SUPPORTING STUDENTS' CONCEPTUAL UNDERSTANDING OF ADDITION INVOLVING NEGATIVE NUMBERS

MASTER THESIS



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UNIVERSITAS NEGERI SURABAYA PROGRAM PASCASARJANA PROGRAM STUDI PENDIDIKAN MATEMATIKA 2013

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A Thesis submitted to Surabaya State University Postgraduate Program as a Partial Fulfillment of the Requirement for the Degree of Master of Science in Mathematics Education Program

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Mulay in

Prof. I Ketut Budayasa, Ph.D NIP. 195712041994021001 FOR MY PARENTS,

MY EXCELLENT FATHER, SUPRATMAN YUSUF

AND MY INCREDIBLE MOTHER, SITI NURYANI

THANK YOU FOR LOVE, MOTIVATION, AND STRENGTH. THANK YOU FOR ALWAYS BEING ON MY SIDE EVEN WHEN I WAS WRONG,

EVERYDAY I LOVE YOU.



ABSTRACT

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Negative number is widely accepted as abstract concept that students struggle to understand. Providing some rules of addition or subtraction involving negative numbers could not successfully help the students to grasp the meaning of negative numbers. The abstractness of negative numbers should be bridged by some contexts which can help the students to discover the notion of negative numbers. Moreover, the contexts in teaching should be carefully chosen because not all context experienced by students in Indonesia. The present study aims at developing the learning activities about negative numbers with the use of games which can improve students' conceptual understanding of addition involving negative numbers.

The study use Design Research approach in conducting a range of process including the preparation phases, the pilot experiment, the real teaching experiment, data collection and data analysis. The teaching experiment is conducted to test the Hypothetical Learning Trajectory (HLT) that has been designed. The HLT is compared to the actual students' learning and improved based on the analysis of supporting data during the study including interview with the teacher, classroom observation, written works, and the video recording of the lessons. The students involved in this study were the third grade students in SDIT Ghilmani Surabaya.

The findings suggested that the designed activities which include some games can improve students' conceptual understanding of addition involving negative numbers in a way that students can discover by themselves in giving meaning to negative numbers within the context given. Number lines activities were a helpful context as well as the model of thinking which assist the students to learn addition of integers. Also, the activities support the students to shift from context to concepts and vice versa because it enable student to achieve understanding about addition involving negative numbers in more formal way. It is also recommended to develop activities regarding negative numbers in various contexts to strengthen the acknowledgement of students about the concepts.

ABSTRAK

Pratiwi, Weni Dwi. 2013. *Supporting Students' Conceptual Understanding of Addition involving Negative Numbers*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Siti Maghfirotun Amin, M.Pd. dan (II) Dr. Agung Lukito, M.S.

Kata Kunci: Bilangan Bulat, Bilangan Negatif, Penjumlahan, Penelitian, Konteks *Utang-Bayar*, Garis Bilangan, Permainan Dadu, Design Research

Bilangan negatif dipandang sebagai konsep abstrak yang sangat sulit dipahami siswa. Memberikan beberapa aturan penjumlahan atau pengurangan yang melibatkan bilangan negatif tidak akan membantu siswa memahami dan memaknai bilangan negatif. Hal ini harus dijembatani oleh beberapa konteks yang dapat membantu siswa menemukan makna bilangan negatif . Terlebih lagi, konteks yang dipilih pada saat mengajarkan materi ini harus benar-benar dipertimbangkan mengingat tidak semua konteks mengenai bilangan negatif 'dekat' dengan siswa. Penelitian ini bertujuan dalam mengembangkan aktivitas pembelajaran kelas tentang bilangan negatif dengan menggunakan permainan yang dapat membantu meningkatkan pemahaman konseptual siswa tentang penjumlahan yang melibatkan bilangan negatif.

Studi ini menggunakan pendekatan Design Research dalam pelaksanaan serangkaian proses penelitian mencakup tahap persiapan, pilot experiment, teaching experiment, pengumpulan data, dan analisis data. Teaching Experiment bertujuan untuk mengetes Hypothetical Learning Trajectory (HLT) yang sudah didesain sebelumnya. Pada tahap ini HLT akan dibandingkan dengan respon siswa pada pembelajaran sebenarnya dan kemudian diperbaiki berdasarkan analisis data dari observasi kelas, wawancara dengan guru, hasil pekerjaan siswa, dan rekaman pembelajaran. Siswa yang terlibat pada penelitian ini adalah siswa kelas 3 SDIT Ghilmani Surabaya.

Hasil dari studi ini menunjukkan bahwa pembelajaran dengan menggunakan konteks permainan dapat meningkatkan pemahaman konseptual siswa mengenai penjumlahan yang melibatkan bilangan negatif dengan memungkinkan siswa untuk dapat menemukan sendiri makna dari bilangan negatif yang sesuai dengan konteks yang diberikan pada pembelajaran. Pembelajaran menggunakan garis bilangan yang berperan sebagai konteks sekaligus sebagai model berpikir siswa dapat membantu siswa memahami penjumlahan yang melibatkan bilangan negatif. Selain itu,serangkaian desain pembelajaran pada penelitian ini juga dapat membantu siswa mentransfer pemahaman dari konteks ke konsep dan juga sebaliknya. Rekomendasi dari penelitian ini adalah agar pembelajaran mengenai bilangan negatif yang melibatkan banyak konteks dapat lebih dikembangkan sehingga dapat memantapkan pemahaman siswa tentang bilangan negatif.

PREFACE

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CHAPTER I

INTRODUCTION

A. Research Background

During school life, students will experience the process of expanding their mathematical knowledge to include negative numbers. The students are challenged to use their existing concepts about natural numbers to make sense of negative numbers. The students may accept that negative numbers as the inverse of positive numbers. However, this idea is insufficient. There are many ideas about the notion of negative numbers which should be discussed with students. Many students are confused about different types of numbers and do not understand that all different types of numbers are part of the system of real numbers (Bruno, et. al.,1999). Therefore, the notion of negative numbers must be well-understood to make children prepare to learn and understand the bigger numerical system.

Children aged nine to ten years old should know and understand the reason why they learn negative numbers. They may find the notion of negative numbers surround them and know that there is another type of numbers. They try to interpret negative numbers by considering what they already know about natural numbers. Their previous knowledge about natural numbers can help as well as confuse them because they must become aware that the sign in the numbers gives meaning to it. Children wonder why 5 is not equal with -5. They should be able to reason and describe this understanding by themselves before they experience more complicated situations relating to negative numbers.

The concept of negative numbers can be complicated for children if it is not presented in a meaningful way to students. In dealing with negative numbers, students have specific difficulties, such as conceptualizing numbers less than zero; treating negative numbers as mathematical objects; and formalizing rules for integer arithmetic, particularly the meaning of the fact that the opposite of a negative number is a positive number. (Stephan and Akyuz, 2012). Students struggle how to cope with the meanings of negative numbers which fit to their thinking and intuition. Also, they may find difficult to distinguish the minus sign of negative numbers and sign for the operation.

Children may grasp the idea of negative numbers if it is learned through a familiar context to stimulate their common sense and intuition. Using the context, they will use their existing number concept, and the development of new concepts and strategies will be supported by the contexts (Linchevski, 1999). Whitacre et. al. interviewed a child and found that a child considers negative numbers as belonging to a *ghost-world* in which faint images of object could be seen, although the objects were not actually there. It seems that children are able to reason and interpret negative numbers by using their intuition. The evidence is that a first grader who was not familiar with the notion of negative numbers could reason and do the task correctly within the magnitude context (Whitacre, et. al., 1999). The term magnitude context refers to opposite magnitude contexts, some contexts such as lending and owing money; travelling forward and backward,

moving up and moving down in an elevator, and soon are included as opposite magnitude context (Whitacre, et. al., 2012) Children can have powerful thinking to learn the early concept of negative numbers.

Some contexts in learning negative numbers had already been tried and attempted by researchers such as the context of the temperature, abacus and the dice (Linchevski and Williams, 1999), assets and debts (Stephan and Akyuz, 2012), students' movement along a path (Otten), and the use of the number line to represent the operation in negative numbers (Heefer, 2011). They use contexts to give meaning to negative numbers that students need to understand. Context must be carefully chosen by considering the local culture. An Indonesian context for negative numbers has been proposed by Arwadi (2012), which is an equilibrium context. He used the model of a sequence of marbles to solve the problem of opposite things (opposite magnitude). However, it is argued that although students can operate negative numbers using a sequence of marbles and a mental number line, the students are still confused by interpreting negative numbers in their own words within this context. They need time to understand negative numbers in another context to fully grasp the concept of negative numbers. For example, the use of mathematical symbols should be postponed until they certainly get the idea, and can reason about what they have learnt and construct by themselves an acceptable idea about negative numbers.

At some points, some of the contexts mentioned above could help the learner to better understand negative numbers. However, some of those are addressed for higher grades, and they contain some abstract calculation which can be complicated for children aged 9-10 years old and probably it is not replicable for lower grade. Also, the students need to know why negative numbers appear in their life. They have to experience by themselves that negative numbers appear in their life in a particular situation. Therefore, there is a need to put them in that kind of situation.

By considering this issue, in the study it is intended to use an activity involving a game and number lines to have students in grade three learn negative numbers. The game will help them encounter negative numbers by themselves. The board game is a designed game using money context in which students will move around and stop at the one box containing the item which they have to buy. It is expected that trough this game, students will experience a situation in which they encounter negative numbers. The idea of designing this game is that negative numbers could appear with subtraction process. By making a situation in which students will spend their money and finally have insufficient money to buy something and need to borrow money from somebody else. It causes the students having debts to somebody. They are expected to understand that when they have debts, they have a 'negative' amount of money.

Students are hoped to grasp the idea by having them reflect about the games, so they are able to explain what negative numbers are within this context. Also, some number line activities are included to encourage students to use this model to learn addition of integers.

B. Research Question

Based on the research background elaborated in the previous chapter, it is expected that the conceptual understanding of the students about addition involving negative integer can be supported by designing some activities and implement it in the classroom. To solve this problem; therefore, research question of this study is : *How to develop students' conceptual understanding of addition involving negative numbers?*

C. Aim of the Research

The present study aims at developing the learning activities on negative numbers with the use of money context and the model of number line which can improve students' conceptual understanding of addition involving negative numbers.

D. Definition of key terms

This part describes the definition of some terms underpinned in this study which is used to solve the research questions in this study.

- 1. *Negative number*, The integers which is less than zero.
- 2. *Addition*, is mathematical operation that represents the total amount of numbers together in a collection which is signified by plus sign (+)
- 3. *Concept,* A notion or statement of an idea which is formed in the mind/thinking.

- 4. *Understanding,* is a mental process which requires the connecting facts, the learners need to make connection between the new acquired information to their existing knowledge. (Skemp, 1987)
- 5. *Conceptual Understanding*, the ability of the learner to develop the properties of the concept and other associated concepts to strengthen the knowledge about something. (Chadwick: 2009)
- 6. *Conceptual Understanding of addition*, the ability of the learner to develop the properties of the concept about mathematical operation that represent amount of objects together in a collection and other associated concepts to strengthen the knowledge about it.
- 7. *Supporting*, the process or act to assist or to help. In this study, supporting can be considered as the process or the act to assist which is done by researcher and the teacher in learning. Researcher support in designing instructional activities; meanwhile teacher support in implementing the activities in classroom.

E. Significance of the Research

This present study contributes in promoting Realistic Mathematics Education as the basic foundation to improve teaching and learning process. Also, this study managed to develop the topic of negative numbers involved in the sequence of lesson. Generally, this present study can be taken into consideration in assisting further researchers who want to conduct a study in similar circumstances and can be the source of information in collecting data.

CHAPTER II

THEORETICAL FRAMEWORK

The chapter elaborates some main concepts which underpin in this study. The content of the whole section will describe mostly about the findings of some studies concerning to students' understanding about negative numbers, the teaching strategies in improving the conceptual understanding, and how the use of context in reality support the students' thinking from informal level to more formal level. Specifically, the present study conducted with the aim of improving the students' way of learning negative numbers and promoting Realistic Mathematics Education (RME) as an instructional theory in designing sequence of learning negative numbers. As cited in Gravemeijer and Terwel (2000, p.77) RME is grounded in Freudenthal's (1971,1977) idea that mathematics is a human activity, and consequently must "be connected to reality, stay close to children and should be relevant to society and students should learn mathematics by developing and applying mathematical concepts and tools in daily-life problem situations that make sense to them. Students are seen as reinventors, with teachers guiding and making conscious to student the mathematization of reality, with an eye to encouraging students to reflect on the process. Also, RME gives the possibility to shift the students' thinking, which is less formal to an abstract level of understanding. Some other part of this section is about the topic of negative numbers in Indonesian curriculum and how the sequence of learning designed in consideration that it might fit the curriculum. In this study, it will be conducted

some supporting action of teacher and the research in designing as well as implementing the designed activities in the real teaching experiment. In general, based on Merriem Webster - Dictionary, support means *to assist* or *to help*. Therefore, it can also be described as the act or the process to assist or to help. In this study, support is defined as the process or the act to assist which is done by researcher and the teacher in learning. As explained before, the process of supporting done by the researcher is in designing the set of instructional activities. Meanwhile the process of supporting which is done by the teacher is in implementing the designed instructional activities in real classroom. Also, the teacher does some actions to support the learning by assisting the students in the form of leading the discussion and encouraging the discourse among students.

A. The Notion of Negative Numbers

Historically speaking, in European during the 18th century, negative numbers regarded as less than nothing, but this idea was difficult to accept (Kilhamn, 1999). Some great mathematicians struggled in including negative numbers in numerical system (Whitacre et.al, 2012). This condition brought impacts to teaching and learning negative numbers nowadays. It was difficult for children to make sense negative numbers. Their prior knowledge, however could not help them to understand the meaning of negative numbers.

Negative numbers are the numbers included in numerical system that should be learnt and understood by the students to expand their numerical knowledge. Negative numbers is integers which is less than zero; meanwhile integers is a whole number that can be either greater than zero, called positive, or less than zero called negative. It is also required to define whole number as the set of all counting numbers plus zero. The students' first conception about negative numbers may appear by doing subtraction of unsigned numbers result as signed numbers. This extension of number domain could be the indication of the acceptance of negative difference (Kilhamn, pg. 199). Concerning this issue, the possible task presented is in the form of subtraction of a larger number from a smaller number. Kilhamn suggested that the only context that could be appropriate for this task was the money context with the concept of having and owing. If the task applies for a collection of objects, the subtraction results in negative difference does not make sense. The acceptance of negative difference was considered crucial when defining and interpreting negative numbers. However, many students have difficulties in learning the concept of negative numbers because it cannot be modeled physically and require them to reason in order to make sense of negative numbers (Stephan and Akyuz, 2012).

While exploring the investigation about the difficulties students may have in expanding their numerical knowledge, Bruno and Martinon (1999, pg. 790) elaborated three basic dimensions of numerical extensions. They are abstract dimension, conceptual dimension, and number line dimension. Abstract dimension includes the formal knowledge about numbers, such as operations and algorithms. Contextual dimension relate to in which the negative numbers applied to some environments. For example, owing and lending money, move up-down the elevator, and temperatures. Specifically in money context, the simplest use of numbers is the expression of a state ('I have \$3'). It is useful to view the state as

changeable value. Another use of numbers is the absolute comparison of two states ('I have \$4 more than you'). This dimension includes how children talk about negative numbers within those environments. By supporting students to write mathematical context, they can improve their understanding of the concept because they can reflect on what they learn and do some clarification to it.

The third dimension is number line dimension. Number line dimension relates to how students use a number line as a representation of addition and subtraction involving negative numbers. Supporting the idea about number line, other researcher have describe number line as a representation of numbers on a straight line where points represent integers and the distance between points matches the arithmetical difference between the corresponding numbers. (Heffer, 2011). The number line is widely accepted as a model to learn the operation involving integers.

The operation which will be focused in the learning is addition. The addition that will be learned by the students through the lesson is the operation between two integers that could be the addition of negative and positive numbers. Addition is the process of finding the total, a sum, by combining two or more numbers.

B. Conceptual Understanding

Chadwick (2009) revealed that the concept is the big idea, understanding, or thought embodying the set of things that have one or more properties in common. Chadwick also highlight that concrete concepts are generally easier to understand and can be used to help to find the meaning of more abstract ones. What can be conferred from this statement that the earlier concrete concepts that student might have should have connection with the concept that they want to learn. Concepts stimulate the learners to organize the information according to pattern of the similarity and difference. Skemp (1987, pg. 217) revealed that understanding is an active process which requires the connecting facts, the learners need to make connection between the new acquired information to their existing knowledge. Also, it requires not only having knowledge but also doing something with it. When the learners have understanding about some concept, they should also organize and examine the concept that helps them to strengthen the knowledge. Meanwhile conceptual understandings are what learners know and understand about the concept (Chadwick: 2009, pg 6). Moreover, Chadwick (2009) elaborated that conceptual understanding is the ability of the learner to develop the properties of the concept and other associated concepts to strengthen the knowledge about something. The process of developing the concept can be also similar to exploring and examining the concept and certainly those process not only about making definition of some concepts but more about the process to build different interpretation of a concepts in any circumstances. Besides, conceptual understanding of addition means that the ability of the learner to develop the properties of the concept about combining two or more numbers in order to find the total or the sum and other associated concepts to strengthen the knowledge about it. The students can better learn about the concept if they are already able to improve their conceptual understanding. Chadwick (2009, pg. 23)

Examined that improvement of learner's conceptual understanding can be seen by these criteria:

- the level of their understanding and use of abstract concepts increases (in the topic of negative numbers, the example of abstract concepts could include awareness of the difference between operational sign and number sign);
- they make connections between multiple concepts (manipulating the operation sign in multiplication or division involving negative numbers);
- they apply and transfer their understandings to more complex and distant contexts as well as to those that are familiar (applying negative numbers in various circumstances);
- 4. they take responsible actions and make informed decisions that are based on their new understandings (solving advance mathematics problem);
- they begin to understand that concepts can have different interpretations (broaden number sense by comparing negative number to other type of number in numerical system).

If the learners fulfill these criteria, the conceptual understanding has been apparently improved.

C. Sign Rules and Number Line

It is widely accepted that the minus sign is used both as a sign of operation (subtraction) and as a sign indicating a negative number. Vlassis (2002) noted that many of the difficulties students had when solving equations did not depend on the structure of the equation or the appearance of variables but was a result of the degree of abstraction caused by the presence of negative numbers. In very beginning, it is useful to introduce this to children after they have been able to give their own meaning about negative numbers.

Streefland (1996) suggested using a context in learning negative number involving the minus-sign rules. He believes that context enable students to learn the negative numbers as a real-contextual task rather than a number with simple sign-rules. The understanding of the operation of negative numbers including different signs can be supported by using a number line model. Heffer (2011) defining the number line as a representation of the numbers on a straight line. Furthermore, he describes that the two components of a number line, points and distance between points have distinct functions; points represents integer or real numbers; distance between points indicate the arithmetical difference between numbers. The negatives on the number line are ordered opposites to the positives. The representation of operation of negative numbers usually involves the movement of numbers along the path of the number line (Kilhamn, pg. 145). Movement to right showed the direction to positive numbers; meanwhile the movement to the left showed the direction to negative numbers. Later, in this study, the movements along a number line could use the term 'jumps' resulted in positive or negative numbers.

D. Researches on Negative Numbers

Several researchers have attempted to improve the learning of negative numbers using the environments to minimize the abstractness of the concept of negative numbers. Some of them use manipulation to the contexts. Stephan and Akyuz (2012) have investigated the understanding of the students about negative numbers with the use of financial context by using the terms of assets and debts including the use of vertical number line to represent the calculation. The findings showed that the combination between the finance context and the use of a number line managed to raise students' conceptual understandings of integer and its operation. After the instruction, the result indicate that the instruction help students improve their test on addition and subtraction significantly. Different context was proposed by Whitacre et al. (2012). They use the context which they called opposite magnitude relating to the notions of additive inverses and integers as representatives of equivalence classes of the sums. Children were given a sheet of paper with smiling faces and frowning faces representing happy and sad thoughts, and they were asked to make a comparison between days. Here, the authors realized that children are familiar with the word 'opposites', therefore, the authors, use this prior knowledge of children introduce negative numbers in such a way that children can explain their reasoning concerning the use of negative numbers in that context. Along the same lines, another study of Whitacre et al.(1999) described how children make meanings of negative numbers through interviews. The authors conducted an interview with two children to know their ideas and thinking concerning negative numbers before they are given a formal instruction. The discussion was about the reasoning of two children from a first grader and a fourth grader when they solved open number sentences. Based on the interview, the children who have not received any formal instruction about negative numbers managed to reinvent sign rules for addition and subtraction of negative numbers and can reason the problem by relating the knowledge concerning the properties of integers.

Other researchers have described negative number similarly. Arwadi (2011) conducted a study with RME approach about negative number by using equilibrium context, in which the number line was used as the model. Arwadi described that the equilibrium context is the situation involving two distinct entities which contradict each other. The reality context used in the study is the sequence of marbles which was arranged in a row horizontally. The researcher used the sentence like 'having marbles' and 'lack or marbles'. By relating the sequence to the number line, it helps the students did the reasoning that the larger numbers located in the right and the the 'lack of marbles' means less than having no marbles. The researcher suggested that the combination of the equilibrium context and number line model can support students' understanding of negative numbers. Besides, the researcher also elaborated that the by involving students to discuss the state of having and lack of marbles can help them to get the meaning of negative numbers informally.

Meanwhile, Streefland (1996) reported a course for the group consisting of 11-12 year-old upgraders in primary school by using the context of passenger getting on and getting off the bus. In this study, the researcher emphasize on how the students do the calculation of the number of passengers on the bus, and how they can operate negative numbers with shortages. The researcher revealed that, with this context, the students were able to notate number sentence in the opposite manner. This could be a good starting point for students to notating number formally.

Given the abstractness of negative numbers, Linchevski and William suggested the findings of their study elaborating the disco game using abacus attaches the integer more intuitively to the processes of the situation, but the dice game is more concrete because children can refer the integer into object that it can be easily added and subtracted. (Linchevski and Williams, 1999). Given the contextual approaches that many researchers used to make negative numbers become more understandable, Borba and Nunes (1999) argued that not every meaning is clearly understood, thus, the introduction of negative numbers should be initiated by the most familiar meanings and consider the need for giving distinct marks of positive and negative numbers.

E. RME (Realistic Mathematics Education): The Instructional Design

Approach

The approach in designing the sequence of learning activities is Realistic Mathematics Education (RME). The key idea of RME is mathematics as human activity (Freudenthal, 1973). Freudenthal revealed that the process of doing mathematics is more valuable than mathematics as a ready-made product (Gravemeijer and Terwel, 2000, pg.780). Through the learning process, students are guided to reinvent mathematics by organizing and interpreting realistic context. (Gravemeijer, 1994; Stephan, 2012,pg. 433). This process will help the students to reason the mathematical aspect within the context. Later, they also

realize the usefulness of mathematics concept that they can explain why they should learn mathematics.

In RME, modeling as an activity is further elaborated in a didactical sense meaning that students reorganizing the subject matter with the emergence of informal ways of modeling and symbolizing. (Gravemeijer, 2002, pg.2). The design will emphasize the context of the game that students can organize their thinking through this context because the case of negative numbers is problematic because this extension of numbers involving formal mathematical operation, and because they are 'symbolically written' (Freudenthal, 1999, pg. 432). The activities in the learning negative numbers are design in consideration of five tenets of RME. These are the description of the tenets and how they are applied in the aspects of the design.

1. The use of context in phenomenological exploration

While participating in instructional activities, the students are expected to expand their mathematical reality. (Gravemeijer, 2002). In an instructional sequence, students may explore the informal situation by relating it with their existing knowledge. In the game context, they find themselves that negative numbers can be explained and explored by engaging in such game.

2. Emergent modeling

Another key idea in RME is mathematizing which can also mean as 'make more mathematical' (Gravemeijer, 2000, pg.781). Starting with the contextual problem which is experientially real for them, they transform their thinking into a mathematical model involving symbol and operation. In the instructional activities on negative numbers, students interpret the concept of a negative number in the game context and represent on the number line as a mathematical model. There are four levels of emergent modeling (Gravemeijer, 2002, pg. 2; see also Gravemeijer,1994). This is a description how the instructional sequence involves these levels of modeling

- a. Activity in the task setting, students may interpret the concept of negative numbers in the learning situation they engage. For example, by playing the game, they can state that negative numbers appeared when they have debts to someone because they do not have enough money to buy things (situational level)
- b. Referential activity, the students attempt to model the situation by using their own interpretation and visualization. They may model the situation of having debts by making notes and using the play money.
- c. General activity, students start using the number line as a mathematical tool. The transformation of their models into the number line model is the process to a formal level.
- d. Formal mathematical reasoning, the students who are in this level will no longer depend on the model-for because they manage to work on the numbers and symbolic operation formally.

3. The use of students' contribution

The instructional activities ask the students to play the game as well as to facilitate themselves in discussion. All activities invite them to participate in

doing the investigation about negative numbers in the context and the operation involved.

4. Interactivity

The interactive instruction of design guide the students to cooperate to do the range of tasks, share and discuss the essential concepts, rules, and principle in learning negative numbers.

5. The intertwining of various learning strands or units

Within the context using a board game, various learning could be intertwined to improve the student's understanding. It also gives the possibility to expand the game to learn any other essential mathematical topic.

F. The topic of negative numbers in Indonesian curriculum

In Indonesian curriculum, the topic of negative numbers included in the chapter of integers are taught in the second semester, in fourth grade. For several reasons, the lesson is conducted in third grade to know whether to see how students in lower grade response to the designed activities. Based on the mathematics textbooks used in the classroom (BSE), there are four main topics students should understand about negative numbers. They are (1) Introduction to integers; (2) addition of integers; (3) subtraction of integers; (4) mixed operation (addition and subtraction) of integers. However, in this study, I will not focus to all those core elements of learning negative numbers. I intend to make the learning less formal because at some points, for examples subtraction of negative numbers, which involve two minus sign, (one for operation; one for numbers sign) are presented formally in the mathematical textbook. Because that concept

has no relation to the context of my study, it will be difficult to understand by the students. I limit my study not to elaborate the rules of sign in subtraction of two negative numbers. Therefore, my expectation is that, with the help of games context, students can understand negative numbers by describing it within the context.

