. Munier and Merle (2009) found that the visual field sequence which brings out the sector conception of the angle is beneficial to grade 3 students. However, the mirror and compass sequence appear to be more complex, which suggests that they might be more suitable for students in grade 4 or 5.

Similar to Munier's and Merle's study, Bustang (2013, p. 128-129) used visual field activities to promote students' learning about angle in grade 3. In his study, he found that the activities made the concept of angles meaningful for students and it is breakaway from the conventional teaching method that does not allow students to experience physical situations.

Fyhn (2008) studied students in a higher grade level (grade 7) in recognizing the largest and smallest angles via an indoor climbing activity. She gave three examples of how the students mathematized the climbing activity into the concept of angle; the students could recognize the angles even with only one visible side, could recognize the acute angles, and could recognize the dynamic aspect of angles.

The findings in those studies converge to explain the power of contexts in teaching and learning about angles. The use of contexts and a meaningful learning environment has been used in realistic mathematics education (RME) for decades. The use of contexts in the teaching and learning process plays an important role in successful learning outcomes. Therefore, in the present study we use the context in each part of the learning sequence following one of the RME's characteristics.

2.4 Realistic mathematics education (RME)

In order to explain and investigate how the lesson sequence that we developed in this study helps the students to understand the angle and its magnitude, we use the domain specific instructional theory on the teaching and learning of RME as a heuristic approach. Here we apply the five characteristics of RME that Treffers (1987) described as a framework of thinking about the process of designing the learning sequence. The five characteristics are; the use of context, model, students' own productions and constructions, interactivity, and intertwinement of various learning strands.

2.4.1 The use of contextual problems

Gravemeijer (1994, p. 105) described contextual problems as situations where an everyday life problem was posed. However, the problems are not necessarily to be everyday problems; for the more advanced students mathematics itself will become a context. Therefore, our task is to find the phenomena, contexts, or problem situations about angles that beg to be organized by mathematical means. In order to accomplish this task, in this study we analyze how mathematical knowledge about the concept of angle can help the students in organizing and structuring the real phenomena that relate to it.

In the beginning of every lesson in this study, the contexts are presented explicitly to the students. The contexts that we select are relatively real in students' mind. For instance, in the first lesson we use everyday objects that are strongly related with the attributes of the angle to be investigated by the students. We expect that they can reformulate their own definitions of angle from the context. We also employ hand-on activity and mathematical explorations in the next teaching and learning process to make the topic accessible and meaningful to the students. For instance, we ask the students to construct the upper case letters using matchsticks and then analyze the angles in the letters to make the students grasp the sense of magnitude and similarity of angles. In addition to that, we give the students a mathematical exploration of the angles in the tiled floors in order to allow them to get further justifications and advance their knowledge about angle and its magnitude.

2.4.2 The use of model

Here, the model can be interpreted as a process of concretized expert knowledge. The idea of using a model is to make the abstract concepts concrete in order to make it easier to grasp (Gravemeijer, 2004). A model plays an important role in the process of abstraction. It acts as a bridge between real-world situations and the intended mathematics concepts. Therefore in the present study we develop the models to support students' understanding about the magnitude of angle. For example, we use wooden matchsticks and tiled floors to represent the angles and its magnitude. From the activity, we expect the students to progressively develop more abstract understanding about the concepts of letters-angles (F, X and Z angles).

The models in the present study are used to support students mathematizing the concept of angle from everyday life situations. In RME this process is called mathematization. Treffers (1987) formulated the idea of two types of mathematization; horizontal and vertical. Horizontal mathematization was related to the applied aspect of mathematics (translating the real-world context into a mathematical model or vice versa), and vertical mathematization was related to the pure aspect of mathematics (abstracting the mathematical model into mathematical objects, structures, or methods). One example of horizontal mathematization in this study can be seen in the learning activity of reconstructing the top view of railways where the students use lines and angles in the drawing process. Here, the students translate the real-world context (the railway) into a mathematical model of it (top view of the railway). The vertical mathematization appeared in the activity is the students use their drawings to construct a mathematical structure of similar angles on a straight line (transversal line) that falling across two parallel lines.

2.4.3 Using students' own construction

An ideal condition happens when the students solve a mathematical problem is that they can develop their own strategies to tackle the problem. The role of the teacher in this context is to support the students to progressively escalate the strategies. The students own productions in each learning activity can be used as a valuable source in conducting a fruitful classroom discourse. By conducting the classroom discourses in this way, the teacher can maintain the meaningfulness of the discussions, because the students may attach personal value to their own constructions. Therefore, in this study we suggest to the teacher to provide the students with a room to discuss their own work, strategy, and ideas.

2.4.4 Interactivity

Like any other social interaction, the teaching and learning process involves extensive communication in order to make it effective. In this study, the communication in forms like; negotiating, arguing, and explaining are fostered by the teacher in an intensive way. In this study, classroom discussions are considered to be the core aspect in fostering students' development in the learning process.

2.4.5 Intertwinement

Intertwinement suggests the integration of several mathematics topics in one classroom activity. The concept of angle has strong relations with the concept of line. This means that when one learns about the angle s/he learns about the line simultaneously. Therefore, in this teaching and learning activity we also support the emergence of the concept of line in every lesson.

2.5 The concepts of angle in Indonesia

The concept of angle in the Indonesian curriculum is introduced to the students in the early stages of their mathematics career and then continues to increase in complexity until grade 12. One can immediately see how the Indonesian curriculum gives great appreciation toward this topic. Table 2.1 describes the concept of angle in the Indonesian curriculum chronologically. As we can see from table 2.1, the concepts of angle occur almost in every grade and increase in complexity. However, this study will focus solely on seventh grade students. In grade seven the concept of angle is taught simultaneously with the concept of line. The focus of the teaching and learning in this stage is mainly to make the students understand the relations of angles that are formed by a straight line that is falling across two parallel lines.

Table 2.1. Angle in the Indonesian Curriculum

Grade	**Semester**	**Topic (including angle)**	**Sub-topic**
Second	Even	The parts of simple plane figures	Identifying the angles on the simple plane figures
Third	Even	The types and the sizes of angle	Identifying the angles from several objects Explaining the angle as the space in between two intersecting lines. Ordering the angles based on their sizes Identifying and reproducing three types of angles (acute, right-angle, and obtuse) Identifying angle as rotation and constructing full rotation angle, half rotation angle, and one fourth rotation.

Table 2.2. Angle in the Indonesian Curriculum (Continued)

Fourth	Odd	Measurement	Angle measurement
Fifth	Odd	Using time, angle, distance, and speed in problem solving situation.	Conducting angle measurement
Seventh	Even	Line and angle	Defining angle and their unit of measurement Types of angle Arithmetic operation on angles Redrawing angles using a ruler and compass Right-angles and straight-angles Angles that are formed by parallel lines cut by transversal lines Measuring angles and drawing special angles using a ruler and compass Bisecting angles using a ruler and compass
Eighth	Even	Circle	Inscribed angles
Ninth	Odd	Similarity on plane figures	Embedded throughout the topic
Eleventh	Odd	Trigonometry	Embedded throughout the topic
Twelfth	Odd	Vector	Embedded throughout the topic
	Even	Geometric transformations	Embedded throughout the topic

2.6 Research aims and research questions

The intention of this study is to develop an innovative teaching and learning activity about angle and its magnitude in secondary school level. Since, lack of study that focuses on this topic, this study offers a new insight on this area. It also can give a valuable idea for an educational designer in designing an educational material of this topic. In addition to that, this study will widen the scope of the PMRI (Indonesian RME) to the secondary school level that recently studied the topics in primary school level.

As it has stated before, the aims of this study are to investigate how a teaching and learning sequence that employs the selected angle situations can help students understand the definitions of angle, comprehend the important concepts of angles, and grasp the sense of magnitude of angles. In order to accomplish these aims and answer the research question, we attempted to answer the following sub-research questions.

1. *How do 7^{th} graders define the angle from the everyday life objects that strongly related to the angle?*
2. *How does the alphabets reconstruction activity using wooden matchsticks allow the students to infer the similarity between angles on a straight line that is falling across two parallel lines?*
3. *How does the gaps patterns between tiles can help the students to advance their idea of similarity between angles on a straight line that is falling across two parallel lines?*
4. *How does the pattern on the tiled floor models help the students to enhance the idea of angles magnitude?*
5. *How do students apply the acquired knowledge to reason about angles magnitudes in more general situation?*

CHAPTER 3
METHODOLOGY

3.1 Research approach

In general we can say the aim of this study is to develop a local instructional theory to support students' comprehension about angle and its magnitude in grade seven. In order to reach the purposed aim, we develop innovative educational materials to support students' learning in the intended grade level. In the process of developing those materials, we iteratively calibrate the materials to make it fit with practices. By iterative calibrating, we want to make sure those materials can be used in more general educational practices. Therefore, in this study design research is employed as the appropriate research approach to achieve the aim. Barab and Squire (2004; cited by van den Akker, et al., 2006) define a design research approach as "a series of approaches, with the intent of producing new theories, artifacts, and practices that account for and potentially impact learning and teaching in naturalistic settings".

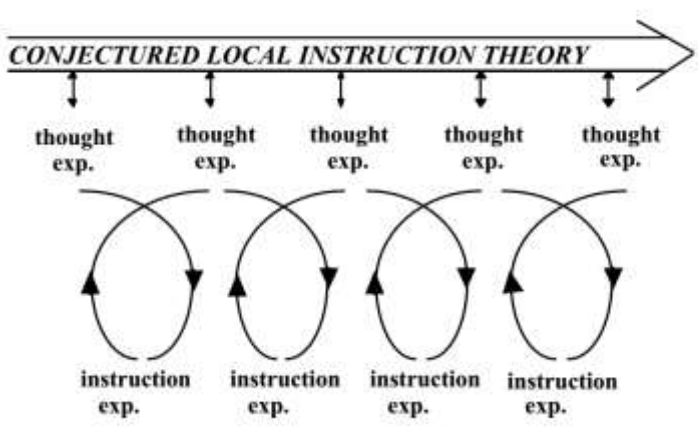


Figure 3.9. Cyclic process of design research (Gravemeijer, 2004).

Design research has a cyclic-iterative character. Typically the cycles consist of three iterative phases: preparation and design, teaching experiment and retrospective analysis. The results of the retrospective analysis normally lead to new designs and a follow-up cycle (van Eerde, 2013). Below we discuss these phases in more detail.

3.1.1 Preparation and design phase

Bakker and van Eerde (2013) explained that the relevant present knowledge about a topic should be studied first in the preparation phase. In the design phase, it is recommended to collect and invent a set of tasks that can be useful and discussed with colleagues who are experienced in designing for mathematics education.

Furthermore, van Eerde (2013) listed the three core steps in a preparation and design phase: a literature review, the formulation of research aim and the general research question, and the development of a Hypothetical Learning Trajectory (HLT). A literature review aims at finding the relevant knowledge about the topic. The results of the literature review allow the researchers to define the knowledge gap and to generate a research aim and general research question. Using the information gathered from the literature review, the researchers develop the sequence of teaching and learning activity and then generate the initial HLT for the sequence. The initial HLT consists of a learning goal, learning activities, and hypothetical learning process. The initial HLT should be tested during the teaching experiment and calibrated iteratively based on the students' actual learning process.

3.1.2 Teaching experiment phase

Generally, in most design research studies, the teaching experiment phase consists of two sub-phases: the first and second cycle. In the first cycle, the researchers 'test' their educational design with a small group of students in order to adjust the content and the design. The aim is to get a better design for the second cycle of the teaching experiment. The second cycle is the actual teaching and learning process in which the educational design is applied in the natural setting (classroom). Here in this study we extend the teaching experiment by adding an extra cycle in order to try some crucial elements of the improved materials. We apply the revised design to another small group of students.

Generally, there are three main steps in the teaching experiment phase; determining what and how the data are collected, a discussion with the teacher, and the teaching experiment (van Eerde, 2013). The data collection typically includes student work, tests before and after instruction, field notes, audio recordings of whole-class discussions, and video recordings of every lesson and of the final interviews with students and teachers (Bakker and van Eerde, 2013). Before the teaching experiment, the researchers discuss about how the teaching and learning process should be conducted with the teacher as described in the teacher guide. The aim is to make clear the crucial aspects of the teaching and learning activity that teacher should focus on. The teaching experiment produces important information to revise and adjust the HLT.

3.1.3 Retrospective analysis phase

The data from the teaching experiment phase are prepared for analysis. During the retrospective phase, the researchers compare the hypothetical learning process with the students' actual learning in order to improve the predictive power of the HLT. In design research, the retrospective analysis can be done with two methods; a task-oriented method and the 'constant comparative method'.

Task-oriented method

Bakker and van Eerde (2013) described this method as a comparison process of data on students' actual learning during the different tasks with the HLT using Dierdorp's analysis matrix. Tables 3.1 and 3.2 were adopted from Bakker's and Eerde's submitted paper (2013). The left side of the Dierdorp's analysis matrix summarizes the tasks and the hypothetical learning process, and the right side is for excerpts from relevant transcripts and clarifying notes from the researcher (Bakker and van Eerde, 2013).

Table 3.1. Dierdorp's Analysis Matrix for comparing Hypotetical Learning Trajectory (HLT) and Actual Learning Trajectory (ALT).

Hypothetical Learning Trajectory			Actual Learning Trajectory		
Task number	The task	Conjecture of how students would respond	Transcript excerpt	Clarification	Match between HLT and ALT: Quantitative impression of how well the conjecture and actual learning matched (e.g., -, 0, +)

Table 3.2. Overview of ALT Result Compared with HLT Conjectures for the Tasks Involving a particular type.

+		x				x	x	x	x				…	…
0	x		X							x	x		…	…
-				x	x							x	…	…
Task	1	2	3	4	5	6	7	8	9	10	11	12	…	…

Note: An x means how well the conjecture accompanying that task matched the observed learning (- refers to confirmation for up to 1/3 of the students, and + to at least 2/3 of the students)

Constant Comparative method

The constant comparative method is additional to the first method (Bakker and van Eerde, 2013). In this method, the researchers read the entire transcript, listen to all the voice recordings and watch all the videotapes chronologically. After that, they select several interesting fragments to generate assumptions. Those assumptions are tested at other episodes of the lessons, in order to find confirmation and counter-examples. The researchers repeat the generated-tested assumptions process several times and perform peer examination in order to reach the final assumptions of the teaching and learning activity.

3.2 Data collection

In this part of the chapter, we describe four data collection phases that we use in this study. The aim is to give an overview about what and how the data are collected. The participants of this study are the teacher and the students in grade seven.

3.2.1 Preparation phase

In the preparation phase, we collect several different data and use different methods to collect them. The table 3.3 describes about what and how the data are collected in this phase:

Table 3.3. Data and Method

Data	Method		
	Semi-structured Interview	**Lesson Observation**	**Written Work**
Teaching method	with the teacher before and after the study	the teacher in the classroom before the teaching experiment	-
Classroom management	with the teacher before and after the study	in the classroom before the teaching experiment	-
Socio-mathematical norms	with the teacher before the study	-	-
Teacher's knowledge about Indonesian realistic mathematics education (PMRI)	with the teacher before the study	-	-
Students' prior knowledge about angle	with the teacher and the focus group before the teaching experiment	-	on pretest before the teaching experiment

Those data are analyzed and the results are used to make necessary calibrations in the planned teaching and learning activity and the teacher's guide.

3.2.2 First teaching experiment (first cycle)

It is appropriate to test the designed materials in advance with a small group of students (6 students) to get an insight into students' reaction to the designed tasks. The researcher acts as the teacher in the first cycle. The data that we collect from this sub-phase are students' definitions of an angle that are derived from the everyday life objects, students' strategies to solve the tasks, students' knowledge about parallel lines, students' knowledge about the magnitude of angles, and students' reasoning about the magnitude of angles on a straight line that falling across two parallel lines. In order to collect these data we make a video recording, and collect field notes, and students' written work. These data are analyzed and the results are used to test the initial HLT, improving the predictive power of the initial HLT, and to make necessary adjustments to the designed learning activities.

3.2.3 Second teaching experiment (second cycle)

In this sub-phase, the improved version of HLT is applied in the classroom environment by the teacher. We collect crucial data that similar with the data in the first cycle, such as; students' definitions of an angle that are derived from the everyday life objects, students' strategies to solve the given tasks, students' knowledge about parallel lines, students' knowledge about angles magnitude (0°, 90°, [180°, 360°], and [360°, ∞)), and students' reasoning about the magnitude of angles on a straight line that falling across two parallel lines. We collect data using video recordings, field notes on teacher's and students' crucial actions, and the students' written work. Then, those data are prepared to be analyzed in retrospective analysis.

3.2.4 Third teaching experiment (third cycle)

The re-improved version of the HLT is tested to a small group of students in order to try some crucial elements of the refined materials. In this sub-phase, our main attentions are to get explanations, justifications and clarifications about students thinking, and to understand how the design helps the students to acquire the intended knowledge. The data that we collect in this sub-phase are similar with the data that we collect in the first and the second cycles. Either the method to collect the data is also similar.

3.2.5 Pretest and posttest

Pretest and posttest are conducted to assess the students' acquisition of knowledge and to provide the 'quantitative' description of students' understanding about the topic. This quantitative description can be acquired from students' answers on items test (pretest and posttest). However, we are also interested in the qualitative description of students' understanding about the topic. Therefore, we designed the test items in such a way that we can observe how students solve the problems. Generally, in the pretest and the posttest we will find multiple choices, numerical problems, exploration questions and diagrammatic problems. The pretest in this study is conducted in the preparation phase. The aim is to inquire students' prior knowledge about angles (what students know and don't know). At the end of the teaching experiment, the posttest is conducted as a follow-up action from the pretest on the preliminary phase. The aim of the posttest is to assess the students' development of understanding about the concept of angle and its magnitude.

The pretest and the posttest are similar but not the same. In order to allow us to compare the results from both tests, we retain a proportion of items in the pretest and blend the retained items with new items which examine the equivalent expected learning outcomes in the posttest, or use different types of questions for an equal item in the pretest and the posttest. Beside the students' written work on the pretest and the posttest, we collect data from the interview as well. Considering the scale of this study, we perform the interview only with the focus group in the second teaching experiment (second cycle). The aims of conducting the interview are to inquire students' understanding on the topic and make an inventory of students' solution procedure to the given problems.

3.2.6 Validity and reliability

Bakker and Eerde (2013) explained about validity and reliability in design research that validity was concerned with whether we really measured what we intended to measure. Reliability was about the independence of the researcher. Since we want to evaluate students' comprehension about the concept of angle, in this study we collect several data, such as students' written works, interview tapes from both the teacher and students, field notes, and video registrations. The use of different types of methods allows us to conduct triangulation that can contribute to the internal validity of the study. Moreover, we employ electronic devices (cameras and tape recorders) to increase the objectivity and the internal reliability of the data collection in this study.

3.3 Data analysis

3.3.1 Pretest

The pretest is given to the students in the preparation phases. The data that we have are students' written work when they are solving the test items and students' verbal explanations (video recordings) in the interview session after they take the test. We develop a rubric (see pre and posttest rubric) to rate the students' works. The data are carefully analyzed according to the rubric in order to investigate students' prior knowledge and to know the starting points of students about the concept of angle. The results of the analysis are used to make some adjustments in the initial HLT to improve the predictive power of it. In addition to that, the results of the pretest are used to select the focus group that consists of students with various level of knowledge about the topic.

3.3.2 First teaching experiment

The aim of the first teaching experiment is to get an insight into how the selected students react on the designed tasks. In this case, the selected students act as a 'miniature' of the students in the second teaching experiment. We analyze the data in this phase using a task-oriented method in order to know how the predictions of the HLT correspond (or don't correspond) with the students' actual learning process. The data analysis is performed in the following steps:

1. Video observation

The videos of a lesson are watched with the research questions and the HLT as guidelines. Here, the focus is to find confirmation and counter-examples for the conjectured learning process in the actual learning process.

2. Video observation notes

The interesting fragments in the videos of a lesson are excerpted. Here, the interesting fragments refer to any observable and interpretable activities in the lesson that can be categorized as confirmation or counter-example of the students' learning.

3. Dierdorp's analysis matrix

The excerpts from the videos of a lesson are analyzed in Dierdorp's analysis matrix in order to know how the predictions of the HLT correspond (or don't correspond) with students' actual learning process.

The results from this analysis are used to calibrate the initial HLT in order to make the HLT ready to use in the second teaching experiment. Ideally, after the task-oriented method, we could perform the 'constant comparative method' to gain more theoretical insight into the learning process. However, since this is a small scale study, we cannot perform the follow-up analysis due to time restrictions.

3.3.3 Second and third teaching experiments

Similar to the analysis in the first teaching experiment, in these sub-phases we analyze the data using a task-oriented method. The results of the analysis from this phase are used to answer the research questions, generate a conclusion, and revise the HLT.

3.3.4 Posttest

The way we analyze the data from the posttest is similar to what we do in the pretest. However, we also compare the posttest results with the pretest results quantitatively to know in general how well the knowledge gained by the students and qualitatively via interviews to evaluate and examine the development of students' learning and understanding of the concept of angle. All the outcomes from this phase are used as additional data for triangulation, answering research questions and drawing the conclusions.

3.3.5 Validity and reliability

According to Bakker's and Eerde's submitted paper (2013), internal and external validity and reliability seem most relevant in the context of design research. Therefore, in this part of this chapter, we will describe these types of validity and reliability related to the data analysis in this study.

1. Internal validity

In the analysis phase, the internal validity refers to the soundness of the reasoning that has led to the conclusions. In order to improve the internal validity of analysis of this study, we take the following steps:

In the retrospective analysis, we analyze the data using a task-oriented method in order to generate and test the hypothetical learning process in the HLT. We also perform data triangulation with other data, such as students' written work, field notes, and video registrations of interviews and lessons in order to strengthen (search for confirmation and counter-examples) the results from the retrospective analysis.

2. External validity

External validity is strongly related to the generalizability of the results. In design research, the generalizability means that others can adjust and perform the current study to their local contingencies. In order to improve the external validity of this study, we utilize the explicit educational materials (HLT, teacher's guide, and students' worksheets) that can be easily followed by others.

3. Internal reliability

Internal reliability refers to the degree of independence of the researcher of the collection and analysis of the data (Bakker and Eerde, 2013). In order to improve the objectivity of the data analysis, during the retrospective analysis we discuss the critical transcript from the actual learning process with colleagues for peer examination.

4. External reliability

External reliability usually denotes replicability, meaning that the conclusions of the study should depend on the subjects and conditions, and not on the researcher (Bakker and Eerde, 2013). For improving the external validity of this study, we present the study in an explicit way (how the study has been carried out and how the data are analyzed and the conclusions have been drawn from the data), so the other researcher can track the whole process of this study.

3.4 Research subject and time line of the research

3.4.1 Research subject

The research was conducted in a secondary public school named SMP Negeri 17 in Palembang. This school has been involved in the Pendidikan Realistik Indonesia or Indonesian Realistic Mathematics Education project before and as a result the mathematics teachers in this school more or less know about RME and design research. In this study we were involving 46 seventh graders (i.e. 6 students in the first teaching experiment and 40 students in the second teaching experiment) and their teacher. The students in the first teaching experiment were selected from another parallel classroom that differs from the students in the second teaching experiment but taught by the same teacher. The students the first teaching experiment consist of 1 female and 5 male students and in the second teaching experiment consist of 19 female and 21 male students. They were about 12 to 13 years old.

3.4.2 Time line of the research

The timeline of the study is summarized in the table 3.4:

Table 3.4. Time line of the study

	Date	Description
Preparation and design phase		
Preparation	September 2013 – January 2014	Studying literatures and designing the initial HLT
Discussion with teacher	3 – 5 February 2014	School and classroom observation. Communicating the detail of the study with the teacher.
Teaching experiment phase (The first cycle)		
First meeting	4 February 2014	Pretest (Initial version)
Second meeting	5 February 2014	Interview to gather students' solution procedures

Table 3.4. Time line of the study (Continued)

Third meeting	11 February 2014	Activity 1: Angle from everyday life situations (Initial version)
Fourth meeting	12 February 2014	Activity 2: Matchsticks, letters, and angles (Initial version)
Fifth meeting	18 February 2014	Activity 3: Letters on the tiled floors (Initial version)
Sixth meeting	19 February 2014	Activity 4: Reason about the magnitude of angles on the tiled floors (Initial version)
Seventh meeting	25 February 2014	Activity 5: Angle related problems (Initial version)
Eighth meeting	26 February 2014	Posttest and Interview to gather students' solution procedures (Initial version)
Teaching experiment phase (The second cycle)		
First meeting	18 February 2014	Pretest (revised version) and interview relate to the pretest to gather students' solution procedures
Second meeting	19 February 2014	Activity 1: Angle from everyday life situations (revised version)
Third meeting	20 February 2014	Activity 2: Matchsticks, letters, and angles (revised version)
Fourth meeting	25 February 2014	Activity 3: Letters on the tiled floors (revised version)
Fifth meeting	26 February 2014	Activity 4: Reason about the magnitude of angles on the tiled floors (revised version)
Sixth meeting	27 February 2014	Activity 5: Angle related problems and posttest (revised version)
Seventh meeting	5 March 2014	Interview relate to posttest to gather students' solution procedures
Teaching experiment phase (The third cycle)		
First meeting	7 April 2014	Pretest (revised version) and interview relate to the pretest to gather students' solution procedures
Second meeting	8 April 2014	Activity 1: Angle from everyday life situations (revised version)
Third meeting	10 April 2014	Activity 2: Matchsticks, letters, and angles (revised version)
Fourth meeting	11 April 2014	Activity 3: Letters on the tiled floors (revised version)
Fifth meeting	12 April 2014	Activity 4: Reason about the magnitude of angles on the tiled floors (revised version)
Sixth meeting	14 April 2014	Activity 5: Angle related problems and posttest (revised version) Interview relate to posttest to gather students' solution procedures

CHAPTER 4
HYPOTHETICAL LEARNING TRAJECTORY

In chapter III we already mentioned about generating a hypothetical learning trajectory (HLT) as one of the three core steps in preparation and design phase. Here we will discuss about the practical aspects of the HLT in the present study. HLT can be viewed as a general plan and predictions about the actual teaching and learning activities. In order to generate a good HLT, we have to envision the mental activities that students might engage in when they would participate in the teaching and learning sequence (Gravemeijer, 2004). Simon (1995) explained that the HLT consist of three components: the learning goals, the learning activities, and the hypothetical learning processes (conjectures on students reactions).

The central learning goal of the lessons is to support the students to build their understanding about angle and its magnitude via reasoning activities. In order to reach the intended learning outcomes, we designed a lessons sequence that consists of five lessons. The five lessons cover the activities such as, redefining angle via ordering the angles magnitudes, hand-on activity with matchsticks (angles on letters), mathematical explorations on the letters like figures on the tiled floor models, reasoning about the angles magnitude on the tiled floor models, and solving the problems related to the angles in more general cases. Each lesson has a specific manner to accomplish the learning outcomes, in which we will discuss in more detail in the next part of this chapter. We also generated the hypothetical learning processes that we think are more likely to be occur in the actual learning process. Here, we will describe the hypothetical learning processes for all learning activities by describing the starting point of students, the learning goals, the mathematical activities, the conjectures of students' reactions and the students' solution procedures.

4.1. Lesson 1: Angles from everyday life situations

4.1.1 Starting points

As we know from the table 2.1 in chapter 2, it is not the first time the seventh grade students in Indonesia encounter the concepts of angle. They have encountered several important concepts of angle before such as, the definitions of angle, the angle measurement, and the classification of angles based on it sizes (acute, right-angle, and obtuse). Therefore, we want to utilize this current knowledge in order to allow them to extend their knowledge to the next level. The following assumptions about students' abilities are the starting point for this lesson:

a. Students can identify and indicate the angles from the everyday life objects.
b. Students can differentiate the magnitude of angles based on several benchmarks (i.e. acute, right-angle, obtuse, straight, reflex, and perigon).
c. Students can work with the static and dynamic situations of angles.
d. Students know about the unit of measurement for the angle magnitude.
e. Students can use a protractor to measure the magnitude of an angle.

4.1.2 The learning goals

Main goal

Students are able to recall the concepts of angle magnitude that they have learnt before and reformulate a definition of angle.

Sub-goals

a. Enable students to identify the angles on the everyday life objects.
b. Enable students to indicate the angles on the everyday life objects.
c. Enable students to classify the angles based on its magnitude.
d. Enable students to analyze and explain the important criteria in order to determine the magnitude of angles.
e. Enable students to contrast the magnitude of angles from the dynamic angles situation.
f. Enable students to explain how the angle formed.
g. Enable students to reformulate a definition of angle.

4.1.3 Description of activity

This lesson includes four stages. In the first stage, the students should analyze, identify, and indicate an angle on each picture of the everyday life objects. In the second stage, the students make a poster which sort the indicated angles based on their magnitude (from the smallest to the largest). In the third stage, the students discuss about the important criteria in determining the magnitude of angles according on their own production in the second stage. In the last stage, the students should discuss about how the angle formed and what the most satisfied definition of angle according to the students' judgment.

First stage: identify and indicate an angle on the everyday life objects

The teacher starts the lesson by distributing the picture of everyday life objects (see figure 4.1) and asks the students to analyze the objects from mathematical point of view. The teacher then gives several indirect guided questions in order to lead the students to recognize the existence of angles on each object. The questions that teacher ask might be; Do you familiar with the objects on the card? What are those objects have in common? and What geometrical concepts that embedded on the objects that you can figure out?

It might be happen that the students do not immediately recognize the existence of angles on the objects. If so, we can simplify the situation by focusing the discussion on some simple objects such as, football field corner, roof top, or tiled floors. In Indonesian classroom we can support students to retrieve their memory about angle using the nature of their language. Therefore, in this case the teacher can utilize the picture of football field corner (object A) to support students. Since, the word 'corner' in bahasa (Sudut) is literally translate as angle and most of the time students perceive and associate the word 'sudut' with right-angle.

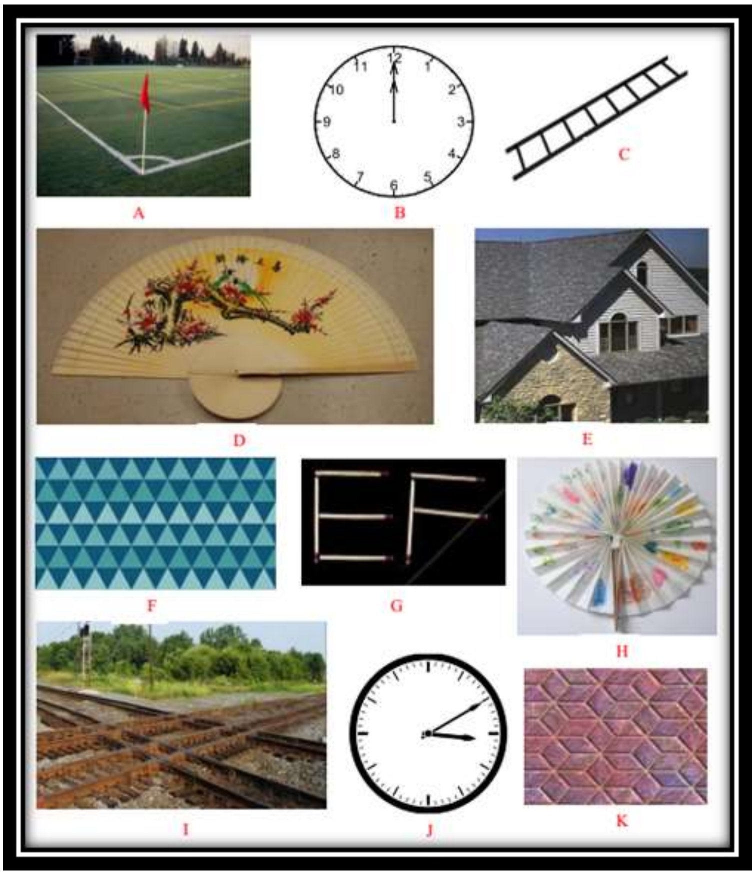


Figure 4.1. The pictures of everyday life objects that related with the angles.

After the teacher think his/her students know the mathematical topic that they will encounter during this lesson. The teacher can distribute the worksheet (see worksheet 1 in the appendix) and ask the students to work in group of four. The worksheet consists of several tasks and questions. Before the students start to work on the worksheet, the teacher has to make sure his/her students fully understand the instructions in the worksheet. The teacher can ask the students to read it out loud and ask them if there are some instructions that they don't understand. The teacher can also reformulate the problems, give definition of a term on the problems that students do not understand, or give students simple situation to provide them the ground for thinking. It is important to know that, the clarifications for the instructions in a worksheet should be performed consistently by the teacher throughout the teaching experiments in this study.

Indicating an angle in each object is the first task that students should do after analyzing the given pictures. Considering the perspective appearance, orientation and the scale of the given pictures, therefore, it is important for the teacher to warn the students to see the angles in each object as it is in the real world not as the appearance in the pictures.

Second stage: ordering the magnitude of angles

The second task in the worksheet asks students to make a poster which sort the magnitude of indicated angels in an ascending order. They then display their poster to be observed by their fellow students. Each member in every group must observe at least two or three posters and analyzes the differences and similarities between each poster. It is most likely that the posters are different and unique, it depend on which angles that they have indicated from the given pictures. We consider this fact as a good opportunity to start a classroom discussion/debate about the order of angles magnitudes in each poster. Throughout the discourse, by the helps of teacher, students should figure out the important criteria in order to determine the angle magnitude.

Third stage: discussion about the magnitude of angles

After students observe, compare, and analyze the posters, they may find several discrepancies in those posters. In this case, the following instructions for the classroom discussion should be perform by the teacher to help students to communicate their ideas.

a. The teacher select one poster that seems has flaws related to the order of magnitude of angles and asks the students to discuss about it. The teacher can directly ask for explanations from the poster makers and then invite the other groups to give their responds/opinions.
b. In the given pictures, there are three situations that involving the right-angle. Students may not put the right-angles in one cluster in their poster. Thus, the teacher can start a debate by asking his/her students about the name or the degree of these angles. Asking a question like; why if the indicated angles have the same name or equal degree but not in the same level of order.
c. If there is no significant flaw in every poster, the teacher could purpose more advance questions to be discuss such as; What do you think about the angles

on the picture A and B (right-angle and zero/full-angle)? What about C and F (comparing the angle magnitude)? Can someone explain why angle on object E is bigger than angle on object F? and How do you differentiate the magnitude of angles without using a protractor (to see what criteria students use to compare the angle magnitude)?

d. It also useful to ask each group to give some suggestions to the other groups on how to order the angles magnitude.

The teacher can finalize the poster session by give the students time to write their mathematical conclusions related to the activity or ask them to write about what they have learnt from the classroom discussion. In addition to that, the teacher has to be fully aware that this activity has to be brief and straightforward.

The activity continues, in which students should answer the questions in the worksheet. It is favorable if the students work individually at first, and then discuss it in their group before giving the final answers or taking a final conclusion. The first two questions in the worksheet are designed to reintroduce the dynamic angle situations. First the students have to select a picture in which the angle can change its size (e.g. traditional fan, letters from matchsticks, and analog clock). After that they have to draw the two conditions where the selected picture showing the smallest and the biggest angles. These tasks aim at enabling the students to strengthen their understanding on the concept of 0° and 360° angles, where at the same time introduce to them about the duality of a 0° angle.

Four stage: redefining a definition of angle

The two last problems in the worksheet allow the students to explain about how the angles are formed and use their own explanations to reformulate an angle definition. Our intention in asking the students about how the angles are formed is to help students to relate the angle with the concepts of lines, directions, rotation, and regions. If the students realize the relationships between angle, line, directions, rotation and region, it is more likely that they will define the angle in term of line and its direction. In the last two questions, we explicitly ask the students to explain how the angles are formed and what their own definition of angle is. The teacher could perform the following instructions to orchestrate the classroom discussion.

a. When the students explaining the angle formation the teacher should lead the students to reason about angle construction using lines and their directions.
b. In redefining a definition of angle the teacher should make the students reformulate the definition using the current knowledge on this lesson (line, direction, rotation and region).

4.1.4 Conjecture on students' reaction

a. In the first task, some students may give several different signs to indicate an angle in each picture and some may indicate more than one angle in every picture.
b. In the first and the second tasks, some students may encounter difficulties to indicate and ordering the angles on pictures B, D, and H (0°, 180°, and 360° on an analog clock and the traditional fans).
c. In the second and the third tasks (poster), some students may make the unordered list of the magnitude of angles because they judge the magnitude of the angles based on a different criteria (e.g. based on the length of the arms, based on the region of the angle, or based on the scale of the original objects). This may trigger a debate on the classroom about what is mean by the magnitude of an angle.
d. In answering the first and the second questions, some students may draw a small non-zero angle to represent the 0° angle and draw an obtuse non-360° angle as the biggest angle. In addition to that, the students may explain the magnitude of the angles by reason with the number on the analog clock or rely on their rough estimation. However, if the students have the adequate understanding about angle measurement they may not encounter significant difficulties in answering the questions.
e. In answering the third question, the students may explain that an angle is formed by two intersecting lines, or by two lines that rotate their intersection point.
f. In answering the fourth question, the students may make a definition of angle which focuses on one of the following criteria: as space between two lines

which meet in a point, as the difference of direction between two lines, or as the amount of turn.

4.1.5 Discussion

During this lesson, there are three main classroom discourses that teacher should stress on. First, during the discussion in the poster session, teacher has to focus on how students identify the angles and how they put the angles in an order based on its magnitude. The teacher should invite the students to explain their strategy in constructing the list. Some students may use overlapping (copy-paste) strategy to explain how they compare the magnitude of one angle to the other angle, some may use right-angle as the benchmark in comparing the magnitude of angles, and some may rely on their rough estimation about the magnitude of angles.

Second, during the discussion of the first two questions, teacher should invite his/her students into a classroom discourse that negotiate about how students perceive a zero angle and a full angle via diagrammatic approach and approximation strategy to grasp the duality property of these angles. During the discussion it also possible to make them understand that in every angle figure there must be two angles exist (less than 180° angle and its reflex angle). In addition to that, the teacher should invite the students to reason about the ‘special’ angles (0°, 90°, 180°, 270°, and 360°) by extending the previous diagrammatic explanation and approximation strategy.

Third, on the two last questions, the aim is to allow the students to reformulate the definitions of angle via reasoning about how an angle is formed. During this activity, the teacher should realize that there will be no perfect definition of angle on this stage. However, the teacher can expect the students to come up with some acceptable definitions of angle. In reformulating the definition, students may use one of the following criteria; angle as the spaces between two lines which meet in a point, as the difference of direction between two lines, or as the amount of turn.

4.2 Lesson 2: Matchsticks, letters and angles

4.2.1 Starting points

It is a rather simple activity to introduce the similarity between angles that formed by a straight line that falling across two parallel lines (parallel-transversal situation). We introduce the topic by asking the students to reconstruct the uppercase letters using matchsticks and analyze the angles on each letter. In order to be able to perform this activity the students need to have at least an intuitive understanding about angle magnitude.

4.2.2 The learning goals

Main goal

The students are able to infer the similarity between the angles magnitudes that formed by a straight line that falling across two parallel lines.

Sub-goals

a. Enable students to construct the angles in various magnitudes.

b. Enable students to compare and criticize the letters reconstructions related to the angle magnitude.

c. Enable students to describe the concept of reflex angle.

d. Enable students to predict and infer angles similarity in the given situation.

4.2.3 Description of activity

We divide this lesson into three stages according to the nature of the tasks and the questions in the worksheet (see worksheet 2 in the appendix). In the first stage the students are asked to construct the upper case letters using matchsticks. During the second stage, we ask them to observe, analyze, and discuss the constructions that they make in order to make them understand the situation. In the last stage the students should infer the angles similarity in a the parallel-transversal situations during the classroom discussions.

First stage: letters reconstruction using matchsticks

In groups of four, on their table, students reconstruct the uppercase letters using matchsticks without breaking the matchsticks. It is important to inform students in advance that they only have limited amount of sticks, so they have to use it wisely in order to be able to reconstruct the entire letter. The intention of this activity is to give the students a hand on activity to construct the angles in various magnitudes. This task also provides the ground for students to strengthen their sense of angle magnitude.

Figure 4.2. Letters from the wooden matchsticks

Second stage: constructions comparison

The teacher inform to the students that, in each group, two students will stay near their work to answer the questions from other students, and the other two students walk around to observe the other groups' works. Alternately, the two students that previously stay now walk around and the other that previously walks around now stays near their work. In this stage, it is important to encourage the students take notes on their finding during the observation. In addition to that, the teacher should ask students to give some suggestions or questions to the other groups' works. The main aspects from the reconstructions that students should focus on are; the amount of matchsticks that used to make the construction, and the shape of each individual letter. The information that students acquire throughout the observation is crucial for explaining the concept of reflex angles and to infer the similarity between angle magnitudes.

Third stage: inferring angles similarity

After students answering the questions on the worksheet, teacher invites them into a classroom discussion. During the discussion, teacher should discuss about how to compare the angle magnitudes in order to determine which angle is bigger/smaller than another angle. The discussion about the smaller and the bigger angles should lead the students to the conclusion that both angles have to be in a same letter (i.e. the concept of reflex angles). The discussion can help the students to make sense that the 0° angle and 360° angle have to be in the same figure (duality). Furthermore, the discussion about the relation between the parallelity and the angle magnitudes should lead the students to infer that some angles on the letters that have parallel sticks will have the same magnitude. Teacher can also invites students to negotiate this concept, by comparing the angles on the letters that have parallel sticks with the letters that doesn't have parallel sticks in order to help them to arrive at the intended knowledge. In addition to that, teacher needs to conduct a small discussion that focuses solely on the similarity between angles on the letter X. It is important because the students will need this fact in order to allow them to perform the tasks in the next lesson.

4.2.4 Conjecture on students' reaction

a. When the students work with the tasks, some groups may make some letters using way too many matchsticks and find out that some letters are appear in different shape in the other groups' works.

b. In answering the first and the second questions, some students may use the sharpness of a vertex, and some may use the opening of the letter to determine the size of angles on a letter. The students may also select two different letters to represent the smallest and the biggest angles and not realize the fact that those angles have to be in the same letter (acute angle and its reflex angle).

c. In answering the third question, some students may misinterpret the term parallel as something else (e.g. symmetry, perpendicular, intersects, etc.).

d. In answering the fourth question, it is possible that we can observe students' understanding about the similarity between angles magnitude limited to the right-angle situation. In addition to that the students may use the sharpness of the vertices as the benchmark to determine the similarity between angles.

e. In answering the fifth question, the students cannot find the similar angles in the letters that don't have parallel sticks on them. It may generate students' recognition of the necessary condition of similarity.

4.2.5 Discussion

This lesson was designed in order to allow the students to predict and to infer the similarity between angles on a straight line that falling across two parallel lines (parallel-transversal situation). We use letters as a raw model for introducing the concept because of its simplicity. In this lesson, we expect students to make a conjecture about the angles similarity after they analyze the sizes of angles on the letters. However, during this stage, we don't expect the students will have the sophisticated explanations about this concept. We limit the outcomes of this activity, in which students can give the acceptable explanations for angles similarity.

The core of this lesson is on the discussion of the two last questions. The questions ask students to analyze the magnitude of angles on the letters that formed by parallel sticks and write down their findings. There are three possible outcomes related to this activity. First, the students successfully infer the similarity between the angles. In this case, the teacher has to invite the students to discuss and explain how the students arrive at that claim. A good discourse should make students' strategies observable.

Second, if the students cannot infer the similarity between angles. In this case, the teacher should help the students by grouping those letters from the simple to the complex (from the letters that have right-angles to the letters that haven't). After that, teacher asks students to focus on the simplest cases such as, letters E, F, and H where the right-angles are obvious. Teacher should extend the exploration on these simple cases by tilt one or two matchsticks in order to make several variations of the letters. The exploration can help students to move from the trivial situations to the non-trivial cases.

After students realize the angles are similar using right-angle as a benchmark, teacher should move progressively to the more complex cases such as, letters N, Z, and M. Furthermore, teacher should ask for generalization about the situation. Third, if there is a portion of students that not yet infer the similarity between angles. This is the most likely situation that will occur in the classroom environment. In this case, the teacher should ask some students from both groups to explain their findings in front of classroom and orchestrate a discussion that compare those findings to help students to arrive at the intended conclusion.

As it stated before, in this lesson students may not have an adequate explanation about why there will be the similarities between angles magnitude when a straight line falling across two parallel lines. In fact, in the next lesson we provide students with a suitable context/situation in order to allow them to get further justifications of the angles similarity in a parallel-transversal situation.

4.3 Lesson 3: Letters on the tiled floor models

4.3.1 Starting points

This lesson intended to give students a further justification about angles similarity in a parallel-transversal situation. We chose mathematical explorations on the tiled floor models as a way for the students to be able to prove their conjectures about angles similarity that they have acquired from the previous lesson. We assume the students can perform the following activities before they work with the tasks and the questions in the worksheet (see worksheet 3 in the appendix).

a. The students can reason with the line patterns from the given geometrical figures.
b. The students understand the terms of lines such as, parallel, perpendicular, and intersect each other.
c. The students know that a full angle is equivalent to 360°.

4.3.2 The learning goals

Main goal

The students are able to explain angles similarity by utilizing the uniformity of tiles on the tiled floor models.

Sub-goals

a. Enable students to identify the lines patterns on the tiled floor models by analyzing the gaps between adjacent tiles.
b. Enable students to examine the angles on the tiled floor models.
c. Enable students to determine the magnitude of angles on the tiled floor models to get further justification of angles similarity on the letters that have parallel sticks on them (students' conjecture from the second lesson).
d. Enable students to relate the magnitudes of angles on two situations; letters from matchsticks and letters on a tiled floor model.
e. Enable students to describe the parallel lines using the similarity of angles and vice versa.

4.3.3 Description of activity

The teacher start the lesson by telling a story about a girl named Ana that found the patterns of her name on a tiled floor when she observed the gaps between adjacent tiles in her kitchen. After telling the story, the teacher display two pictures of tiled floors and ask the students to determine which floor that Ana refer to (see figure 4.3). Our intention in presenting the story is to raise students' expectation that they will do some explorations on the presented situation. At this moment, it is not obligatory for students to have the sophisticated explanations for their opinions. When working with the worksheet (see worksheet 3 in the appendix) the students will have more room to explain their idea related to the presented situation.

We divide this lesson into three stages. In the first stage, students should perform a mathematical exploration related to the patterns like letters on the two floor models. The second stage, students compare the letters on a tiled floor model (kitchen floor) with the letters on the matchsticks activity (second lesson) to justify angles similarity in a parallel-transversal situation by using the uniformity of tiles. In the last stage, students should explain about angles similarity that they have justified. Students can utilize the uniformity of tiles and connect it to their knowledge about angles magnitudes on some letters (F, X, and Z) to justify their claims from the second meeting (letters from matchsticks).

First stage: exploring the angles on the tiled floors

There are several instructions in the worksheet that ask students to perform the tasks such as, showing their opinions to the story that presented earlier, finding as many letters as possible from the kitchen floor, and comparing the angles magnitudes on the letters on the kitchen floor with the angles on the letters from matchsticks activity. Teacher can orchestrate a classroom discourse that simultaneously covers these tasks in one compact discussion. The main goal of the discussion is to make students aware that they can calculate the magnitude of an angle without using a protractor in some special situations.

Figure 4.3. Tiled floor models.

Second stage: justify angles similarity using the uniformity of tiles

Students should work in their previous group on the lesson two to perform this task. The task requires students to compare, analyze, and explain the angles on the letters in two situations; matchsticks and the kitchen floor. In this stage, teacher should stress the discussion on comparing the shape of some letters (E, F, N, X, and Z) from the poster in lesson two with the letters on the kitchen floor. Teacher also should help students to justify their previous conjectures about the similarity between angles on these letters. Conducting a classroom discourse that focus on the fact that the shape and the orientation of the lines do not affect the similarity between corresponding angles may help students to justify their conjectures. In addition to that, it is also important to ask students to recall why the vertical angles (X-angle) have the same magnitude, even this not really related to the task on this stage. However, the students need to understand this fact in order to be able to explain the similarity between angles on a straight line that falling across two parallel lines.

Third stage: explaining the similarity between angles magnitudes using the uniformity of the tiles

In the worksheet 3, there is another picture of a tiled floor model (Figure 4.4) and some questions related to this floor model. Students will carry out simple mathematical explorations that beg them to applying their current knowledge. It is rather more complex situation compare with the previous activities, where the patterns of the gaps between tiles not clearly depict the shape of letters. However, if the concepts from the previous explorations are well understood, then it is more likely that they will arrive at a consensus where they are agree that parallelity and angles similarity are strongly connected.

