# DEVELOPING STUDENTS UNDERSTANDING OF THE CONCEPT OF CONSERVATION OF AREA AS A PREPARATORY FOR LEARNING AREA MEASUREMENT

# **MASTER THESIS**



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THE STATE UNIVERSITY OF SURABAYA POSTGRADUATE STUDIES MATHEMATICS EDUCATION

2013

# DEVELOPING STUDENTS' UNDERSTANDING OF THE CONCEPT OF CONSERVATION OF AREA AS A PREPARATORY FOR LEARNING AREA MEASUREMENT

#### MASTER THESIS

A Thesis Submitted to Surabaya State University Postgraduate Program as a Partial fulfillment of the requirements for the Degree of Master of Science in

Mathematics Education Program

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2013

This thesis is dedicated to **my parents** For their endless love, support and encouragement Also to **my sisters** for their support and understanding And last is for all **my big family** who always supports me to have a higher education. **Thanks** 

#### ABSTRACT

Funny, R.A. 2013. Developing Students Understanding of the Concept of Conservation of Area as A Preparatory for Learning Area Measurement. Master Thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University. Supervisor: Prof. Dr. Siti. M. Amin M. Pd., and Dr. Agung Lukito, M.S.

# *Keyword:* students' understanding, the concept of conservation of area, preparatory for learning area measurement

The concept of conservation of area is belongs to the concept which must be learned by the students in order to understand the concept of area measurement. Piaget argued that the concept of conservation of area should be taught as preparatory to learn the concept of area measurement. Hence, to know how the students learn the concept of conservation of area as a preparatory to learn the area measurement, this study will design an instructional learning activity to teach the students in the third grade of Elementary School in Surabaya. This study conducted before the students learn the concept of area measurement.

Design research method is used as an approach in this study. It was designed a sequence of five learning activities by using the Realistic Mathematics Education (RME) approach. Real life context which is making table-cloth and the shape of rice field was chosen in developing a sequence of learning activities to reach the learning goal. The hypothetical learning trajectory of the sequence of five activities was implemented in the teaching and learning process. Six students from class 3C were involved in the first cycle and all the students from class 3A became the participant in the second cycle. Data collection were generated from the video recording during the teaching and learning, the students' work, pre test and post test, and interviewing the students.

In the implementation of the learning activities, making table cloth activity could reveal the idea of recomposing the cloth-rag from the students themselves. Also, from the following activities, comparing rice field and trade up activity, the students built their understanding that recomposing a shape will preserve its area because none of the parts of the shape are wasted or leftover. They also found that two shapes can have the same big although having different form. Somehow, some students were found still struggle with the way to recompose a shape. They could not figure out which parts of the shape to be cut and where to be placed.

#### ABSTRAK

Funny, R.A. 2013. Developing Students Understanding of the Concept of Conservation of Area as A Preparatory for Learning Area Measurement. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Siti. M. Amin M. Pd., dan (II) Dr. Agung Lukito, M.S

# *Kata kunci:* pemahaman siswa, konsep konservasi luas, persiapan belajar konsep pengukuran luas

Konsep konservasi luas termasuk ke dapam konsep-konsep yang harus dipelajari siswa untuk memahami konsep pengukuran luas. Piaget berpendapat bahwa konsep konservasi luas ini harusnya diajarkan sebagai persiapan siswa mempelajari konsep pengukuran luas. Sehingga, untuk mengetahui bagaimana siswa mempelajari konsep konservasi luas sebagai persiapan mempelajari konsep pengukuran luas, penelitian ini membuat aktivitas pembelajaran instruksional untuk mengajarkan kepada siswa kelas 3 Sekolah Dasar di Surabaya. Penelitian ini dilakukan sebelum siswa mempelajari konsep pengukuran luas.

Penelitian ini menggunakan design research sebagai metode penelitiannya. Penelitian ini membuat lima aktivitas pembelajaran dengan menggunakan pendekatan Realictis Mathematics Education (RME). Konteks dalam kehidupan sehari-hari seperti membuat taplak meja dan bentuk-bentuk dari sawah dipilih untuk mengembangan aktivitas pembelajaran yang dapat mencapai tujuan pembelajaran. Hipotesis trayektori pembelajaran (HLT) dari kelima aktivitas tersebut diimplementasikan dalam proses belajar mengajar. Enam siswa kelas 3C dilibatkan dalam siklus 1 sedangkan seluruh siswa kelas 3A menjadi subjek dari siklus 2. Pengambilan data dilakukan melalui rekaman video selama proses belajar mengajar, hasil kerja siswa, pre tes and post tes dan wawancara dengan siswa.

Dalam implementasi dari aktivitas pembelajaran, aktivitas membuat taplak meja dapat memunculkan ide untuk mengubah komposisi dari kain perca berdasarkan ide siswa sendiri. Selain itu, dari aktivitas membanding bentuk sawah dan tukar-tambah sawah, siswa dapat membangun pemahaman bahwa pengubahan komposisi dari suatu bangun akan mengawetkan luas dari bangun tersebut karena tak ada bagian yang terbuang atau bertambah. Siswa juga menyadari bahwa dua bangun yang bentuknya berbeda dapat mempunyai luas yang sama. Bagaimanapun juga, beberapa siswa masih kesulitan untuk mengubah komposisi dari suatu bangun ke bangun lain. Mereka tidak dapat menentukan bagian mana yang harus dipotong dan dimana harus meletakkannya.

#### **PREFACE OF THESIS**

Praise be to Allah SWT, The cherisher and sustainer of the worlds; God who has been giving His blessing and mercy to the writer to complete the thesis entitled "Developing Students' Understanding of the Concept of Conservation of Area as A Preparatory for Learning Area Measurement".

This thesis is submitted to fulfill one of the requirements to gain Master degree of Master of Science (M.Sc,) of International Master Program of Mathematics Education (IMPoME) in State University of Surabaya.

In finishing this thesis, the writer really gives his regards and thanks for people who has given guidance and help; they are

- 1. My lovely mommy, Candrawati (Mamchan) and daddy, Agus Nurhadiyanto who always give me support, pray for me and take care of me during finishing the thesis.
- 2. My lovely sisters, Ruris Testi Navalia and Relancari Ade Wiranti who always pray for me and help me with housework when I was busy with my thesis.
- 3. Prof. Dr. Siti. M. Amin, M.Pd, the first supervisor, who has given her best guidance to write a quality content of the thesis.
- 4. Dr. Agung Lukito, M.s, the second supervisor, who has given his help to show the best way to research starting from Netherland.
- 5. Drs. M. M. Monica Wijers as my supervisor when conducting the proposal of thesis in Utrecht University.
- 6. Dr. M.L.A.M Marteen Dolk as coordinator of the IMPoME from Utrecht University.
- 7. All lecturers in Utrecht University (drs. Frans Van Galen, dr. HAA. Dolly van Eerde, dr. Arthur Bakker, Dr. Paul HM Drijvers, Martin Kindt, Dr. Steven Wepster, Ir. Henk van der Kooij, Betty and all staff which helped the writer when doing her study in Netherland)
- 8. Prof. Drs. I Ketut Budayasa, Ph.D and Prof. Dr. Dwi Juniati, M.Si, the examiners, who give their suggestion to improve this thesis.
- 9. Prof. Dr. R.K. Sembiring as the examiner and Chairman of PMRI (Pendidikan Matematika Realistik Indonesia).
- 10. Ustad Anwar as the Headmaster, Ustadzah Endah as the Vice Headmaster and Ustadzah Lila Prasanti as the mathematics teacher of SD AL-Hikmah Surabaya which help me to do research there. Also, all students in grade 3C and 3A which be ready to be studied.
- 11. Mbak Martha Metrica Hutagalung and Pak Pontas Hutagalung as PMRI team who always help and support me about the scholarship issue.

- 12. My friends in IMPoME (Weni, Puji, Febrian, Sis, Shofan, Ismi, Rini, Bustang, Elika, Eva, Mbak Syl, and Sakinah) for the friendship in Indonesia – Netherland. Ishariyadi and Umam who accompanied us when having the thesis defense. Also, all BI-IMPoME in UNSRI and in UNESA, thanks.
- 13. My friend, Wuli which help me to conduct pilot project in Indonesia while I was in Netherland, An-nisa Latief which always listens me when complaining with something.
- 14. Mbak Eha, Mas Andy, Mbak Risda, Mas Pardi, Bunda Hafizah, Ayah Hafiz, Mbak Tika, Mas Arie, Nazhifa, Salsabila, Mbak Mila, Mas Richo (Chairman of PPI UTRECHT), Mas Eko (Dutch Teacher), Mas Sisvanto, Mbak Intan and Mbak Fanny (Saman Dance Teacher), Bang Novrizal, Bang Boy, Mas Andi Yani, Loresta Lintang, Bu Dewi and all Indonesian Society and Students Society in Utrecht, Netherland for the familial warmth.
- 15. All my big family (my aunties, my uncles, my cousins, my grandpa and grandma) who always pray for me while I was in Netherland and preparing the thesis.
- 16. And all people who support the writer in writing this thesis which could not be mentioned one by one.

Finally, the writer realizes there are unintended errors in writing this thesis. It's really allows all readers to give their suggestion to improve its content in order to be made as one of the good examples for the next thesis.

Surabaya, 31 August 2013

Rindu Alriavindra Funny

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#### **CHAPTER I**

#### **INTRODUCTION**

#### A. Background

The term "conservation of area" might be not really familiar for students in Indonesia. But the idea that two different shapes could have the same area has been learned by the students. But for the lower grade students, the reason why and how two different shapes could have the same area is still being a big question. For answering how and why two different shapes could have the same area, the students need to understand the conservation of area. The idea of conservation of area is that particular shape could be modified into different form with the area remains the same. Actually, the term "conservation of area" has been known since 1965 when a psychologist and philosopher, Piaget, conducted his conservation tasks among children to investigate children cognitive development and conservation of area is belongs on the conservation tasks. From his research, Piaget argued that the concept of conservation should be mastered by the children on age 7-12 years old. Since in Indonesia, children on age 7-12 years old also learn about area measurement so it is the right time for them to learn the concept of conservation of area.

Furthermore, Piaget (1981 as cited in Kordaki, 2003) argued that conservation of area is a fundamental and preliminary aspect in students understanding of the concept of area measurement. With regard to Piaget, there were some researches on the conservation of area for learning the area

1

measurement. But the existing research used to be preceded by determining the area of irregular shape, such as an island or a leaf, by grid paper. For instance, Fauzan [2002] used the concept of conservation of area to introduce the area and perimeter to fourth grade students in Indonesia (10-year-old) with the limitation of grid paper. Students experienced that the half of a square can fit another half of square, which is the idea of compensation. Meanwhile, compensation is an important idea for understanding the concept of conservation of area. This means that Fauzan [2002] only investigated the concept of conservation of area on the isolation of the use of grid paper. Despite the fact that grid paper is very helpful for introducing area measurement, it can make the students too dependent on the grid and can lead the students just to count or to estimate the number of squares in grid paper. The students tend to lose the fundamental concepts of the conservation of area and the unit of area measurement in their higher level (Maher and Beattys, 1986; Liebeck, 1987 as cited in Kordaki, 2003). Therefore, the investigation of students' understanding on the concept of conservation of area without grid paper, especially in Indonesia, still needs to be studied.

Meanwhile, conservation means that the quantitative value remains unaltered while its figure can be qualitatively new (Piaget et al, 1981 as cited in Kordaki, 2003). Thus, conservation of area keeps the area of a figure remains the same while it figure can be change into different form. Recently, individual shapes of non-convex polygon have been used to investigate students' thinking on the concept of the conservation of area by Kordaki [2003]. Kordaki used some equivalent triangles or parallelograms with common bases and equal heights. But the characteristic of these shapes will be learned by the students in Indonesia in the secondary level. Hence, the students' understanding on the conservation of area for irregular polygons for the primary level needs to be investigated.

Moreover, Realistic Geometry also suggests the young children to learn the concept of area measurement in a more informal way (Gravemeijer, 1990 as cited in Fauzan, 2002), without the use of number that generally called a qualitative approach (Liebeck, 1987; Carpenter et al., 1975; Hiebert, 1981 as cited in Kordaki 2003). By omitting the use of number in teaching the concept of conservation of area, the students could be only focused on the process of recomposing the shape and explore the effect of recomposing the shape without disturbing by proving it with number.

Based on the aforementioned discussion, the need of investigating of the students' understanding of the concept of conservation of area with qualitative approach and omitting the use of grid paper for the lower grade students is strongly recommended and this study will do it. Since the use of grid paper usually learned when students are learning the area measurement, so this study conducted the teaching learning activities of the concept of conservation of area before the students learn area measurement.

#### **B.** Research Question

According to the background of this study, the research question is stated as follows;

"How can students learn about the concept of conservation of area as preparatory for learning area measurement?"

To support us answering the general research question, we derive research-subquestions as follows

- 1. How can the students develop the understanding that recomposing a shape will preserve the area of the shape?
- 2. How can the students learn that the sum of the parts will be equal to the area of the whole shape (part-whole relation)?

#### C. Aim of the Research

In an attempt to support the students' understanding of the concept of conservation of area, this study developed a hypothetical learning trajectory for third grader students in Indonesia to learn the concept of conservation of area before they learn area measurement. This hypothetical learning trajectory will be designed, tested, analyzed and redesigned to contribute to the development of the local instructional theory for area measurement as the second aim for this study.

#### **D.** Definition of Key Terms

In order to avoid misinterpretation for the readers, the researcher defines some terms used in this study as listed below.

1. Developing students' understanding

Developing is defined as a progress of doing something. While, understanding is an active process of assimilating new knowledge to be fit and to be appropriate with the existing knowledge. If knowledge is the possession of an idea, then understanding is a measure of how well this idea is integrated with or connected to other existing ideas in the learner's cognitive framework.

So, developing students' understanding could be defined as a progress of assimilating the new knowledge to be fit with the existing knowledge. The new knowledge in this study is the concept of conservation of area. The existing knowledge could be derived from what the students have learned formally in the school (for detail see the Curriculum of the third grade of Elementary School).

2. Concept of Conservation of area

Concept mean as an idea or thought which inferred from particular instance. Idea or though is something formed in mind.

Meanwhile, conservation of area means modification in form cannot produce change in area (Kordaki, 2003). Thus, concept of conservation of area is an idea or thought that modification in form of a shape cannot produce change in its area.

3. Developing Students' Understanding of Concept of Conservation of Area

Since before we have defined that developing students' understanding could be defined as a progress of assimilating the new knowledge to be fit with the existing knowledge and progress usually consists of several phase or steps which support each other, in this study we would try to analyze the step or the phase that the students pass to understand the conservation of area. In order to know the development of the students' understanding of the concept of conservation of area, we analyze the students' thinking process while learning the concept of conservation of area.

4. Preparatory for Learning Area Measurement

Preparatory is obtained from the word 'prepare'. Prepare means to make ready beforehand for a specific purpose, as for an event or occasion. In this study, the term preparatory is defined as "before". So, the concept of conservation of area will be learned by the students before they learn about area measurement formally in the school.

5. Learning Area Measurement

Area measurement is seeking to determine the quantity of a surface enclosed within a two-dimensional (2-D) region, including the areas of basic shapes and irregular polygons or embedded figures (Structhens et al., 2003 as cited in Huang, 2008). Meanwhile, learning is fitting new experiences into existing cognitive structures and revising theses structures fit the new data.

Hence, learning area measurement means fitting new experience about determining the quantity of a surface enclosed within 2-D region with existing knowledge. Since, in this study the concept of conservation of area will be learned before learning the area measurement, so the existing knowledge of the students before learning area measurement must be the concept of conservation of area.

6. Recomposing a shape

Recomposing a shape is an action of modifying a shape into another form of shape with preserve its area.

#### 7. Part-Whole Relation

Part-whole relation is an idea that the sum of area for all parts will be same with the area of the whole shape.

#### E. Significance of the Research

This study expected two significances to be achieved. The first significance is to elaborate a local instruction theories on the concept of conservation of area as support for develop local instruction theories on the area measurement. The second significance is providing a clear idea to mathematics teacher on how to develop learning activities especially for topic conservation of area. This study is aimed to give an example of the design process of instructional activities and some consideration that must be taken into account in the design process.

#### **CHAPTER II**

#### THEORETICAL FRAMEWORK

#### A. The Concept of Conservation of Area

From the literature, conservation means preservation. So, conservation of area means preserving the area. As well, **conservation of area is defined as a modification in form that cannot produce change in area** (Piaget, Inhelder, Sheminska, 1981 as cited in Kordaki, 2003). Whatever happens to the figure, like dividing into some parts and the rearranging the parts to form another figure as long as it is not added or subtracted.

The important point in understanding the conservation of area is the concept of compensation and the part-whole relation (Kospentaris, Spyrou, Lappas, 2011). As well as the notions of changing only its position and splitting into some parts, recomposing the parts to produce an equivalent figure needs to be learned by students (Kordaki, 2003). In the same way, the notions of reversibility and transitivity also become fundamental aspect in the conservation of area (Piaget et al., 1960 as cited in Kospentaris, Spyrou and Lappas, 2011).

#### **B.** Piaget's Conservation of Area

The concept of conservation of area has been introduced on 1965 by Piaget. This concept is included in the seven conservation task of Piaget (1965 as cited in Baruch, Conway and Jordan). They are conservation of number, length, liquid, mass, area, weight and volume.



Figure 2.1 The Piaget conversation area test

Piaget used a geometrical experiment called "cows on a farm" to test conservation of area (see Figure 2.1). To illustrate this, Piaget used green cardboard to represent farmland. Two identical green farms were shown to the children; each had a little wooden cow placed upon it. The children were asked to think whether the cows had the same amount of grass to feed upon. Piaget would proceed to add equally little cubic farmhouses to the models. In one farmyard the houses where all arranged in a tidy row, on the other farmland the houses were all scattered about. Piaget found that until the ages of 8-9, children are irresistibly inclined to think there is less grazing land on the farmland which contains the scattered houses.. It will be same when you show a child four marbles in a row, then spread them out; the preoperational child will focus on the spread, and will tend to believe that there are now more marbles than before.

# C. Developing Student's Understanding of the Concept of Conservation of Area.

Skemp (1976 as cited in Barmby, Haried, Higgins and Suggate, 2007) argued that understanding is knowing what to do and building up a conceptual structure. Knowing what to do could be done when students get the knowledge and this knowledge will be processed by the students to fit with their existing knowledge. Thus, the students could build the conceptual structure between their knowledge (new and existing knowledge).

Understanding could not be inferred from someone to the other since it is an active process of individual. A task or activities could be able to facilitate students to understand, but any individual task can be performed correctly without understanding. Therefore, we need a good task which could facilitate the students' understanding into a concept of conservation of area. But how to say that the students have understood the concept? Based on Barmby, Harries, Higgins & Suggate [2007], people can never have complete understanding, they can always develop understanding by developing more links for example between apparently very different concept that we have not associated together previously. However, if students have come across a concept in any way, then they will have some understanding of that concept, but only limited or inappropriate links within their understanding might be.

In line with developing which means as the progress of doing something, so developing students' understanding of the concept of conservation of area could be defined as a progress of assimilating the new knowledge to be fit with the existing knowledge in order to build the conceptual structure of conservation of area. The new knowledge in this study is the concept of conservation of area. The existing knowledge could be derived from what the students have learned formally in the school which supports the process of students' understanding. Therefore, in this study to measure the understanding of the students about the concept of conservation of area is limited by two important ideas. First is recomposing a shape will preserve the area of the shape and second is part-whole relation, the sum of area of parts will be same with the whole. An instructional activity which will be designed by this study is used as a tool for facilitating students' understanding. So, what we want to know from developing students' understanding in this study is not only having the knowledge and using it in the new way, but also the way of build up the conceptual structure about the concept of conservation of area.

Teaching the concept of conservation of area will mostly use cut-and-paste strategy in re-arranging the parts of a figure to produce another shape with an equal area for recomposing the shape (Clement and Stephan, 2003). A useful tool to help students understand this concept was created by Kordaki [2003]. She developed a computer microword such that students can investigate the geometrical shape called CaRMe for secondary students. In her study, she focused on the transformation of a non-convex polygon to another polygon with an equal area and the comparison of a non-convex polygon to a square. The microworld gives the students an opportunity to manipulate the shape and to see what the effect on another shape. Kordaki [2003] found that the visualization she made could help the student to find the proof of conservation of area. Moreover, this visualization can easily convince the students. Therefore, visualizing conservation of area for teaching the concept for lower grade students is necessary. Hence, this study will concern to use visualization to teach the concept of conservation of area for lower grade students in Indonesia.

#### D. The Concept of Conservation of Area in Learning Area Measurement

The conversation tasks of Piaget have been done outside of the topic of the learning process. Thus the students' understanding will become far from the use of the concept of conservation regarding to the relating topic, such as number, area, volume and etc. Some researches were conducted to include the concept of conservation in one of the relating topic. For instance, Clement and Stephan (2001 as cited in Haris & Ilma, 2011) argued that the concept of conservation of area belongs to five basic ideas for understanding the area measurement for the early age students.

The first idea is partitioning which means as the mental of act of cutting two dimensional space without a two dimensional unit. The second idea, unit iteration is covering region with area unit with no gaps or overlapping. Then the third idea, conservation of area means to understand that the area of a region does not change although cut and re-arranged to another shape. Moreover, the fourth idea, structuring array is defined as understanding that an array is truly two dimensional. And the last idea, linear measurement makes the students know that area measurement is the product of two linear measurements.

Although, Clements and Stephan [2003] proposed conservation of area into the five basic ideas to learn the area measurement, but there was no such concept in their instructional design activities. Their instructional activities just focused on the following; investigate covering regions with a unit of measure, understand how to structure arrays, understand how the length of the sides of rectangle can determine the number of units in each row and the number of rows in the array, and the last is understand how to multiply the two dimensions as a shortcut for determining the total number of squares. From these instructional activities, we can see how the conservation of area is still neglected in the instruction of the concept of area measurement. The absence of activities for the concept of conservation of area or even the neglecting to emphasize the concept of conservation of area in the area measurement causes most of the children's difficulties in this topic (Kordaki, M. & Potari, D.1998; Piaget J. , Inhelder B., Sheminska A.1981 as cited in Kordaki, 2003).

#### E. Conservation of Area for Irregular Polygon

The concept of conservation of area has been considered a prerequisite to area measurement (Piaget, 1981 as cited in Kospentaris, Spyrus, Lappas, 2011). Thus, when we measure an area, we assume that the partial units are conserved and can be combined in many ways in order to form the invariant whole. Later on when students have learned the area formula, the concept of conservation of area will help them to derive the area formula for some figures and to show relation between them. Like the formula of a parallelogram which can be derived from the formula of a rectangle or square, combined with the fact that a parallelogram can be manipulated into a rectangle or a square. This derivation of formula has recently been introduced in the learning process in Indonesia (Mustaqiem & Astuty, 2008). It is usually done in the fourth grade when the students learn about the area formula for triangles and parallelograms. Unfortunately, the teacher

usually does not introduce the term "conservation of area" in this case. Indeed, the teacher did not discuss deeper about this concept.

As a consequence, when they are given the approach to derive an area formula from another shape, they might fail. Perhaps, by the instruction of the teacher, students will able to find it, but it will not be meaningful for them. Students will wonder how come it has the same area, or why the area of a parallelogram is the same as a rectangle and other curious questions will appears in their mind. Most of the time, the students were taught to use grid paper to know why they have the same area. Although grid paper is used to help them see the area or count that these figures exactly have the same amount of grid inside (Mustaqiem & Astuty, 2008), the concept of conservation of area is not as simple as counting the grid paper. Since for some figures, it is difficult to count the exact grid, such as the square with different position (see Figure 2.2).



Figure 2.2 The square with different position

Students who have learned with grid paper to count the area, will be very dependent on the grid. So, when they are given this problem, they will try to just count the number of square inside the blue and red figures. But counting the number of square in the red figure is difficult for students since they will just use an estimation to find the exact area. Also, Van Hiele in the secondary education (1973 as cited in Fauzan, 2002), concluded that the students usually follow what the teacher give to them. So, the teacher introduces the approach to find the area of a shape from the first time just by using grid paper, then the students will always use grid paper to count the area, especially the area of an irregular shape. They might never try to find another strategy for approaching the problem. In any case, it seems that the grid is just the proof when the area of two figures is the same, but does not help to build the notion of conservation of area.

There is some hypothesis about why the concept of conservation of area cannot be easily understood by the students. Piaget (1981 as cited in Gold 2001) attributed the failure of conservation to the lack of the concept involved. Meanwhile, another hypothesis that came up is that it is caused by misinterpretation of the question asked. However, no answer can yet be given regarding to the fundamental question about the failure of conservation task (Gold, 2001). Thus, the initial concept of conservation of area must be carefully introduced before the use of grid to measure the area (Piaget, 1981 as cited in Kordaki, 2003). Therefore, this study will design a learning sequence to support students' understanding of the concept of conservation of area before the students learn about area measurement, without the help of the grid. In the end of our design, the students will be able to use their understanding of conservation of area to find the area of irregular polygons.

#### F. The Indonesian Curriculum

In the Indonesia curriculum for the lower grade students, the concept of conservation of area is not explicitly stated in the standard competence when area measurement firstly introduced in the third grader (see Table 2.1). However, in the teaching and learning process in the school the idea of conservation of area is usually being used and being taught by the teacher without using the term "conservation of area".

Table 2.1 The standard competence of the students in the third grade,  $2^{nd}$ semester in Indonesia

Standard Competence	Basic Competence
Geometry and Measurement	
4. Understanding the characteristic of asimple 2-dimensional shape.	<ul><li>4.1 Identify the type of a simple 2- dimensional shape based on its properties or characteristic.</li><li>4.2 Identify many types of angle measurement.</li></ul>
5. Calculating the perimeter and area of asquare and rectangle and their implementation to solve the problem.	<ul> <li>5.1 Calculating the perimeter of a square and rectangle</li> <li>5.2 Calculating the area of a square and rectangle.</li> <li>5.3 Solving a problem which relate to the perimeter and area of a square and rectangle.</li> </ul>

This nescience of the term usually leads the students to ignore the existence of the concept of conservation of area. Meanwhile, the standard competence of the sixth grader needs the emergence of the concept of conservation of area to find the area of irregular polygon (see Table 2.2).

Standard Competence	Basic Competence
3. Calculating the area of a simple polygon the area of a circle and the	3.1 Calculating the area of a polygon which is a combination of two simple
volume of a prism with triangle	2-dimensional shapes.
base.	3.2 Calculating the area of a circle.
	3.3 Calculating the volume of a prism
	with a triangle base and the cylinder.

Table 2.2 The standard competence of the students in the sixth grade,  $2^{nd}$ semester in Indonesia

Indeed, the concept of conservation area is the basic ideas for finding the area of an irregular polygon. Also, the fact that concept of conservation of area is a preliminary concept for learning area measurement (Piaget, 1981 as cited in Kordaki, 2003) make this concept being very importance for students to learn in their first time of learning area measurement or before they learn about area measurement.

Based on the Kordaki [2003] and Kospentaris, Spyrou, Lappas [2011] that the importance idea on the concept of conservation of area are compensation and part-whole relation, this study tried to formulate the basic competence for teaching the concept of conservation of area. This study proposed three specific basic competences for the students in the third grade to learn the concept of conservation of area. First, understanding the idea that recomposing shape, cut and reform a shape into another form will preserve the area. Second, understanding the identity in the concept of conservation of area, if none of parts are wasted or added then the area of the shape will be the same although the form is changed. Last, understanding the idea of part-whole relation, the sum of the area of the parts will be equal to the area of the whole shape. The three basic competences involve the reversibility and transitivity which occur in the conservation of area.

Furthermore, the concept of conservation of area needs to be related to the topic of measurement of area and should be introduced as early as possible, but it should have the same attention as the other big ideas. Thus, this study will make a learning sequence that gives full attention to the students' understanding of the concept of conservation of area which also relates to the understanding of area measurement for the third grade students in Indonesia.

# G. The Use of RME (Realistic Mathematics Education) in Developing Student's Understanding of the Concept of Conservation of Area.

It is very important to introduce the concept of conservation of area in the learning phase of area measurement (Kordaki & Potari, 1998). Furthermore, the notion of conservation of area is much related to the real world, for instance how to purchase a piece of land that has the same area as another piece of land.

The knowledge about the concept of conservation of area cannot be transferred by the teacher to the students. The students must experience it by themselves. The concept of quantitative value (area) which presupposed in understanding conservation, is a high level abstraction which is inferred from immediate experience rather than directly perceived (Baruch, Conway and Jordan, n.d). This idea actually belongs to one of the principles of RME (Realistic Mathematics Education), a realistic approach to mathematics education based on the Freudenthal's interpretation of mathematics as human activity (Freudenthal, 1973 as cited in Fauzan, 2002). Therefore, based on the need of experience and the relation with the real world, RME is felt as the most appropriate approach to learn the concept of conservation of area.

RME has three principles which support the learning process of conservation of area, which are guided reinvention, didactical phenomenology, and emergent models (Gravemeijer, 1994 as cited in Wintarti, 2011). In the guided reinvention principle, the students should be given the opportunity to experience a process of reinventing the concept of conservation of area (Gravemeijer 1994, 1999 as cited in Fauzan, 2002). The way to make the students re-invent the concept is by giving them meaningful phenomena to be organized and to stimulate the learning processes, which is defined by didactical phenomenology (Fauzan, 2002). Subsequently, the meaningful situation should lead the students to find a model, self-developed models, in order to bridge the informal knowledge (from the situation) with the formal mathematics (the concept of conservation of area). The self-developed model that students' found may become the model of informal mathematical activity and over time may develop into a model for more formal mathematical reasoning (Gravemeijer, 1994 as cited in Wintarti 2011).

The role of the teacher is very important to support the guided reinvention of the concept of conservation of area except the design itself. Since Treffers (1987 as cited in Al-Jupri and Turmudi, 2009) said that the students could reinvent mathematics under the guidance of the teacher of the instructional design. Hence for support the teacher to guide the students, we provide a teacher guide which explains what the teacher should do in facing the students' response. The teacher guide will assist the teacher about what they should do and not to do with the direct guidance, such as the question or the story to be given to the children.

Furthermore, the context which used in the instructional design will support the students to reinvent the concept of conservation of area. The context makes the students to experience the concept of conservation of area by making a tablecloth from a cloth rag. When the students recompose the cloth rag to fit with the table, they will realize that the area of the cloth-rag remains the same. This is the start of the students to understand the concept of conservation of area. Then the students understanding about the concept will develop in the sequence of the activities.

Moreover, this context will give the students an opportunity to find and develop the cut and paste strategy by themselves. This strategy then will be used as a model for bridging the informal mathematical activities, cut and paste, into the formal mathematical knowledge, the notion of conservation of area.

In order to develop the learning sequence based on RME's principles, the instructional activities should be derived from the five tenets on RME which are defined by Treffers in Bakker [2004]. The following description explains how these tenets are adopted in this study.

1. Phenomenological exploration

A concrete meaningful context is elaborated as the starting point for the lesson sequence in this study. The 'cloth rag' context will bring the students to use the strategy cut and paste for making the table cloth. This context is very meaningful since the students can imagine, touch, manipulate and recompose the cloth into the wanted shape of the table cloth.

2. Using models and symbols for progressive mathematization

Models and symbols are used to facilitate the students' progressive mathematization, from the intuitive into more formal mathematical concepts. These instruments should be meaningful for the students and have the potential for generalization and abstraction. In this study, the cut and paste of the cloth rag will show to the students how to conserve the area. Once they have understood the use of cut and paste, they will begin to use it as the mathematical reasoning. Later on, the students do not need to really cut the shape. They will be able to imagine which part should be cut and paste to another part and to give the reason why it is so.

3. Using students' own constructions and productions

During the activities, the students are given the opportunity to explore their way to reshape the cloth rag into the wanted shape. The students will produce their own models when cutting and pasting the cloth as the meaningful strategy for them.

4. Interactivity

The students' own production can be used to compare and reflect on the merits of the different models. There will be a class discussion to facilitate the understanding of students' own production. In this study, the design will provide a working group environment in which students communicate, compare and justify their ideas with each other.
### 5. Intertwinement

Intertwinement suggests the instructional activities to have relations with other domains. Also, the concept of conservation of area should not be taught independently since it is belongs to the five basic ideas for learning the concept of area measurement. Therefore, the design of this study will connect the notion of conservation of area with the concept of area measurement. In the end, the students will see the use and the importance of this concept to find the area of irregular polygons by reshaping it into regular polygons under the concept of conservation of area.

This study hypothesized that through the idea of recomposing shape and partwhole relation have been able to build the students' understanding about the concept of conservation of area. These important ideas, recomposing shape and part-whole relation will be divided into several instructional activities which explained in Chapter IV. We also conjectured that recomposing shape will take longer time rather than part-whole relation to be understood since it is the initial idea of the concept of conservation of area.

#### **CHAPTER III**

### **RESEARCH METHODS**

This study will use design research as the research method since it is appropriate for the aim of the study which is to develop a local instruction theory which will contribute to improve mathematics education in Indonesia. The innovative design of the learning sequence on the concept of conservation of area will put more attention into the students learning process, not on the students' result.

## A. Research Approach

The purpose of this study is to improve the mathematics education toward the concept of conservation of area in the third grade of students in Indonesia. Therefore, this study used the design research. The purpose of design research is to improve the mathematics education. In other word, design research is an approach for discovering ways to develop a design, such as learning activity, based on theories and to determine the effectiveness of this design in practice.

The innovative design in this study will give such an intervention for the students to realize the importance of conservation of area, focusing on two big ideas which are recomposing the shape and the part-whole relation. There are three concrete phases to conduct design research. They are 1) preparing for the experiment, 2) experimenting in the classroom, and 3) conducting retrospective analyses. The goal of preparing the experiment is formulating the learning goals, instructional end points to which one is aiming, instructional starting points,

conjectured local instructional theories that underlie the study, conjectured about the learning process that anticipates how students' thinking and understanding might evolve when the instructional activities are employed in the classroom. In this study, it will be done by preparing the Hypothetical Learning Trajectory (HLT) as a link between an instruction theory and a concrete teaching experiment.

When all the preparation work has been done, the overall endpoints are specified, the starting points are defined, and a conjectured local instruction theory is formulated, the design experiment can be started. The purpose of the second phase, design experiment, is both to test and to improve the conjectured local instruction theory which has developed in the preliminary phase, and to develop an understanding of how it works. In this phase, in order to observe the students' response toward the design activities, the HLT of this study will be implemented in a classroom setting.

