EDUCATIONAL DESIGN RESEARCH:
DEVELOPING STUDENTS' UNDERSTANDING OF THE MULTIPLICATION STRATEGY IN AREA MEASUREMENT

MASTER THESIS

UNESA
Universitas Negeri Surabaya

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UNIVERSITAS NEGERI SURABAYA
PROGRAM PASCASARJANA
PROGRAM STUDI PENDIDIKAN MATEMATIKA
2013

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A Thesis Submitted to
Surabaya State University Postgraduate Program
as a Partial Fulfillment of the Requirement for the Degree of
Master of Science in Mathematics Education Program

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2013

APPROVAL OF SUPERVISORS

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DEDICATION

This thesis is dedicated to my parents, my teachers, my sisters, my brothers, and my friends who give me never ending love and support.

It is also dedicated to my lovely fiancée, Uswa, who has been a great source of my motivation and inspiration.

I love you all

Berani Hidup tak Takut Mati. Takut Mati Jangan Hidup.
Takut Hidup Mati Saja!!!
Karena Hidup ini Hanya Sekali, Hiduplah yang Berarti.

ABSTRACT

Putrawangsa, Susilahudin. 2013. *Educational Design Research: Developing Students' Understanding of the Multiplication Strategy in Area Measurement.* Thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University. Supervisors: (I) Prof. Dr. Siti Maghfirotun Amin, M.Pd. dan (II) Dr. Agung Lukito, M.S.

Keywords: Area, Area Measurement, Multiplication Strategy, Design Research

Multiplication strategy or area formula, length times width, is commonly introduced to Indonesian students as the strategy in measuring area of a plane figure. However, many studies reveal that the use of the multiplication strategy *'without understanding'* on the concepts underpinning the strategy mainly contributes to students' difficulties and poor understanding in area measurement. Therefore, this study intends to develop a local instructional theory that supports students in developing their understanding of the concepts underpinning the multiplication strategy in area measurement.

This study is conducted based on the view of Design Research approach consisting of two cycles of teaching experiments. The aim of the study is to contribute to the development of a local instructional theory in developing students' understanding of the multiplication strategy in area measurement about both the process of learning and the means designed to support that learning. The data are collected from observation of teaching experiments, students' work, pretest and posttest, teacher and student interview, and classroom observation. The data are analyzed by testing the hypothetical learning trajectory to the actual learning trajectory. The study is conducted among the third grade students in two primary schools in Surabaya Indonesia.

The findings of this study suggest that to help students in developing their understanding of the multiplication strategy in area measurement, the students need to be able to see the relations among the concepts underpinning the strategy, such as the measurement units of area, the array structure of the units, the multiplicative structure of the array, and the role of dimensions as the representation of the array. Those concepts should be learned consecutively since each of the concepts is built on one after another consecutively. The learning activities that can be used to develop the understanding on those concepts are: (1) comparing area activity by using hands or books as the measurement tools could help students in developing their understanding of the measurement units of area, (2) Structuring array activity of square units is an excellent activity to help students in understanding the array structure of area units, (3) Investigating the area of rectangular surfaces having less and less graphic information of clues of units can be used to help students in developing their understanding of the multiplicative structure of the array structure, and (4) Investigating the area of plane figures where their dimensions are given can be used to introduce the role of dimensions as the representation of the array structure in applying the multiplication strategy in area measurement.

ABSTRAK

Putrawangsa, Susilahudin. 2013. *Educational Design Research: Developing Students' Understanding of the Multiplication Strategy in Area Measurement*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Siti Maghfirotun Amin, M.Pd. dan (II) Dr. Agung Lukito, M.S.

Kata Kunci: Luas, Pengukuran Luas, Strategy Perkalian, Penelitian Pengembangan

Strategi perkalian atau rumus luas, panjang kali lebar, biasanya diperkenalkan kepada siswa di Indonesia sebagai cara mengukur luas bangun datar. Akan tetapi banyak penelitian menunjukkan bahwa penggunakan strategi perkalian ini tanpa memahami konsep yang mendasarinya adalah penyebab utama kesulitan siswa dan buruknya pemahaman siswa terhadap pengukuran luas. Oleh karean itu, penelitian ini bertujuan untuk mengembangkan teori pengajaran yang membantu siswa dalam mengembangkan pemahamannya terhadap konsep yang mendasari strategi perkalian dalam pengukuran luas.

Penelitian ini dilakukan berdasarkan pendekatan Design Research yang terdiri atas dua siklus uji coba pengajaran. Tujuan dari penelitian ini adalah berkontribusi terhadap pengembangan teori pengajaran dalam mengembangkan pemahaman siswa terhadap strategi perkalian dalam pengukuran luas, yaitu mengenai proses pembelajaran dan hal lainnya yang dirancang untuk mendukung proses pengajaran tersebut. Data dalam penelitian ini didapatkan dari pengamatan terhadap uji coba pengajaran, hasil kerja siswa, tes awal dan tes akhir, wawancara guru dan siswa, dan pengamatan kelas. Data-data tersebut dianalisa dengan menguji hipotesa lintasan belajar dengan lintasan belajar yang sesungguhnya terjadi di lapangan. Penelitian ini di selenggarakan pada siswa kelas tiga di dua Sekolah Dasar di Surabaya Indonesia.

Temuan dari penelitian ini menyarankan bahwa untuk membantu siswa dalam mengembangkan pemahaman mereka mengenai strategi perkalian dalam pengukuran luas siswa tersebut membutuhkan kemampuan untuk melihat hubungan antara konsep yang membangun strategi tersebut, yaitu: konsep satuan luas, struktur array satuan tersebut (susunan satuan dalam baris dan kolom), struktur perkalian dari dari struktur array tersebut, dan peranan dimensi sebagai representasi dari struktur array tersebut. Konsep-konsep tersebut harus dipelajari secara berurutan karena setiap konsep dibangun di atas konsep lainnya secara beraturan. Kegiatan belajar yang dapat diterapkan untuk mengembangkan pemahaman terhadap konsep tersebut antara lain: (1) kegitan membandingkan luas dengan menggunakan telapak tangan atau buku sebagai alat pengukuran dapat membantu siswa dalam mengembangkan pemahaman mereka terhadap satuan pengukuran luas, (2) kegiatan menyusun array dari satuan persegi adalah kegiatan yang tepat untuk membantu siswa dalam memahami struktur array dari satuan luas, (3) investigasi terhadap luas permukaan persegi empat yang memiliki informasi grafik tanda satuan yang semakin berkurang dapat di gunakan untuk membantu siswa dalam mengembangkan pemahaman mereka mengenai struktur perkalian dari strukutr array tersebut, dan (4) investigasi luas bangun datar dimana dimensinya diberikan dapat digunakan untuk memperkenalkan peranan dimensi sebagai representasi dari struktur array ketika menerapkan strategi perkalian untuk pengukuran luas.

PREFACE

Grateful thank to Allah the Almighty for His blessing so that this thesis can be finished in due time. Blessings and greetings to the Prophet Muhammad, peace be upon him, for his motivation through his wise teaching and advices.

Accomplishing this thesis is a great achievement for me. I will not be able to do it well without guidance, assistances, supports, and motivations from people around me. Therefore, I would like to express my great gratitude to:

1. Prof. Dr. Siti Maghfirotun Amin, M.Pd. and Dr. Agung Lukito, M.S. as my supervisors who kindly guides and assists me during the period of collecting data, writing the analyses and preparing the report of my research.
2. Drs. Monica Wijers and Dr. Dolly van Eerde from Freudenthal Institute of Utrecht University who genially guides and assists me during the period of establishing instructional theory, designing learning materials, and completing research methodology.
3. Dr. M.L.A.M. Dolk, the coordinator of the International Master Program on Mathematics Education in the Netherlands, for his support and motivation during my study period in Utrecht University the Netherlands.
4. PMRI Center Board for the opportunity given to me as one of the grantees of the International Master Program on Mathematics Education.
5. All lecturers and staff of postgraduate program of Surabaya State University and Utrecht University for their supporting during conducting my master.
6. The mathematics teachers and the third grade students of SD LAB UNESA and SDIT Ghilmani Surabaya for their excellent cooperation during the period of collecting data.
7. My parents, my sisters, my brothers, and my friends for their motivation and support. Especially my fiancée, Uswa, for her supports, motivation, and inspiration.

Finally, I consciously understand that this thesis is far from being perfect. Thus, any critics and constructive ideas will be gladly accepted.

Susilahudin Putrawangsa

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CHAPTER I

INTRODUCTION

A. Background

Measuring area is one of the most commonly utilized forms of measurement that is closely associated with real world applications, science, and technology (Hirstein, Lamb, & Osborne in Huang & Witz, 2009). Students in early age need to learn area measurement to help them seeing the usefulness of mathematics in everyday life (Reys et al. in Yuberta, 2011).

Multiplication strategy is commonly introduced to students to measure the area of a plane figure, especially rectangular plane figure, by multiplying the length and the width of the figures. This strategy is also known as the multiplicative formula (Clement & Stephen, 2004) which is intensively used in measuring the area of a rectangular plane figure.

Multiplication strategy provides an efficient way in finding the area by just multiplying the length and the width of the surface being measured, especially a rectangular surface. Moreover, the multiplication strategy becomes the foundational idea for the students to understand other area formulas. For example, understanding the formula for finding the area of triangles and parallelograms involves the idea of the multiplication strategy (why we have to multiply the based and the height of a parallelogram when finding its area). That is why the multiplication strategy plays an important role in learning area measurement.

There are two concepts are mainly involved in applying the multiplication strategy, such as the array structure of area units and the array notion of multiplication of the array structure (Clement & Stephen, 2004). Therefore, understanding the area units of area and the array structure of the area units becomes the foundational concepts that need to be understood beforehand in order to be able applying the multiplication strategy meaningfully.

In Indonesian classrooms, the multiplication strategy or length-time-width formula is commonly introduced to students as the strategy in measuring area of a plane figure (Fauzan, 2002). However, most of them apply the formula without understanding why the formula works (Fauzan, 2002). This contributes to students' poor understanding in area measurement, such as being difficult to measure the area of non rectangular surfaces and having no idea of the notion of unit of area.

There are many reasons why the multiplication strategy contributes to students' poor understanding in area measurement. A study of classroom instructions in Indonesia by Fauzan (2002) found that the classroom instructions which are dominated by remembering facts and concepts verbally, studying computational aspects, and applying formulas or strategy without understanding cause students' poor understanding in many concepts of mathematics, including area measurement. In studying area measurement, for example, the multiplication strategy, length-time-width formula, is usually given right away to students without developing their understanding of the concepts underpinning the strategy. Students just apply the formula without knowing the idea behind the formula and why they have to apply the formula.

Moreover, a study by Zacharos (2006) suggests that students tend to generalize the multiplication strategy in measuring the area of a plane figure regardless the shape of the figure being measured due to the excessive use of the multiplication strategy without understanding. When students are asked to measure the area of a non rectangular figure, for example, they just multiply the length and the width of the figure without considering that the multiplication strategy can only be applied in rectangular plane figure. Moreover, the idea of the measurement units of area tends to be neglected by the students due to introducing the multiplication strategy in early stage in learning area measurement (Bonotto, 2003). Students just consider the product of the multiplication strategy, length time width, as merely the product of arithmetic operation (multiplication). They do not consider it as the number of the measurement units.

Regarding to the students' poor understanding in area measurement explained above, it raises the need of developing students' understanding on the multiplication strategy in area measurement to help them gain a better understanding on the use of the strategy in finding area.

B. Research Question

Regarding the problems explained above, the intention of this study is to develop a series of learning activities that support students in developing their understanding of the multiplication strategy in area measurement. Therefore, the research question is formulated as the following: *"How can we help students in developing their understanding of the multiplication strategy in area measurement?"*

C. Aims of the Research

The purpose of this research is to contribute to the development of a local instructional theory in developing students' understanding of the multiplication strategy in area measurement about both the process of learning and the means (learning materials) designed to support that learning.

D. Definition of Key Terms

In this section, the important key terms are defined to avoid misinterpretation on the terms used in this study among the readers. Those key terms are defined as in the list below;

1. Understanding

   Understanding is defined as the ability to relate a new knowledge (such as ideas, concepts, principles, information etc.) to an appropriate existing knowledge.

2. Developing of understanding

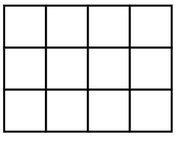
   Developing is defined as a progress of doing something. Meanwhile, understanding is defined as the ability to relate a new knowledge to an appropriate existing knowledge. So, developing of understanding could be defined as a progress of relating a new knowledge to an appropriate existing knowledge. For example, in this study the new knowledge is the concepts underpinning the multiplication strategy. The existing knowledge could be derived from students' prior knowledge as the result of the previous instructions or knowledge that they got from daily-life experience.

3. Area Measurement

Area is the number of area units covering a plane figure. Here, the area units are two-dimensional and identical shapes used as the unit measurement; while, a plane figure is a flat two-dimensional (2D) shape which is made of straight lines, curved lines, or both straight and curved lines. Meanwhile, measurement is the process of quantifying an object or event into specific units. For example, instead of saying that someone is tall, we can specify a measurement and specify that the individual is 6 feet tall. So, area measurement is the process quantifying area of a plane figure into area units.

4. The multiplication strategy in area measurement

The multiplication strategy is the strategy in finding the area of a rectangular plane figure (such as rectangle or square) by multiplying the number of units in a raw with the number of rows in an array. Here, the array is the structure of units in rows and columns. For example, the following figure shows an array of square units in three rows and four columns. The number of units in a row is 4; meanwhile the number of rows in the array is 3. So, by using the multiplication strategy the area of the array is the multiplication of 4 and 3 which is 12 in square units.



5. A right-angle plane figure, in this study, refers to a close shape that all the edges are straight and all the angles are right angles.
6. A rectangular plane figure, in this study, refers to a plane figure that has four straight edges and four right angles.

E. Significance of the Research

There are three significances of this study. First, the study contributes to the development of a local instructional theory in domain of area measurement both the process of learning and the means designed to support that learning. It is expected that the instructional theory can be used by mathematics teachers or as inspirations for the teacher in teaching area measurement.

Second, the study provides a clear view for mathematics teacher in designing and developing learning activities, particularly in the domain of area measurement. It is expected that this study can be a good example for mathematic teachers in designing and developing mathematics learning activities and the means to support the learning activities.

Third, this study provides an example of conducting design research in mathematics education. It is expected that this study can be a good guidance for other researchers in implementing educational design research, especially in mathematics, in the future.

CHAPTER II

THEORETICAL FRAMEWORK

In this chapter the theoretical frameworks underpinning this study are elaborated. The theories about understanding, including how to develop it, the area measurement and the multiplication strategy in area measurement are discussed first. Afterward, the elaboration about PMRI (the Indonesian Realistic Mathematics Education) as the idea underpinning the instructional design of this study is explained. In the end, an overview about the Indonesian curriculum on area measurement is explained as the basis to understand the classroom situation in teaching and learning on area measurement in Indonesia.

A. Understanding in mathematics

According to Skemp (1982), to understand something means to assimilate it into an appropriate schema. The schema is a cognitive map or an intellectual structure or a mental model that represent the relationships between concepts and processes, at one level, and between selected schemas, themselves, at another. The schema can also represent the existing knowledge structure.

Once a student can assimilate something (such as experiences, ideas, facts, etc.) in an appropriate schema, the student will be able to use the thing flexibly to other situation or to relate it to other things. This implies that when a student has an understanding on one thing, he/she can use the understanding flexibly to address or make relations to other things. For example, when students are asked to mention the following sequence of number “1, 4, 7, 10, 13, 16, 19, 22” after being presented to them for a couple minutes, the students who have a lack understanding on the sequence of the numbers will just memorize the numbers. Meanwhile, the students who have a good understanding of the sequence are able to see the relationship among the numbers (the difference of each two consecutive number is 3) and they do not need to memorize the number, but only realize the relationship (pattern) among the numbers. In this state, Skemp (1982) does not say that the one who memorizes the numbers has no understanding on the numbers anymore. But, he preferred to say that he/she has a different type of understanding with the students who recognize the pattern of the numbers. Therefore, Skemp (1982) categorized to types of understanding: instrumental and relational understanding in mathematics.

Skemp (1982) defines instrumental understanding as the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. If a student has an instrumental understanding, he/she only can take an appropriate procedure in solving a problem but he/she does not know why he/she has to take the procedure. This kind of understanding is simply described as applying “rule without reason”.

Relational understanding, on the other hand, is the ability to deduce specific rules or procedures from more general mathematical relationship. A student who has this type of understanding can explain what procedure and why he/she has to use the procedure in solving a problem. Skemp (1982) simply described this type of understanding as "knowing both what to do and why".

By considering the theories about understanding explained above, we define understanding in this study as the ability of relating knowledge (such as ideas, concepts, principles, information, etc.) to other existing knowledge.

Therefore, in this study, we defined understanding of the multiplication strategy in area measurement as the ability to see the relation among the concepts underpinning the strategy, such as area units, array structure, and the multiplicative structure, such that the students can see why the strategy works.

B. Developing an understanding

Skemp highlighted the importance of the relations between the subject being understood (the new knowledge) and the existing relating knowledge in emerging or developing an understanding (Barmby et al, 2007). The process of understanding has something to do with a new knowledge being understood and the existing relating knowledge. The new knowledge is assimilated into an appropriate existing knowledge building an ability to recognize the new knowledge. If the existing knowledge provides enough information to assimilate the new knowledge, consequently it will build relations among them that emerge an understanding. Nickerson (1985) described this relation as "the more one knows about a subject, the better one understands it, the richer the conceptual context in which one can embed a new fact, the more one can be said to understand the fact." (p. 235-236). This implies that to emerge an understanding of a subject it requires establishing as many as possible relations among the subject being understood with the existing relating knowledge for example by associating between two apparently different ideas that they have not associated together previously.

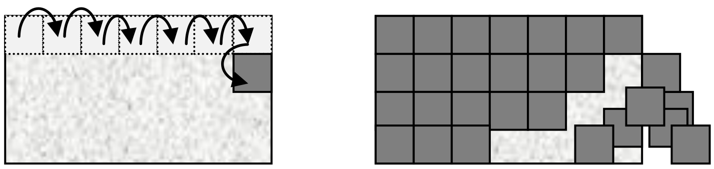
Since developing is defined as a progress of doing something and understanding itself is defined as the ability to assimilate a new knowledge to an appropriate existing knowledge. So, developing understanding could be defined as a progress of assimilating a new knowledge to the appropriate existing knowledge. The new knowledge in this study is the concepts underpinning the multiplication strategy. The existing knowledge could be derived from what the students have learned formally in the school or knowledge that they got from daily-life experience.

C. Area Measurement and Multiplication Strategy

According to Fauzan (2002), area is a number of measurement units needed to cover a plane figure. Meanwhile, Baturo and Nason (1996) equate area with the quantified amount of plane figure that is enclosed within a boundary.

Measuring area process takes place when there is a need to determine the size of a plane figure. This need often arises when direct perceptual comparison is prevented to obtain the size of the plane figure or expected to be ineffective (Nunes, Light & Mason in Baturo & Nason, 1996). According to Baturo and Nason (1996), measuring area becomes the matter of portioning a plane figure being measured into discrete units of the same size and then counting those units. This quantification of the units gives rise to area measurement.

Measurement units of area can be utilized iteratively in two different ways to generate an area measurement. The first is selecting a unit by taking one element out of a whole and then transposing this unit by continuously changing its position, without overlapping or leaving gaps, on the remainder of the whole (see Figure 2.1a). The second is determining a suitable measurement units and using as many of these units as are required to cover the whole (see Figure 2.1b) (Baturo & Nason, 1996).



(a) (b)

Figure 2.1
The two different ways of iteration of the unit of area

According to Outhred & Mitchelmore in Sari (2012), there are four basic principles that constitute children's intuitive understanding of area measurement. These principles are complete covering (covering surface being measure using measurement units), spatial structure (the measurement units can be arrange in many different ways), size relation (the bigger the units the smaller the number of units needed), and multiplicative structure (the structure of units in rectangular figure that allow the multiplication strategy in counting the units). The four principles successively show the children's acquisition in learning area measurement.

Meanwhile, Clements and Stephen (2004) argue that there are at least four important concepts are involved in measuring area, namely partitioning, unit iteration, conservation, and structuring an array. Those four concepts are described in the following table:

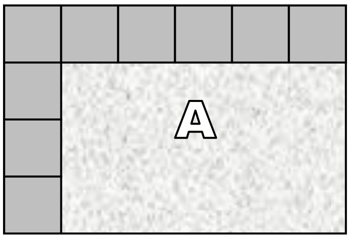
Table 2.1
The Four Basic Concept of Area Measurement

No	**Concepts**	**Descriptions**
1	Partitioning	The mental act of dividing a plane figure to be some identical figures that can be counted.
2	Unit iteration	Covering a plane figure with the measurement units of area without gap or overlapping among the units and the covering does not extend over the boundaries of the figure.
3	Conservation	Understanding that the area of a plane figure is conserved although the figure is decomposed or reconstructed into other form of figures.
4	Structuring an array	Arranging the units of area in rows and columns.

Multiplication strategy is commonly introduced to students to measure the area of a plane figure, especially rectangular plane figure, by multiplying the length and the width of the figures. The length of a rectangular figure is the longest dimension of the figure. Meanwhile, the width is the shortest dimension of the figure.

Clements and Stephen (2004) associated the multiplication strategy with the multiplicative formula of area (length times width) where the concepts underpinning the multiplicative formula are based on the array structure of area units and the array notion of multiplication of the array structure.

When the area units are arranged in columns and rows, the arrangement yields an array structure of area units. In the array structure of a rectangular plane figure, the number of units constituting each row or column is always the same. This array structure yields a multiplicative structure that allows multiplication strategy in finding the number of the units covering the rectangular plane figure (the area of the plane figure) by multiplying the number of the columns and the rows. Let's see Figure 2.2 for example. The area of the plane is 4 x 6 that are 24 squares since there are 4 rows and 6 columns of the square units that can be used to cover the whole plane figure.



There are 4 rows and 6 columns of square units covering the plane figure. Therefore the area of figure is 4×6 = 24 squares

Figure 2.2
The multiplication strategy in area measurement

Clements and Stephen (2004) propose four instructional activities that students need to be engaged in to help them grasping conceptual understanding on the multiplicative formula in area measurement:

1. First, students should experience of covering plane figures with units of measure. They should realize that there are to be no gaps or overlapping and that the entire plane figure should be covered.
2. Second, they should learn how to structure arrays. Figuring out how many squares in pictures of arrays, with less and less graphic information of clues, is an excellent task.

3. Third, students should learn that the length of the sides of a rectangle can determine the number of area units in each row and consecutively tell the number of rows in the array. This will help students to understand the role of dimensions as the representation of the array structure.
4. Fourth, students who can structure an array can meaningfully learn to multiply the length and the width of the plane figure as a shortcut for determining the total number of the area units covering the plane figure.

Regarding to the aim of this study, developing students understanding of the multiplication strategy in area measurement, the learning trajectory proposed by Clements and Stephen (2004) above is an appropriate learning trajectory for that purpose. However, the study by Zacharos (2006) and Bonotto (2003) is also considered as the part of the learning trajectory. Zacharos (2006) and Bonotto (2003) suggest that the students need to understand area as the extent of the region firstly before they are introduced to the measurement unit of area.

D. PMRI (Pendidikan Matematika Realistik Indonesia)

The instructional theory in this study is designed based on the view of PMRI. PMRI is a teaching and learning approach of the Indonesian version of the Realistic Mathematics Educations (Fauzan, 2002). Therefore, the instructional theory in this study is based on the view of the Realistic Mathematics Education.

According to Gravemeijer in Fauzan (2002) there are three key heuristic principles of RME that are used in instructional design; namely *guided* *reinvention through progressive mathematization*, *didactical phenomenology*, and *self developed models or emergent models*.

1. **Guided Reinvention through Progressive Mathematization**

In the guided reinvention principle, the students should be given the opportunity to experience a process similar to that by which mathematics was invented. With regard to this principle, a learning route (learning trajectory) has to be mapped out that allow the students to find the intended mathematics by themselves. The learning trajectory should be emphasized on the nature of the learning process rather than on inventing mathematics concepts/results. It means that students have to be given the opportunity to gain knowledge so that it becomes their own private knowledge, knowledge for which they themselves are responsible with. This implies that in the teaching learning process students should be given the opportunity to build their own mathematical knowledge on the basis of such a learning process.

2 **Didactical Phenomenology**

The didactical phenomenology implies that a learning mathematics should be started from phenomena that are meaningful for the student, that beg to be organized and that stimulate learning processes.

According to Gravemeijer in Fauzan (2002), the goal of a phenomenological investigation is to find problem situations for which situation-specific approaches can be generalized, and to find situations that can evoke paradigmatic solution procedures that can be taken as the basis for *‘vertical mathematization’* (the process of gaining formal mathematics).

An implication of the didactical phenomenology principle is that the instructional designer has to provide students with contextual problems taken from phenomena that are real and meaningful for them.

According to Figueiredo in Fauzan (2002), the contextual problems in RME fulfill a number of functions, such as:

a. Help students to understand the purpose of the problem;
b. Provide students with strategies based on their own experience and informal knowledge;
c. Offer students more opportunities to demonstrate their abilities; and
d. Invite students to solve the problems.

3 Self-Developed Models

The third key principle for instructional design in RME is self-developed models or emergent models. This principle plays an important role in bridging the gap between informal knowledge and formal knowledge. It implies that students have to be given the opportunity to use and develop their own models when they are solving mathematical problems.

At the beginning the students will develop a model which is familiar to them. After the process of generalizing and formalizing, the model gradually becomes an entity on its own. Gravemeijer (1994) in Fauzan (2002) calls this process a transition from model-of to model-for. After the transition, the model may be used as a model for mathematical reasoning.

E. Indonesian Curriculum on Area Measurement

The content of mathematics in Indonesian primary school includes three main aspects: numbers, geometry, and data analysis (BSNP, 2006). The topic about area measurement is included in the aspect of geometry. This topic is taught in the same chapter with the topic of perimeter measurement in the third grade of primary school, precisely on the second semester.

The standard competence and the basic competences that are expected to be achieved by students by the end of the learning process are explained in the following table.

Table 2.2
*Standard Competence and Basic Competences of Indonesian Curriculum Relating to the Area Measurement**

Standard Competence	**Basic Competence**
5. Calculating perimeter and area of square and rectangular, and their application in problem solving	5.1 Calculating perimeter of square and rectangle 5.2 Calculating area of square and rectangle 5.3 Solving problems related to perimeter and area of square and rectangle

* (BSNP, 2006).

Actually, the goals of mathematics curriculum for Indonesian primary schools pay much attention to several important aspects of mathematics education, such as developing pupils' reasoning, activity, creativity and attitude, and providing pupils with mathematics skills so that they can handle real life problems mathematically (Fauzan, 2002).

However, in the classroom implementation the goals above seem to be blurred since the specific instructional objectives from Grade 1 till Grade 6 of primary school are dominated by remembering facts and concepts verbally, studying computational aspects, and applying formulas (Fauzan, 2002). In geometry instruction, for example, when it comes to learning topic of areas and perimeters, the objectives are dominated by remembering facts and applying formulas. Furthermore, the instructional activities in mathematics textbooks in Indonesia are also a far away from the goals in the curriculum. The contents of the mathematics textbooks emphasizes on introducing facts, concepts and formulas as well as practicing computation skills or applying the formulas (Fauzan, 2002). According to Karnasih & Soeparno (1999) and Soedjadi (2000) in Fauzan (2002), many abstract concepts are introduced without paying much attention to aspects such as logic, reasoning, and understanding. The topics that are taught seem far removed from pupils' daily life and even the teachers themselves sometimes do not know the usefulness of the topics that they teach (Fauzan, 2002).

CHAPTER III

RESEARCH METHOD

A. Research Approach

Design research approach is used in this study since the intention of this study is to contribute to the development of a local instructional theory, both the process of the learning and the means (such as the learning materials and the learning trajectory) to support that learning.

In conducting the design research, the researcher goes through the following three phases, such as (1) preparation for the experiment, (2) experimenting in the classroom, and (3) conducting retrospective analyses.

1. Preparation for the experiment

In this phase a local instructional theory is formulated that consists of conjectures about a possible learning process, together with the conjecture about possible means (materials) of supporting that learning process. It is important to be aware that the local instructional theory can be elaborated and refined while conducting the experiment.

Literatures and documents relating to the domain of the study are studied and reviewed to support in formulating the instructional theory. Interview and discussion with educators, mathematicians, and other researchers are also conducted to improve the instructional theory

In this phase, the learning goals (the endpoint) are clarified that are expected to achieve, the students' prior knowledge (the instructional starting points) as the consequences of the earlier teaching instructions, the desired learning culture (classroom norms and discourse), the role of teacher and students required to execute the instructional theory (the classroom norms), and also the theoretical intent of this study. Each point of these is explained in the following paragraphs.

In clarifying the learning goals, the learning goals are scrutinized from a disciplinary point of view in order to establish what the most relevant goals are, not simply adopt the educational goals that are current in a given domain. It is studied in depth what actually students need to understand the multiplication strategy in area measurement through reviewing research literatures and documents relating to the domain of the study, area measurement.

In clarifying the instructional starting point, the target students' prior knowledge are investigated relating to the domain of the study through several ways, such as reviewing existing related research literatures, conducting pretest on the domain and conducting student interview.

The learning culture and the role of teacher are discussed in the same time since each of them is related. It is considered that 'the way the instructional activities enacted in the classroom' considerably influences the effectiveness of the designed instructional theory. Therefore, the classroom cultures are scrutinized including classroom norm and discourse that are desired to support the achievement of the learning goals. In this case, teacher is needed to be proactive in establishing such cultures. Teacher is also needed to be proactive in introducing the instructional activities, selecting possible topics for discussion, and orchestrate whole class discussion.

The theoretical intent of this study is to contribute to the development of an empirically grounded local instruction theory in developing students' understanding of the multiplication strategy in area measurement.

2 Experimenting in the classroom

The instructional theory which has been developed in preparation phase is tested to improve the conjectures of the local instructional theory and to develop an understanding of how it works (see van den Akker et al, 2006).

A cyclic process is one of the characteristic of this study in this phase. This process consists of the process of designing, redesigning and testing instructional activities and other aspects of the design through cyclic process of thought experiments and instruction experiments

In each cycle, the researcher conduct an anticipatory thought experiment by envisioning how the proposed instructional activities might be realized in interaction in classroom, and what students may learn as they participate in them. During the enactment of the instructional activities in the classroom, the researcher tries to analyze the actual process of student participation and learning. On the basis of this analysis, the researcher makes decisions about the validity of the conjectures that are embodied in the instructional activity, establishment of particular norms, and the revision of those specific aspects of the design. The design research therefore consists of cyclic processes of thought experiments and instruction experiments (see Figure 3.1).

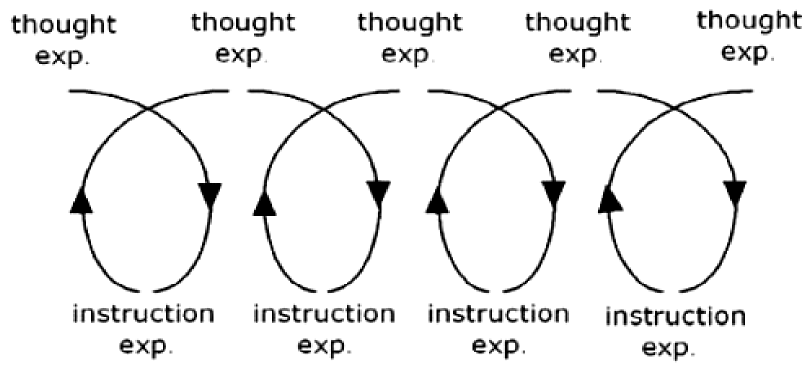


Figure 3.1
The cyclic process of thought experiments and instruction experiments in design research

The cycles of design and analysis in this phase can be associated with Simon's (1995) idea about mathematical teaching cycle. According to this idea, a mathematic teacher will first try to anticipate what the mental activities of the students will be when they participate in some envisioned instructional activities. Then the teacher will try to find out to what extent the actual thinking processes of the students correspond with the hypothesized ones during the enactment of those activities to finally reconsider potential or revised follow-up activities. The term "hypothetical learning trajectory" is used to characterize the teacher's thinking which is describe as the consideration of the learning goal, the learning activities, and the thinking and the learning in which the students might engage. The mathematical teaching cycle, then, may be describe as conjecturing, enacting, and revising hypothetical learning trajectories.

In design research, the cyclic process of thought and instruction experiment serves the development of the local instructional theory. This cyclic process require that the researcher engages in an ongoing analysis of individual students' activity and of classroom social processes to inform new anticipatory thought experiments, the design or revision of instructional activities, and sometimes the modification of the learning goals.

Actually there is no limitation of the number of cycles of the experiments. However, a researcher may stop the cyclic process of the experiments (the researcher may satisfied) if the instructional design (the intervention) appear to be effective where the quality of the realized outcome are close enough to the quality of the intended outcome based on the analysis of the evaluation data.

3. Conducting retrospective analysis

The goal of retrospective analyses will depend on the theoretical intent of the design research. In this study, for example, its theoretical intent is to contribute to the development of a local instructional theory which is developing students' understanding of the multiplication strategy in area measurement.

To do retrospective analysis with respect to the theoretical intent, the entire data set collected during the classroom experiment, such as the videotape of all lessons, video-recorded of all student interview, copies of all students' work and artifacts, all field notes, and other supportive data are analyzed as the basis to test the conjectures developed in the instructional theory.

In the retrospective analysis, the data are analyzed through two phases. In the first phase, the data are studied chronologically, activity by activity. At each activity the conjectures in the instructional theory are tested. If one of the conjectures does not occur, the conjectures have to be revised or provided an argument that the conjecture have evolved. In the end of this analysis, a sequence of conjectures and refutations that are tied to specific activity are obtained.

In the next phase, the sequence of conjectures and refutations are analyzed further. The findings from these analyses become the basis to draw conclusion, answer the research question and establishing a local instructional theory. This phase of analyses is included in the research report of this study.

The overview of the three phases above is shown in the following table.

Table 3.1
The overview of the phases of the design research in this study

Phase	**Activity**	**Product**
Preparation phase	Studying literature about the topic of the study	An initial local instructional theory based on literatures with its hypothetical learning trajectory.
	Clarifying the theoretical intents (developing an instructional theory).	
	Clarifying end point (learning goals)	
	Clarifying starting point (students' prior knowledge as the consequences of the earlier teaching instructions)	

	Clarifying the classroom norms to support the implementation of the instructional theory.	
Experimenting in the classroom	Cyclic process of experiment consisting of ongoing testing and refining of the initial instructional theory based on the findings of the experiments.	The data telling how the local instructional theory works during cyclic process of experiments.
Retrospective Analysis	Analyzing the whole entire set of data as the basis to answer the research question as well as to establish the local instructional theory.	A local instructional theory which has been tested and refined through cyclic process of experiments.

B. Subject of the study

The subjects of this study are a number of students taken from the third grade students of Indonesian primary school. They are around 9 to 10 years old. The experiment is planned to be conducted in two cycles. In each cycle, it involves a different group of participants/subjects. There will be six participants involving in the first cycle. They are from the third grade students of SDIT Ghilmani Surabaya. The selection of the students is based on the level of the students and the recommendation of the teacher. The students are categorized as the students who have average mathematics ability among the students in their class since they can be the representation of their classmate. The small group of participants is used in this cycle since the purpose of this cycle is mainly to adjust and improve the initial instructional theory for the next cycle.

Meanwhile, in the second cycle, the instructional theory will be experimented in one class of the third grade students of SD Laboratorium UNESA Surabaya. Among the whole students, two of them are selected as the focus students. The selection of the focus students will be based on their mathematical ability. The average students will be chosen in order to obtain the students that can be the representation of the other students.

C. Data Collection

In each cycle, the data are collected in four phases: preparation phase, pretest, teaching experiment, and posttest. Each of those phases are elaborated in the following paragraphs.

1. Preparation phase

Before conducting the experiments, the researcher conducts a preliminary investigation to the classroom of the subjects where the learning design will be implemented to get the preliminary insight of the classroom environment (situations) and about the subject as the basis to adjust and improve the initial instructional theory. To do that, the researcher will conduct a classroom observation and a teacher interview.

- Classroom observation

  Here, the researcher will observe the classroom of the subjects during a normal teaching process. In this state, the data about the teacher (how he/she conduct the teaching in mathematics, student or teacher cantered), about the students/subjects (how the students behave in the classroom, active or passive), and about the discourse (whether inquiry or direct instruction) are collected. To collect those data, an observation scheme is made as the guidance for the researcher during the observation and the teaching process during the observation is also recorded (video recording) as the documentation to help researcher to restudy the data obtaining from the observation.

- Teacher interview

  The mathematics teacher of the subject is interviewed by the researcher to obtain the data about students' difficulties in learning mathematics, teacher's teaching experience, and how the teacher usually conducts a teaching. To collect those data, an interview scheme is made to guide the researcher during the interview and the interview is recorded (audio recording) as the documentation to help the researcher in restudying the data.

2. Pretests

After the preparation phase, the subjects of the study are tested to gain information about subjects' prior knowledge relating to the topic of the study, area measurement. Then, to obtain deeper understanding of students' prior knowledge as well as to enhance researcher's understanding on students' written works of the test, the researcher will interview the students about their works on the test. Here, the interviews are recorded (video recording) to help the researcher in restudying the data.

The data intended to be collected in this phase are: students' prior understanding of area, how students solve area comparison problem, how students measure area of a rectangle, and how students measure the area of right-angle plane figure, and how students solve multiplication problem. Students' written work and the interview recording are collected to be analyzed further. The findings of the analysis in this phase will be used to improve and adjust the initial instructional theory.

3. Teaching experiments

In this phase, the initial instructional theory (consisting of the learning trajectory, activities, and materials) which has been designed and refined is then tested to the subject of the study.

Here, the researcher will observe the implementation of the instructional theory either by both direct observation and video recording. The observation will focus on how the instructional theory works and helps students' in developing their understanding of the intended learning goals. During the observation, some students are also interviewed (and it is recorded) to clarify their thinking and their understanding toward the topic of the teaching or toward the given problems during the teaching. Students' written works during the teaching experiment are collected as well as the basis to clarify students' thinking and understanding. The researcher also makes important notes (filed note) to record the important issues during the teaching experiments.

In this phase, the researcher intends to collect the data about how students engage in designed learning activities, how the learning trajectory and activities helps students in developing their understanding of the multiplication strategy.

So, in the end the researcher will obtain a various type of data, such as video observation, interview record, students' written work, and field note. Those data will be analyzed all together further and the findings of the analysis will be used to evaluate the validity, the effectiveness, and the practicality (see Nieveen, 1999) of the instructional design as the basis to adjust and improve the design.

4. Postest

In the next phase, to examine students' knowledge and development after the teaching experiments, a posttest is conducted. Here, the subjects are tested relating to the topic of the study, area measurement. Then, to obtain deeper understanding of students' knowledge and development after the teaching experiment as well as to enhance researcher's understanding on students' written works on the test, the researcher will interview the students about their works on the test. Here, the interviews are recorded (video recording) to help the researcher in restudying the data.

The data intended to be collected in this phase are: how students solve area comparison problem, how students measure area of a surface, and how students apply the multiplication strategy in area measurement after the experiments.

The overview of the process of data collection the four phases is shown in the following table.

Table 3.2
Overview of the process of data collection

Phase and purpose	Data to be collected	Method of collection
Preparation phase "to clarify the classroom environment and about the subject"	Classroom observation: - How the teacher conducts a teaching in mathematics, student or teacher centered? - How student behave in the classroom, active or passive? - How the classroom discourse, inquiry of directs instruction? Teacher interview: - students' difficulties in learning mathematics, - teacher's teaching experience, - how the teacher usually conducts a teaching.	- **Classroom observation** The teacher and the students are observed during a normal teaching process. The observation is recorded (video) - **Teacher interview** The teacher is interviewed. The interview is recorded (audio)
Pretest "to clarify students' prior knowledge"	- Students' prior understanding of area, - how students solve area comparison problem, - how students measure area of a rectangle, - how students measure the area of right-angle plane figure, - how students solve multiplication problem.	- **Test** Students' are given a written essay test. - **Student interview** Students are interviewed on their answer on the test to clarify their thinking. The interview is recorded (video)
Teaching experiment "to test the instructional design"	- Students' thinking and responses toward the design instructional activities, - how the learning activities and materials helps students in developing their understanding of the multiplication strategy in area measurement.	- **Teaching observation** The teacher and the students are observed during the experiments. The observation is recorded (video) - **Student interview** Students are interviewed during or after the experiments to clarify their thinking. The interview is recorded (video) - **Document of students' written work** Students' written works on the worksheet during the experiment are collected.

Posttest “to clarify students’ knowledge after experiment”	- how students solve area comparison problem, - how students measure area of a surface, and - how students apply the multiplication strategy in area measurement.	- **Test** Students’ are given a written essay test. - **Student interview** Students are interviewed on their answer on the test to clarify their thinking. The interview is recorded (video)

5. Validity and reliability of data collection

Either in planning a research or interpreting the findings of a research, determining the impact of the results is dependent upon two concepts: validity and reliability. Essentially, validity of data collection entails the question “Does the measurement tools actually measure what we intend it to measure?” Meanwhile, the issues of reliability addresses the question “Does repeated measurements provide a consistent result given the same initial circumstances?”

The theoretical intent of this study is to develop a local instruction theory in developing students’ understanding of the multiplication strategy in area measurement. Therefore, observation on the learning activity during the experiments, conducting pre and posttest, interviewing with students and collecting all students’ work and artifacts are the valid data and methods of collecting data as the basis for developing such an instruction theory (see van den Akker et al, 2006).

Reliability in data collection refers to the degree of the consistency of a measure when using a certain method or instrument in collecting data (Denscombe, 2010). There are three methods of data collection used in this study: observation, interview, and tests (pretests and posttests). In collecting the data from classroom observation during the teaching experiments and the student interview, the data are obtained through video recording. This way of collecting data will increase the reliability of data since the data are independent which does not greatly influence by the interpretation of the observer or the interviewer.

D. Data Analysis

The data obtained during the data collection will be analyzed qualitatively. The following paragraphs will describe how the data are analyzed.

1. Preliminary phase

The video from the classroom observation are studied intensively to find the information about the classroom environment, including the classroom norms, the role of the teacher and the students, and the interaction among them. Meanwhile, the recording from the teacher interview is studied carefully to obtain further information relating to the classroom environment and about the subjects.

In the end of the analyses, remarks about the classroom environment are established as the consideration in adjusting and improving the initial instructional theory.

2. Pretest

The data of the pretest are analysed by studying students' answers on the tests. The students' tendencies in each question are categorized by checking the number of questions solved correctly or incorrectly and how students deal with the problems.

Each problem in the tests is designed to checks a specific concept relating to the topic of the study. Once a student fails to answer a certain problem it implies that the student fail to understand the concept within the problem. To get deeper view of students' understanding on their answer on the test, the data from the students' interview is studied as well. Here, the interview recording is carefully studied. In the end, remarks on students' prior knowledge are established. The remarks will be used to adjust and improve the initial instructional theory.

3. Teaching experiment

In analyzing the data obtained from the observation, there are two phases of analyses are followed. In the first phase, the whole videos are watched and studied lesson by lesson to obtain the overview of students' responses in each lesson. Then, some fragments are selected from the video and are transcribed. The selection of the fragments is based on whether the fragments can be used to test the conjectures in the hypothetical learning trajectory or not.

Afterwards, the conjectures in the hypothetical learning trajectory are tested by comparing them with the findings in the actual learning trajectory. In this moment, the fragments are analyzed intensively to really understand what actually happen in the fragments to know whether the fragments support or refuse the conjectures in the hypothetical learning trajectory. If a certain conjecture does not occur, it means that the conjecture have to be revised or provided an argument that the conjecture has evolved.

A sequence of conjectures and refutations (conjectures which are not occurred) that are tied to a specific activity are obtained in the end of the analyses of this phase.

In the next phase, the sequence of the conjectures and the refutations as the fundamental data are analyzed further and deeper. In this moment, understanding the reason why the conjectures in the hypothetical learning trajectory are accepted or refused are studied further. Here, the researcher builds his assumptions based on the data. The findings from the other sources of data, such as pretests and student interview, are used to support the assumptions. This phase of analyses is presented in the report of this study.

The findings from the analyses of this phase become the basis to refine and improve the initial instructional theory in the first cycle and become the basis to draw conclusion, answer research question, and developing a local instructional theory in the second cycle.

2. Posttests

As well as the data from the pretest, The data of the postttes are analysed by studying students' answers on the tests. The students' tendencies in each question are categorized by checking the number of questions solved correctly or incorrectly and how students deal with the problems. Each problem in the tests is designed to checks a specific concept relating to the topic of the study. Once a student fails to answer a certain problem it implies that the student fail to understand the concept within the problem.

To get deeper view of students' understanding on their answer on the test, the data from the students' interview is studied as well. Here, the interview recording is carefully studied. In the end, remarks on students' knowledge are established. The remarks will be used to see students' development after the experiment and as the consideration in adjusting and improving the instructional theory for the next cycle.

4. Validity and reliability of data analysis

There two methods are used in this study to increase the validity of the findings of the data analysis: triangulation and grounding data.

a. Triangulation

Triangulation involves the practice of viewing thing from more then one perspective. This can mean the use of different methods, different sources of data, or different researcher within the study. There are two purposes of using triangulation: to improve accuracy of the findings and to obtain fuller picture of the findings.

In this study, the researcher uses methodological triangulation to increase the validity of the findings. In this study, the researcher uses various methods of data collection to obtain the data, such as observation, tests, and interview. Those data then are used together to see the subject of the study to get either a fuller picture of the subject as well as to increase the confidence (validity) of the findings of the analysis.

b. Grounding data

The findings that are extensively grounded in the fieldwork and empirical data provides a solid foundation for the conclusion based on the data and adds to the credibility (validity) of a research. In this study, the data from all methods of data collections (observation, interview and test), including video, audio, picture, field note, subject artefacts, etc. are extensively used to deduce conclusions.

To increase the reliability of the analysis in this study, the researcher accommodates the idea of transparency that allowing other people (such as other researchers) to follow the path and the key decisions taken by the researcher from conception of the research through the findings and conclusion derived from the research. This fairly detailed record of the process of the research decisions may increase the reliability of the research. In this study, the researcher considers the transparency of data analysis by explaining how the data are analysed and used to obtain findings, such that the readers can easily follow how the data were analysed.

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Simon (1995) coined the term, "hypothetical learning trajectory", which is describe as the consideration of the learning goal, the learning activities, and the thinking and the learning in which the students might engage. In the hypothetical learning trajectory, a teacher will first try to anticipate what the mental activities of the students will be when they participate in some envisioned instructional activities. Then the teacher will try to find out to what extent the actual thinking processes of the students correspond with the hypothesized ones during the enactment of those activities to finally reconsider potential or revised follow-up activities (see van den Akker et al, 2006).

A. Formulating the big ideas underpinning the multiplication strategy in area measurement

In designing the hypothetical learning trajectory, the learning activities proposed by Clements and Stephen (2004) which is intended to help students obtaining conceptual understanding on the multiplicative formula (multiplication strategy) in area measurement are mainly considered since they are appropriate with the purpose of this study. Those learning activities are:

First, students should experience of covering plane figures with units of measure. They should realize that there are to be no gaps or overlapping and that the entire plane figure should be covered. Here, the measurement unit of area is introduced to the students.

Second, they should learn how to structure arrays. Structuring array activity will help students to realize the regularity of the arrangement of the units of area that can help them in counting the area units efficiently. Clements and Stephen (2004) suggested that figuring out how many squares in pictures of arrays, with less and less graphic information of clues, is an excellent task to support students' understanding of the array structure of units of area.

Third, students should learn that the length of the sides of a rectangle can determine the number of area units in each row and consecutively tell the number of rows in the array. Knowing the number of units in a row and the number of row in the array yield the multiplicative structure of the area units.

Fourth, students who can structure an array can meaningfully learn to multiply the length and the width of the plane figure as a shortcut for determining the total number of the area units covering the plane figure. The students who realize the array structure and the multiplicative structure of area units will understand why the multiplication of the length and the width of a rectangle yield the area of the rectangle.

However, before the researcher starts the learning activities proposed by Clements and Stephen (2004) above, the researcher also considers the studies by Zacharos (2006) and Bonotto (2003). Those studies suggest that students need to understand area as the extent of region before they are introduced to area as the number of measurement units (calculation approach of finding area). Having understanding of area as the extent of the region will help students to understand the meaning of the numbers obtained from measuring area through the arithmetical calculation, such as using formula (Zacharos, 2006; Bonotto, 2003).

Therefore, by considering the theories above (Clements & Stephen 2004, Zacharos, 2006; Bonotto, 2003), we conjecture that to help students in developing their understanding of the multiplication strategy in area measurement, the students need to learn the following big ideas consecutively;

1. Understanding area as the extent of a region.
2. Understanding the measurement unit of area.
3. Understanding the array structure of unit of area.
4. Understanding the multiplicative structure of the array structure.
5. Understanding the role of dimensions as the representation of the array in applying the multiplication strategy in area measurement.

Each big idea is related one to each other. For example, to understand the array structure the students need to understand the area units of area, and to understand the area unit of area the students to realize area as the measurement of the extent of a region. Understanding those big ideas will help students to understand why the multiplication strategy can be used to measure the area of a surface, especially rectangular surface.

B. The Hypothetical Learning Trajectory

By considering the big ideas underpinning the multiplication strategy in area measurement, in the following paragraph we are going to explain our conjectures of the learning activities that can be used to address those big ideas. Each big idea is set to be the learning objective of a lesson. The following table shows the overview of the learning goal, learning activities, and the conjecture of students' thinking.

