SUPPORTING INDONESIAN FIFTH-GRADE STUDENTS TO LEARN ABOUT PROPORTION

MASTER THESIS

Andrea Arifsyah Nasution

NIM 127785079

STATE UNIVERSITY OF SURABAYA

POSTGRADUATE PROGRAM

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SUPPORTING INDONESIAN FIFTH-GRADE STUDENTS TO LEARN ABOUT PROPORTION

MASTER THESIS

A Thesis Submitted to Surabaya State University Postgraduate Program as a Partial fulfillment of the requirements for the Degree of Master of Science in Mathematics Education Program

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APPROVAL OF SUPERVISORS

Thesis by Andrea Arifsyah Nasution, NIM 127785079, under the title *Supporting Indonesian Fifth-Grade Students to Learn about Proportion* has been qualified and approved to be tested.

Supervisor I, Date,

Prof. Dr. Siti Magfirotun Amin, M.pd. 25 Juni 2014

Supervisor II, Date,

Dr. Agung Lukito, M.S. 24 Juni 2014

Acknowledged by
Head of the Mathematics Education Program,

Dr. Agung Lukito, M.S.
NIP. 196201041991031002

APPROVAL

Thesis by Andrea Arifsyah Nasution , NIM 127785079, had been defended in front of the Board Examiners on July 17th, 2014.

Board of examiners

Name	**Signature**	**Position**
Dr. Agung Lukito, M.S.	Signed	Chairman/Supervisor II
Prof. Dr. Siti M. Amin, M.Pd.	Signed	Member/Supervisor I
Prof. I Ketut Budayasa, Ph.D.	Signed	Member
Dr. Siti Khabibah, M.Pd.	Signed	Member
Prof. Dr. R. K. Sembiring	Signed	Member

Acknowledged by

Director of Post Graduate Program,

Signed & Stamped

Prof. I Ketut Budayasa, Ph.D.

NIP. 195712041994021001

DEDICATION

This thesis is dedicated to my parents, my lovely wife and son, my brother, my sister and my lecturers who have always supported during my study at UNESA, Indonesia and Utrecht University, the Netherlands

I will always love you

When I was a child, I was dreaming to stand on snow and play with it
Last year, I felt like I was still a child who stood and played when the snow was falling down

ABSTRACT

Nasution, A. A. 2014. *Supporting Indonesian Fifth-Grade Students to Learn about Proportion*. Master thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University. Supervisors: Prof. Dr. Siti M. Amin, M.Pd. and Dr. Agung Lukito, M.S.

Keywords: Students' Visualization, Proportion, Proportional Reasoning, Realistic Mathematics Education (RME), Design Research

Generally, the way of teachers in teaching proportion usually stands at giving the students a ready-made formula, such as cross multiplication, to solve proportional problems involving fractions. As a result, the students tend to memorize the formula and just apply it in order to solve the problems. This will be meaningless for them because they do not even know and do not understand the meaning of proportionality. Additionally, proportion is also a difficult topic for students to study. Therefore, there is a need for us to support the students to learn about proportion and to develop their proportional reasoning to solve problems through innovations in teaching proportion.

To deal with this situation, we conducted a study which was aimed to support the fifth-grade students to learn about proportion and to develop their proportional reasoning. In order to achieve the aim of this study, we designed five instructional activities which were based on the heuristic of *Realistic Mathematics Education* (RME) within a learning trajectory. In this case, *design research* was chosen as an approach in order to investigate how the design works in the classroom.

In this study, HLT played an important role as a guideline to carry out the teaching and learning process. The HLT was implemented through six fifth-grade students from SD Al – Hikmah in the first cycle. All the students' activities during the teaching and learning process were recorded so that they can be analyzed in a retrospective analysis. In this case, the retrospective analysis of the first cycle showed that the aims of the teaching and learning process had been achieved yet. Thus, the HLT was revised based on the retrospective analysis before they were applied in the second cycle. Subsequently, the improved HLT was used in the next cycle within 30 students from class 5C. In this case, the results of the teaching and learning process indicated that the designed students' activities could help students learn about proportion and developed their proportional reasoning. Therefore, the cyclic process of this study ended in the second cycle. This is because the aims of the teaching and learning process had been achieved within two cycles.

ABSTRAK

Nasution, A. A. 2014. *Supporting Indonesian Fifth-Grade Students to Learn about Proportion*. Master thesis, Program Studi Pendidikan Matematika, Program Pasca Sarjana Universitas Negeri Surabaya. Pembimbing: Prof. Dr. Siti M. Amin, M.Pd. dan Dr. Agung Lukito, M.S.

Kata kunci: Visualisasi/model siswa, Perbandingan, Penalaran perbandingan, Realistic Mathematics Education, Design Research

Pada umumnya, cara guru mengajarkan perbandingan biasanya hanya memberikan siswa rumusan yang telah tersedia, seperti perkalian silang, untuk menyelesaikan masalah perbandingan. Hasilnya, siswa cenderung menghafal dan hanya mengaplikasikannya dalam menyelesaikan masalah. Hal ini akan menyebabkan siswa tidak mengetahui dan tidak mengerti arti dari perbandingan itu sendiri. Disamping itu, perbandingan juga merupakan topik yang sulit bagi siswa untuk dipelajari. Oleh karena itu, kita perlu membantu siswa untuk belajar mengenai perbandingan dan mengembangkan penalaran mereka dalam menyelesaikan masalah – masalah perbandingan melalui inovasi – inovasi dalam mengajarkan perbandingan.

Untuk menyikapi hal tersebut, dilaksanakanlah suatu penelitian yang bertujuan membantu siswa – siswa kelas lima sekolah dasar dalam belajar perbandingan. Supaya mencapai tujuan tersebut, lima aktivitas belajar mengajar didesain dengan berdasarkan prinsip *Realistic Mathematics Education* (RME) yang dirangkai dalam suatu proses pembelajaran. Oleh karena itu, *design research* digunakan sebagai suatu pendekatan untuk menginvestigasi bagaimana desain tersebut bekerja di kelas.

Dalam penelitian ini, HLT berperan penting sebagai sebuah pedoman untuk melaksanakan penelitian. HLT tersebut kemudian diterapkan kepada enam orang siswa kelas 5B SD Al – Hikmah di siklus pertama. Semua aktivitas belajar siswa selama siklus pertama direkam sehingga dapat dianalisis dalam "*retrospective analysis*". Hasil analisis dari siklus pertama menunjukkan bahwa tujuan dari proses pembelajaran belum tercapai. Oleh karena itu, HLT tersebut direvisi dalam "retrospective analysis" sebelum diterapkan pada siklus yang kedua. HLT yang telah direvisi kemudian diterapkan pada siklus berikutnya dengan subjek 30 orang siswa kelas 5C. Hasil analisis dari siklus kedua menunjukkan bahwa kegiatan pembelajaran yang didesain dapat membantu siswa dalam belajar perbandingan. Oleh karena itu, penelitian ini berhenti pada siklus kedua karena tujuan dari kegiatan pembelajaran telah tercapai dalam 2 siklus.

PREFACE

My very special thanks I gratitude to Allah SWT for his blessing so that I can finish this thesis. In addition, my special thanks I gratitude to my wife, Sahri Ullya, S.Pd., my son and all my families that have helped and supported me during my study.

To begin with, my journey of doing research started when I was studying for my bachelor degree at State University of Medan (UNIMED). At that time, I was conducting a study which took place in secondary school since I was interested in that level. I got so much help from the lectures and my supervisor about how to conduct a research and how to write the thesis of my research. Furthermore, two years after my graduation in 2010, I got a chance to continue my study at State University of Surabaya and Freudenthal Institute, Utrecht University, the Netherlands. In order to fulfill the requirements of the regulations, I got much help from my lectures in UNIMED, Prof. Dian Armanto, M.Pd., M.A., M.Sc., Ph.D. and Dr. Hasratuddin Siregar, M.Pd. During my study at both universities, I was involved in PMRI as a student to implement Realistic Mathematics Education (RME) in Indonesia. In order to get my master degree, I needed to conduct a study which was aimed to investigate and to develop mathematics education in Indonesia especially in primary and secondary education.

In order to conduct my study, I needed to prepare for the design and learning materials that would be implemented in that study. The preparation started in the second semester of my study in Utrecht University, the Netherlands. During the preparation, I got so much help from my supervisor, Mieke Ables. She guided me how to create a good learning design but it was not easy for me. However, I know that there will be no progress if there is no struggle. Therefore, I would like to give my gratitude to her and to all the lectures and stuffs that helped me during my study at Utrecht University, such as: Maarten Dolk, Dolly van Eerde, Frans van Galen, Martin Kindt and others.

After finishing my study and making the preparation at Utrecht University, I, subsequently, needed to conduct the research in primary education in Surabaya, Indonesia. During the research in Indonesia, I also got so much help from my supervisors, Prof. Dr. Siti M. Amin and Dr. Agung Lukito, M.S., the principle, Ustadzah Endah, the staffs and the mathematics teacher, Ustadz Anwar Musyaddat, of the SD Al – Hikmah where this study was conducted. I also express my gratitude to the director of post graduate program, Prof. Dr. I Ketut Budayasa, M.Sc., for his support to my study.

Last but not least, my special thanks I gratitude to all my IMPoME 2012 friends in UNESA, such as Ronal Riffandi, Said Fachry Assagaf, Herani Tri Lestiana, Cici Tri Wanita and Yoga Dwi Windi Kusumaningtyas that had helped me record the teaching and learning process and supported in carrying out this study at SD Al – Hikmah.

Finally, I realize that there are still many errors and mistakes in writing this thesis. Thus, critics, commend and suggestions from the readers are expected in

order to improve this thesis so that this thesis can be a good and helpful reference for the other researchers to conduct their studies.

Surabaya, July 2nd, 2014

Andrea Arifsyah Nasution

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CHAPTER I

INTRODUCTION

1.1. Research Background

It is a truism that proportion is very close to children's daily life since they can find it everywhere. An example of this situation is that when they want to buy milk in a supermarket, when we want to buy t-shirts in a department store, when we want to cook rice at home and other situations. Thus, they can apply the concept proportion so often in those situations. For instance, the price of one box of milk is Rp. 45.000,00 what is the price of 5 boxes of milk? In this case, the statement "one box of milk costs Rp. 45.000,00" describes a proportion between an amount of boxes of milk that can be bought and the amount of money (Lo & Watanabe, 1997). Based on this situation, it can be seen see that proportion has a wide scope in a real-life situation. However, they do not know what proportion is.

In this case, many mathematicians, generally, define a proportional situation as the equality between two ratios (e.g. Tourniaire & Pulos, 1985; Langrall & Swafford, 2000; Lo & Watanabe, 1997; Piaget & Inhelder, 1975). According to van Galen et al. (2008) pointed out that a proportion can be defined as a comprehensive concept since it includes fractions, percentages and decimals. For instance, one family buys 20 kg of rice from a rice mill. For how many days it will last if they consume $\frac{3}{4}$ kg of rice in one day? Based on this situation, we can see that to solve proportional problem requires more knowledge about the relation two or more units, such as number of t-shirts and the price, the number of

kilogram of rice and the number of days and others. Thus, it is necessary for the children to learn about this topic.

In Indonesian educational system, children start learning about proportion when they are in the fifth grade of elementary schools. At this level, most of the young children are required to use the fixed mathematical procedures and algorithms in order to solve problems related to proportion (e.g. Soenaryo , 2007; Sumanto et al., 2008; cited in Sumarto, 2013). The reason of this situation is that many mathematics text books used in Indonesia which mostly contains a set of formal rules and algorithms and they are lack of applications based on realistic problems (Zulkardi, 2002). For instance, Sumarto (2013) stated that most of the mathematics teachers teach cross multiplication to solve proportional problems. She also stated that when the students are taught formal procedure, they will not be able to develop their proportional reasoning. Thus, it is necessary to help children develop their mathematical thinking about proportion.

Relating to this situation, a number of educational studies have been conducted to investigate the development of students' proportional reasoning and to support those children to solve proportional problems (e.g. Widjaja et al., 2010; van Dooren et al., 2010; Tourniaire & Pulos, 1985; Lo & Watanabe, 1997). Moreover, these studies also investigated the preliminary knowledge of the students about proportion and their strategies to solve proportional problems. Therefore, it is not surprising that proportional reasoning becomes one of the major topics in the school mathematics from elementary schools to secondary schools (van Dooren et al., 2005).

Moreover, the results of recent studies also identified that it is very difficult for fifth-grade students to solve proportional problems and their proportional reasoning (e.g. Hiebert and Behr, 1988; van Dooren et al., 2010; Lo & Watanabe, 1997; Widjaja et al., 2010; Silvestre & da Ponte, 2012). The possible reason for this situation is that proportional reasoning involves an understanding of the "relation between relation" (Piaget, 1975; cited in Boyer, 2008). Additionally, students feel difficult to deal with fractions since fractions, itself, are the most complicated numbers to deal with arithmetic (Ma, 1999).

Furthermore, these studies also identified three factors that can lead most students to errors, such as: the existence of an integer ratio, the complexity of the numbers, such as: fractions and decimals, and the order of the number. This can be a reason why the development of proportional reasoning which includes fractions is one of the most difficult aspects for students' mathematical thinking. For these reasons, we attempt to conduct a study which is aimed to investigate the development of students' proportional reasoning which includes fractions by creating a meaningful teaching and learning situation, such as: making the topic real in students' mind through the use of concrete models.

Dealing with this, many researchers suggest the mathematics teachers to use concrete models to develop students' mathematical thinking about proportion that includes fraction, such as: the use of pictures or drawings, (Widjaja et al., 2010), sets of discrete objects and the linear models, such as number lines, are the models most commonly used to represent fractions in the elementary and junior high school (Behr & Post, 1992, cited in Bulgar, 2009) and the use of ratio table

(Widjaja et al., 2010; Middleton & van den Heuvel-Panhuizen, 1995). Relating to these circumstances, one way of assisting the students to develop mental strategies for solving proportion problems is through the use of ratio tables (Middleton & van den Heuvel-Panhuizen, 1995, cited in Dole, 2008). Moreover, Dole (2008) pointed out that using ratio table is an efficient way to symbolize the elements within proportion situations, and for supporting thinking strategies for solutions. However, it is rarely found that the researchers use a ratio table as the concrete model in their studies in order to support students' proportional reasoning in terms of fractions. Therefore, it is necessary to conduct more researches in order to find out how the ratio table can develop the students' proportional reasoning which includes fractions.

1.2. Research question

To deal with the problems, this study is conducted in order to develop a number of learning activities which are designed to support students to learn about proportion. Therefore, the research question of this study is formulated as follows:

How can models support students to develop their proportional reasoning involving fractions?

To answer the research question above, we also attempt to formulate research questions, such as:

1. How do the 5^{th} grade students solve proportional problem including fraction?
2. How do model, such as: students' visualization, support students' proportional reasoning?

3. How can ratio table support students to develop their proportional reasoning?

1.3. Research aim

In order to help students learn about proportion, we conducted a research which is aimed to contribute to the classroom practices in learning about proportion which involves fractions. To be more specific, this research is purposed to investigate the development of students' proportional reasoning by using models.

1.4. Definition of key terms

In order to clarify the key terms used in this study, we, in this case, provide the descriptions of each of them, such as:

1. Proportion can be stated as a statement of equality of two ratios $\frac{a}{b} = \frac{c}{d}$ (Tourniaire & Pulos, 1985) and a comprehensive concept since it includes fractions, percentages and decimals (Van Galen, 2008).
2. Reasoning refers to the students' ability to think about and to use mathematics in a meaningful way (NCTM, 2007). Thus, the students can give their opinion to look for solutions to the problem and they can justify or prove their solutions or even they can evaluate the solutions. In relation to this study, proportional reasoning signifies as the students' ability to think about and the way how the students use mathematics to solve proportional problem in a meaningful way.

3. Basically, model is a representation which is used to solve problems. Furthermore, Sumarto (2013) pointed out that models are important to bridge the students' informal knowledge into formal mathematics.
4. Supporting, in general, can be defined as a process of giving help to somebody (merriam-webster dictionary). In relation of this study, supporting students to learn refers to a process of giving help for students to learn about proportion through designing learning activities and educational materials.

1.5. Relevant research

There are many researchers have conducted studies to help students learn about proportion, such as: Sumarto (2013) conducted a study to develop students' proportional reasoning by using ratio table. This study took place in the fifth grade of elementary school in Palembang. The result of this study shows that model, such as ratio table, plays an important role to support students develop their proportional reasoning.

CHAPTER II

THEORETICAL FRAMEWORK

This chapter will provide the structure of the main theories that contribute to the groundwork of this study. In this case, several studies, related to the teaching and learning about developing students' proportional reasoning, were studied to find out a meaningful way in order to support students to develop their proportional reasoning which includes fractions and to design the instructional materials for the curriculum of Indonesian primary education within the fifth grade.

Furthermore, a proportional situation is used as a starting point to investigate the students' mathematical thinking in terms of proportion. In this case, the Realistic Mathematics Education approach is embedded in this study in order to design and to examine situations in which these instructional activities are expected to lead the students to a more formal mathematical thinking.

2.1. Proportion

Proportion can be stated as a familiar mathematics topic for children. This is because they can find it everywhere. For instance, enlarging and reducing photos or maps, price comparisons, making recipes, comparing probabilities, how shadows change during the course of the day, graphs and diagrams (van Galen et al., 2008). Although children are familiar with proportion, the concept of proportion, however, is still difficult for children (Tourniaire & Pulos, 1985; Boyer et al. 2008).

Meanwhile, a proportion, in mathematics perspective, can be stated as a statement of equality of two ratios $\frac{a}{b} = \frac{c}{d}$ (Tourniaire & Pulos, 1985) and can be defined as a comprehensive mathematical concept (Galen et al., 2005). This means that understanding proportionality is also important in mathematics since it is a foundation of rational operations, unit partitioning in problem solving (Empson, 1999; Fuson & Abramamson, 2005; Haseman, 1981, Pitkethly & Hunting, 1999; Saxe et al., 1999; Sopian et al., 1997; cited in Boyer et al., 2008).

Dealing with this situation, Shield and Dole (2002) stated that students' understanding of proportion generally poor since it is not easy for them to solve proportional problems. Dealing with this, there are several factors that influence students' performance to solve proportional problems, such as:

1. Tourniaire & Pulos (1985) identified two common factors, such as:
   - Structural variables of the proportional problem

     Structural variables of the proportional problem can be stated as a factor that comes from the chosen number, such as: presence of integer ratio, numerical complexity, and the order of the number, the presence of a unit and the presence of unequal ratio for comparison problem.
   - Context variables of the proportional problem

     The contextual factor indicates the familiarity of the context, complexity of the problem, the content of the problem and the understandable problem. In this case, familiarity of the context means that students will get easier to solve proportional problem if they are familiar with the context. Complexity of the problem signifies the chosen number in

the problem, such as the problems which contain mix numbers will be more difficult than the problems that include no mix numbers. Understandable problem means that it will be easier for students to solve proportional problems if they understand the problem.

2. Steinthorsdottir (2006) distinguished two variables that contribute to the difficulties for students to solve proportional problems, such as: number structure and problem contextual structure.

2.2. Proportional reasoning

In general, reasoning refers to the students' ability to think about and to use mathematics in a meaningful way (National Council of Teachers of Mathematics, 2007). Thus, the students can give their opinion to look for solutions to the problem and they can justify/prove their solutions or even they can evaluate the solutions.

Furthermore, proportional reasoning is used as a major tool for human being to interpret those everyday-life phenomena (Centre de Recherchesor l'Enseignement des Mathematiques, 2002; Lesh et al., 1988; cited in van Dooren et al., 2009). Besides, proportional reasoning has been called the backbone, the cornerstone, the gateway to higher level of mathematics success and is considered as a "capstone" of primary school mathematics (Kilpatrick, Swafford & Findell, 2001; Lamon, 1999; Lesh et al. 1988; cited in Parish, 2010). According to Piagetian theory, proportional reasoning involves understanding the "relation between relation", and is a hallmark of formal operation (cited in Boyer, 2008). In relation to this study, proportional reasoning signifies as the students' ability to

think about and the way how the students use mathematics to solve proportional problem in a meaningful way.

Relating to this situation, the development of proportional reasoning, then, can be seen as an important goal of primary school mathematics (Shield & Dole, 2002). This is because many topics in mathematics require students' proportional reasoning and thinking. For instance, it is used when students solve geometry, percentages, fractions, ratio, decimal, scale, algebra and probability problems (Dole et al., 2008). In this study, we attempt to investigate students' proportional reasoning when they are given proportional problems which include fractions.

2.3. Fraction

In the earliest stage of the introduction of fraction, one of the commonly used meanings of fraction is the part-whole relation (Van de Walle et al., 2010 & Streefland, 1991). For instance, one-second is commonly represented by one part over the two equal parts. However, there are still many mathematics teachers, who teach in elementary schools, do not realize that there are other representations of fraction.

Van De Walle et al. (2010) distinguished 5 interpretations of fraction, such as: part-whole, measurement, division, operator and ratio. When students are introduced fractions with part-whole manner, the teacher should also give them the other interpretations. This is because many researchers who research fraction understanding believe that students would understand better with more emphasis across other meaning of fractions (Clarke et al., 2008; Siebert & Gaskin, 2006; cited in Van De Walle et al., 2010).

2.4. Students' strategies to solve proportional problems

Many studies have been conducted to investigate students' proportional reasoning in primary and secondary schools (e.g. Tourniaire & Pulos, 1985; Shield & Dole, 2002; Dole et al., 2008; van Dooren et al., 2009; Langrall & Swafford, 2000; Widjaja et al., 2010; Sumarto, 2013 and others). The results of these studies have shown that there are several strategies that students mostly do, such as:

1. Students incline to use additive methods to solve proportional problems (Hart 1981; Karplus et al., 1983; Misailidou & Williams, 2003; Tourniaire & Pulos, 1985; cited in van Dooren 2010).
2. Students' initial attempts to solve proportional problems suggest their tendency to use procedural algorithms, such as the division algorithm and multiplication (Widjaja et al., 2010).
3. Students, in a certain case, will use repeated subtraction to solve proportional which indicates a better understanding of the problem (Widjaja et al., 2010).
4. Students use standards solution procedure for solving proportional problems which is the "cross-multiply and solve for x" (Post et al., 1988; cited in Shield & Dole, 2002).

2.5. Ratio Table

2.5.1. What is ratio table?

Ratio table is a suitable mathematical model to facilitate and to support students to learn about proportion. It is an appropriate tool to make handy

calculations and to make students reason with proportion. Middleton & van den Heuvel-Panhuizen (1995, cited in Dole 2008) pointed out that a ratio table is one way of assisting the students to develop mental strategies for solving proportion problems. For instance, to determine how many kilometers that we have travelled in 50 minutes if we can reach 42 km in 60 minutes by a car. In this case, ratio table shows clear visualizations of the calculations since we can add more columns as we need (see figure 2.1.).

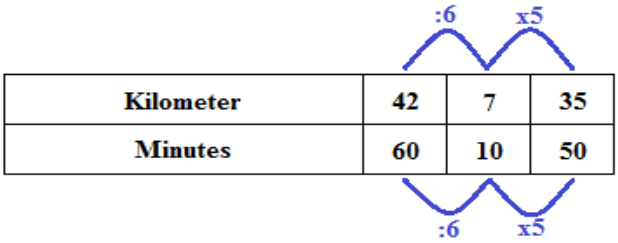


Figure 2.1. Ratio table

Beside ratio table can be used to deal with proportional problems, it can also be applied to solve division problems. Streefland (1991) argued that "division can be handled fairly smoothly with the aid of ratio table". This is because it is very often that division tasks are represented in a unit-rate form, which is a special form of ratio and proportion (Lo & Watanabe, 1997). For instance, in order to make one cake, mother needs $2\frac{1}{2}$ cups of flour. How many cakes can mother make if she has 45 cups of flour? Figure 2.2. shows how ratio table solve this proportional problem problem by using multiplicative strategy.

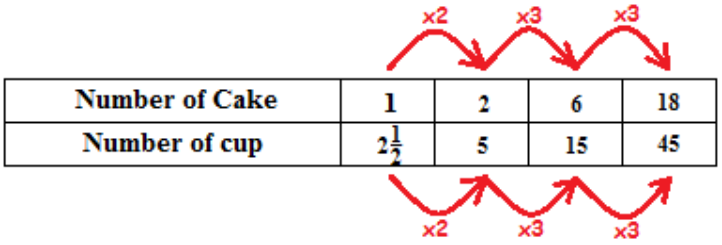


Figure 2.2. Ratio table with multiplicative strategy

Thus, it is obvious that a ratio table can be used by the teachers to teach proportion in the classroom.

2.5.2. How to operate ratio table?

When teachers want to use ratio table in teaching proportions including fractions at the elementary level, then it is necessary for them to discuss what can be done in a ratio table. Indeed, context plays an important role to make the students realize what mathematical operations that they can use, such as the number of paper bee and the number of its legs. In this case, student can multiply both units with the same number; they, however, can not add them with the same number. This is because each unit, in a ratio table, must have the same proportion. Van Galen et al. (2008) proposed that "the advantage of ratio table is that all numbers have their own place and that the unit measurement must stay the same". Thus, teachers need to state clearly what units are used and the arrows to see the operations.

Furthermore, there are several rules that students have to consider when they are working with ratio table, such as: adding, subtracting, doubling, halving, multiplying with the same number and dividing by the same number. On the one hand, adding, doubling and multiplying are used to determine a larger unit

measurement. On the other hand, subtracting, halving and dividing are used to determine a smaller unit measurement. The strategies of using a ratio table can bee seen in table 2.1.

Strategies	**Explanation**
Addition	+ 2 / + 2 { Kg of rice | Persons: 2 | 10; 4 | 20; 6 | 30 } + 10 / + 10
Doubling	x 2 / x 2 { Kg of rice | Persons: 2 | 10; 4 | 20; 8 | 40 } x 10 / x 10
Multiplying with the same number	x 2 / x 3 { Kg of rice | Persons: 2 | 10; 4 | 20; 12 | 60 } x 2 / x 3
Halving	x 1/2 / x 1/2 { Kg of rice | Persons: 4 | 20; 2 | 10; 1 | 5 } x ½ / x ½
Subtracting	x 2 / – 1 { Kg of rice | Persons: 2 | 10; 4 | 20; 3 | 15 } x 2 / – 5
Dividing by the same number	: 3 / : 4 { Kg of rice | Persons: 12 | 60; 4 | 20; 1 | 5 } : 3 / : 4

Table 2.1. Strategies to operate a ratio table

2.6. Realistic Mathematics Education (RME)

2.6.1. What is Realistic Mathematics Education?

Realistic Mathematics Education (RME) is a domain-specific teaching and learning instruction theory for mathematics education (e.g. Treffers, 1987; De Lange 1987; Streefland 1991; Gravemeijer, 1994; Van den Heuvel-Panhuizen 1996; Van den Heuvel-Panhuizen, 2003) and originally adopted from The Netherlands. According to Van den Heuvel-Panhuizen (2003), the word '*realistic*' comes from '*zich realiseren'* which means 'to imagine'.

In Realistic Mathematics Education, mathematics means that it must be related to the reality and is viewed as a human activity (Freudenthal, 1991). In this case, the term "realistic" does not always refer to the students' real word. The word '*realistic*', however, denotes more to problems situation that students can imagine and are meaningful for them (Van den Brink, 1973; Wijdeveld, 1980; cited in Van den Heuvel-Panhuizen, 2003). In other words, mathematics, in classroom practices, can be related to problem situations as long as these situations are real in students' mind.

2.6.2. The Five Tenets of Realistic Mathematics Education

In this study, the structure of designing the teaching and learning instructional activities is based on the idea of the five tenets of RME which were introduced by Treffers (1991). In this case, we describe the five tenets and relate them to this study.

1. Phenomenological exploration

From the point of view of Realistic Mathematics Education, the preliminary instructional mathematical activities must be concrete and real to students. This means that the mathematical activity should not be started at a formal level. Therefore, this study employs the proportional situation to determine 20 kg of rice are enough for how many days.

2. The use of models

In order to bridge the students' informal to more formal level understanding, mathematical models are needed. In this study, the preliminary knowledge of the students towards proportion which includes fractions needs to be developed. Thus, it is expected that models, such as: students' visualization and ratio table, can bridge students' thinking from a concrete to a more formal level in this study.

3. The use of students' own creation and contributions

This is the most crucial part in teaching mathematics based on Realistic Mathematics Education approach. Students' creations and contributions during the teaching and learning activity are used in order to promote the next steps in the learning process. In relation of this study, the various solutions of students' strategies when they are working with a ratio table can be used to develop the next learning activity. In this case, the students' strategies to work with a ratio table during each activity will be discussed in the class discussions in order to get their understanding and contributions to the topic.

4. The interactivity

The learning activities in Realistic Mathematics Education approach do not stand only between teacher and students which is called a one way of teaching method. However, the interactivity, in this case, means the interactions between students and also between students and teacher. In relation to this study, the interaction between students takes place when they present their work to the whole class and in discussions. Moreover, the interaction between the students and the teacher also happens during the class discussion. Therefore, teacher plays an important role in leading the discussion.

5. The intertwinement of various mathematics strands

Intertwinement of various mathematics strands signifies the relation of a mathematics topic which is being observed to other topics, such as geometry, percentages, fractions, ratio, decimal, scale, algebra and probability problems (Dole et al., 2008). In this study, the proportional situations in terms of fractions, in this study, are intertwined with the use of ratio table. In this case, the informal way of working with ratio table is emerged through the proportional activities since they embed the use of ratio table.

2.6.3. Emergent modeling

According to Gravemeijer (2004), instead of trying to help children make connection with ready-made mathematics, we should try to help students construct their mathematical knowledge in a more bottom-up manner. He also argued that emergent modeling signifies the notion of a model that may come to the fore as a model of informal mathematical activity and over time may develop into a model

for more formal mathematical reasoning. Thus, the function of the model in designing teaching and learning based on Realistic Mathematics Education is that it should help students develop their thinking from an informal to a more formal level.

Gravemeijer (1994) proposed that there are four levels of mathematical activities used in teaching RME in classrooms, situational activities, referential activities, general activities and formal mathematics reasoning. The sequence of the model from a *model-of* thinking that represents a problem to *model-for* more formal reasoning can be seen in figure 2.3.

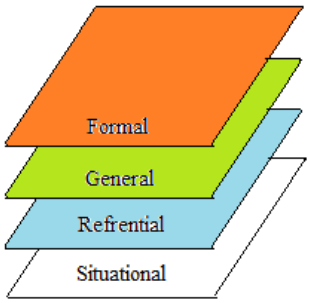


Figure 2.3. Four levels models in designing RME approach

In relation to this study, the implementation of these four levels of emergent models can be seen as the following.

1. Situational activity

The situational level is the starting point of the emergent modeling where preliminary understanding, situational knowledge and strategies are applied to deal with a contextual problem. The setting of the situational level in this study takes place when the students deal with proportional problems within two

quantities, the number of kilograms of rice and the number of days. The students will use their prior understanding about comparing whole numbers and fraction, such as: 20 kg of rice are enough for how many days if one day Mrs. Ani's family consumes $\frac{3}{4}$ kg of rice. At this level, they may draw pictures to explain their answer.

2. Referential activity

The referential level can be stated as the level of *model-of* among the whole learning sequence. In this case, a class discussion will encourage students to move from the situational to the referential level by promoting students to make the representations of the problem.

In this study, students will write their strategies that represent the proportional problems and present it in class discussion. Indeed, the students will not automatically come up with the ratio table which is the *model-of* the situation. Thus, teacher plays an important role during the discussion. In this case, the teacher will discuss and guide the students to more structured representations.

3. General activity

The general activity can be stated as an activity where students start working with the *model-for* thinking. In this activity, the model is used to show a framework of mathematical relations which focus on strategies.

Based on this situation, this study also employs ratio table as the *model-for* to investigate the relations to a more mathematical reasoning. The *model-for* is emerged where students have to make their own ratio table in order to solve proportional problems which include fractions. The students are asked to describe

the relations among units which they can see in the ratio table. In order to do this, they need to remember what they have learned in order to operate ratio table.

4. Formal mathematics reasoning

Formal mathematics reasoning signifies the mathematical activities which do not depend on the support of the *model for*. In this case, the design of the instructional activities, in this study, does not reach this formal mathematics reasoning. This is because this study only focuses on students' reasoning to solve proportional problems which include fractions by using ratio table.

2.6.4. Guided reinvention

One of the characteristics of RME teaching is the guided reinvention. In this case, guided reinvention can be defined as experiencing the process of learning mathematics as a process which is similar to the process of the mathematics when it was invented (Gravemeijer, 1994). A process of guided reinvention then would have to ensure that this mathematical activity would foster the construal of mathematics as a body of knowledge by the students (Gravemeijer and Cobb, 2006). Therefore, the main objective of the guided reinvention is that the students experience real towards the mathematics that they have created.

2.7. Socio Norms and socio-mathematics norms

In conducting this study, we also concerned to the socio norms and the socio-mathematics norms of the classroom. In this case, socio norms can be defined as the expected way of acting and explaining that become instantiated through a process of mutual negotiation between the teacher and the students (Gravemeijer and Cobb, 2006). They also pointed out that the examples of these

social norms can be seen when they explained and justified their solutions, attempted to make sense of explanations, show an agreement or a disagreement and give alternative solutions when a conflict or disagreement happened.

Meanwhile, the socio-mathematics norm can be differentiated from socio norms as the ways of clarifying and acting in the whole-class discussions which are specific to mathematics (Gravemeijer and Cobb, 2006). They also mentioned the example of the socio-mathematics norms that it includes what count as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, an acceptable mathematical solution and justification. Therefore, socio mathematics norms were very important in this study where the students could make the socio norms when a mathematics conflicts happened.

In investigating the development of students' proportional reasoning, the socio mathematics norm and socio-mathematics norm needed to be established in the classroom. In creating the socio norms and the socio-mathematics in the classroom, the teacher played an important role to create the socio norm and the socio mathematics norm in the classroom. The teachers needed to change their traditional teaching into the new teaching method. Indeed, this will take more time to set such learning situation.

However, the socio norms and the socio-mathematics norms, basically, had been established at the school where we conducted this study. When we conducted a classroom observation, we could see that the students were not afraid to explain their solutions when the teacher asked them. Additionally, the students

were familiar to work together in group. Therefore, it was not difficult to develop the socio and socio-mathematics norms in the classroom.

2.8. The role of the teacher in the classroom

In conducting design research in the classroom, we have to pay attention to the role of the teacher. In this case, the teacher plays an important role to establish the expected classroom situation. According to Gravemeijer and Cobb (2006) stated that:

> "The role of teacher will include introducing the instructional activities, guiding the process of talking. Further, the teacher will have to select possible topics for discussion and orchestrate the whole-class discussions on these topics".

In addition, the teacher played an important role to guide students to reinvent the desired mathematics notions. In this case, guiding students mean directing the students to reinvent the mathematical phenomenology based on their own way of thinking. The teacher should create such learning situation so that the students can experience the process of learning mathematics as the similar process when mathematics was invented (Gravemeijer, 1994).

2.9. Proportion in the Indonesian primary school curriculum

In the Indonesian primary educational system, students start learning about proportion when they are in the lower grades. At these levels, the students learn about simple proportion. In the fifth grade, the students also learn about proportion. At this level, the students learn about kinds of proportion which include integer numbers and fractions. Additionally, they also learn the operations in proportion. However, they have learned the informal or the lower level of proportion in the previous grades.

In this case, table 2.2. shows that the proportion is included in the Indonesian primary education.

Standard Competence	Basic Competence
5. Using fraction in order to solve mathematical problems	5.1. Changing fraction into percents, decimals and vice versa. 5.2. Adding and subtracting fractions. 5.3. Multiplying and dividing fractions. **5.4. Using fraction to solve problems related to ratio and scale.**

Table 2.2. Short fifth-grade syllabus of Indonesian primary school

CHAPTER III

RESEARCH METHODOLOGY

3.1. Participants

This study was conducted at SD Al – Hikmah in Surabaya, Indonesia. In this case, this study will take place in the second semester of the fifth grade. In this semester, students in the fifth grade will learn about proportion and kinds of proportions, such as: direct proportion and indirect proportion.

The participants of this study are the students who study in the fifth grade and the mathematics teacher. In this case, the fifth-grade students of SD Al – Hikmah would be envolved to conduct the teaching and learning process of this study.

3.2. Research Approach

Basically, the aim of this study is to contribute to the development of the local instructional theory about students' proportional reasoning which includes fractions. To be more specific, the purpose of this research is to investigate how a mathematical model, such as ratio table, can be used in order to support students' proportional reasoning which includes fractions. These two conditions imply that there exists a need to design several instructional activities and to conduct a research in order to see how these instructional materials support students' proportional reasoning which includes fractions. Moreover, a-ready-made local instruction theory about students' proportional reasoning which includes fractions in RME is still difficult to find. Thus, there is a need to give a contribution in

order to make an improvement and to make an innovation to the development of local instruction theory in mathematics education related to this topic.

For these purposes, the design-based research is selected as an appropriate approach in order to answer the research question and to reach the research aims. In this case, Bakker and Van Eerde (2013) proposed that "design-based research is claimed to have potential to bridge between educational practice and theory". This is because the aim of design research is to develop local instruction theories about both the process of learning and the means designed in order to support that learning (Bakker & Van Eerde, 2013; Gravemeijer & Cobb, 2006). In the relation to this study, a number of instructional activities are designed to enhance and to make innovations in educational practices, especially the 5^{th} grade, about proportion in Indonesian elementary schools. This indicates that this study concentrates not only to know whether mathematical model can support students to learn proportion which includes fractions, but also more specific on how it can support the 5^{th} grade students in Indonesia.

According to Gravemeijer and Cobb (2006), conducting a design study consists of three fundamental phases, such as (1) preparing for the experiment, (2) experimenting in the classroom, and (3) conducting retrospective analysis. Based on this situation, these three phases in this research will be described as follows:

(1) Phase one, preparing for the experiment

Based on a design experiment, the purpose of preliminary phase of a design research is to create a local instruction theory which can be elaborated and refined the design experiment (Gravemeijer & Cobb, 2006). In this case, the preliminary

ideas of this study are inspired by studying literatures about the concepts of proportion which includes fractions before designing the learning activities. During studying these literatures, the researcher started designing the learning activities. The order of these learning activities is arranged and described more detail in the learning line which is part of a Hypothetical Learning Trajectory (HLT).

Basically, a Hypothetical Learning Trajectory (HLT) involves four components: the starting point, the learning goals which define the directions, the learning activities and the hypothetical learning process which includes the conjectures of students' thinking and understanding in the context of learning activities (Simon, 1995; Van Eerder, 2013). In this case, these conjectures of hypothetical learning trajectory are dynamic. In other words, the term dynamic means the conjectures of hypothetical learning trajectory can be changed and modified depending on the actual situations of students' learning experience during the teaching and learning process.

(2) Phase two, experimenting in the classroom

The second phase of the design research is the experimenting in the classroom. Basically, there is a process of (re)designing, testing the instructional activities, conjectures of students' thinking and other aspects in a design research which is called a cyclic process (Gravemeijer et al., 2006). In a design research, a researcher makes a reflection of the instructional activities which is aimed to improve the HLT as the preparation for the next cycle. In orther words, there is no

limitation in conducting the number of cycles. In this case, Gravemeijer (2004) visualized the cyclic process in design research in the figure 3.1.

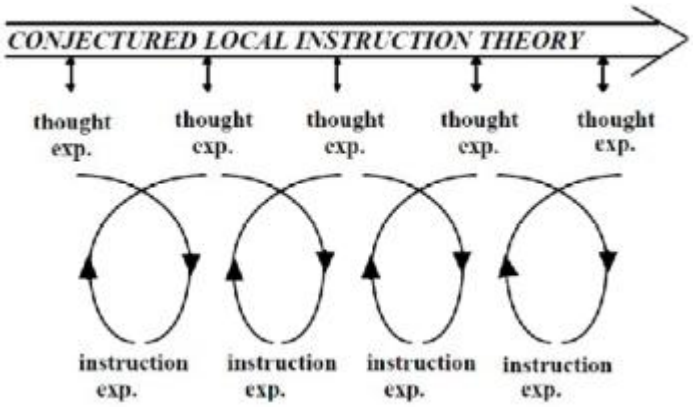


Figure 3.1. A cyclic process in Design-Based Research

In relation to this study, we did not know the number of cycles that this research would take place in the actual teaching experiment. To deal with this situation, we conducted a pilot experiment which was also considered as the first cycle of the design study. It could be considered as a bridge that intertwines the preliminary design and the teaching experiment. In this case, the aim of this pilot experiment is to know the prior knowledge of the students and to investigate whether the instructional activities in the Hypothetical Learning Trajectory (HLT) were doable for students. Thus, the preliminary hypothetical learning trajectory is tested. Subsequently, the investigated actual teaching and learning experiment was used in order to make adaptations in the HLT. After improving the initial HLT, the new HLT could be tested in the next cycle.

The next cycle was conducted as the next step after conducting the pilot experiment in which the mathematical activities of the classroom experiment is the revision from the previous cycle which was also called the second cycle.

Initially, the aim of teaching experiment is to collect data in order to answer the research question. In fact, it is more than just answering the research question. According to Gravemeijer and Cobb (2006), the purpose of the teaching experiment is both to test and to improve the conjectured local instruction theory developed during the preliminary phase and to develop an understanding of how it works. In order to investigate how it works, a discussion between the researcher and the teacher is important to make an agreement about the upcoming activities on how the lesson will be delivered based on the point of view of the researcher and the teacher. Moreover, there is also a reflection about the whole series that focuses on the strong and the weak points of the teaching and learning process.

After conducting the second cycle, we did reflections to the teaching experiment whether the actual teaching experiment had met the learning goals in the HLT that had beed made by the researcher before. On the one hand, if the actual teaching experiment had corresponded to the HLT, then we could stop the teaching and learning, within two cycles. On the other hand, if the actual teaching experiment did not correspond to the HLT, then we needed to conduct more cycles to revise the HLT and to see how the HLT worked in the actual teaching experiment.

(3) Phase three, conducting retrospective analysis

In this phase, all the data during the teaching and learning process are analyzed. According to Gravemeijer and Cobb (cited in Van den Akker et al., 2006), the form of the analysis will involve an iterative process of analyzing the entire data. In this case, the aim of this retrospective analysis is to develop a local

instruction theory. Gravemeijer and Cobb (cited in Van den Akker et al., 2006) pointed out that one of the primary aims of a retrospective analysis is to contribute to the local instruction theory. In making this retrospective analysis, the Hypothetical Learning Trajectory played an important role as a guideline for us in order to conduct this study. In this case, we compared the HLT to the actual teaching and learning process in the classroom in order to improve the initial HLT.

3.3. Data Collection

In this study, we use different types of data in order to answer the research question. The data obtained from different phases, such as: pre-test, preliminary phase, teaching experiments and post-test. The description of data collecting data for each phase can be seen as follows.

3.3.1. Preliminary Phase

Collecting data in the preliminary phase is aiming to investigate the present knowledge of the students that will become the starting point in the teaching experiment and to gain information about the teaching and learning environment in the observed classroom. During this phase, we will conduct classroom observation, pre-test, interview with the teacher and the students and video recording in order to collect the data in this study.

3.3.1.1. Pre-test

During this phase, we will conduct two pre-tests that will be held before the first cycle and before conducting the second cycle. The aim of these two pre-tests

is to get an insight about the prior knowledge of the students related to the proportion. Thus, we get the students' written work as our data collection.

3.3.1.2. Interview with the teacher and the students

After conducting pre-test, we conduct an interview with the teachers and the students. Before the teaching experiment, the researcher will conduct some interviews with the teacher. This interview aims to get the information about the students' prior knowledge, classroom norms which include socio norms and socio mathematics norms and the understanding of the students towards the topic. Moreover, the researcher will also conduct an interview with several students in order to get the data about the level of students' thinking and ability before the experiment is carried out. In this phase, the video recording is used to record the interview with the teacher and the students.