The next phase, the retrospective analyses, is conducting analysis of the entire data set collected during the experiment. The goal of the retrospective analysis depends on the theoretical intent of the design experiment. In this phase, the HLT will be compared to the actual learning process of the students when it is implemented. The analysis should be transparent, not only explanation about the instance that support the conjecture, but also the examples that contradict the conjecture. The research question of this study will be answered in this analysis.

Furthermore, this study also fulfills the five characteristic of design research which are;

#### a) Interventionist

The intervention of the researcher has a big role in this study. The design of the instructional activities on HLT, all have interfered purposively to make better understanding among students to the concept of conservation of area.

b) Iterative

There will be a cyclic process, from the design preparation (HLT) then implemented or being trial in a small class (4 or 6 pupils) to be evaluated as the first cycle. Next the revision of the design to really implemented in the next classroom setting as Cycle 2. Then it will happen continuously, the teaching experiment Cycle 3, revision of HLT, teaching experiment Cycle 4, revision of HLT and so on until the learning goal has been achieved and the general actual learning trajectory has been appropriate with the newest HLT.

c) Process-oriented

The design activities will focus to support students understanding of the concept of conservation of area, not to examine the input-output of the design. The students' learning process will be captured rather than students' result. The development of the design as the improvement of students learning process is emphasized in this characteristic.

d) Utility-oriented

The context of the design is close to the students' real life, "cloth rag", since many Indonesian women can sew and they might have several cloth rags to be re-used. So, the practicality for users in real context can be seen.

e) Theory-oriented

The needs for students in the early grade to learn the concept of conservation of area is supported by the study of Piaget about the conservation task since 1981. Also, this concept is underlying the students' understanding of area measurement (Kordaki, 2003; Clement & Stephan, 2003). So, this design is based upon theoretical propositions; and field testing of the design contributes to the development of local instructional theory on the topic of area measurement.

#### **B.** Data Collection

#### **1.** Preparation Phase

This study will be conducted for Indonesian students in the third grade (9-10 year-old). The data which want to be collected from the preparation phase is kind of students' activity and students' thinking when they are tested (pilot test) and interviewed in this study. While, the ways to collect data in the preparation phase are;

## • (Written work) Pilot test

Since there is lack research about the concept of conservation of area yet in Indonesia, so this study needs to know how the students in Indonesia solve the conservation of area's task. The pilot project test conducted for the fourth grader Indonesian student in order to know how the student solve the problem to find the area of irregular polygon. We tried to make the students use his existing knowledge and we hoped it is the knowledge of conservation of area. The fourth grader student is tested since he has learned the area measurement. In this case, we wanted to investigate whether the concept of conservation of area is emphasized in his learning process of area measurement in the third grade or not. The result of this pilot test will be used as consideration in designing the Hypothetical Learning Trajectory (HLT). Based on the information of Pilot test, then the researcher can design the activities that fit with the needs of the students.

• Interview with teacher

Interview with teacher is very important in order to know the students' habit in learning area measurement, the students capacity to understand the concept of conservation of area and the socio-norm in the classroom. All of these information was very useful for determining the appropriate activities and concept to be write in the HLT.

### 2. Teaching Experiment (Iterative Cycle)

The teaching experiment of this study conducted in Indonesia students in the third grade. There should be several cycle to be conducted in teaching experiment. The first cycle usually conducts with a small group of students (6 pupils) to test and to improve the initial HLT in the classroom setting. The heterogeneous of the students' level of achiever will be considered in choosing the participants, but not for the students who have a far gap like the smartest and the weakest students in the class. We are afraid that the high achiever will dominate the learning process while the low achiever just will follow the other. If it is happened then the effectiveness of the HLT could not be measured. Hence, we need the students who are able to tell what they are thinking, not too shy but also not too talkative will be recommended so we could track their thinking in order to improve the HLT.

The researcher will be the teacher on first cycle in order to try out the HLT, to investigate students' thinking of the activities in the HLT and to test the conjectures on the HLT. The researcher is the one who know more about the intention of the HLT since she is the one who make the HLT. Moreover, the data collected from a cycle is going to be used to revise the HLT for next cycle.

The next cycle will be conducted in whole classroom setting of the third grade students in Indonesia with their own mathematics teacher, not the researcher. The participant in next cycle must be different with the previous cycle. The revision HLT will be used in teaching in this cycle. At least for the second cycle, we need to have a focus group to be focused in tracking the students' development of understanding. The focus group is a group consists of 4-5 students that recommended to be observed based on the close gap of their level achiever, not too smart and not too weak, and based on their ability to communicate their thinking to each other.

Type of data collection during the teaching experiments varied. In each cycle, we collected data from student work, pretest and posttest, field notes, video recording of whole-class discussion and classroom activity in each lesson and interviews.

• Written work

Written work will reveal the student's activity to solve the problem about the concept of conservation of area. Most of activities in each lesson will use written work, so the students' written work will have a big contribution for

collecting the data about how the students' activity as a response for solving the problem in written work.

• Field note of the observation

Field note is an observation written by the researcher while conducting the study. Field note usually contains only important issue which happens along the study. Field note is used to collect the data about spontaneous thinking of the researcher (researcher thinking about the work of the activity in the real setting) to solve the particular problem which happened in the learning process at that time.

Observation of the classroom activity by video recording

The video recorded is used to record all of the learning process activity start to the beginning until the end of lesson sequence. The video recorded will be moved along the class not only for recording the whole class activity, but also the group work activity and the focus group activity. From all the video recording, only the most interesting video fragment (not always support the HLT, sometime could be reject the HLT) or the video fragment which could be used for building the conclusion will be chosen and transcript to be analyzed.

• Interview

Interview is conducted for the teacher and also the students. The teacher will be interviewed relating to his/her opinion about the clarity of the HLT and the teacher guide, the effectiveness of the HLT in the teaching experiment and her suggestion to improve the HLT. While for the students, the interview conducted to check and investigate their thinking process when solving a particular problem. Only focus group which was interviewed about their work in Posttest and their opinion about the teaching process they have had.

### 3. Pretest and Posttest

Pretest will be given for the students before the lesson sequences are implemented in order to collect the data about some knowledge that the students have known (students' prior knowledge). The students' prior knowledge is very important to determine what kind of activity which fit with them and to give insight whether they can follow our HLT or not. The problem in Pretest regards to the concept of conservation of area by Piaget. All the problems in Pretest will be asked again to the students as the some problems in Posttest.

Meanwhile, Posttest will be given to collect the data of assessing students' development in understanding the concept of conservation of area, what they have learned about the topic, not for examining them. This test is given to the students after they are involved in the learning process. The aim of doing Pretest and Posttest is to compare students understanding process and to show what they have learned before and after this study.

Both of Pretest and Posttest will collect the data by using written work problem and video observation. In brief, the type of data which is collected and the methods to collect it in all phases are described as the following table,

Phase	Kind of data to be collected		The way to collect the data	Participant	
Preparation Phase	Dilat	Student's activity to find the area of irregular polygon.	Written work (pilot test) and video observation.		
	test	Student's thinking process to find the area of irregular polygon.	Interview of the students about the reason of answering the pilot test and video observation.	fourth grader.	
	Observation of students' normal activity in teaching and learning process		Pre-observation of the class by video observation, field note and interview with the teacher	All students in class used in Cycle 2	
Teaching Experiment			Cycles		
	Students' prior knowledge		Pretest, written work, video observation.	Cycle 1: 6 students of third grader Cycle 2, 3 and so on: All students in a class which differ with students in Cycle 1	
	Students' activity		Written work and video Observation		
	Students' thinking process		Written work, video observation, interview, and field note.		
	The development of student's understanding		Written work, video observation, posttest and interview of students		
	Clarity of the HLT and the teacher guide, the effectiveness of the HLT in the teaching experiment and suggestion for improving the HLT.		Interview with the teacher, field note.	Teacher	

Table 3.1 Outline of data collection

### 4. Validity and Reliability

Validity refers to the accuracy and precision of the data (Denscombe, 2010). In collecting the data, the validity of data collection measured whether the methods used lead to the correct data. The criteria about the correctness data is determined by the appropriateness of the data collected with the learning goals to be achieved.

In this study, the used of Pretest to investigate students' prior knowledge and the used of Posttest to know the students' development after the learning process could improve the *internal validity*. Also, the use of video recording to record complete of classroom activity helps this study to have the correct data about what is really happened in the class. Moreover, the video recording could be watched repeatedly to ensure the interpretation of the data.

Meanwhile, the clarity of how to collect the data of the first cycle and the next cycle could be generalized to another sample. This improves the *external validity* of data collection of this study.

*Reliability* refers to whether a research instrument is neutral in its effect and consistent across multiple occasions of its use (Densombe, 2010). *Reliability* in data collection relate with the consistency of the method in producing the data. For instance, in this study the use of written work, video observation and interview for the same subject (students) is aimed to check the consistency of the finding. If from all the methods (written work, video observation and interview), the data found is the support each other so this methods have been able to improve the *internal reliability* of this study. Also, when interpreting the students thinking

video observation is felt as the best way to guarantee the consistency of interpretation time by time.

Furthermore, the clear information about how to collect the data will improve the external reliability of this study. By providing information how the researcher has been carried out will make reader able to replicate this study.

#### C. Data Analysis

### **1.** Preparation Phase

The effectiveness of the initial HLT will be analyzed for the first teaching experiment, whether the learning goal achieved, whether the conjectured correct and how the students' response with our HLT. The analysis of the initial HLT used for improvement in the revised HLT to be used in the next cycle.

#### 2. Teaching Experiment (Iterative Cycle)

The result of in the Cycle 1 will be analyzed and be used as the basic knowledge for revise the HLT. The overview of the whole teaching and learning process can be analyzed from the recorded video. The selected fragments of the whole class or the focus group of students which is interested will be transcribed to interpret the students' thinking. These fragments and the written works will be compared with the conjectures in the improved HLT. The interesting fragment which supports or not support the HLT will be chosen to be analyzed with the conjectures in the HLT. The conjectures in the HLT will be assessed whether some parts of them are supported by the students and the other parts need revision. As a result of the analysis in Cycle 1 an improved HLT will be designed and implemented in the next cycle. The HLT functions as a guideline on what to focus on to be analyzed on this cycle. Indeed, the class observation and the interview with teacher and students will complete the analysis. This information will enrich the analysis to draw the conclusion of this study. The analysis of this cycle will be used for answering the research question of this study.

### 3. Retrospective Analysis

In the retrospective analysis, the analysis will be conducted by comparing the conjecture in the HLT with the students' actual learning. The analysis in this phase will determine the continuation of the cycle. When the cycle has been achieved the learning goal and the conjecture of the HLT mostly has appeared or supported by the students' actual learning in the newest HLT so the cycle could be stopped.

### 4. Validity and Reliability

Validity in data analysis means how to analyze the data to be able to measure what to be measured.

• Grounded theory

This study is qualitative research, so in analyzing the data should use the particular findings of the original data (pure data) as the basis for developing statements that apply at general level or which usually called as grounded theory. In analyzing the original data, researcher needs for reading and rereading the transcript from the video to be able to have a conclusion. The way

of deriving conclusion based on the findings of the original data will improve the validity of this study.

• Respondent validation

The researcher can return to the participants with the data and findings as a means of checking the validity of the findings. Confirmation of the students thinking in the written work by conducting the interview could improve the validity when analyze the data.

• Triangulation

Triangulation means viewing things from more than one perspective toward the same object.

This study used methodological triangulation to improve the validity. The comparison of analysis by using written work, observation of class discussion and interview can provide a check on the accuracy of findings. If similar methods produce the same results, it would seem reasonable to conclude that the findings are accurate and that they are authentic.

Reliability in data analysis mostly relate with the independency of the analysis process. Would the research produce the same result when conduction by the other researcher is the initial question to be answered to check the reliability of a study. In this study, the analysis of the data presented in a good order of each activity to provide a clear information of the reader about how the students in this study build their understanding about the concept of conservation of area. The explanation about the finding which is support and also not support the conjecture on each activity will make the reader be able to track the finding analysis to get into the conclusion. This will be improved the transparency of this study. So, the procedure and decision making of this study could be checked by other researcher who would be in a position to confirm the existence of the data and evaluate the decisions made in relation to the data analysis.

Furthermore, due to the clarity and transparency of this study in presenting the data analysis, it has big possibility to be replicate by other researcher. And if the other researcher find a similar result when conduction this study then it could measure the external reliability of data analysis.

#### **CHAPTER IV**

# HYPOTHETICAL LEARNING TRAJECTORY

According to Clement & Sarama (2004 as cited in Daro, Mosher & Corcoran 2011) learning trajectory is a description of children's thinking and learning in a specific mathematical domain including the related conjectured through a set of instructional tasks designed to raise the mental processes and to move children levels of thinking. Furthermore, the hypothetical learning trajectory (HLT) is made up of three components which are the learning goals that define the direction, the learning activities and the prediction of students' thinking and learning process for understanding the context of the learning activities (Simon, 1995, p. 136 as cited in Daro, Mosher & Corcoran, 2011). Since the aim of this study is to develop local instruction theory on the area measurement focusing on the conservation of area, the instructional activities being designed will facilitate the students' understanding about the concept of conservation of area as the preparatory for learning area measurement.

### A. Formulating Learning Goal Activities in HLT

Based on Kordaki [2003], recomposing parts to produce an equivalent figure needs to be learned by the students as well as the notions of changing only its position and splitting into some parts in order to understand the concept of conservation of area. While, the initial idea of part-whole relation is splitting a shape into some parts and found that the sum of area of the parts will be the same or vice versa, arranging some parts to fit in the whole. Therefore, based on the theory what idea needs to be mastered to understand the concept of conservation of area, to design the HLT this study set up two main goals. First, the students are not only able to recompose a shape into another shape, but they also understand that the recomposing they have done will preserve the area. Second, the students can master the part-whole relation in the concept of conservation of area.

Reversibility and identity could measure the students' understanding of the concept of conservation of area (Piaget as cited in Houde, O. & friends 2011). Reversibility means that the ability to see the transformation and then mentally reverse it so that the perceptual change is cancelled out. While, identity related with the modification of the shape which means that no parts are added or wasted in the process of modifying the shape. Looking from the function of reversibility and identity, these ideas could be used for supporting the understanding that recomposing shape will preserve the area. Therefore, in the first goal, there are three big ideas to be learned by the students which supported each other which are recomposing shape, reversibility and identity. In order to elaborate these three big ideas, we need at least three activities to conduct the learning activities for the students to develop their understanding toward the three big ideas. However, since we also wants to investigate students' thinking, so we also need an activity to reveal the students skill for communicating their thinking. As the concept of recomposing shape, reversibility and identity has big contribution for the students' understanding of the concept of conservation of area, so we will conduct math congress for facilitating the students to share their thinking process.

Furthermore, students can master the three big ideas through *cut* and *paste* in order to recompose areas (Beattys&Maher, 1985; Hirstein, Lamb&Osborn, 1978 as cited in Kordaki&Potari, 1998). Hence, almost all of the activities designed in this HLT will use cut and paste strategy.

Meanwhile, for the second goal, part-whole relation which means an idea that the sum of area for all parts will be same with the area of the whole shape could be learned by determining how many parts (smaller shape) which could be used for fulfill the whole shape (bigger shape). Since the concept of conservation of area in this study is conducted before the students learn area measurement, so the students have not learned about the notion of area formally. Therefore, for the part-whole relation this design used the rice field production as the context for substituting the notion of area. From the context of how many ton of rice field produced by particular rice field, we expected the students could use the number as substitution of the area. Hence, the part-whole relation concept is enough for just elaborating in one activity.

Next, we also note the importance of implementing the students' understanding that recomposing shape will preserve the area for the irregular polygon. Since, later on the concept of area be broadened to other shapes, including irregular shapes and surfaces of 3-dimensional object (Fauzan, 2002), we also need to introduce the use of conservation of area for these kind of shapes. However, for finding the area of irregular shape will commonly used approximation of grid paper, so we approached this issue by replacing the irregular shape by irregular polygon. Since the students still could find the area of

irregular polygon by recomposing the shape into a rectangle or a square. It means that the student's could implement their understanding about the conservation of area without disturbing by the need of grid paper. Therefore, we looked that giving the activity to develop the students to recompose the irregular polygon into a rectangle or a square is needed for the students. So, the last one activity in this study will be recomposing irregular polygon activity.

Based on the aforementioned theory and discussion, we will elaborate a sequence of six activities to reach the learning goals of the concept of conservation of area. The six activities will be called as making-table cloth activity, comparing rice field activity, math congress, trade-up rice field, rice field production and recomposing irregular polygon. The detail explanation of each activity will elaborate later in this chapter. The HLT of six activities will be implemented in the third grade of primary school in Indonesia. In each lesson, the HLT will describe the prior knowledge that students have which support the learning goal, the development of students' activities, the expectations and the conjectured of student's answer and the end goal.

In HLT, the focus is not only the learning goal but also the conjectures about how the students' thinking from the learning activities. Therefore, in elaborating the activities to reach the goal, the conjecture of each learning goal and the activities which will design is explained as the following table.

Big idea	Activity	Learning Goal	Conjectures	
	Making table- cloth (a cloth-rag which has twice longer and a half wider than table is asked to cover the table precisely)	<ul> <li>Use the idea of recompose the cloth-rag to fit the table</li> <li>Understanding the effect of recomposing shape (after recomposing the cloth-rag could fit the table, so cloth-rag and table have the same area)</li> <li>Understanding the idea of reversibility (the area of cloth-rag before and after cutting remains the same)</li> </ul>	<ul> <li>The context of making table- cloth could reveal the awareness of the students to recompose the cloth-rag to fit the table.</li> <li>Do cut and paste strategy to recompose the cloth-rag</li> <li>When the students see that the cloth-rag could fit the table, they conclude that both of the cloth-rag and the table are the same bigger which means have the same area.</li> <li>The students could understand that the area of the cloth-rag before and after cutting remains the same by proofing with demonstration.</li> </ul>	
	Comparing rice field	<ul> <li>Identify two different shapes which have the same size by recomposing</li> <li>Investigate two different shapes which have different size by recomposing</li> <li>Form some shapes which have the same size with the given shape.</li> </ul>	<ul> <li>Compare the shape by using superposition then try to recompose the shape to prove that both shapes have the same area.</li> <li>Compare the shape by using superposition and use recomposing shape to prove that both shapes have different area.</li> <li>Recompose an original shape into free different form of shape while understanding that</li> </ul>	
composing shape	Math- congress (discussing the problem in comparing rice field activity)	<ul> <li>Discuss the different way of recomposing the shape.</li> <li>Concluding that recomposing will preserve the size of the new shape</li> </ul>	<ul> <li>Students only used superposition to determine which one is bigger between two rice field</li> <li>Students directly used cut and paste to recompose the shape.</li> <li>Students used combination of superposition and then cut and paste to recompose the shape.</li> <li>Students could communicate their reason why they should cut this part and paste it into the other part by using superposition or just trial and error strategy.</li> <li>Students could communicate that since no parts is added or</li> </ul>	

Table 4.1 The Initial HLT (Learning goal and Conjectures)

			<ul> <li>rice field to different form so the area of the original and the new rice field remains the same.</li> <li>Students could communicate their thinking process, share their experience in solving the problem and defense their argumentation.</li> </ul>
	Trade-up rice field	<ul> <li>Investigate the area of a shape when it is cut into pieces and separated in location," <i>Is it still same with the original shape?</i>".</li> <li>Understand the concept of identity in the conservation of area</li> </ul>	<ul> <li>Students understand the story and could conclude that the area of rice field will be the same when it is cut and separated into location as long no parts is added or wasted during the cutting and separating process.</li> <li>Point out and use the idea of identity as one reason for preserving the area while the</li> </ul>
	Rice field production	- Understand the part-whole relation in the concept of conservation of area.	<ul> <li>Students get the notion of area indirectly from the context of rice field production. They treat the number of rice production (in ton) as the notion of area.</li> <li>Students could reason that the rice field production will be decreased if there is a house is built on the rice field.</li> <li>Arranging several number of small rice field to cover the big rice field. Then adding the number of rice production form all the small rice field to find the number of rice production of the big rice field.</li> </ul>
Part-whole relation		- Start to use the idea of unit to find the rice production of the big rice field.	- Use the smallest rice field which is known its number of rice produces to be arranged in the big rice field. So, the students only count the number of rice field produces by its unit times the number of unit (the smallest rice field) needed to cover the big rice field.

Develop students's skill	Recomposing irregular polygon	- Develop knowing w where to p	students' which parts to blace	skill : be cut a	for - ind -	Determining which part to be cut and where to place by trial and error. Be able to recompose the irregular polygon into a rectangle or a square without adding or wasting any parts of the shape.
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The conformity of the learning goal, the conjecture and the actual learning by the students will determine the number of cycle need to conduct. If the learning goal has achieved and the compatibility between the conjecture and actual learning by the students is high so the last cycle has success and allowed to stop.

### B. Meeting 1 - Making Table Cloth Activity

### **1. Starting Point**

Area measurement is taught in the second semester of third grade of primary school in Indonesia for the first time in the formal learning in the school. So, the concept of conservation of area will be taught as the preparatory to learn the concept of area measurement. The knowledge and the skills that students have learned are;

- Simple 2-dimensional shape such as square, rectangle, triangle, etc and its characteristics.
- Visualizing the 2-dimensional shape.
- Identifying the types of 2-dimensional shape such as rectangle, square, parallelogram and etc.

## 2. Learning Goal

Main goal:

Understand the process of recomposing a shape into another shape for preserving the area.

Sub goals:

- Use the idea of recomposing the cloth-rag to fit the table.
- Understanding the effect of recomposing shape (after recomposing the cloth-rag could fit the table, so cloth-rag and table have the same area)
- Understanding the idea of reversibility (the area of cloth-rag before and after cutting remains the same)

#### 3. Mathematical Activities

Meeting 1 will use the context of cloth-rag. The cloth-rag is chosen because this context can give the students natural situation to cut and paste the cloth. Also, the cloth-rag is usually formed in the non-regular shape due to the rest of making cloth. The non-regular shape of cloth-rag will be asked to be a table cloth which is more regular like square, rectangle or oval. Furthermore, students in Indonesia have been familiar with this context as there are many people who become a tailor or even their parents might be a tailor. A tailor usually has a bunch of cloth-rag. Nowadays many people in Indonesia have modified the cloth-rag to be used as a table cloth, doormat, bag, and etc.

In Meeting 1, the teacher will introduce the context by asking the students familiarity about the cloth-rag and will make sure the students understand the situation well.

### 4. Situation

Tina's mother is a tailor. She has many clothes rag in her house. One day, she wants to make a tablecloth for her dining table. But the cloth rag that she thought it will be possible to be made for tablecloth (see Figure 4.1)



*Figure 4.1 The cloth-rag and the table (Worksheet 1)* 

## a) Worksheet 1 (Individual)

Do you think it is possible to cover the table with the cloth-rag precisely? (Just use your imagination)

## b) Worksheet 2 (Work Group)

The students will be given the actual fabric which is twice longer and a half width of the table. In this case, the size of the table is same with the table in the classroom.

- Proof what your answer in the Worksheet 1
- Which one is bigger, the table or the cloth-rag? Why?

## c) Exercise 1 (Individual)

Another cloth-rag for another table (the students will just get the model of the cloth-rag and the table from the paper, not the real one) will be given as the problem in the exercise.



Figure 4.2 The cloth-rag and the table (Exercise 1)

- Can the cloth-rag cover the table precisely (see Figure 4.2)?
- Which one is bigger? How do you know (see Figure 4.2)?



*Figure 4.3 The cloth-rag and table (Exercise 2 – Problem no 2)* 

- Which cloth-rag 1 or 2 that fit with either the table A or B (see Figure 4.3)?
- Which one is bigger, the cloth-rag 1 or 2(see Figure 4.3)?
- Why? Explain your answer!

# 5. Conjecture

## a) Worksheet 1

To answer this question, some students might answer yes and some will answer no. The possible reason of students to answer no is due to the longer length and the less width of the fabric than the table. Without really holding the real things or the models of the things, students at third grade will be hard to imagine this problem. But for the students who answer yes, they will think to cut it into several parts and paste it to make the perfect rectangle with the precise size with the table. They might get the idea to make the partition from the puzzle or mosaic. Or some students might answer "*I do not know or I do not have any idea*".

### b) Worksheet 2

When the students are given the real fabric to cover the real table, the class table. They will be asked to proof their answer. There are several possible strategies they will use. The most possible strategy that used by the students will be cut and paste. The students might trace the fabric into the table and cut the additional length to be two equal parts.

Other possible strategy is the students cut the fabric into four same pieces by doing twice halving. Then the four pieces are arranged above the table.

Even, the students do the trace of the fabric vertically to the table then cut the additional part, and again trace beside the first trace then cut the additional part. This is done until the small rest of the fabric which should be divided into two rectangular pieces to be put on the remaining space on the table to be covered. This strategy is illustrated as the Figure 4.4



Figure 4.4 The trace strategy to cut and paste a shape

The students might think that the fabric is bigger since it is longer without considering the less width than the table. Or they may think that the table and the fabric are same big. But some students will get difficulties to find the reason why they are the same. The students might be able to reason that the table and the fabric are the same due to the complete covering.

In other word, the covering of the fabric into the table without give remainder will be used by the students as the proof of it. Or the students might answer this question like the expectation of this HLT, which is the table and the fabric are same big because what they have done, cut and paste, still preserve the fabric. Or even the students answer that the manipulation of the fabric can fit the table precisely, so it means they have the same size.

#### c) Exercise 1

Based on their experience with the actual cloth-rag and the table, the students might use the cut and paste strategy to solve the problem in exercise. Since this is such an individual work, so the explanation and the way the students get the answer needs to be written in the worksheet.

Some of students might think that this problem is same with the previous problem in the worksheet, so they might follow their previous strategy to cover the table with the cloth-rag.

Most of the students will do the trace horizontally rather than vertically since it is the easier. By doing tracing horizontally, the student might see that the cloth is three time longer than the table but it is also one-third less wide than the table. Then the students might cut the clothrag into three equal parts. Therefore, it can cover each other precisely.

If the students can come to the conclusion that cut and paste' strategy preserve the quantitative value inside the figure (area), it is awesome, but do not expect too much. As long as the students can show the covering process and argue which one is bigger, it is enough for Meeting 1. For the way covering the table, the students might not face a problem, but they might find difficulties in determining which one is bigger and in giving the reason why it is so.

The next problem in the exercise has more challenge for the students. The high achiever students will easily see the relation of this problem with the previous problem. They might just put the cloth above the table and see the possibilities to cut and paste the part of the cloth to cover the table. First they will do superposition, then cut and paste, and finally they can get the answer. For the low achiever, they might struggle in determining which one should be cut and where to be put, but since the shape and the strategy used in this problem is still simple and easy, the low and middle achiever can afford it.

The students will start by trying the cloth 1 and the table A. When they see it cover each other, the students will do the rest, cloth 2 and the table B. Since the table B cannot be covered by the cloth 2 precisely, then some students might try the cloth 2 with table A, but some might be not and just answer that cloth 1 cover the table A, while cloth 2 does not cover the table 2. If it is happen, teacher can encourage the students for also looking the table A for being covered by cloth 2. After the student finish matching the cloth 1 and 2 to table A or B, some of the students will do superposition and cut-paste again to see which one is bigger, cloth 1 or cloth 2. But might for some high achiever, they can conclude that they cloth 1 is the same big as cloth 2 because they can cover precisely the same thing, table A.

Overall, to sum up, there are three big ideas in Meeting 1. First is whether the students can determine the possibility of the problem just by using imagination. Second, the students experience the handy activity to prove and to convince other and themselves. The last is the students realize that if two things cover each other precisely, they have the same size. But, do not expect too much that the students will come to the conclusion of conservation of area in Meeting 1 because it will be too soon. Just let it goes as the way it is and the more concept of conservation of area will be learned in the next lesson.

### C. Meeting 2 - Comparing Rice Field Activity

# 1. Starting Point

The students have known the use of cut and paste strategy to recompose a shape into another shape which has the same size.

#### 2. Learning Goals

- Identify two different shapes which have the same size by recomposing
- Investigate two different shapes which have different size by recomposing
- Form some shapes which have the same size with the given shape.

#### 3. Mathematical Activities

The students will be brought in the context of rice field in Indonesia. We use the rice field context due to its close relation to the students in Indonesia. It is well known that Indonesia has many rice fields. All of the students in Indonesia know about rice field and have ever seen rice field in the journey of their life.

Some of the students might not be aware that the shape of rice field is not always rectangular, it can be a little bit parallelogram, have curve on some side or being irregular polygondue to the combination of several shape.

## a) Problem 1

Problem 1 ask the students to figure out whether these two shapes of rice field, Pak Badrun' rice field and Pak Darman' rice field, have the same size or not (see Figure 4.5).



Figure 4.5 The rice field of Pak Badrun and Pak Darma

# b) Problem 2

The Problem 2 is asked the students to proof that Pak Salman rice field is the biggest size among the other, Pak Badrun and Pak Darma (see Figure 4.6).



Figure 4.6 The rice field of Pak Salman

### c) Problem 3

Discover another shape of rice field which has the same size with Pak Salman rice field

### 4. Conjecture:

Students might follow their previous experience, using cut and paste strategy to figure out which one is bigger between those three rice fields. Students will cut the salient part of the rice field to move it into not salient part.

#### a) Problem 1

For Problem 1, the most possible strategy that used by the students is cut the salient rectangular part on the right of Pak Badrun's rice field and paste it in the bottom of the rice field, the left-bottom. But if the students do this they still will have a rectangular shape which is a little bit longer but also less width than Pak Darma. Some students might stop on this step and conclude that one of the rice fields is bigger than the other, can be Pak Badrun's rice field is bigger than Pak Darma's rice field or vice versa. They will use the approximation of little bit longer and little bit less width if they conclude one of the rice field is bigger than the other (see Figure 4.7)



Figure 4.7 The use of approximation after superposition

Meanwhile, some students maybe still struggle to find until the end whether these two shapes have the same size or not. If they use the strategy below (see Figure 4.8), they will cut again the part which is little longer and use it to cover the less width.



Figure 4.8 The struggle of the students to recompose the rice field until the tiny rest of the parts

Instead of using cut and paste strategy directly, the students might do **superposition, by putting the shape above each other** and see which one

is bigger. This is such a good way to compare things, but the weakness is you cannot know precisely which one is bigger of their non-overlapping part. Therefore, if the students use superposition to solve this problem they cannot give the exact proof about the size of the rice field.

Moreover, perhaps there are students who use the combination of superposition and cut and paste strategy. First they will do superposition to find which part is should be cut and where it should be pasted. Then they cut the part and paste it into the appropriate place. In the end they can easily figure out the answer. This strategy is very helpful when you want to find the way to recompose the shape into another shape which has the same size.

#### b) Problem 2

Similar with Problem 1, in solving the Problem 2 the students might use cut and paste strategy or superposition, or even the combination of both strategies. In recomposing the shape, the students might try to make it as closest as possible into the rectangle shape like Pak Darma (see Figure 4.9). Then they started to compare with one of the rice field, either Pak Badrun or Pak Darma rice fields. But most of the students will choose Pak Darma rice field to be compared with the rice field of Pak Salman since it is easier.



Figure 4.9 The comparison between the rice field of Pak Salman and Pak Darma

# c) Problem 3

The Problem 3 might be easy for students who have mastered the cut and paste strategy from the previous lesson. In this case, there will be just one strategy to be used by the students which is cut and paste, no superposition or even estimation. They really have to recompose the shape.

Students are free to make the new shape from recomposing the rice field. But most of the possible shape will be irregular shape which tends to be abstract shape (see Figure 4.10).







Figure 4.10 The possible students' answer of the problem 3 worksheet 3 In the end of lesson 2, make sure that the students have written some of their work in the poster paper. It will be better if the students have finished making their poster about Problems 1, 2, and 3. But if it has not finished yet, ask them to continue their work in the next meeting.

### **D.** Meeting 3 - Math Congress

### **1. Starting Point**

The students have solved Problems 1, 2 and 3 from the previous meeting. They also have made the poster to show the result of their group in Math Congress in Meeting 3

### 2. Learning Goals

- Identify two different shapes which have the same size by recomposing
- Investigate two different shapes which have different size by recomposing
- Form some shapes which have the same size with the given shape.
- Discuss the different way of recomposing the shape and concluding that recomposing will preserve the size of the new shape from the old one.
### 3. Mathematical Activities and Conjectures

The students will be given some time to finished and to prepare their poster before presenting in Math Congress. In Math Congress, the students will discuss about some possible strategy that used the groups.

Some possible strategies used by students for Problem 1 are:

- Directly using cut and paste by predicting which part should be cut and where to be put to compare the shapes.
- Just using superposition to compare the shapes.
- Using the combination of superposition and cut-paste strategy to compare exactly the shapes.

All of the strategies are allowed to be used in this problem, but each of them has its own weakness. Point out the weakness of the strategy and let the students to determine which strategy beneficially for them. For solving Problem 2, the students usually follow the strategy they have used in Problem 1. Therefore, the possible strategies which might occur in solving Problem 2 are similar with Problem 1.

Meanwhile, Problem 3 just gives students a choice to use direct cut-paste strategy. The students should predict which cut to be cut and where to locate it. Before they do cut and paste, they should imagine the outcome shape. Or maybe they can just do it by trial and error. The conjecture of the new shape made by the students is very various and if it is look from the logical of rice field' point of view the shape is more or less being not logical shape of rice field.

But it is okay. This is what expected from Meeting 3, without considering about how does the new shape look like, the size of the original and the new shape is remain the same. The question is very important to be asked to the students for clarifying their thinking. "*Although the shape is changed into unfamiliar shape, why the size is still same*?", Some students might argue with the concept of identity, they realize that material remains the same if nothing is added to or subtracted from the original. If none of the students come up with that reasoning, the teacher can provoke a question what the students think if some of the cut pieces they have is missing or someone accidentally add some cut pieces into your puzzle shape, "*Does the size of the new shape same with the original shape*?".

In the end of Meeting 3, the students are expected to master the idea of identity and to have deep understanding about the concept of conservation of area. Meeting 3 has big contribution to the students' understanding about the concept of conservation of area. Therefore, the teacher should give big attention about how far the students understand about the concept of conservation of area from the experiment and also from the class discussion.

