EDUCATIONAL DESIGN RESEARCH:
DEVELOPING STUDENTS' UNDERSTANDING OF AREA MEASUREMENT THROUGH THE UNIT OF AREA

MASTER THESIS

Febrian
NIM 117785029

UNIVERSITAS NEGERI SURABAYA
PROGRAM PASCASARJANA
PROGRAM STUDI PENDIDIKAN MATEMATIKA
2013

EDUCATIONAL DESIGN RESEARCH:
DEVELOPING STUDENTS' UNDERSTANDING OF THE AREA MEASUREMENT THROUGH THE UNIT OF AREA

MASTER THESIS

A Thesis submitted to
Surabaya State University Postgraduate Program
as a Partial Fulfillment of the Requirement for the Degree of
Master of Science in Mathematics Education Program

Febrian
NIM 117785029

UNIVERSITAS NEGERI SURABAYA
PROGRAM PASCASARJANA
PROGRAM STUDI PENDIDIKAN MATEMATIKA
2013

APPROVAL OF SUPERVISORS

Thesis by Febrian, NIM 117785029, with the title *Educational Design Research: Developing Students' Understanding of the Area Measurement through the Unit of Area* as been qualified and approved to be tested.

Supervisor I,

Prof. Dr. Dwi Juniati, M.Si

Date,

19/07/2013

Supervisor II,

Dr. Agung Lukito, M.S

Date,

24-5-2013

Acknowledge by

Chairman of the Mathematics Education Study Program

Dr. Agung Lukito, M.S

NIP 196201041991031002

APPROVAL

Thesis by Febrian, NIM 117785029, had been defended in front of the Board of Examiners on August, 23rd 2013.

Board of Examiners

Name	Signature	Position
Dr. Agung Lukito, M.S	....	Chairman/Supervisor II
Prof. Dr. Dwi Juniati, M.Si	....	Member/Supervisor I
Prof. I Ketut Budayasa, Ph.D	....	Member
Prof. Dr. Siti M. Amin, M.Pd	....	Member
Prof. Dr. R. K. Sembiring	....	Member

Acknowledged by
Director of Post Graduate Program,

Prof. I Ketut Budayasa, Ph.D
NIP. 195712041994021001

345 … Thanks God … you showed me that nothing great's achieved without great pain …

This thesis is dedicated for my beloved family and friend

ABSTRACT

Febrian. 2013. *Educational Design Research: Developing Students' Understanding of theArea Measurement through the Unit of Area.*Thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University. Supervisors: (I) Prof. Dr. Dwi Juniati, M.Si. dan (II) Dr. Agung Lukito, M.S.

Keywords. Area measurement, Unit of area issues, Pendidikan Matematika Realistik Indonesia, Hypothetical Learning Trajectory, Local Instruction Theory

Many researches found that students have a poor understanding of the tool and process used for the area measurement of a plane figure. One of the reasons on this poor understanding is the lack of concern on the construction of the concept of appropriate unit for area measurement. Hence, the need to construct the understanding of area measurement through the concept of unit of area becomes crucial. In this study, some important ideas like area unit issues: unit overlapping, gap between unit,and leftover; the appropriate shape of unit to cover region; the use of identical units to measure and to compare the area of region; the inverse relation between unit size and number of units used are concerned to help the students to construct their understanding on the use of unit of area on area measurement. The students are then expected to flexibly use the square grids which represent the model of unit of area measurement.

For this reason, a sequence of instructional activities is designed. *Pendidikan Matematika Realistik Indonesia*(PMRI) approach was used in our design research. The Hypothetical Learning Trajectory (HLT) becomes source for design and research instruments. Two cyclic design research is conducted. The result of preliminary teaching experiment is used to improve our HLT for teaching experiment. At the final of our study, Local Instruction Theory on area measurment trhough the area unit is proposed.

The study shows that activity with cookie and cashewnut topping could emerge the issues on area mesurement unit: overlapping; gap in between unit; leftover and could give students the initial understanding of relation between the number of units and the area from measuring top of cookie with cashewnuts. The extension of cookie context, choosing the topping shape, could promote the students to understand the best shape of unit to cover the surface. The photoframe context in which students measured the frame size with the photos could emerge the concept of unit consistency (the use of identical units) either to measure and to compare the area. This context could also support students' understanding of the inverse relation between the size of unit and the number of units used in measurement. The experience of finding the cookie's size by the use of topping conducted by hands on activity and prior lesson sequence to it could help students to understand the use of square grids, a model used to find the area. We found that this sequence of lessons generally could support the students' understanding of the area measurement through the unit of area.

ABSTRAK

Febrian. 2013. *Educational Design Research: Developing Students' Understanding of theArea Measurement through the Unit of Area*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Dwi Juniati, M.Si. dan (II) Dr. Agung Lukito, M.S.

Kata Kunci. Pengukuran luas, Isu unit luas, Pendidikan Matematika Realistik Indonesia, Hypothetical Learning Trajectory, Local Instruction Theory

Banyak penelitian menemukan bahwa siswa memiliki pemahaman yang kurang mengenai alat dan proses yang digunakan dalam mengukur luas bidang datar. Salah satu alasan kurangnya pemahaman ini adalah kurangnya perhatian pada pembangunan konsep unit yang tepat untuk pengukuran luas. Dengan demikian, kebutuhan untuk pembangunan pemahaman pengukuran luas melalui unit luas menjadi hal yang penting.Dalam penelitian ini, beberapa konsep penting seperti isu unit luas: unit bertumpukan, gap diantara unit, dan bagian yang belum tertutupi pada sebuah permukaan; bentuk unit yang tepat untuk menutupi permukaan; penggunaan unit identik untuk mengukur dan membandingkan luas permukaan; relasi invers antara ukuran unit dan jumlah unit yang digunakan ditekankan untuk membantu siswa mengkonstruksi pemahaman mereka terhadap penggunaan unit luas dalam pengukuran luas. Siswa kemudian diharapkan dapat menggunakan grid sebagai model unit pengukuran luas yang terbaik secara fleksibel.

Untuk tujuan tersebut, beberapa aktifitas pembelajaran dirancang. Pendekatan Pendidikan Matematika Realistik Indonesia (PMRI) digunakan dalam penelitian desain dua siklus ini. Hypothetical Learning Trajectory (HLT) menjadi sumber untuk mendesain pembelajaran dan instrumen pembelajaran. Hasil dari siklus pertama digunakan untuk perbaikan HLT untuk siklus kedua. Selanjutnya, Local Instruction Theory pengukuran luas melalui unit luas kemudian diajukan.

Penelitian menunjukkan bahwa kegiatan dengan kue kacang mede dapat memunculkan unit isu: tumpukan antar unit; celah antar unit; bagian yang belum terttutupi pada permukaan dan dapat meberikan siswa pemahaman awal hubungan antara jumlah unit dan luas dari aktifitas mengukur permukaan atas kue dengan kacang mede. Kelanjutan konteks kue ini, bentuk topping untuk menutupi permukaan kue dapat membantu siswa memahami bentuk unit yang tepat.Konteks foto frame dapat memunculkan konsep unit konsistensi (penggunaan unit identik) baik dalam mengukur maupun membandingkan luas. Konteks ini juga mendukung pemahaman siswa terhadap relasi invers antara ukuran unit dan jumlah unit dalam pengukuran luas. Pengalaman dalam menemukan ukuran kue dengan menggunakan topping dilkukan dengan metode *hands-on activity* bersama dengan pelajaran sebelumnya dapat membantu siswa memahami penggunaan grid petak, sebuah model yang digunakan untuk menemukan luas. Kami temukan bahwa secara umum, rancangan aktifitas pemebelajaran ini dapat mendukung pemahaman siswa terhadap pengukuran luas melalui unit luas.

PREFACE

Thanks to Allah for its blessing on me to finish my greatest job in my recent education. Accomplishing this thesis has been such great experience that was not even out of priceless guidance and support from my truly blessed people around me. Hence, I would like to appreciate and thank to:

1. Prof. Dr. Dwi Juniati, M.Si. and Dr. Agung Lukito, M.S. as my supervisors who guided and assisted me during the period of collecting data, writing the analyses and preparing the report of my research.
2. Drs. Monica Wijers and Dr. Dolly van Eerde from Freudenthal Institute of Utrecht University who warmly guided and assisted me during the period of establishing instructional theory, designing learning materials, and completing research methodology.
3. Dr. M.L.A.M. Dolk, the coordinator of the International Master Program on Mathematics Education in the Netherlands, for his warm support and motivation during our study period in Utrecht University the Netherlands.
4. PMRI Center Board for the opportunity given to me as one of the grantees of the International Master Program on Mathematics Education.
5. All lecturers and staff of postgraduate program of Surabaya State University and Utrecht University for their support during conducting my master study.
6. The principles, the mathematics teacher and the third grade students of SD LAB UNESA and SDIT Al Ghilmani Surabaya for their excellent cooperation during the period of collecting data.
7. My parents, my sisters, my brothers, and my friends for their motivation and support. Especially my soul friend for her supports, motivation, and inspiration.

Finally, I consciously understand that this thesis is far from being perfect. Thus, any critics and kindly constructive ideas will be warmly accepted.

Febrian

TABLE OF CONTENT

LIST OF TABLE

LIST OF FIGURES

LIST OF DIAGRAM

LIST OF APPENDICES

CHAPTER I

INTRODUCTION

A. Background

Area measurement, one of the most commonly utilized forms of measurement, is a part of mathematics that is closely connected to real-world applications. The study of measurement indeed becomes an important part of the elementary school curriculum starting from the third grade. Furthermore, it is important since there is a relation between the basic concepts of area measurement with other mathematical ideas (Haris, 2011). Hence, this study focuses on supporting the understanding of area measurement by students in the third grade of elementary school.

Research in the field of education often reveals poor understanding of the processes used for the area measurement of a plane figure (Zacharos, 2006). One potential aspect of the poor understanding towards the area measurement is the way teachers teach the concept of area directly by length-times-width formula. In Indonesia, it is commonly found that the teachers and textbooks tend to move quickly to the use of formulas for the areas of basic shapes without giving the students the opportunity to study the pattern and structure of array (Fauzan in Haris, 2011). The teachers often demonstrate the length-time-width formula to get the area of given plane. This issue is not appropriate for the aspect of measurement in which the unit used must be related to the aspect of the object to be measured. Using this formula in area measurement means generating a wrong understanding since it is not appropriate to measure the area with a linear tool. This also provides the students with the difficulty in measuring area, because they sometimes treat length as a space-filling attribute, and they measure linearly, instead of area units (Nitabach and Lehrer, 1996). Hence, this kind of learning might confuse the students or even brings them to produce misconceptions about the concept of area and its relation to the length and the width.

Another way to teach area measurement happening in Indonesia is that the students are asked to partition the rectangular plane figure into several square units, and then count them by multiplying the number of square units vertically and horizontally, to finally obtain the formula for the area. This last procedure only makes sense, however, if the students understand that this calculation is equivalent to dividing the rectangular shape into identical square units and counting them (Nitabach and Lehrer, 1996).

There is one consideration that can be derived from this observation which is the idea of understanding of the area unit. In order to become proficient in measuring an area, students need to understand two fundamental concepts which are the idea of an area unit and the process of iterating an area unit to complete an area measurement (Nitabach and Lehrer, 1996). In line with this idea, Fauzan (in Haris, 2011) stressed on the understanding of specific unit, unit pattern and unit structure on the area measurement.

Many studies have focused on the enhancement of the students' understandings towards the area measurement starting with the understanding of the area unit. One of study conducted by Haris (2011) has focused on the students' understanding of characteristics of the area unit by introducing the students to the context of the traditional Indonesian handicraft called *anyaman. Anyaman* used in this learning already showed up several iterative rectangular shapes on the body of *anyaman* itself. In his study, the researcher got the students to understand several important aspects related to unit issues of the area concept such as an identical unit, a different size of unit used, and others like unit iteration, and area conservation. What becomes highlighted is that the context used being considered already presents the idea of a rectangular-shaped of a unit area arranged in an array form which can be regarded as the necessary condition for the invention of area formula.

We rather think that introducing this array form from beginning of the lesson could seemingly close the possibility for children to recognise the informal unit and its irregular shape, since irregular shaped units are more revealing in everyday life related to area concept from the perspective of an area unit. We also regard that giving array from the beginning of the lesson would also seemingly close the possiblity for children to discuss about the unit issues: overlapping unit, leftover, and gap between unit.

Therefore, in our study, we come with several ideas that can be discussed further; that is covering or iterating. The question is what if the study does not start from introducing an already rectangular-shaped unit area like *anyaman.* Another question is how the students themselves are guided into understanding of the area unit so they find their *'anyaman-shaped'* look through a covering or unit iterating process by themselves. In other words, we try to convey the idea of units that include shapes other than rectangular or square units since our focus is to help the students understand the coordination of the units in a particular setting instead of just generating particular formulae. We also keep in mind that complete covering is generally more difficult than covering a length, since it is not that clear how to choose and arrange units, especially for the students who have little or no sense of “square” units. They often leave small holes or use inconsistent units when covering an area (Nitabach and Lehrer, 1996).

This study addresses the issues on the designed hypothetical learning trajectory in learning area measurement through the area unit for the third grade elementary school students. A learning sequence with Pendidikan Matematika Realistik Indonesia (PMRI) approach is used in this study.

B. Research Question

Based on the background of the study, we propose our research question as follows;

How can students’ understanding of unit of area help them to understand the concept of area measurement?

To support us answering the general research questions, we derive research sub-questions as follows;

1. *What activities focusing the understanding of an area unit can be used to support students understand the area measurement?*
2. *What kinds of understanding of area measurement that students could reach from the learning process?*

C. Aim of the Research

The first aim of our study is to support students' learning process on area measurement through the area unit. The second aim of our study is to contribute to the development of a local instruction theory for the topic of area measurement. In effort to reach this aims, a hypothetical learning trajectory is designed, tested, analyzed, and redesigned to fullfil the needs of our research aims. Included in the process, some classroom activities based on PMRI approach are designed. Specifically, this study focuses on the third grade of elementary school in Indonesia.

D. Definitions of Key Terms

In order to avoid readers from misleading, some operational terms are defined as follow.

1. The understanding is defined as the ability to perceive and comprehend something into an appropriate schema. Schema is an organized structure of knowledge, into which new knowledge and experience might fit. It also can be perceived as the ability to comprehend something or particular knowledge. The root of understanding is to comprehend or to understand.

   To make it more operational, we describe 'to understand' in our study as follows "to understand x means to give appropriate response to x". For instance, the students will have the understanding of inverse relationship of unit of area and the area of region if the students could response that the inverse relationship is an opposite relation between the numbers of unit obtained with the size of the unit used in measuring activity.

2. Surface in this study particularly refers to the bounded region or just region. However, surface is only used in context-related situation. For example, top surface of cookie.
3. Bounded region or just region refers to plane figure that is defined to include all the points on its boundary and all the points in its interior (Jacob, 1974). For example is a triangular region which is different with triangle. Triangle is defined as a set of three line segments. Meanwhile, triangular region is plane figure that contains both triangle and all the points in the interior.

   In our study, we extensively use the term rectangular region for instance to describe the plane figure that contains both a rectangle and all the points in the interior. Hence, in this study we extensively use it as in the expression "the area of top surface of rectangular cookie".
4. Area is defined as a spatial extend and the quality of space (Lehrer, 2003). Area can also be defined as space contained within flat and enclosed shape. Then, we have area measurement as the measurement of space contained within flat and enclosed shape.

   The result of measurement of an area is a number that represents the amount of two-dimentional surface contained. For example, we have a rectangular paper as flat surface. Then, we could do the area measurement on the rectangular paper that gives us the amount or the number as the result of measurement.

To shorten the situation, we propose the aggrement as follows; if we state the area of rectangular paper, it will mean that we are searching for the result of area measurement of rectangular paper which is the number.

5. The understanding of unit of area in this study belongs to the understanding of unit issues (overlapping units, gap between units, leftover); the appropriate shape of unit to cover a surface (bounded region); the idea of identical units, the inverse relation between unit size and the number of units used in covering a region (Nitabach and Lehrer, 1996); and finally how this unit understanding can lead to the use of square or rectangular units in measuring and comparing area of both rectangular and non-rectangular plane figures.
6. Overlapping unit is a unit that covers part of another unit.
7. Gap between units refers to the situation in which space occurs between units. The units are not arranged really next to each other.
8. Leftover means the part of bounded region near the edge that is not yet covered by unit.
9. Uncovered part of region referred to what students represented as both the gap in between unit and leftover during the study.
10. Identical units refers to the units which have the same size and shape as well
11. Area determiner refers to the units of area that are used to derive the area of particular region.
12. Inverse relationship implies the opposite relation occuring between the size of unit used with the number of units used while covering plane to get the area.

For example, with the same plane figure or region, the area will be less if the unit used is larger. The area will be more if the unit used smaller.

13. Array is rectangular arrangement of unit in rows and columns
14. Interior partial array refers to the array constructed inside the plane figure when looking for area of plane figure.

E. Significance of the Research

Two significances are expected to be achieved in our study. The first significance is to give grounded instruction theory on area measurement through the area unit. The second significance is to provide mathematics teacher with an insight on how to enhance mathematics teaching and learning process that can support students' understanding on area measurement through the area unit. This study also offers researcher with an overview of how to design instructional activities and some consideration taken into account in the process.

CHAPTER II

THEORETICAL BACKGROUND

A. Area Measurement

The area measurement is defined as the measurement of the space contained within flat and enclosed shape (if we refer area as the spatial extend). Area on the other hand is defined as an amount of two-dimensional surface that is contained within the boundary and that can be quantified in some manner (Baturo and Nason, 1996). Moreover, in regard to the idea of quantifying, in everyday life, the idea of the area of a shape or an object can be defined as the *"amount of stuff"* needed to cover the shape (Konya and Tarcsi, 2010). It is commonly implied that the concept of area is the idea of finding the amount of tiles, for example, needed to cover a surface.

Stephan and Clements (2003) show that there are at least four foundational concepts that are involved in learning to measure area: (1) partitioning, (2) unit iteration, (3) conservation, (4) structuring an array.

Partitioning is a mental act of cutting two-dimensional space with a two-dimensional unit. Firstly, the experience given to students are activites including tilling a region and discussion on overlapping units, gap between units, and leftover. Then this experience gives the opportunity to the idea of partition a region mentally into sub-regions that can be counted.

Unit iteration is an another concept that students construct as they cover regions with area units. There is a big concern on issues: overlapping units, gap between units, and leftover. Then, the students are led to the idea that an area is number of a unit in total after iteratively used in covering the region.

Conservation, is another concept related to the idea of cutting a given region and rearranging its parts to form another shape resulting in the same area. In this case, the students need to experience the exploration and discussion on the folding and rearranging pieces to establish that one region, cut and reassembled, covers the same space.

Another concept is structuring an array which is an extremely sophisticated process of finding area of rectangular shape.In this case, the students need to learn such structuring to understand area.

Regarding these four foundational concepts in learning area mesurement, some thoughts are drawn. To understand area means to assimilate the area in an appropriate schema (Skemp, 1987). Appropriate schema refers to these foundational concepts. Mostly these concepts highlight the importance of unit area in area measurement. Specifically, the concept of unit iteration suggests that the understanding of area unit could contribute to the understanding of area measurement through tilling activity that concerns on the issues of units and the iteration itself that leads to the idea of counting units. In consequence, the area unit and its isues take important role in area measurement. Therefore, a concern on building up students' understanding of area measurement through the area unit becomes main part in this study.

B. The Understanding of the Unit of Area Measurement

Many researchers have described the importance of the unit concepts in measurement (e.g., Battista, 2006; Kamii, 2006; Mulligan, Mitchelmore, & Prescott, 2005 in Barrett, 2011). Reynolds and Wheatley (1996) contend that to understand area a child must construct and coordinate units. This idea implies that once the idea of constructing an area unit is formed, and the coordination of all constructed area units is made, the children are possibly assisted when they are dealing with the area measurement. According to these literatures, the need of understanding area unit becomes more centralized in the topic of area measurement since it takes a worthwhile role during area measurement learning.

There are several consideration related to the understanding of area unit. The first thing is the unit appropriateness; a unit must be related to the aspect of the object to be measured. Several researchers have described young students' difficulty measuring area (Nitabach and Lehrer, 1996), demonstrating how they sometimes treat the length as space-filling attribute. In this occasion, the students measure the length of a side, then move the ruler a bit and measure the length between the sides again, and so on, and finally treat the length as a space-filling attribute.

Another case will be finding the area by using linear tool which is a ruler. Sometimes, instruction is given by using a ruler. Both these situations clearly describe on how inappropriate it is to use linear tool in dealing with an area measurement. Therefore, the concept of unit appropriateness, the concept implying the approriate use of tools to the obejct being measured is broken. If instruction begins with a ruler, one of the most common mistakes is for children to measure the length of each side and add the two linear measures together, or the students may easily multiply the length and the width of the rectangle to get an area, and still it is a misconception.

To deal with this misconception as well as to promote the concern on the appropriateness of unit feature, a particular learning set is highly needed. One of important aspects to consider is the idea of unit iteration. Unit iteration is an important concept that students construct as they cover regions with area units (Stephan and Clements, 2003). In addition to that, Lehrer (2003) explains that students' first experience with area might include tiling a region with two-dimensional unit of choice and, in the process, discuss issues of leftover space, overlapping units, and precision to name a few. Clearly, the most highlighted in this case is the unit of choice. A special issue occurs when the students deal with tiling particular regions or several manipulative or irregular figures. The two most commonly observed strategies with the use of manipulations are *boundedness* and *resemblance* (Lehrer, 2003). This idea reveals how the students tend to think of boundary of a surface and do the tiling in which the unit used never exceeds the boundary itself. The students also choose the unit resembling the shape of surface being tiled. However, young children thinking in this way may ignore the idea of gaps (space-filling), they intend to respect on the idea of boundary. As a consequence, an instruction that helps students focus on the idea of leftover space, gap in between, and overlapping is highly required.

Moreover, determining the area can be thought of essentially as a tiling of a plane with congruent regions that become units of measure (Reynolds &Wheatley, 1996, p. 567). This idea implies the consistency of the units used when measuring the area through the covering activity which is also important to consider. The consistency implies the use of consistent units throughout the measuring task. When doing covering, the students often use a variety of shapes to cover an area and then calculate the area by counting the total number of shapes without consideration on how many units each shape represents (Nitabach and Lehrer, 1996). This occasion is proven by the situation when provided manipulative (i.e. squares, triangles, circles, and rectangles) for use in finding the area measure of variety of forms, most children in grades 1 through 3 freely mixed units and reported the total count of units. To a certain degree, the idea of mixed units is allowed as long as students report the number of each shape used, and not treat all the shapes inasingle amount of used units. Hence, the learning set must be carefully designed to address this situation. Students must be guided to 'see' the idea of unit consistency. Once the students have chosena unit to cover an area, they must iteratively use it until the surface entirely covered without overlapping, gaps in between or leftover space.

In addition, the idea of unit consistency triggers the idea of identical units used in measuring the area. It is hoped that the students will find the idea of identical units since it gives an important contribution to the next implication. If the units used are identical, a count will represent the measure (Lehrer, 2003).

This implies the idea of quantifying the area surface. The number of identical units used represents the measured area.

Meanwhile, the result of iterating units ought to signify an array structure for students (Stephan and Clements, 2003). While the students at a particular stage seem unsure how to deal with measuring area, the teacher introduces a square grid as a measurement device. At later phase, the students will gradually accept this notation and use it to estimate and combine partial units.

After all, teachers should always be attentive to creating young children's initial spatial intuitions and appreciate the need for students (Stephan and Clements, 2003):

1. to construct the idea of measurement units
2. to have many experiences in covering quantities with appropriate measurement units and counting those units
3. to spatially structure the object they are to measure
4. to construct the inverse relationship between the size of a unit and the number of units used in a measurement.

C. Area Measurement in Indonesian Curriculum

In Indonesia, the concept of area is first taught to students in the third grade of elementary school. The concept of area, which is in the geometry and measurement domain, in Indonesian curriculum, is contained at the second semester of the third grade as shown in the following table.

Table 2.1 . Indonesian mathematics curriculum of area measurement for third graders Elementary School

Third Grade of Elementary School, Second Semester

Standar Kompetensi (Standard of Competence)	**Kompetensi Dasar (Basic Competence)**
Geometri dan Pengukuran 5. Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah	5.1 Menghitung keliling persegi dan persegi panjang 5.2 Menghitung luas persegi dan persegi panjang 5.3 Menyelesaikan masalah yang berkaitan dengan keliling, luas persegi dan persegi panjang
Geometry and Measurement 5. Counting the perimeter, the area of square and rectangle, and their use in problem solving	5.1 Counting the perimeter of square and rectangle 5.2 Counting the area of square and rectangle 5.3 Solving the problem relating to the perimeter, the area of square and rectangle

Source: Badan Standar Nasional Pendidikan (BSNP), 2006

At this level, the concept of area is introduced from square and rectangular plane. The concept of area is introduced as *'counting'*. In this level also, the students need not only to grasp the concept of area and how to count it, but also need to implement this concept of area to particular mathematical problems.

However, the concept of area as we can see on the table is more stressed on the *'counting'* that leads to the use of particular formulae, known as multiplication strategy on length and width, for the area of particular planes: rectangle and squares. We noted that the textbook and the teaching tend to directly move to the formula without regarding the appropriate way of unit used to measure the area. There is almost no regard that the area cannot be measured by linear tool, length and width.

Moreover, it can be caused by the lack of understanding of the connection between the formula and the partition of planes into an array of square. The formula is supposed to appear after the understanding of plane which can be partitioned into an array of square. It is because the number of squares will be easily found by multiplying the number of squares itself in vertical and horizontal way. Finally, to reach an understanding of why square can be precise unit used to measure the area, the students firstly need to experience the learning sequences that create their development of understanding on the unit of area itself.

Based on this occasion, there is consideration of the importance of unit in area measurement that emerges. The learning process that builds up students' understanding of the area unit in Indonesia becomes very highly demanded to support students' understanding in area measurement. We intend to derive some innovation to the learning of area measurement that can improve the learning process itself. We consider that the use of context in learning process can be the solution.

D. Building Up the Understanding of a Unit of Area Measurement through the Contexts

One skill possibly all people have is measuring. Measure has become a part of people's life. Our ancestors even used their body parts to do measurement. Before the tool for measuring an area was found, people used seeds spread on a particular surface as a unit of area measurement (Haris, 2011). Therefore, using this historical aspect on area measurement can be a good inspiration in designing particular learning.

In this study, we try to pursue the answer to our research question by firstly designing and then testing the learning sequence on area measurement through the area unit. It starts from building up the understanding of the area unit through the learning using irregular shape of unit in covering activities. The context appropriate for starting the learning is cashewnut cookie which is brought into the class. Cashewnuts have nearly irregular shapes that can be used to fit the idea of *resemblance* aforementioned. The use of irregular shape is also potential enough to build the initial area unit understanding: overlapping units, gap between units and leftover. The instruction makes them estimate how many nuts used to entirely cover the top of cookie. This context is also possible for generating the initial idea of relation between the number of unit and the area.

Another context we consider potential to use after what students have grasped in the previous learning with nuts is about making a photo frame. Photo frame is a handicraft used for a particular decorative aim. In this context students will learn about the idea of unit consistency and the relationship between the sizes of units used with the number of units used itself (inverse relation). Yet, they will also set to learn the idea of area in relationship to the number of units (photos) used in this context. We consider the need to efficiently count the number of units in this learning by photo frame context is important. Hence, we formulate the quick image on unfinished photo frame activity. The context of tiled floor is also used. The students will try to figure out which carpet or other thing is larger by laying them on the tiled floor.

Hands-on activity on measuring the area of both rectangular and non-rectangular cookie by the use of rectangular square chocochips is set to generate the initial understanding of measuring non-rectangular plane figure. Finally, we guide the students into a more precise model of the area unit like square units, and we let the students develop their understanding of area unit as the measurement of the area. The multiplication on interior partial array is also concerned to help the students deal with quick counting of the units.

By this designed learning sequence, we research on whether the students will learn the idea of a unit of area measurement and its contribution to the area measurement without giving too much stress on that length-times-width formula.

E. Indonesian Version of Realistic Mathematics Education (RME)

The concept in mathematics is abstract which makes it difficult for students to study. Hence, the need to make it concrete and 'real' becomes an issue. Realistic Mathematics Education can address this issue since the idea is to create a situation in which what children's learning becomes more sensible and realistic for them. The meaning of realistic in this kind of instruction can also be realistically imagined by the students themselves. RME itself has been developed in the Netherlands since the 1970s (de Lange, 1996).

The Indonesian version of RME is called Pendidikan Matematika Realistik Indonesia (PMRI). PMRI has been developed this past ten years in several elementary schools in several areas in Indonesia. With this approach, students are supported to see mathematics which becomes more tangible for them to study, yet the students will take an active role since it evokes the students' contribution towards learning process.

Realistic Mathematics Education has five basic tenets defined by Treffers (1987, in Bakker, 2004). The following is a description of each tenet followed with the explanation of the use of the tenets in our designed learning.

1. *Phenomenological exploration or the use of context*

   The students explore mathematics from a real situation that has meaning to develop basic concepts of mathematics. In our study, the students will face several realistic contexts such as cashewnuts. We conjecture the context can promote the students' understanding of unit overlapping, gap in between unit and leftover space. We also use the context of making a photo frame as handicraft which is familiar with children's life. We conjecture that this context can evoke students' understanding of the idea of unit consistency and relationship between unit size and the number of unit used. To support students' understading on the use of unit to measure the size of thing, we use the tiled floor context.

2. *Using models and symbols for progressive mathematization*

   This level promotes the bridge from a concrete level to a more formal level. In our study, the students will extensively face several models of units such as nuts, irregular shapes, regular shapes and even tiled floor as well. We conjecture to support students' development of thinking from a concrete to a more formal level. We also expect in the end that students can finally use grid to figure out the area of plane figure.

3. *Using students' own constructions and productions*

   Students will use their own strategies when solving the problems. In this level students will actively construct and produce their understanding of several important ideas like unit characteristics during the activities.

4. *Interactivity*

   This characteristic makes the students to become 'social learners', not only an individual learner. The studentsshare their strategy and work with others. Hence, in the learning, the students work in several groups of children to facilitate this purpose.

5. *Intertwinement*

   This highlights the consideration on the relation of recent focused domain, area measurement with other domains in time, multiplication for example. In this learning we invite students' strategy in using of estimation that is closely related to number sense like multiplication and addition. The geometry in general will also be concerned.

CHAPTER III

METHODS

This chapter describes the method which is used to reach the research aim as well as to answer the research questions. In this chapter, there are three issues which are discussed. The issues are the research approach, the data collection, and the data analysis.

A. Research Approach

We intend to make innovation on mathematics education especially in topic of area measurement through anarea unit based on the problem in our country aforementioned. One of the studies which have aims for making innovation in learning process of mathematics and improving the mathematics education especially regarding our topic, area measurement, is called design research.

Design research is a research approach that consists of planning and creating educational settings, and analysing teaching and learning process (Doorman, 2005). Design research or also known as developmental research is aimed to develop theories, instructional materials and empirical grounded understanding of how the learning process works (Bakker, 2004; Drijvers, 2003; Gravemeijer, 1994 in Putra 2011). This implies that understanding of how the learning process goes in classroom activity becomes heart of design research.

There are three phases of design research in our study (Gravemeijer & Cobb, 2006)

1. Preliminary design

In this phase, we studied some literatures related to area measurement, the unit of area measurement, realistic mathematics education and design research. After getting knowledge, we tried to create the students' learning sequence or trajectories focusing on the understanding of unit area issues and the area unit for measuring area. We formulated lessons in the learning sequence. For each lesson in this sequence, we formulated the learning goals for students, mathematical activities, and hypothesis about the thinking process of students' learning. These three aspects are known as the components of hypothetical learning trajectory (Simon, 1995). We also tried to find the appropriate context to support our lesson design. We also discussed about the context issue and the learning design with the expert, our supervisor, who experienced in designing the learning. For generating students' understanding on the unit area issues and the idea of area unit to measure the area we used the context of cashewnuts cookies and photo frame, tiled floor and the grid paper. We set the the mathematical activities and tried to make the conjectures of learning process that happens in the classroom.

2. Teaching experiment

We derived teaching experiment to test our hypothetical learning trajectory on the unit of area and area measurement. During the teaching experiments, we collected the data to support our analysis. The result of this analysis was used to improve our learning trajectories. We proposed cycles of teaching experiment in this design research study namely preliminary teaching experiment or commonly called pilot study as the first cycle and teaching experiment(s). The pilot study was conducted in small group containing six students of third grade with the researcher as the teacher. The aim of doing the pilot study in a small group of students is to get ourself focus on adjustment of the HLT during the execusion. The analysis and the result of this pilot study were used to revise and to improve our hypothetical learning trajectory for the next cycle. The next cycle(s) was conducted in larger group which was group of students of one class with the mathematics teacher as the teacher and the researcher as the observer. The result of the latest teaching experiment in which the learning goals were pursued was analysed to develop the *local instruction theory* for the area measurement domain. It implies that we stopped the cycle of the study whenever we already reached the learning goals and he hypothetical learning trajectory was supported by actual learning trajectory. Local instruction theory is a theory that describes the envisioned learning route, successive patterns in the teaching and learning process (Cobb et al., 2003; Gravemeijer.,1994; Gravemeijer, et al. 2003).

3. Retrospective analysis

Retrospective analysis was done after each teaching experiment. The data of mathematical activity obtained from each cycle was analysed and comparison between what conjectured in hypothetical learning trajectory and what had happened in real classroom activity was done. The analysis was used to make improvement of the hypothetical learning trajectory by redesigning instructional activities for the next cycle. If we already reached learning goals and our hypothetical learning trajectory was supported by actual learning trajectory, we stopped the cycle. The result obtained through the latest retrospective analysis contributed to the local instruction theory.

McKenney & Reeves (2012) imply that each cycle slightly changes indicating that the time and the number of participant which might increase. Based on this consideration, we proposed cycles of the study in this design research portrayed as preliminary teaching experiment or pilot study which was conducted in small group of students. The aim of doing the pilot study in a small group of students is to get ourself focus on adjustment of the HLT during the execusion. The next cycle(s) is teaching experiment which was conducted in a classroom as described previously. The phase of the design research in this study can be shown in this following scheme.

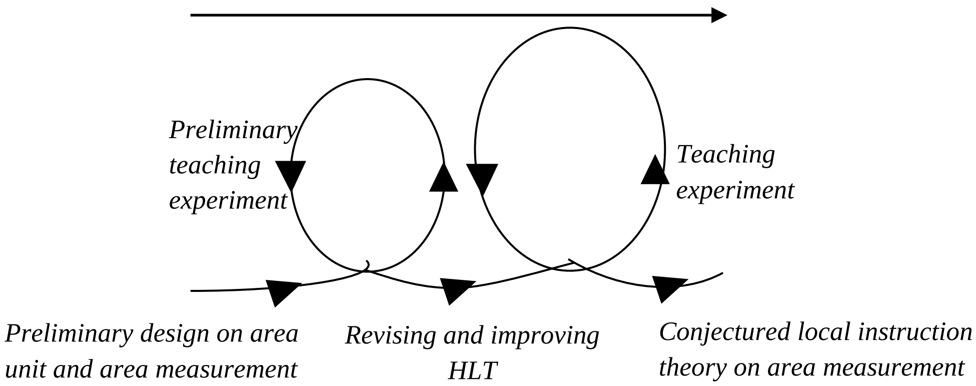


Figure 3.1. The cyclic processes on area unit and area measurement learning design

In this research we propose the criteria to stop our cyclical process of design research. Stopping the cycle of this research is under the following circumstances

1. We generally already achieve the intended learing goals we set up in every activity of our designed lesson sequences.
2. Our hypothetical learning trajectory is supported by the actual learning trajectory.

B. Data Collection

The study of area unit and the area measurement was conducted in two of elementary schools in Surabaya, Indonesia, which are already partners of Surabaya State University in implementing Pendidikan Matematika Realistik Indonesia (PMRI). The school is called Ghilmani Elementary School (during preliminary teaching experiment with six students of class IIIB and the researcher as the teacher) and Laboratory School of Surabaya State University (during teaching experiment with students of class IIIA, the mathematics teacher as the teacher and the researcher as the observer). The target group of this study is the third graders (age around 9 to 10 years) as well as the mathematics teacher. During the study, we collected the data through several phases described below.

1. Preparation Phase

The data collected in this phase are the data o sociomathematical norms, students' activity, students' thinking process, and teacher activity during the teaching and learning process. The aim for data collection in this preparation phase is to get insight into the classroom norms and sociomathematical norms as well as to observe the activity of the teacher and the students during the observed learning process. The teacher and the students in this phase are the participants of the teaching experiment. The students are the third graders.

Firstly, the data of socimathematical norms, students' activity, and teacher's activity were obtained through observation on the classroom (*Appendix C*). The data collection was conducted by using the video recording. During the observation, we also interviewed several students focusing on their mathematical thinking while they were solving the problem. Field notes were also used to record the result of small interview.

The second data were sociomathematical norm, classroom culture, students' proficiency spread, students' activity, and the teacher's teaching approach (*Appendix B*). These data were obtained through the interview with the teacher of teaching experiment after doing the observation. The interview was recorded with the audio recorder. The data obtained through this interview was triangulated with the data of the classroom observation. In this interview also, the researcher informed the teacher about the study and asked the teacher for helping the researcher to pick up several focused students in the class to be extensively concerned in the second cycle of the study.

The pretest (*Appendix F*) was also conducted. The pretest aimed to get the data about students' initial understanding of area concept and the test contains several tasks testing the students' prior knowledge on the use of unit to measure area and the use of units to compare area as well. Generally, we look for dat of: What the students know about area and how the students find the area of region. Therefore we derive several data through the pretest:

1. Students' recognition on the units as the area determiner. For example in our pretes, do the students recognize the use of units (strawberry problem, tiled floor problem, table and book problem) to derive area?
2. Students' conception on the amount of units as the area. For example in pretest, do the students count the units (strawberry, tile, and book) and report the number of units as the area?
3. Strategy on representing area with amount of different units (unit consistency). For example, in our pretets, how the students derive the area of floor which contains different sized tiles?
4. Students' conception on the use of unit to measure and compare area. For example, do the students use square in paperquilt to derive the area and compare the area?
5. The students' use and exploration of ruler in finding the area?

However, the pretest was firstly given to some students of preliminary teaching experiment from other class and other school in same grade pointed by mathematics teacher. The pretest was intentionally conducted firstly for preliminary teaching experiment students since we also wanted to know whether designed tasks are understandood by the students. The result informed us to make upgrade for the pretest of the students in teaching experiment. For those cycles, the small interview to the students (focused group in second cycle) will be conducted after pretest to get clarification on their thinking process.

Finally, the data of classroom observation, teacher interview, and the pretest for teaching experiment class were altogether used to be analysed. This informed us to take appropriate adjustment to our HLT relating to initial point of lesson sequence.

2. Preliminary Teaching Experiment (first cycle)

This preliminary teaching experiment also served as the pilot study. We treat preliminary teaching experiment as the first cycle. The participants of this preliminary teaching experiment were six third grade students pointed by the mathematics teacher. The students in this preliminary teaching experiment were not from the class of next cycle (teaching experiment). The students in this experiment have middle level of understanding. The aim for this data collection is to get the data of students' activity and students' thinking during the lessons (see tables in the next pages). These data was used in analysis and comparison of what we conjecture in the HLT of cycle 1.

During the preliminary teaching experiment, students' activity was recorded by video recorder. In addition, the researcher also did some small interview to get insight on what the students thought. The result was written in fieldnotes. The data of students' written work was also used for analysis. All the data gained from video recorder and field notes and students' written work were altogether analysed and compared to our conjecture in HLT of cycle 1 through the retrospective analysis of the first cycle.

3. Teaching Experiment (s)

The improved HLT was conducted in this teaching experiment. We called this teaching experiment as second cycle. The participants of this second cycle were the mathematics teacher and the third grade studentsof one class coming from different school. There were two kinds of observation during the learning process of second cycle. The first one was the overall classroom observation focusing on the teacher's and students' activities. For this occasion, we used the static video recording to catch the classroom environment. The second observation was focused in several students in a group who had middle level of understanding as discussed during the teacher interview in preparation phase. The activities of these focused students were recorded with dynamic second video recorder. In addition, small interview was also conducted to record what these focused students think during the activity. Then the result was recorded in field notes. Finally, students' written work focusing in the concerned students was collected for further analysis.

The aim of the data collection in this teaching experiment(s) is to get the information (see the next pages) important to be analysed during the next retrospective analysis in which our conjecture in second HLT and the next one was compared with the actual situation during the lesson. For this occasion, we needed to triangulate all the data (same data with ones from preliminary teaching experiment) from two classroom observations, the data from small interview, and the data from students' written work to get broader image on the learning situation.

4. Pretest and Posttest

As elaborated above, the pretest was conducted before the cycles but with different importance. The first pretest was conducted to several students of the preliminary teaching experiment. This aims to get the students' initial understanding of the use of unit of area to measure and compare the area. This test contained five different tasks (*Appendix F and H*). The first pretest conducted in the preliminary teaching experiment was also used to see whether the problems designed in the test understandable for the students. This was very important for us to derive some revision towards this pretest. The revised version of the first pretest (second pretest) was conducted in teaching experiment (second cycle) aiming at getting the students' prior knowledge of the area units and its use in measuring and comparing the planes. The revised version of second pretest was conducted before next cycle and so on. In all pretests, the result was the students' written works that were very important to the quantitative and qualitative analysis. After the pretest, in all cycles, small interview was conducted to get the clarification of the students' thinking process towards the tasks. Finally, the result helped us take appropriate adjustment on our HLT in cycles regarding the initial point of lesson in the sequence.

We also used the posttest (*see Apendix G and I*) conducted in every end of the cycles. This aims at getting the insight of the students' development of understanding during the lesson sequence. Hence, the students' written work was collected to be analysed. For the focused students in the teaching experiment, the interview was also conducted to get the clarification of their thinking process during accomplishing the test.

