DESIGNING TANGRAM GAME ACTIVITY AS AN INTRODUCTION TO THE CONCEPT OF AREA CONSERVATION IN THE TOPIC OF AREA MEASUREMENT

MASTER THESIS

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Shofan Fiangga
NIM 117785018

UNIVERSITAS NEGERI SURABAYA
PROGRAM PASCASARJANA
PROGRAM STUDI PENDIDIKAN MATEMATIKA
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MASTER THESIS

A Thesis submitted to
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Shofan Fiangga
NIM 117785018

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APPROVAL OF SUPERVISOR

Thesis by Shofan Fiangga, NIM 117785018, with the title *Designing Tangram Game Activity as an Introduction to the Concept of Area Conservation in the Topic of Area Measurement* has fulfilled the requirement and been agreed for the defend.

Supervisor I,	Date,
Prof. I Ketut Budayasa Ph.D.	-----------------------------------
Supervisor II,	Date,
Prof. Dr. Siti Magfirotun Amin, M.Pd	-----------------------------------

Acknowledge by
Head of the Mathematics Education Study Program

Dr. Agung Lukito, M.S
NIP 196201041991031002

APPROVAL

Thesis by Shofan Fiangga, NIM 117785018, had been defended in front of the Board of Examiners on ………. 2013.

Board of Examiners

Name	**Signature**	**Position**
Dr. Agung Lukito, M.S	……………………	Chairman/Member
Prof. Dr. Siti M. Amin, M.Pd	……………………	Member/Supervisor I
Prof. I Ketut Budayasa, P.hD	……………………	Member/SupervisorII
……………………………	……………………	Member
Prof. Dr. R.K. Sambiring	……………………	Member

Acknowledge by
Director of Post Graduate Program,

Prof. I Ketut Budayasa, Ph.D
NIP. 195712041994021001

DEDICATION

This Hard Work

This Sacrifice

This Struggle

This Prayer

This Thesis

I'll dedicate all of them

For me, for my family, for my lovely

For Allah the Al-Mighty

"God Never Play Dice"

Albert Einstein

ABSTRACT

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Keywords: Area Conservation, Tangram, Van Hiele, RME, PMRI

Most of the teachings in the topic of area measurement tend to give the formula too early to the students. This situation results what van Hiele called as mismatch situation, the condition where the teaching of the teacher is not appropriate with what the students understand in their state results. Consequently, the students could not have the meaning of measuring area except remembering formula. This situation should be administered properly by developing a well design learning activities that could help the students to understand area conservation in area measurement. To develop a good local instructional theory, RME theory suggests that we must apply the parts of each constructed among guided reinvention, didactical phenomenology, and emergent modeling well. Here, the model of tangram by using "price" is developed to reason with the concept of area especially for the big idea of conservation. Then, we expect that it will become the model for reasoning the concepts of area and help them to solve area related problems. The conclusions on the result of the students' learning on each activity in our design provide supporting evidence that by using the tangram we could invite the students to experience the idea of measuring an area using the concept of conservation. The tangram with its conservation properties could be used as supporting model for understanding the area conservation in area measurement.

ABSTRAK

Fiangga, Shofan. 2013. *Pengembangan Desain Pembelajaran Menggunakan Aktivitas Permainan Tangram sebagai Pengantar kepada Konsep Konservasi Luas pada Pengukuran Luas*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. I ketut Budayasa Ph.D. dan (II) Prof. Dr. Siti Maghfirotun Amin, M.Pd.

Kata Kunci: Konservasi Luas, Tangram, van Hiele, RME, PMRI

Sebagian besar pembelajaran di topic pengukuran luas cenderung memberi rumus terlalu dini pada siswa-siswa. Situasi ini menghasilkan kondisi dimana pengajaran guru tidak sesuai dengan apa yang sebenarnya siswa mampu pahami pada tingkat tertentu itu.Situasi ini disebut *mismatch condition* oleh van Hiele. Siswa tidak bisa memahami apa yang dimaksud mengukur luas selain menggunakan rumus. Situasi ini harus diperbaiki sesuai dengan apa yang seharusnya siswa bisa pahami dengan mengembangkan aktivitas pembelajaran yang didesain dengan baik yang mampu membantu siswa untuk memahami konservasi luas dalam pengukuran luas. Untuk mengembangkan *local instructional theory* yang baik, teori RME menyarankan bahwa kita harus menerapkan beberapa hal penting berikut ini, yaitu: *guided reinvention, didactical phenomenology,* dan *emergent modeling* dengan baik. Di sini, model tangram dengan harga dikembangkan sebagai penalaran dengan konsep luas sebagai topiknya khususnya pada ide utama konservasi luas. Kemudian, kita berharap agar model tangram dengan harga ini mampu menjadi *the model for* yang bisa digunakan siswa untuk menalar berbagai permasalahan yang berhubungan dengan luas. Kesimpulan pada hasil dari pembelajaran siswa pada setiap aktivitas pada desain kami menyimpulkan bukti pendukung bahwa dengan menggunakan tangram kita bisa mengajak siswa untuk mempelajari ide mengukur luas dengan menggunakan konsep konservasi. Tangram dengan karakteristik kinservasinya dapat digunakan untuk mendukung model untuk memahami konservasi luas di pengukuran luas.

PREFACE

Alhamdulillah to Allah the Almighty for His blessing so that this thesis can be finished in due time. Blessings and greetings to the Prophet Muhammad, peace is upon him, for his motivation through his wise teaching and advices.

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Finally, I consciously understand that this thesis is far from being perfect. Thus, any critics and constructive ideas will be gladly accepted.

Shofan Fiangga

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CHAPTER I
INTRODUCTION

A. Research Background

Most of the teachings in the topic of area measurement tend to give the formula too early to the students (e.g. Kordaki & Balomenou, 2006; Kospentaris et al., 2011; Papadopoulos, 2010). This results in the students understanding of area as merely procedural, as the use of the formula. Nevertheless, these studies provide a suggestion of the importance of learning the concept of area conservation in the school curriculum. They agreed with the idea of why we need to focus on conservation as the core in understanding area. Accordingly, the understanding of area conservation is an important prerequisite concept for the understanding of area measurement using area-units and area-formulae. In fact, most of the pupils deal with confusion about rearrangement problems (Kordaki & Balomenou, 2006). They cannot see that if we decompose a shape into another form, then the area remains invariant. The consequence of this is that the students will have difficulty while dealing with the area of more complex 2 dimensional figures. Kordaki & Balomenou (2006) concludes from several studies that, in teaching the topic of area, the integration of teaching conservation area, unit iteration, and area formula is important for children.

The fact that the geometry topic in Indonesia heavily relies on the use of formula can cause a detrimental effect. Learning geometry in

Indonesia is defined as learning only the formula and the properties of geometrical concepts without any meaningful knowledge (Fauzan, 2002b). S. M. Amin (personal communication, Augusts 9, 2012) stated that the learning activities in Indonesia teach the students about the properties and the formula of geometrical figures without considering the students' spatial ability and their environment. This idea is in line with what Soedjadi (in Fauzan, 2002a) found which is that the students in elementary and secondary school have difficulties in recognizing and knowing geometrical objects. In fact, according to my experience, the students tend to be interested only in the formula and remembering it

In Indonesia, especially area measurement topic, there is no space provided in the school curriculum to teach the students about area conservation (Departemen Pendidikan Nasional [Depdiknas], 2003). Meanwhile, the teaching in the topic of area requires the integration of teaching conservation area, unit iteration, and area formula which is important for children. The students should learn the concept area conservation appropriately by considering to what extent they could accept this abstract concept. Unless, the condition where the teaching of the teacher is not appropriate with what the students understand in their state will occur. This condition is what van Hiele called as mismatch situation (van Hiele, 1985). This forces the students who do not fully understand the area measurement to remember the formula. Thus, in this study we will design a sequence of learning activities which could provide appropriate experience for the students to learn about area conservation.

Nevertheless, to change the package of mathematics into interesting and fun activities such that that mismatch situation does not occur is not an easy task. However, the use of games or puzzles seems promising as non routine problem for the students. Van Hiele (1999) claimed that playful exploration could occur when using a mosaic puzzle in dealing with certain shapes and their properties, symmetry, parallelism, and area. In his design, the students are given a full opportunity to freely play with the puzzle and use their imagination to create everything they want. If we can design an activity using a mosaic puzzle game, we expected that the student could explore more about area conservation using their own methods in a fun ways.

One of the famous mosaic puzzle games is Tangram, an ancient Chinese puzzle. Its seven geometric movable pieces can be assembled to make more elaborated shapes (Jovanovic et al, 2009). Bohning and Althouse (1997) introduced the word “tangramming” as the activity of assembling a figure using tangram pieces. The tangram experience is essential for the students’ development of a positive attitude towards mathematics, especially for recognizing and appreciating geometry in their natural world, because the students deal with the geometrical vocabulary, shape identification and classification, by which the hands on activities allows the discovering of the relationship among those seven pieces. Furthermore, the property of tangram namely that among those seven pieces a relationship can easily be drawn as comparison where we can compare the size of each tangram pieces. This can be used for a promising activity in teaching about area conservation.

To develop a well design learning activities that could help the students to understand area measurement better, RME theory suggest that we must apply the parts of each constructed among guided reinvention, didactical phenomenology, and emergent modeling well (Gravemeijer, 2004). Freudenthal (1991) said that guiding the learners to reinvent the both valuable knowledge and abilities will help them to learn those concepts easier. The use of tangram puzzle will lead the students to deal with the reinvention of area conservation by rearranging and restructuring the pieces of tangram. The didactical phenomenology emerges along the activities where the students experience rich stimulating geometric activities by playing with the tangram. We expect that the students will gain the insight of area conservation in area measurement through the contextual learning activities. Here, the model of tangram by using using price is developed to reason with the concept of area especially for the big idea of conservation. Then, we expected that it will become the model for reasoning the concepts of area and help them to solve area-related problems

Area of an object remain invariant in size regardless whatever changes on its shape or positions (Fauzan, 2002). Egocentric thinking makes the reality seen subjectively by the children. The children experience the process from which they cannot consider that the measurement is invariant, it is the process of de-centering thinking because the main point of measurement is that the object's size remain invariant no matter the positions are changed (Piaget, 1960). Conservation means that the quantitative value of an area remains unaltered while its figure can be qualitatively new (Piaget et al., 1981). Piaget also noted the difficulties of the students in understanding the area conservation. He found that (1) that subtracting and adding of the same parts operationally and (2) moving a portion of shape to make it into another shape are hardly understood by the students, that these will make the area invariant, because its continuity is lost.

Understanding the concept of area conservation is a process of giving meaning to its different representations: for example numerical, visual and symbolic. Students have the opportunity of expressing their own knowledge of the above concept by selecting from among representations those most appropriate to their cognitive development and of constructing a broader and more abstract view of this concept by selecting more than one representation system. Moreover, the meaning that students can give to the concept of the conservation of area is closely related to the tools, the tangram pieces, that they use and to the shapes, irregular polygon, on which they have to study. Therefore, we define understanding of area conservation as activities of (1) using the tools, tangram, to examine the occurrence of area conservation by rearranging the shape formed by the tangram pieces and (2) using the concept of area conservation to quantify the area of irregular polygon.

The teaching and learning of geometry in Indonesia is conducted in a formal way. The tangram with its properties in which we can freely manipulate it in various meaningful ways might break the formal way of learning geometry in Indonesia and make a fun and meaningful activity for children. The tangram pieces as geometrical plane figures can support the students' understanding of plane figure properties as well as their understanding of plane figures' area. The main focus will be how to measure the area of polygon using the idea of area conservation. The use of hands on activities in grasping the concept of area is seems promising that the students can do something real with the notion area conservation. Then, we propose the following research question, **how can tangram game activities help the students in understanding the concept of area conservation in area measurement?**

In answering that question, we use design research approach as the framework in developing local instructional theory. Because this study concerns about geometrical thinking, we also consider how Van Hiele level of geometry reasoning development could suggest a good learning trajectory. By which and the Realistic Mathematics Education Domain specific theory as the design guidance, the hypothetical learning trajectory was designed.

B. Research Question

From the problem in teaching area measurement especially in Indonesia and how important the concept of area conservation is which have been elaborated in the background, we propose the main research question which is **how can tangram game activities help students to understand the concept of area conservation in area measurement?**

C. Research Objective

The objective of this design research is to contribute on the development of a local instructional theory for area measurement. In this research, the researcher will implement RME approach by designing a classroom activity that can support students' to understand the concept of area conservation in area measurement.

D. Definition of Key Terms

1. ***Tangram game acitivties (Tangramming),***

   "Tangramming" is the activity in assembling a figure using tangram pieces where manipulating the geometric shapes can be so much fun and amazement that the seven pieces can be arranged to make so many different figures. (Bohning and Althouse, 1997)

2. ***Area conservation***

   Area is a number of measurement units needed to fully cover a region without gap or overlap. Area of an object is remain invariant regardless whatever changes on its shape or positions. Area conservation means that the quantitative value, the number of measurement unit, of an area remains unaltered while its figure can be qualitatively new (Piaget et al., 1981).

3. Understanding of area conservation

Piaget also noted the difficulties of the students in understanding the area conservation. He found that (1) that subtracting and adding of the same parts operationally and (2) moving a portion of shape to make it into another shape are hardly understood by the students, that these will make the area invariant, because its continuity is lost.

Understanding the concept of area conservation is a process of giving meaning to the different representations: for example numerical, visual and symbolic. Students have the opportunity of expressing their own knowledge of the above concept by selecting from among representations those most appropriate to their cognitive development and of constructing a broader and more abstract view of this concept by selecting more than one representation system. Moreover, the meaning that students can give to the concept of the conservation of area is closely related to the tools, the tangram pieces, that they use and to the shapes, irregular polygon, on which they have to study. Therefore, we define understanding of area conservation as follow:

a. Using the tools, tangram, to examine the occurrence of area conservation by rearranging the shape formed by the tangram pieces

b. Using the concept of area conservation to quantifying the area of irregular polygon

4. ***Area measurement***

Area measurement is a process to quantifying the size of a surface area by using discrete units of the same size and then counting those units. Using the concept of area conservation in measuring area is quantifying by measuring the conserved shape

5. ***How tangram game activities can help students understand the concept of area conservation in area measurement***

Bakker and van Eerde (submitted 2012) suggest that the design research is mainly aimed to design and develop educational materials. These educational materials are aimed to help the student in understanding a certain concept. Hence, to help the students here can be inferred as designing and developing both learning trajectory and educational materials.

To help the students understand the concept of area conservation in area measurement, a suitable educational material should be administered. Thence, in this research, designing and developing both learning trajectory and educational material on the topic of area conservation.

E. Significance of Research

As we mention in the background, the main problem in area measurement is that the students tend to heavily rely on the use of formula to measure an area. Remembering a certain formula for a certain figure could be detrimental for the students when they can no longer find numbers to multiply or incorrectly use the available numbers within the formula. The ability in composing and decomposing shapes will simplify the complex shapes problem into simpler measurable shape. Furthermore, the understanding of area conservation assures the students to be able solving area measurement problem without bounded with the formula. This is what the students will find in their reality.

CHAPTER II
THEORETICAL FRAMEWORK

A. Area Measurement and the Students' Difficulties in Learning Area Measurement

According to Fauzan (2002), area is a number of measurement units needed to fully cover a region without gap or overlap. Then, measuring area is a process to quantifying the size of a surface area by using discrete units of the same size and then counting those units

Piaget (1960) defines three level of development of Students' understanding of area measurement concept. The first level is elementary conservation, which allows the students to think of area of an object is remain invariant in size regardless whatever changes on its shape or positions. The second level is linear measurement of area, which provides the children a linear tool to count the area. Here, the idea of unit as reference is highlighted. The third level is calculating the area, which allows the students to use metrical procedure to the area. Here, structuring array is one of the big ideas. This development of the children concept of measurement takes place by the time of the de-centering process of early age children from their egocentric thinking. This egocentric thinking makes the reality seen subjectively by the children. When they could see the measurement are just the same from any point of views, it is the process of de-centering thinking because the main point of measurement is that the object's size is remain invariant no matter the positions are changed (Piaget, 1960). Here, why the concept of area conservation is really important for the basic progression of area measurement (see also Kordaki & Balomenou, 2006; Kospentaris et al., 2011; Papadopoulos, 2010). Furthermore, Piaget (1960) also said that the discovery of concept of conservation in early age is fundamental as the basic concept for further development of mathematical knowledge for the children.

Several studies suggest that most of the teachings in the topic of area measurement tend to give the formula too early to the students (e.g. Kordaki & Balomenou, 2006; Kospentaris et al., 2011; Papadopoulos, 2010). This results in the students understanding of area as merely procedural, as the use of the formula. Nevertheless, these studies provide a suggestion of the importance of learning the concept of area conservation. They agreed with the idea of why we need to focus on conservation as the core in understanding area. Accordingly, the understanding of conservation of area is an important prerequisite concept for the understanding of area measurement using area-units and area-formulae. Furthermore, Piaget (1960) said:

> "…. In any domain whatever, the discovery of conservation implies the construction of a logical 'grouping' or else of a (mathematical) 'group' of operations, which is why the study of these invariants and their first appearance is important"(Piaget, 1960, pg. 278)

The above argument implies that for further development of the concept, especially mathematics, the idea of conservation is also important as the basic knowledge to proceed. Piaget also noted the difficulties of the students in understanding the area conservation. He found that (1) that subtracting and adding of the same parts operationally and (2) moving a portion of shape to make it into another shape are hardly understood by the students, that these will make the area invariant, because its continuity is lost. Furthermore, understanding the concept of area conservation is a process of giving meaning to its different representations: for example numerical, visual and symbolic. Moreover, the meaning that students can give to the concept of the conservation of area is closely related to the tools, the tangram pieces, that they use and to the shapes, irregular polygon, on which they have to study.

The fact that most of the pupils deal with confusion about rearrangement problems is undeniable (Kordaki & Balomenou, 2006). They cannot see that if we decompose a shape into another form, then the area remains invariant. The consequence of this is that the students will have difficulty while dealing with the area of more complex 2 dimensional figures. In measuring the area of triangle, parallelogram, rhombus, and so on, the decomposition of those plane figures in such a way that have the same area but more feasible to be measured cannot be meaningful for the students unless they already have had an understanding of area conservation.

Kospentaris et al. (2011) suggest that it is crucial to include the area conservation issue in an appropriate manner in the school curriculum. Kordaki and Balomenou (2006) claim the construction of the identity of area implies a sufficient understanding of its conservation. They also conclude from several studies that, in teaching in the topic of area, the integration of teaching of conservation area, unit iteration, and area formula is important for children. Therefore, it is important to pay more attention on the idea of conservation as the concept of area.

B. Indonesian Classroom Situation and the Use of Formula in Geometry

The fact that the use of formula in dealing with geometry concepts in Indonesia becomes more important than the conceptual understanding on it is undeniable. Learning geometry in Indonesia is defined as learning only the formula and the properties of geometrical concepts without any meaningful knowledge. (Fauzan, 2002b). S. M. Amin (personal communication, Augusts 9, 2012) stated that the learning activities teach the students about the properties and the formula of geometrical figures without considering the students' spatial ability and their environment. They just accept what are being the properties of the shapes and how to calculate the area of a perimeter of a certain shape.

Nevertheless, it is actually what the government asked through their description of competency standard (see Depdiknas, 2003). The competency standard for 3rd grader in the topic of area measurement is to calculate the area and the perimeter of square and rectangle. That is the reason why most of the teachings on area measurement topic heavily rely on the use of formula only. It seems that the teacher just accept this standard as product oriented competency regardless the need of conceptual understanding process along it. Furthermore, about the general topic in geometry, Soedjadi (in Fauzan, 2002a) found that students in elementary and secondary school have difficulties in recognizing and knowing geometrical objects. Moreover, Widjaja and Heck (2003) stated that, in Indonesia, a negative attitude toward mathematics is showed by most of Indonesian pupils, because they perceive mathematics lesson, especially geometry, as a difficult task for them. Fauzan (2002b) also mentions the unfavorable situation for mathematics classroom in Indonesia as follow: (1) the passiveness of the pupils; (2) invariant teaching style, like direct introduction; (3) tend to only focus on factual knowledge; (4) the use of non-extended and closed questions; (5) lack of critical reasoning; and (6) no hands-on activity. In fact, according to my experience, the students tend to care only the formula and remember it. This makes a non-meaningful learning that lessens their understanding of the concepts of area. As the result, they are demanded to remember all stuffs like the formula and the properties of the concepts which really forces them to work really hard on remembering those stuffs.

These difficulties faced by the students in the class make the students cannot enjoy the mathematics lesson. They just do remembering the mathematical facts. Also, there is no place for wrong answer. The result is that the students are afraid when they enter the mathematics classes. Zulkardi (2002) stated that the problem is not the mathematical contents, but, in several cases, the lack of the content of teaching skills of

Indonesian teacher makes the mathematical content seen more difficult than it should be. Swadener and Soedjadi (1988) suggest that the teacher in all countries should have a wide insight of education knowledge and what issues could affect them and their students.

As the conclusion, we can say that why the pupils in Indonesia perform a negative attitude toward mathematics like their tendency to always remember or use formula is caused by an appropriate manner of teaching done by the teacher. The teaching should not only care about what the students should learn but also how the students think when learn about it. Otherwise, the knowledge will only remain as something that must be remembered and recalled.

C. The Mismatch Condition on the Area Measurement Topic

The problem in teaching geometry in the class has raised significantly in the matter of which the student cannot perform a geometry skill well, for example calculating the area, define the characteristic of a shape, and so on. Van Hiele (1985) pointed out that the mismatch situation is the main problem of the students' difficulty in reasoning in geometry. According to van Hiele level of geometry reasoning, he proposed the level where geometry reasoning development takes place.

1. Level 0 (Visualization)

Visual appearance of an object is being the main feature that can be achieved by the students. Mostly, they try to relate the new object with the similar object that they have had in their mind. Furthermore, characteristic of the object are mostly not being perceived by them.

2. ***Level 1 (Analysis)***

   Students could see properties of a geometry object. They could elicit a collection of properties from a given geometric figures. However, they still could not see the relationships between these properties. Moreover, the students in this level could not determine which properties is the important one or just the same and/ or equivalent with another properties..

3. ***Level 2 (Abstraction)***

   Students could make relationships between properties and between figures. Meaningful definition and informal arguments of proofing can be created by the students using logical implications and class inclusions.

4. ***Level 3 (Deduction)***

   Students can construct proofs and showing an understanding of the role of axioms and definitions. They also could determine the necessary and sufficient conditions.

5. ***Level 4 (Rigor)***

   At this level, students understand the formal aspects of deduction, such as establishing and comparing mathematical systems.

A mismatch situation is a situation where level of which the student think while learning in the class is different, or lower, compare to the level of which the teacher teach geometry lesson in the class; thus, all the students can do is only remembering.

The example of how mismatch occur is when the teacher give the area formula too early. Formula for calculation an area of certain shape is in the deduction level in Van Hiele's level because the area formula of a plane figure needs relation between mathematical concepts. This makes when the teacher give a formula to the students in early school, probably they are still in visual level, they cannot understand about the concept of area well, and they force themselves to just remember the formula in their heads. They can just answer the problem given, but to reason with it is impossible since it is only a fact that is remembered. The same situation also happens when the teacher asks his/her pupils to elicit the properties of a plane figure, which is the 2nd level of van Hiele. For the students who already had attained in this level, this task is not a big problem, but for others, this could become a hard work for them. Since they cannot understand the properties of the concepts of those plane figures, what they can do is only remembering the facts and doing nothing with the reasoning.

D. Tangram Mosaic Puzzle for Area Conservation on Non-regular Shape

To change the package of mathematics into interesting and fun activities is not an easy task. The use of game or puzzle is seemed promising to do so. Van Hiele (1999) claimed that playful exploration can occur when using mosaic puzzle in dealing with certain shape and their properties, symmetry, parallelism, and area. In his design, the students are given a full opportunity to freely play with the puzzle and use their imagination to create everything they want. He designed the activities in which the students encounter with the properties and the symmetry of the shape and explore the area by using grid paper as the basic for comparing each other piece. However, the introduction of gridline to introduce the idea of conservation is useless if we can design an activity using mosaic puzzle game in which the student can explore more in the area conservation using their own ways.

One of the famous mosaic puzzle game is Tangram, an ancient Chinese puzzle. Its seven geometric movable pieces can be assembled to make more elaborated shape (Jovanovic et al, 2009). Bohning and Althouse (1997) introduced the word "tangramming" as the activity in assembling a figure using tangram pieces. The tangram experience is essential for the students' development in positive attitude towards mathematics especially for recognizing and appreciating geometry in their natural world because the students deal with the geometrical vocabulary, shape identification and classification, and discovering the relationship among those seven pieces. Furthermore, the properties of tangram that the relationship between among those seven pieces can be easily drawn as comparison can be used as a promising activity in area conservation.

Moreover, the exploration of complex and non-regular shapes generated from the tangram activities could helps the students to better understand about the whole concept of area, especially concerning the use of units and sub-units and the conservation of area (Papadopoulos, 2008)

The teaching and learning of geometry in Indonesia is conducted in a formal way. The tangram with its properties in which we can freely manipulate it in various meaningful ways might break the formal way of learning geometry in Indonesia and make a fun and meaningful activity for children. The tangram pieces as geometrical plane figures can support the students' understanding of plane figure properties as well as about plane figures' area especially for non-regular polygon. The use of hands on activities in grasping the concept of area is seems promising that the students can do something real with the notion area as conservation and the unit iteration.

E. Implementing Realistic Mathematics Education Approach to Bridge the Mismatch Condition in Teaching the Concept of Area

A mismatch situation in the topic of area measurement occur when the teacher force the students to understand a formal concept, in this case is the formula of area of certain figure, while the students are still working on informal level. The Realistic Mathematics Education (RME) approach suggests that we should apply the concept of emergent modeling from RME to bridge the students understanding from informal into a formal one. This concept can be seen as the solution of how to help the mismatch situation where using the formula to find the area of a given shape, which is the part of the competency standard in Indonesian curriculum, is the formal level of measuring area.

Freudenthal (in Gravemeijer & Terwel, 2000) stated that the students should experience the usefulness of mathematics in their lesson. It does not mean that we can just throw the formula to the students and ask them to use the formula in a context problem. We need to guide them to the need of the use of mathematical concepts. Inspired by several works like Decroly, van Hiele, and Geldof (Gravemeijer & Terwel, 2000), he developed a theory on teaching mathematics called Realistic Mathematics Education (RME). We need to guide them to the need of the use of mathematical concepts. The Realistic Mathematics Education (RME) theory can help the students to really get the idea of which we really need this concept of area.

To develop a well design teaching activity based on RME theory, we must apply the parts of each constructed among guided reinvention, didactical phenomenology, and emergent modeling as the design heuristic well (Gravemeijer, 2004). Regarding how important the area conservation concept is and what the difficulties of children dealing with it are, RME could be the suitable learning design heuristic. Freudenthal (1991) said that guiding the learners to reinvent the both valuable knowledge and abilities will help them to learn those concepts easier. How those three design heuristics evolved is described as follow:

1. According to Gravemeijer (2004), the guided reinvention emerges as a learning sequence by which the students will experience a certain process where they could reinvent the mathematics in the lesson. Streefland (in Gravemeijer, 2004) stated that informal procedure of the students could be considered as the process to go to more formal mathematics. The use of tangram puzzle, as the informal procedure, will lead the students to deal with the reinvention of area conservation by rearranging and restructuring the pieces of tangram.
2. The phenomenology of how the mathematical ideas need to be organized may be used by the designer in developing a sequence of learning activities (Freudhental in Gravemeijer, 2004). Here, the mathematics' concepts, procedures or tools could be derived unintentionally by the students who experience the learning. The didactical phenomenology emerges along the activities where the students experience rich stimulating geometric activities by playing with the tangram. We expect that the students will gain the insight of area conservation in area measurement through the contextual learning activities.
3. The model is used to help in solving the problem. Gravemeijer (2004) define an emergent model as a model that cannot

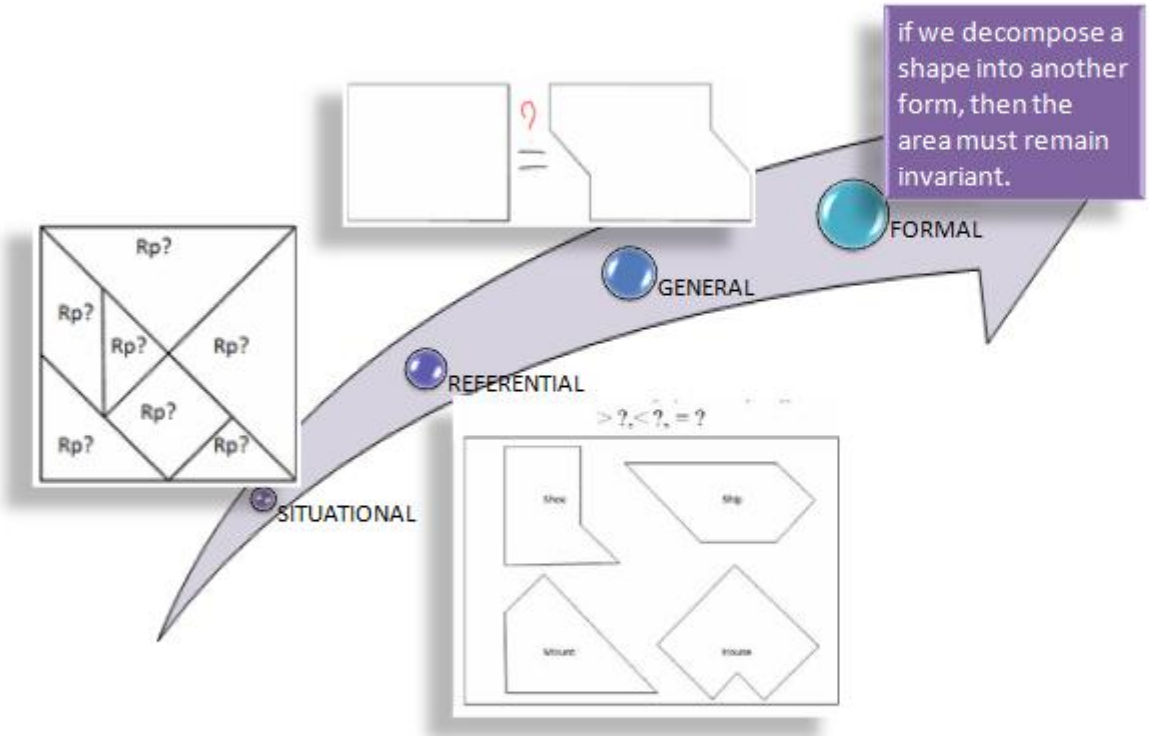
distinguished clearly with the concept which is being modeled. The resulted model from informal knowledge is evolved into formal mathematical concept. Furthermore, Gravemeijer define this process as the development of the *model-of* informal mathematical knowledge into *model-for* formal mathematical reasoning. Here, the model of tangram by using price is developed to reason with the concept of area especially for the big idea of conservation. Then, we expected that it will become the model for reasoning the concepts of area and help them to solve area-related problems. Gravemejer define the development of the model will pass through 4 levels (see picture 2.1). Those levels are described as follow:

1. ***Situational level***

   The informal model will arise from the contextual problem given in the beginning of model development. The interpretation of the model is really bounded into the context given as the background of the problem. Here, the use of the “Pricing the tangram pieces” context is considered as situational level modeling

2. ***Referential level***

   The model becomes available to be used for other context. In referential level, the interpretation of the model is no longer bounded with the former context. Here, the flexibility usage of tangram pricing model in determines which shape has the same price or not. The *model-of* the activity is formed.



Picture 2.1. The Emergent Model of Tangram Pricing Model on Area Conservation

3. General level

The Mathematical concepts start to appear as a part of the model which is no longer need a context to be interpret. The model involves the mathematical relationship to derive its meaning. Here, the conjectured relationship between the price of a shape and its area start to arise as the generalization of tangram pricing model. The *model-for* mathematical reasoning emerges

4. Formal level

When the reasoning become independent from the model means that the formal level is obtained. A new structure of mathematical reality is formed and ready to be used for reasoning. Here, the discussion of area conservation becomes salient as the main goal of the learning sequence.

Nowadays, there is a big movement on mathematics teaching and learning in Indonesia. The RME approach that provides many opportunities to develop a meaningful mathematics activity is starting to be adopted in Indonesia. We, then in ongoing process, try to develop local instruction theory which is aimed to develop *Pendidikan Matematika Realistik Indonesia* (PMRI)

F. Tangram-Mozaic Puzzle: Meaningful PMRI Activities to Help the Students' Mismatch Condition in Learning the Concept of Area Conservation

The teaching and learning of geometry in Indonesia is conducted in a formal way. The tangram with its properties in which we can freely manipulate it in various meaningful ways might break the formal way of learning geometry in Indonesia and make a fun and meaningful activity for children. The mismatch situation where the teaching of the teacher is not in the same level as the learning of the students could be minimize by making the children really engage to play with the classroom activities.

The Van Hiele level, which consists of a sequence of 5 consecutive levels, is developed with experience of the students (van Hiele, 1985). The levels are respectively developed from visualization, analytic, abstraction, formal deduction, and mathematical rigor. Thus, for 3rd grader, the use of visualization level as the basic level in the teaching could help the students' understanding. This visualization level serve as informal knowledge by which we used in the use of tangram as visualization media on the concept of area conservation. Then, there is no room anymore for the students to always memorize the concepts of geometry.

The tangram pieces as geometrical plane figures can support the students' understanding of plane figure properties as well as their understanding of plane figures' area especially the idea of area conservation as the main focus of this study. The use of PMRI approach insists the use of suitable context and problem that really ask the students to use the concept of area conservation in solving the problem. The activities should start from the students own creations by which we then discuss each other solutions to come to the class conclusion. The design introduces the model to help the students to reason about area by comparing the price as mean as comparing the area.

CHAPTER III
METHODS

A. Research Approach

Implementing the RME theory and van Hiele level of geometric thinking to bridge the students into conceptual understanding rather than memorizing formula results this study which aimed to describe how we can help the students understanding of the concept of area conservation. The main concern of the researcher is to improve education in Indonesia by designing innovation and change on development of PMRI especially on the topic of area measurement. To reach this objective, we, then, decide to conduct design research for answering the question. Here, we not only can elicit the way of the methods work well, but also we can understand why they work well. The resulting outcome of this study is a sequence of learning activities which is crucial as the answer of the research question (Bakker & van Eerde, submitted). Accordingly, we are interested in designing a learning activities sequence using tangram in the scope of RME theory regarding van Hiele level of geometrical thinking to help the students to understand the concept of area conservation.

Gravemeijer & Cobb (2006) define 3 phases of conducting design experiment. These phases are described as follow,

1. *Preparing the Experiment*

In the preparation phase, the first thing that we prepare to design such learning activities that can really help the students is literature review. Relevant present knowledge about a topic should be studied first (Bakker & van Eerde, submitted). This preliminary phase has a goal which is formulating hypothesized local instruction theory which will be carried out and tested in the experiment (Gravemeijer & Cobb, 2006). From those literatures, we hope that we can find the information about what is the students' difficulties dealing with the area measurement topic and what might be the solution for them to develop a good learning sequence. Then, the sequence of learning activities as well as the hypothesized students' thinking and students' strategies, which is called the Hypothetical Learning Trajectory (HLT), is resulted. This HLT will be carried out and tested during the design experiment and could be adjusted to the students' actual learning.

2. *The Design Experiment*

After finishing the design of HLT in preliminary phase, it is tested and improved in experimental phase. According to Gravemeijer & Cobb (2006), the design experimental phase is aimed both to test and to improve the hypothesized local instructional activity. In this phase, the learning sequence of 4 lessons developed in the previous phase will be carried out in teaching experiment. The teaching experiment will be conducted in cycles iteratively. In the first cycle, the experiment will be implemented in a small class consist of 6 students with the researcher itself as the teacher. This pilot experiment is aimed to adjust the HLT with actual situation in Indonesia classroom before it will be carried out in the actual classroom experiment. The second the next cycle is the actual classroom design experiment. The learning sequence will be implemented by the teacher in his/her class. There will be discussion before conducting the activities to make sure that the teacher understand about what the researcher expects and after the lesson as reflection on what have happened as means for adjusting and improving the HLT.

3. *Retrospective Analysis*

The retrospective analysis runs thoroughly during the design experiment after each lesson is conducted by comparing the HLT with the Actual Learning Trajectory. According to Gravemeijer & Cobb (2006), this analysis is aimed to contribute on the development of the local instructional theory. From the data obtained from the lesson series, we consider which hypothesized learning occurs and which one does not. This is deliberately done to improve the HLT and to answer the research question. it is important to keep track how the HLT being adjusted and improved to make the analysis more transparent and tractable (Bakker & van Eerde, submitted).

B. Data Collections

We are going to conduct the study in Indonesian School in Surabaya, Indonesia. The subject participants of this study are 3rd grade students. Applying design research means that we will conduct consecutive phases in which we collect data. The phases will be described as follow

1. Preparation Phase

The data collection in preparation phase will be conducted in 2 methods. They are using teacher interview and classroom observation. Here, the data collection is aimed to gain an insight of both students' prerequisite knowledge and classroom situation by which the HLT for the first time being adjusted.

We try to elicit that information directly from the teacher. Hence, we conducted interview with the teacher to gain insights into his/her opinions, feelings, emotions and experiences of his/her class teaching. Video recording is conducted to get more authentic data collection. We also collect the data from the class observation. In addition with interview, observation provides an opportunity to compare what we have already had from interview. It does not rely on what people say about what they do or they think. During this observation, we draw the direct evidence to observe what actually happens in the classroom activities on both the classroom activities and the teacher and the students. Video recording is conducted to get more authentic data collection.

During the data collection in preparation phase, we also improve the learning activities according to the available data. We will revise as long as there is new information that can be used for developing the learning.

2. The First Cycle

Firstly, we conduct pre-test to know the students' starting points for 5 3rd graders with various achievements chosen from the teacher's recommendation. They are asked to solve several problems of area measurement. Here, the first draft of the learning activities is adjusted regarding the students' actual starting points. Secondly, the lesson sequence will be carried out fully by the researcher in the class of those 5 students. During the lessons, observation will be conducted to elicit data from the activities about the students' thinking. Here, video recording is conducted to get more authentic data collection and the researcher will be the teacher for this first cycle. The data collected here is mainly aimed to revise the teacher guide such that it is ready to be used by the teacher for the Second Cycle. In the end of this cycle, there will be post-test which is aimed to know how the students will solve the problem compare to the pre-test after experiencing a whole learning sequence. In detail, the questions in pre-test and post-test are mostly the same. However, certain questions are in the different difficulty but still in the same objective. This is aimed to find out where the learning takes place.

For the first cycle, the researcher's observations of the lessons and the written works of the students are documented in reports. The reports will emphasize on classroom discussions, reflections and interesting individual contributions by students to support whether the conjectures really fulfilled or some issues come out to revise the learning activities in some extend. These data are required to qualify the HLT before carrying out to the whole class experiment.

3. *The Second Cycle and the Next Cycle*

The pre-test in the beginning of the second cycle is also aimed to know the students' starting points and with which the HLT is adjusted. Then, we carry out the final design of our learning activities into real classroom teaching in the 3rd grade class with the actual classroom teacher and see how it will work. In the class observation, there are 2 focuses of the data collection which are on the class discussion and the focus group discussion. During the lessons sequence, observation will be conducted to elicit data from the activities to support whether the conjectures really fulfilled or some issues come out to revise the learning activities in some extend. Here, video recording is conducted to get more authentic data collection. The data collected here will be the basis conclusion to answer the research question and improve the Instructional Theory. The actual teacher will be the one who responsible of the learning activities in the whole meetings. In the end of this cycle, there will be post-test which is aimed to know how the students will solve the problem compare to the pre-test after experiencing a whole learning sequence. In detail, the questions in pre-test and post-test are mostly the same. However, certain questions are in the different difficulty but still in the same objective.. This is aimed to find out where the learning takes place.

Furthermore, we will focus on recording one group in the class as the focus group which will be recorded by additional video tape. The discussion with the teacher is needed to decide which group could have interesting outcomes on how they react to the learning activities in the class. It is important to have a focus group here that we cannot capture and analyze all of the students in the class, but by focusing only on one group, we will have a complete reaction of what students do during the teaching.

The data collected from this second cycle will be analyzed and used to final revise of the HLT. Also, to answer our research question, how our activities could help the students' understanding of the area conservation in area measurement.

This cycle of teaching experiment would be conducted until there are no significant differences between the last study and the study before the last one. This condition suggests that the learning trajectory is established which guarantee the reliability of this research. Also, the cycle should be stopped if the goal of this research is reached.

4. Validity and Reliability

Dealing with the data reliability, data registration is important to be considered. The interview will be recorded using video tape tool as well as the class observation. The triangulation can be used as references where we analyze the data which are relate with the students learning process (see APPENDIX A5 for the collected data) from interview and class observation along with the students' written work. This triangulation by method contributes the internal validity of the data. We also try to maintain the tractability of the method as clear as possible to follow by giving the clear description on what we do for collecting the data like providing the interview and observation schemes and also the Hypothetical Learning Trajectory will be guidance for collecting the data. The written product like students works on pre-test and post-test, poster, and worksheet will also be captured as data registration without any change to increase the reliability.

The expert review on how we can collect what we want to measure will also be considered as an effort to improve the validity of this study. In addition, interventions in the design research have better ecological validity because the place where the interventions are implemented is in the actual schools environment (Bakker & van Eerde, submitted). The results from this design research would not have to be translated much compare to experimental research one.

C. Data Analysis

Data collection phase results the student work, video of the class, interviews, questionnaires and tests which are still the raw materials before ready to be analyzed. In analyzing the data, we do retrospective analysis in which the observations will be compared with the conjectures on the HLT. The result of the analysis will lead to revisions and improvements of the both unit and the theoretical and empirical foundations. Detail of the analysis for each phase will be provided as follow:

1. *Preliminary Teaching Experiment (First Cycle)*

The data of the first cycle *pre-test* will be analysed to determine starting points of students' knowledge. The analysis will include both a quantitative analysis and a qualitative analysis. We do quantitative analysis of the number of correct answer. Beside, we categorize the students' work according to the use of strategy and symbolizing as a qualitative analysis of the strategies students have used. The analysis is also aimed to know whether the students understand about the questions or not.

After we conducted the first trial, we must analyze the data from observation and do retrospective analysis. Firstly, we will watch the video registration of the small group where the preliminary teaching experiment would be conducted to select interesting fragments where the learning occurred as conjectured or unexpected outcomes occurred. The chosen fragments will be transcribed and used along with the students' written works and field notes to test the HLT. Here, we must really consider which activity should be removed or revised or, even, add a new activity. We also must pay attention to the written teacher guide because it should be described quite clear for the teacher. Then, we ready for carrying the new design into big scale class.

In analyzing the data of the *post-test*, firstly, we will do the same things like in *pre*-test analysis where we do quantitative and qualitative analysis. This will result description on both the students' answers and strategies. Then, the results of *post-test* and *pre*-test will be compared to see what students have learned. The purpose of this comparison is to investigate students' development in understanding the concept of area measurement and their strategies in solving problems in area measurement topics after they experience the tangram activities. The result of the comparison will be enrichment of the analysis of the teaching experiment which is aimed to draw a conclusion.

2. ***Teaching experiment (Second Cycle)***

The data analysis of the *classroom observation* and *teacher interview* will result an insight of the classroom situations including the overview of the teacher and the students where the lessons will be conducted. Before starting the classroom experiment, those results can be used when discussing the lessons with the teacher to make sure that the teacher really understands about the learning design. In addition, they also could be used to elaborate the teacher guide.

The data of the second cycle *pre-test* will be analysed to determine starting points of students' knowledge. The analysis will include both a quantitative analysis and a qualitative analysis. We do quantitative analysis of the number of correct answer. Beside, we categorize the students' work according to the use of strategy and symbolizing as a qualitative analysis of the strategies students have used. The analysis is also aimed to know whether the students understand about the questions or not.

The resulted videos from most lessons in the final teaching experiment which are also observed and recorded on audio and/or video tape by the researcher and an assistant are analyzed consecutively. The data analysis of the *classroom observations* will start by watching the video-registrations chronologically. Firstly, the video registrations of the whole group will be examined to get an overview of each lesson. Then the observations of the focus group will be examined. The observation reports focus on various points of attention which were identified for each lesson in advance: advanced reasoning, reflections, sudden moments of insight, expected obstacles, matters of attitude, reactions to tangram game activities etc. This categorization has facilitated the sorting and listing of classroom activities for each of these points of action, enabling us to elicit trends which cannot be detected from written work alone.

In analyzing the observation, we must really consider the fragment in which conjectured activities are occurred or not. Here, after the fragments are transcribed, we compare what we can elicit from the fragments of teaching experiment on both video recording and the students' works chronologically. This is conducted to test the conjectures in the HLT. We also need to generalize how our activities could help the students understanding the concept of area conservation. To support the validity and reliability of the data analysis in analyzing the data, discussions with colleagues and supervisors are conducted beside they can also help to understand the data more.

In detail, while implementing hypothesized learning, the researchers confront the conjectures activities with the actual learning activities that are observed. This reflection should be done after each lesson because it may lead to changes to the original plan for the next lesson. The results of the analysis will be used to draw conclusions, to answer the research question and to revise the original HLT.

3. *Validity and Reliability*

It is expected that there will be sufficient data to see how the tangram activity could develop the students to understand area conservation and by which could help them in understanding area measurement in order to validate the research internally or the HLT validation. It includes the video recording of the whole lessons series, students' works, video recording of the interview, and field notes. Furthermore, the data triangulation of different methods of data collections will also be added for the internal validity. For external validity, transferability will be seen as the generalization of this research as means of implementing the research into populations which has similar characteristic.

We will maintain the tractability, as external reliability, as high as possible by retrospective analysis. This means we will provide clear descriptions of how the teaching experiment is happening, how we interpret it, and how we give conclusion (transparency). Furthermore, to gain more reliable analysis, we will discuss the interpretations of the data resulted from this research with the colleagues and supervisors. This inter-subjectivity discussion is aimed for common ideas on data collection and better understanding on the interpretation.

CHAPTER IV
HYPOTHETICAL LEARNING TRAJECTORY

A. Background Theory of the Hypothetical Learning Trajectory

The first thing that we have to prepare to design such learning activities that can really help the students is literature review. There are a lot of researches on how the students learn about area measurement (e.g. Kordaki and Balomenou, 2006; Kospentaris, Spyrou & Lappas, 2011; Piaget & Inhelder, 2001). Beside, the research on how geometry topic should be taught (e.g. van Hiele, 1985; Freudenthal, 1991). From those literatures, they suggest that.

In my HypotheticalLearning Trajectory, area conservation becomes the main concern. Piaget (1960) defines three level of development of Students' understanding of area measurement concept. The first level is elementary conservation. Then, the linear measurement of area is the second level. The last level is calculating the area (see page 11). Furthermore, in the basic progression of area measurement, the concept of area conservation is really important (see also Kordaki & Balomenou, 2006; Kospentaris et al., 2011; Papadopoulos, 2010). Piaget (1960) said that the discovery of concept of conservation in early age is fundamental as the basic concept for further development of mathematical knowledge for the children. Hence, the researcher decides to focus more on the concept of area conservation.

Piaget (1960) defines understanding the concept of area conservation as a process of giving meaning to its different representations. In detail, the meaning that students can give to the concept of the conservation of area is closely related to the tools that they use, which will be underpinning my first 4 activities, and to the shapes on which they have to study, which will be underpinning my last 2 activities,. Nevertheless, in the reality, the mismatch situation on the teaching of area measurement leaves a gap between informal level, where the students still struggles, and the formal level, where the teacher trying to deliver the topic. Thus, implementing van Hiele perspective and RME approach could be promising solutions.

The Van Hiele level, which consists of a sequence of 5 consecutive levels, is developed with experience of the students (van Hiele, 1985). The levels are respectively developed from visualization, analytic, abstraction, formal deduction, and mathematical rigor. Thus, for 3rd grader, the use of visualization level as the basic level in the teaching could help the students' understanding. This visualization level serve as informal knowledge by which we used in the use of tangram as visualization media on the concept of area conservation.

The implementation of RME could bridge the gap occure between teacher and students. Gravemeijer (2004) define an emergent model as a model that cannot distinguished clearly with the concept which is being modeled. The resulted model from informal knowledge is evolved into formal mathematical concept. Furthermore, Gravemeijer define this process as the development of the *model-of* informal mathematical knowledge into *model-for* formal mathematical reasoning. Here, the model of tangram by using price is developed to reason with the concept of area especially for the big idea of conservation. Then, we expected that it will become the model for reasoning the concepts of area and help them to solve area-related problems. Gravemejer define the development of the model will pass through 4 levels). Those levels are described as follow:

1. ***Situational level***

   The informal model will arise from the contextual problem given in the beginning of model development. The interpretation of the model is really bounded into the context given as the background of the problem. Here, the use of the “Pricing the tangram pieces” context is considered as situational level modeling. The price is used as a realistic representation of area for the students because they could have reasoned that the price represent also the magnitude of an object. In the other words, it is clear for them to accept that to get more rice from grocery store they need more money.

2. ***Referential level***

   The model becomes available to be used for other context. In referential level, the interpretation of the model is no longer bounded with the former context. Here, the flexibility usage of tangram pricing model in determines which shape has the same price or not. The *model-of* the activity is formed.

3. ***General level***

The Mathematical concepts start to appear as a part of the model which is no longer need a context to be interpret. The model involves the mathematical relationship to derive its meaning. Here, the conjectured relationship between the price of a shape and its area start to arise as the generalization of tangram pricing model. The *model-for* mathematical reasoning emerges

4. ***Formal level***

When the reasoning become independent from the model means that the formal level is obtained. A new structure of mathematical reality is formed and ready to be used for reasoning. Here, the discussion of area conservation becomes salient as the main goal of the learning sequence.

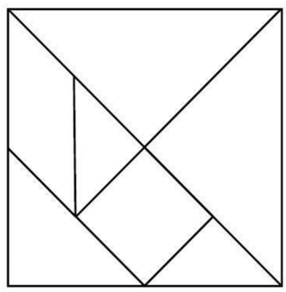
The designed Hypothetical Learning Trajectory (HLT) for the first implementation will be elaborated in the following pages. Simon and Tzur (2004) describes that HLT consist of 3 parts which are the goals of learning, the description of the learning activities and the hypothesized students thinking. In the end of the description, there is a plan for implementing these 6 activities in around 4-5 meetings with 2x35 minutes for each meeting. It is really dependent with the actual condition later. The guidance for the teacher (in the appendix) that I have designed is aimed for

4 meetings learning sequences. However, there is still possibility to compose and decompose the activities but still in the right order.

During the design research, there are also adding, revising, improving the learning design. After we have had information about what are the students' difficulties and the possible solution on them, we, then, try to elicit that information directly from the teacher. We also can use that information to develop the suitable learning activities for the students. Hence, we conducted interview with the teacher to gain insights into his/her opinions, feelings, emotions and experiences of his/her class teaching. Furthermore, after the implementation of the Learning Design, the Hypothetical Learning Trajectory is compared with the Actual Learning Trajectory to figure out which conjectures are perfectly applied or need revision in the end.

B. HLT - Free Explore Playing Tangram (Orientation- 1st Activity)

Students are given a free opportunity to play with mosaic puzzle. We expected that the students could do composing, arranging, and shaping easily through this activity. They form a variety of shapes from the tangram. The students are also asked to trace the shape they drawn.



Picture 4.1. A set of Tangram

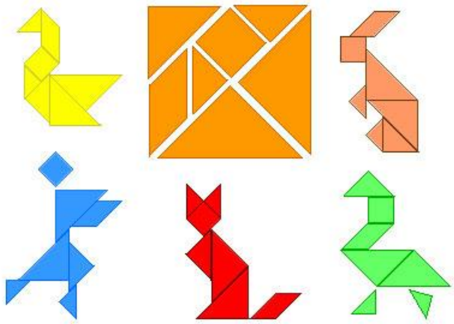
Learning Goal

1. The students are be able to play with tangram in composing, arranging, and shaping the tangram pieces.

Learning Activities and Conjectured Students' Learning

TASK 0 (INDIVIDUAL)

For warming up tasks, the students will play freely with the mosaic puzzle. Each of them could make many forms of puzzle shapes but then they are asked to choose one of them and trace it in a blank paper



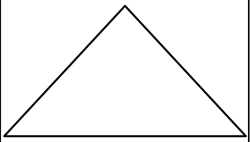
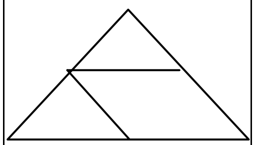
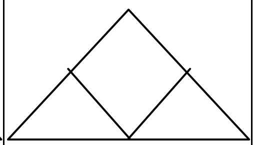
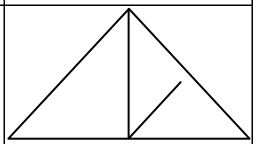
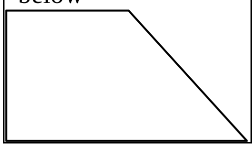
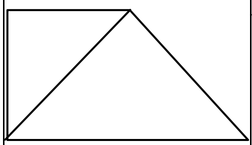
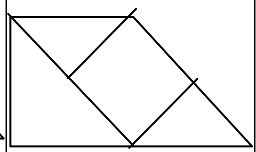
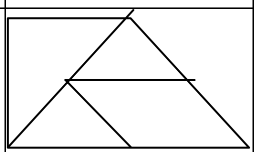
Source: http://www.bcps.org/offices/lis/models/Tangram%20Gr1/images/headingpicture.jpg
Picture 4.2. Possible shapes of tangram

The students are expected to draw a silhouette of his/her puzzle shape. In addition, the teacher can ask some of them to tell something about the shape on which they have been working. Then, they can also draw an illustration of what they have already made using those tangram pieces.

TASK 1(INDIVIDUAL)

Then, the students will be asked to form a given silhouette of geometrical shape using several pieces. The tasks are designed from using only 2 pieces into using more pieces. Here, the teacher tries to make the students to get used to play with the tangram. Moreover, we hope that the students will encounter the notion of area conservation in this sub-activity. Below is the example of the shape in the task. However, even they have similar constructions; the students will probably consider that those are different regarding the orientation of the construction.

Table 4.1. Example of Playing Shapes

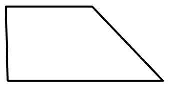
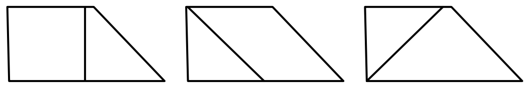
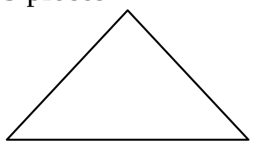
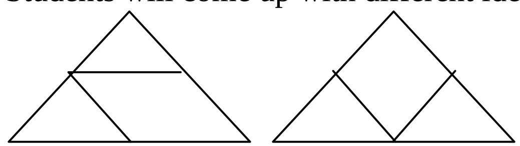
The task	Possible answer 1	Possible answer 2	Possible answer 3
Use three pieces to construct the shape below 			 And some others possibility
Use three pieces to construct the shape below 			 And some others possibility

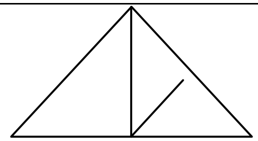
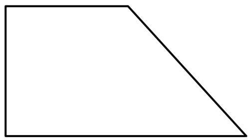
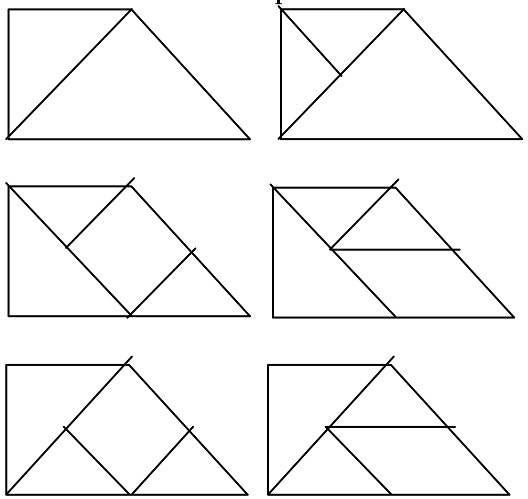
Here, the students are expected to deal with the idea with the conservation in the concept of area. By knowing that the students could use different pieces to make a same shape or even they can also use different number of pieces to make a same shape, we hope they will start to grasp the idea of area conservation.

The mosaic puzzle potency gives an opportunity to the students to experience a rich and stimulating activity in geometry. They deal directly with several geometric features. Angles, sides, symmetry, and equality are some geometric features that the students experience through the hands-on activity using tangram. The students would not realize if they are studying but playing puzzle game. They will start their observation of area concept in informal way before they enter more formal phase on it. In the end, as an introduction activity, we hope that the students will get used to play with tangram in composing, arranging, and shaping.

The shape that the students have traced in the first task will be used for the upcoming 2 activities. One optional activity is challenge games which will be conducted after all of the students form their own shapes. They will challenge another student by exchanging the shapes and try to figure out the construction. Another activity that uses the traced shape will be conducted later. This one will focus on the task which asks the students to compare their shapes which one is the biggest. But, I will not discuss this activity here because it is in the 3rd activity of my design.

TABLE 4.2. THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Guidance for teacher
Constructing own shape using some pieces	The students are accustomed to play with tangram	The students will try to construct their own shape by adjusting side by side and the angles as well.	Teacher should check whether the students are familiar with the tangram, The teacher can start by telling the story about what tangram is and what you could do with tangram
Constructing own shape using more pieces and record it	The students are freely construct their own shapes using more pieces	The students will try to construct their own shape by adjusting side by side and the angles as well.	Teacher can ask about what is the name of your shape? Or why do you like that? Could you draw your design of tangram in a paper? Elicit the students' reasoning on the geometrical feature
Constructing a given shape using 2 pieces (1A)	The students are able to construct a certain shape using 2 pieces 	Students will come up with different idea  Guidance is needed for the students who have difficulty in solving it.	The teacher guides the students' investigation of the problem. The teacher can ask about which side of the pieces fit with the side on the shape Asking about which construction is the biggest is proposed to know how far the students understand the area conservation
Constructing a given shape using 3 pieces (1B)	The students are able to construct a certain shape using 3 pieces 	Students will come up with different idea 	The teacher guides the students' investigation of the problem. The teacher can ask about which side of the pieces fit with the side on the shape The teacher can ask the students to look back to previous activity; can they extend the result of it?

Activity	Learning Goal	Conjectured of Students' thinking	Guidance for teacher
		 There will be another forms similar with previous construction but different orientation Guidance is needed for the students who have difficulty in solving it.	Asking about which construction is the biggest is proposed to know how far the students understand the area conservation
Constructing a given shape using 3 pieces (1B)	The students are able to construct a certain shape using undetermined pieces 	Students will come up with different idea  There will be another forms similar with previous construction but different orientation Guidance is needed for the students who have difficulty in solving it.	The teacher guides the students' investigation of the problem. The teacher can ask about which side of the pieces fit with the side on the shape The teacher can ask the students to look back to previous activity; can they extend the result of it? Asking about which construction is the biggest is proposed to know how far the students understand the area conservation
Challenging game (optional)	The students are able to do partition using the pieces	A student challenge another student to exchange the traced shaped and try to figure out the construction of it	In the end of this 1st activity, we expect that the students able not only to construct a shape from pieces, but also to do partition on a shape into pieces

C. HLT - Selling the Tangram Pieces (2^{nd} Activity)

This is the design of my second activity of my learning sequence in introduction to the area concept. Here, we would like to guide the students to build the model of tangram with price from the context given to reason with.

Learning Goal

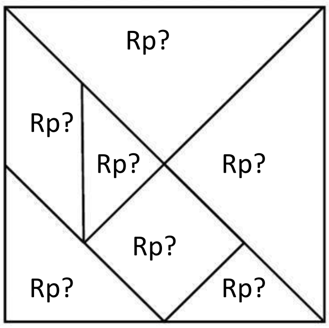
2. The students are be able to build the model of tangram with price from the context given to reason with
   a. The students are able to determine the price of each piece using their own strategy
   b. The students are able to argue about the strategy that they used to find the price
   c. The students are able to understand the money model to reasoning about the area

Learning Activities and Conjectured Students' Learning

TASK 1 (GROUP)

SELLING THE TANGRAM PIECES

Giving context "Selling the Tangram Pieces"



Picture 4.3a. Pricing Tangram

After the students have fun playing tangram, they are invited to think about a toyshop of teacher's brother who sells a tangram puzzle. The price for a complete set of tangram puzzle is Rp 40.000,-. The owner of the toyshop also wants to sell the tangram puzzle for a piece, because he figure out that many children who have bought a set of tangram will lose one or more pieces of the tangram on the upcoming months. The teacher then asks the students to help his/her brother to put prices in every different piece.

During the group discussion, the teacher should guide the students by keeping them in the right track. However, the teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like "why do you choose this way?"" could you explain why it is possible?", or "could you explain to your friends about what have you done?" The teacher should also encourage discussion in a group rather than just following a leader in a group.

The teacher should make sure that the students discuss this matter carefully. The students are asked to put their solutions on a paper for presentation session.

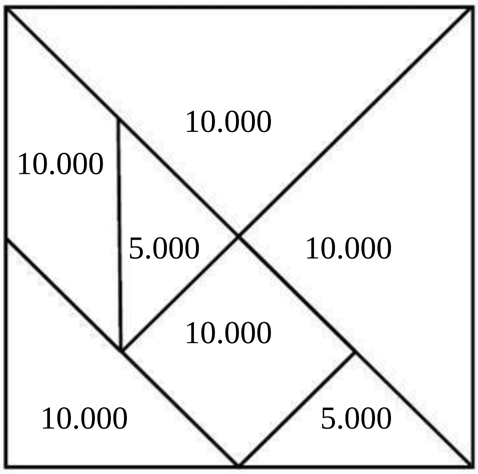
TASK 2 (CLASS DISCUSSION)

Each group will present their solution in a poster in front of the class. After the students find out the price of each piece, we expect that some of them make a list of those pieces by which the teacher can use it as the discussion starting point in math congress. They will share how they find the price for each price and discuss the possibility of various solutions from one of them by presenting their answer on a poster. In presenting their own solution, we expect that the students will be able to argue about the strategy that they used to find the price.

Here, the conjectures of how the students will react to the problem and how the teacher guides them. The price for each tangram piece should respect to the size of the piece. Here the expected solution from the students:

1. Notice that 2 of the biggest triangle are taking half of the whole tangram set, we hope that the students can come up with the idea that those two triangles are worth Rp. 20.000,-
2. Notice that those 2 biggest triangle which worth Rp. 20.000,- are same size (they see in the triangle grid that both of them consist of 4 grid triangles), we hope that the students can come up with the idea that each of them is worth Rp. 10.000,-
3. Notice that 1 medium size triangle is taking half of the biggest triangle (a medium size triangle consist of 2 grid triangles), we hope that the students can come up with the idea that it is worth half of biggest triangle price, Rp. 5.000,-
4. Notice that 1 square is same with the medium size triangle (a square consist of 2 grid triangles), we hope that the students can come up with the idea that it is worth as expensive as the medium size triangle, Rp. 5.000,-

5. Notice that 1 parallelogram is same with the medium size triangle (a square consist of 2 grid triangles), we hope that the students can come up with the idea that it is worth as expensive as the medium size triangle, Rp. 5.000,-
6. Notice that 1 smallest triangle is taking half of the medium size triangle (a smallest triangle consist of 1 grid triangle), we hope that the students can come up with the idea that it is worth half of medium size triangle price, Rp. 2.500,-



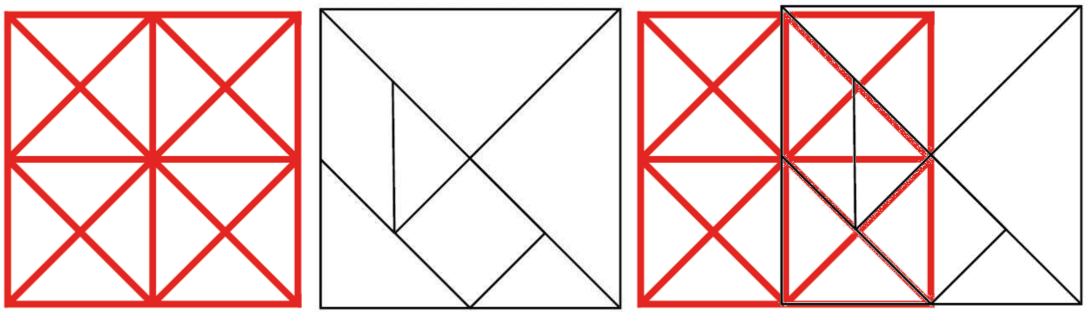
Picture 4.3b. Already Priced Tangram

Another strategy to reason in putting price in each piece is using overlapping strategy. Here, pupils will put smaller piece on the bigger piece and see what is the comparison. However, by using this strategy, pupil will not be able to come to the price of each piece because the known price is for the whole tangram set. Nevertheless, this strategy is quite something if many students will come up with this strategy. The teacher can assist them by giving a clue about the price of one of the piece, for example the price for the square.

There is possibility that the students will combine those two strategies in order to faster compare by overlapping some pieces in each. The will make halve the initial shape to halve the price as well until they could calculate the price for the small triangle. Then, using overlap strategy, they sum up 2 small triangles to form a square of a parallelogram.

We also consider that the students will possibly just only estimate and guess the price of each piece. They just mention the price from what they see visually. However, this strategy could not guarantee the correctness of the answer. There are 2 estimations that students could do. The first one is the students just divide Rp. 40.000,- by 7 because there are 7 pieces of tangram and estimate the result. Here, the teacher could ask whether it is reasonable for those different size pieces has the same price. The second is the students will regard the size but they decide the price roughly. Here, the teacher could ask them to sum up all of those prices and see whether it has also summed into Rp 40.000,-. Then, the teacher could bring this idea to the first strategy by asking the students to do hands on activity on the provided tangram pieces.

Other strategy but not essential for now is using gridlines. If it is the case, either they use square gridlines or equilateral-triangles gridlines, both of them are not suitable for this kind of tangram. The teacher could provide an isosceles triangle grid respecting to the tangram puzzle to help the students in reasoning about how to put a price in each tangram piece. The area can also be explored by using grid paper as the basic for comparing each other piece. Here, the students can use grid strategy to solve the problem. However, the use of grid paper will be given later if students still cannot solve it using the conjecture.



Picture 4.4. The Use of Gridlines in Tangram

Regardless the size, perhaps, the students will argue about the material of the pieces will influence the price. The teacher should lead that the price is related to the area of the pieces, or at least got the idea of it. However, it is not necessary to force the student to understand this relation. If so, just proceed to the next activity.

TASK 2-A (CLASS DISCUSSION)

In this activity, pupils will also discuss about the cheapest and the most expensive piece. We expect that the students will grasp the notion of area by comparing price of each piece. Here, they use money model to reason about area. What the students should understand is that at least they agree on bigger the shape more expensive the price will be. We expect that the students will reason that the size determine the price of the shape.

This topic of discussion may occur in the previous discussion. The teacher should guide into a conclusion that bigger the size of the piece more expensive the price is. There is also a possibility that the students will discuss that some different pieces have the same price. The class may discuss this matter, but if the time is not enough the teacher could skip this topic of discussion for the next activity.

TABLE 4.3, THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Guidance for teacher
Finding the price of each piece from the context given	The students are able to determine the price of each piece using their own strategy	From the given context, the students may come up with some strategies: 1. Using halving/ doubling 2. Using rearrangement-overlapping 3. Using estimation a. Divide 40.000 by 7 b. Estimate the price regarding the size roughly 4. Using gridline For the students who have difficulty in determining the price, teacher guidance is needed	Here, the teacher responses respect to the student's ways: 1. Teacher can ask the students about 2 pieces which are one of them is twice of the other and let the students to reason about it. 2. The teacher can assist them by giving a clue about the price of one of the piece, for example the price for the square. (To progress in the problem, the combination of these 2 strategies seems to be used by the students) 3. The main thing is that teacher could bring this idea to the first strategy by asking the students to do hands on activity on the provided tangram pieces. a. the teacher could ask whether it is reasonable for those different size pieces has the same price. b. the teacher could ask them to sum up all of those prices and see whether it has also summed into Rp 40.000,-. 4. The teacher can provide guidance for the students to make an appropriate gridlines. The teacher should make sure that each group prepares their presentation in a poster that will be conducted in the last session
Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price	Regardless the size, perhaps, the students will argue about the material of the pieces will influence the price	The teacher should lead that the price is related to the area of the pieces, or at least got the idea of it.
Determine which piece is the most expensive or the cheapest	The students are able to understand the money model to reasoning about the area	The students will reason that the size determine the price of the shape	Actually, this task can occur in the previous task when they are looking for the price of each piece. The idea of this task is getting insight about the price of each piece before using the price in the next activity about finding the price of the shapes that they have already traced in the first activity

D. HLT - Comparing Whose Shape is More Expensive (3rd Activity)

After the students solving the problem, they are expected to know the notion of relationship between size of the tangram piece and its price. However, the teacher should not assume that all of the students have already understood. The following mini discussion is provided to help the students understanding of the relation between the size and the price

Learning Goal

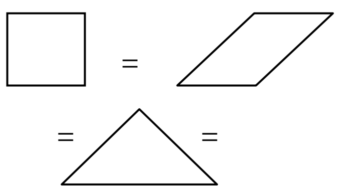
3. The students are be able to understand the relation between the price and the size..

Learning Activities and Conjectured Students' Learning

TASK 1 (PAIR – FOLLOWED BY CLASS DISCUSSION)

Discussion on 3 different shapes with the same price.

From the second activity, finding the price for each piece, there are 3 different shapes which have same price.



Picture 4.5. The Same Price Pieces

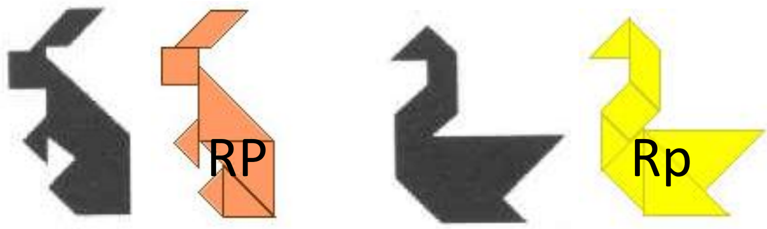
The discussion will be about how those three different shapes have same price. Here, the conjectured students' thinking

1. The students will cut and paste while comparing the pieces one into another
2. The students will consider that those shapes can be divided into 2 triangles

The discussion should be guided into a conclusion that those three shapes have the same size or area. The students will accept that there is possibility of different shapes have the same price. The decomposition of 2 triangles could be considered as the reason of area conservation in this mini discussion; this is the conclusion that should be addressed in teacher guidance in this class discussion.

After the students experience the small scale area conservation, then, the students will be asked to do bigger area conservation. We asked the students to play with the game of whose shape is the most expensive. They are asked to use the tangram pieces that have been put price on it to determine the price of a silhouette trace or the sketch of their made shape.

TASK 2 (INDIVIDUAL)



Picture 4.6. Pricing my Shape

In finding the price of each shape which is consist of several tangram pieces, it is a common sense that the students will just sum up the price for each pieces to get the total price. If it does not work, then the teacher can ask them to calculate the total price by adding one by one piece. The teacher should make sure that they do calculation correctly.

Another strategy is that they use the fact that the price of the whole pieces is Rp 40.000,-. Hence, if they use all of the pieces in making their own shapes, then they may just say its price is Rp 40.000,-. Or, if they do not use all of the pieces in making their own shapes, then they may just say its price is Rp 40.000, - minus the price of unused piece(s). The teacher should guide these students about which piece's price should be subtracted and proceed to the prices.(Part Whole relation)

TASK 3 (CLASS DISCUSSION)

The following activity is about math congress. They are asked to paste their traced shapes, which are already put price on it, in front of the class. If they use all of the pieces of the tangram, they will find that the price of a shape that they made will be same. However, there will be a possibility that the students did not use all of the tangram pieces and figure out that the price for his/her shapes will be less than the one who use all of the pieces. Here, the teacher could trigger a discussion how to relate the use of price in understanding the area. The examples of discussion topic that can be used in the discussion are:

1. All of the shapes have same prices.

   The teacher guides the discussion on why different shapes have the same price. The discussion will evolve around what characteristic that make those different shapes have the same price. Here, we conjecture that the students will come up with the idea of area. Then, the teacher may just continue to the next activity using the shapes given by the teacher. However, it not necessarily if the students get to the idea of area. If so, there will be more activity regarding this issue in the next activity. If only several different shapes with same price occur, the following discussion should be administered

   Using this issue as the discussion topic will trigger a need of a notion that is constant. Considering 2 shapes with different form but have same price, the students will try to define a characteristic of those 2 shapes which are same. Here, the teacher should guide them carefully until they can come to the word "area". During the discussion, there is a possibility that the students consider about the size of the material used to make a piece of the tangram pieces. In this case, teacher can lead those students to the situation in which they can imagine themselves made the tangram pieces by cutting a plate of wood. If they must buy a plate of wood, then they can notice that bigger the plate will make the price more expensive.

2. How to determine which one is cheaper or more expensive?

Using this issue as the discussion topic will trigger a need of a notion that becomes parameter with which the students can define a shape is cheaper than the other. Considering 2 shapes with different price, the students will try to define a parameter of those 2 shapes which can be reference for them to determine which shape is cheaper or more expensive. Here, the teacher should guide them carefully until they can come to the word "area". During the discussion, there is a possibility that the students consider about the size of the material used to make a piece of the tangram pieces. In this case, teacher can lead those students to the situation in which they can imagine themselves made the tangram pieces by cutting a plate of wood. If they must buy a plate of wood, then they can notice that bigger the plate will make the price more expensive.

3. Different size but same number of pieces

Posting this issue is quite important because otherwise the students will only reason why 2 different shapes have the same price is because they have the same number of pieces. To help the students from this misconception, the teacher could provide the idea from the previous activity when they deal with different shape that not always if he number of the formed pieces are the same the shapes are in the same size

In the end of this activity, we expect that the students will grasp the notion of "area" which is used as the parameter by the students to determine which shape is cheaper or more expensive. Or, in another words, understanding the relationship between the price and the used pieces, or beyond this, the relationship between the price and the size

TABLE 4.4, THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Guidance for teacher
Mini discussion	The students understand the relation between the size and the price among pieces.	1. The students will cut and paste while comparing the pieces one into another 2. The students will consider that those shapes can be divided into 2 triangles	Teacher should guide the discussion into the idea of the decomposition of 2 triangles could be considered as the reason of area conservation
Finding the price of each own shapes	The students are able to determine the price of each own shapes using their own strategy	1. They sum up the price for each pieces to get the total price 2. They use the fact that the price of the whole pieces is Rp 40.000,-. 3. if they do not use all of the pieces in making their own shapes, then they may just say its price is Rp 40.000,- minus the price of unused piece(s)	If the students do not have any idea how to proceed, then the teacher can ask them to calculate the total price by adding one by one piece. The teacher should guide these students about which piece's price should be subtracted and proceed to the prices (Part whole relation). To progress in the problem, the combination of the strategy seems to be used by the students The teacher should make sure that each student prepares their picture of shape which has already been put price.
Math Congress	The students are able to argue and to reason in understanding the relationship between the price and the used pieces, or beyond this, the relationship between the price and the size	Here, the teacher can trigger a discussion how to relate the use of price in understanding the area. The examples of discussion topic that can be used in the discussion are: 1. Different shape but same price? 2. How to determine which one is cheaper or more expensive? 3. All of the shapes have same prices.	The teacher should lead that the price is related to the area of the pieces, or at least got the idea of it. The teacher guides the discussion on why different shapes have the same price. The discussion will evolve around what characteristic that make those different shapes have the same price There is possibility that the students will only reason using the number of pieces, not the size. If so, the teacher can show the students that different size but same number of pieces

E. HLT - Compare the Given Shapes (4^{th} Activity)

The follow up activity from comparing shapes using tangram-pricing model, here the researcher develop the activity such that the students will be able to understand the concept of area conservation without heavily rely on the use of price as the bridge

Learning Goal

4. The students are be able to understand the one important properties area which is area conservation (if we compose and decompose the area into another form the area remain invariant) .

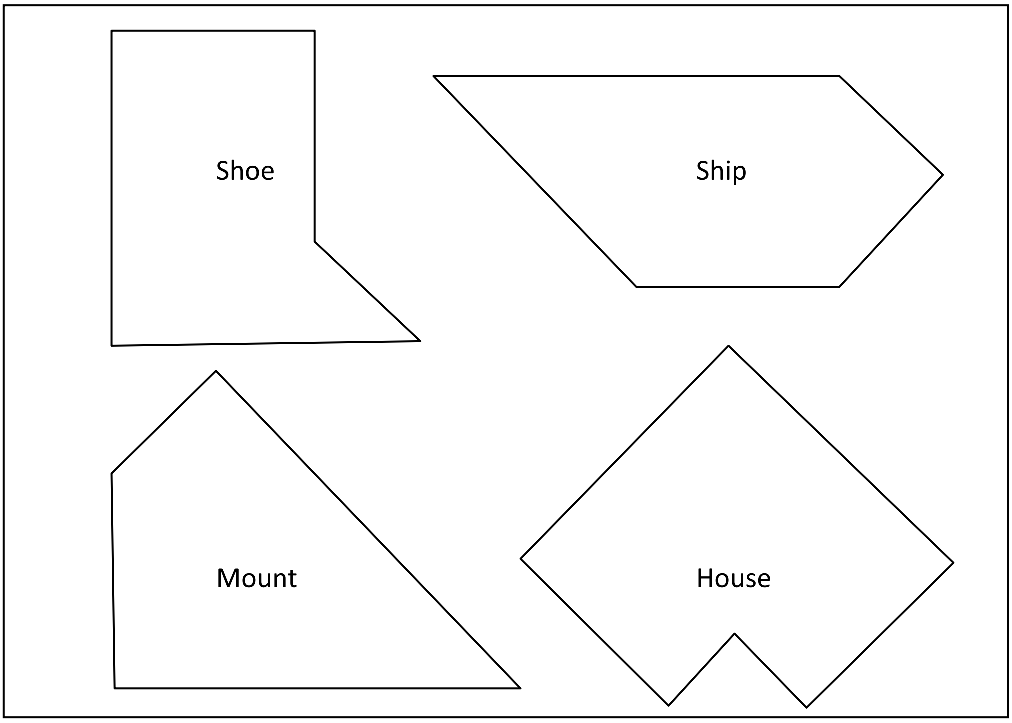
Learning Activities and Conjectured Students' Learning

TASK 1 (GROUP DISCUSSION)

Similar with the previous activity, but in this work, student will play with the shapes given by the teacher in group. The shapes used in this activity are designed feasible for the students to solve in such a time as a group. Here the shapes:

During the group discussion, the teacher should guide the students by keeping them in the right track. However, the teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like "why do you choose this way?", " could you explain why it is possible?", or "could you explain to your friends about what have you done?". The teacher should also encourage discussion in a group rather than just following a leader in a group.

$$> ?, < ?, = ?$$



Picture 4.7. The Shapes to be Compared

The teacher should make sure that each group prepares their presentation in a poster that will be conducted in the last session.

TASK 2 (CLASS DISCUSSION)

Each group will present their solution on a poster in front of the class.

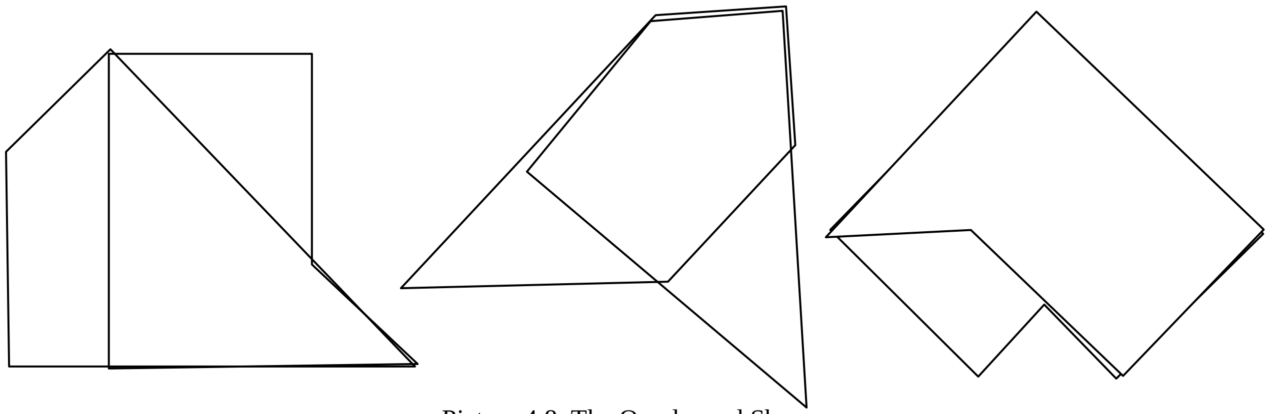
They are asked to determine which shape is cheaper or more expensive or same price. Here, the students are really guided to use the notion of area in each shape as the parameter to determine which shape is cheaper. By only using the fact that the shape can be arranged using several pieces, we conjecture that they are expected not to use the reasoning using the total of the price but using overlapping strategy. Here, the money model will not be a main tool to reason with. They will notice from how the pieces are arranged into those shapes. Slowly, the teacher guides the students to accept a notion of area as one of the parameter of shapes. However, if the concept of area does not come out in the discussion, the teacher may post question the role of area and its relation with the price.

Here the conjectured students solutions of the problem

1. ***Estimate the shape by using overlapping strategy to compare***

We anticipate the idea of estimation here. Some of the students might see the shapes are similar in certain parts. So, they will try to compare those similar parts and try to put one shape on top of another shape and see how the discrepancy occurs there.

Here, the example of how the students try to compare using overlapping strategy.

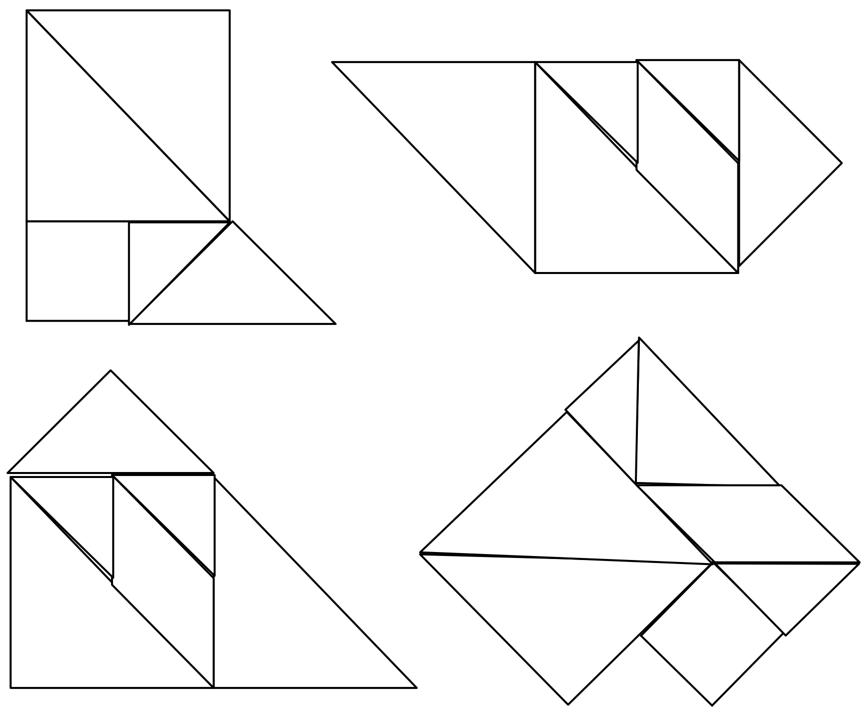


Picture 4.8. The Overlapped Shapes

By comparing in such a way, they will find the arrangement. However, the students do not use the price to reasoning that will make the discussion about area could occur. Here, the teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. And, area conservation occurs here. The teacher can ask the price of each shape later. The students can use what they have already got from this overlapping strategy and use the pieces for the differences in each shape that they could calculate the price followed by the same discussion. The main idea is that when the student could understand the idea of those shapes could be composed and decomposed by looking at the left over part while overlapping each other and still the area remain invariant.

2. *The use of tangram pieces to reconstruct and put price to compare*

Here the example of how the students will arrange the tangram pieces.



Picture 4.9. The Restructured Shapes

They will, then, calculate the price and then make the order of the shapes from the cheapest to the most expensive one. After the students finish making the order, the discussion on what determine whether the prices are the same or not is conducted. We expect that the students will use the term of area as the reasoning. By comparing in such a way, they will find the arrangement. The students might not use the price to reasoning but the fact that those shape consist of same pieces will be their reason to determine this shape has the same price with another. Here, if the discussion of the area of the shape does not occur, the teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. The teacher could ask them to compare the size, and then the discussion will follow in a line with the first conjecture.

In the end of this activity, we expect that the students will understand and accept that every shape has what so called area by which we can determine which one is bigger or smaller. As the result, they could understand that those areas if we compose and decompose the area into another form the area remain invariant (area conservation) (1st goal of my design)

TABLE 4.5, THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Guidance for teacher
Compare the Given Shapes	The students are able compare the given shapes using the idea of area or price	Here the conjectured students solutions of the problem 1. Estimate the shape by using overlapping strategy to compare 2. The use of tangram pieces to reconstruct and put price to compare	The teacher should also encourage discussion in a group rather than just following a leader in a group. The teacher should make sure that each group prepares their presentation in a poster that will be conducted in the last session
Math Congress: Share the group work	The students are able to argue about the strategy that they used to compare the given shapes	During the discussion, these ideas are conjectured from students thinking: 1. Compare using overlapping, they will find the arrangement. However, the students do not use the price to reasoning that will make the discussion about area could occur 2. calculate the price and then make the order of the shapes from the cheapest to the most expensive one	Respective to what the topic of the discussion, here is the guidance for the teacher: 1. The teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. And, area conservation occurs here 2. If the discussion of the area of the shape does not occur, the teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. The teacher could ask them to compare the size, and then the discussion will follow in a line with the first conjecture.

F. HLT - Covering Area by Using Only One Kind of Piece of Tangram

After the students have already understood about the concept of area conservation, then, the learning activity moves to the area measurement issue. Here, the starting of measuring activity as covering like what the students have done in the previous activities but, here, they are only allowed to use one kind of tangram pieces

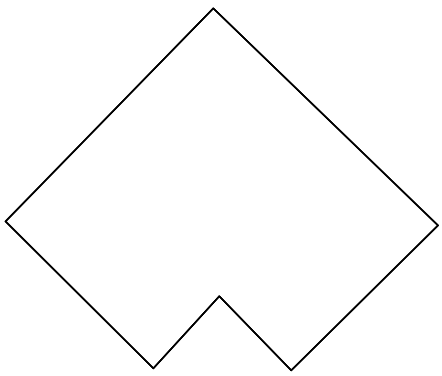
Learning Goal

5. The students are be able to do covering the area as measurement activity by using different kind of tangram piece.
   a. The students are able to use one kind only tangram pieces from the available possible tangram pieces to fully cover the given shape.
   b. The students are able to understand the relation covering the shape using triangles or squares only as measurement activity.