Table 4.1
The initial Hypothetical Learning Trajectory

No	**Learning Goal**	**Learning activity**	**Conjectures of Students' Thinking**
1	Understanding area as the extent of a region	Comparing area activities of two irregular non-identical plane figures.	- Students will, at first, use their perception in examining the larger area. Then, turning using the superposition strategy by putting one figure on top of another. They, finally decompose the figure to find the larger figure. - The students just really examine the region inside of the figure. They will not really care of the length or the width of the figures during comparing the area. - The students can see that the bigger the region of a figure, the larger the figure is. - In this state, the students see area as the extent of a region or space.
2	Understanding the measurement unit of area	Comparing area activity by using other smaller figures/shapes as the tool.	- The students will realize the importance of the tool as the measurement units in area measurement to tell area. - The students will arranged the given tool, as the units, on the figure being compared and compare the number of tool used to determine the larger area. - The idea of unit consistency will be discussed. - The idea of gap and overlapping in units as well will be a part of students' discussion. - The students will express area as the number of measurement units. - The students will understand that the more units covering a figure, the larger the figure is.
3	Understanding the array structure of area units	Measuring area of a plane figure by using limited square units (structuring array activity).	- The students will treated the square units as the measurement units to find the area of the plane figure. - The students will realize the structure of the square units in columns or in rows, that is in a rectangular surface the number of square units in rows or in columns is always the same. - The students take benefit of the structure in counting the square units.
4	Understanding the multiplicative structure of the array	Investigating the area of rectangular surfaces with less and less	- In finding the area of the surfaces, the students will count the units. - Since the surfaces getting less and less graphic information of clues of units, the students will experience the evolution of

	structure of units in rectangular surface.	graphic information of clues of units.	counting from counting the area units one by one, then row by row (or column by column), and finally by multiplication. - Before coming up with the multiplication strategy, the students realize the consistent number of units in each row or columns that allows the students to do repeated addition in counting the units which brings them to the multiplication strategy in counting the units. - Here, the students realize why the multiplication strategy can be used in counting the area units.
5	Understanding the role of dimensions as the representation of the array in applying the multiplication strategy in area measurement	Investigating the area of a surface where the array structure of the area units of the surface is given through the dimensions of the surface.	- The students realize that the dimensions tell the number of units in each row and the number of rows in the array of a rectangular surface (realizing the dimensions as the representation of the array structure). - Once the students realize the dimensions as the representation of the array structure, they then meaningfully multiply the length and the width of the sides of a rectangular surface to determine the area of the surface. -

The following paragraphs will explain more details about the activities designed above.

Lesson 1: Introducing Students to the Notion of Area as the extent of region

Lesson 1 is designed to introduce students to the preliminary concept of area as the extent of a region through area comparison problems. Students need to understand area as the extent of region before they are introduced to area as the number of measurement units (Zacharos, 2006; Bonotto, 2003). Here, students will experience of using perception, superposition, or decomposing strategy in solving area comparison problem. These strategies are the preliminary strategies that students need to experience with before they are introduced to measuring area. In this lesson, it is conjectured that the students will experience of expressing area in terms of wording, such as *large*, *larger*, *small,* and *smaller*.

1. Learning goals

In this lesson it is expected that students experience of using perception, superposition, or decomposing strategy as approach in solving area comparison problems.

2. Learning activity

There are two main problems as the learning activities. In the first problem, students are asked to determine which grassland provides more grasses (see Figure 4.1). To deal with the problem, students are given the cut of the two grasslands, so they can easily move the figure of the grassland during exploration to lead them to discover many possible ways of solving the problem.

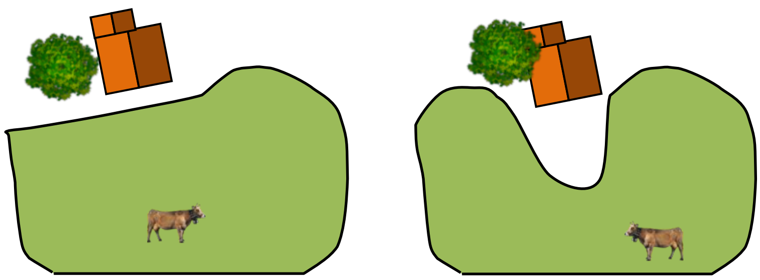


Figure 4.1
A pair of grassland: Which grassland provides more grasses?

In the second problem, students will solve a problem of another pair of grasslands (see Figure 4.2). These grasslands are more regular form of shape (right angle shapes) than the previous pair.

Different from the previous problem, the students are asked to determine the larger grassland, instead of asking the grassland provides more grasses, to support the progressive mathematization. So, it is closer to the concept of area. Students are also given the cut of the two grasslands, so that they can flexibly find their strategies to solve the problem.

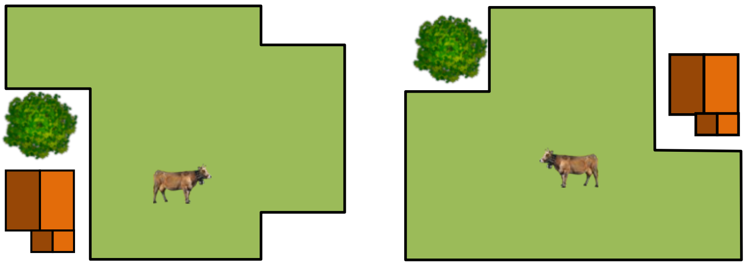


Figure 4.2
A pair of grasslands: Which grassland is larger?

The learning materials are purposively designed to trigger the students to the idea of superposition and decomposing strategy in solving area comparison problem. The students are given the cards of the plane figure being compared. The cards help them to experience of superposition by putting each card on top of each other and to experience of decomposing by cutting the cards.

Moreover, the shape and the size of the plane figures being compared are also in purpose. In the first problem, the similarity and size difference among the plane figures being compared is more obvious than those in the second problem. This is purposively designed to trigger the students to experience at least superposition strategy in solving the first problem; and in the second problem the idea of decomposing is forced to emerge since the students deal with more complicated situation.

3. Hypotheses of the learning process

In solving both problems explained before, students may use the perception strategy as their preliminary strategy. They then come up with the conjecture that one must be bigger than another one.

To involve students to keep exploring of more convincing ways of solving the problems, teacher asks students of how to show and convince other students about their conjectures. In this moment, the superposition or the decomposing strategy potentially emerge since students are provided with the cut of the grasslands.

The first problem potentially leads students to the use of superposition strategy instead of decomposing strategy since the figures being explored are not easy to be decomposed. Meanwhile, the second problem purposively designed to lead students to the use of decomposing strategy since the figures that students work with are easy to be decomposed.

However, the use of superposition strategy may also be possible but it seems to be difficult to emerge because there are several places of leftover area.

Lesson 2: Developing Students' Understanding of the Measurements units of Area

This lesson is purposively designed to lead students to the idea of unit measurement of area. In this lesson, students will experience of covering two parking areas with three different cards as the references (units) in order to be able to find the size of the two parking lots. Here, the indirect measurement strategy through a reference is expected to be learned in this lesson.

1. Learning goals

Students discover the measurement units of area through area comparison problem.

2. Learning activity

As the main activity, students will be engaged in an area comparison problem. The students are asked to find the larger parking lots of two neighbour parking lots. They are provided with three different sizes of cards as the tools to do exploration.

First of all, students are given the figure of two parking lots on a paper (see Figure 4.3). They are asked to determine which parking lot is larger.

Since the superposition or decomposing strategy seems to be difficult to emerge. They probably rely on their perception to build up conjectures of the size of the parking lots.

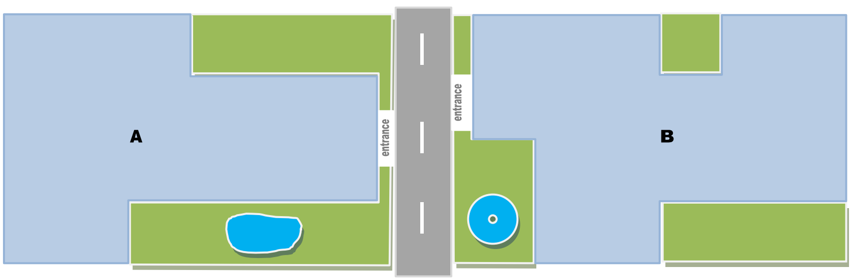


Figure 4.3
A pair of parking lots: Which parking lot is larger?

However, the finding from the perception strategy is not convincing enough because the plane figures being compared have slightly different in size. Therefore, students are asked to find other strategies to prove their conjectures. In this moment, students then are given the three different sizes of cards: 18 squares, 9 rectangles that have double in size with the square, and 9 triangles that also have double in size with the square (see Figure 4.4). Students then are asked whether the three cards may help them or not.

The number of each card is provided that enough for students to explore of covering activity. There is possibility that the students used mixed cards as the references. In this case, teacher asked students to think weather it is possible to use one type of card as the reference.

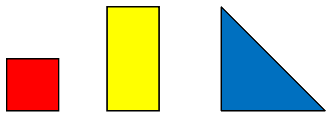


Figure 4.4
The three cards as the references

To lead students to the idea of the unit of area measurement, students are involved in classroom discussion on the rule of the cards as the reference that telling the size of the parking area. The reference is then called as the unit measurement.

The context and the learning materials are purposively to guide students to the idea of measurement units of area. Area comparisons problem triggers the need of measuring the plane figure being compared with a certain identical measurement units. The existence of the cards helps the students to see the need of the measurement units. The number of the cards is purposively given to allow the students to experience of unit iteration which is one of the foundational concepts in area measurement.

The students are given three different cards. It is purposively design to give experience of choosing the best measurement units. This also provide the room for discussion on the unit consistency since there is possibility that the students using more than one types of cards to cover the plane figure.

Different from area comparison problems in lesson 1, the area comparison problem in this lesson is not provided with the cards of the plane figure being compared. This situation does not allow the students to used superposition or decomposing strategy in comparing the plane figure. Here, students will learn that superposition and decomposing strategy are not the only strategies that can be used to solve area comparison problem. The students also will see the limitation of the use of the strategies.

3. Hypotheses of the learning process

When students are given the problem, determining the larger parking lot, without giving them the three cards, it is conjectured that students will use their perception as the foundation in deciding the larger figure since they are not provided with the cuts of the area being explored which allow them to use superposition or decomposing strategy.

After student experience of the need of a reference when measuring process, they are provided with the three cards and are asked whether the cards help them. It is conjectured that students will use one or some of the three cards as the reference. They cover both parking lots with the cards and find that one of the parking area need more cards than another one. For that reason, that parking lot is larger.

Another possible strategy is that students will draw grid lines or equal sizes of squares that cover the parking lots then compare the number of created squares in each area. However, this strategy is rare to be emerged since students have no prior experience of constructing grid lines or squares. If this solution is emerged, it is suggested to discuss about the rule of the square as the reference in area measurement. In this moment, it is also suggest to introduce them with the three cards as other possible references and discuss it with students.

It is conjectured that the idea of unit of area measurement is obtained during the classroom discussion. During the classroom discussion, students with teacher discuss about the rule of the cards as the reference that telling the size of the parking area. The reference is then introduced as the unit measurement.

Lesson 3: Developing Students' Understanding of the Array Structure of Units of Area

The purpose of this lesson is to help the students seeing the array structure of units of area. After students learn the unit of area, they then are engage in a learning activity that asks students in exploring the array structure of units of area through structuring array activity.

It is expected that they are able to see the array structure of the units of area in columns and rows (where in a rectangular shape each row or column contains the same number of units) and use the structure in determining the area of a plane figure.

1. Learning goals

Students are able to discover the array structure of units of area and use the structure to measure the area of a plane figure.

2. Learning activity

Students are provided with a floor that is intended to be tiled (see Figure 4.5). The problem is finding the area of the floor in squares of the tiles.

In the previous lesson, students have already learned area as the number of the area units covering a plane figure. So, the area of the floor can be found if they know the number of the tiles covering the whole part of the floor. Therefore, students are provided with ten square tiles (see Figure 4.6). Here, it is expected that students treat the squares as measurement units.

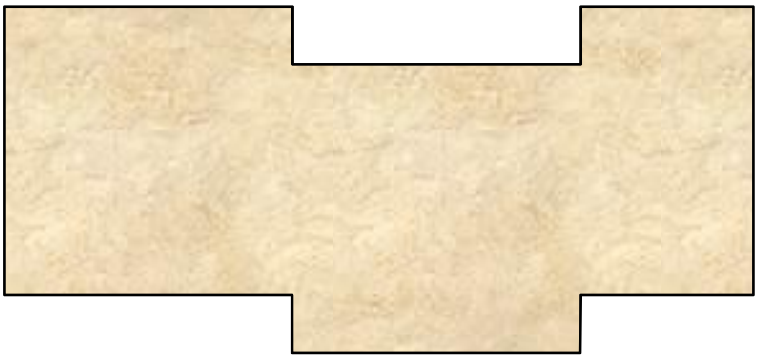


Figure 4.5
Untilled floor that intends to be tilled

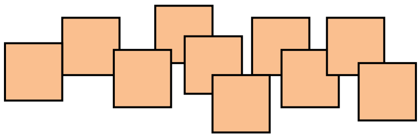


Figure 4.6
Ten square tiles as the unit measurement

In addition, the number of the given squares (tiles) and the size of the floor being measured are purposively design to help students in seeing the array structure of the squares when they are arranged on the floor. If the squares are arranged on the floor, it is found that each column contain the same number of squares which is five squares. Counting by five is familiar among the students in this age. Since they know that in each column contains the same number of squares, they will find the number of squares in rows. Here, skip counting by five will emerge to count the whole squares if the students are aware of the array structure.

The limited number of the give squares is also purposively designed. This limitation forces students to think of the way to use the squares effectively in measuring the floor. Working by considering the array structure is one of the effective ways of using the squares to measure the floor.

Moreover, the shape of the floor is also designed to help students to recognize the array structure. The shape allows the students to split it to be some smaller rectangular shapes. Working in a smaller rectangular shape is easier to see the array structure.

3. Hypotheses of the learning process

As the students have learned about the measurement units of area, it is conjectured the students will treat the ten squares as the measurement units in finding the area of the plane figure. They will find the number of the squares covering the whole plane figure of the floor without gap and overlapping. In finding the number of the squares, they will arrange the squares on the floor.

It is conjectured that there are several strategies used by the students in finding the number of the squares covering the floor. First, the students will cover the floor part by part using the ten squares until the whole plane figure of the floor is covered. Since the number of squares is limited, they will keep in record of counting the squares after every one part is done. As the students covering the floor by using the squares part by part, it is expected that they are aware of the array structure of the squares. They are able to see that in every column (if they cover the plane figure column by column) contains the same number of squares.

Second, it is conjectured that the students will split the plane figure to be smaller parts (see Figure 4.7). Then, students investigate the area of each part using the ten given squares. They cover each part with the given ten squares. Since they working on a smaller part with limited number of squares, the array structure of units of area is easier to be recognized. There is possibility that the students only cover the first column and the top row of a part of the plane figure and by repeated addition they will obtain the whole possible squares covering the part since they have already recognize the array structure of the units of area.

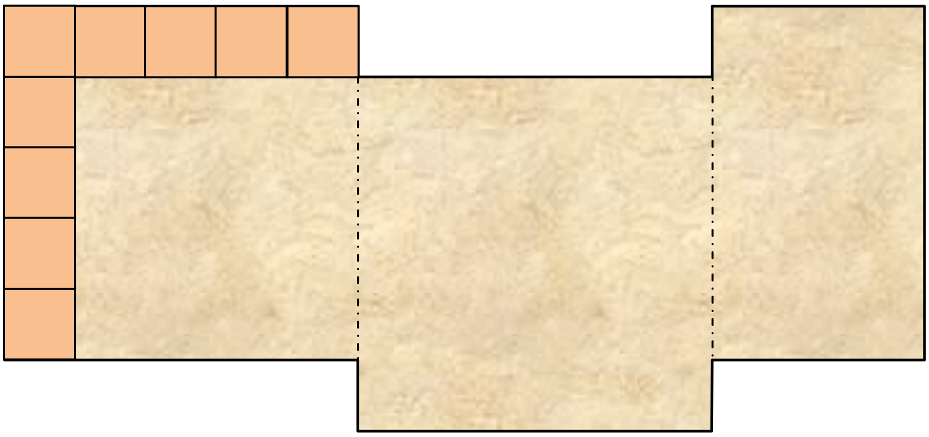


Figure 4.7
An example of splitting the floor into three parts and an example of covering the first column and the top row of a part

Third, there is possibility that the students use some squares to help them in drawing gridlines as the same size as the squares. During drawing the gridlines it is expected that the students can see the array structure of the units of area either in columns or in rows and the array structure will help them in counting the squares covering the floor.

Fourth, there is also a possibility that the students will use only one or some squares and trace those squares on the plane figure. When the tracing is done, they will have gridlines representing the process of the tracing. They then counted the grids to obtain the area of the plane figure. Here, it is expected that the students are able to see the array structure of the units during tracing process.

Lesson 4: Developing Students Understanding of the Multiplicative Structure of Units of Area

After the students have been introduced to the array structure of units of area, in this lesson it is expected that the students can see the multiplicative structure of units of area through measuring the area of tiled floors.

1. **Learning goals**

   Students are able to discover the multiplicative structure of units of area and use the structure to measure the area of a plane figure.

2. **Learning activity**

   In this lesson, students are given a top view of a patio and a living room (see Figure 4.8). Both the patio and the living room are covered with tiles that have the same size. There are carpets, tables, and chairs, etc. on the floors that cover the tiles. Students are then asked to find the area of the patio and the living room respectively. Here, it is expected that the students treat the tiles as the measurement units.

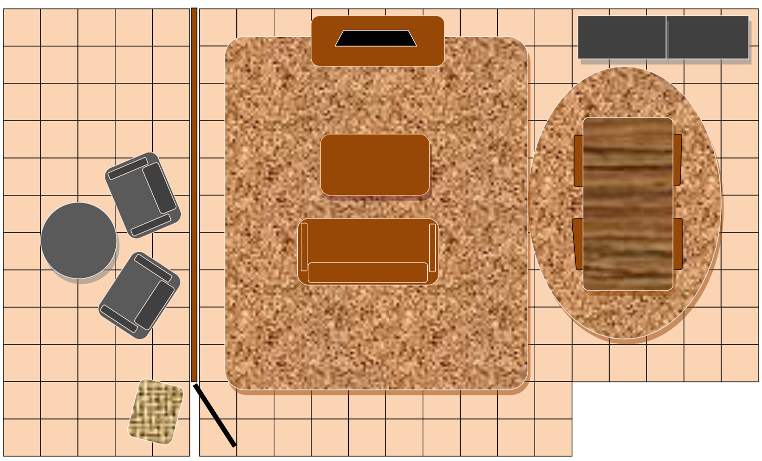


Figure 4.8
A tiled patio and living room

The setting of the floor is purposively design to help students seeing the multiplicative structure of area units. For example, some tiles are covered by carpets or tables. This situation does not allow students to count the tiles one by one, instead triggering the emergence of counting by considering the array structure. From the array structure students can be brought to count by considering the multiplicative structure of the array structure. Moreover, the number of tiles being counted which is relatively many for students lead to the emergence of more effective way of counting.

The shape of the floor is also design purposively. The shape allows the students to split the plane figure into some rectangular plane figures. This situation will help students to see the multiplicative structure of the units since the multiplicative structure of area units is exist in rectangular plane figure.

The number of tiles in rows and columns leads students to deal with a familiar numbers, such as 5 and 10, especially after the floor is split. Therefore, when the students are aware of with the multiplicative structure they will deal with a familiar multiplication, such as multiplication of 5 or 10. These multiplications can trigger students to use multiplication instead of counting one by one.

3. Hypotheses of the learning process

Since they have learned about measurement unit of area, it is conjectured that the students will treat the tiles as the measurement units.

Therefore, they will find the number of the tiles covering the floor to know the area of the floor. It is also conjectured that the students will recognize the array structure of the tiles since they have learned this idea in the previous lesson.

By considering the students' prior knowledge explained above, there will be several strategies used by the students to find the area of the floor. The first possible strategy is counting the tiles by considering the array structure of the tiles. For example, they know that there are five tiles in each row of the patio. To count the whole tiles in the patio, they will count the tiles five by five (repeated addition by five). To lead the students to see the multiplicative structure, the teacher can orchestrate a discussion among the students about the relation between repeated addition and multiplication, such as 5 + 5 + 5 + 5 as 4 x 5.

Other possible strategy is splitting the floor to be some smaller floors. In the living room for example, the shape of the plane figure allows the students to split the plane figure to be some rectangular plane figures (see Figure 4.9). They then counted the tiles in each smaller plane figure one by one or by considering the array structure. If the students count the tiles one by one, the teacher remain the students to see the array structure of the tiles that they have learned before. If the students recognize the array structure, they will count the tiles row by row or column by column by doing repeated addition. The teacher then can bring the students to the multiplicative structure by discussing about the relation between repeated addition and multiplication.

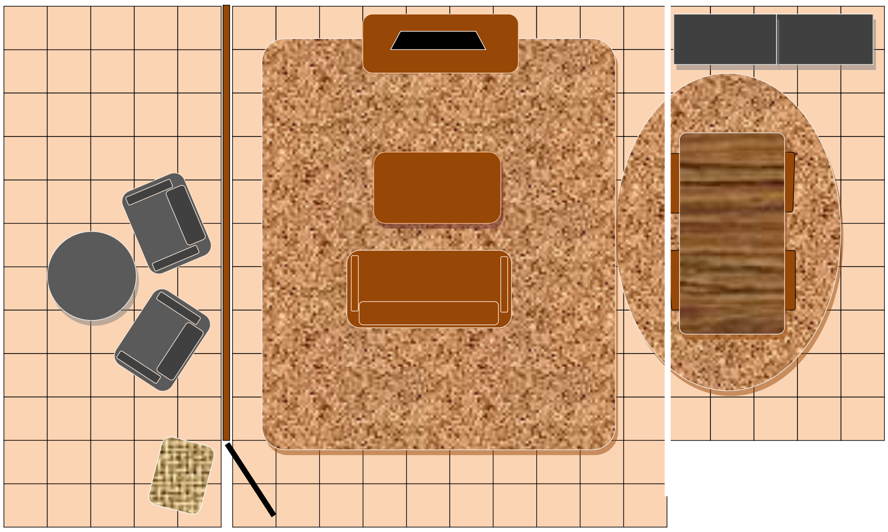


Figure 4.9
The living room is split into two parts

Lesson 5: Developing Students' Understanding of the Role of the Dimensions in Applying Multiplication Strategy in Area Measurement

In this lesson, the students are brought one step forward to more formal mathematics where the role of the dimensions of a plane figure in applying multiplication strategy is introduced and developed by the students in this lesson.

1. Learning goals

Students are able to understand the role of the dimensions of a plane figure in applying multiplication strategy in measuring the area of the plane figure

2. Learning activity

To help student to achieve the learning goals, the following problems are designed. In the first problem, students are asked to determine the area of five rectangular plane figures (see Figure 4.10). The number of tiles in each plane figure is getting less and less to support students to be aware of the array structure and to see the role of the dimensions as the representation of the array structure. From this activity it is expected that the students learn the role of dimensions as the representation of the array structure of the units of area.

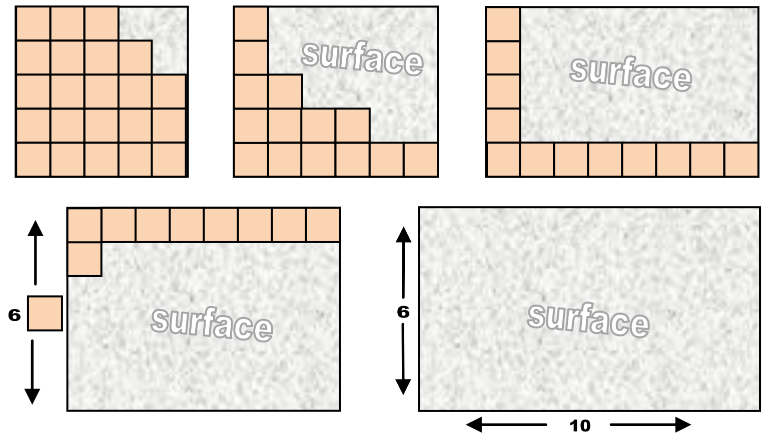


Figure 4.10
The five rectangular plane figures

In the second problem, students are asked to determine the area of a plane figure (see Figure 4.11). In this problem, the role of the dimensions as the representation of the array structure is introduced. It is expected that students are able to use the concepts that they have learned from the previous activities and the previous lessons, such as consider the array structure in counting units, use splitting technique to split the plane figure into some smaller parts, look at the dimensions to know the array structure, and consider the multiplicative structure in applying the multiplication strategy in counting the tiles.

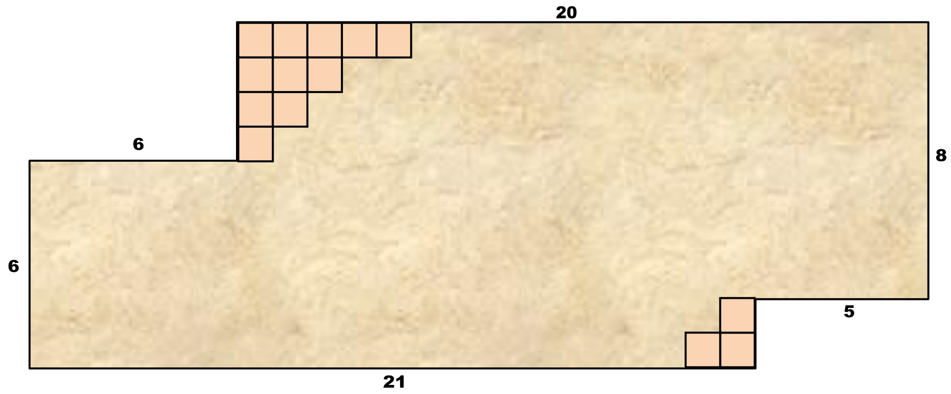


Figure 4.11
Investigating the area of an uncompleted tiled floor

3. Hypotheses of the learning process

In dealing with the first problem, it is conjectured that the students will apply the multiplication strategy in counting the tiles since they have learned about the multiplicative structure of units of area. When they are dealing with the last two plane figures, they will be aware of the role of the dimensions. It is conjectured that they will see that the dimensions represent the array structure of the squares.

However, there is possibility that the students will count the squares by considering the array structure only. They count the squares column by column or row by row. Once the students deal with the last to plane figures they will be aware of the multiplicative structure and then applied the multiplication strategy.

In dealing with the second problem, it is conjectured that the students will get difficulties in applying the multiplication strategy. The teacher can suggest the students to split the plane figure into some rectangular plane figures (see Figure 4.5.3); or there is possibility that the splitting technique will emerge among the students since they have experienced of this technique since lesson 2. Once the plane figure has been split to be rectangular plane figures, it is conjectured that the students will consider the dimensions in each rectangular plane figure. By their experience in the previous problem, they know that the product of the multiplication of the dimensions of a rectangular plane figure will give the area of the rectangular plane figure.

However, there is possibility that the dimensions are interpreted incorrectly by the students, especially after splitting. For example, 20 can be misinterpreted in two ways. First, when the plane figure is split, 20 is actually split to be15 and 5 (see Figure 4.12), but the students keep consider 20 as still 20 although it has been split to be 15 and 5. Second, students interpret 20 as the number of units goes along that edge only and do not considered it as the number of the columns along the edge.

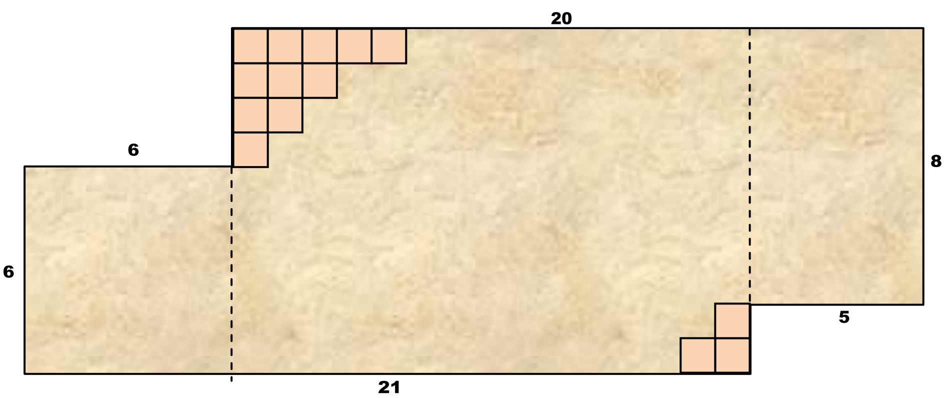


Figure 4.12
An example of how the plane figure is split into three parts

CHAPTER V

RETROSPECTIVE ANALYSES

Considering the limitation of the time, the experiments were conducted in two cycles. Those two cycles are the preliminary teaching experiment and the teaching experiment. In this chapter, the analyses on the findings from the first cycle and the second cycle are elaborated. The analyses focus on the emergence of the mathematical ideas or concepts during the experiments and the means to support the emergence.

In the first cycle, the analyses are started by providing the remarks of the classroom environment based on the findings in the classroom observation and the teacher interview and then followed by the remarks on students' prior knowledge from the pretest and the student interview. Then, the findings from the preliminary teaching experiments consisting of five lessons are elaborated. Then, the remarks from the posttest and the students interview are elaborated to see students' development after the experiments. Then, the conclusions are stated based on the findings from the teaching experiment, the pretest, the posttest, and, the student interviews. The analyses of the first cycle are ended by providing the refinement of the initial hypothetical learning trajectory based on the findings in this cycle.

In the second cycle, as well as the first cycle the analyses are start by providing the remarks on the classroom environment based on the findings in the classroom observation and the teacher interview and then followed by the remarks on students' prior knowledge from the pretest and the student interview. Then, the findings in the teaching experiments consisting of four lessons are elaborated. Different from the previous cycle, the analyses of this cycle are ended by providing the remarks of students' knowledge and development after the interventions based on the findings in the posttest of this cycle.

The results of the analyses of the first cycle are used to improve and refine the hypothetical learning trajectory (HLT) and teaching materials of the initial instructional theory. Meanwhile, the results of the analyses of the second cycle are used to answer the research question. In establishing a local instructional theory, the findings from the second cycle are mainly considered and also supported by the findings from the first cycle.

It is important to note that the agreement between the researcher and the other involving analysts, such as mathematicians, educators, etc. have been considered in the report of the analyses. Almost there is no a big difference between the researcher and the other involving analysts during the analysis. Therefore, the difference will be not intensively discussed in this chapter to keep the focus of the analysis.

A. The Research Timeline

The research timeline is necessary to provide readers with the information on when the research is conducted and to increase the track-ability of the study. The following table shows the research timeline of this study.

Table 5.1
The Research Timeline

Dates	Activities
Preparation for the experiments Place: Utrecht University, The Netherlands.	
September to December 2012	Formulating the initial local instructional theory and designing classroom materials.
Preliminary classroom experiment (The first cycle) Place: SDIT Al Ghilmani Surabaya, Indonesia.	
Monday, February 11th 2013	Classroom observation before the experiments
Monday, February 11th 2013	Teacher Interview
Wednesday, February 13th 2013	Pretest
Wednesday, February 13th 2013	Students interview
Friday, February 15th 2013	Classroom observation during experiment on lesson 1: Introducing the students to the concept of area through area comparison problems
Monday, February 18th 2013	Classroom observation during experiment on lesson 2: Introducing and developing students' understanding of the unit measurements of area
Wednesday, February 20th 2013	Classroom observation during experiment on lesson 3: Developing students' understanding of the array structure of units of area
Wednesday, February 20th 2013	Classroom observation during experiment on lesson 4: Developing students understanding of the multiplicative structure of units of area
Thursday, February 21st 2013	Classroom observation during experiment on lesson 5: Developing students' understanding of the role of the dimensions in applying multiplication strategy in area measurement
Friday, February 22nd 2013	Posttest and student interview
February 23nd – March 3rd 2013	Analyses of the first cycle including refinements of the hypothetical learning trajectory of the initial local instructional theory and classroom materials.
Classroom experiment (The second cycle) Place: SD Laboratorium Universitas Negeri Surabaya, Indonesia.	
Monday,	Classroom observation before the experiments

March 4th 2013	
Friday, March 8th 2013	Teacher Interview
Monday, March 11th 2013	Pretest
Friday, March 15th 2013	Students interview
March 16th – April 7th 2013	Refinements of the initial hypothetical learning trajectory and classroom materials.
Monday, April 8th 2013	Classroom observation during experiment on lesson 1: Developing students' understanding of the measurement unit of area
Tuesday, April 9th 2013	Classroom observation during experiment on lesson 2: Developing students' awareness of the array structure of units of area
Thursday, April 11th 2013	Classroom observation during experiment on lesson 3: Developing students' awareness of the multiplicative structure of units of area
Thursday, April 11th 2013	Classroom observation during experiment on lesson 4: Developing students' understanding of the role of the dimensions in applying multiplication strategy in area measurement
Friday, April 12th 2013	Posttest
April 13th – Mei 20th 2013	Analyses of the second cycle and writing report

B. Remarks on Classroom Environment Based on the Classroom Observation and Teacher Interview of the First Cycle

There are some remarks about the students and the teacher that influence the classroom environment according to the findings from the classroom observation and the teacher interview.

First, although the teacher often engage the students in group activity, the students still got difficulties in participating in classroom discussion or group discussion. They tend to work individually instead of in group. It is conjectured that the group activity conducted by the teacher less likely trigger the students to work in group.

Second, the teacher often asks students in participating in classroom discussion, but only for answering questions. The questions asking ‘why’ and ‘how’ are rare used by the teacher. Therefore, the students are not accustomed in giving or producing their opinion in the classroom.

Third, the teacher often gives the students real context mathematics when teaching mathematics. For example, investigating corner around their classroom when they learn about angles.

Fourth, the classroom is not strongly a teacher-centred classroom but also not a student-centred classroom. It is something in the middle.

By considering the remarks above, some adjustments made in the instructional theory, such as:

1. Talking about the context of a problem before the problem are given to help students to be accustomed in expressing their idea as well as to introduce them to the context of the problem.
2. Students are given time to work individually before they work in group. This plan provides the students with some ideas before they discuss in group.
3. Students are asked to discussed in small group first before classroom discussion. This plan can increase students’ confidence when doing classroom discussion.

C. Remarks on Students' Prior Knowledge Based on the Pretest and Student Interview of the First Cycle

There are six students involved in the pretest. They are categorized as the students who have an average mathematical level among the students in their class. There are five questions being asked which each question asks different concepts in area measurement. Here are the remarks obtained based on the findings in the pretest and supported by the findings in the student interviews.

1. Students have a prior understanding of area as a region

Here students are asked to select the shapes that have area (see Figure 5.1). The purpose of this question is to examine students' understanding of area, whether they see area as a region or as a line.

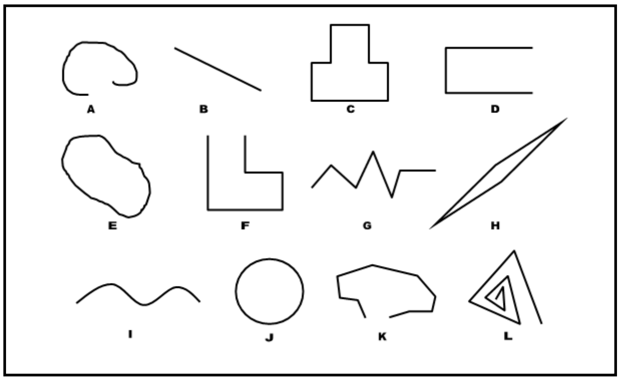


Figure 5.1
The 12 shapes: students are asked to select the shapes that have area

The findings show that all of the students have a prior understanding of area as a region although there are inconsistencies of the shapes chosen by the students since they still have a lack understanding of the kind of shapes that have area.

For example, most of the students (five out of six students) chose close curves (such as C, E, H, or J) and open curves (A, B, D, F, G, I, K, or L) in the same time. Here, there is an inconsistency of the shapes chosen by the students that have area. But, when they are interviewed, they said that the area of the shapes is the region inside the boundaries. It seems that these students actually have a prior understanding of area as a region, but they have no idea yet about the kind of shapes that have area.

Only one student chose open curves as the shapes that have area, such as A, B, D, F, G, I, K, or L. But, in the interview this student said that the area of the shapes is the region around the shapes. Here, he still saw area as a region. As well as the other students, he has no idea yet about the kind of shapes that have area.

2. Most of the students have no idea about area as the number of area units covering a plane figure

Here, the students are given three plane figures that are covered by square units (see Figure 5.2). They are asked to sort the plane figures from the largest to the smallest. The problem are intended to check whether students could see area as the number of the squares (units) covering the plane figures or not.

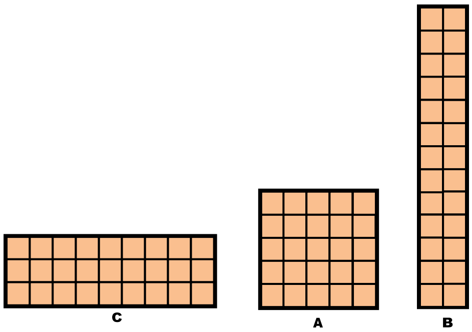


Figure 5.2
Comparing the area of three plane figures

In the second problem, the students are asked to compare the area of two incomplete-tiled floors (see Figure 5.3). The purpose of this problem is to check students' consistency between their answer in the first problem and in the second problem. In addition, this problem was intended to see students' prior understanding of the array structure of units of area (the squares units).

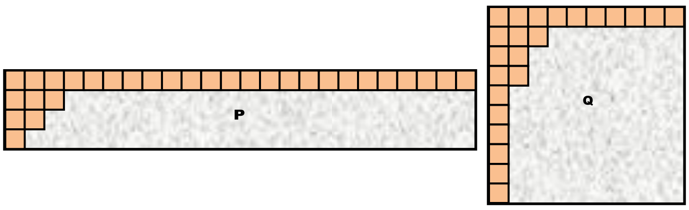


Figure 5.3
Two plane figures covering by square tiles

It is found that most of the students have no idea yet how to deal with area comparison problem. Most of them compared the length or the width of the plane figures when comparing the area of the plane figures. Only a few of them could see that the squares can be treated as the units to do comparison.

The findings from the first problem show that most of them (four out of six students) sorted the plane figures based on the length or the width of the plane figures. When they considered the length, they stated that *b* is the larger and then c and a. Meanwhile, the sort would be *a*, *c*, and *b* when they considered the width. Other two students sorted the plane figures based on the number of the squares (units) covering the plane figures. These students counted the squares constituting the plane figures and then compared the numbers they obtain. Therefore, they stated that the larger is c since it contains 27 squares, and then *b* and *a* which have 26 and 25 squares respectively. It seems that these students have a prior understanding of area as the number of square units covering the plane figures.

In the second problem most of the students (five out of six students) compared the length or the width the plane figures when comparing the area of the figures although in the first problem there are some of them who could compare the area of plane figures by considering the square units. Only one student compared the area of the plane figure by considering the square units on the plane figures. But, he only compared the visible squares, not compare the whole possible squares covering the plane figures, It seems that this student has a prior understanding of area as counting the square units, but he did not understand yet about area as the number of area units covering a surface completely. This also implies that the students have no idea yet about the array structure of the square units.

3. Students have no idea yet about measuring area of a rectangular plane figure

Here, the students are shown a rectangular plane figure and are asked to measure the area of the plane figure (see figure 5.4). The purpose of the problem is to find out what students will do when they are asked to measure the area of a rectangular plane figure.

Figure 5.4
A rectangular plane figure: students are asked to measure the area of the plane figure

The findings show that three out of the six students measured the area of the plane figure by measuring the length and the height of the plane figure by ruler. They presented the area of the plane figure in length and in width, for example 8 cm of length and 4 cm of width. The other two remaining students stated that the area of the plane figure is the length of the plane figure only, such as 8 cm. The last another remaining student measured the area of the plane figure by measuring all of the dimensions of the plane figure. He defined every side of the plane figure by letter, such as a, b, c, and d. He then said that the area of the plane figure is a=4 cm, b=8 cm, c=4 cm, and d=8 cm.

According to the findings, it is concluded that the students have no idea yet how to deal with measuring area. They assumed measuring area as measuring the dimensions of the plane figure. It implies that in this level the students only knew length measurement and has no idea yet about area measurement.

4. Students have no idea yet about area measurement of a right-angle plane figure

Students are given a right-angle plane figure with its dimensions in each side (see figure 5.5). Students are asked to determine the area of the plane figure. The purpose of the problem is to investigate how the students will deal with measuring the area of a right-angle plane figure and how they will interpret the dimensions.

Figure 5.5
Measuring the area of a right-angle plane figure

The findings show that two of the six students measured the perimeter instead of the area when they are asked to measure the area of the plane figure. They measured the length of edge by a ruler. They summed the result of their measurement to obtain the area. Here, they saw area as perimeter. Meanwhile, two other students stated the area as the width of the plane figure. They looked at the width (height) of the plane figure and by using a ruler he found that the length of the width is 6cm. Another student stated it as the length of the plane figure, which is 10cm. The last one remaining student has no idea yet of finding the area of the plane figure.

Grounding to the findings above, the students have no idea yet how to measure area. Most of them thought of measuring area as measuring the perimeter of a plane figure and some others considered measuring area as finding the width or the length of a plane figure. The role of the dimensions as the representation of the array structure of the area units was not understood by the students.

D. The Preliminary Teaching Experiment (The First Cycle)

There are six students of the third grade (around 9 years old) are involved in this experiment. They are Zidan, Rizal, Amiq, Alif, Haikal, and Opik. They are from the average students regarding to their mathematics ability among the students in their class. In most of the lessons, Opik did not present in the classroom due to sick. Therefore, he was not consider in these analyses.

There five lesson in this experiment. The purposes of the five lessons are shown in the following tables.

Table 5.2
The purposes of each lesson in the preliminary teaching experiment

Lessons	Purposes
1	Introducing the students to the concept of area through area comparison problems
2	Introducing and developing students’ understanding of the unit measurements of area

3	Developing students' understanding of the array structure of units of area
4	Developing students understanding of the multiplicative structure of units of area
5	Developing students' understanding of the role of the dimensions in applying multiplication strategy in area measurement

1. Lesson 1: Introducing a preliminary concept of area through area comparison problems

In this lesson, it is expected that the students learn the preliminary concepts of area, such as determining the larger or smaller area through area comparison problems. The learning activities in this lesson are designed to provide students with the experience of using perception, superposition, and decomposing strategy in solving area comparison problems. It is argued that area comparison problem provide a preliminary insight of the concept of area where the students may express the size of plane figures in terms of 'large' or 'small'.

The learning activity in this lesson consists of a preliminary problem, two main problems, and a follow-up problem. The findings from those problems are explained in the following paragraphs.

The preliminary problem

The purpose of the preliminary problem is to activate students in learning environment as well as to provide them with the experience of using perception in area comparison. The students are given a sheet containing five frames and six cuts of pass-photos (see Figure 5.6). They are asked to frame the photos to the appropriate given frames.

Afterwards, there are asked to arrange the frames based on their sizes, from the smallest to the biggest.

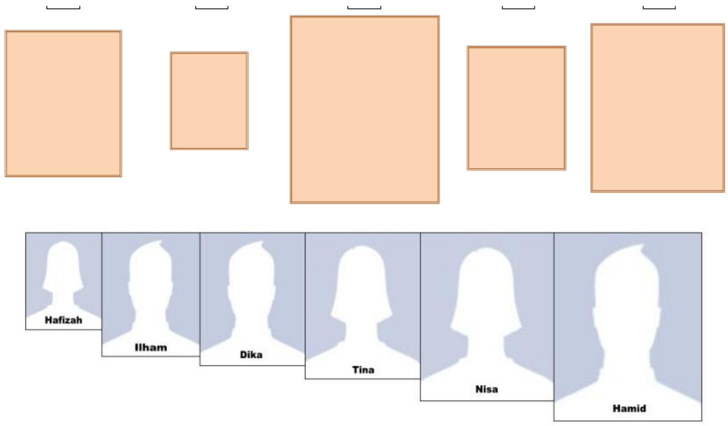


Figure 5.6
The five frames and the six pass-photos

As the conjecture, all students used their perception in solving this problem. However, perception seems to be not convincing enough for them. Therefore, there are some disputes among the students even in one group. To convince themselves sometimes they used the superposition strategy by trying-out putting a photo more than once on the frames until the photo is fit with a certain frame.

In arranging the frame, they also relied on their perception. Let's consider the following transcript among the students.

Students: *(they read the question by heart together that asks to arrange the frame from the smallest to the biggest)*
Opik: This, this, this, this, and this *(he directly points Hafizah's, Ilham's, Tina's, Nisa's and Hamid's frame respectively)*

The transcript shows that the students relied on their perception in arranging the frames. They just pointed the frames from the smallest to the largest without doing any measurement but just by looking at the frames.

Considering the findings, it is argued that the existence of the perception strategy is due to the differences are big enough among the size of the photos and frames that allow the students to compare them just by looking at them without conducting any measurement. Furthermore, the use of perceptual comparison is triggered by the shape of the frames and the photos. All of them are rectangular shapes with the same ratios of length and width. This similarity allows the students to only use their perception by looking at the figures since it does not strongly force them to come up with other strategies.

The main problems

The main problems are designed to introduce students to the use of superposition and decomposing strategy in area comparison. There are two main problems. In the first problem, students are asked to determine which grassland provides more grasses (figure 5.7). Then, to support the progressive mathematization, in the second problem students are asked to determine the larger grassland of another pair of grasslands, instead of asking the grassland that provides more grass (see Figure 5.8).

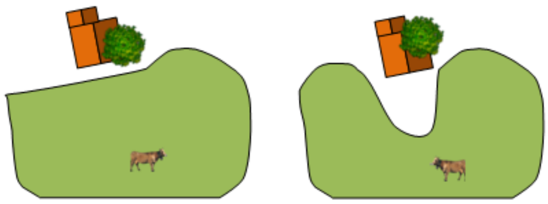


Figure 5.7
A pair of grassland for the first problem

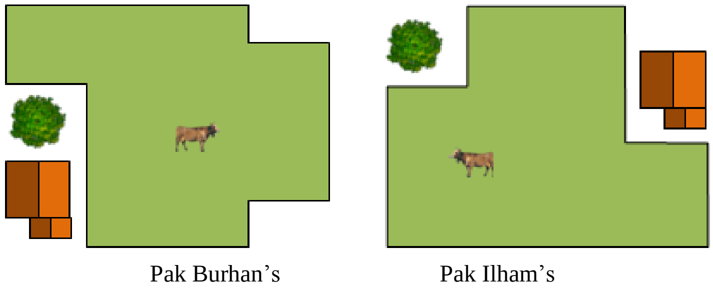


Figure 5.8
A pair of grassland for the second problem

In dealing with the first problem, the students started comparing the length of the two grasslands, but they found that both grasslands have the same length. Therefore they were failed and there was a dispute among them on their answer. They then looked at another dimensions, the width of the grasslands. He found that Pak Karim's grassland is looked slimmer than pak Ilham's. Therefore, they argued that Pak Ilham's provides more. In this moment, the students still relied on their perception. They looked the grasslands and examined the differences. This is probably because they have no idea yet of comparing or measuring area.

Then the students are given the cards of the grasslands and are asked whether the cards could help them. As the conjecture, the students will develop other strategies, such as superposition and decomposing strategy.

The findings show that all groups used the superposition strategy first before they decomposed the cards. Let's consider the following transcript among the students and the teacher.

Rizal: Here! (*he point B. see figure 5.9*) there is a hole in Pak Karim's
Zidan: The hole is small if this one (*A*) is bent and we put it here (*B*).
Teacher: How will you do that *(bending)*, Zidan?

Zidan: We cut this *(A)*. If we put this *(A)* here *(B)*, this *(Pak Karim's)* will have a small hole.
Teacher: So, which one is bigger?
Amiq: If we cut this *(A)*, it will be like this (*he draws a figure like A on B by his finger*).
Zidan: There is a small hole (*refers to pak Karim's grassland*).
Teacher: So, do you want to cut it?
Amiq: yea…

Then they cut A and put it on B. They conclude that Pak Ilham's is larger. The teacher then asked them to explain it. Here what the student said:

Zidan : This is a hole (*refers to B*), a small hole.
Teacher : Which figure has the small hole?
Zidan : Pak Karim
Teacher : What about Pak Ilham?
Zidan : No hole.

The transcript shows how the idea of decomposing emerges among the students. They cut the leftover area of only one plane figure and they put the cut leftover area on the top of the leftover area of another plane figure. Then, to determine the larger plane figure they examined which plane figure has more leftover area. Here, they expressed the leftover area with the word 'hole'. For example, Rizal said *"Here! There is a hole in Pak Karim's"*. The hole in the sentences implies that there is a leftover area on Pak Ilham's grassland.

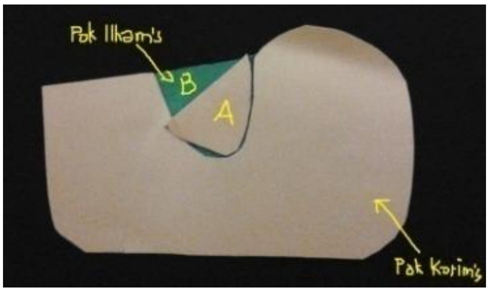


Figure 5.9
Decomposing strategy: An example of students' solution

Another group of the students develop another ways of decomposing as it is shown in Figure 5.10. They cut the leftover area of both plane figures. To know the larger plane figure, they compared the size of the cut leftover areas by putting one on top of each other.

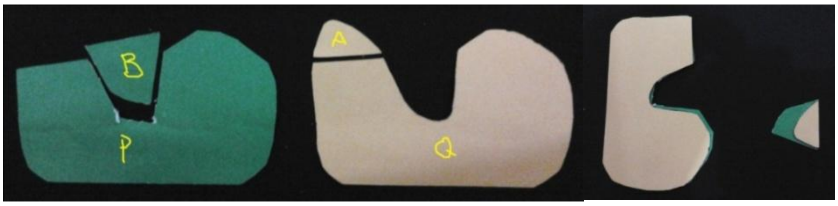


Figure 5.10
Decomposing strategy: Another example of students' solution

From the findings above, there are two different ways of decomposing developed by the students (see Figure 5.9 and 5.10). However, all students came to the same result that Pak Ilham's grassland is larger than Pak Karim's. Here, the existence of the cards triggers the students to come with superposition and decomposing strategy. Moreover, the structures of the two grasslands, where most part of the two grasslands are the same. It stimulates students to come up with superposition and decomposing strategies.

In the second problem, as well as the previous problem, the students were given the cards of the grasslands. As the conjecture, the students used the decomposing strategy to solve the problem. There are three steps the students went through before they decompose the cards. First, the students looked at the card carefully to examine the larger card but they failed. Second, they then put one card on top of each other. Third, they examined the leftover area in each cards.

Each group came up with a different ways of decomposing. Let's consider the following figure 5.11 showing the decomposing strategy used by one group of the students.