G. Mathematical Classroom Practice and Social Mathematical Norms

Mathematical classroom practice and social mathematical norms will be discussed in this section because in conducting the study, the observation should not be limited in the individual learner development and the classroom discourse (Kilhamn, pg. 127). The other crucial factor, which may influence the activities, is the setting in which the teaching and learning process takes place. These are the following activities might happen in the classroom:

1. Whole class discussion; whole class discussion mostly be started by the teacher by explaining the core of learning, by asking some questions to students, and posing some problem which may stimulate the students to participate. The discussion usually happens in the beginning of the lesson or after finishing the students' group work which need to be discussed in a larger group. The questions from a student can also be a whole discussion. If the situation happens, students may ask someone else to answer before she/he comes to the final answer. Teacher has a role to give decision on how long the discussion and she/he also has a right to make a final decision. However, the students also can argue or make other statements concerning teacher's explanation.

- 2. Individual work; individual work usually happen at the end of the discussion to make an individual assessment of students. The students sit in their own seat and do some task quietly. During this activity, students are allowed to ask some questions by raising their hands or some to teacher's desk, but not asking to their friends. In this activity, students are expected to do the tasks by themselves to measure their knowledge after the lesson. Teacher may circle around the classroom to see directly what students are doing or help them if they have something to ask, but they are afraid to go to teacher's desk to ask or feel shy to raise their hands.
- 3. Working in pairs; working in pairs mostly takes less time that the workgroup does because students have many things to discuss in pair, some may ask questions and some others may response their friends' questions. In pair, they usually do the activity with their desk mates. The activity, which is done in pairs, could be a small activity, for example, in discussing questions from the teacher.
- 4. Work in a group; the bigger assignment is done in a group of 3, 4, 5 people. The students may choose by themselves the member of their group or for some reason, the teacher may do that, for instance because the teacher wants the students to work with people they never interact with or because the teacher want to have the mixed group in which there are students with different ability. In group work, the teacher will circle around the classroom to monitor and observe the discussion of the students, and how they behave to each other, whether ones is too dominant

than other or too silent. The teacher might find the solution if there is a student has not participated yet by giving her/his chance to speak, and everybody in her/his group must stop talking. In this study. In this study, the workgroup may take place during the students play the games about integers or when the students do the worksheet.

5. Non-mathematical activities; beside the mathematical activities, there are some non mathematical activities such as the teacher may give some information about homework, test, the lesson for next meeting, or other practical things relating to classroom.

Social mathematical norms relate to the judgments on the students' explanation whether it is acceptable mathematically. Cobb (1996, pg. 461) introduced social mathematical norms as an acceptable mathematical explanation and justification including what counts as mathematical difference and mathematical sophistication. Differences of student's opinion, solution, and argumentation may happen in whole class discussion or group work. By sharing as much as possible answers they can discuss, they will learn and notice which of those is the most acceptable as the sophisticated solution, opinion, or argumentation. For example in this study, In learning negative numbers, students may share their interpretation and finally come to the judgments whether which of those interpretation or meaning of negative numbers could be more acceptable as sophisticated explanation.
CHAPTER III

RESEARCH METHODS

The chapter is about the method used in the study include the description of the study approach, data collection, and data analysis.

A. Research approach

The purpose of this study is to improve mathematics education specifically in supporting students' conceptual understanding of addition involving negative numbers in Indonesian primary school. Thus, this present study uses design research. Design research is chosen as the research approach because an intervention should be conducted in mathematics education by designing a learning sequence in understanding negative numbers, which are based on Realistic Mathematics Education. Through this study, it is expected that to give a contribution in developing teaching materials for mathematics teachers in supporting students in learning the concept.

B. Data collection

1. Preparation phase

Preparation phase includes classroom observation and interview with the teacher. All these data collections are conducted as the preparation before conducting the classroom teaching experiment. The purpose of preparing these two types of data is to get insight and clarify about the classroom situation, and how the *sociomathematical norms* happen during the lesson. The interview with

the teacher will support the classroom observation meaning that some information obtained from the classroom observation need clarification from the teacher.

a. Classroom observation

Conducting a classroom observation is aimed at getting information about the classroom, the setting of the classroom, social norms, the interaction between the teacher and the students. The focus of the observation is not only the practical things, but also the situation which can help the researcher to get a prediction about what will happen in the classroom. Before conducting the classroom observation, the researcher makes a list which includes all relevant points that should be observed by the researcher.

During this observation, some data which is needed to be collected are the participation of the students, how the teacher handles the classroom discussion, setting of the classroom, the interaction among students during the discussion, what they discuss, and how much the contribution of the teacher in guiding the students during the lesson or discussion. The purpose of knowing this situation that the researcher can relate the data to HLT to check whether the HLT include the particular behavior or reaction of the students. Also, the classroom observation is useful for the readiness of students as well as the researcher because it is an introduction before doing the experiment. The observation is conducted by making a video registration of the lesson. The researcher is also assisted by some other colleagues to make some notes about the lesson.

b. Interview with the teacher

An interview with the teacher is conducted after conducting the classroom observation. The purpose of doing the interview with the teacher is to get information and clarification of what the teacher did during the observation because probably, there some points that need to be explained by the teacher. The interview with the teacher is also conducted to talk about the lesson with the teacher, to communicate how the lesson be implemented in the classroom. The Hypothetical Learning Trajectory as well as the teacher guide is discussed to the teacher. The researcher may explain briefly about the expectation of teaching and learning process during the lesson to the teacher. However, the teacher is the one who know the characteristics and the abilities of the students better than the researcher do that she/he can adjust the HLT to the classroom. It is necessary for the researcher to keep the respectful relationship with the teachers because the improvement of the teaching and learning process needs cooperation between both of the researcher and teacher. Besides, specifically in teaching negative numbers, the interview also useful to know how the teacher interpret negative numbers and how she will introduce it to the students. Teacher's experience is helpful for the researcher to as the preparation before the researcher conducts the experiment in the classroom.

To sum up, some data collected during this phase such as information about teacher's personal experience in teaching concerning some difficulties and challenge in conducting discussion among students and other aspects of socio norms and socio mathematical norms. Also, in this phase, some clarification about classroom observation is required to be asked to the teacher to enrich the description of certain situation that have been observed.

2. Pre-test

The pre-tests are conducted in two cycles: (1) the first cycle: preliminary teaching experiment and (2) the second cycle, classroom experiment. the purpose of doing the pre-test in the first cycle is not only for checking the starting points of the students, but also to make sure whether or not the students can understand the questions. In the second cycle, the purpose of the pre-test is to know the starting point of the students about negative numbers. The researcher will give the pre test for students; although the students have not learned about negative numbers, the researcher assumes that they have an informal knowledge about the concept. In the second cycle,. The participants of this test are 15 students of fourth grade. The outcome of this test will give the first impression about the performance of the students before the lesson.

By seeing the result of written test, the students' prior knowledge about negative numbers and addition concept is investigated.

3. Preliminary teaching experiment

The preliminary teaching experiment is the first cycle of testing the HLT. The participant of this experiment is the small group of the students consists of at least five students. In this phase, the researcher acts as the teacher. The outcome of this phase is the written work of the students, the discussions, argumentations and reasoning of the students in interpreting negative numbers, and what difficulties students might have concerning to the activities.

Some data collected during this phase are in this following description:

a. Written test

Students' work related to the activities are collected to get information about the students' knowledge after the learning process. Written test is important document to support the whole data contributing to the restrospective analysis.

b. Video recording

Video recording of students' learning process during the lesson also play important role. It is required to collect some interesting discussion among students. The transcripts are made to investigate the situation clearly. Some students' strategies and responses are also collected by this method.

c. Observation

Observation method collect some situation about students' strategies, the teacher activities, the role of the teacher in opening discussion, and how the students give response toward the teacher's instruction. Some field notes are made from the observation to support other method of collecting data.

d. Interview with the students

Interview helps in collecting information about student's thinking process. For example, while students do the problem, sometimes they explain something which is not quite clear. Therefore, it is needed to ask them some follow up questions. Also, interview can collect data about the students' clarification concerning to the situation which has been observed previously. It relates to some strategies they use to solve the problem or some important discussion among them.

4. Teaching experiment

In experimenting the next cycle(s), the participant is different from those in the previous cycle. In this phase, the researcher tests the HLT which has been designed in the preparation phase. The purpose of doing the cycle is to test the HLT and the conceptual understanding of the students about negative numbers. During the experiment(s), the researcher pays attention to one group of the students as the focus of the observation. The focus group consists of the students which have been selected by the teacher. In selecting the students for the focus group, the teacher considers making group of the students who can cooperate. The purpose of doing this is that to get deep information about the students. Besides, in this cycle, the researcher can so as much as interaction with the students to know their thinking. The classroom experiment (s) can be recorded in considering that the existence of a video camera will not influence the students' natural behavior. A researcher's role is as a participant observer who assists the teacher in interacting with the students. While doing the observation, the researcher can make some notes about what kind of activity, what the teacher talks about in the lesson, who is participating in the lesson, and what is written in the whiteboard. The researcher will not only focus the observation in the focus group, but also the researcher walks around the classroom to talk with some individual students.

The data collection method in this cycle(s) are the individual written test, the written group work, the recorded observation including the discussion among the students the reaction from the teacher, and some notes about important things such as some unexpected situation, obstacle in managing the classroom, and so on. To sum up, the role of the researcher in this phase is as the observer of some group of the students in following the activities. The outcome of this cycle is a learning trajectory that has been revised which has been the end product of the learning. This cycle can be repeated until it reach some requirements. The requirements of obtaining it caused by some situation: (1) Learning goals of each lesson are already achieved, (2) The similarity condition found between Hypothetical Learning Trajectory and actual learning trajectory in teaching experiment(s). Briefly explained, some data collected during this phase is quite similar to those in preliminary teaching experiment. They are in this following description:

a. Written test

Students' work related to the activities are collected to get information about the students' knowledge after the learning process. Written test is important document to support the whole data contributing to the retrospective analysis.

b. Video recording

Video recording of students' learning process during the lesson also play important role. It is required to collect some interesting discussion among students. The transcripts are made to investigate the situation clearly. Some students' strategies and responses are also collected by this method.

c. Observation

Observation method collect some situation about students' strategies, the teacher activities, the role of the teacher in opening discussion, and how the students give response toward the teacher's instruction. Some field notes are made from the observation to support other method of collecting data.

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Interview helps in collecting information about student's thinking process. For example, while students do the problem, sometimes they explain something which is not quite clear. Therefore, it is needed to ask them some follow up questions. Also, interview can collect data about the students' clarification concerning to the situation which has been observed previously. It relates to some strategies they use to solve the problem or some important discussion among them.

5. Post test

After conducting all the learning sequences, the researcher gives the posttest to the students which are involved in classroom experiment. The post tests are conducted in the first cycle (preliminary teaching experiment) and the second cycle (teaching experiment). The purpose of conducting this test is to get information whether or not the students make a progress after experiencing the sequence of activities in classroom experiment. Also, the post test is helpful to know if the students use the model to solve the problem and how the context used in the activities can support their understanding. The outcome of this test is the written test of the students which will be analyzed and compared to the pre-test of the previous phase.

C. Validity and Reliability

To improve the internal validity of the study, the researcher will triangulate three types of methods in collecting data during the phases of this study such as tests, the classroom observation and interview with the teacher. The researcher expects that with the triangulation of the methods of data collection, the conclusion could be strengthened. To increase the external validity, the researcher presents all the data collection transparently to make sure that the readers can follow how to adjust their own hypothetical learning trajectory to the context of the study.

To increase the internal reliability of the study, the researcher has already discussed the instrument of the study with the expert to minimize the subjectivity of the researcher. The instrument is in the form of written test of the students, both for the pre test and the post test. Therefore, it will provide more information about the data to expect the reader can have the same interpretation and conclusion. Besides, the researcher have elaborated the learning process step by step as well as the important parts of the lesson during the classroom experiment to increase the external reliability (*trackability*) to help ones who replicate this study can have the same conclusion.

The Outline of data collection					
Phase	Activities	Participants	Data collection method	Data which is needed to be collected	
Pre test		All studens in preliminary and teaching experiment	written test	students' prior knowledge	
Preliminary teaching experiment	Students play a game with money context (having- owing) and how they investigate negative numbers in representing amount of debt.	6 third- grader students from SDIT Ghilmani Surabaya	-observation -video record	students' discussion; students' learning process some responses of the students about how they explain negative numbers for the first time; students' activity during the lesson students' thinking from some interesting situation and ask some students	
	Students visualize the problem of having debt or		-observation	students' knowledge about number line; students' activity	
	having money on the number line.		-video record	students' strategies in solving problem using number line; students' discussion and responses	
			-interview	students' understanding about debt or if they have experience having debt; students' understanding about debt	
	Students locate numbers in correct order on the number line.		-observation	students' strategies in connecting the random number on the number line	
1			-video record	students' discussion about	

Table 3. 1 The Outline of Data Collection in

				-interview	the position of numbers and compare those numbers ask some additional questions about two numbers and how the
					students can compare them
	Students play a game which shows the			-observation	students' strategies on how they make movement along the number line
	adding integers.			-video record	students' discussion in which students discuss the result of two different numbers and make jumps in different direction
				-interview	students' understanding in interpreting the game
	Students do some			-observation	
	related to context of the game.			-video record	students' idea of the problem ; some discussion
				-interview	students' difficulties and strategies
	Students make their own number line to visualize			-observation	students' way to draw the number line
	to jumps.			-video record	some discussion about making jumps; and if they make jumps in correct direction when they pay or accept money
				-interview	some students' clarification about the answers they give
	Students exercise to do some			-observation	students' strategies in writing numbers of addition
	problems related to the previous problem and wrote			-video record	some interesting discussion to see what they can get from the activities
	the addition in more formal way.			-interview	students's way in concluding the lesson
	Refine initi	al HLT			
Teaching	Activities	10 +	hird		
experiment(s)	1 1011 11105	grader	iniu-		
		~			

		-	1	
		students		
		from SDIT		
		Ghilmani		
		Surabaya		
	Students play a		-observation	students' discussion;
	game through			students' learning process
	money context			
	(having-owing)		-video record	some responses of the
	and investigate			students about how they
	negative			explain negative numbers
	numbers in			for the first time, students
	representing			activity during the lesson
	amount of debt.		intorviou	students' thinking from
			-Interview	some interesting situation
				and ask some students
				personally
	Students		-observation	students' knowledge about
	visualize the			number line: students'
	problem of			activity
	having debt or			5
	having wonay		-video record	students' strategies in
	naving money			solving problem using
	on the number			number line; students'
	line.			discussion and responses
			-interview	students' understanding
				about debt or if they have
				experience having debt;
				students' understanding
				about debt
	Students locate		-observation)	students' strategies in
	numbers in			connecting the random
	correct order on			number on the number line
	the number line.		video record	students' discussion about
				the position of numbers and
				compare those numbers
			-interview	ask some additional
				questions about two
				numbers and how the
				students can compare them
	Students play a		-observation	students' strategies on how
	game which			they make movement along
	shows the			the number line
	simulation of			
	adding integers.		-video record	students discussion in
				which students discuss the
1			1	result of two different

			-interview	numbers and make jumps in different direction students' understanding in interpreting the game
	Students make their own number line to		-observation	students' strategies in drawing the number line
	problems related to jumps.		-video record	some students' discussion about making jumps; and if they make jumps in correct direction when they pay or
			-interview	students' clarification about the answers they give
	Students exercise to do some problems		-observation	students' strategies in writing numbers of addition process
	previous problem and wrote the		-video record	interesting discussion to see what they can get from the activities
	addition in more formal way.		-interview	students' way of concluding the lesson
Post test		All studens in preliminary and teaching experiment	written test	students'learningprogress,students'developmentinunderstandingtheconceptin

D. Data analysis

1. Pre test

The result of pre-test is analyzed to get some information about the starting point of the students. The considerations in analyzing the result of the pre-test are about the students' interpretation about the problem whether they could reason the problem using their existing knowledge and some strategies that used by the students in solving problems in the pre-test.

2. Preliminary teaching experiment

The result of the pre-test in the preliminary teaching experiment will be analyzed by checking the correct written answers of the students to know the starting points of the students and to test whether or not they understand what being asked in the test. Another type of data collected in this cycle is the result of classroom observation and some field notes about some noteworthy things happen during the lesson. Also, the video registration of the lesson will be analyzed by selecting some interesting video fragment about students' discussions or students' performance. The researcher compares what happened during the classroom to the expectation of the teaching and learning process in Hypothetical Learning Trajectory. The purpose of analyzing the data of the whole lesson is to test the conjectures of students' responses to the actual learning process. The transcript of the selected fragments will be used to test the conjectures of the HLT. The researcher will use the analyzed data to revise the original HLT for the teaching classroom experiment. The results of the post-test are analyzed by checking whether the students make progress after the lesson. This can be seen in the written work considering the strategies and the model used in getting the answers of the problems.

3. Teaching experiment

The result of the classroom observation and interview with the teacher will be analyzed to get information about the situation of the classroom. The analysis of the pre-test in the second cycle includes a quantitative analysis and a qualitative analysis. A quantitative analysis relates to the numbers of correct answers made by the students; meanwhile a qualitative analysis relates to the strategies used by the students to solve the problems on the test. The data analysis in the teaching experiment is quite similar to those in the first cycle. The classroom observation, the video registration, and the field notes are analyzed to see whether they fit to the conjectures in Hypothetical learning trajectory. After analyzing the video lesson of the whole class, the analysis emphasize on the focus group of students which are selected by the researcher. The focus group will be the representation of the whole class. The researcher will analyze the discussion among them, the written worksheet of group work, and interesting students' argument might appear during the lesson. Some interesting fragments of video lesson will be analyzed to compare the prediction of teaching and learning process and the actual teaching and learning process. The analysis of these data will contribute in producing conclusion of the study whether or not the research questions answered based on the data and to revise the original hypothetical learning trajectory.

4. Post-test

The post-test is analyzed by checking all students' answer and see the change or progress they made compared to the pre-test. The analysis is done by considering that students make progress of their conceptual understanding after the lesson. Besides, the analysis pay attention to their work whether or not they used the model to solve the problems in the test. The data analysis from the posttest also contributes in answering the research questions.

E. Validity and Reliability of Data Analysis

The internal validity of the data analysis can be improved by triangulating three types of data such as; the written test of the pre test and post test, the result of classroom observation, and the result of interview data. To increase the external validity, the researcher presents the description of analyzed data transparently that the readers can use the information to test their own hypothetical learning trajectory to the context of the study. To improve the internal reliability, the researcher discussed some essential parts of the lesson with supervisors and colleagues to minimize the subjectivity of the researcher in interpreting the situation during the lesson. For instance, the video fragment of interesting scene in the lesson will be discussed together with supervisors or colleagues to get the same interpretation. To improve the external reliability, the researcher make a video of the lesson that the reader can follow the teaching and learning process.

F. Research Subject and Timeline of the Research

The research subject of this study is the students from the third grade aged 8-9 years old. In the preliminary teaching experiment 6 students from the third grade classroom which are all boys from SDIT Ghilmani Surabaya were involved . The preliminary teaching experiment is aimed at investigating the responses of the students to the designed tasks and to test the initial HLT. The real teaching experiment involves 19 students from the girls classroom of third grade in SDIT Ghilmani Surabaya. The purpose of real teaching experiment is to test the refined HLT and to observe the students' learning development. The students involve in real teaching experiment were not the students involved in the preliminary teaching experiment. This following table is the timeline of the research.

Table 3. 1. Timeline of the research

Date	Activities	Data Collection	
October- January 2012	Studying literatures and designing initial HLT		
February 2013	Discussion with the teacher	Interviewing	
	Classroom observation	Video recording	
	Pre-Assessment	students' work	
18 February 2013	Lesson 1 (The "Hop Shop" game)	Video recording and students' work	
20 February 2013	Lesson 2 (How much money/debts will I have? (drawing jumps on the number line) Lesson 3 (Ordering	Video recording and students' work	
21 February 2013	Lesson 5 ("What is your position now?")	Video recording and students' work	
22 February 2013	Lesson 6 (Empty number line)	Video recording and students' work	
February-March 2013	Redesigning the HLT		
1 April 2013	Lesson 1(The "Hop Shop" game)	Video recording and students' work	

5April 2013	Lesson 2 (Drawing jumps on the number line) Mini lesson (ordering integers)	Video recording and students' work
8 April 2013	Lesson 3 (The Number Line Game)	Video recording and students' work
10 April 2013	Lesson 4 ("What is your position now?")	Video recording and students' work
12 April 2013	Lesson 5(Empty number line)	Video recording and students' work
12 April 2013	End-Assessment	Video recording and students' work

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Gravemeijer (2004) revealed that Hypothetical Learning Trajectory (HLT) is the prediction on how students may react to some instructional activities. Teacher should investigate that response of the students fit on the conjectures or the she has to do some adjustment of the conjectured to some findings during the classroom session. HLT can also be guideline for the teacher to keep the track about what students might respond. To improve the HLT, it requires some activities that can support the HLT and enable the researcher to make some revision on that. In the description below, there will be seven activities. They are designed in considering some theory about learning negative numbers. The activities which have been designed as the tool to support the HLT are: (1) the game using money context in which students reinvent the concept of negative number, (2) number line activity with the use of money context where payingaccepting money is modeled by jumping along the number line, (3) locating negative and positive numbers on the number line, (4) & (5) Another game involving number line model in which students are introduced about addition of integers, (6) determining the final position of one's movement along the number line, (7) using empty number line and learn how to write addition formally.

There are some theories underpinning this sequence that support the whole learning process. The theory has been the consideration in designing the order of each activities. It is hoped that this sequence can help students better understand the concept of addition involving negative numbers. To describe the first activity, Mc Gullick, et. al (2012) explained that it is difficult for students to accept negative numbers because it requires an acceptance of quantities worth less than zero; it is easy to speak that "I have two books" but it is not easy to say "I have negative two books". With this elaboration, it is needed to start the learning with created situation in which students can speak about the negativity of certain object for example, using money context. Under the circumstances, students can learn how to make representation of some object. Kilhamn revealed that the simplest form of representation is the identification of a concrete object, an attribute, an activity etc. by a name, a word or a sign. It is considered that the students could come up with giving sign to some object to represent. The activity aims at giving opportunity to students to achieve this stage where they can explain by themselves about some object, although it relates to some object in "negative" sense which cannot be modeled physically.

To support some number line activities, Kilhamn revealed about arithmetic as motion along path; addition as well as subtraction can be seen as motions of this path. The path here means number line. Dealing with number line, it is also describe some representation of the component of number line to state some situation which can be seen in this following figure.

No movement	Zero
A centre location or starting point	Zero
A location along the path where you start or end up after having moved	A number.
A movement	A number
The length of a movement (distance)	Magnitude ²³ of a number
Further to the right or up is higher value	Value of a number
Moving to the right or up (or forward)	Addition
Moving to the left or down (or backward)	Subtraction

Figure 4.1 The Representation of Component of Number line in some

Situated-problem.

The picture above bring some ideas to create some number line activities in the sequence to help students to learn addition using motion along number line. Also, he says that number processing can be reflected in the form of number line, because the student can compare numbers by seeing the distance. To fulfill this, it is offered some number line activities in which the students can compare number as well as locate number in appropriate order. Afterwards, it is expected that the students can improve their understanding through these deep processes before they get ready for the next steps; the formal addition and subtraction.

HLT will be described in each lesson which consists of the learning goals of the lesson, the description about instructional activities and the conjectures on how students may react to the lessons.

A. Lesson 1: The "Hop Shop" game

1. Learning Goal:

Students improve their understanding about negative numbers

2. Description of the activity;

The conceptual understanding of students about negative numbers is influenced by their previous knowledge about natural numbers. Students may manage to compute negative numbers as they do in natural numbers. The main focus of doing this activity is that to make the students aware of the existence of negative numbers in their lives and can make them reason why a big negative numbers has smaller value than small positive numbers.

In the board game, there will be the pictures of favorite things children may like. Such as toys, dolls, cute stationaries, etc. There will the label of the price of each item. In the boxes of the board games, there will be chance for them to add their money by getting the shopping voucher or get bonuses from the bank or the shopkeeper. All players will get \$15 from the bank. One dice is used to determine the movement of each player. Play money is used for transaction. By using dice, they have to move to know where they will land the counter; afterwards they have to buy the items in the box in their last stop.

One of four students will be the bank unit to manage the money, lend the money to those who have insufficient money. Before all the shoppers start playing the games, the bank gives them the same amount of money for shopping. The one who cannot manage his money properly will lose the game. The bank will record all the transaction of the shoppers.

3. The conjectures of students' thinking:

a. All the students will move the counter based on the number on the dice.

- b. The shopkeeper might write on the transaction sheet that the all the players will get \$15 for shopping.
- c. If students' counters land in a space of an item, they have to pay the amount of money to the shopkeeper.
- d. The students will realize that their money is spent more and more because they have to pay to the shopkeeper if the counter land on the space of an item.
- e. The shopkeeper will record the amount of money the players might have after the transaction.
- f. After several turns, the students might have no more money in their hands. They might say "Now, I cannot pay for the item because I have no more money", then the students who act as the shopkeeper might say "I owe you money, then I have to write that you now have negative (the amount of debts) here"
- g. The shopkeeper will record the amount of money/debts the players have after the transaction.
- h. If a student's counter land on the space of soccer t-shirt costing \$10; meanwhile, his money is only 4, then the shopkeeper should tell to him "Now you do not have enough money to pay, you only have 4, then I owe you \$6", "I write here -6, your debts now is \$6"
- i. Another important situation might happen is that when the students pass the starting point and get \$3, the student playing as the bank might say "This is your \$3" and give the play money to each of the player.

- j. For example, the students already has a debt of \$10 and he pass the starting point, he might think that he get more money to pay the debt and says, "I have less debt now, because I have more money", or "I still have a debt of \$7 to the shopkeeper", or "I have minus 7"
- k. The shopkeeper could declare the situation by saying that "You have paid\$3, you still have a debt of \$7 to me", or "I wrote down -7 here"
- 4. Discussion:

During the game, the students will have discussion about owing-having money. The students will talk about why should negative numbers used in the game and how they could say about it. The teacher should assist them in finding the meaning of negative numbers in the context. For example, they could say -5 as the amount of money they borrow from the shopkeeper. When they only have \$7, but they have to pay \$9, then there will be an important discussion about how could they write and say about it. One could say "you need \$2 more". "you have debt of \$2 to the shopkeeper" or one could emphasize that you should put minus sign in front of the number of debt.

