DESIGN RESEARCH ON MATHEMATICS EDUCATION: RATIO TABLE IN DEVELOPING THE STUDENTS' PROPORTIONAL REASONING

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:

SYLVANA NOVILIA SUMARTO
NIM. 20112812012

Universiteit Utrecht

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
JUNE 2013

APPROVAL PAGE

Research Title	: Design Research on Mathematics Education: Ratio Table in Developing the Students' Proportional Reasoning
Student Name	: Sylvana Novilia Sumarto
Student Number	: 20112812012
Study Program	: Mathematics Education

Approved by:

Prof. Dr. Zulkardi, M.I.Komp., M.Sc. Supervisor I	Dr. Darmawijoyo Supervisor II
Dean of Faculty of Teacher Training and Education	Head of Mathematics Education Department
Sofendi, M.A., Ph.D. NIP 19600907 198703 1 002	Prof. Dr. Zulkardi, M.I.Komp., M.Sc. NIP 19610420 198603 1 002

Date of Approval: June 2013

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By:
Sylvana Novilia Sumarto
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Approved by Examination Committee	**Signature**
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. **Sriwijaya University**	----------
Dr. Darmawijoyo **Sriwijaya University**	----------
Dr. Maarten Dolk **Freudenthal Institute for Science and Mathematics Education, Utrecht University**	----------
Dr. Yusuf Hartono, M.Sc. **Sriwijaya University**	----------
Dr. Ratu Ilma Indra Putri, M.Si. **Sriwijaya University**	----------

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
JUNE 2013

STATEMENT PAGE

I hereby:

Name : Sylvana Novilia Sumarto

Place of birth : Pasuruan

Date of birth : November 22, 1987

Academic Major : Mathematics Education

State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.

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Palembang, June 2013

The one with the statement

Sylvana Novilia Sumarto

NIM. 20112812012

ABSTRACT

The way to teach proportion by giving a ready used- formula remains meaningless for the students. They may just memorize the procedure of cross multiplication without understand about the insight of proportionality itself. Thus, we need a learning design which is not only emphasizing the ability to solve the proportional problem but also can develop the students' proportional reasoning. Within a learning trajectory, five activities were designed based on the principle of *Pendidikan Matematika Realistik Indonesia* (PMRI)- the Indonesian version of Realistic Mathematics Education (RME). In this study, design research was carried out to investigate how the ratio table can support the students to develop their proportional reasoning as well as to contribute to the development of the local instructional theory. Thirty five students Grade 4 from SDN 179 Palembang were involved in this study. The analysis of students' works and the video of the learning process in the classroom show that wrapped in the real contextual problem, the ratio table may develop the students' proportional reasoning as well as the strategy to solve proportional problem.

Keywords: Ratio Table, Proportion, Proportional Reasoning, *Pendidikan Matematika Realistik Indonesia* (PMRI), Design Research.

ABSTRAK

Mengajarkan materi perbandingan dengan memberikan rumus-siap pakai membuat proses pembelajaran menjadi tidak bermakna bagi siswa. Siswa mungkin hanya akan menghapalkan prosedur perkalian silang tanpa memahami inti dari konsep perbandingan. Oleh karena itu, kita membutuhkan suatu desain pembelajaran yang tidak hanya menekankan pada kemampuan siswa dalam menyelesaikan masalah perbandingan tetapi juga dapat mengembangkan kemampuan bernalar siswa tentang konsep perbandingan (*proportional reasoning*). Terintegrasi dalam suatu lintasan belajar, lima aktivitas didesain berdasarkan prinsip dalam Pendidikan Matematika Realistik Indonesia (PMRI). Dalam penelitian ini, *design research* diterapkan untuk menginvestigasi bagaimana tabel perbandingan (*ratio table*) dapat mendukung perkembangan nalar siswa tentang perbandingan sekaligus untuk berkontribusi pada perkembangan teori instruksi lokal (*local instructional theory*). Sebanyak tiga puluh lima siswa kelas 4 dari SDN 179 Palembang terlibat dalam penelitian ini. Hasil analisis dari hasil kerja siswa dan video rekaman proses pembelajaran di kelas menunjukkan bahwa bersama dengan masalah kontekstual, ratio table dapat mengembangkan kemampuan bernalar siswa sekaligus mengembangkan strategi untuk menyelesaikan masalah perbandingan.

Kata kunci: Tabel perbandingan, perbandingan, kemampuan bernalar siswa tentang perbandingan (*proportional reasoning*), Pendidikan Matematika Realistik Indonesia (PMRI), *design research*.

SUMMARY

Sylvana Novilia Sumarto. Design Research on Mathematics Education: Ratio Table in Developing the Students' Proportional Reasoning.

In Indonesia, the proportion was taught since Grade 5 elementary school which was integrated in the lesson about fraction. However, the teacher taught by giving the ready-used formula. By using the algebra notation and the cross multiplication, the students may solve the proportional problem but it is not a guarantee that they understand about the concept of proportion itself. In line with this, van Galen &van Eerde (2013) stated that if the procedures were not understood well by the students then they will become vulnerable tricks, meanwhile the proportional reasoning is a "capstone" of elementary school mathematics (Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr as cited in Parish, 2010). Therefore, we need a learning design which supports the development of students' proportional reasoning.

Within a design research, we developed a learning trajectory of proportion which may facilitate the students to develop of students' proportional reasoning. We tested the Hypothetical learning trajectory (HLT) to the 35 students Grade 4 of SDN 179 Palembang, Indonesia in two cycles. There were six students involved in the first cycle. Here, the researcher had a role as the teacher. The result from the first cycle was used to adjust the initial HLT. Meanwhile, the other 29 students were participated on cycle 2. The students were taught by their own teacher based on the refined HLT.

Five activities were designed based on the principle of *Pendidikan Matematika Realistik Indonesia* (PMRI)- the Indonesian version of Realistic Mathematics Education. As the characteristics of PMRI, we started the lesson by proposing contextual problems which were familiar for the students and used the ratio table as the model here.

During the first cycle, we found that we need to change some sentence in the problems in order to make it more understandable for the students. We also change the order of some activities; the activity of exploring the ratio table would be given before the students worked on the simple missing value problem on the cycle 2. Thus, after be refined the order of the activities became as follow.

The first activity was Make Butterflies. In this activity, the students were asked to determine the number of wings, bodies, and antennas to make certain numbers of models of butterfly if known that to make a model of butterfly we need 4 wings, 1 body and 2 antennas. This activity aims to make the students understand the notion of ratios in proportional problems. The second activity was My Chocolate. In this activity the students explore the ratio table through making a list of chocolate price. They determine the number of chocolate by themselves then found the price. The goal of this activity was the students may be able to gain some strategies that can be done by using ratio table and may lead them to

reinvent the rule 1 (unit 1 strategy). These strategies would help them to solve the proportional problems. The third activity was Feeding Caterpillars. This activity proposed a missing value problem. The students were asked determine the number of leaves which are needed to feed caterpillars in 64 days if known that we need 5 leaves to feed the caterpillars in two days. The fourth activity was "What do you think?" In this activity, we proposed some different contextual problems in order to strengthen the students' ability in using the techniques in solving simple proportional problem as well as to improve the students' proportional reasoning. The last activity was Best Price. The problem which was posed in this activity was a comparison problem. Here, we want to build up the students' idea about proportionality in comparison situation.

In line with the aforementioned background of this research, the present study aims to facilitate the development of students' proportional reasoning as well as to contribute to the development of local instructional theory. Accordingly, we proposed a research question as: *"How can the ratio table supports pupils to develop proportional reasoning?"*

To address the main research question in this study, first we answered sub questions, based on the findings on cycle 2 of this study, as follow.

"*How do the students in Grade 4 use their intuitive understanding to solve proportional problem?"*

On the pretest and the interview in the beginning of cycle 2, we see that some students were able to gain the idea of unit 1 method to solve the simple missing value problem while the others multiplied with the wrong number. The others came up with the idea of doubling and repeated addition to look for the fourth number in missing value problem. However, some students still need guidance to deal with this problem. By proposing a similar question with smaller numbers, the lower achievement students, who have no proficient multiplication, may be able to think about the repeated addition as the way to solve the original problem.

Instead of using the proportionality, some students used the absolute value to solve the comparison problem. They may not have idea about the relation between two factors which influence the comparison situation, such as the price and the weight. Basically, the context which used in problem and the questions which were posed during the interviews may influence the students' reaction. When the context which used in the problems was experienced by the students, they may be able to give the right answer based on what happened in their daily life. Proposing the similar problems and change the number may lead the students to get the idea to solve the problem.

"*How can the ratio table support pupils to develop proportional reasoning in missing value problem?"*

Based on the analysis data from the activity 1 until activity 4, we may conclude that by using the ratio table, the students can organize the information from the problem because the table classifies each item in systematic way. Sometimes the students do not aware of what number represent is; the students just directly multiply it or divided it without any consideration. The ratio table may prevent the students to do this. The ratio table also allows the students to write the intermediate step of the calculation. First, it facilitates the students who cannot do the calculation well. Second, by writing the intermediate steps, the

students may be able to see the relation among the numbers. Here, the students can construct their proportional reasoning.

"How can the ratio table support pupils to develop proportional reasoning in comparison problem?"

Through the data analysis on the activity 5, we can say that the students may use the table to look for the proportions which give a good comparison situation. They find that the table helps them to find the number of items that they got for the same price or to find the price for the same number of items and then compare the price. Of course, it can be happened after the students master the use of ratio table in simple proportional problem. It is in line with what stated in Tourniere & Pulos (1985), "Comparing strategy is an advanced method, and the ability to choose the arithmetically easier comparison is acquired long after the proportional techniques mastered."

Based on the answer to the sub research question, we may formulate the answer for the main research question as follow.

The ratio table together with the context can help the students to develop their proportional understanding as well as the strategies to solve the proportional problems; both in missing value and comparison problems. By using ratio table, the students may organize the information from the question. It helps them to identify what the numbers represent for. The intermediate steps which are showed on the ratio table allow the students to see the relationship among the numbers. Hence, they can develop the proportional relation among the numbers. Once they realize about this relation, they may use that as the reason to think proportionally. When the students already mastered the techniques to solve simple proportional problem, they may developed the ratio table as a tool to do calculation and reasoning in comparison situation. The ratio table allowed them to gain the proportions which show the comparison situation.

RINGKASAN

Sylvana Novilia Sumarto. Design Research on Mathematics Education: Ratio Table in Developing the Students' Proportional Reasoning.

Di Indonesia, materi tentang perbandingan diajarkan sejak kelas 5 SD, terintegrasi dengan bab tentang pecahan. Namun dalam mengajarkannya, Guru langsung memberikan rumus jadi, yaitu dengan menggunakan notasi aljabar dan perkalian silang. Melalui metode pembelajaran semacam ini, siswa mungkin dapat mengerjakan soal-soal perbandingan dengan menggunakan rumus yang telah diajarkan di sekolah, tetapi ini bukanlah jaminan bahwa siswa memahami benar tentang konsep perbandingan itu sendiri. Sejalan dengan ini, van Galen dan van Eerde (2013) menyatakan bahwa suatu prosedur atau cara yang tidak dipahami secara benar oleh siswa hanya akan menjadi suatu trik. Hal ini menyebabkan kemampuan bernalar siswa tentang perbandingan (*proportional reasoning*) menjadi tidak berkembang, padahal *proportional reasoning* merupakan "*capstone*" (batu penjuru) dari matematika di sekolah dasar (Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr seperti dikutip dalam Parish, 2010). Oleh karena itu, kita membutuhkan suatu desain pembelajaran yang dapat mendukung perkembangan kemampuan bernalar siswa dalam masalah perbandingan.

Dengan menggunakan metode *design research*, kami mengembangkan sutau lintasan belajar yang dapat memfasilitasi siswa dalam mengembangkan penalaran tentang konsep perbandingan. Kami mencobakan suatu hipotesis lintasan belajar-*Hypothetical learning trajectory* (HLT) kepada 35 siswa kelas 4 SDN 179 Palembang, Indonesia dalam dua siklus. Enam orang siswa dilibatkan dalam siklus pertama. Disini, peneliti berperan sebagai guru pengajar. Hasil dari siklus yang pertama ini digunakan untuk merevisi HLT versi awal. Untuk 29 siswa lainnya berpartisipasi dalam siklus yang kedua. Pada siklus kedua ini, siswa diajar oleh guru mereka sendiri berdasarkan HLT yang telah direvisi.

Terdapat lima aktivitas dalam desain pembelajaran ini. Kelima aktivitas tersebut didesain berdasarkan prinsip-prinsip dalam Pendidikan Matematika Realistik Indonesia (PMRI). Sesuai karakteristik dalam PMRI, kita memulai pembelajaran dengan masalah kontekstual yang familiar bagi siswa dan menggunakan tabel perbandingan (*ratio table*) sebagai modelnya.

Selama mejalankan siklus pertama, kami menemukan bahwa kita perlu mengubah beberapa kalimat dalam soal agar lebih mudah dimengerti oleh para siswa. Kami juga mengganti urutan aktivitas pada siklus kedua; aktivitas mengeksplorasi tabel diberikan sebelum siswa mengerjakan masalah *missing value* (tipe soal yang mencari salah satu nilai yang belum diketahui). Setelah diperbaiki, maka urutan aktivitas dalam desain pembelajaran ini sebagai berikut.

Aktivitas pertama adalah Membuat Kupu-kupu. Pada aktivitas ini, para siswa diminta untuk menentukan banyak sayap, badan, dan antena yang dibutuhkan

untuk membuat sejumlah model kupu-kupu jika diketahui bahwa untuk membuat sebuah model kita membutuhkan 4 sayap, 1 badan, dan 2 antena. Tujuan dari aktivitas ini adalah untuk membuat siswa memahami konsep rasio dalam perbandingan. Aktivitas yang kedua adalah Cokelatku. Pada aktivitas ini, siswa mengeksplorasi tabel perbandingan dengan cara membuat daftar harga cokelat. Siswa menentukan sendiri banyak cokelatnya kemudian siswa menentukan harganya. Aktivitas ini bertujuan untuk membantu siswa mengembangkan beberapa strategi yang dapat dilakukan dengan mengggunakan tabel perbandingan dan memnugkinkan mereka untuk menemukan kembali aturan mencari nilai 1. Strategi-strategi ini nantinya dapat membantu mereka dalam menyelesaikan masalah perbandingan. Aktivitas yang ketiga adalah Memberi Makan Ulat. Pertanyaan dalam aktivitas ini merupakan *missing value*. Siswa diminta untuk menentukan banyak daun yang dibutuhkan untuk memberi makan ulat selama 64 hari jika diketahui kita membutuhkan 5 daun untuk memberi makan ulat selama 2 hari. Aktivitas keempat adalah "Bagaimana menurutmu?" Dalam aktivitas ini, siswa diberi beberapa masalah kontekstual yang berbeda dengan tujuan untuk memperkuat kemampuan siswa dalam menggunakan teknik-teknik untuk menyelesaikan masalah perbandingan sederhana sekaligus untuk mengembangkan kemampuan bernalar siswa. Aktivitas yang terakhir adalah Harga Terbaik. Masalah yang diberikan kepada siswa dalam aktivitas ini merupakan masalah *comparison* (tipe soal yang membandingkan dua atau lebih situasi perbandingan). Disini, kamu ingin membangun ide siswa tentang adanya konsep perbandingan dalam membandingkan sesuatu.

Berangkat dari latar belakang penelitian, maka tujuan dari penelitian ini adalah untuk memfasilitasi perkembangan kemampuan bernalar siswa dalam konsep perbandingan sekaligus berkontribusi dalam perkembangan teori instruksi lokal (*local instructional theory*). Sejalan dengan hal ini, maka rumusan masalah dalam penelitian ini adalah *"Bagaimana tabel perbandingan dapat mendukung perkembangan kemampuan bernalar siswa dalam perbandingan?"*

Untuk menjawab masalah utama dalam penelitian ini, pertama kami akan menjawab sub pertanyaan, berdasarkan temuan pada siklus kedua, sebagai berikut.

"Bagaimana siswa kelas 4 menggunakan pemahaman intuisi mereka untuk menyelesaikan masalah perbandingan?"

Dalam tes awal dan wawancara di awal siklus kedua, kita mengamati bahwa ada siswa yang dapat menemukan ide unit satuan untuk menyelesaikan masalah *missing value* yang sederhana ketika yang lain mengalikan dengan bilangan yang salah. Beberapa siswa yang lain menggunakan ide melipatgandakan (*doubling*) dan penjumlahan berulang untuk mencari bilangan keempat pada masalah *missing value*. Namun, beberapa siswa masih memerlukan bimbingan dalam menyelesaikan masalah ini. Dengan menanyakan pertanyaan serupa yang menggunakan bilangan yang lebih kecil dapat membantu siswa yang lemah di pelajaran matematika, yang mungkin tidak memiliki kemampuan dan pemahaman

yang cukup tentang konsep perkalian, untuk berpikir tentang penjumlahan berulang sebagai cara untuk menyelesaikan masalah semula.

Bukannya menggunakan konsep perbandingan, beberapa siswa justru menggunakan nilai mutlak untuk menyelesaikan masalah *comparison*. Mungkin mereka tidak memiliki pemahaman tentang relasi antara dua factor yang memepengaruhi situasi perbandingan, misalnya harga dan berat. Pada dasarnya, konteks yang digunakan dalam soal dan pertanyaan yang diajukan selama wawancara mempengaruhi respon siswa. Ketika konteks yang digunakan pernah dialami sendiri oleh siswa, mereka dapat memberikan jawaban yang benar berdasarkan apa yang terjadi dalam kehidupan mereka sehari-hari. Dengan mengajukan soal yang serupa dan mengganti bilangan yang digunakan dalam soal mampu membawa siswa untuk mendapatkan ide pemecahan masalah.

"Bagaimana tabel perbandingan dapat mendukung perkembangan nalar perbandingan siswa dalam masalah missing value?"

Berdasarkan analisis data dari akivitas 1 hingga aktivitas 4, kita dapat menyimpulkan bahwa dengan menggunakan tabel perbandingan siswa dapat mengorganisasi informasi dari soal yang diberikan karena tabel menggolongkan masing-masing benda dalam cara yang sistematis. Terkadang siswa tidak memperhatikan sebenarnya bilangan yang ada di dalam soal itu merepresentasikan apa; siswa hanya mengalikan atau membagi bilangan-bilangan tersebut tanpa pertimbangan apapun. Hal ini dapat dicegah dengan menggunakan tabel perbandingan. Tabel perbandingan juga memungkinkan siswa untuk menuliskan proses penghitungan langkah demi langkah. Pertama, ini bermanfaat untuk memfasilitasi siswa yang tidak terlalu bisa berhitung dengan baik. Kedua, dengan menuliskan proses penghitungan langkah demi langkah membuat para siswa dapat melihat relasi diantara bilangan-bilangan yang ada. Disini, siswa dapat mengkonstruk penalaran perbandingan mereka.

"*Bagaimana tabel perbandingan dapat mendukung perkembangan nalar perbandingan siswa dalam masalah comparison?*"

Melalui analisa data yang diperoleh dari aktivitas 5, kita dapat mengatakan bahwa siswa dapat menggunakan tabel untuk mencari perbandingan yang dapat digunakan untuk membandingkan dengan baik. Mereka menyadari bahwa tabel dapat membantu mereka dalam menemukan banyaknya benda ynang mereka peroleh untuk harga yang sama atau untuk mencari harga untuk dua atau lebih benda dengan banyak yang sama dan kemudian membandingkan harganya. Tentunya, ini dapat terjadi setelah siswa menguasai penggunaan tabel perbandingan dalam masalah perbandingan yang sederhana. Hal ini senada dengan apa yang dinyatakan dalam Tourniere & Pulos (1985), strategi yang digunakan dalam membandingkan sesuatu adalah suatu metode tingkat lanjutan, dan kemampuan untuk memilih perbandingan mana yang lebih mudah, diperoleh lama setelah teknik penyelesaian masalah perbandingan dikuasai.

Berdasarkan jawaban untuk sub pertanyaan sebelumnya, kita dapat menjawab pertanyaan utama dalam penelitian ini sebagai berikut.

Tabel perbandingan bersama dengan konteks dapat membantu siswa mengembangkan penalaran perbandingan sekaligus strategi pemecahan masalah perbandingan; baik dalam masalah *missing value* maupun *comparison*. Dengan menggunakan tabel perbandingan, siswa dimungkinkan untuk mengorganisasi informasi dari pertanyaan yang diberikan. Tabel membantu mereka dalam mengidentifikasi apa yang direpresentasikan oleh bilangan-bilangan tersebut. Langkah per langkah yang ditunjukkan oleh tabel perbandingan memungkinkan siswa untuk melihat relasi diantara bilangan-bilangan yang ada. Dari sini mereka dapat mengembangkan hubungan proporsional diantara bilangan-bilangan tersebut. Pada saat mereka menyadari hubungan ini, mereka dapat menggunakannya untuk bernalar secara proporsional. Lebih lanjut, ketika siswa telah menguasai teknik pemecahan masalah perbandingan yang sederhana, mereka dapat menggunakan tabel perbandingan sebagai alat untuk menghitung sekaligus bernalar dalam situasi proporsional. Tabel perbandingan memunculkan perbandingan yang menunjukkan situasi proporsional.

PREFACE

By the gracious of God, I could finish writing this thesis; thank you Lord, for all your blessings to me.

This thesis was written to fulfill the requirements for the degree of Master of Science (M.Sc) in International Master Program on Mathematics Education (IMPoME), Sriwijaya University, Palembang- Indonesia in collaboration with Utrecht University, Utrecht- the Netherlands. I hope the results from this thesis may contribute to the development of the mathematics education, especially the implementation of *Pendidikan Matematika Realistik Indonesia* (PMRI) in Indonesia.

First, in this opportunity I would like to express my deepest gratitude to my wonderful supervisors, Frans van Galen, Prof. Zulkardi and Dr. Darmowijoyo, for their never ending support, helpful advice, sincere help, serious guidance, great motivation and encouragement during writing this thesis. The idea of this research was developed through my discussion with Marteen Dolk and Dolly van Eerde. Thank you very much to both of you. Special thanks also for Mieke Abels, Marteen Kindt, Paul Drijvers, Dirk Jan, Arthur Bakker, Steven Wepster, Barbara van Amerom, Elwin Savelsbergh, Henk van der Kooij, Betty Heijman, Mark Uwland, and all staffs in Freudhental Institute, Utrecht Univesity for all your help and guidance during my study in Utrecht. In memoriam, Jaap den Hertog. Thank you for assisting me during the preparation to go to the Netherlands.

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Palembang, June 2013

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CHAPTER I

INTRODUCTION

1.1 Background

Proportion can be found everywhere. For instance, we use the concept of proportion in rate problems. If we need 6 litres to drive 15 km then we need 12 litres to drive 30 km. We also use the concept of proportion in a shopping context to compare prices. If there are two kinds of peanut butter, one costing 2 euro for a 150 ml bottle and the other costing 3 euro for a 250 ml bottle, we can use the concept of proportion to determine that the second bottle is cheaper. In addition, we also find the concept of proportion in scale. Most mathematicians defined a proportion as the equality of two ratios, for example $- = -$ (Tourniaire & Pulos, 1985; Langrall & Swafford, 2000) but in a certain sense, a proportion can be defined as a more general concept, which is specifically reflected in fractions, percentages and decimals (Galen *et al*, 2008). In this sense, we can expand the list of the use of proportion in our everyday life. However, in this study we will use the sense of proportion as in the first definition. Regarding the usefulness of the concept of proportion in our daily life, we may say that proportion is one of important concepts in mathematics. Therefore, it is necessary to learn about this topic.

In Indonesia, the students learn about proportion on the 5^{th} until 8^{th} grade (Depdiknas, 2006). During 5^{th} grade they will learn about direct proportion and in the higher level they will learn about indirect proportion as well. According to Zulkardi (2002), most of textbook that used in Indonesia contain mainly the set of rules and algorithm which is already formal and they lack of application which is

needed by the students in order to make the concept be real for them. The lesson of proportion in grade 5 started by giving the fixed procedures to solve the problems (e.g. Soenaryo, 2007; Sumanto *et al*, 2008). In line with that, the teachers teach the cross multiplication to solve the problem. Although the students are able to solve the proportional problem by applying this method, they may just memorize the procedure and not develop their proportional reasoning. In addition, van Galen & van Eerde (2013) also stated that if the procedures were not understood well by the students then they will become vulnerable tricks.

"Proportional reasoning is defined as a term that denotes reasoning in a system of two variables between which there exists a linear functional relationship..." (Karplus *et al*, 1983, p.219). The proportional reasoning is a "capstone" of elementary school mathematics (Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr as cited In Parish, 2010). Many studies have been conducted to observe the development of children' proportional reasoning (e.g. Karplus *et al*, 1983; Tourniaire & Pulos, 1985; Tourniaire, 1986; Langrall & Swafford, 2000; Silvestre & da Ponte, 2012). These studies observe the students' informal knowledge about proportion, what kind of strategies they used to solve proportional problems and what factors from the problems that influence their performance. Based on the studies, there are three difficulty factors of a proportional problem which influence students' performance: presence of an integer ratio, numerical complexity, and the order of the number that the students have been looking for. The problems with small, integer ratios and questions about what is the biggest number to be found are easier for the students (Rupley, 1981 in Tourniaire, 1986). These studies contributed to the developmental of

theory about the students' proportional thinking and reasoning. However, these theories are not applicable in the practical situation. The teachers in the school may not have any idea about the real application in the teaching practice in their classroom. Therefore, it still need more research to improve the theory development at the level of the instructional activities, the course (local instruction theories) and at the level of the domain-specific instruction theory. With regard to this, in this study we proposed a learning trajectory and the instructional learning activities which can be used and implemented by the teachers.

As mentioned above, the students in Indonesia learn about proportion formally in Grade 5. However, the sense of proportion has been learned since the earlier grade although it is not listed clearly in the curriculum. It is developed gradually since that. For instance, the children in kindergarten knew that the giant's chair is bigger than the normal human's chair or the students in the second grade knew that the number of steps from a man is smaller than the number of steps a 5 years old-girl for the same distance. The students in 4^{th} grade learned about pattern and relationship which are also related with the proportional thinking. It means that they have some prior knowledge about the concept of proportion in some extent. Accordingly, we initiate to try out our design for students Grade 4 with the consideration that through this design they may able to prepare themselves before they learn proportion more formally in Grade 5. Moreover, it may help them to lay the groundwork for future work in the higher level.

Regarding the purpose of this study which is to facilitate the students in developing their proportional reasoning, five activities were designed based on the

principle of Pendidikan Matematika Realistik Indonesia (PMRI) - which is adapted from the Realistic Mathematics Education (RME). In line with the statement by Langrall & Swafford (2000, p. 260), "Instruction with proportional reasoning should begin with situations that can be visualized or modelled", the activities in this study also were initialized by the contextual problems which can be visualized by the students. The problems used their daily activities as the context, so they may be able to imagine the real situation as described on the questions. In this design, we also proposed a ratio table as the model that may facilitate them to develop their proportional reasoning.

The ratio table was choosen as the model in this design because it shows the proportions clearly. We can expand the number of columns to suit our needs. It is an ideal aid for making handy calculations, gaining insight, and students reason with proportion because the table invites the students to write down intermediate steps. For instance, to find how many km which can be reached in 80 minutes if we can go for 15 km in 60 minutes, the students may use 5 in 20 as intermediate step or 30 in 120 then 10 in 40 to get 20 km in 80 minutes. The strength of a ratio table is that the students can reason with a number relationship which they already know (van Galen *et al*, 2008). Accordingly, in this study a learning sequence was designed with focus on the developing of students' proportional reasoning by using a ratio table as model.

1.2 Research Aims

According to the background of this study, the purposes of this study were:

1. To develop the learning design which was not only emphasized the students' ability in solving the proportional problem but also may develop the students' proportional reasoning.
2. To contribute to the development of local instructional theory in proportion.

1.3 Research Questions

Regarding to the aforementioned background of this study, we proposed the main research question for this study as follow.

How can the ratio table support pupils to develop proportional reasoning?

To address the main research question, we proposed three sub research questions as:

1. *How do the students in Grade 4 use their intuitive understanding to solve proportional problem?*

Before we investigated how the ratio table may help the students to develop their proportional reasoning, we should know how they may solve the proportional problem by using their intuitive understanding and what the students' earlier idea about proportion. This information also may help us to improve the designed learning trajectory.