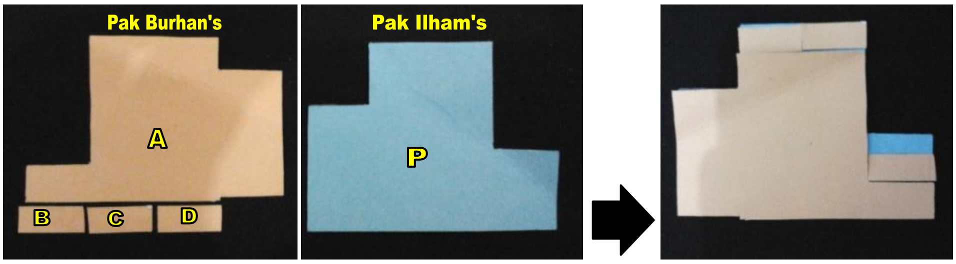


Figure 5.11
Decomposing strategy: An example of students' solution of the second problem

The figure above shows that the students used the decomposing strategy. They first looked at the card carefully (perception), put one card on top of another (superposition) to examine the leftover area in both cards, and then decompose one card to another card (decomposing). Their explanation is shown in the following conversation.

Zidan: We cut this one (*Pak Burhan's grassland is cut to be A, B, C, and D*).
And we put this (B) here (P) and this one also (C) we put it here (*P*) and this one (D) we add here (*P*) *(He puts B, C, and D on P respectively).*
So, this one is bigger *(he points P since there is still leftover area on P).*
Teacher : So, which one is bigger?
Rizal : Pak Ilham's
Teacher : the green one or the blue one.
Amiq : the blue one.

Another decomposing strategy shows by another group (see Figure 5.12). As well as the previous group, they looked at the two cards carefully; they then put one of the cards on top of another card; then they examined the leftover areas of both cards, then they cut the leftover area in both cards; and they compare the area of each corresponding leftover. Their explanation is shown in the following transcript.

After they cut the leftover area in both cards, Haikal said:

Haikal: These are same *(he takes B and Q, and puts B on Q but there is no leftover area).*
These are also same *(he takes C and R, and puts C on R but there is no leftover area).*

Opik These are different. (*as Haikal takes A and R, puts A on R but there is a leftover area on P*).

Teacher : So, which one is bigger?

Students : Pak Ilham's (*as one of them points on the leftover area of P*)

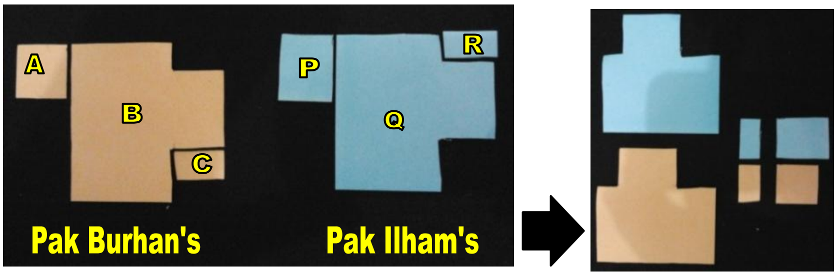


Figure 5.12
Decomposing strategy: Another example of students' solution of the second problem

Grounding to the findings, it is assumed the existence of the cards and triggered the students to do the superposition by putting one card on top of another card before they decompose the cards to examine the leftover area. Moreover, the shape of the cards itself triggers students to decompose them since they are easier to decompose since they contain straight-line edges.

The follow-up problem

The follow up activity has two functions. The first is to examine students' understanding of concepts learned in the previous activities. The second is to provide students with exercise to strengthen their understanding of concepts learned before. In this activity, students are still engaged in comparing activity, but the situation is more complicated. The students are given three shapes and they are asked to sort the shapes from the biggest to the smallest.

As the conjecture, both groups applied the decomposing strategy. Before they decompose the shapes, they first looked at the three shapes carefully and then they tried putting one shape on top of other shapes to examine the leftover area. Although both groups used the same strategies, there were differences between the two groups on the way they applied the strategy. The Figure 5.13 shows each group's way of decomposing. Both groups came to the same result that shape B is the largest followed by shape A and then shape C.

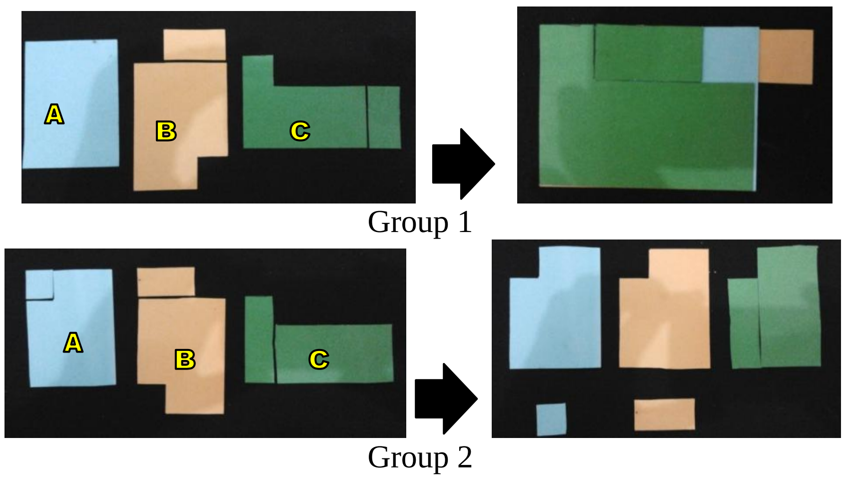


Figure 5.13
Two different ways of decomposing occurred among the students

Conclusion and discussion

Grounding to the findings, it is argued that the area comparison problems in this lesson could lead to the emergence of perception, superposition, and decomposing strategy in solving area comparison problems.

The emergence of perception, superposition, or decomposing strategy is due to the shape of the grasslands. In the first pair of the grasslands, for example, the students dealt with irregular shapes, where most of the parts of the shapes are identical (see Figure 5.7). The irregularity triggered students to use the superposition strategy to find the larger shape since the perception strategy provides unconvincing result. Once the students put one shape on top of another shape (superposition), they are then triggered to decompose the shapes since most parts of the shapes are identical. The same things also happened when the students dealt with the second pair of the grasslands (see Figure 5.8).

The existence of the cards of the grasslands also triggers the emergence of superposition or decomposing strategy. The cards allows students to put one card on top of each other when examining and comparing the size of the cards. Moreover, it also allows students to decompose the card by cutting them.

Experiencing the three strategies in the same time and in the same problem helps students to see the strength and the weaknesses of each strategy. In solving the first pair of grassland, for example, the students firstly experienced of using perception in comparing the area of the grasslands, but the result did not convince them. They then used the superposition strategy, but some of them are not convinced with the result of the strategy. Finally, they came to the decomposing strategy. Here, they could easily see the result of the strategy provide a more convincing result.

The process of moving from one strategy to another strategy provides students with a rich experience of solving area comparison problems without conducting calculation. The students could see why a certain strategy is better and yields a stronger result than other strategies.

The words 'large' and 'small' are used by the students to express the size of an area. When the students want to say that one shape has more or less area than another shape, they stated it in the words 'larger' or 'smaller' respectively. The words 'same' and 'different' are also used to express area. For example, Haikal said "*these (two shapes) are same*". He actually wants to say that both shapes have the same area. When Opik said "*these (two shapes) are different"*, he actually wants to say that those shapes have different area.

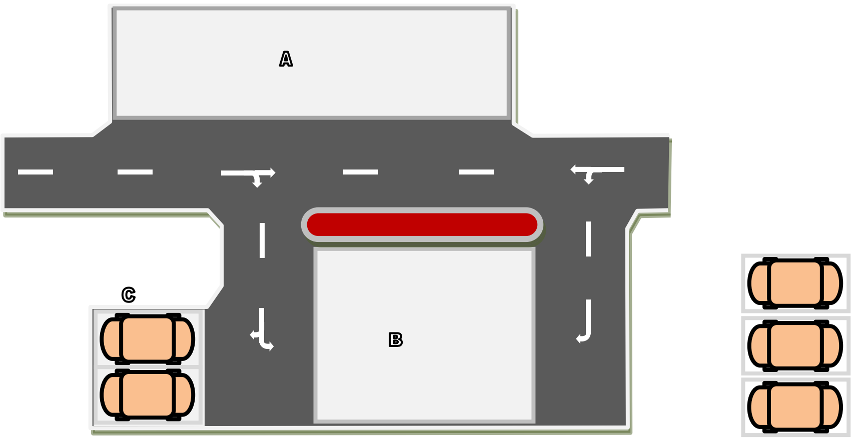
Here, the area comparison problem lead to the emergence of using the words 'large', 'small', 'same' and 'different' in expressing area. Expressing area in terms of numbers is occurred yet in this problem since the problem did not ask to measure area but to compare area.

2. Lesson 2: Introducing and developing students' understanding of the unit measurements of area

This lesson is purposively designed to introduce students to the idea of measurement units of area. To reach the purpose, the students are given three problems: a preliminary problem, a main problem, and a follow-up problem.

The Preliminary Problem

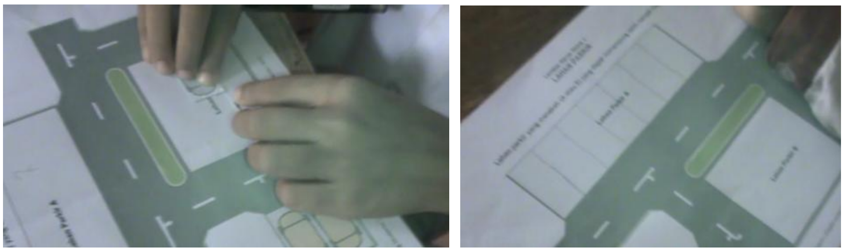
This problem is designed to provide students with an experience of doing unit iteration through covering area activities. The students are given a picture containing of two parking lots and three pieces of rectangular cards representing cars (see Figure 5.14). Students are asked to determine which parking lot occupies more cars, A or B.



(a) (b)

Figure 5.14
(a) Parking lots and (b) cards representing cars

As the conjecture, the students covered the parking lots by iterating the given cars on the surface of the parking lots. There are two different ways of iteration occurred. First, the students took two cars and iterated them by moving them on a parking lot one by one (see Figure 5.15a). Second, the students took only one car and traced it on a parking lot till cover the whole surface of the parking lot (see Figure 5.15b). Here, none of the students use the given three cars to do iteration. It is probably because the size of the parking lots which is relatively small that allows the students to use only one or two cars.



(a) (b)

Figure 5.15
Two different ways of iterating the cards to cover the parking lots

The way they counted the cars occupying the parking lots is influenced by the way they iterated the cars. The students who iterated the cars by moving them one by one counted the cars one by one as they moved the cars. Meanwhile, the students who traced the cars on the parking lots counted the cars one by one after the tracing is done.

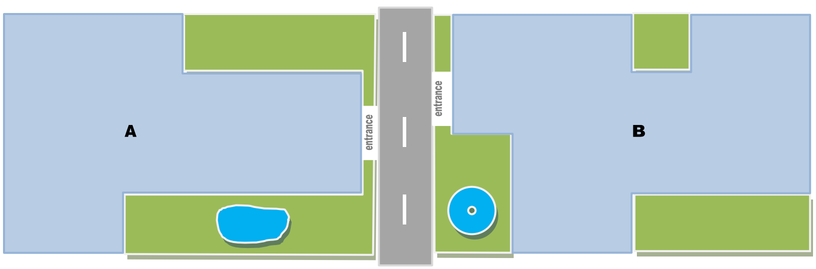
In this activity, the idea of area then introduce by asking the students the larger parking lot. They all agreed that parking lot A is larger since it occupies more cars than parking lot B (7 and 6 cars respectively). Here, the preliminary idea of unit measurement of area is introduced where the students could see the larger area by considering the number of cars as the units covering the area and the number of the cars is gotten from doing unit iteration (car iteration).

The Main Problem

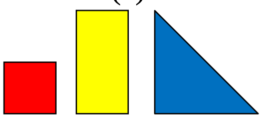
The purpose of the problems is to introduce students to the measurement units of area where students will be engaged in area comparison problem. As the problem, the students are asked to find the larger area of two parking lots (see figure 5.16a).

Students are given three different sizes and shapes of cards (see figure 5.16b). It is expected that the students use the cards as the tools in comparing the area of the parking lots. The three cards consist of 18 squares, 9 rectangles (the size of each rectangle is twice of the square) and 9 triangles (the size of a triangle is twice of the square).

However, the cards are not given in the first place. Firstly, the students are asked to find the larger parking lot by using their perception or other strategies. It is conjectured that the students will get difficulties. Then, the students are given the cards to help them in doing investigation. This experience is purposively designed to help students to understand the usefulness of the cards in solving the problem.



(a)



(b)

Figure 5.16
Two parking lots and three cards

The findings show that before the students are given the three cards they used their perception in determining the larger parking lot. They looked at the parking lots and then pointed one of the parking lots as the larger parking lot. However, as the conjecture there were disputes among them in deciding the larger parking lot.

They then are given the three cards and are asked whether the cards could help them. As the conjecture, the students used them to compare the area of the parking lots as it is shown in the following conversation between Zidan, Rizal and the teacher.

Zidan : We have done.
Teacher : So, which one is larger?
Rizal : B
Teacher : Could you explain?
Zidan : In B, we used 9 rectangles. Meanwhile, in A, we used 8 rectangles and 1 square.
Teacher : 8 rectangles and 1 square? So, how could you say B is larger?
Zidan : Because it has 9 rectangles.
Teacher : Ooooh… So?
Zidan : We know that the square is smaller than the rectangle. The rectangle is bigger.

The transcript shows that the students could use the cards as the unit measurement in comparing the area of the parking lots. They knew that parking lot B is larger than A since B occupies 9 rectangles meanwhile A occupies 8 rectangles and 1 square where the size of the square is a half of the rectangle. Considering the cards they used, it seems that the students tried to used only one type of cards which the rectangle. They only used the square to cover the last remaining uncovered plane figure in parking lot A which is too big if they used a rectangle.

After they discussed about the possibility of using one type of cards, this group chose the squares as the units since the squares could cover either in parking lot A or B completely without the need of other cards. Based in this finding, it is assumed that the students have an understanding of unit consistency in comparing area. But, the idea of unit consistency is not directly seen in the eyes of the students. The teacher still needs to guide them.

However, the idea of unit consistency is not easy for the other group, Opik and Alif. They compared the area of the parking lots using more than one type of cards (see figure 5.17). They used squares to cover parking lot A and the rectangle in parking lot B. Before that they used the squares and the rectangles in parking lot A and the triangles in parking lot B, but they got difficulties in arranging the triangles. Therefore, they did not use the triangles any longer.

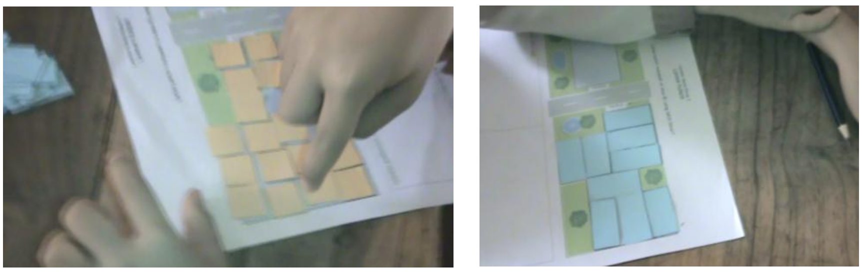


Figure 5.17
Amiq and Alif used the squares to cover parking lot A and the rectangles in parking lot B

Let’s consider the following conversation between Alif and the teacher.

Teacher : Well, how many all together [in parking lot B]?
Alif : 9
Teacher : 9?
Alif : squares.
Teacher : squares?
Alif : Oh no… 9 rectangles.
Teacher : So, you have 17 squares *(in parking lot A)* and 9 rectangles *(parking lot B).* So, which one is larger?
Alif : This one *(He points parking lot A).*
Teacher : How do you know it?
Alif : Here *(he points the 17 squares on parking lot A).* We have counted it.

The transcript shows that the student concluded that parking lot A is larger than B since A occupies 17 squares meanwhile B only 9 rectangles. It seems that he compared the area of the parking lots by considering the number of cards used, but he did not consider the size of the cards he used. It is assumed that the student did not realize yet that the size of the units will influence their measurement.

After discussion with the teacher, they were aware that the different size of cards they used will influence their conclusion. The teacher guided them to realize that the size of the rectangle and the squares is not same; the size of a rectangle is two squares. Let's consider the following transcript.

Teacher : Well, how many rectangles here *(parking lot B)*?
Amiq : 9
Teacher : So, how many squares will be here *(parking lot B)*?
Students: mmmm…. *(They seem to have no idea).*
Teacher : Here, one rectangle is equal to two squares. So, if there are 9 rectangles, how many squares are there?
Amiq : I don't know *(He tries arranging the squares on the parking lot B to find the number of squares are needed to cover it).*
Teacher : Well, two squares are for one rectangle. If it is 9 rectangles, how many [squares]?

Alif : 18
Teacher : How many?
Alif : 18
Teacher : So, how many squares here?
Alif : 18 squares.
Teacher : How do you get 18 squares?
Amiq : We add it.
Alif : add it? No! We multiply it.
Teacher : Multiply by what?
Amiq : 9
Alif : 9 times 2.
Teacher : Good, 9 is multiplied by…?
Alif : 2.
Teacher : so, what is the product [9 x 2]?
Students : 18.
Teacher : So, which parking lot is larger? Here *(parking lot A)* you have 17 squares. And here *(parking lot B)* you have 18 squares. Which one is larger?
Students : this *(they point together parking lot B).*

The transcript shows how the teacher guided the students to the idea of unit consistency. After the students got that there are 17 squares in parking lot A and 9 rectangles in parking lot B, the teacher then guided to use only one type of card by asking students to transform the rectangles to be squares. In the end the students were aware that the 9 rectangles imply 18 squares since the size of one rectangle is two squares. They just multiplied 9 by 2 to get 18 squares. Then, by considering the 17 squares in parking lot A and 18 squares in parking lot B, the students realized that parking lot B is larger. Here, the teacher played significant role in guiding the students to see the idea of unit consistency in area measurement.

The Follow-up problem

The purpose of the follow-up problem is to bridge the students from area comparison problem to area measurement problem. Therefore, the students are asked to measure the area of a plane figure (see Figure 5.18) by using the given squares. It is expected that the students will treat the squares as the unit measurement.

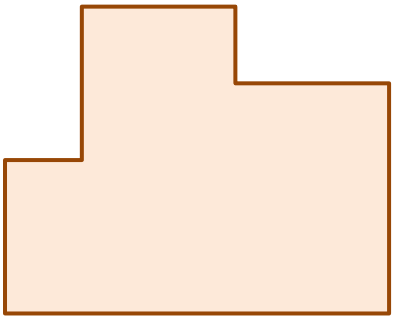


Figure 5.18
A plane figure: students' are asked to measure the area of the plane figure as the follow-up problem

As the conjecture, the students used the given squares as the unit measurement to measure the area of the plane figure. They covered the plane figure using the squares. They then counted the number of the squares covering the plane figure. They found that the area of the plane figure is 16 squares.

It seems that their experience of solving the previous problem, the parking lot, influenced their strategy in solving the follow-up problem. In this problem, the students got the idea of what they have to do if they want to measure the area of a plane figure.

Conclusion and Discussion

Grounding to the findings above, it is concluded that the activity in this lesson helps students to see the role of the square units in telling the area of a plane figure. In solving the parking lot problem, for example, the students could determine the larger parking lot by considering the number of square units covering the parking lots. Even in the follow-up problem, their understanding of units is clarified. They could measure the area of a plane figure. They iterated the square units on the plane figure and they found that the area of the plane figure is the number of squares needed to cover the figure completely.

It is argued that the sequence of learning activities helps students to understand the measurement units of area. In the preliminary problem, the students experienced of unit iteration and using of a reference (the cars) in determining the larger parking lot. Then, in the main problem they are engaged with area comparison problem. Their experience of unit iteration in the previous problem help them in this problem. In this problem students learned the measurement units of area. In the next problem, the follow-up problem, students are given an experience of using the measurement units of area in measuring the area of a surface.

However, the idea of unit consistency is not easy to be understood by the students. To guide them to understand the idea of unit consistency, the teacher needed considerably a long discussion with the students. Here, the role of the teacher is necessary to help the students seeing the idea of unit consistency.

3. **Lesson 3: Developing students' understanding of the array structure of units of area**

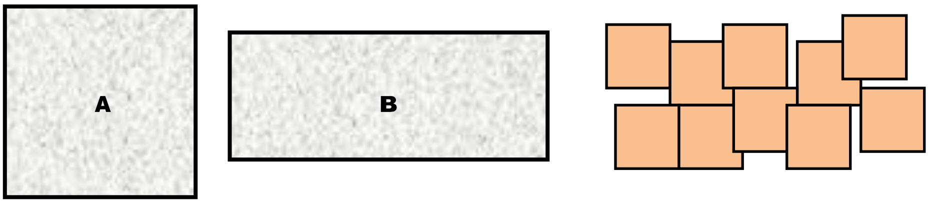
After students learn the unit of area, they are engaged in exploring the array structure of units of area through structuring array activity. It is expected that they are able to see the array structure of the units of area in columns and rows and use the structure in determining the area of a plane figure.

There are three problems designed in this lesson: a preliminary problem, a main problem, and a follow-up problem.

The Preliminary Problem

In this problem students are asked to find the larger plane figure between plane figure A and B (see figure 5.19a) by using 10 given squares (see Figure 5.19b).

The purpose of the problem is to remind them on their understanding of measurement units of area that they have learned in the previous lesson as well as to introduce them a preliminary experience of structuring array of the area units.



(a) (b)

Figure 5.19
(a) Plane figure A and B and (b) the ten square units

As the conjecture, the students used the 10 squares as the unit measurement in comparing the area of the two plane figures. They covered the plane figure with the squares. To find the larger plane figure, they compared the number of the squares needed to cover each plane figure. Let's consider the following transcript.

Teacher : Which one is larger?
Rizal : [plane figure] B
Teacher : Why B?
Rizal : Because it has 10 squares.
Teacher : What about A?
Rizal : It has 9 squares.

The transcript shows that the student could compare the area of the plane figures by considering the number of squares covering each figure. Here, the student treated the squares as the measurement units. It seems that the student could see area as the number of measurement units covering a plane figure.

In this problem, the array structure of units of area in rows and columns can be seen by the students during iterating the square units. It is found that there are three different ways of iteration emerged among the students as it is explained in the following paragraphs.

The first way of iteration is by considering the array structure of units in rows. Zidan and Rizal took the advantage of the structure of the units in rows in counting the number of squares covering the plane figures. In plane figure B, for example, they knew that in each row of the arrangement of the squares there are 5 squares. Since there are two rows, therefore the whole squares are 5 + 5 equal to 10 squares. Meanwhile, in plane figure A they said that the number of the squares is 3 + 3 + 3 equal to 9 squares because there are three rows and in each row contains three squares.

The second way of iteration is by tracing one square to the whole plane figure. Amiq and Alif used one square and trace the square on the plane figures as they transposed it by continuously changing its position without overlapping or leaving gaps on the remainder area of the plane figure. One the tracing is done, as the result they got gridlines on the plane figures (see Figure 5.20). They then counted the squares on the gridlines row by row to tell the area of the figures. Looking at the way they counted the squares, counting row by row, it is assumed that they could see the array structure of the area units.

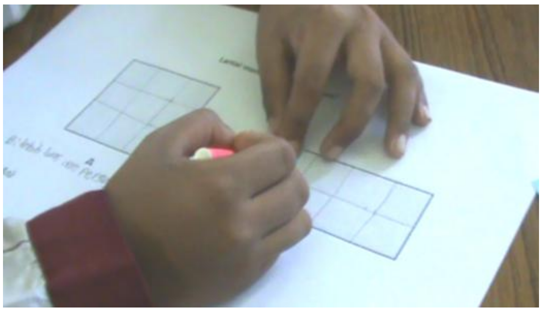


Figure 5.20
Tracing one squares to the whole plane figures

The third way of iteration is by taking as many of the squares as are required to cover the whole plane figures (see Figure 5.21). Haikal used this type of iterations he took as many of the squares as are required to cover one of the plane figures. He compared the size of the plane figures by considering the number of squares covering the figures. However, the array structure of the square units was not recognized yet by this student.

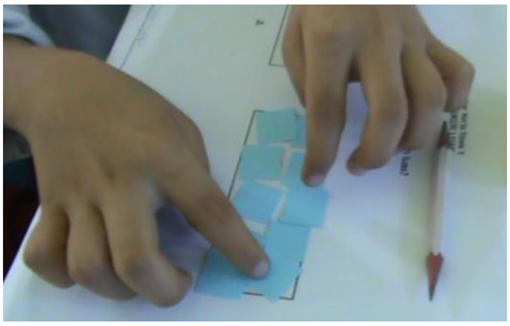


Figure 5.21
Square iteration by taking as many of the squares as are required to cover a plane figure

The Main Problem

In this problem, it is expected that the students could see the array structure of area units. To reach the purpose, the students are asked to measure the area of a plane figure (see Figure 5.22) by using ten given squares (see figure 5.19b). Here, the students will experience of structuring the array of area units by using the ten given squares.

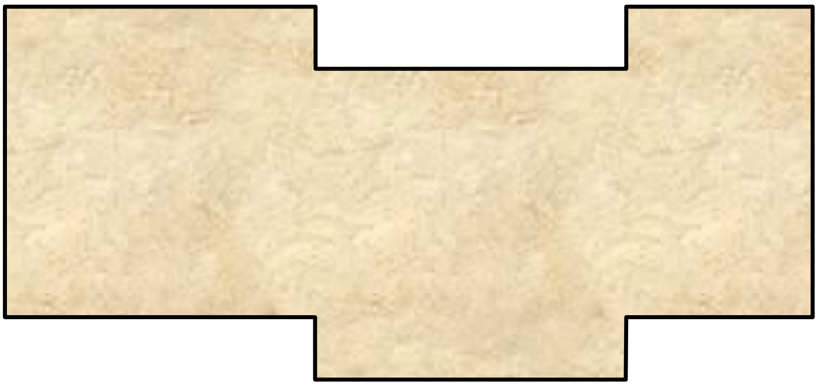


Figure 5.22
A plane figure: students are asked to measure the area of the plane figure by using the given ten squares

The findings show that to deal with the problem the students tried to find the number of squares covering the plane figure. To find the number of the squares, all students split the plane figure into some smaller rectangular figures to help them finding the number of the squares covering the plane figure (see Figure 5.23).

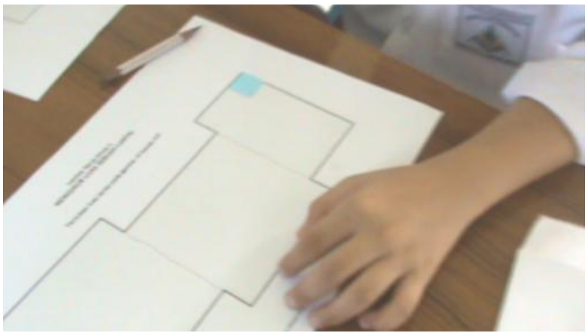


Figure 5.23
Splitting Technique: Students split the plane figure into some smaller plane figures

However, the reasons underpinning the splitting were different among the students. Some of them split the plane figure in order to work with smaller parts when iterate the square units by tracing one of the square units (see Figure 5.24). Some other students split the plane figure to help them drawing gridlines (see Figure 5.25).

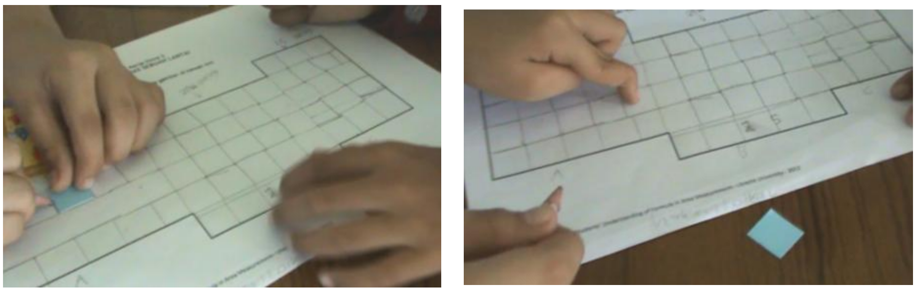


Figure 5.24
Unit iteration: the students take one square and trace it over the whole plane figure

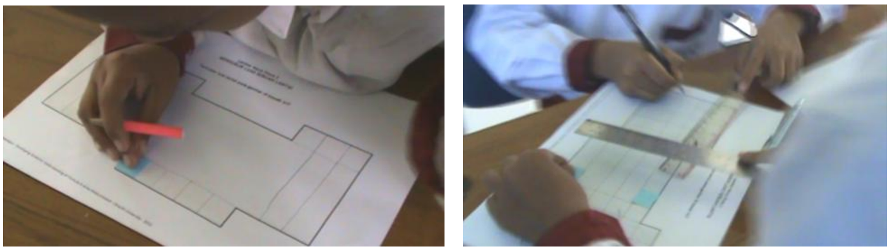


Figure 5.25
Drawing gridlines by using one or some squares as a reference

Grounding to the findings above, it is conjectured that the emergence of the splitting technique as well as the unit iteration and drawing gridlines is due to some factors, such as the size of the plane figure being measured and the number of the given squares.

The size of the plane figure did not allow the ten given squares to cover it completely or squares could not be used to cover the whole plane figure completely. This situation triggered the students to split the figure into some smaller figures.

Although the students have split the figure into smaller figures, the number of the given squares still was not adequate to cover the smaller figures. This condition triggered the students to iterate the squares by tracing one square over the whole figures or by drawing gridlines.

The way the students iterated the square units influences their strategies in counting the square units.

The students who iterated the square units by tracing one square counted the squares one by one without considering the array structure. It seems that the experience of the tracing process influences their strategy in counting. Unfortunately, these students could not see the array structure of the area units yet. It is conjectured that the tracing causes students' difficulty in seeing the array structure since during the tracing the students just focused on the square used to trace but did not consider the array structure. Moreover, if the tracing process is not done well, the result of the tracing fade the array structure (see Figure 5.26).

Figure 5.26
The result of tracing process: the array structure of area units is faded

Meanwhile, the students who drew the gridlines counted the squares by considering the array structure of square units as it is shown in the following transcript.

Teacher : How did you count the units?
Zidan : One, two, three, four, five *(He counts the squares on the first column, see figure 5.27)*
Then, together with Rezal, he counted by skip counting of 5:
Students: 5, 10, 15, 20 , 25, 30, 35, 40, 45, 50, 55, 60, 65. *(They count as they point each column one by one)*

The transcript shows that the students counted the squares column by column. They know that there are five squares in each column. Therefore, they counted the squares by skip counting of five. It seems that the students were aware of the array structure of the units of area.

It is conjectured that their experience of drawing gridlines help them in counting the squares by considering the array structure because when drawing the gridlines they needed to consider the arrangement of the squares. Moreover, the gridlines provides a better view of the arrangement of the squares that helps students to see the array structure of the square units (see Figure 5.27).

In addition, the shape of the plane figure triggers the students to count column by column. There are five squares in every column of the plane figure which triggers the students to think of counting five-by-five (skip counting of five). The number of the squares being counted which is quite many also triggers the students to think of finding an effective way of counting the squares. If there are only a few squares, the students probably count the square one by one as it is found in the preliminary problem.

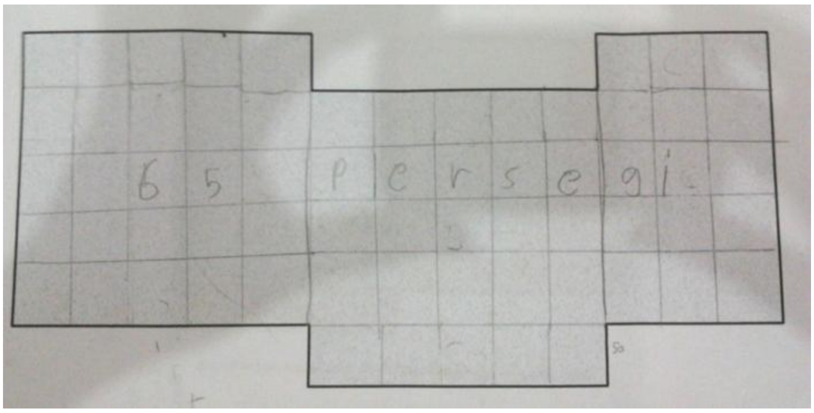


Figure 5.27
The result of drawing gridlines

The follow-up problem

The problem in the follow up activities is designed to assess students understanding of the array structure of area units that they have learned in the previous problem. Students are asked to find the area of an incomplete-tiled floor (see Figure 5.29).

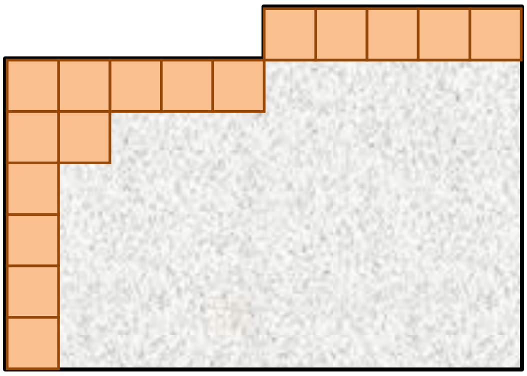


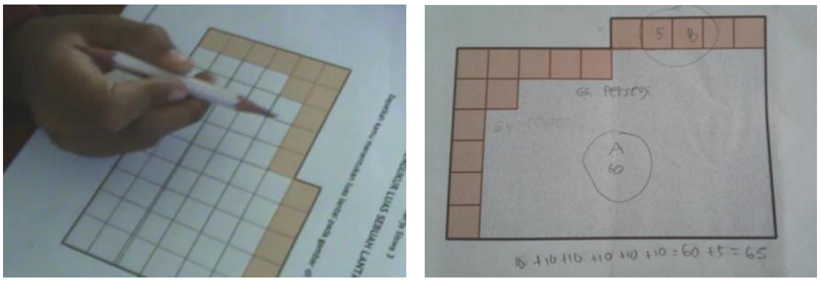
Figure 5.29
A follow-up problem to assess students' understanding of the array structure of area units

The findings shows that none of the students counted the squares one by one. Instead, they counted the squares row by row, column by column and counting by grouping .

Counting row by row was used by Amiq and Alif. Before counting the squares they drew the gridlines on the plane figure to complete the squares (see Figure 5.30a). They then counted the squares column by column as it is shown in the following conversation.

Amiq : 1, 2, 3, 4, 5, 6 *(He counts the squares on the first column)*
Teacher : Well, those are 6 [squares] and then?
Amiq : 6, 12, 18, … *(He counts as he points the first, the second, and the third column respectively)*
Alif : and 24, … *(He counts the squares on the first four columns as Amiq points the fourth column).*

The transcript shows that they counted the squares by skip counting of 6 since they knew that there are 6 squares in each column. Regarding to the way they counted the squares, it is argued that these students could see the array structure of the units of area.



(a) (b)

Figure 5.30
(a) Drawing gridlines to complete the squares and (b) splitting the plane figure into two plane figures before counting the squares

Another group of the students, Zidan and Rizal, they counted the squares by considering the number of the squares in rows, instead in column. Different from the previous group these students did not drew gridlines. Instead, they split the plane figure into two parts, A and B (see Figure 5.30b). Let's consider the way they count the squares as it is shown in the following transcript.

Rizal : 10, 20, 30, 40, 50, … *(He counts the units row by row from the second row until the sixth row. Then the teacher interrupts him)*
Teacher : Why skip counting by 10?
Zidan : by 10? Here, we have 1, 2, 3, 4, 5, and 1, 2, 3, 4, 5 *(He counts the visible five squares on the second row and the first row consecutively, see figure 5.30b)*
Teacher : So, how many (squares) all here? *(He points the second row)*
Zidan : 10
Teacher : and then?
Zidan : 20, 30 … *(He counts as he points the squares on the third and the fourth row)*
Students: … 40, 50, 60, and 5 *(They count together as one of them points the squares on fifth until the seventh row. When they say 5, they point the five remaining squares on the first row).*

The transcript shows that the students counted the squares by considering the number of squares in each row. They counted the squares by skip counting of 10. Regarding to the way they counted the squares it seems that these students are aware of the array structure of the area units.

Meanwhile Haikal developed another counting strategy, so called counting by grouping. After they drew gridlines to complete the squares on the plane figure, they grouped the squares five by five. In grouping the squares, they did not consider the array structure of the units. It seems that these students were not aware of the array structure of area units yet. In the end, these students are asked to consider the number of the squares in rows or in columns and are asked whether it could help them in counting the square units. They finally could counts the squares by considering the square units in rows. Here, the guide of the teacher is necessary for these kinds of students to help them seeing the array structure of area units.

Conclusion and Discussion

Grounding to the findings, it is concluded that structuring array activities in this lesson could help students to see the array structure of the units of area. In dealing with the main and the follow-up problem, for example, most of the students could count the squares by considering the array structure of the square units. They counted the squares by looking at the number of squares in columns or rows. However, there are some students that need an extra guide from the teacher to help them seeing the array structure of the units. Asking them to look at the number of the squares in columns or in rows can be a good guidance to help them seeing the array structure of the square units.

Based on the findings, it is also found that the way the students iterated the square units influenced their strategy in counting the units. The students who iterated the square units by tracing one square counted the squares one by one without considering the array structure Meanwhile, the students who drew the gridlines counted the squares by considering the array structure of square units.

Considering the findings in this lesson, structuring activity can be a good task to help students seeing the array structure of units of area. However, there are some factors relating to the task that need to be considered, such as the shape of the figure being measured and the number of units being counted. Measuring the area of rectangular shapes of right-angle shapes could help students to see the array structure of units of area since those shapes provide a good arrangement of units in columns and in rows. The number of units being counted also influences the strategy in counting the square units. A few units will lead to count one by one, meanwhile, many units triggers the emergence of a strategy in counting the units efficiently, such as counting column by column or row by row.

4. Lesson 4: Developing students understanding of the multiplicative structure of units of area

After the students have been introduced to the array structure of unit of area, in this lesson it is expected that the students learn the multiplicative structure of the array structure of the units of area. For that purpose, students are given two problems consisting of a main problem and a follow-up problem to be solved in this lesson.

The main problem

Here, students are asked to find the area of a patio and a living room (see Figure 5.31). it is expected that the students treat the tiles as the measurement units of area.

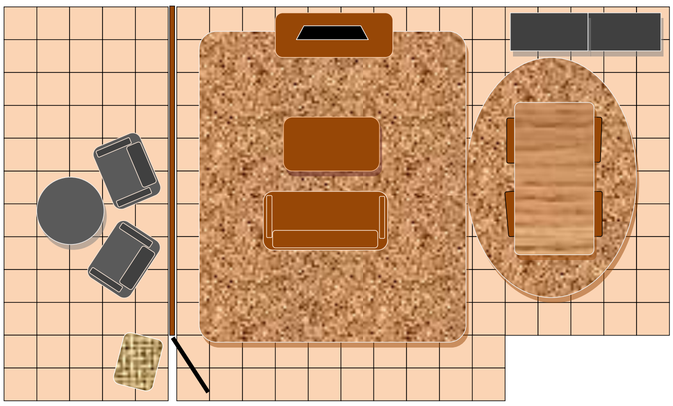


Figure 5.31
The patio and the living room: students are asked to find the area of the patio and the living room

The findings show that only a few students could see the multiplicative structure of unit of area in counting the tiles. Most of them counted the tiles by considering the array structure of the units.

Only Zidan and Rizal considered the multiplicative structure of the tiles as it is shown in the way they counted the tiles. Let’s consider the following transcript.

Teacher : How [did you count the tiles]?
Zidan : Here are 12 x 5 *(He points the first column and the last row of the tiles in the patio respectively).*
Teacher : Yes, how many [all the tiles] then?
Zidan : 60 squares

The transcript shows that the student could see the multiplicative structure of the tiles. He knew that there are 12 rows of tiles in the patio and in each row there are five tiles. So, the whole tiles covering the patio are 12 x 5 equal to 60 tiles.

In finding the number of tiles in the living room, they split the room into two rectangular plane figures and then applied the multiplication in finding the area of each rectangular figures (see Figure 5.32) as it is shown in the following transcript.

Zidan : Here are 12 and here are 10. So, it will be 120.
(He points the tiles on the first column and the last row of the first part respectively. See figure 5.32).
Teacher : Yes.
Zidan : Here are 10 and here are 5. So it will be 50.
(He points the tiles on the first column and the last row of the second part respectively)
Teacher : Yes.
Zidan : Then we summed them [120 and 50] to be 170.
Teacher : Well.
What is the area of the patio that you have found before?
Rizal : 60
Zidan : So, the living room has 170 squares, and the patio has 60 squares.

The fragment shows that the students split the plane figure into two rectangular parts and then applied the multiplication strategy in counting the tiles in each part. The emergence of the multiplication in counting the tiles implies that the students could see the multiplicative structure of units of area. Moreover, they knew that the multiplication can only be applied in rectangular plane figure. Therefore, they split the living room into two rectangular plane figures.

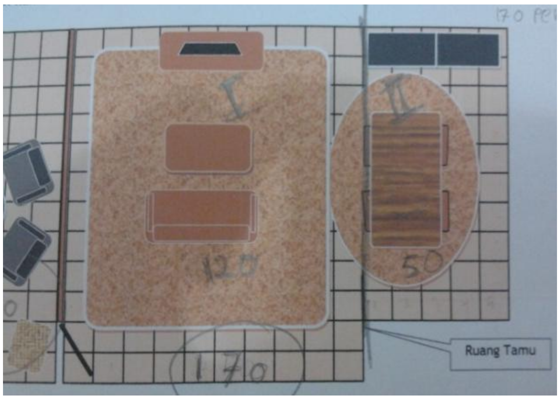


Figure 5.32
Zidan and Rizal strategy: They split the floor into two parts

However, most of the students counted the tiles by considering the number of tiles in column or in rows, instead of seeing the multiplicative structure of the tiles, for example, Haikal's strategy in counting the tiles of the patio showing in the following transcript.

Teacher : Could you show me how did you count the squares?
Haikal : We know that here are 5 [tiles], and this are 10 [tiles] *(He points the first five tiles on the first row and then the first ten tiles on the first and the second row together).*
Teacher : Yes.
Haikal : Add them by 10 again. We got 20 *(He points two following rows).*
Haikal : Add them by 10 again. It is 30 *(He points again two following rows).*
He kept adding by 10 until all the tiles are counted. In the end, he got 60 tiles.

The transcript shows that Haikal considered the number of tiles in rows when counting the tiles. He knew that in the patio there are five tiles in each row and ten tiles in each two rows. They then counted the tiles two rows by two rows; or skip counting by 10 since there are 10 tiles in each two rows.

Considering the way he counted the tiles, it is assumed that the student could see the array structure of the units of area, but he could not see yet the multiplicative structure of the units yet.

As well as Haikal, Amiq counted the tiles by considering the array structure instead of the multiplicative structure.. In dealing with counting the tiles of the living room, for example, he counted the tiles row by row after he split the living room to be two rectangular floors (see Figure 5.33). In the first rectangular floor, for example, he knew that there are 10 tiles in each row. Therefore, there are 20 tiles in each two rows. Then, in counting the tiles covering the floor 20 by 20 or skip counting of 20.

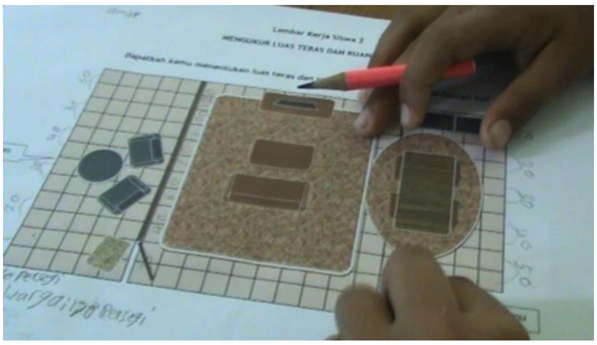
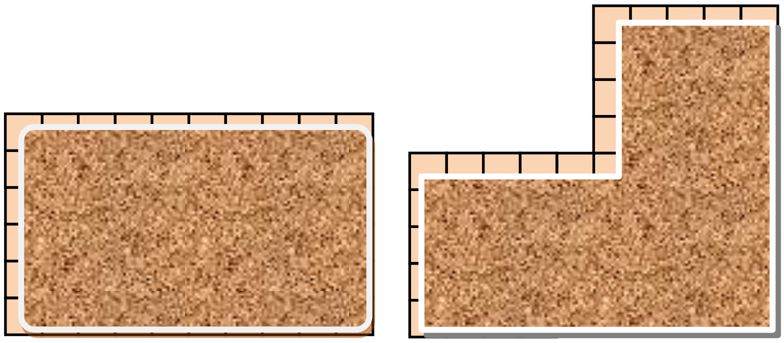


Figure 5.33
Amiq's strategy: Considering the array structure of the tiles

The follow-up problem

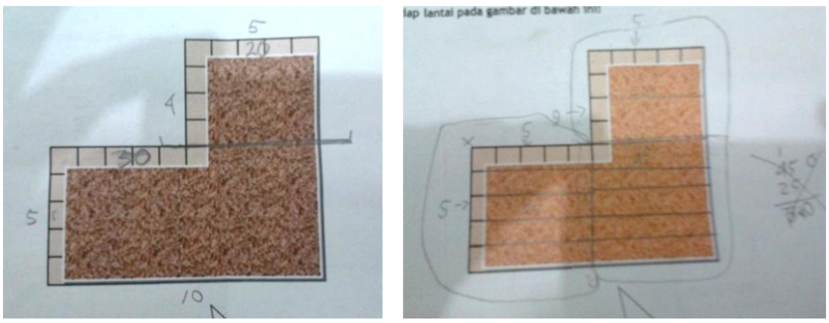
In the follow-up problem, students are asked to find the area of two floors that each of the floors is covered by a carpet (see Figure 5.34).



(a) (b)

Figure 5.34
Two floors covered by carpets

Different from the findings in the main problem, most of the students (four out of five students) used the multiplication strategy in counting the tiles (see Figure 5.35) to find the area of the floors. It seems that they knew that finding area means finding the number of area units.



(a) (b)

Figure 5.35
Two different multiplication strategies in counting the tiles

The figure above shows two different strategies in applying the multiplication strategy among the students. Both students split the floor into two partitions, but the partitions are different. Zidan and Rizal split the floor to be 5*x*10 tiles and 4*x*5 tiles. Meanwhile, Haikal and Alif split the floor to be 5*x*5 tiles and 9x5 tiles. In counting the tiles, Zidan and Rizal, for example, multiplied 5 and 10 to obtain the area of the first partition and multiplied 4 and 5 to obtain the area of the second partition.

However, not all of the students used the multiplication strategy in finding the area of the floors. Amiq, for example, preferred counting the tiles column by column or row by row strategy, instead of working with the multiplication strategy. In dealing with the first floor he knew that in every row there are 10 squares. Then, he grouped every two rows to get 20 squares. He then did skip counting of 20, such as 20, 40, and 60 to obtain the tiles covering the whole floor (see Figure 5.36).

Figure 5.36
Counting the tiles row by row

Conclusion and Discussion

Grounding to the findings above, it is concluded that the multiplicative structure of area units is not easy to be seen by the students through the problems in this lesson. In the main problem, for example, only two students could see the multiplicative structure.

However, more students could see the multiplicative structure when they dealt with the second problem, measuring area of floors covered with carpets. It is conjectured that is because of the visibility of the tiles and the size of the floor being measured.

Measuring the area of the patio in the first problem, for example, triggered students counting the tiles one by one or row by row because almost all the tiles are visible. As well as in the patio, the living room also did not really trigger the student to come up with counting by multiplication since there are still many tiles are visible that lead the students to count the tiles column by column or row by row, instead of using multiplication strategy.

Meanwhile, in the carpeted floor (the second problem) almost all the tiles are covered by the carpets and only one row and one column of the tiles are visible. This situation triggered the students to see the multiplicative structure of the tiles. The arrangement of the tiles forces them to think that each column or row consists the same number of tiles.

As well as the visibility of the tiles, the size of the floor being measure is also conjectured triggering the students to see the multiplicative structure of the area units. In the first problems, the student dealt with a big size of floors. The size of the floors could fade the multiplicative structure of the tiles in the view of the students. It is because the number of the tiles in floor leads to the multiplication of big numbers. In the patio, for example, the students have to be able solve the multiplication of 12 x 5. Students in this age usually are not master yet with the multiplication of relatively big numbers.

However, dealing with a small size of floor leads students to count the tiles one by one without considering the multiplicative structure. Here, it is needed the size of floor which is not too big and not too small. The floor in the second problem can be a good example of the size of the floor. In the first floor, for example, they only deal with the multiplication of 6 x 10. Meanwhile, in the second floor they will deal with the multiplication of 5 x 10 and 4 x 5 after the floor is split.

5. Lesson 5: Developing students' understanding of the role of the dimensions in applying multiplication strategy in area measurement

In this lesson, the students are brought one step forward to more formal mathematics where the role of the dimensions of a plane figure in applying multiplication strategy in measuring area is introduced and developed by the students.

There are three problems in this lesson: a preliminary problem, a main problem, and a follow-up problem. The findings from those problems are explained in the following sections.

The Preliminary problem

In this problem the students are given five floors and are asked to find the area of the floor (see Figure 5.37). The purpose of the problem is to introduce the students to the role of the dimensions as the representation of the array structure of the area units.

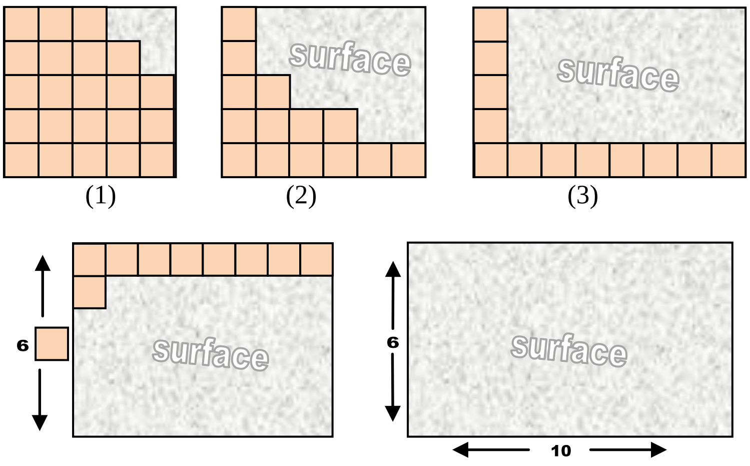


Figure 5.37
Five incomplete-tiled rectangular floors

In finding the area of the floors, every group experienced different learning experience, but in the end all of them could use the role of the dimensions as the representation of the array structure of area units and used the multiplication strategy in measuring area of the floors.

On one hand, Zidan and Rizal directly applied the multiplication strategy in measuring the area of the figures as it is shown in the following fragment.