The more they play the game, the more they understand the negative numbers as well as how to compute negative numbers. Bringing the concept of negative numbers in the context of a game could be very meaningful for students. The role of the teacher while students play the game is to monitor and encourage some important discussion. The teacher could say "if you have debts, and you should buy more items, then what will happen?" or "if you have debt of \$5 with you and you get paid by the bank \$3, what happen with your debt?" This guidance questions could help the students in understanding negative numbers in better way through the context of the game.

B. Lesson 2: How much money/debts will I have? (drawing jumps on the number line)

1. Learning goal:

Students can draw jumps on number line to solve problems relating to the context in the game.

2. Description of the activity:

In this activity, the students will be given a problem card. The problem is the situation which is taken from the situation in the game context. For example "I have \$6, I want to buy a school bag which costs \$8". The group will discuss how they can write the situation on the number line by making jumps and figure out how much money/debts someone has after paying the money. The purpose of doing this activity is that in order to know how the students can do the calculation of negative numbers, whether or not they understand negative numbers through the board game. They will do this activity in a group of four students, each group will get 5 cards, they will discuss the problems on the cards together with their groups. Afterwards, one group will exchange their cards to other groups and draw jumps again in another worksheet. Afterwards, one group will present their work; they check their answer and explain how they can get the answer. Their explanation should relate to the context of the board game.

- 3. The conjectures of students' thinking:
 - a. The students will read the problem, for example "I have \$6, I want to buy a school bag which costs \$8. The students know that 6 is the first position on the number line, because they have to pay 8, so they make 8 jumps to the left. The students realize that this situation will give them a negative numbers, which is -2 as a result. Some students also know that they have to subtract a bigger number from a smaller number. They may think that they have insufficient money. Therefore, they owe \$2 from the shopkeeper. Students will be able to say that -2 has the same meaning as someone owe money \$2 from the shopkeeper because they know that debts should be written in negative numbers.
 - b. The other problem which may lead the discussion among the students is the problem: "Now I have\$ -12, but I have just passed the starting point to get \$3". The students may think that he already has debts to the shopkeeper but now he gets more money. They will draw the jumps to the right on the number line because they get money from the bank. Some students probably know that they will have less debt because they have money. The students may answer that the debts now becomes -9. Some students will subtract the debts \$12 by \$3. The discussion will be about the operation of two numbers; the negative number and the positive number and how can they draw the operation on the number line.

- c. The problem on the card is "I have \$ -5, I have to buy a cupcake which costs \$5". Given this problem, the students will think that they already have debts to the shopkeeper but they have to pay for another item. The students will draw the jumps to the left and realize that they have more debt to the shopkeeper, which is \$10.
- 4.. Discussion:

In this activity, probably it is the first time for the students to use a number line as calculating tool. Therefore, the teacher could explain briefly to the students why they should use a number line and what is the usefulness of using number line to calculate number and understand the problem of the context. Also, the teacher could explain why the students should make jumps on the number line. Students could investigate by themselves about the condition when they borrow money from the shopkeeper; they have to move to the left, which moves to the negative numbers because they subtract money. And they could also reason that they move to the right, the jumps getting closer to positive numbers when they get more money from the bank. Teacher could facilitate the discussion during the students do the worksheet together in groups.

C. Lesson 3: Ordering Integers

1. Learning goal:

Students understand how to order negative numbers

2. Description of the activity:

In this activity, students are asked to relate the experience they have after playing the game in the previous meeting. The task is about comparing and ordering numbers. There will be some numbers; positive and negative numbers. They are asked to arrange the number in a good order to determine who will be the winner, the runner up1, the runner up 2, and the loser. In pairs, they will discuss how they decide that a number is more/less than another numbers.

- 3. The conjectures of students' thinking:
 - a. Student might reason why bigger negative numbers less than smaller positive number. They might think that the bigger negative number, the smaller value it has.
 - b. In the first task, they may put biggest positive number in the position of the winner, the second biggest in the runner up, the third bigger number in the second runner-up position, and the smallest negative numbers in the position of the loser.
 - c. In the second task, students will response the task by connecting the number to the correct position. They may put the biggest positive numbers to the last right position from zero. For example, because the number 20 is already connected to the line as an example, they may find that 15 is the biggest positive number which should be put in exactly the left side of 20.

Figure 4.1 Number line activities



- d. Students may put the positive numbers in the right side of zero and negative numbers in the left side of zero
- e. Students may have discussion why 5 are more than -6, and why -1 is more than -13.

4. Discussion:

Doing this activity, students may confuse why small positive number is greater than big negative numbers. Teacher could explain this to the students by relating it to the situation in the context. For example, explaining to the students that in the previous activity, in the owing-having context, teacher could say "we use negative numbers to say that we have no money and we borrow money from someone else" or remind the student about the game the played "when you have debt to the shopkeeper, it means that you have no more money, that's why you use negative numbers". Teacher can also say that "if you have \$7, then you have money, you could say it with positive numbers". Teacher could emphasize that negative numbers is used to say something less than zero—less than nothing. In the second task, students learn about positioning numbers. They may put negative numbers in the left side of zero and positive number in the right side of zero. Teacher makes sure that students know that -3 is much closer to zero than -13.

D. Lesson 4: The Number Line Game (Part 1)

1. Learning Goal:

Students get familiar with a number line as a tool for calculating numbers

2. Description of the activity:

The number line game is the game involving negative numbers in which the students will learn how to do simple operation of negative numbers based on the movement along the number line. The movement on the number lines is determined by the number of the dice. This game will use two-colored dice: a blue dice and a red dice. The teacher will present some rules for this game. The blue dice will represent positive direction which means that the students must move to the right. And the red dice will represent negative direction which means that students will move to the left. In this activity, students will play in one group, they students will toss the two dice the same time, the blue dice show the students the number of boxes they move to the right, and the red dice show the number boxes in the direction to the left . The number of boxes they must jump is according to the number which is on the dice. They student must always record each movement by writing down the number on the paper. Note that in writing the number in the red dice (negative direction), they should always put minus sign before the number.

The students will write down each movement in this following table:

The number on the blue	The number on the red	Your position now
dice	dice	
5	-6	-1

Table 4. 1 Table for number line activities

Note that in writing the number in the red dice (negative direction), they should always put minus sign before the number.

- 3. The conjectures of students' thinking:
 - a. Students might think that the more they jump to the left, they will get to negative numbers, the more they jump to the right, and they will get to positive numbers. In comparing numbers, they might think that the more they go to the left, they will have smaller number.
 - b. The student may write the numbers on the red dice as negative numbers they put minus sign in front of the numbers , because they have to move to the left, get closer to the negative numbers.
 - c. The student may write the numbers on the blue dice as positive numbers put plus sign in front of the numbers or just let the numbers without any sign.
- 4. Discussion:

This activity could be another form of number line activity which previously done in the lesson 2. The difference is that in this game, students try to treat the number more formally, not as the numbers relating to the context. Here, the students should think about the operation of the number formally. Teacher can also ask the students to think about how they can translate the number in mathematical expression and ask them what kind operation they should use in playing this game. Later, after having some discussion with the students about the operation, teacher may explain that in the context of number line, the addition relates to the sum of the two movements; to the right and to the left. Also, students can say that it is the case of subtraction because it shows the difference of two numbers; the number on the red dice and the number on the blue dice.

E. Lesson 5: The Number Line (part 2)

1. Learning goal

Student can solve missing value problem after experiencing the number line game.

The number on the blue dice	The number on the red dice	Your position now
5	-6	-1
6		2
	-6	-3
4	-5	
5		2
3	-3	

Table 4. 2 The missing value problem

	-3	-1
1		0
4	-6	
	-6	-3

2. Description of the activity:

After students experience the number line game, they will be asked to figure out the appropriate number to complete the table.

- 3. The conjectures of students' thinking:
- a. Students might think that if the number of jumps to the right is 6, and the position is -2, then the students understand that the number of the jumps to the left should be 8, written as -8 because if they move to the right 6 to the left, then they have to move 8 jumps to the left to reach -2.
- b. Students might think that if the number of the jumps to the left is -6, and the position is -3, then they have to think that the number of the jumps to the right should be 3, because to get to 3, they need to jumps 3 to the right from -6.
- c. Students could get the pattern and conclude that if they have more jumps to the right that those to the left, then the position could be in the right of zero, which is positive numbers. If they have more jumps to the left than those to the right, then the position could be in the left of zero, which is negative numbers.

4. Discussion:

This task could be more challenging for students because they have to solve missing value problem. For example, by looking at the blank of the table, students can clarify the problem and say "the number of the jumps to the right is 6 and, and the final position is 2, how many jumps to the left needed?". The teacher could ask them to make their own number line to figure out the problem or use the number line game to make jumps. The students could say that they need 4 jumps to the left to reach 2. Other discussion is that students can conclude that the more they move to the right, the bigger the possibility to get the final position in positive numbers and the more the move to the left, the bigger the possibility to get the final position in negative numbers. The teacher has to remind the students that the number on the red dice should always be written with the minus sign in front of the numbers.

F. Lesson 6: "What is your position now?"

1. Learning Goal:

Students can draw their own number line and operate negative numbers using that number line.

2. Description of the activity:

The students will do the activity in pair, one student will get 5 cards of the problem. The students are asked to determine the position after the jump. They can use the number line board in the number line game to help them to count the position.

Figure 4.2 Problems of determining the final position after jumping along the number line

I am in the position 3

I made jump 5 to the left

In what position am I now?

I am in the position -8

I made jump 3 to the right

In what position am I now?

- 3. The conjecture of students' thinking :
 - a. Students in pair may use the number line of the number line game to determine the position. They may find that if they jump to the left, they may get negative numbers. If they jump to the right, they may get positive numbers. They also discuss how the initial position in positive number can be negative numbers; also, the initial position in negative numbers can be positive if they move to the right.
 - b. Later, after they try the cards using the number line. They may try the cards without using the number line boards. The teacher will provide the students with a number line to visualize the number of the jumps they could make and the result they may get after the jumps.

This is the possible response of students:

I am in the position 3 I made jump 5 to the left In what position am I now?
And they could make a number line of this problem:



Figure 4.3 Possible students' answer in using number line The number line is the drawing of the problem which shows the position 3 and 5 jumps to the left to go to -2.

4. Discussion:

In doing this activity, students are expected to make their own number line and try to draw the process of calculating on that number line. The teacher observes the discussion among students in pairs.

G. Lesson 7: Empty Number Line

1. Learning goal:

Students can draw the jumps on the number line to understand the operation.

2. Description of the activity:

This activity will be the same as the activity 3 'the number line game'. The student will do the activity with the same rules. The black die represent the positive numbers and the jump to the right; meanwhile the red die represent the negative numbers and the jump to the left. The students will themselves draw on the number line the jumps they have to make according to the number on the dice. The starting jumps is always started from 0



Figure 4.4 Dice game activity

3. The conjecture of students' thinking:

The students may start from zero and make jumps to the left, and continue to the right, or start with the right jump, and then to the left. The may be confused about the starting point, some students may think that the starting position is one of the numbers on the dice. If students have difficulties by using empty number line, they may look back to the number line game in the third activity.

4. Discussion:

In this activity, students will try to do the task individually. After each of the students already finish the task, the teacher could start the discussion about how they solve the problems. Since in the previous lesson, they already learn about making jumps along the number line, probably they do not find any difficulties in doing this activity, but here, the task presented as a formal form of the number line game. There are only two dice with numbers on it. The teacher could say that it is different dice because the students may find some numbers bigger than 6 to challenge them to compute bigger number. Besides, the teacher can also explain about the numbers which could make zero. The teacher can say that "if you move 3 jumps to the left, and then you move 3 jumps to the right, you will come to zero". It is also the same when you move to the right first and the continued by the

movement to the left. Teacher can explain that it is because the distance between 0 to 5 is the same as the distance from 0 to -5.

CHAPTER V

RETROSPECTIVE ANALYSIS

In this chapter, the analyses of teaching and learning process during pilot experiment and teaching experiment are elaborated. The chapter will include how the designed activities work in the classroom and how the Hypothetical Learning Trajectory tested and revised based on the learning of the students. The students' development in learning the mathematical concept, which is about integers, will be also discussed.

A. Pilot Experiment

Pilot experiment in this study was conducted in some steps: (1) the preassessment, (2) Try out the activities. Six students (3 high level students, 2 average level students, 1 low level student) from the third grade classroom were selected by the teacher. They are Hamdi, Adi, Angga (high level students), Willy, Misfah (average level students), and Kemal(low level student). All students that had been selected were boys because there was a separation between girls and boys in the school. Therefore, all students in their classroom are all boys. The reason in choosing the small group of students in this phase is that because this experiment will emphasize the tested HLT and some changes need to make in improving it as the preparation for teaching experiment.

1. Students' Prior Knowledge and Pre-assesment

The pre-assessment was conducted to check the readiness of the students to follow the lesson about negative numbers. There are five problems in the preassessment; this test is aimed at checking the pre-knowledge of the students about the addition and subtraction of whole numbers. The purpose is to make sure that students know how to add and subtract two or more numbers for later they can use this knowledge to operate integers. Four of five problems are missing value problems to ask the students to find the appropriate numbers if the result of adding and subtracting is already known.

Most of the students managed to do the problems of the test. Five students who followed the test did the same mistake which was in the fourth problem about the missing value problem of subtraction. Among five students, two students did all problems correctly and three of them did one mistake. They had difficulty in determining a number which should be subtracted by eleven to get four. It was not easy for them because it was confusing. Most of the students do counting by their fingers.

It was observed that most of the students still count by their fingers. They also managed to do multiple operations; therefore, it is concluded that they were able to follow the activities. Also, it was also asked to the students about money and debt to make sure they already familiar with those terms. Some students admitted that they had no experiences owing money from someone. However, they knew the condition in which one owe money to someone else. Given some situation, for example, "If you want to buy 5 books costing Rp5000,00, then you only have Rp3000,00, so how much money you owe to the seller?", they could easily answered this question. With the result of the investigation of students' prior knowledge, the students were ready enough to do next activities regarding negative numbers.

2. Try out the activities

a. Activity 1: The "Hop Shop" game

1) Mathematical goal:

- a) Students improve their understanding about negative numbers.
- b) Students can develop the notion of negative numbers

2) Overview of the activity:

Students played the game twice; in the first meeting and the second meeting. In meeting 1, students played the hop shop game in one group of 5 students (Kemal, Angga, Adi, Willy, and Hamdi). Misfah did not attend the class in this meeting. They decided by themselves who will be the shopkeeper and the bank. Kemal chose himself to be the shopkeeper and as the player at the same time. The teacher reminded the player who played as the bank and the shopkeeper to separate their money from the shopkeeper's money and bank money. During the game, the students got confused about the money in their hands and the money recorded in the transaction sheet written by the shopkeeper. The children had no problem to play the game until they were asked to represent debt in other way to make it different with the amount of money the player still have. Because the time was not enough for another round, the students only play the game once. Another round was done in the second meeting. The excerpt of students' discussion below showed how students begin to play the

game:

Researcher	: "How much do you have to pay Akmal?
Akmal	: "I don't want to buy"
Researcher	: "In this game, you have to buy the things and pay the money to the shopkeeper"
Angga	: "Come on, Malyou have to pay for the accessories"
Researcher	: "If you don't pay, you will be the richest person of all"
Akmal	: "ooohthat's good" (laughing) ok, I should pay 5."
Resarcher	: "So, how much money do you have now?"
Akmal	: " <i>Five</i> "
Researcher	: "Are you sure?"
Wildan	: "18 minus 5"
Akmal	: (counting with his finger)
Angga	:"13, Mal"
Akmal	: "13 (writing on the transaction sheet). Hurrah! I still have much money"

From the conversation, it seemed that students understood how to play the game based on the rule. Since in this game, the students are challenged to count the amount of money during the whole game. Therefore, Kemal struggled to count numbers because he needed more times to subtract and add numbers at the same time. Angga and Willy were already able to count numbers quickly. This conversation is chosen to show how the mathematical knowledge of students relating to the game context. The following excerpt will be discussed to describe how the notion of negative numbers as well as the students' interpretation concerning to that appear during the game. The concept of negative number through this game will appear when the students are subtracting a smaller number by a bigger number. In this context, the students will use money to operate the subtraction process that negative sign will be introduced. Before directly introducing the signed number to student, it is necessary to know the pre

knowledge of students in representing debt and the way they talk about it. This short conversation gives some information relating to that. They were obliged to pay the money for the things the bought. At first, they argued that whether they bought the things or not should be optional based on their experience playing similar board games. However, the students were asked to follow the rule of the game.



Figure 5.1 The students played the 'Hop Shop Game'

During the whole game, it was realized that they did some calculation of simple number. Some of the students still count by fingers. Sometimes, it took longer time for them to count the numbers in more complicated situation; for example subtracting and adding money for the same time.

Willy	:Getting money \$1 from each player" (reading the card)
Researcher	: "Now all the players should give money to Willy."
Angga, Hamdi	: "Okay" (giving money to Willy)
Angga	:"Have you received your \$3?"
Willy	: "Yes."
Willy	:"Now I have to pay for glasses." (giving money to
Kemal)	
Researcher	: "How much does Willy have now?"
Kemal	: "\$12."
Researcher	:"Are you sure?"
Willy	: "15, then I got 3, so hmm. 18, hmmmmthen get money
	from you guys became 22, subtracted by 6, okayhmmm
	so 16."

"Hmmm..so you have..hmm "(counting by finger)
"22 subtracted by 6?"
"22, 21,...(counting by finger)
"16"



Kemal

Willy Hamdi

Kemal

Figure 5.2 Hamdi counted by fingers

The most important section in this lesson was the time when the students tried to make representation about debt. At the first time, they had no idea how to make the amount of debt differ from the amount of money. It was suggested to them to make representation for debt. Kemal who became the shopkeeper wrote "1 utang" (debt of 1) to represent debt. It was also needed to make sure that he understood that it means that a player already borrowed \$1 from him. Afterwards, one student gave suggestion to make shorter representation. Therefore, they replaced it into for example "1 u" or "9 u" to say debt of 1 and debt of 9. They needed process until they came up with another way to write down debt in the worksheet.

Considering that the students did not have enough knowledge about negative numbers, it was important to introduce it to the students to replace the representation "9 u" (debt of 9) into -9. The students could easily accept the representation using negative sign because sometimes they referred it as "kurang 9" which means that somebody needed to pay 9 to the shopkeeper. After they ended the game, the students were asked to put the numbers on the transaction sheet in a good order. The least number was the number of the biggest debt (negative number). The intention of ordering number was to find the winner and the loser of the game. This group of students played the game twice in the first, in the first round, Kemal was chosen to be the shopkeeper; in the second round, Misfah was the shopkeeper. These following pictures are the transaction sheet from the first round and the second round, respectively:

Permain I Nama:	Parnain 2 Nama:	Pomain 3 Nama:	Pernain & Numa:	Pemain 5 Nama 15
15 July	+ Hichnal	HL15 P-	RANJON	Wildan
9	13	R9	13	16
18	12	6	7	112
4	29	Constant /	1	1
Stating-	R	ANT MIL	12 4	13
- 10	3	3.V	_N	5
	-9	900	- 11 1	Perch 1
-3	A	-11		-13
2	-8	- 14		74 W
7	77	-15 3		
64	20 5			

Figure 5.3 The transaction sheet of Akmal's group

It was quite complicated to analyze the handwriting of Akmal. The starting point was easy to see. He wrote the process of subtracting the money, then he tried to make sign that one has debt to the shopkeeper by writing it as "1 utang" (debt of 1). Below it, he also wrote "0 "to give sign that one has no more money. It was suggested to them to use shorter way to write the debt. After discussing with other students, they agreed to write it down as "3. U" (debt of 3). Right after this, negative sign was suggested to be used in it.

3) Reflection of the Activities:

Because the players sometimes miscalculate the money that in the transaction sheet showed different amount of money recorded. The solution is that to let the students play the game without using the real play money in the second round. The students will only imagine and calculate the amount of money based on what has been written by the shop keeper. The game will be played once in the first meeting and if there is enough time for another round then let the students to play it without the play money with the aim of avoiding the confusion and challenging them to improve their calculating skill. Also, it was necessary to ask the student some questions relating to the game whether or not they can interpret the game in correct way. The teacher in the next cycle probably can give students more opportunity to say something about the representation of debt and stimulate them to say more what they know about negative numbers in daily life. However, it is also important not to give too much guidance too early that students had not much opportunity to present their own thinking about representation of debts.

b. Activity 2: How much money/debts will I have? (drawing jumps on the number line)

1) Mathematical goal:

Students can draw jumps on number line to solve problems relating to the context in the game.

2) Overview of the activity

In second meeting, the students were asked to do 10 problems relating to a number line. The task was focused on how the students can make jumps along the number lines. The given information was that the students have to make jumps to the right if they get money from the bank and make jumps to the left if they pay money to the shop keeper. There are two parts of the task given to the students. The first part was 5 problems which were already on the paper. Other 5 problems were taken from the cards which enable the students to exchange the cards of the problem and make some discussion on how they do the jumps along the number lines. Some questions relating to the previous lesson were asked "*What does -2 mean to you?*" One of the students responded that -2 was the amount of money he borrowed from the shopkeeper. Several questions were asked to the students to make sure that they have enough knowledge to continue to another activity involving number line.



Figure 5.4 Students work out the problem individually

The students located 'debts 4' (of the problem) in the left of the number lines. They interpreted "get 7 from the bank" by making 7 jumps to the right and stop in 3. However, the work of some students were observed that they gave an arrow to mark the starting position. However, it was not clear about the arrows; whether it marked the starting position or not. Therefore, some extra questions were needed to the students as the confirmation about the answers.



Figure 5.5 The solution of number line problem

The picture below was the work of Adi, he wrote "ambil (take)" to represent the amount of money he got from the bank and say "bayar (pay)" to represent the amount of money the paid to the shop keeper. Those representations helped him to determine the direction of the moves he should make



Figure 5.6 A students represent the picture as "take" and "pay"

In this activity, the students worked with a number line for the first time. Later in the teaching experiment, other students were asked to work the problems in a group to stimulate more discussion among them.

3) Reflection of the activities:

There were some findings on the lesson that outside our expectation and needed to be improved for the teaching experiment.

- a) The practical things such as arranging the problems and the pictures (the picture of the number lines) that students will not be confused and giving the number for every problem.
- b) The instruction of the problem should be explained more clearly by the teacher. For example, by asking the students to make an arrow to identify in which numbers they start to make jumps. Because some of the students did not make any different about the starting numbers and the numbers after jumps.
- c) The class discussion should be more facilitated. It should be clarified by the students how they know about the numbers in a problem. For instance, which numbers are the starting numbers, which of those shown the jumps, and which numbers are the numbers after making jumps. The thing is to make sure that teacher can get much information about students' knowledge through class discussion. Students can get prediction about students' understanding about the solution of the problem and how they can talk about that. The discussion among students should be observed.
- d) The problem sheet should contain a space in which the students can write something as a conclusion of the problem. Because it is not clear whether the students really understand the problem or not. The concluding sentence can clarify the students' thinking about what they already write or draw in

the task. It is also essential to know how students justify the answers of the problems.

c. Activity 3: Mini Lesson—Ordering Integers

1) Mathematical goal:

Students can order and position integers on a number line

2) Overview of the activity:

This activity helped students to understand how to locate and position integers along the number line. There were two parts of this activity, the first one was the task to order some numbers and put them in boxes to determine the winner, the second winner, the third winner and the loser. Most of the students had no difficulty doing this problem. Some students wrote both names and the numbers; other just wrote the numbers. The experience from the previous game helped them to solve all problems correctly. They knew that -22 represent amount of debt which is bigger that -15. Meanwhile 7 could be the biggest of all since it represent money.



Figure 5.7 A student managed to order the number but he did not put the negative sign



Figure 5.8 The solution of ordering and positioning integers

The second part of this activity was about connecting numbers to the correct position of the number line. Some students managed to solve the problem without having to put numbers along the number line. However, in the picture above, writing all numbers really help them to find the exact position of certain numbers. It took longer time for the students to do this task because they needed to figure out comparison between numbers.



Figure 5.9 Conclusion made by a student

The figure above shows the conclusion of the activity. Actually it is expected that students could answer the blank as "getting bigger" or "getting smaller" or they may say "the more to the right, the bigger positive number" and "the more to the left, the smaller negative numbers". Nevertheless, this student use other word to describe the situation since what they know was that -18 was at the left

side of -15, therefore -18 is smaller that -15. Similar situation happen to positive numbers.

3) Reflection of the Activity

Most of the students responses these activities as expected. The result of students work suggested that this activity can be quickly finished by the students. However, the important part which is whole classroom discussion should be more encouraged. It is also possible to change this activity into a mini lesson since students can do it well and fast. In teaching experiment, probably, teacher should observe each student regarding the way they use to solve the problem. The students should be able to explain the solution given to review their knowledge about the concept of negative numbers within the context.

d. Activity 4: The Number Line Game (Part 1)

1) Mathematical goal:

Students can use number line as a tool for calculating number

2) Overview of the activity:

The students played the game involving number line. By using two dice with different colors and a counter, they played the simulation of the addition of two numbers. The blue dice and the red dice showed the number of moves should be made on the number line. The number on the blue dice is the number of jumps to the right. The number on the red dice is the number of the left.



Figure 5.10 Students worked on the number line activities in a group of three students

The students were divided into two groups. They worked in the group of three students. Adi, Hamdi, and Misfah were in the same group. Akmal, Angga, Willywere in another group. At first, the group of Angga misunderstood the rule of the game. In each turn, they should back to the start, roll the dice and make moves. However, they did not put the dice back to the start. Therefore, the moves they made were actually did not mean anything since it failed to show the simulation of the addition of integers.

Angga	: "1, 2, 3, 4(stop the counter) 1, 2, 3, 4, 5 (change the
	direction)
Kemal	: "Where do you stop?"
Angga	: "1"
Kemal	: "Lohhow did you get 1?"
Angga	: "The blue dice was 4 subtracted by 5 (the number on the red
	dice), then I got 1"

In the conversation, Angga did not start from zero that he got the answer, he started from 2. The answer should be -1. Kemal also believed that what Angga did was right, so he repeated the same mistake.