Learning Activities and Conjectured Students' Learning

TASK 1 (INDIVIDUAL)

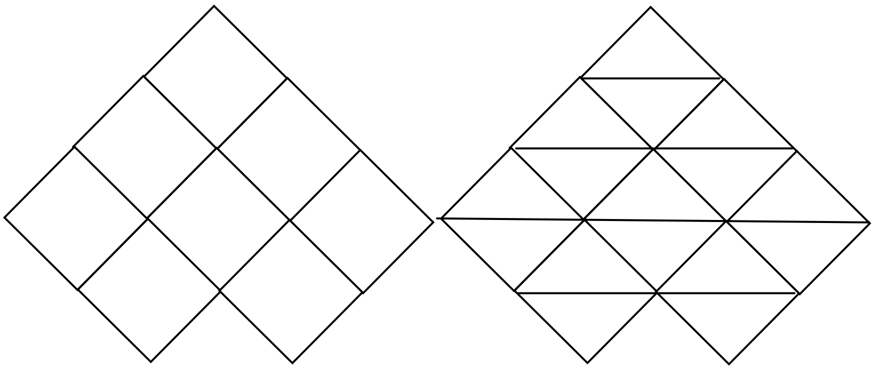
Using only one kind of shape of tangram piece, the students are asked to use it to cover a certain area.



Picture 4.10. The Shape which must be explored

Because they use only one kind to cover the shape, the fact that the number of pieces is not enough will make the student to think hard about it. If he students could not overcome this problem and ask to the teacher, the teacher may ask them to borrow another group pieces or provides more pieces for them. There is also possibility that the students will not have difficulties dealing with insufficient number of pieces to be used to cover the shape. they could draw the pieces on the shape as reference and proceed to cover the whole.

In covering the shape, we conjecture that they will use small triangle shape only or square shape only. They can use it as area unit, by the chance to enter the notion of area unit as the properties of area. The shapes given in previous activity are also design for this activity of which the students can play by covering those shape using only one kind of shape. The teacher should make sure that the students cover the shape completely.



Picture 4.11. The Possible Covered Shape Using One Kind only

TASK 2 (CLASS DISCUSSION)

There will be a discussion in the end of the activity. If, in fact, all of the students use squares only or triangles only, the teacher could ask them how if using the triangles or squares. We can put them in conflict to discuss each other solution. Here, the teacher should insert issues about the students' preferences of which shape they like and why. The idea of one square can be divided into 2 triangles should be highlight because we would like this idea to proceed on the next activity. Another idea is that the square and the small triangle are suitable for covering such shapes.

Here, the teacher could also ask why others pieces are not suitable to cover such shape.

In the end of this activity, we expect that the students will understand covering the area as measurement activity by using different kind of tangram piece may result same area.

TABLE 4.6, THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Guidance for teacher
Covering area by using only one kind of piece of tangram	The students are able to use one kind only tangram pieces from the available possible tangram pieces to fully cover the given shape.	The students have difficulty dealing with the insufficient number of pieces to be used: If not, then here the possible solutions in covering the shapes by the students: 1. the students may use square pieces only 2. the students may use triangle pieces only	The teacher could ask them to borrow another group pieces or provide them with additional pieces the teacher should make sure that the students cover the shape completely
Math Congress: Share the group work	The students are able to understand the relation covering the shape using triangles or squares only as measurement activity.	Some idea may occur in this class discussion: 1. one square can be divided into 2 triangles 2. the square and the small triangle are suitable for covering such shapes	If all of the students use squares only or triangles only, the teacher could ask them how if we use the triangles or squares and compare the results The teacher could also ask why others pieces are not suitable for covering such shape

G. HLT - Playing with the area of polygon (6th Activity)

This activity is designed such that the students are able to use what they have explored in the previous activities which are the concept of area conservation and measuring area by covering. Then, the teacher may post a problem of area conservation from rectangle to parallelogram using tangram to remind the student understanding of area conservation from previous lesson..

Learning Goal

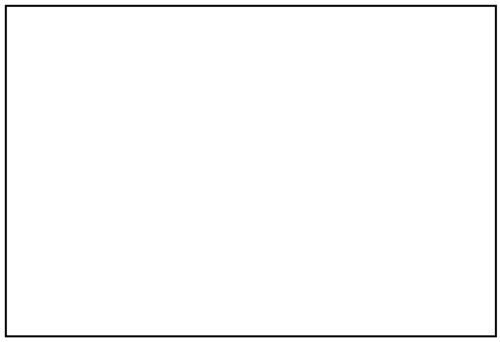
6. The students are able to apply the concept of conservation to measure the area of polygon.

Learning Activities and Conjectured Students' Learning

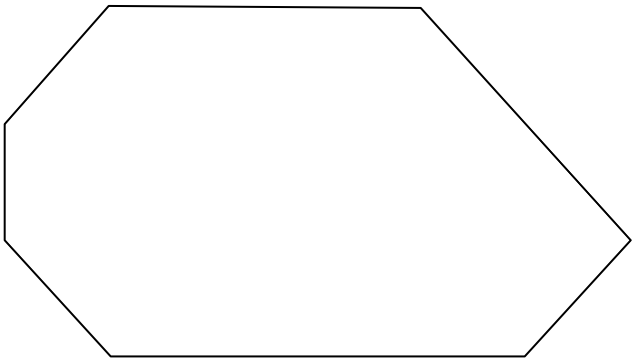
TASK 1 (GROUP)

The last activity is how the students will use what they have got to measure polygon shapes. The shape is designed in such a way that the students will have difficulty due to the number of the squares and triangles are not enough to cover by using only one type. They will need to use both shapes respectively to measure the area of the polygon.

During the group discussion, the teacher should guide the students by keeping them in the right track. However, the teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like "why do you choose this way?"" could you explain why it is possible?", or "could you explain to your friends about what have you done?" The teacher should also encourage discussion in a group rather than just following a leader in a group.



1st polygon



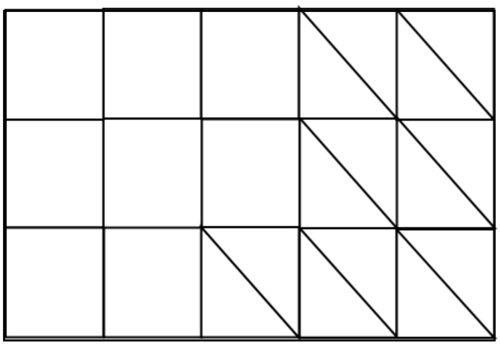
2nd polygon

Picture 4.12. The Polygons Which Must be Explored

TASK 2 (CLASS DISCUSSION)

There will be a discussion in the end of the activity. The conjectured students' solutions are described as follow:

1. 1st polygon
   - If they cover using pieces one by one, here an example of what the students will do:



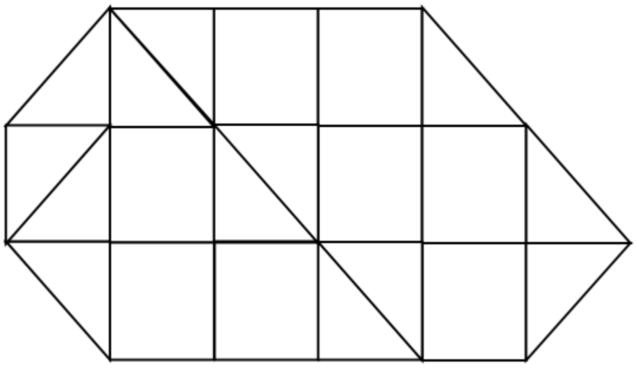
Picture 4.13. The Possible Covered 1st Polygon

Then, they will count the inscribed pieces 8 squares and 14 triangles

- They realize that there will be 5 columns and 3 squares are in a column, so, they find out the number of squares is 5x3 = 15 squares
- They realize that there will be 5 columns and 6 triangles are in a column, so, they find out the number of triangles is 5x6 = 30 triangles

The students should realize that those answers are actually the same, and then the discussion about the relation between triangles and squares should do it.

2. 2^{nd} polygon
   - If they cover using pieces one by one, here an example of what the students will do:



Picture 4.14. The Possible Covered 2^{nd} Polygon

Then, they will count the inscribed pieces 8 squares and 13 triangles

- They realize that there is a connection of this 2^{nd} polygon with the 1^{st} polygon. They could say that if the students do rearrangement on the 1^{st} polygon, the number of pieces on the 2^{nd} polygon is the same as on the 1^{st} polygon but minus one triangle.
- They realize that there is a connection of this 2^{nd} polygon with the 1^{st} polygon. But then, the students cut the 1^{st} polygon into several pieces and rearrange it such that those pieces form the 2^{nd} polygon.

Here, the area conservation is used in doing area measurement. If the students do not come out with the second conjecture, that the students could use the 1st polygon to measure the 2nd polygon, the teacher could ask them to compare the 2nd polygon with the 1st polygon. The discussion also guided into the concept of area conservation.

In fact, the students must use more than one kind of tangram shape, because of the limited number of squares or triangles if they decide to only use one kind of piece. Various combinations of triangles and squares will occur; the teacher could put them in conflict to discuss each other solution. Here, they can mention the area of this polygon as triangles only, squares only, or both (for 1st shape). Then, we continue with the shape which has odd number of triangles (for 2nd shape). They should regard about the fact that a square is same area with 2 triangles and use it in the next discussion of the measurement of the polygon shape, and then the teacher asks the students to state the area using squares only.

Discussion on different solutions of arrangement between groups should be carried out clearly. Difference arrangement of one shape still result same measurement.

In the end of this activity, after the students use the tangram pieces (triangles and squares) to cover the polygon**, we expect that they will understand the idea of conservation to measure the area of polygon -- the conservation of area -- (2nd goal of my design).** By what I mean, when measuring an arbitrary polygon which is hard to be measured, the students could rearrange the shape into another shape which is easy to be measured.

TABLE 4.6, TABLE OF THE HLT OF THE SIXTH ACTIVITY

Activity	Learning Goal	Conjectured of Students' thinking	Guidance for teacher
Playing with the area of polygon Measuring polygon shapes	The students are able to use the pieces from the previous activity to measure given polygon shapes	The students may use several strategies: 1. Using all pieces to cover up the shape one by one 2. Using only several pieces and do estimation then 3. Using the previous shape that has been covered and use it as starting point to cover	The teacher should guide the students by keeping them in the right track The teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should also encourage discussion in a group rather than just following a leader in a group.
Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price. In the end, the students are able to use the concept of area conservation in measuring the area of a shape	During the discussion, here the conjectured topic discussed: For the 1st polygon 1. they cover using pieces one by one 2. They realize that there will be 5 columns and 3 squares are in a column 3. They realize that there will be 5 columns and 6 triangles are in a column, For the 2nd polygon 1. they cover using pieces one by one 2. They realize that there is a connection of this 2nd polygon with the 1st polygon	For the discussion on the 1st polygon, the relationship between triangles and squares. The students should realize that those answers are actually the same For the discussion on the 2nd polygon, if the students do not come out with the second conjecture, that we can use the 1st polygon to measure the 2nd polygon, the teacher could ask them to compare the 2nd polygon with the 1st polygon. The discussion also guided into the concept of area conservation. Discussion on different solutions of arrangement between groups should be carried out clearly. Difference arrangement of one shape still result same measurement. **In the end of this activity, we expect that the students will understand that, in measuring the area, the area of certain figure is remain the same even though we use different arrangement (the conservation of area). (2nd goal of my design)**

CHAPTER V
RETROSPECTIVE ANALYSIS

In this chapter, the retrospective analysis of the data collected from the first and the second cycle is provided. The analysis will be divided into 2 parts, A summary of the analysis of the first cycle (see the appendix for the complete analysis) and the analysis of the second cycle.

The first part describes about the analysis from the first cycle. In this part, we will describe chronologically starting from the students' pre-knowledge from the pre test. The following analyses are the students' learning processes from the data of the learning activity and the post test. In the end of this part, we provide the important remarks from the first cycle that the researcher used to improve the HLT and how to improve it. Here, the main purpose of the first cycle is described which is adjusting the raw HLT that the researcher had designed empirically before applying it in the real class situation. The second part describes about the analysis from the second cycle.

Similar structure with the first part, the researcher describes the analysis of the second cycle chronologically. The difference between there 2 parts is that the focus of the analysis is more likely about the mathematical idea in the real classroom situation. That is the reason also why the researcher provided the analysis of the classroom situation from the classroom observation and teacher's interview in the beginning of the second part as additions for the students' pre knowledge section beside from the pre test data. The discussions on the mathematical idea during the experiment are elicited from either focus group or the remaining students in the class.

A. The Result from the First Cycle

The first cycle was conducted in 4 meetings. The following table shows the schedule in the first cycle.

Table 5.1, The Schedule of the First Cycle

Data Collection	Date
Pre test	16 February 2013
Activity 1	23 February 2013
Activity 2	23 Februari 2013
Activity 3	14 March 2013
Activity 4	14 March 2013
Activity 5	11 April 2013
Activity 6	12 April 2013
Post test	12 April 2013

The first cycle was conducted in SD Laboratorium Unesa. in detail, it was class 3B with bu Fitri as the homeroom teacher. The researcher selected 6 students from the class for the preliminary teaching experiment. The election of those 6 students was based on teacher recommendation on the students who have low, medium, and high achievement. They are Akmal, Joscha, Kezra, Syafira, Sisca, and Andika. This group has an active characteristic. They are quite talk active and have no difficulty in playing with game activity.

The preliminary teaching was done to implement the first adapted HLT and the first version of the teacher guide. The collected data will be analyzed to elicit how the teaching and learning might happen based on those both HLT and teacher guide. In the end, the HLT will be refined such that it is applicable in the real classroom situation as well as the teacher guide.

The retrospective analysis for the first cycle that had been conducted by analyzing the data from video, note, and students' worksheets results the fact that they have difficulty in reasoning using geometrical picture or drawing (for complete analysis see APPENDIX). The students could not easily compare given shapes. In fact, the use of tangram puzzle and pricing context during the learning sequence was quite helpful. They could use the price and calculate the total as the reasoning. However, the learning sequence still needs to be revised especially in measuring activity.

1. Important Remarks on the Students Pre-Knowledge

In the beginning of the 1^{st} cycle, the researcher conducted the pre test to elicit the students' initial position. The results of the pre test are very helpful in a way that the researcher can find out many valuable information of how the pupils will reacts with the learning sequence activities that has been designed.

The pre-test consisted of 2 main questions. The first problem is asking the students to compare 2 different rectangles but actually have the same area which have been cut into 8 equal smaller rectangles. Here, we would like to know whether the students could compare the shapes by using its partition because the use of tangram will be heavily rely on making composition of a shape. the second problem is asking the students to decompose and compose irregular shapes into a square.

This suggests that when comparing raw shapes (the shapes asked in the 2nd problem), most of them were failed to answer correctly because of the tendency of them to make a comparable shape, which is good, but ignoring the area inside the shape. This implies that they do not understand the concept of area conservation which is in line with what Piaget (2001), Kordaki (2006), and Papolodous (2011). This learning sequence that I have designed is precisely could help them to understand the concept of area conservation.

However, the result from 1st problem implies that if the pupils notice the idea that the shapes could be decompose into smaller pieces and those pieces can be compared easily, then they will compare which shape is bigger correctly. Thus, the use of tangram pieces as easy comparable objects that can be decomposed from the given shape would give benefit for the pupils to help them. The idea is that when the pupils cannot answer the comparing problem, by decomposing shapes into pieces using tangram pieces, they should be able to compare the pieces and figure out the correct answer in comparing those shapes. (see appendix C-2 – C-5)

2. Data and the Analysis of the First Cycle Learning Activities

The first activity was designed to give an opportunity for the students to get accustomed with the tangram puzzle. Various activities of composing, arranging, and shaping were experienced by the students easily. However, we expected that they would come out with more solutions as we conjectured. In fact, most of them had a tendency of finding similar solutions with what their friend did. (see appendix C-6 – C-17)

Lack of creativity in reasoning also occurred in the second activity. This activity was designed to guide the students to build the model of tangram with price from the context given to reason with. The context was about finding how to sell tangram in pieces if the price of a set of tangram was Rp. 40.000. The conjectured solutions from students were not taken place as expected. Using simple geometrical reasoning like comparing the pieces which is the area is twice of another could not easily be done by the students. They preferred to use numerical reasoning which made them to guess and check the price of each piece until the total amount of them was Rp. 40.000. Guidance from the teacher was quite intensive here. Yet, some of them still could not understand correctly about the idea of halving and doubling the shapes which was intended to guide them in entering the idea of area conservation. (see appendix C-18 – C-28)

In the third activity, the students dealt with various shapes constructed by each of them and discussed which shape is cheaper or more expensive. We expected, by experiencing this comparing activity, that they could notice that the concept of area determine how expensive or how cheap the price is. Even though not all conjectured solutions were taken place, the students successfully understood that there is a relation between price and size. Moreover, they also considered that bigger the shape more expensive the price. The idea of shapes whose pieces are the same but it is not necessarily mean that the shapes have the same size was not discussed thoroughly. (see appendix C-29 – C-36)

The fourth activity was intended to guide the students in using area conservation to compare the given area without heavily rely on the use of pricing model. This phase was designed to guide from ‘model of’ to ‘model for’. The fact that the students could not solve the problem unless they use the price as reasoning was undeniable. This suggests that more mini lesson should be added properly as the scaffolding activities. This addition successfully helped the students to do area conservation on the given objects in the next activity. They were able to compare the given shapes without using price as reasoning. (see appendix C-37 – C-49)

In the fifth activity, the students were invited to use the tangram pieces as measurement unit. The researcher expected them to be able in understanding the fact that, in conserving area, the use of different construction or measurement will result the same area. Choosing the best piece that could be used to measure a given shape was done correctly by the students. They also did not have difficulty in comparing which measurement was correct because either using square piece or small triangle piece was the same measurement. (see appendix C-50 – C-58)

The last activity was designed to guide the students to use their knowledge in area conservation to measure a given area. The shapes of polygon, actually, could be decomposed easily by the students after experiencing the series of conserving activities. Yet, they had difficulty in measuring the area using the tangram piece. Conserving one shape into another shape such that they could measure easily was not occurred. They deliberately covered the area of the given shapes by using the tangram pieces. Small mini lesson should be administered for the students to bridge the knowledge slowly. Asking them to measure directly without providing enough scaffolding was the main reason why they unable to perform area conservation. (see appendix C-59 – C-65)

The use of tangram as media to do area conservation was quite helpful for the students. The price model also provided an opportunity for them to understand that the area remain the same if we compose and decompose the shape into another shape by relating the price with size of the shape. The second, the third, and the fourth activities show that the students could do freely reasoning using price.

In the other side, the students had a tendency to use price as reasoning than using geometrical reasoning. In the second and the fourth activity, we can see that price model had become more salient than reasoning in geometrical area conservation. The activity to support the students' development of knowledge seems not enough. The scaffolding activities should be provided more extensively.

3. Conclusion from the First Cycle

The analysis on this study suggests that area conservation for 9-10 year-old pupils is a difficult task. The use of price context is really helpful to help the student in dealing with the notion of area conservation. However, unfortunately, they tend to still use mechanical approach to solve the problem than geometrical approach. By what I mean, they have difficulty in reasoning using geometrical picture or drawing. Nevertheless, they could use the price and calculate the total as the reasoning. This implies that the students were accustomed with mechanistic teaching approach. The conclusive standard procedure is heavily used by the students which result the fact that they could not solve the problem independently (Nelissen & Tomic, 1993). The context was also not considered as helpful idea in proceeding to solve the problem.

This finding is also parallel with the first 2 levels development of van Hiele's level of geometrical reasoning (see pg 16)s. The dealt with visualization level quite extensively. They did trial and error regardless the fact of geometrical fact of halving and combining shapes that can be used to help them in reasoning about the problem. However, they have a better performance in doing area conservation using geometrical reasoning when they deal directly with hands-on activity using geometrical figure (tangram puzzle) by giving the students the real feeling of area conservation. The use of tangram puzzle appends an experiential knowledge for them to enter the *analysis* level, second level of van Hiele's level.

The analysis also suggests that the use of scaffolding is important to help the students in measuring an area using the concept of area conservation. Asking a problem followed by supporting small activities as the scaffolding would help the students to understand the concept easily regarding the fact the students are accustomed with mechanistic teaching approach. This finding supports what Gagne believes (in Nelissen & Tomic, 1993) that, in mechanistic class; the students should experience a series of smaller progressive tasks before dealing with the complex one.

4. Summary Revision from the 1st Cycle

The first activity is about orientation activity that the pupils play with the puzzle, construct a given shape, and draw a shape on it. The problems in the first activity could be divided into 2 big ideas. The first is the problem in drawing activity. Here, the pupils had difficulty in drawing the shape that they had made. Some of them forgot the shape because it was broken, others just draw the simplified picture of the shape (for example, they made a house shape but they just draw a simple silhouette of house). This simplified made them harder to be recalled in the next activity. The teacher could help by changing to tangram into stronger one such that it would not break and ask the teacher to really help the pupils in drawing. The second problem is the limited number of the solutions. Each student only came up with one solution. It would be better if they could find more solutions which help them to get the variation of their creativity. Perhaps, providing fewer problems but the pupils could explore more on these problems is better.

The second activity in my cycle did not take place well. This activity is about finding the price of each piece of tangram from a given set of tangram. The common mistake in finding price is that the pupils did trial and error. The try a number for each piece and doing calculation again and again. This is done until the total price of the pieces is Rp. 80.000. About the learning goal for this activity, the pupils did not solve the problem well. The conjecture of how the pupils will solve the problem using tangram pieces was not occur successfully. The teacher needed to give them clue. For the revision, perhaps at the beginning, the problem should start by finding the price for the triangle only because it only involves halving method. Then, the problem will be continued for the remaining pieces, square and parallelogram. There will be additional guidance from the teacher like drawing a square and cut it into a half. Then, the teacher could ask the pupils the price of these halved portions which should be related to the picture of a set of tangram. In addition, perhaps the teacher could provide the worksheet with the table of the piece list. Such that they could be lead to consider another piece.

The third activity asked the pupils to put a price on their own shape that had been made in the first activity using the price that they had discovered in the second activity. In fact, what we got is that during the discussion the pupils could build an understanding of the idea of more expensive the price bigger the size. The discussion on several conjectured big idea were successfully occurred. Furthermore, from the discussion above we can see that the pupils were dealing with the following big idea: (1) Different shape but same prize and (2) How to determine which one is cheaper or more expensive? No essential learning trajectory revision is added here. The sequence of the activities in this session can bring the pupils to understand the relationship between size and price. However, one big idea was missed. The idea of shapes whose pieces are the same but it is not necessarily mean that the shapes have the same size was not discussed thoroughly. Perhaps, the teacher could provide 2 shapes and their constructions to trigger the discussion on this manner.

The fourth activity is about comparing the given shapes. The shapes have been designed such that they could use tangram to conserve the area and compare which one is bigger and no longer rely on the use of price. However, they still could not use the concept of area conservation directly. What they did was that they reconstruct the shape with their tangram and find the price for it, but once they had had help about how to divide the shape. They could progress in finding the price. This is not the intended solution because it shows that the model still could not evolve from *model of* to *model for*. This additional activity was quite helpful to achieve the learning goal that has been established before. The problem to measure certain shapes using a shape as preference is an operational activity of the big idea of using the concept of area conservation in measuring area. For the next cycle, this activity should be more elaborated and planned. Moreover, the use of money should be omitted to suppress bounded context situation.

The fifth and the sixth activities were originally designed for measuring a shape using a certain piece and use the idea of the area conservation on it. Unfortunately, the pupils did not do the conservation but focused on covering activity one by one using certain shapes. This took a long time that the class did not have enough time to discuss the conservation idea. The researcher decided to add an activity here where the pupils should do conservation in making the shape. Even though the pupils struggling with conserving task, they could do it using the pieces of tangram. Perhaps, we should omitted the covering activity and provide the pupils with readily covered shapes. This will save time for other discussion. Also, changing the problem with more scaffolding one might help because my former problem was quite big jump to the pupils who have not experienced measuring a shape by area conservation.

The significant changes here is that the merger of the fifth and the sixth activities into one activity. This new fifth activity is designed for guiding the students to use the idea of area conservation to measure certain polygons. The researcher will design an activity to invite the students to conserve a shape into another shape first. Then, the problem is about inviting the students to use area conservation to measure non-regular polygon.

B. The Result from the Second Cycle

The second cycle was conducted in 3 meetings excluding the pre-test and the post test meeting. 6 activities that had been planned to be implemented for the second cycle could not be done completely. In detail, we omitted the 3rd activity due to several reasons. The following table shows the schedule in this cycle.

Table 5.2, The Schedule of the Secod Cycle

Data Collection	Date
Pre test	30 April 2013
Activity 1	2 May 2013
Activity 2	2 May 2013
Activity 3	Omitted
Activity 4	15 May 2013
Activity 5	23 May 2013
Post test	24 May 2013

The second cycle took place in different school of the first cycle. it is SD Negeri Kebraon 1 Surabaya in the class 3 with bu Erna as the homeroom teacher. During the data collection, a focus group had been decided to be completely captured during the whole learning activities. This group consisted of 4 students. They are Sabrina, Ari, Lukas, and Sari. They are recommended by the homeroom teacher as the middle and high achiever in the class. Also, furthermore, this focus group which has a similar characteristic with the students in the first cycle is formed by considering the teacher’s advice. An active group which could have a lively discussion is expected from the focus group. The researcher also expected that they would have no difficulty in playing with tangram game activities.

. The second cycle was intended to capture the emerging of the mathematical idea from the informal situation from the students until they reach the formal level of area conservation. We could see that how the whole learning sequence could support the students learning processes.

The retrospective analysis for the first cycle that had been conducted by analyzing the data from video, note, and students' worksheets results the fact that in general, the students could decompose a shape based on decomposing tangram pieces and composehave difficulty in reasoning using geometrical picture or drawing. The students could not easily compare given shapes. In fact, the use of tangram puzzle and pricing context during the learning sequence was quite helpful. They could use the price and calculate the total as the reasoning. However, the learning sequence still needs to be revised especially in measuring activity.

1. Analysis from the Classroom Observation and Teacher Interview

In the following, we will discuss what the condition of the class is. The data we got from the classroom observation and the teacher interview. In the end we also provide how our revised design might work on this class.

a. Classroom Observation

The observation in cycle 2 was conducted to know how the teacher carries out the teaching and learning activities in her class. The focuses of this observation are about the teaching and learning process, how the RME use or not use by the teacher, and how the condition of the students. In general, the class is not accustomed with teaching and learning using RME approach. The observation was conducted in a 3rd grade class with 36 students. Most of the classroom activities are conducted using direct instruction without any discussion at all.

From the observation, the researcher notices that the teacher was accustomed with direct instruction. She also use question and answer strategy to the students in posing a problem. However, her preparation is not good that she just picked up a number of problems from a workbook. However, the students followed the teacher's instruction carefully. They listened to the teacher explanation and answered if the teacher asked a problem. Here, the students seemed to be more interested to answer the teacher's question. They competed each other to raise their hands and answer the problems. Unfortunately, they did not correct her teacher when she did a mistake in writing the answer on the whiteboard. Furthermore, they also seemed not to accustom to work in group.

Talking about how far the teacher experience about Realistic Mathematics Education, from the observation, we can conclude that the teacher did not use RME approach at all. The problem that she gave was a bare problem which is really has a bad connection with the students' life. It gave no meaning for the students about the problem. Furthermore, she also directly gave the students the formula of area without giving the students opportunity to build their understanding of area. In addition, the teacher mechanistically demonstrates how they should solve the problem by putting the number into the formula and do multiplication to get the answer. We can find no discussion at all that could bring the students' own creation to build the knowledge of area.

b. *Teacher Interview*

We did an interview to the teacher to elicit the teacher's experience in teaching in the class. The teacher is still not a public servant employee even though she has been teaching for 10 years. In general, she does not have much experience in various model and strategy in teaching. Most of her teaching is using direct instruction with simulation and demonstration sometimes. Furthermore, she does not know about what PMRI is. In addition, the teacher really believes that the use of formula, especially in area topic, is important.

From the interview, we could conclude that the teacher is not accustomed in conducting discussion activity in the class. The teacher said that she has been being a homeroom teacher since 10 years ago. Previously, she just teaches in the school only for art subject. She does not have variation in teaching subjects. Mostly, she does direct instruction to teach mathematics. Furthermore, she also said that discussion is a rarely happened in the class because the condition of the class seats is too narrow to be managed. However, sometimes the teacher also asks the students to work in pairs.

What is unexpected fact that we got from the interview is that the teacher does not know anything about *Pendidikan Matematika Realistik Indonesia* (*PMRI*), an Indonesian version of RME. The reason of why the teacher does not know anything about PMRI is that because her background is not mathematics. She is originally from art subject. When the researcher asked about what is the important issue in teaching area measurement topic, she believes that the use of formula is really important. Furthermore, she also claimed that if the students do not know or forget about the formula, then the students would not be able to solve a problem in the topic of area.

c. How my Design might Work on the Class

Implementing our design in such a class is not an easy task because the teacher and the students seem not to be accustomed with RME approach. The common problem in their class is a bare problem that can be solved by putting numbers in the formula without the need of discussion in building the knowledge. However, the students enthusiastic could be used as the main fuel for having a nice discussion type classroom. The teacher could ask them to express their own idea first by giving them a playful activity using tangram puzzle. The researcher believes that it could increase the students' confidence. Also, the use of contextual problem could bring a meaningful learning because they could do something in the world that is accessed by the students.

2. Data and the Analysis of the Second Cycle Learning Activities

The second cycle was designed to find out the mathematical thinkigof the students. How they will react toward our revised HLT will be presented in the following sections.

a. Pre-Test Analysis

The test was conducted to elicit the students' prior knowledge. The test was about 30 minutes done by 34 students in the class.

!st Problem - Comparing 2 Different Shapes by Decomposing the Shapes

Here the 1st problem

2 different cakes, cake A and cake B, are cut as the following, which cake is the biggest?

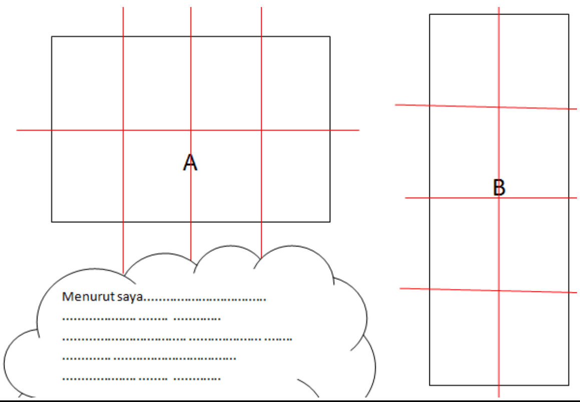


Figure 5.1, the picture to compare

The first problem is about comparing 2 different cakes but actually in the same size of area. Both of the cakes have been cut into 8 pieces which are also equal in size. The students answers are vary from those who answer that cake A is bigger than cake B or vice versa. There are also students who answer correctly that those 2 cakes have the same size. In general, the students regard the length and width in determining which cake is the biggest. However, there are also a number of students who are able to compare those shapes correctly by considering the decomposition of the shapes into equal number and size pieces.

16 students had answered that cake A is bigger than cake B. Some students who gave the reason why cake A is bigger than cake B, wrote that the cake A is wider than cake B. However, there are also many students who did not provide clear reason. For the correct answer, there are also quite many students who answered correctly. 13 students in the class wrote that those 2 cakes have the same area. There are students who reasoned that those cakes are cut into equal part with equal in size; hence, cake A is as big as cake B. Other students did not provide a clear reason. About the remaining students, 5 pupils answered that cake B is bigger than cake A. In general, they reasoned that cake

B is longer than cake A. Hence, they concluded that cake B is the biggest.

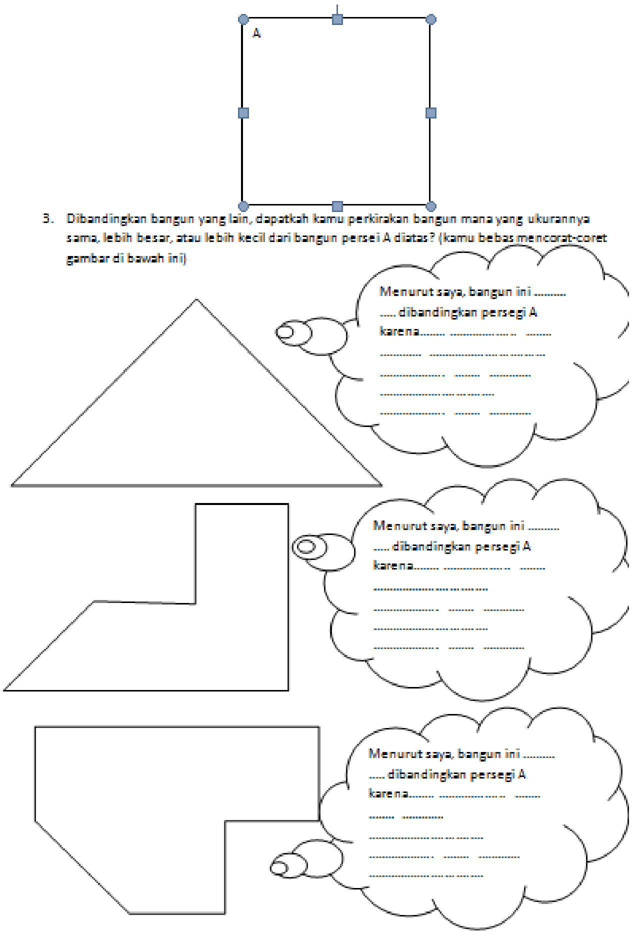
This suggests that the students are actually able to compare the shapes correctly by using the decomposition of shapes. If the pieces have already divided into congruent shape the students could compare the size of the whole shape correctly. They see each fraction has the same shape. When those same number fractions are combined into 2 different shapes they know that those shape (cake) just the same in size.

2ND PROBLEM – Conserving Shapes into a Square

The 2nd problem is on the following page

The problem asks the students to decompose the given shapes and compose it into a given square. We expect that the students would do mental cut by drawing the cut path on the picture of shapes into pieces and explain how to construct those pieces. The answers of the students show that the students are unable to do area conservation. None of the students are able to conserve the area of the three given shapes into comparable composition such that they could compare the shapes easily. Furthermore, none of the students do some drawing or sketch on the pictures of the shapes given.

Compare with another shape, can you guess, which shape has the same size, smaller size, or bigger size with this square A? (you are free to make a scratch on the picture to solve)



Picture 5.2, 2nd problem

The students' answers for this problem suggest that when comparing a raw shape that they must decided how to compare by them, most of them were failed to answer correctly because of they were unaware that they could make a same size shape of the given shapes by using the idea of area conservation. It is in line with what Piaget, Kordaki, and Papolodous thought that pupils in this age are having difficulties dealing with the concept of area conservation. Thus, we expect that after they experience the whole learning sequence that the researcher had designed they will be able to use the idea of area conservation in measuring certain area.

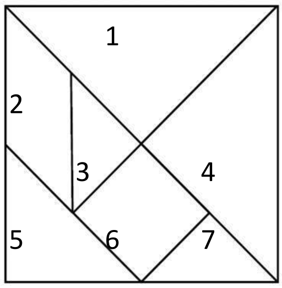
Remarks on Pupils' Pre-Knowledge in the 1st Cycle

This discussion suggests that when comparing shapes (the shapes asked in the 2nd problem), most of them were failed to answer correctly because they even do not try something significant like drawing or writing something to explain the shapes. This implies that they do not understand how to compare one shape with another shape. Furthermore, this also suggests that they do not know about the concept of area conservation which is in line with what Piaget (2001), Kordaki (2006), and Papolodous (2011). However, the result from 1st problem implies that if the pupils notice the idea that the shapes could be decompose into smaller pieces and those pieces can be compared easily, then they will compare which shape is bigger correctly. Thus, the use of tangram pieces as easy comparable objects that can be decomposed from the given shape would give benefit for the pupils to help them.

Hopefully, this learning sequence that I have designed is could help them to understand the concept of area conservation.

b. Free Explore Playing Tangram (Orientation Activity)

Students are given a free opportunity to play with mosaic puzzle. We expected that the students could do composing, arranging, and shaping easily through this activity. They form a variety of shapes from the tangram. The students are also asked to trace the shape they drawn. Before the researcher starts the analysis, we define the pieces as follow: big triangle (1, 4), parallelogram (2), small triangle (3, 7), medium triangle (5), and square (6) (see picture 5.3)



5.3 the pieces of tangram

Learning Goal

1. The students are able to play with tangram in composing, arranging, and shaping the tangram pieces.

Analysis On Learning Activities And Actual Students' Learning

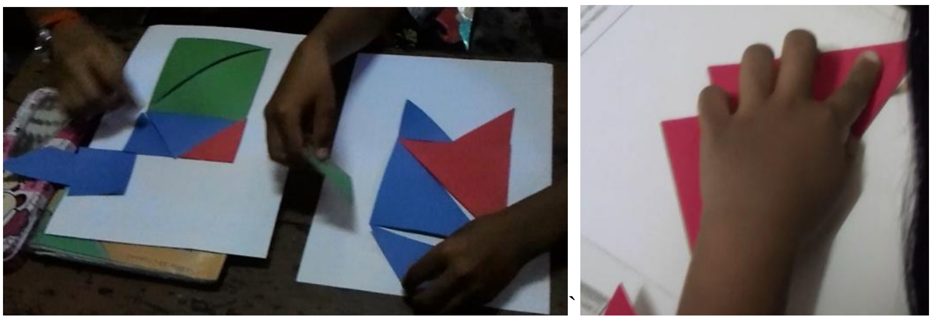
ACTIVITY	SUB-LEARNING GOAL
Orientation with the tangram	The students are accustomed to play with tangram
Constructing own shape	The students are freely construct

using more pieces and record it	their own shapes using more pieces

The teacher started the lesson by telling a tale of Grandfather Tang (see picture 5.4). The students listened to the teacher's story enthusiastically. They followed the teacher explanation about the origin of tangram puzzle. This story prepared the students to know what tangram puzzle is.

Picture 5.4, Teacher told a story of Tangram

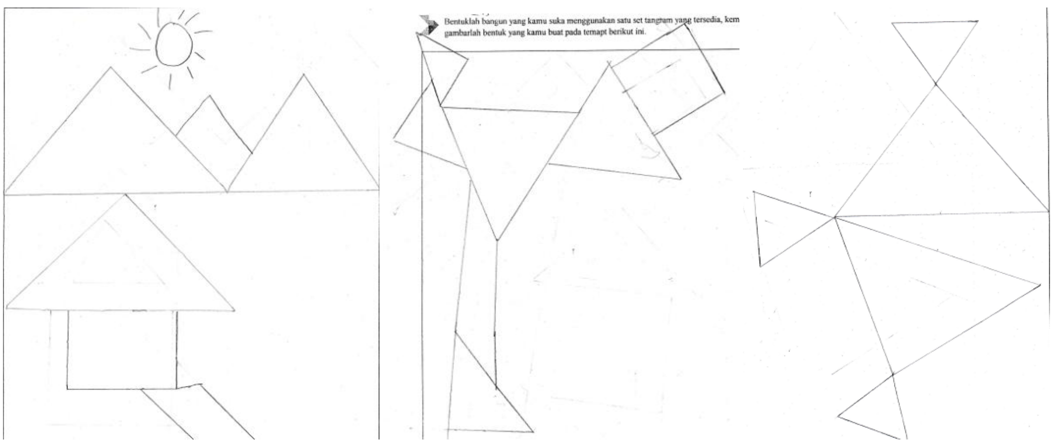
The first activity was that the students are asked to make own shape. They were freely playing by constructing a shape using a set of tangram. There were students who made a shape by overlapping one piece into another piece. Various shapes were made by those 32 students in the class.



Picture 5.5, The Students constructed a shape (a) and drew the Shape (b)

The problem here is that the students had difficulty in drawing what they had made into a piece of paper. There were students who drew directly by putting the pieces one by one on the paper and then they drew it. Others tried to make a sketch of the shapes that had been made (see 5.5b).

The shapes that had been made by the students could be distinguished into 3 different categories. They are mountain scenery shapes, concrete shapes, and abstract shapes. Most of the students made a shape which similar with a painting of a scenery of nature with mountain shapes. They made the 2 big triangles as the mountains (see picture 5.6a). Other students made a concrete shapes like human or house using a set of tangram (se picture 5.6b). The ones who made a human-like shape imitated the shapes that were presented on the worksheet. The last group, the minority group, made an abstract shape (see picture 5.6c).



Picture 5.6, examples of students' drawing: scenery (a), concrete (b), and abstract (c)

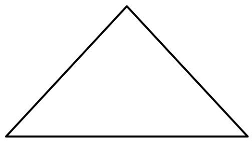
The pupils could draw the sketch for their own shapes. There were 2 ways in drawing the sketch occur at that time. Some of them took the pieces one by one on the paper and drew it. The rest of the pupils just drew directly onto the paper. However, the drawings were not visible enough. This result also happened in the first cycle. It suggest that drawing such picture of a shape for their ages is quite difficult. They could draw the shapes but the pictures are not easy to be recognized as the sketch of a shape. We cannot reconstruct which piece is used to form the shape by looking at the student's picture.

In the analysis of this sub-activity, we see that this sub-activity can support the students to play with tangram in composing, arranging, and shaping the tangram pieces especially for constructing students' own shapes. The teacher also supported the students in orientating with the tangram puzzle with a nice story telling. However, the drawing activity did not take place as expected. The students had difficulty in making a picture of the shape that they had drawn. The analysis also suggests that drawing into a piece of paper is not an easy task. Therefore, the activity in finding the price of each own shape could not be done because most of the pictures were not clear to be measured. The activity of making and drawing own shapes is really important for the students. As the orientation activity, it could help the students to be accustomed with the tangram puzzle and improve the students’ creativity in constructing a shape which is important for the next activity.

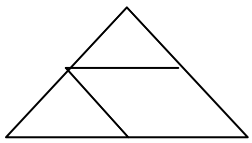
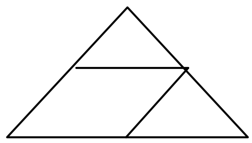
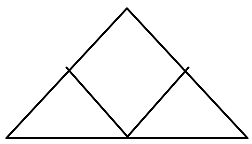
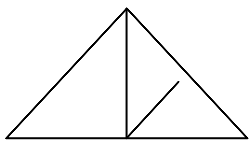
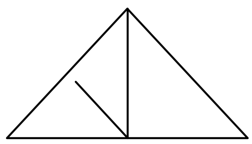
TASK 1(INDIVIDUAL)

ACTIVITY	SUB-LEARNING GOAL
Constructing a given shape using 3 pieces	The students are able to construct a certain shape using 3 pieces

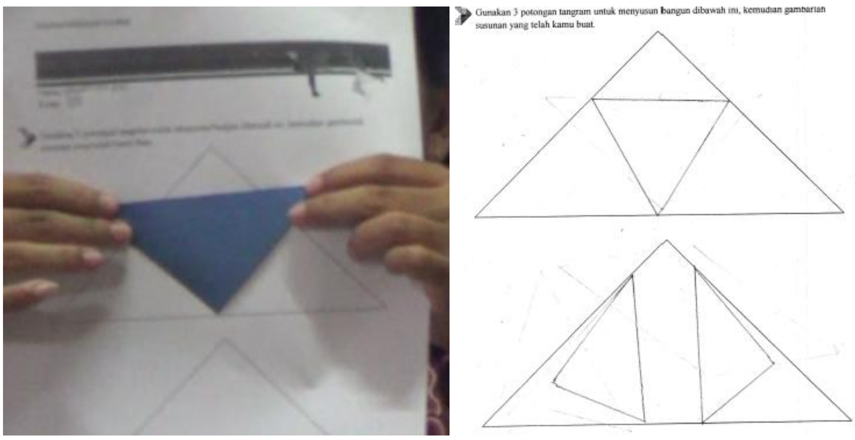
The students are asked to find a construction of the following shape using 3 pieces of tangram. They are asked to give 2 solutions here.



Students could come up with different idea

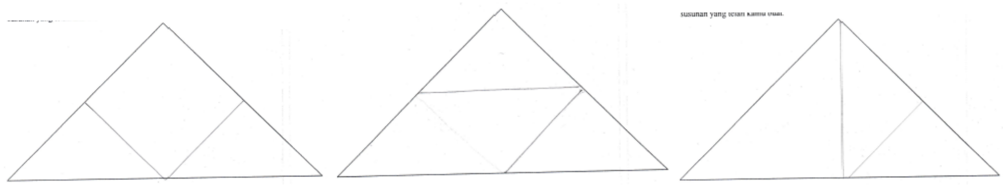
 1st solution,  2nd solution,  3rd solution,  4th solution, and  5th solution

The teacher explained the problem in the beginning to make sure the students would not misunderstand about the problem. But, unfortunately, the teacher gave a wrong example (picture 5.7a). She put a piece of tangram which was overlap with the shape.



Picture 5.7, Teacher's incorrect example (a) and the student's incorrect solution (b)

The teacher's wrong example leaded the students into an incorrect construction. Then, the teacher corrected herself by noting to the students to make a construction with no gap and overlap. In the end, almost all of the students were able to make constructions of the given shape. Only one student was unable to understand the problem correctly. It seems that the teacher wrong example influence his answer (picture 5.7b). The students did trial and error when trying to find out the correct construction. They took one by one piece of tangram and try to put the piece into the shape.



(a) (b) (c)

Picture 5.8, Categories of the students' solutions

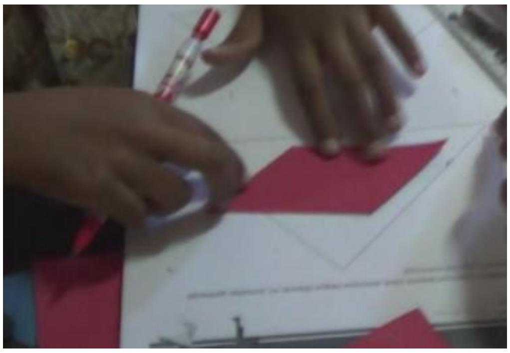
The use of square and parallelogram as a part of construction was an easy hint for the solution resulted from the students. All of the students, except the one who answered in the picture 4b, answered using square in the construction (picture 5.8a) beside their solutions using parallelogram (picture 5.8b). Only 1 student answered using triangle (picture 5.8c). This fact probably is triggered because the teacher showed a student correct answer in front of the class. The solution consists of square and parallelogram.

The idea of gap and overlap also come up during the teacher and student interview when the students working on their construction. The teacher questioned whether his/her construction was fixed without overlapped pieces or not.

Transcript 1, Teacher and Student Discussion

Situation	**The student had already found one construction using square**
Teacher	Ok, let's find another solution You could use another piece (pointing on other pieces)
S1	(taking parallelogram piece and putting it on the shape)
Teacher	Ok, try to fix the piece into the shape, could you?
S1	(putting into fixed angle and side) ya…
Teacher	See, it is fix, isn't it? What else should you put to

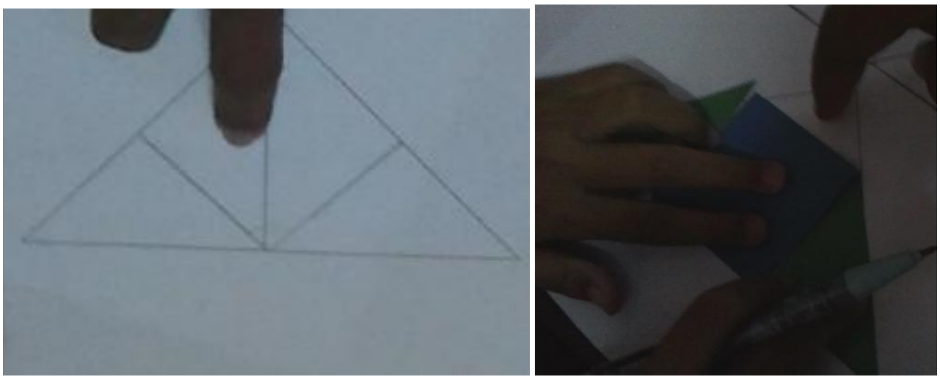
	complete it?
S1	(Taking a small triangle and covering the right gap)
Teacher	Right, what should we do for the remaining gap?
S1	(Using the other small triangle to cover the upper gap)
Teacher	Ok L125548-027



Picture 5.9, Illustration of Transcript 1

The transcript suggests that teacher's role is important to make sure that the students will make a correct construction. In detail, the teacher instruction of asking whether the construction has no gap of overlap is the important point here. The instruction will guide the students to make a correct construction which has no gap and overlap.

During the students' working, several incorrect solutions occurred (picture 5.10). Here, the teacher questioned the students' reason why they made such construction. In the end, they corrected the solution and tried to find the correct ones.



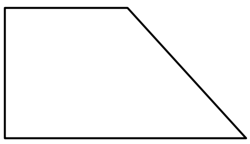
Picture 5.10, Students' incorrect solutions which came up during their working

In the discussion on this part, we see that the problem can invite the students to construct a certain shape using 3 pieces. The incorrect example given by the teacher has a detrimental effect for the students that the students follow it. However, the teacher's instruction about avoiding gap and overlap is important here to make a good construction and could correct the mistakes that had been done previously.

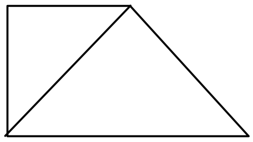
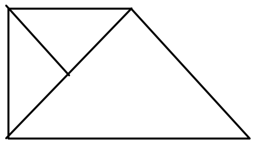
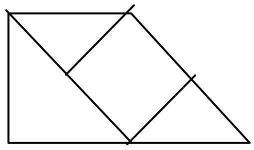
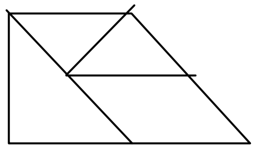
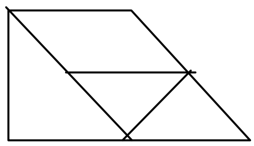
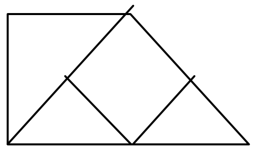
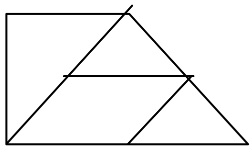
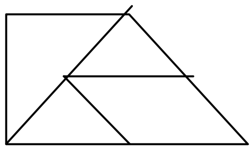
TASK 2 (INDIVIDUAL)

ACTIVITY	SUB-LEARNING GOAL
Constructing a given shape using needed pieces	The students are able to construct a certain shape using various number of pieces

The students are asked to find a construction of the following shape using 2 or more pieces of tangram. They are asked to give 2 solutions here.

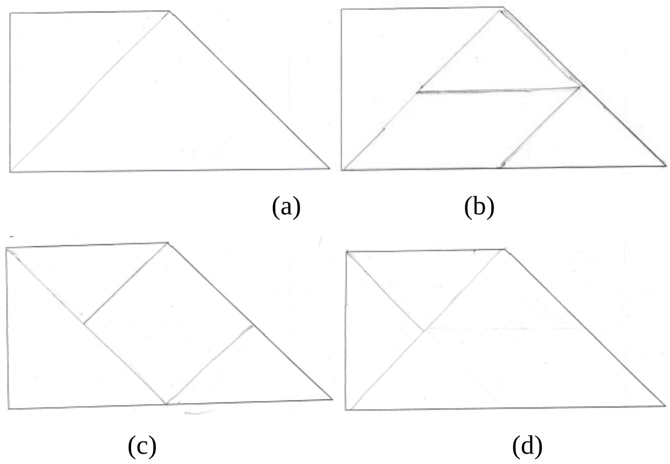


Students would come up with different solution

 1st solution,  2nd solution,  3rd solution,  4th solution,  5th solution,  6th solution,  7th solution, and  8th solution

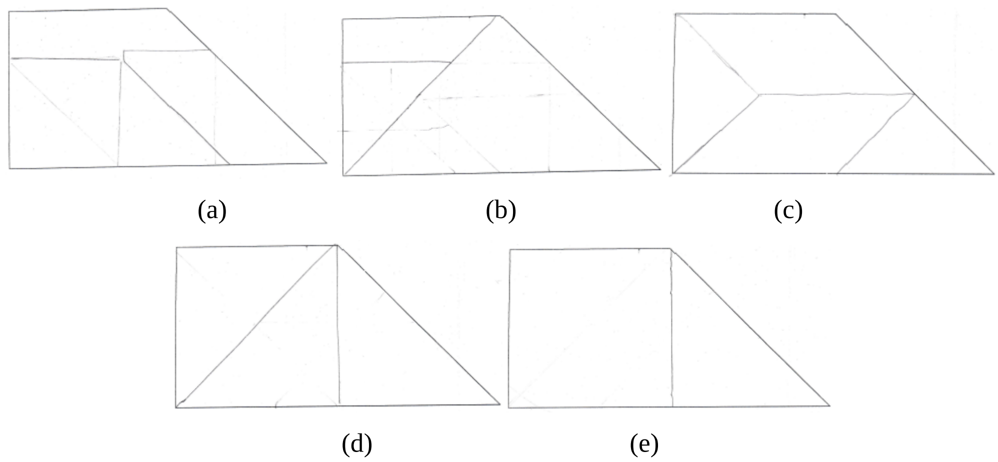
This task was started with a brief explanation from the teacher. To make sure that the students would understand the task, the teacher asked the students to look back at the previous task. She highlighted covering shape without gap or overlap to the students. But here, the students chose freely the number of pieces needed to cover the shape.

Most of the students did not have difficulty in finishing this task. They could cover the shape using available pieces mostly by using 2 and 4 pieces. Almost all of the students find the solution using 2 pieces, medium triangle and big triangle (picture 5.11a). Other solutions which sometimes occurred on the students answer are using 4 pieces either using a parallelogram or a square (picture 5.11b and 5.11c). The solution using only pieces was hard to find by the students (picture 5.11d). However, several students succeeded to find the solution using 2 small triangle and 1 medium triangle pieces.



Picture 5.11, Categories of the students' correct solutions

Only a few students were unable to finish this task. One of them gave incorrect solutions which leave a gap (picture 5.12a and 5.12b). Others used repeated pieces like 2 parallelogram and medium triangle (picture 5.12c and 5.12d). One student gave unclear solution because he/she used bigger square than available square



Picture 5.12, Categories of the students' incorrect solutions

Here, the students were dealing with the idea with the conservation in the concept of area. By knowing that the students could use different pieces to make a same shape or even they can also using different number of pieces to make a same shape, we hope they will start to grasp the idea of area conservation. Unfortunately, they still did not trigger any conclusion of area conservation here because the teacher did not ask the students about whether those different constructions could result the same area. Guidance from the teacher like asking them to take a look at the previous task in order to get an idea how to proceed with the problem was no use. They could not gain the idea of using the result from previous task and adding some pieces to form the shape in the problem.

In the discussion on this part, we see that the problem can invite the students to construct a certain shape using various pieces. Only a few students gave incorrect solutions. However, the teacher's instruction about using the previous task is not working well. The students faced difficulty in using the construction from the previous task to complete the shape in this task

Important Remarks from the First Activity

To test the credibility, we would compare the data from different resources in triangulation to find out the data consistency.

Table 5.3, The Table of Data Triangulation on the 1st Activity

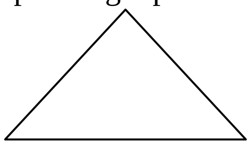
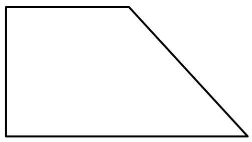
Activity	Collected Data	Worksheet	Observation
Orientation with the tangram Constructing own shape using more pieces and record it	How the students construct shape using the given tangram	The students draw various construction of tangram. The drawings are mostly not clear such that the students will have difficulty when they are asked to make the same construction later	The students actively make one construction into another construction. The students keep doing drawing and redrawing because they are seems confuse about how to draw
Constructing a given shape	How the students construct a given shape using provided pieces	The students draw 2 constructions. Some of them submit more than 2 constructions.	They put the piece one by one and trying to find out which piece is not overlapping or producing gap

From the table, it provides evidence of the data consistency of how the students would react when they are asked to construct shapes using the tangram pieces. They are seems not to have difficulty in making the shape. But, drawing the shape is not an easy task.

In the analysis of the first activity, we see that how the students react with the problem in this activity support most of the conjectures in the HLT. However, the unexpected result comes from the first task where the students are asked to make their own shape and draw the shape. Most of the students could not draw the shapes that they had drawn well. It suggests that drawing a sketch of a real object for their age is still a hard job. Consequently, the teacher could not use their picture for the third activity. Thus we should find another way to capture the students drawing if we still want to use it for the third activity. Nevertheless, the mosaic puzzle, the teacher guidance, and tasks give an opportunity to the students to experience a rich and stimulating activity in geometry. By constructing the right pieces, they dealt directly with several geometric features. Angles, sides, symmetry, and equality are some geometric features while siding, rotating, and flipping the tangram pieces. The students would not realize if they are studying but playing puzzle game.

The following table summarizes the result on what the students react from the activities.

Table 5.4, The Table of Actual Students' Responses on the 1st Activity

Activity	Learning Goal	Actual Students' Responses
Orientation with the tangram Constructing own shape using more pieces and record it	The students are accustomed to play with tangram The students are freely construct their own shapes using 7 given pieces and then draw a sketch of it.	The students can play with the tangram well and made more than one shape. They cannot draw the sketch of the shape that they have made well. There are mountain scenery shapes, a people-like shape, and others abstract shapes.
Constructing a given shape using 3 pieces	The students are able to construct a certain shape using 3 pieces 	Most of the students are able to produce 2 different solutions. Several of them are able to make more than 2 solutions Some of them are unable to provide a correct solution
Constructing a given shape using 2 or more pieces.	The students are able to construct a certain shape using 2 or more pieces 	The students can cover the shape using available pieces mostly by using 2 and 4 pieces. Only a few students were unable to finish this task. One of them gave incorrect solutions which leave a gap and others use the pieces twice. The teacher's instruction about avoiding gap and overlap is important here to make a good construction and could correct the mistakes that had been done previously

c. *Finding The Price of Tangram Pieces (2nd Activity)*

We would like to guide the students to build the model of tangram with price from the context given to reason with.

Learning Goal

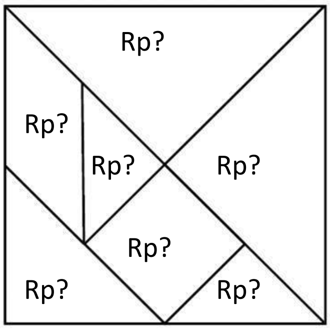
2. The students are be able to build the model of tangram with price from the context given to reason

Analysis on Learning Activities and Actual Students' Learning

TASK 1 (Group Discussion)

SELLING THE TANGRAM PIECES

Giving context "Selling the Tangram Pieces"



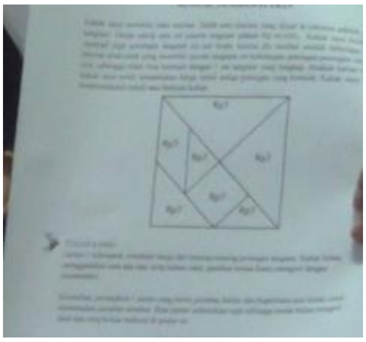
5.13. The tangram puzzle

After the students had fun playing tangram, they were invited to think about a toyshop of teacher's brother who sells a tangram puzzle. The prize for a complete set of tangram puzzle is Rp 40.000,-. The owner of the toyshop also wants to sell the tangram puzzle for a piece, because he figure out that many children who have bought a set of tangram will lose one or more pieces of the tangram on the upcoming months. The teacher then asks the students to help his/her brother to put prizes in every different piece.

ACTIVITY	SUB-LEARNING GOAL
Finding the price of each piece from the context given	The students are able to determine the price of each piece using their own strategy

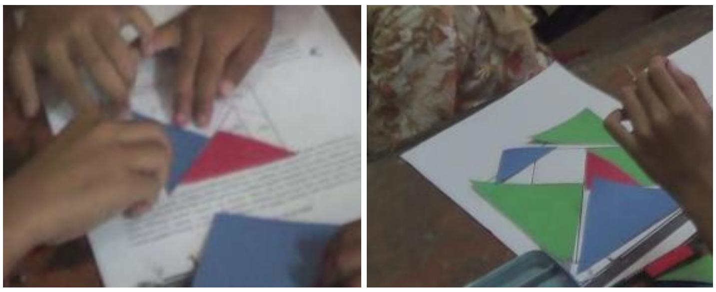
To start the class, the teacher told the students to make the construction of square using the whole tangram pieces like the picture on the worksheet (picture 8), unfortunately, the picture on the worksheet was too small.

Hence, several students had difficulty in constructing the square. However, in the end, all of the groups were able to make the correct construction.



Picture 5.14, The picture of the tangram on the worksheet

The teacher made sure that all of the students are able to make a square shape. in the beginning, the students started to make a square which was too small to be constructed if they use the given tangram piece. Some of them used the small picture on the worksheet as their original size which made them difficult to construct the square that was meant by the teacher, therefore, they made an incorrect square (picture 5.15a). However, others used the original size of the pieces and constructed the correct square (picture 5.15b)



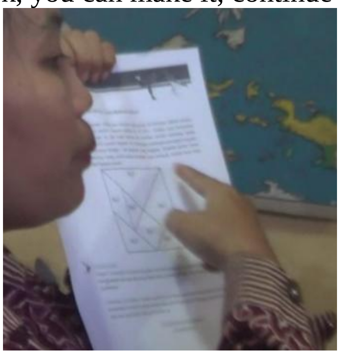
(a) (b)

Picture 5.15, Students' responses for constructing square

The teacher started to tell the students about the problem in this worksheet. She read aloud the problem and asked the students whether they understood the problem well or not. Surprisingly and unfortunately, the teacher gave a clue to the students to solve the problem (picture 10) even though she said that the students could use their own ways as far as they are logical solutions. The clue given by the teacher really directed the students to use halving strategy.

Transcript 2, Teacher Directed the Solution

Situation	**The teacher explained the problem and directed the solution**	
Teacher	So, a set of tangram is 80.000, ok Let's see these triangles (pointing at triangles formed by halving the square) These triangles must be a half of the square, mustn't they? Then, how much the price for a triangle then?	
Students	(Silent)	
Teacher	How much? If this whole set is 80.000, if we divide into 2 parts (pointing on the half of square), then how much?	
Students	40.000	
Teacher	40, then divide again into 2 triangles? (pointing on 2 big triangles) then how much?	
Students	20	
Teacher	Ok, you can make it, continue with your group	L132017-228



Picture 5.16, Illustration of teacher gave a clue to solve the problem

In the group discussion, there were several methods used by the students to solve the problem. In the beginning, they did not follow the teacher clue. Some of them estimate directly by guessing the appropriate price for each piece regarding the fact that the total price must be 80.000. Others just put price regardless the fact that the total must be 80.000. The transcript below shows how the focus group tried to solve it.

Transcript 3, The Focus Group Discussion in Solving the Problem

Situation	**When the teacher explained the problem, this group had already started to discuss the problem**
S_LUK1	Each of them must be 20, so this this this this this and this (pointing on each piece) must be 20
S_SAB1	No, this is 20 and 20 (pointing on 2 big triangles), then the rest is 10 10 10 10 10 (pointing on the remaining pieces one by one)
S_ARI1	50 60 70 80 90 (counting by starting from 40 because it had already counted as 40.000 for 2 big triangles for the remaining pieces)
S_SAR1	Hmmm, it should be 80 right, for the total price?
S_ARI2	How if 8000, 8000, 8000, … (pointing on each piece except the 2 big triangles)
S_SAB2	No, better like this. These are 20 (pointing on 2 big triangles). And the remaining are 5000, 5000, 5000, … (pointing on each piece except the 2 big triangles)
S_ARI3	Noooo, it would be too cheap. 8000 is better
S_SAB3	Its ok
S_ARI3	Wait, what is 40:8 ehh no, 40:5, how much is it?
Students	(Silent)
S_LUK2	Let’s do something with with the tangram construction (pointing on the construction of square using tangram because previously they just discussed the picture of it on the worksheet)
Teacher	Yeah, what you can do using the tangram? Make sure your friend understand what you said, ok
S_SAB4	These 2 are 20 (pointing on the big triangles) so they will 40 with the remaining will be 45 50 …
S_SAR2	Hmmm, it is divided like this (showing to cut the square into 4 equal triangles through the square’s diagonals) it will be 20 20 20 20

Students	(Silent)
S_LUK3	Hmmm, the small pieces should be 5000, aren't they? (small pieces means the tangram pieces except the 2 big triangles)
S_ARI4	The total is 90
S_LUK4	Heh, no (disagree), this 2 are 40 (pointing at 2 big triangles) then each of this are 5000, so 50 and 80 ..
S_SAR3	Let me write whatever we got, then we continue … H132130-128

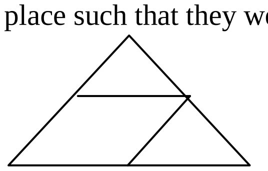
The excerpt shows that the problem gave the students opportunity to think. Most of their solution were coming from guessing and trial and error, and, they could check whether their answer was correct or not by relating to the total price. Even thought they could not justify their answer geometrically using the fact that each of pieces could compose or be composed another piece, they simply justify by checking the total sum of the pieces whether 80.000 or not.

In the excerpt of the discussion of the focus group above, we see that the students argue about the price for the remaining 5 pieces (1 medium triangle, 2 small triangles, square, and parallelogram). One of them insisted that each of those should be 5000 (see S_SAB2). She argued with another student (see S_ARI2) that the remaining 5 pieces should be priced 8000 each, since those 5 pieces form the same area with the 2 big triangles do. Because those 2 big triangles worth 40.000, then to find the price for each of those 5 remaining pieces he divided 40.000 by 5 and he got

8000 (see S_ARI3). Unfortunately, they miscalculated when they were checking the total price (see S_LUK4). Nevertheless, they still continued the discussion.

Several minutes later, the focus group had managed to put price for each piece correctly. The following transcript is about the teacher and the focus group discussion on explaining the group answer.

Transcript 4, The Focus Group Explain their Answer

Situation	**The teacher tried to elicit the reasoning. previously, they had discussed why the medium triangles is 10.000 because it is a half of the big triangle**	
Teacher	This is 10.000 right (pointing on parallelogram)	
Students	Ya	
Teacher	So, why did you put 10.000 for this piece	
S_LUK1	Because it is also a half of this one (pointing on a big triangle)	
Teacher	Are you sure? Is it really a half of it? (putting the parallelogram on top of the big triangle)	
S_LUK2	Hmmmm,..	
S_ARI1	I don’t think so	
Teacher	Ok, do you remember how to construct a big triangle using a parallelogram?	
S_ARI2	(putting the parallelogram on top of the big triangle	
Teacher	Which pieces do we need to complete this?	
S_ARI3	(taking 2 small triangles and put on the remaining place such that they were formed as follow) 	
Teacher	Correct, so these 2 small triangles and a parallelogram are the same with a big triangle, so their total price is the same with a big triangle 20.000, isn’t it? Then how do we find the price of parallelogram?	
S_LUK3	It is 10.000 because 20 – 5 – 5	
Teacher	Good, you could find it by using this why	H132130-128

From the transcript, we can see that there was an unexpected solution given by the focus group (see

S_ARI3). In finding the price of the parallelogram, we conjecture that they would use 2 small triangles which can be composed into a parallelogram. Thus, the price of a parallelogram is equal with 2 small triangles which is 2x5000=10.000. Surprisingly, they used their previous activity to solve it (see S_ARI3). Using the fact that the big triangle is 20.000 and the small triangle is 5000, they calculated the price for the parallelogram if, together with 2 small triangles, they formed a big triangle. Hence, they subtracted 20.000 – 5000 – 5000 = 10.000 (see S_LUK3). Here, we can see that the students could use the geometrical shape of tangram pieces to check their answer. It is not impossible that with the correct guidance, the students would use the geometrical shape of tangram pieces rather than guessing only. However, in their group report, they did not put this solution.

During the group discussion, the teacher move around from one group to another group to make sure that the students solution were clear and correct because most of their answer were coming from guessing strategy. The teacher explained in each group that they should use halving strategy where the constructed square is halved into 2 equal triangles and continue halving the triangle into smaller triangles. The teacher instruction was not fully understood by the students. They had difficulty in proceeding to the remaining pieces where they could also apply halving strategy. This is similar with what happened in the focus group. Their solutions were based only on guessing. This fact supports our conjecture that the students would use guessing to solve this problem. However, the researcher expects that the students will try to use another strategy when the teacher questioned them how to justify the answer, unfortunately, the teacher just directed the students to use another strategy and it results that the students went back again in using guessing strategy for the remaining 5 pieces.

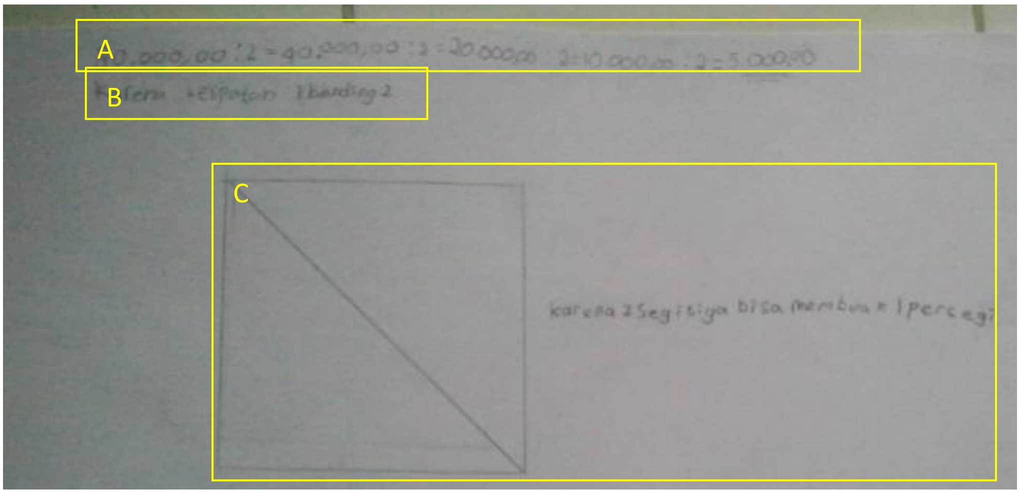
In the discussion for this task, we can see that the contextual problem invited the students to think reasonably. Many ways could be used to find the price for each piece of tangram reasonably. However, the problem was presented in a directed way. In the other words, the picture of the square construction on the worksheet limits the students to produce more creative solutions. About the teacher role, the teacher guidance which tended to direct the students to use halving strategy was not work well. The students could not understand fully the teacher explanation and they prefer to guess the answer by using trial and error number until they got 80.000 as the total price.

TASK 2 (CLASS DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price

In this activity, some groups are expected to explain their answer in front of the class. We also conjectured that there will be classroom discussion about various ways to calculate the price for each. However, in the reality, the presentation of the answer cannot invite a classroom discussion. In detail, the group only read the answer that they had written in the group paper. None of the students questioned the presented answer. It seems that to trigger a live discussion in such a discussion is quite hard because, according to the teacher explanation, the class is not accustomed with discussion class.

One of the group who presented their answer was the focus group. They wrote how they find the price for the big triangle, the medium triangle, the small triangle, and the square on the group's answer paper (see picture 5.17 section A). In fact, in the previous activity, the teacher had a discussion with this group about finding the price of the parallelogram, but they did not mention it in this paper. The answer on the paper is not clear enough. They just wrote the division of 80.000 by 2 repeatedly until it came to 5000. The available explanation on the paper is just saying that because multiplication 1 and 2” (see picture 5.17 section B). Our conjectured is that this section was intended to explain why they divided 80.000 by 2 repeatedly. There is also their explanation of why the square is 10.000 (see picture 5.17 section C). They drew a picture of a square which is divided into 2 triangles. They also noted that the 2 triangles could be composed into a square.



Picture 5.17, The Focus Group’s Answer on Their Worksheet

In presenting their answer, this group just read their answer on the paper directly without any clear explanation on it. One of them read the paper in front of the class (see picture 5.18). After she finished reading the paper, the teacher opened the opportunity to discuss this group answer. However, none of the students post their questions to this group, the classroom discussion did not occur well.

Picture 5.18, The Focus Group's Presentation

For other group, the similar thing also happened. They also just read their written answer on the paper. Also, the expected discussion did not occur. The teacher tried to invite students to take place in the discussion by posting a question to a certain student. However, it still could not trigger a live discussion.

In general, the students answer of this problem use the idea of halving strategy. They consider a set of tangram as a square construction first then they halved consecutively the square into 2 triangles. They continued to halve until they got the small triangle. No other solution given by the students except this strategy. it because the teacher mostly guide the students to use this strategy. Moreover, the picture of the set of tangram on the worksheet seems to guide the students to use halving strategy as well.

In the discussion on this part, we see that the problem can invite the students to think mathematically about the price of a set of tangram by regarding the size of each piece. However, the teacher guidance and the illustration picture given on the worksheet limited the students' creativity to solve this problem using various ways. In addition, during the classroom discussion, the students were not eager enough to involve in a discussion. The teacher question to the students in the discussion was not enough to invite the students to have a discussion.

Important Remarks from the Second Activity

To test the credibility, we would compare the data from different resources in triangulation to find out the data consistency.

Table 5.5, The Table of Data Triangulation on the 2nd Activity

Activity	Collected Data	Worksheet/Students' work	Observation
Finding the price of each piece from the context given	How the students determine the price of each pieces if the total price of those 7 pieces is Rp 80.000	A lot of addition calculation is written on the students' worksheet.	In group they discuss how to make the total price 80.000. they keep changing the number until they get the correct combination.
Math Congress: Share the	How the students present and defend their idea	Mostly, they only write the price of each	When explaining their solution, most of the students only read the

Activity	Collected Data	Worksheet/Students' work	Observation
group work	in the class discussion	piece	paper about the price of each piece

From the table, it provides evidence of the data consistency of how the students solve the problem. They tend to use trial and error in calculating the number repeatedly until the total sum is correct. Moreover, how the students present their solutions is also consistent. They had difficulty in presenting their own idea. It shows that the class is not accustomed into a discussion class.

In the analysis of the second activity, we see that how the students solve the contextual problem about finding the price for each piece of tangram does not support our conjecture. We expect that the students will answer with vary strategies, but, in fact, all of the students solved the problem by using halving strategy. It occurred because the teacher directly guided students to use halving strategy. Furthermore, the illustration of a set of tangram in a square shape only allows the students to use halving strategy. For the improvement, it would be better if the teacher does not provide the picture of a set of tangram on the worksheet. Only giving the contextual problem itself seems could open more opportunity for the students to discover more possible strategies. For the teacher, he/she should provide a clear teacher guide such that the teacher would not give a direct guidance for the students to use specific solution. The researcher also notices that it would be better to change the 80.000 with 80 ribu (ribu is Indonesian word for thousand). It will help the students to work on smaller number than use 5 digits number to work on.

About the analysis on the classroom discussion, we can see that how the students behave does not support our hypothesis. Arguing by questioning and answering did not happen well. The presenter only read the answer that they had written on the paper. Moreover, the teacher intervention by posting a question to other group to react about the group presentation was not effective to invite a live discussion. Perhaps, it is because all of the groups have the same answer. This situation is not suitable for having a good classroom discussion. As if the students could discover several different strategies, then a live discussion would occur.

The following table summarizes the result on what the students react from the activities.

Table 5.6, The Table of Actual Students' Responses on the 2nd Activity

Activity	Learning Goal	Actual Students' Learning
Finding the price of each piece from	The students are able to determine the price of each piece	The students can find the price for each piece. The students could not understand fully

Activity	Learning Goal	Actual Students' Learning
the context given	using their own strategy	the teacher explanation and they prefer to guess the answer by using trial and error number
Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price	Only one type of solution discussed by the students. The students were not eager enough to involve in a discussion.

d. *Comparing Whose Shape Is More Expensive (3rd Activity)*

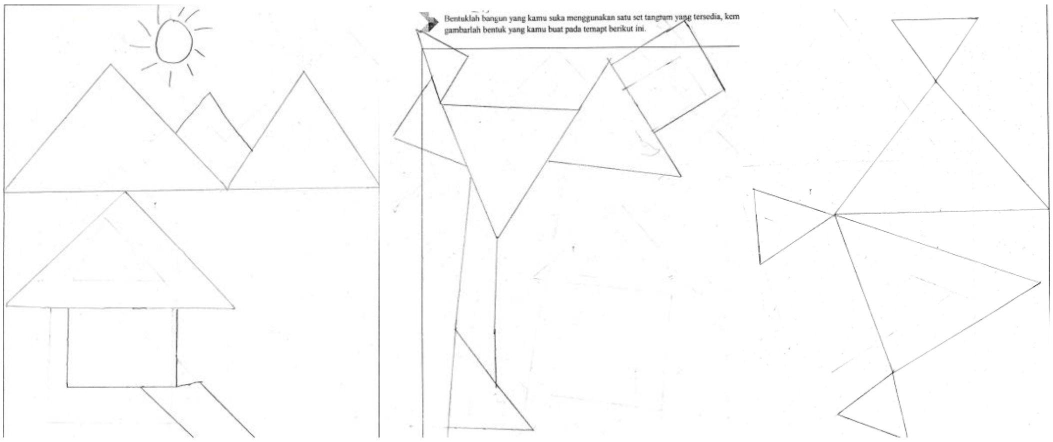
After the students solving the problem, they are expected to know the notion of relationship between size of the tangram piece and its price. However, the teacher should not assume that all of the students have already understood. The following mini discussion is provided to help the students understanding of the relation between the size and the price

Learning Goal

3. The students are be able to understand the relation between the price and the size..

Analysis On Learning Activities And Actual Students' Learning

In the first activity, after the students drew each own shape, the researcher decided to omit this activity of comparing shapes. The main reason is that the pictures or sketches drawn by the students could not be analyzed well. Most of the pictures or sketches are not proportionally drawn (see picture 5.19). This situation makes the students difficult to experience the idea of the relation between price and size



Picture 5.19, Examples of Students' Drawings

Another supporting reason is that the nest activity will deal with the relation of price and size as well. Thus, the deletion of this activity would not change the learning trajectory significantly. However, originally, the next activity was intended to focus on the conservation issue. This situation makes the 4th activity become more intense that the teacher should focus on 2 main issues in the class. They are (1) the relation between price and size and (2) the idea of conserving a shape into another comparable shape.

e. *Compare The Given Shapes (4th Activtiy)*

The follow up activity from comparing shapes using tangram-pricing model, here the researcher develops the activity such that the students will be able to understand the concept of area conservation without heavily rely on the use of price as the bridge

Learning Goal

4. The students are be able to understand the one important properties area which is area conservation without using price as reasoning (if we compose and decompose the area into another form the area remain invariant).

Analysis On Learning Activities And Actual Students' Learning

TASK 0 (ADDITIONAL MINI LESSON)

ACTIVITY	SUB-LEARNING GOAL
Why square and parallelogram have the same price	The students are able to understand that 2 or more shapes which have the same price have the same size of area

In this mini lesson, the teacher will ask to the students which one is bigger, the square piece or the parallelogram piece. By discussing the fact that those 2 shapes can be decomposed and composed each other, we hope that the students would understand that different shapes which have the same price are also in the same size.

(a) (b)

Picture 5.20, Teacher's Demonstration in Conserving Area

The teacher started this mini lesson by showing the students 2 triangles. She demonstrated that the square can be decomposed and composed into a parallelogram (see picture 5.20). During the demonstration, the teacher guided the students to understand that as long as the composition is the same the area of different shapes will be still the same. the following is the discussion transcript.

Transcript 5, The Teacher Demonstrated and Guided the Students

Situation	Using 2 triangles, the teacher demonstrated how we could conserve an area of a square into parallelogram.
Teacher	Ok, what we have here? (showing the media of 2 triangles)
S_CLS1	2 equilateral triangles (they miscall the name of piece, it should be isosceles triangles)
Teacher	Ya, if I construct like this, what will it be? (see picture 15a)
S_CLS2	It will be square
Teacher	Correct, then I decompose it again into what (separating the construction)
S_CLS3	They become 2 equilateral triangles again
Teacher	Then, I construct again into this position, what will it be? (see picture 15a)
S_CLS4	Parallelogram mom.
Teacher	See, what do you think about the area Which one has the biggest area, when they become square or parallelogram?
S_CLS5	They have the same area mom
Teacher	Why do you think like this?
S_CLS6	Because they have the same pieces
Teacher	Good

L150559-047

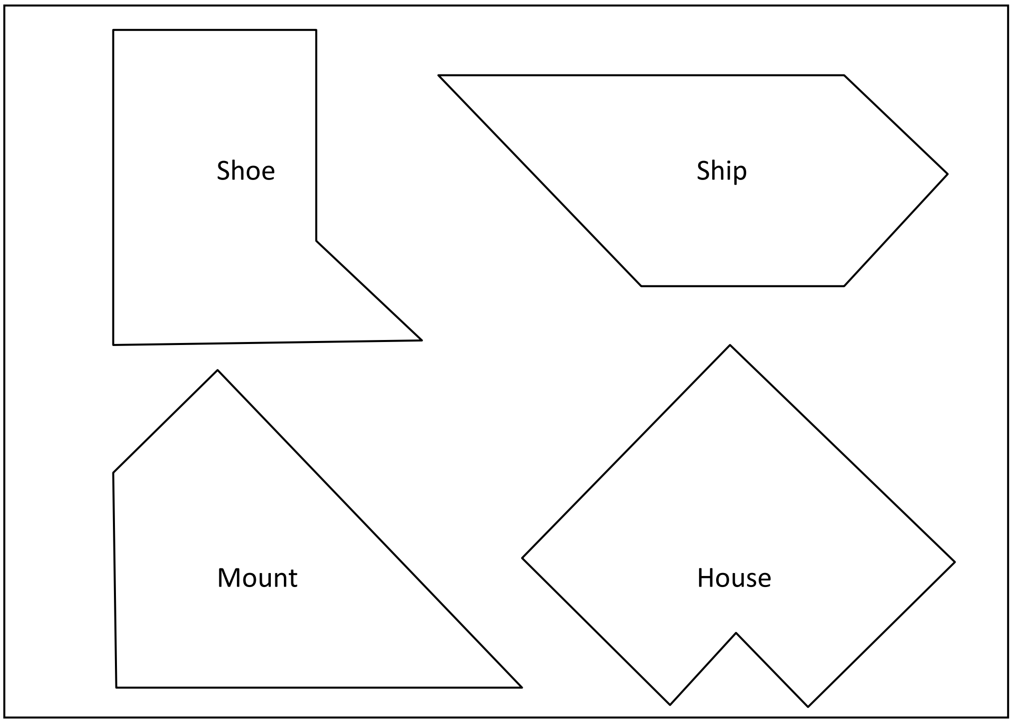
The transcript shows that the student could accept that conservation of shapes make the area of those shapes remains invariant (see S_CLS5). However, they could not give the correct reason why the areas of those shapes remain invariant. It could go to the incorrect reason that they use the number of pieces as the reason why those shapes have equal area (see S_CLS6).

From the discussion of this task, we can see that the students could understand the idea of area conservation. With teacher guidance using a demonstration of decomposing and composing, the students get the insight of of what area conservation is. However, the students still could not understand fully why those shapes have the same area or the still have limited vocabulary to explain why the area remains invariant. Thus, they said that the number of pieces is the same as the reason.

TASK 1 (GROUP DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Compare the Given Shapes	The students are able to compare the given shapes using the idea of area or price

In this activity, student will play with the shapes given by the teacher in group. Each group will be given a worksheet and a set of pieces contain 4 different shapes (see picture 5.21). They are asked to determine which shape has the biggest, the smallest, or equal area. Here, we expect that the students could gain the relation between price and size. Also, we expect that, in the end, the students would be able to use the idea of area conservation to compare between shapes.



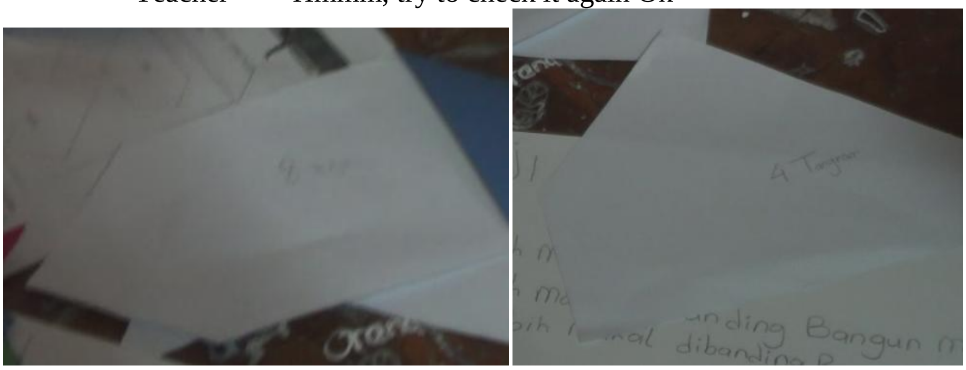
Picture 5.21, The Shapes Compared in this Activity

During the group discussion, none of them could progress in solving this problem except by using tangram pieces. In the beginning, most of the students answer which shape is the biggest by finding the number of pieces which construct a shape regardless what shape and size of the pieces. This is not what the researcher intended the students to experience. Hence, the teacher gave the students guidance to consider also the price as well. In the end, the relationship between price and size become important to solve this comparing problem.

Almost the whole class compared the shapes by using the number of pieces which construct the shapes regardless the size of each piece. At the beginning, they spent their time to figure out the possible construction for each given shape. After they discover the construction for each shape, they arrange the shapes from the biggest one according their discussion based on the number of pieces.

Transcript 6, The Teacher Discuss the Students Answer

Situation	**This group had written the answer on the paper group. They use the number of pieces to answer which shape is the biggest.**	
Teacher	Ok, let see your answer (pointing the students paper answer) Which shape is the biggest?	
S_GA1	This one sir (pointing at Mount shape)	
Teacher	Why do you think so?	
S_GA2	Because is has the biggest number of pieces. it has 8 pieces to construct this shape (see pict 5.22a)	
Teacher	So, if you compare with this shape (pointing at Ship shape). Which shape is bigger?	
S_GA3	This shape (pointing at Mount shape) because it has more pieces than Ship only 4.(see picture 5.22b)	
Teacher	Hmmm, try to check it again Ok	L153719-115



(a) (b)

Picture 5.22, Students' Answer Using the Number of Pieces

The transcript above shows that using the number of pieces regardless the size is quite trivial for the students to be used by the students as the consideration in comparing shapes. They consider more pieces to construct a shape will determine the size of a shape (see S_GA2). Here, the teacher made sure that the students should check whether their answer is reasonable or not.

Transcript 6, The Students Discussion in Comparing Shapes

Situation	**This group tried to figure out how to compare the shape.**	
Teacher	Ok, what do you mean 8 pieces here? (pointing on House shape)	
S_GB1	Here, (trying to reconstruct 8 pieces for House shape)	
Teacher	Then, what will you do?	
S_GB2	Hmmm	
Teacher	What will you do to compare the shapes?	
S_GB3	(Putting the shapes around)	
S_GB4	May be we could measure it?	
S_GB5	How??	
Teacher	Let see, each piece has its own price, doesn't it? Why don't you try to calculate the total price	
S_GB6	Ya, like previous task, we find the price	H152825-115

The excerpt shows that the idea of measuring occurred from the students' discussion (see S_GB4). However, they did not know how to proceed. In the end, the teacher suggested them to use the price of each piece as comparison besides considering the number of pieces.

Similar with the previous group, in the discussion of the focus group, initially, they also used the number of pieces in each shape as comparison factor to determine which shape is the biggest. Nevertheless, the discussion with the teacher guided the students to think another way because the use of the number of pieces could not be used.