5. Validity and Reliability of Data Collection

The validity of this study can be seen in two ways: internal validity and external validity. To observe students' understanding on area measurement, we collect the data related to students' (mathematical) activity and students' thinking process. The data are collected through video recording, the students' written work and the interview relating to student's activity and thinking process. Hence, we provide the valid data with the valid collecting method. These data became the rich sources for triangulation that allows us to see the data consistency or data complementary to get fuller picture of what observed. This data together with the method and triangulation on the data by the use of different tools will contribute to our internal validity of the study. We provide the transparent way of data collection part so that the reader not only can follow through the process of collecting data, but also can do adjustment to their study relating to the data collection method. This will contribute to our external validity of the study. Finally, the data registration from different methods in collecting the data will contribute to the reliability of the data. Collecting data with video recording for example, adds to internal reliability for it is used to collect the data more or less objectively, independent of the researcher. The data collection in this study can be presented in the following outline.

Table 3.1 The outline of the data collection of the study

Phase	Activities	Participants	Data Collection Methods	Goals
Preparation	Observing the classroom	The students and the teacher of teaching experiment phase (cycle 2)	Video recording, field notes, and small interview with the students	To get insight into the classroom norms (whether the class is silence or not, space for students to talk/discuss, see Appendix B and C) sociomathematical norms (discussion on possible solution), the teacher and students' activities (teacher's rule, management of the class, do the students explain their answer, write, talk, read behaviour, group, etc)
	Interviewing the teacher	The teacher of teaching experiment phase (cycle 2)	Audio recording, semi-structured interview form/field notes	To get useful information towards the classroom norm, sociomathematical aspect, the students' ability (the same data as described in classroom observation above) and so forth Orientation to the designed lessons
Pretest before cycle 1	Conducting pretest before cycle 1	A small group of students in preliminary teaching experiment (cycle1)	Written test (students work)	To get the information of students' insight on the concept of area and area unit. To investigate whether the students understand the problem and the questions in the test
Small	Conducting		Video recording	To get

interview	small interview for pretest clarification		and students' work	clarification on what students think on the tasks in pretest
Cycle 1	Observing students' activity in lessons of preliminary teaching experiment	A small group of students in preliminary teaching experiment (cycle 1)	Video recording, students' work	To get the data for purpose of retrospective analysis of Cycle 1 and improving the HLT for second cycle
	Asking the students through small interview		Field notes	To get additional data for purpose of retrospective analysis of cycle 1
Posttest after cycle 1	Conducting posttest after cycle 1	The students of the preliminary teaching experiment	Written test (students work)	To get the information of students' development of understanding in cycle 1
Small interview	Conducting small interview for posttest clarification		Video recording and students' work	To get clarification on what students think on the tasks in posttest
Pretestbefore cycle 2	Conducting pretest before cycle 2	The students of the teaching experiment (cycle 2)	Written test (students work)	To get the information of students' insight on the concept of area and area unit
Small interview	Conducting small interview for pretest clarification	The focused group of students of cycle 2	Video recording and students' work	To get clarification on what students think on the tasks in pretest
Cycle 2	Observing students' and teacher's activity in lessons of teaching experiment	The teacher and the students of the teaching experiment and the focus group of the students in the teaching experiment (cycle 2)	Video recording, students' work	To get the data for purpose of retrospective analysis of cycle 2
	Asking the students through small interview	The focus group of the students in the teaching experiment (cycle 2)	Field notes	To get additional data for purpose of retrospective analysis of cycle 2
Posttest after Cycle 2	Conducting posttest after cycle 2	The students of the teaching experiment (cycle 2)	Written test (students work)	To get the information of students' development of understanding on the designed lesson series

Small interview	Conducting small interview for posttest clarification	The focused group of students of cycle 2	Video recording and students' work	To get clarification on what students think on the tasks in pos-test

The table shows the openness for continuation of the study (the next cycle process). The activities, data collection method, and the goals will be the same way as the previous teaching experiment (cycle 2). However, we keep respect to our criteria for stopping the cycle of our design research study (see page 24). However, we would like to propose the data in our study during the cycles. We noted that we stopped the cycle after the second one since we already achieved the criteria set on stopping the cycle. Then, based on our designed hypothetical learning trajectory (both initial one in chapter IV and refined one on second cycle see Appendix D) we entail the data during the cycles as follows.

The first table displays the data we collected through the preparataion phase (pretest), preliminary teaching experiment and posttest relating to students' preknowledge (pretest), students' mathematical activity, and students' thinking process based on our initial hypothetical learning trajectory (entailed in chapter IV).

The Phase	**Data**	**Collection Method**
Pretetst	The use of ruler to find the area	Video recording, written work, interview
	Students' recognition on the units as area determiner through strawberry, tiled floor, and book and table problem)	Video recording, written work, interview
	Students' conception on the amount of units (through strawberry, tile, and book problem) as the area	Video recording, written work, interview
	Strategy on representing area with amount of different units (unit consistency) through the tiled floor problem	Video recording, written work, interview
	Students' conception on the use of unit (tiled floor and paperquilt problem) to measure and compare area	Video recording, written work, interview

Meeting 1	The students' arrangement of the nuts (overlapping, gap, and leftover) and the understanding of each arrangement to the size of cookie	Video recording, written work, interview
	Students' investigation on ideal number (by avoiding the overlapping, the gap, and the leftover) of nuts to represent the area of ookie	Video recording, written work, interview
	Students state the relationship between number of unit (nuts) and the area of cookie	Video recording, written work, interview
Meeting 2	Students put the toppings in regard of overlapping, gap, and leftover	Video recording, written work, interview
	Students discuss the structure of topping to decide which topping better cover the cookie	Video recording, written work, interview
Meeting 3	Whether the students use or do not use different sized photos to cover frame	Video recording, written work, interview
	The students strategy to report the number of photos used (consistent or not)	Video recording, written work, interview
	The students already use identical photos or not	Video recording, written work, interview
	The students draw connection between the number of units and unit size (inverse relation)	Video recording, written work, interview
	Students' counting strategy on the number of photos	Video recording, written work, interview
Meeting 4	Use of photos as area determiner of frame	Video recording, written work, interview
	State that different frame can have the same area (number of photos)	Video recording, written work, interview
	Conclude that more photos, larger the frame	Video recording, written work, interview
Meeting 5	Perception on the area measurement and comparison by the use of identical units (tiled floor context)	Video recording, written work, interview
	Multiplication strategy construction	Video recording, written work, interview
Meeting 6	The use of tiled floor as the unit to derive area of oval table	Video recording, written work, interview
	The use of multiplication to count the tiles	Video recording, written work, interview
	The way of estimating the area of non-rectangular region (oval table by the use of inner array and splitting joining strategy)	Video recording, written work, interview
Meeting 7	The use of transparent square grid to derive the area	Video recording, written work, interview
	Estimation of area by the use of transparent square grid (by the use of inner array and splitting joining strategy)	Video recording, written work, interview
Posttest	Students' recognition on the units as area determiner through nuts, tiled floor, book and table, grid problem	Video recording, written work, interview
	Students' conception on the amount of units (nuts, tiled floor, book and table, grid problem) as the area	Video recording, written work, interview
	Strategy on representing area with amount of different units (unit consistency) through the tiled floor problem	Video recording, written work, interview
	Students' conception on the use of unit (square grids problem) to measure and compare area	Video recording, written work, interview
	How the students construct their own grid to derive a figure's area	Video recording, written work, interview

The second table displays the data we collected through the preparation phase (pretest), teaching experiment and posttest relating to the students' preknowledge (through pretest), students' masthematical activity, and students' thinking process based on our second cycle hypothetical learning trajectory (*see Appendix D*).

The Phase	Data	Collection Method
Pretetst	Students' recognition on the units as area determiner through banana, tiled floor, and book and table problem)	Video recording, written work, interview
	Students' conception on the amount of units (through banana, tile, and book problem) as the the area	Video recording, written work, interview
	Strategy on representing area with the amount of different units (unit consistency) through the tiled floor problem	Video recording, written work, interview
	Students' conception on the use of unit (tiled floor and paperquilt problem) to measure and compare area	Video recording, written work, interview
Meeting 1	The students' arrangement of the nuts (overlapping, gap, and leftover) and the understanding of each arrangement to the size of cookie	Video recording, written work, interview
	Students' investigation on ideal number (by avoiding the overlapping, the gap, and the leftover) of nuts to represent the area of ookie	Video recording, written work, interview
	Students state the relationship between number of unit (nuts) and the area of cookie	Video recording, written work, interview
Meeting 2	Students put the toppings in regard of overlapping, gap, and leftover	Video recording, written work, interview
	Students discuss the structure of topping to decide which topping better cover the cookie	Video recording, written work, interview
Meeting 3 Meeting 4	Whether the students use or do not use different sized photos to cover frame	Video recording, written work, interview
	The students strategy to report the number of photos used (consistent or not)	Video recording, written work, interview
	The students already use identical photos or not	Video recording, written work, interview
	The students draw connection between the number of units and unit size (inverse relation)	Video recording, written work, interview
	Students' counting strategy (multiplication) on the number of photos in unfinished frame shown through quik image	Video recording, written work, interview

	Use of photos as area determiner of frame	Video recording, written work, interview
	Conclude that more photos, larger the frame	Video recording, written work, interview
	Perception on the area measurement and comparison by the use of identical units (tiled floor context)	Video recording, written work, interview
Meeting 5	The use of square chocochips to derive area of cookie	Video recording, written work, interview
	The use of multiplication to count the tiles	Video recording, written work, interview
	The way of estimating the area of non-rectangular region (oval table by the use of inner array and splitting joining strategy)	Video recording, written work, interview
Meeting 6	The use of transparent square grid to derive the area	Video recording, written work, interview
	Estimation of area by the use of transparent square grid (by the use of inner array and splitting joining strategy)	Video recording, written work, interview
Posttest	Students' recognition on the units as area determiner through nuts, tiled floor, book and table, grid problem	Video recording, written work, interview
	Students' conception on the amount of units (nuts, tiled floor, book and table, grid problem) as the area	Video recording, written work, interview
	Strategy on representing area with amount of different units (unit consistency) through the tiled floor problem	Video recording, written work, interview
	Students' conception on the use of unit (square grids problem) to measure and compare area	Video recording, written work, interview
	How the students construct their own grid to derive a figure's area	Video recording, written work, interview

C. Data Analysis

1. Preparation Phase

The first data analysis was conducted in the preparation phase. The data (classroom norms, sociomathematical norms, mathematics teacher's activity, students' activity and thinking process) collected from classroom observation on the class of teaching experiment was analysed. The fragment of the interview with the teacher of cycle two was transcribed and triangulated with the data from classroom observation. Then, we tried to give interpretation on the initial situation of the classroom norms and sociomathematical norms, teacher and students' activities and thinking process.

2. Preliminary Teaching Experiment

To get insight on the students' initial understanding relating to the concept of area before preliminary teaching experiment, the pretest was conducted. The data (see page 27) obtained was analysed to get the insight on the students' initial understanding of area concept as well as to analyse whether the students understand the problems or questions about area given in the test. The analysis included: the quantitative analysis for the number of good answers and the qualitative analysis for the students' strategy in solving the problem. This is important for revising the pretest which would be given to students in cycle two prior to the teaching experiment.

The data collected from preliminary teaching experiment was analysed through retrospective analysis. In this analysis, the Hypothetical Learning Trajectory (HLT) was compared to the actual students' learning process based on the video recording, the field notes, and the students' work. The data from video recording was analysed. However, not all parts of the video were analysed. The fragment of the video which was relevant and relating to the students' learning process was chosen. The fragment chosen containing the conversation between students and students and between the teacher and the students was also chosen from both cycles. These fragments were then transcribed. Then, we tried to give our interpretation focusing on the students' thinking process through the learning process they were engaged in.

The data from small interview during the learning was also analysed to give more insight on their mathematical thinking. This data was complemented with the notes from field notes and we gave interpretation also to these data. These data were then triangulated with the data from students' written work. At the final of cycle one, the posttest result was analysed to see how far the students develop their understanding through the lessons in cycle 1. The result of the data analysis in preliminary teaching was then used to revise our hypothetical learning trajectory for the second cycle; teaching experiment.

3. Teaching Experiment(s)

Before teaching experiment, the pretest was conducted. The data obtained was analysed to get the insight on the students' initial understanding of area concept. The analysis included: the quantitative analysis for the number of good answer and the qualitative analysis for the students' strategy in solving the problem.

The data collected from teaching experiment was analysed through retrospective analysis. In this analysis, the Hypothetical Learning Trajectory (HLT) was compared to the actual students' learning process based on the video recording, the field notes, and the students' work. The data from video recording was analysed. The fragment of the video which was relevant and relating to the students' learning process was chosen. The fragment chosen containing the conversation between students and students and the teacher and the students would be focused. These fragments were then transcribed. Then, we tried to give our interpretation focusing on the students' thinking process through the learning process they are engaged in.

The data from small interview during the learning was also analysed to give more insight on their mathematical thinking. This data was complemented with field notes and we will give interpretation also to these data. These data were then triangulated with the data from students' written work. At the final of cycle one, the posttest result was analysed to see how far the students develop their understanding through the lessons in cycle 2. The result of the data analysis in teaching experiment was then used to revise our hypothetical learning trajectory for the second cycle. The same analysis process is conducted for the possible next cycle. This final revision of HLT for the last cycle contributed to the local instruction theory in the domain of area measurement.

For every cycle, we triangulated the data analysis through the video, field notes, and students' written work to improve the validity of our study relating to the data analysis. We also wanted to have discussion about these data analysis with our supervisor to get the quality of analysis improved.

We conducted the analysis on teaching experiment in two ways, analysis of each daily lesson, and the analysis of all lessons. The analysis on each lesson aimed to get insight on students' thinking and learning process on the respected activities designed in each lesson. Meanwhile, the analysis of all lessons was conducted more to get insight of intertwinement between the lessons to find out how far the succession or development of the students' thinking and learning process during the teaching experiment. At the end, we drew a conclusion based on the retrospective analysis and answer our research questions. We also provide some recommendation to improve the hypothetical learning trajectory in domain of area measurement.

4. Validity and Reliability of Data Analysis

We intend to have our internal validity in this study improved. The data obtained through students' activity by using video was analysed and compared with what we conjecture in HLT. This data analysis was triangulated with analysis from the data gained by small interview (field notes) and pupil's written work. This aims for getting more valid analysis about the students' mathematical activities and their mathematical thinking. This analysis was carried out in both cycles.

We also wanted to present the analysis in such a way so that the other people who also want to do study can find out if and how they will adjust the HLT in this study to their own local setting. In this occasion, we tried to improve our external validity of the study.

During the analysis, we also discussed with the expert (supervisor) to have the same track on analysis process. We equated our interpretation to derive such conclusions. This contributed to our internal reliability of the study.

Finally, we presented as clearly as possible the process of analysis of the data such that the reader can follow the track of our analysis. We did this to keep our external reliability improved through the transparency of data analysis.

CHAPTER IV

INITIAL HYPOTHETICAL LEARNING TRAJECTORY

For each lesson in this sequence, we formulated the learning goals for students, mathematical activities, and hypothesis about the process of students' learning. These three aspects are known as the components of hypothetical learning trajectory (Simon, 1995). Hypothetical Learning Trajectory is used to plan mathematic lesson.

In this study, we aim to contribute to the development of a local instruction theory for area measurement which focuses on the understanding of area units. For this reason we build up instructional activities designed in a learning trajectory to promote the students to learn the area measurement through the understanding of area units. The students will develop their understanding on unit characteristics to grasp the concept of area of surface.

Hypothetical learning trajectory is constructed based on literature study. In our study, mostly the area measurement is approached by the use of area unit. Stephan and Clement (2003) imply that one of the foundations that can be used in learning area measurement is the idea of area unit and unit iteration. There is a big concern on issues: overlapping units, gap between units, and leftover. Then, the students are led to the idea that an area is number of a unit in total after iteratively used in covering the region. This is the theory that promotes us to design the first lesson: the idea of unit issues (gap, overlapping, and leftover). There are several considerations of why these unit issues become important to learn in initial phase of learning area measurement. Learning to cover is the activity used in learning to measure area. This aims to generate the idea of unit iteration. Nitabach and Lehrer (1996) revealed that the students have difficulty to see the concept of unit iteration. The students often leave small holes between units, arrange unit overlapping each other which is not allowed during covering the region to derive the area. This is the reason why the learning that addresses these unit issues becomes so important. Furthermore, students need to be guided to see the connection between unit and area. In other way, the students need to have in mind that unit of area is the area determiner. In addition, the unit of choice is also becoming issue to address so that the students understand why particular shape of unit can be better as area unit based on the argument on unit issues: overlapping, gap, and leftover. This is our consideration to derive the next lesson in our designed learning sequence.

Another important concept that develops afterwards is the concept of identical units not only to measure but also to compare the area. The theory suggested by Nitabach and Lehrer (1996) imply that the students has less sense of unit consistency in area measurement compared to in linier measurement. Hence, the students often show inconsistency use of unit either by shape or size and report all the number of mixed units in a single amount that they think represent the area of region being measured. Therefore, some acitivity need to address this unit consistency issue. We proposed photoframe activity in lessons where students could explore measuring area by the use of photos.

We assume that the students will move forward if they learn the inverse relationship between the number of units used and the unit size while covering the region to derive the area. Inverse relationship entails the possibility for difference area obtained. The students are expected to have the flexible way of measuring area by the unit they choose.

Meanwhile, the result of iterating units ought to signify an array structure for students (Stephan and Clements, 2003). Students are provided the learning in which they start to construct the idea of array in regard to unit arrangement. One of the possible uses of array is to create the efficient way of counting. Finally, the students are set to experience measuring any rectangular or non-rectangular region by the use of rectangular unit. One of the tools used to derive area of region is square grid. All the previous concepts will accumulate in this final stage.

Hence, in this chapter, we propose at least seven lessons that promote the understanding of unit issue, the initial connection between unit and area, and unit as the area determiner (lesson 1); unit of choice (lesson 2); unit consistency and inverse relationship (lesson 3 and 4), unit to measure and to compare area (lesson 5); measuring the area of rectangular and non-rectangular region by the use of of rectangular unit (lesson 6); and measuring and comparing region by the use of square grid (lesson 7), to carry out our hypothetical learning trajectory based on literature study. We set several activities with several contexts like cashewnuts and cookies, photo frame, tiled floor, and transparent grid. Each lesson contains particular activity designed to address particularly specific goals that will accumulate to our main goal of the study. The HLT will be implemented in third grade students of elementary school in Indonesia.

A. Meeting 1

In this lesson the students will learn the issues of area unit related to unit overlapping, gap in between unit, and leftover through the covering surface activity by irregular-shaped units (in this case, we use cashewnuts on top of cookie).

1. Students' Background Knowledge

The students have lack of sense on how to measure area (based on pretest result). The students also have the lack of knowledge about area unit and its relationship to the area measurement. The potential knowledge about area will be gained through covering task in which they have the idea of *boundedness* and *resemblance* on their own. The *boundedness* means respecting to the outline of region being covered while covering. The *resemblance* means choosing the shape of unit that resembles the region being covered. Hence, the cashewnut cookie context is regarded potential to address the puplis' initial learning on area in this study.

2. Learning Goals

a. The students understand the idea of overlapping, gap in between, and leftover between the unit and the unit size through the context of cashewnuts cookies

b. The students investigate how many units used for given planes through the context of cashewnuts cookies

c. Students get insight on the relationship between the number of unit and the size (area) of plane through the cashewnuts cookie context.

3. Description of Activities and Possible Students' Thinking

The teacher starts the lesson by introducing the cookie context. Then, the teacher can show several pictures of top view of cookie with the toppings. The teacher might also show the real nuts and quickly demonstrate putting the nuts on top of outline just to give a glimpse to students about what they will do. Then the first task is given.

a. Activity 1 - How Many Cashewnuts are Needed?

In this part, students in each group will get their materials and they will cover the top of cookie with the cashewnuts and record the number of nuts they use to do it . Then, the students will discuss the number of the nuts they used with others.

At the beginning of the activity of part 1, the teacher presents the story of Bu Handoyo and her cookie

Help Bu Handoyo with the cookie....

Bu Handoyo loves the cookies very much. She will make cookies with the top full of cashewnuts. Can you imagine that? The picture above is top view of her cookie. For 100 cookies she makes, she needs to figure out how many cashewnuts that will be needed in total(you know that it is important to estimate the nuts needed for particular number of cookies you make, since you do not want to have so many remaining nuts and the price for cashewnuts are relatively expensive in Indonesia). That is Mrs. Handoyo is also thinking about. So, what Bu Handoyo does then.....
She only needs to count the number of nuts on one cookie. She puts those nuts on that cookie. After she knows how many nuts needed, she will know how many cashewnuts needed for all her cookies by estimating. Do you understand that?
Now, can you help Bu Handoyo to find how many cashewnuts needed on top of a cookie ?

While working on cookie and cashewnuts, the students are also set to discuss and answer several questions in their worksheets. The students might choose the nuts with complete body and avoid the incomplete ones. In some degree the students might also regard the size of the nuts. While they arrange the nuts on the top of cookie, here are the possible things that might occur

1. The students arrange the nuts so that they create the gap/space between the nuts and do not consider about that.
2. The students might form overlapping nuts in several spots on top of cookie but they do not realize that
3. The students might realize the boundary of outline, but create gap and or overlapping on nuts in the middle part
4. The students might not regard the leftover near the edge
5. The students might not realize overlapping, gap, and even leftover

After the students finish on the experiment, they need to count how many nuts there are. They might use their calculation

1. Count one by one
2. Count in group and skip counting

Then, the students need to record the number of nuts on the given place and show to other groups and the teacher the number itself. This is large possibility for different numbers to come out among the group. Hence, this is the good situation in which the teacher set the students to visit and see each others' works. The curious students will confront with the question *"why are the numbers are different", "so how will we report to Bu Handoyo then?", "can you figure out first what happened?"*. The students at first try to find the answer towards the question in their worksheet.

After that, the teacher set the students talk about this situation in class discussion. The first possible thing that students think is about the size of the nuts. They might regard that the size does matter in making the number of nuts used different. Hence, the teacher bring the students to further discussion.

1. The teacher invites the group which formed the gap in between the nuts and the group that created the overlapping nuts. Have the children *'see'*the effect of these kind of arrangement by asking *"do you see that?", "what happen if you have the arrangement like this/that?".*

   The students might think that if they have gap in between the nuts, then the number of nuts used will be less than it is supposed to be. Otherwise, if they have overlapping nuts, the number of nuts will be more.

2. The teacher also invites the group with the leftover on the cookie top. The teacher might ask *"can you do this?", "what will happen if you do this?", "does not that affect the number of nuts?"*

   The students might even think that they do not have to do that and realize that leftover parts should be then covered.

After discussion, the students set to complete their discussion notes given. Hence, they might even think to rearrange the nuts on the top of the cookie. They break the overlapping, they put the nuts really next to each other to minimize the gap, and cover the leftover.

After that, the students are then asked about *"can we really have no gaps in between the nuts?why do you think?"*. The students might think that sometimes it is hard to avoid the gap because of the shape of the nuts.

Then the discussion is directed to the question *"so, can we really have the same number of nuts at the final to report to Bu Handoyo?"*

The students might respond that it can be impossible to have the same result in regard of the size of the nuts and possible overlapping, gap, and small leftover near the edge. When talking about the counting, the studentsmight argue that skip counting will lessen their effort in finding the number of nuts on the cookie.

b. Activity 2 – The Further Investigation

To sharpen the students' vision on the idea of overlapping, gap between the nuts, and possible leftover, and the initial insight on the relationship between the number of nuts and the cookie's size (related to area case), the second activity is set. This activity is an investigation on the effect of the number of nuts used to cover particular size of cookie.The groups are given some nuts (around 12) and two outlines of cookie.

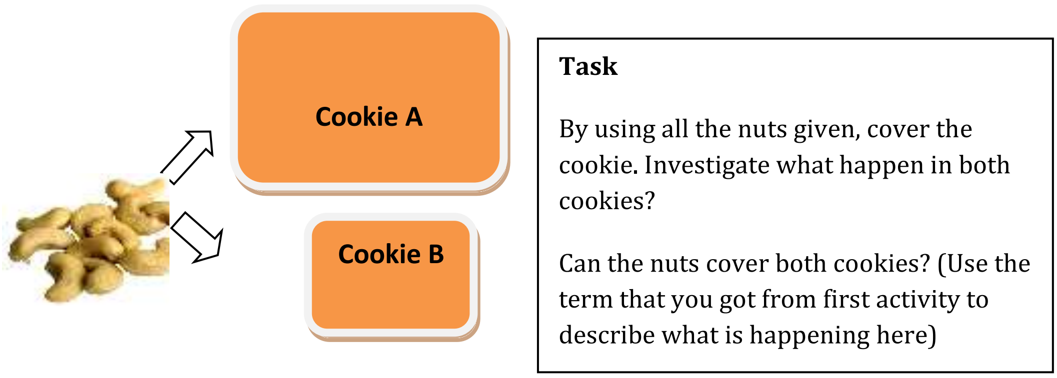


Figure 4.1 The cookie and 12 nuts

The following are the possible responses

- The students might easily recognize that the number of nuts will not be sufficient to cover the cookie since the size of cookie is designed larger than it is supposed to be covered with the given nuts. Hence, the gap and leftover occur. The students then say that there should be more than 15 nuts used to cover this cookie. Otherwise, make the cookie a little bit smaller.
- The students might argue that the nuts will overlapp since all the nuts used is too many for the smaller cookie. The students might also say that the number of nuts must be smaller than given. Otherwise, the cookie is made a little bit larger.
- The students might also think to avoid the gap, overlapping, leftover when covering

The teacher then stresses the last idea. The students are then asked to make the outline of the cookie that can be covered by the number of nuts given to them. The teacher stresses to students that they should avoid as much as possible the gap, the overlapping, and leftover. This following is one of the example of what students do.

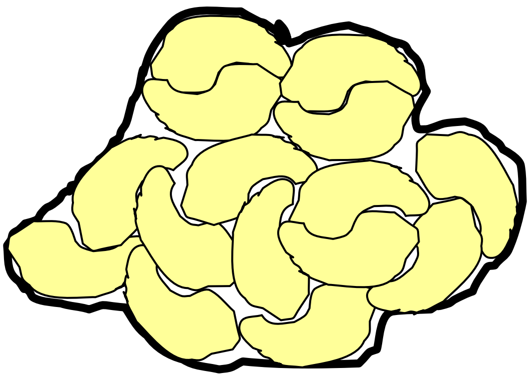


Figure 4.2 The conjectured pupils' work

The groups might come with different outlines of cookies. The short discussion is then derived. The teacher builds the discussion on the relationship between the size of cookie and the number of nuts. Firstly, all the groups show their cookie

- The students might argue that with 12 nuts, they can have different shape of cookie
- They might also say that their cookie's size can be the same since they use the same number of nuts
- They can add that the size can be the same in if each nuts have the same size
- The cookie of one group seems larger than others since this group created many gaps.

c. Activity 3 –Ordering the Cookies Based on the Size

The students will work individually in this task. They will order the cookie based on the size (without necessarily putting nuts again on top of the cookie, the students will figure out the size and the order).

In this task, the studentsmight have in their mind that one cookie will have more nuts rather than others. Finally they will deal with order those cookies in regard of the nuts without necessarily putting nuts on top of the cookie. At this moment, the students might possibly regard that more nuts, larger the cookies but not that much. A small discussion is then conducted. Then the discussion is directed to the order of the cookies based on the number of the nuts used. The teacher might ask : *"do you all have the same* *order?", "what do you think about the cookie size?", "Can you say something about the size?"*. If the students seem not really sure, it is also possible to check their answer by putting the nuts on top of cookies.The students might realize that more nuts, larger cookie.

B. Meeting 2

In this lesson, the students will do the covering activity by the use of rectangular and non-rectangular units. They will learn on what will be appropriate shape used to do perfect covering on the particular surface through the context of *covering cookie with various topping shapes*. They will choose one appropriate unit to cover the surface.

1. Students' Prior Knowledge

In the previous lesson, the students already learned the idea of unit overlapping, gap in between units and leftover. They will probably make use these terms while discovering such situation. The students have also learned from previous lesson that the gap between the nuts sometimes cannot be avoided. In this lesson, the students might regard the overlapping, gap, and leftover while covering the cookie with given toppings.

2. Learning Goals

a. The students will investigate which unit covers the region (entirely cover) through the context of cookies' topping

b. The students discuss whether rectangular or non-rectangular shape is better to use to cover the region through the context of cookie's topping.

c. The students understand that square unit can be best unit to cover a region through the context of cookie's topping

3. Description of Activities and Possible Students' Thinking

The teacher provides the story of Bu Handoyo who wants to cover the top of cookie by putting four kinds of topping. This is the problem to be given

> *"Bu Handoyo will make two kinds of cookie. She intends to fully cover the top of the cookies with several toppings, cashewnuts, rectangular chocochips, M&M chocolate, and star chocochips. She wonders how she will do that with those toppings. Can you help her? How would you do that?"*

The students are given the individual worksheet and they need to work on the group. The teacher gives the groups the sheets containing picture of cookies' outline. The students are also given the replica of square chocochips, star chocochips, and M&M chocolate while the real nuts used are from the lesson 1.

The groups then do the experiment of covering the top of the cookie by each topping in every cookie. In this activity the students can invite the ideas like overlapping toppings, as well as possible gap and also the leftover.

While the students do this activity several possible things might come up. The teacher observe these as well

1. The students just cover the cookie without any regard on the overlapping, the gap, and leftover. The teacher can have the students regard on those aspects and reinstruct them to cover again in another way.
2. While working with cookie A, the students find it difficult to leave out the leftover near the edge. Some groups might just leave it in; otherwise they

keep trying to close the space near the edge by cutting the topping and put them. Otherwise they make the toppings overlap near the edge.

3. In cookie A, the students might also find it difficult to not create gap between the nuts, some of them might be aware of it since they already know something about it in lesson 1. The students might also find it hard not to create gap between the star chocochips, and M&M chocolate. Hence, to cover fully, some overlaps should occur. They might find that square chocochips cannot create gap.
4. In cookie B, the students find it easy to cover the entire top with square chocochips, but not with others since they still find leftover and gap.

After the group finish the experiment, the students in each group discuss each other. Then, each pupil fills the worksheet by themselves. The students then move around to see others' works and might compare each other. The students come back again to reach their group and discuss about the issue, then, they attempt to answer the question in the worksheet.

Discussion

The teacher then puts the students in class discussion. The first discussion is about the cookie A with those four toppings. Some students might react that

1. They cannot avoid the gap in between the topping other than square chocochips, unless they overlap the topping to close that gap.
2. Leftover is always created in cookie A for four of the toppings, they can easily handle this, but they think that they need to split the topping and use it to close the uncovered area near the edge.

The teacher can use these students' argument. The teacher must be aware of this idea and ask the students whether it is true or not. However, the studentsmust be directed again to the problem, they need to only find which topping covers the top of cookie better without necessarily doing splitting the topping at this phase. The discussion is continued to the cookie B and the students might argue that

3. They cannot avoid leftover and gap as well in topping other than square chocochips. Otherwise some overlapps should be made.

Then the teacher ask the students*"which topping does cover the top of cookie better and why?".* The teacher might stress 'better' in this case meaning that the topping can entirely cover the surface of cookie. At first the students might say that it is the square chocochips, but it might leave the leftover in cookie A, not in cookie B. Hence, the teacher might bring the second question asking *"so, which cookie is better covered by the topping?"And which topping is that and tell us why?"*

The students react that it is still the square chocochips and the cookie which is entirely covered is the cookie B. The teacher then has the students to 'see' that the topping better to cover is square and the surface covered must also be the same shape by questioning *"why do you think cookie B is entirely covered by square chocochips?"* On the other hand the students are also asked *"why do you think square chocochips will leave the leftover near the edge of the cookie?", "can you say something about it?".* The students might say that the shape of the cookie must be rectangular like the topping. The teacher might also ask*, "why do you think square chocochips is better to cover the cookie?".*The students might just react that it will not create the gap; the only matter is if they find the shape not like cookie B, the leftover will be always created.

The students finally conclude that the better topping to cover the cookies is the square chocochips and it will be perfectly cover the cookie shaped like cookie B, not cookie A. The teacher might propose the word the same shape in this case and the students should be set aware of that.

C. Meeting 3

In this lesson, students will learn to express the amount of consistent units used to cover a surface through photo frame context. The students are expected to realize that expressing the total number of different sized unit used in single amount of number is not appropriate way to quantify the surface. The students will also learn the relationship between the size of the consistent units used and the number of the units itself (inverse relationship) to cover a surface through photo frame context.

1. The Students' Prior Knowledge

The students have already grasped the idea of overlapping, gap in between, and the leftover and the effect of these things to the number of units used to cover the surface. The students have also had the insight towards the better shape of unit used to cover particular surface. That knowledge is conjectured to support students in context of photo frame (both unit and surface covered have the rectangular shape)

2. Learning Goals

a. The students understand the idea of consistent unit used to cover a region through the photo frame making

b. The students understand the inverse relationship between the size of unit and the number of unit used to cover a region through the photo frame making

c. The students develop the efficient way of counting by multiplication

3. Description of Activities and Possible Students' Thinking

a. Activity 1 – How Many Photos are Needed?

The students are given this story

Photo frame maker
Pak Rudi is a photo frame maker. He produces many kinds of photo frame in his shop and sells them. Pak Rudi makes frame with beautiful photos like animal, flower, and fruit. Children so much like his frame.
Today Pak Rudi wants to make frame full with the pictures of cat. He gets frame now in the hand and the pictures to be attach.
Now, he wonders how many pictures he will use on one frame. Do you have any idea to help him?
With you now are the frame of Pak Rudi with a basket of photos to be attach on the frame. Let's help pak Rudi. Finally, report to him how many photos you use

The groups are given Pak Rudi's identical frame and basket of different sized photos (it is intentionally set to check students' insight whether they know about the concept of identical unit, yet the photos provided are not sufficient to cover the frame), but the number of photos given is not sufficient to cover both frame. We intend to trigger students to produce efficient counting strategy and not to take long time attaching photos on frames. Each member of group will get their own worksheet. The studentsthen work in group to create the photo frame, attach the picture in the frame and they report to Pak Rudi as they accomplish making the frame.

Since there is no instruction to only use identical photos, the students possibly attach those two different sized photos. Complete the photos possibly by drawing it. Here is one of the examples that might come up when they accomplish the task.

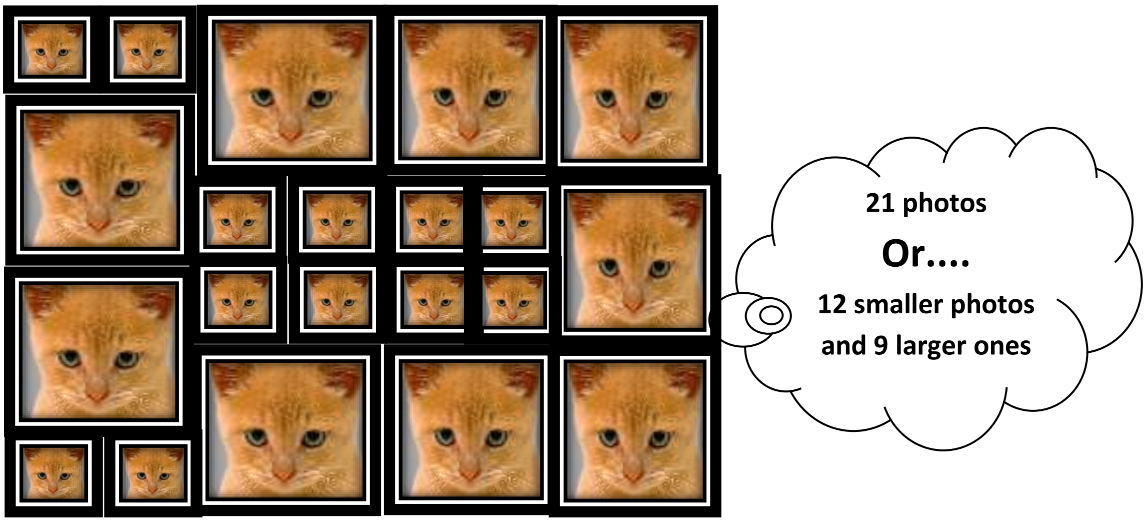


Figure 4.3. The conjectured pupils' work

There is also possibility for the group to use only larger photos or the smaller photos. However, they find that the number of the photos used is not enough. After the students are done with the frame, they will report the number and fill the sheet. The students are then put into class discussion.

Discussion

The students show their work and report the number of photo they used. The picture above is one of the possible example of students' works. The students might confront with the situation in which the number of photos used in each group is different for the same frame. They are facilitated with teacher's question like *"Do you have any idea?"*

Here are the possible students' thinking on the difference. The first one is seeing to the different sized photos. They might realize that the arrangement of the larger and smaller photos in each frame is quite different. Finally, the students then can argue that the number of larger and smaller in each frame different. They finally can see that the difference is caused bythe different number of larger photos and smaller photos used in each group, hence, the number of photos in total becomes different. The teacher can ask, *"How do you know that?".*

The students might count both photos in each frame. Then, each group will count again each different sized photo in their frame and report that again. The students make such clarification. The students might realize that it is not sufficient to just report all the photos used in single amount of number, hence, they have to be precise telling how many larger photos and smaller photos used.

The students who use identical photos already, they might just realize that their work can be reported in single amount for that use of identical units. The students then prepare to work out the identical photos.

The studentswho already used identical at first, might do it again but they use different size.

b. Activity 2 – How Many Identical Photos?

The students attach the frame with identical photos. They can use larger or smaller one. The students then report the number of photos used. They answer the question in the worksheet. While the students try to find the number, the students might develop the efficient way of counting like multiplication, skip countingby row and column.

The students are then put into small discussion, but firstly they need to report the number they got. The students see each other works and do some comparison. They might find some relationship. They can see that smaller photos fill the frame in larger number than the larger photo does. They might derive inverse relationship (the size of photos to the number of photos used) in this moment. The students might argue that smaller photo will take less space than the larger does, so in this frame there will be more smaller photos attached compared to the larger one. At the final the students grasped the relationship between the number of photos used in respect to the size of the photo itself. For the final report, the students might come up with these following suggestion

1. m smaller photos, n larger photos, m and n depend on how many is each of them or
2. 12 larger photos or
3. 48 smaller photos

D. Meeting 4

In this lesson, the students will help Pak Rudi to make and sell some photo frames. The students will attach photos in different size of photo frame. They will help Pak Rudi to determine the price for each frame. The price is finally related to the number of photos used and the use of size term *'which one larger'*. In this lesson the students will learn the idea of units to measure the size.

1. Students' Prior Knowledge

Students have grasped the idea of unit consistency or the use of identical unit to measure area quantitatively. They also have insight into the inverse relationship between the size of the units and the number of units used to cover a surface.

2. Learning Goals

a. The students understand that the unit is used to measure the size (area) through the context of photo frame.

b. The students understand that different region shape can have the same number of unit through the context of photo frame

c. The students understand that more the units, larger the size

3. Description of Activities and the Possible Students' Thinking

a. Activity 1 – Complete the Frame and Determine the Price

The teacher tells the story about Pak Rudi, the frame maker.

Help pak Rudi to make and sale the photo frames

Pak Rudi thanked to you for helping him with previous work. Now, he wants to make several frames with photos. He has several different frames to be attached with the photos. Then, he also wants to determine the price from frames he makes. He has a photo pricing 100 rupiahs.

Can you help him make the frames? Then, attach the price for the frame you work with...

Finally, report the frame you make together with the price of it to pak Rudi.

The students work in group. Every pupil gets the worksheet. There are five unfinished frames displayed in the sheet. The size of the frame provided are the frames which size 8x3 , 6x6, 4x7, 4x6, and 4x9 photos, but the students are not informed with this size. The students not only set to finish the frame, in other words they need to find the total number of photos used in each frame, but they also need to attach the price for the frame, given a photo prices Rp 1000.Here are the possible students' thinking and strategy

1. They complete the frame by picture, and then count the photos by
   - One by one
   - Skip counting by row
   - Skip counting by column
   - Multiply the row and column
2. The students do not complete but try to have complete one in their mind and do counting even there are no pictures in the frame. Some counting strategies elaborated in point 1 can also be evoked by the students.

After the students discuss with other members in their group, the students then answer questions in worksheet. The students then set to have class discussion.

Discussion

The first step, the students might demonstrate the way they do the calculation. The teacher does comparison on the calculation strategy starting from possible one by one to the multiplication. The pupil might see that their exhausting calculation one by one can be handled by more precise way of counting strategy. The teacher should direct the students 'see' this simple way of calculation.

After they finished getting the number of picture and attach the price for each frame, they might realize that there are different shapes of frame pricing the same. They might point out the frame with size 3x8 and 4x6 first. The students then face the question *"What do you notice?".* The students might react that there can be more than one shape of the frame in which 24 photos can be attached just like what they discovered with 3x8 and 4x6. The other students might also point out the frame 6x6 and 4x9 in which the number of photos used is the same.

After being questioned*"So what is the relationship can you get from this?"*by the teacher, the students might say that if the number of photo used in frame is same, then, the price will be the same. The studentsare supposed to react by being questioned*"What else can you say about price and the number of picture?", "Let's say that I have more pictures in my frame, so what happen with the price?"* The students might argue that the price will depend on the number of photos, if they have more photos on it, the more the price is. Otherwise, the less the price is.

The teacher then directs the students to the idea of size, and price. The students are asked what the relationship between the size of frame and the price is. The students might say based on what they already know that the more photos used, the more the price. The more photos used means the larger the size, as the effect is the price will be more. So students argue that the larger the size, the more it prices. Other students might also express in another way around. They might say that if we have smaller size of frame, then the price will also be lower. They also can add that the same price can mean the same size.

As the final step, the students are asked to conclude what the relationship between the number of photos, the price and the size in more precise and understandable way possible to all the audience.

b. Activity 2 – Find the Number of Photos and the Price for the Frame

Each pupil will work individually. They are given the second part of worksheet containing several unfinished frames as in the part 1. They need to find the number of photos used to cover the frame and they also need to attach the price for these frame.

As the students work individually, the teacher moves around and sees the students' work. The counting strategy of the students are recorded and observed. The point of this is whether the students still reveal the exhausting calculation or not. If they still evoke this, the students will be guided to do simpler counting.

At the end of the part, the students can report the number of the photos used in each frame together with the price and make the order based on the price. In this lesson the students are conjectured to have vision on the use of unit to measure the area. They also have the idea more units, larger the surface (more photos, larger).

E. Meeting 5

In the previous lesson, the students has learned the use of photos to determine the size of frame. More photos used, larger the frame. It implies that students has already insight into the use of unit to find the area.

