# INTRODUCING MULTIPLICATION STRATEGIES USING ARRAYS

MASTER THESIS

Ismi Ridha Asy-Syifaa
117785046

STATE UNIVERSITY OF SURABAYA
POSTGRADUATE
MATHEMATICS EDUCATION
2016

# INTRODUCING MULTIPLICATION STRATEGIES USING ARRAYS

MASTER THESIS

A thesis submitted in partial fulfillment of the requirement for the degree of
Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
(in collaboration between State University of Surabaya and Utrecht University)

Ismi Ridha Asy-Syifaa
117785046

STATE UNIVERSITY OF SURABAYA
POSTGRADUATE
MATHEMATICS EDUCATION
2016

**APPROVAL OF SUPERVISORS**

Thesis by Ismi Ridha Asy-Syifaa (117785046) with the title *Introducing Multiplication Strategies Using Arrays* has been qualified and approved to be tested.

Supervisor I, | Date,

Dr. Agung Lukito, M.S. | 25 July 2016

Supervisor II, | Date,

Dr. Siti Khabibah, M.Pd. | 25 July 2016

Acknowledge by,
Head of Mathematics Education Program

Dr. Agung Lukito, M.S.
NIP 196201041991031002

**APPROVAL**

Thesis by Ismi Ridha Asy-Syifaa (117785046) with the title *Introducing Multiplication Strategies Using Arrays* has been defended in front of the Board of Examiners on 18 August 2016.

Board of Examiners

| Name | Signature | Position |
|---|---|---|
| Dr. Agung Lukito, M.S | | Chairman/Supervisor I |
| Dr. Siti Khabibah, M.Pd | | Member/Supervisor II |
| Prof. Dr. Mega T Budiarto, M.Pd | | Member |
| Dr. Manuharawati, M.Si | | Member |
| Dr. Masriyah, M.Pd | | Member |

Acknowledged by,
Director of Post Graduate Program

Prof. I Ketut Budayasa, Ph.D
NIP. 195712041994021001

## ABSTRACT

Asy-syifaa, Ismi R. 2016. *Introducing Multiplication Strategies Using Arrays.* Master Thesis, Mathematics Education, Postgraduate, State University of Surabaya. Supervisors: (I) Dr. Agung Lukito, M.S., and (II) Dr. Siti Khabibah, M.Pd.

**Keywords**: multiplication strategies; arrays; multiplication models; commutative property; doubling strategy; one-less strategy; one-more strategy.

Elementary students are usually asked to memorize the basic multiplication facts after they are briefly introduced to the multiplication. However, some studies showed that students need to learn about multiplication strategies first before they are asked to memorize the facts. Regarding to this suggestion, this study is conducted to develop educational materials on introducing multiplication strategies using arrays, especially on introducing the commutative property as a multiplication strategy, the doubling strategy, the one-less strategy, and the one-more strategy. Also, it contributes to the development of a local instructional theory in multiplication, especially on introducing multiplication strategy using arrays.

To develop the educational materials, a hypothetical learning trajectory (HLT), consisted of the materials and the conjectures' of students learning, was developed and tried out on two-cycles teaching experiments using design research in 2013. The participantswere the second-grade-students in SD. LAB UNESA, Surabaya, Indonesia.The data collected were mainly the observation of teaching experiment and the students' worksheet. These data were analyzed to compare the conjecture and the actual students' answers and learning processes. The results of analysis were the source to construct the conclusion about how to introduce multiplication strategies using arrays and to generate a local instruction theory on introducing multiplication strategies using arrays.

For this study, the arrays were designed so that it could elicit the strategies. Findings showed that not all strategies being introduced were elicited from the designs since the students did not use the designs as a means to help them determine the unknown multiplication products. If a strategy could be elicited from the designs, the students were under the teacher's guidance that encouraged them to use a faster way to derive the unknown facts from a known fact. Nevertheless, to be able to introduce the multiplication strategies using arrays, the students needto understand the idea of arrays as multiplication models first.

## ABSTRACT (in Bahasa Indonesia)

Asy-syifaa, Ismi R. 2016. *Memperkenalkan Strategi-strategi Perkalian MenggunakanArray*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana, Universitas Negeri Surabaya. Pembimbing: (I) Dr. Agung Lukito, M.S., and (II) Dr. Siti Khabibah, M.Pd.

**Kata-kata kunci**: strategi-strategi perkalian; model array; model perkalian; sifat komutatif perkalian; strategi *doubling*; strategi *one-less*/*one-more*.

Siswa Sekolah Dasar (SD) biasanya diminta untuk menghafalkan perkalian dasar setelah perkalian diperkenalkan secara singkat. Namun, beberapa penelitian menunjukan bahwa mereka perlu untuk mempelajari strategi-strategi perkalian sebelum diminta untuk menghafalkan perkalian dasar. Berdasarkan saran tersebut, penelitian ini dilakukan untuk mengembangkan instrumen pembelajaran untuk memperkenalkan strategi-strategi perkalian dengan menggunakan *array*untuk memperkenalkan sifat komutatif sebagai salah satu strategi perkalian, strategi *doubling*, strategi *one-less*, dan strategi *one-more*. Selain itu, penelitian ini pun berkontribusi pada pengembangan teori pengajaranlokal (*local instuction theory*) pada topik perkalian, khususnya dalam pengenalan strategi perkalian.

Untuk mengembangkan instrumen pembelajaran, sebuah hipotesis mengenai lintasan pembelajaran (*hypothetical learning trajectory*, disingkat HLT) yang berisi instrumen pembelajaran berserta dugaan mengenai proses berpikir siswa dikembangkan dan diujicobakan dalam dua kali siklus penelitian di tahun 2013. Penelitian ini menggunakan metode *design researh*. Subyek penelitian adalah siswa kelas 2-B, SD LAB UNESA, Surabaya, Indonesia. Data yang dikumpulkan sebagaian besar merupakan hasil observasi dari penelitian di kelas dan lembar hasil kerja siswa. Data-data tersebut dianalisa untuk membandingkan jawaban siswa dan proses belajar yang terjadi dengan apa yang telah diasumsikan. Hasil dari analisis disusun untuk mendapatkan kesimpulan mengenai bagaimana cara untuk memperkenalkan strategi-strategi perkalian menggunakan *array* dan untuk memperoleh teori pengajaran lokal (*local instuction theory*) untuk mengenalkan strategi perkalian menggunakan *array*.

Pada penelitian ini, strategi perkalian diharapkan dapat muncul dari *array* yang telah didesain. Hasil penelitian menujukan bahwa tidak semua strategi dapat muncul karena siswa tidak menggunakan desain-desain tersebut sebagai alat untuk menemukan hasil dari suatu perkalian. Jikapunsekelompok siswa dapat memunculkan strategi yang ada pada desain dan menggunakannya, siswa-siswa tersebut bekerja di bawah arahan guru yang meminta mereka untuk menggunakan strategi yang lebih cepat untuk menemukan hasil dari suatu perkalian. Selain itu, untuk dapat mengenalkan strategi-strategi perkalian menggunakan *array*, siswa harus dapat mengerti bahwa *array* adalah salah satu model dari perkalian terlebih dahulu.

## PREFACE

Praise and thank to Allah SWT for allowing me to finish this thesisas partial fulfillment for completing my master study on Mathematics Education Program in State University of Surabaya.Thetitleis: "*Introducing Multiplication Strategies Using Arrays*", and it serves as the topicstudy reported in this thesis. This topic was chosen considering two ideas;(1) multiplication strategies need to be introduced first before students memorize the basic multiplication facts and (2) arrays visualize the strategies better.

This thesis consists of six chapters. Chapter 1 presents background, aim, and question of the study. Chapter 2 presents theoretical framework as foundations on developing the educational materials. Chapter 3 presentsmethodological aspect on how the study was conducted. Chapter 4 presents objects of the study. Chapter 5 presents retrospective analysis of the data collected. And, Chapter 6 presents conclusion and discussion resulted from the study.

By writing this thesis, I consciously understand this report is far frombeing a perfect piece of work. Thus, any critics and feedback are gladly welcomed.

Ismi Ridha Asy-Syifaa
(ismiridhaa@gmail.com)

## ACKNOWLEDGEMENT

Acknowledgment is usually written as a part from a preface, however in this thesis; please excuse me for excluding it and presenting it as a new chapter. For me, this thesis isthe end of my almost-six-years journey. I saw it as a journey because it had brought me travel from one place to another place and from one condition to another condition. Istartedthis journeyin the end of December 2010, and now, it is almost in the end of 2016. Such a long journey! Therefore, I really want to give my gratitude ina rather personal way, even I know that words could never be enough to express it.

I started this journey when I was searching for a scholarship to continue my study after I had an experience teaching mathematics and found myself did not know much about mathematics and how to teach it (a former student even said that I taught biology than mathematics since the whiteboard was full of notes, like a biology teacher did). From the internet, I found an opportunity and thenbecame interested to apply it. The scholarship was on mathematics education and named:IMPoME (International Master Program on Mathematics Education).

I felt like the requirements were at my reach and I wanted to apply it, but it was not an instant decision for me to decide if I was going to apply it for real. I still had a doubt and was not confident considering my background was not on education. I tried to have a conversation with my former lecturer whom I had experienced working with before I graduated: Dr. Nana NawawiGaos. He said to me: "You will do not know if it is what you want if you do not try to do it." Then, after that meeting, I gathered my confident and started to collect the application documents. For this, I thank to him for the encouragement although: "Pak, I think, I probably still only get another 'paper' from another university."

On the beginning of March 2010, after a long process of collecting the documents and then finally could submit the application before the deadline, I gota call for the interview. I was interviewed by the chief of the program, Prof. Sembiring, and I guessed it went well since I got the scholarship! For this, I thank to PMRI team (which IMPoME was part of their program), especially PakSembiring for the chances.I know I feel like I am letting all of you down since I took a long road to finish it and still not contribute on anything, but I am so grateful that you gave me that chance!

After being accepted as one of IMPoME students, I started the journey in Yogyakarta. For about four months, I enrolled in an IELTS preparation classes. Although most of the time I could not see the benefit of joining the classes, I met many new people with many different stories and I also met the other IMPoME students for the first time here. Meeting them made me learn to see things from different perspectives. For this, I thank them for being a part of my journey; thanks for the laughter, the stories, and the friendship!

The real journey as anIMPoME student wasstarted when I enrolled classes in State University of Surabaya.For the first semester, I learned some things about mathematics and being introduced for the first time to mathematics education.The experience of learning was different from what I had felt when I wasan undergraduate students so that it also made me learn to see things from different perspectives. For this, I thank my lecturers: Prof. I KetutBudayasa, Ph.D, Prof. Dr. Siti Maghfirotun Amin, M.Pd., Prof. Dr. Dwi Juniati, M.Si., Dr. Agung Lukito, M.S., Dr. Tatag Yuli Eko Siswono, S.Pd., M.Pd., and Dr. Abadi, M.Sc. for being a part in my learning process.

I did not know if it was my luck because I never thought that I could get IELTS score that made me having the chance to enroll the second and third semesters in Utrecht University, The Netherlands. Thus, I also just saw it as my fate to learn more about seeing things from other different perspectives in another country. Although, honestly, I could not keep up quite well with the learning conditions there, but the experience I had when I was living there could open my eyes to see things that I never thought about or experienced before. Such an unforgettable experience!

For the experience of learning in Utrecht University, I thank to all of my lecturers (I got their full names from the university website, so I hope I did not make mistakes to address them): dr. M.L.A.M (Maarten) Dolk, M. J. (Mieke)Abels, drs. F.H.J (Frans) van Galen, dr. S.A. (Steven) Wepster, prof. dr. P.H.M (Paul)Drijvers, dr. E.R. (Elwin)Savelsbergh,dr. Dirk Jan Boerwinkel, drs. M. (Martin) Kindt, dr. Arthur Bakker, dr. H.A.A. (Dolly) van Eerde, ir. H (Henk) van der Kooij, and also Barbara van Amerom and Mark Uwland.

I am aware that I already mentioned their name, but I still want togive more sentences to express my gratitude since I thought I took their time more than it should be. To Marteen, who was the coordinator of the program, I saw that you tried to take care of us like a parent; you provided time to discuss and to find a solution for our problems, even the problems were not study related; I thank you for this. This experience also made me see a role of a teacher from another level. I know it was not only you who tried their best to provide time when their students needed it, but since you were the coordinator, I saw you acted more.

To Frans and Dolly, who supervised me when I was preparing my teaching experiment, I knew that I sometimes missed your classes since I was so confusedwith everything and only tried to solve those riddles by myself; I did not know how to explain the problems I encountered when preparing my experiment. However, I thank you for always being so patient, trying to approach me and giving your hand to help me. Moreover, I thank you for always providing time to answer my questions; even after I never contacted you inthe last three years and then I suddenly sent you emails asking for help, both of you still welcomed me, responded it,  and always encouraged me; I thank you for this, it meant a lot!

Returning back to Indonesia, I conductedtheexperiment. I was so lucky to find a school that was willing to provide time so that I could try out my designs. Although I encountered many difficulties and was not quite enjoying the process and then made the experiment developed so dreadfully, this work was valuable for me, as it also my first experience, so that I tried my best to present it on this thesis. For this, I thank all students and the teacher, Ibu Haning, who helped me and participated in the experiment; without their participation, this thesis could probably not exist.

When conducting the experiment and writing this thesis, I was supervised by Dr. Agung Lukito, M.S. and Dr. Siti Khabibah, M. Pd. For this, I thank you for trying to help and encourage me. To Pak Agung, I know I have troubled you so much since I did not finish this thesis on time; I am sorry for that, and thank you for always providing time and being there when I encountered the administration problems. To Ibu Bibah, I remembered that I was afraid to meet you in the beginning; regarding the rumor. However, after I met you, although it was too late and only for a few meetings, I learned a lot from your questions, explanations, and also your life stories; I thank you for that.

Even I had finished writing this thesis, I still could not end this journey before I defended it in front the examiners: Prof. Dr. Mega T Budiarto, M.Pd, Dr. Manuharawati, M.Si, and Dr. Masriyah, M.Pd. For this, I thank you for taking the time to examine and assess my thesis.I was so afraid to defend this thesis since my thesis did not quite provide clear findings and give any fruitful results, or even contributed to an important thing in mathematics education. Therefore, thank you for letting me to finish this journey. Thus, yeay (!), I could get my master degree and end this journey so I could take my steps for another journey.

Last but not least, my gratitude is for so many special persons who have been there even before I started this journey: my close friends, my siblings, and, of course, my parents. Thank you for always be there and always trying to helpand understand me. You might not directly help me to finish this thesis, but your existence still making me keep being here. Also, especially for my parents, thank you for still supporting me financiallyafter the scholarship fund was ended in the fourth semester so I couldhave a chance toend this journey in the right way although it took a long time; I am sorry for the mess I have done.

In the end, I could only say that I am being grateful for everything happened in my life. This journey is truly made me learn to see things from other different perspectives. I am learning that enjoying the process is more important than only worrying about the result. I am learning that making mistake is fine because life is an on-going learning process.I amlearning that the truth is sometimes relative since different people have gone through different life experiences. Many things I could not seem to see, through this journey, I try to see it. I am learning many things that I never taught about or even I knew if they existed in this world. However, the most important is I am learning to cherish every moment and live in the moment so that I can enjoy this life! Thank you! ☺

# CONTENTS

## LIST OF TABLES

## LIST OF FIGURES

## LIST OF APPENDICES

# CHAPTER I

# INTRODUCTION

This chapter presents the introduction for the study elaborated in this report. The study reported is a part of anotherstudy, thus a preliminary remark is presented first. After that, background, aim, question and significance of the study elaborated in this report are presented.

## A. A Preliminary Remark

A study was conducted to support second-grade-students with learning basic multiplication facts by introducing some multiplication strategiesand rules. The end-goal of that study was to find out how the students use these strategies and rules to solve multiplication bare problems.In practice, that studydeveloped dreadfully andprovided extremely limited dataon showing students used the introduced strategies and rules. To put it simply, reporting the study on that topic was considered difficult.

Nonetheless, as a part of that aforementioned study, the study on introducing multiplication strategies using arrays provided slightly more data to be presented and reported.Therefore, this report attempts to focus on elaboratingthis topic. To depict a smooth overview and also to focus only on reporting the study on introducing multiplication strategies using arrays, only some materials of the aforementioned study are chosen to be reported.

The background, aim, question, and significanceof the study elaborated in this report, which is the study onintroducing multiplication strategies using arrays, are presented in the following subchapters.

## B. Background

### 1. Students Have to Learn Multiplication

Multiplication serves as a foundation of higher-level mathematics topics, such as division, ratio, fraction, decimal, etc(Wong & Evans, 2007) and thus elementary students have to learn it (Chapin, 2006). However, most of them frequently find multiplication to be a hindrance in their mathematical progress (Wong & Evans, 2007) because they encounter difficulties on memorizing multiplication tables (Wallace & Gurganus, 2005).

Based on this issue, in order to explain why students are having difficulties on memorizing the multiplication tables, how multiplication facts usually being taught are briefly explained below.

### 2. How the Basic Multiplication Facts Being Taught

Students are usually given the multiplication tables and then are asked to practice the facts by writing down the series of numbers, "looking at them", reciting them, or listening to tapes, in order to memorize the facts (Steel & Funnell, 2001). However, this instruction is not an effective way (Woodward, 2006; Caron, 2007) since there are students who had mastered it during the third grade performing poorly in the following years (Smith & Smith, 2006).

Based on this issue, how multiplication should be taught is briefly explained below.

### 3. How Multiplication Should Be Taught

In earlier times, Ter Heege (1985) mentioned that students need to increase their skills of calculating multiplication using strategies, such as doubling or deriving facts,before memorizing the basic multiplication facts. This instruction will support them to acquire a flexible mental structure of the facts instead of a collection of rules. Another studyalso mentioned that students gain stronger concept of multiplication understanding through learning strategies (Wallace & Gurganus, 2005).

**4. Premise 1**

From the explanations in Subchapters 3, regarding the issues mentioned in Subchapter 1 and 2, learning multiplication strategies could be a solution to overcome students' difficulties on multiplication topic, especially memorizing multiplication tables. Moreover, a study showed how adults solve single-digit multiplication problems not only by retrieving the answer from a network of stored facts but also using rules, repeated addition, number series, or deriving facts (LeFevre, Bisanz, Daley, Buffone, Greenham, & Sadesky, 1996).

Therefore, learning multiplication strategies will give benefits for students since it helps on gaining stronger concept of multiplication understanding and later on memorizing the basic multiplication facts.

**5. Condition in Indonesia (1)**

The similar condition, as presented in Subchapter 1 and 2, also occur in Indonesia showing most of the elementary students fail to do multiplication multi-digit because they lack of memorizing basic multiplication facts(Armanto, 2002).However, based on several mathematics text books (Purnomosidi, Wiyanto, & Supadminingsih, 2008; Anam, Pretty Tj,

&Suryono, 2009; Mustoha, Buchori, Juliatun, & Hidayah, 2008), second-grade-students are firstly introduced to the concept of multiplication as repeated addition.

## 6. Conclusion 1

Based on the explanation in Subchapter 5,similar conditions regarding to the students encountering difficulties on memorizing multiplication tables also occur in Indonesia. But, there is a possibility that second-grader-students in Indonesia can use repeated addition as a strategy to solve multiplication bare problems, as they are taught about repeated addition as multiplication in the beginning of learning.

However, although the students probably learn about a multiplication strategy before memorizing multiplication tables, this strategy is probably the only strategy introduced. Yet, as mentioned in Subchapter 3, there are other multiplication strategies that could be introduced. Therefore, by considering the premise in Subchapter 4, there is a need to design education materials to introduce other multiplication strategies.

Based on this conclusion, in order to design the educational materials, an approach on how to design educational materials is chosen and briefly explored below.

## 7. Realistic Mathematics Education (RME)

Teaching in mathematics education has shifted away from “teaching by telling” toward “learning as constructing knowledge” (Kroesbergen & Van Luit, 2002; Gravemeijer, 2010). Therefore, the educational materials should not be designed for the teacher to tell or show multiplication strategies directly, but to let the students discover the strategies. Based on this idea, RME is a suitable approch to use (Cobb, Zhao, & Visnovska, 2010).

RME is a domain-specific instructional theory that provides an approach on how mathematics should be taught based on the view that students construct their own mathematical knowledge (Van den Heuvel-Panhuizen, 1996; Gravemeijer, 2008). One of the essential features of RME is the didactical use of models (Van den Heuvel-Panhuizen, 2003). Models are seen as representation of problems situations that could elicit students' informal strategies (Van den Heuvel-Panhuizen, 2003).

## 8. Premise 2

Based on the conclusion in Subchapter 6 and the explanation in Subchapter7, the educational materials are designed to introduce multiplication strategies and RME suggestschoosing a multiplication model so that it could represent situation of problems and also elicit the use of strategies naturally. Therefore, a multiplication model ischosen and briefly explained in the following subchapter.

## 9. Arrays as Multiplication Models

Arrays are one of the representation models of multiplication that support students to visualize multiplication strategies (Chinnappan, 2005). Calculation using strategies also occurs when multiplication problems presented in arrays (Barmby, Harries, Higgins, & Suggate, 2009). For long-term use, arrays can be useful model for enhancing students' understanding of multi-digit multiplication and other mathematics topic (Young-Loveridge & Mills, 2009).

### 10. Condition in Indonesia (2)

Based on several mathematics text books, ‘group of’ model is commonly used as a model of multiplication (Purnomosidi, Wiyanto, & Supadminingsih, 2008; Anam, Pretty Tj, & Suryono, 2009; Mustoha, Buchori, Juliatun, & Hidayah, 2008). Whereas, a study conducted in Indonesia showed instruction using arrays can better help students understand the concept of multiplication (Tasman, 2010).

### 11. Premise 3

Considering the explanation in Subchapter 9 and 10, arrays are the preferable models to use for introducing multiplication strategies.

### 12. Conclusion 2

Based on the conclusion in Subchapter 6 and the premise in Subchapter 11, there is a need for educational materials to introduce multiplication strategies using arrays. In order to that, a study is conducted with an aim presenting below.

## C. Research Aim

Considering the issues mentioned in the background, this study aims to develop educational materials for introducing multiplication strategies using arrays and thus this study contributes to the development of a local instructional theory in multiplication, especially on introducing multiplication strategies using arrays.

With the aforementioned aims, a research question is defined below.

## D. Research Question

Based on the research aim, a research question of this study is: "*How can arrays support students in learning multiplication strategies*?"

## E. Definition of Key Terms

To minimize differences of perceptions, some key terms used in the title, research aim, and research question are described below.

(1) To introduce

To help someone experience something for the first time.

(2) Educational Materials

Equipments that are neededfor an educational activity.

(3) Multiplication strategies

Ways to solve multiplication bare problems.

(4) Arrays

A rectangular arrangement consists of units in rows and columns.

(5) Local Instruction Theory

Educational materials and its instruction with envisioned of students' possible answers to the materials that have been tried out (summarized from Armanto (2002) and Gravemeijer K (2004)).

## F. Research Significance

As mentioned in the research aim, this study develops educational materials on introducing multiplication strategies using arrays and local instructional theory on introducing multiplication strategies using arrays.

Therefore, this studyprovides an overview on how the educational materials being designed and tried out and also a grounded local instructional theory on introducing multiplication strategies. Moreover, with these overviews and theories, this study can provide inputs for educators or researchers who want to introduce multiplication strategies using arrays.

# CHAPTER II

# THEORETICAL FRAMEWORK

This chapter presents the theoretical frameworks that serve as foundations on developingthe educational materials for introducing multiplication strategies using arrays.

## A. Students' Starting Points

As mentioned in Chapter1, this study aims to design educational materialsto introducemultiplication strategies. Since learning multiplication strategies needs to be conducted before students memorize the basic multiplication facts, this study aims for second-grade-students in Indonesia. Therefore, the assumption of students' starting points are: students have been introduced to multiplication and see multiplication as repeated addition and vice versa.

## B. Strategies to Be Introduced

As mentioned in Chapter1, this study aims to introduce multiplication strategies. Therefore, there is a need to find out about the strategies and then later choose which ones to be introduced.

Studies reported that there are more strategies students used, instead of using repeated addition, to solve multiplication bare problems (Ter Heege, 1985; Heirddsfield, Cooper, Mulligan, & Irons, 1999; Wallace & Gurganus, 2005; Watson & Mulligan, 1998; Sherin & Fuson, 2005; LeFevre, Bisanz, Daley, Buffone, Greenham, & Sadesky, 1996).

Ter Heege (1985) described six informal strategies students usedto calculate the basic multiplication problems. The strategies are:

(1) Applying the commutative property: $6 \times 7 = 7 \times 6$;

(2) Making use of the fact that multiplying by $10$ is so simple: $10 \times 7 = 70$, that is $7$ with a $0$ added.

(3) Calculating certain products by doubling: use $2 \times 7 = 14$ as a support to calculate $4 \times 7$ by doubling $14$.

(4) Halving familiar multiplications: use $7 \times 10 = 70$ as a support to calculate $5 \times 7$ by halving $70$.

(5) Increasing a familiar product by adding the multiplicand once: use $5 \times 7 = 35$ as a support to work out $6 \times 7$ by calculating $35 + 7$.

(6) Decreasing a familiar product by subtracting the multiplicand once: use $10 \times 7 = 70$ as a support to work out $9 \times 7$ by calculating $70 - 7$.

From these strategies, the doubling (3), the one-more (5), and the one-less (6) strategies could cover almost all calculation to determine multiplication facts in the table. For example, in multiplication table of $8$, multiplication $3 \times 8$, $4 \times 8$, $6 \times 8$, $7 \times 8$, $8 \times 8$, and $9 \times 8$ could be determined by doubling the product, adding the multiplicand once (one-more), or subtracting the multiplicand once (one less) of multiplication $1 \times 8$, $2 \times 8$, $5 \times 8$, or $10 \times 8$.

Therefore, introducing those three strategies could give benefit for students, especially when they need to memorize the basic multiplication facts. Other researchers also mention that students use doubling strategy effectively (Braddock, 2010) and adults use deriving facts (LeFevre, Bisanz, Daley, Buffone, Greenham, & Sadesky, 1996) when solving single-digit multiplication. Here, one-less and one-more strategies are considered as deriving strategies.

Besides those three strategies, understanding the commutative property also gives more benefit because students could cover almost half of the multiplication tables by applying this property. For this study, determining a product using this property is seen as a strategy. Therefore, the strategies being introduced are the commutative property, the doubling, the one-more, and the one-less.

## C. Array as Multiplication Models

As mentioned in Chapter1, arrays are used as the models to introduce multiplication strategies. Therefore, some information about the use of arrays as multiplication models is briefly explored below.

Arrays are powerful models of multiplication since they could represent the idea of multiplication nicely and visualize the idea of factors, grouping, properties of multiplication (commutative, associative, and distributive) andmultiplication algorithm(Fuson, 2003; Chinnappan, 2005). Moreover, calculation using and visualization of strategies occurs when multiplication problems presented in arrays (Chinnappan, 2005; Barmby, Harries, Higgins, & Suggate, 2009).

However, arrays are often difficult for some students to understand since there are students who could fail to see an object in a row and a column simultaneously (Battista, Clements, Arnoff, Battista, & Van Auken Borrow, 1998). Therefore, there is a need to pay attention to this situation.

Wallace & Gurganus (2005) mentioned that an array contains a specified number of items that is repeatedly arranged a given number of times in rows and columns. This means array models also represent repeated addition. Therefore, since the students are assumed have learned multiplication as repeated addition, a guidance showing repeated addition in arrays could be used to help students who fail to see an objects simultaneously in a row and a column.

## D. Using RME to Design the Materials

As mentioned in Chapter1, RME is the approach used in designing the educational materials. Therefore, the materials should represent the characteristics of RME (Zulkardi, 2010). These characteristics reflect ideas that could be used to help students construct their own knowledge (Van den Heuvel-Panhuizen, 2000; Gravemeijer, 2008). The characteristics are:

(1) The use of contexts

In RME, students are given problems in contexts so that they could come up and develop mathematical tools and understanding the mathematical concepts by themselves. Therefore, the contexts are needed to be close to students’ reality.

(2) The use of models

RME uses models as the representation of problems situations. A model serves as an important device for bridging the gap between informal (context-related) and more formal mathematics. In the beginning of lessons, a model aims to elicit and develop students' informal strategies.

(3) The use of students own productions

RME treats students as active participants instead of being receivers of ready-made mathematics. Thus, students are confronted with problem situations. Through these problems, they have the opportunity to develop all sorts of mathematical tools and insights by themselves.

(4) The interactive character of the teaching process

RME sees teacher-student or student-student interaction as a means that could help students to construct their mathematical understanding. When working on problems, students need to be invited to explain and to discuss their ideas, strategies, and struggles. They also are expected to justify their own answers so that they could reflect on what they are doing in the end of lessons.

(5) The intertwining of various learning strands

RME sees mathematics topics as a unity. Various subjects are supposed not to be taught separately or neglecting the cross-connection.

All these characteristics are better explained when they are showed in an example. Therefore, some problems designed based on RME approach are briefly explained below.

## E. Examples on RME Materials

Van Galen & Fosnot (2007) designed some materials using arrays as models that could elicit the use of multiplication strategies. They mentioned that covering some parts of objects presented in arrays could provide a built-in constraint to counting one by one and also to support doubling (Figure 2.1). For example, in the unfolded curtain at the top right, there are four rows of three diamonds visible, but when the other curtain are drawn shut, there will be four rows of six. To determine the total diamonds, the result of $4 \times 6$ is double $4 \times 3$. The window with a cat provides a visible $4 \times 4$ array, but students need to determine $4 \times 8$. Meanwhile, the fourth images offers $2 \times 7$ as an anchor fact to determine $4 \times 7$.

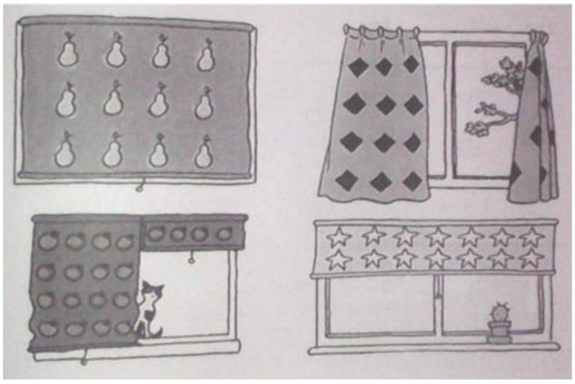
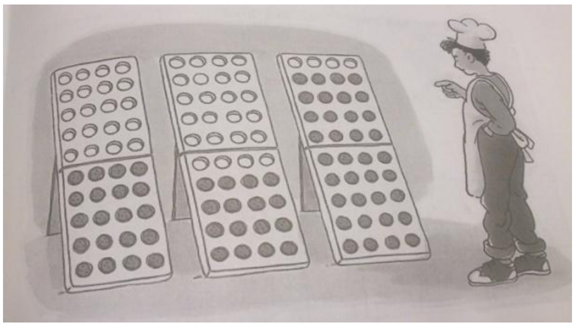


Figure 2.1: An example of RME materials to support doubling.

