DESIGN RESEARCH ON MATHEMATICS EDUCATION: RATIO TABLE IN DEVELOPING THE STUDENTS' PROPORTIONAL REASONING

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

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SRIWIJAYA UNIVERSITY
JUNE 2013

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STATEMENT PAGE

I hereby:

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State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.

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ABSTRACT

The way to teach proportion by giving a ready used- formula remains meaningless for the students. They may just memorize the procedure of cross multiplication without understand about the insight of proportionality itself. Thus, we need a learning design which is not only emphasizing the ability to solve the proportional problem but also can develop the students' proportional reasoning. Within a learning trajectory, five activities were designed based on the principle of *Pendidikan Matematika Realistik Indonesia* (PMRI)- the Indonesian version of Realistic Mathematics Education (RME). In this study, design research was carried out to investigate how the ratio table can support the students to develop their proportional reasoning as well as to contribute to the development of the local instructional theory. Thirty five students Grade 4 from SDN 179 Palembang were involved in this study. The analysis of students' works and the video of the learning process in the classroom show that wrapped in the real contextual problem, the ratio table may develop the students' proportional reasoning as well as the strategy to solve proportional problem.

Keywords: Ratio Table, Proportion, Proportional Reasoning, *Pendidikan Matematika Realistik Indonesia* (PMRI), Design Research.

ABSTRAK

Mengajarkan materi perbandingan dengan memberikan rumus-siap pakai membuat proses pembelajaran menjadi tidak bermakna bagi siswa. Siswa mungkin hanya akan menghapalkan prosedur perkalian silang tanpa memahami inti dari konsep perbandingan. Oleh karena itu, kita membutuhkan suatu desain pembelajaran yang tidak hanya menekankan pada kemampuan siswa dalam menyelesaikan masalah perbandingan tetapi juga dapat mengembangkan kemampuan bernalar siswa tentang konsep perbandingan (*proportional reasoning*). Terintegrasi dalam suatu lintasan belajar, lima aktivitas didesain berdasarkan prinsip dalam Pendidikan Matematika Realistik Indonesia (PMRI). Dalam penelitian ini, *design research* diterapkan untuk menginvestigasi bagaimana tabel perbandingan (*ratio table*) dapat mendukung perkembangan nalar siswa tentang perbandingan sekaligus untuk berkontribusi pada perkembangan teori instruksi lokal (*local instructional theory*). Sebanyak tiga puluh lima siswa kelas 4 dari SDN 179 Palembang terlibat dalam penelitian ini. Hasil analisis dari hasil kerja siswa dan video rekaman proses pembelajaran di kelas menunjukkan bahwa bersama dengan masalah kontekstual, ratio table dapat mengembangkan kemampuan bernalar siswa sekaligus mengembangkan strategi untuk menyelesaikan masalah perbandingan.

Kata kunci: Tabel perbandingan, perbandingan, kemampuan bernalar siswa tentang perbandingan (*proportional reasoning*), Pendidikan Matematika Realistik Indonesia (PMRI), *design research*.

SUMMARY

Sylvana Novilia Sumarto. Design Research on Mathematics Education: Ratio Table in Developing the Students' Proportional Reasoning.

In Indonesia, the proportion was taught since Grade 5 elementary school which was integrated in the lesson about fraction. However, the teacher taught by giving the ready-used formula. By using the algebra notation and the cross multiplication, the students may solve the proportional problem but it is not a guarantee that they understand about the concept of proportion itself. In line with this, van Galen &van Eerde (2013) stated that if the procedures were not understood well by the students then they will become vulnerable tricks, meanwhile the proportional reasoning is a "capstone" of elementary school mathematics (Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr as cited in Parish, 2010). Therefore, we need a learning design which supports the development of students' proportional reasoning.

Within a design research, we developed a learning trajectory of proportion which may facilitate the students to develop of students' proportional reasoning. We tested the Hypothetical learning trajectory (HLT) to the 35 students Grade 4 of SDN 179 Palembang, Indonesia in two cycles. There were six students involved in the first cycle. Here, the researcher had a role as the teacher. The result from the first cycle was used to adjust the initial HLT. Meanwhile, the other 29 students were participated on cycle 2. The students were taught by their own teacher based on the refined HLT.

Five activities were designed based on the principle of *Pendidikan Matematika Realistik Indonesia* (PMRI)- the Indonesian version of Realistic Mathematics Education. As the characteristics of PMRI, we started the lesson by proposing contextual problems which were familiar for the students and used the ratio table as the model here.

During the first cycle, we found that we need to change some sentence in the problems in order to make it more understandable for the students. We also change the order of some activities; the activity of exploring the ratio table would be given before the students worked on the simple missing value problem on the cycle 2. Thus, after be refined the order of the activities became as follow.

The first activity was Make Butterflies. In this activity, the students were asked to determine the number of wings, bodies, and antennas to make certain numbers of models of butterfly if known that to make a model of butterfly we need 4 wings, 1 body and 2 antennas. This activity aims to make the students understand the notion of ratios in proportional problems. The second activity was My Chocolate. In this activity the students explore the ratio table through making a list of chocolate price. They determine the number of chocolate by themselves then found the price. The goal of this activity was the students may be able to gain some strategies that can be done by using ratio table and may lead them to

reinvent the rule 1 (unit 1 strategy). These strategies would help them to solve the proportional problems. The third activity was Feeding Caterpillars. This activity proposed a missing value problem. The students were asked determine the number of leaves which are needed to feed caterpillars in 64 days if known that we need 5 leaves to feed the caterpillars in two days. The fourth activity was "What do you think?" In this activity, we proposed some different contextual problems in order to strengthen the students' ability in using the techniques in solving simple proportional problem as well as to improve the students' proportional reasoning. The last activity was Best Price. The problem which was posed in this activity was a comparison problem. Here, we want to build up the students' idea about proportionality in comparison situation.

In line with the aforementioned background of this research, the present study aims to facilitate the development of students' proportional reasoning as well as to contribute to the development of local instructional theory. Accordingly, we proposed a research question as: *"How can the ratio table supports pupils to develop proportional reasoning?"*

To address the main research question in this study, first we answered sub questions, based on the findings on cycle 2 of this study, as follow.

"*How do the students in Grade 4 use their intuitive understanding to solve proportional problem?"*

On the pretest and the interview in the beginning of cycle 2, we see that some students were able to gain the idea of unit 1 method to solve the simple missing value problem while the others multiplied with the wrong number. The others came up with the idea of doubling and repeated addition to look for the fourth number in missing value problem. However, some students still need guidance to deal with this problem. By proposing a similar question with smaller numbers, the lower achievement students, who have no proficient multiplication, may be able to think about the repeated addition as the way to solve the original problem.

Instead of using the proportionality, some students used the absolute value to solve the comparison problem. They may not have idea about the relation between two factors which influence the comparison situation, such as the price and the weight. Basically, the context which used in problem and the questions which were posed during the interviews may influence the students' reaction. When the context which used in the problems was experienced by the students, they may be able to give the right answer based on what happened in their daily life. Proposing the similar problems and change the number may lead the students to get the idea to solve the problem.

"*How can the ratio table support pupils to develop proportional reasoning in missing value problem?"*

Based on the analysis data from the activity 1 until activity 4, we may conclude that by using the ratio table, the students can organize the information from the problem because the table classifies each item in systematic way. Sometimes the students do not aware of what number represent is; the students just directly multiply it or divided it without any consideration. The ratio table may prevent the students to do this. The ratio table also allows the students to write the intermediate step of the calculation. First, it facilitates the students who cannot do the calculation well. Second, by writing the intermediate steps, the

students may be able to see the relation among the numbers. Here, the students can construct their proportional reasoning.

"How can the ratio table support pupils to develop proportional reasoning in comparison problem?"

Through the data analysis on the activity 5, we can say that the students may use the table to look for the proportions which give a good comparison situation. They find that the table helps them to find the number of items that they got for the same price or to find the price for the same number of items and then compare the price. Of course, it can be happened after the students master the use of ratio table in simple proportional problem. It is in line with what stated in Tourniere & Pulos (1985), "Comparing strategy is an advanced method, and the ability to choose the arithmetically easier comparison is acquired long after the proportional techniques mastered."

Based on the answer to the sub research question, we may formulate the answer for the main research question as follow.

The ratio table together with the context can help the students to develop their proportional understanding as well as the strategies to solve the proportional problems; both in missing value and comparison problems. By using ratio table, the students may organize the information from the question. It helps them to identify what the numbers represent for. The intermediate steps which are showed on the ratio table allow the students to see the relationship among the numbers. Hence, they can develop the proportional relation among the numbers. Once they realize about this relation, they may use that as the reason to think proportionally. When the students already mastered the techniques to solve simple proportional problem, they may developed the ratio table as a tool to do calculation and reasoning in comparison situation. The ratio table allowed them to gain the proportions which show the comparison situation.