In this lesson, the term ‘size’ representing area is extensively highlighted. The students will learn to use the identical units to measure and compare the size/area. First, the students will confront with the situation in which they need to compare the size of the floor but the units size used are different. This sets the students to think about the idea of identical units to measure and compare the area.

1. Students’ Prior Knowledge

The students have already had insight into the use of units to measure the quantity of surface through the photo frame context, but they do not have yet the idea of the use of identicalunits to not only measure a surface but also to compare more than one surfaces. They also have grasped the idea more units, larger the surface.

2. Learning Goals

a. The students understand that the measurement and comparison of the size (area) by the use of units should include the identical unit throught the tiled floor context

b. Students derive the efficient way of counting (multiplication)

3. Description of Activity and the Possible Students’ Thinking

a. Activity 1 – Find the Size of the Floor and Compare Them

The teacher tells the story of Bu Kartono's new house

> ***Tiled floor of Bu Kartono's new house***
>
> *Bu Kartono just moved in the new house. There are some rooms displayed on the following top view picture.*
>
> *Can you see that the floor is tiled?*
> *What else can you see?*
> *Can you find how large each room in the house is? What will be your way? Can you then compare how large is the floor?*

The students work in group. Every pupil gets the worksheet. There is picture showing top view of three rooms of Bu Kartono's new house. Each room has the floor tiled. The tile size in those three rooms is different to each other. This tiled floor context will trigger students to use the tile to figure out the size of the floor. However, they will confront with the result.

First they will discuss in group the size of floor for each room.They possibly count the tiles by skip counting by row or column, multiplying the row and column, or just do the calculation in mind and report the number of tiles. Then, the students will confront with the situation in which they need to compare the size between the floors. They are asked to order the floor based on the size.

At first, the students might confuse why the floor which seen larger have the smaller number of tiles while the smaller floor has the larger number of tiles. They might also say that they believe that the larger floor will remain the larger, so they think that there can be something they do not yet understand. Some students might also see that the size of tiles may play role in this case, but they do not argue in appropriate way. They are asked to write about what they are thinking in the thinking cloud. The teacher then brings the students into class discussion.

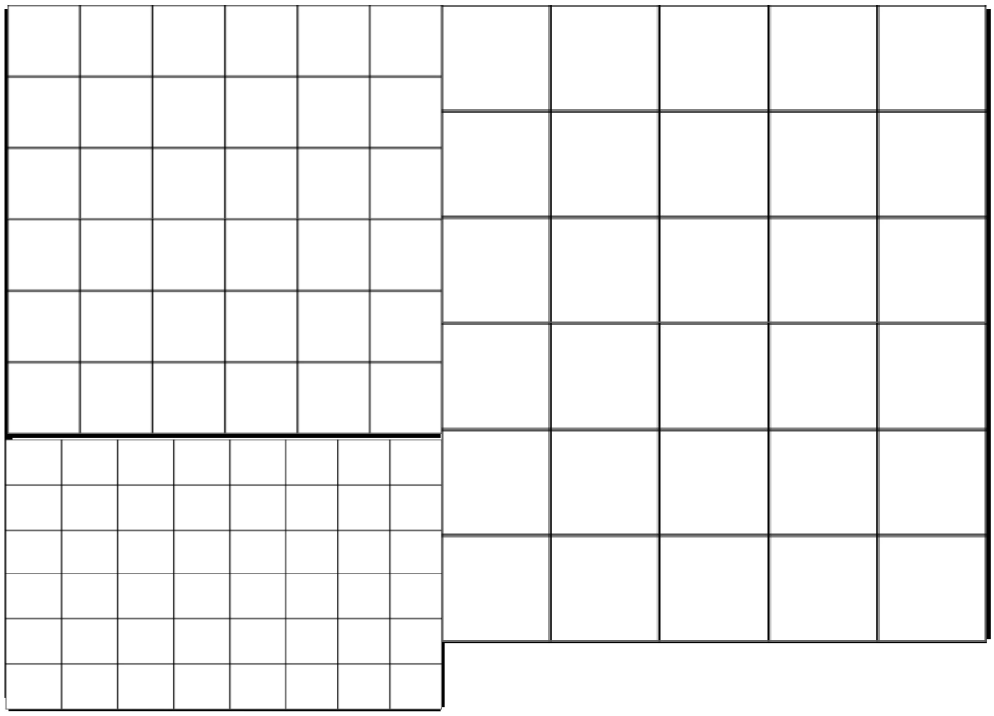


Figure 4.4 The top view of tiled floors

Discussion

The first discussion is focused on the students' counting strategy. The students might demonstrate the efficient way of counting and have the others understand their own way to do that.Then, the discussion is directed into the problem why the larger floor contains the smaller number of tiles while the smaller floor has the larger number of tiles. Some students might recall the third lesson in which they got insight into the inverse relation between the size of unit and the number of unit used. The students might argue that the tile size in larger floor is larger than the tile size in smaller floor, hence, if the tile becomeslarger, then the number of tiles used will be less. They also argue that this is because the matter of tile size.The students' focus might be directed into comparison situation by being asked*"So can we now compare those three floor size then?, "What do you think?"* The students might think that without the tile they can see which floor is larger and which one is smaller. However, they need also to speak about the size in term of tiles used. Perhaps, the students might say that unit does not help them to compare. The teacher can ask *"So what will be appropriate situation for the tiles?"* The students might say that the size of tile in each room must be in the same size so that they can compare between the floors. The students can also say the use of only one of those different sized tiles.

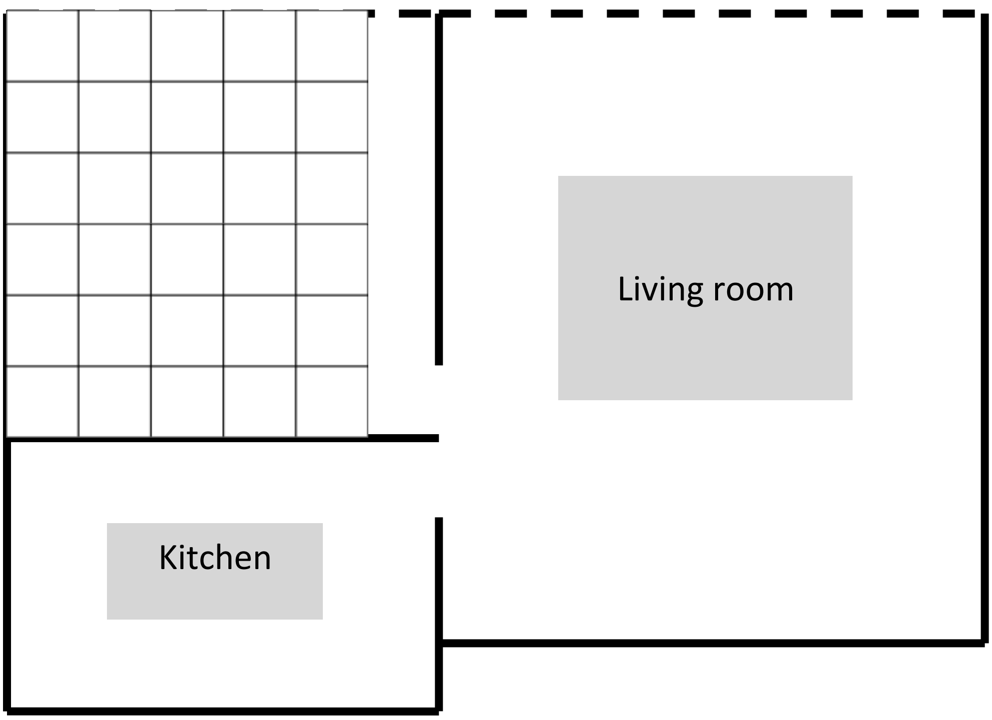


Figure 4.5 The unfinished tiled floors

The students are then set to execute their idea by giving the same picture as in the worksheet part one, but in this moment, all the floor are not completely tiled, so that they can imagine tiling those floor as well as to count how many each rooms contains the tiles. At the final of part one, the students might come up with identical use of tiles in each room and report the number obtained. They finally come with the order of the floor size by the use of the units.

b. Activity 2 –Bu Kartono's other rooms - Can you find how large each room's floor is?

The students will work individually to find the size of each floor by the use of the tiles as the unit. While they are finding out the number of tiles used, they possibly generate the efficient way of counting strategy by the multiplication on the row and column.

The students' individual work will be discussed a little in the end of part 2. They share their result and compare to each other. They might come up with the same order of floor by the use of tiles as the unit.

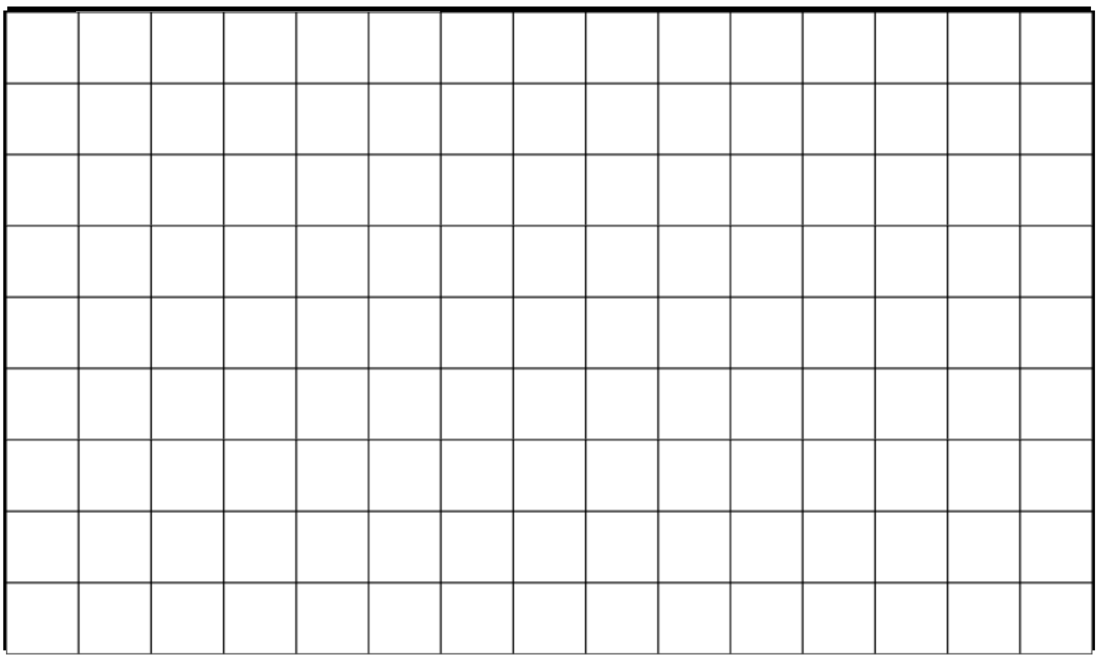


Figure 4.6 The unfinished tiled floors (next problem)

The lesson is ended by small reflection on the use of identical units to not only measure but also to compare several surface. The students might argue that they need to have the same unit, both shape and size to fairly compare the size of surface of things.

3. Mini lesson – Which Carpet is Larger?

The students will discuss which carpet is larger through this extended

Which carpet is larger ?
Bu Kartono wants to buy a new carpet for her new house. She is looking for the large size possible for her living room. In the carpet store, she found two interesting carpets with different size and pattern. She finally decides to take only the larger one among them. The store owner puts those carpets on the tiled floor.
Can you help Bu Kartono to decide which carpet is the larger? What will be your way? (see picture in teacher guide Appendix D)

The students at first might just point out the larger one based on their visual perception. Since they are asked to show how that one can be larger, the students might find a way. The students might directly wonder that the tiles can be used to find the size of carpet. The students use their counting strategy to get the size of each carpet. The students might direcly use the multiplication to get the number of tiles each carpet covers. Hence, they obtain which carpet is larger.

F. Meeting6

In this lesson, the students will learn to measure the area of irregular shape by the use of regular units. The context used is table on the tiled floor.

A minilesson starts the lesson.

1. Students'Prior Knowledge

The students have already learned how to measure the 'size' of surface by the use of identical units and use that units to do comparison. They recognizes that the square unit is the best unit to do covering activity. However, they just dealt with the rectangular shape of surface. Hence, they will start to move to measure the irregular shape of surface. The knowledge they got in lesson 2 about the cookies and the topping (cookie A and square chocochips) will help them in this lesson.

2. Learning Goals

a. The students measure the area of non-rectangular shape by the use of rectangular shape through the context of the table on the tiled floor

b. Students use the multiplication on counting the units

c. Students can do estimation in finding the area of non-rectangular shape

3. Description of Activities and Possible Students' Thinking

a. Mini Lesson

The students will work in their groups to tackle two questions. The students are asked to find how large the cookies are by the use of square chocochips. This task combines what they learned in lesson 2 (topping for covering) and lesson about tiled floor and measure. The students are provided two cookie pictures.

In cookie A students might easily finding out the number of square chocochips used to find how large the cookie is. The students might complete the drawing or just directly do counting by multiplication or skip counting. They report how large the cookie A is by the use of chochips, like what they knew about the size of carpet by the use of tiles.

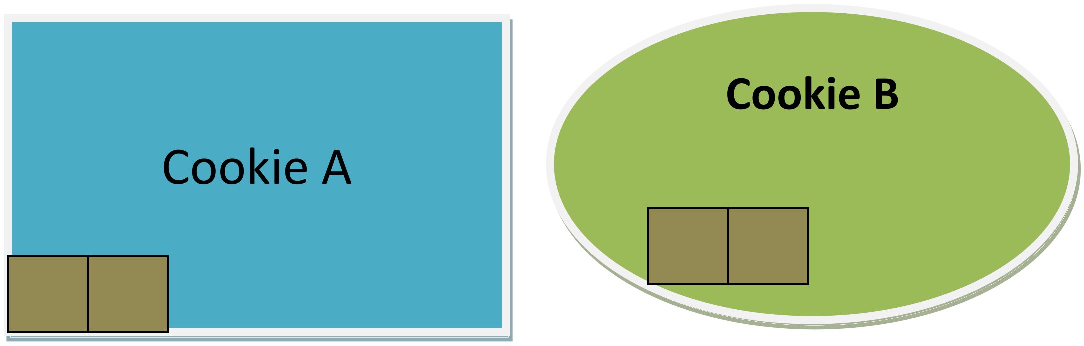


Figure 4.7 The oval and rectangular cookie

The experience in lesson 2 might be very useful for the students. They knew that the chocochips will not cover the cookie perfectly, they leave in the leftover (near the edge). It is conjectured that the pupil will say this when finding out the size of cookie.

The other possible reaction might be splitting out one chocochips, put one part here and another one there. They might then say that splitted chips can cover leftover. They do the same until the cookie is covered.

Another might be by estimation, they say that the cookie is around.........chips.

b. Discussion of Minilesson

The students will report their result on the size of both cookies. In cookie A, the students might find it easy to find the size by simply multiplication on the chips. In cookie B, the students with the estimation will firstly present their estimation idea. It will be complemented by the idea of splitting some chips to totally cover the cookie. The discussion about the number of chips used to find the size of cookie is then conducted, by estimation and by splitting.

c. The task – How Large is the Table?

The students will work in this problem

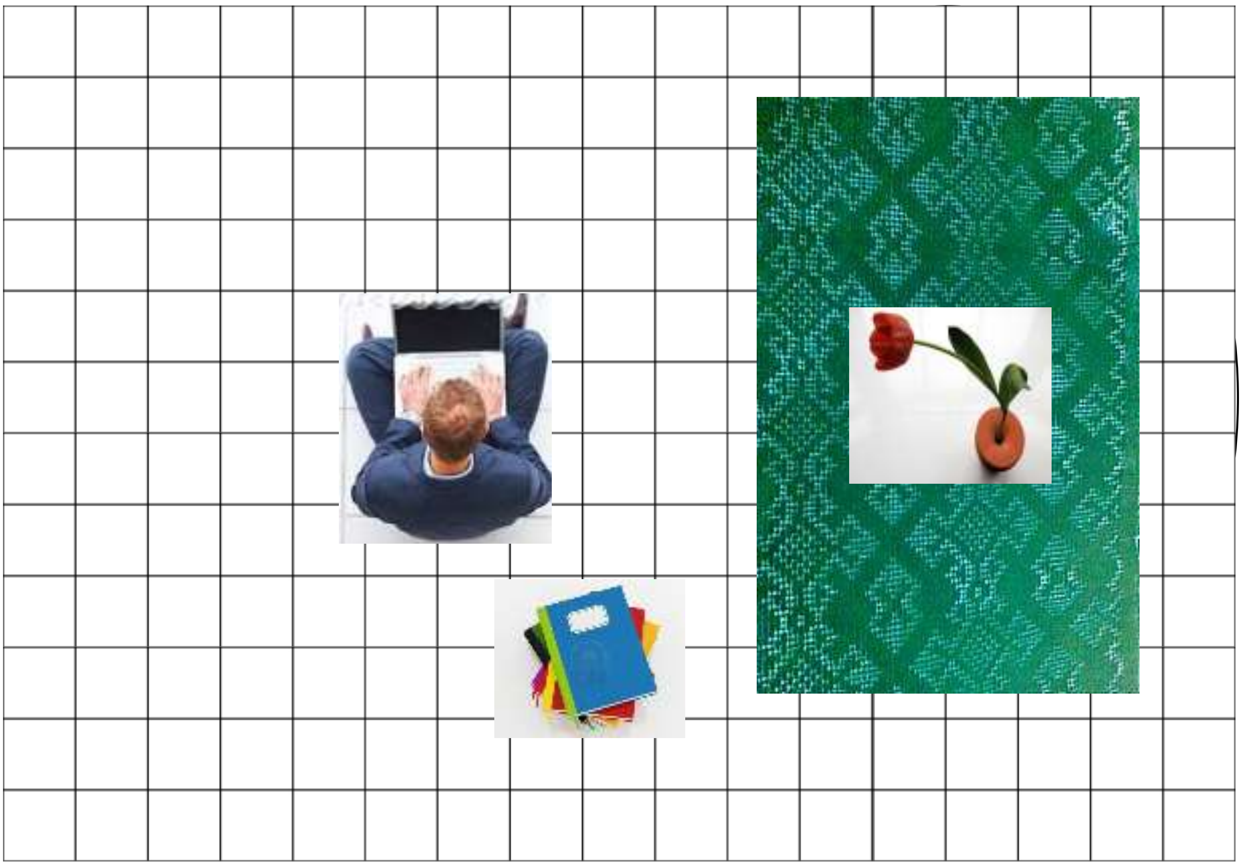


Figure 4.8 The oval table on tiled floor

The students will make use the experiment on the cookie B in measuring the table. They also recall the carpet problem. The students will use the multiplication to find the number of tiles covered by the table cloth and count the rest. When dealing with the unfully covered tiles, the students might come up with severeal idea

1. They find one part and find another one precisely, and think that both can be joined/counted as one tiles
2. They only estimate that there are several pieces of uncomplete covered tiles and count them as one.

d. Discussion

At the final, the students might come up with a little difference on the size of the table, then a discussion is conducted around the result. The students are expected to the precise estimation of the size of the table by the use of tiles.

G. Meeting 7

In this lesson, the students will use the grid as the model to find the area of both rectangular and non-rectangular shape of region. The students will do estimation for the non-rectangular shape, discuss what happens if they have different shape of grid units, and finally compare which is the better, finding the area of region by nuts or by the grid.

1. Students' Prior Knowledge

The students already knew to find out the area of region by the use of units. They also know the inverse relationship between the unit size and the number of units used to cover particular surface. They also get insight on what the area is. The students have already known that comparing the area should include the identical units. Now, they will be introduced to the grid as the good model of finding the area of surface.

2. Learning Goals

a. The students use the grid to find the area of rectangular and non-rectangularregion

b. The students do estimation for the area of irregular shape

3. Description of Activities and Students' Possible Thinking

The students are firstly shown how to use the grid by the teacher. The teacher uses a book for example and finds the area of it. The students are asked what the area for the book is. The students are also instructed to use colour marker when they are measuring the non-rectangular region (the marker with the same colour is attached to the incompletely covered grids that make one). The students are shown the picture of the table in lesson 6 by the teacher, and the teacher attaches red for example in one of the pieces of incompletely covered tile and find other pieces, and then attach red also, joined with previous one to get 1 single grid.

a. Activity 1- Whose Palm is Larger?

Each group decides who will be the one whose palm is traced on the outline paper provided. At the final the students will make the comparison on their palm size.

This is one of the examples of what students make during measuring their palm. For the grids inside the outline, the students probably do multiplication to find how many the grids are. Students might also do several multiplications on grids. It depends on how they partition the completely covered grids inside.

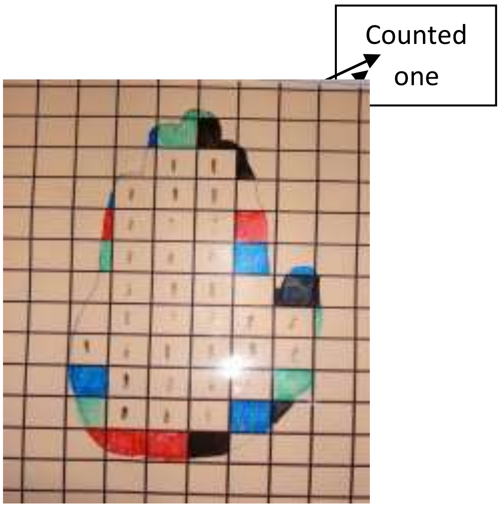


Figure 4.9 Measuring the area of palm

They also count the remaining complete grids. Hence, the discussion among the students in each group might come when they deal with the incompletely covered grids. The students might argue each other about the parts that make one grid.

Discussion

The students will show up their measurement result and report how large their palmsareby the use of grid. Might be at the first time students will not be so easy to determine which palm is larger, since there can be a miscalculation or something. Hence, the students will do investigation again. There can also be situation in which students doubt which one is larger (since the number of grid estimated the same), the students might directly compare each other palm. The students then face the question *"How if they measure with another one in which every grid is larger than this one?"* The students might relate this in what they learn about photo frame and the relation between the photo size and the number of photo used (inverse relationship). They might argue that the number of grid will be less compare to the number we just got by the first grid. To make it sure, students set to do measure with the second grid (containing larger unit grid). Later on, they also might find the situation in which joining parts becomes more difficult (it is because that the single unit is too large, especially when they only have part of it extremely small being covered). Finally the students see that using the smaller one, the area obtained is more precise. However, they will have to see that the number of grid used is totally different for the use of different sized grid. At the final, they conclude that the area of the palm is depended on the grid that they use.

b. Activity 2 – Measuring the Area of Cookie

The students will be given again the cookies they learned in the lesson 1 and 2 on a paper. They will realize that what they did in lesson 1 is similar to this one, which is measuring how large is the cookie. However, they might realize that nuts are not that good to be used to find the area of cookie since the gap, leftover might occur. Hence, they might come up to say that grid is better one.

The students will use both different sized grids to measure their cookie. They are asked to report the area of it both obtained from two different sized grids. At final, they compare what they got with other students and do some check to their own and others’ result. The lesson is ended up with reflection from the students by guided several questions.

CHAPTER V

RETROSPECTIVE ANALYSIS

This chapter provides the analysis on the collected data during two cyles of our study. The analysis will be started from prior knowledge of students of first cycle about the area measurement and their conception about area unit through pretest and interview. This analysis enabled us to do some adjusment before conducting our first cycle study. The analysis is followed with the description of students' learning process during the first cycle and some remarks we obtain from posttest. These chronological analysis will lead to the general conclusion about students' learning process and their development during the first cycle study.

Based on the result of cycle 1 study, we do some adjustment and refinement. Firstly, we adjust the pretest content for our cycle two students based on pretest execussion in cyle 1 study. Secondly, we refine our HLT based on finding and some weaknesses during the execussion of our cycle 1 study. Then, the refined HLT (for cycle 2 study) is used to do some analysis during the second cycle study. The chronological way of analysis is then presented in same order as explained in cycle 1 study. During analysis of both cycles, students' learning process, mathematical ideas, and their development become main issues of the analysis. In the first cycle, we extensively observed six students, while in the second cycle, we especially observe five students in our focus group, and generally the rest of the students in real classroom.

A. Research Timeline

Table 5.1 The research timeline

Phase	Activities	Participants	Date
Preparation	Studying area measurement and designing initial HLT	The researcher and Dutch supervisor	Oktober 2012 – January 2013
	Observing the classroom	The students and the teacher of teaching experiment phase (cycle 2) from Laboratory Elementary School of Unesa, Surabaya	13rd February 2013
	Interviewing the teacher	The teacher of teaching experiment phase (cycle 2) from Laboratory Elementary School of Unesa, Surabaya	
Pretest before Cycle 1	Conducting pretest before cycle 1	A small group of 6 students in preliminary teaching experiment (Cycle 1) from Elementary School of Ghilmani Surabaya	14th February 2013
Small interview	Conducting small interview for pretest clarification		
Cycle 1	Meeting 1 : Cashewnut cookie	A small group of 6 students in preliminary teaching experiment (Cycle 1) from Elementary School of Ghilmani Surabaya	19th February 2013
	Meeting 2 : Cookie and toppings		21st February 2013
	Meeting 3 : Photo frame		22nd February 2013
	Meeting 4 : Photo frame		22nd February 2013
	Meeting 5 : Tiled floor		4th March 2013
	Meeting 6 : Measure area of cookie and oval table		5th March 2013
	Meeting 7: Measure with transparent grid		6th March 2013
Posttest after Cycle 1	Conducting posttest after cycle 1	The 6 students of the preliminary teaching experiment from Elementary School of Ghilmani Surabaya	6th March 2013
Small interview	Conducting small interview for posttest clarification		
Pretest before Cycle 2	Conducting pretest before cycle 2	The students of the teaching experiment (Cycle 2) from Laboratory Elementary School of Unesa, Surabaya	15th March 2013
Small interview	Conducting small interview for pretest clarification	The focused group of students of cycle 2 from Laboratory Elementary School of Unesa,	

		Surabaya	
Cycle 2	Meeting 1 : Cashewnut cookie	The teacher and the students of the teaching experiment and the focus group of the students in the teaching experiment (Cycle 2) from Laboratory Elementary School of Unesa, Surabaya	3rd April 2013
	Meeting 2 : Cookie and toppings		5th April 2013
	Meeting 3 : Photo frame		8th April 2013
	Meeting 4 : Photo frame quick image		10th April 2013
	Meeting 5 : Measure area of cookie and oval table		29th April 2013
	Meeting 6: Measure with transparent grid		1st May 2013
Posttest after Cycle 2	Conducting posttest after cycle 2	The students of the teaching experiment (Cycle 2) from Laboratory Elementary School of Unesa, Surabaya	3rd May 2013
Small interview	Conducting small interview for posttest clarification	The focused group of students of cycle 2 from Laboratory Elementary School of Unesa, Surabaya	

B. The Information Obtained from the Classroom Observation and Teacher's Interview (Teaching Experiment Class/Cycle 2)

We used video recording while observing the classroom activity. We also took notes during the observation. Meanwhile, we just took notes while interviewing the teacher. We used several questions in appendix B and C as guidance to do analysis.

The culture of the classroom IIIA of UNESA Laboratory Elementary School and class organization

Generally, the teacher has rule as demonstrator of the knowledge, facilitator of the activity, and the leader of discussion during the learning process. We found that the rule of the mathematics teacher is not dominant. The teacher just has little time of telling the students what they are going to learn, then the teacher gives the students such activity. For example, when students have the task about calculating the money for some goods sold. The students by themselves explored the things to sell, put the price, and do some calculation for the things bought in their shop. While having discussion, the students are given more time to speak and the teacher let the students discuss among each other. Sometimes, the classroom is pretty noisy for students' activity while discussion in group or other activities done during the discussion. However, the teacher could manage the class mess.

The students' ability

We got some information from the teacher that the students in class IIIA are good learners. They are middle-and-up-competency-level students. We also find this information quite supported by our seeing during the observation that the students do not really have difficulty while studying the money problems for example. Some students seemed understand of doing dificult calculation. Therefore, the teacher does not really struggle to handle students' difference on the competency. There is one case that is the students usually can learn well in pair they themselves find comfortable in. We observed that the students could easily share their thought each other. The students also have competitive soul between others. Hence, we find this interesting because the students are motivated to learn better and better.

The teacher's approaching on teaching and activity

Generally, the teacher gives problem to the students to solve in pair. The teacher do not really show dominant rule for example too much explaining and demonstrating the knowledge. The students have big part on the learning process. Students are also given a lot of task to bring home. We noted that the students could keep up with this kind of approach. The teacher also uses LCD sometimes to teach particular concepts.

The possible ethnomatematics teacher probably does and sociomathematical norms

Quite sometimes we found the teacher asks her students to give their answer their way. For example, students are asked to give their answer and their own reason. However, the teacher does not really go deeply in discussing possible different strategy and solution. Still the students are directed into one particular answer the class will deal with.

About Pendidikan Matematika Realistik Indonesia

Generally, this approach is still new for the teacher. Therefore, she thinks she never does such approach. However, we noted that the teacher sometimes uses the problem in teching which we think good enough to be the start of PMRI approach.

Based on this observation and interview during the preparation phase, we are quite informed to do such adjustment to our designed teaching and learning in our study. Some of them are as follows

1. We use the information that the students could learn better in group they are comfortable with. Hence we create group based on this consideration. We discuss possible arrangement of group's member not only based on the pretest result, students' competency, but also with comfort factor. This we assume will help us having students to learn better and contribute during the groups' activity and discussion.

2. We found that the students get used to work on the problem. So we find our designed lessons appropriate to conduct.
3. We make the teacher's rule during the process more dominant by facilitating and guiding students to construct their own knowledge (see some teacher's rule we entailed in teacher guide)
4. Knowing that the students are competitive, we set and bring the lesson so challenging by using reward at the final (for example, announcing which group does better at the end, etc).
5. We make use LCD in our particular activity during the study.

C. PriorKnowledge of First Cycle Students on the Area Measurement and Area Unit

We conducted our preliminary teaching experiment with six students of Elementary School of Ghilmani, Surabaya, East Java. They are Nana, Naila, Yasmin, Lilis, Lana, and Rohmah. They are the third grade students who are in the middle level of competency in their class. We consulted about this choice of students with mathematics teacher and classroom teacher as well. These students do not yet have formal study of area mesurement in their grade.

We obtain the prior knowledge of our first cycle students through the pretest and interview. In pretest, we gave five problems related to area measurement and area unit issue. Afterwards, we interviewed the students to get the clarification of their answer from pretest as well as collected as many as possible the information on what students know about area and unit conception during the interview. The followings are the analysis of pretest results. The order of content is started from *'measuring'* to *'comparing'* by the use of unit (see *Appendix F*).

In the first problem the students were asked to figure out how *'large'* was the surface of tart, and they were conjectured to use the strawberries to quantify the surface. Based on the pretest result, none of the students used the fruit to figure out the area of the surface. Instead, all of the students used the ruler and measured the tart in picture horizontally from edge to edge. Nana obtained little larger than 10 cm but still less than 11 cm. Naila got 9 cm from her measurement. Yasmin answered 10 cm, while Lilis wrote 9 cm. Lana and Miftahul Rohmah obtained 10 cm for the tart's area.

From the students' answer, it is implied that they still have no sense on how to measure the area or to measure how large is the region. They also seem to have no knowledge on the concept of unit of area. That is why we assume that they did not use the strawberry to quantify the tart surface. Hence, they used ruler. We assume that they already get used to ruler in measuring length and used this tool to measure area. Finally, they treated area measurement just the same with length measurement, in other words to say, expressing how long is just the same with expressing how large (in cm), eventhought maybe they still see that length and area is clearly different. This is we assume since they do not know how to measure the area.

The following is the conversation with Nana after the test on the problem of tiled floor.

Researcher : What is the area of the tiled floor? How is your way to get it? What is the area?
Nana : Hmm, 16 cm
Researcher : 16 cm.?How did you get that?

Nana : Ruler
Researcher : How?
Nana : [measuring the length of floor (9cm) and the widht (6 cm), the result is supposed to be 15 cm, not 16 as she wrote] ... 9 is added with 6
Researcher : Okay....what do you think the area is?
Nana : Area?
Researcher : What is the area?
Nana : The area is this...[pointing out by her fingers and did some sweep on the floor picture to show what indicates the area based on her opinion]

Based on this interview, it is clear that Nana knew something related to area which is surface. However, she did not know how to deal with measuring the area itself. That is why she still used the ruler and did some exploration as she could. Naila, Yasmin, Lilis, Lana also did the same thing, but they only measured the lenght of the floor as they did with the cake, measuring from edge to edge to get the area of tiled floor. There was no consideration on the number of the tiles amongst these students.Rohmah did different among others. To respond the question, she regarded that there are different sized tiles on the floor. She counted that there are 16 small tiles and 2 larger tiles. However, she expressed the area of floor in fraction notation 16/2. There was no particular reason on why she wrote the area of the floor in that way. In a particular degree, Rohmah seemed to have a little insight on area as quantifying the floor by the use of tiles contained on the floor. Even , she clearly regarded the tile size. However, she still did not deal with area as the total number of the tiles. This indicates that Rohmah will be helped if she passed the upcoming lesson designed she is about to join.

In the third question, Nana chose the second paperquilt to be the larger one. At first, she kept using the ruler and did some measurement. Finally she declared that the first one was longer and the second one was larger and picked the second one as the answer. Naila answered the second paperquilt but she had no reason. It was the same with Yasmin but she chose the first one to be the larger. Only Lilis who answered the second paperquilt to be larger based on the number of square figures contained in the paper. She counted them and found that second paperquilt contains more square. Hence, the second was the larger one. Lana kept using the ruler and decided that the second is the larger one. Rohmah chose the second one but revealed no reason.

Based on these students' answer, Lilis has the potential seeing the area unit concept in this paperquilt problem while other still do not yet grasp the idea of the unit to measure how large is the surface.

In the fourth question, Nana drew some lines on the table indicating the making of other books that cover the table. However, she did not end up with the right answer since she wrote the area for the table to be 30. However, when asked how her way was to find the area, she directly measured by ruler, then showed the area from the ruler. This was an inconsistency which she showed. She then said that she forget what 30 was. This implies that Nana still did not recognize the way to measure the area.

The same happened with Naila, Yasmin, Lilis, and Lana. They still used the ruler and did the same way of measure with the tart and the floor. Rohmah was different among other, she wrote 10/3 as the area like she did in tiled floor, 16/2. In this table case, it was clear that she knew there will be 10 books on the table, might be, she knewthat with that 10 table, she could cover the table. However, she still did not know what that 10 was referred to. She still had no idea that the area was the number of the books used to cover the table. That was why she wrote 10/3 with no reason. Nevertheless, Rohmah has already considered the use of books to tackle this area problem.

The last problem is about unfinished tiled floor. At first Nana still found the area by ruler. Finally, she complemented her answer by saying that floor A is rectangle while floor B is square. Then floor B was larger. There is no further explanation why but she believed that floor B remainsthe larger. In this case, still she did not take the tiles into account as the unit to find how large the floors are. Almost the same with Nana's answer, Lilis argued that floor B is the larger since it is square. From her answer, it is implied that square is always larger than rectangle. She did not also regard the number of the tiles in this problem.

Meanwhile, Naila and Yasmin gave the answer floor B but they had no reason on their answer. Lana kept using ruler as she thinks it was a tool to find how large the floor was. Rohmah also gave answer floor B but revealed no reason. Generally, there was no consideration on the number of tiles used to express the area of the unfinished tiled floors.

To sum up, we provide the following table to show how we triangulated the data obtained from different sources during the pretest analysis.

Activity	Data Collected	Video Recording	Students' Written Work	Interview
Answering the questions on the pretest	The use of ruler to find the area	The students used ruler and measure from edge to edge of region. For example, to respond the strawberry cake area, the students used the ruler and measured the cake in picture from edge to edge	Expressed the area linearly (in cm). A student answered the area of strawberry cake as 9 cm, another answered between 10 cm and 11 cm	No information
	Students' recognition on the	Students did not use the unit to derive	Students answered linearly, for	Used and demonstrated

	Units as area determiner through strawberry, tiled floor, and book and table problem)	Area. For example, Nana used and demonstrated how ruler used to find the area of tiled floor	Example Nana's answer 16cm for the area of tiled floor	Ruler to derive area of tiled floor. Measure the length and the width the add them
	Students' conception on the amount of units (through strawberry, tile, and book problem) as the area	Used the ruler and drew units (books) through book on the table problem indicating all books needed to cover table	Represented 30 as the area of table	Used the ruler and measured the table and reported the area to be 30
	Strategy on representing area with amount of different units (unit consistency) through the tiled floor problem	No information	Represented 16/2 as the area of tiled floor	No information
	Students' conception of unit (tiled floor and paperquilt problem) to measure and compare area	No information	A student answered paperquilt B was larger compared to A for having more squares	A student answered which paper was larger by the number of squares contained in each paperquilt

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' prior knowledge based on this pretest.

Conclusion on students' prior knowledge on area measurement and area unit conception

For the question 1 (strawberry tart) and 5 (unfinished tiled floors), all the students seemed to have no consideration on the use of unit to find the area. Meanwhile the problem 2 (tiled floor), 3 (paperquilt) and 4 (table) at least 2 students could reveal the consideration of the units (tiles, square papers, and books) in area measurement eventhough they did not directly see the use of these unit to derive such area. Hence, they just ‘saw’ it but still did not know what to do with it. Only Lilis could see the use of square paper to have contribution on the decision which paperquilt was larger. Her response suggested that she might have ‘strated to ‘see’ the use of unit to the area. However she did not expand this idea to other problems.

Generally, the students still used the ruler to find the area eventhough in particular degree they could refer area to given surface (see what Nana said when talking about tiled floor area in conversation). It entails that they might have idea of area but they still did not know how to measure it. The only thing that they knew, might be is how to measure length by the use of ruler. This is what we assume why students used the ruler in measuring area. What students didwas exactly inappropriate, but this was because they did not know yet what on the surface to measure and how to measure the area itself. We assume that they could see difference between measure the line and measure the area, but since they did not know how to measure area, they might think that the best way to approach the area was by using ruler with any exploration they could do. To some extend, they seemingly just treated measuring line and measuring area in the same way.

Only small regard about the use of unit revealed in the pretest. However, this is a good situation to introduce the concept of area through the unit to the students. Finally, what we can say from the test that the students can be helped we assume in the upcoming designed lessons that concern on the use of unit to measure the area.

We are informed by the result of this test to do some improvement to our pretest of cycle 2 study. Some improvement to the pretest for Cycle 2 are as following

1. Students are not allowed to use ruler during the test. We regard students not to derive misconception from earlier phase of learning area through unit.
2. We change the order of questions. We start from comparing area task by the use of unit, then the measuring task by unit. We would like to know whether by changing this order students can make use comparing task (based on number of unit) to measuring task (counting unit)
3. Some students at comparing task still regard that square form is larger than rectangular form. Hence, we avoid giving them square at the first, we will try to provide two different sized rectangular planes.

D. Preliminary Teaching Experiment

There were six students of Elementary School of Ghilmani, Surabaya who participated in our first cycle study. They were third grade students.We made two groups of students and each group consisted of three students, namely blueberry group (Nana, Yasmin, and Naila) and cherry group (Rohmah, Lana, and Lilis).

1. Meeting 1

a. Activity 1 – Help Bu Handoyo with the Cookie

The students seemingly understood that they were required to help Bu Handoyo to find the number of cashewnuts on top of one cookie of hers.

When given the nuts, Rohmah, Lana, and Lilis directly put a stack of nuts on the top of cookie, while Nana, Yasmin, and Naila chose to put them one by one starting from the edge of the cookie. Slightly Rohmah's group took a look at the moment what Nana's group did. Rohmah's group finally did the same way as Nana's group did.

When trying to find out the number of the nuts, cherry group broke the arrangement they made and counted the nuts used then arranged them again. Cherry recorded that they had 25 nuts on top of cookie by simply counting them just one by one. Blueberry did not need to break the arrangement. They just counted the nuts put on the top of cookie directly and got 26 nuts afterall. The groups found the result different each other. Hence, they were required to observe what the cause was, then to write some answers in their worksheet.

Figure 5.1 Discussion on the number of nuts

The students did not yet regard about the nuts arrangement (overlap, gap or leftover) in this phase. They also did not yet see that easily the effect of the size of the nuts used to the number of the nuts itself in total. They just see that cherry's nuts arrangement was neater than blueberry's was.

The students disscused together about what would be the case on the difference that emerged. Since no one argued about the size of the nuts which did matter, we proposed a question.

Researcher : Now, try to take a look at it, this(reffering to blueberry's cookie) holds more than that (reffering to cherry's cookie) does. See, what do you think will be the cause? Look at this,how do you think the size of the nuts? (pointing out some nuts)
Lana : These are smaller (reffering to nuts on the blueberry's cookie)
Researcher : O this one, there is smaller one, larger one, see? What about these? (Referring to the nuts on cherry's cookie)
Lana & Lilis: They are larger
Researcher : So, there are larger nuts here (cherry's cookie) and there is smaller one right here (blueberry's cookie), so what will be the effect, if you have larger or smaller nuts?
Rohmah : Not the same
Researcher : Smaller is here
Lilis : If it is smaller, there will be more...

We see that it was not hard to trigger the students talking about the effect of the nuts size to the number of nuts itself. Lana and Lilis could easily figure out the size of the nuts. When asked about the effect, initially Rohmah said that it would not be same. We can refer this into the number of the nuts based on what Rohmah said. According to this problem, it is true that they found the different number of nuts used. Hence, Rohmah could say that different size of the nuts could affect the total number of the nuts used on top of both cookies. She sounded like getting some insight into the relationship between the size of the nuts and the number of the nuts used. Lilis came with a good idea. She argued that if smaller nuts are used on the cookie, then the number of nuts used wouldl be more. From this moment, we conjecture that she could see what comes after it. Yasmin said that if the cookie contained the smaller nuts, the number of nuts used would be more, if the nuts are larger, then the number of nuts will be less.

In this first activity, the students could not yet derive the idea of overlapping, gap or left over. We tried by asking nuts arrangement. However, they responded by connecting arrangement with how neat their nuts put on the cookie were. They also could not accomplish the sheet filled after the discussion. We directly moved the students to second activity.

b. Activity 2 – Cookie and 12 Nuts

Each group put 12 nuts on each cookie. Both cookies are different in size. Both group almost did the same way putting the nuts on top of cookie.