It was interesting from this group that Angga used the word "subtract". He interpreted that the movements to the left and to the right was the subtraction

process of two numbers. It was because he read the problem as subtracting the positive number (the number on the blue dice) with negative numbers (the number on the red dice). Since the number on the red dice was always be written in negative sign, he read the sign not as the negative sign but as the subtraction sign. Probably Angga can read the pattern of the number of the tables. When he saw 6 on the blue dice and 4 on the red dice, he directly could see the numbers as 6 - 4 which he knew that is equal to 2. Nevertheless, it was not certain that he understood that 5 - 6 is equal to -1. It is quite complicated since students were expected to see the problem as addition problem, the addition of negative and positive numbers. The discussion about the role of the sign was postpone in this case.

It was too early for them to grasp that 5 + (-6) becomes -1 is equal to 5 - 6 becomes -1 since those operation was the equal but how they can understood this within the context. It was kind of big jumps for students in third grade, giving them abstractness of the concept will give them more confusion. Our intention was to introduce that the combination of the two movements was the example of addition of two integers.

Hamdi, Adi, and Misfah were in the right way. Before starting his turn, Hamdi has some confusion because he asked about the example given; why the final position of 5 on the blue dice and 6 on the red dice ended on -1. He assumed that the final position of each turn could be written in negative. He was asked to investigate it by making moves. It was quite surprising that he write all final position of each turn in negative, some incorrect answers were found and he put negative sign for zero although in very beginning they were already reminded not to put negative sign for zero because zero does not belong to negative numbers. In this group; however Misfah managed to fill the tables correctly.



Figure 5.11 Hamdi's worksheet

Kegiatan 4: Permainan garis bilangan (Part 1) Media: papan permainan paris bilangan Telekantak segika pada dada kire dire nersh peda teled yang tersedu, telekan pela pada angka berapa peminini melah telambah in kiri dan ke kanan.		
Anghe pasts doubt Sire	- Angles page darks second	Facial analysis multistighteds
	-4	Be Line that he hanne
5	-4	
	-5	0
4	- 9	1
4	-3	
i i		-5
6		-5
4 5 1	-1 -2 -1	-5
4 5 1		-5 -3 -1 -3

Figure 5.12 Misfah's worksheet

3) Reflection of the Activities:

Some remarks from this lesson is the way to overcome the confusion of Angga's group. The teacher should emphasize that after each turn, the players should put the counter back to the start to avoid meaningless activity. To solve this, it is considered to make one column which reminds the students the starting point of each turn. Besides, the discussion should be more facilitated to know how the students interpret the activities either they use the key word "subtraction" or any other important term. The way the students move the counter should be observed whether or not they know that moving to the left-then right is equal to moving to the right-then left.

e. Activity 5: The Number Line (part 2)

1) Mathematical goal:

Students can solve missing value problem after experiencing the dice game.

2) Overview of the activity:

The activity challenged the students to play the dice game to fill another table. The table was quite similar. However, the students were not only asked to determine the final position but also to find out the number on the red dice or that of the blue dice to get to certain position.



Figure 5. 13. A student used a number line to fill the problem on the table

There were two kinds of table, the first table was the table consisting of number 1-6 and the second was the table consisting of numbers bigger. Similar to the previous work of the dice game, the group of Hamdi, Adi, and Willy interpreted the problem as subtraction problem. Therefore, they answered incorrectly. In the table below, some answers were correct, some answers were incorrect. When given 7 on the blue dice, and the final position at -3, they answered that the number on the red dice should be -4. The correct answer is 10. What they actually did was that they subtracted 7 by 3 to get 4, afterwards they put negative sign following 4. That was not acceptable way to do the counting. Observing the conclusion they gave in the worksheet, they actually did not use the number line. Instead, they did subtraction in a way similar to the subtraction process they did to positive numbers. Differently, the group of Angga, Akmal, and Willy did the counting using the number line. As the result, they could solve the problem all correctly.

3) Reflection of the activities:

The students' understanding should be clarified to make sure that they understand the problem by asking "Do you know what the problem is?", or "Where should you start to make jumps", or "How many jumps do you need to get to the final position?"?", the last questions may be too guide them but it could be the last option if the students already too much far away from the instruction. They need to confirm what exactly the questions and be able to explain it with the help of the context. Some student might think that they no longer need the number line to solve the problem because they could be easier to count the numbers because the know subtraction of positive number less than 20, they might think that counting by finger could be better way to count. However, the teacher should encourage the students in using the number line to see the situation and to calculate the numbers on it. In the end of the lesson, the teacher should emphasize that the activity is the example of addition of positive numbers.

f. Activity 6: What is your position now?

1) Mathematical goal:

Students are able to draw a number line and to operate negative numbers using that number line.

2) Overview of the Activity:

The students were asked to make their own number line to model the situation of the problem. The problems were about to find the final position of a person after making certain number of jumps from initial position. Angga, Akmal, and Willywere in the same group; Adi, Misfah and Hamdi worked together in another group. To start this, Angga tried to make a number line, he made boxes to be filled by numbers. At very beginning, the number line was not introduced yet to the students. The intention was to give opportunity for the students to make their own number line with their own style. He changed his number line into a straight line in the following problems.



Figure 5.14 A students made a number line in a shape of a rectangle



Figure 5.15 Angga's number line

The number line above was the response to the question "*I am in number 3, I move 9 jumps to the left, Where is my position?*" Angga could do the problem correctly using his number line; he put the answer in the box, which was 6. Kemal made a number line different form that of Angga. He drew a line and wrote the numbers on it, making jumps, and did some checking on that. He tried to figure out the final position after making 11 jumps to the left from 5. He realized that he made excess jumps and then made jumps to right get to 5 counted it until11 to check his answer. After he was convinced to his answer, she put -6 in the box.



Figure 5.16 Kemal's number line

3) Reflection of the activities:

In this activity, some good responses from students appeared because they could use the number line properly although they never worked with number line before. Two out of 6 students manage to make use the number line fast and efficiently since they only wrote the number given and tried to make moves according to the problem given. Others used the line, put some marks on it and wrote as many numbers on that number line. A problem appeared during the lesson that the student did not have much discussion; they chose to do it individually, so one student responsible for one problem. It was difficult to stimulate students' discussion since they were only provided with one worksheet.

f. Activity 7: Empty Number Line

1) Mathematical goal:

Students can draw the jumps on the number line to understand the operation of bigger numbers

2) Overview of the Activity

In this activity, students were asked to solve problems relating to the dice game they played in previous meeting. There were 8 problems with the number lines given and the pictures of the numbered dice. The students were asked to make jumps according to the numbers on the dice. The number lines given are the empty number lines that students can the number themselves. Most of the students started by writing all numbers along the number line; meanwhile some of them realized that they only needed to write down numbers which are necessary for making jumps.



Figure 5.17 students used the marked number line to make jumps based on the dice

One strategy of the students in making jumps was that they began to make jumps to the right above the number line and make jumps to the left below the number line. They thought that by making different jumps for different direction, they can see the process easily. Besides, in making jumps they found that it does not matter to start making jumps from left to right or from right to left. Neverthelss, most of them started to make jumps from right to left. When students were asked to draw conclusion about the activity, they could easily figure out that "the result of adding bigger number on red dice and smaller number on the blue dice" is negative numbers.

Researcher : "Why is it negative?" Adi : "Because negative is bigger, so if I have 3 (making jumps to the right), 1, 2, 3 and then 1, 2, 3, 4 (making jumps to the left) and I stop on negative numbers.

It may be concluded that in this activity, students can use number line as the model of as the situation form the dice game and can also use it as the model of thinking they make jumps to add the two numbers on the dice.

3) Reflection of the activities

Students gave good responses in doing this activity, some of them could do all problems correctly, the students will be given more challenges; Students can use the number line correctly using their own strategy. Also, the students were given chance to write down the numbers in more formal way meaning that they can express the situation in mathematical sentence. Also, some more problems in which they can move from problem to context will be added. In the teaching experiment, the teacher should remind the students to give marks at which they stop making jumps. Probably, teacher could facilitate more discussion among students since in this activity, the students did the activity individually, thus they did not have opportunity to discuss the activity with other students.

C. The Improvement of Hypothetical Learning Trajectory

Considering the students' background knowledge and the result of the analysis of the preliminary teaching experiment, some changes relates to the order

of the activity and the number of problem in each activity will be made. There will be no significant changes in the first lesson, in which students played "Hop Shop" game involving the money context and the process of subtracting money causing the appearance of the number line. In this activity, students had various representations about debt. Therefore, the students will be given freedom to make that representation until they have good understanding to accept negative numbers as new representation about debt. Teacher, in this case should wait until the students have discussion among their group about it and stimulate them to think about negative sign. The second lesson includes the introduction of number line as the model of thinking. In the preliminary teaching experiment, it was not clear how the students make jumps and what interpretation they made regarding those jumps. To solve this, besides asking students while they make jumps, there will be some questions leading them to some confirmation about jumps they made based on the problem given. For example by adding the some questions about "In which number do you start making jumps?", "in which number do you end your jumps?", "How much money/debt do you have now after you make jumps?". Those questions could help the students to better understand the problem and the way to use number line properly.

The third lesson will be changed into the mini lesson by considering that all students in preliminary teaching experiment could the problem all correctly in the first part. Therefore the number of problem is also reduced by only presenting 2 of them. (Five problems were presented in the preliminary teaching experiment). The Focus is on the second part which is about connecting the random integers to the number line given, the students were asked to estimate the position of numbers along the number line. The mini lesson will be done in the second meeting before the students do the third activity.

The third activity, is the dice game. In this activity, the students should clearly mention the rule of the game to prevent the misinterpretation of the students. It happened in the preliminary teaching experiment that the students did not follow the game according to the rule. What should be underlined is that after the students finished the game, they should return the counter to the starting position. It was completely wrong that one group of students continued the turn without putting back the counter to the starting position. Therefore, little changes will be made by adding one column to remind the students that the they have to come back to the starting position each turn.

In the fourth activity, the improvement of the activity is by adding some illustration of children make jumps and expect that students can model the situation using the number line. With the help of the picture, the visualization will be more interesting to students.



Figure 5.18 Revised version of the fourth activity

Besides, instead of providing the blank space, the straight line is provided. Given this number line, they will make some marks and numbers on it to model the situation given. Consider the preliminary teaching experiment, the students had difficulty in making a straight line and it took much time for them to make a line. Therefore, the straight line is given in the first place.



Figure 5.19 A straight line and zero were already provided in the worksheet

The second part of the dice game is omitted because the student in third grade could not imagine the situation. In the preliminary teaching, the students had difficulty in answering the problem and they ended up estimating the answer without further thinking. Most of them interpreted the problem as subtracting process. There is modification of the last lesson about empty number line by combining some variety of problems. The first type of the problem is the visualization of the dice game including the space provided for the students to write the addition in more formal way. The second type of the problem include the missing value problem in which the students need to determine how many jumps should be taken to reach certain number. Students will use two types of number line, marked number line and empty number line.



Figure 5. 20 The changes of activities for teaching experiment

B. Teaching Experiment

Generally, the phases in teaching experiment are quite similar to those in pilot experiment. There are some changes in the series of the activities, but not too many. In this teaching experiment, the focus of the study is the development of the students' thinking about the concept learned, in each activity. The investigation of students' learning is emphasized and analyzed to see the how the design work and support the teaching and learning process.

1. Try Out the Activities

a. Activity 1: The 'Hop Shop' Game.

1) Mathematical Goal:

- c) Students improve their understanding about negative numbers.
- d) Students can develop the notion of negative numbers

2) Overview of the activity

In this teaching experiment, the activity will be the same. However, the whole class participated in this activity. The students were divided into some groups of students. In this study, the focus group should be chosen in each to see their learning. However, because the teacher reported that the students in his classroom were not accustomed to work in group. The teacher suggested changing the group in each lesson, because most of the students complained and did not feel comfortable working with some students. Actually, the intention of choosing one focus group of students is that because deep investigation is needed in students' learning in a group to see the development of the group during the lesson. However; considering the condition of his class, the teacher was convinced that it could not be done in his classroom. Therefore, the solution given by the teacher was that the students still work in group but they may change partner/member of their group to avoid the 'fight' among them. Moreover, the teacher helped them to choose friends as the member of their group themselves.

Given this obstacle, it is believed that the learning of the former focus group could be investigated as long as the recording and some data of the written work to collect the information to support the analysis and the interpretation of the students' development are available. Before going to the main lesson, the teacher prepare the time for about ten minutes for the students to choose the member of their group. 5 students were chosen by the teacher. Afterwards, the five students chose their friends to be in the same group with them. The teacher believed this way of grouping the students could probably be effective to encourage them to work cooperatively.

Students work in group of 5 students. 19 students were divided into 4 group consist of 4-5 students. Similar to what have been described in pilot experiment, the students in this activity played the game with money context. The goal of the activity is as an introduction of the concept of negative numbers. The students are expected to be able to find the representation of debt. Prior to this, we make sure that all students know what debt is. The notion of negative numbers appeared when students subtract money to pay some things in each turn. After several term, students spend all of their money and need more money to pay things. In this condition, they will borrow money; consequently, they will have debt. The

important thing of this activity was the representation of debt given by the students.

The teacher explained the rule of playing the game. The teacher was assisted to circle around the classroom for making sure that students understand how to play the game. Before describing that part, this part of conversation among students is the part in interpreting an instruction of the activity.

Canya	: "What is free parking?"
Researcher	: "It means that you may stop wherever you want"
Canya	: "How to do that?"
Researcher :	"You may stop at community chest (dana umum), chance (kesempatan), or wherever you like."
Canya	: "Come on, you can stop at chance (kesempatan)
Nanda	: "Okay (opening the chance card) Huaaaa, I should pay"
Canya, Firda	
Ayoeng	: "hahahahaha"
Canya	: "What is in the card?"
Nanda	: "Pay money in any amount for charity" (read the card)
Firda	: "How much?"
Researcher	: "You can pay in any amount"
Nanda	: "I want pay \$2" (giving money to Ayoeng)
Ayoeng	:"Give it to the bank"
Researcher	: "How much money does Nanda have now?"
Ayoeng	: "Nanda, how much do you have?"
Reseacher	: "Count it"
Nanda	: "15, 9, subtracted by 2, 7
Ayoeng	: (writing on the transaction sheet)

Based on the conversation above, what students did was they discussed to work on the game based on the rules. Since not all students understood how to play the game. The activity students often did was doing some calculation to determine the money they had after paying to the shopkeeper. After observing some group of students doing the calculation, most of them had no difficulty until they found that they had no money and they had to borrow some money to the shopkeeper. It was confusing for some students to write down the debt on the transaction sheet. The teacher realized this, and students came to the some group suggesting that they may give some representation on that for example by giving the sign "utang 5(debt of 5)". This guidance helped the students a lot. However one group of students who had not get the suggestion yet had an idea that debt might be represented by a sign to which they call "subtraction sign". It is seen that they had no idea in interpreting the sign they added to the number as a negative sign because it is not yet introduced to the students. Here a conversation of the group described above.

Researcher	: "Who already have no money? Who already have debt?"
Egi	: "I did, but I have paid it"
Dzakia	: "I have debt, I haven't paid it yet."
Researcher	: "How did you write the debt?"
Egi	: "subtract" (using the gesture as if she write a subtraction sign)
Researcher	: "What did you use to represent debt?" (repeating the question)
Egi	: "We used the subtraction sign"
Researcher	: "How did you write that?"
Egi	: "How much did I debt to the bank?Hmm(thinking)3 and 2
	-3(kurang 3), -2(kurang 2), so5(kurang 5)
Dzaki	: "I have debt of 1, so -1(kurang 1) (asking Gita to write down the
	debt on the transaction sheet)
Researcher	O_{kav} remember if anyone of you get debt of more than 20 then

Researcher : *Okay* , *remember if anyone of you get debt of more than 20, then she lose the game and we have to stop the game.*"



Figure 5. 21 One group of students play the 'hop shop' game
Our interpretation why students came up with the idea of 'subtraction sign' was that because of the language. They knew that 'debt' could decrease the amount of money they had. That was the reason they interpreted the sign as 'subtract' (kurang). However, this could be the big jump for their thinking because it can lead them to further concept of negative numbers. While the other groups still need to be guided in making the representation of debt, this group have a good start to learn the role of the negative sign.

Gita	"Yes, we're already done"
Researcher	"Who is the winner of the game? Who is the loser?"
Gita	"I win the game."
Dzakia	"Gita wins the game."
Researcher	<i>"Let's see the transaction sheet (pointing the numbers on the paper) and who is the second winner?"</i>
Egi	"This one." (pointing the amount of debt of Dzakia)
Researcher	"Whv?"
Egi	"Because she added more debts."
Researcher	"How much is it?" (pointing the numbers on the paper)
Egi	"This one was already paid. (pointing the number -1), 7 plus 5 (pointing -7 and -5) is 12, 12 plus 6 (pointing -6) is 18, 18 plus 6 (pointing -6) and -5 she lose
	-5) is 12. 12 plus 6 (pointing -6) is 16. 16 plus 6 (pointing-6) andshe lose the same
Researcher	"So, who is the loser among these three." (pointing the numbers (the last amount of money of Gita, Egi, and Dzakia)
Gita	"This one." (pointing Egi's amount of debt)
Researcher	"How much is it?" (pointing the amount of Egi's debt)
Egi	"20" (counting the amount of debt of herself)
Researcher	"If we compare these two (Dzakia's debt and Egi's debt) who is the loser?
Dzakia	"I am the loser."
Egi	"It means that I am the second winner."
Researcher	"Why is Dzakia a loser in this game?"
Dzakia	"Because I have more debts than Egi has".
Researcher	"How much your debt?"
Dzakia	"24"

The conversation was among the researcher, Egi, Dzakia, Gita, and Alya. This group was the only group who represent debt with negative signs. What they knew about the sign and how they gave meaning to it was investigated. They were comparing the debt of Dzakia and that of Egi. Egi has \$20 (written as -20) and Dzakia has \$24 (written as -24). It was not easy for Dzakia and Egi to compare those numbers. After counting the numbers and got 20, she knew that the number was smaller that that of Dzakia, which was 24. Therefore, she said that Dzakia had bigger number that she had. Unfortunately, although the teacher always said that the loser is the person who had bigger amount of debt. They failed to realize that 20 and 24 are the amount of debt. It is assumed that the sign means debt. Dzakia was the one who can convince Egi that actually Egi win and she loosed because she had bigger number of amount.



Figure 5.22 Egi tried to compare -20 and -24 on the transaction sheet

There was one similar thing done by the students. In the beginning, they wrote the numbers on the transaction sheet by writing on the last money they had after paying the money to the shopkeeper, However, this was changed after they reached debt. Instead of writing the amount of debt directly on the transaction sheet, they wrote the debt they got. Therefore, they should count first to know the total debt they had in the end of the game.



Figure 5.23 One group presented debt in negative sign

In the written work above, when they still had money, they wrote the last amount of money they had after paying things. Probably since they still had the real play money with them, they could figure out how much money they had easily. When they reached negative number they did not directly calculate the amount of debt. Instead, they wrote how much the debt. See the figure in the red circle, the students were expected to write those number as "-8" (after adding the previous debt -1 to the current debt-7) Consequently, they need to count all debt after finishing the whole game. Probably that was because they were interfered with the negative sign following the number making them cannot be easily counted. Compared to another group, the group of Arina, Nana, Wanda, and Fafa, they still wrote down the sum of the debt. They represented debt as "hutang x".

Nama: 1 15	Namaties 2 15	Permain 3 Nama: Xerna 19	Pemain 4 Nama:
10	10	M	13
151	18-	9	3
5	17	7	8
2	0	2	0
telong a	Blackman 1	O Hotora 2	8
lating 5	Mulang 2	Rang M	3
dang 4		History 77	Hobing 6
		Bidang Zas	

Figure 5.24 The work of Arina's group

This group calculated the amount of debt in the third column. However, it was not clear about the numbers in the first column whether they did some miscalculation or they did not make sum of the amount of debt. Another thing that was observed from the written work of the students was that they wrote "0" (zero) to note that one had spent all the money.

Pemain 1 Nama: Rohnuk	Pemain 2 Nama(A)	Pernain 3 Nama(Pemain 4 Nanva:
	12	10	1.0
		10	
	4	Q	
	0	helap os.	
7	4- 1	heren 19	
		hotan 18.	

Figure 5. 25 Yasmine's group work

Some of the group of the students wrote zero to give a sign that somebody has run out of money. The teacher in this lesson let the students to play the game without giving too much intervention. The students could make the representation of debt with their own way. The teacher knew that one group managed to use negative numbers as expected; therefore the teacher use this to start discussion with the whole class.



Figure 5.26 The teacher asked the students to order the numbers



Figure 5.27 changing the representation using "debt of x" into representation using negative sign

The teacher asked the students some problems to know the magnitude of the numbers. He wrote some numbers on the blackboard, they were taken from the transaction sheet from students' work. He asked the students to order the number from the biggest to the smallest. On the blackboard, the teacher wrote -12, -15, -2, and -7. When the teacher asked *"Which one is the biggest?"*, most of the students

answered that -15 is the biggest of all. Afterwards, the teacher said that the biggest number showed less debt than others. Immediately, the students change corrected their answer, they changed it into -2. This explanation made them easy to decide the correct order of those numbers more easily. To make sure that all students understood the problem, the teacher gave another example to compare two numbers. To give more challenge to the students, the teacher asked the students to order the numbers: -4, -1, 5, 2, 15. From the observation, comparing two numbers for most of the students was easier than ordering series of numbers. They still considered -4 is bigger than 2. They will get the same activity in the mini lesson in the next meeting.

3) Discussion

Here is the description of the comparison of students' actual learning with the Hypothetical Learning Trajectory. Previously in Chapter 4, some prediction on how students response the task have been made. This following table shows the overview about the actual students' thinking compared to those in HLT. Some conjectures happened in actual learning, but some others differ to what has been conjectured.

 Table 5.1 Comparison between Hypothetical Learning Trajectory and The Actual Students' Learning of Lesson 1

The Hypothetical Learning	The actual students' learning
Trajectory	
The students will realize that their	Some students thought that buying
money is spent more and more because	things was optional. That they do not
they have to pay to the shopkeeper if	spent money in each turn.
the counter land on the space of an	
item.	

The shopkeeper will record the amount of money the players might have after the transaction.	The shopkeeper wrote the amount of money the players should pay.
After several turns, the students might have no more money in their hands. They might say "Now, I cannot pay for the item because I have no more money", then the students who act as the shopkeeper might say "I owe you money, then I have to write that you now have negative (the amount of debts) here"	Most of the students came up with representation of debt using "hutang" (debt of) or "u" (d) There was only one group of student who made representation using negative number.
Another important situation might happen is that when the students pass the starting point and get \$3, the student playing as the bank might say "This is your \$3" and give the play money to each of the player.	The bank did not give the money of passing the start and directly count the money the player should have after paying things.
The shopkeeper could declare the situation by saying that "You have paid \$3, you still have a debt of \$7 to me", or "I wrote down -7 here"	Not all the group wrote down the numbers as negative number.

b. Activity 2: How much money/debts will I have? (drawing jumps on the number line)

1) Mathematical goal:

Students can draw jumps on number line to solve problems relating to the context

in the game.

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2) Overview of the Activity

The lesson was started by the teacher by asking the students about the game they played in the previous meeting. The teacher asked some students who became the winner who became the loser in the game.

> Teacher : "Who lose the game we played yesterday?" Egi : "Dzakia was the loser" Dzakia : "Yes, I was the loser". Teacher : "Why?" Dzakia : "Because I have so much debt"

After several conversations with some students, the teacher told the students about the game. He drew a long number line on the blackboard and put some boxes for placing the number in it. The teacher had prepared 21 number cards from -10 to 10. The teacher put zero in the middle of the number line to begin. Student seemed understood the instruction. The teacher explaining the rule of the game that they must order the numbers from the smallest to the biggest, the smallest and the biggest number referring to the amount of biggest debt to the biggest money. The teacher tried to come back to the previous context to make sure that students can relate the activity to what they got in the previous meeting.

One student became the volunteer to order the first number given by the teacher. She was asked to put number 4 and put it in the right box on the number line. She counted the box with her finger from zero to find the correct box for number 4.



Figure 5.28 Canya put number four by counting one by one from zero

After seeing Canya managed to put the number correctly, other students came to the teacher to get other number cards and tried to put them in the right place. There were some different strategies appeared when students locate the number. Similar to Canya, the strategy of Egi was to count by her finger form zero either to to the left side (negative numbers) or to right side (positive numbers). Rohma showed different strategies, she counted form the left or from the right toward to zero. Since Rohma got -9, she counted from -10 to -9.

Rohma : "I got 9" Firda : "1,2,3,4...".(counting from zero) Rohma : "10, 9...." (stick the number on the blackboard)



Figure 5.29 Students are ordering numbers on the number line

What Rohma meant actually was negative 9, not positive 9. However she said the number as 9, most of the students did the same thing. This happened because they still consider negative 9 similar to positive 9 or it is because they were not familiar yet with number and the negative sign. However, it was not a difficult activity to arrange the numbers on in the good order. Talking about the creativity, a student created her own cards of 11 and -11 and directly put those number lines after the number 10 and -10, respectively. Still, it is necessary to investigate their understanding about the reason why the numbers should be arranged that way and what is the meaning of the position of each number.



Figure 5.30 The teacher introduced the number line to students.

After all students managed to put and arrange number in correct order, the teacher asked the students to come back to their sit and the teacher started to introduce number line. He said that the number line on the blackboard is the extension of the positive number line. Also, the teacher said that zero which is in the middle position does not belong to either positive of negative numbers

Teacher	: "This is a number line" (pointing the arrangement of the number cards done by the students) What is that? (asking the students)
Students	: "Number line"
Teacher	: "Usually we do not have this part (pointing the the left side from zero), we only have this part (pointing the right part from zero), so the number line is extended. Do you understand?"
Students	: "Yesss"
Teacher	: "Can we extend this part" (pointing the left part from zero, negative numbers)
Student 1	: "Yes, we can."
Teacher	: "Okay, now what is the number after this?" (pointing number -11)
Student 2	: "Twelve"
Teacher	: "Not 12"
Student 3	: "-12"
Teacher	: "Yes, and what is after that?"
Student 3	: "-13, -14"
Teacher	: "And what is the number in this place" (pointing the wall quite far from the blackboard on which the position of the number line)
Student 4	: "-20"

After the session of introducing the number line, the teacher reviewed the previous lesson about debt and demonstrated the use of the number line to students to solve the problem of the previous activity. The teacher created a situation of having money \$5 and must pay \$10 for an item. This was done by the teacher to give an example to students of the use of a number line in solving similar situation that will be given afterwards in the form of worksheet. Given the situation, most of the students already understood that the result was having debt, because they knew that it was not enough to pay \$10 with \$5. The teacher told the students the rules that when they get money, they need to move to the right. Also, if they have to pay, they need to move to the left.