RINGKASAN

Sylvana Novilia Sumarto. Design Research on Mathematics Education: Ratio Table in Developing the Students' Proportional Reasoning.

Di Indonesia, materi tentang perbandingan diajarkan sejak kelas 5 SD, terintegrasi dengan bab tentang pecahan. Namun dalam mengajarkannya, Guru langsung memberikan rumus jadi, yaitu dengan menggunakan notasi aljabar dan perkalian silang. Melalui metode pembelajaran semacam ini, siswa mungkin dapat mengerjakan soal-soal perbandingan dengan menggunakan rumus yang telah diajarkan di sekolah, tetapi ini bukanlah jaminan bahwa siswa memahami benar tentang konsep perbandingan itu sendiri. Sejalan dengan ini, van Galen dan van Eerde (2013) menyatakan bahwa suatu prosedur atau cara yang tidak dipahami secara benar oleh siswa hanya akan menjadi suatu trik. Hal ini menyebabkan kemampuan bernalar siswa tentang perbandingan (*proportional reasoning*) menjadi tidak berkembang, padahal *proportional reasoning* merupakan "*capstone*" (batu penjuru) dari matematika di sekolah dasar (Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr seperti dikutip dalam Parish, 2010). Oleh karena itu, kita membutuhkan suatu desain pembelajaran yang dapat mendukung perkembangan kemampuan bernalar siswa dalam masalah perbandingan.

Dengan menggunakan metode *design research*, kami mengembangkan sutau lintasan belajar yang dapat memfasilitasi siswa dalam mengembangkan penalaran tentang konsep perbandingan. Kami mencobakan suatu hipotesis lintasan belajar-*Hypothetical learning trajectory* (HLT) kepada 35 siswa kelas 4 SDN 179 Palembang, Indonesia dalam dua siklus. Enam orang siswa dilibatkan dalam siklus pertama. Disini, peneliti berperan sebagai guru pengajar. Hasil dari siklus yang pertama ini digunakan untuk merevisi HLT versi awal. Untuk 29 siswa lainnya berpartisipasi dalam siklus yang kedua. Pada siklus kedua ini, siswa diajar oleh guru mereka sendiri berdasarkan HLT yang telah direvisi.

Terdapat lima aktivitas dalam desain pembelajaran ini. Kelima aktivitas tersebut didesain berdasarkan prinsip-prinsip dalam Pendidikan Matematika Realistik Indonesia (PMRI). Sesuai karakteristik dalam PMRI, kita memulai pembelajaran dengan masalah kontekstual yang familiar bagi siswa dan menggunakan tabel perbandingan (*ratio table*) sebagai modelnya.

Selama mejalankan siklus pertama, kami menemukan bahwa kita perlu mengubah beberapa kalimat dalam soal agar lebih mudah dimengerti oleh para siswa. Kami juga mengganti urutan aktivitas pada siklus kedua; aktivitas mengeksplorasi tabel diberikan sebelum siswa mengerjakan masalah *missing value* (tipe soal yang mencari salah satu nilai yang belum diketahui). Setelah diperbaiki, maka urutan aktivitas dalam desain pembelajaran ini sebagai berikut.

Aktivitas pertama adalah Membuat Kupu-kupu. Pada aktivitas ini, para siswa diminta untuk menentukan banyak sayap, badan, dan antena yang dibutuhkan

untuk membuat sejumlah model kupu-kupu jika diketahui bahwa untuk membuat sebuah model kita membutuhkan 4 sayap, 1 badan, dan 2 antena. Tujuan dari aktivitas ini adalah untuk membuat siswa memahami konsep rasio dalam perbandingan. Aktivitas yang kedua adalah Cokelatku. Pada aktivitas ini, siswa mengeksplorasi tabel perbandingan dengan cara membuat daftar harga cokelat. Siswa menentukan sendiri banyak cokelatnya kemudian siswa menentukan harganya. Aktivitas ini bertujuan untuk membantu siswa mengembangkan beberapa strategi yang dapat dilakukan dengan mengggunakan tabel perbandingan dan memnugkinkan mereka untuk menemukan kembali aturan mencari nilai 1. Strategi-strategi ini nantinya dapat membantu mereka dalam menyelesaikan masalah perbandingan. Aktivitas yang ketiga adalah Memberi Makan Ulat. Pertanyaan dalam aktivitas ini merupakan *missing value*. Siswa diminta untuk menentukan banyak daun yang dibutuhkan untuk memberi makan ulat selama 64 hari jika diketahui kita membutuhkan 5 daun untuk memberi makan ulat selama 2 hari. Aktivitas keempat adalah "Bagaimana menurutmu?" Dalam aktivitas ini, siswa diberi beberapa masalah kontekstual yang berbeda dengan tujuan untuk memperkuat kemampuan siswa dalam menggunakan teknik-teknik untuk menyelesaikan masalah perbandingan sederhana sekaligus untuk mengembangkan kemampuan bernalar siswa. Aktivitas yang terakhir adalah Harga Terbaik. Masalah yang diberikan kepada siswa dalam aktivitas ini merupakan masalah *comparison* (tipe soal yang membandingkan dua atau lebih situasi perbandingan). Disini, kamu ingin membangun ide siswa tentang adanya konsep perbandingan dalam membandingkan sesuatu.

Berangkat dari latar belakang penelitian, maka tujuan dari penelitian ini adalah untuk memfasilitasi perkembangan kemampuan bernalar siswa dalam konsep perbandingan sekaligus berkontribusi dalam perkembangan teori instruksi lokal (*local instructional theory*). Sejalan dengan hal ini, maka rumusan masalah dalam penelitian ini adalah *"Bagaimana tabel perbandingan dapat mendukung perkembangan kemampuan bernalar siswa dalam perbandingan?"*

Untuk menjawab masalah utama dalam penelitian ini, pertama kami akan menjawab sub pertanyaan, berdasarkan temuan pada siklus kedua, sebagai berikut.

"Bagaimana siswa kelas 4 menggunakan pemahaman intuisi mereka untuk menyelesaikan masalah perbandingan?"

Dalam tes awal dan wawancara di awal siklus kedua, kita mengamati bahwa ada siswa yang dapat menemukan ide unit satuan untuk menyelesaikan masalah *missing value* yang sederhana ketika yang lain mengalikan dengan bilangan yang salah. Beberapa siswa yang lain menggunakan ide melipatgandakan (*doubling*) dan penjumlahan berulang untuk mencari bilangan keempat pada masalah *missing value*. Namun, beberapa siswa masih memerlukan bimbingan dalam menyelesaikan masalah ini. Dengan menanyakan pertanyaan serupa yang menggunakan bilangan yang lebih kecil dapat membantu siswa yang lemah di pelajaran matematika, yang mungkin tidak memiliki kemampuan dan pemahaman

yang cukup tentang konsep perkalian, untuk berpikir tentang penjumlahan berulang sebagai cara untuk menyelesaikan masalah semula.

Bukannya menggunakan konsep perbandingan, beberapa siswa justru menggunakan nilai mutlak untuk menyelesaikan masalah *comparison*. Mungkin mereka tidak memiliki pemahaman tentang relasi antara dua factor yang memepengaruhi situasi perbandingan, misalnya harga dan berat. Pada dasarnya, konteks yang digunakan dalam soal dan pertanyaan yang diajukan selama wawancara mempengaruhi respon siswa. Ketika konteks yang digunakan pernah dialami sendiri oleh siswa, mereka dapat memberikan jawaban yang benar berdasarkan apa yang terjadi dalam kehidupan mereka sehari-hari. Dengan mengajukan soal yang serupa dan mengganti bilangan yang digunakan dalam soal mampu membawa siswa untuk mendapatkan ide pemecahan masalah.

"Bagaimana tabel perbandingan dapat mendukung perkembangan nalar perbandingan siswa dalam masalah missing value?"

Berdasarkan analisis data dari akivitas 1 hingga aktivitas 4, kita dapat menyimpulkan bahwa dengan menggunakan tabel perbandingan siswa dapat mengorganisasi informasi dari soal yang diberikan karena tabel menggolongkan masing-masing benda dalam cara yang sistematis. Terkadang siswa tidak memperhatikan sebenarnya bilangan yang ada di dalam soal itu merepresentasikan apa; siswa hanya mengalikan atau membagi bilangan-bilangan tersebut tanpa pertimbangan apapun. Hal ini dapat dicegah dengan menggunakan tabel perbandingan. Tabel perbandingan juga memungkinkan siswa untuk menuliskan proses penghitungan langkah demi langkah. Pertama, ini bermanfaat untuk memfasilitasi siswa yang tidak terlalu bisa berhitung dengan baik. Kedua, dengan menuliskan proses penghitungan langkah demi langkah membuat para siswa dapat melihat relasi diantara bilangan-bilangan yang ada. Disini, siswa dapat mengkonstruk penalaran perbandingan mereka.