3.3.1.3. Classroom observation

Classroom observation is conducted before starting the teaching experiment (the first cycle). The aim of conducting classroom observation is to investigate the situation of students' learning, how the teacher teaches the students and to investigate the socio norms and the socio mathematics norms during the teaching and learning process. Thus, it is necessary to record the whole teaching and learning process during the classroom observation and make some field notes to see what happens in the classroom.

In general, all the collected informations, during this preliminary phase, were used to adjust the initial HLT to the situation of the observed classroom. Subsequently, They were used to improve the initial Hypothetical Learning

Trajectory (HLT) in terms of the students' prior knowledge and the starting point of the design experiment for the next cycles.

3.3.2. Teaching experiments

In this study, we conducted the first phase of teaching experiments which was purposed to pilot the HLT in the actual teaching experiment. In other words, the first cycle was not only purposed to know the students' thinking and reasoning, but also to test the conjecture in the Hypothetical Learning Trajectory. This pilot teaching experiment was carried out with a small group of students by trying out the initial HLT that had been designed before. In this case, we selected students from the fifth grade who had high, middle and lower level of understanding as the participants in conducting this pilot teaching and those students were different from the students who would work in the second cycle.

In this phase, the data would be collected through students' written work, by making a video recording of the whole activities in this lesson. Furthermore, the researcher would teach the students during this phase. After the data had been gained, then they were used in order to make reflections aimed to revise and to improve the initial HLT. Subsequently, we used the revised HLT as the guideline to conduct the next cycle.

During the next teaching experiments, the researcher collected the data in the form of video registration, students' written work and field note. The video recording was used to record the whole series of students' activities in order to get the data of the students' thinking and reasoning during the teaching experiment. Therefore, we use two camera recorders that would record the activities of the

whole teaching and learning process and the students' activities of the focus group. In this case, the students also had a high, middle and lower levels of understanding which were obtained based on the interview with the teacher.

Moreover, we also used field note in order to collect data in this study. This was because field note was also an important aspect to collect the data in this study. In this case, although two cameras had been installed to record the whole class activities, the field note, however, was still needed to investigate a certain moment of the student's activities and strategies which might not be recorded properly by the video cameras.

3.3.3. Post-Test

In the end of the learning activities, the students will be given post-test within the whole students in the classroom. The aim of this post-test is to evaluate the students' understanding towards what they have learned within the whole teaching experiment. In this case, the data collected from the students' work. After conducting the post-test, we continue conducting an interview with the students about their work in the post-test. The aim of the interview is to investigate the students' thinking and reasoning to solve the problems in the post-test. In order to record the students' reasoning during the interview, a video recording is needed.

3.4. Validity and reliability

Basically, a researcher wants to analyze the data in a reliable way and to gain valid conclusions. Hence, the terms validity and reliability are two important concerns in conducting a research. Dealing with this, the term validity basically refers to "whether you really measure what you want to measure"and reliability

refers to the independence of the researcher. In this study, the description of validity and reliability can be seen as follows.

3.4.1. Validity

The data gathered through the learning activities which are designed in order to help students learn about proportion which involves fractions. In a design based research, the validity is distinguished into two aspects, the internal and external validity (Bakker and Van Eerde, 2013). The internal validity stands for the quality of the data collections and the soundness of the reasoning which will lead to the conclusions. In the relation to this study, the internal validity of this study, on the one hand, includes the data collection, the method of data analysis and triangulation. This is because the internal validity refers to the quality of the data collections and the power of argumentations which led to the conclusions (Bakker et. al, 2012). Meanwhile, the external validity can be simply defined as the "generalizability" or "transferability" of a study. In other words, the external validity of this study extends to which findings are transferable to some broader domain (Van den Akker et. al, 2006).

In order to contribute to the internal validity in this study, the researcher will collect the different data during the teaching experiment, such as video recording of observations will be made during the teaching and learning process of each lesson, student's work, interview with the teacher and the students and field notes. In the case of external validity of this study, it is expected that this study will be enrolled successfully so that the results of this study can be helpful for other researchers as a source for them to make an adjustment towards their studies.

3.4.2. Reliability

The term reliability refers to the consistency of the results that are drawn based on this study. In this case, reliability is also distinguished in to two guidelines namely internal reliability and external reliability (Miles & Huberman, 1994; cited in Bakker and Van Eerde, 2013; & Bakker, 2004).

In order to improve the internal reliability of a research, there are several methods that a researcher can do. Bakker and Van Eerde (2013) pointed out that data collection by objective devices such as audio and video registrations contribute to the internal reliability of a research. Thus, in this research the researcher uses video recorder in order to make the analysis reliable internally. Furthermore, the external reliability usually refers to the replicability of the results of this study. Additionally, Bakker and Van Eerde (2013) argued that external reliability usually denotes replicability, meaning that the conclusions of the study should depend on the subjects and conditions and not on the researcher. In this case, the term replicability is commonly illuminated as virtual replicability in which a criterion of this is trackability (Gravemeijer & Cobb, 2006; Bakker & Gravemeijer, 2004; Bakker & Van Eerde, 2013; Maso & Smaling, 1998).

In other words, this research must be well documented so that it is clear how this research is conducted and how the conclusions have been drawn from the data (Bakker & Van Eerde, 2013). Therefore, the researcher realizes that the data analysis must be made transparently in order improve the external reliability of this study.

3.5. Data Analysis

There are different kinds of data involved in this study, such as data from interview with the teacher and the students, data from classroom observation, students' written work obtained from pre-test, teaching experiments, and post-test and field notes. In this session, we analyze these data by using the Hypothetical Learning Trajectory as a guideline in order to answer the research question and to draw conclusions of this study.

3.5.1. Pre-Test

Pre-test is designed in order to investigate the present relevant knowledge and to identify the starting point of the students towards the proportion which includes fractions. The result of the test is expected to uncover students' prior knowledge, their thinking and also their errors solving proportional problems which include fractions. Therefore, the preliminary Hypothetical Learning Trajectory needs to be adjusted based on the outcomes of the pre-test so that it fits the level of students thinking and understanding.

3.5.2. Teaching Experiments

The data gathered from the students' written works and video recording of the whole activities during the preliminary teaching are analyzed to see the development of students' learning. In this case, the conjectures of students' thinking in the Hypothetical Learning Trajectory are compared to the actual situation of the students' learning process. It can be seen which conjecture happens in the classroom, which conjectures does not and even to see unexpected moments that students will react in different way from the conjectures. Therefore,

it can be seen whether the preliminary Hypothetical Learning Trajectory can support students to develop their proportional reasoning including fractions or not based on the analyses of this cycle. As a result, the Hypothetical Learning Trajectory needs to be revised and to be improved so that it can be implemented in the next cycle.

In the next cycle, the video recordings are analyzed to investigate the moments happened during the teaching experiment. Some interesting short fragments from the video recording are selected and transcribed to show the readers about students' thinking and reasoning. In this case, an interesting fragment means a moment when students get the idea of the crucial parts of a learning process. Then, these fragments and the students' written work are compared to the new Hypothetical Learning Trajectory which has been improved from the previous cycle. Once the researchers have achieved the goals of the HLT in the actual teaching experiment, they can stop conducting the cycles. In addition, the analysis of the cycle can be used to answer the research question and to make conclusions about students' development in learning about proportion.

3.5.3. Post-Test

At the end of the whole lessons, we give the students a post-test. Basically, a post-test is made to see the development of students' understanding and the achievement from the learning activities. In this case, the post-test will have the similar questions with the pre-test. This is because the researcher wants to see the development of students' understanding about the topic and whether the students learn from this lesson.

After getting the result of the post-test, the researcher compares it to the results of the pre-test. This analysis aims to see the development of students' thinking and understanding about the division of a whole number by a fraction. The result of analyzing post-test will produce additional results that can support the analysis of teaching experiment in order to draw conclusions of this study.

3.5.5. Validity and reliability

There are different kinds of data collected in this study, such as video recording, students' written work and field notes. Thus, we use data triangulation in order to analyze these different kinds of data. These various types of data collection together with the testing the HLT will contribute to the internal validity of the data analysis of this research.

In this conducting this study, we realize that making the data analysis transparent will contribute to the external reliability of this study. Therefore, it is very important to present a clear and detailed description of how we analyze the collected data so that if other researchers follow the data analysis, they will yield the same result.

4.6. Timeline of the study

As we had stated in the early part of this chapter, this study was conducted at SD Al – Hikmah, Surabaya. It involved fifth-grade students from class 5B and class 5C. In this case, the first cycle involved 6 students from class 5B and the second cycle involved 30 students from class 5C. Additionally, we also provided the time line when we conducted this study in table 3.1. below.

	Date	Description
Preparing the teaching experiment		
Visiting the school (Discussing with the school and the teacher)	24^{th} – 28^{th} February 2014	Discussing the schedule of the research which included the plans of the research with the teacher and the principle and conducting the classroom observation.
Preliminary teaching (Cycle I)		
	6^{th} – 7^{th} March, 2014	Conducting pretest and interview with the students
Meeting I	March 10^{th}, 2014	Conducting activity I: Helping the teacher solve rice problem.
Meeting II	March 11^{th}, 2014	Conducting activity II: Helping the teacher solve rice problem.
Meeting III	March 12^{th}, 2014	Conducting activity III: Making cakes to sell
Meeting IV	March 13^{th}, 2014	Conducting activity IV: Proportion in unit fraction
Meeting V	March 14^{th}, 2014	Conducting activity V: Proportion in non-unit Fraction
Teaching experiment (Cycle II)		
	13^{th} – 14^{th} March, 2014	Conducting pretest and interview with the students
Meeting I	March 17^{th}, 2014	Conducting activity I: Helping the teacher solve rice problem.
Meeting II	March 18^{th}, 2014	Conducting activity II: Helping the teacher solve rice problem.
Meeting III	March 19^{th}, 2014	Conducting activity III: Making cakes to sell
Meeting IV	March 20^{th}, 2014	Conducting activity IV: Proportion in unit fraction and and non-unit fraction

Table 3.1. The timeline of this study

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

In conducting this study, we develop a hypothesis of teaching and learning process which is commonly called a Hypothetical Learning Trajectory (HLT). An HLT is a useful instrument to manage an instructional theory and a real teaching experiment (Bakker & van Eerde, 2013). In this case, Simon (1995, p.136) outlined a Hypothetical Learning Trajectory as follows:

> "The Hypothetical Learning Trajectory is made up of three components: the learning goal that defines the direction, the learning activities and the hypothetical learning process – a prediction of how students' thinking and understanding will evolve in the context of the learning activities".

It has been mentioned that the aim of this research is to investigate how a ratio table can support students' proportional reasoning which includes fractions. In order to reach this aim, we also design instructional activities which are aimed to introduce ratio table and to develop students' proportional reasoning by using ratio table. In a design based research, the Hypothetical Learning Trajectory needs to be tested (Bakker & van Eerde, 2013). This is because the results of this study are expected to give a contribution to the development of local instructional theory of proportion.

In this chapter we provide the HLT that is used in this study. This study will be conducted within five lessons in one of the Indonesian primary schools, in Surabaya. Therefore, the general overview of the HLT which also includes its teaching and learning series can be seen in the table 4.1. below.

Meeting	Activities	Learning Goal(s)	Description
1	Solving rice problem by using student's model.	- Make models (pictures) to represent the situational problem - Use the models to solve problem.	In this meeting, the students will work on proportional problem which includes fraction. In this problem, they are asked to determine for how many days that 20 boxes of 1 kilogram of rice will last if one family consumes $\frac{3}{4}$ kg of rice.
2	Solving rice problem with a ratio table.	- Make sense of proportion - Work with ratio table.	During this meeting, the students will also work on proportional problem which involves fraction. This meeting, they will be asked to determine for how many days that 20 boxes of 1 kilogram of rice will last if one family consumes $\frac{3}{4}$ kg of rice in an early ratio table.
3	Making cakes to sell	- Use ratio table to solve proportional problem. - Describe how to use ratio table to solve proportional problem.	In this meeting, will be asked to try to use ratio table in solving proportional problem, such as: To make 1 chocolate cake Mrs. Ani has 45 cups of chocolate powder. To

			make 1 cake, she needs $2\frac{1}{2}$ cups of chocolate powder. How many cakes can she make? After solving the problem, they will discuss it in the class discussion.
4	Proportion in unit fration.	Make ratio table and use it to solve proportional problems.	In this meeting, the students will be asked to solve several proportional problems which include fraction by using a ratio table. For example, Miss. Rika goes to supermarket to buy 15 kg of rice to supply her daily need. Everyday, she cooks $\frac{1}{2}$ kg of rice. Can you determine 3 kg of rice are enough in how many days? Then, they will discuss it in the class discussion.
5	Proportion in non-unit fraction.	Solve proportional problem by using model.	In this meeting, the students will be asked to solve a problem which is similar to the problem in the first meeting, such as: determining for how many days that 25

			kilograms of rice will be enough for how many days if one family consumes $\frac{3}{4}$ kg of rice.

Table 4.1. The overview of Hypothetical Learning Trajectory (HLT)

To be more specific, the description of the Hypothetical Learning Trajectory of this study can be seen as follows:

4.1. Activity 1: "Solving rice problem by using student's model"

a. Learning goals

- Use the models to solve problem.

b. Starting points

- Students in the fourth grade have learned about the mathematical operations, such as: addition, subtraction, multiplication and division with whole numbers.
- Students have learned about fraction and how to represent a fraction when they were in the fourth grade.
- Students have learned about how to operate fractions in the fifth grade before this topic is taught.

c. Description of the activity

At the beginning of this meeting, the teacher groups the students where each group consists of three or four students. Then, the teacher shows a picture of a box of one-kilogram rice and asks them the following problem.

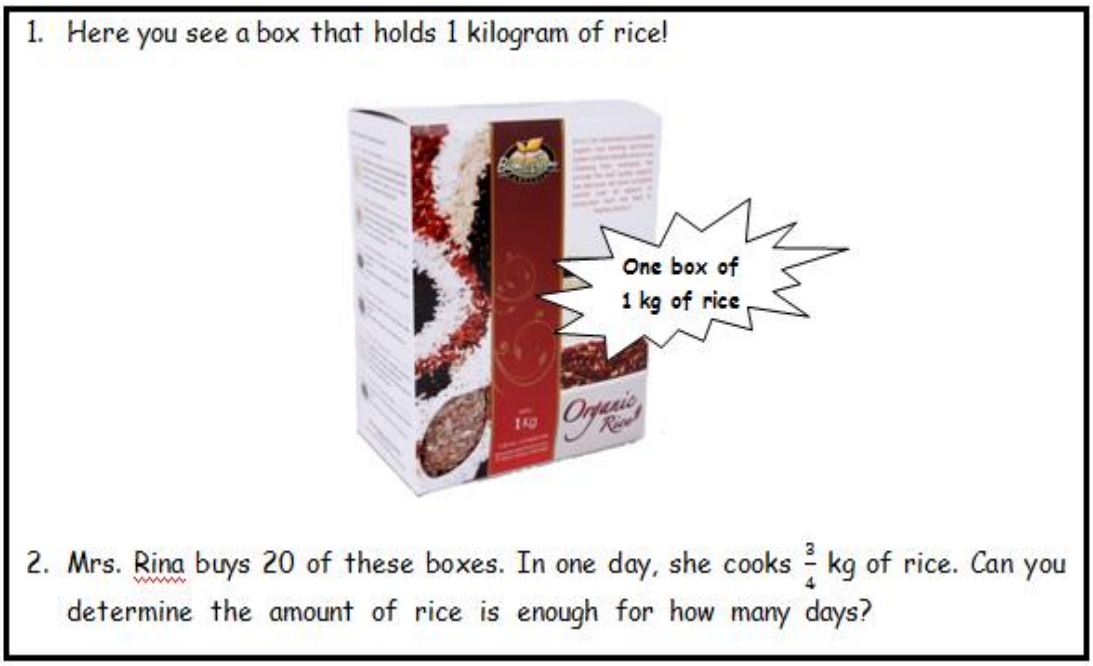


Figure 4.1. Student's worksheet 1

After that, the teacher asks students to solve the problem individually (about 10 minutes) and then discuss it in group.

When the students discuss the problem in group, the teacher can walk around to see how the students solve the problem. If there are students draw a rectangle to explain their strategy, then the teacher can make a discussion based on this answer. However, if there are no students can draw rectangle or they do not know how to represent $\frac{3}{4}$ in a picture, then the teacher can draw a box or a rectangle that represents 1 kg of rice in the white board.

d. Conjecture expectations of the students' strategies

In determining 20 boxes of one-kilogram rice are enough for how many days, there are several possible strategies that the students might do (Widjaja et al., 2010).

1. The students will apply the repeated subtraction strategy. For instance, $\frac{80}{4} - \frac{3}{4}$ $= \frac{77}{4} - \frac{3}{4} = \frac{74}{4} - \frac{3}{4} = \frac{71}{4} - \frac{3}{4} = \frac{68}{4} - \frac{3}{4} = \frac{65}{4} - \frac{3}{4} = \frac{62}{4} - \frac{3}{4} = \frac{59}{4} - \frac{3}{4} = \frac{56}{4} - \frac{3}{4} = \frac{53}{4} - \frac{3}{4} = \frac{50}{4} - \frac{3}{4} = \frac{47}{4} - \frac{3}{4} = \frac{44}{4} - \frac{3}{4} = \frac{41}{4} - \frac{3}{4} = \frac{38}{4} - \frac{3}{4} = \frac{35}{4} - \frac{3}{4} = \frac{32}{4} - \frac{3}{4} = \frac{29}{4} - \frac{3}{4} = \frac{26}{4} - \frac{3}{4} = \frac{23}{4} - \frac{3}{4} = \frac{20}{4} - \frac{3}{4} = \frac{17}{4} - \frac{3}{4} = \frac{14}{4} - \frac{3}{4} = \frac{11}{4} - \frac{3}{4} = \frac{8}{4} - \frac{3}{4} = \frac{5}{4} - \frac{3}{4} = \frac{2}{4}$

   So, 20 boxes of one-kilogram rice are enough for 26 days, remains $\frac{2}{4}$.

2. The students attempt to multiply the two numbers, such as: $\frac{3}{4}$ x 20 = 15 days.

3. The students draw $\frac{3}{4}$ as a fragmented notion of collections of three things over four things. An example of this strategy can be seen in the figure below.

Figure 4.2. Student's representation of $\frac{3}{4}$

4. The students may spontaneously come up with the idea of drawing a rectangle. Then, they use mark in order to group three boxes or four days, such as the following figure.

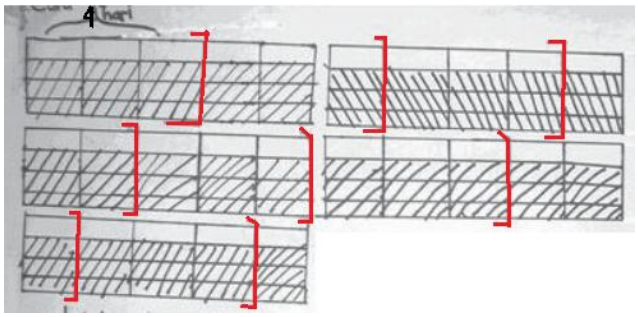


Figure 4.3. Student's strategy to solve problem

4.2. Activity 2 : "Solving rice problem by using a ratio table"

a. Learning goals

- Make sense of proportion
- Work with ratio table.

b. Starting points

The students have already discussed the sense of proportion which includes fraction during the first meeting and they know how to represent fractions in a picture, such as: rectangle or box, as the teacher writes on the white board during the discussion.

c. Description of the activity

At the beginning of this meeting, the teacher lets the students finish their poster about 15 minutes. After the students finish their poster, the teacher starts a discussion to show students' strategies. To be more specific, the aim of the discussion is to investigate students' reasoning in order to solve the problem.

d. Discussion 1

After the students finish their poster, the teacher starts a discussion session within the whole groups in the classroom. In this discussion, the teacher asks one group to present what they do in order to solve the problem. In this case, the teacher can choose the students work which is not totally correct so that there will be a conversation in the whole class discussion.

After the students present their work, the teacher can ask other students to give their comments by asking "Do you have any questions or do you want to add something to this group?" and then ask them "Do you understand what they have explained?".

e. Students' reaction during discussion

1. When the teacher asks the students whether they understand about the explanation, there are several possible answers, such as:
   - They will say that they understand what their friends have just explained.
   - They will say that they do not understand.
   - They will keep silent.
2. When the teacher asks one of the students to repeat what they have heard from the presentation, the students may possibly say something which is not said by their friends, such as: one box is enough for four days, two boxes are enough for four days, etc.

f. Discussion 2

The second discussion is intended to let the teacher and the students talk about shifting students answer to ratio table. To start the discussion, the teacher can ask the student to write it on the white board that three kilograms of rice are enough for four days. Then, the teacher can ask them a question "Can you determine 6 boxes of rice are enough for how many days?". After one of the students write the answer on the ratio table, the teacher can ask them another question "Can you determine 6 boxes of rice are enough for how many days?". Finally, the teacher askw them to determine 20 boxes of rice are enough for how many days.

4.3. Activity 3 : "Making cakes to sell"

a. Learning goals

- Use ratio table to solve proportional problem.
- Describe how to use ratio table to solve proportional problem.

b. Starting points

The students have already discussed about the early steps of operating ratio table with teacher's help in meeting 2.

c. Description of the activity

In this meeting, the students will be given a problem which is aimed to let the students try to use ratio table to solve proportional problem which includes fraction. In this case, the students are asked to solve the following question:

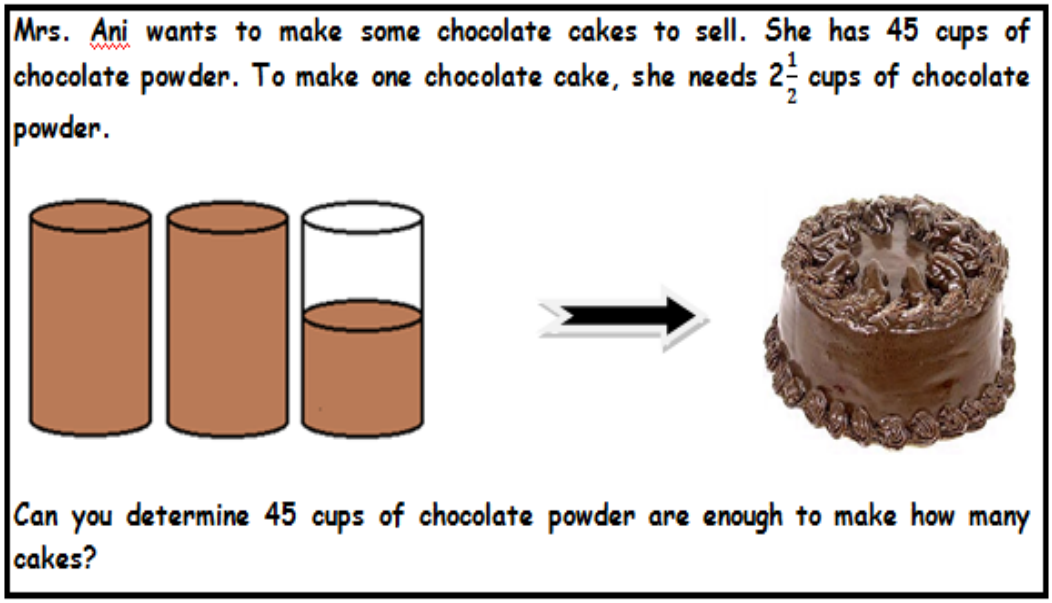


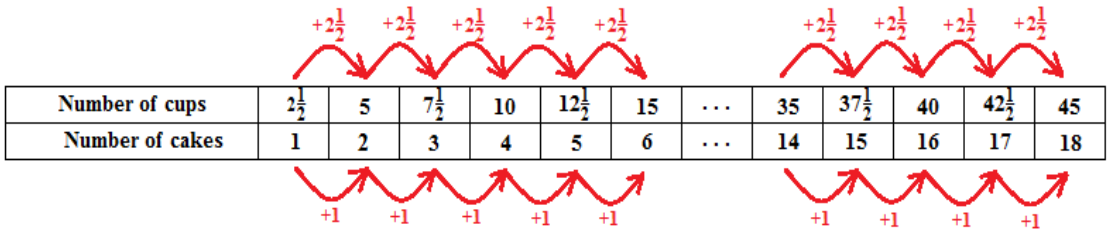
Figure 4.4. Student's worksheet 3

To solve this problem, the teacher asks the students to work individually (about 10 minutes). After that, the teacher asks them to discuss it in group. During the discussion in group, the teacher needs to walk around to see whether the students can solve the problem.

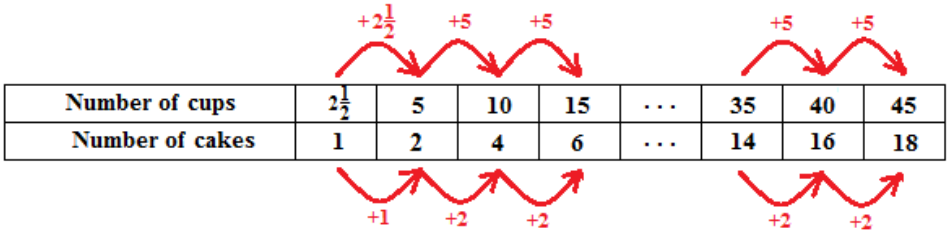
d. Conjecture expectations of the students' strategies

In determining how many cakes Mrs. Ani can make, there are several strategies that the students may do, such as:

1. The students still use pictures to answer the problem. They will draw rectangle to represent the cup. In this case, they will make several rectangles and shade each $2\frac{1}{2}$ rectangles until they get 45 cups.
2. The students will use ratio table. In using the ratio table, the students will do repeated addition or combination that they will count by one cake. In this case there are several strategies that the students possibly use, such as:
   - They will add "$2\frac{1}{2}$ cups for 1 cake" and "$2\frac{1}{2}$ cups for 1 cake" to get the proportion "5 cups for 2 cakes". Then, they keep adding "$2\frac{1}{2}$ cups for 1 cake" to each result until they get the proportion "45 cups for 18 cakes". Thus, they will reason that 45 cups of chocolate powder are enough to make 18 cakes.



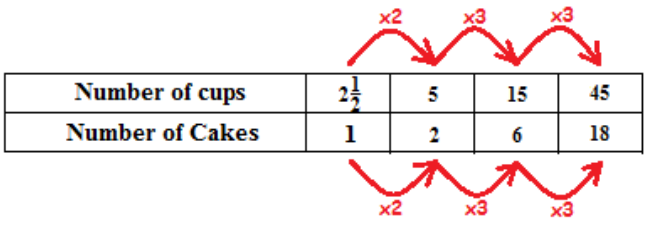
- They will add "$2\frac{1}{2}$ cups for 1 cake" and "$2\frac{1}{2}$ cups for 1 cake" to get the proportion "5 cups for 2 cakes". Then, they add "5 cups for 2 cakes" and "5 cups for 2 cakes" to get the proportion "10 cups for 4 cakes". They keep adding "5 cups for 2 cake" to each result until they get the proportion "45 cups for 18 cakes". Thus, they will reason that 45 cups of chocolate powder are enough to make 18 cakes.



Number of cups	$2\frac{1}{2}$	5	10	15	...	35	40	45
Number of cakes	1	2	4	6	...	14	16	18

3. They will do multiplicative strategy. In this case, there are many strategies that the students possibly use, such as:

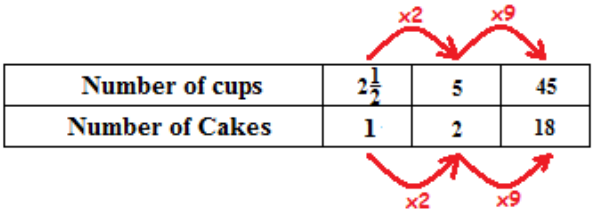
- They will double "$2\frac{1}{2}$ cups for 1 cake" to get the proportion "5 cups for 2 cakes". After that, they multiply "5 cups for 2 cakes" with 3 to get the proportion "15 cups for 6 cakes". Then, they multiply "15 cups for 6 cakes" with 3 to get "45 cups for 18 cakes". Thus, they will argue that 45 cups of chocolate powder are enough to make 18 cakes.



Number of cups	$2\frac{1}{2}$	5	15	45
Number of Cakes	1	2	6	18

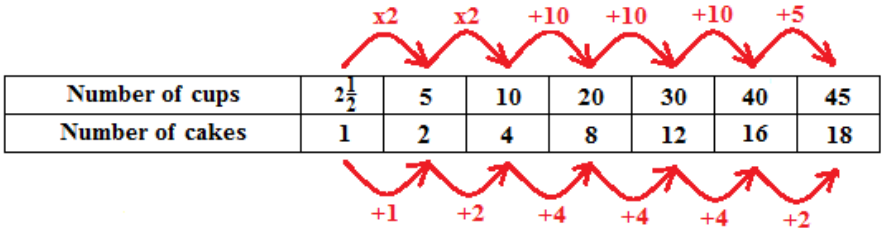
- The students double "$2\frac{1}{2}$ cups for 1 cake" to get the proportion "5 cups for 2 cakes". Then, they will multiply "5 cups for 2 cakes" with 9 in

order to get the proportion "45 cups for 18 cakes". Thus, they will say that 45 cups of chocolate powder are enough to make 18 cakes.



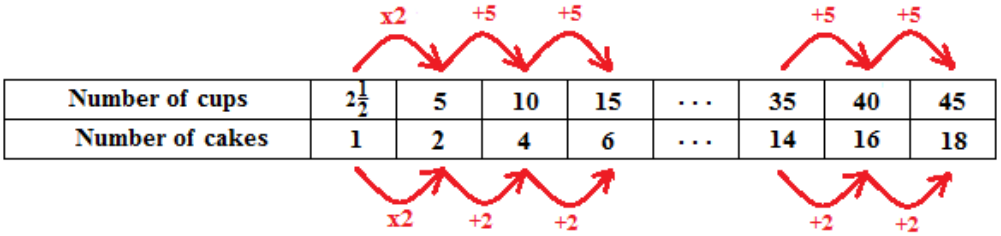
4. The students will do combination strategy. This means that they use more that one calculation to solve the problem. In this case, there are also several strategies that the students possibly use, such as:

- They will double "$2\frac{1}{2}$ cups for 1 cake" to get the proportion "5 cups for 2 cakes". They double it again to get "10 cups for 4 cakes". Then, they count by "10 cups for 4 cakes", which is easy to count by 10, to get 40 cups. In order to determine how many cakes that Mrs. Ani can make with 45 cups, the students will combine the number of cakes that Mrs. Ani can make within 40 cups and 5 cups. Thus, they will get the result which is 18 cakes.



- They will do the same strategy as the first point. However, they, at the end, will get that 40 cups are enough 16 cakes and there are 5 cups remains.

- They will double $2\frac{1}{2}$ to get 5. Then, they will add 5 repeatedly until they get 45 cups are enough to make 18 cakes.



Number of cups	$2\frac{1}{2}$	5	10	15	...	35	40	45
Number of cakes	1	2	4	6	...	14	16	18

e. Discussion

To begin the discussion session, the teacher can ask one of the groups to present their work in front of the class. If there are students make mistakes in solving the problem or their answer is not totally correct, such as: there are 5 remaining cups, the teacher, then, can start the discussion from this point. Thus, there will be a discussion between the teacher and the students or among the students.

4.4. Activity 4 : "Proportion in unit fraction"

1. Learning goals

- Use ratio table to solve proportional problems.

2. Starting point

The students have already discussed how to use ratio table in order to solve proportional problem which includes fractions.

3. Learning Materials

Student's worksheet4, chart paper, marker and poster of students' work in meeting 3.

4. Time Allocation

70 minutes

5. Description of the activity

In this meeting, the students will be given several problems which are aimed to let the students make their own ratio table and use it to solve proportional problems which include fractions. In this case, the students are asked to solve the following questions:

1. *Miss. Rika goes to supermarket to buy 15 kg of rice to supply her daily need. Everyday, she cooks* $\frac{1}{2}$ *kg of rice. Can you determine 3 kg of rice are enough in how many days?*
2. *Miss. Sri goes to supermarket to buy 25 kg of rice to supply her daily need. Everyday, she cooks* $\frac{1}{3}$ *kg of rice. Can you determine 3 kg of rice are enough in how many days?*
3. *Miss. Rani goes to supermarket to buy 15 kg of rice to supply her daily need. Everyday, she cooks* $\frac{1}{5}$ *kg of rice. Can you determine 3 kg of rice are enough in how many days?*

In this case, the students are asked to determine the result of the problems. At first, the teacher asks the students to work individually (about 10 minutes). After that, the teacher asks them to discuss it in group. After the students have

finished doing the problems in groups, the teacher makes a discussion in order to see the students' strategies to solve the problems by using ratio table.

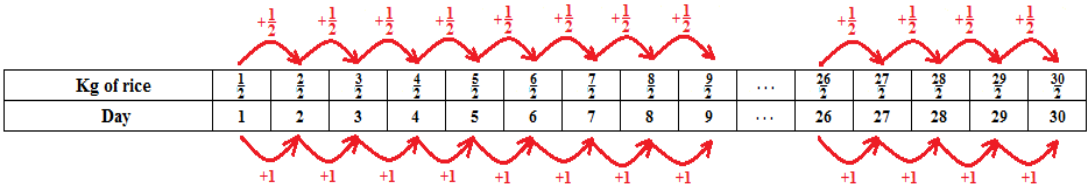
The intension of the discussion session does not focus on seeing the results that the students get, but it should focus on how the students can give their arguments to describe their strategies to answer the problems. Thus, the teacher, during the discussion, should be able to encourage the students to give their contribution to the discussion.

6. Conjecture of the students' strategies

In order to solve the first problem, there are several possible strategies that the students possibly use, such as:

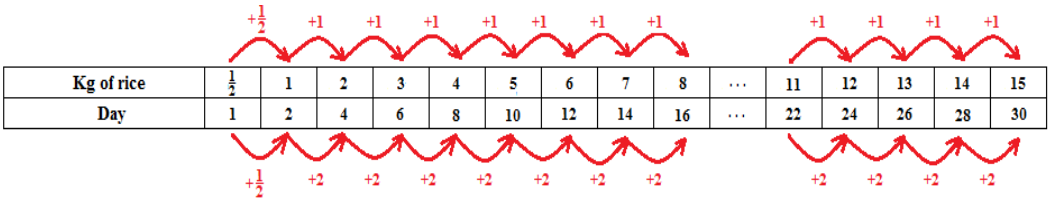
1. They will use repeated addition. In this case, there are several strategies that the students may possibly use, such as:

   - They will add "$\frac{1}{2}$ kg for 1 day" and "$\frac{1}{2}$ kg for 1 day" to get the proportion "1 kg for 2 days". Then, they keep adding "$\frac{1}{2}$ kg for 1 day" to each result until they get 15 kg are enough for 30 days. Thus, they will argue that 15 kg of rice will be enough for 30 days.



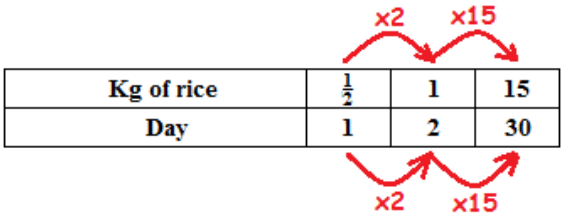
   - They will add "$\frac{1}{2}$ kg for 1 day" and "$\frac{1}{2}$ kg for 1 day" to get proportion "1 kg for 2 days". Then, they will add "1 kg for 2 days" continuously until they get 15 kg of rice and they get the portion of days which is

30 days. Thus, they will argue that 15 kg of rice will be enough for 30 days.

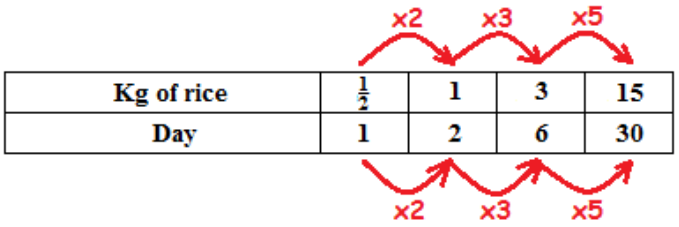


2. They will do multiplicative strategy. In this case, there are several strategies that the students may possibly use, such as:

- They will double "$\frac{1}{2}$ kg for 1 day" to get the proportion "1 kg for 2 days".Then, they multiply "1 kg for 2 days" with 15 and they get that 15 kg of rice are enough to supply for 30 days. Thus, they will say that 15 kg of rice are enough for 30 days.

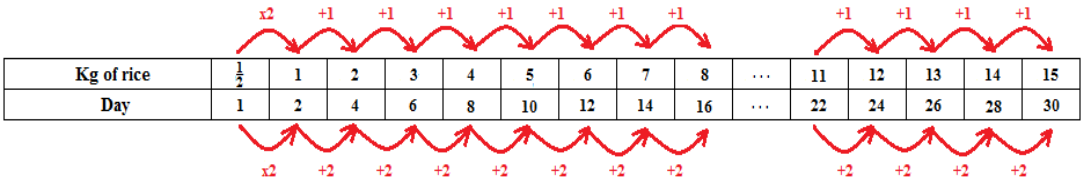


- They will double "$\frac{1}{2}$ kg for 1 day" to get the proportion "1 kg for 2 days". Then, they multiply "1 kg for 2 days" with 3 and get the proportion "3 kg for 6 days". At last, they multiply the result with 5 to get what is asked by the question, which is 15 kg of rice are enough for 30 days. Thus, they will say that 15 kg of rice are enough for 30 days.



3. Students will do "combination" strategy. This means that they use more that one strategy to solve the problem. Dealing with this, there are several strategies that the students possible use, such as:

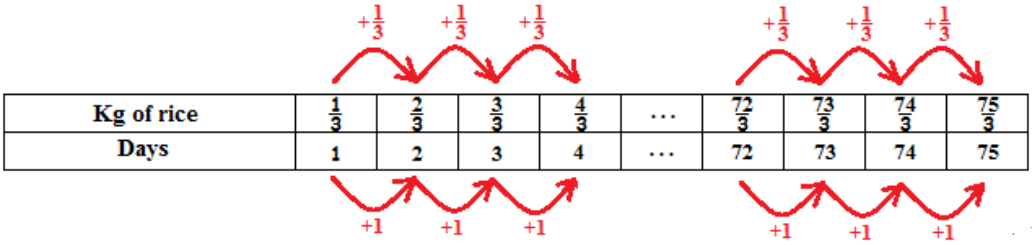
   - They will add "$\frac{1}{2}$ kg for 1 day" and "$\frac{1}{2}$ kg for 1 day" to get proportion "1 kg for 2 days". Then, they will add "1 kg for 2 days" and "1 kg for 2 days" to get the proportion "2 kg for 4 days". Then, they keep adding "2 kg for 4 days" until they get the proportion "16 kg for 32 days". Since they realize that they have to find the proportion 15 kg, they just subtract "1 kg for 2 days" from "16 kg for 32 days". Thus, they will say that 15 kg of rice will be enough for 30 days.
   - They will double "$\frac{1}{2}$ kg for 1 day" to get the proportion "1 kg for 2 days". Then, they add "1 kg for 2 days" and "1 kg for 2 days" to get the proportion "2 kg for 4 days". After that, they keep adding "2 kg for 4 days" until they get "16 kg for 32 days". At the end, they subtract "1 kg for 2 days" from the proportion "16 kg for 32 days" to get the answer that 15 kg of rice are enough for 30 days.



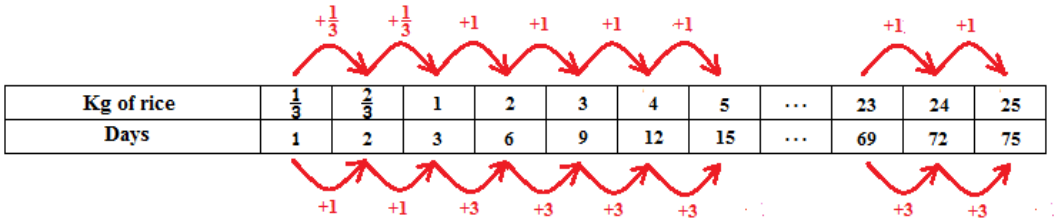
In order to solve the second problem, the students will do similar strategies to solve the first problem. To be more specific, the strategies, that the students possibly use, can be seen as follows:

1. They will do repeated addition strategy. In doing this strategy, they may come up with several possible calculations, such as:

   - They will add "$\frac{1}{3}$ kg for 1 day" to get the proportion "$\frac{2}{3}$ kg for 2 days". Then, they keep adding "$\frac{1}{3}$ kg for 1 day" until they get "$\frac{75}{3}$ kg for 25 day" or "25 kg for 75 days". Thus, they will say that 25 kg of rice will be enough for 75 days.

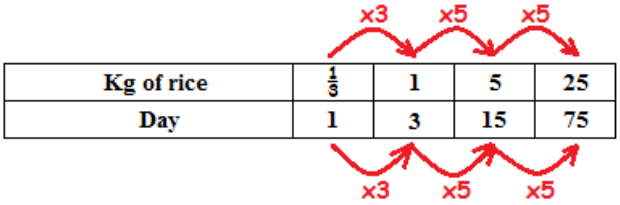


   - They will add "$\frac{1}{3}$ kg for 1 day" for three times to get the proportion "1 kg for 3 days". Then, they will add "1 kg for 3 days" continuously until they get the proportion "25 kg for 75 days". Thus, they will say that 25 kg of rice will be enough for 75 days.

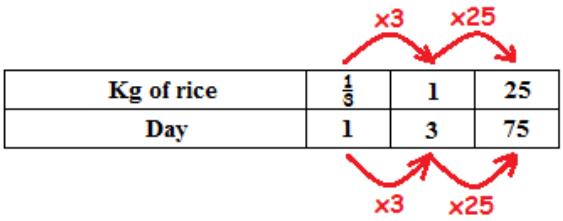


2. They will do multiplication strategy. In this case, there are several strategies that the students may possibly use, such as:

   - Students will multiply "$\frac{1}{3}$ kg for 1 day" with 3 to get the proportion "1 kg for 3 days". Then, they multiply "1 kg for 3 days" with 5 and multiply the result again with 5 to get the objective "25 kg for 75 days. Thus, they will argue that 25 kg of rice are enough for 75 days.

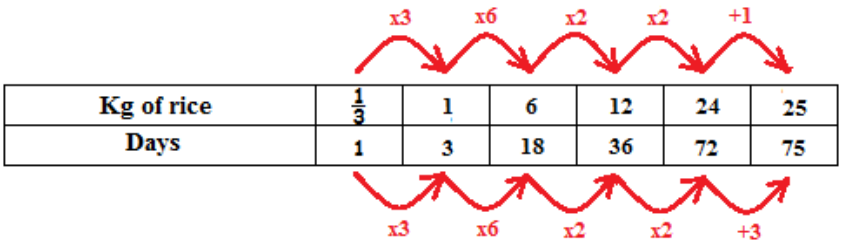


   - Students will multiply "$\frac{1}{3}$ kg for 1 day" with 3 to get the proportion "1 kg for 3 days". Then, they will straightly multiply "1 kg for 3 days" of each unit with 25. Thus, they get the result of the problem, which is 25 kg of rice are enough for 75 days.