From what they have experienced to find the shape which has the same size, the students might realize the concept of conservation on it. From Problem 1, the students might think that you can recompose a shape to fit with another shape which has the same size. Meanwhile, on Problem 2, the students realize if you cannot recompose a shape to be fit into another shape, it means they do not have the same size; one must be bigger than the other. Perhaps, in solving Problem 2, the students aware with the fact that the recomposing shape will always have the same size with the original shape .

Based on the information they get from Problems 1 and 2, the students are expected to come to the concept of conservation of area. They probably can conclude that if you recompose a shape and fit precisely with another shape, so they have the same area. Another conclusion is that when you recompose a shape, whatever the new shape look like, it will have the same size with the original one if nothing added or subtracted.

So, in the end of Meeting 3, the students can have some description about what is the concept of conservation of area. Recomposing can preserve the size of the shape between the original shape and the new shape, and between one shapes into another shape which can be covered precisely by each other.

#### a) Exercise 2

As the closing of Meeting 3, having such an exercise is needed for the students to check their understanding about the concept of conservation of area. Exercise 2 is challenging enough for the students since the students have to be carefully with the questions. The question is asked the students to compare which rice field is bigger (see Figure 4.11). Meanwhile, both of the rice field had built some houses inside.



Figure 4.11 The problems in the exercise 2

Exercise 2 is such an additional one. If there is more time, the teacher can asked the students to do this exercise in pairs. But do not expect the students to finish it at that time. This exercise also can be such a warming up for facing the next lesson. Ask the students to submit their work, even with unfinished result. Convince them that it will be not score and it will be discussed in the next meeting. Use the result of this exercise to predict how difficult the next meeting will be conducted.

# E. Meeting 4 - Trade Up Rice Field Activity

## 1. Starting Point

The students have understood that recomposing a shape belongs into one of the concept of conservation of area. They also have learned how to recompose a shape without change any area on it. But the meaning of the word *"area"* for the students is still understood as the size of the shape.

## A. Learning Goals

- Investigate the area of a shape when it is cut into pieces and separated in location, "*Is it still same with the original shape?*".

- Understand the concept of identity in the conservation of area, if nothing is added or subtracted from the pieces of shape so although the pieces have spread out they still have the same area, the same size with the whole.

# 2. Mathematical Activities

The Meeting 4 will use the fact that sometime rice field of one people is not located adjacent. A collection of some separated rice field perhaps has the same size with a whole big rice field. Moreover, the use of rice production on this problem is aimed to make the students think about something inside the shape, not only just the size by comparing the shapes. Therefore, the rice production can lead the students to the notion of area which means the inside of given figure. But it does not mean that the students should know the meaning of area and should use the word of area in Meeting 4.

a) Problem 1

Pak Sudirman is a rich man. He has many rice fields in the some cities, Surabaya and Kediri. The shape of Pak Sudirman's rice field is as the following. Someday, Pak. Sudirman suddenly curious does his two rice fields in the Kediri and Surabaya produce the same amount of rice (see Figure 4.12).



Figure 4.12 The rice field for the problem in the trade-up activity

# 3. Conjectures

The students might use their experience to cut the small pieces to rearrange it into the whole or big shapes. So, in Problem 1, the students will cut the part on separate rice field. Then like a puzzle they try to cover the whole rice field If they can cover it fully, so they conclude that these two rice fields will produce the same amount of rice. Meanwhile, if they can not cover it fully, so it means that one of the rice field is bigger than another. As consequence, the bigger rice field, the more amount of rice will be produced by the rice field and the rice production should be emphasized in the discussion. The bigger rice field, the more rice production by the rice field or vice versa.

#### a) Exercise 3 (Re-do Exercise 2)

In the end of Meeting 4, it is better to discuss Exercise 2 on Meeting 3, as Exercise 3. Ask the students to re-do Exercise 2 (as Exercise 3). Give the new exercise paper and give some house model and the two rice field paper for the students to try it on. The house model and rice field paper can help the students to check and to proof their previous answer. These tools also can help the students as a tool for reasoning. Check what students' thinking in the previous meeting with they are thinking now. Investigate whether the students thinking on the previous meeting help the students to pass Meeting 4. Ask them to write their thinking in the worksheet.

From Exercise 3, the students might know that the land for the house in rice field A is same in size with the land used for house in the rice field B by using recomposing. But the problem is the fact that the size of the rice field should be subtracted by the land might still difficult to be understood by the students in the third grade.

The students might find that the land that use for the house in rice field A and rice field B is actually the same size although the shape is different. From this on, the students can think that the rice field A is same with rice field B because the land that use for the house inside the rice field is the same. So the remaining land for the rice field is also the same. Or even, after knowing that the land of the house is same in size, the students suddenly come to the conclusion that the rice field also the same without considering the subtraction of the rice field with the land of the house.

## F. Meeting 5 - Rice Production Activity

## **1. Starting Point**

Students have learned about how to preserve the area by recomposing the shape. Also, they have known that separating some pieces of a shape as long as nothing added or subtracted, the concept of identity, will make the area remains the same.

#### 2. Learning Goals

- Understand the part-whole relation in the concept of conservation of area.
- Determine the area of a shape which is same and twice bigger as the given shape.
- Start to use the pieces of the shape as the units to measure the area of the big shape.
- Recompose an abstract shape into perfect rectangle

### 3. Mathematical Activities

The context of rice production is still used in Meeting 5 because it is ideal for facilitating the students understanding about the part-whole relation. Students can feel the need to count 'something' inside the given shape. Instead of using the word 'area', this design will use the rice production. Actually, this design does not avoid or does not hide the meaning of area for the students, but because this design is as a preparatory to learn about area measurement so it is better to postpone the introduction of the word 'area' for students. Also, due to the worried if we give the instruction to find the area, we afraid the students will go to the multiplication of length time width. Meanwhile, we do not expect that multiplication of length and width now.

However, the context used in this design can lead the students to the notion of area. When the students are asked to compare the size actually they are comparing the area, when the students are asked to count the amount of rice produces by a rice field, they also area comparing the area of the rice field unconsciously. So, do not forbid them to use the word 'area' if they think what they are doing now is relating with area, but also do not introduce them the word 'area' first.

## a) Problem 1

Pak Sudirman just knows from their experience that particular size of rice field terrace will produce particular amount of the rice as the following (see Figure 4.13);



Figure 4.13 The square rice field with one ton of rice produces

Now, find how many tons the below rice field will produce rice (see Figure 4.14)?



Figure 4.14 The three shapes of rice field which has not known the rice production

# b) Problem 2

Now, he is wonder if he has two rice fields which are big enough as the following figure, how many rice it will produces (see Figure 4.15)?



Figure 4.15 The two big shapes of rice field with unknown rice production

# c) Exercise 4

Reform the shape below into a rectangle or a square (see Figure 4.16)!



Figure 4.16 The irregular polygon in Exercise 4

#### 4. Conjecture

#### a) Cut and Paste to Find the Same Size and the Bigger Size.

Since the students have been familiar with cut and paste, they probably will directly cut the pieces and try to paste it in the right place in order to compare with the given square rice field. Here is some students' strategy when cut and paste the shape (see Figure 4.17). When they know that the parallelogram rice field is actually have the same size with the square rice field, so they can conclude that it produces the same amount of rice, which is 1 ton of rice. Meanwhile, there might be some students, who still struggle to understand that the parallelogram will produce the same of rice as the square. Ask and recall their knowledge about recomposing that can form the same size of the shape. Also emphasize the fact that the parallelogram and the square have the same size (see Figure 4.17).



*Figure 4.17 The possible way to recompose the parallelogram into a square* When the students have understood it, then they could understand the next problem.

For the big rectangle rice field, they find it is twice bigger as the square rice field (see Figure 4.18). Thus, the students will conclude that it will produce twice amount of rice than the square rice field which is 2 ton of rice.



Figure 4.18 The big rectangles is twice bigger the square

Moreover, when the students can show that the little and long rectangle is actually have the same size with the square rice field (see Figure 4.19), so the amount of rice produced by this shape will be same with the square, 1 ton of rice.



Figure 4.19 The little and long rectangle is same with a square

For Problem 2, the students have to build a shape from the small pieces of given shapes. The given shapes in Problem 1 will be used for forming the big rectangular and parallelogram rice field. The students can used several and different given shape to form the big rice fields. Here is several possibilities strategy that will be done by the students (see Figure 4.20).



Figure 4.20 The students possible answer to arrange the small rice field into the big rice field
From this activity, the students will understand that the amount of rice production in a rice field depend on the sum of the rice production in the small part of rice field terrace. In the other word, the sum of the terrace rice field is equal to the rice production of the whole rice field.

In the end of Meeting 5, there will be an Exercise 4 (see Worksheet 5) not only to check the students' understanding but also to give deep understanding about the implementation of the concept of conservation of area for shape in abstract and without context. The Exercise will ask the students to reform the irregular polygon into a perfect rectangle.

To solve the problem, most of the students might use cut and paste strategy. The difficulties is just on determining which part should be cut to be fit with the protrude part to make it rectangle. Some students might need more than once shape to do trial and error. The way of cutting the shape will influence the accuracy of the rectangle. But at least the students know the reason why they do it, just let it go. Here are some possibilities of students' way of cutting the shape (see Figure 4.21).





While the students are working with this shape, the teacher can ask the students whether the new shape and the original shape still the same bigger. This question is necessary to check students understanding of the concept of conservation of area. The students might use their previous activities to answer and to reason this question. Like the cloth-rag or the rice field shape.

## G. Meeting 6 – Recomposing Irregular Polygon Activity

# 1. Situation

## a) Exercise 5

The students will be given such an Exercise 4 for warming up and recall their knowledge before they do the Posttest. The exercise is similar with the previous meeting, reforming the irregular polygon into the perfect rectangle (see Figure 4.22).



Figure 4.22 The irregular polygon which must be recomposed into a rectangle

# 2. Conjecture:

Exercise 4 is set to be more difficult that the previous exercise to challenge the students. The difficulties that always faces by the students is to determine the part to be recompose into the rectangle. Perhaps, some students will do trials and error to reform this shape into the rectangle. Therefore, the teacher should prepare more additional shape to try on. Here are some possible strategies to cut and paste by the students on each shapes (see Figure 4.23).



Figure 4.23 The possible way to recompose the irregular polygon into a rectangle

### **CHAPTER V**

## **RETROSPECTIVE ANALYSIS**

Based on Gravemeijer & Cobb (2006) in Akker, et.al (2010), retrospective analysis is studying the entire data set to contribute to the development of a local instruction theory and (improvement of) the interpretative framework. The retrospective analysis in this chapter was done based on the observation of Cycle 1 and Cycle 2 of this study. This study was conducted in SD Al-Hikmah Surabaya. Cycle 1 was conducted in the class 3C while the second was in the class 3A. The teacher is Bu Lila Prasanti. She is a mathematics teacher in this school who taught in the grade 4 last year. This is the first time of Bu Lila to teach the topic of area measurement in the grade 3. But because of her experience in teaching mathematics for Elementary School for 10 years, we do not need to worry about her work.

In this study, the analysis will be conducted in two cycles, Cycle 1 and Cycle 2. Cycle 2 has to take the findings of Cycle 1 into account in order to improve the HLT in Cycle 1. Therefore, the design of this study usually has to change (or develop) from one cycle to the other and this change will be recorded in the HLT. The schedule of the observation is explained in Table 5.1.

	Date - Month	Activities
Preparation of thesis proposal in Netherland	August – December 2012	Consultation with the Netherland supervisor
Preparation for Cycle 1 in Indonesia	28 – 29 January 2012	Consultation with the Indonesian supervisor about the readiness for Cycle 1.
School preparation I	30 January 2012	Communicate with the headmaster for permission
School preparation II	4 February 2013	Communicating with the teacher about the research and arranging the schedule
CYCLE 1		
1 <sup>st</sup> meeting	6 February 2013	Pretest and worksheet 1,2 and exercise 1
2 <sup>nd</sup> meeting	7 February 2013	Mini lesson 1, Worksheet 3, math congress and exercise 2
3 <sup>rd</sup> meeting	8 February 2013	Mini lesson 2, Worksheet 4, exercise 3 (re-do) and a half of Worksheet 5
4 <sup>th</sup> meeting	13 February 2013	MiniLesson3,FinishWorksheet 5 and Exercise 5
5 <sup>th</sup> meeting	14 February 2013	Mini Lesson 4, Exercise 4 and Posttest
CYCLE 2		
1 <sup>st</sup> meeting	21 February 2013	Pretest
2 <sup>nd</sup> meeting	25 February 2013	Worksheet 1 and Worksheet 2
3 <sup>rd</sup> meeting	26 February 2013	Mini Lesson 1, Exercise 1 and Worksheet 3
4 <sup>th</sup> meeting	1 March 2013	Interperception1 and Math Congress
5 <sup>th</sup> meeting	4 March 2013	Mini Lesson 2, Worksheet 4 and Exercise 2
6 <sup>th</sup> meeting	5 March 2013	Inter-perception2andWorksheet 5
7 <sup>th</sup> meeting	6 March 2013	Exercise 3 and 4
8 <sup>th</sup> meeting	8 March 2013	Mini Lesson 3 and Posttest

Table 5.1. The schedule of the observation

## A. Analysis of Cycle 1 of HLT Implementation

In Cycle 1, there were six students from the class 3C who are Sasya, Iin, Ais, Fatil, Mail and Adit. The students were chosen based on their academic achievement, Aik and Fatil are the high achievers so that they are little bit dominant in the discussion. Iin and Mail are the average achievers; both of them are shy students. Iin is a little bit brave to say her answer but when she is asked to tell the reason she is afraid. Meanwhile, Mail needs to be forced to speak and to tell his opinion. Moreover, Sasya is very brave and talkative although she is the low achiever in this group with Adit. She usually says the answer although sometimes she cannot find the good reason. In Cycle 1, it was the first time for them to be in a group.

The HLT divided the lesson sequence into 6 meetings, but in Cycle 1 we will be not too strict or too free about the number of the meeting. As in Cycle 1 we will follow the students' responses on each activity in order to know how the HLT works for the students. Thus, in Cycle 1 we just needed 5 meetings to finish all of the activities because Meeting 2 and 3 are combined. Meeting 3 is designed for Math congress, since there were just two groups so the time needed was shorter than the prediction.

As information, the teacher in Cycle 1 is the researcher herself, while for Cycle 2 the teacher is their own mathematics teacher, Bu Lila.

### 1. Meeting 1 – Making table cloth activity

## a) Pretest

After short introduction about what the students should do, teacher gave little explanation about the instruction of each question in the Pretest. After the students were cleared about the question, they started working. Fatil thought that the shape A and B are same since B can be made just by rotate the shape A. However, the other students thought that B is bigger than A. This students might be still use their perceptual justification, focusing only to dominant dimension to determine that B looks bigger than A.

But suddenly Fatil put out a ruler and measured each length of the shape (see Figure 5.1). Then he added all of the length and used that number to determine which shape is bigger. He misused the concept of perimeter as the concept of area to solve this question.





Figure 5.1 using ruler to measure the shape

He also thought that C is bigger than D since the sum of the length and width is 22 while D is 20. Also for E and F, he did the same thing, measuring and adding all of the length and width on each shape. Then he concluded that E is bigger than F since the sum of E is 18.9 while F is 17.4. It is caused by the fact that Fatil has learned about the concept of perimeter and area indirectly when he joined the mathematics olimpiade's class. Unfortunately, we could not make sure how he was taught this concept at that time. But based on the curriculum, he should have not learned the concept of area measurement yet. Therefore, we assumed that his understanding about area measurement has not strong enough.

For Problem 2, there are three students who have thought about counting the number of rice field terrace. These students found that both of the rice field, in Surabaya and Bojonegoro, have the same number of rice field terrace, 12. They might think that although the rice-field in Surabaya is separated, but it is made up of the identical rice-field terrace. Therefore, they could conclude that the rice field in Surabaya and Bojonegoro will produce the same amount of rice since both have the 12 identical rice-field's terrace.

Meanwhile, the rest of the students answered that rice field in Bojonegoro will produce more rice because it is intact. These students also argued that the rice field in Surabaya will produce less rice since the location is separated and could not be integrated. It showed that these students did not consider the identical rice-field terrace for determining the area of the rice field.

## Discussion

Actually, in Pretest we do not expect that the students will use ruler to measure the length. Since there is no rule about the use of ruler in this Pretest so it was better to let the students do what they want to do and observed what they are doing. Then the student showed that they still swapper between the concept of area and perimeter. This is one of misconception that usually being found among students although they have learned the concept of area measurement. This misconception indirectly confirmed the need to introduce the conservation of area before the students learn the concept of area measurement. So the students can be more concerned into the shape itself, do not being disturbed by the measurement of the length.

The instruction on Problem 1 is clear enough, but still the teacher need to explain the instruction for some students who still have difficulties to understand it. The time, 15 minutes, for finishing two questions in Pretest was appropriate for them not too short to figure out the answer, but also not too long to reduce the boredom for students.

#### b) Worksheet 1

The students easily understood the question in Worksheet 1 since all of them have been familiar with the context. In the introduction of the context they said that they knew what is a tailor, what is her job, what is cloth-rag, what do people usually use the cloth-rag for and etc.

In Worksheet 1, the students have to imagine the possibility to make a table-cloth from a cloth-rag which has twice longer and a half width of the size of the table. The table and the cloth-rag were shown in Worksheet 1. Since the story in the Worksheet 1 said that Bu Tina has many cloth-rags, there were some students who think that they can use the other cloth-rag which are not

shown in the worksheet. Actually the story of Problem 1 has been explained that Bu Tina now just has the cloth-rag showing in the worksheet for making the table-cloth.

All of the students in Cycle 1 answered that the cloth-rag can be made for a table-cloth with different reasons. Mostly students argued that Bu Tina still has many cloth-rags to be used for covering the table precisely. But there was a student who answered that the cloth-rag can be cut and can be combined to cover the table. Hence, the idea to do recomposing shape by cut and paste strategy was revealed from the students themselves in the making table-cloth activity.

#### Discussion

The story is seemed too long for being understood by the students. Also, they were too depended on the teacher' explanation for understanding the problem. Therefore, when the students read the story, *"Because Bu Tina is a tailor, she had many cloth-rag in her house."*, they paid less attention into the next sentence which is, *"But Bu Tina now just has cloth-rag that twice longer and a half wider than the table."*. So they thought that Bu Tina has many cloth-rags to make table-cloth.

Hence, we can delete the story that Bu Tina has many cloth-rags so the students will be directed to the fact that the only cloth-rag is showed in the worksheet. We will emphasize that Bu Tina now just has one and only one cloth-rag which is shown in the worksheet. Moreover, the students' capability to understand the concept of half of the width of the cloth rag which is a half of the table was not discovered yet. It happened because none of the students talked about the measurement of the cloth-rag of the table in Worksheet 1.

## c) Worksheet 2

Worksheet 2 is used for proving the students' answer in the Worksheet 1. Thus, the students were given the fabric of the cloth-rag with the size twice longer and a half wider than the table. The table used in Cycle 1 was the Barbie's table. The students could use these tools as visual aids to answer the questions in the worksheet. Problem 1 asked them to prove whether it is possible to make a table cloth from the cloth-rag, the students can easily find the way to make the table cloth. They asked whether it is okay if they cut the cloth. Then they placed the cloth-rag horizontally in the table and found where to cut (see Figure 5.2). From this activity, the students knew that they have to cut the cloth-rag into two equal parts. Next, the students combined these two parts together and sewed on the table.



Figure 5.2 The process of students answer (superposition, know where to cut, and cover the table with the cloth-rag)

The students' conclusion stopped on their hands-on activity after the clothrag can be made for the table-cloth. But when they were asked which one is bigger, they were confused.

# Transcript 5.1 The confusion of the students about which one is bigger between the cloth-rag or the table

Teacher	: Which one is bigger, the table or the cloth-rag?
Students	: The table
Teacher	: Why do you think that table is bigger than the cloth-rag?
Aik	: Becausebecause the cloth-rag is small and long (while using her hand to show her thinking)

While suddenly, Sasya said that the cloth-rag is bigger than the table.

Aik : Haduh (annoyed expression)... Are you sure? Look, this is the table and this is the cloth-rag (pointing in the worksheet).

The other students were disturbing their conversation. So Aik stopped her explanation while Sasya looked still confused. Sasya looked dilemma between hold on her opinion or followed Aik is answer since Aik is cleverer than her.

Teacher : Ok, once again. Which one is bigger, the table or the cloth-rag?

Students : The table

The conversation shows that the girl tried to dominate the group. We can see from the fact that the boy did not give their contribution to determine the group answer. The boy just let the girls answer the problem. Generally, the students were confused with the word bigger and the size. They thought that big is related with size, then when you cut the cloth the size changes, the length and the width change. So, they thought that the table is bigger than the cloth-rag which has been cut into smaller parts.

### Discussion

Therefore, the teacher asked the students whether the cloth-rag is same or not before and after cutting. Some of the students answered same, some answered not. They argued the cutting process that makes the cloth-rag before and after cutting are not the same. Also, the smaller size of the parts of the cloth-rag makes the students think that it were different before and after. Also, when asked which one is bigger, students should be provided with the option, the cloth-rag, the table or both same bigger. If not, students will think that they have to choose either table or the cloth-rag. They will not think about another possible answer, like both of them have the same area or none of them bigger than the others.

Overall, the students still could not understand the idea that the cloth-rag will have the same area as the table if and only if they cover each other precisely. Actually, this was like the prediction on the HLT that this idea might not easily occur in the Meeting 1. But the teacher should be aware incase the students still could not get the idea that of recomposing a shape will preserve its area for the next meeting. If it is so, teacher should take a step by giving more guidance focusing on this problem.

## d) Exercise 1

In Exercise 1 students should do it individually. There were four problems (see Appendix page 254-255). In problem number 1, students could easily find the way to cover the table by cutting the cloth-rag into three equal parts. But when answering which one is bigger, most of them answer table is bigger than the cloth, just one answer that table and cloth have the same area. Unfortunately, this answer cannot tell us more about the students thinking since the question did not ask the students to give the reason.

In problem number 2, students could show their answer but they did not try all the cloth-rag. After they found that the cloth-rag 1 fit with table, they stopped trying and left the cloth-rag 2. When students compare which one is bigger, cloth-rag 1 or cloth-rag 2, they placed the shape beside each other.

We believed that the students were intended to do superposition, placing two shapes on top of each other to compare the size but they have never done that. So, they did their own way to compare the cloth-rag 1 and 2 (see Figure 5.3).



Figure 5.3 The superposition did by the students

For Problem 3, first the students tried the cloth-rag 2 then the cloth-rag 1. Since none of the cloth-rag can fit the table, students concluded that none of the cloth-rag can be used a table-cloth for the table 2.

## Discussion

Hence, problem number 3 should be changed to make the students try both of the cloth-rag 1 and cloth-rag 2 so in the end they might conclude that cloth-rag 1 has the same area with cloth-rag 2 since both can fit the same table, table 1. If not, then the teacher can give additional guidance about it. Meanwhile, problem number 2 is a perfect problem to make the students understand that for different shapes can have the same area.

Moreover, when giving the model of the shape as the visual aids for the students, make sure the students just are given one of each shape since when they are given more, the students will use all of them. Meanwhile, the use of more than one cloth-rag to cover the table will tell them if they are wrong in cutting or doing error they can ask another shape to the teacher. Do not forget to tell the students to bring their own scissors and glue to avoid them spending too much time in borrowing stuff from others.

#### 2. Meeting 2

## a) Mini Lesson 1

Mini lesson 1 was used for emphasizing the students' understanding towards the concept of conservation of area from Meeting 1. Actually this mini lesson has designed using the shape in the Exercise 1 as Figure 5.4.



Figure 5.4 The shape on the mini lesson 1

By using the shape in Figure 5.4, the students will recall their understanding in Meeting 1. The questions asked for this mini lesson is about the possibility to make the cloth-rags cover the table and how to do that.

However, since the students have not understood that the cloth-rag before and after recomposing have the same area, we decided to tackle this problem first. We changed Mini lesson 1 using the cloth-rag and the Barbie's table by repeating the hands-on activity in Meeting 1.

Transcript 5.2 The conversation about cloth-rag fit into the table

Teacher : Yesterday, we have this cloth-rag, right?
Students : Yes
Teacher : And we have to cut this cloth-rag to make the table-cloth. Now look at this table-cloth, I have sewed it with tape and it looks like this. Does this table-cloth have the same size as table (*while showing Figure 5.5*)?



Figure 5.5 Mini lesson 1, comparing the sewed cloth-rag with table

Students	: Yes, same.
Teacher	: Why?
Fatil	: Because it has been designed to be same
Aik	: Because the size is same with the table.

From the transcript above, it was surprising that yesterday Aik was confident saying the table is bigger than the cloth-rag, but in Meeting 2 she answered that the cloth-rag is same with the table since the size is same. Although in her answer of Exercise 1, she has never answered about the two shapes can be same because of the size.

Transcript 5.3 The conversation about the concept of reversibility Teacher : I have another cloth-rag having the same size as the cloth-rag before we were cutting (showing Figure 5.6). Now, which one is bigger, the sewed cloth-rag or the original cloth-rag? And why?



Figure 5.6 Mini lesson 1 comparing the sewed cloth-rag with the original

Students	: This is ( <i>pointing the original cloth-rag</i> ) bigger since it is longer.
Teacher	: Are you sure this is longer? Now I will also cut the longer cloth-rag into two parts. And look, you can cover the sewed cloth-rag by the two piece parts. Then what do you think? Which one is bigger?
Students	: Loh,( <i>expression of surprise</i> )same. But how does it come?
Teacher	: Now if I combined again the two piece parts like the original one, which one is bigger (see the figure 5.1.6 of the original shape-the right one)?
Student	: (pointing the original cloth-rag)
Fatil	: Ahhhh ( <i>confused expression</i> )How can it be? Why it is contradictory? I am confused.
Teacher	: Nah ( <i>expression of happy</i> ), who can answer Fatil's question?
Sasya	: I know. Because this has been cut and this has been sewed.

# Discussion

From the transcript above, we can see that the students still cannot get the conclusion about conservation. The students still thought that cutting the shape will always change the size of the shape. So, we need an extra work to make the students switch their mind from the size into the area without telling them about the word 'area' directly.

### b) Worksheet 3 – Comparing Rice Field Activity

In Meeting 2, the students were divided into two groups, three girls and three boys in order to explore their thinking because when the six students being one group, the girls were dominant than the boys. Therefore, in this meeting they were split up to make sure no one is dominant in the group so every student could explore their opinion.

When the students were introduced the context about rice field in Indonesia, they have known about it. There was no difficulty for them to understand and to relate the context into the problem in Worksheet 3. Also, they knew that the shape of the rice field in Indonesia is not always rectangular.

For Problem 1, the two groups seemed enthusiastic to solve the problem. First of all, the group of boys was initiative to measure the length of the rice field using ruler, then Fatil added all of the length on each rice field (using the concept of perimeter as the concept of area) and concluded that Pak Darma's rice field is bigger than Pak Badrun's.

Meanwhile, the group of girls compared Pak Darma's rice field and Pak Badrun's by placing the two shapes side by side (see Figure 5.7). We assume that these students intended to do superposition, but they did it in a different way.



Figure 5.7 The struggling of the group of girl compare two rice fields

Later on, both of the group found the way to prove that the two rice field is the same. They have used the correct way to place the shape in top of each other and they found which part to cut and where to place (see Figure 5.8).



*Figure 5.8 The way of the groups to find which part to cut and where to place* 

Furthermore, in Problem 2, when students were asked to show that Pak Salman's rice field is the biggest among others, they used one of the Pak Badrun or Pak Darma' rice field to be compared. But unfortunately they did it without giving confirmation whether they understood that they could do that because of Pak Badrun's rice field has the same area with Pak Darma's. Students tend to choose Pak Darma's rice field to be compared with Pak Salman's since it is easier to be compared. They also did not need to recompose the shape, they conclude that the rice field of Pak Salman's is bigger than Pak Darma's by using their perceptual justification after superposition.

For Problem 3, students got difficulties in understanding the question. They could not get the idea that they have to form other shape which is different from Pak Salman's rice field. They did not know how to form another shape. Both of the groups tried to cut Pak Salman's rice field to fit the previous one.

#### Discussion

From the result of Worksheet 3, the students needed more guidance to understand what they should do especially when they do Problem 3. The students need additional help to understand the meaning of "different shape" and "change". Therefore, we can give the students such a mental process in order to recognize and interpreted a word or an instruction through one or more of the senses stimulated by a physical object or we called it as interperception

The interpretation will use several pairs of shapes to show the similarity and difference of the forms. The similar shapes will be showed by rotation and flipping of the shapes. Meanwhile, the other two pairs will show a little and a much difference on the form. By giving this inter-perception before the students doing Problem 3, they are expected to be able to change the Pak Salman's rice field into another form, not forming the original one and without adding or cutting down any part of it. Next, the students will be easy to lead to the correct answer of the question, "Does the new rice field of Pak Salman have the same area with the original one?".

## c) Math Congress

Since there were just two groups Math Congress could not held effectively. Both of the groups have understood quickly each other's strategy to solve the problem. But in Math Congress, the group of girl's has successfully found the basic idea of conservation of area.

When the teacher took the part that they cut from Pak Salman' rice field to form the new rice field then put it in another side (see Figure 5.9), the group of boys still thought that it was different but the group of girls concluded that it has the same area except the pattern is different.



Figure 5.9 The new form of Pak Salman's rice field

# Discussion

We could conclude that what students' thought as 'pattern' is actually the form of the shape. Finally, these girls have got the idea of recomposing shape will preserve the area in Meeting 2. This was the start of the students to learn the conservation of area. Therefore Math Congress was done in the short time, less than 15 minutes. Hence the number of meetings in Cycle 1 could be less than the planning in the HLT, from 6 meetings become 5 meetings. However, we think that it will be different when we held this math congress in the real class, with the whole class (Cycle 2). There should be many students to react with his friend's answer, asking, giving opinion or criticizing.

#### d) Exercise 2

After the students did Math Congress, they were given an Exercise 2 which was intended as an introduction for the next meeting. There was just one problem in the exercise that is to determine which rice field will produce more rice, A or B if in both of them were built houses with different shape but used the same area of the land.

There is no other clue like the length of the houses or the length of the rice field. Also, the students were not provided the model of the rice field. They should solve it just by looking at the figure. All of the students answered that rice field A is bigger than B. These students argued about the number of the house in both rice fields. Since the rice field A just has one house, so the rice field A must be bigger than the rice field B which has three houses.

From these reasons, we can conclude that these students have understood that the house built in the rice field will decrease the area of the rice field. Meanwhile, they still could not see that the area of the big house in the rice field A and the three small houses in rice field B actually equal.

### Discussion

Actually, after the students passed through the next meeting, Meeting 3, and learned about the identity in the concept of conservation of area, they were expected to be able to think that the area used for the one big house is equal to the four small houses. So the students will re-do this exercise in the next meeting with follow-up questions.

But we think it was impossible for them to know that the area of one big house and four small houses are the same without any attribute (e.g. length) on it or any tools (model of the shapes) to help. Therefore in Exercise 3, the redo exercise, the students will be given model of the rice fields and the houses to answer the question.

## 3. Meeting 3 – Trade Up Rice Field Activity

#### a) Mini Lesson 2

The purpose of Mini lesson 2 is to make the students really understand the concept of recomposing before they go to the concept of part and whole. Since some of the students have found that Pak Salman's rice field can have different pattern but still has the same area, we needed to explore this idea. In Mini lesson 2, this understanding will be explored by using paper A4. Paper A4 is very familiar with the students. Also it is easy for them to understand that two identical paperA4 have the same size, so no one is bigger than each other. Moreover, when one of the papers A4 was cut in pieces and was arranged to form different shape, the teacher could use the fact of two
identical A4 to push the students to think that before and after cutting, the shapes still have the same area.

The students were given two A4 papers and convinced that all are the same, when the teacher cut one of the papers into four different parts and asked them whether the two papers still have the same area or not, all of the students answered they have the same area except one boy. The girls have understood that those papers have the same area just different in shape.

Since there is one student who still did not get this concept, but the teacher gave context in this activity. Assume that one paper A4 need 100 ml watercolor to color the paper. As the two papers A4 are same, each of the paper A4 need100 ml of watercolor. When one of the paper A4 is cut into three parts, the students were asked, *"How many ml of watercolor needs to color this paper?"*. The students have known that to color the paper they still need 100 ml. They answered it very confidently. Although when the teacher asked the question for several times, the students still hold on with their answer and argued that it was the same but different in the pattern (form).

# Discussion

Thus, we could say that students are easier to understand the concept of recomposing by using the numeral than the word bigger or the size. Although, Fatil , a student who from the first got difficulty to understand that the two identical papers A4 one of them has been cut and arranged differently were having the same area, finally could understand that two identical papers A4 will need the same amount of watercolor even when one of them has been torn in pieces.

Therefore in Cycle 2, Mini lesson 2 using A4 paper could be used as a strong and helpful activity in case the students still have not got the concept of conservation of area. As this activity has too strong response to the students in understanding the concept of recomposing, we decided to use it as an alternative mini lesson when the students do not understand the concept or too late to understand the concept.

#### b) Worksheet 4

When the students were given Worksheet 4, they should be given the model of the rice field. But since the group of the girl's showed their enthusiasm to solve the problem by imagining how to combine the separate rice field terrace in Surabaya, they were free to work without the model first.

Unfortunately, the lines made by imperfect printing were used by the girl's to determine which one of the rice field will produce more rice (see Figure 5.10)



They tried to measure the terrace by using her hand and to predict where to place. Meanwhile, the boys' again used ruler to measure the terrace. First they measured the length of the big rice field. Then they added all of the length. Next, they subtracted this number by each length of the terrace rice field. Unfortunately, we did not confirm how the rule works for these boys.

By doing their strategies, the students could not find the way to solve the problem. The group of girl's thought that Surabaya will produce more rice because the checkered rice field will make this whole rice field when it is combined. On other hand, the group of boys also argued about the checkered rice field in Surabaya which is 5 square more than in Kediri. But they answered that Kediri will produce more rice. We thought this group was confused to answer this problem.

After the students were given the model of the rice field, they easily understand what to do. But they still could not get the answer that both of the rice fields, Surabaya and Kediri will produce the same amount of rice since these rice fields have the same area. They have tried to figure out whether these rice fields have the same area or not, but they failed (see Figure 5.11).