Teacher : Could you show me how you measured the area of the plane figures?

Rizal started explaining how they measure the area of plane figure 1.

Rizal : 1, 2, 3, 4, 5, and 1, 2, 3, 4, 5.
5 x 5 is 20. Oh no! But it is 25.
(He counts the tiles covering the first column and the last row of plane figure 1 respectively).

Zidan : 1, 2, 3, 4, 5, and 1, 2, 3, 4, 5, 6.
6 x 5 is 30.
(He counts the tiles covering the first column and the last row of plane figure 2 respectively)

Students: 1, 2, 3, 4, 5, and 1, 2, 3, 4, 5, 6, 7, 8.
5 x 8 is 40.
(He counts the tiles covering the first column and the last row of plane figure 3 respectively)

Zidan : 6!
(He points the width dimension of the plane figure 4)
Students : 1, 2, 3, 4, 5, 6, 7, 8.
6 x 8 is 48
(They count the tiles on the first row and multiplied with the width dimension of the plane figure 4).
Zidan : 6 x 10 is 60.
(He points the dimensions of the plane figure 5 and multiplies them).

The fragment shows the students understood the role of the dimensions as the representation of the array structure of the tiles and used the dimensions in measuring the area of the figures by using the multiplication strategy. For example, in measuring the area of the fifth figure, they multiplied 6 and 10 since they knew that there are 6 rows of tiles where in each row consists of 10 tiles.

On the other hand, Haikal and Alif drew gridlines to complete the tiles of the floors and then used the multiplication strategy in counting the whole tiles. Although they drew the gridlines, they could see the role of the dimensions in measuring area as they could say the area of the fifth floor is 6 x 10 which is 60 squares. Here, the purpose of drawing gridlines is to convince themselves.

Meanwhile, Amiq at the first time measured the area of the floor by counting the tiles column by column. But, in the end he was aware of the multiplicative structure of the tiles and then he used the multiplication strategy.

However, the guide from the teacher is necessary to help Haikal, Alif, and Amiq to count the tiles by the multiplication strategy. For example, when Haikal and Alif kept drawing the tiles, the teacher asked them whether it is necessary to draw the tiles if the dimensions told the array structure. When Amiq count the tiles column by column, the teacher asked him to think of the relation between repeated addition and multiplication.

The main problem

In this problem, the students face with more complicated and formal mathematics problem than the problems in the previous lessons. Here, the students are asked to determine the area of a right-angle floor (see figure 5.38).

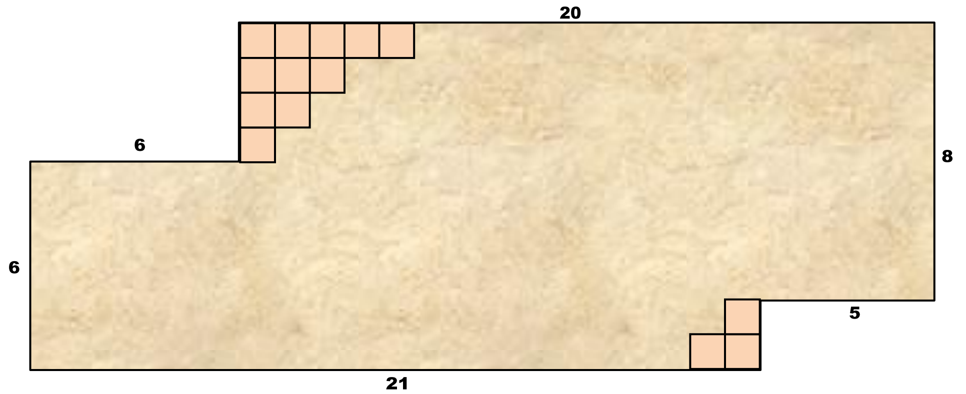


Figure 5.38
A right-angle plane figure

On one hand, Zidan and Rizal could measure the area of the floor by applying the multiplication strategy. Before they applied the strategy they split floor to be there rectangular floors. The use of multiplication after the floor has been split into rectangular floor indicates that they are aware of the multiplicative structure of the area units.

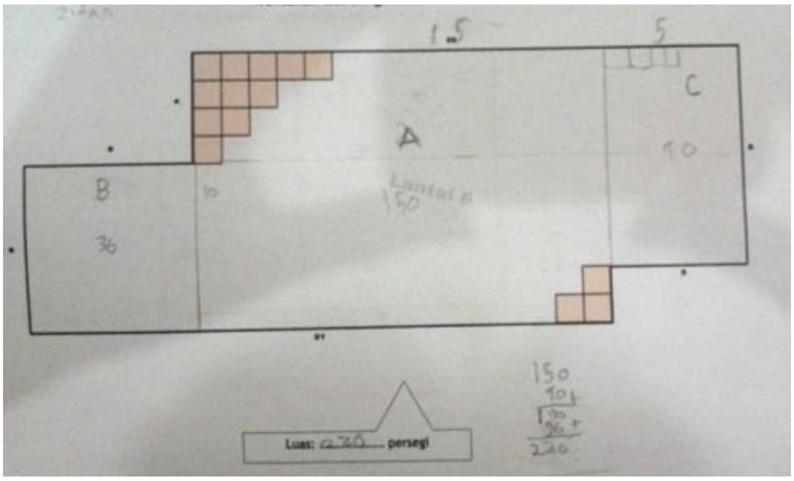


Figure 5.39
Zidan's and Rizal's solution

The figure above shows Zidan and Rizal's strategy in finding the area of the floor. They split the floor to be three rectangular surface (A, B, and C) they knew that the area of B is 6 x 6 which is 36 squares. They also knew that the area of A is 15 x 10 which is 150 tiles and the area of C is 8 x 5 which is 40 tiles. They summed the area of A, B, and C to know the area of the floor.

These students show their understanding of the role of the dimensions very well. They not only realized that the dimensions tell the array structure of the squares, but also realized the effect on the dimension after the splitting. For example, along the top side of the combination of A and C (see Figure 5.39 and 5.38) there are 20 tiles. After the side is split to be A and C, they knew that there are 15 tiles on the side of A and 5 tiles on the side of C.

On the other hand, the other three students were struggle before finally they could apply the multiplication strategy in measuring the area of the floor. Haikal, Alif, and Amiq split the floor to be three rectangular floors. They used the multiplication strategy to measure the area of B and C, but they were struggle when measuring the area of A (see Figure 5.39). They failed in interpreting the dimensions to tell them the array structure of the units after they split the floor. For example, after the student split the floor to be A and B (see Figure 5.40), they still assumed that QC is still 21 squares. They did not realize that the dimension BC also have to be split into BQ and QC. This change is not easy for the students to understand which causes the students multiplied the wrong numbers when applying the multiplication strategy.

Moreover, they also lack understanding of the equivalent of two dimensions. They did not realize that AB is equivalent with PQ. The students also did not realize that the dimension from R to Q is the sum of the dimension from R to P and P to Q.

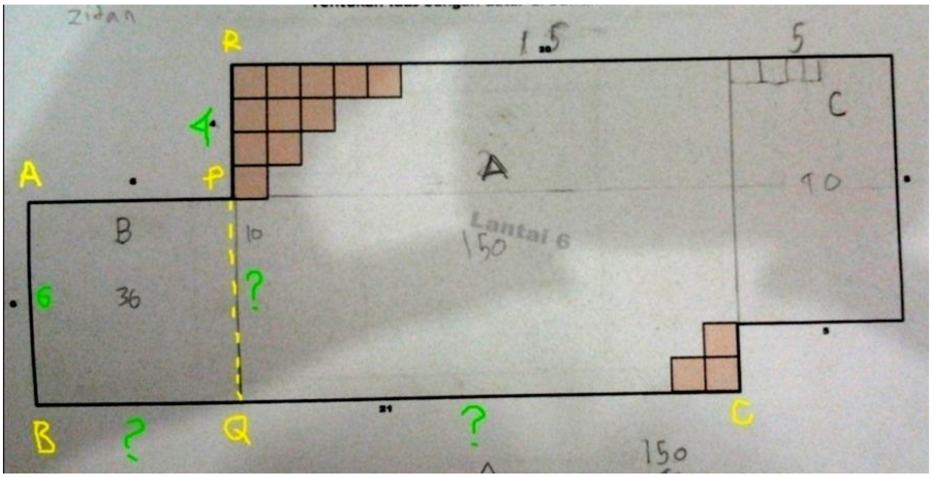


Figure 5.40
Splitting: the students did not consider the change occurring on the dimensions after they split the plane figure

The follow-up problem

The follow-up problem is designed to assess students' understanding of the role of the dimensions and the multiplication strategy in measuring area. In this problem, the students are asked to measure the area of a right-angle plane figure (see Figure 5.41).

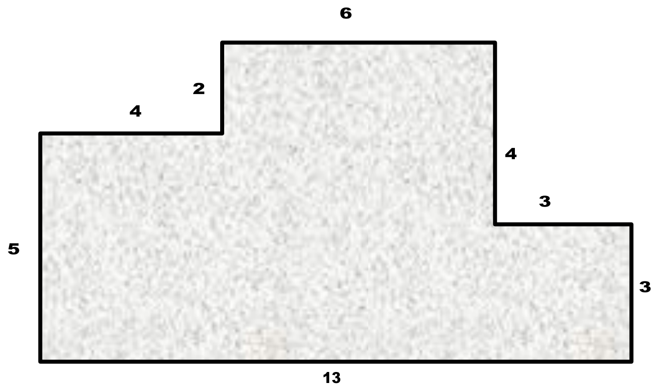


Figure 5.41
The follow-up problem: The students are asked to determine the area of the plane figure

The findings shows that all of the students could apply the multiplication strategy in finding the area of the figure. They split the figure into some rectangular figures before they applied the multiplication strategy (see Figure 5.42)

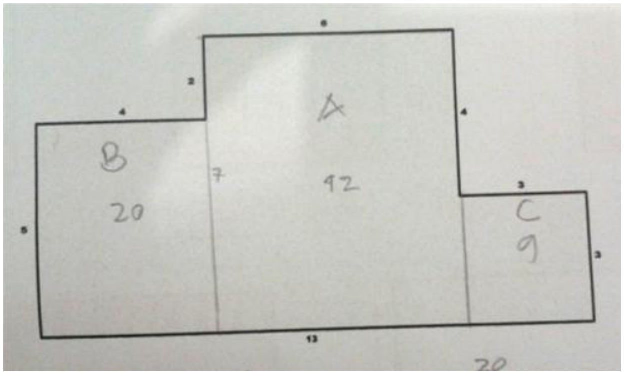


Figure 5.42
An example of students' solution

However, before they could use the multiplication strategy some of the students still got the difficulties relating to issues of dimensions as it has been explained before, but the difficulties were not as serious as the previous problem. Here, the guidance from the teacher again played a significant role to guide students to see the change on the dimensions after splitting.

Conclusion and Discussion

Grounding to the findings above, in general it is concluded that the learning activity in this lesson could develop students' understanding of the role of the dimensions in applying multiplication strategy to measure the area of a plane figure.

In the preliminary problem, for example, the students were indirectly guided to see the role of the dimensions as the representation of the array structure of area units. All students could see the role of the dimensions and use the information given by the dimensions in applying the multiplication strategy to measure area.

In the main problem, all students could measure the area of the plane figure by applying the multiplication strategy although initially some of them were struggle due to their difficulties in interpreting the dimensions (Here, the guidance from teacher to address students' difficulties is necessary).

Typically students' difficulties in understanding the dimensions based on this problem are struggling in interpreting the change on the dimensions after splitting and struggling in seeing the equivalence of two dimensions.

E. Remarks on Students' Knowledge and Development Based on the Posttest and Student Interview of the First Cycle

A posttest is conducted after the interventions is done to check students' knowledge and development after the interventions. The remarks obtained from the findings of the posttest and are supported by the findings from the student interviews are elaborated in the following paragraphs.

1. Students have better understanding of area as a region inside of a boundary

Here students are asked to select the figures that have area (see Figure 5.43). The purpose of this question is to examine students understanding of area, whether they conceive it as a region or as a line.

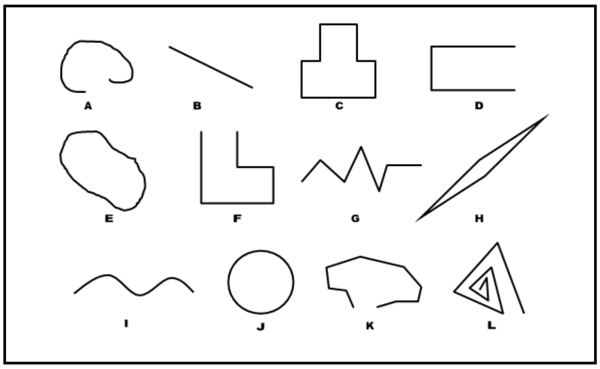


Figure 5.43
Students are asked to decide the figures that have area

In the pretest, it is found that all of the students have a prior understanding of area as a region although there are inconsistencies of the figures being voted.

The findings from the posttest show a more consistent vote than the finding in the pretest. Three of them voted consistently the close curves, such as C, E, H, and J. The other two remaining students chose close curves and also some of open curves except D, G, and I. They did not chose D, G, and I probably because they are looked like lines.

Comparing the findings in the pre and the posttest, it is found that there is a development among the students on their understanding of area. In the pretest, it is found that some students voted lines as the figures that have area; meanwhile, in the posttest shows all of them voted close curves or a curve that is looked like a close curve as the figures that have area. Moreover, as well as in the pretest the findings from the student interview show that all of the students could see area as the a region inside a boundary; but none of them see area as the boundary lines as it is found in the pretest.

2. Students see area as the number of area units covering a plane figure

There are two problems used to check students' understanding of area as the number of area units. In the first problem, the students are given three plane figures that are covered by square units (see Figure 5.44). They are asked to sort the plane figures from the largest to the smallest.

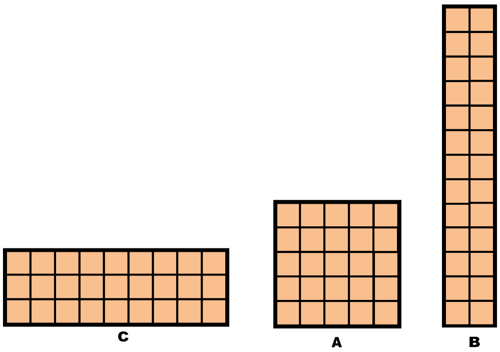


Figure 5.44
Comparing the area of three plane figures

Meanwhile, in the second problem the students are asked to compare the area of the two floors (see Figure 5.45). This problem has dual function. The first is to check students' consistency between their answer in the first problem and in the second problem. The second is to see students' understanding of the array structure of units of area.

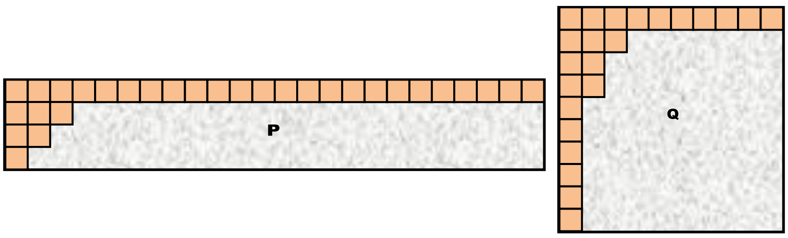


Figure 5.45
Two plane figures covering by square tiles

In the first problem, the findings show that all of the students could sort the plane figures by considering the number of the area units (squares) covering the plane figure. They even could use the multiplication strategy in counting the squares. For example, to obtain the number of the squares of the plane figure C (see Figure 5.44), they multiplied 7 and 3 which is 27. 7 is the number of the columns of plane figure C and 3 is the number of the rows. This findings are contrast with the findings in the pretest where most of the students could not see area as the number of area units (squares) covering a plane figure.

In the second problem, the findings show that all students could compare the area of floors by considering the number of the square units covering the floors. They could see the array structure of the square units since they applied the multiplication strategy in counting the squares. For example, They then multiplied 24 and 4 which 96 in finding the area of P since they knew that there are 12 columns and 4 rows of squares in P (see Figure 5.45). These findings are contrary with the findings in the pretest where all of the students did not compare the number of the square units when comparing the area of the floors.

Grounding to the findings above, it is concluded that there is a development on students' understanding regarding to the concept of area before and after the interventions. In the pretest most of them could not see the concept of area as the number of area units covering a plane figure, but in posttest they could see the concept well.

3. Students could measure the area of a rectangular plane figure

Here, the students are asked to measure the area of a rectangular plane figure (see figure 5.46). The purpose of the problem is to assess students' understanding of measuring the area of a rectangular plane figure.

Figure 5.46
A rectangular plane figure: students are asked to measure the area of the plane figure

The findings show that all students could apply the multiplication strategy in measuring the area of the plane figure. In this problem they showed their understanding of the role of the dimensions as the representation of the array structure. They knew that there are 4 rows and 8 columns of units. To obtain the whole units, they multiplied 4 and 8 which is 32. Therefore, they said that the area of the plane figure is 32 area units.

In contrast with the findings in the pretest, they have no idea yet how to measure the area of a rectangular plane figure. All of them measured the length or the width of the plane figure, instead of the area. Therefore, it is concluded that there is a development on students' understanding of area measurement, where initially they were not able measuring the area of the rectangular plane figure, but after the interventions they showed their understanding of the use of multiplication strategy in measuring a rectangular plane figure.

4. Students could measure the area of a right-angle plane figure

Students are given a right-angle plane figure with its dimensions in each side (see Figure 5.47). Students are asked to determine the area of the plane figure. The purpose of the problem is to investigate how the students will deal with measuring the area of a right-angle plane figure and how they will interpret the dimensions.

Figure 5.47
Measuring the area of a right-angle plane figure

In contrast with the findings in the pretest where they were not able to measure the area of the similar surface, the findings from the posttest show that all students could measure the area of the plane figure by applying the multiplication strategy (see figure 5.48).

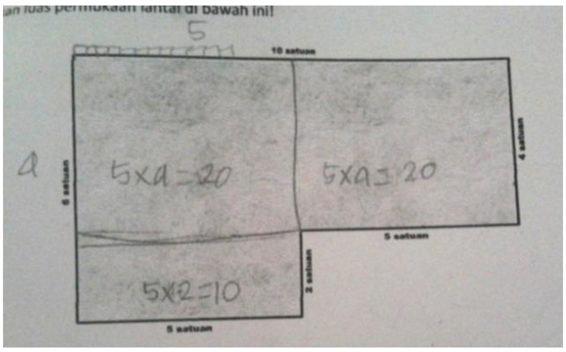


Figure 5.48
An example of students' answer

The figure shows that the students split the plane figure into some rectangular plane figures. By considering the dimensions, they applied the multiplication strategy. It seems that the students understood that the multiplication strategy can only be applied in a rectangular plane figure. Moreover, they also could see the role of the dimensions telling the array structure of the area units.

F. Conclusion of the Preliminary Teaching Experiments (the First Cycle)

In this section the conclusion based on the findings from the preliminary teaching experiment (the first cycle) will be elaborated.

Lesson 1: Although the pretest shows that the students could see area as a region, the activities in this lesson provide a wider understanding about area. The area comparison problems (asking students to compare area and find the larger area) in context of grassland in this lesson could lead to the emergence of perception, superposition, and decomposing strategy in solving area comparison problems, especially when the cards of the grasslands are provided. Experiencing the three strategies in the same time and in the same problem helps students to see the strength and the weaknesses of each strategy. In addition, the words *'large'* and *'small'* are used by the students to express the size of the area of two plane figures. They also used the words *'same'* and *'different'* to express the area of two plane figures when the two figures have the same or different size of area respectively.

Lesson 2: If the finding from the pretest is considered, it is found that before the experiments most of the students have no idea yet about the measurement unit of area as well as area as the number of the measurement units. Meanwhile, the finding in the posttest shows that the students could consider the measurement units of area and look area as the number of the measurement units. This implies that the activity in this lesson could help students to see the role of the units in telling the area of a plane figure. In solving the parking lot problem, for example, the students could say that one parking lot is larger than the other parking lot since the parking lot occupies more area units. However, the idea of unit consistency is not easy to be understood by the students at first time. In dealing with the parking lot problem, for example, they used more than one type of cards to compare the area of the parking lots. Here, the guidance of the teacher is necessary to help the students seeing the idea of unit consistency.

Lesson 3: Before the experiment, the students have no idea yet about the measurement units as well as the array structure of the units (see pretest). On the other hand, the students could consider the array structure of area units when counting the units during measuring area. It suggests that structuring array activities in this lesson could help students to see the array structure of the units of area. In dealing with the main and the follow-up problem, for example, most of the students could count the squares by considering the array structure of the squares. They counted the squares by looking at the number of squares in columns or rows. However, there are some students that need an extra guide. Haikal, for example, preferred of counting the squares one by one without considering the array structure of the squares. Asking them to look at the number of the area units in columns or in rows can be a good guidance to help them seeing the array structure.

Lesson 4: It is concluded that the multiplicative structure of the tiles is not easy to be seen by the students through the problems in this lesson. In the main problem, the patio and the living-room problem, only two students (from five students) could see the multiplicative structure. However, more students could see the multiplicative structure when they dealt with the second problem, the carpet problem. This ability, moreover, is even shown when the students measure the area of rectangular figure during solving the problem in the posttest. It is conjectured that the visibility of the tiles and the size of the floor being measured influence students in seeing the multiplicative structure.

Lesson 5: It is concluded that the learning activity in this lesson could develop students' understanding of the role of the dimensions in applying the multiplication strategy to measure the area of a plane figure. Although the students could interpret the dimensions and used them in applying the multiplication strategy, the findings show that most of the students experienced of struggling in interpreting the change on the dimensions after a plane figure is split to be rectangular figures.

G. Hypothetical Learning Trajectory (HLT) for the Second Cycle as the Refinement of the HLT in the First Cycle

The interventions (the initial instructional theory consisting of the hypothetical learning trajectory and teaching materials) are revised and refined based on the findings in the preliminary teaching experiment and the three main issues: practicality and effectiveness (see Nieveen, 1999). The intervention is called practical if the end-user (for instance the teacher and the learner) consider the intervention to be usable and that is easy for them to use the materials in a way that is largely compatible with the developers' intention. Meanwhile, the intervention is called effective if the result of the intervention is in the desired outcome.

As it has been elaborated in Chapter IV, the initial instructional theory consists of five lessons and each lesson has its own purpose. By looking at the research question, developing students' understanding of the multiplication strategy in measuring area, it is found that the purpose of lesson 1 in the initial learning trajectory slightly contributes to answer the research question. Therefore, the initial learning trajectory is revised to be only four lessons. However, the mathematical ideas in lesson 1 are integrated in the four lessons.

For the other four remaining lessons, lesson 2 to 5, in the initial instructional theory, some changes and adjustment have been made by considering the issues of the effectiveness and the practicality of the interventions according to Nieveen (1999).

Since too many small changes have been made in the instructional theory and in the learning materials that take considerably time to elaborate it, the researcher directly provides the revised instructional theory consisting of a hypothetical learning trajectory and the teaching materials as are elaborated in the following paragraphs.

1. **Lesson 1: Developing students' understanding on the measurement unit of area**

   The purpose of the activities in this lesson is to introduce the measurement unit of area. Through understanding the measurement units of area, the students will learn about physically quantity of area, seeing area as the number of measurement units covering a plane figure. In this lesson, the informal units, such as hands and books, are used to help students having a preliminary insight of the unit of area.

   The activity in this lesson contains three problems: hand problem, book problem, and floors problem as a follow-up problem. In the hand problem, students are asked to determine the number of hands (palms) needed to cover the surface of students' desks. The purpose of this problem is to introduce the students to the informal measurement units of area. In the hypothetical learning trajectory, the students will cover the desk with their hands and come up with different results of hands since they have different size of hands and different ways of doing covering. It is expected that the teacher use the differences to discuss about the unit consistency and the idea of gap and overlapping.

   In the book problem, the idea of measurement units of area is learned further. The students are engaged in using books as the measurement unit to measure the area of some plane figures in their classroom, such as teacher table, whiteboard, announcement board, door, etc. They then are asked to sort those plane figures based on their size (area). This problem intends to provide the students with the experience iterating unit measurement in measuring area to enrich their understanding of the measurement units of area and area measurement. As the hypothetical learning trajectory, there will be different ways of putting the books on the surfaces being measured. Some of them may be used one book and transpose the book over the surfaces or take as many books as required to cover the surfaces. In sorting the surfaces, it is conjectured that they will consider on the number of books needed to cover the surfaces.

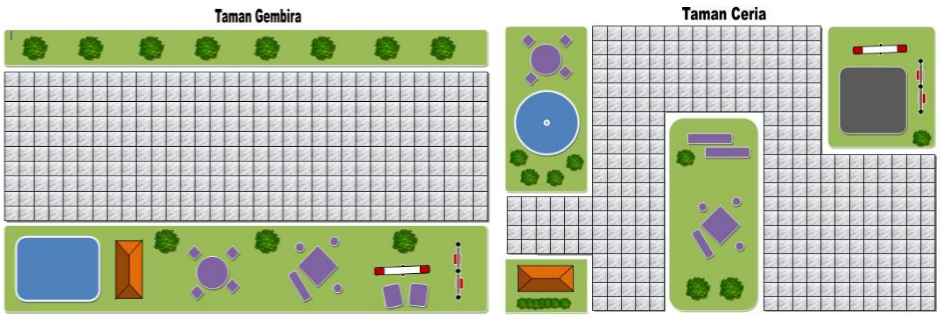
In the third problem, the floor problem, students are given three tiled floors and they are asked to sort the floors based on the area of the floors. The purpose of the problem is to check students understanding of the physical quantity of area and the role of units in telling the area of floors.

2. Lesson 2: Developing students' awareness of the array structure of units of area

After the students have been introduced to the measurement unit of area, this lesson intends to build students' awareness of the array structure of the units of area on rectangular plane figures as the basis to introduce them to the multiplicative structure of the area units.

To gain the purpose, students are given two problems. In the first problem the students are asked to compare the area of two parking lots (see the tiled plane figures of Figure 5.49). This problem is purposively designed to help students seeing the array structure of the area units on a rectangular plane figure.

It is conjecture that students will compare the number of tiles in the parking lots to find the larger parking lot. In counting the tiles, they will notice that the number of tiles in each row or columns of the first parking lot is always the same. This finding help them in counting the tiles row by row or column by column. Here, the students are aware of the array structure of the tiles. In dealing with the second parking lot, it is conjectured that the students will split the parking lot to be some rectangular parts and they will count the tiles by considering the array structure that they have known from the previous experience, counting the tiles in the first parking lot.



(a) (b)

Figure 5.49
Two parking lots which are being compared their area

In the second problem the students are asked to measure the area of a floor which is covered by a carpet (see Figure 5.50). This problem intends to check weather students really see the array structure of area units or not. If the students really notice the array structure of area units, they will realize that the visible units can tell them the units under the carpet.

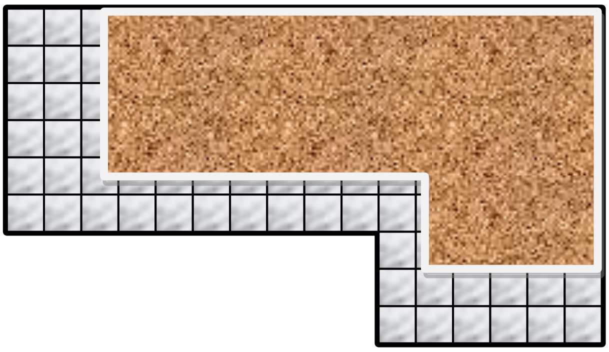


Figure 5.50
Two parking lots (the tiled plane figures) which are being compared their area

It is conjectured that the students will count the tiles as the measurement units in determining the area of the floor. Since most of the tiles are invisible, the students will notice the array structure of the tiles during counting the tiles. They may look at the tiles column by column or row by row. There is possibility that the students split the floor to be rectangular floors to help them counting the tiles.

3. Lesson 3: Developing students' awareness of the multiplicative structure of units of area

This lesson is designed to help students discovering the multiplicative structure of area units through investigating the area of some rectangular tiled floors. There are two problems in this lesson: a house-sketch problem and a follow-up problem.

The house-sketch problem is designed to discover the multiplicative structure of units of area. Here, the students are asked to discuss in group to find the area of each floor in a house sketch (see Figure 5.51).

It is conjectured that to find the area of floors students will treat the tiles as the measurement units. it is expected that the students can see the multiplicative structure of the units when they count the tiles. The awareness of the multiplicative structure can be seen from the way they count the tiles. For example, it is assumed that the students can see the multiplicative structure of units of area if they count the tiles by applying the multiplication strategy.

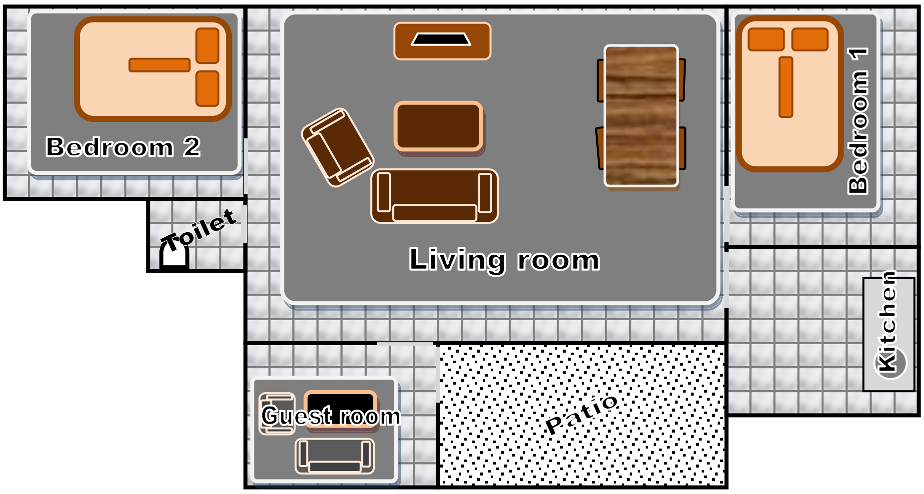


Figure 5.51
House-sketch problem

Meanwhile, the follow-up problem is set to check students' awareness of the multiplicative structure. In this problem, the students are asked to determine the area of a floor as it is shown in Figure 5.52.

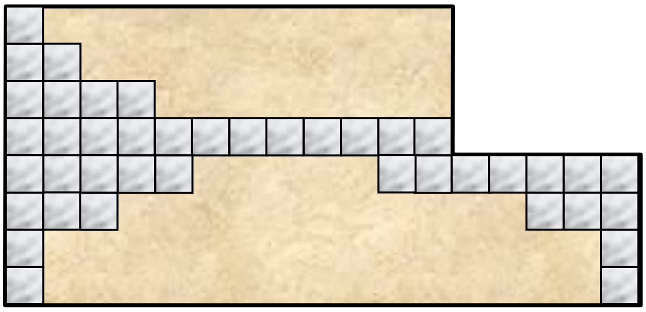


Figure 5.52
Incomplete-tiled-floor problem

To find the area of the floor, it is conjectured that the students will split the floor to be some rectangular plane figures and then applied the multiplication strategy. If the students do so, it implies that the students could see the multiplicative structure of units of area.

4. Lesson 4: Developing students' understanding of the role of the dimensions of a plane figure in applying the multiplication strategy in area measurement

This lesson intends to bring students one step forward to more formal mathematics where the idea of dimensions of a plane figure is introduced. It is expected that they understand the function of the dimensions as the representation of the array structure and use them in applying multiplication strategy in area measurement.

There are two problems proposed in this lesson. In the first problem, students are asked to measure the area of a floor (see Figure 5.53). This problem intends to introduce students about dimensions as the representation of the array structure of area units. It is expected that the students can use the dimensions as the shortcut in applying multiplication strategy, instead of drawing gridlines.

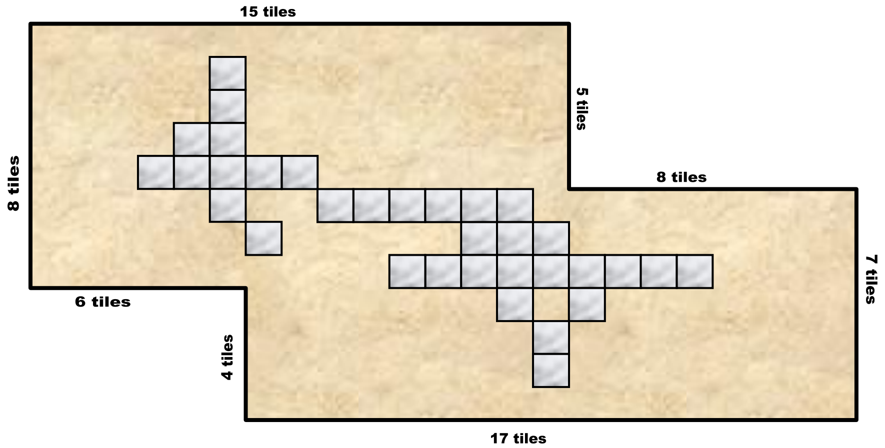


Figure 5.53
Measuring the area of a right-angle plane figure

In second problem, the students are asked to measure the area of another right-angle plane figure. Different from the previous problem, there is no any visible tile on this plane figure. The students are just given the dimensions of the figure (see figure 5.54). This problem intends to check students' understanding of the dimensions in applying the multiplication strategy.

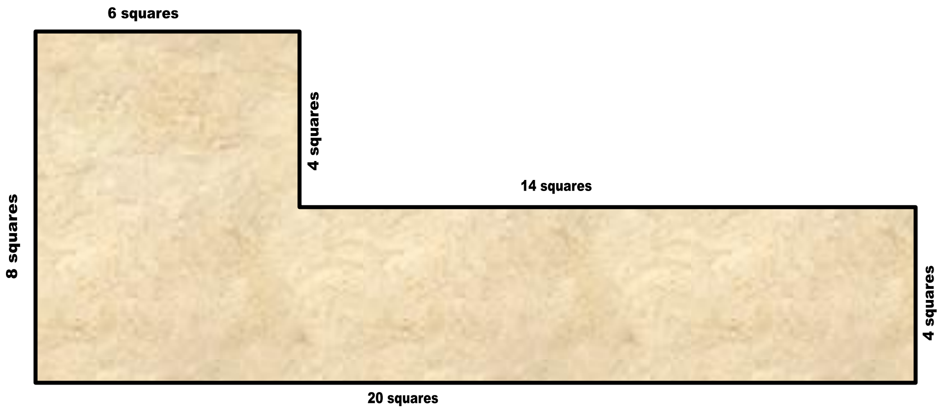


Figure 5.54
The second students' problem in measuring the area of a right-angle plane figure

To solve both problems, it is conjectured that the students will split the figure to be some rectangular figures. They then look at the dimensions as the representation of the array structure of area units. By considering the array structure, it is conjectured that the students will apply the multiplication strategy to find the area of the rectangular plane figures.

H. Remarks on Classroom Environment Based on the Classroom Observation and Teacher Interview of the Second Cycle

There are some remarks about the students and the teacher that influence the classroom environment according to the findings from the classroom observation and the teacher interview.

First, the teacher seems to dominated the classroom when teaching mathematics. Here, the teacher-centred classroom environment really obvious. As the consequences, the students tend to be a passive learner and tend to wait an instruction or permission from the teacher before doing something.

Second, the teacher mostly depend on the textbook in teaching mathematics. Here, the problems propose to the students are often taken from the textbook that sometimes is not real and familiar for the students.

Third, teacher and students interaction is only occurred when giving instructions, questioning-answering questions and giving guidance when the students get difficulties in answering the questions.

Fourth, The teacher is rare to orchestrated a group discussion or a classroom discussion. Therefore, the students tend and prefer to work individually instead of in group. Consequences, the students are not accustomed in expressing their ideas during a discussion.

By considering the remarks above, some adjustments made in the instructional theory, such as:

1. Provide the students with a real context problem and talking about the context of a problem before the problem are given to help students to be accustomed in expressing their idea as well as to introduce them to the context of the problem.
2. Students are given time to work individually before they work in group. This plan provides the students with some ideas before they discuss in group.
3. Students are asked to discussed in small group first before classroom discussion. This plan can increase students' confidence when doing classroom discussion.

I. Remarks on Students' Prior Knowledge Based on the Pretest and Student Interview of the Second Cycle

There are seven questions being asked in the pretest to investigate students' prior understanding of area, area measurement and the concepts that have a relation to area measurement, such as multiplication as repeated addition, array structure, etc. Four days after the pretest, two students were interviewed to clarify further and deeper on their answers in the pretest.

From 21 students, there 16 students participated in this tests. The remaining five students were absent due to sick.

The following paragraph elaborate the remarks relating to students' prior knowledge as the result of the analyses of the findings in the pretest and the student interview. The general remarks from the pretest of the whole students will be elaborated first, then the remarks from the interview, and is ended with the remarks on the focus students.

1. Students could see area as a region inside a boundary

In the pretest, the students are asked to choose the figures that have area from eight figures (see Figure 5.55). The problem intends to investigate students' prior conception of area.

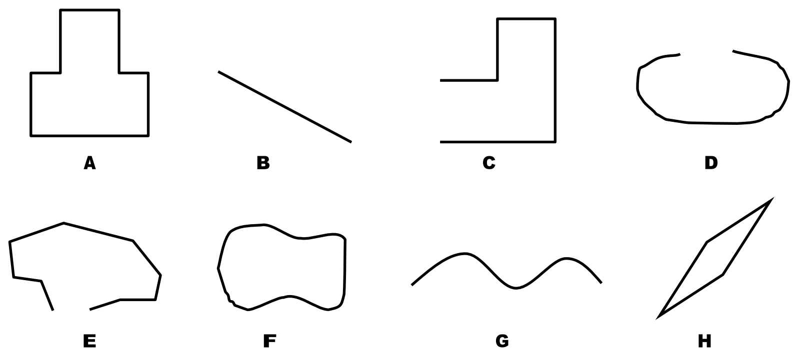


Figure 5.55
Students are asked to determine the figures that have area

The findings show that all of the students have conception of area as a region inside a boundary. None of them chose lines, such as B and G. It is assumed that the students did not choose those figures since it is obvious that those figures have no region.

Six out of 16 students chose close curve figures, such as A, F and H. These students showed their understanding of area as a region in a closed boundary.

The other nine remaining students also see area as a region, but they have no idea of close or open curve figures. Therefore, they not only chose close curve figures but also open curve figures, such as C, D, and E, which they assume that those figures have area.

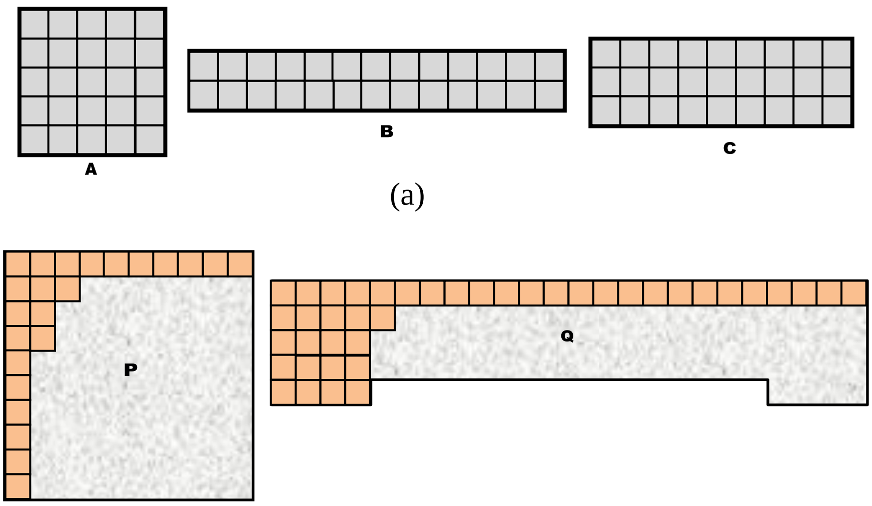
In the interview, the interviewed students shows their understanding of area as a region although they select open and close curve figures as the figures that have area. They said that the area of those figure is the region inside of the boundary. This findings implies that the students have a prior understanding of area as region but they have no idea yet about open or close curve figure.

As well as the majority of the students, the focus students, Kevin and Giri, showed their understanding of area as the region inside a boundary. Both of them chose figure A, F and H which are close curve figures as the figures that have area. It seems they knew that open curve figures have no area.

2. Students could not see area as the number of measurement unit yet

In the pretest, the students are asked to solve two area comparison problems. The purpose of the problems is to clarify students' prior understanding of area as the number of the measurement units of area. The first problem intends to investigate students' understanding of area as the number of the measurement units of area. Meanwhile, the second problem intends to clarify students' understanding on the structure of measurement units of area in columns and rows if they have already understood area as the number of the measurement units of area.

In the first problem the students are asked to sort three plane figures based on their area from the largest to the smallest (see Figure 5.56a). In the second problem, the students are asked to determine the larger floor from two incomplete-tiled floors (see Figure 5.56b).



(b)

Figure 5.56
Area comparison problem

The findings from both problems are elaborated in the following paragraphs.

In the first problem, there are 9 out of 16 students, which is the majority, considered area as the width or the length of the plane figures. When they considered the width, they said that figure A is the largest since it is the widest figure, and then C and D; or when they considered the length they said that B is the largest since it has the longest length, and then C and A. They did not consider the number squares covering the plane figures yet since they has no idea yet about area as the number the measurement units of area covering a plane figure. Only three students sort the plane figures by considering the number of squares covering the figures. They sated that C is the largest since it has 27 squares and then B (26 squares) and A (25 squares). These students seems to understand area as the number of the measurement units of area. However, the remaining four students have no idea yet in comparing area those figures. They wrote answers which have no relation to the question.

The same tendency is also occurred in the second problem. The students could not see yet area as the number of measurement units covering a plane figure. Instead, they looked at the length or the width of the figures being measured. The findings show that 11 students considered the width of the figures. Therefore, they said that P is larger than Q. Meanwhile, 4 students considered the length. Therefore, they sated that Q is larger than P. These students did not see yet area as the number of the measurement units of area. Therefore, they have no idea yet about the array structure of the units of area. Another remaining student showed no clear answer on the problem. It seems that he has no idea yet how to compare the figures.

As well as the findings in the pretest, the student interview also shows the similar tendency among the students. They considered the width or the length when comparing the figures.

The focus students, Kevin and Giri, show a different level of understanding in both problems. Giri has a better understanding of area than Kevin. In solving the first problem, for example, Giri compared the number of squares constituting on the three plane figures; meanwhile, Kevin compared the width of the plane figures.

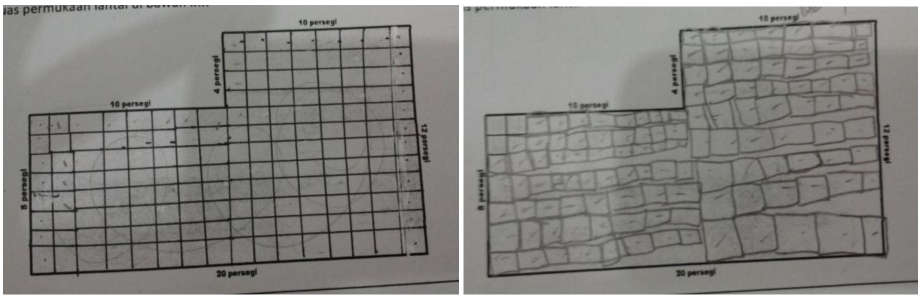
3. **Students measuring perimeter instead of area when asking to measure area**

In the pretest, the students are asked to measure the area of a floor where the dimensions of the floor are given (see Figure 5.57). The purpose of the problem is to know how they will deal with measuring area problem.

Figure 5.57
Students' problem in measuring area of a floor

The findings from the pretest and the student interview show the similar tendency. It is found that the majority of the students (11 out of 16 students) measured the perimeter of the floor instead of the area of the floor. They added up the numbers representing the dimensions of the floor. For example, Anya stated that the area of the floor is 8 + 10 + 4 + 10 + 12 + 20 = 64.

Other four students show a better understanding of measuring area. They drew square units on the plane figure to help them seeing the number of tiles occupied by the floor. But they drew the squares wrongly because they failed in interpreting the dimensions as the representation of the array structure. For example, Widya drew eight columns of squares where she should draw ten columns of squares (see figure 5.58a). Other students drew the squares with inconsistent size (see figure 5.58b).



(a) (b)

Figure 5.58
Two examples of students' answers

Another remaining student shows an unclear answer relating to the problem. It is conjectured that this student has no idea yet in dealing with measuring area problems.

The focus students, Kevin and Giri, show different level of understanding toward the problem. As well as the majority of the students Kevin measured the perimeter of the floor when asking to measure the area of the floor. Meanwhile, Giri tried drawing tiles (gridlines) to complete the tiles of the floor, but he failed.

4. Students not seeing the array structure of units of area yet

In the pretest, the student are asked to determine the number of tiles covering a floor (see Figure 5.59). The purpose of the problem is to investigate students' awareness of the array structure of the units of area.

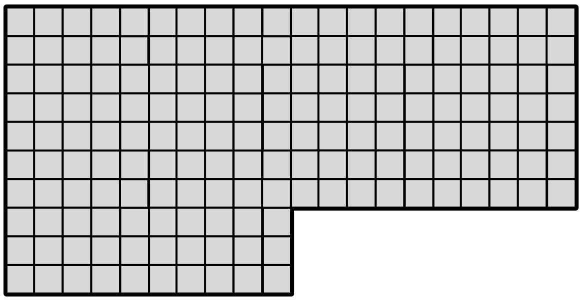


Figure 5.59
A problem to investigate students' awareness of the structure of units of area

The findings from the pretest and supported by the findings from student interview show that most of the students (11 out of 16 students) did not see the array structure of the tiles yet. They counted the tiles one by one. Therefore, some of them did miss counting. Mita and Mahida, for example, counted the tiles one by one and they got 172 tiles instead of 170. They probably counted the same tiles twice.

Only five students counted the tiles by considering the array structure of the tiles in columns or in rows. Widya, for example, counted the tiles row by row by doing repeated addition, such as 20 + 20 + 20 + etc. Another strategy is shown by Winot. Instead of looking the tiles in rows, he considered the array structure of the tiles in columns and did repeated addition on the number of tiles column by column.

As well as the majority of the students, the focus students, Kevin, did not consider the array structure of the tiles yet when counting the tiles. He counted the tiles one by one. Meanwhile, Giri, another focus student, showed no idea yet in dealing with this problem since he has no answer.

5. Students seeing repeated addition as multiplication

In the pretest, the students are asked to transform repeated addition operations into multiplication operations. The purpose of the problem is to investigate students' prior understanding of a repeated addition as a multiplication.

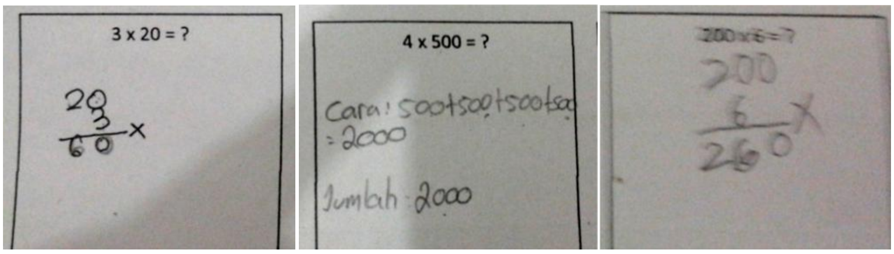
The findings in the pretest and supported by the findings in the student interview show that almost all of the students (14 out of 16 students) including the focus students could see repeated addition as multiplication. Kevin and Zaki, for example, could state 6 + 6 + 6 and 5 + 5 + 5 + 5 + 5 + 5 as 3 x 6 and 6 x 5 respectively.

Two remaining students, Nazwa and Derry, could not see repeated addition as multiplication. Nazwa saw 6 + 6 + 6 as 6 x 6 x 6; meanwhile, Derry transformed 6 + 6 + 6 to be 7 x 6. This findings notifies that these students needed to be guided during the intervention relating to transforming a repeated addition into a multiplication.

6. Students solving multiplication problem using column multiplication strategy

In the pretest, students are asked to solve three multiplication problems, such as 3 x 20, 4 x 500, and 200 x 6. The purpose of the question is to investigate students' understanding of multiplication, whether they look multiplication as repeated addition or as memorizing fact.

The findings from the pretests and supported by the findings in the interview show that 10 out of 16 students solve multiplication problem by using column multiplication (see Figure 5.60a). Five of them, including the focus students, Kevin and Giri, solve the multiplication problems by using repeated addition (see Figure 5.60b). Especially for 200 x 6, they did it commutatively. They transformed 200 x 6 as 6 x 200 and they wrote 200 + 200 + 200 + 200 + 200 + 200 on the worksheet. Another remaining student failed in solving the problem. He actually intended to do column multiplication, but he added the number column by column instead of multiplied them (see 5.60c).



(a) (b) (c)

Figure 5.60
Students' different strategies in solving multiplication problem

J. Teaching Experiments (The Second Cycle)

In this section, the analyses on the findings from the teaching experiments of the second cycle are elaborated. The refinement of the hypothetical learning trajectory is applied in this experiment. There four lessons in this experiment and the purposes of each lessons is shown in the following table.

Table 5.3
The purposes of each lesson in the teaching experiment of the second cycle

Lessons	Purposes
1	Developing students' understanding on the measurement unit of area
2	Developing students' awareness of the array structure of units of area
3	Developing students' awareness of the multiplicative structure of units of area
4	Developing students' understanding of the role of the dimensions in applying multiplication strategy in area measurement

There are 21 students of the third grade (around 9 years old) are involved in this experiment. Two students, Kevin and Giri, are selected as the focus students among 21 students. They are from the average students regarding to their mathematics skill among the students in their class.

The analyses will focus on the emergence of the mathematical ideas and concepts during the teaching experiments and the means to support the emergence. The analyses are based on the time chronological on the activities that the students are engaged in. In each activity, the findings from the whole students are analyzed first and then on the focus students. The analyses are then ended with a conclusion.

1. Lesson 1: Developing students' understanding on the measurement unit of area

The purpose of the activities in this lesson is to introduce the measurement unit of area. Through understanding the measurement units of area, the students will learn about physically quantity of area, seeing area as the number of measurement units covering a plane figure. In this lesson, the informal units, such as hands and books, are used to help students having a preliminary insight of the unit of area.

The activity in this lesson contains three problems: hand problem, book problem, and floors problem as a follow-up problem. The analyses on the findings from the those problems are explained in the following paragraphs.

First problem: Hand Problem

In this problem students are asked to determine the number of hands (palms) needed to cover the surface of their own desks. The purpose of this problem is to introduce students to the measurement units of area as the basis to understand the idea of the physical quantity of area, where the students could tell the size of a plane figure by telling the number of units covering the plane figure.