The worksheet were distributed to all students, they had to do the problem on the worksheet in a group of 3-4 students. The number line already provided on the worksheet. Therefore, the students only need to use the number line properly based on the problem given. The goal of the activity that the students can draw jumps to help them to calculate the numbers. The students are expected to understand the problem and know when to start to make jumps and when to stop.

Arina	: "The debt is 4" (pointing -4 to make start), getting money 7, so
	the money ishmm7 subtracted by 4, so the money now is
	3. "(counting by her finger)
Researcher	: "How to draw it o the number line? Look at this, if we get money,
	where should we move?"
Arina	: "Move forward"
Researcher	: "How many should you make jumps?"
Arina	: "7"
Researcher	: "Come on, you have to help Arina to work the problem, discuss it
	together."
Arina	: "1234567"
Researcher	: "So?"
Arina	: "3"
Researcher	: "What is 3?"
Arina	: "Money"

From the conversation above, it was noticeable that Arina was dominant in her group. Instead of asking her friend to work on the problem together and did some discussion together, she asked her team to work each problem one by one individually until they finished all the problems. It was found that this kind of situation happen to most of the group in the class. It was also difficult to ask the student to sit comfortably in the work table together and concentrate on the problems given.

Despite the obstacle, Arina had already have good understanding about the context because she could count the number before trying to use the number line to help her but there was still tendency in considering the negative number as positive numbers, for example, saying "4" instead of "-4" and this case happened

to most of the students. Unlike Arina, Canya and Egi who were in the same group

struggled to understand the meaning of the problems.

: "How to do this problem?"
: "I think it is similar to the previous problem. Let's see, 'I have
money \$10', so where should we start?"
: "10 (writing on the worksheet, draw a point on number 10 of the
number line). And then?
: "And then it says 'I want to pay So, where should you move
then. Look at the illustration of the number line. There are the nicture of the shop and the hank "
: To the shop
: "Make your jumps then."
: " 1234567 " (counting while Canya draw the jumps)
"So when should you stop?"
. So when should you slop?
: "At zero"
: "What does it mean? I need answer from others."
: "Hmmthe money has run out."

It was noticeable that it was important to ask the students the meaning of the numbers they write on the worksheet. The problem in this group was the same as the group of Arina. Not all members of the group participate in doing the problem. It was difficult to handle the situation since they were only provided with one worksheet that made some students only looked at some others work the problems without any relating discussion.



Figure 5.31 Canya and her group tried to understand the problem on the worksheet

Observing other group of students, it was important that all students understand the problem well and can explain how they solve the problem in certain ways. The group of Lana, Firda, and Wanda discussed the part about deciding on which number they started, they stopped, and how many debt/money they had.

Lana	: "At which number you start?" (reading the problem)
Wanda	: "4"
Lana	: "3" (write on the worksheet). "At which number you stop?"
Wanda	: "We stop at 3"
Researcher	: "Where was the position when you start?"
Wanda	: "4"
Researcher	: "Are you guys sure? What is the difference between this and this?" (Pointing A and A)
Lana	: "That is debt. That is not." (pointing A and A respectively)
	$\begin{array}{c} 1 \text{ mai is uebi. 1 mai is noi. (pointing -4 and 4, respectively).} \\ (4 11 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2$
Researcher	: "And how should you write that?"
Lana	: "Use negative sign. So we start at -4"

Similar to the excerpt above the group of Rohma, Nana, and Ayoeng still considered -4 is the same as 4. When the researcher asked them the starting position, they answered 4 instead of -4. After we remind them that those numbers is different then they realize to say it as "4 min" there was no clue why the students came with the same mistake. One possible reason is that because in the activity, the teacher did not emphasize the difference between those numbers, unsigned and signed numbers. However, later students were expected to be able to say the number correctly to make them aware about the differences.



Figure 5. 32 The student wrote both the amount of debt and the amount of money

In the class discussion, Lilis was chosen by the teacher to present the group of their group. Using the number line on the blackboard, She explained clearly about where they started, stop, and the amount of money they finally had. Still, the other group still had confusion about the context given. The context used was about shopping voucher. Some of them considered shopping voucher as paying some money to the shopkeeper although the real meaning of this was that if one gets a shopping voucher, one should get money. Consequently, the students should make moves to the right side if they get some money. It was realized that the confusion was reasonable, students may interpret that they know that shopping voucher should belonged to the shop that cause they move to the left. However, the illustration did not assist them to get the real meaning of the shopping voucher itself. To handle similar confusion, the teacher always reminded the students that the jumps to the left is for paying money; the jumps to the right is for getting money. If it is related to the starting position, the position in the left of zero is the state of having debt whereas the position in the right of zero is the state of having money.

3) Discussion

Table 5.2 Comparison between Hypothetical Learning Trajectory and The Actual Students' Learning of Lesson 2

The Hypothetical Learning	The actual students' learning
Trajectory	
Given the problem "I have debt of \$4,	- Some students did not directly know
I get shopping voucher \$7". The	the starting point, they started from
students know that -6 is the first	zero.
position on the number line, because	-Some students thought that "shopping
they get 7, so they make 7 jumps to the	voucher" means paying money instead
left. The students realize that this	of getting money.
situation will give 3 as a result.	
Students will be able to say that -2 has	Some students still used the word
the same meaning as someone borrow	"utang" instead of using negative
money \$2 from the shopkeeper	sign.
because they know that debts should	
be written in negative numbers.	
They will draw the jumps to the right	Students sometimes misinterpret the
on the number line because they get	problem that paying money always
money from the bank. Some students	ended up with debt; therefore they
probably know that they will have less	jumped until they reach negative
debt because they have money to pay.	numbers. However, it was not always
	the case if one has bigger amount of
	money than the amount of money he
	has to pay.

Students sometimes did something which was outside the expectation. As written in the first conjecture, the confusion of some students was sometimes understandable because in the worksheet, there were two illustrations helping students to make moves. The picture of the shop is in the left; the picture of the bank is in the right. The context "shopping voucher" was quite complicated to what they saw in the picture. Because they interpret "voucher" here as paying money to the shop. Also, it was realized that the illustration distracted them. Therefore, this kind of confusion can be caused by the context and the problem given. Afterwards, the teacher explained a bit about the role of zero in this activity. The teacher asked what the students know if they stop at zero. Then the teacher explained that zero means the state when one has no money and has no debt as well.

c. Mini Lesson—Ordering Integers

1) Mathematical goal:

Students can order and position integers on a number line

2) Overview of the activity

The mini lesson was conducted directly after the second activity. The mini lesson consisted of some problems in ordering integers. The context used was the score of the game for each player. The task of the students was to order the number and determine who the winner and the loser are in the game. They were asked to rank the players as well as the numbers. The activity was done in a group of students similar to those of the second activity. The good thing was the students managed to relate the numbers to the previous context in the previous activity. They use debt and money to decide the winner, the second winner, the third winner, and the loser.



Figure 5.33 The students did the mini lesson in a group of three students

Most of the groups can do this task easily and correctly. They put the name of the payers in appropriate boxes based on the order of the numbers. What they found difficult was that the second activity which included the number line and some numbers provided. They were asked to put a number in certain and approximated position on the number line. The clue of doing this task was that the students must aware of the position of negative and positive numbers because there were still some students who did not really understand that negative numbers should be put in the left of zero and the positive numbers should be put in the right of zero.



Figure 5.34 The students estimated the position of numbers on the number line

Different strategies appeared in determining the position of the number on the number line. The group of Canya, Egi, and Lilis were quite fast in understanding the problem. However, they still need extra explanation from the teacher because two examples (20 in the positive numbers; -18 in the negative numbers) given in the worksheet did not help them much. To know the position, Canya directly made line linking the number to the position in the number line. She just approximated it by based on the position of 20 in the positive number and -18 in the negative number. If it was small number she pointed forward from zero, if it was quite big number, she pointed backward from 20. Afterwards, she linked the number to her approximated point. Nana who was in the same group of Rochma and Ayoeng did the problem in the different way. To help her determining the position of the numbers in the number line, she pointed from zero approaching the certain number and then made a line linking them to the number line. After linking the number to the number line, Nana wrote the number below the number line to help her finding the position of other numbers more easily and to remind her if the number already linked or not.

Another different strategy in finding the position of numbers in the number line was to predict the position first and gave them marks. Afterwards, the line linking them to the number line was drawn.



Figure 5.35 Arina's group work



Figure 5.36 Canya's group work

3) Discussion

This part will elaborate the comparison between the actual learning of the students and the conjectures of the students' responses.

The Hypothetical Learning	The actual students' learning
Trajectory	
They might think that the bigger negative number, the smaller value it has.	Some students thought that -18 was much bigger that 5; they came to realize when they relate it to debt context. They said that -18 was smaller because it meant more debt someone had.
In the first task, they may put biggest positive number in the position of the winner, the second biggest in the runner up, the third bigger number in the second runner-up position, and the smallest negative numbers in the position of the loser.	The students order the name of the person based on the amount of debt the person had. The biggest amount of debt must be put in the box of the "loser". And the least amount of debt put in the box of the "first winner"
They may put the biggest positive numbers to the last right position from zero. For example, because the number 20 is already connected to the line as an example, they may find that 15 is the biggest positive number which should be connected to exactly the left side of 20.	Some students did not directly connect the number to the correct position unless they wrote and order the number below the number line at first.
Students may put the positive numbers in the right side of zero and negative numbers in the left side of zero	Some students put -18 next to 20 because they thought before 20, there should be -18 which close to it; some others put -18 in the left of zero, exactly in the most left part of the number line.
Students may have discussion why 5 are more than -6, and why -1 is more than -13.	The discussed those number by relate them with the debt context. In this case, 5 is bigger because it represent money, not debt.

Table 5.3 Comparison between Hypothetical Learning Trajectory and The Actual Students' Learning of Mini Lesson

It can be concluded that this mini lesson help the students to arrange the negative numbers in a good order. Their understanding about the magnitude about negative numbers still be associated with the previous context, which was debt context.

d. Activity 3: The Number Line Game

1) Mathematical goal:

Students can use number line as a tool for calculating number

2) Overview of the activity

The teacher started the class by reviewing the previous activity about number line. One student was asked to make a straight line. Although there is no such activity in HLT, the teacher wanted to repeat the same opening activity to make students familiar with a number line. It is understandable because the upcoming activity was related to number line. Also, the teacher said that he wanted the students will be able to make their own number line. Therefore, it was agreed about the importance to emphasize this part about number line as a good start of the activity. Students were called one by one to put numbers on the line.

The teacher demonstrated the example of previous problem in the number line as a review to previous lesson. Afterwards, the teacher took two dice to demonstrate the example of the activity about dice game. The teacher used the dice with different colors, red and blue. The red dice showed the number of jumps to the left and the blue dice showed the number of jumps to the right. In this activity, students were challenged to play a game involving a number line. Students worked in the group of 4-5 students. The teacher explained that he had two kinds of dice; red and blue dice. The blue dice is used for getting money. If the number on the red dice is 5. So the students jumped to the right 5 jumps on the number line; the red dice is used to jumps to the left. If the number on the red dice is 2, then the students should move 2 jumps to the left. Given that example, students were expected to be able to do the activity about the dice game. As explained previously that in each lesson, the teacher asked the students to choose the members of their groups themselves to avoid the fight among them and to encourage more discussion. After spending several times in selecting the group, the teacher was assisted to distribute the worksheet and the game set including the dice and the board of a number line used in this game. The worksheet of this activity is the table of the game on which the students fill the number on the red dice, the number on the blue dice, also the number where they stop at. The teacher circle around the classroom to monitor what students did in their group. The teacher was helped to make sure that the students understood how to play the game.

Arina : (tossing the dice). "This is 5. That is 4. (writing on her worksheet)..
So 1, 2, 3, 4, 5,..1, 2, 3, 4. Yes, I stop at 1."
Teacher : "Okay, well done. Put the number on the last column."

It seemed that Arina understood how to play the game well. While in other group of Dzakia, Jasmine, and Wanda, they confused to start the game because they did nit know which move should be done at first; the move to the left of the move to right. What they got in the first round was 3 on the blue dice.

Jasmine	: "1, 2, 3,"
Dzakia	: "Go to the negative first"
Jasmine	: "1, 2, 3" (counting to the left)

Dzakia : "What number is on the red dice?" Okay..2, So move two jumps to negative first.

It should be observed where this group got the information about making moves to the left side (negative numbers should be done first). They were suggested to investigate by themselves which moves should be done first and what is the difference of making the either the left or the right moves at first. It is important for students to know for the next lesson to introduce them about the commutative property of addition and subtraction. What can be reflected from this activity that this case lead to the discussion to differ 3 + (-2) and -2 + 3. This discussion was postponed because the students were not ready yet to receive such information.

It was observed from Arina's group that Arina still could not cooperate with her group well because she seemed to dominate her group. What she did was to tell everyone in her group to write down the number in the worksheet and let herself to make moves on the number line. It was a difficult situation because other students in her group agreed on what she said. Consequently, no discussion happened. Also, it was not good for the other students' learning since they did not experience by themselves.



Figure 5.37 Arina filled the table of the problem

Researcher	: "Whose turn?"
Egi	: "My turn" (toss the dice)
Researcher	: "So, what do you get?"
Egi	: "The red dice is 5, the blue one is 5(while writing on the
worksheet),	so the final position is 0 (answer convincingly)
Researcher	: "Wow, how do you get the answer so quickly?"
Gita	:"Yes, you have to check it"
Egi	: "Okay, 12345 (pointing on the number line and change
the move to	another direction) 12345. ves it's 0."



Figure 5.38 The students figure out the final position after making movements

It was noticeable that Egi in this case probably could see the pattern of the numbers or guessed it that she can predict that 5 jumps to the left and 5 to the right could result in zero. Still, her friends were not convinced yet and thought Egi needed to check by making moves on the number line. Dzakia's group also experienced the same situation. Jasmine got 1 both for red dice and the blue dice. She found it needs to be check although other friend told her that the answer should be zero. The next turn of Dzakia getting 4 both for red dice and the blue dice, they had been finally convinced that the answer of the same number for red dice and the blue dice resulted zero. However, most of the students were able to see the pattern only for the same number of the two dice. Given different number on the dice, they still did some checking by making moves on the number line.

Since the group of Dzakia were done with the tables, we came to the students to talk about the conclusion about the activity. When reading the statement of the conclusion, they did not directly understand. Therefore, the situation concerning to that was demonstrated. The statements in the conclusion asked about (1) If the number of the red dice is bigger than that of the blue dice, what number (positive or negative) the final position would be, and (2) If the number of the red dice is smaller than that of the blue dice, what number (positive or negative) the final position would be. They were expected to be able to deduce from the tables for example, by knowing that "6 on the red dice, 2 on the blue dice resulted in negative number, which is -4". The similar thing happened when they judge that "5 on the blue dice, 3 on the red dice, resulted in the positive number, which is 2". To accommodate this, the dice was used to show them the situation, given 6 on the red and 2 on the blue will give them 4. It takes severalminute-discussion for them to reach the conclusion that 4 is positive number. This because it seemed that they felt any other answer but positive number was expected. Afterwards, It was easy to continue the statement for them. They did the same thing to investigate it by playing some numbers from the two dice.

Jasmine : "If the number on the blue dice smaller.." (taking two dice) Dzakia : "This one is 6 (taking red dice)." Jasmine : "This is 1." (taking the blue dice) Dzakia : "1, 2, 3, 4, 5, 6. And then...1." Researcher : "So, what is the result?" Dzakia : "5 min. Negative." Students struggled to decide whether 5 is positive and -7 is negative number. They did not come to that at first. They even said that 5 is big number and -2 is the small number. They meant positive numbers are the big numbers whereas negative numbers are the small numbers. Apparently, the students did not have any idea that what they were doing was related to the addition of integers. In the next lesson, an activity in which they can engage with more formal but simple addition of integers will be introduced.

Posisi Awai	Angka poda dadu biru	Angka pada alaau miniah	Posisi setelah melangkah ke kiri dan ke kanan
START	5	-	-1
START	6	-6	0
START	4	<i>a-</i>	-1
START	5	2	3
START	2	-2	0
START	2	- 2	0
TART		.7	-1

Figure 5.39 Egi's work

Kesin Penju	npulan: Imlahan angka-angka pada dadu biru dan merah adalah
posisi	setelah melangkan ke kara seri dari seri seri dari seri
Apabi	la bilangan negatif lebih besar daripada bilangan positif, menghasilkan bilangan.
Apabila	a bilangan negatif lebih kecil daripada bilangan positif,
Maka n	nenghasilkan bilangan. Qasidis

Figure 5.40 The conclusion of the Egi's group work

3) Discussion

Table	5.4 Comparison between Hypothetical Learning Trajectory and The Actua	1
	Students' Learning of Lesson 3	

The Hypothetical Learning Trajectory	The actual students' learning
Students might think that the more	The students knew that they will have
they jump to the left, they will get to	smaller value of the negative numbers
negative numbers, the more they jump	as they move to the left because they
to the right, and they will get to	thought thee spent money in that
positive numbers. In comparing	direction causing more and more debt
numbers, they might think that the	they could have. Therefore, the more
more they go to the left, they will have	to the left, the value of the negative
smaller number.	numbers will get smaller.
The student may write the numbers on the red dice as negative numbers— they put minus sign in front of the numbers , because they have to move to the left, get closer to the negative numbers	Some students interpreted the negative sign following the number of the red dice as the subtraction sign since most of them thought that the game showed them how to subtract number rather that as the simulation of addition involving negative numbers
The student may write the numbers on	The student just let the positive
the blue dice as positive numbers—put	numbers without any sign because it
plus sign in front of the numbers or	did not really bother them in
just let the numbers without any sign.	calculating the numbers.

e. Activity 4: What is your position now?

1) Mathematical goal:

Students can draw a number line and to operate negative numbers using that number line.

2) Overview of the Activity:

The teacher started the lesson by asking the students one by one to counts from -10 to 10. Afterwards, the students were reminded to the number line activities they had done in the previous meetings. The teacher took sometime to have discussion about the dice game since there was no time for class discussion the previous meeting. He simulated the game by taking the two-colored dice and demonstrate some situation. Using the number line prepared on the blackboard, the teacher showed the students how to process the number by making jumps to the right and to the left.



Figure 5. 41 The teacher asked the students to arrange number cards in a good order The students were asked to answer and demonstrate the problems given by the teacher. The activity was not in the HLT, the teacher did kind of quick activity in which the students need to think fast about the answer—the final position of making jumps based on the number of two dice. The teacher managed to get the students' attention by giving such activity. Surprisingly, the students showed that they could do the problem as well as demonstrate the jumps on the number line. The teacher wrote the problem he had given to the students on the blackboard and by giving mark to each number as 'awal' (start), 'loncat' (the number of jumps), and 'akhir' (final position). The teacher introduced the way to write down the numbers on the dice mathematically. The addition sign represent the combination of two movements based on the numbers on the dice.



Figure 5.42 The teacher used plus sign to represent the combination of two movements



Figure 5.43 The teacher wrote the addition in more formal way

Then the activity continued by the teacher by giving more formal way of writing the numbers. It was surprised that the teacher did such thing earlier that was expected before. In HLT, the written expression of the dice game would be in the last lesson. However, the students seemed had no problem dealing with that because it was quite easy for them to follow the problem based on what they did before. For example, for the problem 2 + (-6), the students understood if it is said as 2 on the blue dice and 6 on the red dice. Given that situation, students could imagine having jumps along the number line and it was not so difficult that they could get -4.

After spending around 20 minutes for class discussion, the students will work in pair to do the worksheet. The worksheet was another activity involving the number line. In this worksheet, they were given the empty number line and they needed to make their own jumps to determine the final position of a person jumping along the number line.



Figure 5.44 The students used a ruler to make distance between numbers on the number line

What they did for the first time was that they made marks on the line and gave them numbers. The students had no experience in using number line as a calculating tool. Therefore, it confused them how to use the empty number line for the first time. Arina and Gita started to use a ruler and marked the number line. Other students spent their time dealing with deciding how many numbers should they made on the number line. I was suggested that some students use only the number they needed and asked them to look at the problem and see the numbers carefully.

The classroom was monitored to see what students did in pair. It was surprising because a student who has low performance could do the problem quite well although she worked the problem alone without any friend.

Researcher	: "If I asked you what is the result of $-2 + 7$?
Fafa	: "-2 + 7?"
Researcher	: "What would be your answer?"
Fafa	: (counting on the air, then looking at the number line) 1,2,4,
-	5,,6 ,7hmm 5."

	Saya berjalan 3 langkah ke kanan Dimana posisi saya sekarana?
Andre	-£
	912 5
	-0+3=-0
Gambarkanlah gerakan Andre pad	da garis bilangan di bawah inil
Gambarkanlah gerakan Andre pad	da garis bilangan di bawah inil
Gambarkanlah gerakan Andre pac	da garis bilangan di bawah inil
Gambarkanlah gerakan Andre pad	

Figure 5.45 Fafa's work



Figure 5.46 Arina's work

3) Discussion

Table 5.5 Comparison between Hypothetical Learning Trajectory and The ActualStudents' Learning of Lesson 4

The Hypothetical Learning Trajectory	The actual students' learning
They may find that if they make more jumps to the left, they may get negative numbers. If they make more jumps to the right, they may get positive numbers.	Some students investigated the some situation by using the dice to understand that more jumps to the left make negative number; vice versa.
They also discuss how the initial position in positive number can be negative numbers; also, the initial position in negative numbers can be positive if they move to the right.	Some students already knew the situation through the dice game in the previous activity.

g. Activity 5: Empty Number Line

1) Mathematical goal:

Students can draw the jumps on the number line to understand the operation of bigger numbers.

2) Overview of the Activity:

In this activity, the students will have a worksheet be done in group. The worksheet was about dice game. The worksheet consisted of 8 problems; 3 of them was the problem with mark number line; 3 others was the problem with empty number line; the rest was the problem of missing values in which the students should determine how many jumps should be made to get to certain numbers. Students will work on this problem in a group of 3-4 students. To start the lesson, the teacher presented an example of the problem in the worksheet. The intention was to check whether the students could understand the problem well. The number lines were provided in each problem to help students to make moves and count. The teacher was assisted to move around the classroom and saw what the students did.



Figure 5.47 the students discussed used marks on number line to make jumps

The problem number 2 showing the picture of dice; 8 on the blue dice and 11

on the red dice was asked to the group of Jasmine, Gita, Nana, and Fafa.

Gita	: "Here is -11." (pointing the first box)
Jasmine	: (writing on the worksheet). "Here?"
Nana	: "8 is in this box."
Jasmine	: (writing on the worksheet). "Here?"
Gita	: (talking to Jasmine) "I think you should jump from 8"
Nana	: "It's should be from here (pointing -11)
Fafa	: "From 8, and make 11 jumps to there" (pointing the left side)

They discussed about which move should be made in the beginning. A student suggested that they should move from 8 and should make 11 jumps to the left. Another suggested that they should move from -11 and should make 8 jumps to the right. This group started to answer the problem by filling the boxes and start counting using number line. At this stage, it was assumed that the students relied on the number line in answering the questions because it was sort of problems they could not count by their fingers. It was expected that they will make jumps on the number line given. In fact, they only made points on that. However, it was not necessary since they understand the problem.

Rohma	: "1, 2, 3, 4, 5, 6, 7, 8." (making jumps on the number line and
	change direction) "1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11." (counting and
	<i>making to 11 jumps to the left).</i> "-1, -2, -3, -4." (count from 0).
	"-4." (writing the answer on the box)
Researcher	: "Are you sure?"
Rohma	: "0,-1, -2, -33." (changing the answer)

Instead of making points, Rohma made jumps along the number line. Since the number line given was a marked number line, they needed to find the number at which they have to start. Also, to know the exact number in which they stop, they needed to count from zero. Most of the students use that kind of counting strategy using number line; however, Rohma made better representation by using the real jumps instead of points.

Focusing on students' discussion about the seventh problem; missing value problem, the students had some confusion on how to put the given number on the number line. The problem asked about 1 + (...) = -4. Here, they needed to find the correct number should be filled in the box. However, not all students could interpret the problem properly.



Figure 5.48 Canya's group work



Figure 5.49 Arina's group work
Learning	Activities	Goals	Actual
Sequence			
The Hop Shop' Game.	Students play a game through money context (having-owing) and investigate negative numbers in representing amount of debt.	-Students can develop the notion of negative numbers	Students make representation of debt by saying "debt of 5" or "5 u". Some others represent debt as "-5" (kurang 5)
How much money/debts will I have? (drawing jumps on the number line)	Students visualize the problem of having debt or having money on the number line.	Students can draw jumps on number line to solve problems relating to the context in the game.	Students understand that "debt of 5" should be put in the number of -5. Having money should be put in positive numbers. Also, students can make jumps in different direction based on the situation of paying money and accepting money.
Mini Lesson— Ordering Integers	Students locate numbers in correct order on the number line.	Students can order and position integers on a number line	Students understand to put numbers in appropriate position on the number line.
The Number Line Game	Students play a game which shows the simulation of adding integers.	Students can use number line as a tool for calculating number	Students understand how to determine the final position (result of addition) of two numbers.
What is your position now?	Students make their own number line to visualize problems related to jumps.	Students are able to draw a number line and to operate negative numbers using that number line.	Students understand that adding integers can be done by moving forward and moving backward along

Table 5.6 The Achievement of the Learning Goal based on the Actual Learning

				the number line
Empty	Number	Students exercise	Students can draw	Students managed
Line		to do some	the jumps on the	to write numbers
•		problems related	number line to	in more formal
		to the previous	understand the	way and translate
		problem and wrote	operation of	them back to the
		the addition in	bigger numbers	context.
		more formal way.		

The overview above shows that the conjectures of the learning in each session are already achieved in some ways. After the learning process, students better understand the concept indicating that they achieve the learning goals. Therefore, the teaching experiment can be stopped in this stage.

C. Analysis of Pre-test and Post-test

The problem of the pre-test and the post-test are the same, the students were asked to solve four problems. The difference between the pre-test and the post test is the order of the problem given.