2. *How can the ratio table supports pupils to develop proportional reasoning in missing value problem?*
3. *How can the ratio table supports pupils to develop proportional reasoning in comparison problem?*

As stated by Karplus *et al* (1983), Tournaire & Pulos (1985) and Silvestre & da Ponte (2012), in general the proportional problems were categorized as missing value problem and comparison problem. Thus, the second and the third sub research questions were addressed in order to investigate how the ratio table may facilitate the students to develop their proportional reasoning in both types of proportional problems.

CHAPTER II

THEORETICAL BACKGROUND

2. 1 Proportion

Proportion is an important concept in mathematics. We encounter this concept everywhere, for example in enlarging and reducing photos, price comparisons, the recipes, comparing probabilities, graphs and diagrams (van Galen et al, 2008). However, Tourniaire and Pulos (1985) stated that this concept is difficult. The concept is acquired late and many adults do not master the concept. In formal mathematics, a proportion is defined as the equality of two ratios, for example $\frac{a}{b} = \frac{c}{d}$ (Tourniaire & Pulos, 1985; Langrall & Swafford, 2000).

In general proportional problems are categorized in two types, namely missing value problem and comparison problem (Karplus *et al*, 1983; Tournaire & Pulos, 1985; Silvestre & da Ponte, 2012). Missing value problems present three numbers and ask for the fourth one, such as how many candies that we get for 3 euro if we know that 6 candies costing 2 euro. Meanwhile, the comparison problems present two or more pairs of numbers and ask about their comparison. For instance, if there are two kinds of peanut butter, one costing 2 euro for a 150 ml bottle and the other costing 3 euro for a 250 ml bottle, we can determine the second butter as the cheaper one by comparing the price for 1 euro for each of them. Basically, the comparison problem is more difficult than the missing value problem. It requires the students to be able to solve the simple proportional problem before they can work on the comparison situation. As stated in Tourniere & Pulos (1985), "Comparing strategy is an advanced method, and the ability to choose the arithmetically easier comparison is acquired long after the proportional techniques

mastered." In line with that, the activities which proposed the missing value problems were presented before the activities about the comparison were given.

According to a literature review by Tournaire & Pulos (1985), the factors from the problems that may influence students' performance are

1. *Structural variables of the proportional problem*
    - The number structure of the proportion problems influences the subjects' performance. There are three main difficulty factors: presence of an integer ratio, numerical complexity, and order of number that have been looking for. The presence of integer ratios makes a problem easier. The difficulty introduced by non-integer ratios is widely recognized. The problems with small, integer ratios and ask the biggest number to be found are easier for the students
    - The presence of a unit
    - The presence of unequal ratios, for comparison problem
2. *Context variables of the proportional problem*
    - The familiarity of the context. The familiar problem is easier than unfamiliar ones.
    - The mixture problems is more difficult than non-mixture problems
    - People can more easily visualize discrete content than continuous content.
    - The mode of delivery of the problem.

2. 2 Proportional Reasoning

Karplus *et al* (1983) defined the proportional reasoning as a term that denotes reasoning in a system of two variables between which there exists a linear functional relationship. Proportional reasoning is an important ability to build a

foundation for high school mathematics and algebraic reasoning (Langrall & Swafford, 2000). Furthermore, as cited in Parish (2010), Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr consider the proportional reasoning as a "capstone" of elementary school mathematics. It is needed to learn the important topics in mathematics such as fraction equivalence, place value, long division, percentage calculation, measurement conversions, and derivation of rates which are troublesome for the students.

According to Boyer & Levine (2012), considering the value and the effectiveness of intuitive capacities, the development of mathematical understanding is characterized not as ''all or none'' but it is mowing from partial to more complete and become less contextually dependent. The intuitive understandings influence the people's mathematical skills even after they emerge the formal mathematics. In line with that, giving the instruction that builds on the students' intuitive understanding of proportional relations may support the development of their proportional thinking.

2. 3 Knowledge Gap

Many studies have been conducted to observe the development of children's' proportional reasoning (e.g. Karplus *et al*, 1983; Tourniaire & Pulos, 1985; Tourniaire, 1986; Langrall & Swafford, 2000). These studies observe the students' informal knowledge about proportion, what kind of strategies they used to solve proportional problems and what factors that influence their performance. These studies give contribution to the developmental of theory about the students' proportional thinking and reasoning. However, those studies are more in theoretical way; less contribute to the practical side. Thus, we still need to do

more research to develop learning sequences which can be implemented directly by the teacher at school and also can be used to improve the theory development at the level of the instructional activities, the course (local instruction theories) and at the level of the domain-specific instruction theory as well. Therefore, within a design research, a learning trajectory and the instructional design were developed in this study. The designed activities were implemented and analysed based on the hypotethical learning trajectory.

2. 4 *Pendidikan Matematika Realistik Indonesia* (PMRI)

In the past, teachers always taught their students in traditional way, gave explanation about the concepts and procedures to solve the given problem. Because of that, most students in mathematics classroom did not see mathematics as creative instead as something to be explained by their teacher, then practiced and applied. It made mathematics as a dead language (Fosnot & Dolk, 2001). In order to make the learning process become more meaningful for the students, we should not give them the ready-made product; they should reinvent the idea of mathematical concept by themselves (Gravemeijer &Terwel, 2000; Grevemeijer, 1994). Mathematics should be understood as a human activity (Freudenthal, 1973), which is creative and alive. Accordingly, about 1970s the Realistic Mathematics Education (RME) was developed in the Netherlands.

RME then was adapted as *Pendidikan Matematika Realistik Indonesia* (PMRI) in Indonesia. The PMRI was implemented in Indonesia since 2001(Zulkardi, 2002). These both approaches are the same in principle but may be different in the context which was chosen for the problem. The context in PMRI was adapted so that fits into the Indonesian culture.

There are five characteristics of RME that should be considered as a guideline to make a learning sequence (Gravemeijer, 1994). The following explanation will describe about how the characteristics of RME fits the design in this study.

1. The use of contexts

The starting point of mathematics instruction should be real to the student. This process will allow them to investigate the situation. Through the process of finding and identifying the relevant mathematical elements, schematizing and visualizing, they can discover the patterns and develop a model in a mathematical concept. Then by reflecting and generalizing, the students will develop a more complete concept. While they apply mathematical concepts to other aspects on their daily life, they reinforce and strengthen the concept (Zulkardi, 2002). With regards to this, the activities in this study used the contextual problem as the starting point of the lesson, such as the money context. The students are familiar with the money context. They find it in their daily life. They buy things with money.They also familiar with situation which requires them to compare the price to have the best buying. This context allows them to use their experience to gain the "elementary" strategies to solve the given problems.

2. The use of models

Models are important for the students to bridge them from informal into formal mathematic. In this study, the ratio table will be the model to develop the students' proportional reasoning. At the first, the students just will see the ratio table as a usual table which is used to make a systematic list of certain items. Then they will realize that the ratio table can be used to do the calculation in proportional way. Then, in the end of the learning sequence, the students are able

to use the ratio table as a tool to thinking and reasoning in more formal level. The intermediate steps which are showed by the ratio table let the students to see the proportional relation among the numbers. Together with the context, the ratio table may develop the students' proportional reasoning.

3. The pupils' creations and contributions

In this study, the teacher will give the students time to think and explore the rules on ratio table by their selves. Their finding of the rules will help them to reach the formal algorithm for the higher grade. The role of the teacher is very important here. The teacher should encourage the students to involve in the discussion. Related with this, the teacher also should consider the social norm in the class.

4. The interactivity

The interaction between pupils and between pupils and teachers is an essential part in RME instructional processes. Explicit negotiation, intervention, discussion, cooperation and evaluation are essential elements in a constructive learning process. In this interactive instruction, pupils are engaged in explaining, justifying, agreeing and disagreeing, questioning alternatives and reflecting (Zulkardi, 2002). Regarding to this, on each lesson, the teacher will conduct the discussion in the classroom. The teacher should give chance to the students in developing their understanding as a taken-as-shared- pedagogy (Yackel & Cobb, 1996). Oce again, the teacher has the big role here. The teacher should create the situation inwhich the students can be interactive among each other.

5. The intertwining

The integration of mathematical strands or units is essential. The intertwining of learning strands is exploited in solving real life problems (Zulkardi, 2002).This characteristic is implied in this study. The topic of proportion is also related with the notion of other mathematical operation, such as linear function, fraction, and numbers operation.

2. 5 Ratio Table

Ratio table is an appropriate model to facilitate students in learning about proportion. It is an ideal aid for making handy calculations, gaining insight, and students reason with proportions because the table invites the students to write down intermediate steps. For instance, to find how many km which can be reached in 80 minutes if we can go for 15 km in 60 minutes, the students may use 5 in 20 as intermediate step or 30 in 120 then 10 in 40 to get 20 km in 80 minutes. The ratio table shows the proportion clearly because we can expand the number of columns to suit our needs. The strength of ratio table is that the students can reason with number relationship which they already know.

It is important for the teacher to discuss what can be done in ratio table. Context makes the students realize about what kind of operation that you can do, such as price and weight, you can multiply with same number but cannot add with the same number. It is important to state units are used and arrows to help the students see the operation. But it doesn't mean to introduce the ratio table from above without any discussion- the students may reinvent by themselves. Reinvent here is not about the notation but the choice of systematic notation method and discovering for themselves how useful a ratio table to do calculation. The

transition is from systematic list (the items under each category) to a ratio table. “The advantage of ratio table is that all numbers have their own place and that the unit of measurement must stay the same.” (van Galen et al, 2008)

Some rules in calculating with a ratio table that should be reinvented by the student were doubling, halving, adding, taking away, multiplying by the same number, dividing by the same number. Doubling, adding and multiplying by the same number were used to find the bigger number as the answer while halving, taking away and dividing by the same number were used to find the smaller number as the answer.

Table 2.1 *The Strategies on the Ratio Table*

Strategies	Description
Doubling	chocolates | Price ×2 { 2 | 1500 } ×2 4 | 3000 8 | 6000
Halving	chocolates | Price ×1/2 { 8 | 6000 } ×1/2 4 | 3000 2 | 1500
Adding	chocolates | Price + { 8 | 6000 } + 4 | 3000 12 | 9000
Taking away	chocolates | Price - { 8 | 6000 } - 2 | 1500 6 | 4500
Multiplying by the same number	chocolates | Price ×4 { 8 | 6000 } ×4 32 | 24000

Dividing by the same number		chocolates	Price	
	: 6	24	18000	: 6
		4	3000	

Beside these the strategies, the students may be able to use repeated addition and the unit strategy as the way to solve proportional problem. The unit strategy involved the division and multiplication. The students should find the unit which was used as the multiplication factor to find the answer, such as the price per kg to find the price of 15 kg of rice, the number of books per box to find the total number of books in 5 boxes, and the distance that can be driven for 1 liter of fuel to find the distance that can be driven for 6 liters of fuel.

2. 6 Social norms and socio-mathematical norms

"Social norms refer to expected ways of acting and explaining that become instantiated through a process of mutual negotiation between the teacher and students" (Gravemeijer& Cobb, 2006). In specific way, Yackel & Cobb (1996) defined the socio-mathematical norms as classroom norms which emphasize the mathematical aspects of student's activity, such as the understanding of what counts as a mathematical difference, a mathematical sophistication, an acceptable mathematical explanation and justification in classroom.Both of them have an important role to reach the learning goal of this research. By considering these factors, the teacher may establish the environmental in the classroom which is conducive to improve students' mathematical conceptual development (Yackel & Cobb, 1996). For instance, by discussing what can be count as different solution or give the students time to think about the other possible strategy may lead the

students to reinvent the rules on the ratio table, such as halving or doubling besides using the repeated addition.

In line with that, the aims of this study are not only to give a contribution to the development of the theoretical instruction, but also to improve the quality of the learning situation in the class. Through the teacher guide which developed in this study, we may guide the teacher to more concern in the learning process in the class. The teacher should realize that the social norms are important to make the class be conducive. The social norms is formed through a long process, it cannot be changed instantly. Thus, the teacher should be consistent in enforcing the "rules" in the class.

2. 7 Proportion in Indonesian Curriculum

Based on the Indonesian curriculum, the students learn about proportion on the 5th until 8th grade. During 5th grade they will learn about direct proportion and indirect proportion in the higher level. The following table shows how the proportions are taught as a part of fraction during the second semester in grade 5.

Table 2.2 *The Curriculum Grade 5 on Second Semester* (Depdiknas, 2006)

Standard Competence	**Basic Competence**
Numbers	
5. Using the fractions to solve problems	5.1 Converting the fractions into the decimals and vice versa. 5.2 Adding and subtracting fractions and decimals. 5.3 Multiplying and dividing fractions and decimals. ***5.4 Using the fractions in solving proportional problems and scale.***

According to Zulkardi (2002), most of textbook that used in Indonesia contain mainly the set of rules and algorithm which is already formal and they lack of application which is needed by the students in order to make the concept be real for them. In line with that, the lesson of proportion in grade 5 started by giving the fixed procedures to solve the problems (e.g. Soenarjo, 2007; Sumanto *et al*, 2008), less attention is given to students' understanding of the concept itself. As cited by Silvestre and Ponte (2012), Stanley, McGowan, and Hull stated that the usual teaching method in which teacher asked the students to solve proportion by using representations, equalities between ratios, and linear function should be replaced by other methods in which pupils engage in activities that help them to discover that proportion is the variation of two quantities related to each other. Thus, we need to propose a learning sequence that starts from the informal level and rich of contexts that may lead them to develop the proportional thinking.

Although proportional reasoning is not stated explicitly in the curriculum for the students under fifth grade, the sense of proportion itself has been learned since that. For instance, the children in kindergarten knew that the giant's chair is bigger than the normal human's chair or the students in the second grade knew that the number of steps from a man is smaller than the number of steps a 5 years old girl for the same distance. The students in 4^{th} grade learned about pattern and relationship which are also related with the proportional thinking. It means that they have some prior knowledge about the concept of proportion. Thus, the proportions are also appropriate for the 4^{th} grade students. In addition, this concept is important as the groundwork for them for the higher level.

Considering the mathematics curriculum, we can see that the topic of fraction also given in grade 4 (see Table 2.2). Although the problems in basic competence of "solving the problems which involves fraction" usually only involved the operations in the fraction, we can expand it to the simple proportional problem. It means that we can teach the topic of proportion integrated with the fraction as in grade 5. Accordingly, teaching proportion in grade 4 is reasonable and reliable.

Table 2.3 *The Curriculum Grade 4 on Second Semester* (Depdiknas, 2006)

Standard Competence	**Basic Competence**
Numbers	
5.Using the fractions to solve problems	5.1 Explaining the meaning of fractions and their order 5.2 Simplifying the fractions 5.3 Adding fractions 5.4 Subtracting fractions 5.5 ***Solving the problems which involves fractions***

CHAPTER III

METHODOLOGY

3.1 Participants

The present study was a part of nternational Master Program on Mathematics Education (IMPoME) project in Indonesia. This program is a collabartion between Sriwijaya University, Surabaya State University and Utrecht University which aims to develop the implementation of PMRI in Indonesia. In line with the goals of the IMPoME program, the present study was conducted in SDN 179 Palembang, Indonesia, which is one of the partner schools of PMRI in Palembang, Indonesia.

There were thirty five students grade 4 participated in this study. Six students from class 4 A were involved in the first cycle on this study while the remains, 29 students from class 4D, participated in the second cycle. The six students from class 4A were chosen based on their achievement in mathematics, namely low, middle and high. The information about the level of students' ability was gathered through the informal interview with their homeroom teacher, Bu Lisa.

3.2 Research Approach

The general aim of this study is to facilitate the pupils in developing their proportional reasoning. Within a design research, a learning trajectory was developed and tested. The result of this study also contributes to the development of local instruction theory. Design research is chosen in this study because it is an approach which can bridge between the practical side and the theoretical side (Bakker & van Eerde, 2012). Furthermore, according to Bakker (2004),

> "Design research is evaluated against the metrics of innovation and usefulness, and its strength comes from its explanatory power and grounding in experience. Moreover, it often leads to products that are useful in educational practice because they have been developed in practice."

Gravemeijer and Cobb (2006) defined three phases in a design research experiment. The description of each phase and how the different roles of a Hypothetical Learning Trajectory (HLT) in each phase are given as follows.

1. Preparing for the experiment

From the design point of view, the purpose of this phase is to formulate a local instruction theory which can be elaborated during conducting the intended design experiment. Meanwhile from the research point of view, the aim of this phase is to clarify the theoretical intent. Regarding this, we started the preliminary phase by developing the sequence of instructional activities. There were 5 activities that were conducted in 4 meetings, each 70 minutes. The conjectures of students' thinking and reaction were also included. We elaborated all of these in a Hypothetical Learning Trajectory (HLT). As mentioned in Bakker (2004), the use of HLT in this phase is as a guide in designing and developing the instructional activities. As the preparation for the class observation and the interview with the teacher, we also provided a classroom observation scheme and an interview scheme (see appendix 1 and 2). These schemes were used as the remainder so we did not miss the important aspect during the interview or the observation.

2. The design experiment

The design experiment phase aims to test and improve the conjectures in our HLT and to understand how it works. In this phase, the learning sequence was

implemented in two cycles. The first cycle was done as the pilot study. A small group consists of 6 students from class 4A were participated in the first cycle. The findings from the first cycle were used to adjust and to improve the first version of HLT. Then the revised HLT was implemented in the experimental class during the second cycle. Twenty nine students Grade 4 were involved in this teaching experiment. In this phase, the HLT has a role as a guideline to conduct the teaching and learning process (Bakker, 2004).

3. Retrospective analysis

The retrospective analysis phase is important to do in design research. The goal of this phase is to contribute in the development of a local instructional theory. The analysis was done by focusing on the conjectures in HLT. In line with this, Bakker (2004) stated that the third function of HLT in design research is as a guideline to do the analysis.

3.3 Data Collection

In this study, there were some data that we used to answer the research question. The data were collected in different way and for different purpose. The more explanation is described in the following statements.

3.3.1 Preparation phase

Before the researcher conduct the teaching experiment, the researcher need to gain some information as the starting point for the research. During the preparation phase, the researcher did classroom observations in the experimental class, an interview with the teacher and pretest respectively.

1. Classroom observation

The classroom observation was conducted as a preparation before the teaching experiment on the second cycle. We need to do classroom observation in the experimental class due to know how the teacher teaches the students, the classroom' culture and the socio-mathematics norms in the classroom, the practical thing about the classroom setting, and more or less the behaviour and characteristics of the students in the class. During the classroom observation, the researcher recorded the whole learning process in video/ camera and also made some field notes based on the observation scheme.

2. Interview with the teacher

The semi-structured interview with the teacher is also important to support or compare the findings during the classroom observation. The researcher asked the questions to the homeroom teacher from class 4D, Bu Intan based on the interview scheme (see appendix 2) and also some follow up questions. The whole interview was recorded by recorder. During the interview, the researcher also made some notes for the important points.

3. Pretest

The pretest was conducted twice. The first written test held in the beginning of the first cycle. The participants of the first pretest are 6 students from 4A. The aim of the first pretest is to see whether the problems are clear and understandable for the pupils or not. Besides, it also used to check whether the problems on the pretest can measure the students' prior knowledge like we want or not. The other advantage of doing the small pretest with the small group is that we can ask directly to the pupils about their interpretation of the problems, also about their

thinking, how they solve the problem, why they solve like that, what is the idea behind their strategy. Then we can use this to improve our problems for the second pretest.

The second pretest was conducted in the beginning of second cycle, before the teaching experiment. The participants were all students from the experimental class. The teacher gave the test and the students work individually for the given time. The aim to do the second pretest is to know the prior knowledge of the students. We used these findings to determine the starting point for the experiment and to do the first adjustment of the revised HLT after conducting the first cycle.

3.3.2 Preliminary teaching experiment (the first cycle)

The first cycle was conducted in a small group of 6 students from class 4A. The students were chosen based on the information from the teacher about their achievement in mathematics. They are Ferdy as the representation of higher achievement, 4 middle ones namely: Rati, Lani, Ando, and Falah, and 1 lower achiever namely Nurul.

During the first cycle, the researcher taught the lesson based on the initial HLT as in chapter 4. We recorded each lesson by video/ camera. We did small interview with the students during the lesson to investigate their understanding about the topic. We also made field notes and collect all students' written work. This cycle is an important step because the results of the first cycle were used to improve and revise the HLT for the second cycle.

3.3.3 Teaching experiment (the second cycle)

The participant of the second cycle was all students from the experimental class. Twenty nine students of class 4D participated in this cycle. The class was taught

by their own teacher based on the revised HLT. Instead of teaching, the researcher was observing the learning process and making some field notes. The whole learning process was recorded by video/ camera. The students' written works also were used as the data collection.

During the second cycle, we also have a small group of 2 as the focus group. We recorded the discussion in that group. The data from the focus group was used to get more detail picture about what happened during the learning process.

3.3.4 Posttest

To know the development of the students' understanding about proportion, the researcher should conduct a pretest before the teaching experiment and a posttest after the teaching experiment. As the pretest, the posttest also were conducted twice. First, in the end of the pilot experiment and the second one was held in the end of the second cycle. The other aim of conducting the first posttest is to check the structure of the problems whether it is understandable for the students or not and then use the result to improve the problems on the second posttest.

During the posttest, the teacher gave some proportional problems (see appendix 3) and the students worked individually for the given time. The problems on the posttest were different with the problems on pretest but still with the same indicators. We compared the students' answer and reason from the pretest and the post test. Although the main goal of this study is not to evaluate the teaching method, the comparison between the students' result from the pretest and the pos test can be used as impression of the result from the lessons.

The participants of the first posttest were the 6 students on the first cycle. Meanwhile the participants of the second posttest were all students from the experimental class.

3.3.5 Validity and reliability

The methods which are used to collect the data during this study are not limited in one method. The data from the interview are collected by using an audio device while the data from the classroom observation are collected by using video recorder and also by making some field notes. The variation of how we collect all the data makes the methods triangulation possible. It contributes to the validity of data collection.

The clear description on how the data collected may let the reader to follow the process of data collection. It improves the external reliability of the data collection because of the traceability. In addition, the reliability also refers to the reasonableness and the power of arguments for the inferences. To improve that, we also did the peer examination by discussing the critical issue with the teacher and the colleagues.

3.4 Data Analysis

This study involves different kind of data. We have data from the classroom observation, interview with the teacher, the written test before and after the teaching experiment, and also the students' written work during the lessons. We analyzed them by using HLT as the guideline to derive a conclusion and answer the research question in this study.

3.4.1 Pretest

The analysis data of the pretest involved a quantitative and a qualitative analysis. We examined the students' work and count the number of correct answer to determine the level of students' understanding. Indeed, we also looked in the students' scratch and their strategy to get more details picture about the student's thinking. We used these findings to determine the starting point for the experiment.

3.4.2 Preliminary teaching experiment (the first cycle)

The data from the first cycle are the video from the whole lessons, field notes and the pupils' written work. The data analyses of the first cycle were started by watching the video-registration and choose some fragments of them. We should consider the fragments in where the videos show that the learning process is success and reach the goal as we expected or otherwise, it is not happened like our expectation. Then the selected fragments were transcribed and were used to test the conjectures in the first version of HLT. To support the result of video analysis, we used the data from pupils' written work and the field notes as well. We used these findings to revise and improve the initial version of HLT for the second cycle.

3.4.3 Teaching experiment (the second cycle)

The data from the classroom observation and teacher interview during the preparation phase were analysed to get the basic information about the context in which the teaching experiment was conducted. The information can be used when discussing the lessons with the teacher and to elaborate the teacher guide.

The data of the second cycle are the video of the whole lessons, the video of the focus group and the pupils' written work. The analysis of the video observation was started by watching the video registration. Some interesting fragments of the video were transcribed and used to test the conjectures on HLT. The analysis from the video of the whole lessons in the classroom was used to get an overview of the lesson, how the learning process happened in the classroom, what is missing during the teaching process or the discussion part. Meanwhile the analysis from the video of focus group was used to get more detail picture about the development of students' understanding. In addition, the students' written work was used as additional data. Overall, the results of the analysis were used to derive conclusions, to answer the research question and to improve the original HLT as well.

3.4.4 Posttest

As in the analysis data of the pretest, in the analysis data of posttest also involved a quantitative and a qualitative analysis. We examine the students' work, to count the number of correct answer and to see their strategy. We use these result and compare it with the result of pretest to see what students have learned. Thus, we get the impression of how the learning sequences work, how the ratio table may help the pupils in developing their proportional understanding.

3.4.5 Validity and reliability

The data collection of this study is varying, namely the videos, the field notes, and the students' written works. The analysis of more than one type of data makes data triangulation possible. As consequence, it improves the internal validity of

the data analysis. In addition, the testing conjectures of the HLT with the collected data also contribute to the internal validity of data analysis.

By presenting the clear and detailed description of how we analysis the collected data and testing our HLT may let the other people to follow the way of analysis and be able to adjust the HLT to their own local setting. It contributes to the external validity and also the external reliability of data analysis as well, in the sense of transparency.

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Hypothetical learning trajectory (HLT) is a way to describe an important aspect of pedagogical thinking in teaching mathematics (Simon, 1995 in Simon and Tzur, 2004). An HLT contains the mathematics goal, the mathematics activities, and the students' conjecture which describes the learning process of the students.

The aim of this study is to know how the ratio table can support the pupils to develop proportional reasoning. In line with the aims, the instructional activities are designed to introduce the ratio table to the students, to lead the students reinvent the rule on ratio table and to develop the students' proportional reasoning by using ratio table. Within a design research, the learning trajectory was tested. As consequence, the result of this study contributes to the development of a local instruction theory about proportion.

In this chapter, we present the HLT which was used in this study. There are five activities which were implemented in grade 4 of SDN 179 Palembang in Indonesia. The general overview of the HLT is described in the following table and the more detail information is presented in the following sections.

Table 4.1 *The Brief Description of HLT*

No	Activity	Description	Learning Goal
1	Make Butterflies	The students determine the number of wings, bodies, and antennas to make certain numbers of butterfly if known that to make a model of butterfly we need 4 wings, 1 body and 2 antennas.	• Students understand the notion of ratios in proportional problems.
2	Feeding Caterpillars	The students determine the number of leaves which are needed to feed caterpillars in certain number of days if	• To introduce the ratio table to the students - To introduce the table as a systematic list to

		known that we need 5 leaves to feed the caterpillars in two days. The type of problem is missing values.	the students. - Students explore the strategies that can be done with ratio table.
3	My Chocolate	The students make a list of chocolate bars' price with their own number. They determine the number of chocolate bars by themselves.	• Students explore the strategies that can be done with ratio table. • The students find the rule of 1
4	What do you think?	The students should solve the given problem and discuss the incorrect strategy as well. *"If we get 3 glasses of chocolate milk for 2 coupons then how many coupons we need to buy 24 glasses of chocolate milk."*	• The students know and understand what rules that cannot be applied on the ratio table and why it so.
5	Which is the cheaper one?	There are 2 items which have different prices for different number of items. The students should determine which one is the cheaper one between the two.	• The students use the ratio table to solve proportional problem. • The students use the ratio table as a tool to thinking and reasoning.

4.1 Activity 1: Make Butterflies

A. Mathematics Goal

The goal of this lesson is to lead the students to understand the sense of ratio in proportional problem.

B. Starting points

The students in grade 4 already learned about some number operation such as addition, subtraction, multiplication and division in the previous grade.

C. Mathematical Activities and Students' Conjectures

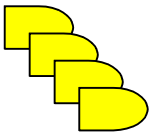
Step 1: Working in pairs (10-15 minutes)

The teacher will ask the following question:

Make Butterflies

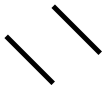
There are four parallel class of 4th grade who visit a science fair. During the workshop in there, each student will build models of butterflies.

To make a model of butterfly, they will need the following items.



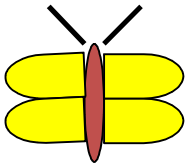
4 wing pieces

1 body



2 antennas

When the model put together, it looks like this:



Determine the number of wings, bodies and antennas which are needed for each class if we know that the number of student in each class as follows:

a. 4A consists of 27 students
b. 4B consists of 26 students
c. 4C consists of 25 students
d. 4D consists of 24 students

Step 2: Classroom discussion (45-50 minutes)

- The teacher makes a table on the blackboard and gives the title for each column. The discussion about the reason why the number of students in the class and the number of butterflies is equal should be happened in the class. It's also important to make the students realize that 1 student makes 1 butterfly is also a ratio 1:1.
- The teacher asks the students about their answer. The teacher writes the students' answer on the table then discuss how they can get that.