Figure 5.2 Pupils arranging 12 nuts

In a larger cookie, nuts were arranged close to each other. The groups left part of cookie top surface uncovered. Meanwhile in smaller cookie, they obtained some nuts put on top of other nuts makingoverlapping idea was possible to derive.

Researcher : What do you see? Can you see whether it (referring to larger cookie, A)is entirely closed?
Students : No
Researcher : What about this one (referring to smaller cookie, B)?
Students : All closed
Researcher : In this cookie (B), what do you see about nuts arrangement? What do you call it?

Lilis : Overlap
Researcher : Overlap, so now write on your paper from the observation

From students' answer on the worksheet, it was shown that cherry understood the situation observed. They clearly said that with given 12 nuts, cookie A would not be entirely closed, and no overlap occurs, meanwhile cookie B will be closed and has overlapping nuts. This group already brought the idea of overlapping and uncovered surface (we assume as the situation representing what we conjectured as gap and leftover). The group regarded gap and leftover as the uncovered part. Blueberry answered the question just based on whether the cookie is covered or not by the nuts, without regarding the overlapping nuts.

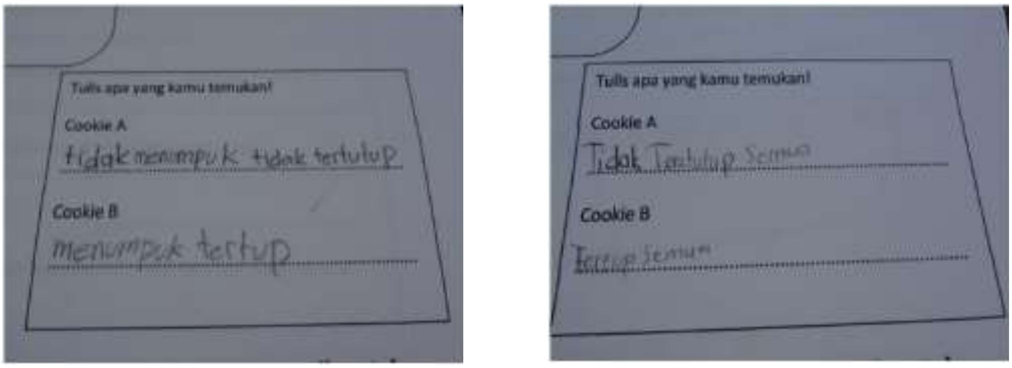


Figure 5.3 Groups' responses on activity 2

We interpret that cherry already had insight into the connection between the size of the cookie and the number of the nuts. In cookie A, the number of nuts was inadequate to cover the cookie, hence, there should be more than 12. Meanwhile, in cookie B, the number of nuts exceeded so that overlapping occured.

In last part of activity 2, students made their own cookie and they needed to avoid bringing overlapping nuts. However, during students were creating their own cookie, the students firstly modelled the shape like semi circle (cherry group) and right angled triangle (blueberry group) in which they used ruler to create the shapes. This was not like what we conjectured (students first put the nuts, avoid the gap, overlapping, then make outline of the cookie). Hence, some gaps and leftover were still created. They named gap and leftover as uncovered surface by then.


Figure 5.4 Groups' arranging nuts on their own cookie

When discussing about the last part of activity 2, several students emerged some ideas.

Researcher : What does this cookie look like?
Students : Semicircle
Researcher : What about this?
Students : Right angled triangle
Researcher : What about the number of nuts on each of cookie?
Naila : Same
Lana : Different
Researcher : Same?How many on this one?
Lana : (Counting one by one) 12
Researcher : What about this?
Lana : 12
Researcher : 12, the number of nuts is the same, but how is the shape of both cookie?
Students : Different
Researcher : Okay, do you think that both cookie is same size?

Students : No
Researcher : Why not?
Lilis : It is because this one is too small (semicircle cookie) and that one too large (right angled triangle cookie)
Researcher : There is part of it uncovered?
Students : Yes
Researcher : So which one is larger?
Students : This one
Researcher : Because?
Lilis : There is uncovered part
Researcher : Yes, the uncovered part in this triangle cookie seems larger, what about the uncovered part in the semicircle cookie?
Lana : There are some..here...here...here
Researcher : Which one is larger
Students : This one (triangle cookie)

We try to interpret what Lilis meant by *'too small'* and *'too large'* in her argument. What we assume is that *'too small'*could mean that the semicircle cookie seems small for those 12 nuts while it will look larger for triangle cookie ('*too large'*). Lana also regarded the nuts and uncovered surface to decide which cookie was larger. We summarised these puplis' thinking as follow: more uncovered surface available, larger the cookie, in another way around the number of nuts is less than it is supposed to be.

We conjecture that the students could see the contribution of the nuts in this case with the cookie size. They also already regarded the overlap and uncovered surface (gap and leftover) while deciding which cookie is larger.

c. Activity 3 – How Many Nuts Put and Ordering the Cookie

The students used only one nut to find the the total number of nuts on each given cookie. They made such prediction. Nana even made a model of nuts by drawing the nuts on the cookie. From her drawing, it was shown that Nana ignored nuts size and uncovered surface. That was why she came with the wrong order of cookie size based on the number of nuts. Lilis and Naila did not draw at all, they just predicted. But the order seemed also wrong. Rohmah, Lana, and Yasmin did not also do some drawings, their prediction was almost correct and they came with the correct order. From this activity, Nana seemed the one who still struggled with this learning while other could slowly kept up and grasped some important ideas about the effect of nuts size to the number of nuts used, the overlapping and uncovered space and its relationship to the cookie size.

The last question about what will happen to the cookie size if the number of nuts become more ended up the meeting. On the worksheet, Lilis answered the cookie will no longer be able to hold the nuts, the nuts will overlap. To particular degree, Lilis' answer seemed correct, but she still could not get into the idea of cookie size which becomes larger. It was because we did not sound that the overlap should be always avoided. Nana seemed confusing since she answered 'enough'. Rohmah answered the size remains the same, while Yasmin, Lana and Naila answered that the cookie becomes larger/larger.

Finally we closed the meeting by asking again on what will happen to the cookie size if the nuts is getting more. Rohmah finally answered the cookie will be larger. All the students understood what Rohmah meant. We changed the question to be what happen if the nuts becomes less. Yasmin ended up with answer the cookie will be smaller.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 1 analysis.

Activity	Data Collected	Video Recording	Students' Written Work	Interview
Activity 1 – Help Bu Handoyo with the cookie	The students' arrangement of the nuts (overlapping, gap, and leftover) and the understanding of each arrangement to the size of cookie	Each group puts nuts on top of cookie that make nutsoverlapping in some parts. However, they did not yet regard these issues until the next activity	The students wrote the number of nuts they used to cover the cookie	Based on the conversation between teacher, Lana, and Lilis, they already understand that smaller nuts used, more nuts used
Activity 2 – Cookie and 12 Nuts	Students' investigation on ideal number (by avoiding the overlapping, the gap, and leftover) of nuts to represent the area of cookie.	Groups made the outline of cookie firstly by ruler then put nuts on leaving some parts uncovered	The semicircle shaped cookie and triangular shaped cookie for given 12 nuts	Students argued that more uncovered parts of the cookie while being covered by given nuts, larger the cookie
Activity 3 – How Many Nuts Put and Ordering the Cookie	Students state the relationship between number of unit (nuts) and the area of cookie	Used one nut s to derive the area of cookie. The students drew some nuts indicating full cover	Drawing of covered cookie with some nuts, the students reported the number of nuts as the size of cookie	No information

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on student' understanding based on this meeting 1. In activity 1, students did not yet see clearly the issues: overlapping, gap, and leftover and each effect to the number of nuts. The students then just realized these issue through the activity 2 that led them to the idea of number of nuts to cover the cookie. The students started to realize the connection between the number of nuts and the size of cookie which was entailed in activity 3.

We recap some findings and weaknesses from this meeting 1 as following.

Findings
Activity 1
Students could not easily derive the concept of overlapping, gap, and leftover in activity 1. They just regarded the size of the nuts and the effect of it
In the first activity, especially in discussion part to generate the concept of overlapping, gap, and leftover by being asked the nut arrangement, the students connected ‘arrangement’ with how ’neat’ the nuts are put on the top of cookie
Activity 2
Gap and leftover were regarded uncovered surface/part
There are students firstly made the outline of cookie by ruler, then put the nuts on top of it. Hence, they become not concerned at this phase on the uncovered part created. **The possible cause : We gave more concern on *“make the cookie!!!”*, then we assume students will freely create their outline**
When asked which cookie is larger, the students could argue by the use of uncovered parts. More uncovered parts available, larger the cookie
Activity 3
There are still three students could not respond well to the question *“what will happen to the cookie size if the number of nuts becoming more?”* Some students’ responses : Enough; the cookie could not hold more nuts meaning the nuts overlap; the same

Weaknesses
Activity 1
Take a long time to read for students about Bu Handoyo’s cookie and for students to understand. The students’ focus was also distratced with name of Bu Handoyo
There was group making stack of nuts at first on the cookie, but later on changing the arrangement by putting the nuts one by one after seeing other group’s work
There are groups who already arranged nuts on the cookie, but later on they broke the arrangement to count the nuts. Hence, some nuts were not counted.
Students could not accomplish the sheet filled after discussion of activity 1

2. Meeting 2

Putting toppings on top of cookie

The students learnt about the overlapping at the previous lesson and brought this idea when they arranged the topping on top of cookies. They were aware about the overlap and tried to avoid the overlapping toppings on cookie.

Figure 5.5 Groups' work on putting toppings on top of cookies

We observed the way the students put the M&M chocolate on both cookies. The blueberry got the oval cookie and arranged the M&M chocolate starting from the edge and covered the rest. From their work, it was clear that they already avoided overlap. The cherry worked on rectangular cookie and they arranged every three M&M chocolates in columns until they got five columns of three M&M chocolates. We observed that only in a small part was overlap created even they already said that overlap was not allowed.

When working with the star chocochips, the blueberry did the same as what they did to previous topping. We assume that this group respected to the idea of boundedness. They tried not to exceed the outline while covering. It was clear that the stars they arranged left so many gaps and leftover. The cherry did a little messy. In some places above the cookie, the stars were randomly arranged, even exceeding the outline of cookie. However, some gaps and leftover were displayed. We assume that both groups already saw that these topping arrangement should give uncovered part, but we did not yet hear any argument during this activity.

The next topping was the square chocochips. The blueberry started to firstly glue the cookie and then they attached the topping. While they put the chip, the chip was directly glued and hard to move. This was we assume to be the cause why in some places, the square chips were arranged so that tiny gaps were created. We just then did question to the blueberry about whether they were really able to cover the cookie with the square chips. We also asked why the gap happened and let them think on what would happen if they little bit drag one chip to the next one and asked them whether the gaps still occured. They seemed understand that the chip would perfectly cover by then. The cherry did nice with their cookie. They arranged not that perfect but it was shown that square chips could cover the cookie.

Lastly, the hexagonal chips. From students' work, it was shown that both groups arranged this chips so that gaps were created. We interpret that the students could not yet coordinate the hexagonal chip arrangement so that gaps were avoided. They could not yet see that hexagonal shape can also perfectly tile a surface.

Filling the worksheet

We set the students to observe their own cookie and answer some questions in the worksheet. They also would observe the second cookie (done by other group) and answer some questions about the second cookie. We let them at first discuss to each other about the answer towards the questions, and set them fill the answer individually. For both cookies, the questions are all the same. The students were asked whether each topping can cover the cookie and they were asked why.

M&M chocolate

For both cookies, oval and rectangular cookie, Blueberry had a various answer towards their observation. Nana and Yasmin answered M&M chocolate could not cover since it is circle, while Naila argued that M&M would not cover the part of the cookie surface. We interpret that Yasmin and Nana has visual reasoning towards the shape of topping while Naila referred to whether the uncovered parts were available or not. However, their answer complemented each other.

Star chocochips

For both cookies, A and B, Naila answered that stars could not cover the cookie since some what she called 'holes' appeared. She referred holes to the uncovered parts. Yasmin still regarded the shape. For both cookies she argued that the stars could not cover based on what she looked at the work. Nana had more precise reason on why stars could not cover both cookie. We also did some small interview towards blueberry group (Yasmin, Nana, and Naila)

Researcher : Take a look at the stars (pointing out cookie with stars topping). Do you think that it can cover entirely the cookie?
Nana : Yes it can
Researcher : Sure?
Nana : No, it cant (changing her answer)
Researcher : Why cant?
Yasmin : Because, it leaves...
Nana : Because star has lancip-lancipnya

Yasmin seemed to argue about the uncovered parts while Nana saw something on the star that make it disable to cover the surface. She argued that the star has *'lancip-lancipnya'* (something we interpret as acute arms) so that she thought that it was impossible to have stars entirely covering the cookie.

Square chocochips

For cookie A, Naila still regarded the holes around the edge. Yasmin regarded that the square shape did matter so that it could not cover entirely the cookie. Nana regarded the uncovered parts on the cookie. Blueberry regarded that square chocochips could cover the cookie B for it covers all the surface. We interpret that Blueberry already saw that square chocochips was better covering the surface, especially on cookie B

Hexagonal chocochips

Based on their arrangement, they regarded that hexagonal could not cover entirely the surface of cookie. From their work, we assume that they could not yet organize the arrangement of hexagonal chips so that no gaps in between. In fact they not only left leftover, but also gap in between the hexagonal chips.

Meanwhile, cherry had totally different observation. Rohmah, Lilis and Lana argued that if all four toppings are arranged so that overlap was avoided, then no toppings would cover the cookie A perfectly. We interpret this based on their response to questions in worksheet. When asked whether those four toppings could cover cookie B, they responded that the square chips could cover and the other toppings could cover if the toppings were well arranged/rearranged. We assume that they think those three toppings can cover if only the overlap is allowed. This idea would be clearer when the discussion was conducted.

Discussion

At this moment, the students could argue that M&M chocolate and star chocochips could not cover both cookie A and B perfectly since they could see some uncovered parts (they said *bolong-bolong*). While in hexagonal chocochips, students at first said that this chip could also not cover entirely. However, they were still not aware that hexagonal could not create gaps. They still could not organize this topping. Hence, we tried to give condition to students, if one chip was dragged and asked them whether dragging some hexagonal could close some holes. Students then argued that it can. However, they still realized that there would be leftover near the edge.

There was an interesting thing in what Yasmin argued. When asked whether square chocochips could cover the cookie A, she directly said no since the cookie was oval, so the square chips could not cover it. There was *bolong-bolong* (uncovered parts) created. This was what we found in the theory. Yasmin seemed talking about resemblance. The square could not cover the oval.

Finally, the students could argue that square chocochips could cover entirely the cookie B. Eventhough some gaps were still created (it was because the children at first glued the cookie then attached the chips making it hard to move if the chip already put, it was directly glued), they still could see if the chips were dragged, then all of the chips could perfectly cover the cookie B.

Hence, the students could decide that the topping shape which could cover a surface better was the square chips. Meanwhile the shape that can be better covered by square chips was the rectangular (cookie).

We assume from this lesson, that students already had in their mind the idea of appropriate shape of unit to cover a surface and in the same time they knew the shape that can be better covered by that square chips. This will be helpful to deliver students to the next lesson.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 2 analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Cookies and toppings (covering cookie with different topping to decide which topping size better covering the cookie)	Students put the toppings in regard of overlapping, gap, and leftover	The stduents put each topping really next to each other to avoid such gap as they can. They also avoid to create such overlap	Based on their work, they answered that the only topping better cover the cookie was square chocochips	Some topping regarded the overlapping, gap, and leftover
	Students discuss the stucture of topping to decide which topping better cover the cookie	The star chocochips is pointed especially in its arms part which make star cannot cover cookie	Star chocochips cannot cover the surface	Students argued that star chocochip cannot cover the cookie for it has they named the acute arms (se conversation between researcher, Nana, and Yasmin

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on student's understanding based on this meeting. In this activity, the stduents already considered to avoid overlapping, and gap during putting topping on top of cookie. This activity enables them to observe each topping and reported which one better covering the cookie. They used such topping structure argument to really show why particular topping, like star, cannot really cover better the cookie since it has some acute arms that make it hard to cover part between another star's arms. Hence, gap will always occur. At final, students could argue why square chocochips can be better topping and rectangular cookie can be better covered.

We recap some findings and weaknesses from this meeting 2 as following.

Findings
Students could not organize the arrangement of hexagonal chocochips (so that there are no gaps)
Nana could argue that star could not cover the cookie better for it has acute arms
The idea of resemblance emerged, Yasmin and Lana argued that square chips could not cover cookie A for it shapes ovel, so chip could not entirely cover it
When asked why square is better to cover, some of them just said because it is square, the others said because it can cover better without gaps

Weaknesses
It took a long time for the students to put the topping on top of cookie.
The students firstly glued the cookie, then just put the toppings then. It was hard to drag the toppings which were already glued to avoid the gaps
Students' discussion about how to arrange the toppings was so minimum
The questions in worksheet was poorly made. Avoid double question like Can ..... ? why? ... give it first space for can ... ? .... then ask why?......, some confusion happened when students filled this and we would face difficulty when analysing

Some students still ask whether that overlap is allowed when covering

3. Meeting 3

a. Activity 1 – How many photos are used?

When starting to attach the photos on the frame, Lana asked whether she could use the smaller photos on her frame. Lilis also asked about how to arrange the photos on the frame meaning that whether she could start attaching photos really from the edge of the frame. We regard these questions came since we did not really clearly give the instruction to them. Hence, we need to be very clear in the next lesson (Cycle 2).

Both groups have different arrangement of photos on the frame. To get the total number of photos on the frame, blueberry and cherry at the first time used the doubling strategy.

Researcher : How many photos are used ?
Lana : 30
Yasmin : 30
Researcher : Can you show me ?
Yasmin : (pointing out the frame) 15 plus 15

Taking Yasmin's doubling strategy, we asked both groups to prove whether 30 photos was correct.

Figure 5.6 Groups' work on putting photos on frames

Yasmin, Nana, and Naila decided to drew lines on the uncovered bottom half of the frame. They modelled the upper half arrangement of the photos at the bottom half by those lines. Hence, they figured out all the total number of photos by doubling.

Meanwhile, Lana and her group, cherry, seemed out of the doubling idea like Lana's argument about 30. This group finally drew some lines on the bottom half of the frame. However, they made four rows of smaller photos. They did not seem to model the arrangement of photos of the upper half of the frame. We thought this was a nice situation to derive discussion since both groups would have different number of photos. This group finally counted the photos one by one and got 39 totally.

We put both groups in discussion. The students were asked why the number was different. Lana firstly reacted that it was because the arrangement was different (she termed this as *'tataan'nya berbeda*). The students seemed to agree with this first argument. However, they did not yet *'see'* size of photos doing matter in this frame case. We tried to stimulate students arguing about the size by asking

Researcher : What about the photo size? Each frame has the smaller and larger ones. Does each frame hold the same number of those different sized photos?
Students : No
Lana : Those (pointing out to the blueberry's frame) holds more larger photos. It is six

At this moment, students started to realize that the size had something to do with the difference created. We then asked for more about it. We generated another question to see whether the students kept up with the size of photos.

Researcher : So, why cherry's frame holds more photos?
Rohmah : It is because measured by the smaller photos

Researcher : Well, is that true? Is that why so many photos used here?

We interpreted what Rohmah said which was for the uncovered bottom half of the frame, they used only smaller photoswhile the other group kept mixing up the photos. She could see that if only smaller ones are used, the number of photos attached will be more. However, we did not ask further on why the result on only using smaller photos wouldreveal more than the mixed one’s would. We did not guide students further seeing that smaller photos occupied smaller surface, then the attachment of smaller photos will be more. Hence, there was no further explanation from students. We took this as the input for next round of cycle. We will create further question to deliver students on the argument.

After this session, we showed the students our own frame. We asked them to find how many photos used on it. They found that it was the same with blueberry’s. Then we asked why. At the first students did not that easily responded towards the situation. We then asked them to see the smaller and the larger photos. They then argued that the number of each smaller and larger photos on each frame were the same. That was why then they know why both frame holds the same number of photos.

Researcher : So, how to have the frame with the same number of photos?
Lilis : Just use the larger one
Researcher : Use the larger one, or? How about the number of the larger photos?
Lilis : It should be the same on both frame.

We took Lilis’ argument to concern on. All the students seemed at the moment understand what Lilis just said. We interpret that the studentsknew that if they got different sized photos and a same size frames, both frames will hold the same only if each frame contains the same number of larger and smaller photos, or each frame just contains larger or just contains smaller. This was the nice point where we tried to bring students further with the investigation.

We noticed that it is not easy to set up a question that can trigger students to think that reporting the number of different sized photos used in a single number is inappropriate. Hence, we sometimes ask directly *"Don't you think that we also need to regard the number of smaller photos and larger photos since we have different number each other? So how to report then?"*

b. Activity 2 – How Many Larger Photos / Smaller Photos

With the frame they worked on previously, the students were asked to figure out the number of larger photos to be only used in the frame as Lilis argued.

Both group had the idea to find the number of larger photos on the frame. Nana, Naila, and Yasmin derived four smaller photos to be one larger photo. Hence, they counted every four smaller photos as one larger photo. Then, they got 6 larger photos on the upper half of the frame then doubled it to get 12. We could see that blueberry kept holding the idea of doubling. They already escaped from counting one by one again. We assume this development because of our set on the number of photos provided.

Rohmah and Lilis also could see that one larger photo occupies four smaller photos. Lana at first seemed not really understand this. Hence, we slowly asked her and she finally could figure out the number of larger photos. However, they kept counting one by one by their sweeping hands on the frame showing those 12 larger photos. We concluded that they might not really consider the efficient way of counting, probably by doubling strategy. However, both groups could accomplish finding the number of larger photos on the frame.

We also asked the students to find out the number of smaller photos on the frame. Nana found the easier way to count the number of smaller photos. She counted the upper half, then doubled it to get 48. Yasmin and Naila received this counting way. Meanwhile, Rohmah, Lana, and Lilis together firstly counted it one by one to get 48. After joining our discussion with blueberry group, they changed their counting strategy to become 24 plus 24 meaning they now could see the advantage of doubling. In class discussion, we tried to stimulate students to count easier. It is not easy to deliver them to multiplication. Hence, we started from counting by column and row. Then, Nana derived multiplication, 6x8 for the frame with smaller photos. At the final of the class, all the students reported three ways of reporting the number of photos used on the frame.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 3 analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Activity 1 – How many photos are	Whether the students use or do not use different	Groups oftudents used two different sized photos on the	They wrote thet they used those two different	

used?	sized photos to cover frame	frame	sized photos	
	The students strategy to report the number of photos used (consistency or not)	One group used doubling strategy for deriving the total number. Another group counted one by one	Both group drew the line indicating the arranged photos to get the total number of photos	One group used doubling strategy because they could see same structure of photos arrangement on the upper half and bottom half of the frame. Another group just counted one by one with no reason
	The students already use identical photos or not	None of both groups used the idenctical photos on their frame	No information	They just used both photos
	The students draw connection between the number of units and units size (inverse relation)	Students srguing why smaller photos result more photos to appear	No information	A student explained the use of smaller photos will result more photos used
Activity 2 – How many larger and smaller photos?	Students' counting strategy on the number of photos	Students counted every one bigger photos as four smaller ones. Doubling strategy is still used. Counting one by one by sweeping hands	12 larger photos or 48 smaller photos as the frame size	No information

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting. In the first activity, the students did not yet deal with how they were gonna report the frame size by the use of different sized photos. Hence, they just counted all them to become a single amount. However, the students then started to realize that the difference occuring forced them to think that reporting in a single amount was not appropriate. Hence, they then reported the frame size in how many smaller photos and how many bigger photos. They also derived the inverse relationship between unit size and the number of units. At the second activity, they could find a way to efficiently count the number of photos.

We recap the findings and weaknesses in this meeting 3 as follow

Findings
With the unsufficient number of photos provided in the pocket (number of photos caould cover half of frame), the students can emerge the doubling/slowly escape from conting one by one
The students do not easily emerge the multiplication at this phase
The students modeled the uncovered part of frame by drawing lines and squares of photos
When asked why the number of photos on both frames is different, the students at first reacted with the word 'tataan'
Students should know that both frame is the same in size. They just became focus on the photos, and regardless to the area of frame.

Weaknesses
The students took a relatively a long time to attach the photos to the frame
The instruction is not really clear for students
We could not create questions at the moment so that students could not keep up with the idea of reporting the number of different photos used in single amount / number is not appropriate
We did not go further asking the students about why smaller photos used will appear more than the larger does. The students just have in their mind that smaller photos will be more to appear.

4. Meeting 4

We asked the students individually to complete the frame with the photos and find out how many photos were needed. Then, they would attach the price for each frame. We found that all the students drew the model of the photos for the uncovered part of the frame. They drew lines to form the photos. Only Lilis chose not to draw by ruler. She rather used only pencil and directly drew the line. We found that the drawing was not good enough.

She revealed same inconsistency of the photos size, some gap, and leftover. However, we assumed this because she decided not to take ruler.

Meanwhile five students accomplished the frame by drawing with ruler. Hence, they took long time to do it. First was because completing drawing, the second was because we gave them so many frames. We will consider to decrease the number of frame for the next lesson in next Cycle. However, none of the students just used their visual reasoning, probably just estimated without needed any drawings. At the final, some students still needed to count one by one the number of frame. Nana was the first who used the easier way of counting. She used multiplication strategy. There is also another students who counted by columns and did skip counting.

At the final of the session, the students had same number of photos in each frame and the same price for each frame as well. We brought them into discussion. We first asked students to see again the frame they accomplished. We brought the students to see whether there were frames with the same price. They easily recognised that there were frame 1 and 4 with the same price (Rp.2400) and frame 2 and 5 (Rp.3600). They completely argued that all frames were different in shape. We then tried to provoke students to talk about the size. We asked them to see the number of photos in frame 1 and 4, then we asked whether they have same size. For the first time, they said no. They seemed not convinced that both frames were the same size. Hence, we tried to have the students to look back at the number of photos in each frame. They found each frame contains the same number of photos. But still they were not easily see that both of them were equal in size. We assume that the students did not yet clearly see the connection between the number of photos (the size determiner) and the size. We then tried to relate this with the price. We asked what happen with the frame size if the price is the same. We found that relating the price to the frame size quite convinced them to see that different shape of frames containing the same number of photos have the same size.

Hence, we asked them again about frame 3 and 5 and whether they had the same size. They easily remarked that both frames had the same price, then they were equal in size. Finally, we tried to bring students again to see the relationship between the number of photos and the frame size. We asked what happen to the frame size if both frames had the same number of photos. They finally could argue that the frame should equal in size.

We ended up the meeting by asking them what happen if the number of photos in one frame becoming more. The students responded that the frame size will be larger. We asked in another way around, what happen if the number of photos in frame becoming less. The students effortlessly argued that the frame size will be smaller.

We found that generally meeting 4 went as we conjectured to happen. We marked that students were still slow to work on the frame accomplishment. They took a quite long time so that the discussion time went shortly. We assumed also that the number of frames we provided was also too many. So we consider to decrease it.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 4 analysis.

Activity	Data Collected	Video Recording	Students' Written Work	Interview
Completing the frame and put the price	Use of photos as area determiner of frame	The students were not easy to see the photos as the area determiner	Students wrote the number of photos on each frame	No information
	State that different frame can have the same area (number of photos)	The students just realized the same number of photos means the same prize, but not yet the same size	No information	Firstly, they did not yet conclude sama number of photos, same size. However, after being asked to check the price and the number of photos, they started to see same number of photos, same size
	Conclude that more photos, larger the frame	The students conclude that more photos, larger the frame	The students conclude that more photos, larger the frame	The students conclude that more photos, larger the frame

From the table , we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting. In the first activity, it was not easy for the students to directly relate the number of photos to the area of frame unless it was related to the price context. However, the students started to realize that more price, more photos attached meaning larger the frames. Finally, the students could conclude that more photos, larger the frames.

We recap our findings and weaknesses during the meeting 4 as follow

Findings
None of the students who did not necessarily draw the line to make photos. There was no estimation appearing.
There was students who drew the photos but with no ruler drawing with the inconsistent size of photos, even appearing some gaps between photos and leftover
There were still students performing counting one by one eventhough we did not provide the uncomplete photos.

Weaknesses
Too long time to accomplish the task
Discussion about the order of frame was not accomplished
The sheet filled after class discussion was not done because of running out of time

5. Meeting 5

In this lesson, the students learned to compare the area of surface by the use of identical units.

a. Activity 1 – Tiled Floor

The context used was tiled floor (see figure 4.4). The students would try to compare the size of floor by the use of tiles. Lana could see that family room was the largest one. When we asked why, Rohmah added that it was the largest for it’s shape or look.She did not take tiles into account. We did not stop asking clarification at that point, then we asked how large the size of that largest floor was. Lana could argue that she could use multiplication on tiles to get the number of tiles. We intrepret that Lana has already had the idea of unit to find the area of floor, then she made use of it to find how large it was. She moved in front of the class and used the picture to demonstrate the multiplication she meant. She said that the floor sized 6x5 tiles. However, she did not yet deal with remembering the product of multiplication, even for that 6x5. Hence, she counted the tiles one by one and obtained 30.

When we asked which floor was the largest, Naila could directly see that kitchen was the smallest one without seeing the tiles. When we asked her on how large it was, she came in front of the class and directly counted the number of tiles one by one to get 48. However, we did not intend students to count it one by one anymore, so we decided to have little time discussing about the multiplication.

We started by asking what multiplication it was. Lana and Rohmah came in front of the class, and did some sweep by their hands in respect to rows to show there were six rows of eight, then she declared 8x6. However, they did miscalculation. They said 47. We asked them again what will be the product. Nana came with answer that : 6+6 = 12, 12+12=24, 24+24=48, and others also could see it. We assume this because they still did not master the product of multiplication of two numbers.

When they got 48 for the kitchen floor, we asked them to compare it with the size of family room floor and we asked *"How could that happen?"*.Naila answered that it was because the size of tiles in family room is larger than the size of tiles in kitchen room. We interpreted that Naila understood the situation. Perhaps she recalled the lesson about the frame, the smaller photos and larger photos and used the idea of unit size and the number of units in this tiled floor context. We also asked for the size of bedroom floor. They could answer that it was 6x6=36 tiles.

We found that the students at first could compare and order the floor easily based on their size without regarding the tiles. Meanwhile, when asked how large each floor was, they found something totally different. The largest floor contains the tiles at least while the smallest floor, kitchen contains tiles at most. At first the students were hard to think about the situation. We kept asking about what would be the appropriate situation in order to get the correct order. Then, Lana had important initial idea that each floor has to have the same small tiles. However, she could not think further about this idea. We, in fact hoped that there would be students to argue that the tiles were the same size, then they could compare the floor correctly. However, we found it hard for students to receive and conclude it. Hence, we decided to conclude it together with them.

We conclude that the students had already known that the cause was the size that did matter, so they got different order. But they could not yet conclude, maybe, that every tile should be the same size so that they can compare in fair way. Hence, we are not really sure whether the students has in their mind that in comparing area, they got to use the same size unit.

b. Activity 2 – Which carpet is larger?

Nevertheless, the next task about deciding which carpet was the larger quite informed what students already dealt with. This task already brought the same tile size to compare which one was larger. In this situation, students found it easy to compare directly. They directly regarded the number of tiles covered by the carpet and compared them. They used multiplication and found the result that can be easily compared to obtain which carpet was larger.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 5 analysis.

Activity	Data Collected	Video Recording	Students' Written Work	Interview
Tiled floor activity	Perception on the area measurement and comparison ny the use of identical units (tiled floor context)	The students chose the room regarded larger based on the visual reasoning, no use on the tiles	No information	Started to see the different sized tiles on each floor was different that makes them hard to compare. For example, Lana said the tiles on kitchen floor were far too smaller, so that many tiles contained
Which carpet is larger?	Multiplication strategy construction	The students could easily apply multiplication on identical sized tiles on the floor	Carpet covering more tiles is larger. Area was obtained by multiplication	Students applied multiplication to easily count the number of tiles

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis,we could derive some conclusion on students' understanding based o this meeting. In teh first activity, it was not easy fro the students to compare the area of floors by the use of different size of tiles contained on each floor. Students could just argue that the smaller tiles will result more. We needed to help them to conclude that the same tiles should be used to compare each floor. The next activity showed students already able to apply multiplication to easily count the number of tiles.

We recap our findings and weaknesses during the meeting 5 as follow

Findings
Students could see the order of floor visually without regarding to the tiles Students could find how large was each the floor Students conflict with the tile size. Based on the number of tiles they got different order. The students regarded that the smallest floor contains smallest tiles, then the number of tiles was the most while in the largegest floor the tiles were the largegest, then the number of tiles was the least. It was difficult for them to argue that all the floor should contain the same size of tiles. We directly asked *"So, what should it be, the size of all floor?"* Some students said the tiles should be the same size. However, in the next task, there were still students drawing different sized tiles.
The task about choosing the larger carpet went well. The students could regard the tiles for finding which one was the larger one. The multiplication emerged.
Eventhough the students already knew multiplication strategy, they did not easiliy find the product of particular multiplication

Weaknesses
Sheet was not entirely accomplished

6. Meeting 6

a. Activity – Mini Lesson on the Size of Cookie

The students were asked to find out the area of cookies' top surface by the use of square chocochips. It was not hard for the students to find out the number of chocochips to cover the rectangular cookie by multiplication, 5x4. They could express the number to be size of the cookie itself. However, the students struggled to find the number of chips used to cover the cookie B. We found that students arranged the chips with no gaps in between, but left the uncovered parts near the edge. Even, some chips was arranged so that it exceeded the outline. We then asked *"How is your way to cover the uncovered parts?".* We found that Nana had idea to break or crack/split any possible chip and those splitted parts were spread over the edge to get the uncovered parts covered.

We took this Nana's idea really important to be concerned by other students. We found that the other students agreed with this Nana's idea. We let the students did this way, and observed how they were going to do the counting. However, they could find it easily to figure out the number. Nana and her group thought that some parts of the chips could be counted as one, for example, this part put here and that part put there could be counted as one chip because these parts came from one chip.

We found that students already had the idea of making parts into possible one unit which was important and enough to set the students to work on the next task.

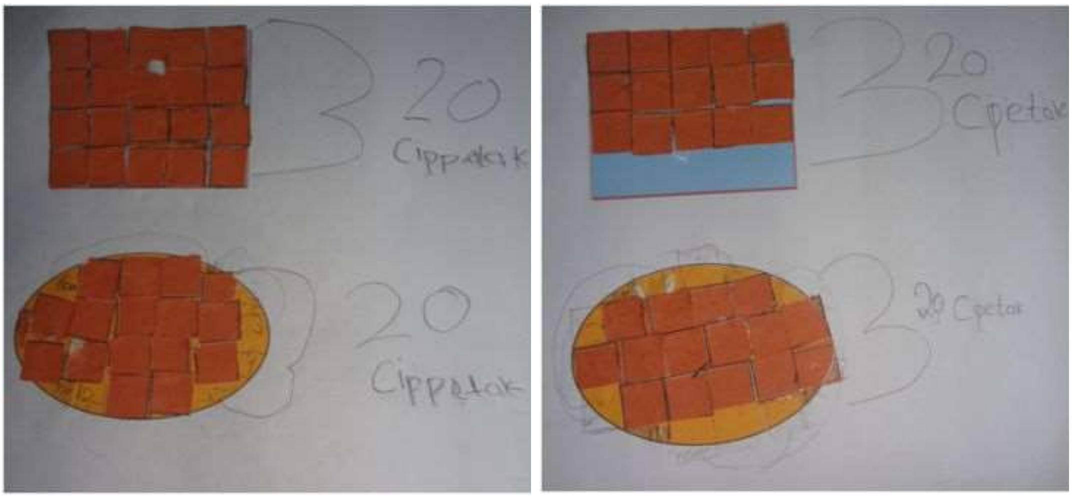


Figure 5.7 Groups' work on measuring cookies with square chips

Eventhough that they did not really perform splitting the chips, they had idea to draw some curved arrows around the cookie to indicate the spread of splitted chips. For example, they marked the exceeding part near outline (see the cherry's picture above) and imaginatively cracked and moved to other place to cover the uncovered part. The blueberry even gave sign for the splitted chips, for example number 5 right there joined with 5 right here to get 1 chip to count. They finally came with the estimation of the number of chips used to cover the oval cookie which was 20 square chips.

b. Activity 2 – What is the Area of oval Table?

When asked how their way was to find the area of the table, Naila could say that tiles there in the picture could be directly used, like the chips on the cookie. Naila's idea towards the use of unit to find out the area seemed stronger at this level. The following pictures are the work of each group to find out the area of the oval table.

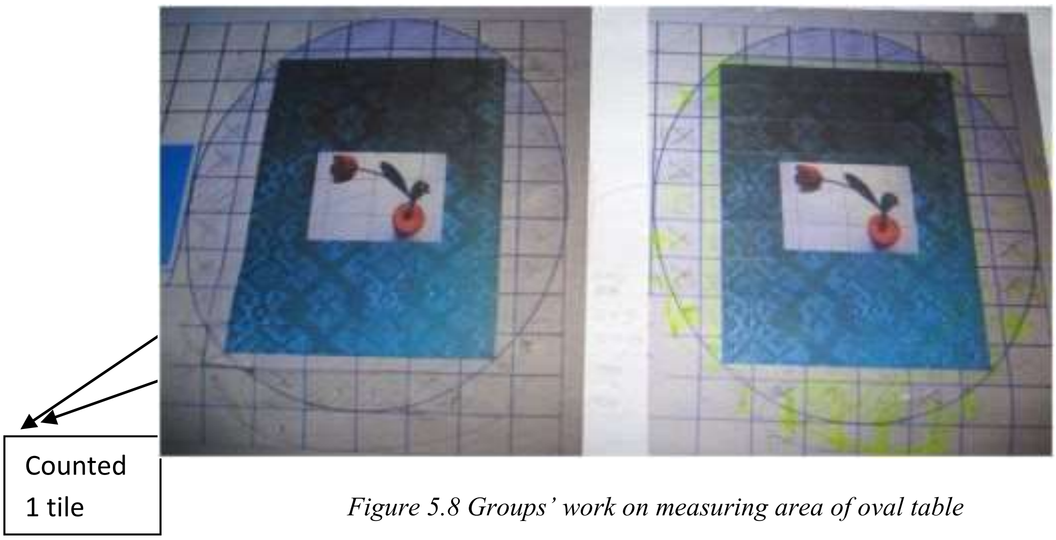


Figure 5.8 Groups' work on measuring area of oval table

The first thing that students did was to find out the number of tiles hidden by the table cloth. We found that both group did not directly apply the multiplication to find out the number of tiles. They instead drew some lines horizontally and vertically to display the hidden tiles. We assume that students could not yet think efficiently at this moment. Even, one group came again with conting one by one those tiles.

We asked again about the way to efficiently count the tiles. They could respond that they needed to multiply. However, we found that students took their time to 'locate' the area for multiplication/interior partial array, whether they need to multiply 6 and 9 or 4 and 7. We assume that the table cloth position which was not accurately cover the tiles confused the students to apply multipilication. However, they made their own choice to use multiplication 6 and 9. They saw that in the area 6x9, each square on the corner was not entirely covered by the table. The students then thought to subtract those four tiles from 54 tiles (6x9), they obtained 50 at first.

For the remaining tiles, they found that there were some fully covered tiles and they added to that 50. For the other tiles, they used the strategy they invented in the cookie task. They advanced the idea to derive such strategy for joining splitted units. From the picture above, one of both parts of not fully covered tiles complemented to other one and joined to become one unit. The students did this strategy to other tiles and came up with the estimation of area of oval table. We found that students came with 68 for the area of the oval table.

We conclude that students already understand how to deal with the area of non-rectangular surface. The experience about finding the area of cookie by the use of chips gave them knowledge how to deal with finding the area of non rectangular surface. They know they will deal with the not fully covered units and they need to do some estimation of the number of those units. We assume that their new knowledge will be extensively used in the last lesson. However, some weaknesses still occured, especially when students tried to multiply to get the number of units. They seemed still not flexible to choose the 'area' of multiplication/interior partial array. They also deal with remembering the product of two numbers. However, this part gives us information how to deal with the multiplication task. We take this really matter to refine our next design.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 6 analysis.

Activity	Data Collected	Video Recording	Students' Written Work	Interview
Finding the area of oval table	The use of tiled floor as the unit to derive area of oval table	Students found the tiles hidden under the table cloth	No information	A student reacted that the tiles can be units counted to represent area of table
	The use of multiplication to count the tiles	Students tried to locate the inner array to apply the multiplication	The drawing of inner array	Use multiplication on 6 and 9 inner array
	The way of estimating the area of non-rectangular region (oval table by the use of inner array and splitting joining strategy)	Students applied splitting joining strategy to count the tiles	Some curved arrows made to pair some pieces counted ones	No information

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting. We concluded that the students already regarded the use of tiles to find the area of oval table. They firstly struggled to locate their inner array to apply the multiplication. However, they could find the area by inviting the idea of splitting and joining strategy to derive the area of oval table.