"*Bagaimana tabel perbandingan dapat mendukung perkembangan nalar perbandingan siswa dalam masalah comparison?*"

Melalui analisa data yang diperoleh dari aktivitas 5, kita dapat mengatakan bahwa siswa dapat menggunakan tabel untuk mencari perbandingan yang dapat digunakan untuk membandingkan dengan baik. Mereka menyadari bahwa tabel dapat membantu mereka dalam menemukan banyaknya benda ynang mereka peroleh untuk harga yang sama atau untuk mencari harga untuk dua atau lebih benda dengan banyak yang sama dan kemudian membandingkan harganya. Tentunya, ini dapat terjadi setelah siswa menguasai penggunaan tabel perbandingan dalam masalah perbandingan yang sederhana. Hal ini senada dengan apa yang dinyatakan dalam Tourniere & Pulos (1985), strategi yang digunakan dalam membandingkan sesuatu adalah suatu metode tingkat lanjutan, dan kemampuan untuk memilih perbandingan mana yang lebih mudah, diperoleh lama setelah teknik penyelesaian masalah perbandingan dikuasai.

Berdasarkan jawaban untuk sub pertanyaan sebelumnya, kita dapat menjawab pertanyaan utama dalam penelitian ini sebagai berikut.

Tabel perbandingan bersama dengan konteks dapat membantu siswa mengembangkan penalaran perbandingan sekaligus strategi pemecahan masalah perbandingan; baik dalam masalah *missing value* maupun *comparison*. Dengan menggunakan tabel perbandingan, siswa dimungkinkan untuk mengorganisasi informasi dari pertanyaan yang diberikan. Tabel membantu mereka dalam mengidentifikasi apa yang direpresentasikan oleh bilangan-bilangan tersebut. Langkah per langkah yang ditunjukkan oleh tabel perbandingan memungkinkan siswa untuk melihat relasi diantara bilangan-bilangan yang ada. Dari sini mereka dapat mengembangkan hubungan proporsional diantara bilangan-bilangan tersebut. Pada saat mereka menyadari hubungan ini, mereka dapat menggunakannya untuk bernalar secara proporsional. Lebih lanjut, ketika siswa telah menguasai teknik pemecahan masalah perbandingan yang sederhana, mereka dapat menggunakan tabel perbandingan sebagai alat untuk menghitung sekaligus bernalar dalam situasi proporsional. Tabel perbandingan memunculkan perbandingan yang menunjukkan situasi proporsional.

PREFACE

By the gracious of God, I could finish writing this thesis; thank you Lord, for all your blessings to me.

This thesis was written to fulfill the requirements for the degree of Master of Science (M.Sc) in International Master Program on Mathematics Education (IMPoME), Sriwijaya University, Palembang- Indonesia in collaboration with Utrecht University, Utrecht- the Netherlands. I hope the results from this thesis may contribute to the development of the mathematics education, especially the implementation of *Pendidikan Matematika Realistik Indonesia* (PMRI) in Indonesia.

First, in this opportunity I would like to express my deepest gratitude to my wonderful supervisors, Frans van Galen, Prof. Zulkardi and Dr. Darmowijoyo, for their never ending support, helpful advice, sincere help, serious guidance, great motivation and encouragement during writing this thesis. The idea of this research was developed through my discussion with Marteen Dolk and Dolly van Eerde. Thank you very much to both of you. Special thanks also for Mieke Abels, Marteen Kindt, Paul Drijvers, Dirk Jan, Arthur Bakker, Steven Wepster, Barbara van Amerom, Elwin Savelsbergh, Henk van der Kooij, Betty Heijman, Mark Uwland, and all staffs in Freudhental Institute, Utrecht Univesity for all your help and guidance during my study in Utrecht. In memoriam, Jaap den Hertog. Thank you for assisting me during the preparation to go to the Netherlands.

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Palembang, June 2013

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CHAPTER I

INTRODUCTION

1.1 Background

Proportion can be found everywhere. For instance, we use the concept of proportion in rate problems. If we need 6 litres to drive 15 km then we need 12 litres to drive 30 km. We also use the concept of proportion in a shopping context to compare prices. If there are two kinds of peanut butter, one costing 2 euro for a 150 ml bottle and the other costing 3 euro for a 250 ml bottle, we can use the concept of proportion to determine that the second bottle is cheaper. In addition, we also find the concept of proportion in scale. Most mathematicians defined a proportion as the equality of two ratios, for example $- = -$ (Tourniaire & Pulos, 1985; Langrall & Swafford, 2000) but in a certain sense, a proportion can be defined as a more general concept, which is specifically reflected in fractions, percentages and decimals (Galen *et al*, 2008). In this sense, we can expand the list of the use of proportion in our everyday life. However, in this study we will use the sense of proportion as in the first definition. Regarding the usefulness of the concept of proportion in our daily life, we may say that proportion is one of important concepts in mathematics. Therefore, it is necessary to learn about this topic.

In Indonesia, the students learn about proportion on the 5^{th} until 8^{th} grade (Depdiknas, 2006). During 5^{th} grade they will learn about direct proportion and in the higher level they will learn about indirect proportion as well. According to Zulkardi (2002), most of textbook that used in Indonesia contain mainly the set of rules and algorithm which is already formal and they lack of application which is

needed by the students in order to make the concept be real for them. The lesson of proportion in grade 5 started by giving the fixed procedures to solve the problems (e.g. Soenaryo, 2007; Sumanto *et al*, 2008). In line with that, the teachers teach the cross multiplication to solve the problem. Although the students are able to solve the proportional problem by applying this method, they may just memorize the procedure and not develop their proportional reasoning. In addition, van Galen & van Eerde (2013) also stated that if the procedures were not understood well by the students then they will become vulnerable tricks.

"Proportional reasoning is defined as a term that denotes reasoning in a system of two variables between which there exists a linear functional relationship..." (Karplus *et al*, 1983, p.219). The proportional reasoning is a "capstone" of elementary school mathematics (Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr as cited In Parish, 2010). Many studies have been conducted to observe the development of children' proportional reasoning (e.g. Karplus *et al*, 1983; Tourniaire & Pulos, 1985; Tourniaire, 1986; Langrall & Swafford, 2000; Silvestre & da Ponte, 2012). These studies observe the students' informal knowledge about proportion, what kind of strategies they used to solve proportional problems and what factors from the problems that influence their performance. Based on the studies, there are three difficulty factors of a proportional problem which influence students' performance: presence of an integer ratio, numerical complexity, and the order of the number that the students have been looking for. The problems with small, integer ratios and questions about what is the biggest number to be found are easier for the students (Rupley, 1981 in Tourniaire, 1986). These studies contributed to the developmental of

theory about the students' proportional thinking and reasoning. However, these theories are not applicable in the practical situation. The teachers in the school may not have any idea about the real application in the teaching practice in their classroom. Therefore, it still need more research to improve the theory development at the level of the instructional activities, the course (local instruction theories) and at the level of the domain-specific instruction theory. With regard to this, in this study we proposed a learning trajectory and the instructional learning activities which can be used and implemented by the teachers.

As mentioned above, the students in Indonesia learn about proportion formally in Grade 5. However, the sense of proportion has been learned since the earlier grade although it is not listed clearly in the curriculum. It is developed gradually since that. For instance, the children in kindergarten knew that the giant's chair is bigger than the normal human's chair or the students in the second grade knew that the number of steps from a man is smaller than the number of steps a 5 years old-girl for the same distance. The students in 4^{th} grade learned about pattern and relationship which are also related with the proportional thinking. It means that they have some prior knowledge about the concept of proportion in some extent. Accordingly, we initiate to try out our design for students Grade 4 with the consideration that through this design they may able to prepare themselves before they learn proportion more formally in Grade 5. Moreover, it may help them to lay the groundwork for future work in the higher level.

Regarding the purpose of this study which is to facilitate the students in developing their proportional reasoning, five activities were designed based on the

principle of Pendidikan Matematika Realistik Indonesia (PMRI) - which is adapted from the Realistic Mathematics Education (RME). In line with the statement by Langrall & Swafford (2000, p. 260), "Instruction with proportional reasoning should begin with situations that can be visualized or modelled", the activities in this study also were initialized by the contextual problems which can be visualized by the students. The problems used their daily activities as the context, so they may be able to imagine the real situation as described on the questions. In this design, we also proposed a ratio table as the model that may facilitate them to develop their proportional reasoning.