The number of butterflies	Wings	Bodies	Antennae
27	108	27	54
26	104	26	52
25	100	25	50
24	96	24	48

A prediction of students' response:

- Using multiplication by 4 to find the number of wings, multiplication by 1 to find the number of bodies and multiplication by 2 to find the number of antennas for all number of butterflies (the question a to d).
- The students may using the multiplication by 4, 1 and 2 just for the two first number and look the pattern which is if the number of butterfly decrease by one then the number of wings will decrease by 4. Then for the next, they just add or subtract to find the answer.
- Using repeated addition to solve the problem for questions a to d.
- Find the number of wings, bodies and antennas for 25 butterflies first because it's the easiest to be found and some of them may already realize about the strategy to add or subtract by 4, 1 or 2 to find the number of wings, bodies, and antennas respectively.

To solve the problems, most students may use multiplication as their strategy because they learned it in the previous grade. However, we hope that the students also will come up with the idea of subtracting by 4 or adding by 4 to get number of wings if the number of butterflies decreases or increases by 1(by using the pattern of the ratio).

- During the discussion, the teacher also asks some follow up questions to help the students see the pattern of adding by 4 for every adding 1 butterfly, such as

  "*What about 23 butterflies?"*

  "What about 28 butterflies?"

- The teacher asks other follow up question which aims to let the students realize the idea of division in proportional problem.

  "*How many complete butterflies that we can make from 90 wings, 23 bodies and 45 antennas?"*

A prediction of students' response:

- Dividing the 90 by 4, 23 by 1 and 45 by 2 and give the different number of complete butterfly for each item.
- Dividing the 90 by 4, 23 by 1 and 45 by 2 and give the smaller number of complete butterfly from each item as the answer.
- Using the ratio 4:1:2 to find the numbers of each item which are the nearest number from the given problem. For instance, by multiplying each items by 20 we get 80:20:40 and by adding two more butterfly we get 88:22:44
- Using the number of body as the starting point then divide the 90 and 45 by 23 to see whether there is enough wings and antennas to make 23 butterflies or not. If not, then they ca subtract the number of butterfly one by one.

4.2 Activity 2: Feeding Caterpillars

A. Mathematics Goal

The goal of this lesson is introduce the ratio table as a systematic list and a tool to do calculation and to lead the students find the strategies that can be done with the ratio table.

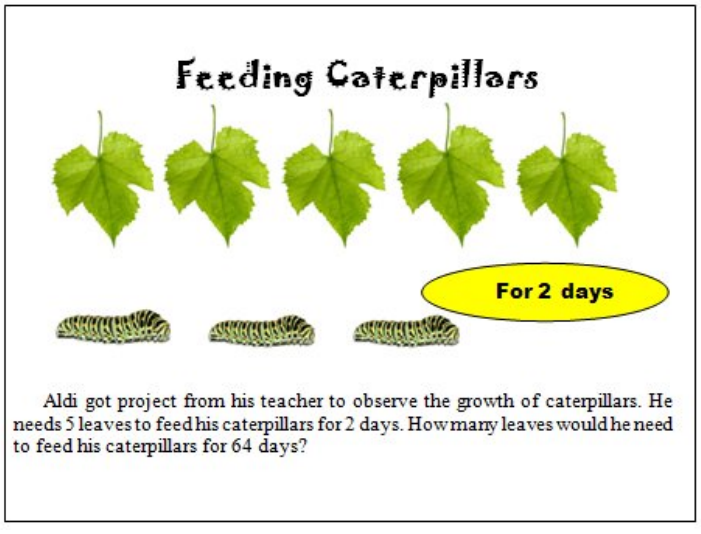
B. Starting points

The students already discussed the sense of ratio during lesson 1 and they already knew how to write on the table in systematic way as the teacher wrote on the blackboard during the discussion section.

C. Mathematical Activities and Students' Conjectures

Step 1: Working in pairs (10 minutes)

- The teacher will ask the following question:



Step 2: Classroom discussion (60 minutes)

- The teacher asks the students to present their answer and their strategy. Discuss the students' strategies

A prediction of students' response:

- Using division by 2 and multiplication by 5 to find the number of leaves.
- Using doubling strategy by using ratio table.
- Adding by 5 more to get the number of leaves for 2 days, adding by 5 more to get the number of leaves for 4 days (repeated addition).
- Using the strategies grouping by 2 and then counting how many two in 64. Then multiply the number by 5.

- If there are students who come up with the idea of using list, the teacher can start the discussion about the ratio table from here. If not, the teacher starts the discussion by making the following table on the blackboard to check whether the presented answer is correct or not. For this problem, the main strategy that would be reached is doubling. The idea about how to make the list in systematic way should be emphasized. The teacher also says that the table is called ratio table.

Days	Leaves
2	5
4	
8	
16	
32	
64	

- During the discussion, the teacher also can ask some follow up questions like

  "*What about 6 days?"*

  "What about 10 days?"

  "What about 24 days?"

A prediction of students' response:

- The number of leaves for 6 days can be found by add the number of leaves for 2 days and 4 days
- The number of leaves for 6 days can be found by subtract the number of leaves for 8 days by the number of leaves for 2 days.
- The number of leaves for 10 days can be found by add the number of leaves for 8 days and the number of eaves for 2 days
- The number of leaves for 10 days can be found by find the half of the number of leaves for16 days plus 4 leaves
- The number of leaves for 10 days can be found by multiplying the number of leaves for 2 days by 5.

- The discussion can be continued to gain other strategies, such as the halving, adding and taking away or multiplying and dividing by the same numbers. For each question, the teacher should give the students time to think and discuss in their group. The student can make a note or scratch on the blank answer sheet which be given during the discussion. The teacher should encourage the students to find different strategies.
- The teacher also can propose the question the other way around

"For how many days 70 leaves can be used?"

A prediction of students' response:

- The students may divide the 70 by 5 to find the number of days.
- The students divide 70 by 5 and multiply it by 2.
- Using the group of 5 and multiply the number of group by 2

- Using the number of leaves for 4 days, which is 10 as the starting point. Thus, they can find easily by multiply the 4 by 7.

4.3 Activity 3: My Chocolate

A. Mathematics Goal

The goal of this lesson is to do more exploration about the strategies that can be done with ratio table and to lead the students to reinvent the rule of 1.

B. Starting points

The students already discussed some rules that can be used by using ratio table on lesson 2.

C. Mathematical Activities and Students' Conjectures

Step 1: Classroom discussion (15 minutes)

- The teacher shows the following figure to the students.

- The teacher asks to the students for some prices when draws a ratio table on the blackboard.

Chocolate bars	Money
4	3000
?	6000
12	?

- First the teacher asks "How many chocolate bars that we will get for Rp6.000?" Discuss how they can find it.

A prediction of students' response:

- Using doubling strategy
- Adding by 4 more
- Divide the 6000 by 3000 and multiply it by 4.

- Then the teacher asks "How much we paid for 12 chocolate bars?"

A prediction of students' response:

- Adding the price for 4 and 8 chocolate bars.
- Multiply the price for 4 chocolate bars by 3.
- Dividing the 12 by 4 and multiply it by 3000.

Step 2: Work in pairs (15 minutes)

- The teacher asks the students to complete the list of chocolate bars' price on the worksheet. The teacher should encourage the students to choose the bigger or more difficult numbers. They can explore by using the strategies that they knew from the previous lesson.

Chocolate bars	Rupiah

From this activity we can see whether they use some manipulative rules or not.

A prediction of students' response:

- The students just using doubling strategy until a certain number of chocolate.
- The students choose the number of chocolate bars randomly and they calculate to find the price by using multiplication or division.
- Find the price of 1 chocolate bar and multiply any number that they choose with the price.
- They combine two or more different manipulative ways to determine the number of chocolate bars and the price as well.

Step 3: Classroom Discussion (40 minutes)

- The teacher will discuss some numbers that proposed by the students and asked how they can get that numbers.
- The teacher can give some follow up questions to the students in order to guide them reinvent the strategies in case if they cannot generate the strategies by using the numbers that they proposed. The possible number to be used as the follow up questions as follows.

1. *How many chocolate bars for Rp12.000?*
2. *How many chocolate bars for Rp21.000?*
3. *How many chocolate bars for Rp27.000?*
4. *How much we should pay for 40 chocolate bars?*
5. *How much we should pay for 44 chocolate bars?*
6. *How much we should pay for 48chocolate bars?*

7. *How much 13 chocolate bars?* (to gain the idea of rule 1).

8. *How many chocolate bars that we get for Rp.108.000?*

(Sometimes, by giving the difficult number, we can stimulate the children to use the manipulative strategy).

4.4 Activity 4: What do you think?

A. Mathematics Goal

The goal of this lesson is to discuss the rules that can be applied and cannot be applied on the ratio table and why it is so.

B. Starting points

The students already discussed and explored the rules that can be applied on ratio table during lesson 2 and 3.

C. Mathematical Activities and Students' Conjectures

Step 1: Work in pairs (10 minutes)

- The teacher proposes the following problem in students' worksheet.

> In the bazaar, we can buy 3 glasses of chocolate milk with 2 coupons. How many coupon that we need to get 24 glasses of chocolate milk?

Step 2: Discussion to the whole class (50 minutes).

- The teacher discuss about the students' strategies.

A prediction of students' response:

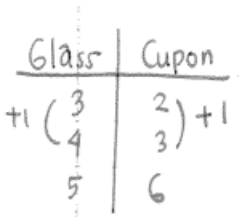
- The students may use the multiplication and division to find the answer, 24 is divided by 3 and multiplied by 2.
- The students use correct manipulative strategies, such as doubling, halving, adding, taking away, or multiplying and dividing with the same number. For instance, the students may double the 3 glasses-2

coupons to get 6 glasses-4 coupons and multiply it by 4 to get 24 glasses-16 coupons. The students may use multiplication 3 until they get 27 glasses- 18 coupons and realize that they cannot use multiplication of 3 anymore. They may subtract the 27 by 3 and 18 by 2.

- The students maybe using repeated addition until they get the number of coupons for 24 glasses.
- The students who use incorrect strategies such as adding and subtracting with the same numbers.

- The teacher asks the students with incorrect strategy to present their work (if there is) and asks the other students to response about the answer. The teacher should underline that the students should try to understand the strategy which are proposed by their friends before they can argument about it.
- If there is no student who comes up with the wrong strategy then the teacher may propose the idea of incorrect strategy. The teacher may show it from LCD or just write it on the blackboard. The teacher may say that

"There is a student from another class who answer as follows. The student didn't finish his work yet. But, can you understand what he tried to do? What do you think about his way?"



It is also important for the teacher to ask why a certain rule should not be applied on ratio table, such as adding by 1 for both coupon and the number of glass. It is not because the teacher says so, or because that is how a ratio table works, but because the context tells us that for each 2 coupons you get 3 glasses. So the children should not learn a rule (multiplication yes, addition no), but they should learn always to keep the context in mind; if in doubt think about the proportional context.

A prediction of students' response:

- The students may say that we cannot add for the same number like in that answer because they realize that for each 2 coupons you get 3 glasses.
- The students may say that it is possible to add 1 on each because for 1 more glass, we need 1 more coupon.

- In the end of this lesson, the teacher gives some remarks about what strategies are allowed to be applied on the ratio table and what are not.

4.5 Activity 5: Which is the cheaper one?

A. Mathematics Goal

The goal of this lesson is to introduce the ratio table as tools to solve proportional problem. For this lesson, the type of the problems is comparison problem. In this lesson, the ratio table is used as a tool to thinking and reasoning.

B. Starting points

The students are able to use ratio table as a tool to do calculation and they learned the rules on the ratio table that may be used to do the calculation.

C. Mathematical Activities and Students' Conjectures

Step 1: Work in pairs (10 minutes)

Step 2: Discuss to the whole class (20 minutes)

- During the discussion, the teacher will discuss about the students' thinking, how they will compare the two different things. It is important to discuss why we used proportion instead of the absolute value in comparison problem.

A prediction of students' response:

- The students may answer that the left is more expensive than the soft drink on the right side because 4.000 is bigger than 3.000.
- The students may realize that the number of cans also influence the price.

- The teacher discusses some ideas to find the cheaper prize.

A prediction of students' response:

- Look the number of cans for the same price, such as for Rp12.000.
- Look the price for the same number of cans, such as the price for 6 cans.

- By unitizing, looking for the price per 1 can. However the calculation may be difficult for the students because they may have difficulties to divide 4.000 by 3.

Step 3: Work in pairs for the second problem (10 minutes).

2. There are two type of rice in the shop. The cost of the first type is Rp120.000 per 15 kg and the cost of second type is Rp45.000 per 5 kg. Which is the cheaper one?

Step 4: Discuss to the whole class (25 minutes)

- The teacher discusses the students' answer. Based on the first problem, we can assume that the students already realize and understand the idea of proportional in comparison problem. Thus, the teacher can recall about this idea only if needed.

A prediction of students' response:

- Look the number of kg for the same price, such as for Rp120.000.
- Look the price for per kg rice (by unitizing the weight).
- Look the price for the same weight such as the price for 5 kg rice.

- If there is remaining, time, the teacher can asks a missing value problem with more complex situation which involves the mixture problem.

"If the seller mix the 2 sacks of the first type rice and 3 sacks of the second type of rice, how much the price of the mixture rice per kg?"

A prediction of students' response:

- By using the price per kg, they multiply it with the weight of each type then divide it by the total weight of mixture rice.

- They multiply the price by number of sacks and divide it by number of sacks.

CHAPTER V

TESTING HYPOTHETICAL LEARNING TRAJECTORY

In Chapter 4, we saw an initial HLT which consists of 5 activities. These learning sequences were designed based on some consideration related to the existing theories about the ratio table in developing the students' proportional reasoning as stated in the chapter 2. The activities were designed such that the students learned continuously. The previous activity supported the students to do the following activities.

The present study was conducted in Grade 4, SDN 179 Palembang, Indonesia for 9 February - 25 March 2013. During this time, the design was implemented twice, in the first cycle as the preliminary teaching experiment and in the second cycle as the teaching experiment. The first cycle lasted in two weeks within 4 meetings, each about 70 minutes. Meanwhile the second cycle took longer time because there were some events at school. The interval time between the first lesson and the second lesson was about a week. However, in general it did not influence the learning process in the class.

In this chapter we present some data which were collected during the first and the second cycle. Here, we also describe how this data were analyzed to improve the activities and the HLT as well.

5.1 Design Experiment

5.1.1 Preliminary Teaching Experiment (First Cycle)

The first cycle has been held in the library of SDN 179 Palembang. There were 6 students from Class 4A SDN 179 Palembang involved in the first cycle. They were Rati, Lani, Nurul, Ando, Ferdy, and Falah. The students were chosen by

considering their grade. Ferdy was a student who got the first rank in the class. Rati, Lani, Ando, and Falah have about the same ability as the middle achievers. Meanwhile, Nurul was considered as the lower achiever.

The cycle 1 was initiated by conducting a pretest and the student interviews. Then, the learning design was implemented based on the HLT in Chapter 4. There were five activities for five meetings, 70 minutes on each. Some activities were done in group. The groups on cycle 1 were not fixed for each meeting because some students were absent on certain days. Besides, there were some students who didn't want to work together in the same group. During the cycle 1, the students were taught by the researcher herself so the researcher had an opportunity to think about the improvement for HLT on cycle 2.

The first activity was Make Butterflies. In this activity, the students were asked to determine the number of wings, bodies, and antennas to make certain numbers of models of butterfly if known that to make a model of butterfly we need 4 wings, 1 body and 2 antennas. This activity aimed to make the students understand the notion of ratios in proportional problems. There were two worksheets in this activity. The first worksheet was done in pairs while the second worksheet was done individually.

The second activity was Feeding Caterpillars. This activity proposed a missing value problem. The students were asked determine the number of leaves which are needed to feed caterpillars in 64 days if known that we need 5 leaves to feed the caterpillars in two days. This activity was conducted in order to know how the students solved the missing value problem after they learned the notion of ratio

table during the first meeting and to see whether they may be able to gain other strategies to solve the problem.

The third activity was My Chocolate. In this activity the students explore the ratio table through making a list of chocolate price. They determined the number of chocolate by themselves then found the price. The goal of this activity was the students may be able to gain some strategies that can be done by using ratio table and may lead them to reinvent the rule 1 (unit strategy). These strategies would help them to solve the proportional problems.

The fourth activity was "What do you think?" In this activity, we proposed some different contextual problems in order to discuss about some rules that could not be applied on the ratio table and why it so.

The last activity was Best Price. The problem which was posed in this activity was a comparison problem. Here, we want to build up the students' idea about proportionality in comparison situation. In the end of the learning sequence, the posttest was given to the students.

5.1.2 Teaching Experiment (Second Cycle)

Before implemented the learning design on the cycle 2, the researcher did the classroom observation and the teacher interview. There were 29 students from class 4D SDN 179 Palembang participated on this cycle. As on the cycle 1, the teaching experiment on cycle 2 also was started by conducting pretest and followed by the student interviews. All students joined on the pretest but only 18 of them who were interviewed on the following week after the test.

During the lesson, they were taught by their own homeroom teacher. There were some changes on the activities which used in this cycle. Beside the revision

of the worksheet, the order of the activity 2 and 3 also were switched and were put together in the same meeting. Thus, there were only four meetings, 70 minutes on each, in this cycle. In the end of these learning sequences, the posttest was conducted in order to get the impression of how the design may support the students to develop their proportional reasoning.

5.2 Retrospective Analysis

5.2.1 Preliminary Teaching Experiment (First Cycle)

1. Pretest and student interview

In cycle 1, the pretest was conducted twice because the problems on the first pretest couldn't give the appropriate information about the students' prior knowledge, then we used the second version to gain impression of the students' prior knowledge about proportion. For both pretests, the students worked individually in about 10 minutes. After each pretest, the researcher also did the short interview with all students to crosscheck the interpretation of the students' answer.

Pretest version 1

Pretest version 1 consists of two questions. The first question was asking about the number of glasses of mineral water in some number of boxes if given that there were 24 glasses in 1 box. The 6 students could give the correct answer. Some of them wrote the way to get the answer, which was the multiplication, and the others just gave the answer directly without wrote the way. Based on the students' answer we can say that the students have a good basic in multiplication. However, this problem didn't help us to know how the students' ability to solve

proportional problem because the number that used in the problem was 1 which was more likely to the multiplication problem than the proportional ones.

The second problem asked about the number of boxes of mineral water that we should buy to get 100 glasses of mineral water if known that there were 24 glasses in 1 box. This problem required the students' understanding about division and the ratio of the number glasses and the number of boxes. There was no student who could figure out the answer. It may because they used to work with divisible number and have never met the problem which needed more logical thinking like this. This problem asked the students to think not only about the procedure of division but also the proportional reasoning. If we just buy 4 then we just have 96. It means we need 4 more glasses. So, we should add 1 more box. However, we cannot get 100 glasses exactly because the number of glasses in 1 box was 24. This problem may be too difficult for the students who weren't used to work with the reasoning problem. Instead of thinking how to solve the problem, they thought that the number was wrong because it was not divisible by 24. In addition, same with the reason on problem 1, because of the ratio of the number of boxes and the number of glasses was 1: 24, it was more likely to multiplication problem than proportional problem. Thus, the problems on the second pretest were changed.

Pretest version 2

The second version of pretest contains three problems which aimed to know the students' prior knowledge. This version was used on the cycle 2 as well.

Problem 1

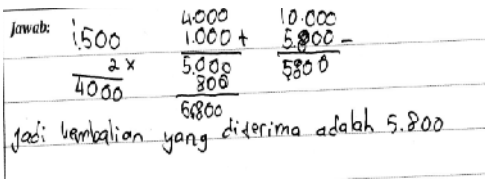
Problem 1 was about looking for the change in the buying context. This problem was given to measure the students' ability in addition, subtraction, multiplication and division.

The price list of some items in school store was given as follows.

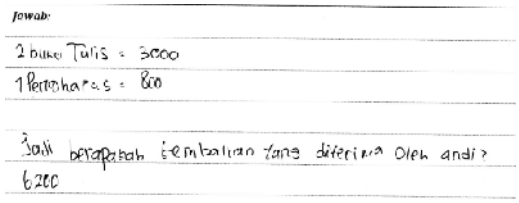
Items	Price
Ballpoint	Rp1.200
Drawing book	Rp1.800
Book	Rp1.500
Eraser	Rp 800
Pencil	Rp 1.000
Sharpener	Rp 900

Figure 5.1 Problem 1 on Pretest Cycle 1

The students were asked to determine the change which will be received by Andi, if he bought 2 books, 1 pencil, and 1 eraser and paid Rp10.000. From six students, only two students gave the wrong answer.



Translation: So, the change is 5.800



Translation: So, how much the change that received by Andi? 6200

Figure 5.2 Lani's Written Work (left) and Nurul's Written Work (right)

From Lani's work, we can see that she knew how to solve the problem. She multiplied the price of book by 2 and added it with the price of pencil and eraser. She also knew that to find the change, she should subtract the 10.000 by the total cost. But, she did a miscalculation on 1500 times 2. She also could not get the right answer for the subtraction of 10.000 by 5.800. It may because that she has

difficulties in calculation with big numbers, thousands, or it also may because her carelessness. Meanwhile from Nurul's written work that she forgot to add the price of pencil, so the change was Rp1.000 more than what was supposed to be.

Problem 2

For problem 2, the students were asked to find the number of donuts in 4 boxes if it is known that there were 24 donuts in 2 boxes and also were asked to explain their answer.

From 6 students, there were two students who gave the correct answer, Ferdy and Nurul. Nurul didn't write the way, she just wrote the answer directly while Ferdy wrote the multiplication of 24 times 2. Ferdy may realize that 4 boxes was twice of 2 boxes, then he just multiply the number of donuts in two boxes by 2 as well.

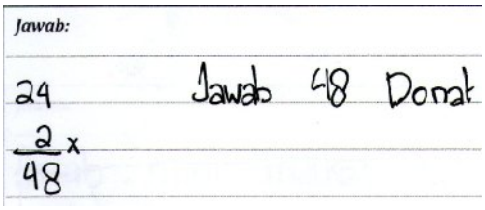


Figure 5.3 Ferdy's Answer

The other students multiplied 24 by 4 to get the number of donuts in 4 boxes. It maybe because they didn't pay attention to the number of boxes in which contains 24 donuts. But it also may because they did not aware the sense of proportion here. They used the multiplication to solve the problem because they only thought that to get the bigger result; it should be multiplied.

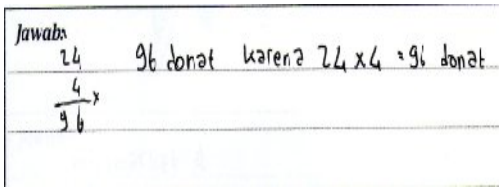


Figure 5.4 Ando's Answer

Translation: 96 donuts, because 24×4=96 donuts

There was also one student who multiplied 24 by 4 and by 2 to get 192 as the result.

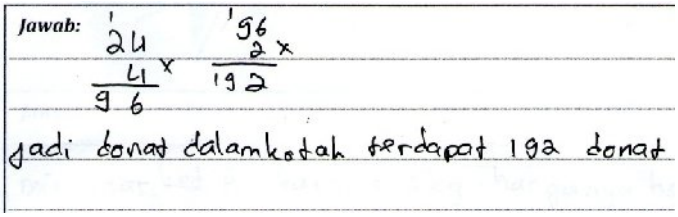


Figure 5.5 Lani's Anwer

It was not clear why she did that, because like other students, she also did not write the reason to explain her answer. The researcher assumed that it may be because the students did not read the problem in proper way and she just saw the numbers instead of tried to understand the problem itself. She knew that she should gain the bigger numbers as the result, thus she just multiplied all the numbers together.

There was no students' answer which gave an appropriate explanation or reason about how and why they used the strategy to look for the answer. Some students' answer just repeated the strategy as the reason without any explanation. It may be because it was hard for them to express their reason or it also may be because they have never thought about the reason and just did the work procedurally. Thus, we need to encourage the students to do reasoning.

Problem 3

Problem 3 was about the comparison of apple cost in two different markets, market A and market B. It was given that the apples in market A costs Rp50.000 for 2 kg and the apples in market B costs Rp60.000 for 3 kg. Then the students were asked to determine which the cheaper one is.

For this problem, all students didn't answer correctly. They didn't aware about the different weight of apple in both markets. Although the price of apple in market A was cheaper than market B, the weight of apple in market A also was lighter than in market B then they shouldn't determine the cheaper by just notifying the absolute price.

Based on the analysis of students' written work, we may conclude that in general the students understand about the arithmetic operation and could solve the problem which related to arithmetic. They knew when they should use the multiplication or when they should use the subtraction as required by the context. Although sometimes they do miscalculation, they have an appropriate understanding about arithmetic operation as the prior knowledge to learn about proportion.

For the missing value problems, the students know that it have relation with multiplication. However, the majority of the students were struggling to solve it correctly. It was the same for the comparison problems; the students were also not thinking proportionally, they compare the price by absolute value. It means that they still need to learn more in order to grasp the insight of proportion and for the implementation of the design on the next cycle; the teacher should encourage the students to participate actively in the discussion.

Through the implementation of the five designed activities on the first cycle, we investigated how the learning design fits the students' need. Furthermore, we used the findings here to refine the HLT as well, especially for the context or the sentence of the problems and the order of the activities.

2. Activity 1: Make Butterflies

There were two worksheets in this activity. The first worksheet was done in pairs while the second worksheet was done individually.

The first worksheet asked the students to determine the number of wings, bodies, and antennas to make 2, 4, 8 and 16 butterflies if known that to make a model of butterfly, we need 4 wings, 1 body, and 2 antennas. This activity was designed in order to make the students aware of the sense of ratio in proportional problem.

The problem here used 1 as the multiplication factor in order to fit the students' prior knowledge, which was the concept of multiplication. If we used the bigger number, for instance two, we afraid there was a big jump for them because they never learned about proportion before. In addition, based on the result of the pretest the students had an appropriate ability in multiplication, so at least they may be able to use this strategy to solve the problem.

As predicted in initial HLT, all groups used the multiplication to solve the problem. The group of Falah and Nurul also used the same method to find the number of wings, bodies, and antennas, but in the end, they added all of the items. It was possible that they just had the different interpretation about what the problem asked about. Hence, the researcher asked them to reread the problem in order to correct their interpretation of the problem.

1. Researcher :Why do you add that?
2. (Falah looked at the question)
3. Researcher :What does the problem ask about?
4. Falah :Class 4A consist of 27 students…
5. Researcher :Read the problem first…
6. Falah :Determine the number of wings, bodies, and antennas which
7. were needed for each class if known the number of students
8. in each class as follows.
9. Researcher :So, what does the problem ask about?
10. Falah :The result…
11. Researcher :What is asked by the problem? The sum? Or the number of
12. each item?
13. Falah :The sum.
14. Researcher :Where is the word sum?
15. Falah :Not there…so, each.

Transcript 5.1

Falah still assumed that the problem asked about the total number of all items before the researcher asked him to check if there was the word 'sum' in the problem. From what Falah said on line 4, we can see that he tend to see just the numbers when he read the problem. Thus, we should make the problem as simple as possible; not put too much story on it. Accordingly, the sentence of the problem for cycle 2 and the story which wrapped the context were changed in order to make the students understand easier.

The activity was continued by the discussion with all six students. During the discussion, the teacher (the researcher) made a table to list the students answer. The discussion was held to check the answer and the strategies that used by each group. It also aimed to gain different strategy to solve the proportional problem because so far all of them just use multiplication without knowing why they did

the multiplication. They only said that it was like what they learned. However, it was not easy to invite all the students join the discussion and bring up the idea of "different strategies". They were shy to give their argument and they seem afraid that their answer was wrong. Although the researcher questioned the number of wings, bodies and antennas for some other numbers of butterflies, the students were stick on the multiplication strategy, as they learned in the previous grade. It indicates the students did not get used to the socio-mathematics norm, such as the presence of different strategies.