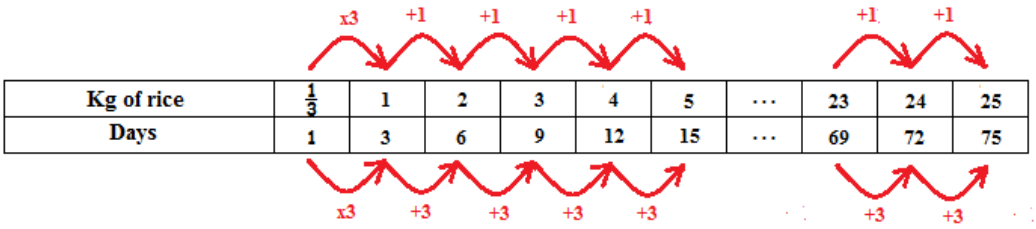


3. Students will do combination strategy. In this case, there are also several strategies that the students possibly use, such as:

- They will multiply "$\frac{1}{3}$ kg for 1 day" with 3 to get "1 kg for 3 days". Then, they multiply "1 kg for 3 days" with 6 and double the result again. They get the proportion "12 kg of for 36 days". Then, they continue doubling "12 kg of for 36 days" to get "24 kg for 72 days". Since they have known that 1 kg is enough for 3 days and 24 kg of rice are enough for 72 days, then they just combine the number of days within 24 kg of rice and the days within 1 kg of rice to get 25 kg of rice are enough for 75 days.



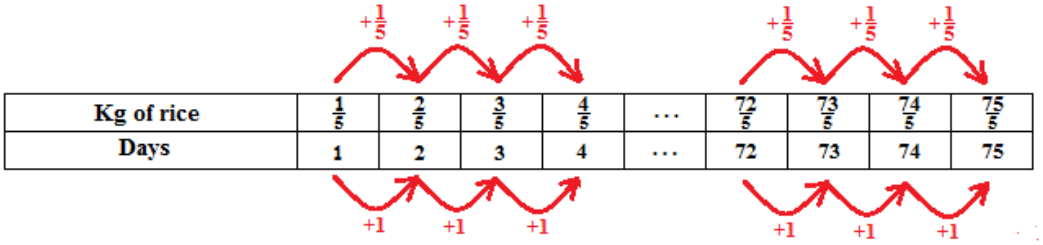
- They will multiply "$\frac{1}{3}$ kg for 1 day" with 3 to get "1 kg for 3 days". Then, they add "1 kg for 3 days" and "1 kg for 3 days" to get "2 kg for 6 days". They keep adding "1 kg for 3 days" until they get "25 kg for 75 days". Thus, they will say that 25 kg of rice will be enough for 75 days.



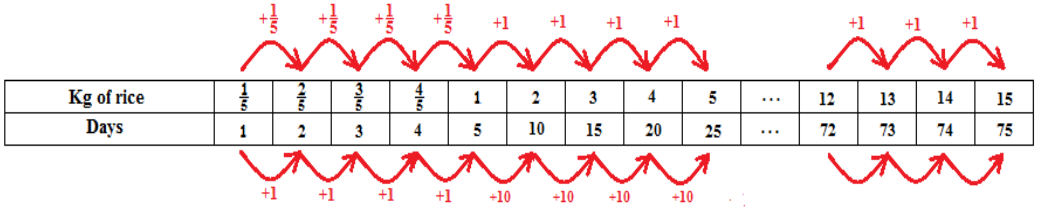
In order to determine the answer of the third problem, there are several strategies that the students will come up, such as:

1. Students will use addition strategy. In this case, there are several strategies that the students may possibly use, such as:

   - They will add "$\frac{1}{5}$ kg for 1 day" to get the proportion "$\frac{2}{5}$ kg for 2 days". Then, they keep adding "$\frac{1}{5}$ kg for 1 day" until they get "$\frac{75}{5}$ kg for 75 day" or "15 kg for 75 days". Thus, they will say that 15 kg of rice will be enough for 75 days.

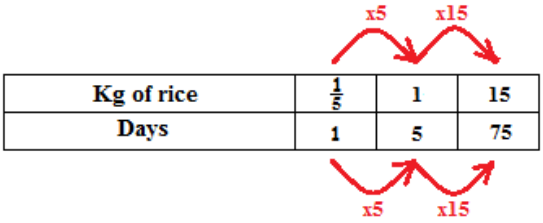


   - They will add "$\frac{1}{5}$ kg for 1 day" for five times to get the proportion "1 kg for 5 days". Then, they will add "1 kg for 5 days" continuously until they get the proportion "55 kg for 75 days". Thus, they will say that 15 kg of rice will be enough for 75 days.

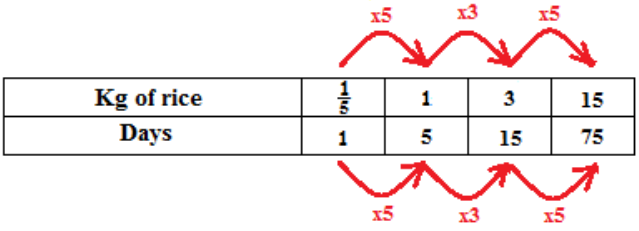


2. Students will use multiplication strategy. In this case, there are several strategies that the students may possibly use, such as:

   - They will multiply "$\frac{1}{5}$ kg for 1 day" with 5 and they get that 1 kg of rice is enough for 5 days. Then, they straightly multiply the result with 15. Thus, they obtain that 25 kg of rice are enough for 75 days.

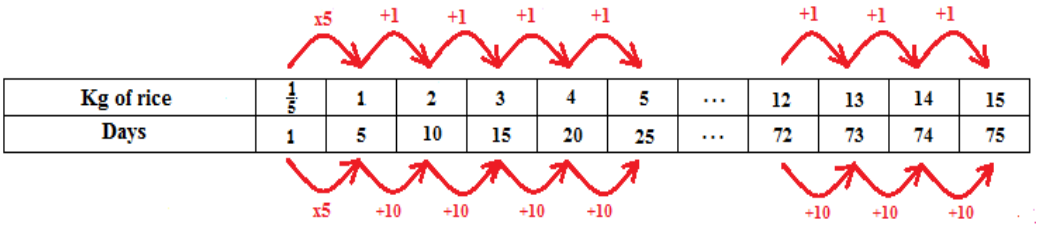


- Students, firstly, multiply "$\frac{1}{5}$ kg for 1 day" with 5 and get the proportion "1 kg for 5 days". Then, they multiply "1 kg for 5 days" with 3 to get "3 kg for 15 days". At the end, multiply the result again with 5. Hence, they get that 15 kg of rice can supply for 75 days.

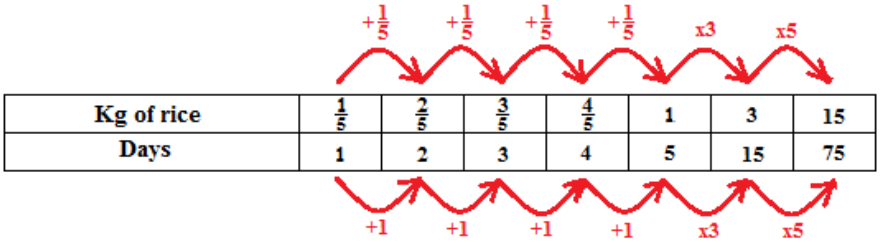


3. Students will do combination strategy. In this case, there are also several strategies that the students possibly use, such as:

- They will multiply "$\frac{1}{5}$ kg for 1 day" with 5 to get "1 kg for 5 days". Then, they add "1 kg for 5 days" and "1 kg for 5 days" to get "2 kg for 10 days". They keep adding "1 kg for 5 days" until they get "15 kg for 75 days". Thus, they will say that 15 kg of rice will be enough for 75 days.



- They will add "$\frac{1}{5}$ kg for 1 day" for five times to get the proportion "1 kg for 5 days". Then, they multiply "1 kg for 5 days" with 5 and multiply the result again with 3. Thus, they will get the proportion "15 kg for 75 days".



7. Discussion

After the students have worked in group to solve the problems, then the teacher makes a discussion which is aimed to discuss their work and strategies how they solve the problems. The discussion is intended to investigate the students' reasoning towards the given problem. Thus, the teacher needs to lead the students to a discussion where they can contribute their arguments and opinions about the problem.

To deal with this situation, the teacher can choose the group of students who makes mistakes or their answer is interesting to be discussed, such as: combination strategies. This strategy can make the students reason about what they have done. For instance, when the students use repeated addition and then they subtract it to get the answer. In this case, the teacher can start the discussion by asking the students to explain this.

4.5. Activity 5 : "Proportion in non-unit fraction"

a. Learning goals

Solve proportional problem by using model.

b. Starting point

- The students have learned how to represent fraction in a picture
- The students have learned how to use ratio table to solve proportional problem which includes fractions

c. Description of the activity

In beginning of this meeting, teacher asks the students to solve the following proportional problem.

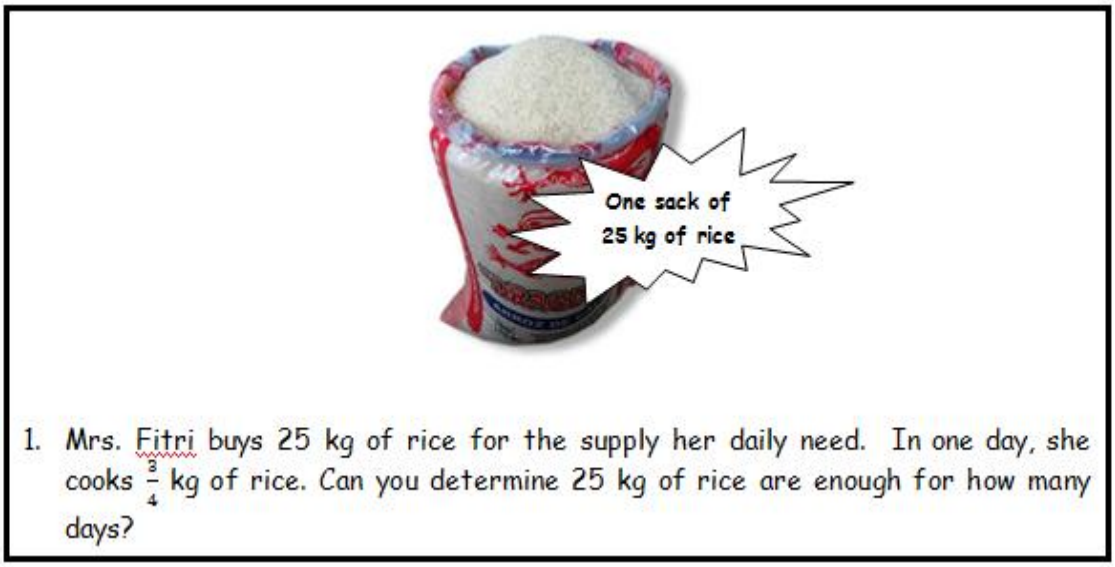


Figure 4.5. Student's worksheet 5

In this case, the students are asked to solve the problems in the students' worksheet. In solving the problems, the students are expected to use ratio table. After the students have finished doing the problems, the teacher makes a

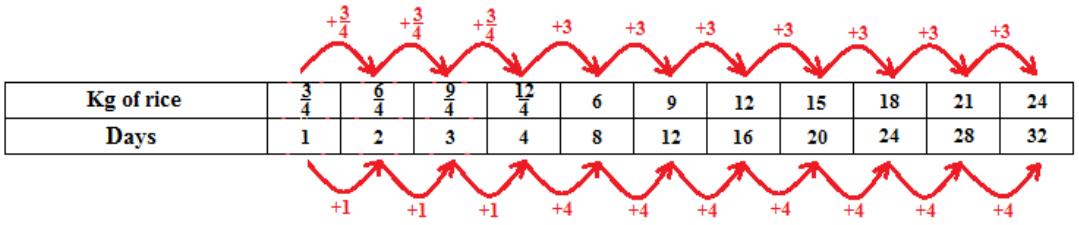
discussion in order to see the students' strategies to solve the problems by using ratio table.

d. Conjecture expectations of the students' strategies

In order to answer the problem, there are several strategies that the students may possibly use, such as:

1. They will use repeated "addition" strategy. In doing this, there are several strategies that the students possible use, such as:

   - They will add "$\frac{3}{4}$ for 1 day" and "$\frac{3}{4}$ for 1 day" to get $\frac{6}{4}$ for 2 days. They keep adding "$\frac{3}{4}$ for 1 day" to the result until they get "$\frac{12}{4}$ for 4 days" or "3 kg of rice for 4 days". Then, they add "3 kg for 4 days" and "3 kg for 4 days" to get "6 kg for 8 days". Then, they keep adding "3 kg for 4 days" until they get 24 kg of rice are enough for 32 days. To be more specific, see the following figure.



Kg of rice	3/4	6/4	9/4	12/4	6	9	12	15	18	21	24
Days	1	2	3	4	8	12	16	20	24	28	32

   In this case, they get the result that 25 kg of rice will be enough for 32 days and remains 1 kg of rice.

   - They will do the same strategy as the previous one. However, they will possibly use pictures automatically, such as: rectangle, to determine 1 kg of rice is still enough for 1 day. At the end, they get the result that 25 kg of rice will be enough for 33 days and $\frac{1}{4}$ remains.

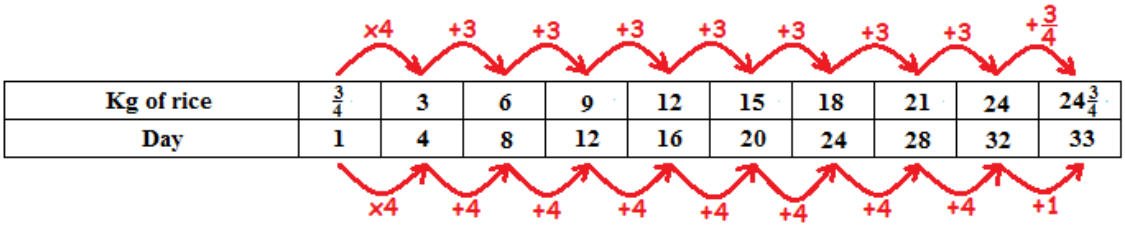
2. The students will possibly come up with the idea of "multiplication" strategy. In this case, there are several possible strategies that the students use, such as:

   - They will multiply "$\frac{3}{4}$ kg of rice for 1 day" with 4 to get the proportion "3 kg of rice for 4 days". Then, they will multiply "3 kg of rice for 4 days" with 2 to get the proportion "6 kg of rice for 8 days". They keep multiplying it with 3, 4, 5 … until they get the proportion "24 kg of rice for 32 days". Thus, they will get the answer that 25 kg of rice will be enough for 32 days with 1 kg remains.
   - They will multiply "$\frac{3}{4}$ kg of rice for 1 day" with 4 to get the proportion "3 kg of rice for 4 days". Then, they multiply with 4 to get the proportion "12 kg of rice for 16 days" and multiply it again with 2 to get the proportion "24 kg of rice for 32 days". Thus, they will get the result of the problem which is 25 kg of rice will be enough for 32 days with one kg of rice remains.

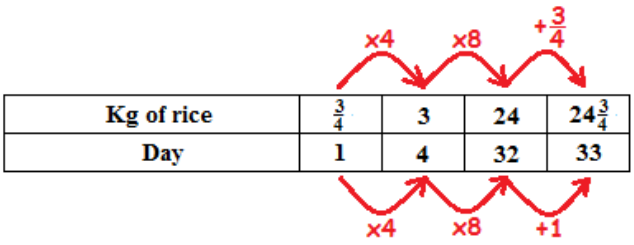
3. The students will do "combination" strategy. In this case, there are several strategies that the students may possibly use, such as:

   - Firstly, they will multiply "$\frac{3}{4}$ kg of rice for 1 day" with 4 to get the proportion "3 kg of rice for 4 days". Then, they will add the result with "3 kg of rice for 4 days" to get the proportion "6 kg of rice for 8 days". They keep doing this until they get "24 kg of rice for 32 days". Since they have to determine the proportion of 25 kg of rice for how

many days, they add or combine "24 kg of rice for 32 days" and "$\frac{3}{4}$ kg of rice for 1 day" that they have got before. Thus, they get the result that 24 $\frac{3}{4}$ kg are enough for 33 days with kg $\frac{1}{4}$ remains.



- Students will multiply each unit with 4 and they get the result that 3 kg of rice are enough for 4 days. Then, they straightly multiply the result with 8 and get that 24 kg of rice are enough for 32 days. Then, they will do the same strategy as no 1.



e. Discussion

In this meeting, the discussion session is conducted after the students finish working with their works. The discussion is intended to investigate the students' reasoning towards the given problem. Thus, the teacher need to lead the students to a discussion where they can contribute their arguments and opinions about the problem.

Dealing with this situation, the teacher can start the discussion by asking one group of students who makes mistakes in solving the problem or the group of students whose answer is not totally correct to present their work. For instance: they say that “25 kg of rice will be enough for 32 days with 1 kg remains”. This answer is a good way to start a discussion. In this case, teacher can ask the opinions of other students first about the answer by asking them “What do you think about their answer? Or do you agree with the answer? Or do you have something to add?”. If there are reactions from the students to explain about 1 kg remains, the teacher can ask him/her to explain it in front of the class. If there are no students who do not recognize it, the teacher can start a discussion by saying “Ok, your friend say that 25 kg of rice are enough for 32 days with 1 kg remains. What do you think of that 1 kg? Is still enough to cook rice?”. If the students say “yes”, then they teacher can challenge them “Can you determine 1 kg of rice is enough for how many days? Or you can draw picture to explain it”.

CHAPTER V

TESTING HYPOTHETICAL LEARNING TRAJECTORY

In the 4th chapter, we had explained how we designed the Hypothetical Learning Trajectory (HLT) to support the fifth-grade students to learn about proportion which includes fractions. In this chapter we present the retrospective analysis of the data that we had collected from the first cycle and the second cycle which were conducted at SD Al – Hikmah, Surabaya, within March 3rd, 2014 – March 14th, 2014. In this study, we conducted the first cycle for five meetings, in which each meeting consisted of 70 minutes, although the school regulation only provided us four meetings. To deal with this situation, we carried out the fifth meeting right after the fourth meeting was conducted. However, after making several revisions, we decided to conduct the second cycle for four meetings, in which each meeting also consisted of 70 minutes.

In this chapter, we also provide the results and our findings of the teaching experiment during the first cycle where we tested the HLT that we had designed within a small group of students, which were 6 students. Dealing with this, we will explain the results of pretest in order to get an overview of the students' starting points and to see students' strategies to solve proportional problems which include fractions. The remarks in this pretest were made into the HLT as a consideration to improve it. Then, we conducted the teaching experiment of the first cycle and analyzed it. Later on, we presented the results of the posttest. In this case, the remarks and findings during the first cycle were also used in order to

make an improvement to the HLT and then implemented it in the second cycle. The description and the analysis of the second cycle will be presented in the same order as the first cycle. At the end of the analysis, the conclusion of testing the HLT will contribute to answer the research question of this study.

5.1. Teaching Experiment

In this study, the teaching experiment was conducted within two cycles, the preliminary teaching (the first cycle) and the teaching experiment (the second cycle). The first cycle was conducted in order to make improvements to the Hypothetical Learning Trajectory (HLT) and the instructional activities based on the remarks and findings in this cycle. After improving the HLT and the instructional activities, then they were implemented in the second cycle. To be more detail, we will describe the teaching and learning process of the first and the second cycle and we will also provide the retrospective analysis of each cycle.

5.1.1. Preliminary Teaching Experiment (First Cycle)

The first cycle of this study was held on March 3rd, 2014 at SD Al – Hikmah, Surabaya. In conducting this cycle, we involved six fifth-grade students from class 5B. They were Galih, Akmal, Adit, Dhika, Hafiz and Raihan. In choosing these students, we conducted an interview with their mathematics teacher. In this case, these students were chosen by their mathematics teacher within a consideration of their level of understanding. Akmal was a smart student. Galih, Hafiz, and Raya were students who had a middle level. However, Adit and Dhika had a low level of understanding.

Furthermore, the process of enrolling this first cycle was started by conducting a classroom observation, doing a pretest, conducting an interview with the students, conducting the teaching and learning experiment and doing a posttest. Here, we will present a short description of each process of the first cycle.

5.1.1.1. Classroom observation

At the beginning of this study, we conducted the classroom observation in class 5B a week before conducting the lesson I. The aim of this observation was to investigate the teaching and learning process of the students when they learn mathematics with their teacher.

5.1.1.2. Pretest

After doing classroom observation, we conducted a 20-minute pretest which consisted of three problems to the students before conducting the first cycle. The aim of the pretest was to know the prior knowledge of the six students and to know how they solved kinds of problems related to proportions which include fractions. In doing this test, all students were asked to solve the problems by themselves.

5.1.1.3. Interview with the students

After doing the pretest, we conducted an interview with the students. This interview was made in order to check the strategies what they did during the pretest. Moreover, it was also aimed to get an impression of the starting point of each student in this cycle. To deal with this, we, therefore, interviewed all students to get these data.

5.1.1.4. Teaching and learning process

1. Lesson I : Solving rice problem by using students' models

In the first meeting, the teacher conducted a teaching and learning process which was aimed to make models as the representation of the situation and to use the models in order to solve problem. In this meeting, the students were given a worksheet which consisted of one problem and they were asked to solve it individually (about 10 minutes). In that problem, the teacher told his story of buying 20 one-kilogram packs of rice. Then, the students were asked to help the teacher determine for how many days that the teacher can consume 20 packs of rice if he cooks $\frac{3}{4}$ kg of rice in one day. In solving this problem, there were several strategies and different answers that the students did. In order to solve the problem, some students, on the one hand, were able to represent $\frac{3}{4}$ in a drawing, such as: representing each pack of rice by a square. Then, they divided the square into four equal parts and shaded three parts of it to get $\frac{3}{4}$. On the other hand, other students did not know how to deal with the problem.

After doing the work, they were asked to work in group, where each group consisted of 3 students, to find out how to solve the problem. During the discussion, the students can share their ideas and strategies with their friends and put their strategy on a poster. After doing discussion in group, they need to show and to present their work in front of the class. After the presentation, the teacher and the students made a conclusion of the problem at the end of the teaching and learning process. This conclusion will be used in the second meeting to remind the students about what they did in the first meeting.

2. Lesson II : Solving rice problem with a ratio table

In second lesson, the teacher continued the teaching and learning process. This time, the goals of the learning process were making sense of proportion, introducing ratio table and start working with the ratio table. Dealing with this, the students were given the same problem as the previous meeting. They were asked to determine for how many days that the teacher can consume 20 packs of rice if he cooks $\frac{3}{4}$ kg of rice in one day. Indeed, the students had known the answer of the problem since they had learned it in meeting I. However, they were encouraged to get the impression of an early stage of ratio table where they will use it in the next meetings. Then, the teacher can put the students' answer in the form of table which we call "ratio table" in this study.

In doing the activity, the students were asked to work in group to find the solution of the problem by using ratio table and put it on a poster paper. When the students worked in group, they were asked to solve the problem by with ratio table by continuing the table that has been discussed at the beginning of the lesson. After the students worked in group, the teacher carried out a whole-class discussion where one group presents their poster and another group can ask questions. After the students in each group presented their poster, the teacher guided the students to conclude about the solution of the problem.

At the end of the lesson, the teacher introduced that the table that they were working with was called ratio table. They will learn more about this ratio table in the next meetings.

3. Lesson III : Making cakes to sell

In the third lesson, the teaching and learning process continues. The aims of this meeting were to use ratio table that they had learned in the second meeting to solve proportional problem and to describe how to use ratio table to solve proportional problem. The students were given a worksheet that consisted of one problem which was designed to let the students practice to solve proportion problem by using ratio table and to make them able to use ratio table. Furthermore, the problem was about helping a cake seller to determine the number of chocolate cakes that can be made within 45 cups of chocolate powder if she needs $2\frac{1}{2}$ cups of chocolate powder.

In doing the activity, the students are asked to work individually about 10 minutes. Then, the teacher asks them to discuss the problem in group and put their strategy in a poster paper. After working in group, the teacher holds a whole-class discussion. During this discussion, they students need to present their work in front of the class and their friends can ask questions if they do not understand something about the presentation.

After the students presenting their poster, the teacher guides the students to make the conclusion of the problem and what mathematical operations that they can use in a ratio table. At the end of this meeting, the teacher gives them a home work where they can practice to operate ratio table at home.

4. Lesson IV: Proportion in unit fraction

In this study, we conducted the fourth meeting about 70 minutes the $day after the third lesson was carried out. The aims of this meeting were to make ratio

table and use it to solve proportional problem. In this meeting, the teacher gave the students three problems to solve, such as:

1. *Miss Rika goes to supermarket to buy 15 kg of rice to supply her daily needs. Everyday, she cooks* $\frac{1}{2}$ *kg of rice. Can you determine 15 kg of rice are enough for how many days?*
2. *Miss Sri goes to supermarket to buy 25 kg of rice to supply her daily needs. In one day, she cooks* $\frac{1}{3}$ *kg of rice. Can you determine 25 kg of rice are enough for how many days?*
3. *Miss Rani goes to supermarket to buy 15 kg of rice to supply her daily needs. Everyday, she cooks* $\frac{1}{5}$ *kg of rice. Can you determine 15 kg of rice are enough for how many days?*

In order to solve the first problem, the teacher gave the students time to think individually about 5 minutes and discuss in group. During the discussion, the teacher provided poster paper so they can write their strategy on it. Then, the teacher asked them to talk in group how to solve the problem. After the students discussed the problem, they need to share and to explain their poster to another group. Furthermore, the students worked on the second problem after the presentations were over. As usual, all students did the same activity when they worked with the first problem.

Since the time was limited for us to discuss the third problem, we asked the students to solve the third problem at home at the end of the lesson.

5. Lesson V: Proportion in non-unit fraction

Since the school only gave us four meetings to enroll this study, we, therefore, conducted this lesson 5 right after the fourth lesson was conducted. The aim of this lesson was to investigate how the students answer proportional problem which includes fractions by using models. In this meeting, the students were given a problem in student's worksheet 5 to solve. The problem was solving a rice problem that Mrs Fitri buys 25 kg of rice and she cooks $\frac{3}{4}$ kg of rice in one day. In this case, they were asked to determine 25 kg of rice were enough for how many days.

In order to solve the problem, the teacher gave them time to think by themselves about 10 minutes and discussed it in group. When they worked in group, they need to write down their strategy in a peace poster paper. After they worked in group, the teacher held a whole-class discussion where the students in each group had to present their poster. During the discussion, other students were free to give comments, to add something and even to ask for questions.

After they presented their work, the teacher guided them to make a conclusion of the problem and how they solved it. At the end of the meeting, the teacher asked them what strategy that they thought best way to solve proportional problem.

5.1.1.5. Posttest

In this study, we carried out a 35-minute test for the students after the teaching and learning process was conducted. The test consisted of three problems which were aimed to evaluate the development of students' understanding

towards what they had learned within the whole teaching and learning experiment. In doing this test, the students were asked to solve the problems individually.

5.1.2. Teaching Experiment (Second cycle)

In this study, we carried out the second cycle on March 10, 2014 at SD Al – Hikmah, Surabaya. Before conducting the second cycle, we interviewed the teacher and carried out a classroom observation. In this cycle, we involved 30 students from class 5C. In conducting the second cycle, we also conducted the pretest, did the teaching experiment and made the post test in that class. Before conducting the teaching experiment, we made several improvements based on the remarks and findings in the first cycle after testing the Hypothetical Learning Trajectory in the real teaching and learning process within a small group of students. Subsequently, the improvements were applied in the second cycle. In conducting the teaching experiment, the students were taught by their own mathematics teacher for four lessons.

In this cycle, there were several revisions of the worksheet and the lesson. Based on the findings in the first cycle, the third question in the student's worksheet 4 was removed since the time allocation was not enough to do the third question. Moreover, the students were able to operate the model, such as ratio table, to solve problem. In addition, the order of lesson 5 and 6 were set within one meeting. Thus, there were four meetings in the second cycle where each lesson took 70 minutes. In order to evaluate the students' understanding towards the topic, we made a posttest at the end of the teaching and learning process.

5.2. Retrospective Analysis

The data that we had collected in the teaching experiments, the first cycle and the second cycle, will be analyzed in this retrospective analysis. In this case, we will analyze the data for each cycle and also provide several transcripts of the video fragments for each lesson which are interested to be analyzed. The transcript was made to give us information about the students' thinking when they were learning this topic. At the end, the results and the findings of this retrospective analysis were used to make an improvement to the Hypothetical Learning Trajectory and to answer the research question of this study.

5.2.1. Preliminary teaching experiment (Cycle I)

5.2.1.1. Pretest

At the beginning of cycle 1, we conducted a pretest which consisted of three questions to the six students. The aim of the pretest was to get the information about the students' prior knowledge and to get the data students' strategies in order to solve proportional problem which includes fractions. In doing the pretest, the students worked individually about 20 minutes. In this test, Galih was able to answer two questions correctly among the three questions given which was considered as the highest score among all the students that also followed the test. Akmal was able to answer one problem correctly. Meanwhile, Hafiz, Dhika, Adit and Raya were trying to answer all problems but they did not succeed it.

Furthermore, the students' pretest showed that most of the students still did not know how to deal with the problems. Thus, they just tried to solve the

problem by using their understanding about the operation of fraction. Here, we will describe the students' strategies to solve each problem in the pretest

Problem 1

This problem asked the students to determine for how many days a family consumes 20 packs of rice if they cook $\frac{3}{4}$ kg of rice in one day. Among the six students, two students, Galih and Akmal, can give a meaningful strategy to solve the problem. However, only one student, Galih, can answer this problem correctly. On the one hand, Galih drew twenty boxes where each box was considered as the representation of one pack of rice. On the other hand, Akmal did not realize that he drew 21 boxes. They did the same way that they divided each box into four equal parts and shaded three parts of it to make $\frac{3}{4}$ kg. In this case, Akmal divided the box horizontally and Galih divided the box in vertical and horizontal. After that, they counted each $\frac{3}{4}$ kg continuously from the first box until the last box. At the end, Galih found that 20 packs of rice will be enough for 27 days and Akmal noticed that 20 packs of rice are enough for 28 days (see figure 5.1. and figure 5.2.).

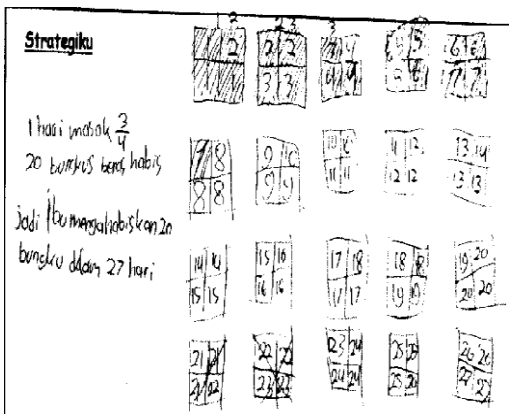


Figure 5.1. Galih's strategy

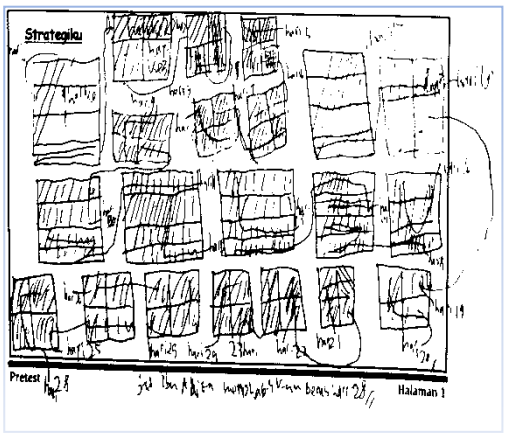


Figure 5.2. Akmal's strategy

At the same time, other students tried to answer the problems but their answer was still incorrect. For instance, Hafiz transformed $\frac{3}{4}$ into percent which was 75%. Then, he subtracted 25% from 100%. After that, he divided 25% by 5 to get 5 kg of rice. He noticed that one day Mrs. Anita cooks 5 kg of rice. In order to get 20 boxes, he just multiplied 5 kg with 4. Thus, he found that Mrs. Anita can cook 20 boxes of rice in 4 days. Dhika answered the problem by using multiplication strategy. He straightly multiplied the denominator and the numerator of $\frac{3}{4}$ with 20 to $\frac{60}{80}$ = 1,3. Thus, he got that Mrs. Anita consumes 20 packs of rice in 1,3 days. Raya tried to solve this problem by using subtraction strategy. In this case, he subtracted $\frac{3}{4}$ from 20 (he wrote $\frac{20}{1} - \frac{3}{4}$). He divided 20 by 4 and got $\frac{2}{1}$ = 2. Therefore, he found out that Mrs. Anita can consume the rice in 2 days. Meanwhile, Adia did multiplication and division strategies. Firstly, he multiplied 20 with 3 to get 60. Then, he divided 60 by 4 and he got the answer 15 days. He also made a drawing, but it was not clear what those pictures mean since he did not give any explanations about them. Figure 5.3. shows Adit's strategy in order to solve the problem.

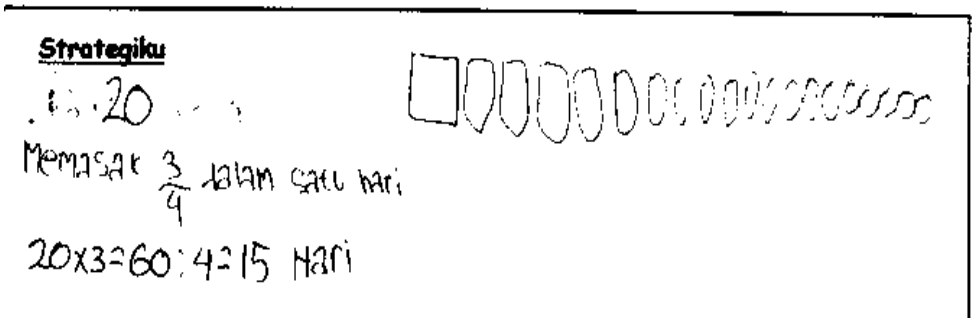


Figure 5.3. Adit's strategy

Problem 2

This problem asked the students to determine the number of chocolate cakes that can be made within 30 cups of chocolate powder, if the cake seller needs $2\frac{1}{2}$ cups to make one cake. This time, only Galih can solve this problem. He drew 30 boxes that represent 30 cups of chocolate powder. Then, he counted each $2\frac{1}{2}$ boxes and made a sign of it as the representation of the number of chocolate cakes. Finally, he found out that 30 cups of chocolate powder were enough to make chocolate 12 cakes.

At the same time, Akmal, Hafiz, Dhika, Adia and Raya did not succeed to solve this problem. Akmal, firstly, was trying to solve the problem by drawing pictures. However, he could not continue it and he crossed it out. Then, he made a new solution which was multiplying $2\frac{1}{2}$ with 2 and got 5 cups. He multiplied it again with 6 and he got 30 cups. At the end, he noticed that 30 cups of chocolate powder were enough to make 6 cakes.

Meanwhile, Raya, ADhika and Adia did the same strategy as they did in order to solve the first problem. Raya transformed $2\frac{1}{2}$ into $\frac{5}{2}$ and subtracted it from 30 (he wrote $\frac{30}{1} - \frac{5}{2}$). Then, he divided 30 by 2 and he got $\frac{10}{1}$ = 10. Thus, he found that 30 cups of chocolate powder were enough to make 10 cakes. ADhika also transformed $2\frac{1}{2}$ into $\frac{5}{2}$ and multiplied the denominator and numerator with 30 and he got $\frac{150}{60}$ = 2,5. Thus, he noticed that 30 cups of chocolate powder were enough to make 2,5 cakes. Adit drew 30 circles and he transformed $2\frac{1}{2}$ into $\frac{5}{2}$. Then, he

straightly wrote 30 cups were enough to make 10 cakes. As usual, it was not clear how he got the answer since he did not explain it. Furthermore, Hafiz's solution was different from his solution in problem 1. This time, he wrote 30 is the chocolate and $2\frac{1}{2}$ = 5 chocolates. Then, he divided 30 by 5 and he got 6. Thus, he found out that 30 cups of chocolate powder were enough to make 6 chocolate cakes. To be more detail, see figures 5.4., 5.5., 5.6., 5.7., 5.8. and 5.9. below.

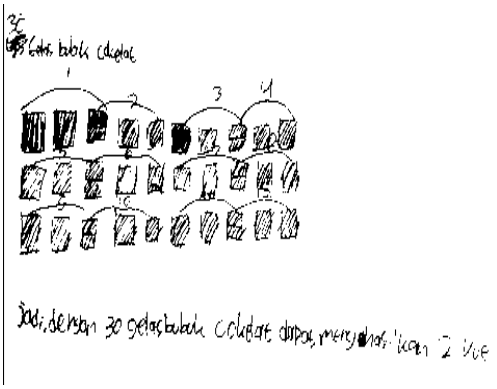


Figure 5.4. Galih's strategy

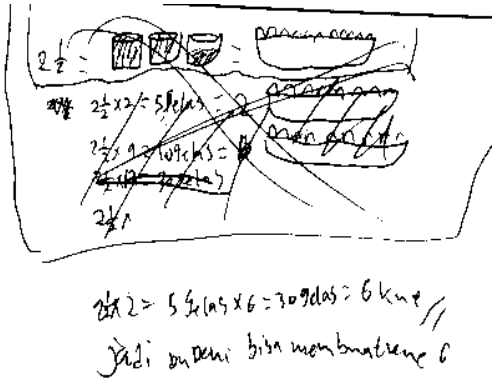


Figure 5.5. Akmal's strategy

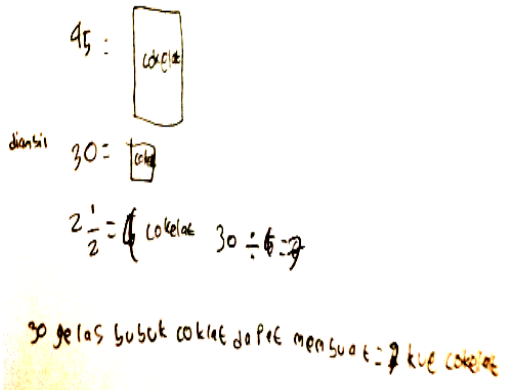


Figure 5.6. Hafiz's strategy

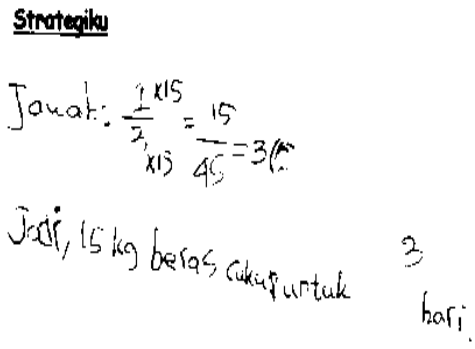


Figure 5.7. Dhika's strategy

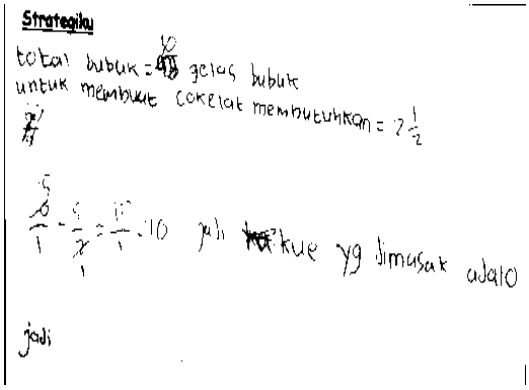


Figure 5.8. Raya's strategy

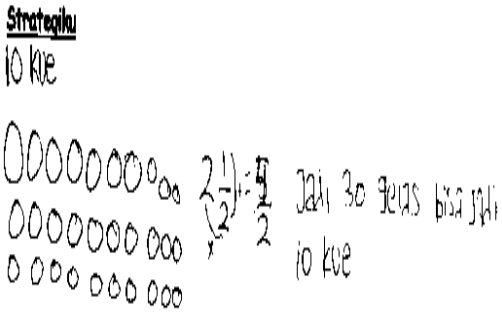


Figure 5.9. Adit's strategy

Problem 3

The problem 3 was also about solving a rice problem. This time, they were asked to determine for how many days that a 15-kilogram sack of rice are enough if someone cooks $\frac{1}{3}$ kg of rice in one day. In this problem, only one student, Akmal, can answer it correctly. He used his proportional reasoning in order to solve this problem. In this case, he wrote 1 kg of rice was enough for 3 days or 3 days = 1 kg of rice. Then, he straightly multiplied 3 with 15 and he got 45 and wrote 45 days = 15 kg of rice. Thus, he wrote 45 as the answer of the problem.

Furthermore, other students did not succeed to answer this problem. On the one hand, some students just repeated what they did in order to solve the first problem and the second problem. For instance, Hafiz changed 15 kg into 15% and divided it by 5 and got $\frac{3}{5}$. Then, he multiplied $\frac{1}{3}$ with $\frac{3}{5}$ and got $\frac{4}{8}$. Thus, he wrote that 15 kg of rice was enough for 4 days. As usual, it was not clear how he got 15%, $\frac{3}{5}$ and the conclusion since there were no explanations about those.

On the other hand, Galih tried to solve the problem but his answer was not right. In order to solve this problem, he drew a box which represents one sack of

15 kg of rice. Then, he divided the box into three parts and shaded it. Thus, he made a conclusion that the amount of rice will enough for three days. To be more detail, see figures 5.10, 5.11, 5.12., 5.13.,5.14 and 5.15 below.

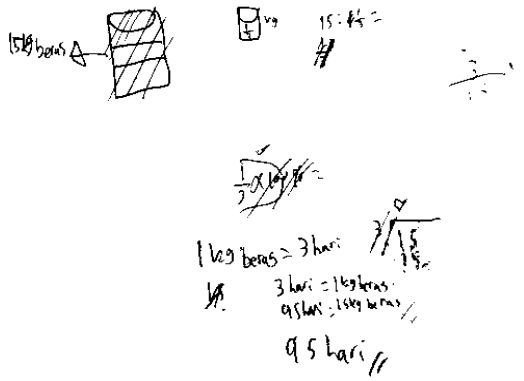


Figure 5.10. Akmal's strategy

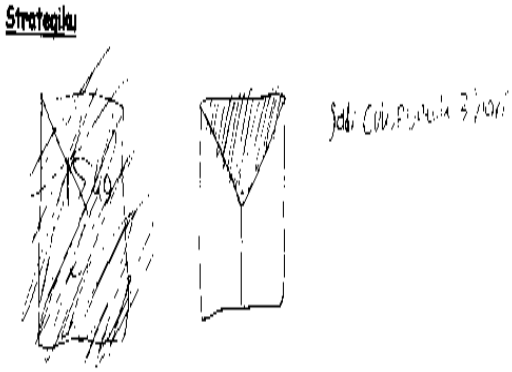


Figure 5.11. Galih's strategy

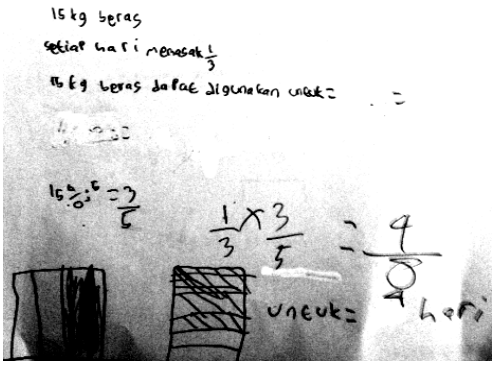


Figure 5.12.. Hafiz's strategy

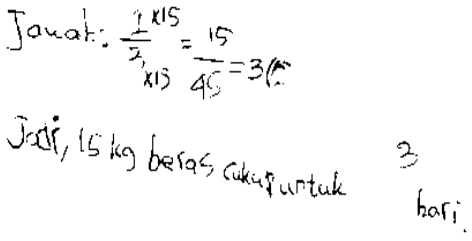


Figure 5.13. Dhika's strategy

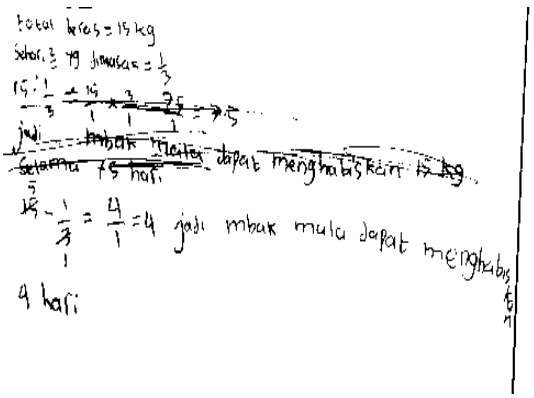


Figure 5.14. Raya's strategy

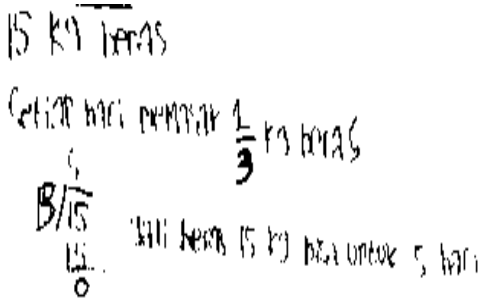


Figure 5.15 Adit's strategy

5.2.1.2. Interview with the students

The interview, with the six students, was carried out one day after the pretest was conducted. This interview was carried out in order to know students' strategies to solve proportion problems which include fractions. During this interview, they were asked to explain what they did in the pretest. Here, we can not put all the transcripts of the interview, so we select a moment of the interview which is interesting to be discussed. In this case, the word "interesting moment" means a moment where the students struggle to find the answer of the problem but they did succeed it. Thus, the following short fragment shows how Galih struggled to interpret the third problem and tried to answer it.