They also designed a material that supports the use of multiplication by five and ten as anchor facts and also one-less strategy (Figure 2.2). The problem shows three backing trays and asks students to determine the total cookies left in the trays. To determine the cookies in the first tray, they could use $10 \times 4$ as an anchor fact to determine the product of $5 \times 4$ that represents the total cookies in the first tray. To determine the cookies in the second tray, they could realize that the total cookies is one less row from the first tray and thus they could subtract the product of $5 \times 4$ represent the total cookies in the first array with the total cookies in a row; this strategy is called one-less.

Meanwhile, to determine the total cookies in the third array, they could add the total cookies in the first and second arrays or derive the product of $10 \times 4$ using the one-less strategy.



(b)

Figure 2.2: An example of RME materials to support one-less.

Besides providing materials, they also explained on how to conduct the lessons. They suggested the teacher asks the students to imagine the problems before mentioned the questions. They put emphasize on inviting students to discuss the strategies to solve the problems. For these two problems, they noted that students mainly used skip-counting (or on writing it is called as repeated addition) before they realize the idea of doubling or the one-less/one-more.

For this study, these materials give substantial influences in the process of designing the materials for introducing multiplication strategies usingarrays. Together with inputs in the previous subchapters, the results of designing the materials are explained below.

## F. Conclusion: Educational Material Designs

The educational materials for introducing multiplication strategies using arrays are designed considering the inputs described in the previous subchapters. Based on that, lesson sequence, contexts, problems, conjectures of students' thinking and learning instruction are constructed and explained in the following subchapters.

**1. Lesson Sequence**

As mentioned in SubchapterB, the multiplication strategies introduced in this study are the commutative property, the doubling, the one-more, and the one-less. Also, as mentioned in Chapter 1, these strategies are introduced using arrays as the multiplication models and thus are considered as a new representation for the students. Therefore, there is a need to introduce arrays as the representation of multiplication before introducing multiplication strategies using arrays.

When representing multiplication, an array could represent two forms of multiplication where the multiplicand and multiplier are interchangeable. That means the total number of objects in a row could simultaneously represent as multiplicand or multiplier, and so could a column. In order to give the flexibility to see these two representations, the use of commutative property is introduced after introducing arrays as the representation of multiplication.

Also, based on the input described in SubchapterB, students effectively use doubling as a strategy to solve multiplication problems. Therefore, for Lesson 3, the idea of doubling as a multiplication strategy is introduced. Then, the idea of one-less and one-more as multiplication strategies are introduced in the same lesson, which is in Lesson 4, since these strategies share a similar idea: add/subtract one row of an array.

Therefore, there are four lessons conducted in this study. Lesson 1 is about introducing arrays as multiplication models. Then, respectively, Lesson 2, 3, and 4, aim to introduce the commutative property, doubling, and one-less/one-more as multiplication strategies.

**2. Contexts**

For this study, eggs, toy cars, and stickers are chosen as the contexts considering that they are close to students' reality. Eggs could be found easily in traditional and modern markets. Meanwhile, toy cars and stickers are sold in mini- markets or supermarkets. Also, the arrangements of these contexts are usually presented in arrays. Therefore, thestudents could easily imagine the situation of the problems.

**3. Mathematical Problems and Conjectures of Students' Thinking**

As mentionedpreviously, thereare four lessons constructed. Lesson 1 is about introducing arrays as multiplication models and Lesson 2, 3, and 4 are for introducing multiplication strategies. For every lesson, there is a mathematical problem given. The idea of problems is to present an array and asking students to determine the total number of objects in it. The problems are designed based on the aforementioned inputs in previous SubchaptersA, C, D, and E. Through the problems, it is expected that the idea of arrays as multiplication models and multiplication strategies could be elicited. The explanation about each problem in every lesson is described below.

**a. Lesson 1**

Figure 2.3 shows the material designed for introducing arrays as multiplicationmodels. The problem asks students to determine the total number of eggs in a box. The eggs are put in a box in $9 \times 10$ arrangement. These numbers are chosen since this multiplication is represented in a quite big array, but the product could be determined easily as repeated addition of $10$.


Figure 2.3: Material design for Lesson 1.

Some eggs in the box are covered to minimize the use of counting one by one and focus on seeing the array as a multiplication representation. The eggs in the top and left sides are uncovered as its total represents the multiplicand and multiplier of the multiplication represented in the array. The eggs in the right and bottom sides are also uncovered in order to inform the students that there are eggs below the cover.

When working on this problem, although most of the eggs are covered, there are students who will try to count through all eggs one by one by visualizing the covered eggs in their mind. However, since the eggs are put in a quite big arrayand also most of them are covered, the students are expected to fail getting the correct answer so that they need to learn how to find the multiplication represented in the array.

From that stage, only able to use counting one by one strategy, the students need to be guided to realize that every row has the same total numbers of eggs.After realizing it, they are expected will write repeated addition to determine the answer. For students who realize that repeated addition could be represented as multiplication, they will reform the repeated addition to its multiplication and then find the product using repeated addition.

After the students got the multiplication in their answer, to help themmore picturing the idea of arrays as multiplication representations, the teacher needs to ask two related questions: " How many objects in a row?" and "How many objects in a column?" The answers represent the factors of the multiplication. Thus, with these questions, the students will associate the answers to the multiplication factors so that they couldsee the idea of multiplication represented in the array.

For students who have better understanding that multiplication could be used to determine total number of objects, they will only try to find the total number of objects in a row and also in a column. Then, to find the answer, they put those two numbers into a multiplication form and find the product of it.

**b. Mathematical Problem for Lesson 2**

Figure 2.4 shows the material designed for introducing the commutative property of multiplication. The problem asks students to determine who has more collection: Race or his brother. Since the commutative property is represented by two different structured arrays presentingthe same total number of objects, uncovering all objects in the arrays is expected to help students focusing to visualize this idea.

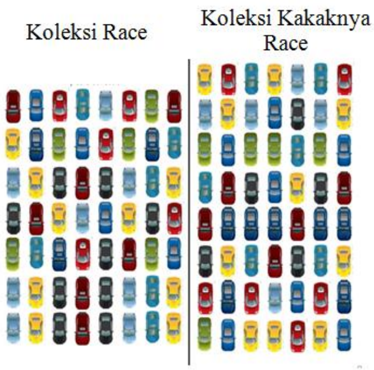



Figure 2.4: Material design for Lesson 2.

Although there is no input mentioned about this before, in order to confuse students if they use counting one by one and also to implicitly force them to find the multiplication in the arrays, the objects presented are chosen to be rectangle-ish shape. Therefore, toy car figures are chosen to be the objects presented in the arrays. For Race's collection, the toy cars are arranged in $7 \times 8$, and his brother are in $8 \times 7$.

When working on this problem, since the students have learned to find multiplication in an array in the previous lesson, they are expected to find the multiplication in each array directly and then find its product. By finding the products, they will realize that two multiplicationswith the same factor are having the same product.

For the students who see that the total number of rows and columns in those two arrays are interchangeable, they could conclude that the total numbers of toy cars are the same without determining the product of each multiplication. However, there will be students who will still try to count one by one or repeated addition.

**c. Multiplication Problem for Lesson 3**

Figure 2.5 shows the material designed for introducing the idea of doubling as a multiplication strategy. The problem asks students to determine the total number of stickers in a special package; the package covers some stickers. Some stickers are covered because it was mentioned in SubchapterEthat covering some part could support the use of doubling. However, in order to tell students that there are stickers below the package, stickers in a column are exposed.

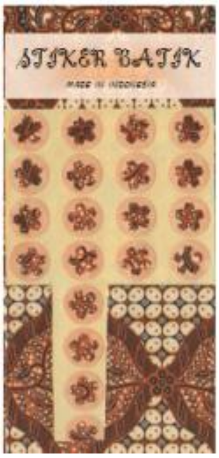


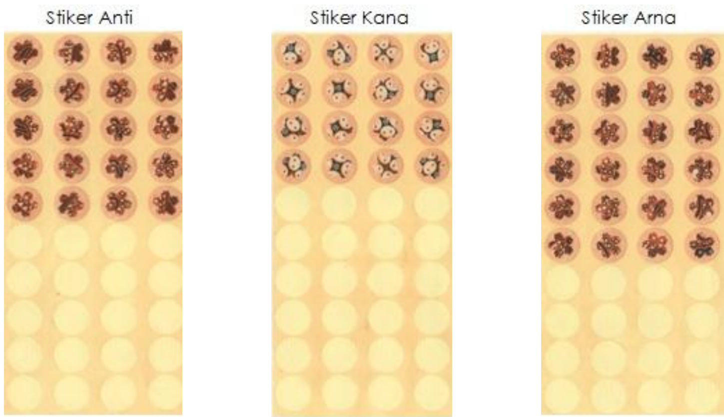

Figure 2.5: Material design for Lesson 3.

The stickers are divided in two equal parts: uncovered and covered. All stickers are arranged in $8 \times 4$ and all stickers in uncovered parts are in $4 \times 4$. These numbers are chosen since multiplication by 8 is considered not easy to determine, but multiplication by 4 could be seen as easier facts to determine.

Therefore, students are expected to use the product of multiplication by 4 and then double it to determine the product of multiplication by 8.

**d. Multiplication Problem for Lesson 4**

Figure 2.6 shows the material designed for introducing the idea of one-less/one-more as a multiplication strategy. The problem asks students to determine the total number of stickers in three different packages. The multiplication represented in the first package serves as the anchor fact and so the multiplication products in the other two packages are determined from the anchor fact.




(a) (b) (c)

Figure 2.6: Material design for Lesson 4.

This material could actually support the use of multiplication by 10as anchor facts to determine the product of multiplication by 5 and also the use of halving strategy. However, for this study, the use of multiplication 5 as anchor facts is emphasized more in determining the multiplication products represented in the last two arrays.

Multiplication $5 \times 4$ is chosen as the anchor fact to determine the product of multiplication $4 \times 4$ and $6 \times 4$. The anchor fact is chosen as this multiplication is considered as an easier fact to determine, since it is a multiplication by five. Also, through the visualization of the arrays, the students are expected to obtain the other two products easier using the idea of the one-less/one-more strategy.

For this study, the idea of one-less/one-more strategy is introduced by showing that the other two arrays are either one row less or one row more from the first array. Therefore, when determining the total number of stickers in the last two arrays, students are expected to add or subtract the total number stickers in a row to the total number of stickers in the first array. In order to show this idea explicitly, the stickers in the arrays are chosen to be uncovered.

**4. Instructions**

Before students are given the problems, the teacher needs to elaborate the contexts orsituations related to the problems. After that, in order to support students to construct their own productions and also students' interaction, students are expected to work in a small group. Each student gets its own worksheet to reflect their work. The teacher moves around to see how the students work and what struggles they are having when the students working on the problems.

Based on Subchapter 4, the students will not directly realize the idea of the multiplication strategies being introduced. Therefore, classroom discussions are conducted after the students finish their work. This discussion also intended to foster interaction in the learning process. The teacher orchestrates the discussion and decides which group needs to explain first. The groups that could get the idea of the introduced strategies are expected to present their solution in the last order.

As mentioned in the previous SubchapterC,there is a condition where students could not see an object simultaneously in a row and a column. The guidance to represent repeated addition in arrays firstly and then progress to multiplication isconsidered as the acceptable move to overcome this condition. This guidance also could be used if there are students who still not get the idea of repeated addition as a multiplication strategy.

## G. Remark

All aforementioned ideas and designs are combined together as educational materials for introducing multiplication strategies using arrays. In order to try them out, explanations of the materialspresented in ChapterIV as guidance to conduct theexperiment.

# CHAPTER III

# METHODOLOGY

This chapter describes the methodological aspects on how to conduct this study.

## A. Design Research as a Research Approach

The aim of this study is to develop educational materials for introducing multiplication strategies using arrays and also to find out how these materials support the students' learning. For this purpose, design research appears as an appropriate methodology to be used(Gravemeijer, 2004; Gravemeijer & Cobb, 2006; van Eerde, 2013).

There are three phases to conduct an experiment using design research, namely: 1) preparing a design experiment, 2) conducting a design experiment, and 3) carrying out a retrospective analysis (Gravemeijer K., 2004; Gravemeijer & Cobb, 2006; van Eerde, 2013). These three phases serve as a cycle. For this study, there were two cycles conducted for two different purposes.

| Cycle 1 | | | |
|---|---|---|---|
| Date | | | Activity |
| 13 | February | 2013 | Pretest |
| 14 | February | 2013 | Interview |
| 20 | February | 2013 | Lesson 1 |
| 06 | March | 2013 | Lesson 2 |
| 06 | March | 2013 | Lesson 3 |
| 07 | March | 2013 | Lesson 4 |

| Cycle 2 | | | |
|---|---|---|---|
| Date | | | Activity |
| 18 | March | 2013 | Classroom-Observation |
| 19 | March | 2013 | Pretest |
| 20 | March | 2013 | Interview |
| 21 | March | 2013 | Lesson 1 |
| 25 | March | 2013 | Lesson 2 |
| 27 | March | 2013 | Lesson 3 |
| 27 | March | 2013 | Lesson 4 |
| 01 | April | 2013 | Posttest |

Table 3.1: Experiment timeline.

Design experiment in Cycle 1 acted as a preliminary experiment for adjusting the educational materials and the conjectures of students' learning. Meanwhile, the design experiment in Cycle 2 was conducted as the main experiment to answer the research question. These two design experiments were conducted in SD Lab UNESA Surabaya in February and March 2013 (Table 3.1).

In the subchapters below, the explanations on who participated in each cycle and how each phase conducted are described.

## B. Participants

The experimental participants were the second grader in SD Lab UNESA Surabaya; class 2A and 2B. There were six students from class 2B: Fira, Mitha, Vina, Samuel, Prima, and Mike, who were chosen by the homeroom teacher and initially intended to participate in all activities of Cycle 1. However, two students (Prima and Mike) were hardly to manage so they did not take part after Lesson 1 and their work did not count in the analysis process.

All of students in class 2A were initially intended to participate in Cycle 2 but some students were absence during some lessons in the experiment. The students were also assumed to work in a pair but they moved to a different class so the layout of the seating adjusted the total number of students working together. There was a group chosen as the focus group.

For Lesson 1 and 2, the students worked in a group of three. The students in the focus group were Divan, Ranuh, and Rizal. The members of the group were not chosen intentionally since the researcher thought that students would act as usual when they worked with their own close friends. The researcher only chose a student, i.e. Divan, and then the other two students were instantly being the member of the group.

Divan was chosen based on the student interview finding that showed he used repeated addition as a strategy, instead of only writing it as a confirmation. He also was not familiar to multiplication tables. Therefore, it was assumed that he would make use of the strategies after it was being introduced. However, the interaction among these focused group students was not enough to show how they worked on the problems.

To overcome this situation, the researcher asked the teacher to merge this focused groupwith another group sat behind them, and so werethe other groups in the class. This decision was made since the group sat behind the focused group was assumed more active. Therefore, for Lesson 3 and 4, students worked in a group of six. The students in the focus group were Divan, Rizal, Mazta, Satria, Faiz, and Hamed.

## C. Preparing the Design Experiment of Cycle 1

This phase was a section devoted to do literature review, formulate research aim and question, and develop a hypothetical learning trajectory. The explanation of activities in this phase is presented below:

### 1. Carrying out the literature review

Once a multiplication topic was chosen, literature about multiplication teaching, multiplication strategies, multiplication problems, multiplication models (especially arrays), and Realistic Mathematics Education (RME) were studied to determine the research aim, the research question and mainly to get the ideas of the educational materials would be.Furthermore, literature about design research, as the research approach, was also studied.

The work resulted from this activity is presented in Chapter I, II, and III.

**2. Developing the Hypothetical Learning Trajectory (HLT)**

During a design study, a HLT plays an essential instrument: after it is being developed, it is used in carrying out the experiment and then serve as guidance on conducting the analysis process (Van Eerde, 2013). A HLT is a prediction on how learning might proceed and it consists of: students' starting points, learning goals, mathematical problems, and hypotheses on learning process (Simon, 1995; Van Eerde, 2013). For this study, the mathematical problems were included on the explanation of the hypothesis on learning process.

In order to introduce multiplication strategies using arrays, there are four lesson designed in this study. Lesson 1 aims to introduce arrays as models to represent multiplication. Meanwhile, Lesson 2, 3, and 4 lessons attempt to introduce the multiplication strategies: the use of commutative property, the doubling, and the one-less/one more. The content of the HLTdeveloped based on literature reviewpresented in Chapter II and the HLT itself is presented in Chapter IV.

**3. Assessing the Actual Students' Prior Knowledge**

Since the HLT was developed only based on literature review, there was a need to find out the consistency between the students' prior knowledge and the students' starting point in the HLT. There were three preliminary activities conducted to find out the students' prior knowledge: pretest, student interview, and teacher interview. These activities were conducted in three different days.

All students, who were intended as the experimental students, took part in the pretest; their worksheets were collected. After the pretest, the students were interviewed in a semi-structured style; this activity was video-recorded and students' calculation scratches were also collected. After the pretest and student interview, their mathematics was interviewed in a semi-structured style to verify the findings. Some field notes were taken during the process. There was no video collected in order to make the teachers felt comfortable answering the questions.

All the data collected were analyzed. The strategies students used in the pretest were listed and then were confirmed with the students' explanation in the interview. Through these two activities, the conclusion about students' prior knowledge was taken. This conclusion was also verified by the teacher's clarification on what multiplication topic had been taught. The explanation of the retrospective analysis from this activity is presented in Chapter V.

For this study, the conclusion generated in the preliminary activities did not ask for a modification on the HLT. Therefore, the HLT developed from the literature review was directly used to conduct the experiment in Cycle 1.

## D. Conducting the Design Experiment of Cycle 1

The experiment in Cycle 1 served as the preliminary experiment that functioned for adjusting the HLT before implementing it in the main experiment. The readability of the problems and the conjectures of the students' thinking were tested. The researcher conducted the whole teaching-learning process in Cycle 1 using the HLT as guideline.

To collect the experiment data, a static video recorder was placed in front of the classroom to record the whole teaching-learning process of the group. Another video recorder was also used to record some interesting situations, or to conduct small interviews asking on how the students worked. The students' worksheet and field notes were also collected.

## E. Carrying out the Retrospective Analysis of Cycle 1

In design research, hypotheses are tested continuously during the experiment which makes it enable to change some or all components in a HLT in order to adapt the real situation (Simon, 1995; Van Eerde, 2013). Therefore, a retrospective analysis was conducted not only after a cycle ended but also after a lesson conducted.

For this study, a brief analysis was conducted to adjust the next activities based on the situation happened in the previous lesson. Then, after all lessons were conducted, the retrospective analysis was carried out to adjust some or all components in the initial HLT. The analysis was focused on how the students read the problems and what other strategies occurred.

In the beginning of the analysis, the videos were watched to compare the actual classroom practice and the hypotheses presented in the HLT. Fragments contained students' conversation and gestures when solving the problems were noted. These fragments were traced back over and over to confirm the findings. Any situations related to the way of students' solving the problem were taken into consideration, whereas the non-relevant parts were ignored.

The analysis was also supported by the written data collected: students' worksheet and field notes. Selected fragments were transcribed to support the findings. The conclusion was generated from the findings that were relevant to the aim of the experiment. This conclusion decided the need for a modification to the HLT. The findings about strategies students used which were not in the HLT were included in the revised HLT.

The explanation of the retrospective analysis is presented in Chapter V. For this study, the findings in this cycleasked for a revision of the HLT. The modification was conducted in the phase of preparing the design experiment for Cycle 2.

## F. Preparing the Design Experiment of Cycle 2

For this study, as the result from the retrospective analysis in Cycle 1, there was some revision and addition on some parts of the initial HLT. This revision was executed after conducting the preliminary activities for Cycle 2 in order to find out about the students' prior knowledge that could contribute to the modification process. The activities were classroom observation, pretest, student interview, and teacher interview. All these activities were conducted in different days.

The classroom situation was video-recorded during the observation, in order to get insight on how the mathematics teaching-learning usually occurred. Some field notes were taken during the process. Meanwhile, for the pretest, student interview, and teacher interview, they were conducted with similar method for collecting and analyzing data in Cycle 1.

All students in the experimental class participated in the pretest. Six students (Aia, Shafa, Divan, Falah, Krishna, and Nabil) were selected to explain their pretest solution in an interview. The interview was to confirm if the students used repeated addition as a strategy to determine the multiplication products oronly as an explanation to the facts that they remembered. After the pretest and student interview, their mathematics teacher was also interviewed to verify the findings.

The retrospective analysis of the preliminary activities is presented in Chapter V. These findings, together with the experiment findings in Cycle 1, contributed to the modification of the HLT. The revised version of the HLT is presented in ChapterIV.

## G. Conducting the Design Experiment of Cycle 2

The experiment in Cycle 2 was a main design experiment that aimed to find out how the educational materials work and to answer the research question. In this cycle, the revised HLT was implemented in the real classroom situation. The mathematics teacher of class 2A conducted the whole teaching-learning process using the revised HLT as guideline. The researcher acted as observer, but could intervene the students' learning when they were stuck.

To collect the experiment data, a static video recorder was placed in front of the classroom. This video was focused to record all students' responds and activities, and the teacher's explanation. The position of the recorder was assumed could record the conversation and activities the students in the focus group. Another video recorder was also used to record some interesting situations, or to conduct small interviews asking the students' work. Besides video recording, the students' worksheet and field notes were also collected.

## H. Carrying out the Retrospective Analysis of Cycle 2

Similar method to analysis the experiment data in Cycle 1 was applied in this cycle. After all lesson were conducted, the retrospective analysis were carried out to answer the research question. The revised HLT was used as a guidance to compare the actual learning andthe hypotheses to find out how the materials work in practice. The analysis was focused on how the students work on the problem and how the materials support the learning goals.

The videos were watched in the beginning of the analysis process. The static video captured the whole teaching-learning activities so that the general classroom activities could be analyzed and noted. When the students worked in a group, the focus group's works were analyzed. However, the data taken from the moving andstatic video-recorded is limited and slightly unclear. Fortunately, these data still could be used to generate the conclusion of this study.

Through the videos, fragments that contained students' conversation and gestures when solving the problems were noted. These fragments were traced back over and over to confirm the findings. Any situations related to the way of students' solving the problem were taken into consideration, whereas the non-relevant parts were ignored.

The retrospective analysis was also supported by the written data collected: students' worksheet and field notes. Selected fragments were transcribed to support the findings. The conclusion was generated from the findings that were relevant on answering the research question. This conclusion is presented in Chapter VI. Meanwhile, the explanation of the retrospective analysis is presented in Chapter V.

# CHAPTER IV

# HYPOTHETICAL LEARNING TRAJECTORY (HLT)

This chapter presents the HLTs that were used to conduct the experiments in Cycle 1 and Cycle 2 of this study.

## A. Hypothetical Learning Trajectory (HLT) for Cycle 1

This HLT is constructed based on inputs presented in Chapter II, and elaborated for each lesson below.

### 1. Lesson 1: Introducing arrays as models to represent multiplication

#### a. Students' Starting Point

The students know multiplication as repeated addition and vice versa, and could use repeated addition as a strategy to solve multiplication bare problems.

#### b. Learning Goal

Finding multiplication represented in an array, the students are introduced to the idea of arrays as models to represent multiplication.

#### c. Hypothesized Learning Process

The teacher shows a picture of eggs with some are covered by a label (Figure 4.1); the eggs are in array of $9 \times 10$. The teacher then introduces 'How many eggs are there?' context: tell a story about a seller who cannot figure out the total number of eggs in a carton pack because some are covered by a label. The teacher then asks the students to help the seller to determine the total number of eggs in the carton pack because the seller does not want to unpack the eggs.


Figure 4.1: Mathematical Problem in Lesson 1 – Cycle 1.

After explaining the context and the problem, the teacher asks the students to work in a group and distributes the worksheet to each student. Regarding to the problem, there are several conjectures on how students answer the question:

(1) The students count through all eggs one by one while visualizing the covered eggs in their mind;

(2) The students find out that the total numbers of eggs in every row are the same and then:

   (a) use repeated addition:

   $$10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 = 90, \text{ or}$$

   (b) write repeated addition, reform it to multiplication, and find product of the multiplication,

   $$10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 = 9 \times 10 = 90;$$

(3) The students find out that there are $10$ eggs in a row and $9$ eggs in a column, and then put those two numbers into multiplication form and find the product of it, $9 \times 10 = 90$.

After the students finish and collect their work, the teacher conducts a classroom discussion. If there are students who count one by one, the teacher guides them to realize that the total numbers of eggs in every row are the same so they could use repeated addition. If the students have used repeated addition, the teacher asks them to reform it into its related multiplication. Lastly, the teacher encourages the students to find multiplication in arrangement by asking how many eggs in a row and in a column.

**2. Lesson 2: Introducing the use of commutative property**

**a. Students' Starting Point**

The students could find multiplication represented in arrays.

**b. Learning Goal**

Through the material designed, the idea of commutative property of multiplication could be elicited and introduced.

**c. Hypothesized of Learning Process**

The teacher shows a picture of two toy car arrangements of Race and his brother (Figure 4.2); the Race's are in array of 7 × 8 and his brother's are in array of 8 × 7. The teacher then introduces 'Who has more toy cars?' context: tell a story about Race and his brother who place their car toys in two different arrangements and are confused to decide who has more cars. The teacher then asks the students to help Race and his brother to decide who has more cars.

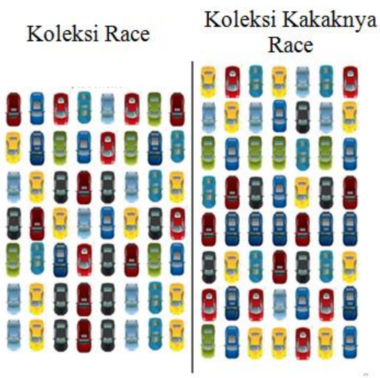



Figure 4.2: Mathematical Problem in Lesson 2 – Cycle 1.

After explaining the context and the problem, the teacher asks the students to work in a group and then distributes the worksheet to each student. Regarding to the problem, there are several conjectures on how students answer the question:

(1) The students count all cars on Race's and his brother's arrangements one by one, and then conclude that the total numbers of cars are the same.
(2) The students use repeated addition $(7+7+7+7+7+7+7+7=56)$ and $(8+8+8+8+8+8+8=56)$ to determine total numbers of Race's and his brother's car toys respectively and then conclude that the total numbers of cars are the same.
(3) The students find multiplication in Race's arrangement and determine the product of it $(7\times 8=56)$. They then find multiplication in Race's brother's arrangement and determine the product of it $(8\times 7=56)$. Then, since they find that the products of the multiplications are the same, they conclude that the total numbers of Race's and his brother's cars are the same.
(4) The students find the multiplication in Race's and his brother's array and realize the multiplication having the same factors $(8\times 7=7\times 8)$. Without determining the products, since the total number of rows and columns in those two arrays are interchangeable, they conclude the total numbers of Race's and his brother's cars are the same.

After the students finish and collect their work, the teacher conducts a classroom discussion. If there are students who count one by one or use repeated addition, the teacher guides them to find multiplications represented in the arrays. If the students havefound the multiplications, the teacher asks them to find the products together so that the students see that multiplications having the same factors will have the same products.

## 3. Lesson 3: Introducing the idea of doubling

### a. Students' Starting Point

The students could find multiplication represented in arrays.

### b. Learning Goal

Through the material designed, the idea of the doubling strategy could be elicited and introduced.

### c. Hypothesized of Learning Process

The teacher shows a picture of stickers with some are covered by the package (Figure 4.3); the stickers are actually in array of $8 \times 4$. The teacher then introduce 'How many stickers are there?' context: tell a story about a little girl who cannot figure out the total number of stickers in a special package because some stickers covered by the package. The teacher then asks the students to help the little girl to determine the total number of stickers in the special package.

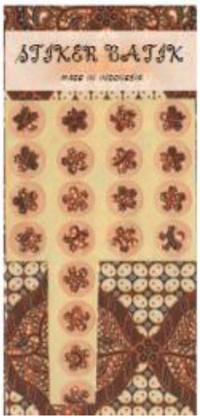



Figure 4.3: Mathematical Problem in Lesson 3 – Cycle 1.

After explaining the context and problem, the teacher asks the students to work in a group and then distributes the worksheet to each student. Regarding to the problem, there are several conjectures on how students answer the question:

(1) There will be students who still try to count one by one or use repeated addition although they have experienced find multiplication represented in arrays to determine the total number of images presented.

(2) The students find the multiplication represented the arrangement, $8 \times 6$. To determine the product, the students realize that the stickers are divided in two equally parts; half-uncovered and half-covered. The students could easily find the multiplication and product in the uncovered part; that is $4 \times 6 = 24$. Since the stickers are in two equally parts that means $8 \times 6$ is two $4 \times 6$ and the product of $8 \times 6$ is double of the product $4 \times 6$.

After the students finish and collect their work, the teacher conducts a classroom discussion. If there are students who count one by one or use repeated addition, the teacher guides them to find multiplication represented in the array by asking how many eggs in a row and in a column.If the students havefound the multiplications, the teacher discusses how the product of $8 \times 6$ could be derived from the known fact of $4 \times 6$using the doubling strategy.

**4. Lesson 4: Introducing theidea of one-less/one-more**

**a. Students' Starting Point**

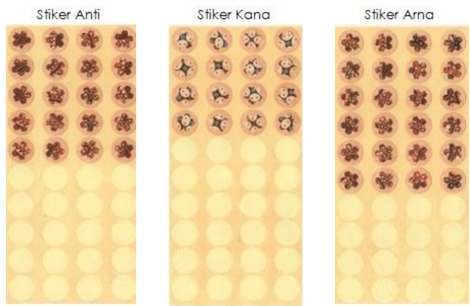
The students could find multiplication represented in arrays.

**b. Learning Goal**

Through the material designed, the idea of the one-less/one-more strategy could be elicited and introduced.

**c. Hypothesized Learning Process**

The teacher shows a picture of three sticker packages placed next to each other in array of $5 \times 4$, $4 \times 4$, and $6 \times 4$ respectively (Figure 4.4). The teacher then introduces 'How many stickers do I have?' context: tell a story about three girls: Anti, Kana, and Arna who have similar sticker packages and want to figure out the total number of their stickers. The teacher then asks the students to help the girls to determine the total number of each sticker left in the packages.