We recap our findings and weaknesses duringas follow

Findings
While in mini lesson, the pupil (Nana) could emerge the idea of splitting the single chips to cover leftover
While in minilesson, students could emerge the idea of compensate

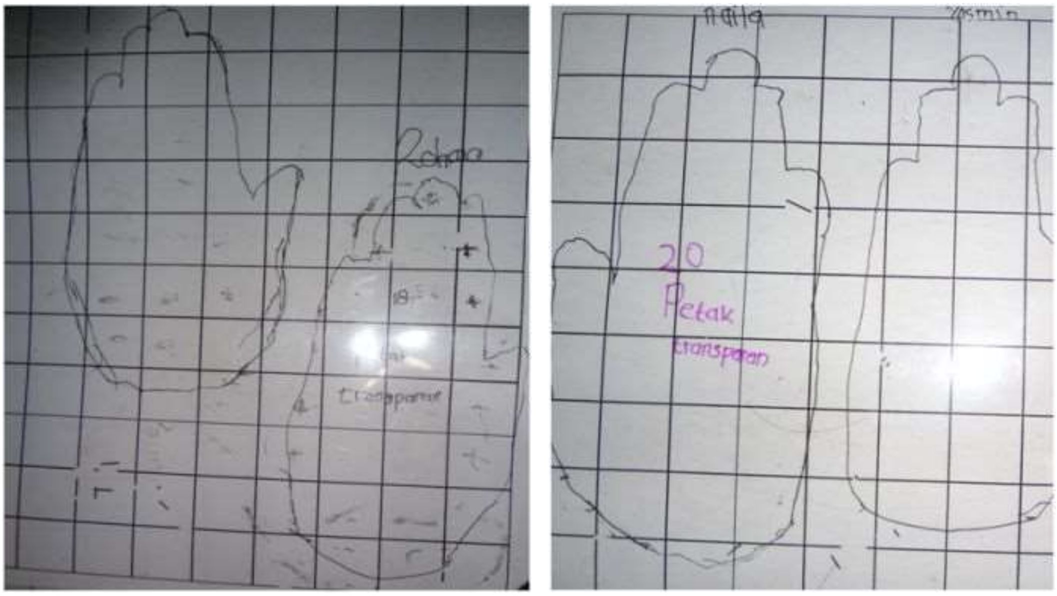
Weaknesses
Lack of discussion on efficient counting strategy in oval table task.
It takes long time for students to measure the size of cookies by the chips

7. Meeting 7

In this meeting, the students used transparent grid as the model to find the area of particular surface. The studentsfigured out the number of grid squares as the area of region.

a. Activity 1 – The area of your palm

Firstly, we set the students to compare each other's palm size. The students could perform superposition/put each palm each other, and the students could argue which one was larger. We then asked the students about how large each's palm was and how they are going to measure that palm. Hence, they had their palm outline on paper each of them. Then, we introduced the tool to measure the area of their palm which we call transparent of grid paper. We asked the students to put it on the top of paper with palm outline. We asked them to figure out how large they were. The pupil remembered finding the area of oval table with the tiles in previous lesson and they used their experience including the strategy to count the tiles in this activity.



Cherry's work — Blueberry's work

Figure 5.9 Groups' work on measuring area of palm by the transparent grid

The students gave mark '+' for the not fully covered grid square. The students matched every possible + and another + to possibly form one grid to count. We found that the students did this carefully and discussed the possibility. However, the students at first did not directly apply multiplication to find some number of grid squares inside the palm. We saw that students were hard to create the area for applying multiplication inside the palm/interior partial array. That was why we came back to guide them to do efficient way of counting. One of way Rohmah did on her palm was getting 8 grids by multiplying 4 and 2 inside the palm, then she counted the remaining gridsquares.We saw that the previous lesson really helped them in this phase. At the final the students could come up with the estimation of palm area by the use of transparent grid. They wrote the area down in the middle of the palm outline.

However, we did not set the students discuss more about possibility to derive such multiplication as one of the easy way to count the grid to find the area of palm. We directly move to the next discussion.

We also showed the students the second transparent grid which contains smaller squares. We asked the students on what would happen if the second grid was used. The students could argue that the result would be different. We clarified what students meant. We asked whether the area remained the same or not. The students added that the result would be more. They said that it was because the size of the grid was smaller, then the the number of smaller squares occupying the palm will be larger than the one used before.

We saw that the students already grasped the idea of inverse relationship between the unit size and the number of units used from the lesson about photo frame. We then asked them to prove what they said. Rohmah took the second grid and put it on the paper of palm outline, then she counted. She found that it was indeed more than the number of squares on grid used before.

We then asked them about the possibility of having another size of transparent grid. They could easily say that it could be another one and the result was based on the size of the grid. We saw that the students could see that the result of area measurement by the use of grid depends on the size of the grid squares.

b. Activity 2 – Measuring the Cookie with Grid

The students came with their own choice of grid used to find out the area of cookie. Both groups used the smaller squared grid. Both group came with the same estimation of the cookie area which was 29 squares.

At the final, we asked the students to compare the use of nuts and the use of grid to find the area of region. The students argued that the grid provides more precise result of area measurement since the nuts they said contains gap and sometimes could not perfectly cover the surface.

From this lesson we conclude that the students already had knowledge how to figure out the area of both rectangular or non rectangular by the use of grid, the precise model of unit. They also could see that the result of the measurement depended on the size of the grid used.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 7 analysis.

Activity	Data Collected	Video Recording	Students' Written Work	Interview
The area of your palm, the cookie	The use of transparent square grid to derive the area	The students counted the tiles, gave such sign + for the unfully covered tiles for pairing	The area of each palm by the use of tiles	The use of grid was the same with tiles in oval table problem
	Estimation of area by the use of transparent square grid (by the use of inner array and splitting joining strategy)	Pairing the + sign to get some full tiles	The area of palm	The students called the strategy they learned in meeting 6 to apply for finding the area of palm

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting. We concluded that the students could relate the way to find the area of palmby the use of grids with the the use of tiles to derive the area of table. The students could also apply splitting joining strategy to find the area of the palm.

We recap our findings and weaknesses as follow

Findings
The students seem hard to derive and use multiplication in palm task as the way added to find the area of non rectangular region

Weaknesses
We did not do asking and answering to check whether the students already seen each grid has the same size
We did not set the students explore more the grid

E. The Remarks of Knowledge of Students after Cycle 1 Study

We conducted posttest in the end of learning process of Cycle 1 study. It aims to clarify students' development of learning during the Cycle 1 study.
It does not aim to compare students recent knowledge with their prior knowledge.

The posttest presented seven problems (see *Appendix G*). Several questions asked the students to compare the area of particular surface by the use of nuts, books, and grid. Other questions asked the students to find the area of planes by the use of object like tiles and grid. We also tried to check whether the students could create their own grid to figure out the area of given planes.

For questions number one, all of the studentsanswered that cookie A was the larger one. They got the same reason. They argued that cookie A was the larger since it holds more nuts. It seemed that these students already applied what they learnt in lesson one which was more nuts larger the cookie. We also notice that these students already regarded the nuts as the units of area that contribute to the area of cookie itself in these comparison of area context of problem.

For question number two, Yasmin did wrong result for 26. Neither did Lana for 30. We assumed that they did miscalculation. In this question Rohmah answered that this floor's area was 18 tiles. It was clear that she counted all the different sized tiles as one single amount that represents area.

Figure 5.10 Nana's strategy to find the area of tiled floor

Meanwhile, Nana and Naila got the answer 24 tiles for the area of the given floor. It was also clear that Nana drew some lines in the larger tiles displaying that she divided this larger tile into four smaller ones. She got all the identical tiles then counted 24 totally. We assumed that Nana and Naila already had the idea of expressing the area of surface by the quantity of identical units.

In question number three, all of five students answered correctly by choosing table A as the larger one. However, they got different reasons on their answers.Yasmin, Naila, and Rohmah reffered to the shape of the table. Yasmin and Naila considered the table A larger since it looked longer than table B. We interpreted that Yasmin and Naila did not take the book into consideration to determine which table was larger. They instead used their visual reasoning. Rohmah argued that table A was larger since it was rectangle and table B square. We interpreted that Rohmah also looked that table A seemed longer than B. All in all, these three students did not take books as the tools/units to find the size of table to compare.

Meanwhile Lana and Nana had already regarded the number of books on each table and used this to compare which table was larger. We interpreted that Nana and Lana already seen that books can be used as units to find out how 'large' the table was.

Meanwhile, the best approximation for the area of the plane figure in number four given by the use of grids is 32 square units either by 8x4 or the joining spitted units strategy. The best approximation for the plane figure in number five given by the use of grids is 33 square units.

Yasmin and Lana still seemed not able to make use the grid to figure out the area of given plane figures. Yasmin got 8 for plane number three and 6 for plane number four. We interpreted each number belongs to the one of the sides of each plane. While in number three there were eight squares on two pairs of parallel sides (that was why Yasmin answered 8) and in number four there were 6 squares. She did not know how to find the area by the use of grid after all. Same with Yasmin's case, Lana got 16 squares for number 3 and 14 squares on number 4. We recognised that 16 was obtained from 8+8, the number of squares in total in two parallel longest sides. We could not recognise why she obtained 14 for number 4. However, Lana also did not know yet how the grid can be used to find out the area of given plan.

Nana, Naila, and Rohmah already used the grid provided to find the area of given figure. Nana used the joining strategy for both planes, joined some pieces of splitted squares to make one full square. However, she came with the inadequate approximation that led her to wrong choice of larger plane (question number 6). Naila almost reached the good approximation, 30 and 32 and she decided the plane number four was the larger one. Rohmah also reached the good approximation. While she got 32 in plane number four, she got little bit exceeding number of squares which was 35 for the area of plane number five, but she could answer question six as the consequence.

We interprete that these three students already could manage the area of given plane by the use of grid. We conjecture that they now have in mind that grid is one of the tools used to find the area. In addition to that, they can represent the area of plane as the quantity of the squares covered by that plane (see following picture).

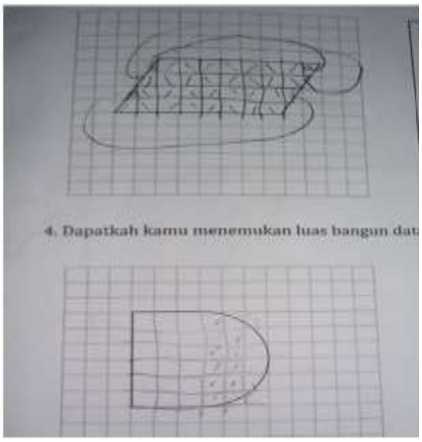


Figure 5.11 Pupil's strategy to find the area of plane question number 4 and 5

For the last question, Yasmin and Naila left the last question blank. They did not manage to do it.

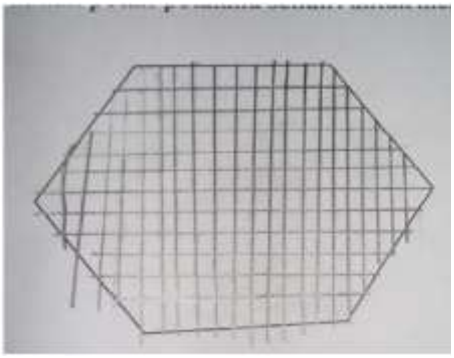


Figure 5.12 Pupil's constructed grid to find the area

Nana, Lana, and Rohmah interpreted the question correctly and they created their own grid to find the area of hexagonal figure. These three students obtained different result since the grid they made different in size each other.

Rohmah drew her own grid, small enough and obtained 92 squares. It was clear that squares she made were identical. In Lana's case, she created some different sized squares. She drew much larger than Rohmah did.

This inconsistency of the unit size led her to the wrong approximation of area by the use of square units. Nana drew grid so much smaller than both did. However she came with 78 squares which was the wrong calculation after all. We noted that Nana run out of time to accomplish this task so that she just provided the answer 78.Hence, these students seemed already understand how to find the area by the use of grid. Nevertheless, they need to learn more that the grid they make should include the identical squares.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the posttest.

Activity	**Data Collected**	**Students' Written Work**	**Interview**
Posttest	Students' recognition on the units as area determiner through nuts, tiled floor, book and table, grid problem	Students could measure and compare area of cookie by the use of nuts	More nuts, larger the cookie
	Students' conception on the amount of units (nuts, tiled floor, book and table, grid problem) as the area	Students already represent the unit amount as the area of region	Area can be derived by the use of units
	Strategy on representing area with amount of different units (unit consistency) through the tiled floor	Some students already regarded identical units in deriving the area	Larger unit can be regarded some smaller units
	Students' conception on the use of unit (square grid problem) to measure and compare area	Students made use of grid to derive area as well as compare area. They also applied the strategy splitting joining strategy to deal with non-rectangular region	Unit used to derive and compare area
	How the students construct their own grid	The students picked themselves inner array to	No information

		apply the multiplication	

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through diferent tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data.

From this posttest, we obtain some remarks. The students already regarded the contribution of unit to derive area of plane. Some of them also regarded the idea of identical units when trying to quantify the plane by the use of particular units. For some students, the use of grid as the model to find the area of plane was already understood. It is proven by the written work of the students from the test. They also could do some estimation for the non-rectangular plane by the idea of joining pieces of splitted square.

From this posttest, we see in general that there are some development of understanding of the students towards area measurement through the unit conception. For some students, the development is clearly seen. For example is Nana. She and some other students already understood of the use of area unit to derive the area; understood the connection between the number of units and the area; understood the concept of identical units; understood the inverse relationship between unit size and the number of unit used; understood to find the area of rectangular and non-rectangular by the use of grid and do some estimation.

F. The Conclusion of the Students' Learning Process in First Cycle

We derive some conclusion on students' learning process starting from their prior knowledge, their participation on each lesson of our cycle 1 study and supported by the result of the posttest.

Experiencing the lesson on the cashewnut cookie provided the students with comprehension on the unit issue: overlap and gap between unit within the situation of counting the unit. The students could even see the effect of each issue to the number of units used. We also noted that the students built their understanding towards the connection between the ideal number of nuts and the area of plane after they understand the unit issue: overlapping and gap and they think of making ideal size of plane by the use of particular number of units without possible gap and overlap. This is followed by the fact that the students finally could derive the connection between the size of plane and the number of units used to cover the plane itself. The next learning process the students experienced is finding the appropriate shape of unit to cover the region. We noted that the students already had in their mind that square units is the best shape to cover the surface. They also even could relate it to the shape of region being covered. The students understood that rectangular region would be better covered while non-rectangular one will remain some part uncovered.

The photo frame context, the students are able to recognize the idea of identical units and the inverse relationship between the units size and the number of units used to cover surface. However, the students still need to be guided to see clearly the contribution of photo as area determiner in this phase. Moreover, the students could grasp the idea of multiplication to find the area of rectangular surface with units arranged in array form. However, the activity still consumed a lot of time and students did not really go far to discuss the structure of array and how multiplication (row-column) really work there. Hence, for quite sometime, students still applied counting one by one.

The students in particular degree could understand that they will be in fair comparison of area if the planes being compared contain the same size of units through the tiled floor lesson. However, the students will be helped more if the lesson is refined to support students' comprehension on the use of identical units to compare the area. We also noted that the students built their stronger understanding on the use of units when they accomplish the hands on activity on area with rectangular and oval cookie by the use of square chips. The students also figured out the approach to find the area of non-rectangular region. They understand that they need to split the chip to really cover the edge of the non-rectangular region, then use the remaining part of chip to cover other part. We found that these students emerge the strategy of joining splitted chips to find the area. This understanding we assume helps the students when they deal with finding the area of surface by the use of grid. They also use the estimation when they find the area of non-rectangular region.

However, we found from the posttest that few students revealed the inconsistency of understanding. We noted that some students revealed their understanding clearly during the lessons on cycle 1 we observed. Nevertheless, their understanding seemed not revealing when they faced the posttest. For example, we noted that Rohmah already understood the idea of unit to derive area, like in cashewnut cookie lesson and tiled floor lesson. However, she did not reveal this idea when she faced the question about table size on posttest. She instead argued about the shape of table, not the use of books as area determiner. We assume that the students might forget particular concept they already grasped. For some degree, the concept they already learned seemed either revealing and sometime not revealing. We also think Rohmah might not really grounded on her comprehension. Based on this conclusion and situation found on students' learning process of cycle 1 study, we did some adjustment and refinement on our HLT.

G. The Improved HLT for Teaching Experiment (Cycle 2)

Based on the findings and the weaknesses found in every meeting of the first cycle experiment, we make refinement of the HLT for the teaching experiment. The followingis our new HLT. It reveals what activities from cycle 1 were revised and why we revise those activities in respect to students' learning trajectory.

Meeting 1

We minimize the story about the cookie context and eliminate the name of figure in the story with cookieman since we found that the students' focus was distracted during the session by the name Bu Handoyo. We also minimize the questions in worksheet and communicate this with Indonesian supervisor.

For activity 2, we found that with the size of cookie we made, 12 nuts do not really have overlapping, so we added the number of nuts become 14. We also set the question *“what is the ideal size for 14 nuts”* as the replacement of direct questioning “*make your own cookie with 14 nuts”* in task of making own cookie since we found that the students just focused in making own cookie, nut the ideal size for 14 given nuts. Hence, they simply ignored the state of overlapping, and even created large gaps (cookie made is larger than it is supposed to be for 14 nuts)

In cycle 1, we only provided students with one nut and let them figured out how many nuts on each cookie. We found that they drew some nuts but revealed gap and inconsistent arrangement. Hence, we set the students to have all nuts used to cover each cookie in third task and worked together. We wanted them to have in mind *‘measuring cookie size by the use of nuts’.*

Meeting 2

In cycle 1 we also provided the students with hexagonal chips. However, they did not know yet how to structure that hexagonal on the cookie so that no gap existing. We also noted that this shape of chips is not really matter to bring in the next cycle, so we omit this hexagonal shpaed chips.

We also changed the question in the worksheet like the following

“Can this square chips........? Why?...”

To become

“Can this square chips....? .... Why ? ....”

because we found that the students just focus on part *why* and left the question part *“Can...?”* when answering. It is difficult to analyse whether what they wrote on the part *“why”* means “*it can”* or *“it cant”*

Meeting 3

We still use the photo frame context. However, we changed the order of activity and the instruction. In first cycle, the students were given the same size of frame and they need to figure out *'how many photos they have to cover the frame'*in activity of lesson 3 of first cycle. We found this task gave little sense of unit as the area determiner since the idea of identical unit, inverse relationship is more stressed in the end. Hence, we got that not all the students at the end connected the size of frame with the number of photos that clearly.

Hence, we changed the instruction. We set the students firstly to experience *'comparing'* the frame we set different in size. We conjecture that the students will think the size of each frame, then the need to find the size emerges. Then we set the students to *'measure'* the frame by the use of photos. The findings including the use of inconsistent photos or identical photos become important to discuss in this lesson. By this activity, the students will also compare the area by the use of identical unit. Hence, we regard to omit the activity about comparing the area of tiled floor (activity 1 in meeting 5 of Cycle 1) since it is hard enough for students to derive the idea of using identical unit to compare the size.

We omit the activity about helping Pak Rudi to complete and sell the frame in meeting 4 of cycle 1 since this activity was regarded wasting the time for small aim we try to achieve which was different shape could have the same area. The students just focused drawing photos on the uncomplete frame we gave. We also found that this task did not really promote students to understand the unit as the area determiner. So we think other activity to address this aim.

We then regard to strengthen students with the relation between *'the frame size'* and *'the number of photos'*. Then, we give them new activity in mini lesson about making own frame with given number of photos and different arrangement and look of frame is discussed to promote the idea of different shape of frame can have same size/area. Not so much time is used during this activity.

Meeting 4

We found that giving the students worksheet containing pictures of unfinished photo frame in Pak Rudi task in meeting 4 of cycle 1 could also not generate the quick counting *'array multiplication'* since the students only completing the frame and counting one by one the photos. We found that we finally direct them to the multiplication in this way. We could not guarantee so much that they could understand why multiplication works there. Then, we change the way by creating another activity. We give them quick frame displayed by using projector to support students escape from counting one by one. We set them thingking about counting by row, by column, and finally by multiplication (row-column). The lesson about carpet is still used in the second cycle.

Meeting 5

We keep the mini lesson about the cookie size by the use of square chips and activity about oval table as well. However, we add activity about discussing the way to generate the idea of counting unit by the use of partial array inside the figure (multiplication)

Meeting 6

We add activity *'making your own grid to measure the size of palm'* to strengthen students' insight into the grid, the size of grid, and the number of grid they get as the size of palm. Hence, we support further students' understanding on the inverse relationship between the size of unit and the number of units used.

The following diagram displays the change we make from our HLT of cycle 1 to our HLT of cycle 2

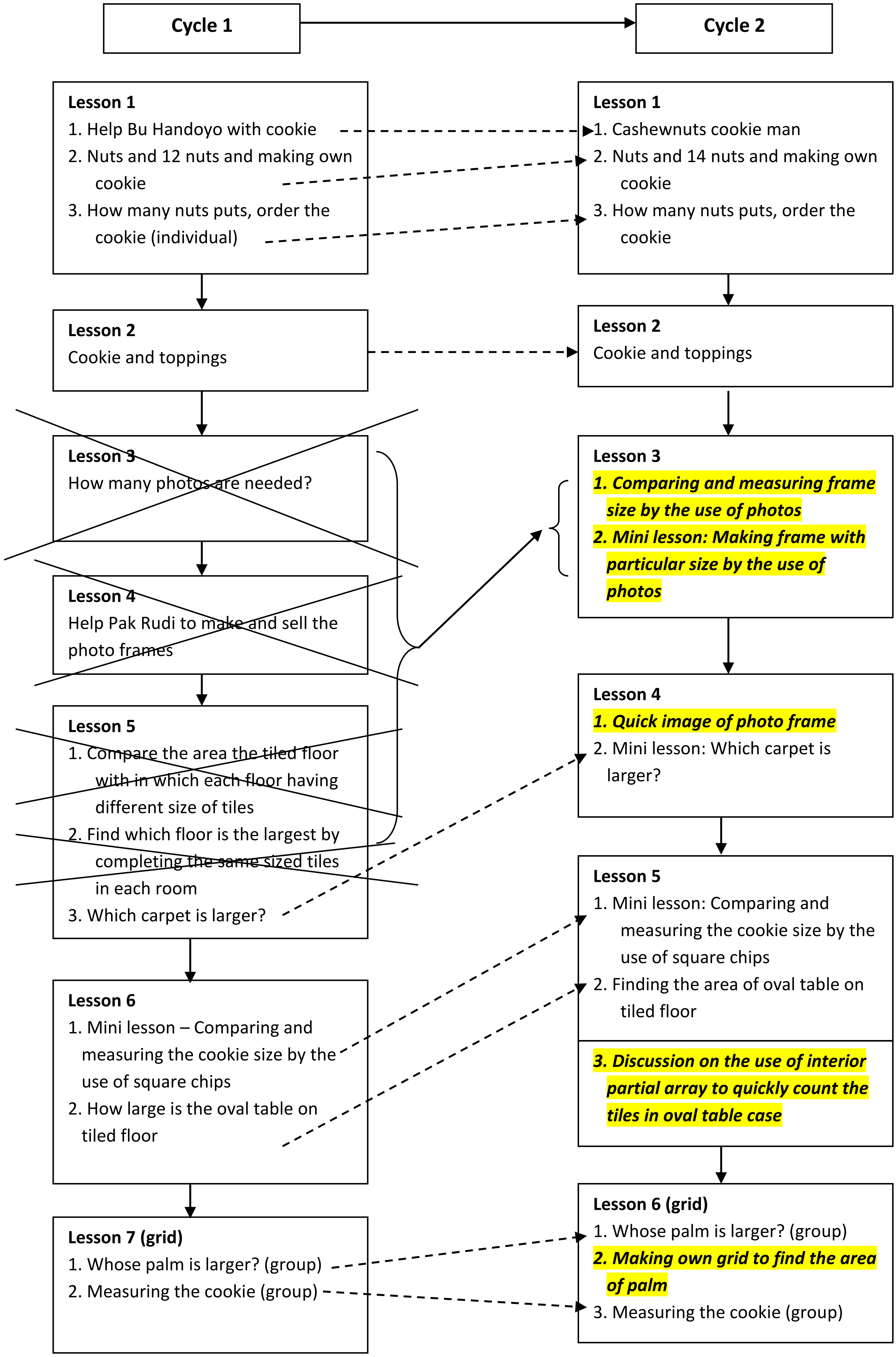
Diagram 5.1 Refinement of HLT

Still be used

Changed

The improved HLT for Cycle 2

Activity written bold mean new changed activty



H. PriorKnowledge of Cycle 2 Students on Area Measurement and Area Unit

The participants of our cycle two pretest were fifteen students of class IIIA of UNESA Laboratory Elementary School, Surabaya. They were also students of cycle two study. We posed pretest to get the insight of these students on the area measurement and the area unit. We gave them five problems. The contents of cycle two pretest (see *Appendix H*) were improved regarding the points we derived from the result of cycle 1 pretest. We analysed students' prior knowledge based on the overview of students' response on cycle two pretest.

In banana rack problem, generally more than half of students could see the banana as the tool to derive which tray was larger. The students could see that more banana, larger the rack. In particular degree, these students seemed to have initial idea of area unit. There were also students choosing which tray was the larger not by the use of banana. They instead used their visual reasoning and did direct comparison by what they observed. For example, one pupil could argue that rack A was the larger since it looked longer. Hence, it was clear that banana was not yet seen as the unit in this case. For the remaining students, the arrangement of banana seemed matter. They argued that the arrangement of banana in form of those on rack A implied that rack A was the larger. However, we saw that the students could not really see that arrangement not really take important role.

Table and books took a measuring context. However, we noted that only five students could really see the use of books as the unit used to measure the table in this case. Meanwhile other students used their imaginary ruler (since they were not allowed to use ruler) and did some exploration by it to measure the lenght, or the width or even the perimeter. We conclude that generally students could not yet see the contribution of the books on the table. Yet, they, we assume, had not yet have the vision towards the area unit in this level and did not yet know what and how to measure.

For paperquilt case, we found eight students who already used the square papers on the quilt to determine which one was larger. These students seemed to have strong vision on the use of square papers as the unit of area. However, the remaining six students argued by the use of the shape, not the unit. They argued that paperquilt A was the larger since it looked longer than B. We triangulated this pretest data with the interview. We also found that these students did not yet speak about each square on the quilt. Therefore, we can conclude that for these students, the conception of units did not yet emerge.

We saw many students failed in the tiled floor problem. Only one pupil could accomplish the task by the use of the tiles to derive the area. Meanwhile two other students have also had vision towards the tiles but they came with the wrong calculation. The exploration by the use of imaginary ruler happened with the remaining students. They measured all the possible sides on the floor to derive what they think as the area of that floor. For some students who did this way also did the same way as they accomplished the second problem about the table and the books. Therefore, we conclude that generally the students still did not see the use of tiles as the area determiner in this problem.

For the last problem, comparing two unfinished tiled floor, five students already used the tiles and they revealed the tiles on the floor and counted them.

They chose the floor with the larger number of tiles as the larger floor. While seven students kept using their visual reasoning instead the tiles to derive which floor was larger. The other two students saw the floor just the same size.

To sum up, we provide the following table to show how we triangulated the data of students' prior knowledge obtained from different sources during the pretets analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Answering the questions on the pretest	Students' recognition on the units as area determiner through banana, tiled floor, and book and table problem)	More than half students used banana to derive the area of rack	Use of banana as the unit to derive and compare area	Banana can be used to amount the rack
	Students' conception on the amount of units (through banana, tile, and book problem) as the area	Counting the banana and the amount represent area of the rack	The larger rack contain more banana	The number of banana on each rack determine which rack was larger
	Strategy on representing area with amount of different units (unit consistency) through the tiled floor problem	Some students approached the tiled floor area by drawing the ruler along the sides	They stated the area in term of liniarity (cm)	They argued that the area of tiled floor can be found with the ruler
	Students'conception on the use of unit (tiled floor and paperquilt problem) to measure and compare area	Some students used square figures number on each paperquilt to derive the area and the comparison	More squares, larger the paperquilt	The larger paperquilt holds more square figures

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' prior knowledge based on this pretest.

Conclusion on students' prior knowledge on area measurement and area unit conception

From the pretest result, we can conclude that

1. Most of the students already could see the use of units in every comparing problem. They could compare which plane was the larger by the use of units.
2. The use of visual reasoning in which unit idea was not yet emerged was sometimes quite often to occur especially in the comparing task.
3. For some students, the use of unit was still revealing even in the measuring problem. Hovewer, most of them did not yet see the unit concept in this case.
4. For few students who already seen the unit idea in area measurement, the need of identical units emerged in the tiled floor context.

From this conclusion, we can say that students have the potential to emerge the unit idea in the area measurement, especially for the comparing task. For somestudents who already had vision on the importance of unit to area, they even developed the idea of identical units. Nevertheless, for some other students, the insight into area and how to measure it is seemingly weak.

By this conclusion, added to the analysis ofstudents' learning from cycle 1study, we were informed to do some adjustments in our HLT. From the analysis of cycle 1 study and supported by the result of this pre-tets, we did take comparing-measuring as the large issue in our HLT cycle 2. We found that from cycle 1 study and this pretest result, the need of unit became majority coming from the comparing task.

I. Teaching Experiment

The students of teaching experiment (cycle two study) are fifteen third graders from class IIIA of UNESA Laboratory Elementary School, Surabaya. The focus group cointains five students namely: Asti, Raffa, Rama, Ranti, and Sasi (for complete list of groups, see *Appendix J*). They were chosen based on our discussion with mathematics teacher. They were students from middle level. This information was also supported from our pretest result (see table in *Appendix J*).

1. Meeting 1

a. Activity 1 – Cashewnut Cookieman

The goal of this first activity is that the students could recognise the issue of overlapping units, and uncovered part of plane (gap and leftover) and the effect of each issue to the number of unit used through the cashewnut cookie context. We gave the students the following context

A cookie man will make cookies with the top of each cookie full of cashewnuts. Because cashewnuts is relatively expensive in market, the cookie man does not want to have possible remaining nuts after he finish making all cookies.

Before working, we set that these following discussion points come after the problem. The teacher asks some questions and the students are asked to respond.

- *What do you think the cookie man will do?*

  Students can be guided to say that the man needs to estimate the number of nuts on each cookie

- *How does the cookie man find the number of nuts for his entire cookies that he makes?*

Students can be guided to say that the cookie man multiply the number of nuts on each cookie with the number of cookies.

After students understand the problem, they will work to find out the number of nuts used on one cookie.

After reading the problem, the teacher added statement *"the cookie man does not want to use so many nuts, so how do you help him so that he does not experience loss"* which we did not think her to do. However, she then asked the students what would be the better idea to help the cookieman.

*Teacher : **What is your idea how to help the cookie man to make the cookie (making circle with fingers indicating the cookie) but the nuts used is not so many? ****
Some students: Divide Ma'am
Teacher : Asti
Asti : Divide
Teacher : What is divided?
Asti : So there will be cashewnuts and chocolate for example
Other students raised their hands
Teacher : uhmm, so not overwhole full [she came to board and pictured Asti's idea, the picture was cookie with a cashewnut put on center of cookie and then the chocolate on other place]
Bunga raised hand
Teacher : Bunga, what is your opinion?
Bunga : Split the nuts
The teacher pictured Bunga's idea, other students kept raising the hand, however
Teacher : Keep your answer...

From the conversation, it is implied that what these students had in mind was not *'find the number of nuts on one fully covered cookie'* like we expected them to have, but *'put not too many nuts and reduce the nuts on each cookie as possible as they can do to get the small number of nuts'*. In other word to say is that they ignored or just forgot the statement of *'full cover'* on the problem. We assume that this happened because teacher's statement (*) on the conversation above was interpreted differently by the students. Hence, that large shift happened.

Furthermore, we noted that it was also because the teacher did not do further discussion we set and she did not direct the students to the important point:

The students estimate the number of nuts on one cookie, so that the cookie man will know the total number of nuts used for all of his cookies made.

Hence, she did not sound the problem in which students need to find the number of nuts on one cookie. The absence of this important point clearly led the students to the misunderstanding. In addition, the teacher did not do any clarification and further discussion about the important point and the real task instead of rereading the problem and giving stress to the word *'full'*. Hence, the students became even more confused.

We moved around the groups and recorded what students discussed and argued about. The first group we observed was the third group (Hiu, Munawar, Dinda, Ersa, and Olga)

They are putting nuts on top of cookie
Olga : Do not put too many
Hiu : Just put three?
Dinda : Put on the centre
Hiu : But it is asked to be full
*Munawar : **Full but not much, so how?***

The other group we observed was the first group (Ranti, Raffa, Rama, Sasi, and Asti), our focus group.

Asti : Full
Ranti : Full?...so it means like this [putting nuts on top of cookie]
*Asti : **No, not like that, it is full, but not too many***
On the background was sound of teacher rereading the problem "the cookie is full of nuts but he does not want to have many"

We conluded that these students confused about the task poorly given by the teacher. Teacher also did not do discussion about the point we set so that she gave the unclear task. What they had in mind that confused them about the task was *"full but not too many"*. The following are the work of groups.

Figure 5.13 Groups' work in activity 1

The teacher and the students did discussion about the difference between the number of the nuts each group used. However, the students argued the difference occuring since that every group has different idea and thinking. They did not even talk about the nuts arrangement, gap for example. We assume that based on groups' work, it was hard to see the issue of overlapping or even discussion about the uncovered part (gap and leftover). Furthermore, we found that the teacher missed asking the students about the arrangement itself that can bring students talk about those issues. We concluded that from activity 1, students did not yet figure out the issue of overlapping, gap, and leftover. Hence, they could not yet 'see' the effect of overlapping and uncovered parts to the number of nuts used to cover the cookie.

However, the students were helped to see the issue of overlapping and uncovered parts in the activity 2 we anticipated.

b. Activity 2 – Cookie and 14 Nuts

The second activity aims to get students recognising the issues of overlapping and uncovered parts (gap and leftover) and also seeing the connection between the number of units to the size of plane through the context of cashewnuts cookie. We observed students in focus group while they were busy working on putting those 14 nuts on each cookie.

Ranti : Write what you found based on your observation! [reading the instruction] Uhm ... the nuts here [pointing out to the cookie A] do not overlap, but here [pointing out to the cookie B] overlap
Raffa : Because this is small [pointing out to the cookie B]
Ranti : Well, why these overlap? Since the cookie is small. Why these do not overlap? Since the cookie is large
Asti : Yes, that is it. That's what I think. Yes
Raffa : I also think the same.

Figure 5.14. Pupils discussing about the difference of the arrangement of 14 nuts on both cookies

We found that these students had already connected the idea of the size of cookie given and the effect of number of nuts given. Then, they observed the overlapping nuts. We interprete that these students could see that cookie A is too large for those given 14 nuts so that no nuts overlapping.

Meanwhile, they thought that the cookie B was too small for those given 14 nuts so that some overlapping nuts were created. We then observed another group.

(Busy putting the 14 nuts on top of cookie B)
Ersa : Here
Olga : Put it in between these
Hiu : There is exactly gap between the nuts

This group emerged the term *'gap'* while they tried to accomplish putting those fourteen nuts on top of cookie B. They tried to put the nuts between two other nuts as well as made some nuts overlapping since the cookie B size was too small for those fourteen given nuts. We also did small interview with this group after couple of minutes.

Observer : Hiu, have you already finished?
Hiu : Finished
Observer : What if I ask you something?
Hiu : What?
Observer : How many nuts here? [pointing out cookie A]
Hiu and Olga : Fourteen
Observer : How many nuts here? [pointing out cookie B]
Hiu : Fourteen
Observer : So, do you think that the number of nuts here enough for cookie A?
Students : Yes, enough
Observer : Enough? Can I put another nut here [pointing out some gaps in cookie A to put other nuts], can I do that?
Students: You can
Observer : So, fourteen nuts is enough for cookie A?
Dinda : Not really
Observer : Why?
Dinda : Because on some uncovered parts, more other nuts can be put
Observer : That's good, do you understand about what Dinda said?
Students : Yes
Observer : And then, is that enough for fourteen nuts to be put on the cookie B?
Olga : No
Observer : Why so?
Olga : Because, it's already full
Ersa : The place is small
Observer : So, do you think the nuts are too many for the cookie B?
Hiu : Nope, more less
Observer : What do you mean?
Hiu and other students : They fit there
Observe : What if I add one on this one, to be 15, one on this one

Students : 16
Observer : Can I really do that? Do you think that 14 is already too many for cookie B?
Dinda : The cookie A can have no room later on
Ersa : I think it's too many
Observer : So how is it supposed to be?
Students : 11

We conclude that these students understand that if some gaps exist, then the number of nuts can be more than what they have since they can put another nut in between. Hence, they can see the effect of existing gap between the nuts on top of cookie. Based on small interview, we can see that firstly students did not directly relate the number of nuts to the size of given cookie. We tried to ask to get the students into it. We found that they finally said that fourteen nuts are less for cookie A because they can put other nuts. They found fourteen is little bit more for the cookie B.

We found that this important idea, connection between the number of nuts and the cookie size, worked out while the students set up to the next session, making ideal cookie size for fourteen nuts. The teacher asked the students to make their own cookie for those fourteen given nuts and informed them about the ideal cookie size and shape for those fourteen given nuts.

From this session we found that each group already understood the situation in which they need to make their own outline of cookie to those fourteen given nuts. They already considered overlapping and gaps not to occur or minimally to occur. We moved around to see each group's work.

In Ranti's group, we found that they firstlymade the outline, rectangular shaped cookie. After that, they put the nuts on top of that cookie and we observed that the cookie they made was little bit larger for those fourteen nuts. However, we tried to remind the students whether that was the proper cookie size for those fourteen nuts and restated the task. They then shrank the cookie a bit to get those fourteen nuts really fit to the cookie.

In Bunga's group, we found that they did the same way as the Ranti's group. Generally, this was not exactly what we predicted to happen. Nevertheless, the students already got an idea about the ideal size of cookie for those fourteen given nuts with strict consideration to possible gap and overlapping nuts.

We found that Olga's group worked differently amongst others. We saw them firstly arranging the nuts. They seemed to avoid overlapping nuts as well as minimized the gap in between.

Figure 5.15 Pupil making a representation of cookie with ideal size of given fourteen nuts

We found that they then traced the outline of cookie exactly from the edge of nuts, made a circular-liked figure surrounding the nuts and ended up with the shape formed by the arrangement of the cookie. We interpreted that they just have created the representation of ideal size of cookie for those 14 given nuts.

From this activity, we conclude that the students already understand some issues like unit overlapping, and uncovered parts (gap and leftover) through this activity and the effect of each issue to the number of nuts used to cover the cookie. They even could make connection between the number of nuts used to the size of given cookie and did some observation whether the size really ideal for the given number of nuts through the consideration of unit issues: overlapping and uncovered parts (gap and leftover).

Furthermore, they finally could make figure that was approprite in size for the given number of nuts by the strict consideration on overlapping nuts and possible gap and leftover. We assume that they have now an insight into the area in term of number of unit through the cashewnuts cookie. We assume that they can have those idea now since they understand the issue of overlapping and gaps as well: for particular number of nuts given, the outline of cookie will not be proper for those nuts if some gaps or overlapping occur. They think that the nuts should be in situation such that they are close to each other, no overlapping and no possible gaps. Hence, the ideal size can be formed. We made the next task such that students can really see the clear relation between the unit (nuts) and the area (cookie size). We put the students further to realize the use of nuts to figure out the area.We already used the term ‘measure’ in this task.

c. Activity 3 – Measure and Order the Cookies

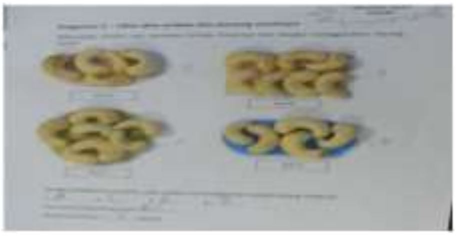


Fig 5.16 Measuring and comparing the cookie size

In this activity, the students measured the cookie by the use of nuts. They reported the number of nuts on each cookie and made an order of cookie based on their size.We found that all the students already avoided overlap and minimized the gap in between the nuts. We interpreted this situation as finding the ideal number of nuts without overlapping and gaps to fit the cookie. Here, the students clearly already figured out the use of nuts to find the size of cookie.

While asked the order of cookie based on the size, Ranti and some other students argued that the size was regarded based on the number of nuts. She meant that the largest cookie would hold nuts at the most.We found that Ranti and her group had already understood the use of nuts to derive the size or area of the cookie. They even could argue that more nuts used, larger the cookie they have.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 1 analysis.

Activity	Data Collected	Video Recording	Students' Written Work	Interview
Cashewnut cookieman – find out how many nuts used	The students' arrangement of the nuts(overlapping, gap, and leftover) and the understanding of each arrangement to the size of cookie	The students arranged the nuts so that they left much gap/uncovered (see picture)	Students' arrangement of the nuts and the number of nuts	They did the arragement because they thought that the nuts used full but not so many
Cookie and 14 nuts – determining the ideal size of cookie for given 14 nuts	Students' investigation on ideal number (by avoiding the overlapping, the gap, and leftover) of nuts to represent the area of cookie	Students putnuts first and arranged them so that gap, overlapping at best avoided	Students drew outline of cookie for those given 14 nuts	The students declared that the cookie they made is ideal for given 14 nuts
Measure and order the cookies	Students state the relationship between number of units (nuts) and the area of cookie	Students argued that the size of cookie was regarded based on the number of nuts	More the nuts, larger the cookie. The students wrote the order of cookie based on the size	More nuts, larger the cookies

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting.

All in all, in this meeting, we conclude that giving clear task and discussion part during the activity 1 is very important to avoid students' confusion around the task. Having the discussion with the students about finding the number of nuts on top of fully covered cookie should be conducted prior to finding the number of nuts itself. It is due to avoid the misunderstanding like we experienced in the class. Hence, the role of the teacher is clearly strong in this phase to make the task clear for the students.

We found it also hard to have students emerging the unit issues: overlapping, gaps under the students' work – full but not many. They could not directly see the phenomenon of overlapping nuts, gaps in between the nuts from the difference of the number of nuts in each group they observed. For this reason, the teacher must be able to orchestrate the discussion around this task and trigger some questions to have students to talk about the unit issues.

However, the teacher successfully anticipated the lack of implementation on the first activity in the second activity. Although in the first activity, students could not easily emerge the unit issues, this concept could be triggered from activity of putting fourteen nuts on each different sized cookies we found succesful. From the analysis, we found that

1. The students emerged the state of overlapping nuts and gap in between the nuts from their observation. We noted that they could argue about the connection between the number of nuts and the size of the cookie provided. They also already connected the size to the ideal number of nuts to cover the cookie A and B.
2. We found that students already had in mind the connection between the number of nuts and the possible ideal size of cookie. We interpret this situation as they found the connection between the concept of unit to area measurement.

3. In the activity 3, we found that the students already developed the understanding of the use of the nuts to find how large the cookies were.

2. Meeting 2

The goal of this meeting is that students can investigate and determine the appropriate shape of unit used to cover the region as well as the shape of region better covered by the unit through the cookies and toppings context.

A cookie man will make two kinds of cookie, oval cookie and rectangular cookie. He will cover the top of cookie with four toppings : cashewnuts, star chocochips, M&M chocolate, and square chocochips without overlapping. Can you help him out to put the toppings?
Then determine
1) Which topping can cover entirely the top of cookie?
2) Which cookie is better covered by the topping?

Without the overlap, the students attached those toppings on each cookie, investigated, and determined the shape of cookie better to cover the cookie as well as the cookie better covered by the topping.

Group work

We noted that the focus group (Raffa, Rama, Ranti, Sasi, and Asti) were arranging the topping, they found that topping like M&M chocolate and star chocochips cannot cover entirely the surface of cookie unless they make the toppings overlap.