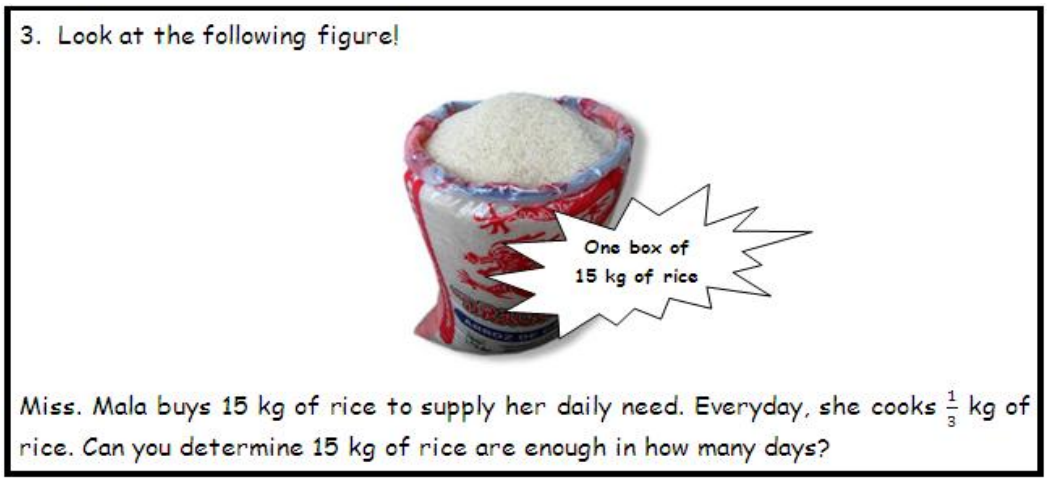


Figure 5.16. Question of interview

Fragment 5.1. Galih's struggle to answer the third problem of pretest

1. Researcher : Can you read the question?
2. Galih : Yes. (*reading the question*)
3. Researcher : Do you understand the problem?
4. Galih : Yes
5. Researcher : Can you say it with your own words what the problem is?
6. Galih : Oke… Mrs. Mala buys 15 kg of rice. She cooks $\frac{1}{3}$ kg of rice in one day.
   : In how many days 15 kg of rice will be enough?
7. Researcher : What did you do to solve the problem?

8. Galih : I drew a box. I divided it into three parts and I shaded one part of it.
9. Researcher : What does this box mean?
: (*Pointing to the first box*)
10. Galih : It represents one sack of 15 kg of rice.
11. Researcher : Ohh… Why do you divide the box into three parts?
: (*Asking to explain the second box*).
12. Galih : Hmm… Because everyday Mrs. Mala cooks $\frac{1}{3}$ kg of rice.
: So, I divided it into three parts and shaded one part of it.
13. Researcher : Why do you shade one part?
14. Galih : To get $\frac{1}{3}$ kg.
15. Researcher : What is the conclusion?
16. Galih : So, 15 kg of rice of rice are enough for 3 days.

Based on the students' pretest and the fragment 5.1 above, it can be seen that Galih understands how to represent fractions, such as: $\frac{3}{4}$, $2\frac{1}{2}$ and $\frac{1}{3}$, in a picture. For instance, he draws a box, divides it into three parts and shades one part of it in order to represent $\frac{1}{3}$ in a picture. However, it was still difficult for him to interpret the third problem and to solve it. Therefore, there is a need to support those students to learn about proportion which includes fractions.

Conclusion of pretest and interview

Based on the pretest and the interview above, it can be seen that there are several strategies that fifth-grade students may possibly use. Although some students can give a meaningful strategy, such as drawing pictures, most of the students, however, tend to apply what they understand to solve proportional problem, such as doing multiplication, subtraction, division and even using percentages. This is because they do not get the sense of the problem. For instance, Galih represents one sack of 15 kg of rice with a box and he just divides it into three parts. He states that this because she cooks $\frac{1}{3}$ kg (see line 11 and 12).

Moreover, the results of the pretest and the interview with the students show about the starting point of each student. Galih has an understanding to how to represent a fraction of "one unit" in a picture and knows the part of the fraction. However, he does not understand to interpret the part of a fraction within a unit of things, such as one sack of 15 kg of rice. For example, Galih draws a box to represent one sack of 15 kg of rice. Then, he divides the box into three parts and shades one part to get $\frac{1}{3}$ kg. Thus, he gets that one sack of 15 kg rice is enough for 3 days (see lines 8 – 15). Based on this example, it can be explicitly seen that he does not draws the conclusion based on $\frac{1}{3}$ of "15 kg" but he sees $\frac{1}{3}$ of "one sack".

5.2.1.3. Teaching and learning process

1. Meeting 1: Solving rice problem by using students' models

The first meeting was conducted on Monday, March 3rd, 2014 in the archive room. In this lesson, the six children, Akmal, Galih, Dhika, Adit, Raya and Hafiz, were taught by the researcher himself for about 70 minutes. The purpose of this lesson was to make models as the representation of the situation and to use the models in order to solve problem. In general, this lesson indicates that most of the students were able to use model as a tool to answer proportional problem. Dealing with this, the following retrospective analysis shows how the six students reason during the teaching and learning process.

1. Initial activity

At the beginning of this lesson, the teacher showed them a one-kilogram box of rice and told a story to the students. He told the students the following story.

"*My wife bought twenty packs of rice yesterday in a supermarket. In one day, she cooked* $\frac{3}{4}$ *kg of rice for our daily needs*".

After the teacher told the story, he asked the students to determine the number of days that 20 packs of rice will last. In order to do that, the students were asked to solve the problem individually in the students' worksheet 1 about 10 minutes and discussed it together in group.

In our Hypothetical Learning Trajectory (HLT), we conjectured the students did not know how to deal with the problem. Thus, the teacher needed to guide the students to get the idea of determining the solution in a meaningful way. Additionally, we also conjectured that there were some students could give their answer by using their own model, such as drawing pictures. In the actual teaching and learning process, both conjectures happened. Some students did not know how to deal with the problem and some students could give their answer by using their own model. Since there were some students could give their answer by drawing pictures, the teacher, subsequently, let the students work individually. These different strategies will lead them into a discussion.

2. Main activity

After 10 minutes working individually, the teacher asked the students to work in group about 15 minutes. The teacher asked the students to make group where each group consisted of three students. Thus, there were two groups in which the first group consisted of Galih, Raya and Adit and the second group consisted of Akmal, Hafiz and Dhika. In this case, they need to put their strategy in a poster

paper. During the discussion, each student shared their strategy with their friends in the group.

In this case, while the first group was still discussing what strategy that they wanted to put in the paper, the second group had decided it. Akmal, a member of the second group, straightly drew 20 boxes and he divided each of them in to four parts horizontally and vertically. Then, he counted each three little parts and make a sign above it, such as: day 1, day2, day3, …, day 26 and they noticed that there was $\frac{2}{4}$ kg remains. Thus, the got the conclusion that 20 packs of rice will be enough for 26 days with $\frac{2}{4}$ remains.

At the same time, the first group was still working with their poster. They decided to put each strategy on the paper. This is because Galih's answer was different with his two friends Raya and Adit. In this case, Raya and Adit came up with the strategy of multiplication. They straightly multiplied $\frac{3}{4}$ with 20 and got the answer that 20 packs of rice were enough for 15 days. However, Galih did not agree with his friends' answer. Then, he also wrote his strategy in that paper. To solve the problem, he drew 20 boxes as the representation of 20 one-kilogram packs of rice. Then, he divided each box into four parts and put the same number for each three little parts as the representation of the number of days (see figure 5.17. and figure 5.18. below).

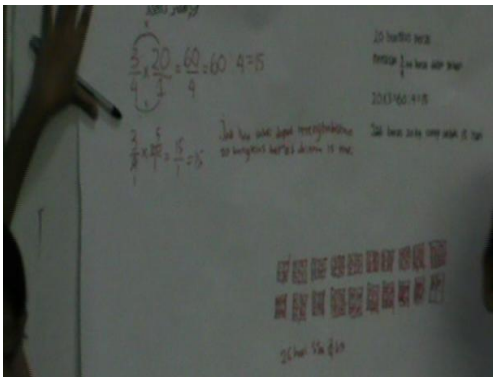


Figure 5.17. The strategy of group 1

Figure 5.18. The strategy of group 2

Whole-class discussion

After the students worked in group, the teacher asked them to present their strategy to the whole group in front of the class. In this case, short fragment is provided to show the first group reasons how to solve the problem.

Fragment 5.2. The strategies of group 1 to solve the first problem.

1. Teacher : What did you do to solve the problem?
2. Adit : So… This is $\frac{3}{4}$ times $\frac{20}{1}$. (*Explaining the first strategy*).
   : Three times 20 is 60 and 4 times 1 is 4. Then, 60 is divided by 4 is 15.
   : So, Mrs. Sahri can consume 20 packs of rice in 15 days.
3. Teacher : Next…
4. Adit : And this one… (*Explaining the second strategy*)
   : Hmmm… Mrs. Sahri buys 20 boxes of rice and $\frac{3}{4}$ kg in one day.
   : I just multiplied 20 with 3, which is 60 and 60 is divided by 4, which is 15.
   : So, 20 packs of rice are enough for 15 days.
5. Teacher : Wait… Do you have any questions? (*Asking another group*)
6. Akmal : Yes… How do get 3? (*Showing* $20 \times 3 = 60 \div 4 = 15$)
7. Adit : Since it is $\frac{3}{4}$.
8. Raya : Why don't you write $\frac{3}{4}$?
9. Group 1 : Hmmm… (*Thinking*)
10. Raya : Hah… This way… (*Asking Adit to explain Galih's strategy*)
11. Adit : And this one… I do not know… (*Showing Galih's strategy*).
12. Raya : Galih… Please explain it… (*Asking Galih to explain it*).
13. Teacher : Ok… Please Galih…
14. Galih : Mrs. Sahri has 20 packs of rice. In one day, she cooks $\frac{3}{4}$.

: So, one – one – one, two – two – two, three – three – three, … , twenty six –twenty six – twenty six. (*Counting each three little parts*).
: So, there are 26 days and $\frac{2}{4}$ kg of rice remains.

15. Teacher : Oh… Do you have any questions?
(*Asking another group for clarification*)

16. Group 2 : Hmmm… No…

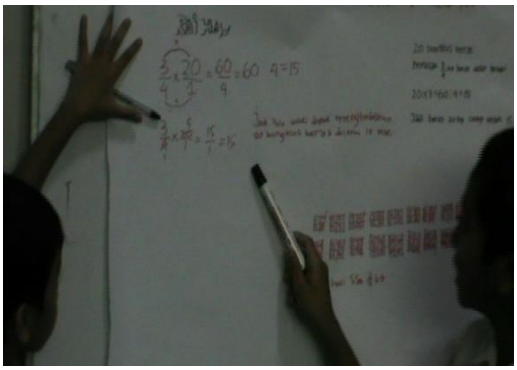


Figure 5.19. Strategy of group 1

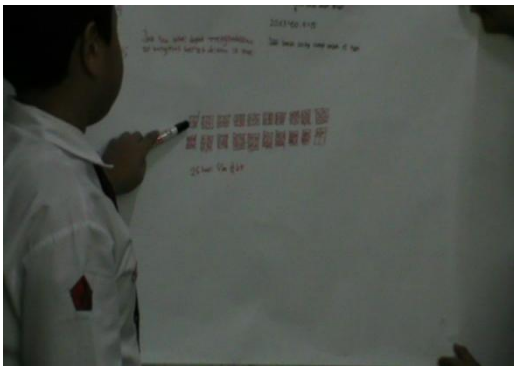


Figure 5.20. Strategy of group 2

Since there were two different answers, the teacher, at the end of the presentation, asked other students which strategy that they agreed. On the one hand, Hafiz and Dhika agreed with the strategies of Raya and Adit. This is because they did the same strategy in his their own worksheet. On the other hand, Akmal agreed with Galih's strategy because he stated that it is clear and we can count it straightly. Since they still debated the strategy, the teacher asked the second group to present their work. To make it clear, we provide a short fragment how Akmal presents his group's work by using model (pictures).

Fragment 5.3. Akmal uses his own model in solving problem

17. Teacher : How about your strategy? Can you explain it?
: (*Asking the second group to present their work*)

18. Akmal : Mrs. Sahri has 20 packs. One, two, three, four, five, six, … , twenty.
: (*Counting the number of boxes that they made*)
: Everyday, she takes $\frac{3}{4}$. So, each box is divided into four parts.

: So, it is divided by 4. (*Pointing one box*)

19. Teacher : Can you write one day for $\frac{3}{4}$? So, your friends understand it.

20. Akmal : (*Writing One day for* $\frac{3}{4}$)

: So, each box is divided by 4. (*Showing all the boxes*)

: $\frac{3}{4}$… So, three… one-two-three is for one day, one-two-three is for two days, one-: two-three is for the third day, and so on until the 26^{th} day. (*Pointing the last box*)

: There is one box left if we want no remaining. (*Imagining one box left*).

21. Teacher : The remaining?

22. Akmal : So, there is the remaining here $\frac{2}{4}$ kg.

After making the whole-class discussion, the teacher guide the students to make a conclusion of the problem based on the students' answers. When the teacher asked the students to raise their hand if they agreed with the solution, there was a student, Hafiz, did not do it. In this case, he stated that this is because his answer was different. Thus, the teacher asked him to write his strategy in the white board. He transformed $\frac{3}{4}$ into percents and he got 75%. Then, he subtracted 75% from 100% and he got 25%. Then, he multiplied $\frac{3}{4}$ with 20 to get 15 and he multiplied the result with 3 to get 45 (*Writing* $\frac{3}{4} \times 20 = 15 \times 3 = 45$). Therefore, he concluded that 20 packs of rice were enough for 45 days.

Dealing with this, the teacher made a short discussion to discuss Hafiz's solution. During the discussion, other students asked him some questions about what 75% and 25% meant and why he multiplied $\frac{3}{4}$ with 20 and multiplied the result with 3 to get 45. In this case, he argued that 75% was the percentage of the used rice everyday and 25% was the percentage of the remaining. However, when he was asked about what "$\frac{3}{4} \times 20 = 15 \times 3 = 45$" meant, he can not explain it.

Therefore, the teacher asked other students whether they agreed with Hafiz's solution. In this case, they did not agree with Hafiz's solution. Then, the teacher, together with the students, made a conclusion of the problem and asked one of them to write it on the white board that 20 one-kilogram packs of rice were enough for 26 days with $\frac{2}{4}$ kg remains.

Conclusion

Based on the retrospective analysis of meeting 1 above, the conjectures of students' thinking happened in the actual teaching and learning process. In this case, we see explicitly that there were some students did not know how to solve proportional problem. They tended to apply what they understand to deal with the problem. For instance, the student used percentage, subtraction and multiplication as they did in their pretest (see lines 1 – 7).

Furthermore, some students were able to solve the problem in a meaningful way, such as Akmal and Galih. In this case, it can be seen that Akmal and Galih drew pictures in order to solve problem. For instance, they draw a box as the representation of one pack of rice. Then, they divide each box into four parts and shade three parts of each box in order to get the fraction $\frac{3}{4}$. At the end, they can determine the solution of the problem by using this model (see line 14 and lines 18 – 20).

2. Meeting 2: Solving rice problem with a ratio table

The next day, we carried out the second lesson for about 70 minutes. This time, the purposes of this lesson were were making sense of proportion, introducing ratio table and start working with the ratio table. In this lesson, it will

be described how the teacher guided the students to the early stage of ratio table by asking them questions. Dealing with this, the following retrospective analysis shows how the teacher posed questions in order to guide the students to the idea of ratio table and how the students used the ratio table to solve the problem.

In conducting the second meeting, the teacher asked the students to solve the following problem.

> *Mrs. Sahri buys 20 one-kilogram packs of rice. In one day, she cooks* $\frac{3}{4}$ *kg of rice. Can you determine 20 one-kilogram packs of rice are enough for how many days?*

In our Hypothetical Learning Trajectory (HLT), we expect the students get the idea of proportion with their own understanding and the teacher can directly guide the students to the idea of ratio table. However, it, in this study, will be described how the teacher guided the students to the early stage of ratio table by asking them questions.

1. Initial activity

In our Hypothetical Learning Trajectory, we assumed that the students were still working on their work in the previous meeting. Thus, the beginning of this lesson stood at finishing the students' work and discussing the work within the whole-class discussion. However, this situation did not happen in the actual teaching and learning process. This is because the students work faster than we expect. Moreover, some students can give a meaningful way such as they used their own model, such as: their own visualization, to solve problem. Therefore, the teacher only told the students that he would continue the lesson at the beginning

of this meeting. In order to do that, he asked them to recall what they had learned in the previous meeting by asking them several questions. The question was aimed to remind the students about the problem that they solved in the previous lesson and to discuss the way how they solved it.

Dealing with this, some students still remembered that they were working with a problem which asked them to determine the number of days that 20 packs of rice would be enough if $\frac{3}{4}$ kg was cooked everyday. Subsequently, the teacher continued asking them the second question. The second question was intended to remind them about how they solve the problem yesterday. In this case, all the students also still recognized how to deal with the problem. They said that drawing 20 boxes and each box was divided into four parts. We took $\frac{3}{4}$ part from each box and so on. The next question was asking the students about the conclusion that they had agreed in the previous meeting. Raya came to the front of the class and stated that "so the conclusion is that Mrs. Sahri can consume 20 packs of rice for 26 days with $\frac{2}{4}$ kg remains". Then, the teacher asked one of them to write it on the white board.

2. Main Activity

After making a short conversation, the teacher tried to guide the students to an early stage of ratio table by making short discussion. At the beginning of the discussion, the teacher asked them to explain how to draw $\frac{3}{4}$ of one kilogram. Afterward, the teacher asked them 3 kg = … days and 6 kg = … days. In our Hypothetical Learning Trajectory, we conjecture that the students still do not

know how to deal with those problems as what happened in lesson 1 or they will apply what they understand to deal with those problems. For instance, the students use multiplication, subtraction and division. However, we also assume that the students used their model that they have learned in the first lesson.

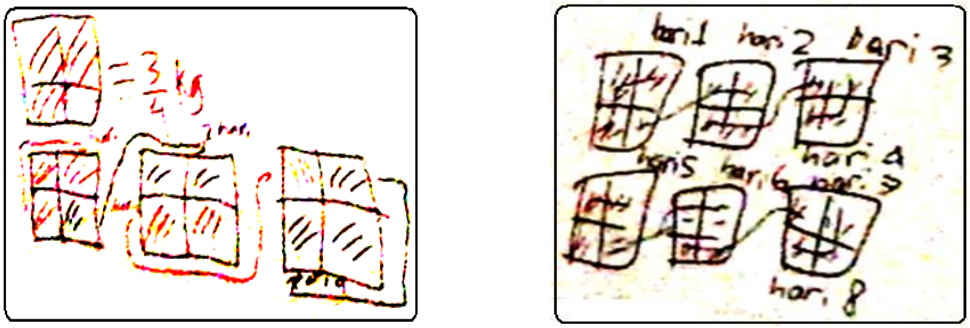
Here, the description of what happened in classroom. In this case, we asked several students to solve the three problems by using their understanding in front of the class. We found that all the students that came to the front were able to answer the problems correctly by using their own model. For instance, Raya can explain it by using picture to represent $\frac{3}{4}$. He drew a box divided into four parts and shaded three parts of it. Subsequently, the teacher continued the questions if he had 3 kg of rice and 6 kg of rice. He asked the students that these amounts of rice are enough for how many days. To deal with this, the following short fragment showed how the teacher guided the students to the idea of proportion.

Fragment 5.4. The teacher guides the students to get the idea of proportion

1. Teacher : Now, if I have 3 packs of rice, then for how many days 3 packs of rice will last?
2. Hafiz : Four days.
3. Akmal : How come?
4. Teacher : Can you explain it?
5. Hafiz : (*Start drawing three boxes, divide each box into four parts, and shade each* : *three parts of it with different color*)
   : This is $\frac{3}{4}$ and I draw $\frac{3}{4}$ where each box is divided into four parts and three parts of : it are shaded. So, this is the first day, this is the second day, this is the third day : and this is the fourth day.
   : (*Pointing from the first three parts until the last three parts*)
6. Teacher : So, can you write the conclusion?
7. Hafiz : So, three packs of rice can be consumed for four days.
8. Teacher : Ok, now… If there are six packs of rice, then for how many days they will last?
   : Dhika… (*Asking Dhika to explain it*)
9. Dhika : (*Drawing six boxes, dividing each part into four parts and shading each three* : *parts*)

: Each kilogram of rice is divided into four. Then, one day there must be $\frac{3}{4}$. So, $\frac{3}{4}$ is taken from one pack of rice. The rest can be cooked for the second day and I just take two parts from the second box. Then, the rest can be cooked for the : third day. And so on until the last three parts can be cooked in the eighth day.

10. Teacher : What is the conclusion?
11. Dhika : Six packs of rice are enough for eight days
: (*Writing 6 packs are enough for 8 days*)



A B

Figure 5.21. Students' visualization

The transcript 5.4 designates that the students used model, such as: their own visualization, to solve the problems. To determine the answer of 3 kg = … days, the second student used the same way as the first student did. Based on the figure 5.21A., it can be explicitly seen that the student draw three boxes. He divided each box into four parts and shaded three parts of each box. Subsequently, he counted every three parts to get the answer that 3 kg = 4 days (see also lines 1 – 7). At the same time, the second figure also illustrates the student's strategy to solve the third problem which is 6 kg = … days. To solve this problem, the third student did the same strategy as the two students before. Based on the figure 5.21B, we can explicitly see that the student recognized that 6 kg = 8 days (see also lines 9 – 11).

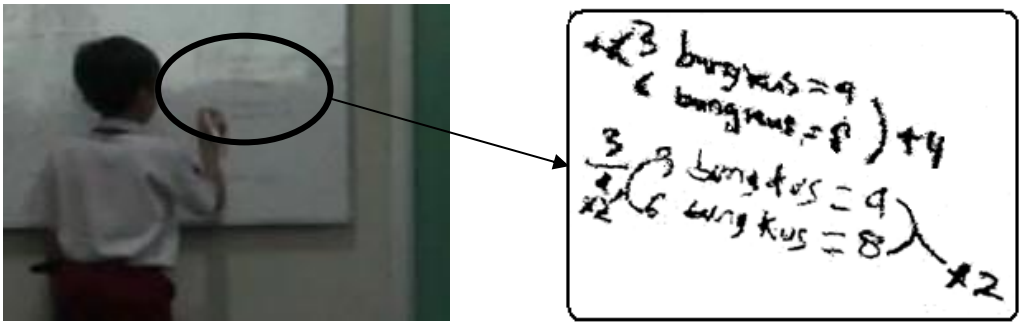
Based on the transcript and figure 5.21, we can notice that model plays an important role in order to help students solve proportional problem. It helps the students visualize their thinking towards the way how to solve proportional problem. In this case, we can explicitly see that the students automatically used model, such as: their own visualization, to solve the three problems. Therefore, we can say that the students' visualization helps them when they were asked to reason how they get the answer of the proportional problems.

After the students got the idea of proportion, the teacher continued the lesson by guiding the students to the early stage of ratio table. In this case, the early stage of ratio table means the moment when the students build the sense of proportion by using their model. Thus, the fragment 5.5 below shows how the teacher did it.

Fragment 5.5. The students build the sense of proportion

12. Teacher : Now, I want to ask another question. Do you see a pattern?
: 3 kg of rice are enough for 4 days. Dhika gets that 6 kg are enough for 8 days.
: Do you see the way?
13. Hafiz : Just add them. (*Going to the white board to explain it*)
: Three packs are equal to four days and six packs are equal to eight days.
: (*Writing 3 packs = 4 days, 6 packs = 8 days*)
: And 12 packs are equal to… (*Thinking*)
14. Teacher : Are there any one able to help him?
: 3 packs are enough for 4 days and 6 packs are enough for 8 days.
: 3 packs become 6 packs. How come?
15. Akmal : Multiply them.
16. Raya : We can also add them.
17. Teacher : Can you write it on the white board? (*Asking Akmal to explain his answer*).
18. Akmal : This one and this one are multiplied with 2. (*Pointing 3 and 6*)
19. Teacher : Do you agree with his answer?
20. Raya : I also want to try it. Why don't we just add them?
21. Teacher : Can you write it on the white board?

22. Raihn : From 3 to 6 we can add 3 and from 4 to 8 we add 4. (*Pointing 3 to 6 and 4 to 8*)



A B

Figure 5.22. Students get the idea of proportion

The transcript 5.5 and the pictures show how the students build the sense of proportion by using their own model. In this case, we can explicitly see that Hafiz concluded *3 packs = 4* and *6 packs = 8* based on the model he saw. For instance, he saw "*three boxes and six* $\frac{3}{4}$*s*" and "*six boxes and eight* $\frac{3}{4}$*s*" (see line 13 and figure 5.22B). Based on the transcript and figure 5.22, we can notice that model also plays an important role in order to help the students build the sense of proportion. It helps the students visualize the proportional situations.

After the students wrote their thinking, the teacher managed those answers in a table. In this case, he drew a table which consisted of two rows where the first row was filled with kilograms and the second one was filled with days. Since the first conclusion was "3 kilograms of rice are enough for 4 days" and he put 3 in kilogram and 4 in days". Then, he continued to the second conclusion that was "6 kilograms of rice are enough for 8 days" and put 6 in kilogram and 8 in days. After that, he asked the students to continue it in group.

After the students wrote their thinking, the teacher managed those answers in a table. In this case, he drew a table which consisted of two rows where the first row was filled with kilograms and the second one was filled with days. Since the first conclusion was “3 kilograms of rice are enough for 4 days” and he put 3 in kilogram and 4 in days”. Then, he continued to the second conclusion that was “6 kilograms of rice are enough for 8 days” and put 6 in kilogram and 8 in days. After that, he asked the students to continue it in group about 15 minutes.

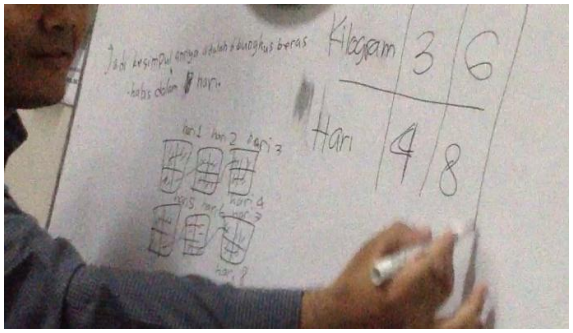


Figure 5.23. Rewriting students’ answers in a table

Whole-class Discussion

After working in group, it is the time for the students to present their work in front of the class. The presentation was aimed to discuss students’ strategies to determine the answer of the problem by applying ratio table. In general, the students tend to use addition strategy when the use ratio table. In this case, the following short fragment shows how the students reason with their own ratio table.

Fragment 5.6. The students start using the ratio table

1. Teacher : Ok… you can start now.
2. Raya : 3 kg can be cooked for 4 days.
   : So, we add 3 to get 6 and we add 4 to the days to get 8.

: 6 plus 3 is equal to 9 and 8 plus 4 is 12. 9 plus 3 is 12 and 12 plus 4 is 16
: 12 plus 3 is 15 and 16 plus 4 is 20. Then, 15 plus 3 is 18 and 20 plus 4 is 24.
: (*Explaining the numbers in the ratio table*)
: And it remains 2 kg. We still can cook with 2 kg.
: So, these are the two kilograms. (*Pointing 2 boxes*)
: This one is divided into four parts and three parts of it are shaded
: And these parts are not shaded. So, the remaining is $\frac{2}{4}$ kg.

3. Teacher : So, the remaining is $\frac{2}{4}$ kg.
: So, what is the conclusion?
4. Group 1 : 18 boxes can be spent in 24 days.
5. Teacher : That is the first conclusion that 18 boxes can be spent in 24 days.
: And it remains 2 kg. Then, …
6. Raya : We can still cook 2 kg of rice.
7. Teacher : For how many days that 2 kg of rice are enough?
8. Raya : 2 days
9. Teacher : Is there the remaining?
10. Group 1 : Yes… $\frac{2}{4}$ kg.

After group 1 finished their presentation, it was the time for group 2 presented their poster. Dealing with this, group 2 did the same strategy as the previous group. They added 3 to the kilogram and added 4 to the days. At the end, they found that 18 kg are equal to 24 days. Then, the students got confused in order to determine 20 since they can not add 3 to the kilograms and 4 to the days. Therefore, straightly wrote 24 days = 18 kg with $\frac{2}{4}$ kg or 0,5 kg remains. The fragment 5.7 below shows how they explain this.

Fragment 5.7. Student's struggle to operate ratio table

11. Hafiz : So, 18 kg can be spent in 24 days and it remains 2 kg.
: Hmmm… $\frac{2}{4}$ kg. Then, $\frac{2}{4}$ times $\frac{25}{25}$ is equal to $\frac{50}{100}$ kg.
: We change it into decimal and we get 0,5.
: So, the conclusion is 24 days = 18 kg with $\frac{2}{4}$ kg or 0,5 kg remains.
12. Raya : I want to ask question
: How do you get $\frac{2}{4}$?
13. Hafiz : Hmmm… Because the remaining is 2 kg.
14. Raya : Oh… Because the remaining is 2 kg?

15. Hafiz : No…

Dealing with this, Akmal tried to help Hafiz by doing the same strategy as the first group which was by drawing pictures. He drew the remaining 2 kg of rice with two boxes and divided was box into four. Then, he shaded three parts of each box in order to get $\frac{3}{4}$. In this case, he recognized that there were still two parts which were not shaded. So, he got the same conclusion as group 1 that 20 packs of rice were enough for 26 days with $\frac{2}{4}$ remained.

After the students presented their poster, the teacher asked the students to compare the conclusion that they had got today to the result in the previous meeting. They recognized that the conclusion was the same with their conclusion in the previous meeting that 20 one-kilogram packs of rice were enough for 26 days with $\frac{2}{4}$ remains. Then, the teacher, at the end of the lesson, introduced ratio table to the students. At the same time, he also gave the students homework which was to try ratio table by making another way of operating it.

Conclusion

Based on the retrospective analysis above, teacher plays an important role in order to guide students to the idea of ratio table. In order to do that, teacher can ask them questions to scaffold their thinking to a moment where the teacher expects the students to be. The examples of questions and how the teacher do it in a teaching and learning can be seen from line 1 until line 22.

Additionally, it can be seen that most students were able to operate a ratio table although there are some moments that they still struggle to do it. For instance, it is easy for the students to get the proportion 18 kg = 24 days by adding

3 to the “kilogram” unit and adding 4 to the “days” unit. However, they still struggle how to get 20 kg from 18 kg in that table. To deal with this problem, they explain it by drawing two boxes as the representation of 2 kg of rice. To sum up, they use the two models in order to deal with the problem.

3. Meeting 3 : Making cakes to sell

The third meeting was conducted the next day after conducting the second meeting for about70 minutes. This meeting was aimed to use ratio table to solve proportional problem which includes fraction and to describe how to use the ratio table.

1. Initial activity

As usual, the teacher started the teaching and learning process by conducting a short conversation with the students by asking them three questions about what they had learned in the previous two meetings. In this case, Raya described the problem that they solved in the first meeting and Hafiz explained what they did yesterday. Then, the teacher asked Galih to explain what they did to solve the problem in both meetings. Galih stated that we drew boxes in the first day. Each box was divided into four parts and three parts were taken away. So, the result was 20 packs were enough for 26 days with $\frac{2}{4}$ remains. The second day, we used another way besides drawing pictures, which was ratio table.

After conducting a short conversation with the students, the teacher discussed the homework within the whole class. Based on the discussion, two students misinterpreted the question. They made another way to solve the problem which was not asked by the teacher. They solved the problem by using division

strategy. However, some students tried to solve it by using ratio table. However, they still confused how to operate it. An example of this can be seen in figure 5.24 below.

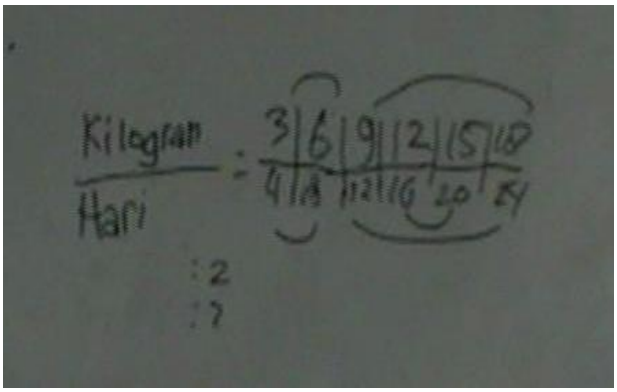


Figure 5.24. Dhika's ratio table

Dealing with this, the students need some practices to operate ratio table. Therefore, he gave them new problem in student's worksheet to solve. In this case, the students were asked to solve the following problem.

> *Mrs. Ani wants to make some chocolate cakes to sell. She has 45 cups of chocolate powder. To make one chocolate cake, she needs* $2\frac{1}{2}$ *cups of chocolate powder. Can you determine 45 cups of chocolate powder are enough to make how many cakes?*

Before started doing the worksheet, the teacher asked one of them to read it. Then, they were asked to work individually about 5 minutes and discussed it in group. In our HLT, we assumed that some students would solve the problem by using ratio table. Additionally, we also assumed that some students still draw pictures to deal with the problem. In the actual teaching and learning process, most of the

students, in general, still drew pictures to solve the problem. They represented a glass of chocolate powder with a box.

2. Main activity

The teacher asked them to work in group. At first, the two groups made pictures to solve the problem after 5 minutes working individually. For instance, the second group drew 45 boxes as the representation of 45 glasses of chocolate powder. Then, they changed $2\frac{1}{2}$ into fraction which was $\frac{5}{2}$. After they got the fraction, they straightly shaded each five parts with different color and made the mark above it, such as: 1 cake, 2 cakes, 3 cakes, … , 18 cakes. Thus, they got the conclusion that 45 chocolate powders were enough to make 18 cakes. Meanwhile, the first group also drew 45 boxes. They grouped each $2\frac{1}{2}$ boxes and made a mark for each of them, such as: 1, 2, 3, … , 18.

Since one of the aims of this meeting was to let the students try ratio table, then the teacher come to each group and guide them to start using it. The following short fragments shows how those children worked with ratio table in each group.

Fragment 5.8. Students in Group 1 try ratio table.

1. Teacher : What did you learned yesterday?
2. Students : Ratio table
3. Teacher : Can you make your answer by using ratio table?
4. Raya : We will try
5. Teacher : Ok… What is the problem?
6. Students : $2\frac{1}{2}$ cups of chocolate powder can make 1 cake.
7. Teacher : That's right… If there are five cups…
8. Galih : Two.
9. Teacher : Hah… You can continue it…
10. Galih : So, it will be cakes and glasses. (*Making the names of each unit*).
    : 2 cakes 5 glasses.

: Let's add it. 4 cakes 10 glasses, 6 cakes 15 glasses, …, 16 cakes… (*Thingking*)
11. Raya : 40 glasses
12. Galih : 18 cakes 45 glasses.
13. Raya : That is enough!!!

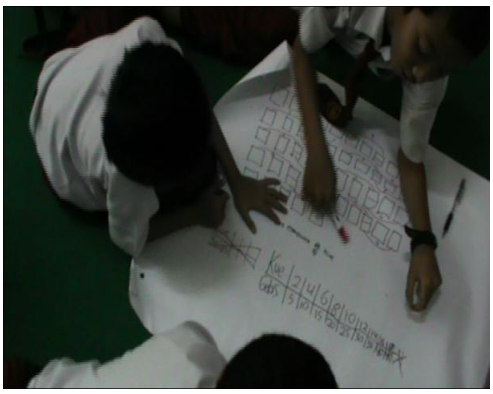


Figure 5.25. Strategy of group 1

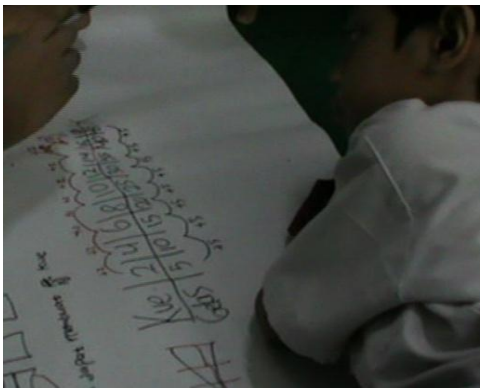


Figure 5.26. Ratio table of group 1

The teacher moved to the second group after having a short conversation with the first group. He also did the same with the second group. The following short fragment shows how the students in the second group discuss how to solve the problem by using their own ratio table.

Fragment 5.9. Students in group 2 try ratio table
14. Hafiz : One cake is $\frac{5}{2}$. So, two cakes are $\frac{10}{2}$.
15. Dhika : Two cakes?
16. Hafiz : $\frac{10}{2} = \frac{5}{1} = 5$ cups
17. Dhika : 3 cakes are $\frac{15}{2}$ cups
18. Hafiz : 4 cakes are 20
19. Dhika : Five cakes?
20. Hafiz : Five cakes $\frac{25}{2}$
21. Dhika : We have to make it until 45 cups
22. Hafiz : Yes.

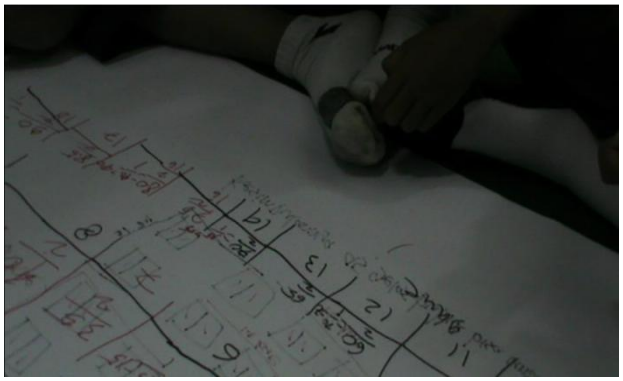


Figure 5.27. Discussion of group 2

Whole-class discussion

After discussing the problem in group, the teacher started the discussion for the whole groups. During the discussion the each group had to present their work to another group. The second group got the first turn to present their work. The following fragment 5.9 shows how the two groups reason to solve the by using ratio table.

Fragment 5.9. The strategy of the students in group 2 to use ratio table

23. Dhika : To make one cake we need $\frac{5}{2}$ cups.
: Two cakes need $\frac{10}{2} = \frac{5}{1} = 5$, $\frac{15}{2}$ for 3 cakes, $\frac{20}{2} = \frac{10}{1} = 10$ for 4 cakes.
24. Teacher : That is until,,,
25. Dhika : $\frac{90}{2} = \frac{45}{1} = 45$ for 18 cakes
26. Teacher : So, what is the conclusion?
27. Group 2 : Mrs. Ani can make 18 cakes by using 45 chocolate powders.

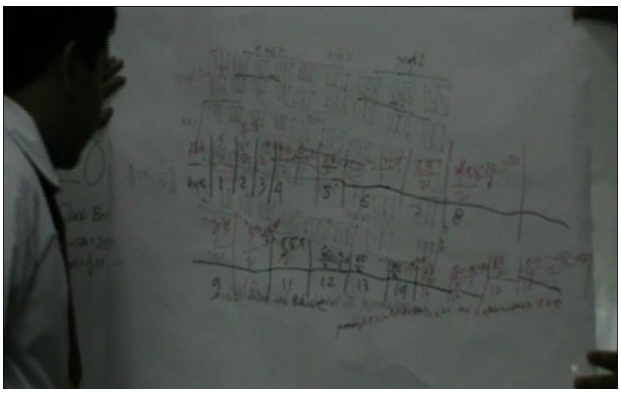


Figure 5.28. Ratio table of group 2

After the second group presented their work, it is the time for the first group to present their poster. In this case, the following short fragment shows how the first group reason to solve the problem by using their own ratio table.

Fragment 5.10. Students in group 1 use repeated addition strategy

28. Teacher : Can you explain your strategy?
29. Galih : In 5 glasses, it can be used to make 2 cakes. So, 10 glasses are equal to 4 cakes.
: 15 glasses for 6 cakes, 20 for 8 cakes, 25 for 10 cakes, 30 for 12 cakes, 35 for 14 cakes, 40 for 16 cakes and 45 for 18 cakes.
: So, Mrs. Ani can make 18 cakes.
30. Teacher : Do you have any questions?
31. Students : No…
32. Teacher : Do you find any differences between Galih's strategy and Dhika's strategy?
33. Raya : The difference is using fraction.
: Dhika uses fraction but Galih does not use fraction. (*Pointing Galih's strategy*)

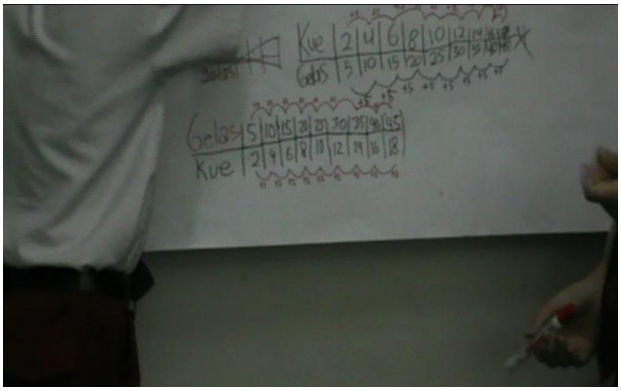


Figure 5.29. Ratio table of group 1

Conclusion

Based on the retrospective analysis above, we can explicitly see that the students are getting familiar with the ratio table. When the students discuss in group, they are able to solve the problem by using ratio table. For instance, students in group 1, on the one hand, use addition strategy within whole number. On the other hand, group two works on fractions that they add those fractions continuously with $\frac{5}{2}$ (see lines 23 – 25 above)

Furthermore, we can see that the students show their proportional reasoning by using ratio table and it can make the students think how to solve proportional problem. An example of the students' proportional reasoning and struggle can be seen in the presentation of students in group 1 and 2. In this case, students in group 1 know that $2\frac{1}{2}$ cups can make 1 cake, then they can determine that 2 cakes are equal to 5 cups by adding $2\frac{1}{2}$ and 1 to the cups and the cake. They did this way until the get the answer of the problem (see line 29).

4. Meeting 4 : Proportion in unit fraction

We carried out the fourth meeting one day for about 70 minutes after the third meeting was conducted. This meeting was intended to let the students make their own ratio table and use it to solve proportional problem.

1. Initial activity

At the beginning of this meeting, the students were asked to solve the following questions.