The ratio table was choosen as the model in this design because it shows the proportions clearly. We can expand the number of columns to suit our needs. It is an ideal aid for making handy calculations, gaining insight, and students reason with proportion because the table invites the students to write down intermediate steps. For instance, to find how many km which can be reached in 80 minutes if we can go for 15 km in 60 minutes, the students may use 5 in 20 as intermediate step or 30 in 120 then 10 in 40 to get 20 km in 80 minutes. The strength of a ratio table is that the students can reason with a number relationship which they already know (van Galen *et al*, 2008). Accordingly, in this study a learning sequence was designed with focus on the developing of students' proportional reasoning by using a ratio table as model.

1.2 Research Aims

According to the background of this study, the purposes of this study were:

1. To develop the learning design which was not only emphasized the students' ability in solving the proportional problem but also may develop the students' proportional reasoning.
2. To contribute to the development of local instructional theory in proportion.

1.3 Research Questions

Regarding to the aforementioned background of this study, we proposed the main research question for this study as follow.

How can the ratio table support pupils to develop proportional reasoning?

To address the main research question, we proposed three sub research questions as:

1. *How do the students in Grade 4 use their intuitive understanding to solve proportional problem?*

Before we investigated how the ratio table may help the students to develop their proportional reasoning, we should know how they may solve the proportional problem by using their intuitive understanding and what the students' earlier idea about proportion. This information also may help us to improve the designed learning trajectory.

2. *How can the ratio table supports pupils to develop proportional reasoning in missing value problem?*
3. *How can the ratio table supports pupils to develop proportional reasoning in comparison problem?*

As stated by Karplus *et al* (1983), Tournaire & Pulos (1985) and Silvestre & da Ponte (2012), in general the proportional problems were categorized as missing value problem and comparison problem. Thus, the second and the third sub research questions were addressed in order to investigate how the ratio table may facilitate the students to develop their proportional reasoning in both types of proportional problems.

CHAPTER II

THEORETICAL BACKGROUND

2. 1 Proportion

Proportion is an important concept in mathematics. We encounter this concept everywhere, for example in enlarging and reducing photos, price comparisons, the recipes, comparing probabilities, graphs and diagrams (van Galen et al, 2008). However, Tourniaire and Pulos (1985) stated that this concept is difficult. The concept is acquired late and many adults do not master the concept. In formal mathematics, a proportion is defined as the equality of two ratios, for example – = – (Tourniaire & Pulos, 1985; Langrall & Swafford, 2000).

In general proportional problems are categorized in two types, namely missing value problem and comparison problem (Karplus *et al*, 1983; Tournaire & Pulos, 1985; Silvestre & da Ponte, 2012). Missing value problems present three numbers and ask for the fourth one, such as how many candies that we get for 3 euro if we know that 6 candies costing 2 euro. Meanwhile, the comparison problems present two or more pairs of numbers and ask about their comparison. For instance, if there are two kinds of peanut butter, one costing 2 euro for a 150 ml bottle and the other costing 3 euro for a 250 ml bottle, we can determine the second butter as the cheaper one by comparing the price for 1 euro for each of them. Basically, the comparison problem is more difficult than the missing value problem. It requires the students to be able to solve the simple proportional problem before they can work on the comparison situation. As stated in Tourniere & Pulos (1985), "Comparing strategy is an advanced method, and the ability to choose the arithmetically easier comparison is acquired long after the proportional techniques

mastered." In line with that, the activities which proposed the missing value problems were presented before the activities about the comparison were given.

According to a literature review by Tournaire & Pulos (1985), the factors from the problems that may influence students' performance are

1. *Structural variables of the proportional problem*
    - The number structure of the proportion problems influences the subjects' performance. There are three main difficulty factors: presence of an integer ratio, numerical complexity, and order of number that have been looking for. The presence of integer ratios makes a problem easier. The difficulty introduced by non-integer ratios is widely recognized. The problems with small, integer ratios and ask the biggest number to be found are easier for the students
    - The presence of a unit
    - The presence of unequal ratios, for comparison problem
2. *Context variables of the proportional problem*
    - The familiarity of the context. The familiar problem is easier than unfamiliar ones.
    - The mixture problems is more difficult than non-mixture problems
    - People can more easily visualize discrete content than continuous content.
    - The mode of delivery of the problem.

2. 2 Proportional Reasoning

Karplus *et al* (1983) defined the proportional reasoning as a term that denotes reasoning in a system of two variables between which there exists a linear functional relationship. Proportional reasoning is an important ability to build a

foundation for high school mathematics and algebraic reasoning (Langrall & Swafford, 2000). Furthermore, as cited in Parish (2010), Kilpatrick, Swafford & Findell; Lamon; Lesh, Post & Behr consider the proportional reasoning as a "capstone" of elementary school mathematics. It is needed to learn the important topics in mathematics such as fraction equivalence, place value, long division, percentage calculation, measurement conversions, and derivation of rates which are troublesome for the students.

According to Boyer & Levine (2012), considering the value and the effectiveness of intuitive capacities, the development of mathematical understanding is characterized not as ''all or none'' but it is mowing from partial to more complete and become less contextually dependent. The intuitive understandings influence the people's mathematical skills even after they emerge the formal mathematics. In line with that, giving the instruction that builds on the students' intuitive understanding of proportional relations may support the development of their proportional thinking.

2. 3 Knowledge Gap

Many studies have been conducted to observe the development of children's' proportional reasoning (e.g. Karplus *et al*, 1983; Tourniaire & Pulos, 1985; Tourniaire, 1986; Langrall & Swafford, 2000). These studies observe the students' informal knowledge about proportion, what kind of strategies they used to solve proportional problems and what factors that influence their performance. These studies give contribution to the developmental of theory about the students' proportional thinking and reasoning. However, those studies are more in theoretical way; less contribute to the practical side. Thus, we still need to do

more research to develop learning sequences which can be implemented directly by the teacher at school and also can be used to improve the theory development at the level of the instructional activities, the course (local instruction theories) and at the level of the domain-specific instruction theory as well. Therefore, within a design research, a learning trajectory and the instructional design were developed in this study. The designed activities were implemented and analysed based on the hypotethical learning trajectory.

2. 4 *Pendidikan Matematika Realistik Indonesia* (PMRI)

In the past, teachers always taught their students in traditional way, gave explanation about the concepts and procedures to solve the given problem. Because of that, most students in mathematics classroom did not see mathematics as creative instead as something to be explained by their teacher, then practiced and applied. It made mathematics as a dead language (Fosnot & Dolk, 2001). In order to make the learning process become more meaningful for the students, we should not give them the ready-made product; they should reinvent the idea of mathematical concept by themselves (Gravemeijer &Terwel, 2000; Grevemeijer, 1994). Mathematics should be understood as a human activity (Freudenthal, 1973), which is creative and alive. Accordingly, about 1970s the Realistic Mathematics Education (RME) was developed in the Netherlands.

RME then was adapted as *Pendidikan Matematika Realistik Indonesia* (PMRI) in Indonesia. The PMRI was implemented in Indonesia since 2001(Zulkardi, 2002). These both approaches are the same in principle but may be different in the context which was chosen for the problem. The context in PMRI was adapted so that fits into the Indonesian culture.

There are five characteristics of RME that should be considered as a guideline to make a learning sequence (Gravemeijer, 1994). The following explanation will describe about how the characteristics of RME fits the design in this study.

1. The use of contexts

The starting point of mathematics instruction should be real to the student. This process will allow them to investigate the situation. Through the process of finding and identifying the relevant mathematical elements, schematizing and visualizing, they can discover the patterns and develop a model in a mathematical concept. Then by reflecting and generalizing, the students will develop a more complete concept. While they apply mathematical concepts to other aspects on their daily life, they reinforce and strengthen the concept (Zulkardi, 2002). With regards to this, the activities in this study used the contextual problem as the starting point of the lesson, such as the money context. The students are familiar with the money context. They find it in their daily life. They buy things with money.They also familiar with situation which requires them to compare the price to have the best buying. This context allows them to use their experience to gain the "elementary" strategies to solve the given problems.

2. The use of models

Models are important for the students to bridge them from informal into formal mathematic. In this study, the ratio table will be the model to develop the students' proportional reasoning. At the first, the students just will see the ratio table as a usual table which is used to make a systematic list of certain items. Then they will realize that the ratio table can be used to do the calculation in proportional way. Then, in the end of the learning sequence, the students are able

to use the ratio table as a tool to thinking and reasoning in more formal level. The intermediate steps which are showed by the ratio table let the students to see the proportional relation among the numbers. Together with the context, the ratio table may develop the students' proportional reasoning.

3. The pupils' creations and contributions

In this study, the teacher will give the students time to think and explore the rules on ratio table by their selves. Their finding of the rules will help them to reach the formal algorithm for the higher grade. The role of the teacher is very important here. The teacher should encourage the students to involve in the discussion. Related with this, the teacher also should consider the social norm in the class.