(a) (b) (c)

Figure 4.4: Mathematical Problem in Lesson 4 – Cycle 1.

After explaining the context and the problem, the teacher asks the students to work in a group and then distributes the worksheet to each student. Regarding to the problem, there are several conjectures on how students answer the question:

(1) There will be students who still try to count one by one or use repeated addition although they have experienced find multiplication represented in arrays to determine the total number of images presented.
(2) The students find multiplication represented in the arrays, $5 \times 4$, $4 \times 4$, and $6 \times 4$, and then to determine the product:
    (a) the students find every product of multiplications without using the previous known product of $5 \times 4$;

(b) the students use $5 \times 4 = 20$ as a known fact, and then use the one-less strategy to derive the product of $4 \times 4$ ($4 \times 4 = (5 \times 4) - 4 = 20 - 4 = 16$) and use the one-more strategy to derive the product of $6 \times 4$ ($6 \times 4 = (5 \times 4) + 4 = 20 + 4 = 24$).

After the students finish and collect their work, the teacher conducts a classroom. If the students still count one by one or use repeated addition, the teacher guides them to find multiplication represented in the arrayby asking how many eggs in a row and in a column. If the students have found the multiplications, the teacher then discusses how the total number Anti's sticker can be used as a known fact to derive the total number of Kana's and Arna's using the one-less/one-more strategy.

## B. Hypothetical Learning Trajectory (HLT) for Cycle 2

This HLT is a revised HLT constructed based on the findingsin Cycle 1 presented in Chapter V and elaboratedfor each lesson below.

### 1. Lesson 1: Introducing arrays as models to represent multiplication

To introduce arrays as multiplication models, there are two activities designed. Activity 1 is a classroom activity. Activity 2 is a student activity. The students' starting point, learning goal, and hypothesized of learning process are described below:

#### a. Students' Starting Point

Students have been introduced to multiplication and know multiplication as repeated addition and vice versa.

#### b. Learning Goal

Finding multiplication represented in arrays, the students are introduced to the idea of arrays as models to represent multiplication.

**c. Hypothesized Learning Process**

**1) Activity 1**

The teacher tapes a covered poster in the whiteboard. The poster consists of square images arranged in array of $10 \times 10$ (Figure 4.5). The teacher explains the activity: uncover the poster in a short time and ask to determine the total number of square images shown.

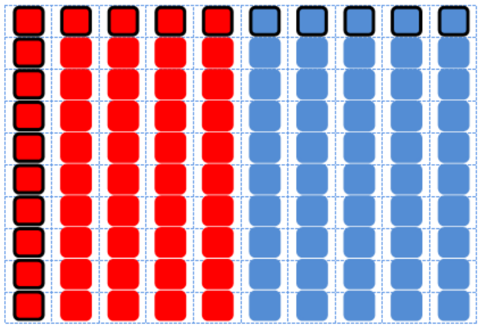


Figure 4.5: Quick images in Activity 1 – Lesson 1 – Cycle 2.

After that, the teacher starts to uncover the first row for about ten seconds. At the very first seconds, the teacher asks the students to determine the total number of the square images shown. Regarding to this question, the students will mostly do counting one by one. After getting the answer, the teacherwill make the students aware of the multiplication represented in the array by asking some guiding questions below:

*In the whiteboard, the first row is uncovered.*

Teacher : "How many rows are there?"
Students : "One."
Teacher : "How many squares in a row are there?"
Students : "Ten."
Teacher : "What is one times ten?"
Students : "Ten."
Teacher : "So, one time ten is ten.

The teacher then covers the poster and uncovers until the second row for about five second. At the very first second, the teacher asks the students to determine the total number of the square images shown. Regarding to this question, there are several conjectures on how students answer the question:

(1) The students use repeated addition, $10 + 10 = 20$

(2) The students find out that there are $10$ eggs in a row and there are $2$ rows, and then put those two numbers into multiplication form and find the product of it, $2 \times 10 = 20$.

The teacher also discusses the students' solution to determine the answer after asking the students' answer. If there are students using repeated addition, the teacher guides them to reform the repeated addition into its multiplication and then tries to make the students aware of the multiplication represented in the array by asking similar questions and doing similar gestures as mentioned before.

The teacher then gradually uncovers the poster until the third row, the fourth row and so on until all squares images uncovered. The teacher does the same cycle explained above every time uncover the poster.

**2) Activity 2**

The teacher shows a picture of eggs with some are covered by a label (Figure 4.6); the eggs are actually in array of $9 \times 10$. The teacher then introduces 'How many eggs are there?' context: tell a story about a seller who cannot figure out the total number of eggs in a carton pack because some are covered by a label. The teacher asks the students to help the seller to determine the total number of eggs in the carton pack because the seller does not want to unpack the eggs.


Figure 4.6: Mathematical Problem in Activity 2 – Lesson 1 – Cycle 2.

After explaining the context and the problem, the teacher asks the students to work in a group and distributes the worksheet to each student. Regarding to the problem, there are several conjectures on how students answer the question:

(1) The students count through all eggs one by one while visualizing the covered eggs in their mind;

(2) The students find out that the total numbers of eggs in every row are the same and then:

   (a) use repeated addition,

   $$10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 = 90, \text{ or}$$

   (b) write repeated addition, reform it to multiplication, and find product of the multiplication,

   $$10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 = 9 \times 10 = 90;$$

(3) The students find out that there are $10$ eggs in a row and $9$ eggs in a column, and then put those two numbers into multiplication form and find the product of it, $9 \times 10 = 90$.

After the students finish and collect their work, the teacher conducts a classroom discussion. Since the students could answer in different strategies, several situations could happen in the discussion. The teacher uses guides below to conduct the discussion:

(1) If there are students who count one by one, the teacher starts the discussion asking how many eggs in every row so the students realize that the total numbers of eggs in every row are the same and then will come up to repeated addition.
(2) If the students have used repeated addition, the teacher asks them to reform it into its related multiplication.
(3) If the students have used repeated addition and came up to its multiplication, the teacher emphasizes how to get the multiplication from the array by asking some guiding questions:

*The teacher shows the picture of the problem.*
Teacher : “How many eggs are there in a row?”
Students : “Ten.”
Teacher : “How many rows are there?”
Students : “Nine.
Teacher : “So, what is nine times ten?”
Students : ”Ninety.”
Teacher : “So, the total number of eggs in this pack is nine times ten and equal to ninety.”

## 2. Lesson 2: Introducing the use of commutative property

Lesson 2 is about introducing the commutative property of multiplication. There are two activities presented. Activity 1 was a classroom activity. Activity 2 was a student activity. The students’ starting point, learning goal, and hypothesized learning process are described below:

### a. Students’ Starting Point

The students could find multiplication represented in arrays.

### b. Learning Goal

Through the material designed, the idea of commutative property of multiplication could be elicited and introduced.

### c. Hypothesized of Learning Process

### 1) Activity 1

The classroom activity uses a poster and its $90^0$-rotated poster to introduce the idea of the commutative property (Figure 4.7). The initial arrangement presented flower images in $5 \times 4$. The activity was about presenting the posters as quick images so the students need to find two multiplications having the same factors but are getting from two different arrangements, and then making them see that the products are the same.

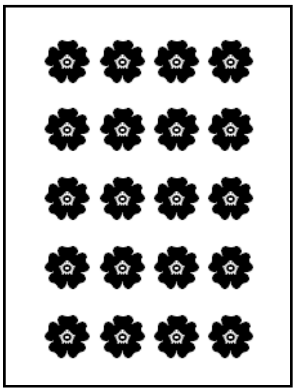


Figure 4.7: Quick images in Activity 1 – Lesson 2 – Cycle 2.

The teacher starts the activity by showing poster presented flower images arranged in $5 \times 4$ in a short time. At the very first second, the teacher asks the students to determine the total number of flower images presented. There are several conjectures on how students answer the question:

(1) There will be students who still try to count one by one or use repeated addition although they have experienced find multiplication represented in arrays to determine the total number of images presented in the previous lesson.

(2) The students find multiplication represented in the array, and then determine the product of the multiplication ($5 \times 4 = 20$).

After showing the poster, the teacher asks the answer, asks how the students determine the answer, and discusses how the students determine the answer: if the students still hardly to find the multiplication represented in the array, the teacher conducts a discussion as in Activity 2 – Lesson 1.

After showing the first arrangement, the teacher rotates the poster $90^0$ and then asks the students to determine the flowers images presented (Figure 4.8). There are several students' thinking conjectures to answer the problem:

(1) The students use similar aforementioned strategies conjectured.
(2) The students realize that it is the same poster but has different arrangement, so the total number of flower images will be the same.
(3) The students realize that it is the same poster but has different arrangement, so they interchange the factors of the previous multiplication and then realize that the product will be the same with the previous one.

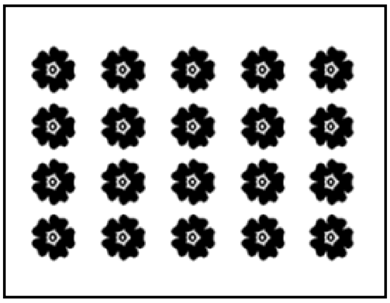


Figure 4.8: Rotated quick images in Activity 1 – Lesson 2 – Cycle 2.

After showing the rotated poster, the teacher asks the answer, asks how the students determine the answer, and discusses how the students determine the answer:

(1) If the students still hardly to find the multiplication represented in the array, the teacher conducts a discussion as in Activity 2 – Lesson 1.
(2) If the students have found the multiplication, the teacher can start the

discussion about the idea of the commutative property of multiplication:

(a) first, the teacher and the students find the products of the both multiplications;

(b) the teacher and the students then compare the form of both multiplications and realize that the multiplications have the same factors;

(c) in the end, the teacher and the students concluded that the multiplication having the same factor will have the same products.

(3) If there are students who have usedthe idea of the commutative property to get the answer, the teacher could start by asking these students to explain on how they get to the answer and then conduct thesimilar activity in (2) to strengthen the students' explanation about the commutative property.

**2) Activity 2**

The activity is about determining who has more toy cars, Race or his brother. There were two toy car arrangements (Figure 4.9). The first arrangement is Race's which arranged his toy cars in array of $7 \times 8$. The second arrangement is his brother's which arranged his toy cars in array of $8 \times 7$. The first arrangment was the anchor array to determine the total toy cars in the second arrangement using the commutative property of multiplication.

The teacher starts the activity by showing a picture of two toy car arrangements of Race and his brother. The teacher then introduces 'Who has more toy cars?' context: tell a story about Race and his brother who places their car toys in two different arrangements and are confused to decide who has more cars. The teacher then asks the students to help Race and his brother to decide who has more cars.

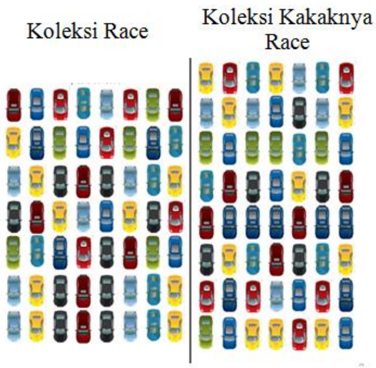



Figure 4.9: Mathematical Problem in Activity 2 – Lesson 2 – Cycle 2

After explaining the context and the problem, the teacher asks the students to work in-group and then distributes the worksheet to each student. Regarding to the problem, there are several conjectures on how students answer the question:

(1) The students count all cars on Race's and his brother's arrangements one by one, and then conclude that the total numbers of cars are the same.

(2) The students use repeated addition $(7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 56)$ and $(8 + 8 + 8 + 8 + 8 + 8 + 8 = 56)$ to determine total numbers of Race's and his brother's car toys respectively and then conclude that the total numbers of cars are the same.

(3) The students find the multiplication in Race's array and then determine the product of it $(7 \times 8 = 56)$. After that, they find multiplication in Race's brother's array and then determine the product of it $(8 \times 7 = 56)$. Since they find the products are the same, they conclude that the total numbers of Race's and his brother's cars are the same.

(4) The students find the multiplication in Race's and his brother's array and realize the multiplication having the same factors $(8 \times 7 = 7 \times 8)$. Without determining the products, since the total number of rows and columns in those two arrays are interchangeable, they conclude the total

numbers of Race's and his brother's cars are the same.

After the students finish and collect their work, the teacher conducts a classroom discussion.

(1) If the students still hardly to find multiplication represented in the array, the teacher can start the discussion by doing the similar activity in Lesson 1 – Activity 2.

(2) If the students have found the multiplication, the teacher can start the discussion about the idea of the commutative property of multiplication:

   (a) first, the teacher and the students find the products of the both multiplications;

   (b) the teacher and the students then compare the form of both multiplications and realize that the multiplications have the same factors;

   (c) in the end, the teacher and the students concluded that the multiplication having the same factor will have the same products.

(3) If there are students who have used the idea of the commutative property to get the answer, the teacher could start by asking these students to explain on how they get to the answer and then conduct the similar activity in (2) to strengthen the students' explanation about the commutative property.

### 3. Lesson 3: Introducing theidea of one-less/one-more

Lesson 3 is about introducing the one-less/one-more strategy. There are two activities presented. Activity 1 is a classroom activity. Activity 2 is a student activity. The students' starting point, learning goal, learning task, and hypothesized of learning process are described below.

#### a. Students' Starting Point

The students could find multiplication represented in arrays.

**b. Learning Goal**

Through the material designed, the idea of the one-less/one-more strategy could be elicited and introduced.

**c. Hypothesized of Learning Process**

**1) Activity 1**

The classroom activity uses three posters to introduce the idea of the one-less/one-more strategy of multiplication. Each poster presents computer images, respectively, in $5 \times 5$, $4 \times 5$, and $6 \times 5$ (Figure 4.10). The activity is about presenting the posters as quick images one after the other. The first poster was the anchor array for determining the total computer images in the second and third posters using the one-less/one-more strategy.

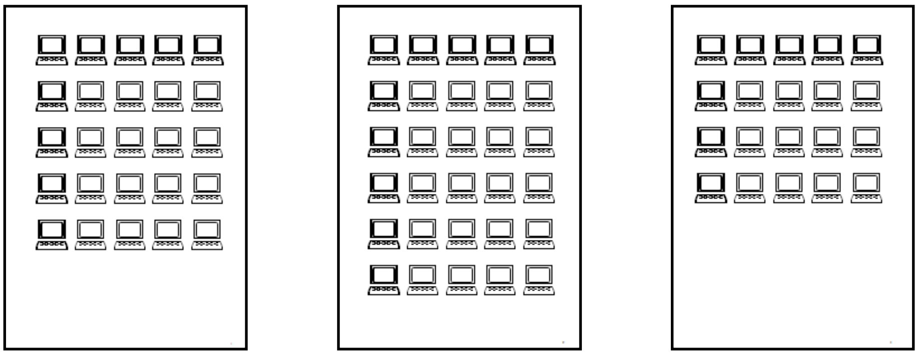

(a) (b) (c)

Figure 4.10: Quick images in Activity 1 – Lesson 3 – Cycle 2

The teacher starts the activity by showing the first posters in a short time and then asking the students to determine the total number of computer images presented. There are several conjectures on how students answer the question after the teacher shows the first poster:

(1) There will be students who still try to count one by one or use repeated addition although they have experienced find multiplication represented

in arrays to determine total number of images presented in the previous lessons.

(2) The students find the multiplication represented in the array and then determine the product of the multiplication ($5 \times 5 = 25$).

After showing the first poster, the teacher asks the answer, asks how the students determine the answer,and discusses how the students determine the answer: if the students still hardly to find the multiplication represented in the array, the teacher conducts a discussion as in Activity 2 – Lesson 1.

The teacher then tapes the first poster in the whiteboard after the discussion, shows the second poster next to the first poster, and asks the students to determine the computer images presented. Regarding to this problem, there are several students' thinking conjecture to answer the question:

(1) The students still count one by one or use repeated addition, without realizing multiplications represented in the array.
(2) The students find multiplication represented in the array, that is $6 \times 5$, and then determine the product without using the known previous fact of $5 \times 5$.
(3) The students realize that the computer images are one row more, so they add 5 to previous answer.
(4) The students find multiplication represented in the array, that is $6 \times 5$, and realize the array is one row more, so the product of $6 \times 5$ is only adding 5 to the product of $5 \times 5$ that previously mentioned.

After showing the second poster, the teacher asks the answer and asks how the students determine the answer. The teacher then tapes the second poster next to the first poster after the discussion and discusses on how to determine the answer:

(1) If the students still hardly to find the multiplication represented in the array, the teacher start the discussion doing the similar activity in Lesson 1 – Activity 2.

(2) If the students have found the multiplications but there is no student who come up to the idea of the one-more strategy, the teacher start the discussion comparing the first and second poster:

   (a) makes the students realize that the second poster is one row more than the first poster;

   (b) emphasizes the multiplication and product represented the arrangement and the total number of computer images in the first poster as a known fact.

   (c) asks the students to derive the product of multiplication in the second poster using the known fact or without counting all over again: “If the first poster is $5 \times 5$, and the second poster is one row more from the first one, that is $6 \times 5$, and if the product of $5 \times 5 = 25$, so how to get the product of $6 \times 5$ using the known fact of $5 \times 5$?”

   (d) concludes the idea of the one-more strategy: “So, if you know the product of $5 \times 5$ is $25$, to find the product of $6 \times 5$, you can add 5 to 25, that is $25 + 5 = 30$. This process uses the one-more strategy.”

(3) If there are students who have used the idea of the one-more strategy to get the answer, the teacher could start by asking these students to explain on how they get to the answer and then conduct the similar activity in (2) to strengthen the students’ explanation about the one-more strategy.

After discussing about the one-more strategy, the teacher takes off the second poster, shows the third poster next to the first poster to introduce the one-less strategy, and then asks the students to determine the computer images presented. The students’ thinking conjectures and the discussion’s guides are similar although it is about the one-less strategy.

**2) Activity 2**

The activity is about determining the total stickers in three three sticker packages placed next to each other: Alin's, Belinda's, and Carla's (Figure 4.11). The stickers were arranged in 15 × 8, 14 × 8, and 16 ×8 respectively. The first array was the anchor array for determining the total stickers in the second and the third arrays using the one-less/one-more strategy.

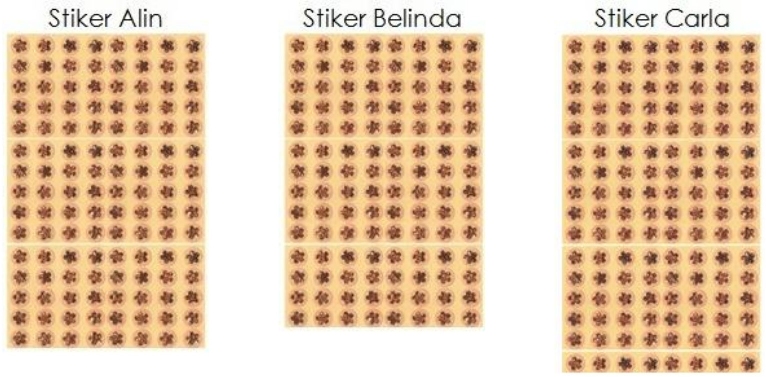



Figure 4.11: Mathematical Problem in Activity 2 – Lesson 3 – Cycle 2.

The teacher starts the activity by showing a picture presenting the three sticker packages. The teacher then introduces 'How many stickers do I have?' context: tell a story about three girls: Alin, Belinda, and Carla, who have similar sticker packages and want to figure out the total number of their stickers left. Theteacher then asks the students to help the girls to determine the total number of each sticker left in the packages.

After explaining the context and the problem, the teacher asks the students to work in-group, and then distributes the worksheet to each student. Regarding to the problem, there are several conjectures on how students answer the question:

(1) There will be students who still try to count one by one or use repeated

addition although they have experienced find multiplication represented in arrays to determine the total number of images presented.

(2) The students find the multiplications represented in the arrays: $5 \times 4$, $4 \times 4$, and $6 \times 4$, and then:
   (a) the students find every product of the last two multiplications without using the previous known fact of $5 \times 4$; or
   (b) the students use $5 \times 4 = 20$ as a known fact, and then use the one-less strategy to derive the product of $4 \times 4$: $4 \times 4 = (5 \times 4) - 4 = 20 - 4 = 16$, and use the one-more strategy to derive the product of $6 \times 4$: $6 \times 4 = (5 \times 4) + 4 = 20 + 4 = 24$, to determine the total number of computer images presented in each arrangement.

After the students finish and collect their work, the teacher conducts a classroom discussion:

(1) If the students still hardly to find the multiplication represented in the array, the teacher start the discussion doing the similar activity in Lesson 1 – Activity 2.
(2) If the students have found the multiplications, the teacher start to discuss about the idea of the one-less/one-more strategy of multiplication:
   (a) makes the students realize Belinda's stickers in one row less than Alin's, and Carla's is one row more than Alin's;
   (b) emphasizes the multiplication and product represented the arrangement and the total number in Alin's as a known fact;
   (c) asks the students to derive the product of multiplication in Belinda's and Carla's using the known fact, or without counting all over again.
   (d) concludes the use of the one-less/one-more strategy: "So, if you know the product of $5 \times 4$ is $20$. To find the product of $4 \times 4$, you can subtract 4 from 20, that is $20 - 4 = 16$. This process uses the one-less strategy. To find the product of $6 \times 4$, you can add 4 to 20, that is $20 + 4 = 24$. This process uses the one-more strategy."

(3) If there are students who have used the idea of the one-less/one-more strategy to get the answer, the teacher could start by asking these students to explain on how they get to the answer and then conduct the similar activity in (2) to strengthen the students' explanation about the one-less/one-more strategy.

**4. Lesson 4: Introducing the idea of doubling**

Lesson 4 is about introducing the doubling strategy. There are two activities presented. Activity 1 is a classroom activity. Activity 2 is a student activity. The students' starting point, learning goal, learning task, and hypothesized of learning process are described below.

**a. Students' Starting Point**

The students could find multiplication represented in arrays.

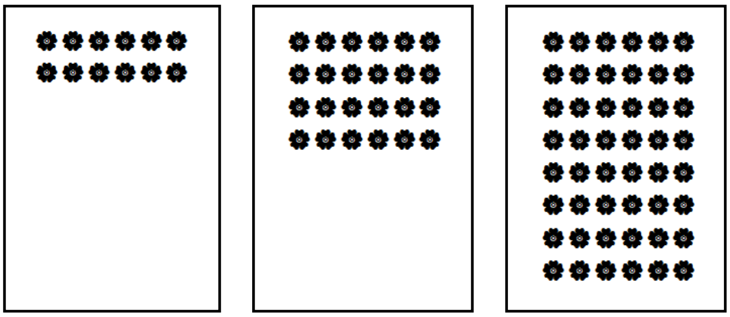
**b. Learning Goal**

Through the material designed, the idea of doubling strategy could be elicited and introduced.

**c. Hypothesized Learning Process**

**1) Activity 1**

The classroom activity uses three posters to introduce the idea of the doubling strategy of multiplication. Each poster presents flower images, respectively, in $2 \times 6$, $4 \times 6$, and $8 \times 6$ (Figure 4.12). The activity is about presenting the posters as quick images one after the other. The first poster is the anchor array for determining the total flower images in the second poster using the doubling strategy. The second poster is the anchor array for determining the flower images in the third poster using the doubling strategy.


(a) (b) (c)
Figure 4.12: Quick images in Activity 1 – Lesson 4 – Cycle 2

The teacher starts the activity by showing the first posters in a short time and then asking the students to determine the total number of computer images presented. There are several conjectures on how students answer the question after the teacher shows the first poster:

(1) There will be students who still try to count one by one or use repeated addition although they have experienced find multiplication represented in arrays to determine the total number of images presented in the previous lessons
(2) The students find the multiplication represented in the array and then determine the product of the multiplication ($2 \times 6 = 12$).

After showing the first poster, the teacher asks the answer, asks how the students determine the answer, and discusses on how to get the answer: if the students still hardly to find the multiplication represented in the array, the teacher conducts a discussion as in Activity 2 – Lesson 1.

The teacher then tapes the first poster in the whiteboard, shows the second poster next to the first poster, and asks the students to determine the flower images presented. Regarding to this problem, there are several students' thinking conjecture to answer the question:

(1) The students still count one by one or use repeated addition, without

realizing multiplications in the array.

(2) The students find multiplication represented in the array, that is $4 \times 6$, and determine the product without getting from the known previous product of $2 \times 6$.

(3) The students realize that the flowers images are two rows more, so they add 12 to previous answer.

(4) The students find multiplication represented in the array, that is $4 \times 6$, and realize the array is two row more, so the product of $4 \times 6$ is double of the product of $2 \times 6$ that previously mentioned.

After showing the second poster, the teacher asks the answer and asks how the students determine the answer. The teacher then tapes the second poster next to the first poster after the discussion and discusses on how to determine the answer:

(1) If the students still hardly to find the multiplication represented in the array, the teacher conducts a discussion as in Activity 2 – Lesson 1.

(2) If the students have found the multiplications but there is no student who come up to the idea of the doubling strategy, the teacher start the discussion comparing the first and second poster:

   (a) makes the students realize that the rows in the second poster is double than the rows in the first poster;

   (b) emphasizes the multiplication and product represented the arrangement and the total number of computer images in the first poster as a known fact.

   (c) asks the students to derive the product of multiplication in the second poster using the known fact or without counting all over again: "If the first poster arrangement is $2 \times 6$, and the second poster is two rows more from the first one, that is $4 \times 6$, and if the product of $2 \times 6 = 12$, so how to get the product of $4 \times 6$ using the known previous product of $2 \times 6$?"

(d) concludes the idea of the doubling strategy: "So, if you know the product of $2 \times 6$ is $12$, to find the product of $4 \times 6$, you can double $12$, that is $12 + 12 = 24$. This process uses the doubling strategy."

(3) If there are students who have used the idea of the doubling strategy to get the answer, the teacher could start by asking these students to explain on how they get to the answer and then conduct the similar activity in (2) to strengthen the students' explanation about the doubling strategy.

The teacher then takes off the first poster, shows the third poster next to the second poster, and then asks them to determine the flower images presented. The students' thinking conjectures and the discussion's guides are the same as when showing the second poster.

**2) Activity 2**

The activity is about determining the total stickers in a special package (Figure 4.13). The special package covered some stickers by the label. The stickers are arranged in $8 \times 6$ in total, but the label make the stickers are divided in two $4 \times 6$. All stickers in the first part of $4 \times 6$ are uncovered. Meanwhile, only stickers in the left side are uncovered in the second part of $4 \times 6$ to elicit and introduce the idea of doubling strategy.

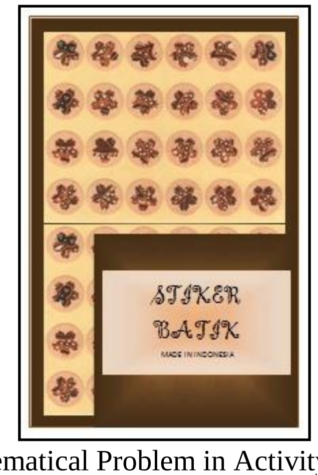



Figure 4.13: Mathematical Problem in Activity 2 – Lesson 4 – Cycle 2

The teacher starts the activity by showing a picture of stickers with some covered by a special package. The teacher then introduce 'How many stickers are there?' context: tell a story about a little girl who cannot figure out the total number of stickers in a special package because some stickers covered by the package. The teacher then asks the students to help the little girl determines the total number of stickers in the special package.

After explaining the context and the problem, the teacher asks the students to work in-group and then distributes the worksheet to each student. Regarding to this problem, there are several conjectures on how students answer the question:

(1) There will be students who still try to count one by one or use repeated addition although they have experienced find multiplication represented in arrays to determine the total number of images presented.
(2) The students find the multiplication represented the arrangement, $8 \times 6$. To determine the product:
    (a) the students use repeated addition; or
    (b) the students realize that the stickers are in two parts in; half-uncovered and half-covered. The students could easily find the multiplication and product in the uncovered part; that is $4 \times 6 = 24$. Since the stickers are in two equally parts that means $8 \times 6$ is two $4 \times 6$ and the product of $8 \times 6$ is double of the product $4 \times 6$.

After the students finish and collect their work, the teacher conducts a classroom discussion:

(1) If the students still hardly to find the multiplication represented in the array, the teacher conducts a discussion as in Activity 2 – Lesson 1.

(2) If the students have found the multiplication, the teacher start the discussion about the idea of the doubling strategy of multiplication:

   (a) makes the students realize that stickers are divided in two equally parts: uncovered and covered;

   (b) asks the students to find the multiplication and product in the uncovered part; that is $4 \times 6 = 24$, and emphasizes it as a known fact;

   (c) asks the students to derive the product of $8 \times 6$ using the known fact;

   (d) concludes the solution as the doubling strategy.

(3) If there are students who have used the idea of the doubling strategy to get the answer, the teacher could start by asking these students to explain on how they get to the answer and then conduct the similar activity in (2) to strengthen the students' explanation about the doubling strategy.

# CHAPTER V

# RESTROSPECTIVE ANALYSIS

This chapter presents the retrospective analysis of data collected in Cycle 1 and Cycle 2.

## A. Retrospective Analysis of Cycle 1: Preliminary Activities

Pretest, student interview, and teacher interview were thepreliminary activities conducted to find out the actualstudents'prior knowledge. The experimental students were Fira, Mita, Samuel, and Vina. Theyworked on multiplication bare problems in the pretest (Figure 5.1), and then explained how they solved some of those problems in the interview. The pretest and interview were in two different days.

| | | | |
|---|---|---|---|
| 1) $2 \times 3 =$ | 4) $9 \times 2 =$ | 6) $5 \times 4 =$ | 9) $10 \times 8$ |
| 2) $4 \times 3 =$ | 5) $3 \times 8 =$ | 7) $4 \times 4 =$ | 10) $9 \times 8 =$ |
| 3) $8 \times 3 =$ | | 8) $6 \times 4 =$ | |

Figure 5.1: Pretest Questions in Cycle 1.