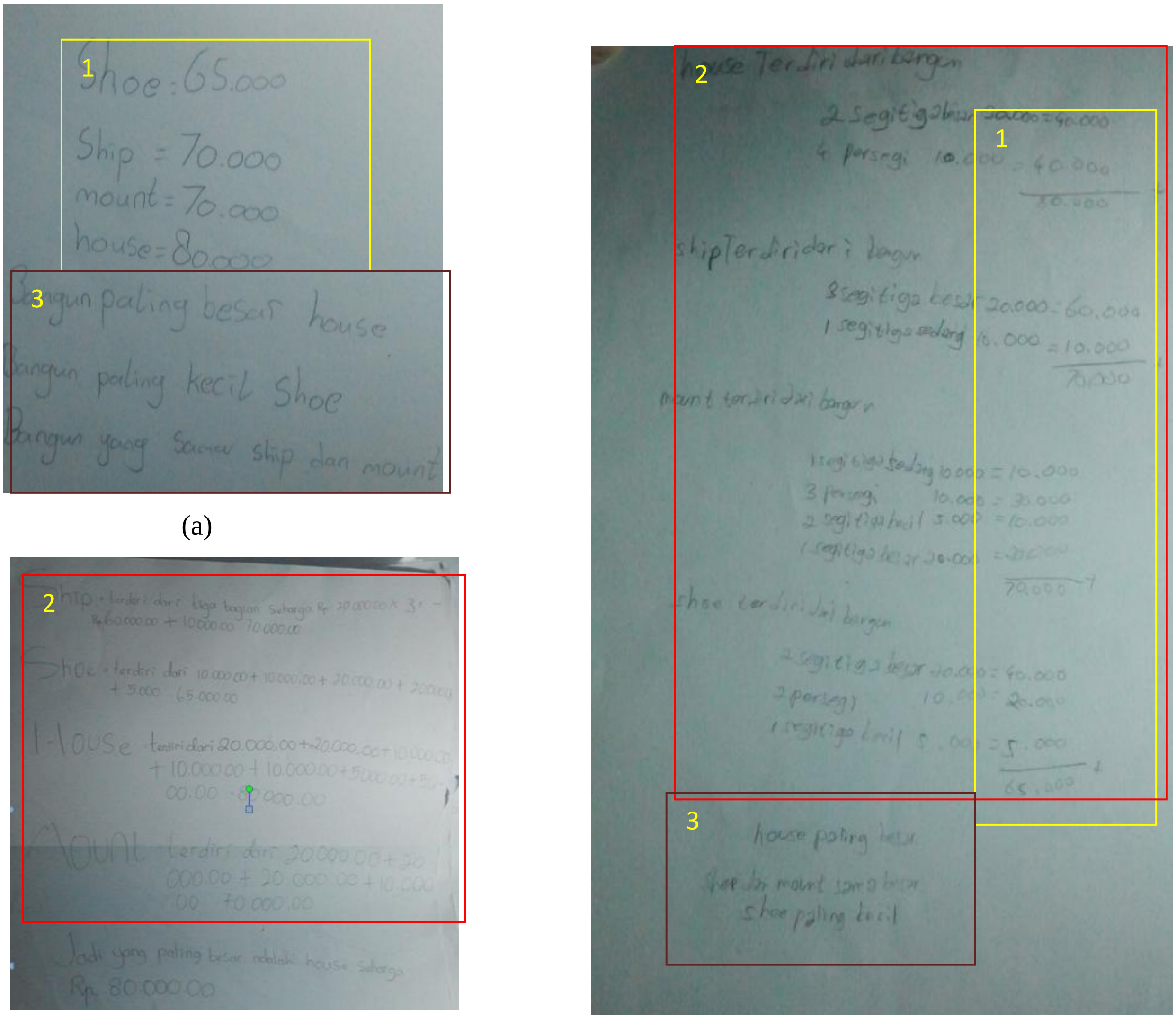
They noticed that using the number of pieces in shape could differ depend on which pieces they used. The teacher then suggested to the students to use the idea of pricing on each shape as the main consideration to determine the size of shape, which one is the biggest or the smallest.

In the discussion of this activity, we can see how the students solve this problem does not support our conjecture. We expect that the students will consider the price without guided by the teacher. In fact, the teacher should guide the students to consider the price for each piece besides using the number of pieces on each shape only. Perhaps, this happened because in the previous mini lesson, the teacher did not give a clear and final explanation how to determine the size of a shape. She also did not gave an appropriate response when the students stated that the number of pieces determine the size of shape.

TASK 2 (CLASS DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Math Congress: Share the group work	The students are able to argue about the strategy that they used to compare the given shapes

This task is intended to share the result of each other group's work. We expected that various solution using tangram pieces to compare which shape is the biggest or the smallest could be discussed. Unfortunately, the time was



(a) (b) (c)

Picture 5.23, Some Students' Group Answer in Comparing the Shapes

up. The class spent too much time in discussing the problem in group. In the following are some examples of the students' group works. In the following are the examples of the students' works.

In general, all of the students could find the correct price for each shape. However, not all of them could finish their report on a paper completely. From three components,

(1) the price for each shape, (2) the calculation of the price on each shape, and (3) the conclusion of the problem. There are only 2 groups who could provide those 3 answers, once again. Due to the fact that this class is not accustomed with discussion and reasoning, providing the reason for the answer is sometimes quite difficult. We can see from the students' answers that only few group could not only provide the calculation, but also give the reason of each shape.

Important Remarks from the Fourth Activity

To test the credibility, we would compare the data from different resources in triangulation to find out the data consistency.

Table 5.7, The Table of Data Triangulation on the 4th Activity

Activity	Collected Data	Worksheet/Students' work	Observation
Mini Lesson: Why square and parallelogram have the same price	What the students reason of why the square and the parallelogram have the same price is	No information	The students use the number of pieces to determine whether the shapes are equal or not.
Compare the Given Shapes	How the students determine which shapes have equal size or more.	At first, they reconstruct the shape into pieces using the tangram pieces. they write the number of pieces in each shape. After discussion with the teacher, most of the students change the worksheet with the new solutions where they use the amount of money for	The students are busy to find the reconstruction of the shape using tangram pieces. they number the pieces one by one After discussion with the teacher, they reviewed whether the use of the number of pieces only is

Activity	Collected Data	Worksheet/Students' work	Observation
		each pieces. on the worksheet, the write the calculation of each shape. and determine the biggest shape which has the most expensive price	enough. Then, they use the price to represent the different size on each pieces.

From the table, it provides no evidence of the data consistency of how the students reason using the number of pieces only. However, in the second activity, we can clearly see that it is consistently showing that the students regard the number of pieces to compare the shape. later, after the guidance from the teacher, they change their approach by not only using the number of pieces, but also regarding the price on each pieces.

From the discussion on this activity, we can see that how the students solve and explain their answer do not support our full HLT. In solving comparing problem, the students could not compare without using the tangram pieces which is not in line with our expectation that the students would compare by overlap and cut and paste. In fact, they decomposed completely the shape using the tangram pieces. However, to some extent, the students react according to our conjectures. We expect that the students would regard the price on each shape or piece has relationship with the size. The discussion during the class suggest that the students are no longer reason using the number of pieces to represent the size but using the total price in a shape to determine which shape has the biggest or smallest area.

The teacher intervention also really important here to guide the students in both understanding the relation between price and size and guiding the students from reasoning using the number of pieces into reasoning using the total price of the pieces. About the problem in this activity, it is really suitable to invite the students to think about price and size comparison.

For the revision, perhaps the activity and the problem do not need to be changed. But the HLT and the teacher guide regarding the suitable conjectures on what the students will react and do in solving the problem. Also, important addition on what the teacher should react to support the students thinking, especially in the idea of suing pieces to compare, should be administered more completely.

The following table summarizes the result on what the students react from the activities.

Table 5.8, The Table of Actual Students' Responses on the 4th Activity

Activity	Learning Goal	Actual Students' Learning
Mini Lesson: Why square and parallelogram have the same price	The students are able to understand that 2 or more shapes which have the same price have the same size of area	The students misunderstand that the parallelogram has the same price with the square because they have the same pieces construction.
Compare the Given Shapes	The students are able to compare the given shapes using the idea of area or price	The students still consider the number of pieces determine the size of the shape. The teacher guide them to find out that using the number of pieces only is not enough
Math Congress: Share the group work	The students are able to argue about the strategy that they used to compare the given shapes	In general, all of the students could find the correct price for each shape. However, not all of them could finish their report on a paper completely

f. Covering Area By Using Only One Kind Of Piece Of Tangram (5th Activity)

After the students have already understood about the concept of area conservation take place in the relation with price, then, the learning activity moves to the area measurement issue. Hence, in the beginning, the researcher develops guiding activity in conserving the area. Then, it will be followed by measuring activity which invite the students to use the idea of area conservation to make it easier.

Learning Goal

5. The students are be able to measure the area of certain polygons using the concept of area conservation

Analysis On Learning Activities And Actual Students' Learning

TASK 1 (INDIVIDUAL – CLASSROOM DISCUSSION)

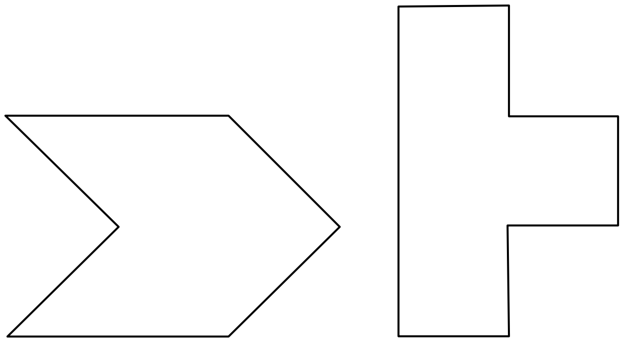
ACTIVITY	SUB-LEARNING GOAL
Decompose and compose 2 given polygons into a square	The students are able to do conservation of a shape into another shape which has the same area

In this activity, the students are asked to do simple area conservation by cutting and pasting in some parts of the shapes. However, the result is surprisingly not the same with what we had expected. The completely do decomposition of the shapes and recompose it into a square. Here, the researcher also adds more additional media to invite the students more actively in the discussion. The researcher made a presentation to make the teacher explanation more interesting.

The teacher explained the problem using the presentation (see picture 5.24). She explained that we can decompose and compose a triangle into a square by cutting it.

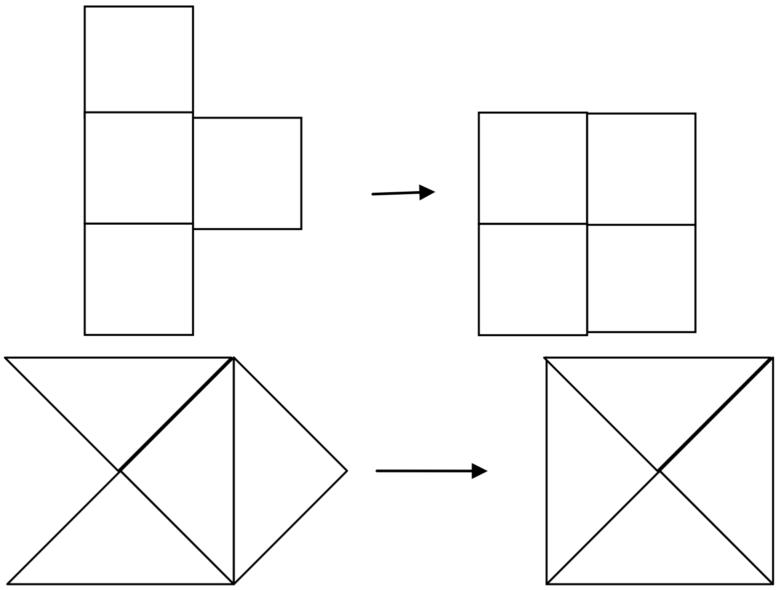
Picture 5.24, Teacher Explained the Problem using Presentation

The following is the picture of the polygons which must be decomposed by the students.



Picture 5.25, The polygons in Conserving Problem

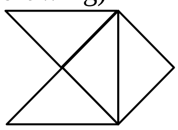
All of the students could answer correctly. They decomposed each of the shapes into 4 equal parts (see picture 5.26). Then, they compose those 4 equal parts into a square.



Picture 5.26, The Illustration of the Solution.

The following transcript shows how the students could do conservation without using the real objects. They just make a drawing to illustrate which position is cut and compose it into a square.

Transcript 6, The Students Discussion in Comparing Shapes

Situation	**The discussion is about how to arrange the shape that had been cut**
Teacher	Ok, can you explain your answer?
S_GC1	We divide first the shape like this (pointing on her drawing) 
Teacher	Then, what will you do?
S_GB2	Hmmm
Teacher	What will you do to make it to be square?
S_GB3	Like this may be (putting one by one on the square) (see picture 22b)
Teacher	What do you do for these 2, they have different position. (pointing on the pieces in the red square)
S_GB5	Hmmm
Teacher	Could you do something, may be flip it, rotate it or else?
S_GB6	Ya, we could rotate it such that it fits with the remaining place

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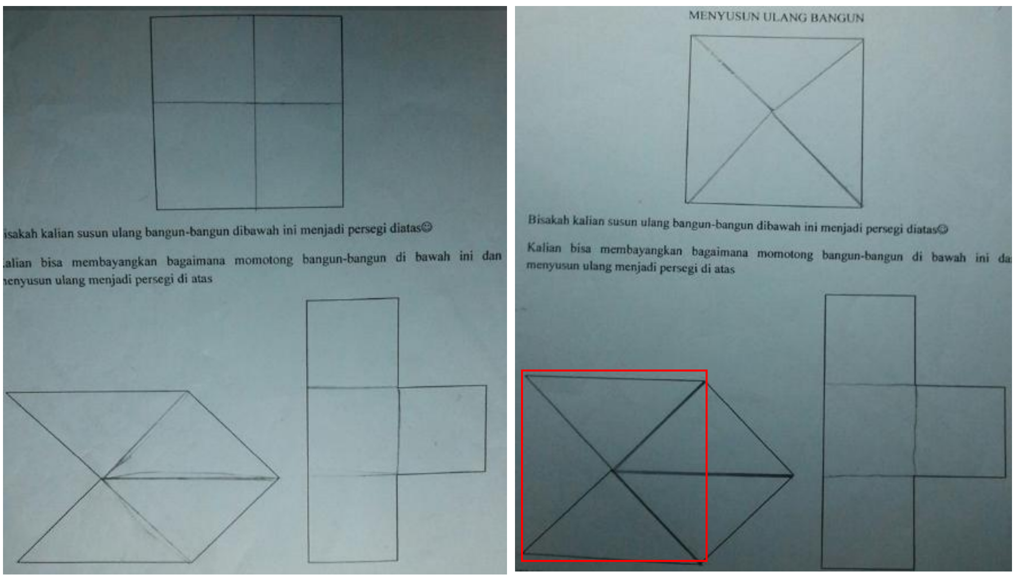


(a) (b)

Picture 5.27, The Students Solutions.

In general, we can see that the students are able to do area conservation from a given polygon into a square. Surprisingly, none of them came up with the solution that the researcher had been conjectured. We conjectured that the students will cut the polygon in the certain area and compose it into a square, in fact, the idea of decomposing using tangram piece is really meaningful for the students. Thus, they did not cut and paste the polygon but they decomposed into pieces. From here, they recomposed the pieces such that it forms a square.

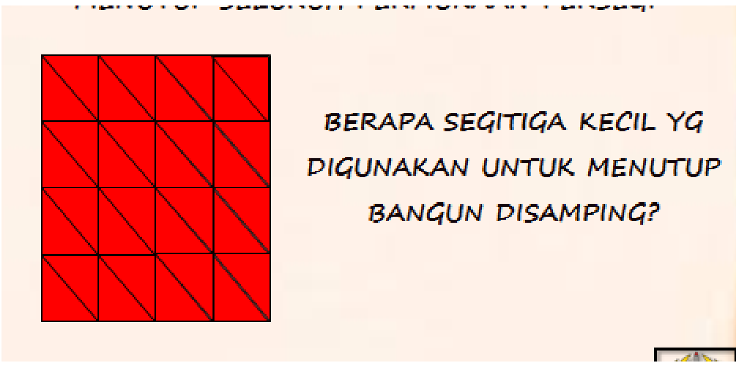
The teacher had an important guidance here. She invited the students to freely use whatever possible strategies. The problem itself successfully gave the students the experience about conserving. Nevertheless, this activity is not enough to make sure that the students understand the concept well. Furthermore, we need more complex polygon by which the student can work on. Otherwise, the understanding will not be understood well.

TASK 1 (INDIVIDUAL)

ACTIVITY	SUB-LEARNING GOAL
Covering A Given Shape Using given pieces	The students are be able to measure the area of certain polygons using the concept of area conservation

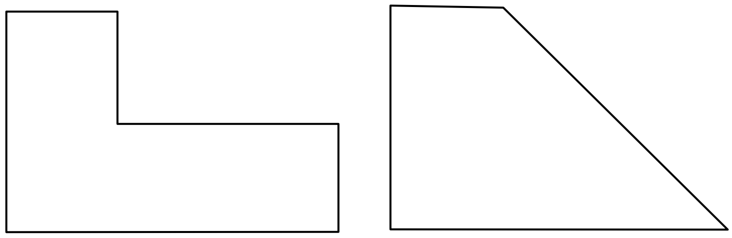
In this activity, after they dealt with conserving shapes in the previous activity, we expect that the students would use the conserving idea to measure the given polygon. Here, the teacher asks the students to find the number of triangles needed to cover a shape. We conjectured that the students would not do counting to find out how many triangles needed as the measurement of the shape. in addition, here, the teacher still use the media to invite the students in the discussion.

The teacher explained the problem using the presentation (see picture 5.28). She explained that we can decompose and compose a triangle into a square by cutting it.



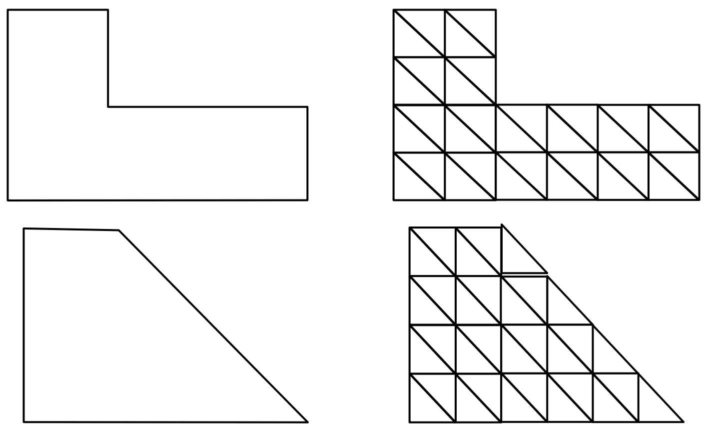
Picture 5.28, The Slide Explained the Problem by the Teacher

The following is the picture of the polygons which must be measured by the students using a certain triangle.



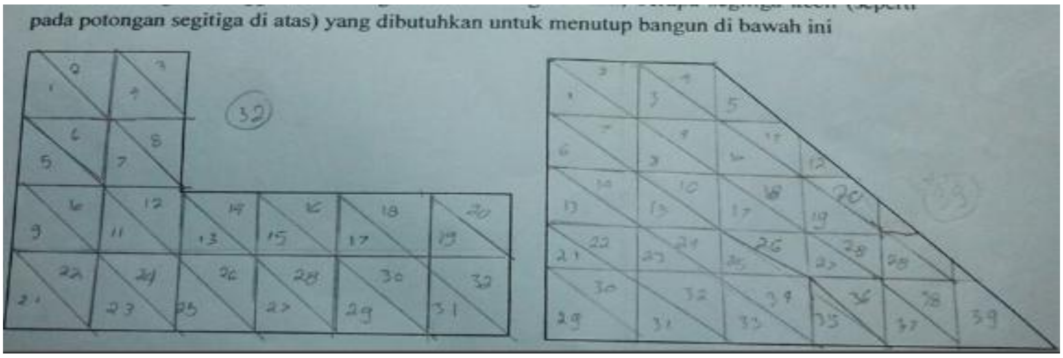
Picture 5.28a, The polygons in Measuring Problem

Most of the students could answer correctly. They decomposed each of the shapes into 4 equal parts (see picture 5.29). Then, they compose those 4 equal parts into a square.

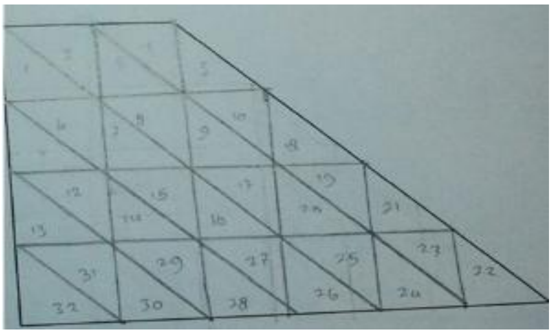


Picture 5.29, Examples Solution.

In the following, we will discuss the focus group work in this problem. In general, the class also did similar mistake with this focus group. One general thing is that, most of the students did counting in finding the number of pieces. None of them came up with other strategy.



(a)



(b)

Picture 5.30, The Students Solutions.

From the picture 5.30a, we can see that the students have no difficulty to solve the first shape. the mistake occurred when they try to draw the small triangles inside the second shape. Unlike in the first shape where they can divide first the shape into 4 equal squares then try to draw the triangles in each square, for the second shape, they could not easily find similar way. What they did was just trying to measure it sing ruler in such way that they will make the triangles. However, it seems that making equal triangles in this shape is not easy.

In the next minutes, teacher asked them whether the triangles there are really the same. One student of the group tried to do something like in the previous shape. He made squares first in the second shape (see picture 5.30b). In the end, they could find the correct number of triangles for the second shape.

In the end, the teacher asked about whether they could use another solution. She guided the students to use conservation to measure the shape. One of the students could present the answer in front of the class how we use conservation to measure the polygon by relating the shape with the previous shape that had been measured.

Important Remarks from the Fifth Activity

To test the credibility, we would compare the data from different resources in triangulation to find out the data consistency.

Table 5.9, The Table of Data Triangulation on the 5th Activity

Activity	Collected Data	Worksheet/Students' work	Observation
Decompose and compose 2 given polygons into a square	How the students find the size relation between shapes	They put the division line to divide the shape into 4 triangles which could be decomposed into a square. The similar solutions also appear for the second shape	When the students are asked to show how they solve the problem, they demonstrate how they divide the shape into several pieces. then, they make the relation of each pieces until it forms a square
Covering A Given Shape Using given pieces	How the students find the number of measurement unit in the	The students draw several lines parallel to divide the given shape into several small triangles. Some of them	The students use the ruler to divide the given shape into smaller triangles. Then, they count it one by one to

Activity	Collected Data	Worksheet/Students' work	Observation
	given shapes.	miss draw the line such that it result inconsistent triangles.	figure out how many triangles could fit in a shape.

From the table, it provides no evidence of the data consistency of how the students use area conservation in comparing shapes. They decompose shape into pieces then try to construct it again into another shape as comparison. In the next activity, it shows that the students still do not use the idea of conservation here. They just try covering one by one while drawing small triangles and then they do counting. It suggest that the students still could not use area conservation in measuring area.

In general, we can see that the students are not able to implement the idea of area conservation to measure the shapes directly. They need guidance to think more like what the teacher did here. She asked the students to think another way using area conservation.

From the discussion on this activity, we can see that how the students react to the problem support our first conjecture in another way. They used the idea of decomposing into tangram pieces to conserve a shape into another shape. In fact, actually, we expect the students would just do cutting and pasting to conserve it.

About the second conjecture that the students could use area conservation to measure the area, the students' reactions do not support our HLT. All of them use counting one by one to find the number of triangles there. However, teacher questioning in the end of the lesson open opportunity for the students to use the idea of area conservation.

The following table summarizes the result on what the students react from the activities.

Table 5.10, The Table of Actual Students' Responses on the 5th Activity

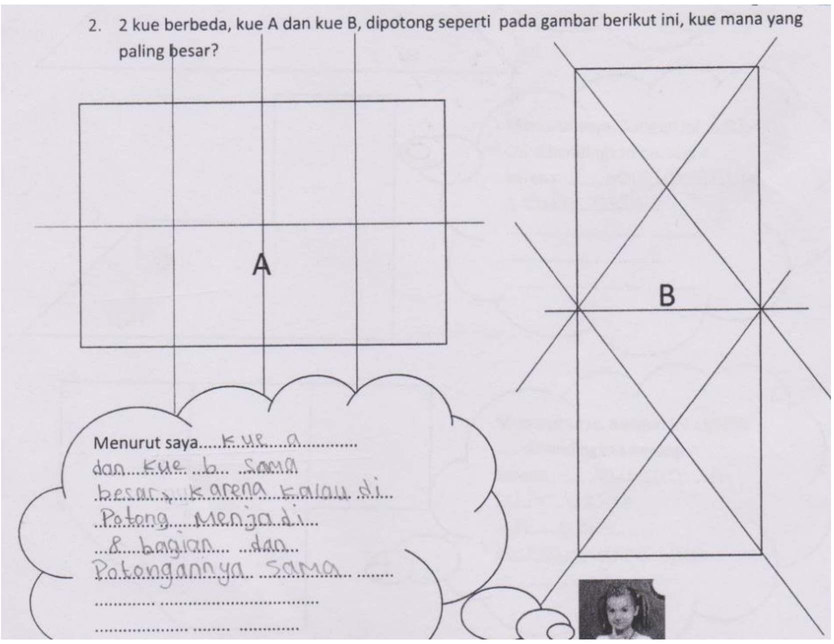
Activity	Learning Goal	Actual Students' Learning
Decompose and compose 2 given polygons into a square	The students are able to do conservation of a shape into another shape which has the same area	The students are able to do area conservation from a given polygon into a square. They did not cut and paste the polygon but they decomposed into pieces and recomposed the pieces such that it forms a square
Covering A Given Shape Using given pieces	The students are be able to measure the area of certain polygons using the concept of area conservation	In general, all of the students cannot directly use the idea of conservation in measuring except do counting on it.

g. *Remarks from the post test*

Post test was conducted after the whole learning sequence had done. Generally, the questions in pre test and post test are similar. The only different aspect is that make a question of comparing cakes harder by making the cut in different shape but still has the same number of pieces.

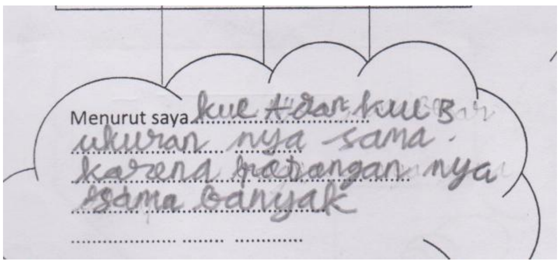
1) Considering the Number of Pieces in Comparing

In this problem, we ask the students to compare 2 given cakes, which are actually have the same size, that have been cut into the same number of pieces but different shape.



Picture 5.31, The Question and one of Students Solutions.

Most of the students answer that those cakes have the same size. However, their reasons are not quite clear. They only mention that the number of cut pieces is the same (see picture 5.32).



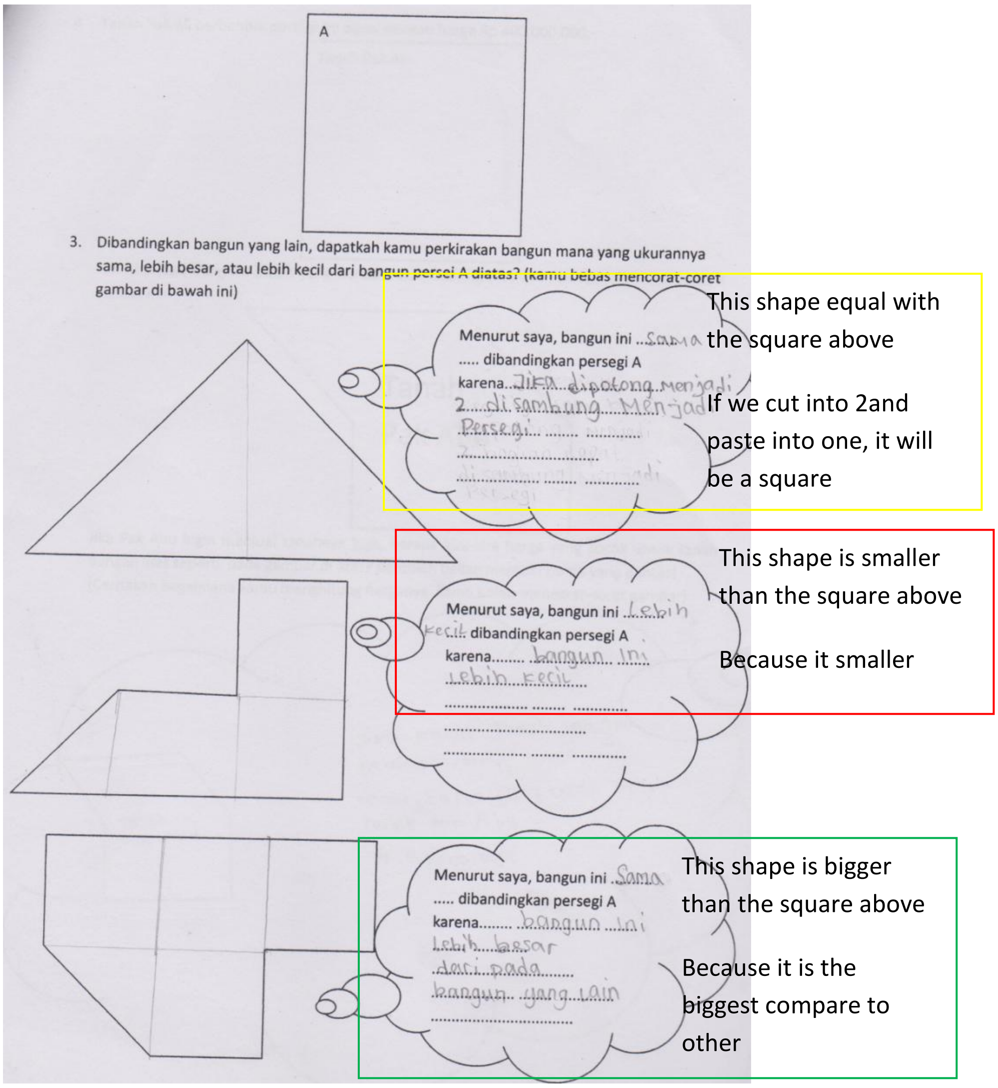
Picture 5.32, One of Students Solutions using the number only.

Nevertheless, there are some students who are able to consider also the shape of the size. They mentioned that they should compare the pieces as well. From an answer on picture 5.31, we also can see the student's answer. She reasoned not only considering the fact that those cakes are cut into the same number of pieces but also regarding the cut itself, she said also that the cut pieces are actually the same.

In general, the students still have lack of reasoning in comparing the shapes. They only regard the number of pieces, which is the easiest thing to compare, to determine which cake is the biggest. However, they have improved their reasoning since in the pre test they there were quite many students who unable to compare correctly. They just regard how long the length and width of the cakes given.

2) Decomposing to Compare

In this problem, we ask the students to compare three different shapes with a given square (see picture 24). We expect them to do area conservation here to find out whether they are the same or not.



Picture 5.33, The problem and One of Students Solutions

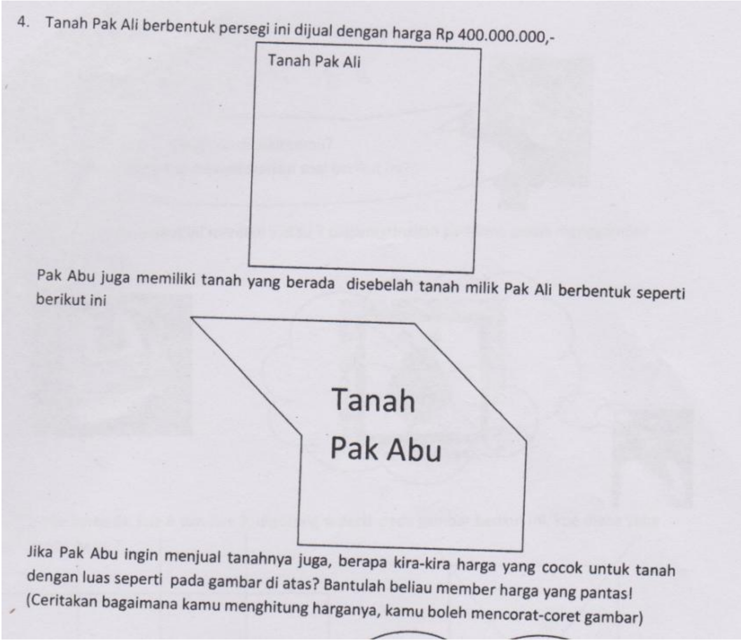
A significant number of students could answer correctly if we compare with the pre test where none of them are able to figure out the correct answers. From the picture 5.33, we can see how the students reasoning using the picture and explaining it. They cut the shapes into several pieces which are unexpected solutions. They then try to compose those pieces whether they could be a square or not.

However, there still some students who unable to do conserving here. They still answer incorrectly because they could not make square with the same size from the given shapes.

In general, they did unexpected answer. By what I mean they did not cut in small portion only but they decomposed the shapes into pieces like what they did in the 5th activity. And then, they try to recompose it and see whether they could be composed into a square or not.

3) *Using Area Conservation to Measure the Area.*

In this problem, we ask the student to measure a given shape by comparing with the square which has been measured (see picture 26). We expect the students would do conservation to find out the price for the shape



4. Tanah Pak Ali berbentuk persegi ini dijual dengan harga Rp 400.000.000,-

Pak Abu juga memiliki tanah yang berada disebelah tanah milik Pak Ali berbentuk seperti berikut ini

Jika Pak Abu ingin menjual tanahnya juga, berapa kira-kira harga yang cocok untuk tanah dengan luas seperti pada gambar di atas? Bantulah beliau member harga yang pantas!
(Ceritakan bagaimana kamu menghitung harganya, kamu boleh mencorat-coret gambar)

Picture 5.34, The Problem of Measuring

In this problem, a significant number of students also are able to answer the question correctly, even though not the majority of the students. They answer that if they sell the field, it will be 800.000.000 for those 2 fields where 400.000.000 for each of Pak Ali's field and Pak Abu's field.

C. *HLT Validation.*

In this section, the result from different sources are confronted to see how consistent the result in each activity as the HLT validation. The sources that are used are the result from the first cycle and the second cycle. Furthermore, this analysis is aimed to get fuller picture about what is going on and how we should consider it (see APPENDIX C63 – C66 for the table).

The first activity shows that the student can do construction activity well. They can make several constructions of the given shapes. Moreover, in the second cycle, more solutions are produced by the students. However, when they are asked to draw the shape that they have made, consistently, from those 2 cycles, the drawings are mostly cannot represent clearly about their picture. Thus, the drawing activity would be better if it is omitted from the learning trajectory.

In the second activity, most of them do trial and error in choosing certain number and calculate it such that their sum would be 40.000 (in the 1^{st} cycle) or 80.000 (in the 2^{nd} cycle). This solution appears extensively during the lesson. Hence, we should change the Learning Trajectory using this idea as the starting point to guide the students into using the intended solutions, which is the solution regarding the area of shapes.

The next is the fourth activity because the third activity is omitted due to the problem in drawing a clear drawing of shapes. In the fourth activity, we conjecture that the students will use superposition or overlapping strategy to compare the given shapes. In fact, in 2 cycles, the students heavily rely on the use of tangram pieces. They still cannot compare the shape without using the tangram pieces. However, the teacher should till make sure that the students would not come into misunderstanding of the use of the number of pieces only regardless the size.

In the fifth activity, unexpected solutions are given by the students in the second cycle. It seems that providing an open opportunity for the students to react to the problem could allow them to think more clearly. We expect that they will cut and paste the shape into another shape such that they can compare the shape clearly. But, the students decompose the shape in pieces and then compose those pieces into another shape. Here, we notice that the students use the area conservation to figure out whether those shapes are equal in size or not.

Still in the fifth activity, we expect that the students would use area conservation in measuring the size of the given shape. Unfortunately, form those 2 cycles, none of them can invite the students successfully. However, in the second cycle after they success to conserve the area, guidance from the teacher could help them to reason also using area conservation in measuring the given shape. It seems that the students would be able if the supportive activities are provided more.

D. General Final Remark

After analyzing the whole learning sequence on what the students and teacher have done during the teaching experiment. We conclude several remarks on the HLT, the teacher role, and the students thinking.

The whole conclusion on the teaching experiment in the 2nd cycle suggests that our HLT still need to be revised. Unexpected solution given by the students should be administered appropriately. The important ones are in the second activity where the students try to find the price for each tangram and the fifth activity where the students measure the shapes using triangles. The teacher guide should be also revised accordingly.

The teacher really supports how to make a nice class discussion. Several media which are used by the teacher like the picture of the tangram tale illustration, the big tangram model, and the use of power point to raise the students' interest work quite well. However, couple times the teacher did not do like what we have discussed based on the teacher guide. It is hard to ask the teacher to read our teacher guide carefully and perhaps to post some questions if the teacher guide is not clear enough.

About the students learning, our conjecture that tangram could help the student in understanding the concept of area conservation is correct. They could do decomposing a shape and recomposing it into another shape by dividing the shape into pieces like tangram pieces. However, to guide the student to use the idea of area conservation, it seems that our learning sequence still needs to discuss this issue more. But we believe that it would work due to the fact that the students could apply this idea when the teacher asked them how if we use are conservation in measuring area. Here, the guidance from the teacher is really important. The result from the post test also suggests a positive trend. The learning sequence would be more promising if we could add more activity for area measurement using conservation.

CHAPTER VI

CONCLUSION

In this chapter, we will discuss 3 important issues which are conclusion section, suggestion section, and recommendation section. In the conclusion section, we will try to answer the research question and how the local instruction theory on the learning of area conservation in area measurement topic especially in the use of tangram game. Important discussion issue will be elaborated I the second section of suggestion. Here, we will describe several interesting remarks which should be administered properly for better result. The last section, the recommendation, contains important remarks from the whole learning sequence of my design. These remarks are intended for the upcoming study of the educational research in the area measurement domain especially in area conservation.

A. Conclusion

In this section, we will try to answer the research questions that had been proposed and, accordingly, to elaborate the local instructional theory on the topic of area measurement especially in area conservation using tangram game. In details, firstly, we will answer the 2 sub research questions consecutively. Then, we will try to draw general conclusion from those to answer the main research question afterward. In the end, the local instructional theory on teaching the concept of area conservation in area measurement topic is established.

The use of tangram in area conservation could provide a well learning sequence using the important properties of tangram. The tangram can be decomposed and composed from one shape into another shape. This is the essential properties from tangram that can be used to guide the students to understand the concept of area conservation.

The idea of area conservation is actually difficult to be understood by the students in that young age. Area of an object remains invariant in size regardless whatever changes on its shape or positions. Egocentric thinking makes the reality seen subjectively by the children. When they could see the measurement are just the same from any point of views, it is the process of de-centering thinking because the main point of measurement is that the object's size is remain invariant no matter the positions are changed (Piaget, 1960). Conservation means that the quantitative value of an area remains unaltered while its figure can be qualitatively new (Piaget et al., 1981).

The result from the pre test suggests that if the pupils could notice the idea that the shapes could be decompose into smaller pieces, then those pieces can be compared easily. Thus, the use of tangram pieces as easy comparable objects that can be decomposed from the given shape would give benefit for the pupils to help them (see pg 67 – 70). The first four activities were intended to invite the students to deal with the idea of area conservation started from informal activity, playing with shapes of tangram construction, until reaching the quite formal one, conserving a shape into a square. In the analysis of the first activity, we see that how the students play informally with the idea of conservation. Here, they try to find various constructions for one shape. The constructing activity give the students the experience of decomposing a shape into constructible pieces. The students are playing puzzle game but learning as well. (see pg 71-79)

After the students get the insight about divide a shape into pieces, we insert the notion of area by putting price on each piece. In the second activity, we invite the students to make the relation, by themselves, of price and size. Here, we expect that the tangram priced model starts to emerge. From the context of "selling tangram in pieces", the students experience the fact that more expensive the price will make the area bigger. Furthermore, the students not only learn about decomposing shape geometrically, but they also reason using the notion of area behind the idea of pricing. (see pg 80 - 89). In the following activity, the fourth activity because the third activity was omitted, we introduce the students how to apply tangram priced model to compare various given shapes. In solving comparing problem, the students could not compare without using the tangram pieces which is not in line with our expectation that the students would compare by overlap and cut and paste. In fact, they decomposed completely the shape using the tangram pieces. We expect that the students would regard the price on each shape or piece has relationship with the size. The discussion during the class suggest that the students are no longer reason using the number of pieces to represent the size but using the total price in a shape to determine which shape has the biggest or smallest area. In the end, the students have already understand how to use tangram and price to compare the area, (see pg91 – 97)

Regarding the research question, in general, the tangram provides a nice opportunity for the students to understand the decomposition and the composition area. Furthermore, it also provides the students to relate the idea of price as the notion of area.

Measuring area is a process to quantifying the size of a surface area by using discrete units of the same size and then counting those units. Using the concept of area conservation in measuring area is quantifying the area by measuring the conserved area. In general, the learning sequence, especially the last 2 activities, provide the idea of using area conservation in measuring a certain area.

The, fifth activity is intended to initialize the implementation of area conservation to measure an area. In the discussion, we see how the students react to the problem. All of them use counting one by one to find the number of triangles there. However, teacher questioning in the end of the lesson open opportunity for the students to use the idea of area conservation. From the discussion on this activity, we can see that how the students react to the problem support our first conjecture in another way. They used the idea of decomposing into tangram pieces without using the physical object of tangram pieces to conserve a shape into another shape.

in fact, actually, we expect the students would just do cutting and pasting to conserve it. (see pg 98 – 104)

Moreover, still answering the research question, in general, the designed learning sequence still could not invite the students to use area conservation as a big idea for helping them in measuring an area. It seems that we need more activities to help the students in using area conservation to measure area. By using the fact that the students could implement the partitioning of shape using the tangram pieces without the use of tangram pieces physical objects, we believe that the students will be able to use the tangram model to measure the area.

The tangram priced model emerges to support the students understanding on area conservation. Furthermore, the students are expected to be invited to measure area easily by applying the concept of area conservation. From the previous 2 sub research questions, the tangram could provide the students an opportunity for the students to experience the conservation informally. The students could decompose a shape and compose into another shape by considering tangram pieces as the framework. Furthermore, the use of price on each piece as a bridge for the students to relate the notion of area with real world is successfully guiding the students on it. They could reason that more expensive the price will make the size bigger also. However, we should pay attention to the possibility that the students would consider only the number of pieces as comparison.

Nevertheless, with a good starting point on the students understanding on area conservation, we try to invite the student into more formal conservation for measuring area. The students could use his knowledge on conserving a shape into another shape. By using the fact that the students could implement the partitioning of shape using the tangram pieces without the use of tangram pieces physical objects, we believe that the students will be able to use the tangram model to measure the area. However, more intensive learning sequence on this issue should be administered properly. One meeting only in this activity suggests that it is not enough to support the students.

The aim of this study is to develop a local instructional theory for area conservation in the topic of area measurement for 3rd grade students. Gravemeijer and Cobb (2006) describe local instructional theory as collections of both the conjectures of the learning process and the responsive support for each conjecture. The following table summarize how the instructional theory on the local topic of area measurement.

Table 6.1. The Local Instructional Theory on Area Conservation in Measuring Area

Activity	Tool	Imagery	Practice	Concept
Free Exploring Tangram Playing	A set of tangram, a shape to be constructed	Invite the students to be accostumed in composing and decomposing tangram.	Making given shapes, making own favourite shape	Composing and decomposing shape in small scale
Finding The Price of	A set of Tangram	Invite the students to	Dividing a given set of	Geometrically dividing the

Tangram Pieces		determine the price for each tangram piece logically.	tangram in reasonable price for each piece	price and its relation with area.
Comparing the Given Shape	4 sets of tangram and 4 different shapes to be compared	Invite the students to use the tangram pieces as comparison tools regarding the price for each pieces as well	Comparing 4 different shapes by using the tangram pieces and their price as comparison tools	Area conservation on different shapes and the relationship between size and price
Covering Area By Using Only One Kind Of Piece Of Tangram	2 given shapes for conservation and 2 given shapes for measuring	Invite the students to use the concept of area conservation in finding the number of triangles in certain shapes.	Finding how many triangles could fill a certain shape by considering the result on familiar shape	The idea of using the concept of area conservation in measuring an area of a shape.
Additional suggested activity: Finding the area of certain shape by comparing with more easy shape		Invite the students to use the concept of area conservation in finding area of certain shape	Finding the area of a certain shape by considering the result on familiar shape	The idea of using the concept of area conservation in measuring an area of a shape.

In summary, the conclusions on the result of the students' learning on each activity in our design provide supporting evidence that by using the tangram we could invite the students to experience the idea of measuring an area using the concept of conservation. Because, tangram puzzle itself has the properties of conservation which is important to relate the students informal understanding into a formal concept of area conservation..

B. Suggestion

In this section, we will reflect on how the study was conducted. The weakness on my study and some important remarks will be elaborated in the following. There are also recommendations for better teaching experiment in this topic.

1. *The Difficulty in Inviting Classroom Discussion.*

During the implementation of the learning sequence, the expected classroom discussion could not occur successfully as we can see in several classroom discussion. This is really different if we compare with the discussion in the group where the students could share their opinion freely without any hesitation. According to the teacher explanation, the class is not accustomed with discussion method. The teacher mostly uses the direct instruction with the teacher as the center of discussion. We suggest that, in the teacher guide, we must provide several guiding question for the teacher to invite more students in the discussion.

2. *The Role of the Teacher Guide*

In general, the teacher support in the discussion phase in the learning sequence was not help the students much. The fact that the teacher had difficulty in understanding the whole teaching guide is undeniable. Mostly, the teacher did not do many important small remarks suggested in the teacher guide. She missed the detail and too focused on the final goal. The important thing here is that we need enough time to discuss the teacher guide completely with the teacher. We could give the teacher guide to the teacher some times before the teaching experiment to give the teacher opportunity to read and make remark on the unclear explanation in the teacher guide.

3. *The Cycle in this Research*

The number of cycle done in this study is only twice. Methodologically, we should redo the cycle until we do not find any significant differences between the latest cycle and a cycle before. However, in this master thesis study, we can find, in the first four activities about the use of tangram in introduction of area conservation, no significant update between the 2 cycles. Unfortunately, it does not also apply for the last activity about triggering the student to use the idea of area conservation in measuring area. More cycle should be administered properly to figure out what activities are suitable for helping the students in this big idea.

4. *The Students Difficulty in Sharing their Idea*

This happened when we asked the students to write their solution of the problem on the paper to be presented. As we can see in several students' group papers, unfortunately, most of the students' answers on their paper were just mostly about the final answer. Thus, we usually question the students on what they write actually. Actually, the students could better share clearly their strategy orally than literally. Teacher guidance on this issue also did not work well. It would be better if the students are asked to write whatever they think at that time. It is better than asking the student to write the solution because it can capture the real students thinking.

C. Recommendation

In this section, we will reflect on how the study was conducted. Some important useful remarks will be elaborated in the following. We recommend for the upcoming study in this topic to use these specific recommendations.

1. The Use of Tangram

Tangram is the main media used here. Hence, providing a good and precision tangram could help the students understanding the shapes better. Otherwise, most of the construction activity will lead to misunderstanding if the tangram used does not have a good precision with the shapes on which the students are asked to construct. The misunderstanding is like the students still regard the shapes are equal even though they have small overlapped or gap.

2. The Teacher's Support

Even though the teacher could not fully understand the teacher guide, in certain aspect, the teacher provides much effort in conducting the activity and made them became interesting ones. The teacher could make the students interest with the activity by demonstrating the story in the first activity or having slide presentation in the last activity. The classroom management by the teacher works well here. The teacher could manage the class such that the class is not too noisy. During the classroom discussion, several teacher instructions could maintain the group discussion.

3. *The use of Information Technology Media*

During the teaching experiment, the teacher suggests that if we use a media of presentation, it would make the students more enthusiastic in following the instruction in the learning sequences. Hence, in the last meeting, we try to make an interactive visual media and implement it. The result on using visual media to guide the students is really promising. Most of the students could follow the idea presented in the media. The concept of conserving area is followed clearly by the students about the idea of decomposing and composing a shape into another shape.

4. *The Students Enthusiasm*

The use of tangram puzzle is a promising activity. Van Hiele (1999) guarantee that the use of puzzle, especially a puzzle like tangram, could be acceptable by most of the students in this context. The data in this study also support the fact that the students really enthusiastic in the activities using tangram (see the analysis of the activities). For more study in this area conservation, the researcher recommends to elicit more possibility in the use of tangram.

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APPENDIX 1

Interview Scheme

The background information:

- How long have you been teaching?
- Are you specialized in teaching mathematics or the entire subject I the 3rd grade?

The teaching and learning process:

- What method and/or approach do you frequently use in teaching mathematics?
- How the responses of the students when learning mathematics by this approach?
- How do you prepare your teaching?
- How does the discussion take place in the classroom? And how does it go?
- How the responses of the students if they have discussion?
- Are the students accustomed in share their opinion or idea?
- Are the students accustomed in using poster to explain their opinion or idea?
- Have you taught the students by giving the worksheet?
- Are there a classroom rules? Like how the students asking questions or permits or when you want to ask attention while students make a noise.
- How critic is your class?

The RME and Area measurement topic:

- How and what do you know about RME?
- How often do you usually use context in teaching material?
- Have you ever implemented the RME approach?
    - If yes, what topics are? How are they going?
    - If no, why don't you implement it?
- How does it work in students understanding to the topics?
- Based on your experience in teaching by RME approach, what is your opinion about this approach?
- How do you teach area measurement topic?
- What are your difficulties in teaching the area measurement topic?
- What are the students' difficulties in learning area measurement topic?
- What do you think about the use of RME in this topic?

About the students:

- How many students in your classroom? The proportion of boys and girls
- What are their ages?
- How are their grades?
- Who are the higher achiever, medium, and lower achiever?
- Are they typically active or passive students? Tell me more about it, are they accustomed to think critically and giving their opinion?
- Are they accustomed to work in group?
- How do you group them?

Observation Scheme

The teaching and learning process:

- How does the teacher preparation?
- What method and/or approach does the teacher use in teaching mathematics?
- How the responses of the students when learning mathematics?
- How does the discussion take place in the classroom? And how does it go?
- How the responses of the students if they have discussion?
- Are the students accustomed in share their opinion or idea?
- How does the teacher use the teaching material? (The whiteboard, worksheet, textbook, learning media, discourse materials)
- How the teacher interact with the students?
- Are there a classroom rules? Like how the students asking questions or permits or how does the teacher do for attention while students make a noise.
- How critic is the class?
- How do the students work? Individually or in a group?
- How does the teacher manage the time?

The RME:

- Does the teacher use RME approach?
- Does the teacher use contextual problem?
- Does the teacher use models?
- Do the students reveal a smart strategy?
- Do the students interact to further discuss the problem?
- Does the teacher relate the topic with other topic or subject?

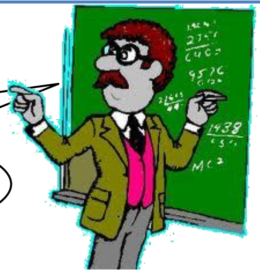
About the students:

- How many students in the classroom? What are their ages?
- Who are the students who talk actively or too much silent?
- How they participate in class discussion?
- How are the students in working in group?
- How does the teacher group them?

APPENDIX 2

Pre-test

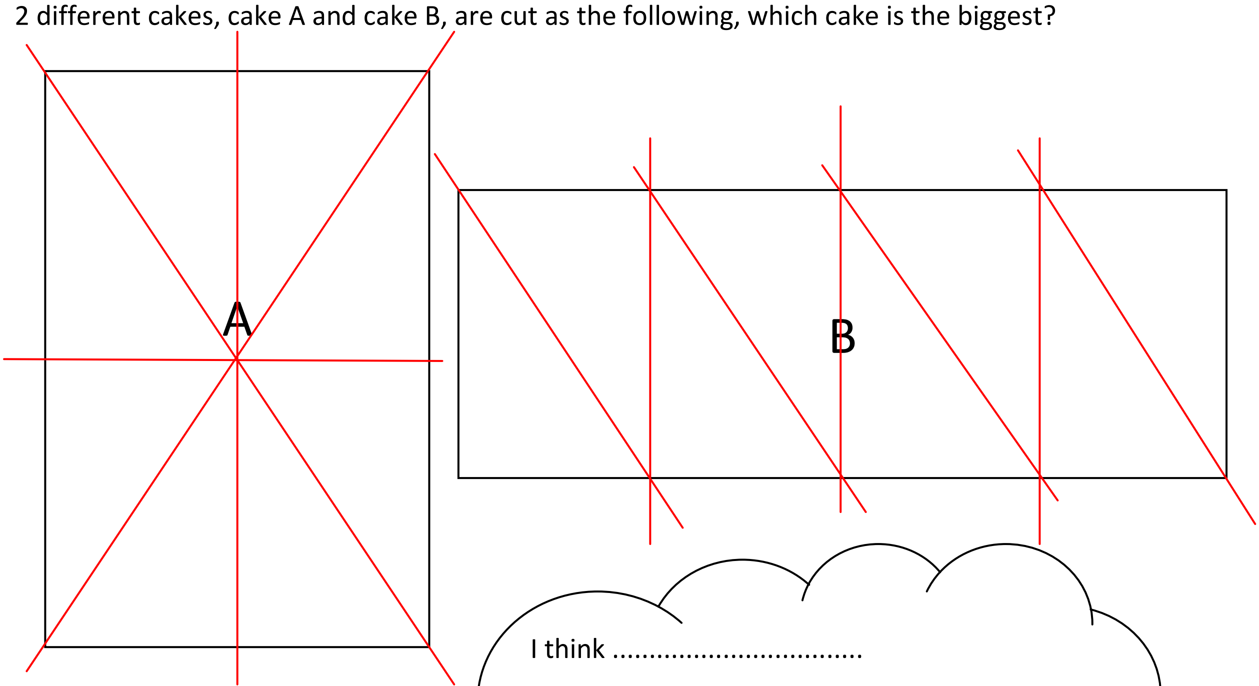
Name/Grade: …………………..



Tell me what are you thinking?
How do you solve the following problem?

1. Cut the following cake into 5 or 7 pieces(using your pencil draw the cutting line)

2. 2 different cakes, cake A and cake B, are cut as the following, which cake is the biggest?



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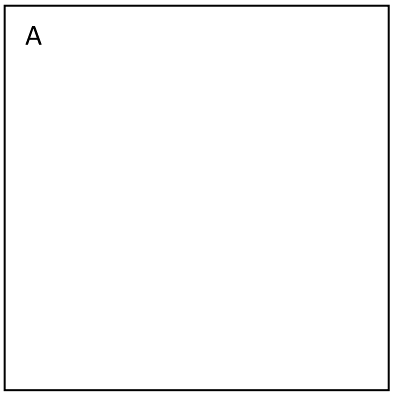
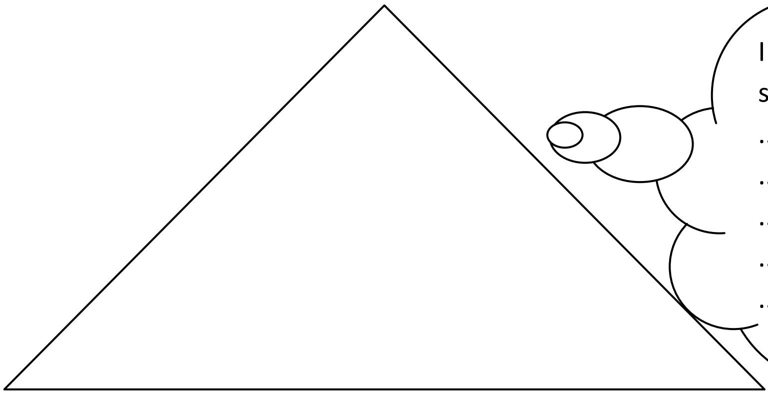
3. Compare with another shape, can you guess, which shape has the same size, smaller size, or bigger size with this square A? (you are free to make a scratch on the picture to solve)





I think this shape is .... than square A because........
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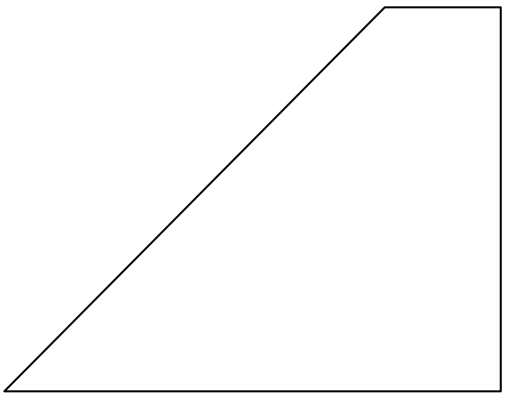

I think this shape is .... than square A because........
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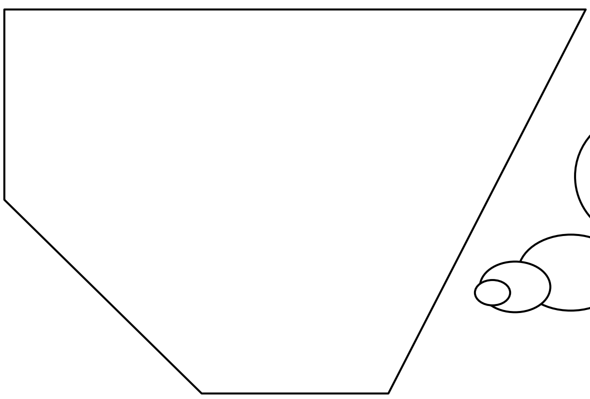

I think this shape is .... than square A because........
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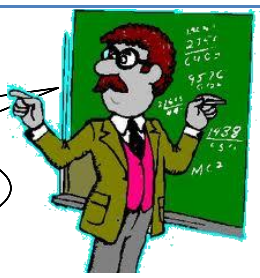
Post-test

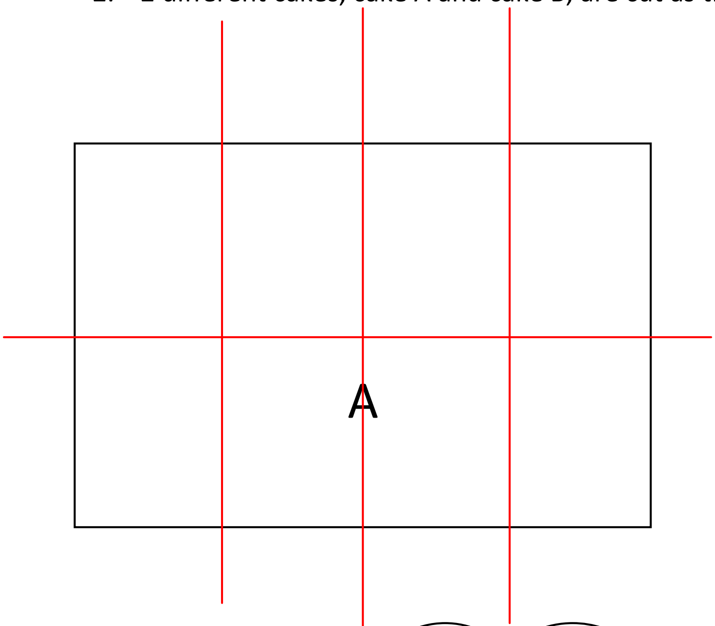
Name/Grade: ....

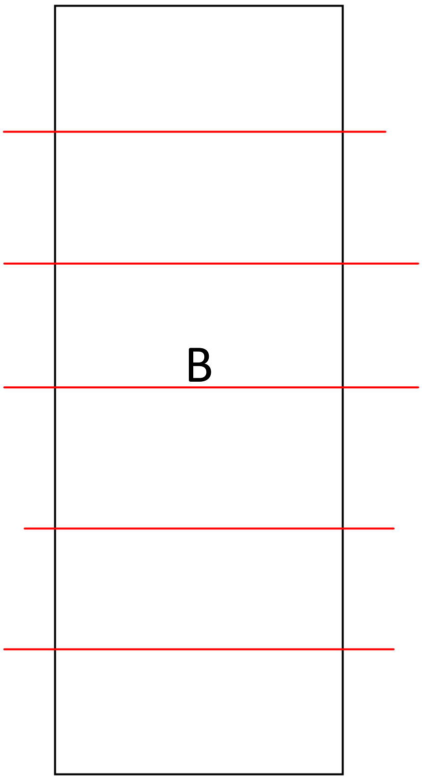


Tell me what are you thinking?
How do you solve the following problem?

1. Cut the following cake into 5 or 7 pieces(using your pencil draw the cutting line)

2. 2 different cakes, cake A and cake B, are cut as the following, which cake is the biggest?





I think ....
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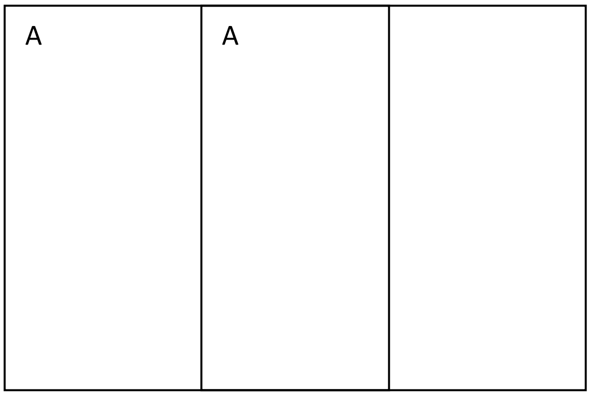
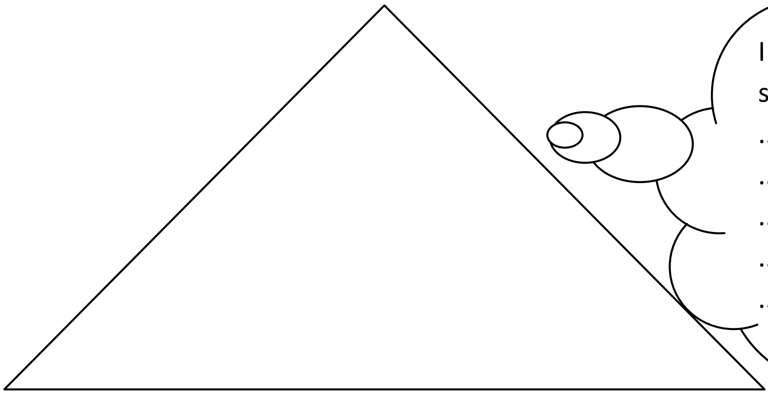
3. Compare with another shape, can you guess, which shape has the same size, smaller size, or bigger size with this square A? (you are free to make a scratch on the picture to solve)





I think this shape is .... than square A because........
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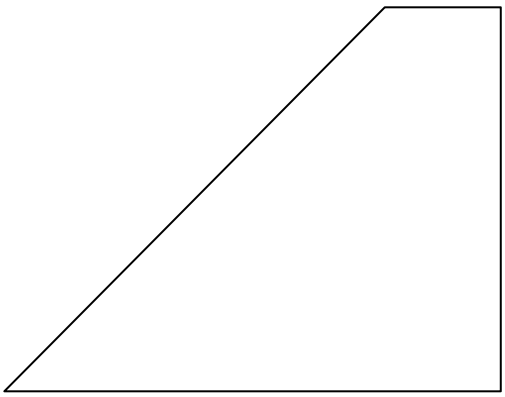

I think this shape is .... than square A because........
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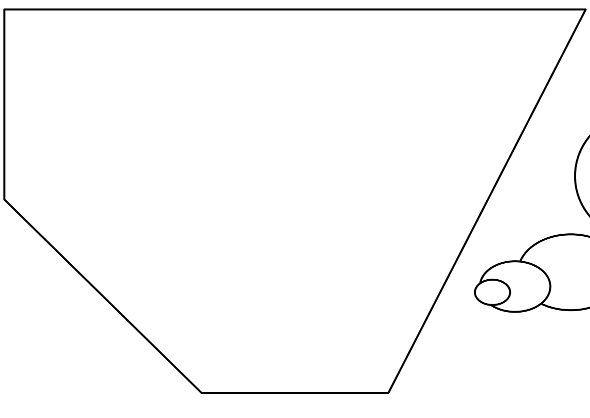

I think this shape is .... than square A because........
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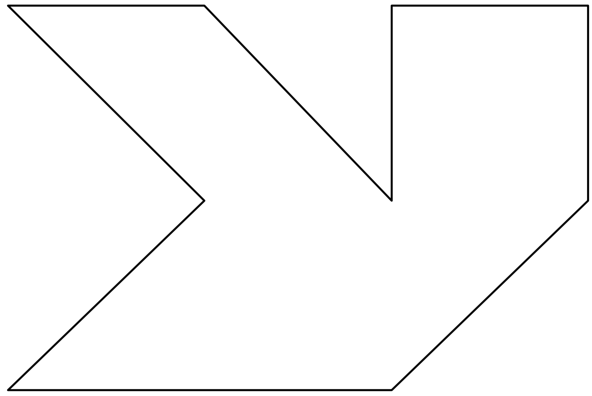
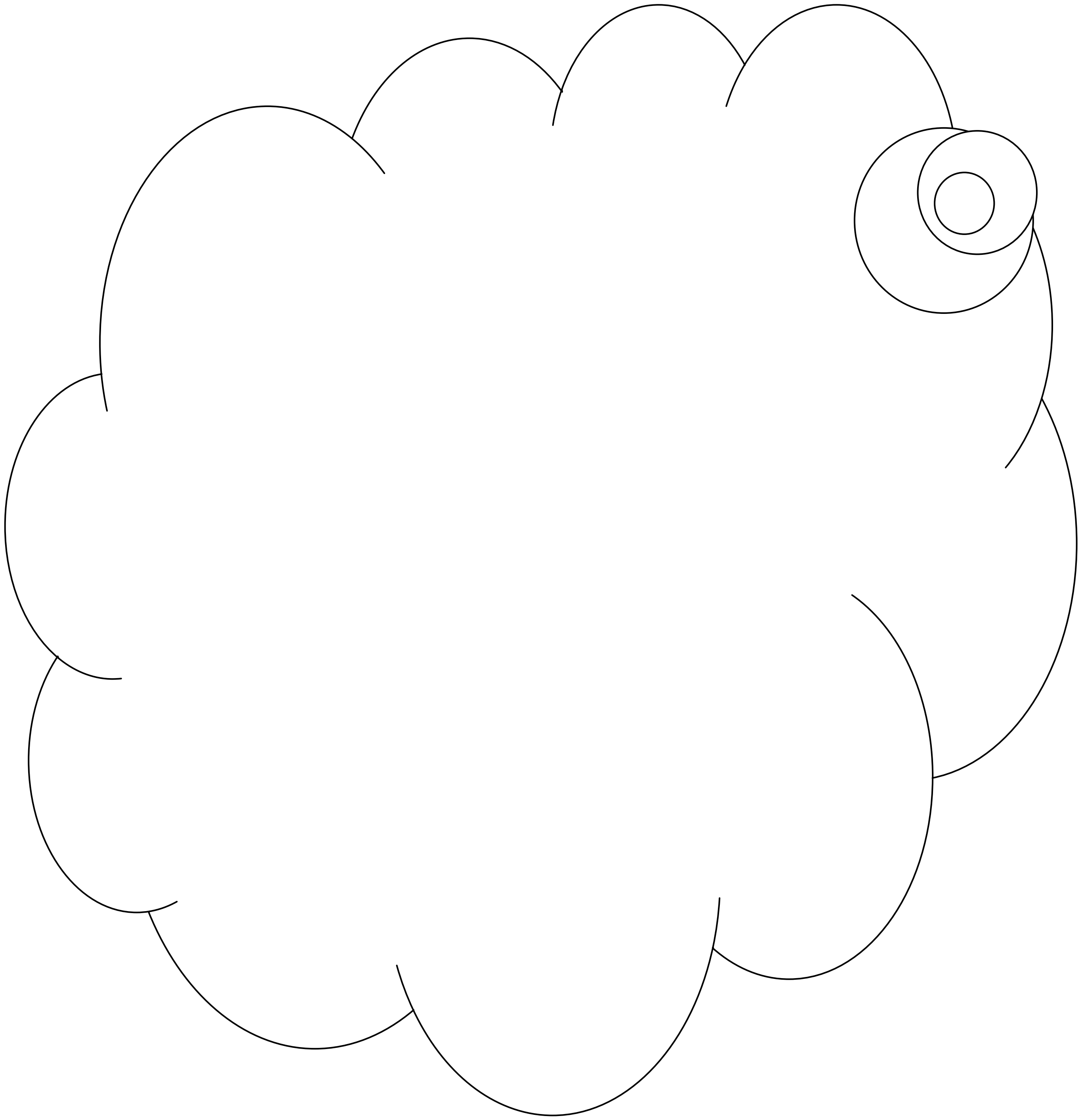
4. Could you draw another figure which has same area with the following shape? you may make a scratch in this picture

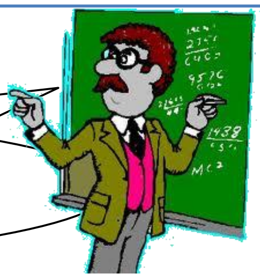




APPENDIX 2.5

Pre-tes

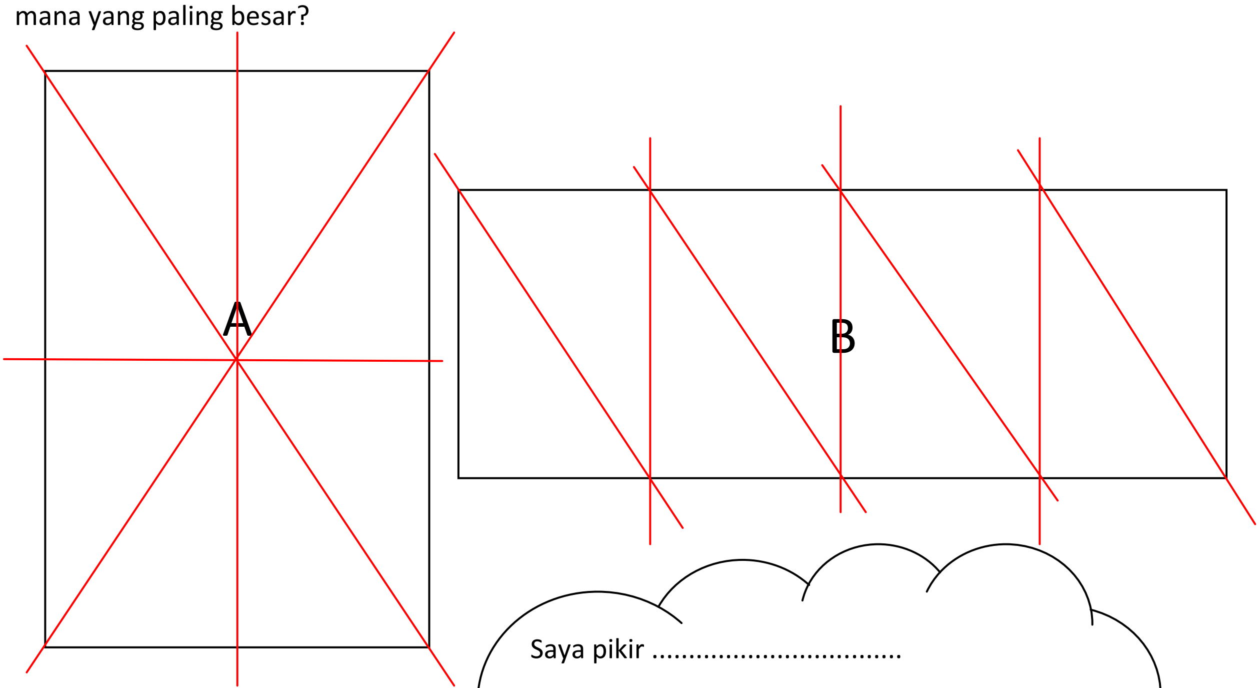
Nama/Kelas: …………………..



Beritahukan kepada saya, apa yang ada di pikiranmu?
Bagaimanakah kamu memecahkan permasalahan berikut ini?

1. Potonglah kue tar berikut ini menjadi 5 atau 7 potong (gunakan pensilmu untuk menggambar garis potong pada gambar kue berikut!)

2. 2 buah kue tar berbeda, kue A dan kue B, dipotong seperti pada gambar berikut ini.kue mana yang paling besar?



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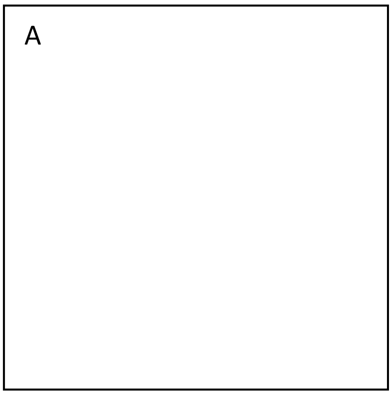
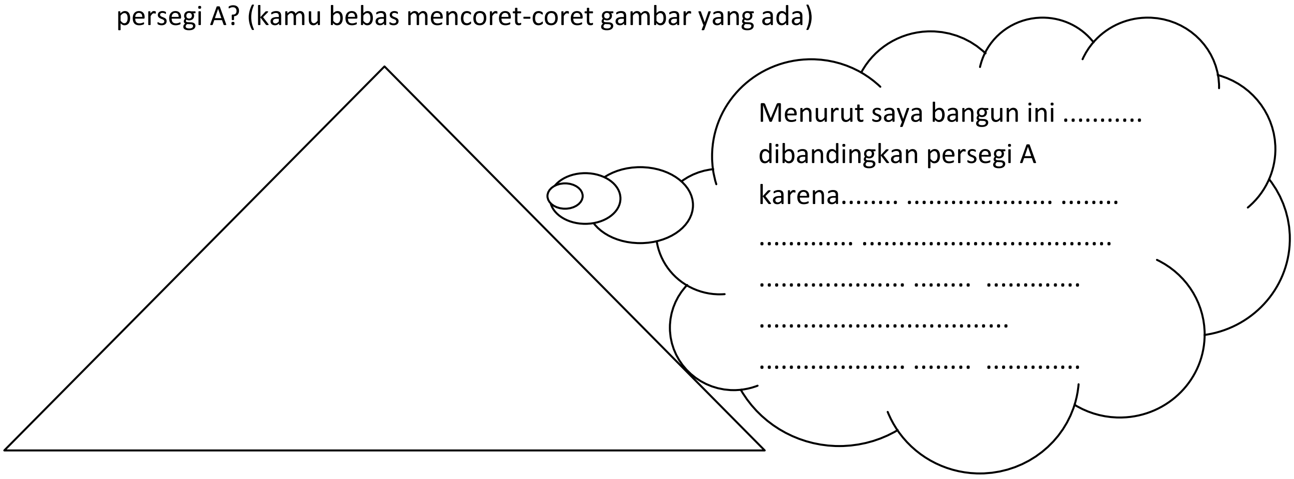
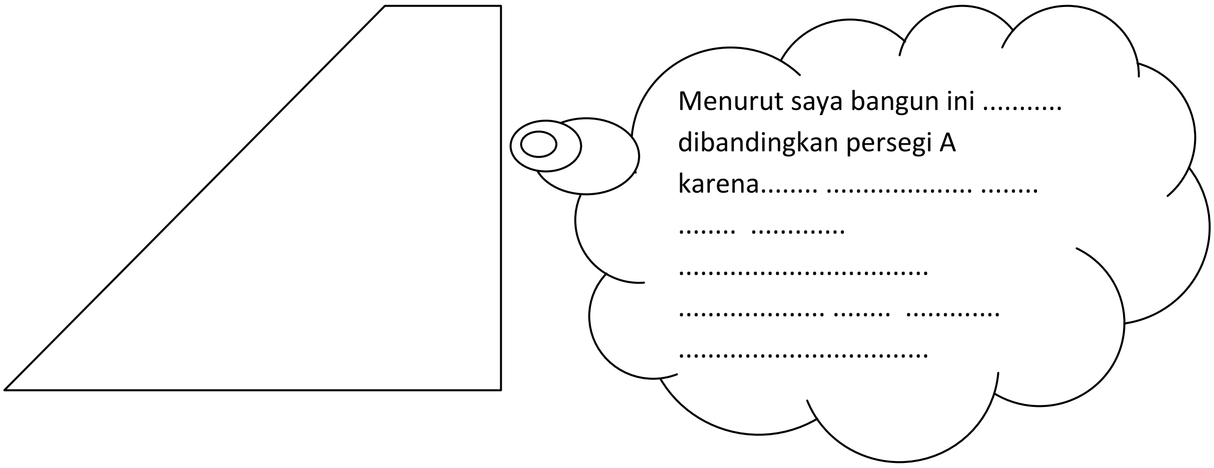
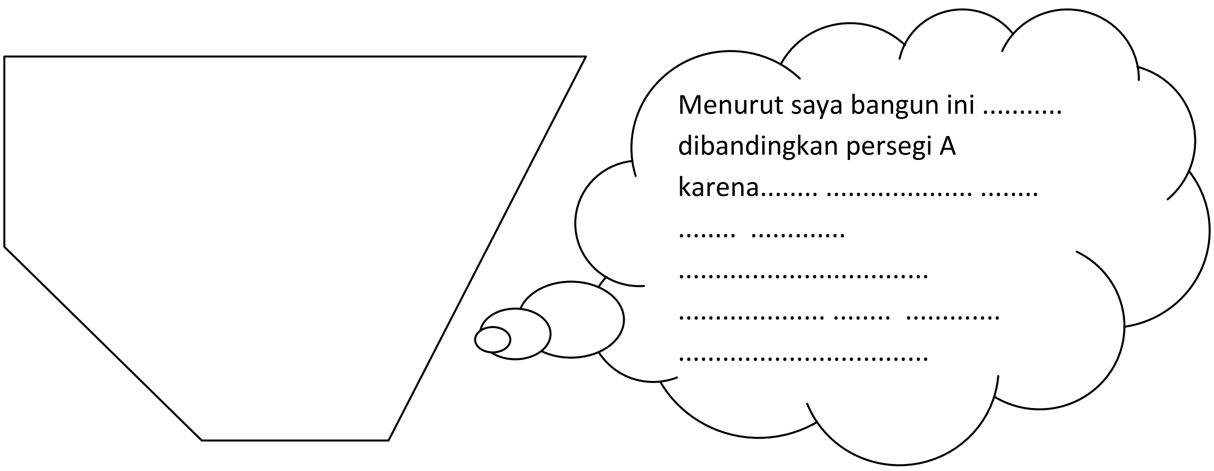


3. Perhatikan gambar persegi diatas, bila dibandingkan dengan gambar-gambar berikut ini, perkirakanlah mana yang sama besar, lebih kecil, taua lebih besar jika dibandingkan dengan persegi A? (kamu bebas mencoret-coret gambar yang ada)







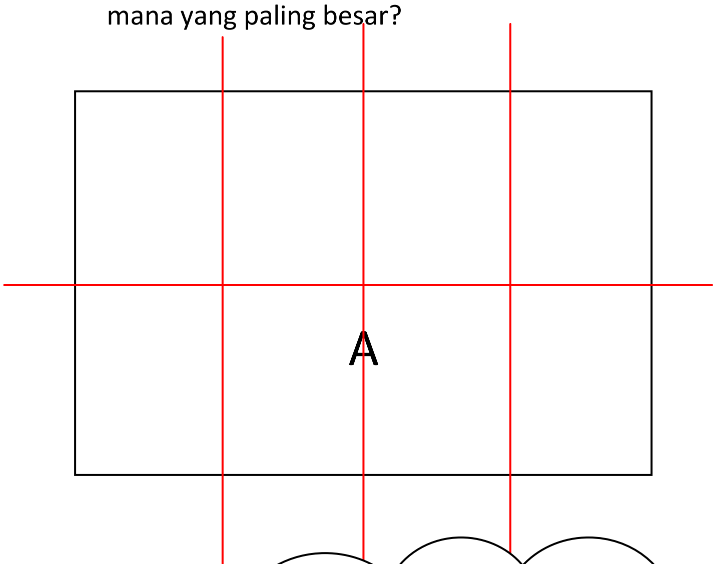
Post-tes

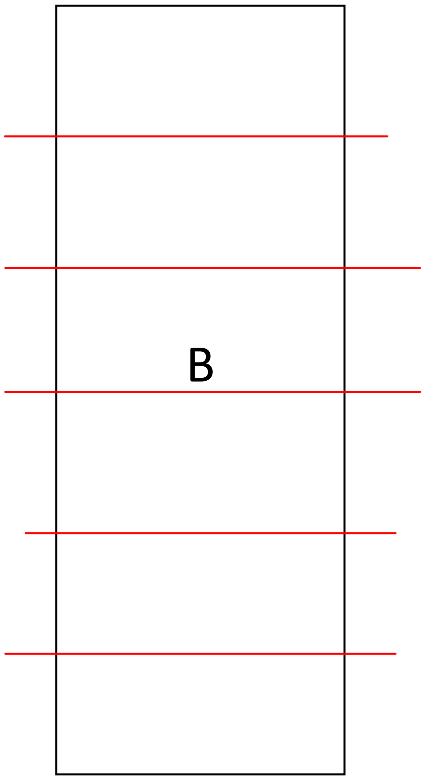
Nama/Kelas: …………………..

Beritahukan kepada saya, apa yang ada di pikiranmu?
Bagaimanakah kamu memecahkan permasalahan berikut ini?

1. Potonglah kue tar berikut ini menjadi 5 atau 7 potong (gunakan pensilmu untuk menggambar garis potong pada gambar kue berikut!)

2. 2 buah kue tar berbeda, kue A dan kue B, dipotong seperti pada gambar berikut ini.kue mana yang paling besar?