Problem is about ordering the series 5 numbers in a good order. The series consists of positive numbers and negative numbers. The problem is aimed to see students' understanding about the magnitude of the problem. In pre-test, students did not see any difference between the positive numbers and negative numbers. They did not get that the sign of the numbers will make different value. The students only relied on their prior knowledge about positive numbers to reason the problem. In answering the problem, most of the students omit the negative sign because they thought that the signed numbers are the same as unsigned numbers.



Figure 5.50 A student's solution of problem 1 in pre-test



Figure 5. 51 A student's solution of problem 1 in post-test

The analysis of post-test showed that 14 out of 19 students could answer this correctly. However in the pre-test no one could answer the problem correctly. In the post test, some students already connected the numbers to the context of having-owing money to order the numbers. Given the number with negative sign following it, some students could understand that "—" means debt. Therefore they were already able to say that -12 is less than -5 because -12 mean having more debt.

In the second problem, the students are asked to make sense of negative numbers. Actually, the aim of this problem is to see how the students interpret the problem using their own prior knowledge about numbers. In the pre-test, most of the students answered the problem incorrectly. The question is "What is the result of 7-10?". Most of the students thought that there was no correct answer of that problem because it was impossible to subtract 7 from 10. In the post test 15 out of 19 students could answer the problem correctly.



Figure 5.52 A student's solution of problem 2 in pre-test



Figure 5.53 A student's solution of problem 2 in post-test

The third problem asked the students to complete the number line involving positive and negative numbers. In the number line, some numbers are already provided.



Figure 5.54 The number line with some unknown numbers

In the pre-test, there were only 3 students who can complete the number line correctly. Others only filled the number with all positive numbers without any sign. In the post-test 18.students can complete the table correctly. In the classroom, some students knew number line; but they did not use that number line as a tool for calculating. Some of them admitted that they did not know anything about number line.



Figure 5.55 The confusion of a student in completing the number line

Some students were confused by negative sign in front of the numbers. The figure above showed a response of a students who could not see the connection between the numbers in the right side of zero and the left side of zero.

The last problem is about the addition which is written formally. The students are asked to calculate the result of addition involving negative numbers. In this problem, there are 3 problems.

In the pre-test, most of the students answer the problem in the same way of adding positive numbers. In the post test 15 out of 19 students could already answer the problem correctly. Some of the students use number line to calculate the result; some of them referred back to the having –owing context to find the answer. When being asked about the problem (a), some students prefer to use the context "having debt of 5 plus debt of 5 becomes debt of 10". Some students found it was difficult to answer problem (a) using dice game. It was reasonable because in the previous game, the students were only presented the combination of a positive and a negative number, not a negative number plus negative number.

Probably later in the next stage, it is suggested to develop the dice game to learn not only the addition of a positive number and a negative number, it is expected that the game cal also be used to solve problem consisting of addition a negative numbers and a negative number

a.
$$-5 + (-5) = ...^{0}$$

b. $-2 + 3 = ...$
c. $7 + (-9) = ...^{2}$.

Figure 5.56 A student's solution of problem 4 in post-test

From the result of the post test, it can be concluded that students can get some insight about addition involving negative numbers. Given some problems about addition involving negative numbers, they could refer back to some contexts given form the activities to help them to solve the problems. At this stage of learning development, they are ready enough to learn more abstract topic of negative numbers, for example subtraction involving negative numbers.

Overall, giving students a good basic knowledge of integers in very beginning could help them improve their number sense. As the result, they manage to challenge themselves for advance learning about numbers

D. The Overview of Data Triangulation in Collecting Data

In chapter 3, it is mentioned that the type of data collected during the experiment are observation, field notes in interviewing the students, written work of the students, and video record of all sessions. Some data support each other. It is also found the need to investigate the data triangulation in order to check on the validity and reliability. There some evidences from the lesson will be collected and triangulated. This following table will elaborate some findings from the lesson.

	Type of data		
Evidences in each lesson	written work	video record	interview with the students
Lesson 1 Students managed to give meaning to negative numbers	Students wrote "-9" to state that they borrow \$9	In the video record, a student said that negative sign meant the amount of money that need to pay.	The student said that "-9" as lack of 9 (kurang 9),
Students compare negative numbers		Students discussed that -21 should be bigger than -24	Students discussed that -21 is bigger than -24 because it means bigger amount of debt
Lesson 2 -Student can locate the number in the context in appropriate place on the number line	Students wrote debt of 4 in the place -4 of the number line.	The students marks number -4 on the number line to state debt of 4	A student explained that debt of 4 should be located in the left of 0. To be exact in -4.
-Students determine the meaning of the number of jumps	Students draw jumps to the left 7 jumps.	One of the students in group argue that they should move to	

Table 5.1 Data Triangulation of Teaching Experiment

in the context		the left 7 jumps	
		because they pay	
		money.	
Mini lesson	Student	Students put -13 in	The student
	connecting	the right of -18	explained that -18
-Students	number -13		of the problem is
connected random	between -18 and -		the least.
numbers in correct	6		
position of the			
number line			
-Students estimate	Students write 20	Students said that	
numbers on the	in the right of	20 is the biggest	
number line.	zero; the furthest	positive number in	
	position of zero	the problem.	
	among all		
	numbers.		
Lesson 4	Student marked -4	Students make 9	Students repeated
-Students	as the final	jumps to the left	the action of
determine the final	position of initial	and stopped at -4	counting the
position of some	position 5		9 jumps
movements	followed by 9		y Jumps.
	jumps to the left.		
	Student write that		The student said
- Students	the final position		that she does not
determine the final	of -5 (number of		need to make
position of some	red dice) and 5		she already know
movements	(number of blue		that 0 must be the
	dice)		final position.
Lesson 5	Student write	The students use +	
Students realize	-8+3 = 5 to say	to combine the	
that addition	that he already	number; the	
process can be	make 3 jumps to	starting position	
visualize with the	initial position	and the number of	
Inumber line	initial position -8.	Jumps Studente melve	The student that
Lesson o		jumps in the sir	he could imagine
number line to		jumps in the air	his number line in
lumber line to		it in the worksheet	his own mind.
know the result of		it in the worksheet	

addition		

Some evidences from the teaching experiment above showed the data triangulation to strengthen the interpretation about the learning process of the students. The elaboration of each data shows the consistency among them which provide full information about some investigation. It is clearly seen that by triangulating data, the internal validity will be increased. Also, the chosen fragment of the video record used in the analysis contributes in increasing the internal reliability to minimize the subjectivity of researcher's point of view.

CHAPTER VI

CONCLUSION

The aim of the research is to develop the learning activities on negative numbers with the use of games which can improve students' conceptual understanding on negative numbers. To answer the research question and research sub questions above, some evidences from the sequences of learning will be elaborated. Later, some suggestion and recommendation for further studies will be described to contribute to improvement in learning addition of negative numbers.

A. Answer to Research Question

To answer the research question above, it is essential to answer some specific research sub questions by considering on what happened during the lesson. Some important situations relating to students learning were summarized and the actual of students thinking as comparison to the conjectures which have been made relating to students' thinking and learning were investigated.

1. Summary of each sequence of students' learning

The first sub research question is related to whether having-owing context support students' understanding about negative numbers. To answer this question, the focus is on the first lesson about the game with having-owing context. The game was designed as an activity to support students' first conception about negative numbers. The chosen context which is about having and owing money was intentionally selected since most of the student already familiar with money. Through this context students could manipulate the situation involving negative numbers in it. Although the way they represent debt by using negative numbers should be introduced by the teacher, Students could relate the concept with having-owing money easily. The emphasis is on the process how they can build some representation of debt as well as the way the talk about it. For third grade students, negative number is something new for them. Specifically in our research subject, it is found that they were not familiar at all with the notion of negative numbers. Through the lesson, students already managed to associate the negative numbers as the money they have to pay to somebody. They made many ways in representing amount of debt; for example by saying it as "debt of 1" or "1 U". And some brilliant students also made remarkable way by representing amount of debt as "-1". Moreover, they also managed to explain it with the help some guidance questions. In this lesson, the students also learned to order negative numbers. It can be concluded that in this activity, the students can get the meaning of the negative numbers within the context of having-owing and they could have some representation of the context for example representing amount of debt using negative numbers.

In mini lesson, the students were asked to order integers and to put some given integers. After experiencing they can order integers in the good order by considering the having-owing context. The students also managed to explain that - 12 is less than -1 because -12 represent the bigger amount of debt. They also noticed that 2 is much bigger than -10 because 2 represent amount of money. The students also learned how to put numbers in correct position. This will help students to get familiar with number line involving positive as well as negative numbers. By knowing the position of the number, the students understand that the

value of the positive numbers will get bigger as they are move away from zero; the value of negative numbers will get smaller as they move away from zero. This understanding will encourage the student in learning further concept about addition of negative numbers. It can be concluded from this activity that students learn how to arrange integers on the number line and can compare the value of those numbers.

In the second activity, the students were asked to use number line as a model to add numbers based on the context. The addition involved positive and negative numbers. Using the number line, they were expected to make jumps along the number line. They should understand in which number they should make jumps and how many jumps they should move to determine the final position. The context used in this activity was having-owing context. It was found that students can model the situation given in the problem by using number line. The context also supported the students to understand negative numbers. In the third lesson, the student played a game in which they learned about addition of integers. By using two dice they made some situation of adding two movements along the number line. They could determine the final position of the two movements. The simulation of the game can assist them to learn addition of integers by making jumps based on the numbers on the two dice. It can be concluded that in this lesson students managed to understand the simulation of addition process of integers.

In the fourth lesson, students were encouraged to work with empty number lines. The context given was the situation of a person jumping along the number line. The task for the students was to determine the position of the person if he moved from the certain position. The students could make their own number line and make jumps based on the problem. Here, the students learn how to make their own number line and use it to draw jumps. Using this model, they understood that negative numbers show the move to the left and positive numbers show the move to the right. It can be concluded that this activity support them to use model in leaning addition of integers.

The fifth lesson asked the student to do some different tasks including the simulation of dice game using marked number line and using the empty number lines, and the task of the modification of the dice game in which they needed to determine the number of jumps to reach certain position. The more advance learning were also asked to students that they needed to write down the numbers in formal way. The students were encouraged to put number into mathematical sentence showing that the simulation of the dice game is the process of adding positive numbers and negative numbers. It was found that students could use the context to learn the further concept of addition. It is expected that they can come back to the context if they are given the similar problems. This lesson helped the students to learn higher level of addition of integers with the help of model and context of the problem. This activity also promotes number line as *a model for* as they could use it as the tool for representing the situation in pre-formal level.

2. Answer to the Main Research Question

Before interpreting the evidences throughout the teaching experiment, it needed to reveal that according to description of chapter 5 about the comparison

of HLT and actual learning, the teaching experiment stop at cycle two meaning that the HLT and actual learning is considered similar and the learning goals has been achieved. Therefore, the conclusion can be reviewed form the learning process. The main research question for this study is "How to improve students' conceptual understanding of negative integers". To answer this research question, it is necessary to refer back to the analysis of each lesson in answering the sub research question. It can be concluded that the sequence of learning can improve student's conceptual understanding of negative integers by support them to use the model of number line to solve problems within the context. The representation they made in each activity (in a game involving having-owing context and in dice game) can be a model of situation. The activities also promote number line as a model for determining the final result of addition of two integers. However, some activities are required to be improved for better learning of the students. The activity in the dice game for instance probably can only be used for learning addition. It needs much modification that it can be use for further learning, such as subtraction of integers. There were also students' responses which were not listed in our conjectures. Nevertheless it helps to encourage more classroom discussion.

3. Discussion

This part will emphasize about the contribution of RME to the sequence of teaching and learning process and the role of the teacher in conducting the designed activities.

Local instruction theory on learning addition involving negative numbers

The aim of the present study is to contribute to the development of local

instruction theory for the addition involving negative numbers in the grade three of primary school.According to Gravemeijer (2004) local instruction theory refers to the description of, and rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topic. The tool and the activities proposed in the instructional design are summarized in the following table.

Activity	Description of activity	Learning goal
The Hop Shop' Game.	Students play a game through money context (having-owing) and investigate negative numbers in representing amount of debt.	-Students can develop the notion of negative numbers
How much money/debts will I have? (drawing jumps on the number line)	Students visualize the problem of having debt or having money on the number line.	Students can draw jumps on number line to solve problems relating to the context in the game.
Mini Lesson—Ordering Integers	Students locate numbers in correct order on the number line.	Students can order and position integers on a number line
The Number Line Game	Students play a game which shows the simulation of adding integers.	Students can use number line as a tool for calculating number
What is your position now?	Students make their own number line to visualize problems related to jumps.	Students are able to draw a number line and to operate negative numbers using that number line.
Empty Number Line .	Students exercise to do some problems related to the previous problem and wrote the addition in more formal way.	Students can draw the jumps on the number line to understand the operation of bigger numbers

a. Realistic Mathematics Education (RME)

The activities were designed with consideration of basic tenets of RME. The first tenet of RME is *phenomenological exploration*. As already described earlier in chapter 2 of this report, that in improving the conceptual understanding, the students should be involved in a meaningful experience to reinvent the characteristic of the concept. The experience in learning addition of integers was preceded with the lesson aimed at introducing negative numbers. The activity of playing the game "Hop Shop" involving having-owing context helps students to explore the meaning of negative numbers within the context. The students learn how to make representation of the amount of debt by using negative numbers. For students in grade three with no pre-knowledge about negative number, they could make big jumps in shifting the informal level to pre-formal level. The basic counting included in this activity also can be a good starting point for them to learn more advanced learning of addition of integers. Through the game, the students also experience by themselves to know how negative number appear in their life. They learn to understand that negative numbers can be used for representing amount of debt while amount of money is represented by positive numbers.

The second tenet of RME is using models and symbol for progressive mathematization. The progressive mathematization is the development from informal level through more formal mathematical concept. The designed activities encourage students to develop their own model of situation and lead them to the self-developed model or number line. The students in one activity were invited by the teacher to locate numbers on the big number line on the blackboard; here they tried to construct the number line to be used for the next learning sequence. The third tenet of RME is the use of students' contribution. In the first lesson, there was a group of students who could represent amount of debt by using negative numbers. Although at first they did not interpret the sign as negative numbers, the thinking process of the students can lead to classroom discussion for the whole class. Besides, a number line used in some activities was constructed by the students as they did the activity of ordering number cars on the blackboard.

The interactivity of the sequences of learning appeared when students did some discussion in a group or pair of students. They shared their opinion and considering others' opinion to decide the solution of some problems. The role of the teacher plays important roles in promoting guided reinvention and support the interaction between teacher and students and the interaction among students. The fifth tenet, the intertwining of various learning strands or units enable the student in later stage of learning could integrate the knowledge of addition of negative numbers to understand any other mathematical concept including in number sense and higher level of number computation; for example, to learn negative decimal and fraction. The good conceptual understanding in this concept can be good preparation for students to understand other kind of numbers higher education. Immediately, the students also need the experience of learning addition of integers to grasp the subtraction of integers.

b. Classroom Discussion

The socio mathematical norms run properly during the sequence of learning. The discussion between teacher and students appeared and the discussion among students held during the lesson as expected. The teacher facilitated the discussion among students. The teacher played a role as a facilitator and encouraged each student to come up with some arguments that lead to discussion. It was noticed that the teacher also asked the students who rarely participate in the classroom. The teacher anticipated that one students became very dominant that others. Therefore, the teacher enable all students got their own turn to present something or answer a question during the lesson.

c. The Role of the Teacher

The students' learning and classroom circumstance around depend on the skill of the teacher in managing the situation of the classroom, encouraging the discussion and improvise the learning adjusted to the characteristic of the students.

1) Encouraging Guided Reinvention

The teacher brought the students to understand the pre-formal concept through the process of exploring the context which is bridged with the emergent modeling. In learning negative numbers, the teacher asked some questions to stimulate students' thinking. These were the questions posed by the teacher to open the discussion.

- 1) "How did you get -3? What do -3 mean? Do you still have money or not?"
- 2) "Which one is bigger? 0 or -1?"

- 3) "Where is the position of -8? At the left or the right of zero?"
- "If you have 6, then you have to pay 10 for T-Shirt? How much is your debt now? How do know?"

The questions asked by the teacher stimulated the students to think about context more deeply that it could contribute to good understanding to the basic concept of learning negative numbers.

2) Managing the Classroom Situation

The weakness of this research is the problem about inconsistent focus group. The same focus group for each lesson is essential in a study to make sure that the investigation of a students' development in a group can be explored that a researcher can get deep information thoroughly. However, the teacher found the need to change the group in each lesson that the former focus group now separately divided into other groups. It is understandable that the teacher felt that the students in the classroom were not accustomed to group work. The teacher found that it was difficult to divide the group since the students requested that they wanted to choose their own group. The situation of coping with this problem took longer time than what has been expected. The teacher offered the solution to students give some changes to the group in each lesson. Also, that it was quite fair enough for the student as long as the students could have discussion amongst them and can cooperate well with the group chosen by them. Here, the teacher was challenged with complicated situation about the students and tried to mediate the situation democratically. Although, it was outside our plan in conducting the

study, it is found that it was no problem as long as the written work and the video observation of students in our former focus group are available to be observed.

B. The Weaknesses of the Research

In conducting this present study, there are some unavoidable flaws while doing cycles, collecting data, analysis process, or in conclusion making. The researcher responsibly found the need to reveal some of them for transparency and the improvement of the study. Some issues regarding to this weakness will be elaborated as follows;

1. The characteristics of the students in the first cycle and those in the second is quite different

The researcher realize the important of choosing the comparable research subject because if the subject involved in both cycles have different characteristic, it is really difficult to give the same conclusion about their learning. Some aspect such as the difference of ability of the subject involved, the style of the learning, gender, should be also considered as the key point. In this study, there are some differences of the subject such as gender and the style of the learning of the students which is not included in the analysis of the learning. As the result, some conjectures is achieved by the student from the first cycle, but not by the students in the second cycle and vice versa. Therefore, it is concluded that the cyclic process(es) need comparable research subject to support consistent interpretation. 2. Overlooking the learning style of the student

In designing instructional activities, it is admitted that the learning style of the students is not taken into consideration. The learning style, to some extent influence the result of experiment and if it is not included when designing the activities, it cause some weak findings. Therefore, the activities should consider the way the students learn and what kind of activity that help them the most in learning.

3. Some evidences from the data collection do not strongly support the triangulation.

Triangulation is required to get full picture of information and to strengthen the conclusion of the findings. Nevertheless; in this study, there are some weak source of complementary data, therefore it influence the consistency of data collection methods used in this research. The researcher realize that it take more time to analyze the entire data to check the validity of the findings.

C. Recommendations

Realistic Mathematics Education (RME) is recommended to further studies as a fundamental principle in designing a sequence of learning and conducting teaching and learning process for better education. In this study, RME could improve the socio norms, especially the classroom discussion. Concerning to the topic on this study, negative numbers is widely known as abstract concept for primary school, to tackle this problem, RME offer a meaningful learning that the abstractness of this topic can be bridged with the use of the appropriate context. Money context with having-owing situation could be one way to introduce students about negative numbers. Moreover, learning through game activity could be an option for enjoyable and fun learning. It is obviously true that student in grade four even in grade three had no problem in doing operation of integers if they were given some rules to do so. However, applying the rules of counting without basic knowledge will not help students for advance learning and had no contribution for their number sense. Therefore, another recommendation is that the teacher in grade four should design an activity in which they opportunity to learn giving the meaning to negative numbers, not immediately giving the formal approach to deliver this concept.

REFERENCES

- Arwadi, Fajar. 2012." Design Research on Mathematics Education: Developing a model and using context to support students' understanding in the topic of addition involving negative numbers for fourth grade students in primary school". Unpublished master's thesis, Sriwijaya University, Palembang.
- Bruno, A., Martinon, A. 1999. "The teaching of numerical extensions: the case of negative numbers". *International Journal Mathematics, Science, and Technology. Vol.* 30 No.6, pp. 789-809.
- Borba, R., Nunes, T. 1999. "Young children's representation of negative numbers". *British Society for research into learning mathematics*. Vol. 19(2), pp. 7-12.
- Chadwick, David. 2009. *Approaches to Building Conceptual Understanding*. New Zealand: Learning Media for the Ministry of Education.
- Cobb, P., Yackel, E. 1996. "Sociomathematical norms, argumentation, and autonomy in mathematics". *Journal for research in mathematics education*, Vol. 27, pp.458-477.
- Freudenthal, H. 1999. *Didactical Phenomenology of Mathematical Structure*. New York: Kluwer Academic Publisher.
- Gravemeijer, Koeno. 2002. Emergent modeling as the basis for an instructional sequence on data analysis. Retrieved December, 1, 2011, from The University of Auckland, Department of Statistic website: http://www.stat.auckland.ac.nz/~iase/publications/1/2d5_grav.pdf
- Gravemeijer, K., Terwel, J. 2000. "Hans: Freudenthal: a mathematician on didactics and curriculum theory". *Journal Curriculum Studies* Vol. 32 No. 6, pp. 777-796.
- Gullick, M., Wolford, G., Temple, E. 2012. "Understanding less than nothing: Neural distance effect for negative numbers". *NeuroImage*, Vol 62, pp 542-554. doi:10.1016/j.neuroimage.2012.04.058
- *Kilhamn, Cecilia*.2011. "Making sense of negative numbers" Master's thesis, University of Gothenburg, 2011. Retrieved from <u>https://gupea.ub.gu.se/handle/2077/24151</u>

- Linchevski, L., Williams, J.1999. "Using intuition from everyday life in filling the gap in children's extension of their number concept to include the negative numbers". *Educational Studies in Mathematics*. Vol. 39, pp. 131-147
- Heeffer, A. 2011. "Historical objections against the number line". *Science and Education*. Vol. 20, pp. 863-880.
- Stephan, M., Akyuz, D. 2012. "A proposed instructional theory for integer addition and subtraction". *Journal for Research in Mathematics Education*. Vol. 43 No. 4, pp. 428-464.
- Streefland, L. 1993. "Negative numbers: concrete and formal in conflict?: Contexts in mathematics education". Proceedings of the Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA), Brisbane, 1993, 62376- 62376.
- Streefland, Leen. 1996. "Negative numbers: reflection of a learning researcher". Journal of Mathematical Behavior. Vol. 15, pp. 57-77.
- Vlassis, J. 2004. "Making sense of the minus sign or becoming flexible in negativity". *Learning and Instruction*. Vol.14 No.5, pp. 469-484.
- Whitacre, I., Bishop, J.P., Phillip, R. A., et. al. 2011." Integer: history, textbook approaches, and children's productive mathematical intuitions". Retrieved December 9, 2011, from San Diego State University, College of Education Web site:http://coe.sdsu.edu/ste/projectz/documents/2011PME-NA% 20Integers% 20Textbook% 20Approaches% 20History.pdf.
- Whitacre, I. Bishop, J.P., Lamb, Lisa L.C., et. al. 2012. "Happy and sad thoughts: An exploration of children's integer reasoning". *Journal of Mathematics Behaviour*. Vol. 31, 356-365.

Appendices

Teacher Guide. Student materials. Scheme of interview and observation. Overview of Hypothetical Learning Trajectory. Pre-test and Post-test

Teacher Guide

Standar Kompetensi

Menjumlahkan dan mengurangkan bilangan bulat

Kompetensi Dasar

- Mengurutkan bilangan bulat
- Menumlahkan bilangan bulat

1. Tujuan Pembelajaran

Siswa dapat memahami bilangan negatif melalui konteks permainan dengan konteks *utang -bayar*

2. Indikator pencapaian belajar siswa :

- Memberikan contoh kegunaan bilangan negatif dalam kehidupan sehari-hari
- Mengenal bilangam negatif dalam konteks pembelajaran
- Menerapkan bilangan negatif dalam konteks *owing-having (utang-bayar)*
- Menggunakan tanda negatif pada bilangan

3. Kegiatan

A. Awal

Pembelajaran dimulai dengan berdiskusi bersama siswa mengenai pengetahuan awal mereka mengenai bilangan negatif. Guru dapat mengajukan beberapa pertanyaan sebagai pertanyaan penggiring yang membantu siswa sampai pada pemahaman bilangan negatif. Pertanyaan tersebut di antaranya sebagai berikut.

- Sekarang saya mau bertanya apa yang kalian ketahui tentang bilangan positif, dapatkah kalian memberikan contoh bilangan positif?
 - (siswa dapat merespon dengan menyebutkan angka 1, 2, 3, 4 dan seterusnya)
- Bagaimana dengan bilangan negatif? Bagaimana bilangan negatif digunakan dalam kehidupan sehari-hari?
- Siapa yang sudah mengenal tanda negatif? Bagaimana menuliskan tanda negatif pada bilangan?

B. Inti

- Guru memperkenalkan permainan kepada siswa, membaca, dan menjelaskan aturan permainan kepada siswa.
- Siswa dan guru berdiskusi mengenai pengalaman anak-anak berbelanja.
- Guru menanyakan kepada siswa apakah mereka sudah mengenal istilah berhutang dan menjelaskan kepada siswa bahwa permainan yang akan mereka mainkan dalam

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Rencana Pelaksanaan Pembelajaran (RPP)

- kelompok berkaitan dengan utang. Guru juga tidak lupa memberikan pemahaman kepada siswa bahwa berhutang sebenarnya tidak baik untuk dilakukan.
- Siswa duduk dalam kelompoknya masing-masing.
- Guru membagikan satu set permainan kepada masing-masing kelompok.
- Masing-masing kelompok memilih teman mereka yang akan menjadi bank dan yang akan menjadi pemilik toko.
- Siswa yang menjadi pemilik toko akan mendapatkan lembar transaksi berupa catatan mengenai jumlah uang yang tersisa dari masing-masing pemain setelah mereka membayar sejumlah uang sesuai dengan yang tertera pada kotak berlabel.
- Siswa yang menjadi bank sekaligus menjadi pemain harus memisahkan keuangannya.
- Siswa menentukan sendiri siapa yang mendapat giliran pertama hingga yang mendapat giliran terakhir.
- Guru dan siswa berdiskusi tentang jalannya permainan diantaranya mengenai cara siswa menuliskan sisa uang dan menyimbolkan utang.
- Siswa yang pemilik toko dibantu oleh guru pada saat menuliskan lembar transaksi dan bersama-sama pemain lainnya melakukan pengecekan apakah siswa (pemilik toko0 mengisi lembar transaksi dengan benar.
- Guru memastikan bahwa sisa uang yang mereka pegang saat itu harus sama jumlahnya dengan uang yang dituliskan oleh pemilik toko pada lembar transaksi.
- Pada saat pembelajaran berlangsung, guru memastikan bahwa pemain yang mendapat giliran berikutnya harus benar-benar menunggu pemain yang lain menyelesaikan transaksi agar siswa yang menjadi pemilik toko tidak kebingungan.
- Apabila permainan siswa telah sampai pada situasi "utang" kemudian guru mengajak siswa untuk berdiskusi mengenai situasi masing-masing pemain, misalnya "Ali sudah kehabisan uang, ini berarti Ali harus berhutang pada Rangga(pemilik toko) sebesar \$3", kemudian setelah Ali mendapat giliran selanjutnya, Siswa dibantu oleh guru untuk dapat memahami situasi selanjutnya misalnya "Ali sudah berhutang \$3, kemudian Ali berhutang lagi \$5, berapa banyak utang Ali sekarang?"
- Apabila ada kesalahan perhitungan uang, guru menjelaskan bahwa yang bertanggung jawab adalah semua pemain, bukan hanya tanggung jawab siswa yang menjadi pemilik toko.