From the pretest worksheet, three students wrote repeated addition as their solution and one student wrote finger technique as her strategy to solve all the questions (Appendix C).Fragment 5.1 shows how typically they explained on how to get the multiplication products when they were being interviewed. Their mathematics teacher verified that she had taught about repeated addition as multiplication and had asked them to memorize basic multiplication facts.

| | | | |
|---|---|---|---|
| | | | *The researcher asked Samuel in the interview.* |
| 1 | Researcher | : | "What is the product of $2 \times 3$?" |
| 2 | Samuel | : | "Six." |
| 3 | Researcher | : | "How did you get (the product of ) it?" |
| 4 | Samuel | : | "Three plus three." |

| | | |
|---|---|---|
| 5 | Researcher | : "What is the product of 4 × 3?" |
| 6 | Samuel | : "12" |
| 7 | Researcher | : "How did you get (the product of ) it?" |
| 8 | Samuel | : "There are four of 3. Then, I add the 3s." |
| 9 | Researcher | : "What is the product of 8 × 3?" |
| 10 | Samuel | : "24" |
| 11 | Researcher | : "How did you get (the product of ) it?" |
| 12 | Samuel | : "There are eight of 3. Then, I add the 3s." |

Fragment 5.1: Samuel used repeated addition to explain how he got the products.

From these preliminary activities, there are some findings could be taken:

[1] The students are familiar to repeated addition as multiplication since most of them wrote repeated addition as their solution to get the multiplication products, and also the interviews supported this finding.

[2] The students probably have familiar to the basic multiplication facts since, from the teacher interview, they have already asked to memorize the facts by their teacher.

## B. Remark 1

There was no interference to conduct the teaching experiment for Lesson 1 since the students' prior knowledge was consistent with the hypothesized students' starting point to conduct the lesson: students already familiar to repeated addition as multiplication.

## C. Retrospective Analysis of Cycle 1: Lesson 1

Lesson 1 was about introducing arrays as multiplication models. The activity was about determining the total number of eggs in a cartoon pack (Figure 5.2). Some eggs were covered on purpose by a label. The eggs were in array of 9 × 10. By presenting the array, the students are expected to find the multiplication represented in it.


Figure 5.2: Mathematical Problem in Lesson 1 – Cycle 1.

**1. Looking Back: Video Recording and Students' Worksheet**

The researcher started the lesson, introduced 'How many eggs are there?' context, and then distributed the worksheet while explained the problem: determining the total number of eggs in a carton pack where a label covered some eggs. After the students got the worksheet, there were three different situations occurred:

(1) Mita counted the uncovered eggs starting from the right edge in the top row (Figure 5.3). She got 34 as the total number of all eggs in the pack.

Figure 5.3: Mita only counted the covered eggs: "1, 2, 3, ..., 31, 32, 33, ..., 34".

(2) Fira and Samuel could directly find the correct multiplication represented in the eggs arrangements. They actually worked separately, but then they discussed the strategy after heard Mita's answer (Fragment 5.2).

| | | | |
|---|---|---|---|
| | | | *Fira approached Samuel to discuss the answer, since Mita has different answer, and then started to count the uncovered eggs.* |
| 13 | Fira | : | "1, 2, 3, ..., 34." *After counted the uncovered eggs, Fira realized how Mita got wrong answer, and then Samuel showed how to get the answer correctly to Fira who then confirmed it as the correct solution.* |
| 14 | Samuel | : | "It is easy. 1, 2, 3, ..., 10 (*he counted the eggs in the top row*), 1, 2, 3, ..., 9 (*he counted the eggs in the left side*). So, it is 9 times ten. |
| 15 | Fira | : | "Yes, (*it is*). Mita was wrong." |

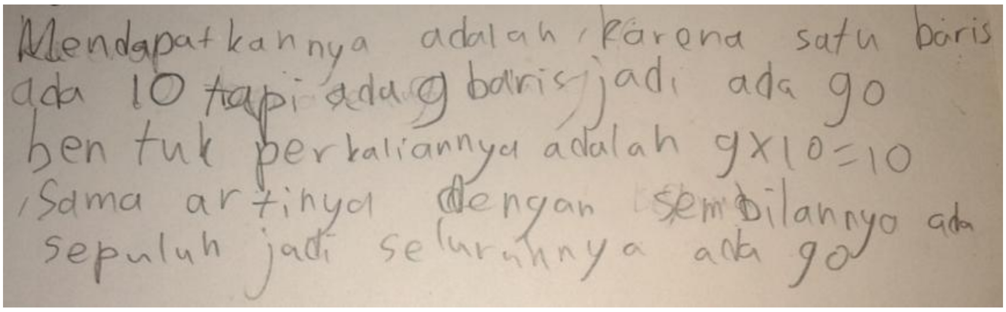
Fragment 5.2: Discussion between Fira and Samuel.

(3) Vina, who previously saw how Fira worked on the problem (Figure 5.4), struggled trying to find the multiplication represented in the arrangement. However, she concluded the answer is $10 \times 4$ after changing from $8 \times 4$.


Figure 5.4: Vina showed how Fira solved the problem.

The classroom discussion conducted after the students had theiranswer. The discussion confirmed that Mita and Vina only focused to determine the uncovered eggs. They were failed to realize that there were eggs under the label. After the confused students got the clear idea about what the question is, Samuel explained how he solved the problem; his explanation is the same as he wrote on his worksheet (Figure 5.5).

After Samuel finished his explanation, Mita looked confused. Samuel then tried to explain again but now he also showed how he got 10 eggs in a row and nine rows in the carton pack. After seeing how Samuel pointed the picture while counted to get the nine rows, Mita asked “There are 10 (*in the first row*), why do you count (*the egg in the top edge*) again when you count downward (*to get the total number of rows*)?”


Mendapatkannya adalah, karena satu baris ada 10 tapi ada 9 baris, jadi ada 90 bentuk perkaliannya adalah 9x10=10 ,Sama artinya dengan sembilannya ada sepuluh jadi seluruhnya ada 90

Samuel wrote: “The solution is, because there are 10 (*eggs*) in a row, but there are 9 rows, so there, (*the total number of eggs*), are 90. The multiplication (*represented the arrangement*) is $9 \times 10 = 90$, means there are ten 9’s, so there, (*all eggs*), are 90.”

Figure 5.5: Samuel’s solution in his worksheet:

Corresponding to Mita’s question, the researcher tried to make the students see repeated addition in the problem and then reform it to its multiplication as planned. However, in the middle of explanation, Samuel got in the way and restated what he said before. Fortunately, Mita realized that she was mistaken approaching the problem andthe rest of the students seemed accepting Samuel’s explanation on how to find the multiplication represented in the array.

**2. Findings**

From this lesson, there are some findings could be taken:

[3] The instruction of the problem was still not quite clear since Mita and Vina read the problem differently from what had expected; they only counted the uncovered eggs

[4] The problem could make students find the multiplication represented in the array since Fira and Samuel could instantly find the multiplication represented in the array.

[5] To explain on how to get the multiplication, Samuel mentioned that "there are 10 eggs in a row, but there are nine rows, so ...".

## D. Remark 2

As mentioned in Chapter III, a thorough retrospective analysis was conducted after all lessons were conducted, but a brief analysis was conducted after every lesson. After Lesson 1, a brief analysis was conducted to find out if the students' knowledge was consistent to the starting point for conducting the next lessons. Providing the evidences on the students' worksheets and their answer in the discussion, it was concluded that the students could find multiplication represented in arrays. Therefore, there was no adjustment to the design of materials used and analyzed in the following lessons.

## E. Retrospective Analysis of Cycle 1: Lesson 2

Lesson 2 was about introducing the commutative property. The activity was about determining who has more toy cars, Race or his brother. There were two toy car arrangements (Figure 5.30): the left one was Race's which arranged in $7 \times 8$ and the right one was his brother's which arranged in $8 \times 7$. The multiplication fact represented in the first arrangement was the anchor fact to determine the total toy cars in the second arrangement. By presenting these two toy cars arrangements, the commutative property of multiplication was expected to be elicited and introduced.

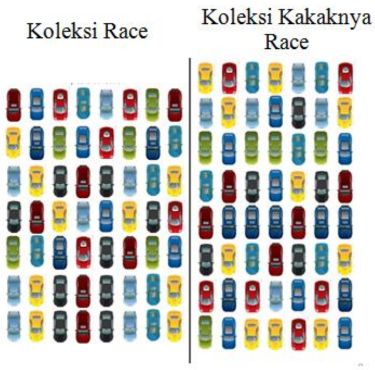



Figure 5.6: Mathematical Problem in Lesson 2 – Cycle 1.

**1. Looking Back: Video Recording and Students' Worksheet**

The researcher started the lesson, introduced 'Who has more toy cars?' context, and thendistributed the worksheet while explained the problem: determining who has more toy carsbased on two cars arrangements, Race's or his brother's. After the students got the worksheet, there were three different situations occurred:

(1) Fira and Samuel, independently, found the multiplications represented in the both toy car arrangements: $7 \times 8$ and $8 \times 7$, and concluded that the total numbers of Race's and his brother' toy cars were the same.

(2) Vina also found that Race and his brother had the same total number of toy cars, but she did not get the correct multiplications. She got multiplication $7 \times 7$ represented in the both toy car arrangements.

(3) Meanwhile, Mita only looked Samuel's work.

The classroom discussion conducted after the students had their own answer. The researcher asked the students to explain their solution. Samuel gave similar explanation on how he got to the multiplications as he explainedin Lesson 1. He then concluded that the product of the two multiplications were the same and so were the total number of toy cars in the two arrangements (Fragment 5.3).

| | | |
|---|---|---|
| | | *Samuel explained how he got the answer.* |
| 16 | Samuel | : "Because one row in Race's collection, there are eight (*cars*), the total number of rows is seven. For Race's brother, there are seven (*cars*) in one row, but the total number of rows is eight. Therefore, the way to count (*the total number of cars for each collection*), $8 \times 7$ for this (*pointing to the Race's arrangement*), and this (*pointing to the Race's brother's arrangement*), $7 \times 8$. The result are the same, $56$." |

Fragment 5.3: Samuel's solutionof problem in Lesson 2.

Different from Samuel's explanation, Fira realized that the total cars in the top row of Race's and in a column of his brother's arrangement were the sameand it hold conversely. This led her to the conclusion that the total numbers of cars were the same although the arrangements were different. From her worksheet, she also managed to write $8 \times 7 = 7 \times 8 = 56$.

After Fira explained her solution, the researcher asked Mita to explain her solution but she refused it. Vina then explained she found that Race and his brother have the same total number of toy cars because the multiplication$7 \times 7$represented the both arrangements. When the researcher asked how she got to the $7 \times 7$, she failed to show it, she looked confused to count the car that simultaneously in a row and a column.

## 2. Findings

From this lesson, there are some findings could be taken:

[6] The instruction of the problem was clear since all students understand the problem as expected.

[7] The problem could make some students got the idea of the commutative property since Fira and Samuel were managed to concluded that $8 \times 7 = 7 \times 8 = 56$.

[8] Finding multiplication represented in an array was still difficult for some students since Vina was in difficulty to count the car that simultaneously in a row and a column and Mita only showed Samuel's work.

## F. Retrospective Analysis of Cycle 1: Lesson 3

Lesson 3 was about introducing the doubling strategy of multiplication. The activity was about determining the total stickers in a special package (Figure 5.7). The special package covered some stickers by the label. The stickers are arranged in $8 \times 4$ in total, but the label make the stickers are divided in two $4 \times 4$. All stickers in the first part of $4 \times 4$ are uncovered. Meanwhile, only stickers in a column are uncovered in the second part of $4 \times 4$. By using this special package, the idea of doubling strategy was expected to be elicited and introduced.

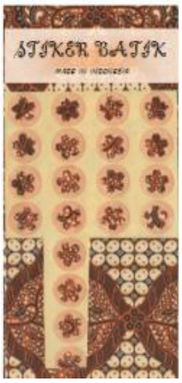



Figure 5.7: Mathematical Problem in Lesson 3 – Cycle 1.

### 1. Looking Back: Video Recording and Students' Worksheet

The researcher started the lesson, introduced 'how many stickers are there?' context, and then distributed the worksheet while explained the problem: determining the total number of stickers where the package covered some stickers. After got the worksheet, there were three different situations occurred:

(1) Fira and Samuel, who worked independently, instantly found the multiplication represented in the sticker arrangements, that is $8 \times 4$, and seemed familiar with the fact as they answered the question swiftly. From Samuel's worksheet, he also added an explanation how to get to the product using repeated addition.

(2) Vina was showed counting all stickers one by onealthough there were stickers covered (Figure 5.8), but she managed to add multiplication as the solution in her worksheet.

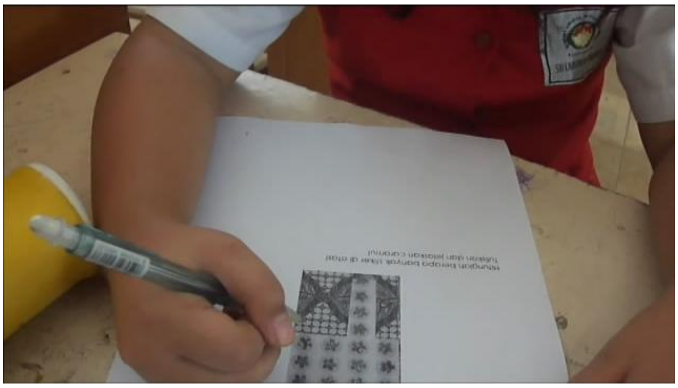


Figure 5.8: Vina pointed the covered sticker when she was counting all stickers one by one

(3) Mita counted the uncovered stickers one by one, and then use doubling to get to the answer, but she failed to do the addition (Figure 5.9).

The classroom discussion conducted after the students had their answer. Based on the situations occurred, the researcher wanted to bring up the idea of the doubling strategy, but the students refused to hear more explanation about it. They said they already got the correct answer using their approach. Thus, the researcher just ended the activity.

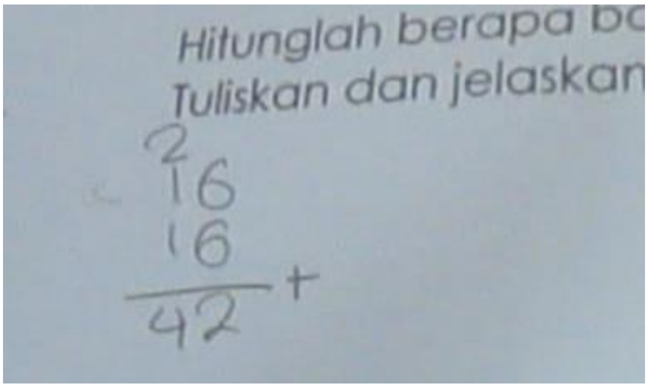



Figure 5.9: Mita used doubling but failed to do the addition and related it to multiplication.

## 2. Findings

From this lesson, there are some findings could be taken:

[9] The instruction of the problem was clear since all students understand the problem as expected.

[10] The problem could not quite elicit the doubling strategy if the students were familiar to the multiplication fact; as what Fira and Samuel did after they found the multiplication represented in the array.

[11] Counting one by one was occurredsince Vina managed to get the correct answer by doing it, see Figure 5.8.

[12] The problem could elicit the idea of the doubling strategy since Mita got the idea of using the doubling strategy to solve the problem, but she did not use it as a strategy to find the multiplication product, see Figure 5.9.

[13] Students refused to hear more explanation about the doubling

strategyafter they got their correct answer.

## G. Remark 3

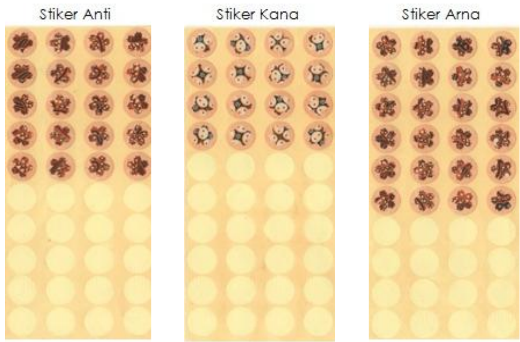
From Lesson 3, delivering the idea of doubling strategy was intended and actually possible conducted through the classroom discussion. However, there was a condition where the students refused to hear the explanation about the doubling strategy got in the way, see Findings [13].

Regarding to that condition, the researcher assumed that conducting a group activity before giving the problemwas a solution to eliminate the condition where the students refused to hear the explanation about the introduced strategies. Therefore, for Lesson 4, a group activity was conducted before asking the students to work on the problem by themselves. The activity was about solvingthe same intended problem together as a group.The problem was: determining the total number of stickers of Anti's, Kana's and Arna's, seeFigure 5.10.

## H. Retrospective Analysis of Cycle 1: Lesson 4

Lesson 4 is about introducing the one-less/one-more strategy. The activity was about determining the total stickers in three three sticker packages placed next to each other: Anti's, Kana's, and Arna's (Figure 5.10). The stickers were arranged in $5 \times 4$, $4 \times 4$, and $6 \times 4$ respectively. The multiplication fact in the first array was the anchor fact for determining the total stickers in the second and the third arrays. By showing the arrays next to each other, the idea of one-less/one-more strategy was expected to be elicited or introduced.

Based on what previously mentioned inRemark 3, this activity was conducted twice, as a group activity and also as an individual activity.




(a) (b) (c)

Figure 5.10: Mathematical Problem in Lesson 4 – Cycle 1.

**1. Looking Back: Video Recording and Students' Worksheet**

Based on the finding in the previous lesson, to eliminate the condition when the students refused to perceive the idea of other strategy, the researcher conducted a group activity beforehand. This activity was actually used the same intended problem for this lesson but the problem was solved together.

The researcher started the lesson, introduced "How many sticker do I have" context, presented the problem with Kana's and Arna's sticker arrangements covered, and then asked the students to determine the total number of Anti's stickers (Figure 5.11).

Figure 5.11: The students tried to determine the total number of Anti's stickers.

Fira and Samuel instantly got the answer; it was 20. The researcher confirmed that they found the multiplication $5 \times 4$ to get the answer. Meanwhile, Mitacounted the stickers one by one and Vina refused to tell how she solved it.

After the students got the total number of Anti's stickers, the researcher showed the Kana's sticker in a short time while Arna's stickers still covered. Fira, Mita, and Vina instantly shouted the multiplication $4 \times 4$ represented in Kana's sticker, but only Samuel who realize that there was one row missing (Figure 5.12).

Samuel said:
"Because this is missing (*pointing the last row in Anti's stickers*), it is missing one row, so (*Kana's sticker arrangements*) is $4 \times 4$."

Figure 5.12: Samuel realized one row is missing.

After discussing how the product of $4 \times 4$ could be derived from the product $5 \times 4$ using the one-less strategy, the researcher then showed the last sticker arrangement, Arna's sticker. Samuel instantly realized that there were 24 stickers and he then explained his solution (Fragment 5.4).

| | | | |
|---|---|---|---|
| | | | *After the researcher showed the arrangement of Arna's sticker, Samuel explained how he got the answer.* |
| 17 | Samuel | : | "I know it! I know it! 24! 24!" |
| 18 | Researcher | : | "Why is it 24?" |
| 19 | Samuel | : | "In here (*pointing Kana's sticker arrangement*), you add one (from Anti's sticker arrangement). (*Pointing Arna's sticker arrangement*), add two rows (from Kana's sticker arrangement). So, it is 16 added by 8, so it is 24." |

| | | | |
|---|---|---|---|
| | | | *Since Samuel compared Arna and Kana sticker arrangement instead of Arna and Anti, the researcher asked whether it is easier to count.* |
| 20 | Researcher | : | "How about if you start from this (*pointing Anti's sticker arrangement*)?" |
| 21 | Samuel | : | "It is easier, (only) add 4!" |

Fragment 5.4: Samuel's explanation of solution in Lesson 4.

After confirming if the other students could perceive the idea of the one-less/one-more strategy from Samuel's explanation, the researcher continued to the main activity, distributed the worksheet, and asked the student to work on the same problem. After the students got the worksheet, there were four different situations occurred:

(1) Samuel used the one-less/one more strategy as he explained before in the group activity;

(2) Firafound the multiplication represented in each three sticker arrangements, but did not use the one-less/one-more strategy to calculate the products;

(3) Vina counted the stickers in each three arrangements one by one, and then tried to relate them to its multiplication.

(4) Meanwhile, Mitaonly counted the stickers in the three arrangements one by one.

**2. Findings**

From this lesson, there are some findings could be taken:

[14] The instruction of the problem was clear since all students understand the problem correctly.

[15] The problem could elicit the idea of one-less/one-more strategy since

Samuel found the strategy when determining the multiplications in the arrays, see Figure 5.12 and Fragment 5.4.

[16] The group activity made the idea of the one-less/one-more strategy emerged so that a student could use it when solving the problem, as what Samuel did.

[17] Counting one by one was occurred since Vina and Mita managed to get the correct answer by doing it.

## I. Retrospective Analysis of Cycle 2: Preliminary Activities

Classroom observation, pretest, student interview and teacher interview were the preliminary activities conducted to find outthe actual students' prior knowledge.Classroom observation was conducted first before the pretest, student interview and teacher interview. All activities are conducted in different days.

When the class was being observed, the mathematics teacher conducted a hands-on activity and then askedthe students to work in pair to solve some division bare problems. The students sometimes were hardly to manage. The teacher even had a special treatment to control the students: letting them released their energy by drumming the table for some time.

The teacher assisted and guided the students directly when they were solving the problemsor getting the wrong answer. There was no classroom discussion after the students finished working on every problem; the teacher only verified whether the students had wrong or right answer.

After conducting the class observation, the pretest and student interviewwere conducted in two different days. All students worked on multiplication bare problems in pretest(Figure 5.13), and then only six selected students explained their solution to solve some pretest problem in the interview.

| | | | |
|---|---|---|---|
| 1) $2 \times 6 =$ | 4) $9 \times 2 =$ | 6) $5 \times 4 =$ | 9) $10 \times 8 =$ |
| 2) $4 \times 6 =$ | 5) $6 \times 8 =$ | 7) $4 \times 4 =$ | 10) $9 \times 8 =$ |
| 3) $8 \times 6 =$ | | 8) $6 \times 4 =$ | |

Figure 5.13: Pretest Questions in Cycle 2

From the pretest worksheet, all students wrote repeated addition to solve some or all multiplication bare problems (Appendix C). Some students left their calculation scratches showing how they make use of the repeated addition as a strategy to solve the problems.

Their mathematics teacher alsoverified that she had only taught about repeated addition as multiplication intensively and had not asked them to memorize basic multiplication facts or introduced other multiplication techniques or properties.

From these preliminary activities, there are some findings could be taken:

[18] The students are familiar to repeated addition as multiplication since most of them wrote repeated addition as their solution to get the multiplication products, and also the interviews supported this finding.

[19] Most of the students probably have not familiar to the basic multiplication facts since the teacher has not asked them to memorize the facts.

[20] The students were accustomed working in-group, but there was no

classroom discussion afterward.

## J. Conclusion 1

Based on the findings generated from the teaching experiment in Cycle 1 and the preliminary activities in Cycle 2, there are some conclusions could be taken:

[21] Based on Finding [2], the students in Cycle 1 have been asked to memorize the multiplication basic facts. Thus, some of them were familiar to the facts and could not come up to the introduced strategies, see Finding [10]. Therefore, there is a need to change the multiplication facts represented in the arrays so that they did not represent the basic facts.Nevertheless, based on Finding[19], the students in Cycle 2, have not been asked to memorize so most of them are expected not familiar to the basic facts. Therefore, the suggestion to changethe multiplication represented in the array is not necessary to be followed. However, to minimize the condition when the students are already familiar to some easier basic facts (below multiplication by five), the multiplications represented in all arrays are better the bigger multiplications (above multiplication by five). This applies for the problems in Lesson 3 and 4. Also, choosing bigger arrays are expected to minimize the use of counting one by one, as showed in Finding [11] and [17].

[22] Based on Finding[4], the problems designed could make the students find the multiplication represented in the array. However, Finding [3] shows how there are students who understand the problem differently.

Therefore, for Lesson 1, the problem needs to be modified so that it could minimize students' confusion, especially clarifying the existence of eggs below the label.

[23] From Finding [5], instead of mentioning: there are 10 eggs in a column and 9eggs in a row, a student mentioned: there are 10 eggs in a row, but there are nine rows. Therefore, instead of asking "How many eggs in a column?" and "How many eggs in a row?", the questions asking "How many eggs in a row?" and "How many rows are there?" are closer to the students' thinking.

[24] Based on Finding [7], the problem designed could elicit the commutative property, and also Finding [6] shows that all student understand the problem correctly. Therefore, for Lesson 2, there is no modification for the problem.

[25] Based on Finding [12] and [15], the problems designed could elicit the introduced strategies, and also Finding [9] and [14] show that all student understand the problem correctly. However, based on Conclusion [21], there is a need to change the multiplication represented in the arrays. Therefore, for Lesson 3 and 4, the problems need to be modified to present bigger arrays.

[26] Based on Finding [8], there are students who could not get the correct multiplication correctly after Lesson 1 was conducted. This probably the discussion could not provide more time to deliver the idea.Meanwhile, from Finding [13], the students refused to hear the explanation about the

introduced strategy and,from Finding [16], conducting a preliminary activity made the introduced strategy could be elicited in the discussion. Therefore, there is a need for designing an activity before students work on the problem so that it could give more time for students to perceive the introduced strategies and minimize the condition where the students refuse to perceive the idea of other strategies

[27] Comparing Finding [4], [7], [12], and [15], all materials designed had a chance to work for its intended purpose, except for the problem in Lesson 3 that gave a quite small chance to elicit the doubling strategy of multiplication. This suggests reconsidering the learning sequences for Lesson 3 and 4 so that the students work on problems that gave more chance to elicit the introduced strategy first.

In general, from theaforementioned conclusions, there are suggestions for modifying the initial problems and designing a preliminary activity for each lesson. Therefore, before conducting the teaching experiment for Cycle 2, the HLT used in Cycle 1 (or the initial HLT) needs to be revised. The results of modifications and designing processes are presented in the subchapter below.

## K. Revising Hypothetical Learning Trajectory (HLT)

Based on Conclusion 1, there were some components in the initial HLT that needed to be revised. The results of modifications and designing processes are explained below.

### 1. Changing Lesson sequences

Based on Conclusion [27], there is a suggestion to change the learning sequence between Lesson 3 and 4 to make the students work on problems that gave more chance to elicitthe introduced strategy first. Besides that, introducing the idea of doubling using arrays is also considered a bigger step to visualize since it double rows instead of add one row or subtract one row. Therefore, for Cycle 2, the one-less/one-more strategy is introduced in Lesson 3 and the doubling strategy is in Lesson 4.

**2. Redesigning the mathematics problemfor Lesson 1**

Lesson 1 is about introducing the idea of arrays as multiplication models. Based on Conclusion [22], there is a suggestion to modify the initial problem so that it could minimize students' confusion, especially clarifying the existence of eggs below the label. Thus, the question was inserted in the problem implicitly.Also, in order to focus on finding the total eggs in a row and a column, the label was made to cover almost part of the box, except the left and top sides (Figure 5.14b).

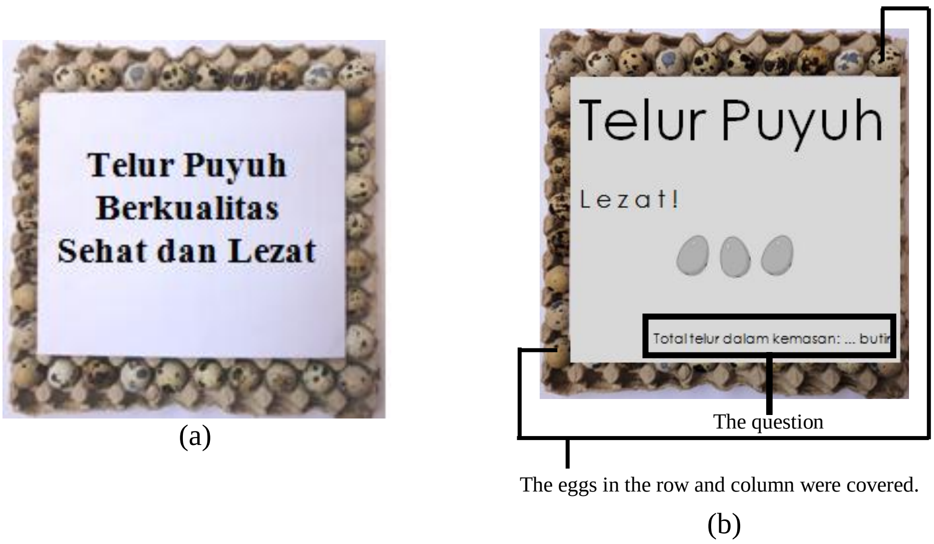
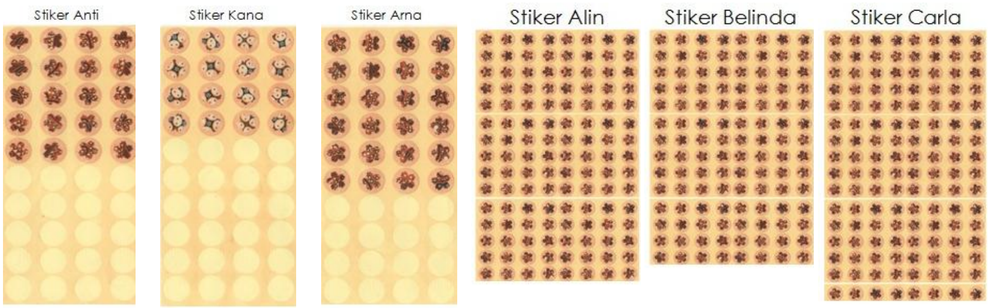



Figure 5.14: Redesigning the problem in Lesson 1; (a) before and (b) after.

**3. Redesigning the mathematics problem for Lesson 3**

As mentioned previously, Lesson 3 for Cycle 2 was about introducing the one-less/one-more strategy. Based on Finding [25], there is a suggestion to change the array size of the initial problem; it suggests to present bigger arrays (above multiplication by 5). Since the arrays in the initial problem were uncovered, the bigger arrays represented multi-digit multiplications was chosen and so the implicit attempt to prompt the halving strategywas diminished (Figure 5.15).




(a) (b)

Figure 5.15: Redesigned the problem context in Lesson 3; (a) before and (b) after.

**4. Redesigning the mathematics problem for Lesson 4**

As mentioned previously, Lesson 4 for Cycle 2 was about introducing the doubling strategy. Based on Finding [25], there is a suggestion to change the array size of the initial problem; it suggests to present bigger arrays (above multiplication by 5). Instead of representing multi-digit multiplication as in Lesson 3, since the array in the initial problem was partially uncovered, the bigger array chosen still represent the multiplication basic fact, but above multiplication by 5. Also, following the modification for the problem inLesson 1, a column uncovered was in the left side (Figure 5.16).


(a) (b)

Figure 5.16: Redesigning the problem in Lesson4; (a) before and (b) after.