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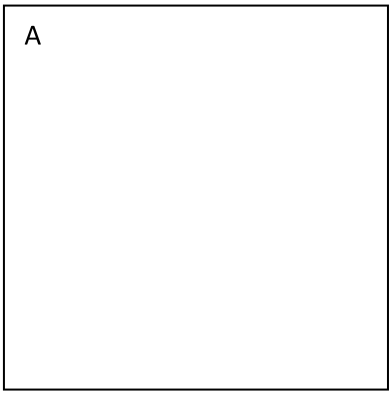
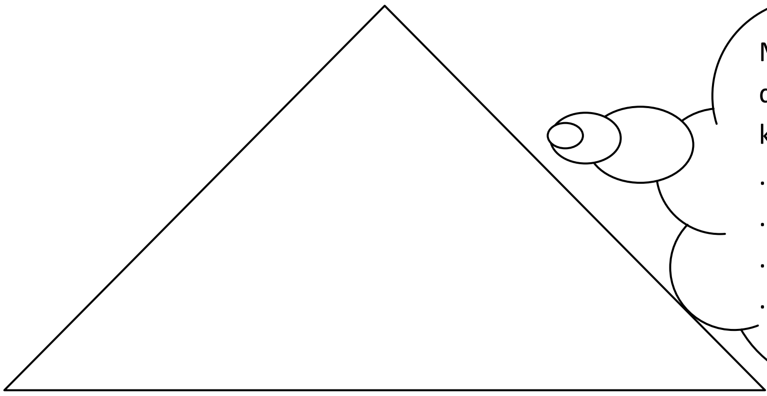


3. Perhatikan gambar persegi diatas, bila dibandingkan dengan gambar-gambar berikut ini, perkirakanlah mana yang sama besar, lebih kecil, taua lebih besar jika dibandingkan dengan persegi A? (kamu bebas mencoret-coret gambar yang ada)



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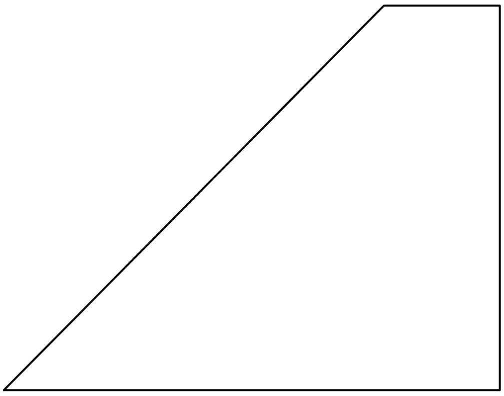

Menurut saya bangun ini ....
dibandingkan persegi A
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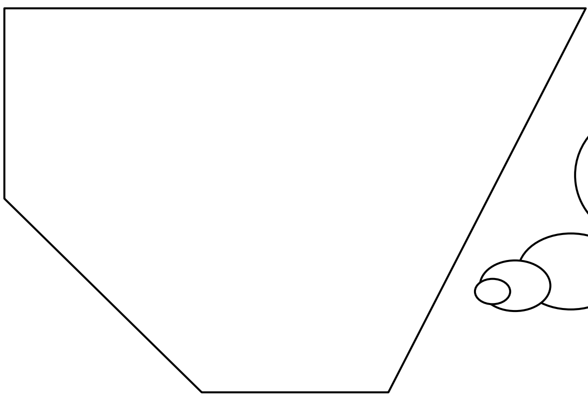

Menurut saya bangun ini ....
dibandingkan persegi A
karena........ ....
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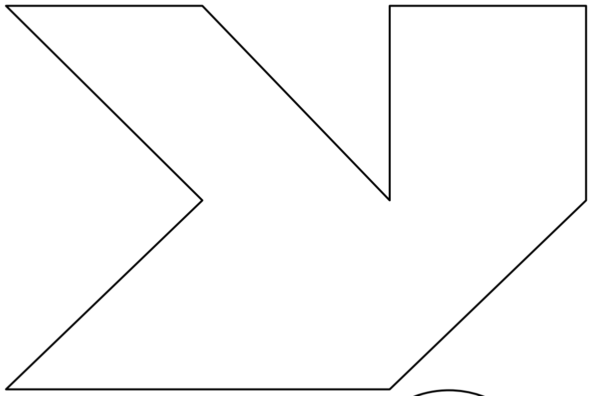
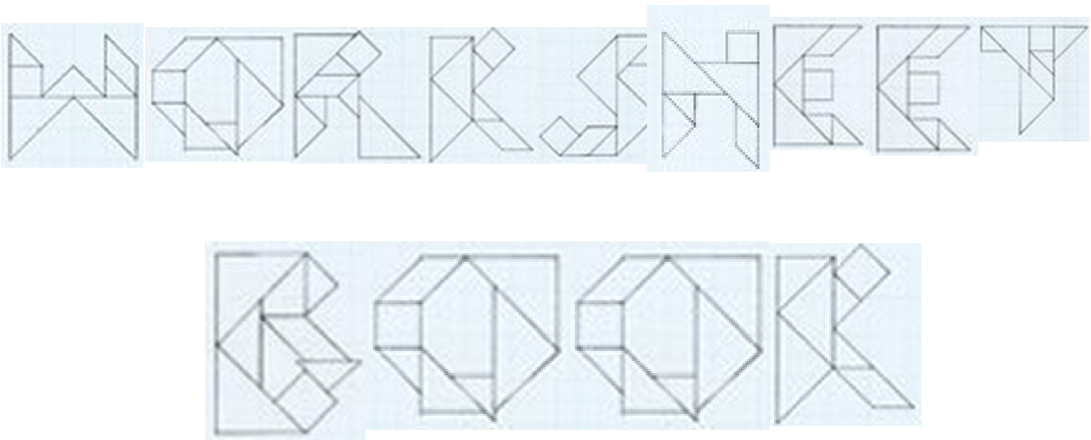
4. Dapatkah kamu menggambar bentuk/bangun lain yang memiliki luas yang sama dengan bangun berikut ini? Jika bisa, gambarlah bangun tersebut pada tempat yang telah disediakan

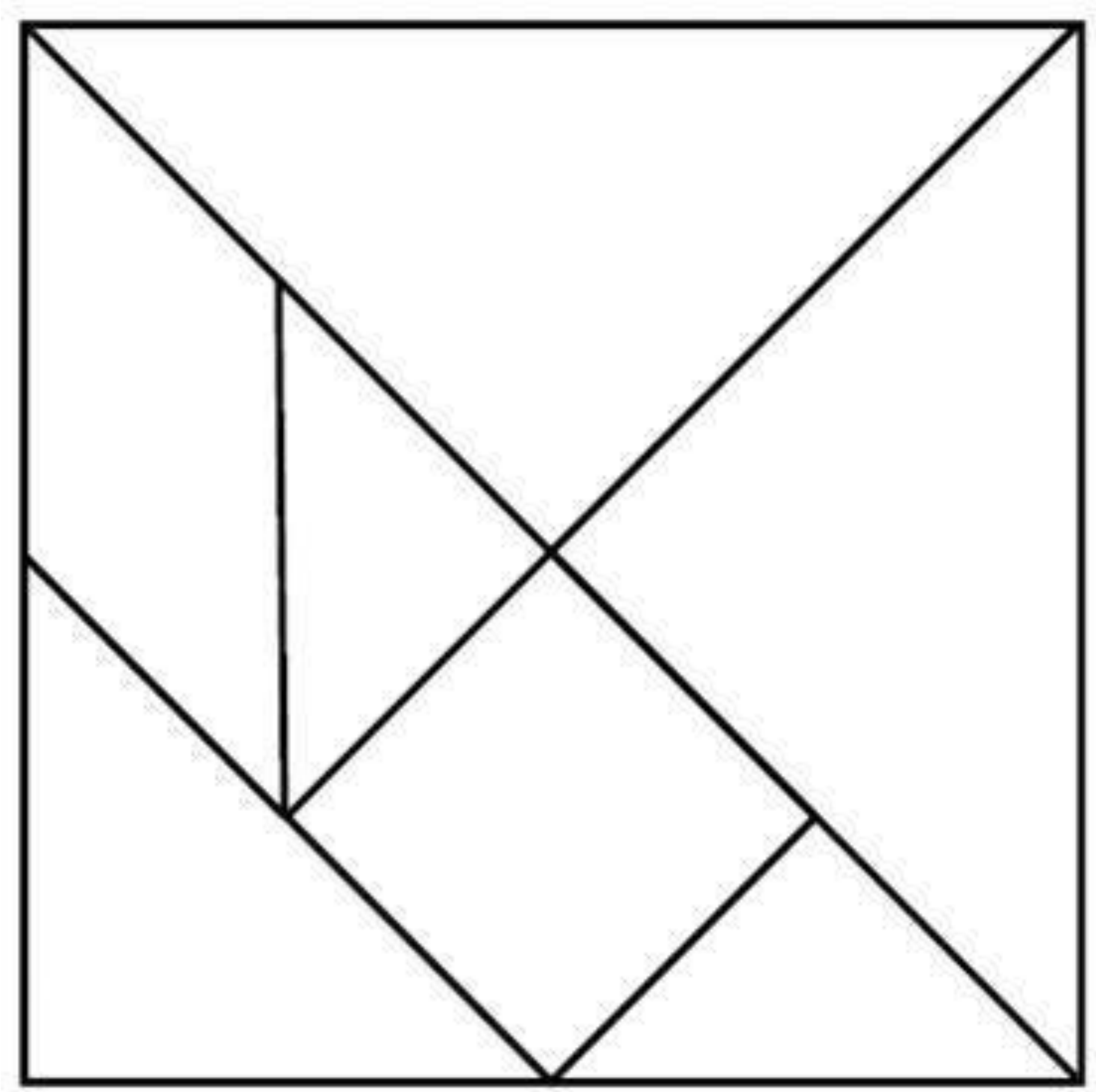


APPENDIX 3

Tangram Abstract World Ma...
Michael Tompsett
Buy a Print Starting at: $27.00
fineartamerica.com



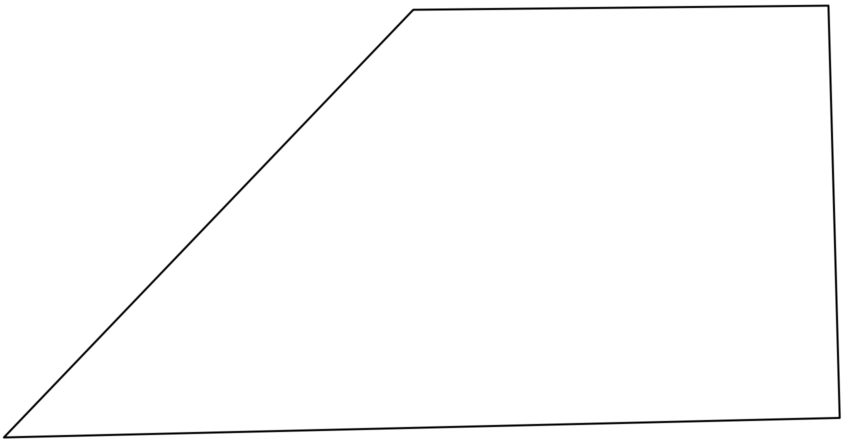
THE TANGRAM SET THAT IS USED IN THE WHOLE ACTIVITIES



Meeting 1 – Worksheet A Session 1

Name :
Class :

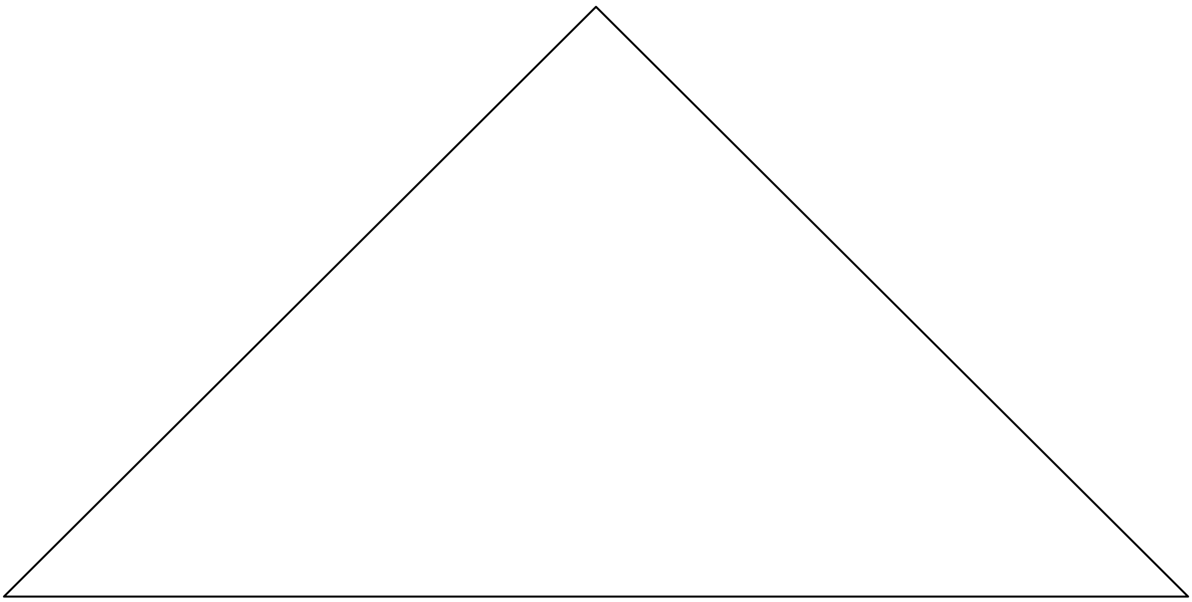
Use 2 pieces of tangram pieces to construct the following figure, then draw your construction here.



Meeting 1 – Worksheet B Session 1

Name :
Class :

Use 2 pieces of tangram pieces to construct the following figure, then draw your construction here.

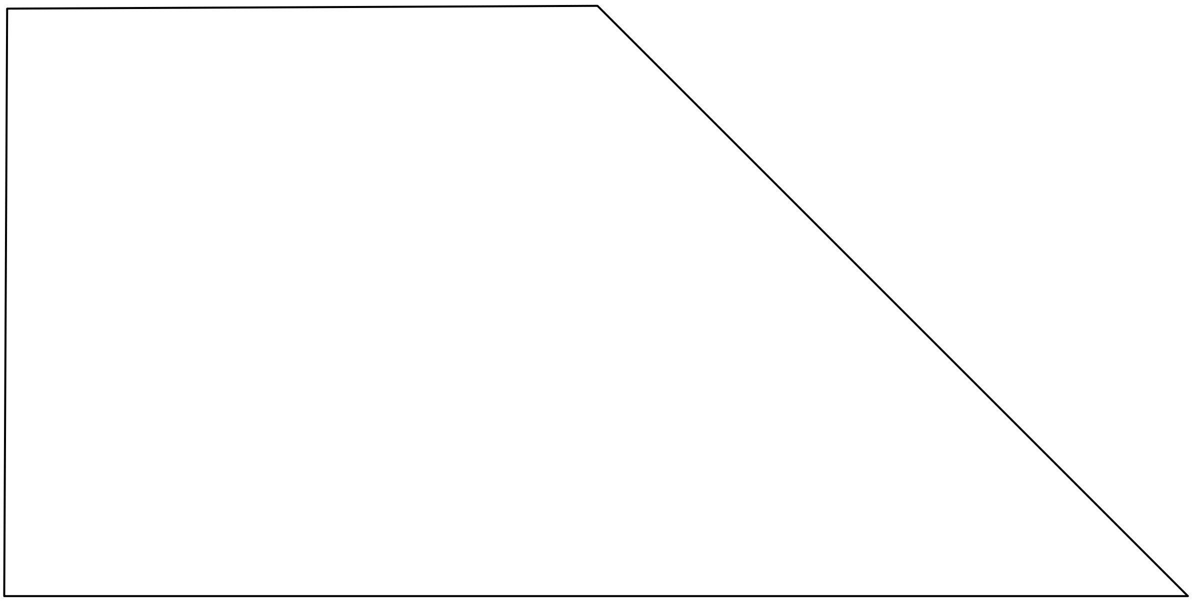


Meeting 1 – Worksheet C Session 1

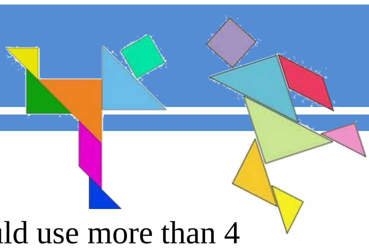
Name :

Class :

Use several pieces of tangram pieces to construct the following figure, then draw your construction here.



Meeting 1 – Worksheet 0 Session 1



Name :

Class :

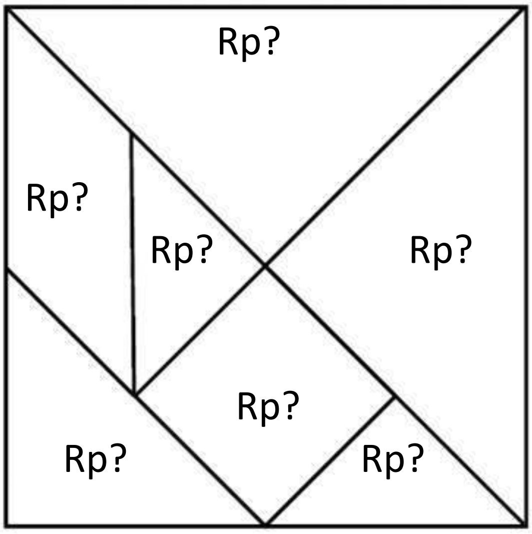
Make your own favourite shapes using the one set tangram. You could use more than 4 pieces or the whole pieces. Then, draw or sketch or trace your shapes in the following space.

Meeting 1 – Worksheet Session 2

Name:
Class:

Selling the Tangram in Pieces

My brother owns a toyshop. Have you ever come to a toyshop? What did you see there? Did the toyshop sell tangram? In my brother toyshop, he sell tangram. Kakak saya memiliki took mainan. Salah satu mainan yang dijual di tokonya adalah puzzle tangram with Rp. 40.000,- per set. The owner of the toyshop also wants to sell the tangram puzzle for a piece, because he figure out that many children who have bought a set of tangram will lose one or more pieces of the tangram on the upcoming months, which makes the children could not play fully with a incomplete a tangram set. Could you help him to put a price on each pieces. My brother will really grateful of your help.



Your Task

In a group determine the price for each tangram pieces. You are freely to choose how you will solve this problem using the method that really convinces you?

Then, prepare a poster containing the solution and how your strategy that have you discussed in a group. Make the poster as clear as possible that your other friends from different group could understand

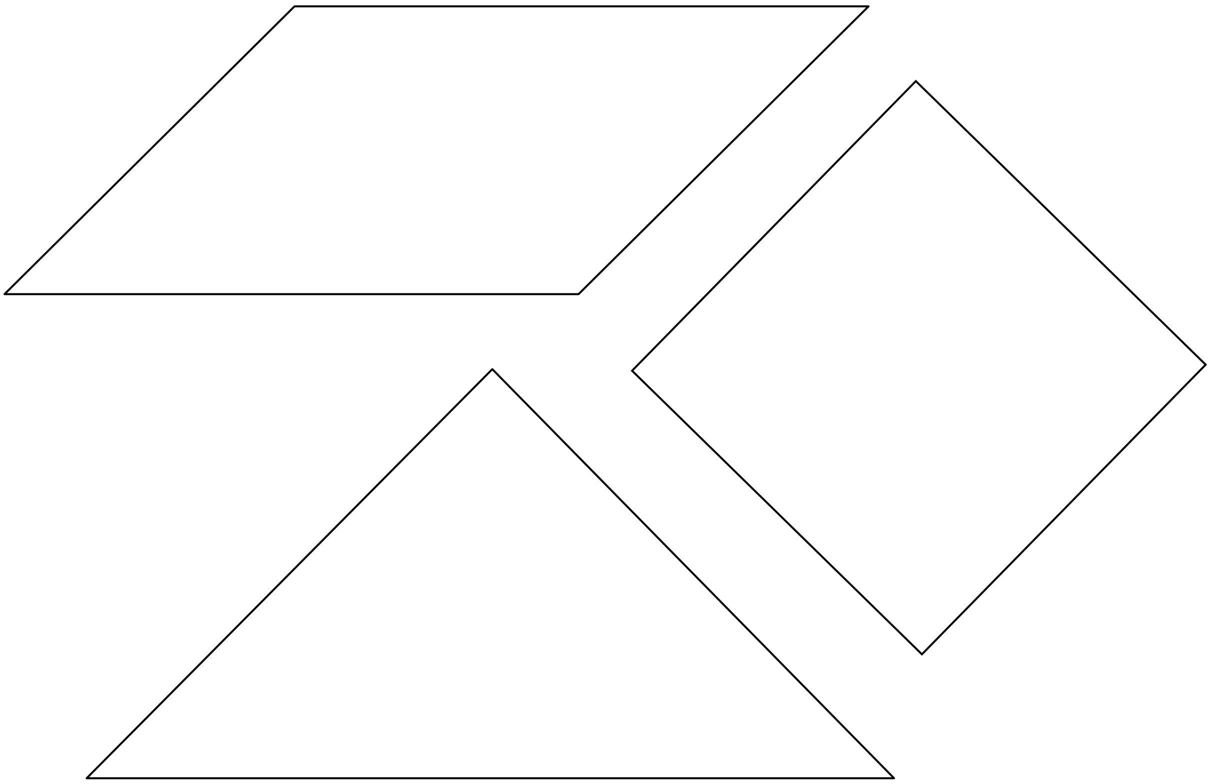
SELAMAT MENGERJAKAN
SEMANGAT☺

Meeting 2 – Worksheet 1

Name :
Class :

Why the price are the same?

In the following, there are several pieces that you have put priced on it yesterday ehihc have the same price. Why can it be? Could you explain to your table mate? You could make a draw on these picture to discuss.



In my opinion, those three shapes have the same price because...................................

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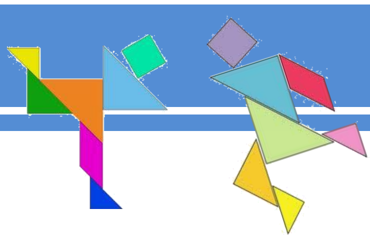
Meeting 2 – Worksheet 2



Name :
Class :

Whose Shape is the Most Expensive?

Do you remember the shap that you had madein the 1st meeting? Now put price for your shape.

YOUR TASK

How you put price on your own shape. Explain the strategy below and make sure that your friends could understand what you write.

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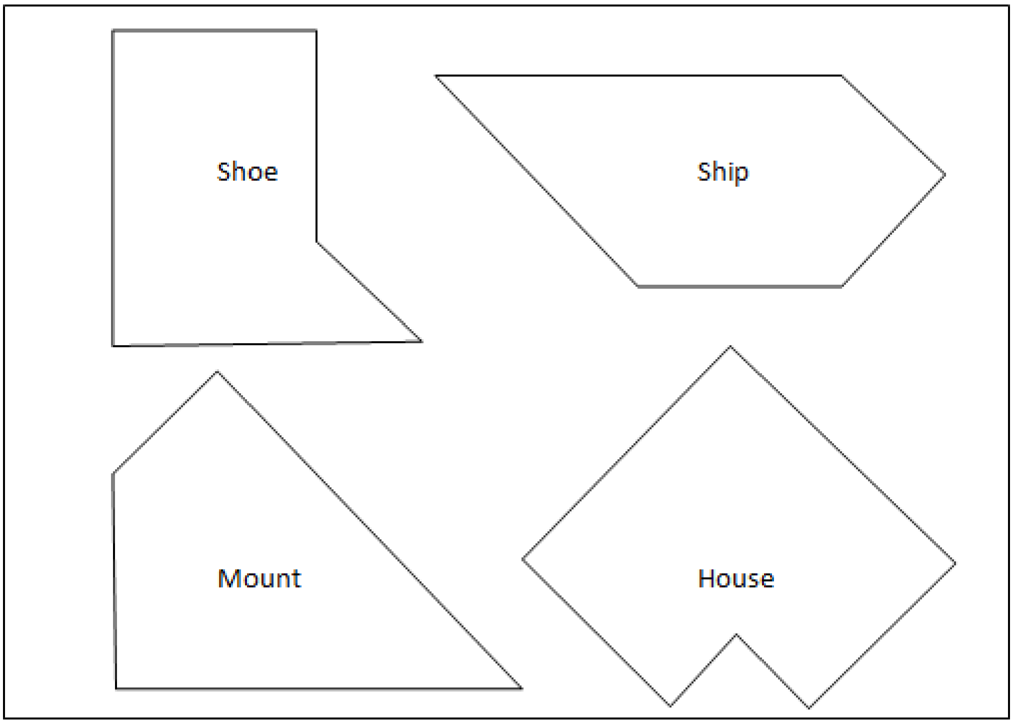
Meeting 3 – Worksheet 2

Name :
Class :

COMPARE THE GIVEN SHAPES WHICH ONE IS THE MOST EXPENSIVE, CHEAPEST, THE SAME PRICE

$> ?, < ?, = ?$



Now, we will play with the given shapes. The picture above is the illustration of the shapes that you will be working on. The actual shapes you can find in the following pages

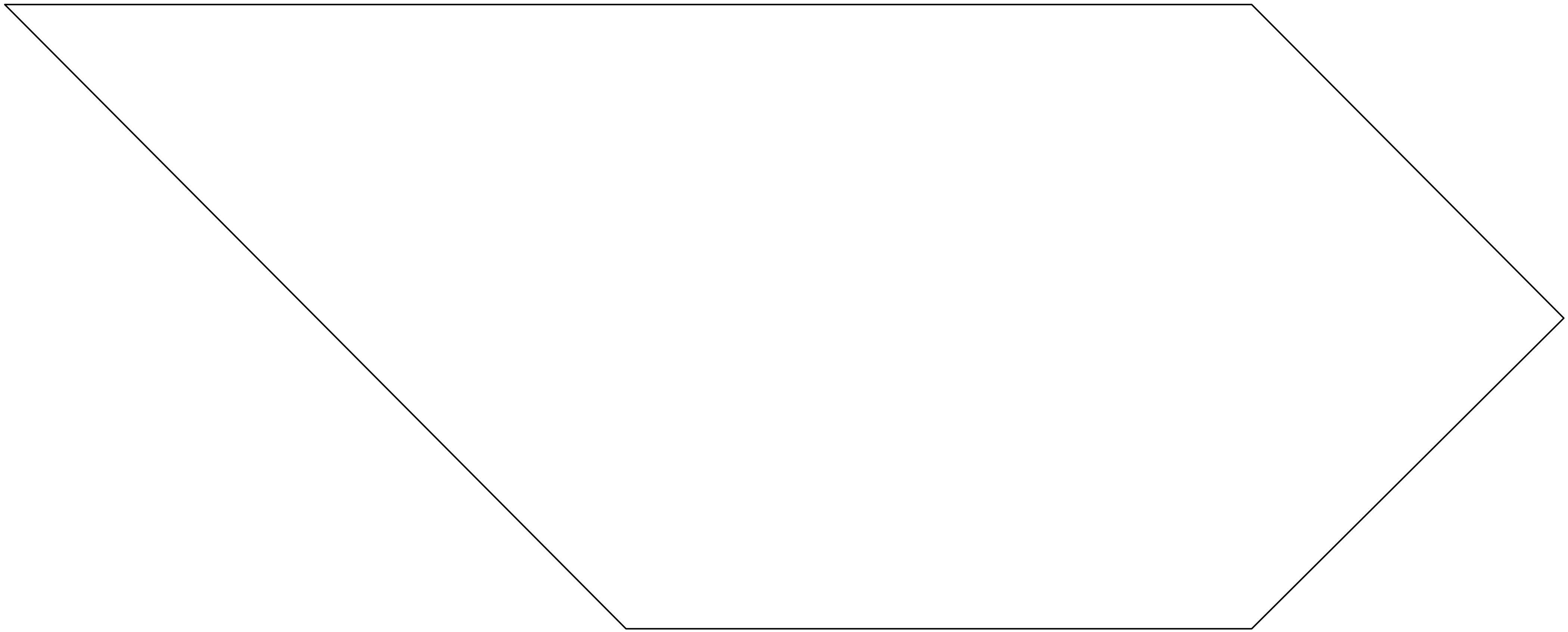
YOUR TASK

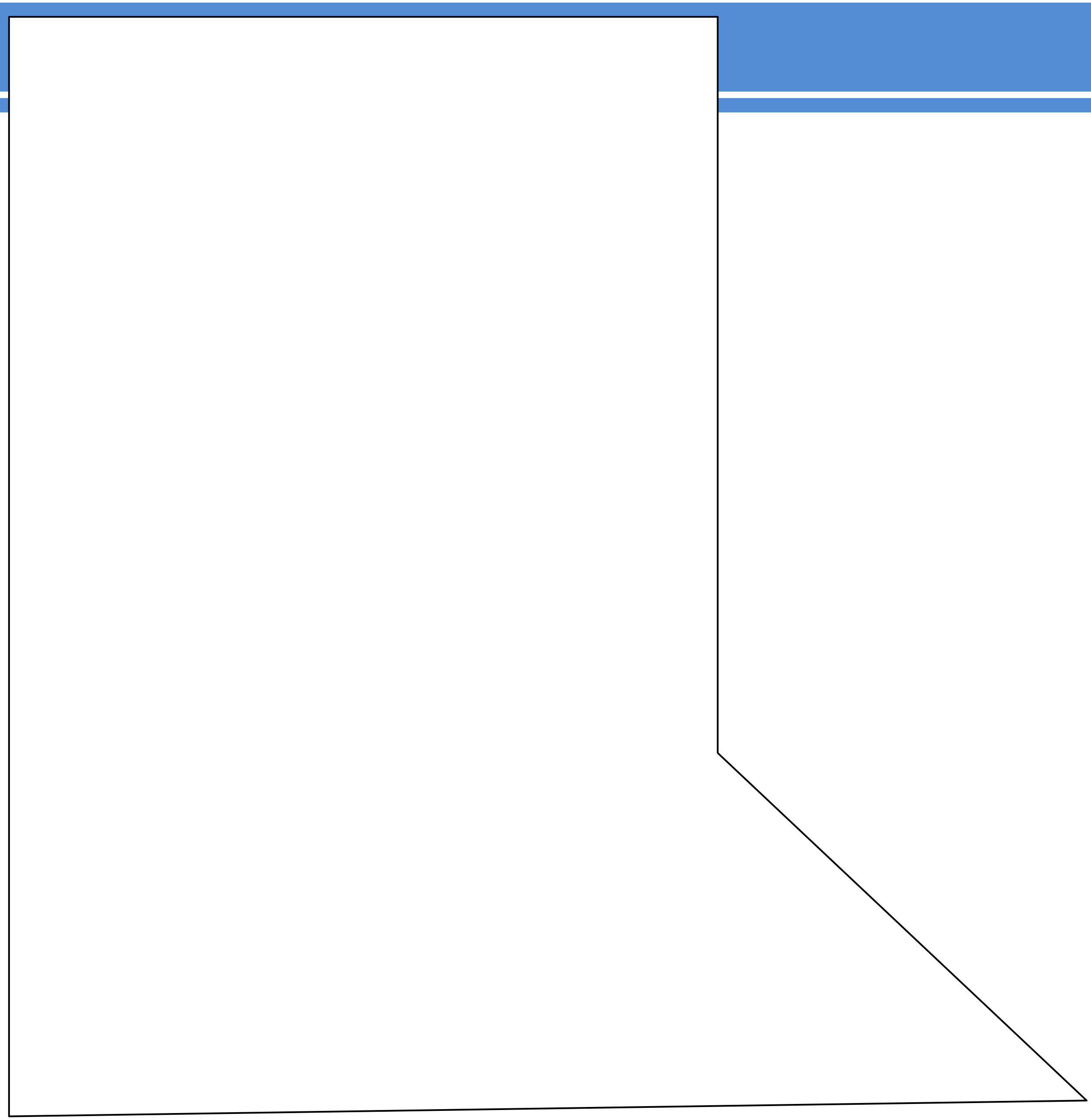
In a group determine which shape has the most expensive, the cheapest, and the same price. You are freely to choose how you will solve this problem using the method that really convinces you?

Then, prepare a poster containing the solution and how your strategy that have you discussed in a group. Make the poster as clear as possible that your other friends from different group could understand

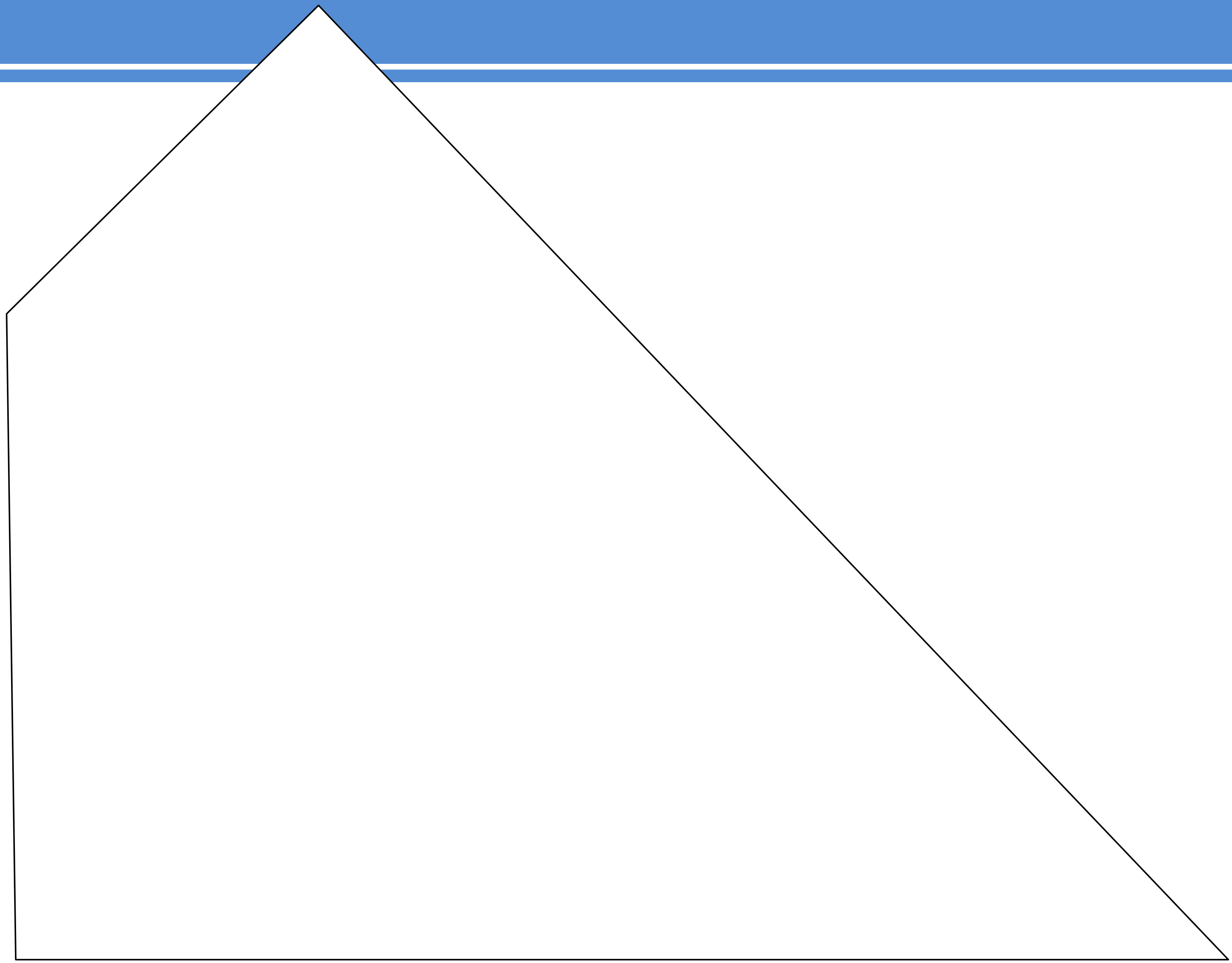
SELAMAT MENGERJAKAN
SEMANGAT☺

SHAPE A

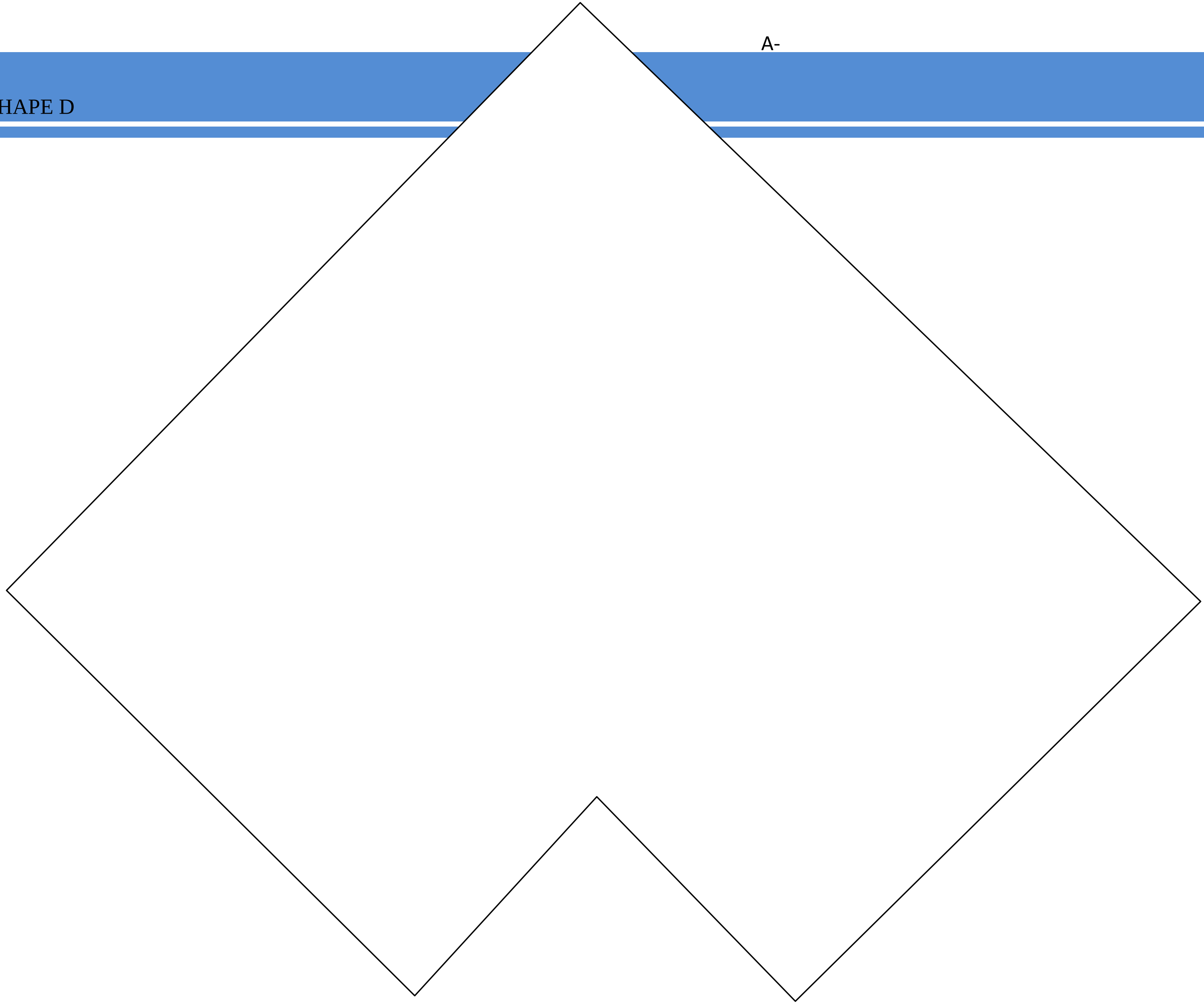




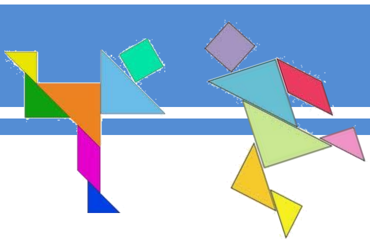
SHAPE C



SHAPE D

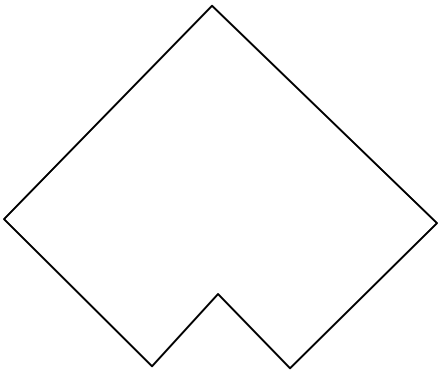


Meeting 5 – Worksheet



Name :
Class

COVERING AREA BY USING ONLY ONE KIND OF PIECE OF TANGRAM



Do you still remember the picture above? In the previous meeting we play with this shape. We could rearrange this shape into tangram pieces. Now your task is a bit different but really challengging ☺

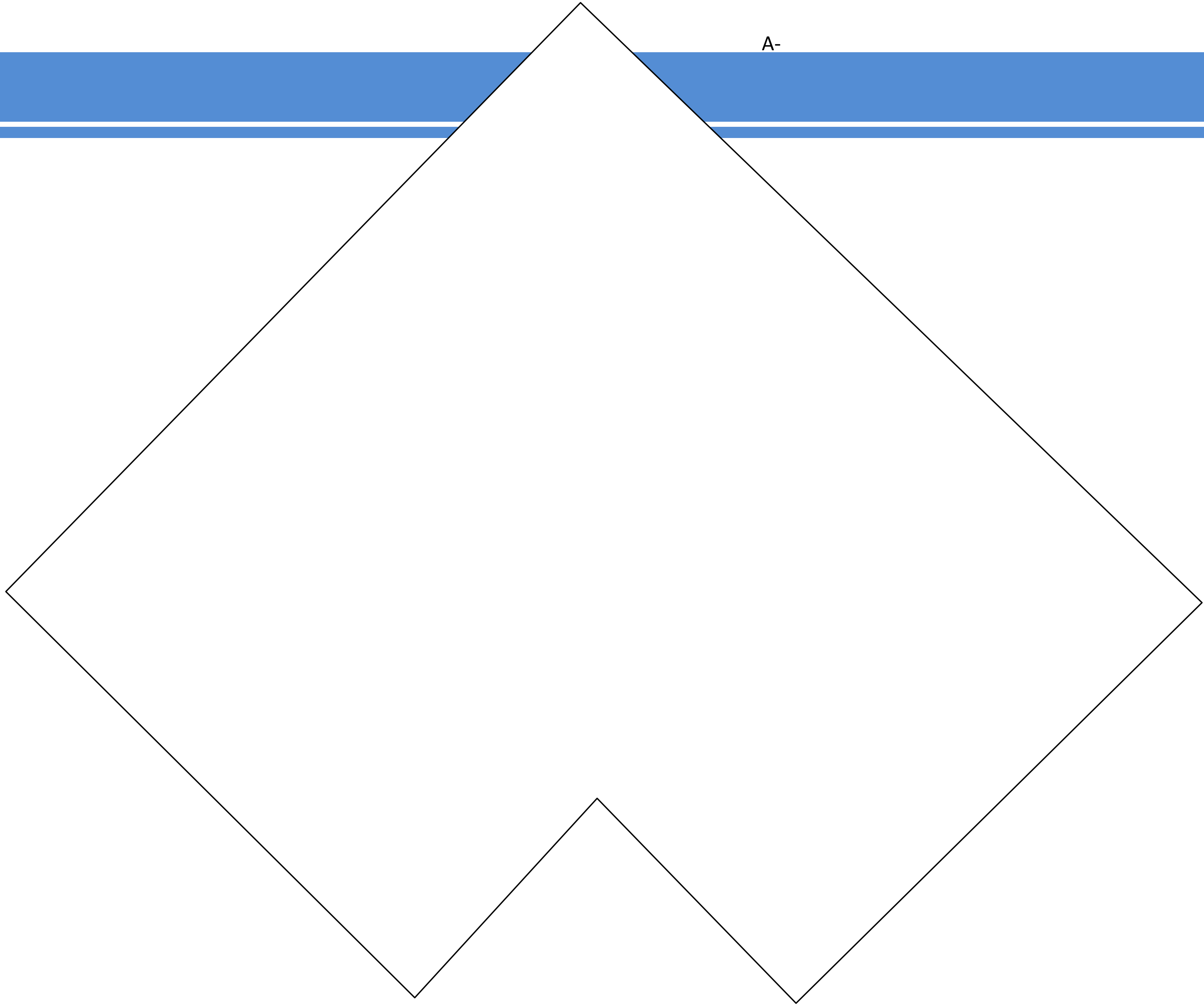
YOUR TASK

Use only one kind tangram pieces from 7 available tangram pieces to cover the whole shape in the following page. If needed, draw the structur how you cover the shape using one kind of pieces.

SELAMAT MENGERJAKAN
SEMANGAT☺

A-

A-31

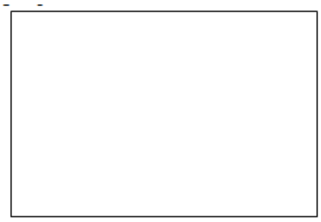


Meeting 6 – Worksheet

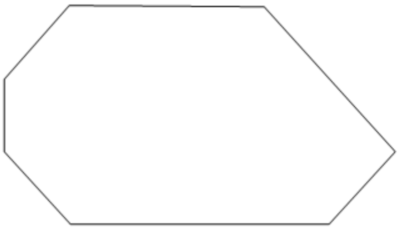
Name :
Class

PLAYING WITH THE AREA OF POLYGON

After we play covering a shape using one kind only pieces, now we try to use those 2 possible tangram pieces which are square and small triangle to cover these polygon (the actual shape are in the following pages).



Segi banyak 1



Segi banyak 2

YOUR TASK

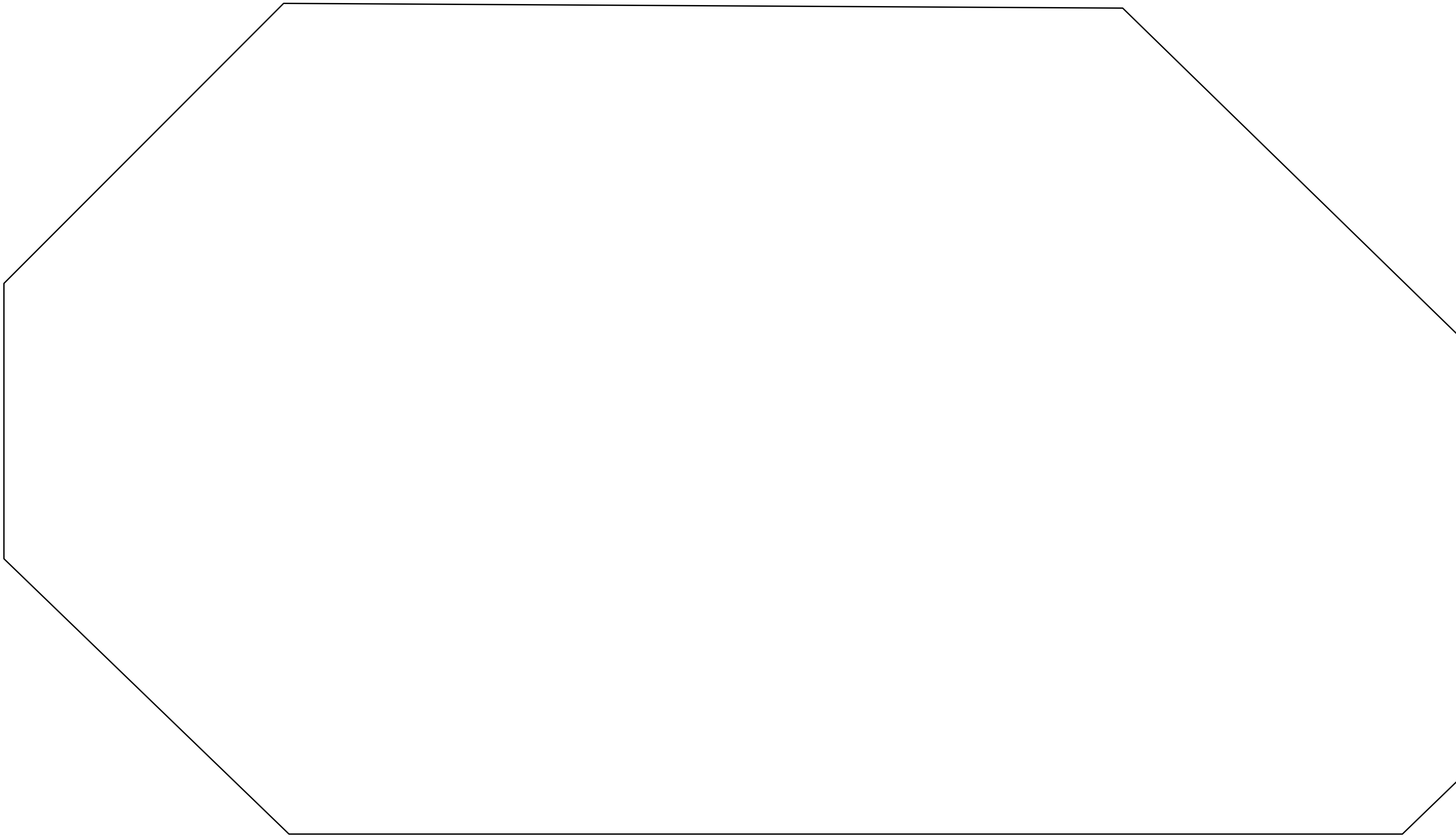
In a group discuss how you determine the area of the given polygons in squares. You are freely to choose how you will solve this problem using the method that really convinces you?

Then, prepare a poster containing the solution and how your strategy that have you discussed in a group. Make the poster as clear as possible that your other friends from different group could understand

SELAMAT MENGERJAKAN
SEMANGAT☺

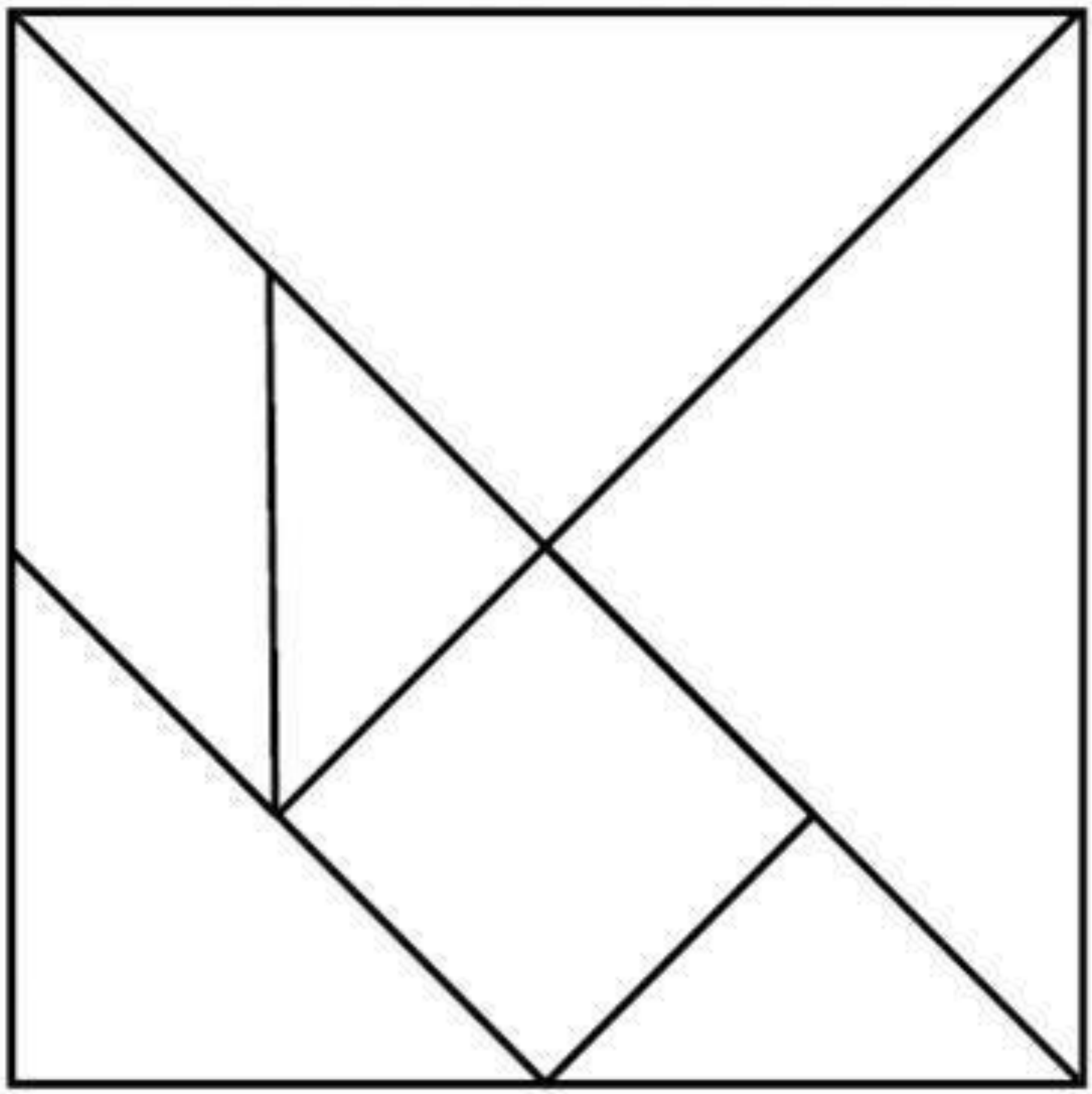
BANGUN D

BANGUN D



APPENDIX 3
Bahasa version

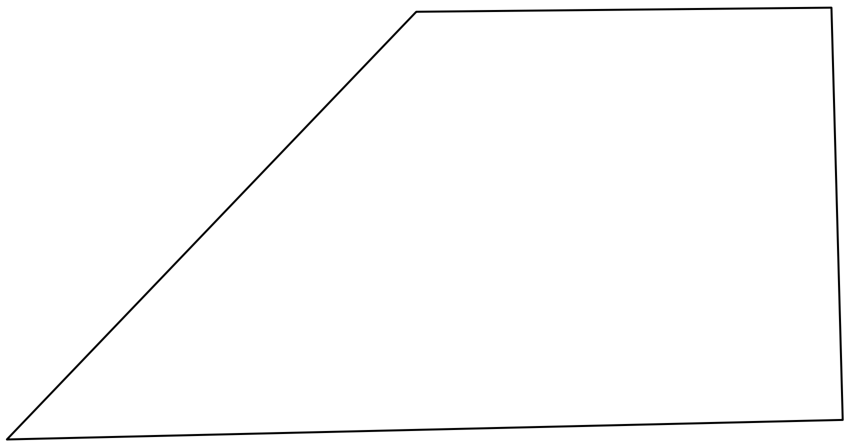
SET TANGRAM YANG DIGUNAKAN



Pertemuan 1 – Lembar Kerja Siswa A Sesi 1

Nama :
Kelas :

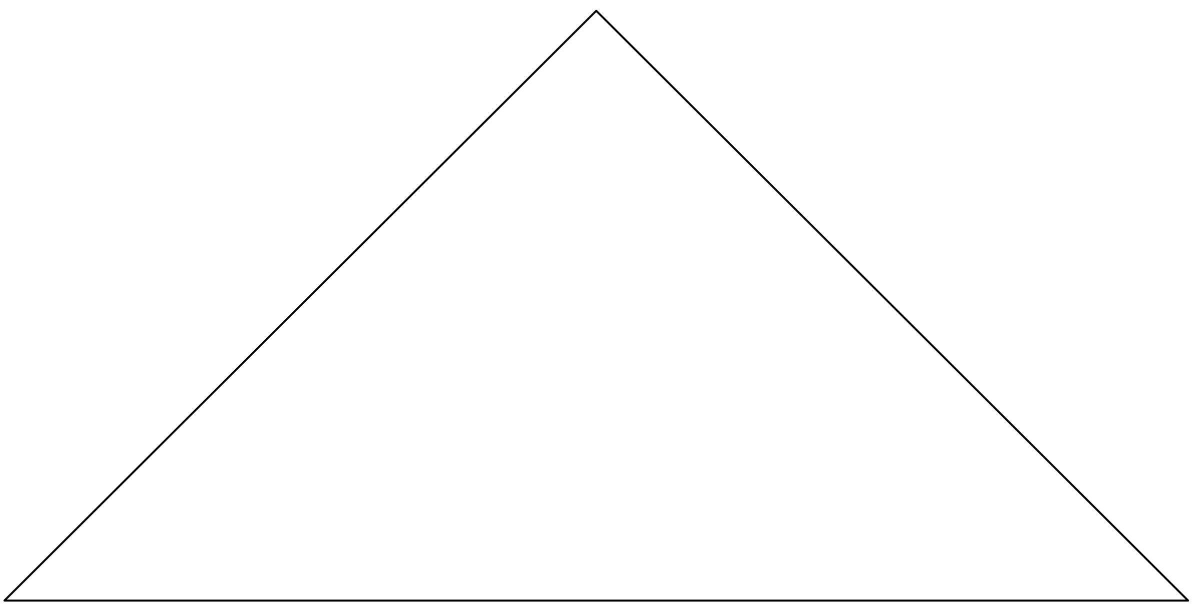
Gunakan 2 potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat.



Pertemuan 1 – Lembar Kerja Siswa B Sesi 1

Nama :
Kelas :

Gunakan 3 potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat.

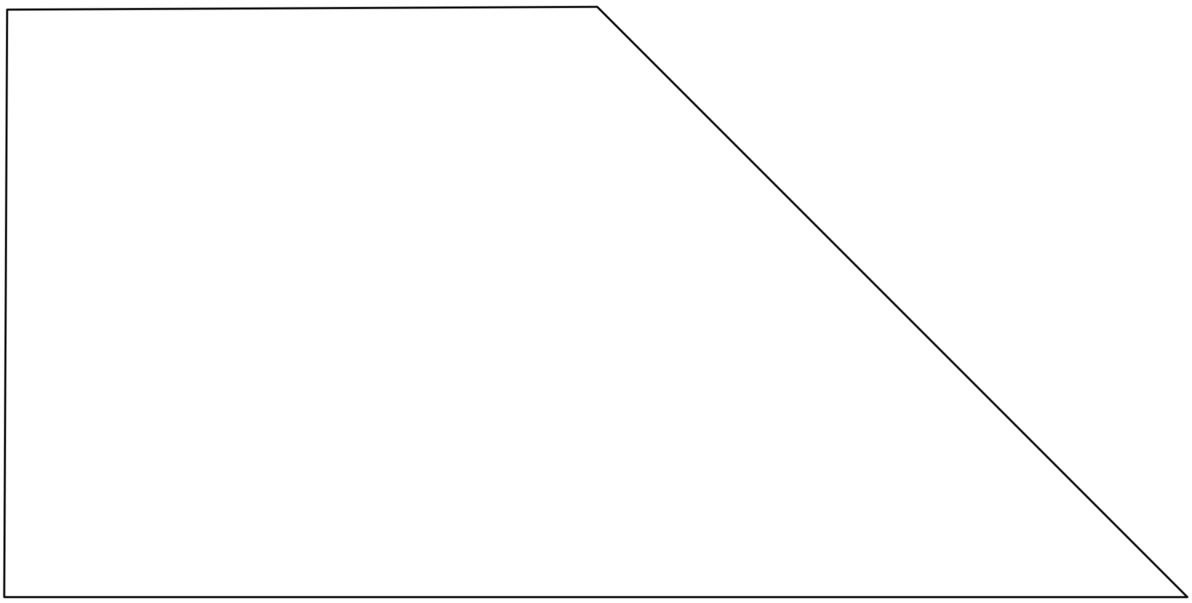


Pertemuan 1 – Lembar Kerja Siswa C Sesi 1

Nama :

Kelas :

Gunakan beberapa potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat.



Pertemuan 1 – Lembar Kerja Siswa 0 Sesi 1

Nama :

kELAS :

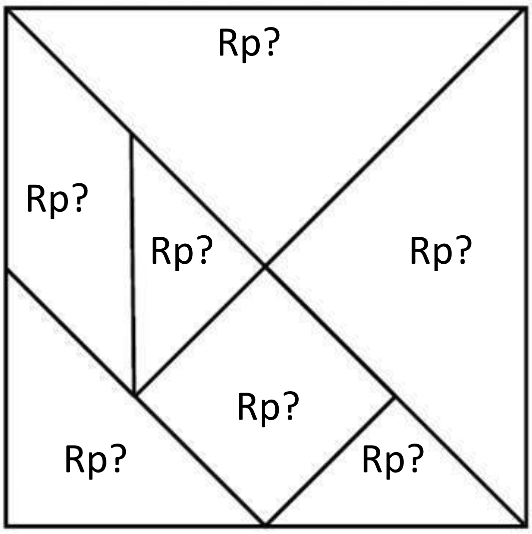
Bentuklah bangun yang kamu suka menggunakan satu set tangram yang tersedia, kemudian gambarlah bentuk yang kamu buat pada temapt berikut ini.

Pertemuan 1 – Lembar Kerja Siswa Sesi 2

Nama :
Kelas :

MENJUAL TANGRAM ECERAN

Kakak saya memiliki took mainan. Salah satu mainan yang dijual di tokonya adalah puzzle tangram. Harga untuk satu set puzzle tangram adalah Rp 40.000,-. Kakak saya berencana menjual juga potongan tangram itu per buah, karena dia melihat setelah beberapa bulan banyak anak-anak yang memiliki puzzle tangram ini kehilangan potongan-potongan tangram nya, sehingga tidak bisa bermain dengan 1 set tangram yang lengkap. Bisakah kalian bantu kakak saya dalam menentukan harga untuk tiap potongan yang berbeda. Kakak saya akan berterimakasih sekali atas bantuannya.



TUGAS KAMU
Dalam 1 kelompok, tentukan harga dari masing-masing potongan tangram. Kalian bebas menggunakan cara apa saja yang kalian perkirakan paling meyakinkan.

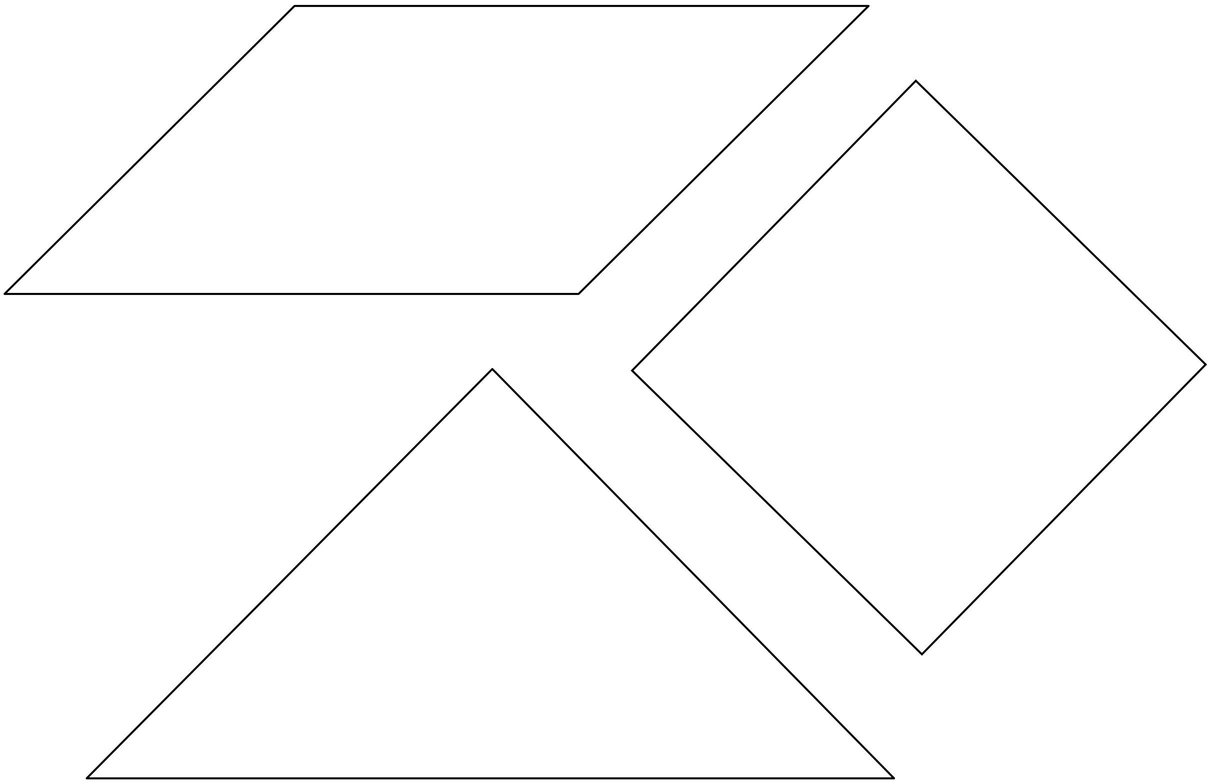
Kemudian, persiapkan 1 poster yang berisi jawaban kalian dan bagaimana cara kalian untuk menentukan jawaban tersebut. Buat poster sedemikian rupa sehingga teman kalian mengerti akan apa yang kalian maksud di poster ini

SELAMAT MENGERJAKAN
SEMANGAT☺

Pertemuan 2 – Lembar Kerja Siswa 1
Nama :
Kelas :

MENGAPA HARGANYA SAMA?

Di bawah ini adalah eberapa potongan tangram yang ternyata harganya sama. Mengapa bisa demikian? Bisakah kamu jelaskan kepada teman sebangkumu? Kamu bisa mencorat-coret gambar berikut ini untuk menjelaskannya



Ketiga potongan itu berharga sama karena menurut saya ………………………………………

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Pertemuan 2 – Lembar Kerja Siswa 2

Nama :

Kelas :

MENTAKSIR HARGA BENTUK MASING-MASING

Ingat bentuk yang telah kalian buat di pertemuan pertama? Sekarang bagaimana jika kita taksir harga dari masing-masing bentuk yang telah kalian buat.

TUGAS KAMU

Bagaimana cara kalian dalam menentaksir harga dari bentuk yang telah kalian buat. Tuliskan strategy kalian pada tempat yang telah disediakan berikut ini dan usahakan teman kalian memahami apa yang kalian tulis

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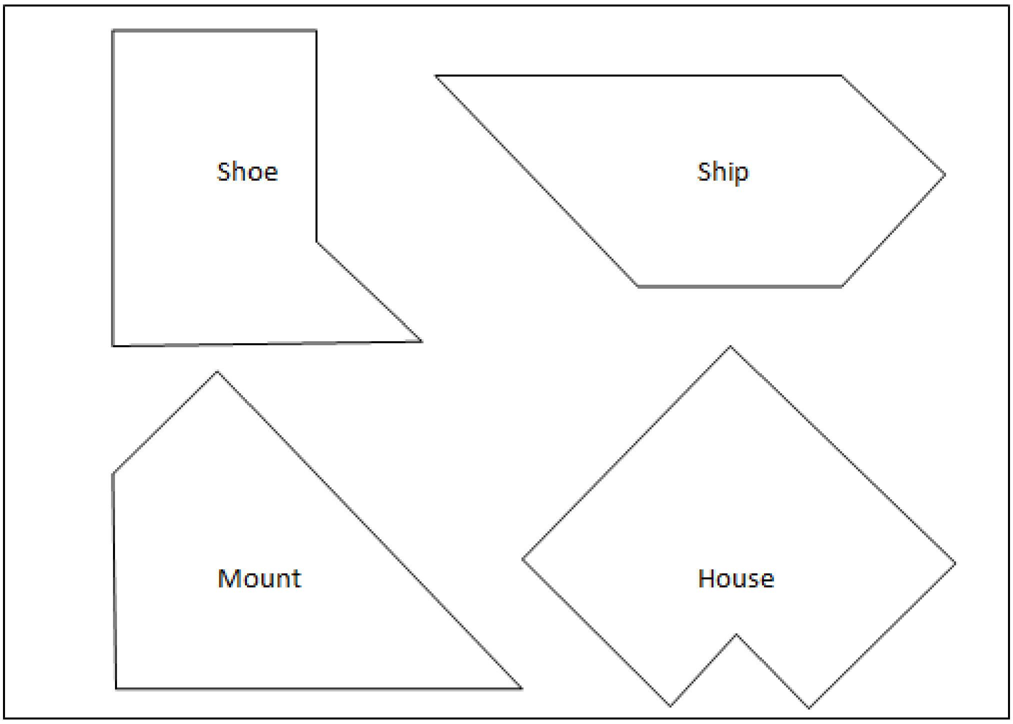
Pertemuan 3 – Lembar Kerja Siswa 2

Nama :
Kelas :

MEMBANDINGKAN BENTUK MANA YANG HARGANYA LEBIH MURAH, LEBIH MAHAL, ATAU SAMA

$$> ?, < ?, = ?$$



Sekarang, kita akan bermain-main dengan beberapa bentuk yang telah disediakan. Gambar diatas adalah ilustrasi dari bentuk-bentuk yang akan kita mainkan. Ukuran sebenarnya ada di halaman selanjutnya.

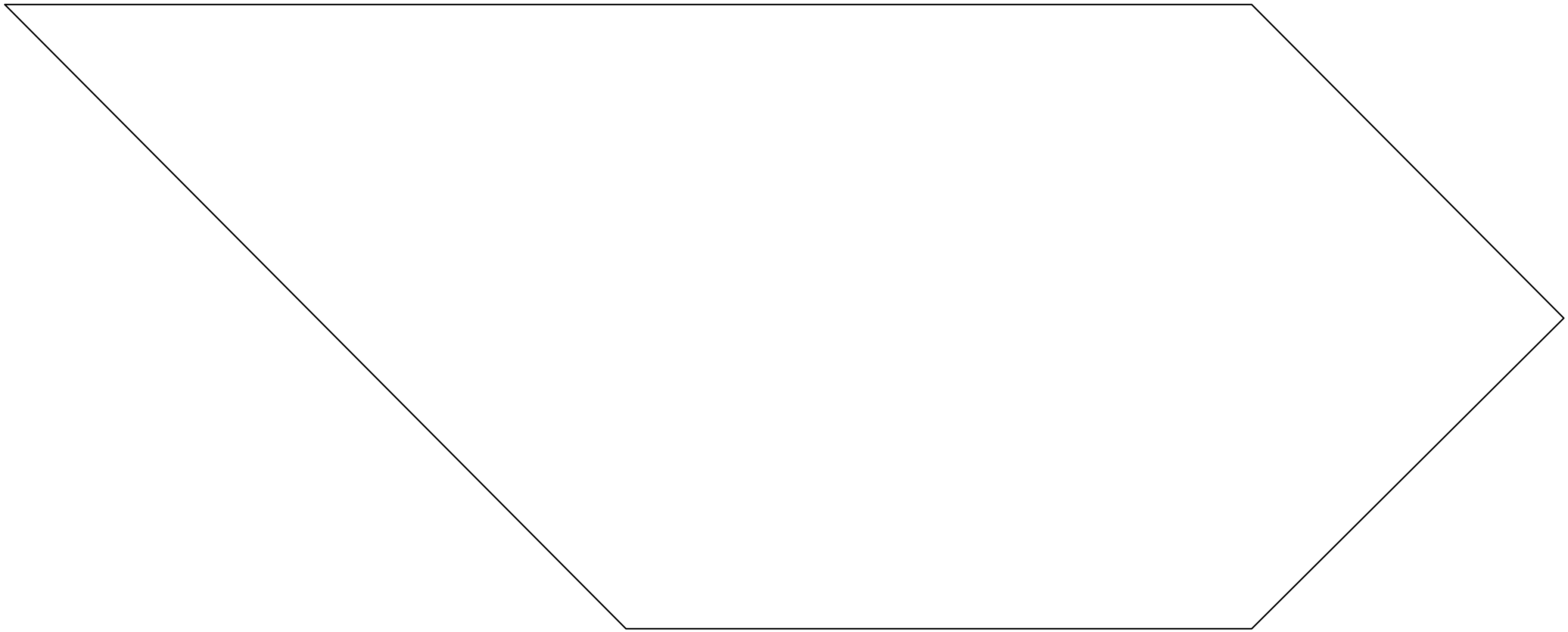
TUGAS KAMU

Dalam 1 kelompok, diskusikan bagaimana untuk menentukan bentuk mana yang harganya paling mahal, paling murah, atau sama. Kalian bebas menggunakan cara apa saja yang kalian perkirakan paling meyakinkan.

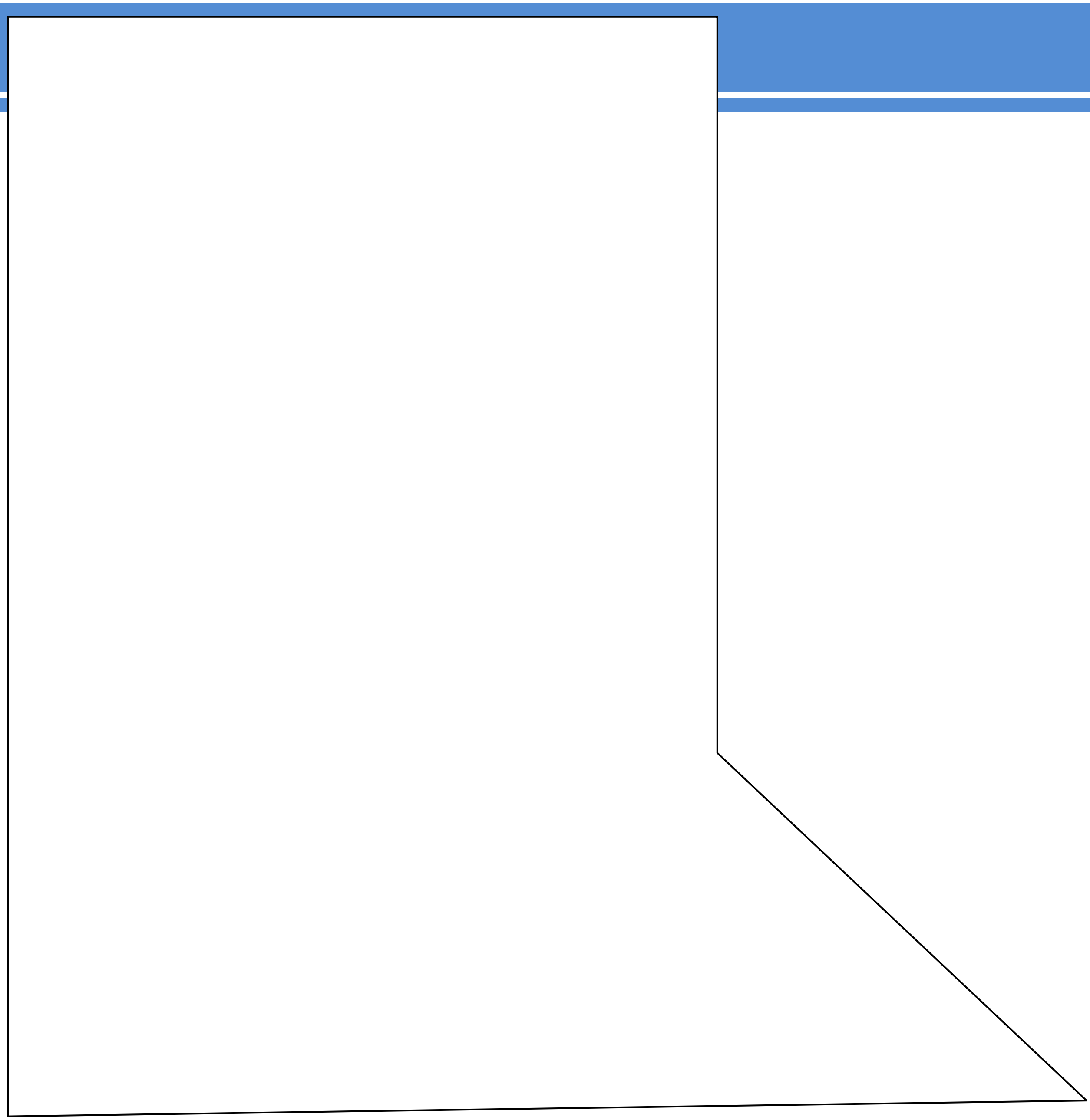
Kemudian, persiapkan 1 poster yang berisi jawaban kalian dan bagaimana cara kalian untuk menentukan jawaban tersebut. Buat poster sedemikian rupa sehingga teman kalian mengerti akan apa yang kalian maksud di poster ini

SELAMAT MENGERJAKAN
SEMANGAT☺

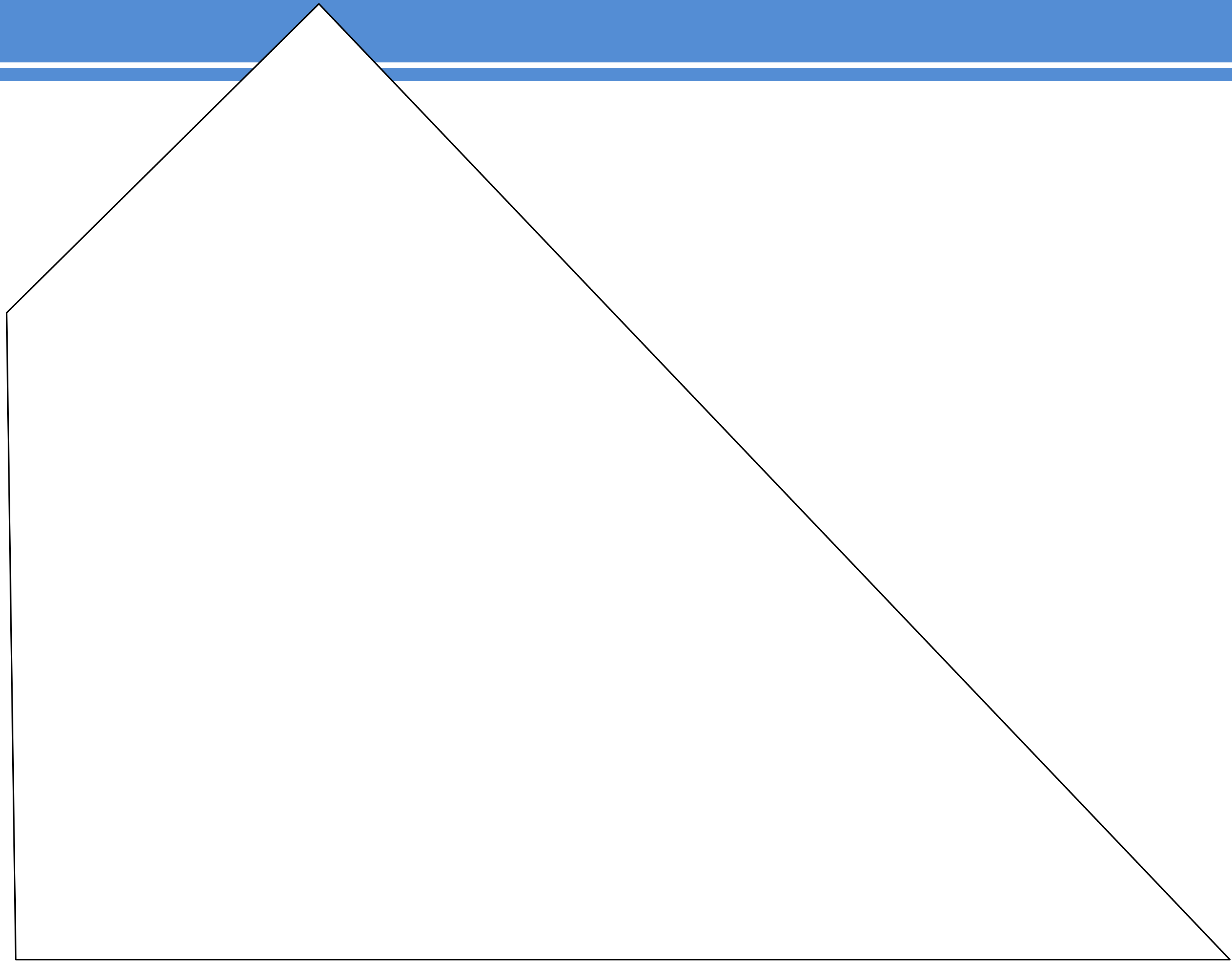
BANGUN A



BANGUN B

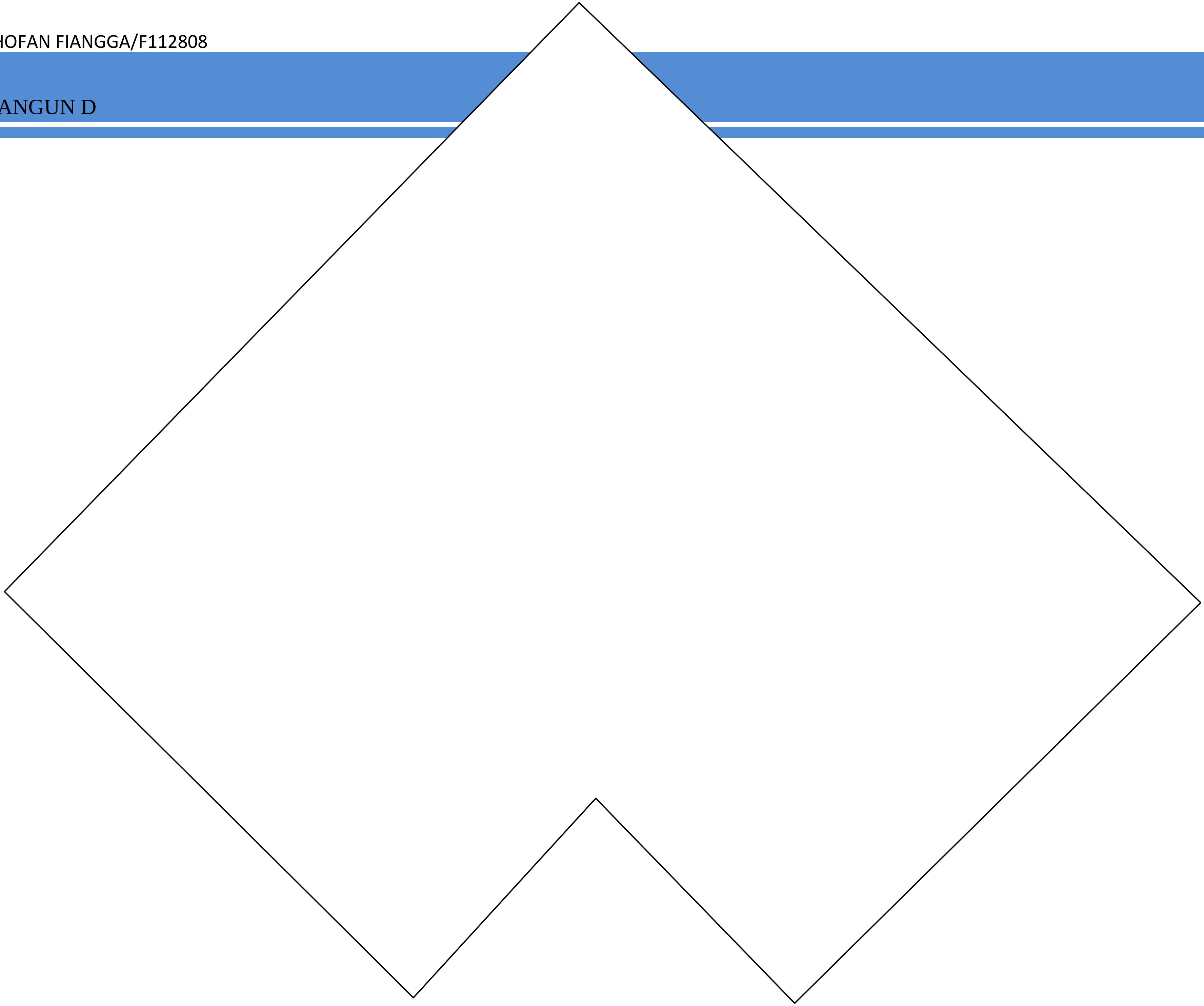


BANGUN C

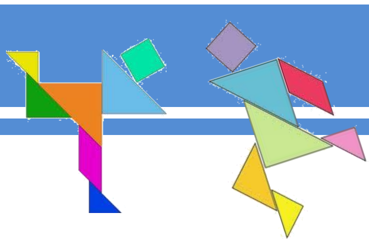


SHOFAN FIANGGA/F112808

BANGUN D



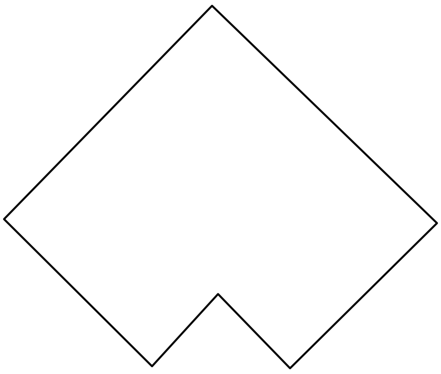
Pertemuan 5 – Lembar Kerja Siswa



Nama :

Kelas

MENUTUP SEBUAH BANGUN DENGAN SATU JENIS TANGRAM



Masih ingat bangun diatas kan? Di pertemuan sebelumnya kita telah bermain dengan bangun ini. Kita telah dapat menyusun potongan-potongan tangram menjadi bentuk bangun seperti diatas. Sekarang tugasnya sedikit berbeda dan lebih menantang ☺

TUGAS KAMU

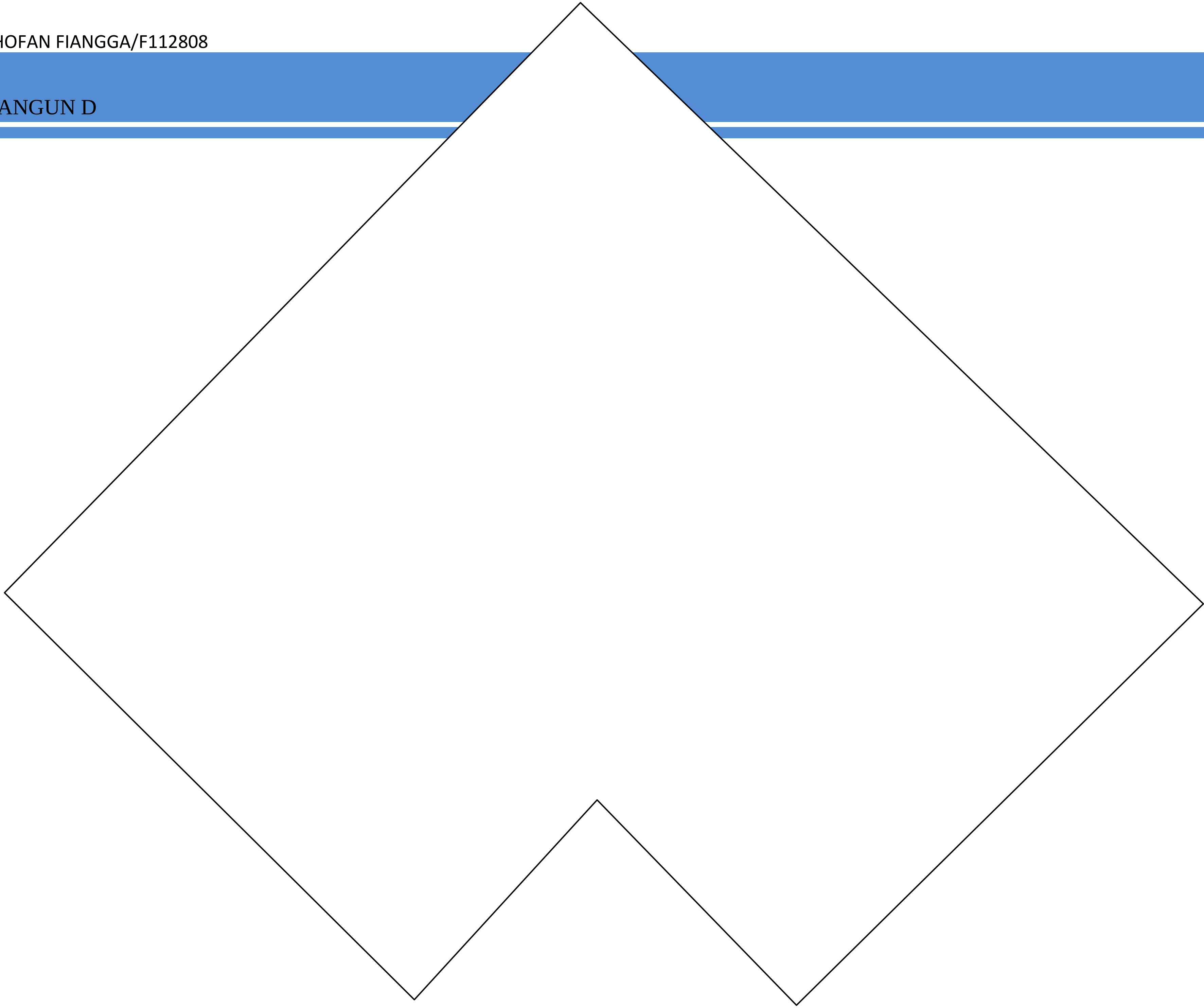
Gunakan saah satu jenis dari ke 7 potongan tangram yang ada untuk menutup seluruh permukaan bangun pada halaman selanjutnya. Jika perlu,eri tanda pada bentuk bangun di halaman berikut ini untuk menandai cara kalian dalam menutup seluruh bangun dengan menggunakan satu jenis potongan tangram saja.

SELAMAT MENGERJAKAN

SEMANGAT☺

SHOFAN FIANGGA/F112808

BANGUN D

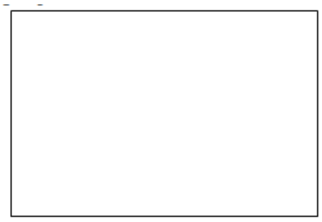


Pertemuan 6 – Lembar Kerja Siswa

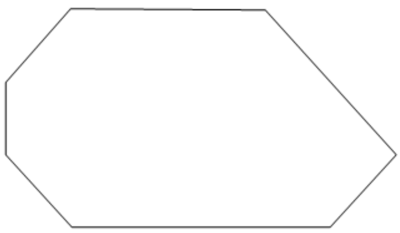
Nama :
Kelas

MENENTUKAN LUAS BANGUN SEGI BANYAK

Setelah kita bermain-main menentukan luas sebuah bentuk dengan menggunakan satu jenis potonga tangram. Sekarang, kita coba menggunakan dua jenis potongan tangram itu, yaitu potongan berbentuk persegi dan segitiga kebil, untuk mengukur segi banyak berikut ini (ukuran sebenarnya ada di halaman berikutnya)



Segi banyak 1



Segi banyak 2

TUGAS KAMU

Dalam 1 kelompok, diskusikan bagaimana untuk menentukan luas bangun segi banyak tersebut (pada halaman berikut ini) dengan menggunakan 2 jenis potongan tangram yaitu bentuk persegi dan segitiga.