Selama permainan berlangsung, siswa kemudian diajak berdiskusi mengenai cara mereka dalam menuliskan utang dan bagaimana mereka melakukan perhitungan.

• Guru mengamati apa yang didiskusikan anak dalam kelompok, bagaimana mereka memaknai utang, bagaimana mereka menghitung "utang 2 ditambah utang 3" dan sebagainya.

• Apabila salah satu pemain telah berhutang lebih dari \$20 kepada pemilik toko kemudian guru menjelaskan kepada semua siswa dalam diskusi kelas bahwa setelah permainan selesai siswa diminta untuk menentukan siapa pemenang dari permainan dan siapa yang kalah. Siswa diharapkan dapat mengurutkan angka-angka yang tertulis pada lembar transaksi yang merupakan angka-angka negatif. Siswa harus dapat menjelaskan mengapa pemain yang mendapatkan skor -4 lebih "baik" daripada pemain yang memperoleh skor -11. Guru dapat membantu siswa menjelaskan bahwa -11 berarti "berhutang 11"; -4 berarti "berhutang 4" sehingga yang hutangnya paling sedikit akan menjadi pemenangnya.

C. Penutup

Pembelajaran diakhiri dengan diskusi antara siswa dan guru. Guru membimbing siswa untuk dapat menyimpulkan apa yang mereka dapatkan dari kegiatan belajar sambil bermain yang baru saja mereka lakukan. Guru menutup pembelajaran dengan menjelaskan bahwa siswa tidak diperbolehkan berhutang dalam kehidupan mereka sehari-hari karena berhutang tidak baik. Mereka hanya boleh berhutang dalam permainanan "Hop Shop" saja.

4. Alokasi waktu : 2 x 35 menit

5. Media Pembelajaran:

Media pembelajaran yang digunakan adalah satu set permainan yaitu:

- Papan permainan 'Hop Shop'
- Dua buah dadu
- Uang permainan
- 5 buah pion permainan (5 orang pemain)
- 5 buah kartu 'kesempatan' dan 5 buah kartu 'dana umum'
- 1 lembar transaksi
- Aturan permainan (terlampir)

6. Materi Pembelajaran: Bilangan Bulat

Pengenalan bilangan bulat negatif

Pengenalan dan pemahaman siswa mengenai konsep bilangan negatif akan sangat dipengaruhi oleh pengetahuan awal siswa mengenai bilangan asli. Siswa pada kelas 3 mungkin tidak lagi mengalami kesulitan dalam menjumlahkan dan mengurangkan bilangan asli. Sebelum memperkenalkan kepada siswa mengenai operasi penjumlahan dan

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pengurangan bilangan negatif, dibutuhkan sebuah kegiatan pembelajaran yang dapat membantu siswa memahami bilangan negatif. Pembekalan siswa dengan pemahaman mengenai bilangan negatif melalui pengalaman belajar yang bernakna dapat membantu siswa dalam mempelajari bilangan yang lebih lanjut di masa yang akan datang. Dalam pembelajaran ini, konteks yang digunakan adalah konsep utang dan bayar. Konteks ini dipilih karena melalui pengalaman siswa dalam menggunakan uang, siswa dapat membayangkan situasi yang terjadi apabila seseorang sudah tidak punya uang dan harus meminjam kepada orang lain. Situasi seperti ini disajikan dalam bentuk permainan yang sebenarnya sudah familiar bagi siswa (permainan monopoly). Selama permainan berlangsung, guru akan membantu siswa merepresentasikan utang dengan angka negatif yang digunakan apabila seorang pemain meminjam uang kepada pemain lainnya.

7.Lintasan Belajar Siswa

(Hypothetical Learning Trajectory)

- Siswa menentukan sendiri siapa yang akan menjadi pemilik toko ataupun yang menajadi bank pada permainan.
- Siswa yang menjadi pemilik toko akan menuliskan \$15 pada lembar transaksi sebagai modal awal bagi setiap pemain.
- Pada saat pion salah satu pemain berhenti pada salah satu kotak berlabel barang berikut harganya, siswa membayar sejumlah uang yang tertera di dalam kotak.
- Siswa yang menjadi pemilik toko akan berkata "kamu berhenti di kotak berlabel tas sekolah seharga \$8, kamu harus bayar \$8 ke pemilik toko".
- Siswa yang menjadi bank akan memberikan uang \$3 setiap kali pemain melewati kotak start, dan berkata kepada salah satu pemain, "sekarang uang kamu akan bertambah \$3 menjadi 18\$".
- Setelah bermain beberapa putaran, seorang akan kehabisan uang sehingga mereka tidak bisa membayar apa pun lagi. Semua pemain termasuk pemain yang menjadi pemilik toko akan memahami kondisi seperti ini dan mengatakan kepada pemain tersebut bahwa pemain yang kehabisan uang tersebut akan berhutang kepada pemilik toko.
- Pemilik toko akan menuliskan pada lembar transaksi apabila seorang pemain telah kehabisan uang. Siswa akan menuliskan angka 0 atau menuliskan kata 'habis'.
- Apabila uang para pemain habis dan salah satu pemain akan merespon "saya sudah tidak bisa lagi membayar barang karena sudah tidak punya uang", pemilik toko menyadari bahwa para pemain berhutang pada pemilik toko.

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Rencana Pelaksanaan Pembelajaran (RPP)

- Para pemain mendiskusikan cara menuliskan utang para pemain kepada pemilik toko pada lembar transaksi. Cara siswa menuliskan utang dapat bermacam-macam. Siswa mungkin menuliskan utang sebagai "utang 3" atau "kurang 3" atau " harus bayar 3" yang memiliki arti yang sama yaitu pemain berhutang 3 kepada pemilik toko.
- Apabila seorang pemain mempunyai utang -4, dan baru saja melewati kotak start, maka pemain tersebut mendapatkan \$3. Siswa tersebut harus memahami situasi ini dengan menyatakan "Saya baru saja mendapakan uang \$3 dari bank, artinya aku bisa bayar \$3 ke pemilik toko, dan utang saya bersisa 1". Pemilik toko menuliskan "utang 1" sebagai -1.
- Siswa yang sudah mempunyai utang \$5 (pemilik toko menuliskan -5 pada lembar transaksi) dan berhenti pada kotak berlabel "tas sekolah" seharga \$8 memahami bahwa utangnya bertambah "Saya sudah punya \$5, ditambah lagi utang \$8, sehingga saya punya utang \$13. Pemilik toko menuliskannya sebagai "-13"
- Siswa mengurutkan bilangan negatif yang tertulis pada lembar transaksi: Siswa mungkin berpikir bahwa -21 lebih besar daripada -15, sehingga pemain yang mempunyai -21 menang dan yang mempunyai -15 kalah. Beberapa siswa mungkin juga berpendapat bahwa pemain yang skornya -21 kalah dan pemain dengan -15 menang karena memiliki utang yang lebih sedikit.

Standar Kompetensi

Menjumlahkan dan mengurangkan bilangan bulat

Kompetensi Dasar

- Mengurutkan bilangan bulat
- Menumlahkan bilangan bulat

1. Tujuan Pembelajaran

Siswa dapat mengunakan garis bilangan untuk menyelesaikan yang berkaitan dengan permasalahan pinjam-bayar

2. Indikator pencapaian belajar siswa

• Menggunakan garis bilangan untuk menyelesaikan masalah yang berkaitan dengan permasalahan pinjam-bayar (Menggambar lompatan sepanjang garis bilangan)

3. Kegiatan

• Menggambar lompatan pada garis bilangan

Menggambar Lompatan Pada Garis Bilangan

A. Awal

Pembelajaran dimulai oleh guru dengan mereview kembali permainan yang sudah dimainkan oleh anak-anak dan meminta salah satu anak untuk dapat menceritakan apa yang mereka dapatkan setelah belajar sambil bermain pada pertemuan sebelumnya. Guru kemudian dapat memperkenalkan garis bilangan sebagai model/alat yang dapat mengoperasikan bilangan (penjumlahan dan pengurangan). Guru dapat menayakan pertanyaan-pertanyaan sebagai berikut.

• Apa yang kalian ketahui tentang garis bilangan?

• Apakah di antara kalian ada yang dapat menggambarkan garis bilangan di papan tulis Kemudian guru menjelaskan mengenai garis bilangan yang merupakan perpanjangan dari garis bilangan positif yang mungkin sudah diketahui oleh siswa.

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B. Inti

- Guru mendistribusikan lembar kerja siswa yang terdiri dari 5 soal yang harus dikerjakan secara berkelompok oleh siswa.
- Soal-soal tersebut berupa gambar-gambar garis bilangan dan siswa diinstruksikan untuk dapat membuat lompatan-lompatan pada garis bilangan sesuai dengan soal yang ada.
- Guru menjelaskan bagaimana cara mengerjakan soal dengan memberikan satu contoh
- Guru berkeliling kelas untuk memastikan bahwa setiap siswa memahami instruksi yang diberikan oleh guru.
- Setelah beberapa saat, guru meminta salah satu siswa sebagai perwakilan dari kelompok untuk dapat menyebutkan jawaban dan hasil diskusi mereka.
- Guru meminta kelompok lai n untuk memberikan tanggapan. Kemudian guru dapat mengajak seluruh kelas untuk mendiskusikan apakah jawaban mereka berbeda dari yang lain dan bagaimana mereka menjelaskanannya.

C. Penutup

• Guru meminta salah satu siswa untuk dapat menyimpulkan pembelajaran tentang garis bilangan. Siswa juga dapat menjelaskan pendapat mereka mengerjakan soal menggunakan garis bilangan untuk mengetahui apakah model garis bilangan membantu mereka dalam memahami permasalahan yang diberikan khususnya soal-soal yang berkaitan dengan pinjam dan bayar.

4. Alokasi waktu : 2 x 35 menit

5. Media Pembelajaran:Lembar Kerja Siswa6. Materi Pembelajaran: Bilangan Bulat

Siswa kelas 3 sekolah dasar mungkin sudah mengenal garis bilangan dan sudah menggunakan garis bilangan tersebut untuk mengoperasikan (menjumlahkan dan mengurangkan bilangan asli). Pengetahuan awal ini dapat dimanfaatkan oleh guru untuk mengantarkan siswa pada pengoperasian bilangan negatif. Garis bilangan untuk bilangan negaif adalah perpanjangan garis bilangan positif. Pada garis bilangan ini, angka-angka bertanda negatif kemudian ditempatkan sesuai urutannya, semakin ke kiri, angkanya semakin

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besar, namun nilai angkanya semakin kecil. Seperti konteks sebelumnya, pada pertemuan kedua ini, siswa akan menggunakan garis bilangan untuk menyelesaikan soal-soal yang berkaitan dengan pinjam dan bayar. Garis bilangan sudah tersedia pada lembar kerja siswa, yang harus dilakukan oleh siswa adalah menentukan banyak lompatan yang harus mereka pada garis bilangan sesuai dengan soal yang diberikan. Banyak lompatan ke kiri adalah banyaknya uang yang mareka dapatkan dari bank. Misalnya pada contoh soal "Saya punya \$6, saya mau membeli tas seharga \$8". Kelompok siswa akan mendiskusikan bagaimana mereka menggambarkan situasi ini pada garis bilangan yang tersedia dan menentukan berapa banyak uang/utang yang mereka dapat menentukan jumlah utang yang dimiliki atau sisa uang yang masih dimiliki seseorang. Pengalaman belajar seperti ini dapat melatih mereka menggunakan garis bilangan, memvisualisasikan masalah dengan menggunakan model, dan memudahkan mereka dalam mengoperasikan bilangan.

7.Lintasan Belajar Siswa (Hypothetical Learning Trajectory)

- Siswa membaca masalah dengan seksama, contohnya " saya punya \$6, saya mau membeli tas seharga \$8. Siswa memahami bahwa 6 adalah posisi awal mereka pada saat menggambarkannya pada garis bilangan. Karena mereka membayar \$8, mereka mungkin akan membuat lompatan sebanyak 8 lompatan ke kiri. Siswa menyadari bahwa situasi seperti ini akan menghasilkan bilangan negatif. Hasil akhir dari lompatan ini adalah -2 yang berarti bahwa seseorang berhutang 2 kepada pemilik toko. Setelah menyelesaikan gambar, siswa dapat menjelaskan bahwa -2 dapat berarti seseorang meminjam uang \$2 kepada pemilik toko. Pada pelajaran sebelumnya, mereka mengerti bahwa tanda negatif merepresentasikan utang, jadi siswa tidak akan kesulitan untuk menjelaskan hasil akhir dari perhitungan mereka.
- Siswa mungkin akan menggunakan kata kunci yang membantu mereka mengingat kapan mereka harus membuat lompatan ke kanan dan kapan mereka harus membuat lompatan ke kanan. Kata kunci yang mereka gunakan adalah "ambil" dan "bayar". Mereka akan mengatakan "ambil" untuk menyatakan banyaknya uang yang diperoleh dari bank dan menandainya sebagai banyaknya lompatan ke kanan. Begitu pula dengan "bayar" yang menunjukkan banyaknya uang yang dibayarkan ke pemilik toko dan menandainya sebagai banyaknya lompatan ke kiri.
- Untuk tipe soal yang berbeda misalnya, "sekarang saya mempunyai \$-5, saya membeli cupcake seharga \$5". Siswa akan berpikir bahwa -5 sebagai posisi awal dan menempatkannya pada -5 pada garis bilangan yang berarti "berhutang 5 pada pemilik toko". Namun setelah membaca kembali soalnya, siswa memahami bahwa ia harus membeli lagi sebuah cupcake yang berarti harus berhutang lagi \$5 kepada pemilik

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toko. Siswa membuat lompatan ke kiri yang berarti "bayar" ke toko. Setelah menggambarkan lompatan, siswa akan berhenti pada angka -10. Siswa harus memahaminya sebagai "berhutang \$10 kepada pemilik toko"

Diskusi:

Pada kegiatan ini mungkin beberapa siswa belum terlalu sering atau sama sekali belum pernah menggunakan garis bilangan untuk menghitung bilangan. Oleh karena itu, guru dapat menjelaskan kepada siswa alasan mereka harus menggunakan garis bilangan untuk permasalah yang telah dideskripsikan di atas. Guru dapat menjelaskan kepada siswa tentang kegunaan garus bilangan pada kegiatan ini. Misalnya, dapat membantu siswa dalam mengoperasikan bilangan, dengan bantuan garis bilangan, siswa juga dapat membayangkan situasi pada soal karena siswa bebas memvisualisasikan pemikiran mereka tentang permasalahan.
Standar Kompetensi

Menjumlahkan dan mengurangkan bilangan bulat

Kompetensi Dasar

- Mengurutkan bilangan bulat
- Menumlahkan bilangan bulat

1. Tujuan Pembelajaran

Siswa dapat menggunakan garis bilangan pada konteks permainan dadu

2. Indikator pencapaian belajar siswa

Menggunakan garis bilangan pada konteks permainan dadu.

3. Kegiatan

Permainan Dadu

A. Awal

- Guru mereview kembali pelajaran sebelumnya dan mengingatkan para siswa tentang garis bilangan.
- Guru menjelaskan kegiatan yang akan siswa lakukan pada pertemuan ini masih berkaitan dengan garis bilangan.

B. Inti

- Guru mendistribusikan set permainan (papan permainan garis bilangan dan pion untuk masing-masing pemain).
- Setiap anak mendapatkan tabel permainan untuk mencatat hasil lompatan ke kanan, lompatan ke kiri, dan posisi terakhir.
- Guru menjelaskan aturan permainan. Siswa diberikan kesempatan untuk menanyakan hal-hal yang berkaitan dengan permainan apabila masih ada yang kurang jelas.
- Guru memastikan bahwa setiap pemain harus selalu mengembalikan pion ke START setelah menyelesaikan gilirannya.
- Guru berkeliling kelas, mengawasi siswa untuk mengecek apakah siswa memahami peraturan permainan dengan baik dan dapat mengisi tabel permainan dengan tepat.

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Rencana Pelaksanaan Pembelajaran (RPP)

- Siswa berdiskusi dalam kelompok, guru mengamati diskusi siswa dan mengajukan beberapa pertanyaan misalnya "bagaimana kamu menuliskan angka pada dadu merah?", "Apakah kamu bisa menebak posisi terakhir tanpa menggunakan garis bilangan?". Selain itu, guru juga menginstruksikan kepada siswa untuk membantu teman-temannya yang lain dan mengecek hasil dari tabel mereka.
- Setelah siswa mengisi semua kolom tabel, guru meminta siswa untuk mengumpulkan angka terakhir pada posisi terakhir siswa dan membandingkan angka-angka tersebut untuk mengurutkan angka-angka tersebut dari yang terbesar hingga yang terkecil.
- Guru dan siswa berdiskusi tentang permainan dan menjelaskan bagaimana caranya menuliskan angka-angka pada dadu tersebut sebagai operasi penjumlahan yang lebih formal. Guru menyampaikan kepada siswa bahwa permainan yang baru saja mainkan adalah salah satu cara untuk mempelajari operasi penjumlahan pada bilangan bulat. Guru dapat mengambil contoh dari salah satu pekerjaan siswa. Contohnya, apabila angka pada dadu biru 6, dan angka pada dadu merah 4 (dituliskan sebagai -4) maka posisi terakhir dapat ditentukan dengan menjumlahkan 6 + (-4) yang menghasilkan 2. Contoh lain adalah apabila angka pada dadu biru 3, dan angka pada dadu merah 5 (dituliskan sebagai -5), maka posisi terakhir dapat ditentukan dengan menjumlahkan kedua angka tersebut menjadi 3 + (-5) menghasilkan -2.
- Guru meminta perwakilan dari masing-masing siswa untuk maju ke depan kelas menuliskan hal yang sama di papan tulis dan menjelaskan kepada teman-teman lainnya.

C. Penutup

• Siswa dan guru membuat kesimpulan dari kegiatan yang sudah mereka lakukan.

4. Alokasi waktu : 2 x 35 menit

5. Media Pembelajaran:

Lembar Kerja Siswa (tabel permainan) Papan permainan 2 buah Dadu yang berbeda warna (merah dan biru) 5 buah pion

6. Materi Pembelajaran: Bilangan Bulat (konteks permainan dadu)

Permainan dadu adalah permainan yang melibatkan garis bilangan. Melalui permainan ini, siswa diharapkan dapat belajar mengenai penjumlahan bilangan bulat. Dengan menggunakan dua buah dadu yang berbeda warna, siswa akan bermain membuat lompatan pada garis bilangan. Dadu berwarna merah akan menunjukkan banyak lomptan ke kiri, sedangkan dadu

berwarna biru akan menunjukkkan banyak lompatan ke kanan. Pada kegiatan ini siswa akan menuliskan setiap angka yang ditunjukkan pada dadu biru dan merah serta posisi terakhir setelah mereka membuat lompatan.

Lintasan Belajar Siswa (Hypothetical Learning Trajectory)

- Siswa berpikir bahwa semakin mereka bergerak ke kiri, mereka semakin menuju ke bilangan negatif. Semakin bergerak ke kanan, mereka akan semakin menuju bilangan positif.
- Setiap siswa akan memulai gilirannya dari START. Apabila pemain telah menyelesaikan gilirannya, pemain tersebut akan mengembalikan pionnya ke START untuk giliran berikutnya.
- Kesalahan yang mungkin dilakukan oleh siswa adalah melanjutkan gilirannya sebelum memindahkan pion ke posisi START.
- Siswa akan menuliskan angka pada dadu merah sebagai angka dengan tanda negatif di depannya.
- Siswa akan menuliskan angka pada dadu biru sebagai bilangan positif yang menunjukkan gerakan ke kanan.
- Setelah mendapat giliran beberapa kali, siswa dapat menganggap bahwa posisi terakhir setelah melangkah ke kanan dan ke kiri adalah hasil dari pengurangan dari angka pada dadu merah dan dadu biru. Misalnya siswa mengatakan, "angka pada dadu biru 6, dan angka pada dadu merah 2 (siswa menuliskan -2 pada tabel sesuai dengan perintah soal), dan posisi terakhir adalah 6 dikurangi 2, yaitu 4". Siswa dapat menyimpulkan hal ini setelah mempelajari beberapa pola bilangan yang dituliskannya pada tabel.
- Setelah permainan berakhir, masing-masing kelompok akan menghitung angka pada posisi terakhir mereka dan siswa dapat menyatakan bahwa "Aku mempunyai angka yang lebih besar daripada kamu, karena aku berhenti di -3, sedangkan kamu berhenti pada angka -5". Siswa yang lain mungkin merespon "angka yang aku punya daripada angka yang kamu punya karena 1 lebih besar dari -1"
- Siswa yang telah melihat pola angka-angka pada posisi terakhir mungkin akan mencoba untuk mencari tahu posisi terakhir tanpa harus menggunakan dadu dan garis bilangan, namun menganggapnya sebagai operasi pengurangan.

• Siswa akan berpendapat bahwa apabila angka pada dadu biru "6" dan angka pada dadu merah "-5" akan menghasilkan "1" karena "6-5 = 1"

Diskusi kelas:

Permainan ini merupakan bentuk lain dari kegiatan garis bilangan yang sudah dipelajari pada pelajaran sebelumnya. Kegiatan ini akan membantu siswa untuk memahami pengoperasian bilangan bulat khususnya untuk operasi penjumlahan. Penjumlahan bilangan bulat pada kegiatan ini adalah penjumlahan banyaknya lompatan ke kanan dan lompatan ke kiri.

Misalnya, apabila angka pada dadu biru 6, dan angka pada dadu merah 4 (dituliskan sebagai -4) maka posisi terakhir dapat ditentukan dengan menjumlahkan 6 + (-4) yang menghasilkan 2. Contoh lain adalah apabila angka pada dadu biru 3, dan angka pada dadu merah 5 (dituliskan sebagai -5), maka posisi terakhir dapat ditentukan dengan menjumlahkan kedua angka tersebut menjadi 3 + (-5) menghasilkan -2. Guru dapat menjelaskan kepada siswa bahwa hasil dari penjumlahan dengan menggunakan bilangan bulat dapat ditentukan dengan bantuan garis bilangan. Pada sesi diskusi ini, guru dapat memberikan contoh yang lain dan mendemostrasikan bagaimana menuliskan penjumlahan angka pada dua dadu tersebut secara formal dan mengecek pemahaman siswa dengan menunjuk beberapa siswa untuk dapat menuliskannya ke papan tulis serta menentukan hasilnya dengan menggunakan garis bilangan.

Selain itu, hal lain yang perlu didiskusikan guru bersama siswa adalah sifat komutatif penjumlahan yang dalam hal ini akan dijelaskan dengan angka pada dadu merah dan angka pada dadu biru. Siswa harus memahami bahwa 6 + (-4) dan -4 + 6 menghasilkan angka yang sama yaitu 2. Jadi tidak ada perbedaannya apabila siswa membuat lompatan ke kanan terlebih dahulu atau membuat lompatan ke kiri terlebih dahulu.

Standar Kompetensi

Menjumlahkan dan mengurangkan bilangan bulat

Kompetensi Dasar

- Mengurutkan bilangan bulat
- Menumlahkan bilangan bulat

1. Tujuan Pembelajaran

Siswa dapat menggambar lompatan pada garis bilangan yang kosong untuk menyelesaikan masalah penjumlahan bilangan bulat.

2. Indikator pencapaian belajar siswa :

Menggambar lompatan pada garis bilangan yang kosong untuk menyelesaikan masalah penjumlahan bilangan bulat.

3. Kegiatan

A. Awal

Guru mengingatkan kembali tentang pelajaran sebelumnya.

B. Inti

- Karena pada kegiatan ini siswa akan mengerjakan soal-soal secara berpasangan dengan teman sebangkunya, guru memilihkan kelompok siswa atau memilih siswa memilih partnernya sendiri.
- Guru membagikan lembar kerja siswa kepada masing-masing kelompok yang terdiri dari 2 orang anak.
- Siswa diberikan waktu untuk dapat mengerjakan soal-soal. Setelah semua siswa menyelesaikan soal, salah satu siswa mempresentasikan hasil jawaban mereka. Guru meminta siswa lain untuk menanggapi hasil diskusi temannya.
- Guru memimpin diskusi kelas tentang penulisan operasi penjumlahan dalam bentuk yang lebih formal.

C. Penutup

• Guru membantu siswa untuk dapat mengambil kesimpulan dari kegiatan yang mereka lakukan.

4. Alokasi waktu : 2 x 35 menit

5. Media Pembelajaran:

Lembar Kerja Siswa

6. Lintasan Belajar Siswa (Hypothetical Learning Trajectory)

- Siswa akan brpikir untuk menggunakan permainan garis bilangan untuk menentukan jawaban dari permasalah.
- Siswa mungkin langsung dapat menggambarkan situasi pada garis bilangan yang telah disediakan.
- Misalnya pada contoh di bawah ini,



Pada lembar jawaban, siswa akan memberikan respon sebagai berikut. 3 merupakan posisi awal. Setelah itu, siswa akan membuat lompatan ke kiri sebanyak 5 kali hingga mencapai angka -2. Angka 0 yang sudah tersedia pada garis bilangan akan membantu mereka dalam menempatkan posisi memulai dan mengakhiri lompatan pada angka yang tepat.