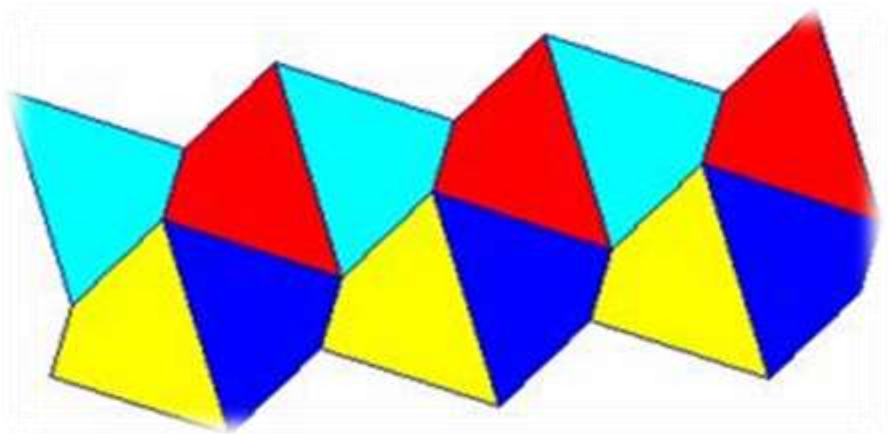


Figure 4.4. Tiled floor model in the third lesson.

4.3.4 Conjecture on students' reaction

a. In the first task, students will highlight the gaps between tiles that form a word 'ANA' but they may use different amount of gaps to construct the word.
b. In the second task, some students may find all the letters on the kitchen floor and some may not.
c. In the third task, students may find out the relation between the parallel orientation of the gaps and the parallel orientation of the matchsticks resulting the same consequence; similarity between angles in both situations. They may also figure out that they can easily see the similarity of angles on the tiled floors situation compare with the letters from the matchsticks activity.
d. In answering the first question, students may indicate all the angles with the same mark (symbol) and produce the ambiguity when we ask them which angle that equal to which angle.
e. In answering the second question, some students may use equal length symbol to indicate the parallelity.
f. In answering the third question, students would have different opinion related to the existence of the right-angle on the figure.
g. In answering the fourth question, students may realize that there is a connection between the parallelity and the similarity of angles on a situation when a straight line falling across a pair of parallel lines.

4.3.5 Discussion

This lesson is designed to create an adequate learning environment to allow students to test their own conjectures related to the angles similarity in a parallel-transversal situation. The magnitudes of angles in the lesson two are uncertain and limit the possibility for students to have satisfied proofs about angles similarity. However, in this lesson, the context is more suitable for the students to justify what they already infer from the lesson two. The magnitudes of angles on the tiled floor models are easy to determine. For instance, if there are six tiles that have a common point, students can carry out some simple calculations to find out that each corner of the tile will be 60°. The certainty of angles magnitudes can help students in the process of justification. In addition to that, the appearance of the letters on both situations also can help students to justify their conjectures.

It is important to understand that the focus of this lesson is on the aspect of reasoning about angle magnitude. We focus our attention mainly on how students' reasoning about angles magnitudes helps them to prove their previous conjectures. As we can see, students should perform some calculations related to the angles magnitudes. We are fully aware that, students need to have some strategies on how to calculate the magnitude of angles in the presented situations. Therefore, in the next lesson we provide the students with a learning context that will help the students to sharpen their mathematical ability in reasoning about angles magnitude.

4.4 Lesson 4: Reason about angles magnitudes on the tiled floor models

4.4.1 Starting points

In this lesson, we still use a similar learning situation with the previous lesson (lesson 3). However, the focus of this lesson is more on the numerical aspects of students reasoning about angles magnitudes. We assume the students know the following facts before they work with the tasks in the worksheet.

a. The students know about a reflex angle.
b. The students know that a right-angle is equal to 90°.
c. The students know that a straight angle is equal to 180°.
d. The students know that a full angle is equal to 360°.

4.4.2 The learning goals

Main goal

The students are able to reason about angles magnitudes using the uniformity of the tiles.

Sub-goals

a. Enable students to predict the magnitude of angles on each corner of a tile.
b. Enable students to calculate the magnitude of angles on each corner of a tile using the concept of similarity.

c. Enable students to realize the uncertainty related to the magnitude of angles in certain situations.

4.4.3 Description of activity

We divide this lesson into three stages. In the first stage, the students investigate the magnitude of angles from a simple situation (angles on a bricked wall). In the second stage, the students analyze several tiled floor models and mark the angles that have the same magnitude. In the final stage, the students calculate the magnitude of each corner of the tiles by utilize the uniformity of the tiles.

First stage: investigate the magnitude of angles on a bricked wall

In this stage, teacher orchestrates a discussion that leads students to find as many as angles with different magnitude on the picture of a bricked wall (see figure 4.5) and explicitly mention the numerical values of those angles. The goal of discussion is to provide a context for students to make sense the sum of angles. This activity also provides the students with a context that can allow them to make sense the straight-angle is 180°, full-angle is 360°, and reflex angle from the classroom discussion. The teacher can post the following questions in the classroom discussion:

a. The angle on the corners of each brick is in the same size. What do you know about its degree?
b. If we put the bricks side by side, we can see the joint of two corners form a bigger angle. On the figure, can you determine the size of all angles on the joint of the bricks? Explain how you do the calculation?
c. How many different magnitudes of angles that you can find?

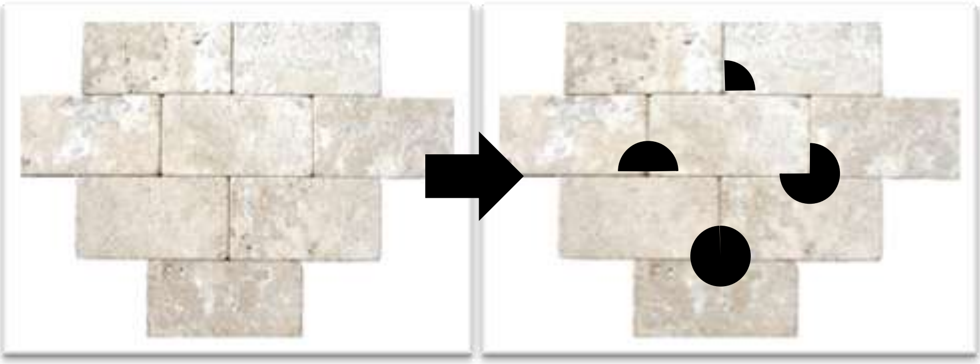


Figure 4.5. Bricked wall picture in the fourth lesson.

Second stage: analyze the angles on the tiled floors

Students should work in group of four to perform this task. They have to compare and analyze the corners of each tile on each floor model in order to get a general overview of the situation. Students may produce several possible overviews from their investigation; the number of different shape of tiles on each floor model, the number of different angles magnitudes, and the certainty and the uncertainty related to the angles magnitudes on each corner.

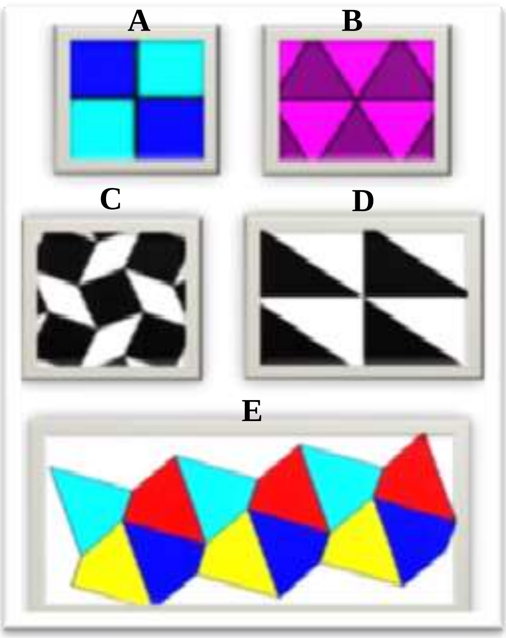


Figure 4.6. Various tiled floor models in the fourth lesson.

Third stage: calculating the magnitude of angles on each corner of the tiles

Before students performing the calculations to find the numerical value of angles magnitude, first they will work on two more simple problems. The first problem asks students to determine the corners that have the same angle. The second problem asks them to give some explains related to their answer for the first problem. By doing so, we expect the students to have an in-depth understanding related to the situation presented.

In order to make students calculating the angles magnitudes, we ask them to investigate the angles on a meeting point of tiles in the tiled floor models. Students have to determine the numerical values of each angle in a meeting point in every tiled floor models. The task requires students to be aware of the uncertainty of some numerical values of the angles on the present situation. For instance, for floor models C, D, and E some angles on them cannot be obtained with certainty using only reasoning (see figure 4.6). Therefore, teacher should encourage his/her students to make some educational guesses that based on several assumptions in order to fit some numerical values of angles magnitudes to the assumed situation.

4.4.4 Conjecture on students' reaction

a. In the first task, after indicating the angles that have the same magnitude, students may give a general descriptions about the magnitude of angles for each floor model related to the type of the tiles without any numerical values of the angles (e.g. right-angle, acute angle, obtuse angle, smallest or biggest angles, and sharp corners). However, it is also possible that they will give the numerical values for each angle on the corners despite there are uncertainties about the magnitude of angles in some floors (C, D, and E).
b. In the second task, students may explain the similarity of the angles as a logical consequence of uniformity of the tiles. However, some students may explain the similarity using the concept that they already learnt from the previous meeting (letters-angles).
c. In the third task, students may conclude that, the sum of angles on a common point is 360°, the magnitude of angles on each common point can easily be

obtained when all the corner are similar, and in some situation (A, B, D, and F) the concept of letters-angles can be applied.

d. In the fourth task, some students may divide the 360° with the number of the tiles that meet in a point in order to determine the angle magnitudes of each tile's vertex.
e. In the fifth task, some students may guess the magnitude of the unknown angles, some may claim that the problems do not have any solution due to the lack of information, and some may claim that the problem have too many solutions depend on their assumptions.

4.4.5 Discussion

This lesson is designed to prepare students to the more general situations in reasoning about angles magnitudes. In other words, this lesson act as a bridge that allows students to make a progressive generalization of the knowledge. In the first three lessons, students can only reason about the magnitude of angles in some special cases but in the last teaching experiment we want them to be able to tackle the more general problems. It is also important for students to realize the uncertainty about the angles magnitudes in some situations. By working with uncertain situations, we want them to make an educational guess that based on some assumptions. We presume when students work with uncertain situations, it is more likely that they will acquire in-depth understanding about the topic.

4.5 Lesson 5: Angles related problems

4.5.1 Starting points

In this lesson, students should be able to solve some problems that related to the angles magnitudes in more general cases. We employ everyday life contexts to serve our goals. We assume the students can perform the following actions before they work with the tasks and the questions in the worksheet (see worksheet 5 in the appendix).

a. The students can draw a top view of an object.
b. The students know the concept of letters-angles (F, X, and Z angles).

c. The students can make an educational guess based on certain assumptions.

4.5.2 The learning goals

Main goal

The students are able to apply the properties of letters angles (F, Z, and X-angles) in the angle related problems.

Sub-goals

a. Enable students to translate given information into a diagram.
b. Enable students to show angle similarity on a straight line that falling across two parallel lines.
c. Enable students to use their current knowledge to solve the angle related problems.
d. Enable students to use their current knowledge to give reasonable explanations related to their computations.
e. Enable students to figure out the uncertainty in a problem.

4.5.3 Description of activity

We divide this lesson into three stages. In the first stage, students investigating the angles on the intersections of the railways after they make the top view drawing of the railways in advance. The second stage, students have to apply their knowledge about letter angles (F, X, and Z angles) to explain the similarity between angles in their railways drawing. In the third stage, students will encounter more general mathematical problems that require them to apply their knowledge about letters-angles.

First stage: angles of railways

The lesson begins when teacher displaying a perspective picture of a railway where the bars seem meet each other in the horizon (Figure 4.7). The teacher then asks students to determine a point of view where they will see the bars so that the bars don't meet each other. The teacher should lead students to understand the top view of the situation in a classroom discussion. After the discussion the teacher distributes the worksheet (see worksheet 5 in the appendix), in this stage students have to indicate the angles on the intersection of the railways that have the same magnitude. To perform this task, first students have to draw the top view of the railways and then identify the angles (see figure 4.8 and 4.9). The teacher should also ask students to explain why those angles in the same magnitudes.

Figure 4.7. Perspective picture of a railway.

Second stage: letters-angles in general

In the previous lessons, students have justified their conjecture about the similarity between angles on some letters (F, X, and Z). In this stage, we want the students to generalize that concept by asking them to explain why the concept also hold true in this context and ask for generalization.

Third stage: solving the problems related to the magnitudes of angles

We present four problems that related to the angles magnitudes for students to solve. In the first problem, we implicitly ask students to apply their knowledge about letters-angles to figure out the relation between angles on a straight line that falling across two parallel lines. The second problem, students have to assign the numerical values for the angles in parallel-transversal situation from the given information. In the third problem, students need to apply the concept of straight-angle to tackle the problem. The fourth problem, encourage students to make an assumption to answer the given problem.

Figure 4.8. The picture of railways intersection.

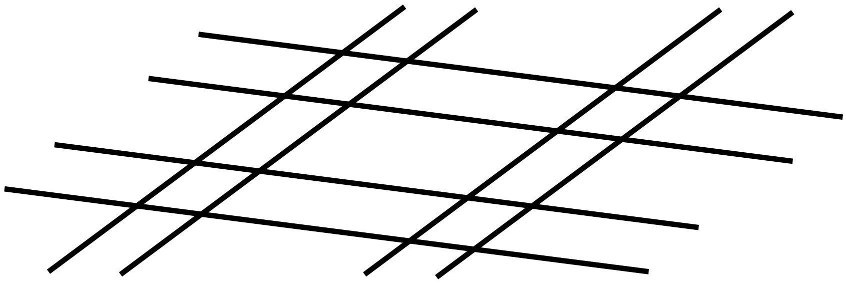


Figure 4.9. A top view sketch of the railways.

4.5.4 Conjecture on students' reaction

a. In the first task, some students may draw a trivial condition of the intersection where all railways are perpendicular. However, the ideal condition is when students draw the top view of the railway that varies in shape.

b. In the second task, students may indicate the angles on the railway that have the same magnitude and give explanations using letters-angles concepts without help from the geometrical patterns or grids.

c. In answering the first question, students may find out that angle 1 and angle 3 are equal, find out that angle 2 and angle 4 are equal, find out that the sum of angle 1 and 4 or 1 and 2 is $180°$, or find out that the sum of four angles is $360°$.
d. In answering the second question, students may apply their understanding about the properties of angles in parallel-transversal situation in the first question to find the solutions.
e. In answering the third question, some students may conclude that $70°$ is the rights answer ($180°$ as a benchmark) and some may conclude that $250°$ is the rights answer ($360°$ as a benchmark).
f. In answering the fourth question, students may give different combination for the size of two angles where the sum of both angles is $130°$.

4.5.5 Discussion

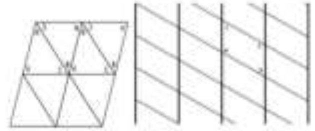
During the constructing of the top view of the railways' intersection, it is possible that some students draw the railways intersection that perpendicular to each other and we consider it as a trivial construction. In order to avoid the superficial understanding toward the intended knowledge, the teacher and the students have to conduct a further discussion about another possible arrangement of the railways intersection, so that the non-trivial constructions emerge. By doing so, all of students' constructions are unique and it may create a supportive learning environment to help them to generalize the concept of similarity between angles in parallel-transversal situations.

In this final teaching and learning activity, we want students to arrive at the formal level of understanding toward the topic. The four questions in the worksheet allow the students to transfer their current knowledge to the more abstract situations. The first two questions were designed to support students understanding about the concept of similarity between angles on a straight line that fall across a pair of parallel lines and the last two questions were designed to provide students with an alternative situation where they have to make several assumptions to solve the problems.

In the first question, students have to explain how they determine the similarity between angles when there is a straight line fall across two parallel lines. In the classroom discussion we might observe some students apply their current knowledge about letter-angles (F, X, and Z-angles) to explain the similarity between angles. In the second question, we ask students to determine the magnitude of unknown angles from the given information. In this particular case, we want them to apply their current knowledge in a numerical context. The third question begs students to reason about the magnitude of straight-angle to find the magnitude of an unknown angle. However, some students may also use full-angle instead of straight-angle to solve this problem. In the last question, we give students a variation of the third question where they will encounter uncertainty condition. Students' mathematical explorations on this problem can be considered as an important learning activity that enhance their current knowledge and give them a better understanding toward the topic.

GENERAL LEVEL

Learning line of the development of reasoning and understanding about the magnitude of angle in grade 7



FLOATING CAPACITY

1st Lesson

Indicating the angles from the real-world objects ⟶ Ordering the angles from the smallest to the biggest ⟶ Drawing the extreme situations of a dynamic angle ⟶ Discuss about how an angle is formed ⟶ Defining the angle

Students encounter the real-world objects that associate with angles

Carry out simple observations and reasoning via angles size ordering activity

The students are able to recall and redefining the angle

2nd Lesson

Reconstructing the upper case letters using wooden matches ⟶ Compare the results of the reconstructions ⟶ Observe and analyze the size of angles on the letters that have parallel sticks

Students reconstructing the alphabets from wooden matches

Observing and investigating the size of angles on the letters that have parallel sticks

The students are able to infer the similarity between angles that formed by a straight line that falling across two parallel lines

3rd Lesson

Finding the letters like figures ⟶ Compare the letters on a tilled floor with the letters on the alphabets from matchsticks ⟶ Indicating the angles and parallel lines on a tiled floor ⟶ Analyze the relation between parallel lines and the size of angles

Students analyzing the angles on the tiled floors

Comparing the letters on a tilled floor with the letters on the alphabets reconstruction activity to justify about the angles similarity

The students are able to describe the similarity between angles that formed by parallel and transversal lines

4th Lesson

Students analyze the size of angles on the tilled floors

Investigating the sizes of angles on the tiled floors

The students are able to reason about the size of angles using tiles' patterns

Investigating the sizes of angles on the tiled floors ⟶ Analyzing and explaining the size of angles on a common point of a tiled floor ⟶ Showing the 360 degrees angle ⟶ Calculate the size of each angle on a tile

5th Lesson

Students solve the angle related problems

Computing the unknown angles size from the given information

The students are able to applying the properties of letters angles (F, Z, and X-angles) in the angle problems

Determine the top view of the railways ⟶ Identify the angles that have the same size ⟶ Observe and reason about the size of angles on a tiled floor ⟶ Compute the unknown angles size from the given information ⟶ Reason with straight angles ⟶ Reason about uncertainty

CHAPTER 5
RETROSPECTIVE ANALYSIS

Throughout this chapter we will compare the hypothetical learning process with students' actual learning in order to improve the predictive power of the HLT. The process we called as the retrospective analysis. The results of this analysis are used to answer research question, sub-research questions, and to give a contribution to the local instruction theory for understanding angle and its magnitude. In addition, the results are considered as the underlying principles that explain how and why this design works. The retrospective analysis in the current study consists of three steps; analyzing the first teaching experiment (first cycle), analyzing the second teaching experiment (second cycle), and analyzing the third teaching experiment (third cycle). In the beginning of each step we will describe students' prior knowledge and in the end of it we will describe students' current knowledge (acquired knowledge). Throughout this chapter, we triangulate the data that we gathered from the pre and post assessments (test and the interview) with our findings in the actual learning process. This process helps us to explain students' understanding toward the concept, provide us with an inventory of students' solution procedures, advance our design, and answer the research questions. The short version of the retrospective analysis (Dierdorp's Analysis Matrix) for the teaching experiments can be found in the appendix of this paper.

5.1 First teaching experiment (first cycle)

There are 6 seventh graders that involved in the first cycle, with 1 female student and 5 male students in composition. During the first teaching experiment, the researcher acted as the teacher. In this phase, we 'test' our educational design with these students in order to adjust the content of the design and make a revised version of the design. The revised version of the design will be used as a guideline for the next teaching experiments. The detail of the observations, analyses, and evaluations of the first teaching experiment describe as follow in a chronological sequence.

5.1.1 Pre-assessment

The students took a 20-minute pretest before going into the entire lesson sequence. The pretest items were designed to assess students' prior knowledge about angle and its magnitude. Due to the limitation in evaluating students' gained scores for describing their understanding, we conducted a further analysis on the students' written work to inquire what students had known and hadn't known about the mathematical topic before they went into the lessons sequence. Therefore, in this study the scores that students gained from the pretest are not the absolute indicator of students' prior knowledge.

Most of the students were unable to reach 50% of the total score in the pretest, this may indicates that the students have limited understanding about angle and its magnitude. Based on the students' written work and interview, we found that all those 6 students perceived angle as the spaces in between two lines in the plane which meet in a point. We also observed that they mastered to use a protractor for measuring the angle magnitude and knew the unit for angle was degree (°). Most of the students could identify angles from any geometrical figures. However, some of them used non-standard symbols to indicate the angles (e.g. strip, check mark, and circle) instead of arc symbol (∡) which is commonly used. Fifty percent of them could identify right-angles in the given figures of L-shape that varied in size, this indicates that some of them have already had good understanding about right-angle. The students categorized angles in three different categories based on it magnitude; acute, right-angle, and obtuse. Some students were able to infer similarity of angle magnitude from the given geometrical figures but currently their observations and analyses are less detail.

We found one student (Alif) in this small group had flexibility in understanding angle definitions. He accepted all three definitions of angle that we presented in one of the test item as the right definitions of angle. However, his understanding about angle magnitudes in some sense was still limited for angles that were less than 180° (so did other students). In other words, he (they) didn't perceive reflex angle as an angle. For instance, when we asked him to indicate the smallest and the biggest angles in the given figure, he (as most of the students did) gave marks to some shape that he (they) thought as angles and gave no clear distinctions between the smallest and the biggest ones (see figure 5.1). Interestingly, from his written work, it is obvious that he hesitated to accept the fact that a reflex angle was also an angle, which could be seen from the usage of pencil instead of ink to indicate the reflex angle.

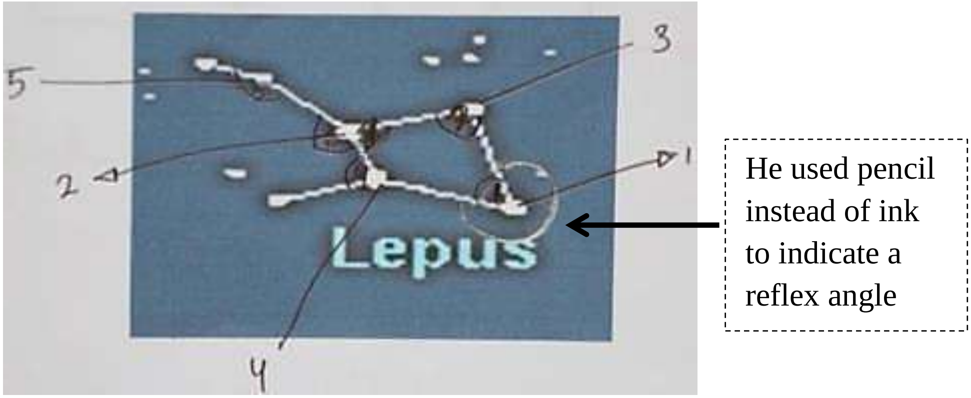


Figure 5.1. Alif's written work indicates his hesitation about reflex angle.

We found a student (Ajeng) that had already known about the angles categorization based on its magnitudes (acute, right-angle, and obtuse) and could reason about the angle magnitudes on the analog clock (1 hour equal to 30°). However, the interview reveals that his competency was on the level of remembering the subject matter (relied on her capability in retrieving information). For instance, one of the questions in the test asked the students to determine an unknown angle magnitude in a straight-angle situation provided with a known angle magnitude (50°). Instead of analyzing the situation and applying the knowledge, she solved the problem on her way (i.e. $x° + 50° = 180°$), she preferred to randomly present the information that she had already known before, as a result, she generated an irrelevant respond to the given problem (see figure 5.2). In this case, we don't know for sure the reason why this student gave such irrelevant respond for the presented problem.

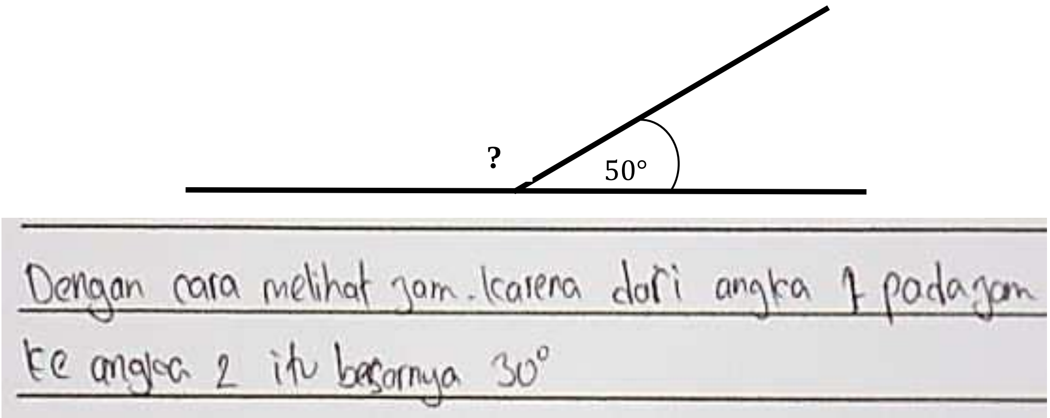


Figure 5.2. Ajeng wrote, “On a clock, from 1 to 2 the size of the angle is 30°.

Her limited understanding about angle magnitude can also be observed in her answer to the question that asked her to put seven polygons in an ascending order based on the magnitude of internal angle. It seems that she made an order of the polygons based on their area instead of the order of the polygons based on their internal angle. Without hesitation we can conclude that, she perceived the angle as the area between two intersecting lines. Unfortunately, her conception about angle hindered herself to perceive the concept of angle magnitude.

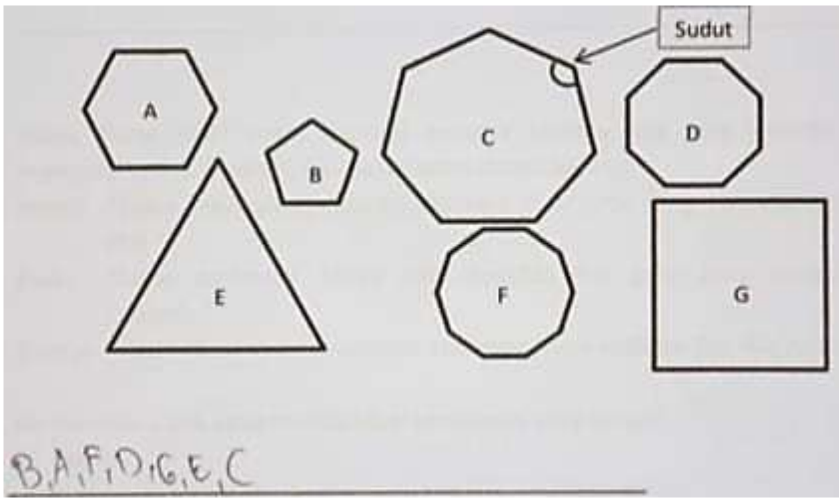


Figure 5.3. Ajeng ordered the given shapes based on their area.

Five out of six students encountered a difficulty to perceive some special angles (e.g. 0°, 180°, or any angle that greater than 180°) due to their limited inventory of angle definition. They only accepted the angle as the space in between two lines in the plane which meet in a point. One of the students named Giga clearly showed an effect of his limited inventory of angle definition. When we asked him to explain what he knew about the angles magnitude in a vertical angles situation, his judgment about angles magnitude seemed affected by the size of the arcs that indicates the angles in the given situation. The designed problem is about vertical angles where one of the arcs that indicates the angle is slightly narrow compare with its pair. Students' solutions to the presented problem indicate that they are less capable to infer similarity between angles in this particular context because most of them gave wrong answers or gave no answer at all.

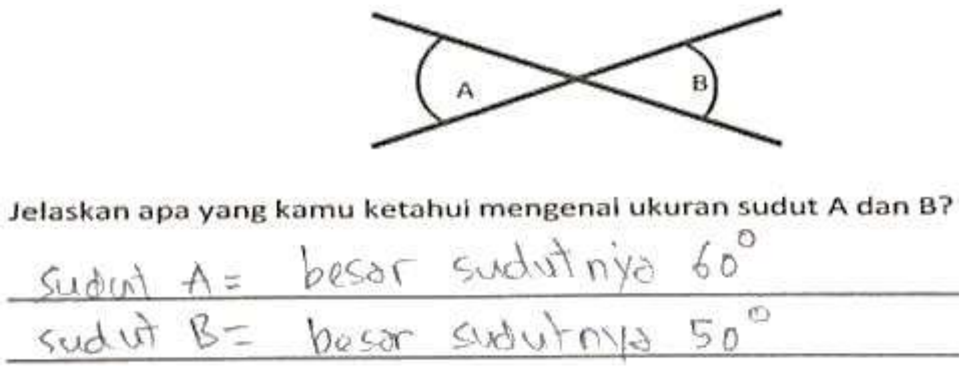


Figure 5.4. Giga's solution to the vertical angles problem, it says $\boldsymbol{A = 60}°$ and $\boldsymbol{B = 50}°$.

Fifty percent of the students were unable to recognize the right-angle in the three given figures of L-shape that differed in size. They tended to claim that an L-shape that could cover the largest area represented the greater angle. In the other words, students claimed that if they made a quadrilateral by adding two extra line segments that paralleled to the two arms, they could decide the angle magnitude by evaluating the area of the quadrilaterals. The definition of angle that strongly related to the concept of area that students embraced also produce another consequence. Since the area that they understood has to be bounded and without any line segment in between the coverage area of an angle. Thus, they did not see the possibility to add or subtract angles in some angle situations. For instance, we asked the students to determine how many angles that they could see in the X-shape and all of them only saw 4 angles instead of more than 4 angles (13 angles). We also found that, all of six students were unable to solve the straight-angle problem in the test due to the lack of reasoning.

From the description above, we can infer that although the students had learnt about the concept before, their understanding toward the concept is still limited and fuzzy. It can be observed in their attempts to indicate the biggest and the smallest angles in given geometric figures, most of them were unable to give adequate responses. We claim the root of the problem is lying on the definition of angle that students embraced. They perceived the angle as the coverage area between two angle arms. An additional information that we got from the observation is most of the students were reluctant to read in order to understand the instructions in given problems and if they read it, they did it carelessly. We also conclude that, it is one of the factors that sometimes make the students misinterpret the instruction in the test.

Using this information in hand, we decide to make small adjustments in the pretest items in order to increase the prediction power of the test. The revised version of the pretest, mainly focus on the technical aspect instead of content aspect, because we don’t see any significant flaw related to the content of the test. For instance, in every item test we printed in bold the key words to make the students immediately focus on the main aspects of the problems. We make the instructions shorter and understandable as well. Based on their written work, we know that many of them were reluctant to explain what they were thinking. By changing the word ‘explain’ with ‘write down’, we expect the students are willing to show what they know from the given situations. A bamboo fence problem in the pretest that asked about, how many angles that exist in given figures is considered to be redundant. It has the same intention with the problem of X-shape either asked about the same thing but differed in complexity. Therefore, we removed the bamboo fence problem from the test. In order to increase the reliability of the test, we also conducted a peer examination of the test items with colleagues.

5.1.2 Lesson 1: Angle from everyday life situations

As it explained in chapter IV, the first lesson includes four stages. The aims are to make the students retrieve their knowledge about angle and at the same time enable them to redefine the angle. We performed each stage in such a way as to generate a supportive learning environment in order to strengthen students' understanding on the very basic concepts of angle.

First stage

In the first stage, the students were asked to indicate an angle in a set of pictures of everyday life objects (see figure 5.5). Students' reactions to the given task were matched with our predictions. These are the examples of students' reactions that are in line with our conjectures in the HLT; (a) all of the students could indicate the angles in the given figures but some of them didn't use the formal symbol (∡) to indicate the angles, (b) most of them indicated more than one angles in each figure, and (c) didn't recognize the existence of a 0° angle in some objects. In the actual teaching and learning process, we asked the students to focus only on one angle in each figure although they had indicated more than one angles in each figure, we did it in order to avoid the perplexity when the students worked with the second task.

Second stage

In the second stage the students worked in groups of two to sort the indicated angles based on their magnitude and made a poster (see figure 5.6). We predicted that some of the students might encounter difficulties to indicate and order the angles in pictures B, D, and H (0°, 180°, and 360° on an analog clock and the traditional fans) but all of them showed good understanding about the magnitude of those angles except the 0° angle. It can be observed from the way they sorted the angles from the given figures based on their magnitude (figure 5.6). All of them put the 360° angles on the very end of the sequence. We also found an interesting finding in the students' construction. In every sequence that students made, the figures with 90° angle or looked like 90° angle clustered in the middle of the sequence, and the figure with an acute angle clustered in the beginning of the sequence.

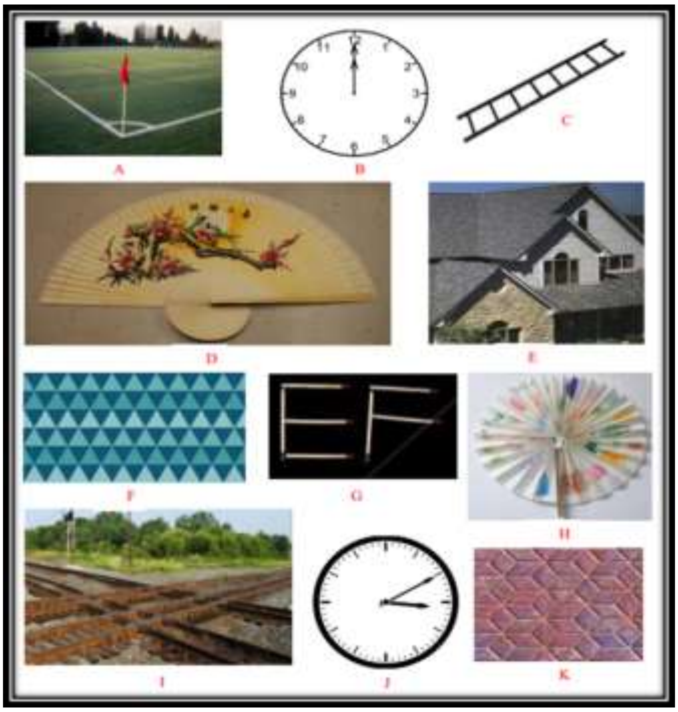


Figure 5.5. Pictures of everyday life objects.

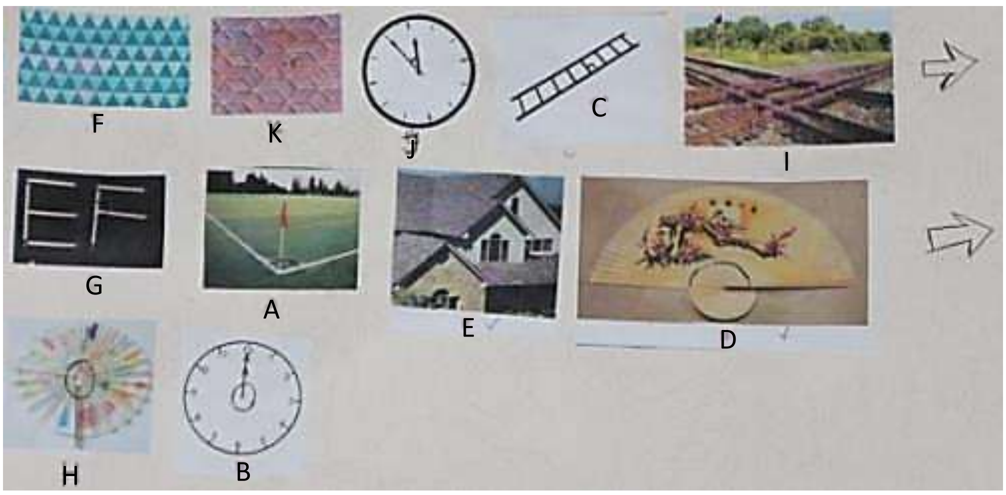


Figure 5.6. Ajeng and Giga sorted the angles magnitudes based on acute, right-angle, and obtuse as benchmarks.

Third stage

In the third stage, the students observed, compared, analyzed, and discussed the posters related to the order of the indicated angles. Students found several discrepancies related to the order of the angles. The classroom discussion revealed that although Ajeng and Giga grouped the right-angles nearly in the same cluster, they assumed that the magnitude was different. The following fragment from the classroom discussion depicts how Ajeng and Giga interpreted the right-angle situations.

[1]Researcher:	*"I found an interesting thing in your poster. Let us observe the angles on the football field corner, ladder, and matchsticks! (Pointing to the right-angle in each picture) What do you think about their sizes in the real life if we measure them by using a protractor?"*
[2]Giga:	*"90."*
[3]Rafli:	*"It will be 90° if it is in the real-world."*
[4]Researcher:	*"So A is 90° (Pointing to the right-angle of the football field corner). How about C?" (Pointing to the right-angle in a ladder)?*
[5]Giga & Ajeng:	*"90" (Give answer at the same time)*
[6]Ajeng:	*"90 if you erect it" (Made hand gestures of vertical ladder)*
[7]Researcher:	*"How about G?" (Pointing to the right-angle in letters E and F)*
[8]Giga:	*"90"*
[9]Ajeng:	*"That's right-angle." (Justifying Giga's answer)*
[10]Researcher:	*"You knew that they have the same size, but why you don't put them side by side?" (Pointing along the sequence of Ajeng's and Giga's poster)*
[11]Ajeng:	*"If you see A in the picture, it is not 90° but it is 90° in the real-world." (Tried to explain her way in perceiving the angle in the picture)*
[12]Researcher:	*"So you see the angle as it is in the picture." (Summarizing)*
[13]Ajeng:	*"Yes"*

The classroom discussion revealed that the students comprehended the presented situation but they embraced two different interpretations related to the given situations (real-world or picture). Although, both Ajeng and Giga agreed to sort the angles by seeing the angles as their appearance in the picture, we cannot clearly see what references that they used to cluster those angles. In addition to that, by applying the same strategy to the situation, we found several inconsistencies in their construction. For examples, it is clear that the angle which they had indicated in the floor with parallelogram tiles (120°) was larger than the indicated angle in the analog clock (30°) but they sorted them in the other way around. The same thing happened with the angles that they indicated in the pictures of railways intersection and ladder.

In contrast with Ajeng's and Giga's construction, Alif and Hilal saw the angles as their appearance in the real-world to sort the indicated angles and produced a well-constructed poster (figure 5.7). It is because if we use the same strategy to sort the indicated angles we will produce a similar result. However, it is clear that the students didn't anticipate the existence of a 0° angle in the presented situation even they had known the 0° angle is the smallest angle. We are fully aware that the concept of zero angle is a dual concept. The 0° angle conflicted with the concept of full angle and therefore hindered students' recognition of the concept. Due to the duality of the 0° angle and full angle, a further discussion was conducted to help students to comprehend the concept.

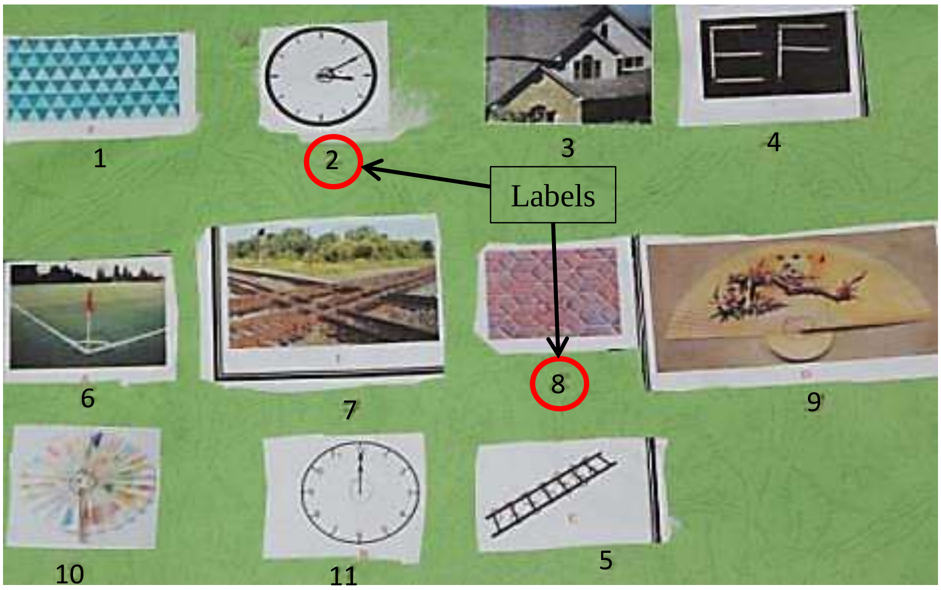


Figure 5.7. Alif and Hilal labeled the pictures to sort the angles magnitudes using real-world interpretation.

From the discourse about the duality of a 0° angle and a full angle we also found that the students didn't realize the existence of reflex angles in every angle figure. We believe that the use of static angle situations that is frequently presented in every mathematics text book in elementary schools has built this conception. In the classroom discussions about duality of a 0° angle, the researcher tried to embed the concept of reflex angles using a dynamic angle situation. In order to engage the students into the discussion, the researcher arranged two pens perpendicular to one another and asked the students what angles that they could see (see figure 5.8). As we expected, they recognized the right-angle from the presented arrangement. The researcher then moved one of the pens gradually to make the angle bigger, when the situation reached the angle that more than 180° it forced the students to accept and realize the existence of reflex angles.

Figure 5.8. The researcher utilizes a dynamic angle situation in order to make sense the duality of the 0° angle.

Fourth stage

In the fourth and last stage the students worked individually. We presented 4 questions to investigate what are students' definitions about angle evolve during the lesson. In addition to that, we also inquired about how the students grasped the sense of angle magnitude via drawing the extreme conditions of dynamic angle situations. In the first two questions, we asked the students to draw the extreme conditions of the dynamic angle situations (i.e. analog clock) and then asked them to give some explanations related to the magnitude of each condition. The actual teaching and learning process matched with our conjectures in the HLT in which we argued some students might draw a small non-zero angle to represent the 0° angle and draw an obtuse non-360° angle as the biggest angle. During the learning activity, 4 out of 6 students agreed that 360° was the biggest angle in analog clock situation and all of them claimed that the angle between two consecutive numbers on the clock represented the smallest angle (30°). A discourse to discuss about the smallest angle on the analog clock was conducted to clarify students' conception. The following fragment from the discourse depicts the clarification of this conception.

[14]Researcher:	*"How you draw a smallest angle? Can somebody explain it?"*
[15]Giga:	*(Raised his hand) "The hour hands on 3 and minute hands on 2."*
[16]Researcher:	*(Made a drawing based on Giga's description and show it to the other students) "Is it what he means?"*
[17]Alif:	*"Hour hands on 3!!!" (Figured out that the researcher swaps the hands of the clock on his drawing)*
[18]Researcher:	*"... (Waiting for the responses from the other students)"*
[19]Other students :	*"(Rambled) It doesn't matter, that is the same, 30°"*
[20]Researcher:	*"Okay, do some of you have different opinion about its size?"*
[21]Students:	*"No…"*
[22]Researcher:	*"Is it possible for us to construct an angle that is smaller than this one?"*
[23]Students:	*"Yes (Giving their answer at the same time)"*
[24]Researcher:	*"So there is another smaller angle, how do you draw it?*
[25]Rafli:	*"That will be very small"*
[26]Giga:	*"More (He meant 'less') than 1 minute, (Made hand gestures for small thing) in one minute"*
[27]Researcher:	*"Draw it!" (Students drew the situation, see figure 5.9)*

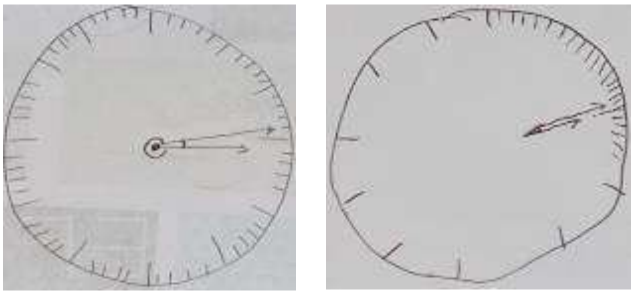


Figure 5.9. Giga’s and Rafli’s attempted to draw the smallest angle.

[28]Researcher: “Giga, can you show us your drawing! (Giga showed his drawing) Can someone else draw another smaller angles than this?”

Students try to draw another smaller angles that are approaching a 0° angle. The researcher realizes the difficulties that the students encountered so, we uses different approaches to help them.

[29]Researcher: “Okay, what about the biggest one?” (Asked the students to think about the dual possibilities of the situation)

[30]Students: “360” (Giving an answer at the same time)

[31]Giga: “12 o’clock”

[32]Researcher: “If you know 360° is the biggest angle, so what can you say about the smallest angle?”

[32] Students: “0” (Giving an answer at the same time)

[33]Researcher: “Okay, so 0° is the smallest angle. Can you draw it?

When Alif drew and claimed a straight line as a picture of 0°, the other students think the straight line represent 180°. The discussion showed that the students still struggled to draw the 0° angle, because the 180° and 360° angles can always be pointed out in every drawing attempt. Since the focus of the first meeting was to recall the angle concepts and redefine the angle definitions, we postponed the clarification of this debate to the fourth meeting where we mainly stress our attention to the magnitude of angles.

We also asked each student to write down a definition of angle according to them. From their work we can observe the change that occurred in their understanding about the angle. At the beginning of the lesson most of them defined the angle as the spaces between two intersecting lines, but after doing the activities in this lesson they defined angle as the difference of direction between two lines (Figure 5.10). None of the students defined the angle as the amount of rotation between two lines, even the analog clock context emphasizes the relation between angle and rotation. The actual teaching and learning activities in this lesson could help the students to retrieve their prior knowledge about angle and its magnitude. The activities also allowed the students to inductively redefine the angle using the ideas that they got from observing, comparing, analyzing, and discussing the angles from everyday life objects.

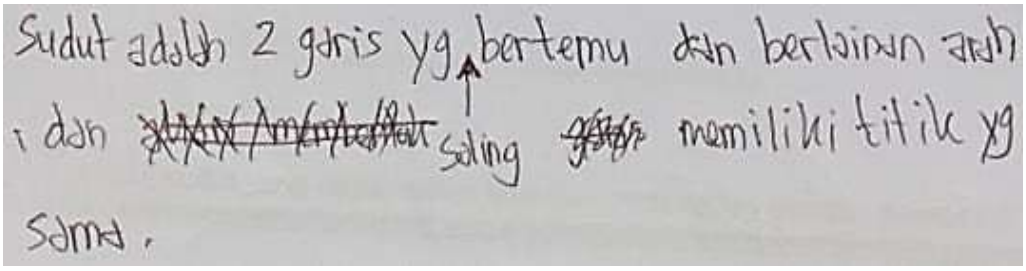


Figure 5.10. It says, “Angle is two lines that meet each other with different directions and have a common point”.

The analysis of this lesson allows us to improve our design for the first lesson. The improved version of the first lesson included the following things:

1. Removing unnecessary empty boxes for the first three instructions in the worksheet 1 (see worksheet 1) where the verbal explanations in the classroom discussion are more effective compared with the written explanations.
2. Splitting the empty box for the second question into two boxes in order to lead the students to give only two intended answers.
3. Adding more details in the teacher guide for classroom discussion related to comparing the magnitude of two or more angles in order to reveal students’ references (benchmarks) in classifying the angle magnitudes.
4. The guided questions for the classroom discussion about a 0° angle need revision. The discussion should allow the students to use the approximation strategy to realize that the 0° angle is in the same figure with 360° angle (dual of a 0° angle).

5.1.3 Lesson 2: Matchsticks, letters, and angles

There are 3 stages in this lesson with the aim to help the students to infer angles similarity between angles that formed by a straight line that falling across two parallel lines (parallel-transversal situation). During the lesson, we asked the students to make a poster of upper case letters using matchsticks, analyze the angles on the letters that have parallel sticks, and inferre the similarity between those angles.

First stage

In the first stage, we put the students into two groups and asked them to make a poster of upper case letters using matchsticks. The aim of this activity is to give the students a hand on experience in constructing the angles with various magnitudes. The students performed well during the activity. They could easily reconstruct the upper case letters without hesitations (see figure 5.11). However, there was a technical difficulty when the students performed this task. The students found it difficult to glue the matchsticks on the poster paper, as a result, one of the groups lagged behind and we immediately asked this group to arrange the matchsticks on their table instead of gluing it on their poster paper.