**5. Designing a preliminary activity for each Lesson**

Based on Conclusion [26], there is a suggestion to conduct a preliminary activity in every lesson.For this study, these activities were designed for a classroom setting. Therefore, quick images were chosen to be the appropriate means (Van Galen & Fosnot, 2007). Quick images, typically presented through a configuration of dots, are a rich activity to promote conceptual of subitizing: instantly seeing how many (Clements, 1999).

A quick image asks students to explain how the dots organized in order to calculate the total number of dots in the image, after briefly viewing the image(Clements, 1999). Therefore, these materials would enforce students to find the multiplication and the idea of the multiplicationstrategies represented in the arrays instantly.

For Lesson 1, the quick imagewas a poster presented $10 \times 10$ square images(Figure 5.17). They were divided in five columnsof red and blue squares. The outers of squares in the top and right sides were boldfor focus of attention. The instruction was designed to find the multiplication factors represented inarrangements that were shown gradually, so the students could move from counting one by one to repeated addition and then last to multiplication. The pattern of multiplication ten and five could be recognizedimplicitly through this activity.

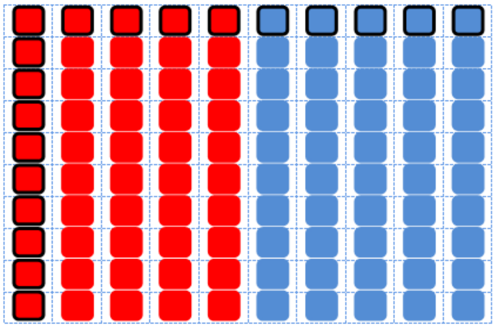
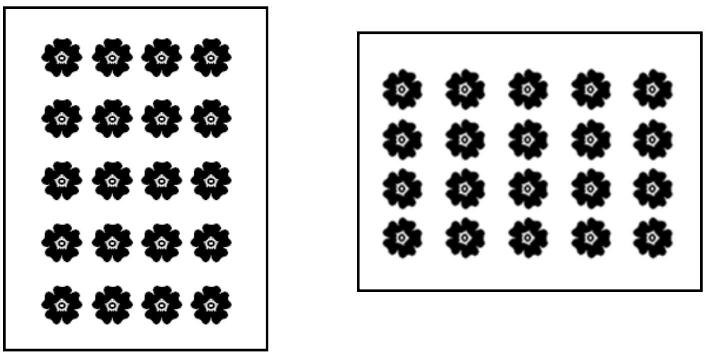


Figure 5.17: Quick image for Lesson 1 – Cycle 2.

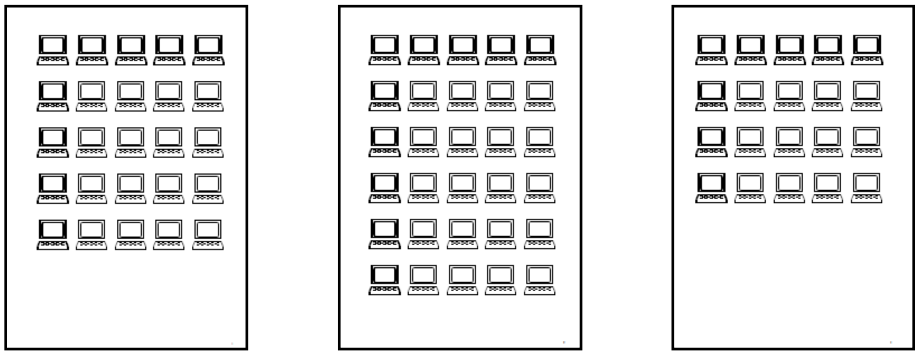
For Lesson 2, the quick images were a poster presented flower images in array of $5 \times 4$ and the same picture turned 90° (Figure 5.18). The array sizewas chosen since this multiplication was considered as one of the easier facts to calculate so that determining the products would not be the focus. The instruction was designed to provide a visual representation of the commutative property.



(a) (b)

Figure 5.18: Quick image for Lesson 2 – Cycle 2; (a) initial state and (b) rotated $90^0$

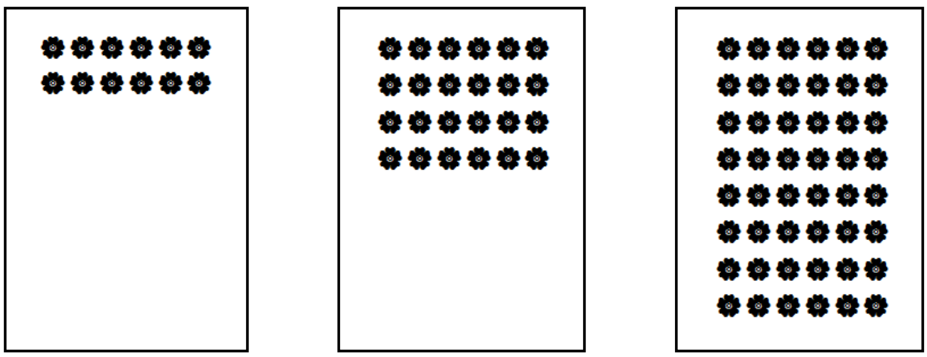
For Lesson 3, the quick images were three posters presented computer images in array of 5 × 5, 6 × 5, and 4 × 5 (Figure 5.19). The array sizes were chosen since delivering the idea of one-less/one-more strategy using multiplication by five was considered easier; adding and subtracting five were easy to calculate. Besides, implicitly, they introduced the pattern of multiplication by five.



(a) (b) (c)

Figure 5.19: Quick images for Lesson 3 – Cycle 2.

For Lesson 4, the quick images were three posters presented flower images in array of 2 × 6, 4 × 6, and 8 × 6 (Figure 5.20). The array sizes were chosen assuming that 2 × 6 was considered as an anchor fact, where there are many students who will be familiar to this multiplication. Therefore, determining the other products using the doubling strategy could occur.



(a) (b) (c)

Figure 5.20: Quick images for Lesson 4 – Cycle 2.

### 6. Adjustingan Instruction

Based on Conclusion[23], instead of asking how many eggs in a row and in a column, the guiding questions used to encourage the students to find the multiplication represented in arrays are changed to “How many eggs in a row?” and “How many rows are there?”.

## L. Remark 4

All modifications and designs presented in the previous subchapter were implemented in the revised HLT; presented in Chapter IV. Since the hypothesized of the students’ starting point to conduct all lessons in the revised HLT were not changed from the intial HLT, based on Finding [18], the students’ prior knowledge in Cycle 2 was consistent with the hypothesis of the students’ starting point to conduct Lesson 1. Therefore, Lesson 1was conducted directly after revising the initial HLT.

The retrospective analysis of the data collected from Lesson 1 in Cycle 2, using the revised HLT, is presented in the subchapter below.

## M. Retrospective Analysis: Lesson 1 – Cycle 2

Lesson 1 was about introducing arrays as multiplication models. There were two activities analyzed to provide an overview on how to introduce arrays as multiplication models. Activity 1 was a classroom activity. Activity 2 was a student activity.

The retrospective analysis of these activities is described below:

### 1. Looking Back: Video Recording of Activity 1

The classroom activityused a poster consisting of square images in

$10 \times 10$ to introduce arrays as multiplication models(Figure 5.21). The activity was about showing rows of squares in a short time (or, as quick images) and then asking the students to determine the total squares presented. The rows were conjectured to be uncovered progressively. By presenting these arrays, the students were expected to find the multiplications represented in the arrays.

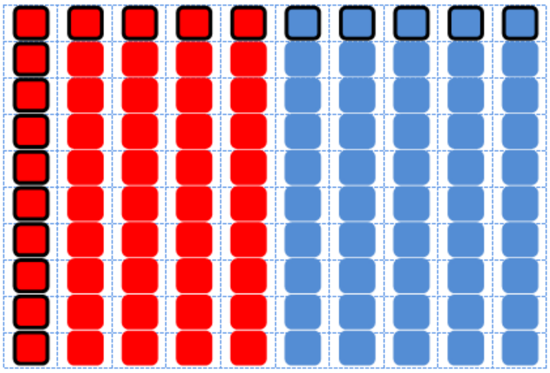


Figure 5.21: Quick images in Activity 1 – Lesson 1 – Cycle 2

Figure 5.22: The poster when it was still uncovered.

The teacher started the activity whilecovering theposter in the whiteboard (Figure 5.22). She uncovered the first row for about ten seconds and asked the students to determine the total number of thesquares shown.The students shouted the answer, it was 10. A studentwas seen counted by pointing his index finger along the row side (Figure 5.23). After confirming the students' answer, the teacher then started to make the students aware of the multiplication represented in the first row (Fragment 5.5).

(a) (b)

Figure 5.23: A student was pointing his index finger; (a) in the beginning of the counting, (b) in the end of the counting.

| | | | |
|---|---|---|---|
| | | | *In the whiteboard, the first row is uncovered.* |
| 22 | Teacher | : | "So, one row consists of (how many squares)?" |
| 23 | Students | : | "Ten." |
| 24 | Teacher | : | "One times ten is?" |
| 25 | Students | : | "Ten." |

Fragment 5.5: The teacher related the array in the first row to its multiplication.

The teacher continued the activity. She covered the poster, uncovered it until the second row, and asked how many squares shown. The students got the correct answer; it was 20. When the teacher asked how they got to the answer, there was a student, named Ranuh, mentioned the repeated additionThe teacher then related it to its multiplication (Fragment 5.6).

| | | | |
|---|---|---|---|
| | | | *The teacher asked the students how they got the total number of squares shown in two rows.* |
| 26 | Teacher | : | "Twenty? Where did you get it?" |
| 27 | Students | : | "Ten plus ten." |
| 28 | Teacher | : | "Yes, it is ten plus ten. As we have known that one row consists of?" |
| 29 | Students | : | "Ten." |
| 30 | Teacher | : | "So, two times (ten) is?" |
| 31 | Students | : | "Twenty." |

Fragment 5.6: The teacher tried to relate the array, repeated addition, and its multiplication.

The teacher covered the poster again, uncovered it until the third row, and asked how many squares shown. Most of the students shouted the answer was30, but there was a student, named Nabil,mentioning the multiplication represented in that square arrangement(Fragment 5.7). There was no discussion on how the student got the multiplication.

| | | |
|---|---|---|
| 32 | Teacher | : “Try (to determine how many squares are they)!” (*uncovering the poster until the third row*) |
| 33 | Nabil | : “Three times ten.” |

Fragment 5.7: A student found the multiplication represented in the shown array.

The teacher covered the poster again, uncovered it until the fourth row, and asked how many squares shown. After some students shouted the correct answer, the teacher directly uncovered all squares in the poster and asked how many squares shown. There were students pointing their index finger from the top row to figure out the answer (Figure 5.24). When the teacher asked how they got to the answer, some students used multiplication to explain their answer (Fragment 5.8).

Figure 5.24: Some students were trying to get the total number of all rows.

| | | |
|---|---|---|
| | | *The teacher uncovered all squares in the poster.* |
| 34 | Teacher | : “How many (squares)?”<br>*There were some students tried to count the total number of all rows by pointing their index finger from the top row.* |
| 35 | Students | : “Hundred!” |

| | | |
|---|---|---|
| 36 | Teacher | : "Where did you get the hundred?" |
| 37 | Students | : "Ten times ten." |

Fragment 5.8: The students used multiplication to explain how they got the answer.

After showing all squares in the poster, the teacher asked the students to determine the total number of the red squares shown only. Some students started to mention the multiplications to explain on how they got the total number of red squares shown (Fragment 5.9).

| | | |
|---|---|---|
| | | *The teacher uncovered the two first rows and asked the students to determine the total number of all red squares shown.* |
| 38 | Teacher | : "Ten? How did you get it?" |
| 39 | Yusuf | : "Two times five." |
| 40 | Teacher | : "Yes, two times five. What is the product of two times five?" |
| 41 | Students | : "Ten." |
| | | *The teacher uncovered the four first rows and asked how many red squares shown.* |
| 42 | Teacher | : "How many are there (the red squares shown)?" |
| 43 | Students | : "Twenty." |
| 44 | Teacher | : "Twenty? How did you get it?" |
| 45 | Ryan | : "Four times five." |
| | | *The teacher uncovered all rows and asked how many red squares shown.* |
| 46 | Teacher | : "How many are there (the red squares shown)?" |
| 47 | Students | : "Ten times five." |

Fragment 5.9: Some students mentioned multiplication to explain how they got the total squares presented.

## 2. Findings: Students' Answers in Activity 1 – Lesson 1

From the looking back of the video-recording of Activity 1 in Lesson 1, there are some findings on how the students answered the problem:

[28] Counting one by one was a strategy to determine the total squares in the first row since Mazta pointed his index finger along the row side; see Figure 38.

[29] When determining the total squares in the second row, repeated addition was evident as a strategy since Ranuh mentioned it; see Fragment 6: 27.

[30] When determining the total squares for other bigger arrays, some students gradually managed to mention the related multiplication; see Fragment 7: 33, Fragment 8: 37, Fragment 9: 39, 45, 47.

Based on the aforementioned findings, all students' actual answers are compared to the conjectures in the Table 5.1.

Table 5.1: Comparison between conjectures of students' answers and students' actual answers of Activity 1 – Lesson 1.

| Conjectures of Students' Answers | Students' Actual Answers |
|---|---|
| (1) Counting one by one.<br>(2) Using repeated addition.<br>(3) Representing the total objects in a row and a column into a multiplication and then finding the product of it. | • There was a student who used counting one by one to determine the total squares in the first row.<br>• There was a student who mentioned repeated addition to explain on how to determine the total squares in the second row.<br>• There were students who gradually mentioned the multiplications to explain on how to determine the total squares in the other bigger arrays. |

From thecomparison, the actual students' answers were not much different from the conjectures.

### 3. Findings: How Activity 1 – Lesson 1 Conducted

From the looking back of the video in Activity 1 – Lesson 1, there are some remarks on how the activity conducted:

[31] The teacher asked students to determine the total squares presented when one, two, three, four, and ten rows uncovered; see Fragment 5, Fragment 6, Fragment 7, and Fragment 8.

[32] After uncovering one and getting students' answer, the teacher related the arrays presented to its multiplication by asking "How many squares in a row?" and "What the product of ...?"; see Fragment 10: 22, 24. The

teacher also conducted similar activities when uncovering two rows; see Fragment 6: 28, 30.

[33] The teacher asked students to determine the total red squares presented when two, four, and ten rows uncovered; see Fragment 9.

Based on the aforementioned remarks, all activities are compared to the conjectures in the following table.

Table 5.2: Comparison between conjectures of activities and classroom actual activities of Activity 1 – Lesson 1.

| Conjectures of Activities | Classroom's Actual Activities |
|---|---|
| (1) Uncover rows as quick images, ask students to determine the total objects presented, and after that ask students how they got the answer.<br>(2) Relate the array to its multiplication by asking some guiding questions: "How many rows?", "How many objects in a row?", and "What is the product of ... ?"<br>(3) Gradually uncover the poster from only one row presented, two rows, three rows, four rows, and so on until all rows uncovered. | • The teacher asked students to determine the total squares presented when one, two, three, four, and ten rows uncovered<br>• The teacher related the arrays and the multiplications when one and two rows uncovered.<br>• The teacher related the arrays presented to its multiplication by asking "How many squares in a row?" and "What the product of ...?"<br>• The teacher asked students to determine the total red squares presented when two, four, and ten rows uncovered |

From the comparison, there are some findings could be taken:

[34] All rows were uncovered only after uncovering one, two, three, and four rows.

[35] Not all arrays presented were being related to its multiplication by asking some guiding questions, only two first arrays.

[36] The question "How many rows are there?" was not being asked when explaining on how the array and the multiplication were related.

**4. Looking Back: Video Recording of Activity 2in Lesson 1**

The activity was about determining the total number of eggs in a cartoon pack (Figure 5.25). Some eggs were covered on purpose by a label. The eggs were in array of $9 \times 10$.

Figure 5.25: Mathematical Problem in Activity 2 – Lesson 1 – Cycle 2.

The teacher started the activity: explained the problem, asked the students to work in a group, and then distributed the worksheet.The students in the focus group were Divan, Ranuh, and Rizal. After they got the worksheet, Rizal counted the eggs in the left side starting from the top. He got nine eggs. He then counted the eggs in the top row from the left, but starting from the second column (Figure 5.26). Thus, he also got nine eggs. He then concluded that the total number of eggs was the product of $9 \times 9$.

Figure 5.26: Rizal started to count the total number of eggs in the top row from the second column.

Rizal then told Divan and Ranuhthat the answer was $9 \times 9$, but he did not know the product of this multiplication and so did Divan and Ranuh.

Since they were still not certain with the answer, they asked the teacher if there were eggs below the cover. However, after some time, they started not to pay attention for solving the problem. The researcher, who sat closely, tried to help them (Fragment 5.11).

| | | | |
|---|---|---|---|
| | | | *Since they got the wrong multiplication expression, the researcher tried to make them see the repeated addition in the array first.* |
| 48 | Researcher | : | "How do you solve it?" |
| 49 | Students | : | *(no answer)* |
| 50 | Researcher | : | "How many eggs in a row?" |
| 51 | Rizal | : | "Nine." |
| 52 | Researcher | | "Really? Count it." |
| 53 | Rizal | | (*Pointing while counting the eggs in the top row starting from the right side of the picture and then stopped in the second column from the left*) "One, two, three, four, five, six, seven, eight, nine." |
| 54 | Researcher | | "How about this?" (*pointing the egg in the corner left*) |
| 55 | Rizal | | (*no answer*) |
| 56 | Researcher | | "So, how many eggs in this row?" (*pointing the top row of the picture*) |
| 57 | Rizal | | "Ten." |
| 58 | Researcher | | "Nine or ten?" |
| 59 | Rizal | | "Ten." |
| 60 | Researcher | | "How about (the total number of eggs) in the second row? Is it the same (number of eggs) as in the first row? |
| 61 | Rizal | | (*Counting the eggs in the left column starting from the second row*) "Eight." |
| | | | *The researcher asked the question again since Rizal did not pay attention to the question* |
| 62 | Researcher | | "If the first row, there are ten (eggs). How many are they in the second row?" |
| 63 | Divan | | "Ten." |
| 64 | Researcher | | "Although it is covered (*pointing the second row.*), do you think is it (having) the same (total number of eggs)?" |
| 65 | Divan | | "Yes." |
| 66 | Researcher | | "Why?" |
| 67 | Divan | | "Because the first row is ten." |
| 68 | Researcher | | "How about the third row?" |
| 69 | Divan | | "Ten." |
| 70 | Researcher | | "How about the fourth row?" |
| 71 | Divan | | "Ten." |
| 72 | Researcher | | "How about the fifth row?" |
| 73 | Divan | | "Ten." |
| 74 | Researcher | | "How about the sixth row?" |
| 75 | Divan | | "Ten." |

| | | |
|---|---|---|
| 76 | Researcher | "How about the seventh row?" |
| 77 | Divan | "Ten." |
| 78 | Researcher | "How about the eighth row?" |
| 79 | Divan | "Ten." |
| 80 | Researcher | "How about the ninth row?" |
| 81 | Divan | "Ten." |
| 82 | Researcher | "So, how many are they together?" |
| 83 | Rizal and Divan | "Ninety!" |

Fragment 5.11: The researcher helped the students to relate the array andits repeated addition.

Finally, they managed to get the answer; it was $90$. When they were writing down the answer in the worksheet, the researcher asked them on how they got to the answer. Divan tried to explain it using the repeated addition, but Ranuh interrupted and said it in multiplication expression (Fragment 5.12).

| | | |
|---|---|---|
| 84 | Researcher | "How did you get ninety?" |
| 85 | Divan | : "Ten plus ten plus ten ... (*Ranuh interrupted*)." |
| 86 | Ranuh | : "Ten times nine." |

Fragment 5.12: Ranuh find multiplication represented in the problem.

After all of students finished their work, the teacher asked the answer from each group and then wrote the students' solution in the whiteboard. All groups used multiplication,$9 \times 10$ or $10 \times 9$, to explain how they got to the total number of eggs in the carton pack. To close the activity, the teacher made a statement that all students got the correct answer.

## 5. Looking Back: Students' Worksheet of Activity 2in Lesson 1

Most of the students wrote multiplication to explain on how they got to the answer (Appendix D). Some of them also provided repeated addition to their explanation on how they got the product of the multiplication. Figure 5.27 is an example of students' answer written on the worksheet.

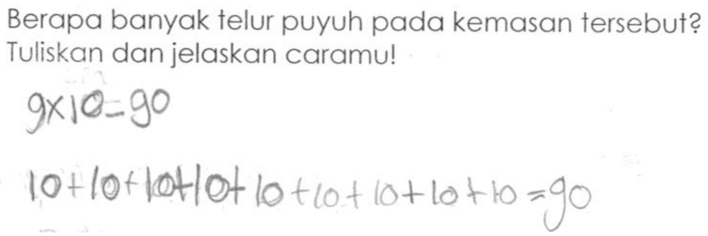



Figure 5.27: A student’s worksheet in Activity 2, Lesson 1, Cycle 2.

**6. Findings: Students’ Answers in Activity 2 – Lesson 1**

From the looking back of the video and the students’ worksheets in Activity 2 – Lesson 1, there are some findings on how the students answered the problem:

[37] When a nearly covered array presented, Rizal focused only on counting the total eggs in the top and left sides; see Figure 5.26.

[38] Rizal could not get the correct multiplication since he started to count the total eggs in the top row from the second column; see Figure 5.26.

[39] By implementing the guide presented in the HLT, all focused students could realize the repeated addition represented in the array and then finally led them to find the multiplication in the array; see Fragment 5.11 and Fragment 5.12.

[40] When a classroom discussion was conducted, all groups mentioned the multiplication represented in the array to explain on how they determined the total eggs. This condition was also evident in students’ worksheet; see AppendixD.

[41] To explain on how they determined the total eggs, most of the students wrote the multiplication in their worksheet. Some of them also provided

repeated addition to their explanation on how they determined the product of the multiplication; see AppendixD.

Based on the aforementioned findings, all students' actual answers are compared to the conjectures in the following table

Table 5.3: Comparison between conjectures of students' answers and students' actual answers of Activity 2 – Lesson 1.

| Conjectures of Students' Answers | Students' Actual Answers |
|---|---|
| (1) Counting through all eggs one by one.<br>(2) Using repeated addition.<br>(3) Writing repeated addition, reforming it to its multiplication, and then finding the product of the multiplication.<br>(4) Finding the total eggs in a row and a column and putting those two numbers into the multiplication and then finding the product of the multiplication. | • A focused student directly tried to finding out the multiplication represented in the array by counting the total number of eggs in a row and a column.<br>• Most of the students wrote multiplication as their solution to explain on how they determined the total eggs. Some of them also provided repeated addition as their explanation on how they got the product of the multiplication. |

From the comparison, there are some findings could be taken:

[42] There is no evidence showing a student counting through all eggs one by one.

[43] There is no evidence showing a student using repeated addition only or using repeated addition and then came up to the multiplication.

## 7. Findings: How Activity 2 – Lesson 1 Conducted

From the looking back of the video in Activity 2 – Lesson 1, there are some remarks on how the activity conducted:

[44] The teacher explained the problem, asked the students to work in a group, and then distributed the worksheet.

[45] The teacher conducted the classroom discussion to find out how the students determined the answer.

[46] Since the students already mentioned the multiplication to explain how they determined the answer, the teacher only verified that their answer was correct.

Based on the aforementioned findings, all activities are compared to the conjectures in Table 5.4.

Table 5.4: Comparison between conjectures of activities and classroom actual activities of Activity 2 – Lesson 1.

| Conjectures of Activities | Classroom Actual Activities |
|---|---|
| (1) Explain the context of the problem.<br>(2) Ask students to work on the problem in-group.<br>(3) Discuss the students' answers. | The teacher explained the problem, asked the students to work in-group, and then distributed the worksheet. After that, the teacher conducted the classroom discussion to verify students' answers. |

From the comparison, the actual activities were not much different from the conjectures so the activity was generally conducted as planned.

## N. Remark 5

As mentioned inChapter III, the retrospective analysis was conducted after all lessons were conducted. However, after Lesson 1, a brief analysis was conducted to find out if the students' knowledge was consistent to the starting point for conducting the next lessons. Providing the evidences on the students' worksheets and their answer in the discussion, it was concluded that the students could find multiplication represented in arrays. Therefore, there was no adjustment to the materials used. The retrospective analysis of the data collected inLesson 2, 3, and 4 is presented in the next subchapters.

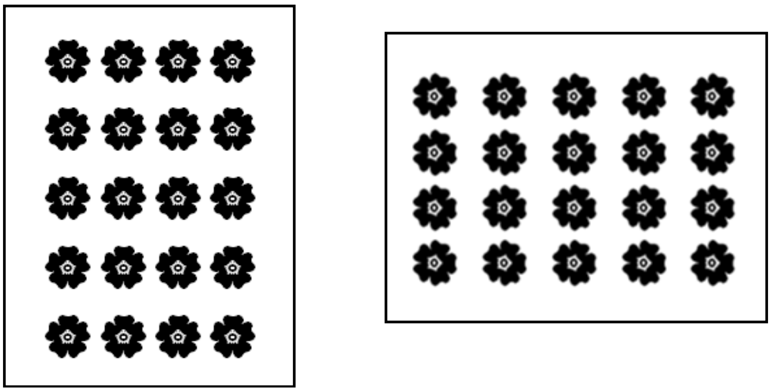
## O. Retrospective Analysis: Lesson 2 – Cycle 2

Lesson 2 was about introducing the commutative property. There were two activities analyzed to provide an overview on how to introduce the commutative property of multiplication. Activity 1 was a classroom activity. Activity 2 was astudent activity.

The retrospective analysis of these activities is described below:

**1. Looking Back: Video Recording of Activity 1 in Lesson 2**

The classroom activity used a poster and its $90^0$ -rotated posterto introduce the idea of the commutative property (Figure 5.28). The initial arrangement presented flower images in $5 \times 4$ .The activity was about presenting the posters as quick imagesso the students need to find two multiplications having the same factors but are getting from two different arrangements, and then making them see that the products are the same.



(a) (b)

Figure 5.28:Quick images in Activity 1 – Lesson 2 – Cycle 2; (a) initial state and (b) rotated $90^0$.

The teacher started the activity showing the first poster in a short time and asked the students to determine the total number of flower images in the poster. The teacher then pointed three students to tell their answer and then write their solution in the whiteboard. The three students use multiplication to explain how they got the answer. However, one student wrote a wrong answer: $4 \times 6 = 24$ (Figure 5.29).

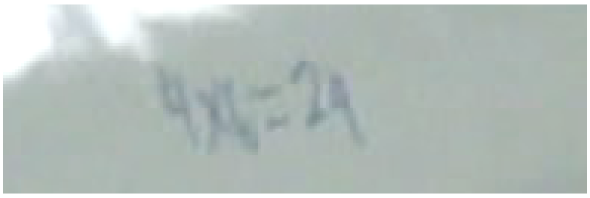


Figure 5.29: A student wrote a wrong answer to determine the total flower images.

Since there was a wrong answer, the teacher showed the poster once again, but this time in a longer time. The teacher noticed that there were students who tried to count one by one and so she told them that counting one by one would not help them to determine the answer. After making it clear that the answer was $5 \times 4 = 20$, the teacher continued the activity.

The teacher rotated the poster, showed it in a short time, and asked the students to determine the total flower images presented. The teacher then pointed two students to tell their answer and write their solution in the whiteboard. The two students use multiplication to explain how they got to the answer, it was $4 \times 5 = 20$.

After that, the teacher explainedhow to get those two multiplications from the two arrangementspresented in the poster. She then showed howthe product of $4 \times 5$ will be the same as the product of $5 \times 4$ and mentioned it because of the commutative property (Fragment 5.13). To conclude the activity, together with the students, the teacher calculated the products of $4 \times 5$ and $5 \times 4$ using repeated addition to show those two multiplications have the same products.

| | | |
|---|---|---|
| 87 | Teacher | "(The product of) four times five is the same with (the product of) five times ... ?" |

| | | | |
|---|---|---|---|
| 88 | Students | : | "Four." |
| 89 | Teacher | : | "That is (because of) the commutative property (of multiplication). So, if we revers (the position of the factors), the product is still the same. Like we did before (using the poster), (*the teacher showed the poster*), on this arrangement (the total objects are) 20, (*the teacher rotated the poster*), on this arrangement (the total objects are also) 20. No image missing (when the poster being rotated). Okay? Clear? |
| 90 | Students | : | Clear! |

Fragment 5.13: The teacher discussed about the commutative property.

**2. Findings: Students' Answers in Activity 1 – Lesson 2**

From the looking back of the video-recording of Activity 1 in Lesson 2, there are some findings on how the students answered the problem:

[47] To determine the total flower images in the first array, there was a student who still tried to count one by one. Meanwhile, some students, who were pointed by the teacher to write their answer in the whiteboard, wrote the multiplication as their explanation on how they determined to the answer.

[48] To determine the total flower images in the second array, some students, who were pointed by the teacher to write their answer in the whiteboard, also only wrote the multiplication as their explanation on how they determined to the answer.

Based on the aforementioned findings, all students' actual answers are compared to the conjectures in Table 5.5. From the comparison, there are some findings could be taken:

[49] There is no evidence showing a student using repeated addition.

[50] There is no evidence showing a student mentioning the posters are the same to explain the answer or to explain how to determine the

multiplication product.

Table 5.5: Comparison between conjectures of students' answers and students' actual answers of Activity 1 – Lesson 2.

<table>
<tr><th colspan="3">Conjectures of Students' Answers</th><th>Students' Actual Answers</th></tr>
<tr><td colspan="4">To determine the total objects presented in:</td></tr>
<tr><td rowspan="4">Second array</td><td rowspan="3">First array</td><td rowspan="3">(1) Attempting to count one by one or using repeated addition.<br>(2) Finding the multiplication represented in the array and then determining its product.</td><td>First array</td></tr>
<tr><td>• Attempting to count one by one.<br>• Only mentioning the multiplication to explain on how to get the answer.</td></tr>
<tr><td>Second array</td></tr>
<tr><td></td><td>(3) Realizing the poster are the same but only have different arrangements, so the answer will be the same with the previous one.<br>(4) Finding multiplication of this array, realizing that the poster are the same so the product of the multiplication is the same with the previous one.</td><td>Only mentioning the multiplication to explain on how to get the answer</td></tr>
</table>

## 3. Findings: How Activity 1 – Lesson 2 Conducted

From the looking back of the video in in Activity 1 – Lesson 2, there are some findings on how the activity conducted:

[51] The teacher showed the first array as a quick image, asked the students to determine the total flower images presented, and then pointed some students to tell the answer and wrote how they determine the answer.