Next, the researcher asked the students to see the number pattern. It might help them to realize the relationship among the numbers beside the multiplication by 4, 2 or 1. At first, the students did not aware about the pattern in numbers. But, when the researcher asked them to focus only on the number of wings, one of the students came up with the pattern of "the difference of 4". It was a good starting point for the students. Since then, they started to notice the relation between the previous numbers to the next. In the end, the students also realized the pattern of "the difference of 2" for the number of antennas, and "the difference of 1" for the number of bodies. But, they did not aware that the difference of antennas and the difference of the number of butterfly influenced each other.

During the discussion, we also discussed about the ratio in proportion through the following problem.

> *If we have 90 wings, 23 bodies, 45 antennas, then how many complete butterfly which we can make?*

This problem aimed to lead the students to the idea of ratio between wings, bodies, and antennas. They may realize that to make 1 complete butterfly, they

need 4, 1, 2, wings, body, and antennas respectively. Although it did not state clearly that they were called ratio, the students could figure out that we only could make 22 butterflies from them. They would have not enough antennas to make 23 butterflies.

After the discussion, the lesson was continued by working on the second worksheet. The second worksheet was done individually. The second worksheet was used to measure the individual understanding of the lesson from the first worksheet and from the discussion in the class. In addition, it was conducted on cycle 1 in order to test whether the problems were understandable.

There were five questions in the second worksheet. The context which used in the second worksheet was the same as the context on the first worksheet. It was about making the model of butterflies.

Problem 1

How many wings, bodies and antennas that we need to make 9 models?

Problem 2

If we have 45 wings and we want to make the complete model of butterfly, how many model of butterfly that we can make with those wings?

Problem 3

If we have 29 wings, 7 bodies, and 13 antennas then how many complete butterflies that we can make?

Problem 4

We have 9 broken models of butterfly. Fortunately, some parts of the models can be reused to make the new model. From those 9 broken model, we know that 12

wings, 3 bodies, and 5 antennas broken. How many wings, bodies, and antennas should be bought to make exactly 15 butterflies?

Problem 5

To make 107 models of butterfly, we need 428 wings, 107 bodies, and 214 antennas. Find the number of wings, bodies, and antennas that we need to make 106, 108, and 214 models of butterfly!

The remarks for the second worksheet

For problem 1, all students used the multiplication to solve the problem. Only Nurul who made a mistake for the number of wings, instead of 39 wings, she answered 44 wings. It was not clear how she got the 44 because no calculation on her answer sheet. It may be because she just memorized the multiplication facts and forgot what the result of 9 times 4 was. However, in general the students understood the problem and there was no ambiguous sentence on it.

Problem number 2, 3, and 4 actually had the same goal, which was to make the students realize about the remains from indivisible numbers. But, during the discussion it seems that the students were hard to understand the problem. They got confused because problem 2 only mentioned the wings and no other component which were needed to make complete butterfly. Meanwhile, the story in problem 4 made the students confused. Thus, we revised these problems.

The last problem used big number, hundreds, and it was not successful to make the students notice the relation between the given number and the number which was asked. The numbers in this problem also were changed with the smaller one, and were presented in a table.

Minilesson

It was not the same as planned in HLT, we inserted a mini lesson before we started the activity 2. The aim of the minilesson was to verify whether using the smaller number may lead the students to gain the different strategy from the students.

In the minilesson, the students were asked the following problem.

Every student in Grade 5 should make 2 models of butterfly. Make a list of the possible number of students and the number of butterflies they made!

Through this problem, the students worked with smaller number. In fact, the students were able to find other strategies beside multiplication. There was a student who added the number of models by 10 every adding the number of students by 5, and there were two students who added the number of models by 4, every adding the number of students by 2. While the other three students used multiplication by two to find the result.

Based on the students' work and the discussion in the minilesson, we may conclude that worked with the small number allowed the students to see the pattern of the number and to think other method beside multiplication. Furthermore, it also led the students to see the relation among numbers which can be used as the starting point to do the proportional reasoning. According to this finding, the researcher changed the number for the first problem to the smaller ones.

3. Activity 2:Feeding Caterpillars

In activity 2, the students worked in pairs. They were asked to determine the number of leaves which were needed for 64 days if it was known that for 2 days we need 5 leaves.

There was a group of students who multiplied the number of leaves with 64 days without divided it with 2. It was the same mistake as in the pretest. The other group used a repeated addition without using a table. They just wrote the number of days was equal to the certain numbers of leaves, but they already wrote in the same column for each item. However, they did not add in proportional way. It seems that they need more time to explore the strategies to solve the proportional problem before they worked on the missing value problem. Therefore, we thought to give the activity of exploring ratio table before they tried to solve the simple missing value problem in order to help them in solving the problem. Through exploring the ratio table, they may gain some strategies to do calculation in solving the missing value problem on this activity.

Surprisingly, during the discussion there was a student who was able to think of the unit 1. However, he also could not give the correct answer. It was because he couldn't multiply or divide by a fraction/ decimal number. Regarding to the students' prior knowledge, they were not ready to use the multiplication and division with fraction/ decimals because they have not learned it before. Although there was a student who brought up the idea of unit 1, it was too formal to be discussed for that time. Thus, we only discussed about the repeated addition strategy.

In addition, the figure on this problem was changed. We do not use the caterpillars anymore, because we assume that the students would be tricked by the number of caterpillar in the figure.

4. Activity 3:My Favorite Chocolate

During the third activity, the students were asked to list the chocolate price. First, the students were told that the price of 4 chocolates is Rp3000. Then they may choose the number of chocolate as they want and find the price of it. This activity intended to let the students explore the ratio table.

Here, the activity was held a little bit different with initial HLT. Instead of working in group, the students worked individually. And, the result was not good. No discussion in pairs let them work fast but without much thinking on it. There was no different strategy occurred. It may because we discussed much about the repeated addition in the previous lesson, then they just used it for this activity as well.

During the discussion, the students were asked, why they added the price by 3000 for every adding the number of chocolate by 4. Here, the context helps them to realize that why they need to add that. They said that it because if they buy 4 more chocolate, they should pay Rp4000 more.

Regarding the time for activity 2 and 3 which was relative short, we decided to do both activities in one meeting, for 70 minutes.

5. Activity 4: What do you think?

After the students explored the ratio table and gained some strategies to work with, the lesson was continued by applying these strategies on some missing value

problems. In the activity 4, there were two worksheets given. The first worksheet was done in pairs while the second worksheet was done individually.

The problem on the first worksheet asked about the number of coupon that we need to buy 24 glasses of chocolate milk if we know that we get 3 glasses of chocolate milk for 2 coupons. This problem was proposed in order to lead the discussion about what kind of strategy that we cannot do with the ratio table, such as added the both side by the same number, or added it without considering the ratio of both items. However, this problem was too easy and the students could solve it well; only one group who made a miscalculation.

The second worksheet consists of three problems to let the students practice with the missing value problem individually. These three problems used different contexts in order to expand the implementation of idea proportionality in other contexts.

Problem 1

In a secondhand shop, we can change our 5 comics with their 4 comics. How many their comics which were given to us for 35 our comics?

Problem 2

There is a promo in the supermarket. For every purchase Rp100,000 and the multiple, we get 1 coupon. We can change the 3 coupons with 4 coins to play in the Game Zone. How many coupons which we need to get 24 coins?

Problem 3

In an adventure game, we may change 5 golden balls with 2 lives, how many lives that we can get if we had 23 golden balls?

The remarks for the second worksheet on activity 4

Problem 1 and 2 were appropriate for the students. They were able to solve it well. But, the sentence in problem 1 was changed a little bit because the same utterance of comics, for both the secondhand shop and for the customer, made the students got confused. Thus, for the next, it changed to be (old) magazines and (new) comics.

For the problem 3, the students could not give the correct answer. After the discussion, they knew how to deal with this problem.

From the students' works, it seems that the problem in the first worksheet was too easy for the students. On the other hand, problem 3 on the second worksheet was too difficult for the students and it lead the students to make mistakes. It was good to be discussed. Thus, the problems were switched.

6. Activity 5: Which is the cheaper one?

Through the activity 1 until 4, the students were introduced to the ratio table and had some discussion on how to use the ratio table to solve the missing value problems. In addition, these four activities also helped the students to construct the idea of proportionality in certain contexts. In activity 5, they may use this idea and the strategies for the comparison situation. For this activity, the students worked in pairs.

There were two questions proposed in the separate sheets. The second was done after the discussion of the first problem. Each question was about comparing the price of something but in different level of difficulty. The first question used soda which was countable while the second one used the honey with the weight unit of gram.

Problem 1

Problem 1 asked the students to determine which one the cheaper is, soda A or soda B, if it was known that 3 sodas A cost Rp4000 while 2 sodas B cost Rp3000.

At first, the students used the estimation to determine the price of 1 soda A while the other only said that "Rp4,000 for 3 is cheaper than Rp3,000 for 2" without giving strong argumentation. But, in the end of the lesson the students can compare easier and be more convince after we suggest them to make the table of price for each item. This finding is in line with the statement by van Galen (2008), ratio table is an ideal aid for making handy calculations, gaining insight, and students reason with proportions because the table invites the students to write down intermediate steps.

Problem 2

On problem 2, the students were asked to determine which one the cheaper is, first type rice or the second type rice. The first rice costs Rp120,000 for 15 kg while the second rice costs Rp45,000 for 5 kg.

The students also used the same strategy as the first problem. However, when the students used table to find the equal price or equal amount for problem 2, they had difficulties in calculation the price 120.000 for 15 kg rice. One of the groups want to look for the price of 5 kg rice and they did a mistake when they divide the 120,000 by 3, maybe it because too much zeros on the price. Another group who want to use unit 1 method also found the difficulties with this number. They divide the price with the weight and found 9,000 as the result for 45,000 divided by 5, but when they should calculate the division of 120,000 by 15, they could not finish their calculation. Thus, the numbers for problem 2 was modified to be

simpler one. In addition, the way to give the name of each item was not good; first type rice and second type rice. As consequence, the context was changed as well.

7. Posttest

In the end of the learning sequences, we conducted a posttest for the students. A-25 minutes test taken. There were five problems in the posttest. The questions were different from what we used in the pretest.

The first four problems were the missing value problems. It was known that to make 4 chocolates cupcakes, we need 80 grams of flour and 40 grams of chocolate powder and the students were asked to determine the number of flour and chocolate powder to make 24, 12, 10, and 13 cupcakes. In principle, these problems were similar as the missing value problems on the lesson but the 3 items on the problems make a lot of work from the students. Besides, the ratio table that they used during the lesson was the two columns table, but here they should work with three columns. Furthermore, the context was used in these problem less of variation. Thus, these problems were revised.

The last problem asked about the comparison of 3 cars. The students should determine which one the most efficient car. Car A need 5 liters of fuel to drive 75 km, car B need 12 liters of fuel to drive 144 km, and car C need 13 liters of fuel to drive 182 km. As the same reason for the previous problem, it required the students to work with three columns ratio table. Thus, the problems need to be revised. In addition, the numbers on this problem could not gain different solution from the students.

Conclusion of the analysis on cycle 1

Based on the students' work and the discussion during the lesson we may say that the ratio table can help the students to more understand about proportionality and also help the students to gain more strategy to deal with proportional problem. The ratio table helps the students to organize information from the problem. It also helps them to see the relation among numbers which can be used as a starting point to developed proportional reasoning. The ratio table also facilitates the students to do calculation especially through the repeated addition strategy for the students who have no good ability in multiplication. However, some changes should be done in order to improve the learning design. The details of the changing were described on section 5.2.3

5.2.2 Classroom Observation and Teacher Interview

The class observation and the interview with the teacher were conducted before the teaching experiment begun. Based on the observation in the class, the teacher already got used to conduct the discussion in the class. The teacher could handle the class well. The students paid attention when the teacher gave the explanation to them. However, it seems that the teacher gave to much explanation and less participation from the students. She also told the correct answer and explained the procedure directly to the students if the students gave the wrong idea or the wrong answer. In addition, it seems that the teacher had never discussed the issue about different solution or different answer, thus the students just applied the same strategy for the same problem.

According to the interview with the teacher, she has some experience about PMRI. She joined some PMRI workshop and the teacher training. Sometimes she

used learning media in the classroom, such as power point to explain the material and plat model of pizza when she introduced about fraction. She also had some songs or ice breaking games which were spontaneously could bring up the enthusiastic of the students. She said that the students were used to work in group and in general they were active during the discussion. But, they never used worksheet during the lesson. They were only working on their own books.

During the interview, we also discussed about the possibility to make a bigger group in order to reduce the number of working groups but both of us agree that it is better to ask the students in pairs. Otherwise they would not work and just had a talk to each other. Because of the number of students in the class are 29, so there is one group which consists of 3 students.

5.2.3 The Refined Hypothetical Learning Trajectory (HLT)

Based on the findings of the first cycle, the class observation and the interview with the teacher, the researcher refined the initial HLT with the following considerations.

1. Activity 1

Based on the students' work on the minilesson which was given before activity 2, the students were able to gain the other different solution when they work with the smaller numbers. Therefore, the researcher changed the number on the first worksheet for activity 1 with the smaller one. The sentence for the problem also was changed to be the more familiar and simple ones. According to the trial during the preliminary teaching experiment, the students didn't know some terms which written in Bahasa Indonesia. Some students also didn't read the whole

problem carefully. It may because the context which wrapped the problem was too long for them. Thus, we made it shorter

Problem 2 and problem 4 on the second worksheet for activity one was changed. For problem 1 and 3, we just changed the number.

2. Activity 2 and activity 3

In the initial HLT, the activity 2 and 3 will be given on different meeting with allocation time 2× 35 minutes for each meeting. However, in the first cycle, the researcher thought that is possible to make both activities on the same day because the time that needed for the activity 2 was relative short. The students may get the idea from each activity in short time. Considering the students difficulties in answering the proportional problem on activity 2, the order of the activity 2 and 3 was changed. We let the students to explore the ratio table first before they start to solve the proportional problem.

3. Activity 4

The problem for the first and the second worksheet were changed. The problem 3 on the second worksheet became the problem on the first worksheet. The problem on the first worksheet became problem 2 on the second worksheet and the previous problem 2 on the second worksheet became problem 3. The change was made because the problem about the game adventure was more difficult. The number used in that problem was not divisible and the context also prohibited the existence of fraction or decimals; the lives should be an integer.

In initial HLT, the main aim of this lesson was to make the students know and understand what rules that cannot be applied on the ratio table and why it so. However, during the first cycle, we realized that it can be reached through the

whole lessons, thus the focus here should be to expand the implementation of proportionality in different context. We expect that by doing this, the students would not limit the use of proportionality in only certain context.

Furthermore, on cycle 2, the second worksheet was done in group, not individually like in cycle 1, because the implementation of the proportionality in varies contexts was good to be discussed in the small group before the students brought their idea to the whole classroom discussion.

4. Activity 5

The number for problem 2 on activity 5 was changed because the previous numbers were too big for the students in Grade 4; hundred thousand.

5. The use of power point

To adjust with the habit of the teacher and to help the teacher with the follow up questions, the researcher made a power point for some activities, especially for the activities which posed many follow up questions, such as activity 1 and activity 2.

5.2.4 Teaching Experiment (Second Cycle)

1. Pretest and student interview

The pretest on the second cycle was conducted to gain an insight about the initial ability of the students in class 4D about proportion. This pretest used the same problems as the pretest version 2 on cycle 1. Twenty nine students worked on pretest for 15 minutes in maximum. Then 18 of 29 students were interviewed on the following week after the pretest.

Problem 1

Problem 1 is looking for the change in the buying context. This problem aims to investigate the students' ability in addition, subtraction, multiplication and division.

The price list of some items in school store was given as follows.

Items	Price
Ballpoint	Rp1.200
Drawing book	Rp1.800
Book	Rp1.500
Eraser	Rp 800
Pencil	Rp 1.000
Sharpener	Rp 900

Figure 5.6 Problem 1 on Pretest Cycle 2

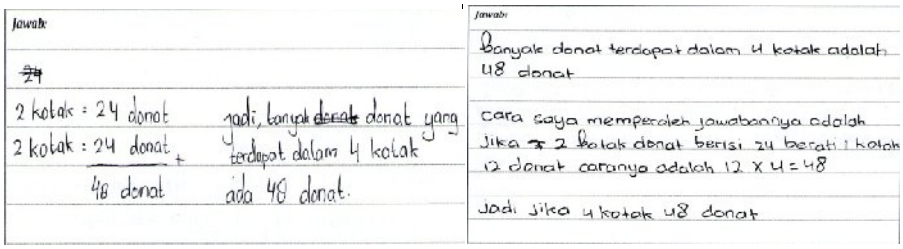
The students were asked to determine the change which will be received by Andi, if he bought 2 books, 1 pencil, and 1 eraser and paid Rp10.000.

There were only 9 students who could not answer correctly. From those 9 students only 4 of them who had mistakes about the concept, such as used the addition or subtraction in wrong way, while the other 5 did the miscalculation. It is not clear for these 5 students, they did miscalculation because of their carelessness or because they have problems in doing calculation. However, in general the students in class 4D fulfilled the requirement to learn proportion.

Problem 2

Problem 2 is a missing value problem. The students were asked to find the number of donuts in 4 boxes if it was known that there were 24 donuts in 2 boxes. The students should explain their answer.

Twenty of twenty nine students gave the correct answer for this problem. They used repeated addition, doubling, multiplication, and also the unit strategy to get the answer. However, there were only a half of them who can give a clear argument on what they did to find the answer. They argued that if they have 2 boxes of 24 donuts then if they have 4 boxes now, it means that they add two more boxes. Thus, they added 24 more donuts (see figure 5.7a). The others also argued that if 2 boxes contained 24 donuts then 1 box contained 12 donuts, then if they had 4 boxes now, it means they could multiply the 12 donuts by 4 to get the result (see figure 5.7b). Although these students were able to give an appropriate reason for this problem, we need more investigation through more complex problems because the problem 2 was too simple. In addition, the majority of the students still have difficulties to express their reasoning. Therefore, the learning design which emphasized the development of students' proportional reasoning is needed.



(a) (b)

Figure 5.7 Lailatul's Answer (left) and Livina's Answer (right)

Meanwhile, the students who gave the wrong answer used the multiplication or division in the wrong way. They multiply by the wrong number. It may be because they were not really understood about the sense of proportion in the problem and they may just know that the problem in somehow related to the

multiplication or division as they learned in the previous grade. They did not realize that if the number of boxes there was not 1 then he could not directly multiply it by 4. There was also a student who used division to answer the problem as shown in figure 5.8

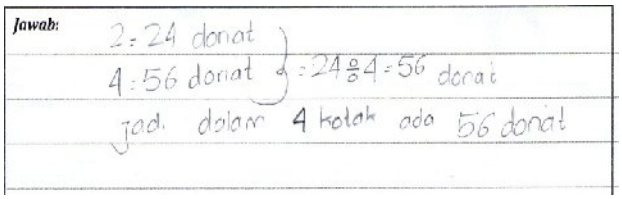


Figure 5.8 Imanah's Answer

Imanah wrote the division by 4 to answer the problem. However it is not clear why her answer was 56 instead of 6. It is maybe that she wanted to multiplied it instead of divided it by 4. But, if that so, then she did miscalculation. It should be 96 instead of 56. However, it is also possible that she mix up with the multiplication by 2, then she got the 5 there.

During the interview, surprisingly, she gave the correct answer directly. She said that it was twenty four plus twenty four. However, when the researcher asked her reason, she got confused. She was not sure why it should be added. It was not clear how she could change her mind, from the division/ multiplication during pretest and the repeated addition during the interviews. Although she gave the correct answer during the interviews, it needs more investigation to make sure how her proportional reasoning was.

In addition, there were two students who used addition by 4 to find the answer.

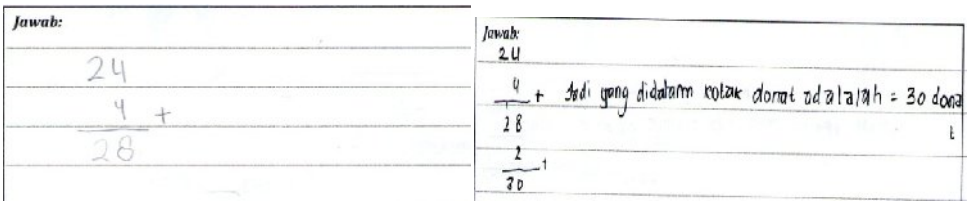


Figure 5.9 Lulut's Answer (left) and Aqila's Answer (right)

They used addition instead of multiplication to solve this problem. It seems that they did not have sufficient understanding about the concept of multiplication itself.

During the interviews, Lulut still had the same idea to find the answer; she added 24 donuts with 4 boxes. However, after the researcher posing the similar question which used smaller number, "If there are two pencils in 1 pencil case and there is another one which contains the same number of pencils, how many pencils in those two pencil case?" She can answer directly, that there are four pencils in those two's. She also can explain well that she got the answer by repeated addition of two. Then, when she was asked to answer the original problem, she did 24 plus 24 to get 48 donuts. It needs more investigation to know whether this repeated addition also can be developed as the doubling strategy or not. Although the difference between doubling and the repeated addition is slight, we cannot say that Lulut have already got the idea of double here. It may be difficult for Lulut to grasp the idea of multiplicative strategy in proportion because it seems that she did not have enough understanding about the concept of multiplication itself. However, it does not mean that she cannot deal with the proportional problem. She still can use the idea of repeated addition to solve the problem.

It is different for Aqila. During the interview, she brought up the idea of division by two to find the number of donuts in one box. It is not clear how she

could think of unit strategy. But, she could not do the calculation. It is difficult for her to figure out 24 divided by 2. She need some help in order to find the result. Then, when she already got 12 as the result, she used repeated addition instead of multiplication to find the number of donuts in 4 boxes.

Based on what happened to Aqila, sometimes the students may have an idea about proportional thinking. They may get it from their own experience but the proficient ability in arithmetic is also a factor that determines the students' ability in solving proportional problem. The students who were insufficient in prerequisite material of proportional may have obstacles to solve the proportional problem. Although we facilitate them with a model, such as the ratio table in this study, it also required the students to have sufficient ability in arithmetic.

Problem 3

Problem 3 is comparing the price of apple in two different markets, market A and market B. The apples in market A costs Rp50.000 for 2 kg and the apples in market B costs Rp60.000 for 3 kg. The students should determine which market sells the cheaper apples.

Some students could not answer this problem correctly. They also gave unclear reason.

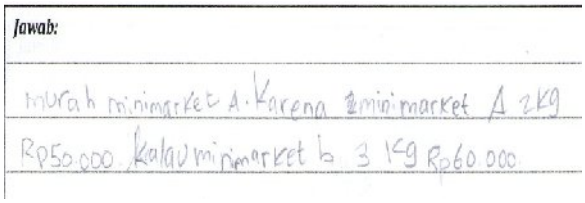


Figure 5.10 Lulut's Answer on Problem 3

Translation: Minimarket A is cheaper. Because minimarket A, 2 kg Rp50000 while minimarket B 3 kg Rp60 000.

When the researcher proposed the same question during the interview, Lulut gave the same answer as she wrote on the pretest; minimarket A is cheaper than minimarket B. When the researcher asked her reason, she said that 60000 is 10000 more than 50000. It indicates that she thought the problem as additive situation. Furthermore, she realized that the price of the heavier apple is more expensive. It made her concluded that the heavier weight was the more expensive and vice versa. According to her answer, it seems that she did not consider the idea of proportionality in comparison situation. She could not relate the price and the weight to build up the comparison situation.

To bring up the idea of relation between the price and the weight, the researcher used the other numbers for the same problem. The first modified problem was “Given that the price of apples in minimarket A is Rp50000 for 2 kg while in minimarket B is Rp50000 for 3 kg”. The price in this problem is the same but it has different weight. For this problem, Lulut could determine that minimarket B is cheaper than minimarket A. It was not clear how she can come to that thinking. Maybe, it because she had that experience in real situation instead of used some mathematics strategy; if we buy something with the same price, the one with the big amount is the cheaper.

The second modified problem was “Given that the price of apples in minimarket A is Rp50000 for 2 kg while in minimarket B is Rp60000 for 2 kg”. Now, the problem proposed the same weight but with the different answer. Together with the first modified problem, we expect that the students can realize the relation between the price and the weight in comparison situation. For this problem, Lulut was able to answer correctly. However, Lulut could not explain

how she can come to that conclusion. It may indicate that her experience in the real situation which lead her to the conclusion and not her mathematical thinking.

The interview was continued with the discussion to determine the price of 4 kg apples in minimarket A. The question was intended to lead her to the idea of which proportion that can give a good comparison. Actually, we want to guide her to compare the price of 6 kg apples in minimarket A and minimarket B. Through working on this, we expect that she could generalize the idea that we need to make the weight from both apples to be equal before comparing the price. However, she was struggling to determine the price for 4 kg apples in minimarket A. It maybe because the number is big, in thousands, and she just did it in her mind without scratch paper. But, it is also possible that she does not really master the strategy to solve the simple proportional problem yet. It is in line with the statement by Tourniere & Pulos (1985), that the comparing ratios is an advanced method, the way to choose the easier proportion to do comparison will be acquired after the proportional techniques are mastered.

The other students gave a correct answer but the unclear reason.

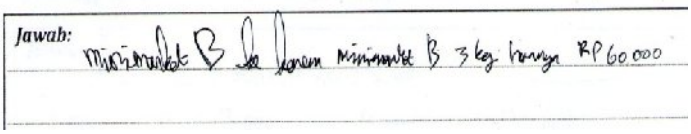


Figure 5.11 Ghafar's Answer on Problem 3

Translation: Minimarket B because in minimarket B 3 kg just Rp60 000

Ghafar gave different answer during the interview session. He believed that the price in minimarket A is cheaper than minimarket B. He just considered the weight of apple to answer this problem. He argued that the smaller weight means

the cheaper one. It was also happened when the researcher asked the follow-up questions.

1.	Researcher	: If the price in minimarket A Rp20 000 for 1 kg (the price in
2.		minimarket B remains the same as the original)
3.	Ghafar	: Minimarket A, because it is only 1 kg
4.	Researcher	: If the price in minimarket B Rp30 000 for 1 kg (the price in
5.		minimarket A remains the same as the original)
6.	Ghafar	: Minimarket B because it is 1 kg for Rp30 000 while in
7.		minimarket A 2 kg for Rp50 000

Transcript 5.2

Actually, for the first question in the transcript (line 1 and 2), the price was the same. But Ghafar said that minimarket A is cheaper than minimarket B, because he consider the weight in minimarket A is 1 kg while minimarket B 3 kg. It is also the same for the second question in the transcript, he still believed that 1 kg apple which costs Rp30,000 is cheaper than 2 kg apples cost Rp50 000. Then, the researcher made the problem with the same price but different weight, Rp50 000 for both 2 kg apples in minimarket A and 3 kg apples in minimarket B. This question aims to make him realize that he cannot only consider the weight to determine the cheaper apples. For the first, he answered the price is the same, because both of them costs Rp50,000, but when the researcher remarked that the weight is different, Ghafar changed his answer. He realized that the price of apples in minimarket B is cheaper. Then, he did not assume that the smaller weight as the cheaper one anymore.

The next question was about the situation in which the apples have the same weight but different price. The question just changed the price of apples in minimarket B to be Rp40 000 and left the price in minimarket A remained the

same. For this question, Ghafar was able to see that the price in minimarket B is cheaper than minimarket A.

Through the discussion, Ghafar had a conclusion as follow. If the price is the same, then we can determine which one the cheaper is by compare the weight. The heavier weight is the cheaper. If the weight the same then we can see the price to determine which one the cheaper.

Although in the end, he cannot solve the original problem 2, we may say that the context here has a big role to lead the students to think further. Because of the context familiar for the students, the students may relate the problem to their own experience. Thus, the students may able to construct their own idea to solve the problem.

Based on the discussion with Ghafar, we may say that proposing the same problem with different price and weight can help the students to consider the price and weight at the same time to do comparison. We can propose the problem which have different price but have the same weight and have different weight but different price. However, it needs further discussion to be able to compare something with the different price and different weight. The students need to do more exploration to relate the two sets of numbers and make the comparison between them.

Conclusion from pretest and student interview

Based on the pretest and the interviews, we may conclude that in general the students had proficient ability in arithmetic. It means that the students in class 4D fulfilled the prerequisite material to learn proportion. Furthermore, the majority of the students could solve the simple missing value problem by the repeated

addition, doubling, and unit strategy. Some students who could not solve this problem should be guided by posing some similar questions by using the smaller numbers.