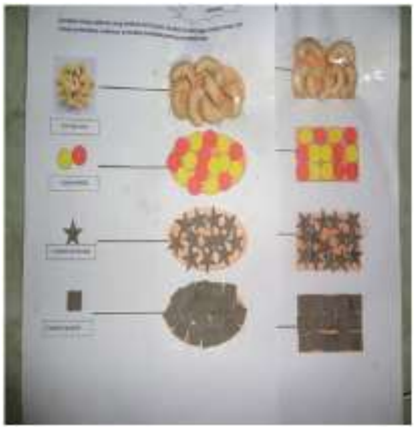


Figure 5.17Focus group's work on arranging topping on cookie

However, we found that in oval cookie, square chocochips were put following the outline of oval, so that they got chips in the middle a little messy overlap. We noted the discussion they had during this session.

Sasi : The nuts cannot cover entirely the cookie because they are large
Asti : That's wrong. Think the shape is irregular
Ranti : Because the shape of cookie is oval. Wait I think it can, because the nuts has circular figure so it can cover oval cookie, while the nuts cannot cover the rectangular cookie

What Ranti said means that the shape of nuts which is circular-like can cover the oval cookie which is also circular-like. Ranti seemed figuring out the relation of the shape between the unit and the regon and decided that the same look of unit and region will make the unit able to cover the surface. We interprete this as the idea of *resemblance* implying that the shape of unit resemble the surface to cover.At first this group seemed doubtful about Ranti's argument because others could see that not all parts of oval cookie was covered by the nuts. We started to do small interview.

Observer : Please take a look at this, can the stars cover entirely the surface of cookie?
Sasi : It can't
Observer : What is it because? What can you see from this?
Asti : There is it [pointing out to uncovered parts]
Ranti : Star has the corner
Observer : Rama, Raffa, can these stars cover entirely the cookie?
Rama, Raffa : Nope
Observer : Can you see there is uncovered part here
Ranti : Yes, there is
Observer : What do you think will be the cause?
Raffa : Because .. uhmm
Asti : The shape is irregular
Observer : What shape is irregular?
Raffa : Oval
Ranti, Asti : The star
Sasi : There is 'lancip-lancip'nya (the arms)
Observer : Hear what Sasi said, she said lancip-lancip shape. Which do you think it is?
Students : Here [pointing out the arms of star]
Observer : So, what will be the effect of the lancip-lancip?
Ranti : This [pointing out cookie] shapes oval
Sasi : The shape is oval so, because the star has lancip-lancip, so it cant cover the oval because they get different shape
Obsever : Rama, Raffa, please attention. So Sasi please explain to us again
Sasi : The star cannot cover the oval cookie because it has lancip-lancip here

Observer : Can this lancip cover this part [pointing out the area between another star's arms]?
Ranti : I think it can if it is dragged here, so it covers here
Sasi : But I think it is not enough to cover it
....

From this conversation, we concluded that the students in our focus group needed to be triggered to talk more about the cause why particular topping could not cover the cookie. At first they only regarded the uncovered part and somehow about the connection between the shape of cookie and topping. However, they did not yet talk about the structure of topping further. However, we noticed that Sasi could argue that stars have the arms which make it disable to cover the cookie. The arm of one star could not cover the area between two arms of another stars like she argued. We also figured out that she also made connection between star and oval shape. Since they are different so that star with arms could not cover the oval cookie. We found that she and her group already considered the reason why some toppings could not cover cookie at structure level, not only talking about whether there was uncovered part or not. They could argue the M&M chocolate also could not cover both cookie after all.

We also realized that Ranti started to concern not only about the idea of resemblance, but also the uncovered part existing on the cookie. So that she changed her mind about the nuts and oval cookie. She found that the nuts could not cover the oval cookie after all.

Nevertheless, this group seemed ignoring the state of no overlapping topping since at the last oval cookie with square chocochips, they made some overlap. We notice this to happen because of the arrangement of the chips itself. We found that these students followed the circular outline of oval cookie and put the square chip around it like the picture above so that at some places, these chips overlapped each other. We asked them about this and they finally realized that they ignored the rule and they found that their arrangement was not good enough.

However, we noted that this group came with an answer that none of topping could cover the oval cookie, and rectangular chips was the only one which can cover the rectangular cookie. We also found the similar arrangement to other two groups. The group of Bunga, Adhif, Adit, Aisyah, and Tita seemed to argue that nuts could cover both cookies. They firstly did not yet regard the uncovered parts of each cookie. They also argued that stars do not fit in both of cookie so that it cannot cover the cookie. The M&M chocolate could cover the cookies only if they are made overlapping.

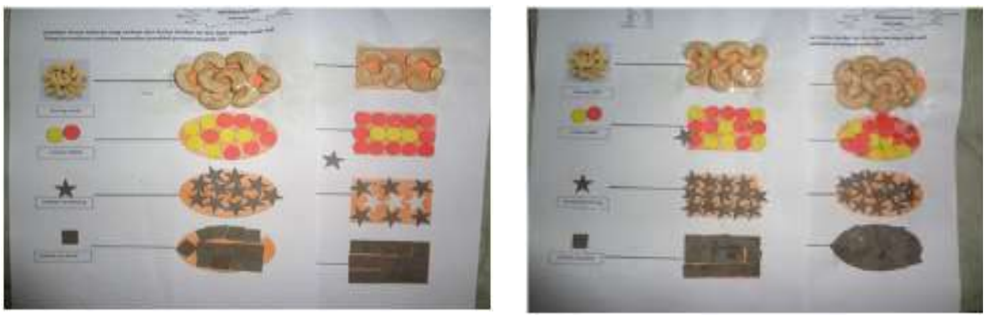


Figure 5.18 Other groups' works on topping arrangement

We also noted that they made overlapping square chip on top of oval cookie to cover that cookie. We found that this group somehow did not regard the task rule.

The group of Hiu, Munawar, Ersa, Olga, and Dinda could argue that none of the toppings could cover the oval cookie for some uncovered parts reason. We notice that they argued square chocohips was the only one which could cover that rectangular cookie for they get the same rectangular-like shape.

Class discussion

We notice that the teacher tried to have students concluding their answer in respect to the given task. We found that all students argued that the square chips was the only topping that better covered the cookie because it had no gap especially when they referred to rectangular cookie. We also found that students answered that the rectangular cookie was the better covered cookie by those square chips. Another reason was that they saw that both cookie and square chips had almost the same rectangular-like shape, so that square chips could cover the rectangular cookie. We found that the students by the end of the session concluded about the connection between the shape of surface to cover and the shape of unit to cover which we interpreted the idea of resemblance.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 2 analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Cookie and toppings	Students put the toppings in regard of overlapping, gap, and leftover	Students arranged the nuts so that at best overlapping, gap were avoided	They chose the square chocochips to be the best covering cookie	For the topping other than square chocochips, the gap could not be avoided

	Students discuss the structure of topping to decide which topping better cover the cookie	Students discussed why stars could not cover for it has the arms	Stars could not cover the cookie entirely because its arms	An arm of a star could not cover the space between two arms of another star

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting.

We conclude that the students had in their mind that for covering task: the square or rectangular shape unit was best shaped unit to have and they could cover another rectangular surface might be perfectly and they might be unable to cover the non rectangular surface since they will leave uncovered parts near the edge. Hence, we assume that the students can make sense the next lesson about the photo frame context which already bring the shape of rectangular for both units (photos) and surface (frames).

3. Meeting 3

This meeting aims to emerge the understanding of the identical unit to cover region through the photo frame context. It also aims to have students to investigate the relation between the size of unit with the number of unit used to cover the region. Finally, the students understand the use of identical units to measure and to compare the area of region.

Firstly, we provided the students with three different sized and shaped frames (6x6, 5x7, and 9x4 of smaller photos consecutively for each frame). The teacher showed the frames and asked the students to determine which frame was the largest. We let them first getting into *'comparison'* situation before talking about *'measuring'*, the core of the task.

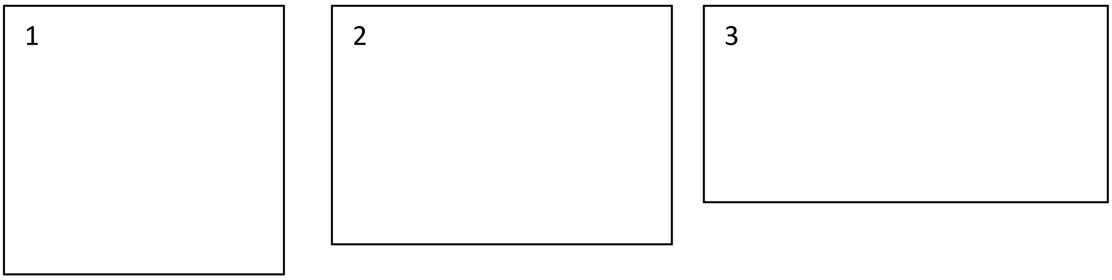


Figure5.19 The frames

Teacher : Which one is the largest? Give your reason!
Students : One
Ranti raises her hand
Teacher : Okay Ranti
Ranti : One (without giving reason)
Teacher : What about you Rama?
Other pupil is saying two
Teacher : Rama...
Rama is exploring his fingers to make rectangular figure while his eye looking through the midle of figure to the frames the teacher is holding
Rama : That's the square (referring to the first frame), the longer one (referring to the second frame) or another one?
Tecaher : Which one is the largest Ma?
Rama : One
Tecaher : What about you Ais?
Aisya : One
Teacher : One also
Hiu : One
Teacher : One, Sashi?
Sashi : Two
Teacher : Why is it two Sashi?
Sashi : I can see that is larger ...
Observer : Show us, can you show us ?
Sashi : This part is longer and also that one [she compared the longer side on the second frame with the side of the first frame, and compare another side of second frame with the side of the third frame]
Teacher : Adhif
Adhif : Number one
Teacher : Why is it number one?
Adhif : (touching the side of square frame and compare it the side of the second frame), this one is longer than this
Teacher : So, it's longer
.....

Asti : One (showing the same way as Adhif did)
The other students also argued that frame 1 is the largest

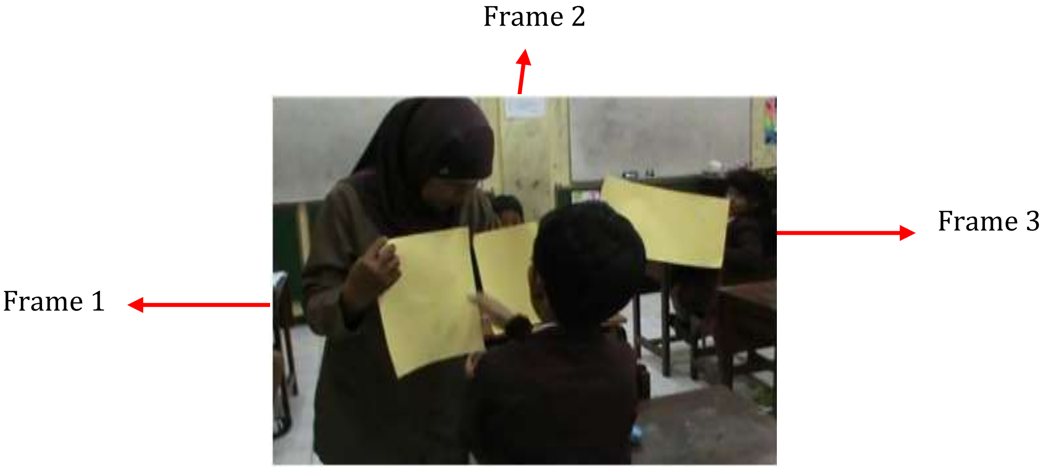


Figure5.20 Adhif showed why he thinks the frame one is the largest

The students directly argued which one was the largest through their visual reasoning. Most of them chose the first frame as the largest one. There was no exact superposition (putting one on top other to find which one is larger) they made. In fact they only argued based on whether particular side of frame was longer than others'. For example, they chose frame one to be the largest one since it has longer side as what Adhif explained.

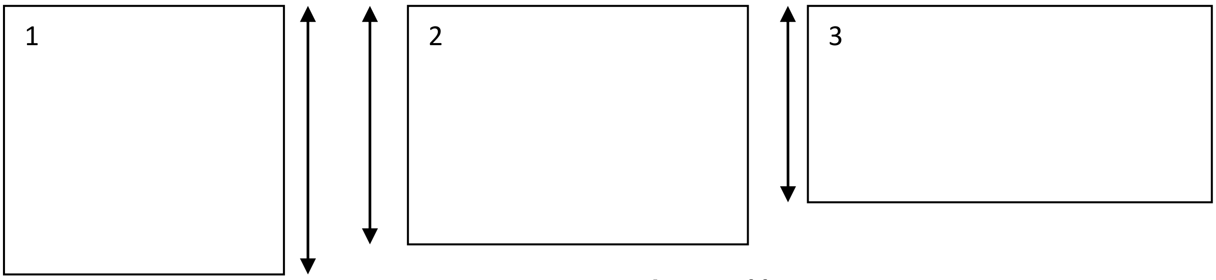


Figure5.21 Comparing the size of frame by the sides

What we found was that some of the students seemed not aware of the lack on their argument. Eventhough they argued about the sides, they did not seem to take into consideration other possible lenght of sides (e.g the longer side in frame 3 compared to the side of first frame). They used only one side of each frame to compare and found which one is larger. We also assume that the first frame was treated the largest since for them it looked larger while the other two did not really look larger but longer eventhought they do not use the longer side of these two frames to compare with the side of the first frame.

Based on the situation, the teacher moved the students from *'comparison'* situation to the *'measuring'* situation. The teacher asked the students to convince themselves by measuring the size of each frame to really show their answers were correct. We noted that firstly some students said that size could be obtained by measuring by the use of ruler. However, we avoided the students to work the idea out and asked them another possibility to do to find the size.

Students remembered that the size of cookie could be obtained by the use of nuts. Some students could argue that they can use photo attached on frame to figure out the size of frame. Hence, we observed that the students shifted successfully from *'comparison'* situation to *'measuring'* situation and they knew already what they were doing. They needed to find the size/area of each frame by the use of photos and determine the order of frame.

Measuring the frame

The teacher distributed the frame and the bunch of photos for each group. We intentionally provided different sized photos in the bunch to see whether the students mix up the photos attached on the frame. In fact, we found this to happen while observing each group's work. Each group used those two different sized photos on their frames.

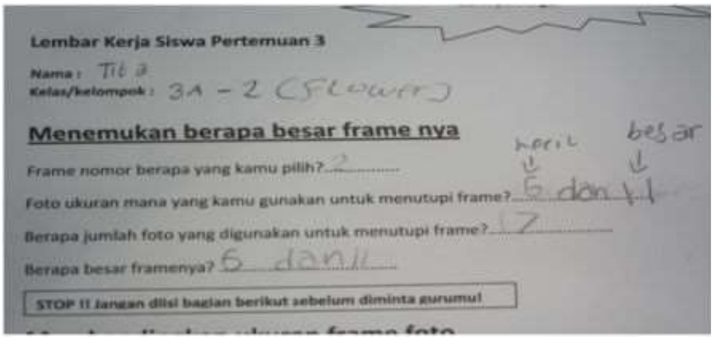
Frame 2
by group 2

Frame 1 by focus group

Frame 3
by group 3

Figure5.22 Groups' work on frame

Before the class discussion, the students accomplished the questions on the worksheet about the frame they made. The following pictures are the representative of the answer of each group.


Lembar Kerja Siswa Pertemuan 3
Nama : Tita
Kelas/kelompok : 3A – 2 (Flower)
Menemukan berapa besar frame nya
kecil besar
Frame nomor berapa yang kamu pilih? 2
Foto ukuran mana yang kamu gunakan untuk menutupi frame? 6 dan 11
Berapa jumlah foto yang digunakan untuk menutupi frame? 17
Berapa besar framenya? 6 dan 11
STOP !! Jangan diisi bagian berikut sebelum diminta gurumu!

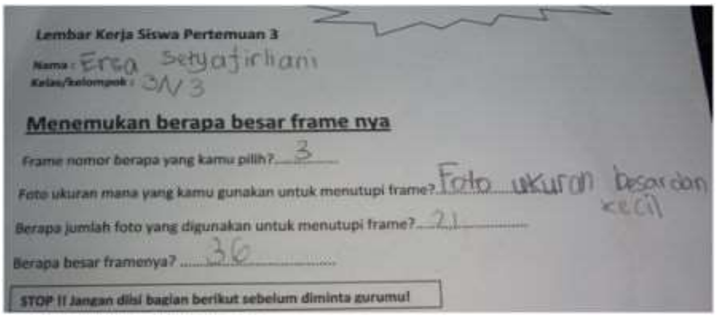
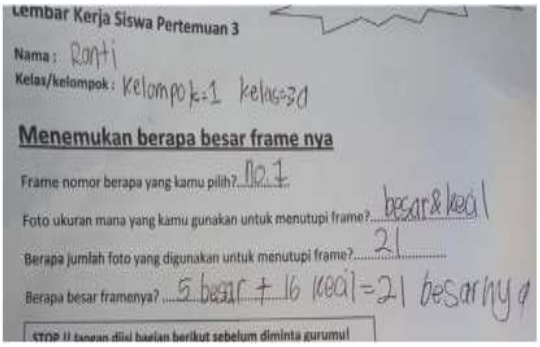

Lembar Kerja Siswa Pertemuan 3
Nama : Ersa Setyafirliani
Kelas/kelompok : 3A/3
Menemukan berapa besar frame nya
Frame nomor berapa yang kamu pilih? 3
Foto ukuran mana yang kamu gunakan untuk menutupi frame? Foto ukuran besar dan kecil
Berapa jumlah foto yang digunakan untuk menutupi frame? 21
Berapa besar framenya? 36
STOP !! Jangan diisi bagian berikut sebelum diminta gurumu!

Figure5.23 The groups' answers on the questions of the worksheet

We found that the students in these two groups responded the question *"How many photos are used to cover the frame?"* with an appropiate interpretation by counting both different sized photos to get a particular number. They treated those different sized photos the same in the process of counting the number of photos used. Tita's group used 17 photos on the frame consisting 6 larger photos and 11 smaller photos while Ersa's group attached 21 photos consisting 5 larger photos and 16 smaller ones.

When the question turned to finding the size of their frame, these both group came with idea of the amount of photos as the size of frame and they got different idea in expressing their frame size. Tita's group expressed the size of frame by the use of those both different sized photos (6 and 11) while Ersa's group obtained 36 as the size of their frame. We interpreted that Ersa's group came with an idea to express the size of the frame by the use of only smaller photos. We interpreted this situation as Ersa's group treated larger photos as four smaller photos since they could see that four smaller photos could occupy one larger photo. Then they used those idea to find the size of the frame. Hence, we assume at this phase they started to derive the idea of identical unit to measure the area.

In our focus group, we found that the students interpreted the question about the number of photos used adequately. They used 21 photos consisting 5 larger photos and 16 smaller ones. However, we found that a pupil named Rama had different answer amongs other in his group about the question of the size of the frame. Overall, the students in our focus group did not yet derive the idea of identical unit in this task.


Lembar Kerja Siswa Pertemuan 3

Nama : Ranti

Kelas/kelompok : Kelompok=1 kelas=3d

Menemukan berapa besar frame nya

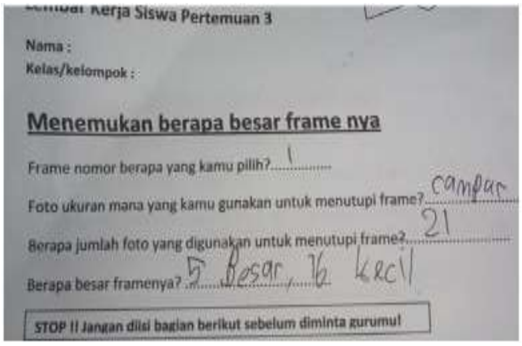
Frame nomor berapa yang kamu pilih? no. 1

Foto ukuran mana yang kamu gunakan untuk menutupi frame? besar & kecil

Berapa jumlah foto yang digunakan untuk menutupi frame? 21

Berapa besar framenya? 5 besar + 16 kecil = 21 besarnya

STOP !! Jangan diisi bagian berikut sebelum diminta gurumu!


Lembar Kerja Siswa Pertemuan 3

Nama :

Kelas/kelompok :

Menemukan berapa besar frame nya

Frame nomor berapa yang kamu pilih? 1

Foto ukuran mana yang kamu gunakan untuk menutupi frame? campur

Berapa jumlah foto yang digunakan untuk menutupi frame? 21

Berapa besar framenya? 5 besar, 16 kecil

STOP !! Jangan diisi bagian berikut sebelum diminta gurumu!

Figure5.24 Our focus group's answers on the worksheet

Ranti, Raffa, Sasi, and Asti expressed the size of frame to be '5 larger + 16 smaller = 21'. We know that they already considered the number of each larger and smaller photos in this measuring area task at the beginning. However, the addition operation summing up the total photos (larger and smaller) to get 21 as the size of the frame implied something else. It indicated, we assume, that they finally treated 21 as the size of the frame in which they ignore the fact that both photos has different size so that summing up the number is not approprite to do to express the size of the frame. We derive based on this fact that they will get another size for example 15, as the size of frame which 12 smaller photos and 3 larger ones. Based on this analysis, we conclude that these students could not yet express the area in how many unit **a** is used and **b** is used, if a and b is different. They instead treated the sum of **a** and **b,** let say **c**, to be the area.

Rama, on his own responded to the size question with an answer **5 larger photos, 16 smaller ones** as the size of the frame. He did not at all sum up the number of photos. We conclude that he knew that both photos are different in size so that summing up the number of each photos is inadequate to do. Hence, he just left the size to be 5 larger photos, 16 smaller ones instead. We conjecture for another case, he will not sum **a** and **b** to get **c** as the area if **a** and **b** is not the same. He leaves **a** and **b** to be the area. During the group discussion, we saw that Rama did not do so much discussion with others. Rather, he just listened and sometimes complained about other's opinion but he did not give his idea directly to them. That was why we conclude that his answer was by his own different with other students in his group.

Nevertheless, we found that the students in our focus group did not yet reveal the idea of identical unit in this task. They might not yet derive the connection between larger and smaller photos or at least they seemingly ignored this connection. That was why we assume that they could not yet express the size of frame by the use of identical photos, for example their frame size to be 36 smaller photos.

Comparing the frames

The students went back to the initial problem about determining which frame was the largest. The students would also come with the order of the frame based on the size.During the group discussion, we found that students came with the right order of the frame based on the size and determined the largest frame which was frame 1 and 3 by group 1 and 3 respectively. We noted that the group came with different way of solution. The level of solution was also different.

We found that the solution of Ersa's group was based on the idea of identical unit, measuring the frame by the use of only smaller photos eventhough they used both photos. The following is the conversation between observer and Ersa's group.

Observer : To be able to compare each frame, what is it supposed to be? How is the size of the photos?
Olga : Count the photos, the larger photo is counted four.
Observer : The larger photo is counted four, then?
Olga : The smaller one is counted one
Observer : Counted one, then you get which one is the larger and smaller? (referring to the frame). Do you understand what Olga said?
Ersa nods her head impyling that she agrees with Olga's statement
Observer : Hiu, do you understand what Olga said?
Hiu : No
Observer : Olga...Olga can you explain again your opinion?
Olga : Let's Esa do it
Ersa : I think it is better to use just smaller photos or just larger photos
Observer : Okay
....
Observer : Okay Hiu, what is the way to make the frame comparable?
Hiu : Yes. The larger one is counted four, the smaller one is counted one.

We assume that they think it would be fair to compare the frame if they counted only one type of photo which was the smaller photos they chose. By counting the larger as four smaller photos, they will express all the frame size in term of smaller photos, so that the result of counting will enable them to compare the size of those frame. They got frame 1 and 3 as the largest and both frame were the same size, 36 smaller photos and then frame 2 with 35 smaller photos. Based on this situation, we concluded that this group used the idea of identical photos to compare the frame. Hence, they already moved to the next level, from the use identical photos to measure the frame to the use of identical photos to compare the frame.

The other groups, Tita's group and our focus group had seemingly the same solution into the frame comparison. Tita's group argued that both frame 1 and 3 as the same and the largest ones. We found that they decided 1 and 3 because both frames had the same number of photos which was 21 photos (both for larger and smaller ones) while frame 2 they made only consisted 17 photos (for both smaller and larger ones). We assume that both frame, 1 and 3 which contained the same number of larger and smaller photos (5 and 16 respectively) might give the insight to them that both frame had the same size and they both were the largest ones.

We had a conversation with our focus group during this section.

Observer : How is the size of photos in each frame so that you can compare them?
Asti : Same
Observer : In that situation, each frame contains mixed photos, can you compare them?
Asti : We can
Raffa : Different
Observer : Different? Okay, if they are different, can you compare them?
Raffa : Yes we can. Look at the larger and smaller ones (referring to number of each photos)

At first Asti seemingly tried to regard the use of the same photos on each frame. However, she then did not go back to it when asked whether they can compare each frame if each has different sized photos on it. She argued that the comparison still could be done in that situation. Her opinion implied that she just left her potential idea on the use of the identical photos as she stated before. In addition, her latest opinion was supported by Raffa's argument that the comparison was still conductable by considering the number of each smaller and larger photos on each frame. Since the frame 1 they made and the frame 3 the Ersa's group made contained the same number of photos, 21 (for both smaller and larger one) and each frame even contained the same number of larger and smaller photos (5 and 16 respectively), they concluded that the frame were the same size and both were the largest.

We try to give our interpretation based on our focus group that this group might not yet reveal or might not yet regard the idea of identical photos to do comparison. In respect to this situation, frame 1 and 3, their way might be true. However, we conjecture that their strategy will probably no longer be applicable if both frame contain different number of smaller and larger photos. They might think step further. They might figure out another way to compare the frame size. We also conjecture that a little guidance given to this group might trigger Asti to recall her idea about the use of identical photos she just stated. We assume that this group will be helped as like Ersa's group if they work the idea of identical photos out.

Class Discussion

The teacher headed the discussion. She tried to bring the students to the idea of identical unit to help them to compare the frame size.

....
Teacher : How is our way so we can count them all, all larger or just all small?
Students : The photos should be the same
Teacher : Yes, the photos should be the same. What if the larger photos replaced by the smaller ones? Can we do that ?
Rama : We can
Students : We can
Students in group focus : Yes we can, we need four
Teacher : Yes we need 4. 1, 2, 3, and 4 (putting smaller photos on the one larger photo). What if we replace all the larger photos on this frame with the smaller ones? So which one of you who already found the number of photos on this frame? (referring to the first frame)
Bunga : 21
Ranti : 21
Teacher : Ersa, what did your group find if we use smaller photos?
Ersa : 36
Teacher : What about Ranti's group?
Ranti : Wait ma'am. (counting)
Asti : 36 (smiling)
.....
Observer : What if I ask you about the size of frame 1?
Bunga : 36
Teacher : Frame 2?
Observer : What is the size of frame 2?
Teacher : The way is just the same, replace the larger with smaller ones
Ersa's group : 35
....
Teacher : What about frame 3?
Some students move in front of the the board to count the photos in frame 3
Rama : 36
Bunga : 36 Ma'am
Observer : Okay pay attention please. So what is the area of this frame? (referring to the first frame)
Students : 36
Observer : What if I change the smaller ones in this frame with the larger photos?
Ranti : yes we can, so there are 9 photos there
Observer : So , how many ways do we have to find the area of this frame?
Students: Two, by using the larger photos or smaller ones. If we use smaller ones, it will be 36 and we use the larger ones, it will be 9

From the conversation, we can see that some students, especially students coming not from Ersa's group finally realized that they could change the larger with smaller ones, so they only had smaller photos in the frames. These students were brought by the teacher to the idea of identical unit. We found that these students finally understood that they could compare by the use of identical photos. Especially our focus group students, Asti, who finally realized that her idea about using the same photos could be used in this comparison of the frames. She showed that she got the better way to compare the frame which is by the use of identical photos, smaller photos.

We also found that the students, especially our focus group, Rama, Ranti and others could express the size of frame more than one way depending on the photos they used. They could use the larger one so that they get 9. Using the smaller ones gave them 36. We interprete that the students could see that the size depended on the photos they use. We also assume that they understand why the area by the use of larger photos is smaller than the area by the use of smaller photos. We conclude that the focus group understand the idea of inverse relationship.

Based on this analysis, we conclude that

1. The students understand the idea of identical units to measure and to compare area
2. The students understand the idea of inverse relationship between the size of the units and the area

At this meeting, the mini lesson about making own frame was not conducted since the class already finished. However, we intended to have this mini lesson in the beginning of the meeting 4.

4. Meeting 4

a. Mini lesson – Making Frame

The aim of this mini lesson is to have students understand that with the same number of unit (photos), the shape of frames could be diffferent eventhought the frame size is the same. Then, the puplis would create three frames with the size consecutively: 12 smaller photos; 4 larger photos; 2 larger photos and 4 smaller photos. The students were asked to pick up the photos, attached the photos on large paper, and cut to get the intended frame. We found that there were some differences of frame shape between group (see the picture above). The following conversation was the fragment when class discussion about making own frame activity.

Observer : What is the difference between this frame and that frame (pointing out the frame with 12 smaller photos of group1 and group 2)
Ranti : Arranged horizontally and vertically. The first one was arranged horizontal (frame of group 1) and the second one was arranged vertically
Observer : Ranti said this one is horizontal and that one is vertical, so what can you speak about ***the shape*** *of these frame?*
Bunga : ***They are different***
Observer : The shape is different each other, however, what about ***the size****?*
Students : ***They are the same***
Observer : What makes the size the same?
Students : ***The number of photos on both frames are the same***
....
Observer : Now, what is the difference between the second frame (frame with size 4 larger photos) of group 1, 2, and 3? I want to hear Munawar's voice
Munawar : The second frame of the third group was arranged horizontally
Observer : What about the frame of group 1 and 2?
Munawar : They both are the same
Observer : Can you figure out what they look like?
Students : Like square
Observer : Then, what can you say about the second frames of all groups?
Ranti : ***They are different in shape, but the size is the same***
....
Observer : Now, take a look at the last frame
Students : They are the same size

Some students already understood that photos as the size determiner of the frame like Ranti tought. The same number of photos on frames makes the frames the same in size. Some students already made clear connection between the size, the number of photos, and the possible shape of frame. Based on this observation, some students could see that the frame with the same number of photos, have the same size eventhough the shapes are different.

b. Quick image – The Unfinished Frames

The aim of this activity is that the students can be forced to count in an efficient and quick way and slowly leave counting one by one for photos on frame arranged in an array form. In order to promote this aim, we showed the frame one by one quickly. Firstly, the students were asked to write the answer and their strategy on the worksheet individually. Then, the discussion was conducted shortly. We continued to the second frame and do the same procedure and so on.

From the first frame, we found that most of the students answered 8 photos as the size of frame 1. Generally, their strategy was adding another 4 on top of displayed four photos on the frame like what our focus group student Rama did. While another strategy was multiplying 4 with 2, since some students saw there were two '4 photos' on the frame. Bunga for example, saw that there were 4 photos on the bottom, and another four on top, then she concluded there were 4x2 photos on the frame, so that the size of frame 1 was 8 photos.In this phase, there was no special term used to describe 'top' and 'bottom' yet.

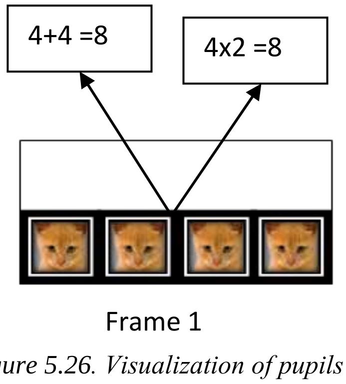
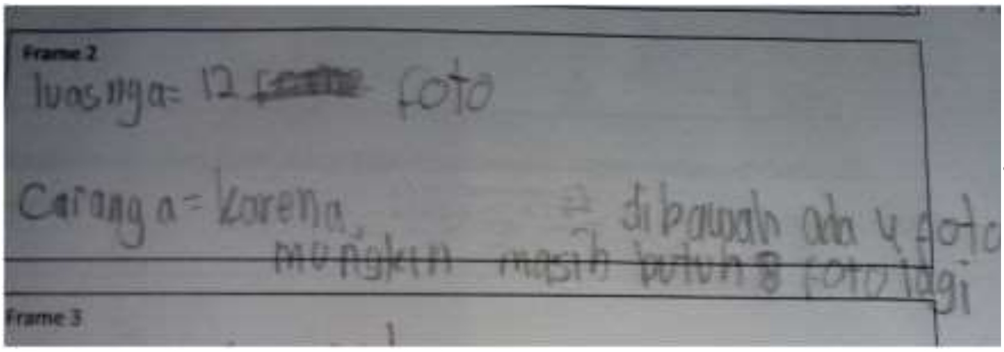


Figure 5.26. Visualization of pupils' strategy on first frame

In the second frame, we found that several students kept ‘adding another 8’. Other students directly multiplied 4 and 3, while some others elaborated their strategy to reveal the multiplication between 3 and 4. Ranti, a pupil of our focus group firstly wrote her answer down on the paper implying that she needed to add another 8 to the frame based on what she saw quickly on LCD so that she had 12 in total as the size of the frame. The following picture is her written work on the second frame.



another 8

Figure 5.27 Ranti’s written work for second frame size

When discussing about the area of the frame, Ranti presented her strategy as what she wrote on the worksheet. However, she expanded her answer by connecting the second frame with the first one and by revealing the multiplication strategy. Here is the conversation happening during the discussion on the second frame.

Observer : So, what is the area of the second frame?
Ranti : 12
Observer : 12, 12 of what?
Ranti : 12 photos
Observer : How is your way Ranti ?
Ranti : ***I saw from the first frame there were 8 photos, but it was smaller than the second, I think I need to add another 4 on this second frame. Because the second frame there was 4 photos, so it means I add another 8***
Observer : So, how is your way to get the total of photos?
Ranti : 4 x 3

Observer : 4 x 3, Uhmm .. Okay is there any other answers?
Raffa raises his hand
Observer : Yes raffa, do you have another strategy?
*Raffa : **I saw from the first frame there was 2, but now there was 3, so I had 4 x 3***
Observer : 4 x 3, do you agree with Raffa?
Students say yes
Observer : Is there any other strategy?
Adhif raises his hand
Observer : Okay, Adhif?
*Adhif : **Because 4 photos at the bottom equals with four photos in the middle and equals to four at the top***
Observer : Hmm, do you mean that there are four at the bottom, at the middle, and at the top consecutively?
Adhif : Ya
Observer : So what is it in total?
Adhif : 12
Observer : Where does it come from?
Adhif : 3 x 4

Fig 5.28 Visualization of pupils' strategy on second frame

From this conversation, we can see that Ranti switched her way from adding into a multiplication strategy. She seemed like seeing at first there were three groups of four like there were two groups of four in the first frame. Then, we assume that what was in her mind was that she just simply added 8 to 4 in the second frame. This idea seemed to contribute to her latest answer by multiplication 4 and 3. Her frame of thinking about three groups of 4 seemed bringing her into the idea of multipying that 4 with 3. The same idea occured with our pupil of focus group, Raffa. He saw that the second frame could be derived from the first frame by adding another 4. It is proven by this phrase "*I saw from the first frame there was 2, but now there was 3, so I had 4 x 3"*.

From the conversation above, Adhif said that "*Because 4 photos at the bottom equals with four photos in the middle and equals to four at the top"*.We took this idea further by looking the terms *'bottom', 'middle'*, and *'top'* as Adhif said. We introduced the term *'row'* to represent the terms Adhif produced in his argument.

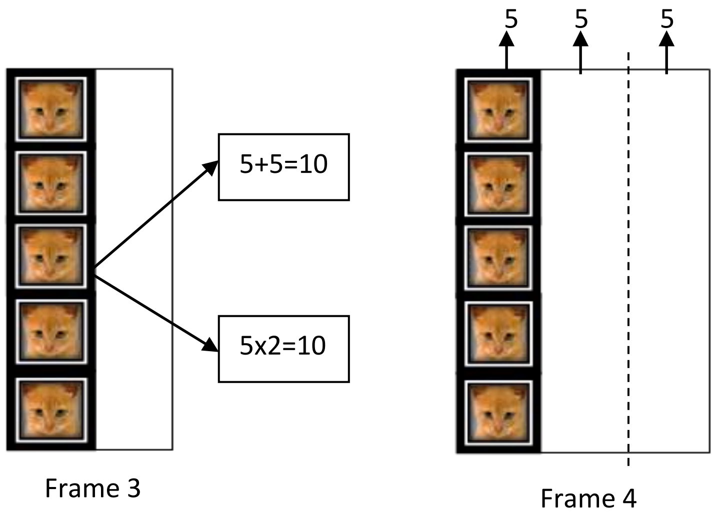
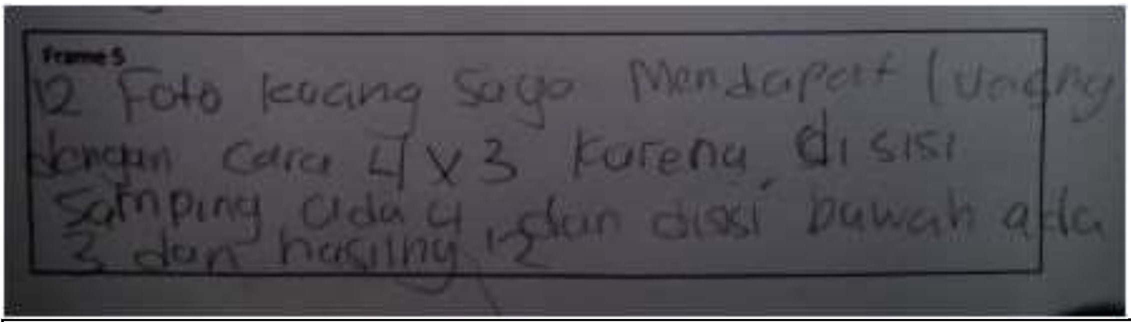


Fig 5.29 Visualization of pupils' strategy on third and forth frame

In the third frame, generally the students still occupied 'adding another photos'. Our focus group students, Ranti, Asti, Rama used this way. The other strategy students used was simply multiply 5 with 2. Raffa, our focus group pupil was the example of the students who simply used this strategy. The answer of fourth frame was derived from the third frame. The strategy revealed were 'adding another 5 photos' (Ranti, Asti, Rama) and multiplication of 5 and 3 (Raffa and Sasi).The term *'column'*was then introduced to the students so that they could use this term later on.

On the fifth frame, generally the students used 'adding another photos' and multiplication 3 and 4, and there was interesting multiplication between 6 and 2. Two students of our focus group revealed different strategy by the use of multiplication to get the area of the fifth frame.



12 cat photos, I get the area by 4 x 3, because on one side there are 4, and in bottom side there are 3, and the product is 12

Fig 5.30 Asti and Raffa strategy to answer the area of the fifth frame

The following conversation happening during the discussion makes what these both students wrote on the worksheet clearer.

Observer : So, what is the area of the fifth frame? I want to hear Raffa's voice
Other students raise their hands
Raffa : The area is 12 coming from 6 x 2
Observer : 12, Uhmm, I will try to understand what Raffa said. He said 12. Where it comes from?
Raffa : 6 x 2

Observer : 6 x 2? Which do you mean 6? Which do you mean 2? Please pay attention to Raffa
*Raffa : **Six ... 1, 2, 3, 4, 5, 6 (counting the displayed photos on the fifth frame)***
Observer : Then ?
*Raffa : **1, 2, 3, 4, 5, 6 (counting the photos need to be added) So 6 x 2***
Observer : O.....Okay, is there any other answer?
Students raise their hands
Observer : Yep, Dinda..
Dinda : The frames contains what it is called? Column ... and this one called row..in every column there were .. hmmm (forgetting the number), uhmm... so (unclear voice)
Observer : Okay, can anybody help Dinda?
Ranti : 3 x 4
Observer : How do you get that ? What does it mean?
Olga : 3 is the column
*Asti : **4 is the row. So it means 4 x 3***
Olga : 4 x 3

We can see that Asti already understood the way the multiplication works on finding the number of photos. She already used the column and the row and multiply the number of photos on one row with the number of photos in one column eventhough in her written work, she did not really used the term row and column. We assume that Asti, another pupil like Ranti and some other students who used this way already derived the efficient way of counting by the use of array multiplication. Meanwhile, for some other students, 'adding another photos' sometimes was still quite often to appear. However, our focus group pupil, Raffa still had to struggle about the column-row multiplication. He derived the 6 x 2 for the area of the fifth frame. The following picture is the visualization of Raffa's thinking.

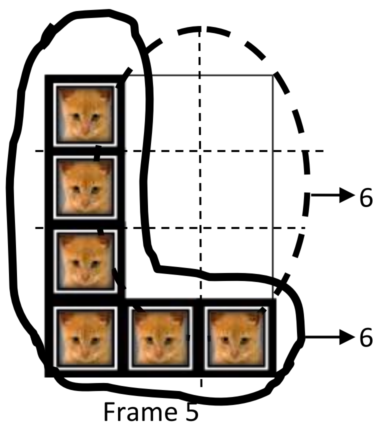


Fig 5.31 Visualization of Raffa's thinking

It is kind of fortunate that Raffa found the number of photos that need to be added was the same with the number of photos availabe on the frame. That was why we assume he multiplied 6 with 2. Raffa’s thinking also implied that he still did not really see column-row structure in this frame. There was one thing we assume this to happen. We think that it was because this frame was different with the series of frame displayed before in which the photos available were arranged only in a row or in column. For this fifth frame, we revealed both, column and row.

We noted that there was no further discussion about Raffa’s strategy in the class. Hence, he became confused in the next task about measuring and comparing when he found his way was no longer applicable. We then tried to help him out in the discussion.

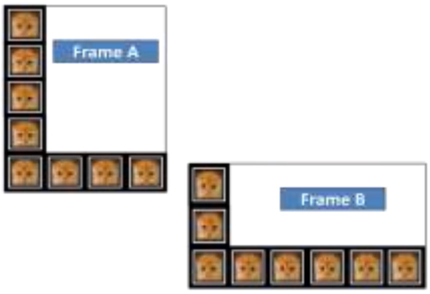


Fig 5.32 Measuring and comparing the area of frame A and B

The following picture is the written work of our focus group pupil.