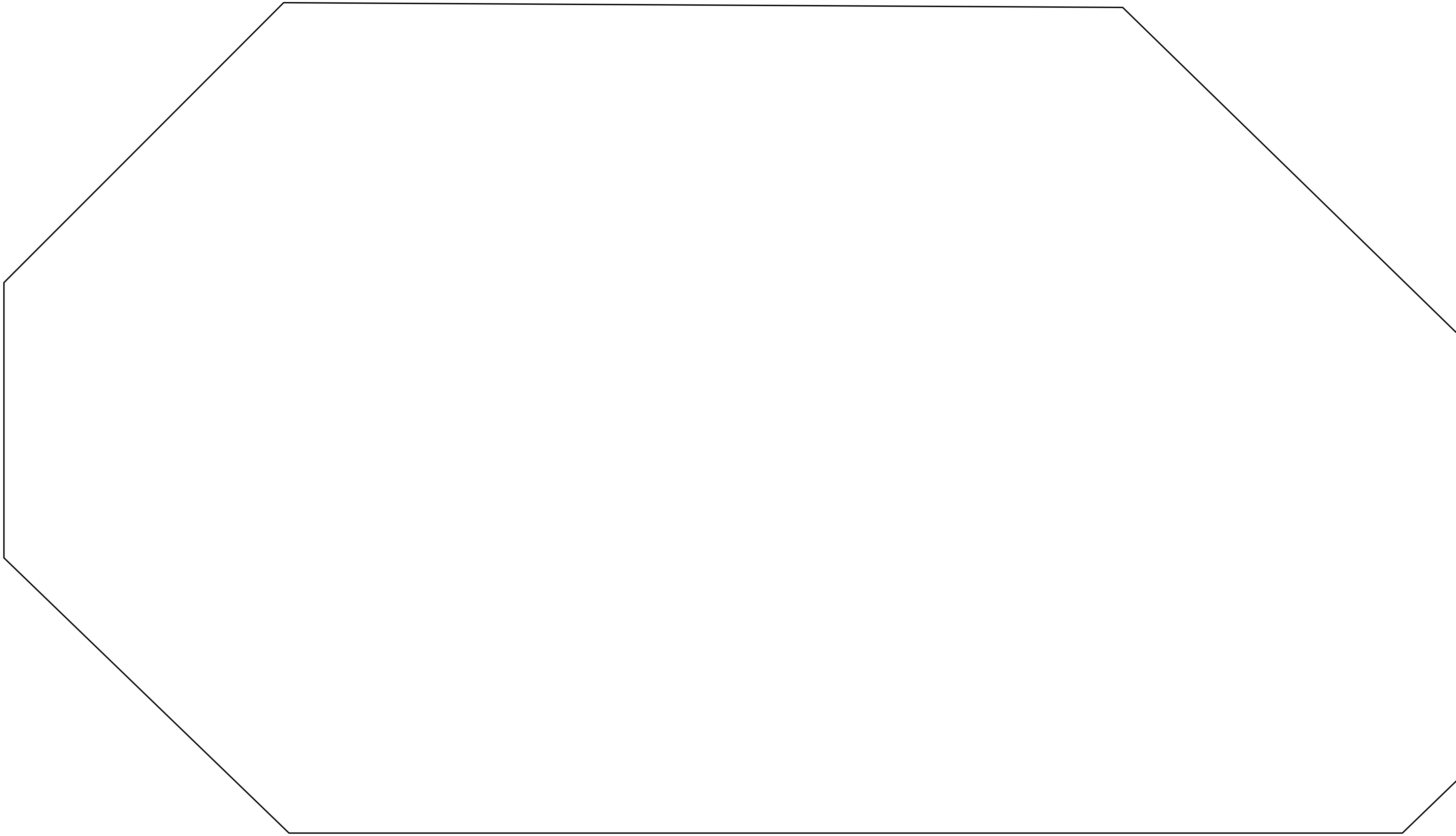
Kemudian, persiapkan 1 poster yang berisi jawaban kalian dan bagaimana cara kalian untuk menentukan jawaban tersebut. Buat poster sedemikian rupa sehingga teman kalian mengerti akan apa yang kalian maksud di poster ini

SELAMAT MENGERJAKAN
SEMANGAT☺

SHOFAN FIANGGA/F112808

BANGUN D

BANGUN D



APPENDIX 4

Mosaic-Puzzle Game
INTRODUCTION TO THE CONCEPT OF AREA

Game Activity for 3rd grade (9-10 years old)

Teacher Guide

1st Lesson

INTRODUCTION OF TANGRAM AND THE CONTEXT

Duration of lesson:

2x35 minutes

Material:

Sets of tangram mosaic puzzle (The number of sets depend on how many pupils in the class such that each pupil has one set)

A picture of various shapes made from a set of tangram

Worksheet for lesson 1

Learning goal

1. The students are be able to play with tangram in composing, arranging, and shaping the tangram pieces.
   1.1. The students are accustomed to play with tangram
   1.2. The students are freely construct their own shapes using more pieces
   1.3. The students are able to construct a certain shape using 2 pieces
   1.4. The students are able to construct a certain shape using 3 pieces
   1.5. The students are able to construct a certain shape using undetermined pieces
2. The students are be able to build the model of tangram with price from the context given to reason with
   2.1. The students are able to determine the price of each piece using their own strategy
   2.2. The students are able to argue about the strategy that they used to find the price
   2.3. The students are able to understand the money model to reasoning about the area

The Teaching and Learning Activities

IST SESSION (20 MINUTES)

1. Orientation Activity

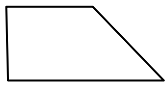
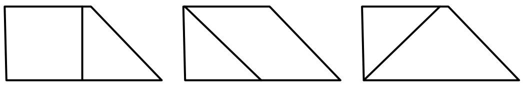
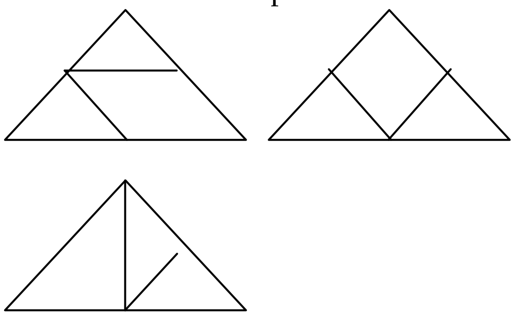
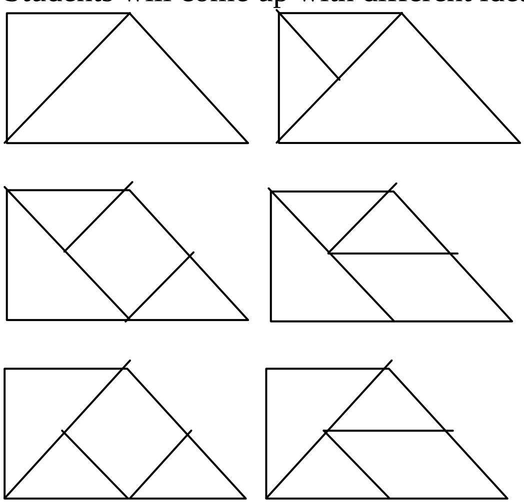
   Students are given a free opportunity to play with mosaic puzzle. We expected that the students could do composing, arranging, and shaping easily through this activity. The teacher should introduce what the tangram is into the students by giving several pictures and telling a story about it. Then, the students are given a set of tangram for each to play with. For warming up tasks, the students will play freely with the mosaic puzzle. Each of them could make many forms of puzzle shapes but then they are asked to choose one of them and trace it in a blank paper. The shape that the students have traced in the first task will be used for the upcoming 2 activities.

The Activity	**The Role of the teacher**
Introduction of the tangram:	The teacher introduces the tangram into the students. The teacher could also tell a story about the origin of tangram. http://library.thinkquest.org/J002441F/tangrams.htm
Making own shape from a set of tangram (INDIVIDUAL) The students are expected to draw a silhouette of his/her puzzle shape. In addition, Then, they can also draw an illustration of what they have already made using those tangram pieces.	Teacher should check whether the students are familiar with the tangram, The teacher can start by telling the story about what tangram is and what you could do with tangram Teacher can ask about *"what is the name of your shape? Or why do you like that? Could you draw your design of tangram in a paper?"* Elicit the students' reasoning on the geometrical feature. "*why you side this piece and this piece here, not like this?"*(putting in not-sided sides)

2. Constructing Given Shapes Activity

   (INDIVIDUAL)

   The students will be asked to form a given silhouette of geometrical shape using several pieces. The tasks are designed from using only 2 pieces into using more pieces. Here, we try to make the students to get used to play with the tangram. Moreover, we hope that the students will encounter the notion of area conservation in this sub-activity. By knowing that we can use different pieces to make a same shape or even we can also using different number of pieces to make a same shape, we hope they will start to grasp the idea of area conservation. In the end, as an introduction activity, we hope that the students will get used to play with tangram in composing, arranging, and shaping. **However,** the teacher is not supposed to talk the area conservation explicitly. Let them think and develop this idea by themselves.

The Activity	**The Role of the teacher**
Constructing a given shape using 2 pieces (Worksheet 1A)  Students will come up with different idea  Guidance is needed for the students who have difficulty in solving it.	The teacher guides the students' investigation of the problem. The teacher can ask about which side of the pieces fit with the side on the shape Asking about which construction is the biggest is proposed to know how far the students understand the area conservation
Students will come up with different idea  There will be another forms similar with previous construction but different orientation Guidance is needed for the students who have difficulty in solving it.	The teacher guides the students' investigation of the problem. The teacher can ask about which side of the pieces fit with the side on the shape The teacher can ask the students to look back to previous activity; can they extend the result of it? Asking about which construction is the biggest is proposed to know how far the students understand the area conservation However, the teacher should not explaining the concept of area conservation yet
Students will come up with different idea  There will be another forms similar with previous construction but different orientation	The teacher guides the students' investigation of the problem. The teacher can ask about which side of the pieces fit with the side on the shape The teacher can ask the students to look back to previous activity; can they extend the result of it? Asking about which construction is the biggest is proposed to know how far the students understand the area conservation However, the teacher should not explaining the concept of area conservation yet

Guidance is needed for the students who have difficulty in solving it.	

3. Challenge Other Students to Construct (OPTIONAL)

   (PAIR)

   One optional activity is challenge games which will be conducted after all of the students form their own shapes. They will challenge another student by exchanging the shapes and try to figure out the construction. Another activity that uses the traced shape will be conducted later.

The Activity	The Role of the teacher
A student challenge another student to exchange the traced shaped and try to figure out the construction of it	Manage the challenge game between students

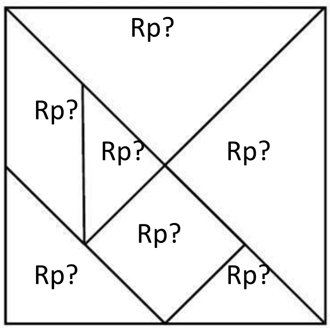
2ND SESSION (45 MINUTES)

1. Introduction of the Context (GROUP)

   Here, the teacher guides the students to build the model of tangram with price from the context given to reason with.

SELLING THE TANGRAM PIECES

"Selling the Tangram Pieces"



After the students have fun playing tangram, they are invited to think about a toyshop of teacher's brother who sells a tangram puzzle. The prize for a complete set of tangram puzzle is Rp 40.000,-. The owner of the toyshop also wants to sell the tangram puzzle for a piece, because he figure out that many children who have bought a set of tangram will lose one or more pieces of the tangram on the upcoming months. The teacher then asks the students to help his/her brother to put prizes in every different piece.

After a couple minutes thinking about the problem, the students are asked to solve the problem in a group of 3 or 4. In the end, the students are supposed to make a poster for each group to show how they solve the problem.

The Activity	The Role of the teacher
Group Discussion: The students may use several strategies 1. Halving pieces 2. Combining pieces 3. Estimate the price a. divide 40.000 by 7 b. estimate roughly regarding size 4. Use gridlines	The teacher should guide the students by keeping them in the right track. If the students do not know how to proceed, the teacher could suggest “how about the price if we divide the set into 2 equal parts?”, “Could you think the combination of the pieces will sum the price of each?” Here, the teacher responses respect to the student’s ways: 1. Teacher can ask the students about 2 pieces which are one of them is twice of the other and let the students to reason about it. 2. The teacher can assist them by giving a clue about the prize of one of the piece, for example the prize for the square. (To progress in the problem, the combination of these 2 strategies seems to be used by the students) 3. The main thing is that teacher could bring this idea to the first strategy by asking the students to do hands on activity on the provided tangram pieces. a. the teacher could ask whether it is reasonable for those different size pieces has the same price. b. the teacher could ask them to sum up all of those prices and see whether it has also summed into Rp 40.000,-. 4. The teacher can provide guidance for the students to make an appropriate gridlines. The teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like “why do you choose this way?”” could you explain why it is possible?”, or “could you explain to your friends about what have you done?” The teacher should also encourage discussion in a group rather than just following a leader in a group.

2. Show of the Poster

   (CLASS DISCUSSION)

   Each group will present their solution in a poster in front of the class. After the students find out the price of each piece, we expect that some of them make a list of those pieces by which the teacher can use it as the discussion starting point in math congress. They will share how they find the prize for each prize and discuss the possibility of various solutions from one of them by presenting their answer on a poster. In presenting their own solution, we expect that the students will be able to argue about the strategy that they used to find the price. How the students will solve the problems and how the students will be the main idea when the students presenting their poster.

Regardless the size, perhaps, the students will argue about the material of the pieces will influence the price. The teacher should lead that the price is related to the area of the pieces, or at least got the idea of it. However, it is not necessary to force the student to understand this relation. If so, just proceed to the next activity. **However,** the teacher is not supposed to talk the area conservation explicitly. Let them think and develop this idea by themselves.

<table>
<tr><th>The Activity</th><th>The Role of the teacher</th></tr>
<tr><td>During the discussion, here the conjectured topic discussed:
the students may come up with some strategies:
1. Using halving/ doubling
Halving the pieces, the price halved as well
2. Using rearrangement-overlapping
Put smaller pieces into bigger piece to compare, then summing the price
3. Using estimation
a. Divide 40.000 by 7
b. Estimate the price regarding the size roughly
4. Using gridline
Make a gridlines as preference to compare the pieces
This conjecture is not necessarily used by the teacher as order of presentation.
For the students who have difficulty in determining the price, teacher guidance is needed</td><td>Here, the teacher responses respect to the student’s ways:
1. Teacher can ask the students about 2 pieces which are one of them is twice of the other and let the students to reason about it.
2.The teacher can assist them by giving a clue about the prize of one of the piece, for example the prize for the square.
(To proceed in the problem, the combination of these 2 strategies seems to be used by the students)
3.The teacher could bring this idea to the first strategy by asking the students to do hands on activity on the provided tangram pieces. Or asking “is it make sense if all of these pieces have the same price but different size?”
4.The main thing is that teacher could bring this idea to the first strategy by asking the students to do hands on activity on the provided tangram pieces.
a. the teacher could ask whether it is reasonable for those different size pieces has the same price.
b. the teacher could ask them to sum up all of those prices and see whether it has also summed into Rp 40.000,-.
5.The teacher can provide guidance for the students to make an appropriate gridlines.
The teacher should make sure that each group prepares their presentation in a poster that will be conducted in the last session</td></tr>
</table>

3. Ordering the pieces according to the price

   (CLASS DISCUSSION)

   In this activity, pupils will also discuss about the cheapest and the most expensive piece. We expect that the students will grasp the notion of area by comparing prize of each piece. Here, they use money model to reason about area. What the students should understand is that at least they agree on bigger the shape more expensive the price will be. We expect that the students will reason that the size determine the price of the shape. Firstly, the teacher could ask one students to write down a sequence of the pieces order. If the group had shown an order in previous discussion,the teacher could use that result to start with. This topic of discussion may occur in the previous discussion. The teacher should guide into a conclusion that bigger the size of the piece more expensive the price is. There is also a possibility that the students will discuss that some different pieces have the same price. The class may discuss this matter, but if the time is not enough the teacher could skip this topic of discussion for the next activity.

The Activity	**The Role of the teacher**
The students try to arrange the pieces according to the price The students will reason that the size determine the price of the shape	Actually, this task can occur in the previous task when they are looking for the price of each piece. The idea of this task is getting insight about the price of each piece The teacher should talk about the 3 same price pieces a bit not too much because it has own activity on the upcoming lesson. The teacher should guide them to grasp the relationship between size and price but do not force the students to understand immediately.

CLOSING (5 MINUTES)

The teacher reviews the activities which have been done in that day. How the students feeling after doing such activities are? And to what extend mathematical concepts, the relation between price and size, is understood by the students. The following question could be options for the teacher to proceed:

"What do you think by playing tangram in school? Do you like it?"

"What do you think about the price? Why this one is the cheapest and the other is the most expensive?"

The teacher can reformulate what the summary of the students' talk in this section are. The teacher also could ask the students itself to summarize the main idea. However, it is important that the teacher must not summary that the price has relation with the size. Let the students find it by themselves through the activities.

Teacher Guide

2nd Lesson

COMPARING WHOSE SHAPE IS MORE EXPENSIVE

Duration of lesson:

2x35 minutes

Material:

Sets of tangram mosaic puzzle (The number of sets depend on how many pupils in the class such that each pupil has one set)

The picture or the sketch or the silhouette of the students' own shape from the 1st meeting

Worksheet for 2nd meeting

Learning goal

3. The students are be able to understand the relation between the price and the size..
   3.1. The students understand the relation between the size and the price among pieces that there could be different price have the same price.
   3.2. The students are able to determine the price of each own shapes using their own strategy
   3.3. The students are able to argue and to reason in understanding the relationship between the price and the used pieces, or beyond this, the relationship between the price and the size

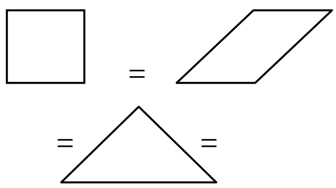
The Teaching and Learning Activities

The picture, sketch, or silhouette that has been made from the first meeting will be used here, but after a mini lesson in the 1st session.

IST SESSION (20 MINUTES)

1. Mini Discussion

   From the second activity, finding the price for each piece, there are 3 different shapes which have same price.



Here, we expect that the students experience the small scale area conservation. A small pair discussion will occur as respond to answer how these 3 different shape pieces could have the same price.

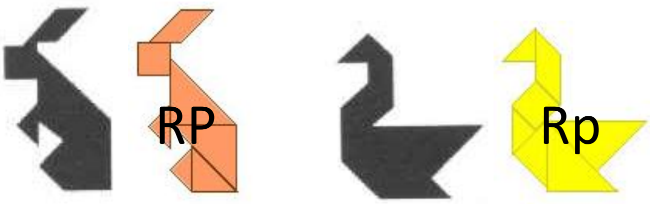
The Activity	**The Role of the teacher**
Delivering the problem	The teacher explains about the problem as clear as possible. The teacher asks the students to talk with their pair discussing the problem
Pair discussion between students	The teacher should wait and let the students to come up with their own idea. The teacher may also walking around to check whether the students need guidance.
Class discussion: The students will show their thinking. Here, the example of how they answer: 1. The students will cut and paste while comparing the pieces one into another 2. The students will consider that those shapes can be divided into 2 triangles	Teacher should guide the discussion into the idea of the decomposition of 2 triangles could be considered as the reason of area conservation *"could you compare the square/parallelogram/medium triangle with small triangles? What do you get?"* Summarize that there is possibility of different shapes have the same price

SECTION 2 (45 MINUTES)

2. Finding the price of each own shapes

(INDIVIDUAL)

The picture, sketch, or silhouette that has been made from the first meeting will be used here. Each student will bring their own shape and, by themselves, finding the total price for the shape.



In finding the price of each shape which is consist of several tangram pieces, it is a common sense that the students will just sum up the price for each pieces to get the total price. If it does not work, then the teacher can ask them to calculate the total price by adding one by one piece. The teacher should make sure that they do calculation correctly.

Another strategy is that they use the fact that the price of the whole pieces is Rp 40.000,-. Hence, if they use all of the pieces in making their own shapes, then they may just say its price is Rp 40.000,-. Or, if they do not use all of the pieces in making their own shapes, then they may just say its price is Rp 40.000, - minus the price of unused piece(s). The teacher should guide these students about which piece's price should be subtracted and proceed to the prices.(Part Whole relation). **However,** the teacher is not supposed to talk the area conservation explicitly. Let them think and develop this idea by themselves.

<table>
<tr><th>The Activity</th><th>The Role of the teacher</th></tr>
<tr><td>The students calculate their own shape using their own strategy. Examples of strategies as follow:
1. They sum up the price for each pieces to get the total price
2. They use the fact that the price of the whole pieces is Rp 40.000,-.
3. if they do not use all of the pieces in making their own shapes, then they may just say its price is Rp 40.000,- minus the price of unused piece(s)</td><td>If the students do not have any idea how to proceed, then the teacher can ask them to calculate the total price by adding one by one piece. "Suppose you are seller, how much should I pay for the whole pieces?"
The teacher should guide these students about which piece's price should be subtracted and proceed to the prices (Part whole relation).
To progress in the problem, the combination of the strategy seems to be used by the students. The teacher could lead the students to use this strategy by asking "could you use the other way around"
The teacher should make sure that each group prepares their presentation in a poster that will be conducted in the last session</td></tr>
<tr><td colspan="2">If they forget to draw the pieces in the picture/silhouette/sketch of their shape, then they will need time to reconstruct the shape using the tangram set. After that, the teacher can use the above guidance.</td></tr>
</table>

4. The Show of the Sketch of the Shape.

   (CLASS DISCUSSION)

   Each student will present his/her own pictures of shapes that have been put price in front of the class. By comparing the price of all shapes, then the teacher together with the whole class must group the shapes that have the same price. After they have done grouping the shapes according to the price, the class discussion should be started by guidance by the teacher. The main issue is about the relation between price and size.

   However, there will be a possibility that the students did not use all of the tangram pieces and figure out that the prize for his/her shapes will be less than the one who use all of the pieces. Here, we can trigger a discussion how to relate the use of prize in understanding the area. The examples of discussion topic (if applicable) that can be used in the discussion are:

1. Different shape but same prize?

   Using this issue as the discussion topic will trigger a need of a notion that is constant.
2. How to determine which one is cheaper or more expensive?

   Using this issue as the discussion topic will trigger a need of a notion that becomes parameter with which we can define a shape is cheaper than the other.
3. All of the shapes have same prices.

   This is the situation where all of the shapes have the same prices. What the class could discuss is why those shapes have the same price. Do they have a similar characteristic that relate each other
4. Different size but same number of pieces

   Posting this issue is quite important because otherwise the students will only reason why 2 different shapes have the same price is because they have the same number of pieces.

Regardless the size, perhaps, the students will argue about the material of the pieces will influence the price.

The Activity	**The Role of the teacher**
Grouping the shapes	Guiding the students in putting the shapes in a group of the same price
Discussion on the 1st issue (if applicable) Different shape but same prize? 1. Considering 2 shapes with different form but have same price, the students will try to define a characteristic of those 2 shapes which are same. 2. During the discussion, there is a possibility that the students consider about the size of the material used to make a piece of the tangram pieces.	1. The teacher should guide them carefully until they can come to the word “area”. The teacher could ask “*the shapes are different but the prices are the same, what do you think responsible for it?* 2. Teacher can lead those students to the situation in which they can imagine themselves made the tangram pieces by cutting a plate of wood. If they must buy a plate of wood, then they can notice that bigger the plate will make the prize more expensive.
Discussion on the 2nd issue (if applicable) How to determine which one is cheaper or more expensive? 1. Considering 2 shapes with different	1. The teacher should guide them carefully until they can come to the word “area”. The teacher could ask “*the shapes are different but the*

price, the students will try to define a parameter of those 2 shapes which can be reference for them to determine which shape is cheaper or more expensive. 2. During the discussion, there is a possibility that the students consider about the size of the material used to make a piece of the tangram pieces.	*prices are the same, what do you think responsible for it?* 2. Teacher can lead those students to the situation in which they can imagine themselves made the tangram pieces by cutting a plate of wood. If they must buy a plate of wood, then they can notice that bigger the plate will make the prize more expensive.
Discussion on the 3rd issue (if applicable) All of the shapes have same prices. Here, we conjecture that the students will come up with the idea of area. Then, we may just continue to the next activity. However, it not necessarily if the students get to the idea of area. If so, there will be more activity regarding this issue in the next activity.	The teacher guides the discussion on why different shapes have the same price. The discussion will evolve around what characteristic that make those different shapes have the same price.
Discussion on the 4th issue (if applicable) Different size but same number of pieces The students could have misconception that different shapes will not always if he number of the formed pieces are the same the shapes are in the same size	The teacher could provide the idea from the previous activity when they deal with different shape will not always if he number of the formed pieces are the same the shapes are in the same size The teacher could showing the previous activity about constructing shape with the same number of pieces but results different size. Then, the teacher could ask *"if the number of pieces does not matter her, what do you think determining the size comparison"*

CLOSING (5 MINUTES)

The teacher reviews the activities which have been done in that day. The teacher should lead that the price is related to the area of the pieces, or at least got the idea of it. However, it is not necessary to force the student to understand this relation. If so, just proceed to the next activity.

"What the parameter do you think by which determining the price of those various shape?"

"Could you think about the size of it" or if still hard for the students to understand a bit about the relation of size and price the teacher could ask *"Is there any relationship between the size and the price?"*

In the end of this activity, we expect that the students will grasp the notion of "area" which is used as the parameter by the students to determine which shape is cheaper or more expensive. Or, in another words, understanding the relationship between the price and the used pieces, or beyond this, the relationship between the price and the size.

The teacher can reformulate what the summary of the students' talk in this section are. The teacher also could ask the students itself to summarize the main idea.

Teacher Guide
3rd Lesson
COMPARE THE GIVEN SHAPES

Duration of lesson:

2x35 minutes

Material:

Sets of tangram mosaic puzzle (The number of sets depend on how many pupils in the class such that each pupil has one set)

More square and small triangles pieces for measuring activity if needed(2nd SESSION)

Worksheet for 3rd meeting

Learning goal

4. The students are be able to understand the one important properties area which is area conservation (if we compose and decompose the area into another form the area remain invariant) .
   4.1. The students are able compare the given shapes using the idea of area or price.
   4.2. The students are able to argue about the strategy that they used to compare the given shapes.
5. The students are be able to do covering the area as measurement activity by using different kind of tangram piece.
   5.1. The students are able to use one kind only tangram pieces from the available possible tangram pieces to fully cover the given shape.
   5.2. The students are able to understand the relation covering the shape using triangles or squares only as measurement activity.

The Teaching and Learning Activities

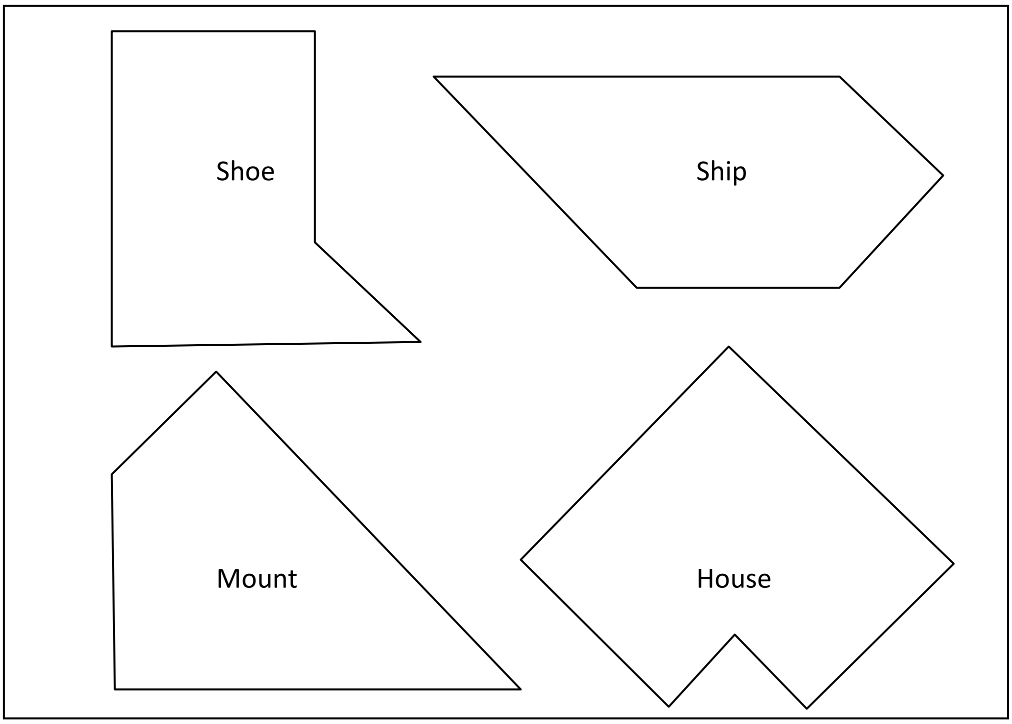
SECTION 1 (45 MINUTES)

1. Compare the Given Shapes
   (GROUP DISCUSSION)
   In the beginning, the teacher reminds about the previous activity whose shape is the most expensive. The teacher should highlight the main issues that have been

discussed in the previous discussion as well like *"What are the relationship between the size and the price"*

Similar with the previous activity, but in this work, student will play with the shapes given by the teacher and discuss it in a group of 3 or 4. The shapes used in this activity are designed feasible enough for the students to solve in such a time as a group. Here the shapes:

$$> ?, < ?, = ?$$



The comparison between those shapes is important. The fact that there will be 2 shapes which have the same area should come up in the students' discussion; otherwise, let the teacher highlight it in the next class discussion. Each group is asked to make a poster as the result of their discussion which will be presented in the next class discussion.

The Activity	The Role of the teacher
Group Discussion:	The teacher should guide the students by keeping them in the right track. If the students do not know what to do the teacher could suggest "could you think about using the tangram?" "how are the price" If the students already finish comparing the shapes the teacher could

	suggest “could you use another strategy to compare without price?”, “What do you think about the area?” The teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like “why do you choose this way?”” could you explain why it is possible?”, or “could you explain to your friends about what have you done?” The teacher should also encourage discussion in a group rather than just following a leader in a group. If the group has difficulty in solving the problem, the teacher could provide assistants or suggestions like “could you use the tangram to help your problem?”, “what about the left over part?” While moving around, the teacher could note which group will present for the next discussion by choosing which group did overlapping which group did structuring tangram.

2. The Show of the Poster

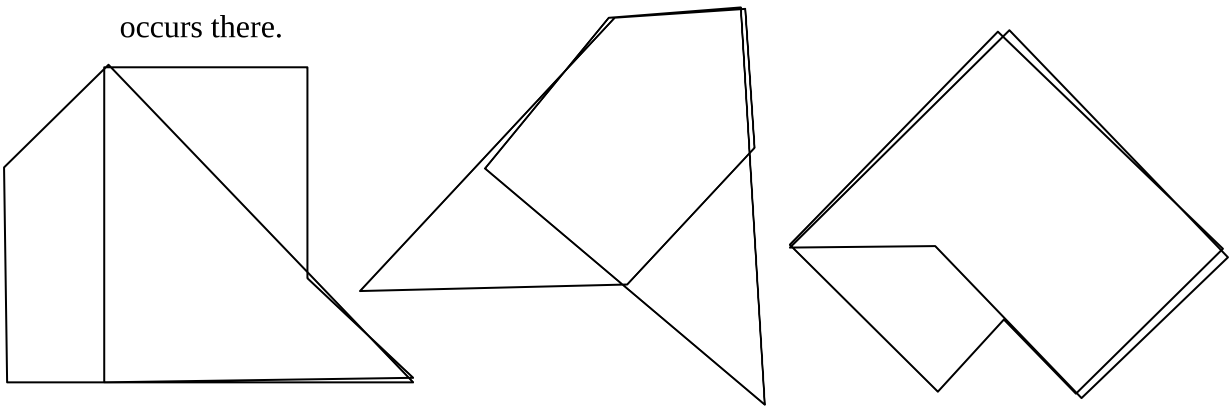
   (CLASS DISCUSSION)

   Each group will present their solution on a poster in front of the class one by one. They are asked to determine which shape is cheaper or more expensive or same price. Here, the students are really guided to use the notion of area in each shape as the parameter to determine which shape is cheaper. By only using the fact that the shape can be arranged using several pieces, we conjecture that they are expected not to use the reasoning using the total of the price but using overlapping strategy. Here, the money model will not be a main tool to reason with. They will notice from how the pieces are arranged into those shapes.

   Here the conjectured students solutions of the problem

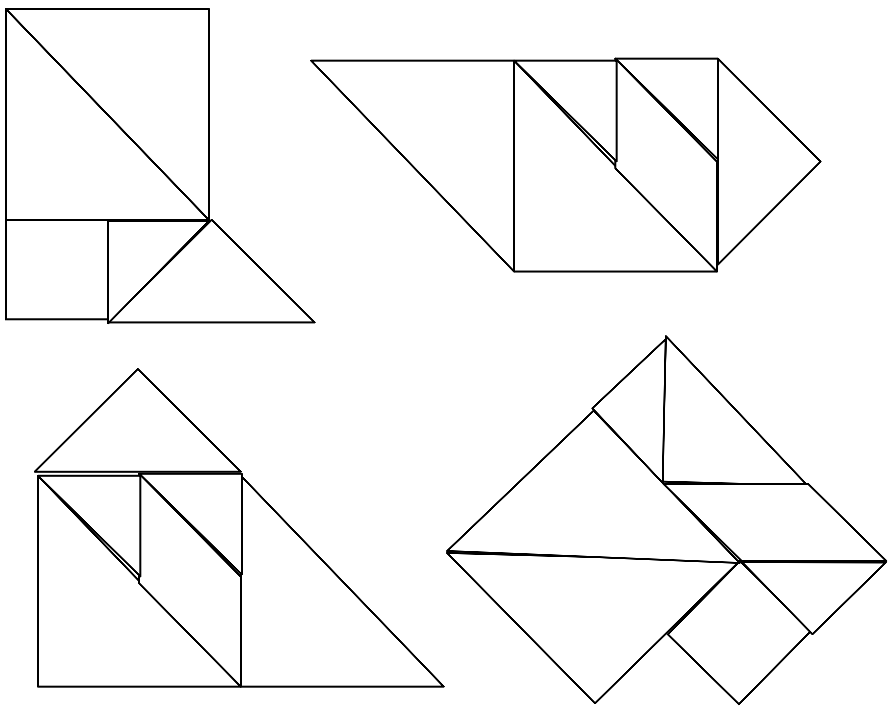
   1. Estimate the shape by using overlapping strategy to compare

      We anticipate the idea of estimation here. Some of the students might see the shapes are similar in certain parts. So, they will try to compare those similar parts and try to put one shape on top of another shape and see how the discrepancy occurs there.



2. The use of tangram pieces to reconstruct and put price to compare

   Here the example of how the students will arrange the tangram pieces.



   They will, then, calculate the price and then make the order of the shapes from the cheapest to the most expensive one.

On each group presentation, the main conclusion should be that the idea of composing and decomposing the shape from those 2 conjectures is understood by the students that it would not change the area as long as they use the same composition.

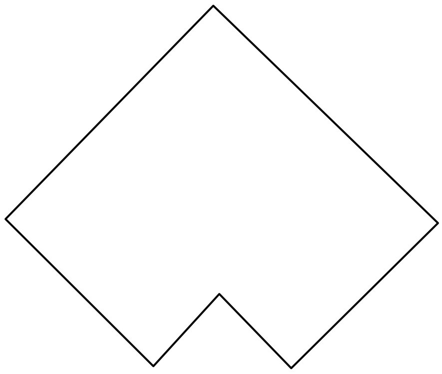
The Activity	The Role of the teacher
Group presentation There will be 2 main issues about the solution from the students for each group. Here the 2 activity respect to the 2 main issues that should be carefully treated	which group who will present have been chosen from previous group discussion
1. When the students using overlapping strategy By comparing in such a way, they will find the arrangement. However, the students do not use the price to reasoning that will make the discussion about area could occur	Here, the teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. And, area conservation occurs here. The teacher can ask the price of each shape later. The main idea is that when the student could understand the idea of those shapes could be

	composed and decomposed by looking at the left over part while overlapping each other and still the area remain invariant.
2. The use of tangram pieces to reconstruct and put price to compare They will, then, calculate the price and then make the order of the shapes from the cheapest to the most expensive one. We expect that the students will use the term of area as the reasoning. By comparing in such a way, they will find the arrangement. The students might not use the price to reasoning but the fact that those shape consist of same pieces will be their reason to determine this shape has the same price with another.	if the discussion of the area of the shape does not occur, the teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. The teacher could ask them to compare the size, and then the discussion will follow in a line with the first conjecture. The main idea is that when the student could understand the idea of those shapes could be composed and decomposed by looking at the pieces that can be put in and out and still the areas remain invariant.
The conclusion of the discussion	The teacher should summary from those 2 strategies that whatever the left over or the pieces which can be put in or out, the main idea is composing and decomposing a shape into another shape with the areas remain invariant

2ND SESSION (20 MINUTES)

3. Mini Discussion (INDIVIDUAL)

   After the discussion finish, each students go back to their own groups seats and then the next problem is given. Using only one kind of shape of tangram piece, the students are asked to use it to cover a certain area of shape. the shape itself is already given to the students in the previous session. Here, they must use the available tangram that they had as a group and decide which shape that they are going to be used to cover the shape.



The Activity	The Role of the teacher
Delivering the problem	The teacher explains about the problem as clear as possible. The teacher asks the students to talk with their pair discussing the problem.
Group discussion	The teacher should wait and let the students to come up with their own idea To make sure, the teacher could ask "is it ok if you cover using this and this (refer to the shapes except square and small triangle)?", "what do you think if your pieces are not enough?" But if the students really have difficulty with the fact that the number of pieces is not enough for covering the shape, The teacher could ask them to borrow another group pieces or provide them with additional pieces The teacher should make sure that the students cover the shape completely.
Class discussion: In the class discussion, the students might come up with 2 idea 1. one square can be divided into 2 triangles 2. the square and the small triangle are suitable for covering such shapes	Teacher should guide the discussion into the idea of the decomposition of 2 triangles could be considered as the reason of area conservation If all of the students use squares only or triangles only, the teacher could ask them how if we use the triangles or squares and compare the results The teacher could also ask why others pieces are not suitable for covering such shape

CLOSING (5 MINUTES)

In the end of this activity, the teacher reviews the activities which have been done in that day. We expect that the students will understand and accept that every shape has what so called area by which we can determine which one is bigger or smaller. As the result, they could understand that those areas if we compose and decompose the area into another form the area remain invariant (area conservation) from the 1st session activities.

From the 2nd session of activity, we also expect that the students will understand covering the area as measurement activity by using different kind of tangram piece may result same area. Here, the possible question that could be posted by the teacher.

"What will happen with the area if I cut this part and paste it into another position?"
"Could you think about another shape which has the same area with it"

If the students answer these questions using the tangram as reasoning it is ok

The teacher can reformulate what the summary of the students' talk in this section are. The teacher also could ask the students itself to summarize the main idea. **The important achievement for the students here is that the students are no longer need to use the price to reason why this shape has the same area with another shape.**

Teacher Guide

4th Lesson

PLAYING WITH THE AREA OF POLYGON

Duration of lesson:

2x35 minutes

Material:

Sets of tangram mosaic puzzle for each student

More square and small triangles pieces for measuring activity if needed

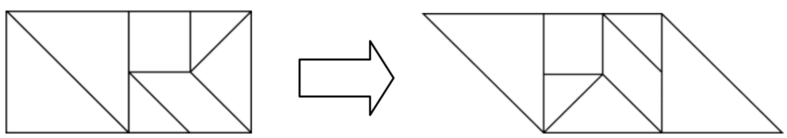
Worksheet for 4th meeting

Learning goal

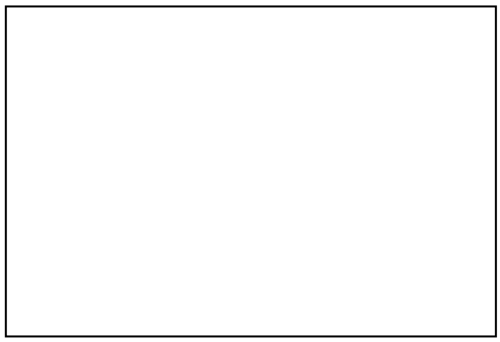
6. The students are be able to apply the concept of conservation to measure the area of polygon.
   6.1. The students are able to use the pieces from the previous activity to measure given polygon shapes.
   6.2. The students are able to argue about the strategy that they used to find the price. In the end, the students are able to use the concept of area conservation in measuring the area of a shape.

The Teaching and Learning Activities

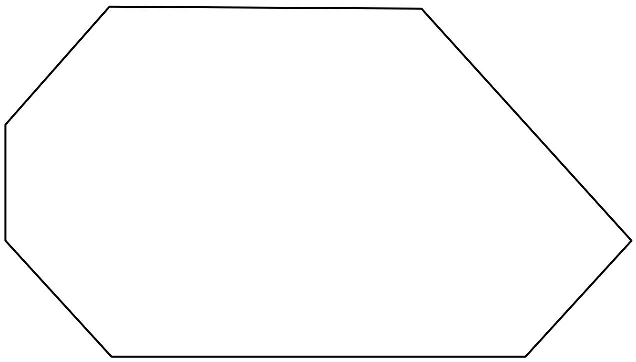
1. Compare the Given Shapes
   (GROUP DISCUSSION)
   In the beginning, the teacher reminds about the previous activity comparing the 4 given shapes. The teacher should highlight the idea of composing and decomposing the shape into another shape will have the area remained invariant. The teacher could demonstrate the example of compose and decomposing a rectangle into parallelogram using tangram and ask:
   "What are the relationship between the previous rectangle and this parallelogram?",
   "What do you think about the area? Which one has the largest area?"



The last activity is how the students will use what they have got to measuring polygon shapes. The student will play with the shapes given by the teacher and discuss it in a group of 3 or 4. The shape is designed in such a way that the students will have difficulty because of the number of the squares and triangles are not enough to cover by using only one type. They will need to use both shapes respectively to measure the area of the polygon. The students are asked to measure the 1st polygon firstly, and then after they have measured it, they could proceed into the next polygon. Here the pictures:



1st polygon



2nd polygon

The use of square and small triangles pieces in the appropriate manner is important. The students should notice that they could decompose the 1st polygon and compose the pieces to form the 2nd polygon. In the end, they are asked to state the measurement in squares. How many squares is the size of the 1st and the 2nd polygon?

The Activity	**The Role of the teacher**
Group Discussion: The students may use several strategies: 1. Using all pieces to cover up the shape one by one	The teacher should guide the students by keeping them in the right track. If the students do not know what to do the teacher could suggest “could you think about using the a preference pieces” If the students already finish measuring the polygon without comparing the size of those polygons the teacher could

2. Using only several pieces and do estimation then 3. Using the previous shape that has been covered and use it as starting point to cover	suggest “could you use another strategy to compare without covering one by one”, “What do you think about the area comparison of those polygons?” The teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like “why do you choose this way?”” could you explain why it is possible?”, or “could you explain to your friends about what have you done?” The teacher should also encourage discussion in a group rather than just following a leader in a group.

The teacher should make sure that each group has written down their answer in a poster which will be presented on the next classroom discussion.

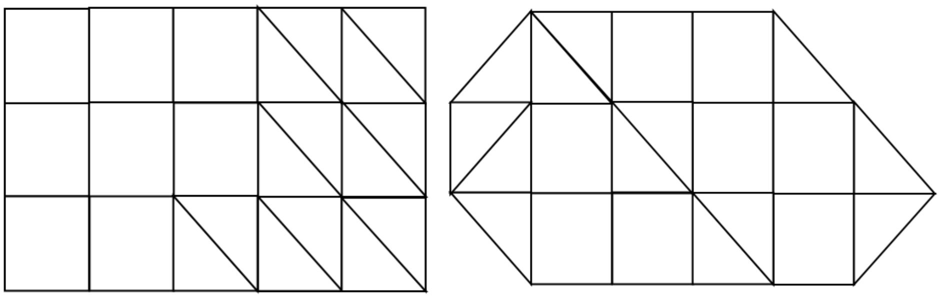
2. The Show of the Poster

(CLASS DISCUSSION)

Each group will present their solution on a poster in front of the class one by one. They are asked to determine which shape is cheaper or more expensive or same price. Here, the students are really guided to use the area conservation in measuring plane figure by only using the allowed pieces.

Here the conjectured students solutions of the problem

1. The students could do covering one by one to know the measurement of the polygon



2. The students could use array to measure the polygon
3. The students could use the former shapes as the reference and proceed to the remaining area to measure.

In this class discussion, the teacher should lead the students’ idea to the last conjecture where the students are able to use the idea of area conservation to measure the area of a plane figure. No matter from which conjecture the students start, the role of the teacher is important here.

The Activity	The Role of the teacher

During the discussion, here the conjectured topic discussed: For the 1st polygon 1. they cover using pieces one by one 2. They realize that there will be 5 columns and 3 squares are in a column 3. They realize that there will be 5 columns and 6 triangles are in a column, For the 2nd polygon 1. they cover using pieces one by one 2. They realize that there is a connection of this 2nd polygon with the 1st polygon a. They will use the available pieces, either the square or the small triangle pieces to measure and compare those 2 polygons b. They will cut and paste the polygon into another polygon to compare those 2 polygons	For the discussion on the 1st polygon, the relationship between triangles and squares. The students should realize that those answers are actually the same The teacher could ask: *“what do you think about the use of square of small triangles, which one is better?”, “Do you see the relationship between those 2 different answer?”* For the discussion on the 2nd polygon, if the students do not come out with the second conjecture, that we can use the 1st polygon to measure the 2nd polygon, the teacher could ask them to compare the 2nd polygon with the 1st polygon. The teacher could ask: *“What do you think which polygon is bigger?”, “Do you see the relationship between those 2 polygons?”* The discussion also guided into the use of the concept of area conservation either the students use the pieces to compare or cut and paste the polygon to compare. Discussion on different solutions of arrangement between groups should be carried out clearly. Difference arrangement of one shape still result same measurement.

In fact, the students must use more than one kind of tangram shape, because of the limited number of squares or triangles if they decide to only use one kind of piece. Various combinations of triangles and squares will occur; we can put them in conflict to discuss each other solution. Here, they can mention the area of this polygon as triangles only, squares only, or both (for 1st shape). Then, we continue with the shape which has odd number of triangles (for 2nd shape). They should regard about the fact that a square is same area with 2 triangles and use it in the next discussion of the measurement of the polygon shape, and then we ask the students to state the area using squares only.

Discussion on different solutions of arrangement between groups should be carried out clearly. Difference arrangement of one shape still result same measurement.

CLOSING (5 MINUTES)

In the end of this activity, the teacher reviews the activities which have been done in that day. We expect that the students will understand and be able to use the idea of area conservation in area measurement. As the result, they could measure the area by only using a former shape as reference because it has the same area then do the remaining parts.

"What will happen with the area if I cut this part and paste it into another position?"

"Could you think about another shape which has the same area with it"

> *If the students answer these question using the tangram as reasoning, then the teacher should ask them another case like curved shape that students cannot tangram*

The teacher can reformulate what the summary of the students' talk in this section are. The teacher also could ask the students itself to summarize the main idea.

In the end of this activity, we expect that the students will use the tangram pieces (triangles and squares) and the idea of conservation to measure the area of polygon (the conservation of area).

SHOFAN FIANGGA/F112808

THE PLANNING OF THE MEETING

Because I have not tested the whole learning sequence in real context, in case that it go smoothly, it will only take 4 meetings. Otherwise, my estimation is that 5 meetings are enough.

NO	THE ACTIVITY	MEETING 1					MEETING 2					MEETING 3					M	
		15'	15'	15'	15'	10'	15'	15'	15'	15'	10'	15'	15'	15'	15'	10'	15'	15'
1	ACTIVITY 1		*															
2	ACTIVITY 2						*											
3	ACTIVITY 3																	
4	ACTIVITY 4																	
5	ACTIVITY 5															* *		
6	ACTIVITY 6																	

* If activity 1 needs more time, activity 1 could take over the first quarter of the activity 2 and the discussion of activity 2 could be more focus in the next meeting
** If activity 4 takes less time, activity 5 could take over the last quarter and the discussion of activity 5 could be more focus in the next meeting

Permainan Mosaic-Puzzle

PENGANTAR KONSEP KONSERVASI LUAS

Aktivitas Permainan untuk kelas 3 (Umur 9-10 tahun)

APPENDIX 4.5

Panduan Guru

Petemuan 1

PERKENALAN TANGRAM DAN PENGANTAR KONTEKS

Lama Pertemuan:

2x35 menit

Material:

Beberapa set Tangram (tergantung pda jumlah murid di kelas tersebut)

Sebuah gambar tentang berbagai bentuk yang disusun dari 1 set tangram

Lembar Kerja Siswa untuk pertemuan 1

Tujuan Pembelajaran:

1. Siswa dapat bermain dengan tangram dalam al memnyusun dan membentuk potongan-potongan tangram.
   1.1. Siswa dapat bermain bebas menggunakan tangram
   1.2. Siswa dapat menkonstruk sebuah bentuk menggunakan potongan-potongan tangram yang ada
   1.3. Siswa dapat membentuk sebuah bangun tertentu dengan menggunakan 2 potongan tangram yang telah disediakan
   1.4. Siswa dapat membentuk sebuah bangun tertentu dengan menggunakan 3 potongan tangram yang telah disediakan
   1.5. Siswa dapat membentuk sebuah bangun tertentu dengan menggunakan sebarang potongan tangram yang telah disediakan
2. Siswa dapat mengembangkan model tangram berharga dari konteks yang telah diberikan sebagai model berpikir tentang luas.
   2.1. Siswa dapat menentukan harga dari tiap potongan tangram dengan menggunakan caranya sendiri.
   2.2. SIswa dapat menjelaskan strategynya unutk mencari harga dari tiap potongan
   2.3. Siswa dapat memahami model tangram berharga sebagai model berpikir untuk memecahkan masalah

Kegiatan Belajar Mengajar

SESI 1 (20 MENIT)

1. Aktivitas Orientasi

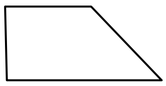
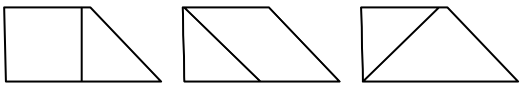
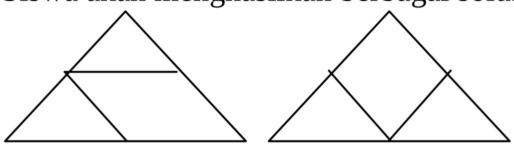
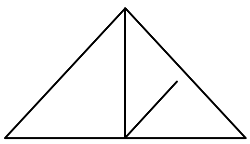
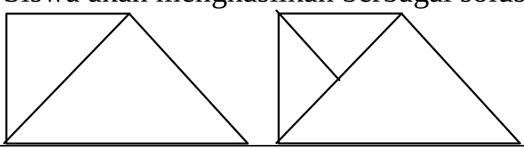
   Siswa diberi kesempatan bebas untuk bermain dengan tangram. Diharapkan siswa mampu menyusun dan membentuk dengan mudah setelah menjalani aktivitas-aktivitas berikut ini. Guru diharapkan memperkenalkan mengenai tangram degan memberikan beberapa contoh bentuk dari tangram dan sedikit cerita mengenai asal-usul tangram. Kemudia, tiap siswa diberikan 1 set tangram untuk bermain. Sebagai permulaan, siswa diminta untuk bebasbermain dan menyusun bentuk-bentuk yang mereka inginkan. Kemudian, siswa diminta memilih 1 bentuk yang paling dia suka dan mengambar membuat sketsanya di kertas yang telah disediakan. Guru diharapkan menyimpan bentuk-bentuk yang telah dibuat siswa karena akan digunakan pada aktivitas mendatag.

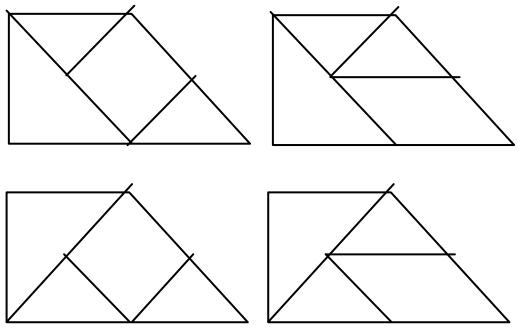
Aktivitas	**Peran Guru**
Perkenalan Tangram	Guru memperkenalkan tangram ke siswa, apa yang bias kita lakukan dengan tangram. bisa juga menceritakan sedikit sejarah megenai tangram. http://library.thinkquest.org/J002441F/tangrams.htm
Membuat 1 bentuk tangram yang disukai (INDIVIDU) Siswa diharapkan menggambar silhouet/sketsa dari bentuk yang telah dia pilih. Mungkin juga mereka menggambar bagaimana potongan2 itu disusun menjadi bentuk yang telah mereka buat.	Guru diharapkan mengecek apakah siswa terbiasa bermain dengan tangram, Guru dapat bertanya mengenai *“apa nama bentuk yang kamu buat? Atau mengapa kamu suka membuat bentuk itu? Dapatkah kamu menggambar bentuk tangram itu di kertas?”* Telusuri penalran siswa terutama secara geometris. “*mengapa kamu susu sisi ini dengan sisi ini, tidak terbalik seperti ini?”*(meletakkan susunan tidak pas pada sisinya)

2. Aktivitas Merekonstruk Bentuk yang Diberikan

   (INDIVIDU)

   Siswa diminta untuk membentuk sebuah silhouette dari bentuk-bentuk geometris yang telah diberikan menggunakan beberapa potong tangram. Ttugas di aktivitas ini didesain mulai dari menggunakan 2 potong hingga lebih. Disini, kita berharap siswa terbiasa untuk menyusun dan menkonstruk bermain dengan tangram. Lebih jauh lagi, kita berharap siswa akan menyadari akan adanya konservasi luas disni dengan menyadari bahwa kita dapat menggunakan potongan-potongan tangram berbeda untuk menyusun sebuah bentuk yang sama atau bahkan jumlah yang berbeda juga. Oleh karena itu, siswa diharapkan mulai menyadari akan konservasi luas. Di akhir aktivitas ini, kita harapkan siswa dapat dengan leluasa bermain menyusun dan membentuk menggunakan potongan-potongan tangram yang ada. **Namun**, guru janganlah memaksakan siswa untuk memahami konsep konservasi luas seperti yang telah disebutkan sebelumnya. Biarkan mereka membangun pemahaman mereka sendiri melalui aktivitas-aktivitas yang ada.

Aktivitas	**Peran Guru**
Menkonstruk bangun yang diberikan dengan menggunakan 2 potong (LKS 1A)  Siswa akan menghasilkan berbagai solusi  Bimbingan diperlukan untuk siswa yang mengalami kesulitan.	Guru membimbing siswa untuk menginvestigasi persoalan. Guru dapat menanyakan mengenai bagaimana menyusun sisi-sisi potongan tangram dalam bangunan tersebut. Guru dapat menanyakan konstruksi mana yang paling besar. Hal ini dilakukan untuk mengetahui sejauh mana siswa mengetahui tentang konsep konservasi area.
Siswa akan menghasilkan berbagai solusi   Kemungkinana ada jawaban lain yang mirip dengan jawaban di atas namun dengan urutan terbalik. Bimbingan diperlukan untuk siswa yang mengalami kesulitan.	Guru membimbing siswa untuk menginvestigasi persoalan. Guru dapat menanyakan mengenai bagaimana menyusun sisi-sisi potongan tangram dalam bangunan tersebut. Guru dapat meminta siswa untuk melihat kembali permasalahan sebelumnya. Mungkin mereka bias memperluas solusi yang merekamiliki dari permasalahan sebelumnya. Guru dapat menanyakan konstruksi mana yang paling besar. Hal ini dilakukan untuk mengetahui sejauh mana siswa mengetahui tentang konsep konservasi area. Namun,guru diharapkan jangan terburu-buru menjelaskan konsep konservasi area.
Siswa akan menghasilkan berbagai solusi 	Guru membimbing siswa untuk menginvestigasi persoalan. Guru dapat menanyakan mengenai

 Kemungkinana ada jawaban lain yang mirip dengan jawaban di atas namun dengan urutan terbalik. Bimbingan diperlukan untuk siswa yang mengalami kesulitan.	bagaimana menyusun sisi-sisi potongan tangram dalam bangunan tersebut. Guru dapat meminta siswa untuk melihat kembali permasalahan sebelumnya. Mungkin mereka bias memperluas solusi yang merekamiliki dari permasalahan sebelumnya. Guru dapat menanyakan konstruksi mana yang paling besar. Hal ini dilakukan untuk mengetahui sejauh mana siswa mengetahui tentang konsep konservasi area. Namun,guru diharapkan jangan terburu-buru menjelaskan konsep konservasi area.

3. Menantang Siswa Lain untuk Menkonstruk bangun yang Dibentuk (PILIHAN) (BERPASANGAN)

   Aktivitas tambahan ini adalah menantang siswa lain untuk menebak konstruksi masing-masing. Mereka akan saling bertukan gambar siluet dan saling menebak konstruksi masing-masing.

Aktivitas	Peran Guru
Tiap siswa menantang siswa lain untuk bertukar siluet bentuk dan saling menebak konstruksi potongan tangra yang digunakan.	Mengatur permainan di kelas antara siswa.

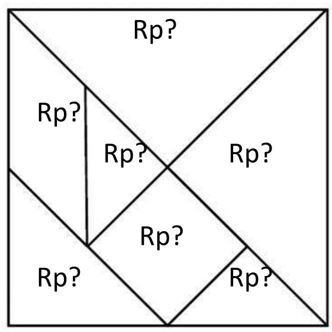
SESI 2 (45 MENIT)

4. Perkenalan Konteks (GRUP)

   Disini, guru membimbing siswa untuk membangun model tangram berharga dari konteks yang diberikan sebagai model berpikir untuk memecahkan masalah.

MENJUAL POTONGAN TANGRAM

“Menjual Potongan Tangram”



Setelah siswa puas bermain tangram, mereka diminta untuk memecahkan permasalahan di took mainan milik saudara laki-laki guru yang menjual puzzle tangram. Harga dari 1 set tangram lengkap adalah Rp 40.000,-. Pemilik took juga ingin menjual tangram itu eceran per potong karena kadang beberapa anak kehilangan satu atau dua potongan tangram yang mereka miliki. Guru kemudian meminta siswa membantu pemilik took untuk menentukan harga dari masing-masing potongan tangram yang tersebut.

Setelah beberapa menit berpikir mengenai permasalahan tersebut secara individu, kemudian siswa diminta menyelesaikan dalam grup 3 atau 4 orang. Diakhir kegiatan kelompok, siswa diminta membuat poster untuk dipresentasikan mengenai bagaimana mereka memecahkan persoalan konteks diatas.

Aktivitas	**Peran Guru**
Diskusi Grup: Siswa mungkin menggunakan strategy berikut ini: 1.membagi separuh potongan tangram 2. menggabung potongan tangram 3. Meperkirakan harga kasar a. membagi 40.000 : 7 b.estimasi kasar berdasarkan ukuran 4. menggunakan grid	Guru sebaiknya membimbing siswa. Jika siswa kebingungan bagaimana cara memecahkan permasalahn ini, guru dapat menyarankan siswa dengan mengarahkan “bagaimana harganya jika kita bagi potongan ini menjadi 2 bagian sama besar?”, “Bagaimanakan harganya jika kita gabung 2 potong tangram ini?” Berikut ini respon guru sesuai dengan strategy siswa: 1. Guru dapat menanyakan kepada siswa untuk membanding 2 potongan mana yang harganya 2 kali lipat berdasarkan ukurannya dan tanyakan alasannya kepada siswa. 2. Guru dapat membantu siswa dengan memberikan petunjuk untuk perkiraan harga potongna kecil (2 strategy diatas bisa digunakan bersamaan untuk mempermudah pemecahan masalah) 3. Ide utama di strategy ini adalah siswa diarahkan untuk menggunakan 2 strategy sebelumnya. a. Guru menanyakan apakah logis potongan-potongan yang berbeda itu memilikiharga yang sama. b. Guru dapat menanyakan apakah jumlah semua potogan itu juga Rp 40.000,-. 4. Guru dapat membimbing siswa untuk membuat grid yang sesuai. Guru harus membiarkan pemikiran siswa sendiri utuk berkreasi memecahkan masalah Guru diharapkan juga menanyakan beberapa hal yang memancing pemikiran kritis siswa seperti “memangapa kamu memilih strategy ini?”, ” dapatkah kamu menjelaskan solusimu yang mungkin?” atau “dapatkah kamu menjelaskan strategimu kepada temanmu?” Guru diharapkan juga memancing diskusi dalam grup daripada hanya mengikuti satu pendapat.

5. Presentasi Poster

   (DISKUSI KELAS)

   Eberapa grup akan mempresentasikan solusinya dalam poster. Guru memilih solusi yang beragam dan usahakan tidak sama. Seteah diskusi tentang bagaimana menentukan harga per potong tangram ini selesai, diarapkan ada siswa yang membuat daftar harga dari potongan tangram tersebut. Pada diskusi kelas, berbagai kemungkinan solusi permaslahan konteks akan didiskusikan. Diharapkan siswa mampu berargumentasi mempertahankan jawaban masing-masing terutama mengenai bagaimana mereka mencari harga dari tiap potongan tangram.

Selain ukuran, mungkin siswa akan mempermasalahkan mengenai bahan pembuatan akan berpengaruh terhadap harga potongan tangram. Guru diharapkan dapat mengarahkan siswa untuk memahami bahwa harga tiap potongan dipengaruhi oleh luas atau paling tidak memahami bahwa ukuran yag menentukan harganya. Namun, pemaham area belum begitu penting disini dan jangan sampai guru memaksakan siswa untuk mengerti karena akan semakin diperdalam di aktivitas-aktivitas selanjutnya. Guru diharapkan juga jangan menekankan mengenai konsep konservasi area,biarkan mereka sendiri membangun konsep tersebut**.**

Aktivitas	Peran Guru
Selama diskusi, berikut ini perkiraan topik yang munculdari siswa: 1. Membagi 2 / menggandakan Membagi 2 potongan maka harganya juga dibagi 2 2. Penyusunan ulang/ mebandingkan tindih Membandingkan potongan yang kecil diatas yang besar untuk membandingkan 3. Estimasi c. Membagi 40.000: 7 d. Estimasi kasar melihat ukuran 4. grid membuat grid sebagai acuan dalam membandingkan ukuran dari potongan perkiraan ini tidak perlu semuanya ada dan diikuti guru. Untuk grup yang kesulitan mempresentasikan, guru membantu mengarahkannya.	Disini, respon guru terhadap masing-masng strategy siswa: 1.Guru dapat meminta siswa untuk membandingkan 2 potongan tangram mana ukurannya 2 kali yang lain dan mengapa. 2.Guru dapat memperjelas ide tentang menggabung potongan dan harga dengan member contohpada potonga tertentu. (pada diskusi, guru juga bisa mengarahkan jak menggunakan 2 strategi bisa mempermudah) 3.Ide utama di respon ini adalah siswa diarahkan untuk menggunakan 2 strategy sebelumnya.. c. Guru dapat meminta siswa untuk memikirkan apakah logis bermacam-macam bentuk potongan itu memiliki harga yang sama. d. Guru dapat menanyakan apakah jumlah semua potogan itu juga Rp 40.000,-. 4.Guru bisa mendiskusikan bersama kelas grid apa yang palig cocok digunakan untuk masalah ini Guru sebaiknya memastian seluruh siswa memahami dan mengikuti diskusi.

6. Menyusun Potongan-potongan Tangram Sesuai Harga

(DISKUSI KELAS)

Di aktivitas ini, siswa juga akan mendiskusikan mengenai harga potongan termurah dan termahal. Kita berharap siswa akan memahami ide tentang konsep luas dengan membandingkan harga dari tiap potongan. Di sini, penggunaan model tangram berharga diharapkan dapat digunakan siswa untuk memahami tentang luas. **Namun, tidak perlu memahai apa itu konservasi luas, tapi siswa paling tidak memahami jika semakin besar ukurannya maka semakin mahal harganya (ukuran menentukan harga).** Pertam-tama, guru dapat meminta siswa untuk menuliskan urutan potongan tangram, namun, jika pembahasan mengani urutan harga sudah dibahas pada diskusi sebelumnya, guru hanya tinggal memperdalam pemahaman siswa mengani ukuran menentukan harga. Kemungkinan juga siswa menyadari jiak ada potongan yang berbeda namun harganya sama. Guru bisa menahan diskusi ini untukpertemuan selanjutanya.

Aktivitas	Peran Guru
Siswa berusaha menyusun potongan-potongan tangram sesuai harga	Jika topik diskusi ini muncul di diskusi sebelumnya,guru hanya tinggal mengarahkan mengenai ide semakin besar ukurannya maka semakin mahal harganya
Mungkin ada siswa yang menyadari jika ada potongan berbeda yang harganya sama	Mengenai 3 potongan tangram yang harganya sama, guru diharapkan sedikit membahas mengenai hal ini namun jangan terlalu dalam, karena akan dibahas pada aktivitas selanjutnyA.
Siswa akan memahami bahwa ukuran menentukan harga	Guru diharapkan dapat membimbing siswa untuk memahami bagaimana hubungan anatara ukuran dan harga dari tiap potongan.

PENUTUP (5 MENIT)

Guru merangkum kembali apa yang telah dibahas bersama-sama kelas. Guru diharapkan menanyakan bagaimana perasaan siswa setelah bermain seharian. Sejauh apa pemahaman siswa mengenai hubungan ukuran dan harga dari tiap potongan. Berikut pertanyaan yang bisa diajukan guru

“bagaimana menurutmu bermain tangram di sekolah? Apakah kalian suka?”

“apa yag kamu pikirkan mengenai harga? Mengapa ptoongan ini ebih mahal daripada yang ini?”

Guru merangkum ulang lagi bersama-sama siswa mengenai apa yang telah dipelajari seharian.

Panduan Guru

Petemuan 2

MEMBANDINGKAN BENTUK MANA YANG PALING MAHAL

Lama Pertemuan:

2x35 menit

Material:

Beberapa set Tangram (tergantung pda jumlah murid di kelas tersebut)

Lembar Kerja Siswa untuk pertemuan 2

Gambar dari sketsa siluet bentuk yang telah dibuat siswa di pertemuan pertama

Tujuan Pembelajaran:

3. Siswa dapat memahami hubungan antara harga dan ukuran sebuah bangun.
   3.1. Siswa dapat memahami hubungan antara harga dan ukuran bangun-bangun yang bentuknya berbeda namun harganya sama.
   3.2. Siswa dapat menentukan harga dari bangun yang telah mereka buat masing-masing dengan strategi mereka sendiri
   3.3. Siswa dapat berargumen dan memahami hubungan antara harga dan potongan yang digunakan.

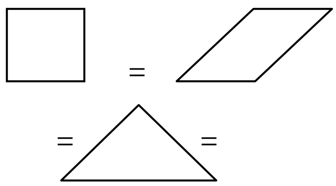
Kegiatan Belajar Mengajar

Gambar dari siluet bangun yang telah dibuat siswa akan digunakan setelah aktivitas kecil pedahuluan berikut ini.

SESI 1 (20 MENIT)

1. Diskusi kecil

   Dari aktivitas pertemuan sebelumnya, mungkin siswa akan menyadari beberapa potongan berbeda yang harganya sama. Berikut potongannya.



Di sini, kita harapkan siswa mengalami skala kecil konservasi area. Diskusi berpasangan diharapkan muncul untuk membahas mengapa 3 potongan berbeda tersebut berharga sama.

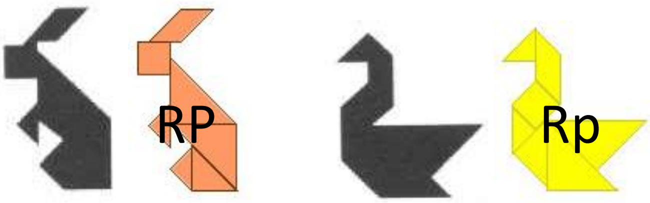
Aktivitas	**Peran Guru**
Menjelaskan permasalahan	Guru menjelaskan permasalahan sejelas mungkin. Guru meminta siswa untuk mendiskusikan dengan pasangannya.
Diskusi berpasangan antar siswa	Guru diharapan menunggu siswa memikirkan sendiri dahuu mengenai solusinya Guru juga bisa berkeliling memantau diskusi masing-masing pasanagn mungkin ada yang memerlukan bimbingan.
Diskusi Kelas: Siswa akan menyelesaikan dengan beberapa strategi berikut ini: 1. Siswa akan memotong dan menempel saat membandingkan potongannya 2. Siswa akan memperhitungkan juga bahwa bangun-bangun itu dapat dipecah menjadi 2 segitiga	Guru sebaiknya membimbing diskusi ke ide dekomposisi dari pada 2 segitiga yang dapat dikenal juag sebagai konservasi luas *“dapatkah kamu membandingkan persegi/ jajargenjang/ segitiga sedang dengan segitiga kecil? Apa yang kamu peroleh?”* Menyimpulkan bahwa adanya kemungkinan bentuk yang berbeda namun harganya sama

SESI 2 (45 MENIT)

2. Menghitung Harga Bangun Masing-masing yang telah Dibuat

   (INDIVIDU)

   Gambar, sketsa, atau siluet yang telah dibuat pada petemuan pertama digunakan disini. Tiap siswa akan membawa bangunnya masing-masing dan mentaksir harga total masing-masing bangunya.



Dalam menentukan harga dari tiap bangun yang terdiri dari beberap potongan tangram, logis jika siswa akan menjumlah saja harg tiap potongan sebagai harga total. Jika ada siswa yang kesulitan, guru dapat meminta mereka untuk menjumlah satu persatu harga masing-masing potongan tangram yang ada dan pastikan perhitungannya tepat.

Strategi lain adalah mereka memahami bahwa jika merekamenggunakan semua potongan dalam 1 set tangram tersebut adalah 40.000,maka jika mereka menggunakan semuanya atau menggunakan 6 potongan tangram saja, maka harganya sama atau 40.000 dikurangi potongan yang tidak digunakan. Gurur diharapkan membimbing siswa untuk menentukan potongan mana yang harganya harus dikurangkan untuk mendapatkan harga total dari bangun tersebut (Part Whole relation).

<table>
<tr><th>Aktivitas</th><th>Peran Guru</th></tr>
<tr><td>Siswa mentaksir harga bangunnya masig-maisng menggunakan strateginya masing-masing. Berikut adalah contoh strateggi yang dilakukan:
1. Menjumlah harga masing-masing ptotongan untuk memperoleh harga total
2. Menggunakan fakta bahwa total 1 set tangram berharga Rp 40.000,-.
3. Jika mereka tidak menggunakan semua potongan dalam 1 set maka hanya mengurangi Rp 40.000,- dikurangi harga potongan yang tak digunakan</td><td>Jika siswa kesulitan mengenai apa yang akan dilakukan, guru dapat memancing siswa dengan menanyakan “Jika kamu penjual bangun ini, berapa yang harus saya bayar untuk bangun ini?”
Guru sebaiknya membimbing siswa untuk menentukan potongan mana yang harus dikurangi harganya (Part whole relation).
Selanjutnya, kombinasi 2 strategi bisa mempermudah siswa menyelesaikan permaslahan ini. The teacher could lead the students to use this strategy by asking “could you use the other way around”
The teacher should make sure that each group prepares their presentation in a poster that will be conducted in the last session</td></tr>
<tr><td colspan="2">If they forget to draw the pieces in the picture/silhouette/sketch of their shape, then they will need time to reconstruct the shape using the tangram set. After that, the teacher can use the above guidance.</td></tr>
</table>

7. The Show of the Sketch of the Shape.

   (CLASS DISCUSSION)

   Each student will present his/her own pictures of shapes that have been put price in front of the class. By comparing the price of all shapes, then the teacher together with the whole class must group the shapes that have the same price. After they have done grouping the shapes according to the price, the class discussion should be started by guidance by the teacher. The main issue is about the relation between price and size.

   However, there will be a possibility that the students did not use all of the tangram pieces and figure out that the prize for his/her shapes will be less than the one who use all of the pieces. Here, we can trigger a discussion how to relate the use of prize in understanding the area. The examples of discussion topic (if applicable) that can be used in the discussion are:

5. Different shape but same prize?

   Using this issue as the discussion topic will trigger a need of a notion that is constant.
6. How to determine which one is cheaper or more expensive?

   Using this issue as the discussion topic will trigger a need of a notion that becomes parameter with which we can define a shape is cheaper than the other.
7. All of the shapes have same prices.

   This is the situation where all of the shapes have the same prices. What the class could discuss is why those shapes have the same price. Do they have a similar characteristic that relate each other
8. Different size but same number of pieces

   Posting this issue is quite important because otherwise the students will only reason why 2 different shapes have the same price is because they have the same number of pieces.

Regardless the size, perhaps, the students will argue about the material of the pieces will influence the price.

Aktivitas	**Peran Guru**
Grouping the shapes	Guiding the students in putting the shapes in a group of the same price
Discussion on the 1st issue (if applicable) Different shape but same prize? 3. Considering 2 shapes with different form but have same price, the students will try to define a characteristic of those 2 shapes which are same. 4. During the discussion, there is a possibility that the students consider about the size of the material used to make a piece of the tangram pieces.	3. The teacher should guide them carefully until they can come to the word “area”. The teacher could ask “*the shapes are different but the prices are the same, what do you think responsible for it?* 4. Teacher can lead those students to the situation in which they can imagine themselves made the tangram pieces by cutting a plate of wood. If they must buy a plate of wood, then they can notice that bigger the plate will make the prize more expensive.
Discussion on the 2nd issue (if applicable) How to determine which one is cheaper or more expensive? 3. Considering 2 shapes with different price, the students will try to define a parameter of those 2 shapes which can be reference for them to	3. The teacher should guide them carefully until they can come to the word “area”. The teacher could ask “*the shapes are different but the prices are the same, what do you think responsible for it?* 4. Teacher can lead those students to the

determine which shape is cheaper or more expensive. 4. During the discussion, there is a possibility that the students consider about the size of the material used to make a piece of the tangram pieces.	situation in which they can imagine themselves made the tangram pieces by cutting a plate of wood. If they must buy a plate of wood, then they can notice that bigger the plate will make the prize more expensive.
Discussion on the 3rd issue (if applicable) All of the shapes have same prices. Here, we conjecture that the students will come up with the idea of area. Then, we may just continue to the next activity. However, it not necessarily if the students get to the idea of area. If so, there will be more activity regarding this issue in the next activity.	The teacher guides the discussion on why different shapes have the same price. The discussion will evolve around what characteristic that make those different shapes have the same price.
Discussion on the 4th issue (if applicable) Different size but same number of pieces The students could have misconception that different shapes will not always if he number of the formed pieces are the same the shapes are in the same size	The teacher could provide the idea from the previous activity when they deal with different shape will not always if he number of the formed pieces are the same the shapes are in the same size The teacher could showing the previous activity about constructing shape with the same number of pieces but results different size. Then, the teacher could ask *"if the number of pieces does not matter her, what do you think determining the size comparison"*

CLOSING (5 MINUTES)

The teacher reviews the activities which have been done in that day. The teacher should lead that the price is related to the area of the pieces, or at least got the idea of it. However, it is not necessary to force the student to understand this relation. If so, just proceed to the next activity.

"What the parameter do you think by which determining the price of those various shape?"
"Could you think about the size of it" or if still hard for the students to understand a bit about the relation of size and price the teacher could ask *"Is there any relationship between the size and the price?"*

In the end of this activity, we expect that the students will grasp the notion of "area" which is used as the parameter by the students to determine which shape is cheaper or more expensive. Or, in another words, understanding the relationship between the price and the used pieces, or beyond this, the relationship between the price and the size.

The teacher can reformulate what the summary of the students' talk in this section are. The teacher also could ask the students itself to summarize the main idea.

Teacher Guide

3rd Lesson

COMPARE THE GIVEN SHAPES

Duration of lesson:

2x35 minutes

Material:

Sets of tangram mosaic puzzle (The number of sets depend on how many pupils in the class such that each pupil has one set)

More square and small triangles pieces for measuring activity if needed(2nd SESSION)

Worksheet for 3rd meeting

Learning goal

4. The students are be able to understand the one important properties area which is area conservation (if we compose and decompose the area into another form the area remain invariant) .
   4.1. The students are able compare the given shapes using the idea of area or price.
   4.2. The students are able to argue about the strategy that they used to compare the given shapes.
5. The students are be able to do covering the area as measurement activity by using different kind of tangram piece.
   5.1. The students are able to use one kind only tangram pieces from the available possible tangram pieces to fully cover the given shape.
   5.2. The students are able to understand the relation covering the shape using triangles or squares only as measurement activity.

The Teaching and Learning Activities

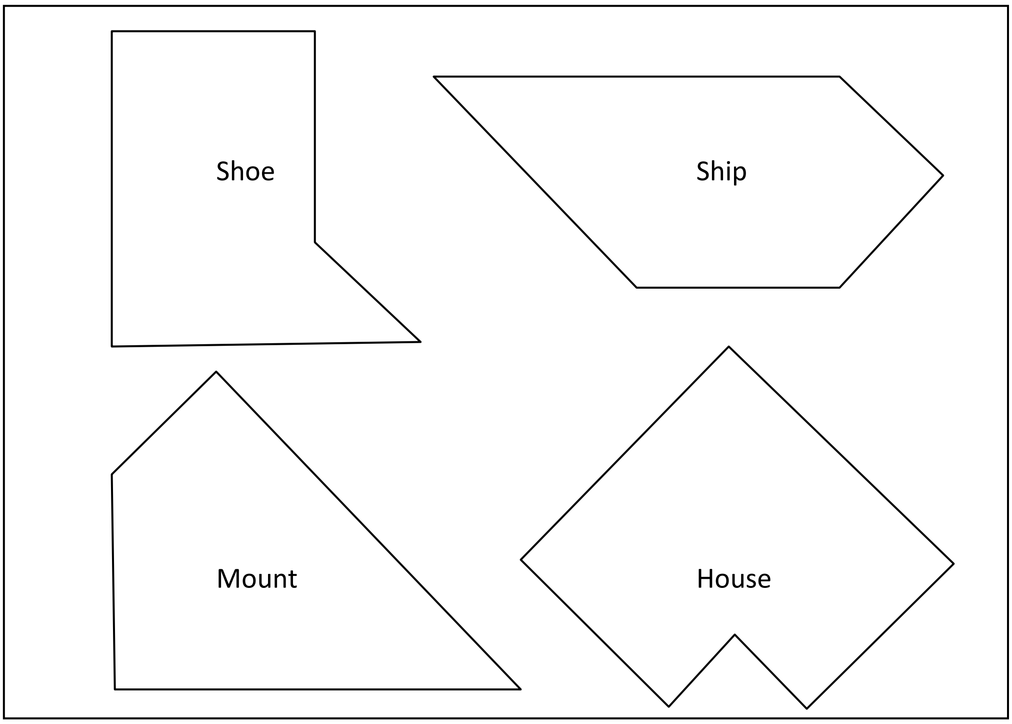
SECTION 1 (45 MINUTES)

4. Compare the Given Shapes
   (GROUP DISCUSSION)
   In the beginning, the teacher reminds about the previous activity whose shape is the most expensive. The teacher should highlight the main issues that have been

discussed in the previous discussion as well like *"What are the relationship between the size and the price"*

Similar with the previous activity, but in this work, student will play with the shapes given by the teacher and discuss it in a group of 3 or 4. The shapes used in this activity are designed feasible enough for the students to solve in such a time as a group. Here the shapes:

$$> ?, < ?, = ?$$



The comparison between those shapes is important. The fact that there will be 2 shapes which have the same area should come up in the students' discussion; otherwise, let the teacher highlight it in the next class discussion. Each group is asked to make a poster as the result of their discussion which will be presented in the next class discussion.

The Activity	The Role of the teacher
Group Discussion:	The teacher should guide the students by keeping them in the right track. If the students do not know what to do the teacher could suggest "could you think about using the tangram?" "how are the price" If the students already finish comparing the shapes the teacher could

	suggest “could you use another strategy to compare without price?”, “What do you think about the area?” The teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like “why do you choose this way?”” could you explain why it is possible?”, or “could you explain to your friends about what have you done?” The teacher should also encourage discussion in a group rather than just following a leader in a group. If the group has difficulty in solving the problem, the teacher could provide assistants or suggestions like “could you use the tangram to help your problem?”, “what about the left over part?” While moving around, the teacher could note which group will present for the next discussion by choosing which group did overlapping which group did structuring tangram.

5. The Show of the Poster

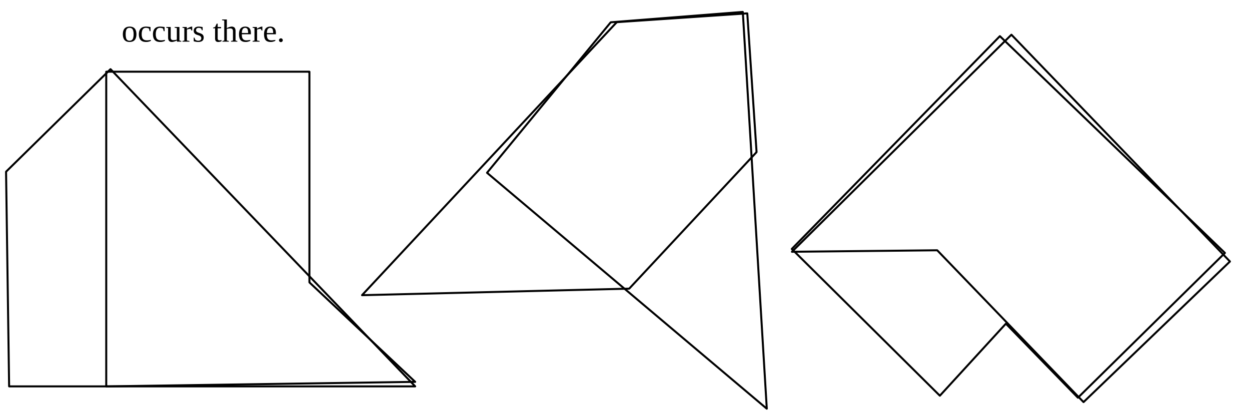
   (CLASS DISCUSSION)

   Each group will present their solution on a poster in front of the class one by one. They are asked to determine which shape is cheaper or more expensive or same price. Here, the students are really guided to use the notion of area in each shape as the parameter to determine which shape is cheaper. By only using the fact that the shape can be arranged using several pieces, we conjecture that they are expected not to use the reasoning using the total of the price but using overlapping strategy. Here, the money model will not be a main tool to reason with. They will notice from how the pieces are arranged into those shapes.

   Here the conjectured students solutions of the problem

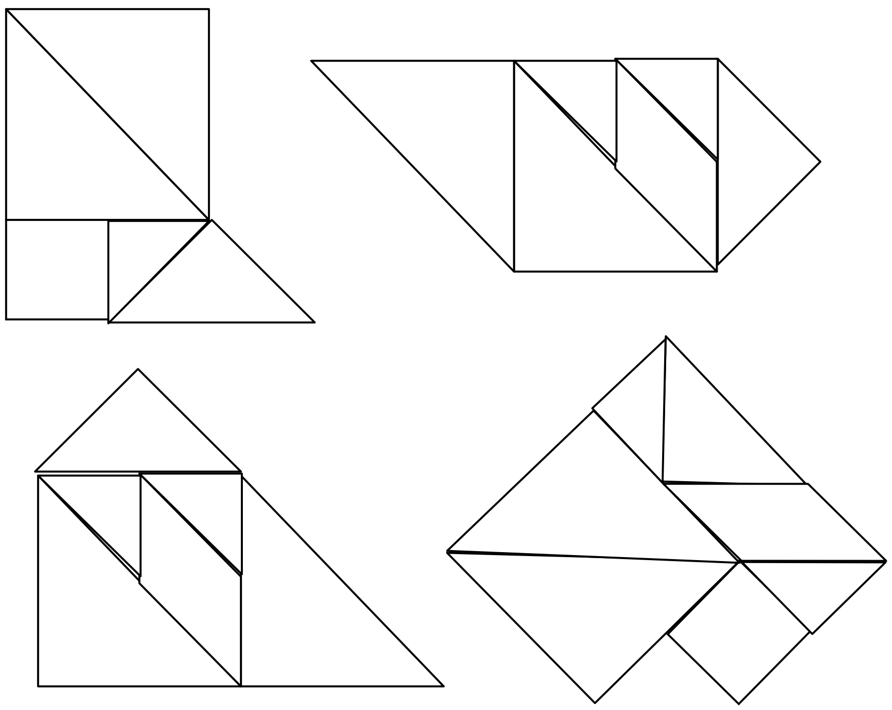
   3. Estimate the shape by using overlapping strategy to compare

      We anticipate the idea of estimation here. Some of the students might see the shapes are similar in certain parts. So, they will try to compare those similar parts and try to put one shape on top of another shape and see how the discrepancy occurs there.



4. The use of tangram pieces to reconstruct and put price to compare

   Here the example of how the students will arrange the tangram pieces.



   They will, then, calculate the price and then make the order of the shapes from the cheapest to the most expensive one.

On each group presentation, the main conclusion should be that the idea of composing and decomposing the shape from those 2 conjectures is understood by the students that it would not change the area as long as they use the same composition.

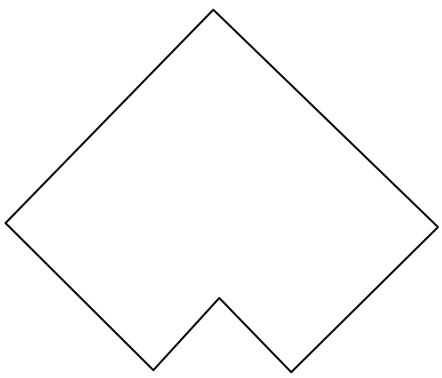
The Activity	The Role of the teacher
Group presentation There will be 2 main issues about the solution from the students for each group. Here the 2 activity respect to the 2 main issues that should be carefully treated	which group who will present have been chosen from previous group discussion
3. When the students using overlapping strategy By comparing in such a way, they will find the arrangement. However, the students do not use the price to reasoning that will make the discussion about area could occur	Here, the teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. And, area conservation occurs here. The teacher can ask the price of each shape later. The main idea is that when the student could understand the idea of those shapes could be

	composed and decomposed by looking at the left over part while overlapping each other and still the area remain invariant.
4. The use of tangram pieces to reconstruct and put price to compare They will, then, calculate the price and then make the order of the shapes from the cheapest to the most expensive one. We expect that the students will use the term of area as the reasoning. By comparing in such a way, they will find the arrangement. The students might not use the price to reasoning but the fact that those shape consist of same pieces will be their reason to determine this shape has the same price with another.	if the discussion of the area of the shape does not occur, the teacher should guide the student to conclude that the parameter with which we can determine the price is same or not is area. The teacher could ask them to compare the size, and then the discussion will follow in a line with the first conjecture. The main idea is that when the student could understand the idea of those shapes could be composed and decomposed by looking at the pieces that can be put in and out and still the areas remain invariant.
The conclusion of the discussion	The teacher should summary from those 2 strategies that whatever the left over or the pieces which can be put in or out, the main idea is composing and decomposing a shape into another shape with the areas remain invariant

2ND SESSION (20 MINUTES)

6. Mini Discussion (INDIVIDUAL)

   After the discussion finish, each students go back to their own groups seats and then the next problem is given. Using only one kind of shape of tangram piece, the students are asked to use it to cover a certain area of shape. the shape itself is already given to the students in the previous session. Here, they must use the available tangram that they had as a group and decide which shape that they are going to be used to cover the shape.



The Activity	The Role of the teacher
Delivering the problem	The teacher explains about the problem as clear as possible. The teacher asks the students to talk with their pair discussing the problem.
Group discussion	The teacher should wait and let the students to come up with their own idea To make sure, the teacher could ask "is it ok if you cover using this and this (refer to the shapes except square and small triangle)?", "what do you think if your pieces are not enough?" But if the students really have difficulty with the fact that the number of pieces is not enough for covering the shape, The teacher could ask them to borrow another group pieces or provide them with additional pieces The teacher should make sure that the students cover the shape completely.
Class discussion: In the class discussion, the students might come up with 2 idea 3. one square can be divided into 2 triangles 4. the square and the small triangle are suitable for covering such shapes	Teacher should guide the discussion into the idea of the decomposition of 2 triangles could be considered as the reason of area conservation If all of the students use squares only or triangles only, the teacher could ask them how if we use the triangles or squares and compare the results The teacher could also ask why others pieces are not suitable for covering such shape

CLOSING (5 MINUTES)

In the end of this activity, the teacher reviews the activities which have been done in that day. We expect that the students will understand and accept that every shape has what so called area by which we can determine which one is bigger or smaller. As the result, they could understand that those areas if we compose and decompose the area into another form the area remain invariant (area conservation) from the 1st session activities.

From the 2nd session of activity, we also expect that the students will understand covering the area as measurement activity by using different kind of tangram piece may result same area. Here, the possible question that could be posted by the teacher.

"What will happen with the area if I cut this part and paste it into another position?"
"Could you think about another shape which has the same area with it"

If the students answer these questions using the tangram as reasoning it is ok

The teacher can reformulate what the summary of the students' talk in this section are. The teacher also could ask the students itself to summarize the main idea. **The important achievement for the students here is that the students are no longer need to use the price to reason why this shape has the same area with another shape.**

Teacher Guide

4th Lesson

PLAYING WITH THE AREA OF POLYGON

Duration of lesson:

2x35 minutes

Material:

Sets of tangram mosaic puzzle for each student

More square and small triangles pieces for measuring activity if needed

Worksheet for 4th meeting

Learning goal

6. The students are be able to apply the concept of conservation to measure the area of polygon.
   6.1. The students are able to use the pieces from the previous activity to measure given polygon shapes.
   6.2. The students are able to argue about the strategy that they used to find the price. In the end, the students are able to use the concept of area conservation in measuring the area of a shape.