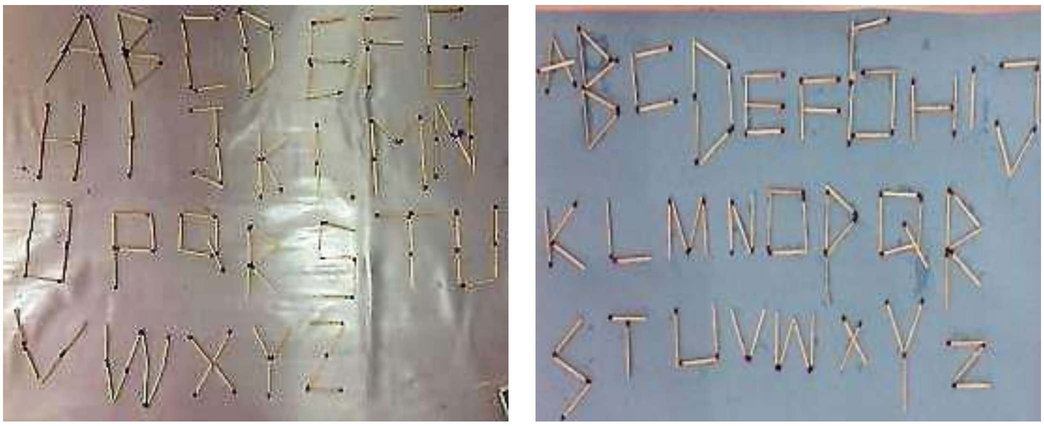
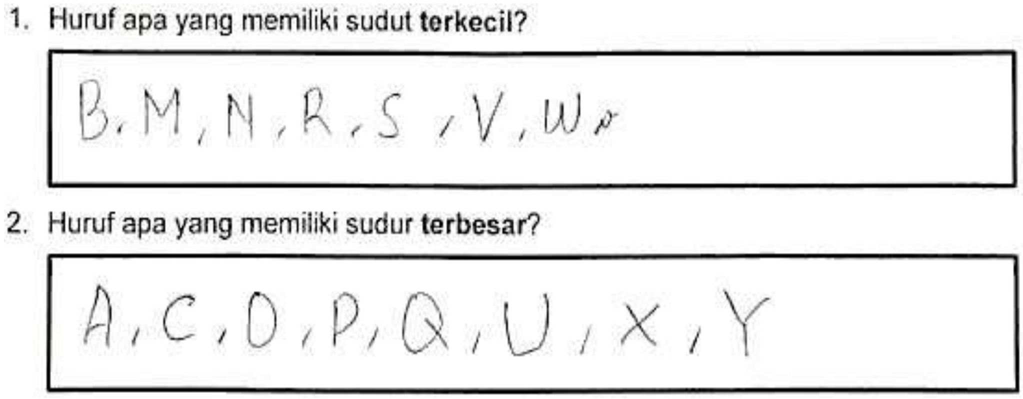


Figure 5.11. Students' constructions.

Second stage

We gave students time to observe each other poster in the second stage. Up to this point, the students found no significant finding related to the angles magnitude on the letters. Mainly they found differences in technical aspects such as, the number of sticks to construct each letter, the shape of the letters, and the appearance of the posters. In order to keep the students on the track, we asked them about letters that have the smallest angle and the biggest angle. Unfortunately, all of the students misinterpreted the instruction and gave the plural answer for this singular question (see figure 5.12). From the discourse we found that they had difficulty to distinguish between singular and plural in the instruction. A discourse was performed to clarify this misinterpretation. The following fragment from the discourse describes how students interpreted the instruction and how we as a teacher could help them throughout a classroom discussion.

[1]Giga:	*"Which letter do you think that has the smallest angle? (Read the question out loud and immediately gave the answer) A, B, K, M, N, P, R, V, W, X, Y, and Z"*
[2]Rafli:	*"B is 90°" (Criticized Giga's answer)*
[3]Giga:	*(Lifted his shoulders)*
[4]Researcher:	*(The researcher realized the unintended responses from Giga and provided an analogy for the situation) "If I ask you, who is the shortest student in your classroom? (The students were pointing to Hilal and giggling at the same time) Is that possible to have more than one solution for this kind of question? Think about it for a moment!"*
[5]Alif:	*"One" (Talked to Rafli to convince him)*
[6]Researcher:	*"Back to the question, 'What letter do you think that has the smallest angle?' how many solution will it have?"*
[7]Abell:	*"One!"*
[8]Researcher:	*"So why did all of you give more than one solution?"*
[9]Rafli:	*"Yeah…how that happened?" (Realize about the misinterpretation)*



1. Huruf apa yang memiliki sudut **terkecil**?

B, M, N, R, S, V, W

2. Huruf apa yang memiliki sudur **terbesar**?

A, C, D, P, Q, U, X, Y

Figure 5.12. Students' plural answers for singular questions.

After the students realized their misinterpretation we asked them to decide which letter that had the smallest angle. The students came up with different solutions. For examples, Giga and Hilal chose A, Alif chose V, Rafli chose W, Abell chose N, and Ajeng chose M. The researcher used these different solutions as a starting point for a classroom discussion. The researcher drew again all those letters and asked the students to indicate which angle that they refer to. We realized that the students had good sense about angles magnitudes. The following fragment from the classroom discussion reveals how students used their sense of angle magnitude to explain the similarity between angles.

[10]Researcher: *"Between V, W, N, M, and A, how do we compare the angle sizes in order to know which one has the smallest angle?" (Started the discussion)*

[11]Rafli: *"By finding the acute and the obtuse angles"*

[12]Ajeng: *"No…You can compare it with the analog clock!"*

[13]Researcher: *"Okay, between V and A (Reconstructed the letters according to the students constructions; V with 4 sticks and A with 3 sticks) How we compare the sizes of these angles?"*

The students gave their argumentations, but generally they were unable to convince their fellow students about their claims. After few moments of thought, Giga came up with a strategy. He removed two sticks from the very ends of the

V’s arms and put one of the stick to turn it into a letter A. He managed to convince their fellow students that the angles on letters V and A were in the same magnitude.

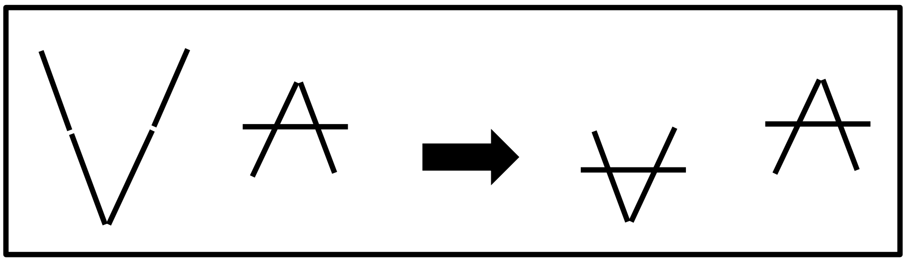


Figure 5.13. Giga’s strategy to show both angles are in the same magnitude.

[14]Researcher:	*“Now we agree that the angles on A and V have the same magnitude. How about the letters W, N, and M?”*
[15]Abell:	*“N and M are equal”*
[16]Giga:	*“N and M are equal!” (Pointing out to the angles in the tops of both letters)*
[17]Rafli:	*“W and M are the same, because W is the upside down version of M.”*
[18]Researcher:	*“But first, how do you compare N and M?” (Rearrange the sticks into the letters according to the students’ construction)*

After few moments, Abell came up with a similar strategy to show the angles were in the same size, he removed two sticks from M and one stick from N to make both letters appeared in the same shapes.

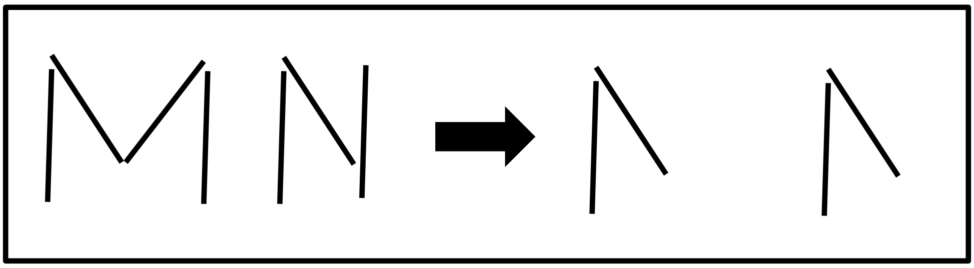


Figure 5.14. Abell’s strategy to show both angles are in the same magnitude

[19]Rafli: *"Yeah…that's the same."*
[20]Hilal: *"They become the same now."*
[21]Researcher: *"How about the angles on it?"*
[22]Giga: *"The angles are in the same size as well."*
[23]Researcher: *"Now we have two groups of letters that have different angles magnitudes. The first group consists of A and V, and the second group consists of M, N, and W. Therefore, we only need to compare two letters, which letters do you want to compare?" (The students chose to compare V and N)*
[24]Abell: *"N is smaller than V." (Ajeng made a claim and Abell indicated the angles)*
[25]Giga: *"It is an acute angle." (Other students were measuring the opening of the letters using a matchstick to compare the angles)*
[26]Researcher: *"N has the smallest angle? Can some of you explain it?!"*
[27]Alif: *(Removed a stick from the letter N and drew the imaginary line segment on the opening of each letter)*
[28]Researcher: *"Do you want to say that the opening on letter V is bigger than the opening on letter N?"*
[29]Alif: *(Nodding)*
[30]Researcher: *"So what is your conclusion about the letter that has the smallest angle?"*
[31]Students: *"N"*

We performed the same approach to make the students use their reasoning in order to reinvent the concept of reflex angle. The students gave different answers related to which letter that had the biggest angle. In the discussion the students agreed that the biggest angle and the smallest angle have to be in the same figure (N), if they take into account the reflex angles. It was evidence that the students have grasped the concept of reflex angles at this point.

Third stage

In the last stage, the students analyzed the angles on the letters that had parallel sticks such as, E, F, H, N, U, and Z. In general, the actual students' reactions meet our conjectures. We observed that the students could easily give an explanation about angle similarities when 90° angles were involved (E, F, H, and U). Although they were able to infer the angle similarities when 90° angles weren't involved, they needed some guidance to explain their claims properly. The students were able to reason using their existing knowledge in the attempt to show the similar angles in the letter Z. They argued that, they could reshape the letter Z into a diamond shape in order to make clear the similar angles. The students' explanations were based on the fact that the opposite angles in a parallelogram are in the same size.

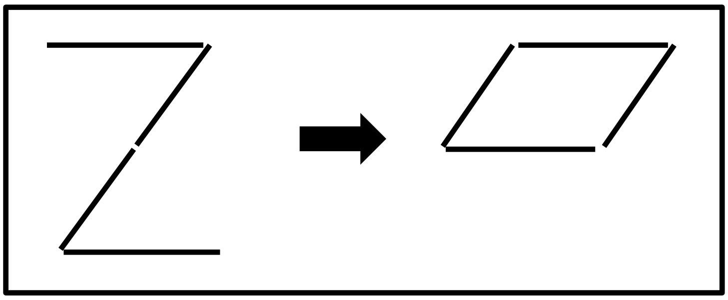


Figure 5.15. Students employed a property of parallelogram to explain the similarity between angles.

Based on the actual teaching and learning activities, we argue that the activities in this lesson could support students' learning to infer angles similarity in the parallel-transversal situations. Justification of this claim can be found in the students' written work when they indicated the angles that had the same magnitude (see figure 5.16).

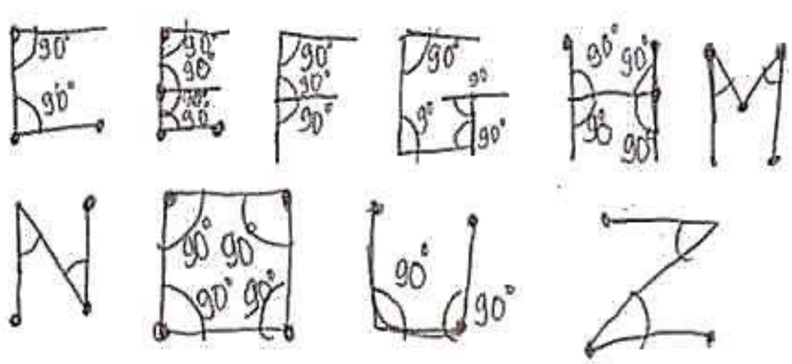


Figure 5.16. Students inferred the angles similarity.

We evaluate the second lesson based on the observations and analyses of the students' actual reactions throughout this lesson. The evaluation of this lesson allows us to improve our design. The improved version for the second lesson included the following things:

1. We ask the students to arrange the matchsticks on their table instead of using glue and paper to make a poster.
2. We print in bold the key words in the worksheet in order to avoid misinterpretation.
3. We restructure the teacher guide to effectively guide the students to compare the letters reconstructions.
4. In the teacher guide we add a discussion that aims to make a bridge between 0° and 360° (duality: reflex angles).

5.1.4 Lesson 3: Letters on the tiled floor models

As it stated before in chapter 4, the core of this lesson is to provide a supportive learning environment for the students to justify their conjectures about angles similarity that they have inferred in the lesson 2. There are 3 stages in this lesson.

First stage

During the first stage of the actual teaching and learning process, students performed a mathematical exploration on the patterns like letters on the two given pictures of the tiled floor models. Students' reactions in the actual process were in line with our conjectures in the HLT in which we argued the students will highlight the gaps between tiles that form a word 'ANA' but they use different amount of gaps to construct the word. We also found that, the follow-up task that requires the students to find the letters in the second floor model (bedroom floor) is redundant. Although, they were able to work with the task, due to the repetition of the instruction, most of them found that the task was tedious and time consuming.

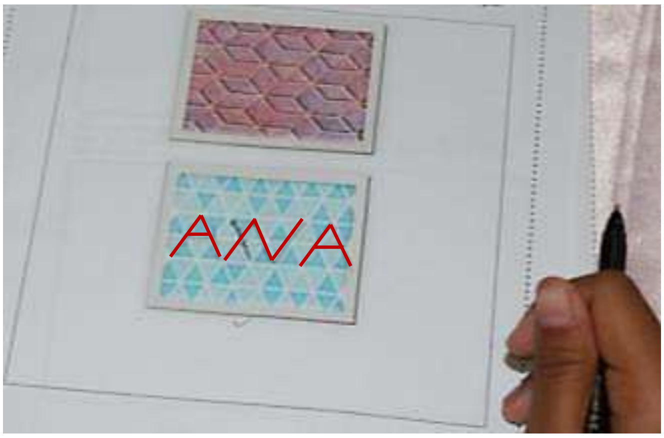


Figure 5.17. Ajeng showed the word ANA on the kitchen floor model.

Second stage

In the second stage, the students compared the letters on the kitchen floor model (first floor) with the letters from matchsticks activity (lesson 2). The comparison process allowed the students to justify the angles similarity on some letters (i.e. E, F, N, X, and Z) by using the uniformity of the tiles. We observed that, most of the students were able to infer the similarity between the angles in the classroom discussion.

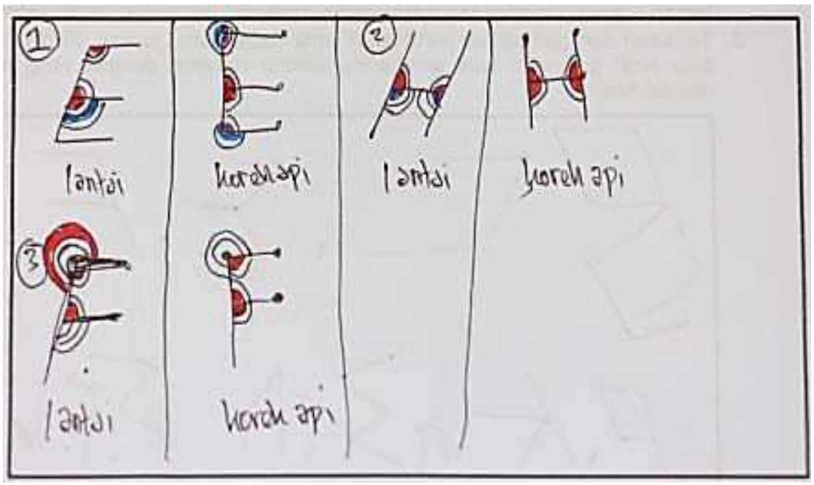


Figure 5.18. Giga and Alif were unable to infer angles similarity when no right-angle involved.

The following fragment from the classroom discussion depicts how the researcher supported the students to explain their ideas about angles similarity.

[1]Researcher:	*"As you know, the tiles on the floor are in the same shape but differ in their arrangement. It allows them to fill up the floor. Maybe you can use this fact to explain about which angle that has the same size."*
[2]Giga:	*(Highlighted the letter F on the picture of kitchen floor and made claim about the similar angles)*
[3]Abell:	*"But it is tilt! (Comparing Giga's drawing with letter from matchsticks)*
[4]Giga:	*"No… it is the same"*
[5]The students:	*"It is tilt! (Tryng to convince Giga)*
[6]Researcher:	*"Let us focus on Giga's drawing! He drew the F like this (Draw Giga's drawing, see figure 5.18) and he claimed that these angles were the same (Pointing to the adjacent angles that Giga highlighted) do you agree with that?"*
[7]Alif:	*"That's wrong (Whispering)"*
[8]Researcher:	*"One of your friends said it's not right!"*
[9]Giga:	*"This one is obtuse and this one is acute (Pointing to the angles that he had indicated before as the similar angles)*

The students realized that Giga had indicated the wrong pair of angles. The researcher asked the students to focus on the obtuse angle and asked them to find which angle in the F figure that has the same magnitude with it. They were able to show the intended angles after a brief discussion.

[10]Researcher:	*"Okay, Abell claimed that this angle equal to this angle (Pointing out to a pair of corresponding obtuse angles on the letter F) can anybody give a reason, why these angles are in the same size?"*
[11]Alif:	*"The angles have the common line" (Pointing along the vertical arm of letter F)*
[12]Giga:	*"In one line" (Justifying Alif's claim)*
[13]Researcher:	*"What do you mean by 'one line'?"*
[14]Alif:	*"In this line (repointing to the vertical arm of letter F)*
[15]Researcher	*(Realized that the students struggled to give verbal explanations) "Can you give the reasons by using the fact that the tiles are uniform? How many tiles there?" (Pointing to the obtuse angles on F)*
[16]Alif:	*"Two" (Circling the obtuse angles on letter F)*
[17]Researcher:	*"Now compare it to the acute one! We know there are two tiles here. (Pointing to the obtuse angle) How about on this angle? (Pointing to the acute angle)*

[18]Abell:	*"One"*
[19]Rafli:	*"Oh…yaa…I see it now" (Realized that the number of the tile's vertex that involved could be used to explain the similarity)*

From the discussion the students have grasped the concept of angles similarity by reasoning with the fact that the floor is formed by uniform triangular tiles. At this stage, the students' conjecture about angles similarity in the parallel-transversal situation have clarified.

Third stage

In the last stage, the students showed the similarity between the magnitudes of angles on the floor that formed when a straight line falling across two parallel lines. In general, the actual process meets our conjectures in the HLT in which we argued the students may realize that there was a connection between the parallelity and the similarity of angles on a situation when a straight line falling across a pair of parallel lines. The students realized that there was a connection between parallelity and angles similarity on a situation when a straight line falling across a pair of parallel lines. The students' written work clearly shows this comprehension (see figure 5.19).

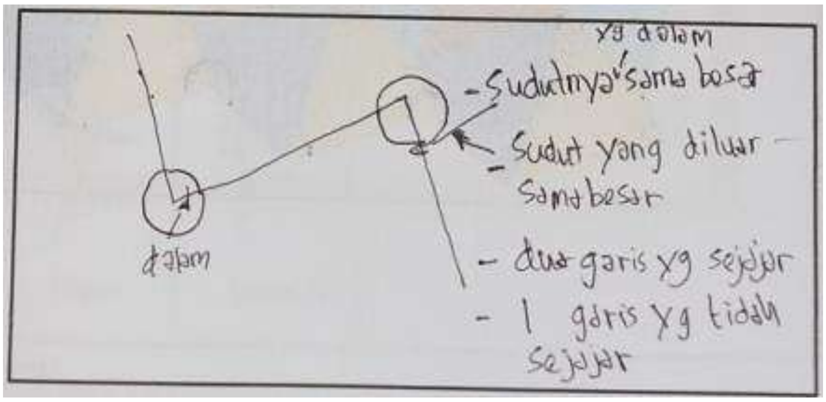


Figure 5.19. It says, "The internal angles are in the same size, the external angles are in the same size, two parallel lines, and one non-parallel line.

The analysis of this lesson allows us to improve our design for the third lesson. The improved version of the third lesson included the following things:

1. In order to maintain the effectiveness of the activity, we decide to omit the instruction that ask the students to find the letters in the bedroom floor model. As a consequence, we also omit a follow-up instruction of this task, which ask the students to compare the letters in the kitchen floor with the letters in the bedroom floor.
2. Instead of asking the students to find and compare the angles in the letters that formed by parallel line segments in both kitchen floor and letters from the matchsticks, we reformulate the instruction so that the students only focus on the letters that we specified in the instruction (E, F, N, and Z).

5.1.5 Lesson 4: Reason about angles magnitudes on the tiled floor models

The main purpose of this lesson is to support students in order to be able to give a reasonable estimation of angle magnitude from a given angle situation. At the beginning of the lesson, the researcher invited the students to explore the angles magnitude on a figure of a brick wall. During their exploration we observed most of the students accepted the possibility to add right-angles to make the bigger angles such as, 180°, 270°, and 360°.

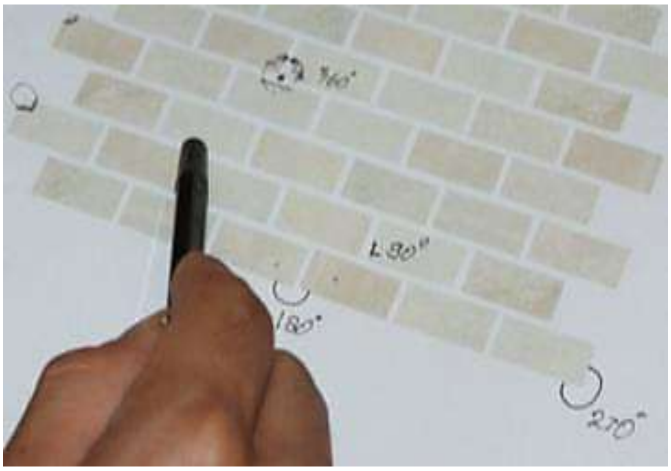


Figure 5.20. Students saw the possibility to add tight-angles to form a bigger angle.

After the exploration, the researcher displayed 6 different models of tiled floors and asked the students to carry out simple analysis and calculations. At first, all the students immediately recognized the right-angles in some of the given situations, even the right-angles were in the tilted position.

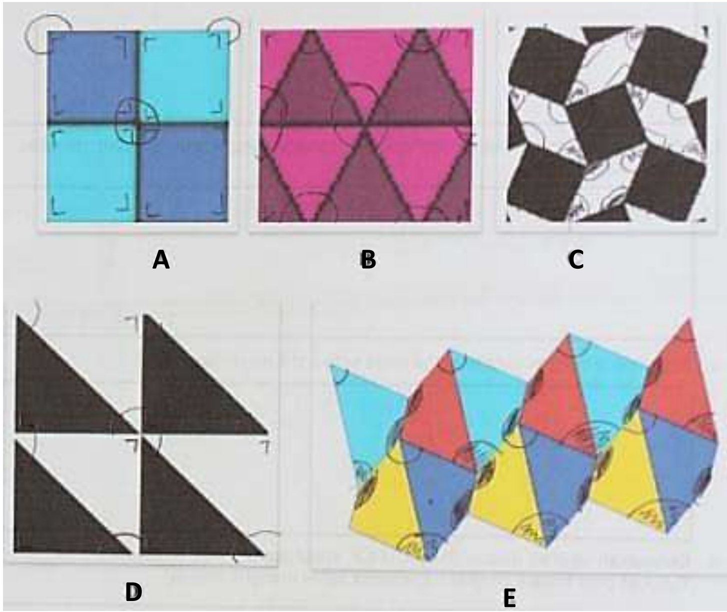


Figure 5.21. Students indicated the angles that have the same magnitude.

In addition to that, they encountered no significant difficulty in determining the angles that have the same magnitude due to the uniformity of the tiles in every given floor eased their analysis.

They also figured out that in every meeting point of the tiles, the total angle is 360°. The students' claim was based on the fact that they can draw a circle to indicate the angle on every meeting point of the tiles. Although they know about this fact, the students still struggled to derive this fact in order to help them to calculate the angle magnitude on the corner of every individual tile. The following fragment from classroom discussion shows how the students struggled to apply this knowledge to solve the relevant problems.

[1]Researcher: "Let us focus our attention on the size of angles on floor C! Who wants to say something about the size of the angles?"
[2]Hilal: "90…"
[3]Giga: "90, 130,…." (Overlapped answers of Hilal and Giga)
[4]Researcher: "I can barely hear you! Can you do it one after another! Who wants first?"
[5]Hilal: "90°, 145°,and 30°"
[6]Researcher: "Do you agree with that?" (Asking other students' opinions)
[7]Rafli: "No…"
[8]Researcher: "Okay, not all of you agree with Hilal. So is there any other opinion?" (Students rumbled)

It took few moments for the other students to give their answers.

[9]Researcher: "On the C floor, beside 90°, what else?
[10]Giga: "30 and 130!"
[11]Researcher: "Anyone else? Abell?!"
[12]Abell: (Shook his head)
[13]Researcher: "Consider the angles on floor C! At this moment we know there are two right-angles there. Beside the 90° angles, can we be sure about the sizes of acute and the obtuse angles?"
[14]Alif & Rafli: "No…"
[15]Researcher: "The only thing we can do is to make a guess. But first, can you predict the total size of the acute and the obtuse angles?"
[16]Abell: "180"
[17]Researcher: "So the total sum of acute and obtuse angles is 180°. But how is about the size of each individual angle? If I want to know it, what should I do?"
[18]Abell: "Use a protractor!" (Other students were giggling)
[19]Researcher: "Well…we are not allowed using a protractor here. Okay, let say that the acute is 30°, what is about the obtuse one?"(Students rumble)

The students attempted to calculate the value of unknown angle.

[20]Rafli: "100…em…150"
[21]Researcher: "How do you calculate that?"
[22]Giga: "First, 180 and the remainder is 150." (Other students nodded their head)
[23]Researcher: "Okay, let us see Abell's work. (Using Abell's work to invite the other students into the discussion) He claimed that the acute angle is 45°. (Abell and other students were giggling) That's fine, I also guess 30° as well. If it is 45°, what is about the obtuse one?"
[24]Alif: "105"
[25]Abell: "No…it is 130" (Other students shook their heads)
[26]Alif: (Recalculating his answer) "135"

From the classroom discussion, we observed how students struggled to apply the concept in order to solve the given numerical problems. However, after the researcher provided the students with guidance, they were able to apply their knowledge. In general the actual teaching and learning process is in line with our conjectures in the HLT in which we predicted some students may guess the magnitude of the unknown angles, some may claim that the problem do not have any solution due to the lack of information, and some may claim that the problem have too many solutions depend on their assumptions. A discussion about calculating the angles magnitude on the other floor models showed that, the students have acquired the strategy to calculate the angles magnitudes on every given floor model. Therefore, we argue that, the lesson is appropriate to help the students to reason about the magnitude of angles using the uniformity of the tiles.

The analysis of this lesson allows us to improve our design for the fourth lesson. The improved version of the fourth lesson included the following things:

1. In the first task we will ask the students to indicate the angles that have the same magnitude instead of general instruction that asked the students to analyze the angles on the given floor models. It is because, during the activity to find the angles that have the same magnitude, simultaneously, they will perform the analysis on the angles in each floor.
2. The students have to work in group instead of individually.
3. A classroom discussion that encourages the students to test their assumptions about the angles magnitude is added in the teacher guide.

5.1.6 Lesson 5: Angle related problems

The goal of this lesson is to provide a supportive learning environment for the students to apply their acquired knowledge to solve the problems related to the angles magnitudes in more general cases. To begin with, the researcher presented a simple problem related to the angles magnitudes. Here, the students have to figure out the same angles that formed by 4 line segments that intersect in a point. The actual learning process showed that the students were able to figure out which angle that wasequal with another angle using the concept of vertical angles.

After the students analyzed the given problem, the researcher posted a how-if question. The problem is to find the size of all angles in the 4 line segment problem if all of the angles are in the same size. The students applied the fact that the total of angles has to be 360° in order to solve the problem. They claimed that, each angle had to be 45° in order to satisfy the original situation. They also checked whether the answer was right or wrong by adding 45° angles repeatedly and found that all eight 45° angles added up to 360°.

Before the students worked with the problems in the worksheet, the researcher presented a perspective picture of a railway. In the picture, the bars of the railway seem to intersect each other in the horizon. The researcher then asked the students to determine a point of view how they saw the bars so that the bars were parallel to each other. It is quite surprising that some students immediately gave responses about top view. They claimed that, they would get parallel bars in the picture if they saw the railways from above. Since, the next tasks required the students to draw the top views of the given railways pictures, therefore, the researcher concluded that they were ready to work with the problems in the worksheet (see worksheet 5 in the appendix).

Our conjecture about students' reactions on the first task matched with the actual learning process. All of the students drew the trivial condition of the situation (see figure 5.22) where all the angles in the railways intersections were in the same size (90°). When the students worked with this task, they were reluctant to draw another possible arrangement of the railways intersection. The students didn't see the reason why the intersection had to be in the non-trivial condition. In order to avoid superficial understanding toward the concept, we conducted a follow-up activity of this problem. In the follow-up activity, the researcher asked the students to draw another railways intersection in non-trivial condition, give a value for an angle on their drawing, and ask their fellows to determine the unknown angles.

We observed that, all of the students struggled to determine the unknown angles. For example, Giga and Abell attempted to solve a non-trivial problem by applying the fact that the sum of internal angles in a quadrilateral is 360°. Their strategy produced inconsistencies in their answers due to both of them started by guessing the size of an angle and then derived the guessed value to find the unknown angles, without considering the properties of angles in the parallel-transversal situation. From the previous activities, we know the students have the knowledge about the concepts such as; reflex angles, straight-angle, full angle, and corresponding angles. However, when the tasks became more complicated, the students were unable to apply these concepts to help them to solve the problems.

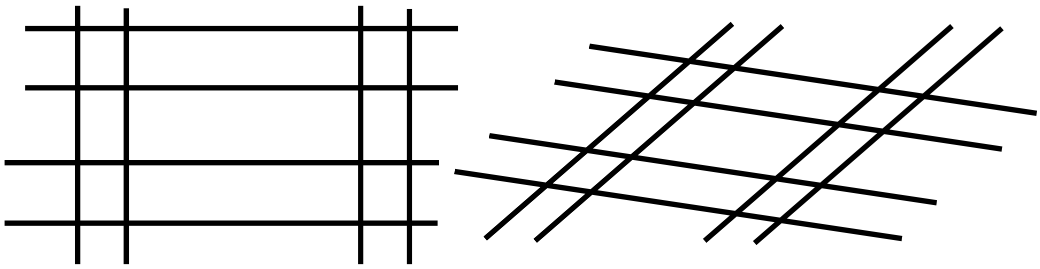


Figure 5.22. A trivial and a non-trivial conditions of the railways intersections.

When the students worked with the questions in the worksheet, most of them performed well in the first three questions. They applied the key concepts in solving the given problems. When the researcher asked the students to explain about their solutions, their strategies were observable during the discussion. For example, only Alif and Hilal gave general description about angles magnitude in the first problem, other students gave specific description (numerical estimations). Although they gave specific description, their solution for the second problem suggested a generalization about the condition. We also observed that, all of the students were able to solve the third question in the worksheet, in which they had to calculate an unknown angle magnitude in a triangular tiles situation. Many of them tried to apply the fact that the sum of internal angles in a triangle was 180°. Despite students' capability to solve the given problem, a brief discussion with the students showed that even some of them knew about the fact (and some were still confused with 360°) they still struggled to find a good strategy to attack the problem. The researcher encouraged the students to focus their attention on the alignment of the angles in order to allow them to use the concept of straight angle to solve the problem.

In the last question, most of the students were unable to see the uncertainty in the given problem. We asked them about how sure they were with their own predictions of the sizes of two unknown angles in the triangle context when one angle size was given. Mainly there are two different approaches that students used to solve this problem. First, the students assumed that the two unknown angles were in the same magnitude. Second, the students used the unrelated information in the previous problem as extra information to reduce the number of unknown variables.

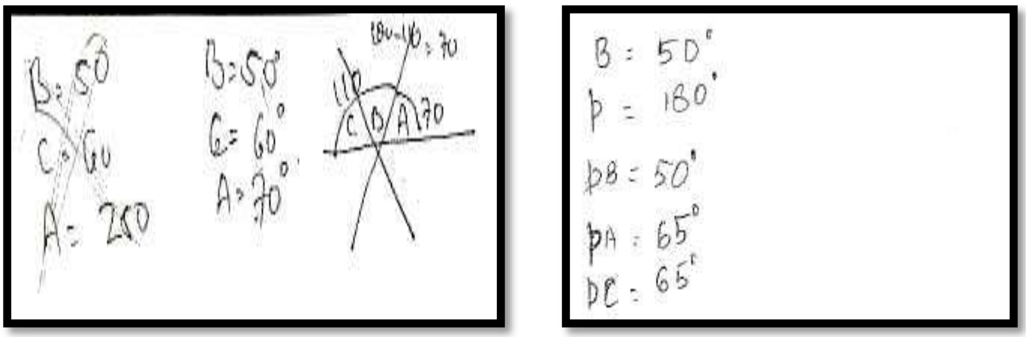


Figure 5.23. Students' two different approaches when they encountered an uncertainty situation.

The analysis of this lesson allows us to improve our design for the fifth lesson. The improved version of the fifth lesson included in the following things:

1. Change the railway intersections picture so that the intersections do not look like in a perpendicular formation.
2. Add some details on the teacher guide related to the classroom discussion that discuss about the way to determine the angles magnitude in a students' own construction of the railway intersections.

5.1.7 Post-assessment

The students took a 20-minute posttest after went into the entire lesson sequence. The posttest items were designed to assess students' current knowledge about angle and its magnitude. The following table summarizes the gained scores of those 6 students:

Table 5.1. Small group's pre and posttest scores

No	Name	Pretest Score	Posttest Score
1	Abell Ricardo. O (Abell)	4.38	8.75
2	Ajeng Ayu Puspita Sari (Ajeng)	3.44	9.4
3	M. Alif Zhafar. G (Alif)	6.25	9.06
4	M. Hilal Naufal (Hilal)	3.12	8.44
5	M. Muqsith Giga Saputra (Giga)	4.4	8.75
6	Rafli Dwiyanda (Rafli)	2.5	7.18
	M (SD)	4.01 (1.2)	8.59 (.69)

If we compare the gained scores from both pre and posttests (table 5.1), we can clearly see a significant increase in students test scores. However, our main intention is to use the pre and posttest results as a resource for clarification of students' development throughout the lessons sequence. Due to the limitation in evaluating students' gained scores for describing their development, we conducted a further analysis on the students' written work. The analysis revealed which knowledge that students acquired and in what aspect of students understanding toward the concept has changed after following the lessons sequence.

Based on the analysis on students' written work and video registrations of the interview, we noted several important remarks as follow:

a. Angle definitions that students embraced

In the end of the learning process, the students perceived the angle was not just as the space in between two lines in the plane which meet in a point. They also perceived the angle as the difference of direction between two lines. The clarifications of this claim can be found in students' written work and their verbal justifications. For example, in one of the test items, we presented a set of angle figures, in which of the magnitudes of the angles are different and the lengths of the arms are varied in size. All of the students encountered no difficulty when we asked them to compare the sizes of those angles; even when we displayed a bigger angle with the shortest arms. Their verbal explanations clearly indicated that they perceived the angle as the difference of direction between two lines. In addition to that, we also presented a set of right-angle figures that varied in orientation and also varied in the length of their arms. The students were able to recognize the angles as the right-angle figures and this justified our claim about angle definitions that students embraced.

We argue the development of students' inventory of angle definitions is a cumulative result of the activities in the lessons sequence. For instance, in the first lesson, we asked the students to explain how an angle was formed. Mainly the students came up with the explanation that used the difference of direction between two line segments in order to explain about angle formation. The activity in the second lesson strengthens students' comprehension of the angle as the difference of direction between two lines. A particular activity that promotes students understanding about angle as the difference of direction between two lines is when the students constructed the upper case letters using matchsticks. In the activity, the students realized that the angle also could be defined using the direction of the lines.

b. Students' comprehension about angle magnitude

The students have developed their understanding about angle magnitude. Ordering the angle magnitude on the real-world objects and to reason with the angle magnitudes on the tiled floor models proved to be the fruitful ways to promote students' development. In the posttest, we presented a problem that asked the students to reordering the given angle figures into an ascending order. Due to their adequate understanding about angle magnitude, all of the students had no difficulty in performing this task.

The understanding about angles similarity had developed as well. The activities that had impact to this development are the activities of angles on the letters from the matchsticks and letters on the tiled floor models. From those particular activities, the students understand that the corresponding angles on letters like F, X, and Z are similar. Some problems in the posttest required the students to have the comprehension of the concept of angles similarity. For instance, in the test we presented an X like figure and asked the students to write down what they knew about the magnitude of the angles on it. Almost all of the students could recognize the angles had the same magnitude. They explained that the X shape figure represented a vertical angles situation.

c. Students' capability to apply the concepts to solve the problems

From the lesson sequence, we observed that the students acquired the knowledge about vertical angles, straight angle, full-angle, and corresponding angles in the parallel-transversal situation. Two problems in the posttest put these understanding into a test. The first problem on this context asked the students to determine an unknown angle magnitude from a known angle magnitude in a straight angle situation. Only one student that made a mistake by assuming the straight angle is 360°. However, from the interview with this student, he reconsidered his answer and figured out that he had made a mistake. He said that he overlooked the problem and as a result he thought that the figure was circular instead of straight.

The second problem asked the students to find out the unknown angles magnitudes in parallel-transversal situation. We provided a numerical value of an angle, and asked the students to deduce the values of the other angles. From their written work and their verbal explanations during the interview session, revealed that the students had good understanding about the concept of corresponding angles. As a result the students could solve the problem without any significant difficulty.

5.1.8 Conclusion for the first teaching experiment

The first teaching experiment showed that the students had already acquired the important knowledge about angle and its magnitude. The students accepted the fact that the angle could be defined in many different ways depends on the context. According to the actual teaching and learning process in the first teaching experiment, we found that the students had two different ways in defining the angle (i.e. as the space and as the difference of direction between lines). However, we realized that the students did not explicitly show a tendency to define the angle as the amount of rotation between two lines. Therefore, in the next teaching experiment we attempted to help the students to add the definition of angle as the amount of rotation in their inventory of angle definitions.

The actual teaching and learning process also showed how the students inferred angles similarity in the given contexts, perceive some special angles (0°, 90°, 180°, 270°, and 360°), made some justifications related to the angles similarity in the parallel-transversal situations, and solved the problems related to angle and its magnitude. However, there are several parts in the teaching and learning process that need to be revised in order to deepen students' understanding toward the intended knowledge. Therefore, we make some revisions and improvements of our HLT. To make such improvements, we discuss our findings from the actual teaching and learning process with teacher and colleagues. This process produces a revised version of students' worksheet, teacher guide and the HLT. These instruments will be used in the next teaching experiment, namely the second cycle.

5.2 Second teaching experiment (second cycle)

In this sub-phase of the teaching experiment, we test our revised design in the classroom environment. The process involved 40 students (i.e. 21 male students and 19 female students) and their teacher. Considering the number of the students that involved in the process, the researcher selected a group of students (4 students) to be a focus group. Throughout this sub-phase, the researcher acted as an observer to gather all important information from the actual teaching and learning activities. The aims are to investigate how the design help the students learn the intended knowledge, make an inventory of students' reactions, and revise the HLT. The details of the observations, analyses, and evaluations of the second teaching experiment described as follow in a chronological sequence.

5.2.1 Pre-assessment

The forty students also took a 20-minute pretest in the beginning of the second teaching experiment as the six students did in the first teaching experiment. The aim of the test is to gather information related to the students' prior knowledge about angle and its magnitude. We also used the result from this test as a base to select the focus group. After they took the test, we conducted a follow-up interview with 4 students from the focus group to get verbal justifications of their answers. Analyses of the students' written works revealed several important findings related to the students existing knowledge.

a. Frame of reference about angle

After analyzing students' written work we found that each student embraced some frames of reference about the angle. They used 3 different frames of reference in order to decide which geometrical figures that could be categorized as the angles. The frames of reference that students used such as; angle as the area between two intersecting lines, angle as the difference in direction between two lines radiate from a single point, and angle as the amount of rotation between two intersecting lines.

Sixty percent of the students used area as a frame of reference. Ten percent of them used difference in direction as a frame of reference. Less than ten percent of them used rotation as a frame of reference. In addition, there were twenty percent of the students that can flexibly use the three frames of reference depend on the presented angle situations.

b. Symbol to indicate the angles

An item in the test asked the students to indicate the smallest and the biggest angles from a given figure. Most of the students only recognized the angles that less than 180° and didn't anticipate the existence of the reflex angles. From the symbols that students used to indicate the angles, we found that at least fifty percent of them perceived the angles in the figure as the amount of opening between the two arms. They used the arc (∡) symbol to indicate the angles.

Twenty five percent of the students thought that the vertices on the figure were the angles. They gave the symbols like dot, circle, or tick on the vertices that they thought as the angles. By using such symbols we presume that those students perceived the angles as the difference of direction between two lines that radiated from a single point. In addition to that, there were 6 students that used unusual symbols to indicate the angles. The 6 students highlighted or marked one of the arms of the angle and claimed the arm as the angle. As a consequence, the longer the arms the bigger the angle becomes. It clearly showed that the 6 students (and the other 2 students that didn't give any responses) have inadequate knowledge about angle.

c. The sense of angle magnitude

There are two test items that can be used as the indicators of students' sense of angle magnitude. The first item is the task that asked the students to sort seven polygons based on their internal angles in an ascending order. There were forty percent of the students that were unable to produce the right answer. Most of the students in this group sorted the polygons based on their area instead of their internal angle (figure 5.24a). We found some students that made the order based on the length of the arms as well (figure 5.24b).

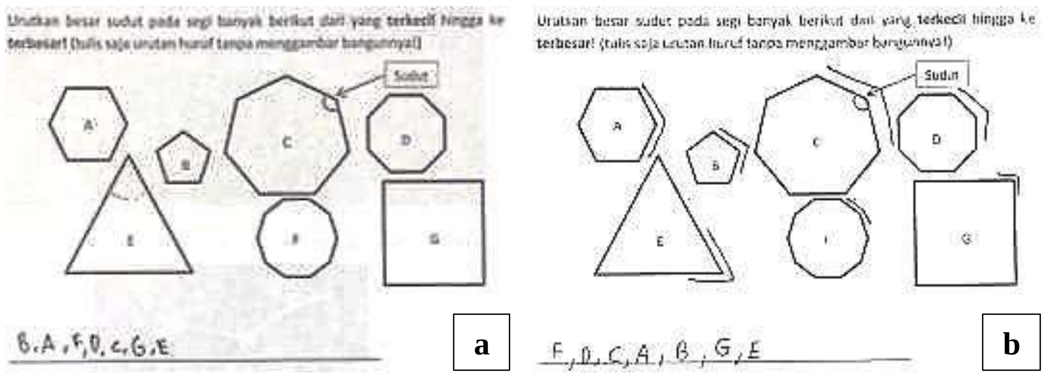


Figure 5.24. In the left figure, student sorted the angle based on the area of the polygon and in the right figure, student sorted the angle based on the length of the arms.

The second item is a problem about vertical angles where one of the arcs that indicated the angle was slightly narrow compare to its pair. The students had to decide the two angles were in the same or different magnitude. Only twenty percent of the students recognized the similarity between the two angles. Some students realized that both angles were in the same magnitude. However, they had some doubt about this fact due to the difference of the arcs that indicated the angles. They claimed that, both angles was less than 45° and both in the same magnitude were due to it generated from two intersecting lines, but angle A had the larger ‘angle area’ compared with angle B although they had the same measurement (figure 5.25a).

Most of the students believed both angles were different in magnitude. They claimed the angle that indicated by the narrower arc was the smaller angle (figure 5.25b & 5.25c). In addition to that, we also found that some students knew about the vertical angles from their text book. However, when we asked them why they chose 60°, they were unable to produce adequate explanation due to their competency was on the level of memorizing (figure 5.25d).

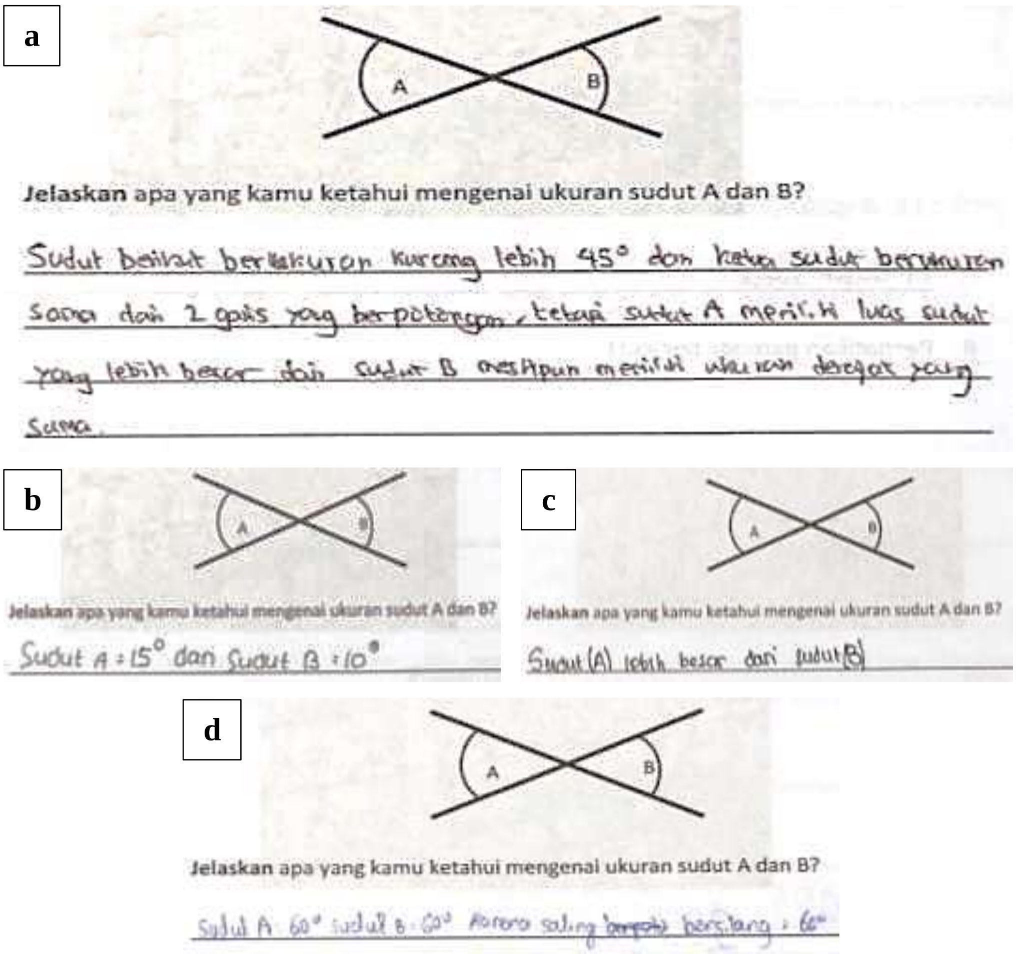


Figure 5.25. Students’ answers to the problem about angles similarity in vertical angles situation.

d. Knowledge about right-angle and straight-angle

Almost fifty percent of the students in this classroom didn't recognize the right-angle figures. In the test, we presented a set of right-angled figures and an opinions pool related to the given figures. The students had to select which one from the three opinions in the pool was the right opinion. It is clear that students' judgment was affected by the size of the given figures. Since most of them agreed, the right-angle that could cover the largest area if we drew other lines that were parallel to the both arms was the largest right-angle.

We also designed a test about straight-angle problem. The problem asked the students to determine the unknown angle magnitude from an alignment of two angles, in which one of the angle magnitudes was given. Only forty percent of the students were able to solve the problem. Their strategy is based on the fact that the sum of both angles is 180°. The students who didn't know about this fact were unable to solve the problem. Some of them attempted to tackle the problem by making a rough estimation about the unknown angle relative to the known angle. According to their estimation they claimed that the unknown angle was three times bigger that the given angle. We also found the students who didn't have an adequate understanding about angle magnitude were unable to solve the given problem, as a consequence their responses were based on the guess without any adequate explanation.

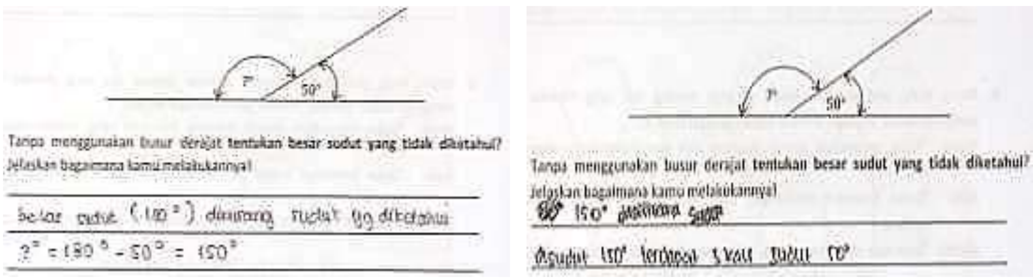


Figure 5.26. In the left figure, the student was able to derive the fact that the sum of both angles is 180°, and in the right figure, the student estimated that the unknown angle (150°) was three times bigger that the given angle (50°).