[52] The teacher then rotated the array $90^0$, showed it as a quick image, asked the students to determine the total flower images presented, and then pointed some students to tell the answer and wrote how they determine the answer.

[53] Since the students have found the multiplications and their products, the teacher concluded that the products of the multiplication are the same;

see Fragment 5.13.

[54] The teacher explained the commutative property by rotating the poster and telling the students there was no object missing so that two arrangements could have the same total objects presented; see Fragment 5.13.

[55] Together with the students, the teacher also calculated the product of the multiplication using repeated addition.

Based on the aforementioned remarks, all activities are compared to the conjectures in Table 5.6.

Table 5.6: Comparison between conjectures of activities and classroom actual activities of Activity 1 – Lesson 2.

| Conjectures of Activities | Classroom's Actual Activities |
|---|---|
| (1) Show a poster as a quick image and ask students to determine the total objects presented, ask how the students got the answer, and discuss on how to get the answer.<br>(2) Rotate the poster, and conduct the activities as mentioned in (1).<br>(3) If the students still hardly to find multiplication represented in the arrays, guide them using similar instruction in Lesson 1.<br>(4) If the students have found the multiplications, discuss about the commutative property by calculating the product of the multiplications and comparing the factors of the multiplications.<br>(5) If there are students who use the idea of the commutative property, ask them to explain it first before explaining about the commutative property. | • The teacher showed the first array as a quick image, asked the students to determine the total flower images presented, and then pointed some students to tell the answer and wrote how they determine the answer.<br>• Similar activities were also conducted when its $90^0$-rotated poster was being showed.<br>• Since the students have found the multiplications and their products, teacher concluded that the products of the multiplication are the same and explained the commutative property by rotating the poster and telling the students there was no object missing so that two arrangements could have the same total objects presented before calculating the product of the multiplication using repeated addition. |

From the comparison, there are some remarks could be taken:

[56] There were not much different from the conjectures when the actual

activities were conducted so the activity was generally conducted as planned.

[57] Instead of only calculating the product of the multiplication, the teacher showed the idea of the commutative property by rotating the poster and mentioning there were no images missing so that two arrangements could have the same objects presented.

**4. Looking Back: Video Recording of Activity 2 in Lesson 2**

The activity was about determining who has more toy cars, Race or his brother. There were two toy car arrangements (Figure 5.30): the left one was Race's which arranged in $7 \times 8$ and the right one was his brother's which arranged in $8 \times 7$. The multiplication fact represented in the first arrangement was the anchor fact to determine the total toy cars in the second arrangement. By presenting these two toy cars arrangements, the commutative property of multiplication was expected to be elicited and introduced.

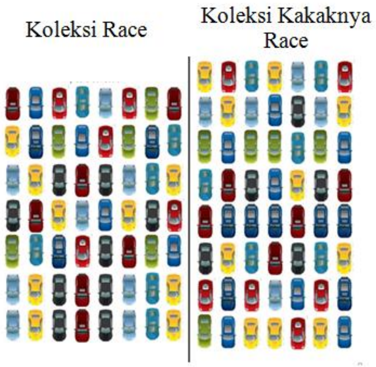



Figure 5.30: Mathematical Problem in Activity 2 – Lesson 2 – Cycle 2.

The teacher started the activity: distributed the worksheet to students, asked them to work in a group, and explained the problem. The students in the focus group are Divan, Ranuh, and Rizal. After they got the worksheet and read the problem, Divan and Ranuh started individually and then counted the Race's cars one by one, meanwhile Rizal only stared the worksheet. After counting until the second row, Divan gave up and asked Ranuh who was still counting the cars one by one (Figure 5.31).

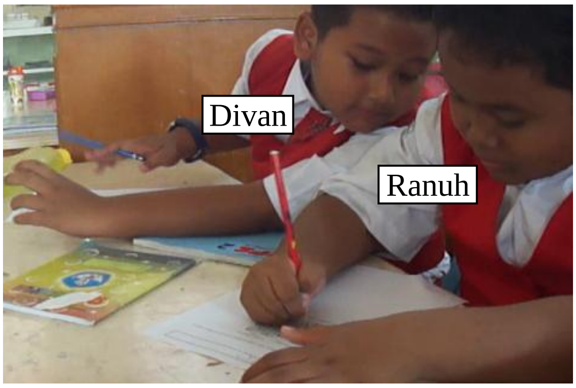



Figure 5.31: Divan looked Ranuh who were still counting the toy cars one by one.

Based on Ranuh's calculation, there were 57 cars of Race's. After finished counting Race's cars, Ranuh continued counting Race's brother's cars; and got 46 cars. He then concluded that Race had more cars. Rizal who was lookedcounting the first row of each car collections agreed to Ranuh's answer, and so did Divan.

When they were discussing how to write thestrategy on how they got to the answer, the teacher overheard them and realized that they got the wrong answer. She then approached them and asked them to find the multiplication for both toy car arrangements (Figure 5.33).The teacherhelped them to find the multiplication represented in Race's arrangement. The students thenfound the product using the finger technique (Figure 5.33). They did the same approach to find the total number of Race's brother's toy cars. They then concluded that the total number of Race's and his brother's cars are the same.

Figure 5.32: The teacher guided Divan and Ranuh to find the multiplications.

Figure 5.33: Divan and Ranuh determined the product using finger-technique.

The teacher conducted a discussion after all students finished their work and before collected the students' worksheet. The teacher started by pointing out that the idea of counting one by one was taking time, and then lead to the idea of the commutative property (Fragment 5.14).

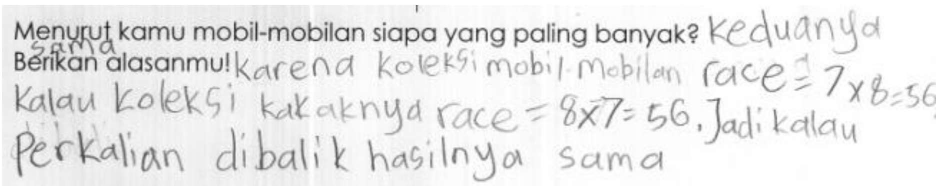
| | | |
|---|---|---|
| 91 | Teacher | : "I saw there were students who still counted one by one (to determine the total number of cars). It took a long time (to get the answer), didn't it?" |
| 92 | Students | : "Yes, it took a long time." |
| 93 | Teacher | : "Ajeng, how did you do it?" |
| 94 | Ajeng | : "I counted (the cars in) the top row one by one, ..." *Using Ajeng's explanation as based of conversation, the teacher started to discuss the solution with all students.* |
| 95 | Teacher | : "Yes. Take a look on Race's, what is the total number of cars in the top row?" |
| 96 | Students | : "Eight!" |
| 97 | Teacher | : "And what is the total number of cars in one column?" |
| 98 | Students | : "Seven!" |
| 99 | Teacher | : "So?" |

| | | |
|---|---|---|
| 100 | Students | : "Fifty six."<br>*The teacher changed the students answer into multiplication.* |
| 101 | Teacher | : "Seven times eight, what about his brother's, (*what the multiplication representation is*) ...?" |
| 102 | Students | : "Eight times seven." |
| 103 | Teacher | : "Seven times eight, eight times seven, are the products the same?" |
| 104 | Students | : "They are the same." |
| 105 | Teacher | : "So, the (factors of) multiplication can be swapped. This is called the commutative property (of multiplication)." |

Fragment 5.14: The discussion lead to the idea of the commutative property.

## 5. Looking Back: Students' Worksheet of Activity 2 in Lesson 2

Most of the students wrote multiplication to explain on how they got to the answer (Appendix D). There were two students, Shafa and Krishna, who wrote the conclusion that led to the idea of the commutative property; Figure 5.34.




The student wrote: "Because Race's toy cars is $7 \times 8 = 56$, and his brother's is $8 \times 7 = 56$. So, if the (factors of) multiplication are swapped, the product will be the same."

Figure 5.34: Shafa's worksheet in Activity 2, Lesson 2, Cycle 2.

## 6. Findings: Students' Answers in Activity 2 – Lesson 2

From the looking back of the video-recording and students' worksheet in Activity 2 – Lesson 2, there are some findings on how the students answered the problem:

[58] To determine the total toy cars of Race's and his brother's, Divan and Ranuh tried to count the objects one by one, but failed to get the correct result; seeFigure 5.31.

[59] Under the teacher's guidance, Divan and Ranuh could find the

correctmultiplications in the arrays; seeFigure 5.32.

[60] After Divan and Ranuh found the multiplication in the first array, they determined the product using finger-technique, and they also did to determine the total toy cars in the second array; see Figure 5.33.

[61] Divan and Ranuh concluded that both arrangements having the same total toy cars after they showed the products of the multiplications are the same.

[62] From students' worksheet, there were students, who wrote the conclusion that led to the idea of the commutative property; see Figure 5.34.

Based on the aforementioned findings, all students' actual answers are compared to the conjectures in Table 5.7.

Table 5.7: Comparison between conjectures of students' answers and students' actual answers of Activity 2 – Lesson 2.

| Conjectures of Students' Answers | Students' Actual Answers |
|---|---|
| (1) Attempting to count one by one.<br>(2) Using repeated addition.<br>(3) Finding the multiplication represented in each array, determining each product, and then concluding the total toy cars in each arrangement are the same.<br>(4) Finding the multiplications in the both arrays, realizing the factors are the same, and then concluding the products are the same and so are the toy cars in each arrangement. | • Using counting one by one but determines the wrong answer.<br>• Finding the multiplication in the first array and calculating its product using finger techniques. After that, doing the same activities to determine the total objects in the second array. |

From the comparison, there are some findings could be taken:

[63] Counting one by one was evident, but it was failed to help students to get the correct answer.

[64] There is no evidence showing a student realizing the factors of the multiplication are the same.

## 7. Findings: How Activity 2 – Lesson 2 Conducted

From the looking back of the video in Activity 2 – Lesson 2, there are some findings on how the activity conducted:

[65] The teacher distributed the worksheet to the students, asked them to work in a group, and explained the problem.

[66] The teacher wandered around to see the focused students' work so that she could help the focused students to find multiplication for each array; see Figure 5.32.

[67] When conducting a classroom discussion, the teacher pointed out that counting one by one will take a longer time to get the answer; see Fragment 5.14:91.

Based on the aforementioned remarks, all activities are compared to the conjectures in Table 5.8.

Table 5.8: Comparison between conjectures of activities and classroom actual activities of Activity 2 – Lesson 2.

| Conjectures of Activities | Classroom's Actual Activities |
|---|---|
| (1) Show the problem and introduce 'Who has more toy cars?' context. Then, ask students to work in-group and distribute the worksheet.<br>(2) If the students still hardly to find multiplication represented in the arrays, guide them using similar instruction in Lesson 1.<br>(3) If the students have found the multiplications, discuss about the commutative property.<br>(4) If there are students who use the idea of the commutative property, ask them to explain it first before explaining about the commutative property. | • The teacher distributed the worksheet to the students, asked them to work in a group, and explained the problem.<br>• The teacher wanders around so she could guide the focused students to find multiplication in the arrays.<br>• The teacher pointed out that counting one by one will take a longer time to get the answer.<br>• The teacher used a student's explanation as a base of conversation to make the students responded that two multiplications had the same product. |

From the comparison, there are some findings could be taken:

[68] There were not much different from the conjectures when the actual activities were conducted.

[69] The focused students were helped by the teacher's guidance when they solved the problem.

## P. Retrospective Analysis: Lesson 3 – Cycle 2

Lesson 3 was about introducing the one-less/one-more strategy of multiplication. There were two activities analyzed to provide an overview on how to introduce the one-less/one-more strategy of multiplication. Activity 1 was a classroom activity. Activity 2 was a student activity.

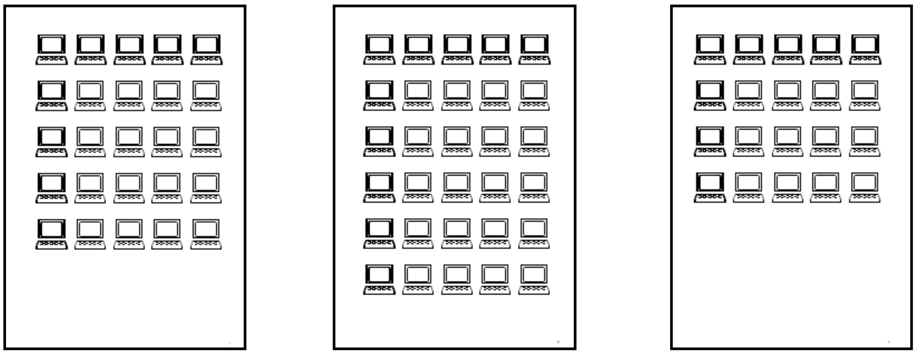
The retrospective analysis of these activities is described below:

### 1. Looking Back: Video Recording of Activity 1

The classroom activity used three posters to introduce the idea of the one-less/one-more strategy of multiplication. Each poster presented computer images, respectively, in $5 \times 5$, $4 \times 5$, and $6 \times 5$ (Figure 5.35). The activity was about presenting the postersas quick images one after the other. The multiplication fact in the first array was the anchor fact for determining the total stickers in the second and the third arrays.By showing the second and then the third arrays next to the first one and then relating the total computer images presented, the one-less/one-more strategy was expected to be elicited and introduced.

. The teacher started the activity showing the first poster in a short time and asked the students to determine the total computer images presented. A student, named Dyah, raised her hand, and was asked to explain her solution

(Fragment 5.15). Building on what Dyah had said, the teacher explained how to get to the multiplication. After that, the teacher put the first poster in the whiteboard.



(a) (b) (c)

Figure 5.35: Quick images in Activity 1 – Lesson 3 – Cycle 2.

| | | | |
|---|---|---|---|
| | | | *Dyah raised her hand after the teacher asked for the answer and the teacher pointed her to tell her answer.* |
| 106 | Dyah | : | "Twenty five!" |
| 107 | Teacher | : | "How did you get it? How did you determine it?" |
| 108 | Dyah | : | "Five times five." |

Fragment 5.15: Dyah found multiplication represented in the poster presented computer images in array of $5 \times 5$.

The teacher continued the activity and showed the second poster. The students got the answer; it was 30. When the teacher asked how they got to the answer, there were two multiplication mentioned, $6 \times 5$ and $5 \times 6$. Although the two answers were correct, it was important to see the array as $6 \times 5$, so the teacher explained how to get the expected multiplication.

After that, the teacher put the second poster next to the first poster (Figure 5.36), compared the total rows in the two posters, and that related that the total objects in the second poster could be derived by adding the total objects in a row to the total objects in the first poster(Fragment 5.16).

The teacher continued the activity showing the third poster. After the students got the total computer images and the multiplication represented in the poster, the teacher taped the third poster next to the first poster and discussed how the product of $4 \times 5$ could be derived from the product of $5 \times 5$ (Fragment 5.17).

Figure 5.36: The teacher showed there was one row more from the previous poster.

| | | | |
|---|---|---|---|
| | | | *The teacher asked the students to compare the computer arrangements in those two posters.* |
| 109 | Teacher | : | “You can see from this (*pointing the first picture*). It is five (rows). How many are they? Twenty five. So, you just add one row (to the second picture). How many are they?” |
| 110 | Students | : | “Five (computer images).” |
| 111 | Teacher | : | “So, twenty five plus five is ...?” |
| 112 | Students | : | “Thirty.” |

Fragment 5.16: The teacher discussed the one-more strategy.

| | | | |
|---|---|---|---|
| | | | *The teacher taped the third poster next to the first poster and then discussed the one-less strategy.* |
| 113 | Teacher | : | “Twenty five is five times five(*pointing the first poster*). Then, how about four times five (*pointing the third picture*)? You just have to ...” |
| 114 | Mazta | : | “You can erase it, or cut it.” |
| 115 | Teacher | : | “What else can you do?” |
| 116 | Students | : | (*no answer*) |
| 117 | Teacher | : | “You can subtract it, can’t you?” |
| 118 | Mazta | : | “Yes, you can scratch it also.” |

Fragment 5.17: The teacher discussed the one-less strategy.

## 2. Findings: Students’ Answers in Activity 1 – Lesson 3

From the looking back of the video-recording in Activity 1 – Lesson 3,

[70] The students only mentioned the multiplication to explain on how they

got the the total computer images in the first, second, and third arrays; seeFragment 5.15.

This finding is compared to its conjectures in Table 5.9. From the comparison, there are some findings could be taken:

[71] There is no evidence showing a student counting the objects one by one or using repeated addition to determine the total objects in the arrays.

[72] There is no evidence showing a student realizing the second and third arrays are one row less and one row more from the first one on determining the total objects in the second and third arrays.

Table 5.9: Comparison between conjectures of students' answers and students' actual answers of Activity 1 – Lesson 3.

<table>
<tr><th colspan="3">Conjectures of Students' Answers</th><th>Students' Actual Answers</th></tr>
<tr><td colspan="4">To determine the total objects presented in:</td></tr>
<tr><td rowspan="6">Second array</td><td rowspan="3">First array</td><td rowspan="3">(1) Attempting to count one by one or using repeated addition.<br>(2) Finding the multiplication represented in the array and then determining its product.</td><td>First array</td></tr>
<tr><td>Using multiplication to explain on how they got the total objects in the first array.</td></tr>
<tr><td>Second array</td></tr>
<tr><td rowspan="3"></td><td rowspan="3">(3) Realizing the array is one row more/one row less from the first array, and then adding/subtracting the total objects in one row to the total objects in the first array.<br>(4) Finding multiplication in the array presented, realizing the array is one row more/one row less from the first array, and then adding/subtracting the total objects in one row to the total objects in the first array to get the product of the multiplication (using one-more/one-less strategy).</td><td>Using multiplication to explain on how they got the total objects in the second array.</td></tr>
<tr><td>Third Array</td></tr>
<tr><td>Using multiplication to explain on how they got the total objects in the third array.</td></tr>
</table>

## 3. Findings: How Activity 1 – Lesson 3 Conducted

From the looking back of the video in Activity 1 – Lesson 3, there are some findings on how the activity conducted:

[73] The teacher showed the first poster as a quick image, asked the students to determine the total flower images presented, asked what the answer and then asked how the students got the answer. After that, she conducted similar activities to show the second poster and then conducted a discussion to compare the first and the second posters to introduce the idea of one-more strategy since the students already got the multiplication represented in the arrays; see Fragment 5.16 and Figure 5.36. She also conducted the similar activities when showing the third poster and then conducted a discussion to compare the first and the third posters to introduce the idea of one-less strategy, see Fragment 5.17.

[74] In the discussion, the teacher managed to get a response from a student about the idea of one-less strategy. The student said that to get the total objects in the third array, they could erase or cut or scratch the last row of the first arrays; see Fragment 5.17: 114, 118.

Based on the aforementioned remarks, all activities are compared to the conjectures in Table 5.10.

Table 5.10: Comparison between conjectures of activities and classroom actual activities of Activity 1 – Lesson 3.

| Conjectures of Activities | Classroom's Actual Activities |
|---|---|
| (1) Show the first poster as a quick image, ask students to determine the total objects presented, ask what the answer is, ask how the students got the answer, and discuss on how to get the answer.<br>(2) Show the second poster as a quick image next to the first poster, and conduct the activities asmentioned in (1). | • The teacher showed the first poster as a quick image, asked the students to determine the total flower images presented, asked what the answer and then discussed on how the students got the answer.<br>• Similar activities conducted for the second and third posters. |

| Conjectures of Activities (cont) | Classroom's Actual Activities (cont) |
|---|---|
| (3) Show the third poster as a quick image next to the first poster, and and conduct the activities as mentioned in (1).<br>(4) If the students still hardly to find multiplication represented in the arrays, guide them using similar instruction in Lesson 1.<br>(5) If the students have found the multiplication but there is no student who come up to the idea of the one-less/one-more strategies, start the discussion by comparing the total objects between the first and the second poster and also between the first and the third poster.<br>(6) If there are students who use the idea of the one-less/one-more strategy, ask them to explain it first before explaining about the one-less/one-more strategy. | • Since the students have found the multiplication in the arrays but there is no one who came up to the idea of the one-less/one-more strategy, the teacher directly explained about the idea of the one-less/one-more strategy. |

From the comparison, the actual activities were not much different from the conjectures so the activity was generally conducted as planned.

**4. Looking Back: Video Recording of Activity 2**

The activity was about determining the total stickers in three three sticker packages placed next to each other: Alin's, Belinda's, and Carla's (Figure 5.37). The stickers were arranged in $15 \times 8$, $14 \times 8$, and $16 \times 8$ respectively. The multiplication fact in the first array was the anchor fact for determining the total stickers in the second and the third arrays. By showing the arrays next to each other, the idea of one-less/one-more strategy was expected to be elicited or introduced.

The teacher explained the problem, distributed the worksheet to each student, and then asked the students to work in-group. The students in the focus group are Divan, Rizal, Mazta, Satria. Faiz and Hamed.Mazta and Satria, individually, started to count the stickers in the top row and the leftside column to find the multiplication represented in Alin's; they got $15 \times 8$ and used short multiplication to find the product; they got $120$. Meanwhile, the rest of students were only looking on Mazta and Satria's discussion, sometime gave comments, or worked on the problem alone.

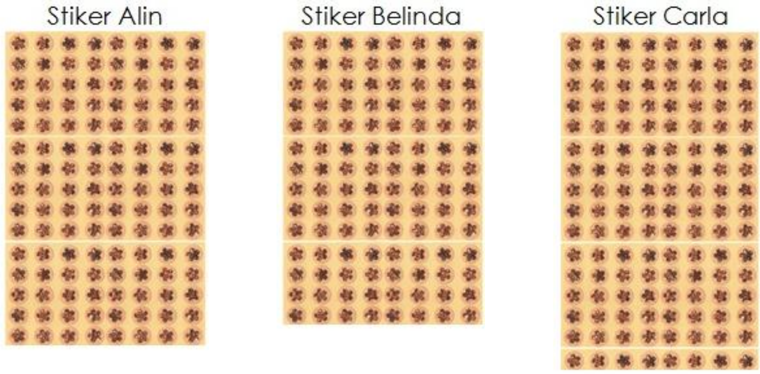



Figure 5.37: Mathematical Problem in Activity 2 – Lesson 3 – Cycle 2.

Figure 5.38: The teacher interrupted Satria who tried to get the product of $5 \times 8$ when use short multiplication to find the product of $15 \times 8$

Mazta and Satria continued to work on Belinda's. The teacher accidentally approached them and observed what they were doing. They used the same approach as before to get to the multiplication. They were also going to use short multiplication to determine the product, but the teacher interrupted them (Figure 5.38) and asked them to find a faster solution (Fragment 5.18).

| | | |
|---|---|---|
| | | *Mazta and Satria tried to find the product of* $14 \times 8$. |
| 119 | Satria | : "So, it is fourteen times eight?" |
| 120 | Mazta | : "One hundred and twenty one." (*Mazta tried to guess the product of fourteen times eight randomly*) |
| 121 | Satria | : "Fourteen times eight? Fourteen times eight? Four times eight? If eight times four ...?" (*Satria tried to find the product of fourteen times eight using short multiplication*) |
| 122 | Mazta | : "Eight times four? Sixteen plus sixteen." |
| | | *The teacher interrupted theMazta and Satria's work.* |
| 123 | Teacher | : "This (*pointing the students' work for the first problem*). You have found the product of $15 \times 8$, haven't you?" |
| 124 | Satria | : "Yes, we have." |
| 125 | Teacher | : "Now, $14 \times 8$. Can you find the product of fourteen times eight faster? How?" |
| 126 | Satria | : "By subtracting it." |
| 127 | Teacher | : "How many do you have to subtract?" |
| 128 | Satria | : "One row." |
| 129 | Teacher | : "How many things in a row?" |
| 130 | Satria | : "Eight." |
| 131 | Teacher | : "So? This (*pointing the product of* $15 \times 8$) subtracted by ...?" |
| 132 | Satria | : "One hundred and twenty subtracted by eight?" |
| | | *Satria raised his eight fingers and then counted backward to get the answer.* |
| 133 | Satria | : "120. 119, 118, 117, 116, 115, 114, 113, 112." |

Fragment 5.18: The teacher guided Satria and Mazta to use the one-less strategy.

Mazta and Satria continued to work on Carla's. Theyfound the multiplication represented was$16 \times 8$. When they tried to find the product, Mazta immediately realized the answer was $128$, but Satria kept tried to use short multiplication. The teacher asked him why he used short multiplication and he answered because he needed a solution to write (Fragment 5.19). Therefore, even he determined the Carla's stickers by adding $8$ to $120$, he wrote short multiplication as his solution to solve the problems (Figure 5.39).

| | | |
|---|---|---|
| 134 | Teacher | : "Why you use this (short multiplication) (*pointing to short multiplication as their solution on Belinda's in Satria's work*)?" |
| 135 | Satria | : "Because, it is the solution (to write)." |
| 136 | Teacher | : "Yeah, but you subtracted 8 from $120$, did you?" |
| 137 | Satria | : "Yes, but how (to write) that solution?" |

Fragment 5.19: Satria did not know how to write the actual solution he used in practice to determine the answer.

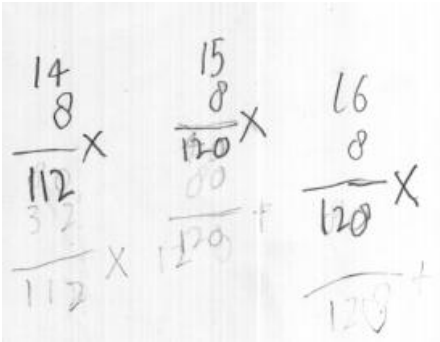


Figure 5.39: Satria's worksheet in Activity 2, Lesson 3, Cycle 2.

After the students finished their work, the teacher asked them to collect their worksheet as the end of the activity.

**5. Looking Back: Students' Worksheet of Activity 2**

Most of the students wrote multiplication to explain on how they got to the answer (Appendix D). There was a student, named Fika, who seemed used the one-less/one-more strategy to determine the total number of Belinda's and Carla's stickers. From her worksheet (Figure 5.40), she calculated the product of $15 \times 8$ using repeated addition, she got 112. Since her answer was wrong and also there was no other calculations using repeated addition to get the product $14 \times 8$ and $16 \times 8$, it showed that she subtracted 8 from 112 and added 8 to 112 to get the product of the two multiplications.

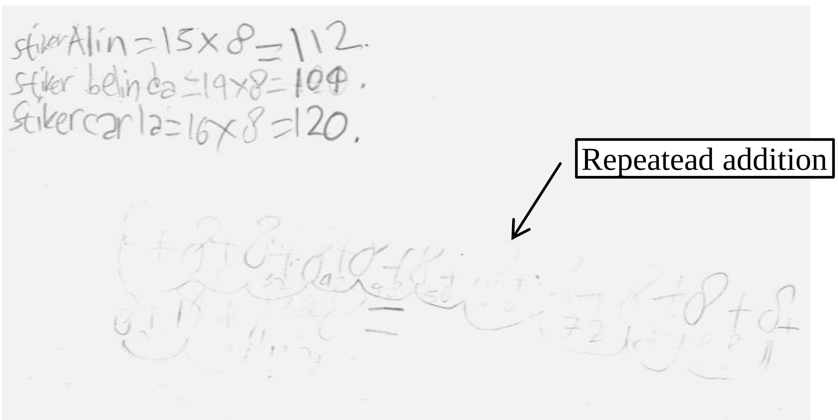



Figure 5.40: Fika's worksheet.

## 6. Findings: Students' Answers in Activity 2 – Lesson 3

From the looking back of the video-recording and students' worksheet in Activity 2 – Lesson 3:

[75] To determine the total stickers in the first array, Mazta and Satria found the multiplication first, and then determined the product using short-multiplication.

[76] Under the teacher's guidance; which is asking them to find a faster way, Mazta and Satria managed to use one-less/one-more strategy to determine the total stickers in the second and third array; see Fragment 5.18.

[77] After getting the product of the multiplication in the second array using the one-less strategy, Satria still tried to use short-multiplication as he said he did not know how to write the solution if he used the one-less strategy; see Fragment 5.19: 137.

[78] After using the one-less/one-more strategy, Mazta and Satria still wrote short-multiplication as their solution, see Figure 5.39.

[79] From a non-focused student's worksheet, Fika showed how she used repeated addition to determine the product in the first array and then used the one-less/one-more strategy to determine the total stickers in the second and the third array; see Figure 5.40.

Based on the aforementioned findings, all students' actual answers are compared to the conjectures in Table 5.11.From the comparison, there are some findings could be taken:

[80] There is no evidence showing a student counting the objects one by one or using repeated addition to determine the total objects in the first, second, or third array.

[81] There is no evidence showing a student only adding/subtracting the total objects in the first array to get the total objects in the second/third arrays.

Table 5.11: Comparison between conjectures of students' answers and students' actual answers of Activity 2 – Lesson 3.

<table>
<tr><th colspan="3">Conjectures of Students' Answers</th><th>Students' Actual Answers</th></tr>
<tr><td colspan="4">To determine the total objects presented in:</td></tr>
<tr><td rowspan="5">Second array</td><td rowspan="4">First array</td><td rowspan="4">(1) Attempting to count one by one or using repeated addition.<br>(2) Finding the multiplication represented in the array and then determining its product.</td><td>First array</td></tr>
<tr><td>Using multiplication to explain on how they got the total objects in the first array.</td></tr>
<tr><td>Second/Third array</td></tr>
<tr><td rowspan="2">• Finding the multiplication represented in the second array and then trying to determine its product using short multiplication, but got interrupted by the teacher. Under the teacher's guidance; which is asking them to find a faster way, managing to find the product of multiplication represented using the one-less/one more. But, still writing short multiplication on the worksheet.<br>• Finding multiplication represented in the first array, then determining its product using repeated addition. After that, using the one-less/one-more strategy to find the product in the second /third array.</td></tr>
<tr><td></td><td>(3) Realizing the array is one row more/one row less from the first array, and then adding/subtracting the total objects in one row to the total objects in the first array.<br>(4) Finding multiplication in the array presented, realizing the array is one row more/one row less from the first array, and then adding/subtracting the total objects in one row to the total objects in the first array to get the product of the multiplication (using one-more/one-less strategy).</td></tr>
</table>

## 7. Findings: How Activity 2 – Lesson 3 Conducted

From the looking back of the video in Activity 2 – Lesson 3, there are some findings on how the activity conducted:

[82] The teacher explained the problem, distributed the worksheet to each student, and then asked the students to work in-group.

[83] The teacher wandered around to see the focused students' work so that

she could help them to come up to the idea of the one-less/one-more strategy when trying to determine the product of the multiplication represented in the second and third arrays. She was asked them to find a faster way to get to the products by making use the designs; see Fragment 5.18: 125.