For comparison problem, the majority of the students solve by using the unit strategy. They were able to compare the price by look for the price for 1 kg apple. Even for them who at first used the absolute value to compare the price, they may realize their mistake, by questioning the same problem with different number; we made the same price for the different numbers of item or the same number of items with different price. They may conclude that both the price and the weight influence the comparing situation.

2. Activity 1: Make Butterflies

As the same as in cycle 1, the Activity Make Butterflies on cycle 2 also consists of two worksheets. Before the students worked in their group, the teacher gave an apperception. The apperception was not planned in HLT, but it was a good idea because it could bring the idea of proportionality and bring up the enthusiastic of the students at once. What the teacher did was asked the students to take a role as butterfly. The teacher asked the students to put the hand up to the head with the index finger up resembles the antenna. Every time the teacher pointed certain number of students to act as butterflies. Accordingly, the students may see that we added the number of antennas by two for every 1 butterfly was added.

The lesson was continued by working on the first worksheet. The students were grouped in two. Because the number of students was 29, thus there was one group who consists of 3 students.

The first worksheet asked the students to determine the number of wings, bodies, and antennas for 2,4, 8, and 16 butterflies if it was known that to make 1 butterfly we need 4 wings, 1 body and 2 antennas.

The majority of students used the multiplication to solve the problem. However, there was a student who came up with the repeated addition, and it became the starting point of the classroom discussion. During the discussion, the students also grasped the reason behind the repeated addition, "If we add 1 more butterfly then the number of antennas will increase by 2.It means if we add 2 butterflies, the number of antennas will increase as 2+2, than if we add 3 more butterflies then the number of antennas will increase as 2+2+2". The apperception in the beginning of the lesson also helped the students to grasp this idea.

During the classroom discussion, the teacher also proposed the following problem.

How many butterflies that we can make from 29 wings, 8 bodies, and 13 antennas?

The teacher asked the students to put their hand in front of their mouth if they knew the answer. It was one of the norms in that class. Some students did that. But the teacher pointed Adistha, who did not put her hand in front of her mouth. However, before Adistha wrote her answer on the blackboard, the other students said that the number of wings was not divisible and the teacher said, "If it remained, just left out the remainder. Otherwise, that item would be not enough to make the complete butterfly." It should be concluded by the students through some discussion but the teacher directly told the students about that.

The same thing happened when Adistha wrote 29:4=6 remains 5. The teacher directly asked her to correct her answer. The teacher told her that the 5 still could be divided by 4, so the result should be 7. It was better if the correction came from the other students instead of from the teacher herself.

At first, Adistha wrote 8 butterflies, the biggest number, as the final answer. At that time some students said that it was wrong. But the teacher prohibited them to tell the right answer. Instead, the teacher asked Ardha to help her in front of the class. Ardha told her that it was 6.

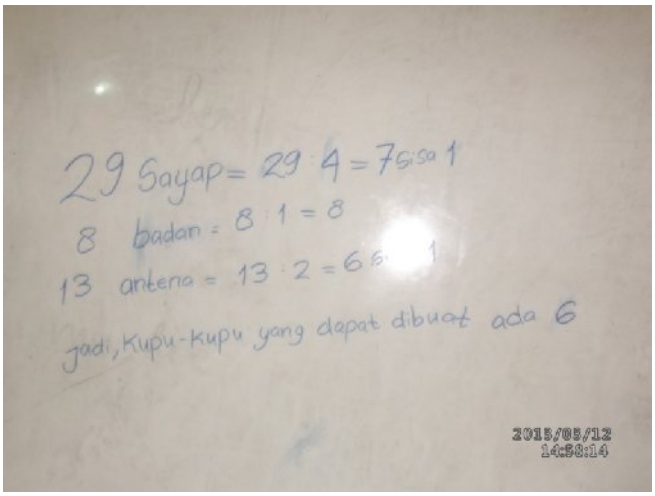


Figure 5.12 Students' Work

Then the teacher asked Ardha to explain to the others why it was six. However, Ardha was too shy to speak in front of the class, thus the teacher continued the discussion as follow.

1.	Teacher	: If he answered 7 butterflies then the number of
2.		antennas would be not enough, right?
3.	Students	:Yes
4.	Teacher	:If he answered 8, how?
5.	Students	:Not enough
6.	Teacher	:What about the wings?

7.	Students	:Not enough.
8.	Teacher	:Antennas?
9.	Students	:Not enough
10.	Teacher	:If six, the butterfly complete or not?
11.	Students	:Complete
12.	Teacher	:Now, Mulyadi look at the blackboard. There is 29
13.		wings. How many wings that needed for 6 butterflies?
14.		Are there enough wings?
15.	Mulyadi	:24
16.	Teacher	:Now, Fadhil. How many bodies for 6 butterflies?
17.	Fadhil	:6
18.	Teacher	:So, how many are left?
19.	Fadhil	:2
20.	Teacher	:Do you think for these 2 more bodies we can make the
21.		complete butterflies if there are only 29 wings?
22.	Fadhil	:No
23.	Teacher	:Now, Saeed. From 13 antennas, we want to take for 6
24.		butterflies. Can we?
25.	Saeed	:Can
26.	Teacher	:So, the six butterflies as Adistha's and Ardha's anwer
27.		are complete butterflies or not?
28.	Students	:Complete

Transcript 5.3

Based on the fragment, the teacher wanted to make sure that the students were understand why we choose the smallest number, 6, instead of 7 and 8. And it seems worked. She emphasized the term "not enough", "less", and "more" for each items. However, it would be better, if the teacher not proposed the questions which required the short answer, like "yes", "can", "complete" but more emphasized the students' reasoning, like "what do you think if…?", "why do you think so?". In that way, the students would be more speak up and gave more arguments.

In the end of the lesson, the students worked on the second worksheet. There were 4 questions here. The students worked individually and no discussion after this. The purpose of giving the second worksheet was for the students' practice. The four problems on the second worksheet used the same context as the first worksheet.

Problem 1

How many wings, bodies, and antennas that we need to make 6 butterflies?

All students used multiplication by 4 to find the number of wings, by 1 to find the number of bodies and by 2 to find the number of antennas.

Problem 2

If we have 48 wings, 12 bodies, and 24 antennas then how many butterflies that we can make?

In general the student used division. They divided the number of wings by 4, the number of bodies by 1, and the number of antennas by 2. Only a few students who used the repeated addition (4+4+4+4+4+4+4+4+4+4+4+4=48) and then count how many times of 4 to find the answer.

Problem 3

If we have 36 wings, 10 bodies, and 17 antennas then how many butterflies that we can make?

Problem 3 was similar to problem 2, but the numbers used here could not be used to make the same number of butterflies.

It was the same as on the problem 2, the students divided the numbers of wings by 4, the number of bodies by 1, and the number of antennas by 2. But after that, some students got confused to determine the number of butterflies because

each division gained different number of butterflies. Some chose the smallest number of butterflies while the other chose the biggest one.

Problem 4

Complete the following table! Explain your way to get the answer!

Butterflies	Wings	Bodies	Antennas
11	a) …	11	22
b)…	52	c)…	d)…
24	e) …	24	48

Figure 5.13 Problem 4 from the Second Worksheet on Activity 1

Actually this problem intended to show the relation among the numbers thus we made it on the table form. However, it seems that the students did not realize about it and still were not able to use the relation among the numbers to fill the blank. They only used multiplication or division to find the answer.

3. Activity 2: My Favorite Chocolate

In this activity, the students made a price list of chocolate. It was given that 4 chocolates cost Rp3000 and then the students were asked to make other possible pairs of numbers of chocolate and its price. In this activity, the students worked in group of 2-3.

During the group discussion, the researcher came to Belda's group because she looked confused when she made the list of the price.

First, Belda found the price of 8 and 12 chocolates by using repeated addition (4+4 chocolates cost Rp3.000+Rp3.000; 8+4 chocolates cost Rp6.000+Rp3.000). It may because they learned it during the first activity. When she wrote the number of chocolates as 24, she was confused to find the price. She tried to count

by using hand but still couldn't find the answer. Reihan also tried to figure out and by seeing the table which showed the list of the price, he got the idea to double the price. He said that the price of 12 chocolates was Rp9.000, so the price of 12 more chocolates was Rp9.000 more. It means that he realized about the proportion of every 12 chocolate costs Rp9.000 and he developed it into doubling strategy. He also used the same method to figure out the price of 48 chocolate as shown in the following transcript (line 7-8, 11-12).

1.	Raihan	:48 thousands… 48 chocolates [*Raihan corrects the name of*
2.		*item*] 48…that 48 plus 18…36 [*Raihan mentioned 48 instead of*
3.		*18*]
4.	Belda	:No. How was your way? (…?) [*untranscribable*]
5.	Raihan	:18… Here…24 plus 24 [*Raihan pointing the 18.000*]
6.	Belda	:Oh… So?
7.	Raihan	:18 (thousands) plus 18 (thousands) is 36 (thousands).
8.		24 (chocolates) plus 24 (chocolates)
9.		36 (thousands)
10.	Belda	:How?
11.	Raihan	:It says that 24 (chocolates) is 18 (thousands). So, 18(thousands)
12.		: plus 18 (thousands).
13.	Belda	:18 (thousands) plus 18(thousands)…
14.	Raihan	:36(thousands)

Transcript 5.4

It supports the HLT which stated that the students may gain other strategy (the doubling strategy) by exploring the ratio table using this problem. The money context helps the students to think about proportional reasoning (if you buy 12 more, you should pay Rp9.000 more). In addition, the ratio table helps the students to organize the information so that the boy may realize about the sense of

ratio. Thus, we may say that the ratio table together with the context helps the pupils to develop the proportional reasoning.

The next session was the discussion in the whole class. During the discussion, the teacher asked some students with different strategies to present their work. One of the groups who were asked to present their work was the focus group in this study. The group of Salwa and Adilla did a good work. They explored some possible way to determine the price of the number chosen, but unfortunately it was not explicitly written on their table.

Banyak cokelat	Rupiah
8 coklat	Rp. 6.000,-
10 cokelat	Rp. 7.500,-
12 cokelat	Rp.9.000,-
16 cokelat	Rp. 12.000,-
24 cokelat	Rp. 18.000,-
32 cokelat	Rp.24.000,-
48 cokelat	Rp. 36.000,-
100 Cokelat	Rp. 75.000,-

Figure 5.14 The Answer from the Group of Salwa and Adilla

Based on their written work, it seems that they used the ratio table alternately. The odd rows were done by Salwa while the even rows were done by Adilla. It seems that Salwa used the repeated addition to get the price of 8 and 12 chocolates, and then she used doubling to find out the price of 48 chocolates. Meanwhile Adilla used the repeated addition by adding the price of 12 +4 chocolates to get the price for 16 chocolates. She did doubling to get the price of 32 chocolates, and did multiplication by 10 to get the price of 100 chocolates. It was unclear how she could get the price of 10 chocolates without her explanation.

Thus, during the classroom discussion, the teacher asked her to explain how both of them may get that.

However, it was not easy to encourage the students to explain their idea. The following fragment showed how hard it was.

1.	Teacher	:How did you get (the price for) 10 (chocolates)?
2.	Adilla	: 8 plus 2 (chocolates)
3.	Teacher	:Yes, the price of 4 chocolates was Rp3000 so the price of
4.		2 chocolates was…? How you get the (price of)10?
5.		You said that for 10 you paid Rp7500. How you get that?
6.		It's correct. Do not be afraid. It is okay if it is wrong.
7.		10 per? 10 per what? 10 per 4. (And then) times?
8.	Adilla	: 3000 divided by 2 [whispering]
9.	Teacher	: Oh… 3000 divided by 2. So, 10 is gotten from 8 plus 2
10.		[Adilla nodding] while the price of 2 chocolates is 1500
11.		How much the price for 8 chocolates? Six…
12.	Adilla	:Six thousands
13.	Teacher	:And then plus the price of 2, so 6000 plus 1500
14.	Adilla	:7500

Transcript 5.5

From the fragment above, it was not easy to make the students express their idea. Although the teacher tried to encourage her by saying that her answer was true (line 6), she still did not say much. However, the teacher also took over the students' words too much even she tended to direct to the formal procedure;

multiplication and division (line 7). Fortunately, the student would like to clarify her answer (line 8).

In addition, from the fragment we knew that she divided the price of 4 chocolates by 2 to get the price of 2 chocolates before added it to the price of 8 chocolates. However, it was not clear whether the division by 2 there was the idea of halving or just division by the same number because there was only slightly difference between these two strategies.

To summarize, in general the students used the repeated addition to determine the number of chocolate and find the price. However, there were some groups who came up with the idea of doubling, multiply by the same number and adding. Based on the students' work and the fragment of the students' discussion, we can conclude that through exploring the ratio table, the students may gain some strategies which are useful to solve the proportional problems.

4. Activity 3: Feeding Caterpillars

As aforementioned, the activity 2 and 3 were conducted in the same lesson. Thus, after the discussion about some strategies on the ratio table, the students worked on activity 3.

In this activity, the students were asked to determine the number of leaves which were needed for 64 days if we knew that we need 5 leaves for 2 days. Some students used doubling, multiply by the same number, and repeated addition while the majority of the students used multiplication and division. It was different from the students on the cycle 1, who used the unit strategy; divided the numbers before they multiplied it. Here, they multiplied the numbers first then divided it,

so they could do the calculation easily because there was no fraction or decimal were involved in the calculation.

The change of order activity 2 and activity 3 on the cycle 2

In initial HLT, the activity of Feeding Caterpillar was put as the second activity based on the assumption that the student would be familiar with the ratio table after they did the first activity. Therefore, the activity 2 began with the simple missing value problem which aimed to see whether the students used the strategy from the first lesson or maybe gain other strategy to solve the missing value problem. Then it was continued by exploring the ratio table through activity My Favorite Chocolate to develop the strategies that they used on the activity 2. However, based on analysis on cycle 1, the activity of Feeding Caterpillars seems difficult for the students. Although it may allow the students to gain the idea of unit 1, the number was not divisible and the students had no prior knowledge about the division or multiplication with fraction so there was no students who could give the correct answer. Nobody tried to multiply first before they divided the numbers, otherwise they may solve the problem because in that way, there was no fraction or decimal involved. Here, the ratio table may help them to gain more strategies, such as doubling, repeated addition, or multiplying by the same number. Thus, it was better to reorder the activities and changed the order of both activities. The Feeding Caterpillar Activity became the 3rd activity and the activity of My Chocolate became the 2nd activity on cycle 2.

As consequence, most of students could give the right answer on activity Feeding Caterpillar. Furthermore, on cycle 1, there was a group who used the repeated addition not in proportional way. It did not happened in cycle 2. All the

students who used the repeated addition to solve this problem may do in the right way. Therefore, we may say the activity of exploring the ratio table should be given before the students deal with the missing value problem.

5. Activity 4: What do you think?

It was necessary to expand the context for the proportional problem, so that the students didn't limit their idea of proportionality just in certain context, especially money context because almost all activities in this learning design used the money as the contextual problems. Therefore, in this activity the students were given some problems with different contexts. There were two worksheets in this activity and the students worked it consecutively. The second worksheet was done after the first problem was discussed.

The first worksheet contained one problem which asked about the proportionality in adventure game as follow.

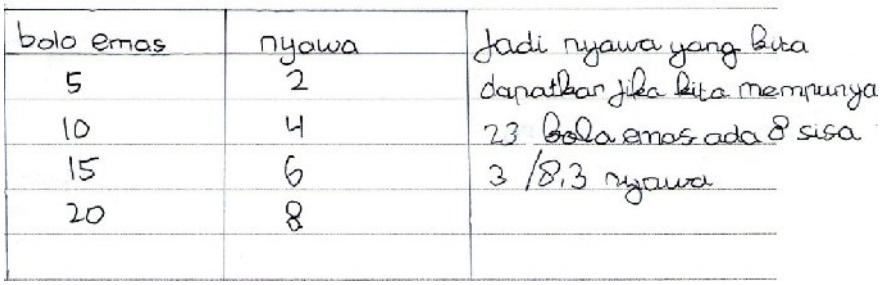
In an adventure game, there was a rule that we can change 5 golden balls with 2 lives. How many lives that we get for 23 golden balls?

Actually, if we back to the activity 1, they already did the similar problem within the context of making butterfly. But, the majority of the students could not figure out the right answer. Some of them forgot about the idea of proportionality here. Although they did in the "right track" to get the proportion for 8 lives for 20 golden balls, they back to add 1 live for every adding by 1 golden balls and got 11 lives for 23 golden balls.

During the discussion in the class, we discussed why we could not do it. Some students said that it was because if we added 1 live for every golden ball then it broke the rules on the games. Another student said that we could not do

that because the remains of the golden balls less than 5, which indicated that we could not change it with the lives yet. One of the groups who did in that way was the group of Livia and Hudhi. They also became the group who did the presentation in the class.

However, in their answer sheet they did not only write as they wrote on the blackboard during the presentation, namely 8 lives for 20 golden balls and remained 3 more balls. They also wrote the lives in decimals as 8.3 which were not correct. It was not clear about the notion of decimals there, because they have never learned the decimals yet. They will get it later in Grade 5. It seems the symbol of decimals there was only for representing the lives and the remained balls.



bolo emas	nyawa	jadi nyawa yang kita dapatkan jika kita mempunya 23 bola emas ada 8 sisa 3 /8,3 nyawa
5	2	
10	4	
15	6	
20	8	

Figure 5.15 The Answer from the Group of Livia and Hudhi

The second worksheet contained three problems as follow.

Problem 1

Problem 1 asked about the proportion in barter activity. The students were given the following question.

> In the second hand shop, we can change 5 old magazines with 4 story books. How many story books we get for 35 old magazines?

In general the students used the repeated addition to solve this problem while others used the multiplication and division, and multiplied by the same number to get the answer.

However, the students who used the multiplication and division to solve this problem were not guaranteed that they understood about the proportionality. The group of Ghafar and Nasywa was one of the groups who used the multiplication to solve the problem (see figure 5.16).

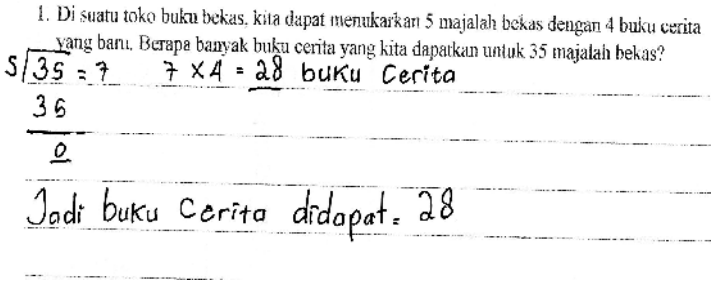


Figure 5.16 The Answer from the Group of Ghafar and Nasywa

Translation: So, the story books that we got are 28.

However, when the teacher asked them to make the ratio table as well, it seems that they had difficulty to work on it. Based on their answer sheet, it was clear that before they had the discussion with the researcher, they used the repeated addition not in proportional way. They wrote 6 instead of 10 old magazines to get 8 new books.


Tabel 1:

Majalah Bekas	buku Cerita
5	4
10	8
15	12
20	16
25	20
30	24
35	28

Figure 5.17 Ghafar's and Nasywa's Ratio Table

Actually, Ghafar refused to make the ratio table again because he already got the right answer. But Nasywa, who was an obedient student, insisted to make the table. Therefore, Nasywa was more active during the discussion with the researcher.

1.	Nasywa	:It means that if 6 (old magazines) you get 8 (story
2.		books) right?
3.	Researcher	:Why? 5 (old magazines) get 4 (story books)…If 6 get 8?
4.		Where did it come from?
5.	Nasywa	:I added it.
6.	Researcher	:Added by 1?
7.	Nasywa	:Yes, it added by 1 and this… 4 added by 4 [*Pointing the*
8.		*number of story books*]
9.	Researcher	:This is added by 4 [*pointing the number of story books*],
10.		but why this is added by 1 [*pointing the number of old*
11.		*magazines*]?
12.	*(Nasywa looks confused and cannot give the answer)*	
13.	Researcher	:It said that for 5 old magazines you get 4 story books…
14.	Nasywa	:So, it also added by 4? [*pointing the number of old*
15.		*magazine*]
16.	Researcher	:Really? See, first you have 5 (old magazines) and it can

17.		be changed with 4 (story books).So, if you just add 1
18.		(magazine)…Do you think the shop will add 4 (story
19.		books)? If so then the shop will be in loss.
20.	Nasywa	:So, it means that it added by 5? [*not clear which one is*
21.		*added by 5 but it seems that she talked about the story*
22.		*books*]
23.	Researcher	:What do you think? The shop wants to be loss?
24.		It is similar with when you were buying, like the money
25.		Yesterday. You bought for Rp 3000 and got 4 chocolates
26.		and you wanted to buy 5 chocolates and should pay Rp
27.		6000. Do you want it?
28.	Nasywa	:Buy 5 chocolates…? No, I don't.
29.	Researcher	:Why? You are loss, aren't you? You just add 1 more
30.		chocolate. But you should pay Rp3000 more… Whereas,
31.		if you pay Rp3000 more, how many chocolates you
32.		should get? 4 more chocolates right? Now, you have 5
33.		magazines and you get 4 story books. Then, if you want
34.		to get 4 more story books. How many magazines you
35.		should add?
36.	Nasywa	:Five

Transcript 5.6

As Nasywa learned from the previous lesson, she used the repeated addition to solve this problem. However, she didn't do it in the right way. At the first, Nasywa added 4 more story books every time she added 1 magazine. But after the researcher reminded her of the money context (line 20-23; 25-28), she understood that the repeated addition should be done as a ratio. Therefore, we can say that the money context may be used to remind the students of the rule of proportionality in other contexts.

On the other side, to be able to use the ratio table, a student should have ability of some manipulative strategies.

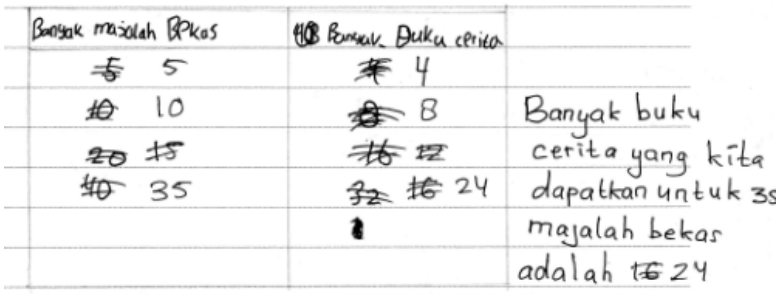


Figure 5.18 The Answer from the Group of Lailatul and Mulia

Translation: The number of story books that we get for 35 old magazines is 24.

The work this group showed that, in principle, they could do the doubling well. There was no mistake in their calculation. However, the problem asked the number of story books for 35 old magazines and not for 40. Hence, they changed their strategy. It seems that they were confused how to get the 35. They also tried to do the repeated addition by 5, and they already figure out the number of new books for 15 old magazines as 12. Maybe they did not realize that if they continued to do that they may find the answer after 4 additions more. Then they changed again their strategy. It was not clear how they can get the 24. Maybe they just double the number of story books for 15 old magazines and got 24 as the result. But, if that so, it should be 30 instead of 35 old magazines.

It indicates that work with ratio table required the ability to do some manipulative strategy. Thus, it was important for the students to master some manipulative strategies.

Problem 2

The students were asked to solve the following problem.

At a fair, we can buy 3 glasses of chocolate milk by 2 coupons. How many coupons that we need to buy 24 glasses of chocolate milk?

In principle, this problem also used the concept of buying like in the money context but not in Rupiah, so it would be different for the students. The number used here was small and the ratio was small, 2:3, not like in money context.

The majority of the students used the multiplication by the same number to find the answer while some others did multiplication and division, the repeated addition or doubling. There was only one group who gave the wrong answer.

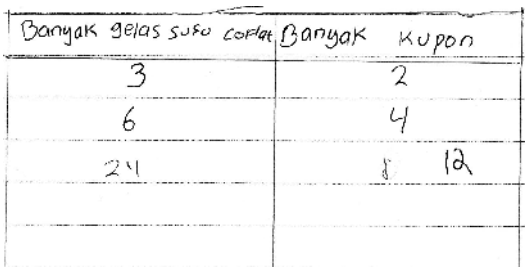


Figure 5.19 The Answer from the Group of Faiq, Lulut and Altih

Based on their written work, it seems that for the first they used the doubling to find the answer because they wrote 8 after 4. But then, they may change their mind and want to get the number of coupons for 24 glasses of chocolate milk. It was not clear how they get 12 as the answer. It may because they did miscalculation. They thought that 24 was the multiplication of 6 and 3, so they multiply the 4 by 3.

Problem 3

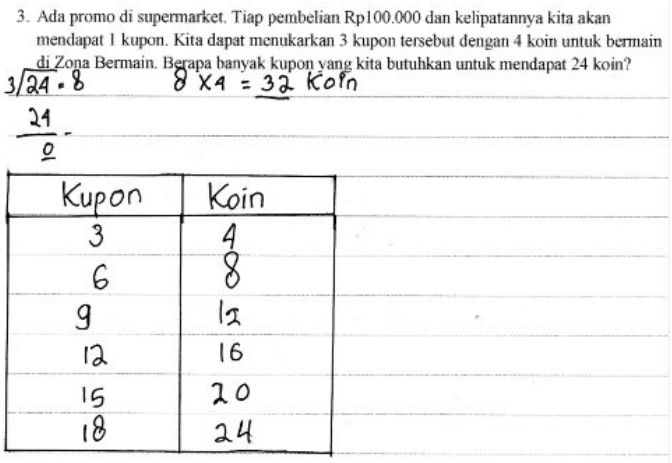
The students were asked to solve the following problem.

> There is a promo in supermarket. Every purchase of Rp900.000 and the multiples, we get 1 coupon. We can change the 3 coupons with 4 coins to play at Game Zone. How many coupons we need to get 24 coins?

The majority of the students used the repeated addition to solve this problem. Some other strategies occurred, such as multiplication and division, multiplying

by the same number and the combination of multiplying and adding. The group of Ghafar and Nasywa was one of the groups who used the multiplication and division. But, after saw that their answer was not correct, the researcher asked them to make the ratio table as they did for problem 1.

On the analysis of problem 1 on this activity, we discussed about how the group of Nasywa and Ghafar made the wrong repeated addition on the ratio table and how the money context helped them to realize their mistake. In this section, we described how the ratio table may prevent the mistake which was possible to occur if we used the multiplication and the division method.


3. Ada promo di supermarket. Tiap pembelian Rp100.000 dan kelipatannya kita akan mendapat 1 kupon. Kita dapat menukarkan 3 kupon tersebut dengan 4 koin untuk bermain di Zona Bermain. Berapa banyak kupon yang kita butuhkan untuk mendapat 24 koin?

3/24 • 8 — 8 x 4 = 32 koin

24
0

Kupon	Koin
3	4
6	8
9	12
12	16
15	20
18	24

Figure 5.20 The Answer from the Group of Ghafar and Nasywa (2)

Based on the figure above, we saw that before the students made a table they had the wrong multiplication and division. They switched the number of multiplication factor and the division factor.

We knew that the student tend to read the problem not by the whole sentence but just the part of the numbers. It made them not aware of what the number represent was and just did the calculation, especially if the number could be divided by both numbers. For instance, 24 could be divided by 3 and 4. Thus, they

may make a mistake such multiply or divide by the wrong number. The ratio table may prevent the students to make this kind of mistake because the table can be used to organize the data that known from the question. The table placed the number on each category. It made the students knew what the number represented was.

Vice versa, from work with the ratio table, the students may grasp the idea to work with more formal procedure.

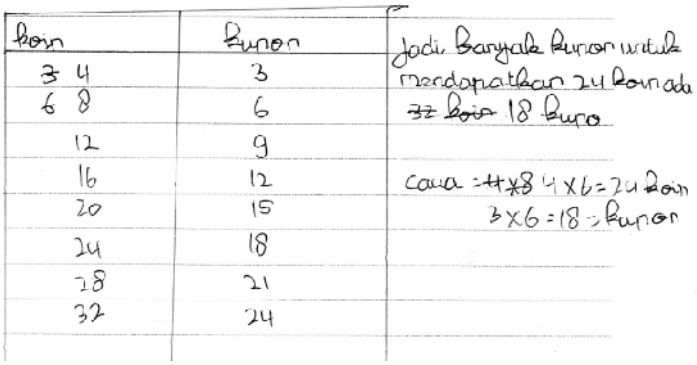


Figure 5.21 The Answer from the Group of Livia and Hudhi (2)

Translation: So, the number of coupon to get 24 coins is 18 coupons.