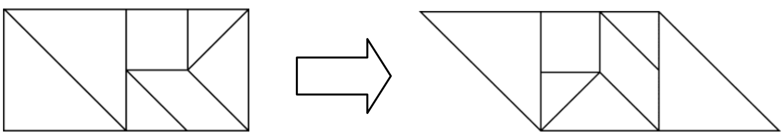
The Teaching and Learning Activities

3. Compare the Given Shapes

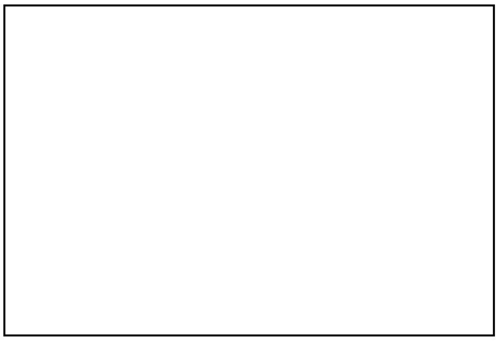
   (GROUP DISCUSSION)

   In the beginning, the teacher reminds about the previous activity comparing the 4 given shapes. The teacher should highlight the idea of composing and decomposing the shape into another shape will have the area remained invariant. The teacher could demonstrate the example of compose and decomposing a rectangle into parallelogram using tangram and ask:

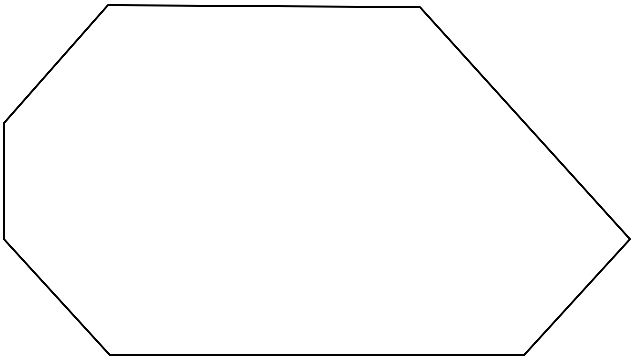
   "What are the relationship between the previous rectangle and this parallelogram?", "What do you think about the area? Which one has the largest area?"



The last activity is how the students will use what they have got to measuring polygon shapes. The student will play with the shapes given by the teacher and discuss it in a group of 3 or 4. The shape is designed in such a way that the students will have difficulty because of the number of the squares and triangles are not enough to cover by using only one type. They will need to use both shapes respectively to measure the area of the polygon. The students are asked to measure the 1st polygon firstly, and then after they have measured it, they could proceed into the next polygon. Here the pictures:



1st polygon



2nd polygon

The use of square and small triangles pieces in the appropriate manner is important. The students should notice that they could decompose the 1st polygon and compose the pieces to form the 2nd polygon. In the end, they are asked to state the measurement in squares. How many squares is the size of the 1st and the 2nd polygon?

The Activity	The Role of the teacher
Group Discussion: The students may use several strategies: 4. Using all pieces to cover up the shape one by one	The teacher should guide the students by keeping them in the right track. If the students do not know what to do the teacher could suggest "could you think about using the a preference pieces" If the students already finish measuring the polygon without comparing the size of those polygons the teacher could

5. Using only several pieces and do estimation then 6. Using the previous shape that has been covered and use it as starting point to cover	suggest “could you use another strategy to compare without covering one by one”, “What do you think about the area comparison of those polygons?” The teacher must also regard the students thinking by letting them in using their own creation for the problem. The teacher should address several question about the students reasoning like “why do you choose this way?”” could you explain why it is possible?”, or “could you explain to your friends about what have you done?” The teacher should also encourage discussion in a group rather than just following a leader in a group.

The teacher should make sure that each group has written down their answer in a poster which will be presented on the next classroom discussion.

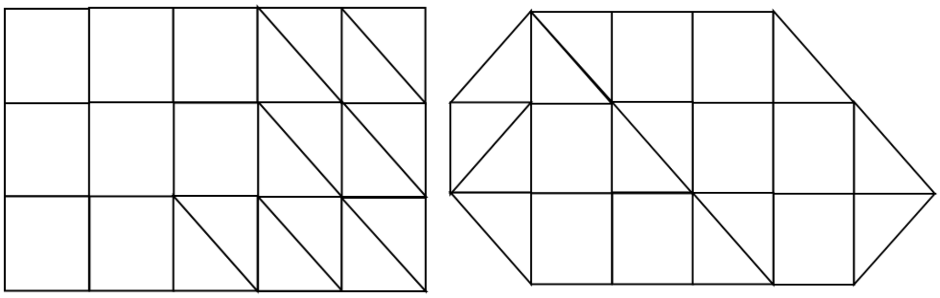
4. The Show of the Poster

(CLASS DISCUSSION)

Each group will present their solution on a poster in front of the class one by one. They are asked to determine which shape is cheaper or more expensive or same price. Here, the students are really guided to use the area conservation in measuring plane figure by only using the allowed pieces.

Here the conjectured students solutions of the problem

4. The students could do covering one by one to know the measurement of the polygon



5. The students could use array to measure the polygon
6. The students could use the former shapes as the reference and proceed to the remaining area to measure.

In this class discussion, the teacher should lead the students’ idea to the last conjecture where the students are able to use the idea of area conservation to measure the area of a plane figure. No matter from which conjecture the students start, the role of the teacher is important here.

The Activity	The Role of the teacher

During the discussion, here the conjectured topic discussed:
For the 1st polygon
4. they cover using pieces one by one
5. They realize that there will be 5 columns and 3 squares are in a column
6. They realize that there will be 5 columns and 6 triangles are in a column,
For the 2nd polygon
3. they cover using pieces one by one
4. They realize that there is a connection of this 2nd polygon with the 1st polygon
    a. They will use the available pieces, either the square or the small triangle pieces to measure and compare those 2 polygons
    c. They will cut and paste the polygon into another polygon to compare those 2 polygons
	For the discussion on the 1st polygon, the relationship between triangles and squares. The students should realize that those answers are actually the same
	The teacher could ask:
	"what do you think about the use of square of small triangles, which one is better?", "Do you see the relationship between those 2 different answer?"
	
	For the discussion on the 2nd polygon, if the students do not come out with the second conjecture, that we can use the 1st polygon to measure the 2nd polygon, the teacher could ask them to compare the 2nd polygon with the 1st polygon.
	The teacher could ask:
	"What do you think which polygon is bigger?", "Do you see the relationship between those 2 polygons?"
	The discussion also guided into the use of the concept of area conservation either the students use the pieces to compare or cut and paste the polygon to compare.
	
	Discussion on different solutions of arrangement between groups should be carried out clearly. Difference arrangement of one shape still result same measurement.

In fact, the students must use more than one kind of tangram shape, because of the limited number of squares or triangles if they decide to only use one kind of piece. Various combinations of triangles and squares will occur; we can put them in conflict to discuss each other solution. Here, they can mention the area of this polygon as triangles only, squares only, or both (for 1st shape). Then, we continue with the shape which has odd number of triangles (for 2nd shape). They should regard about the fact that a square is same area with 2 triangles and use it in the next discussion of the measurement of the polygon shape, and then we ask the students to state the area using squares only.

Discussion on different solutions of arrangement between groups should be carried out clearly. Difference arrangement of one shape still result same measurement.

CLOSING (5 MINUTES)

In the end of this activity, the teacher reviews the activities which have been done in that day. We expect that the students will understand and be able to use the idea of area conservation in area measurement. As the result, they could measure the area by only using a former shape as reference because it has the same area then do the remaining parts.

"What will happen with the area if I cut this part and paste it into another position?"

"Could you think about another shape which has the same area with it"

If the students answer these question using the tangram as reasoning, then the teacher should ask them another case like curved shape that students cannot tangram

The teacher can reformulate what the summary of the students' talk in this section are. The teacher also could ask the students itself to summarize the main idea.

In the end of this activity, we expect that the students will use the tangram pieces (triangles and squares) and the idea of conservation to measure the area of polygon (the conservation of area).

APPENDIX

THE PLANNING OF THE MEETING

Because I have not tested the whole learning sequence in real context, in case that it go smoothly, it will only take 4 meetings. Otherwise, my estimation is that 5 meetings are enough.

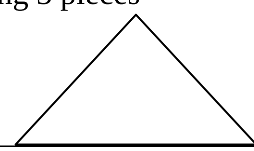
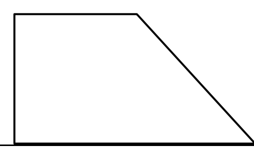
NO	THE ACTIVITY	MEETING 1					MEETING 2					MEETING 3					MEETING 4				
		15'	15'	15'	15'	10'	15'	15'	15'	15'	10'	15'	15'	15'	15'	10'	15'	15'	15'	15'	10'
1	ACTIVITY 1		*																		
2	ACTIVITY 2						*														
3	ACTIVITY 3																				
4	ACTIVITY 4																				
5	ACTIVITY 5															* *					
6	ACTIVITY 6																				

* If activity 1 needs more time, activity 1 could take over the first quarter of the activity 2 and the discussion of activity 2 could be more focus in the next meeting

** If activity 4 takes less time, activity 5 could take over the last quarter and the discussion of activity 5 could be more focus in the next meeting

TABLE OF DATA COLLECTION ON EACH ACTIVITY

In the following table, we will describe the data which are collected on each activity based on the background theory that we have already defined. The mathematical activities described here is the observable students' learning process

Activity		Learning Goal	Collected Data
1	Orientation with the tangram Constructing own shape using more pieces and record it	The students are accustomed to play with tangram The students are freely construct their own shapes using 7 given pieces and then draw a sketch of it.	How the students construct shape using the given tangram
	Constructing a given shape using 3 pieces	The students are able to construct a certain shape using 3 pieces 	How the students construct a given shape using 3 piecese only
	Constructing a given shape using 2 or more pieces.	The students are able to construct a certain shape using 2 or more pieces 	How the students construct a given shape using various pieces
2	Finding the price of each piece from the context given	The students are able to determine the price of each piece using their own strategy	How the students determine the price of each pieces if the total price of those 7 pieces is Rp 80.000
	Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price	How the students present and defend their idea in the class discussion

Activity		Learning Goal	Collected Data
4	Mini Lesson: Why square and parallelogram have the same price	The students are able to understand that 2 or more shapes which have the same price have the same size of area	What the students reason of why the square and the parallelogram have the same price is
	Compare the Given Shapes	The students are able to compare the given shapes using the idea of area or price	How the students determine which shapes have equal size or more.
	Math Congress: Share the group work	The students are able to argue about the strategy that they used to compare the given shapes	How the students present and defend their idea in the class discussion
5	Decompose and compose 2 given polygons into a square	The students are able to do conservation of a shape into another shape which has the same area	How the students find the size relation between shapes
	Covering A Given Shape Using given pieces	The students are be able to measure the area of certain polygons using the concept of area conservation	How the students find the number of measurement unit in the given shapes.

Students' Learning Process

Learning is an active, constructive process to acquire new information, ideas or skills. Hence, the students learning process is the process of the students active process to constructively acquire new information, ideas or skills through the classroom activity. In the classroom context, the constructive task as the guidance for the student to do constructive process in purposeful ways. The students need to integrate this new material with what they already know, in this study context, by using mathematical activity in the topic of area measurement.

What is Collaborative Learning? *

by Barbara Leigh Smith and Jean T. MacGregor

*This is an abbreviation of Smith and MacGregor's article, "What Is Collaborative Learning?" in Collaborative Learning: A Sourcebook for Higher Education, by Anne Goodsell, Michelle Maher, Vincent Tinto, Barbara Leigh Smith and Jean MacGregor. It was published In 1992 by the National Center on Postsecondary Teaching, Learning, and Assessment at Pennsylvania State University.

What is Collaborative Learning?

Washington Center for Improving the Quality of Undergraduate Education

Panduan Guru

Pertemuan 1

PERKENALAN TANGRAM DAN PENGANTAR KONTEKS

Identitas:

Satuan Pendidikan: Sekolah Dasar
Mata Pelajaran : Matematika
Kelas/Semester : III/ semester 2
Alokasi Waktu : 2x35 menit

Standar kompetensi:

5. Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

Kompetensi dasar

5.2. Menghitung luas persegi dan persegi panjang

Tujuan Pembelajaran:

Siswa dapat bermain dengan tangram dalam hal menyusun dan menkonstruk suatu bangun menggunakan potongan-potongan tangram yang disediakan.

- Siswa dapat bermain tangram dengan bebas.
- Siswa dapat menkonstruksi sebuah bangun atau gambar menggunakan potongan-2 atau 3 potongan tangram yang tersedia
- Siswa dapat menkonstruksi bangun tertentu dengan menggunakan sebarang potongan tangram yang disediakan

Siswa dapat mengembangkan model tangram berharga dari permasalahan kontekstual yang telah diberikan sebagai model berpikir tentang luas.

- Siswa dapat menentukan harga dari tiap potongan tangram dengan menggunakan caranya sendiri
- Siswa dapat menjelaskan strateginya untuk mencari harga dari tiap potongan .

Bahan Ajar:

Beberapa set Tangram (setiap anak mendapat 1set tangram)
Lembar Kerja Siswa untuk pertemuan 1
Sebuah gambar tentang berbagai bentuk yang disusun dari 1 set tangram

Kegiatan Belajar Mengajar

KEGIATAN AWAL (20 MENIT)

1. Kegiatan Pengantar

   Siswa diberi kesempatan bebas untuk bermain dengan tangram. Diharapkan siswa mampu menyusun dan membentuk bangun-bangun yang tersusun dari potongan-potongan tangram dengan mudah setelah menjalani aktivitas-aktivitas berikut.

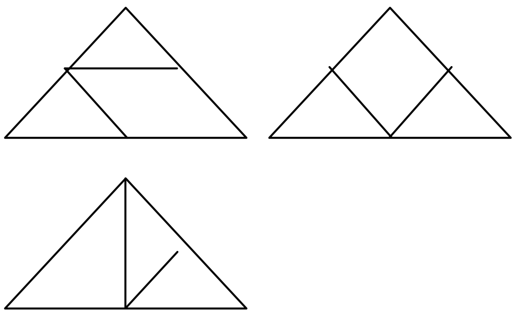
   - Guru diharapkan memperkenalkan tangram dengan menunjukkan kepada siswa dan menceritakan mengenai asal-usul tangram.
   - Kemudian, tiap siswa diberikan 1 set tangram untuk bermain. Sebagai pendahuluan, siswa diminta untuk bermain dan menyusun bentuk-bentuk yang mereka inginkan.
   - Kemudian siswa diminta memilih 1 bentuk yang paling dia suka dan menggambar sketsanya di kertas yang telah disediakan.
   - guru diharapkan menyimpan bentuk-bentuk yang telah dibuat siswa karena akan digunakan pada aktivitas mendatang.

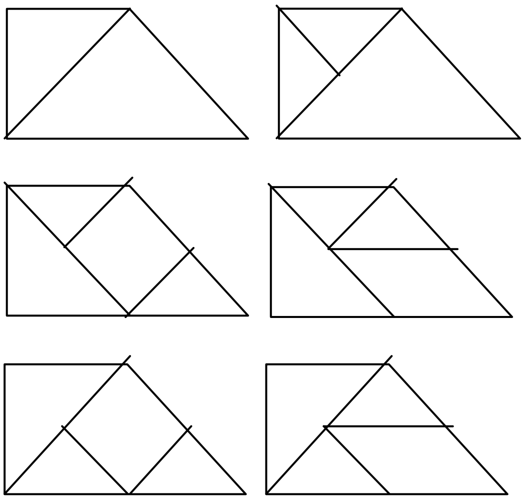
Aktivitas	Peran Guru
Perkenalan Tangram	Guru memperkenalkan tangram ke siswa, menjelaskan kegiatanyang dapat dilakukan menggunakan tangram serta menceritakan asal usul. http://library.thinkquest.org/J002441F/tangrams.htm “ Seorang kakek bernama kakek Tang memecahkan sebuah vas bunga menjadi 7 potongan, dia kemudian ingin memperbaiki dengan menyusun ulang potongan-potongan tersebut. Namun, yang terjadi malah banyak sekali bangun yang bisa kakek Tang bentuk. Kakek Tang malah keasyikan bermain hingga lupa kalau dia harus menyusun ulang potongan itu menjadi vas bunga lagi”
Membuat 1 bentuk tangram yang disukai (INDIVIDU) Siswa diharapkan menggambar siluet/sketsa dari bentuk yang telah dia pilih atau menggambar bagaimana potongan2 itu disusun menjadi bentuk yang telah mereka buat.	Guru diharapkan mengecek kemampuan siswa dalam menggunakan tangram dalam permainan, Guru dapat bertanya mengenai *“apa nama bentuk yang kamu buat? Atau mengapa kamu suka membuat bentuk itu? Dapatkah kamu menggambar bentuk tangram itu di kertas?”* Mengecek penalaran siswa dalam menyusun tangram terutama secara geometris. “*mengapa kamu susun sisi ini dengan sisi ini, tidak terbalik seperti ini?”*(meletakkan susunan tidak pas pada sisinya)

REVISI	Karena meja kursi yang digunakan siswa untuk bermain cukup kecil hasil susunan tangram siswa cenderung mudah rusak. Hal ini

	menyebabkan siswa kesulitan menggambar bangun yang telah disusun Guru sebaiknya menekankan kepada siswa untuk berkreasi sesuai kreativitasnya masing-masing dan tidak meniru hasil temannya. Lembar yang disediakan untuk siswa menggambar tetap karena jika bangun yang dibuat oleh siswa cukup besar dan tidak cukup diharapkan dia akan memikirkan bagaimana menggambar ukuran yang lebih kecil agar muat di lembar yang diberikan

2. Aktivitas Merekonstruki (membentuk kembali) Bangun yang Diberikan (INDIVIDU)
    - ➢ Siswa diminta untuk membentuk sebuah siluet dari bentuk-bentuk geometris yang telah diberikan menggunakan beberapa potong tangram.
    - ➢ Pada kegiatan ini, siswa akan mengkonstruksi mulai dari menggunakan 2 potong hingga lebih secara berturut-turut.
    - ➢ Disini, diharapkan siswa terbiasa untuk menyusun dan mengkonstruk bermain dengan tangram. Lebih jauh lagi, diharapkan siswa akan menyadari akan adanya konsep konservasi luas disni dengan menyadari bahwa kita dapat menggunakan potongan-potongan tangram berbeda untuk menyusun sebuah bentuk yang sama atau bahkan jumlah yang berbeda juga. Oleh karena itu, siswa diharapkan mulai menyadari akan konsep konservasi luas.
    - ➢ Di akhir aktivitas ini, kita harapkan siswa dapat dengan leluasa bermain menyusun dan membentuk menggunakan potongan-potongan tangram yang ada. **Namun**, guru tidak boleh memaksakan siswa untuk memahami konsep konservasi luas seperti yang telah disebutkan sebelumnya. Biarkan mereka membangun pemahaman mereka sendiri melalui aktivitas-aktivitas yang ada.

Aktivitas	Peran Guru
Alternative jawaban yang dibuat oleh siswa  Kemungkinana ada jawaban lain yang mirip dengan jawaban di atas hanya bedanya pada urutan yang terbalik.	Guru membimbing siswa untuk menginvestigasi masalah yang diberikan. Guru dapat menanyakan rencana siswa dalam menyusun sisi-sisi potongan tangram dalam bangunan tersebut. Guru dapat meminta siswa untuk melihat kembali permasalahan sebelumnya. Hal ini memungkinkan siswa memperluas hasil penyelesaian yang mereka miliki dari solusi sebelumnya.

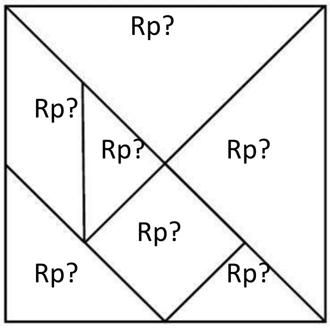
Bimbingan diperlukan untuk siswa yang mengalami kesulitan.	Guru dapat menanyakan bangun mana yang paling besar. Hal ini dilakukan untuk mengetahui pengetahuan siswa tentang konsep konservasi area. Namun, guru diharapkan tidak terburu-biri menjelaskan konsep konservasi luas.
Alternatif penyelsaian yang dapat dibuat siswa  Kemungkinana ada jawaban lain yang mirip dengan jawaban di atas hanya bedanya pada urutan yang terbalik. Bimbingan diperlukan untuk siswa yang mengalami kesulitan.	Guru membimbing siswa untuk menginvestigasi masalah yang diberikan. Guru dapat menanyakan rencana siswa dalam menyusun sisi-sisi potongan tangram dalam bangunan tersebut. Guru dapat meminta siswa untuk melihat kembali permasalahan sebelumnya. Hal ini memungkinkan siswa memperluas hasil penyelesaian yang mereka miliki dari solusi sebelumnya. Guru dapat menanyakan bangun mana yang paling besar. Hal ini dilakukan untuk mengetahui pengetahuan siswa tentang konsep konservasi area. Namun, guru diharapkan tidak terburu-biri menjelaskan konsep konservasi luas.
REVISI	Siswa cenderung hanya mau membuat yang mudah saja. Mereka tidak mau memikirkan berbagai kemungkinan susunan yang ada. Guru bisa meminta siswa untuk menemukan paling tidak 2 susunan. Dan jika perkiraan waktunya masih cukup, guru bisa meminta siswa untuk mencari susunan lain yang lebih rumit. Tujuan utama kegiatan ini adalah siswa akan menyadari bahwa susunan dari suatu bangun tidak hanya memiliki 1 kemungkinan. Diharapkan siswa akan menyadari bahwa 1 bangun bisa disusun dari lebih dari 1 susunan. Dan, berbagai susunan berbeda tersebut bisa memiliki satu daerah yang sama

KEGIATAN INTI (45 MENIT)

3. Perkenalan Permasalahan Kontekstual (GRUP)
   - Disini, guru akan membimbing siswa dalam membangun model tangram berharga dari masalah kontekstual yang diberikan sebagai model berpikir untuk memecahkan masalah.
   - Beriku permasalahannya, guru diharapkan meminta siswa membaca dan memastikan tidak ada siswa yang kesulitan memahaminya.

MENJUAL POTONGAN TANGRAM

"Menjual Potongan Tangram"



Setelah siswa selesai bermain tangram, mereka diminta untuk memecahkan permasalahan di toko mainan milik saudara laki-laki guru yang menjual puzzle tangram. Harga dari 1 set tangram lengkap adalah Rp 80.000,-. Pemilik toko juga ingin menjual tangram itu eceran per potong karena kadang beberapa anak kehilangan satu atau dua potongan tangram yang mereka miliki. Guru kemudian meminta siswa membantu pemilik toko untuk menentukan harga dari masing-masing potongan tangram yang tersebut.

- Siswa diberikan waktu selama beberapa menit untuk memahami permasalahan tersebut secara individu, kemudian baru dibagikan LKS
- kemudian siswa diminta menyelesaikan dalam grup 3 atau 4 orang.
- Di akhir kegiatan kelompok, setiap kelompok diminta membuat poster untuk dipresentasikan mengenai bagaimana mereka memecahkan persoalan kontekstual diatas.

Aktivitas	Peran Guru
Diskusi Grup: Alternatif rencana penyelesaian atau strategi penyelesaian yang digunakan oleh siswa: 1.membagi separuh potongan tangram 2. menggabung potongan tangram 3. Meperkirakan harga a. membagi 40.000 : 7 b.estimasi kasar berdasarkan ukuran	Guru sebaiknya membimbing siswa. Jika siswa kebingungan, guru dapat menyarankan siswa dengan mengarahkan "bagaimana harganya jika kita bagi potongan ini menjadi 2 bagian sama besar?", "Bagaimanakan harganya jika kita gabung 2 potong tangram ini?" Berikut ini respon guru sesuai dengan strategy siswa: 1. Guru dapat meminta siswa untuk membanding 2 potongan mana yang harganya 2 kali lipat berdasarkan ukurannya dan menanyakan alasannya kepada siswa. 2. Guru dapat membantu siswa dengan memberikan petunjuk untuk memperkirakan harga potongna kecil (2 strategy diatas bisa digunakan bersamaan untuk mempermudah pemecahan masalah) 3. Ide utama pada strategi ini adalah siswa diarahkan untuk menggunakan 2 strategi sebelumnya. a. Guru menanyakan apakah mungkin potongan-potongan yang berbeda itu memiliki harga yang sama. b. Guru dapat menanyakan apakah jumlah semua potongan itu adalah Rp 40.000,-. 4. Guru dapat membimbing siswa untuk membuat garis grid yang

4. menggunakan grid	sesuai. Guru memberi kesempatan siswa sendiri untuk berkreasi memecahkan masalah Guru diharapkan juga menanyakan beberapa hal yang merangsang siswa untuk berfikir kritis seperti “memangapa kamu memilih strategy ini?”, ”dapatkah kamu menjelaskan?” atau “dapatkah kamu menjelaskan strategimu kepada temanmu?” Guru diharapkan juga memancing diskusi dalam grup daripada hanya mengikuti satu pendapat.
REVISI	Pada diskusi kelompok ini, siswa akan muncul dengan tebakan yang bisa benar bisa juga salah. Siswa menebak harga masing-masing dan berusaha supaya harga totalnya adalah 40.000 dengan merubah-rubah harga masing-masing potongan. Untuk siswa yang seperti ini, guru diharapkan tetap menanyakan kepada siswa kenapa harga potongan ini 2 kali atau sama dengan potongan yang lainnya. Diharapkan siswa dapat memahami strategi pertama walaupun dengan bimbingan guru. Yang penting disini adalah ide tentang membagi dua suatu bangun maka harganya juga akan tmenjadi setengahnya. Selain itu, diharapkan siswa juga memahami strategi kedua yaitu jika 2 potongan tangram digabung menjadi sebuah potongan tangram yang lain, maka harganya sama dengan jumlah dari masing-masing potongan penyusunnya.

4. Presentasi Poster

(DISKUSI KELAS)

- Beberapa grup akan mempresentasikan solusinya dalam poster.
- Guru memilih solusi yang beragam dan tidak sama.
- Setelah diskusi tentang bagaimana menentukan harga per potong tangram ini selesai, diharapkan ada siswa yang membuat daftar harga dari potongan tangram tersebut.
- Pada diskusi kelas, berbagai kemungkinan solusi permasalahan kontekstual akan didiskusikan.
- Diharapkan siswa mampu berargumentasi mempertahankan jawaban masing-masing terutama mengenai bagaimana mereka mencari harga dari tiap potongan tangram.
- Guru diharapkan dapat mengarahkan siswa untuk memahami bahwa harga tiap potongan dipengaruhi oleh luas atau paling tidak memahami bahwa ukuran yag menentukan harganya.
- Namun, pemaham luas belum begitu penting pada kegiatan ini dan guru diharapkan tidak memaksakan siswa untuk memahami hal tersebut karena akan semakin diperdalam pada aktivitas-aktivitas selanjutnya.

- Guru diharapkan juga tidak menekankan mengenai konsep konservasi luas, dan membiarkan siswa membangun sendiri konsep tersebut**.**

Aktivitas	**Peran Guru**
Selama diskusi, berikut ini perkiraan topik diskusi yang akan muncul dari siswa: 1. Membagi 2 / menggandakan Membagi 2 potongan maka harganya juga dibagi 2 2. Penyusunan ulang/ membandingkan dengan menumpuk bangun Membandingkan potongan yang kecil diatas yang besar untuk membandingkan 3. Estimasi a. Membagi 40.000: 7 b. Estimasi kasar melihat ukuran 4. grid membuat grid sebagai acuan dalam membandingkan ukuran dari potongan perkiraan ini tidak perlu semuanya ada dan diikuti guru. Untuk kelompok yang kesulitan mempresentasikan, guru membantu mengarahkannya.	Respon guru terhadap masing-masng strategy siswa: 1.Guru dapat meminta siswa untuk membandingkan 2 potongan tangram mana ukurannya 2 kali yang lain dan mengapa. 2.Guru dapat memperjelas ide tentang menggabungkan potongan dan harga dengan member contoh pada potongan tertentu, missal menggabung potongan segitiga menjadi persegi. (pada diskusi, guru juga bisa mengarahkan jak menggunakan 2 strategi bisa mempermudah) 3.Ide utama di respon ini adalah siswa diarahkan untuk menggunakan 2 strategy sebelumnya.. a. Guru menanyakan apakah mungkin potongan-potongan yang mempunyai ukuran berbeda itu memiliki harga yang sama. b. Guru dapat menanyakan apakah jumlah semua potogan itu Rp 40.000,-. 4.Guru bisa mendiskusikan bersama kelas grid apa yang palig cocok digunakan untuk masalah ini Guru sebaiknya memastian seluruh siswa memahami dan mengikuti diskusi.
REVISI	Jika yang dilakukan siswa hanya menuliskan harga masing-masing potongan dan tidak menjelaskan dari mana harga tiap potongan tersebut berasal, guru bisa menanyakan kepada mereka bagaimana mereka menemukan jawaban tersebut. Namun, jika siswa masih kesulitan dalam menjelaskan dari mana harga dari tiap potongan tersebut, guru bisa memberikan potongan tangram kepada siswa dan meminta siswa menjelaskan hubungan harga dengan ukurannya menggunakan potongan tangram yang ada Guru harus memastikan siswa memahami strategi 1 dan 2 di akhir diskusi ini.

5. Menyusun Potongan-potongan Tangram Sesuai Harga

   (DISKUSI KELAS)

   - Di aktivitas ini, siswa juga akan mendiskusikan mengenai harga potongan termurah dan termahal.
   - Diharapkan siswa akan memahami ide tentang konsep luas dengan membandingkan harga dari tiap potongan.

- Di sini, penggunaan model tangram berharga diharapkan dapat digunakan siswa untuk memahami tentang luas.
- **Namun, tidak perlu memahami apa itu konservasi luas, tapi siswa paling tidak memahami jika semakin besar ukurannya maka semakin mahal harganya (ukuran menentukan harga).**
- Pertama-tama, guru dapat meminta siswa untuk menuliskan urutan potongan tangram, namun, jika pembahasan mengani urutan harga sudah dibahas pada diskusi sebelumnya, guru hanya tinggal memperdalam pemahaman siswa mengenai ukuran menentukan harga. Kemungkinan juga siswa menyadari jika ada potongan yang berbeda namun harganya sama. Guru bisa menahan diskusi ini untuk pertemuan selanjutanya.

Aktivitas	**Peran Guru**
Siswa berusaha menyusun potongan-potongan tangram sesuai harga	Jika topik diskusi ini muncul di diskusi sebelumnya,guru hanya tinggal mengarahkan mengenai ide semakin besar ukurannya maka semakin mahal harganya
Mungkin ada siswa yang menyadari jika ada potongan berbeda yang harganya sama	Mengenai 3 potongan tangram yang harganya sama, guru diharapkan sedikit membahas mengenai hal ini namun jangan terlalu dalam, karena akan dibahas pada aktivitas selanjutnyA.
Siswa akan memahami bahwa ukuran menentukan harga	Guru diharapkan dapat membimbing siswa untuk memahami bagaimana hubungan anatara ukuran dan harga dari tiap potongan.
REVISI	Jika waktu sudah mendesak dan tidak ada siswa yang member urutan harga dari yang termahal ke yang murah, guru bisa segera meggambarkan masing-masing potongan di papan tulis dan meminta siswa member harga masing-masing. Kemudian, guru membimbing siswa untuk menyimpulkan bahwa semakin besar ukurannya maka harganya semakin mahal dan sebaliknya

PENUTUP (5 MENIT)

Guru merangkum kembali apa yang telah dibahas bersama-sama kelas. Guru diharapkan menanyakan bagaimana perasaan siswa setelah bermain seharian. Sejauh apa pemahaman siswa mengenai hubungan ukuran dan harga dari tiap potongan. Berikut pertanyaan yang bisa diajukan guru

"bagaimana menurutmu bermain tangram di sekolah? Apakah kalian suka?"

"apa yang kamu pikirkan mengenai harga? Mengapa potongan ini lebih mahal daripada yang ini?"

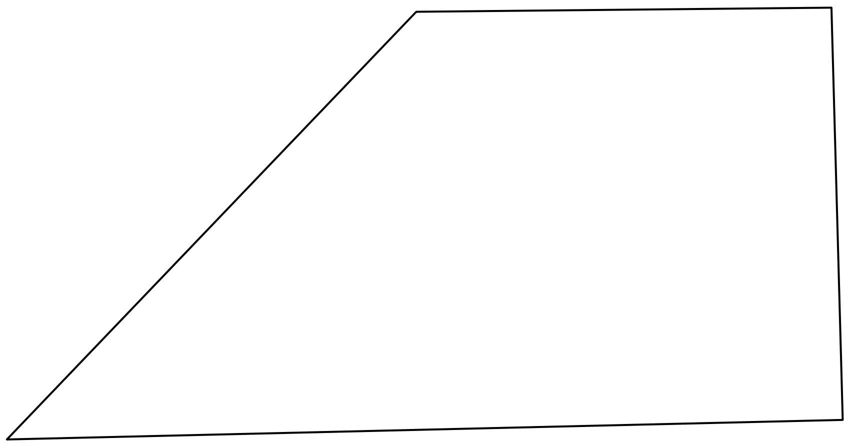
Guru merangkum ulang lagi bersama-sama siswa mengenai apa yang telah dipelajari seharian.

Pertemuan 1 – Lembar Kerja Siswa A Sesi 1

Nama :

Kelas :

Gunakan 2 potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat.

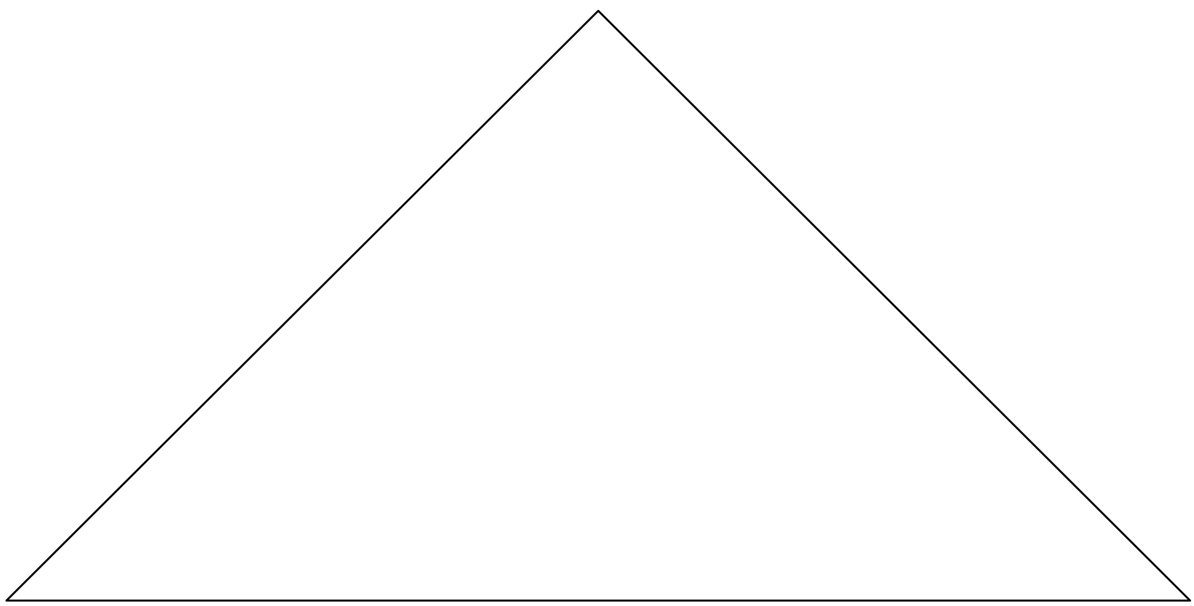


Pertemuan 1 – Lembar Kerja Siswa B Sesi 1

Nama :

Kelas :

Gunakan 3 potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat.

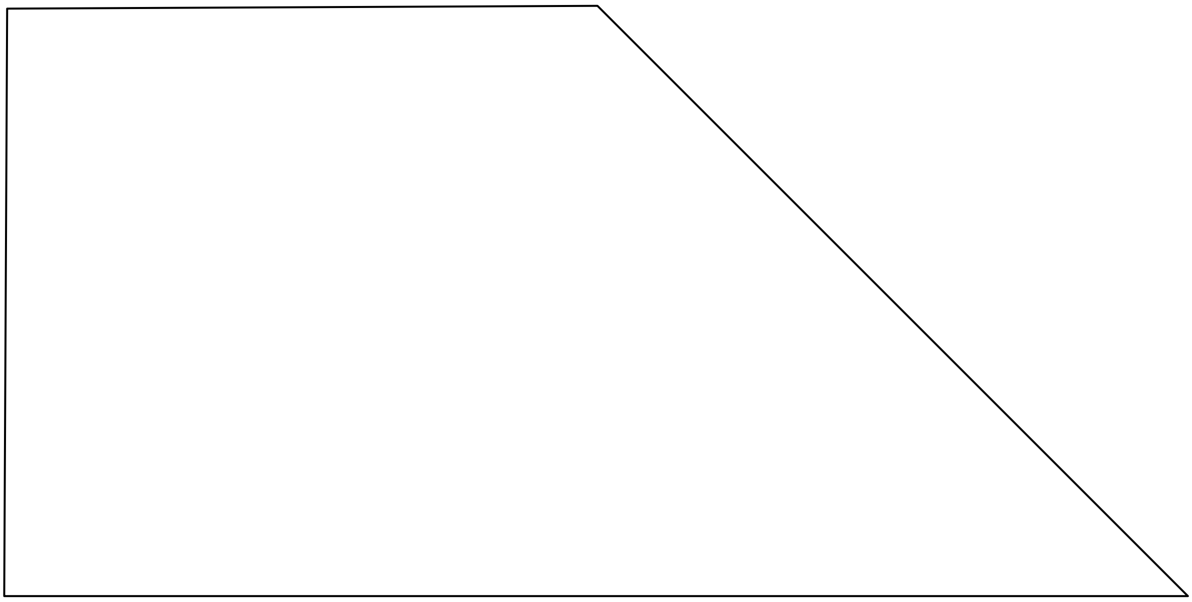


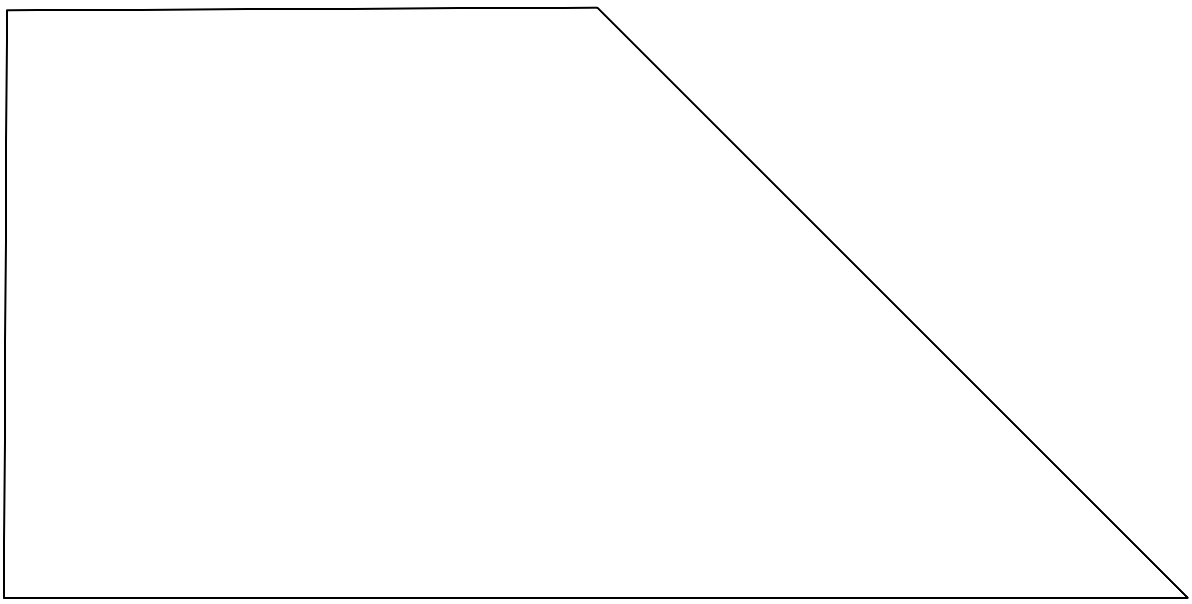
Pertemuan 1 – Lembar Kerja Siswa C Sesi 1

Nama :

Kelas :

Gunakan beberapa potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat, buatlah 2 susunan berbeda.





Pertemuan 1 – Lembar Kerja Siswa 0 Sesi 1

Nama :

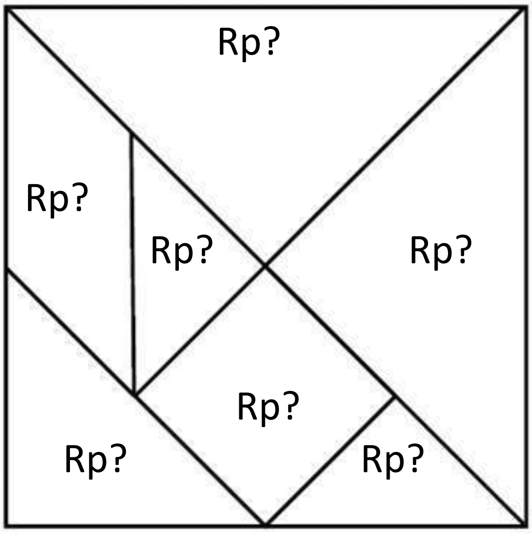
Kelas :

Bentuklah bangun yang kamu suka menggunakan satu set tangram yang tersedia, kemudian gambarlah bentuk yang kamu buat pada tempat berikut ini. Jika tidak cukup gambar di bagian belakang kertas ini.

Nama :
Kelas :

MENJUAL TANGRAM ECERAN

Kakak saya memiliki toko mainan. Salah satu mainan yang dijual di tokonya adalah puzzle tangram. Harga untuk satu set puzzle tangram adalah Rp 40.000,-. Kakak saya berencana menjual juga potongan tangram itu per buah, karena dia melihat setelah beberapa bulan banyak anak-anak yang memiliki puzzle tangram ini kehilangan potongan-potongan tangram nya, sehingga tidak bisa bermain dengan 1 set tangram yang lengkap. Bisakah kalian bantu kakak saya untuk menentukan harga untuk setiap potongan yang berbeda. Kakak saya akan berterimakasih sekali atas bantuan kalian.



TUGAS KAMU

Dalam 1 kelompok, tentukan harga dari masing-masing potongan tangram. Kalian bebas menggunakan cara apa saja yang kalian perkirakan paling meyakinkan.

Kemudian, persiapkan 1 poster yang berisi jawaban kalian dan bagaimana cara kalian untuk menentukan jawaban tersebut. Buat poster sedemikian rupa sehingga teman kalian mengerti akan apa yang kalian maksud di poster ini

SELAMAT MENGERJAKAN
SEMANGAT☺

Panduan Guru

Pertemuan 2

MEMBANDINGKAN BENTUK MANA YANG PALING MAHAL

Identitas:

Satuan Pendidikan: Sekolah Dasar
Mata Pelajaran : Matematika
Kelas/Semester : III/ semester 2
Alokasi Waktu : 2x35 menit

Standar kompetensi:

5. Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

Kompetensi dasar

5.2. Menghitung luas persegi dan persegi panjang

Tujuan Pembelajaran:

Siswa dapat memahami hubungan antara harga dan ukuran sebuah bangun.

- ❖ Siswa dapat memahami hubungan antara harga dan ukuran bangun-bangun yang bentuknya berbeda namun harganya sama.
- ❖ Siswa dapat menentukan harga dari bangun yang telah mereka buat masing-masing dengan strategi mereka sendiri
- ❖ Siswa dapat mengajukan pendapat dan memahami tentang hubungan antara harga dan potongan yang digunakan.

Bahan Ajar:

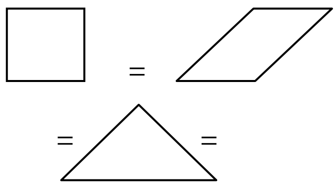
Beberapa set Tangram (tergantung pada jumlah murid di kelas tersebut)
Lembar Kerja Siswa untuk pertemuan 2
Gambar dari sketsa siluet bentuk yang telah dibuat siswa di pertemuan pertama

Kegiatan Belajar Mengajar

Gambar dari siluet bangun yang telah dibuat siswa pada pertemuan pertama akan digunakan setelah Kegiatan pendahuluan berikut ini.

SESI 1 (20 MENIT)

6. Diskusi kecil
    - Dari kegiatan yang telah dilakukan pada pertemuan sebelumnya, mungkin siswa akan menyadari beberapa potongan berbeda yang harganya sama.
    - Guru membagikan LKS siswa untuk pertemuan 2, dan menjelaskan mengenai permasalahan yang ada di LKS.



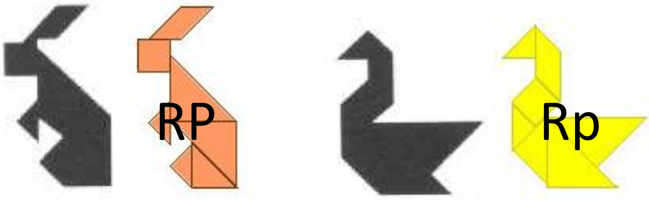
    - Guru harus memastikan bahwa siswa cukup jelas dengan permasalahannya.
    - Di sini, kita harapkan siswa menemukan konsep konservasi luas dalam skala kecil (potongan-potongan persegi, segitiga, dan jajaran genjang). Siswa
    - diharapkan melakukan diskusi secara berpasangan untuk membahas mengapa 3 potongan berbeda tersebut berharga sama.

Kegiatan	**Peran Guru**
Menjelaskan permasalahan	Guru menjelaskan permasalahan sejelas mungkin. Guru meminta siswa untuk mendiskusikan dengan pasangannya.
Diskusi berpasangan antar siswa	Guru diharapan menunggu respon siswa dalam memecahkan masalah tersebut Guru memantau diskusi masing-masing pasangan jika ada yang memerlukan bimbingan.
Diskusi Kelas: Siswa akan menyelesaikan permasalahan dengan menggunakan beberapa strategi berikut ini: 1. Siswa memotong dan menempel tangram saat membandingkan potongannya 2. Siswa akan memperhitungkan juga bahwa bangun-bangun itu dapat dipecah menjadi 2 segitiga	Guru sebaiknya membimbing diskusi ke ide dekomposisi (potongan-potogan tersebut tersusun dari 2 potongan) pada 2 segitiga yang dapat dikenal juga sebagai konservasi luas *“dapatkah kamu membandingkan persegi/ jajargenjang/ segitiga sedang dengan segitiga kecil? Apa yang kamu peroleh?”* Menyimpulkan bahwa adanya kemungkinan bentuk yang berbeda namun harganya sama
REVISI Siswa tidak memotong bangun yang ada tapi melipatnya sehingga mengetahui jika setengahnya menjadi segitiga kecil	

SESI 2 (45 MENIT)

7. Menghitung Harga Bangun Masing-masing yang telah Dibuat (INDIVIDU)

- Gambar, sketsa, atau siluet yang telah dibuat pada petemuan pertama digunakan disini.
- Tiap siswa akan membawa bangunnya masing-masing dan menaksir harga total masing-masing bangunya.



- Dalam menentukan harga dari tiap bangun yang terdiri dari beberapa potongan tangram, kecenderungan untuk aktivitas ini adalah siswa hanya akan menjumlahkan harga tiap potongan sebagai harga total.
- Jika ada siswa yang kesulitan, guru dapat meminta mereka untuk menjumlah satu persatu harga masing-masing potongan tangram yang ada dan memastikan bahwa perhitungannya tepat.
- Strategi lain adalah siswa memahami bahwa jika mereka menggunakan semua potongan dalam 1 set tangram tersebut adalah 40.000,
- Atau jika mereka menggunakan semuanya atau menggunakan 6 potongan tangram saja, maka harganya sama atau 40.000 dikurangi potongan yang tidak digunakan.
- Guru diharapkan membimbing siswa untuk menentukan potongan mana yang harganya harus dikurangkan untuk mendapatkan harga total dari bangun tersebut (Part Whole relation).

Kegiatan	Peran Guru
Siswa menaksir harga bangunnya masing-maisng menggunakan strateginya masing-masing. Berikut adalah contoh strategi yang dilakukan: 1. Menjumlah harga masing-masing potongan untuk memperoleh harga total 2. Menggunakan fakta bahwa total 1 set tangram berharga Rp 40.000,-. 3. Jika mereka tidak menggunakan semua potongan dalam 1 set maka hanya mengurangi Rp	Jika siswa kesulitan mengenai apa yang akan dilakukan, guru dapat memancing siswa dengan menanyakan "*Jika kamu penjual bangun ini, berapa yang harus saya bayar untuk bangun ini?*" Guru sebaiknya membimbing siswa untuk menentukan potongan mana yang harus dikurangi harganya (Part whole relation). Selanjutnya, kombinasi 2 strategi bisa mempermudah siswa menyelesaikan permasalahan ini dengan menanyakan "*Bisakah kamu gunakan strategi lain?*" Guru sebaiknya memastikan bahwa setiap siswa mempersiapkan bangunnya yang akan

40.000,- dikurangi harga potongan yang tak digunakan	dipamerkan di sesi terakhir.
Jika mereka lupa untuk menyusun ulang potongan-potongan tangram pada gambar sketsa atau siluet yang telah dibuat, guru memberikan siswa waktu untuk memikirkannya lagi pasti ada bagian-bagian yang diingat. Kemudian guru bisa melanjutkan kegiatan yg selanjutnya.	
REVISI Siswa ternyata lebih cenderung menghitung harga dari tiap bangun dengan strategi pertama, yaitu menjumlah harga dari masing-masing potongan penyusun bangun tersebut. Namun, untuk hipotesis kedua dan ketiga, diharapkan siswa juga dapat memahami strategi ini.	

1. Pameran Sketsa/ Siluet Bangun.

   (DISKUSI KELAS)

   - Tiap siswa akan menampilkan gambar bangunnya yang telah diberi harga dengan menempelkannya di depan kelas.
   - Dengan saling membandingkan harganya, guru dan siswa mencoba mengelompokkan bangun-bangun mana yang memiliki harga yang sama secara bersama-sama.
   - Setelah mereka selesai mengelompokkannya, diskusi sesuai dengan situasi tersebut akan dimulai dengan bimbingan guru untuk memahami akan adanya hubungan antara harga dan ukuran bangun (luas, walaupun belum sampai pada pengenalan kata luas)

Kemungkinan-kemungkinan yang terjadi selama diskusi akan sangat terbuka. Di sini, kita dapat mengarahkan diskusi siswa pada penggunaan konsep harga dalam memahami konsep luasan. Berikut beberapa kemungkinan tema diskusi yang bisa diarahkan oleh guru:

1. Bentuk berbeda tapi harga sama?

   Dalam hal ini, diharapkan siswa akan memikirkan perlunya akan adanya sesuatu yang tetap yang membuat harga dari berbagai bentuk yang berbeda tersebut memiliki harga yang sama.
2. Bagaimana menentukan bentuk mana yang memiliki harga paling mahal dan paling murah?

   Dengan menggunakan topik ini dalam diskusi, guru diharapkan mengarahkan pemahaman siswa untuk memikirkan sebuah konsep yang menentukan bangun mana yang memiliki harga lebih mahal dari bangun lainnya tanpa harus menghitung harga masing-masing potongan tangram.

3. Semua bangun yang ditampilkan memiliki harga yang sama.

   Keadaan ini terjadi dimana semua bangun memiliki harga sama baik bangun yang sama maupun berbeda bentuk. Apa yang dapat didiskusikan pada bagian ini adalah mengapa semua bangun tersebut memiliki harga sama? Apakah mereka memiliki karakteristik tertentu yang menentukan harga semuanya sama?

4. Bentuk atau ukuran berbeda namun jumlah potongan penyusunnya sama

   Topik cukup penting, bila tidak siswa akan cenderung menggunakan alasan bahwa harga beberapa bentuk itu sama karena jumlah potongan tangram penyusunnya juga sama. Padahal hal ini belum tentu benar kecuali potongan tangram yang digunakan identik/sama.

Di samping ukuran, siswa mungkin juga berpendapat bahwa material penyusun potongan tangram juga menentukan harganya.

Kegiatan	**Peran Guru**
Mengelompokkan bangun	Membimbing siswa dalam mengelompokkan bangun yang memiliki harga yang sama
Mendiskusikan topik pertama (jika memungkinkan) Bentuk berbeda tapi memiliki harga yang sama? 1. Menimbang 2 bangun berbeda berharga sama, siswa akan mencoba mencari karakteristik dari 2 bangun tersebut. 2. Selama diskusi, ada kemungkinan siswa membahas mengenai material yang digunakan.	1. Guru sebaiknya berhati-hati dalam membimbing siswa agar mereka dapat memahami tentang perlunya konsep "luas" di sini. Guru dapat bertanya "*bentuk dari bangun-bangun ini berbeda tapi harganya sama, menurutmu apa yang menyebabkannya demikian?"* 2. Guru dapat membimbing siswa untuk membayangkan jika mereka membuat potongan tangram itu dengan menggunakan potongan kayu. Maka harusnyalah jika semakin besar potongan tangram dari kayu tersebut harganya juga semakin mahal.
Mendiskusikan topik kedua (jika memungkinkan) Bagaimana menentukan bangun mana yang memiliki harga yang lebih mahal atau yang lebih murah? 1. Menimbang 2 bangun berbeda tetapi berharga sama, siswa akan mencoba mencari karakteristik dari 2 bangun tersebut yang dapat digunakan untuk menentukan bangun mana yang lebih	1. Guru sebaiknya berhati-hati dalam membimbing siswa sampai mereka dapat memahami tentang perlunya konsep "luas" di sini. Guru dapat bertanya "*bentuk dari bangun-bangun ini berbeda tapi harganya sama, menurutmu apa yang menyebabkannya demikian?"* 2. Guru dapat membimbing siswa untuk membayangkan jika mereka membuat

<table>
<tr><td>mahal atau murah tanpa menghitung harga potongannya satu persatu
2. Selama diskusi, ada kemungkinan siswa membahas mengenai material yang digunakan.</td><td>potongan tangram itu dengan menggunakan potongan kayu. Maka seharusnya jika semakin besar potongan tangram dari kayu tersebut harganya juga semakin mahal.</td></tr>
<tr><td>Mendiskusikan topik ketiga (jika memungkinkan)
Semua bangun harganya sama.
Di sini, guru memperkirakan siswa akan memunculkan ide tentang luas/area. Kemudian, guru bisa langsung melanjutkan ke Kegiatan selanjutnya. Namun, jika siswa masih belum bisa memunculkan ide tentang luas jangan dipaksa, karena akan ada Kegiatan selanjutnya untuk membantu siswa memahamkan konsep ini.</td><td>Guru membimbing diskusi tentang mengapa bangun-bangun yang berbeda itu memiliki harga yang sama. Diskusi diusahakan membahas tentang karakteristik apa yang membuat bangun-bangun berbeda itu memiliki harga yang sama.</td></tr>
<tr><td>Mendiskusikan topik keeempat (jika memungkinkan)
Ukuran dan bangun yang berbeda namun memiliki jumlah potongan sama.
Siswa kemungkinan bisa salah dalam memahami bahwa bangun yang berbeda tidak selalu sama walaupun potongan penyusunnya berjumlah sama karena potonganya bisa berbeda.</td><td>Guru dapat memberikan ide dari kegiatan sebelumnya ketika mereka berurusan dengan bangun-bangun yang berbeda tidak akan selalu sama jika jumlah potongan tangram yang dibentuk sama selama ukuran potongannya diperhitungkan.
Guru dapat menunjukkan lagi Kegiatan sebelumnya mengenai menyusun bangun dengan potongan-potongan yang berjumlah sama tapi menghasilkan bangun-bangun yang berbeda.
“jika jumah potongan tidak mempengaruhi harga, menurutmu, apa yang menentukan harganya?”</td></tr>
<tr><td colspan="2">REVISI
➢ Siswa mampu memahami hubungan antara harga dan luasan dari suatu bangun. Semakin murah harganya, maka semakin kecil luasannya.
➢ Lebih jauh lagi, alasan mereka mengapa dua bangun memiliki harga sama adalah karena dua bangun tersebut terdiri dari potongan tangram yang sama. Guru diharapkan mengarahkan siswa agar tidak menggunakan alasan ini, namun bombing siswa agar dapat memahami jika harganya sama karena luas bangunya sama (ini yang penting)
➢ Dari sini, mungkin kita bisa memulai diskusi dari alasan siswa di atas yaitu harga sama karena potongn penyusunnya sama. Kemudian kita bisa mengembangkan diskusi kepada topic yang lain.</td></tr>
</table>

PENUTUP (5 MENIT)

- Guru merefleksi kegiatan yang telah dilakukan hari ini.

- Guru sebaiknya mengarahkan siswa bahwa harga dari suatu bangun berhubungan dengan luas dari potongan penyusunnya atau setidaknya siswa mendapatkan sedikit ide tentang hal tersebut.
- Namun, guru tidak terlalu memaksa siswa untuk memahami
- Kegiatan ini secara jelas, sehingga siswa dapat membangun sendiri konsepnya karena masih ada kegiatan-kegiatan penguat selanjutnya.
- Berikut bisa dijadikan pertanyaan untuk membimbing siswa di akhir pelajaran.

  “parameter apa yang menentukan harga dari berbagai macam bangun tersebut”
  “dapatkah kamu melihat hubungannya dengan ukurannya” atau jika siswa masih kesulitan untuk memahami hubungan atara ukuran dan harganya, guru bisa menanyakan *“Apakah ada hubungan antara ukuran bangun itu dengan harganya?”*

Di akhir kegiatan, kami berharap siswa akan mendapat pemahaman tentang ide mengenai luas/area yang digunakan sebagai parameter penentu bangun mana yang lebih murah atau lebih mahal. Atau, dalam kata lain, siswa memahami hubungan antara harga dan potongan tangram yang digunakan, atau lebih jauh lagi, hubungan antara harga dan ukuran dari suatu bangun.

Guru dapat menyusun ulang kesimpulan dari kegiatan hari itu berdasarkan hasil diskusi siswa di akhir sesi ini. Guru juga bisa meminta siswa untuk menyimpulkan dengan menggunakan kata-kata mereka sendiri.

Panduan Guru

Pertemuan 3

MEMBANDINGKAN BENTUK DISEDIAKAN

Identitas:

Satuan Pendidikan : Sekolah Dasar
Mata Pelajaran : Matematika
Kelas/Semester : III/ semester 2
Alokasi Waktu : 2x35 menit

Standar kompetensi:

5. Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

Kompetensi dasar

5.2. Menghitung luas persegi dan persegi panjang

Tujuan Pembelajaran:

- Siswa dapat memahami hubungan antara harga dan ukuran bangun-bangun yang bentuknya berbeda namun harganya sama.
- Siswa dapat membandingkan bangun yang diberikan menggunakan ide luas/area atau harganya
- Siswa dapat mempertahankan pendapatnya dalam diskusi mengenai strategi yang siswa gunakan untuk membandingkan bangun-bangun tersebut

Bahan Ajar:

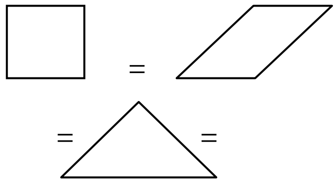
- Beberapa set Tangram (tergantung pada jumlah murid di kelas tersebut)
- Lembar Kerja Siswa untuk pertemuan 3
  Persediaan potongan tangram bentuk persegi dan segitida untuk Kegiatan mengukur jika diperlukan (sesi 2)

Kegiatan Belajar Mengajar

SESI 1 (15 MENIT)

8. Diskusi kecil

Dari kegiatan yang telah dilakukan pada pertemuan sebelumnya, mungkin siswa akan menyadari beberapa potongan berbeda yang harganya sama. Berikut potongannya.



Di sini, kita harapkan siswa menemukan konsep konservasi luas dalam skala kecil (potongan-potongan persegi, segitiga, dan jajaran genjang). Siswa diharapkan melakukan diskusi secara berpasangan untuk membahas mengapa 3 potongan berbeda tersebut berharga sama.

<table>
<tr><th>Kegiatan</th><th>Peran Guru</th></tr>
<tr><td>Menjelaskan permasalahan</td><td>Guru menjelaskan permasalahan sejelas mungkin.
Guru meminta siswa untuk mendiskusikan dengan pasangannya.</td></tr>
<tr><td>Diskusi Kelas:
Siswa akan menyelesaikan permasalahan dengan menggunakan beberapa strategi berikut ini:
3. Siswa memotong dan menempel tangram saat membandingkan potongannya
4. Siswa akan memperhitungkan juga bahwa bangun-bangun itu dapat dipecah menjadi 2 segitiga</td><td>Guru sebaiknya membimbing diskusi ke ide dekomposisi (potongan-potogan tersebut tersusun dari 2 potongan) pada 2 segitiga yang dapat dikenal juga sebagai konservasi luas
“dapatkah kamu membandingkan persegi/ jajargenjang/ segitiga sedang dengan segitiga kecil? Apa yang kamu peroleh?”
Menyimpulkan bahwa adanya kemungkinan bentuk yang berbeda namun harganya sama</td></tr>
<tr><td colspan="2">REVISI
Siswa tidak memotong bangun yang ada tapi melipatnya sehingga mengetahui jika setengahnya menjadi segitiga kecil</td></tr>
</table>

SESI 2 (45 MENIT)

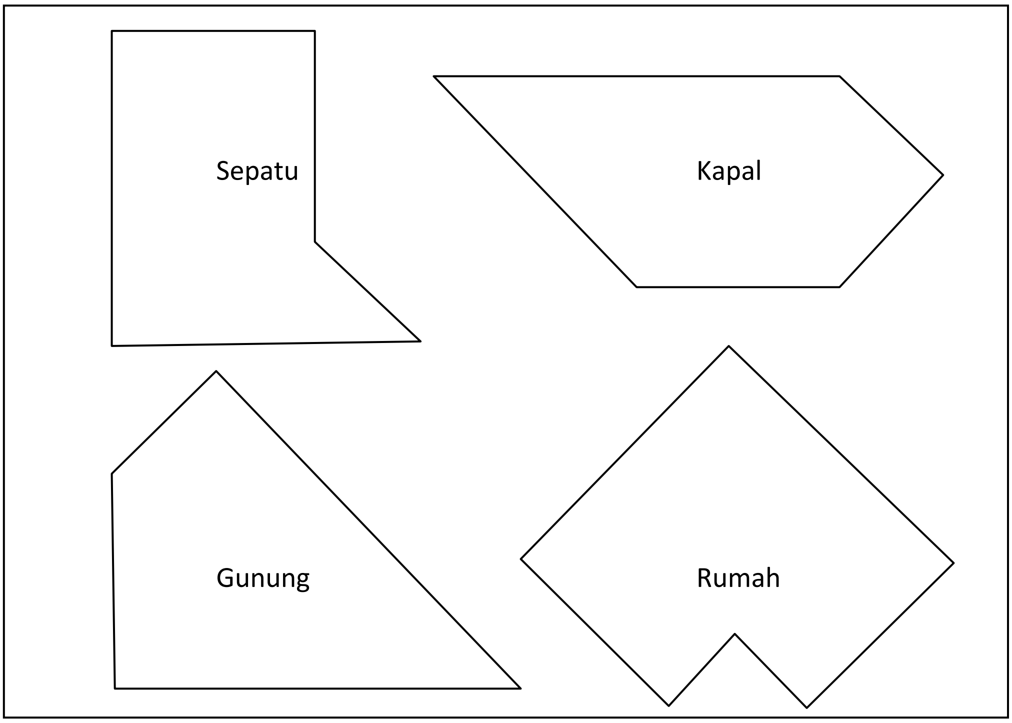
1. Membandingkan Bangun yang Diberikan

   (DISKUSI GRUP)

   Pada awal kegiatan pembelajaran, guru mengingatkan kembali tentang topik yang telah dipelajari pada pertemuan sebelumnya yaitu “bangun milik siapa yang paling mahal”. Guru sebaiknya lebih menekankan masalah yang telah dibahas pada diskusi sebelumnya seperti “*Apakah hubungan antara harga dan ukuran dari suatu bangun?*”

   Seperti pada kegiatan sebelumnya, guru membagi siswa dalam kelompok yang terdiri dari 3-4 orang namun pada kali ini siswa akan bermain dengan bangun yang telah disediakan oleh guru dan mendiskusikannya dalam kelompok. Guru menggunakan bangun-bangun yang telah didesain serealistis mungkin untuk dapat diselesaikan oleh siswa sesuai waktu yang telah diberikan dan tingkat kesulitan yang sesuai. Berikut adalah bentuk dari bangun-bangun yang digunakan:

$$> ?, < ?, = ?$$



Pada kegiatan ini, siswa akan membandingkan bangun-bangun yang telah diberikan diatas. Diharapkan siswa menemukan ada 2 bangun yang memiliki luas yang sama. Jika tidak, maka guru yang akan menekankannya pada diskusi selanjutnya (diskusi kelas).

Aktivitas	**Peran Guru**
Diskusi Grup	Guru sebaiknya membimbing siswa untuk tetap pada topik bahasan yang benar yaitu membandingkan bangun-bangun yang tepat. Jika siswa masih belum tahu apa yang harus dilakukan, guru bisa memancing dengan memberikan pertanyaan "*dapatkah kamu gunakan tangram untuk* membandingkannya", "*bagaimana harganya?*" Jika siswa telah selesai membandingkan bagun-bangun tersebut, guru dapat memberikan saran "*dapatkah kamu menggunakan strategi lain untuk membandingkan tanpa menggunakan harga?*", "*apa yang kamu temukan mengenai luas/area-nya?*" Guru harus memberikan penghargaan terhadap hasil pemikiran siswa sendiri untuk menyelesaikan masalah ini, jangan terlalu memaksakan jawaban yang benar, tapi dari jawaban siswa yang kurang tepat, guru dapat menggiringnya ke jawaban yang benar. Guru juga sebaiknya menambah pertanyaan-pertanyaan yang melatih siswa untuk berfikir kritis seperti "*Kenapa kamu gunakan*

	cara ini?”,”*dapatkah kamu menjelaskan kenapa bisa menggunakan cara ini bukan yang itu?*”, atau “*dapatkah kamu menjelaskan ke temanmu apa yang telah kamu kerjakan?*”. Selain itu guru membimbing siswa untuk benar-benar berdiskusi dan jangan sampai mereka hanya mengikuti 1 orang. Jika siswa mebandingkan bangun dengan saling menumpuk, guru bisa membantu mereka jika mereka kesulitan dengan memancingnya dengan “*Dapatkah kamu memikirkan sesuatu dari tangram*”, “*Apa yang kamu lakukan dengan bagian yang berlebih ini*” Ketika berkeliling, guru dapat memperhatikan kelompok mana yang menarik jika hasil kerjanya disajikan di presentasi kelas, siapa yang saling menumpuk dan siapa yang membuat struktur tangram.

2. Pameran Poster dan Presentasi

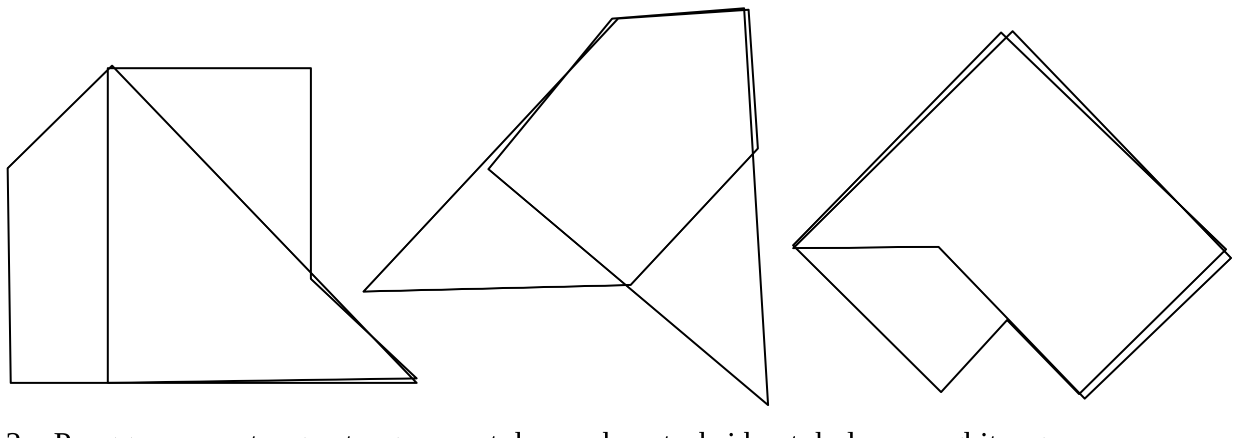
   (DISKUSI KELAS)

   - Guru memimpin diskusi kelas membahas hasil diskusi kelompok membandingkan bangun.
   - Guru memilih 1 hasil jawaban kelompok dan membahasnya di depan kelas.
   - Guru menempel 4 bangun berebda yang dibandingkan di papan tulis dan membandingkan ukurannya. Ada 2 solusi yang mungkin akan siswa gunakan

   Berikut perkiraan alternatif jawaban yang akan digunakan siswa:

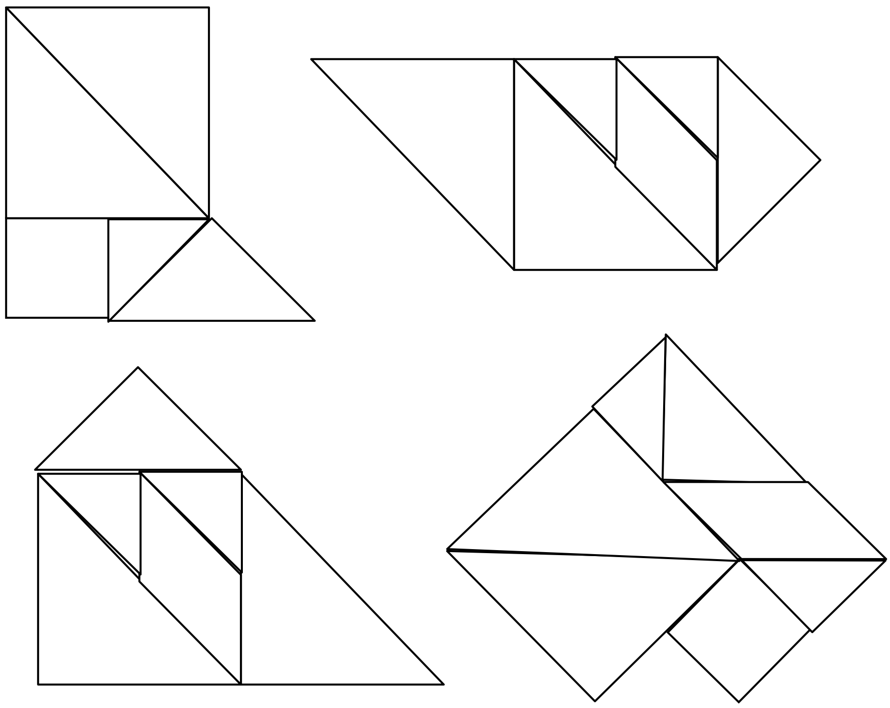
   1. Mengestimasi bangun dengan menggunakan strategi saling menumpuk.

      Guru mengantisipasi ide estimasi disini. Beberapa siswa mungkin melihat bahwa bangun-bangun tersebut mirip di bagian tertentu. Jadi, siswa akan mencoba untuk membandingkan bagian-bagian yang mirip tersebut dan menumpuk satu bangun ke bangun lainnya dan melihat selisih bangun yang terjadi.



   2. Penggunaan potongan tangram untuk merekonstruksi bentuk dan menghitung harganya untuk saling membandingkan.

Berikut contoh bagaimana siswa menyusun potongan tangram pada bangun.



Kemudian siswa akan menghitung harga dari masing-masing bangun dan mengurutkan bangun mana yang harganya sama, paling murah atau yang paling mahal.

Pada saat presentasi, kesimpulan utama sebaiknya adalah bahwa ide menyusun dan menyusun ulang bangun-bangun tersebut, baik dengan menggunakan 2 kemungkinan baik menumpuk atau menggunakan potongan tangram, tidak akan merubah luas selama mereka menggunakan komposisi yang sama.

Aktivitas	Peran Guru
Presentasi Grup Terdapat 2 topik utama jawaban dari permasalahan yang dapat muncul di tiap presentasi. Berikut respon yang bisa diberikan sesuai solusi yang dipresentasikan siswa.	Guru memilih kelompok mana yang akan mempresentasikan jawaban telah dipilih selama diskusi kelompok sebelumnya
1. Ketika siswa menggunakan strategi menumpuk. Membandingkan dengan cara demikian, siswa akan menemukan susunan yang tepat. Kenyataan bahwa siswa tidak lagi membutuhkan harga dar tiap bangun untuk membanding sangat penting untuk memulai diskusi tentang luas suatu bangun	Di sini, guru sebaiknya membimbing siswa untuk menyimpulkan bahwa parameter yang dapat digunakan untuk menentukan harga bangun mana yang lebih mahal tanpa harus menghitung harganya dan juga ide tentang konservasi luas pada perbandingan bangun. Untuk hrga dari tiap bangun dapat dibahas di isu kedua. Ide utama adalah bahwa ketika siswa dapat memahami bagun-bangun tersebut dapat disusun ulang satu sama lain dengan

	memperhatikan bagian berlebih yang sebagai selisih namun ada bagian yang luasnya tetap sama
2. Penggunaan potongan tangram untuk merekonstruksi dan kemudian menggunakannya sebagai pembanding Siswa akan menghitung harga masing-masing bangun dengan bantuan harga dari masing-masig potongan tangram dan menggunakannya untuk membandingkannya. Konsep luas diharapkan muncul juga. Namun, siswa mungkin juga menggunakan alasan bahwa bangun tersebut tersusun atas potongan yang sama bukan membahas luas dari bangun tersebut.	Jika diskusi mengenai luas dari bangun tidak muncul, guru sebaiknya membimbing siswa untuk memikirkan perlunya parameter yaitu luas untuk menentukan harganya sama atau tidak. Guru juga dapat meminta siswa untuk membandingkan bangun dengan cara lain seperti pada cara 1, kemudian diskusi mengikuti alur yang sama. Ide utama di sini adalah bahwa ketika siswa dapat memahami penyusunan ulang potongan-potongan tangram tersebut tidak merubah luas bangun tersebut selama menggunakan potongan yang sama.
Kesimpulan dari Diskusi.	Guru sebaiknya menyimpulkan dari 2 strategi tersebut bahwa potongan lebih manapun yang dapat ditambahkan atau dilepas, akan menuju ke ide bahwa penyusunan ulang bangun satu ke bangun lainnya tidak merubah luasnya.

PENUTUP (5 MENIT)

Di akhir aktivitas ini, guru dan siswa merefleksi kegiatan-kegiatan yang telah dilakukan di hari ini. Guru membimbing siswa untuk memahami dan menerima bahwa setiap bangun memiliki apa yang kita sebut luasan yang dapat digunakan sebagai penentu bangun mana yang lebih besar atau kecil. Sebagai hasilnya, mereka dapat memahami mengenai luas tersebut yang jika disusun ulang maka luasnya akan tetap sama (konservasi luas) dari kegiatan.

Berikut, pertanyaan yang mungkin guru tanyakan ke siswa.

"jika saya memotong bagian ini dan memasangkannya di sisi lain, apakah luasnya tetap sama?", "dapatkah kamu memikirkan bangun lain yang memiliki luasa sama dengan bangun ini"

Jika siswa menjawab menggunakan tangram tidak masalah

Guru dapat merumuskan ulang apa yang menjadi kesimpulan dari yang siswa bahas di sesi ini.guru juga bisa meminta siswa sendiri untuk menyimpulkanya. **Hal yang penting untuk siswa capai di tahap ini adalah siswa tidak lagi perlu menggunakan harga untuk membandingkan bangun mana yang lebih besar atau mengapa bangun ini dan lainnya memiliki luasan yang sama.**

Panduan Guru
Pertemuan 4
MEMBANDINGKAN BENTUK DISEDIAKAN

Identitas:

Satuan Pendidikan : Sekolah Dasar

Mata Pelajaran : Matematika

Kelas/Semester : III/ semester 2

Alokasi Waktu : 2x35 menit

Standar kompetensi:

5. Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

Kompetensi dasar

5.2. Menghitung luas persegi dan persegi panjang

Tujuan Pembelajaran:

- Siswa dapat menggunakan konsep konservasi luas untuk menyusun ulang suatu bangun ke bangun lain yang luasnya sama
- Siswa dapat menggunakan konsep konservasi luas untuk menentukan luasan sebuah bangun.

Bahan Ajar:

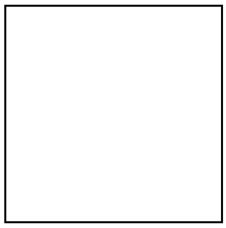
- Beberapa set Tangram (tergantung pada jumlah murid di kelas tersebut)
- Lembar Kerja Siswa untuk pertemuan 4 beserta bangun yang akan digunakan di LKS

Kegiatan Belajar Mengajar

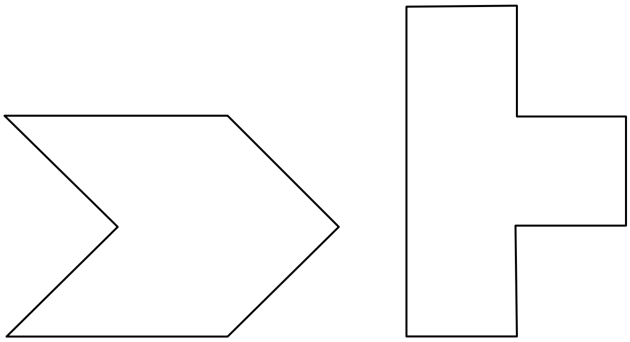
SESI 1 (20 MENIT)

3. Menyusun Ulang Bangun Menjadi Persegi
   (DISKUSI BERPASANGAN)
   - Pada awal kegiatan pembelajaran, guru mengingatkan kembali tentang topik yang telah dipelajari pada pertemuan sebelumnya yaitu “Bagaimana menentukan bangun mana yang paling mahal”.

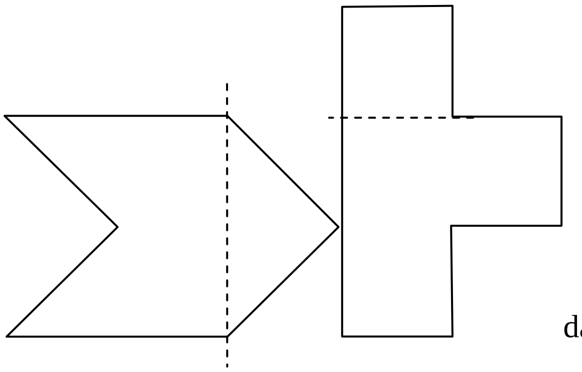
- Guru sebaiknya lebih menekankan masalah yang telah dibahas pada diskusi sebelumnya seperti "*Apakah hubungan antara harga dan ukuran dari suatu bangun?*"
- Guru membagi siswa berpasangan dalam satu bangku untuk mendiskusikan masalah di LKS. Permasalahan yang akan dibahas mengenai bagaimana menyusun ulang sebuah bangun menjadi sebuah persegi



- Berikut ini bangun-bangun yang harus siswa susun ulang menjadi persegi di atas.



- Pada kegiatan ini, diharapkan siswa dapat menemukan bagaimana memotong dan menyusun ulang bangun gambar di atas. Jika dalam 10 menit siswa masih kesulitan, maka guru membagikan potongan bangun yang sama dengan gambar bangun di atas dan meminta siswa bekerja menggunakan gunting pada bangun tersebut.
- Diharapkan siswa akan memotong sebagai berikut



dan menyusunya menjadi persegi

-

Aktivitas	Peran Guru

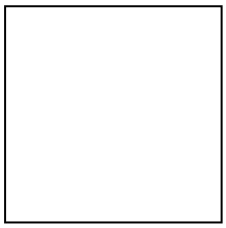
Diskusi berpasangan Siswa diberikan LKS dan bekerja menyusun ulang gambar yang diberikan	Guru membimbing siswa untuk menyelesaikan permasalahan. Guru bisa menyarankan *“ coba kalian bayangkan jika kalian menggunting bangun ini, bagaimana kalian harus mengguntingnya supaya bisa kalian susun ulang menjadi bangun persegi ini”*
Jika masih kesulitan, guru member siswa potongan bangun yang sesuai dengan bangun yang harus disusun ulang	Guru membimbing siswa menggunakan gunting untuk memotong bagian yang tepat. Sebelumnya guru harus memastikan bagian yang akan dipotong tepat. Guru bisa menanyakan *“dengan potongan seperti ini apakah kamu yakin akan bisa menjadi sebuah persegi”*

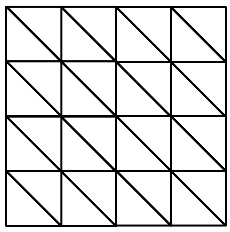
SESI 2 (40 MENIT)

4. Menutup Bangun dengan Menggunakan Satu Jenis Potongan Tangram

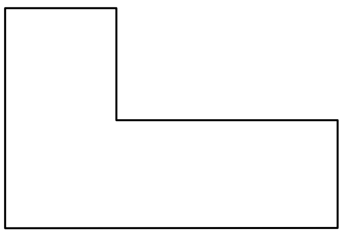
   (DISKUSI KELOMPOK)

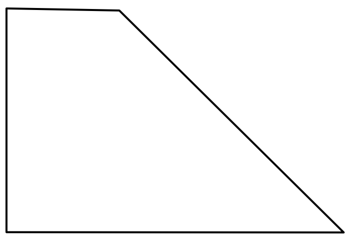
   - Guru membagikan LKS yang kedua dan meminta siswa untuk mendiskusikan permasalahan secara berkelompok.
   - Pertama-tama, guru meminta siswa menghitung berapa segitiga yang digunakan pada persegi.





   - Kemudian, guru menekankan pada siswa untuk menggunakan bangun diatas sebagai acuan, berapa segitiga kecil (seperti pada potongan segitiga di atas) yang dibutuhkan untuk menutup bangun di bawah ini



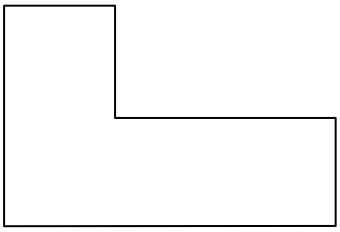
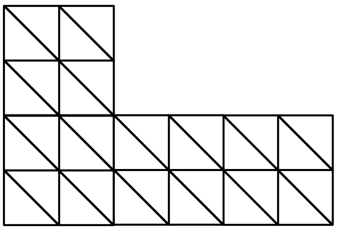


   - Siswa diminta menyelesaikan permasalahan di atas dan menuliskan hasil kerja mereka pada poster yang diberikan
   - Kemudian, jika waktu masih cukup, guru akan memilih beberapa kelompok yang jawabanya menarik untuk mendiskusikannya di depan kelas.

Berikut perkiraan alternatif jawaban yang akan digunakan siswa:

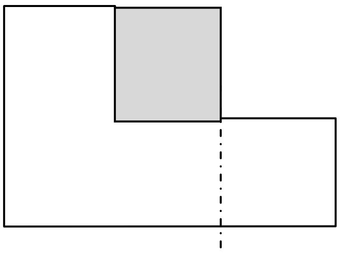
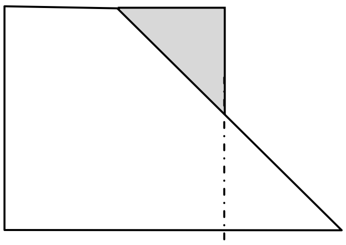
3. Siswa akan menggambar segitiga2 kecil pada bangun yang ditanyakan.

Guru mengantisipasi ide ini dengan meminta siswa mengingat kembali kegiatan sebelumnya dah apakah kita bisa menyusun ulang suatu bangun menjadi bangun lain pada kasus ini.

4. Siswa mencoba meyusun ulang bangu dan menemukan ternyata bangun-bagun tersebut dapat disusun ulang menjadi sebuah persegi.

   Di sini, guru hanya memastika siswa menyusun ulang atau memotong bangun pada bagian yang tepat. Sehingga bisa terlihat dengan jelas bahwa bangun-bangun tersebut memiliki luas yang sama dengan persegi yang telah diberikan sebelumnya.

Pada saat presentasi, kesimpulan utama sebaiknya adalah bahwa ide menyusun dan menyusun ulang bangun-bangun tersebut tidak akan merubah luas selama mereka menggunakan komposisi yang sama. Sehingga mereka dapat menggunakannya untuk mengukur luas suatu bangun dengan lebih mudah yaitu dengan mengubahnya ke bentuk bangun yang lebih mudah diukur luasnya

Aktivitas	Peran Guru
Membagikan LKS dan menjelaskan permasalahan kepada siswa	Guru bersama-sama siswa menghitung dahulu berapa segitiga yang ada di gambar. Kemudian guru memastikan siswa memahami tugas pada LKS ini. Guru bisa menanyakan *"apa yang harus kalian lakukan?", "kira-kira menurut kalian berapa segitiga yang diperlukan untuk bangun ini?"*
Siswa mendiskusikan permasalahan secara berkelompok dan menuliskan jawaban di poster	Guru berkeliling dan membimbing diskusi kelompok agar menemukan metode yang logis. 1. Menggambar segitiga di dalam bangun 2. Menyusun ulang bangun menjadi persegi Jika siswa kesulitan, guru bisa menanyakan *"coba kamu bandingkan dengan kegiatan sebelumnya, bisakah kamu gunakan metode yang sama?"*

Diskusi kelas	guru akan memilih beberapa kelompok yang jawabanya menarik untuk mendiskusikannya di depan kelas. Guru memastikan jawaban siswa logis. Guru bisa menanyakan *“bagaimana kamu bisa memperoleh cara seperti itu?” “ apakah semuanya setuju dengan pendapat kelompok ini”*

PENUTUP (5 MENIT)

Di akhir aktivitas ini, guru dan siswa merefleksi kegiatan-kegiatan yang telah dilakukan di hari ini. Guru membimbing siswa untuk memahami bahwa kita dapat mengukur luas suatu bangun dengan menyusun ulang bangun itu menjadi bangun lain yang luasnya sama. Bangun tersebut tetp tidak berubah luasnya selama hanya disusun ulang tanpa memunculkan tumpukan atau lubang

Berikut, pertanyaan yang mungkin guru tanyakan ke siswa.