Based on the aforementioned remarks, all activities are compared to the conjectures in Table 5.12.From the comparison, there are some remarks could be taken:

[84] There was no a classroom discussion to discuss about the one-less/one-more strategy.

[85] The focused students were helped by the teacher's guidance when they use the one-less/one-more strategy.

Table 5.12: Comparison between conjectures of activities and classroom actual activities of Activity 2 – Lesson 3.

| Conjectures of Activities | Classroom's Actual Activities |
|---|---|
| (1) Show the problem, introduce 'How many stickers are there?' context, ask students to work in-group, and then distribute the worksheet.<br>(2) Collect the worksheets after the students finish their work, and then conduct a discussion.<br>(3) If the students still hardly to find multiplication represented in the arrays, guide them using similar instruction in Lesson 1.<br>(4) If the students have found the multiplications, discuss about the idea of the doubling strategy. | • The teacher explained the problem, distributed the worksheet to each student, and then asked the students to work in-group.<br>• The teacher wanders around so she could help the focused students.<br>• The teacher asked the focused students to find a faster way to determine the product of the second and third arrays using the design that led to the idea of one-less/one-more strategy.<br>• After the students finished their work, the teacher asked them to collect their worksheet to end the activity. |

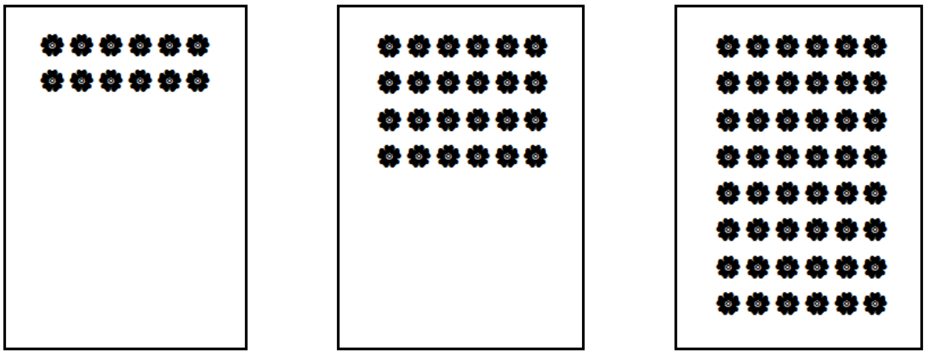
## Q. Retrospective Analysis: Lesson 4 – Cycle 2

Lesson 4 was about introducing the doubling strategy of multiplication. There were two activities analyzed to provide an overview on how to introduce the doubling strategy of multiplication. Activity 1 was a classroom activity. Activity 2 was a student activity.

The retrospective analysis forthese two activities is described below.

**1. Looking Back: Video-Recording ofActivity 1**

The classroom activity used three posters to introduce the idea of the doubling strategy of multiplication. Each poster presented flower images, respectively, in 2 × 6, 4 × 6, and 8 × 6 (Figure 5.41). The activity was about presenting the posters as quick images one after the other. The multiplication fact in the first array was the anchor fact for determining the total stickers in the second and the multiplication fact in the second array for determining the total stickers in the third array. By showing the second array next to the first array and the third array next to the second array and also relating the total flower images presented, the idea of doubling strategy was expected to be elicited and introduced.



(a) (b) (c)

Figure 5.41: Quick images in Activity 1 – Lesson 4 – Cycle 2

The teacher started the activity showing the first posterin a short time and asked the student to determine the total number of flower images. The students got the answer; it was 12. The teacher then asked the students to verify the answer together (Fragment 5.20). After that, the teacher put the first picture in the whiteboard.

| | | |
|---|---|---|
| 138 | Teacher | : "Count it." (*pointing the first row*) |
| 139 | Students | : "One, two, three, four, five, six." |
| 140 | Teacher | : "How many (group of) six are they?" |
| 141 | Students | : "Two." |
| 142 | Teacher | : "So, how many (flower images) are they?" |
| 143 | Students | : "12! Two times six!" |

Fragment 5.20: The teacher and the students found the multiplication represented in the array to verify the answer.

The teacher continued the activity and showed the second poster. The students got the answer; it was 24. The teacher thenput the second poster next to the first posterand tried to discuss how to get the total number of flower images using the doubling strategy (Fragment 5.21).

| | | |
|---|---|---|
| 144 | Teacher | "You have known that this, (the total number of flower images), is twelve (*pointing the first poster*). So, here (*pointing the second poster*), you just have to add ... ?" |
| 145 | Mazta | : "Add two rows." |

Fragment 5.21: The teacher tried to introduce the doubling strategy (1).

The teacher continued the activity and showed the third poster. The students got the answer; it was 48. The teacher then discussed how to get the total number of flower images in the second and third posters using the doubling strategy (Fragment 5.22).

| | | |
|---|---|---|
| | | *The teacher discussed the answers by comparing all pictures.* |
| 146 | Teacher | : "If you know that it is twelve (*pointing the first poster*). Then, (*pointing the second poster*), how many rows added?" |
| 147 | Students | "Two (rows)." |
| 148 | Teacher | "Two rows are twelve, so we add twelve to this (*pointing the second poster*). If you know that this is twenty four (flowers) (*pointing the second poster*), you just have to add (twenty four flowers) to this (*pointing the third poster*), haven't you? Which one faster (to determine the total number of the flowers in the second and third poster), add it or count it all over again?" |
| 149 | Students | "Adding it." |

Fragment 5.22: The teacher tried to introduce the doubling strategy (2).

## 2. Findings: Students' Answers in Activity 1 – Lesson 4

From the looking back of the video-recording in Activity 1 – Lesson 4:

[86] To determine the total flower images in the first array, a student mentioned the multiplication to explain on how they got the answer; see Fragment 5.20.

[87] The students only mentioned the correct answer of the total flower images in the second and third arrays.

Based on the aforementioned findings, all students' actual answers are compared to the conjectures in Table 5.13.

Table 5.13: Comparison between conjectures of students' answers and students' actual answers of Activity 1 – Lesson 4.

<table>
<tr><td colspan="3">Conjectures of Students' Answers</td><td>Students' Actual Answers</td></tr>
<tr><td colspan="4">To determine the total objects presented in:</td></tr>
<tr><td rowspan="4">Second array</td><td rowspan="3">First array</td><td rowspan="3">(1) Attempting to count one by one or using repeated addition.<br>(2) Finding the multiplication represented in the array and then determining its product.</td><td>First array</td></tr>
<tr><td>• There is evidence that the students mentioned the multiplication to explain on how they got the answer.</td></tr>
<tr><td>Second/Third array</td></tr>
<tr><td></td><td>(3) Realizing the array is two rows more from the first array, and then doubling the total objects from the previous one.<br>(4) Finding multiplication in the array presented, realizing that the array is two rows more from the first array, and then doubling the product of the previous one.</td><td>• Only got students' correct answer.</td></tr>
</table>

From the comparison, there are some findings could be taken:

[88] There is no evidence showing a student counting the objects one by one or using repeated addition to determine the total objects.

[89] There is no evidence showing a student realizing the second array is two rows more from the first array and the third array is two more rows from

the second array when determining the total objects or the multiplication products represented in the second and third arrays.

**3. Findings: How Activity 1 – Lesson 4 Conducted**

From the looking back of the video in Activity 1 – Lesson 3, there are some findings on how the activity conducted:

[90] The teacher showed the first poster as a quick image, asked the students to determine the total flower images presented, asked what the answer was, and then discussed on how to get the answer; see Fragment 5.20. After that, she conducted similar activities to show the second and third poster.

[91] When conducting the discussion after showing the second poster, she explained the use of doubling strategy to determine the answer and got a response from a student to add two rows from the array in the first poster to get the total objects in the second poster; see Fragment 5.21.

Based on the aforementioned remarks, all activities are compared to the conjectures in Table 5.14.

Table 5.14: Comparison between conjectures of activities and classroom actual activities of Activity 1 – Lesson 4.

| Conjectures of Activities | Classroom's Actual Activities |
|---|---|
| (1) Show the first poster as a quick image, ask students to determine the total objects presented, ask what the answer it, ask how the students got the answer, and discuss on how to get the answer.<br>(2) Show the second poster as a quick image next to the first poster, and conduct the activities as mentioned in (1).<br>(3) Show the third poster as a quick image next to the second poster, and conduct the activities as mentioned in (1). | • The teacher showed the first poster as a quick image, asked the students to determine the total flower images presented, asked what the answer was, and then discussed on how to get the answer.<br>• Similar activities conducted for the second and third arrays.<br>. |

| Conjectures of Activities (cont) | Classroom's Actual Activities (cont) |
| --- | --- |
| (4) If the students still hardly to find multiplication represented in the arrays, guide them using similar instruction in lesson 1.<br>(5) If the students have found the multiplication but there is no student who come up to the idea of the doubling strategy, start the discussion by comparing the total objects between the first and the second poster and also between the second and the third poster.<br>(6) If there are students who use the idea of the doubling strategy, ask them to explain it first before explaining about the doubling strategy. | |

From the comparisons, the actual activities were not much different from the conjectures so the activity was generally conducted as planned.

**4. Looking Back: Video-Recording ofActivity 2**

The activity was about determining the total stickers in a special package (Figure 5.42). The special package covered some stickers by the label. The stickers are arranged in $8 \times 6$ in total, but the label make the stickers are divided in two $4 \times 6$ . All stickers in the above part are uncovered. Meanwhile, only stickers in the left side are uncovered in the bellow part.By using this special package, the idea of doubling strategy was expected to be elicited and introduced.


Figure 5.42: Mathematical Problem in Activity 2 – Lesson 4 – Cycle 2.

The teacher distributed the worksheet to each student and then explained the problem. The students in the focus group are Divan, Rizal, Mazta, Satria, Faiz, and Hamed. After got the problem, Rizal counted the stickers in the left column, he got 8, and then counted the stickers in the top row, he got 6.He then looked Divan's work instead of finishing his work. Divan counted the entire sticker one by one. Although there were some stickers covered, he managed to get the correct answer (Figure 5.43).

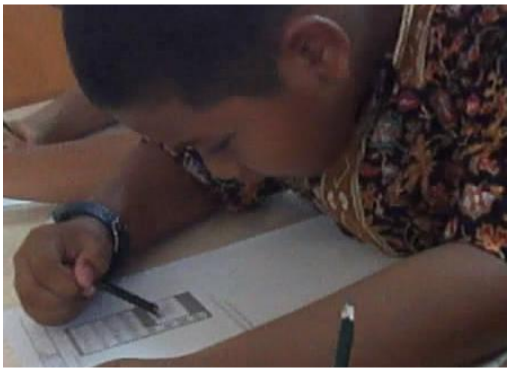


Figure 5.43: Divan counted the stickers covered by the label.

Satria andMaztamanaged to get the multiplication represented in the arrangement. Mazta seemed familiar with the factsince he got the product directly. Satriaused repeated addition to get the product(Figure 5.44). Meanwhile, Faiz and Hamed's work could not be seen from the video. After the students finished their work, the teacher asked them to collect their worksheet as the end the activity.

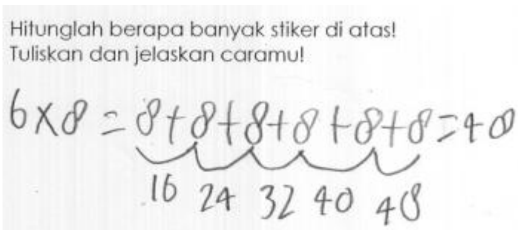



Figure 5.44: Satria using repeated addition to determine the multiplication product.

**5. Looking Back: Students' Worksheetof Activity 2**

Most of the students wrote multiplication to explain on how they got to the answer (Appendix D). One student from the focus group, named Faiz, seemed got the idea of doubling from the design, although he could not managed to relate this strategy in finding the multiplication product(Figure 5.45).


Figure 5.45: A student's worksheet in Activity 2 – Lesson 4 – Cycle 2.

**6. Findings: Students' Answers in Activity 2 – Lesson 4**

From the looking back of the video-recording and students' worksheets in Activity 2 – Lesson 4:

[92] To determine the total sticker, Divan managed to get the answer by counting one by one even there were objects covered; see Figure 5.43.

[93] To determine the total sticker, Satria found the multiplication represented in the array and used repeated addition to determine the product; see Figure 5.44.

[94] From a student's worksheet, Faiz used the doubling strategy but not as a way to determine the product of the multiplication; see Figure 5.45.

Based on the aforementioned findings, all students' actual answers are compared to the conjectures in Table 5.15.

Table 5.15: Comparison between conjectures of students' answers and students' actual answers of Activity 2 – Lesson 4.

| Conjectures of Students' Answers | Students' Actual Answers |
|---|---|
| (1) Attempting to count one by one.<br>(2) Using repeated addition.<br>(3) Finding the multiplication represented in the array and determining the product using repeated addition.<br>(4) Finding the multiplication represented in the array and determining the product using the doubling strategy. | • Counting one by one was evident to determine the correct answer.<br>• Finding the multiplication in the array and calculating the product using repeated addition.<br>• Using doubling but not as a strategy to find the product of multiplication. |

From the comparison, there are some findings could be taken:

[95] There is no evidence showing a student used repeated addition to determine the total stickers.

[96] There is no evidence showing a student used the doubling strategy to determine the product of the multiplication.

**7. Findings: How Activity 2 – Lesson 4 Conducted**

From the looking back of the video in Activity 2 – Lesson 4, the teacher distributed the worksheet to the students, explained the problem, asked the students to work on the problem. After that, she asked them to collect their worksheet as the end of the activity. This finding is compared to its conjectures in the following Table 5.16. From the comparison, there was no a classroom discussion to find out how the students solved the problem.

Table 5.16: Comparison between conjectures of activities and classroom actual activities of Activity 2 – Lesson 4.

| Conjectures of Activities | Classroom's Actual Activities |
|---|---|
| (1) Show the problem, introduce 'How many stickers are there?' context, ask | The teacher distributed the worksheet to the students, explained the problem, asked the |

| Conjectures of Activities (cont) | Classroom's Actual Activities (cont) |
|---|---|
| students to work in-group, and then distribute the worksheet.<br>(2) Collect the worksheets after the students finish their work, and then conduct a discussion.<br>(3) If the students still hardly to find multiplication represented in the arrays, guide them using similar instruction in lesson 1.<br>(4) If the students have found the multiplications, discuss about the idea of the doubling strategy. | students to work on the problem. After that, she asked them to collect their worksheet as the end of the activity. |

## R. Retrospective Analysis of Cycle 2: All Lessons

From the explanations in all lessons, there are some findings could be taken.

[97] From Activity 2 in Lesson 1, there was a student, named Rizal, who failed to find the correct multiplication represented in the array since he could not see an object simultaneously is in a row and a column. However, from Activity 2 in Lesson 4, there was evidence that he could get the correct multiplication represented in the array by himself.

[98] From Activity 2 in Lesson 2, there was a student, named Divan, who was gave up to determine the total 'rectangle-ish' objects in uncovered array since he lost the counting. However, from Activity 2 in Lesson 4, he managed to determine the total object presented in a nearly covered array by counting through the cover.

[99] From Activity 1 in Lesson 3, there is a student, named Mazta, who gave a response when a teacher explained about the one-less strategy. From Activity 2 in the same lesson, although under the teacher's guidance, he managed to use the strategy compared to his friend, Satria.

[100] From Activity 2 in Lesson 3, there is a student, named Satria, who directly gave a response that lead to the idea of one-more strategy after the teacher asked to use a faster way to determine the multiplication product by pointing the design. However, from Activity 1 in Lesson 3, Satria did not give any response to the students' explanation.

[101] From Activity 1 in Lesson 4, there is a student, named Mazta, who gave a response when a teacher explained about the doubling strategy. However, from Activity 2 in the same lesson, he did not use the strategy to determine the multiplication.

## S. Retrospective Analysis of Cycle 2: Posttest

From the students' posttest worksheets, there are some findings on how the students answered the problem:

[102] Most of the students used repeated addition as a strategy or an explanation to determine multiplication bare problems; see Appendix E.

[103] There are only few students who use other strategies; mainly they implemented the use of commutative property. Some of them showed how they elaborated repeated addition differently. For example, they wrote $5 \times 4 = 4 + 4 + 4 + 4 + 4$ in the pretest and then they changed it to $5 \times 4 = 5 + 5 + 5 + 5$ in the posttest, which is much easier to calculate; see Appendix E.

## T. Conclusion 2

Based on the findings generated from the teaching experiment in Cycle 2, there are some conclusions could be taken:

[104] Based on Finding[92] and[98], there were students who still determine the total objects in arrays using counting one by one so that they could not come up to the introduced strategies for determining the multiplication products. Meanwhile, from Finding[75], [76], and [93], the students who were able to find multiplication represented in the arrays were not having trouble when the teacher asked them to find strategies represented implicitly in the arrays. Therefore, these findings confirm the assumption saying that the students need to perceive the idea of arrays as multiplication models first before using arrays to introduce multiplication strategies.

[105] Based on Finding [28] and [29], when arrays presented as quick images for the first time, there were students who determine the total objects in smaller arrays by counting one by one or using repeated addition. But, when the arrays got bigger, these strategies could not be used as the arrays presented as quick images. From Finding[35], the teacher showed how the arrays and the multiplications were related when presenting the first two arrays, but from Finding [32] and [36], she forgot to ask “How many rows are there?” question. However, even the teacher only guiding them two times and forget to ask one of the guiding questions, there were students who gradually mentioned multiplication represented in the arrays, as can be seen in Finding [30]. Therefore, by asking students to determine the total objects in arrays presenting as quick images and gradually show bigger arrays and then showing how the arrays and the

multiplications were related, there are students who will determine the total objects in smaller arrays by counting one by one or using repeated addition, but then there are students who gradually mention the multiplication represented in bigger arrays.

[106] Based on Finding [39] and [41], the focused students could find the multiplication represented in the nearly-covered array that only uncovers the objects in the top and left sides. Therefore, the students can also be introduced to the idea of arrays as multiplication models by giving a problem that is presenting a nearly-covered array.

[107] Based on [37], after an activity presenting arrays as quick images and showing the arrays and the multiplication were related, a student focused to find the multiplication when a nearly-covered array presented. Therefore, when showing how arrays and multiplications were related and then presenting a nearly-covered array, there are students who will focus on counting the objects in the left and top sides to find the multiplication represented in the array.

[108] Based on Finding [38], the student got the wrong multiplication since he started to count the total eggs in the top row from the second column. That means this student could not see an object simultaneously is in a row and a column. Meanwhile, the problem was given after the classroom activity that was assumed could eliminate this condition. From Finding [32], [34], and [35], the teacher only showed how the arrays and the multiplications were related two times and did not ask one guiding

question when conducting the classroom activity. Therefore, this condition could be the reason why this student could not find the correct multiplication in the array. This conclusion is supported by Finding[97] where the same student finally got the correct multiplication in the last lesson. Therefore, to overcome the situation when a student could not see an object simultaneously in a row and a column, the teacher needs provide times on guiding students to realize the relation between the arrays and the multiplications. Also, the students need to be helped moving from seeing a repeated addition in an array to seeing the multiplication in the same array, as can be seen in Finding [39].

[109] From Finding [76], it showed how the students were able to mention the introduced strategies and did not reluctant to do it, even they have already used another strategy, that is the short-multiplication, but when the teacher asked them to find a faster way, they were open to use it. Although it is not direct evidence, this condition showed that conducting a classroom activity before students worked on the problem made the students did not refuse to the teacher's suggestion. Therefore, before the students work on a problem in the student activity, the classroom activity needs to be conducted first and being lead by the teacher so that all students can have a primary knowledge about the introduced strategies and also minimize a condition when there are students who refuse to hear the explanation about the introduced strategies.

[110] Finding [48], [70], and [86] showed that the students only mentioned the

multiplications to explain on how they got the total objects presented in the arrays so that there is no evidence showing whether the students came up to the strategies or could use the multiplication facts represented in the previous arrays to determine the total objects in the next arrays. Therefore, only asking on how the students get the answer is not enough. There is a need to ask about how the students determine the multiplication products to find out if they make use the previous fact or come up to the strategies.

[111] From Finding [54] and [57], instead of only determining the products of two multiplications with the same factors, the teacher used the array to show the idea of the commutative property by rotating the array. From Finding [74] and [91], there were students who gave response related to the introduced strategies when the teacher compared two related arrays to deliver the idea of the one-less/one-more strategy and the doubling strategy. Therefore, the arrays can serve as a visualization aid of the multiplication strategies.

[112] From Finding[58], the rectangle-ish objects made the students failed to count one by one so that they needed to find the multiplications represented in the arrays and then they could see two multiplications with the same factors are having the same product. Meanwhile, from Finding [76] and [94], using bigger arrays representing multi-digit multiplication made the students came up to the one-less/one-more strategies when the teacher asked them to find the faster way. From Finding [94], the

doubling strategy was elicited from the partially covered array. Therefore, the arrays need to be designed considering the size, the object shape, and the appearance (partially-covered/uncovered) to elicit the strategies that want to be introduced.

[113] From Finding [59], [92], and [98], there was a student who could find the multiplication in the array presented in Lesson 2 after being guided by the teacher and from Lesson 4, this students still used counting one by one to determine the total object in a partially covered array. This means this student still did not understand the idea of arrays as multiplication models. Therefore, if there is a student who still counts one by one after being introduced to the idea of arrays as multiplication model, this is an indication that the students still cannot find the multiplication represented in the array.

[114] After the students could find the multiplication represented in arrays. From Finding[60] and [93], the students who did not familiar to the facts, tried to use repeated addition, finger technique, or even try to use short-multiplication that have not been taught to determine the multiplication products. That means they did not see the arrays as a means to determine the unknown multiplication products so that the multiplication strategies cannot be elicited; they only saw the arrays as a source of the problem

[115] Finding [76], [77], and [78] showed that there were students who could used the one-less strategy after the teacher asked them to find a faster way by asking them to consider the arrays. However, they still did not try

to use the one-more strategy to determine the other array. They said that they did not know how to write the solution if they used the one-less/one-more strategy, so they chose to use short-multiplication. This means that there was a possibility that the students got the idea of the one-less/one-more strategy from the previous activity or from the designs they were working on, but they were reluctant to use it since this strategy was not formally introduced and explicitly showed on how to use it and how to write it. Therefore, to make the students confident to use the introduced strategies they find from the designs, the teacher needs to encourage them to write their own strategies and also emphasize on using the strategies they find from the problems/the designs.

[116] Finding [76] showed that the students could come up to the strategies being introduced after the teacher asked them to find a faster way to determine the total objects in the second and third arrays. This means the teacher had a great role to make the students used the strategies. Therefore, to encourage students to find the strategies from the designs/problems, the teacher needs to guide the students to use the introduced strategies, for example by asking them to find a faster way to derive other unknown facts from a known fact

## U. Remark 6

All aforementioned conclusions were constructed to answer the research question presented in Chapter VI.

# CHAPTER VI

# CONCLUSION AND DISCUSSION

This chapter presents answerto the research question, local instruction theory, reflection of this study, and some recommendations for further studies.

## A. Answer to the Research Question

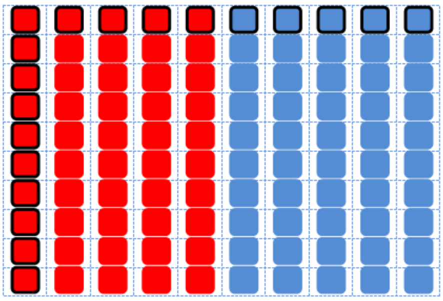
"*How can arrays be used to introduce multiplication strategies*?"

Before using arrays to help students learning multiplication strategies, arrays as multiplication models must be introduced first. To introduce it, students can be asked to determine the total objects in arrays through a classroom activity and a student activity. The classroom activity needs to be conducted first and being lead by the teacher so that all students can have a primary knowledge about the idea before they work on a problem in the student activity. The student activity is conducted so that the students can interact and work with each other to find the multiplication represented in the arrays.

For the classroom activity, the students can be asked to determine the total objects in arrays that are presenting as quick images. The arrays can be presented gradually starting from small arrays, like $1 \times 10$ and $2 \times 10$, to other bigger arrays, like $10 \times 5$ or $10 \times 10$. After the students determine the total object in an array, the array must directly be related to its multiplication by asking some guiding question, like "How many objects in a row?", "How many rows are they?" and "What the product of multiplication ... times ...?"

Through this activity, there are students who will determine the total objects in smaller arrays by counting one by one or using repeated addition, but then there are students who gradually mention the multiplication represented in bigger arrays.

After conducting the classroom activity, the students can also be introduced to the idea of arrays as multiplication models by giving a problem that is presenting a nearly-covered array that only uncovers the objects in the top and left sides; see Figure 6.1(b) for an example. The students are expected to work in-group. When solving the problem, the students who can follow the previous activity will focus on counting the total objects in the left and top sides, but it does not mean that they will get the correct multiplication. There are students who will find it difficult to see an object simultaneously in a row and a column on their counting. Therefore, to overcome this situation, the teacher needs to provide more times on guiding the students to show how the arrays and the multiplications are related and also on making them move from seeing a repeated addition to seeing its multiplication in an array.



(a)


(b)

Figure 6.1: Examples of materials to introduce the idea of arrays as models presented as (a) quick images and (b) a nearly-covered array.

After students are able to find the multiplications represented in arrays, they can be introduced to multiplication strategies using arrays. To introduce it, the students can also be asked to determine the total objects in arrays through a classroom activity and a student activity. Before the students work on a problem in the student activity, the classroom activity needs to be conducted first and being lead by the teacher so that all students can have a primary knowledge about the introduced strategies and also can minimize a condition when there are students who refuse to hear the explanation about the introduced strategies.

For the classroom activity, arrays are showed as quick images. The activity is about determining the total objects presented in arrays, where the total objects in the following arrays can be derived from the total object in the previous arrays, see Figure 6.2for an example. By showing arrays as quick images, the students were expected to find a faster way to determine the total objects, especially for the following arrays, so that the introduced strategies could be elicited. When conducting this activity, there are students who will only mention the multiplications to explain on how they determine the total objects in the following arrays so that it cannot show if the students make use the previous fact or come up to the strategies. Therefore, there is a need to ask about how the students determine the multiplication products.

After those two arrays are presented as quick images, the teacher needs to discuss how to determine the total objects in the following arrays using the strategies being introduced. Here, the arrays serve as a visualization aid of the multiplication strategies. For example, to introduce the commutative property, the teacher can rotate the array to show that two related arrangements could have the same total objects as a representation of two multiplications with the same factors have the same product. Also, to introduce the one-less/one-more strategy or the doubling strategy, putting those two arrays next to each other will make the students see that there were one row more, one row less, or two row more from the total rows in the previous array. When conducting this activity, there are students who will give responses leading to the idea of the multiplication strategies, such as saying “scratch a row” or “add two rows” from the previous array to get the total objects in the following arrays.

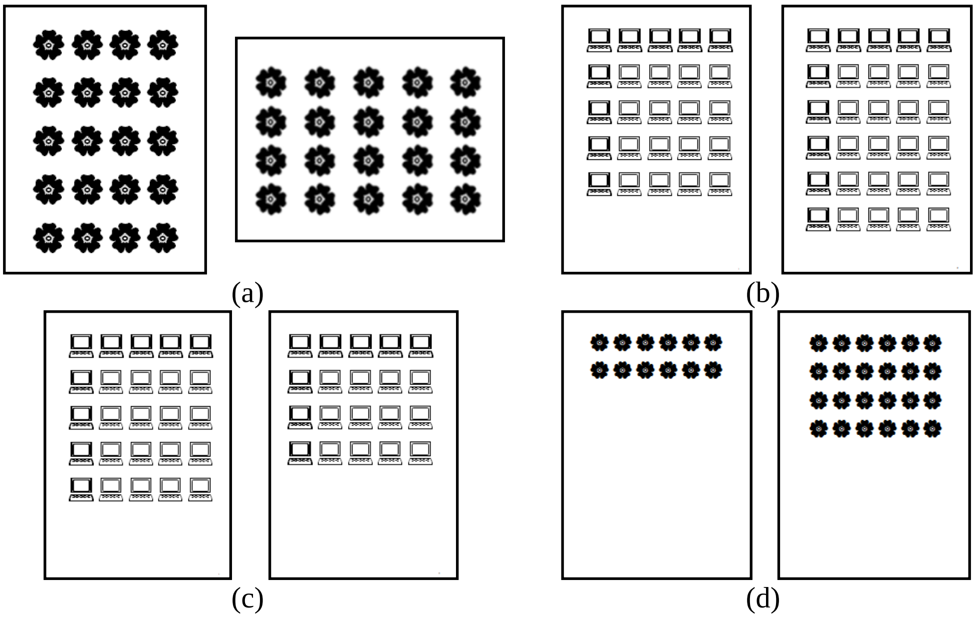
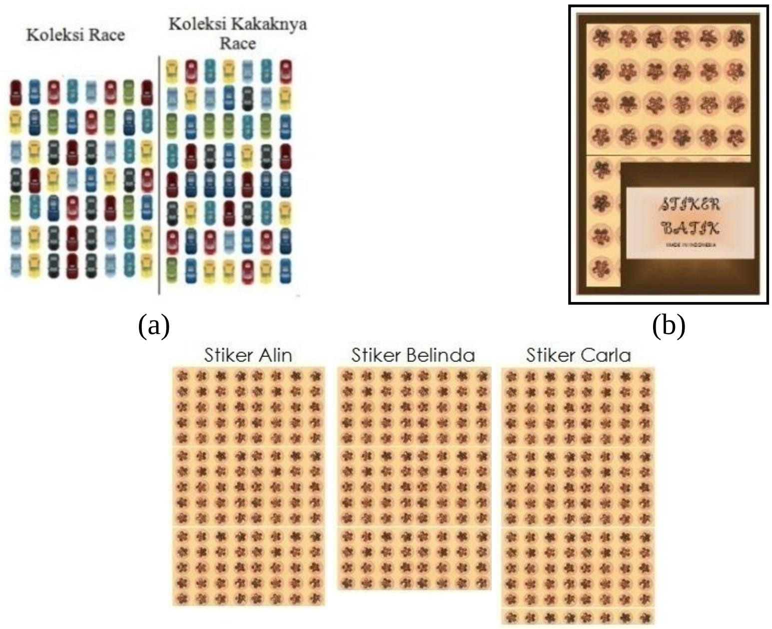


Figure 6.2: Examples of materials presented as quick images to introduce: (a) the commutative property, (b) the one more strategy, (c) the one less strategy, and (d) the doubling strategy.