Figure 5.11 The answer of the girl's and boy's group

# Discussion

From the observation, we assumed that the form of the rice field in Surabaya was complicated for the students. The students have thought to cover the rice field in Bojonegoro with in Surabaya, but they got difficulties in arranging the pieces of the rice field. The arrangement which they choose did not help them to cover the rice field in Bojonegoro perfectly. They also did not use the terrace rice field (the smallest rectangular rice field in Surabaya) as a helpful tools. Actually, by using the terrace rice field, students could find which part of the bigger rice field in Surabaya should be cut. Then they can arrange it precisely with the rice field in Kediri. Thus in Cycle 2, the teacher needs to give guidance orally about the possibility of the students to cut the rice field in Surabaya in order to cover the rice field in Kediri precisely.

#### c) Exercise 3 (Re-do Exercise 2)

Since the students have demonstrated that actually the area of the house in rice field A and rice field B have the same area although the dimension is different, the students changed their mind. But what they understood was not the rice field which has the same area, but they answered that the area of the house in rice field A and B are same. Hence, this misunderstanding has not been clarified yet for the students in Cycle 1.

### Discussion

Looking from this answer, we can conclude that the problem and the instruction on this exercise need to be revised. Especially, the fact that there is no sense of the students to be able to solve this problem due to there is no attribute (e.g. length of the house or model of houses to be try it on) to help them figure out the answer.

#### d) Worksheet 5 Problem 1 – Rice Production Activity

The Worksheet 5 was asked the students to find the number of rice production of some rice field with particular size. The students were given the square rice field which will produce one ton of rice and asked to find the rice production of the other shape of rice fields, a parallelogram and two rectangles (thin and big). The students knew that they need to recompose the parallelogram rice field to fit the square rice field to know how many tons it will produce. The students could argue that when the paralelogram shape was recomposed and fit with the square, so the paralelogram rice field will also produce 1 ton of rice (see Figure 5.12). The students' reason on this problem has developed. They showed their understanding toward the concept of conservation of area in this problem better than the previous problem.



Figure 5.12 The result of recomposing parallelogram to fit the square Furthermore, for the big rectangles rice field, the students traced the square, folded the rest and found that the big rectangle was twice the square. Therefore, the students answered that this rice field in rectangle shape will produce two tons of rice.

### Discussion

Accordance with the assumption that the attribute of the shape can help the students understand the concept of conservation of area, it was proven in Exercise 3. By the information that a rice field in a square will produce one ton rice, the student quickly found the way to figure out how many rice produced by the other form of rice field like rectangular and parallelogram rice field. So, this worksheet was placed in the right place, not too early and not too late.

# 4. Meeting 4

# a) Mini Lesson 3

Students were given Mini lesson 3 about the concept of identity they have learned. Mini lesson 3 was about two rice fields which have some of its land for housing. At the first, the students were showed two identical rice fields (rice field A and B) so these rice fields will produce the same amount of rice.

Then the students were showed that in the rice field A was built a house and were asked whether the two rice fields still produce the same amount of rice. The students have answered that these two rice fields will not produce the same amount of rice anymore. They argued that the area of the rice field has decreased since it was used for the house. But Adit still could not get the reason why these rice fields should produce different rice production now. Although in the end he agreed with his friend, but he looked confused.

Next, when the students were showed that in each of rice field was built three small houses neighboring each other with the same size, they have understood that the rice fields will produce the same amount of rice since the land used for the house in rice field A and B are the same. They argued that just the arrangement (vertical and horizontal) of the houses is different, but the land of the houses has the same area since the size of the house was the same. Even though, we separated the three houses in the rice field A and let the three houses in the rice field B be neighboring, the students could understand that the both of rice field still produce the same amount of rice. They argued that the land used for houses have the same area although the location of houses in the rice field B are separated.

## Discussion

There were no difficulties for the students to get the idea of the mini lesson since in the previous meeting almost all of the students have understood the concept of conservation of area. Only several students such as Adit who often ignore the learning process and still cannot answer the mini lesson satisfactorily. Generally speaking, from this mini lesson we can assume that the students have been able to follow and have got the purpose of the learning activities.

# b) Worksheet 5 Problem 2

Students recalled the work they have done yesterday, Worksheet 5 Problem 1. They were showed their work on that problem and were asked by the teacher how they find the answer. After the students remembered the problem and the way to solve Problem 1, they continued their work to finish Worksheet 5 in two groups, boy and girl. By using their previous answer in Problem 1 the students automatically arranged the model of rice field given to cover the big rice field. The group of girls used the entire model of rice field to cover the big rice field. Meanwhile, the boys just used several models of rice field then gave a line as the sign how many parts of the big rice field can be covered by the model. So just by using one or two models of rice field, the group of boys could find the number of rice produced by the big rice field.

The big rice field whose shape is parallelogram was not in the perfect dimension to be covered by the model of rice field. Therefore, the students assumed that it was the designer fault and continued their work with the conclusion that it was the same.

#### Discussion

There was no specific problem for students in solving Worksheet 5. Like in Meeting 3, the students were easier to understand the concept conservation of area with the attribute given on it. But, the decision to take Worksheet 5 in the end of the lesson activities was sufficient. We have designed that the students will experience the lesson activities from the only geometric task then goes to the intertwined between geometric and number task. It was important for the students to be able to work with just only geometry because the introduction of number as the attribute of the geometry in the beginning will disturb students' progress of understanding. For instance, the use of perimeter as area by one of the students in Cycle 1 was caused by the introduction of number too early for that student.

#### c) Exercise 5 – Recompose Irregular Polygon Activity

Because of the negligence of the teacher, Exercises 5 which reformed an irregular shape into square or rectangle were given too early. In this meeting, the students were doing the Exercise 5 that should be done in the last meeting (before Posttest). Actually, the students should get the Exercise 4 which have the same purpose and the same question but differ in the shape and the level of difficulties. Of course, the Exercise 4 is easier than Exercise 5.

Therefore, the students looked having trouble with the Exercise 5. The irregular shape made the students being frustrated. They did not know how to change that irregular shape into square or rectangle. Even the students did not know what is a square or a rectangle. They also still could not understand the instruction to reform or to make the irregular shape into a square or a rectangle.

Although the teacher has tried to explain to them about what they should do, but there was still some students who got misunderstanding. They are the low achievers. They cut a shape of square in the middle of the irregular shape and paste it in Exercise 5's paper. Also, there was a student who tried to cut the irregular shape into the small squares and tried to arrange those small squares into s rectangle. But it did not work since this student cut the shape randomly. Many of them got stuck on the first problem. But after a student, lin, found a way to reform the second shape to be a rectangle, some other students started to get enlightenment what they should do.

### Discussion

Actually, the purpose of Exercise 5 is just for enrichment for the students before dealing with the similar problem in the Posttest. Since in the reality, this exercise was too difficult for the students and also need too much time for them to solve it, so we decided that the kind of shape and the level of difficulties of the shape needed to be reconsidered again for Cycle 2.

We did it since the main purpose of this study is not to test the students but to help them learn and understand the concept of conservation of area. Therefore, the decreasing of difficulty level in Exercise 5 seemed logically to do.

# 5. Meeting 5 - Posttest

### a) Mini lesson 4

Since the students still could not get the idea of transitive thinking in the Meeting 1, the students were given Mini lesson 4 which repeated problem of Exercise 1. This was done in order to check students' understanding on transitive thinking. In Exercise 1, the students did not show their understanding about transitive thinking because of the less guided and less detail of the instruction. As we know, students on that age still need direct guidance for doing or thinking like what we expect. Therefore, the teacher did Mini lesson 4 to check whether the students could think like the expectation with this direct guidance:

- Does the cloth-rag rag 1 can be made for the table cloth-rag in table A?
- Does the cloth-rag rag 2 can be made for the table cloth-rag in table A?

• Does the cloth-rag rag 1 same big as the cloth-rag 2?

The shapes of the table A and cloth-rag 1 and 2 are same as Exercise 1. Students were trying to re-do the exercise, They were trying to check whether cloth rag 1 can cover the table A and cloth rag 2 can cover the table A also or not. They were given the model of table A, cloth rag 1 and cloth rag 2 to be used for proof their answer.

After the students found that both cloth rag 1 and cloth rag 2 can cover the table A, teacher (researcher) asked whether cloth rag 1 and 2 were have the same area or not.

Transcript 5.4 The conversation about transitive thinking among students

Teacher	: Does the cloth rag 1 same big as the table A?
Students	: Yes, same.
Teacher	: Does the cloth rag 2 same big as the table A?
Students	: Yes, same.
Teacher	: Ok. Now, the question, does the cloth rag 1 same big as
	the cloth rag 2?
Students	: Same, because both of the cloth rags can be used for
	making the cloth table of the same table, table A.

Moreover, we also gave the students a problem about how to change a shape into a perfect rectangle or square. First, the students were asked to change the shape of parallelogram and trapezium into a rectangle. Fatil was changing the shape of parallelogram into a rectangle, while Iin was changing the shape of trapezium into a rectangle. The students were not told the name of the shapes (parallelogram and trapezium), they will just call it as shape 1 and shape 2 since the shape are numbered. The other students were looking what their friends' doing while checking and helping them. The students easily find the way to change the shape 1 and 2 into a rectangle. Therefore, to check the students' understanding is asking question such as; *"How do you get that? Why you should cut that part? Where do you place that part?"*. The students did not answer the question. They just showed what they have done with the shape.

## Discussion

The students did not show any difficulties to follow Mini lesson 4 about transitive thinking. It might be caused by their understanding toward the concept of conservation of area has been built. But it proved that the revised question on this problem has succeeded to guide the students in the transitive thinking. Thus, we could use this question for Exercise 1 whose purpose is to facilitate the understanding of transitive thinking to the students.

#### b) Exercise 4

In the beginning, the students were asked to look the shape on the problem 1 in the Exercise 4. The teacher asked the students to solve this problem together. One of the students answered and showed his way to change the irregular rectangular shape into a rectangle. Then this answer was discussed together with all of the six students to give such an example what actually the problem wanted them to do. After the students understood of they should do, the teacher let them to work on Exercise 4.

# Discussion

The given example was very helpful for the students to understand what they should do in Exercise 4, especially for the low achievers. But we needed to consider the number of the shapes and the level of difficulty. As we have said before, this design was not intended to test the students with difficult problem. We wanted them to learn the concept.

Thus, we will think of changing one of the shapes which is the most difficult one into the easier. We hope by doing this, many students will be able to solve the problem.

#### c) Posttest

Posttest was given to measure the students' understanding about the concept of conservation of area. The problem number 1 and 2 in the Pretest were also given in the Posttest in order to check how the students' thinking change after following the design activities we have made. The rest problems, number 3 and 4, will evaluate the students' competences to capture the big idea of the concept of conservation of area which is recomposing.

Four of six students changed their answer for the problem number1 and 2 in the Posttest, but most of their changed answer is still incorrect. Just Adit, who is the low achiever, did the correct changes for problem number 2. He argued that both of rice field have the same area since it has been proven. But what did he mean by it has been proven was not clear enough. Who have proved it and how to prove it was not identified from his argument. Before, in the Pretest, he answered that the rice field in Bojonegoro will produce more rice since rice field in Surabaya is separated and could not be combined. He also argued that the rice field in Bojonegoro produced more rice since it cannot be separated. Thus, we could not conclude whether Adit' answer could represent that he has understand the concept of conservation of area.

For the other problems number 3 and 4, Aik and Iin have been able to solve correctly. They showed their understanding about the recomposing shape well. They also found the way to communicate their thinking in recomposing the shape when answering question number 3 (see Figure 5.13).



Figure 5.13 The good way to communicate the answer from Aik

However, the other students also used the same way to show their work, but not as clear as Aik. Although some of them have added some sentence to explain their work, but it was not clear yet where to place the cutting part. Even there was a student who measured the length with a ruler, but he found nothing since the length of the shapes was different due to its gradient (see Figure 5.14).



Figure 5.14 Measuring the length to compare two shapes.

From his answer, we can conclude that this student compared the number of each length to figure which shape is bigger. Although, there was no question about which one is bigger.

# Discussion

Overall, in the Posttest few students changed their mind from their answer in the Pretest. Even if they changed it, their answer has not yet been correct especially for the problem number 1. We think we needed more directly treatment for the problem number 1. The students need to experience working with the problem similar to problem number 1.

The new problem in this test were problem no 3 and 4. Actually, the students have been directed to solve the problem no 3 and 4 by learning Exercise 4 and 5. Thus, all of them should understand the problem but two of them still could not find the correct answer. This might be caused by the differences of student's ability to capture the information.

### **B.** Conclusion of the HLT Implementation in Cycle 1

From the experience teaching several students about the concept of conservation of area before they learn about the concept of area measurement, it looks fine. The students just need more time to understand the idea of recomposing shape. It was natural if the students could not easily get the idea of conservation of area since it was the first time for the students learning about the concept of conservation of area directly and independently. Even the students could do the problem sufficiently, but their understanding did not grow as fast as predicted. We need some extra works to bring out the understanding about the concept of conservation of area.

Starting from Meeting 1, the students were given experience to cut and paste a cloth-rag for covering a table. The activities in Meeting 1 was successfully stimulating students to recompose the cloth-rag to fit the table.

As they were given the long fabric (cloth-rag) to cover the Barbie's table, they directly thought to cut the cloth-rag. It showed that students at this age have had the idea of recompose a shape but they have not understood the reason and the effect of recomposing. Therefore, the activities in Meeting 1 will help the students to have at least a reason to recompose the cloth-rag, to make a table-cloth. Although the understanding of the effect of recomposing did not happen in Cycle 1, but we believed that the activities in Meeting 1 will bring out the understanding of recomposing, at least can be used as the foundation to understand the concept of conservation of area. Also, one point that should be noticed in Meeting 1 was that the question must be written explicitly. If it was not, so the students will respond unpredictably or even respond with an unexpected answer.

The process of understanding the idea of recomposing continued to the next activities in Meeting 2. Problem 1 and Problem 2 which asked the students to compare two shape of rice field with different shapes taught the students another function of recomposing shape. Actually, Problem 3 was used to help the students to understand that recomposing will preserve the area but for the two groups it seemed difficult to catch. Even for the instruction, the students could not do it well. Therefore, for Cycle 2, the students need extra guidance to understand Problem 3.

Math Congress has helped the students to confirm their work with the other. Since the answer of two groups is almost the same so they did not need much time to agree on their answers. But the answer for Problem 3 was incorrect. It was caused by their difficulties to understand the instruction. They were asked to change the form of the rice field into another form. But they cut the shape to fit again the original one. They did not understand the instruction to make the form of the shape being different. But after the teacher cut the part of the shape and moved it into another side, the students could answer that those two rice fields have the same area but just different in pattern. We can see that, in math congress the role of the teacher was very important to lead the discussion into the right track.

At the first time the students did Exercise 2, they got difficulties to relate the exercise with the concept of conservation of area. It was caused by the problem

which did not provide enough information to make the students think about conservation of area. There was no numeric information on the shape of the rice field or the houses. Also it was not provided with model of rice field and the house. But after the students were given the model of the rice field and the house in Exercise 3 (redo Exercise 2), they could relate it with the concept of conservation of area.

It is showed that the students have followed the sequence of activities with the understanding of the idea of recomposing. When students were given the worksheet of part-whole relation, they learned it faster since they used the numeric on the shape to prove that the shapes are the same. There was no problem found in the activity to teach the idea of part-whole relation on the concept of conservation of area.

However, when the students were asked to reform an irregular polygon into a rectangle or a square, it seemed difficult for them. There were some students who showed that they struggled to find the way to change the shape, which to cut and where to place. Meanwhile, there were also some students who showed that they still did not understand the instruction. They cut inside the shape a little square or rectangle and used it as the answer.

The last but not least, in the Posttest the students of Cycle 1 still cannot answer the problem number1 correctly. They still used their previous answer which was incorrect. Maybe they were less attention the problems number 1 and number 2 in the Posttest because it was repeated from the Pretest. Furthermore, for the problem number 3 and number 4, a half of the students has understood the instruction and has done the correct answer. If we compare Pretest and Posttest of the students in Cycle 1, more than half showed significant development about the concept of conservation of area. Although the rest of students could follow the learning process well, but in the end they could not show their understanding into Posttest.

From this we can conclude that the instructional learning activities in Cycle 1 still leave problem for the low achievers. They still could not transform their understanding about the concept of conservation of area from their learning process. They still need more guidance.

#### C. Revision HLT

The HLT we made for Cycle 1 was revised based on the retrospective analysis. We make a revision almost in all activities, but not a big change just making the instruction more direct and adding an inter-perception to help the students understand the instruction given.

For instance, in the Pretest, in the last pair of shape in the problem number 1 (shape E and F) was changed with the simpler shape (shape C and D) because the shapes are too complicated for the students to check their understanding of the concept before they learn the concept of conservation of area. The position of the shapes also re-arranged based on the difficulty level. The original shape C and D will be placed as the last pair as the shape E and F. Meanwhile, the new shape as the substitution of shape E and F before will be placed as the shape C and D.

The story of problem number 2 was deleted to reduce the students' confusion when understanding the story because the students need too much time to understand the story of problem number 2. Hence the students can directly focus on the shape and the problem.

In Worksheet 1, the story that Bu Tina has many cloth-rags in her house was deleted because many students answer that it was possible to make the table cloth since Bu Tina still has many cloth-rags. Meanwhile, actually the story has told the students that now Bu Tina just has one cloth-rag like shown in the worksheet.

For problem number 2 in Worksheet 2, it should provide the alternative answer like, "*Compare the cloth-rag and the table, which one is bigger? The table, the cloth-rag or both of them are the*". It should be done as when the question asked which one is bigger, the students thought they have to choose, either A or B. They were often to think that both of them can be same big or none of both is bigger than each other. Moreover, when look from the meaning of the question, it was less sufficient to lead the students to answer like our expectation.

In Exercise 1, the question should be made more explicit since based students at age 9-11 years old still needs direct instruction to understand the problems. For instance,

"Does the cloth-rag 1 can be used to cover table A precisely? Show it! Does the cloth-rag 2 can be used to cover table A precisely? Show it! Which one of the cloth rag 1 or cloth rag 2 that can cover table B precisely? Show and Explain it! Which is bigger, cloth rag 1 or cloth rag 2? Why? Explain your answer!"

For Worksheet 3, in order to solve the Problem 3 accordance with the instruction, the students need to be given such **an inter-perception**. They need to

be introduced and to be experienced with the meaning of the sentence "*change the shape into different from*". The inter-perception will give the students several pairs of shapes then they have to identify which pairs of them are similar and which pairs are not. By doing this, the students are expected to understand that they should change the shapes into different form, not similar with the original shapes.

In Worksheet 4, the story will be revised since it has been many problems who asked the students to compare two things by recompose the shape. Meanwhile, these questions actually less reflected the concept of conservation of area. The story will be contains the concept of conservation of area, so by just understanding the story the students could answer the questions. The story is changed like the following;

"Pak Sudirman is a rich man. Several years ago he has a rice field in Kediri as the big and whole land. Later on, every year Pak Sudirman sells his land of the rice field part by part. A month later he will always buy the land of rice field with the same size that he has sold.

- Therefore, the rice field of Pak Sudirman now (like the figure given) is separated each other.
- Does the rice field of pak Sudirman now still produce the same amount of rice? "

For Exercise 2, the students will also be given the model of the rice field while before was not because without the model and also the absence of attribute on the figure in this exercise was means nothing for the students. Exercise 2 will also do once. There will no re-do exercise (Exercise 3) since it spent too much time.

In Exercise 4, the number of the problem will be reduced from 4 become 3 and the problem number 1 will be used as the example to be discussed in front of the class. Also in Exercise 5, the number of the problem will be reduced from 4 become 2. It is done because just two out of six students in Cycle 1 who could solve two out of four problems. Therefore, we will keep the two problems which could be solved by the students.

Finally, in the Posttest we will make several changes. In the problem number 3, we will change the shapes since the previous shape will be used in the interperception 3. Also, we will add a problem to check the students' understanding to the part-whole relation. Thus, the Posttest will have five problems while before just four problems.

#### D. Analysis of Cycle 2 of HLT implementation

The revised HLT from Cycle 1 will be implemented in Cycle 2. Similar with Cycle 1, the analysis of the implementation of the HLT in Cycle 2 will be focused on the students' learning processes than the result. But, in Cycle 2 we would also focus in the learning process of a chosen group of students, which is a called focus group. The focus group was chosen based on the teacher consideration in order to get a group which could show their thinking during the group activities. The five students in the focus group are Iman, Eky, Fadel, Arka, and Rayan. Fadel is the high achiever, while Iman, Eky and Arka are the middle achiever and Rayan is the low achiever. Two out of five students in the focus group are Iman and Eky.

Cycle 2 was conducted in the class 3A which consisted of 32 students, 16 boys and 16 girls by their own mathematics teacher, Bu Lila. During the teaching

experiment, we did not only observe the learning process of students but also paid attention on the teacher's role during the class experiment. In this analysis, we would briefly review the work of all instruments in each activity. First we will show the analysis of the whole class then we will compare with the analysis of the focus group or vice versa.

# 1. Meeting 1 – Pretest

#### a) Pretest

There were 25 students took the Pretest in Cycle 2. The problems of the Pretest in Cycle 2 were the same with the problem of the Pretest in Cycle 1. We just made change one pairs out of three pairs shape in the problem number 1 with the easier shapes and also we modified the story in the problem number 2 become shorter and more direct. This was done to lead the students to think the concept of conservation of area.

When the students did the problem number 1, they did not allow using ruler by the teacher. Since there were no tools to help, so the students tended to use their perceptual justification, focusing on the dominant dimension. The students observed the length or the width of the shape. Then they determined which one looked longer or wider as the bigger shape just from their observation.

Unfortunately, just two out of five students in the focus group who took the Pretest, which were Fadel and Arka . The other were absent at the day Pretest was conducting. In the first problem, Fadel used his finger to compare the length of the shapes. For instance, he used his finger to measure the width of the shape A and the shape B. He measured the width of the shape A by using two out of five fingers (thumb and index finger) of his left hand and the width of the shape B by using two out of five fingers of his right hand. Then he compared it by associating the measurement of the fingers in the left hand with the right hand to see which one is bigger.

Not only Fadel who used his finger to compare the shapes, there were some students who also used finger to solve the problem. But they used different strategy. For example, one of students tried to use both of the thumb and index finger from the left and the right hand to measure the width of the shape A. Then he moved his finger with the same position into the shape B and found which one is bigger (see Figure 5.15).



Figure 5.15 A student was using finger to compare two shapes

We believed that there were many strategies used by students to compare the shapes in the first problem without the help of ruler. However, since the limitation of the researcher to observe all of the students, so we could not detect all strategies that used by the students. Moreover, in the problem number 2 of the Pretest, more than a half of the students answered correctly. These students argued by using the number of terrace on both rice fields which is same 12 terraces. Unfortunately, we could not conclude that these students thought the terraces as the units since there was no enough proof to show. We believed that they just saw the terrace as something which can be counted. So they used it as a tool to prove. Also, they might be not care whether the size of the terrace in both of the rice field was same or not.

From the analysis of the Pretest, we could conclude that the students in the class 3A have more familiar with the idea of conservation of area when the area can be counted like the problem number 2. However, when the area could not be count, they need additional tool to help such as the model of the shape. Actually, we will provide the model of every shape in the next activity. We will give the students the model of the cloth-rag, the table, the rice field and the shape of irregular polygon to be manipulated by the students in order to get the understanding of the concept of conservation of area.

# 2. Meeting 2 – Making Table-Cloth Activity

### a) Worksheet 1

Worksheet 1 asked the students to think the possibility of making a table cloth from a cloth-rag which is twice longer and a half wider from the table. Before they were given the worksheet, the teacher introduced the context of making table cloth for the student. The introduction happened naturally, most the students have been familiar with the tailor, their work and the cloth-rag. They also discussed the meaning of the cloth-rag for them. A student also answered that one function of the cloth-rag is to be made as a table cloth.

After Worksheet 1 was given to the students, the teacher emphasized that the cloth-rag in Worksheet 1 was just one and only one. So the students could not think that Bu Tina has another cloth-rag in her house. Then the teacher discussed with the students the meaning of a half. The students seemed to know what is a half, but they could not communicate it. Therefore the teacher used an illustration as the following, "*If the width of the paper is 10 cm, so what is the half of it?*". The students answered correctly which is 5 cm. From this activity we could see that the students have known the meaning of half. The teacher also gave comprehension to the students about the meaning of twice longer by the illustration of paper and using measurement (cm). The role of the teacher in Worksheet 1 determined the students' understanding of the problems. When the students understood the problems, then it will lead the students to answer as the expectation.

Furthermore, some students could think of the strategy to cut and paste the cloth-rag to fit the table. The modified story in Worksheet 1 made the students to directly go to the problem, not too much attention to the story. Most of the students answered that it is possible to make the table cloth from the given cloth-rag because the cloth-rag can be cut and be arranged to fit the table. For instance, the cloth-rag should be measured to the table then cut the leftover then sewed it beside the previous cloth-rag. All the students in the focus groups answered that the cloth-rag could be used to make the table cloth but they had different argument. Some of them argued about the size of the cloth-rag which is longer than the table but also a half wider than the table, so it could be fit. Even, Iman have thought to cut the cloth-rag into two and to sew the top and the bottom of the parts. We believed that Iman thought about the correct answer, but he might be not able to communicate his thinking clearly in his answer.

From the students' understanding about twice and half, they thought to cut the cloth-rag in a half then combined the two equal parts to cover the table. But there were a few students who thought that it was impossible to make the table cloth since the table is wider. However, these students have changed their mind after they did Worksheet 2 later on.

Similar with the Pretest, in Worksheet 1 the students were not allowed to use ruler. Therefore, some of the students again used their finger or pencil to compare the length of the table and the cloth-rag. Some students used the strategy to compare the cloth-rag and the table by their finger because they did not have any tool to compare (ruler) or even the model of the cloth-rag to be manipulated. This shows that children, in this case is the students, will use their creativity when they did not have any tool and anyone to help. Sometimes, the students need to be forced to think by giving a problem which they could not answered directly and easily. Therefore, the problem in Worksheet 1 could indicate what the students could do to solve a problem when they do not get any help from the story in the problem, from the teacher or even from the measurement tool (ruler). However, we could say that comparing the shapes, thinking to cut the clothrag and to paste to the table and even planning to sew the pieces of the clothrag to fit the table, was the starting point to understand the concept of conservation of area for the students.

#### b) Worksheet 2

In Worksheet 2, the students were given the model of the cloth-rag to be made as the table cloth of their own table (in the class). They were asked to prove their answer in Worksheet 1 by using the model of cloth-rag and the table, so they could try it on. But unintentionally the set up of the group sitting was using combination of two tables. So, most of the groups used the combination of two tables to measure the cloth-rag. They put the cloth-rag into the two tables and found that they have to cut the cloth-rag into two equal parts. The focus group also did that.

The students in the focus group have known that the cloth-rag can fit the table from their experiment, but in the group discussion they still tent to focus on the dominant dimension. Also, the focus group answered that *'the table is wider while the cloth-rag is longer'*. But the focus group also wrote that dividing the cloth-rag into two could make the width same with the table. So we could see that the students still confused with their experiment. They knew

that recomposing cloth-rag could fit the table, but they still thought that the cloth-rag is bigger than the table because it was longer than the table. Hence, it shows that the students still confused with the size of the shapes.

Therefore, in Cycle 2, we gave several additional activities which called inter-perception in order to strengthen the new concept they have just had. The **inter-perception 1** aimed to strengthen the students understanding about the concept of reversibility in the conservation of area. In the inter-perception 1, the students were showed a figure of three cloth-rags in the LCD. The first cloth-rag was the cloth-rag before cutting. Then the cloth-rag was cut into two equal pieces and separated in the middle to be numbered as the second cloth-rag (see Figure 5.16). The third cloth-rag was the cloth-rag to be fit with the table.



Figure 5.16 Inter-perception 1 for understanding the concept of reversibility

From the figure that showed in the LCD, the teacher asked to the students, "Which one is bigger, the cloth-rag in number 1 or in number 3? Why?". The intention of the question was to ask whether the cloth-rag in number 1 and number 3 has the same area or not. We could not use the term 'area' in this learning process, so we used the question "which one is bigger".

At the first, the students had different answer. Some of them said that the area of the cloth-rag before and after cutting was not the same, some said the same. They looked confuse and not confident with their own answer. In order to check the students' answer, the teacher repeated the question for several time. But the class discussion still sounded off two answers, the same and not the same. Therefore, the teacher gave some clues, such as, *"Is there any part of the cloth-rag that being wasted? or is there any part that added in the cloth-rag?"*. The students answered, *"No"*. From this clue, the students could get some insight about the fact that the cutting of the cloth-rag will preserve the area as long there are not parts that wasted.

In the end of the inter-perception 1, students have been able to conclude that although two shapes have different form, but it is possible to have the same area. From this activity, we could see that the role of the teacher was very important to lead the students understand the purpose of the discussion. However, it was the last hour of the day, so the students were not fully concentrated. They looked agree with the result of the discussion but it can be seen from their face that some of the students were still confused. But we do not need to worry since the next activity will help them to understand the concept of reversibility.

# 3. Meeting 3 – Comparing Rice Field Activity

### a) Exercise 1

Exercise 1 was given in order to check and to support the students understanding of the making-table cloth activities. This exercise should be done in the Meeting 2, directly before the students experienced recomposing the cloth-rag to become table cloth. But due to the limited time, Exercise 1 did in the next meeting, Meeting 3.

There were two problems with five questions in Exercise 1 (see Appendix page 254-255). The first problem asked the students to make a table cloth from a cloth-rag which is three times longer and one-third wider than the table. In fact, most of the students using the strategy to cut the cloth rag into three equal parts to cover the table when solving the first problem, just one student who cut it vertically by measuring the cloth with the table in vertical. There was also student who cut the cloth-rag into two equal parts, then cut again the leftover of the parts to cover the rest of the table. The students could solve the problem easily because of their previous experience in Meeting 1. Therefore, at the first the students tried the same way, cutting the cloth-rag into two equal parts. But after realizing that it was not the answer, the students started to rethink the other way to cover the table with the cloth-rag they have had now.

As the conjecture, the students was followed their previous strategy to recompose the cloth-rag to fit the table in this exercise. Also, the way of cutting the cloth-rag into three equal parts was predicted for being rarely used since the students will be difficult to divide the cloth-rag into three equal parts. But the fact was many students did that. Some of the students directly thought to divide it into three equal parts based on their comparison when put the model of the cloth-rag in top of the table. Meanwhile, some of the students had to do trial and error to get the answer that the cloth-rag should be cut into three equal parts.

Then second problem in Exercise 1 asked the students to compared two cloth-rags (cloth-rag A and B) with two tables (table 1 and 2). This problem lead the students to use their transitive thinking as both of the cloth-rag A and B could fit the table 1. In the other word, the students could think that none of the cloth-rag A or B which will be bigger than each other or they have the same area.

The students was spent too much time in solving the first problem. So for the second problem, they just have just little time to spend. It makes the students could not finish to solve all the questions in the second problem. Since this exercise was done in the first session on Meeting 2, so the teacher was strict on the time. When the time was up, the teacher asked the students to submit their individual work in front of the class Meanwhile, there were just a few students who answered that the cloth rag 1 and the cloth rag 2 have the same area. For instance, Yami argued that the cloth-rag 1 and 2 have the same area because both of them could cover the same table, which is table A. Meanwhile, Riyo argued when the cloth-rag 1 and cloth-rag 2 were taken in top of each other they would have the same area. Unfortunately, we could not track his thinking since he did not write any clue in the exercise paper. From his answer, he might not use transitive thinking to get the answer, but he tent to use superposition, put a shape in top of the other to compare the shapes.

Therefore, from Exercise 1 we could see that it was not easy to make the students think transitively. Actually, this exercise had a chance to explore students' transitive thinking from the answer of Yami. The teacher could discuss the Yami's answer as a short class discussion so the other students could have an idea about what is transitive thinking is. But since this exercise was done in the first session and the limited time, the teacher did not have enough time to do discussion.

#### b) Mini lesson 1

Mini Lesson 1 was used to explore the students' perceptual thinking. The students will be asked to determine whether the table of Bu Ana has the same area with the table of Bu Tina and also the table of Bu Yuyun. When the students were shown the table of Bu Tina and Bu Ana, they were agreed that both tables have the same area by just seeing the picture. Meanwhile, when the table Bu Yuyun was shown, the students answered that the table of Bu Yuyun was bigger than Bu Tina and even Bu Ana. They knew by their naked eyes that the table of Bu Yuyun was extremely big since the table of Bu Yuyun indeed was bigger than the others.

From Mini lesson 1, we could conclude that the students have high sensitivity to the dominant dimension. If the students see that one of the dimension of a shape was dominant than the other shape, they will easily conclude that it was the bigger shape. Thus, comparing the cloth-rag which has dominant dimension on length with the table which is also dominant on width that actually has the same area could help the students to recognize that the dominant dimension could not use to measure the area of a shape.

#### c) Worksheet 3 – Comparing Rice Field

Worksheet 3 asked the students to compare the rice field. There were three problems in Worksheet 3. The first problem asked the students to prove that the rice field of Pak Darma has the same area with the rice field of Pak Badrun. The rice field of Pak Darma and Pak Badrun have different form, Pak Darma's rice field formed is rectangle while Pak Badrun's is L shape (see Appendix page 256-258)

The students were given the model of the two rice fields to be used as a tool to prove. Worksheet 3 was done in a group. All of the groups tried to recompose the model of the two rice fields to be fit one another. As well the focus group, at a glance we saw that the focus group also tried to recompose the model of the rice field. But we found an interesting discussion between the students in the focus group. There were five students in the focus group, Iman, Eky, Fadel, Arka, and Rayan. In the discussion, Eky and Iman dominated the conversation. Arka was following the discussion while giving his support to the argument which is analogue with his understanding. Meanwhile, Rayan was moving around their groups while sometimes helping their friend by lending his ruler or scissor. Fadel just kept silent during the discussion since he has just fight with Iman before. But he still followed the discussion.