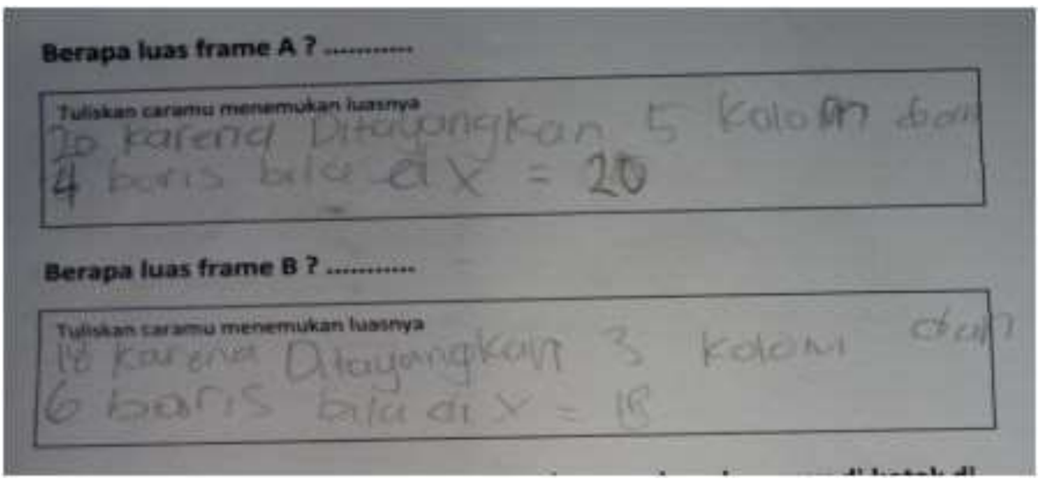


Fig 5.33 Pupils’ reasoning on the measuring and comparing frames task

This pupil already used row-column multiplication to get the area of each frame and based on the product this pupil could directly compare the area and determined which frame was larger. However, Raffa in this task could not yet figure out the area of each frame. He just argued that frame A was the larger compared to frame B with no reasoning on the number of photos on his worksheet. We assume that he did confuse to derive the number of photos on each frame since he found his strategy was not applicable on both frames. Nevertheless, we find out that in the discussion session of this task, he said that the frame B to be 18 photos. His way was counting the photos that need to be added on the frame (10) and the number of photos availabe on the frame (8), so that he got 18. He did not yet use row-column multiplication.

The following picture was shown to the students as additional task. We gave this complete frame to see whether the students able or not to directly apply the multiplication to find the number of photos.

Fig 5.34 Frame with complete photos

We found that some students quickly found that the area of the frame was 36. Rama, one of our students in focus group gave his argument that he only needed to count 6 horizontally on first row and 6 vertically on the first column and he got 6x6=36 photos.We also noted that Ranti also stated the same way.We also found that Raffa was really helped by Rama's argument. He then came with the same answer 36 by multiplying 6 and 6 (the number of photos in one row and one colum respectively).He started to undertand the row-column multiplication.Our argument is supported by the fact that Raffa already used row-column multiplication on the next task about comparing the area of carpet.

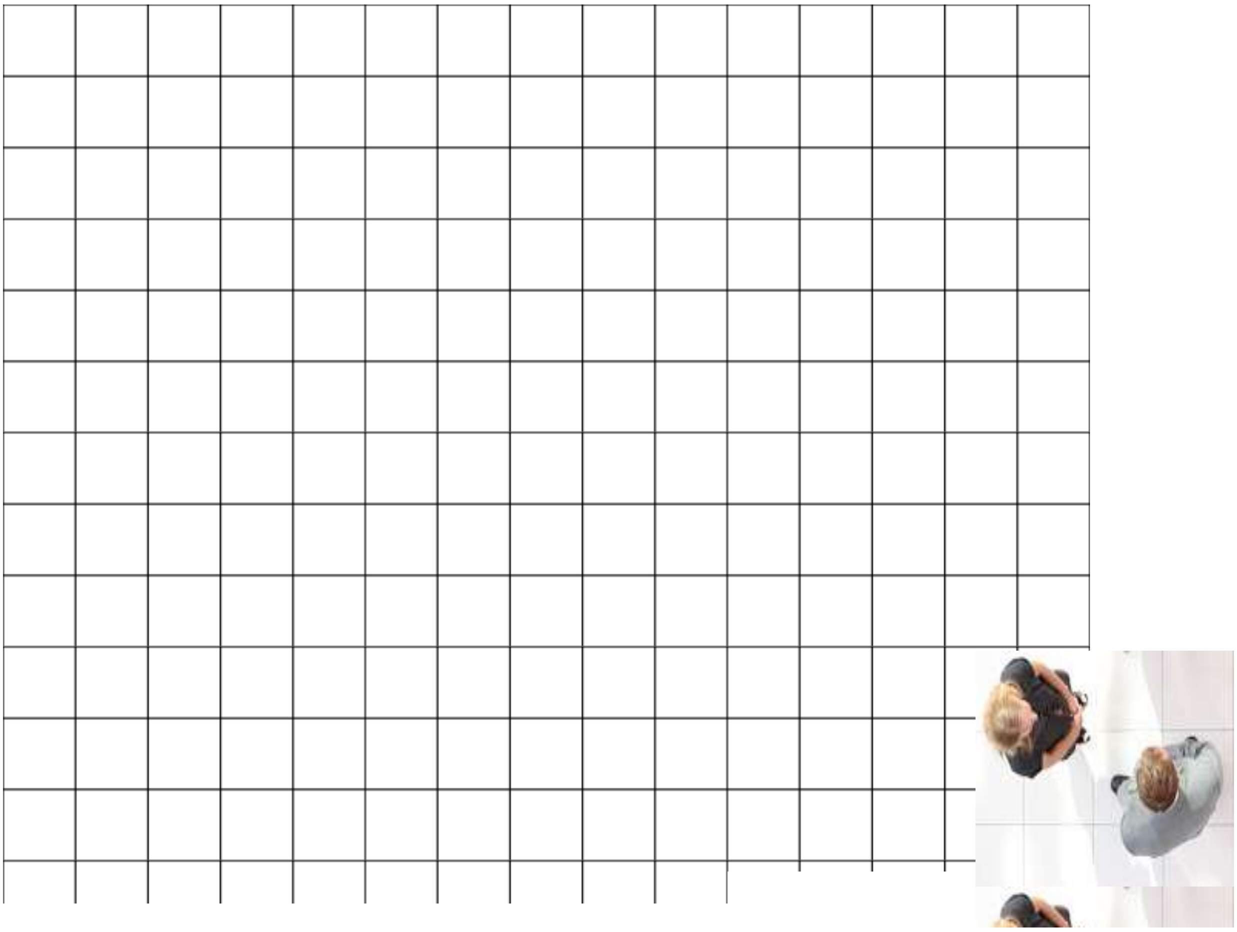


Figure 5.35 The carpet task

Raffa came to us and show us his answer towards the carpet task. He found that the carpet A was 60 tiles while carpet B was 56. He demonstarted the way he calculated the number of tiles. He counted that there were 6 tiles in a row and 10 tiles in a column, then he got the number of tiles by multipying 6 and 10 to get 60. When asked why he needed to multiply that 6 with 10, he argued that there were ten 6's on the carpet. He did the same with carpet B to get 56 (7x8) and found the larger carpet at the end.

Generally the students did also the same with Raffa's strategy. Hence, we can conclude that the students already developed the effiecient way of counting units by the use of multiplication by row and column. However, we still cannot guarantee that sometimes studentsdid not use adding or counting one by one. However, this counting strategy has been a new strategy to them in counting the number of unit efficiently and will help them deal with enormous number of units.Finally, from this meeting, we can conclude that

1. The students understand the idea that different shape of surface can have the same area if they have the same number of units through making frame activity
2. The studentsunderstand and perform an efficient way of counting units by applying multiplication (row and column) through the quick image of unfinished frame

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 1 analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Finding the area of frame	Whether the students use or do not use different sized photos to cover frame	The students used both different sized photos on frame and arranged them	No information	No information
	The students strategy to report the number of photos used (consistency or not)	One group already expressed the area in an amount of smaller photos. Another group counted all smaller and bigger photos and reported as single number	Area just in smaller photos. Area as the amount of mixed photos stated in single number	No information
	The students already used identical photos or not	No. But they just use smaller one to determine the area	The area in number of just smaller photos	Bigger photos counted as four smaller photos when counting the number of photos used
	The students draw connection between the number ofunits and unit size (inverse relation)	Students argued that larger photos, smaller number of photos obtained, vice versa	No information	Students argued that larger photos, smaller of photos obtained, vice versa
Quick image of unfinished frames	Students' counting strategy (multiplication) on the number of photos in unfinished frame shown through quick image	Students counted by column, by rows, and by column and rows	Students counted by column, by rows, and by column and rows	Students counted by column, by rows, and by column and rows
Makin own frames	Use of photos as area determiner of frame	Students regarded the number of photos to derive the area of frames	No information	Students regarded the number of photos to derive the area
	Conclude that more photos, larger the frame	Students argued that more identical photos, larger the frame	Students argued that more identical photos, larger the frame	Students argued that more identical photos, larger the frame
	Perception on the area measurement and comparison by the use of identical units (tiled floor context)	The students used identical photos not only to measure but also compare the area	No information	The students used identical photos not only to measure but also compare the area

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting.

5. Meeting 5

a. Finding the Area of Cookie

The students in each group found the area of rectangular cookie (A) and oval cookie (B) by the use of square chocochips. The main aim of this activity is that the students can estimate the area of non-rectangular plane by the use of rectangular units. Hence, the main focus of this activity is finding the area of oval cookie by the use of square chochips. We also look for the use of multiplication strategy during finding the area of both cookies.

In this activity, students firstly worked on the rectangular cookie. They were given only a piece of square chocochips. All the groups used one chip to figure out the area of rectangular cookie. We observed that there were different strategies between group. All the groups found that the area of rectangular cookie was 20 chocochips.

Generally, all the groups put a chip on one of the corner of the cake, gave the mark, then moved it. What they did after that was finding how many chips on one row and on one column, then they applied the multiplication to derive the number of chips in total. Ersa's group and Tita's group applied this strategy (see the following picture on the left side).

The students of our focus group did a little different. They put a chip on the corner then used the ruler to make the horizontal line indicating the row. Then the chip was moved down, then second horizontal line was drawn, and so on.

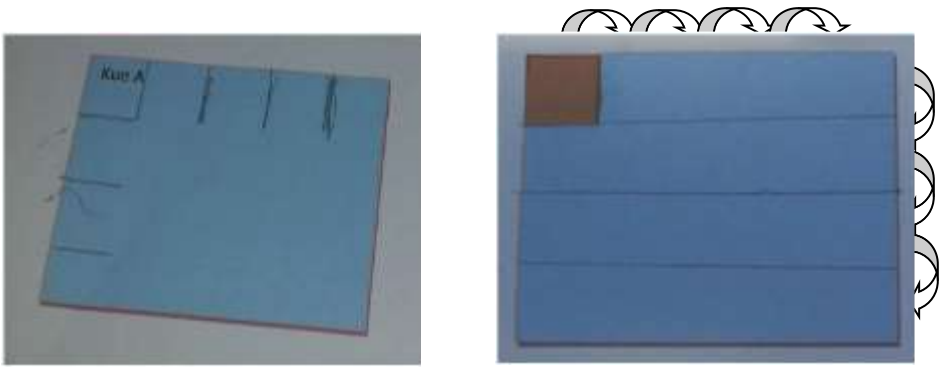


Figure 5.36 From left to right: Other group's strategy and our focus group strategy

After creating the lines indicating the rows, the students of our focus group explained how they got the area of the rectangular cookie as shown in the following conversation. Teacher and observer got closer to focus group and asked question

Observer : Have you found the area of cookie A?
Raffa : We have, 20
Observer : How did you get that?
Ranti : Like this, tit ... tit ... tit ... tit .. tit... (producing voice while moving the chips horizontally) and tit ... tit ... tit .... tit .... (producing voice while moving the chips verticallly)
Observer : So how many you have here (pointing out a row) ?
Raffa : 4...oops..5
Ranti : 4 here (referring to a column)
Asti : So we have 20 in total by multiplying 4 and 5.

They found that there were four rows which were between the lines. The group found that in every row, there were five chips as like Ranti demonstrated. Based on this conversation we saw that this group applied multiplication strategy (row-column) on the chips to get the total number as the area of cookie A. They did not necessarily to count one by one on that chip nor did they need all the chips to cover the rectangular cookie. Therefore, we conclude that these students already understood and able to apply multiplication to find the area of rectangular surface.

The groups then measured the oval cookie by the use of square chips. The groups were given some rectangular chips used to cover the oval cookie. Strategy of the group was observed. We found that two groups (Ersa's group and our focus group) arranged the square chips on top of oval cookie so that several chips exceeded the outline of cookie.

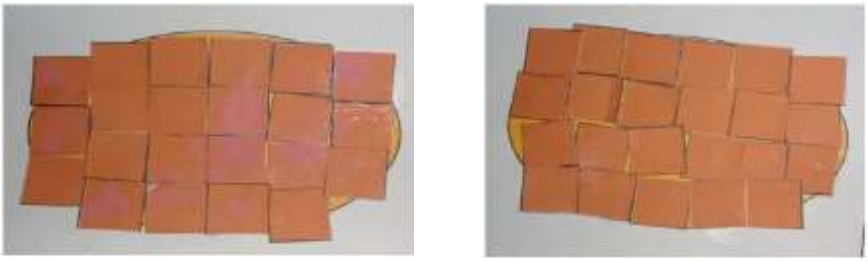


Figure 5.37 From left to right: Other group's work and our focus group's

When asked what is the area for the cookie, these students counted the number of chips they used (based on the pictures above). However they were aware of the exceeding parts of chips around the outline of cookie. We conclude that these students did not yet have any idea how to deal with the area of oval shape by the use of square chips. We assume that this oval shape is something else different with rectangular shape they had ever seen in the previous tasks. They might see that some parts of chips exceeded the outline, but they seemed do not know what to do with those exceeding parts. They might not regard to split the chips yet. That was why we assume they just included those exceeding parts in the process of counting the chips to get the area of oval cookie. Moreover, the different area between these two groups was seemingly caused by the fact that their arrangement of chips was different each other. Our focus group arranged a little bit more tight, so they got 1 more than that other group did.

However, we found that these students became finally aware of the lack on their works when they saw Adit from Tita's group did something interesting. We directly took this moment to put students into discussion on Tita's group's strategy towards the area of oval cookie. We asked students to compare what Adit did to what they did.

Figure 5.38 Adit's group's work

Figure 5.39 Adit's strategy to cover the oval

Observer : What do you think Adit is doing?
Ranti : He is splitting the chips
Observer : Try to compare what he is doing with what you did!
(Asti's voice in the background telling Rama that she thinks that what they did was wrong)
Observer : I see that Adit splits the chip, part of one chip is used to cover the uncovered part near the edge. Do you have any idea what about the remaining part?
Bunga : The remaining part is used to cover other uncovered part.
Observer : Okay, I want to hear someone to explain again what exactly Adit is doing here? Okay
(Ranti raises her hand)
*Ranti : **Use one chip and measure how much the part of one chip used to cover small uncovered part of cookie near this edge, then split the chip up by using scissor, then glue one part on that place so that the part does not exceed the outline.***

Observer : Okay, what about the remaining part after that one chip splitted?
*Ranti : **We can put that here (pointing another uncovered part of the cookie near the edge)***
Observer : Okay, see Adit is doing what Ranti just said. He puts one part of splitted chip on here, and another one, where do you think Adit you need to put it?
(Adit finds the place to put another part of splitted chip)
Observer : Okay, let's give mark on this first part with 1, and the second part as 2. Do you know how much we count those two parts?
*Ranti : **We count them as 1***

What Adit did during this discussion seemed helpful for Ranti (our focus group pupil) to deal with finding the area of oval cookie by the use of square chocochips. Her argument implies that she realized that all the chips exceededing the outline should be splitted so that no chips exceeding the outline and all the chips were really cover the surface of oval cookie. We also notice that the phrase "*We count them as 1"* implies that Ranti already had reasonable thinking towards the counting of parts of one chip that was splitted. Hence, by her thinking, we conclude that she could finally come up with the number of chips in total used to cover the oval cookie probably by estimation.

From this conversation, we found that the students were helped to understand the situation by Ranti's explanation on what Adit did. Especially for those coming from our focus group, they also seemed realized that what they did with their oval cookie was not yet done. This conversation implies that this group needed to accomplish their work. We conjecture that these students would split all the chips that exceeded the outline of oval cookie. Finally they came with an estimation on the number of chips used to cover the oval cookie. Therefore, they got the area for the oval cookie.

We found that this hands on activity gives insight to the students in the next task. In the next task, the students find the area of oval table by the use of tiles.

b. Finding the Area of Oval Table

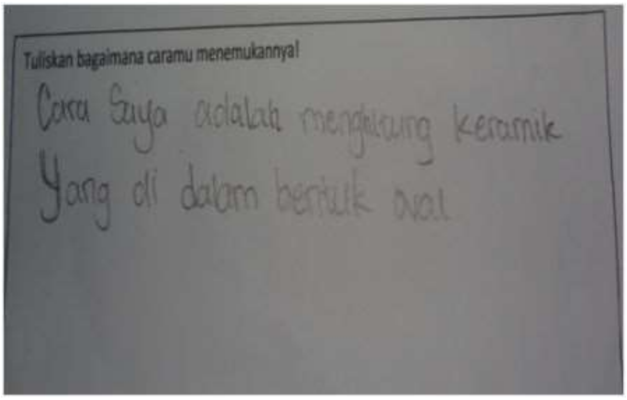


Figure 5.40 The area of oval table task and pupil's response

The students worked individually to find the area of oval table. We found that the students already tought that they could use tiles to figure out the area of oval table. The area of the oval table is obtained by counting the number of tiles inside the oval (as what explained by one of the students in picture). Hence, we conclude that the students knew what refers to the area of oval table and knew how to derive the area by the use of the tiles.

We also found the pupil's argumentation when a small discussion happening helped the other students to accomplish the task.

Dinda : I saw that the table is oval, there are some half parts there (pointing out some unfully covered tiles near the edge of oval table)
Observer : Okay, you hear what Dinda said? She said what about the half parts. Look at this picture.
Ranti : Those parts can be be joined to get one tile
Observer : Do you understand what she means?
Raffa : She means that that part is half but the larger one is one tile.

Ranti : I mean if one tile splitted then we had that part.
Observer : Okay, try to relate what Ranti said to what Adit did when he was looking for the area of oval cookie.
Students : Split
Observer : Then? Do you remember that one part is put in one place and another part is put in another place?
Students : Yes
Observer : Hence, those two parts are counted how many?
Aisyah : One
Observer : So can you finally find the area of this oval table?
Students: Yes, we can

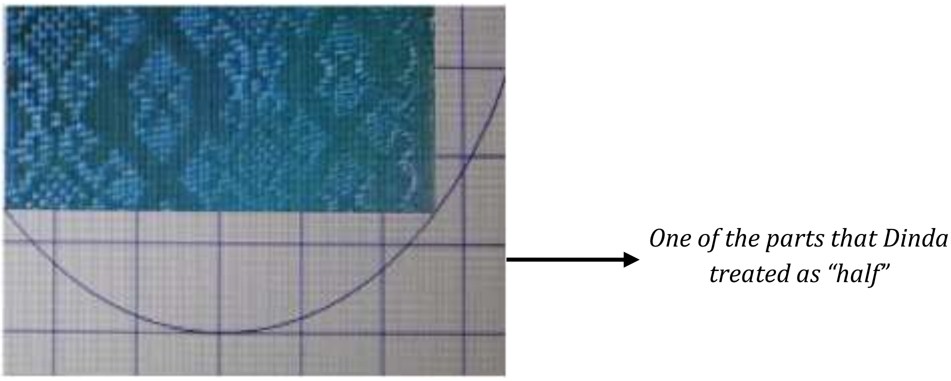


Figure 5.41 Dinda's argumentation on a what treated as half part

Based on this conversation, Ranti seemingly applied what she learned during finding the area of oval cookie. The idea of joining parts to make one was the large idea she revealed on this task. By this idea, she and other students could seemingly perform estimation on the area of oval table. We then let students to find out the area.

We found that several students firstly tried to reveal the tiles hidden by the carpet. Ranti, our focus group pupil for example, drew exactly all the hidden tiles under the carpets. Meanwhile, some other students only revealed tiles on one column and tiles on one row. The example was the work of our another focus group pupil, Asti. After both of these students drew the tiles, they seemingly applied multiplication to get the number of tiles of the area of array they made in the outline of oval table. The following pictures are the ilustration of works of Ranti and Asti.

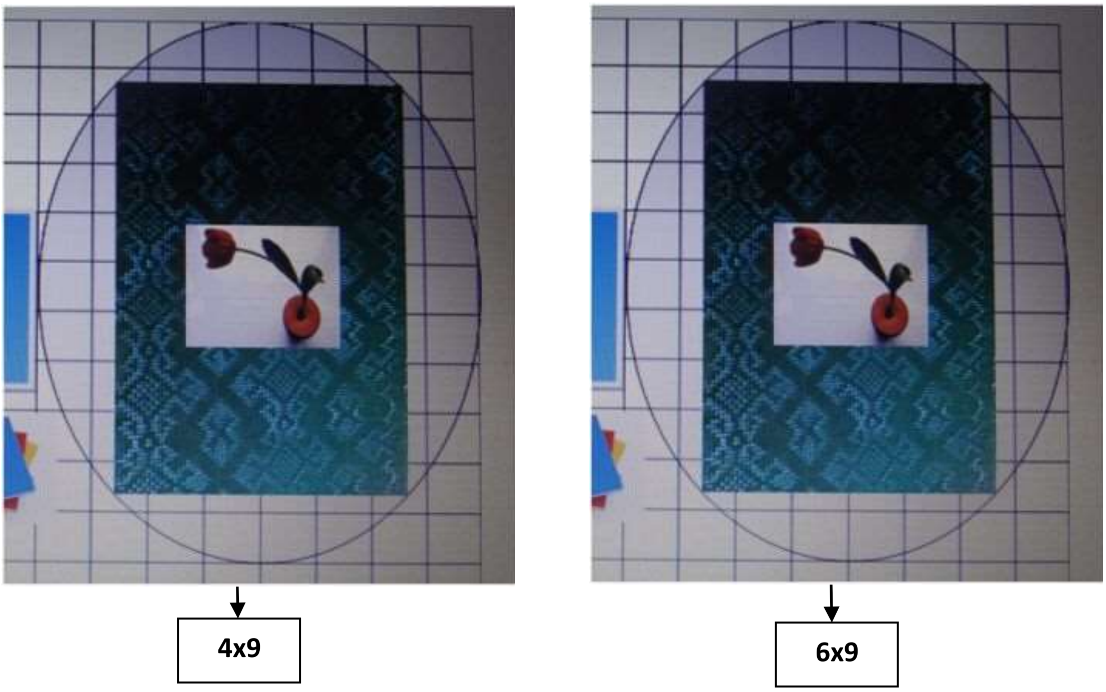


Figure 5.42 From left to right: Ranti's strategy and Asti's strategy

Ranti chose to construct 4x9 array in the oval shape, then derived 36 for the area of her array. It seemed that she regarded all the squares contained in array she made that should be fully included in the oval shape. Meanwhile Asti constructed 6x9 array. This construction of array area was marked by Asti with dots. She seemed to regard the maximal coverage of tablecloth on the table over the tiles, so that she initially looked not really regarding the square on every corner of the array to be not fully covered that moment. However, we found that she did not include every square on corner in the counting process. She found 54 for that 6x9, but she subtracted 4 from 54 to get 50 indicating that she excluded 4 tiles on the corner.

We noted that both students at the remaining process of figuring out the area of oval table, they looked for possible fully covered tiles on the other places outside the array they made. We observed that some students including Ranti made en effort to count the unfully covered tiles in total to be added to the previous counting. They did some estimation. We saw that Ranti came with the estimation of 66 tiles for the area of oval table.

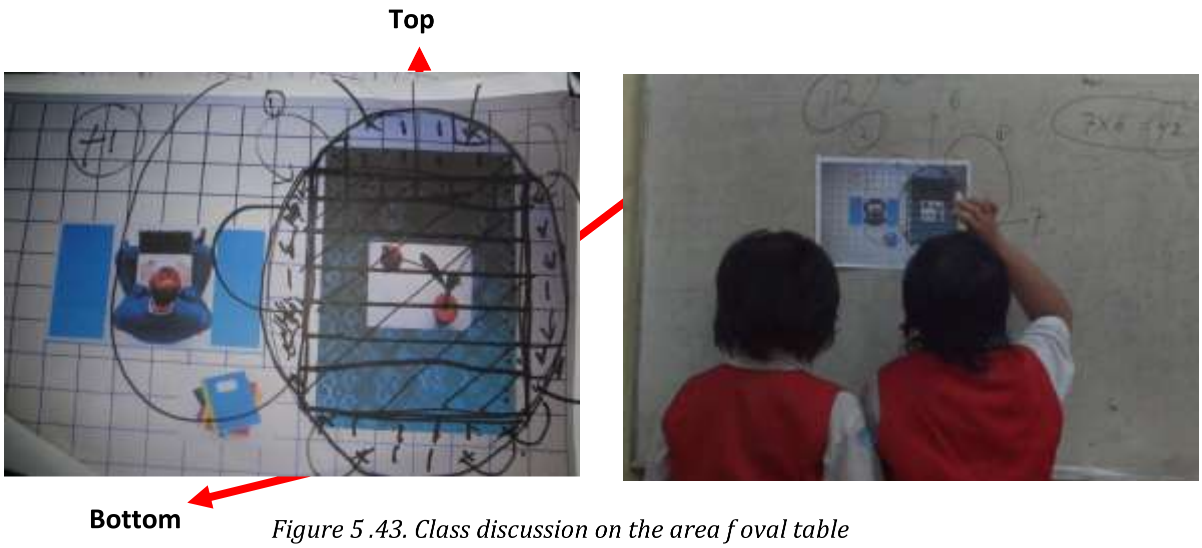


Figure 5 .43. Class discussion on the area f oval table

For some other students including Asti, we found that they still struggled to count the unfully covered tiles around the edge of tiles. Therefore, we conducted the class discussion.

The students together made construction of 6x7 array inside the oval shape. We saw that the students tried to make array in which all the squares were inside the oval. Dinda and Asti then moved in front of the class and performed the drawing to entail some hidden tiles while other students looked at what they did. We found that these students then tried to apply the multiplication to derive the area of array which was 42.

Asti, Dinda, and other students continued to find the area of the remaining parts of the table. We show that the students named 4 places on oval shape outside the array in which the remaining tiles should be counted: the top, the the bottom, the left, and the right.

After that, Raffa, Ranti, Sasi, and Ersa moved in front of the class pointing out the tiles on the bottom and top parts that could be counted one for they were fully covered. They found that there were 6 tiles for each part. The other students found one tile fully covered in each left and right parts. They got 42+12+2=56 tiles for all the fully covered tiles. The students discussed together and determined the number of estimated unfully covered tiles near the edge of the oval. We found that students applied the strategy that they did during finding the oval cookie area. They looked for one piece of part of one tile and looked for another possible piece. They joined those two parts and treated as one tile. We found that all students came with an estimation of 14 tiles and they added it to 56 they found previously to get 70 as the estimation of area of oval table.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 5 analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Finding the area of cookies	The use of square chocochips to derive area of cookie	The students put the chips on the cookie so that overlapping and gap avoided. The students did some splitting and joining some pieces of unfully covering the	Making arrow indicating pairing some pieces counted ones	The students needs to split up the chips around the edge into pieces and put one piece on one place and another on

		part of cookie		another place
Finding the area of table	The use of multiplication to count the tiles	The students created inner array to derive the area of oval table	Students' inner array and counting result by multiplication	Inner array to help them efficiently count the tiles
	The way of estimating the area of non-rectangular region (oval table by the use of inner array and splitting joining strategy)	The students used splitting joining strategy to count the tiles near the edge of table	Making arrows indicating pairing some pieces of unfully covering tiles	They argued to do the same way as they did when finding the area of oval cookie

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting.

From this activity we could conclude that the students, especially some students of our focus group already able to derive the area of oval table by the use of tiles. We also noticed that they used the strategy they emerged during the activity of finding the area of oval cookie by the use of square chocochips. The strategy of splitting the unit to cover the edge of outline they developed on the oval cookie task was used to derive the counting strategy of joining the splitted units by estimation when they tried to find out the area of that oval cookie. By this experience, the students seemingly applied the same strategy when they tried to find the area of oval table by the use of tiles.

All in all, in this meeting, we assume that the experience they got through hands on activity, finding the area of oval cookie by the use of square chocochips, could help them understand the way to find the area of non-rectangular shape by the use of rectangular units. We conjecture that by this knowledge, the students could follow the last lesson about finding the area of palm by the use of grid model.

6. Meeting 6

In this meeting, the students measured the area of palm by the use of transparent grid. The aim of this lesson is that students could figure out the area of rectangular and non-rectangular plane by the use of rectangular grid. The students are also hoped to do estimation when figuring out the area of non-rectangular plane. The estimation is applied when joining some unfully covered units. The students will also also build up the partial array on the non-rectangular plane and apply the multiplication strategy to find the unit in that array.

Before the students worked in the group, they were asked to compare each other's palm. They did superposition when determining whose palm was larger. All the students could determine which one was larger by this way. We turned the task to become measuring *'how large'* those palms. Hence, the students worked in group and they were given paper to trace one palm and transparent grid to derive the area of palm. We gave them the first transparent grid to work on.

Each group pointed out the pupil whose palm was used to be traced on the paper. The transparent grid was attached above the paper with the outline of palm. Firstly generally all groups focused on the fully covered squares inside the palm outline. The students gave marks to show that each square was counted one. Mostly students counted one by one. Therefore, we came to remind them about the easier way to count the number of squares. The students could remind multiplication. However, we trigger them to construct the array inside the palm. Then, the students continued the work themselves.

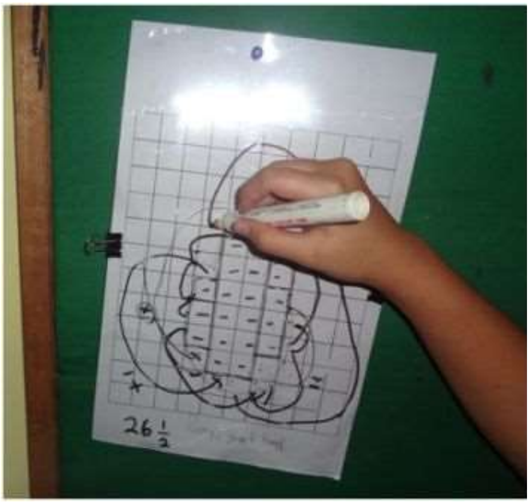


Figure 5.44 Pupils of focus group joined the unfully covered tiles in counting the tiles

The students then continued to find the number of remaining tiles that should also be included in counting process. We found that our focus group used the same way as they did when they counted the area of oval table. Our focus group students tried to make join of two or more pieces of tile to get tile counted one. They made curvature arrow to indicate that one piece was joined with another and they added +1 meaning they found one tile to count. They kept doing the same until all the unfully covered tiles completely joined and counted. Our focus group finally came with the estimation of 26 and half squares as the area of the palm of Rama.

For two other groups, we found that they still needed to be guided to find the number of unfully covered tiles. We also needed to remind them again about the previous lesson, the oval table. However, with little guidance, they could manage themselves to do some estimation of number of unfully covered tiles near the edge of palm outline. While one group came with 34 squares for Bunga's palm, another group came with 17 squares for area of Olga'palm.

Figure 5.45 Groups' result of palm area measurement

From this activity, we finally assume that the students already able to find the area of non rectangular plane by the use of rectangular units which is the square grid. They even used estimation to derive the number of unfully covered squares near the edge of palm outline. It indicates that these students already had the understanding that unfully covered unit could not be counted as one, so they got to find the other piece to make the uncomplete becoming complete meaning counted one. Moreover, we noted that they also had been able to apply multiplication to derive easy counting for partial array they made on the palm outline.

The teacher then asked them to compare whose palm was larger based on the result of groups. We noted that the students chose Bunga's palm to be largest one since it contained number of squares at most compared to Rama's and Olga's. It implies that the students had already had strong vision towards the connection between the area and the number of units included at this moment.

After finishing the discussion, we showed the students the second transparent grid which squares contained were smaller than the squares on the first grid. The following conversation happened when we were showing the second transparent grid.

Observer : What do you think will be the result if we use the second transparent grid (showing the second grid with smaller squares). So might be tell first what is the difference between this and the frist grid?
Students : The squares in it are smaller
Observer : One by one please
*Hiu : **More squares in it***
Observer : Okay
*Adit : **The squares are smaller***
*Ersa : **The squares are smaller and grid contains more squares in it***
Observer : I understand, so we see that the squares are smaller so what will happen?
*Students : **More squares are contained in that grid***
Observer : Okay then, now try to imagine what will be the result if we measure the area of each palm by the use of the second grid?
*Adit : **We will get more***
Observer : Why?
*Adit : **Because the squares are smaller***
Observer : Do you agree with him?
Students : Ya
Observer : Okay, so let's say if we measure Rama's hand by the second grid, whta do you think will be the result?
*Ranti : **The result will be more than 26 and half because the squares contained are more and they are smaller.***

We noted that the first noticable difference recognized by the students was the size of squares which was smaller than the first grid. However, for some students, further argumentation was entailed. These students, especially Ersa, could argue that for the smaller size of squares, then the second grid would contain more squares. She seemed comparing the number of square in first and the second grid. She clearly implied the connection between the size of unit and the unit and the number of unit existing. This is what we assume as the inverse relationship that Ersa was trying to reveal. We noted that the students agreed about what Ersa was speaking about. Adit was one of the students.

We also notice that Ranti, our focus group pupil understood about the situation based on what discussed about the second grid. She seemed understand that the result of measurement by the use of second grid would give her more square than what she got by the use of first grid. It implies that she already understood that by the use smaller squares, then withe same size of palm, the area of palm would get more compared to the frist one using the larger squares. In another words, she understood about inverse relationship at this moment.

From this conversation, we could conclude that the students, especially some of our focus group students already could express the area of plane depending on the size of grid they got. They understood that no ‘just one’ grid was used. The grid used could contain any size of squares in it. Hence, the result of area measurement depended on the grid with particular size of squares they used. Furthermore, we are supported by situation in which the students could receive the fact that the result of their area measurement could different absed on the grid with particular size of squares they used.

In addition, we found that some students already used the knowledge they built in lesson 3 within the context of photo frame. They found the inverse relationship between the size of unit and the number of unit used in covering task. We realize that the students tried to bring this idea when we had discussion about the result of area measurement on palm by the use of second grid. We assume by knowing this relationship, the students could be supported more to receive the fact that the area mesurement result could be different for the same subject or plane measured based on the size of units used.

However, by the time limit, we could not accomplish all the task set in this last meeting. We could not conduct the activity about making own grids and comparing the use of nuts and the use of grid in area measurement.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 6 analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Students measure the area of palm by the grid	The use of transparent square grid to derive the area	The students made use the grid to find the area of palm	No information	The grid could be used as tiles in the oval table case
	Estimation of area by the use of transparent square grid (by the use of inner array and splitting joining strategy)	The students applied splitting joining strategy by the use of grid to derive the area of palm	Some arrows were drawn to indicate pairing some pieces counted one	The use of grid was same like the tiles

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting.

So far, we could conclude from this meeting that the students already able to do area mesurement of rectangular and non-rectangular plane by the use of rectangular unit which was grid and they could do estimation on the area of non-rectangular plane. The students could also make partial array to help themselves counting fast and deriving the multiplication on that array. The students could also use the inverse relationship between the size of unit and the number of unit in measuring area. Therefore, they could receive the fact that the result of area measuremet could be different based on the units size they used.

J. Remarks on the Knowledge of Students after Cycle 2 Study

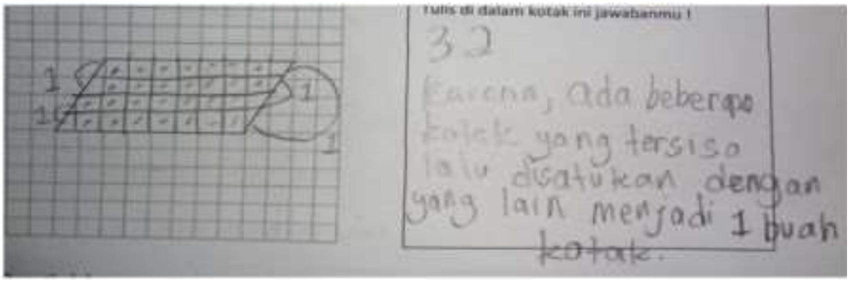
This test was conducted in the end of the teaching experiement to see how far the students' development of understanding during the teaching experiement. There were seven questions in the test. The questions used were the same with the posttest we used in the first cycle but with little adjustment. Some questions are about comparing area and the remaining question are about measuring area by the use of unit (see cycle two post-tets, *Appendix L)*. We provide an overview of students' response on post-tets in *Appendix N*.

While comparing which cookie was larger, almost all of the students including our focus group: Rama, Raffa, Asti, Ranti, and Sasi could compare the area by the use of nuts as the unit. However, we notice that there was still one pupil to use her visual reasoning instead of the nuts.

There were ten students including Rama, Raffa, Asti and Sasi responded correctly to the area of tiled floor. These students expressed the area of floor by the use of identical tiles which was the smaller tiles and derived the area by multiplication between 5 and 6 which was 30. For some degree, some of the students still applied counting one by one. One pupil used the smaller tiles but obtained the wrong result for he had miscalculation. Meanwhile two other students including Ranti used the larger tiles to derive the area of floor. The use of imaginary ruler was still applied by one pupil. This pupil measured by her ruler the perimeter of floor. We notice that one pupil did not use the tiles but derived the result. She provided unclear reasoning. Nevertheless, from question number two we can conclude that generally students already had the understanding of the use of identical unit to measure the area.

There were two kind of responses to the table and books question. Thirteen students including Rama, Asti, Ranti, and Sasi clearly used the books to derive the area of table and compared the table area. Hence, they already used units to tackle this area problem. Meanwhile two other students including Raffa still used their visual reasoning, for example table A was the larger since it looks longer.

In general students could respond to question of area of figures in question number 4 and 5 correctly. They could drew the grid inside the outline precisely and performed the accurate estimation of number of squares. They already understood the joining strategy between unfully covered squares. Meanwhile other students could drew the grid but came with miscalculation. The remaining students could not make the structured grid inside the outline that deliver them to the wrong answer.



32, because there are some remaining squares that need to be joined with others, so some 1 square can be obtained

Figure 5.46 Ranti's response to question number 4

We noted that the students could manage answering the question number 6 asking them which figure was larger. They could refer to the number of square.Generally students could drew the grid to derive the area of given plane(question number 4 and 5). However, some of them could not really make the grid precisely. These students also performed poor counting and estimation of the number of squares as the area of plane. Meanwhile six of them including Asti and Ranti could almost drew the grid precisely and performed the accurate counting and area estimation. Her drawing implies that she understood how to deal with the remaining parts of several squares that need to be joined. Her counting strategy is also reasonable to deliver her to the accurate estimation.However, we noted that three students still struggled in drawing the grid. Their picture imply that they still do not yet make the good structure of grid. Hence, they came with the wrong answer.

Figure 5.47 Asti's response to question number 7

For the last question, six students could make precisely their own grid to derive the area of hexagonal regon. Asti's work showed that she already constructed the well structured array on the figure and performed the accurate estimation n the area of the given figure. Meanwhile, the remaining students still showed the poor constructed grid and estimation of area.

To complement the analysis above, we provide the following table to show how we triangulated the data obtained from different sources during the meeting 5 analysis.

Activity	**Data Collected**	**Video Recording**	**Students' Written Work**	**Interview**
Students working on the posttest	Studnets' recognition on the units as area determiner through nuts, tiled floor, book and table, grid problem	No information	Students used nuts, tiles to derive the area	No information
	Students'	No information	Students	Area as the

	conception on the amount of units (nuts, tiled floor, book and table, grid problem) as the area		represented the area as the amount of units (nuts, tiles)	amount of units
	Strategy on representing area with amount of different units (unit consistency) through the tiled floor problem	Some students already used identical tiles to derive area of floor	Students treated bigger tiles four smaller ones then counted them to get the area of floor	They used only smaller ones to find the area of floor. Bigger counted four smaller omes
	Students' conception on the use of unit (square grid problem) to measure and compare area	Students compared the region by the identical grids	Drew the hidden identical grid to complete finding area and compare regions	No information
	How the students construct their own grid to derive a figure's area	Used ruler and made their own grid identically	Made use the strategy of splitting joining to get area	No information

From the table, we could see that some data obtained through different tools are consistent. Some data obtained through different tools could also add each other enabling us to derive a fuller picture of what observed. This will contribute to our internal validity of the data. Based on this triangulation on analysis, we could derive some conclusion on students' understanding based on this meeting. From this posttest analysis, we can conclude that

1. Generally students already understood the use of units not only to measure the area but also to compare the area, eventhough in some degree 'comparison' setting is stronger to derive unit conception compared to 'measuring'
2. The students already grasped the idea of identical unit to find the area of surface. The different result obtained seemed clearly understood for they have freedom to chose which unit size is used. Hence, we conjecture that these students also grasped the idea of inverse relationship

3. The students generally have had understanding of the use of grid to derive the area and know how to use it. For some degree they have already been capable of making structured grid.
4. The students generally could perform counting strategy (joining several unfully covered squares on grid) to derive the area in total. For some students, multiplication was easily applied. While for some students, accurate estimation is also large issue that students already dealt with.
5. Some studentshave already been able to build their own grid to find the area of given plane and derive the area by the use of squares in that grid.

K. The Conclusion of the Students' Learning Process in Second Cycle

We derive some conclusion on students' learning process starting from their prior knowledge, their participation on each lesson of our cycle2 study and supported by the result of the posttest.

The students have built the initial understanding of area throught the unit issue: overlapping, gap, and leftover they studied in the first meeting (started from activity 2). Experiencing the activity with the nuts to derive unit issues seemingly helped them to start to recognise the use of nuts to derive the size of surface. A connection was started to be built. The students could think that an ideal number of nuts represented as how large the surface was, was the nuts arrangement without possible overlapping and gaps. They understand that use the nuts as the size determiner should in strict regard of overlapping and gap. They could even expand their knowledge by figuring out the connection between the number of nuts and the size of surface they got. More nuts used means larger the surface they got within the cookie context. This we assume as the strong initial insight the students got during the first lesson in our lesson sequence, especially our focus group students.