4. The interactivity

The interaction between pupils and between pupils and teachers is an essential part in RME instructional processes. Explicit negotiation, intervention, discussion, cooperation and evaluation are essential elements in a constructive learning process. In this interactive instruction, pupils are engaged in explaining, justifying, agreeing and disagreeing, questioning alternatives and reflecting (Zulkardi, 2002). Regarding to this, on each lesson, the teacher will conduct the discussion in the classroom. The teacher should give chance to the students in developing their understanding as a taken-as-shared- pedagogy (Yackel & Cobb, 1996). Oce again, the teacher has the big role here. The teacher should create the situation inwhich the students can be interactive among each other.

5. The intertwining

The integration of mathematical strands or units is essential. The intertwining of learning strands is exploited in solving real life problems (Zulkardi, 2002).This characteristic is implied in this study. The topic of proportion is also related with the notion of other mathematical operation, such as linear function, fraction, and numbers operation.

2. 5 Ratio Table

Ratio table is an appropriate model to facilitate students in learning about proportion. It is an ideal aid for making handy calculations, gaining insight, and students reason with proportions because the table invites the students to write down intermediate steps. For instance, to find how many km which can be reached in 80 minutes if we can go for 15 km in 60 minutes, the students may use 5 in 20 as intermediate step or 30 in 120 then 10 in 40 to get 20 km in 80 minutes. The ratio table shows the proportion clearly because we can expand the number of columns to suit our needs. The strength of ratio table is that the students can reason with number relationship which they already know.

It is important for the teacher to discuss what can be done in ratio table. Context makes the students realize about what kind of operation that you can do, such as price and weight, you can multiply with same number but cannot add with the same number. It is important to state units are used and arrows to help the students see the operation. But it doesn't mean to introduce the ratio table from above without any discussion- the students may reinvent by themselves. Reinvent here is not about the notation but the choice of systematic notation method and discovering for themselves how useful a ratio table to do calculation. The

transition is from systematic list (the items under each category) to a ratio table. “The advantage of ratio table is that all numbers have their own place and that the unit of measurement must stay the same.” (van Galen et al, 2008)

Some rules in calculating with a ratio table that should be reinvented by the student were doubling, halving, adding, taking away, multiplying by the same number, dividing by the same number. Doubling, adding and multiplying by the same number were used to find the bigger number as the answer while halving, taking away and dividing by the same number were used to find the smaller number as the answer.

Table 2.1 *The Strategies on the Ratio Table*

Strategies	Description
Doubling	chocolates | Price ×2 { 2 | 1500 } ×2 4 | 3000 8 | 6000
Halving	chocolates | Price ×1/2 { 8 | 6000 } ×1/2 4 | 3000 2 | 1500
Adding	chocolates | Price + { 8 | 6000 } + 4 | 3000 12 | 9000
Taking away	chocolates | Price - { 8 | 6000 } - 2 | 1500 6 | 4500
Multiplying by the same number	chocolates | Price ×4 { 8 | 6000 } ×4 32 | 24000

Dividing by the same number		chocolates	Price	
	: 6	24	18000	: 6
		4	3000	

Beside these the strategies, the students may be able to use repeated addition and the unit strategy as the way to solve proportional problem. The unit strategy involved the division and multiplication. The students should find the unit which was used as the multiplication factor to find the answer, such as the price per kg to find the price of 15 kg of rice, the number of books per box to find the total number of books in 5 boxes, and the distance that can be driven for 1 liter of fuel to find the distance that can be driven for 6 liters of fuel.

2. 6 Social norms and socio-mathematical norms

"Social norms refer to expected ways of acting and explaining that become instantiated through a process of mutual negotiation between the teacher and students" (Gravemeijer& Cobb, 2006). In specific way, Yackel & Cobb (1996) defined the socio-mathematical norms as classroom norms which emphasize the mathematical aspects of student's activity, such as the understanding of what counts as a mathematical difference, a mathematical sophistication, an acceptable mathematical explanation and justification in classroom.Both of them have an important role to reach the learning goal of this research. By considering these factors, the teacher may establish the environmental in the classroom which is conducive to improve students' mathematical conceptual development (Yackel & Cobb, 1996). For instance, by discussing what can be count as different solution or give the students time to think about the other possible strategy may lead the

students to reinvent the rules on the ratio table, such as halving or doubling besides using the repeated addition.

In line with that, the aims of this study are not only to give a contribution to the development of the theoretical instruction, but also to improve the quality of the learning situation in the class. Through the teacher guide which developed in this study, we may guide the teacher to more concern in the learning process in the class. The teacher should realize that the social norms are important to make the class be conducive. The social norms is formed through a long process, it cannot be changed instantly. Thus, the teacher should be consistent in enforcing the "rules" in the class.

2. 7 Proportion in Indonesian Curriculum

Based on the Indonesian curriculum, the students learn about proportion on the 5th until 8th grade. During 5th grade they will learn about direct proportion and indirect proportion in the higher level. The following table shows how the proportions are taught as a part of fraction during the second semester in grade 5.

Table 2.2 *The Curriculum Grade 5 on Second Semester* (Depdiknas, 2006)

Standard Competence	**Basic Competence**
Numbers	
5. Using the fractions to solve problems	5.1 Converting the fractions into the decimals and vice versa. 5.2 Adding and subtracting fractions and decimals. 5.3 Multiplying and dividing fractions and decimals. ***5.4 Using the fractions in solving proportional problems and scale.***

According to Zulkardi (2002), most of textbook that used in Indonesia contain mainly the set of rules and algorithm which is already formal and they lack of application which is needed by the students in order to make the concept be real for them. In line with that, the lesson of proportion in grade 5 started by giving the fixed procedures to solve the problems (e.g. Soenarjo, 2007; Sumanto *et al*, 2008), less attention is given to students' understanding of the concept itself. As cited by Silvestre and Ponte (2012), Stanley, McGowan, and Hull stated that the usual teaching method in which teacher asked the students to solve proportion by using representations, equalities between ratios, and linear function should be replaced by other methods in which pupils engage in activities that help them to discover that proportion is the variation of two quantities related to each other. Thus, we need to propose a learning sequence that starts from the informal level and rich of contexts that may lead them to develop the proportional thinking.

Although proportional reasoning is not stated explicitly in the curriculum for the students under fifth grade, the sense of proportion itself has been learned since that. For instance, the children in kindergarten knew that the giant's chair is bigger than the normal human's chair or the students in the second grade knew that the number of steps from a man is smaller than the number of steps a 5 years old girl for the same distance. The students in 4^{th} grade learned about pattern and relationship which are also related with the proportional thinking. It means that they have some prior knowledge about the concept of proportion. Thus, the proportions are also appropriate for the 4^{th} grade students. In addition, this concept is important as the groundwork for them for the higher level.

Considering the mathematics curriculum, we can see that the topic of fraction also given in grade 4 (see Table 2.2). Although the problems in basic competence of "solving the problems which involves fraction" usually only involved the operations in the fraction, we can expand it to the simple proportional problem. It means that we can teach the topic of proportion integrated with the fraction as in grade 5. Accordingly, teaching proportion in grade 4 is reasonable and reliable.

Table 2.3 *The Curriculum Grade 4 on Second Semester* (Depdiknas, 2006)

Standard Competence	**Basic Competence**
Numbers	
5.Using the fractions to solve problems	5.1 Explaining the meaning of fractions and their order 5.2 Simplifying the fractions 5.3 Adding fractions 5.4 Subtracting fractions 5.5 ***Solving the problems which involves fractions***

CHAPTER III

METHODOLOGY

3.1 Participants

The present study was a part of nternational Master Program on Mathematics Education (IMPoME) project in Indonesia. This program is a collabartion between Sriwijaya University, Surabaya State University and Utrecht University which aims to develop the implementation of PMRI in Indonesia. In line with the goals of the IMPoME program, the present study was conducted in SDN 179 Palembang, Indonesia, which is one of the partner schools of PMRI in Palembang, Indonesia.

There were thirty five students grade 4 participated in this study. Six students from class 4 A were involved in the first cycle on this study while the remains, 29 students from class 4D, participated in the second cycle. The six students from class 4A were chosen based on their achievement in mathematics, namely low, middle and high. The information about the level of students' ability was gathered through the informal interview with their homeroom teacher, Bu Lisa.

3.2 Research Approach

The general aim of this study is to facilitate the pupils in developing their proportional reasoning. Within a design research, a learning trajectory was developed and tested. The result of this study also contributes to the development of local instruction theory. Design research is chosen in this study because it is an approach which can bridge between the practical side and the theoretical side (Bakker & van Eerde, 2012). Furthermore, according to Bakker (2004),

> "Design research is evaluated against the metrics of innovation and usefulness, and its strength comes from its explanatory power and grounding in experience. Moreover, it often leads to products that are useful in educational practice because they have been developed in practice."