As the conjectures, the students will cover the surface of their desks by using their hands and come up with different results of the number of hands required to cover the desks completely since they have difference in size of their hands and ways of doing covering. It is expected that the teacher takes benefit from this situation to discuss about the unit consistency and the idea of gap and overlapping.

As the conjecture, the findings in the actual learning trajectory shows that the students came up with different number of hands. The difference is quite large. The smallest number is 26 hands and the biggest number is 59 hands but most of the students shouted around 30s hands.

Teacher: Anya, how many hands?
Anya: 46 ½.
Teacher: Winot?
Winot: 28.
The teacher keeps asking the students one by one including Agnita and Raihan.
Teacher: Aginta?
Aginta: 26…
Teacher: Raihan?
Raihan: 59.
Then, the teacher invites the students to discuss why they have different results.
Teacher: Well, here you have different results. Why do you have difference results? Who want to answer?
Giri: Because the palms (their hands) are different.
Student: Because the sizes of the palms are different.
Teacher: Well, because the sizes of the palms are different.
What else?
Bila: Because the different ways in putting the fingers (palms) on the desk.
Teacher: Yes. Because the different ways of measuring.

The conversation shows that the students could see why they came up with different results of measurement. They knew that the differences are due to the different size of hands used to cover the desks and the different ways of putting their hands on the desks (some of them put their hands by leaving gap between two hands and some others leaving overlap) (see figure 5.61). It seems that the students knew that the size of their hands and the way they arrange their hands yield different result of measurement.

Figure 5.61
Students' different size of palms and ways in covering the identical desk

However, none of the students argued that the differences potentially occurred due to the size of the surface of the desks being measured, whether their desks are actually the same size or not. In the teaching guide, it is expected that the teacher brings the students to the discussion on this issue as the follow up discussion. But, the teacher failed to bring the issue to the discussion. In this discussion, it is conjectured that the idea about unit measurement and its consistency and the idea of gap and overlapping can be introduced. Students will argued that their desks have to have the same size. To prove their argumentation, they have to use the same size of hands to cover the desks without gap and overlapping. Here, an informal unit, such as hand, is introduced as the measurement unit.

Second Problem: Book problem

In this problem, the students are engaged in using books as the measurement unit to measure the area of some plane figures in their classroom, such as teacher table, whiteboard, announcement board, door, etc. They then are asked to sort those plane figures based on their size (area).

This problem intends to provide the students with the experience of iterating measurement unit of area in measuring area to enrich their understanding of the idea of the physically quantity of area (area as the number of measurements units covering a plane figure).

As the conjectures, in the actual learning trajectory the students show different ways of iterating the books to find the number of books needed to cover the whole surface being measured. The way they iterate the books influences their strategy in counting the books being used.

There are three different strategies of iteration emerge among the students. The first strategy is taking some books and then transposed those books one by one by continuously changing their position on the remainder of the whole surfaces (see Figure 5.62a). The students who used this strategy counted the books being used one by one as they iterated the books one by one. There are two groups of the students used this strategy.

(a) (b) (c)

Figure 5.62
Different ways of iterating measurement units

The second strategy is using as many of the books as are required to cover the surface being measured (see Figure 5.62b). The students who used this strategy counted the books being used one by one but arbitrarily after they have already covered the whole surface. There are two groups of students used this strategy.

The third strategy is transposing some books to find the number of books needed to cover the length and the width of the surfaces. In this strategy. The students who used could see the array structure of the books in columns and rows, therefore they considered the array structure when counting the books –they added the number of books in each column as many times as the number of the columns (see Figure 5.62c). There is only one group used this strategy.

It is conjectured that the differences in iterating and counting the books occurred due to the position of the surfaces being measured. In the horizontal surfaces, such as teacher's table and students' desks, the students used as many of books as are required to cover the surfaces (see Figure 5.62b). This position allows the students to put the books without holding them together. Therefore, they could put as many of books as are required to cover the whole surface.

Meanwhile, in the vertical surfaces, such as the whiteboard, the door, and the announcement board, the students could not use the previous way to cover the surfaces because they have a limitation of the number of books that are able to be hold (see Figure 5.62a and 5.62c). For example, if they are three students in a group, so they could hold six books maximally. This limitation forced the students to take a few number of books and transposed them one by one as well as forced them to come up different counting strategy.

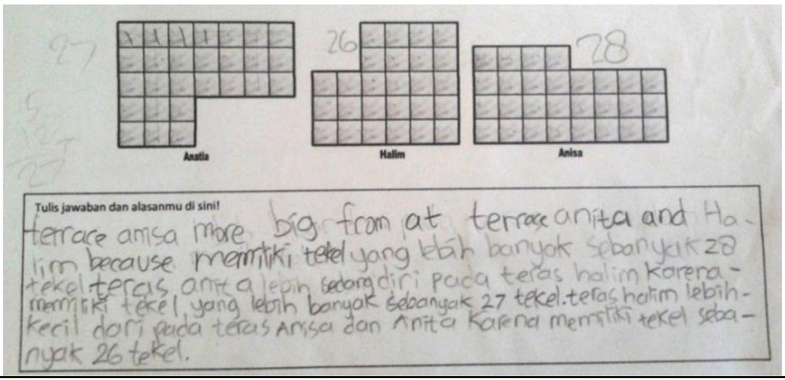
In the end of the activity in this problem, the students could determine the larger and the smaller surfaces that have been measured by considering the number of books needed to cover the surfaces. The students even knew that two plane figures have the same area if they have the same number of books. The students saw the area of the surfaces as the number of books needed to cover the surfaces. Here, the students could see the area as physical quantity –the number of measurement units covering a surface.

What happened with the focus students? The focus students iterated the books by using the first strategy, taking some books and then transposed those books one by one by continuously changing their position on the remainder of the whole surface being measured. It is conjectured that they use this strategy because the position of the surface. They measured the surface of a door which is located vertically. In counting the books, they counted the whole books being used one by one as they iterated the books one by one. By looking at the way they iterated and counted the books, it seems that the students were not aware yet about the array structure of area units.

Follow-up problem: Floor problem

In this problem students are given three tiled floors and they are asked to sort the floors based on the area of the floors. The purpose of the problem is to check students understanding of the physical quantity of area –area as the number of measurement units.

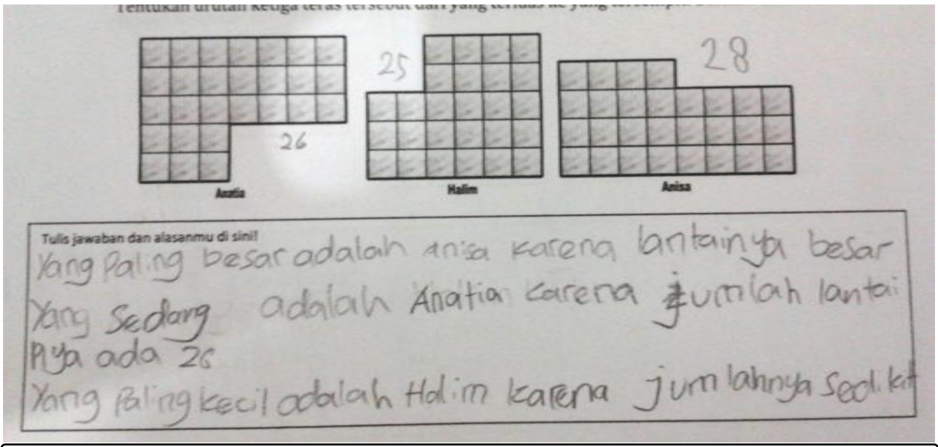
The findings yield that almost all of the students sort the floors correctly. They considered the number of tiles covering the floors. They knew that the more tiles covering a floor, the larger the floor is. Widya, Fachrus, and Derry, for example, stated that Anisa’s floor is the largest since it has 28 tiles, then Anita’s floor since it has 27 tiles, and the smallest is Halim’s floor since it contains 26 tiles (see Figure 5.63). Here, they sort the area of the floors based on the number of the tiles covering the floors. They could see the area as the number of measurement units covering a surface since they treated the squares as the measurement units.



Anisa’s floor is bigger than Anita’s and Halim’s because it has more tiles which are 28 tiles. Anita’s floor is bigger than Halim’s because it has more tiles which are 27 tiles. Halim’s floor is smaller than Anisa and Anita because it has fewer tiles which are 26 tiles.

Figure 5.63
Widya, Fachrus, Dery’s answer

As well as the other students, the focus students also show their understanding of the physical quantity of area. They sort the floors based on the number of tiles covering the floors as it is shown in Figure 5.64.



The largest is Anisa because its tiles are big [many]. The medium is Anita because it has 26 tiles. The smallest is Halim because its tiles are few.

Figure 5.64
Focus students' answer in the follow-up problem

The figure above shows that the focus students could sort the area of the floor from the largest to the smallest by considering the number of tiles. They treated the tiles as the measurement units. Here, the area as the physical quantity is understood by the students. It seems that the previous problems, hand problem and note-book problem, help them to deal with this problem.

Conclusion and Discussion

By grounding to the findings in this lesson the following remarks are established:

First, based on the findings in the hand problem, providing students with the experience of measuring identical size of surface, such as student desk, by using students' hands could lead to the discussion on the idea of unit of area, its consistency, and the idea of gap and overlapping on the units. The students could see that the size of hands used to cover the desk and the way they arrange the hands will yield different result of measurement. These understandings can be a good starting point to lead the students to the discussion on the measurement unit of area and its consistency and the idea of gap and overlapping on the units.

Second, the way the students iterate the measurement units (the books) will influence the strategy they used in counting the units. For example, they will count the books one by one if they iterated the books one by one by.

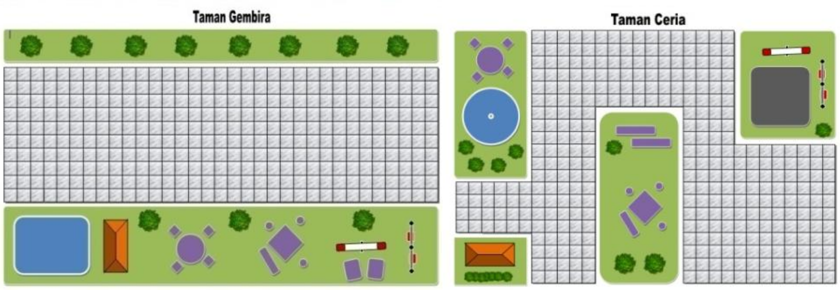
It is also found that the way they iterated the units (the books) is influenced by the position of the surface being measured. On the horizontal surfaces, the students iterated the books by using as many of the books as are required to cover the whole surface. Meanwhile on the vertical surfaces, students used only some books and transposed those book on the remainder surface. It is conjectured that the different way of iteration is due to the possibility of holding the books. In a horizontal surface, they could put as many books as required to cover a surface since the surface allows them to just put the books on the surface without holding them. Meanwhile, in a vertical surface they have to transpose the some books because they were not able to put the books on the surface without holding them. They have a limitation on number of books that they could put together on the surface in the same time.

Thus, it is concluded that the learning activity that provides the students with the experiences of covering and comparing activity in this lesson could help students to understand the measurement units of area and the physical quantity of area. This conclusion is supported by the findings in the follow-up problem, the floor problem. The students could sort the three floors by considering the number of tiles occupying the floors. Here, they treated the tiles as the measurement units.

2. Lesson 2: Developing students' awareness of the array structure of units of area

After the students have been introduced to the measurement unit of area, this lesson intends to build students' awareness of the array structure of the units.

There are two problems in this lesson. In the first problem the students are asked to compare the area of two parking lots (see the tiled surface on Figure 5.65). This problem is purposively designed to help students seeing the array structure of area units in columns and rows and introduce students to the idea of splitting (dividing a plane figure to be rectangular plane figures).



(a) (b)

Figure 5.65
Two parking lots (the tiled surfaces)

In the second problem the students are asked to measure the area of a floor which is covered by a carpet (see Figure 5.66). This problem intends to check weather students really see the array structure of area units or not. If the students really notice the array structure of area units, they will realize that the visible units can tell them the units under the carpet.

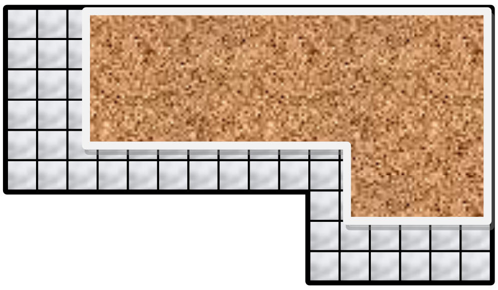


Figure 5.66
A floor covered a carpet

The findings from the both problem are elaborated in the following paragraphs.

First Problem: Comparing area problem

In dealing with the first problem, as the conjectures almost all students compare the area of the parking lots by considering the number of tiles in each parking lot.

Surprisingly and it is out of the conjecture that almost all of the students counted the tiles by using the multiplication strategy. For example, in the parking lot of Taman Gembira the students only counted the tiles covering the width and the length of the parking lot which is 10 tiles and 33 tiles respectively. Then, they multiplied those numbers (10 x 33) to obtain the whole tiles which are 330 tiles. When they are asked about the 330 tiles, they knew that 330 tiles refers to the number of tiles covering the parking lot.

As well as in Taman Gembira, the use of multiplication strategy is also occurred among almost all of the students when they counted the tiles in the parking lot of Taman Ceria. As the conjecture, they split the parking lot into some rectangular surfaces before applying the strategy.

The emergence of the multiplication strategy is actually out of the conjecture. It is expected that the students notice the array structure of the units (tiles) in columns or in rows and do repeated addition to enumerate the number of the whole tiles. It is assumed that the number of tiles which is relatively too many triggers the emergence of the multiplication strategy and the visibility of the tiles, where all the tiles can be seen, contributed to not really force students to see the array structure of the tiles.

However, there are some students still counted the tiles one by one and some others experienced of counting the tiles one by one before they used the multiplication strategy. It seems that the students who count the tile one by one were not aware yet about the array structure of the tiles.

What happened with the focus students? As well as the majority of the students, the focus students show the same strategy in solving the problem, using multiplication strategy, as it is shown in the following transcript.

Teacher: Could you explain your calculation?
Kevin: Here are 10 [tiles] and here are 33 [tiles], and then we multiplied them, 10 times 33 *(pointing the tiles covering the width and the length of the parking lot in Taman Gembira respectively).*
Teacher: How many?
Kevin: 330.
Teacher: OK. What does 330 mean?
Kevin: It is in Taman Gembira.
Teacher: What does 330 refer to?
Giri: The tiles.
Teacher: What?
Giri: 330 tiles.

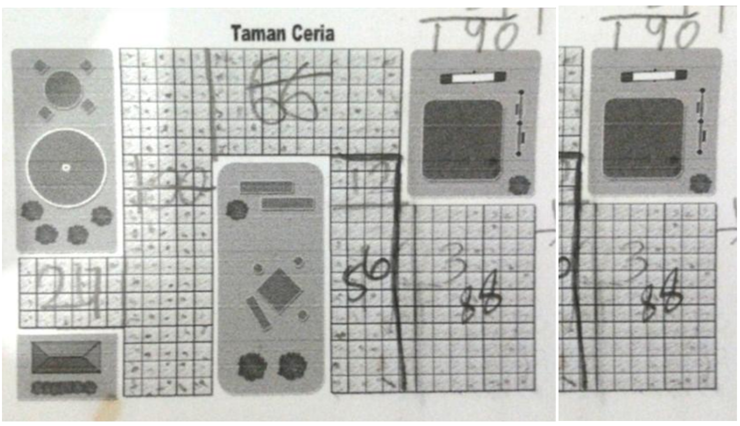
The transcript shows that the idea of the multiplication strategy in counting the tiles is emerged among the focus students. They knew that the product of 10 times 33, which is 330, refers to the number of tiles covering the parking lot of Taman Gembira where 10 tiles are the width and 33 tiles are the length of the parking lot (see Figure 5.65a).

Their understanding of the use of the multiplication strategy is even verified by their strategy in dealing with the second parking lot, Taman Ceria, as it is shown in the following transcript.

Kevin: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 *(He counts the tiles constituting the length of a plane figure, see figure 5.8.2.3b, after they split the plane figure into some rectangular plane figures)*

Kevin: 1, 2, 3, 4, 5, 6, 7, 8 *(He counts the tiles constituting the width of the plane figure).*
So, 11 times 8?
Giri: 88

The transcript shows that the focus students split the parking lot into some rectangular surfaces and one of them is seen in Figure 5.67b. It is conjectured that the emergence the splitting technique is due to the shape of the parking lot, which is right-angle shape that allows the students to split it into some rectangular shapes. They then used the multiplication strategy in counting the tiles covering the surface. They knew that the multiplication strategy can only be used in a rectangular surface. It seems that the use of the multiplication strategy in dealing with this problem is inspired by their strategies in dealing with the previous parking lot, Taman Gembira.



(a) (b)

Figure 5.67
Focus students' used multiplication strategy in measuring the area of the second parking lot, Taman Ceria

In determining the larger parking lot, the focus students consider the number of tiles in the parking lots as it is shown in the following transcript.

Teacher: What is the area of this? *(He points the parking lot in Taman Ceria)*
Giri: 331 [tiles].
Teacher: So, which parking lot is larger?
Kevin: In Taman Ceria.
Teacher: Why?
Giri: It has more squares [square tiles].
Teacher: It has more what?
Giri: Squares [square tiles].
Teacher: Ooo, I see. Here is 331 [tiles] and here *(pointing the 330 tiles in Taman Gembira).*
The actual number of tiles covering the parking lot in Taman Ceria are 334 tiles. Here the students made miscalculation.

The transcript shows that the students compare the area of the parking lots by considering the number of tiles covering the parking lots. Here, they treated the tiles as the measurement units in doing comparison. They knew that the parking lot in Taman Ceria is larger since it contains a more tiles, which is 331 tiles, if it is compared to the parking lot in Taman Gembira (330 tiles).

Grounding to findings above, it shows that the focus students could apply the multiplication strategy in counting the measurement units of area. They also understood that the multiplication strategy can only be done in a rectangular plane figure. Moreover, they could grasp area as the number of the measurement units covering a plane figure. They knew that the more units need to cover a plane figure, the larger the plane figure is.

Second Problem: Measuring area of a floor

The findings shows that most of students' strategies in dealing with the second problem are influenced by their strategies in the first problem, the parking lot problem. Before they applied the multiplication strategy most of them draw the hidden tiles on the carpet as the same size as the visible tiles. They then split the floor into two rectangular parts and then counted the tiles in each part by using the multiplication strategy (see Figure 5.68).

As well as the majority of the students, the focus students also drew the invisible tiles (the tiles under the carpet) and then counted the tiles using the multiplication strategy. Their strategy is shown in the following transcript.

They found that the area of the floor is 114 tiles. It is from 60 tiles plus 54 tiles after they split the floor to be two parts, A and B.

Teacher: Well, it is 114 [tiles]. Where did you get 114?
Kevin: From multiplication.
Teacher: Well, can you show us? Which one did you multiply?
Students: *(The students are silent for a while, thinking how to start the explanation).*
Teacher: Where did you get 60 [tiles]? *(He points 60 on the students' calculation).*
Giri: It is from here *(pointing the area of A, see figure 5.68)* and here is the border *(pointing the border line splitting A and B).*
Teacher: Yes.

Meanwhile, Kevin counted the tiles constituting the length and the width of A.

Kevin: 1, 2, 3, 4, 5, 6 *(counting the width).*
1, 2, 3, 4, 5, 6, 7, 8, 9, 10 *(counting the length).*
Teacher: Ooooh… It is until here *(pointing the border line).*
So, draw a border line here.
Then, what is the area of the surface? *(referring to surface A).*
Kevin: It is 6 x 10.
Teacher: How many then?
Giri: 60.
Teacher: Yes.
Widya: But, it takes long time, Sir. We can just split it here to be two surfaces *(splitting the floor to be 6 x 16 and 3 x 6).*
But here *(referring to Kevin' solution)*, it will be three surfaces.
(Widya, a student next to the focus students, tried to interrupt)

Teacher: Yes *(replying Widya's solution)*.
So what will you count next *(asking the focus students)*?
Giri: This one *(pointing B).* It is 54.
Teacher: Where did you get 54?
Giri: From the multiplication of these *(pointing the tiles on B).*
Meanwhile, Kevin counted the tiles constituting the length and the width of B.
Kevin: 1, 2, 3, 4, 5, 6, 7, 8, 9 (*counting the length).*
1, 2, 3, 4, 5, 6 *(counting the width).*
Teacher: How many?
Kevin: What is 9 x 6? *(asking Giri).*
Giri: It is 54, isn't it?
Teacher: Yes, 54.
Giri: Then, 54 + 60.
Teacher: How many?
Kevin: 114.

The transcript shows that the students used the multiplication strategy in counting the tiles after they split the floor into two rectangular floors, A and B (see Figure 5.68). In A, for example, they knew that the number of tiles covering A are 6 x 10 tiles which are 60 tiles where 6 is taken from the number of tiles constituting the width of surface A and 10 is from the length.

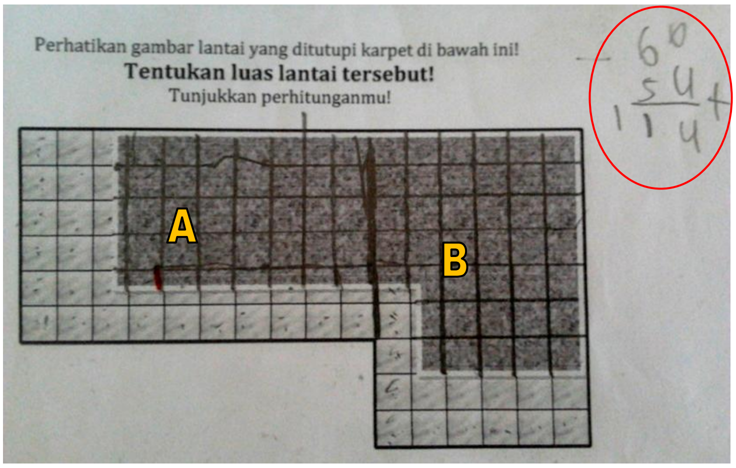


Figure 5.68
Focus students' solution and calculation in measuring the area of the second parking lot, Taman Ceria

Grounding to the students' answer explained above, it is conjectured that the used of the multiplication strategy in this problem is influenced by their strategy in solving the previous problem, the parking lot problem. In the parking lot problem, the students have experienced of using the multiplication in counting tiles covering a rectangular surface.

Furthermore, it is interesting to know why the students keep drawing the hidden tiles if they could use the multiplication strategy in counting the tiles? The findings show that there is no clear purpose of the drawing the hidden tiles among the students since there is no finding showing that the students used the drawing to help them in counting the tiles. It is assumed that the purpose of drawing the hidden tiles is to help them in applying the multiplication strategy where drawing helps them in counting the tiles constituting the length and the width of the floor. This assumption is supported by the findings that the students drew the tiles before applying the multiplication.

Another assumption is that the students drew the hidden tiles to convince themselves. Although they knew that the multiplication tells them the number of the whole tiles, including the hidden tiles, the students in this age are not easy to be convinced except they are provided with visual evidences.

Hence, grounding to the findings in this problem it is concluded that the students are able to use the multiplication strategy in measuring area.

Conclusion and Discussion

As it is stated that the purpose of the problems in this lesson is to build students' awareness of the array structure of the units of area on rectangular plane figures as the basis to introduce them to the multiplicative structure of the units. The array structure refers to the arrangement of the units of area in columns and rows in rectangular plane figures, where the number of units constituting each column or each row remain the same.

No finding clearly shows that the students, especially the focus students, see the array structure of the units. However, the researcher and some other investigators agreed that the emergence of the multiplication strategy and the splitting technique can be the reasons that the students were aware of the array structure. The students knew that multiplication strategy can only be used in rectangular plane figures; therefore, they split a non rectangular plane figure into rectangular plane figures to allow them using the strategy. Here, the students should understand the arrangement of the tiles, including the array structure of the tiles, to be able splitting a non rectangular plane figure to rectangular plane figures.

Another finding that can be the reason that the students are aware of the array structure is the way they used the multiplication strategy. In applying the multiplication strategy the students just counted the tiles constituting the length and the width of a rectangular floor. They also knew that the product of the multiplication refers to the number of the tiles covering the rectangular floor. It seems that the students knew that in each row or in each column of floor contains the same number of tiles, or in other words the students aware of the array structure of the tiles. By considering these findings, it is argued that the students actually were aware of the array structure of the units of area.

It is conjectured that the first parking lot, Taman Gembira, greatly influences students to come up with the multiplication strategy in counting the tiles, instead of considering the array structure of the units. The way the students counted the tiles in this parking lot influences their strategy in counting the tiles of the other problems. The array structure of the tiles in the parking lot, which is 10 rows and 33 columns, triggered the students to do multiplication in counting the tiles because counting by multiplication which the multiplication of tens (33 x 10) is easier than counting by considering the array structure, adding 33 ten times. Moreover, the rectangular shape of the parking lot allows the students to apply the multiplication directly without experiencing of investigating the array structure.

Hence, grounding to the findings it is concluded that the problems in this lesson less likely triggered the students to investigate the array structure of the units of area; but more likely triggered them to the multiplicative structure of units of area.

3. Lesson 3: Developing students' awareness of the multiplicative structure of units of area

The multiplication strategy is actually based on the multiplicative structure of units of area. Although the strategy has already emerged in the previous meeting, lesson 2: parking lot and floor problems, it is not enough to guarantee that the students aware of the multiplicative structure. It is still necessary to provide students with the experience of discovering the multiplicative structure of units of area as the basis to understand the multiplication strategy in measuring area.

Therefore, this lesson is designed to help students discovering the multiplicative structure of area units through investigating the area of some rectangular tiled plane figures in context of house sketch.

There are two problems in this lesson. The first problem, the house-sketch problem, is designed to discover the multiplicative structure. Meanwhile, the second problem is set as a follow-up problem to strengthen as well as to check students' awareness of the multiplicative structure. The following paragraphs will describe the findings from the two problems.

First Problem: House-sketch Problem

The classroom is started by talking about rooms in the students' house to guide students to be familiar with the context of problems in this lesson. Then, the students are given a house sketch with some rooms and a patio in it (see figure 5.69). Students are asked to discuss in group to find the area of each floor in the sketch.

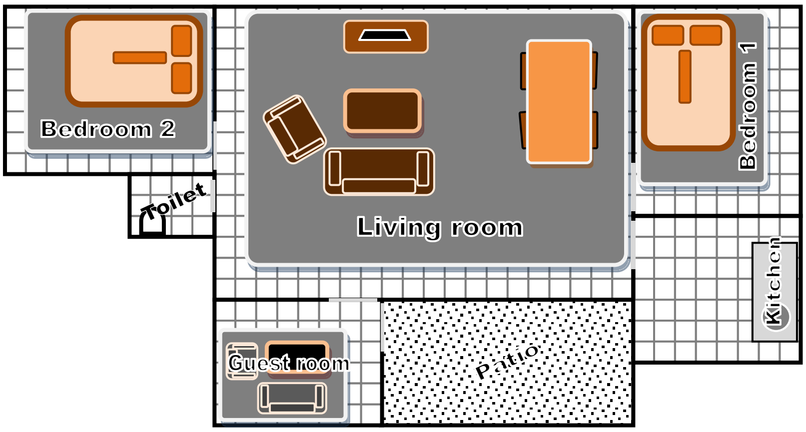


Figure 5.69
A House-sketch problem: students are asked to determine the area of each floor in the house

As the conjecture, the findings show that most of the students treated the tiles as the measurement units and applied the multiplication strategy in counting the tiles.

However, before they finally used the multiplication in counting the tiles it is found that the strategies used by the students are evolved from counting one by one to counting by using the multiplication. In measuring the area of the toilet and the kitchen, for example, Bila and Shafa preferred counting the tiles one by one since almost all the tiles are visible that allows counting one by one. But, they then turned to count the tiles column by column when dealing with the second bedroom since there are many tiles in the room are hidden. They then turned counting the tiles by using the multiplication strategy when they deal with the living room since not only most of the tiles in the living room are hidden but also many. Here, the hidden tiles and the number of the tiles being counted forced them to use the multiplication strategy since it is difficult to count the tiles one by one because the floors being measured is getting less and less graphic information of clues.

Different from the evolution of the counting strategy of Bila and Shafa, Genaro and Facruz started counting the tiles one by one when they dealt with toilet and the kitchen. Then, they keep counting one by one when dealing with the guest room, but here they construct gridlines to helm them counting the tiles one by one. When they dealt with the first bedroom, they kept drawing the gridlines but they did not count the tiles one by one. Instead, they used the multiplication strategy. They kept using the multiplication strategy in counting the tiles in the second bedroom, the living room, and the patio.

Some other students also experience of the evolution of counting the tiles as what Bila, Shafa, Genaro, and Fachruz have. It is conjectured that these students could see the multiplicative structure of the units of area since they have the experience of evolution of counting. Through the evolution, they knew that the multiplication can be used in enumerating the number of the tiles. It seems that the problems that asked students to investigate the area of rectangular-tiled surface with less and less graphic information of clues could help the students in seeing the multiplicative structure of the units of area.

However, there some students that directly counting the tiles by applying the multiplication strategy. Although the students understood that the product of the multiplication in the strategy refers to the number of the measurement units or the area of a plane figure, no clear evidence showing the students grasp the multiplicative structure of the units of area. For example, Widya, and some other students said that the area of the living room is 14 x 20 which is 280 and 280 is the number of the whole tiles covering the living room.

Actually, the multiplicative structure is based on the array structure of the units that allows doing repeated addition in enumerating the tiles. The repeated addition then can be transformed to be multiplication. For example, the area of the toilet can be counted by considering the array structure of the units that allows doing repeated addition 4 + 4 + 4. The operation 4 + 4 + 4 raises the multiplicative structure of the units where it can be transformed to be 3 x 4 to obtain 12. 12 tiles is then the area of the toilet.

It is conjectured that the previous meeting, lesson 2, influence students' strategy in solving this problem and contributes to the students' difficulties in seeing the relation between the array structure and the multiplicative structure. They have not experience of discovering the array structure yet.

In this situation, the guidance from the teacher is necessary. In the end of this activity the students could see the relation after the teacher discussed why the multiplication can be used to enumerate the number of the units. The discussion was started by asking the students to see the tiles column by column or row by row. For example, there are three tiles in each column of the toilet and there are four columns all together. Then they are asked to determine the whole tiles by considering the number of the tiles in each column or in each row. The students said that whole tiles are 3 + 3 + 3 + 3. The teacher then asked them to think about the relation between the repeated addition 3 + 3 + 3 + 3 and the multiplication representation. It was not difficult for the students. They knew that 3 + 3 + 3 + 3 means 4 x 3. Here, they could see why the multiplication can be used to determine the number of the tiles.

Here, the guidance from the teacher is necessary to bridge students to understand the relation between the array structure and the multiplicative structure.

What happened with the focus students? The focus students directly used the multiplication strategy to determine the area of the floors and did not clearly show his understanding of the relation between the array structure and the multiplicative structure before the discussion.

Let's consider the following discussion between the teacher and one of the focus students.

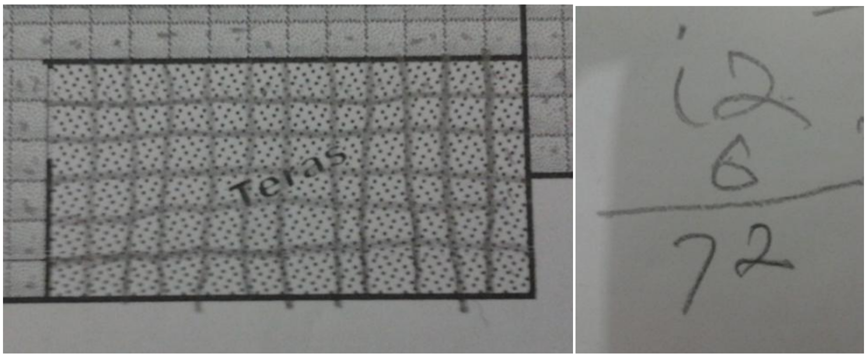
Teacher: What is the area of the toilet?
Kevin: Toilet! It is 12.
Teacher: How did you get it?
Kevin: 1, 2, 3; 1, 2, 3, 4 *(He counts the tiles constituting the width and the length of the toilet).*
and it is 3 x 4.

Teacher: What is 3 x 4?
Kevin: 12
Teacher: Well. What number do 12 refer to?
Kevin: The number of the tiles.
Teacher: What do you think why we can use multiplication to determine the number of the tiles?
Kevin: mmm…. *(He has no idea).*
Teacher: Well. Let's see again. What is 4 x 3?
Kevin: It is 12
Teacher: What do 12 refer to?
Kevin: The number of the tiles.
Teacher: Where did you get 4?
Kevin: It is from the top [row], 1, 2, 3, 4 *(He counts the tiles on the top row).*
Teacher: and 3?
Kevin: It is from this side [the first column], 1, 2, 3 (*He counts the tiles on the first column).*
Teacher: and then if you multiply those numbers you will get the number of the whole tiles. Why can we do that?
Kevin: mmm…. *(He still has no idea).*

The transcript shows that the students could use multiplication to determine the area of the toilet and he knew that the product of the multiplication refers to the number of the whole tiles in the toilet. However, he did not show yet why multiplication can be used to determine the number of the tiles. Based on the transcript, it seems that the student could not see yet the relation between the array structure and the multiplicative structure.

Nevertheless, the finding as the result of the investigator triangulation suggests that the students actually could see the relation between the array structure and the multiplicative structure, but he has no idea yet to represent his thinking. This assumption is supported by the findings that he could find the area of the patio where no tile on the patio.

In measuring the area of the patio, he created his own tiles by regarding the tiles of the neighboring floors (see Figure 5.70). Here, he could draw the tiles by considering the array structure of the tiles. The number of the tiles in each row or in each column is always the same. It seems that he used the arrangement of the neighboring tiles to help him in drawing the tiles, such as he drew an arrangement of tiles with 6 rows and 12 columns where 6 rows and 12 columns is taken from the number of the rows and the columns on the neighboring floors, guest room and living room, respectively (see Figure 5.70a). He then used multiplication to determine the area of the patio (see Figure 5.70b). Here, he combined his understanding of the array structure of the area units and the multiplicative structure of the units to determine the area of the patio.



(a) (b)

Figure 5.70
Students' solution in finding the area of the patio

Moreover, in the previous meeting, lesson 2, this student also knew that the multiplication can only be done in rectangular plane figures and the product of the multiplication refers to the number of the whole tiles.

In addition, the way he applied the multiplication by counting the tiles constituting the length and the width of the plane figure being measure also tells about his understanding of the array structure of the units. It means that he actually knew the array structure of the area units but he has no idea yet to represent it in words.

Second Problem: A follow-up problem

In this problem, the students are asked to determine the area of an incomplete-tiled floor (see Figure 5.71). The purpose of the problem is to check students' understanding of the multiplicative structure of area units.

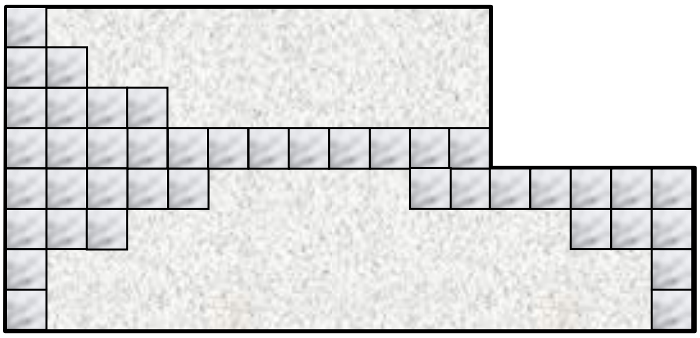


Figure 5.71
Incomplete-tiled-floor problem

To solve the problem, all of the students used the multiplication strategy in determine the area of the floor. Most of them started solving the problem by completing the tiles (drawing gridlines), and then they split the floor into two or three rectangular floors. They applied the multiplication strategy to measure the area (counting the tiles) of each rectangular floor to obtain the area of the whole floor.

It is not expected that the students drew the gridlines because the arrangement of the tiles is actually purposively designed to help them applying the multiplication without drawing gridlines. Instead, it is expected that the students looked at the multiplicative structure of the tiles when applying the multiplication strategy. It is conjectured that they kept drawing gridlines to help them counting the tiles covering the top row and the first column to know what numbers should be multiplied when applying the multiplication strategy.

Students' worksheet of the focus students, for example, shows that they drew gridlines to help them seeing the tiles covering the top and the first column of each rectangular floor being measured (see Figure 5.72). The dots on the tiles along the top row and the first column on the floor indicate students' record of such counting, counting the tiles constituting the length and the width of a rectangular floor. It seems that they have no idea yet that any arrangement of the tiles as long as it tells the length or the width of the floor, can be used to obtain the length and the width of the floor. Here, they could not see yet that the arrangement of the tiles on the floor can be used to determine the length and the width of the rectangular floors.

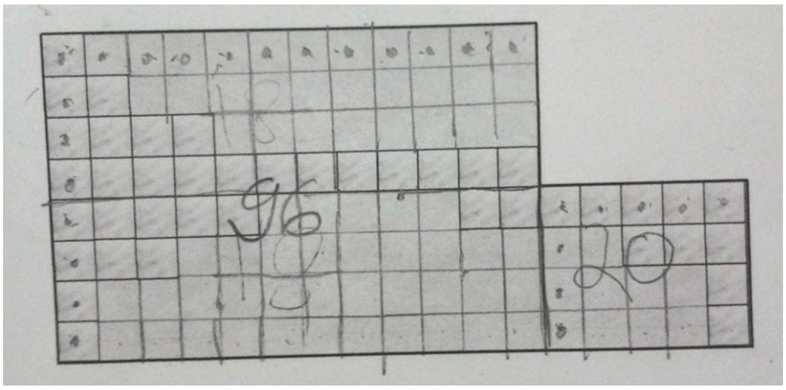


Figure 5.72
The gridlines helps students seeing the tiles covering the top and the first column of each rectangular floor

Considering the focus students' answer in Figure 5.72, the used of the multiplication strategy and the splitting technique indicate that the students see the multiplicative structure of the area units since those two concepts are underpinned by the multiplicative structure of the area units.

Conclusion and Discussion

Grounding to the findings in this lesson, it is found that the problems that asked students to investigate the area of rectangular-tiled surfaces (floors) with less and less graphic information of clues could help the students in seeing the multiplicative structure of the units of area. Here, the students experience of the evolution of counting strategy from counting one by one to counting by using multiplication. For example, Bila and Shafa preferred counting the tiles one by one at the first time. But, they then turned to count the tiles column by column, and then turned to count the tiles by using multiplication. Here, the hidden tiles on the floors and the number of the tiles being counted forced them to use the multiplication strategy since it is difficult to count the tiles one by one.

However, not all students experience of the evolution of counting strategy. Some of them directly counted the tiles by using the multiplication. In this situation, it is difficult to see whether the students are aware of the multiplicative structure or not.

Nevertheless, discussion with other investigators met an agreement that the students actually were aware of the multiplicative structure based on some indications showing that the students are aware of the multiplicative structure, such as: First, the students could use the multiplication strategy in determining the area of a rectangular plane figure They knew that the product of the multiplication refers to the whole units covering the plane figure. There is a strong relation between the multiplication strategy and the multiplicative structure of units of area. The multiplication strategy is based on the multiplicative structure due to the array structure of area units. The array structure allows doing repeated addition in enumerating the tiles. The repeated addition then can be transformed to be multiplication. Therefore, understanding the multiplication strategy can be the indication that the students could see the multiplicative structure.

Second, before applying the multiplication strategy they split a non-rectangular plane figure to be rectangular plane figures. They knew that the multiplication strategy can only be done in a rectangular plane figure. Understanding this prerequisite can be an indication that the students are aware of the multiplicative structure of units of area.

Third, most of the students could draw the arrangements of units by considering the array structure of units of area when they deal with finding the area of the patio and then used the multiplication strategy to determine the area of the patio (see Figure 5.70). Their drawing and the strategy indicate that they have an awareness of the multiplicative structure of units of area.

4. Lesson 4: Developing students' understanding of the role of the dimensions in applying the multiplication strategy in area measurement

In this lesson, the students are brought one step forward to more formal mathematics. The idea of dimensions of a plane figure is introduced in this lesson. It is expected that they are able to use the information given on the dimensions in measuring area.

There are two problems proposed in this lesson. In the first problem, students are asked to measure the area of a floor (see Figure 5.73). This problem intends to introduce students about dimensions as the representation of the array structure of area units. It is expected that the students can use the dimensions as the shortcut in applying multiplication strategy, instead of drawing gridlines.

In second problem, the students are asked to measure the area of another right-angle plane figure. Different from the previous problem, there is no any visible tile on this plane figure. The students are just given the dimensions of the figure. This problem intends to check students' understanding of the dimensions in applying the multiplication strategy.

First Problem: Measuring the area of a floor

In this problem, the students are given a picture of a floor where some parts of the floor are covered by tiles (see Figure 5.73). They are told that the numbers on each edge of the floor represent the number of tiles needed to cover the edges (dimensions). They then are asked to discuss in group how to measure the area of the floor.

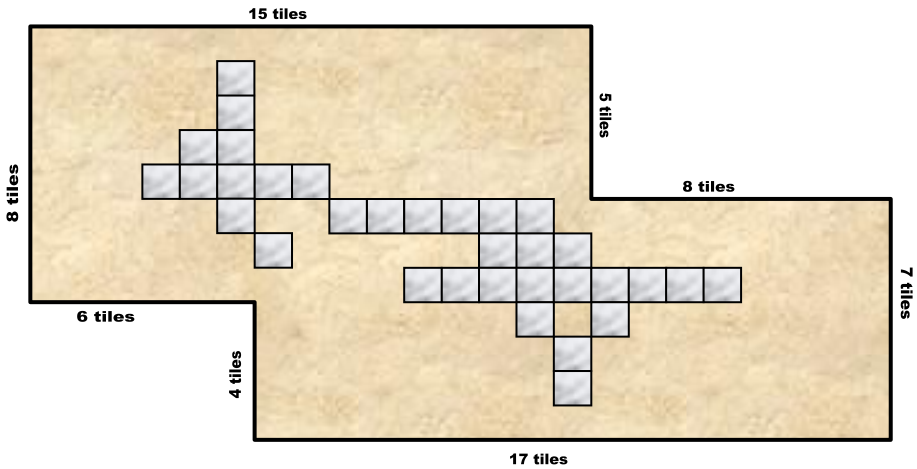
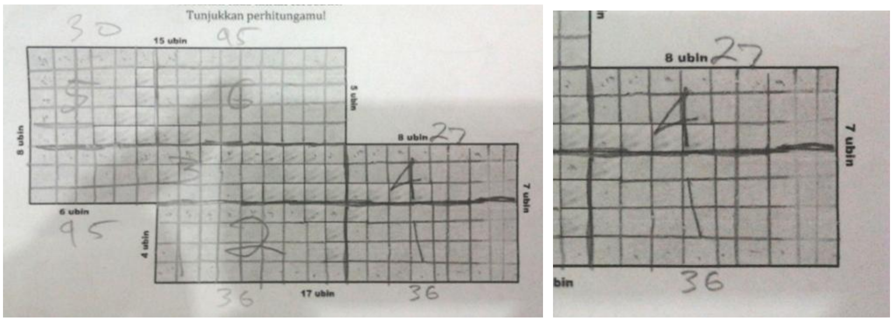


Figure 5.73
Measuring the area of a floor

The findings show that most of the students could answer the problem correctly. They started answering the problem by completing the tiles on the floor (only one group of the students did not complete the tiles). They determined the area of the floor by counting the tiles on the floor.

To start the counting the tiles, as the conjecture they split the floor into some rectangular floors and then most of them used the multiplication strategy in counting the tiles in each rectangular floor. There only two students kept counting the tiles one by one but then one of them turned using the multiplication.

Although the students could measure the area of the floor correctly by using multiplication strategy, the role of dimensions did not play significantly yet. The students did not take benefit from the information about the array structure given in the dimensions. They kept drawing gridlines to help them seeing the array structure. Once the gridline is constructed, the role of the dimensions is ineffectual. They did not look at the dimensions any longer. Therefore, they sometimes drew the gridlines incorrectly like in figure 5.74 below.



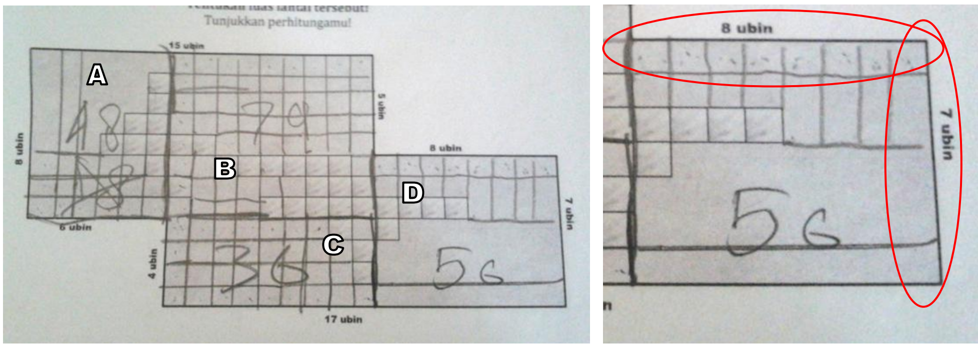
(a) (b)

Figure 5.74

The focus students' solution: They drew gridlines without considering the dimensions

The figure shows the focus students' solution in dealing with the problem. They drew gridlines to complete the tiles and then split the floor to be five rectangular plane figures. Looking at the gridlines carefully, such as Figure 5.74b, it is found that the students made mistake in drawing the gridlines where the size of the square grids (tiles) is inconsistent and the number of the square grids is not the same as it should be. For example, the worksheet stated that it should be 8 tiles constituting one of the edges of the plane figure, but the students drew 9 tiles.

To guide the students to see the students are asked the meaning of the numbers on the edges of the floor. They then realized that they have made mistake. After the students realized the mistake they made, they then revised their solution. The revised solution is shown in figure 5.75 below.



(a) (b)

Figure 5.75
The revised solution of the focus students

As well as the previous solution, the students kept drawing gridlines but not as complete as the previous solution. The students then are asked to explain their solution. Here is the conversation between the focus students and the teacher.

Teacher: Where did you get 56? *(pointing plane figure D on Figure 5.75a).*
Kevin: 8 times 7 *(pointing the dimensions of the plane figure directly respectively without counting the tiles on the edge of the plane figure one by one).*
Teacher: Yes.
What about 36? *(pointing plane figure C on Figure 5.75a).*
Kevin: It is 4 times 9.
Teacher: What is 4 times 9?
Kevin: 36.
He then worked on plane figure A. He split plane figure A into two parts.
Kevin: 1, 2, 3, 4, 5, 6. 1, 2, 3 *(counting the tiles constituting the width of the plane figure and some tiles constituting the length of the plane figure).*
Teacher: Don't you know that the number of the tiles has been told here *(pointing the dimension 6 and 8; Kevin keeps multiplying 3 and 6 to gets 18).*
Teacher: Well.
But, can you measure the whole this plane figure? *(pointing the plane figure A).*
Kevin: All of this? *(pointing the plane figure A).*

Teacher: Yes. And we know there are 6 [tiles] here and 8 [tiles] here.
Kevin: Ooo… 8 times 6!
Giri: 8 x 6? mmm… 48. *(counting with his finger).*
Teacher: Yes. So, which plane figure is 48, Kevin?
Kevin: This one *(pointing plane figure A).*

The transcript shows that the students understood how to treat the dimensions in determining the area of a plane figure, but there was inconsistency on their understanding. On one hand, they knew that the dimensions told them about the array structure when applying the multiplication since they did not count the tiles of the length and the width when finding the area of C and D. Here, they directly pointed the dimensions and multiplied them. On the other hand, they did not use the dimensions but counts the tiles of the length and the width when measuring the area of A. Here, the role of the dimensions faded again until the teacher reminded them about the dimensions. It seems that the role of dimensions in measuring area is still difficult and abstract for the students to understand in this level.

Another students' difficulty in interpreting the dimensions was occurred among another group, Tasha and Yasmin. But, their understanding of the dimensions is one step forward than the focus students. They did not complete the tiles any longer (drawing gridlines) (see Figure 5.76). Here, they understood the dimensions give them the information they need when doing multiplication, but have difficulty in interpreting the change on the dimensions after the floor is split.

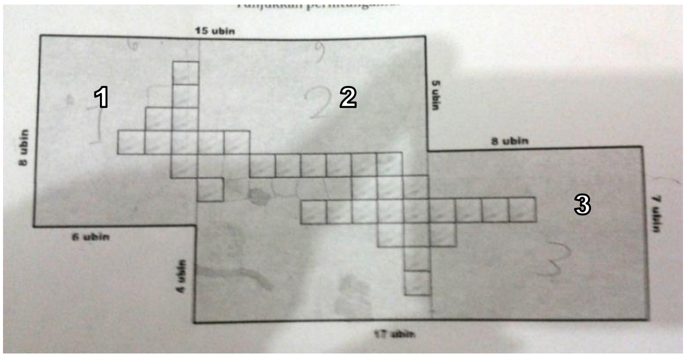
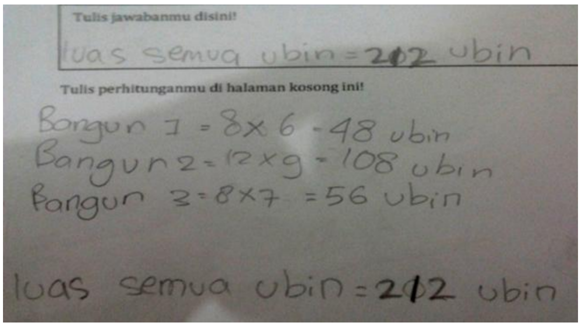


Figure 5.76
Tasha's and Yasmin's solution

The figure above shows Tasha's and Yasmin's solution. They split the floor into three rectangular plane figures. Let's consider the following conversation between Tasha and the teacher.