From the description above, we can infer that although the students had learnt about the concept before, their understanding toward the concept is still limited and vague. Most of the students showed some degree of inconsistency in their knowledge about the angle and its magnitude. Although each student has a frame of reference about angle, still they are unable to hold their conception about angle in the situation where the conception applies (i.e. vertical angles, right-angle, ordering angle magnitude, and straight-angle). It is evident that the students applied their frame of reference about angle without further consideration. As a consequence they struggled to have a clear judgment about what an angle is.

Using the above information in hand, we decide to make several adjustments in the pretest items in order to increase the prediction and evaluation power of the test. For examples, in the first problem we asked students to indicate the smallest and the biggest angles on the *'Lepus'* constellation, however the using of black background for the picture compounded our analysis. Therefore, we reproduce the same picture in white background. The set of right-angle figure in the second problem is revised so that it includes the figures of right-angle without horizontal arm. In order to make students understanding about angle magnitude observable, we asked the students to explain their frame of reference in ordering the angle magnitude in the third problem (sorting the seven polygons based on their internal angle) as a follow-up question. We also reproduce the figure in the vertical angles problem into a figure where the one of the arcs that indicates the angle is narrower compared with its pair. The aim is to test the consistency of students' conception about angle and its magnitude. We remake the last problem that test students' understanding about angles similarity. We utilize numerical problem instead of asking students' opinions about the angles similarity in the given parallel-transversal situation. Furthermore, in order to increase the reliability of the test, we also conduct a peer examination of the test items with colleagues.

5.2.2 Lesson 1: Angle from everyday life situations

The teacher began the lesson by presenting the angle situations, invited students to analyze the angle magnitude, asked students to sort the angle magnitude, gave some questions for students to answer, and conducted several classroom discussions. In this section of the chapter, we will describe, analyze, and evaluate the actual teaching and learning process.

First stage

In the first task, the teacher asked her students to indicate an angle in each figure that she had distributed to the students (see figure 5.5). The teacher had clearly explained the instructions before students worked with the tasks and asked if there were some instructions that students didn't understand. However, most of the students still indicated more than one angles on some figures, especially on the figures that have several similar angles (i.e. tiled floors, ladder, letters E and F, railways intersection, and fan). The students also claimed that the indicated angles in one figure were in the same size (see figure 5.27). This indicates that the students already have the sense about angles similarity. As we had predicted in the HLT, 20% of the students encountered difficulties to indicate the angles that bigger than 180°. It is because their understanding about the angle magnitude were limited to the angles that less than 180°. In addition to that we also found that only 10% of the students that realized the existence of a 0° angle in the given figures. It is reasonable since as we all know the 0° angle is hard to point out in every given figure.

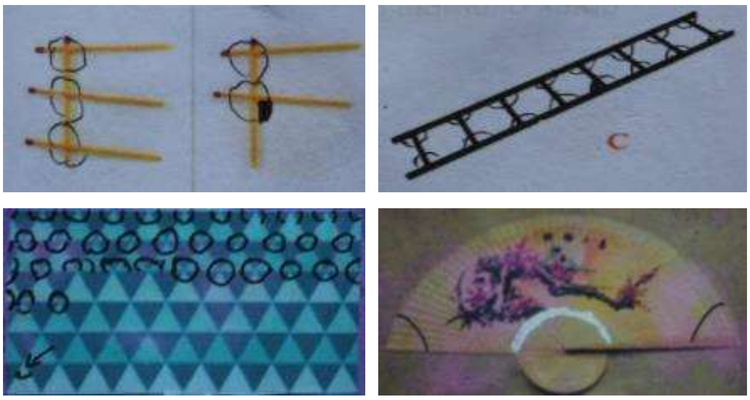


Figure 5.27. Students' recognition about angles similarity.

Second stage

The second task asked the students to sort the angles that they had indicated in an ascending order. At least 60% of the students were able to make the acceptable constructions. In general, the students sorted the angles into three clusters. The figures that had an acute angle clustered in the beginning of the sequence, the figures with 90° angle or looked like 90° angle clustered in the middle of the sequence, and the figures that had the angles that were bigger than 180° clustered in the very end of the sequence (see figure 5.28). The teacher invited the students to give comments and suggestions to the other group's construction. The activity allowed the students to revise their understanding about angles magnitude by observing and analyze each other work.

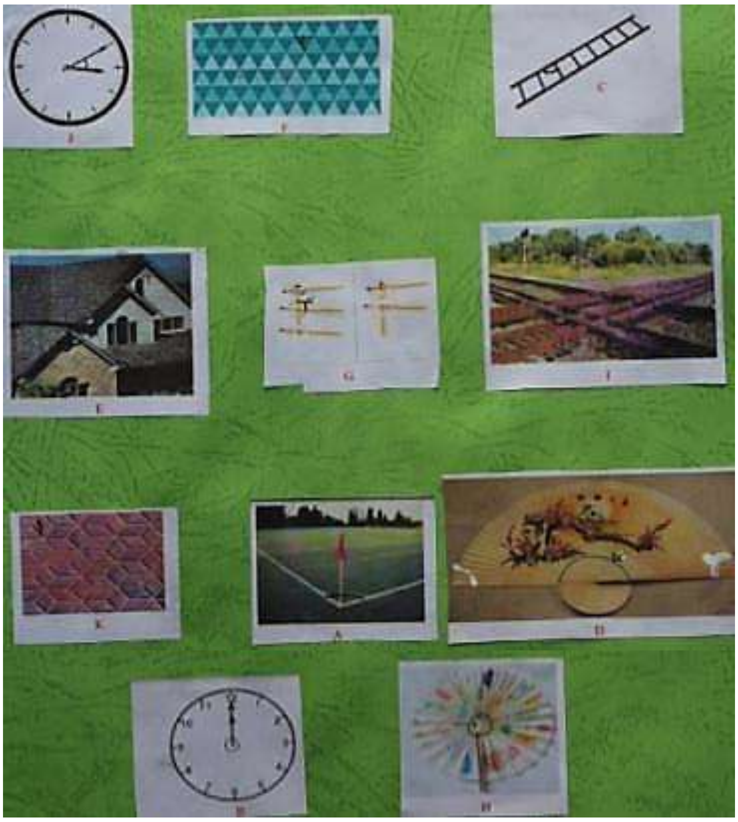


Figure 5.28. A construction of Zaky's group.

Third stage

The following is a fragment from the classroom discourse where a group of students gave comments and suggestions for the other group work.

[1]Teacher: *(Approaching a group of students who analyzing their fellows' work) "What are your group's comments for this poster? Can you read it out loud?"*

[2]Students: *(Re-read their group's comment) "The angle in figure K is bigger than the angle in figure J." (In the poster, the other group put K before J)*

[3]Teacher: *"Which angles do you mean?"*

[4]Students: *(Pointing out to the indicated angles in figures K and J)*

[5]Teacher: *"K is bigger than J! So which one that has to come first?"*

[6]Rozan: *"J" (Point out to the indicated angle in the figure J)*

[7]Teacher: *"Okay…what else?"*

[8]Giri: *"Angle in figure A is bigger than angle in figure I" (In the poster, the other group put A before I)*

[9]Teacher: *"How big is the angle in A?"*

[10]Zaky: *"Obtuse angle"*

[11]Teacher: *"Obtuse??? What is in the picture?"*

[12]Zaky: *"A football field corner" (Students in the group seem to agree with Zaky's answer)*

[13]Teacher: *"The corner of a football field! How big is the angle of a football field corner? As boys, all of you must know how big it is!"*

[14]Zaky: *"90°"*

[15]Giri: *"Right-angle" (Made a hand gesture of right angle)*

[16]Teacher: *"What is about the angle in figure I?"*

[17]Zaky: *"That's a right angle"*

[18]Teacher: *"So the angle in figure I is a right-angle as well?!"*

[19]Giri: *"See I told you the angles in both figures are the same!" (Blamed Zaky for declining his opinion)*

[20]Teacher:	*"So, is that a problem? Is it right or wrong to put both angles in this way?"*
[21]Zaky:	*"That okay"*
[22]Teacher:	*"Okay....what else?"*
[23]Zaky:	*"This is right-angle, this is not" (Pointing out to the indicated angles in figures G and E)*

[24]Teacher:	*"G is a right-angle, what is about E?"*
[25]Zaky:	*"E is an acute angle" (Giri highlighted the angle in figure E that Zaky meant)*
[26]Giri:	*"Roof top is a right-angle Zaky!"*
[27]Teacher:	*"So what do you think?" (Inviting the students to analyze the indicated angle on the roof top)*
[28]Hazlift:	*"Hmmm…it is confusing!"*

The students struggled to decide what angle that a roof top formed. They tilted the figure to see whether the angle was a right-angle or not but some of them were doubt about Giri's claim. In the end of the discussion the students agreed that the angle in the roof top was an obtuse angle.

Throughout the actual teaching and learning process, most of the students used right-angle as a benchmark to sort the angle magnitudes and some even used acute and obtuse angles as the criteria to sort the angles magnitude. At this stage, most of the students had rough understanding about angle magnitude and how to put them in an order.

Fourth stage

The activity continued when teacher asked the students to answer two questions about dynamic angle situations. The aim of the tasks is to provide students with a suitable environment where they can make sense the duality of the concept of 0° and 360° angles. The students' responses related to the task can be categorized into three different groups (see figure 5.29). 50% of the students' responses can be categorized into the first category. The students in this group claimed that the acute non-zero angle as the smallest angle and the obtuse angle that was less than 180° angle as the biggest angle. The second group claimed that the acute non-zero angle as the smallest angle and the obtuse angle that was more than 180° but less than 360° angle as the biggest angle. The second group consists of at least 10% of the students. The third group consists of 20% of the students, this group claimed that the acute non-zero angle as the smallest angle and the 360° angle as the biggest angle. However, we also found that almost 20% of the students were unable to give adequate responds.

In the classroom discussion, the teacher was able to convince the students that the full angle is the biggest angle using approximation strategy. However, to make sense the 0° angle as the smallest angle became problematic for the students and the teacher. It is because the figure of a 0° angle is in the same figure of full angle (duality). We agreed to postpone the justification of this duality in the fourth lesson where the main focus is about angle magnitude. Therefore, at this stage we were fully aware that the students only knew the 0° angle as the smallest angle but didn't have any reasonable explanations toward the concept and its figure.

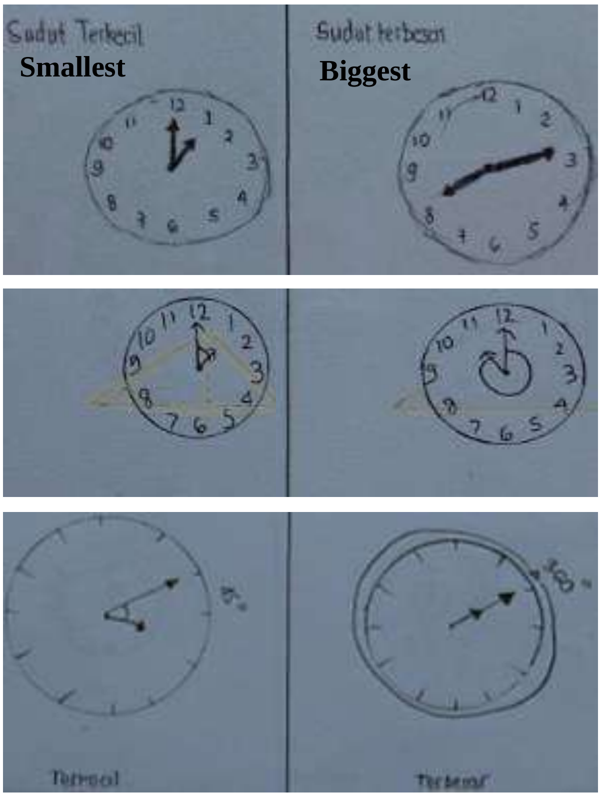


Figure 5.29. From top to bottom, the first, second and third groups of students' responses.

In the end of the lesson, the teacher distributed two questions that asked students to explain how an angle was formed and what were their definitions about angle. When the students attempted to explain how an angle was formed, they tended to explain that an angle was formed when two lines with different directions met in a point (see figure 5.30).

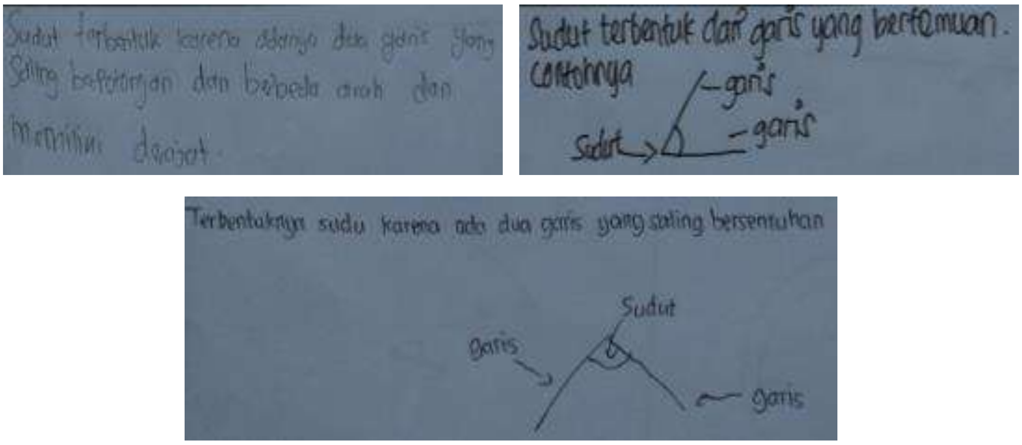


Figure 5.30. Students explained that an angle was formed when two lines intersected each other in a point.

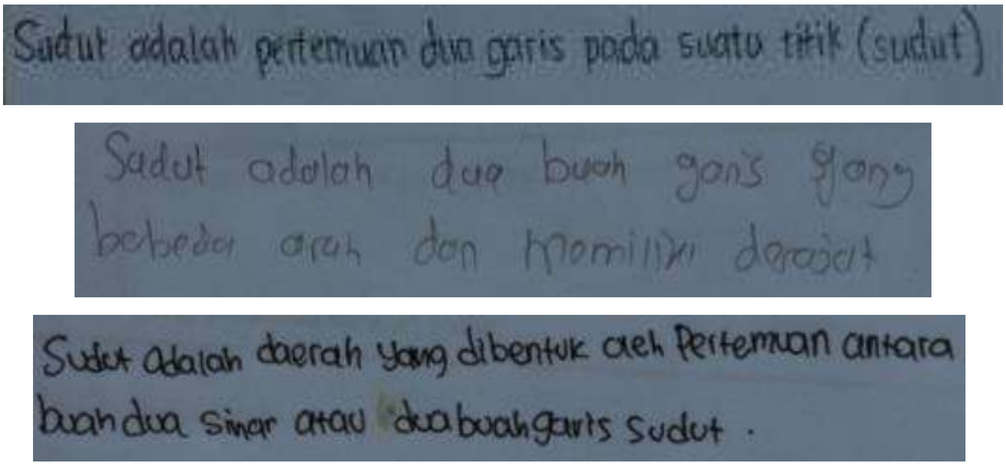


Figure 5.31. Students' definitions of angle. From top to bottom; two lines met in a point, two lines with different directions and had degree, and area between two intersecting lines.

The way students defined the angle was strongly related to the way they explained how an angle was formed. Most of the students defined the angle as the difference in direction between two lines/rays (see figure 5.31). In students' written work we also found that some groups defined the angle as the area between two intersecting lines.

From the description above, we infer that the teaching and learning activities could help students to recall their knowledge about angle that they had learnt before. Although the students were able to recall their memories about angle concepts, we are fully aware that their prior knowledge about angle was limited. For instances, in comparing angle magnitudes activity there were significant number of students that struggled to sort the angles based on their magnitudes. Students' perplexity is a result of how they interpreted the presented angle situations. The students had two different interpretations on how they saw the angles in the presented pictures during poster construction. Unfortunately, the teacher didn't conduct a classroom discussion that discusses about which interpretation that suit best for ordering the angles magnitude. We also figured out that, most of the students struggled to accept the angles that were larger than 180°. Therefore, the students need more supports in order to be able to master the subject matter in the next lessons.

The analysis of this lesson allows us to improve our design for the first lesson. The re-improved version of the first lesson included in the following things:

1. The third instruction in the task asked students to find differences and similarities between the posters. However, in the actual teaching and learning process, this task disorientated the students from the main aim of the task. Therefore, we reformulate the task in order to make students focus on how the other groups order the angle magnitude.
2. Revised the guided questions for classroom discussion about the 0° angle that allows the students to realize that the 0° angle is in the same figure with 360° angle (dual of a 0° angle) by using diagrammatic approximation strategy.
3. Conduct a classroom discussion that discusses about which interpretation that suits best for ordering the angles magnitude.

4. Adding more details in the teacher guide for classroom discussion of angle definitions that students form in order to enrich students' inventory of angle definitions.

5.2.3 Lesson 2: Matchsticks, letters, and angles

The students constructed the upper case letters using matchsticks in the beginning of the learning process. The students encountered no difficulty in performing this task because the teacher explained the detail of the instructions in advance. Interestingly, students' constructions were quite similar to each other.

Figure 5.32. Students work in group to construct the letters from matchsticks.

After the students completed the construction activity, the teacher asked the students to observe, analyze, and criticize each other construction. The amount of matchsticks for each letter, and the shape of each individual letter were the main aspects that most of the students discussed during the activity. Figure 5.33 depicts the differences that students made in some of their letters constructions. The negotiation about the differences in some letters produced the agreement among the students. They agreed that the construction was acceptable if the observer could recognize the letters.

There were three classroom discussions that teacher performed in order to help students to reorganize their knowledge. The first discussion discussed about which letter that had the smallest angle. Most of the students agreed that the angles in letters A and B were the smallest angle. They also concluded that the angles in both letters were in the same size. However, when the teacher asked about which letter that had the biggest angle, the students had several different opinions. The following fragment from the classroom discussion shows how the teacher fostered the emergent of students understanding about reflex angles.

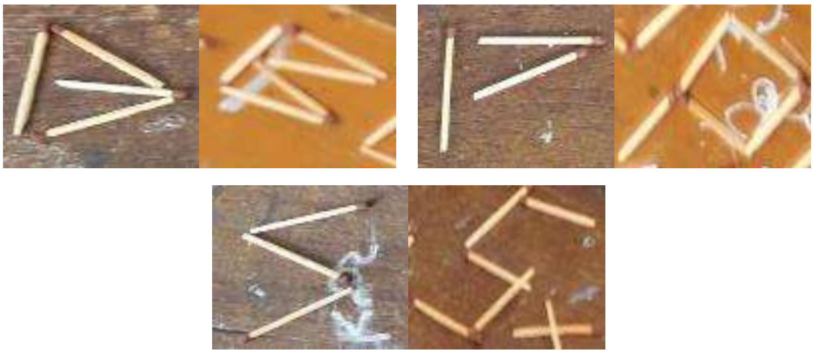
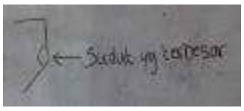


Figure 5.33. Different letters constructions that students produced.

[1]Teacher: *"For the question number two, who wants to present their answer?"*

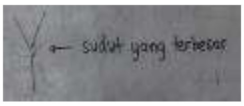
[2]Rozan: *(Raised his hand and indicated the angle in J as the biggest angle)*



[3]Student: *"I have the same solution!" (A student showed his agreement to the Rozan's group solution)*

[4]Teacher: *"Okay, who has different solution from Rozan?"*

[5]Irvan: *(Writing his solution on the whiteboard, he indicated the angle in Y as the biggest angle)*

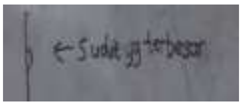


[6]Students: *"Ohhh…Yeah…that's bigger" (Realized the angle that Irvan indicated is bigger than what Rozan had indicated)*

[7]Teacher: *"Anyone else?"*

[8]Adil: *(Writing his solution on the whiteboard, he indicated the right-angle in L as the biggest angle)*

[9]Students: *"That's wrong, angle in L is smaller."*
[10]Reza: *"That's a small angle."*
[11]Teacher: *"Okay, Reza please tell us your solution!"*
[12]Reza: *(Writing his solution on the whiteboard, he indicated the angle in I as the biggest angle)*



[13]Teacher: *"Reza why do choose I?"*
[14]Reza: *"Because that is 180°"*
[15]Teacher: *"Compare it with the angle in L! How big is the angle in L?"*
[16]Reza: *"L is 90°."*
[17]Zaky: *"L is 90°, but J and Y we are not sure."*
[18]Teacher: *"Are you sure that the biggest angle is in I?" Do any of you have another solution?*
[19]Giri: *(Raising his hand)*
[20]Teacher: *"Okay…Giri!"*
[21]Giri: *(Writing his solution on the whiteboard, he indicated the reflex angle in A as the biggest angle)*

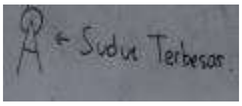


Figure 5.34. The sequence of figures that showed students' attempts to find the biggest angle.

From the classroom discussion, we know that at this point the students were aware about the existence of the reflex angles. However, when the teacher asked why the reflex angle was the biggest angle in the letters, most of the students struggled to give adequate explanation due to the obviousness of the angle magnitude in the sequence of angles figures on the whiteboard. The only reason that students had was the reflex angle was bigger than 180°.

The second classroom discussion discussed about the similar angles in every letters that had parallel sticks. After students selected the letters that had parallel sticks, they indicated the similar angles in each letter (see figure 5.35). Most of the students used classification strategy to categorize their solutions into two different categories. The letters that only had right-angles as the similar angles grouped into the first category. In the second category, the students grouped the letters that had the acute angles as the similar angles. Students' written works and classroom discourses showed that the students were able to infer angles similarity.

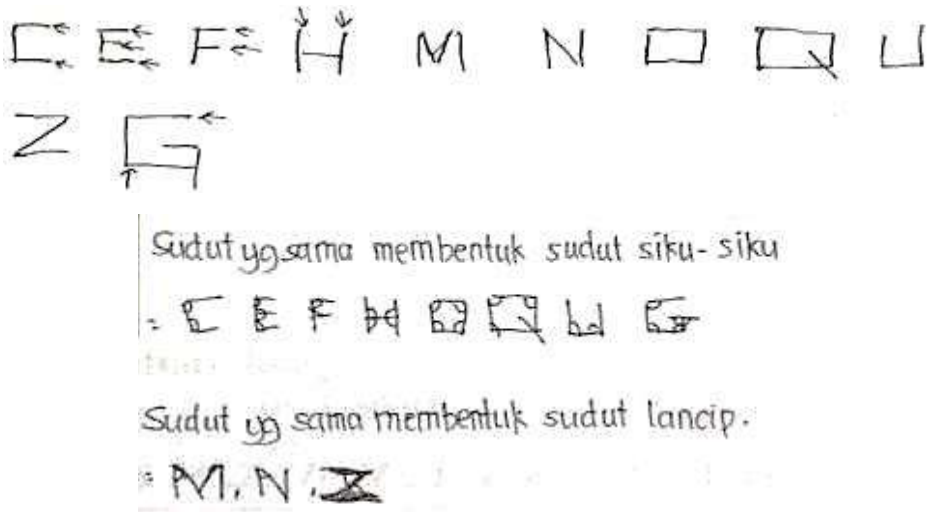


Figure 5.35. Students selected the letters that had parallel sticks and indicated the similar angles.

In the third classroom discussion, the teacher invited the students to analyze the angles in the letters that didn't have the parallel sticks. Students' solutions showed that they couldn't find the similar angles in each individual letter. However, they found that an angle in a letter was similar to the other angle in another letter (see figure 5.36). The students' recognition to the angles similarity indicates their ability to infer similarity between angles magnitudes.

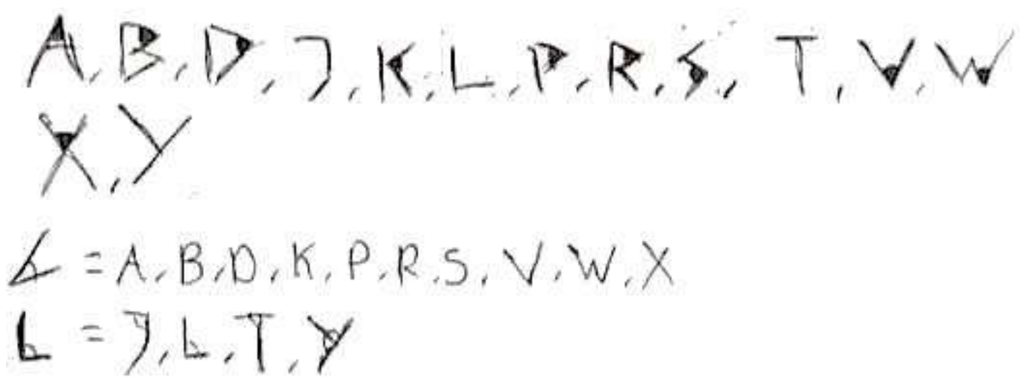


Figure 5.36. Students' recognition about similar angles in different letters.

The analysis of this lesson allows us to improve our design for the second lesson. The re-improved version of the second lesson included the following things:

1. Simplify some of the instructions in the worksheet.
2. Add some details in the teacher guide to lead the students to realize that the biggest and the smallest angles have to be in the same letter.
3. Add a final conclusion as a classroom discussion to conclude about angles similarity in the letters that have parallel sticks.

5.2.4 Lesson 3: Letters on the tiled floor models

In the actual teaching and learning process the students were able to give the adequate responses to the first task. The responses were in line with our conjecture in the HLT, where the students highlighted the different amount of gaps to construct the word 'ANA'. There were 3 out of 10 groups of students that able to find all the letters in the kitchen floor. By using students' own construction in the classroom discussion, the teacher was able to convince the students that they could find all letters in the kitchen floor.

Students' responses to the third task showed the counter-examples to our conjecture about students' reactions to the given task. The teacher asked the students to find the differences and similarities between some letters (i.e. E, F, N, X, and Z) in matchsticks situation and tiled floor situation. The aim is to allow the students to find out that the parallel orientation of the gaps/sticks produce the same consequence; similarity between angles on both situations. There were only 50% of students that gave their answers to the given question. From their answers we realized that the students were reluctant to solve the given problem. Most of them only figured out the similarity of the shape of the letters in both situations where there are parallel lines segments exist in each situation. Students' insufficient observations towards the situations made them unable to reach the expected conclusion. As a result, the teacher prolonged the classroom discussion that discussed about the relation between parallelity and angle similarity. The following fragment from the classroom discourse showed how the teacher helped students to reach the expected conclusion.

[1]Teacher: *"What kind of triangle is in the kitchen floor?"*

[2]Students: *"Isosceles triangle"*

[3]Teacher: *"Isosceles?"* (*Doubting students' answer)*

[4]Giri: *"Equilateral triangle"*

[5]Zaky: *"Isosceles or Equilateral?" (Students defended their answers by shouting 'isosceles' repeatedly)*

[6]Teacher: *"If the triangle is equilateral, what can you say about the angles?" (Trying to end the debate)*

[7]Students: *"The angles will be in the same size if the triangle is equilateral triangle."*

[8]Teacher: *"How big the angle is?"*

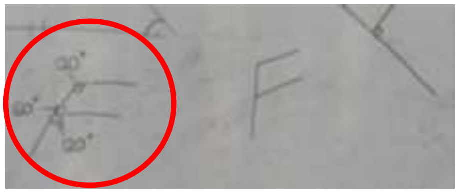
[9]Reza: *"We know that they all in the same size, thus we only need to divide 180 by 3 that is 60°."*

[10]Teacher: *"Yeah…60°. Now how is about the angles in letter F in the kitchen floor? It is different with the F from the matchsticks right? Who can draw the letters?"*

Zaky drew F and Z from the kitchen floor situation and claimed that the angles in F were right-angles.

[11]Teacher: "Are you sure the angles are 90°?"
[12]Students: "Yes…Those angles are 90°."
[13]Teacher: "You said that the angles in the equilateral triangle are 60°! You also said that the right-angle formed by perpendicular lines! Now try to reconsider your answer!"
[14]Students: "But Zaky drew the perpendicular lines, so that must be 90°."
[15]Teacher: "All of you please think about it for a moment!"

After the students reconsidered their answer, Reza realized the flaw in Zaky's solution. He drew the letter F and claimed that the corresponding angles were 120°



[16]Teacher: "The rest of you please pay attention to Reza's solution! He claimed that the upper angle in the letter F is formed by two angles from the equilateral triangles. Therefore, the size is 120°. Now who wants to explain about the angles in the letter Z?"

The students used the same reasoning to explain the similarity between angles magnitude on the letter Z.

At the end of the discussion, we observed that the students figured out the relation between parallelity and the angles similarity. Students' implicit understanding toward the intended conclusion can be observed from their answers to the last problem in this lesson. There were roughly 50% of the students that could give the adequate responses for the last problem.

In the last question, the teacher asked the students to write down at least three facts about the angles in the letter Z in the given tiled floor. In order to provide the students with the appropriate ground for thinking, the teacher gave them three guided questions. The first question asked the students to indicate the angles that had the same magnitude in the given picture of tiled floor. Students' reactions to the given task were in line with our prediction in the HLT in which some of the students used a same mark (symbol) to indicate the angles. This produced the ambiguity when the teacher asked them about which angle that was equal to another angle. Although they used a same mark (symbol) to indicate the angles, from their verbal explanations we know that they knew which angles that they thought to have the same magnitude.

For the second guided question, at least 50% of the students recognized the parallelity in the given situation. Their reactions were in line with our prediction in the HLT, where most of them used equal length symbol to indicate the parallelity. Their understanding about parallelity considered to be an important aspect of their knowledge. The third guided question asked the students about the existence of right-angle in the given tiled floor. Most of the students stated that there was no right-angle in the given picture of tiled floor. It shows that students already grasp the concept of right-angle.

In the end, students' responses to the last question indicate that they realized the connection between the parallelity and the similarity of angles from the given situation. At least 50% of the students showed their understanding about the relation. Most of them claimed three facts about the given situation; there are two parallel line segments, the three line segments are intersecting each other in two intersection points, and there are two angles that have the same magnitude (see figure 5.37). Although, the students didn't explicitly claim about the relation, their responses showed their comprehension about the important aspects of angles similarity in the parallel-transversal situation.

The analysis of this lesson allows us to improve our design for the third lesson. The re-improved version of the third lesson included in the following things:

1. We split the answer box for the third question that ask students to compare the situations of letters E, F, N, and Z in letters from matchsticks and letters on a tiled floor model.

2. Adding a classroom discussion that focuses on supporting students to find the relation between angles in some letters in matchsticks and kitchen floor (E, F, N, X, and Z).

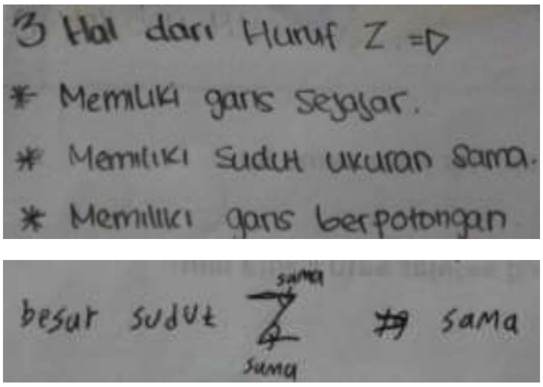


Figure 5.37. Students' responses that showed their comprehension about the relationship between parallelity and angles similarity.

5.2.5 Lesson 4: Reason about the angles magnitudes on the tiled floor models

The teacher started the lesson by invited the students to investigate the magnitude of angles from a simple situation (right-angles on a bricked wall). The students encountered no difficulty in recognizing the right-angles in the given situation. The context also proved to be helpful for the students to make sense the straight-angle, full-angle, and reflex angles. The following fragment from the classroom discourse depicts how the students added several right-angles to form another bigger angle.

[1]Teacher:	*"How do you know that angle is 270°?"*
[2]Zaky:	*"Because 90° subtracted from the 360° from the reflex angle."*
[3]Teacher:	*"Which one is the 360°?"*
[4]Zaky:	*"Emm…(Drawing an imaginary circle around the angle) Emm…What do we call it? Emm…Full rotation."*
[5]Teacher:	*"So…a full rotation is 360°?"*
[6]Zaky:	*"Yes…"(Nodding his head)*

[7]Teacher:	*"So if it is 270° (Pointing to the indicated angle) How big is the inner angle?" (Pointing to the right-angle)*
[8]Zaky:	*"The inner angle is 90°." (Pointing to the right-angle and one of his friends wrote down the measurement of the inner angle)*
[9]Teacher:	*"How about this angle?" (Pointing to a straight angle between two adjacent bricks)*
[10]Ichsan:	*"180°."*
[11]Teacher:	*"How about this one?" (Pointing to an indicated straight angle which students made on one side of the brick)*
[12]Zaky:	*"This one is wrong."*
[13]Teacher:	*"Why is this wrong?" (Students stared at each other)*
[14]Ichsan:	*"Why? (Encouraging his friend to explain it)*
[15]Zaky:	*"These angles are the same." (Pointing to the straight angles that formed by one line segment and two lines segments)*
[16]Teacher:	*"So…this angle is 180° as well?" (Pointing to the straight angle that formed by one line segment)*
[17]Zaky:	*"Yeah…this is 180°, because it is a straight angle."*
[18]Teacher:	*"But this angle only has one line segment."*
[19]Zaky:	*"Oh…this one is not 180° (Pointing to the straight angle that formed by one line segment). This one is the right one." (Pointing to the straight angles that formed by two lines segments)*

From the group discussion above, it shows that the presented situation had provided the students with the appropriate ground for reasoning about the angle magnitudes. In addition to that, the teacher had helped the students to confirm their definition about angle by asking the students to justify their claim about straight angle. The students defined the angle as the difference of direction between two lines. In the group discussion, the students were able to distinguish the figure that can be categorized as an angle and the figure that cannot be categorized as angle according to their definition of angle.

After the mathematical exploration, the teacher distributed the sheets that had 6 different models of tiled floors and asked the students to carry out simple observations and calculations. In the first task the students have to indicate the angles on the given floors that have the same magnitude. Most of the students immediately recognized the right-angles in some of the given tiled floor models, even the right-angles were in the tilted position. Due to the uniformity of the tiles in every given floor, the students encountered no significant difficulty in determining the angles that had the same magnitude. In the second task, the teacher asked the students to explain how they know for sure the indicated angles are in the same size. Students' answers to the second task indicated that they realized the similarity of the angles as a logical consequence of uniformity of the tiles.

In the third task, most of the students were able to explain about the angle magnitude on every meeting point of the tiled floor. All of the students connected the concept of full angle to the given problem. The students concluded that, the sum of angles on every common point was added up to 360. The previous task about angles magnitudes on the brick wall proved to be a fruitful activity that supported students to explain the total angle on each meeting point of the tiled floor. Although the students knew the fact that the sum of angles on every common point is added up to 360, the students still struggled when they encountered the uncertain numerical problems. The students hesitated to make their own assumptions related to the angles measurement of the unknown angle. The students seemed not confident when the teacher asked them to estimate the measurement of the uncertain angles. The following fragment from the classroom discussion about angles magnitude in figure C shows that some of the students employed educated guess strategy to predict the unknown angles magnitude on the given floor model (see figure 5.38).

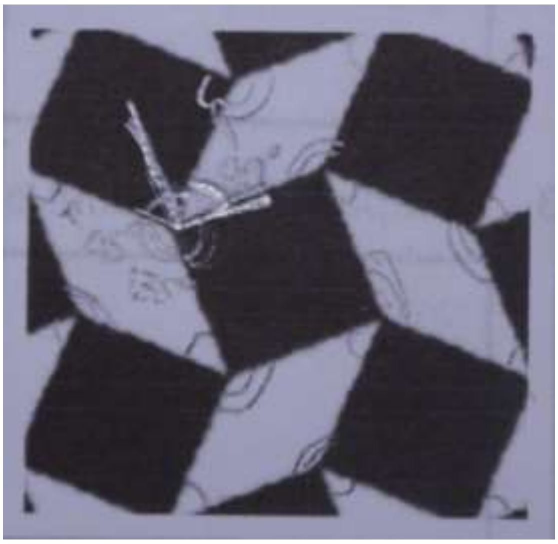


Figure 5.38. Students' strategy to solve the uncertain angle problem.

[20]Teacher: *"How did you find 135° and 45°?" (Pointing to the students' written work)*

[21]Reza: *"This one is 90°, (Mark one of the vertices of the square tile) this one is 135° and this one is 45°." (Pointing to the acute and obtuse angles of the diamond shape tile)*

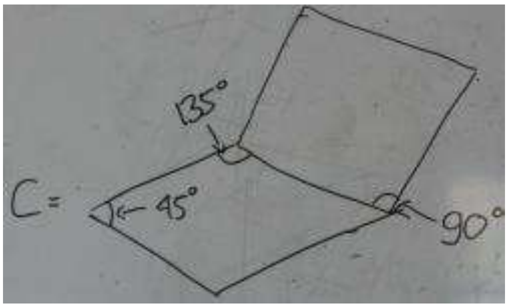
[22]Teacher: *"How do you know that the last two angles are 135° and 45°?"*

[23]Reza: *(In silent he drew an extra line segment on the acute angle of the diamond shape tile to form a right-angle) "If you draw a line here (pointing to the line segment that he just made) this angle will become 90°. Since, this one (Pointing to the acute angle) is half of the 90°, so the angle is 45°."*

[24]Teacher: *"How is about the 135°?"*

[25Reza: *"90° plus 90° plus 45°, (Pointing to the angles in a meeting point of the tiles) you take the sum of the three angles from the 360°. Because the whole angles must add up to 360°, therefore, this angle is 135°."*

Students' solutions to the last problem indicated that they implicitly realized the uncertain condition of the given problem. For instances, 40% of the students only guessed the uncertain angles magnitude and 60% of the students were able to predict all angle in every meeting point. The students who were able to predict the angles magnitude didn't realize the problem had infinite many solutions (see figure 5.39). Unfortunately, the teacher didn't conduct a classroom discussion that supports the students to figure out the uncertainty in the presented problem.



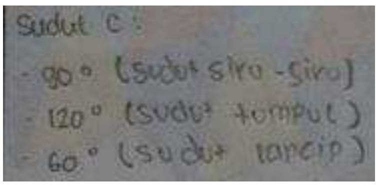


Figure 5.39. Students predicted the angles magnitude in figure C, but didn't realize the problem had infinite many solutions.

The analysis of this lesson allows us to improve our design for the fourth lesson. The re-improved version of the fourth lesson included in the following things:

1. We split the fourth problem into two parts. In the first part the students will deal with certain situations (floors A, B, and F) and in the second part the students will deal with uncertain situations (floors C, D, and E).
2. A classroom discussion that discusses about making assumptions for the angles magnitude on the last problem is added to the teacher guide.

5.2.6 Lesson 5: Angle related problems

Throughout this lesson, we attempted to provide a supportive learning environment for the students to apply their current knowledge to solve problems related to the angles magnitudes in more general cases. During the actual teaching experiment the teacher started the lesson by posting two simple questions that begged the students to apply the concepts of straight-angle, full-angle, and vertical angles. The teacher drew two figures of several lines that intersect in a point and asked the students to calculate the angles magnitude. The first figure consists of four lines and the second figure consists of three lines. Most of the students could calculate the angles magnitude with assumption; all the lines divided the plane into equal parts (see figure 5.40). Students based their calculation on the fact that the number of angles in each figure divides full-angle evenly. The following fragment from the classroom discussion shows how students employed the full-angle concept.

[1]Rozan:	*(Writing down 45° on one of angles in the first figure)*
[2]Teacher:	*"How about the rest of it?"*
[3]Reza:	*"The entire angles are 45°."*
[4]Teacher:	*"All 45°?! (Rozan filled up the rest of the angles) How do you calculate it?"*
[5]Reza:	*"You only need to divide the 360° with 8."*
[6]Teacher:	*"Why 360°?"*
[7]Reza:	*"Because you can draw a circle around the intersection point."*

The students used the same reasoning to calculate the angles in the second figure.

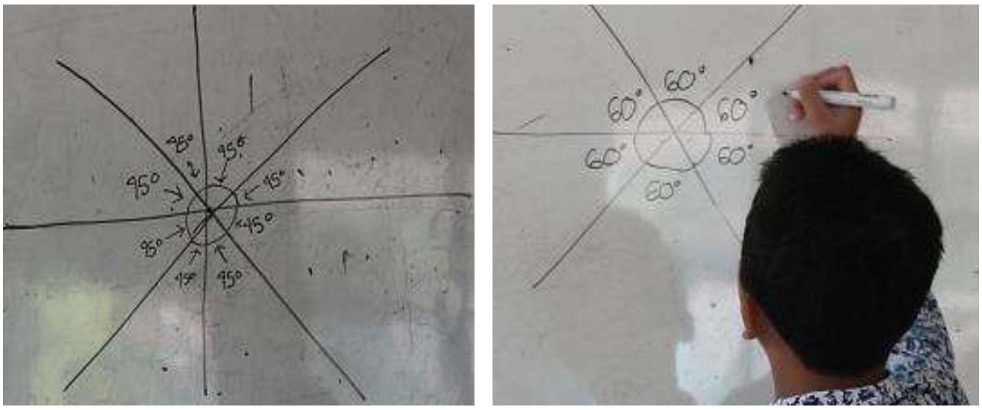


Figure 5.40. Students applied the full-angle concept to calculate the angles magnitude.

The teacher continued the activity by distributing the worksheet and asked the students to work in group of four. The first task required the students to sketch the top view of two pictures of railways intersections. Most of the students didn't see the two pictures as two different things if they sketched the top views of them. As a result almost all of the students drew the trivial condition of the situation where all the angles in the railways intersections were in the same size (90°).

[8]Giri: *(Sketching a top view of the railways)*
[9]Teacher: *"You only made a sketch for these railways. So do you think both railways are the same?"*
[10]Sri: *"They are the same if you see them from above"*

However, some groups of students perceived the railways would have two different top view sketches. In addition to that, their written works indicate that they were aware about the similarity of the angles on each sketch by giving some numerical values of the angles (see figure 5.41). Unfortunately, the teacher forgot to conduct a classroom activity (second task) where the students have to draw a different version of the railways intersection, give a numerical value of an angle on it, and dare the other groups to fill the unknown values. This activity will allow the students to apply the letters-angles concepts without a help from the geometrical patterns or grids to calculate the unknown angles magnitude.

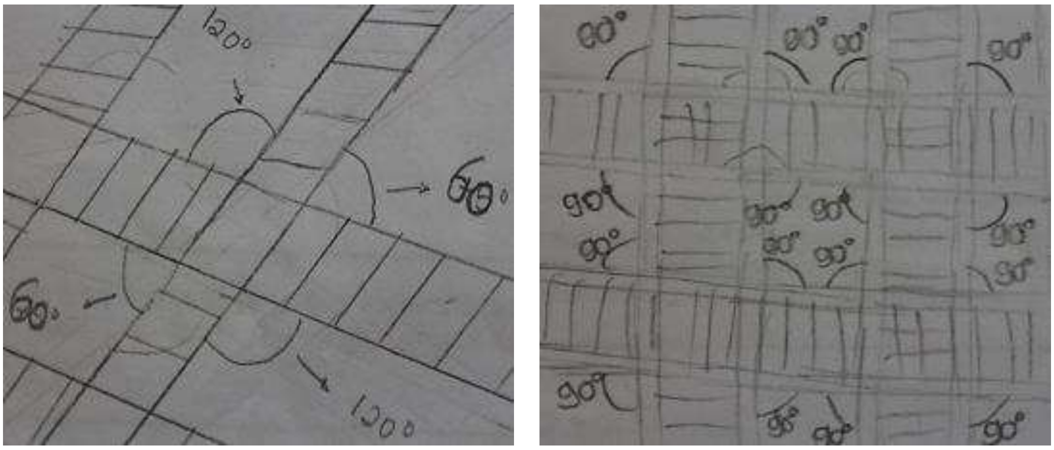


Figure 5.41. Students were aware about the similarity of the angles in their sketches by giving numerical values of the angles.

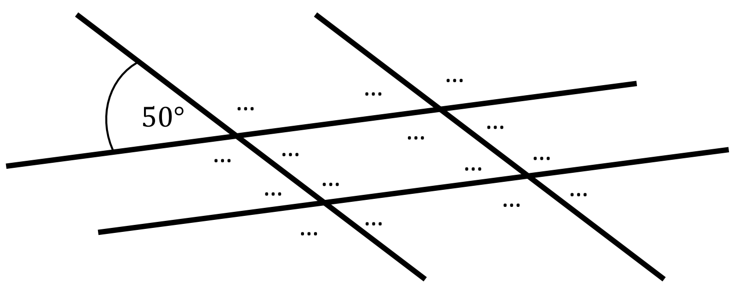
There were several questions about angles related problems in students' worksheet. The first question required the students to determine the pairs of similar angles on a given parallelogram tiled floor model. All of the students were able to find the pairs of similar angles. Some of the students gave general description about the similarity of angles magnitude and the other gave numerical estimations of each pair of similar angles. The second question is a 'what-if question', this question is an extension of the first question. The students have to calculate the unknown angles magnitude from a known angle magnitude. Almost all of the students were able to calculate the unknown angles magnitude. Mainly their strategies involved the use of concepts such as, straight-angle and full-angle, however, this differed with our conjecture on students strategy in solving the given problem. We predicted that the students might apply their understanding about the properties of angles in parallel-transversal situation from the first question to solve the second question.

The third question is also a 'what-if question' where the students have to determine the unknown angle on a given triangle tiled floor model. Students' reactions to the given problem were in line with our prediction in the HLT in which we predicted some students might conclude that 70° was the rights answer (180° as a benchmark) and some might conclude that 250° was the rights answer

(360° as a benchmark). All of the students applied the fact that the total angle in a triangle is 180° and derived this fact to determine the unknown angle. The fourth question can be reformulated as $a + 50° + c = 180°$. Students' solutions to the fourth question produced a debate among the students. Due to the classroom habit that can only accept a single right answer to each question, even for this kind of problem, the students encountered difficulty to accept the fact that the problem had infinite many solutions. There were two categories of students' solutions: (1) the students divided the 130° into two equal parts and claimed the parts as the angles in the question, and (2) the students guessed the sizes of angles in the question in which the sum of both angles was 130°. Although the teacher had orchestrated a classroom discussion that discussed about the possibility to have so many different solutions in this context, the students were still reluctant to accept this fact.

In the end of the lesson, the teacher invited the students to fill up the unknown angles magnitude from a parallel-transversal situation. The aim of the activity is to check whether the students were able to apply their knowledge about angle and its magnitude in a more general case. The following fragment from the classroom discourse depicts the actual teaching and learning activity.

[11]Teacher: *(After drawing a parallel-transversal figure, teacher gave the instruction) "One after another, please complete the angles in the figure on the whiteboard!"*



[12]Students: *"Yes mam." (Rozan were approaching the whiteboard and filled up one of the unknown angles, he wrote 130° to fill up a blank)*

[13]Teacher: *"Is that right?"*

[14]Students: *"Yes.."*

[15]Teacher: *"Rozan, how do you know if the answer is right?*

[16]Rozan: *(unclear voices)*

[17]Teacher: *"What does Rozan state about that angle? "*

[18]Students: *"Straight angle."*

[19]Teacher: *"Straight angle, who knows about the size of a straight angle?" (Pointing to the figure on the whiteboard)*

[20]Reza: *"180 degrees."*

[21]Teacher: *"Yeah...180 degrees. Therefore, 130 degrees plus 50 degrees add up to 180 degrees. Who next? (Students chattered). What is your name? (Asking a student to give his answer)*

[22]Ichsan: *(Students were chattering when Ichsan gave the measurement of one of the unknown angles, he wrote 130° to fill up another blank)*

[23]Teacher: *"Do all of you agree with that? Explain why your answer is 130 degrees! Please tell me! (Holding Ichsan's arm and ask him to give the explanation to his answer)*

[24]Student: *"He guessed!"*

[25]Ichsan: *"Because, it's the same." (Attempting to give an explanation)*

[26]Teacher: *"Same with which one?"*

[27]Ichsan: *"With the 130 degrees from Rozan's answer!"*

[28]Teacher: *What do we call those angles? Who still remember?*

[29]Students: *"Vertical angles."*

[30]Teacher: *"So that.... " (Asking for more explanations)*

[31]Ichsan: *"The angles are the same."*

[32]Teacher: *"Good! (Let Ichsan back to his seat) Next... Zaky!" (Students were mumbling)*

[33]Zaky: *(Approaching the whiteboard and he wrote 50° to fill up a blank)*

[34]Teacher: *"What is your reason?" (Asking for clarifications from Zaky)*

[35]Zaky: *"That's because that 50 equals to that 50." (Pointing to the angles that he had indicated)*

[36]Teacher: *"What do you call those angles?"*

[37]Zaky: *"Vertical angles."*

[38]Teacher: *(Irfan wrote his answer on the whiteboard and at the same time the teacher chatted with other students) "Can you solve it? Do you understand? Good!"*

[39]Irfan: *(Irfan wrote his answer on the whiteboard)*

[40]Teacher: *"Irfan, which angle that has the same size with that angle?" (Asking for clarifications from Irfan after he wrote his answer)*

[41] Irfan: *(Pointing to the similar angles that he had indicated)*

[42]Teacher: *"We call those angles as corresponding angles." (Pointing to the angles that Irfan indicated)*

This fragment shows that the students were able recognize the similarity between angles in a parallel-transversal situation. Unfortunately, in the actual teaching and learning activity, we didn't observe the students applied the concept of letter-angles (F, X, Z-angles). The teacher also didn't encourage students to employ the alternative concept to justify their claim about angles similarity in a parallel-transversal situation. The teacher seemed satisfied with students' answers that mainly applied the concept of straight-angle and vertical angles.