From this group's work, we could see that they use repeated addition with ratio table to solve the problem. The students added the number of coins by 4 and the number of coupons by 3. First, they wrote the number in the wrong position. The number of coins should be 4 instead of 3.

The 3 and 6 on the coin columns indicated that they may write the list of coins first and not fill the both columns at the same time. It also showed that at the first they also misplaced the numbers on the wrong column.

They also may have misread the problem because they wrote the number of coupons instead of coins until 24. However, they wrote the right answer. Here, table helped them to check the answer because it recorded the whole calculation.

In the last sentence, they also wrote the multiplication by 6 as the way. It was because the teacher also asked them to explain their way to get the answer. It was possible that they realized the idea of multiplying by the same number. It also maybe they count how many times they did the repeated addition. However, both of thinking may lead the students to the more formal strategy.

After the students finished their work, the teacher conducted the classroom discussion, but it was just for checking the students' answer.

Based on the analysis of activity 4, we may say that the ratio table helped the students to organize and place the numbers based on the category. It prevents the students who used the multiplication and division to multiply or divide by the wrong number. In this activity, some context were tricky for the students and it may lead the students used a wrong strategy. However, after we reminded them about the money context, they were able to correct their mistake by themselves. Here, we may say that the context was important to build up the students' proportional reasoning. In addition, introduce the ratio table should use the appropriate context in order to keep the students to remind the rules.

6. Activity 5: Which is the cheaper one?

The last activity in this learning design emphasized the use of ratio table in solving comparison problem. There were two questions were proposed in this lesson. The first problem is simpler than the second one because the item which was used is the countable noun while the second one used the uncountable noun which presented the gram as the unit. The two problems were done consecutively. After the students finished the first problem, the teacher discussed it first before the students worked on the second one.

Problem 1

Problem 1 asked the students to determine which one the cheaper is, soda A or soda B, if it was known that 3 sodas A cost Rp4000 while 2 sodas B cost Rp3000.

The majority of the students used the repeated addition to solve the problem while the others used the combination of doubling and multiply by the same number to get the same price or the same number of sodas. The group of Livia and Hudhi was the one of the groups who decided to use the repeated addition to solve the problem after at first they intended to use the unit strategy to solve the problem. The following fragment showed how they changed their mind from using the unit strategy to the repeated addition strategy.

1.	Researcher	:Why?
2.	Livia	:How much for each? [*pointing the price of soda A, Rp4000 for*
3.		*3 sodas*]. It should be divided…
4.	Hudhi	:It is not divisible [*Take out a book to write the calculation*]
5.	Livia	:Soda A. We should divide first…
6.	Hudhi	:[*Do the division of 4000 by 3*] It remains Liv… What I said. It
7.		remains 1.
8.	Livia	:So, it starts with 3 right? [*Make the ratio table*]

Transcript 5.7

The above fragment described that Livia wanted to solve the problem by looking the price of 1 soda for each type at first but Hudhi said that they couldn't divide the 4.000 by 3, because it remained 1. They may also think that 1.333 was not a price, because it was not common in Rupiah. Thus, instead of using the unit strategy, Hudhi suggested making table for each soda as they learned on the previous lesson.

Soda A	Harga	Soda B	Harga
3	RP.4.000	2	RP.3.000
(6)	RP.8.000	4	RP.6.000
9	RP.12.000	(6)	RP.9.000
(12)	RP.16.000	8	RP.12.000
15	RP.20.000	10	RP15.000
(18)	RP.24.000	(12)	RP.18.000
21	RP.28.000	14	RP.21.000
24	RP.32.000	16	RP.24.000
27	RP.36.000	(18)	RP.27000
30	RP40.000	20	RP.30000

Figure 5.22 The Answer from the Group of Livia and Hudhi (3)

After they finished make the table and it's more than what they need (they make the long table), they discuss how to determine which the expensive one or the cheaper one is.

1. Livia :So, which one is cheap?
2. Hudhi :We should look for the equality.
3. Twelve and twelve. This is cheap [*pointing soda A*].
4. Livia :So, which one is cheap?
5. Hudhi :We should look for the equality.
6. Twelve and twelve. This is cheap [*pointing soda A*].
7. Livia :16 (thousands) isn't it?
8. Hudhi :Yes, and this is 18 (thousands) [*pointing soda B*].

Transcript 5.8

Hudhi got an idea that they can find out the cheaper soda by comparing the price of the equal quantity (line 5). He saw both lists and realized that soda A was cheaper than soda B (line 6).

However, the way they used the table to figure out the cheaper sodas was not efficient. As mentioned before, the ratio table which they wrote was already more than they need to determine the cheaper soda (see figure 5.22) but Livia was still

looking for the price of other number of cans. She said that they need to continue the table, but when the researcher asked whether they could determine from the existing table which one was cheaper, they answered yes. Thus, in the end of the discussion, the researcher remarked that there was no need to continue the table if they already knew which the cheaper one was.

1.	Livia	:It is until 30 [*she is pointing the table for soda A*].
2.		[*Pointing the soda B*] For 30 (sodas B), how much? [*She asks*
3.		*Hudhi to continue until 30*] add 2, add 2, add 2… Use that book
4.		first.
5.	Researcher	:[*Hudhi takes out his book*, *maybe to continue the table*] Just
6.		make it behind.
7.	Hudhi	:So, we make a table again then?
8.	Livia	:Yes. We make until…
9.	Hudhi	:Until 100 [*kidding*]
10.	Livia	:30.
11.	Researcher	:Is it necessary to continue the table?
12.	(*Livia and Hudhi are just smilling*)	
13.	Livia	:[she whispers to Hudhi] Is it necessary?
14.	Researcher	:From the table here, can you say which the cheaper one is?
15.	Hudhi	:This [*pointing soda A*]
16.	Livia	:This [*she agrees with Hudhi*]
17.	Researcher	:If so, why you want to continue the table?
18.	Livia	:[*Livia smiles then she writes the answer*]…

Transcript 5.9

It was not clear why Livia wanted to continue the table. It may be that she thinks that the result can be different for a different number of items, or it may be that she just wants to end the table with the same number of sodas (line 1, 2).

After all students solved the problem, the teacher held the whole class discussion. The teacher asked some groups to go in front of the class and present

the answer. The students make a drawing on the blackboard and the other students were asked to see what their friend wrote on the blackboard. Here, there was no much discussion occurred because the strategies which were written on the blackboard were not too different from the other students' work.

Based on the pretest, the students knew that they have to make the same price or the same number of item in order to compare the price. Thus, the discussion about the use of proportionality instead of the absolute value in comparison situation was not the focus in this lesson. The discussion more emphasized the strategy to make that happened. Maybe, it would be better if the teacher also gave a remark about the use of proportionality in the comparison situation during the discussion. So, the students may give more argumentation on why they should use proportion in comparing situation.

Problem 2

After the discussion in the whole class, the students worked on the second problem. Problem 2 asked the students to determine which one the cheaper is, honey A or honey B, if it was known that 600 gram of honey A cost Rp60,000 while 300 gram of honey B cost Rp45,000.

For the second problem, all students were doing well. They were able to solve the problems without any obstacle.

There were six groups who used repeated addition to find the answer. They find the price for the same amount of honey A and honey B. The other three groups used halving to find the price of 300 gram honey A and one group who used doubling to find the price for 600 gram of honey A and honey B.

Meanwhile, the other four groups looked for the price for100 gram honey A and B. It was kind of the unit strategy; they found the price for per 100 gram of honey. The reason why unit strategy became the second favorite after the repeated addition was because the number of weight for both honey A and B were the multiple of 100. Thus, they could easily do the calculation; divided the price of honey A by 6 and the price of the honey B by 3. However, three of these groups still wrote other proportion in the table although they already knew which honey was cheaper (see figure 5.23).

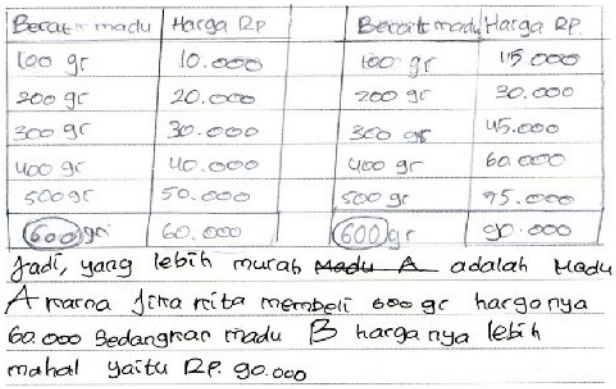


Figure 5.23 The Answer from the Group of Mutia and Imanah

It was not clear why Mutia and Imanah continued to make the table although they have already knew which honey was the cheaper. It may because they want to get the price of 600 gram of honey B as the original weight of the honey A. But it was exaggerated.

Regarding the fact that there were some groups who did the same thing, the teacher should emphasize the aim of making the ratio table and the efficiency of the use of the ratio table.

Unfortunately, the class discussion only focused on the variation of students' strategies. The teacher did not discuss how to use the ratio table efficiently. As on

the discussion of the first problem, the teacher asked some students to present their answer in front of class. Then the other students could evaluate whether their answer was true or false. Meanwhile, according to the students' answer which was more than what they need- seen how long they made the table- it was the important issue for the students. The teacher should bring up the questions what the aim of making the ratio table, what we want to look for, how long we should make the table, when we should stop to write down the proportion.

Based on the students' work and the students' discussion on the activity 5, we may conclude that the ratio table helps the student in comparing situation. The students were able to determine the cheaper price of two different items. Based on the pretest and interview, the students already had some idea of comparing something. They could compare which one cheaper if they had the same price for different number of the item or had the same number of items for different price. The students' difficulty was they didn't know how to convert the given sets of number to be the same price or the same number of item; they could not see it as solving two different missing value problems. Here, the ratio table can help them to solve this matter.

7. Posttest

The posttest was conducted in the end of activity 5. There were only 28 students who participated in the posttest, 1 student was absent at that time.

The problems which were used in posttest on the cycle 2 were different with the problems in the posttest on cycle 1. There were 4 problems proposed here.

Problem 1

Problem 1 asked the amount of chocolate powder that used to make 12 chocolate cakes if it is known that we need 30 gram chocolate powder to make 4 chocolate cakes. The number in this problem is the easiest one, because the question asked for the biggest number and 12 is also the multiple of 4.

There were only 3 students who could not answer this problem correctly while the majority of the students were able to solve it well using the repeated addition. Some students used the multiplication to solve this problem.

Problem 2

Problem 2 asked about the number of servings of fried rice that we can make from 5 kg rice if it is known that we can make 12 servings of fried rice for 3 kg rice. The number in this problem is not really easy because it requires the unit 1 method to find the answer. We should look for the number of servings for 1 kg first then we can multiply the result by 5.

There were 12 students who could not answer correctly while the remains used the unit strategy to find the answer.

Problem 3

Problem 3 asked about the price of 4 balloons if it is known that 24 balloons cost Rp36,000. The number on this problem is more difficult than problem 1. Although the students already got used to solve the problem with money context and 4 is also the factor of 24, the problem requires the smaller numbers for the answer.

There were 13 students who could not give the correct answer. Some of them have correct calculation but have different interpretation of the problem. They

thought that the question asked the total price of 28 balloons. In general, the students who gave the correct answer used the division by the same number using the ratio table, they divided the 24 to get 4 and divide the price of 24 balloons by 4. Meanwhile, the others used the unit strategy to figure out the price of 1 balloon first. Of course, it was more difficult in calculation because the price of balloon contains the 500 as the price while if using the division the price was in thousands.

Problem 4

Problem 4 was the comparison problem. The students were asked to determine which car is the more economical one, if it is known the car A need 6 liter of fuel to drive 80 km and car B need 4 liter of fuel to drive 60 km. The number in this problem is friendly numbers. There were many ways to get the proportion which shows the comparison situation for this problem.

There were 13 students who gave wrong answer for this problem. However, 9 from these 13 students actually had the correct calculation for the price of the similar amount of fuel. They just got confused to determine which car is more economical. They answer that for the same fuel, the less distance was the more economical car. It was different with the students who look for the similar distance; they could determine the more economical car without any confusion. It maybe because they were tricked by the thinking of a price in which for the same amount of fuel, the much money to be paid was the more expensive. But for them who looked for the amount of fuel for the same distance was not tricked because it was clear that for the same distance, the car which used the smaller amount of the fuel was the more economical one.

Conclusion from the posttest

Based on the students' answer on the posttest, we may conclude that some students still have difficulty to solve the missing value problem which looked for the smaller number. It seems that students were not familiar to use the ratio table to look for the smaller ones. In general, they only used the doubling, repeated addition, multiply by the same numbers which were strategies to find the bigger number. Some of the students also still need more discussion to grasp the idea of unit strategy. However, almost all students could give the correct answer for the comparison problem. The ratio table helps them to make the proportion which shows the comparison situation. It means that the ratio table may support the students in developing their proportional reasoning as well as facilitate them to solve the proportional problem. In addition, it also indicates the students were influenced by the lesson through the designed learning sequence so they could use the ratio table to solve the proportional problem.

In addition, some students used the ratio table not as the main tool to do the calculation. First, they used the ratio table to organize the information from the problem and after they using division and multiplication which was a formal method; they used the table just to check their answer.

Conclusion of teaching experiment (cycle 2)

Based on the teaching experiment on cycle 2, we could draw conclusion as follow. The ratio table may help the students to do reasoning and also as a tool to do calculation in solving the proportional problems. The table helps the students to organize the information from the given problems. Besides, it also helps the students who has problem with multiplication to use repeated addition or doubling

as alternative strategy. In addition, the ratio table also allows the students to see the relation among numbers and together with the context; it can build up the students' proportional reasoning. Furthermore, the ratio table may help the students to present the proportion which shows the comparison situation.

Regarding the classroom discussion which happened during the lesson on cycle 2, it seems that it was not too effective. It was just to clarify the students' answer rather than sharing the idea; less discussion about the argumentation. The students presented the different answer but less feedback or response from others. Maybe by posting the questions like "what do you think?" or "do you understand what they said?" may encourage the students to give their argumentation or their opinion. Moreover, some important points were missing during the discussion, such as why we use the idea of proportionality instead of using the absolute value in comparison situation and how to use the ratio table in more efficient way. Moreover, most students were passive during the discussion. If the teacher did not point on them, they would not say anything; only certain students who participated actively during the discussion.

CHAPTER VI

CONCLUSSION AND SUGGESTION

6.1 Conclusion

The purpose of this study was to support the students in developing the proportional reasoning as well as to contribute to the development of the local instructional theory. In line with that, we developed and tested an HLT as described in the previous chapters. Through the data analysis, we may answer the research questions in this study as follow.

Answer to the first sub question

"How do the students in Grade 4 use their intuitive understanding to solve proportional problem?"

The answer for the first sub questions were related to the data collection from the pretest and the student interviews in the beginning of the cycle 2. Here, we gathered the information about the students' prior knowledge and the impression on how the students may use their intuitive understanding to solve the proportional problems.

The pretest and the students' interviews which were conducting on the following week after the pretest showed that in general the students used the multiplication and/ or division to solve missing value problem. Some of them were able to gain the idea of unit 1 method while the others multiplied with the wrong number. The others came up with the idea of doubling and repeated addition to look for the fourth number in missing value problem. However, some students still need guidance to deal with this problem. By proposing a similar question with smaller numbers, the lower achievement students, who have no

proficient multiplication, may be able to think about the repeated addition as the way to solve the original problem.

The comparison problem seems more difficult for the students. The students should be able to solve the simple proportional problem before they work on comparison situation. Otherwise, the students may not able to deal with the new set of numbers which is appeared in comparison problem. Instead of using the proportionality, some students used the absolute value to solve the comparison problem. They may not have idea about the relation between two items which used in comparison situation, such as the price and the weight. However, the role of context which used in problem and the questions which were posed during the interviews may influence the students' reaction. When the context which used in the problems was experienced by the students, they may be able to give the right answer based on what happened in their daily life. Proposing the similar problems and change the number may lead the students to get the idea to solve the problem.

Answer to the second sub question

"How can the ratio table support pupils to develop proportional reasoning in missing value problem?"

The activities in this study emphasized the use of ratio table in solving proportional problems. The learning sequence were designed to introduce the ratio table to the students and allowed them to explore the possible strategy by using the ratio table. Based on analysis of the students' work and the videos of learning process of the activity 1 until activity 4, we may say that the ratio table can help the students to solve the proportional problem as well as develop their proportional reasoning. By using the ratio table, the students can organize the

information from the problem because the table classifies each item in systematic way. Sometimes the students do not aware of what number represent is; the students just directly multiply it or divided it without any consideration. The ratio table may prevent the students to do this. The ratio table also allows the students to write the intermediate step of the calculation. First, it facilitates the students who cannot do the calculation well. Second, by writing the intermediate steps, the students may be able to see the relation among the numbers. Here, the students can build up their proportional reasoning.

Furthermore the activity of exploring the ratio table in My Chocolate may let the students gain the different strategy to do the calculation, such as the doubling, multiply with the same number, repeated addition. In addition, the money context which used to introduce the ratio table in this activity may help them to build up the proportional reasoning based on their experience in daily life; if the price of 4 chocolates is Rp3000 then if we buy 4 more, we should pay Rp3000 more.

Answer to the third sub question

"How can the ratio table support pupils to develop proportional reasoning in comparison problem?"

From the pretest and the student interview in the beginning of cycle 2, we got impression that in general the students had the sense of proportionality in comparison situation. The money context as the problem 3 on the pretest may guide them to realize that both of the price and the number of items should be considered to determine which one the cheaper item is. They could figure out that for the same price, the cheaper item is the one which has the more weight and for the same weight, they just need to consider the price. However, it seems difficult

for them if the price or the weight were not the same. Therefore, in the last activity of the learning sequence in this study, we proposed two comparison problems which had no same price or weight.

From activity 5, we can see that the table can help the students to solve the comparison problem. For instance, by making table of the price of soda A and the price of soda B, the students are able to determine which the cheaper one is. They may use the table to look for the proportions which give a good comparison situation. They find that the table helps them to find the number of items that they got for the same price or to find the price for the same number of items and then compare the price. Of course, it can be happened after the students master the use of ratio table in simple proportional problem. It is in line with what stated in Tourniere & Pulos (1985), "Comparing strategy is an advanced method, and the ability to choose the arithmetically easier comparison is acquired long after the proportional techniques mastered."

Answer to the main research question

"How can the ratio table support pupils to develop proportional reasoning?"

Based on the analysis and the answers of the sub questions, we may say that the ratio table together with the context can help the students to develop their proportional understanding as well as the strategies to solve the proportional problems; both in missing value and comparison problems. By using ratio table, the students may organize the information from the question. It helps them to identify what the numbers represent for. The intermediate steps which are showed on the ratio table allow the students to see the relationship among the numbers. Hence, they can develop the proportional relation among the numbers. Once they

realize about this relation, they may use that as the reason to think proportionally. When the students already mastered the techniques to solve simple proportional problem, they may developed the ratio table as a tool to do calculation and reasoning in comparison situation. The ratio table allowed them to gain the proportions which show the comparison situation.

6.2 Reflection

The implementation of PMRI on the learning design

As the first characteristic in PMRI, we used the familiar context as the starting point of the learning process. Together with the context, we introduced the ratio table as a tool to do the calculation and do the reasoning. The group discussion and the classroom discussion during the lesson may enable the interaction among the students which lead them to develop their proportional reasoning. Moreover, the teacher gave time to the students to think and to explore the ratio table by themselves. It may lead the students to reinvent some strategies that can be used to solve the proportional problems. Last, the contexts were used in this design also intertwined with other field, such as art lesson, science, and daily life problem.

The contribution to the local instructional theory of proportion

Based on the previous study (Zulkardi, 2002), most of textbook that used in Indonesia contain mainly the set of rules and algorithm which is already formal and they lack of application which is needed by the students in order to make the concept be real for them. In some textbook which used to teach proportion in Indonesia (e.g. Soenarjo, 2007; Sumanto *et al*, 2008), the students were taught the cross multiplication directly. The learning process only emphasized the

procedure to solve the problems and less attention to the development of students' proportional reasoning.

In this study, we proposed a learning trajectory and the designed activities which emphasized the insight of proportion. Together with the contextual problems, the ratio table, which used as a model in this learning design, helped the students to develop their proportional reasoning. Through the teacher guide which was provided in this learning design, we may help the teachers to implement the HLT in their class. Because of the contextual problems which used in this design were, in general, familiar for the students, we may say that this design could be implemented in other school in Indonesia.

6.3 Suggestion

Suggestion for the teacher

It is not easy to enforce the new social norm in the classroom. It takes time to introduce and make the students accustom to the new learning situation. As we can see from the description on chapter 5, some discussion in the classroom was not going well because only certain students who actively participated in the discussion. Although the teacher encouraged the other students to speak, they were being passive. Most of them felt shy or they were afraid that their answer was wrong. However, it does not mean that we cannot change the social norm in the class. If the teacher never stops to encourage the students in every lesson, the social norms may be changed.

Moreover, most of the teacher tends to explain the answer or the way to the students and directly correct the answer of the students. It also one of the factors that cause the students becomes passive. They will just accept the explanation

from the teacher or just copy the procedure. Instead of telling the students the right answer, it is better to guide the students to find the right answer by themselves. It can be done by posing the questions which can lead the students to gain an idea to solve the problem. In that way, the discussion in the class also becomes more alive.

Furthermore, regarding the implementation socio-mathematical norms in the class, the teacher should emphasize the normative understanding about what counts as a mathematical difference, a mathematical sophistication, and an acceptable mathematical explanation and justification in the classroom. According to Yackel & Cobb (1996), the teacher may establish the environmental in the classroom which is conducive to improve students' mathematical conceptual development by considering these factors.

Based on the aforementioned statements, we may say that the teacher has an important role to create a good learning situation in the class. Thus, the teachers should do reflection to her/his self, always upgrade their knowledge, and have to change the "bad habit" in order to improve the learning situation in the class.

Recommendations for the future research

The ratio table may help the students to deal with the proportional problem, both in missing value problems and in comparison problems. However, the students tend to use the ratio table only to look for the bigger number. Sometimes the students still face the difficulties to find the smaller number. They more get used to use the strategy which gain the bigger number than the smaller ones. For instance, the students are more familiar to do doubling instead of halving, or build up than moving down. In line with that, the study by Rupley as cited in Tourniere

(1986) also stated that the proportional problems in which number to be found is the biggest are easiest. Therefore, this study can be extended to situation in which requires the smaller numbers.

Moreover, the contexts which used in the problems on each activity were familiar for the students, especially the money context. However, it is necessary to expand the variation of the context so that the students do not limit the application of the proportionality just in certain context. Although on activity 4, we proposed some different contexts, it still needs to be expanded. Thus, the other suggestion for the next study is give some more contexts as the enrichment material to the students, both in missing value problem and in comparison problem.

REFERENCES

Bakker, A. (2004). *Design Research in Statistics Education: on symbolizing and computer Tools*. Amersfoort: Wilco Press.

Bakker, A., & Van Eerde, D. (2012). *An introduction to design-based research for Masters and PhD students*. Submitted.

Boyer, Ty.W & Levine, Susan. C.(2012). Child proportional scaling: Is 1/3=2/6=3/9=4/12? *Journal of Experimental Child Psychology, 111* (2012), 516-533.

Depdiknas. (2006). *Kurikulum Tingkat Satuan Pendidikan Sekolah Dasar*. Jakarta: Depdiknas.

Fosnot, C.T & Dolk. M. (2001). *Young Mathematicians at Work: Constructing fraction, decimals and percents*. Heinemman. Porthsmoth, Netherlands.

Freudenthal, H. 1973. *Mathematics as an educational task.* Dordrecht, the Netherlands: Reidel.

van Galen, F., Feijs, E., Figueiredo, N., Gravemeijer, K., Herpen, E., &Keijzer, R. (2008). *Fractions, percentages, decimals, and proportions: A learning-teaching trajectory for Grade 4,5 and 6*. Sense Publisher, The Netherlands.

van Galen, F. & van Eerde, D. (2013). Solving problem with the percentage bar. *IndoMS. Journal on Mathematics Education, 4* (1), 4-8. Retrieved from http://jims-b.org/

Gravemeijer, K. (1994). *Developing realistic mathematics education*. Utrecht: CD-ß Press /Freudenthal Institute.

Gravemeijer, K., Terwel, J. (2000). Hans: Freudenthal: a mathematician on didactics and curriculum theory. *Journal Curriculum Studies, 32*(6), 777-796.

Gravemeijer,K.,& Cobb,P. (2006). Design research from the learning design perspective. In Van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N (Eds.), Educational Design Research. London: Routledge.

Karplus, R., Pulos, S., &Stage, E.K. (1983). Early adolescents' proportional reasoning on 'rate' problems. *Educational Studies in Mathematics, 14*,219-233.

Langrall, Cynthia. W. & Swafford, J. (2000). Three Balloons for Two Dollars: Developing Proportional Reasoning. *Mathematics teaching in the middle school , 6* (4), 254-261.

Parish, L. (2010). Facilitating the Development of Proportional Reasoning through Teaching Ratio. In L. Sparrow, B.Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceeding of the 33rd annual conference of the Mathematics Education Research Group of Australasia*. Fremantle: MERGA.

Simon, M.A., & Tzur. R. (2004). Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. *Mathematical Thinking and learning, 6* (2), 91-104.

Silvestre,A.I.,& da Ponte J.P. (2012). Missing value and comparison problems: What pupils know before the teaching of proportion. *PNA, 6* (3), 73-83.

Soenarjo, R.J. (2007). *Matematika 5*. Jakarta: Pusat Perbukuan Depdiknas.

Sumanto, Y.D, Kusumawati, H., & Aksin. N. (2008). *Gemar Matematika 5*. Jakarta: Pusat Perbukuan Depdiknas.

Tourniaire,F.& Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational studies in Mathematics, 16*, 181-204.

Tourniaire, F. (1986). Proportion in elementary school. *Educational Studies in Mathematics, 17*, 401-412.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in Mathematics. *Journal for Research in Mathematics Education 27*, 458-477.

Zulkardi. (2002). *Developing a learning environment on Realistic Mathematics Education for Indonesian student teacher*. PrintPartners Ipskamp: Enschede. (available online http://doc.utwente.nl/58718/1/thesis_Zulkardi.pdf)

APPENDIX 1

THE CLASSROOM OBSERVATION SCHEME

1. **The practical situation in the classroom:**
   - How is the class organized? How do the students sit in the classroom?
   - How is the atmosphere of the class? Are the students enthusiastic during the lesson?
   - How is the interaction between the teacher and the students?
2. **The teaching and learning process:**
   - How does the teacher teach the class? Whether she proposes problem or explain the theory?
   - Where is the position of the teacher? Always in front of the class or moving around the classroom?
   - What is the textbook used by the teacher?
   - What kind of question that the teacher gives to the students? Can it lead the discussion?
   - How does the teacher manage the discussion? How much the discussion is set up?
   - Does the teacher give the students time to think before react to the question?
   - Is there any effort from the teacher to generate different solution?
   - What about the time management of the lesson?
3. **The social norms in the classroom:**
   - How does the students response during the discussion?
   - How does the teacher manage with the passive or dominant students?
   - How is the rule in the class? Whether the students are allowed to stop the teacher talking when she explains and give their opinion? Or they may raise their finger or something else?
4. **The students:**
   - What students do when the teacher explains?
   - Whether the students are passive or active?
   - Is it the same student who always involves the discussion?
   - How do they response when they work in group? How the students react among others? Do they work together or solve the problem by their selves?
   - Is there any student who is bossy in the group work? Or otherwise, the passive students in the group work?
5. **PMRI:**
   - How far PMRI can be observed in the classroom?