Here is the conversation between Iman and Eky shortly after they put the rice field of Pak Badrun on top of Pak Darma's rice field (see Figure 5.17)



Figure 5.17 Cutting line of Pak Badrun;s rice field to fit Pak Darma's

Transcript 5.5 The transcript of Eky and Iman in deciding where to cut the rice field

: I will cut the shape (pointing the scissor on the Pak
Badrun' rice field in top of Pak Darma's)
: But how much of this shape will be cut?
: FitFit (try to convince his friends)
: Are you sure? Do you think where to cut? Here
(pointing the line 1 to be cut)
: Loh (expressing surprise and disagreement)
: Therefore, it should be cut here (pointing the line 2 to
be cut).
Eky: Ya...it should be cut there (showing agreement with<br/>Iman)Iman: Ya.. it should be cut here. I need a ruler.

After this conversation, Iman tried to measure the length of the part C to be compared with the length of the part D (see Figure 5.18). From his measurement, Iman found that the length of part C was 6,6 cm while the length of part D was 6,9 cm. Surprisingly, Iman has been able to read the decimal number in the ruler. As well as his friends who agreed with Iman' measurement. Iman, Eky and Arka also understood that 6.6 cm is smaller than 6.9 cm as showed in Transcript 5.6.

Transcript 5.6 The transcript of Iman's idea to recompose the rice field of Pak Badrun

Iman : Do not fit! Do not fit, because we need additional rice field to fit this part (*pointing the part D on Figure 5.18*).



Figure 5.18 Iman's idea to recompose the rice field

Figure 5.18 represented Iman's idea in the discussion. Iman's idea was to cut the part C then put it to cover the part D. He thought it was impossible. Since the length of part D was longer than part C, so he will need another part to cover the rest. His thinking was correct, but he was not aware with the width of the part C which is wider than part D.

Meanwhile, Eky has different idea with Iman. Eky considered about the width of the part C, not the length. The interesting thing was that Eky tried to make Iman understand with his thinking. Transcript 5.7 is the discussion between Iman and Eky about Eky's idea.

Transcript 5.7 The transcript of Eky's idea to recompose the rice field of Pak Badrun

*Eky was asking Iman to measure the length of 'a' and 'b' (see Figure 5.19).* 



Figure 5.19 Eky's idea to recompose the rice field

Eky : See! That is just ...... (*inaudible*). It (*refers to the length of b* which is smaller than a) could be fit.

By showing Iman that the length of 'b' is less than 'a', Eky tried to convinced Iman that the part C could be used to cover the part D.

- Iman : Loh (*cranky expression*) it is not like that. It is for this edge (*part D*).
- Eky : Lah ya (convinving expression) this (pointing the part E) just need small space to cover, then the rest could be used for the edge (part D).
- Iman : It could not be like that.

Arya was looking at Eky and Iman with an open mouthed. He looked confuse by what was going on in front of him.

- Eky : Loh (*disagreement expression*). The most important is the length, is not it?
- Iman : Okay, but the length of this side is (*pointing the 6.6 cm of lengths*) not enough for that side (*pointing the 6.9 cm of the length*).

The conversation shows that the goal of Iman and Eky was actually the same. Both of them wanted to cover the part D. They just have different way to cover it. Both of them agreed to cut the part C. But Iman wanted to use the part C to cover the part D first. He found that the length of the part C which is 6.6 cm was not enough to cover the part D which is 6.9 cm. In this case, Iman just consider the length of the parts C and D without considering the width of the part C which is bigger than part D (see Figure 5.18)

Meanwhile, Eky would use the part C to cover the part E first. He considered the width of part C which is wider than the part E, so the rest of the part C could be used to cover the part D (see Figure 5.19). Actually, these students, Iman and Eky have just had miscommunication. They tried to defend their own idea which in fact they can prove it by trying to cut and paste the model of the rice field.

Our conjecture was that the students will used the model of the rice field just as a tool to prove. We predicted that the students will do trial and error to recompose the model of the rice field. We also conjectured that the students will put the model of the rice field in top of each other to determine which part should be cut and where to place. But we did not predict that the students used the model of the rice field as a tool to think instead of as a tool to prove. The students in the focus group used the model as a tool to think the possibility to recompose the model of rice field just by using their logical thinking toward the measurement. At that moment, they have not tried to cut the shape and proved which idea, Eky's idea or Iman's idea, was correct. Hence, we concluded that the first problem in the comparing rice field activity could support the students based on their level of understanding. It was kind of a good problem since it could be solved by the average and low achiever easily, but also could give an accidentally challenge for the high achiever students. Actually, there were several possible ways to cut and to paste the rice field of Pak Badrun into Pak Darma, starting from the easiest and the most difficult.

The role of the teacher in helping the students to solve this problem needs to be appreciated. Since at the end, the teacher came to the focus group and asked them about their progress in solving the problems. Knowing that this group has difficulties to recompose the model of the rice fields, the teacher gave a little hint to help them as Transcript 5.8

# *Transcript 5.8 The transcript of the teacher assistance for the first presentation in the math congress*

The teacher came and asked whether they could solve the problem.Teacher: Can you solve it?Eggy: Still thinkingTeacher: You can rotated it if you wantEggy and Ikmal was rotating the model of Pak Badrun's rice field in 270°.Ikmal: Like this (see Figure 5.20)?Eggy: Nah (happy expression).Nah, we have just thought (he means by rotating the shape) we find the answer.

The transcript shows that the guide from the teacher to rotate the shapes could open the students thinking. The students was rotating the rice field of Pak Badrun into 270°. Then they found that it was easier to see that the rice field of Pak Badrun can cover the rice field of Pak Darma precisely (see Figure 5.20)



Figure 5.20 The rotation of Pak Badrun's rice field that made by the focus group to find the answer.

The second problem in the Worksheet 3 asked the students to determine the biggest rice field between the rice field of Pak Badrun, Pak Darma and Pak Salman. Since the rice field of Pak Badrun and Pak Darmas have been shown to have the same area, so the students just need to compare the rice field of Pak Salman with either the rice field of Pak Badrun or Pak Darma.

Meanwhile, there were no students who just did one comparison, Pak Salman's with Pak Badrun's rice field **or** Pak Salman's with Pak Darma's rice field. Most of the students did two comparisons, Pak Salman's with Pak Badrun's rice field **and** Pak Salman's with Pak Darma's rice field. But, when the students knew that the rice field of Pak Salman was bigger than Pak Badrun and Pak Darma, the students directly concluded that Pak Salman's rice field was the biggest one. The students have been known that Pak Badrun's rice field has the same area with Pak Darma's. Thus, the rice field of Pak Salman must be the biggest. What students did in solving the second problem showed that the students' transitive thinking has not fully developed. They could not use the idea that they need to compare either the rice field of Pak Badrun or Pak Darma with the rice field of Pak Salman. Hence, the students still need more time and more experience to develop their transitive thinking.

Furthermore, before the students solved the third problem in Worksheet 3, the students were given an **inter-perception 2** about the similar and difference form of a shape. The students were shown in the LCD four pairs (A-B, C-D, E-F, and G-H – see Appendix page 211-212) of shapes which are similar and not similar. The shape B was made by rotating the shape A. The shape D was made by flipping the shape C. The shape F was made by cutting the corner of the shape E. Meanwhile the shape H was really different with the shape G.

The students were asked to determine whether the shape of each pairs was the same or not. The role of the teacher was very important in this class discussion. The teacher was not only leading the discussion, but she also has to give the insight of the students that this activity relates to solve the next problem.

In the discussion, some students have been able to answer that the shape A was the same with the shape B since you can get the shape B by just rotating the shape A. Also, for the shape C and D, the students have understood that they were the same. One student argued that the shapes were the same, but he did not know the term used to represent the way of shapes D made by the shape C. He modeled his thinking by turning over his hand (showing that the shape D was made by flipping the shape C).Moreover, for

the shape E and F, the students easily found that they were different. The cut in the corner showed the students markedly. Also, the shape G and H showed clearly that they were not same in form.

After the students answered the entire question, the teacher started to ask the students to read the third problem in Worksheet 3. The teacher asked what students understand about the problem. The third problem asked the students to reform the rice field of Pak Salman into another different form of rice field and to determine whether the area of the original and the new rice field was the same or not. The teacher emphasized the word '*reform*' and '*another different form*' on the problem by relating to the inter-perception (A-B, C-D, E-F and G-H).

Actually, the **inter-perception 2** was made as the revision of the HLT in Cycle 1. Due to the help of inter-perception 2, the students did not have misunderstanding in the third problem like in Cycle 1. The entire groups reformed the rice field of Pak Salman into different form. But, each of the group has different way to reform the rice field. There was a group who divided the rice field of Pak Salman into two big parts, but also there was a group who divided the rice field of Pak Salman into several small pieces. Then the pieces of the Pak Salman's rice field arranged to form another shape.

The number of pieces that the students cut from the rice field was not become a problem as long as the students used all of the pieces to form the new shape of the Pak Salman's rice field. But this issue could be used to strengthen the students understanding about the recomposing concept. Since, after the students reformed the rice field of Pak Salman into another different, they should determine whether the area of the new rice field was the same with the original rice field of Pak Salman. However, the students seemed to be understood that recomposing a shape will preserve its area as all the students' group answered that the new rice field of Pak Salman has the same area with the original one.

So far, the students' response in Worksheet 3 supported the conjecture in the HLT. The students got the argument that reforming the Pak Salman rice field will not change the area of the rice field from the discussion in the interperception 1. The way of recomposing a shape was being developed by the students in Worksheet 3. Before, in Worksheet 1 and Worksheet 2, the students were given a closed question. They had to recompose a shape to the given shape such as a cloth-rag to fit the given table. Meanwhile, in Worksheet 3, the students were asked to recompose a shape into another different from, free shape. So, in this worksheet the students could get the notion of recomposing concept. This was supported by the students' answer that the original and the new rice field of Pak Salman have the same area since no parts was added or wasted.

Furthermore, the difficulties which predicted to be occurred in Worksheet 3 have been handled by giving the inter-perception 2. Thus, the implementation of Worksheet 3 succeeded to help the students to get better understanding about the concept of conservation of area.

## 4. Meeting 4

# a) Math congress

In Meeting 4, we held Math Congress, such a class discussion to discuss the students work in the previous meeting, Meeting 3. The focus of Math Congress was not the final answer, but how the students get their answer and how the students communicate their thinking into their friends. Math Congress also used to facilitate the development of students' argument toward the concept of conservation of area. Math Congress was conducted by the teacher. All of the students sat on the floor and formed a small circle. This was done to make the students being more focus into their friends who presented their group work. As based on the experience of the teacher, when the students were sitting in the own chair, they tend to be disturbed with their stuff. With the distance between the chair and the board were too far, the students tend to not pay attention to the presenter. Therefore, we made the students to come closer to the presenter and left behind all their stuff.

For the presentation, the researcher and the teacher have discussed to choose three groups include the focus group. The teacher chose three groups based on their answer in each problem. The three groups were Riyo's group, Eky's group (focus group) and Caby's group. Riyo's group was chosen since they were the only group who used different way of cutting the shape in the first problem. This group could answer the third problem but they have not finished answering the second problem. Meanwhile, Eky's group was chosen because of their unique answer in the second problem. On the third problem, Eky's group divided the rice field of Pak Salman into two big parts. Furthermore, Caby's group cut the rice field of Pak Salman into small pieces.

The different way to solve the problem was interesting to be clarified to the students in Math Congress. Unfortunately, most of the students were shame to give their opinion although the teacher has attracted the students with the score but just a few students who being active. However, the students question in Math Congress was monotone. The students just asked "*Why it can be like that? Prove it!*". There were no other questions other the questions above.

The first presentation came from Riyo's group since this group has not finished answering all of the problems. Because it was the first presentation, this group was still same to share what they have done to solve the problem. For these students, it was difficult to communicate their thinking. They did not know what to tell. The teacher really helped this group to communicate their thinking by asking many questions like Transcript 5.9. Iksan, and Raqy are the member of Riyo's group.

Transcript 5.9	The	transcript	of the	teacher	assistanc	e for	the	first
	pre	esentation	in the	math co	ngress			

Teacher	: What is your answer for problem number 1?
Iksan	: Ya the same big. The rice fields of Pak Badrun and Pak
	Darma have the same area.
Teacher	: Just it How come their rice fields have the same area?
	Come on the other could ask to this group! Raise your hand
	of you want to ask.
The class w	pas silent
Teacher	: What is the answer, Iksan? Repeat once again
Iksan repea	ited the answer

Teacher	: Is there any question? ( <i>Silent</i> )
	Okay no question. Do you satisfied with their answer? By
	the answer, "Yes, the same big", you satisfied and be
	silent (with a tone of disappointment).
	Ya Eky,
Eky	: Explain!
Teacher	: Nah (emphasize Eky's question) explain.
The students	s were laughing.
Teacher	: It is true, is not it? The problem asked you to help Pak
	Darma to convince Pak Badrun that their rice fields have
	the same area. So you have to show your way to get the
	answer. Come on!
The group p	resenter was debating about who will answer the question.
Teacher	: It is up to whoever wants to talk. Okay Raqy it is your
	turn. (Raqy was reluctant). Now Ustadzah (an islamic
	nickname for a woman teacher in this school) can assess
	who is the active students, who are the students that in the
	group work just playing not solving the problem. Come on.
	Try to explain, everyone in the group. If just Iksan who
	explained, then only Iksan who will have a good score.

Finally Ragy answered the question.

The conversation shows how the teacher struggled to make the students talk and communicate each other about their answer in the previous meeting, Meeting 3. The help from the teacher happened during the first group presentation. The first group presentation was not enough to show their understanding about the concept of conservation of area although they did the correct answer in the poster but we could not track their thinking due to the communication problem.

However, the presentation of the second group was better than the first. The second group was the focus group. They have finished solving the entire problem. We believed that this group also learned from the previous presentation, so at that time they were better. The focus group started their presentation by explaining the way they compared the rice field of Pak Badrun and Pak Darma. They showed step by step of their way from which to cut and where to place that parts until the rice field of Pak Badrun fit with Pak Darma.

Since the focus group presented enough information, the questions from the other students were more various. For instance, there was a student asked, Irma "Whose rice field that you cut and paste (in the problem number 1)?" or Raqy who asked, "Why you do not cut the salient part of Pak Darma and try to cover Pak Salman's rice field (in the problem number 2)". Transcript 5.10 is showed the discussion of the focus group to answer the above questions.

Transcript 5.10 The transcript to answer Irma's question

Irma	: Whose rice field that you cut and paste?	
Eky	: The rice field of Pak Badrun. We cut the rice field of Pak	
	Badrun to fit the rice field of Pak Darma.	
The students were silent.		
Teacher	: Do you understand Irma?	
Irma	: Yes	
Teacher	: The other students, do you understand Ekys' answer?	
Students	: Yes	

From the transcript we hypothesized that the focus' group has explained well. Irma probably did not hear which rice field that the focus group cut to get the answer for Problem 1. Therefore, after Eky told her that it was the Pak Badrun rice field which has been cut, she easily understood and satisfied. Furthermore, Raqy also asked Eky to clarify the reason he cut the shape on that part as the Transcript 5.11.

### Transcript 5.11 The transcript of Eky to answer the Raqy's question

Raqy : Why do not you cut the salient part of Pak Darma rice field and put it in the Pak Salman rice field (see the figure 5.2.7)?

Eky : We have made it stuck. All the students included the focus group looked confused. The teacher tried to guide the students to get the notion of Ragy's question.

Teacher	: What Raqy means is look the salient part ( <i>pointing the part F, see Figure 5.21</i> ). Why that part (part F) did not use
	to cover the rest of Pak Salman rice field? And how could
	you be sure that the rice field of Pak Salman is bigger than
	Pak Darma?
Eky	: Since if you cut the salient rice field of Pak Darma and put
	into the uncover part of Pak Salman rice field, still there
	will be uncovered area in the Pak Salman rice field.
Teacher	: Is it clear?
Students	: Yes



Figure 5.21 The answer of focus group in math congress

From the conversation above, we could see that this group has able to defend their answer. They looked confident to answer their friend's question since they have mastered the problem although we knew that the focus group really struggled to answer the first problem (see Transcript 5.10 and Transcript 5.11). Thus, from that experience the students in the focus group have mastered the concept better than the other, perhaps. Moreover, the last group in Math Congressdid not bring any new information to the discussion. They just repeated while emphasized what the previous group discussion.

From Math Congress, we could conclude that the discussion still relying on the teacher. The students did not have any initiative to ask, to criticize or even to share their opinion. They just did like what the teacher asked them to do. It might be caused by the socio norm of the students. Previously, the students were rare to have a class discussion like Math Congress. They just used to write their work in front of the class without explaining how they got that answer. Even though sometimes, the students of class 3A must explain their work, but it did not work effectively.

Thus, Math Congress was very helpful to develop students' mathematical thinking. Math Congress in this study not only asked them to share their work, but also to communicate their work. As well, the students learned how to response to their friend work in Math Congress. They learn how to ask, how to criticize and how to give their opinion toward the other's work. Even though, the role of the teacher was still much needed to encourage the students to give their response to their friends' work by giving an extra score for the questioner.

## 5. Meeting 5 – Trade up rice field

## a) Mini Lesson 4

Mini lesson 4 aimed to check the students understanding that two shapes with different size could have the same area and also to develop the notion of area. There were four problems in Mini lesson 4. First, the students were showed in the LCD two identical rice fields, A and B. Then in the further slide showed that the farmer was built a house in one of the rice field, rice field A. The students asked to determine whether the rice production of both rice field still the same or not. Next slide, the farmers again built houses in both of the rice field. Now each of the rice field has three houses with the same size and but different position. In the rice field A the position of the houses was horizontally while in the rice field B was vertically. Moreover, in the next last slide, the location of the three houses in the rice field was different. The house in the rice field A was still located horizontally, but the house in the rice field B was separated.

Mini lesson 4 was done as the class discussion, so the teacher has the role to lead the discussion. When the students showed the slide, all the students agreed that both of the rice field have the same area. Then for the next slide, a house was built in the rice field A, the students answered with one voice that the rice field has different area. One of the student in the focus group, Eky, argued that the rice fields will not have the same area again since the land of one of the rice field has used to build a house. This reason agreed by the other students. From Eky's answer we could say that he has the notion of area. He used the reduction of land as his understanding about the concept of area.

Moreover, when the students were given the figure of the two rice fields with three houses on each field, they directly answered that both of the rice fields will produce the same amount of rice. Some students raised their hands wanted to give their reason. Umar one of the students who was pointed by the teacher argued that both of rice field produces the same amount of rice since the number of houses in each rice field was the same.

Actually, this reason was correct but for checking the students' understanding, the teacher gave follow up questions. The teacher questions guided Umar to also consider the difference of the house position and the size of the house. The teacher wanted to emphasize that although the position of the house are different as long as the size is the same, so the area of the rice field used to built the houses are also the same. But this intention could not catch by the students, especially Umar. Umar still kept the same number of houses as his mainly reason to conclude that both of rice field with three house on each will produce the same amount of rice.

In the last slide of Mini lesson 4, the discussion was heats up. When the teacher showed two rice fields with three houses on each of them but the location of the houses is different, one is union the others is separated.

At the first, there was a student who answered that the number of rice production from the two rice field will be different since the location of the house is different. While the other students looked disagree with her answer. The teacher took this conflict to dig the students' understanding in the discussion. Many students raised their hand to share their opinion. After several students argued that the location of the houses did not influence the rice production, the teacher asked, *"So what influenced the number of rice production in this (pointing the model of the rice field) rice field?"*. Two students answered that the number of rice production will be influenced by the number of houses and the size of the house.

Hence, from Mini lesson 4 we could know what the students thinking about the meaning of area. They used their prior knowledge about the reduction of the land used for the house, the number of the house and the size of the house as the substitute of the word 'area'. This showed that the students still could learn the concept conservation of area without being told what area is.

# b) Worksheet 4

In Worksheet 4, the students were asked to determine whether the rice field of Pak Sudirman will produce the same amount of rice field after the trade-up process. The story of the problem told the students that Pak Sudirman like to trade up his rice field. He has a unique habit. Every time he sold his rice field with particular size, he will buy another rice field with the same size. But at the end the new rice field of Pak Sudirman after trading up was separated in the location (see the appendix page 222)

Our conjecture was that the students will think of the concept of conservation based on the story given. We also hoped that the students could used the fact that Pak Sudirman will buy rice field the same area with the rice field he has sold as the argumentation to answer the problem. But in the implementation, the students did not pay attention to the story. When they looked at the rice fields and read the question, they started to compare the rice field before and after the trade up. Some groups have an idea to cut the rice field of Pak Sudirman after trading up into several pieces to make them easier in arranging the shape into fit the rice field before trading up.

As well, the focus group began their work by arranging the model of the rice field of Pak Sudirman after trading up into the figure of the rice field before trading up in the worksheet. Actually, the way to fit the rice field after

trading up into the rice field before trading up was not so easy. The students would not be able to see the solution at a glance. They have to cut the parts of the rice field after trading up into several small (unit) pieces to ease their way arranging the rice field like the puzzle like the prediction in the HLT (see the appendix page 222)

From the students' answer, there was no one that comparing the rice field as the conjecture. But six out of seven groups have succeeded to arrange the rice field after trading up to fit the rice field before trading up. Hence, from their success the students concluded that both of the rice field still produced the same amount of rice. Meanwhile, the one group which is the focus group could not arrange the rice field after the trading up to fit the rice field before the trading up because of the focus group has lost a piece of the model of the rice field. But they did not realize it until the end of the workgroup. Therefore, the focus group answered that both of the rice field will produce different amount of rice since they found that there is still parts in the rice field before the trading up that empty or has not covered by the rice field after trading up.

Based on the students' answer, we could conclude that the students were still dependent with the experiment to get the answer. When the focus group could not fit the rice field into the other, they will automatically conclude that both of the rice field has different area. In fact, the focus group loosed one piece of the model of rice field after trading up. Thus they could not complete to cover the rice field before trading up with the rice field after trading up. It was indicated that the students still could not move from the model of (experiment using the model of the shapes) to the model for (using the story to answer the problem in Worksheet 4). Meanwhile, in Worksheet 3, the focus groups has been able to show that they could used the cut and paste strategy as the model for thinking. But in Worksheet 4, they reversed to used the cut and paste strategy as the model of to find the answer.

Our expectation, that Worksheet 4 could facilitate the students to move from the model of to model for did not work. The students still used their experiment to determine their answer (model of) rather than used the story of the problem (model for). This caused by the figure of the shapes which took up most the attention from the students. Perhaps, if we did not show the figure at the worksheet the students will consider the story as the help for answer the problem. After the students have an answer just build upon the story, we could show the figure of the rice field. So Worksheet 4 that we made will force the students indirectly to move from the model of to model for.

#### c) Exercise 2

Exercise 2 asked the students to determine whether the rice field A and the rice field B will produce the same amount of rice while both of the rice field was built houses. There was a big house in the rice field A and four small houses in the rice field B. Actually the area that used for building the house in both of the rice field was the same, just different in size.

The aimed of Exercise 2 was to emphasize the idea that two shapes which have different size could have the same area by cutting the one shape into pieces and fitting the pieces into the other shape. The exercise was done individually. Each of the students will be given the model of the rice field and the houses as shown in the worksheet.

The students' skill to arrange the shape to fit with the other shape was developed in this exercise. The students used various ways to compare the area of the house in the rice field A and the house in the rice field B. There were students who cut the four small houses into two of two small houses and put it beside each other so the size was similar with the one big house.

But there were also two students who connected the size of the houses with the used of the land of the rice field. From the original rice field (see the figure 5.2.8), these students cut the land as big as the area of two small houses (part K). Then they put the part K to cover the part L. Finally, these students cover the empty land in the rice field B by the four small houses (placing two parts of each two houses side by side as Figure 5.22).

These students did not only show that the size of the big house and the four small houses was the same but also the area of the land that used for building the houses in both of rice fields was the same.



Figure 5.22 The way of two students to show that the size and the land used of the house in both rice fields was the same.

Hence, we could see that some of the students have developed their notion about area by the context of land used for building the house. The students have known when you build a house in the rice field, so the land of the rice field was reduced. They also knew that the rice production of the rice field also depend on how big the land of the rice field used for the houses. In the other word, the students understood that the area of the land determined the number of rice produces in the rice field.

## 6. Meeting 6 – Rice Production Activity

#### a) Worksheet 5

Worksheet 5 was aimed to teach the students the part-whole relation. There were two problems in Worksheet 5. The first problem asked the students to find how many ton of rice produced by three different shapes of rice field (parallelogram, small rectangular and big rectangular) if the students were just known that the rice field which shape is square with particular size will produce 1 ton of rice. Then in the second problem, the students should find the number of rice produced by two big shapes (rectangle and parallelogram) by using the information in the first problem (for detail see Appendix page 263-265).

We conjectured that the students will adopt the concept of recomposing shape to solve this problem. The different was the students will not recompose the shape, but used the shape to be arranged to cover the big shape precisely. Then the students will count how many shape used to cover the big shape and add each of the number of rice field produced in the each shape (the partwhole relation) to find the rice production of the big shape.

For finding the rice produced of the rice field which shape was parallelogram, the students recomposed the square rice field to fit the parallelogram rice field and found that they have the same area. So the students concluded that the parallelogram will also produce 1 ton of rice. Moreover, for getting the number of rice produced by the small rectangular rice field, the students cut the square rice field into two equal parts and fit it with the small rectangular rice field. They was fit each other. Thus, the students could know that the small rectangular rice field will produce the same amount as the square rice field since they have the same area. Moreover, the students knew if the big rectangular rice field was twice the square rice field, so the rice production of the big rectangular rice field was two tons.

All of the groups were answered as above, also the focus group. However, each group has different way to give their opinion about how they get the answer. For example, the focus group's answer explained they way to get the answer very well. They argued that if they cut the big rectangular in a half, one of the half is same with the square which will produce 1 ton of rice. If the other half connected with this half, then both of them will produce 1 ton of rice which is same with the big rectangular rice field. The focus group showed how they cut the shape and also explained the procedure of their thinking. From the reason of the focus group in this problem, we could track how the students solved this problem. For the first problem in Worksheet 5, the students did not face any problems, also the focus group. They implemented their understanding about recomposing shape will preserve its area when they have to find the number of rice field produce by the parallelogram and small rectangle rice field. Since these rice field have the same area with the square rice field which known to produce 1 ton of rice. All the students tried to recompose the square to fit with the parallelogram or the small rectangle. Then when the parallelogram and small rectangle rice field fit with the square, the students understood that the parallelogram and small rectangle will produce the same amount of rice as the square rice field.

Similarly, for the second problem the students directly understood that they could use the model of rice field in the first problem to be arranged into the two big rice fields in the second problem. Again, they used different way to solve this problem but with the same idea, counting how many model of rice field in the first problem to fit the big rice field in the second problem. The focus group also has an interesting way to find the number of the model of rice field in the first problem to fit with the rice field in the second problem.

Iman and Arka was trying to arrange some square and small rectangular to fit into the rectangle rice field in the second problem. They arranged four square rice fields into the rectangle rice field in the second problem. They gave a strip as a sign for the end of the first row. Also, they did the same thing for the second row, giving a strip sign. By giving the strip sign as the end of the row, these students do not need to use all the model of square rice field to find how many square rice fields could cover this rice field (see Figure 5.23).



Figure 5.23 The strip sign made by the focus group to solve the worksheet 5Based on their work above, the focus group showed their higher level of understanding the concept of conservation of area. They used the model of rice field (the square) as tool for thinking, not as a tool for proofing. Actually, the focus group has shown this signal since in the worksheet 3 when they debated each other about how to cover the rice field of Pak Darma with the rice field of Pak Badrun. For the other group, they might be also used this strategy at the first, trying to predict how many number of the square needed to cover the big rice field. But since at the end, they have covered the big rice field with several number of square rice field so we could not track it.

In the end, we believed that most of the students have got the notion of the part-whole relation. They just get confusion when the arrangement of the rice field in the first problem could not fit the big rice field in their first trial. However, they could handle it after doing several trials and find the way to make the correct arrangement. Sometimes, the imprecise size of the model could lead the students to the confusion. They took it as a big problem when the arrangement could not precisely cover the big rice field. Even though, it used to be the fault of the cutting process.

#### b) Mini Lesson 3

The aimed of Mini lesson 3 was to check the students' skill to see some shape that could be changed into a rectangle without holding the shape. There were four shapes will be shown to the students in the LCD. They are parallelogram; rhombus, trapezium and right-triangle (see the appendix page 227-228). Without any model of the shape were given to the students, just by seeing the figure , the students were asked to change that shapes into a rectangle.

Mini lesson 3 was conducted by the teacher as a class dicussion. We can see that many students have an idea to change the four shapes into a rectangle from how many students raised their hand to answer this problem. As well, the students have been able to communicate their way to recompose the shape into a rectangle. For instance, Rayan, a student in the focus group, answered that the part M should be cut and place into the part N (see Figure 5.24)



Figure 5.24 Rayan's way to recompose a parallelogram to be a rectangle The teacher repeated Rayan's answer several times to make the other students in the class understand what Rayan did. However, there was a

student, Eky (a student of focus group) who seemed to understand this mini lesson at the first. But after he answered how to recompose the trapezium into a rectangle, he showed his misunderstanding on this problem. He said to cut the part O and part P of the shape (see Figure 5.25).



Figure 5.25 The misunderstanding of Eky to recompose trapezium to be a rectangle

Knowing that the student got misunderstanding with the problem, the teacher clarified to Eky whether his way will omit the part O and part P from the trapezium to make it into a rectangle. Eky agreed with the clarification and looked doubt about it. The teacher reminded the students that recomposing a shape must not omit or add a part of the shape.

Then Umar raised his hand and answered that the part O could be moved into the part which has the dashed blue line (see Figure 5.25). Then Eky looked understood about his mistake. Also, he learned from his mistake since in the last shape, right-angle triangle, Eky was able to answer it correctly.

For the next shape, the students have never experienced to work with the right-angle triangle before. But in this case, when many students answered that the right-angle triangle could not be changed into a rectangle, Eky said that it could. By using his fingers, he tried to train the teacher to do like what he

thought. He asked the teacher to cut a tip of the triangle and to place it into the right-below parts of the triangle. But he was confused whether the tip could make the triangle become a rectangle. He was still thinking at that time, but since the time was up, he stopped thinking. Then the teacher gave a clue to cut a half of the triangle and did like what Eky's idea.

All students in the class seemed to understand with the way to recompose the right-angle triangle to be a rectangle. Somehow, it would be better to let Eky thinking and finding his way to get the correct answer. Maybe, the teacher should wait and postpone answering the last problem in this mini lesson tomorrow. But it was the decision of the teacher to give the guidance to the students when they need it. It was not too bad, it is still good.

# 7. Meeting 7 – Recompose Irregular Polygon

#### c) Exercise 3

Exercise 3 was aimed to explore and to develop the students' skill in recomposing s shape. The students were asked to recompose some irregular polygon into a rectangle or a square. They were given the model of the irregular polygon to be manipulated. There were five shapes to be recomposed to a rectangle or a square (for detail see the appendix page 233-235.). The first shape was done together in front of the class led by the teacher as the example to introduce the problem. The students recomposed the first shape into a rectangle by cutting the salient part and putting into the less section.

As a suggestion from the teacher to reduce the boredom of the students, we make a new rule in doing Exercise 3. Previously, the students will be given the model of the shapes together with the worksheet, but at that moment the students should pick by themselves the shape they want to work in front of the class. There was a table in front of the class that occupied by four different shape of irregular polygon. The students were free to choose which shape they will solve first.

This new rule was work. Although the class looked messy since there were students who run or walk to front and go back into their chair, but they looked enjoy. In the journey from their chair into the front of the class, sometime the students chatted with their friends, just for saying hello, asking their progress, *"How many number have you done?"*, or helping their friend to get an idea to recompose the shape that he or she has finished. There were many things happened at the same time in this activity. How wonderful activity it was.

Furthermore, the atmosphere of the students when doing the exercise was promising to give the good result. But it was not occurred in the end when we analyzed the students' answer. There were still several students who could not understand the problem. They recomposed the shape into a rectangle or a square but they piled up the pieces of the shape into smaller square.

Also, there were still a few students who wasted the part of the irregular polygon to make it as a square or a rectangle. When we investigated these students by asking to the teacher, the teacher said that some of them were the low achiever students, but the other was absent in several meeting. Thus they still could not do Exercise 3 perfectly.

Similarly, not all of the students in the focus group were able to solve Exercise 3 perfectly. Rayan, who is the low achiever, could not solve all problems perfectly. He did all the problems, but no one was correct. He just could solve the first problem which has done together in the class as the example. For the second problem, he overlapped the pieces of the shape and forced it to make a rectangle. Meanwhile for the third shape, he cut the salient parts and wasted the parts so the shape becomes a rectangle.

Furthermore, Fadel also has showed his struggle in arranging the parts of the shape he has cut into a rectangle. But he had a good idea to solve the problem. We could track from his answer that at the first he stuck at the first trial (see Figure 5.26). Then he might be get an idea from his failed in the first trial, he cut the shape into several small rectangle following the curvature of the shape and then arranged the small rectangles into a bigger and longer rectangle (see Figure 5.26). It was amazing that Fadel could think this way, only Fadel who recomposed the shape by this way. But he did not do this way for all the problem in Exercise 3. He just did it when he found a rectangular polygon as the problem in the figure 5.2.12. The problem



Figure 5.26 The answer of Fadel using small rectangle to form a perfect rectangle

According to the analysis of the students' answer on Exercise 3, we could conclude that many students have been able to recompose the irregular polygon into a rectangle or a square. They knew that they were not allowed to waste or add any parts of the shape while they were arranging the parts of the shape into a rectangle. Somehow, some students still struggled to find the way to form a rectangle of a square. It means that the students still need to develop the skills to recompose a shape. They should explore their imagination about the geometrical shape.

For instance, the students think how to recompose a triangle into a rectangle, which part should be cut and where to put the parts. When this imagination has developed well as the students 'skill, the students could reduce their dependence into the model of the shape as a tool to prove. The goal of Exercise 3 was reached, this exercise could show the students level of geometrical skills and also supported the development of the geometrical skills for some students.

## 8. Meeting 8

## a) Posttest

Posttest was used to evaluate the students' development in understanding the concept of conservation of area through the learning sequence they have had. We would not examine their work, we would analyze their work to know how far their understanding on the lesson. There were five problems in the Posttest. The first and the second problem were the same problem with the Pretest. The third problem asked the students to recompose a simple hexagonal shape (in the left side) into a given rectangle (in the right side). The shape was made to be easily seen that it could fit the given rectangle. This problem aimed to know the way of students thinking to recompose the problems. Therefore, the first question in this problem asked the students to show their way to recompose the shape. Then for the second question, the students have to decide whether the hexagonal shape has the same area with the given rectangle.