The analysis of this lesson allows us to improve our design for the fifth lesson. The re-improved version of the fifth lesson included in the following things:

1. Reformulate the second question into several numerical problems, where the students should match the numerical problems with the right answers.
2. Make a new version of the last question in order to disable the students to use the unrelated data from the previous problem.

5.2.7 Post-assessment

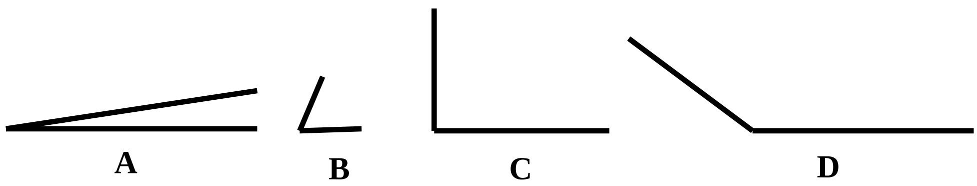
The forty students took a 20-minute posttest after going into the entire lesson sequence. The posttest items were designed to assess students' current knowledge about angle and its magnitude. The gained scores give us a general impression about students' development in understanding about angle and its magnitude (M_{pre}(SD) = 5.09 (1.39) and M_{post}(SD) = 6.5 (1.96)). The results didn't show better development of students understanding toward the intended mathematical concepts. There are two aspects that responsible to the students' learning outcomes in this particular teaching experiment. The first is students' learning habits such as; hesitate to ask (to answer) questions, view the teacher as an absolute authority, be afraid to make a mistake, and rarely encounter production tasks like the tasks in the designed lessons. The second is the roles of teacher in the learning activity such as; the teacher views herself as a distributor of knowledge but not a facilitator of learning process, teacher's classroom management weren't allow the whole classroom to be active in the learning activity, and the teacher didn't assertive in conducting the teaching and learning process.

The classroom culture that students and teacher embraced was not easy to change in only five or six weeks. Unfortunately, this classroom culture is not an ideal condition for this study. This study requires the students to rely on their own productions and actively interact with each other in the discussion to reach the intended knowledge. Most of the proposed teacher's action and students' reactions didn't occur in the second teaching experiment. However, throughout the five lessons in this study, it can be concluded that the students had learnt something about angle and its magnitude. What students had learnt can be deduced from the data that we gathered from the interview session with the focus group and two randomly selected students. Based on the analysis on students' written work and video registrations of the interview, we noted several important remarks as follow:

a. Frame of reference about angle

From the previous interview with the students before they went into the entire lesson sequence, we found that sixty percent of the students used area as a frame of reference. The data from the interview after the students followed the lessons sequence shows that all the interviewed students used difference in direction as a frame of reference. The following fragment from the interview with a student represents the frame of reference about angle that students embraced.

[1]Interviewer:	*"The angle in figure B is the smallest angle, (Read the claim in the problem) why did you claim this is a wrong claim?"*



[2]Interviewee:	*"Because the smallest one is the angle in figure A!"*
[3]Interviewer:	*"So the angle in figure B is bigger than the angle in figure A?"*
[4]Interviewee:	*"Yes."*
[5]Interviewer:	*"But it is clear that the figure B is the smallest one."*
[6]Interviewee:	*"Emm…You must see the angles, not from the size of the figure." (Drawing imaginary lines emanating from the vertex of figure A)*
[7]Interviewer:	*"So the angle in figure A is the smallest angle?!"*
[8]Interviewee:	*"Yes."*

In addition to that, all the interviewed students knew that the smallest and the biggest angles were in the figure A (i.e. acute angle and its reflex angle).

b. Symbol to indicate the angles

Only one interviewed student that still used informal sign to indicate an angle. She used circle and dot instead of the arc (∡) symbols that commonly used to indicate the angles. Although, almost all the interviewed students used the formal symbol to indicate the angle, they understood the meaning of the symbol. They perceived the symbol as an indication symbol and has nothing to do with the angle magnitude attach to it. From students' written works, we found that almost all the students perceived the indicated angles in the figure 5.42 had the same magnitude, even the angles appeared to have different sizes of arcs.

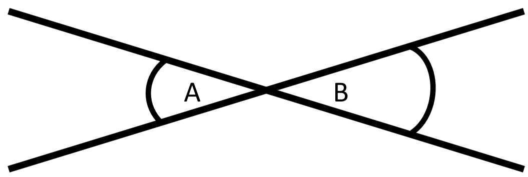
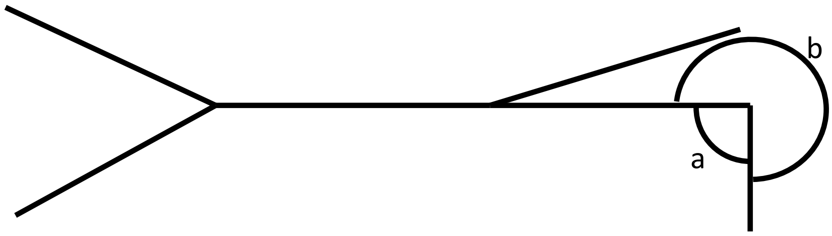


Figure 5.42. Vertical angles where one of the arcs that indicated the angle was narrower compared with its pair.

c. The sense of angle magnitude

One of the test items asked the students to sort seven angle figures in an ascending order and this can be used as an indicator of students' sense of angle magnitude. Most of the interviewed students could sort the angles magnitude. This indicates that most of the interviewed students have good understanding about angle magnitude. In addition to that, a test item that asked the students to indicate the smallest and the biggest angles in a given figure showed that the students could use their sense about angle magnitude in a given problem. The following fragment from the interview with a student represents how students reason with the angle magnitude.

[1]Interviewer:	*"You claimed that 90° is the biggest angle in the figure (angle a), is there an angle that bigger than this 90° angle?"*



[2]Interviewee:	*"This angle! (Pointing to angle b)"*
[3]Interviewer:	*"How big is that angle?"*
[4]Interviewee:	*(Doing calculation in his head) "270"*
[5]Interviewer:	*"How did you do the calculation?"*
[6]Interviewee:	*"This one (angle a) is 90°, and this one (angle b) is 270°"*
[7]Interviewer:	*"How did you know this angle (angle b) is 270°?"*
[8]Interviewee:	*"This angle (angle a) times three."*
[9]Interviewer:	*"So…you mean in this angle (angle b) it is three times of that angles (angle a)?" (Drawing extra lines in angle b that divided it into three equal parts)*
[10]Interviewee:	*"Yes."*

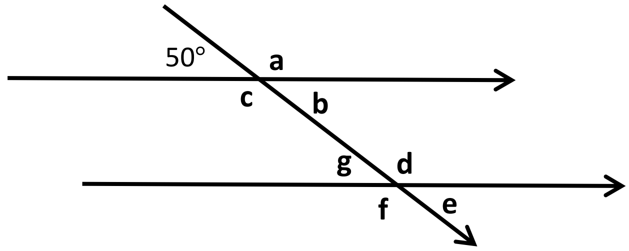
When the student claimed that the angle b was three times the angle a, the student had used the concept of full-angle in advance. He knew that there are four times 90° in a 360°, and based on his calculation fact, he came to the conclusion that the angle b is three times the angle a.

d. Knowledge about right-angle and straight-angle

In order to know students' understanding about right-angle, in one of the test items we presented a set of right-angle figures that differ in size and orientation. Most of the interviewed students could recognize the given figures as the right-angle figures even when there was no horizontal arm in some of the presented right-angle figures. Students claimed that, no matter what the size and the orientation, as long as the arms of the angle were perpendicular to each other, the figure must be a right-angle figure. It is clear that their judgment wasn't affected by the size and the orientation of the given figures anymore. This shows a development in students' understanding. Because before they went into the lessons sequence, most of them agreed that the right-angle figure that could cover the largest area if they drew other lines that were parallel to the both arms was the largest right-angle.

There are two test items that require the students to employ the concept of straight-angle. The first problem asks the students to determine the unknown angle magnitude from an alignment of two angles, in which one of the angle magnitudes is given. Most of the interviewed students could solve the given problem using the straight-angle concept. Their strategy is based on the fact that the sum of both angles is 180°. The second problem requires the students to apply their understanding about angles similarity. The following fragment from the interview with a student represents how students solve the problem about angle similarity.

[1]Interviewer:	*"How big is the angle f?"*



[2]Interviewee:	*"f…em..hundred and…wait (Doing calculation in his head)…130."*
[3]Interviewer:	*"How you calculate it?"*
[4]Interviewer:	*"Because, this straight line is 180° (Pointing to the upper straight-angle) this angle is 50°, so 180° − 50° is 130°." (Pointing to the angle a)*
[5]Interviewer:	*"That's angle a, but not f!"*
[6]Interviewee:	*"Both angles are the same because of this line" (Pointing along the transversal line)*
[7]Interviewer:	*"Can you tell me which angle that is equal to another angle?"*
[8]Interviewee:	*"a, c, d, and f are the same, and b, g, and e are the same."*

From the conversation above, the student employed the straight-angle concept to find the magnitude of a supplementary angle (line 4). After that, he only needed to figure out the pairs of similar angles to solve the whole problem. We can observe students' recognition of similar angles when he stated that angle a and f were in the same magnitude. Student's gesture when he pointed along the transversal line indicates that he knew the necessary condition for angles similarity in a parallel-transversal situation. From all the description above, we can infer that the students had learnt something about angle and its magnitude throughout the lessons sequence. Even though, the pre and posttest results didn’t show better development of students understanding toward the intended mathematical concepts.

5.2.8 Conclusion for the second teaching experiment

The second teaching experiment was conducted in a traditional big size classroom environment. The classroom culture and students’ learning habits created an unfriendly condition for this study. For instances, most of the students didn’t use to express their opinions, were afraid to make mistakes, tended to work individually, and avoided any argumentation. Besides that, the teacher is still new about the RME approach and tended to have different interpretations toward the educational design. Changing the classroom culture, students’ learning habits, and teacher’ belief is favorable before this study was conducted. However, time allocated for this study does not allow that kind of preparation. In addition to that, this study is only a part from a long-term continuation of teaching and learning processes on the concept of angle and its magnitude. The problem that we encountered in this teaching experiment already highlighted by Zulkardi (2002, p.11-12) in his thesis. He stated that, there are at least three main issues in applying RME design in classroom environment. First, most of the RME designs are not readily understood by the teacher. Second, a major change in the roles of teacher is from teaching to ‘un-teaching’. Third, the implementation of an RME design is a long-term project.

Nevertheless, at least the students and their teacher had exposed to a new kind of teaching and learning environment. In this study, we believe that both students and teacher had learnt something. For instance, most of the students before they went into the lessons sequence, judged the angle magnitude based on the length of the arms or based on the area coverage by the arms. It produced some perplexities in recognizing the same angle that have different size figure and different size arcs symbol as the same angles. However, throughout the designed lessons sequence the students accepted the fact that the arc symbol that indicates an angle has nothing to do with the angle magnitude attached to it.

Refering to the actual teaching and learning process in the second teaching experiment, it suggests that the students had acquired the knowledge about angles similarity in parallel-transversal situations. The students could easily recognize the angles on a straight line that falling across two parallel lines without taking the advantage from the grids or any geometrical patterns that can ease the identification process. In the HLT we predicted that they will utilize the concept of letters-angle that they had learnt during the actual teaching process. However, the students didn't use the proposed strategy to reason about the angles similarity. The students perceived the angles similarity in that condition as an obvious geometrical fact. Therefore, in the next teaching cycle we will promote students' reasoning about angles similarity.

5.3 Third teaching experiment (Third cycle)

In this sub-phase of the teaching experiment, we try some crucial elements in improving materials in order to produce an educational design that account for and potentially impact to teaching and learning in naturalistic settings. The process involved 6 seventh grader students (i.e. 3 male students and 3 female students). The students already learnt the subject matter in the previous weeks in their classroom and are willing to become the volunteers in this study. Throughout this sub-phase the researcher acts as the teacher to gather all relevant information for improving the design. The detail of the observations, analyses, and evaluations of the third teaching experiment described as follow in a chronological sequence.

5.3.1 Pre-assessment

The six students in this sub-phase also took a 20-minute pretest and a follow-up interview before going into the entire lesson sequence. In general, there is no significant difference in students' performance compared with the students in the first and the second teaching experiment. Analyses of the students' written works revealed several important remarks related to the students existing knowledge.

a. Frame of reference about angle

After analyzing students' written work and video of the follow-up interview, we still cannot clearly see what kind of frames of reference about the angle that students embraced. The proposed frames of reference that students may use such as; angle as the area between two intersecting lines, angle as the difference in direction between two lines radiate from a single point, and angle as the amount of rotation between two intersecting lines. Those three frames of reference cannot be observed from students' written works as well as their verbal explanations. It seems that the students have their own frames of reference about the angle. The following fragment from conversation with the students depicts how students perceived the angles.

[1]Researcher:	*"When you compare two angles, what features that do you use as the reference to distinguish between big and small?"*
[2]Dina:	*"Their degrees."*
[3]Researcher:	*"Okay, their degrees. How if you don't have a protractor to measure their degrees. What features will you use?"*
[4]Dina:	*"Their shapes."*
[5]Researcher:	*"What do you mean?"*
[6]Dina:	*"I mean the sizes of the shapes, bigger or smaller."*
[7]Researcher:	*"Can you be more specific?"*
[8]Dina:	*(Not give any responses)*
[9]Dela:	*"The sizes."*
[10]Researcher:	*"What sizes?"*
[11]Dela:	*"Degrees….emmm…the angles magnitude." (Pointing to a vertex of a plane figure)*

At this moment we can only infer that the students know the use of a protractor, but their understanding about the angles magnitude are still limited and vague.

b. Symbol to indicate the angles

Although, all the students used the arc (∡) symbols to indicate the angles, some of them seemed not to fully understand the meaning of the symbol itself. They perceived the symbol as an indication of the angle magnitude that attaches to it. So for instance, two angles that have the same magnitude if they are displayed with different size of arcs, some of the students will conclude that the angles have different magnitude.

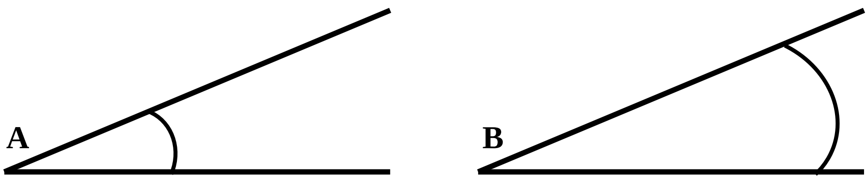


Figure 5.43. Two angles which have the same magnitude but are different in sizes of the arcs leads students to the conclusion that the angle B is bigger than the angle A.

c. The sense of angle magnitude

There are three test items that assess students' sense about angle magnitude. The first item, asked students to indicate the smallest and the biggest angles in a given figure. Some of the students were unable to distinguish the two angles due to their limited idea about what is the meaning of the angles magnitude. Figure 5.44 shows a student' answer which claimed the smallest angle as the biggest angle that indicates his limited understanding about the concept of angles magnitude. The second item, asked the students to sort seven polygons based on their internal angle in an ascending order. Some of the students sorted the given polygons based on the area of the polygons instead of the order of the given polygons based on their internal angle, and some of them didn't show any clear reference in making the order (see figure 5.45).

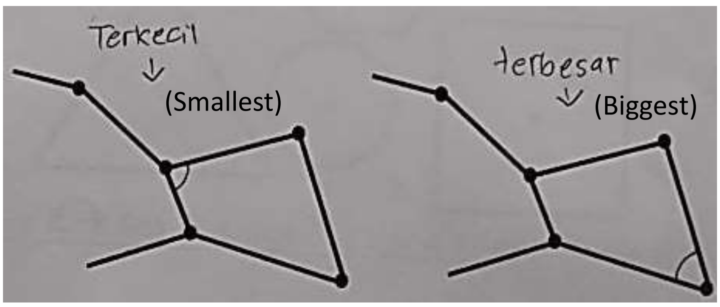


Figure 5.44. A student claimed the smallest angle as the biggest angle.

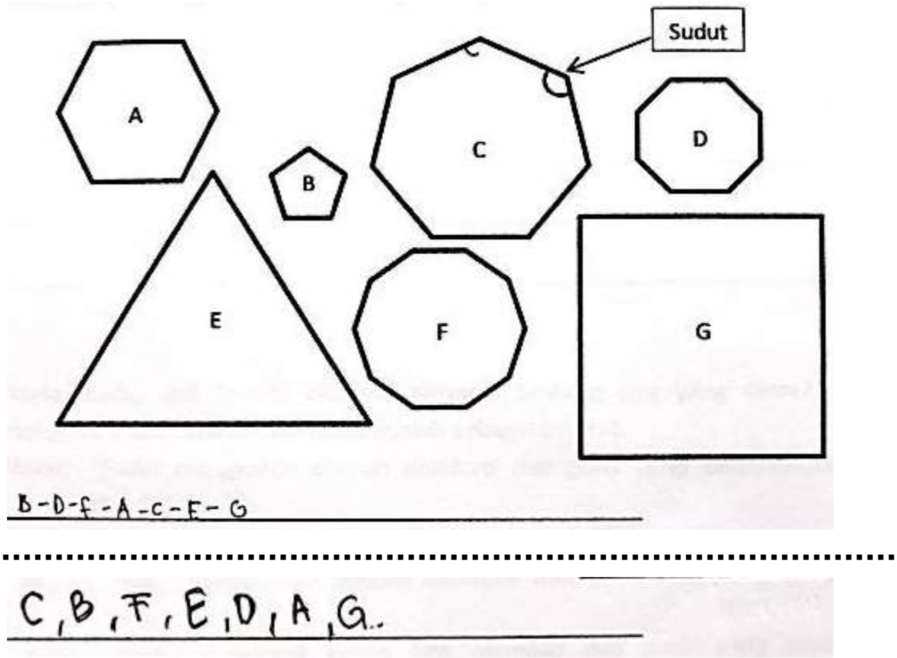


Figure 5.45. Above the dotted line a student sorted the angles based on the area of the polygons, and below the dotted line another student sorted the angles without any clear reference.

In the third test item, we asked the students to explain what they had known about the angles magnitude in a vertical angles situation. Their judgment about angles magnitude seems affected by the size of the arcs that indicates the angles in the given situation (see figure 5.46). All students' solutions to the presented problem indicate that they were less capable to infer similarity between angles in this particular context. It is because all of them concluded that the opposite angles in the vertical angles situation are different in size. Based on the students' written work and the follow-up interview, we can conclude that the students' sense of angle magnitude was limited as well as their understanding about angles magnitude.

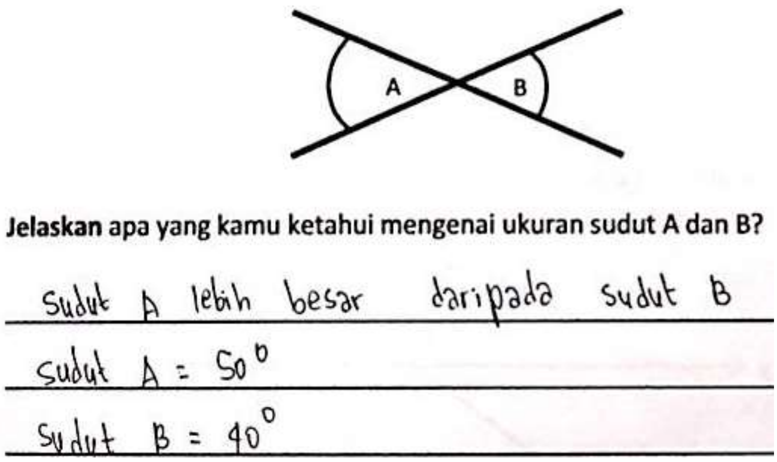


Figure 5.46. All of the students concluded that the opposite angles in the vertical angles situation are different in size.

d. Knowledge about right-angle and straight-angle

There were only two students that could recognize the right-angle figures which differed in size and tilted in orientation in one of the test items. The rest of the group related the sizes of the right-angle figures with the coverage area of the figures. Most of the students agreed that the right-angle figures that can cover the largest area if they draw other lines that parallel to the both arms is the largest right-angle. There is an item test that requires the students to apply the concept of straight angle to solve the given problem. Due to the fact that most of the students still didn't know about straight angle, so, most of the students were unable to calculate the unknown angle magnitude from an alignment of two angles, in which one of the angle magnitudes was given. Mathematically speak, the students were unable to translate the given problem in to the mathematical language (i.e. $50° + x° = 180°$) in order to solve it.

We also presented a follow-up version of the straight angle problem. In that problem, the students should apply not just the straight angle concept but the concepts of similar angles as well. Students' solutions can be categorized into three categories. The first category is the solution where the students were able to deduce the solution from the fact that a straight angle is equal to 180°. The second category is the solution where the students were unable to translate the given problem into a proper mathematical equation (see figure 5.47). The third category is the solution where the students only relied on their rough estimation of the angles magnitude. Based on students' written work most of them employed the rough estimation strategy.

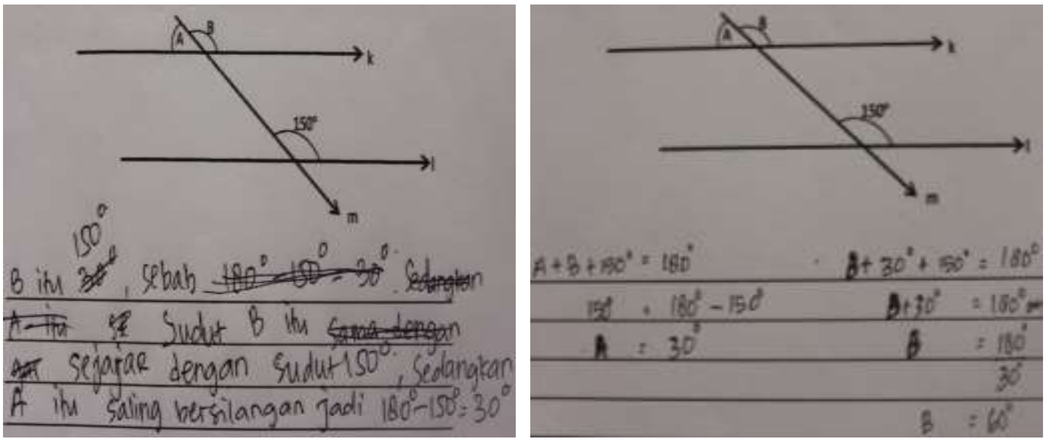


Figure 5.47. The first and second category of students' solutions.

From the description above, we can see some degree of inconsistency in students' knowledge about the angle and its magnitude. Most of the students didn't perform well in several items test that meant to test students' understanding about angle concept. However, in the several test items that meant to assess students' capability to apply their knowledge about angles to solve problems about angle magnitude, suggest that they knew about the key concepts in the presented problems. As we know, the students had learnt about the subject matter in the past few weeks. This indicates even the students had learnt the concepts, but their understanding toward the concept is still limited, and their comprehension about the angle magnitude is still fuzzy.

5.3.2 Lesson 1: Angle from everyday life situations

From the same lesson in the previous teaching experiments we had learnt that most of the students misinterpreted the first two instructions in the worksheet 1. Therefore in this teaching experiment the researcher carefully clarifies the instructions in the worksheet before the students work with the tasks. The first instruction asked the students to indicate an angle in several everyday life figures. All of the students followed the instruction as we expected. In one of the students' written works we found an interesting thing. A group of students had indicated an angle that formed by a line and a curve (tip of a traditional fan). It suggests that they accepted the fact that the curves could form an angle as well.

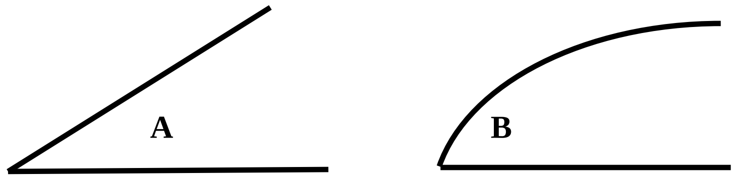
Figure 5.48. A group of students accepted the fact that an angle can be formed by curves.

However, in the whole group discussion, another group argued that the tip of a traditional fan was not an angle. The following fragment from the classroom discussion depicts the discussion.

[1]Della:	*(Give a comment to the other group's work) "In figure D, you make a mistake with your claim. This one is not an angle! (Pointing to a tip of the traditional fan figure)*
[2]Researcher:	*"Your friends stated that the figure that formed by curve line is not an angle. Do you agree with that?"*
[3]Muhammad:	*"Yes we do." (Avoiding further argumentation)*
[4]Researcher:	*"If you think your claim is worth to defense, then please say something about it!" (Muhammad's group looks at each other without any words)*

After few moments in silence, the researcher orchestrates a discussion to make sense that an angle can include a curve line.

[5]Researcher: "Okay, let us observe the following figures. (Drawing two angle figures to contrast the situation) Based on your claim B is an angle."



[6]Aulia: "B isn't an angle!"

[7]Researcher: "Can you explain what makes your group think B is an angle?" (Asking Muhammad's group to defense their claim)

Muhammad's group couldn't explain their claim. The researcher poses several follow-up questions and found out that Muhammad's group opinion now changed, without any explanation they agreed that B was not an angle.

[8]Researcher: "But I think B is an angle as well."

[9]Dhani: "Yes B is an angle!" (One group member in Muhammad's group became confidence)

[10]Researcher: "Nahhh…you are not a persistent person." (Students giggled)

The researcher employed 'zoom in' strategy to explain that the figure B could be an angle if the students saw the tip by using a microscope.

The second task asked the students to sort the indicated angles in an ascending order. All of the students could construct a well ordered poster of the indicated angle magnitudes. We also performed a further discussion related to the order of the angle magnitudes in each poster. The discussion revealed that the students were able to determine the angle magnitudes in the presented figures. The following conversation clarifies this claim.

[11]Researcher: "How do you decide the angle in J is smaller than the angle in F? (Pointing the acute angles in the figures)

F

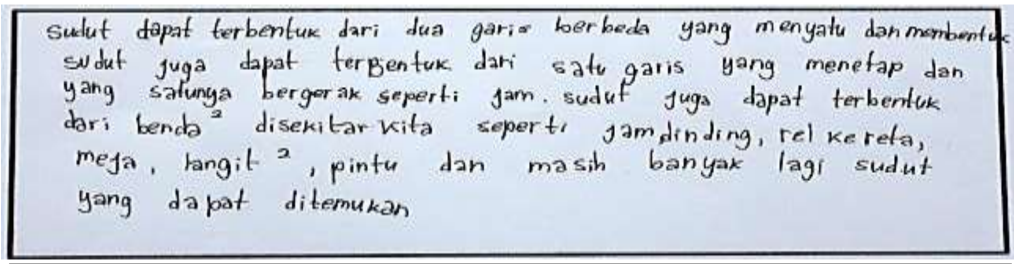
J

[12]Students: (Seeing each other in silent)
[13]Researcher: "Please…think about it for a moment before you give your responses!"
[14]Della: "The F figure is a figure of equilateral triangles, so each angle on it must be 60°. However, the angle in figure J is less than 60°. So J smaller than F"
[15]Researcher: "Can you tell me how big is the angle in J?"
[16]Della: "Roughly 30 or 40."
[17]Researcher: "Dina, can you help us to determine how big is the angle between two consecutive number in J?"
[18]Dina: "That's must be 30°."

Further discussion revealed that in order to know how big the angle between two consecutive numbers is in an analog clock, the students reasoned with the fact that they should divide 360° by 12. They also used the same strategy to explain the angle magnitude in figure F was 60°.

In the worksheet, there are 4 questions which mean to investigate students' understanding about the very basic concepts of the angle and its magnitude. The first two questions, is about a dynamic angle situation where the students should choose an object from their poster. The selected object had to be an object that could change the size of its angle. The students also required to draw two situations where the object representing the biggest and the smallest angles. The students' actual reaction to the given tasks is in line with our conjectures in the HLT in which we predicted some students may draw a small non-zero angle to represent the 0° angle and draw an obtuse non-360° angle as the biggest angle. All of the students drew a small non-zero angle to represent the 0° angle (i.e. angle between two consecutive numbers in an analog clock). There was a difference in students' opinion about the biggest angle in an analog clock. Some of the students claimed that 180° was the biggest angle, and some of them claimed that 360° was the biggest angle. Due to the obviousness of the numerical value of the angles, the students didn't encounter any significant difficulty to accept the fact that the biggest angle is 360°. The researcher was fully aware about the possibility in having the angle with infinite angle magnitude in this context. However, in this stage of students' learning, it is wise to limit the condition in the finite situation. The justification for the smallest angle was performed in a similar way with the strategy in the first two teaching experiments (i.e. approximation strategy; bring one of an angle's arms to the other arm).

The last two questions were designed to investigate students' understanding about angle definitions. Based on students' answers about how an angle was constructed suggest that, the students perceived the angle construction as a result of two lines that intersect in a point. This responses is in line with our conjecture in the HLT. In addition to that, we found a student had realized about the possibility to construct an angle by rotating one of its arm (see figure 5.49). In students' attempts to redefine the angle, most of them defined the angle as two lines that meet in a point or as an arc on the vertex of a pointed figure. Although, one of the students had realized the fact that an angle construction could be explained by using amount of turn, she didn't define the angle as the amount of rotation between two intersecting lines.


Sudut dapat terbentuk dari dua garis berbeda yang menyatu dan membentuk sudut juga dapat terbentuk dari satu garis yang menetap dan yang satunya bergerak seperti jam. sudut juga dapat terbentuk dari benda² disekitar kita seperti jam dinding, rel kereta, meja, langit², pintu dan masih banyak lagi sudut yang dapat ditemukan

"Angle can be constructed by intersecting two different lines, and also can be constructed by rotating one of its arms meanwhile the other arm standstill, like the hands of a clock. Angles which can be found around us such as; clock, railways, table, ceiling, door, etc."

Figure 5.49. Student's explanation of an angle construction that mentioned the dynamic aspect of an angle.

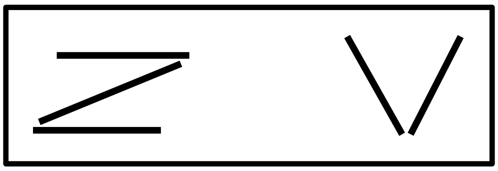
According to the actual teaching and learning activities in this lesson, we can conclude that the proposed activities in this lesson can support students' understanding about angle and its magnitude. The students' written works and their verbal explanations indicate that the students were able to recall the important concepts of angle magnitude that they had learnt before. After analyzing how the students define the angle, we conclude that they were able to reformulate a definition of angle, and the classroom discussion allowed them to add more angle definitions into their inventory of angle definition.

5.3.3 Lesson 2: Matchsticks, letters, and angles

There wasn't any big difference in how students reacted to the presented tasks in this particular lesson compared with the same lesson in the previous teaching experiments. Therefore, here we will focus solely on some crucial elements of the design. After the students reconstructed the upper case letters using wooden matchsticks, the researcher performed a follow-up activity that included several guided questions and classroom discussions. In the guided questions, the students should decide which letters in their reconstruction that have the smallest and the biggest angles. Most of the students claimed that the smallest angle was in letters Z or V, and the biggest angle was in letters I or O. During the classroom discussion, the researcher tried to lead the students to reason about angle magnitudes in those letters. The nature of the discussion allowed the researcher to introduce the concept of reflex angle to the students. The following fragment from the classroom discussion explains how the discussion was conducted.

[1]Researcher:	*"Okay, now I want to collect your opinions about the smallest and the biggest angles in the letters. We start with the smallest angle. Your opinions please!"*
[2]Imam:	*"Z" (Aulia, Dela, and Dina also selected Z letter that formed by three sticks)*
[3]Muhammad:	*"V" (Dhani also selected V letter that formed by four sticks)*
[4]Researcher:	*"So, we have two different opinions. Now the question is howdo we compare the angles in both letters?"*
[5]Imam:	*"That is obvious, Z has the smallest angle."*

[6]Researcher: “Think about it for a moment!” (At the same time on the table reconstructing the letters that students made using the same material)



[7]Muhammad: “Z” (Immediately changed his opinion after the researcher reconstructed the letters)
[8]Researcher: “How do you know that?”
[9]Muhammad: “Because the opening in Z is smaller compared with the opening in V.” (Drew imaginary line segments to represent the amount of opening of these two letters)
[10] Imam: “Yeah…that is obvious.”

The students couldn’t produce an alternative explanation for the situation. Therefore, the researcher, summarized the students’ explanation in order to strengthen their understanding and then continued the discussion.

[12]Researcher: “Now how is about the letter that has the biggest angle?”

Five out of six students chose I as the letter that had the biggest angle and one student (Aulia) chose O. Aulia explained that she picked the letter O because she didn’t read the instruction carefully, and presumed that she had to select an actual letter instead of a letter from the matchsticks.

[13]Researcher: “Okay, let us observe the angles in letters I and O! How big the angles are?”
[14]Della: “180° and 90°.”
[15]Researcher: “Dhani, can you show which angle that Della meant?” (Invited Dhani to actively be involved in the activity)
[16]Dhani: “This one is 180° and this one is 90°.”(Drawing imaginary arcs on the letters I and O)
[17]Researcher: “How if we take the external angle into account?” (Pointing to the reflex angles of both letters)
[18]Imam: “This is 180° and this is also 180°. (Pointing to the opposite angles in letter I) This one is 90° and this one is….emm…(Unable to provide the value)
[19]Researcher: “Can you help Imam to find the magnitude of this angle (reflex angle of 90°)?”
[20]Della: “270°, because if we take 90° from 360° that will be the remainder”
[21]Researcher: “Muhammad, do you understand what she meant?”
[22]Muhammad: “Yes…”
[23]Researcher: “So if we take the external angles into account, what letter that has the biggest angle?”

After brief discussion the students figured out that the biggest angle and the smallest angle were in the same letter.

We also asked the students to observe the angle magnitudes in several letters that had parallel sticks (see figure 5.50). Students' written works and their verbal explanations suggest that the students could easily give an explanation about angles similarity when 90° angles were involved (E, F, H, and U) and used acute angle (sharpness/opening) as a benchmark in their attempts to explain the similarity when there wasn't right-angle involved.

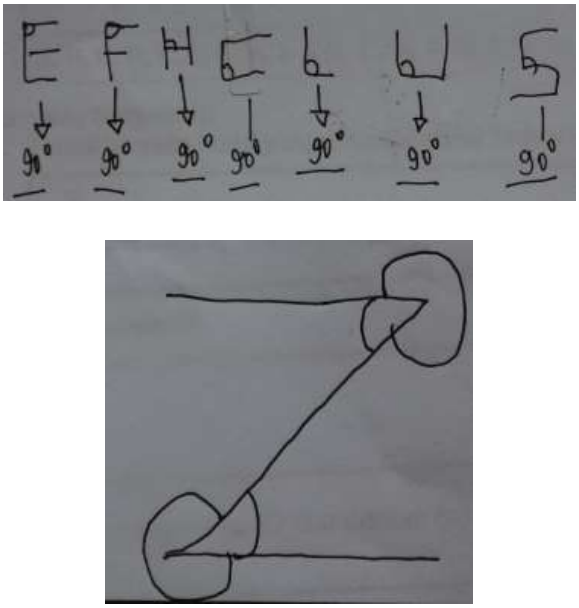


Figure 5.50. Students' written works indicate students' ability to infer angles similarity.

In the end of the lesson, we observed that the students realized the relation between parallelity with angle similarity. It was clear from their verbal explanations during the classroom discussion. In the discussion the students analyzed and compared the angles similarity in two situations (i.e. parallel and non-parallel situations). It was evidence that the students realized that the parallelity is a necessary condition for angles similarity. The following fragment from the classroom discussion and students' written works support our claims.

[24]Researcher: *"How about the angles in the letter Z? I observed that all of you managed to indicate the angles in Z have the same*

magnitude, can you explain how do you know the angles are in the same magnitude?"

[25]Students: "The angles aren't right-angle."(Speak confidentially)

[26]Researcher: "Think about it for a moment!"

Few minutes later, a student came up with an opinion.

[27]Della: "This line with this line are parallel to each other, so the angles must be the same." (Mentioning the necessary condition for angles similarity)

[28]Researcher: "Can you tell us more about it!"

[29]Della: "emmm…" (Unable to provide more explanations)

After giving the students with reasonable amount of time to think the researcher realized that the students accepted the situation as an obvious fact. At this moment the students were only able to infer the similarity between angles that formed by a straight line that falling across two parallel lines, but couldn't produce explanation for this fact.

Based on the actual teaching and learning activities in this lesson, we can conclude that the designed activities in this lesson have potential impacts toward students' understanding about angles similarity. It may become superficial if we claim that the proposed learning activities had made students mastered the concept of angle similarity. The focus of this lesson is only to allow the students to infer the similarity between angles that formed by a straight line falling across two parallel lines. Further justifications of students' conjectures about the concept of angle similarity that occurred during this lesson is promoted in the next lesson.

5.3.4 Lesson 3: Letters on the tiled floor models

Mathematical explorations on several tiled floors models were chosen in order to allow the students to justify their conjectures about angles similarity that they acquired from the second lesson. In general there wasn't any big difference in how students reacted to the given tasks. In this part we focus solely on two core activities of this lesson. The first core activity was to compare the letters from matchsticks with the letters on a tiled floor model, in which the letters formed by parallel line segments. In the comparison process the students overlooked the situations. They only compared the shape and the size of the letters in both situations. Therefore, in order to lead the students to arrive at the intended learning goal the researcher performed a whole group discussion. The goal is to make students realize that in the presented tiled floor situation they can perform exact calculations to calculate the angle magnitude.

[1]Researcher:	*"One of your friends claimed that the letter F in both situations the angles are the same. Do you agree with that?" (Drawing the letters)*

[2]Students:	*"No!"*
[3]Researcher:	*"Can one of you explain it?"*
[4]Della:	*"All the angles here are 90° (angle c), but in this one the angles are roughly 120° (angle a) and 70° (angle b)"*
[5]Researcher:	*"Okay, Della estimated that the angles in the letter F on the tiled floor are 120° and 70°. Can you calculate the exact value of those angles? think about it for a moment!"*
[6]Della:	*"Ahhh…60°" (Seemed very enthusiastic)*
[7]Researcher:	*"Della could you explain to us how you calculated it?"*
[8]Della:	*"The shape of the tiles is equilateral triangle, in which the angles are in the same size. (Explaining it to the researcher)*
[9]Researcher:	*"Please explain it to your friends!"*
[10]Della:	*(Starting her explanation all over again) "The shape of the tiles is equilateral triangle, in which the angles are 60°. So it is clear that this angle (Pointing to the angle a) is 120°."*
[11]Researcher:	*"Good! Can one of you re-explain why this angle (angle a) is 120°? (Imam raised his hand)*
[12]Imam:	*"Because this angle (Pointing to the angle a) consists of two vertices of the triangles, and each vertex is 60°, then the total will be 120°." (Imam utilized the uniformity of the tiles on the floor model)*
[13]Researcher:	*"Do you understand what does he mean?" (Asking other students)*
[14]Students:	*"Yes!"*

In the discussion, the researcher asked the students to calculate angles magnitude in the letter from matchsticks which doesn't have any right-angle on them. The students realized that they could not perform exact calculations in the proposed situation and concluded that the tiled floor model outweigh the matchsticks situation in term of certainty of angles magnitude.

The second core activity was about reinventing the relation between parallel-transversal lines with the angles similarity. Based on the actual teaching and learning activity, the students recognized the necessary condition for angle similarity (i.e. a pair of parallel line). All of them claimed three facts about the necessary condition for angle similarity in parallel-transversal situations; there must be two parallel line segments, a non-parallel line segment must intersect two parallel line segments in two points, and the angles must be in the same magnitude. These claims are similar with students' claims that we can find in the first and second teaching experiments. However, in this particular case the students inferred angle similarity based on the observations on the corresponding angles in the letter Z that vary in shapes, but always have two parallel lines segments on each of them. Therefore, the generalization of this knowledge was not yet achieved in this lesson. In the next lessons, we promote students progressive generalization of this knowledge.

5.3.5 Lesson 4: Reason about the angles magnitudes on the tiled floor models

Throughout this lesson we expected the students to reason about the magnitude of angles on the tiled floor models by utilizing the uniformity of the tiles. The reasoning activities meant to help the students to generalize their current knowledge about angle magnitude. The first two tasks were designed to allow the students to predict the angles magnitude on each corner of a tile. The students reacted to the given task as we expected. They indicated the angles in each floor model by utilizing the uniformity of the tiles and explained that the amount of opening between two lines help them to decide the similar angles. Using the information that they got from the two tasks, the students were able to deduce the fact that the sum of every angle in each meeting point is 360°.

The core activities in this lesson include the two last instructions in the worksheet. The first activity designed to enable the students to calculate the magnitude of angles on each corner of a tile using the concept of similarity. The situation allows the students to perform exact calculations due to the certainty in the presented angle magnitudes. The second activity designed to allow the students to make progressive generalization of the concept of angle similarity. The presented situation has some degree of uncertainty in the presented angle magnitudes. The situation begs the students to make assumptions for one or two angle magnitudes.

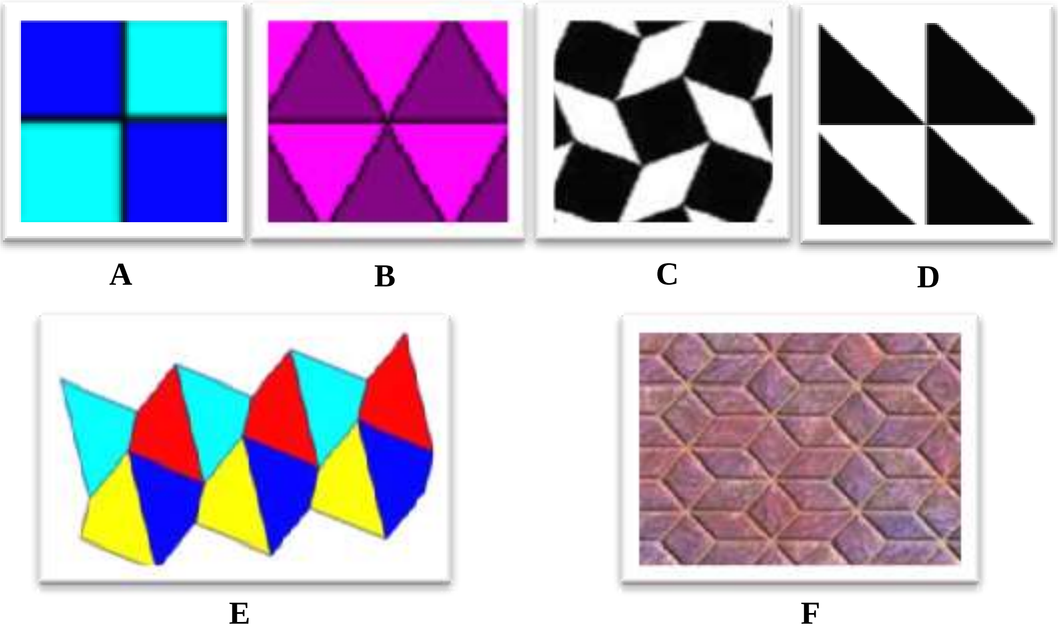


Figure 5.51. The tiled floor models.

In the first core activity, we asked the students to find the angle magnitudes for each vertex of the tiles in the given tiled floor models. The students were able to determine the magnitude of each individual angle. Analyzing students' written works and their verbal explanations revealed their strategy to solve the problem. Students found the angles magnitudes for each vertex of tiles in floor models A, B, and F almost immediately. It wasn't surprising us, because the angles magnitudes in the presented tiled floor models were familiar for the students (i.e. 45°, 60°, and 90°). They also tried to confirm whether their answers were right or wrong by checking whether the total of every angle in each tiled floor model added up to 360° (see figure 5.52). The following fragment from a group discussion depicts students' solution strategy.

[1]Della: *"Look at the angles in floor B! All the angle is 60° right?!" (Asking her friends to justify her claim)*

[2]Aulia: *"One, two, three,…,six. Six of them." (Counting the number of the angles in a meeting point of the tiles)*

[3]Della: *"120, 180, 180 plus 60…(Tried to perform the calculation in her head) may be the sum will be 360°." (She wrote a series of 60° to justify her choice)*

[4]Dina: *"Make it simple, just multiply 60° by 6!" (Offer a way to write their finding)*

[5]Della: *"It is clearer if I do it this way." (Continue writing the series)*

[6]Dina: *"60° times 6 equal to 360°, so the total would be 360°!"*

[7]Della: *(Writing down 60°+60°+60°+60°+60°+60°=360° and also write 60° ×6=360° below her series to satisfy Dina)*

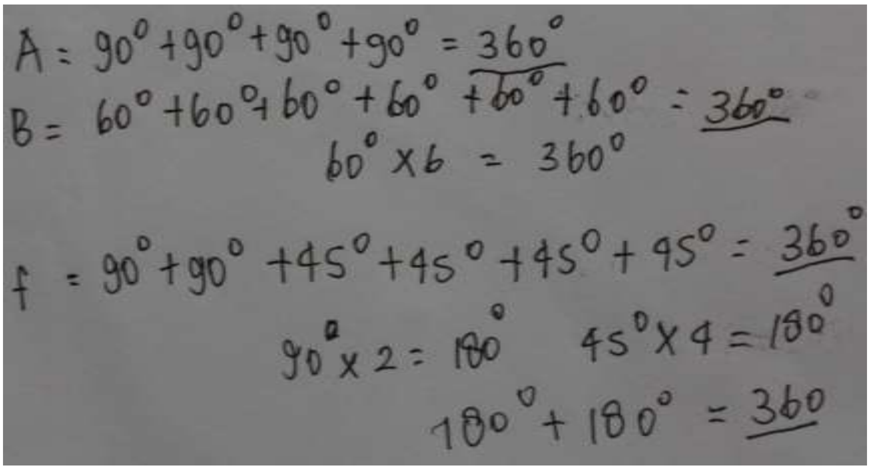


Figure 5.52. Students checked whether the total of every angle in each tiled floor model added up to 360°.

The second core activity proved to be a fruitful activity to promote students to generalize the concept of angle similarity by making assumptions and predictions for the angles magnitudes. The uncertainty in some of the presented angles magnitude in floor models C, D, and E forced the students to make assumptions for one or two angles magnitudes. In the actual teaching and learning activity, we found that the students treated the assumed angle magnitude as an independent variable, and the rest of the unknown angles magnitudes as the dependent variables. The students deduced the values of the dependent variables from the independent variable by employing the concept of angle similarity. They also checked their answers like what they did in the previous problem. The following fragment from the classroom discussion explains how students made their own assumptions and deduced the unknown angles magnitude from the assumed angle magnitude.

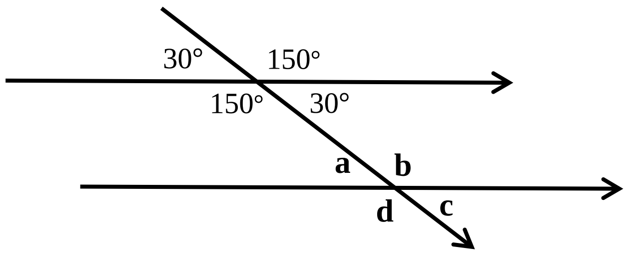
[8]Researcher:	*"Let us calculate the size of each angle in floor C!"*
[9]Della:	*(Showing a series 90° +90° +120° +60° =360°)*
[10]Researcher:	*"Hmm…I want to ask you a question, how did you know one of the angles is 120°?" (Posted a question to check students understanding)*
[11]Della:	*"We know there are two right-angles here (Pointing to the two right-angles in the floor model) the sum of both angles is 180°. This angle (Pointing to the obtuse angle on the floor model) is more than 90° but less than 180°, we predicted the size would be 120°."*
[12]Researcher:	*"How is about the 60°?"*
[13]Della:	*"Because 90+90 is 180, and 180+120 is 300, that 60° less than the 360°."*

Due to the dependency of the solution to the assumption, each group of the students has different opinion in all three situations. In the further discussion the researcher led the students to realize the uncertainty in the presented situations by comparing each of their solutions. Based on the descriptions above, we conclude that the proposed activities in this lesson have potency to support students' understanding about angle magnitude and angle similarity. In the next lesson, we will foster students' generalization of this knowledge by giving them problems about angle magnitude in which their proficiency on applying the concept of angle similarity is needed.