1. *Miss Rika goes to supermarket to buy 15 kg of rice to supply her daily needs. Everyday, she cooks* $\frac{1}{2}$ *kg of rice. Can you determine 15 kg of rice are enough for how many days?*
2. *Miss Sri goes to supermarket to buy 25 kg of rice to supply her daily needs. In one day, she cooks* $\frac{1}{3}$ *kg of rice. Can you determine 25 kg of rice are enough for how many days?*
3. *Miss Rani goes to supermarket to buy 15 kg of rice to supply her daily needs. Everyday, she cooks* $\frac{1}{5}$ *kg of rice. Can you determine 15 kg of rice are enough for how many days?*

To deal with this situation, he gave the first problem to the students and asked them to solve it individually about 5 minutes. Then, discuss it with their friends in group. Before they started working, the teacher asked one of them to read the problem and asked them whether they understood the problem or not. In this case, all the students understood the problem and started working individually.

In our HLT, we predicted the students will use ratio table as the model in order to solve the problems. However, the fact is not the same as what we had conjectured about the students' thinking. Based on the results of the student's

worksheet, there were two strategies that the students did in order to solve the problem. On the one hand, some students still drew pictures to solve the problem as a model to solve proportional problem. They drew a box as the representation of one kilogram of rice. On the other hand, some students solved the problem by using ratio table.

2. Main activity

After working individually, they started working with group. In the group discussion, they had to write their strategies in a poster paper. In this case, the two groups decided to solve the problem with two ways, by using pictures and ratio table.

After the discussion, the teacher asked the students to present their work in front of the class. In this case, the first got the first turn to present it and moved to the second group. To deal with this, the fragment 5.11 below shows how the students in group 1 present their work by using models.

Fragment 5.11. Students in group 1 use their own model to solve problem

1. Teacher : Oke,.. Can you tell us what you did to solve the problem?
2. Galih : Mrs. Rika has 15 kg of rice. Let's say 1 kg as one box.
   : She only cooks $\frac{1}{2}$ kg in one day. So, this one box is divided into two parts.
   : (*Pointing one box*)
   : So, 1 kg is divided by 2. Then, we put 1, 2, 3, 4, 5, 6, … , 30.
   : (*Counting the numbers in the boxes*)
3. Teacher : Ok…
4. Galih : And this one is using ratio table. (*Pointing the ratio table*)
   : 1 kg = 2 days, 2 = 4 days, 3 = 6 days. So, this one kilogram we add 1, add 1, add 1, … (*Pointing the kilogram unit*)
   : The days, we add 2, add 2, add 2 and so on.
   : So, the conclusion is 15 kg are enough for 30 days.
5. Teacher : Do you agree with this group? Was it clear?
6. Dhika : Yes, it is clear…
   (*Akmal raised his hand*)
7. Teacher : Akmal, do you want to ask something?

8. Akmal : What does it meat with 1, 2, 3, 4, 5, …?
9. Galih : This one? (*Pointing the numbers in the boxes*)
10. Akmal : Yes…
11. Galih : This is the day. Hmm,,, Mrs. Rika only uses $\frac{1}{2}$ kg of rice.
: So, this is one day, two days, three days, four days and so on.
: (*Pointing the numbers in the boxes*)
12. Akmal : Oh… Hmm… I have another question.
: How do you know that 2 days needs only 1 kg? (*Pointing the ratio table*)
13. Adit : Of course… One day, she needs $\frac{1}{2}$. (*Adding Galih' explanation*)
14. Galih : It's here… (*Continuing his explanation and pointing the first box*)
: Oh… This one is 1 kg (*Pointing one box*) and this one is 1 kg (*Pointing 1 kg in the ratio table*).
: This is divided by 2. So, one and two (*Pointing 1 and 2 in the first box*). So, two is the answer here. (*Pointing 2 days in the ratio table*)

After the first group presented their work, the teacher asked the second group to present their work. In this case, group 2 did the same strategy as group 1. They also wrote two strategies, by using drawing and ratio table, in their poster. To be more detail, the following figure 5.30 and figure 5.31 below.

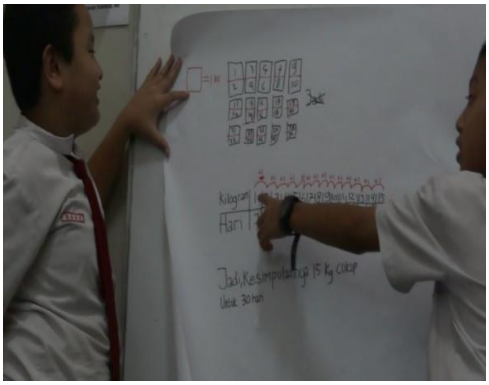


Figure 5.30. Strategy of group 1

Figure 5.31. Strategy of group 2

After finishing the presentation, the teacher gave the students the second problem to discuss. Then, the teacher asked them to solve it individually about five minutes and discuss it together with their friends in group. In general, most of

the students still used pictures as a model to answer the second problem. Some students drew boxes and some of them drew circles was the representations of the number kilograms of rice. The rest used ratio table to answer the problem.

After the students worked individually about 5 minutes, the teacher asked to discuss with their friends in group. During the discussion, each group did the same strategy as they did in the first problem. For instance, the group 1, on the one hand, drew 25 circles as the representation of 25 kilograms of rice. Then, they divided each circle into three parts and put numbers, such as: 1, 2, 3, 4, …, 30 as the representation of the days. On the other hand, group 2 drew 25 boxes as the representation of 25 kilograms of rice. They also divided each box into three parts and put numbers, such as: 1, 2, 3, 4, … , 30, as the representation of the days.

The second strategy was using ratio table. In this case, they did the same way in order to solve problem by using ratio table, which was addition. Group 1 tended not to use fractions in the ratio table. Thus, they recognized that I kg of rice is enough for three days. Thus, they just added one to each column of "kilograms" and added three to each column of "days". At the end, they found that 25 kilograms of rice are enough for 75 days. At the same time, group two also did the same strategy as group 1, which was addition. In this case, they also recognized that 1 kilogram of rice was enough for 3 days. They added 1 to each column of "kilogram" and added 3 to each column of "days". Thus, they also concluded that 25 kilograms of rice are enough for 75 days.

In order to make clear, the teacher asked each group to present their work in the whole class discussion. This time, group 2 got the first turn to present it.

Dealing with this, the fragment 5.12 below shows how group 2 explained their strategies by using models in front of the class.

Fragment 5.12. Students in group 2 use their own model in solving problem

15. Hafiz : Miss. Sri buys 25 kg of rice. So, we draw 25 boxes…
: Then, each box is divided into three parts since it is $\frac{1}{3}$. Divided by 3, divided by 3 and so on. (*Showing each box*)
: One day, she cooks $\frac{1}{3}$ kg. So, this is one day, this is two days, three days and so on until we find 75.
: And this is using ratio table…
(*Akmal changed to explain it*)
16. Akmal : This one is $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{3}{3}.\frac{3}{3}$ is equal to 1 kg and 1 kg is equal to 3 days.
: So, this is 1, 2, 3, 4, 5, … , 25 kg. (*Pointing the unit of kilogram*)
: And this is 3, 6, 9, 12, … , 75 days. (*Pointing the unit of days*)
17. Teacher : Do you have any questions? (*Asking other students for question*)
18. Students : No…
19. Teacher : Ok… I have a question…
:1 kilogram for 3 days and 2 kg for 6 days. How come?
20. Akmal : Oh… because 1 kg times 2 and it is also multiplied with two.
: 3 days is multiplied with 2. So, it is 6 days.
21. Teacher : How do you get 3 kg for 9 days?
22. Dhika : It is added with 3
23. Akmal : we can multiply it with 3
24. Raya : Hah?
25. Akmal : Eh… We add 3

After both groups presented their posters, the teacher asked the students what the conclusion of the first problem and the second problem. Then, the teacher gave them a homework, problem 3 in student worksheet 3, that they had to do at home.

Conclusion

Based on the retrospective analysis above, our conjecture about the students' thinking was not different from the actual situation. In this case, we can see that most students still draw picture in order to solve those problems. This time, they draw boxes as the representation of kilograms of rice, such as: 25 boxes

represent 25 kilograms of rice. Therefore, it can be seen that this model is helpful for the students to solve proportional problem.

Additionally, they can explain what they do in that ratio table. This means that the students can reason proportionally by using ratio table. An example of this situation can be seen in lines 4 – 14 and lines 16 – 25 where each group presents their work by using ratio table. For instance, students in group 1 recognize that 1 kg = 2 days. Then, they can easily determine the next proportion by adding 1 to each kilogram and adding 2 to each day (see line 4).

5. Meeting 5 : Proportion in non-unit fraction

The last meeting was also conducted for about 70 minutes right after we carried out the fourth meeting. The aim of this meeting was to investigate how the students answer proportional problem which includes fractions by using models. Additionally, it was also purposed to see to reason their thinking about their strategy by using models. In general, this meeting indicates that the students in group 1 can use ratio table to solve proportional problem which includes fraction. Therefore, it will be described how the students use the ratio table.

1. Initial activity

At the beginning of this meeting, the teacher told the students that they will continue the previous meeting. In this meeting the students were asked to solve the following problem.

> *Mrs. Fitri buys 25 kg of rice for the supply her daily need. In one day, she cooks* $\frac{3}{4}$ *kg of rice. Can you determine 25 kg of rice are enough for how many days?*

As usual, the teacher asked the students to work individually about 10 minutes to think how to solve the given problem. Then, they can discuss it with their fellow students in their group. In our HLT, we conjectured the students used ratio table as the model to solve the problem. In this case, the actual teaching and learning process was not totally different from the conjectures of the students' thinking. Here is the description of what happened in the actual teaching and learning process.

2. Main activity

During the discussion, group 2 still drew pictures to solve the problem. In this case, they did the same strategy as they solve problems in the previous meeting. For instance, they drew 25 boxes to represent 25 kg of rice. Then, they divided each box into four parts and make a sign for each three little parts. At the end, they got the answer of the problem that 25 kilograms of rice were enough for 33 days with $\frac{3}{4}$ remains.

Meanwhile, the students in group 2 started their discussion by drawing picture. They did the same strategy as they solve problems in the previous meeting. However, they did not finish this and moved to do this by using ratio table. To deal with this, the fragment 5.13 below shows how the students in group 1 discuss how to determine the solution of the problem by using ratio table.

Fragment 5.13. Students in group 1 use ratio table to solve problem

(*Galih started the discussion*)

1. Galih : 1 kg is for how many days? $\frac{3}{4}$? So, do we have to write $\frac{3}{4}$?
2. Raya : $\frac{3}{4}$ is taken from 1 kg, right?
3. Galih : Can you write the way?
   : (*The three students are thinking*)
4. Raya : (*Writing 3 kg are equal to 4 days on the poster*)

Figure 5.32. Starting a ratio table

5. Galih : Hah? 4 days? 3 kilograms?
: 1… 2… 3… (*Counting something on the picture*). Yeah… you are right… Let's continue!!!
: I can not think in that way…
6. Teacher : Very nice Raya…
7. Raya : Huh… I am wrong… (*Cross out the table and make a new one*)
8. Galih : 9 kg is equal to 12 days, 12 kg is equal to 16
: (*Raya continued the ratio table until 18 kg is equal to 24 days and he got confused*)
9. Raya : It remains one.
10. Galih : Hah? This is 25… Oh yeah… It remains one kilogram.
: We can still cook one kilogram.
11. Adit : This is 24… (*Pointing 18 kilograms*)
12. Raya : Of course… This is twenty four… (*Pointing 24 days*)
13. Galih : We are looking for the days… Not the kilogram… (*Pointing 18 kilograms*)
14. Raya : Yes… This is the days… (*Pointing 24 days*)
15. Raya : (*Make a new table*)
: So, it is wrong…
16. Galih : No… we are right…
17. Adit : We are right… (*Cross out the new table*)
18. Galih : How many kilograms are there?
19. Raya : Twenty five
20. Galih : We have just got 18. It is not known… (*Pointing the days*)
: But, this is known… (*Pointing the kilograms*)
: Do you understand?
21. Galih : Let's continue!!!
: 21 kg are equal to 28 days and 24 kg are equal to 32 days.
22. Raya : (*Writing the answer on the table*)
23. Galih : So, there are 25 kg and it remains one kilogram.
24. Raya : Oh ya…ya… (*Get an idea*)
25. Galih : We can still cook 1 kg.
26. Raya : (*Drawing a box and divide it into four parts*)
27. Galih : Shade three parts. So, it remains $\frac{1}{4}$ kg.
28. Raya : Ok… finish

Figure 5.33. Discussion in Group 1

Whole-class discussion

After the students discussed in group, the teacher asked them to present what they did in their poster in front of the class. Group 1 got the first turn to present it with Hafiz as the speaker. In this case, they just drew pictures, such as: boxes, to solve the problem. They drew 25 boxes where each box represented one kilogram of rice. Then, they divided each box into four parts and shaded each three parts of it. At the end, they concluded that 25 kilograms of rice were enough for 33 days with $\frac{1}{4}$ kg remains.

Group 2 started their presentation after group 1 finished doing it. Firstly, they explain their strategy by using their own drawing. Secondly, they continued their presentation by explaining the ratio table that they had discussed in group. To deal with this, the following short fragment shows how group two used models to solve the problem.

Fragment 5.14. Students use repeated addition strategy to solve problem

29. Galih : Miss Fitri buys. Suppose that this one box is one kg of rice.
: Miss Fitri cooks only $\frac{3}{4}$ of this one kilogram. Since there are 25 kg, we make 25 boxes.

: Then, we divide into four parts since $\frac{3}{4}$. Then, we shade three parts. 1…1…1… means one day, 2…2…2… means two days, 3…3…3… means three days, 4…4…4… means four days and so on, until 33 days with $\frac{1}{4}$. So, 25 kg of rice are enough for 33 days with $\frac{1}{4}$ kg remains.

: And this one is using ratio table.

30. Teacher : Wait… Do you have any questions?
(*Akmal raised his hand*)
31. Akmal : You put 3, 4, 4 in one box. What does it mean?
32. Galih : This one is one kilogram. One box is equal to one kilogram. We only use $\frac{3}{4}$ kg.
33. Teacher : Do you understand Akmal?
34. Akmal : Yes…
35. Teacher : Let's continue…
36. Galih : Now, by using ratio table.
: 3 kg = 4 days. 6 kg = 8 days, 9 kg = 12 days and so on until 24 kg = 32 days.
: The remain is 1 kg. We can still cook this 1 kg.
: So, we make one box again here. Then, we take 3 parts of it. So, the remain is $\frac{1}{4}$ kg.
37. Akmal : Do you add, multiply or divide it? (*Pointing the ratio table*)
38. Galih : We add it
(*Adit helped Galih to write the way*)
39. Galih : So, 3 + 3 = 6 + 3 = 9 + 3 = 12 + 3 = 15 + 3 = 18 + 3 = 21 + 3 = 24.
: (*Explaining the kilogram*)
40. Teacher : So, what is the conclusion?
41. Galih : So, the conclusion is 25 kg of rice are enough for 33 days with $\frac{1}{4}$ remains

At the end of the lesson, the teacher asked one of the students to repeat what Group 1 had explained. In this case, Hafiz raised his hand and explained again the explanation. Then, the teacher and the students made the conclusion of the problem.

Conclusion

Students' representations, such as picture, are still used in this meeting. It can be seen from the students' worksheet and the students' poster. This model is used as a way to solve the problem and to reason how to solve it. For instance, students in group 1 use this model to solve proportional problem which include fraction. This model is also used by the students to have a presentation in order to share their thinking (see line 29).

At the same time, student in group 2 also discussed to solve the problem by using ratio table. Based on the retrospective analysis of this meeting, it can be seen that ratio table can be used in order to support students' proportional reasoning. For instance, in order to determine for how many days that 25 kg of rice are enough if Mrs. Fitri cooks $\frac{3}{4}$ kg, they struggle to do that. They add each kilogram with 3 and each day with 4. Then, they get that 24 kg of rice are enough for 32 days. They still have 1 kg left and they use picture to finish this. They make a box that represents one kilogram of rice. They divide it into four parts and shade three parts. Thus, this rice is still enough for one day and the remaining is $\frac{1}{4}$ kg. They conclude that 25 kg of rice are enough for 33 days with $\frac{1}{4}$ kg remains (see lines 1 to 28).

5.2.1.4. Posttest

At the end of the first cycle, we conducted a 30-minute test to five students since one student was sick. This test was purposed to evaluate the development of students' understanding towards what they had learned within the whole teaching

and learning experiment. In this test, the students were asked to solve the following problems.

1. *Mrs. Silvia buys 35 boxes of 1 kg of rice. In one day, she cooks* $\frac{3}{4}$ *kg of rice. Can you determine the amount of rice is enough for how many days?*
2. *Mrs. Mutia wants to make some chocolate cakes to sell. She has 55 cups of chocolate powder. To make one chocolate cake, she needs* $2\frac{1}{2}$ *cups of chocolate powder. Can you determine 55 cups of chocolate powder are enough to make hoe many cakes?*
3. *Miss. Fika buys 20 kg of rice to supply her daily need. Everyday, she cooks* $\frac{2}{3}$ *kg of rice. Can you determine the amount of rice is enough for how many days?*

Based on the result of this posttest, most of the students were able to answer all the questions correctly. Only two students, Hafiz and Dhika, answered two questions correctly since they did not succeed to answer the first problem. Furthermore, they were also able to explain their strategies how they got the answer. In this case, the explanation of students' strategies in order to answer the questions in the posttest can be seen as follows.

Problem 1

To deal with the first problem, two students, Dhika and Hafiz, did not succeed. In this case, Dhika divided 35 by $\frac{3}{4}$ to get the answer. He got that 35 kg packs of rice were enough for 46 days with $\frac{2}{3}$ kg of rice remains. In this case, he did not realize that the fraction $\frac{2}{3}$ still represented the remaining day. Thus, he straightly wrote $\frac{2}{3}$ kg of rice remains without changing it into

kilograms. Meanwhile, Hafiz did the mistakes in calculating the number rice packs. In the question, there were 35 packs of rice, he, however, drew only 30 boxes. That was the reason why he did not get the answer of the problem.

Additionally, the result of the posttest also showed that the variety of students' strategies to solve the first problem. In this case, some students, Adit, Akmal and Hafiz, drew pictures to explain the answer. One student, Galih, used ratio table and the rest, ADhika used division strategy. To be more detail, see the pictures 5.34 – 5.38 below.

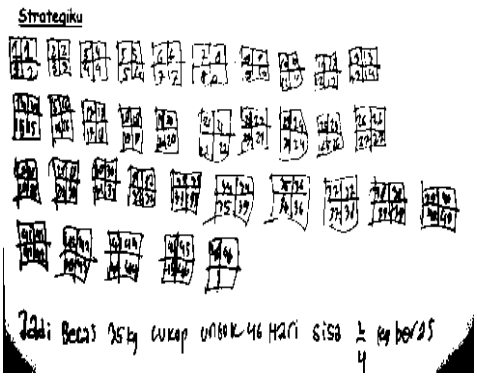


Figure 5.34. Adit's strategy

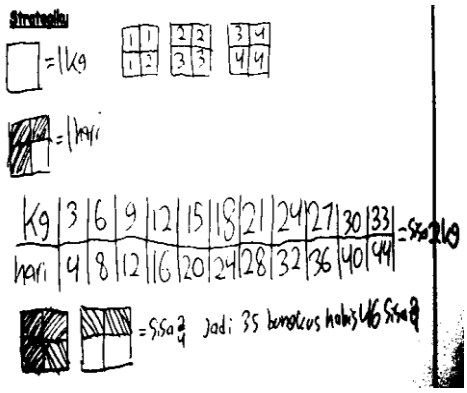


Figure 5.35. Galih's strategy

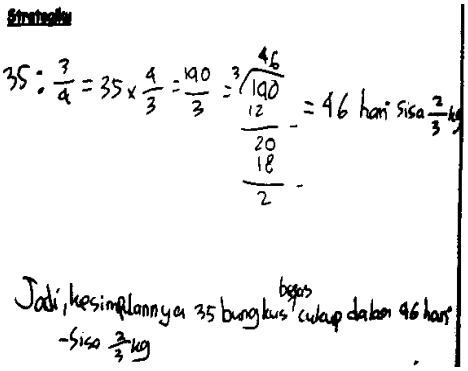


Figure 5.36. Dhika's strategy

Figure 5.37. Akmal's strategy

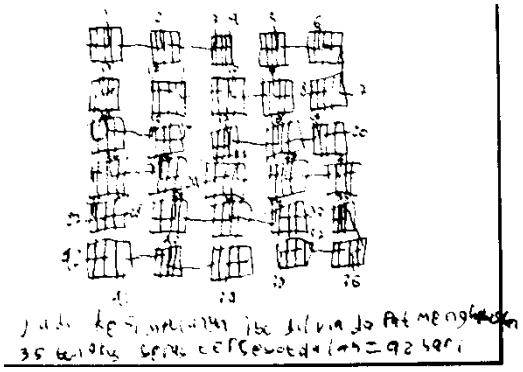


Figure 5.38. Hafiz's strategy

Problem 2

To deal with the second problem, all the students were able to answer it correctly. In this case, each student concluded that 55 cups of chocolate powder were enough to make 22 chocolate cakes.

In terms of the students' strategies, there were no significant changes which meant most of them still used the previous strategy to deal with this problem. For instance, Adit and Hafiz drew 55 boxes that represented 55 cups of chocolate powder. Galih used ratio table to answer the question. In order to start the ratio table, he drew two and a half boxes as the representation of $2\frac{1}{2}$ cups. In this case, he avoided to use fraction since he recognized that 5 cups were enough to make 2 cakes. Thus, he started the ratio table by writing 5 cups = 2 cakes. Then, he just added each unit with this proportion until he got 55 cups = 22 cakes.

Meanwhile, Dhika still used division strategy to answer this problem. In this case, he straightly divided 55 by $\frac{5}{2}$ and did the calculation to draw the conclusion that 55 cups of chocolate powder were enough to make 22 cakes.

However, Akmal did different way from the first problem. This time, he used proportional strategy to answer the second problem. For instance, he added $2\frac{1}{2}$ with $2\frac{1}{2}$ to get 5 cups. Then, he multiplied the result with 11 to get the given number of cups which was 55 cups. Since he knows that 5 cups were able to make 2 cakes, he just multiplied 11 with 2 and got the answer that 55 cups were enough to make 22 cakes. To be more detail, the figures 5.39 – 5.43 below.

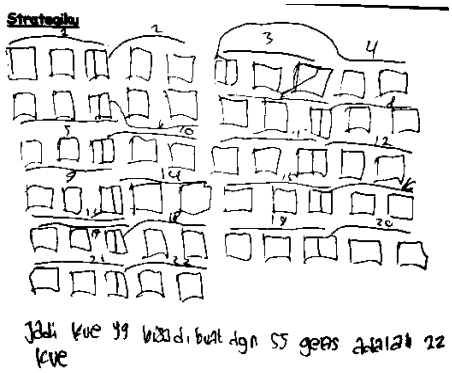


Figure 5.39. Adit's strategy

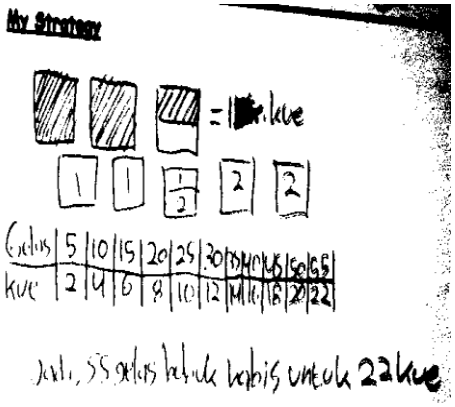


Figure 5.40. Galih's strategy

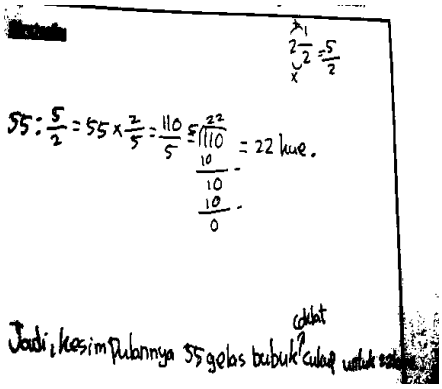


Figure 5.41. Dhika's strategy

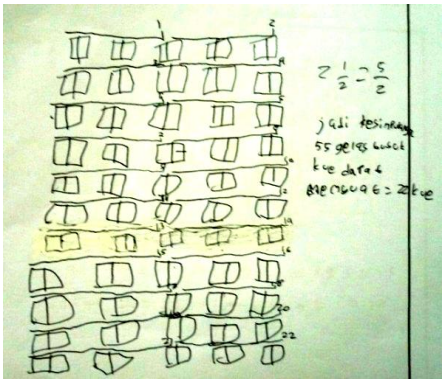


Figure 5.42. Hafiz's strategy

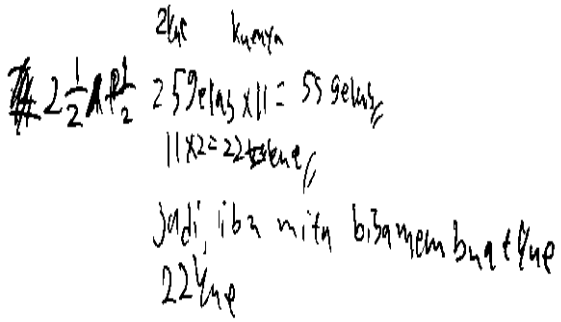


Figure 5.43. Akmal's strategy

Problem 3

In this problem, all the students were able to answer it correctly. Based on the students' worksheet, it can be seen that each students got the same answer that 20 kg of rice were enough for 30 days if $\frac{2}{3}$ kg of rice was cooked everyday. Additionally, most of the students were able to explain how they got the answer of the problem.

When we looked at the students' worksheet, it also can be seen that all students still used the same strategy as they did to solve the previous problems. For instance, Adit and Akmal drew pictures to determine the number of days in 20 kg of rice. This time, they drew 20 circles as the representation of 20 kg of rice. Then, they divided each circle into three parts and he, at the end, got the answer of the problem. At the same time, Galih used ratio table to solve the problem. First, he made pictures to guide him write a ratio table. In this case, he drew two boxes and divided each box into three parts. Then, he counted each two small parts to determine the days. Thus, he got that two kg of rice were enough for three days and started the ratio table with this proportion until he got 20 kg = 30 days. Dhika

also still used division strategy to answer the problem. He divided 20 by $\frac{2}{3}$ and calculated to get the answer of the problem. Hafiz also drew pictures to solve this problem. In this case, he drew 20 boxes as the representation of 20 kg of rice. Then, he divided each box into three parts and counted each two little parts to get the answer. To be more detail, see figures 5.44 – 5.48 below.

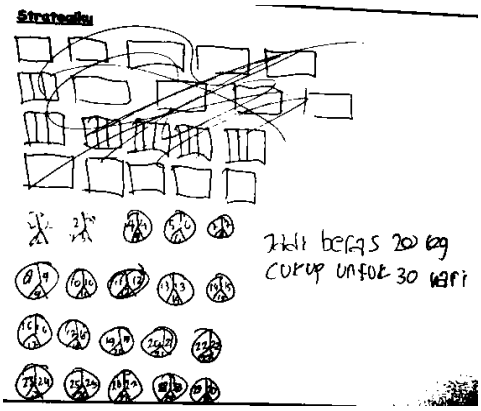


Figure 5.44. Adit's strategy

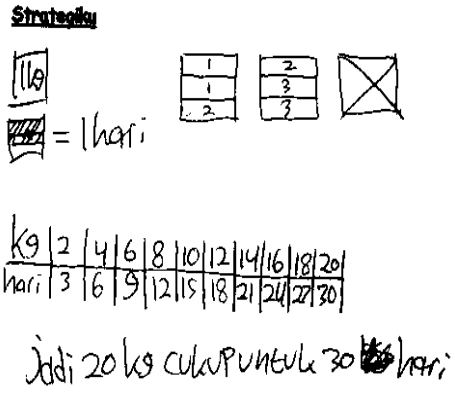


Figure 5.45 Galih's strategy

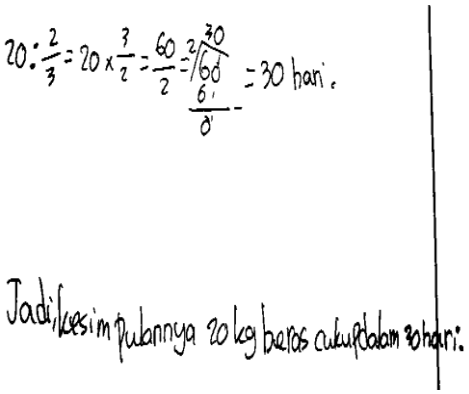


Figure 5.46. Dhika strategy

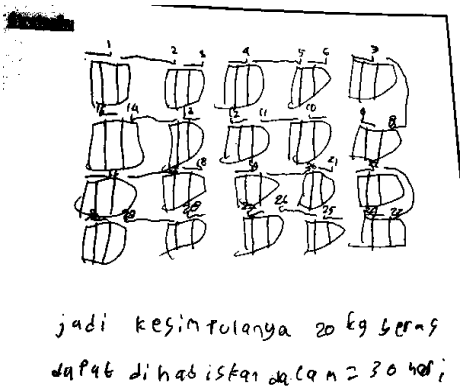


Figure 5.47 Hafiz's strategy

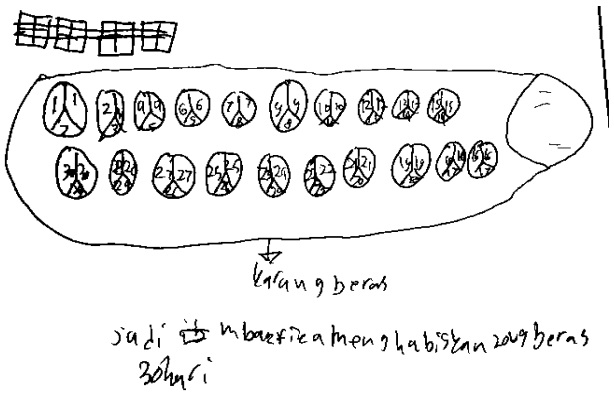


Figure 5.48. Akmal's strategy

5.2.2. Conclusion of cycle 1

Based on the retrospective analysis of the posttest above, it can be explicitly seen that all students are able to use models, such as pictures or ratio table, as a tool to solve proportional problems which include fractions. For instance, they draw pictures, such as boxes or circles, as the metaphor of the packs of rice or each kilogram of rice. Then, they divide the boxes or the circles into several parts in order to get the fractions of the given problems, such as $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{2}{3}$ (see the retrospective analysis of posttest).

In addition, they can give their explanation how they solve the problems in a meaningful way. The expression "meaningful way" means the students can explain their strategy how they get the answer of each problem. For instance, some students draw several boxes and divide each box into four parts. Then, they put numbers in each small part as the representation of the days (e.g. see Adit's strategy). If we see this strategy, we can explicitly see how Adit gets the answer of the problem.

5.2.3. Classroom Observation and Interview with the Teacher

In the second cycle, the classroom observation was also conducted before we carried out the teaching experiment. Based on the classroom observation, we could see that the teacher was used to make the students give their reasoning to explain what they did in solving problems by asking them questions and the teacher could manage the class well. Additionally, the students in that classroom were active and cheerful. In this case, the word "active" means that the students were not diffident to explain their strategy in order to solve problems. During the classroom observation, we, however, could not see that the teacher gave the students a time to discuss to solve problems in group. In this case, the teacher only posed question and let the students solve the problem in front of the class.

Besides observing the teaching and learning process, we also did an interview with the teacher about how he usually taught mathematics in the classroom. Based on the interview, the teacher seldom made a group discussion when he taught mathematics in the classroom. Moreover, the teacher rarely discussed about the different solutions from the students. From the observation, the teacher tended to use formal procedure in solving problems. The interview with the teacher was ended with selecting the students to be the focus group in this study. Based on the interview with the teacher, six students were selected among 30 students in the classroom as the focus group of this study. The students in the focus group were Dekka, Farhan, Ryan, Raka and Nanda. Thus, this cycle will focus on the development of the students in this focus group towards the topic.

5.2.4. Remarks and Improvement of the Hypothetical Learning Trajectory

Based on the remarks and the findings in the first cycle, there were some revisions made to improve the Hypothetical Leaning Trajectory (HLT). The improved HLT was called HLT_1 which was subsequently applied in the second cycle.

In this cycle, there were several revisions of the worksheet and the lesson. Based on the remarks and findings in the first cycle, the third question in the "student's worksheet 4" was removed and it was changed with the problem in the student's worksheet 5. There were two reasons to do this. Firstly, we could explicitly see that the students were able to operate the model, such as ratio table, to solve problem before doing this problem. Secondly, the time allocation was not enough to do the third question because the school committee only provided four meetings to conduct this study.

Since the school committee only provided four meetings to conduct this study, we, subsequently, set the order of lesson 4 and 5 within one meeting. Thus, there were four meetings in conducting the second cycle where each lesson took 70 minutes to conduct. In this case, the HLT_1 can be seen in table 5.1 below.

Meeting	Activities	Learning Goal(s)	Description
1	Solving rice problem by using student's model.	- Make models (pictures) to represent the situational problem - Use the models to solve problem.	In this meeting, the students will work on proportional problem which includes fraction. In this problem, they are asked to determine for how many days that 20

			boxes of 1 kilogram of rice will last if one family consumes $\frac{3}{4}$ kg of rice.
2	Solving rice problem with a ratio table.	- Make sense of proportion - Work with ratio table.	During this meeting, the students will also work on proportional problem which includes fraction. This meeting, they will be asked to determine for how many days that 20 boxes of 1 kilogram of rice will last if one family consumes $\frac{3}{4}$ kg of rice in an early ratio table.
3	Making cakes to sell	- Use ratio table to solve proportional problem. - Describe how to use ratio table to solve proportional problem.	In this meeting, will be asked to try to use ratio table in solving proportional problem, such as: To make 1 chocolate cake Mrs. Ani has 45 cups of chocolate powder. To make 1 cake, she needs $2\frac{1}{2}$ cups of chocolate powder. How many cakes can she make? After solving the problem, they will discuss it in the class discussion.
4	Proportion in unit	- Use ratio table to	In this lesson, the students

	and non-unit fractions.	solve proportional problems. - Solve proportional problem by using model.	learn about determining the solution of proportional problem by using their own models.

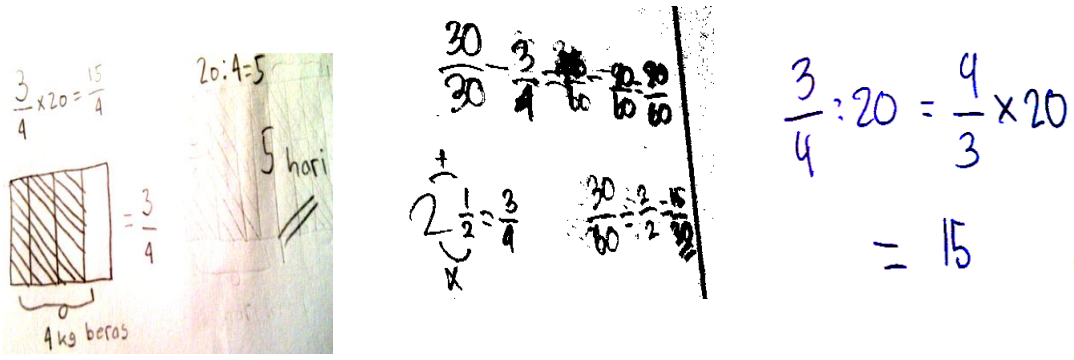
Table 5.1. The improved HLT (HLT_1)

5.2.5. Teaching experiment (Cycle II)

5.2.5.1. Students' Knowledge in the Pretest

At the beginning of cycle 2, we conducted a pretest which consisted of three questions to the students. There were 30 students followed the pretest in this cycle with no changes made. The aim of the pretest was to get the information about the students' prior knowledge, to get the data of students' strategies in order to solve proportional problem and to get the information about the students' starting about proportion. In doing the pretest, the students worked individually about 20 minutes.

In analyzing the pretest, we found that the students' preliminary knowledge in this cycle, in general, was relatively the same with the prior knowledge of the students in the first cycle. In this case, a number of students came up with the idea of addition, multiplication and even the division of fraction. In this case, the students just they just tried to solve the problem by using their understanding about the operation of fraction. In this case, we can see the examples of this situation in the figures 5.49A – 5.49C below.

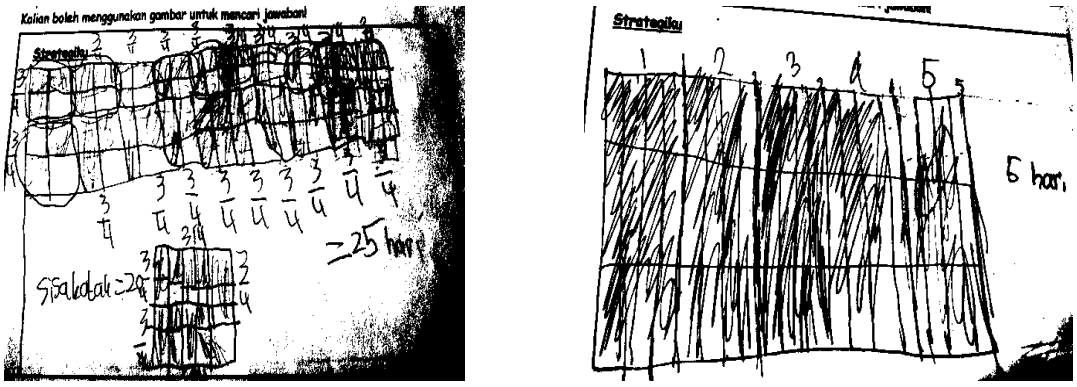


A B C

Figure 5.49 Students' strategies in the pretest

In figure 5.49A, we can see that the students tried to solve the problem the first problem by doing several calculations, such as multiplication and division. He, firstly, tried to multiply $\frac{3}{4}$ with 20 and got $\frac{15}{4}$. Secondly, he divided 20 by 5 and he got the answer which is 5 days. In this case, we did not know why he did these calculations since he did not explain them. Furthermore, the second and the third figures illustrate how students attempted to solve problem 2 and problem 3. In figure 5.49B, we can explicitly see that the student did subtraction strategy. To deal with this, he, firstly, transform $2\frac{1}{2}$ into $\frac{3}{4}$ and subtracted $\frac{3}{4}$ from 30. In this case, this student made a mistake in changing mixed number into fraction. In addition, we did not know why he subtracted $\frac{3}{4}$ from 30 since there were no reasons for this situation. Meanwhile, figure 5.49C visualized how the student tried to solve the first problem by using division strategy. In this case, we can explicitly see that the student straightly divided $\frac{3}{4}$ by 20 and got the answer which is 15.

At the same time, several students used their own model, such as: their visualization, to solve the problems. In this case, the students' model means they drew their own visualization or representation in order to solve the problems. For instance, in order to solve the first problem, they drew 20 boxes and they divided each box into four parts. Subsequently, they shaded or made a sign for each three little parts to represent $\frac{3}{4}$ (see figures 5.50A). At the same time, figure 5.50B shows the student's struggle to solve the first problem. Based on the picture, we can see that this student did not know how to deal with the problem. This student just drew as he understood to deal with the problem.



A B

Figure 5.50. Students' strategies in the pretest

Based on the figures above, we can see the examples of student's mistakes to deal with proportional problems. In figure 5.50A, the student knows how to represent a fraction. For instance, he draws a box, divides it into four parts and shades three parts of it to represent $\frac{3}{4}$. However, he miscalculates the number of days. Meanwhile, the student, in figure 5.50B, does not know how to deal with the

problem. Subsequently, he struggles to solve the problem based on his own understanding.

Furthermore, there were also a number of students that left those problems blank. This is because they did not know how to solve the problems and even did not know how to deal with the problems. To sum up, the findings in the pretest showed that the students had difficulties to solve proportional problems.

5.2.5.2. Interview with the students

After the pretest was conducted, we carried out an interview with the students in our focus group. The intention of this interview was to know students' strategies to solve proportion problems which include fractions and to get the information about the students' starting point about proportion. Thus, the students, during the interview, were asked to explain what they did in the pretest. In this case, we provide an example of the interview which is interesting to be discussed where the students struggle to determine the answer of the problem but did not succeed it. Therefore, the following short fragment shows how Raka struggled to deal with the third problem.

Figure 5.51. Question for interview

Fragment 5.15. Interview with Raka

1. Researcher : Read the question first.
2. Raka : Ok. (*Reading the question*)
   : This is 15 kg of rice and she cooks $\frac{1}{3}$ kg.
   : So, 15 is divided by 3… which is 5. Thus, $\frac{1}{3}$ kg of one kilogram will be enough for five days. (*Pointing 1/3 = 5*)
3. Researcher : How do you know that $\frac{1}{3}$ kg of ice will be enough for five days?
   : How did you do it?
4. Raka : Hmm… I think I am wrong…
5. Researcher : And then?
6. Raka : Everyday she cooks $\frac{1}{3}$. So, this is divided by three. (*Pointing the box*)
   : Thus, this is $\frac{1}{3}$, this is $\frac{2}{3}$ and this is $\frac{3}{3}$, which is equal to 15.
   : (*Explaining the picture that he made*)
   : So, 15 kg of rice will be enough for three days.
7. Researcher : Ohh…
   : What does this picture show about? (*Pointing the box*)
8. Raka : This is the picture of one sack of 15 kg of rice.
9. Researcher : So, what is the conclusion?
10. Raka : 15 kilograms of rice are enough for 3 days.

Based on the students' pretest and the fragment 5.15 above, we can explicitly see that Raka has an understanding to represent fractions, such as: $\frac{3}{4}$, $2\frac{1}{2}$

and $\frac{1}{3}$, in a picture. An example of this situation can be seen when he draws several boxes then he divided each box into four equal parts. Subsequently, he shaded three parts to get $\frac{3}{4}$.

However, it is still difficult for him to interpret the third problem. Based on the short fragment 5.15 above, he understands about fraction as a part-whole relationship. In this case, he understands about the part of a fraction within one unit, such as: understand $\frac{1}{3}$ of one kilogram of rice. However, he struggles to understand $\frac{1}{3}$ of one kilogram of rice within 15 kg of rice. For example, he represents 15 kg of rice with one box and he just divides into three parts. Subsequently, he gets the answer that 15 kg of rice are enough for 3 days (see line 6).

Conclusion of Pretest and Interview

Based on the data of pretest and the interview with the students, it can be explicitly seen that the students' strategies, between the first cycle and the second cycle, do not have significant different. Some students use their own model, such as: drawing pictures, to solve the problems. However, most of them tend to apply what they understand to deal with those problems. For instance, the students use multiplication, subtraction and division. The possible reason for this is that they do not understand the intention of the problems. We can see this situation in the short fragment when we conduct an interview with Raka.

5.2.5.3. Teaching and Learning Process

1. Lesson 1: Solving rice problem by using student's model

The first meeting was conducted on Monday, March 10th, 2014 in the class 5C. In this lesson, all students in class 5C were taught by Ustadz Syawal, their own mathematics teacher, for about 70 minutes. In this meeting, there were no changes made within the worksheet and the activity from the first cycle. In general, this lesson indicates that most of the students were able to use model, such as: their own visualization, as a tool to answer proportional problem. Dealing with this, the following retrospective analysis shows how the six students reason during the teaching and learning process.

1. Initial activity

At the beginning of this lesson, the teacher showed a one-kilogram box of rice and posed the following problem to the students.

> *Ustadz Syaddat buys 20 one-kilogram packs of rice. In one day, she cooks* $\frac{3}{4}$ *kg of rice. Can you determine 20 one-kilogram packs of rice are enough for how many days?*

Then, the students were asked to solve the problem individually in the students' worksheet 1 about 10 minutes and discussed it together in group. In this meeting there were no significant different of students' conjectures to deal with the problem.