APPENDIX 2

THE TEACHER INTERVIEW SCHEME

1. **Teacher's experience:**
   - How long have you been teaching?
   - In which grade(s) you have experienced in teaching mathematics?
2. **The students:**
   - How many students in your classroom? (please mention the number of girls and boys)
   - Who are the higher achiever and lower achievers?
   - In your opinion, are they typically active or passive students?
   - Who are the active students? Do they tend to be the boss in the group work?
   - Who are the passive students?
   - Who always do make a noise in the classroom?
   - Are they accustomed to think critically and giving their opinion?
   - Do they have problem in reading and understanding the text?
   - Are they accustomed to work with worksheet?
   - Are they accustomed to work in group?
   - How do they response when they work in group? How do the students react among others? Do they work together or solve the problem by their selves?
3. **The teaching and learning process:**
   - What method do you frequently use in teaching mathematics?
   - Do the students always have discussion in the classroom?
   - How do you manage the discussion? What will you do to make all the students participate into the discussion?
   - Can you tell me how the discussion goes in the classroom? Do you give the students time to think before they react to your question?
   - How is the response of the students if they have discussion?
   - Are the students accustomed in telling their opinion or idea?
   - What is the textbook or the worksheet which you use to teach in the classroom?
   - Do you arrange group work during your class?
   - How many students that you always grouped in one group?
   - How do you group them? What is your consideration to do it?

4. **The social norm in the classroom:**
   - Is there any rule in the class? What students always do if they want to say something or ask something? Do they come to your desk or raise their hand and then wait until you come?
   - If the students make a noise, how do you always do to get their attention?
5. **PMRI:**
   - Do you have heard about PMRI?
   - What do you know about PMRI? Tell me about PMRI that you know.
   - What is your opinion about PMRI?
   - Do you think it is possible to implement the PMRI approach in your class?

APPENDIX 3

PRETEST AND POSTTEST

Pretest Problem

Nama	:
Kelas/ No absen	:
Tanggal	:

1. Diketahui daftar harga barang-barang di koperasi sekolah sebagai berikut.

Nama Barang	Harga
Bolpoin	Rp1.200
Buku gambar	Rp1.800
Buku tulis	Rp1.500
Penghapus	Rp 800
Pensil	Rp 1.000
Rautan	Rp 900

Andi membeli 2 buku tulis, 1 pensil dan 1 penghapus. Jika Andi membayar Rp10.000, berapakah kembalian yang diterima oleh Andi?

Jawab:

2. 

2 kotak 24 donat

Berapa banyak donat yang terdapat dalam 4 kotak? Jelaskan bagaimana caramu memperoleh jawabannya!

Jawab:

3. Manakah yang lebih murah, harga apel di minimarket A atau minimarket B? Jelaskan alasanmu!

Minimarket A

Minimarket B

Jawab:

Posttest Problems

Nama :

Kelas :

Tanggal:

1. Untuk membuat 4 kue cokelat dibutuhkan 30 gram cokelat bubuk. Berapa banyak cokelat bubuk yang dibutuhkan untuk membuat 12 kue cokelat? Jelaskan jawabanmu!

Jawab:

2. Untuk membuat 12 piring nasi goreng dibutuhkan 3 kg beras. Berapa banyak nasi goreng yang dapat dibuat dari 5 kg beras? Jelaskan jawabanmu!

Jawab:

3.Hadi membeli 24 balon untuk dekorasi pesta ulang tahunnya dengan harga Rp36.000. Ternyata balon yang dibutuhkan masih kurang, sehingga ia membeli 4 balon lagi. Berapa rupiah yang harus dibayar Hadi untuk 4 balon itu? Jelaskan caramu memperolehnya!

Jawab:

4. Perhatikan gambar mobil di bawah ini! Tentukan mobil manakah yang paling irit menggunakan bensin! Jelaskan alasanmu!

Mobil A	Mobil B
	
6 liter bensin untuk 80 km	4 liter bensin untuk 60 km

Jawab:

APPENDIX 4

TEACHER GUIDE

Panduan Guru

Pendahuluan

Ketika siswa belajar tentang perbandingan, mereka seharusnya tidak hanya belajar tentang prosedur penyelesaian soal saja melainkan juga harus memahami alasan matematis yang mendasari prosedur formal tersebut. Kemampuan bernalar dalam masalah perbandingan (*proportional reasoning*) merupakan kemampuan yang penting untuk membangun pondasi matematika tingkatan selanjutnya dan kemampuan bernalar dalam aljabar (Langrall & Swafford, 2000) dan *ratio table* merupakan model yang ideal untuk mendukung pengembangan *proportional reasoning* siswa. Tabel ini merupakan sarana yang bagus untuk melalukan penghitungan serta membantu siswa memahami inti materi. Selain itu table ini juga dapat mengembangkan cara berpikir siswa tentang perbandingan karena dengan menggunakan tabel ini siswa dapat menuliskan langkah-langkah penghitungan mereka. Sebagai contoh, untuk menemukan berapa kilometer yang dapat ditempuh dalam waktu 80 menit jika kita dapat menempuh jarak 15 km dalam waktu 60 menit, siswa dapat menggunakan 5 km dalam 20 menit atau 30 km dalam 120 menit dan 10 km dalam 40 menit untuk mendapatkan selesaian 20 km dalam waktu 80 menit. Tabel perbandingan ini menunjukkan perbandingan secara jelas karena kita dapat menambah banyak baris atau kolom sesuai dengan kebutuhan kita. Kelebihan dari tabel perbandingan ini adalah bahwa siswa dapat menggunakan hubungan bilangan-bilangan sebelumnya untuk memberi alasan bagaimana cara mereka mendapatkan solusinya.

PERTEMUAN 1

AKTIVITAS 1: MEMBUAT MODEL KUPU-KUPU

Alokasi Waktu

2×35 menit

Bahan

- Power point
- Lembar Kerja Siswa (LKS 1.1 dan 1.2)

Tujuan Pembelajaran

- Siswa memahami adanya rasio dalam masalah perbandingan (searah).

Deskripsi Aktivitas Pembelajaran

- Guru membuat kelompok yang terdiri atas 2-3 orang siswa dan meminta mereka untuk selalu dalam kelompok tersebut setiap kali diminta untuk kerja kelompok.

- Guru membagikan LKS 1.1 kepada masing-masing kelompok. Guru meminta masing-masing kelompok untuk menuliskan nama anggota kelompok mereka pada LKS yang diberikan.
- Guru memberikan waktu kepada siswa untuk membaca dan memikirkan soal tersebut selama 2 menit sebelum mereka berdiskusi dan bekerjasama dalam kelompok. Total waktu yang diberikan untuk menyelesaikan tugas tersebut adalah 8-13 menit.
- Guru memimpin diskusi di kelas.
- Guru memberikan LKS 1.2 sebagai latihan individu bagi para siswa.
- Guru mencocokkan jawaban dan Guru dapat membahas beberapa nomor jika dirasa banyak siswa yang mengalami kesulitan.

Peranan Guru

Selama siswa bekerja:

- Setelah 2 menit pertama, guru menanyakan kepada siswa apakah mereka memahami soal yang telah mereka baca. Guru dapat menanyakan pertanyaan berikut.

  "Apakah kalian memahami apa yang dimaksud oleh soal"
  "Dapatkah kamu mengulang pertanyaan itu dengan bahasamu sendiri?"
  "Pertanyaan ini tentang apa?"

  Memastikan pemahaman siswa terhadap soal yang diberikan penting untuk dilakukan karena terkadang hambatan siswa dalam menyelesaikan suatu soal bukan disebabkan siswa tidak bisa memecahkan persoalan it namun justru dikarenakan kurangnya pemahaman siswa terhadap soal yang diberikan.
- Ketika para siswa sedang bekerja, guru berkeliling di dalam kelas untuk melihat bagaimana proses diskusi yang terjadi di dalam kelompok. Guru juga harus melihat jawaban siswa dan mulai menentukan siswa mana yang akan diminta untuk mempresentasikan jawaban mereka. Beberapa pertimbangan untuk memilih jawaban siswa.
  - Perkalian
  - Penjumlahan berulang
  - Menjumlahkan atau mengurangkan bilangan-bilangan sebelumnya untuk menentukan solusinya.

Selama diskusi kelas:

- Guru meminta siswa yang sudah beliau pilih selama siswa bekerja untuk menuliskan jawaban mereka dan menjelaskan bagaimana mereka memperolehnya. Guru menanyakan pada siswa yang lain apakah setuju dengan jawaban yang dituliskan oleh teman mereka dan menanyakan apakah ada yang menggunakan cara lain untuk mendapatkan jawaban tersebut.
- Guru membuat tabel di papan tulis untuk mendata jawaban siswa dalam bentuk yang lebih sistematis.

Kelas	Banyak Kupu-kupu	Sayap	Badan	Antena
4A	27	108	27	54

4B	26	104	26	52
4C	25	100	25	50
4D	24	96	24	48

- Guru mendiskusikan mengapa banyak siswa sama dengan banyak kupu-kupu. Siswa harus memahami bahwa hal ini terjadi karena 1 siswa membuat 1 kupu-kupu, sehingga perbandingannya adalah 1:1.
- Jika siswa hanya mempunyai 1 strategi saja, yang mana paling banyak muncul adalah menggunakan perkalian maka guru hendaknya membimbing siswa dengan mengajukan beberapa pertanyaan tambahan dengan fokus pada banyaknya kupu-kupu dan banyaknya sayap. Hal yang penting untuk diperhatikan dalam setiap diskusi adalah menerapkan peraturan untuk menjawab pertanyaan dari guru. Siswa harus berpikir terlebih dahulu dan bagi mereka yang sudah mengetahui jawabannya hendaknya meletakkan ibu jari pada mulut. Guru yang akan menentukan siapa yang akan menjawab. Hal ini bertujuan agar yang berpartisipasi dalam diskusi nantinya tidak hanya para siswa yang aktif saja, sekaligus untuk memberi waktu berpikir pada siswa yang mungkin membutuhkan waktu berpikir lebih lama. Beberapa pertanyaan yang dapat diajukan (dapat dilihat dalam power point yang terlampir dan dapat dikembangkan sendiri dengan menggunakan bilangan lainnya).

  "Bagaimana dengan 23 kupu-kupu? Berapa banyak sayap yang kita butuhkan?"

  "Bagaimana dengan 28 kupu-kupu? Berapa banyak sayap yang kita butuhkan?"

  Kita harapkan bahwa beberapa siswa akan menyadari pola sayap yang ada yaitu dengan menambahkan 4 atau mengurangi dengan 4 untuk setiap penambahan atau pengurangan 1 kupu-kupu. Jika tidak, guru dapat bertanya dan menekankan pola 108-104-100-96.
- Jika siswa sudah menyadari adanya pola tersebut, maka guru dapat meminta siswa untuk menjawab pertanyaan selanjutnya untuk memastikan apakah mereka juga dapat menerapkan pola tersebut untuk menjawab soal berikutnya. Untuk soal-soal yang diajukan hendaknya bukan bilangan yang besar.

  "Berapa banyak sayap yang kita butuhkan untuk membuat 30 kupu-kupu? 32 kupu-kupu?"

  Jika mereka memahami konsep rasio yanga ada dalam soal tersebut maka mereka dapat menarik kesimpulan sebagai berikut.

  "Jika kita menambah 1 kupu-kupu, kita menambah 4 sayap tetapi jika kita mengurangi 1 kupu-kupu maka kita harus mengurangi banyak sayap dengan 4"

  "Jika kita menambah 1 kupu-kupu, maka kita menambah 4 sayap maka untuk menambah 2 kupu-kupu kita harus menambahkan 8 sayap"
- Pertanyaan sebelumnya mengenai perkalian dalam masalah perbandingan, sekarang Guru akan memberikan pertanyaan yang berkaitan dengan pembagian dalam masalah perbandingan (dapat dilihat dalam power point dan dapat dikembangkan dengan menggunakan kombinasi bilangan lainnya).

  "Sekarang, jika saya memiliki 90 sayap, 23 badan, dan 45 antena, berapa banyak kupu-kupu lengkap yang dapat saya buat?"

Guru meminta siswa untuk mendiskusikan dulu hal ini dengan kelompok mereka sebelum menjawab pertanyaan ini. Siswa dapat membuat coretan pada balik LKS kelompok masing-masing.

Dari soal yang memiliki banyak tidak sesuai dengan rasio sayap: badan: antenna 4:1:2, kita bisa melihat mengarahkan siswa untuk memperhatikan rasio dalam 1 model kupu-kupu.

- Di akhir pembelajaran, Guru hendaknya memberi penekanan tentang perbandingan antara banyak kupu-kupu-sayap-badan-antena selalu sama.

PERTEMUAN 2

AKTIVITAS 2: COKELATKU

Alokasi Waktu

1×35 minutes

Bahan

- Power point
- Lembar Kerja Siswa (LKS 2)

Tujuan Pembelajaran

- Untuk memperkenalkan *ratio table* (tabel perbandingan) pada siswa.
  - Menekankan kembali bahwa *ratio table* merupakan daftar yang tersusun secara sistematis.
  - Siswa mengeksplorasi strategi yang dapat dilakukan dengan menggunakan *ratio table.*

Deskripsi Aktivitas Pembelajaran

- Guru meminta siswa untuk duduk dalam kelompok masing-masing seperti yang sudah dibagi pada pertemuan pertama.
- Guru menempelkan gambar cokelat dan harganya pada papan tulis dan menanyakan pertanyaan sebagai berikut pada siswa atau Guru dapat memperlihatkan gambar cokelat dan harganya pada slide power point yang tersedia.

Cokelat	Rupiah
4	3000
8	?
16	?

- Guru berdiskusi tentang jawaban siswa dan strategi yang mereka gunakan. Pada saat ini, Guru dapat melihat apakah siswa menggunakan strategi manipulasi atau tidak, seperti dobel untuk mendapatkan harga dari 8 cokelat atau mengalikan dengan 3 untuk mendapatkan harga dari 12 cokelat. Diskusi ini dilakukan selama 10-15 menit. Pada saat ini perlu bagi guru untuk menuliskan operasi bilangan yang dilakukan pada "*ratio table*".
- Guru membagikan LKS pada tiap kelompok. Masing-masing kelompok mendapat 1 LKS.

- Guru meminta masing-masing kelompok untuk menuliskan nama anggota kelompok mereka pada LKS yang disediakan. Siswa dapat menuliskan proses penghitungan pada lembar LKS ini. Halaman kosong di baliknya dapat digunakan untuk coretan dalam menjawab soal-soal lanjutan selama proses diskusi di kelas.
- Siswa bekerja selama 10 menit.
- Guru memimpin diskusi tentang banyak cokelat dan harga yang diusulkan oleh para siswa.

Peranan Guru

Selama diskusi kelas (bagian pertama):

- Guru menanyakan pertanyaan yang ada dalam slide power point atau dapat mengajukan pertanyaan dengan bilangan lainnya.

  "Berapa harga 8 cokelat?"

  Berikan waktu kepada siswa untuk berpikir, dan ingatkan kembali para siswa untuk memberi tanda dengan meletakkan ibu jari di depan bibir jika mereka sudah menemukan jawabannya. Guru yang akan menentukan siswa mana yang akan menjawab. Kemudian Guru mendiskusikan bagaimana cara mereka menemukannya. Yang perlu diperhatikan selama diskusi adalah pemahaman atau cara berpikir mereka ketika menyelesaikan soal tersebut.
- Guru mengajukan pertanyaan berikutnya.

  "Berapa harga 16 cokelat?"

  Berikan waktu kepada siswa untuk berpikir, dan ingatkan kembali para siswa untuk memberi tanda dengan meletakkan ibu jari di depan bibir jika mereka sudah menemukan jawabannya. Guru yang akan menentukan siswa mana yang akan menjawab. Kemudian Guru mendiskusikan bagaimana cara mereka menemukannya. Yang perlu diperhatikan selama diskusi adalah pemahaman atau cara berpikir mereka ketika menyelesaikan soal tersebut.

Selama siswa bekerja:

- Guru harus memberikan keberanian kepada siswa untuk memilih variasi bilangan dan berani mengeksplorasi strategi yang mereka gunakan agar tidak terbatas pada cara dobel saja atau penjumlahan berulang saja. Guru juga meminta para siswa untuk menuliskan langkah penghitungan mereka, misalnya dikalikan 3 pada masing-masing kolom atau dijumlahkan, dan sebagainya.
- Ketika siswa bekerja, guru dapat menggunakan kesempatan ini untuk berkeliling kelas, untuk melihat bagaimana proses diskusi yang terjadi dalam masing-masing kelompok. Sekaligus Guru melihat jawaban siswa dan menentukan siswa mana yang akan diminta untuk memperentasikan jawaban mereka di depan. Selama proses diskusi dalam kelompok berlangsung hendaknya Guru menekankan proses kerjasama yang baik, bahwa dalam 1 kelompok harus sama-sama mengerti. Guru dapat mengatakan bahwa nanti yang diminta mewakili belum tentu mereka yang mengerjakan, bisa saja teman yang lain dalam kelompok itu. Untuk itu diperlukan diskusi yang baik selama bekerja dalam kelompok.
- Beberapa hal yang dapat digunakan sebagai bahan pertimbangan dalam memilih siswa mana yang akan maju ke depan adalah Guru dapat memilih

siswa yang memilih bilangan yang cukup sulit atau menggunakan strategi yang belum pernah digunakan sebelumnya, seperti kombinasi antara 2 strategi manipulasi atau siswa yang muncul dengan ide "aturan 1".

Selama diskusi kelas (bagian kedua):

- Guru meminta 2 atau 3 siswa untuk membuat tabel mereka di papan tulis dan meminta mereka untuk menjelaskan langkah-langkah yang mereka lakukan. Guru memilih siswa yang ada dalam pikirannya selama ia berkeliling kelas sebelumnya.
- Jika selama diskusi, kurang munculnya strategi-strategi baru maka Guru dapat menanyakan pertanyaan tambahan yang dapat digunakan untuk mengarahkan siswa pada suatu strategi tertentu.

  Sebagai contoh, dengan diketahui harga 4 cokelat Rp 3.000,00 dan 8 cokelat Rp 6.000,00, Guru dapat menanyakan *"Berapa cokelat yang kita dapat untuk Rp9.000,00?"*. Pertanyaan ini dapat digunakan untuk memunculkan strategi penjumlahan 4 cokelat dan 8 cokelat maupun perkalian dengan 3 dari 4 cokelat.
- Guru dapat pula menanyakan untuk bilangan yang lebih besar dan meminta mereka untuk berdiskusi dulu di dalam kelompok masing-masing. Terkadang, dengan memberikan soal yang sulit, kita dapat menstimulasi siswa untuk menggunakan manipulasi strategi.

  "Berapa banyak cokelat yang kita dapatkan untuk Rp.108.000?"
- Di akhir diskusi, Guru memberikan kesimpulan tentang macam-macam strategi dengan *ratio table* yang sudah mereka pelajari pada hari itu dan bagaimana ini dapat membantu mereka dalam penghitungan bilangan yang lebih besar.

Pembelajaran dilanjutkan dengan Aktivitas 3.

AKTIVITAS 3: MEMBERI MAKAN ULAT

Alokasi Waktu

1×35 minutes

Bahan

- Lembar Kerja Siswa (LKS 3)

Tujuan Pembelajaran

- Siswa mengeksplorasi strategi yang dapat dilakukan dengan menggunakan *ratio table*
- Siswa menemukan aturan 1

Deskripsi Aktivitas Pembelajaran

- Guru meminta siswa berkelompok sesuai dengan kelompok pada pertemuan pertama.
- Guru membagikan LKS kepada setiap kelompok. Masing-masing kelompok mendapat 1 LKS. Guru meminta mereka untuk membaca dan memikirkannya terlebih dahulu selama 2 menit sebelum mereka berdiskusi dan bekerjasama

untuk 8 menit berikutnya. Hal ini perlu dilakukan dengan tujuan membiasakan siswa lebih teliti dalam membaca soal. Terkadang siswa cenderung membaca soal matematika setengah-setengah dan hanya fokus pada angka tanpa pemahaman mendalam pada soal.

- Guru meminta siswa untuk menuliskan nama mereka pada LKS.
- Setelah waktu yang ditentukan (8 menit waktu pengerjaan), Guru memimpim diskusi di dalam kelas.

Peranan Guru

Selama siswa bekerja:

- Setelah 2 menit pertama, Guru memastikan apakah siswa menginterpretasikan soal dengan benar. Guru dapat menanyakan beberapa pertanyaan berikut.

  "Apakah kalian memahami apa yang dimaksud oleh soal"
  "Dapatkah kamu mengulang pertanyaan itu dengan bahasamu sendiri?"
  "Pertanyaan ini tentang apa?"

  Memastikan pemahaman siswa terhadap soal yang diberikan penting untuk dilakukan karena terkadang hambatan siswa dalam menyelesaikan suatu soal bukan disebabkan siswa tidak bisa memecahkan persoalan it namun justru dikarenakan kurangnya pemahaman siswa terhadap soal yang diberikan.
- Ketika siswa sedang bekerja, Guru dapat berkeliling kelas untuk melihat bagaimana diskusi yang terjadi dalam kelompok. Sekaligus Guru melihat jawaban siswa dan menentukan siswa mana yang akan diminta untuk memperentasikan jawaban mereka di depan. Selama proses diskusi dalam kelompok berlangsung hendaknya Guru menekankan proses kerjasama yang baik, bahwa dalam 1 kelompok harus sama-sama mengerti. Guru dapat mengatakan bahwa nanti yang diminta mewakili belum tentu mereka yang mengerjakan, bisa saja teman yang lain dalam kelompok itu. Untuk itu diperlukan diskusi yang baik selama bekerja dalam kelompok. Beberapa pertimbangan yang dapat digunakan untuk menetukan siswa yang akan ditunjuk sebagai berikut.
  - Strategi menggunakan perkalian dan pembagian.
  - Strategi menggunakan penjumlahan berulang.
  - Pengelompokan oleh 2.
  - Menggunakan manipulasi strategi dengan *ratio table*.

Selama berdiskusi di dalam kelas:

- Guru meminta perwakilan dari 2-3 kelompok untuk menuliskan jawaban mereka di papan tulis dan juga menjelaskan cara mereka mendapatkan jawaban tersebut. Meminta siswa untuk tampil ke depan dan menjelaskan kepada teman-temannya yang lain perlu dilakukan untuk melatih keberanian siswa untuk mengungkapkan pemikirannya sekaligus melatih kemampuannya dalam berkomunikasi.
- Ada kemungkinan bahwa siswa menggunkan strategi yang berbeda-beda maupun mungkin jawaban yang juga berbeda-beda.
- Jika ada siswa yang menggunakan ide *ratio table*, Guru dapat menggunakan kesempatan ini untuk mendiskusikan kelebihan dari *ratio table* dibanding strategi lain yang digunakan oleh siswa lainnya.

- Jika tidak, Guru dapat memulai diskusi dengan membuat tabel berikut untuk mengecek apakah jawaban yang diberikan oleh siswa benar atau tidak. Untuk masalah ini, strategi yang dapat digunakan adalah dobel atau mengalikan dengan bilangan yang sama.

Hari	Daun
2	5
4	
8	
16	
32	
64	

- Ide tentang bagaimana membuat daftar dengan cara yang sistematis ini haruslah ditekankan kembali.
- Diskusi dapat dilanjutkan untuk mengarahkan siswa memikirkan strategi-strategi yang lain yang sekiranya mungkin dapat dilakukan untuk menyelesaikan beberapa pertanyaan lanjutan yang ditanyakan oleh Guru, misalnya setengah, menjumlahkan, mengurangi atau mengalikan dan membagi dengan bilangan yang sama. Beberapa pertanyaan yang dapat diajukan oleh Guru sebagai berikut.

  "Berapa banyak daun yang dibutuhkan Aldi selama 6 hari? 10 hari? 24 hari?"

  Banyaknya daun yang dibutuhkan selama 6 hari dapat ditemukan dengan menambah banyak daun yang dibutuhkan selama 2 hari dan 4 hari atau mengurangi kebutuhan daun selama 8 hari dengan banyaknya daun yang dibutuhkan selama 2 hari.

  Banyaknya daun selama 10 hari dapat ditentukan dengan menambah banyaknya daun yang dibutuhkan selama 8 hari dan 2 hari atau dengan mencari separuh dari kebutuhan daun selam 16 hari atau mengalikan kebutuhan daun selama 2 hari dengan 5.

  "Persediaan 70 daun cukup untuk memberi makan ulat selama berapa lama?"

Untuk masing-masing pertanyaan, Guru hendaknya memberikan waktu kepada siswa untuk berpikir dan memberi kesempatan pada mereka untuk berdiskusi dalam kelompok mereka. Para siswa dapat membuat coretan buram pada balik LKS yang masih kosong. Guru harus memeberi dukungan kepada siswa untuk mau mengungkapkan strategi yang berbeda.

PERTEMUAN 3

AKTIVITAS 4: BAGAIMANA MENURUTMU?

Alokasi Waktu

2×35 menit

Bahan

- Lembar Kerja Siswa (LKS 4.1 dan LKS 4.2)

Tujuan Pembelajaran

- Siswa mengetahui dan memahami aturan yang tidak dapat diaplikasikan pada *ratio table* dan mengapa demikian.

Deskripsi Aktivitas Pembelajaran

- Guru membagikan LKS 4.1 kepada para siswa dan meminta mereka untuk membaca dan berpikir selama 2 menit sebelum mereka berdiskusi di dalam kelompok selama 8 menit. Masing-masing siswa mendapat 1 LKS.
- Guru meminta para siswa untuk menuliskan nama mereka masing-masing pada LKS.
- Guru memimpin diskusi dalam kelas.
- Siswa mengerjakan LKS 4.2 secara berkelompok sebagai latihan
- Guru mendiskusikannya di akhir pembelajaran. Penekanan dalam diskusi di kelas untuk LKS 4.2 ini adalah mengenai beragam konteks yang menerapkan perbandingan.

Peranan Guru

Selama siswa bekerja:

- Setelah 2 menit pertama, Guru memastikan apakah siswa menginterpretasikan soal dengan benar. Guru dapat menanyakan beberapa pertanyaan berikut.

  "Apakah kalian memahami apa yang dimaksud oleh soal"

  "Dapatkah kamu mengulang pertanyaan itu dengan bahasamu sendiri?"

  "Pertanyaan ini tentang apa?"

  Memastikan pemahaman siswa terhadap soal yang diberikan penting untuk dilakukan karena terkadang hambatan siswa dalam menyelesaikan suatu soal bukan disebabkan siswa tidak bisa memecahkan persoalan it namun justru dikarenakan kurangnya pemahaman siswa terhadap soal yang diberikan.
- Ketika siswa sedang bekerja, Guru dapat berkeliling kelas untuk melihat bagaimana diskusi yang terjadi dalam kelompok. Sekaligus Guru melihat jawaban siswa dan menentukan siswa mana yang akan diminta untuk memperentasikan jawaban mereka di depan. Selama proses diskusi dalam kelompok berlangsung hendaknya Guru menekankan proses kerjasama yang baik, bahwa dalam 1 kelompok harus sama-sama mengerti. Guru dapat mengatakan bahwa nanti yang diminta mewakili belum tentu mereka yang mengerjakan, bisa saja teman yang lain dalam kelompok itu. Untuk itu diperlukan diskusi yang baik selama bekerja dalam kelompok. Beberapa pertimbangan yang dapat digunakan untuk menetukan siswa yang akan ditunjuk sebagai berikut.
  - Siswa yang menggunakan strategi manipulasi, seperti dobel, setengah, penjumlahan, pengurangan atau mengalikan dan membagi dengan bilangan yang sama.
  - Siswa yang menggunakan strategi yang salah, misalnya menambah atau mengurangi dengan bilangan yang sama.

Selama diskusi di kelas:

- Guru mendiskusikan tentang penerapan konsep perbandingan dalam masalah yang disediakan serta membahas strategi yang digunakan oleh siswa. Guru meminta siswa yang menggunakan strategi yang salah untuk mempresentasikan pekerjaan mereka dan meminta siswa yang lain untuk memberikan tanggapan terhadap jawaban mereka. Guru harus menekankan pada siswa bahwa siswa yang lain harus mencoba untuk memahami strategi yang dipresentasikan oleh teman mereka terlebih dahulu sebelum mereka dapat memberikan tanggapan tentang strategi tersebut.
- Jika tidak ada siswa yang menggunakan strategi yang salah maka Guru dapat mengawali diskusi dengan mengajukan ide yang salah tersebut. Guru dapat mengatakan bahwa di ada siswa lain yang menggunakan cara tersebut.

  "Ada siswa lain yang memiliki ide sebagai berikut. Bagaimana menurut kalian?"