5.3.6 Lesson 5: Angle related problems

We presented four problems in order to investigate students' comprehension about angle concepts that they had learnt so far in this teaching experiment. The designed problems require students to apply the properties of letters angles (F, Z, and X-angles) or any compatible concept of angle and its magnitude. Before the four problems were presented, we asked the students to investigate the angles on railways intersections. The students were asked to sketch the top views of the given railways pictures and carried out some simple analysis to find the relations between the angles. In the actual teaching experiment, the students were able to find the values of the angles in one of the intersection point of their sketches. They applied the concepts of straight-angle, full-angle, and vertical angles to deduce the similarities. However, the students encountered difficultly to explain about the values of the angles for another intersection point that they had stated as the exact copy of the previous intersection point. The following fragment from the classroom discourse captures students' idea about the situation.

[1]Researcher: *(Draw one of the students' works and posting some questions) "How big is the angle a?"*



[2]Aulia: *"30°, that's the same with this one and this one!" (Pointing to the 30° angles in the upper intersection point)*

[3]Researcher: *"How did you know angle a is also 30°?"*

[4]Students: *(Discussing with their neighbor about the possibility to apply the concept of vertical angles)*

[5]Della: *"May be because the angles are straight angle, I don't know."*

[6]Dhina: *"Alternate angles, I think!." (Recalling her knowledge that she had learnt in her classroom previous weeks ago)*

[7]Researcher: *"Okay, let me put it in this way. Do you remember about the similarity of angles in some letters that we had learnt in previous lessons?" (Tried to lead the students to apply the properties of letters angles)*

[8]Aulia: *"X and Z."*

[9]Researcher: *"In this context which letter that you can see?"*

[10]Della: *"Z." (Hesitantly)*

[11]Researcher: *"Okay, Z. So?"*

[12]Della: *"So, the angles must be the same."*

[13]Researcher: *"Now, how is about the angle d?"*

[14]Aulia: *"That's must be 130°."*

[15]Researcher: *"Can you explain why!"*

[16]Aulia: *"Because it looks like F."*

After the discussion, the students continued to work with the four core problems in this final lesson. Students' reaction to the first problem indicated that they already acquired the knowledge about angle similarity. The problem requires them to describe the relation between the angles on a picture of two groups of parallel lines that cross each other (see figure 5.53). The students were able to give specific (numerical estimations) and general description about the angles magnitude in the presented situation.

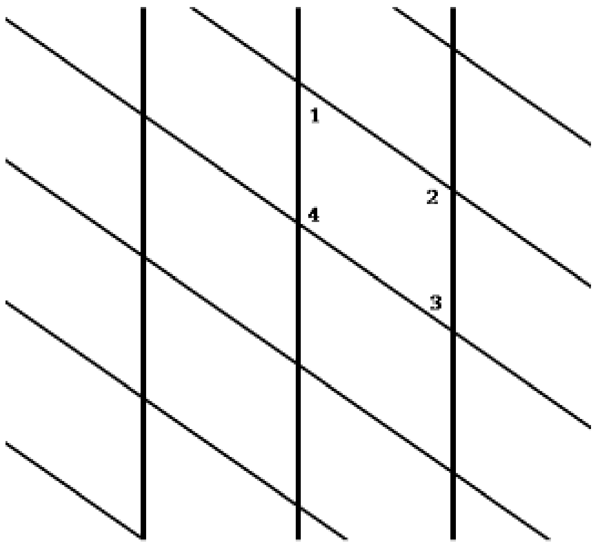


Figure 5.53. Picture from the first problem.

The second problem includes five sub-numerical-questions related to the first problem. The sub-questions were designed to extend students' understanding about the properties and relations of the angles in parallel-transversal situation. In general, the students performed well during the actual teaching experiment. However, we found that the students still lack of confidence when encountered a distraction in the sub-question. The following fragment from a group discussion depicts how students reacted to a distraction in the sub-questions.

[17]Della: *(Students started to work after they wrote the assumed values of the angles) "Angle 1 plus angle 4 is…em…(Checking the assumed values in their list) 60 plus 120, em that is…180°."*
[18]Aulia: *(Continued with the second sub-question) "Angle 3 plus angle 4 is …em…(Checking the assumed values in their list) that is also 180°, how come?"*
[19]Dhina: *"We already used the 180°, now there is no option anymore."*
They checked all the options to find an option that was equal to the 180°.
[20]Della: *"Just skip it for a moment! Let us solve the next questions!*
After few moments, they got back to the second sub-question.
[21]Della: *"The only option now is 270°. Now what?"*
[22]Aulia: *"Fine…just write 270° as the answer!" (Chose the wrong option even they knew the answer)*

The following fragment shows how a classroom discussion can help the students to justify their doubt and nurture their confidence.

[23]Researcher: *"Della tells us your answer for the second question!"*
[24]Della: *"Angle 3 plus angle 4 (Read the question and hesitantly gave her explanation), angle 3 is 60°, and angle 4 is 120°. So the answer is 180°." (The answer was different from her previous answer)*
[25]Researcher: *"How is about your group Imam?"*
[26]Imam: *"We made a mistake, our answer was 270°."*
[27]Della: *"But our group also made the same mistake, we thought 270° was the right answer. It was because we already used the 180° option for the first question and we ran out option."*

Actually, the classroom discussion above also helped the students to realize the relations between the angles in a parallel-transversal situation. The proposed numerical problems allowed the students to explore the problems that exemplify the relations.

The third question is a 'what-if question' where the students have to determine the unknown angle on the given triangle floor model. All of the students easily deduced the solution from the fact that the sum of three angles is 180°. It wasn't surprising us, because the problem that students should solve only has one variable, that can be reformulated as $a + 110° = 180°$. The fourth question can be reformulated as $a + 50° + c = 180°$. Since the problem has two unknown variables the further discussion was conducted to make students accept the fact that the problem doesn't have a unique solution. Students tried to make some assumptions based on the fact that the sum of both angles was 130°. However, because the unknown angles almost looked the same in size they decided to assume that the unknown angles were the same.

5.3.7 Post-assessment

The students took a 20-minute posttest after went into the entire lesson sequence. The posttest items were designed to assess students' current knowledge about angle and its magnitude. The outcome of the test showed a significant increase in students' test scores (table 5.2). It justifies students development in comprehend the concepts of angle and its magnitude.

Table 5.2. Pre and posttest result from the third teaching experiment

No	Name	Pretest Score	Posttest Score
1	Aulia Ramadhani (Aulia)	5.56	9.4
2	Della Puspa Anggraini (Della	6.11	9.68
3	Dhina Aulia (Dhina)	5.56	8.75
4	Imam Kurniawan (Imam)	2.22	8.75
5	Muhammad Chandra (Muhammad)	1.67	7.18
6	Ramadhani Saputra (Dhani)	5	7.18
	M (SD)	4.35 (1.74)	8.49 (0.97)

The clarification of students' development can be deduced from the further analysis on students' written work and video registrations of the interview. Based on the analysis, we noted several important remarks as follow:

a. Frame of reference about angle

Before the students went into the lessons sequence, we know it wasn't clear what frame of reference about angle that students embrace. However, after following this teaching experiment they tend to see the angle as the difference of direction between two lines or as the amount of turn between two lines. Defining angle in these ways had removed students' tendency to see the length of arms affects the angle magnitude. As a result, the students encountered no difficulty to distinguish the angles based on their magnitudes. In addition to that, we found that the students used a word ‘opening’ as a synonym for the angle magnitude.

b. Symbol to indicate the angles

As we know from the pretest outcome and in the early stages of the teaching experiment, the students used the arc (∡) symbol to indicate an angle. At that early stage, they perceived the symbol as an indication of the angle magnitude that attaches to it. In other words, bigger arc means bigger angle. However, after the lessons sequence they perceived the symbol as an indication symbol and has nothing to do with the angle magnitude attaches to it.

c. The sense of angle magnitude

The lessons sequence had promoted students’ sense about angle magnitude. It is evidence that the students had grasped the important attributes of angle in order to help them to compare the angles based on their magnitudes. For instances, students’ answers to a test item that asked them to indicate the smallest and the biggest angles in a given figure, showed that the students were able to distinguish the angles based on their magnitude. We also observed that, the students were able to sort several angles figures based on their magnitudes without any hesitation. It suggests that the design had supported students learning about angle magnitude.

d. Knowledge about right-angle and straight-angle

In the interview session, the students could recognize the tilted right-angle figures as the valid representations of right-angle. Students claimed that, no matter what the size and the orientation are, as long as the arms of the angle are perpendicular to each other the figure must represent a right-angle figure. It is clear that their judgment wasn’t affected by the size and the orientation of the given figures anymore. In one of the items test, we gave the students a numerical problem that required them to apply the straight angle concept. The problem asked the students to determine the unknown angle magnitude from an alignment of two angles, in which one of the angle magnitudes was given. Based on their written works and their verbal explanations in the interview session, we found that the students deduced the solution based on the fact that the sum of both angles is 180°.

5.3.8 Conclusion for the third teaching experiment

According to the actual teaching and learning activities throughout this particular teaching experiment, we had observed a positive trend of students' development in learning about angle and its magnitude. The designed activities that employed the selected angle situations proved to be a fruitful way to deliver the concept of angle and it magnitude to the students. Undoubtedly, in mathematics, a complete understanding on a definition of a mathematical object holds a crucial role in the process of knowledge acquisition. In this teaching experiment, we had promoted students comprehension on angle by utilizing everyday life objects that possess the attributes of angle. Before the students went into the whole lesson, most of them didn't have a clear understanding about what an angle was. Their vague understanding led to the several obvious inconsistencies when they performed the instructions that required the implementation of angle definition. It was evidenced that the students had added some angle definitions to their inventory of angle definition after following the first lesson. In the end of this teaching experiment we had asked the students to write down their definitions of angle. Most of the students had added one or two angle definitions to their inventory of angle definition. The designed activities had led the students to define an angle as the difference of direction between two lines or as the amount of turn between two lines.

Understanding what the angle is has become a stepping stone for the students to grasp the concept of angle magnitude and to comprehend the important concepts of angle. We observed that, although the students had learnt about the angle and it magnitude in their classroom few weeks ago, it was obvious that their understanding toward the subject were limited and superficial. Even for a simple problem like deciding whether an angle figure is a right-angle or not, some of the students still failed. They perceived a right-angle as a figure that had a particular shape or orientation. Rotating and resizing a right-angle figure proved to be an effective way to test students' understanding. The activities in the second and the third lessons have helped the students to revise their conceptions about angle magnitude. Investigating the angles in the tiled floor models was a particular activity that responsibled in improving students' understanding about the angle magnitude. The proposed activities have helped the students to rebuild their conceptions about angle magnitude by utilizing the uniformity and similarity of the tiles. The presented situations have created a reasonable condition where the orientation doesn't affect the angle magnitude. For instance, in a squared tiled floor model the students could easily see why the orientation didn't affect the size of a right-angle figure.

We utilized students' understanding of angle magnitude to lead them to comprehend several important concepts of angle. In particular, we are interested in promoting students' learning about angles similarity in a situation where a straight line that falling across two parallel lines. The pretest results have showed that the students could not determine a pair of similar angles or assigned a value to an angle in the parallel-transversal situation. It seemed that, the students were unable to deduce the solutions from the fact that a straight angle is 180°. In the fourth and the fifth lesson, the students showed a positive development in their understanding on the important concepts of angle. They were able to calculate the entire angle in an intersection point of the parallel-transversal situation by employing at least three key concepts (i.e. straight angle, vertical angles, and full angle). In addition to that, by applying the concept of letter angles (i.e. F, X, and Z angles) that they have learnt in the second and third lessons, the students could explained that the angles in another intersection point are similar to the one that they had calculated. Furthermore, we also deepened students' understanding by inviting them to solve several numerical problems that pave the way to the recognition of the relation between angles in a parallel-transversal situation. The students performed well in those numerical problems without encountered any significant difficulty.

CHAPTER 6
CONCLUSION AND SUGGESTION

The central question of this study was how we support 7th graders to comprehend the magnitude of angles through reasoning activities. To answer this question, five sub-research questions were proposed in chapter 2. In five stages, we showed how the designed activities, supported by the selected angle situations, stimulated students to reason about important aspects of angle and its magnitude. After a summary of the results, we discuss limitation of this study and suggestions for further study.

6.1 Conclusion

As we stated before in the end of chapter 2, we attempted to answer the following sub-research questions, in order to help us to answer the central question of this study.

1. How do 7th graders define the angle from the everyday life objects that is strongly related to the angle?
2. How does the alphabets reconstruction activity using wooden matchsticks allow the students to infer the similarity between angles on a straight line that is falling across two parallel lines?
3. How do the gaps patterns between tiles can help the students to advance their idea of similarity between angles on a straight line that is falling across two parallel lines?
4. How does the pattern on the tiled floors help the students to enhance the idea of angles magnitude?
5. How do students apply the acquired knowledge to reason about the magnitude of angles in more general situation?

After we answer these questions, in the next part of this chapter, we draw the conclusions of this study.

6.1.1 Answer to the sub-research question and research question

When students worked with the tasks in the first lesson, they used informal words such as, opening, corner, and degree to describe the angles in the given everyday life pictures. The actual teaching and learning activity showed that students reasoned about the important aspects of angle from the very start of the lesson. Ordering the angles magnitude from everyday life pictures proved to be a fruitful way to enhance students understanding about angle, where at the same time accommodated students' learning about angle magnitude. The students constructed the extreme situations of angle magnitude on the dynamic angle situations (i.e. analog clock and traditional fan) in order to visualize the 0°, 180° and 360° angles. We argued that letting students encounter angles from everyday life objects could stimulate them to explain how an angle is formed and produce their own definitions of angle. When students explained how an angle is formed, they used terms such as; lines, meeting point, and direction. The terms that students used strongly affected their own definitions of angle. The term that students employed suggest a generalization and abstraction of the real situations. The selected angle situations such as, football field corner, roof top, and tiles embody the angle as space between two lines which meet in a point. Letters from matchsticks and railways intersection embody the angle as the difference of direction between two lines. The analog clock and traditional fan resemble the angle as the amount of turn between two lines on a fix point. Most of the students found that the best way to define the angle is as the difference of direction between two lines. Despite students' claim about the 'best' definition of angle, the students have added some angle definitions to their inventory of angle definition.

In the second lesson, they constructed the upper case letters using matchsticks and reasoned about the angles magnitudes in those letters. Again, the word opening appeared when students argued about how they selected the letters that have the smallest and the biggest angles. At first, students didn't take into account the reflex angles of the letters that they chose. In the classroom discussion, students reconsidered their selections and claimed that the biggest and the smallest angles in this context have to be in a same letter. This showed how students grasp the concept of reflex angle by seeing an angle figure as representation of two angles. We claimed that letting students investigate the angles in the letters that have parallel sticks could support their comprehension about angle similarity. In the simple situation where the letters only have right-angles on them (e.g. E, F, H, U, etc.), the students found it easier to explain about the similarity. For the letters that doesn't have the right-angle on them (e.g. N, M, S and Z), most of the students were still able to indicate the similar angles in those letters. Students also tried to show that the corresponding angles in those letters are in the same magnitude, by applying reshaping and comparing the opening strategies. The actual teaching and learning activities in this lesson, suggest that the situation allowed the students to infer the similarity between angles on a parallel-transversal situation.

When students worked with the tasks in the third lesson, they enhanced their quantitative understanding about angle magnitude. Students were able to reason about angle magnitude using numerical approach. In the actual teaching and learning activity, students figured out that the angles in the letters on the tiled floor models offer a certainty of angle magnitude compare with the letters from matchsticks. The skewed letters in the tiled floor models offer a variation of the previous selected angle situation (i.e. upper case letters from matchsticks). We argued that, letting students comparing the angles magnitude from both situations (i.e. matchsticks and tiled floor models) could help students to justify their conjecture about angle similarity in the letters that formed by some parallel line segments. Students reasoned about angle similarity by utilizing the uniformity of the tiles to show that the corresponding angles in some letters are in the same magnitude. After students reasoned about the angle magnitude quantitatively, most of them highlighted three main necessary conditions for the angle similarity such as, there are two parallel line segments, the three line segments are intersect each other in two points, and there are two angles that have the same magnitude as a consequence. In addition to that, the letters on the tiled floor models stimulated two numerical strategies of finding the angles magnitude for each corner of a tile. In the first strategy, students deduced the angle magnitude for each corner by finding an alignment of corners and divide 180° by how many corners in the alignment. The second strategy was a similar strategy. The students selected a meeting point of the tiles and divide 360° with how many corners in the meeting point.

In the fourth lesson, the students applied their numerical strategies to determine the angles magnitudes of various types of tiles. We argued that letting students performed calculations with various tiled floor models could strengthen students' understanding of angle magnitude. Where at the same time provided them with more examples of parallel-transversal situation. The tiled floor models that consist of one type of tiles that uniform (e.g. equilateral triangle, square, and parallelogram) supported the development of an understanding of the corresponding angles. The tiled floor models that consist of different types of tiles helped the students to reason with uncertain situations and to make some assumptions in order to simplify the situation. When the students worked with the uncertain situation, they made an assumption (estimation) for the value of one angle and then solved the simplify situation. The reasoning activity occurred when students checked their solution to the original situation. They argued that the obtained values for each angle should match with the properties of angle magnitude that possessed by the original situation. For instance, the sum of every angle in a meeting point of the tiled floor model should add up to 360°, and the corresponding angles should be in the same magnitude.

In the fifth lesson, students advanced their understanding of angle similarity by reinvented the relations between corresponding angles in a parallel-transversal situation. Solving numerical problems that exemplified the relations between those angles and followed by a classroom discussion that generalized the idea have helped them to reason about the magnitude of angles in more general cases. The students applied the previous concepts such as, vertical angles, straight angle, full angle, and letters angles to explain about similarity between those corresponding angles. We argued that solving the numerical problems about corresponding angles that have two unknown variables are useful to foster a more general understanding toward the relations between corresponding angles. The students have made an assumption for one unknown variable to allow them to simplify the situation, solving the problem and check whether their solution met the properties of angle magnitude that possessed by the original situation. The actual teaching and learning activities in this lesson, suggest that the presented situation allowed the students to generalize the idea of angle similarity in a parallel-transversal situation.

According to the expositions above, we can conclude that, a teaching and learning sequence that employs the selected angle situations can help students understand the definitions of angle, grasp the sense of angle magnitude, and comprehend the important concepts of angles. The results of this study also suggest that the used of contextual problems/situations play a crucial role in the process of knowledge acquisition. Based on our findings, the used of contextual problems/situations in the teaching and learning process provided students with ground for thinking and prepared them for the advancement of knowledge. In addition to that, we also found that students' own ideas in the learning process have an important contribution to the students' development. However, to generate a learning process that based on students' own ideas, extensive discussion and communication during the learning process is needed.

6.2 Suggestion

Although we concluded that, a teaching and learning sequence that employs the selected angle situations can help students to develop the kind of reasoning about angle and its magnitude that is shown in this chapter, it should be understood that the interventions of the researcher in some of the crucial activities of the teaching experiments may interfere with students' actual learning process. As we know, the teacher that involved in this study has less time to study the design before she performed the teaching experiment (second cycle). Therefore, for the teachers that have interest in applying this design in their classroom, we suggest to study the teacher guide and student worksheet thoroughly.

It might because of the time limitation for the teacher to study the design. She reported that the presented problems in the design were too difficult for her students. She also found it difficult to orchestrate the classroom discussions, especially a discussion that discuss about a problem that has no unique solution. Therefore, another question for further research is how we can help the teacher to successfully teach this topic.

We noticed that, when the students justified the similarity between angles magnitudes in a parallel-transversal situation, their reasoning strategies were unique for each teaching experiment. For examples, the students in the first teaching experiment reshaped the letter Z into a parallelogram, and the students in the third teaching experiment measured the amount of opening to justify the same thing. The strategy that students employed in the first teaching experiment is considered as a better strategy. However, when we tried to encourage the students from the second and the third teaching experiments to use the reshaping strategy, we found that the reasoning process that follow after the reshaping process didn't automatically emerge. This study doesn't intent to make the students follow a certain path in their reasoning activity. Therefore, it is up to the teacher to use the most appropriate heuristics for allowing the students to learn from their own experiences rather than by telling them.

Classroom culture that doesn't compatible with the design is another limitation of this study. The subjects of this study have used to the traditional learning environment. For instances, the students not used to express their opinions, afraid to make mistakes, tend to work individually, and avoid any argumentation. Since the classroom discussion considered as the core aspect of students' learning in this study, thus the classroom condition had created an unfriendly condition for the implementation of the design. Changing the classroom culture is favorable before the implementation of the design and we are fully aware that the transition process will take time. Therefore, we suggest that before implementing this design the teacher and his/her students should agree to embrace the same belief about the classroom culture.

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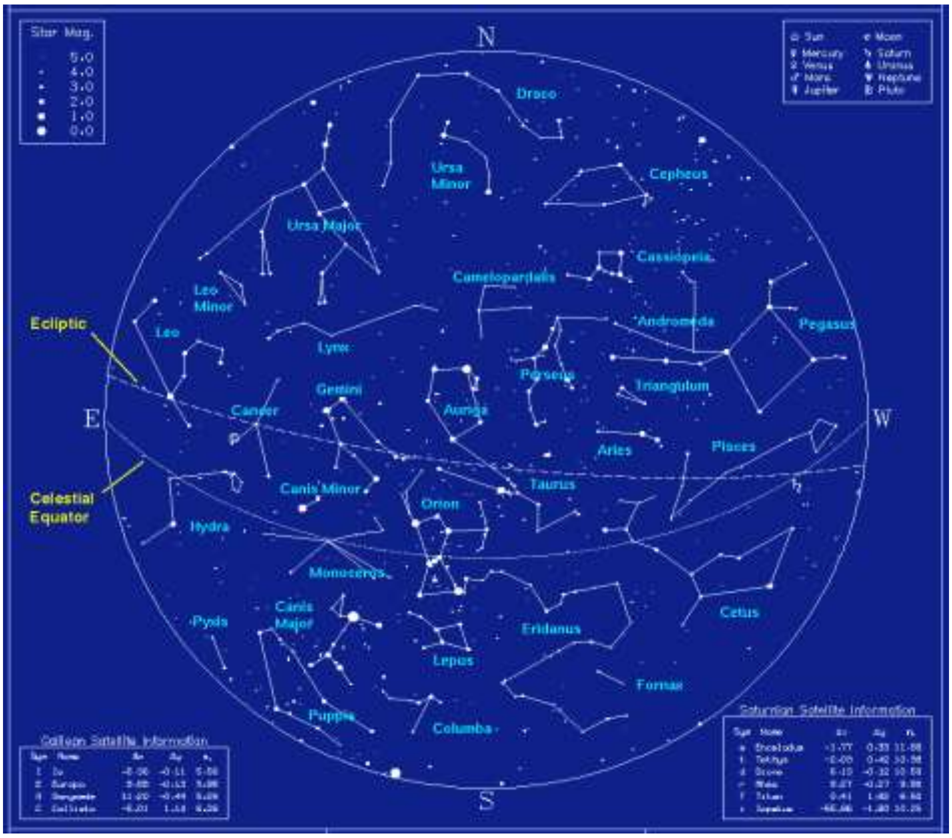
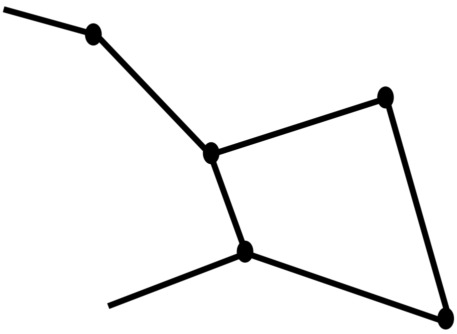
PRETEST

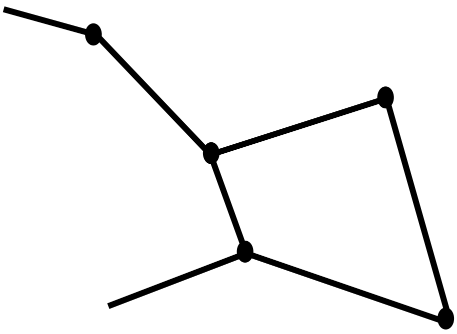
NAME:____

1. The following is a diagram of the constellations of the stars on the night sky.

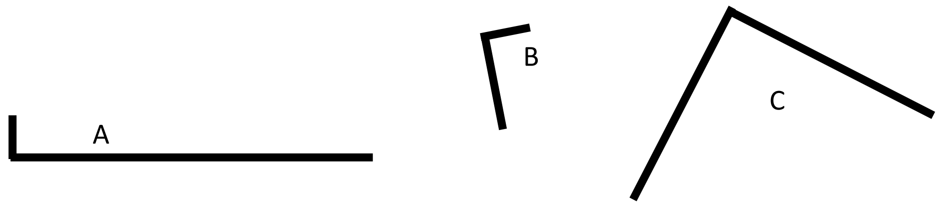


On the constellation of *Lepus* **indicate** the smallest and biggest angles!





2.



From the figures of L shape above Nayla, Rudy, and Shanty state the following statements relate to the size of angle:

Nayla: *"In my opinion figure B showing the smallest angle because it is the smallest L."*

Rudy: *"Wait… I think the figures showing the same size of angle because all of them are right-angle."*

Shanty: *"No Rudy… it is obvious that C is the biggest angle because it will cover the largest area if I draw other lines to make square from it."*

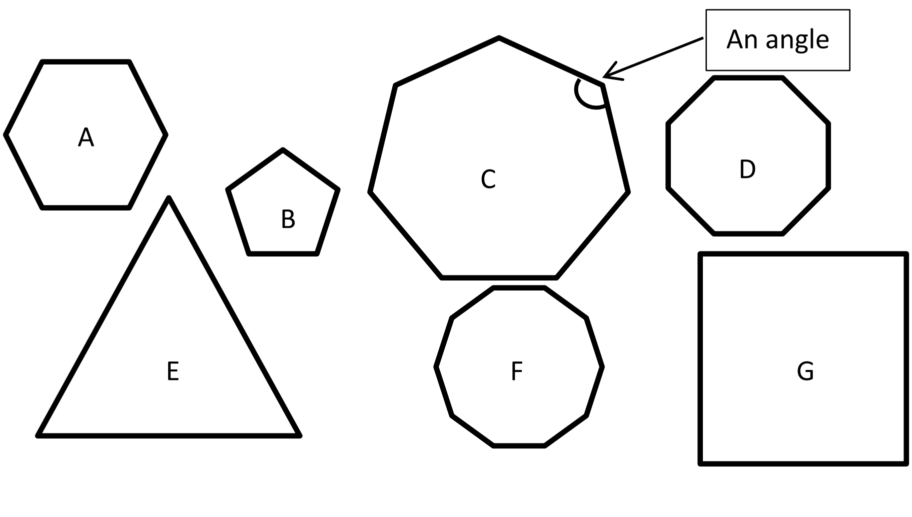
Who do you think offer **the right statement**?

a) Nayla

b) Rudy

c) Shanty

3. **Sort** the size of an angle on the following polygon figures **from the smallest to the biggest**!



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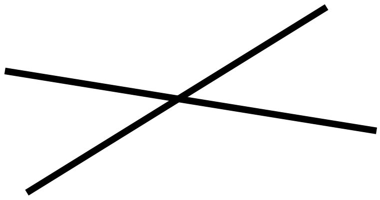
4. If you draw two line segment that intersect each other in the middle. **How many angles that you can see**?



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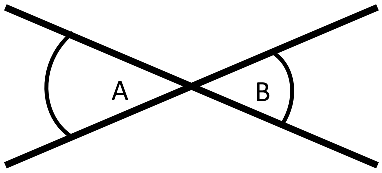
5. Look carefully the following angles!



What do you know about the **size** of angles A and B in the figure above? **Write down** your reason!

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6. The teacher asked Nayla, Rudy, and Shanty on what they know about the angle. Each of them replied as follow:

Nayla: *"Angle is the space between two lines that intersect in a point."*

Rudy: *"Well… I think angle is formed when we have two lines with different directions."*

Shanty: *"Hmm… in my opinion angle is the amount of turn between two lines."*

Who do you think gave the **right explanation** about angle?

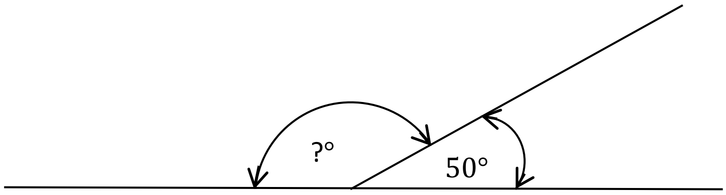
a) Nayla

b) Rudy

c) Shanty

d) Nayla, Rudy, and Shanty

7. Look at the following figure!



Andy measure one of the angle using a protractor and he read 50 degrees on the protractor. Without using a protractor can you **determine the unknown angle**? How do you do that?

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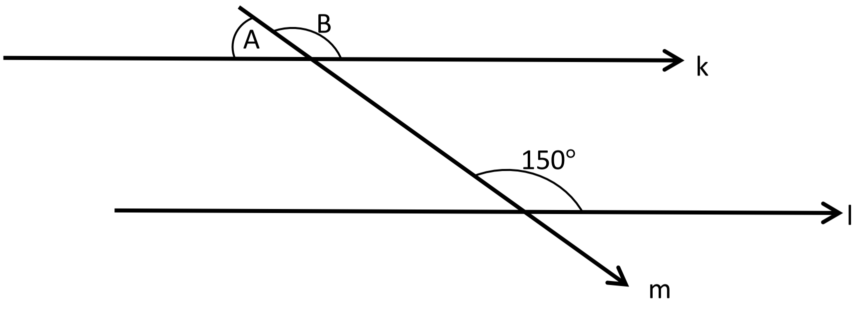
8. Lines k and l are parallel to each other. A line m cuts lines k and l in two points. Can you **calculate** the magnitude of angles A and B?



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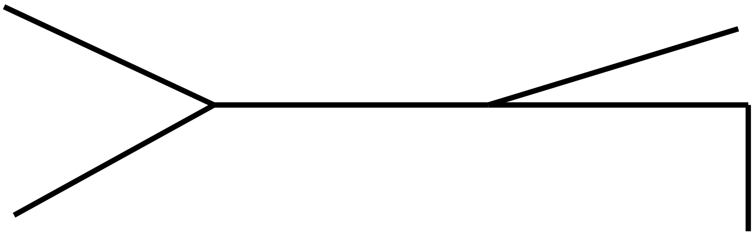
POSTTEST

NAME:____

1. From the figure below, mark the smallest and the largest angle!



2. Observe the following figures:

Which from the following statement is true?

a) Every figure consists of two angles; inside and outside the vertex.

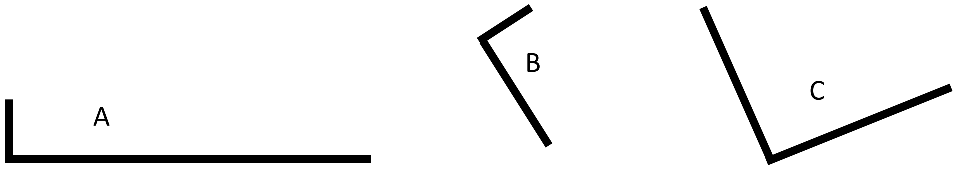
b) Figure B have the smallest angle.

c) Both the smallest and the largest angles can be found in figure A.

d) Statements (a) and (b) are true.

e) Statements (a) and (c) are true.

3. The following are the set of geometric figures.



What do you know about the size of angles in these figures?

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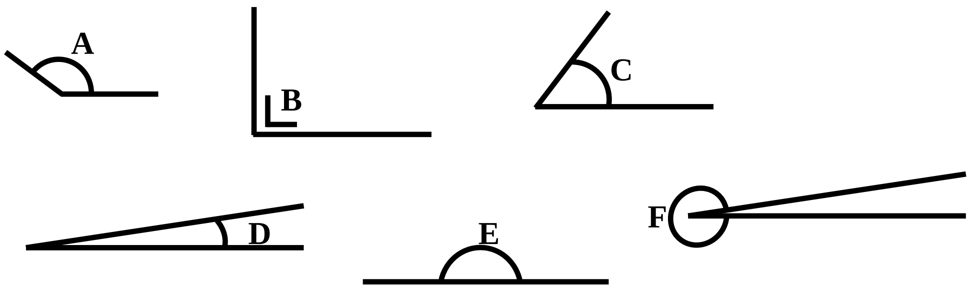
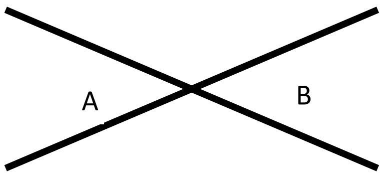
4. Sort the size of the indicate angles on the following figures from the smallest to the biggest!



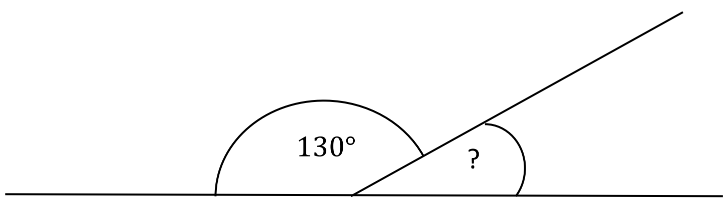
5. Look carefully the following angles!



How many angles that you can see?

6. Explain what do you know about the size of angles A and B in the figure 5?

7. Look at the following figure!



Andy measure one of the angle using a protractor and he read 130 degrees on the protractor. Without using a protractor can you determine the unknown angle? How do you do that?

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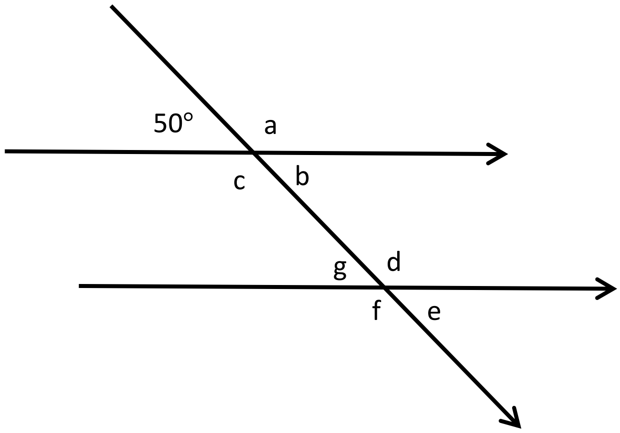
8. Complete the values of the indicate angles on the following figure!



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Pretest and posttest scoring rubric

Pretest

Item	**Score**	**Solutions**
1	4	Clearly indicate a smallest acute angle and its reflex angle
	3	Indicate two angles and give clear distinctions between small and big
	2	Indicate two angles
	1	Give many marks without adequate explanation/indication
	0	Without answer
2	4	Option B
	0	Options A, C, or D
3	4	E, G, B, A, C, D, and F
	3	Make a pair of mistake
	2	Make two pair mistakes
	1	Make more than two pair of mistakes
	0	Without answer
4	4	Sort the angles magnitude by using acute, right-angle, and obtuse as benchmarks
	3	Sort the angles magnitude by counting the number of the vertices in each figure
	2	Sort the angles magnitude by using the sharpness of each vertex
	1	Sort the angles magnitude by using the area of each figure
	0	Without answer
5	4	More than 4 angles
	3	4 angles
	2	3 angles
	1	2 angles
	0	Without answer
6	4	A and B is equal with adequate explanation
	3	A and B is acute angles and give an impression that suggest both angles are the same
	2	Angle A and B is acute angles without explanation
	1	Wrong answer
	0	Without answer
7	4	D
	3	C
	2	B
	1	A
	0	Without answer
8	4	The answer is 130° and provide adequate explanation for the calculation
	3	Right answer but without any explanation
	2	Right answer but wrong explanation
	1	Wrong answer
	0	Without answer
9	4	A=30° and B=150° and provide adequate explanation for the calculation
	3	Right answer but without any explanation
	2	Right answer but wrong explanation
	1	Wrong answer
	0	Without answer

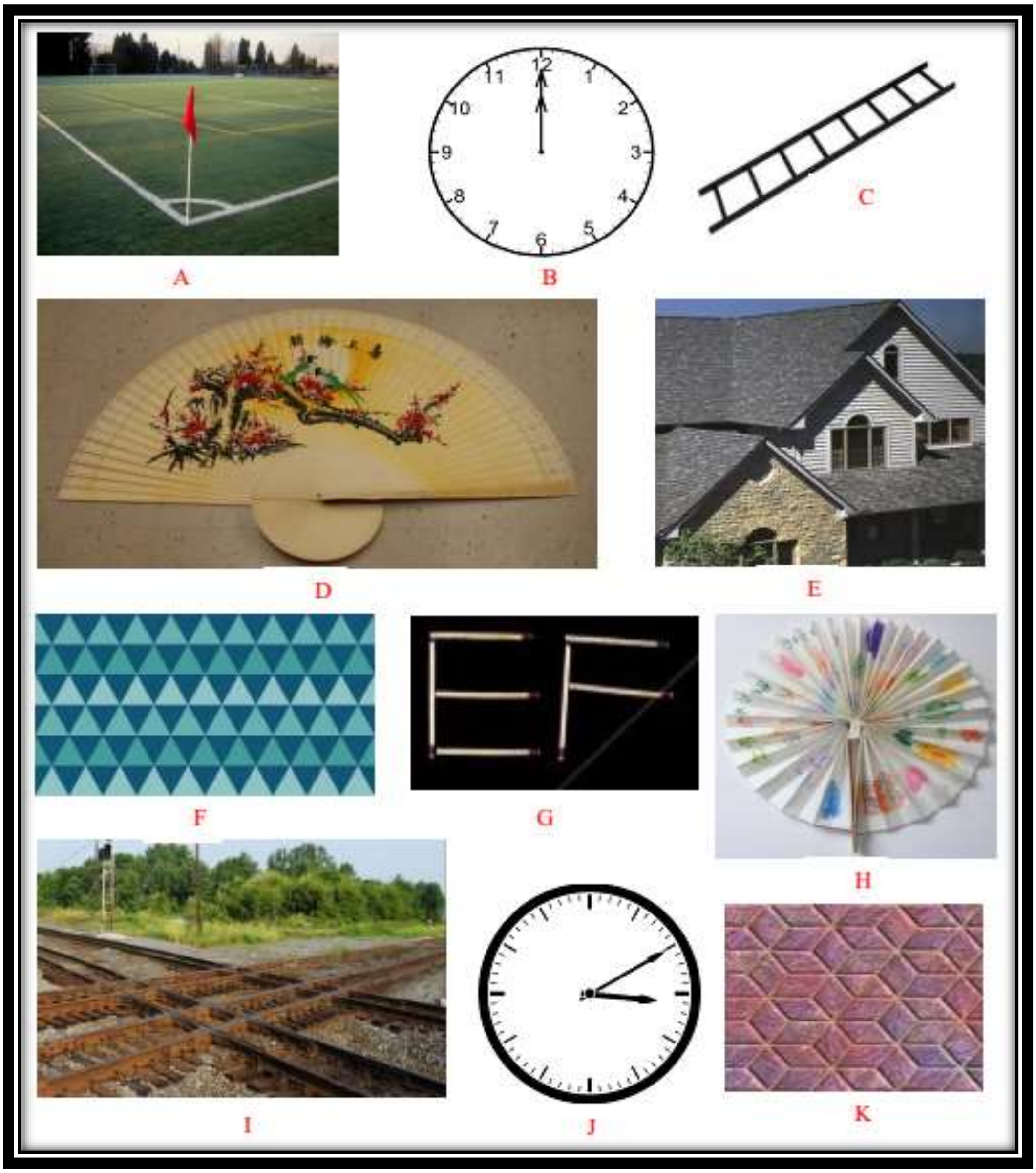
Posttest

Item	Score	Solutions
1	4	Clearly indicate a smallest acute angle and its reflex angle
	3	Indicate two angles and give clear distinctions between small and big
	2	Indicate two angles
	1	Give marks without adequate explanation/indication
	0	Without answer
2	4	Option E
	2	Options A or C
	0	Options B or D or give no answer
3	4	All three figure are right-angle so they are in the same size
	3	A is a right-angle but B and C aren't right-angle
	2	All three figure are right-angle but C is the larger one
	1	B has the smallest angle
	0	Without answer
4	4	D, C, B, A, E, and F
	3	Make a pair of mistake
	2	Make two pair mistakes
	1	Make more than two pair of mistakes
	0	Without answer
5	4	More than 4 angles
	3	4 angles
	2	3 angles
	1	2 angles
	0	Without answer
6	4	Angle A and B is equal with adequate explanation
	3	Angle A and B is acute angles with adequate explanation
	2	Angle A and B is acute angles without explanation
	1	Wrong answer
	0	Without answer
7	4	The answer is 50 and provide adequate explanation
	3	Right answer but without any explanation
	2	Right answer but wrong explanation
	1	Wrong answer
	0	Without answer
8	4	$a=c=d=f=130$ degrees, and $b=e=g=50$ degrees
	3	$a=c=d=f\neq130$ degrees, and $b=e=g=50$ degrees
	2	$a=c\neq d=f\neq130$ degrees, and $b=e=g=50$ degrees
	1	$a\neq c\neq d\neq f\neq130$ degrees, and $b=e=g=50$ degrees
	0	Without answer

WORKSHEET 1

The tasks (In group of four):

1. Indicate **an angle** on each object!



2. Make an ascending order for the angle magnitude that you have chosen! Make a poster of it and display it in the classroom!

3. Observe the posters from the other groups! What makes your poster different from the other posters relate to the order of the angle magnitude and how it can be improve!

The questions:

1. Which objects on your poster that can change the size of their angle?

2. Please draw two situations where an object in question 1 forming the biggest angle and the smallest angle!

Discussion: Now compare your work with the other groups' works! Is it possible to make another angle that smaller or bigger compare with your angles in question 2?

3. How is an angle formed?

4. Therefore, an angle is…

WORKSHEET 2

The tasks (In group of four):

1. Reconstruct the following upper case letters using wooden sticks! Each member of the group selects a set of the letters to be reconstructed. (Remember do not break the sticks!)

A, B, C, D, E, F	G, H, I, J, K, L	M, N, O, P, Q, R, S

T, U, V, W, X, Y, Z

The questions:

1. Which letter that has the smallest angle?

2. Which letter that has the biggest angle?

3. Observe the orientation of the sticks! List all the letters that formed by parallel sticks!

4. Observe the size of the angles on the question 3! Mark the angles that have the same size! Note at least three things!

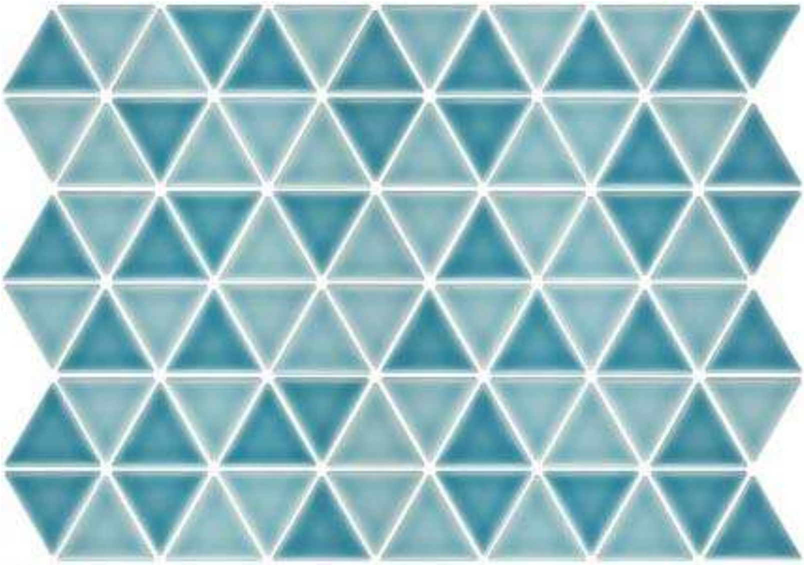
Classroom discussion:

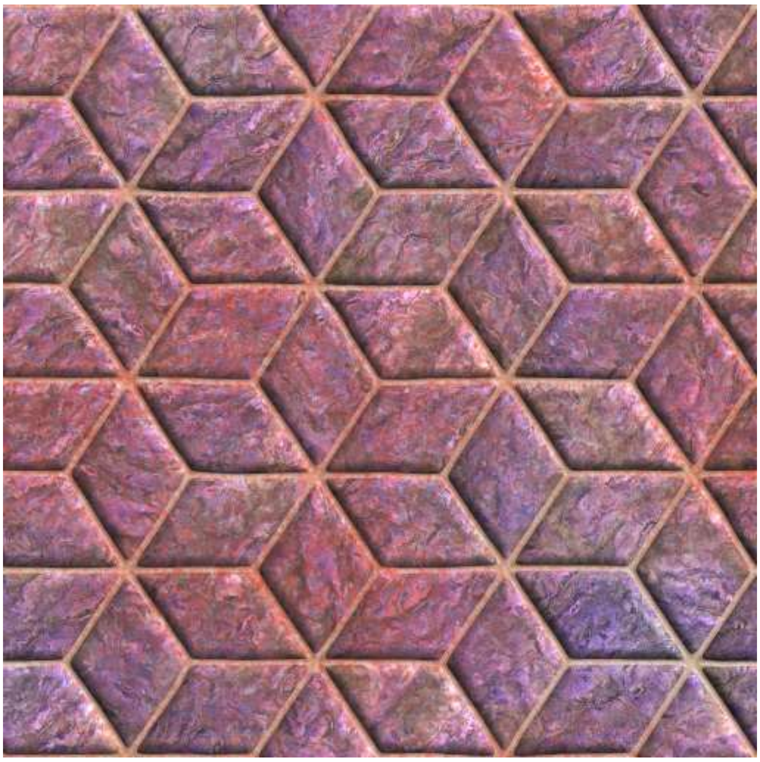
How about the letters that don’t have parallel sticks? Can you say something about it? (Remember to write down the important things that you get from the discussion).

WORKSHEET 3

The situation

Ana had decided to select two kinds of tiles to be used in her house, in the kitchen and in the bedroom. One day when she was in the kitchen, she figured out that with the lines on the tiles in the kitchen, she can form her name.





The tasks (In group of four):

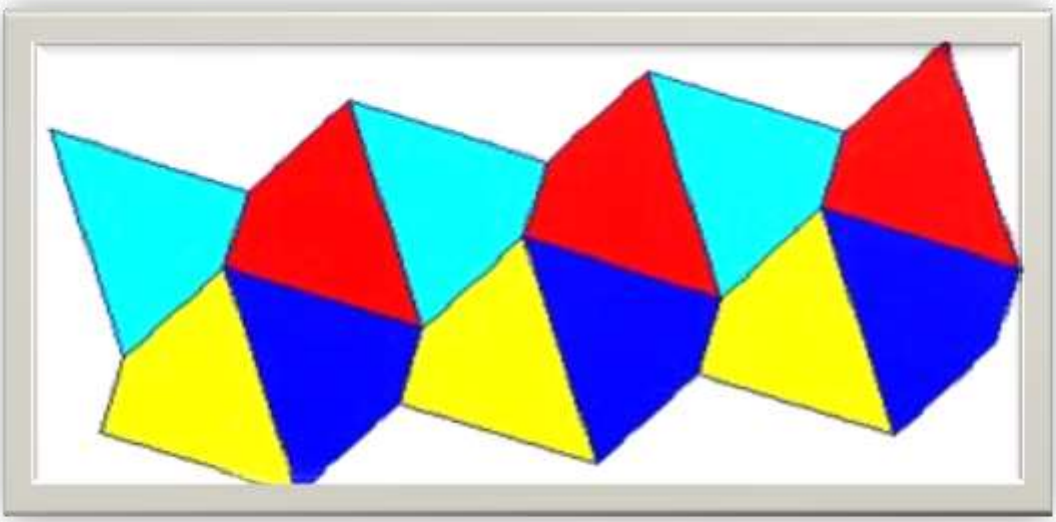
1. Which one from the displayed floors is the kitchen floor? Can you show it?

2. Draw another letters that you can find on the kitchen floor (keep the drawing as precise as possible with what you find on that floor)!

3. Look back at your letters reconstruction in the matchsticks activity! Compare the letters that have parallel sticks on them in that situation with the same letters in kitchen floor!