2. Main activity

In order to solve the problem, the students needed to discuss it in group. In this case, the following fragment illustrates the moment when the students in the focus group discussed the problem.

Fragment 5.16. Students use their own model to solve problem

1. Teacher : Now, you can discuss with your friends in your group how to solve the problem.
   : Write your answer on the poster.
2. Farhan : I draw 20 boxes. (*Pointing all the boxes that he wrote on his worksheet*)
   : And each box is divided into four parts. So, this is one, two, three, four, five, six,
   seven, eight, nine, ten, eleven, twelve, thirteen fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty one, twenty two, twenty three, twenty four, twenty five and twenty six.
   (*Counting each three little parts*)
   : Here, there is still two parts remain. So, it will be $\frac{2}{4}$ which is equal to $\frac{1}{2}$.
   : $\frac{1}{2}$ is close to one. Thus, these two parts are equal to one day. So, the answer is 27 days.
3. Raka : Can you repeat how you do it?
   (*Asking Farhan to repeat his explanation*)
4. Farhan : (*Repeating his explanation once again*)
5. Dekka : I think we can do like this… $\frac{3}{4}$ times 20.
   (*Calculating* $\frac{3}{4} \times 20$)
   : What is the answer?
6. Farhan : 27 days.

After the discussion, all students agreed with Farhan's answer. Subsequently, they wrote their strategy in the poster. At the end of doing the poster, the students recognized something which was not discussed in before. The following short fragment shows the students made a short discussion to discuss the answer of the problem.

Fragment 5.17. Students' mistake to solve proportional problem

7. Dekka : I think the answer is wrong.
8. Nanda : 26,5 days.
9. Dekka : (*Cross out the previous answer and write 26 + 0,5 = 26,5*)
10. Raka : 26,5 days. And the remaining?
11. Farhan : The remaining,,,
12. Ryan : The remaining is 0,5.

This group had an interesting strategy to be discussed in the whole-class discussion. Thus, the teacher asked them to explain their strategy in front of the

class. To deal with this, the following fragment shows how the discussion takes place

Fragment 5.18. Variety of students' strategies

13. Teacher : Ok… Say it loudly…
14. Farhan : There are 20 packs, so we make 20 boxes.
: $\frac{3}{4}$ of each pack is used, so one box is divided by 4… (*Pointing all the boxes*)
: Since $\frac{3}{4}$ of each pack is used, then the first day, the second day, the third day, the fourth day, the fifth day, the sixth day, the seventh day, the eighth day, the ninth day, until the 26^{th} day. (*Counting each* $\frac{3}{4}$)
: And it remains $\frac{2}{4}$ which is equal to $\frac{1}{2}$ or 0,5.
: So, 26 plus 0.5 is equal to 26,5 days
15. Teacher : 26?
16. Farhan : 26 days.
17. Teacher : What is the remaining?
18. Farhan : 0,5
19. Teacher : 0,5.
: Zaki got that 26 days with $\frac{2}{3}$ remains. And Abyan gets the same result.
: Do you have any comments? (*Asking other students for questions*)
20. Zaki : That 0,5 means 0,5 from rice.
21. Teacher : What about you?
22. Zaki : From days.
23. Teacher : What about Abyan?
24. Abyan : From days.
25. Teacher : What is the problem?
26. Students : For how many days Ustadz Syaddat can cook the rice?
27. Teacher : Abyan said that 26 $\frac{2}{3}$ days, right? Because he counted based on the days.
: And Zaki got the same result.

The fragments 5.17 and 5.18 indicate two interesting moments happen during the first lesson. First, they show the variety of students' strategies that lead to different answers. Additionally, they, secondly, contain a mistake which is interested to be discussed. For instance, Abyan's group recognizes that 20 packs

of rice are enough for $26\frac{2}{3}$ days and Zaki's group also gets the same result. However, the answer of Farhan's group is different. In this case, they get that 20 packs of rice are enough for 26 days with $\frac{2}{4}$ or 0,5 remains. The mistake occurs when they add 26 with 0,5 which is 26,5 days. In this case, they do not recognize that the 26 and 0,5 are in different unit (see line 2 and line 7).

To deal with this situation, the teacher asked the students to compare all the different strategies and them to give their opinion towards the answers. Some students recognized that the answer of the Farhan's group was not right. Based on the answer of Farhan's group, they, however, agreed that 20 packs of rice were enough for 26 days with $\frac{2}{4}$ kg remained. Additionally, other students also agreed that 20 packs of rice were enough for $26\frac{2}{3}$ days. Therefore, the students, at the end, agreed that 20 packs of rice were enough for $26\frac{2}{3}$ days or 26 days with $\frac{2}{4}$ kg remained.

Conclusion

Based on the description of the first meeting above, we can see the different strategies that the students possibly use to solve proportional problem. Our expectations towards the students' thinking also happened in this meeting. Some students use their own representation, such as: drawing pictures, to solve the problem. In this case, the way how they solve using those pictures are not different with the students' strategies in the first cycle. An example of this situation can be seen in line 2 and line 14. Furthermore, some students, at the beginning of the lesson, use their own prior knowledge to deal with the problem.

For instance, several students multiply 20 with $\frac{3}{4}$ to get the solution of the problem. Some students divide the two numbers to get their solution.

In this lesson, we can also see that some students' misconceptions to solve proportional problem although they know how to represent fraction in a drawing. The students mix the number of kilograms and the number of days. The mistake, for example, occurs when they add 26 with 0,5 which is 26,5 days. In this case, they do not recognize that the 26 and 0,5 are in different unit (see line 2 and line 7).

2. Lesson 2: Solving rice problem with a ratio table

The next day, the teacher continued the teaching and learning process for about 70 minutes. Basically, there were no changes made in this meeting. Thus, the aim of this lesson also stood the same. As we stated before, we, in this meeting, assumed that the students the idea of proportion with their own understanding and the teacher can directly guide the students to the idea of ratio table. However, it, in this study, will be described how the teacher guided the students to the early stage of ratio table by asking them questions.

1. Initial activity

In our Hypothetical Learning Trajectory, we assumed that the students were still working on their work in the previous meeting. Thus, the beginning of this lesson stood at finishing the students' work and discussing the work within the whole-class discussion. However, this situation also did not happen in the actual teaching and learning process in the second cycle. The reason for this situation

was because the students worked faster than we expected in the previous meeting. For instance, all students, in our HLT, did not know how to deal with the problem given. On the contrary, this did not happen in the classroom activity. In this case, some students could give a meaningful way such as they used their own model, such as: their own visualization, to solve problem. During the first lesson, the students learn how to deal with the problem by using model.

At the beginning of this lesson, the teacher, therefore, recalled what they had learned in the previous meeting by asking them several questions. The questions were aimed to remind the students about the problem that they solved in the previous lesson and to discuss the way how they solved it.

2. Main Activity

After making a short discussion, the teacher attempted to guide the students to get the idea of proportion by using their own model by making a short discussion. In other words, the teacher wanted to guide the students to a moment where the early stage of ratio table occurred automatically.

At the beginning of the discussion, the teacher asked them to explain how to draw $\frac{3}{4}$ of one kilogram. Afterward, the teacher asked them the following up questions, such as: 3 kg = … days, 6 kg = … days and 9 kg = … days. In our HLT, we conjectured that the students still did not know how to deal with those problems as what happened in lesson 1 or they will apply what they understand to deal with those problems. For instance, the students use multiplication, subtraction and division. However, we also assumed that the students used their model that they had learned in the first lesson.

Here, the description of what happened in classroom. In this case, the teacher asked several students to solve the four problems by using their understanding in front of the class. We found that there were no significant differences between the students' strategies in the first cycle and this cycle. They could give their explanation by using their own model. To deal with this, the short fragment 5.19 below shows how the teacher guided the students to the idea of proportion.

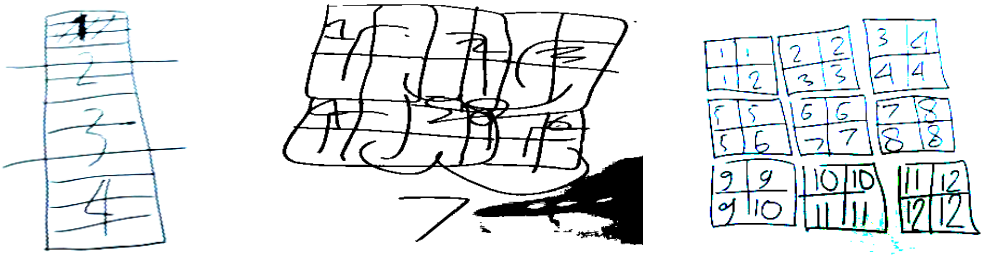
Fragment 5.19. Students' visualization to solve problem

1. Teacher : If Ustadz has three packs of rice, then Ustadz can cook them in how many days? Oke, Adit please…
2. Adin : (*Drawing a rectangle and divided into three parts. Subsequently, he divided each part into four parts*)
   : One day uses $\frac{3}{4}$ kg then this is 1 day. (*Pointing the first three parts*)
3. Teacher : So, that is one day. Can you make it clear?
4. Adin : And this is two, this is three and this is four.
   (*Pointing the second, the third and the fourth three parts*)
5. Teacher : So, they will last in four days.
6. Adin : Yes.
7. Khariz : Ustadz cooks 3 packs of rice in 4 days
8. Teacher : Now, if there are six packs of rice?
   (*Farhan raised his hand*)
9. Teacher : How many days will they be enough, Farhan?
10. Farhan : Eight days.
11. Teacher : How do you know that? Can you explain it?
12. Farhan : (*Going to the white board to explain his answer*)
    : So I drew six boxes. One day, he uses $\frac{3}{4}$ kg so I marked three parts. So, this is one. If there are six pack then they can be consumed for six days. The remaining can be cooked in two days. So, this is seven days and this is eight days.
13. Teacher : Eight days. Now, Ustadz has nine packs of rice. How many days will they be enough?
    (*Dimas raised his hand*)
14. Didit : (*Going to the white board to explain his answer*)
    : Ustadz buys nine packs of rice. : So, these are the nine packs of rice. (*Showing the nine boxes he drew*)

: And all boxed are divided into four parts.

: Ustadz uses $\frac{3}{4}$ kg of rice each day. So, this is one-one-one, two-two-two, three-three-three and it goes until 12.

15. Teacher : So, what is the conclusion?
16. Didit : If Ustadz has nine packs of rice, they will last in 12 days.



A B C

Figure 5.52. Students' visualizations

The short transcript 5.19 shows that students used model, such as: their own visualization, to solve the problems. For instance, to determine the answer of 3 kg = … days, based on the figure 5.52A, the first student draws one rectangle and divided it into three parts, which represent three packs of rice. He divided each part into four parts to represent $\frac{3}{4}$ of rice. Subsequently, he counts each three little parts and gets the answer that 3 kg = 4 days (see lines 2 – 6). At the same time, the second and the third figures also illustrate the student's strategies to solve the second and the third problems which are 6 kg = … days and 9 kg = … days. To solve these problems, the second and the third students, basically, did the same strategies as the first student before. However, they are different in terms of presenting it. Based on the figures 5.52B and 5.52C, we can explicitly see that the students recognized that 6 kg = 8 days and 9 kg = 12 days (see also lines 8 – 15).

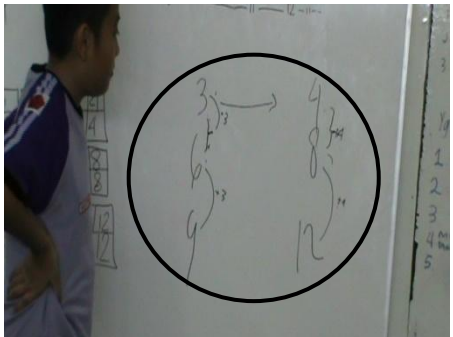
Based on the transcript 5.19 and figure 5.52, we can notice that model plays an important role in order to help students solve proportional problem. It helps the students visualize their thinking towards the way how to solve proportional

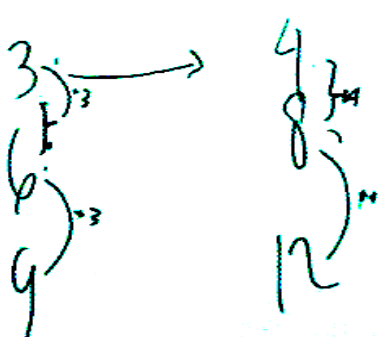
problem. In this case, we can explicitly see that the students automatically used model, such as: their own visualization, to solve the three problems. Therefore, we can say that the students' visualization helps them when they were asked to reason how they get the answer of the proportional problems.

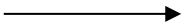
After the students got the idea of proportion, the teacher continued the lesson by guiding the students to the early stage of ratio table. As we had stated before, the early stage of ratio table means the moment when the students build the sense of proportion by using their model. Thus, the fragment 5.20 below shows how the teacher did it.

Fragment 5.20. The teacher guides the students to the idea of proportion

17. Teacher : Now, I want to ask another question. Do you see a pattern?
: Kharis said that Ustadz used 3 kg of rice in 4 days. Haikal got that Ustadz used 6 kg packs of rice in 8 days and Dimas determined that Ustadz used 9 packs of rice in 12 days.
: Do you see the way?
18. Hari : It is a multiplication.
19. Teacher : Can you explain it?
20. Hari : (*Going to the white board to explain it*)
: So, 3 will be equal to 4 and 6 will be equal to 8. We multiply 3 with 2 to get six and we also multiply 4 with 2 to get 8.
21. Teacher : Write it.
22. Hari : Ok
23. Teacher : From 6 to 9?
: (*Pointing to the second and the third proportional problem*)
24. Hari : (*Smile*)
25. Teacher : Who can help him?
(*All students raised their hands*)
26. Teacher : Oke, Adit please…
27. Adin : (*Going to the white board and writing from 3 to 6 to 9 is added with 3 and from 4 to 8 to 12 is added with 4*)
28. Teacher : This one is adding up with 3 and this one is adding up with 4.
: Do you agree?
29. Students : Yes







A B

Figure 5.53. Students get the idea of proportion

The transcript 5.20 and the pictures 5.53 show how the students build the sense of proportion by using their own model. In this case, we can explicitly see that Hari concluded *3 packs = 4* and *6 packs = 8* and 9 packs = 12 days based on the model he saw. For instance, he saw "*three boxes and four* $\frac{3}{4}$*s*", "*six boxes and eight* $\frac{3}{4}$*s*" and "nine boxes and twelve $\frac{3}{4}$s" (see lines 1 – 16 and figure 5.53B). Based on the transcript and figure 5.52, we can notice that model also plays an important role in order to help the students build the sense of proportion. It helps the students visualize the proportional situations.

After the students wrote their thinking, the teacher managed those answers in a table. In this case, he drew a table which consisted of two rows where the first row was filled with kilograms and the second one was filled with days. Since the first conclusion was "3 packs of rice are enough for 4 days" and he put 3 in kilogram and 4 in days. He continued to the second and the third conclusions that were "6 packs of rice are enough for 8 days" and "9 packs of rice for 12 days". Subsequently, he put these results in the ratio table. After that, he asked the students to continue it in group about 15 minutes.

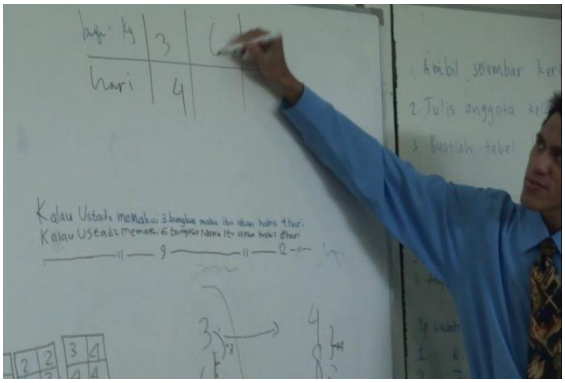


Figure 5.54. Putting students' answers in a table

Whole-class discussion

After working in group, it is the time for the students to present their work in front of the class. The presentation was aimed to discuss students' strategies to determine the answer of the problem by applying ratio table. In general, the students tend to use addition strategy when the use ratio table. In general, the discussion shows that most students were able to determine the proportion between each unit.

However, it was still confusing for the students, including the students in the focus group, to determine the proportion of the remaining. For instance, the students could determine that if Ustadz cooks $\frac{3}{4}$ kg in one day, then he cooks 18 packs in 24 days. In this case, there were two packs remains. Most of the students did not know how to deal with this number. However, there was a group that could give how to determine the answer by using model. In this case, the following fragment 5.21 shows how the students use their own ratio table in order to solve problem.

Fragment 5.21. Students use ratio table with multiplication strategy

30. Teacher : Please Zaki's group.
31. Zaki : Ustadz cooks 3 packs in 4 days
: The first number is multiplied with 1.

: (*Pointing the numbers in kilogram unit*)
: The second column is multiplied with 2. The third column is multiplied with 3.
: The fourth is 3 times 4 is 12. The fifth is 3 times 5 is 15 and the sixth is 3 times 6 is 18.

32. Teacher : What about the "days"?
33. Group : It's the same.
34. Teacher : Do you have questions? (*Asking other students for questions*) (*No questions from the students*)
35. Teacher : Zaki, what about the remaining?
36. Zaki : We can see that there are still 18 packs.
: There are two packs more. So, we draw the two packs.
: It's $\frac{3}{4}$. So, we divided each box into four parts.
: We use three parts. So, this is the 25th day and this is the 26th day.
: It must be $\frac{3}{4}$, but it remains $\frac{2}{4}$ which is equal to $\frac{1}{2}$.
: (*Pointing the remaining little boxes*)
37. Teacher : So, what is your conclusion?
38. Zaki : 26 days with $\frac{1}{2}$ remains.

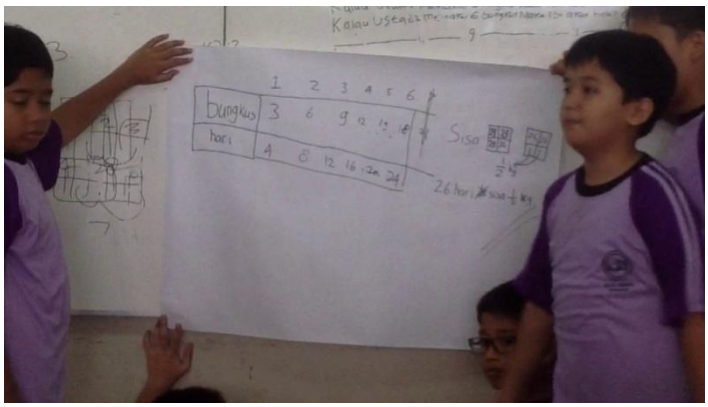


Figure 5.55. Students' ratio table

After Zaki's group presented their work, the teacher repeated what Zaki's group to deal with the reminder. The teacher emphasized that Zaki's group drew two boxes to represent the two packs of rice and they did the same as yesterday.

Based on the fragment 5.21 above, we can explicitly see that most students were still struggling to use model, such as ratio table, to deal with proportional problem with reminder. For instance, it is difficult for students to determine 20

packs for how many days after they got the proportion 18 packs for 24 days. At the same time, some students come up to use both models that they have learned, such as: ratio table and picture. For instance, we can see that they use the ratio table until they get the proportion 18 packs = 24 days. For the two packs, they draw pictures as the representation of the two packs and solve as they solve the problem in first lesson (see lines 30 – 38 and figure 5.55).

After the students presented their poster, the teacher asked the students to compare the conclusion that they had got today to the result in the previous meeting. They recognized that the conclusion was the same with their conclusion in the previous meeting that 20 one-kilogram packs of rice were enough for 26 days with $\frac{2}{4}$ remains. Then, the teacher, at the end of the lesson, introduced ratio table to the students that they will learn for the upcoming days.

Conclusion

Based on the retrospective analysis above, teacher plays an important role in order to guide students to the idea of ratio table. In order to do that, teacher can ask them questions to scaffold their thinking to a moment where the teacher expects the students to be. The examples of questions and how the teacher do it in a teaching and learning can be seen from line 1 until line 28.

In addition, the picture 5.52 shows how the students build the sense of proportion by using their own model. In this case, we can explicitly see that Hari concluded *3 packs = 4* and *6 packs = 8* and 9 packs = 12 days based on the model he saw. For instance, he saw "*three boxes and six* $\frac{3}{4}$*s*", "*six boxes and eight* $\frac{3}{4}$*s*" and "nine boxes wand twelve $\frac{3}{4}$*s*" (see lines 1 – 16 and figure 5.52B). Based on the

figure 5.52, we can notice that model also plays an important role in order to help the students build the sense of proportion. It helps the students visualize the proportional situations.

At the same time, we, based on the fragment above, can explicitly see that most students were still struggling to use model, such as ratio table, to deal with proportional problem with reminder. For instance, it is difficult for students to determine 20 packs for how many days after they got the proportion 18 packs for 24 days. At the same time, some students come up to use both models that they have learned, such as: ratio table and picture. For instance, we can see that they use the ratio table until they get the proportion 18 packs = 24 days. For the two packs, they draw pictures as the representation of the two packs and solve as they solve the problem in first lesson.

3. Lesson 3: Making cakes to sell

After conducting the second lesson, the teacher carried out the third meeting for 70 minutes. The setting of the teaching and learning process of the third lesson in the second cycle was not different from the first cycle. In general, the situation happened in this lesson was not different from the first cycle. In other words, the conjectures of students' thinking in the first cycle also happened in this cycle.

1. Initial activity

At the beginning of this meeting, the teacher did a short conversation with the students about what they had learned in the previous meetings. The aim of this activity was to remind the students the problems that they had worked on and the strategy how they solved it. Based on the previous meeting, the teacher saw that it

was still confusing for the students to operate ratio table. In this case, the students needed to practice to operate ratio table. Thus, the teacher gave a new problem to the students to solve. They were asked to solve the following problem.

> *Mrs. Ani wants to make some chocolate cakes to sell. She has 45 cups of chocolate powder. To make one chocolate cake, she needs* $2\frac{1}{2}$ *cups of chocolate powder. Can you determine 45 cups of chocolate powder are enough to make how many cakes?*

To deal with this problem, the teacher asked the students to solve this problem individually for about 5 minutes and later on they discussed it in group. Based on the activity of this meeting, some students, in general, still drew pictures to solve the problem. They did the same strategy, in representing the cups of chocolate powder and dividing those pictures, as they did in previous meeting.

At the same time, some students used ratio table to solve the problem. In this case, the students in the focus group made their own ratio table and used it to solve the problem. The following fragment shows how the students in the focus group discussed the problem by busing ratio table.

Fragment 5.22. Students use ratio table with repeated addition strategy

1. Dekka : Ok, let's start. It's $2\frac{1}{2}$,
2. Nanda : 5
3. Dekka : Then, $7\frac{1}{2}$
4. Farhan : Dekka, I think you can write in sequence like this.
   : (*Showing his answer*)
5. Dekka : Oh ya…
   : (*Adding up the number of cups with* $2\frac{1}{2}$ *from* $2\frac{1}{2}$ *to 45*)
6. Farhan : Now, $2\frac{1}{2}$ cups = 1 cake.
7. Dekka : Oh ya… (*Adding the number of cakes with from 1 to 18*)

After the students discussed the problem in group, the teacher asked them to present their work in front of the class. In this case, the fragment 5.23 below shows how the students in the focus group present their work.

Fragment 5.23. Students' presentation in the whole-class discussion

8. Teacher : Please Farhan…
9. Farhan : This is two and a half, five, seven and a half, ten, twelve and a half, fifteen, seventeen and a half, twenty, twenty two and a half, twenty five, twenty seven and a half, thirty, thirty two and a half, thirty five, thirty seven and a half, forty, forty two and a half, forty five. (*Pointing all the numbers in the first row*)
: This amount is enough to make how many cakes?
: And this is one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen. (*Pointing all the numbers in the first row*)
10. Teacher : So, the answer is for how many?
11. Farhan : Eighteen
12. Teacher : What is eighteen?
13. Farhan : Eighteen cakes
14. Teacher : Oh… 1, 2, 3, 4, 5, … , 18 are for the number of cakes, right?
: So, what about $2\frac{1}{2}$, 5, $7\frac{1}{2}$, 10, …, 45?
15. Farhan : The number of chocolate powders.

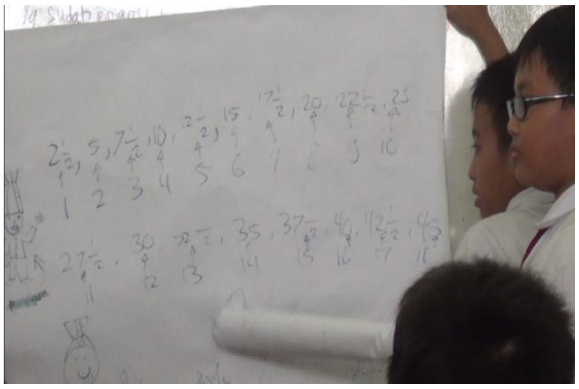


Figure 5.56. Students' ratio table

The fragment 5.23 shows that the students are able to explain their strategy by using ratio table. We can see this situation when Farhan explained what he and his friends made in their poster. In this case, they did repeated addition in operating the ratio table. For instance, they know that $2\frac{1}{2}$ cups of chocolate

powders = 1 cake. They add the number of chocolate powder with $2\frac{1}{2}$ and add the number of cakes with 1 until they get 45 cups of chocolate powder. Subsequently, they get the answer of the problem which is 18.

After this group presenting their work, the teacher gave other students to ask questions and to give comments if there was something that they did not understand. Thus, the teacher did a small discussion after the presentation. Subsequently, he asked other groups to present their work in front of the class.

After conducting the whole-class discussion, the teacher, at the end of this lesson, asked the students to conclude what they had got from the problem. In this case, all the students agreed that the answer of the problem stayed 45 cups of chocolate powder were enough to make 18 chocolate cakes.

Conclusion

Based on the retrospective analysis of lesson 3 above, the fragment shows that the students are able to explain their strategy by using ratio table. We can see this situation when Farhan explained what he and his friends made in their poster. In this case, they did repeated addition in operating the ratio table. For instance, they know that $2\frac{1}{2}$ cups of chocolate powders = 1 cake. They add the number of chocolate powder with $2\frac{1}{2}$ and add the number of cakes with 1 until they get 45 cups of chocolate powder. Subsequently, they get the answer of the problem which is 18.

4. Lesson 4: Proportion in unit fraction and non-unit fraction

In the fourth day, we conducted the last teaching process for about 70 minutes. As we stated before, we put the fifth meeting into the fourth meeting

since the school only provided us four days to conduct this study. Thus, the aim of this lesson was to let the students try ratio table and use their own ratio table to solve proportional problem and to see the students to reason their thinking about their strategy by using models or to see whether they used models to solve problem or not.

1. Initial activity

At the beginning of this lesson, the teacher asked the students to answer the following questions.

1. *Miss Rika goes to supermarket to buy 15 kg of rice to supply her daily needs. Everyday, she cooks* $\frac{1}{2}$ *kg of rice. Can you determine 15 kg of rice are enough for how many days?*
2. *Miss Sri goes to supermarket to buy 25 kg of rice to supply her daily needs. In one day, she cooks* $\frac{1}{3}$ *kg of rice. Can you determine 25 kg of rice are enough for how many days?*
3. *Mrs. Fitri buys 25 kg of rice for the supply her daily need. In one day, she cooks* $\frac{3}{4}$ *kg of rice. Can you determine 25 kg of rice are enough for how many days?*

To deal with these problems, the teacher gave the first problem to the students and asked them to solve it individually about 5 minutes. Then, they discussed it with their friends in group. Before they started working, the teacher asked one of the students to read the problem and asked them whether they understood the problem or not. In this case, all the students understood the problem and started working individually.

Basically, there were three strategies that the students did in order to solve the problem based on the results of the student's works. Some students still drew pictures to solve the problem as a model to solve proportional problem. They drew a box as the representation of one kilogram of rice. Some students solved the problem by using ratio table. Meanwhile, there were also students solved the problem by using division.

2. Main activity

After working individually, they started working with group. In the group discussion, they had to write their strategies in a poster paper. In this case, the two groups decided to solve the problem with two ways, by using pictures and ratio table. During the discussion, the focus group still drew pictures to solve the first problem.

In this case, the students in the focus group, basically, solved the problem by drawing pictures. In order to check whether their answer was right they also other strategies, such as: proportion and division. Thus, the short fragment 5.24 below illustrates how these students discuss this problem in group by using model.

Fragment 5.24. Students use their own model to solve proportional problem

1. Dekka : Ok, let's do it
2. Farhan : (*Explaining his strategy*)
3. Dekka : Let's start with Ryan' strategy.
   : (*Drawing 15 boxes*)
   : One, two, three, four, five, six. (*Counting the number of boxes*)
   : (*Dividing each box into two parts*)
4. Raka : This is one day. (*Pointing one little part*)
5. Nanda : Start from here.
6. Raka : OK. So, this is one day, two days, three days.

7. Dekka : (*Continue the numbering until he got 30 days*)

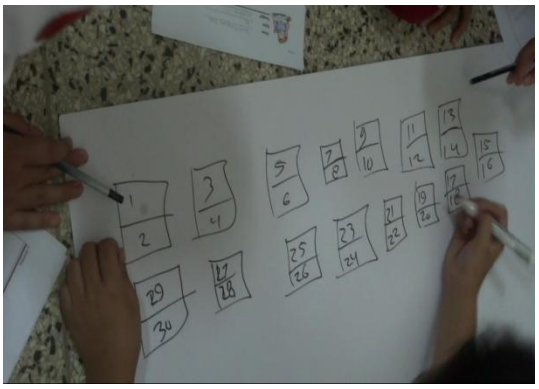


Figure 5.57. Students' representation

Based on the short fragment above, we can explicitly see that these students use model to solve the problem. In this case, they did the same strategy as the first meeting. For instance, they drew 15 boxes as the representation of 15 kg of rice. They divided each box into two parts. Subsequently, they put numbers as the representation of the days (see lines 1 – 7).

After the discussion, the teacher asked the students to present their work in front of the class. In this case, two groups got a chance to present their work. However, the students in the focus group did not present their work because the time limited the teacher to do that.

Once the students presented their works, the teacher asked them to solve the second problem. In this case, they wee also asked to work individually for five minutes and they could discuss it in group. Basically, there was no different of the students' strategy to solve this problem and the strategy to deal with the first one. The fragment 5.25 below shows how the students in the focus group discuss the second problem in group.

Fragment 5.25. Students use their own model to solve proportional problem

8. Dekka : (*Start drawing boxes*)
9. Farhan : We can divide the box like this. (*Showing his strategy*)
10. Dekka : No, like this. (*Dividing the box horizontally*)
11. Raka : One, two, three.
12. Dekka : Yes.
: One, two, three, four, five, … , fifteen. (*Counting the numbers in the row*)
: So, this is 15 days.
13. Raka : I think it is wrong.
14. Farhan : It is true. This one is 15, this is also 15 and so on.
15. Raka : Oh ya…
16. Dekka : So, this will be 15×15 which is 75 days.
: Now, let's check with ratio table.
: (*Suddenly giving the marker to Farhan*)
17. Raka : Wait.
: 1 kg for 3 days, 2 kg for 6 days, … , 25 kg = 75 days.
18. Farhan : (*Writing the ratio table*)

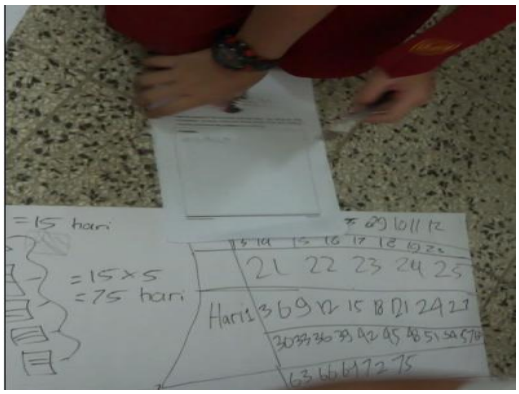


Figure 5.58. Students' ratio table

The fragment 5.25 above shows the students were used to use model to solve proportional problem. In this case, the students started by drawing pictures and used ratio table. Based on the fragment 5.25, we can also see the way how the students operate the ratio table. In this case, the students notice that 1 kg = 3 days. Subsequently, they keep adding the proportion until they get 25 kg = 75 days (see lines 16 – 18).

After the students work in group, the teacher asked them to present their work in front of the class. Basically, the way of students describe their strategy was the same when they discussed it in group. Based on the students' presentation, the students were getting used to use model to solve problems.

Once the students presented their works in front of the class, the teacher asked them to solve the next problem. In this case, this problem was aimed to evaluate what the students had learned from the teaching and learning process. In order to solve the third problem, the teacher asked the students to work individually for five minutes. Later on, they could discuss it in group for 15 minutes.

The students in the focus group did not really discuss the problem. This situation can be seen explicitly in their work that they only did division strategy to solve this problem. To deal with this, there were two possible reasons for this situation. Firstly, the students, based on the teaching and learning process, in the focus group took a long time in doing the problem individually. Thus, they just have about 8 minutes for a discussion. Secondly, they just wrote the simplest way to solve the problem. In this case, they possibly thought that division strategy was the simplest way to solve the problem.

At the same time, Zaki's group showed their way in order to solve the problem in a meaningful way. In this case, they used models, such as ratio table, to solve the problem. Before doing using this model, the students had tried to solve the problem by using division strategy, they, however, were not sure about

their answer. Thus, they used ratio table to solve the problem. The following fragment shows how these students presented their work by using ratio table.

Fragment 5.26. Students use ratio table to solve proportional problem

19. Zaki : This is the rice and this is the days. (*Pointing both units*)
: So, 3 kg = 4 days, 6 kg = 8 days, 9 kg = 12 days, 12 kg = 16 days, 15 kg = 20, 18 kg = 24, 21 kg = 28 days and 24 kg = 32 days.
: The rice remains 1 kg. Thus, this is the rice. (*Pointing one box*)
: There are three parts which are enough for one day. So, this is the 33rd day.
: And the remainder is $\frac{1}{4}$ kg.
20. Teacher : So, what is your conclusion?
21. Zaki : 33 days with $\frac{1}{4}$ kg remains.

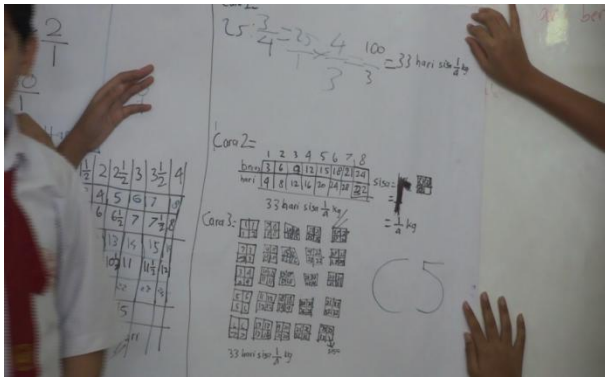


Figure 5.59. Students' ratio table

Based on the short fragment 5.26 above, we can explicitly see that models help these students to solve proportional problem. They use ratio table to see the proportion and pictures to deal with the remainder. For instance, the students determine 3 kg = 4 days, 6 kg = 8 days, 9 kg = 12 days, … , 24 kg = 32 days by using ratio table (See line 19). They notice that there is 1 kg remains. To deal with this remainder, the students draw pictures. For instance, they draw one box that represents one kilogram of rice. They divide the box into four parts and shade

three parts of it. By shading three parts of the box, they students get the 33rd day and they also get the remainder which is $\frac{1}{4}$ kg (See line 19).

The short fragment 5.26 also shows that how models develop their proportional reasoning. In this case, the students used ratio table to get the number of each proportion. At the end of operating the ratio table, they get the remainder. In this case, they notice that they can not deal the remainder with the ratio table. Thus, they used their own representation to deal with the remainder.

Conclusion

Based on the retrospective analysis of lesson 4 above, we can conclude that models help these students to solve proportional problem. They use ratio table to see the proportion and pictures to deal with the remainder. For instance, the students determine 3 kg = 4 days, 6 kg = 8 days, 9 kg = 12 days, … , 24 kg = 32 days by using ratio table (see line 19). They notice that there is 1 kg remains. To deal with this remainder, the students draw pictures. For instance, they draw one box that represents one kilogram of rice. They divide the box into four parts and shade three parts of it. By shading three parts of the box, they students get the 33rd day and they also get the remainder which is $\frac{1}{4}$ kg (see line 19).

In addition, the retrospective analysis that how models develop their proportional reasoning. In this case, the students used ratio table to get the number of each proportion. At the end of operating the ratio table, they get the remainder.

In this case, they notice that they can not deal the remainder with the ratio table. Thus, they used their own representation to deal with the remainder.

5.2.5.4. Posttest

At the end of the second cycle, we also conducted a 30-minute test to all students in class 5C. This test was purposed to evaluate the development of students' understanding towards what they had learned within the whole teaching and learning experiment. In this test, the students were asked to solve the following problems.

1. *Mrs. Silvia buys 35 boxes of 1 kg of rice. In one day, she cooks* $\frac{3}{4}$ *kg of rice. Can you determine the amount of rice is enough for how many days?*
2. *Mrs. Mutia wants to make some chocolate cakes to sell. She has 55 cups of chocolate powder. To make one chocolate cake, she needs* $2\frac{1}{2}$ *cups of chocolate powder. Can you determine 55 cups of chocolate powder are enough to make hoe many cakes?*
3. *Miss. Fika buys 20 kg of rice to supply her daily need. Everyday, she cooks* $\frac{2}{3}$ *kg of rice. Can you determine the amount of rice is enough for how many days?*

Based on the result of this posttest, most of the students in the focus group were able to answer all the questions correctly. In this case, only one student in the focus group, Ryan, answered two questions correctly since they did not succeed to answer the first problem. Furthermore, they were also able to explain their strategies how they got the answer.

Basically, there were no differences between the students' strategies in the posttest for both cycles. The students' strategies in first cycle also occurred in the

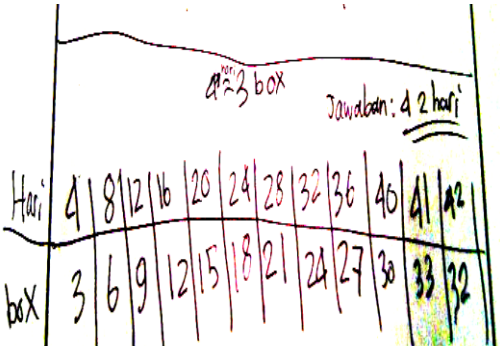
second cycle. The explanation of students' strategies in order to answer the questions in the posttest can be seen as follows. In this case, we only show several examples of the results of posttest for each problem.

Problem 1

To deal with the first problem, some students used ratio table. We can see an example of this strategy in Ryan worksheet. In this case, he knew that 3 boxes of rice can be cooked for 4 days since one day Mrs. Silvia cooked $\frac{3}{4}$ kg of rice. Subsequently, he continued his ratio table by adding up 3 to the number of boxes and 4 to the number of days until he got 33 boxes = 41 days. The question asked him to determine 35 boxes = … days, but he wrote 32 boxes = 42 days. The possible reason why he was not right to solve the problem was that he miscalculated the proportional numbers in his ratio table.

Furthermore, Farhan, Nanda and Reyhan did division strategy to deal with the first problem. They divided 35 by $\frac{3}{4}$ to get the answer. In this case, Raka got that 35 kg packs of rice were enough for 46 $\frac{3}{2}$ days and the other two students determined that 35 kg packs of rice were enough for 46 days with $\frac{2}{4}$ kg remains. We can explicitly see that both answers are correct.

Additionally, the result of the posttest also showed that the variety of students' strategies to solve the first problem. In this case, one student, Dekka drew pictures to explain the answer. We can see these students' works in pictures 69 – 72 below.



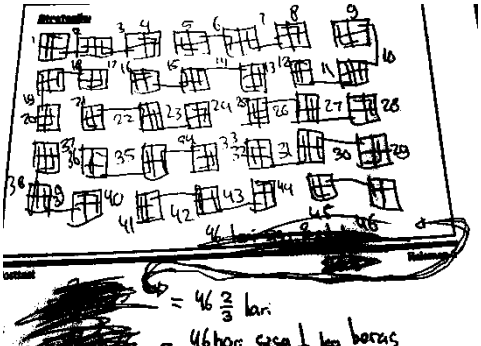


Figure 5.60. Ryan' strategy

Figure 5.61. Dekka's strategy

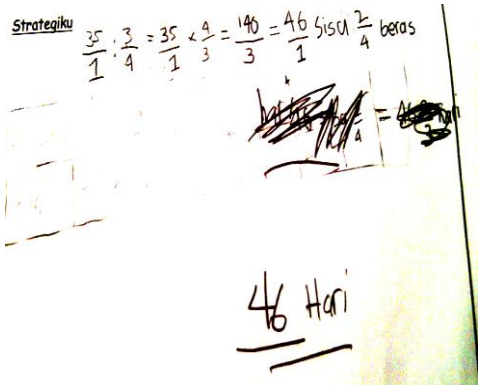


Figure 5.62. Farhan's strategy

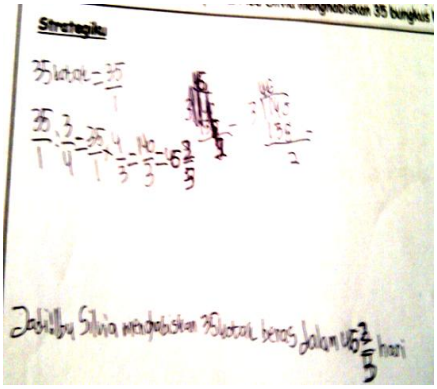


Figure 5.63. Nanda's strategy

Problem 2

To deal with the second problem, all the students in the focus group were able to answer it correctly. In this case, each student concluded that 55 cups of chocolate powder were enough to make 22 chocolate cakes.

In terms of the students' strategies, there no one drew pictures to solve the problem. It means that most of the students used ratio table and the rest used division strategy to solve the problem. For instance, Ryan, Farhan and Raka used ratio table to answer the question. In order to start the ratio table,

they knew that 5 cups chocolate powder = 2 cakes. Thus, they started the ratio table by writing 5 cups = 2 cakes. In this case, they avoided to use fraction since he recognized that 5 cups were enough to make 2 cakes. Then, they just added each unit with its proportion until he got 55 cups = 22 cakes.

Meanwhile, Dekka and Nanda used division strategy to answer this problem. In this case, they straightly divided 55 by $\frac{5}{2}$ and did the calculation to draw the conclusion that 55 cups of chocolate powder were enough to make 22 cakes. To be more detail, the figures 5.64 – 5.67 below.