Teacher: What is the area of surface 1?
Tasha: It is 48.
Teacher: How did you know it is 48?
Tasha: From 8 x 6.
Teacher: Where 8 from?
Tasha: From here *(pointing one of the edges of surface 1 having eight tiles).*
Teacher: and 6?
Tasha: From here *(pointing another edge of plane figure 1 having six tiles).*
Yasmin then wrote that the area of plane figure 2. She wrote 15 x 5.
Teacher: Well.
What is the area of plane figure 2?
Yasmin: 15 x 5.
Teacher: 15 x 5?
How can it be 15?
The fifteen is for all here *(pointing along the edge of 15 tiles).*
Tasha: mmm… well, I need to measure it first *(she takes ruler and then measure the length of the edge 15 tiles).*

The transcript shows that Tasha and Yasmin knew that the area of plane figure 1 is 8 x 6. Eight and six are taken from the dimensions of plane figure 1. However, they got confused when dealing with plane figure 2. They said that the area of plane figure 2 is 15 x 5. They did not realize yet that 15 as well as 5 is not the dimensions of plane figure 2. They did not see yet that the edge (15 tiles) has been split into 6 tiles and 9 tiles. They could not see yet that the edge (5 tiles) is a small part of the dimensions of plane figure 2. However, after the teacher carefully discussed about the change occurred on the dimensions after splitting they got an insight how to treat the dimensions in measuring the area of the floor (see Figure 5.77). The following calculation shows their final solution.



The area of all tiles = 212 tiles.

Surface 1 = 8x6 = 48 tiles
Surface 2 = 12x9 = 108 tiles
Surface 3 = 8x7 56 tiles

The area of all tiles = 212 tiles

Figure 5.77
Tasha's and Yasmin's calculation in measuring the area of the floor

The figure shows that they knew that the dimensions of plane figure 2 are 12 tiles and 9 tiles. 12 tiles are taken from 8 tiles added by 4 tiles. Meanwhile, 9 tiles are taken from 15 tiles subtracted by 6 tiles. Here, they showed a better understanding of the role of the dimensions in applying multiplication strategy in area measurement.

Second Problem: Measuring the area of a right-angle plane figure

In second problem, the students are asked to measure the area of another right-angle plane figure. Different from the previous problem, there is no any visible tile on this plane figure. The students are just given the dimensions of the figure (see Figure 5.78). This problem intends to check students' understanding of the dimensions in applying the multiplication strategy.

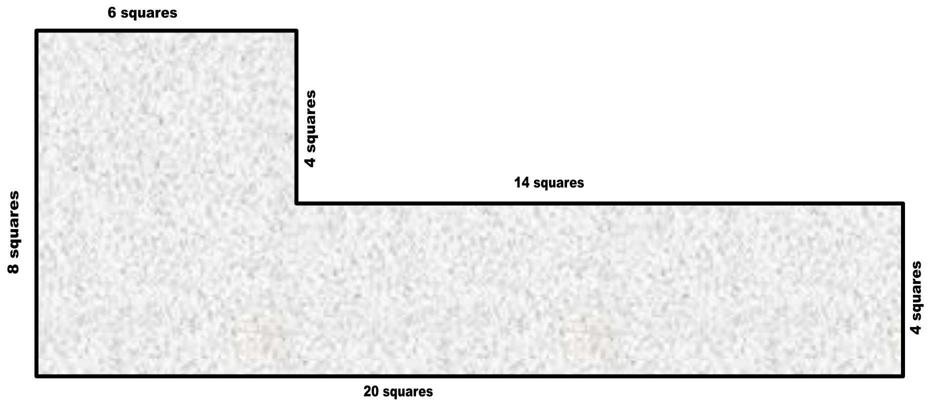


Figure 5.78
The second students' problem in measuring the area of a right-angle plane figure

In presenting the problem, the teacher started by explaining the students that there is a plane figure where each edge of the plane figure consists of a certain number of squares as it is shown in the figure 5.78. The students then are asked to discuss in pairs to determine the area of the plane figure.

As the conjecture, the findings show that most of the students (11 out of 15 collected students' worksheets) solve the problem by combining the splitting technique and the multiplication strategy. They first split the plane figure into two rectangular plane figures and then by considering the dimensions they applied the multiplication strategy to determine the area of the rectangular figures.

However, there are four students' worksheets that kept drawing gridlines on the plane figures. Although the students drew the gridlines, two of them counted the squares by using multiplication and the remaining two counted the squares one by one.

As well as the majority of the students, the focus students started solving the problem by splitting the plane figure into two rectangular plane figures. They then used the multiplication strategy in counting the square units constituting the rectangular figures to obtain the area of the figure (see Figure 5.79).

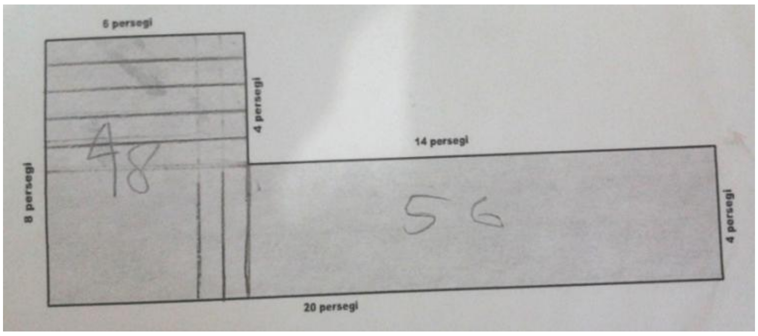


Figure 5.79
The focus students' solution to the second problem

Figure 5.79 shows that the students intended to draw gridlines but they cancelled. Instead, they split the plane figure into two rectangular plane figures.

Surprisingly, there are two strategies used by the focus students in counting the square units. First, they counted the tiles by using multiplication. They multiplied 8 and 6, and 14 and 4. The emergence of this strategy is not surprising since they are already familiar with this strategy before. But, it is surprising that in the second strategy which is used to verify their calculation they apply the repeated addition strategy, counting column by column and row by row. In the first rectangular plane figure, for example, they added 8 six times, 8+8+8+8+8+8, and they obtained 16+16+16, and then 32+16, and finally 48 (see Figure 5.80). This counting strategy did not emerge before. In the hypothetical learning trajectory, it is expected that this strategy emerges in the lesson 2: parking lot problem.

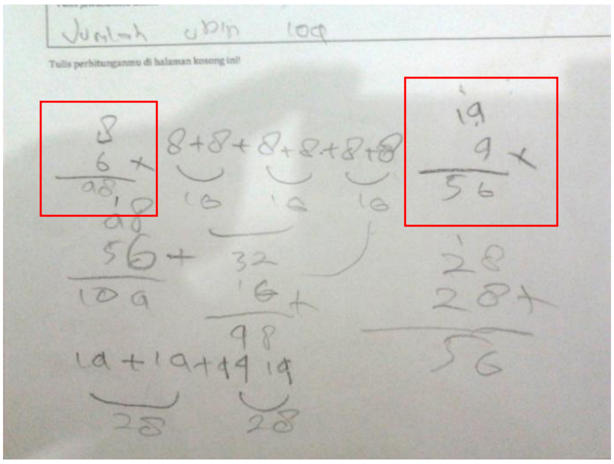
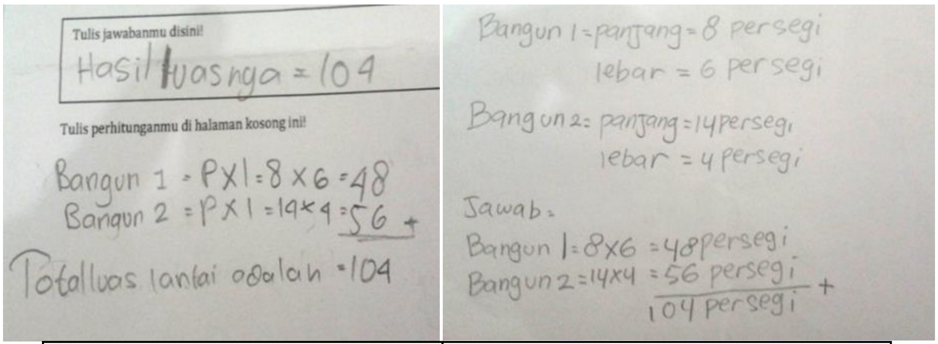


Figure 5.80
Our focus students' calculation to the second problem

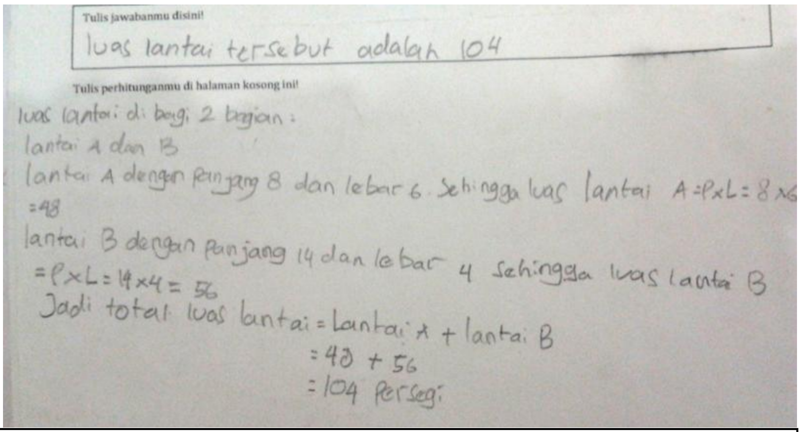
Considering the findings in this problem where most of the students could solve this problem well by applying splitting technique and multiplication strategy, it is argued that the students have a better understanding about the function of the dimensions in applying multiplication strategy in area measurement. Some of them even reach more formal mathematics, such as Tasha, Belva, and Yasmin (see Figure 5.81 and Figure 5.82)


Tulis jawabanmu disini!
Hasil luasnya = 104
Tulis perhitunganmu di halaman kosong ini!
Bangun 1 = PxL = 8 x 6 = 48
Bangun 2 = PxL = 14x4 = 56 +
Total luas lantai adalah = 104

Bangun 1 = panjang = 8 persegi
lebar = 6 persegi
Bangun 2: panjang = 14 persegi
lebar = 4 persegi
Jawab:
Bangun 1 = 8x6 = 48 persegi
Bangun 2 = 14x4 = 56 persegi +
104 persegi

The area = 104 Surface 1 = L x W = 8 x 6 = 48 Surface 2 = L x W = 14 x 4 = 56 The total area of the regions = 104	Surface 1 = length = 8 squares Width = 6 squares Surface 2 = length = 14 squares Length = 4 squares Answer: Surface 1 = 8 x 6 = 48 squares Surface 2 = 14 x 4 = 56 squares 104 squares

Figure 5.81
Tasha's and Belva's solution


Tulis jawabanmu disini!
luas lantai tersebut adalah 104
Tulis perhitunganmu di halaman kosong ini!
luas lantai di bagi 2 bagian:
lantai A dan B
lantai A dengan panjang 8 dan lebar 6 sehingga luas lantai A = PxL = 8x6
= 48
lantai B dengan panjang 14 dan lebar 4 sehingga luas lantai B
= PxL = 14x4 = 56
Jadi total luas lantai = lantai A + lantai B
= 48 + 56
= 104 persegi

The area of the region is split into 2 parts. Surface A and B Surface A has length 8 and width 6. So, its area is A=LxW = 8x6 = 48 Surface B has length 14 and width 4. So, its area is A=LxW = 14x4 = 56 So, the total area = Surface A + Surface B = 48 + 56 = 104 squares

Figure 5.82
Yasmin's solution

Nevertheless, some students still made mistake in solving the second problem. Most of the students' mistakes are in interpreting the dimensions. The students who drew gridlines made mistake in interpreting the dimensions. For example, the task stated that there are six square on an edge (dimension), but Fachruz drew 4 squares on it or Irgi and Shafa drew 8 and 7 squares respectively on it.

However, their understanding of area as the number of measurement units constituting a plane figure is preserved since Fachruz as well as Irgi and Shafa counted the squares constituting the plane figure to obtain the area (see figure 5.83).

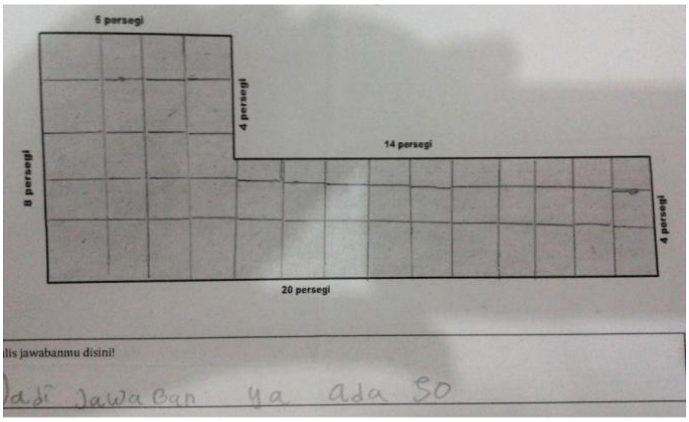


Figure 5.83
Fachruz' solution to the second problem

Multiplying wrong numbers of the dimensions in applying the multiplication strategy is another example of students' mistake. Nazwa's worksheets, for example, shows that she multiplied 6 and 8, 14 and 4, 20 and 4, and then added the product of those multiplication to obtain the area of the plane figure (see Figure 5.84). It seems that this student did not really understand how to use multiplication in obtaining area.

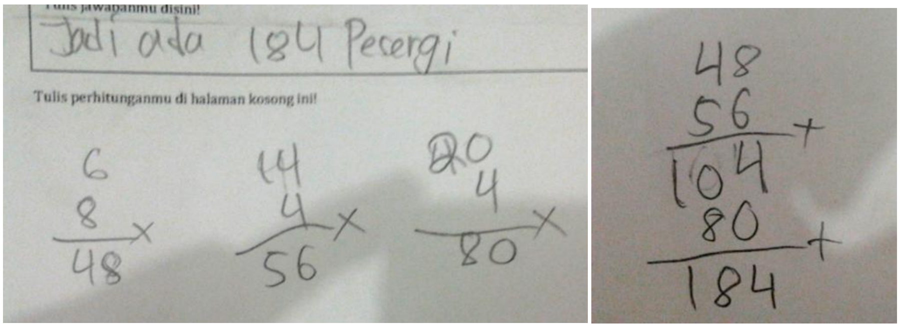


Figure 5.84
Nazwa's solution to the second problem

Reflections and Discussion

Grounding to the findings explained above, most of the students' approach in measuring the area of plane figures in this lesson, especially the first problem, consists of the following procedure: first drawing gridlines (completing tiles), then splitting into rectangular plane figures, and ended by multiplication strategy in counting the measurement units.

It is conjectured that the emergence of drawing the gridlines when measuring area is due to the existence of tiles on the plane figure being measured. For example, in measuring the area of a floor of the first problem most of the students drew the gridlines to complete the tiles. It is because there are some tiles on the floor. But, in the second problem most of the students did not draw the gridlines when measuring the area of the second problem since there are no tiles on the floor.

Meanwhile, the emergence of the splitting is triggered by students' understanding that the multiplication strategy can only be applied in a rectangular plane figure. Therefore, the students tried splitting a non rectangular plane figure to be some rectangular plane figures before applying the multiplication strategy.

It is also found that the students could consider the dimensions of a plane figure when measuring the area of the figure by using the multiplication strategy. Here, they looked at the dimensions as the representation of the array structure. Since the array structure represent the number of units in rows and columns, they could multiply the dimensions representing the array structure of area units of a rectangular plane figure when measuring the area of the figure. They knew that the product of the multiplication refers to the number of area units covering the figures. This implies that the students could see the relationship among the ideas of area units, array structure, and the multiplicative structure.

Nevertheless, it is found that for some students the role of dimensions is not easy to be understood. Either in the first or the second problem, there are some students interpreted the dimensions incorrectly when applying the multiplication strategy. At least there are two kinds of students' mistake in interpreting the dimensions:

First, the students did not understand that the dimensions represent the array structure of the units of area. Therefore, when drawing gridlines the students drew the gridlines that is as not many as the numbers of tiles informed by the dimensions (see Figure 5.83).

Second, they were not aware of the change occurred on the dimensions after splitting. Therefore, they were confused to find dimensions that they have to multiply when applying multiplication strategy after the figure being measured is split.

The discussion with other investigators conjectured that the idea of dimensions is still an abstract concept for some students in this age. They need more activity to guide them in understanding the role of the dimensions in applying multiplication strategy in area measurement.

In addition, it is found that some of the students reached more formal mathematical notation in measuring area. In the second problem, for example, they could note measuring area as the multiplication of the length and the width of a rectangular plane figure being measured (see Figure 5.81 and 5.82).

K. Remarks on Students' Knowledge and Development Based on the Posttest of the Second Cycle

There are five questions being asked in the posttest to investigate students' understanding and development after the interventions. From 21 students, there were 18 students participating in this test. The remaining 3 students were absent due to sick.

The results and the remarks from the posttest are explained in the following paragraphs.

1. Students could see area as the number of measurement units

The students are asked to solve two area comparison problems. The purpose of the problem is to check whether the students can see area as the number of measurement units or not. The problems are the same as the area comparison problem in the pretest. In the first problem, the students are given three plane figures and are asked to sort the three plane figures from the largest to the smallest (see Figure 5.85).

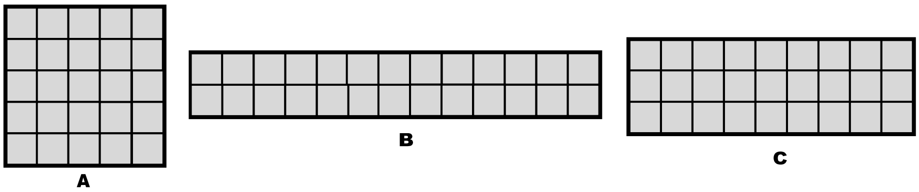


Figure 5.85
The first area comparison problem

Meanwhile, in the second problem the students are given two uncompleted-tiled floors and are asked to determine the larger floor (see Figure 5.86).

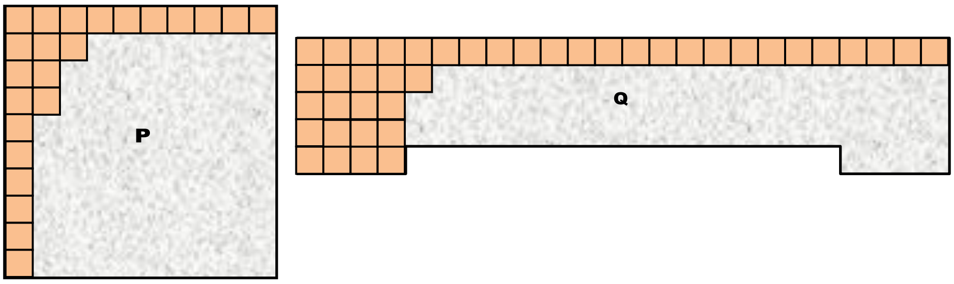


Figure 5.86
The second area comparison problem

The findings from both problems are elaborated in the following paragraphs.

In the first problem, before the interventions the findings from the pretest show that most of the students considered area as the width or the length of a plane figure. Therefore, they compare the width or the length of the plane figures when sorting them.

However, after the intervention the findings from the posttest shows that most of the students (15 out of 18 students) see area as the number of units covering a plane figure. They compare the area of the three plane figures by considering the number of squares covering the figures. They stated that C is the largest since it contains 27 squares and B is larger than A since they contain 26 and 25 squares respectively. The remaining three students still compare the area based on the length or the width of the plane figures. Two of them compared the plane figures based on the length of the plane figure and one of them based on the width of the plane figures.

The focus students, Kevin and Giri, show different level of understanding in solving this problem based on the findings in the pretest. Kevin saw area as width meanwhile Giri seems to see area as the number of area units covering a plane figure. But in this posttest, both of them could see area as the number of area units covering a plane figure.

In the second problem, the findings from the pretest show that most of the students chose the larger floor by considering the width or the length of the floor. In contrast with the pretest, most of the students (15 out of 18 students), including the focus students, compared the floor by considering the number of tiles covering the floors. They sated that Q is larger than P since it contains more tiles, 104 and 100 tiles respectively.

Grounding to the findings between the pretest and the posttest in these problems, it is concluded that there is a development among the students in understanding the meaning of area after the interventions. In the pretest, they saw area as the width or the length of a plane figure, but after the interventions they looked at area as the number of area units covering a plane figure.

2. Students see measuring area as counting the number of area units covering a plane figure

Here, the students are asked to determine the area of a tiled floor (see Figure 5.87). The purpose of the problem is to check students' understanding of what it is called as measuring area, whether they understand it as finding the number the measurement units, or finding the perimeter of a plane figure, or as finding the length or the width of a plane figure.

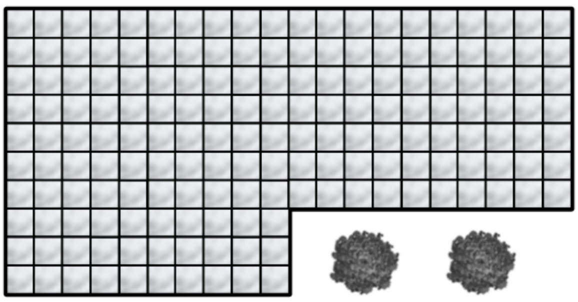


Figure 5.87
The second area comparison problem

In the pretest, most of the students thought of measuring area as counting the perimeter of a plane figure or measuring the width or the length of a plane figure. But, after the intervention the findings from the posttest show that most of the students (16 out of 18 students) could see measuring area as finding the number of area units covering a plane figure.

The focus students, Kevin and Giri, show different level of understanding of what it is called as measuring area based on the findings in the pretest. Kevin thought of measuring area as measuring the width of a plane figure. Meanwhile, Giri had an indication of understanding measuring area as counting the area units covering a plane figure. However, after the intervention both of them could see measuring area as finding the number of area unit covering a plane figure.

Grounding to the findings above it is concluded that there is a development on students' understanding from understanding measuring area as measuring a linear measurement (measuring length, width, or perimeter) to understanding measuring area as area measurement (finding the number of area units covering a plane figure).

3. Student could apply multiplication strategy to measure the area of rectangular plane figure

To check students' understanding of using multiplication in area measurement, the students are asked to find the area of two rectangular plane figures (see Figure 5.88). Here, students' understanding of the role of the dimensions is also tested in this problem.

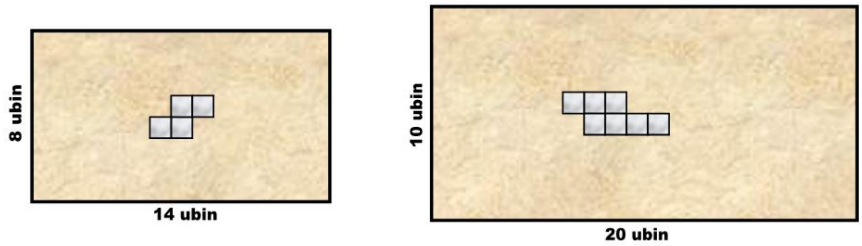


Figure 5.88
Finding the area of a rectangular plane figure

Before the intervention, the findings from the pretest show that none of the students could measure area of such figures. They tended to measure perimeter instead of area. They also could not be able to interpret the information about the array structure given by the dimensions.

In contrast, the findings from the posttest show that most of the students (12 out of 18 students) could measure the area of the plane figures. They even applied the multiplication strategy. They multiplied 8 and 14 when measuring the area of the first rectangular plane figure and multiplied 10 and 20 for the second rectangular plane figure.

It is also found that 11 students solved the problems without drawing gridlines. Here, they knew that the dimensions given on the plane figure tell about the array structure of the tiles which can be used to determine the area of the plane figures.

However, there are some students (6 out of 18 students) still got difficulties in finding the area of such figures. Some of them completed the tiles by drawing gridlines but then they have no idea to proceed. Some other added the dimensions of the plane figure. Zaki, for example, added 14 and 8 to obtain the area of the first rectangular plane figure and 10 and 20 for the second plane figure.

As well as the majority of the students, the focus students, Kevin and Giri, could measure the area of the figures by applying the multiplication strategy. Before the intervention, Kevin measured the perimeter when dealing with the similar problem; meanwhile Giri tried completing the tiles by drawing gridlines but he then had no idea to proceed.

Grounding to the findings above, it is conclude that there is a development on students' understanding in area measurement. Before the intervention most of the students could not be able to measure the area of a rectangular plane figure, but after the intervention most of them could do it and even applied the multiplication strategy.

4. **Student could apply multiplication strategy to measure the area of right-angle plane figure**

To check how the students will deal with measuring the area of a right-angle plane figure, they are asked to measure the area of a right-angle plane figure (see Figure 5.89). Here, students' understanding of the role of the dimensions is also tested in this problem.

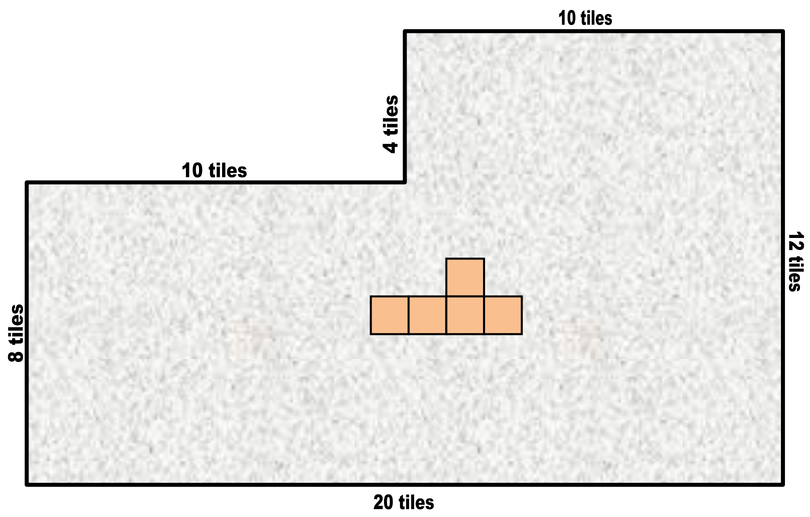


Figure 5.89
Finding the area of a right-angle plane figure

In the pretest, it is found that most of the students measured perimeter instead of area when they are proposed with the similar problem. Moreover, they did not know how to deal with the dimensions measuring area.

Conversely, the findings from the posttest show that most of the students (12 out of 18 students) could measure the area of the plane figure. All of them used the multiplication strategy. They started by splitting the plane figure into some rectangular plane figures and used the multiplication strategy in measuring the area of the rectangular plane figures by considering the array structure given in the dimensions. For example, Kevin (one of the focus students) split the plane figure into three rectangular plane figures and then applied the multiplication strategy to measure the area of each rectangular plane figure (see Figure 5.90). The area of the whole plane figure is the sum of the area of the whole rectangular plane figures. Here, Kevin could use the information of the array structure given by the dimensions in applying the multiplication strategy. In other words, Kevin knew the role of the dimensions in area measurement.

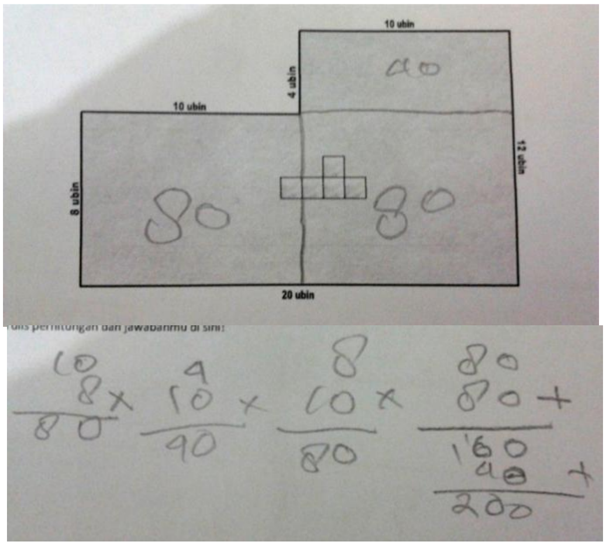


Figure 5.90
Kevin's strategy in measuring the area of a plane figure

Another strategy is shown by Giri (another one of the focus students). Different from Kevin, he split the plane figure into two rectangular plane figures instead of three rectangular plane figures. They then used the multiplication strategy to count the area of each rectangular plane figure. The area of the whole plane figure is the sum of the area of the rectangular plane figures. Different from Kevin, Giri drew the gridlines to complete the tiles by considering the array structure of the tiles given by the dimensions. But, he did not use the gridlines to help him in measuring the area of the plane figure. He just did it to complete his answer.

As well as Kevin, he showed his understanding of the role of the dimensions in measuring area. He knew which numbers to be multiplied to obtain the area of a plane figure. Moreover, his drawing of the tiles indicates of his understanding of the array structure of the tiles which is based on the dimensions (see Figure 5.91).

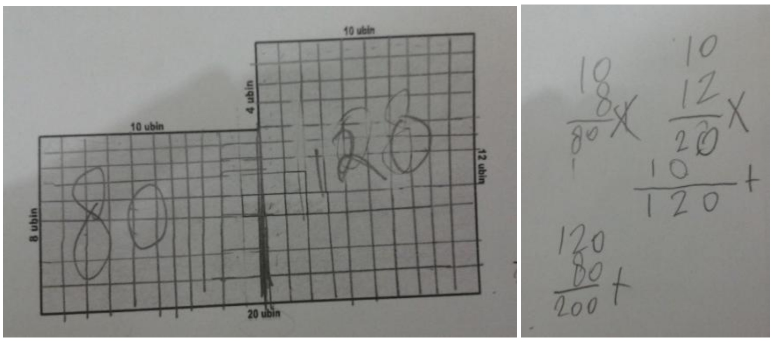


Figure 5.91
Giri's strategy in measuring the area of a plane figure

Nevertheless, some students (6 out of 18 students) still got difficulties in measuring the area of the plane figure. Some of them multiplied the whole dimensions when applying multiplication; some other drew gridlines without considering the dimensions and counted the square grids one by one; and some others still measure perimeter instead of area.

Grounding to the findings above, there is a development of students understanding in area measurement. The development can be traced from the finding in the pre and posttest. In pretest most of them measured perimeter when they are asked to measure area, but in the posttest most of them could measure the area of a plane figure by applying the multiplication strategy. Moreover, in the pretest almost all of them have no idea about dimensions meanwhile in the posttest most of the students could use the dimensions to help them in applying the multiplication strategy in measuring area.

L. Conclusion of the Teaching Experiment (The Second Cycle)

In the pretest, it is found that the students could not be able seeing area as the number of the area units covering a plane figure yet although they could see area as a region inside a boundary. In this moment, they have no idea yet how to measure the area of a plane figure. They measured perimeter instead of area when they were asked to measure area. They also could not be able seeing the array structure of units of area yet.

In lesson 1, it is found that the learning activity that provides students with the experiences of covering and comparing area activity in this lesson could help students to understand the measurement units of area and the physical quantity of area. This conclusion is supported by the findings in the follow-up problem, the floor problem. The students could sort the three floors by considering the number of tiles occupying the floors. Here, they treated the tiles as the measurement units. Moreover, if the findings in the pretest and the posttest are considered, it is found that before the experiment most of the students could not be able seeing area as the number of the area units covering a plane figure although they could see area as a region inside a boundary. On the other hand, after the intervention the students could see area as the number of the measurement units and used the units in comparing the area of two plane figures.

In lesson 2, the problem in this lesson less likely triggered the students to investigate the array structure of the units of area; but more likely triggered them to the multiplicative structure of units of area. However, the emergence of the multiplication strategy and the splitting technique can be the reasons that the students were aware of the array structure. In measuring an area, splitting a right-angle plane figure to be rectangular plane figures in applying the multiplication strategy requires an understanding of the arrangement of units of area, including the array structure of the units. Moreover, when applying the multiplication strategy, they just counted the tiles constituting the top row and the first column of the array structure of the area units. It seems that the students knew that in each row or in each column of the rectangular region contains the same number of area units. In other words, the students indirectly were aware of the array structure of the area units.

In lesson 3, it is found that the problems that asked students to investigate the area of rectangular-tiled surfaces (floors) with less and less graphic information of clues could help the students in seeing the multiplicative structure of the units of area. Here, the students experience of the evolution of counting strategy from counting one by one to counting by using multiplication. For example, Bila and Shafa preferred counting the tiles one by one at the first time. But, they then turned to count the tiles column by column, and then turned to count the tiles by using multiplication. Here, the hidden tiles on the floors and the number of the floor being counted forced them to change their strategies in counting the tiles since it is difficult to keep counting the tiles one by one because the floors being measured are getting less and less graphic information of clues.

In lesson 4, it is found that most of the students' approach in measuring the area of plane figures in this lesson consists of the following procedure: first drawing gridlines (completing tiles), then splitting into rectangular plane figures, and ended by multiplication strategy in counting the measurement units. It is also found that the students could consider the dimensions of a plane figure when measuring the area of the figure by using the multiplication strategy. Nevertheless, it is found that for some students the role of dimensions is not easy to be understood. They did not know that the dimensions represent the array structure of the area units and were not aware of the change occurred on the dimensions after splitting. It is conjectured that the idea of dimensions is still an abstract concept for some students in this age. They need more activity to guide them in understanding the role of the dimensions in applying multiplication strategy in area measurement.

In the post test, in contrast with the findings in the pretest it is found that the students could grasp *'area'* as the number of area units and *'measuring area'* as counting the number of the area units covering a plane figure. They could measure the area of rectangular plane figures and right-angle plane figures by applying the multiplication strategy. Here, they could use the dimensions of a plane figure in applying the multiplication strategy.

M. Checking the Validity of the Data Analysis

Checking the validity of the analysis is only conducted in the second cycle since the result of the analysis of the second cycle will be used as the consideration in answering the research question and establishing a local instructional theory; meanwhile the analysis of the first cycle is used to refine and improve the instructional theory and design.

In checking the validity of the data analysis, the findings from the methodological triangulation are considered during the analysis and by grounding to the data. Here, in establishing a conclusion the researcher not only sees the object being analyzed from only one source of data, but also involves other resources of data. The researcher also mostly grounded the analysis to the original data (the primer data) when interpreting the data to come with a conclusion as it is shown in the analysis, such as looking at the transcript of the original video recording, original students' work, and original interview transcripts.

The following table show how the researcher involves several of sources of data to increase the validity and the reliability of data analysis in each phase.

Table: 5.3
Checking the validity of data analysis of the second cycle

Phase	Data	Finding
Preparation phase	The data about the classroom environment (teaching and learning situation) during a normal teaching process - Observation field note and video observation of the teaching process. - Teacher interview filed note and recording to the interview.	**Observation filed note:** - The classroom is dominated by the teacher. - Students tend to be passive learners. - Direct instruction of teaching **Video observation:** - No discussion during the teaching process. - The teacher starts the classroom activity by giving the students some concepts of mathematics directly and then asks students to do some exercises. - The students just listen to the teacher's explanation and sometimes give some response when the teacher asks something to them. - It seems that the teacher dominates the classroom activity; meanwhile the students just follow the instructions given by the teacher. - Only a few students are active in the classroom activity. **Teacher interview field note and recording:** - There are many students have problem in mathematics, such lack of understanding in multiplication and division. - The teacher actually is not a mathematics teacher, but she is asked to teach mathematics. - In teaching mathematics, the teacher just explains the mathematical concepts and then asks students to solve some problems. - The teacher just uses the textbook as the source of materials in teaching.

Pre-test	The data about students' prior knowledge - Students' written work. - Video recording of the student interview	**Students' written work:** - Not all students have a background understanding of area as the extent of a region. There are some students made mistake in choosing the figures that having area region. - Most of students have no idea yet about area as the number of area units covering a plane figure. When comparing the area of two tiled floors, the students did not consider the tiles as the measurement units; instead they looked at the length or the width of the floors. - The students have a lack understanding of measuring area. Students measured perimeter instead of area when they are asked to measure area. It seems that the students treated area the same as perimeter. - Many students were not really aware of the array structure of the units of area. When counting the squares in an array, the students counted the square one by one instead of taking benefit of the structure of the squares in the array. - Many students could see repeated addition as multiplication. They could say that 6+6+6 as 3x6. **Students' interview:** - Although some students made mistake in choosing the right figures, they know area as the extent of a region or space, but they have no idea the figures that having area. - Most of them consider area as the width of a figure. They said that the wider a figure is, the larger it is. - The students never learned yet how to measure area, therefore they measure perimeter instead of area. They realized that they made mistake, but they did not know what to do if they are asked to measure area. - They did not know yet the array structure of the squares in the array. Therefore, the counted the square one by one.

Teaching experiments	The data about the actual learning trajectory. - Video observation - Interview video recording. - Students' written work.	**Video observation:** - The learning activity that provides students with the experience of covering and comparing area activity in lesson 1 could help students to develop their understanding of the measurement units of area. The students could say that floor B is larger than A since B has more square tiles. - The activity of comparing the area of tiled surface in lesson 2 less likely triggered students to develop their understanding of the array structure of area units. The students preferred counting the square tiles by using multiplication. There is no clear evidence that the students investigate the array structure. - The activity in lesson 3 which asked students to find the area of some rectangular surfaces with less and less graphic information of clues of area units could help students to develop their understanding of the multiplicative structure of the area units. Here, the students experience of evolution in counting strategy. Firstly, they counted the square one by one since all the units are visible. They then counted the units row by row or column by column since some units are hidden. Finally, the counted the units by using the multiplication strategy since they realized that the structure of the units allow to do that and almost all the units are hidden. - Although some students still got difficult in understanding the role of dimensions as the representation of the array structure, many students could consider the dimensions in measuring area by using the multiplication strategy. They knew that if a rectangle has 8units as the length and 7 units as the width it implies that there are 8 rows of units and in each row there are 7 units therefore the whole units is 8x7 which is 56 units. **Students' interview:** - In lesson 1, the students could say that the surface of the blackboard is larger than the tables since there are more books that can be arranged on the blackboard. Here, they treated the book as the measurement units. - In lesson 2, the students preferred using multiplication in counting the tiles since it is more efficient. For example, in measuring the area of a surface with 10 tiles as the width and the 33 tiles as the length, the students said "here is 10 and here is 33, and then we multiply them, 10 times 33". They then multiplied 10 and 33 which is 330. They then said "the area of the surface is 330 tiles. That is faster." - In lesson 3, the students knew why multiplication can be used to count the units in a rectangular surface. For example, in finding the area of a floor which has 3 tiles as the width and 4 tiles as the length, the students could say that the area of the floor is 3 x 4 which is 12 tiles and knew that 12 tiles refers to the number of the tiles within the floor. - In lesson 4, many students could consider the dimensions of a rectangular surface in finding the area of the surface. For example, when measuring the area of a rectangular surface with dimensions 7 square tiles as the width and 8 square tiles as the length, the students could say that the area of the surface is 7 x 8 which is 56 square tiles.

		Students' written work: - In lesson 1, figure 5.63 is a student's written work showing the students' understanding of the measurement units of area. They compare the area of the floors by considering the number of tiles. - In lesson 2, figure 5.67 shows students' work on using the multiplication strategy in counting the area units, instead of investigating the array structure. Here, the student preferred of using multiplication since it is more efficient. - In lesson 3, figure 5.70 is a student's work which shows students' awareness of the multiplicative structure of units of area. In finding the area of plane figures, they multiplied the length and the width of the figures. - In lesson 4, figure 5.75 shows students ability in considering the dimensions of a figure to find the area of the figure by applying the multiplication strategy. Here the students could realize that the dimensions provide the information of the array structure of the units.
Post-test	The data about students' knowledge after the experiments - Students' written work. - Video recording of the student interview	**Students' written work:** - The students could see area as the number of measurement units of area. For example, the students could compare the area of two tiled floors by considering the number of tiles covering the floors. Here, the students treated the tiles as the measurement units. - Students see measuring area as counting the number of area units covering a plane figure. For example, when the students were asked to measure the area of a surface, they tried to count the number of area units covering the surface. - Students could apply the multiplication strategy in finding the area of rectangular plane figure. For example, when the students were given a rectangle with 20 square units as the length and 10 square units as the width, they multiplied 20 and 10 to obtain the area of the rectangle. They knew that the product of 20 times 10 is the number of units covering the rectangle. **Students' interview:** - In comparing two floors that have different number of tiles, the students said that floor B is larger than A since B has more tiles. Here, the students could see area as the number of measurement units of area. They treated the tiles as the measurement units. - When the students were asked "what did you do to measure the area of this figure?" most of the students said that they counted the number of square units covering the figure. Here, the students see measuring area as counting the number of area units covering a plane figure. - In finding the area of a rectangle with 20 square units as the length and 10 square units as the width, the students said that the area of the rectangle is 20 times 10 which is 200 square units. They then were asked further, "what is 200 means?" they said that 200 square units is number of squares within the rectangle and it is the area of the rectangle.

The information given by the table above is considered in checking the validity of the data analysis. To check the validity of the analysis, the researcher proves the findings of the analysis from not only one source of data but also involves other source of data. For example, to convince the researcher that the students have already understood the measurement unit of area, the researcher not only looks for evidences in the data from the observation, but also from student interview and students' written work.

The table above shows that almost all sources of the data support one to each other in describing the subjects. For example, the finding from the observation found that the covering and comparing area activity could help students to come up with the idea of measurement units of area. This finding is supported by the findings from the students' interview and students' written work. This situation increase the validity of the conclusion saying that the covering and comparing area activity could help students to come up with the idea of measurement units of area.

CHAPTER VI

CONCLUSION AND DISCUSSION

In this chapter, firstly, the research question will be answered. Next, a local instructional theory in developing students understanding of the multiplication strategy in area measurement is established based on the teaching experiments.

Afterwards, some reflections on the important issues relating to the implementation of the instructional theory designed in this study are elaborated. In the end, a recommendation for the future research as the follow-up study is suggested.

A. Answer to Research Question

To answer the research question *"How can we help students to develop their understanding of the multiplication strategy in area measurement?"* we mainly grounded to the findings of the study, especially from the teaching experiments. The following paragraphs will elaborate the answer of the research question.

In the revised local instructional theory (LIT), we conjectured that to help students in developing their understanding of the multiplication strategy in area measurement the students need to learn the following big ideas consecutively;

1. Understanding the measurement unit of area.
2. Understanding the array structure of unit of area.
3. Understanding the multiplicative structure of the array structure.

4. Understanding the role of dimensions as the representation of the array in applying the multiplication strategy in area measurement.

This study suggests that the learning activities that can be used to address the understanding of those big ideas are explain in the following paragraphs.

- Comparing area by using hands or books as the measurement tools

  It is found that comparing area by using hands or books as the measurement tools could help students to develop their understanding of the measurement unit of area. When comparing two surfaces of students' desk by using their hand, for example, the students could say that one desk is bigger than other since it occupies more hands. Here, the students treated their hands as the measurement units. Moreover, the idea of unit consistency, gap, and overlapping on area units are also discussed through this problem since each student has different size of hand and covers the desks differently.

- Structuring array of square units

  The activity that asks students to structure an array by using square units could help them in developing their understanding of the array structure of area units. This activity helps students to realize that if the area units are arranged forming an array, it yields a structure, which is called an array structure. Regularity of the number of units in columns or in rows of an array (array structure) becomes the foundational idea for the students to understand the multiplicative structure of area units.

- Investigating the area of rectangular surfaces having less and less graphic information of clues

  The student who has a background understanding of the array structure of area units could understand the multiplicative structure of area unit easily. Providing students with the problem that asks them to investigate the area of rectangular surfaces with less and less graphic information of clues could help them in developing their understanding of the multiplicative structure. In the first time, the students probably consider the array structure of area units when measuring the area of the rectangular surfaces that has enough graphic information of units. They then will realize the multiplicative structure of area units in measuring area when dealing with the surfaces that have less graphic information of clues of units since the surface requires investigating the number units in a row and the number of rows in the array of the surfaces. Once the students understand the multiplicative structure of area units, they will be able to use the multiplication strategy in area measurement and understand why the strategy works.

- Investigating the area of plane figures where their dimensions are given.

  In real world application, measuring area is mostly related to the dimensions of the surface being measured. Therefore, it is necessary for the students to understand the role of the dimensions in area measurement and consider the dimension when using the multiplication strategy. Dimensions of a plane figure are the representation of the array structure of the figure. Investigating the area of plane figures where their dimensions are given can be a good problem to help students developing their understanding of involving dimensions in using the multiplication strategy in area measurement.

Moreover, it is found that there is a strong relationship among the three big ideas (area units, array structure, and multiplicative structure) to help students to understand the multiplication strategy in area measurement. Those three ideas are related each other building understanding of the multiplication strategy in area measurement. Understanding the units of area is the foundational idea that the students need to comprehend if they want to be able measuring area of a plane figure. When the units of area are arranged in columns and rows, it produces an array structure of the units of area. When the array structure is arranged forming a rectangular plane figure, it yields the multiplicative structure of the units of area that allows the multiplication strategy in counting the whole units covering the figure.

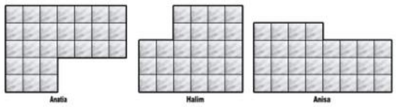
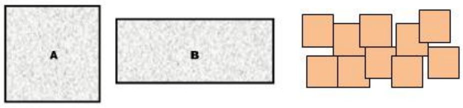
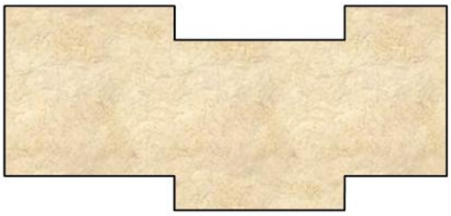
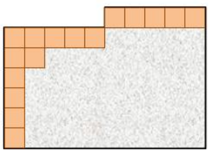
The findings of this study found the relationship among the three ideas (area units, array structure, and multiplicative structure) where each one is built on the others to help students in developing their understanding of the multiplication strategy in area measurement. It suggests that understanding the area units become the foundational idea for students to both: understanding area as the number of measurement units and as the initial idea to introduce them to the array structure of the area unit. Understanding the area units beforehand allows them to do investigation on the array structure of the units. In turns, once the students realize the array structure, they then have a foundational idea for investigating the multiplicative structure of the area units. When the array structure of the units or when the area units are arranged forming a rectangular shape, it emerges the multiplicative structure of the units that allows the multiplication strategy in counting the whole units.

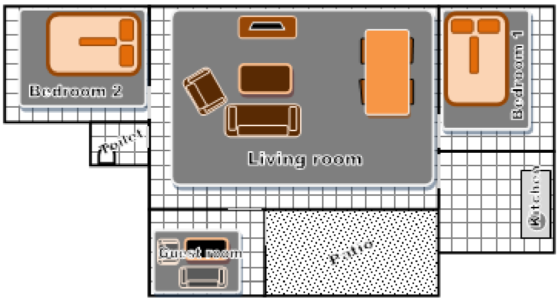
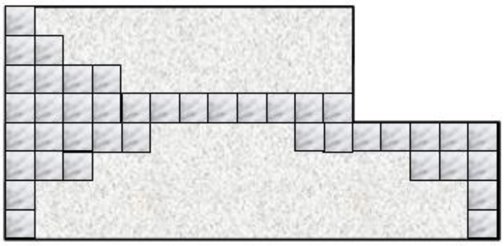
B. The Local Instructional Theory for Classroom Practice

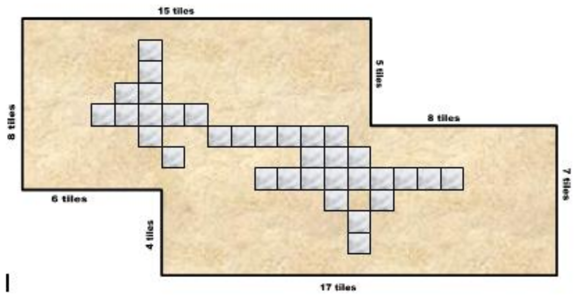
As the product of this study, a local instructional theory in helping students develop their understanding of the multiplication strategy in area measurement is established that can be used in the classroom practice. The instructional theory consists of four steps of learning activities as it is shown in the following table.

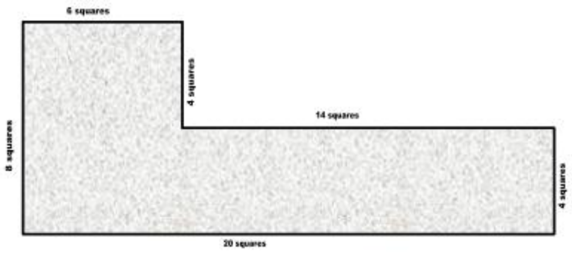
Table 6.1
A Local Instructional Theory in Developing Students' Understanding of the Multiplication Strategy in Area Measurement

Learning Activities	Means (the ways to conduct the learning activity)	The mathematical ideas that students will learn
Step 1 Area comparison activity	Preliminary activity: Students are asked to find the number of hands needed to cover their own desk. It is conjectured that they will yields different result of the number of their hand due to the different size of their hand and the way they cover the desk. Then, they are asked to compare the size of the surface of the desk. It is conjectured that the students will consider the number of the hands in comparing the size of the desk. Then, since they have a different number of hands covering their desks students are asked to discuss why they have different number of hands needed to cover their desks.	Measurement units of area, unit consistency, gap and overlapping.
	Main activity: Students are asked to determine the number of books needed to cover three different surfaces in their classroom, such as door, blackboard, etc. and are asked to compare the area of the surfaces. It is conjectured that the	Physical quantity of area (seeing area as the

	students will consider the number of books needed to cover the surfaces in comparing the area of the surfaces.	number of the area units)
	Follow-up: Students are asked to compare the area of three tiled floors as in the figures below.  It is conjectured that the students will consider the number of the tiles covering the floors in comparing the area of the floors.	Measuring area as counting the number of area units.
Step 2 Structuring Array Activity	Preliminary activity: students are asked to compare the area of two rectangular plane figures and they are given ten squares as the measurement units.  It is conjectured that the students will use the ten squares to cover the figures and compare the number of the squares units needed in comparing the area of the figures.	Structuring array of area units on rectangular shapes (Discovering the array structure)
	Main activity: students are asked to measure the area of a right-angle floor by using the given ten squares as the measurement units.  It is conjectured that the students will use the squares units to cover the floor to find the area of the floor. Since the number of the square unit is limited, they will split the surface into some smaller parts.	(1) Array structure of area units on right-angle shapes. (2) Splitting technique.
	Follow-up: students are asked to measure the area of a right-angle floor by considering the existence structure of tiles on the floor.  It is conjectured that the students will split the floor into some rectangular floors and then count the maximum tiles covering the rectangular floor by considering the array structure of tiles on the floor.	(1) Array structure of area units on right-angle shapes. (2) Splitting technique.

<table>
<tr>
<td rowspan="2">Step 3
Measuring the area of hidden-tiled floors</td>
<td>Main activity: students are asked to measure the area of rectangular tiled floors with less and less graphic information of clues as it is shown in the following figure.

It is conjectured that in counting the square units (tiles) the students will experience of the evolution of counting from counting one by one to counting by considering the array structure and finally to counting by the multiplication strategy.</td>
<td>(1) The multiplicative structure of the array structure of units of area. (2) the multiplication strategy in counting the units of area.</td>
</tr>
<tr>
<td>Follow-up: students are asked to measure the area of a right-angle incomplete-tiled floor as it is shown in the following figure.