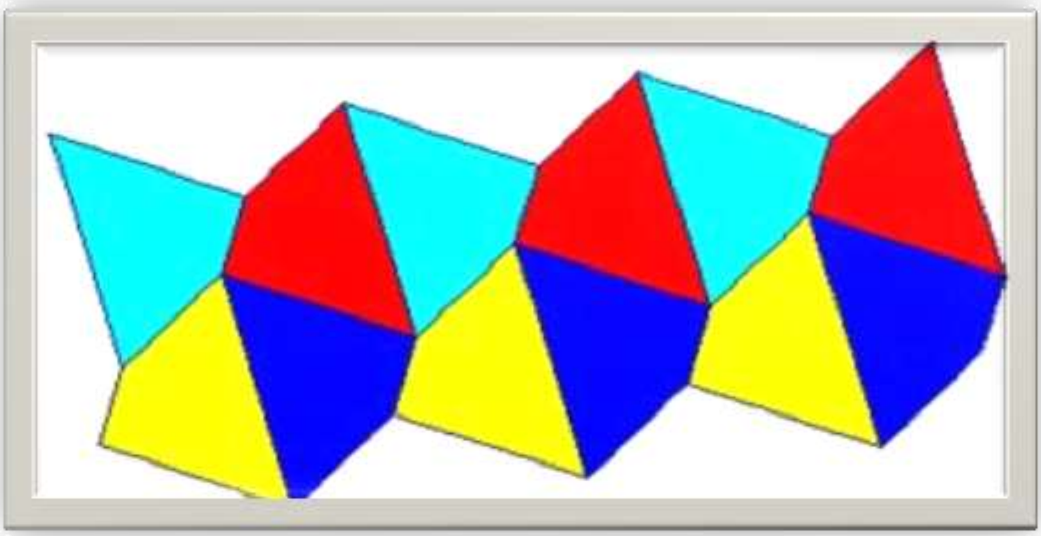
The questions:

On the following tiled floor you barely find a letter. However, you still can find angles and lines on it.



1. Indicate the angles that have the same size with the same mark!

2. Highlight as many as parallel line segments!



3. Are there some line segments that perpendicular to each other? Give a brief explanation why do you think so?

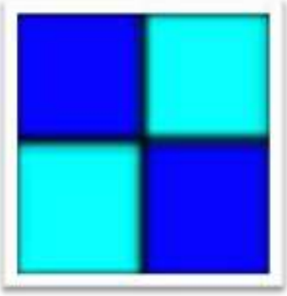
4. On the figure, observe a Z like figure that formed by a pair of parallel line segments that connected by another line segment! Can you tell something about the relations between parallel lines and the size of angles that attach to them? Note at least three things!

The pictures were taken from:

http://theglassfactory.wordpress.com/2011/10/20/more-blue-tiles/
http://www.spiralgraphics.biz/packs/tile/?25

WORKSHEET 4

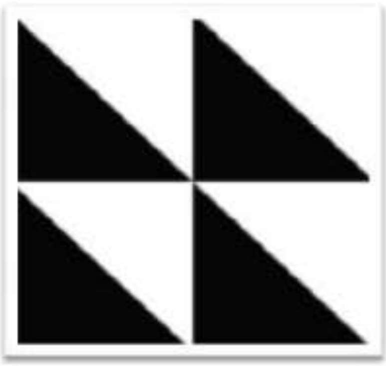
The situation



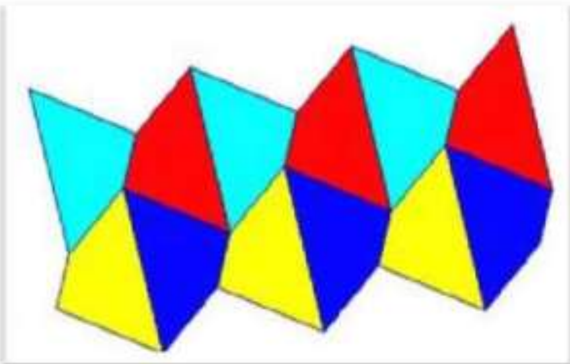
A

B

C



D



E

F

The tasks (In group of four):

1. Observe the pictures of the tiled floors! Indicate the angles that have the same size with the same mark!

2. In each situation, please explain how you know the angles are in the same size!

3. What do you know about the size of the angle on every meeting point of the tiles?

4. Can you give the numerical values for the sizes of each angle on floors A, B, and F? Explain how you determine the sizes!

5. Can you give the numerical values for the sizes of each angle on floors C, D, and E? Explain how you determine the sizes!

Classroom discussion:

Discuses with your friends about their assumptions for the sizes of angles on each tiled floor to compare the results! Remember to write down the important things that you get from the discussion.

WORKSHEET 5

The tasks (In group of four):

1. Observe the following railways intersections!

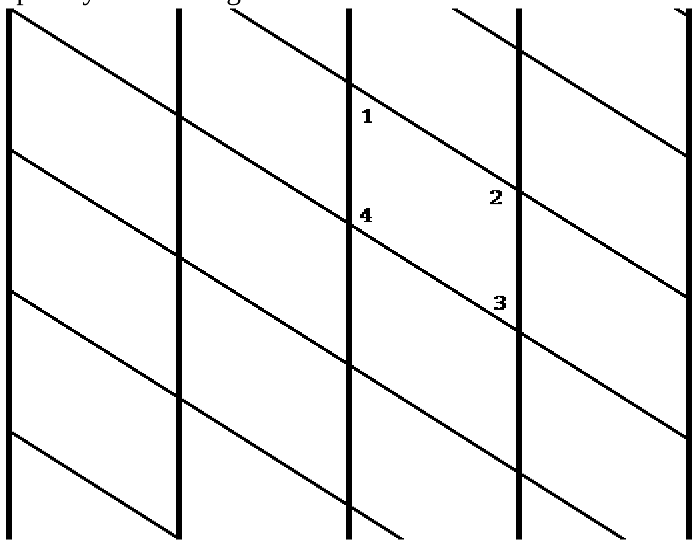
How these railways looks like if you see it from the plane/helicopter? Draw the view in the empty space below!

Classroom discussion:

Do the following activity: draw a different version of the railways intersection, give a numerical value of an angle on it, and dare a friend next to you to fill the unknown values! Do this activity alternately.

The questions:

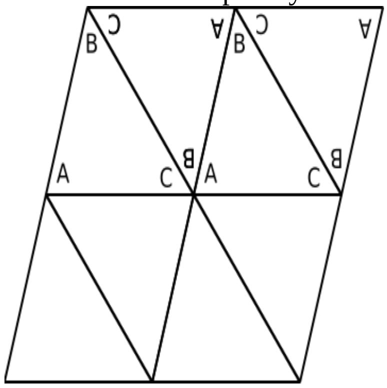
1. Observe the following floor! What can you say about the size of angle 1, 2, 3, and 4? Please explain your thinking!



2. Re-observe the floor in question 1. Match the questions on the left with the appropriate answers on the right!

a. $\angle 1 + \angle 4 = \cdots°$	● $2 \times \angle 2$ (twice the angle 2)
b. $\angle 3 + \angle 4 = \cdots°$	● $360°$
c. $\angle 1 + \angle 3 = \cdots°$	● $2 \times \angle 1$ (twice the angle1)
d. $\angle 2 + \angle 4 = \cdots°$	● $180°$
e. $\angle 1 + \angle 2 + \angle 3 + \angle 4 = \cdots°$	● $270°$

3. Observe the following lines patterns! If angle B and C together are 110 degrees, how large the angle A would be? Please explain your answer!



4. On the lines patterns above (problem 3). If you only know the angle B is 50 degrees. How about the size of angles A and C? Explain your answer!

The pictures were taken from:

http://www.theconstructionindex.co.uk/news/view/atkins-picked-for-usas-busiest-rail-junction
http://euler.slu.edu/escher/index.php/Tessellations_by_Squares,_Rectangles_and_other_Polygons

Teacher's Guide

Fostering Students' Understanding about the Magnitude of Angles through Reasoning

Meeting 1 (80 minutes)

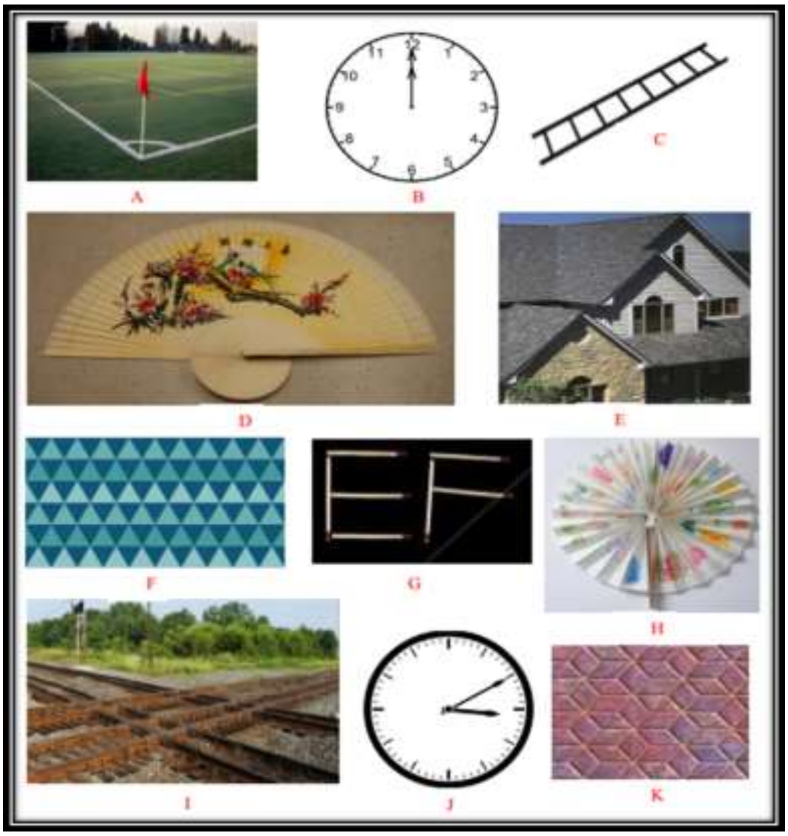
Goal: Students are able to recall the concepts of angle magnitude that they had learnt and reformulate a definition of angle.

Warm up (5 minutes)

Set up the classroom condition to make the students ready to learn.

Lesson part I (35 minutes)

- Starting point and context setup (5 minutes)
  Distribute the following card one for every two students and ask them about the mathematical concepts of the objects in the card that they can figure out.



The guided questions that you might ask:

1. Do you familiar with the objects on the card?
2. Named each object on the card!
3. What do you know about the angles in pictures A, C and G? (*the pictures of right-angle)
4. What is the difference between B and J? (*the expectation is the students realize the duality of zero angles in figure B)
5. What are those objects have in common? (*It is good if they can relate the objects in the card with the concept of angle and line. If they cannot produce the intended answers, you could postpone this problem and move to the next question)
6. What mathematical concepts that embedded on the objects that you can figure out? (*you can help the students to realize the concept of angle by ask them to focus on figure A, where the existence of angle and lines are rather obvious)

- Students at work (30 minutes)

  Distribute the worksheets to each student and ask them to work on the tasks and the questions. Before the students start to work on the worksheet, you have to make sure the students fully understand the instructions in the worksheet. You can ask the students to read it out loud and ask them if there are some instructions that they don't understand. You also can reformulate the problems, give definition of a term on the problems that students do not understand, or give students simple situation to provide them the ground for thinking. You have to walk around to monitor the activity and support the students if it necessary. In this part of the learning activity you only allow to justify students' interpretations on the tasks and questions.

> *NOTE: **<u>The first task</u>** should be solved by students in pair. **<u>The second and the third tasks</u>** should be solved by students in group of four. **<u>The first three questions</u>** should be solved by students individually. **The last question** should be solved by students in group of four.*

Lesson part II (40 minutes)

- Classroom discourses (solutions and strategies)
  1. The first task (~5 minutes)

     The B, D, and H pictures can be the puzzling situations for the students (0, 180 and 360 degrees). However, this condition should be utilized to make students aware about the 0 degree and 360 degrees angles in the real world situations. In addition to that, the students have to be aware that there are 3 pictures that are the right angles (A, C, and G).

NOTE: This task enables students to identify the angles on the real-world objects by recalling their previous knowledge about angles. It also requires the students to raise their awareness that a picture can stretch the size of the angles of an object (contractions/dilatations as the effect of perspective view). For examples; the angles on the picture of the ladder and railway are stretched.

Conjecture of students' reaction	**Guidance for teacher**
Can give a sign on the pictures that they think as angles	Suggest the students to use proper sign to indicate angles
Indicating more than one angles on every picture	It is not a problem because the teacher can ask the students to focus only on one angle in every picture for the next task
Encounter difficulties when indicating angles on pictures B, D, and H	Invite the students into a discussion; Are angles exist on each object? Without using a protractor can you predict the size of the angles in degree as unit of measurement?

2. The second task (~5 minutes)
   In making the order, the solutions are depends on the angles that students selected from each picture. Therefore, you should focus the discussion on the students' explanations about how they order the magnitude of angles.

NOTE: The main purposes of the activity are to see students' comprehension of the angles based on its magnitude, to know how the students distinguish the angles based on its sizes and to understand how the students perceive the angles. You can skip this task as well to the 3rd task for further discussion if the students encounter no significant difficulties.

Conjecture of students' reaction	**Guidance for teacher**
Make the unordered list of angle in the poster	Ask the students how they put the angles into that list in order to know what criteria the students use to determine the size of angle
Judge the size of the angles based on the length of the arms	This could be happen in pictures A, C, and G. Ask the students to name the angles. They may come up with right-angle. Thus, they will realize that all right-angle are in the same size
Judge the size of angles based on the scale of the original objects	Re-explain the question to the students that the task is to compare the angles not compare the size of objects

3. The third task (~5 minutes)
   You have to tell the students to select only one angle on each picture to be display in the poster.

> *NOTE: Through observing and discussing the other groups' posters, this activity aims at enabling students to analyze the important criteria about the size of angle and to infer the properties of angles. The discussion should highlight how the students estimate the magnitude of angles (using the area between arms, the difference in direction between arms, or the amount of rotation). Even though the students can make the intended list of angles, you should encourage them to explain their thinking to make it explicit.*

Conjecture of students' reaction	**Guidance for teacher**
Find discrepancies in the other posters	• Ask the students questions such as; what do you think about the angles on the picture A and B (<, =, or >)? What is the different between angles in picture D and H? • Lead the students to observe the pictures that have right-angle on it. • Invite the students to discuss about the discrepancies on the posters in order to determine the acceptable criteria for the size of angle

4. The first and the second questions (~8 minutes)
   In the discussion you should invite the students to recall the concept of 0 degree and 360 degrees angles.

> *NOTE: The aims of this question are to enable students to contrast the situation of dynamic angle and to make sense the duality of a zero angle.*

Conjecture of students' reaction	**Guidance for teacher**
Drawing a small non-zero angle as the smallest angle	Invite the students into a discussion; Why do you think it is the smallest angle? How do you know the size of the angle? Please explain why do you think so?
Drawing an obtuse non-360 degrees angle as the biggest angle	Invite the students into a discussion; why do you think it is the biggest angle? How do you know the size of the angle? Please explain why do you think so?

5. The third and the fourth questions (~10 minutes)
   Make it as the open discussions where the students have the opportunity to express their thinking. You can scaffold students' responds as well.

   > *NOTE: The goals of the questions are to enable students to explain the angle constructions and to reformulate the definition of angle.*

Conjecture of students' reaction	Guidance for teacher
Explain that an angle is formed by two intersecting lines or explain that an angle is a sub-figure of a polygon	Invite the students to reason about angle construction using lines and its direction
Making a definition of angle which focuses on one of the following criteria: as space between two lines which meet in a point, as the difference of direction between two lines, or as the amount of turn	•Make the three criteria as the valid ways to define angle •Classroom discussion to make the criteria reasonable for the students

- Reflections and conclusions (5 minutes)
  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

Meeting 2 (80 minutes)

Goal: The students are able to infer the similarity between the magnitudes of angles that formed by a straight line that falling across two parallel lines.

Warm up (5 minutes)

Set up the classroom condition and make the students ready to learn. Split the students into groups of 4 and distributes three boxes of wooden matches for each group.

Lesson part I (60 minutes)

- Starting point and context setup (5 minutes)
  Asks the students to guess what they can do with the matchsticks in this learning activity. After the students give their predictions, distributes the worksheet for each group and tells the students that today activity is making the upper case letters using matchsticks.

- Classroom discussions (5 minutes)
  Orchestrate the discussion that orientating the students to the tasks. You have to make clear the restrictions of the letters reconstruction (Do not break the sticks into parts). Provide the students with an opportunity to ask the questions relate to the tasks.

- Students at work (50 minutes)
  You have to walk around to monitor the activity and provide the students with helps if necessary.

> *NOTE: In the first 15 minutes you have to manage to make the students finish their constructions. In the discussion session, the maximum time spend is 10 minutes (Here the focus of the discussion is about the orientation of the sticks; parallel, perpendicular, crossing each other, angles. and the magnitude of angles). The last 25 minutes will be used by the students to solve the questions.*

Lesson part II (25 minutes)

- Classroom discourses (solutions and strategies)

> *NOTE: Focus on the four questions, since the two tasks already discussed in the poster session.*

1. The first and the second questions (~6 minutes)
   In this activity we ask the students to indicate the smallest and the biggest angles on their posters.

> *NOTE: We expected the students to use right-angle as benchmark in order to solve the problems. In addition to that, overlapping strategy can be employ to compare the magnitude of angles.*

Conjecture of students' reaction	Guidance for teacher
Use the sharpness of a vertex to determine the size of angles	Ask the students with a specific question that can advancing students' strategy such as, How about the letter I, is it sharp? How do you explain it? Which one between A and V are sharpest?
Intuitively choose two letters that they think are the answers for the questions, but cannot produce a good explanation about their choice	Ask the students to give further justification on their decision by asking them several questions such as; How do you know this angle is bigger than that angle? Is there any angle that bigger than a right angle?

2. The third question (~4 minutes)
   In order to answering this question, the students have had to know the term parallel.

> *NOTE: The aim of this activity is to enable the students to predict and infer the similarity between angles on parallel lines that cut by transversal lines.*

Conjecture of students' reaction	Guidance for teacher
Misinterpret the term parallel as symmetry and decide that the letters that have symmetry on it fulfill the requirement (A, B, D, V, etc.)	In this case there are two options that teacher can do to support the students. First, by referring to the previous discussion about the orientation of the stick and ask the students to rethink their decision. Second, reformulating the word using plain language (synonym)

3. The fourth question (~12 minutes)
   In this activity, the students have to observe and analyze the size of angles on the letters that have parallel sticks. We expect the problem could enable students to predict and infer the similarity between angles.

Conjecture of students' reaction	Guidance for teacher
Indicate the angles that have the same size but only limited to the right-angle	Encourage the students to observe the other letters that doesn't have right-angle and to predict the size of the angles. In order to justify the similarity between angles on a letter, the teacher can implicitly give hint to the students to employ overlapping strategy. For example: I am not sure if this angle is the same with that angle! But it seems that they are in the same size. How do I 'prove' my conjecture? Maybe it will help if I make another copy of this letter to make the comparison process easier.
Students grasp the important criteria of corresponding angles	The teacher should orchestrate the discussion to make sense the concept of vertical angles in letter X. The teacher could ask other questions about the angles on the letters that doesn't have parallel sticks.

- Reflections and conclusions (3 minutes)
  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

Meeting 3 (80 minutes)

Goal: The students are able to explain the similarity between the magnitudes of angles by utilizing the uniformity of tiles on the floors

Warm up (5 minutes)

Set up the classroom condition and make the students ready to learn. Split the students into groups of 4 and distributes the learning tools.

Lesson part I (45 minutes)

- Starting point and context setup (5 minutes)
  Tells the story of Ana to the students and during the talk displays the pictures of Ana's floors.

  "Ana had decided to select two kinds of tiles to be used in her house, in the kitchen and in the bedroom. One day when she was in the kitchen, she figure out that the lines patterns on those tiles form her name but not as the lines patterns in her bedroom. Can you determine which patterns belong to which floor?"

- Classroom discussions (10 minutes)
  Orchestrate a discussion about the letters on the floors problem. After a classroom consensus about this problem is reached, distribute the worksheets to the groups.

- Students at work (30 minutes)
  The students working in group of 4 and you have to walk around to monitor the activity and provide the students with some helps if necessary.

Lesson part II (40 minutes)

- Classroom discourses (solutions and strategies)
  1. The first and second tasks (~15 minutes)

The discussion should focus on how the students find the letter, the number of line segments that involve in each letter, and the differences in students' approaches.

Conjecture of students' reaction	Guidance for teacher
Highlighting the line between tiles on the floor that forming word "ANA"	Ask the students to copy the letters on their worksheet so it appear in the same shape as on the floors
Drawing another letters that they can find on the kitchen floor	Encourage the students to find as many as letters as they can. In fact all letters can be found on kitchen floor.

2. The third task (~4 minutes)
   The students compare the letters on the tilled floors with the letters on the alphabets reconstruction activity (second meeting).

   NOTE: This activity enables the students to get further justification of the magnitude of angles on the upper case letters (second meeting; letters reconstruction) using angles on tiles.

Conjecture of students' reaction	Guidance for teacher
Figure out that the orientation of line segments on some letters which appear on the kitchen floor are different compare with the letters on the poster but the size of angles still the same	Encourage the students to focus on the angles on each situation and suggest the students to pay attention on the orientation of line segments on each situation
Figure out that they can easily see the similarity of angles on the tiled floors compare with the letters from matchsticks	Invite the students to clarify their explanation about the similarity of angles in the previous meeting using the corners of the tiles

3. The first and second questions (~4 minutes)
   The students indicate the angles that have the same magnitude and grouping the parallel line segments on the tiled floor.

   NOTE: This activity allows the students to build a connection between parallel lines and similarity between angles on it.

Conjecture of students' reaction	Guidance for teacher
Indicate angles that look the same as the same angles dispute the precision of their decision	Ask the students about the precision of their decision by asking them the following questions: How do you know this angle is in the same size with that angle? Even it is look the same but I am not really sure they are in the same size, can you explain to me how do you make your decision?

Come up with more than 4 groups of parallel line segments because the students think the position affect the parallelity	Here the students think in quantitative way instead of qualitative way. In this case, the teacher could ask the students why some line segments even they heading to the same direction count as different group.

4. The third question (~5 minutes)
   The task aim is to make students aware about the concepts of perpendicular lines using the lines patterns on the floor. In this situation there are no perpendicular lines. Therefore, the students should capable to extract the information in the situation.

5. The fourth question (~5 minutes)
   In this activity, the students analyze the relation between parallel lines and the size of angles.

> *NOTE: The aim of this activity is to enable students to describe the parallel lines using the similarity of angles and vice versa using the angles on the tiles.*

Conjecture of students' reaction	**Guidance for teacher**
Figure out that the similarity of angles will appear when parallel lines are exist	Here is the opportunity for the teacher to introduce the mathematical terms (transversal lines, parallel lines, vertical angles, corresponding angles, alternate interior-exterior angles, and consecutive interior angles) in order to make it easier to referring the name of an angle on the parallel lines that cut by transversal lines in the future classroom communication. It is important to know that this activity only giving a name to a specific angle on a specific situation and the students do not have to know the name behind their heads. The intention is to make students realize that it is easier if we have the names for these angles to make communication more efficient.
Figure out that the parallel lines can be checked using the angles attach on them	

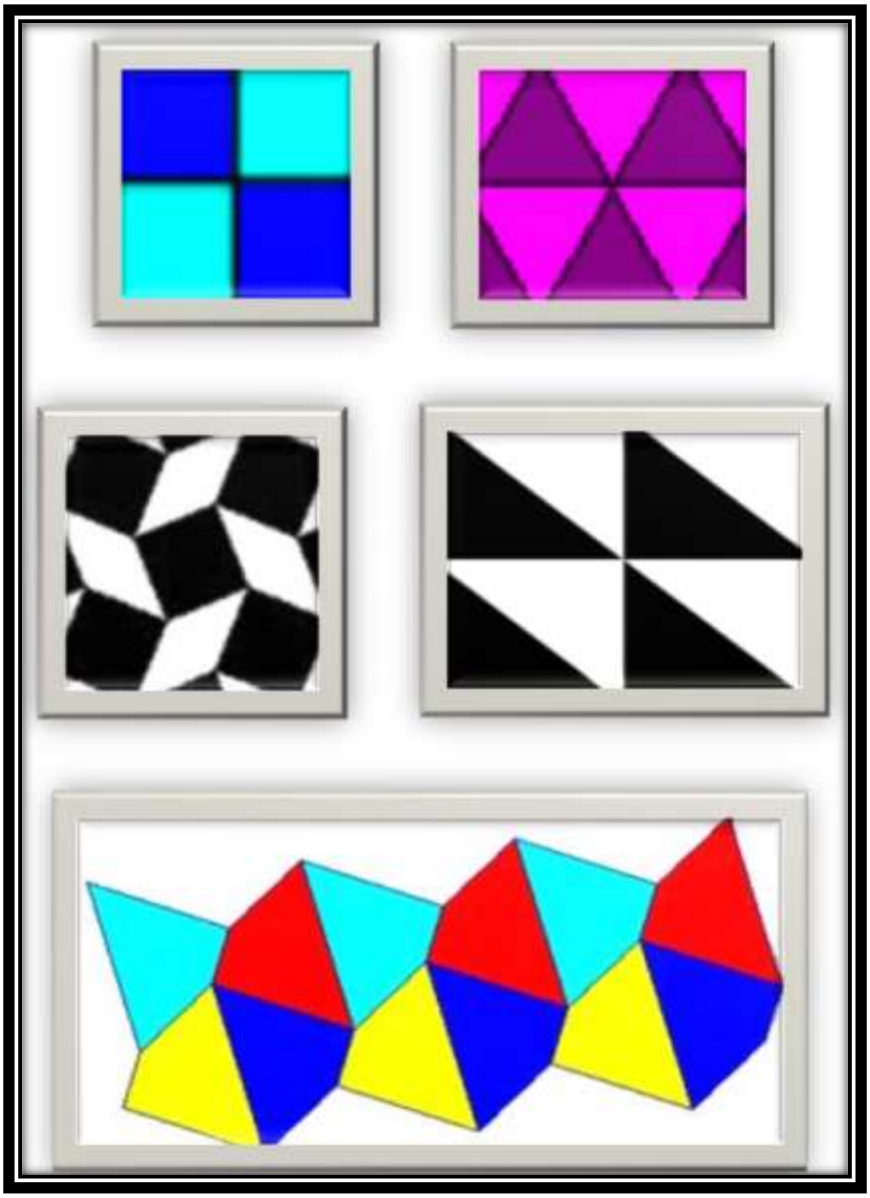
- Reflections and conclusions (3 minutes)
  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

Meeting 4 (80 minutes)

Goal: The students are able to reason about the magnitude of angles using the uniformity of the tiles.

Warm up (5 minutes)

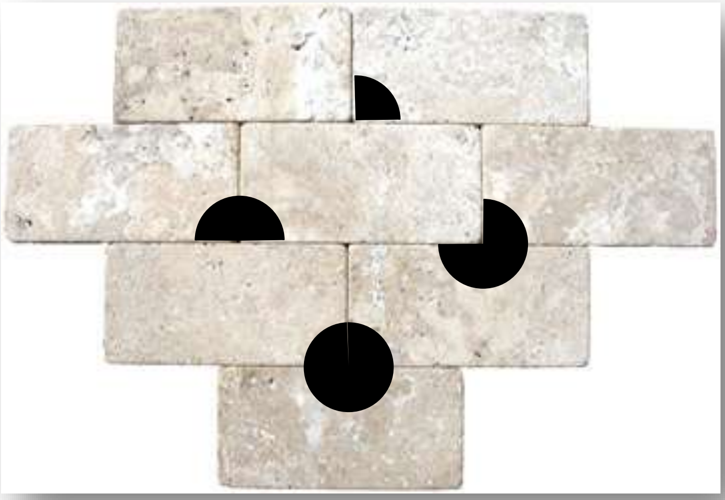
Set up the classroom condition and make the students ready to learn. Split the students into groups of 2 and distributes the learning tools.



Lesson part I (45 minutes)

- Starting point and context setup (5 minutes)
  Ask the students to observe the tiles' patterns on the card and asks them what they think about those tiles.
  The guided questions that you might ask:
  1. How many different types of tiles that needed for build each floor?
  2. How many different magnitudes of angles that you can see in each floor?

- Classroom discussions (10 minutes)
  In this stage, orchestrate a discussion that leads the students to find as many as angle on the picture of bricks. The goal of this discussion is to provide a context for the students in order to make sense the sum of angles.



The following guiding questions can be post in the discussion:

1. As we can see, the angle on the corners of each brick is in the same size. What do you know about the size of the angle on the corners?
2. If we put the bricks side by side, we can see the joint of two corners form a bigger size of angle. On the presented figure, can you determine the size of all angles on the joint of the bricks? Explain how you do the calculation?
3. How many different magnitudes of angles that you can find?

Here, the students have to make sense the straight-angle is 180 degrees and full-angle is 360 degrees from the classroom discussion.

Conjecture of students' reaction	Guidance for teacher
Conclude that the size of the angle on the corners is equal to the size of right-angle	Teacher should encourage the students to give a numerical value for the right-angle
Only come up with explanation of straight-angle (2 right-angles) because the formation of the bricks do not give have 4 corners of the bricks meet	After the students can explain their calculation for straight-angle, the teacher could ask the students about the size of angles from several combinations of joint bricks (see black sector of the circle in the picture). 270 degrees angles could make the situation clearer for the students

- Students at work (30 minutes)
  Distribute the worksheets to each group and ask them to work on it as a group of two. You have to walk around to monitor the activity and provide the students with some helps if necessary.

Lesson part II (40 minutes)

- Classroom discourses (solutions and strategies)
  1. The first question (~6 minutes)
     The students investigate the magnitude of angles on the tiled floors and make an overview of the situation.

> *NOTE: The aim is to make the students predict and calculate the size of angles on each corner of the tile. In order to make that kind of calculation possible the students have to understand the concepts such as, complementary angles, supplementary angles, explementary angles, and vertical angles.*

Conjecture of students' reaction	Guidance for teacher
Give numerical values for each angle on the corners despite there are uncertainty about the size of angles in three floors (C, D, and E)	The numerical values that students give can add up or doesn't add up depend on their assumptions. Therefore, the teacher should orchestrate a classroom discussion in order to justify students' claims. If a claim that students make is right, the teacher should ask for justification. However, if a claim that students make is wrong, the teacher should make it obvious why the claim is wrong via classroom discussion
Give general descriptions about the size of angles for each floor relate to the type of the tiles without any numerical values of the angles	Ask the students to find the differences and the similarities of angles size within a floor and encourage them to apply their knowledge about complementary angles, supplementary angles, explementary angles,

(e.g. right-angle, acute angle, obtuse angle, smallest or biggest angles, and sharp corners)	and vertical angles that they had learned in the bricks investigation

2. The second question (~6 minutes)
   This is a simple and easy question for the students that already arrive at this stage of learning sequence. They can indicate the same angles without hesitations because the tiles obviously tell them about the similarity between corners (i.e. the size of angles). However, you should pay attention on the signs that students use. Here you should encourage the students to be clear and rigor when they give an indication for the same angles. In this activity the crayon or colored markers can be helpful.
   Here the students should explain how they know some angles have the same magnitude. We predict, the students would come up with two different explanations for this question. First, the students utilize the corners of the tiles on each floor in their explanation. Second, the students utilize letters-angles in their explanation (relating the question with the previous activities). You should orchestrate a discussion that allows the students to make a connection between the two explanations.

3. The third question (~5 minutes)
   The students analyze and explain the size of angles on every meeting point of the tiled floors. The goal of this activity is to enable the students to reason about supplementary angles, explementary angles, and vertical angles.

Conjecture of students' reaction	Guidance for teacher
Give the numerical values for each angles but overlook the size of angles on some floors (for instance in floor D the diagonal as angle bisection of the corner of rectangle)	Ask the students how they get the numerical values and ask the students to explain their assumptions
Make a conclusion base on their previous knowledge that on every meeting point, the sum of angles is 360 degrees	Ask for further explanation; How do you know about that? Can you explain to me how you come up with that answer?
Use step-by-step reasoning to arrive at the conclusion. For instance, finding the value of one corner and gradually fill the unknown angles using the properties of angles that they learned	Check students reasoning by ask two or three students to present their work on the blackboard and orchestrate a classroom discussion to remove the flaws in students reasoning (if the flaw exist)

4. The fourth and the fifth questions (~15 minutes)
   The two last questions ask the students to use their knowledge in the numerical problems. The last problem is an uncertainty numerical problem about the size of angles. In this activity, we expect the students can make up

their own assumptions in order to simplify the situations and solve the problems. You should introduce to the students about the assumptions in mathematics. You can use words such as, predict, estimate, or assess before introduce the word assumption.

Conjecture of students' reaction	**Guidance for teacher**
Guessing the size of unknown angles	Discuss with the students about their guesses. The teacher should make the students realize that their guesses can produce a contradiction relate to the situation if the guesses are wrong. If the students guess it right, the teacher should discuss with the students how they guesses can be accurate by reasoning backward in the situation
Claim that the problems do not have any solution due to lack of information	Suggest the students to make up reasonable extra information for each situation (assumptions)
Claim that each situation in the problem have too many solutions	Suggest the students to focus on their selected assumptions relate to the situation

- Reflections and conclusions (3 minutes)
  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

Meeting 5 (80 minutes)

Goal: The students are able to apply the properties of letters angles (F, Z, and X-angles) in the angle related problems.

Warm up (5 minutes)

Set up the classroom condition and make the students ready to learn. Split the students into groups of two.

Lesson part I (45 minutes)

- Starting point and context setup (5 minutes)
  Displaying the following picture and ask the students with the following guided questions:

1. What in is in the picture?
2. What happen with the metal plates in far distance?
3. From which point of view that you can see the railway as it is? (*top view is the intended answer)

- Classroom discussions (10 minutes)
  Displaying the following picture and ask the students with the following questions (*Avoid the respond that only use right-angles in the top view):

1. What is in the picture?
2. Can you see the angles in the picture?
3. How the railways looks like if it views from above? Can you sketch the railways from that point of view!

- Students at work (30 minutes)
  Distribute the worksheets to each group and asks them to work on it. You have to walk around to monitor the activity and provide the students with some helps if necessary.

Lesson part II (40 minutes)

- Classroom discourses (solutions and strategies)
1. The first task (~5 minutes)
   In this task, the students have to determine the top view of the railway. By giving this kind of task, we expect the students to be able reconstruct the given information using diagram.

Conjecture of students' reaction	Guidance for teacher
Drawing the top view of the railway that varies in shape	Give suggestion to the students to make the drawing as accurate as possible and the teacher should lead the students to come up with several unique top view drawings

2. The second task (~5 minutes)
   The students identify the angles on their diagram which have the same size. We repeat this activity in order to make students build the relations between similarity of angles and the orientation of the lines that formed the angles.

Conjecture of students' reaction	Guidance for teacher
Indicating angles on the railway that have the same and give explanations using letters-angles	• Guide the students to figure out more about the similarity between angles by doing the following activities: - Ask the students to present their drawing - Select two different drawings and discuses about what makes the drawing different - Highlight one angle and ask the students to find other angles which in the same size. - Give a value for an arbitrary angle on the drawing and ask the students to find the value of other angles • However, if the students cannot produce an adequate explanation the teacher should encourage the students to recall the letters angles concept (F, Z, and X-angles).

3. The first question (~5 minutes)
   We assume this question can be answer by the students without hesitation. They can answer this question by referring to the previous activities, and use the knowledge from those activities to build an adequate reasoning for the question. In other words, the question allows the students to give a further explanation about similarity between the size of angles without help from geometrical patterns or grids. We expect the students can relate the letters-angles and patterns on a tiled floor with the similarity between angles in more general form.

4. The second question (~5 minutes)
   The students observe and investigate the size of angles on a tiled floor in order to reason about the similarity between angles.

Conjecture of students' reaction	Guidance for teacher
• Find out that angle 1 and angle 3 are equal • Find out that angle 2 and angle 4 are equal • Find out that the sum of angle 1 and 4 or 1 and 2 is 180 degree • Find out that the	Ask the explanations for every finding. Here the students can explain their finding using the corners of the tiles as benchmark. However, the teacher should encourage them to use the concept of similar angles that students had learnt in this teaching and learning activities (F, Z, and X-angles)

sum of four angles is 360 degree	

5. The third question (~5 minutes)
   In answering this question the students have to reason with straight angles. In addition to that, when the students successfully answer this question we expect they will understand the fact that the sum of interior angles of a triangle is 180 degrees.

6. The fourth question (~5 minutes)
   Here we give the students another opportunity to reason with uncertainty in the question by giving them a question that in fact lack of information. Therefore, the answer for this question depends on the assumptions that students make.

Conjecture of students' reaction	**Guidance for teacher**
Give different combination for the size of two angles where the sum of both angles is 130 degrees	Invite the students to discuss about why there is no unique answer for the problem

- Reflections and conclusions (3 minutes)
  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

Dierdorp's Analysis Matrix for Lesson 1 in First Cycle

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Indicate an angle on every given object!	(a) The students may give several different signs to indicate an angle on the pictures (b) Some students may indicate more than one angle on each picture		(a) All of the students could indicate the angles in the given figure but some of them didn't use the formal symbol (∡) to indicate the angles (b) Most of them indicated more than one angle in each figure	+ +
2	Make an ascending order of the indicated angles!	(a) Some students may encounter difficulties to indicate and ordering the angles on pictures B, D, and H (0°, 180°, and 360° on an analog clock and the traditional fans) (b) Some students may make the unordered list of the angles because they judge the magnitude of the angles based on a different criteria/scenario (e.g. based on the length of the arms, based on the region of the angle, or based on the scale of the original objects)	A fragment from the classroom discourse: *[10]Researcher: "You knew that they have the same size, but why you don't put them side by side?" (Pointing along the sequence of Ajeng's and Giga's poster)* *[11]Ajeng: "If you see A in the picture, it is not 90° but it is 90° in the real-world." (Try to explain her way in perceiving the angle in the picture)*	(a) All student showed good understanding about 180° and 360° angles but didn't recognize the existence of 0° angle in some objects (b) The students comprehended the presented situation but they embraced two different interpretations relate to the given situation (real-world or picture) (c) All of them put the 360° angles on the very end of the sequence	0 -

Dierdorp's Analysis Matrix for Lesson 1 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
3	Select an object on the poster that can change the size of its angle and draw two situations where the object forming the biggest and the smallest angle!	(a) Some students may draw a small non-zero angle to represent the 0° angle and draw an obtuse non-360° angle as the biggest angle (b) Some students may explain the angles magnitude by reason with the number on the analog clock or rely on their rough estimation	A fragment from the classroom discourse: *[14]Researcher:"How you draw a smallest angle? Can somebody explain it?"* *[15]Giga: "The hour hand on 3 and minute hand on 2."*	(a) All of them claimed that the angle between two consecutive numbers on the clock represents the smallest angle (30°) (b) Most of the students agreed that 360° is the biggest angle in analog clock situation (c) The students still struggled to draw the 0° angle, because the 180° and 360° angles can always be pointed out in every drawing attempt	- +
4	How is an angle formed?	(a) The students may explain that an angle is formed by two intersecting lines (b) They may explain that an angle is formed by two lines that rotate their intersection point		All the students used terms such as; lines, intersection point, and direction to answer the question	+ 0

Dierdorp's Analysis Matrix for Lesson 1 in First Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
5	An angle is…	The students may make a definition of angle which focuses on one of the following criteria: As space between two lines which meet in a point As the difference of direction between two lines As the amount of turn	Student's written work: *Ajeng: "Angle is two lines that meet each other with different directions and have a common point".*	(a) The students defined the angle as the difference of direction between two lines (b) None of the students defined the angle as amount of rotation between two lines, even the analog clock context emphasize the relation between angle and rotation	+

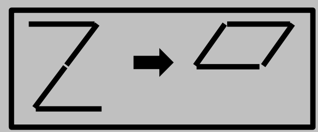
Overview of ALT Result Compared with HLT Conjectures for Lesson 1 in First Cycle

+	x		x	x	x
0		x			
-					
Task	1	2	3	4	5

Dierdorp's Analysis Matrix for Lesson 2 in First Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Reconstruct the upper case letters using wooden sticks!	Some groups may make some letters using way too many matchsticks		(a) The students easily reconstruct the upper case letters using reasonable amount of matchsticks (b) The students found it difficult to gluing the matchsticks on the paper, as a result, one of the groups lagged behind and we immediately asked this group to arrange the matchsticks on their table instead of gluing it on their poster paper	0
2	Observe all the constructions in the classroom! Write down your findings relate to the size, shape, number of matches, similarities, differences, and give the suggestions for improvement of the other construction!	Students find out that some letters are appear in different shape in the other groups' reconstructions		(a) The students found differences in technical aspects of the reconstruction such as, the number of sticks to construct each letter, the shape of the letters, and the appearance of the posters (b) The students found no significant finding relate to the angles magnitude on the letters	+

Dierdorp's Analysis Matrix for Lesson 2 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
3	Which letter that has the smallest angle?	The students may select two different letters to represent the smallest and the biggest angles and not realize the fact that those angles have to be in the same letter (acute angle and its reflex angle)	A fragment from the classroom discourse: *Giga: "What letter that has the smallest angle? (Read the question out loud and immediately give the answer) A, B, K, M, N, P, R, V, W, X, Y, and Z"*	All of the students misinterpreted the instruction and gave the plural respond for the singular question	0
4	Which letter that has the biggest angle?				0
5	Observe the orientation of the sticks! List all the letters that formed by parallel sticks!	Some students may misinterpret the term parallel as something else (e.g. symmetry, perpendicular, intersects, etc.)		(a) Students asked about the definition of parallel in advance (b) Students could list most of the letters that formed by parallel sticks	0
6	Observe the size of the angles on the letters that formed by parallel sticks! Mark the angles that have the same size! Note at least three things!	Students' understanding about the similarity between angles magnitude limited to the right-angle situation. In addition to that the students may use the sharpness of the vertices as the benchmark to determine the similarity between angles		(a) The students could easily give an explanation about angles similarity when 90° angles are involved (E, F, H, and U) (b) The students argued that they can reshape the letter Z into a diamond shape in order to make clear the similar angles	+

Overview of ALT Result Compared with HLT Conjectures for Lesson 2 in First Cycle

+		x				x
0	x		x	x	x	
-						
Task	1	2	3	4	5	6

Dierdorp's Analysis Matrix for Lesson 3 in First Cycle

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Which one from the given floors is the kitchen floor? Can you show it?	The students will highlight the gaps between tiles that form a word 'ANA' but they may highlight the different amount of gaps to construct the word		The students highlighted the word 'ANA' and used different amount of gaps to construct the word	+
2	Draw another letters that you can find on the kitchen floor (keep the drawing as precise as you can with the lines on that floor)!	(a) The students will draw another letters that they can find on the kitchen floor (b) Some students may find all the letters on the kitchen floor and some may not		Most of the groups found all the letters on the kitchen floor	+ +
3	Draw another letters that you can find on the bedroom floor (keep the drawing as precise as you can with the lines on that floor)!	The students only find few letters on bedroom floor		Although, they were able to work with the task, due to the repetition of the instruction, most of them found that the given task was tedious and time consuming	+

Dierdorp's Analysis Matrix for Lesson 3 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
4	Compare the letters in both floors! Note your findings relate to the size of the angles!	(a) Find out that some letters are appear in the same shape (C, D, F, I, J, K, O, P, Q, R, U, V, X, and Y) (b) Find out that some letters are appear in the different shape (B, G, L, and S) (c) Find out that some letters cannot appear on the both floors (A, E, H, M, N, T, W, and Z)		The students only observed the shape of the tiles instead the shape of the letters	0
5	Look back at your letters reconstruction in the matchsticks activity! Can you explain about the size of angles on the letters that have parallel sticks on them in both situations (matchsticks and tiled floors)?	(a) Figure out that they can easily see the similarity of angles on the tiled floors compare with the letters on the poster (b) The students may find out the relation between the parallel orientation of the gaps and the parallel orientation of the matchsticks resulting the same consequence; similarity between angles in both situations	A fragment from the classroom discourse: *[15]Researcher: "Can anybody give a reason, why these angles are in the same size? How many tiles there?" (Pointing to the obtuse angles on F)* *[16]Alif: "Two" (Circling the obtuse angles on letter F)*	(a) The students struggled to give verbal explanations. The researcher gave several supports to help the students to verbalize their ideas (b) Most of the students were able to infer the similarity between the angles	+ +

Dierdorp's Analysis Matrix for Lesson 3 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
5			*[17]Researcher: "Now compare it to the acute one! We know there are two tiles here. (Pointing to the obtuse angle) How about on this angle? (Pointing to the acute angle)* *[18]Abell:"One"* *[19]Rafli:"Oh…yaa…I see it now" (Realize that the amount of the tile's vertex that involve can be used to explain the similarity)*		
6	Indicate the angles that have the same magnitude!	The students may indicate all the angles with the same mark (symbol) and produce the ambiguity when we ask them which angle that equal to which angle		Some students indicated all the angles with the same symbol and produce the ambiguity to distinguish the different pair of angles	+
7	Indicate the line segments that parallel to each other!	Some of the students may use equal length symbol to indicate the parallelity		All of the students used equal length symbol to indicate the parallelity	+
8	Is there a pair of line segment that perpendicular?	The students would have different opinion relate to the existence of the right-angle on the figure		The students debated about the existence of the right-angle on the given figure	+

Dierdorp's Analysis Matrix for Lesson 3 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
9	Observe an adjacent pair of line segment on the given tiled floor! Note at least three things relate to the angles magnitude on them!	The students may realize that there is a connection between the parallelity and the similarity of angles on a situation when a straight line falling across a pair of parallel lines	Student's written work: *Giga: "The internal angles are in the same size, the external angles are in the same size, two parallel lines, and one non-parallel line*	The students realized that there is a connection between parallelity and angles similarity on a situation when a straight line falling across a pair of parallel lines	+

Overview of ALT Result Compared with HLT Conjectures for Lesson 3 in First Cycle

+	x	x	x		x	x	x	x	x
0				x					
-									
Task	1	2	3	4	5	6	7	8	9

Dierdorp's Analysis Matrix for Lesson 4 in First Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Observe the pictures of the tiled floors! Indicate the angles that have the same size with the same mark!	After the students observe the angles that have the same magnitude, they may indicate the angles in each floor relate to the type of the tiles without any numerical values of the angles (e.g. right-angle, acute angle, obtuse angle, smallest or biggest angles, and sharp corners)		The students encountered no significant difficulty in determining the angles that have the same magnitude	+
2	In each situation, please explain how you know the angles are in the same size!	(a) The students may explain the similarity of the angles as a logical consequence of uniformity of the tiles (b) Some students may explain the similarity using the concept that they already learnt from the previous meeting (letters-angles)		Due to the uniformity of the tiles in every given floor, students could easily analysis the similarity	-
3	How about the size of the angles on every meeting point?	The students may conclude that, the sum of angles on every common point is 360		(a) The students figured out that in every common point of tiles on every floor, the total angle is 360° (b) The students' claim was based on the fact that they can draw a circle to indicate the angle on every common point of the tiles	+

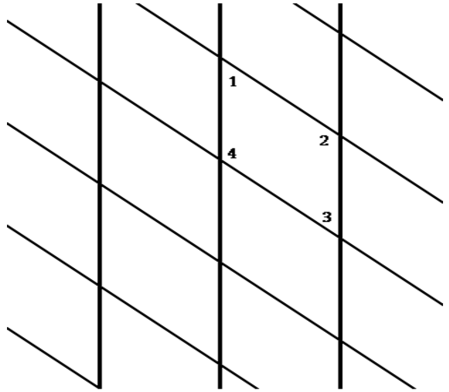
Dierdorp's Analysis Matrix for Lesson 4 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
4	Can you give the numerical values for the sizes of each angle on floors C, D, and E? Explain how you determine the sizes?	(a) Some students may guess the magnitude of the unknown angles (b) Some students may claim that the problems do not have any solution due to the lack of information (c) Some may claim that the problem have too many solutions depend on their assumptions	A fragment from the classroom discourse: *[109]Researcher: "So the total sum of acute and obtuse angles is 180°. But how about the size of each individual angle? If I want to know it, what should I do?"* *[110]Abell: "Use a protractor!"* *(Other students giggling)* *[111]Researcher: "Well…we not allow using a protractor here. Okay, let say that the acute is 30°, what about the obtuse one?"(Students rumble)*	The students guessed the magnitude of the unknown angles	+