Gravemeijer and Cobb (2006) defined three phases in a design research experiment. The description of each phase and how the different roles of a Hypothetical Learning Trajectory (HLT) in each phase are given as follows.

1. Preparing for the experiment

From the design point of view, the purpose of this phase is to formulate a local instruction theory which can be elaborated during conducting the intended design experiment. Meanwhile from the research point of view, the aim of this phase is to clarify the theoretical intent. Regarding this, we started the preliminary phase by developing the sequence of instructional activities. There were 5 activities that were conducted in 4 meetings, each 70 minutes. The conjectures of students' thinking and reaction were also included. We elaborated all of these in a Hypothetical Learning Trajectory (HLT). As mentioned in Bakker (2004), the use of HLT in this phase is as a guide in designing and developing the instructional activities. As the preparation for the class observation and the interview with the teacher, we also provided a classroom observation scheme and an interview scheme (see appendix 1 and 2). These schemes were used as the remainder so we did not miss the important aspect during the interview or the observation.

2. The design experiment

The design experiment phase aims to test and improve the conjectures in our HLT and to understand how it works. In this phase, the learning sequence was

implemented in two cycles. The first cycle was done as the pilot study. A small group consists of 6 students from class 4A were participated in the first cycle. The findings from the first cycle were used to adjust and to improve the first version of HLT. Then the revised HLT was implemented in the experimental class during the second cycle. Twenty nine students Grade 4 were involved in this teaching experiment. In this phase, the HLT has a role as a guideline to conduct the teaching and learning process (Bakker, 2004).

3. Retrospective analysis

The retrospective analysis phase is important to do in design research. The goal of this phase is to contribute in the development of a local instructional theory. The analysis was done by focusing on the conjectures in HLT. In line with this, Bakker (2004) stated that the third function of HLT in design research is as a guideline to do the analysis.

3.3 Data Collection

In this study, there were some data that we used to answer the research question. The data were collected in different way and for different purpose. The more explanation is described in the following statements.

3.3.1 Preparation phase

Before the researcher conduct the teaching experiment, the researcher need to gain some information as the starting point for the research. During the preparation phase, the researcher did classroom observations in the experimental class, an interview with the teacher and pretest respectively.

1. Classroom observation

The classroom observation was conducted as a preparation before the teaching experiment on the second cycle. We need to do classroom observation in the experimental class due to know how the teacher teaches the students, the classroom' culture and the socio-mathematics norms in the classroom, the practical thing about the classroom setting, and more or less the behaviour and characteristics of the students in the class. During the classroom observation, the researcher recorded the whole learning process in video/ camera and also made some field notes based on the observation scheme.

2. Interview with the teacher

The semi-structured interview with the teacher is also important to support or compare the findings during the classroom observation. The researcher asked the questions to the homeroom teacher from class 4D, Bu Intan based on the interview scheme (see appendix 2) and also some follow up questions. The whole interview was recorded by recorder. During the interview, the researcher also made some notes for the important points.

3. Pretest

The pretest was conducted twice. The first written test held in the beginning of the first cycle. The participants of the first pretest are 6 students from 4A. The aim of the first pretest is to see whether the problems are clear and understandable for the pupils or not. Besides, it also used to check whether the problems on the pretest can measure the students' prior knowledge like we want or not. The other advantage of doing the small pretest with the small group is that we can ask directly to the pupils about their interpretation of the problems, also about their

thinking, how they solve the problem, why they solve like that, what is the idea behind their strategy. Then we can use this to improve our problems for the second pretest.

The second pretest was conducted in the beginning of second cycle, before the teaching experiment. The participants were all students from the experimental class. The teacher gave the test and the students work individually for the given time. The aim to do the second pretest is to know the prior knowledge of the students. We used these findings to determine the starting point for the experiment and to do the first adjustment of the revised HLT after conducting the first cycle.

3.3.2 Preliminary teaching experiment (the first cycle)

The first cycle was conducted in a small group of 6 students from class 4A. The students were chosen based on the information from the teacher about their achievement in mathematics. They are Ferdy as the representation of higher achievement, 4 middle ones namely: Rati, Lani, Ando, and Falah, and 1 lower achiever namely Nurul.

During the first cycle, the researcher taught the lesson based on the initial HLT as in chapter 4. We recorded each lesson by video/ camera. We did small interview with the students during the lesson to investigate their understanding about the topic. We also made field notes and collect all students' written work. This cycle is an important step because the results of the first cycle were used to improve and revise the HLT for the second cycle.

3.3.3 Teaching experiment (the second cycle)

The participant of the second cycle was all students from the experimental class. Twenty nine students of class 4D participated in this cycle. The class was taught

by their own teacher based on the revised HLT. Instead of teaching, the researcher was observing the learning process and making some field notes. The whole learning process was recorded by video/ camera. The students' written works also were used as the data collection.

During the second cycle, we also have a small group of 2 as the focus group. We recorded the discussion in that group. The data from the focus group was used to get more detail picture about what happened during the learning process.

3.3.4 Posttest

To know the development of the students' understanding about proportion, the researcher should conduct a pretest before the teaching experiment and a posttest after the teaching experiment. As the pretest, the posttest also were conducted twice. First, in the end of the pilot experiment and the second one was held in the end of the second cycle. The other aim of conducting the first posttest is to check the structure of the problems whether it is understandable for the students or not and then use the result to improve the problems on the second posttest.

During the posttest, the teacher gave some proportional problems (see appendix 3) and the students worked individually for the given time. The problems on the posttest were different with the problems on pretest but still with the same indicators. We compared the students' answer and reason from the pretest and the post test. Although the main goal of this study is not to evaluate the teaching method, the comparison between the students' result from the pretest and the pos test can be used as impression of the result from the lessons.

The participants of the first posttest were the 6 students on the first cycle. Meanwhile the participants of the second posttest were all students from the experimental class.

3.3.5 Validity and reliability

The methods which are used to collect the data during this study are not limited in one method. The data from the interview are collected by using an audio device while the data from the classroom observation are collected by using video recorder and also by making some field notes. The variation of how we collect all the data makes the methods triangulation possible. It contributes to the validity of data collection.

The clear description on how the data collected may let the reader to follow the process of data collection. It improves the external reliability of the data collection because of the traceability. In addition, the reliability also refers to the reasonableness and the power of arguments for the inferences. To improve that, we also did the peer examination by discussing the critical issue with the teacher and the colleagues.

3.4 Data Analysis

This study involves different kind of data. We have data from the classroom observation, interview with the teacher, the written test before and after the teaching experiment, and also the students' written work during the lessons. We analyzed them by using HLT as the guideline to derive a conclusion and answer the research question in this study.

3.4.1 Pretest

The analysis data of the pretest involved a quantitative and a qualitative analysis. We examined the students' work and count the number of correct answer to determine the level of students' understanding. Indeed, we also looked in the students' scratch and their strategy to get more details picture about the student's thinking. We used these findings to determine the starting point for the experiment.

3.4.2 Preliminary teaching experiment (the first cycle)

The data from the first cycle are the video from the whole lessons, field notes and the pupils' written work. The data analyses of the first cycle were started by watching the video-registration and choose some fragments of them. We should consider the fragments in where the videos show that the learning process is success and reach the goal as we expected or otherwise, it is not happened like our expectation. Then the selected fragments were transcribed and were used to test the conjectures in the first version of HLT. To support the result of video analysis, we used the data from pupils' written work and the field notes as well. We used these findings to revise and improve the initial version of HLT for the second cycle.

3.4.3 Teaching experiment (the second cycle)

The data from the classroom observation and teacher interview during the preparation phase were analysed to get the basic information about the context in which the teaching experiment was conducted. The information can be used when discussing the lessons with the teacher and to elaborate the teacher guide.

The data of the second cycle are the video of the whole lessons, the video of the focus group and the pupils' written work. The analysis of the video observation was started by watching the video registration. Some interesting fragments of the video were transcribed and used to test the conjectures on HLT. The analysis from the video of the whole lessons in the classroom was used to get an overview of the lesson, how the learning process happened in the classroom, what is missing during the teaching process or the discussion part. Meanwhile the analysis from the video of focus group was used to get more detail picture about the development of students' understanding. In addition, the students' written work was used as additional data. Overall, the results of the analysis were used to derive conclusions, to answer the research question and to improve the original HLT as well.

3.4.4 Posttest

As in the analysis data of the pretest, in the analysis data of posttest also involved a quantitative and a qualitative analysis. We examine the students' work, to count the number of correct answer and to see their strategy. We use these result and compare it with the result of pretest to see what students have learned. Thus, we get the impression of how the learning sequences work, how the ratio table may help the pupils in developing their proportional understanding.

3.4.5 Validity and reliability

The data collection of this study is varying, namely the videos, the field notes, and the students' written works. The analysis of more than one type of data makes data triangulation possible. As consequence, it improves the internal validity of

the data analysis. In addition, the testing conjectures of the HLT with the collected data also contribute to the internal validity of data analysis.