“jika saya memotong bagian ini dan memasangkannya di sisi lain, apakah luasnya tetap sama?”, “dapatkah kamu memikirkan bangun lain yang memiliki luasa sama dengan bangun ini”

Guru dapat merumuskan ulang apa yang menjadi kesimpulan dari yang siswa bahas di sesi ini.guru juga bisa meminta siswa sendiri untuk menyimpulkanya. **Hal yang penting untuk siswa capai di tahap ini adalah mereka dapat menggunakan konsep konservasi luas untuk mengukur luas suatu bangun dengan lebih mudah yaitu dengan mengubahnya ke bentuk bangun yang lebih mudah diukur luasnya.**

Tangram Abstract World Ma...
Michael Tompsett
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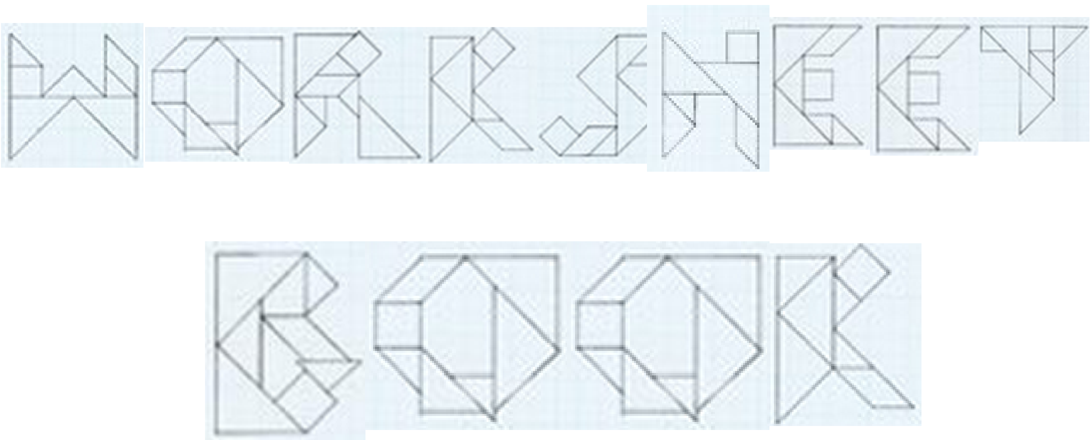
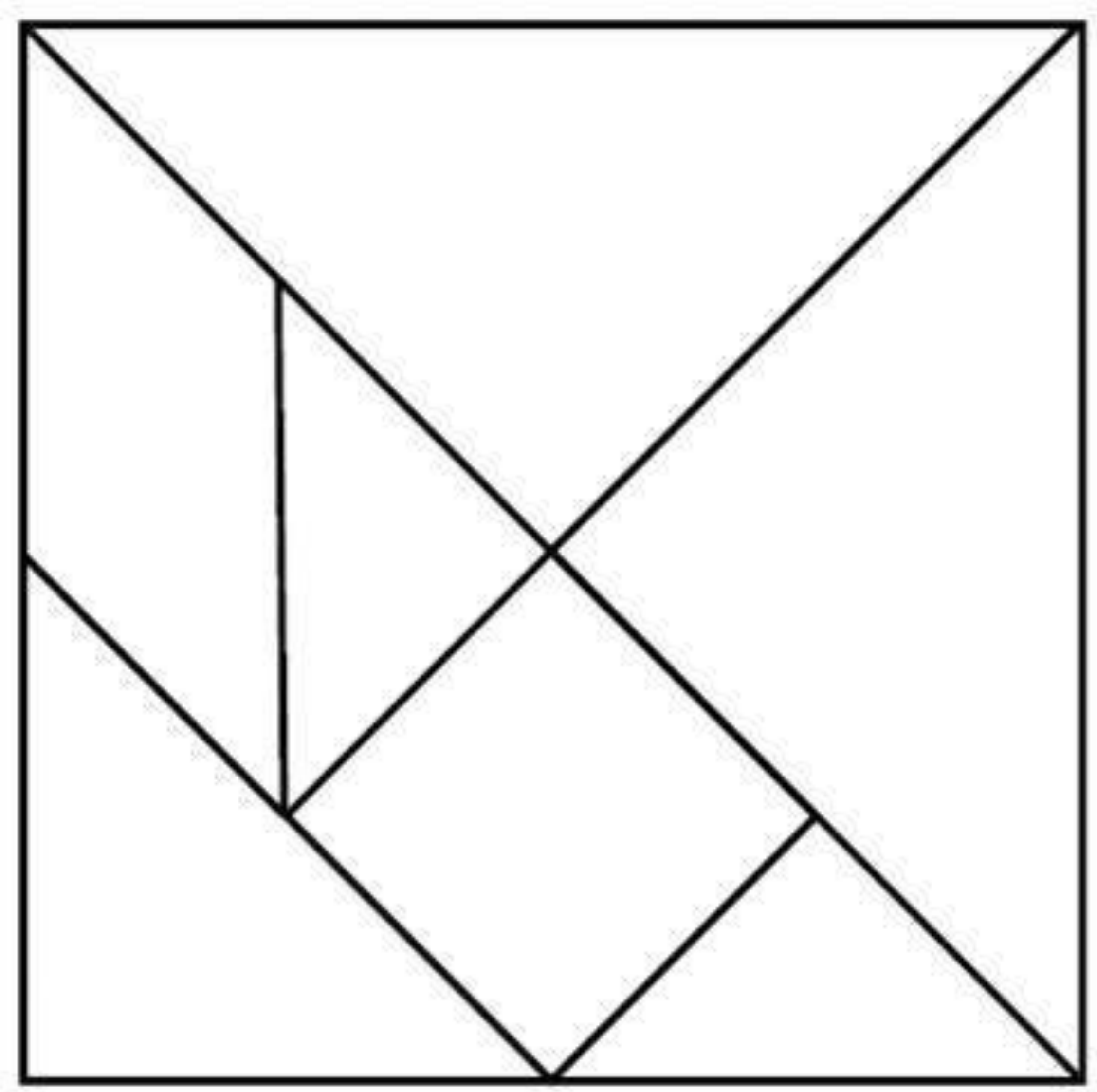


SET TANGRAM YANG DIGUNAKAN

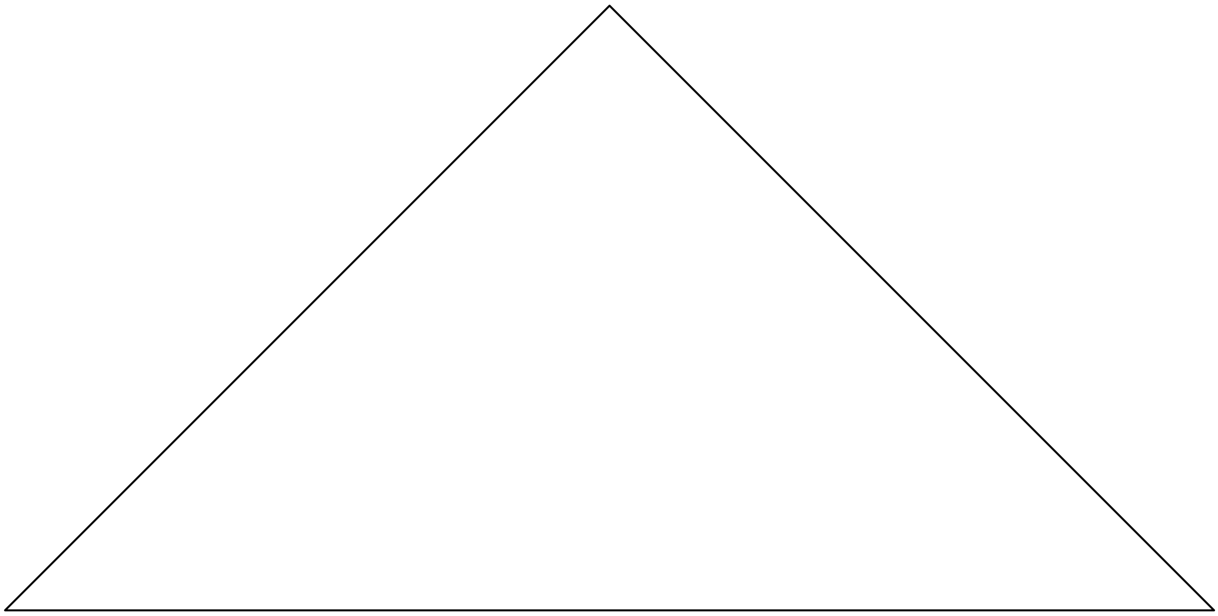


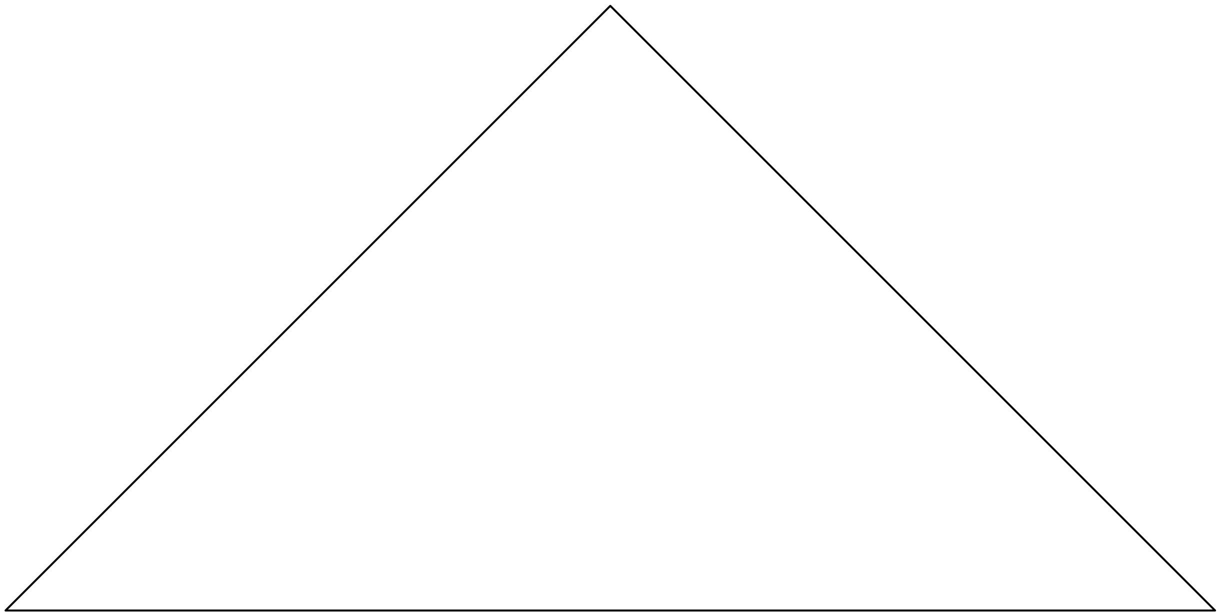
Pertemuan 1 – Lembar Kerja Siswa B Sesi 1

Nama :

Kelas :

Gunakan 3 potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat.



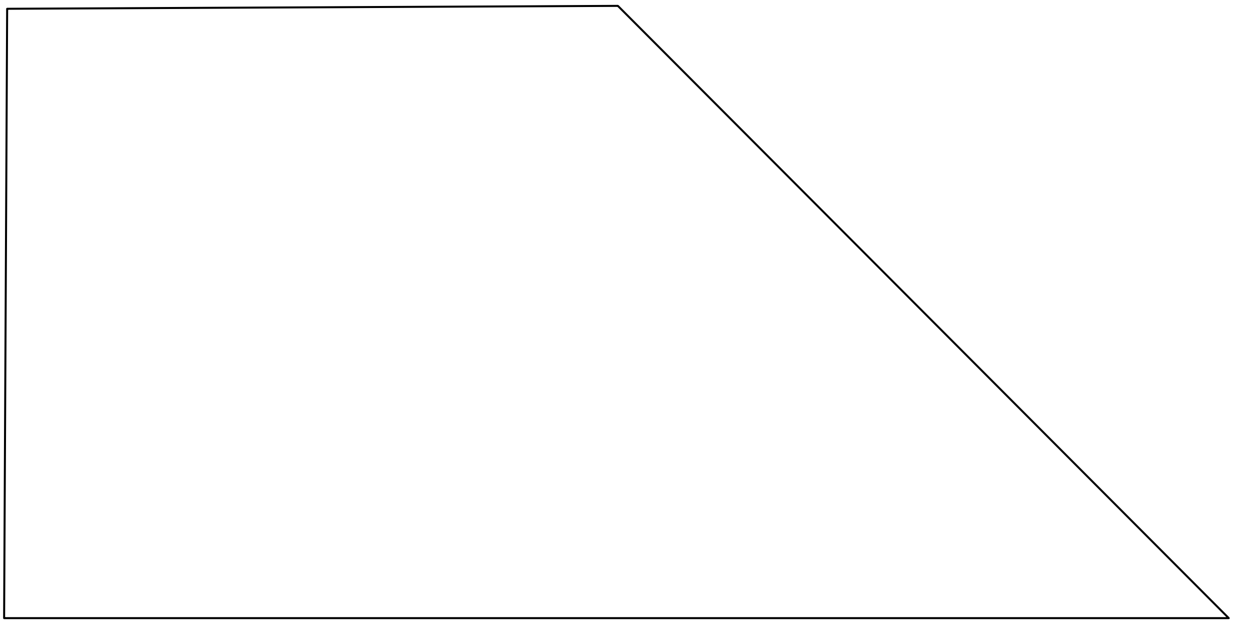


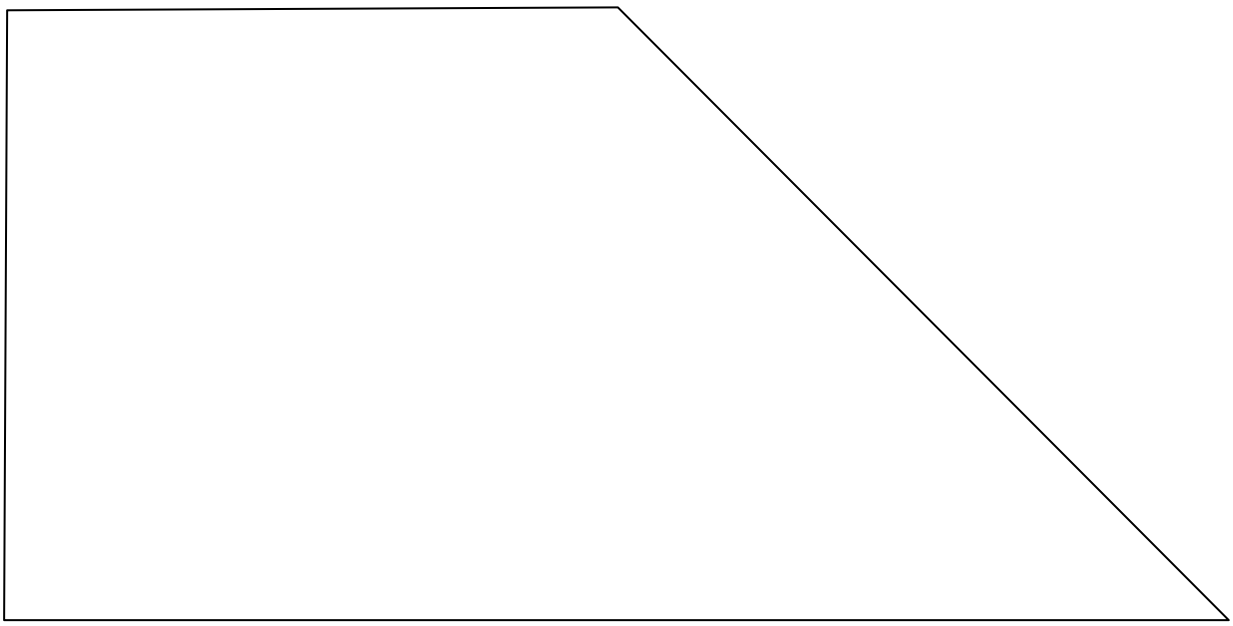
Pertemuan 1 – Lembar Kerja Siswa C Sesi 1

Nama :

Kelas :

Gunakan beberapa potongan tangram untuk menyusun bangun dibawah ini, kemudian gambarlah susunan yang telah kamu buat.





Pertemuan 1 – Lembar Kerja Siswa 0 Sesi 1

Nama :

Kelas :

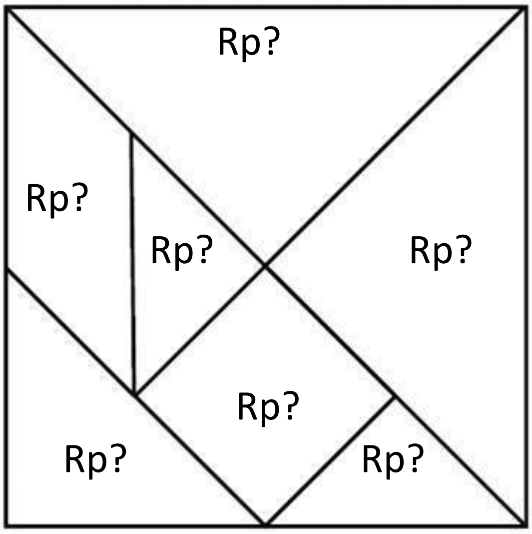
Bentuklah bangun yang kamu suka menggunakan satu set tangram yang tersedia, kemudian gambarlah bentuk yang kamu buat pada temapt berikut ini.

Pertemuan 1 – Lembar Kerja Siswa Sesi 2

Nama :
Kelas :

MENJUAL TANGRAM ECERAN

Kakak saya memiliki toko mainan. Salah satu mainan yang dijual di tokonya adalah puzzle tangram. Harga untuk satu set puzzle tangram adalah Rp 40.000,-. Kakak saya berencana menjual juga potongan tangram itu per buah, karena dia melihat setelah beberapa bulan banyak anak-anak yang memiliki puzzle tangram ini kehilangan potongan-potongan tangram nya, sehingga tidak bisa bermain dengan 1 set tangram yang lengkap. Bisakah kalian bantu kakak saya untuk menentukan harga untuk setiap potongan yang berbeda. Kakak saya akan berterimakasih sekali atas bantuan kalian.



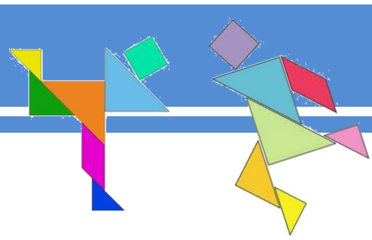
TUGAS KAMU

Dalam 1 kelompok, tentukan harga dari masing-masing potongan tangram. Kalian bebas menggunakan cara apa saja yang kalian suka, pastikan teman kamu mengerti dengan jawabanmu.

Kemudian, persiapkan 1 poster yang berisi jawaban kalian dan bagaimana cara kalian untuk menentukan jawaban tersebut. Buat poster sedemikian rupa sehingga teman kalian mengerti akan apa yang kalian maksud di poster ini

SELAMAT MENGERJAKAN
SEMANGAT☺

Pertemuan 2 – Lembar Kerja Siswa 1

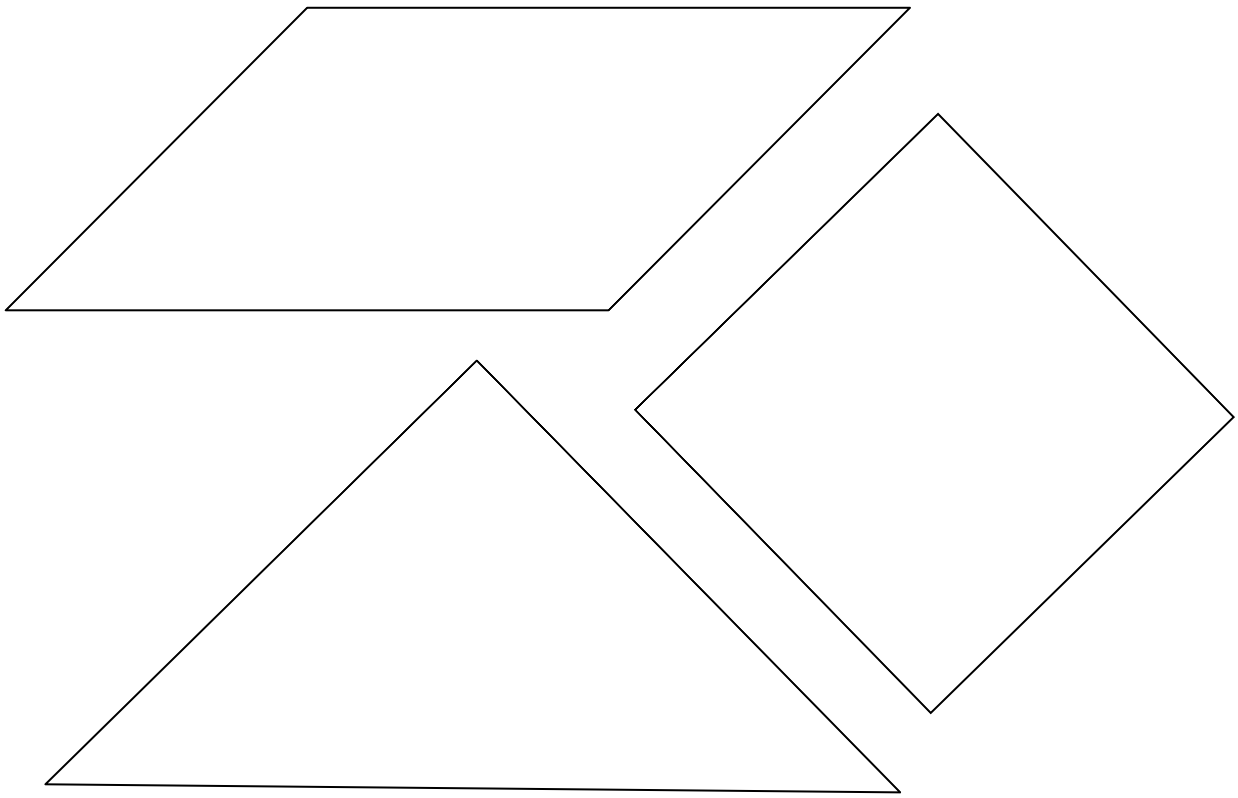


Nama :

Kelas :

MENGAPA HARGANYA SAMA?

Di bawah ini ada beberapa potongan tangram yang ternyata harganya sama. Mengapa bisa demikian? Bisakah kamu menjelaskan kepada teman sebangkumu? Kamu bisa mencorat-coret gambar berikut ini untuk menjelaskannya



Ketiga potongan itu berharga sama karena menurut saya ………………………………………

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Pertemuan 2 – Lembar Kerja Siswa 2

Nama :
Kelas :

MENAKSIR HARGA BENTUK MASING-MASING

Ingat bentuk yang telah kalian buat di pertemuan pertama? Sekarang bagaimana jika kita taksir harga dari masing-masing bentuk yang telah kalian buat.

TUGAS KAMU

Bagaimana cara kalian dalam menaksir harga dari bangun yang telah kalian buat. Tuliskan strategi kalian pada tempat yang telah disediakan berikut ini. Usahakan teman kalian memahami apa yang kalian tulis

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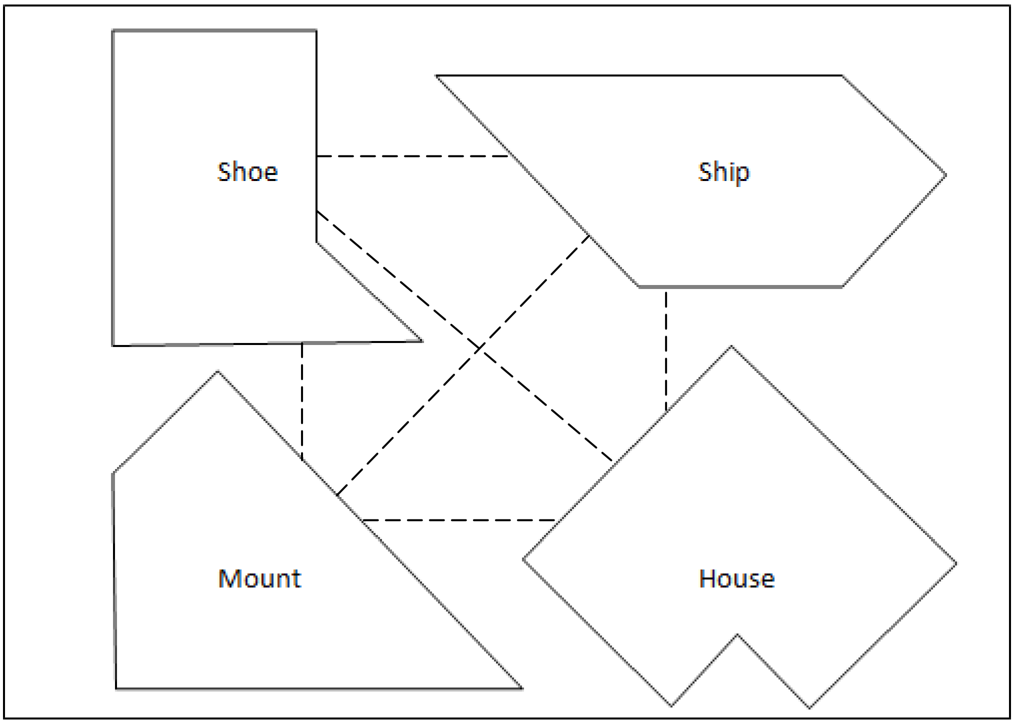
Pertemuan 3 – Lembar Kerja Siswa 2

Nama :
Kelas :

MEMBANDINGKAN BENTUK MANA YANG HARGANYA LEBIH MURAH, LEBIH MAHAL, ATAU SAMA



Sekarang, kita akan bermain-main dengan beberapa bentuk yang telah disediakan. Gambar diatas adalah ilustrasi dari bentuk-bentuk yang akan kita mainkan. Ukuran sebenarnya ada di halaman selanjutnya.

TUGAS KAMU

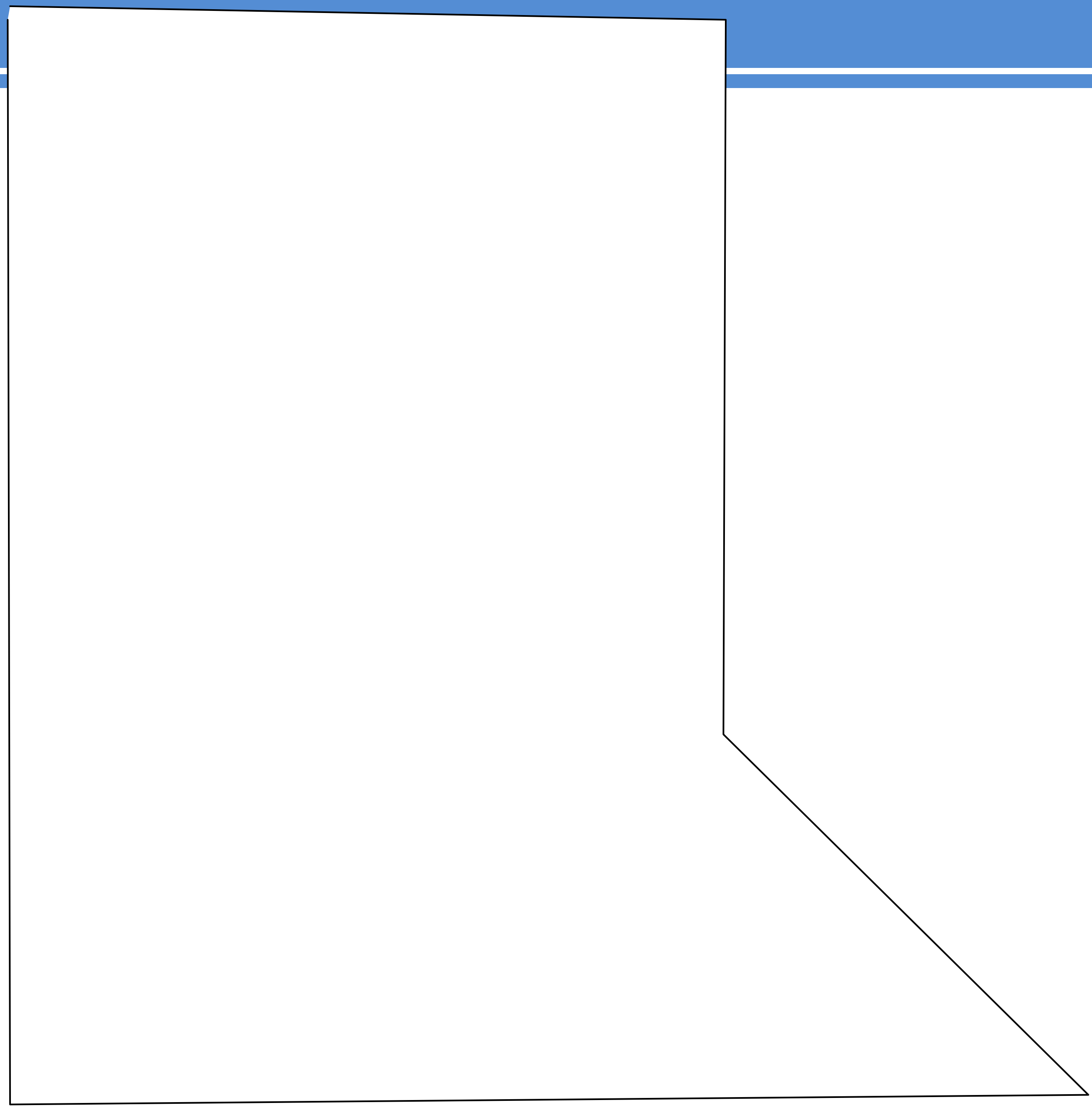
Dalam 1 kelompok, diskusikan bagaimana untuk menentukan bentuk mana yang harganya paling mahal, paling murah, atau sama. Kalian bebas menggunakan cara apa saja yang kalian perkirakan paling meyakinkan.

Kemudian, persiapkan 1 poster yang berisi jawaban kalian dan bagaimana cara kalian untuk menentukan jawaban tersebut. Buat poster sedemikian rupa sehingga teman kalian mengerti maksud di poster itu

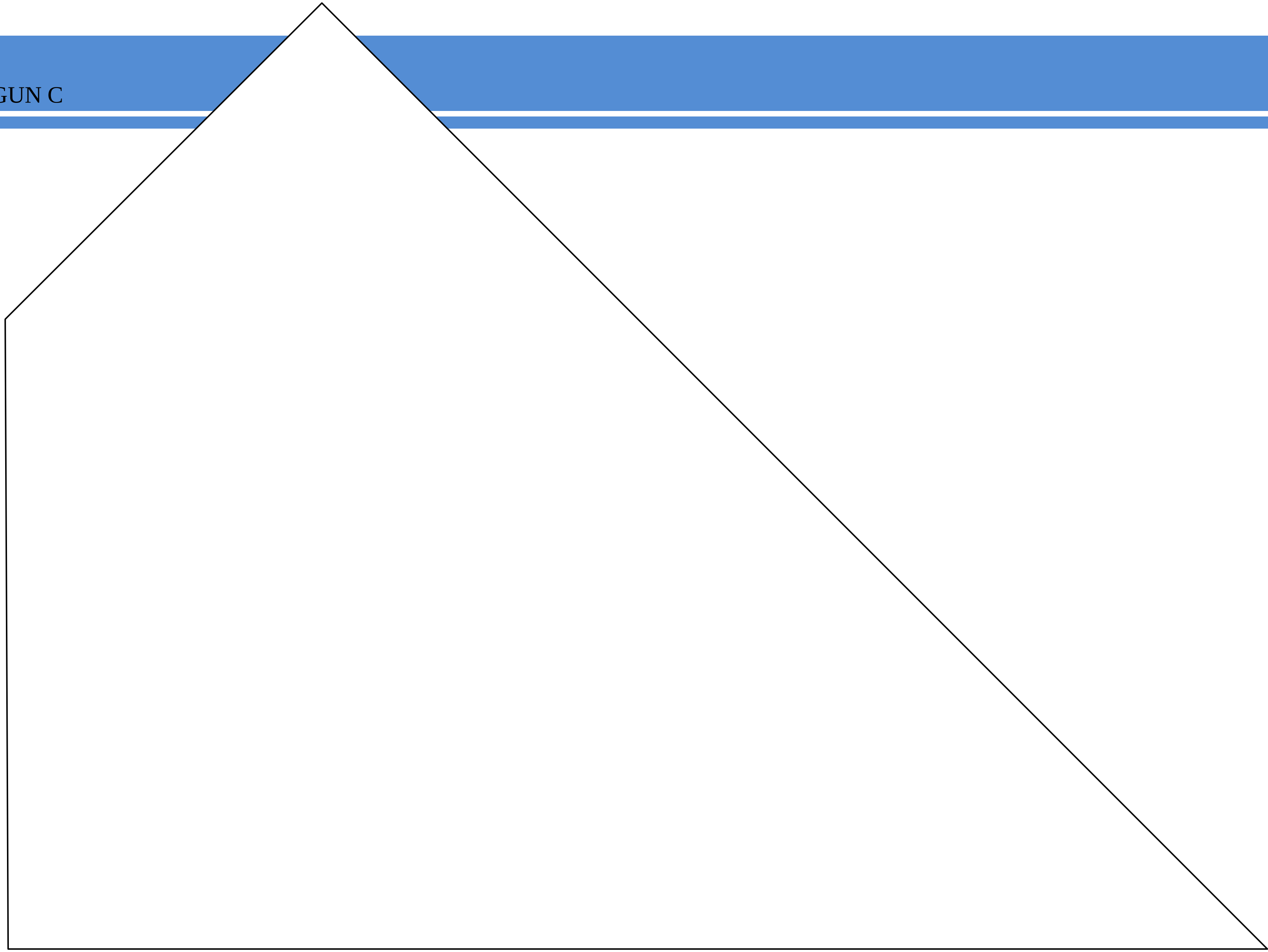
SELAMAT MENGERJAKAN
SEMANGAT☺

BANGUN A

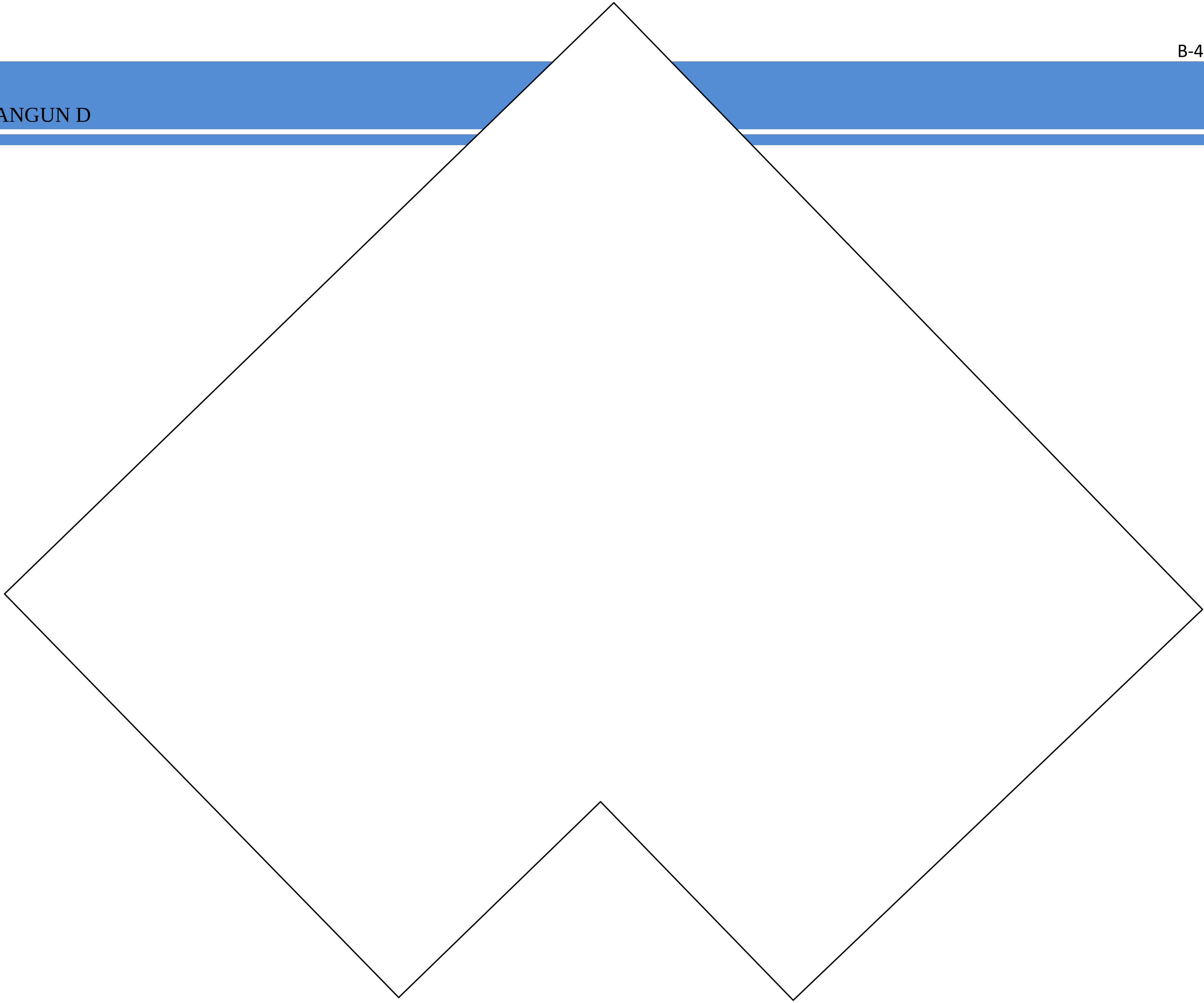
BANGUN B



BANGUN C



BANGUN D

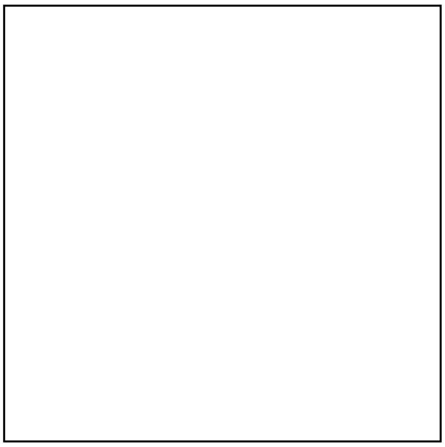


Pertemuan 4 – Lembar Kerja Siswa

Nama :

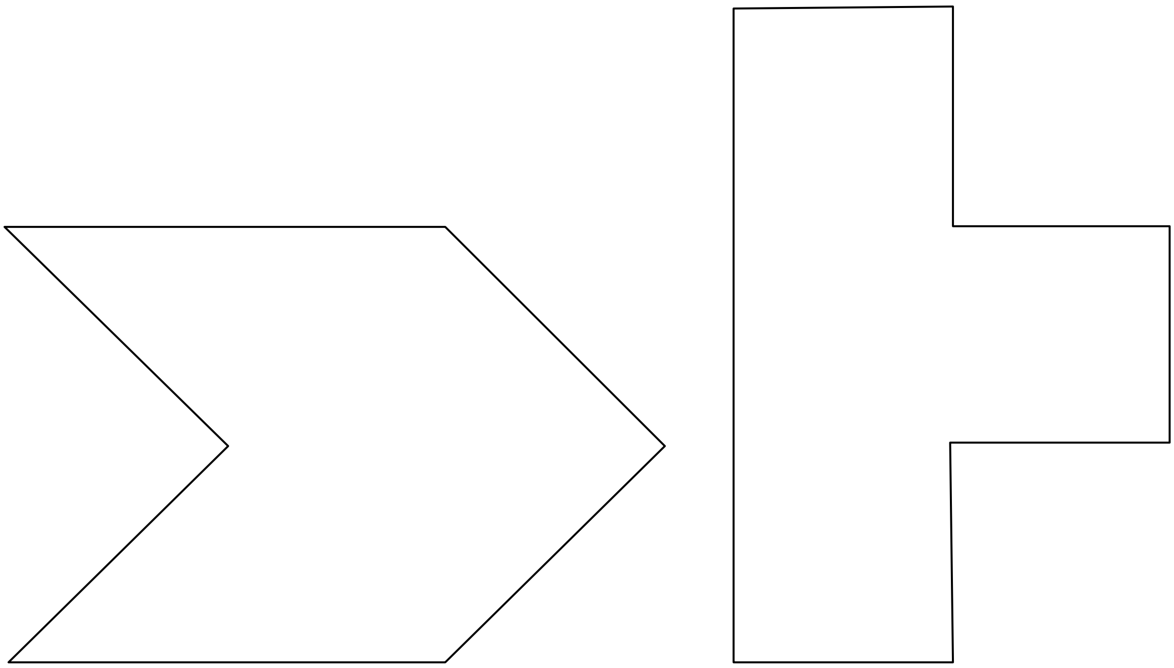
Kelas

MENYUSUN ULANG BANGUN



Bisakah kalian susun ulang bangun-bangun dibawah ini menjadi persegi diatas☺

Kalian bisa membayangkan bagaimana momotong bangun-bangun di bawah ini dan menyusun ulang menjadi persegi di atas



TUGAS KAMU

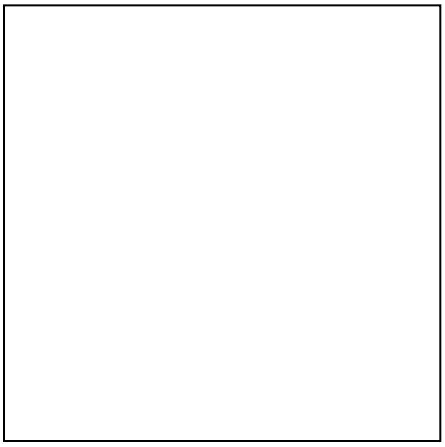
Carilah cara untuk memotong dan menyusun ulang bangun-bangun yang diminta menjadi sebuah persegi

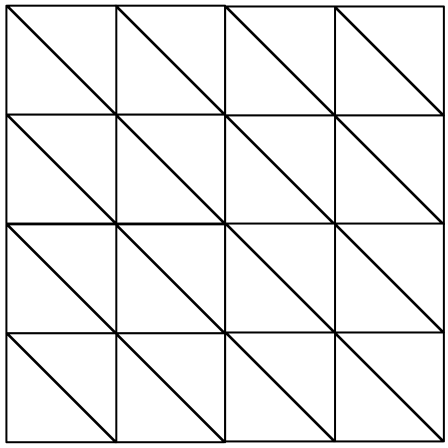
Pertemuan 4 – Lembar Kerja Siswa

Nama :

Kelas

MENUTUP SEBUAH BANGUN DENGAN SATU JENIS TANGRAM

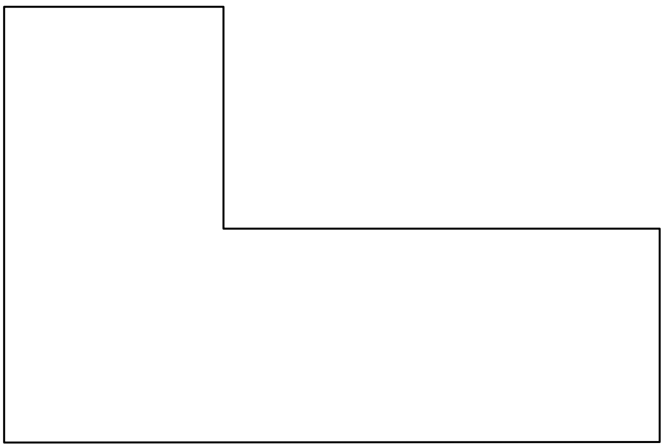


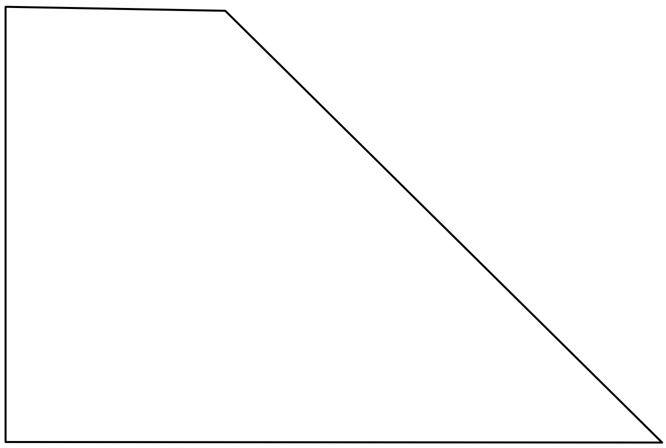


Gambar diatas merupakan bangun persegi yang ditutup oleh potongan tangram segitiga yang kecil, berapa segitiga yag diperlukan untuk menutup bangun diatas? ☺

TUGAS KAMU

Tentukan dengan menggunakan bangun diatas sebagai acuan, berapa segitiga kecil (seperti pada potongan segitiga di atas) yang dibutuhkan untuk menutup bangun di bawah ini





SELAMAT MENGERJAKAN

SEMANGAT☺

ANALYSIS OF THE LESSON DESIGN (1st cycle)

PRE TEST

OBSERVATION

LESSON DESIGN

ACTIVITY 1

ACTIVITY 2

In analyzing the data, the researcher defines a labeling system to keep track of the data consistency and the clearness of the presented data.

(activity code)_(pupil's code).(sequence number)

Activity code	Pupil's code	Sequence number
O1 = pre test data O2 = interview on teacher I = 1st activity II = 2nd activity III = 3rd activity IV = 4th activity V = 5th activity VI = 6th activity F1 = post test data F2 = interview on student's post test	A = Akmal F = Fira D = Dika S = Sisca J = Josca K = Keyzra	1, 2, 3, 4, … The number is depend on the sequence of data in one activity

PRE-TEST ANALYSIS

The test was conducted to elicit the students' prior knowledge. The test was about 30 minutes done by 6 pupils in the first cycle.

1st PROBLEM

Here the problem

2 different cakes, cake A and cake B, are cut as the following, which cake is the biggest?

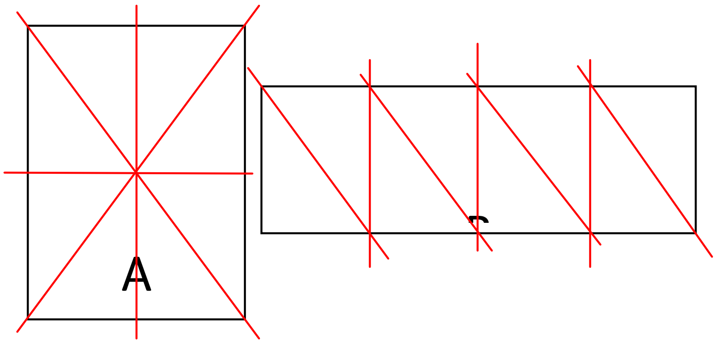
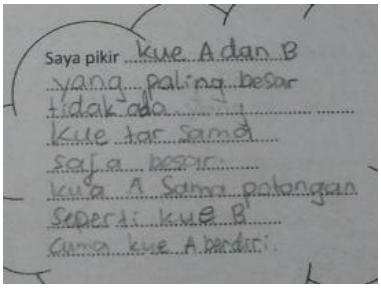
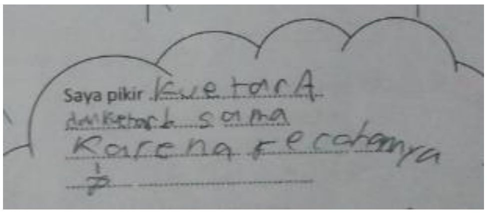


Figure 4.1, the picture to compare

All of the pupils answered that those 2 cakes have the same size. The following are the pupils' answer on why those 2 cakes have the same size.



O1_F.1



O1_A.1

Figure 4.2, pupils' answer

Their reason why those cakes have the same size can be summarized in 2 different ideas. The first is that the cakes are cut into the same shape of pieces which are congruent right triangles (O_F.1). The second is that the cakes are cut into equal fraction which is 1/8. This idea comes up because they had just already learnt about fraction (O_A.1).

This suggests that if the pieces have already divided into congruent shape the students could compare the size of the whole shape correctly. They see each fraction has the same shape. When those same number fractions are combined into 2 different shapes they know that those shape (cake) just the same in size.

2ND PROBLEM

Here the problem

Compare with another shape, can you guess, which shape has the same size, smaller size, or bigger size with this square A? (you are free to make a scratch on the picture to solve)

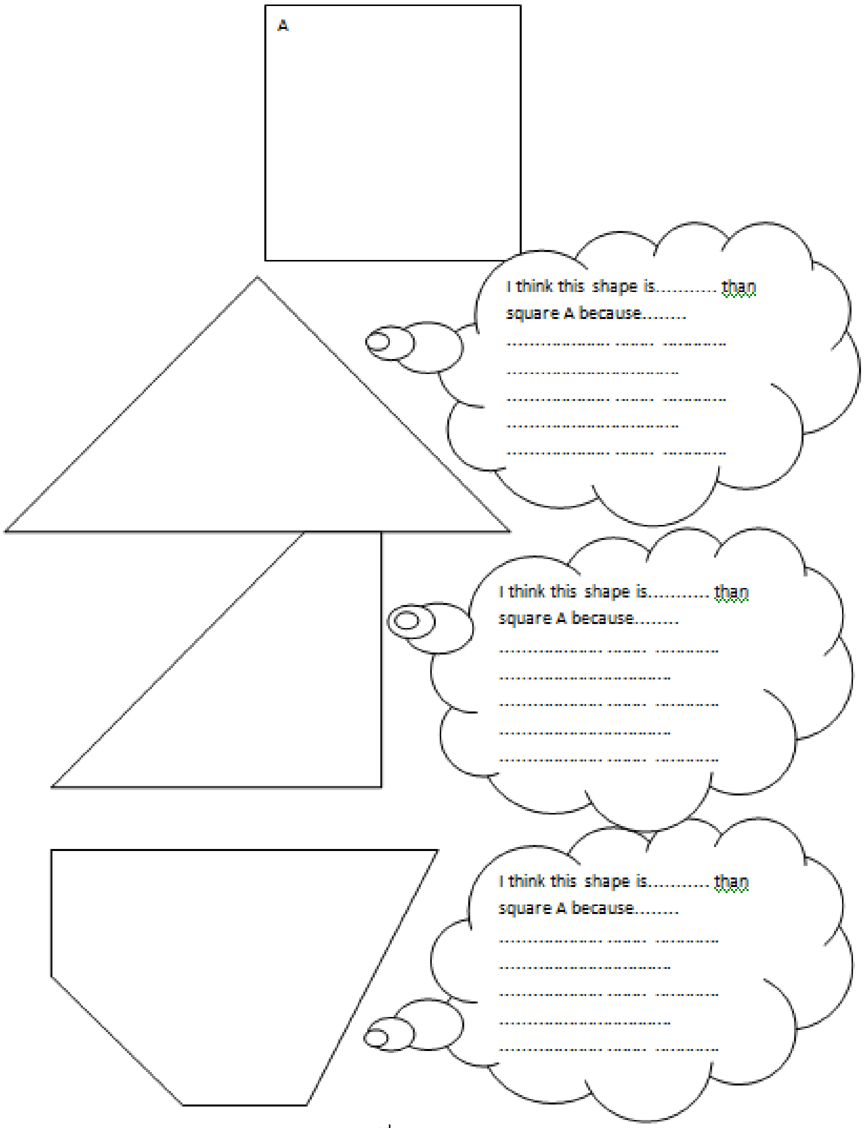
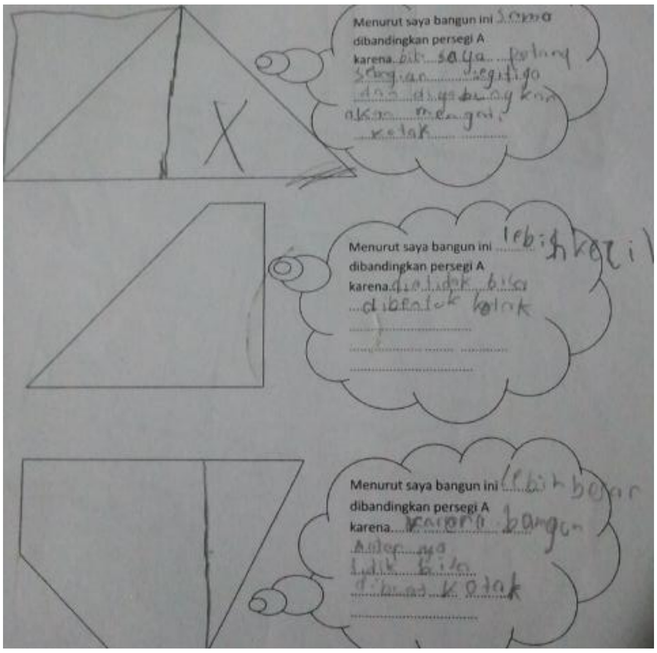


Figure 4.3, 2nd problem of comparing

Only 1 out of 6 pupils answered the question correctly. This pupil has already understood the concept of area conservation. He drew lines as mean as cutting the shape and paste it into another position such that it resembled the square, the shape which they must refer to compare.

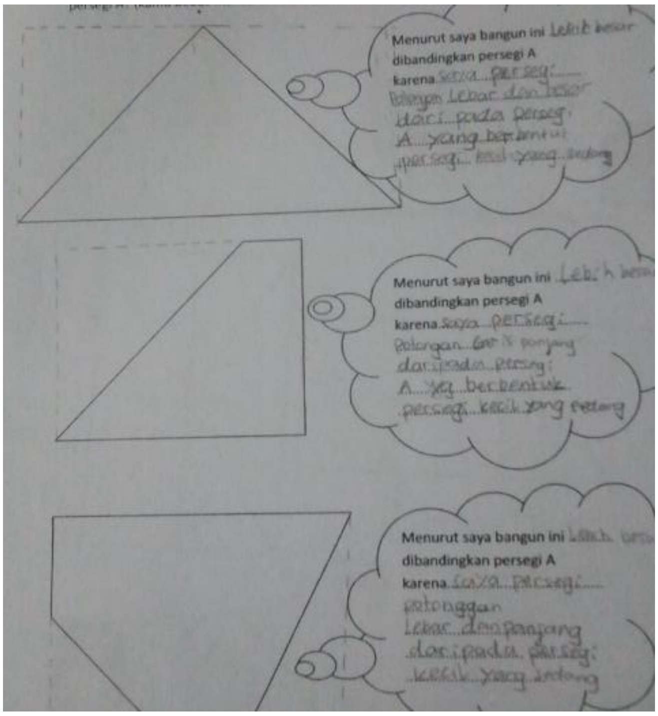


O1_A.2

Figure 4.4, pupils' answer

When he could not make the provided shape into square by cutting and pasting, he then decided that the shape is smaller or bigger.

About the answer of the pupils who answered incorrectly. There is a trend in students' reasons when they reasoned comparing the size of 2 different shapes.



O1_.2

Figure 4.5, pupils' answer

The pupils' tray to compare the square with the other shapes by making out scribed rectangle on the asked shapes. They saw that the out scribed rectangle is bigger than the square in the question and they concluded that the shape is bigger than the square. Here, the pupils were aware that they must make the asked shapes into comparable shape, but, the mistake here is that they ignored the space inside the shape whether they are invariant or not.

This suggests that when comparing a raw shape that they must decided how to compare by them, most of them were failed to answer correctly because of incapable of making a same size shape of the given shapes. It is in line with what Piaget, Kordaki, and Papolodous thought that pupils in this age are having difficulties dealing with the concept of area conservation

REMARKS ON PUPILS' PRE-KNOWLEDGE IN THE 1ST CYCLE

This suggests that when comparing raw shapes (the shapes asked in the 2nd problem), most of them were failed to answer correctly because of the tendency of them to make a comparable shape, which is good, but ignoring the area inside the shape. This implies that they do not understand the concept of area conservation which is in line with what Piaget (2001), Kordaki (2006), and Papolodous (2011). This learning sequence that I have designed is precisely could help them to understand the concept of area conservation.

However, the result from 1st problem implies that if the pupils notice the idea that the shapes could be decompose into smaller pieces and those pieces can be compared easily, then they will compare which shape is bigger correctly. Thus, the use of tangram pieces as easy comparable objects that can be decomposed from the given shape would give benefit for the pupils to help them. The idea is that when the pupils cannot answer the comparing problem, by decomposing shapes into pieces using tangram pieces, they should be able to compare the pieces and figure out the correct answer in comparing those shapes.

Mosaic-Puzzle Game: INTRODUCTION TO THE CONCEPT OF AREA

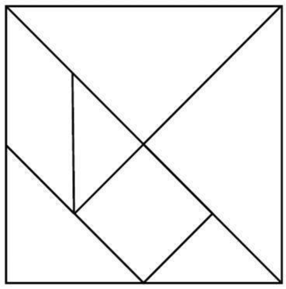
Game Activity for 3rd grade (9-10 years old)

Play with Tangram Mosaic-Puzzle

1ST ACTIVITY

1. FREE EXPLORE PLAYING TANGRAM (ORIENTATION ACTIVITY)

Students are given a free opportunity to play with mosaic puzzle. We expected that the students could do composing, arranging, and shaping easily through this activity. They form a variety of shapes from the tangram. The students are also asked to trace the shape they drawn.



LEARNING GOAL

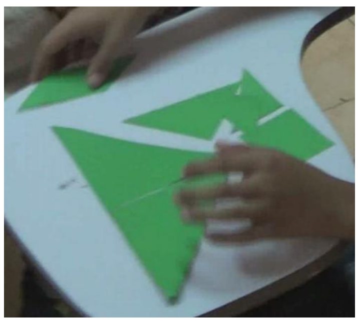
1. The students are able to play with tangram in composing, arranging, and shaping the tangram pieces.

ANALYSIS ON LEARNING ACTIVITIES AND ACTUAL STUDENTS' LEARNING

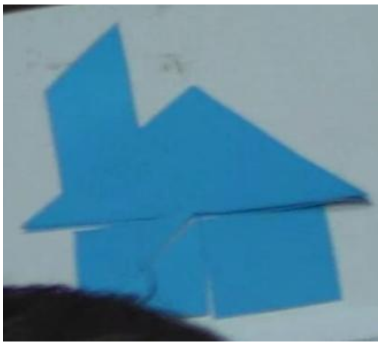
ACTIVITY	SUB-LEARNING GOAL
Orientation with the tangram	The students are accustomed to play with tangram
Constructing own shape using more pieces and record it	The students are freely construct their own shapes using more pieces

For warming up tasks, the students will play freely with the mosaic puzzle. Each of them could make many forms of puzzle shapes (I.AFDSJK_1-8) but then they are asked to choose one of them and trace it in a blank paper

I.AFDSJK_1

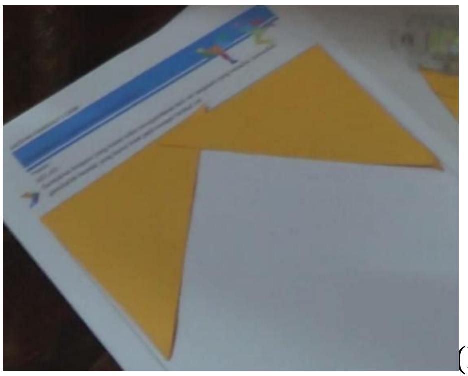


I.AFDSJK_2



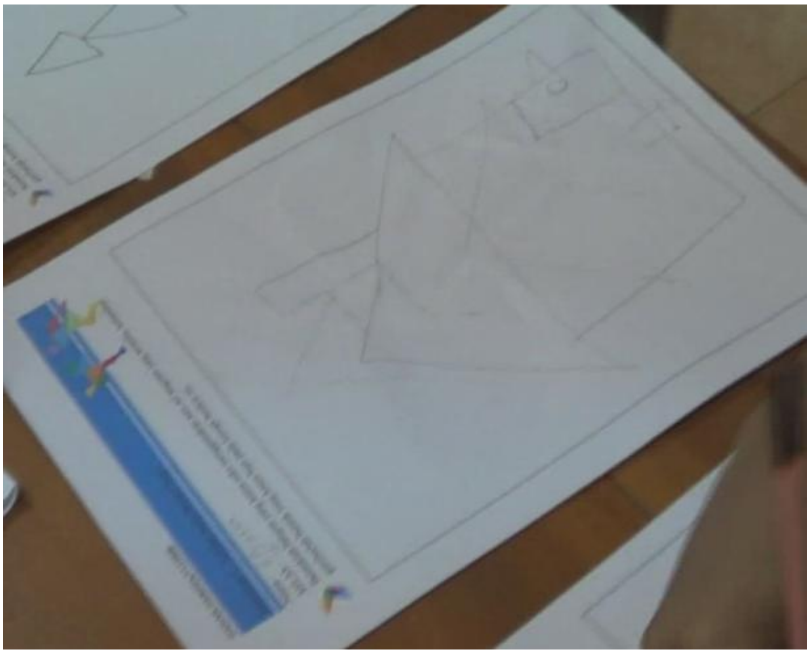
I.AFDSJK_7 I.AFDSJK_8

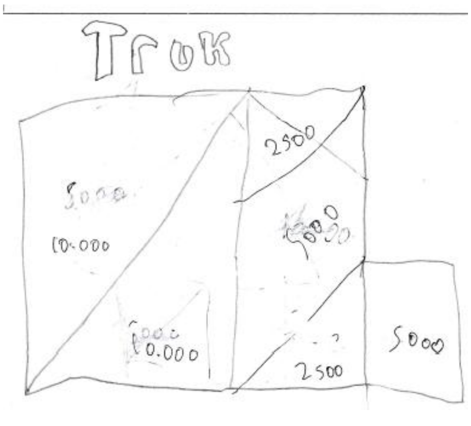
The pupils were not having difficulties in making their own shapes by constructing shapes using available tangram pieces. The problem occurred when they were asked to draw the shape that they had made. Some of them broke the construction before they had drawn the shapes into paper. Some of them also had difficulty in making a sketch of their shapes because it was bigger than the available space. What they did was that they forced the shape side by side can be put into available space (I.F_1).

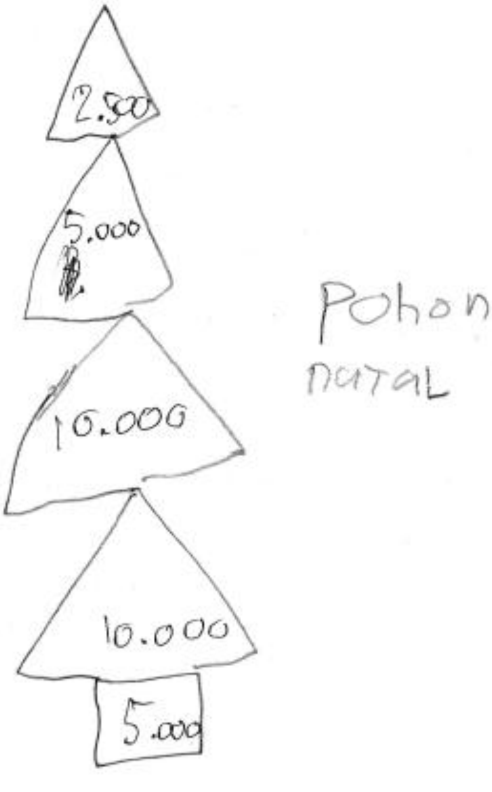


(I.F_1)

The picture of what the pupils had drawn provided as follow:

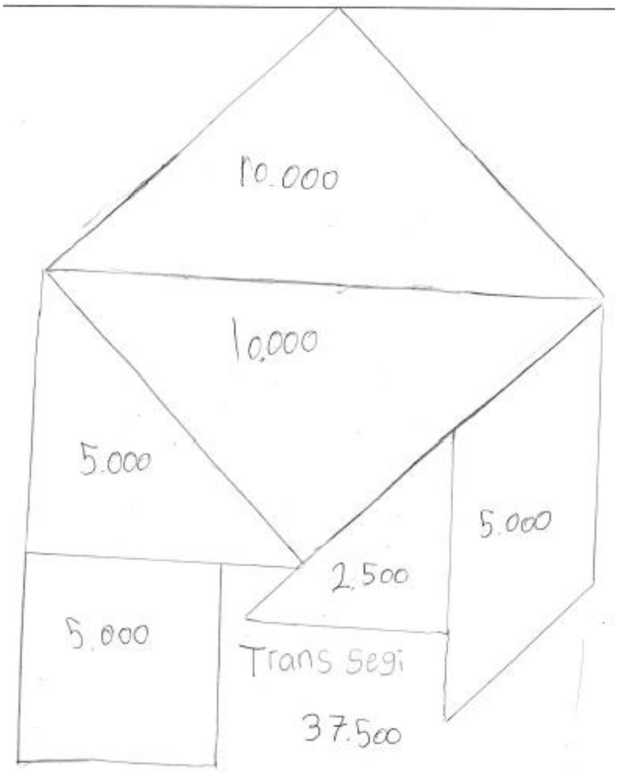




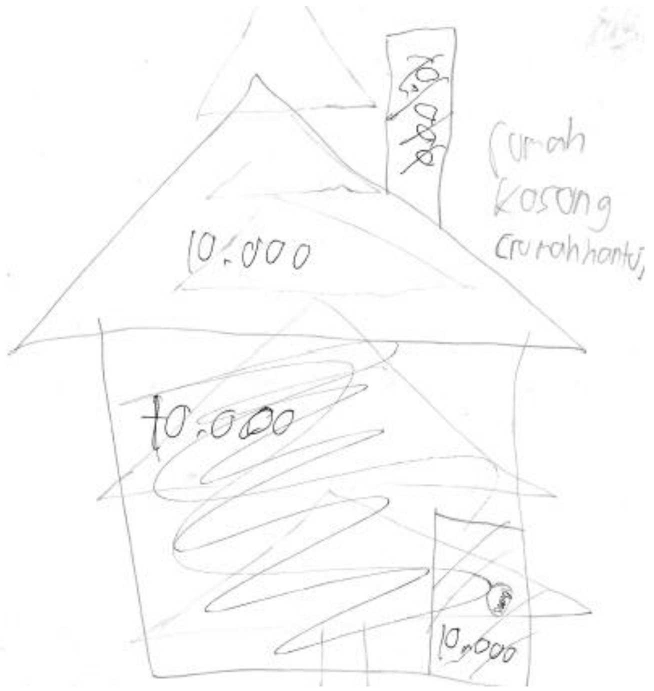


I,A_1

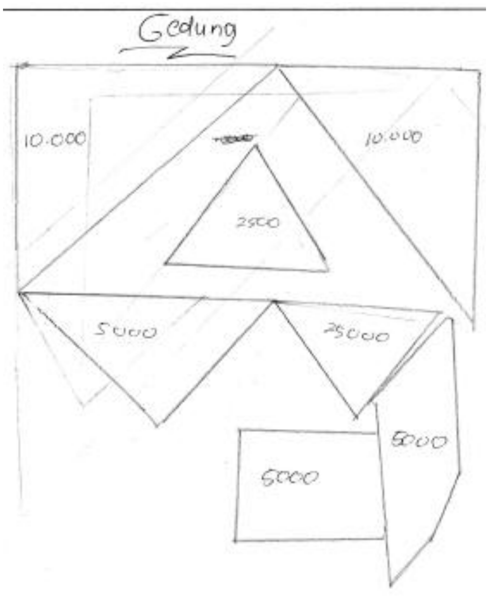
I.J_1



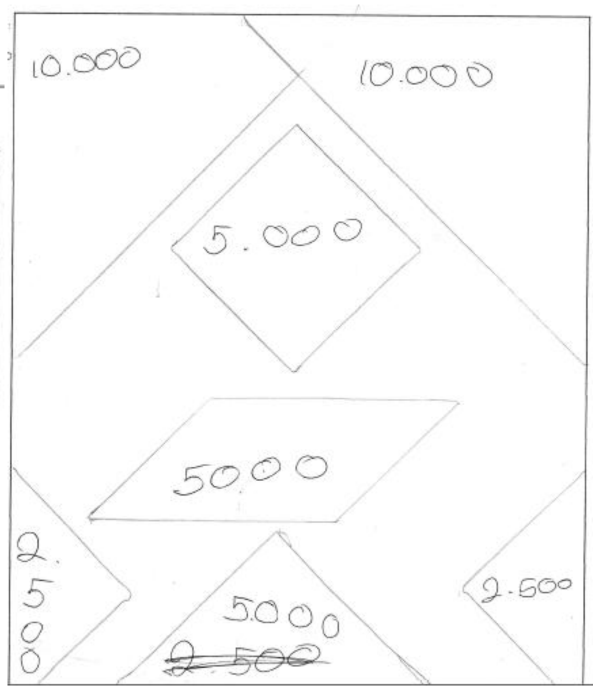
I.D_1



I.K_1



I.F_1



I.S_1

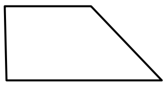
The last 2 sketch are similar because they made them together (I.F_1 and I.S_1). In the end, they tried to make those shapes different by using different construction for bottom shape.

The pupils could draw the sketch for their own shapes. There were 2 ways in drawing the sketch occur at that time. Some of them took the pieces one by one on the paper and drew it. The rest of the pupils just drew directly onto the paper. However, the drawings were not visible enough. For example the pictures above, the pupil just draw a house-like picture for his constructed shape. After they decided which shape to choose, they drew the sketch. Some of them dilated the picture because the space to draw was not enough. Another also put their construction directly and drew it.

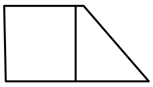
For the improvement, perhaps in order to help the pupils such that they could draw the shapes that they have made, providing a big drawing paper is necessary. Moreover, the use of suitable table to be used in class such that it is big enough for the pupils and they do not need to break the shape before drawing it. This will help them to maintain the shape which will prevent them forgetting the shapes.

TASK 1(INDIVIDUAL)

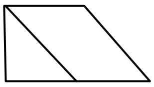
ACTIVITY	SUB-LEARNING GOAL
Constructing a given shape using 2 pieces	The students are able to construct a certain shape using 2 pieces



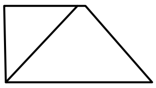
Students will come up with different idea



1st solution

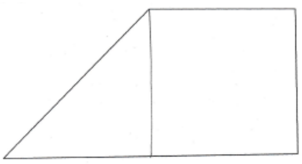


2nd solution

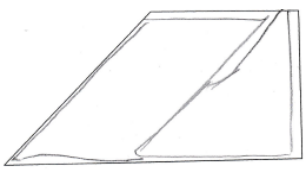


3rd solution

Only the 1st and the 2nd solution appeared in the class. Only one did the 2nd solution (II.J_1) while others did the first (II.D_1). The difficulty was that they could not find the correct pieces but just took one by one without considering which side of certain pieces is actually fixed with the shape.



II.D_1



II.J_1

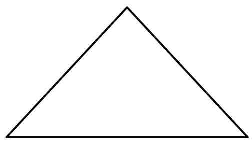
One thing that most pupils did was that they used small triangles to cover this shape. they would need 3 pieces of small triangles here. In fact, each of them only had 2 pieces of small triangle. The pupils must think another construction using more available pieces. Here the excerpt of dialog between teacher on 2 students in solving this problem.

Dika	I lost my pieces, Pak…!!!
Teacher	Really, May I look
	No, you only have 2 pieces for small triangle
Fira	Yeah, I do not have enough pieces pak?
Teacher	Just check it, each of you should have 7 right?
Fira	I am done
Teacher	Hmmm, no, the shape still has a gap right here

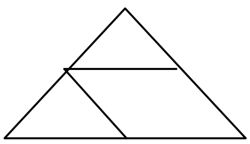
This suggests that the pupils firstly use the simplest form of pieces, which is the small triangle, directly. Moreover, one of them used only 2 small triangle pieces to cover the shape. It made a gap in the construction.

Some of them copied the works of their friend. Some of them saw their friend solutions and then use it. Perhaps the teacher could ask them to submit the pupils solution directly such that others students have no chance to see it.

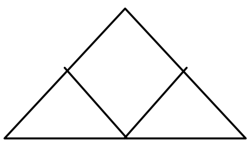
ACTIVITY	SUB-LEARNING GOAL
Constructing a given shape using 3 pieces	The students are able to construct a certain shape using 3 pieces



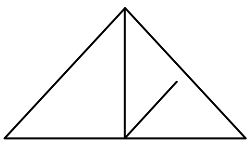
Students will come up with different idea



1st solution

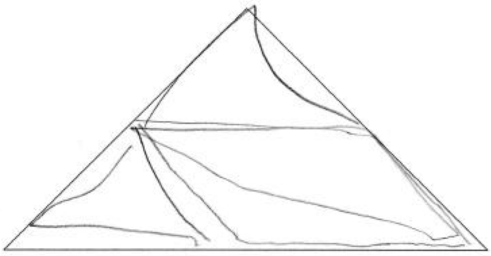


2nd solution

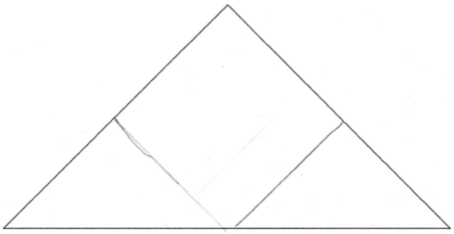


3rd solution

Once again, the pupils just did trial and error in making the correct construction for the shape. in fact, the second problem is designed such that they could use the result from the first problem but most of the pupils did not recognize this relation. However, it seems that 2 of them made use this relation

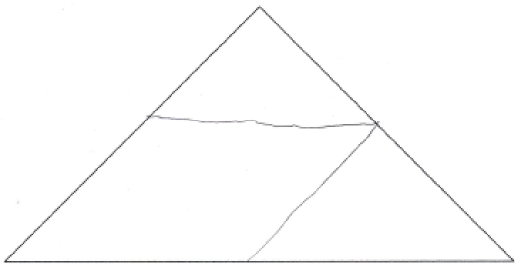


II.J_2



II.F_1

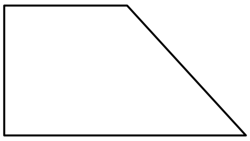
Looking in detail on the constructions, if we omit the small triangle on the left side on both constructions, we will have the same construction for the previous task. Other solution given by the student was similar with the left construction but on flipped arrangement (II.A_1). It seemed that, at that time, the pupil saw their friends that they needed to use the parallelogram pieces. Then, they by themselves tried to arrange the correct construction.



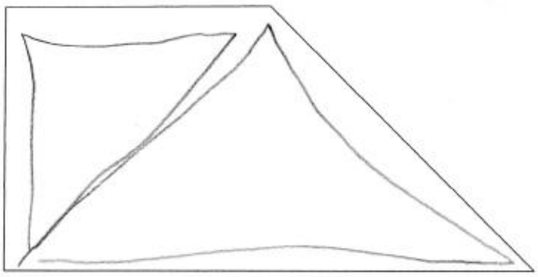
II.A_1

The mosaic puzzle potency gives an opportunity to the students to experience a rich and stimulating activity in geometry. By constructing the right pieces, they dealt directly with several geometric features. Angles, sides, symmetry, and equality are some geometric features while siding, rotating, and flipping the tangram pieces. The students would not realize if they are studying but playing puzzle game. In the end, as an introduction activity, the students seemed to get used to play with tangram in composing, arranging, and shaping but they missed the fact what they could use what they had already made in the first problem to be used in the second problem.

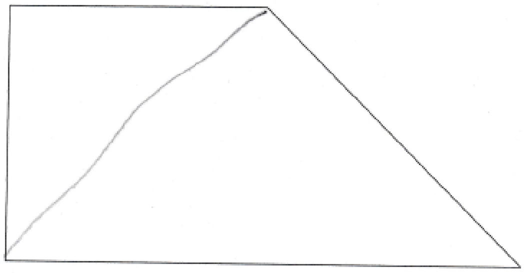
ACTIVITY	SUB-LEARNING GOAL
Constructing a given shape using needed pieces	The students are able to construct a certain shape using undetermined pieces



Here, the students were dealing with the idea with the conservation in the concept of area. By knowing that we could use different pieces to make a same shape or even we can also using different number of pieces to make a same shape, we hope they will start to grasp the idea of area conservation. Unfortunately, they still did not trigger any conclusion of that possible construction because there was only one solution. Perhaps the teacher should ask them to make another construction such that they will discuss about the possibility.

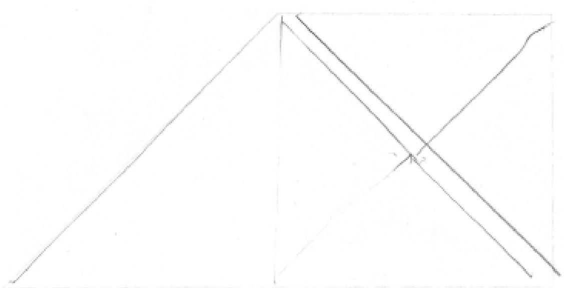


II.J_3



II.A_2

However, there was a pupil who is able to make a different construction. It was started by incorrect solution (II.F_2). By what I mean, she used the small triangles 4 times which was not allowed because there were only 2 small triangle pieces available. Then, she tried to put the piece at once. That was the time when she noticed that he could combine 2 small triangles with medium triangle piece (II.F_3).



II.F_2

II.F_3

Like you can see above, on the left picture, there is a sketch of previous construction of this pupil. She use 4 small triangle to be a square there.

It was too difficult for them to come up with more solutions for this problem. One of the pupils just put several pieces into one. He ignored the sides but tried to cover the shape (II.J_3).

II.J_3

Guidance from the teacher like asking them to take a look at the previous task in order to get an idea how to proceed with the problem was no use. They could not gain the idea of using the result from previous task and adding some pieces to form the shape in the problem.

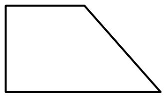
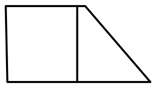
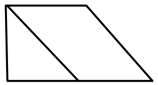
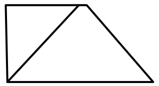
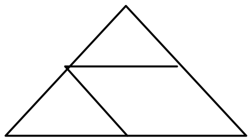
Perhaps, asking them to make more construction would be better because they will think more than only making one simple construction. It will also append more idea about one shape with many constructions.

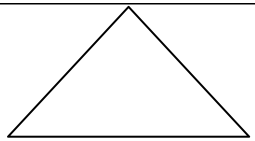
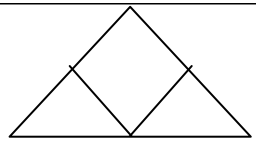
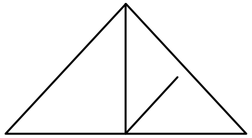
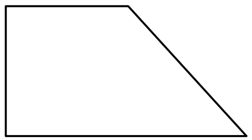
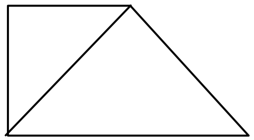
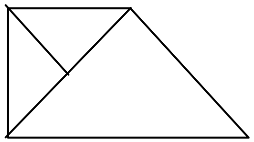
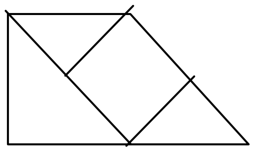
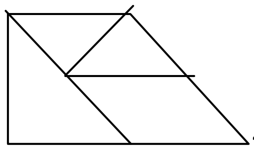
TASK 3

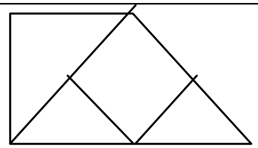
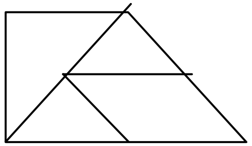
ACTIVITY	SUB-LEARNING GOAL
Challenging game	The students are able to do partition using the pieces

For the last task in this section, since the time was not enough and the fact that the task actually will be repeated, this task was cancelled and the activities was continued to the next main problem.

TABLE OF THE OVERVIEW OF THE ACTIVITY, THE HYPHOTHESES, AND THE ACTUAL LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
Orientation with the tangram Constructing own shape using more pieces and record it	The students are accustomed to play with tangram The students are freely construct their own shapes using 7 given pieces and then draw a sketch of it.	The students will try to construct their own shape by adjusting side by side and the angles as well.	The students directly played with the tangram and made more than one shape. They also saw each other shapes then tried to make another shape inspired by their friend works After they decided which shape to choose, they drew the sketch. Some of them dilated the picture because the space to draw was not enough. Another also put their construction directly and drew it. Others forgot the construction because it was broken. There were a house-like shape, a car-like shape, a tree-like shape, and others abstract shapes.
Constructing a given shape using 2 pieces (1A)	The students are able to construct a given shape using 2 pieces. A given shape is as follow: 	Students will come up with different idea  1st solution  2nd solution  3rd solution Guidance is needed for the students who have difficulty in solving it.	Only the 1st and the 2nd solution appeared in the class. Only one did the 2nd solution while others did the first. The difficulty was that they could not find the correct pieces but just took one by one without considering which side of certain pieces is actually fixed with the shape. Some of them copied the works of their friend
Constructing a given shape using 3 pieces (1B)	The students are able to construct a certain shape using 3 pieces	Students will come up with different idea  1st solution	Only the 1st and the 2nd solutions appeared in the class. Only one did the 2nd solution, while others did the first. Actually, they could use the construction that they had already made for the 1st problem. By rotating and add

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
		 2nd solution  3rd solution There will be another forms similar with previous construction but different orientation Guidance is needed for the students who have difficulty in solving it.	one pieces of it, the solutions for the 1st problem could be used for this task. However, none of them noticed about this fact. Some of them still copied the works of their friend
Constructing a given shape using 3 pieces (1B)	The students are able to construct a certain shape using undetermined pieces 	Students will come up with different idea  1st solution  2nd solution  3rd solution  4th solution	Here, the students took a long time to finish this task. They had a clue how to complete this task after the teacher asked to look into previous shape. And then, the students used the construction for previous task and add Most of the students got the 1st solution because it seemed that it was the easiest construction using only 2 pieces. Only one pupil made the 2nd solution. She made a mistake at the beginning by using 1 medium triangle and 4 small triangles for this shape which was not allowed. Then she found that she can substitute 1 medium triangle and 2 small triangles with 1 big triangle.

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
		 5th solution  6th solution There will be another forms similar with previous construction but different orientation Guidance is needed for the students who have difficulty in solving it.	
Challenging game (optional)	The students are able to do partition using the pieces	A student challenge another student to exchange the traced shaped and try to figure out the construction of it	Not done

HLT OF MY SECOND ACTIVITY

We would like to guide the students to build the model of tangram with price from the context given to reason with.

LEARNING GOAL

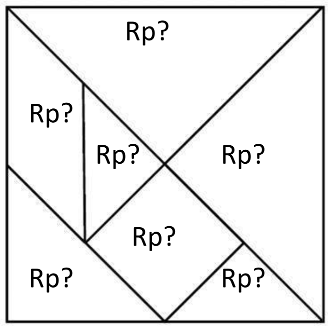
2. The students are be able to build the model of tangram with price from the context given to reason with

ANALYSIS ON LEARNING ACTIVITIES AND ACTUAL STUDENTS' LEARNING

TASK 1 (GROUP)

SELLING THE TANGRAM PIECES

Giving context "Selling the Tangram Pieces"



The tangram puzzle

After the students had fun playing tangram, they were invited to think about a toyshop of teacher's brother who sells a tangram puzzle. The prize for a complete set of tangram puzzle is Rp 40.000,-. The owner of the toyshop also wants to sell the tangram puzzle for a piece, because he figure out that many children who have bought a set of tangram will lose one or more pieces of the tangram on the upcoming months. The teacher then asks the students to help his/her brother to put prizes in every different piece.

ACTIVITY	SUB-LEARNING GOAL
Finding the price of each piece from the context given	The students are able to determine the price of each piece using their own strategy

During the group discussion, most of the pupils did not understand that they must sell the tangram in pieces. However, in detail, they did not understand that they must find the price of each pieces such that the total price of the whole pieces is Rp. 40.000,-.

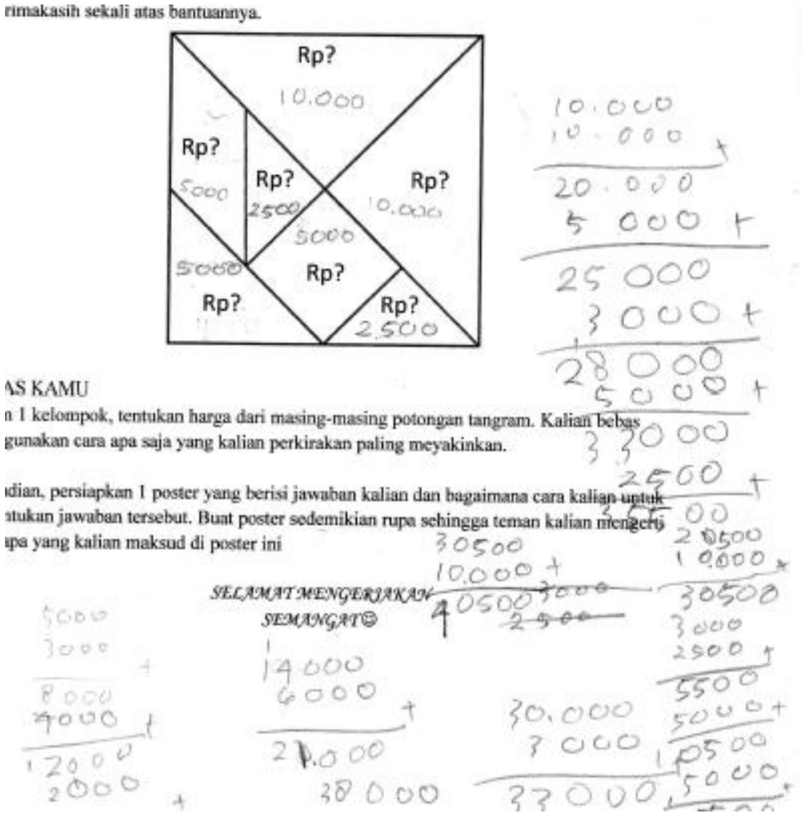
14. One of them considered Rp. 40.000,- was the price for one piece. The following is the conversation with Akmal, who is high achiever.

Teacher	Do you understand the problem?
Akmal	What do you mean by a set?
Teacher	How is the problem written?
	Do you understand? How much for this? (pointed a set of tangram)

Akmal	What is a set look like?	
Teacher	The price for a set is 40.000 (pointing a picture of a set of tangram) This is a set	
Akmal	Ohhh, I was wrong then heheheh	HP-1.3-2.35

From this excerpt, we can see that he did not know what a set of tangram actually is. Misunderstanding of a set of tangram also occurred on another group. When the teacher pointed a picture of a set of tangram and asked about the price of it, they could not answer it. It suggests that it is important to make the contextual problem clear especially about what “a set of tangram” is. It was difficult for them in discussing the problem.

Some of them did not want to think hard about the problem with just putting price whatever they wanted. Then they just calculated the total price. If he total price was not 40.000, then they change the price to make it 40.000. And then, they calculated again (III.D_1).



III.D_1

Guidance from the teacher to the female group about halving the set resulting halving the price was quite helpful for them. However, the halving strategy was only continued until halving the big triangles. They could not continue this halving strategy for the rest pieces. What they did was just putting price without reason. However, they understood that same pieces should be priced equal.

Siska	How much is it? (pointing to the small triangle piece)	
Dika	This must be 2000 and this is also 2000 (pointing both small triangles	
Fira	So this one is 2000	
Siska	Yea, Dika said so	
Dika	This one 2000 (pointing the small triangle) how about the square	
Fira	May be, 6000	
Dika	6000, how come	LP-1.14-5.25

III.DS_1

The fact that the pupils just put price without considering the reason was undeniable. They kept putting price one by one and changing the price such that the total was 40.000. But discussion on female group was more critic than the male group because they asked about why it was priced like that even they could not find the correct reason. Then, guidance was delivered by the teacher by leading the pupils in a group to use halving strategy.

Teacher	How much for the total? (pointing to the set of tangram which had been arranged) If I wanna buy a set of it, how much should I pay then?
Fira	7000
Teacher	How come, is it correct? How much is the price for a set?
Fira	A set is 40.000
Teacher	A set is 40.000, isn’t it? So if I wanna buy the whole pieces, I should pay…
Fira	40.000
Teacher	Ok, this time, if I only buy this (halving the construction, a square became 2 triangles)

	How much should I pay?	
Dika	20.000	
Teacher	Yeah, correct, continue then If I only buy this one (pointing only on one big triangle)	
Group	(silent)	
Teacher	Ok, continue yah for this.	LP-1.14-6.20

Nevertheless, after the teacher explain the strategy and asked them to continue it for the remaining pieces, they did not continue the halving strategy. The same thing also happened when guidance about halving strategy on the male group. It suggests that the activity of halving itself is needed to be added before the pupils experience the pricing activity.

In the group discussion, the pupils did not discuss and share their own idea well. What they did was doing the task by them and telling the answer without discussing it carefully in a group. They also did not make a good report for the presentation but only the answer.

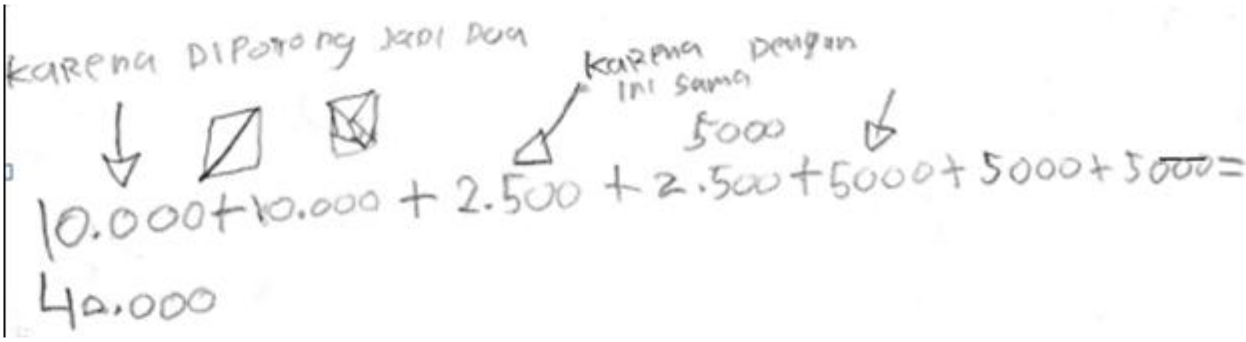
About the learning goal for this activity, the pupils did not solve the problem well. The conjecture of how the pupils will solve the problem using tangram pieces did not occur successfully. The teacher needed to give them clue. For the revision, perhaps at the beginning, the problem should start by finding the price for the triangle only because it only involves halving method. Then, the problem will be continued for the remaining pieces, square and parallelogram. There will be additional guidance from the teacher like drawing a square and cut it into a half. Then, the teacher could ask the pupils the price of these halved portions which should be related to the picture of a set of tangram. In addition, perhaps the teacher could provide the worksheet with the table of the piece list. Such that they could be lead to consider another piece.