Overview of ALT Result Compared with HLT Conjectures for Lesson 4 in First Cycle

+	x		x	x
0				
-		x		
Task	1	2	3	4

Dierdorp's Analysis Matrix for Lesson 5 in First Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	How these railways looks like if you see it from the plane/helicopter? Draw the view in the empty space below!	Some students may draw a trivial condition of the intersection where all railways are perpendicular		All of the students drew the trivial condition of the situation where all the angles in the railways intersections are in the same size (90°)	+
2	Draw a different version of the railway intersection, give a numerical value of an angle on it, and dare a friend next to you to fill the unknown values!	The students may indicate the angles on the railway that have the same magnitude and give explanations using letters-angles concepts without help from the geometrical patterns or grids		The students applied the fact that the sum of internal angles in a quadrilateral is 360°	0
3	Observe the following floor! What can you say about the size of angle 1, 2, 3, and 4? Please explain your thinking! 	The students may find out that: (a) Angle 1 and angle 3 are equal (b) Angle 2 and angle 4 are equal (c) The sum of angle 1 and 4 or 1 and 2 is 180° (d) The sum of four angles is 360°		Some students gave general description about the angles magnitude and the other students gave specific description (numerical estimations)	+

Dierdorp's Analysis Matrix for Lesson 5 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
4	If the angle 1 in question 3 is 60°. Determine the sizes of the other angles! Please explain how you calculate them!	The students may apply their understanding about the properties of angles in parallel-transversal situation in the first question to find the solutions		The students utilized their solution from the third question to solve the problem	+
5	If angle B and C together are 110 degrees, how large the angle A would be? Please explain your answer!	(a) Some students may conclude that 70° is the rights answer (180° as a benchmark) (b) Some students may conclude that 250° is the rights answer (360° as a benchmark)		(a) Many of them tried to apply the fact that the sum of internal angles in a triangle is 180° (b) Some students confused with 360°	+ +

Dierdorp's Analysis Matrix for Lesson 5 in First Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
6	On the lines patterns above (question 5). If you only know the angle B is 50 degrees. How about the size of angles A and C? Explain your answer!	The students may give different combination for the size of two angles where the sum of both angles is 130°		(a) The students were unable to see the uncertainty in the given problem (b) The students assumed that the two unknown angles are in the same magnitude (c) The students used the unrelated information in the previous problem (question 5) as extra information to reduce the number of unknown variables	0

Overview of ALT Result Compared with HLT Conjectures for Lesson 5 in First Cycle

+	x		x	x	x	
0		x				x
-						
Task	1	2	3	4	5	6

Dierdorp's Analysis Matrix for Lesson 1 in Second Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Indicate an angle on every given object!	(a) The students may give several different signs to indicate an angle on the pictures (b) Some students may indicate more than one angle on each picture		(a) All of the students could indicate the angles in the given figures using several different signs (b) Most of them indicated more than one angle in each figure, especially on the figures that have several similar angles	+ +
2	Make an ascending order of the indicated angles!	(a) Some students may encounter difficulties to indicate and ordering the angles on pictures B, D, and H (0°, 180°, and 360° on an analog clock and the traditional fans) (b) Some students may make the unordered list of the angles because they judge the magnitude of the angles based on a different criteria/scenario (e.g. based on the length of the arms, based on the region of the angle, or based on the scale of the original objects)	A fragment from the classroom discourse: *[9]Teacher: "How big the angle in A?"* *[10]Zaky:"Obtuse angle"* *[11]Teacher: "Obtuse??? What is in the picture?"* *[12]Zaky: "A football field corner"* *[13]Teacher: "How big the angle of a football field corner? As boys, all of you must know how big it is!"* *[14]Zaky: "90°"* *[15]Giri:"Right-angle"*	(a) Some students encountered difficulties to indicate the angles that bigger than 180° and most of them didn't recognize the existence of 0° angle in some objects (b) At least 60% of the students were able to make the acceptable constructions (c) Students judged the magnitude of the angles based on acute, obtuse, right-angle benchmarks	+ -

Dierdorp's Analysis Matrix for Lesson 1 in Second Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
3	Select an object on the poster that can change the size of its angle and draw two situations where the object forming the biggest and the smallest angle!	(a) Some students may draw a small non-zero angle to represent the 0° angle and draw an obtuse non-360° angle as the biggest angle (b) Some students may explain the angles magnitude by reason with the number on the analog clock or rely on their rough estimation		(a) All of the students draw a small non-zero angle to represent the 0° angle and only 20% of the students draw the full-angle to represent 360° (b) The students explained the angles magnitude based on acute, obtuse, right-angle benchmarks (rough estimation)	+ +
4	How is an angle formed?	(a) The students may explain that an angle is formed by two intersecting lines (b) They may explain that an angle is formed by two lines that rotate their intersection point		(a) The students explained that an angle is formed when two lines with different direction meet in a point	+ 0

Dierdorp's Analysis Matrix for Lesson 1 in Second Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
5	An angle is…	The students may make a definition of angle which focuses on one of the following criteria: As space between two lines which meet in a point As the difference of direction between two lines As the amount of turn	Students' written work: *"An angle is two lines meet in a point"* *"An angle is two lines with different direction and have degree"* *"An angle is area between two intersecting lines"*	None of the students defined the angle as amount of rotation between two lines, even the dynamic angle situations emphasized the relation between angle and rotation	+

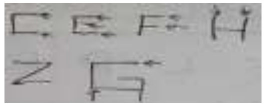
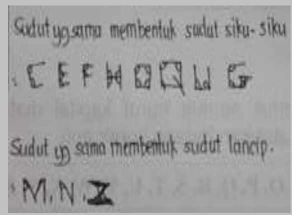
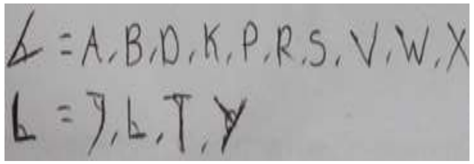
Overview of ALT Result Compared with HLT Conjectures for Lesson 1 in Second Cycle

+	x	x	x	x	x
0					
-					
Task	1	2	3	4	5

Dierdorp's Analysis Matrix for Lesson 2 in Second Cycle

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Reconstruct the upper case letters using wooden sticks!	Some groups may make some letters using way too many matchsticks		(a) The students easily reconstruct the upper case letters using reasonable amount of matchsticks	0
2	Observe all the constructions in the classroom! Write down your findings relate to the size, shape, number of matches, similarities, differences, and give the suggestions for improvement of the other construction!	Students find out that some letters are appear in different shape in the other groups' reconstructions		(a) Students' constructions were quite similar to each other (b) The students found no significant finding relate to the angles magnitude on the letters	0
3	Which letter that has the smallest angle?	The students may select two different letters to represent the smallest and the biggest angles and not realize the fact that those angles have to be in the same letter (acute angle and its reflex angle)	A fragment from the classroom discourse: *[18]Teacher: "Are you sure the biggest angle is in I?" Do any of you have another solution?* *[19]Giri: (Raise his hand)* *[20]Teacher:"Okay…Giri!"* *[21]Giri: (Write his solution on the whiteboard, he indicate the reflex angle in A as the biggest angle)*	Most of the students agreed that the smallest angle and the biggest angle are in letter A	0
4	Which letter that has the biggest angle?				0

Dierdorp's Analysis Matrix for Lesson 2 in Second Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
5	Observe the orientation of the sticks! List all the letters that formed by parallel sticks!	Some students may misinterpret the term parallel as something else (e.g. symmetry, perpendicular, intersects, etc.)		Students could list most of the letters that formed by parallel sticks	0
6	Observe the size of the angles on the letters that formed by parallel sticks! Mark the angles that have the same size! Note at least three things!	(a) Students' understanding about the similarity between angles magnitude limited to the right-angle situation (b) The students may use the sharpness of the vertices as the benchmark to determine the similarity between angles		(a) The students could easily give an explanation about angles similarity when 90° angles are involved (E, F, H, and U) (b) The students used acute angle (sharpness) as a benchmark to determine the similarity	+ +
7	How about the letters that don't have parallel sticks? Can you say something about it?	(a) Students cannot find the similar angles in the letters (b) Students recognize the necessary condition of similarity		(a) Students' solutions showed that they cannot find the similar angles in each individual letter (b) Student found that in a non-parallel situation, an angle in a letter may similar to the other angle in another letter	+ 0

Overview of ALT Result Compared with HLT Conjectures for Lesson 2 in Second Cycle

+						x	
0	x	x	x	x	x		
-							x
Task	1	2	3	4	5	6	7

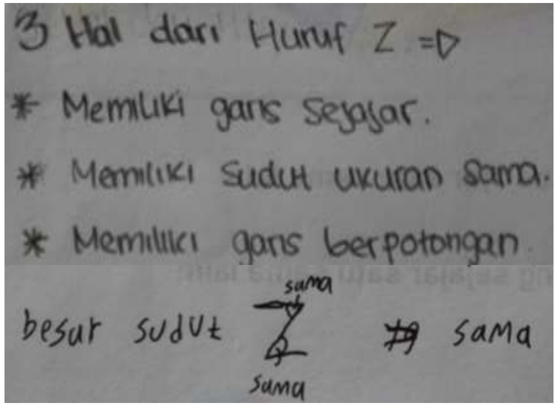
Dierdorp's Analysis Matrix for Lesson 3 in Second Cycle

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Which one from the given floors is the kitchen floor? Can you show it?	The students will highlight the gaps between tiles that form a word 'ANA' but they may highlight the different amount of gaps to construct the word		The students highlighted the word 'ANA' and used different amount of gaps to construct the word	+
2	Draw another letters that you can find on the kitchen floor (keep the drawing as precise as you can with the lines on that floor)!	(a) The students will draw another letters that they can find on the kitchen floor		There were 3 out of 10 groups of students that able to find all the letters in the kitchen floor and it was in line with our prediction in the HLT	+
		(b) Some students may find all the letters on the kitchen floor and some may not			+

Dierdorp's Analysis Matrix for Lesson 3 in Second Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
3	Look back at your letters reconstruction in the matchsticks activity! Can you explain about the size of angles on the letters that have parallel sticks on them in both situations?	(a) The students may find out the relation between the parallel orientation of the gaps and the parallel orientation of the matchsticks produce similarity between angles in both situations	A fragment from the classroom discourse: *[8]Teacher: "How big an angle in a triangle tile?"* *[9]Reza: "We knew that they all in the same size, thus we only need to divide 180 by 3 that is 60°."*	Almost all of the students only figured out the similarity in term of the shape of the letters in both situations	0
		(b) The students may figure out that they can easily see the similarity of angles on the tiled floors situation compare with the letters from the matchsticks activity	*[10]Teacher: "Yeah…60°. Now how about the angles in letter F in the kitchen floor? It is different with the F from the matchsticks right? Who can redraw the letters?*		+
4	Indicate the angles that have the same magnitude!	The students may indicate all the angles with the same mark (symbol) and produce the ambiguity when we ask them which angle that equal to which angle		Some students indicated all the angles with the same symbol and produce the ambiguity to distinguish the different pair of angles	+

Dierdorp's Analysis Matrix for Lesson 3 in Second Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
5	Indicate the line segments that parallel to each other!	Some of the students may use equal length symbol to indicate the parallelity		At least 50% of the students recognized the parallelity in the given situation. Most of them used equal length symbol to indicate the parallelity	+
6	Is there a pair of line segment that perpendicular?	The students would have different opinion relate to the existence of the right-angle on the figure		Most of the students stated that there is no right-angle in the given picture of tiled floor	+
7	On the figure, observe a Z like figure that formed by a pair of parallel line segments that connected by another line segment! Can you tell something about the relations between parallel lines and the size of angles that attach to them? Note at least three things!	The students may realize that there is a connection between the parallelity and the similarity of angles on a situation when a straight line falling across a pair of parallel lines		Most of them claimed three facts about the given situation; there are two parallel line segments, the three line segments are intersect each other in two points, and there are two angles that have the same magnitude	+

Overview of ALT Result Compared with HLT Conjectures for Lesson 3 in Second Cycle

+	x	x		x	x	x	x
0							
-			x				
Task	1	2	3	4	5	6	7

Dierdorp's Analysis Matrix for Lesson 4 in Second Cycle

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Observe the pictures of the tiled floors! Indicate the angles that have the same size with the same mark!	After the students observe the angles that have the same magnitude, they may indicate the angles in each floor relate to the type of the tiles without any numerical values of the angles (e.g. right-angle, acute angle, obtuse angle, smallest or biggest angles, and sharp corners)		Due to the uniformity of the tiles in every given floor, they encountered no significant difficulty in determining the angles that have the same magnitude	+
2	In each situation, please explain how you know the angles are in the same size!	(a) The students may explain the similarity of the angles as a logical consequence of uniformity of the tiles (b) Some students may explain the similarity using the concept that they already learnt from the previous meeting (letters-angles)		Students' responds to the second task indicated that the uniformity of the tiles helped them to give some reasonable responds for the given question	+
3	How about the size of the angles on every meeting point?	The students may conclude that, the sum of angles on every common point is 360		All of the students connected the concept of full angle to the given problem	+

Dierdorp's Analysis Matrix for Lesson 4 in Second Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
4	Can you give the numerical values for the sizes of each angle on floors C, D, and E? Explain how you determine the sizes?	(a) Some students may guess the magnitude of the unknown angles		(a) Some students guessed the unknown angles	+
		(b) Some students may claim that the problems do not have any solution due to the lack of information		(b) Almost all of the students make an educated guess to solve each problem	0
		(c) Some may claim that the problem have too many solutions depend on their assumptions		(c) Students didn't realize the uncertainty in the given problems	0

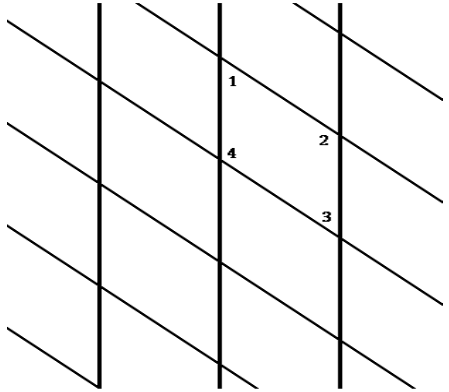
Overview of ALT Result Compared with HLT Conjectures for Lesson 4 in Second Cycle

+	x	x	x	
0				x
-				
Task	1	2	3	4

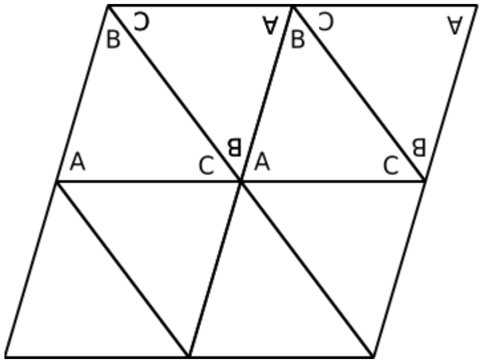
Dierdorp's Analysis Matrix for Lesson 5 in Second Cycle

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	How these railways looks like if you see it from the plane/helicopter? Draw the view in the empty space below!	Some students may draw a trivial condition of the intersection where all railways are perpendicular	A fragment from the classroom discourse: *[8]Giri: (Sketch a top view of the railways)* *[9]Teacher: "You only made a sketch for these railways. So you think both railways are the same?"* *[10]Sri: "They are the same if you see them from above"*	Almost All of the students drew the trivial condition of the situation where all the angles in the railways intersections are in the same size (90°)	+
2	Draw a different version of the railway intersection, give a numerical value of an angle on it, and dare a friend next to you to fill the unknown values!	The students may indicate the angles on the railway that have the same magnitude and give explanations using letters-angles concepts without help from the geometrical patterns or grids		The teacher didn't conduct the activity. However, students' written work indicate that some of the students could determine the numerical value of the angles on their sketch	0

Dierdorp's Analysis Matrix for Lesson 5 in Second Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
3	Observe the following floor! What can you say about the size of angle 1, 2, 3, and 4? Please explain your thinking! 	The students may find out that: (a) Angle 1 and angle 3 are equal (b) Angle 2 and angle 4 are equal (c) The sum of angle 1 and 4 or 1 and 2 is 180° (d) The sum of four angles is 360°		Some students gave general description about the angles magnitude and the other students gave specific description (numerical estimations)	+
4	If the angle 1 in question 3 is 60°. Determine the sizes of the other angles! Please explain how you calculate them!	The students may apply their understanding about the properties of angles in parallel-transversal situation from the first question to find the solutions		Most of the students applied the concept of straight angle and full angle to find the rest of the unknown angles	0

Dierdorp's Analysis Matrix for Lesson 5 in Second Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
5	If angle B and C together are 110 degrees, how large the angle A would be? Please explain your answer! 	(a) Some students may conclude that 70° is the rights answer (180° as a benchmark) (b) Some students may conclude that 250° is the rights answer (360° as a benchmark)		All of the students applied the fact that the total angle in a triangle is 180° and derived this fact to determine the unknown angle	+ 0
6	On the lines patterns above (question 5). If you only know the angle B is 50 degrees. How about the size of angles A and C? Explain your answer!	The students may give different combination for the size of two angles where the sum of both angles is 130°		There are two categories of students' solutions: 1. The students divided the 130° into two equal parts and claimed the parts as the angles in the question 2. The students guessed the sizes of angles in the question in which the sum of both angles is 130°	+

Overview of ALT Result Compared with HLT Conjectures for Lesson 5 in Second Cycle

+	x		x		x	x
0		x		x		
-						
Task	1	2	3	4	5	6

Dierdorp's Analysis Matrix for Lesson 1 in Third Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Indicate an angle on every given object!	(a) The students may give several different signs to indicate an angle on the pictures		(a) All of the students indicated the angles in the given figure by using the formal symbol	0
		(b) Some students may indicate more than one angle on each picture		(b) All of the students indicated one angle in each figure	0
2	Make an ascending order of the indicated angles!	(a) Some students may encounter difficulties to indicate and ordering the angles on pictures B, D, and H (0°, 180°, and 360° on an analog clock and the traditional fans)	A fragment from the classroom discourse: *[14]Della: "The F figure is a figure of equilateral triangles, so each angle on it must be 60°. However, the angle in figure J is less than 60°. So J smaller than F"* *[15]Researcher: "Can you tell me how big the angle in J?"* *[16]Della: "Roughly 30 or 40."* *[17]Researcher: "Dina, can you help us to determine how big the angle between two consecutive number in an analog clock?"* *[18]Dina: "That's must be 30°." (Give the exact value)*	(a) All student showed good understanding about 180° and 360° angles but didn't recognize the existence of 0° angle in some objects	0
		(b) Some students may make the unordered list of the angles because they judge the magnitude of the angles based on a different criteria/scenario (e.g. based on the length of the arms, based on the region of the angle, or based on the scale of the original objects)		(b) The students comprehended the presented situation but they embraced two different interpretations relate to the given situation (real-world or picture)	+
				(c) In the whole group discussion Della's group argued with the other group about the order of the angle on figure F and J. She employed the exact calculation to convince the other group about the order of those angles	

Dierdorp's Analysis Matrix for Lesson 1 in Third Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
3	Select an object on the poster that can change the size of its angle and draw two situations where the object forming the biggest and the smallest angle!	(a) Some students may draw a small non-zero angle to represent the 0° angle and draw an obtuse non-360° angle as the biggest angle		(a) All of the students draw a small non-zero angle to represent the 0° angle and the students draw the full-angle and straight angle to represent 360°	+
		(b) Some students may explain the angles magnitude by reason with the number on the analog clock or rely on their rough estimation		(b) The students explained the angles magnitude based on exact calculation for the angles on the analog clock	0
4	How is an angle formed?	(a) The students may explain that an angle is formed by two intersecting lines	Students' written work: *"Angle can be formed from two intersecting lines which measure in degree and it can be formed when one of the lines move to the other line."*	The students explained that an angle is formed when two lines intersect in a point	+
		(b) They may explain that an angle is formed by two lines that rotate their intersection point			0

Dierdorp's Analysis Matrix for Lesson 1 in Third Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
5	An angle is…	The students may make a definition of angle which focuses on one of the following criteria: As space between two lines which meet in a point As the difference of direction between two lines As the amount of turn	Students' written work: *"An angle is two lines meet in a point"* *"An angle is an arc on the vertex of a pointed figure"*	Only one student that realized the angle as amount of rotation between two lines	+

Overview of ALT Result Compared with HLT Conjectures for Lesson 1 in Third Cycle

+		x	x	x	x
0	x				
-					
Task	1	2	3	4	5

Dierdorp's Analysis Matrix for Lesson 2 in Third Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Reconstruct the upper case letters using wooden sticks!	Some groups may make some letters using way too many matchsticks		(a) The students easily reconstruct the upper case letters using reasonable amount of matchsticks	0
2	Which letter that has the smallest angle?	The students may select two different letters to represent the smallest and the biggest angles and not realize the fact that those angles have to be in the same letter (acute angle and its reflex angle)		Most of the students claimed that the smallest angle was in Z or V, and the biggest angle was in letter I or O.	+
3	Which letter that has the biggest angle?				+
4	Observe the orientation of the sticks! List all the letters that formed by parallel sticks!	Some students may misinterpret the term parallel as something else (e.g. symmetry, perpendicular, intersects, etc.)		Students could list most of the letters that formed by parallel sticks	0
5	Observe the size of the angles on the letters that formed by parallel sticks! Mark the angles that have the same size! Note at least three things!	(a) Students' understanding about the similarity between angles magnitude limited to the right-angle situation (b) The students may use the sharpness of the vertices as the benchmark to determine the similarity between angles		(a) The students could easily give an explanation about angles similarity when 90° angles are involved (E, F, H, and U) (b) The students used acute angle (sharpness/opening) as a benchmark to determine the similarity when there wasn't right-angle involved	+ +

Dierdorp's Analysis Matrix for Lesson 2 in Third Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
6	How about the letters that don't have parallel sticks? Can you say something about it?	(a) Students cannot find the similar angles in the letters		(a) Students' solutions showed that they cannot find the similar angles in each individual letter	+
		(b) Students recognize the necessary condition of similarity		(b) Student found that an angle in a letter without a pair of parallel lines may similar to another angle in another letter	+
				(c) Students realized that the parallelity is a necessary condition for angles similarity	

Overview of ALT Result Compared with HLT Conjectures for Lesson 2 in Third Cycle

+		x	x		x	x
0	x			x		
-						
Task	1	2	3	4	5	6

Dierdorp's Analysis Matrix for Lesson 3 in Third Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Which one from the displayed floors is the kitchen floor? Can you show it?	The students will highlight the gaps between triangular tiles that form a word 'ANA' but they may highlight the different amount of gaps to construct the word		The students highlighted the word 'ANA' and used different amount of gaps to construct the word	+
2	Draw another letters that you can find on the kitchen floor (keep the drawing as precise as possible with what you find on that floor)!	Some of the students may find all the letters on the kitchen floor and some may not		The students were able to find almost all the letters in the kitchen floor	+
3	Look back at your letters reconstruction in the matchsticks activity! Compare the letters that have parallel sticks on them in that situation with the same letters in kitchen floor!	(a) The students may find out the relation between the parallel orientation of the gaps and the parallel orientation of the matchsticks produce similarity between angles in both situations	A fragment from the classroom discourse: *[10]Della: (Start her explanation all over again) "The shape of the tiles is equilateral triangle, in which the angles are 60°. So it is clear that this angle (Pointing to the angle that consists of two vertices) is 120°."*	Almost all of the students only figured out the similarity in term of the shape of the letters in both situations. Further discussion allowed the students to figured out that the tiled floor model outweigh the matchsticks situation in term of certainty of angles magnitude	0

Dierdorp's Analysis Matrix for Lesson 3 in Third Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
		(b) The students may figure out that they can easily see the similarity of angles on the tiled floors situation compare with the letters from the matchsticks activity	*[11]Researcher: "Good! Can one of you re-explain why the angle is 120°? (Imam raises his hand)* *[12]Imam: "Because the angle (Pointing to the angle) consists of two vertices of the triangles, and each vertex is 60°, then the total would be 120°." (Imam utilizing the uniformity of the tiles on the floor model)*		+
4	From the given tiled floor model, indicate the angles that have the same magnitude!	The students may indicate all the angles with the same mark (symbol) and produce the ambiguity when we ask them which angle that equal to which angle		Some students indicated all the angles with the same symbol and produce the ambiguity to distinguish the different pair of angles	+
5	Indicate the line segments that parallel to each other!	Some of the students may use equal length symbol to indicate the parallelity		All of the students highlighted the pairs of parallel line segments	0
6	Is there a pair of line segment that perpendicular?	The students would have different opinion relate to the existence of the right-angle on the figure		All of the students stated that they can find right-angles in the given picture of tiled floor	0

Dierdorp's Analysis Matrix for Lesson 3 in Third Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
7	On the figure, observe a Z like figure that formed by a pair of parallel line segments that connected by another line segment! Can you tell something about the relations between parallel lines and the size of angles that attach to them? Note at least three things!	The students may realize that there is a connection between the parallelity and the similarity of angles on a situation when a straight line falling across a pair of parallel lines		All of them claimed three facts about the given situation; there are two parallel line segments, the three line segments are intersect each other in two points, and there are two angles that have the same magnitude	+

Overview of ALT Result Compared with HLT Conjectures for Lesson 3 in Third Cycle

+	x	x		x			x
0					x	x	
-			x				
Task	1	2	3	4	5	6	7

Dierdorp's Analysis Matrix for Lesson 4 in Third Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	Observe the pictures of the tiled floors! Indicate the angles that have the same size with the same mark!	After the students observe the angles that have the same magnitude, they may indicate the angles in each floor relate to the type of the tiles without any numerical values of the angles (e.g. right-angle, acute angle, obtuse angle, smallest or biggest angles, and sharp corners)		Due to the uniformity of the tiles in every given floor, they encountered no significant difficulty in determining the angles that have the same magnitude	+
2	In each situation, please explain how you know the angles are in the same size!	(a) The students may explain the similarity of the angles as a logical consequence of uniformity of the tiles (b) Some students may explain the similarity using the concept that they already learnt from the previous meeting (letters-angles)	Students' written work: *"Our decision is based on the amount of opening of those angles, because we know in each floor there always be the tiles that have the same shape (triangle, square, etc.)"*	Students' responds to the second task suggested that the uniformity of the tiles helped them to determine the similar angles. Students employed their previous conception that define angle magnitude as the amount of opening between two lines in their explanations	+ +
3	What do you know about the size of the angle on every meeting point of the tiles?	The students may conclude that, the sum of angles on every common point is 360		All of the students connected the concept of full angle to the given problem	+

Dierdorp's Analysis Matrix for Lesson 4 in Third Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
4	Can you give the numerical values for the sizes of each angle on floors A, B, and F? Explain how you determine the sizes?	Students may divide the 360° with the number of the tiles that meet in a point in order to determine the angle magnitudes of each vertex of the tile	Students' written work: *"We calculate the size of the angles by seeing the opening of each angle and guessed the value of one angle."*	The students estimated the numerical value of each angle from every tiled floor and added those numerical value to check whether the total would added up to 360°	+
5	Can you give the numerical values for the sizes of each angle on floors C, D, and E? Explain how you determine the sizes?	(a) Some students may guess the magnitude of the unknown angles (b) Some students may claim that the problems do not have any solution due to the lack of information (c) Some may claim that the problem have too many solutions depend on their assumptions	A fragment from the classroom discourse: *Researcher: "Dina, can you tell us the values of each angle in floor D!"* *Dina: "90+90+45+45+45+45."* *Researcher: "Why 45?"* *Dina: "I know this one is 90°(Pointing to the right-angle figure) and assume this line divide 90° into two equal parts (Making assumption), then the size must be 45°."*	(a) All of the students guessed one of the unknown angles and deduced the value for another unknown angles from this guess (b) Students didn't explicitly realize the uncertainty in the given problems	+ 0 0

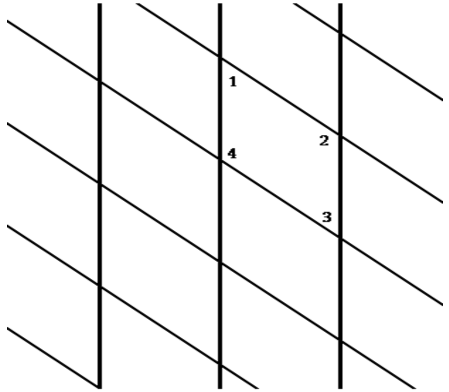
Overview of ALT Result Compared with HLT Conjectures for Lesson 4 in Third Cycle

+	x	x	x	x	
0					
-					x
Task	1	2	3	4	5

Dierdorp's Analysis Matrix for Lesson 5 in Third Cycle

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
1	How these railways looks like if you see it from the plane/helicopter? Draw the view in the empty space below!	Some students may draw a trivial condition of the intersection where all railways are perpendicular		Some students drew the trivial condition of the situation where all the angles in the railways intersections are in the same size (90°)	+
2	Draw a different version of the railway intersection, give a numerical value of an angle on it, and dare a friend next to you to fill the unknown values!	The students may indicate the angles on the railway that have the same magnitude and give explanations using letters-angles concepts without help from the geometrical patterns or grids	A fragment from the classroom discourse *[9]Researcher: "In this context which letter that you can see?"* *[10]Della: "Z." (Hesitantly)* *[11]Researcher: "Okay, Z. So?"* *[12]Della: "So, the angles must be the same."* *[13]Researcher: "Now, how about the angle d?"* *[14]Aulia: "That's must be 130°."* *[15]Researcher: "Can you explain why!"* *[16]Aulia: "Because it looks like F."*		+

Dierdorp's Analysis Matrix for Lesson 5 in Third Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
3	Observe the following floor! What can you say about the size of angle 1, 2, 3, and 4? Please explain your thinking!  	The students may find out that: (a) Angle 1 and angle 3 are equal (b) Angle 2 and angle 4 are equal (c) The sum of angle 1 and 4 or 1 and 2 is 180° (d) The sum of four angles is 360°		Most of the students gave general description about the angles magnitude, the other students gave specific description (numerical estimations) and they stated that the angles in every intersection point is the exact copy of each other	+
4	Re-observe the floor in question 3. Match the questions on the left with the appropriate answers on the right! $\angle 1 + \angle 4 = \cdots°$ — *2 × ∠2 (twice the angle 2) $\angle 3 + \angle 4 = \cdots°$ — *360° $\angle 1 + \angle 3 = \cdots°$ — *2 × ∠1 (twice the angle 2) $\angle 2 + \angle 4 = \cdots°$ — *180° $\angle 1 + \angle 2 + \angle 3 + \angle 4 = \cdots°$ — *270°	The students may apply their understanding about the properties of angles in parallel-transversal situation from the previous questions to find the solutions	A fragment from the group discourse: *[19]Dhina: "We already used the 180°, now there is no option anymore."* *(They check all the option to find an option that equal to the 180°)*	Most of the students applied the concept of straight angle and full angle to find the unknown angles but the students still lack of confidence when they encountered a distractor in the second sub-question	+

Dierdorp's Analysis Matrix for Lesson 5 in Third Cycle (Continued)

	Hypothetical Learning Trajectory (HLT)		Actual Learning Trajectory (ALT)		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
			[20]Della: "Just skip it for a moment! Let us solve the next questions! *(After few moments, they get back to the second sub-question)* *[21]Della: "The only option now is 270°. Now what?"* *[22]Aulia: "Fine…just write 270° as the answer!" (Chose the wrong option even they know the answer)*		
5	If angle B and C together are 110 degrees, how large the angle A would be? Please explain your answer!	(a) Some students may conclude that 70° is the rights answer (180° as a benchmark) (b) Some students may conclude that 250° is the rights answer (360° as a benchmark)		The students applied the fact that straight angle is 180° and deduced the unknown angle from this fact	+ 0

Dierdorp's Analysis Matrix for Lesson 5 in Third Cycle (Continued)

Hypothetical Learning Trajectory (HLT)			**Actual Learning Trajectory (ALT)**		
No	Task	Conjecture	Transcript excerpt	Clarification	Quantitative impression
6	On the lines patterns above (question 5). If you only know the angle B is 50 degrees. How about the size of angles A and C? Explain your answer!	The students may give different combination for the size of two angles where the sum of both angles is 130°		The students divided the 130° into two equal parts and claimed the parts as the angles in the question	0

Overview of ALT Result Compared with HLT Conjectures for Lesson 5 in Third Cycle

+	x	x	x	x	x	
0						x
-						
Task	1	2	3	4	5	6

LESSON PLAN

Topic : Line and Angle
Class : VII
Semester : II
Activity : Angles from Everyday Life Situations
Time allocated : 80 minutes
Meeting : 1

A. Standard Competency

Comprehend the relation between lines and angles and their measurement.

B. Basic Competency

- Determine the relation between two lines, angle magnitude, and angle classification.
- Understanding the properties of angles in a parallel-transversal situation.

C. Indicators

- Students are able to identify the angles on the everyday life objects.
- Students are able to indicate the angles on the everyday life objects.
- Students are able to classify the angles based on its magnitude.
- Students are able to analyze and explain the important criteria in order to determine the magnitude of angles.
- Students are able to contrast the magnitude of angles from the dynamic angles situation.
- Students are able to explain how the angle formed.
- Students are able to reformulate a definition of angle.

D. Goals

Students are able to recall the concepts of angle magnitude that they have learnt before and reformulate a definition of angle.

E. Materials

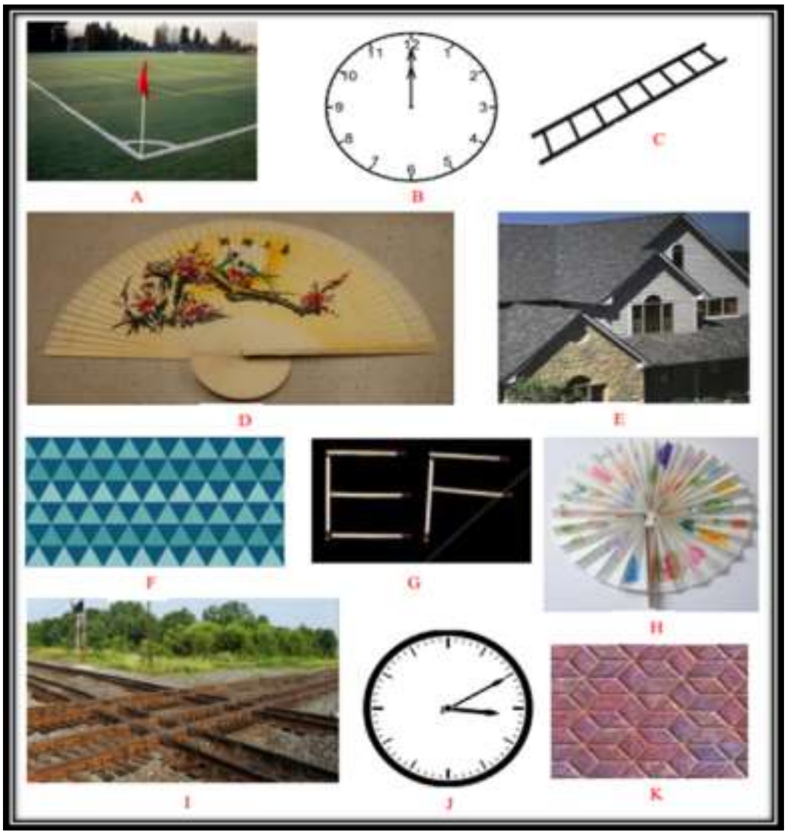
- Picture of the everyday life objects that possess the attributes of angle.
- Students worksheet
- Whiteboard

- Marker
- Scissors
- Glue
- Plain paper

F. Teaching and Learning Activities

Lesson part I

- Starting point and context setup (5 minutes)
  Distribute the following card one for every two students and ask them about the mathematical concepts of the objects in the card that they can figure out.



- Students at work (30 minutes)
  Distribute the worksheets to each student and ask them to work on the tasks and the questions. Before the students start to work on the worksheet, you have to make sure the students fully understand the instructions in the worksheet. You can ask the students to read it out loud and ask them if there are some instructions that they don't

understand. You also can reformulate the problems, give definition of a term on the problems that students do not understand, or give students simple situation to provide them the ground for thinking. You have to walk around to monitor the activity and support the students if it necessary. In this part of the learning activity you only allow to justify students' interpretations on the tasks and questions.

Lesson part II

- Classroom discourses (solutions and strategies)

  The first task (~5 minutes)

  The B, D, and H pictures can be the puzzling situations for the students (0, 180 and 360 degrees). However, this condition should be utilized to make students aware about the 0 degree and 360 degrees angles in the real world situations. In addition to that, the students have to be aware that there are 3 pictures that are the right angles (A, C, and G).

  The second task (~5 minutes)

  In making the order, the solutions are depends on the angles that students selected from each picture. Therefore, you should focus the discussion on the students' explanations about how they order the magnitude of angles

  The third task (~5 minutes)

  You have to tell the students to select only one angle on each picture to be display in the poster.

  The first and the second questions (~8 minutes)

  In the discussion you should invite the students to recall the concept of 0 degree and 360 degrees angles.

  The third and the fourth questions (~10 minutes)

  Make it as the open discussions where the students have the opportunity to express their thinking. You can scaffold students' responds as well.

- Reflections and conclusions (5 minutes)

Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity

G. Assessment

Type of assessment: Students' written works

Palembang, 19 February 2014

Teacher,	Researcher,
Sulastri Hartati, S.Pd NIP 19690627 1991032 007	Boni Fasius Hery NIM 06122802005

Principals of SMP Negeri 17 Palembang

Hj. Mirna, S.Pd., M.M
NIP 19610210 1981102 001

LESSON PLAN

Topic	: Line and Angle
Class	: VII
Semester	: II
Activity	: Matchsticks, Letters and Angles
Time allocated	: 80 minutes
Meeting	: 2

A. Standard Competency

Comprehend the relation between lines and angles and their measurement.

B. Basic Competency

- Determine the relation between two lines, angle magnitude, and angle classification.
- Understanding the properties of angles in a parallel-transversal situation.

C. Indicators

- Students are able to construct the angles in various magnitudes.
- Students are able to compare and criticize the letters reconstructions related to the angle magnitude.
- Students are able to describe the concept of reflex angle.
- Students are able to predict and infer angles similarity in the given situation.

D. Goals

The students are able to infer the similarity between the angles magnitudes that formed by a straight line that falling across two parallel lines.

E. Materials

- Wooden matchsticks
- Students worksheet
- Whiteboard
- Marker
- Plain paper

F. Teaching and Learning Activities

Lesson part I

- Starting point and context setup (5 minutes)

  Asks the students to guess what they can do with the matchsticks in this learning activity. After the students give their predictions, distributes the worksheet for each group and tells the students that today activity is making the upper case letters using matchsticks.

- Classroom discussions (5 minutes)

  Orchestrate the discussion that orientating the students to the tasks. You have to make clear the restrictions of the letters reconstruction (Do not break the sticks into parts). Provide the students with an opportunity to ask the questions relate to the tasks.

- Students at work (50 minutes)

  You have to walk around to monitor the activity and provide the students with helps if necessary.

Lesson part II

- Classroom discourses (solutions and strategies)

  The first and the second questions (~6 minutes)

  In this activity we ask the students to indicate the smallest and the biggest angles on their posters.

  The third question (~4 minutes)

  In order to answering this question, the students have had to know the term parallel

  The fourth question (~12 minutes)

In this activity, the students have to observe and analyze the size of angles on the letters that have parallel sticks. We expect the problem could enable students to predict and infer the similarity between angles.

- Reflections and conclusions (3 minutes)

  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

G. Assessment

Type of assessment: Students' written works

Palembang, 20 February 2014

Teacher,	Researcher,
Sulastri Hartati, S.Pd NIP 19690627 1991032 007	Boni Fasius Hery NIM 06122802005

Principals of SMP Negeri 17 Palembang

Hj. Mirna, S.Pd., M.M
NIP 19610210 1981102 001

LESSON PLAN

Topic	: Line and Angle
Class	: VII
Semester	: II
Activity	: Letters on the Tiled Floor Models
Time allocated	: 80 minutes
Meeting	: 3

A. Standard Competency

Comprehend the relation between lines and angles and their measurement.

B. Basic Competency

- Determine the relation between two lines, angle magnitude, and angle classification.
- Understanding the properties of angles in a parallel-transversal situation.

C. Indicators

- Students are able to identify the lines patterns on the tiled floor models by analyzing the gaps between adjacent tiles.
- Students are able to examine the angles on the tiled floor models.
- Students are able to determine the magnitude of angles on the tiled floor models to get further justification of angles similarity on the letters that have parallel sticks on them (students' conjecture from the second lesson).
- Students are able to relate the magnitudes of angles on two situations; letters from matchsticks and letters on a tiled floor model.
- Students are able to describe the parallel lines using the similarity of angles and vice versa.

D. Goals

The students are able to explain angles similarity by utilizing the uniformity of tiles on the tiled floor models.

E. Materials

- Two pictures of tiled floor models

- Students worksheet
- Whiteboard
- Marker
- Plain paper

F. Teaching and Learning Activities

Lesson part I

- Starting point and context setup (5 minutes)

  Tells the story of Ana to the students and during the talk displays the pictures of Ana's floors.

  > *"Ana had decided to select two kinds of tiles to be used in her house, in the kitchen and in the bedroom. One day when she was in the kitchen, she figure out that the lines patterns on those tiles form her name but not as the lines patterns in her bedroom. Can you determine which patterns belong to which floor?"*


- Classroom discussions (10 minutes)

  Orchestrate a discussion about the letters on the floors problem. After a classroom consensus about this problem is reached, distribute the worksheets to the groups.

- Students at work (30 minutes)

  The students working in group of 4 and you have to walk around to monitor the activity and provide the students with some helps if necessary.

Lesson part II

- Classroom discourses (solutions and strategies)

  The first and second tasks (~15 minutes)

  The discussion should focus on how the students find the letter, the number of line segments that involve in each letter, and the differences in students' approaches.

  The third task (~4 minutes)

  The students compare the letters on the tilled floors with the letters on the alphabets reconstruction activity (second meeting).

  The first and second questions (~4 minutes)

  The students indicate the angles that have the same magnitude and grouping the parallel line segments on the tiled floor.

  The third question (~5 minutes)

  The task aim is to make students aware about the concepts of perpendicular lines using the lines patterns on the floor. In this situation there are no perpendicular lines. Therefore, the students should capable to extract the information in the situation.

  The fourth question (~5 minutes)

  In this activity, the students analyze the relation between parallel lines and the size of angles.

- Reflections and conclusions (3 minutes)

  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

G. Assessment

Type of assessment: Students' written works

Palembang, 25 February 2014

Teacher,	Researcher,
Sulastri Hartati, S.Pd	Boni Fasius Hery
NIP 19690627 1991032 007	NIM 06122802005

Principals of SMP Negeri 17 Palembang

Hj. Mirna, S.Pd., M.M
NIP 19610210 1981102 001

LESSON PLAN

Topic	: Line and Angle
Class	: VII
Semester	: II
Activity	: Reason about angles magnitudes on the tiled floor models
Time allocated	: 80 minutes
Meeting	: 4

A. Standard Competency

Comprehend the relation between lines and angles and their measurement.

B. Basic Competency

- Determine the relation between two lines, angle magnitude, and angle classification.
- Understanding the properties of angles in a parallel-transversal situation.

C. Indicators

- Students are able to predict the magnitude of angles on each corner of a tile.
- Students are able to calculate the magnitude of angles on each corner of a tile using the concept of similarity.
- Students are able to realize the uncertainty related to the magnitude of angles in certain situations.

D. Goals

The students are able to reason about angles magnitudes using the uniformity of the tiles.

E. Materials

- Picture of tiled floor models
- Students worksheet
- Whiteboard
- Marker
- Plain paper

F. Teaching and Learning Activities

Lesson part I

- Starting point and context setup (5 minutes)

  Ask the students to observe the tiles' patterns on the card and asks them what they think about those tiles.

- Classroom discussions (10 minutes)

  In this stage, orchestrate a discussion that leads the students to find as many as angle on the picture of bricks. The goal of this discussion is to provide a context for the students in order to make sense the sum of angles.

- Students at work (30 minutes)

  Distribute the worksheets to each group and ask them to work on it as a group of two. You have to walk around to monitor the activity and provide the students with some helps if necessary.

Lesson part II

- Classroom discourses (solutions and strategies)

  The first question (~6 minutes)

  The students investigate the magnitude of angles on the tiled floors and make an overview of the situation.

  The second question (~6 minutes)

This is a simple and easy question for the students that already arrive at this stage of learning sequence. They can indicate the same angles without hesitations because the tiles obviously tell them about the similarity between corners (i.e. the size of angles). However, you should pay attention on the signs that students use. Here you should encourage the students to be clear and rigor when they give an indication for the same angles. In this activity the crayon or colored markers can be helpful.

Here the students should explain how they know some angles have the same magnitude. We predict, the students would come up with two different explanations for this question. First, the students utilize the corners of the tiles on each floor in their explanation. Second, the students utilize letters-angles in their explanation (relating the question with the previous activities). You should orchestrate a discussion that allows the students to make a connection between the two explanations.

The third question (~5 minutes)

The students analyze and explain the size of angles on every meeting point of the tiled floors. The goal of this activity is to enable the students to reason about supplementary angles, explementary angles, and vertical angles.

The fourth and the fifth questions (~15 minutes)

The two last questions ask the students to use their knowledge in the numerical problems. The last problem is an uncertainty numerical problem about the size of angles. In this activity, we expect the students can make up their own assumptions in order to simplify the situations and solve the problems. You should introduce to the students about the assumptions in mathematics. You can use words such as, predict, estimate, or assess before introduce the word assumption.

- Reflections and conclusions (3 minutes)

Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

G. Assessment

Type of assessment: Students' written works

Palembang, 26 February 2014

Teacher,	Researcher,
Sulastri Hartati, S.Pd NIP 19690627 1991032 007	Boni Fasius Hery NIM 06122802005

Principals of SMP Negeri 17 Palembang

Hj. Mirna, S.Pd., M.M
NIP 19610210 1981102 001

LESSON PLAN

Topic	: Line and Angle
Class	: VII
Semester	: II
Activity	: Angles Related Problems
Time allocated	: 80 minutes
Meeting	: 5

A. Standard Competency

Comprehend the relation between lines and angles and their measurement.

B. Basic Competency

- Determine the relation between two lines, angle magnitude, and angle classification.
- Understanding the properties of angles in a parallel-transversal situation.

C. Indicators

- Students are able to translate given information into a diagram.
- Students are able to show angle similarity on a straight line that falling across two parallel lines.
- Students are able to use their current knowledge to solve the angle related problems.
- Students are able to use their current knowledge to give reasonable explanations related to their computations.
- Students are able to figure out the uncertainty in a problem.

D. Goals

The students are able to apply the properties of letters angles (F, Z, and X-angles) in the angle related problems.

E. Materials

- Picture of railways
- Students worksheet
- Whiteboard
- Marker

- Plain paper

F. Teaching and Learning Activities

Lesson part I

- Starting point and context setup (5 minutes)

  Displaying the following picture and ask the students with the following guided questions:

1. What in is in the picture?
2. What happen with the metal plates in far distance?
3. From which point of view that you can see the railway as it is? (*top view is the intended answer)

- Classroom discussions (10 minutes)

  Displaying the following picture and ask the students with the following questions (*Avoid the respond that only use right-angles in the top view):

1. What is in the picture?
2. Can you see the angles in the picture?
3. How the railways looks like if it views from above? Can you sketch the railways from that point of view!

- Students at work (30 minutes)

Distribute the worksheets to each group and asks them to work on it. You have to walk around to monitor the activity and provide the students with some helps if necessary.

Lesson part II

- Classroom discourses (solutions and strategies)

  The first task (~5 minutes)

  In this task, the students have to determine the top view of the railway. By giving this kind of task, we expect the students to be able reconstruct the given information using diagram.

  The second task (~5 minutes)

  The students identify the angles on their diagram which have the same size. We repeat this activity in order to make students build the relations between similarity of angles and the orientation of the lines that formed the angles.

  The first question (~5 minutes)

  We assume this question can be answer by the students without hesitation. They can answer this question by referring to the previous activities, and use the knowledge from those activities to build an adequate reasoning for the question. In other words, the question allows the students to give a further explanation about similarity between the size of angles without help from geometrical patterns or grids. We expect the students can relate the letters-angles and patterns on a tiled floor with the similarity between angles in more general form.

  The second question (~5 minutes)

  The students observe and investigate the size of angles on a tiled floor in order to reason about the similarity between angles.

  The third question (~5 minutes)

  In answering this question the students have to reason with straight angles. In addition to that, when the students successfully answer this question we expect they will understand the fact that the sum of interior angles of a triangle is 180 degrees.

The fourth question (~5 minutes)

Here we give the students another opportunity to reason with uncertainty in the question by giving them a question that in fact lack of information. Therefore, the answer for this question depends on the assumptions that students make

- Reflections and conclusions (3 minutes)

  Asks students to write down what they had learned so far and what is their mathematical conclusion from the learning activity.

G. Assessment

Type of assessment: Students' written works

Palembang, 27 February 2014

Teacher,

Researcher,

Sulastri Hartati, S.Pd
NIP 19690627 1991032 007

Boni Fasius Hery
NIM 06122802005

Principals of SMP Negeri 17 Palembang

Hj. Mirna, S.Pd., M.M
NIP 19610210 1981102 001