The fourth problem asked the students to recompose an irregular polygon into a rectangle or a square. Then the fifth problem would check the students' understanding about the part-whole relation. The students were asked to determine the number of rice field produced by a big rice field when the students just know the number of rice field produced by particular shapes (for detail see the appendix page 250).

The increase of students who could do the correct answer for the problems which same with Pretest was significant. In the Pretest, less than a half of the students answered correctly in the first problem. But in the Posttest more than a third quarter of the students answered correctly in the first problem. For the second, the increasing was not significant, but the number of students who answered correctly was increased. Moreover, for the other problem in the Posttest, more than a half of students could answer the problem correctly.

Moreover, the focus group showed indicative development of their understanding about the concept of conservation of area. Although three out of five students in the focus group did not join the Pretest because they was absent at that time, but we could observe the development of their understanding through the meeting.

In solving the problem in the Posttest which was the same with the problem in the Pretest, Arka and Fadel, two students of focus group who joined the Pretest, did the improvement into the first problem. Previously, they just answered one out of three questions in the first problem correctly, but in the Posttest they answered two out of three questions perfectly. Then for the second problem, they were consistent with their previous answer which has been correct. However, for the rest of students who did not join the Pretest, Iman and Eky could answered all the questions of the first problem in the Posttest correctly while Rayan just did two out of three questions correctly. Furthermore, these students have answered the second problem perfectly although they have never known the problem before. All the students in the focus group have been aware with the number of terrace rice field in the both of rice field in Bojonegoro and Surabaya which was the same. They argued about the same number of the terrace rice field, so both of the rice field will produce the same amount of rice.

However, just two out of five students in the focus group who could answer the first question in the third problem correctly while for the second question all the students answered correctly. Arka could not solve the first question in the third problem satisfactorily since he wasted the parts that he cut to make a rectangle. As well, Eky and Rayan could not find a way to recompose the shape in this question. They also wasted the parts of the shape they have cut. The first shape in the third problem was difficult for them to be recomposed since there is no protrudes part on the shape. So some students still did not have any idea about where to place the part they have cut. But not for Iman and Fadel, they could think how to handle this shape. They thought to put the parts they have cut into another salient part then it will make a perfect rectangle (see Figure 5.27)



Figure 5.27 Iman and Fadel knew the way to put the parts that they have cut into another salient part to make a rectangle

From Figure 5.27, we could see that these two students have been aware that the parts of the shape which has cut will not always put into non-salient region on the shape. Sometimes, placing the cut parts into the other salient region will make the shape into a rectangle or a square like the first question in the third problem on Posttest. The capability of the students to recognize that they could also put the cut parts into the other salient region was not easily coming into the students' mind. It was proven by the number of students who could not being aware that they have to put the parts they have cut into the other salient region. There were many students who make a mistake in solving the question. Unlike the shape in the first question of the third problem which has both salient parts in its side of the shape, the second question of the problem just has one salient part in one side and protrudes part in the other side. Therefore, the students could easily find the way to recompose that shape into a rectangle. The students have recorded in the learning sequence that it used to cut the salient region of the shape then pasted it into the non-salient region. Almost all of the students could answer this problem correctly. As well, all the students in the focus group have been able to recompose the shape perfectly into a rectangle. Moreover, they used the same way to recompose the shape, cut the salient region and put it into the non-salient region.

The success of students to be able to recompose a shape into a rectangle still relied to the number of the salient and non-salient region in the shape. For instance, the students was able to recompose the shape which just has one salient and non-salient region while they were faced a shape which has more than one salient and non-salient region they was failed to recompose the shape, like what the students did in the fourth problem of Posttest.

The shapes that used as the problem for the fourth problem in Posttest were staircase shapes (for detail see the appendix page 249). Therefore, the number of the salient regions was more than one. Hence many students still was not able to recompose the shape into the rectangle. They finished solving the problem, but it was fault. They confused which was the border of the parts should be cut so it could fill in the non-salient region. Even tough, they thought to answer the problem correctly but it was not correct. Thus, there were eighteen of thirty students who answered correctly. The rest of the students were failed to determine the position (the line) for cutting the shape. For example, three out of five students in the focus group were failed to provide the correct answer. Rayan, one of these students showed that actually he was closer to the correct answer, but he did not considered the area of the non-salient region. He just cut the salient region without considering whether that region could fill the non-salient region precisely. As from his answer, the salient region that he cut was not able to fill the non-salient region completely (see Figure 5.28).



Figure 5.28 The answer of Rayan who could not able to imagine the possibility of the parts he has cut to fill the non-salient region.

Meanwhile, the students who were carefully consider the area of the nonsalient region to be completed will cut the salient region as Figure 5.29. Iman, a student of the focus group, did the picture. He was aware that to fill the nonsalient region completely he needs three layers of the staircase (see the red line in Figure 5.29) since the non-salient region consisted of three layers also (see the blue line in Figure 5.29).



Figure 5.29 The answer of Iman who considered the area of the non-salient part in cutting the salient region of a shape.

From Iman's thinking toward this problem, we could conclude that he has the notion of area and he could use this notion combined with the concept of conservation of area. He used his idea about the area needed to fill the gap of the shape. So he could think in his mind about the area indirectly. Perhaps, the other students also had and reached the same level of understanding as Iman since there were many students who also answered the same way like Iman.

Therefore, it means that the notion of area has been developed in the students' mind now. Indeed, the students have been mastered the concept of conservation area through recomposing shapes. We could see from Posttest answer that when you could cover the new shape completely and you did not waste or add any parts of the shape so whatever the form of the shape will be, the area remains the same.

Furthermore, for the part-whole relation in which represented in the last problem in Posttest also showed an encouraging result. Many students have been able to see the relation of the given shape which has known the number of rice produced with the big shape which is asked. Many students directly
measure how many small rice fields (the rectangle with 3 ton of rice and the triangle with 1 ton of rice) which could complete the big shape. The students tried to combine these given shapes. Many students were able to know that the big shape will need two rectangles and one triangle from their logical thinking since they were not given the model of the shape like in Meeting 6. For instance Eky knew that the big shape could be filled by two rectangles and one triangle. He might use his previous experience in Meeting 6 (rice production activity) to get the idea how to solve the problem. But he also would use his skills to predict, to compare and to measure some 2-dimensional shapes which he has developed since the making table-cloth activity until recomposing irregular polygon.

Unfortunately, when the students have not been able to use their experience and their skills to predict, to measure and to compare the 2-dimensional, their answer in this problem will not far with the correct answer. For example, there was a student answered that the big shape will produce 4 ton of rice since he just added the number of rice production (3 ton + 1 ton = 4 ton) they were given in the problem. Also, there was an answer of Kilman that wrong due to imperfect prediction in measuring the big shape. He said that the big shape will produce 10 ton of rice since the big shape could be covered by three rectangles and 1 triangle (see Figure 5.30). Somehow, it was really normal as the students were not given the model of the shape. So they just used their prediction to justify how many rectangles could fit in the big shape.



*Figure 5.30 The answer of Kilman by using the imperfect prediction to justify the number of rectangle and triangle could cover the big shape.* 

Figure 5.30 shows that the level ability of predicting, comparing and measuring geometrical shapes for each student were different. But, it could be improved by giving the students supporting stimulus as the learning sequence of this study. We believed that this student, Kilman, still could develop his geometrical skills in the future.

Therefore, from all the problems did by the students and the analysis of their answer in the Posttest, we could conclude that these students have developed their understanding about the concept of conservation of area.

#### E. Conclusion of the Implementation of HLT Revision in Cycle 2

Generally, the implementation of the learning sequence in Cycle 2 more or less supported the HLT revision based on Cycle 1. The revision made from the HLT in Cycle 1 was helped the students to understand the purpose of the activity. For instance, Mini lesson 1 helped the students to understand that recomposing s shape will preserve the area since no parts is added or wasted. When the students could understand this idea since the beginning, the students could follow the rest of the learning sequence easily. Since the sequence of the activity was ordered in which the next meeting will improve the students understanding of the previous meeting. Therefore, when the students started their understanding at the beginning of the meeting, so in the end the students will have a continuity understanding until the end of the learning sequence.

The difficulty of the students in determining which parts of the shape should cut and where to place still occurred in Cycle 2. Meanwhile, some of the students have been able to use the recomposing concept as a tool to think. They thought the way to recompose in their mind and used their argumentation to determine whether their way is correct or not. It means that these students have been able to move from the model of (using the recomposing shape as a tool to prove) into the model for (using the recomposing shape as a tool to think).

Furthermore, the flow of the learning sequence in Cycle 2 was better than in Cycle 1. Thus, the students could understand the concept of conservation more in Cycle 2 which conducted based on the revision HLT. Since in the end of Posttest, there was significance increase of the students who answer correctly for the problem which were same with Pretest. It is indicated that these students have changed their mind. Their change was based on the learning sequence that they have passed. Additionally, many students showed their understanding by answering the problems in the Posttest satisfactorily.

Not only looking for Posttest and Pretest, the observation during the learning process also showed the development of the students' understanding into the concept of conservation of area. We could see from the students' argumentation in solving the problem and also in the discussion. Therefore, we could conclude that this study was success to make the students understand the concept of conservation of area as a preparatory to learn the area measurement.

# F. Comparison between Conjectures of HLT with the Actual Learning in Two Cycles.

In order to determine how many cycles we need to achieve the learning goal of this study, comparison between conjectures in HLT with what the students' learned in reality is important. In this study, we used just two cycles since based on Table 5.2 we analyzed that the revising HLT from the result of Cycle 1 make the actual learning in Cycle 2 closer to the conjectures. Therefore, since the actual learning in Cycle 2 has in line with the conjectures of the HLT so the learning goals of the activities have been reached. Thus, we decided to stop in the Cycle 2.

Big idea	Act	Learning Goal	Conjectures	Actual Learning in Cycle 1	Actual Learning in Cycle 2
posing shape	g table- cloth	Use the idea of recompose the cloth-rag to fit the table	The context of making table-cloth could reveal the awareness of the students to recompose the cloth-rag to fit the table.	Students themselves who find the idea to cut the cloth-rag to be fit the table.	Students asked whether it is allowed to cut the cloth-rag and to arrange until it could cover the table.
Recomp	Making t		Do cut and paste strategy to recompose the cloth-rag	Students cut the cloth-rag into two parts and covered the table precisely	All the students cut the cloth-rag into two same parts and cover the table

Table 5.2 The comparison between conjecture in the HLT with the actual learningon Cycle 1 and Cycle 2

			1	1
	Understanding the effect of recomposing shape (after recomposing the cloth-rag could fit the table, so cloth-rag and table have the same area)	When the students see that the cloth-rag could fit the table, they conclude that both of the cloth-rag and the table are the same bigger which means have the same area.	Students still thought that the table is bigger than the cloth-rag as the cloth-rag has become smaller due to cutting process.	Students think that the cloth-rag has the same area as the table as it could fit the table.
	Understanding the idea of reversibility (the area of cloth- rag before and after cutting remains the same)	The students could understand that the area of the cloth-rag before and after cutting remains the same by proofing with demonstration.	Students still confused about reversibility. They thought that the cloth-rag after cutting is become smaller (the size), but they also aware that it come from the same thing before and after cutting.	Students could understand that the cloth-rag before and after cutting have the same area since no parts is added or wasted when cutting the cloth- rag.
	Identify two different shapes which have the same size by recomposing	Compare the shape by using superposition (placing a shape on top of each oher) then try to recompose the shape to prove that both shapes have the same area.	Many students still did the wrong way in using superposition to compare the two rice fields. So they could not get the conclusion that the two rice field have the same area	Students did the correct superposition then found which parts to be cut and where to place in order to prove that the two rice field have the same area
Comparing rice field	Investigate two different shapes which have different size by recomposing	Compare the shape by using superposition and use recomposing shape to prove that both shapes have different area.	Students used superposition to compare the shape but did not do recomposing.	Students used superposition to compare the shape and use recomposing idea as a tool to think if one shape could not fit the other means that the shapes have different area.
	Form some shapes which have the same size with the given shape.	Recompose an original shape into free different form of shape while understanding that the original and the new shape have the same area.	Students recomposed the rice field into the original form. By the help of teacher, students recomposed the rice field into another form and understand that it is preserve the area.	Students recomposed the rice field into different form and directly concluded that the original and the new form of the rice field have the same area.

Math-congress	Discuss the different way of recomposing the shape.	Students only used superposition to determine which one is bigger between two rice field Students directly used cut and paste to recompose the shape. Students used combination of superposition and then cut and paste to recompose the shape. Students communicate their reason why they should cut this part and paste it into the other part by using superposition or just trial and error	All students' group just used superposition to compare two rice field	All students' group used superposition then recomposed the area to compare the rice field. Therefore, the students could answer that they have to cut this part since when it is being superposition this part is excess from the below shape. So this part should be cut and place in the minus of the top shape.
	Concluding that recomposing will preserve the size of the new shape from the old one.	strategy. Students communicate that since no parts is added or wasted when recomposing a rice field to different form so the area of the original and the new rice field remains the same.	Students argued that due to no parts is added or wasted when modifying the shape so it preserves the area.	Students argued that due to no parts is added or wasted when modifying the shape so it preserves the area.
Trade-up rice field	Investigate the area of a shape when it is cut into pieces and separated in location, " Is it still same with the original shape?".	Students understand the story and conclude that the area of rice field will be the same when it is cut and separated into location as long no parts is added or wasted during the cutting and separating process.	Students did not pay attention into the story. They went directly to the shape and do recomposing to fit each other.	Although, the story has revised and shorter, the students still did not care about the story. No one of students used the idea of the story to reason the conservation of area.

		-			
		Understand the concept of identity in the conservation of area	Point out and use the idea of identity as one reason for preserving the area while the shape is recomposed.	Although the story was not success to lead to the identity, but the students used the idea that no parts is added and wasted caused that recomposing will preserve the area	Nevertheless, none of students used the story as the reason of identity, but the recomposing shape in this activity made students understand that identity caused preserving area.
Part-whole relation	Rice field production	Understand the part-whole relation in the concept of conservation of area.	Students get the notion of area indirectly from the context of rice field production. They treat the number of rice production (in ton) as the notion of area.	Students gave a reason that the rice field production will be decreased if there is a house is built on the rice field.	Students gave a reason that the rice field production will be decreased if there is a house is built on the rice field.
			Arranging several number of small rice field to cover the big rice field. Then adding the number of rice production form all the small rice field to find the number of rice production of the big rice field.	Students arranged several small rice field to cover the big rice field then added the number of rice produced by all small rice field to find big rice field	Students arranged several small rice field to cover the big rice field then added the number of rice produced by all small rice field to find big rice field
		Start to use the idea of unit to find the rice production of the big rice field.	Use the smallest rice field which is known its number of rice produces to be arranged in the big rice field. So, the students only count the number of rice field produces by its unit times the number of unit (the smallest rice field) needed to cover the big rice field.	Students used different size of small rice field to be arranged in the big rice field. They did not aware to use the smallest rice field.	Students have used the smallest rice field to be arranged in the big rice field. So, at the end they just need to count how many smallest rice field times the number of rice produced by the smallest rice field.

		r	1	r	1
		Develop students'	Determining which	Students still have	Almost all of the
		skill for	part to be cut and	difficulties in	students have
		recomposing	where to place by	understanding the	understood what
		irregular polygon	trial and error.	instruction.	the problem asked
		into a rectangle or			them to do. Only
		a square	Be able to recompose	Even, some of them	few students who
			the irregular polygon	cut the middle of the	were absent in
	uo		into a rectangle or a square without	irregular polygon.	some previously meeting could not
Π	lyg		adding or wasting	The other students	follow this
ski	od		any parts of the	cut the irregular	activity
,S	lar		shape.	polygon into small	satisfactorily.
lent	nga			square and arranged	
tud	irre			it into square but	Students used to
s d	gu			they wasted some	cut the
evelo	iposi			parts when cutting.	overlapping parts and tried to paste
Ď	noc			Just two out of six	it in the less part.
	Rec			students who	1
	[			success to	They kept try
				recompose irregular	above activity
				polygon into a	until the shape
				rectangle, but it took	they made
				a long time for them	become a
				to understand and to	rectangle or a
				think about the	square.
				problem.	

## G. Checking Validity and Reliability of Data Collection

Based on the formulation of how to check the validity and reliability in Chapter III, the way to check the validity and reliability is happened along the analysis in Chapter V. However, the implication of checking the validity and reliability in Chapter V could not be separated from the analysis. Therefore, to give a clear guidance how this study checked the validity and reliability in Retrospective Analisis, the detail information will be explained as the following table (see Table 5.3).

Pha Data to be Methods of data collection No collected se Student's 1 Written work: activity Student used grid paper to find the area of irregular polygon to find Video Observation : the area Student looked confidently in using grid paper. of irregular Pilot test polygon. 2 Student's **Interview with student:** thinking Since this student was being introduced the use of grid paper Preparation Phase process to find the area of irregular polygon in the school, so when he to find was asked to find the area of irregular polygon he tent to the area dependent on the grid paper. of irregular polygon. 3 Video observation: Students in the class 3A are active students. They like to Observation of answer the teacher's question. They also used to come up in students' front of the class to share and to check their answer. normal activity Field Note: in teaching and Although these students answered teacher's question and learning process shared their answer to others, the focus of the question is not on the students' thinking but more into the correctness of students' answer. 4 Cycles 5 **Pretest:** The test is conducted before the students learn the concept of conservation of area Written work: Students still could not find the correct answer just by Students' prior visualizing that two different form of shape could have the knowledge same area Video Observation: Students still use their perceptual justification, focusing only to the dominant dimension, to judge the size of a shape Interview: Students were not custom to solve such kind of problem 6 Since there are many data about students' activity which was collected, so in here we will just elaborate one example of it. 2. Comparing rice field Written work: The questions of the third problem, "Reform the rice field of Students' Pak Salman into another different from!" could not be easily activity understood by the students Video observation: The students tried to reform the rice field of Pak Salman to fit with the original one. The students copied the strategy they used before.

Table 5.3 Checking for validity and reliability

7	Students' thinking process	Since there are many data about students' thinking process which was collected, so in here we will just elaborate one example of it. <u>1. Making table-cloth Activity</u> <b>Written work:</b> The idea to recompose the shape by cut and paste the cloth-rag into table revealed from the students. <b>Video Observation:</b> By giving the model of the shape to be manipulated, will lead the students to think about cut and paste the shape. The idea of reversibility occurred when the students see the fact that the cloth-rag after and before cutting is come from the same thing in the class discussion. <b>Interview of students:</b> They could understand the idea of reversibility that the cloth- rag before and after cutting has the same area because the students aware that no parts are added or wasted in the cutting process of the cloth-rag. <b>Field Note:</b> The idea of reversibility on the cloth-rag would not appeared by the awareness of students. Teacher must point out this idea by bring it into the class discussion
8	The development of student's understanding	<ul> <li>Video observation:</li> <li>Students could understand the concept of conservation of area after understanding several supporting idea which appeared respectively. They are the effect of recomposing (fit with other means have the same area), why recompose preserve the area is answered by reversibility (before and after cutting the area of the shape remains the same) and identity (no parts is added or wasted).</li> <li>Posttest:: Posttest:: Posttest:: Posttest:: Posttest:: Change of students' answer in the problem which same in Posttest and Pretest. The ability to formulate reasons to solve the problem which related to the concept of conservation of area. Interview of students: The students could communicate their thinking about the way to recompose the shape correctly. They showed the development of their thinking by combining the reason of reversibility and identity.</li></ul>

	Clarity of the HLT and the teacher guide, the effectiveness of the HLT in the teaching experiment and suggestion for improving the HLT.	The teacher understood with what is written in the HLT and teacher guide. Sometime, the teacher just asked about the technical issue such as the reduction of the time. <b>Field Note:</b> Students did not show big difficulties to follow the learning instruction by using this HLT, although sometime they needed to be explained by the teacher what the problem actually asked them to do. So, we can see that the teacher was mastering the HLT and was helped by the teacher guide about how to solve some problem which might happen in the class. The teacher also gave several suggestions to improve the effectiveness of the HLT.
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From explanation in table above, the reader could see how this study checking the validity and reliability of the data collection.

Validity could be checked when the methods have been able to collect the correct data which want to be measured. While reliability is improved when the data which is collected is consistent for all methods used or at least could support each other by providing answer why it is like that. For instance see the table on no 6, students' activity. The finding of written work and video observation is support each other. The problem which occurred in the written work is answered with the observation of what the students did by video.

The complete and clear analysis presenting in this chapter also provide a support for replicating this study, which improve the reliability of the data analysis in this study.

#### **CHAPTER VI**

#### **CONCLUSION AND DISCUSSION**

This chapter contains three parts which are conclusion, discussion and recommendation. Firstly, the conclusion will answer the research question of this study and explain a local instruction theory on the learning the concept of conservation of area. Next, the important issued during the observation of this study will be discussed in the discussion session. The last part of this chapter will elaborate suggestion and recommendation for the future research about the concept of conservation of area to develop the concept of area measurement based on the experience of this study.

## A. Conclusion

This study only used two cycles in conducting the teaching experiment since based on the analysis in Chapter V, the learning goal to understand the concept of conservation of area has been reached. Also, the conjecture of HLT has been correspondence with students' actual learner in Cycle 2. Since in Cycle 2, the learning goal has achieved and the conjecture of HLT is appeared in students' actual learning process then we could stop the teaching experiment just by 2 cycles.

#### 1. Answer to the Research Question

In order to answer the general research question, the research sub-questions will be answered beforehand. Both research sub-questions will be answered by analyzing the process of students' understanding through all activities in this study which has been explained in the Retrospective Analysis in Chapter V.

#### a) The Answer of the First Research Sub-Question

We noted that the students could develop their understanding that recomposing shape will preserve the area of the shape from three ideas. First is the effect of recomposing shape which makes the shape A could fit the shape B. Second is the reversibility concept, the students realizes that a change may be canceled out by mentally reversing the steps and returning to the origin. Third is the identity, the students realizes that if no parts are added or wasted in manipulating the shape so the area remains the same.

#### • The effect of recomposing shape

Students develop their understanding that recomposing a shape will preserve the area of the shape started from the effect of doing recomposing which the cloth-rag could fit the table. By knowing the fact that the cloth-rag could cover table precisely, the students would think about the possibilities of both, cloth-rag and table, to have the same area. Meanwhile, at that time some students still focused with the fact that cutting the cloth-rag will make it smaller made the students could not get the idea that the no one of the table and the cloth-rag is bigger each other. But, these students changed their mind when doing reversibility.

#### • Reversibility

Reversibility means returning back into the original. The reversibility idea was emphasized by inter-perception 1 which asked the students whether the cloth-rag after and before recomposing have the same area. In doing reversibility, the students have aware that the cloth-rag that they cut and combine come from the same cloth-rag. Perhaps they have thought in their mind that it will be the same, the cloth-rag before and after the cutting but they did not know the reason. The idea of identity which could fulfill the students needs to find the reason of reversibility.

• Identity

When the students come into the phase, some students might have had the idea that recomposing shape preserved its area. But they just could not communicate it in the proper way with valid reason. The teacher guidance to remind the students about what happened with their process of arranging the shape, is there any part which added or wasted when the students recompose the shape, opened students mind they did preservation of the shape by not adding or leaving any parts. They used

The idea of the effect of recomposing shape, the concept of reversibility and identity in understanding that recomposing shape will preserve the area involved into the first three activities, making table cloth, comparing rice field and trade-up rice field respectively. Therefore, the learning sequence of the first three activities has been able to build the students understanding about recomposing shape will preserve its area. The order of the concept to be learned by the students also supported the students to understand the concept better.

In conducting the Cycle 1 and Cycle 2, there was some difference in the process of students understanding about recomposing shape preserve the area. Although both of them passed the same order, the effect of recomposing, the reversibility and identity, but the students were easier to understand that recomposing shape preserve its area in the Cycle 2.

In Cycle 1, the students struggled to comprehend that the cloth-rag before and after cutting has the same area (reversibility). They were confused how come the cloth-rag before and after cutting has the same area. The students thought that since the size was different due to the cutting so the area will also change. But after the students were showed the concept of reversibility on the two identical papers A4 which one of it has torn in four pieces, they could understand that the cloth-rag before and after recomposing will have the same area.

However, in Cycle 2 the students could comprehend this idea faster since the help of the teacher in the class discussion. The teacher stimulated the students to think of the parts of the cloth-rag they have cut when rearranged into the original cloth-rag. She asked whether the students used all of the parts and without added a new part to the cloth-rag. From this question, the student realized that the cloth-rag before and after cutting have the same area since no parts that wasted or added (identity). Then by following the activities and solving the problems the students could understand that recomposing a shape will preserve its area.

#### b) The Answer of The Second Research Sub-Question

We noted that the rice field production as a context could lead the students to learn the part-whole relation. Since, the students have not learned the concept of area in formal school, the number of rice production (ton) was used by them to substitute the area ( $cm^2$ ). In general, to learn that the sum of area of the parts will be same with the area of the whole, the students need an attribute to be manipulated, to be summed. Therefore, the rice production could provide a logical and natural attribute (ton) to be summed when the students learn part-whole relation.

In rice field production activity, the students were asked to predict how many ton of rice produced by a big rice field when they were given the information that several smaller shapes (rectangle, parallelogram and square) will produce a particular amount of rice. From the problem, the students are hoped to find how many rice production of the big shape by arranging the given shapes into complete the big rice field and then they sum up all the number of rice field produce by the parts which will be the same as the rice produce by the big rice field.

In both cycles which made in this study, Cycle 1 and also Cycle 2, the students were very easy to solve the problem in the rice production activity. We assumed that it was caused by the numeral attribute which was given into the context. As generally known that students at that age still regard the

number as the core of mathematics. So they were more familiar when solving a problem which is contained numeral information on it.

#### c) The Answer of the General Research Question

Two important ideas to learn the concept of conservation of area are recomposing shape and part-whole relation. Therefore, the students could learn the concept of conservation of area through understanding these two important idea, recomposing and part-whole relation.

How the students understand two important ideas in the concept of conservation of area is by learning supported instructional activity designed in this study. The recomposing idea could be learned by conducting three activities (making table-cloth, comparing rice field and trade-up rice field) which contains reversibility and identity concept. While, part-whole relation could be learned by conduction rice production activity which contain the possibilities to use numeral attribute to understand the notion of area. The rice production can bring to the fact that the number of rice production will decrease when something is built on the rice field.

Furthermore, by the fact that for recomposing shape, students needs the ability to determine which part should cut and where to place to fulfill the demand of the wannabe shapes. Since, on come cases the students have to recompose a shape into particular asked shape. For instance, to find the area of irregular polygon, it will be easier to change the irregular polygon into a square or rectangle. Then we only need to find the area of rectangle which is the easiest one. Therefore, to master the idea of recomposing shape, the students need to practice how to recompose various types of shapes.

Hence, from this analysis to know how the students learn the concept of conservation of area, the students should pass through the instructional activities which have been able to make the students understand the idea of recomposing and part-whole relation. There are making table-cloth, comparing rice field, trade-up activity, rice production and recomposing irregular polygon.

Moreover, the instructional learning activity which was designed in this study was appropriate to be taught into the students before they learn the concept of area measurement since there is no term about area used in the activity, the notion of area is hidden in the rice production context and the less used of number in learning the concept.

## 2. Local Instruction Theory on Learning The Concept of Conservation Of Area

Since the aim of this study is contributing to the development of a local instruction theory to understand the concept of conservation of area which included into the concept of area measurement, below we formulate a table to summary the tool and the sequence of the contextual activities.

Activity	Tool	Imagery	Practice	Concept
Making table-	Cloth-rag,	Signifies the need	Recomposing the	Recomposing
cloth	classroom	of recomposing a	cloth-rag to be fit	a shape will
	table, and the	shape and the	with the table in	make the area

Table 6.1 Local Instruction Theory in learning the conservation of area

	model of the cloth-rag and the table (printed picture of the cloth-rag and the table)	effect of recomposing which preserve the area of the shape	order to make the table-cloth	of the shape remains the same.
Comparing rice field	The model of the Pak Badrun, Pak Darma and Pak Salman's rice field (printed picture of the rice field).	Develop the geometrical skill of the students in order to recompose the shape of the rice field into a particular form and a free form	Comparing the rice field and recomposing a shape into a free form.	Recomposing a shape will preserve the area of the shape although the new form of the shape is far different with the original shape.
Trade-up rice field	The model of the rice field (printed picture of the rice field)	Signifies the reason why recompose a shape could preserve the area of the shape	Investigating the concept of identity by using the story of trade up rice field and the picture of the rice field.	Identity (none of the parts of a shape are wasted or added so the area of the shape remains the same).
Rice production	The model of the rice field (printed picture of the rice field)	Signifies the idea that the sum of area of the parts will be equal to the area of the whole.	Recomposing the shape and arranging the small size of the shape to fit with the big shape.	Part-whole relation (the sum of the area of all parts equal to the area of the whole).
Recomposing irregular polygon	The model of the irregular polygon (printed picture of the irregular polygon)	Sharpen the students' skill to know which parts to cut and where to place in recomposing a shape into a rectangle.	Recomposing the irregular polygon into a rectangle or a square.	Developing the geometrical skills of the students related to the recomposing shapes

#### **B.** Discussion

#### 1. The Weakness Point of The Study

In conducting this study, the researcher realized that there are some weaknesses. The weakness was not only come from the design itself, but also from the researcher. First, the time needed to teach the concept of conservation of area in this study was too long. There were too many activities that the students should pass. Meanwhile, the concept of conservation of area do not stated explicitly in the Indonesian curriculum. Hence, it was not easy to convince the teacher and the school to conduct this study. But by considering the important of the students to master the concept of conservation of area, the teacher and the school could understand the need to pass all of the learning sequence.

Second, the preparation of Cycle 2 was too close after Cycle 1. Thus make the researcher was too hurried in the preparation. For instance, in Pretest of Cycle 2, there were many students who were absent, include three out of five students in the focus group. The researcher did not realize this fact until the next several meetings. So, we could not give these students overtaking pre test to know their prior knowledge. Therefore, in the end we could not compare the students understanding before and after the learning process of these three students in the focus group. But we have done interview with these three students about their initial knowledge after they did Posttest. The three students were asked how they will solve the two problems in Pretest (the same problem of the Posttest, problem number 1 and problem number 2) when they have not learn the concept of conservation of area or about 2 week ago. Unfortunately, it was not work well since these students could not distinguish their knowledge before and after the learning process by themselves. It was the fault of the researcher for this careless. Somehow, we still could track their process of understanding through the learning activities. So we still could make a conclusion on these three students of the focus group who did not join Pretest.

Third, there was no intervention of the researcher in the learning process of Cycle 2. The intervention in here means that the researcher did not give the students some question when observing their learning process. Although the researcher found misunderstanding or contradictory argument in the students's discussion, the researcher only observing. The researcher tried to trace why the students get misunderstanding and how they cope with it. Could the students realize their misunderstanding? Could the students find the good solution for their misunderstanding?. This should be good to keep the originality of the students' answer. Since the researcher is also afraid that after being asked by the researcher, the students will change their mind. Like what happened when you are asked by someone. If after you give the first answer A then you asked by the same question, you intuition will tell you to change your answer become B. Because many people taught if you are asked twice it means your answer is still wrong. But in design research methods, the intervention of the researcher in order to clarify the students' thinking is allowed. As long the question did not directly guide the students into the correct answer.

#### 2. Reflection on The Important Issues

In the Retrospective Analysis, we mainly focused on the students' learning process to understand the concept of conservation of area. We also observed the use of the context in order to support the students understanding of the concept. Meanwhile, some important issues were also noted during the implementation of the design. The reflection of these points is described as follows,

#### a) The Role of Teacher and The Social Norms in The Class 3A

The teacher, Bu Lila, is also the classroom teacher of the class 3A. In the other word, Bu Lila is responsible into the class generally. She has to stay and to manage the class since the students arrive until they go home. Thus, the students are very obedient to Bu Lila since they will always deal with Bu Lila everyday when to take care of everything. The position of Bu Lila which also the classroom teacher gave benefit for the researcher since Bu Lila could know when the empty time for the students so the researcher could use it for this study.

Furthermore, Bu Lila has ever being the best teacher to implement PMRI (Pendidikan Mathematics Realistic Indonesia), the Indonesian version of RME, in her school. Therefore, she was very good and cooperative teacher in her acceptance to conduct this study. She often gave the researcher feedback or suggestion about the lesson plan or the problem based on her experience or based on the condition of the class.

Therefore, in the implementation of the instructional learning activity, Bu Lila was very helpful. Not only she could understand the teacher guide well, but also she could add some important guidance which was not included in the teacher guide. For instance, when the students confused whether the cloth-rag that they have cut will have the same area with the original one, she guided students by asking, "*Does you throw any cut pieces of the cloth-rag?*" and "*Does you add any cut pieces of the cloth-rag?*" From her questions, the students could think that they did not waste or add anything into the cloth-rag when he cut it. Hence, the student could conclude that the area between the cloth-rag before and after cutting was the same. And there were some guidance from the teacher which was not written in teacher guide but she made it by herself based on the situation and the need of the students. The guidance of the teacher was implied in the Retrospective Analysis.

Bu Lila has understood the purpose of this study and also she has learned the teacher guide and the HLT before teaching in the class very well. She also had some discussion with the researcher before conducting the lesson plan in the class. So she knew well what the researcher want from her, her students and her teaching learning process.

In the end, Bu Lila also helped the researcher to solve several problems such as the limitation of time and the effectiveness of the activities in an hour. Generally known, there is 70 minutes was used for mathematics but we just could spend at most 60 minutes. So the time problem makes the meeting of this study which was planned to have six meeting, become seven meeting in Cycle 2. She also suggested the researcher to conduct the recompose irregular polygon activity in the different way, not like usually. Usually, the students will be given the model of the shapes together with the worksheet, but at that moment Bu Lila had an idea to reduce the students' boredom by put the model of the shape in front of the class. Then the students must take the shapes by themselves. They were free to choose which shape they want to work first.

Overall, the teacher was very cooperative and helpful for this study, not only in giving guidance for the students to understand the concept of conservation of area but also in giving supportive suggestion for the effectiveness of this study.