It is conjectured that the students will split the floor into some rectangular floors. Then by considering the multiplicative structure of area units on a rectangular shape, they use the multiplication strategy in counting the tiles to find the area of the rectangular floors.</td>
<td>(1) The multiplicative structure of the array structure of units of area. (2) Splitting technique. (3) the multiplication strategy in counting the units of area</td>
</tr>
</table>

<table>
<tr>
<td rowspan="2">Step 4
Measuring the area of incomplete tiled floors</td>
<td>Main activity: students are asked to measure the area of a right-angle floor where some tiles of the floor are structured on the floor and the numbers of tiles needed to cover each edge of the floor (dimensions) are given.


It is conjectured that the students will split the floor to be some rectangular floors. Then, by considering the dimensions of the floor they will count the tiles covering the rectangular floors by using the multiplication strategy.</td>
<td>(1) Introducing dimensions as the representation of the array structure (2) Splitting technique. (3) the multiplication strategy in counting the units of area</td>
</tr>
<tr>
<td>Follow-up: students are asked to measure the area of a right-angle shape where the numbers of area units needed to cover each edge of the shape (dimensions) are known.


It is conjectured that the students will split the shape to be some rectangular shapes. Then, by considering the dimensions of the shape they will count the area units covering the rectangular shapes by using the multiplication strategy.</td>
<td>(1) Splitting technique. (2) considering dimensions in applying multiplication strategy in counting the units of area</td>
</tr>
</table>

C. The Weaknesses of the research

There are some weaknesses of this research that we are going to elaborate further in this part.

1. The learning styles of the students (such as auditory, visual and kinesthetic learner) are not intensively considered during designing the instructional materials and during the analysis. Actually, for the preliminary students

their learning styles greatly influence their achievement in learning. Not considering the students' learning styles during the designing and the analysis may reduce the quality of the instructional design as well as the findings from the analysis.

2. There is no clear criterion used in this study to evaluate the quality of the interventions (the instructional design). The researcher just compared the hypothetical learning trajectory with the actual learning trajectory in evaluating the quality of the instructional design. This way of evaluation may reduce the convincing result of the analysis.

D. Reflection on the Important Issues

There are several issues discussed in this section relating to the implementation of the instructional theory in the experiment, such as the issues about the students, the role of the teacher, the learning environment, and the issue about the learning style.

The subjects involved in this study are not accustomed with the learning activity that asked them to work in group and having discussion. They also are not accustomed with the inquiry learning activity that asks them to discover by themselves on the solution of the given problem. They used to learn in teacher-centred-classroom norms where the information on how to deal with a problem directly informs by the teacher. These situations to some extent contributed to the difficulties in orchestrating classroom discussion and group discussion in this study. Therefore, the role of the teacher becomes important to establish an inquiry classroom norm and to orchestrate discussion among the students. Providing the students with a mathematical problem before giving mathematical concepts can be a good way to start establishing the inquiry classroom norms.

E. Recommendation for Future Research

This study focus only on applying the multiplication strategy in area measurement by using non-standard measurement units. Here, the students could measure the area of rectangular and right-angle plane figures by applying the multiplication strategy, but the measurement units that they used are still a non-standard measurement unit, such as books, tiles, and squares.

A more formal mathematics concept is needed to be developed as the follow-up research of this study. It is expected that the follow-up research will be about developing students' understanding of the standard measurement units in area measurement. The following research question may be appropriate with this kind of research *"How can we help students to develop their understanding of the standard measurement units in area measurement?"* Here, the standard measurement units, such as centimetre squares and meter squares, are introduced.

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APPENDIX A

CLASSROOM OBSERVATION SCHEME

About Teaching and Learning

1. Is there group discussion?
2. Is there classroom discussion?
3. Is this direct instruction or enquiry of teaching and learning? Give indication!
4. Is there any characteristic of RME occurred in the classroom? If so, what? Give indication!

About Student

1. How they participate in classroom activity? Active or passive? Describe shortly!
2. Estimate the number of active and passive students!
3. In general, do they look motivated during the teaching and learning? Why? Describe shortly!

About Teacher

1. How the teacher starts the classroom activity?
2. How the teacher involves student in classroom activity?
3. How the teacher orchestrates classroom discussion?
4. How the teacher addresses the different opinion between the teacher and students and between students and students?
5. Does the teacher dominate the classroom or give students more opportunity to be active?

About Classroom Norms

1. Is this usual for students to express their idea to the classroom?
2. Is this usual for students to ask teacher when they do not understand something?
3. Is this usual for students to have different opinion among other students?
4. Is this usual for students to have different opinion with teacher?

APPENDIX B

TEACHER INTERVIEW SCHEME

1. What are the mathematical difficulties that your students generally have in your class?
2. How do you solve it?
3. Have you an experience of teaching area measurement?
4. If yes, how did you teach it? Will you change it in future?
5. If no, how will you teach it?
6. Do you often start the classroom activity by asking students opinion or ideas?
7. Do you invite students to be more active in classroom activities? How?
8. Do you give opportunity for students to invent the solution of a given problem by themselves or you explain them the solution? If yes for the first option then do you give also opportunity for students to explain the solution to other students?
9. Do you often use students' solution as the starting idea to explain to other students?
10. Do you use contextual problems from everyday life situation when you teach?
11. What is the source of teaching material you use to teach mathematics? Book? Internet?

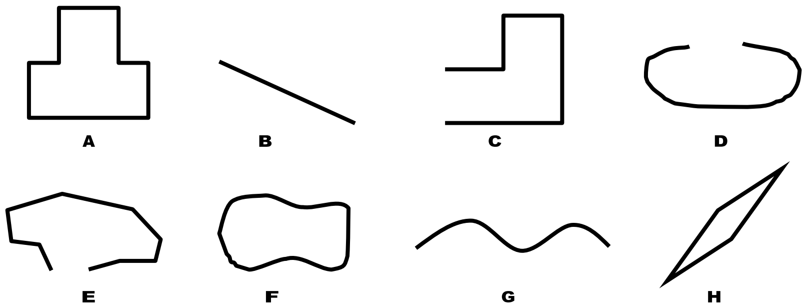
APPENDIX C

PRETEST FOR THE SECOND CYCLE

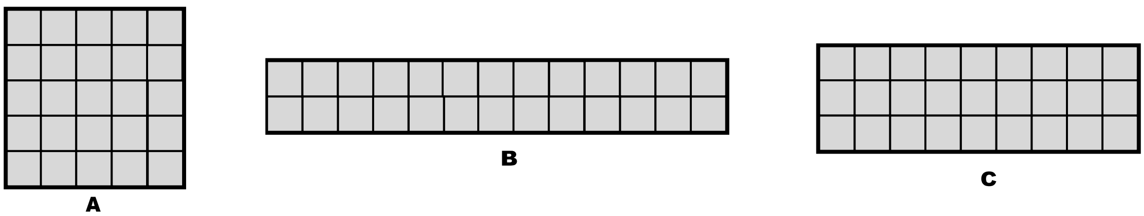
1. Tunjukkan caramu menyelesaikan soal perkalian di bawah ini!

3 x 20 = ?	4 x 500 = ?	200 x 6 = ?

2. Lingkari label gambar-gambar di bawah ini yang memiliki luas!

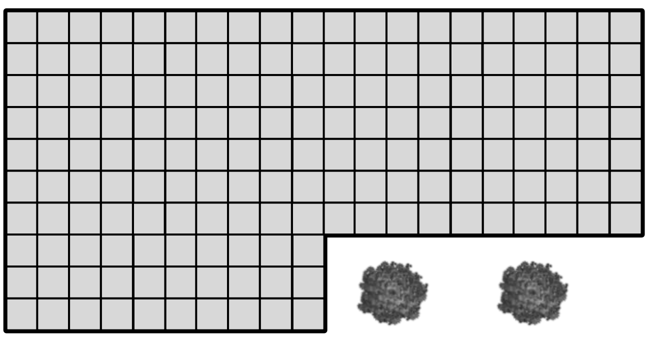


3. Perhatikan ketiga gambar di bawah ini. Urutkan ketiga gambar tersebut dari yang terluas ke yang tersempit! Berikan alasanmu!



Tulis jawaban dan alasanmu di sini:

4. Perhatikan gambar lantai berkeramik di bawah ini. Tentukan banyak keramik di lantai tersebut!



Tulis perhitunganmu dan jawaban di sini!

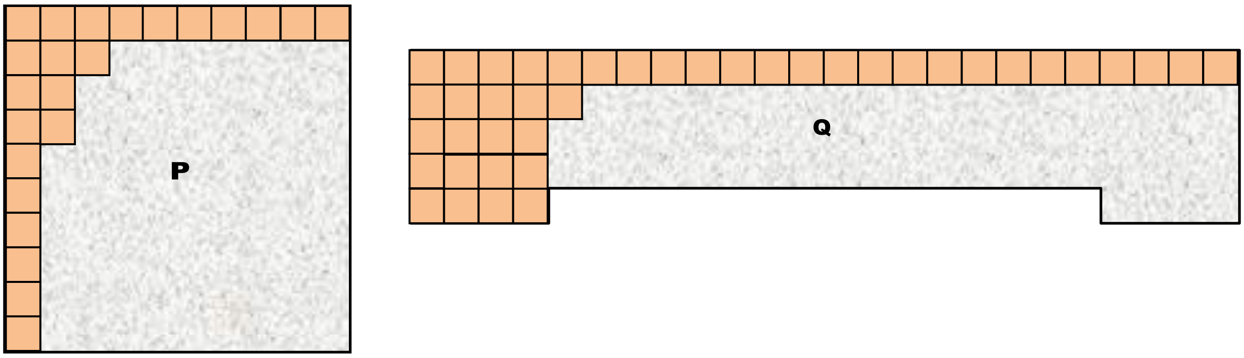
5. Ubahlah operasi pnejumlahan di bawah ini ke dalam bentuk operasi perkalian!

a. 6 + 6 + 6 = ……

b. 5 + 5 + 5 + 5 + 5 + 5 = ……

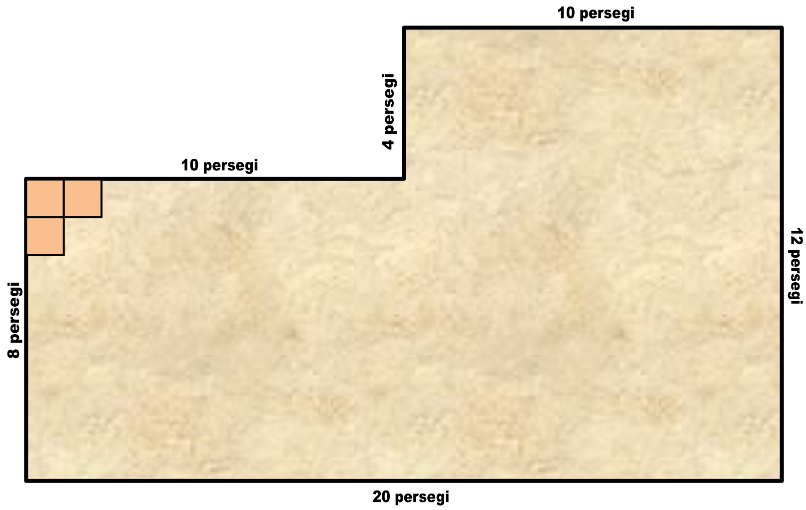
c. 10 + 10 + 10 + 10 = ……

6. Perhatikan kedua gambar lantai P dan Q yang belum selesai dikeramik di bawah ini. Lantai manakah yang lebih luas? Berikan alasanmu!



Tulis perhitungan, jawaban dan alasanmu di sini!

7. Tentukan luas permukaan lantai di bawah ini!

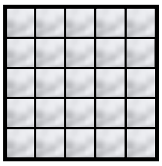


Tulis perhitungan dan jawabanmu di sini!

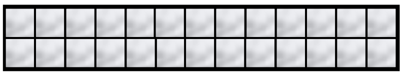
APPENDIX D

POSTTEST FOR THE SECOND CYCLE

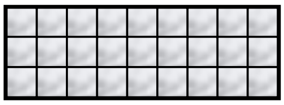
1. Perhatikan ketiga gambar di bawah ini. Urutkan ketiga gambar tersebut dari yang terluas ke yang tersempit! Berikan alasanmu!



A



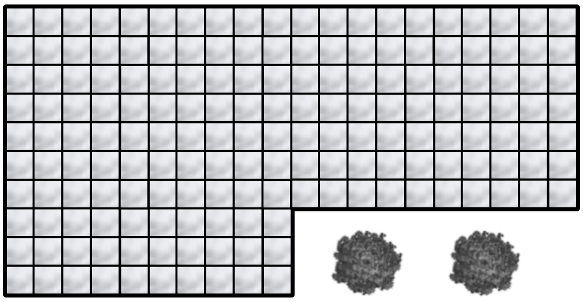
B



C

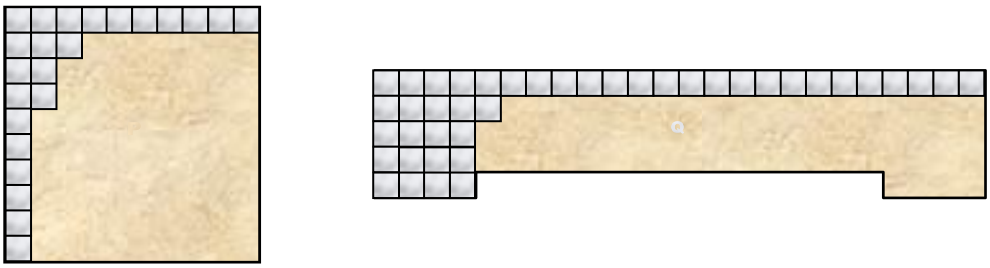
Tulis jawaban dan alasanmu di sini:

2. Perhatikan gambar lantai berkeramik di bawah ini.
Tentukan luas lantai tersebut!



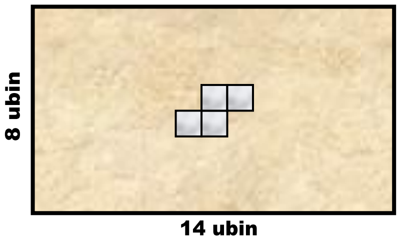
Tulis perhitungan dan jawabanmu di sini!

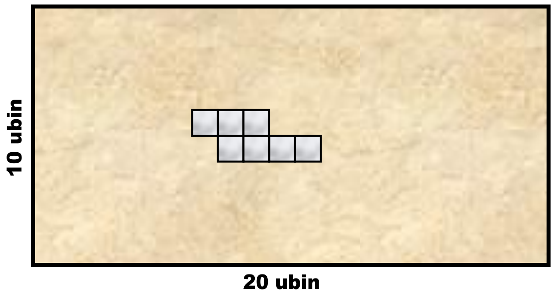
3. Perhatikan kedua gambar lantai P dan Q yang belum selesai dikeramik di bawah ini. Lantai manakah yang lebih luas? Berikan alasanmu!



Tulis jawaban dan alasanmu di sini!

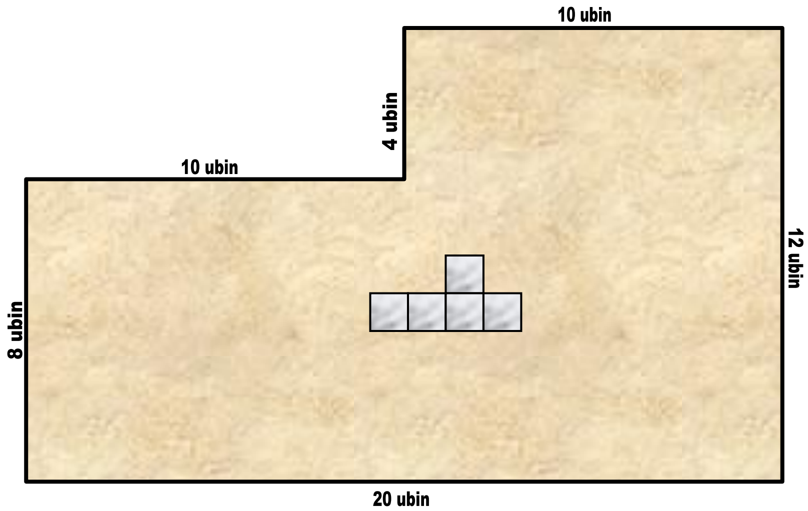
1. Tentukan luas lantai-lantai dibawah ini!





Tulis perhitungan dan jawabanmu di sini!

4. Tentukan luas lantai di bawah ini!



Tulis perhitungan dan jawabanmu di sini!

APPENDIX E

TEACHER GUIDE FOR THE SECOND CYCLE

RENCANA PEMBELAJARAN
PERTEMUAN 1
Kelas 3 Sekolah Dasar

A. Tujuan Pembelajaran

Memperkenalkan siswa pada berbagai satuan pengukuran luas (satuan non standard) dan menggunakannya untuk mengukur luas permukaan bidang datar.

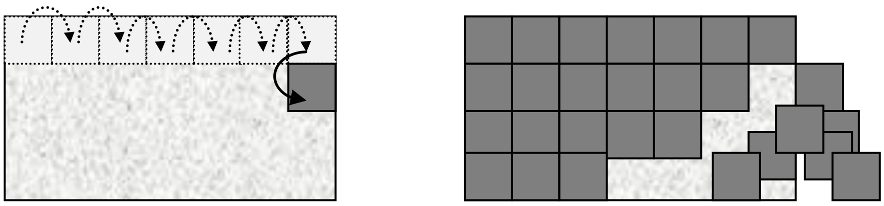
B. Teori Penunjang untuk Guru

Satuan Pengkuran Luas

Luas suatu permukaan adalah ***angka yang menunjukkan banyaknya satuan*** pengukuran luas yang menutupi permukaan tersebut. Karena luas suatu permukaan adalah bidang dua dimensi, maka ***satuan*** dalam pengukuran luas juga merupakan suatu bidang dua dimensi, seperti persegi atau persegi panjang**. Jadi, mengukur luas suatu permukaan berbarti *menentukan (menghitung)* banyak satuan yang menutupi permukaan tersebut.**

Cara Menentukan Banyak Satuan

Menentukan banyak satuan yang dibutuhkan untuk menutupi permukaan dapat dilakukan dengan dua cara. Pertama, dengan cara memindah-mindahkan satu satuan yang telah ditentukan ke seluruh permukaan yang diukur, sehingga setiap kali berpindah lokasi satuan tersebut tidak menumpuk atau berjarak dengan lokasi sebelumnya (lihat gambar 1a). Kedua, menggunakan satuan pengukuran sebanyak yang dibutuhkan untuk menutupi seluruh permukaan yang diukur (lihat gambar 1b).



(a) (b)

Gambar 1
Dua cara dalam menentukan banyak satuan luas yang menutupi suatu permukaan

Ketentuan Satuan

Dalam mengukur luas satu permukaan, satuan yang digunakan dan proses pengukuran harus memenuhi ketentuan sebagai berikut:

- Setiap satuan yang digunakan untuk mengukur luas suatu permukaan harus memiliki ukuran dan bentuk yang sama ***(keseragaman satuan)***.
- Ketika menempatkan satuan di atas permukaan yang diukur harus tidak ada yang saling tumpuk atau berjarak, dalam kata lain harus berhimpit satu sama lain.
- Satuan yang digunakan untuk mengukur luas suatu permukaan tidak boleh menutupi daerah yang bukan daerah permukaan yang diukur.

C. Perangkat

- Lembar Kerja 1

D. Jadwal Kegiatan

- Kegiatan Awal	: 5 menit
- Kegiatan Inti 1	: 18 menit
- Kegiatan Inti 2	: 16 menit
- Kegiatan Inti 3	: 14 menit

- Refleksi Kegiatan Inti : 4 menit
- Kegiatan Akhir : 7 menit
- Kesimpulan dan Penutup : 3 menit
- Waktu tersisa : 2 menit

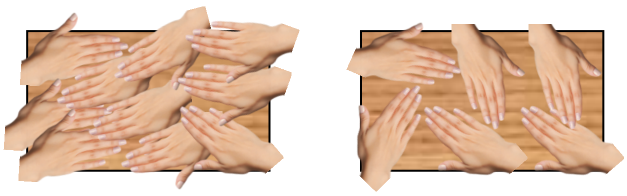
E. Kegiatan Inti

Kegiatan Inti 1: Mengukur luas meja belajar dengan telapak tangan sebagai satuan

- Tujuan dari kegiatan ini adalah untuk memperkenalkan konsep dasar pengukuran luas dengan alat ukur non standard, seperti telapak tangan.
- Untuk memulai kegiatan ini, mintalah setiap siswa untuk menentukan banyak telapak tangan yang dibutuhkan untuk menutupi meja belajar mereka masing-masing.
- Dalam menentukan banyak telapak tangan yang dibutuhkan, kemungkinan siswa melakukan tindakan penaksiran, misalnya dengan mengatakan ‘di sini (permukaan meja) ada yang belum di isi tapi satu tangan saya kebesaran, makanya saya hitung setengah tangan’, atau ‘… hampir 10 telapak tangan, karena ini banyak bagiah-bagian yang gak kerisi, saya hitung aja satu telapak tangan’ atau atau ‘…. Banyak telapak tangan saya di sini sekitar 10 setengah telapak tangan’. Terimalah jawaban siswa yang menggunakan penaksiran seperti ini karena kemampuan menaksirkan ini penting untuk dikembangkan oleh siswa dalam pengukuran.
- Tanyakan hasil temuan mereka. Jawaban mereka mungkin akan berbeda-beda. Ada beberapa faktor yang mempengaruhi perbedaan tersebut:
  1. Karena perbedaan ukuran telapak tangan (lihat gambar a).
  2. Karena adanya perbedaan kerapatan tangan dimana ketika penempatkan telapak tangan di atas meja dilakukan dengan tidak cermat sehingga ‘jejak’ telapak tangan mereka saling tumpuk atau berjarak-jarak (lihat gambar b).
  3. Pengukuran yang kurang tepat dimana telapak tangan mereka mengukur area di luar bidang yang diukur atau ada bagian bidang yang tak terukur (lihat gambar c).
  4. Karena meja yang diukur memang memiliki perbedaan ukuran (lihat gambar d).

Gambar a

Gambar b



Gambar c

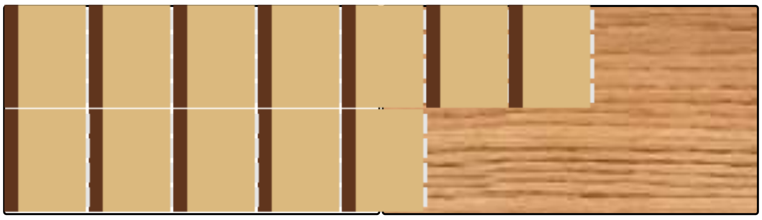
Gambar d

- Pilihlah dua jawaban yang siswa yang paling mencolok perbedaannya. Diskusikan bersama siswa lainnya mengapa perbedaan itu terjadi, apakah karena salah satu atau dua faktor tersebut di atas.

- Untuk memancing respon siswa, guru dapat menanyakan pertanyaan-pertanyaan berikut: mengapa kedua pengukuran tersebut berbeda? Lalu, pengukuran yang manakah dari kedua pengukuran tersebut yang benar? Bagaiamana cara membuktikannya?
- Diskusikan respon siswa terhadapa pertanyaan di atas.
- Jika guru menemukan bahwa perbedaan tersebut diakibatkan karena faktor-faktor tersebut di atas namun siswa tidak menyadari hal itu, maka guru dapat langsung mengarahkan diskusi siswa pada faktor-faktor tersebut.
- Sebelum masuk ke kegiatan selanjutnya, jelaskan kepada siswa bahwa tadi mereka telah melakukan pengukuran luas meja mereka menggunakan telapak tangan sebagai alat ukur.

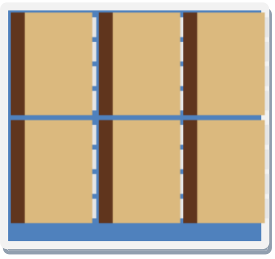
Kegiatan Inti 2: Mengukur luas beberapa permukaan benda-benda di kelas

- Tujuan dari kegiatan ini adalah untuk memperkenalkan siswa satuan pengukuran luas lainnya, yaitu persegi panjang.
- Jelaskan kepada siswa bahwa mereka akan mengukur luas beberapa permukaan bidang benda-benda disekitar mereka tetapi mereka tidak lagi menggunakan telapak tangan sebagai alat ukur, melainkan menggunakan buku tulis atau paket mereka.
- Bagi siswa dalam beberapa kelompok yang berisikan 3-4 siswa.
- Pastikan setiap kelompok siswa menggunakan buku-buku yang memiliki ukuran yang sama sebagai satuan (patokan) pengukuran.
- Dengan menggunakan satuan buku tersebut sebagai alat ukur, mintalah masing-masing kelompok untuk mengukur luas salah satu dari permukaan berikut:
  1. Meja Guru
  2. Dua meja siswa yang digabungkan.
  3. Jendela kelas.
  4. Pintu kelas.
  5. Salah satu poster/gambar yang ada di kelas.
  6. Atau benda-benda lainnya yang dapat diukur luasnya.
- Jika kekurangan media untuk diukur, guru dapat mengambar bidang di papan tulis atau di atas lantai dan mintalah siswa untuk mengukur luas bidang tersebut.



Contoh mengukur luas dua meja siswa dengan buku sebagai satuan

- Ketika mengukur kemungkinan siswa akan menemukan situasi dimana pengukuran mereka tidak selalu bulat. Misalnya, dalam mengukur jendala kemungkinan siswa menemukan bahwa jendela tersebut seukuran dengan 8 buku ditambah sepertiga buku lagi. Maka, luasnya dapat ditulis 8 dan sepetiga buku. Jika siswa sulit melakukan hal ini, mintalah siswa untuk menaksirkannya, misalnya menjadi 8 setengah buku atau 8 ½ buku .



Situasi dimana siswa harus menaksir banyak buku yang seharusnya dapat menempati

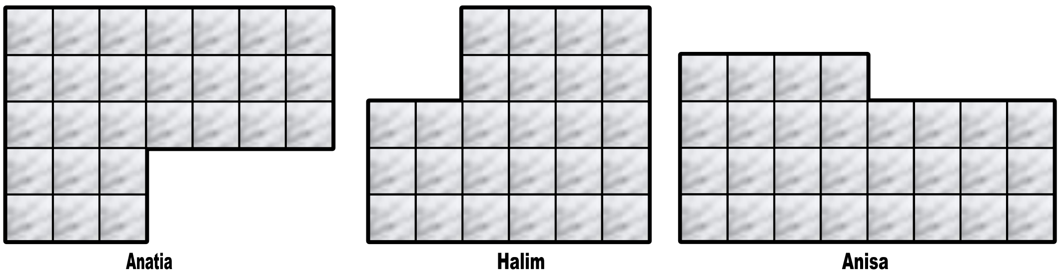
- Untuk merekam temuan siswa, gambar tabel berikut di papan dan himpun jawaban siswa dalam tabel tersebut.

No	Nama Kelompok	Nama Permukaan	Luas
1	Rani, Irman, Tika	Meja guru	12 buku
2	Sohib, Anatia, Della	Dua meja siswa	14 buku
3	Zidan, Dina, Najla	Jendela	8 ½ buku
4	Karim, Salim, Naila, Tio	Pintu	22 buku
5	Salma, Eko, Patrio	Poster/gambar	6 buku
6	Nadia, Sifa, Salim	Benda lainnya	14 buku
….	Dst.	Dst.	….

- Diskusikan temuan siswa tersebut. Misalnya, 'luas meja guru itu sama dengan luas 12 buku yang dijejerkan sehingga tidak ada buku yang saling bertumpukan atau berjarak', atau sebaliknya 'jika 12 buku dijejerkan dimana tidak ada buku yang saling tumpuk atau berjarak, maka jejeran buku tersebut luasnya akan sama dengan luas meja guru'.
- Demonstrasikan (tunjukkan) jejeran buku tersebut sesekali kepada siswa agar konsep tersebut lebih bermakna bagi siswa.
- Diskusikan pula mengenai perbandingan luas permukaan. Misalnya, 'luas permukaan pintu adalah paling luas dibandingkan dengan luas permukaan bidang lainnya, dimana luas pintu tersebut sama dengan 22 jejeran buku', atau 'luas permukaan dua meja siswa lebih luas dibandingkan dengan meja guru, dimana luas dua meja siswa sama dengan 14 jejeran buku, sedangkan meja guru hanya 12 jejeran buku'.

Refleksi Kegiatan Inti
- Mintalah siswa untuk mengingat kembali apa saja yang telah mereka lakukan pada kegiatan sebelumnya, yaitu 'mengukur luas permukaan beberapa benda', seperti mengukur luas meja dengan telapak tangan, mengukur luas benda-benda di kelas dengan buku, dan mengukur luas kelas dengan ubin.
- Jelaskan bahwa telapak tangan, buku, dan ubin yang mereka gunakan untuk mengukur tersebut dinamakan 'satuan pengukuran'. Misalnya, satuan pengukuran yang digunakan untuk mengukur meja mereka disebut satuan telapak tangan, untuk mengukur benda-benda di kelas mereka disebut satuan buku, dan untuk mengukur lantai kelas mereka disebut satuan ubin.
- Khusus untuk buku dan ubin, karena mereka berebentuk persegi panjang dan persegi, maka satuan buku dapat disebut juga dengan satuan persegi panjang sedangkan satuan ubin disebut dengan satuan persegi

G. Kegiatan Akhir
- Untuk menguatkan pemahaman siswa mengenai konsep di pertemuan ini minta siswa untuk bekerja dalam kelompok menyelesaikan LK1 jika ada waktu yang tersisa. Jelaskan kepada mereka bahwa pada LK1 terdapat gambar 3 teras masing-masing di rumah Anisa, Halim dan Anisa. Ketiga teras tersebut telah dipasangkan ubin dengan ukuran ubin yang sama. Mintalah siswa untuk mengurutkan ketiga teras tersebut dari yang terluas ke yang tersempit beserta alasannya.



Gambar 2
Lembar Kerja (LK) 1: Teras di tiga rumah berbeda.
Siswa diminta untuk mengurutkan ketiga teras tersebut berdasarkan luasnya

- Diskusikan jawaban siswa bersama-sama.

- Untuk menyelesaikan masalah ini kemungkinan siswa akan menentukan luas ketiga teras tersebut berdasarkan jumlah ubin (satuan persegi) yang dimuat oleh masing-masing teras.
- Jadi, teras yang terluas adalah teras di rumah Anisa karena luasnya ***28 satuan persegi***, kemudian teras di rumah Antia dengan luas ***27 satuan persegi***, dan yang tersempit adalah teras di rumah Halim dengan luas ***26 satuan persegi***.
- Jika tidak ada waktu, jadikan LK1 ini sebagai pekerjaan rumah yang akan dikumpulkan dipertemuan berikutnya.

H. Kesimpulan dan Penutup

- Simpulkan bahwa hari ini mereka telah belajar mengukur luas permukaan benda-benda yang ada disekitar mereka, seperti meja, jendel, pintu, lantai kelas, dst.
- Jika LK1 dikerjakan siswa di kelas, maka sebagai pekerjaan rumah mintalah mereka untuk mengukur luas ruang tamu mereka di rumah baik menggunakan patokan ubin atau buku atau benda-benda lainnya. Akan tetapi jika LK1 dijadikan sebagai pekerjaan rumah, maka tugas ini ditiadakan.
- Tutup kegiatan pembelajaran.

RENCANA PEMBELAJARAN
PERTEMUAN 2
Kelas 3 Sekolah Dasar

A. Tujuan Pembelajaran

Siswa dapat menemukan struktur susunan satuan luas dalam kolom dan baris dan menggunakan struktur tersebut dalam menentukan luas permukaan bangun datar.

B. Perangkat

- Cetak A4 Gambar 1.
- LK1 dan LK2 untuk masing-masing kelompok

D. Jadwal Kegiatan

- Kegiatan Awal : 5 menit
- Kegiatan Inti (Kerja Kelompok) : 20 menit
- Kegiatan Inti (Konfrensi Matematika Siswa) : 25 menit
- Kegiatan Inti (Refleksi Kegiatan Inti) : 5 menit
- Kegiatan Akhir : 10 menit
- Kesimpulan dan Penutup : 3 menit
- Waktu tersiswa : 2 menit

E. Kegiatan Awal

- Review pembelajaran sebelumnya. Untuk melakukan review ini, siswa dapat diajak untuk menelaah kembali masalah pada LK1 di pertemuan sebelumnya, dimana mereka diminta untuk mengurutkan luas 3 lantai. Review ini dapat difokuskan pada bagaimana ubin (satuan persegi) dijadikan patokan luas ketiga teras tersebut.

F. Kegiatan Inti

Membandingkan dua luas lahan parkir
Memperkenalkan konteks dan masalah

- Sebelum memelui kegiatan ini, stimulus siswa untuk berpikir mengenai luas dengan teknik tanya jawab, contohnya seperti berikut:
  Guru : Ada yang suka main di taman (seperti di taman makam Bungkul)?
  Siswa : Saya buuk!!!
  Guru : Suka main di taman yang kecil (atau sempit) atau yang besar (atau luas)?
  Siswa : yang besar buuuk!!!
  Guru : mengapa?
  Siswa : Kita bisa lari-lari dan loncat-loncatan buk…

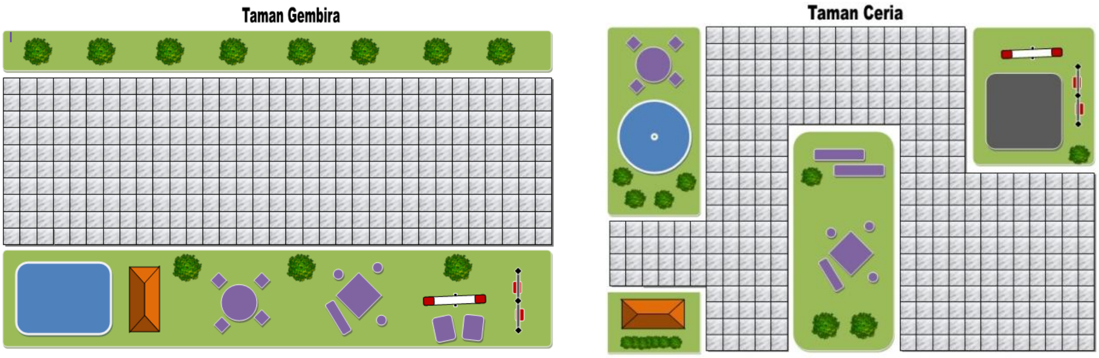
- Kemudian tunjukkan kepada siswa gambar taman yang ada di gambar 1. Jelaskan kepada mereka area berpaving di kedua taman bermain tersebut dan melalui taman tersebut perkenalkan masalah yang ingin siswa selesaikan.
  Guru : Naaah ini ibu punya gambar dua taman di pusat kota.
  Namanya Taman Gembira dan Taman Ceria.
  Coba perhatikan lahan parkir (area berpaving) di kedua taman ini!
  Apakah bentuknya sama?
  Siswa : Yang mana buk!!
  Guru : Yang ini nak.. (guru menunjuk area berpaving di kedua taman)
  Siswa : Ooooh bentuknya beda buuk..
  Guru : Ada yang mau menjelaskan perbedaannya gak?
  Siswa : Saya buk!!! Di taman Gembira bentuknya seperti persegi panjang.
  Kalo di taman Ceria bentuknya memanjang dan gak beraturan, berkelok kelok gitu buk.
  Guru : Kira-kira lahan parkir yang mana yang lebih luas?
  Siswa : Yang berbentuk persegi panjang buk..!!

Siswa : Bukaaan, yang berkelok-kelok itu buk..!!
Guru : Baiklah coba kalian diskusikan bersama dalam kelompok dan tentukan lahan parkir manakah yang lebih luas. Jangan lupa memberikan alasan dari jawabanmu.

- Ingatkan mereka bahwa akan ada Konfrensi Matematika Siswa dimana dua atau tiga kelompok dari mereka diminta untuk mempresentasikan jawaban mereka beserta alasannya di depan kelas.

  Guru : Ibu kasi waktu 20 menit untuk berdiskusi dalam kelompok.
  Setelah itu, ibu minta dua atau tiga kelompok dari kalian untuk menunjukkan hasil diskusinya ke teman-temannya yang lain di depan kelas.
  Jangan lupa jelaskan bagaiaman kamu mendapatkan jawabanmu beserta alasannya.

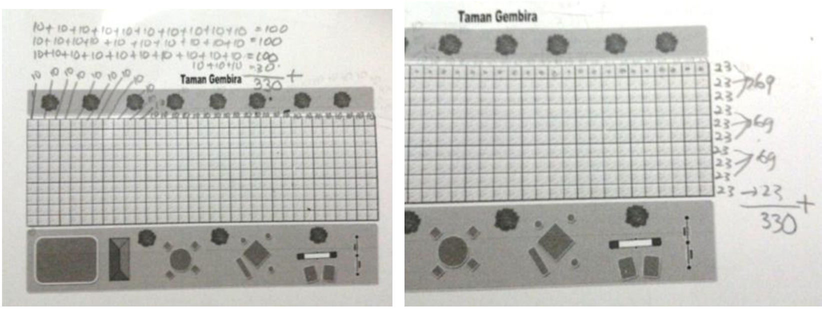
- Selama siswa bekerja dalam kelompok, guru memperhatikan solusi yang digunakan siswa dan pikirkan bagaimana konfrensi matematika siswa akan dilakukan, kelompok manakah yang akan dipilih untuk presentasi, dan bagamaimana diskusi siswa akan di arahkan untuk mencapai tujuan pembelajaran pada pertemuan ini.
- Minta siswa untuk menulis perhitungan mereka pada lembar LK1 atau di lahan kosong yang tersedia di LK1.
- Berikan bantuan jika siswa masih bingung terhadap kegiatan ini.



Gambar 1
Dua taman sebagai konteks pembelajaran siswa. Siswa diminta untuk menentukan lahan parkir (area berpaving) manakah di kedua taman tersebut yang lebih luas.

Kemungkinan Jawaban Siswa

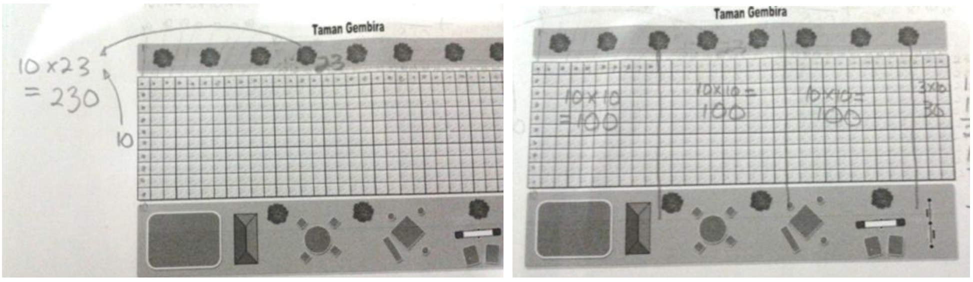
- Berdasarkan pengalaman belajar pada pertemuan sebelumnya, untuk menentukan luas lahan parkir tersebut siswa akan menghitung jumlah paving persegi yang ada di setiap area berpaving tersebut. Mereka menjadikan paving tersebut sebagai satuan pengukuran luas. Dalam menghitung banyak satuan tersebut, siswa kemungkinan akan menggunakan berbagai strategi, diantaranya:
- Strategi yang pertama adalah siswa menghitung paving satuan tersebut dengan cara ***penjumlahan baris per baris atau kolom per kolom***. Misalnya di Taman Gembira, siswa menemukan bahwa setiap kolom dari lahan parkir tersebut ditutupi oleh 10 paving. Karena terdapat 23 kolom, siswa kemudian melakukan penjumlahan sepuluh sebanyak 23 kali, contohnya 10 + 10 + 10 + … dts. Penjumlahan baris per baris juga mungkin dilakukan. Misalnya, karena terdapat 10 baris satuan paving di Taman Gembira dimana setiap baris berisi 23 paving, siswa akan melakukan penjumlahan dua-puluh-tiga sebanyak 10 kali, conothnya 23 + 23 + 23 + … dst. Jika siswa melakukan hal ini berarti siswa telah menemukan struktur satuan dalam kolom atau baris. Ini artinya siswa telah mencapai tujuan pelajaran yang diharapkan pada pertemuan ini. Jika cara ini muncul di salah satu kelompok, maka pilihlah kelompok ini untuk mempresentasikan strategi mereka dan jadikanlah bahan diskusi di Konfrensi Matematika Siswa sebagai modal untuk menjelaskan mengenai struktur satuan tersebut kepada siswa lainnya.



(a) (b)

Gambar 2
Strategi penjumlahan kolom per kolom (a) dan baris per baris (b) dalam menghitung banyak paving satuan yeng menutupi lahan parkir

- Strategi kedua, siswa menghitung banyak paving satuan tersebut dengan ***strategi perkalian baris dan kolom***. Misalnya di Taman Gembira, terdapat 10 baris dan 23 kolom paving. Maka banyak seluruh paving tersebut adalah 10 x 23 atau 23 x 10 yaitu sama dengan 230 paving. Siswa yang menggunakan strategi ini tidak hanya sudah menemukan struktur kolom dan baris dari dari satuan tersebut tetapi juga memahami struktur perkalian dari susunan satuan tersebut. Bisa dikatakan bahwa siswa yang menggunakan strategi ini berada satu level di atas siswa yang menggunakan strategi penjumlahan kolom per kolom atau baris per baris. Jika cara ini muncul di salah satu kelompok, maka pilihlah kelompok ini untuk mempresentasikan strategi mereka dan jadikanlah bahan diskusi di Konfrensi Matematika Siswa sebagai modal untuk menjelaskan mengenai struktur satuan dalam kolom dan baris kepada siswa lainnya, karena strategi perkalian ini sebenarnya didasarkan pada struktur satuan dalam baris dan kolom, yaitu pernjumlahan berulang, misalnya 23 + 23 + 23 + 23 + 23 + 23 + 23 + 23 + 23 + 23 = 10 x 23.



Gambar 3
Strategi perkalian baris dan kolom dalam menghitung banyak paving satuan yeng menutupi lahan parkir

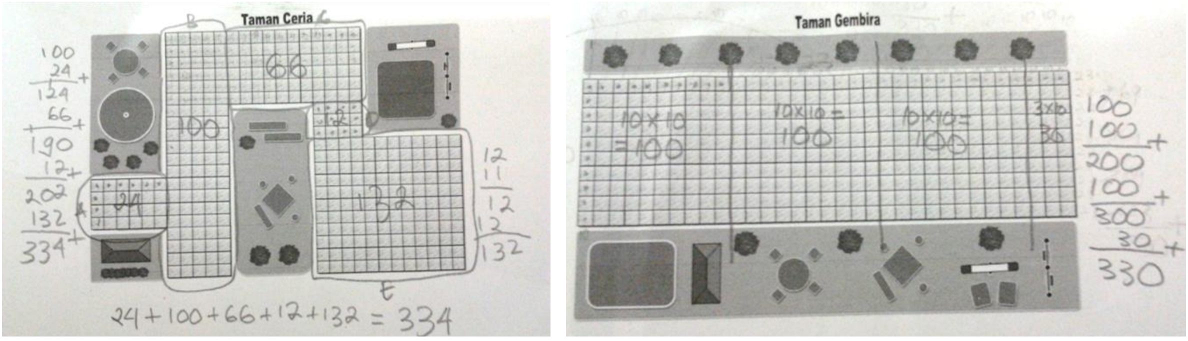
- Strategi lainnya yang mungkin digunakan siswa dalam menghitung paving-paving satuan tersebut adalah dengan cara ***menghitung satu per satu***, misalnya: 1, 2, 3, 4, 5, dst. Seringkali siswa akan mencapai hasil akhir yang kurang tepat karena kemungkinan kesalahan dalam menghitung dengan teknik ini sangat besar mengingat paving-paving yang dihitung berukuran kecil, berhimpitan satu sama lain, dan jumlahnya cukup banyak.

Gambar 4
Strategi menghitung satu per satu dalam menentukan banyak paving satuan yeng menutupi lahan parkir

- Dimungkinkan juga bahwa siswa akan menggunakan campuran dari strategi-strategi yang telah dijelaskan di atas, misalnya ketika siswa menghitung paving tersebut dengan cara satu per satu mereka menemukan struktur paving tersebut dalam baris atau kolom sehingga mereka mengubah strategi penghitungan mereka dengan menggunakan strategi penjumlahan baris per baris atau kolom per kolom.

Membagi-bagi (Splitting)

- Untuk memudahkan siswa dalam proses perhitungan paving satuan tersebut, kemungkinan siswa akan ***membagi-bagi (splitting)*** lahan parkir tersebut ke dalam bebebarapa bagian dan kemudian menghitungnya. Teknik splitting ini kemungkinan akan muncul ketika siswa menyelesaikan masalah perhitungan paving di Taman Ceria. Jika cara ini muncul di salah satu kelompok, maka pilihlah kelompok ini untuk mempresentasikan strategi mereka dan jadikanlah bahan diskusi di Konfrensi Matematika Siswa mengenai teknik splitting dalam memudahkan proses penghitungan satuan.



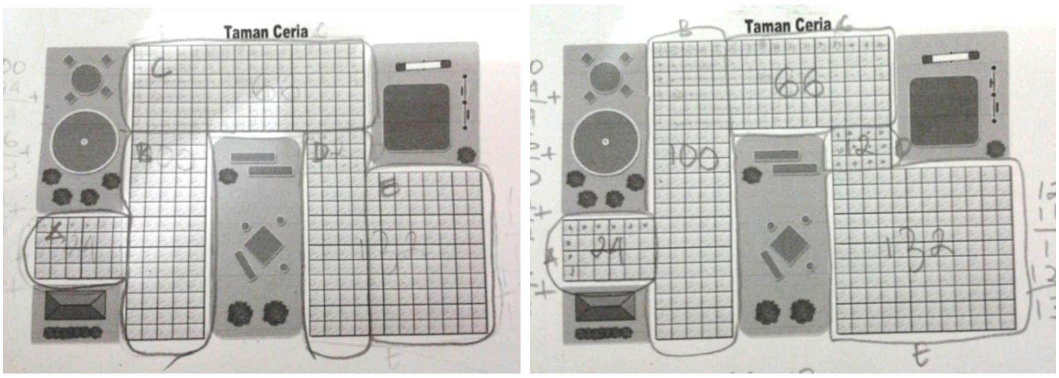
Gambar 5
Teknik splitting: Teknik ini memudahkan dalam menghitung banyak paving satuan

Konfrensi Matematika Siswa

- Konfrensi ini bertujuan sebagai media sharing pengetahuan dan pemahaman antar siswa, dimana solusi siswa yang dipresentasikan dapat dipelajari, dipertanyakan, atau dikembangkan oleh siswa lainnya dalam proses diskusi bersama.
- Dalam konfrensi ini, siswa dipandu oleh guru bersama-sama membangun pemahaman mereka terhadap konsep yang sedang mereka pelajari. Sehingga dalam konfrensi ini, guru tidak hanya bertindak sebagai fasilitator selama konfrensi berlangsung, melainkan juga sebagai pemandu arah diskusi siswa (Apa yang harus didiksuiskan? Siapa? Bagaimana? Kapan?) untuk mencapai tujuan pembelajaran yang telah disepakati.
- Untuk memulai konfrensi, pilih dua atau tiga kelompok untuk mempresentasikan solusi yang mereka temukan di depan siswa lainnya. Pemilihan kelompok ini dapat didasarkan pada pertimbangan-pertimbangan, seperti: (1) solusi kelompok tersebut dapat dijadikan acuan (modal awal) untuk menanamkan atau menjelaskan konsep yang guru ingin siswa ketahui, (2) kelompok tersebut memiliki cara yang berbeda-beda sehingga menarik untuk dibandingkan dalam diskusi bersama, (3) kelompok tersebut memiliki solusi yang kurang tepat atau merupakan kesalahan umum yang dilakuakn kebanyakan siswa sehingga menarik untuk didiskusikan untuk mendapatkan solusi yang tepat, dan berbagai pertimbangan lainnya.
- Setelah siswa mempresentasikan solusi mereka, diskusikan solusi tersebut bersama seluruh siswa. Fasilitasi siswa-siswa yang bertanya, menambahkan, atau menyanggah dan jadikan semua itu sebagai bahan diskusi.
- Sebagai panduan guru bahwa Taman Ceria memiliki lahan parkir yang lebih luas daripada Taman Gembira, dengan perbandingan 334 paving satuan di Taman Ceria dan 330 paving satuan di Taman Gembira. Karena paving tersebut berbentuk persegi, maka guru dapat menjelaskan bahwa luas lahan parkir di Taman Ceria adalah ***334 persegi*** sedangkan di Taman Gembira adalah ***330 persegi***.
- Pastikan salah satu kelompok yang mempresentasikan hasil kerjanya adalah kelompok yang menggunakan strategi penjumlahan kolom per kolom atau baris per beris. Gunakan solusi siswa ini sebagai modal untuk menjelaskan kepada siswa lainnya mengenai struktur satuan dalam baris dan kolom. Jika tidak ditemukan siswa menggunakan strategi tersebut, pilihlah salah satu solusi yang mendekati strategi tersebut atau solusi yang dapat diarahkan ke strategi tersebut.

Jika tidak ada juga solusi siswa yang dapat dijadikan modal untuk menjelaskan struktur tersebut, maka guru dapat mengusulkan strategi penjumalah kolom per kolom atau baris per baris dalam menghitung jumlah paving satuan dan minta pendapat siswa terhadap strategi tersebut. Lalu, gunakan modal tersebut untuk mejelaskan strukur satuan luas dalam baris dan kolom.

- Jika strategi perkalian muncul, mintalah siswa untuk mempresentasikan strategi tersebut. Gunakan strategi ini sebagai modal untuk menjelaskan struktur baris dan kolom dari satuan luas, karena pada dasarnya strategi perkalian baris dan kolom tersebut di dasarkan pada struktur baris dan kolom. Mislanya, 10 x 23 pada kasus Taman Gembira. Hal ini dapat didiskusikan dengan siswa dari mana angka 10 dan 23 itu datang, apakah jumlah paving di setiap kolom dan baris selalu sama, dan seterusnya.
- Berikan perhatian khsus pada siswa yang menggunakan teknik ***splitting***. Ajak siswa berdiskusi mengapa teknik splitting ini sangat membantu untuk memudahkan dalam perhitungan banyak satuan. Beberapa manfaat dari teknik splitting di anataranya: (1) membantu siswa menyederhanakan situasi bangun datar yang dihitung luasnnya, (2) karena siswa bekerja dengan bagian-bagian kecil dari bangun, kemungkinan siswa mengalami kesalahan penghitungan akan semakin kecil, (3) memungkinkan siswa untuk menerapkan strategi lainnya dalam penghitungan satuan, seperti strategi perkalian (lihat gambar 5 dan 6b). Perlu diketahui bahwa teknik splitting ini dapat dilakukan dengan berbagai cara meskipun pada bangun yang sama. Sehingga akan ada kemungkinan berbagai teknik spliiting yang dilakukan siswa pada bangun yang sama (lihat gambar 6).