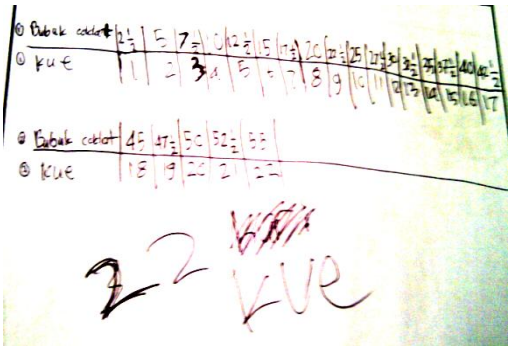


Figure 5.64. Ryan's strategy

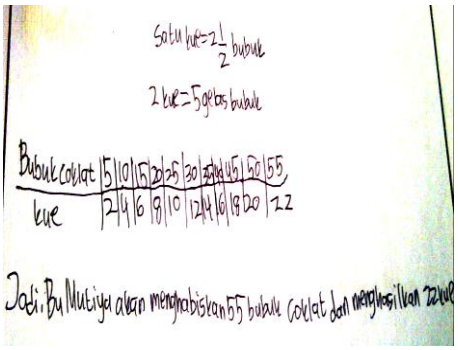


Figure 5.65. Raka's strategy

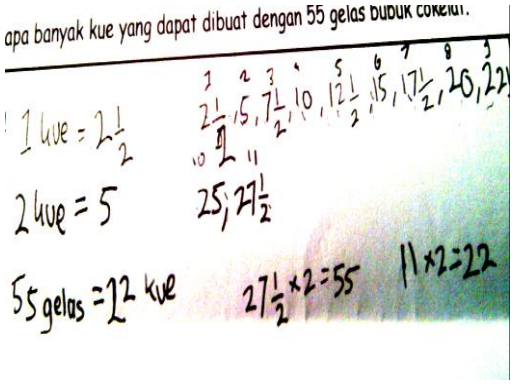


Figure 5.66. Farhan's strategy

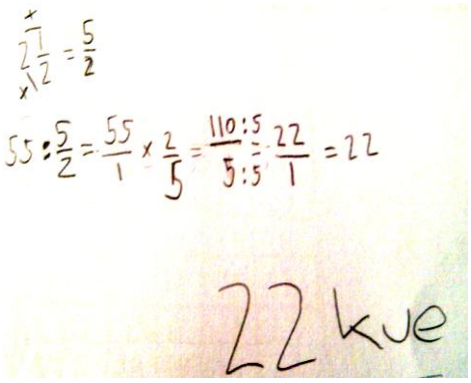


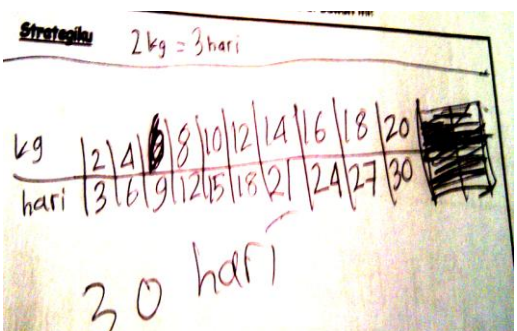
Figure 5.67. Nanda's strategy

Problem 3

In this problem, all the students in the focus group were able to answer it correctly. Based on the students' worksheet, it can be seen that each students got the same answer that 20 kg of rice were enough for 30 days if $\frac{2}{3}$ kg of rice was cooked everyday. Additionally, most of the students were able to explain how they got the answer of the problem.

When we looked at the students' worksheet, it also can be seen thatNanda solved this problem by drawing. In this case, he recognized that he had to make 60 small little boxes since there are 20 kg and the faction $\frac{2}{3}$ consisted of three little boxes. Thus, he multiplied 3 with 20 which is 60 boxes. Subsequently, he put the mark for each two little boxes as the representation of the days. Thus, there were 30 parts that he could make within the 60 little boxes. Thus, his answer was 30 days.

At the same time, Ryan still used ratio table to solve this problem. In this case, he recognized that 2 kg = 3 days since Mbak Fika cooked $\frac{2}{3}$ kg rice in one day. Thus, he started his ratio table with this proportion. At the end, he determined that 20 kg of rice can be cooked for 30 days. Furthermore, Farhan and Dekka proportion to solve this problem. For instance, Farhan wrote that 1 day will be equal to $\frac{2}{3}$ kg. Based on this proportion, he noticed that 3 days are equal to 2 kg. Since there were 20 kg of rice in the problem, it meant that he had to multiply 2 with 10 to get 20. Subsequently, he also had to multiply 3 with 10 and he got the answer which was 30 days. In this case, Dekka did the same strategy with Farhan.



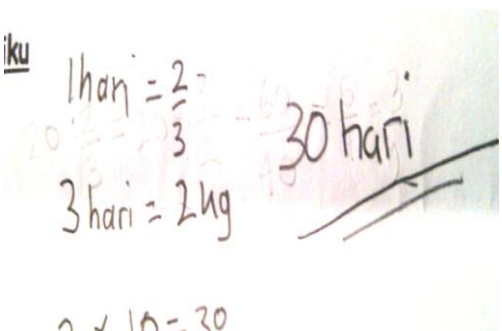


Figure 5.68. Ryan' strategy

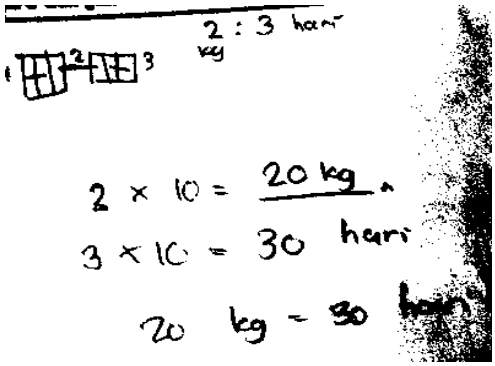


Figure 5.69. Farhan's strategy

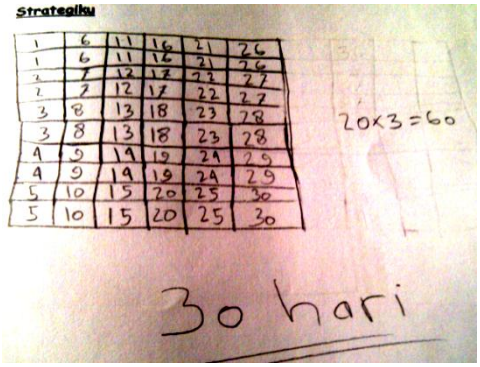


Figure 5.70. Dekka's strategy

Figure 5.71. Nanda's strategy

5.2.6. Conclusion of Students' Learning Process in the Second Cycle

Based on the teaching and learning process of the second cycle, we can summarize that there were several mistakes and obstacles happened in teaching proportion to students. The possible reason for this situation is that the students never met such problems. Subsequently, many students did not know how to deal with those problems. During the teaching and learning process, we can see that

although there were many mistakes the students can learn about proportion by their own understanding.

Starting with a realistic problem, the students can get the sense of the situation that will help them solve the problems. They can imagine how the problems happen in a real-life situation and start the way how to solve those problems. In this case, the students were able to make their own model or representation as the metaphor of the problem. For instance, they drew a box as the representation of one pack of rice. Subsequently, they used these models as a tool to solve the problems.

During the teaching and learning process of the second cycle, we can also explicitly see that model plays an important role to support the students to learn about proportion. In this case, the students, at the beginning of the lesson, solved the problems by making their own model, such as: drawing pictures. In order to determine for how many days that 20 packs of rice are enough if one family cooks $\frac{3}{4}$ kg in one day, the students, for example, drew 20 boxes as the representations of 20 packs of rice. Subsequently, they divided each box into four parts and shaded three parts of each box to represent the fraction "$\frac{3}{4}$". Based on the models, the students, at the end, determined the answer of the problem.

In addition, students' representation also helps not only solve proportional problems, but it also helps them get the sense of proportionality. In this case, we can explicitly see that one student concluded *3 packs = 4* and *6 packs = 8* and 9 packs = 12 days based on the model he saw. For instance, he saw "*three boxes and six* $\frac{3}{4}$*s*", "*six boxes and eight* $\frac{3}{4}$*s*" and "nine boxes wand twelve $\frac{3}{4}$*s*" (see lines 1

– 16 of lesson II and figure 5.52B). Therefore, the models help the students visualize the proportional situations.

Once the student had gained the sense of proportion, ratio table can be used to develop their proportional reasoning. In that ratio table, the students can see the proportion of each unit and they can explain about their answer. For instance, it is difficult for students to determine 20 packs for how many days after they got the proportion 18 packs for 24 days. At the same time, some students come up to use both models that they have learned, such as: ratio table and picture. For instance, we can see that they use the ratio table until they get the proportion 18 packs = 24 days. For the two packs, they draw pictures as the representation of the two packs and solve as they solve the problem in first lesson (see figures 5.52). In this case, we can see that the students can explore many strategies by using ratio table. They can use mathematical operation, such as: addition and multiplication, within a ratio table. However, they can also combine their own model with ratio table. Based on this fact, we can say that ratio table can develop students' proportional reasoning.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

In the previous chapter, we have stated that the aim of this study is to contribute to the classroom practices in learning about proportion which involves fractions in the primary education. In order to reach the goal of this study, we developed a Hypothetical Learning Trajectory (HLT) and tested it in a real teaching and learning process as we explained in the previous chapters. In this chapter, we provide two parts. In the first part, we will describe the conclusions of this study that we have derived from the whole teaching and learning series. By analysing the data of the whole teaching and learning process, we, in this case, may answer the research question and sub-questions of this study. In the second part, we will give suggestions and recommendations for future teaching and study related to developing students' proportional reasoning.

6.1. Conclusion

In this study, we conducted the teaching and learning activities within two cycles. The reason for this situation was that we had achieved the goals of learning in order to support the students to learn about proportion involving fractions. In addition, our expectations in the HLT were in line with the actual students' thinking of the cycle 2. Subsequently, we just conducted the teaching experiment within two cycles.

6.1.1. Answer to sub-questions

In order to answer the research question of this study, we need to answer the three sub-questions beforehand. In this case, we answer the three sub-questions by analysing and summarizing the data of all activities of this study.

6.1.1.1. An answer to the first sub-question

In order to answer the first sub-question, we employed the data collection and the analysis of the students' pretest and the interview with the students at the beginning of both cycles. Based on these activities, we obtained the information about students' prior knowledge and how they use their prior knowledge to solve proportional problems which includes fractions.

Generally, the pretests and the interviews with the students indicated that the students tended to apply what they understand to solve proportional problem, such as doing multiplication, subtraction, division and even using percentages. However, there were also several students solved the problem by using model, such as: making their own representation, to solve the problems. In doing such

multiplication strategy, the students just multiplied the numbers provided in their worksheet to determine the answer of the problem. The same situation also happened to some students that did subtraction and division strategies. They just subtracted or divided the numbers provided in their worksheet to determine the answer of the problems. For instance, several students multiply 20 with $\frac{3}{4}$ to get the solution of the problem. Some students divide the two numbers to get their solution.

At the same time, some students solved proportional problem by using their own model or representation. In this case, the students drew boxes as the metaphor of the real situation of the problem, such as one box represents one pack of rice. In order to represent the fraction, these students divided each box into several parts and shaded some parts of them. For instance, these students drew a box as the representation of one pack of rice. They divided the box into four pars and shaded three parts of them to represent "$\frac{3}{4}$". In order to determine the answer of the problem, they just counted the number of $\frac{3}{4}$s that they had gathered.

6.1.1.2. An answer to the second sub-question

In order to answer the second sub-question, we employed the data collection and the analysis of the teaching and learning process of both cycles and the result of posttest. Based on the activities of the teaching and learning series, we obtained the information about how model, such as students' visualization, support their proportional reasoning.

Based on the analysis of the whole teaching and learning process, we drew two conclusions that model, such as: students' visualization, plays an important

role in two ways. Firstly, model helps students solve proportional problem. It helps the students visualize their thinking towards the way how to make their own representation of a fraction and to solve proportional problem. At the same time, we could also see this situation happened in the students' posttest. In this case, the results of posttest showed that some students still used their own model to solve the problems. For instance, the students draw a box and divide it into four parts. After dividing it, they shade three parts over the four parts of the box in order to represent $\frac{3}{4}$. Subsequently, they use the model, such as their visualization, in order to show the way how they get the proportion, such as: $\frac{3}{4}$ kg = 1 day, 3 kg = 4 days and 6 kg = 8 days.

Secondly, model also plays an important role in order to help the students build the sense of proportion. It helps the students visualize the proportional situations. For instance, the student concluded *3 packs = 4* and *6 packs = 8* based on the model he saw. This is because he saw "*three boxes and six* $\frac{3}{4}$*s*" and "*six boxes and eight* $\frac{3}{4}$*s*".

6.1.1.3. An answer to the third sub-question

In order to answer the third sub-question, we also employed the data collection and the analysis of the teaching and learning process of both cycles and the results of posttest. Based on the activities of the teaching and learning series, we obtained the information about how model, such as ratio table, develop their proportional reasoning. Additionally, the results of posttest gave us an evidence towards the students' strategy to solve proportional problems.

In this study, the model, such as ratio table, was introduced when had acquired the sense of proportion. In this case, the sense of proportion means the moment when the students could determine the part of each unit. For instance, the students could determine the number of days that 6 packs of rice will last if 3 packs of rice will last in 4 days. It was the best moment to introduce ratio table when the students achieved this condition.

Based on the teaching and learning series of both cycles in this study, we also deduced two conclusions that model, such as: ratio table, also supported students in two ways. Firstly, it helped students solve proportional problem. It helped the students visualize their thinking towards the way to determine the number of each unit and explained how they obtained those numbers. In this case, we took two examples of students work on ratio table when they were asked to solve cakes problem. Based on the ratio table, we could see explicitly that some students used addition strategy within whole number. In doing this strategy, the students avoided to operate fractions in their ratio table. They transformed the fractions into the whole numbers. On the other hand, some students worked on fractions that they add those fractions continuously with $\frac{5}{2}$. The students just added the fractions in their ratio table.

Secondly, it can be seen explicitly that model, such as ratio table, supported students to develop their proportional reasoning. It helped the students to see the proportion of each unit that they could determine the next proportion based on the numbers that they had got. For instance, in order to determine for how many days that 25 kg of rice are enough if Mrs. Fitri cooks $\frac{3}{4}$ kg, they struggle to do that.

They add each kilogram with 3 and each day with 4. Then, they get that 24 kg of rice are enough for 32 days. They still have 1 kg left and they use picture to finish this. They make a box that represents one kilogram of rice. They divide it into four parts and shade three parts. Thus, this rice is still enough for one day and the remaining is $\frac{1}{4}$ kg. They conclude that 25 kg of rice are enough for 33 days with $\frac{1}{4}$ kg remains.

6.1.2. An answer to the research question

The answer of the research question is based on the answer of the sub-questions of this study. The answers of the sub-questions indicate that the use of model, firstly, can be explicitly seen when pupils have to solve proportional problems which involves fractions. They use the model because it helps the students visualize their thinking towards the way how to make their own representation of a fraction and to solve proportional problem. Secondly, model also plays an important role in order to help the students build the sense of proportion. It helps the students visualize the proportional situations.

In addition, it can be seen explicitly that model, such as ratio table, supports students to develop their proportional reasoning. It helps the students to see the proportion of each unit that they could determine the next proportion based on the numbers that they had got. It also helps students recognize the relationship within the numbers. Therefore, they can develop their proportional reasoning towards the relationship among those numbers. In an intermediate level, the students may develop the ratio table as a tool to think proportionally. The ratio table gives them a way to determine the proportionality.

6.2. Reflection

6.2.1. The weakness point of this study

In conducting this study, we realized that there were several weaknesses happened. Firstly, we provided less students' strategies to the teacher since it was not easy to predict what the students would do in the actual teaching and learning process. Thus, the teacher was a little bit upset to manage the variety of students' strategies. Subsequently, the teacher was not confident to make a decision which answer would be suitable to the problem.

Secondly, there was a lack of communication between the researcher and the teacher. In this case, it was quite difficult to manage the time for a discussion with the mathematics teacher towards the design. Thus, this situation signifies several significant differences between what the teacher's did during the teaching and learning process and the expectation of the researcher. There was less time to discussions about teacher's reactions in the classroom. It reflects the moment that the teacher did not explore the students which had a formal strategy to solve problems. The teacher did not ask the students the reason why they have to do so.

Thirdly, we did not pay attention to the time allocation. In our lesson plan, we had allocated the time for each activity of this study. In the actual teaching and learning process, the situation, sometimes, happened as we did not expect. For instance, the students took a long time to discuss problem in group or they took a long time to work individually. Thus, they had a short time to discuss in group. This situation influenced the students' to make a discussion. If they had more time for a group discussion, they could discuss more strategies how to solve the

problem. However, if the students had less time for a group discussion, the students tended to solve the problem with a short way without having a discussion.

Lastly, we also considered about the technical errors in writing this thesis. In this case, there are serveral parts of this thesis which were not written in a proper way. An example of the mistakes can be seen in writing the lesson plans. In the lesson plans, some parts of the learning goals were made based on the teacher's point of view and also based on the students. Thus, we did not pay attention to revise the mistakes of writing the lesson plan before they were given to the teacher.

6.2.2. The implementation of PMRI in the design of learning series

Basically, we employed the idea of RME as the main theory in designing the instructional activities and conducting this study. The contextual activities of RME themselves were adopted into the Indonesian context. In this case, the designed instructional activities were the parts of developing the local instructional theory about proportion. Subsequently, the applications of the tenets of RME towards proportion will take more time that this study. Based on the results of this study, the phenomenological exploration, such as packs of rice, could be used as the starting point to teach about proportion and to develop the students' proportional reasoning.

6.2.3. Contribution to the local instructional theory of proportion

According to Zulkardi (2002; cited in Sumarto, 2013), most of mathematics text books in Indonesia mainly consist of set of rules and algorithm which is

already formal and they lack of application which is needed by the students to make the concept real for them. Subsequently, the teaching of proportion only stands at giving the students a ready-made formula, such as: cross multiplication, to solve proportional problems. As a result, the students tend to memorize the formula and just apply it in solving the problems. This will be meaningless for them because they do not even know and do not understand the meaning of proportionality. Additionally, this will make students difficult to develop their proportional reasoning. Therefore, there is a need for us to support the students to learn about proportion and to develop their proportional reasoning to solve problems through innovations in teaching proportion.

This study highlights a new way for fifth grade students in primary education to learn about proportion which includes fractions. Additionally, it underlies an innovative way for mathematics teachers to teach proportion since this study also employs the heuristic of Realistic Mathematics Education (RME) as the main theory in which we designed five instructional activities within a learning trajectory. Because the students were familiar with the contextual problems, we, in this case, might say that this design allow other teacher implemented it in other school in Indonesia. Moreover, the models, such as: students' visualization and ratio table, could be used to help students learn about proportion and to develop their proportional reasoning.

6.2.4. A Local Instruction Theory (LIT) for the development of proportion

In the previous chapter, we have stated that the aim of this study is to contribute to the development of local instruction theory towards proportion for

the fifth grade of elementary school within a design-based research. In this case, the hypothetical learning trajectory is modified and revised by an understanding of the actual learning trajectory through a retrospective analysis (Nickerson and Whitacre, 2010). Nickerson and Whitacre (2010) also pointed out that a retrospective analysis of the actual learning activities informs the revision of HLTs in subsequent classroom teaching experiments and may lead to the underlying LIT.

Basically, a *local instructional theory* was developed in a design research which offers teachers a framework of reference for designing and engaging students in a set of sequenced, exemplary instructional activities that support students' mathematical development of a focused concept (Gravemeijer, 2004). In other words, a local instruction theory refers to "the description of, and rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topic" (Gravemeijer, 2004). In the relation to this study, the local instruction theory can be seen in the following table 6.1.

Activity	**Instructional activities, rationale and assumptions**	**Aims and Envisioned learning route**
Activity 1: Solving rice problem by using student's model	***Preliminary activity:*** In the first lesson, the teacher told a story to the students that he/she had a rice problem which needed to be solved. Thus, he/she asked the students to help him/her solve the problem. ***Main activity:*** In this case, the teacher could ask them to work individually and they, later on, could work in group. Subsequently, they were asked to defend their work in front of other groups in the whole-class discussion.	The aims of this lesson is: To use student's model to solve problem. During this lesson the students will be able to create their own models / representation,

	Rationale: In doing the problem, the students did it with their own model. For example, they drew their own representation/model to deal with the problem. In this case, the models were important for the students since they could explain their strategies by using the model. In other words, there was a need for the students to use their model. The problem should give the students times to think and lead them to a discussion where they could explain and adjusted their answer.	such as: drawing (pictures) and use them to adust their answer to solve the problem.
	Assumptions: In dealing with the problem, there were two possible situations that will happen in the actual teaching and learning process, such as: 8. All students do not know how to deal with the problem. In this case, the teacher can guide the students to a model where they can work on. 9. Some students can give their answer by making their own representations, while other students did other strategies. This situation can be a good situation where it can lead them into different solutions. Thus, the teacher can guide them to determine the solution of the problem.	
Activity 2: Solving rice problem with a ratio table	***Preliminary activity:*** In the second lesson, the students will be asked to determine for how many days that 20 boxes of 1 kilogram of rice will last if one family consumes $\frac{3}{4}$ kg of rice in an early ratio table. However, the teacher needs to guide the students to get the sense of proportion before introducing the early ratio table. ***Main activity:*** In this activity, the teacher should give the students time to think individually and to work in group. Additionally, the teacher should give the students time to adjust their thinking in the whole class discussion. At the end of the lesson, the teacher can introduce ratio table to the students.	The of this lesson is: 1. Making sense of proportion. 2. Working with ratio table. Durig this lesson the students are expected to get the idea of proportion by themselves and to develop their proportional reasoning in a

	Rationale: In this case, the models were important for the students since they could explain their strategies by using the model. In this case, it is expected that the model would guide the students to get the sense of proportion and to develop their proportional reasoning.	ratio table.
	Assumptions: In order to deal with the problem, the students are expected to get the idea of proportion based on the model that they have created. After the students get the idea of proportion, the students can develop their proportional reasoning in a ratio table.	
Activity 3: Making cakes to sell	***Preliminary activity:*** In this meeting, the students are asked to try to use ratio table in solving proportional problem. ***Main activity:*** In this activity, the teacher should give the students time to think individually and to work in group. In solving the main problem of this lesson, some students may still use their strategy in the previous lesson. However, the teacher can guide them to think proportionally then put it in the table.	The aims of this lesson are: 1. Using ratio table to solve proportional problem. 2. Describe how to use ratio table to solve proportional problem. In this lesson, the students learn about how to solve proportional problem by using ratio table.
	Rationale: The model, such as: ratio table is used as a tool to develop students' proportional reasoning. This is because the students can see the proportion in a ratio table and they can also apply many strategies to operate that model to solve proportional problems.	
	Assumptions: In order to deal with the problem, the students are expected to explore the ratio table with their friends. They can apply different kinds of operations in the ratio table, such as: doing repeated addition and multiplication.	

<table>
<tr>
<td rowspan="3">Activity 4:
Proportion in unit and non-unit fraction</td>
<td>Preliminary activity: In this meeting, the students will be asked to solve several proportional problems which include fraction.

Main activity: In the main activity, the teacher can ask the students to work individually and the teacher should also give the students time to discuss in group. In doing group discussion, the students can share their strategies and argue each other to determine the solution of the problem.</td>
<td rowspan="3">The aims of this lesson are:
1. Use ratio table to solve proportional problems
2. Solve proportional problem by using model

In this lesson, the students learn about determining the solution of proportional problem by using their own models.</td>
</tr>
<tr>
<td>Rationale:
When the students try ratio table, the model is expected to develop students' proportional reasoning. The model plays an important role to visualize the students' thinking how they solve proportional problem since we can see explicitly the students' strategies within a ratio table.
Later on, the students are given a problem which can show whether they use the models that they have learned or not.</td>
</tr>
<tr>
<td>Assumptions:
In order to deal with the problems, it is conjectured that models can support the students to deal with those problems and to develop their proportional reasoning.</td>
</tr>
</table>

Table 6.1. Local Instruction Theory (LIT) of Proportion

6.3. Suggestion

6.3.1. Suggestion for the teacher

It is a truism that it is not easy to apply a new way of teaching and to bring new socio norms in the classroom. Teacher needs more time to commence and to make the students get used to learn with the new learning circumstances. Based on the retrospective analysis in chapter 5, we, in this case, can see some discussions are not going as we expected. The students did not really discuss the problems. The students tend to write what they wrote in their worksheet without having a

discussion. Therefore, the teacher needs to encourage the students during the teaching and learning process to change the socio norms of the classroom.

The teacher needs to explore the students' answer in solving problems. In this case, the teacher can explore the students who have wrong answer to explain their thinking instead of going straightly to the right answer. Furthermore, the teacher can also start the exploration with the students who solved the problem in a formal way. The teacher can do this by posing several questions that can guide the students to get an idea how to solve the problems or to solve the problem in a different way.

Based on the explanation those explanations, we can see explicitly that teacher plays an important role to make a meaningful learning situation in the classroom. Therefore, making reflections after teaching, enriching their knowledge and changing their traditional teaching will be the proper ways to improve the teaching and learning process in the classroom

6.3.2. Recommendation for future research

Based on the answer of the first sub-question, we can see explicitly see that students will make theiw own model if the problem situation is real in their mind. Thus, we firtly recommend to start the teaching and learning process with contextual problems which are familiar for the students. This is because it will help them recognize and solve the problems, such as the rice problem. However, the problems do not limit on the context of rice. They can be modified into various conditions.

Based on the answer of the research question of this study, we can explicitly see that models, such as: students' visualization and ratio, play an important role to support students to learn about proportion. In order to introduce ratio table to the students, we, secondly, recommended starting it with their own visualization. The students will easily know how they get the proportion of each unit based on the model that they have made. Once the students get the sense of proportion, ratio table can be introduced to the student very smoothly.

Based on the results of the research question of this study, model, such as: ratio table, can be used to develop the students' proportional reasoning. Thus, we, lastly, also recommend the future research to use ratio table as the model in developing students' proportional reasoning.

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Learning Line of Student's Thinking

Students will explore a proportional which includes fraction

- Make model as the representation of situation.
- Use the model to solve problem.

Lesson I

Solve rice problem by using student's model

Students will start working with ratio table

- Make sense of proportion
- Work with ratio table

Lesson II

Solving rice problem with a ratio table

Students will explore a proportional problem which includes mix numbers

- Use ratio table to solve problem
- Describe how to use ratio table to solve proportional problem which includes fraction

Lesson III

Making cakes to sell

Students will explore asome proportional problems which includes unit and non-unit fractions

- Solve proportional problem which includes fraction by using model.

Lesson IV

Proportion in unit and non-unit fraction

Mathematical idea | Learning goals | Activity

APPENDIX I

Teacher's Interview Scheme

1. **Background**
   - How long have you been teaching in primary school?
   - How long have you been teaching in the fifth grade?
   - Do you teach only mathematics or other subjects?
2. **Teaching and learning process**
   - How many students are there in the classroom?
   - How do you usually teach mathematics to students? Do you let them work individually or they work in group?
   - How is the situation of the classroom? Are the students brave to give their arguments during your lesson?
   - What is the level of the students' thinking?
   - What about students' achievement towards this topic?
   - Is this topic difficult for students to study? If yes, what are their difficulties when they study this topic?
   - How do you usually group the students?
   - How long have you been teaching in PMRI school?
   - What is the last mathematics topic that they have learned?
3. **General questions**
   - What are your obstacles in teaching mathematics to children in the classroom?
   - What do you think about this topic?
   - How do you usually teach this topic?

APPENDIX II

Classroom Observation Scheme

1. **Classroom norms:**
    - How do the students react during the teaching and learning process?
    - How does the teacher open the teaching and learning process?
    - How does the teacher teach this topic in the classroom?
    - Do the students pay attention when the teacher explains a mathematics topic?
    - Do the students always keep silent when the teacher asks questions?
    - How does the teacher end a teaching and learning activity?
    - Are the students active during the teaching and learning process?
2. **Teaching and learning process**
    - How does the teacher teach mathematics in the classroom? Does the teacher demonstrate or the teacher only explains the topics in front of the class?
    - Does the teacher always give students a chance to give their opinions?
    - How does the teacher support the students in learning mathematics?
    - How does the teacher manage the classroom?
    - Does the teacher provide a discussion session when she / he is teaching in the classroom?
    - Does the teacher use mathematical models in teaching mathematics in the classroom?
    - How does the teacher manage the time in teaching?
    - How does the teacher appreciate students' different solution?
    - How does the teacher guide the students when they have problems in learning a mathematics topic?
    - How do the teacher and the students interact to discuss a problem?

APPENDIX III

Teacher Guide

Meeting 1

Activity 1 : "Helping teacher solve rice problem"

a. Learning goals

- Make model as the representation of the situation
- Use the model to solve problem

b. Starting points

- Students in the fourth grade have learned about the mathematical operations, such as: addition, subtraction, multiplication and division with whole numbers.
- Students have learned about fraction and how to represent a fraction when they were in the fourth grade.
- Students have learned about how to operate fractions in the fifth grade before this topic is taught.

c. Learning Materials

Student's worksheet 1, marker and poster paper

d. Time Allocation

70` minutes

e. Description of the activity

At the beginning of this meeting, the teacher groups the students where each group consists of three or four students. Then, the teacher shows a picture of a box of one-kilogram rice and asks them the following question.

1. Here you see a box that holds 1 kilogram of rice!

2. Mrs. Rina buys 20 of these boxes. In one day, she cooks $\frac{3}{4}$ kg of rice. Can you determine the amount of rice is enough for how many days?

After that, the teacher asks students to solve the problem individually (about 10 minutes) and then discuss it in group.

When the students discuss the problem in group, the teacher can walk around to see how the students solve the problem. If there are students draw a rectangle to explain their strategy, then the teacher can make a discussion based on this answer. However, if there are no students can draw rectangle or they do not know how to represent $\frac{3}{4}$ in a picture, then the teacher can draw a box or a rectangle that represents 1 kg of rice in the white board.

f. Reactions for the Teacher

1. If the students do the repeated subtraction strategy, then the teacher can ask the students "can you help me understand what you are doing?". After the students explain what they are doing, the teacher can ask them "Can you explain it by using a picture?". This question is aimed to check their understanding what they are doing.

2. If the students straightly multiply 20 with $\frac{3}{4}$ or 20 x $\frac{3}{4}$, then the teacher can ask them "Can you explain why you multiply 20 with $\frac{3}{4}$?". In this case, the teacher can direct the students to the context by reintroducing the problem. Mrs. Ani cooks $\frac{3}{4}$ kg in one day. If there are 3 kg of rice, can you determine this amount is enough for how many days? After the students answer it, the teacher can ask them "Ok, 3 kg of rice are enough for four days. In the problem, she has 20 kg of rice. Can you determine the amount of rice is enough in how many days?"
3. After teacher walks around and recognizes that the students still do not know how to deal with the problem, the teacher can draw a rectangle that represents one box of one-kilogram rice in the white board. In order to help them realize what they are doing, the teacher can introduce the problem again. This time, the teacher needs to emphasize the context asks the students to represents $\frac{3}{4}$ in the rectangle by asking them a question "how do you make $\frac{3}{4}$ in a rectangle?". Then, the teacher can guide the students by asking them "if there are 3 kg of rice, then in how many days it will last?". After that, the teacher asks them "If there are 20 boxes, then can you determine in how many days it will last?"

Meeting 2

Activity 2 : "Helping the teacher solve rice problem"

a. Learning goals

- Make sense of proportion
- Work with ratio table

b. Learning Materials

Student's worksheet 2, markers, students' poster from meeting 1 and chart paper.

c. Time Allocation

70 minutes

d. Description of the activity

At the beginning of this meeting, the teacher lets the students finish their poster about 15 minutes. After the students finish their poster, the teacher starts a discussion to show students' strategies. To be more specific, the aim of the discussion is to investigate students' reasoning in order to solve the problem.

e. Discussion 1

After the students finish their poster, the teacher starts a discussion session within the whole groups in the classroom. In this discussion, the teacher asks one group to present what they do in order to solve the problem. In this case, the teacher can choose the students work which is not totally correct so that there will be a conversation in the whole class discussion.

After the students present their work, the teacher can ask other students to give their comments by asking "Do you have any questions or do you want to add something to this group?" and then ask them "Do you understand what they have explained?".

f. Reactions for the teacher during the discussion

1. If the students say that they have understood what their friends say during the presentation or they keep silent, the teacher can ask one of the other students to repeat it by asking "Can you repeat what you have heard from your friends' explanation?"

2. If the students say that they do not understand, the teacher can ask the group to repeat their explanation again. Then, ask other students to repeat what the group has explained for the second time.
3. If the students say that one box is enough for four days or two boxes are enough for four days, the teacher can ask other students to respond by posing a question "Do you hear the same as your friend? Or do hear something different?"
4. Widjaja et al. (2010) noticed that students will use mark to group three boxes or four days. In this discussion, the teacher can ask other students what this mark means to make it clear.

After the discussion, reintroduce the problem in the previous meeting. This time, the teacher asks one of the students to explain again what their friends have explained. This conversation should end to a conclusion that three boxes or three kilograms of rice are enough for four days.

g. Discussion 2

The second discussion is intended to let the teacher and the students talk about shifting students answer to ratio table. To start the discussion, the teacher can ask the student to write it on the white board that three kilograms of rice are enough for four days. Then, the teacher can ask them a question "Can you determine 6 boxes of rice are enough for how many days?". After one of the students write the answer on the ratio table, the teacher can ask them another question "Can you determine 6 boxes of rice are enough for how many days?". Finally, the teacher askw them to determine 20 boxes of rice are enough for how many days.

Meeting 3

Activity 3 **:** "Making cakes to sell"

a. Learning goals

- Use ratio table to solve proportional problem which includes fraction
- Describe how to use ratio table to solve proportional problem which involves fraction

b. Learning Materials

Student's worksheet 3, chart paper, markers and poster of students' work in meeting 1 and meeting 2.

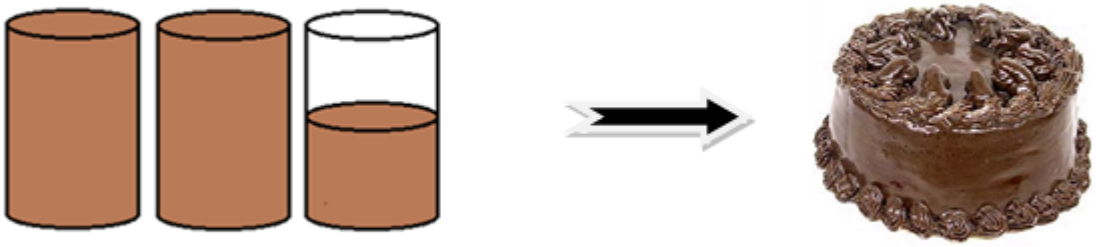
c. Time Allocation

70 minutes

d. Description of the activity

In this meeting, the students will be given a problem which is aimed to let the students try to use ratio table to solve proportional problem which includes fraction. In this case, the students are asked to solve the following question:

Mrs. Ani wants to make some chocolate cakes to sell. She has 45 cups of chocolate powder. To make one chocolate cake, she needs $2\frac{1}{2}$ cups of chocolate powder.



Can you determine 45 cups of chocolate powder are enough to make how many cakes?

To solve this problem, the teacher asks the students to work individually (about 10 minutes). After that, the teacher asks them to discuss it in group. During the

discussion in group, the teacher needs to walk around to see whether the students can solve the problem.

e. Reactions for the teacher

1. If the students still use picture to solve the problem, the teacher should them do it. After the students finish their work, the teacher can ask them "How do you solve the problem?". After the students have explained it, the teacher can ask them "Can you solve it in a different way?" or "Can you solve this problem by using ratio table?"
2. If the answer of the students is not totally correct, such as: there are 5 remaining cups, the teacher can ask other students whether this answer right or not. If other students say that it is right, the teacher can ask them why it is right and vise versa. At the end, the teacher can ask them a question "If there are still 5 cups remains, are these cups still enough to make cakes?" In this case, the teacher should ask these questions during the discussion within the whole group.
3. If the answer of the students is wrong, the teacher can ask other students "Do you agree with this answer?" or "Do you have different way to solve this problem?".
4. If the students make correct answer, the teacher can ask other students whether they agree with this answer. If the students say that they agree with the answer, the teacher can ask them to explain why they agree with the answer. Furthermore, the teacher can ask different solutions from other students and can ask them to write their answer if they have different strategy.

f. Discussion

To begin the discussion session, the teacher can one of the group to present their work in fron of the class. If there are students make mistakes in solving the problem or their answer is not totally correct, such as: there are 5 remaining cups, then the teacher can start the discussion from this point. Thus, there will be a discussion between the teacher and the students or among the students.

Meeting 4

Activity 4 : "Proportion in unit fraction"

a. Learning goals

Make ratio table and use it in order to solve proportional problems

b. Learning Materials

Student's worksheet4, chart paper, marker and poster of students' work in meeting 3

c. Time Allocation

70 minutes

d. Description of the activity

In this meeting, the students will be given several problems which are aimed to let the students make their own ratio table and use it to solve proportional problems which include fractions. In this case, the students are asked to solve the following questions:

4. *Miss. Rika goes to supermarket to buy 15 kg of rice to supply her daily need. Everyday, she cooks* $\frac{1}{2}$ *kg of rice. Can you determine 3 kg of rice are enough in how many days?*
5. *Miss. Sri goes to supermarket to buy 25 kg of rice to supply her daily need. Everyday, she cooks* $\frac{1}{3}$ *kg of rice. Can you determine 3 kg of rice are enough in how many days?*
6. *Miss. Rani goes to supermarket to buy 15 kg of rice to supply her daily need. Everyday, she cooks* $\frac{1}{5}$ *kg of rice. Can you determine 3 kg of rice are enough in how many days?*

In this case, the students are asked to determine the result of the problems. At first, the teacher asks the students to work individually (about 10 minutes). After that, the teacher asks them to discuss it in group. After the students have finished doing the problems in groups, the teacher makes a discussion in order to see the students' strategies to solve the problems by using ratio table.

The intension of the discussion session does not focus on seeing the results that the students get, but it should focus on how the students can give their arguments to describe their strategies to answer the problems. Thus, the teacher, during the discussion, should be able to encourage the students to give their contribution to the discussion.

e. Discussion

f. After the students have worked in group to solve the problems, then the teacher makes a discussion which is aimed to discuss their work and strategies how they solve the problems. The discussion is intended to investigate the students' reasoning towards the given problem. Thus, the teacher needs to lead the students to a discussion where they can contribute their arguments and opinions about the problem.

g. To deal with this situation, the teacher can choose the group of students who makes mistakes or their answer is interesting to be discussed, such as: the students' strategy no. 3. This strategy is interesting to be discussed since it requires the students to contribute their reasoning. To deal with this, the teacher can ask them to explain how they solve the problem. After the students have explained their answer, the teacher can ask other students to give comments and the teacher can also ask other students for different solutions.

h. Reactions for the teacher

1. If the students use counting by half, one third and one fifth, the teacher can ask other students "Do you agree with the answer?" and the teacher can ask the students whether they have different solutions for the problems.
2. If the students make mistakes in solving the problem, such as: doing miscalculation, the teacher can ask other students to react to what their friends do by asking them "What do you think about this? Do you agree with your friends?". If there are students say that the answer is wrong, the teacher can ask them to give

their reasoning why they think that it is wrong. After the students have explained the answer, the teacher can ask other students to students for different answers.

3. The teacher can also start the discussion by asking group of students who make interesting answer, such as: the students' strategy in point 3, to explain their strategy in front of the class by asking them "Ok, can you show us how do you solve the problem?". After the students have explained their strategy, the teacher can ask other students to give their reactions or to repeat what their friends have just said by asking them "If you have understood, can you repeat what your friend has just said?".

Meeting 5

Activity 5 **:** "Proportion in non-unit fraction"

a. Learning goals

Solve proportional problem by using models

b. Learning Materials

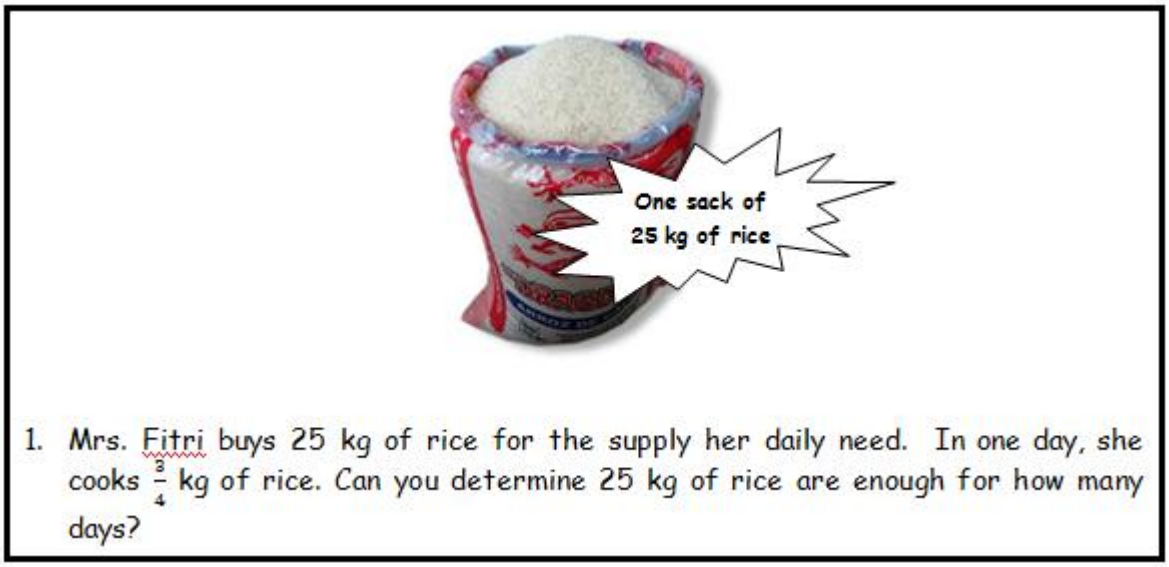
Student's worksheet5, chart paper, marker and poster of students' work in meeting 4.

c. Time Allocation

70 minutes

d. Description of the activity

In beginning of this meeting, teacher asks the students to solve the following proportional problem.



In this case, the students are asked to solve the problems in the students' worksheet. In solving the problems, the students are expected to use ratio table. After the students have finished doing the problems, the teacher makes a discussion in order to see the students' strategies to solve the problems by using ratio table.

e. Reactions to the teacher

1. If the students do not know how to deal with the problem, the teacher can remind them about the problem that they have done in the first meeting by asking them

"Do you still remember about Mrs. Rina's story?". The teacher can challenge the students by asking them "Now, you have 25 kg of rice, can you determine for how many days it will last?".

2. If the answer of the students is not totally correct, for instance: they say that "25 kg of rice will be enough for 32 days with 1 kg remains". This answer is a good way to start a discussion. In this case, teacher can ask the opinions of other students first about the answer by asking them "What do you think about their answer? Or do you agree with the answer? Or do you have something to add?". If there are reactions from the students to explain about 1 kg remains, the teacher can ask him/her to explain it in front of the class. If there are no students who do not recognize it, the teacher can start a discussion by saying "Ok, your friend say that 25 kg of rice are enough for 32 days with 1 kg remains. What do you think of that 1 kg? Is still enough to cook rice?". If the students say "yes", then they teacher can challenge them "Can you determine 1 kg of rice is enough for how many days?".
3. If there are students who come up with the strategies as the third point above, the teacher can keep this to be discussed in the discussion as another way to solve the problem.

f. Discussion

In this meeting, the discussion session is conducted after the students finish working with their works. The discussion is intended to investigate the students' reasoning towards the given problem. Thus, the teacher needs to lead the students to a discussion where they can contribute their arguments and opinions about the problem. Dealing with this situation, the teacher can start the discussion by asking one group of students who makes mistakes in solving the problem or the group of students whose answer is not totally correct to present their work. For instance: they say that "25 kg of rice will be enough for 32 days with 1 kg remains". This answer is a good way to start a discussion. In this case, teacher can ask the opinions of other students first about the answer by asking them "What do you think about their answer? Or do you

agree with the answer? Or do you have something to add?". If there are reactions from the students to explain about 1 kg remains, the teacher can ask him/her to explain it in front of the class. If there are no students who do not recognize it, the teacher can start a discussion by saying "Ok, your friend say that 25 kg of rice are enough for 32 days with 1 kg remains. What do you think of that 1 kg? Is still enough to cook rice?". If the students say "yes", then they teacher can challenge them "Can you determine 1 kg of rice is enough for how many days? Or you can draw picture to explain it".

APPENDIX IV

Pretest

Name : .....

Class : .....