#### b) The Social Norms in The Class 3A

The class 3A consists of 16 girls and 16 boys. The chair of the boys and girls was separated. The left side of the class was for the girls and the right side was for the boys. They were sitting individually, one student one chair and one table, but in each side there were four chairs in a row. The room of the class was small enough so there was no enough space when we want to change the arrangement of the chair in the class. Therefore, when the students worked in a group, the teacher just asked them to combine two tables as their place to work.

The teacher and the students had an ice breaking for refresh their mind when they get bored with the lesson or when they have mathematics lesson in the last session, up to 12 o'clock in the afternoon. There were several ice breakings taught by the teacher, but the researcher just knew several of it. For instance, clapping their hands based on the teacher instruction such as clap1, claps 2, claps 3 and etc. It was just the normal claps, but they have a special rhythm for the claps 5 and its multiples. Also, in the claps 20, the students used all of their body. They clapped five times by his hand, five times by hitting the table, five times hitting the floor and the last five by hitting the table and the floor together. The students also have such a slogan whenever they finished doing the worksheet or the workgroup they will shout "Allahu Akbar". They were such an excellent class with a supportive atmosphere for learning process.

However, the students seem lack of the communication skills. Actually, teacher has told the researcher that in the class 3A there are more students which used to communicate than the other class. In fact, they are just several students who active in the class discussion and most of them are boys.

The students in the class 3A were used to be active but not in communication of mathematics. We mean that, the students liked to go in front of the class and write their answer in the white board or they liked to answer the teacher question about the problem they have worked. Meanwhile, they were not custom in discussion the mathematics concept of the problem or sharing their opinion or their thinking about mathematical concepts. We could see that the socio-mathematical norms of the students still need to be developed. The teacher was struggled to make the students posed a question or just gave their opinion in Math Congress. In order to make the discussion work, the teacher encouraged the students by giving an extra score for the students who asking a question. She also gave an understanding of the students about being active in the discussion. Sometime, the teacher also gave some statements which could stimulate the students to have an idea to ask or even to ask by using the teacher statement. For instance, "*Is the information enough for you? Just by saying, "Ya both of the rice field have the same area", you believe in them*". The above statements purposively said by the teacher to stimulate the students to asked the group of presenter for explaining more about their answer.

So in this case, the way of the group presenter to present their answer in Math Congress also needs to be improved. They tent to just rely on their answer, the poster. Perhaps, they thought by showing the poster their friends will be understood what they were doing at that time to get the answer. Again, the teacher took the role to lead the discussion. She often asked the group presenter to give more information into the discussion.

Furthermore, the students question in Math Congress was monotone. Their question was around the way of the group presenter proved the answer such as, "*How do you get that? Proof it?*" Just a few students asked about the concept of the problem. For instance, "*Why do you cut that part (pointing the parts the students have cut)?*" Moreover, the students were seemed to be easily satisfied with the answer of their question. It can be seen from none of the follow up question asked after the group presenter answered the question for a student. Even from the students who did not ask the question, they did not ask a question to support their understanding of their friend's question.

To sum up, the social norm in the class 3A was good enough. They just need to improve their socio mathematical norm in the class, especially in the mathematical discussion. As we believe that the students in this class have a potential to communicate their thinking better than now.

#### C. Recommendation For Further Research

This study was focusing on the students understanding of the concept of conservation of area as preparatory for learning the area measurement. So this learning sequence should be taught before the students learn about the area measurement. We did this based on Piaget's argument. He recommended the students to learn the concept of conservation of area before learning the concept of area. Also, many researchers argued that the lack of the concept of conservation of area after the students learn the area measurement impacted a bad result on the students' future understanding.

Meanwhile, there was no research about the effect to teach the concept of conservation of area before the area measurement. We just observed how to design an instructional activity to make the students understand the concept of conservation of area as preparatory to learn the area measurement better.

So for the future research, it is needed to also find the effect to teach the concept of conservation of area before the area measurement. The follow up study is expected to answer how the students are facilitated to explore their understanding about the concept of conservation of area when they are learning or after they learn the area measurement.

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## **APPENDIX I**

## **TEACHER INTERVIEW**

## **Background:**

- How long have you been teaching Elementary students?
- In which grade(s) you have experienced in teaching mathematics?
- How long have you been taught the third grade pupils?

## **Teaching process:**

- What do you think about your class? Is it wonderful? Is it difficult?
- What do you think about the students? How active they are?
- What is your experience teaching area measurement for the first time in the third grade student?
- How do you usually teach the concept of area measurement? What kind of tool do you usually used? What concepts should be learned by the students on that topic?
- Based on your experience, what do you think the students will do if they face the problem like this (the area of irregular polygon) in the third grade?
- How do you usually guide the students to find the area of parallelogram? Is it from rectangle or not? If yes, how do you make to change the parallelogram into rectangle? How do you make the students understand that the area of parallelogram is same with the area of rectangle?
- Have you ever teach the concept of conservation of area in the process of learning area measurement?
- Do you know the importance of the concept of conservation of area in the process of learning area measurement?
- Do you think the concept of conservation of area possible to be included in the topic of area measurement?

## **About PMRI**

- Do you know about PMRI?
- Do you have any experience about PMRI, workshop, seminar or teaching with this approach?
- Do you think it is possible to implement PMRI as a teaching approach in your classroom?

## **Class organization:**

- How often you make group work for students?
- How many students per group?
- What is your consideration for grouping the students? Based on the score or performance level?
- Do you make the group heterogenic or homogeny? How is the comparison? Is it 1:2:1, low: middle: high?
- Do you make any special rule for the students during the lesson?
- What is the consideration of pointing a pupil during the class discussion?

## **APPENDIX II**

## THE CLASSROOM OBSERVATION

### **Practical Classroom setting**

- How many students in the class? How many boys and girls?
- How the formation of the students chair, pairs or group?
- (If it is in the group, how many pupils are in the group?)
- Who are the high achiever, medium, and lower achiever students?
- Who is the active student? Who is the passive student?
- Time management of the lesson

## **Teaching process**

- How usually the class happens? Group or pair or individually.
- How does the teacher approach in teaching? Mostly discourse or discussion?
- Does the teacher moving around when the students do their work?
- What usually the students use during the class, textbook or worksheet?
- How the students response to the teacher question? Enthusiastic or not.
- What the teacher usually do to attract the students' interest?
- How does the teacher give question to the students?
- How does the teacher reaction toward the students' answer, opinion or question?
- How does the teacher lead the discussion? Does she always be the boss? Or does she give the students opportunity to tell their idea or their thinking?
- Does the teacher give opportunity to students for thinking for a while before giving response?
- Is there any mathematical model used in the lesson?

## **Class organization**

- How do teacher interact with students?
- How do students interact among others?
- How do the pupils participate in the lesson?
- Does the teacher pointing the same pupil all the time?
- How do the social norms on that class?
- Is there any irrelevant behavior during the lesson?

#### **APPENDIX III**



## VISUALIZATION OF THE HLT

## **APPENDIX IV**

## **TEACHER GUIDE**

#### **PERTEMUAN 1**

## RENCANA PELAKSANAAN PEMBELAJARAN

Satuan Pendidikan	:	SD Al-Hikmah Surabaya
Mata Pelajaran	:	Matematika
Pokok Bahasan	:	Konservasi Luas
Kelas/Semester	:	III/2
Alokasi Waktu	:	2 x 35 menit

## A. Standar Kompetensi

5. Menghitung keliling dan luas persegi dan persegi panjang serta penerapannya dalam menyelesaikan masalah.

#### B. Kompetensi Dasar

Memahami konsep konservasi luas, bahwa luas dari suatu bangun tidak akan berubah jika dipotong dan diubah menjadi bentuk yang lain.

#### C. Indikator

- 1) Menyelidiki kemungkinan kain perca untuk dibuat taplak meja dari informasi yang ada.
- 2) Membandingkan permukaan meja dan kain perca.
- 3) Memberikan jawaban dengan cara berpikir transitif, jika luas bangun A = luas bangun C dan luas bangun B = luas bangun C, maka luas bangun A = luas bangun B

## D. Tujuan Pembelajaran (TP)

Siswa dapat membuat suatu bangun baru dari bangun yang diberikan dengan syarat kedua bangun akan mempunyai luas yang sama.

- Merangsang logika siswa untuk berpikir bagaimana membuat sesuatu yang kelihatannya tidak mungkin menjadi mungkin.
- Memahami manfaat dari komposisi ulang suatu bangun.
- Menggunakan strategi potong dan tempel untuk mengkomposisi ulang bentuk bangun.
- Memberikan alasan mengapa harus memotong di bagian 'tertentu' dan menempelnya di bagian 'tertentu yang lain'.
- Memahami konsep konservasi luas bahwa komposisi ulang suatu bangun dapat mengawetkan luas .

## E. Materi Prasyarat

• Pengukuran panjang, lebar, kecil, besar, sama besar dll.

## F. Alat dan Bahan

- Kain perca
- Gunting (setiap siswa)
- Lem kertas (setiap siswa)
- Isolasi (setiap kelompok)
- Stapler dan isinya (setiap kelompok)
- LCD dan Laptop atau komputer

## G. Kegiatan Pembelajaran

## Kegiatan Awal

(5 menit)

• Siswa ditanya tentang pengetahuan mereka terhadap penjahit, profesi penjahit dan apa saja barang yang berada di sekitar penjahit.

"Pernahkan kalian melihat penjahit? Pernahkah kalian menjahitkan baju di penjahit?"

"Nah biasanya penjahit punya banyak kain sisa dari jahitannya yang disebut kain perca."

"Kain perca itu bisa digunakan lagi tidak? Berikan contoh barang yang bisa dibuat dari kain perca!"

• Siswa ditunjukkan beberapa gambar tentang penjahit melalui LCD.



Kegiatan Tengah

## LKS 1 (5 menit)





Menurutmu, apakah mungkin kain perca yang panjangnya dua kali panjang meja dan lebarnya setengah kali lebar meja bisa dibuat taplak yang pas dengan ukuran meja?

- Siswa diberikan LKS 1.
- Siswa diminta untuk membaca soal dalam LKS 1 dan diberikan waktu untuk berpikir (secara individu) beberapa menit (2-3 menit)
- Siswa menuliskan jawaban nya di LKS 1
- Dalam sesi ini siswa tidak diberikan model dari kain perca. Untuk menjawab soal di LKS 1, siswa harus menggunakan logika mereka.

## Prediksi jawaban siswa LKS 1:

Akan ada tiga jenis jawaban, bisa, tidak bisa dan tidak tahu.

- ✓ Bisa, karena kain perca dapat dipotong-potong kecil dan disambung untuk dijadikan taplak meja.
- ✓ Tidak bisa, karena kain perca terlalu kecil untuk meja atau karena kain perca lebih panjang dari meja.
- ✓ Tidak tahu.

Biarkan siswa meyakini jawaban mereka untuk saat ini. Dorong siswa untuk menuliskan alasannya.

## LKS 2 - Work Group (15 menit)

- Setelah selesai mengerjakan dan mengumpulkan LKS 1, siswa diberikan LKS 2 per kelompok.
- Setiap kelompok terdiri dari 6 siswa dengan kemampuan akademik yang bervariasi.
- Siswa diberikan kain perca yang panjangnya dua kali panjang meja dan lebarnya setengah kali lebar meja. Meja yang digunakan adalah meja kelas siswa.
- Kain perca dan meja di kelas akan digunakan siswa untuk membuktikan jawaban mereka.
- 1. Dapatkah kamu membuat kain perca tersebut menjadi taplak yang pas dengan meja? Jelaskan jawabanmu disini!
- 2. Bandingkan meja dan kain perca yang kamu punya, mana yang lebih besar, kain perca, meja atau keduanya sama besar? Mengapa demikian?

## Prediksi jawaban siswa LKS 2

Soal no 1:

✓ Kebanyakan siswa akan menjiplak kain perca ke meja untuk mengetahui bagian mana yang harus dipotong. Bahkan mungkin ada siswa yang langsung memotong kain tanpa melakukan penjiplakan.
✓ Strategi yang paling banyak digunakan siswa adalah memotongnya



menjadi dua bagian sama panjang. Jika siswa melakukan ini, guru perlu mengklarifikasi kepada siswa apakah mereka memotongnya karena panjang kain dua kali dari meja atau mereka memotongnya didasarkan hasil jiplakan atau hanya

asal saja.

✓ Strategi lain yang mungkin muncul adalah menjiplak kain perca secara horizontal atau memotongnya kain perca langsung menjadi 4 bagian yang sama besar.



✓ Siswa bisa menempel kain perca dengan isolasi atau dengan stapler untuk menggabungnya sehingga bisa menjadi taplak meja yang pas dengan meja kelas.

Soal no 2:

- ✓ Siswa mungkin berpikir bahwa meja lebih besar daripada kain perca, karena kain perca kecil tetapi panjang.
- ✓ Atau siswa berpikir bahwa kain perca lebih besar dari meja karena kain perca panjang.
- ✓ Atau siswa berpikir sesuai dengan harapan pembelajaran ini yaitu kain perca dan meja sama besar karena keduanya pas satu sama lain ketika di tempelkan.
- ✓ Jika ada siswa yang mengukur panjang meja dan kain dengan menggunakan penggaris, biarkan dulu dan amati apa yang mereka lakukan. Kemudian beri pengertian kepada siswa bahwa mereka tidak memerlukan penggaris untuk menyelesaikan soal ini.

# **DISKUSI KELAS (15 menit)**

- Minta perwakilan dari kelompok untuk maju dan menunjukkan taplak meja buatan mereka
- Tanyakan kepada masing-masing kelompok apakah kain perca itu pas dengan meja.

- Tanggapi jawaban siswa dengan selalu menekankan bahwa kain percanya pas dengan ukuran meja. Jika siswa menjawab meja atau kain perca yang lebih besar mereka bisa memikirkan ulang atas fakta bahwa meja dan kain perca pas satu sama lain.
- Demonstrasikan ulang salah satu strategi siswa (memotong menjadi dua sama panjang)



• Tanyakan kepada siswa tentang besar kain perca sebelum dan sesudah di potong, apakah besarnya sama atau tidak. Mengapa?

Ini adalah konsep dasar dari konservasi luas. Jika siswa belum mampu memberikan jawaban yang sesuai maka biarkan saja dahulu untuk pertemuan ini. Karena konsep ini terintegrasi di beberapa pertemuan berikutnya.

Kegiatan Akhir

- Setelah diskusi kelas, berikan siswa lembar LATIHAN 1 beserta model meja dan kain perca untuk di gunakan sebagai alat bantu.
- Model dari meja dan kain perca harus telah di gunting dan berikan secara pas 1 lb meja, 1lb kain perca, 1 lb meja A, 2 lb kain perca 1, 2 lb kain perca 2, dan 1 lb meja B. Lebih baik siswa meminta model lagi ke gurunya jika terjadi kesalahan dalam memotong karena jika siswa diberi jumlah model yang lebih,

mereka cenderung menggunakan semua model yang ada, khususnya untuk soal no 1.

# LATIHAN 1 (Individual) (30 menit)



# Prediksi jawaban siswa:

Soal no 1.

✓ Berdasarkan pengalaman siswa sebelumnya di LKS 2, maka untuk menyelesaikan soal no 1, kebanyakan siswa mungkin akan memotong model dari kain perca menjadi tiga bagian seperti contoh di bawah ini.



Soal no 4

- ✓ Mungkin ada siswa yang bisa menyimpulkan bahwa tidak ada kain perca yang pas untuk meja B setelah mencoba salah satu, kain perca 1 atau kain perca 2. Mereka paham bahwa kain perca 1 dan 2 sama besar. Jadi jika salah satu kain perca tak bisa pas dengan meja B, maka tidak ada kain perca yang pas dengan meja B.
- ✓ Selain itu, beberapa siswa mungkin perlu untuk mencoba memasangkan kain perca 1 dan kain perca 2 ke meja B untuk mengetahui bahwa tidak ada kain perca yang akan pas dengan meja B.



- ✓ Untuk soal no 5, mungkin beberapa siswa sudah tidak perlu membandingkan lagi kain perca1 dan kain perca 2 seperti contoh jawaban di atas. Siswa mungkin sudah bisa menggunakan cara berpikir transitif, yaitu karena kain perca 1 pas dengan meja A dan kain perca 2 juga pas dengan meja A, jadi kain perca 1 dan 2 pasti akan sama besar.
- Jika tidak ada siswa yang berpikir transitif, di akhir pembelajaran guru dapat mengulang kembali soal yang berkaitan dengan cara berpikir transitif ini di depan kelas.
- Sembari menunjukkan gambar (bisa hasil pekerjaan siswa atau jawaban aslinya di LCD), guru mengajak siswa berdiskusi dengan mengajukan pertanyaan di bawah ini

"JIka kedua kain perca, kain perca 1 dan kain perca 2 sama-sama pas untuk meja A, apakah itu berarti kain perca 1 sama besar dengan kain perca 2? Mengapa?"

# INGAT:

Jangan menggunakan kata luas dalam pembalajaran ini karena ditakutkan ada beberapa siswa yang sudah memahami konsep luas dan dimungkinkan akan muncul penggunaaan formula panjang kali lebar. Hal ini sangat tidak diharapkan dalam pembelajaran ini.

# Simpulan

Siswa bersama guru menyimpulkan apa yang telah mereka pelajari hari ini.

- Jika meja dan kain perca pas satu sama lain, maka besarnya sama.
- Jika kain perca 1 pas dengan meja A dan kain perca 2 pas dengan meja A, maka kain perca 1 dan 2 akan pas satu sama lain, dengan kata lain kain perca 1 dan 2 sama besar.

#### PERTEMUAN 2

### RENCANA PELAKSANAAN PEMBELAJARAN

Satuan Pendidikan	:	SD Al-Hikmah Surabaya
Mata Pelajaran	:	Matematika
Pokok Bahasan	:	Konservasi Luas
Kelas/Semester	:	III/2
Alokasi Waktu	:	2 x 35 menit

### A. Standar Kompetensi

5. Menghitung keliling dan luas persegi dan persegi panjang serta penerapannya dalam menyelesaikan masalah.

### B. Kompetensi Dasar

Memahami konsep konservasi luas, bahwa luas dari suatu bangun tidak akan berubah jika dipotong dan diubah menjadi bentuk yang lain.

### C. Indikator

- Membuktikan bahwa dua bangun datar dengan bentuk yang berbeda itu sama besar.
- Menentukan bangun terbesar dari 3 bangun
- Mengubah bentuk suatu bangun datar menjadi bentuk lain yang berbeda tetapi mempunyai besar sama.

# D. Tujuan Pembelajaran

- Mengidentifikasi dua bangun yang bentuknya berbeda tetapi mempunyai luas yang sama dengan strategi komposisi ulang.
- Menginvestigasi dua bangun yang bentuknya berbeda mempunyai luas yang sama dengan strategi komposisi ulang.
- Mengubah bangun menjadi bangun lain dengan bentuk yang berbeda tetapi besarnya masih sama dengan bangun aslinya.
- Mengkomunikasikan tentang strategi yang dipilih oleh siswa.
- Mengidentifikasi cara-cara berbeda dalam mengkomposisi ulang suatu bangun.
- Menyimpulkan bahwa komposisi ulang suatu bangun akan mengawetkan luasnya.

# E. Materi Prasyarat

• Komposisi ulang

# F. Alat dan Bahan

- Gunting (setiap siswa)
- Lem kertas (setiap siswa)
- Kertas poster (setiap kelompok)
- LCD dan Laptop atau komputer
- Puzzle
- Model sawah (setiap kelompok)

# G. Kegiatan Pembelajaran

# Kegiatan Awal

# Pengenalan Konteks (10 menit)

• Mini lesson 1 untuk mengingatkan siswa terhadap pelajaran yang telah mereka pelajari sebelumnya. Selain itu, mini lesson juga digunakan untuk membantu siswa memahami konsep-konsep yang belum mereka pahami



KAIN PERCA BU TINA



KAIN PERCA BU ANA



Ceritakan kepada siswa bahwa Bu Ana juga mempunyai kain perca yang pas dengan meja Bu Tina. Tunjukkan lagi kepada siswa bahwa kain perca Bu Tina pas dengan meja, begitu juga kain perca Bu Ana pas dengan meja. Pertanyaannya: "Apakah kain perca Bu Tina sama besar dengan kain perca Bu Ana?"

"Apa alasannya?"



Meja Bu Yuyun

"Dapatkah kain perca Bu Ana di buat menjadi taplak meja yang pas untuk meja Bu Yuyun? Apakah kain perca Bu Ana sama besar dengan meja Bu Yuyun? Apa alasanmu?"

### Prediksi jawaban siswa:

- ✓ Beberapa siswa mungkin masih menjawab bahwa kain Bu Tina dan Bu Ana tidak sama besar karena kain Bu Tina sebenarnya panjang dan telah dipotong.
- ✓ Mungkin ada beberapa siswa yang menjawab kedua kain perca sama panjang karena keduanya sama-sama pas dengan meja A.
- Menunjukkan gambar-gambar sawah kepada siswa dan mengklarifikasi pengetahuan siswa bahwa tak selamanya bentuk sawah selalu persegi atau persegi panjang (melalui LCD).



# Kegiatan Inti

LKS 3 – Kerja Kelompok (30 menit)

- Setiap kelompok terdiri atas 4/6 siswa dengan beragam kemampuan akademik.
- Setiap kelompok diberi LKS 3 beserta model-model sawah dalam soal untuk di gunakan sebagai alat membuktikan.
- Siswa pada umur ini, 8-9 tahun, dirasa masih mengalami kesulitan dalam memahami maksud dari soal-soal di LKS 3 ini. Sehingga, guru perlu memastikan pemahaman siswa terhadap tiap-tiap soal. Hal ini dapat dilakukan sebelum siswa mengerjakan LKS 3. Untuk soal no 1 dan 2, guru hanya perlu menjelaskan ulang apa yang diinginkan oleh soal. Sedangkan untuk soal no 3, guru perlu memberikan interpersepsi tentang bangun datar persegi dan persegi panjang. Selain itu siswa perlu tahu bagaimana bentuk-bentuk yang berbeda, apa maksud perintah 'ubahlah' dan sampai batas apa suatu bangun dianggap sudah mengalami perubahan.

Tanyakan kepada siswa (melalui layar LCD), apakah bentuk dari dua bangun yang ditunjukkan sama atau berbeda.





- No 1 dan 2, bentuk dari bangun tersebut adalah sama. Siswa dapat mengubah bentuk bangun 1 menjadi bangun 2 dengan memutarnya sejauh 90°.
- No 3 dan 4, juga dua bangun dengan bentuk yang sama. Siswa dapat mengubah bentuk bangun 3 menjadi 4 dengan membaliknya secara vertikal ke kanan.
- No 5 dan 6, adalah dua bangun dengan bentuk yang berbeda sedikit. Walau hanya sedikit tapi bentuk bangun tersebut tetap berbeda karena bangun 6 ada sedikit potongan di bagian kiri atas sedangkan bangun 6 berbentuk persegi yang utuh.
- No 7 dan 8, perbedaan bentuk dari kedua bangun tersebut sangatlah jelas terlihat oleh siswa.

# **INTERPERSEPSI 2**

Dengan menggunakan jawaban siswa pada masalah 1 dan 2, tanyakan kepada siswa apakah bentuk sawah-sawah yang telah mereka potong itu sama atau berbeda. Jika siswa kurang paham, maka tunjukkan lewat LCD gambar berikut:



"Apa yang berubah dari bentuk sawah tersebut? Mana perubahan itu? Tunjukkan!"

"Apakah bentuk sawah tersebut berubah setelah dipotong?"

"Jadi apa yang kalian ketahui tentang kata 'mengubah'? Apa yang harus kalian lakukan jika diminta untuk mengubah sawah Pak Salman"

Guru bisa memberikan pertanyaan-pertanyaan seperti di atas untuk membantu siswa memahami apa arti perintah 'UBAHLAH'.

- Pada soal no 3, cek apakah siswa telah menggunakan semua bagian yang telah mereka potong dari sawah Pak Salman. Minta siswa untuk menunjukkan bagian mana yang dipotong dan dimana bagian tersebut ditempelkan.
- Siswa diberi pertanyaan tentang konsep identitas seperti, "Apakah kalian telah menggunakan seluruh bagian yang terpotong?, Bagaimana jika salah satu bagian hilang? Apakah bentuk sawah baru yang kalian buat sekarang masih akan sama besar dengan sawah Pak Salman?Bagaimana jika salah satu teman kalian secara tidak sengaja menambahkan potongan yang lain?Apakah besar sawah baru yang kalian buat masih sama besar dengan sawah Pak Salman?"
- *"Walaupun bentuknya berubah menjadi bentuk yang abstrak, mengapa besar sawah yang baru dengan sawah Pak Salman sama?",* pertanyaan ini sangat penting untuk siswa, sebagai klarifikasi pemahaman mereka terhadap konsep konservasi luas.
- Beberapa siswa mungkin sudah memahami konsep identitas, mereka tahu bahwa jika tidak ada yang ditambahkan atau dikurangkan dari sawah Pak Salman yang asli maka besar sawah yang baru masih akan sama. Jika tidak ada satupun siswa yang memahami konsep identitas ini minta siswa untuk berpikir bahwa potongan-potongan sawah tersebut seperti puzzle. Jika ada salah satu bagian puzzle yang hilang atau berlebih maka kalian tidak akan bisa membuat puzzle menjadi utuh.
- Guru bisa mengajak siswa untuk memikirkan puzzle yang pernah mereka mainkan pada waktu kecil, atau guru bisa menunjukkan puzzle melalui LCD atau juga bisa menunjukkan puzzle yang sebenarnya untuk membantu siswa memahami konsep identitas ini.
- Beritahu siswa bahwa mereka harus menunjukkan hasil kerja mereka dan di tempelkan di kertas POSTER untuk nantinya di pamerkan pada saat Kongres Matematika.

Beri penjelasan kepada siswa tentang cara menuliskan jawaban mereka pada kertas poster;

- Tuliskan hanya ide pokok dan metode yang kalian gunakan untuk menyelesaikan masalah tersebut.
- Jangan menuliskan kalimat-kalimat panjang, kalian bisa menggantinya dengan gambar. Gambarkan ide kalian di poster.

Gambar dan tulisan jangan terlalu kecil. Buat gambar tersebut bisa dilihat dan dibaca oleh teman-teman yang lain.

- Kalian bisa menggunakna model-model yang diberikan untuk menunjukkan hasil jawaban kalian. Kalian bisa menempelkan hasil jawaban kalian di poster dan menjelaskan apa yang telah kalian lakukan.
- Guru perlu menyiapkan semua yang dibutuhkan siswa untuk membuat poster seperti spidol besar, lem dan kertas poster.
- Amati dan bantu siswa jika mereka masih kurang memahami maksud soal.
- Contohnya, jika pada masalah 1 dan 2 siswa hanya menggunakan superposisi (menjiplak) untuk menunjukkan sawah mana yang lebih besar, tanyakan kepada siswa, "Apakah kalian yakin sawah itu lebih besar daripada sawah ini? Bagaimana kalian yakin bahwa bagian ini (sawah B yang tidak tertutup oleh sawah A) lebih besar dari bagian itu (sawah A yang tidak tertutup oleh sawah B)? Tunjukkan kalau begitu!"
- Ingat jangan pernah berkata bahwa jawaban siswa 'SALAH', tak ada salah dalam belajar, kesalahan adalah proses untuk menuju kebenaran.
- Jika siswa mengkombinasikan superposisi dengan gunting-tempel, tekankan keuntungan kombinasi ini dengan bertanya, "Apa fungsi dari kalian melakukan superposisi terlebih dahulu baru kemudian mengguntingnya? Kapan kalian melakukan penjiplakan, sebelum atau sesudah memotong sawah-sawah tersebut?"
- Dengan bertanya seperti itu, siswa diharapan menyadari kegunaan menggunakan superposisi untuk menentukan posisi mana yang harus dipotong dan dimana bagian tersebut dapat ditempelkan.



#### Prediksi jawaban siswa:



Siswa mungkin melakukan strategi di atas dengan menggunakan sistim coba-coba (*trial and error*). Tetapi juga di mungkinkan ada siswa yang sudah memikirkan cara membandingkan bangun tersebut dengan superposisi (menempelkan bangun di atas bangun yang lain – seperti menjiplak) untuk melihat bagian mana yang berlebih dan bagian mana yang kurang. Seperti contoh dibawah ini.



#### Soal no 2

Bagi beberapa siswa yang sudah bisa berpikir transitif, mereka hanya perlu menunjukkan bahwa salah Pak Salman lebih besar dari salah satu sawah Pak Badrun atau Pak Darma. Hal ini cukup membuktikan bahwa sawah Pak Salman adalah yang paling besar karena sawah Pak Badrun dan Pak Darma sama besar. Tetapi, guru perlu memastikan cara berpikir ini. Tanyakan kepada siswa, "Mengapa kalian hanya membandingkan dengan sawah Pak Badrun saja? Bagaimana dengan sawah Pak Darma?" atau kebalikannya. Bagi siswa yang belum bisa berpikir transitif, maka mereka akan membandingkan lagi dengan sawah lainnya. Dari sini guru bisa membimbing sekali lagi untuk berpikir transitif.



Pada soal no 3 ini, hasil jawaban siswa akan terkesan sangat abstrak. Beri pengertian kepada siswa bahwa jawaban mereka benar asal mereka tetap memakai semua potongan yang ada walau bentuk yang mereka hasilkan aneh.

Pastikan bahwa siswa paham bahwa pemindahan potongan-potongan bangun menjadi bentuk lain akan tetap membuat bangun tersebut sama besar. "Nah tadi kan potongan ini kalian pindah ke sini, apakah besar sawah yang sebelum potongannya dipindah dan setelah potongannya dipindah sama? Mengapa?"

# KONGRES MATEMATIKA (25 menit)

- Pastikan seluruh kelompok telah siap dengan poster masing-masing.
- Ajak semua siswa untuk duduk lesehan di depan kelas, jangan biarkan siswa duduk di bangku mereka masing-masing. Ajak siswa untuk berkumpul di satu titik agar konsentrasi siswa tidak terbagi dan fokus dengan presentasi dari temannya.
- Selama siswa bekerja, guru mengamati proses dan hasil akhir dari jawaban siswa tersebut. Pilih kelompok berdasarkan kepentingan pembelajaran.
- Berikan aturan yang jelas pada saat kongres ini, contohnya:
  - Jika guru berbicara, siswa harus diam.
  - Jika ada temannya yang berbicara dalam rangka mengemukakan pendapat, siswa yang lain harus mendengarkan dan harus bisa mengulang kembali apa yang telah di bicarakan temannya tersebut.
  - Jika ingin berbicara angkat tangan tanpa bersuara.
- Urutkan tampilan siswa berdasarkan hasil jawaban mereka, salah satu contoh seperti di bawah ini;
  - Kelompok yang mempunyai kesalahpahaman dalam menyelesaikan masalah 1,2, atau 3. Sehingga temantemannya bisa mengkoreksi kesalahpahaman tersebut. Selain itu bisa dijadikan pelajaran bagi siswa lainnya
  - 2. Kelompok yang hanya menggunakan superposisi untuk menyelesaikan masalah. Hal ini ditujukan untuk memberikan gambaran kepada siswa kekurangan hasil diskusi jika hanya berdasarkan superposisi.
  - 3. Kelompok yang menggunakan kombinasi superposisi untuk menentukan bagian yg berlebih dan bagian yang kekurangan kemudian dilanjutkan dengan gunting-tempel untuk membuktikan bahwa kedua sawah mempunyai besar yang sama atau yang paling besar.
- Selama kongres matematika, siswa harus didorong untuk bertanya atau memberikan pendapat mereka.
- Jika ada siswa yang sedang bertanya atau memberikan komentar, minta siswa yang lain untuk memperhatikan siswa tersebut bukan melihat ke arah guru.
- Pastikan semua siswa mengerti apa yang teman mereka sedang tanyakan atau komentari, jika perlu tunjuk beberapa siswa untuk

mengulang kembali apa yang telah teman mereka tanyakan atau komentari. Contohnya, "Doni, coba ulang kembali apa yang Rina tanyakan!"

- Dalam kongres ini, guru bertindak sebagai moderator. Jangan terlalu mengintervensi diskusi, tetapi arahkan menuju simpulan.
- Di akhir kongres ajak siswa untuk menyimpulkan apa yang telah mereka pelajari hari ini.

#### Kegiatan Akhir (5 menit)

#### SIMPULAN

.Simpulan yang diharapkan adalah

"Bahwa bangun yang di komposisi ulang akan tetap sama besar hanya bentuknya saja yang berbeda."

Jika siswa belum sampai pada kesimpulan ini, guru bisa memberikan bimbingan dan arahan agar siswa bisa berpikir tentang hal tersebut.



Komposisi ulang = mengawetkan luas dari bangun yang baru dengan bangun yang lama



#### **PERTEMUAN 3**

### RENCANA PELAKSANAAN PEMBELAJARAN

Satuan Pendidikan	:	SD Al-Hikmah Surabaya
Mata Pelajaran	:	Matematika
Pokok Bahasan	:	Konservasi Luas
Kelas/Semester	:	III/2
Alokasi Waktu	:	2 x 35 menit

### A. Standar Kompetensi

5. Menghitung keliling dan luas persegi dan persegi panjang serta penerapannya dalam menyelesaikan masalah.

### B. Kompetensi Dasar

Memahami konservasi luas dalam konsep identitas, jika tidak ada bagian yang dikurangkan atau ditambahkan dati suatu bangun maka luas bangun tersebut akan sama walau bentuknya berubah.

# C. Indikator

- Memasangkan petak-petak sawah kecil ke sawah yang besar (seperti puzzle).
- Menyimpulkan jika petak-petak sawah kecil dapat menutupi semua bagian sawah besar, maka petak-petak sawah kecil dan sawah besar akan menghasilkan padi yang sama banyak.

# D. Tujuan Pembelajaran

- Menginvestigasi luas dari suatu bangun ketika dipotong-potong menjadi beberapa bagian dan disebar ke beberapa lokasi apakah masih tetap sama.
- Memahami konsep identitas pada konservasi luas. Jika tidak ada bagian yang ditambahkan atau dikurangi dari bangun tersebut, walau terpisahpisah, bangun tersebut masih mempunyai luas yang sama.