TASK 2 (CLASS DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price

We had considered that the students will just only estimate and guess the price of each piece. They just mention the price from what they see visually. However, this strategy could not guarantee the correctness of the answer. There are 2 estimations that students could do. The first one is the students just divide Rp. 40.000,- by 7 because there are 7 pieces of tangram and estimate the result. Here, the teacher could ask whether it is reasonable for those different size pieces has the same price. The second is the students will regard the size but they decide the price roughly. Here, the teacher could ask them to sum up all of those prices and see whether it has also summed into Rp 40.000,-. Then, the teacher could bring this idea to the first strategy by asking the students to do hands on activity on the provided tangram pieces.

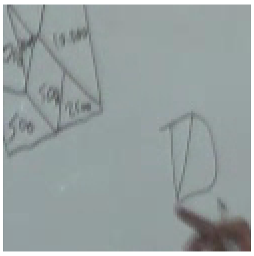
Each group will present their solution in a poster in front of the class. The presentation was started by the male group. Actually, what they had written in the poster was only a calculation for finding price without a clear explanation. But, when the present their group solution, they draw a picture of a set of tangram.



III.AJK_1

In the group poster (III.AJK_1), they wrote their calculation such that the total price of a set was 40.000. in detail, they also wrote some explanations about how they came with the answer. They drew halved figure to illustrate that 10.000 for the big triangle came from the fact that the halved the set of tangram twice. There also was an explanation of how 2.500 was chosen for the small triangle. They stated that the small triangles were equal with the 5.000, but we do not know which 5.000 they referring to.

III.AJK_2



III.AJK_3

Teacher	Now, explain your answer	
Akmal	This 10.000 plus 10.000 is 20.000, isn't it?	
Teacher	Aha	
Akmal	Why these 2 is 20.000, because from the beginning of a set of tangram, when you cut the square into a halved (2 triangles), the price of this side id 20.000. because this side has 2 big triangles, then each of them should be 10.000	
Keyzra	Correct	
Teacher	Please pay attention, I'll ask you later	
Akmal	And for this (pointing to the small triangle) is 2.500. because if you combine these 2 triangles (drawing a square), it will be like this shape. so it will be 5.000	
Teacher	Aha	
Akmal	Then, these also 5.000 (pointing at the parallelogram and the medium triangle) That makes the total of them 40.000	LP-2.3-3.30

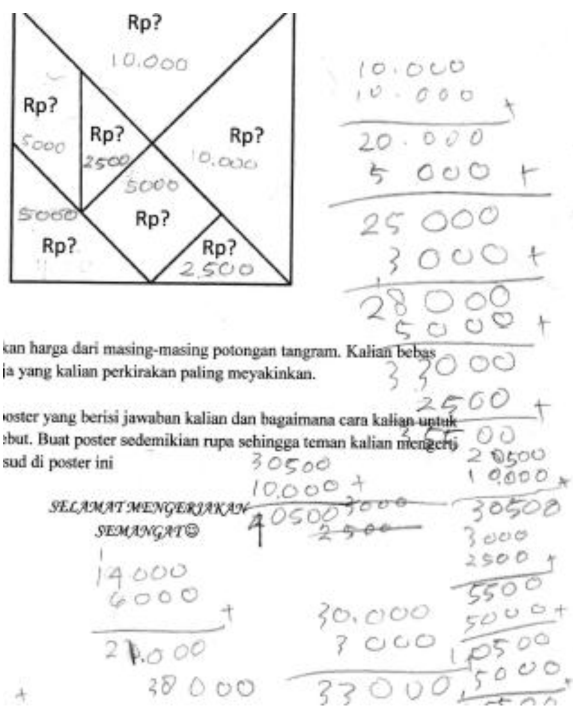
From the conversation above, it seems that they had already used the halving strategy. They knew that if they halve the shape the price will be halved as well. However, they did not apply this strategy for the remaining pieces. But the use of guessing price is salient here. They just counted that the need 20.000 more. So, they guessed that they should be 5.000 for parallelogram, medium triangles, and the square with putting aside the small triangles as the remaining pieces from halving the square.

Teacher	Why the square must be 5.000?	
Akmal	Because it is easy to use 5.000	
Teacher	So?	
Akmal	Ya, if I change into another price, it was difficult to come with 40.000 as the total price	LP-2.4-1.01

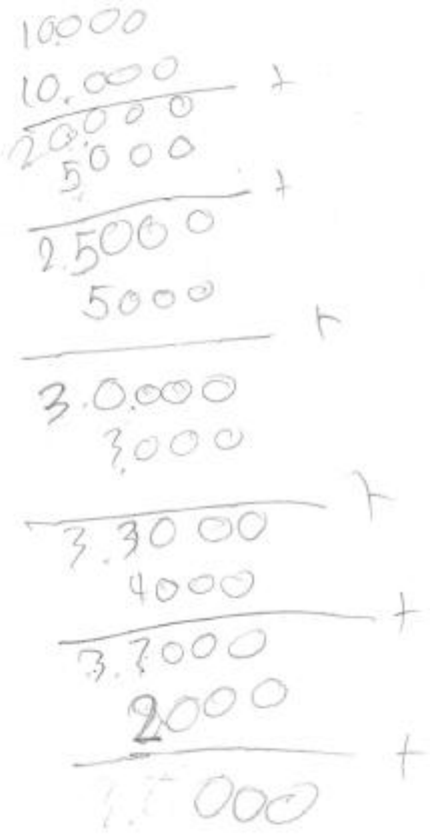
Here, we can see that, numerical reasoning is easier to be used than geometrical reasoning. By what I mean, he used the fact that they must make the total of 40.000, unless they must keep changing the price for the pieces.

Teacher	Now, explain your answer	
Akmal	This 10.000 plus 10.000 is 20.000, isn't it?	
Teacher	Aha	
Akmal	Why these 2 is 20.000, because from the beginning of a set of tangram, when you cut the square into a halved (2 triangles), the price of this side id 20.000. because this side has 2 big triangles, then each of them should be 10.000	
Keyzra	Correct	
Teacher	Please pay attention, I'll ask you later	
Akmal	And for this (pointing to the small triangle) is 2.500. because if you combine these 2 triangles (drawing a square), it will be like this shape. so it will be 5.000	
Teacher	Aha	
Akmal	Then, these also 5.000 (pointing at the parallelogram and the medium triangle) That makes the total of them 40.000	LP-2.4-3.30

For the female group, they also drew a picture of a set of tangram before explaining their strategy. They really use guessing strategy very much. Even after the teacher guided them to use tangram pieces to reason about the price, they did not continue it.

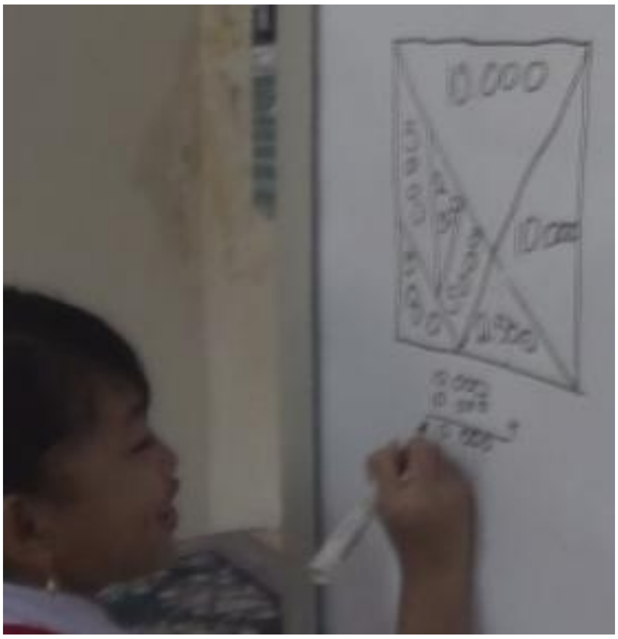


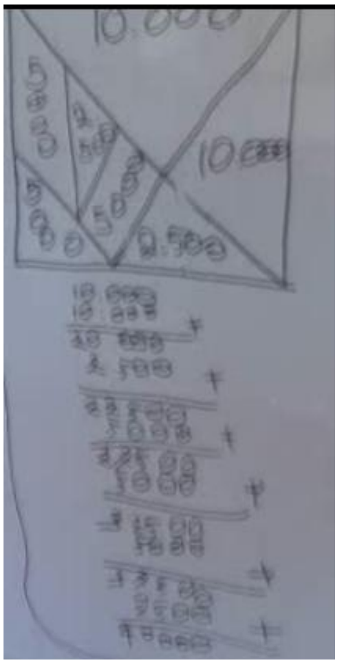
III.DFK_2



III.DFK_3

From the picture above (III.DFK_2-3), we can see that they use calculation from guessed price quite much. However, they still managed to come to the correct answer. There was no clear explanation on how they could come to that answer.





III.DFK_3 III.DFK_4

Teacher	Fira, explain your answer
Fira	This one 10.000 + 10.000 (pointing to the big triangles) is 20.000 (write the calculation below the picture)
Teacher	Pay attention please… If you have something to ask just go ahead
Fira	(continue to make calculation for the rest pieces)
Teacher	Loh, explain to your friend fira…
Fira	Yaa, this one should be added to this one to be summed up into 40.000
Teacher	Done?
Fira	Yah
Teacher	Ok then

LP-2.5-3.00

From the excerpt above, we can see that they did not explain how they came with the answer but how to check whether the answer was correct or not. It is in line with what we have analyzed for their poster group. They only did calculations to make the total 40.000. again, numerical reasoning seems to be more salient here than geometrical reasoning.

From those 2 groups, we can see that for the pupils if they are asked to explain how they can came to the solutions, they explained numerically. By what I mean, they just guessed the price for each pieces and tried to have a total price of 40.000. they had difficulty to explain the geometrical reasoning when using halving strategy. It seems that for their ages, using geometrical reasoning is still a difficult task. However, they could understand that if you halve the shape the price should also be halved.

About the presentation itself, perhaps providing a good poster will be more effective. Asking the pupils to make the picture and the calculation on the poster clearly before submitted will be more effective for the presentation such that they did not need to draw the tangram on the whiteboard. The discussion also did not go smoothly. None of them was asking critical question. Perhaps the teacher could ask more about rephrashing the presentation by another pupil who did not present in front of the class.

TASK 2-A (CLASS DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Determine which piece is the most expensive or the cheapest	The students are able to understand the money model to reasoning about the area


III.T_1

This activity was directly conducted after the class discussion finish. Because none of the group presentation mentioned this topic, the teacher drew the pieces on the blackboard in order from the biggest to the smallest. Then, the teacher asked the pupils about the price of each. The pupils could mention the price for each piece correctly.

Like what we had conjectured, the pupils answered the question of what is the relation between the size and the price by saying that bigger the size bigger also the price. This conclusion is enough as the conclusion for this part.

TABLE OF THE OVERVIEW OF THE ACTIVITY, THE HYPHOTHESES, AND THE ACTUAL LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
Finding the price of each piece from the context given	The students are able to determine the price of each piece using their own strategy	From the given context, the students may come up with some strategies: 1. Using halving/ doubling 2. Using rearrangement-overlapping 3. Using estimation a. Divide 40.000 by 7 b. Estimate the price regarding the size roughly 4. Using gridline For the students who have difficulty in determining the price, teacher guidance is needed	For the first time, the students just put price wherever they one. They just give a price for a piece unreasonably. When the teacher asked them whether the total price is 40.000 or not, they just added and subtracted the price for each piece such that their sum is 40.000 Another idea was just dividing 40.000 by 7 The teacher here tried to guide them by giving example of dividing the shape. with several examples, the students then understood that halving the shape will also halving the price.
Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price	Regardless the size, perhaps, the students will argue about the material of the pieces will influence the price	In this presentation, the students sometimes cannot explain the reason of the price logically. What they can explain is only for triangles-shape pieces. For another square and parallelogram, they only put the price as they like to make the sum of them is 40.000. Moreover, they seemed only using numerical reasoning by using the fact that they must be summed into 40.000. the use of halving strategy was not noticed as the important geometrical reasoning.
Determine which piece is the most expensive or the cheapest	The students are able to understand the money model to reasoning about the area	The students will reason that the size determine the price of the shape	By drawing the order of the shape, the students made an arrangement of the price of the shape from the cheapest to the most expensive. They could understand that bigger the size will result bigger the price as well.

HLT OF MY THIRD ACTIVITY

COMPARING WHOSE SHAPE IS MORE EXPENSIVE

After the students solving the problem, they are expected to know the notion of relationship between size of the tangram piece and its price. However, the teacher should not assume that all of the students have already understood. The following mini discussion is provided to help the students understanding of the relation between the size and the price

LEARNING GOAL

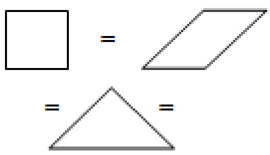
3. The students are be able to understand the relation between the price and the size..

ANALYSIS ON LEARNING ACTIVITIES AND ACTUAL STUDENTS' LEARNING

TASK 1 (PAIR – FOLLOWED BY CLASS DISCUSSION)

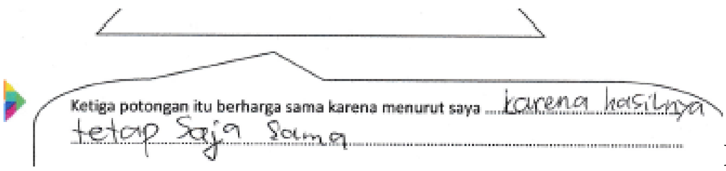
ACTIVITY	SUB-LEARNING GOAL
Discussion on 3 different shapes with the same price.	The students understand the relation between the size and the price among pieces (different shapes may have the same price or size)

From the second activity, finding the price for each piece, there are 3 different shapes which have same price.

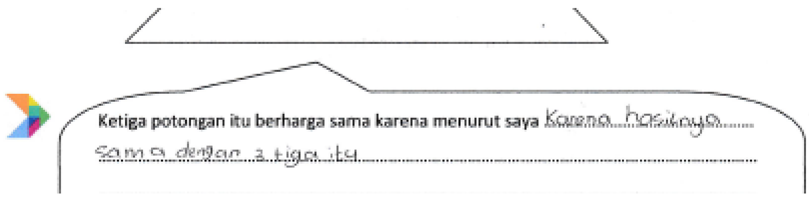


The discussion will be about how those three different shapes have same price.

Here, the following are pupils' answer to the problem.



III_S.1



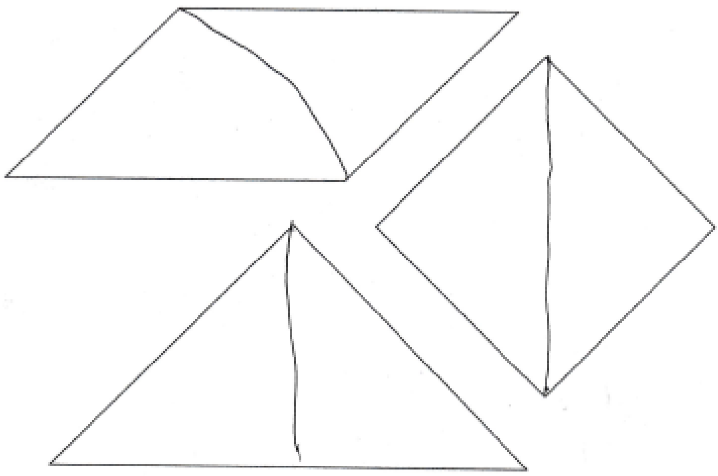
III_F.1

Ketiga potongan itu berharga sama karena menurut saya Karena Ketiga potongan Tangeram harganya Sama

III_J.1

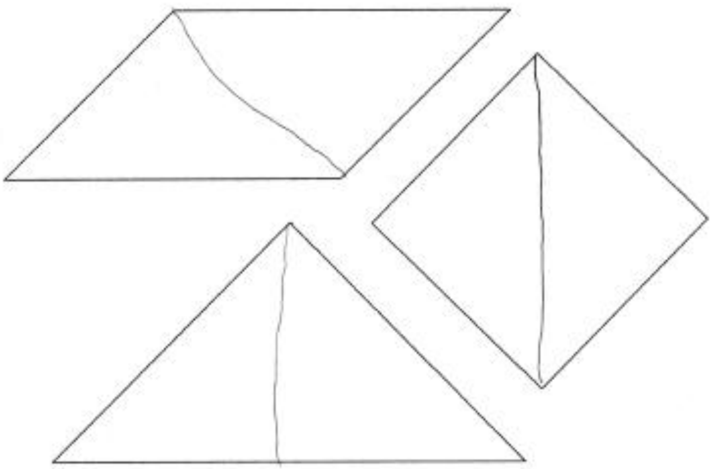
Ketiga potongan itu berharga sama karena menurut saya karena saat di bagikan hasilnya sama

III_D.1



Ketiga potongan itu berharga sama karena menurut saya karena bila di bagi menjadi dua akan sama besar ukuran jadi harganya sama sesuai ukuran dan dibagi 2 akan menjadi sama

III_A.1



Ketiga potongan itu berharga sama karena menurut saya karena kalau dibagi hasilnya sama

III_K.1

There were 2 solution appear on the class discussion. They were:

1. Folding the shape into a half.

   Explain, picture, excerpt 2.25

   This strategy still had weakness. When he wanted to compare the parallelogram

2. Using the small triangles to compare

   Explain, picture, excerpt

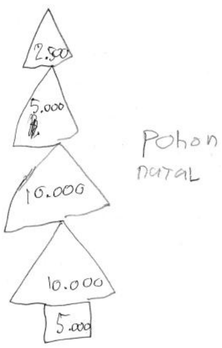
The discussion should be guided into a conclusion that those three shapes have the same size or area. The students will accept that there is possibility of different shapes have the same price. The decomposition of 2 triangles could be considered as the reason of area conservation in this mini discussion; this is the conclusion that should be addressed in teacher guidance in this class discussion.

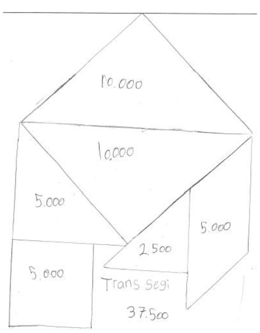
TASK 2 (INDIVIDUAL)

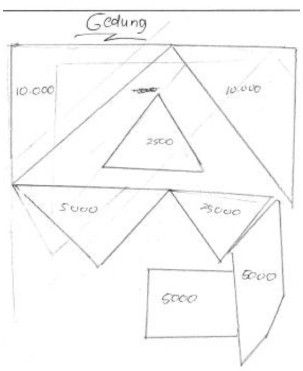
ACTIVITY	SUB-LEARNING GOAL
Finding the price of each own shapes	The students are able to determine the price of each own shapes using their own strategy

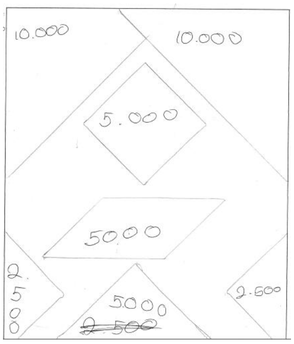
The pictures of the shape that had been recorded by the pupils were used here. Each of them was asked to put price into each own picture.

The pupils used only one strategy to calculate the price for their own shapes. They put price on each piece which constructed the shapes. Then, they calculated the total price. This idea of putting price and finding the total could be seen clearly from the following pupils' works.



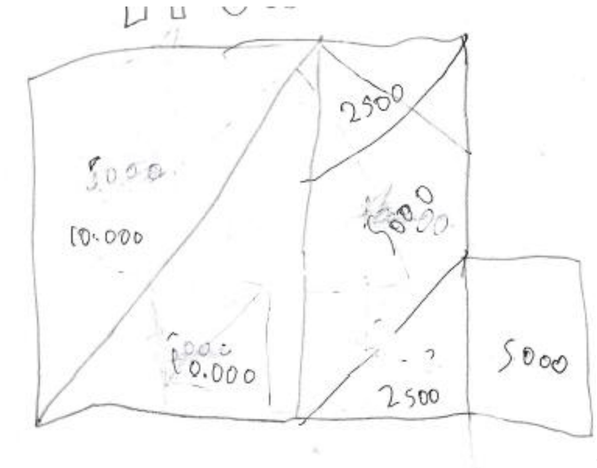






III_J.2 III_D.2 III_F.2 III_S.2

However, some of them had difficulty in determining the price for the pieces because the pictures of shape that they had drawn were not so clear. They tried to reconstruct the shapes by the teacher guidance



III_A.2

III_K.2-3

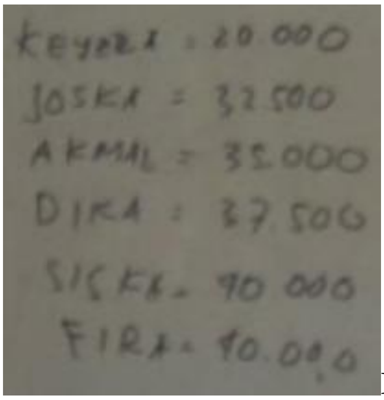
One of the students reconstructed the shape that they had already made. Some scratches which were the previous pupil's work still could be seen (III_A.2). Another pupil could not figure out the construction of his shape. The teacher helped him by asking him to redraw the picture in another paper and giving a clue about what pieces used in his construction (III_K.2-3).

The easiest strategy, counting one by one, was performed by the pupils. It seemed that they did not find out another strategy, especially the pupils who use all 7-pieces of tangram for their construction (III_S.2 and III_F.2). They could not understand that if they use the whole 7-pieces then it will be 40.000. Too much pieces to be recognized may be the reason why they missed this fact. Furthermore, what they had perceived about

TASK 3 (CLASS DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Math Congress	The students are able to argue and to reason in understanding the relationship between the price and the used pieces, or beyond this, the relationship between the price and the size

The math congress was started by the discussion on whose shape was the most expensive. The teacher wrote down the shape's name and the price of each of them.



III_0.1

The teacher tried to guide the pupils to conclude that more expensive the price of a shape then the size would be bigger and vice versa.

Teacher	Whose shape is the most expensive
Joscha	Sisca and Fira
Teacher	And the cheapest
Keyzra	Mine (raise his hand) Hehhee, ya mine is only 20.000 hehehehe
Teacher	So, whose shape is the biggest?
Joscha	Sisca and Fira
Teacher	And whose shape is the smallest?
Keyzra	Mine (raise his hand again)
Teacher	Take a look of their shapes, is that true?
Pupils	(wandering and observe those 3 pupils' shapes) True Pak, yah
Teacher	Ok, then. Pay attention to Fira's and Sisca's shapes. They have the same prices, it means they also have the same size, don't they?
Pupils	(have a look on Fira's and Sisca's shape) (then one of them say) Yes pak, because they have the same pieces.
Teacher	Ok then, let see. Then if the price is bigger then the size is …
Pupils	Bigger as well
Teacher	If the price is cheaper then the size is …
Pupils	Smaller

LP-3.2-2.00

The excerpt above suggest that during the discussion the pupils could build an understanding of the idea of more expensive the price bigger the size. The discussion on several conjectured big idea were successfully occurred. Furthermore, from the discussion above we can see that the pupils were dealing with the following big idea:

1. Different shape but same prize
2. How to determine which one is cheaper or more expensive?

No essential learning trajectory revision is added here. The sequence of the activities in this session can bring the pupils to understand the relationship between size and price. However, one big idea was missed. The idea of shapes whose pieces are the same but it is not necessarily mean that the shapes have the same size was not discussed thoroughly. Perhaps, the teacher could provide 2 shapes and their constructions to trigger the discussion on this manner.

TABLE OF THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
Mini discussion	The students understand the relation between the size and the price among pieces.	1. The students will cut and paste while comparing the pieces one into another 2. The students will consider that those shapes can be divided into 2 triangles	1. The students folded the shapes, the square, the medium triangle, and the parallelogram, into 2 equal small triangles 2. The students found that can be divided into 2 triangles 3. The students understood that those 3 shapes have the same price because formed by same pieces
Finding the price of each own shapes	The students are able to determine the price of each own shapes using their own strategy	1. They sum up the price for each pieces to get the total price 2. They use the fact that the price of the whole pieces is Rp 40.000,-. 3. if they do not use all of the pieces in making their own shapes, then they may just say its price is Rp 40.000,- minus the price of unused piece(s)	1. most of the students forgot about the shape tangram construction, then, the teacher managed to help them to reconstruct the shape and put price on them 2. all of them put price one by one and then summed up all of them
Math Congress	The students are able to argue and to reason in understanding the relationship between the price and the used pieces, or beyond this, the relationship between the price and the size	Here, we can trigger a discussion how to relate the use of prize in understanding the area. The examples of discussion topic that can be used in the discussion are: 1. Different shape but same prize? 2. How to determine which one is cheaper or more expensive? 3. All of the shapes have same prices.	1. The pupils reasoned that the different shapes have the same price because they are composed with the same pieces 2. Teacher tried to guide the students to understand the relation between size and prize

HLT OF MY FOURTH ACTIVITY

COMPARE THE GIVEN SHAPES

The follow up activity from comparing shapes using tangram-pricing model, here we develop the activity such that the students will be able to understand the concept of area conservation without heavily rely on the use of price as the bridge

LEARNING GOAL

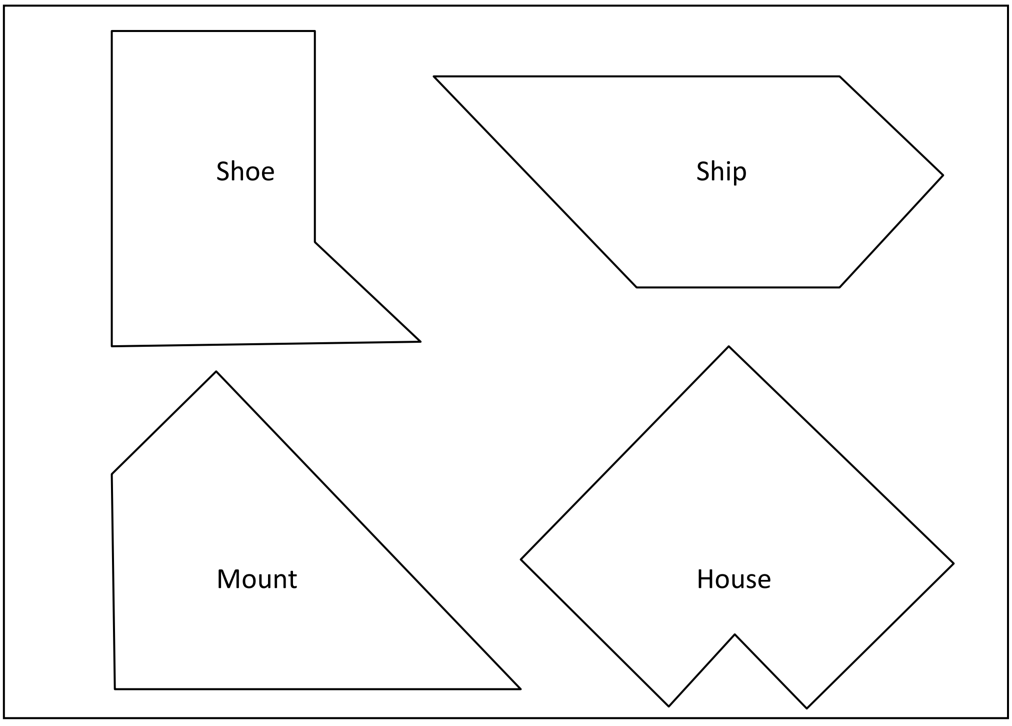
4. The students are be able to understand the one important properties area which is area conservation without using price as reasoning (if we compose and decompose the area into another form the area remain invariant).

ANALYSIS ON LEARNING ACTIVITIES AND ACTUAL STUDENTS' LEARNING

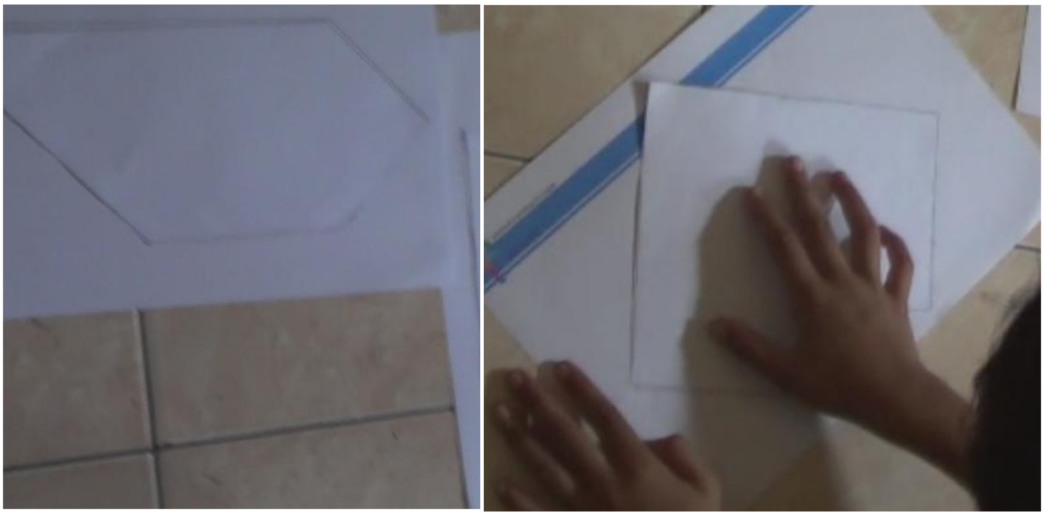
TASK 1 (GROUP DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Compare the Given Shapes	The students are able to compare the given shapes using the idea of area or price

Similar with the previous activity, but in this work, student will play with the shapes given by the teacher in group. The shapes used in this activity are designed feasible for the students to solve in such a time as a group. Here the shapes:



The pupils were provided with the worksheet of the shapes that must be compared. We also provide a cut shape such that they can do hands on activity in comparing the shapes.



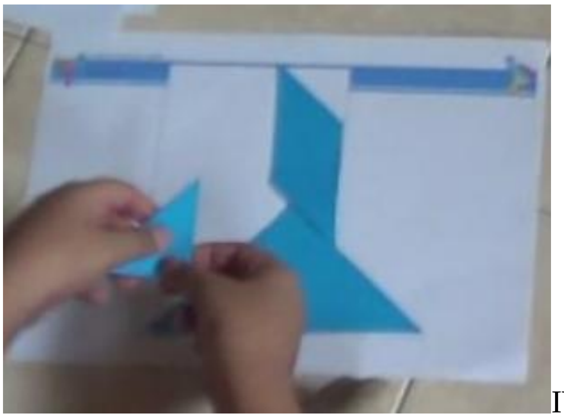
IV_0.1

This worksheet and the provided shapes enabled them to not only explore the shape visually but also compare directly the shapes. They could do overlapping and tangramming strategy. Both of those strategies occur at that time, but, the use of tangramming strategy was being the main concern.

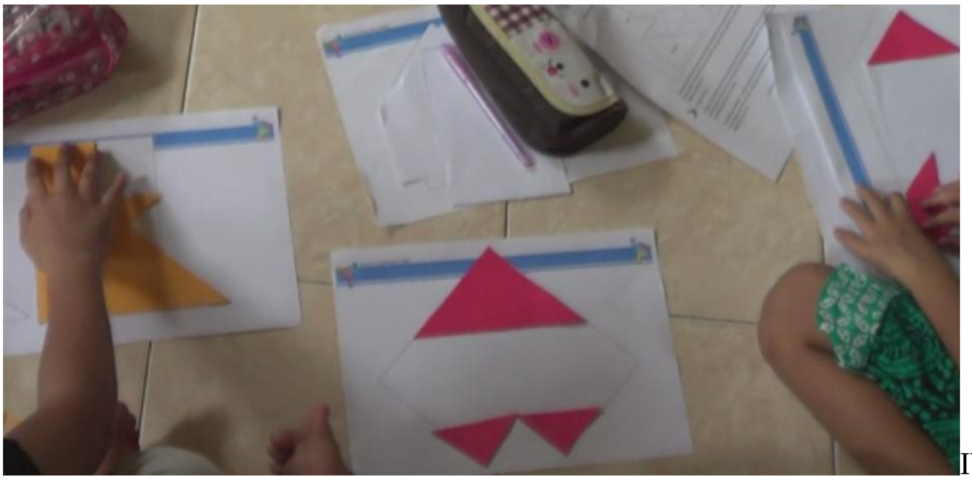
IV_A.1

The picture IV_A.1 suggests that the pupil directly did the overlapping strategy and then he compared the left-over area to determine which shape is bigger. By comparing the shoe and the ship using overlapping strategy, he could find out that those 2 shapes have the same size. However, he got difficulty in comparing the others shapes using overlapping strategy. None of the pupils could make a progress.

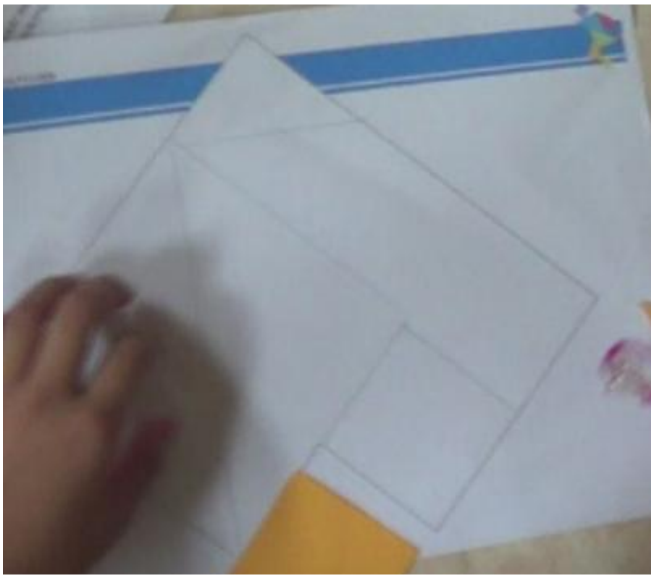
After a couple minutes struggling to compare, the teacher gave a clue for them to use the tangram pieces. In the group discussion, using tangram pieces to compare the shapes was being the main strategy occurred at that time. The use of overlapping strategy was not so salient here because of the difficulty on comparing the overlap and/or left-over area. They spent dozen minutes to figure out the construction of the given shape. Then, they calculated the price by summing the price for each piece.



IV_K.1



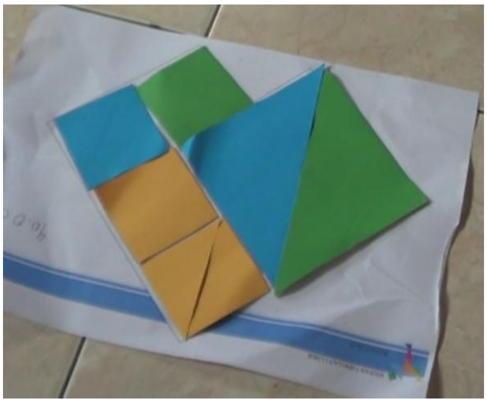
IV_DSF.1



IV_D.1

Once they could find the construction of a shape, they directly drew the construction of the shape. they could not figure out the price directly (IV_D.1). They calculated the price for that shape. Here, they still needed the object as the representation of the price. It suggest that they were really bounded to the context of constructing a shape using tangram pieces. They still could not use the tangram as the model for.

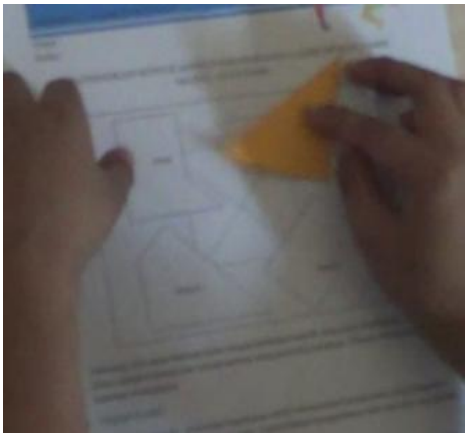
IV_AJK.1



IV_AJK.2

Another strategy was that the pupils used the same pieces from different set. Actually, for a set of tangram, it only has 2 big triangles. In this work (IV_AJK.1), the pupils used 3 big triangles from 3 different sets. Another work was constructed using 3 different squares from 3 different sets (IV_AJK.2). the use of the same pieces from different sets enabled them to construct easily because the pieces can be repeatable easily.

One unique strategy was that one pupil tried to compare the picture of shapes on the worksheet, not the shapes in the real size. Regardless the fact that whether he knew that those pictures were made proportionally, he did not continue his strategy, and joined his friend to construct the shape using tangram.



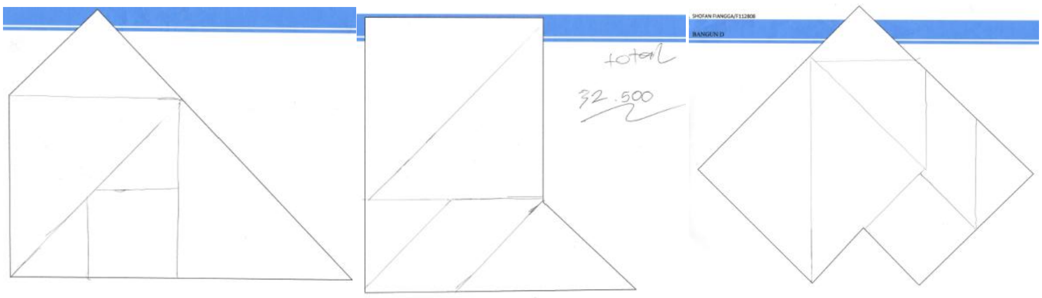
IV_J.2

In this activity, we can see that the pupils heavily relied on the use of the tangram. The conjectured pupils' learning here is that the pupils could reasoning about comparing shapes using the idea of area conservation with/without the use of tangram pieces. But, the fact suggests that the students could not change the tangram pieces model from model of to model for. Regarding the pupils' cognitive development on their age where Piaget, Bruner, and van Hiele, teacher's guidance during the group discussion should also administer decomposing a shape into another shape in this activity to show the pupils about operasionalization of area conservation.

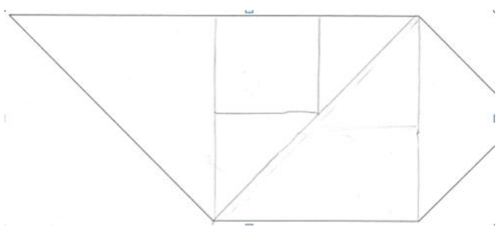
TASK 2 (CLASS DISCUSSION)

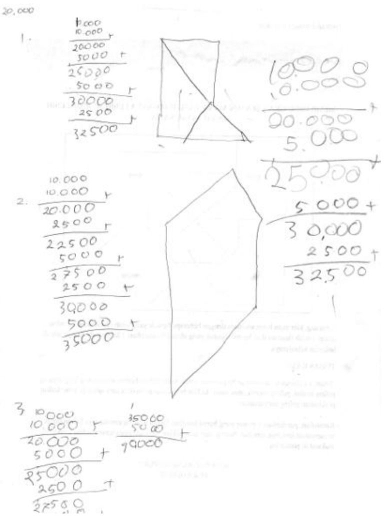
ACTIVITY	SUB-LEARNING GOAL
Math Congress: Share the group work	The students are able to argue about the strategy that they used to compare the given shapes

The presentation was started by the female group. They explained that the house shape was the biggest shape because it had the most expensive price (IV_SDF.1)



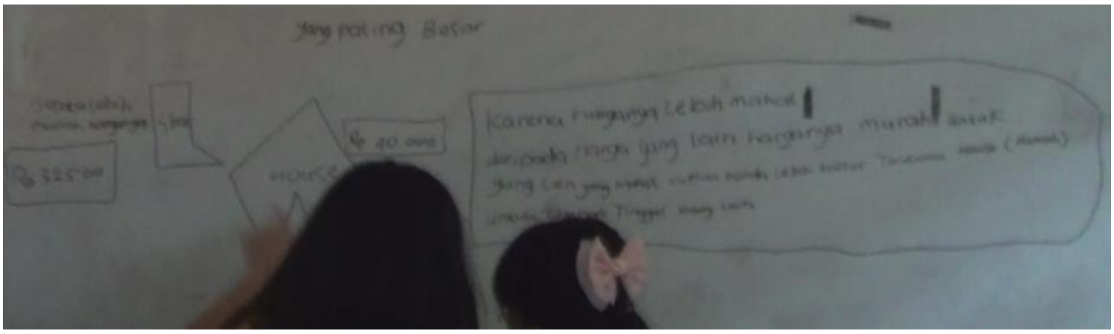
VI_DFS.2 VI_DFS.3 VI_DFS.4





VI_DFS.5 VI_DFS.6

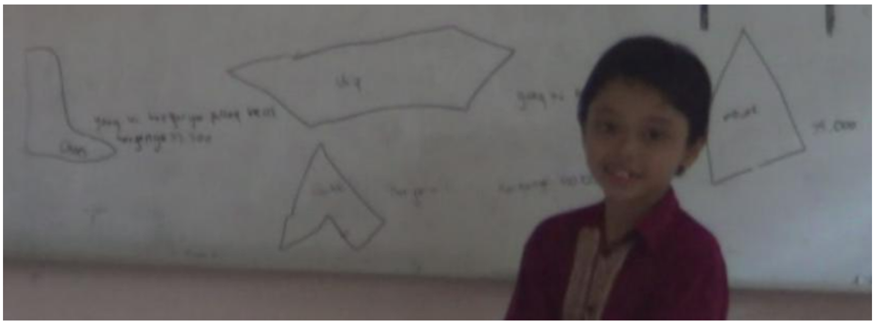
They chose the house-like shape as the biggest shape because it had the most expensive price (IV_DFS.4). They explained that the calculated the price of each shape by constructing first the shape into pieces. They showed their calculation on each shape (IV_DFS.6). They also noted that there were 2 same size shapes because they had the same price (IV_DFS.2 and IV_DFS.5).



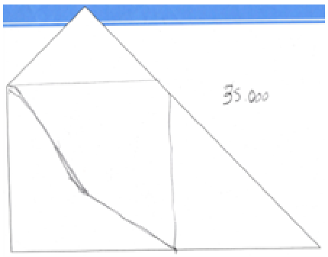
IV_DFS.1

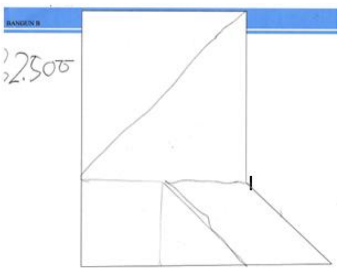
About this female group presentation, they cannot explain their work clearly. What they did was writing their answer only. No discussion took place here because the male group also quite busy playing on their own.

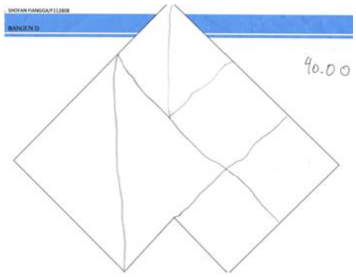
The second was the male group presentation. What they did in the beginning was drawing their works on the whiteboard.



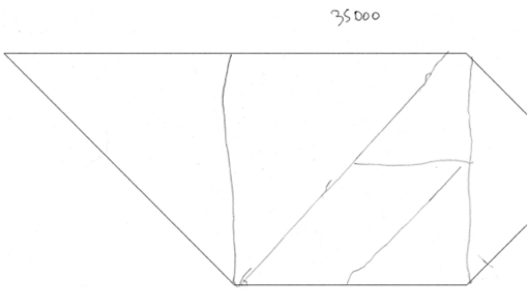
IV_AJK.3

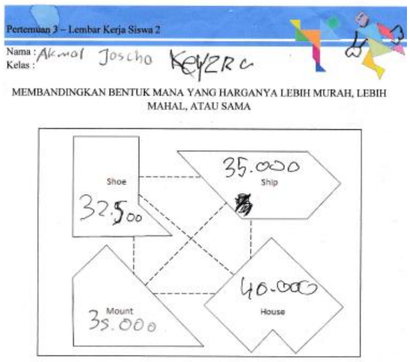






IV_AJK.4 IV_AJK.5 IV_AJK.6





IV_AJK.7 IV_AJK.8

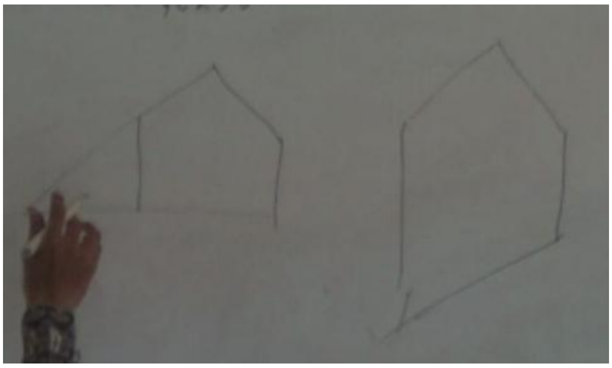
The male group also explained the same solution using the price of each shape as the reason. Actually, in this group, there was a student who compared the shapes using overlapping strategy. But, there was no such strategy explained by this group except tangramming pieces strategy.

In the end of this activity, we expect that the students will understand and accept that every shape has what so called area by which we can determine which one is bigger or smaller. As the result, they could understand that those areas if we compose and decompose the area into another form the area remain invariant (area conservation) (1st goal of my design). However, the teacher feel that the pupils still did not grasp the idea of area conservation after doing comparing activity. Then, the teacher decided to add class discussion about area conservation.

ADDITIONAL TASK IN 4TH ACTIVITY

ACTIVITY	SUB-LEARNING GOAL
Conserve the Given Shapes	The students are able to do area conservation on the given shapes

Before starting the classroom discussion, the teacher gave the overlapping strategy in comparing the shapes in the previous problem. Moreover, he extended the context into composing and decomposing shapes. The following is the discussion on the decomposing idea.



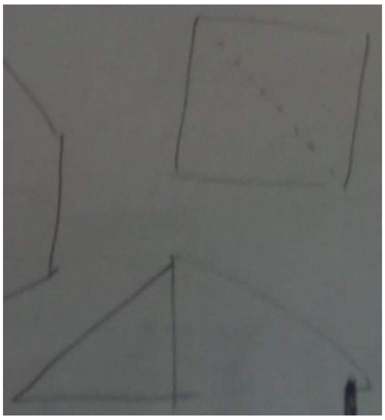
IV_O.4

Teacher	Which shape is the biggest? (showing the shapes)
Akmal	It’s just the same
FIra	No, if decomposing like that, it might change
Teacher	Let see, (draw the 2 shapes and decompose) What do you think, I can decompose this shape into another one, does the area change?
Pupils	Oh yah, just the same

LP-4.8-2.40

From the excerpt above, the pupils had difficulty in understanding that if we decompose a shape the area must remain invariant. However, the use of priced shape context helped the pupils to figure out the conservation property.

The first discussion was to compare 2 shapes, a square and a triangle.



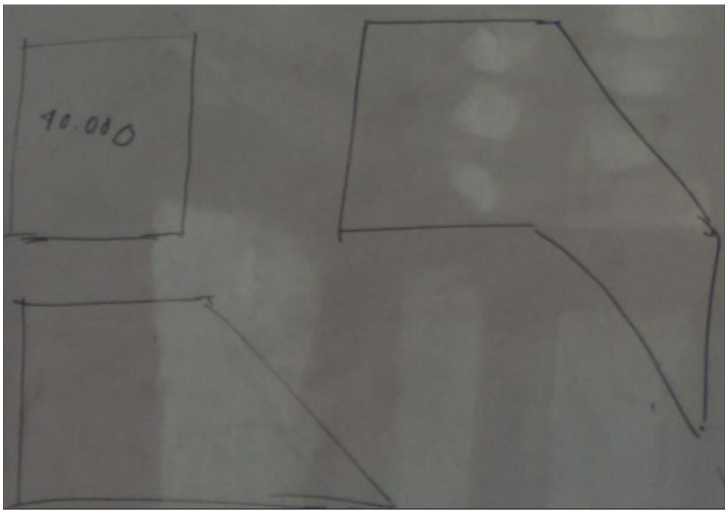
IV_O.5

IV_J.4

Teacher	Yeah, Which shape is the biggest? (drawing square and triangle)	
Akmal	It’s just the same	
Josca	Ya	
Teacher	Ok Josca, could you explain	
Josca	Ok, (walking)	
	Ya, if this part is moved into this place (drawing a triangle into a square)	
Pupils	Oh yah, just the same	LP-4.9-0.10

This excerpt suggests that the pupils had already been able to perform decomposing activity which makes the area invariant. The idea had been grasped by the pupils. The next big idea is using the concept of area conservation to measure an area of certain shape using provided shape as reference.

The use of area conservation in measuring an area was delivered by the teacher using the following problem.



IV_O.7

The problem was given as 3 shapes which are a square, a trapezoid, and a polygon such that the area of trapezoid is 1.5x the area of square and the area of polygon is 2x the area of square. The teacher provided as well the price for a square 40.000. then, the pupils task was to determine the price for other shapes.

Here the discussion between teacher and pupils:

Teacher	Ok, can you find the price of this shape? (pointing trapezoid)	
Akmal	It is 60.000	
teacher	Ya, why is the price 60.000	
Fira	Because, it is bigger than square	
Teacher	Hmmm, but it does not guarantee, you can say 70.000 or 80.000. why must it be 60.000?	
Akmal	It is 50.000, if we cut here (cut the trapezoid into a square and a triangle)	
Teacher	So,the triangle is…	
Akmal	10.000 toh	
Teacher	Loh, are you sure, take a look on this square	
Josca	The triangle is 20.000	
Teacher	Hei, take a look on the triangle and the square. What is the relationship	
Akmal	It is a half of square, so 20.000	
Pupils	So, that shape must be 60.000	LP-4.10-0.10

The excerpt above indicates that constructing a shape into familiar shapes (a trapezoid into a square and a triangle) was not a difficult task. The problem was that they still were bounded into the context of tangram. That is the reason of why they consider the price of the triangle was 10.000 not 20.000.

The following is further discussion on measuring the price of the polygon

Teacher	Ok, then how about this one (pointing on polygon)
Pupils	70.000,80.000
Teacher	Ok,anyone wanna explain it?
Fira	I'll try (walking) Hmmm, I dunno Pak
Teacher	Use the previous shapes
Fira	Hmmmm
Teacher	Yah, anyone wanna help Fira
Fira	Putri DIka here…
Dika	See, this shape is square, so it must be 40.000
Fira	Ya, so it is 40.000, then the rest? Is it 35?
Teacher	Loh, are you sure?
Akmal	See this, this can be cut into triangles So it must be 2x20.000

Fira Oh ya(drawing the line)
So it is 20.000 and 20.000 LP-4.10-2.05

IV_DFK.8

The excerpt suggests that they still could not use the concept of area conservation directly. but once they had had help about how to divide the shape (IV_DFK.8). They could progress in finding the price. Moreover, they did not use 10.000 as the price for triangles because we had already discussed it just before this conversation. If the model for reasoning has established in the beginning of activity, it would be used more as model for reasoning.

This additional activity was quite helpful to achieve the learning goal that has been established before. The problem to measure certain shapes using a shape as preference is an operational activity of the big idea of using the concept of area conservation in measuring area. For the next cycle, this activity should be more elaborated and planned. Moreover, the use of money should be omitted to suppress bounded context situation

TABLE OF THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
Compare the Given Shapes	The students are able compare the given shapes using the idea of area or price	Here the conjectured students solutions of the problem 1. Estimate the shape by using overlapping strategy to compare 2. The use of tangram pieces to reconstruct and put price to compare	The students directly used the strategy that they also used to put price on their own shape (previous activity) They were playing the tangram by constructing the given shapes. Then, they calculated the price by adding the price for each piece. In this activity, we can see that the pupils heavily relied on the use of the tangram. The conjectured pupils' learning here is that the pupils could reasoning about comparing shapes using the idea of area conservation with/without the use of tangram pieces. But, the fact suggests that the students could not change the tangram pieces model from model of to model for
Math Congress: Share the group work	The students are able to argue about the strategy that they used to compare the given shapes	During the discussion, these ideas are conjectured from students thinking: 1. Compare using overlapping, they will find the arrangement. However, the students do not use the price to reasoning that will make the discussion about area could occur 2. calculate the price and then make the order of the shapes from the cheapest to the most expensive one	The students explained that they reconstruct the shape using the available tangram. A group made use the tangram pieces from others such that they had 3 big triangles in his construction (a set tangram only consist of 2 big triangles) No group came up with overlapping strategy, then the teacher try to explain this strategy to compare which shape is the biggest or the smallest. The students seemed to understand it. The teacher gave several object on the whiteboard and ask them to compare and most of the students understood it well. However, the teacher feel that the pupils still did not grasp the idea of area conservation after doing

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
			comparing activity. Then, the teacher decided to add class discussion about area conservation
ADDITIONAL TASK Conserve the Given Shapes	the students are able to use the idea of area conservation in measuring the price of the given shapes	The students will use decomposing using the tangram pieces like triangle, square, and parallelogram to conserve one shape in to another shape	The problem to measure certain shapes using a shape as preference is an operational activity of the big idea of using the concept of area conservation in measuring area The pupils had difficulty in understanding that if we decompose a shape the area must remain invariant. However, the use of priced shape context helped the pupils to figure out the conservation property. Constructing a shape into familiar shapes (a trapezoid into a square and a triangle) was not a difficult task. The problem was that they still were bounded into the context of tangram. That is the reason of why they consider the price of the triangle was 10.000 not 20.000

HLT OF MY FIFTH ACTIVITY

COVERING AREA BY USING ONLY ONE KIND OF PIECE OF TANGRAM

After the students have already understood about the concept of area conservation, then, the learning activity moves to the area measurement issue. Here, the starting of measuring activity as covering like what the students have done in the previous activities but, here, they are only allowed to use one kind of tangram pieces

LEARNING GOAL

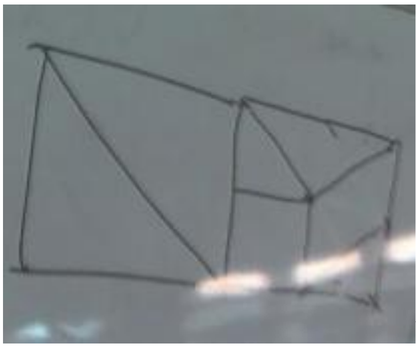
5. The students are be able to do covering the area as measurement activity by using different kind of tangram piece.
   a. The students are able to use one kind only tangram pieces from the available possible tangram pieces to fully cover the given shape.
   b. The students are able to understand the relation covering the shape using triangles or squares only as measurement activity.

ANALYSIS ON LEARNING ACTIVITIES AND ACTUAL STUDENTS' LEARNING

TASK 0 (INDIVIDUAL)

ACTIVITY	SUB-LEARNING GOAL
Decompose the rectangle into parallogram	The students are able to decompose and compose a rectangle shape into parallelogram shape

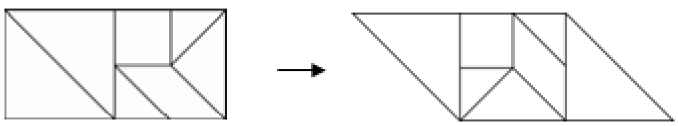
Before entering the main problem, the teacher gave a mini lesson on area conservation. The problem in this mini lesson was decomposing and composing a shape into another shape using the tangram pieces. The first task was that the pupils were asked to make a construction of a rectangle using the available tangram pieces. The pupils could construct the rectangle using the tangram pieces. They had no difficulty in making the construction (V_O.2).


V_O.1


V_O.2

The next task was decomposing a rectangle into a parallelogram. The pupils had difficulty to decompose the shape of rectangle into parallelogram. the teacher then guided them by providing the tangram construction for the rectangle. We expect that the pupils would just take one piece and put it in another position.


V_O.1



The pupils were struggling with the problem. None of them did the same like what we had expected. There was a pupil who constructed the parallelogram using 2 big triangles only (V_K.1). Others pupil broke the construction of rectangle that they had made and rearranged the pieces into parallelogram.


V_K.1


V_S.1

The teacher asked the pupils to decompose the construction from rectangle shape into parallelogram shape by picking up a piece and putting it in another position. They succeeded in picking up the right piece but had difficulty in putting that piece on the right place.

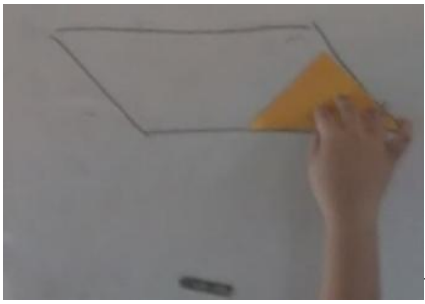

V_S.2


V_D.1

One of the pupil picked up to much pieces. This made the construction was smaller (V_S.2). another pupil also picked to much pieces. That made them difficult to construct the right shape. Here, we can see that, in constructing a shape from one shape, omitting pieces to change the shape into another shape is easier than adding pieces to change the shape.


V_S.3


V_S.4


V_S.5


V_S.6

The pictures above show how S managed to construct the parallelogram shape. She incorrectly made a parallelogram (V_S.3). She did not know how to fill the remaining area with the only remaining piece (V_S.4), and then he compared the last piece into the picture (V_S.5). After she had done constructing the shape (V_S.6), she helped another pupils to finish the task. It seems that the tangram model still had not changed from model of to model for because she still needed to consider the physical objects there. However, the next problem shows that the pupils easily decomposed and composed the parallogram into triangle (V_K.2; V_F.1; V_D.2).


V_K.2


V_F.1

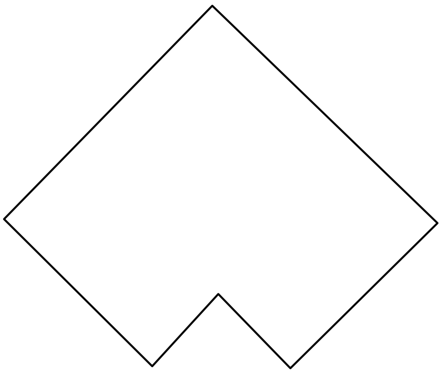

V_D.2

The analysis of this mini lesson suggests that area conservation using physical object, in the beginning, is quite difficult to recall by the pupils. But once they are shown how to decompose and compose a shape into another shape, it becomes routine task.

TASK 1 (INDIVIDUAL)

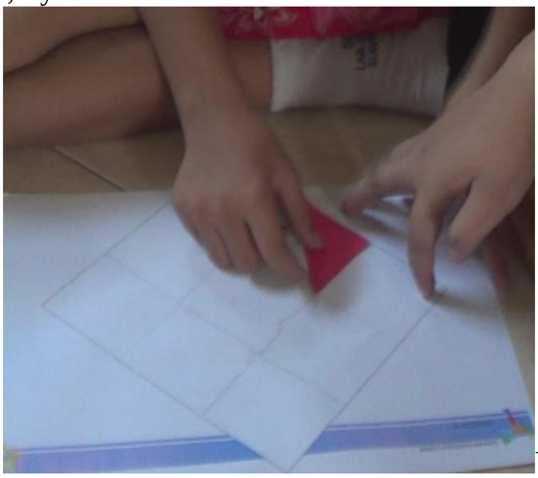
ACTIVITY	SUB-LEARNING GOAL
Covering a given shape using one kind piece only	The students are able to use one kind only tangram pieces from the available possible tangram pieces to fully cover the given shape.

Using only one kind of shape of tangram piece, the students are asked to use it to cover a given area. The area is as follow:



The pupils tried to mix and match finding which shape of piece could be used to cover the whole area. It took them quite sometimes to find the correct shape of piece to cover the given area completely. All of them found that using square piece is the only way to cover. It seems that it is the easiest piece to be used in this shape.

Sisca	I have done Sir	
Teacher	Ok, have you tried to use this shape (pointing small triangle shape)	
Sisca, FIra	Yes, I have. It was overlap here (pointing the shape)	
Teacher	Are you sure?	
Fira, Sisca	Yes, we have used it Sir. It was overlapped here.	
Teacher	Ok, let's try using this piece carefully	
Fira	Here,if I put here and here, hmmmmm (putting the piece in the bottom)	
Sisca	No, it was not overlap there, but here (putting the piece in the left side of shape) hmmmm, was it really overlap? … Yes Sir, it seems that it is possible	
Teacher	Ok, try it out	LP-5.7-0.15



V_FS.1

The excerpt above tells about how the pupils did not cover the area correctly which made them found out that small triangle piece was inappropriate to cover the area. Fira and Siska said that the triangle cannot be used. The teacher asked them to show how they were sure that the small triangle piece could not be used for covering. When they were trying to show that it cannot be used, they changed their main after they figured out the square can be changed by 2 small triangles.

During the individual work, the pupils protested that the teacher explained more to a certain pupils.

Dika	Hmmm,Pak, You were unfair
Teacher	How come?
Dika	You explain to the answer to Fira and Siska And it is different with my answer here
Teacher	Hmmm, don't say like that, your work is also correct
Dika	Really pak?

Teacher	Yes, why can'r. the answer of a problem is not always only one solution	
Dika	OK pak	LP-5.7-1.20

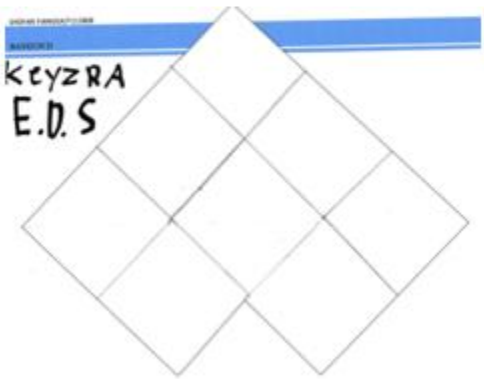
The excerpt above suggests that the pupils are not accustomed by the non-routine task. This task which has more than one solution seems to be non-routine task for them.

The shapes given in previous activity are also design for this activity of which the students can play by covering those shape using only one kind of shape. In covering the shape, we conjecture that they will use small triangle shape only or square shape only. The fact was the same with what we had conjectured. However, it seems that they did not use it as area unit.

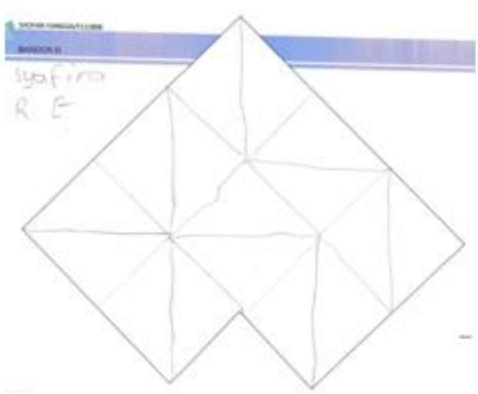
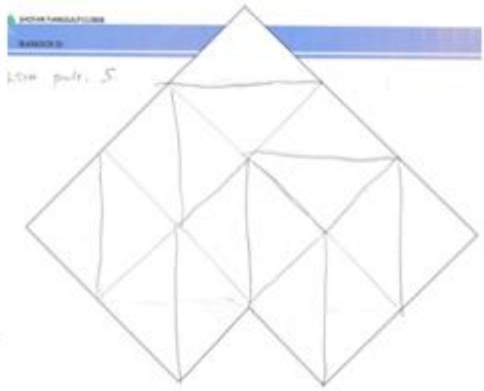
TASK 2 (CLASS DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Math Congress: Share the group work	The students are able to understand the relation covering the shape using triangles or squares only as measurement activity.

There were 2 different solutions appeared from the pupils. The first solution was that they used square piece to cover the shape. The other solution, using small triangle piece, was originated from the first solution. They extended the possibility using the relation that the square can be divided into 2 small triangles.



V_K.3 V_D.3

V_F.3 V_S.7

2 pupils found the answer using square piece. They got that they needed to 8 square pieces to cover the shape (V_K.3, V_D.3). Meanwhile, the remaining pupils succeeded to find out that small triangle shape could also be used for covering the shape. They figured out that they needed 16 small triangle pieces to cover the shape. (V_F.3, V_S.7).

Teacher	Ok, if I ask you,how many pieces needed to cover the shape?
Pupils	1,2,3, … (counting the drawn pieces on their work)
Teacher	Are you sure?
Keyzra	8 sir
Dika	Yes, 8
Teacher	8, yah, are you sure?
Siska	Precisely, 16 Sir
Teacher	Yah, 8 or 16, which one?
Siska	Just the same pak
Teacher	Why?
Siska	Because it was cut pak (showing a square piece and cutting it into triangle pieces) It was cut in to 2 pieces pak So, 16 here is the same as 8 pak
Teacher	Ok, then, this shape consists of
Fira	16 triangle pieces
Teacher	Or…?
Fir, Siska	8 pieces, 8 square pieces
Teacher	Ok

LP-5.8-0.15

The excerpt above suggests that the pupils could reason why there were 2 answers for this problem. They used the relationship between the square pieces and the small triangle pieces to explain that 16 small triangle shapes is as big as 8 square pieces.

In the end of this activity, we expect that the students will understand covering the area as measurement activity by using different kind of tangram piece may result same area.

TABLE OF THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Actual Students' Learning
ADDITIONAL MINI LESSON Decompose the rectangle into parallogram	The students are able to decompose and compose a rectangle shape into parallelogram shape		
Covering area by using only one kind of piece of tangram	The students are able to use one kind only tangram pieces from the available possible tangram pieces to fully cover the given shape.	The students have difficulty dealing with the insufficient number of pieces to be used: If not, then here the possible solutions in covering the shapes by the students: 1. the students may use square pieces only 2. the students may use triangle pieces only	The students had difficulties in doing the task. The students just try to put different pieces on the shape given. (it's better if we give an easier shape to cover first before doing it on bigger one) Firstly, they did not know that they can use triangles because they only see squares were fit. Then, they noticed that it is possible to use small triangle because they had already known the relationship between squares and triangles
Math Congress: Share the group work	The students are able to understand the relation covering the shape using triangles or squares only as measurement activity.	Some idea may occur in this class discussion: 1. one square can be divided into 2 triangles 2. the square and the small triangle are suitable for covering such shapes	The discussion was taking place where they can use the small triangles after dividing their picture of squares into triangles They understood that either 8 squares or 16 small triangles is the same answer

HLT OF MY SIXTH ACTIVITY

Playing with the area of polygon

This activity is designed such that the students are able to use what they have explored in the previous activities which are the concept of area conservation and measuring area by covering. Then, the teacher may post a problem of area conservation from rectangle to parallelogram using tangram to remind the student understanding of area conservation from previous lesson..

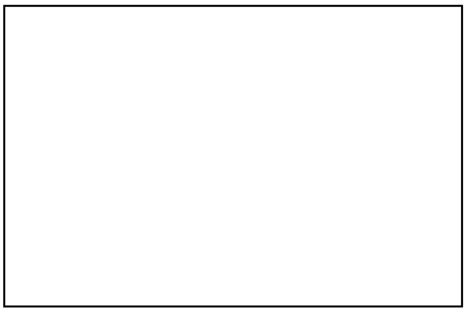
LEARNING GOAL

6. The students are able to apply the concept of conservation to measure the area of polygon.

LEARNING ACTIVITIES AND CONJECTURED STUDENTS' LEARNING

TASK 1 (GROUP)

ACTIVITY	SUB-LEARNING GOAL
Playing with the area of polygon Measuring polygon shapes	The students are able to use the pieces from the previous activity to measure given polygon shapes



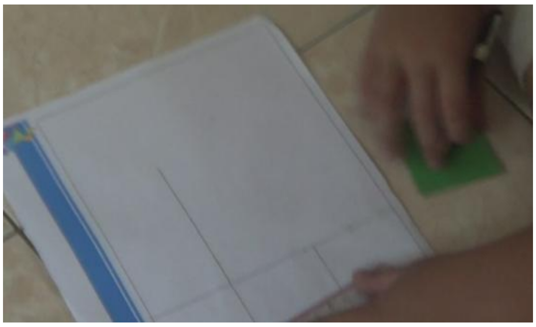
1st polygon shape to becovered

The activity was started by covering the shape. the teacher asked the pupils to find how. Due to the experience that they had in the previous activity, here, they directly used the small triangle pieces and the square pieces. During the covering activity, none of them used small triangle pieces to cover.

Teacher	Why don't you use this small triangle piece?	
Kezra	I don't like	
Fira	Because,the shape is rectangular, so you cannot use it	
Siska	No, you can.	
Fira	Hmmmm, (trying to cover using the small triangle piece) Oh ya, it could. But it is easier using this square	
Teacher	Ok	LP-5.12-0.15

The excerpt above shows that the pupils chose to use the square piece because it is easier to be used as cover. If they used the triangle pieces, it would take more time in covering the shape.

VI_S.1

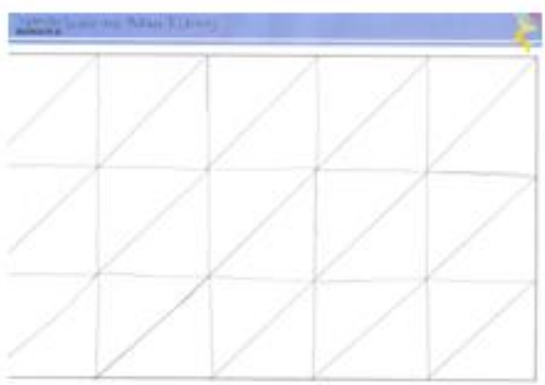


VI_K.1

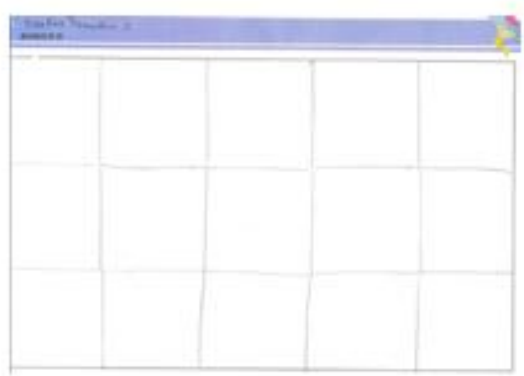
VI_F.1

VI_D.1

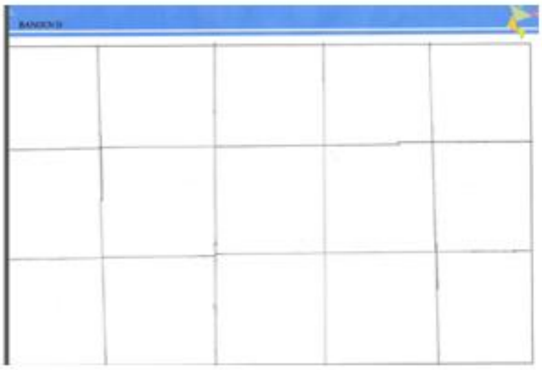
In covering the shape using the square piece, some of them used the square piece directly to draw how many square used in covering the shape (VI_S.1; VI_F.1; VI_S.1). There were a pupil who use ruler to draw the square gridlines with the square piece as the reference (VI_K.1). there was no difficulty faced by the pupils in covering in such a way. However, our conjecture that they will use simple multiplication to find the number of covered shapes did not occur.

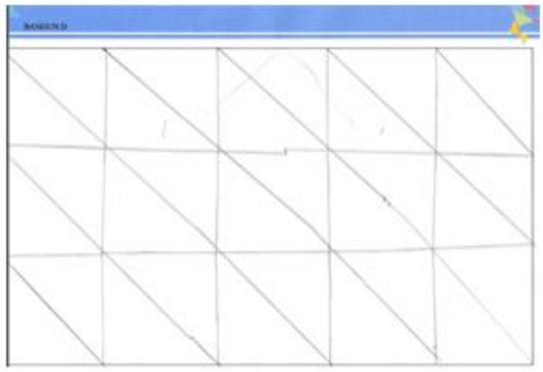


VI_D.2



VI_F.2

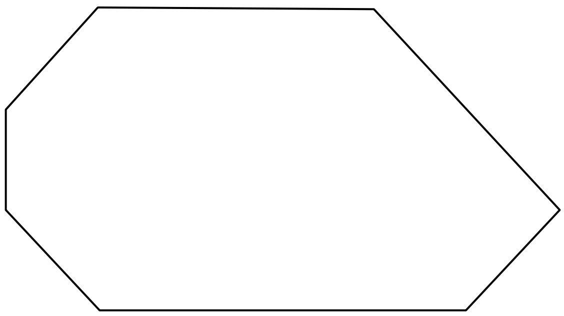




VI_S.2 VI_K.2

The pupils could find that they need 15 square pieces or 30 small triangle pieces. They could find how many small triangle pieces needed even without using the pieces. They covered the shape using square pieces pattern and then they split each square into 2 small triangle pieces (VI_K.2; VI_D.2). The pupils have already been able to use the relationship between the small triangle and the square piece to find out the number of pieces needed.

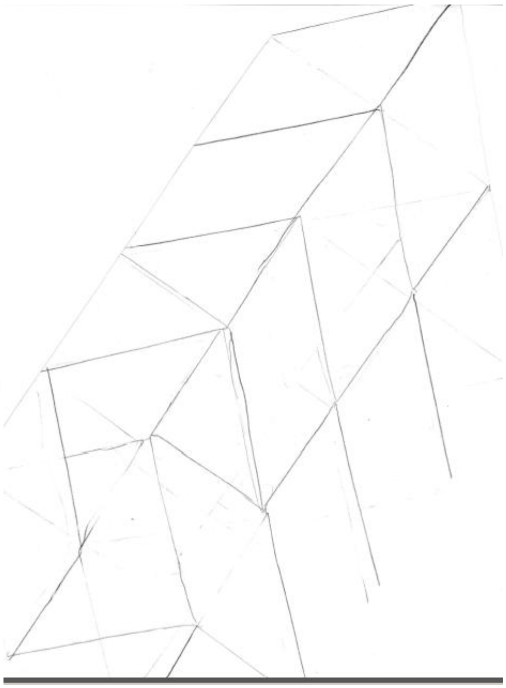
The second task was covering the following shape. This shape was designed such that they pupils could use the 1st polygon shape and conserve the area into the 2nd polygon.



2nd polygon shape to be covered

They spent very long time working in this task. Yet, they could not finish the task. Teacher guidance had been provided to use the 1st polygon shape as the reference to measure this shape in square. The pupils kept busy covering the shape using small triangle pieces.

VI_KS.1



VI_DF.1

One work was not finish because they worked inconsistently in putting the pieces into the shape (VI_KS.1). The different orientation on putting the pieces resulted disproportional side to cover the shape correctly. That made the pieces overlapped each other. Another team tried to use pattern in covering the shape (VI_DF.1). The use of pattern helped them to see the covering pieces using one kind only piece. Even though they were informed that they could extent their solution to the use of small triangle pieces, the still could not get the relationship.

Due to the fact that the pupils took quite long time in doing covering activity on the given shapes, we did not have time to ask the pupils to share their works in Math Congress. It seems that covering activity was being the main pupils' concern at that time regardless the conservation area as the learning objective of this activity.

To make it better, omitting the covering activity seems promising to lead the pupils to be more focused on the idea of area conservation. Furthermore, providing a direct conservation task will bring the pupils to experience the conservation longer.

TASK 2 (CLASS DISCUSSION)

ACTIVITY	SUB-LEARNING GOAL
Math Congress:	The students are able to argue about the strategy that they used to

Share the group work	find the price. In the end, the students are able to use the concept of area conservation in measuring the area of a shape

Due to the fact that the pupils took quite long time in doing covering activity on the given shapes, we did not have time to ask the pupils to share their works in Math Congress. It seems that covering activity was being the main pupils' concern at that time regardless the conservation area as the learning objective of this activity.

To make it better, omitting the covering activity seems promising to lead the pupils to be more focused on the idea of area conservation. Furthermore, providing a direct conservation task will bring the pupils to experience the conservation longer.

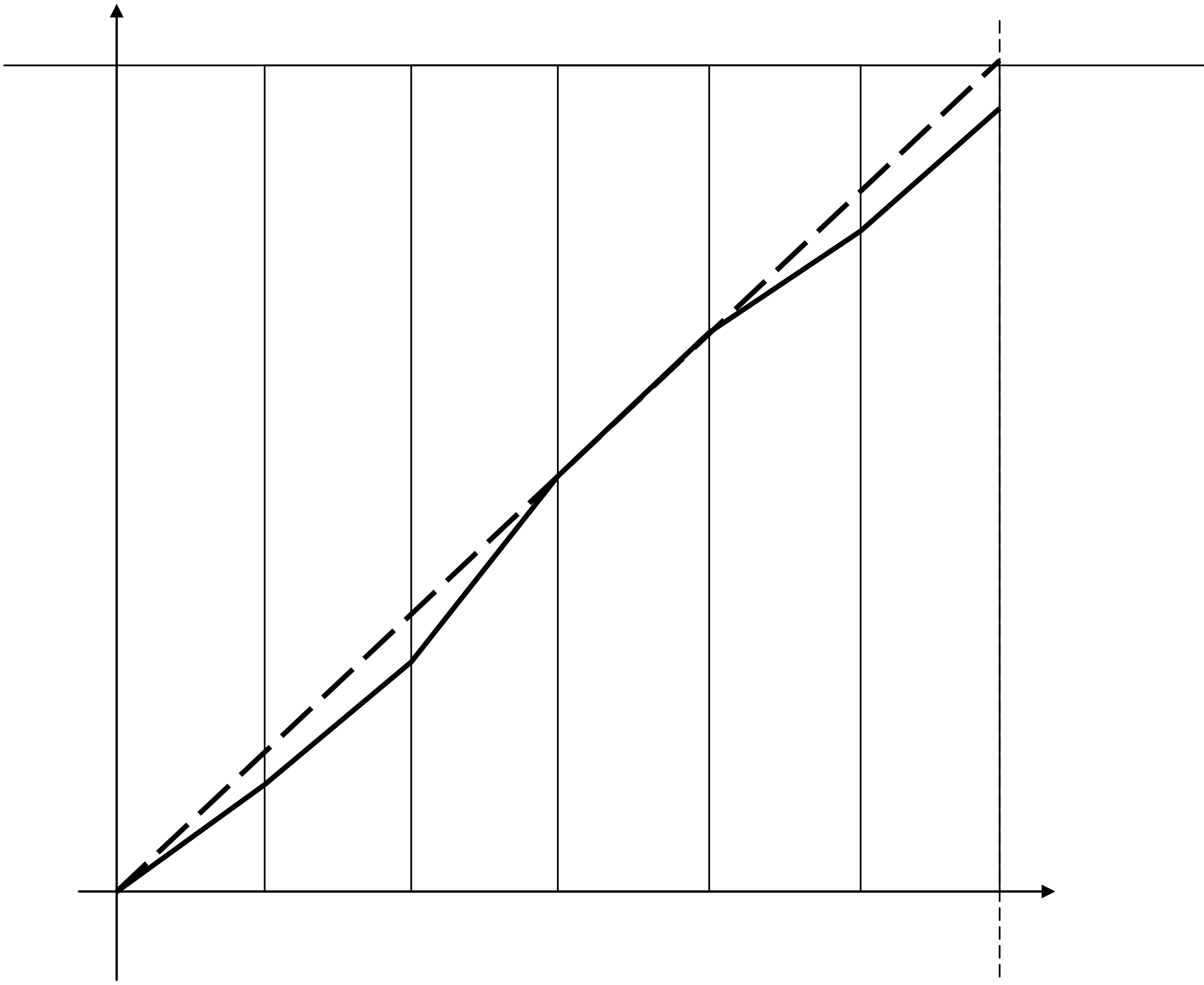
After all, the students had already understood that if we cut a piece from a shape and put it into another position such that a new shape is formed then the area remain invariant. But when measuring the shape in squares or in triangles they failed. Covering activity seemed disturb the pupils learning trajectory. It results that conservation was not being the main issue but covering.

TABLE OF THE OVERVIEW OF THE ACTIVITY AND THE HYPHOTHESES OF LEARNING PROCESS

Activity	Learning Goal	Conjectured of Students' thinking	Actual Student' Learning
Playing with the area of polygon Measuring polygon shapes	The students are able to use the pieces from the previous activity to measure given polygon shapes	The students may use several strategies: 1. Using all pieces to cover up the shape one by one 2. Using only several pieces and do estimation then 3. Using the previous shape that has been covered and use it as starting point to cover	The students had difficulties in doing the task. The students just try to put different pieces on the shape given. After the teacher guidance, the students started to cover up using the 1st strategy. The students understood that the use of both small triangles and square is just the same.
Math Congress: Share the group work	The students are able to argue about the strategy that they used to find the price. In the end, the students are able to use the concept of area conservation in measuring the area of a shape	During the discussion, here the conjectured topic discussed: For the 1st polygon 1. they cover using pieces one by one 2. They realize that there will be 5 columns and 3 squares are in a column 3. They realize that there will be 5 columns and 6 triangles are in a column, For the 2nd polygon 1. they cover using pieces one by one 2. They realize that there is a connection of this 2nd polygon with the 1st polygon. They will use the available pieces, either the square or the small triangle pieces to measure and compare those 2 polygons 3. They realize that there is a connection of this 2nd polygon with the 1st polygon. They will cut and paste the polygon into another polygon	For the 1st polygon After the teacher guidance, the students started to cover up using the 1st strategy. Then they also counted the squares one by one and found that there are 15 squares or 30 small triangles. For the 2nd polygon They cannot find the relationship with the previous shape. What they did was just covering one by one but failed to make it precisely. It seems that we need to add additional activity to bridge this difficulty. A small version of this activity would satisfy. After all, the students had already understood that if we cut a piece from a shape and put it into another position such that a new shape is formed then the area remain invariant But when measuring the shape in squares or in triangles they failed. Covering activity seemed disturb the pupils learning trajectory. It results that conservation was not being the main issue but covering

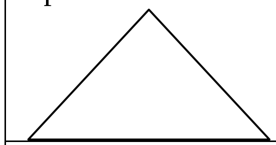
Make clear the description of the activity

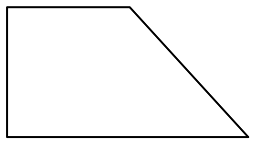
(if refers to CHAPTER 4



HLT OF MY FIFTH ACTIVITY

TABLE OF HLT VALIDATION ON 1ST AND 2ND CYCLES

Activity		Learning Goal	First Cycle	Second Cycle	Remark
Orientation with the tangram Constructing own shape using more pieces and record it	1	The students are accustomed to play with tangram The students are freely construct their own shapes using 7 given pieces and then draw a sketch of it.	The students directly played with the tangram and made more than one shape. After they decided which shape to choose, they drew the sketch but most of them cannot draw clearly. There were a house-like shape, a car-like shape, a tree-like shape, and others abstract shapes.	The students can play with the tangram well and made more than one shape. They cannot draw the sketch of the shape that they have made well. There are mountain scenery shapes, a people-like shape, and others abstract shapes.	The students play informally with the idea of conservation. They try to find various constructions for one shape. The constructing activity give the students the experience of decomposing a shape into constructible pieces.. The students would not realize if they are studying but playing puzzle game. The drawing task should be omitted or revised properly
Constructing a given shape using 3 pieces		The students are able to construct a certain shape using 3 pieces 	Most of the students are able to solve the construction but only give 1 solution Some of them still copied the works of their friend	Most of the students are able to produce 2 different solutions. Several of them are able to make more than 2 solutions Some of them are unable to provide a correct solution	Providing 2 or more empty shape for the students to draw the solutions can trigger them to be more creative to find other solutions.
Constructing a given shape using 2 or		The students are able to construct a certain shape using 2 or more pieces	The students have difficulty in solving this construction. However, only 1 solution produced from each students.	The students can cover the shape using available pieces mostly by using 2 and 4 pieces. Only a few students were unable	Providing 2 or more empty shape for the students to draw the solutions can trigger them to be more

more pieces.				to finish this task. One of them gave incorrect solutions which leave a gap and others use the pieces twice. The teacher's instruction about avoiding gap and overlap is important here to make a good construction and could correct the mistakes that had been done previously	creative to find another solutions.
Finding the price of each piece from the context given	2	The students are able to determine the price of each piece using their own strategy	The students give a price for each piece unreasonably. They just added and subtracted the price for each piece such that their sum is 40.000 Another idea was just dividing 40.000 by 7	The students can find the price for each piece. The students could not understand fully the teacher explanation and they prefer to guess the answer by using trial and error number	Trial and error in choosing the correct price for each piece became more salient than using logical approach regarding the shape. The students tend to use numerical manipulation to find the answer by trial and error in choosing the price to get the total number correctly rather than using logical geometry approach.
Math Congress: Share the group work		The students are able to argue about the strategy that they used to find the price	The students sometimes cannot explain the reason of the price logically. They seemed only using numerical reasoning by using the fact that they must be summed into 40.000	Only one type of solution discussed by the students. The students were not eager enough to involve in a discussion.	Discussing reasonable price regarding its size can make the students find the correct solution rather than doing trial and error strategy.
Mini Lesson: Why	4	The students are able to understand that 2 or more	The students find that those shapes can be divided into 2 triangles	The students misunderstand that the parallelogram has the same price with the square because	The students start to find out that the number of pieces used in constructing

square and parallelogram have the same price		shapes which have the same price have the same size of area	The students understand that those 3 shapes have the same price because formed by same pieces	they have the same pieces construction.	a shape could determine the size.
Compare the Given Shapes		The students are able to compare the given shapes using the idea of area or price	The students directly put price on their own shape. But, then, they construct the given shapes and calculated the price by adding the price for each piece. In this activity, we can see that the pupils heavily relied on the use of the tangram.	The students still consider the number of pieces determine the size of the shape. The teacher guide them to find out that using the number of pieces only is not enough	The use of number of pieces is still used by the students to reason about the size. However, in this activity, the size of pieces must also be considered to determine the size.
Math Congress: Share the group work		The students are able to argue about the strategy that they used to compare the given shapes	The students explained that they reconstruct the shape using the available tangram No group came up with overlapping strategy	In general, all of the students could find the correct price for each shape. However, not all of them could finish their report on a paper completely	Most of them use the price on each shape to compare the shapes. The price relates the abstract concept of area with the students real world fundamentally.
Decompose and compose 2 given polygons into a square	5	The students are able to do conservation of a shape into another shape which has the same area	The students heavily rely on the use of pieces rather than using the idea of conserving the shape to figure out that those shapes have the same area.	The students are able to do area conservation from a given polygon into a square. They did not cut and paste the polygon but they decomposed into pieces and recomposed the pieces such that it forms a square	In the last cycle, the students have already succeeded to apply the concept of conserving area in comparing different shape. The decomposing of tangram is applied by the students to conserve the shape
Covering A Given Shape		The students are be able to measure the area of certain	The students choose to make the unit one by one and count it one by one as well. The idea of	In general, all of the students cannot directly use the idea of conservation in measuring	To invite the students to use area conservation in measuring a certain area

Using given pieces		polygons using the concept of area conservation	conserving shape is not used by the students at all.	except do counting on it.	still need some activities to go. The previous activities given are still insufficient to guide the students to conserve in measuring area.