(a) (b)

Gambar 6

Dua contoh cara teknik splitting pada bangun datar yang sama

- Bagi siswa yang menggunakan strategi penghitungan satu per satu, ajak mereka berdiskusi, misalnya dengan menanyakan strategi lainnya yang dapat dilakukan untuk menghitung paving tersebut dengan lebih efficient. Memberikan kesempatan kepada siswa untuk memikirkan strategi lainnya memungkinkan siswa untuk datang dengan salah satu strategi lainnya seperti strategi penjumlahan baris per baris atau strategi perkalian. Guru sebaiknya jangan memberitahukan langsung kepada siswa mengenai kedua strategi tersebut, tapi berikan kepada siswa untuk menemukannya. Tapi, jika sudah tidak memungkinkan maka guru dapat menunjukkan bantuan agar siswa menemukan strategi tersebut.

Refleksi Kegiatan Inti

- Sebelum masuk ke kegiatan selanjutnya, ajaklah siswa untuk mengingat kembali bagaimana mereka menentukan luas kedua lahan parkir, yaitu bagaimana struktur satuan dalam baris dan kolom memudahkan mereka dalam menentukan luas lahan perkir tersebut.

Guru : Seperti yang kita tahu bahwa mengukur luas permukaan berarti menghitung banyak satuan yang ada di permukaan tersebut. Ada yang masih ingat bagaimana cara si Fulan menghitung paving satuan pada lahan parkir?

Siswa : Saya buk… Si Fulan menjumlahkan paving-paving tersebut kolom per kolom

Siswa : Terkadang juga dia melakukannya dengan menjumlahkan paving-paving itu baris per baris.

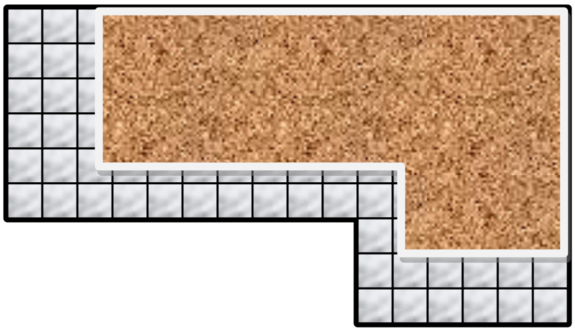
Guru : Iyya betul…

Coba perhatikan paving satuan di taman Gembira! Apakah jumlah paving satuan di setiap kolom satuan ini selalu sama?

Siswa : Iyya buuk..
Guru : Kalau dalam setiap baris?
Siswa : sama juga buk…
Guru : berapa?
Siswa : di setiap baris ada 23 paving satuan buk.

G. Kegiatan Akhir

- Untuk menguatkan pemahaman siswa mengenai konsep struktur satuan dalam kolom dan baris di pertemuan ini, minta siswa dalam kelompok menyelesaikan LK2 jika ada waktu yang tersisa.
- Jelaskan kepada mereka bahwa gambar di LK2 menunjukkan gambar sebuah lantai berubin yang di tutupi oleh sebuah karpet. Mintalah siswa untuk menentukan luas lantai tersebut. Dalam hal ini, keremik persegi dijadikan sebagai satuan pengukuran.
- Minta siswa untuk menulis perhitungan mereka dimanapun di lembar LK2 atau di lahan kosong yang tersedia di LK2.



Gambar 8
Lembar Kerja (LK) 2: Sebuah lantai yang ditutupi oleh karpet.
Siswa diminta untuk menentukan luas permukaan lantai tersebut.

- Jika siswa telah memahami struktur satuan dalam baris dan kolom, maka kemungkinan siswa akan memperhatikan struktur satuan ubin tersebut dalam baris atau kolom meskipun tertutupi karpet.
- Ada kemungkinan siswa akan membagi lantai tersebut menjadi beberapa bagian. Pembagian ini dapat dilakukan dengan berbagai cara, salah satunya membaginya menjadi lantai berukuran 6 x 10 persegi dan 6 x 9 persegi. Kemudian dengan menerapkan strategi penjumlahan baris per baris, atau kolom per kolom, atau strategi perkalian, siswa menemukan luas masing-masing bagian. Luas keseluruhan adalah jumlah dari luas bagian-bagian tersebut.
- Luas tersat tersebut adalah 60 ubin persegi + 54 ubin persegi = 114 ubin persegi.
- Jika tidak ada waktu, jadikan LK2 ini sebagai pekerjaan rumah yang akan dikumpulkan dipertemuan berikutnya.

H. Kesimpulan dan Penutup

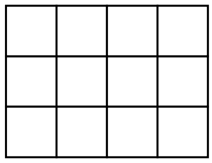
- Sebelum menutup kegiatan pembelajaran, guru mengingatkan kembali mengenai struktur satuan dalam baris dan kolom, serta bagaimana struktur tersebut membantu dalam menentukan luas permukaan bangun datar.
- Jika LK2 dijadikan sebagai tugas rumah, ingatkan siswa untuk mengumpulkannya dipertemuan selanjutnya.
- Guru menutup kegiatan pembelajaran.

RENCANA PEMBELAJARAN
PERTEMUAN 3
Kelas 3 Sekolah Dasar

A. Tujuan Pembelajaran

Siswa dapat menemukan struktur perkalian dalam susunan satuan luas.

Struktur perkalian dalam susunan satuan laus didasarkan pada struktur kolom dan baris dari satuan tersebut. Perhatikan gambar di bawah ini! Banyak satuan persegi pada gambar tersebut dapat di tulis dalam struktur kolom, misalnya 4+4+4, atau baris, misalnya 3+3+3+3. Dimana 4+4+4 dapat ditulis menjadi 3x4 dalam struktur perkalian dan 3+3+3+3 dapat ditulis 4x3. 3x4 dan 4x3 sama-sama menghasilkan 12. Jadi luas gambar tersebut adalah 12 satuan persegi.



B. Jadwal Kegiatan

- Kegiatan Awal : 5 menit
- Kegiatan Inti (Kerja Kelompok) : 25 menit
- Kegiatan Inti (Konfrensi Matematika Siswa) : 25 menit
- Kegiatan Akhir : 10 menit
- Kesimpulan dan Penutup : 5 menit

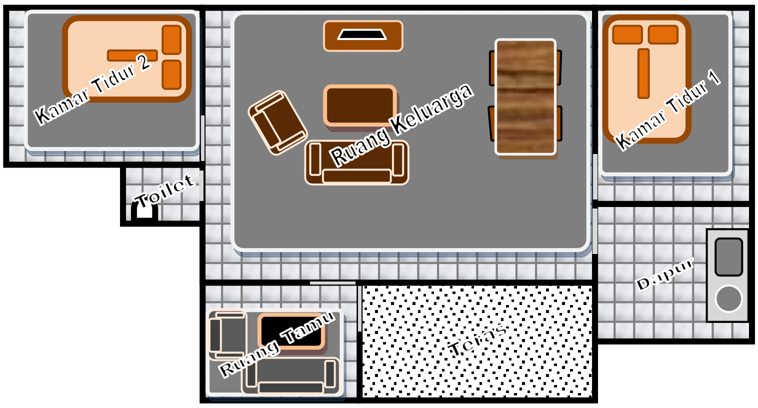
C. Kegiatan Awal

- Guru mereview LK2 pada pertemuan sebelumnya. Guru mengingatkan siswa mengenai struktur susunan satuan dalam baris dan kolom.

D. Kegiatan Inti

Kegiatan 1 : Menentukan Luas Permukaan Lantai
Sifat : Kerja kelompok (3-4 siswa per kelompok)

- **Untuk menstimulus siswa pada situasi belajar, lakukan tanya jawab bersama siswa mengenai ruangan-ruangan yang ada di rumah mereka.** Guru dapat menanyakan ruangang yang paling besar dan paling kecil ukurannya di rumah mereka.
- **Tunjukkan kepada siswa gambar denah sebuah rumah.** Jelaskan bagian-bagian dari rumah tersebut, yang meliputi teras, ruang tamu, ruang keluarga, toilet, dan dua buah kamar tidur.
- **Bagikan masing-masing kelompok LK1 dan mintalah mereka untuk berdiskusi dalam kelompok untuk menentukan luas tiap-tiap ruangan tersebut.**
- Minta siswa untuk **menulis jawaban dan perhitungan** mereka di lahan kosong yang tersedia pada LK tersebut.
- Ingatkan mereka bahwa akan ada **Konfrensi Matematika Siswa** dimana dua atau tiga kelompok dari mereka diminta untuk mempresentasikan jawaban mereka beserta alasannya di depan kelas.
- Selama siswa bekerja dalam kelompok, **guru memperhatikan solusi yang digunakan siswa** dan pikirkan bagaimana konfrensi matematika siswa akan dilakukan, kelompok manakah yang akan dipilih untuk presentasi, dan bagamaimana diskusi siswa akan di arahkan untuk mencapai tujuan pembelajaran pada pertemuan ini. Berikan bantuan jika siswa masih bingung terhadap kegiatan ini.



Gambar 1

Kegiatan 2 : Konfrensi Matematika Siswa
Sifat : Diskusi bersama seluruh siswa

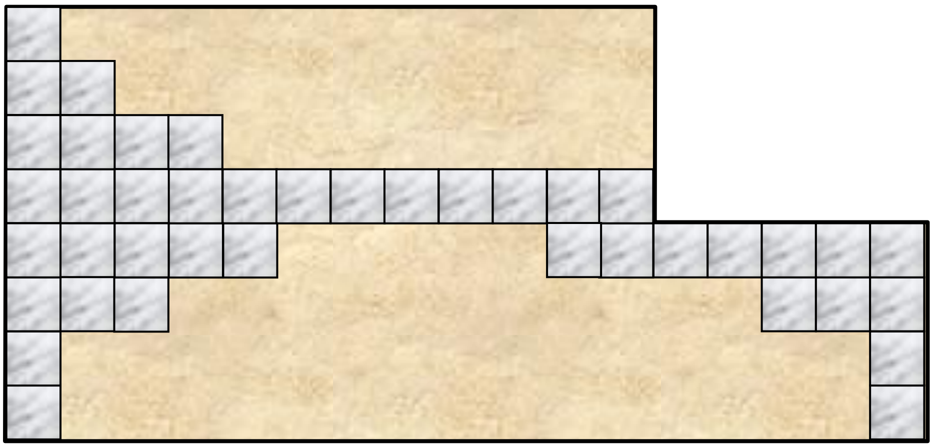
- Mintalah dua atau tiga kelompok siswa yang menggunakan **strategi penjumlahan baris per baris** (atau kolom per kolom) **untuk memaparkan jawaban mereka** di depan siswa lainnya.
- Minta juga kelompok siswa yang menggunakan **strategi perkalian** (jika ada) untuk memaparkan jawaban mereka di depan siswa lainnya.
- **Diskusikan** bersama siswa lainnya mengenai strategi yang digunakan oleh kelompok tersebut.
- **Gunukan solusi siswa tersebut untuk memperkenalkan struktur perkalian pada susunan satuan luas.** Hal ini dapat dilakukan dengan mendiskusikan solusi siswa yang menggunakan strategi penjumlahan baris per baris. Guru dapat menggunakan solusi tersebut sebagai modal awal untuk memperkenalkan siswa strategi perkalian. Sebagai contoh untuk siswa, dengan strategi penjumlahan baris per baris maka luas lantai toilet adalah 4 + 4 + 4 = 12 ubin persegi. Diskusikan bersama siswa bagaimana cara penulisan 4 + 4 + 4 dalam bentuk perkalian (4+4+4 = 3x4). Mintalah siswa untuk menemukan hubungan antara angka 3 dan 4 pada 3x4 pada susunan ubin pada gambar. Diharapkan siswa dapat memahami bahwa 3 adalah banyak baris satuan sedangkan 4 adalah banyak kolom satuan. Jadi, untuk menentukan luas toilet tersebut dapat juga ditentukan dengan cara mengalikan jumlah baris satuan dengan jumlah kolom satuan. Inilah yang disebut dengan strategi perkalian. Mintalah kepada siswa untuk mencoba menggunakan strategi tersebut untuk menentukan luas permukaan lantai lainnya. Sedangkan, bagi siswa yang telah menggunakan strategi perkalian, guru dapat menggunakan modal tersebut secara langsung untuk menjelaskannnya kepada siswa lainnya mengenai struktur perkalian pada susunan satuan luas.
- **Untuk kasus lantai teras** dimana tidak ditemukan informasi mengenai jumlah baris satuan dan kolom satuan, **diskusikan** bersama siswa untuk menemukan bahwa jumlah baris satuan dan kolom satuan dapat diperoleh dengan memperhatikan lantai yang berdekatan dengan teras tersebut, yaitu jumlah baris satuan pada lantai teras sama dengan baris satuan pada ruang tamu; sedangkan jumlah kolom satuan dapat diperolah dengan memperhatikan susunan satuan pada ruang keluarga.
- Ingatkan kepada siswa bahwa **strategi perkalian dapat digunakan untuk mengukur luas hanya pada bangun yang berbentuk persegi atau persegi panjang**.

E. Kegiatan Akhir

Kegiatan : Menentukan Luas Permukaan Lantai
Sifat : Kerja kelompok (3-4 siswa per kelompok)

- **Bagikan setiap kelompok LK2** dan jelaskan bahwa terdapat sebuah lantai yang belum seluruh permukaannya dipasangkan ubin persegi.

- Dengan menggunakan strategi perkalian, **mintalah siswa dalam kelompok untuk menentukan luas permukaan lantai tersebut.**



Gambar 2

F. Kesimpulan dan Penutup

- Sebelum menutup kegiatan pembelajaran, ajak siswa untuk mengingat kembali apa saja yang mereka telah lakukan (pelajari) dalam pertemuan ini.
- Guru mengingatkan kembali mengenai struktur perkalian pada susunan satuan luas.
- Guru menutup kegiatan pembelajaran.

RENCANA PEMBELAJARAN
PERTEMUAN 4
Kelas 3 Sekolah Dasar

A. Tujuan Pembelajaran

Siswa dapat memahami fungsi dimensi pada penggunaan strategi perkalian dalam menentukan luas permukaan bangun datar.

B. Teori Penunjang untuk Guru

C. Perangkat

- Gambar lantai pada LK1 untuk guru; dan LK1, LK2, dan LK3 untuk masing-masing kelompok

D. Jadwal Kegiatan

- Kegiatan Awal	: 10 menit
- Kegiatan Inti (Kerja Kelompok)	: 20 menit
- Kegiatan Inti (Konfrensi Matematika Siswa)	: 25 menit
- Kegiatan Inti (Refleksi Kegiatan Inti)	: 3 menit
- Kegiatan Akhir	: 10 menit
- Kesimpulan dan Penutup	: 2 menit
- Waktu tersisa	: 0 menit

E. Kegiatan Awal

Review

- Review secara singkat pembelajaran sebelumnya, mengenai struktur perkalian dari susunan satuan luas. Review ini dapat dilakukan dengan pembahasan mengenai LK2 pada pertemuan sebelumnya.

F. Kegiatan Inti

Mengukur Luas Permukaan Lantai

Memperkenalkan konteks dan Masalah

- Untuk menstimulus siswa mengenai konteks pembelajaran, ajaklah mereka betanya jawan mengenai lantai dan ubin.

  Guru: Coba kita lihat permukaan lantai kelas ini!
  Masih ingat dipertemuan sebelumnya bahwa kita telah mengukur luas lantai ini.
  Ada yang masih ingat bagaimana kita mengukur luasnya?
  Siswa: Saya ingat buk.
  Waktu itu kita menghitung banyak ubin-ubin yang ada di lantai ini buk.
  Siswa: Iyyaa betul… lantainya dibagi-bagi buk.
  Kita juga dibagi menjadi kelompok dan masing-masing kita mendapatkan bagian lantai
  Untuk dihitung luasnya.
  Guru: Betul…
  Naaah sekarang ibu memiliki sebuah gambar lantai seperti ini.

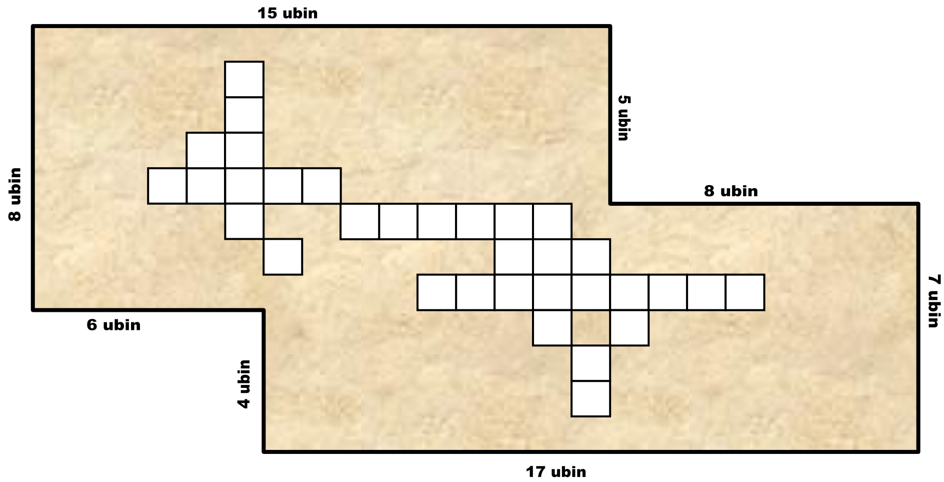
- Guru munjukkan kepada siswa gambar 4 dan jelaskan kepada mereka bahwa gambar itu adalah gambar permukaan sebuah lantai. Sebagian permukaan lantai tersebut telah dipasangkan ubin seperti terlihat pada gambar. Jelaskan kepada siswa maksud angka-angka (dimensi) pada gambar tersebut. Misalnya, 15 ubin pada sisi atas gambar tersebut menunjukkan bahwa di sisi tersebut membutuhkan 15 ubin. Kemudian guru mengajukan permasalahan pada lantai tersebut.

  Guru: Ibu kebingungan bagaimana cara mencari luas lantai ini.
  Dapatkah anak-anakku membantu ibu bagiamana menentukan luas permukaan lantai ini.
  Tapi sebelumnya, ibu ingin kalian berkerja dalam kelompok seperti kelompok kemarin.

- Ingatkan mereka bahwa akan ada Konfrensi Matematika Siswa dimana dua atau tiga kelompok dari mereka diminta untuk mempresentasikan jawaban mereka beserta alasannya di depan kelas.

Guru : Ibu kasi waktu 20 menit untuk berdiskusi dalam kelompok.
Setelah itu, ibu minta dua atau tiga kelompok dari kalian untuk menunjukkan hasil diskusinya ke teman-temannya yang lain di depan kelas.
Jangan lupa jelaskan bagaiamana kamu mendapatkan jawabanmu beserta alasannya.

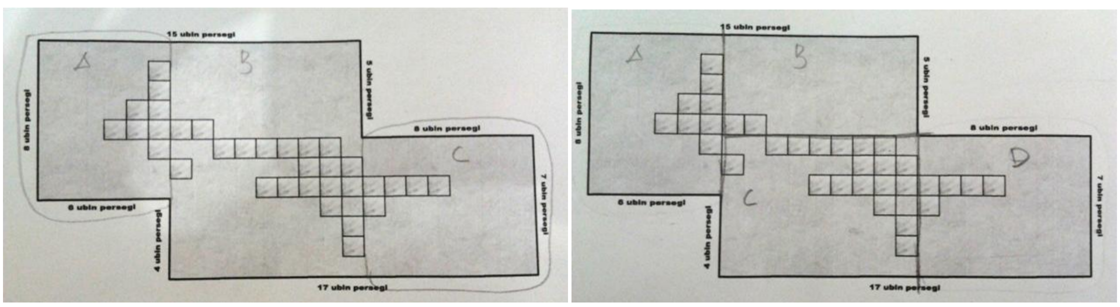
- Bagikan setiap kelompok LK2, dan minta siswa untuk menulis perhitungan mereka di lahan kosong yang tersedia pada LK tersebut.
- Selama siswa bekerja dalam kelompok, guru memperhatikan solusi yang digunakan siswa dan pikirkan bagaimana konfrensi matematika siswa akan dilakukan, kelompok manakah yang akan dipilih untuk presentasi, dan bagamaimana diskusi siswa akan di arahkan untuk mencapai tujuan pembelajaran pada pertemuan ini.
- Berikan bantuan jika siswa masih bingung terhadap kegiatan ini.



Gambar 4
Lembar Kerja (LK) 2: Sebuah permukaan lantai dimana siswa diminta untuk menentukan luas permukaan lantai ini.

Kemungkinan Jawaban Siswa

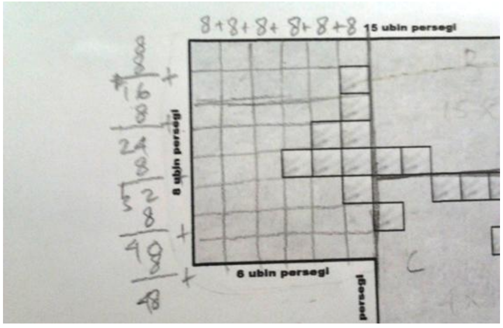
- Seperti yang diketahui sebelumnya bahwa mengukur luas suatu permukaan berarti menghitung banyak satuan yang menutupi permukaan tersebut.
  Splitting dan Gridding
- Sebelum siswa mulai menghitung satuan tersebut, diperkirakan siswa akan membagi (splitting) permukaan lantai tersebut dalam beberapa bagian untuk memudahkan mereka dalam proses penghitungan satuan. Untuk melakukan pembagian ini, ada banyak kemungkinan cara siswa membagi permukaan tersebut. Jika teknik membagi ini muncul di salah satu kelompok, maka pilihlah kelompok ini untuk mempresentasikan strategi mereka dan jadikanlah bahan diskusi di Konfrensi Matematika Siswa mengenai bagaimana teknik splitting memungkinkan siswa untuk menerapkan strategi perkalian dalam menghitung luas (lihat gambar 7).



Gambar 5
Dua contoh pembagian (spliiting) yang mungkin dilakukan siswa

- Selain teknik membagi (spliiting) kemungkinan juga siswa akan melakuakn teknik gridding, yaitu siswa membuat garis garis (grids) untuk melengkapi satuan-satuan yang tidak lengkap.

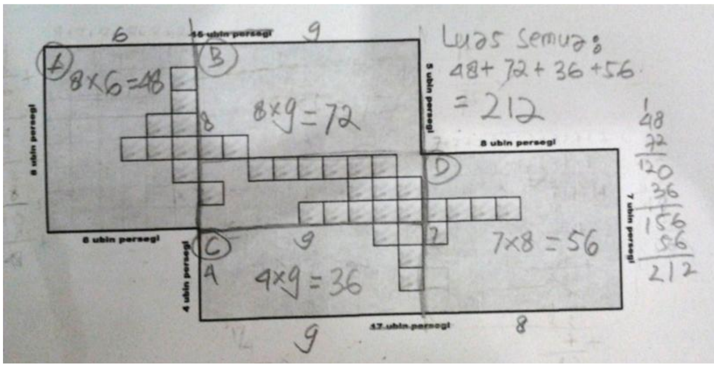
Dengan demikian siswa akan mudah untuk menghitung satuan tersebut baik dengan strategi perkalian atau penghitungan baris per baris, kolom per kolom, atau satu per satu.



Gambar 6
Siswa menggunakan teknik gridding (membuat garis grid) dan kemudian dengan strategi penjumlahan kolom per kolom siswa menentukan luas permukaan tersebut.

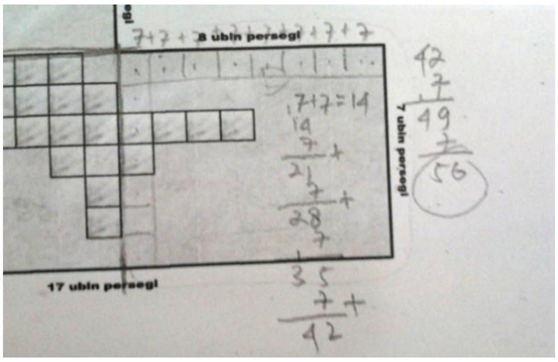
Penghitungan Satuan

- Dalam menghitung jumlah satuan yang terhimpun di dalam lantai tersebut, diperkirikan siswa akan menggunakan strategi perkalian, karena pada pertemaun sebelumnya siswa telah diperkenalkan dengan konsep ini. Setelah siswa membagi lantai tersebut ke dalam beberapa bagian yang berbentuk persegi atau persegi panjang, siswa kemudian menentukan banyak baris dan kolom satuan dari setiap bagian tersebut dengan cara memperhatikan informasi yang sudah di berikan pada gambar. Kemudian, dengan strategi perkalian, siswa menentukan banyak satuan di setiap bagian tersebut. Jika strategi ini muncul di salah satu kelompok, maka pilihlah kelompok ini untuk mempresentasikan strategi mereka dan jadikanlah bahan diskusi di Konfrensi Matematika Siswa sebagai modal untuk menanamkan konsep perkalian dalam mengukur luas permukaan bangun datar.



Gambar 7
Dalam menyelesaikan masalah di LK2 diperkirikan siswa akan menggunakan strategi perkalian, karena pada kegiatan sebelumnya dan juga pada pertemaun sebelumnya siswa telah diperkenalkan dengan konsep ini.

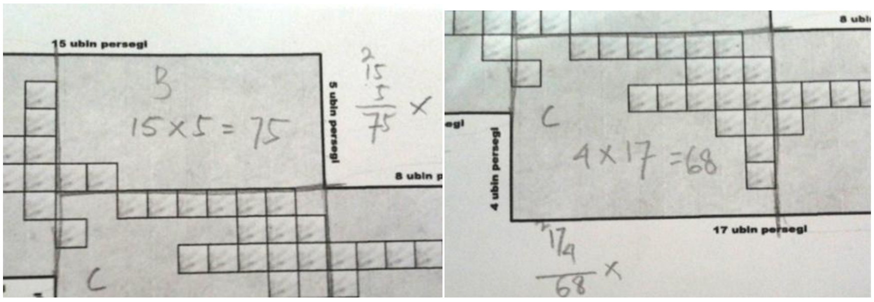
- Strategi lainnya yang kemungkinan muncul juga adalah strategi perhitungan baris per baris atau kolom per kolom. Setelah membagi permukaan lantai tersebut ke dalam beberapa bagian atau setelah melengkapi satuan-satuan yang tidak nampak (membuat garis grid), siswa menghitung banyak satuan tersebut dengan cara menambahkan jumlah satuan yang terdapat di setiap baris atau kolom. Jika strategi ini muncul di salah satu kelompok, maka kelompok ini dapat dipilih untuk mempresentasikan strategi mereka dan jadikanlah bahan diskusi di Konfrensi Matematika Siswa sebagai modal untuk mengarahkan strategi ini ke strategi perkalian, dengan cara mendiskusikan kesamaan antara penjumlahan berulang dan perkalian, misalnya 7+7+7+7+7+7+7+7 = 8 x 7.



Gambar 8
Dalam menyelesaikan masalah di LK2, strategi lainnya yang kemungkinan muncul juga adalah strategi perhitungan baris per baris atau kolom per kolom.

Kesulitan Siswa

- Dalam menyelesaikan masalah ini diperkirakan siswa akan mengalami beberapa kesulitan dan kesalahpahaman, seperti kesulitan dalam membagi-bagi lantai tersebut (splitting) dan juga kesulitan memahami perubahan yang terjadi pada dimensi permukaan lantai setelah lantai tersebut dibagi-bagi. Selain itu dimensi yang tertera dalam gambar kemungkinan salah dimengerti oleh siswa. Jika masalah ini muncul di salah satu kelompok, maka pilihlah kelompok ini untuk mempresentasikan kerja mereka dan jadikanlah bahan diskusi di Konfrensi Matematika Siswa sebagai modal untuk menjelaskan kepada siswa lainnya mengenai kesalahan tersebut dan bagiamana mengatasinya.



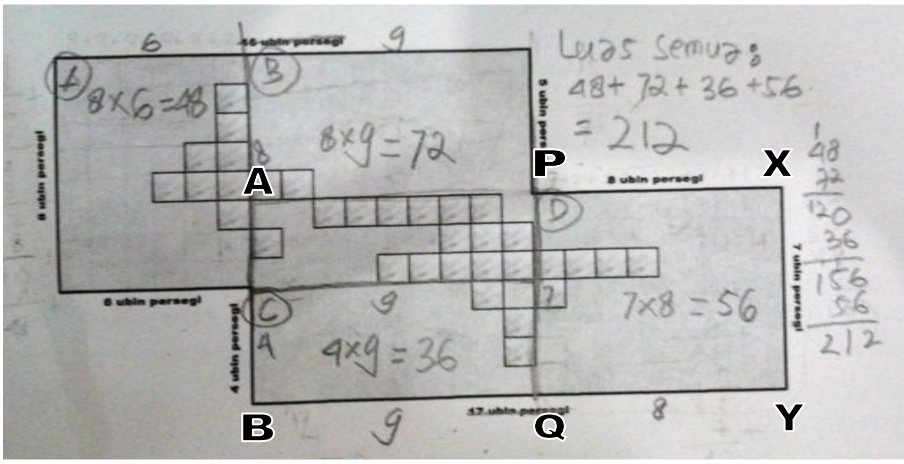
Gambar 9
Kesulitan siswa dalam membagi-bagi lantai tersebut (splitting) dan juga kesulitan memahami perubahan yang terjadi pada dimensi permukaan lantai setelah lantai tersebut dibagi-bagi. Selain itu dimensi yang tertera dalam gambar kemungkinan salah dimengerti oleh siswa.

Konfrensi Matematika Siswa

- Konfrensi ini bertujuan sebagai media sharing pengetahuan dan pemahaman antar siswa, dimana solusi siswa yang dipresentasikan dapat dipelajari, dipertanyakan, atau dikembangkan oleh siswa lainnya dalam proses diskusi bersama.
- Dalam konfrensi ini, siswa dipandu oleh guru bersama-sama membangun pemahaman mereka terhadap konsep yang sedang mereka pelajari. Sehingga dalam konfrensi ini, guru tidak hanya bertindak sebagai fasilitator selama konfrensi berlangsung, melainkan juga sebagai pemandu arah diskusi siswa (Apa yang harus didiksuiskan? Siapa? Bagaimana? Kapan?) untuk mencapai tujuan pembelajaran yang telah disepakati.
- Untuk memulai konfrensi, pilih dua atau tiga kelompok untuk mempresentasikan solusi yang mereka temukan di depan siswa lainnya. Pemilihan kelompok ini dapat didasarkan pada pertimbangan-pertimbangan, seperti: (1) solusi kelompok tersebut dapat dijadikan acuan (modal awal) untuk menanamkan atau menjelaskan konsep yang guru ingin siswa ketahui, (2) kelompok tersebut memiliki cara yang berbeda-beda sehingga menarik untuk dibandingkan dalam diskusi bersama, (3) kelompok tersebut memiliki solusi yang kurang tepat atau merupakan kesalahan umum yang dilakuakn kebanyakan siswa sehingga menarik untuk didiskusikan untuk mendapatkan solusi yang tepat, dan berbagai pertimbangan lainnya.

- Setelah siswa mempresentasikan solusi mereka, diskusikan solusi tersebut bersama seluruh siswa. Fasilitasi siswa-siswa yang bertanya, menambahkan, atau menyanggah dan jadikan semua itu sebagai bahan diskusi.

  Penting untuk diingat
- Dalam diskusi di konfrensi ini, arahkan siswa kepada penggunaan strategi perkalian dalam menyelesaikan masalah di atas. Untuk dapat menggunakan strategi tersebut siswa harus mampu membagi (split) lantai tersebut ke dalam beberapa persegi atau persegi panjang. Dalam hal ini, jika siswa mengalami kesulitan, guru dapat menjelaskannya menggunakan jawaban siswa pada LK1, yaitu kegiatan sebelumnya.
- Teknik splitting (pembagian) sangat penting perananannya dalam menyelesaikan masalah di kegiatan ini. Berikan waktu yang cukup untuk mendiskusikan hal ini bersama siswa. Seperti diketahui ada berbagai cara untuk melakukan pembagian seperti ditunjukkan pada gambar 5 dan 7.
- Kendala yang sering muncul adalah siswa tidak memahami bahwa akan terjadi perubahan dimensi di beberapa bidang baru hasil pembagian tersebut. Misalnya dimensi 15 ubin persegi pada gambar 7, dimensi tersebut berubah menjadi dua ruas setelah pembagian dilakukan yaitu dimensi 6 ubin persegi dan 9 ubin persegi. Perubahan seperti ini tidak semua siswa memahaminya bahkan agak sulit dipahami oleh siswa. Oleh karena itu perlu didiskusikan bersama. Jika ada kelompok yang melakukan kesalahan seperti ini, mintalah mereka untuk mempresentasikan hasil kerja mereka, dan kemudian gunakan jawaban siswa ini sebagai modal untuk menjelaskan kesalahan tersebut kepada siswa lainnya.
- Kendala lainnya yang berkaitan dengan dimensi adalah mengenai 'kesamaan dimensi'. Untuk memahami hal ini perhatikan gambar 10 di bawah. Berdasarkan gambar telah diketahui bahwa ruas XY adalah 7 ubin persegi. Siswa kemungkinan tidak memahami bahwa ruas XY adalah sama dengan ruas PQ dan AB. Masalah ini perlu didiskusikan bersama dalam konfrensi ini. Jika ada kelompok yang melakukan kesalahan seperti ini, mintalah mereka untuk mempresentasikan hasil kerja mereka, dan kemudian gunakan jawaban siswa ini sebagai modal untuk menjelaskan kesalahan tersebut kepada siswa lainnya.

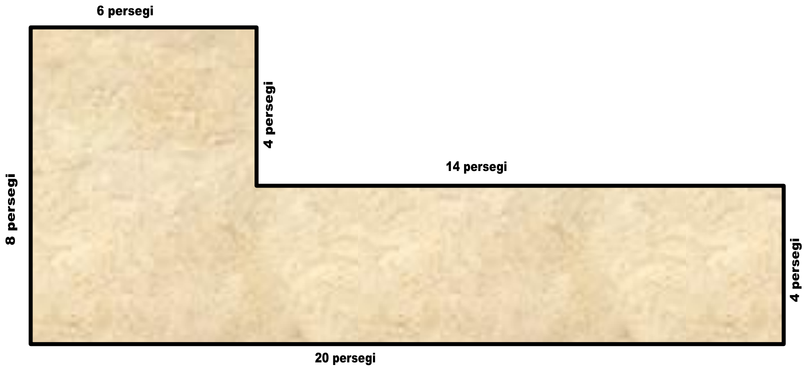


Gambar 10
Kesulitan siswa dalam membagi-bagi lantai tersebut (splitting) dan juga kesulitan memahami perubahan yang terjadi pada dimensi permukaan lantai setelah lantai tersebut dibagi-bagi.

Refleksi Kegiatan Inti
- Ajak siswa berdiskusi bagaiman teknik splitting dan strategi perkalian membantu mereka dalam perhitungan luas suatu permukaan bangun datar yang tak beraturan.

G. Kegiatan Akhir
- Untuk menguatkan pemahaman siswa mengenai strategi perkalian dalam menentukan luas permukaan bangun datar, minta siswa dalam kelompok menyelesaikan LK3 jika ada waktu yang tersisa.

- Jelaskan kepada mereka bahwa gambar di LK2 menunjukkan gambar sebuah lantai. Lantai tersebut akan dipasangkan ubin yang beberbentuk persegi dimana setiap ruasnya (dimensi) membutuhkan ubin persegi seperti tertera pada gambar (Gambar 11).
- Mintalah siswa untuk menunjukkan bagaimana cara menerapkan strategi perkalian dan teknik membagi (splitting) dalam menentukan luas lantai tersebut.



Gambar 11
Lembar Kerja (LK) 3: Siswa diminta untuk menentukan luas permukaan lantai di atas dengan menerapkan strategi perkalian

H. Kesimpulan dan Penutup

- Sebelum menutup kegiatan pembelajaran, ajak siswa untuk mengingat kembali apa saja yang mereka telah lakukan (pelajari) dalam pertemuan ini.
- Guru mengingatkan kembali mengenai teknik membagi (splitting) dan strategi perkalian dalam menentukan luas permukaan bangun datar.
- Jika LK2 dijadikan sebagai tugas rumah, ingatkan siswa untuk mengumpulkannya dipertemuan selanjutnya.
- Guru menutup kegiatan pembelajaran.

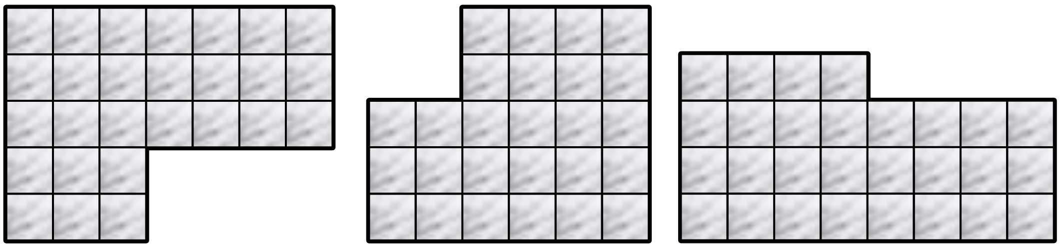
APPENDIX F

STUDENT WORKSHEETS OF THE SECOND CYCLE

Lembar Kerja 1 Pertemuan 1

Nama anggota kelompok:

Perhatikan gambar ketiga lantai teras di bawah ini.
Tentukan urutan ketiga teras tersebut dari yang terluas ke yang tersempit! Berikan alasanmu!



Anatia Halim Anisa

Tulis jawaban dan alasanmu di sini!

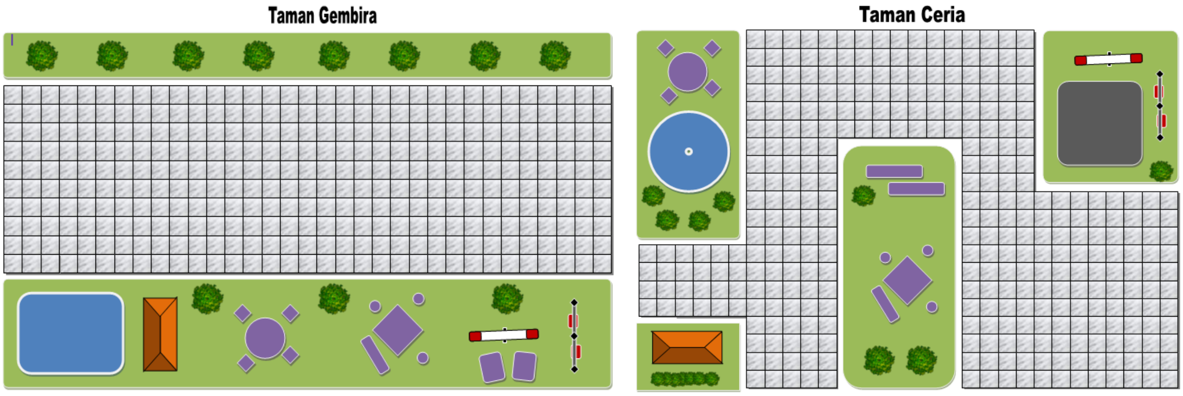
Lembar Kerja 1 Pertemuan 2

Nama anggota kelompok:

Perhatikan lahan parkir di kedua taman di bawah ini!

Manakah yang lebih luas? Berikan alasanmua!

Tunjukkan perhitunganmu!



Tulis jawaban dan alasanmu di sini!

Tulis perhitunganmu di lembar kosong ini!

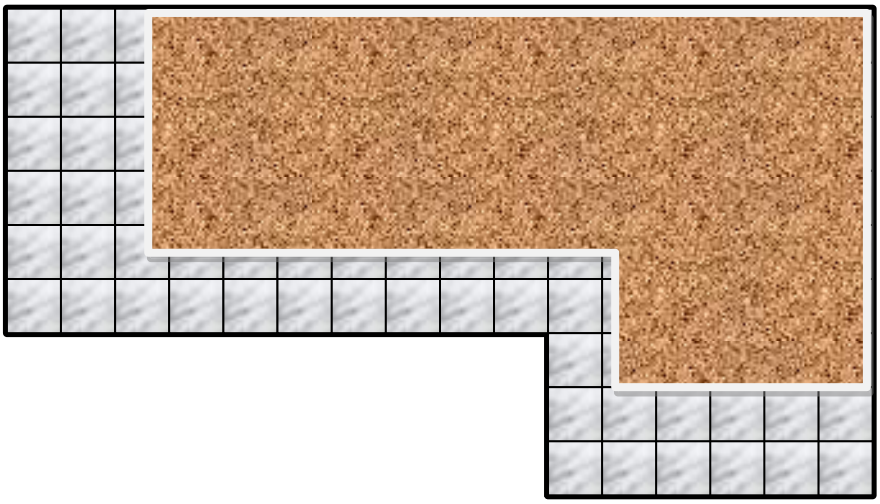
Lembar Kerja 2 Pertemuan 2

Nama anggota kelompok:

Perhatikan gambar lantai yang ditutupi karpet di bawah ini!

Tentukan luas lantai tersebut!

Tunjukkan perhitunganmu!



Tulis jawabanmu disini!

Tulis perhitunganmu di lembar kosong ini!

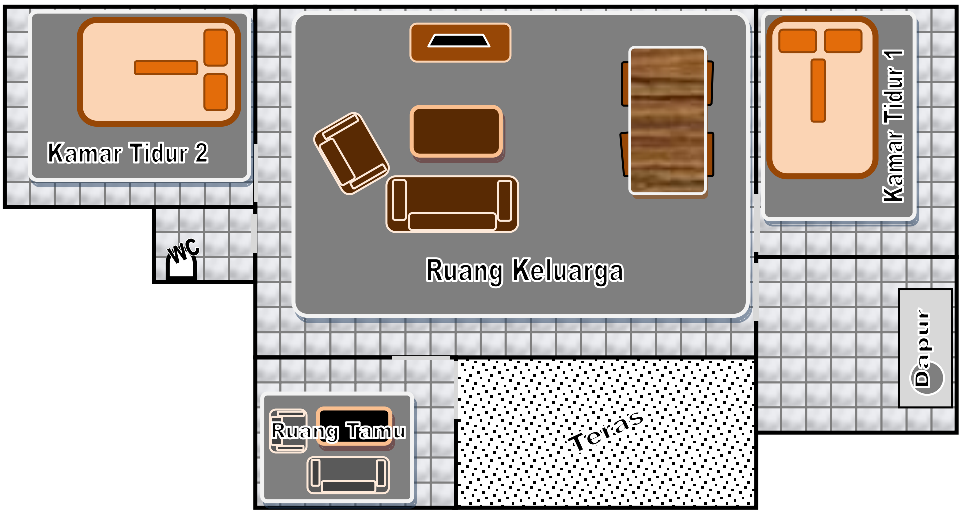
Lembar Kerja 1 Pertemuan 3

Nama anggota kelompok:

Perhatikan lantai yang ada di gambar denah rumah di bawah ini!

Tentukan luas setiap lantai?

Tulis jawabanmu pada tabel! dan Tunjukkan perhitunganmu!



Tulis Jawabanmu pada tabel di bawah ini

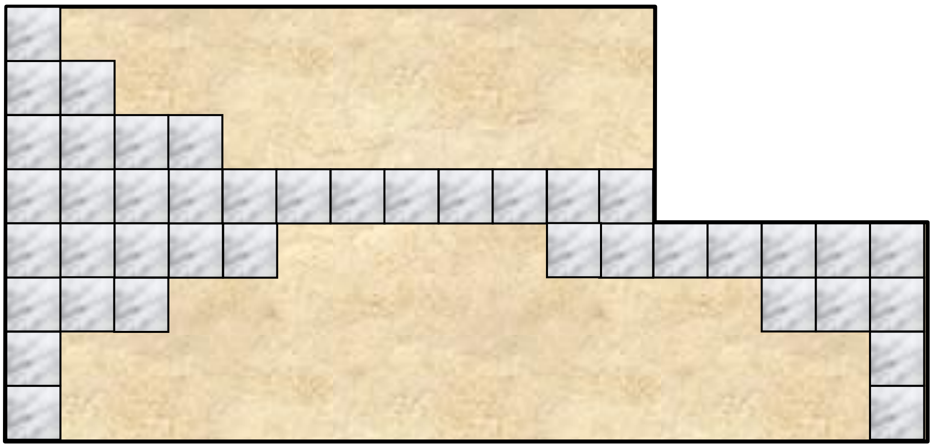
Lantai	Luas
WC	
Dapur	
Ruang Tamu	
Kamar Tidur 1	
Kamar Tidur 2	
Ruang Keluarga	
Teras	
Seluruh lantai	

Tulis perhitunganmu di halaman kosong ini!

Lembar Kerja 2 Pertemuan 3

Nama anggota kelompok:

Perhatikan gambar lantai yang belum selesai dipasangkan ubin di bawah ini!
Tentukan luas lantai tersebut!
Tunjukkan perhitungamu!



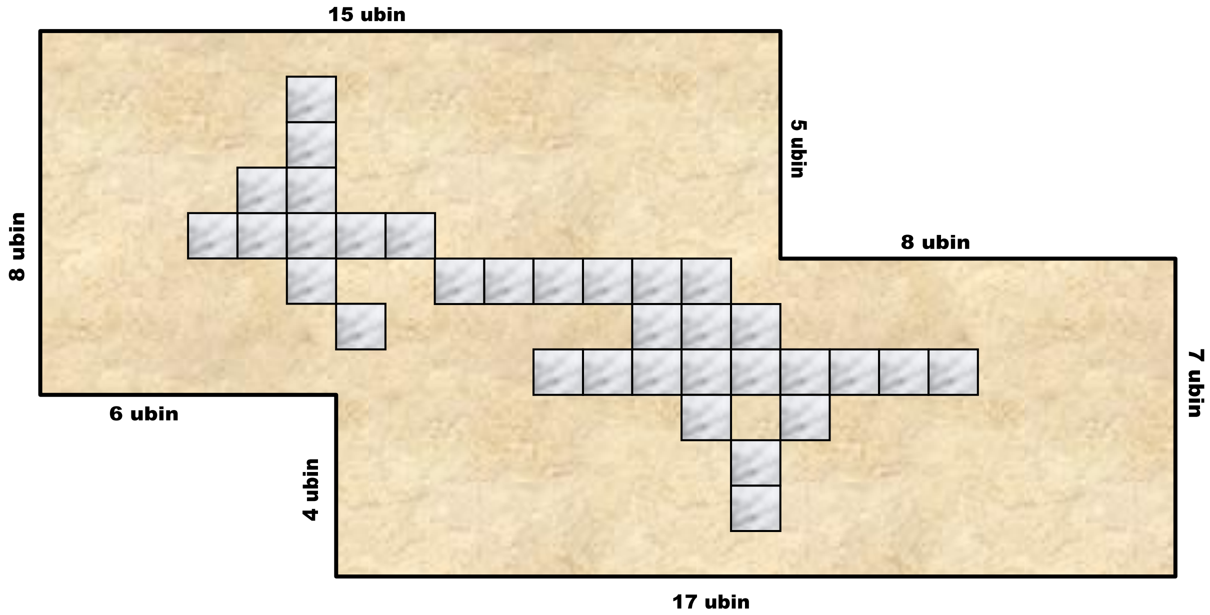
Tulis perhitunganmu di halaman kosong ini!

Tulis jawabanmu disini!

Lembar Kerja 2 Pertemuan 4

Nama anggota kelompok:

Perhatikan gambar lantai yang belum selesai dipasangkan ubin di bawah ini!
Tentukan luas lantai tersebut!
Tunjukkan perhitungamu!



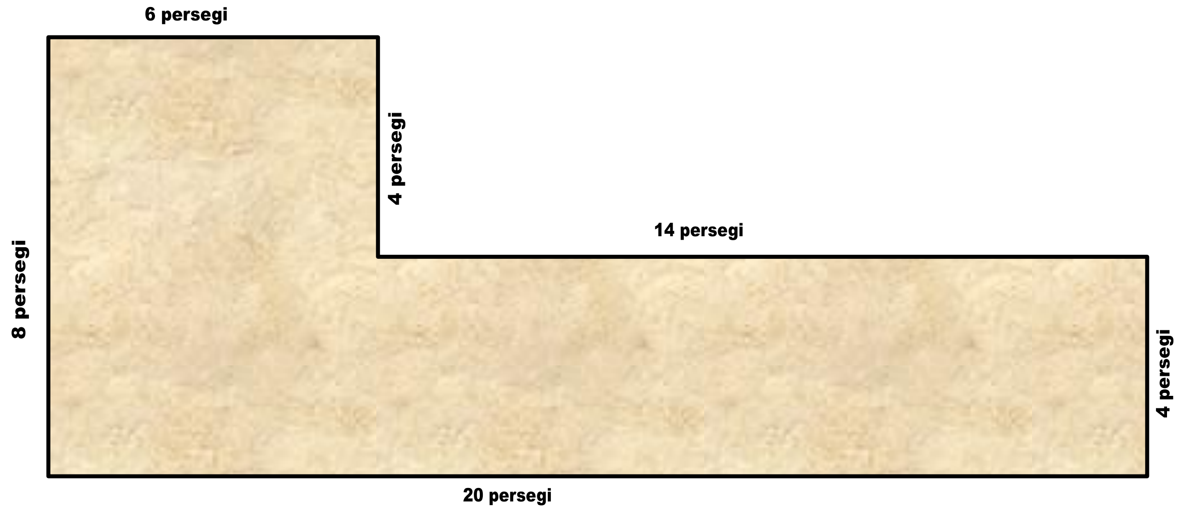
Tulis jawabanmu disini!

Tulis perhitunganmu di halaman kosong ini!

Lembar Kerja 3 Pertemuan 4

Nama anggota kelompok:

Perhatikan gambar lantai yang hendak dipasangkan ubin berbentuk persegi di bawah ini!
Dengan menggunkan cara perkalian, tentukan luas lantai tersebut!
Tunjukkan perhitungamu!



Tulis jawabanmu disini!

Tulis perhitunganmu di halaman kosong ini!

Appendix G

EXAMPLES OF STUDENTS' WORK ON THE WORKSHEET

(See next pages)