# E. Materi Prasyarat

Komposisi ulang

### F. Alat dan Bahan

- Gunting (setiap siswa)
- Lem kertas (setiap siswa)
- Model sawah (setiap kelompok dan setiap siswa)

### G. Kegiatan Pembelajaran

#### Kegiatan Awal

• Mini lesson 2 (10 menit) – CADANGAN

Mini lesson ini perlu diberikan jika setelah pertemuan 1 dan 2, masih banyak siswa (>50%) yang tidak memahami konsep konservasi luas dengan menggunakan komposisi ulang. Jika siswa sudah paham terhadap konsep konservasi luas dengan pembelajaran di pertemuan 1 dan 2, maka mini lesson ini bisa ditiadakan. Guru bisa langsung melakukan mini lesson 3.

Dalam mini lesson ini, masalah konservasi luas akan diberikan dengan menambahkan atribut "pengukuran (ml)" untuk membantu siswa memahami konsep pengawetan luas.

Seperti:

Ada dua kertas putih A4, yakinkan siswa bahwa kedua kertas besarnya sama.

Setiap kertas membutuhkan sekitar 100 ml cat air untuk mewarnai kertasnya.

Nah jika salah satu kertas dipotong-potong oleh guru menjadi beberapa bagian dan disusun menjadi bentuk lain dengan menggunakan semua potongan tersebut, tanyakan kepada siswa "Berapa ml cat air yang dibutuhkan untuk mengecat kertas tersebut?"

Diharapkan dengan adanya atribut (ml) ini, siswa-siswa yang masih mengalami kesulitan memahami konservasi luas dapat memahaminya.

Jika ada beberapa siswa yang belum paham, ulangi kegiatan tersebut beberapa kali sehingga mereka yakin bahwa cat air yang dibutuhkan akan sama walaupun kertas sudah terpotong-potong.

#### • Mini lesson 3 (5 menit) Ada dua sawah.



Tetapi, satu tahun kemudian dibangun rumah di salah satu sawah tersebut.



LKS 4 (30 menit)

- Siswa diberikan lembar kerjas siswa 4 yang juga akan dikerjakan secara berkelompok.
- Siswa akan diberikan model dari sawah-sawah itu sebagai alat bantu. Siswa diperboleh untuk memotong, melipat atau apa saja pada model itu untuk menemukan jawaban.



Apakah sawah Pak Sudirman saat ini masih menghasilkan padi yang sama banyak dengan sawah Pak Sudirman sebelumnya (lihat gambar diatas)?



Prediksi jawaban siswa:

✓ Siswa mungkin menggunting sawah Pak Sudirman yang sekarang menjadi beberapa bagian dan menyusunnya sehingga bisa menutup sawah Pak Sudirman tujuh tahun yang lalu seperti gambar dibawah ini.



- ✓ Jika mereka bisa menutup sawah tujuh tahun yang lalu secara pas maka siswa akan menyimpulkan bahwa sawah tersebut akan menghasilkan padi sama banyak.
- ✓ Jika karena kekurangtepatan siswa dalam menggunting atau menyusun potongan-potongan sawah Pak Sudirman yang sekarang, maka salah satu sawah akan tampak lebih besar dari yang lain. Siswa akan menyimpulkan bahwa sawah yang lebih besar itu akan menghasilkan padi yang lebih banyak dari yang satunya.
- ✓ Jadi hubungan antara besar sawah dan jumlah padi yang dihasilkan akan dipandang secara linear oleh siswa.

DISKUSI KELAS (10 menit)

- Setelah semua kelompok selesai mengerjakan LKS 4, ajak siswa untuk mendiskusikan hasil jawaban kelompok mereka.
- Tunjuk perwakilan kelompok siswa yang menjawab dengan memotong sawah Pak Sudirman yang sekarang dan disusun untuk menutupi sawah Pak Sudirman tujuh tahun yang lalu.
- Tanyakan kepada siswa apakah mereka telah menggunakan semua bagian sawah yang mereka potong. Minta siswa untuk menunjukkannya.
- Tanyakan pertanyaan-pertanyaan di bawah ini untuk mengecek pemahaman siswa tentang konsep identitas dalam konservasi luas;

*"Jika ada bagian yang hilang dari sawah yang telah kalian potong, apakah kedua sawah tersebut masih menghasilkan padi sama banyak?"* 



"JIka ada potongan bagian kelompok lain yang secara tidak sengaja terbawa oleh salah satu teman kalian, apakah sawah Pak Sudirman sekarang akan menghasilkan padi yang sama besar dengan dahulu? Atau malah lebih banyak?



• Kemudian, jika ada siswa yang menyelesaikan masalah ini dengan cara yang berbeda, minta siswa tersebut untuk menunjukkan strategi uniknya

kepada teman-temannya. Ajak siswa yang lain untuk memberikan pendapat mereka tentang strategi yang digunakan temannya tersebut.

• Arahkan siswa untuk tidak memberikan kritik yang negatif atau menyalahkan jawaban temannya. Berikan pengertian bahwa kritik yang mereka utarakan adalah kritik yang membangun. Guru bisa memberikan contoh dengan terlebih dahulu memberikan kritik pada kelompok pertama.

#### Kegiatan Akhir

#### LATIHAN 2 (10 menit)



 Apakah kedua sawah, sawah A dan B menghasilkan padi yang sama banyak? Mengapa?

Jelaskan jawabanmu disini!

(kalian akan diberi model sawah di atas sebagai alat bantu)

• Bimbing siswa untuk menyelidiki soal ini. Contohnya, guru bisa memberikan pertanyaan, "Apakah sawah A dan B mempunyai besar sama? Apakah rumah di sawah A dan sawah B sama besar? Jika tidak ada rumah di kedua sawah, apakah sawah A dan B akan menghasilkan padi yang sama banyak atau tidak?" Prediksi Jawaban Siswa:

- Siswa mungkin berpikir bahwa kedua sawah memiliki ukuran yang sama sehingga pasti menghasilkan padi yang sama banyak juga. Siswa tidak mempertimbangkan tanah yang digunakan untuk mendirikan rumah.
- ✓ Siswa mungkin berpikir bahwa sawah A lebih kecil dari B karena rumah di sawah A lebih besar. Sehingga padi yang dihasilkan sawah A juga lebih sedikit dari sawah B.
- Siswa mungkin berpikir bahwa sawah A dan sawah B sama besar karena tanah yang digunakan untuk rumah di sawah A dan sawah B sebenarnya sama besar. Siswa ini mungkin bisa berpikir bahwa padi yang dihasilkan oleh sawah A atau B terpengaruh oleh besar tanah yang digunakan untuk rumah.

Siswa mungkin sudah membuktikannya dengan menggunakan model sawah dan rumah bahwa kedua sawah luasnya sama dan kedua rumah di masingmasing sawah juga luasnya sama. Siswa dapat melakukan komposisi ulang dalam membuktikannya.

#### Simpulan

Simpulan yang diharapkan pada pertemuan ini adalah

- Konsep identitas juga mengawetkan luas. Jika tidak ada bagian yang ditambahkan atau dikurangkan dari potongan-potongan suatu bangun, maka ketika potongan itu disusun menjadi bentuk lain luasnya akan sama dengan bangun semula.
- Jika ada suatu bangun yang dipotong-potong kemudian ditempatkan menyebar dalam suatu lokasi maka luas keseluruhan bangun yang dipotong-potong akan tetap sama dengan bangun yang utuh.

#### **PERTEMUAN 4**

### RENCANA PELAKSANAAN PEMBELAJARAN

Satuan Pendidikan	:	SD Al-Hikmah Surabaya
Mata Pelajaran	:	Matematika
Pokok Bahasan	:	Konservasi Luas
Kelas/Semester	:	III/2
Alokasi Waktu	:	2 x 35 menit

#### A. Standar Kompetensi

5. Menghitung keliling dan luas persegi dan persegi panjang serta penerapannya dalam menyelesaikan masalah.

#### B. Kompetensi Dasar

Memahami konsep konservasi luas dalam part-whole relation.

#### C. Indikator

- Menyimpulkan bahwa sawah dengan bentuk berbeda bisa menghasilkan padi yang sama banyak.
- Menggunakan satuan luas tidak formal (sawah berbentuk persegi menghasilkan padi 1 ton) untuk mengetahui padi yang dihasilkan oleh sawah bentuk lain.
- Menambahkan jumlah semua satuan luas tidak formal (sawah berbentuk persegi menghasilkan padi 1 ton) yang digunakan untuk menutup sawah besar untuk menghitung padi yang dihasilkan oleh sawah yang besar tersebut.
- Mengubah bangun-bangun dengan bentuk tidak beraturan menjadi persegi atau persegi panjang.

#### D. Tujuan Pembelajaran

- Menentukan luas bangun yang sama atau dua kali lebih besar dari bangun lain yang diketahui luasnya.
- Menginvestigasi bahwa sawah dengan bentuk berbeda bisa menghasilkan padi yang sama banyak.
- Menggunakan potongan terkecil sawah yang diketahui (sawah berbentuk persegi menghasilkan padi 1 ton) sebagai satuan luas tidak formal untuk mengetahui banyak padi yang dihasilkan oleh sawah yang lebih besar.
- Mengkomposisi ulang bangun-bangun tidak beraturan menjadi bentuk persegi atau persegi panjang.

# E. Materi Prasyarat

- Komposisi ulang bangun
- Identitas dalam konservasi luas

# F. Alat dan Bahan

- Gunting (setiap siswa)
- Lem kertas (setiap siswa)
- Model sawah (setiap kelompok)

# G. Kegiatan Pembelajaran

# Kegiatan Awal:

# INTERPERSEPSI (PERSEGI DAN PERSEGI PANJANG) (2 menit)

- Perkenalkan persegi dan persegi panjang. Misalnya, persegi adalah segiempat yang keempat sisinya sama panjang. Persegi panjang adalah segiempat yang panjang dan lebarnya berbeda.
- Mungkin untuk sebagian siswa gambar akan sedikit membantu. Tunjukkan gambar ini kepada siswa dalam berbagai posisi (horizontal, vertikal, miring, dan sebagainya)



Persegi panjang : mempunyai panjang dan lebar yang berbeda

Persegi : mempunya sisi – sisi yang sama panjang.

# MINI LESSON 3 (3 menit)

1. Ubahlah bangun di bawah ini menjadi persegi atau persegi panjang!





2. Dari kedua bangun di bawah ini, manakah yang lebih besar, apakah bangun no 2 atau bangun persegi panjang hasil komposisi ulang bangun no 2 (2a)?

Atau keduanya sama besar? Mengapa?



3. Dari kedua bangun di bawah ini, manakah yang lebih besar, apakah bangun no 4 atau bangun persegi panjang hasil komposisi ulang bangun no 4 (4a)? Atau keduanya sama besar? Mengapa?

> Telah diubah menjadi 4

#### Prediksi jawaban siswa:

• Soal no 1

Karena siswa telah diberi interpersepsi tentang persegi dan persegi panjang, maka siswa akan mudah membayangkan bentuk yang dinginkan yaitu persegi atau persegi panjang.

Cek dan bimbing pemahaman siswa dengan memberikan pertanyaanpertanyaan di bawah ini:

- Apakah kalian tahu bagaimana bentuk persegi atau persegi panjang?
- Kira-kira apa yang bisa kalian lakukan untuk mengubah bangun no 1 menjadi persegi atau persegi panjang? Bisakah bangun tersebut dipotong-potong dan disusun ulang untuk menjadi persegi atau persegi panjang? Tunjukkan!
- ✓ Untuk bangun no 4, mengapa tidak bisa di ubah menjadi bentuk persegi atau persegi panjang? Kalian yakin, pikirkan sekali lagi dan cobalah !

Jika ada siswa yang menjawab, minta siswa memberikan instruksi apa yang harus dilakukan guru untuk mengubah bangun tersebut menjadi persegi atau persegi panjang.

Pastikan semua siswa memperhatikan ketika temannya menjawab bagaimana cara mengubah bangun tersebut menjadi persegi atau persegi panjang, jika perlu minta siswa lain mengulangi cara yang dilakukan teman mereka.

• Soal no 2 dan no 3

Bagi siswa yang sudah memahami konsep konservasi luas, untuk menjawab pertanyaan no 2 ini akan sangat terasa mudah. Siswa dapat menjawab bahwa kedua bangun tersebut sama besar karena bangun persegi panjang terbuat dari bangun no 2.

Tetapi bagi siswa yang belum memahami penuh konsep konservasi luas mungkin masih akan bermasalah dengan ukuran yang berbeda (panjang dan lebar) dari kedua bangun sehingga masih menyimpulkan bahwa salah satu pasti lebih besar dari yang lain.

Jika terjadi hal semacam ini, guru perlu mengingatkan siswa tentang pelajaran pada pertemuan-pertemuan awal, tentang kain perca dan konsep identitas dalam konservasi luas. Ajukan pertanyaan-pertanyaan berikut ini untuk membantu siswa menjawab soal no 2 dan 3 dengan benar.

- ✓ "Masih ingat dengan kain perca Bu Tina? Apa yang terjadi ketika kain perca pas dengan meja Bu Tina? Mana yang lebih besar, kain perca atau meja atau keduanya sama besar?"
- "Ingatkah kalian dengan sawah Pak Salman? Ketika kalian mengubah sawah Pak Salman menjadi bentuk lain, apakah besar sawah Pak Salman berubah? Mengapa?"
- ✓ "Apakah kalian membuat bangun no 2a dari bangun no 2? Jadi apakah bangun no 2 dan no 2a sama besar?
- ✓ Jika bangun no 2 membutuhkan 2 ml cat air untuk mewarnainya, maka ketika bangun no 2 diubah menjadi bangun no 2a, berapa ml cat air yang dibutuhkan untuk mewarnai bangun no 2a?

Kegiatan Inti

ton

LKS 5 (20 menit)

• Setelah itu siswa diberikan LKS 5.

1. Jika sawah yang berbentuk persegi, seperti gambar di samping akan menghasilkan 1 ton padi, tentukan berapa ton padi yang akan dihasilkan oleh petak-petak sawah di bawah ini!



2. Pak Sudirman sekarang penasaran lagi dengan dua sawahnya yang berukuran besar. Berapa ton padi yang akan dihasilkan oleh tiap-tiap sawah tersebut?



• Siswa akan diberikan beberapa model dari sawah pada LKS 5 untuk digunakan sebagai alat bantu.

• Kali ini jumlah model yang diberikan tidak dibuat pas atau terbatas karena siswa tidak akan melakukan komposisi ulang lagi. Dengan banyaknya jumlah model-model sawah siswa mendapatkan kesempatan untuk menyelidiki beberapa strategi untuk menentukan jumlah padi yang dihasilkan oleh sawah-sawah yang besar.

#### Prediksi jawaban siswa:

 Menggunakan strategi gunting dan tempel untuk menentukan jumlah padi yang dihasilkan oleh sawah – sawah Pak Sudirman yang lain. Soal no 1

Berikut ini beberapa kemungkinan susunan yang dilakukan siswa:



Ketika siswa selesai menemukan sawah yang berbentuk jajargenjang ternyata mempunyai besar yang sama menjadi sawah yang berbentuk persegi, maka siswa secara otomatis

akan menyimpulkan bahwa sawah tersebut juga akan menghasilkan 1 ton padi. Kemungkinan besar siswa masih belum tahu bahwa bentuk itu dinamakan jajargenjang, jadi guru tidak perlu mempermasalahkan nama dari bentuk sawah tersebut.



Untuk sawah yang berbentuk persegi panjang di samping, siswa mungkin menemukan dengan cepat bahwa sawah itu dua kali dari sawah persegi. Siswa bisa memotong sawah persegi panjang menjadi dua sama besar sehingga membentuk dua persegi atau siswa bisa menggunakan dua sawah persegi untuk menutupi sawah

persegi panjang. Sehingga secara otomatis siswa akan menyimpulkan sawah persegi panjang itu akan menghasilkan 2 ton padi.



Siswa mungkin akan memotong sawah persegi panjang yang terletak horizontal di samping menjadi dua sama panjang dan membandingkannya dengan persegi yg menghasilkan 1 ton

padi. Atau sebaliknya, sawah persegi dibandingkan dengan sawah persegi panjang horizontal kemudian dipotong bagian yang berlebih dan ditempelkan di bagian yang kurang sehingga akan diketahui bahwa kedua sawah tersebut sama besar. Siswa akan menyimpulkan bahwa sawah ini juga menghasilkan 1 ton padi.

Soal 2

Dalam soal ini, beberapa siswa mungkin akan langsung menutup sawah besar dengan potongan model-model sawah kecil yang telah diketahui jumlah padi yang dihasilkannya. Siswa dapat berpikir demikian karena dirangsang oleh pemberian model-model sawah dalam jumlah yang cukup banyak, lebih dari 5 model untuk setiap bentuk sawah.

Berikut ini adalah beberapa kemungkinan susunan yang dilakukan siswa:



• Dari aktifitas ini siswa diharapkan mendapatkan pemahaman bahwa untuk menentukan jumlah padi yang dihasilkan oleh sawah besar bergantung kepada jumlah padi yang dihasilkan sawah kecil (sawah persegi, sawah persegi panjang atau sawah jajargenjang).

# DISKUSI KELAS (10 MENIT)

- Minta siswa untuk menunjukkan jawaban mereka di depan kelas.
- Tanyakan kepada siswa apakah cara yang berbeda dalam menggunakan macam-macam model sawah untuk menemukan jumlah padi yang dihasilkan sawah besar masuk akal dan bisa dipercaya.
- Tanyakan kepada siswa apakah bisa hanya menggunakan satu macam model sawah untuk menentukan jumlah padi yang dihasilkan sawah besar.

"Jika kalian hanya diberi satu macam model sawah kecil, misal sawah persegi, apakah kalian masih bisa menentukan jumlah padi yang dihasilkan oleh kedua sawah besar?" Analisis argumentasi siswa dalam menjawab pertanyaan ini, terutama untuk sawah besar yang kedua (bentuk jajargenjang).

Siswa mungkin akan mempunyai ide untuk memotong sawah persegi agar pas dengan sawah besar kedua atau siswa akan menggunakan estimasi untuk menentukan jumlah padi pada bagian sawah yang tidak bisa tertutup oleh sawah persegi itu.

Kegiatan Akhir

Nama :

LATIHAN 3 (25 menit)

• Setelah siswa diberikan lembar LATIHAN 3, minta siswa membaca soal dan tanyakan pemahaman siswa terhadap maksud soal tersebut.

LATIHAN 4

Tanggal

1. Ubahlah bangun di bawah ini menjadi persegi atau persegi panjang!

Kelas:



Tempelkan hasilnya di bawah ini!



2. Ubahlah bangun di bawah ini menjadi persegi atau persegi panjang!

# Tempelkan hasilnya di bawah ini!

3. Ubahlah bangun di bawah ini menjadi persegi atau persegi panjang!





- Mungkin beberapa siswa yang berkemampuan akademik tinggi dapat memahami maksud soal tersebut, minta siswa tersebut untuk menjelaskan kepada teman-temannya di depan kelas.
- Beri siswa model bangun no 1 dan ajak siswa menyelesaikan bersama soal tersebut.
- Setelah siswa memahami apa yang harus dilakukan dan telah menemukan jawaban soal no 1, guru bisa menunjukkan contoh cara menjawab soal no 1 pada gambar berikut,



Ajukan pertanyaan tambahan kepada siswa ketika membahas perubahan bangun pada soal no 1 tentang besar kedua bangun, "*Apakah bangun sebelah kanan sama besar dengan bangun sebelah kiri*?"

Jika siswa masih belum dapat memahami bahwa kedua bangun itu sama besar, ingatkan kembali siswa dengan mengajukan pertanyaan-pertanyaan berikut ini;

- "Masih ingat dengan kain perca Bu Tina? Apa yang terjadi ketika kain perca pas dengan meja Bu Tina? Mana yang lebih besar, kain perca atau meja atau keduanya sama besar?"
- "Ingatkah kalian dengan sawah Pak Salman? Ketika kalian mengubah sawah Pak Salman menjadi bentuk lain, apakah besar sawah Pak Salman berubah? Mengapa?"
- "Apakah kalian membuat bangun sebelah kanan dari bangun sebelah kiri? Apakah kalian telah menggunakan seluruh bagian dari bangun di sebelah kanan menjadi bangun di sebelah kiri? Jadi, apakah besar dari bangun sebelah kiri sama dengan bangun sebelah kanan?
- ✓ "Jika bangun sebelah kiri membutuhkan 4ml cat air untuk mewarnainya, berapa ml cat air dibutuhkan oleh bangun sebelah kanan? Samakah dengan bangun sebelah kiri? Mengapa?"

Pertanyaan-pertanyaan di atas sangat perlu untuk diulang agar siswa benar-benar memahami konservasi luas (komposisi ulang, konsep identitas dan *part-whole relation*)

Dengan cara ini siswa bisa menggunakan visualisasi di atas untuk mengkomunikasikan cara dan hasil jawaban mereka dalam lembar LATIHAN 3.

Prediksi jawaban siswa:

#### Soal no 1

Siswa akan menggunakan jawaban yang telah dibahas bersama di depan kelas. Walaupun siswa sudah mengerjakan secara klasikal di kelas, minta siswa untuk menempelkan dan menjelaskan jawaban mereka di lembar LATIHAN 3.

#### Soal no 2

Siswa akan memotong salah satu bagian yang menonjol untuk ditempelkan pada bagian yang kurang, salah satu contohnya seperti gambar berikut.



### Soal no 3

Gambar bangun pada soal no 3 ini mungkin sedikit menyulitkan siswa untuk menentukan bagian mana yang harus dipotong. Siswa akan berpikir bahwa empat bagian yang berlubanglah yang harus ditambal dengan bagian bangun yang lain. Banyak siswa yang akan memotong bagian atas atau bawah bangun, tetapi ketika dirasa hal itu menimbulkan kesulitan yang lain siswa akan meminta tambahan model bangun ini kepada guru untuk diulang.



Mungkin setelah beberapa menit berpikir dan melakukan uji coba siswa akan menemukan bahwa bagian yang bisa dipotong dan ditempelkan ke empat bagian yang berlubang adalah bagian di sisi kanan. Bagian tersebut kemudian dibagi menjadi empat persegi sama besar untuk ditambalkan ke empat lubang pada bangun itu sehingga terbentuklah persegi panjang.

# Simpulan

- Guru bersama siswa menyimpulkan bahwa untuk mengetahui jumlah padi yang dihasilkan pada sawah yang besar, bisa dilakukan dengan membagibagi sawah tersebut menjadi sawah yang lebih kecil yang diketahui banyaknya padi yang dihasilkan.
- Beberapa bangun yang bentuknya tidak beraturan bisa diubah menjadi persegi atau persegi panjang dan besarnya akan sama

#### **PERTEMUAN 5**

### RENCANA PELAKSANAAN PEMBELAJARAN

Satuan Pendidikan	:	SD Al-Hikmah Surabaya
Mata Pelajaran	:	Matematika
Pokok Bahasan	:	Konservasi Luas
Kelas/Semester	:	III/2
Alokasi Waktu	:	2 x 35 menit

#### A. Standar Kompetensi

5. Menghitung keliling dan luas persegi dan persegi panjang serta penerapannya dalam menyelesaikan masalah.

### B. Kompetensi Dasar

Memahami konsep konservasi luas dalam part-whole relation.

# C. Indikator

- Menggunakan konsep *part-whole relation* pada bentuk bangun yang tidak beraturan.
- Memotong dan menempel bentuk-bentuk tidak beraturan untuk mengubahnya menjadi persegi atau persegi panjang.
- Menyelesaikan soal Post-test.

# D. Tujuan Pembelajaran

- Mengubah bangun-bangun yang berbentuk tidak beraturan menjadi persegi atau persegi panjang.
- Mengukur pemahaman siswa tentang konservasi luas yang telah mereka pelajari selama ini melalui POST-TEST.

# E. Materi Prasyarat

- Komposisi ulang
- Identitas dalam konservasi luas

# F. Alat dan Bahan

- Gunting (setiap siswa)
- Lem kertas (setiap siswa)
- Model sawah (setiap kelompok)
- Model bangun (setiap siswa)
• Lembar post-test (setiap siswa)

### G. Kegiatan Pembelajaran

### Kegiatan Awal:

Mini lesson 4 (10 menit)



Mini lesson ini bertujuan untuk mengingatkan siswa tentang *part-whole relation* yang telah mereka pelajari di pertemuan sebelumnya. Perbedaanya adalah soal ini menuntut siswa untuk berpikir apa yang harus dilakukan terlebih dahulu dengan bangun berbentuk tidak beraturan tersebut.

Prediksi jawaban siswa:

• Mengubah menjadi persegi atau persegi panjang terlebih dahulu

Karena siswa telah mempelajari bagaimana mengubah bentuk-bentuk bangun yang abstrak menjadi persegi atau persegi panjang, maka siswa mungkin akan mengubah bangun tersebut menjadi persegi atau persegi panjang. Kemudian menggunakan sawah berbentuk persegi kecil yang menghasilkan 1 ton padi sebagai alat bantu (satuan tak formal) untuk menentukan berapa ton padi yang dihasilkan sawah tersebut.



1	1	1
ton	ton	ton
1	1	1
ton	ton	ton
1	1	1
ton	ton	ton

Jadi, sawah tersebut menghasilkan 9 ton padi. • Tanpa mengubah menjadi persegi atau persegi panjang

Beberapa siswa mungkin akan memotong sawah persegi yang menghasilkan 1 ton padi pada diagonal untuk menutupi bagian segitiga di sawah besar. Sehingga siswa ini tidak perlu mengubah bangun tersebut menjadi persegi atau persegi panjang untuk menentukan berapa ton padi yang dihasilkan.



Kegiatan Inti

Latihan 5 (20 menit)

• Siswa diberikan lembar LATIHAN 5. Perintah pada latihan 5 ini sama dengan perintah pada LATIHAN 4 yang telah dikerjakan siswa pada pertemuan sebelumnya. Sehingga siswa tidak perlu lagi mendapatkan penjelasan ulang tentang maksud soal, hanya tingkat kesulitan pada LATIHAN 5 lebih tinggi daripada LATIHAN 4



Tunjukkan caramu membuat persegi dari bangun tersebut disini!

• Siswa akan diberikan model bangun sebagai alat bantu untuk menunjukkan cara merekamemotong. Siapkan beberapa model cadangan jika ada model yang rusak atau salah pengguntingannya oleh siswa.

Prediksi jawaban siswa:

Soal no 1



Soal no 2



Kegiatan akhir

POST-TEST (30 menit)

- Berikan siswa lembar *Post-test* secara Individu.
- Ingatkan kepada siswa untuk bekerja secara individu.
- Beri pengertian kepada siswa bahwa percaya diri itu penting, jadi tidak perlu melihat atau meminta jawaban dari temannya.
- Karena ini adalah *Post-test* maka diharapkan guru bisa lebih memperhatikan siswa untuk bisa bekerja secara individu.

### SIMPULAN

- Siswa memahami konsep konservasi luas antara lain:
  - Jika suatu bangun dikomposisi ulang menjadi bangun baru dengan bentuk yang lain, maka luas kedua bangun akan sama dengan syarat tidak mengurangi atau menambah bagian dari bangun tersebut ketika proses komposisi.
  - Jumlah luas potongan dari suatu bangun akan sama dengan luas bangun yang utuh.
  - Beberapa bangun yang bentuknya tidak beraturan, bisa diubah menjadi bentuk persegi atau persegi panjang sehingga lebih mudah untuk diketahui atributnya, misal luas atau dimensi.

### **APPENDIX V**

### STUDENTS' WORKSHEET





#### Bangun A atau bangun B?

Lingkari salah satu hurufnya! Jika keduanya sama besar, lingkari kedua hurufnya.



- 2. Pak Hasan mempunyai sawah di Bojonegoro yang berbentuk seperti di bawah





Sawah manakah yang menghasilkan padi lebih banyak? Sawah di Bojonegoro atau sawah di Surabaya atau keduanya menghasilkan padi yang sama banyak? Mengapa?



### **POST-TEST**



1. Manakah yang lebih besar?



besar, lingkari kedua

hurufnya.

- 2. Pak Hasan mempunyai sawah di Bojonegoro yang berbentuk seperti di bawah

Tahun ini Pak Hasan membeli sawah lagi di Surabaya seperti gambar di bawah ini.



Sawah manakah yang menghasilkan padi lebih banyak? Sawah di Bojonegoro atau sawah di Surabaya atau keduanya menghasilkan padi yang sama banyak? Mengapa?





3. Ubahlah bangun di sebelah kiri sehingga menjadi seperti bangun sebelah kanan!

4. Ubahlah tiap - tiap bangun di bawah ini menjadi persegi atau persegi panjang.

a.



Tunjukkan bagaimana kamu mengubahnya!

b.

Tunjukkan bagaimana kamu mengubahnya!



5. Diketahui bahwa sawah dengan bentuk persegi panjang dan segitiga menghasilkan sejumlah padi yang tertera dalam gambar di bawah ini.!



Berapa ton padi yang dihasilkan oleh sawah seperti gambar di bawah ini?



Jelaskan jawabanmu di sini!	

# **LEMBAR KERJA SISWA 1** (Individual)

Nama:

. . . . . . . . .

Tanggal:

Kelas:



# LEMBAR KERJA SISWA 2

(Kelompok)

Nama :

Tanggal:

Kelas :

- MEMBUAT TAPLAK MEJA
- 1. Dapatkah kalian membuat kain perca tersebut menjadi taplak yang pas dengan meja? Jelaskan jawabanmu disini!



kain perca, meja atau keduanya sama besar? Mengapa demikian?





• • • • • • • • • • • • • • • • • • • •	 	• • • • • • • • • • • • • • • • • • • •
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KESIMPULAN

Apa yang dapat kalian simpulkan?

### LATIHAN 1 (Individual)

Tanggal:

Kelas:

1. Ada satu meja lagi yang membutuhkan taplak meja.



Sedangkan sisa kain perca yang dimiliki Ibu Tina adalah seperti dibawah ini.



(Kamu akan di beri model meja dan kain perca untuk kamu gunakan)

Dapatkah kamu membuat taplak yang pas dengan ukuran meja dari kain perca tersebut?

Jelaskan!

Nama:

• • • •			••••	 	• • • • • •	 		• • • • • •		•••••		•••••		• • • • •		•••
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Tunjukkan hasil temuanmu disini!



Manakah yang lebih besar, kain perca, meja atau keduanya sama besar? Mengapa?

Perhatikan gambar di atas.

- 2. Apakah kain perca 1 dapat dibuat taplak untuk meja A? Tunjukkan !
- 3. Apakah kain perca 2 dapat dibuat taplak untuk meja A? Tunjukkan !
- 4. Kain perca mana yang pas dibuat taplak untuk meja B? Tunjukkan dan jelaskan!
- 5. Mana yang lebih besar, kain perca 1 atau 2? ..... Mengapa? Jelaskan jawabanmu! ..... ..... ..... .....

### LEMBAR KERJA SISWA 3

Ada dua sawah yang berbentuk seperti di bawah ini.

# SAWAH PAK BADRUN

# SAWAH PAK DARMA



Perhatikan cerita di atas!

### MASALAH 1

Bantu Pak Darma untuk meyakinkan Pak Badrun bahwa mereka memiliki sawah yang sama besar.

Tunjukkan bagaimana caramu membuktikannya di kertas POSTER





# MASALAH 2

Bantu Pak Salman untuk membuktikan bahwa sawahnya yang paling besar diantara sawah Pak Badrun dan sawah Pak Darma

Tunjukkan bagaimana caramu membuktikannya di kertas POSTER

### MASALAH 3

Pak Salman tiba-tiba penasaran apakah ada sawah lain yang besarnya sama dengan sawahnya. Bisakah kamu membantunya?

a. Ubahlah sawah Pak Salman menjadi bentuk yang lain (berbeda) !



b. Apakah sawah Pak Salman yang asli dengan yang sudah kalian ubah sekarang sama besarnya? Jelaskan?

Tunjukkan bagaimana caramu membuktikannya di kertas POSTER

### LEMBAR KEKJA 515

# Nama :

(Kelompok) Tanggal:

Kelas :

- )
- •
- 1. Pak Sudirman adalah orang kaya. Tujuh tahun yang lalu di Kediri dia mempunyai satu sawah berbentuk seperti di bawah ini.



Setiap tahunnya Pak Sudirman menjual sebagian sawahnya tetapi kemudian ia juga membeli sawah dengan besar yang sama dengan bagian yang ia telah jual. Akhirnya sekarang sawah Pak Sudirman bentuknya menjadi seperti di bawah ini.

Apakah sawah Pak Sudirman saat ini masih menghasilkan padi yang sama banyak dengan sawah Pak Sudirman sebelumnya (lihat gambar diatas)?





Jelaskan jawabanmu di sini!

### **LATIHAN 2**

Nama:

Kelas:



1. Apakah kedua sawah, sawah A dan B menghasilkan padi yang sama banyak? Mengapa?

Jelaskan jawabanmu disini! (kalian akan diberi model dari sawah diatas untuk digunakan sebagai alat bantu)

Tanggal:



a. Sekarang, tentukan berapa ton padi yang akan dihasilkan oleh petak-petak sawah di bawah ini!



Tunjukkan bagaimana kamu mendapatkan jawabanmu.

	Karena
= ton padi	
	Karena
= ton padi	
= ton padi	Karena

b. Pak Sudirman sekarang penasaran lagi dengan dua sawahnya yang berukuran besar. Berapa ton padi yang akan dihasilkan oleh tiap-tiap sawah tersebut?



Sawah ini menghasilkan..... ton padi

Bagaimana kamu mengetahuinya? Tunjukkan caramu di sini!



Jelaskan cara berpikirmu di awan pikiran di bawah ini!

Pertamanya, saya pikir	$\overline{\ }$
	$\wedge$
a to	



Bagaimana kamu mengetahuinya? Tunjukkan caramu di sini!



(	Pertamanya, saya pikir	
		$\mathbf{\lambda}$
	).	
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# LATIHAN 4

Nama :	Kelas :	
Tanggal		

1. Ubahlah bangun di bawah ini menjadi persegi atau persegi panjang!



Tempelkan hasilnya di bawah ini!

- 2. Ubahlah bangun di bawah ini menjadi persegi atau persegi panjang!

Tempelkan hasilnya di bawah ini!



3. Ubahlah bangun di bawah ini menjadi persegi atau persegi panjang!

Tempelkan hasilnya di bawah ini!

# LATIHAN 5

Buatlah persegi dari tiap bangun di bawah ini.



Tunjukkan caramu membuat persegi dari bangun tersebut disini!



Tunjukkan caramu membuat persegi dari bangun tersebut disini!