By presenting the clear and detailed description of how we analysis the collected data and testing our HLT may let the other people to follow the way of analysis and be able to adjust the HLT to their own local setting. It contributes to the external validity and also the external reliability of data analysis as well, in the sense of transparency.

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Hypothetical learning trajectory (HLT) is a way to describe an important aspect of pedagogical thinking in teaching mathematics (Simon, 1995 in Simon and Tzur, 2004). An HLT contains the mathematics goal, the mathematics activities, and the students' conjecture which describes the learning process of the students.

The aim of this study is to know how the ratio table can support the pupils to develop proportional reasoning. In line with the aims, the instructional activities are designed to introduce the ratio table to the students, to lead the students reinvent the rule on ratio table and to develop the students' proportional reasoning by using ratio table. Within a design research, the learning trajectory was tested. As consequence, the result of this study contributes to the development of a local instruction theory about proportion.

In this chapter, we present the HLT which was used in this study. There are five activities which were implemented in grade 4 of SDN 179 Palembang in Indonesia. The general overview of the HLT is described in the following table and the more detail information is presented in the following sections.

Table 4.1 *The Brief Description of HLT*

No	Activity	Description	Learning Goal
1	Make Butterflies	The students determine the number of wings, bodies, and antennas to make certain numbers of butterfly if known that to make a model of butterfly we need 4 wings, 1 body and 2 antennas.	• Students understand the notion of ratios in proportional problems.
2	Feeding Caterpillars	The students determine the number of leaves which are needed to feed caterpillars in certain number of days if	• To introduce the ratio table to the students - To introduce the table as a systematic list to

		known that we need 5 leaves to feed the caterpillars in two days. The type of problem is missing values.	the students. - Students explore the strategies that can be done with ratio table.
3	My Chocolate	The students make a list of chocolate bars' price with their own number. They determine the number of chocolate bars by themselves.	• Students explore the strategies that can be done with ratio table. • The students find the rule of 1
4	What do you think?	The students should solve the given problem and discuss the incorrect strategy as well. *"If we get 3 glasses of chocolate milk for 2 coupons then how many coupons we need to buy 24 glasses of chocolate milk."*	• The students know and understand what rules that cannot be applied on the ratio table and why it so.
5	Which is the cheaper one?	There are 2 items which have different prices for different number of items. The students should determine which one is the cheaper one between the two.	• The students use the ratio table to solve proportional problem. • The students use the ratio table as a tool to thinking and reasoning.

4.1 Activity 1: Make Butterflies

A. Mathematics Goal

The goal of this lesson is to lead the students to understand the sense of ratio in proportional problem.

B. Starting points

The students in grade 4 already learned about some number operation such as addition, subtraction, multiplication and division in the previous grade.

C. Mathematical Activities and Students' Conjectures

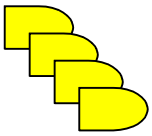
Step 1: Working in pairs (10-15 minutes)

The teacher will ask the following question:

Make Butterflies

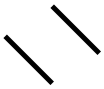
There are four parallel class of 4th grade who visit a science fair. During the workshop in there, each student will build models of butterflies.

To make a model of butterfly, they will need the following items.



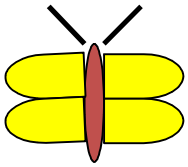
4 wing pieces

1 body



2 antennas

When the model put together, it looks like this:



Determine the number of wings, bodies and antennas which are needed for each class if we know that the number of student in each class as follows:

a. 4A consists of 27 students
b. 4B consists of 26 students
c. 4C consists of 25 students
d. 4D consists of 24 students

Step 2: Classroom discussion (45-50 minutes)

- The teacher makes a table on the blackboard and gives the title for each column. The discussion about the reason why the number of students in the class and the number of butterflies is equal should be happened in the class. It's also important to make the students realize that 1 student makes 1 butterfly is also a ratio 1:1.
- The teacher asks the students about their answer. The teacher writes the students' answer on the table then discuss how they can get that.

The number of butterflies	Wings	Bodies	Antennae
27	108	27	54
26	104	26	52
25	100	25	50
24	96	24	48

A prediction of students' response:

- Using multiplication by 4 to find the number of wings, multiplication by 1 to find the number of bodies and multiplication by 2 to find the number of antennas for all number of butterflies (the question a to d).
- The students may using the multiplication by 4, 1 and 2 just for the two first number and look the pattern which is if the number of butterfly decrease by one then the number of wings will decrease by 4. Then for the next, they just add or subtract to find the answer.
- Using repeated addition to solve the problem for questions a to d.
- Find the number of wings, bodies and antennas for 25 butterflies first because it's the easiest to be found and some of them may already realize about the strategy to add or subtract by 4, 1 or 2 to find the number of wings, bodies, and antennas respectively.

To solve the problems, most students may use multiplication as their strategy because they learned it in the previous grade. However, we hope that the students also will come up with the idea of subtracting by 4 or adding by 4 to get number of wings if the number of butterflies decreases or increases by 1(by using the pattern of the ratio).

- During the discussion, the teacher also asks some follow up questions to help the students see the pattern of adding by 4 for every adding 1 butterfly, such as

  "*What about 23 butterflies?"*

  "What about 28 butterflies?"

- The teacher asks other follow up question which aims to let the students realize the idea of division in proportional problem.

  "*How many complete butterflies that we can make from 90 wings, 23 bodies and 45 antennas?"*

A prediction of students' response:

- Dividing the 90 by 4, 23 by 1 and 45 by 2 and give the different number of complete butterfly for each item.
- Dividing the 90 by 4, 23 by 1 and 45 by 2 and give the smaller number of complete butterfly from each item as the answer.
- Using the ratio 4:1:2 to find the numbers of each item which are the nearest number from the given problem. For instance, by multiplying each items by 20 we get 80:20:40 and by adding two more butterfly we get 88:22:44
- Using the number of body as the starting point then divide the 90 and 45 by 23 to see whether there is enough wings and antennas to make 23 butterflies or not. If not, then they ca subtract the number of butterfly one by one.

4.2 Activity 2: Feeding Caterpillars

A. Mathematics Goal

The goal of this lesson is introduce the ratio table as a systematic list and a tool to do calculation and to lead the students find the strategies that can be done with the ratio table.

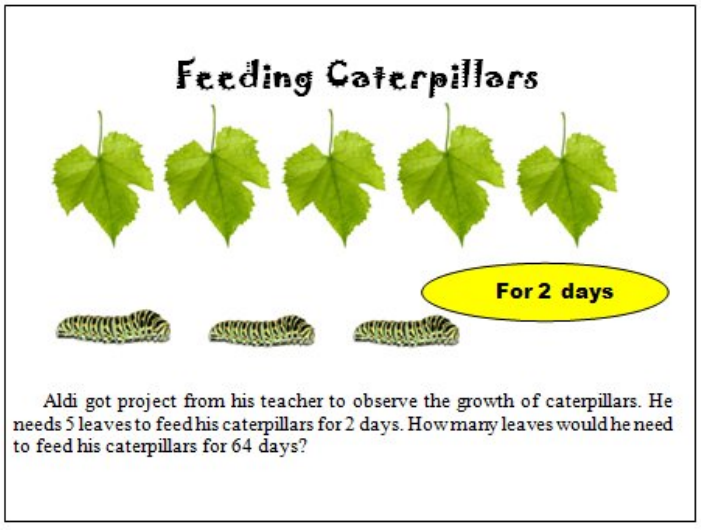
B. Starting points

The students already discussed the sense of ratio during lesson 1 and they already knew how to write on the table in systematic way as the teacher wrote on the blackboard during the discussion section.

C. Mathematical Activities and Students' Conjectures

Step 1: Working in pairs (10 minutes)

- The teacher will ask the following question:



Step 2: Classroom discussion (60 minutes)

- The teacher asks the students to present their answer and their strategy. Discuss the students' strategies

A prediction of students' response:

- Using division by 2 and multiplication by 5 to find the number of leaves.
- Using doubling strategy by using ratio table.
- Adding by 5 more to get the number of leaves for 2 days, adding by 5 more to get the number of leaves for 4 days (repeated addition).
- Using the strategies grouping by 2 and then counting how many two in 64. Then multiply the number by 5.

- If there are students who come up with the idea of using list, the teacher can start the discussion about the ratio table from here. If not, the teacher starts the discussion by making the following table on the blackboard to check whether the presented answer is correct or not. For this problem, the main strategy that would be reached is doubling. The idea about how to make the list in systematic way should be emphasized. The teacher also says that the table is called ratio table.

Days	Leaves
2	5
4	
8	
16	
32	
64	

- During the discussion, the teacher also can ask some follow up questions like

  "*What about 6 days?"*

  "What about 10 days?"