After conducting the classroom activity, the multiplication strategies can also be introduced through the student activity, where the students are asked to work on a problem in-group. The problem presented arrays that were designed to elicit the multiplication strategies. The arrays need to be designed considering the size, the object shape, and the appearance (partially-covered/uncovered). For example, the arrays can be designed by presenting rectangle-ish objects in two related arrays to introduce the commutative property, using bigger arrays that represent multi-digit multiplication to introduce the one-less/one-more strategy, or dividing an array into two equal parts with the last part is partially-covered to introduce the doubling strategy; see Figure 6.3.




(c)

Figure 6.3: Examples of materials to introduce (a) the commutative property, (b) the doubling strategy, and (c) the one-less/one-more strategy.

When students work on this problem, there are students who will still try to count one by one; this is an indication that the students still cannot find the multiplication represented in the array. For the students who are able to find the multiplication represented in the arrays and have known some multiplication strategies, like repeated addition, finger technique, or short-multiplication, they will use these strategies instead of using the arrays as a means to determine the unknown multiplication products so that the multiplication strategies cannot be elicited; they only saw the arrays as a source of the problem. Nevertheless, there will also a condition when there are students who get or find the introduced strategy from the designs or the previous activity but reluctant to use it since they are expecting more ‘formal’ strategies that have been taught how to use it and how to write it. Therefore, the teacher needs to emphasize on using the strategies they found from the problems/the designs and also guide them to use a faster way to derive other unknown facts from a known fact using the introduced strategies.

## B. Local Instruction Theories

The educational materials being tried out in Cycle 2, together with the findings onthe students’ answers, are generated as theories on introducing multiplication strategies using arrays that give overviews of the potential materials to use and how students’ potential answers when they work on the materials.  For this study, there are four theories generated. The first theory is an overview onintroducing arrays as multiplication models.

Table 6.1: A local instruction theory on introducing arrays as multiplication models.

| Materials and Instruction | Students’ Possible Answers |
|---|---|
| Material: quick images<br>Instruction:<br>(1) show an array as a quick image;<br>(2) ask students to determine the total objects in the array;<br>(3) show how the array and its multiplication are related;<br>(4) conduct (1), (2), and (3) for other arrays;<br>(5) conduct as a classroom activity | • There are students who will determine the total objects in smaller arrays by counting one by one or using repeated addition<br>• After the activities are conducted several times, there are students who gradually mention the multiplication every time an array presented. |

| Materials and Instruction (cont) | Students' Possible Answers (cont) |
|---|---|
| Material: a nearly covered array<br>(1) only objects in top and left sides are uncovered;<br>(2) use an array that represent big multiplication, like $9 \times 10$.<br>Instruction:<br>(1) conduct after the quick-image-activity above;<br>(2) ask students to work in-group to determine the total objects in the array. | • There are students who will try to focus on fining the multiplication represented in the array by counting the total objects in the top and left sides representing the factors of the multiplication.<br>• There are students who will find it difficult to see an object simultaneously in a row and a column on their counting. |

The second theory is an overview of the potential educational materials and the students' possible answers on introducing the commutative property as a multiplication strategy.

Table 6.2: A local instruction theory on introducing the commutative property.

| Materials and Instruction | Students' Possible Answers |
|---|---|
| Material: quick images<br>Instructions:<br>(1) show an array as a quick image;<br>(2) ask students to determine the total objects in the array;<br>(3) rotate the array and ask students to determine the total objects in it. | • There are students who can find the multiplication represented in each array, with no indication using the answer from the first array to determine the total objects in the second array |
| Material: two related arrays<br>(1) arrays contain the same number of objects;<br>(2) use rectangle-ish objects;<br>(3) all objects are uncovered;<br>(4) use an array that represent big multiplication, like $7 \times 8$ and $8 \times 7$.<br>(5) the total objects in the second array could be derived from the first array using the commutative property.<br>Instruction:<br>(1) conduct after the quick-image-activity above;<br>(2) ask students to work in-group to determine the total objects in the array. | • There are students who still try to count one by one, but then failed to determine the correct answer since the design misleads it.<br>• There are students who find the multiplication in the first array and then determine the product using finger-technique. After that, they will find the multiplication in the second array and then determine the product using finger-technique. |

The third theory is an overview of the potential educational materials and the students' possible answers on introducing the one-less/one-more strategy as a multiplication strategy.

Table 6.3: A local instruction theory on introducing arrays as multiplication models.

| Materials and Instruction | Students' Possible Answers |
|---|---|
| Material: quick images<br>(1) use two related array;<br>(2) the multiplication fact in the first array serves as an anchor fact to determine the total object in the second array using one-less/one-more strategy.<br>Instructions:<br>(1) show the first array as a quick image;<br>(2) ask to determine the total objects in the array;<br>(3) tape the array in the whiteboard;<br>(4) show the second array as a quick image next to the first array.<br>(5) ask to determine the total objects in the array. | • There are students who can find the multiplication represented in each array, with no indication using the answer from the first array to determine the total objects in the second array |
| Material: three uncovered arrays<br>(1) the arrays are related to each other;<br>(2) the total objects in the second array could be derived using one-less strategy from the first array, and the third using one-more strategy.<br>(3) use an array that represent multi-digit multiplication, like $15 \times 8$, $14 \times 8$, and $16 \times 8$.<br>Instruction:<br>(1) conduct after the quick-image-activity above;<br>(2) ask students to work in-group to determine the total objects in the array. | • There are students who will find the multiplication represented in the arrays and determine the products using short-multiplication.<br>• There are students who will come up to the one-less/one-more strategies after they are asking to use using a faster way to determine the products, instead of using short-multiplication.<br>• There are students who will find the multiplication represented in the first array and determine the product using repeated addition, and after that using the one-less/one-more strategies to determine the total objects in the second and third arrays. |

The fourth theory is an overview of the potential educational materials and the students' possible answers on introducing the doubling strategy as a multiplication strategy.

Table 6.4: A local instruction theory for introducing the doubling strategy.

| Materials and Instruction | Students' Possible Answers |
|---|---|
| Material: quick images<br>(1) use two related array;<br>(2) the multiplication fact in the first array serves as an anchor fact to determine the total object in the second array using one-less/one-more strategy.<br>Instructions:<br>(1) show the first array as a quick image;<br>(2) ask to determine the total objects in the array;<br>(3) tape the array in the whiteboard;<br>(4) show the second array as a quick image next to the first array.<br>(4) ask to determine the total objects in the array. | • There are students who can find the multiplication represented in each array, with no indication using the answer from the first array to determine the total objects in the second array |
| Material: an partially covered array<br>(1) some objects are covered by label;<br>(2) the label make the stickers are divided in two equally parts;<br>(3) all objects in the first part are uncovered;<br>(4) only objects in the left sides of the second part are uncovered.<br>(5) the total objects in the array could be derived from the total objects in the uncovered part using doubling strategy.<br>Instruction:<br>(1) conduct after the quick-image-activity above;<br>(2) ask students to work in-group to determine the total objects in the array. | • There are students who will use the doubling strategy, but not as an indication to find the product of the multiplication.<br>• There are students who will finding the multiplication represented in the array and then determine the product using repeated addition. |

## C. Reflection

There are some reflections were taken relating to how this study was conducted.As mentioned in Chapter III, the data in Cycle 2were limited. This condition was acknowledged when the researcher conducted theretrospective analysis after all the teaching experiments were finished. The researcher realized that the assumption to make the static video-recorder to capture all students' responds and activities, the teacher's explanation, and alsothe students in the focus group was not aquite good decision since the conversation and activities of the focused students could not be heard and seen clearly. However, the moving video-recorder could capture some conversation and activities of the focused students so that they could be used to answer the research question.

Through conducting the analysis, the researcher also realized that conducting the thoroughanalysis in the end of the experiments was making some drawbacks could not be handle in advance; for example like the way to collect the data, as mentioned in the previous paragraph. Also, there were some important decision that missed to be taken in advance; for example there were no of follow-up questions asking the students on how they calculated the multiplication products so that there was an unclear indication whether the students could relate the visualization of the strategies in the arrays in their calculation.Moreover, the limited data made the process on getting the answer of the research question was not easy to conduct. This lack of data was also a reason why this thesis is reported late from the schedule.

## D. Recommendation for Future Researches

This study used arrays as multiplication models. Some researchers considered that this model is difficult for second-grade-students since they need to perceive the idea of an object is simultaneously in a row and a column first. However, this study showed that it is possible for the students to perceive this model at early stage. Therefore, since this study was not focusing on this aspect, a depth research about this topic could give benefit to overcome the students' difficulties more precisely.

This study also tried to introduce one multiplication strategy in one lesson. However, the data generated were limited because the introduced strategies did not always could be elicited from the designs. Therefore, for future researchers who are interested in introducing multiplication strategies, it is recommended to design materials that could elicit several multiplication strategies in a problem. Thus, when a strategy failed to be introduced, this strategy could be discussed in other lessons.

**REFERENCES**

Anam, F., Pretty Tj, M., & Suryono. (2009). *Matematika 2: Untuk Sekolah Dasar/Madrasah Ibtidaiyah Kelas 2.* Jakarta: Pusat Perbukuan, Departemen Pendidikan Nasional.

Armanto, D. (2002). *Teaching Multiplication and Division Realistically in Indonesian Primary Schools: A Prototype of Local Instructional Theory.* Enschede: PrintPartners Ipskamp.

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2009). The array representation and primary children's understanding and reasoning in multiplication. *Educational Studies in Mathematics, 70* (3), 217-241.

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' Spatial Structuring of Arrays of Squares. *Journal for Research in Mathematics Education, 29* (5), 503-532.

Braddock, S. (2010). *Developing Fourth Graders' Profiency in Basic Multiplication Facts Through Strategy Instruction*. Florida: the University of Central Florida.

Caron, T. A. (2007). Learning Multiplication: The Easy Way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 80* (6), 278-282.

Chapin, S. H. (2006). *Math matters: understanding the math you teach, grades K-8.* California: Math Solutions Publications.

Chinnappan, M. (2005). Prospective teacher's representation of multiplication. Dalam P. Clarkson, & A. Downton (Penyunt.), *28th Conference of the Mathematics Education Research Group of Australasia* (hal. 249-256). Sydney: Merga.

Clements, D. H. (1999). Subitizing: what is it? why teach it? *Teaching Children Mathematics* , 400-405.

Cobb, P., Zhao, Q., & Visnovska, J. (2010). Learning from and Adapting the Theory of Realistic Mathematics Education. *Éducation et didactique, 2* (1), 105-124.

Fuson, K. C. (2003). Toward Computational Fluency in Multidigit Multiplication and Division. *Teaching Children Mathematics* , 300-305.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learrning, 6* (2), 105-128.

Gravemeijer, K. (2010). Realistic Mathematics Education Theory as a Guideline for Problem-Centered, Interactive Mathematics Education. Dalam R. Sembiring, K. Hoogland, & M. Dolk (Penyunt.), *A decade of PMRI in Indonesia.*

Gravemeijer, K. (2008). RME Theory and Mathematics Teacher Education. (D. Tirosh, & T. Wood, Penyunt.) *Tools and Processes in Mathematics Teacher Education* , 283-302.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. (K. G. Van den Akker, & N. Nieveen, Penyunt.) *Educational Design Research* , 17-51.

Heirddsfield, A. M., Cooper, T. J., Mulligan, J., & Irons, C. J. (1999). Children's mental multiplication and division strategies. *The 23rd Psychology of Mathematics Education Conference*, (hal. 89-96). Haifa, Israel.

Kroesbergen, E. H., & Van Luit, J. E. (2002). Teaching Multiplication to Low Math Peformers: Guided versus Structured Instruction. *Instructional Science, 30*, 361-378.

LeFevre, J., Bisanz, J., Daley, K. E., Buffone, L., Greenham, S. L., & Sadesky, G. S. (1996). Multiple Routes to Solution of Single-digit Multiplication Problems. *Journal of Experimental Psychology: General, 125* (3), 284-306.

Mustoha, A., Buchori, Juliatun, E., & Hidayah, I. (2008). *Senang Matematika 2: untuk SD/MI kelas 2.* Jakarta: Pusat Perbukuan, Departemen Pendidikan Nasional.

Purnomosidi, Wiyanto, & Supadminingsih, E. (2008). *Matematika 2: untuk SD/MI kelas 2.* Jakarta: Pusat Perbukuan, Departemen Pendidikan Nasional.

Sherin, B., & Fuson, K. (2005). Multiplication strategies and the appropriation of computational resources. *Journal of Research in Mathematics Education, 36* (4), 347-395.

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education, 26* (2), 114-145.

Smith, S. Z., & Smith, M. E. (2006). Assessing Elementary Understanding of Multiplication Concepts. *School Science and Mathematics, 106* (3), 140-149.

Steel, S., & Funnell, E. (2001). Learning Multiplication Facts: A Study of Children Taught by Discovery Methods in England. *Journal of Experimental Child Psychology*, *79,* 37–55.

Tasman, F. (2010). *Design Research on Mathematics Education: Supporting Second Graders' on Learning Multiplication Using Structured Objects In Indonesian Primary School.* Palembang, Indonesia: Universitas Sriwijaya.

Ter Heege, H. (1985). The acquisition of basic multiplication skills. *Educational Studies in Mathematics* , 375-388.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and Realistic Mathematics Education.* Utrecht, The Netherlands: CD-ß Press Utrecht University.

Van den Heuvel-Panhuizen, M. (2000). Mathematics Education in the Netherlands: A Guided Tour. *Freudenthal Institute Cd-rom for ICME9.* Utrecht: Utrecht University.

Van den Heuvel-Panhuizen, M. (2003). The Didactical Use of Models in Realistic Mathematics Education: An Example from A Longitudinal Trajectory on Percentage. *Educational Studies in Mathematics, 54*, 9-35.

Van Eerde, D. (2013). Design Research: Looking into The Heart of Mathematics Education. Dalam Zulkardi (Penyunt.), *The First South East Asia Design/Development Research (SEA-DR) International Conference.* Palembang.

Van Galen, F., & Fosnot, C. T. (2007). *Groceries, Stamps, and Measuring Stamps - Early Mulitplication.* Portsmouth: Firsthand Heinemann.

Wallace, A. H., & Gurganus, S. P. (2005, Agustus). Teaching for mastery of multpilication. *Teaching Children Mathematics* , 26-33.

Watson, J., & Mulligan, J. (1998). A development multimodal model fo multiplication and division. *Mathematics Education Research Journal, 10* (2), 61-86.

Wong, M., & Evans, D. (2007). Improving basic multiplication fact recall for primary school students. *Mathematics Education Research Journal, 19*, 89-106.

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disabilities Quarterly, 29*, 269-289.

Young-Loveridge, J., & Mills, J. (2009). Teaching Multi-digit Multiplication using Array-based Materials. *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia.2.* Palmerston North, NZ: MERGA.

Zulkardi. (2010, 12 30). Retrieved from: www.reocities.com/ratuilma/rme.html

# APPENDICES

## A. Name of Participants

### 1. Cycle 1

| No | Pretest | Lesson 1 | Lesson 2 | Lesson 3 | Lesson 4 |
|---|---|---|---|---|---|
| 1 | Fira | Fira | Fira | Fira | Fira |
| 2 | Mita | Mita | Mita | Mita | Mita |
| 3 | Vina | Vina | Vina | Vina | Vina |
| 4 | Samuel | Samuel | Samuel | Samuel | Samuel |

### 2. Cycle 2

| No | Pretest | Lesson 1 | Lesson 2 | Lesson 3 | Lesson 4 |
|---|---|---|---|---|---|
| 1 | Divan | Divan | Divan | Divan | Divan |
| 2 | Faiz | - | Faiz | Faiz | Faiz |
| 3 | Hamed | Hamed | Hamed | Hamed | Hamed |
| 4 | Mazta | Mazta | Mazta | Mazta | Mazta |
| 5 | Ranuh | Ranuh | Ranuh | - | - |
| 6 | Rizal | Rizal | Rizal | Rizal | Rizal |
| 7 | Satria | Satria | Satria | Satria | Satria |
| 8 | Acha | Acha | Acha | Acha | Acha |
| 9 | Adelia | - | - | - | - |
| 10 | Aia | Aia | Aia | Aia | Aia |
| 11 | Doni | Doni | Doni | Doni | Doni |
| 12 | Dyah | Dyah | Dyah | - | - |
| 13 | Falah | - | Falah | Falah | Falah |
| 14 | Fika | Fika | Fika | Fika | Fika |
| 15 | Intan | Intan | Intan | Intan | Intan |
| 16 | Krishna | - | Krishna | Krishna | Krishna |
| 17 | Maudy | - | Maudy | Maudy | Maudy |
| 18 | Mila | Mila | Mila | Mila | Mila |
| 19 | Nabil | - | Nabil | Nabil | Nabil |
| 20 | Riyan | Riyan | Riyan | Riyan | Riyan |
| 21 | Shafa | Shafa | Shafa | Shafa | Shafa |
| 22 | Silfia | - | Silfia | Silfia | Silfia |
| 23 | Yoga | Yoga | - | Yoga | Yoga |
| 24 | Yusuf | Yusuf | Yusuf | - | - |

## B. Classroom Observation Schemes

Classroom observation aimed to get insight into the learning and teaching processes and norms in the classroom so that it can be used to elaborate the HLT. The list of objects to be observed and its findings are presented below.

| Observations Lists | Details | Findings |
|---|---|---|
| Classroom Settings | (1) What are facilities in the classroom? | Standard facilities of a classroom in city in East Java province: tables, chairs, blackboard, AC, etc. |
| | (2) How students seat? | Two tables were grouped so that a pair of students was facing the other pair. |
| | (3) How many students in the class? | 24 students. |
| Teaching-Learning Process | (4) How does the teacher open the class?Mention the topic that they will learn today? Then, directly start to teach the topic?Discuss the homework?Review the previous class?Mention the rule? | The teacher asked the students what they had learned in the previous lesson and reviewed it briefly. After that, the teacher mentioned the topic they were going to learn on that day, i.e. introduction to division. |
| | (5) How does the teacher explain the topic? Asking students about their experience related to the topic as a starting point for discussion?Directly explain the main topic? | On that day, the teacher conducted a hands-on activity. She asked the students to work in a pair and then distributed straws as learning tools. After that, she mentioned some division bare problems for the students to solve, and also asked them to use the straws as means. |
| Teacher-Student Interaction | (6) How is the interaction between the teacher and the students when the teacher explains the topic? The teacher explains and the students listen and then write what teacher writes in the whiteboard? Or the teacher invite for discussion or sharing opinion? | On that day, the teacher conducted a hands-on activity. So, it could not be observed. |

| | | |
|---|---|---|
| | (7) If a student has a question, what will the teacher do? | (a) The teacher was quite close to her students. The students could easily approach her. The teacher tried to answer and explain if the students were confused.<br>(b) Several times, some students asked the teacher to verify their answer. If the students got the wrong answer, the teacher only told them that it was wrong. |
| | (8) Is the class a mess, without control? | Sometimes, and the teacher had a special treatment as explained below in classroom norms part. |
| Student Interaction | (9) How do the students interact among them when there is a group discussion? | Most of students worked together to solve the problems in their group. |
| Classroom Norms | (10) Is there any special norms used in the class? | - |
| | (11) If the class is not quite, what the teacher do? | The students sometimes were hardly to manage. The teacher even had a special treatment to control the students: letting them released their energy by drumming the table for some time. |
| | (12) If there is a group discussion, how the teacher distributes the students? | There was no a classroom discussion after activity. |

## C. Pretest Result

### 1. Cycle 1

| Questions ↓ | Student Number → | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1. | $2 \times 3 = \cdots$ | RA3 | FT | RA3 | RA3 |
| 2. | $4 \times 3 = \cdots$ | RA3 | FT | RA3 | RA3 |
| 3. | $8 \times 3 = \cdots$ | RA3 | FT | RA3 | RA3 |
| 4. | $9 \times 2 = \cdots$ | RA2 | FT | RA2 | RA2 |
| 5. | $3 \times 8 = \cdots$ | RA8 | FT | RA8 | RA8 |
| 6. | $5 \times 4 = \cdots$ | RA4 | FT | RA4 | RA4 |
| 7. | $4 \times 4 = \cdots$ | RA4 | FT | RA4 | RA4 |
| 8. | $6 \times 4 = \cdots$ | RA4 | FT | RA4 | RA4 |
| 9. | $10 \times 8 = \cdots$ | RA8 | FT | RA8 | RA8 |
| 10. | $9 \times 8 = \cdots$ | RA8 | FT | RA8 | RA8 |

Abbreviation:

| | |
|---|---|
| RA | Use repeated addition, e.g. RA7: $7 + 7 + 7 + \cdots + 7$ |
| RA* | Use repeated addition and writing down the solution (adding every two numbers), e.g. $6 \underbrace{+6}_{12} \underbrace{+6}_{18} \underbrace{+6}_{24} + \cdots \underbrace{+6}_{48} = 8 \times 6 = 48$ |
| $RA^2$ | RA* and write the long addition to calculate the result |
| FT | Finger technique |
| OA | Only answer |
| C | Use the commutative property, e.g. $12 \times 2 = 2 \times 12 = 12 + 12 = 24$ |
| D | Use the doubling strategy, e.g. $4 \times 2 = 8$, $4 \times 4 = 8 + 8 = 16$ |
| — | Abbreviations with this line means the students have wrong answers |

Note:
Student number is based on the list of students in the previous appendix.

### 2. Cycle 2

| Questions ↓ | Student Number → | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | $2 \times 6 = \cdots$ | OA | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | OA | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA2 | RA2 |
| 2. | $4 \times 6 = \cdots$ | RA6 | ~~RA6~~ | RA6 | RA6 | ~~RA6~~ | RA6 | RA6 | RA6 | RA4 | RA6 | RA6 | RA6 | ~~OA~~ | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA4 | RA4 |
| 3. | $8 \times 6 = \cdots$ | RA6* | RA6 | ~~RA6~~ | RA6 | RA6 | RA6 | RA6 | ~~RA6*~~ | ~~RS6~~ | RA6 | ~~RA6~~ | ~~RA6~~ | ~~OA~~ | RA6* | RA6 | RA6 | RA6 | ~~RA6~~ | RA6 | RA6 | RA6 | RA6 | RA8 | RA6 |
| 4. | $9 \times 2 = \cdots$ | ~~RA2~~$^2$ | RA2 | RA2 | RA9 | RA9 | RA2 | RA2 | RA2* | RA9 | RA2 | RA2* | RA2 | OA | RA2* | RA2 | RA2 | RA2 | RA9 | RA9 | RA2 | RA2 | RA2 | RA9 | RA2 |
| 5. | $6 \times 8 = \cdots$ | RA8$^2$ | RA8 | ~~RA8~~ | RA8 | RA8 | RA8 | RA8 | RA8* | RA8 | RA8 | RA8 | RA8 | ~~OA~~ | RA8* | RA8 | C | RA8 | ~~RA8~~ | RA8 | RA8 | RA8 | RA8 | RA5 | RA8 |
| 6. | $5 \times 4 = \cdots$ | RA4 | RA4 | ~~RA4~~ | RA4 | RA4 | RA4 | RA4 | RA4* | RA4 | RA4 | RA4* | RA4 | OA | RA4 | RA4 | RA4 | RA4 | ~~RA4~~ | RA4 | RA4 | RA4 | RA4 | ~~RA4~~ | RA4 |
| 7. | $4 \times 4 = \cdots$ | RA4 | RA4 | ~~RA4~~ | RA4 | RA4 | RA4 | RA4 | RA4 | RA4 | RA4 | RA4* | RA4 | ~~OA~~ | RA4 | RA4 | D | ~~RA4~~ | RA4 | RA4 | RA4 | RA4 | RA4 | ~~RA4~~ | RA4 |
| 8. | $6 \times 4 = \cdots$ | RA4 | RA4 | RA4 | RA4 | ~~RA4~~ | ~~RA4~~ | RA4 | RA4 | RA4 | RA4 | RA4* | RA4 | ~~OA~~ | RA4 | RA4 | D | RA4 | RA4 | RA4 | RA4 | RA4 | RA4 | OA | RA4 |
| 9. | $10 \times 8 = \cdots$ | RA8$^2$ | RA8 | ~~RA8~~ | RA8 | ~~RA8~~ | RA8 | RA8 | RA8 | RA8 | RA8 | RA8 | RA8 | ~~OA~~ | RA8 | RA8 | RA8 | RA8 | RA8 | RA8 | RA8 | RA8 | RA8 | RA10 | RA10 |
| 10. | $9 \times 8 = \cdots$ | RA8$^2$ | RA8 | ~~RA8~~ | OA | RA8 | ~~RA8~~ | RA8 | RA8* | RA8 | RA8 | RA8 | RA8 | ~~OA~~ | RA8 | RA8 | FT | ~~RA8~~ | RA8 | RA8 | RA8 | RA8 | RA8 | RA9 | RA8 |

### D. Strategies Written in theStudents' Worksheet in Cylce 2

| No | Name | Lesson 1 | Lesson 2 | Lesson 3 | Lesson 4 |
|---|---|---|---|---|---|
| 1 | Divan | WM | OA | WM | WM1 |
| 2 | Faiz | - | OA | A, DB | OA |
| 3 | Hamed | OA | ~~OA~~ | ~~WM, RA~~ | OA, RA |
| 4 | Mazta | OA | OA | OA | WM, SM |
| 5 | Ranuh | WM | WM | - | - |
| 6 | Rizal | WM | WM | WM | WM1 |
| 7 | Satria | WM, RA | OA | WM, RA | WM, SM |
| 8 | Acha | - | WM | WM | WM |
| 9 | Adelia | WM, RA | - | - | - |
| 10 | Aia | WM, RA | WM | RA | WM |
| 11 | Doni | OA | OA | ~~WM~~ | WM |
| 12 | Dyah | WM | WM | ~~WM~~ | WM |
| 13 | Falah | - | WM | WM | OA |
| 14 | Fika | - | WM1 | WM, RA | WM*, RA |
| 15 | Intan | WM, RA | WM | WM | WM |
| 16 | Krishna | - | WM→CP | WM | WM |
| 17 | Maudy | - | WM | WM, RA | WM |
| 18 | Mila | - | WM, RA | ~~WM~~ | WM |
| 19 | Nabil | - | WM | WM, RA | WM |
| 20 | Riyan | WM, RA | WM1 | WM, RA | WM |
| 21 | Shafa | WM | WM→CP | WM, CP | WM |
| 22 | Silfia | WM | ~~WM~~ | WM, RA | OA |
| 23 | Yoga | - | - | WM | WM |
| 24 | Yusuf | OA | OA | - | - |

Note

| | |
|---|---|
| OA | Only wrote the answer. |
| RA | Wrote repeated addition. |
| WM | Wrote multiplication. |
| WM1 | Only wrote one multiplication. |
| WM, RA | Wrote multiplication, spot repeated addition. |
| WM, SM | Wrote multiplication, spot short multiplication. |
| WM*, RA | Wrote multiplication, spot repeated addition, led to the use of the one-less/one-more strategy. |
| WM, CP | Wrote multiplication, spot the use of the commutative property. |
| WM→CP | Wrote multiplication, concluded the idea of the commutative property. |
| OA, RA | Only wrote the answer, spot some repeated addition. |
| A, DB | Wrote addition in doubling format. |
| — | Abbreviations with this line means the students have wrong answers |

## E. Posttest Result of Cycle 2

Abbreviations:

| | |
|---|---|
| RA | Use repeated addition, e.g. RA7: $7 + 7 + 7 + \cdots + 7$ |
| RA* | Use repeated addition and writing down the solution (adding every two numbers), e.g. $6 \underbrace{+6}_{12} \underbrace{+6}_{18} \underbrace{+6}_{24} + \cdots \underbrace{+6}_{48} = 8 \times 6 = 48$ |
| RA2 | RA* and write the long addition to calculate the result |
| FT | Finger technique |
| OA | Only answer |
| C | Use the commutative property, e.g. $12 \times 2 = 2 \times 12 = 12 + 12 = 24$ |
| D | Use the doubling strategy, e.g. $4 \times 2 = 8$, $4 \times 4 = 8 + 8 = 16$ |
| OL | Use the one-less strategy |
| OM | Use the one-more strategy |

| Questions ↓ / Student Number → | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $2 \times 6 = \cdots$ | NOT ATTEND | RA2* | NOT ATTEND | OA | NOT ATTEND | RA6 | RA6 | RA6 | NOT ATTEND | RA6 | RA6 | RA6 | RA2 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | RA6 | NOT ATTEND | OA |
| $4 \times 6 = \cdots$ | | RA6* | | OA | | RA6 | RA6 | RA6 | | RA6 | RA6 | RA6 | RA4 | RA6 | RA6 | D | RA6 | OA | RA6 | RA6 | RA6 | RA6 | | OA |
| $8 \times 6 = \cdots$ | | RA8* | | OA | | RA6 | RA6 | RA6 | | RA6 | RA8 | RA6 | RA6 | RA6 | RA6 | D | RA6 | OA | RA8 | RA6 | RA6 | RA6 | | OA |
| $9 \times 2 = \cdots$ | | RA9 | | OA | | RA2 | RA2 | RA2 | | RA2 | RA9 | RA2 | RA2 | C | C | C | C | RA9 | RA9 | RA2 | C | RA2 | | OA |
| $6 \times 8 = \cdots$ | | RA8* | | OA | | RA8 | RA8 | RA8* | | RA8 | RA8 | RA8 | RA6 | RA8* | RA8 | C | RA8 | OA | RA6 | RA8 | RA8 | RA8 | | OA |
| $5 \times 4 = \cdots$ | | RA5* | | OA | | RA4 | RA4 | RA4 | | RA4 | RA5 | RA4 | RA4 | RA4 | RA4 | C | C | OA | RA5 | RA4 | RA4 | RA4 | | OA |
| $4 \times 4 = \cdots$ | | RA4* | | OA | | RA4 | RA4 | RA4 | | RA4 | RA4 | RA4 | OA | RA4 | RA4 | D | RA4 | RA4 | RA4 | RA4 | RA4 | RA4 | | OA |
| $6 \times 4 = \cdots$ | | RA6* | | OA | | RA4 | RA4 | RA4 | | RA4 | RA6 | RA4 | OA | RA4 | RA4 | C | C | OA | RA4 | RA4 | RA4 | RA4 | | OA |
| $10 \times 8 = \cdots$ | | RA10* | | OA | | RA8 | RA8 | RA8 | | RA8 | RA10 | RA8 | OA | RA8 | RA10 | C | C | OA | RA10 | RA8 | C | RA8 | | OA |
| $9 \times 8 = \cdots$ | | RA9* | | OA | | RA8 | RA8 | RA8* | | RA8 | RA9 | RA8 | OA | RA8 | - | OL | C | OA | RA8 | RA8 | RA8 | RA8 | | OA |