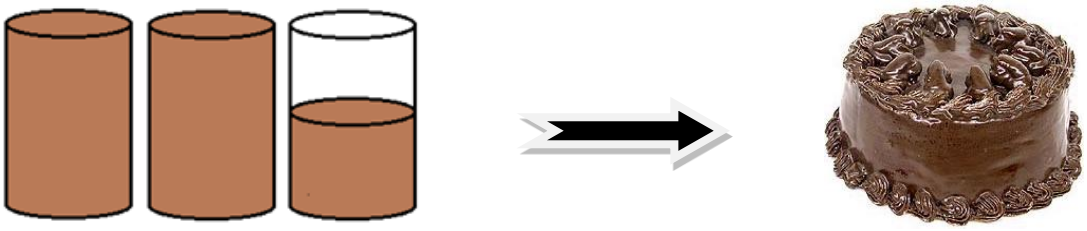
Date : .....

1. Here you see a box that holds 1 kg of rice!

Mrs. Anita buys 15 boxes of 1 kg of rice. In one day, she cooks $\frac{3}{4}$ kg of rice. Can you determine the amount of rice is enough for how many days?

My Strategy

2. Mrs. Dewi wants to make some chocolate cakes to sell. She has 30 cups of chocolate powder. To make one chocolate cake, she needs $2\frac{1}{2}$ cups of chocolate powder.



Can you determine 30 cups of chocolate powder are enough to make how many cakes?

My Strategy

3. Look at the following figure!

Miss. Mala buys 15 kg of rice to supply her daily need. Everyday, she cooks $\frac{1}{3}$ kg of rice. Can you determine 15 kg of rice are enough in how many days?

My Strategy

APPENDIX V

Posttest

Name : .....

Class : .....

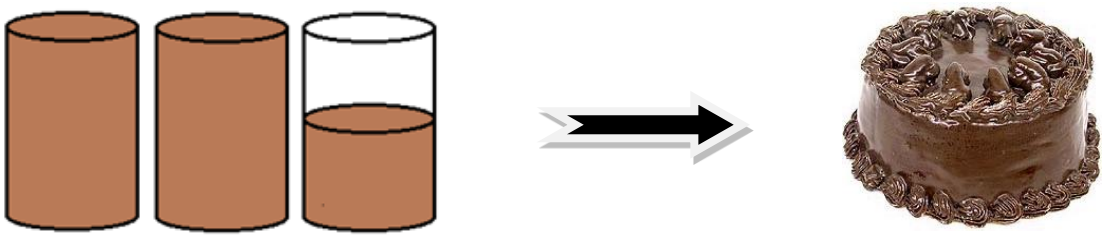
Date : .....

1. Here you see a box that holds 1 kg of rice!

Mrs. Silvia buys 35 boxes of 1 kg of rice. In one day, she cooks $\frac{3}{4}$ kg of rice. Can you determine the amount of rice is enough for how many days?

My Strategy

2. Mrs. Mutia wants to make some chocolate cakes to sell. She has 55 cups of chocolate powder. To make one chocolate cake, she needs $2\frac{1}{2}$ cups of chocolate powder.



Can you determine 55 cups of chocolate powder are enough to make how many cakes?

My Strategy

3. Look at the following figure!

Miss. Fika buys 25 kg of rice to supply her daily need. Everyday, she cooks $\frac{1}{3}$ kg of rice. Can you determine 25 kg of rice are enough in how many days?

My Strategy

APPENDIX VI

Student's Worksheet

Name : .....

Class : .....

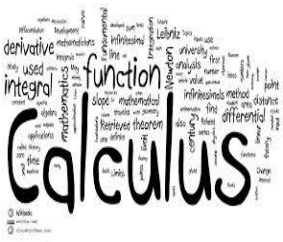
Date : .....

Student's Worksheet 1

1. Here you see a box that holds 1 kilogram of rice!

2. Mrs. Rina buys 20 of these boxes. In one day, she cooks $\frac{3}{4}$ kg of rice. Can you determine the amount of rice is enough for how many days? You may draw pictures to find your answer!

My Strategy



Name : . . . . . .

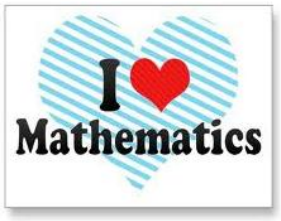
Class : . . . . . .

Date : . . . . . .

Student's Worksheet 2

Mrs. Rina buys 20 of these boxes. In one day, she cooks $\frac{3}{4}$ kg of rice. Can you determine the amount of rice is enough for how many days? You may draw pictures to find your answer!

My Strategy



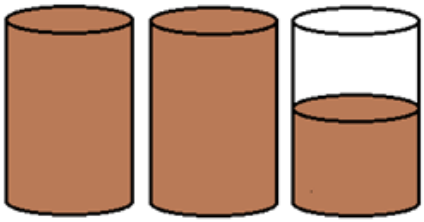
Name : .....

Class : .....

Date : .....

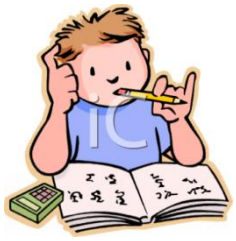
Student's Worksheet 3

Mrs. Ani wants to make some chocolate cakes to sell. She has 45 cups of chocolate powder. To make one chocolate cake, she needs $2\frac{1}{2}$ cups of chocolate powder.



Can you determine 45 cups of chocolate powder are enough to make how many cakes?

My strategy:



Name : .....

Class : .....

Date : .....

Student's Worksheet 4

1. Miss. Rika buys 15 kg of rice to supply her daily need. Everyday, she cooks $\frac{1}{2}$ kg of rice. Can you determine 15 kg of rice are enough in how many days? Write and explain your **strategies and answer** in the following box!

My strategy:

1. Miss. Sri buys 25 kg of rice to supply her daily need. Everyday, she cooks $\frac{1}{3}$ kg of rice. Can you determine 25 kg of rice are enough in how many days? Write and explain your **strategies and answer** in the following box!

My strategy:

2. Miss. Rani buys 15 kg of rice to supply her daily need. Everyday, she cooks $\frac{1}{5}$ kg of rice. Can you determine 15 kg of rice are enough in how many days? Write and explain your **strategies and answer** in the following box!

My answer:

Name : .....

Class : .....

Date : .....

Student's Worksheet 5

1. Mrs. Fitri buys 25 kg of rice for the supply her daily need. In one day, she cooks $\frac{3}{4}$ kg of rice. Can you determine 25 kg of rice are enough for how many days? Write and explain your **strategies and answer**!

My strategy:

APPENDIX VII

RENCANA PELAKSANAAN PEMBELAJARAN (RPP)

Sekolah : SD Al – Hikmah, Surabaya
Mata Pelajaran : Matematika
Kelas/Semester : 5/2
Pertemuan : I (Pertama)
Alokasi Waktu : 2 x 35 menit
Standar Kompetensi : 5. Using fraction in order to solve mathematical problems
Kompetensi Dasar : 5.3. Using fraction to solve problems related to ratio and scale.

A. Aktivitas

Menolong guru menyelesaikan masalah beras.

B. Tujuan Pembelajaran

- Siswa dapat menggunakan model atau gambar mereka sendiri untuk menyelesaikan masalah perbandingan.
- Untuk melihat strategi siswa dalam menyelesaikan masalah perbandingan yang mencakup pecahan.

C. Indikator

- Siswa dapat menyelesaikan masalah perbandingan yang mencakup pecahan dengan menggunakan model yang mereka buat.

D. Materi pembelajaran

Perbandingan yang mencakup pecahan

E. Pendekatan pembelajaran

Pembelajaran dengan menerapkan RME (Realistic Mathematics Education)

F. Kegiatan Pembelajaran

Kegiatan	Kegiatan yang dilakukan oleh guru	Kegiatan yang dilakukan oleh siswa	Alokasi Waktu
Kegiatan awal	- Guru mengkondusifkan kelas ke suasana belajar	- Siswa berdoa sebelum belajar.	15 menit

	- Guru memotivasi siswa untuk berperan aktif dalam pembelajaran. - Guru memulai pembelajaran dengan menunjukkan kepada siswa sebuah kotak berisi 1 kg beras dan menceritakan sebuah masalah. - Guru meminta siswa untuk bekerja sendiri – sendiri. - Guru membagi siswa ke dalam kelompok, dimana setiap kelompok terdiri dari 5 orang.	- Siswa mengikuti apa yang disampai-kan oleh guru. - Siswa mendengar-kan cerita yang disampaikan oleh guru. - Siswa bekerja sendiri setelah guru memberikan masalah - Siswa bekerja dalam kelompok setelah bekerja sendiri – sendiri.	
Kegiatan Inti	<u>Explorasi dalam kelompok</u> - Guru meminta siswa mengerjakan LKS I - Guru meminta siswa untuk berdiskusi dengan kelompok yang sudah ditentukan - Guru meminta siswa menuliskan cara mereka pada kertas poster yang sudah tersedia.	- Bersama kelompok masing – masing, siswa berdiskusi untuk menyelesai-kan masalah yang disampaikan guru - Siswa menuliskan hasil kerja mereka dalam bentuk poster.	15 menit
	<u>Diskusi kelas</u> - Guru meminta siswa untuk menampilkan pekerjaan mereka selama diskusi dengan kelompok masing – masing. - Guru membimbing siswa selama diskusi berlangsung, seperti memberikan kelompok lain untuk bertanya	- Siswa menunjukkan/ mempresentasikan hasil diskusi dengan kelompok mereka masing – masing di depan kelas. - Siswa berperan aktif dalam diskusi kelas dengan cara bertanya jika ada hal yang tidak jelas atau tidak dimengerti.	30 menit

	atau guru memberikan pertanyaan.		
Penutup	- Guru beserta siswa menyimpulkan hasil pembelajaran hari ini, seperti: • Apa kesimpulan kita hari ini? • Dengan apa kita menyelesaikan masalah hari ini? - Guru mengumpulkan poster siswa.	- Siswa menyimpulkan hasil yang mereka dapat pada hari pertama dan bagaimana cara mendapatkan kesimpulan tersebut.	10 menit

G. Media Pembelajaran

- Lembar Kerja Siswa I (LKS I)
- Kertas Poster
- Marker/Spidol

Surabaya, 17 Maret 2014

Peneliti,

Andrea Arifsyah Nasution
NIM. 127785079

RENCANA PELAKSANAAN PEMBELAJARAN
(RPP)

Sekolah : SD Al – Hikmah, Surabaya
Mata Pelajaran : Matematika
Kelas/Semester : 5/2
Pertemuan : II (Kedua)
Alokasi Waktu : 2 x 35 menit
Standar Kompetensi : 5. Using fraction in order to solve mathematical problems
Kompetensi Dasar : 5.3. Using fraction to solve problems related to ratio and scale.

A. Aktivitas

Menolong guru menyelesaikan masalah beras

B. Tujuan Pembelajaran

- Membimbing siswa mendapatkan ide perbandingan.
- Merubah model siswa ke dalam bentuk tabel perbandingan.
- Memperkenalkan tabel perbandingan.

C. Indikator

- Siswa mendapatkan ide perbandingan.
- Siswa belajar mengoperasikan tabel perbandingan.

D. Materi pembelajaran

Perbandingan yang mencakup pecahan

E. Pendekatan Pembelajaran

Pembelajaran dengan menerapkan RME (Realistic Mathematics Education).

F. Kegiatan Pembelajaran

Kegiatan	Kegiatan yang dilakukan oleh guru	Kegiatan yang dilakukan oleh siswa	Alokasi Waktu
Kegiatan Awal	- Guru meminta siswa untuk berdoa sebelum belajar. - Guru mengkondusifkan kelas ke suasana belajar.	- Siswa berdoa sebelum belajar. - Siswa mengikuti apa yang disampaikan oleh guru.	15 menit

	- Guru mengingatkan siswa tentang: • Apa yang telah mereka pelajari pada pertemuan pertama. • Bagaimana cara mereka menyelesai-kan masalah tersebut. • Kesimpulan yang mereka dapatkan pada pertemuan I dan menuliskannya di papan tulis. - Guru merubah jawaban siswa ke dalam bentuk tabel.	- Siswa mengingat kembali tentang apa yang telah mereka pelajari pada pertemuan pertama. - Siswa menuliskan cara untuk menyelesai kan masalah yang diberikan oleh guru dan menuliskan kesimpulan di papan tulis.	
Kegiatan Inti	<u>Explorasi dalam kelompok</u> - Guru meminta siswa berdiskusi untuk meneruskan tabel yang telah dibuatnya dalam kelompok yang telah ditentukan. - Guru meminta siswa menuliskan cara mereka pada kertas poster yang sudah tersedia.	- Bersama kelompok masing – masing, siswa berdiskusi untuk melanjutkan tabel yang telah dibuat guru sebelumnya. - Siswa menuliskan hasil kerja mereka dalam bentuk poster.	15 menit
	<u>Diskusi kelas</u> - Guru meminta siswa untuk menampilkan pekerjaan mereka selama diskusi dengan kelompok masing – masing. - Guru membimbing siswa selama diskusi berlangsung, seperti memberikan kelompok lain untuk bertanya atau guru memberikan pertanyaan.	- Siswa menunjukkan/ mempresentasikan hasil diskusi dengan kelompok mereka masing – masing di depan kelas. - Siswa berperan aktif dalam diskusi kelas dengan cara bertanya jika ada hal yang tidak jelas atau tidak dimengerti.	30 menit

Penutup	- Guru beserta siswa menyimpulkan hasil pembelajaran hari ini. - Guru meminta siswa membandingkan hasil yang mereka dapatkan pada pertemuan ini dengan hasil pada pertemuan pertama. - Guru memperkenalkan kepada siswa bahwa tabel yang dibuatnya disebut dengan tabel perbandingan. - Guru mengumpulkan poster siswa.	- Siswa menyimpulkan hasil pembelajaran hari ini. - Siswa membanding-kan hasil yang mereka dapat pada pertemuan ini dengan hasil pada pertemuan pertama.	10 Menit

G. Media Pembelajaran

- Lembar Kerja Siswa I (LKS I)
- Kertas Poster
- Marker/Spidol

Surabaya, 18 Maret 2014

Peneliti,

Andrea Arifsyah Nasution
NIM. 127785079

RENCANA PELAKSANAAN PEMBELAJARAN
(RPP)

Sekolah	: SD Al – Hikmah, Surabaya
Mata Pelajaran	: Matematika
Kelas/Semester	: 5/2
Pertemuan	: III (Ketiga)
Alokasi Waktu	: 2 x 35 menit
Standar Kompetensi	: 5. Using fraction in order to solve mathematical problems
Kompetensi Dasar	: 5.3. Using fraction to solve problems related to ratio and scale.

A. Aktivitas

Membuat kue untuk dijual.

B. Tujuan Pembelajaran

- Siswa mencoba menggunakan tabel perbandingan untuk menyelesaikan masalah perbandingan yang mencakup pecahan.
- Untuk membuat siswa mengetahui bagaimana cara menggunakan tabel perbandingan dalam menyelesaikan masalah perbandingan yang mencakup pecahan.

C. Indikator

- Siswa belajar mengoperasikan tabel perbandingan.
- Siswa tahu bagaimana cara menggunakan tabel perbandingan dalam menyelesaikan masalah perbandingan yang mencakup pecahan.

D. Materi pembelajaran

Perbandingan yang mencakup pecahan

E. Pendekatan Pembelajaran

Pembelajaran dengan menerapkan RME (Realistic Mathematics Education).

F. Kegiatan Pembelajaran

Kegiatan	Kegiatan yang dilakukan oleh guru	Kegiatan yang dilakukan oleh siswa	Alokasi Waktu
Kegiatan Awal	- Guru meminta siswa untuk berdoa sebelum belajar. - Guru mengkondusifkan kelas ke suasana belajar. - Guru mengingatkan siswa tentang: • Apa yang telah mereka pelajari pada pertemuan pertama dan pertemuan yang kedua. • Bagaimana cara mereka menyelesai-kan masalah - masalah tersebut. • Kesimpulan yang mereka dapatkan pada pertemuan I dan menuliskannya di papan tulis.	- Siswa berdoa sebelum belajar. - Siswa mengikuti apa yang disampai-kan oleh guru. - Siswa mengingat kembali tentang apa yang telah mereka pelajari pada pertemuan pertama dan pertemuan kedua.	10 menit
Kegiatan Inti	<u>Explorasi dalam kelompok</u> - Guru meminta siswa mengerjakan LKS III - Guru meminta siswa untuk berdiskusi dengan kelompok yang sudah ditentukan untuk membahas cara menyelesaikan masalah tersebut. - Guru meminta siswa menuliskan cara mereka pada kertas poster yang sudah tersedia.	- Bersama kelompok masing – masing, siswa berdiskusi untuk melanjutkan tabel yang telah dibuat guru sebelumnya. - Siswa menuliskan hasil kerja mereka dalam bentuk poster.	15 menit
	<u>Diskusi kelas</u> - Guru meminta siswa untuk menampilkan pekerjaan mereka	- Siswa menunjukkan/ mempresentasikan hasil diskusi dengan kelompok mereka	35 menit

	selama diskusi dengan kelompok masing – masing. - Guru membimbing siswa selama diskusi berlangsung, seperti memberikan kelompok lain untuk bertanya atau guru memberikan pertanyaan.	masing – masing di depan kelas. - Siswa berperan aktif dalam diskusi kelas dengan cara bertanya jika ada hal yang tidak jelas atau tidak dimengerti.	
Penutup	- Guru beserta siswa menyimpulkan hasil pembelajaran hari ini, seperti: • Apa kesimpulan kita hari ini? • Dengan apa kita menyelesaikan masalah hari ini? - Guru mengumpulkan poster siswa.	- Siswa menyimpulkan hasil pembelajaran hari ini dan cara yang dilakukan untuk mendapatkan kesimpulan tersebut.	10 menit

H. Media Pembelajaran

- Lembar Kerja Siswa III (LKS III)
- Kertas Poster
- Marker/Spidol

Surabaya, 19 Maret 2014

Peneliti,

Andrea Arifsyah Nasution
NIM. 127785079

RENCANA PELAKSANAAN PEMBELAJARAN
(RPP)

Sekolah	: SD Al – Hikmah, Surabaya
Mata Pelajaran	: Matematika
Kelas/Semester	: 5/2
Pertemuan	: IV (Keempat)
Alokasi Waktu	: 2 x 35 menit
Standar Kompetensi	: 5. Using fraction in order to solve mathematical problems
Kompetensi Dasar	: 5.3. Using fraction to solve problems related to ratio and scale.

A. Aktivitas

Perbandingan pada pecahan

B. Tujuan Pembelajaran

- Siswa menggunakan tabel perbandingan untuk menyelesaikan masalah perbandingan yang mencakup pecahan.
- Siswa membuat model dan menggunakan model tersebut dalam menyelesaikan masalah perbandingan yang mencakup pecahan.

C. Indikator

- Siswa bisa menggunakan tabel perbandingan untuk menyelesaikan masalah perbandingan yang mencakup pecahan.
- Siswa mampu membuat model sendiri dan menggunakan model tersebut untuk menyelesaikan masalah perbandingan yang mencakup pecahan.

D. Materi pembelajaran

Perbandingan yang mencakup pecahan

E. Pendekatan pembelajaran

Pembelajaran dengan menerapkan RME (Realistic Mathematics Education)

F. Kegiatan Pembelajaran

Kegiatan	Kegiatan yang dilakukan oleh guru	Kegiatan yang dilakukan oleh siswa	Alokasi Waktu
Kegiatan awal	- Guru meminta siswa untuk berdoa sebelum belajar. - Guru mengkondusifkan kelas ke suasana belajar. - Guru mengingatkan siswa tentang: • Apa yang telah mereka pelajari pada pertemuan pertama dan pertemuan yang kedua. • Bagaimana cara mereka menyelesai-kan masalah - masalah tersebut. - Kesimpulan yang mereka dapatkan pada pertemuan I dan menuliskannya di papan tulis.	- Siswa berdoa sebelum belajar. - Siswa mengikuti apa yang disampai-kan oleh guru. - Siswa mengingat kembali tentang apa yang telah mereka pelajari pada pertemuan pertama dan pertemuan kedua.	10 menit
Kegiatan Inti	<u>Explorasi dalam kelompok</u> - Guru meminta siswa mengerjakan LKS IV dan LKS V. - Guru meminta siswa untuk berdiskusi dengan kelompok yang sudah ditentukan untuk membahas cara menyelesaikan masalah tersebut. - Guru meminta siswa menuliskan cara mereka pada kertas poster yang sudah tersedia.	- Bersama kelompok masing – masing, siswa berdiskusi untuk melanjutkan tabel yang telah dibuat guru sebelumnya. - Siswa menuliskan hasil kerja mereka dalam bentuk poster.	15 menit
	<u>Diskusi kelas</u> - Guru meminta siswa untuk menampilkan	- Siswa menunjukkan/ mempresentasikan hasil diskusi dengan	35 menit

	pekerjaan mereka selama diskusi dengan kelompok masing – masing. - Guru membimbing siswa selama diskusi berlangsung, seperti memberikan kelompok lain untuk bertanya atau guru memberikan pertanyaan.	kelompok mereka masing – masing di depan kelas. - Siswa berperan aktif dalam diskusi kelas dengan cara bertanya jika ada hal yang tidak jelas atau tidak dimengerti.	
Penutup	- Guru beserta siswa menyimpulkan hasil pembelajaran hari ini. - Guru merefleksi kegiatan – kegiatan pertama hingga terakhir, seperti menanyakan kepada siswa topik yang mereka pelajari selama 4 pertemuan. - Guru mengumpulkan poster yang dibuat siswa.	- Siswa menyimpulkan hasil pembelajaran hari ini. - Siswa merefleksi kegiatan pertama sampai kegiatan terakhir bersama – sama guru.	10 menit

G. Media Pembelajaran

- Lembar Kerja Siswa IV dan V (LKS IV dan V)
- Kertas Poster
- Marker/Spidol

Surabaya, 20 Maret 2014

Peneliti,

Andrea Arifsyah Nasution
NIM. 127785079

APPENDIX VIII

PANDUAN UNTUK GURU
(TEACHER GUIDE)

Pertemuan 1

Aktivitas 1 : "Membantu guru menyelesaikan masalah beras"

A. Tujuan pembelajaran

- Membuat model sebagai representasi dari masalah
- Menggunakan model untuk menyelesaikan masalah

B. Kemampuan awal siswa

- Siswa telah mempelajari operasi matematika, seperti: penjumlahan, pengurangan, perkalian dan pembagian pada bilangan bulat di kelas empat.
- Siswa telah mempelajari tentang pecahan dan bagaimana menggambarkan pecahan ketika mereka belajar di kelas empat.
- Siswa telah mempelajari tentang bagaimana mengooperasikan pecahan di kelas lima sebelum topik ini dipelajari.

C. Bahan – bahan pembelajaran

Lembar Kerja Siswa 1, spidol dan kertas poster

D. Alokasi waktu

70 menit

E. Deskripsi aktivitas

Di awal pertemuan ini, guru mengelompokkan siswa ke dalam beberapa kelompok dimana setiap kelompok terdiri dari tiga atau empat siswa. Kemudian, guru menunjukkan kepada siswa gambar sebuah kotak yang berisi satu kilogram beras dan bertanya kepada mereka pertanyaan berikut ini.

1. Berikut ini adalah gambar sebuah kemasan yang berisi 1 kg beras!

2. Ibu Rina membeli 20 bungkus beras. Dalam satu hari, dia memasak $\frac{3}{4}$ kg beras. Dapatkah kalian tentukan berapa hari ibu Rina menghabiskan 20 bungkus beras?

Setelah itu, guru meminta siswa untuk menyelesaikan masalah tersebut sendiri – sendiri (sekitar 10 menit) dan kemudian mendiskusikannya dalam kelompok.

Ketika siswa mendiskusikan masalah di dalam kelompok, guru dapat berkeliling untuk mengamati bagaimana siswa menyelesaikan pertanyaan tersebut. Jika ada siswa yang menuliskan caranya dengan menggambar sebuah persegi panjang, maka guru dapat membuat diskusi dari jawaban ini. Akan tetapi, jika tidak ada siswa yang menjelaskan caranya dengan meggunakan gambar atau siswa tidak tahu bagaimana merepresentasikan pecahan $\frac{3}{4}$ dengan gambar, maka guru dapat menggambarkan sebuah kotak atau sebuah persegi panjang yang merepresentasikan 1 kg beras di papan tulis.

F. Reaksi bagi guru

1. Jika siswa menulikan cara pengurangan berulang, maka guru dapar mananyakan kepada siswa “Bisa tolong saya menjelaskan apa yang kamu kerjakan?”. Setelah siswa mejelaskan apa yang mereka lakukan, guru dapat bertanya kembali kepada siswa “Kamu bisa menjelaskannya dengan gambar?”. Pertanyaan ini ditujukan untuk mengetahui pemahaman siswa tentang apa yang mereka kerjakan.

2. Jika siswa langsung mengalikan 20 dengan $\frac{3}{4}$ atau 20 x $\frac{3}{4}$, maka guru dapat bertanya kepada mereka "Coba jelaskan sama saya mengapa kamu mengalikan 20 dengan $\frac{3}{4}$?". Dalam hal ini, guru dapat menuntun siswa ke konteks dengan cara menceritakan kembali masalahnya. Ibu Ani memasak $\frac{3}{4}$ kg beras dalam satu hari. Jika ada 3 kg beras, 3 kg beras cukup untuk berapa hari? Setelah siswa menjawab pertanyaan tersebut, guru dapat bertanya kepada mereka kembali "Ok, 3 kg beras cukup untuk 4 hari. Di soal tersebut, ibu Ani mempunyai 20 kg beras. Cukup untuk berapa harikah jumlah beras tersebut?
3. Setelah guru berkeliling untuk mengamati siswa dan mengetahui bahwa siswa tersebut masih tidak mengetahui bagaimana cara menyelesaikan masalah tersebut, guru dapat menggambarkan sebuah persegi panjang yang merepresentasikan sebuah kotak yang berisi satu kilogram beras di papan tulis. Untuk mengetahui apakah siswa mengerti apa yang mereka kerjakan, guru dapat mengulang kembali permasalahan tersebut. Kali ini, guru perlu menekankan konteks tersebut dan meminta siswa untuk merepresentasikan pecahan $\frac{3}{4}$ dengan sebuah gambar persegi panjang dengan menanyakan mereka pertanyaan "Bagaimana cara kamu membuat pecahan $\frac{3}{4}$ pada gambar ini?". Setelah itu, guru dapat mengarahkan siswa dengan menanyakan pertanyaan "Jika ada 3 kotak beras, maka berapa harikah jumlah beras ini akan habis?". Setelah itu, guru dapat menanyakan mereka kembali "Jika ada 20 kotak beras, maka berapa harikah beras ini akan habis?"

Pertemuan 2

Aktivitas 2 : "Memecahkan masalah beras"

A. Tujuan pembelajaran

- Mengenal arti dari perbandingan
- Bekerja pada tabel perbandingan

B. Bahan – bahan pembelajaran

Lembar Kerja Siswa 2, spidol, poster siswa dari pertemuan 1 dan kertas presentasi.

C. Alokasi waktu

70 menit

D. Deskripsi aktivitas

Pada awal pertemuan ini, guru membiarkan siswa menyelesaikan poster mereka sekitar 15 menit. Setelah siswa menyelesaikan poster mereka, guru dapat memulai diskusi untuk membahas strategi siswa menyelesaikan masalah. Untuk lebih jelas, tujuan dari diskusi tersebut adalah untuk mengamati kemampuan bernalar siswa untuk menyelesaikan masalah.

E. Diskusi 1

Setelah siswa menyelesaikan poster mereka, guru dapat memulai sesi diskusi dengan semua kelompok di kelas. Pada diskusi ini, guru meminta satu kelompok untuk menampilkan apa yang telah mereka kerjakan untuk menyelesaikan masalah tersebut. Dalam hal ini, guru dapat memilih pekerjaan siswa yang tidak benar ataupun yang tidak seluruhnya benar sehingga akan terjadi suatu debat atau dialog dalam diskusi kelas.

Setelah siswa menampilakan pekerjaan mereka, guru dapat meminta siswa – siswa yang lain untuk memberikan tanggapan mereka dengan menanyakan pertanyaan "Apakah kalian punya pertanyaan atau kalian ingin menambahkan sesuatu yang belum disampaikan oleh teman – teman kalian di kelompok ini?" dan setelah itu guru dapat bertanya kembali "Apakah kalian mengerti apa yang telah dijelaskan oleh teman – teman kalian ini?"

F. Reaksi guru selama diskusi berlangsung

1. Jika siswa mengatakan bahwa mereka sudah mengerti tentang apa yang telah ditampilkan oleh teman – temannya atau mereka hanya diam, maka guru dapat meminta salah seorang siswa untuk menjelaskan kembali apa yang telah dijelaskan oleh teman – temannya dengan menanyakan "Dapatkah kamu mengulang apa yang telah kamu dengar dari penjelasan teman kamu barusan?"
2. Jika siswa mengatakan bahwa mereka tidak mengerti, guru dapat bertanya kepada kelompok tersebut untuk mengulang kembali penjelasan mereka. Setelah itu, meminta kembali siswa yang lain untuk menjelaskan apa yang telah dijelaskan oleh kelompok tersebut.
3. Jika siswa mengatakan bahwa satu kotak cukup untuk empat hari atau dua kotak cukup untuk empat hari, guru dapat bertanya kepada siswa yang lain untuk merespon dengan menanyaan mereka "Apakan kamu mendengar hal yang sama dengan teman mu? Atau kamu mendengar hal yang lain?".
4. Widjaja et al. (2010) mengemukakan bahwa siswa akan menggunakan tanda untuk mengelompokkan tiga kotak atau empat hari. Dalam diskusi ini, guru dapat meminta siswa untuk menjelaskan tanda tersebut agar siswa yang lain mengerti.
5. Setelah diskusi tersebut, guru memaparkan kembali masalah yang terdapat di LKS. Di sini, guru meminta salah satu siswa untuk menjelaskan kembali apa yang telah dijelaskan oleh teman mereka. Diskusi ini seharusnya berujung pada sebuah kesimpulan bahwa tiga kotak atau tiga kilogram beras cukup untuk 4 hari.

G. Diskusi 2

Diskusi yang kedua ini ditujukan untuk merubah jawaban siswa ke dalam bentuk tabel perbandingan. Untuk memuai diskusi tersebut, guru dapat meminta siswa untuk menuliskan di papan tulis bahwa tiga kilogram cukup untuk empat hari. Kemudian, guru dapat menanyakan mereka pertanyaan "Dapatkah kalian tentukan 6 kotak beras cukup untuk berapa hari?". Setelah salah seorang siswa menjawab pertanyaan tersebut dengan tabel perbandingan, guru dapat menanyakan pertanyaan yang lainnya "Dapatkah kalian menentukan 6 kotak beras cukup untuk berapa hari?". Selanjutnya,

guru dapat meminta mereka untuk menentukan 20 kotak beras cukup untuk beraa hari.

Pertemuan 3

Aktivitas 3 : "Membuat kue untuk dijual"

A. Tujuan pembelajaran

- Menggunakan tabel perbandingan dalam menyelesaikan masalah
- Mendeskripsikan bagaimana cara menggunakan tabel perbandigan untuk menyelesaikan masalah

B. Bahan – bahan pembelajaran

Lembar Kerja Siswa 3, Kertas presentasi, spidol dan poster siswa pada pertemuan 1 dan kedua.

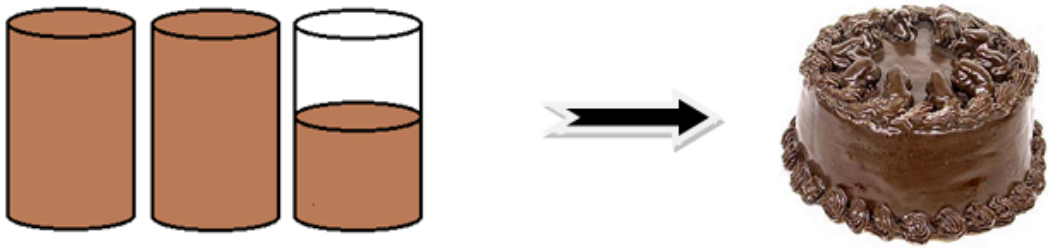
C. Alokasi waktu

70 menit

D. Dekripsi aktivitas

Pada pertemuan ini, siswa akan diberikan sebuah masalah yang ditujukan untuk membiarkan siswa untuk mencoba untuk menggunakan tabel perbandingan untuk menyelesaikan masalah perbandingan yang mencakup pecahan. Dalam hal ini, siswa diminta untuk menyelesaikan masalah di bawah ini:

Ibu Ani ingin membuat kue cokelat untuk dijual. Dia mempunyai 45 gelas bubuk cokelat. Untuk membuat 1 buah kue cokelat, dia membutuhkan $2\frac{1}{2}$ gelas bubuk cokelat.



Tentukan berapa banyak kue yang dapat dibuat dengan 45 gelas bubuk cokelat.

Untuk menyelesaikan masalah ini, guru meminta siswa untuk menyelesaikan masalah secara individu (sekitar 10 menit). Setelah itu, guru meminta siswa untuk

mendiskusikannya dalam kelompok. Selama diskusi kelompok tersebut, guru perlu berkeliling untuk mengamati apakah siswa mampu menyelesaikan masalah tersebut.

E. Reaksi bagi guru

1. Jika siswa masih menggunakan gambar untuk menyelesaikan masalah, guru dapat membiarkan mereka. Setelah siswa menyelesaikan pekerjaan mereka, guru dapat menanyakan mereka "Bagaimana cara kamu menyelesaikan masalah tersebut?". Setelah siswa menjelaskan caranya, guru dapat bertanya kembali "Dapatkah kamu menyelesaikannya dengan cara yang lain?" atau "Dapatkah kamu mengerjakannya dengan menggunakan tabel perbandingan?".
2. Jika jawaban siswa tidak seluruhnya benar, seperti: sisanya 5 gelas, guru dapat bertanya kepada siswa yang lainnya apakah mereka setuju dengan jawaban temannya tersebut. Jika siswa yang lainnya mengatakan bahwa mereka setuju, guru dapat bertanya kepada mereka mengapa mereka setuju dengan jawaban tersebut. Sebaliknya, jika siswa tidak setuju, guru juga dapat menanyakan mengapa mereka tidak setuju dengan jawaban tersebut. Akhirnya, guru dapat menanyakan kepada siswa "Jika ada 5 gelas tersisa, apakah masih cukup untuk membuat kue lagi?". Dalam hal ini, guru hendaknya menanyakan pertanyaan – pertanyaan ini selama diskusi dengan seluruh kelompok berlangsung.
3. Jika jawaban siswa salah, guru dapat bertanya kepada siswa yang lain "Apakan kalian setuju dengan jawaban ini?" atau "Apakah kalian memiliki cara yang berbeda untuk menyelesaikan masalah ini?".
4. Jika siswa menjawab pertanyaannya dengan benar, guru dapat bertanya kepada siswa yang lainnya apakah mereka setuju dengan jawaban tersebut. Jika siswa mengatakan bahwa mereka setuju dengan jawaban temannya, guru dapat meminta mereka untuk menjelaskan mengapa mereka setuju dengan jawaban tersebut. Disamping itu, guru dapat meminta penyelesaian yang berbeda dari siswa dan meminta mereka untuk menuliskan jawaban mereka jika mereka memiliki cara yang berbeda.

F. Diskusi

Untuk memulai sesi diskusi, guru dapat meminta salah satu kelompok untuk menampilkan pekerjaan mereka di depan kelas. Jika siswa membuat kesalahan dalam menyelesaikan permasalahan atau jawaban mereka tidak benar seluruhnya, seperti: sisanya 5 gelas, kemudian guru dapat memulai diskusi dengan situasi ini. Oleh karena itu, diskusi antara guru dan siswa dan antara siswa itu sendiri akan terjadi.

Pertemuan 4

Aktivitas 4 : "Perbandingan pada pecahan"

A. Tujuan pembelajaran

Membuat tabel perbandingan dan menggunakannya dalam menyelesaikan masalah.

B. Bahan – bahan pembelajaran

Lembar Kerja Siswa 4, kertas presentasi, spidol dan poster siswa pada pertemuan 3.

C. Alokasi waktu

70 menit

D. Deskripsi aktivitas

Pada pertemuan ini, siswa akan diberikan beberapa pertanyaan yang bertujuan untuk membiarkan siswa untuk membuat tabel perbandingan mereka sendiri dan menggunakannya untuk menyelesaikna masalah perbandingan yang mencakup pecahan. Dalam hal ini, siswa akan diminta untuk menyelesaikan masalah masalah berikut ini.

1. *Mbak Rika pergi ke supermarket untuk membeli 15 kg beras untuk kebutuhan sehari – hari. Setiap hari, dia memasak* $\frac{1}{2}$ *kg beras. Tentukanlah 15 kg beras cukup untuk berapa hari?*
2. *Mbak Sri pergi ke supermarket untuk membeli 25 kg beras untuk kebutuhan sehari – hari. Setiap hari, dia memasak* $\frac{1}{3}$ *kg beras. Tentukanlah 25 kg beras cukup untuk berapa hari?*
3. *Ibu Fitri membeli 25 kg beras untuk kebutuhan sehari – hari keluarganya. Setiap hari, dia memasak* $\frac{3}{4}$ *kg beras. Cukup untuk berapa harikah 25 kg beras tersebut?*

Dalam hal ini, siswa diminta untuk menentukan hasil dari pertanyaan – pertanyaan tersebut. Pada awalnya, guru meminta siswa untuk berkerja secara individu (sekitar 10 menit). Setelah itu, guru meminta siswa untuk mendiskusikannya dalam kelompok. Setelah siswa selesai menyelesaikan pertanyaan – pertanyaan

tersebut dalam kelompok, guru dapat membuat diskusi untuk melihat cara siswa menyelesaiakan masalah dengan menggunakan tabel perbandingan.

Tujuan dari diskusi tersebut tidak hanya focus pada hasil yang diperoleh oleh siswa, akan tetapi focus pada bagaimana siswa memberikan pendapat dan alasan mereka dalam menjelaskan cara mereka untuk menjawab pertanyaan – pertanyaan tersebut. Oleh karena itu, guru sebaiknya dapat memotivasi siswa supaya mereka memberikan pendapat selama diskusi berlangsung.

E. Diskusi

Setelah siswa bekerja dalam kelompok, kemudian guru membuat diskusi yang bertujuan untuk mendiskusikan pekerjaan dan strategi siswa bagaimana mereka menyelesaikan masalah tersebut. Diskusi tersebut bertujuan untuk menginvestigasi kemampuan bernalar siswa mengenai masalah yang diberikan. Oleh karena itu, guru perlu mengarahkan siswa ke suatu diskusi dimana siswa dapat memberikan pendapat mereka mengenai masalah tersebut.

Dalam hal ini, guru dapat memilih kelompok siswa yang membuat kesalahan atau jawaban siswa yang menarik untuk didiskusikan, seperti: strategi siswa pada nomor 3. Strategi ini menarik untuk didiskusikan karena untuk menjawab pertanyaan tersebut membutuhkan penalaran siswa. Untuk itu, guru dapat meminta mereka menjelaskan bagaimana cara mereka menyelesaikan masalah tersebut. Setelah siswa menjelaskan jawaban mereka, guru dapat meminta siswa yang lainnya untuk memberikan pendapat mereka dan guru juga dapat meminta jawaban yang berbeda dari siswa yang lainnya.

Untuk menyelesaikan pertanyaan yang ketiga, guru dapat memulai diskusi dengan cara meminta salah satu kelompok yang membuat kesalahan dalam menyelesaikan masalah tersebut atau kelompok siswa yang jawabannya kurang tepat untuk menjelaskannya di depan kelas. Sebagai contoh: mereka mengatakan bahwa “25 kg beras cukup untuk 32 hari dan sisa 1 kg beras”. Jawaban ini merupakan cara yang tepat untuk memulai diskusi. Dalam hal ini, guru dapat meminta pendapat siswa yang lainnya dengan menanyakan “Bagaimana pendapat kalian terhadap jawaban

mereka? Apakah kalian setuju dengan jawaban mereka? Atau apakah kalian ingin menambahkan sesuatu yang lain?". Jika ada siswa yang mendapatkannya, guru dapat memulai diskusi dengan mengatakan "Ok, teman kalian mengatakan bahwa 25 kg beras cukup untuk 32 hari dengan sisa 1 kg beras. Bagaimana pendapat kalian tentang 1 kg beras tersebut? Apakah masih cukup untuk memasak nasi?". Jika siswa mengatakan "Ya", maka guru dapat menantang mereka "Dapatkah kalian menentukan 1 kg beras cukup untuk berapa hari? Atau kalian bisa membuat gambar untuk menjelaskannya".

F. Reaksi untuk guru

1. Jika berhitung dengan setengah, sepertiga dan seperlima, guru dapat bertanya kepada siswa yang lain "Apakah kalian setuju dengan jawaban ini?" dan guru dapat meminta strategi yang berbeda untuk masalah tersebut.
2. Jika siswa membuat kesalahan dalam menyelesaikan pertanyaan – pertanyaan tersebut, misalnya: salah dalam perhitungan, guru dapat meminta siswa yang lain untuk memberikan pendapat terhadap yang telah dikerjakan teman mereka dengan menanyakan "Bagaimana menurut pendapat kalian tentang jawaban mereka? Apakah kalian setuju dengan jawaban mereka?". Jika siswa mengatakan bahwa jawaban tersebut salah, guru dapat meminta mereka untuk memberikan alasan mengapa mereka mengatakan salah. Setelah mereka menjelaskan strategi mereka, guru dapat meminta strategi yang berbeda dari siswa – siswa yang lainnya.
3. Guru dapat juga memulai diskusi dengan meminta kelompok siswa yang strateginya menarik untuk didiskusikan, misalnya: strategi siswa pada poin ketiga, untuk menjelaskan cara mereka di depan kelas dengan menanyakan mereka "Ok, dapatkah kalian menunjukkan bagaimana kalian menyelesaikan masalah tersebut?". Setelah siswa menjelaskan strategi mereka, guru dapat meminta siswa yang lainnya untuk memberikan reaksi mereka atau meminta mereka untuk menulang kembali apa yang temannya telah katakan dengan

menanyakan mereka “Jika kalian sudah mengerti, bisakah kalian mengulang apa yang teman kalian katakana tadi?”.

4. Jika siswa tidak tahu bagaimana untuk menyelesaikan masalah yang ketiga, guru dapat mengingatkan mereka kembali masalah yang telah mereka kerjakan pada pertemuan 1 dengan menanyakan “Apakah kalian masih ingat masalah tentang ibu Rina?”. Guru dapat menantang siswa dengan cara menanyakan “Sekarang kalian punya 25 kg beras. Dapatkah kalian tentukan jumlah beras tersebut cukup untuk berapa hari?”.
5. Jika jawaban siswa tidak benar, misalnya: mereka mengatakan bahwa “25 kg beras cukup untuk 32 haridan sisanya 1 kg”. Jawaban ini merupakan langkah yang tepat untuk memulai diskusi. Dalam hal ini, guru dapat meminta startegi yang berbeda dari siswa yang lainnya mengenai jawaban tersbut dengan menanyakan “Bagaimana menurut pendapat kalian tentang jawaban mereka? Apakah kalian setuju dengan jawaban tersebut? Atau apakah kalian memiliki hal yang lain untuk ditambahkan?”. Jika ada siswa yang menjawab 1 kg sisa, guru dapat memintanya untuk menjelaskan di depan kelas. Jika tidak ada siswa yang mendapatkan hal tersebut, guru dapat memulai diskusi dengan menanyakan mereka “Ok, teman kalian megatakan bahwa 25 kg beras cukup untuk 32 hari dan sisa 1 kg. Bagaimana menurut pendapat kalian tentang 1 kg tersebut? Apakah masih cukup untuk memasak nasi lagi?”. Jika siswa mengatakan “ia”, maka guru dapat menantang siswa dengan menanyakan “Dapatkah kalian menentukan 1 kg beras cukup untuk berpa hari?”.
6. Jika ada siswa yang menuliskan cara seperti pada poin ketiga di atas, guru dapat menyimpan ini untuk didiskusikan pada sesi diskusi sebagai cara yang lain untuk menyelesaikan masalah tersebut.