  "What about 24 days?"

A prediction of students' response:

- The number of leaves for 6 days can be found by add the number of leaves for 2 days and 4 days
- The number of leaves for 6 days can be found by subtract the number of leaves for 8 days by the number of leaves for 2 days.
- The number of leaves for 10 days can be found by add the number of leaves for 8 days and the number of eaves for 2 days
- The number of leaves for 10 days can be found by find the half of the number of leaves for16 days plus 4 leaves
- The number of leaves for 10 days can be found by multiplying the number of leaves for 2 days by 5.

- The discussion can be continued to gain other strategies, such as the halving, adding and taking away or multiplying and dividing by the same numbers. For each question, the teacher should give the students time to think and discuss in their group. The student can make a note or scratch on the blank answer sheet which be given during the discussion. The teacher should encourage the students to find different strategies.
- The teacher also can propose the question the other way around

"For how many days 70 leaves can be used?"

A prediction of students' response:

- The students may divide the 70 by 5 to find the number of days.
- The students divide 70 by 5 and multiply it by 2.
- Using the group of 5 and multiply the number of group by 2

- Using the number of leaves for 4 days, which is 10 as the starting point. Thus, they can find easily by multiply the 4 by 7.

4.3 Activity 3: My Chocolate

A. Mathematics Goal

The goal of this lesson is to do more exploration about the strategies that can be done with ratio table and to lead the students to reinvent the rule of 1.

B. Starting points

The students already discussed some rules that can be used by using ratio table on lesson 2.

C. Mathematical Activities and Students' Conjectures

Step 1: Classroom discussion (15 minutes)

- The teacher shows the following figure to the students.

- The teacher asks to the students for some prices when draws a ratio table on the blackboard.

Chocolate bars	Money
4	3000
?	6000
12	?

- First the teacher asks "How many chocolate bars that we will get for Rp6.000?" Discuss how they can find it.

A prediction of students' response:

- Using doubling strategy
- Adding by 4 more
- Divide the 6000 by 3000 and multiply it by 4.

- Then the teacher asks "How much we paid for 12 chocolate bars?"

A prediction of students' response:

- Adding the price for 4 and 8 chocolate bars.
- Multiply the price for 4 chocolate bars by 3.
- Dividing the 12 by 4 and multiply it by 3000.

Step 2: Work in pairs (15 minutes)

- The teacher asks the students to complete the list of chocolate bars' price on the worksheet. The teacher should encourage the students to choose the bigger or more difficult numbers. They can explore by using the strategies that they knew from the previous lesson.

Chocolate bars	Rupiah

From this activity we can see whether they use some manipulative rules or not.

A prediction of students' response:

- The students just using doubling strategy until a certain number of chocolate.
- The students choose the number of chocolate bars randomly and they calculate to find the price by using multiplication or division.
- Find the price of 1 chocolate bar and multiply any number that they choose with the price.
- They combine two or more different manipulative ways to determine the number of chocolate bars and the price as well.

Step 3: Classroom Discussion (40 minutes)

- The teacher will discuss some numbers that proposed by the students and asked how they can get that numbers.
- The teacher can give some follow up questions to the students in order to guide them reinvent the strategies in case if they cannot generate the strategies by using the numbers that they proposed. The possible number to be used as the follow up questions as follows.

1. *How many chocolate bars for Rp12.000?*
2. *How many chocolate bars for Rp21.000?*
3. *How many chocolate bars for Rp27.000?*
4. *How much we should pay for 40 chocolate bars?*
5. *How much we should pay for 44 chocolate bars?*
6. *How much we should pay for 48chocolate bars?*

7. *How much 13 chocolate bars?* (to gain the idea of rule 1).

8. *How many chocolate bars that we get for Rp.108.000?*

(Sometimes, by giving the difficult number, we can stimulate the children to use the manipulative strategy).

4.4 Activity 4: What do you think?

A. Mathematics Goal

The goal of this lesson is to discuss the rules that can be applied and cannot be applied on the ratio table and why it is so.

B. Starting points

The students already discussed and explored the rules that can be applied on ratio table during lesson 2 and 3.

C. Mathematical Activities and Students' Conjectures

Step 1: Work in pairs (10 minutes)

- The teacher proposes the following problem in students' worksheet.

> In the bazaar, we can buy 3 glasses of chocolate milk with 2 coupons. How many coupon that we need to get 24 glasses of chocolate milk?

Step 2: Discussion to the whole class (50 minutes).

- The teacher discuss about the students' strategies.

A prediction of students' response:

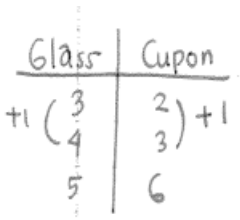
- The students may use the multiplication and division to find the answer, 24 is divided by 3 and multiplied by 2.
- The students use correct manipulative strategies, such as doubling, halving, adding, taking away, or multiplying and dividing with the same number. For instance, the students may double the 3 glasses-2

coupons to get 6 glasses-4 coupons and multiply it by 4 to get 24 glasses-16 coupons. The students may use multiplication 3 until they get 27 glasses- 18 coupons and realize that they cannot use multiplication of 3 anymore. They may subtract the 27 by 3 and 18 by 2.

- The students maybe using repeated addition until they get the number of coupons for 24 glasses.
- The students who use incorrect strategies such as adding and subtracting with the same numbers.

- The teacher asks the students with incorrect strategy to present their work (if there is) and asks the other students to response about the answer. The teacher should underline that the students should try to understand the strategy which are proposed by their friends before they can argument about it.
- If there is no student who comes up with the wrong strategy then the teacher may propose the idea of incorrect strategy. The teacher may show it from LCD or just write it on the blackboard. The teacher may say that

"There is a student from another class who answer as follows. The student didn't finish his work yet. But, can you understand what he tried to do? What do you think about his way?"



It is also important for the teacher to ask why a certain rule should not be applied on ratio table, such as adding by 1 for both coupon and the number of glass. It is not because the teacher says so, or because that is how a ratio table works, but because the context tells us that for each 2 coupons you get 3 glasses. So the children should not learn a rule (multiplication yes, addition no), but they should learn always to keep the context in mind; if in doubt think about the proportional context.

A prediction of students' response:

- The students may say that we cannot add for the same number like in that answer because they realize that for each 2 coupons you get 3 glasses.
- The students may say that it is possible to add 1 on each because for 1 more glass, we need 1 more coupon.

- In the end of this lesson, the teacher gives some remarks about what strategies are allowed to be applied on the ratio table and what are not.

4.5 Activity 5: Which is the cheaper one?

A. Mathematics Goal

The goal of this lesson is to introduce the ratio table as tools to solve proportional problem. For this lesson, the type of the problems is comparison problem. In this lesson, the ratio table is used as a tool to thinking and reasoning.

B. Starting points

The students are able to use ratio table as a tool to do calculation and they learned the rules on the ratio table that may be used to do the calculation.

C. Mathematical Activities and Students' Conjectures

Step 1: Work in pairs (10 minutes)

Step 2: Discuss to the whole class (20 minutes)

- During the discussion, the teacher will discuss about the students' thinking, how they will compare the two different things. It is important to discuss why we used proportion instead of the absolute value in comparison problem.

A prediction of students' response:

- The students may answer that the left is more expensive than the soft drink on the right side because 4.000 is bigger than 3.000.
- The students may realize that the number of cans also influence the price.

- The teacher discusses some ideas to find the cheaper prize.

A prediction of students' response:

- Look the number of cans for the same price, such as for Rp12.000.
- Look the price for the same number of cans, such as the price for 6 cans.

- By unitizing, looking for the price per 1 can. However the calculation may be difficult for the students because they may have difficulties to divide 4.000 by 3.

Step 3: Work in pairs for the second problem (10 minutes).

2. There are two type of rice in the shop. The cost of the first type is Rp120.000 per 15 kg and the cost of second type is Rp45.000 per 5 kg. Which is the cheaper one?

Step 4: Discuss to the whole class (25 minutes)

- The teacher discusses the students' answer. Based on the first problem, we can assume that the students already realize and understand the idea of proportional in comparison problem. Thus, the teacher can recall about this idea only if needed.

A prediction of students' response:

- Look the number of kg for the same price, such as for Rp120.000.
- Look the price for per kg rice (by unitizing the weight).
- Look the price for the same weight such as the price for 5 kg rice.

- If there is remaining, time, the teacher can asks a missing value problem with more complex situation which involves the mixture problem.

"If the seller mix the 2 sacks of the first type rice and 3 sacks of the second type of rice, how much the price of the mixture rice per kg?"

A prediction of students' response:

- By using the price per kg, they multiply it with the weight of each type then divide it by the total weight of mixture rice.

- They multiply the price by number of sacks and divide it by number of sacks.