  Hendaknya Guru menekankan pula alasan mengapa cara seperti yang di atas tidak boleh dilakukan, misalnya menambahkan 1 pada nyawa dan 1 pada banyaknya bola emas.Ini bukan karena Guru mengajarkan demikian atau karena seperti itulah cara kerja *ratio table* namun karena konteks yang mengatakan pada kita bahwa untuk setiap 5 bola emas kamu mendapat 2 nyawa. Jadi, siswa tidak hanya akan belajar tentang aturan (perkalian dengan bilangan yang sama diperbolehkan tetapi penjumlahan dengan bilangan yang sama tidak diperbolehkan), tetapi mereka juga belajar untuk selalu mengingat tentang konteks tersebut disaat mereka memiliki keraguan dalam mengerjakan soal perbandingan.

PERTEMUAN 4

AKTIVITAS 5: HARGA TERBAIK

Alokasi Waktu

2×35 menit

Bahan

- Lembar Kerja Siswa (LKS 5).

Tujuan Pembelajaran

- Siswa menggunakan *ratio table* untuk menyelesaikan masalah perbandingan.
- Siswa menggunakan *ratio table* sebagai alat untuk berpikir, memberi alasan, dan alat hitung.

Deskripsi Aktivitas Pembelajaran

- Guru meminta para siswa untuk duduk di dalam kelompok mereka masing-masing seperti pada pertemuan sebelumnya.
- Guru membagikan LKS 5 kepada setiap kelompok.
- Guru meminta siswa untuk menuliskan nama mereka masing-masing pada LKS.

- Guru meminta siswa untuk membaca dan memahami sejenak soal pertama yang ada di dalam LKS secara seksama selama 2 menit sebelum mereka diskusi dengan teman sekelompoknya.
- Diskusi di dalam kelompok berlangsung selam 8 menit.
- Guru memimpin diskusi soal pertama di kelas.
- Guru meminta siswa untuk membaca dan memahami sejenak soal kedua yang ada di dalam LKS secara seksama selama 2 menit sebelum mereka diskusi dengan teman sekelompoknya.
- Guru memimpin diskusi tentang soal kedua di dalam kelas.

Peranan Guru

Selama siswa bekerja:

- Setelah 2 menit pertama, Guru memastikan apakah siswa menginterpretasikan soal dengan benar. Guru dapat menanyakan beberapa pertanyaan berikut.

  "Apakah kalian memahami apa yang dimaksud oleh soal"

  "Dapatkah kamu mengulang pertanyaan itu dengan bahasamu sendiri?"

  "Pertanyaan ini tentang apa?"

  Memastikan pemahaman siswa terhadap soal yang diberikan penting untuk dilakukan karena terkadang hambatan siswa dalam menyelesaikan suatu soal bukan disebabkan siswa tidak bisa memecahkan persoalan it namun justru dikarenakan kurangnya pemahaman siswa terhadap soal yang diberikan.

Selama diskusi di kelas:

- Soal yang diberikan adalah tentang membandingkan sesuatu hal, misalnya harga yang lebih murah. Selama diskusi terjadi, Guru harus mengutamakan proses berpikir siswa, bagaimana cara mereka untuk membandingkan 2 hal yang berbeda. Hal yang perlu didiskusikan adalah mengapa kita harus menggunakan perbandingan dan bukan nilai mutlak untuk membandingkan 2 hal.

  Prediksi jawaban siswa untuk soal pertama (membandingkan harga soda):

  - Siswa mungkin akan menjawab bahwa soda A lebih mahal daripada soda B karena 4.000 lebih besar dari 3.000.
  - Siswa menyadari bahwa banyak kaleng juga mempengaruhi harga.
- Guru mendiskusikan bagaimana cara untuk menentukan harga mana yang lebih murah.

  Prediksi cara yang digunakan oleh siswa:

  - Siswa mencari banyak kaleng yang didapatkan dengan harga yang sama, misalnya Rp12.000.
  - Siswa mencari harga masing-masing soda untuk banyak kaleng yang sama, misalnya harga untuk 6 kaleng.
  - Dengan menggunakan satuan, mencari harga per kaleng soda. Tetapi untuk harga per kaleng ini mungkin cukup sulit dilakukan oleh siswa karena mereka akan kesulitan membagi 4.000 dengan 3.
- Guru melakukan hal yang sama untuk soal kedua, dimana mengandung satuan berat.

APPENDIX 5

Rencana Pelaksanaan Pembelajaran (RPP)

Sekolah : SDN 179 Palembang

Mata Pelajaran : Matematika

Kelas/Semester : IV/Genap

Pertemuan : 1 (Pertama)

Alokasi Waktu : 2 x 35 menit

Standar Kompetensi : 5. Menggunakan pecahan untuk menyelesaikan masalah

Kompetensi Dasar : 5.5 Menyelesaikan masalah yang melibatkan pecahan (pengayaan)

A. Tujuan Pembelajaran:

- Siswa memahami adanya rasio dalam masalah perbandingan searah.

B. Indikator :

- Siswa dapat menentukan banyak sayap, badan dan antena pada kupu-kupu sesuai dengan rasio yang ada
- Siswa dapat menentukan berapa banyak kupu yang dapat dibuat dari sejumlah sayap, badan, dan antena

C. Materi Pembelajaran

Perbandingan merupakan topik yang penting dalam Matematika di sekolah dasar. Kemampuan bernalar dalam perbandingan dibutuhkan untuk mempelajari banyak materi lain, seperti dalam geometri dan aljabar.

Rasio merupakan dasar dari perbandingan karena berdasarkan pengertiannya, perbandingan merupakan persamaan dua rasio. Untuk itu sebagai titik awal pembelajaran, siswa akan diberikan materi tentang rasio.

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	- Berdoa - Guru mengkondisikan kelas pada situasi belajar *Apersepsi* - Siswa diminta berperan sebagai kupu-kupu -Guru memotivasi dan mengajak siswa untuk	10 menit

	berpartisipasi dalam pembelajaran -Guru menyampaikan kegiatan yang akan dilakukan oleh siswa. - Guru membagi siswa dalam kelompok. Tiap-tiap kelompok terdiri dari 2-3 siswa.	
Kegiatan Inti	Eksplorasi - Siswa diminta mengerjakan LKS 1.1 - Dikusi dalam kelompok Elaborasi - Diskusi bersama dalam kelas tentang beberapa strategi berbeda yang dapat digunakan untuk menyelesaikan masalah - Guru menanyakan beberapa pertanyaan tambahan yang ada di power point	35 menit
Kegiatan akhir	Konfirmasi - Guru memberikan kata pujian bagi siswa yang berpartisipasi dalam diskusi - Dengan bimbingan Guru peserta didik merefleksikan kegiatan pembelajaran yang dilakukan dan menyimpulkan hasil pembelajaran tentang: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? - Guru memotivasi siswa yang belum aktif - Siswa diminta mengerjakan LKS 2 sebagai latihan individu	25 menit

F. Media Pembelajaran

- LKS 1.1 dan LKS 1.2 (terlampir)
- Power point (terlampir)

G.Penilaian:

Intrumen : LKS 1.2 (terlampir)

Bentuk soal : uraian

Rubrik penilaian :

No	Kunci Jawaban	Skor
1.	Banyak sayap=16 Banyak badan=6 Banyak antenna=12	Jawaban benar semua =10 Salah 1 = 8 Salah 2 = 6 Salah semua = 4

		Tidak ada jawaban = 0
2.	12 kupu-kupu	Jawaban benar =10 Jawaban salah = 4 Tidak menjawab =0
3.	8 kupu-kupu	Jawaban benar = 10 Jawaban salah = 4 Tidak menjawab = 0
4.	a. 44 b.13 c.13 d.26 e.96	Jawaban benar semua=10 Salah 1= 8 Salah 2= 6 Salah 3= 4 Salah 4= 2 Salah semua=1 Tidak menjawab=0

Nilai = (skor :4) × 10

Palembang, 14 Maret 2013

Guru kelas IV D

Nurmasintan, S.Pd
NIP.196701201989072001

Peneliti

Sylvana Novilia Sumarto, S.Si
NIM.20112812012

Mengetahui
Kepala SDN 179 Palembang

Maulina, S.Pd, M.M
NIP. 196507101989072003

Rencana Pelaksanaan Pembelajaran
(RPP)

Sekolah : SDN 179 Palembang

Mata Pelajaran : Matematika

Kelas/Semester : IV/Genap

Pertemuan : 2 (Kedua)

Alokasi Waktu : 2 x 35 menit

Standar Kompetensi : 5. Menggunakan pecahan untuk menyelesaikan masalah

Kompetensi Dasar : 5.5 Menyelesaikan masalah yang melibatkan pecahan (pengayaan)

A. Tujuan Pembelajaran:

- Untuk memperkenalkan *ratio table* (tabel perbandingan) pada siswa.
  - Menekankan kembali bahwa *ratio table* merupakan daftar yang tersusun secara sistematis.
  - Siswa mengeksplorasi strategi yang dapat dilakukan dengan menggunakan *ratio table.*

B. Indikator :

- Siswa dapat membuat daftar harga dengan rasio yang benar
- Siswa dapat menyelesaikan masalah *missing value*

C. Materi Pembelajaran

Mengeksplorasi tabel perbandingan untuk menemukan beberapa strategi penting dilakukan sebelum siswa mencoba menggunakan tabel untuk menyelesaikan masalah perbandingan (masalah *missing value*).

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	- Berdoa - Guru mengkondisikan kelas pada situasi belajar *Apersepsi* - Siswa ditanya tentang berapa uang yang harus dibayar untuk membeli barang tertentu -Guru memotivasi dan mengajak siswa untuk berpartisipasi aktif dalam pembelajaran -Guru menyampaikan kegiatan yang akan dilakukan oleh siswa.	20 menit

	- Guru memminta siswa untuk duduk sesuai dengan kelompok yang dibentuk pada pertemuan pertama.	
Kegiatan Inti	Eksplorasi - Siswa diminta duduk dengan kelompok masing-masing. -Siswa diminta untuk mengeksplorasi tabel perbandingan dengan mengerjakan LKS 2 Elaborasi -Diskusi kelas tentang strategi yang berbeda yang dapat dilakukan dengan menggunakan tabel perbandingan - Guru mengajukan beberapa pertanyaan tambahan sesuai dengan yang ada di power point -Siswa mengerjakan LKS 3, untuk mengaplikasikan strategi yang mereka temukan atau kembangkan dalam aktivitas mengeksplorasi tabel perbandingan - Diskusi kelas untuk membahas soal dengan menenkankan tentang bagaimana pengapllikasian strategi yang telah mereka temukan dalam soal perbandingan	40 menit
Kegiatan akhir	Konfirmasi - Guru memberikan kata pujian bagi siswa yang berpartisipasi dalam diskusi - Dengan bimbingan Guru peserta didik merefleksikan kegiatan pembelajaran yang dilakukan dan menyimpulkan hasil pembelajaran tentang: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? • Strategi apa saja yang dapat kita terapkan dengan menggunakan tabel perbandingan? - Guru memotivasi siswa yang belum aktif	10 menit

F. Media Pembelajaran

- LKS 2 dan LKS 3 (terlampir)
- Power point (terlampir)

G.Penilaian:

Intrumen : LKS 3

Bentuk soal : uraian

Rubrik penilaian :

No	Kunci jawaban	Skor
1.	160 daun	Jawaban benar =10 Jawaban salah = 4 Tidak menjawab =0

Nilai = Skor × 10

Palembang, 20 Maret 2013

Guru kelas IV D

Nurmasintan, S.Pd
NIP.196701201989072001

Peneliti

Sylvana Novilia Sumarto, S.Si
NIM.20112812012

Mengetahui
Kepala SDN 179 Palembang

Maulina, S.Pd, M.M
NIP. 196507101989072003

Rencana Pelaksanaan Pembelajaran
(RPP)

Sekolah : SDN 179 Palembang

Mata Pelajaran : Matematika

Kelas/Semester : IV/Genap

Pertemuan : 3 (Ketiga)

Alokasi Waktu : 2 x 35 menit

Standar Kompetensi : 5. Menggunakan pecahan untuk menyelesaikan masalah

Kompetensi Dasar : 5.5 Menyelesaikan masalah yang melibatkan pecahan (pengayaan)

A. Tujuan Pembelajaran:

- Perluasan konteks permasalahan bagi siswa dalam menerapkan konsep perbandingan

B. Indikator :

- Siswa dapat menerapkan ide perbandingan dalam konteks yang berbeda.

C. Materi Pembelajaran

Perbandingan ditemukan dalam berbagai masalah dalam kehidupan sehari-hari. Untuk itu dala pertemuan ini siswa akan diajak untuk menyelesaikan beberapa masalah perbandingan dengan konteks yang berbeda.

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	- Berdoa - Guru mengkondisikan kelas pada situasi belajar *Apersepsi* -Guru mengingatkan kembali apa saja strategi yang telah ditemukan oleh siswa kemarin -Guru memotivasi dan mengajak siswa untuk berpartisipasi aktif dalam pembelajaran -Guru menyampaikan kegiatan yang akan dilakukan oleh siswa. - Guru memminta siswa untuk duduk sesuai dengan kelompok yang dibentuk pada pertemuan pertama.	20 menit
Kegiatan Inti	Eksplorasi	40 menit

	-Siswa mengerjakan LKS 4.1 untuk mengeksplorasi soal missing value dengan bilangan yang tidak habis dibagi - Diskusi dalam kelompok Elaborasi: - Diskusi kelas - Siswa diminta mengerjakan LKS 4.2 sebagai latihan soal perbandingan dengan beberapa konteks yang berbeda. - Diskusi dalam kelompok - Diskusi kelas	
Kegiatan akhir	Konfirmasi - Guru memberikan kata pujian bagi siswa yang berpartisipasi dalam diskusi - Dengan bimbingan Guru peserta didik merefleksikan kegiatan pembelajaran yang dilakukan dan menyimpulkan hasil pembelajaran tentang: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? - Guru memotivasi siswa yang belum aktif	10 menit

F. Media Pembelajaran

- LKS 4.1 dan LKS 4.2 (terlampir)

G. Penilaian:

Intrumen : LKS 4.2

Bentuk soal : uraian

Rubrik penilaian :

No	Kunci Jawaban	Skor
1.	28 komik	Jawaban benar= 10 Jawaban salah = 4 Tudak menjawab= 0
2.	16 kupon	Jawaban benar =10 Jawaban salah = 4 Tidak menjawab = 0

3.	18 koin	Jawaban benar =10 Jawaban salah = 4 Tidak menjawab = 0

Nilai = (Skor : 3) × 10

Palembang, 21 Maret 2013

Guru kelas IV D

Nurmasintan, S.Pd

NIP.196701201989072001

Peneliti

Sylvana Novilia Sumarto, S.Si

NIM.20112812012

Mengetahui

Kepala SDN 179 Palembang

Maulina, S.Pd, M.M

NIP. 196507101989072003

Rencana Pelaksanaan Pembelajaran
(RPP)

Sekolah : SDN 179 Palembang

Mata Pelajaran : Matematika

Kelas/Semester : IV/Genap

Pertemuan : 4 (Keempat)

Alokasi Waktu : 2 x 35 menit

Standar Kompetensi : 5. Menggunakan pecahan untuk menyelesaikan masalah

Kompetensi Dasar : 5.5 Menyelesaikan masalah yang melibatkan pecahan (pengayaan)

A. Tujuan Pembelajaran:

- Siswa memahami bahwa dalam mebandingkan situasi kita menggunakan konsep perbandingan bukan nilai mutlak
- Siswa menggunakan *ratio table* untuk menyelesaikan masalah perbandingan.
- Siswa menggunakan *ratio table* sebagai alat untuk berpikir, memberi alasan, dan alat hitung.

B. Indikator :

- Siswa dapat menyelesaikan masalah *comparison* yang diberikan

C. Materi Pembelajaran

Penyelesaian masalah *comparison* lebih sulit daripada *missing value* karena siswa diminta untuk memperhatikan juga himpunan bilangan baru sebagai perbandingan yang lain. Untuk itu, siswa harus menguasai terlebih dulu strategi untuk menyelesaikan masalah perbandingan sederhana sebelum ia menerapkannya dalam masalah comparison.

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	- Berdoa - Guru mengkondisikan kelas pada situasi belajar *Apersepsi* -Siswa ditanya tentang bagaimana cara membandingkan harga 2 barang yang berbeda -Guru memotivasi dan mengajak siswa untuk berpartisipasi aktif dalam pembelajaran -Guru menyampaikan kegiatan yang akan	20 menit

	dilakukan oleh siswa. - Guru memminta siswa untuk duduk sesuai dengan kelompok yang dibentuk pada pertemuan pertama.	
Kegiatan Inti	Eksplorasi -Siswa mengerjakan LKS 5, sebagai soal *comparison* yang pertama - Diskusi dalam kelompok Elaborasi - Diskusi kelas - Siswa mengerjakan LKS 5, soal *comparison* kedua. Siswa mengelaborasikan konsep perbandingan dalam *comparing situation* dalam situasi perbandingan yang melibatkan unit. -Diskusi kelas	40 menit
Kegiatan akhir	Konfirmasi - Guru memberikan kata pujian bagi siswa yang berpartisipasi dalam diskusi - Dengan bimbingan Guru peserta didik merefleksikan kegiatan pembelajaran yang dilakukan dan menyimpulkan hasil pembelajaran tentang: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? -Guru memotivasi siswa yang belum aktif	10 menit

F. Media Pembelajaran

- LKS 5 (terlampir)
- posttest (terlampir)

G. Penilaian:

Intrumen : posttest

Bentuk soal : uraian

Rubrik penilaian :

No	Kunci Jawaban	Skor
1.	90 gram cokelat bubuk	Jawaban benar= 10 Jawaban salah = 4 Tidak menjawab = 0
2.	20 piring nasi goreng	Jawaban benar = 10 Jawaban salah = 4 Tidak menjawab =0

3.	Rp6000	Jawaban benar= 10 Jawaban salah = 4 Tidak menjawab = 0
4.	Mobil B	Jawaban benar =10 Jawaban salah tetapi penghitungan benar = 8 Jawaban dan penghitungan salah= 4 Tidak menjawab =0

Nilai = (skor: 4)×10

Palembang, 23 Maret 2013

Guru kelas IV D

Nurmasintan, S.Pd
NIP.196701201989072001

Peneliti

Sylvana Novilia Sumarto, S.Si
NIM.20112812012

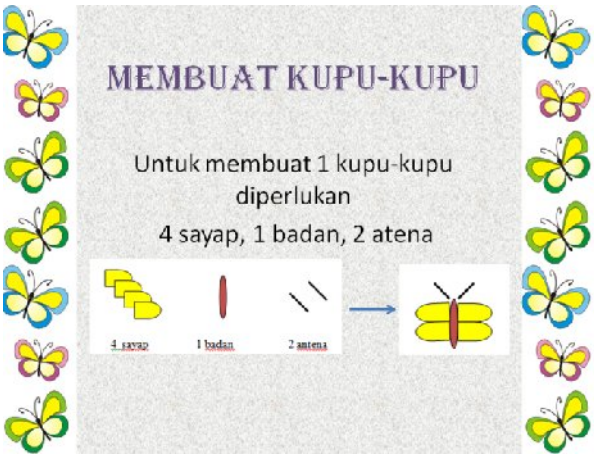
Mengetahui
Kepala SDN 179 Palembang

Maulina, S.Pd, M.M
NIP. 196507101989072003

APPENDIX 6

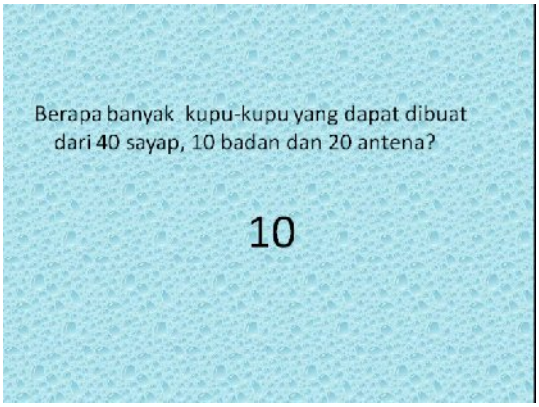
POWER POINT SLIDE

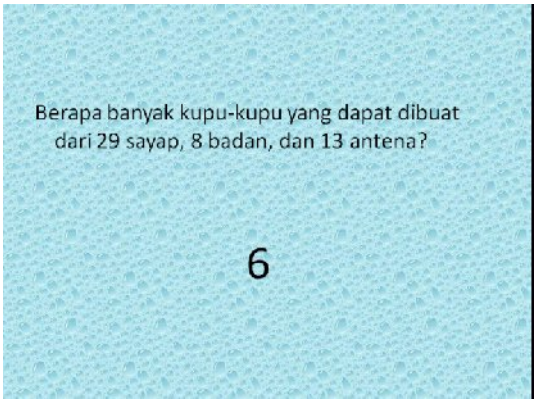
1. Activity 1



Banyak Kupu-kupu	Sayap	Badan	Antena
1	4	1	2
2	8	2	4
4	16	4	8
8	32	8	16
16	64	16	32

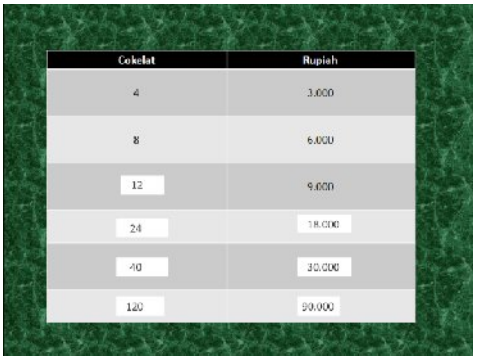
Banyak Kupu-kupu	Sayap	Badan	Antena
1	4	1	2
2	8	2	4
4	16	4	8
8	32	8	16
16	64	16	32
3	12	3	6
5	20	5	10
7	28	7	14


Berapa banyak kupu-kupu yang dapat dibuat dari 40 sayap, 10 badan dan 20 antena?
10


Berapa banyak kupu-kupu yang dapat dibuat dari 29 sayap, 8 badan, dan 13 antena?
6

2. Activity 2

Cokelat	Rupiah
4	3.000
8	6.000
16	12.000



Cokelat	Rupiah
4	3.000
8	6.000
12	9.000
24	18.000
40	30.000
120	90.000

APPENDIX 7

STUDENTS' MATERIAL

1. Activity 1 (Worksheet 1)-LKS 1.1

Nama	:
Kelas/ No absen	:
Tanggal	:

Membuat Kupu-Kupu

Dalam pelajaran SBK di kelas 4, siswa belajar membuat hiasan kupu-kupu. Untuk membuat seekor kupu-kupu, diperlukan bahan sebagai berikut.

4 sayap

1 badan

2 antena

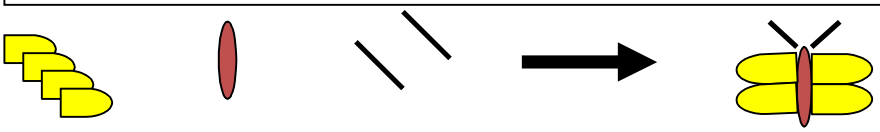
Ketika semua bahan sudah dipasangkan, maka akan terbentuk kupu-kupu seperti di bawah ini:

Tentukan banyaknya sayap, badan, dan antena yang dibutuhkan untuk membuat:

a. 2 kupu-kupu
b. 4 kupu-kupu
c. 8 kupu-kupu
d. 16 kupu-kupu

2. Activity 1 (Worksheet 2)-LKS 1.2

Nama	:
Kelas/ No absen	:
Tanggal	:



4 sayap 1 badan 2 antena 1 kupu-kupu

1. Berapa banyak sayap, badan dan antena yang kita butuhkan untuk membuat 6 kupu-kupu?

2. Jika kita memiliki 48 sayap, 12 badan dan 24 antena, berapa banyak kupu-kupu yang dapat dibuat? Jelaskan jawabanmu!

3. Jika kita mempunyai 36 sayap, 10 badan dan 17 antena, berapa banyak kupu-kupu yang dapat dibuat? Jelaskan alasanmu!

4. Lengkapilah tabel berikut! Jelaskan bagaimana caramu memperoleh jawabannya!

Banyak kupu-kupu	Sayap	Badan	Antena
11	a) …	11	22
b)…	52	c)…	d)…
24	e) …	24	48

3. Activity 2-LKS 2

Nama	:
Kelas	:
Tanggal	:

Cokelat Kesukaanku

4 cokelat Rp3.000,-

Banyak cokelat	Rupiah

4. Activity 3-LKS 3

Nama	:	
Kelas	:	
Tanggal	:	

Memberi Makan Ulat

Aldi mendapat tugas dari gurunya untuk mengamati perkembangan ulat. Dia membutuhkan 5 daun untuk memberi makan ulat-ulatnya selama 2 hari. Berapa banyak daun yang dia butuhkan untuk memberi makan ulat-ulat tersebut selama 64 hari?

4. Activity 4 (Worksheet 1)-LKS 4.1

Nama	:
Kelas	:
Tanggal	:

Bagaimana menurutmu?

Dalam suatu *game* petualangan, kita dapat menukar 5 bola emas dengan 2 nyawa, berapa banyak nyawa yang kita dapatkan jika kita mempunyai 23 bola emas?

5. Activity 4 (Worksheet 2)-LKS 4.2

Nama	:
Kelas	:
Tanggal	:

1. Di suatu toko buku bekas, kita dapat menukarkan 5 majalah bekas dengan 4 buku cerita yang baru. Berapa banyak buku cerita yang kita dapatkan untuk 35 majalah bekas?

2. Di pasar malam, kita dapat membeli 3 gelas susu cokelat dengan 2 kupon. Berapa banyak kupon yang kita butuhkan apabila kita ingin membeli 24 gelas susu cokelat?

3. Ada promo di supermarket. Tiap pembelian Rp100.000 dan kelipatannya kita akan mendapat 1 kupon. Kita dapat menukarkan 3 kupon tersebut dengan 4 koin untuk bermain di Zona Bermain. Berapa banyak kupon yang kita butuhkan untuk mendapat 24 koin?

6. Activity 5-LKS 5

Nama	:
Kelas	:
Tanggal	:

Harga Terbaik

1. Perhatikan gambar soda berikut. Manakah diantara kedua jenis soda ini yang lebih murah? Jelaskan alasanmu!

Soda A

Soda B

Nama	:	
Kelas	:	
Tanggal	:	

2. Perhatikan 2 jenis madu berikut. Manakah diantara kedua jenis madu ini yang lebih murah? Jelaskan alasanmu!

Madu A

Madu B

NOTULEN

THESIS DEFENSE

Student Name : Sylvana Novilia Sumarto

Student Number : 20112812012

Study Program : Mathematics Education

Research Title : Design Research on Mathematics Education: Ratio Table in Developing the Students' Proportional Reasoning

Date of defense : 22 June 2013

Examiner	Questions and Suggestions	Reaction
Dr. Ratu Ilma Indra Putri, M.Si.	I want to know how can you explore students' thinking using the ratio table because in some mathematics class in Indonesia, the students may have difficulties to use the ratio table; directly asked them to use the ratio table or let them used their strategies?	Explained in the thesis defense
	How many strategies that occurred?	Explained in the thesis defense
Dr. Yusuf Hartono	I think your design is focus more on how the students solve the proportional problem. Can you explain about how the ratio table may develop the students' proportional reasoning? Can you show in which part the students' reasoning change?	Explained in the thesis defense
Dr. Maarten Dolk	Based on the students' work of activity 4 that you showed on the slide, it seems that the students may develop the number sense using the ratio table. Is that him doing or your design or was it good starting point to make the students develop about the number sense?	Explained in the thesis defense
	What was the previous local	Explained in the

	instructional theory of the coordinate system and what is your contribution to the local instructional theory?	thesis defense
	In the design research, we talked about the cyclic process. Give me an example of your change from the cycle 1 and cycle 2.	Explained in the thesis defense
Dr. Darmawijoyo	In this design, the ratio was constant. When do you think the non constant ratio should be given to the students? In what Grade?	Explained in the thesis defense
Prof. Dr. Zulkardi, M.I.Komp., M.Sc.	In design research, you took a role as the designer, the researcher, the teacher trainer. Do you think which role is the most difficult for you?	Explained in the thesis defense
	Do some change in the layout: • aim and the research question should be in the chapter I • Make an agreement about where you place the discussion part	Suggestion accepted and the thesis layout was revised as suggested.

Supervisor I

Prof. Dr. Zulkardi, M.I.Komp., M.Sc.

Supervisor II

Dr. Darmawijoyo

Head of

Mathematics Education Department

Prof. Dr. Zulkardi, M.I.Komp., M.Sc.

NIP 19610420 198603 1 002