Diskusi Kelas:

Guru menjelaskan bahwa kegiatan yang baru saja dikerjakan oleh siswa adalah salah satu cara untuk memahami operasi penjumlahan pada bilangan bulat. Guru akan menuliskan ke papan tulis salah satu contoh soal misalnya

Saya berada pada posisi -3

Saya melompat 5 langkah ke kanan

Dimana posisi saya sekarang ?

Guru akan menjelaskan bahwa soal di atas apabila dituliskan ke dalam kaliamat matematika yang formal akan menjadi -3 + 5=...., pada contoh ini -3 merupakan posisi awal sebelum siswa menggambar lompatan, 5 adalah jumlah lompatan ke kanan pada garis bilangan sesuai dengan informasi pada soal. Sedangkan hasil penjumlahan dari kedua bilangan tersebut adalah posisi terakhir setelah melompat. Selain dapat menggambarkan lompatan pada garis bilangan, siswa pada diskusi ini juga dapat menjelaskan hasil pekerjaan mereka. Diantaranya adalah dengan memahami angka yang mana yang merupakan lompatan, angka mana yang menjadi posisi awal, serta dalam menentukan posisi akhir setelah melompat.

April 2013

Rencana Pelaksanaan Pembelajaran (RPP)

Standar Kompetensi

Menjumlahkan dan mengurangkan bilangan bulat

Kompetensi Dasar

- Mengurutkan bilangan bulat
- Menumlahkan bilangan bulat

1. Tujuan Pembelajaran

• Siswa dapat menggambar lompatan pada garis bilangan untuk memahami operasi penjumlahan

:

• Siswa dapat menuliskan operasi penjumlahan dengan lebih formal.

2. Indikator pencapaian belajar siswa

3. Kegiatan

Kegiatan ini adalah sebagai lanjutan dari kegiatan permainan dadu dan garis bilangan yang telah dilakukan siswa pada pertemuan ketiga. Pada lembar kerja siswa, terdapat gambar dua dadu manipulatif yang mirip dengan dadu yang digunakan siswa pada kegiatan ketiga. Namun pada kegiatan ini, siswa ditantang untuk dapat mengoperasikan angka-angka yang lebih besar. Dibantu dengan garis bilangan, siswa juga diharapkan dapat menentukan jumlah dari dua pergerakan ke kiri dan ke kanan.

A. Awal

Guru mengingatkan siswa tentang permainan dadu yang telah dilakukan pada pertemuan sebelumnya.

B. Inti

- Siswa bersama teman sekelompoknya mengerjakan soal secara bersama-sama
- Siswa mendiskusikan hasil jawaban bersama teman sekelomponya
- Guru memfasilitasi diskusi kelas

C. Penutup

Beberapa siswa dipilih untuk menyimpulkan pelajaran

4. Alokasi waktu : 2 x 35 menit

5. Media Pembelajaran: Lembar Kegiatan Siswa

6. Materi Pembelajaran: Bilangan Bulat (konteks permainan dadu)

7. Lintasan Belajar Siswa (Hypothetical Learning Trajectory)

- Pada garis bilangan bertanda, siswa membuat lompatan sesuai dengan angka pada .dadu merah dan angka pada dadu biru
- Siswa menuliskan penjumlahan dengan 2 cara:



Siswa mengerti bahwa walaupun bilangan ditukar tempatnya hasilnya akan tetap sama.

- Untuk soal mengisi angka pada dadu, siswa meletakkan bilangan negatif pada dadu merah; bilangan positif pada dadu biru
- Untuk mengisi satu kotak yang kosong, siswa menggunakan garis bilangan untuk menentukan bilangan yang sesuai.

Student Material



Permainan "THE HOP SHOP"









HOP SHOP

Voucher belanja sebesar \$5



HOP SHOP

Dapat uang \$1 dari masing-masing pemain



HOP SHOP

Bonus giliran, lempar dadu satu kali lagi

Chance Cards





Community Chest Cards





HOP SHOP

Double turn, throw the dice one more time

Uang Mainan





Nama Shopkeeper :

Pemain 1	Pemain 2	Pemain 3	Pemain 4
Nama:	Nama:	Nama:	Nama:
15	15	15	15

Total:	Total :	Total :	Total:

Aturan Permainan 'Hop Shop'

- Permainan dimainkan oleh 5 orang pemain, 1 orang pemain menjadi shopkeeper dan 1 orang menjadi bank (bank sekaligus berperan sebagai pemain)
- Perlengkapan permainan terdiri dari satu papan permainan 'Hop Shop', 1 dadu, 4 buah bidak, uang mainan, lembar transaksi untuk shopkeeper, 1 set kartu dana umum, dan 1 set kartu kesempatan (1 set kartu terdiri dari 5 kartu)
- Set kartu harus diletakkan pada tempat yang tersedia pada papan permainan, semua uang harus terlebih dahulu diserahkan kepada bank, lembar transaksi diserahkan kepada shopkeeper.
- Tugas masing-masing pemain: (1) Bank adalah pemain yang mengatur keuangan para pemain, bank memberikan uang modal sejumlah \$15 kepada semua pemain. Walaupun pada permainan, bank bisa sekaligus menjadi pemain, bank harus memisahkan uang nya sebagai pemain dan uang bank. Bank memberi dan menerima uang dari semua pemain. Misalnya bank memberi uang kepada pemain yang melalui start, dan bank menerima uang dari pemain yang membayar pajak atau denda lainnya.
 (2) Shopkeeper adalah pemain yang berperan sebagai pemilik 'Hop Shop', shopkeeper mencatat semua transaksi pembelian dari semua pemain. Pemain yang bertindak sebagai shopkeeper hanya menjadi shopkeeper, tidak boleh ikut menjadi pemain. Selama permainan berlangsung, shopkeeper mengumumkan jumlah uang yang dimiliki oleh masing-masing pemain pada transaksi terakhir dan juga jumlah

hutang yang harus dibayarkan kembali kepada shopkeeper. Apabila terjadi kesalahan perhitungan, pemain boleh mengajukan protes dan menghitung kembali jumlah uang atau hutang yang dimiliki. (3) Pemain. Pemain terdiri dari 4 orang termasuk pemain yang berperan sebagai bank sekaligus. Pemain harus selalu mengingat jumlah uang yang mereka miliki atau jumlah hutang yang harus mereka bayarkan kembali kepada shopkeeper.

- Giliran pemain memutar sesuai urutan posisi pemain. Masing-masing pemain melempar dadu untuk menentukan jumlah langkah.
- Pada saat bidak pemain melalui start, bank memberikan uang sejumlah \$3 kepada pemain tersebut.
- Pada saat bidak berhenti pada kotak berlabel nama barang dan harga, pemain **harus** membeli barang tersebut dan membayar sejumlah yang tertera pada label barang .
- Apabila pemain tidak mempunyai uang lagi, berarti pemain meminjam uang kepada shopkeeper. Shopkeeper mencatat hutang pemain tersebut. Hutang para pemain akan semakin bertambah dan setiap pemain harus selalu mengingat jumlahnya dan menyesuaikan jumlahnya dengan catatan shopkeeper pada lembar transaksi.
- Permainan berakhir apabila salah seorang pemain memiliki hutang lebih dari \$20, dan pemain tersebut kalah dalam permainan.
- Pemenang pada permainan adalah pemain yang masih memiliki uang atau memiliki hutang paling sedikit.
- Para pemain bersama shopkeeper membandingkan jumlah hutang/uang para pemain dari yang terbesar hingga terkecil untuk menentukan pemenang pada permainan.
- Pada akhir permainan, shopkeeper akan mengumumkan siapa yang menang dan siapa yang kalah.

Kartu Dana Umum dan Kesempatan:

Kartu Dana Umum terdiri dari 5 kartu:

- 1. Maju ke START. Apabila pemain mendapatkan kartu ini, maka pemain harus melangkah ke kotak START.
- 2. Bayar pajak penghasilan \$5. Pemain harus membayar kepada bank sejumlah \$5.
- 3. Parkir bebas. Apabila pemain mendapatkan kartu ini, makan pemain boleh berhenti di kotak mana saja yang diinginkan.
- 4. Mundur 6 langkah.
- 5. Maju 5 langkah.

Kartu kesempatan terdiri dari 5 kartu:

- 1. Bayar buku sekolah \$6. Apabila mendapatkan kartu ini, pemain harus membayar buku sekolah kepada shopkeeper.
- Setor uang ke bank untuk sumbangan sosial. Pemain memberikan uang sejumlah yang diinginkan kepada bank untuk sumbangan sukarela. Namun ini hanya berlaku untuk pemain yang masih memiliki uang. Apabila seorang pemain sudah tidak memiliki uang, pemain tersebut tidak harus membayar sumbangan sukarela.
- 3. Kupon belanja \$5. Pemain mendapatkan bonus belanja sebesar \$5. Kupon ini juga dapat digunakan untuk membayarkan hutang kepada shopkeeper.
- 4. Dapat uang \$1 dari setiap pemain. Apabila seorang pemain mendapatkan kartu ini, maka pemain tersebut akan mendapatkan uang \$1 dari para pemain lainnya termasuk bank dan shopkeeper.
- 5. Bonus giliran, lempar dadu sekali lagi. Pemain mendapatkan kesempatan untuk mendapatkan bonus giliran satu kali lagi.

Aturan mencatat lembar transaksi (untuk shopkeeper)

• Shopkeeper menuliskan namanya pada lembar transaksi.



- Shopkeeper mencatat jumlah uang yang dimiliki oleh masing-masing pemain pada transaksi terakhir.
- Karena uang modal yang diberikan oleh bank adalah \$15, shopkeeper harus mencatat \$15 pada awal permainan.



Nama Shopkeeper :

Pemain 1 Nama:	Pemain 2 Nama:	Pemain 3 Nama:	Pemain 4 Nama:	
15	15	15	15	

• Apabila seorang pemain membeli es krim seharga \$5, maka shopkeeper hanya menuliskan jumlah uang setelah pemain membayar



Nama Shopkeeper :

Pemain 1 Nama:	Pemain 2 Nama:	Pemain 3 Nama:	Pemain 4 Nama:	
15	15	15	15	
10				
			2	3

Rules of the 'Hop Shop' Game



- The game is played by 5 people, 1 player will be a shopkeeper, 1 player will be the bank (a bank can also be the player).
- Equipment of the games consist of a board, 2 dice, 4 tokens as the counters, play money, transaction sheet for the shopkeeper, 5 chance and community chest cards,
- Place the board and the cards on the board, play money will be stored to the bank, transaction sheet should be given to the shopkeeper.
- Roles of each player (1) the banker is the player who manage the play money, one who is chosen as the bank act as a player at the same time but one must separate his/her own money from those of the bank. The banker give and take the money from the player, for example giving money to all player when one pass the starting space, and take money for income tax and other funding. (2) the shopkeeper is the one who keep record the transactions made during the game, one who act as shopkeeper cannot act as a player, it announce the amount of the money each player after transaction and remind the player about the amount of the debt they have to pay. In case that there is a miscalculation of the money, the player can complain to the shopkeeper. (3) the players consist of four people, the player must keep record of their incomes and debts.

- Starting with the banker, each player in turn throws the dice. The player with the highest total starts the play. Play the token. The number of the spaces indicated by the dice. After 1 player have completed the play, turn passes to the left. The token remains on the space occupied.
- Each time a player's token passes over 'start', a banker pays that player \$3.
- Whenever the player land in the space of item, you have to buy the stuff and pay the money to the shopkeeper.
- If the player has no more money, it means that he/she owe money from the shopkeeper.
- The player who has debts more than \$-20,
- The winner of the game is the player who has less debts.
- The shopkeeper announces the winner of the game.
- All the players order the score/the debts of each player to determine who has fewer debts, who has more.

Chance and Community Chest Cards:

Chance cards consist of 5 cards:

- 1. 'Go to start'. If the player gets this card, he should move to the starting point.
- 2. 'Pay income tax \$5'. The player should pay \$5 to the bank.
- 3. 'Shop free parking'. The player can move wherever he wants. If the player stop in a box of item, he does not have to pay.
- 4. 'Move backward six spaces'.
- 5. 'Move forwards 5 spaces'

Community chest cards consist of 5 cards:

- 1. 'Pay school books \$6'. The player has to pay the money to the shopkeeper.
- 2. 'Give any amount of money to the bank for charity'. The player should pay any amount of money, could be \$1, \$2, \$3, etc.
- 3. 'Shopping voucher \$5'. The player gets bonus \$5 from the shopkeeper.
- 4. 'Collect \$1 from each player'. The player get \$1 from every players including the bank and the shopkeeper.
- 5. 'Double turn; throw the dice one more time'. The player get opportunity to throw the die one more time.

How the shopkeeper manages the transactions:

• The shopkeeper writes down her/his name on the transaction sheet

Player 1 Name	Player 2 Name	Player s Name	Player 4 Name:

- The shopkeeper records the amount of money of each player has in the last transaction
- Since all the player will get \$15, the shopkeeper will start with \$15. He/she only need to write 15 instead of \$15.

l c p	<mark>s h</mark>	<mark>O</mark> <mark>P</mark>	
Shopkeeper name:			
Player 1 Name:	Player 2 Name:	Player 3 Name:	Player 4 Name:
15	15	15	15

• If a player takes his turn, and he has to pay ice cream costing \$5, then the shopkeeper only needs to write down the rest of his money after he paid.



Shopkeeper name:

Player 1 Name:	Player 2 Name:	Player 3 Name:	Player 4 Name:
15	15	15	15
10			

• The shopkeeper is responsible with the money but all the players must remember the last transactions and the money they have after that transaction.

Kegiatan 2 : Menggambar lompatan pada garis bilangan

Mari kita mendiskusikan kembali permainan 'Hop Shop' yang baru saja kamu mainkan bersama teman satu kelompokmu. Di bawah ini adalah sebuah garis bilangan , kamu akan menggunakan garis bilangan dibawah ini untuk menentukan jumlah uang/hutang yang kamu miliki setelah berbelanja.

Perhatikan contoh 1.

Saya punya \$8, saya membayar boneka Barbie seharga \$3 (contoh soal pada kartu) Bagaimana kamu menggambarkannya pada garis bilangan? 8 adalah posisi awal, 3 adalah banyaknya lompatan ke kiri.



Jadi, uang yang kamu miliki sekarang adalah \$5

Contoh 2.

Saya punya hutang \$7, saya mendapatkan uang \$3 dari bank. Bagaimana kamu menggambarkannya pada garis bilangan?



Lompatan ke kiri adalah pada saat kamu membayar uang ke shopkeeper Lompatan ke kanan adalah pada saat kamu mendapatkan uang dari bank. Kalian akan mendapatkan 5 kartu soal.

Sekarang kerjakanlah soal-soal pada kartu yang kalian dapat ini sesuai dengan contoh yang diberikan.

Saya punya hutang \$4, saya mendapatkan voucher belanja sebesar \$7

Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?

Saya punya uang\$10, saya mau membayar kaos sepakbola seharga \$10



Saya punya hutang \$8, saya mendapatkan bonus dari bank sebesar \$7

Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Saya punya hutang \$5, saya membeli cupcake seharga \$5. Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?





Sekarang kerjakanlah soal-soal pada kartu itu sesuai dengan contoh yang diberikan.

1.

Saya punya utang \$4, saya mendapatkan voucher belanja sebesar \$7

Bagaimana kamu menggambarkannya pada garis bilangan?



Pada angka berapa kamu memulai lompatan? Pada angka berapa lompatan tersebut berakhir? Berapa uang/utang yang kamu punya sekarang? Saya punya uang\$10, saya mau membayar kaos sepakbola seharga \$10

Bagaimana kamu menggambarkannya pada garis bilangan?



Pada angka berapa kamu memulai lompatan? Pada angka berapa lompatan tersebut berakhir? Berapa uang/utang yang kamu punya sekarang? Saya punya utang \$8, saya mendapatkan bonus dari bank sebesar \$7

Bagaimana kamu menggambarkannya pada garis bilangan?



Pada angka berapa kamu memulai lompatan? Pada angka berapa lompatan tersebut berakhir? Berapa uang/utang yang kamu punya sekarang?
Saya punya utang \$5, saya membeli cupcake seharga \$5.

Bagaimana kamu menggambarkannya pada garis bilangan?



Pada angka berapa kamu memulai lompatan? Pada angka berapa lompatan tersebut berakhir? Berapa uang/utang yang kamu punya sekarang? Saya punya utang \$2, saya ingin membeli tas sekolah seharga \$8

Bagaimana kamu menggambarkannya pada garis bilangan?



Pada angka berapa kamu memulai lompatan? Pada angka berapa lompatan tersebut berakhir? Berapa uang/utang yang kamu punya sekarang? Tukarkanlah kartu yang kalian dapatkan dengan kartu kelompok lain, kemudian kerjakanlah seperti contoh yang sudah diberikan. Diskusikanlah jawaban kalian.

Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



2. Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



3. Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Bagaimana kamu menggambarkannya pada garis bilangan? Berapa uang/hutang yang kamu punya sekarang?



Kegiatan 3

Mengurutkan bilangan bulat: "Siapakah pemenangnya?"

Tujuan Pembelajaran:

Siswa memahami cara megurutkan bilangan bulat

Nama :		
Grup:		

Urutkan angka-angka di bawah ini di tempat yang besuai untuk mengetahui siapa yang menjadi pemenang pertama, pemenang kedua, pemenang ketiga, dan yang kalah.

Grup 1

Tuliskanlah angka dan nama pemain pada kotak yang tersedia dengan posisi yang benar.

Anita : -25

Lala : 5

Raka : -7

Kiko : -10



Grup 2

Lisa : **-21**

Radith; -17

Ana: -25

Marisa: 3



Grup 3

Rahmat: -18

Ardi: -5

Tia: **-8**

Dody: -22



Grup 4 Siti: 1

- Dani: -11
- Aris: -16

Lola: -28



Grup 5

Cinta: -15



Sekarang, dapatkah kamu menyusun bilangan-bilangan ini pada garis bilangan dengan urutan yang tepat. Beberapa bilangan telah diletakkan pada garis bilangan sebagai contoh.



Kesimpulan:

Bilangan negatif yang nilainya besar lebih..... daripada bilangan positif yang nilainya kecil.

Semakin ke kanan, bilangan positif nilainya semakin.....

Semakin ke kiri, bilangan negatif nilainya semakin......



Kegiatan 3: Permainan garis bilangan

Media: papan permainan garis bilangan

Tuliskanlah angka pada dadu biru dan merah pada tabel yang tersedia, tuliskan pula pada angka berapa posisimu setelah melangkah ke kiri dan ke kanan.

Angka pada dadu biru	Angka pada dadu merah	Posisi setelah melangkah ke kiri dan ke kanan
5	-6	-1

Kesimpulan:

Apabila angka pada dadu biru lebih besar daripada angka pada dadu merah, maka posisimu adalah bilangan......

Apabila angka pada dadu biru lebih kecil daripada angka pada dadu merah, maka posisimu adalah bilangan.....

Kegiatan 3: Permainan dadu (Revised Activity for Teaching

Experiment)

Media: papan permainan garis bilangan

Tuliskanlah angka pada dadu biru dan merah pada tabel yang tersedia, tuliskan pula pada angka berapa posisimu setelah melangkah ke kiri dan ke kanan.

Posisi Awal	Angka pada dadu biru	Angka pada dadu merah	Posisi setelah melangkah ke kiri dan ke kanan
START	5	-6	-1
START			

Kesimpulan:

Apabila angka pada dadu biru lebih besar daripada angka pada dadu merah, maka posisimu adalah bilangan.....

Apabila angka pada dadu biru lebih kecil daripada angka pada dadu merah, maka posisimu adalah bilangan.....



Kegiatan 4: "Dimana posisimu sekarang?"

Kita tahu bahwa bilangan terbesar pada dadu adalah 6. Pada kegiatan ini, kita akan bergerak lebih dari 6 langkah, ke kanan mau pun ke kiri. Tentukan dan tuliskan posisimu setelah kamu bergerak ke kiri maupun ke kanan.

Saya berada pada posisi 3 Saya bergerak 9 langkah ke kiri Dimana posisi saya sekarang?	Saya berada pada posisi -8 Saya bergerak 3 langkah ke kanan Dimana posisi saya sekarang?
Saya berada pada posisi -2 Saya bergerak 7 langkah ke kanan Dimana posisi saya sekarang?	Saya berada pada posisi 10 Saya bergerak 7 langkah ke kiri Dimana posisi saya sekarang?
Saya berada pada posisi 5 Saya bergerak 11 langkah ke kiri Dimana posisi saya sekarang?	Saya berada pada posisi -2 Saya bergerak 7 langkah ke kiri Dimana posisi saya sekarang?

Banyak langkah ke kanan	Banyak langkah ke kiri	Posisimu sekarang
10	-5	
	-1	8
7		-3
3	-8	
12	-12	
9		-4
	-8	1
1	-11	
7		5
19		0

Diskusikanlah dengan teman satu grupmu kemudian isilah titik-titik di bawah ini dengan angka yang tepat.

Apa yang dapat kamu simpulkan dari kegiatan di atas

Kegiatan 5 (Revised Activity for Teaching Experiment)

Nama

:

•

Grup

Kegiatan 4: "Dimana posisimu sekarang?"



Gambarkanlah gerakan Andre pada garis bilangan di bawah ini!



2.



Gambarkanlah gerakan MIko pada garis bilangan di bawah ini!





Gambarkanlah gerakan Oka pada garis bilangan di bawah ini!







Gambarkanlah gerakanRangga pada garis bilangan di bawah ini!





Gambarkanlah gerakanDavid pada garis bilangan di bawah ini!

0

Apa yang dapat kamu simpulkan dari kegiatan di atas



Kegiatan 5:

Garis Bilangan kosong

Perhatikan di bawah ini, ada dua buah dadu berwarna hitam dan merah. Gambarlah banyak langkah ke kanan dan ke kiri dimulai dari angka 0.

Tentukan pada angka berapa posisimu setelah melangkah ke kiri dan kekanan.



Kesimpulan:

Penjumlahan angka-angka pada dadu hitam dan merah adalah posisi setelah melangkah ke kiri dan kenan.

Pada penjumlahan tersebut,

Apabila bilangan negatif lebih besar daripada bilangan positif, maka menghasilkan bilangan.....

Apabila bilangan negatif lebih kecil daripada bilangan positif,

Maka menghasilkan bilangan.....

Nama : 1..... 2..... 3..... 4.....

Grup :

Kegiatan 5 (Revised Acitvity for Teaching Experiment)

Tentukan pada angka berapa posisi angka terakhir setelah lompatan ke kiri dan ke kanan serta tuliskanlah penjumlahan angka-angka tersebut pada kotak yang tersedia.





Hitunglah hasil penjumlahan angka-angka di bawah ini! Gunakan garis bilangan yang tersedia.







Kesimpulan:

Penjumlahan angka-angka pada dadu biru dan merah adalah posisi setelah melangkah ke kiri dan kanan.

Pada penjumlahan tersebut,

Apabila bilangan negatif lebih besar daripada bilangan positif, maka menghasilkan bilangan.....

Apabila bilangan negatif lebih kecil daripada bilangan positif,

Maka menghasilkan bilangan.....

Scheme of Interview with the teacher and Classroom Observation

Topic for classroom observation

Classroom setting:

- How many students in the classroom? How many girls? How many boys?
- How is the classroom set? (sitting arrangements, the number of students in groupwork)

Teacher's activities:

- How does the teacher manage the classroom? How does she/he start the class?If there is too much noise?
- How does the teacher facilitate the discussion during the lesson?
- How the teachers react to students answer?
- How many students participate in the classroom? How many students do not participate at all?
- Do the teacher use contextual problem??
- Does the teacher pay attention to each individual when she/he struggle to work on a mathematical problem?
- Does the teacher interact with the students in friendly manner?
- How about the difficulty of mathematical problem given by the teacher? (easy, average, challenging?)

Students' activities:

- Do the students ask some questions relating to the lesson?
- If there is a group work, what happen in each group? Is there any students try to dominate others? Or is there any students who do not pay attention to the teacher?
- Is there any students who criticize the way of teacher deliver the lesson?
- How do the students interact each other? Do they discuss the lesson? Or do they discuss other things?

Interview with the teacher

Teaching method of the teacher:

- What kind of preparation do the teacher do before entering classroom?
- What kind of teaching method and media are usually used by the teacher?
- How does the teacher make the groupwork? (ask the students to make the group by themselves or the teacher decide it)
- What things does the teacher usually consider when choosing the context of a mathematical topic?
- What are the teachers' difficulties in managing classroom?
- How does the teacher lead the discussion among the students? What does the teacher do to ask the silent students to speak or participate?

About the students:

- What kind of classroom? (heterogeneous, or mostly consists of smart students)
- How many hours the students study mathematics in the classroom in one week?
- Do the students like mathematics?
- Do the students like studying in group, in pairs, or individually?
- During the class, do the students tell to the teachers directly if they do not understand or they wait until they are given opportunity to ask some questions?

Some specific questions concerning to the study:

- How will the teacher introduce negative numbers? (what is her/his plan in teaching this topic?, what kind of context?)
- Does the teacher already familiar in teaching mathematics by using a game?

• What does the teacher think about the topic of negative number? Would it be difficult for students? What kind of difficulties the students may have (in teacher's perspective)

Overview of the Learning Trajectory: Introduction of Negative Numbers



Pre Assesment (Preliminary Teaching), Pre-test, and Post-test (Teaching Experiment)

Pre Assesment

Nama:		
Kelas:		

1.



Spongebob mempunyai 10 krabby patty, 4 diberikan kepada Tuan Krab dan 1 dimakan Patrick.

Berapa krabby patty yang tersisa?

2. Isilah kotak di bawah ini dengan angka yang tepat!



$$- 11 = 4$$

Tanggal:

Nama: Kelas:

1. Urutkan angka-angka di bawah ini mulai dari yang terkecil hingga yang terbesar!





2. Berapakah hasil dari 7-10 = ...?

Jawaban:		
Alasan:		

3. Lengkapi garis bilangan di bawah ini.



4. Hitunglah!

- a. -5 + (-5) = ...
- b. -2 + 3 = ...
- c. 7 + (-9) =.....
Nama: Kelas:

1. Berapakah hasil dari 7-10 = ...?

Jawaban:			
Alasan:			

2. Lengkapi garis bilangan di bawah ini.



3. Urutkan angka-angka di bawah ini mulai dari yang terkecil hingga yang terbesar!



Alasan:			

- 4. Hitunglah!
- a. -5 + 5 = ...
- b. -2 + 3 = ...
- c. 7 + (-9) =.....