Having an experience with an hands-on activity on the cookie and the topping, the students were helped to observe and to study by themselves the approprite shape of unit to cover surface. At this momment, they could figure out the square unit as the best shape of unit. In the same time they could argue about what surface shape better covered or not by those square unit. This we assume to help the students familiar with the next lesson about photo frame.

In the lesson about photo frame, the students started the lesson with *'comparing'* area situation. Their initial strategy by comparing visually made them think to find the prove of why particular surface they treated larger was larger. Hence, the students started to be guided to measure by the use of units. Here, we assume the students made use the knowledge about the insight they got in first lesson about the nuts. The students tried to emerge the need of identical units. The context seemed to make them think to find a better way to express the area of frame in identical units (identical photos). The students even derived their understanding on the inverse relationship between the size of unit and the number of unit used to cover surface. All in all, the students even had strong vision on the use of unit as area determiner when they set to accomplish making their own frame and they started to realize that the same number of units implies the same area eventhoguh the plane figures got different shape each other. Through all this, the need to derive efficient counting of the number of units becomes an issue. We note that the students emerged the row-column multiplication when they were provided with the quick frame. In this phase, some students of our focus group have already been able to elaborate why the multiplication works over there. They reasoned by the use of row and column that make multiplication emerged. However, we noted some students in particular degree, still applied the counting one by one. Therefore, we treat this as situational issue. We believe that the students will force themselves to use multiplication whenever they find the enormous number of units.

Their learning track seemed to be continued when they dealt with finding the area of non-rectangular surface. Beforehand, experiencing hands-on activity seemed helpful and effective for them to learn how to deal with finding the area of non-rectangular surface. The oval cookie context seemed contributive to the emergence of students' splitting units large idea when the students tried to cover all of the cookie surface without having units exceeding the outline. Some students and our focus groups even exploring the strategy on how to count all the units including the splitted ones. Hence, the need to join some splitted units while counting simply rose at the learning process. That was what we assume to help the students to deal with finding the area of oval table.

This splitting-joining strategy was extensively used when they were studying to measure their own palm area. For some students including our focus group, the need to efficiently count the grid inside the palm rose. With a simple guidance, the students could construct their own array inside the palm and simply applied the multiplication. They have been able to think efficiently in counting the units. During this palm activity, all the previous concepts were recalled. One of them was the idea of inverse relationship when they discussed about different sized square grid and the different result of using them. They could understand that the result of measuring depended on the grid they use.

However, some activities were not conducted in the last meeting. For example, making their own grid. Nevertheless, we found some of our focus group students could keep up with the question number 7 in posttest implying that they already understand on the grid use in area measurement. From her grid drawn, it was shown that she already understood the structure of array: identical unit, column and rows. Furthermore, by the use of their own grid, they could perform reasonable counting strategy that delivered them to the precise and accurate estimation of area of especially non-rectangular plane figure.

For some students, especially our focus group students, the development of their understanding on area measurement and unit conception seemed observable. For example are Ranti, Asti, Rama, and Raffa. They showed a clear understanding during the lessons in our cycle 2. We also notice that some other students are quite clearly observed when constructing their understanding. They were extensively recorded by the moving camera that enabled us to do observation on them also.

However, some understanding is sometimes quite unrevealing not only in lessons but also in the test. We also note that some students seemed understand some concepts while studying. Nevertheless, they seemed forget about the concept they already learned while they were doing the task. For example, Ersa could understand that she could use just smaller unit to derive area (the use of identical unit). However, she did not recall this concept while she faced the tiled floor question in posttest.

Finally, we can conclude that generally students already passed their learning trajectory on the topic of area measurement through unit area we designed in our study. However, some consideration is still need to be discussed for further improvement.

CHAPTER VI

CONCLUSION AND DISCUSSION

This chapter provides three main parts which are conclusion, discussion, and recommendation. The first one is explanation on the conclusions that consist of the answer to the research questions and a local instructional theory on the learning of area measurement through the area unit. The second one is the discussion providing the information about important issues in this study. The last part is elaboration on further recommendations for further educational research especially in domain of area measurement.

A. Conclusion

1. Answer to the Research Question

The main research question in our study is *"How can students' understanding of unit of area help them understand the concept of area measurement?"*

To support us with finding the answer to our main research question, we derive two research sub-questions that will be elaborated in this chapter .

1. *What activities can support students' understanding of an area unit that can be used to understand the area measurement?*
2. *What kinds of understanding of area measurement that students could reach from the learning process?*

The answer of our first sub-research question

1. ***What activities can support students' understanding of an area unit that can be used to understand the area measurement?***

Some activities set in our refined hypothetical learning trajectory were designed focusing on the understanding of area unit to support students' understanding of the area measurement. Our study suggests that students' understanding of the area measurement can be reached through the understanding of the area unit through these activities. Generally, the activities are covering region by the use of particular units and counting the number of units. The following is the explanation revealing the activities we proposed and the learning goals.

- Covering the region with irregular shaped units

This first activity aims to adress the initial issues on area unit: overlapping units, gap between units, and leftover. The units chosen are irregular shape to enable the state of overlapping, gap in between, and leftover while students covering the region. Our study suggests that cashewnuts can be a choice to derive this aim. In our we design cashewnuts cookie man context to start the activitity. This activity is conducted in group. The students will put the nuts on top of cookie and investigate the number of nuts used to cover the top surface of cookieman's cookie. This context enable different result to occur. Then, the discussion on the difference is conducted to invite the idea of unit issue.

- Covering some regions with the sufficient and or unsufficient number of irregular-shaped units

This activity aims at generating the idea of unit issues' effect to the number of units used to cover region. Based on our study, the context of the cookie and 14 nuts could help students to see the effect of the overlapping nuts, gap between the nuts, leftover to the number of nuts needed to cover the region of cookie. This triggers the idea of ideal number of units to cover region. To address the goal of this activity, two different sized region of cookies are covered by the same number of nuts. The students in group observe the arrangement of the nuts on each cookie to derive the state of overlapping, gap and leftover. The students then discuss about the ideal number of nuts to cover the cookie which later becomes the important idea on the relationship between the number of units and the area. The students will also start to realize the units (nuts) as the area determiner.

- Measuring and ordering the region by the use of irregular-shaped units

After the students understand the idea of unit as area determiner, an activity that brings the concept further to area measurement and comparison is needed. Based on this study, the cashewnuts cookie context could be used to address this aim. We designed an activity of measuring and ordering the cookies by the use of nuts conducted individually. At this phase, the students already avoided overlapping, gap, leftover while putting the nuts on top of each cookie region since they have dealt with ideal number of nuts used as the area determiner for the cookies covered.

- Covering region with some unit of choice and discussing the unit shape better cover the region and the shape better covered by th e unit

In this activity, the students will investigate the shape of unit better covering particular region and the shape of region better covered by those units. This phase becomes important since the students will realize the rectangular shape for example could be appropriate to cover surface. Based on our study, the activity with the cookie and toppings conducted in group can be one way. The students are given two kinds of cookie regions and some different-shaped toppings. They will attach each topping on top of both given cookie region. At this moment, the discussion about the shape structure and area unit issues: overlapping, gap, leftover are generated. The students are expected to find why particular shape of unit can be appropriate to cover region and why particular shape of region can be better covered by particular units. This learning becomes important to conduct prior to the next activity which introduces the photo frame context as medium to study.

- Comparing and measuring the regions by the use of rectangular units

The previous activity gives the opportunity for the students to work on the rectangular-shaped units while they cover regions. Hence, the need to measure and compare several regions by the use rectangular-shaped units become the important activity at this phase. Our study suggests that photo frame can be a potential context. In this activity, the students will cover the frame with the photo. There are two important concepts that need to derive around this activity. The first one is the idea of identical units. The students will represent the number of photos as the area of frame. There are possibility that the children use different sized photos. Hence, the need to express the area of frame becomes important highlight. The students will face the situation in which reporting the area of frame as the number of identical photos can be better way. Further, the students will investigate the difference obtained between measure the frame with several different sized photos. This will ignite the idea of inverse relationship between the unit size and the area.

- Inventing the strategy to efficient counting units by quick image

At this moment, the students already understand that the number of units represents the area of region. Hence, the need to count the units quickly becomes demanded. Quick image can become a way to ignite the quick way of counting units. The students are given a very short moment to see and count the number of units they just saw. Our study suggest that a quick image on unfinished frame can be a way to create row-column multiplication. The students are forced to work only in a row or a column for example and make use the availability of them to quickly count all unit in total. This will bring the students to the idea of row-column multiplication. By discussing the row-column structure displayed by quick image, we found that the students could reason in a logicla way to count the units. Hence, the understanding on why multiplication works can be grasped.

- Mini lesson on the area comparison of some regions by the use hidden identical units

An extension of quick image is a mini lesson on area measurement and comparison on the region with hidden units. This activity becomes important for children to apply the counting strategy they found previously on the region with the hidden units. Our activity: Which carpet is larger? can facilitate the use of counting strategy.

- Hands-on activity on the area measurement and comparison of rectangular and non-rectangular region by the use of rectangular units

To bridge the students from measuring the rectangular region to non-rectangular region by the use of rectangular units, an activity is needed. Our study suggests that providing students with experience of covering non-rectangular region by the rectangular units through hands-on activity could make students deal with the strategy to measurement. Splitting unit is one of the way we found in our study invented by the students when they cover the non-rectangular region. Then, they will apply joining splitted units while counting the total number of units used while covering the region. Mini lesson on covering oval cookie with the square chochips can be one way to address the aim.

- Measuring the area of non-rectangular region by the use of rectangular units

At this phase, the students will apply the strategy they invented in measuring the area of non-rectangular region by the use of rectangular units. Finding the area of oval table on tiled floor can be way to conduct. The students will apply the strategy of splitting and joining the tiles to get the area of oval table possibly by estimation.

- Discussion on the use of interior partial array to quickly count the number of units

The need to overcome the enormous number of units while measuring the area of region becomes important. Previously, the students already grasped the efficient counting strategy on array form. Hence, they will make use of this strategy to find the area of non-rectangular region. While facing non-rectangular region, the students will construct an array form on the interior part of the region and simply apply the multiplication strategy on row-column on it. Altogether with the remaining units are counted to get the total of units representing area. Our study suggests that the use of interior partial array multiplication can be used in oval table on tiled floor context. Discussion on the different construction of interior array can be a good way to bring students to the flexibility of counting units through array form.

- Measuring the regions by the use of square grid

The grid as the precise model of unit is then introduced at this phase. The students will use this grid to measure both rectangular and non-rectangular regions. Some concepts can be recalled such as the inverse relationship and the difference on the result of measurement can be discussed. *'Whose palm is larger?'* can be a way to start the learning. The students traced their palm on the paper and simply attach the transparent grid to derive the area. The students will use all previous concepts they learned when dealing with finding the area of palm.

The answer of our second sub-research question

2. What kinds of understanding of area measurement that students could reach from the learning process?

We noted that the students developed their understanding through our designed learning sequence on area measurement through the area unit. The following explanation reveals what understanding the students developed during each activity portrayed in our hypothetical learning trajectory.

- The initial understanding of area unit issue (overlapping units, gap between units, and leftover); the effect of each issue to the number of units; the idea of relationship between the number of units and the area

While arranging the cashewnuts on top of cookie (activity 1) and on top of two different sized cookies (activity 2), the students observed the different arrangement of nuts revealing during the task. The students themselves then started to recognise the existance of some nuts that overlapped (their term was *bertumpuk*) on one cookie and there was gap (their term was *celah*) between nuts on another cookie. The idea of unit issue was treated by the students as the effect of the number of nuts provided related to the size of cookie on which every 14 nuts were put on. In addition to that, the students started to conclude that the overlapping nuts appeared as the effect that the region being covered is to small for existing nuts, so that nuts overlapped. In other way, the gaps occured since the size of region is to large for given nuts. We concluded that the students grasped the initial understanding of unit issues (overlapping units, gap between units, and leftover) through this cashewnut cookie context.

Furthermore, working with cashewnut cookie particularly in activity 2 part 2 of meeting 1 made the students started to derive the ideal number of nuts used to cover cookie (in which overlapping and gaps already regarded to avoid). Drawing the ideal size for those number of nuts implying that the students started to gain insight into the idea of area as the number of unit. The connection even became stronger when the students tried to derive how large their cookies were (activity 3 meeting 1) by the use of nuts we regarded comprehended by the students from prior activity. They expressed how *'large'* their cookies were in term of number of cookies used. Therefore, we can conclude that students already got insight into the initial connection between unit and area measurement.

- The understanding of unit shape better covers the plane figure and the shape of figure better covered by the units

While covering the cookies with the given toppings and avoiding the overlapping, the students firstly realized that some toppings (M&M chocolate, star chochips, and cashewnuts) remained the gaps. We found the next step was the students tried to investigate why these topping made some gaps. The remarkable discussion was about the star chips (see second conversation page 139). The students said that it got its arms that make it hard to cover fully the surface. We noted that the students started to argue about the structure of the unit to complement their argument about gaps. Hence, they not only talked about whether gaps occured or not, they also argued about the structure of chip. Meanwhile the students could find that square chocochips to be the better shape covering the region. We assume that the students started to realize that square unit was the better covering unit. They also found that rectangular region was the better covered. Therefore, we conclude that the students already understood the unit shape batter covers the region and the shape region better covered through covering activitiy.

- The idea of identical unit and its use

The students already grasped the idea of unit and area. Hence, in the photo frame lesson, the students tried to find the area of frame by the use of photos. Attaching different sized photos forced the students to think on how they were going to express the area of their frame. We noted some group expressed their frame area in way *"how many a and how many b"*, for a and b were different unit. While, another group expressed *"how many a plus how many b"* as like what Nitabach and Lehrer (1996) suggested in the theory of unit consistency. We found later on they realized this mistake. One group already emerged the need of identical unit, so that they just expressed in how many a or how many b. More over, comparing frame triggered the need of identical unit for all the group. It was because all the students felt hard to compare the frame containing both photos. Therefore, they think to just compare in one type of photo. We noted that the students finally derived the idea of comparing the area by the use of identical units. From that moment on, the students had already realized the idea of identical unit not only to measure but also to compare the area.

- The understanding of inverse relationship between the size of unit and the number of units used in area measurement

Having understanding of the use of identical units not only to measure but also to compare area, the students started to realize that the different result by the use different unit occured when they measured the frame. For example, measuring the frame with the larger photos would result smaller area. This triggered the students to think of why such difference could happen. We noted that our focus group students understand that larger photos took more space so that smaller number was obtained. They also argued that using smaller photos would give larger result (see conversation page 153). We saw that the students tried to make connection between the size of units used with the number of units used to cover surface. We interpret that the students made the inverse relation between the unit size and the number of units. This relation was seemingly understood by all the students.

- Generating the efficient way of counting unit by multiplication on rectangular plane figure and understanding why such multiplication works

While seeing the frames very quickly, the students were forced to leave counting one by one. The first strategy the students derived was adding. When they saw a group of photos arranged in one row, the students would think of adding another number of photos in other rows that were not yet attached. The students gradually developed their strategy by multiplying the number of photos in one column for example with the number of column they saw in quick frame. Hence, the students tried to derive multiplication by column. The same happened with muliplication by row. The pupil were then forced to think further when the frame we provided displayed quickly the photos which were arranged in row and column. For some students, the multiplication by row and column was then derived. We noted that these students not only used the multiplication, but they also could elaborate why the multiplication works there. Therefore, we can conclude that the students already generated an efficient way of counting units: multiplication of array (row-column) and understood why multiplication worked.

- Inventing strategy to measure the area of non-rectangular region by the use of rectangular units; the use of grid to measure area of both rectangular and non-rectangular region

We noted that the students' hands on activity on finding the area of oval cookie by the use of square chips triggered students to generate the idea of splitting the unit when covering the edge of oval shape (see picture 5.38). While it turned to determining the area, the students made use the strategy they found while covering the edge of oval shape. They invented the strategy for joining those splitted units. They tried to join again some splitted unit to be counted one. We found that the students also came with estimation in this moment. These strategies were extensively used by the students while they measured the area of oval table with tiles and the area of palm by the use of grid. We can see that the students could manage to find the area of non-rectangular plane by the use of strategy they generated while accomplishing the area of oval cookie task.

- The understanding of the contribution of the use of interrior partial array to figure out the total area of given region

We found that the need to efficiently count the number of tiles on interior part of figure (we called interior units) occured when students tried to find the area of oval table and the palm. They saw that it would be efficient if they applied the multiplication strategy they developed in prior lesson. We saw that the students could construct interior array (see picture 5.44) and applied the multiplication to find the number of units included. Together with the remaining units, the students could obtain the total number of units as the area of plane figure.

The answer of our research question

How can students' understanding of unit of area help them to understand the concept of area measurement?

We noted that understanding the initial issues on unit: overlapping and gap informed the students how to derive the ideal number of nuts to cover the top of cookie. The insight into number of units and the area was thean arisen. The initial area concept was then achieved by the use of unit when students measured the area of cookie with the nuts. This quit informs us that the students started to realize the need of unit to derive area.

Their understanding on the units became more sharpened when they started to realize the need of identical units to measure and to compare the area through the photo frame context. Their flexibility on the area measurement became initiated by the inverse relation they recognized. This give the students the freedom to use different sized shape identically to cover the surface. Both units leading them to the different result was clearly understood.

When the students got their strong vision on the use of unit as area determiner, the students started to realize the efficient way of counting units. Hence, our designed lesson provided them the way to derive it. The understanding of students on unit became enhanced by the concept of array, multiplication by row-column, and how it works. It quite informs us that the students needed to move forward. In addition, working with oval cookie and square chips provided the students with more understanding on how to measure area of non-rectangular by splitting joining unit strategy they even made use in the next sequence of finding the area of non-rectangular surface by the use of grid. Together with multiplication applied in finding the area of interior array, the students could derive the total area. Hence, from this analysis, we can conclude that the students already developed their understanding on the idea of area units. This understanding contributed to the area measurement since they had clear connection between the area and the unit and all the property of units they already learned. Therefore, we can conclude that students' understanding on the unit of area could help the students to understand the area measurement.

2. Local Instruction Theory on Learning Area Measurement

The aim of the present study is to contribute to the development of a local instructional theory for area measurement through area unit for third grade students of elementary school. According to Gravemeijer and Cobb (2006), a local instructional theory consists of conjectures about a possible learning process and possible means of supporting that learning process. The following is the table summarizing the tool and the contextual activity we proposed in instructional design.

Table 6.1: *Local Instruction Theory in learning area measurement through area unit*

Activity	Tool	Imagery	Practice	Concept
Cashewnuts cookie man (group)	Cashewnuts, cookie outline on paper	Signifies the acquisition of unit issue	Putting nuts on top of cookie, Counting, Compare the number of nuts	Overlapping units, gap between unit, leftover and the effect of each when counting unit
Cookie and 14 nuts (group)	14 cashewnuts, two different sized cookie outlines on paper	Signifies acquisition of unit issue and initial connection between the number of unit and area	Putting nuts on top of cookies, observe the arrangement of nuts on both cookie	Overlapping units, gap between unit, leftover . Derive the initial connection between ideal number of unit and size of

				surface
Measure and order the cookies (individual)	Cashewnuts four different shaped cookie outlines,	Signifies the acqusition of unit as area determiner	Putting the nuts on top of cookies, counting the number of nuts, Compare the cookie size	Derive initial unit as area determiner Connection between the number of nuts and area
Cookie and toppings (group)	Two different shaped cookie outlines (rectangular and oval), four toppings: cashewnuts, the replica of square chocochips, star chocochips, and M&M chocolate	Signifies the choice of unit shape to cover	Putting the topping on top of cookie, observe which topping covers the cookie, which surface better covered	Unit appropriate shape to cover surface and the surface shape better covered
Comparing and measuring frame size by the use of photos (group)	Three different shaped frames. Bunch of different sized photos, glue	Signifies the use of unit to measure and compare area	Putting the photos on frame, counting the number of identical photos, compare the area	Unit as area determiner Area comparison by the use of units The identical units The inversee relationship The connection between the number of unit and the area
Mini lesson: Making frame with particular size by the use of photos (group)	Large paper, different sized photos, glue and scissor	Signifies acquisiton of different shape with same area	Make frame, compare the shape of frames with the same area	Different shape of surface can have the same area
Quick image of photo frame (individual)	Five unfinished frame (powerpoint file), LCD/Projector	Signifies the use of multiplication on counting units	Creating quick counting strategy by multiplication on row and column	Multiplication by row and column

Mini lesson: Which carpet is larger? (individual)	Top view of carpets on tiled floor showed with LCD/Projector	Signifies the area measurement and comparison by the use of units	Measure and compare area by applying multiplication	Use multiplication by row column to measure and compare area
Mini lesson: Comparing and measuring the cookie size by the use of square chips (group)	Two outlines of cookie (oval and rectangular cookie), replica of square chocochips, glue and scissor	Signifies the area measurement of rectangular and non-rectangular surface by the use of unit	Hands on activity Measure area, derive the startegy of counting unit, compare the area	Measure the area of non-rectangular area Splitting unit Joining splitted units counting strategy Estimation
Finding the area of oval table on tiled floor(individual)	Picture of top view of oval table on tiled floor	Signifies the area measurement of non-rectangular plane by the use of unit	Measure area of non-rectangular area use countign strategy	Apply oining splitted units counting Estimation
Discussion on the use of partial array to quickly count the tiles in oval table case (large group)	Picture of top view of oval table on tiled floor	Signifies the discussion on construction interior array in outline of surface	Class discussion on the constructing interior array	Use multiplication on interior partial array in counting units
Whose palm is larger? (group)	Paper, pencil to trace the palm, two transparent grids with different size of squares, paper clips, marker	Signifies the area measurement by the use of grid	Measure and compare area	The use of grid as tool to find area of either rectangular plane or non-rectangular plane The inverse relationship

B. Discussion

1. The Weakness Points of Study

Several points became our consideration to be the weaknesses of our present study. Firstly, we noted that the discussion on our designed instructional activities with the teacher did not run effectively. Before conducting the teaching experiment, we shared our designed instructional to the teacher to be read. Then, we had couple weeks to have some discussion about the designed learning with the teacher after she read that. However, we noted there were only three times during that period we had chances to discuss together about the design. In addition to that in every discussion we had, the effectiveness of discussion became an issue since the teacher was in busy time we could not even predict for her teaching schedule and other school-related business. Some important points were even discussed superficially because she needed to do something else. Our discussion run quite shortly. We could not even guarantee that she really understood what we implied in the teacher guide she already read about the instructional activities eventhough she declared that she already understood that when discussion. For example was what happened in the first meeting about cashewnut cookie context (see meeting 1 of cycle 2) where she led the problem into wrong interpretation that made students confused. Hence, from that point on, we tried to become more engaged in the learning process by helping teacher whenever she asked something she forgot and guided her during the class for consecutive next meetings.

In addition to the teacher factor, secondly we noted that the teacher's role gradually decreased in three last meetings. In meeting 4 and 6, the teacher did not fully take her role as the teacher. She rather was observer and observed students' work during the session and some moments she orchestrated the students in small group to have discussion. It was because she was suddenly not really in a good health that moment, so she asked us to handle some roles. As example, we took her role when quick frame lesson. Meanwhile, in the meeting 5, she had a must-handled business outside the school, so that we at the moment were informed to take her role in the class. Regarding the schedule we already arranged together with school and teacher which was considered very tight, therefore, we just conducted these meetings.

Thirdly, in our study, we did not take students' learning style (audio, visual, kinestetic) into consideration while designing hypothetical learning trajectory. We noted that this factor takes an important role in the learning process. We found that there were students that could easily be helped to learn particular concept by visual. For example, by the use of presentation slides displaying with a projector, some students could easily grasp an understanding why multiplication works on determining the total number of units to derive area. We also found that the use of this visual can make some of the students learn easily compared to whenever they learn without the slides. It quite informed us that the learning style has significant effect to students' learning process. Then, the consideration on students' learning style can be very matter in designing hypothetical learning trajectory.

Another important aspect is the choice of context to support students learn particular concept. Our study suggests that the use of cashewnut cookie context could promote the students to learn the unit issues: overlapping units, gap between unit, and leftover; the initial connection between the unit and the area; and the unit as the area determiner. However, there is still discussion and doubt whether the cookie with topping full of cashewnut context can be a real and close enough to students' daily life. Some third persons' opinion imply that this kind of cookie with topping full of nuts is rarely found in everyday life, especially in Indonesia kids' environment. The common cookie with topping usually consists of few nuts. Otherwise, a context like *kempyang* or *tempeyek* full of nuts therefore is more common and preferable to the students. Hence, a consideration to make the context become more real and closer to the students becomes a potential issue in next study.

We had some of activities in last meeting which were not conducted due to time issue. The last meeting of our study was also the last mathematics class before the students had a week off. Unfortunately, in the last meeting, we only had short time because the mathematics class started quite lately. Hence, we run out of time to conduct two remaining activities. Moreover, we did not yet any chance to continue those activities. Eventhough leaving out these two activities did not give significant effect to our students' learning process; we noted that these two activities were still potential to bring students' understanding further than they already had recently.

Then, we realize that the explanation on students' learning behaviour was not further elaborated in our study. For example, we could not give deeper explanation on how students really worked and cooperated during the lesson in their small group. There was a limitation on doing this. We also did not provide any information and explanation that actually the students in our class got used to work in pair rather than small group consisting more than two students. In addition, we did not provide any further analysis on the possible effect of this group setting.

Lastly, we also take consideration on the number of cycle in this study, since we limited our study into two cycle. It would be better if the cyclical process is continued to obtain better refined hypothetical learning trajectory.

2. Reflection on the Important Issues

We reflect some important points in our study relating to preparation and during the implementation described as follow.

a. The Thorough Discussion on the Instructional Activities with the Teacher

We found it very important to have intense discussion with the teacher about the instructional activites designed to be implemented in teaching experiment. We note that lack of the discussion made the teacher did not really master every role she was supposed to perform while in teaching experiment. Including to that was mastering what exactly the task the students try to accomplish and how to guide them. We believe, that good discussion will keep both researcher and teacher on the track of learning design.

b. Teacher and Observer Role

We noted very clearly in the preparation that the teacher will be the the one whose roles are to teach, to guide just like usual but instead based on the guidance. The researcher will be the observer and sometime be the person who come to small group especially focus group to derive interview to clarify what students are thinking about. This is due to teacher's limitation on orgainizing the class. Moreover, sometimes we found teacher forgets particular role or actions on certain occasion during the learning. We found it quite normal. Therefore, the role of the researcher as observer could be as the reminder of what to do for the teacher. For particular degree, the observer can be the partner of teacher during the lesson. Hence, a good cooperation between teacher and researcher could be the way to manage the class. However, we keep minimize the intervention by the observer in order not to play major role possessed by the mathematics teacher.

c. The Students' Observation on Appropriate Shape of Unit to Cover the Region

We think that giving the students freedom to explore the shape of unit better covering the surface becomes important to derive. This will give possiblity to other shape but square treated as approprite unit shape to cover surface. For example: rectangle, hexagonal, triangle, L shape, etc. It is because we found that after students could bring idea of gap as the source of their resoning, the students could gradually move to reason about the structure of unit shape that make it able to cover surface.

d. Making Own Square Grid to Derive Area

The students of teaching experiment did not yet experience making their own grid at the last meeting we had design. The rational of conducting this activity was students could build the structured array or grid used to derive area. We noted that missing this activity did not really have large impact to our students' learning process generaly for some students. However, few students we found in posttest (making own grid used as the last question to test all students' understanding eventhough they did not yet learn it) still struggled to construct structured array. Some of them even drew unstructured squares with different sizes possible and the structure was quite messy. This was what delivered them to the wrong acquisition of area of given plane.

C. Recommendation for Further Research

In this study, we focus on the area measurement through area unit understanding. Some contexts were used to support students' concept acquisition on the area unit relating to area measurement. We derive some recommendations for further research focusing on the area measurement through area unit. Generally there are two kinds of recommendation. The first one is related to teacher-observer cooperation. The second one is more about the designed instructional activities.

For the first recommendation, if it is possible, is try to give more space and time to thoroughly discuss the instructional activities with the teacher before conducting the experiment. This is important to make the teacher to understand her role and how to manage the learning in the study. However, this is not closing the opportunity for teacher and observer to work together especially when the teacher forget and misses occasional part during the learning process. Hence, the observer could be reminder as well as the guide.

The second recommendation is about the designed task. The discussion of context of choice is something important in designing process. Then, the discussion on whether the context is close and real enough for the students with the authorized person can be very helpful to generate more helpful context. In our study, we also found that making the task about finding the appropriate unit shape more open can trigger the possibility for students observing their own appropriate shape unit to cover surface. This is quite important since we conjecture that finding themsleves will expand students' vision on various shape of units possible to cover the surface. They will enrich their knowledge. Moreover, we recommend to conduct the design hypothetical learning trajectory which regards the students' learning style (audio, visual, and kinesthetic). We found that regarding these students' learning trajectory can help the students more in grasping particular concept in learning process. Another recommendation is the activity of making own grid to derive area of plane. We found this important since for some particular students, making structured grid or array is still problematic for them. Hence, the learning that adresses this issue becomes very important and revealing. Lastly, we consider that our students did not yet have very formal and abstract level of mathematics on area measurement since their recent knowledge implies the use of grid to derive area. Hence, the further learning to provoke students' more formal level is quite demanded.

References

Badan Standar Nasional Pendidikan (BSNP). (2006). *Standar kompetensi dan kompetensi dasar SD/MI in standar isi untuk satuan pendidikan dasar menegah*. Jakarta

Bakker, A. (2004). *Design research in statistic education*. On symbolizing and computer tools. Amersfoort: Wilco Press

Barret, J.E et al. (2011). *Children's unit concepts in measurement: a teaching experiment spanning grades 2 through 5*. ZDM Mathematics Education 43: 637-650. USA

Baturo, A., & Nason, R. (1996). *Students teacher's subject matter knowledge within the domain of area measurement.* Educational Studies in Mathematics, 31: 235-68

Cobb, P., Confrey, J., diSessa, A.A., Lehrer, R. & Schauble, L. (2003). *Design experimentsin educational research*. *Educational Researcher*, *32*, 9-13.

de Lange, Jan. (1996). *Using and applying mathematics in education*. In A.J. Bishop et al. (Eds.), International Handbook of Mathematics Education, 49-97. The Netherlands: Kluwer Academic Publishers

Doorman, L.M. (2005). *Modelling motion: from trace graphs to instantaneous change.* Dissertation. Utrecht University

Gravemeijer, K.P.E. (1994). *Developing realistic mathematics education*. Utrecht: CD-βPress.

Gravemeijer, K., Bowers, J. & Stephan, M. (2003). *A hypothetical learning trajectory onmeasurement and flexible arithmetic.* In: M. Stephan, J. Bowers, P. Cobb & K. Gravemeijer (Eds.), Supporting students' development of measuring conceptions: Analyzing students' learning in social context. *Journal for Research in Mathematics Education Monograph,12*, 51-66

Gravemeijer, K.P.E & Cobb, P. (2006). *Design research from a learning design perspective.* In: Van Den Akker, j., Gravemeijer, K.P.E McKenney, S. & Nieveen, N (Eds.). Educational Design Research. New York: Routledge

Haris, D. 2011. *The role of context in third graders' learning of area measurement.* IndoMS. J.M.E, pp. 55-66

Jacob, H.R. (1974). *Geometry*. United State of America

Konya, E.H., & Tarcsi, M. (2010). *The role of tiling , cutting, and rearranging in the formation of the concept of area.* Hungary

Lehrer, R. 2003. *Developing understanding of measurement*. Chapter 12: 179-190. Vanderbilt University

Nitabach, E., & Lehrer, R. (1996). Developing spatial sense through area measurement. *Teaching Children Mathematics*, 2, 473-476

Putra, Z.H. 2011. *Design research on addition: developing mental calculation strategies on addition up to 20.* Thesis. Sriwijaya University

Reynolds, A., & Wheatley, G.H. (1996). *Elementary students' construction and coordination of units in an area*. Journal for Research in Mathematics Education, 27(5), 564-581

Simon, M.A. (1995). *Reconstructing mathematics pedagogy from a constructivist perspective. Journal for Research in Mathematics Education, 26*(2), 114-145.

Stephan, M., & Clement, D.H. (2003). *Linear and area measurement in prekindegarten to grade 2.*

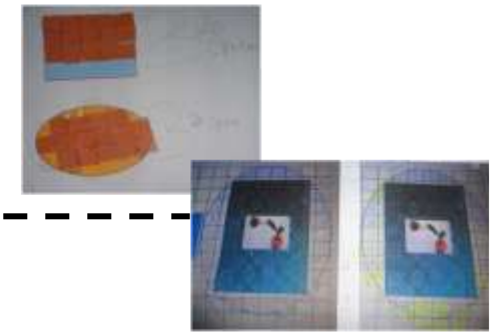
Appendix A
The Visualization of final HLT

Lesson 6 (grid)
1. Whose palm is larger? (group)

→ **The use of grid to measure and compare area of region**

Lesson 5
1. Mini lesson: Comparing and measuring the cookie size by the use of square chips
2. Finding the area of oval table on tiled floor
3. Discussion on the use of interior partial array to quickly count the tiles in oval table case

→ **Measuring non-rectangular region by the use of rectangular units, splitting and joining unit strategy, the use of inner partial array to efficiently counting units**



Lesson 4
1. Quick image of photo frame
2. Mini lesson: Which carpet is larger?

→ **Efficient way of counting unit (mutiplication), and apply it in comparing area by unit**

Lesson 3
1. Comparing and measuring frame size by the use of photos
2. Mini lesson: Making frame with particular size by the use of photos

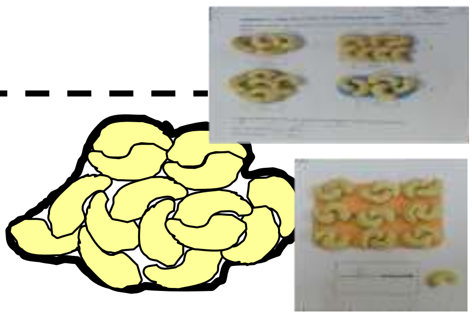
→ **Identical units idea to measuring and comparing area, invers relationship, unit-area connection**

Lesson 2
Cookie and toppings

→ **Appropriate shape of unit to cover region**

Lesson 1
1. Cashewnuts cookie man
2. Nuts and 14 nuts and making own cookie
3. How many nuts puts, order the cookie

→ **Area unit issues (overlapping, gap, leftover), unit-area connection,**



AppendixB

The Teacher's Interview Scheme

1. **The culture of the classroom**
   - What is the rule when teacher explain?
   - Is the any rule when students are given time to speak?
   - How far is the teacher's concern about the time given to students to think, to talk, to discuss?
   - How far the teacher's concern on anticipating different habitual things of students?
   - How does the teacher deal with the noisy situation?
   - Does the teacher apply particular punishment?

2. **The students' ability**
   - What is the spread of the students' ability or proficiency in mathematics?
   - Whether the spread is at average, or too many slow learners, or fast learners?
   - How does the teacher treat different students with different ability?
   - How does the teacher create group of discussion?
   - How much care is given to the slow learners?
   - How does the teacher manage the fast students in the class?

3. **The teacher's approach on teaching**
   - How does the teacher approach particular lesson in mathematics class?
   - Is demonstrating knowledge pretty common?
   - Does the teacher often to give problem?
   - Does the teacher often set the students do the task by drilling technique?
   - How open is the teacher about the students' question that might appear when teaching?
   - How much is the teacher's concern on time proportion between explaining, setting the time for students discussion?

4. **The possible ethnomatematics teacher probably does**
   - How far is the teacher's view towards different kind of solution towards particular problem?
   - How does she usually manage the students' possible different solution?
   - How does she set the students come up with certain solution?

AppendixC
The Classroom Observation Scheme

1. **The mathematics classroom culture and social culture of the classroom, the interaction between teacher and students, and amongs students themselves**
   - How far is the students' willing to face the learning?
   - How is the students' approach when they give their opinion?
   - Whether students are mostly silent or actively talkers?
   - How does the teacher manage with the passive or dominant students?
   - How is the rule developed when teacher explaining something? Whether the students are allowed to stop the teacher talking and give their opinion? Or they may raise their finger or something else?
   - How much discussion is set up?
   - What is the technique used by the teacher when asking something?
   - How does the teacher give space for students to talk their idea?
   - Whether teacher reveals some action like laughing, smiling, hand moving, when reacting to students' idea or joke or when explaining something?
   - How much is the willingness appeared when the teacher explain what students do not understand yet?
   - How close are students to the teacher in classroom setting?
   - Whether the students look enjoying the moment, or look stress?

2. **The teacher's activity**
   - How does the teacher open the class? Whether she directly gives problem? Is it usual? Or demonstrate knowledge/lesson and let the students do some task?
   - What is her dominant role in class discussion?
   - Is there any effort to generate different solution?
   - How is the teacher end the class?
   - How does the teacher manage the discussion? Does she move around while students discussing? Does she do asking to groups to check their work?

3. **The students' activity**
   - What do students do when their teacher explains?
   - What students do while discussing particular thing in group?
   - How they explain their strategy to other group member?
   - Is there any students acting bossy?
   - Is there any silent pupil in the group?
   - What do students write? read? talk?

4. Practical organization of the class

- How is it look like the arrangement of students seats?
- Is there any reason from the teacher to decide such seat arrangement? Why?

5.About pendidikan matematika realistik approach in mathematica learning?

- Do the teacher use real problem?
- Do the teacher use model?
- How much the teacher apply the tenets in PMRI?

AppendixD

Teacher Guide

Rencana Pelaksanaan Pembelajaran

Subjek : Matematika
Kelas : III
Semester : 2
Alokasi waktu : 2 x 35 menit
Jumlah siswa : 15 orang
Pertemuan : 1

A. Standar Kompetensi :

Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

B. Kompetensi Dasar :

Memahami konsep pengukuran luas bidang datar melalui satuan unit luas

C. Tujuan Pembelajaran :

1) Siswa memahami dampak dari tumpukan (overlapping) antar unit luas, permukaan yang tidak tertutupi (gap dan leftover) dan ukuran unit terhadap jumlah unit yang dibutuhkan dalam pengukuran luas melalui konteks kue kacang mede.
2) Siswa menginvestigasi jumlah unit yang dibutuhkan untuk sebuah bidang datar/permukaan melalui konteks kue kacang mede.
3) Siswa mengkonstruksi hubungan antara jumlah unit yang dibutuhkan dan ukuran besarnya permukaan (bakal luas) melalui konteks kue kacang mede.

D. Indikator Pembelajaran :

Di bawah kondisi membandingkan jumlah kacang mede yang digunakan kelompok siswa dalam menutupi permukaan kue, maka

1) Siswa menginvestigasi dan menyimpulkan hubungan antara ukuran kacang mede yang digunakan terhadap jumlah kacang yang dibutuhkan untuk menutupi permukaan atas kue.
2) Siswa menginvestigasi dan menyimpulkan hubungan antara susunan kacang mede yang bertumpukan di atas permukaan atas kue terhadap jumlah kacang mede yang diperlukan.
3) Siswa menginvestigasi dan menyimpulkan hubungan antara susunan kacang mede yang tidak menutupi sebagian permukaan atas kue jumlah kacang mede yang diperlukan.

Dengan memahami ide ukuran kacang serta menghindari tumpukan antar kacang dan permukaan yang tidak ditutupi kacang,

4) Siswa membuat luasan permukaan kue dengan kacang yang diberikan
5) Siswa mendiskusikan berbagai macam bentuk permukaan atas kue dengan jumlah kacang yang diberikan
6) Siswa membandingkan ukuran (bakal luas) kue-kue yang dibentuk

7) Siswa dapat menentukan banyaknya kacang yang dibutuhkan untuk menutupi permukaan kue
8) Siswa mengkonstruksi hubungan antara banyaknya kacang yang dibutuhkan dan luasan kue yang terbentuk

E. Pengetahuan awal siswa :

Siswa dapat mengetahui secara visual konsep awal luas melalui besar atau kecilnya permukaan, membandingkan permukaan mana yang lebih besar atau mana yang lebih kecil namun belum mengetahui bagaimana melakukan pengukuran luas (berdasarkan hasil pretest). Sebagian siswa berpotensi menggunakan penggaris dalam mengukur, dengan demikian hindari siswa melakukan pengukuran luas dengan penggaris ini untuk menghindari miskonsepsi tentang konsep luas sedari awal.

F. Pendekatan : PMRI
G. Metode : Hands on activity, diskusi dan mengerjakan LKS
H. Alat dan bahan: Lembar Kerja Siswa (LKS), kacang mede, gambar kue, LCD

I. Kegiatan Pembelajaran

Kue kacang mede

Pendahuluan

1) Pengenalan konteks – *Kue kacang mede (3 menit)*

Tunjukkan gambar top view kue kepada siswa dan mulailah percakapan dengan menanyakan topping apa yang biasa berada di atas kue. Beberapa siswa mungkin menyebutkan selai, cokelat dan krim, namun fokuskan kepada topping yang dapat dihitung, misalnya kacang dan cokelat cips. Tanyakan kepada siswa apa mereka pernah melihat kue dengan topping penuh dengan kacang?

Kegiatan Inti

2) Kegiatan 1 – *Pembuat kue kacang mede (kelompok - 10 menit)*
Ceritakan kepada siswa tentang pembuat kue kacang mede.

Seorang pembuat kue akan membuat kue penuh dengan kacang mede di atas permukaan kuenya (tunjukkan bentuk outline kuenya). Karena kacang mede tergolong mahal di pasaran, si pembuat kue tidak ingin untuk sejumlah kuenya, banyak kacang mede yang berlebih.

Mulailah tanya jawab dengan siswa mengenai apa yang akan dilakukan si pembuat kue itu

- Apa yang dilakukan si pembuat kue menurutmu?
  Arahkan jawaban siswa kepada ide estimasi jumlah kacang per kue
- Bagaimana cara si pembuat kue menemukan jumlah kacang yang diperlukannya?
  Arahkan jawaban siswa kalau total kacang yang diperlukan adalah jumlah per kue dikalikan banyak kue yang dibuat

Hal ini dimaksudkan agar siswa mengerti atau paham permasalahannya dan membuat siswa menyelami konteks kue kacang mede. Kemudian tekankan permasalahan berikut karena masalah ini adalah inti dari kegiatan pertama ini

Dapatkah kamu membantu menemukan banyak kacang yang dibutuhkan oleh pembuat kue di sebuah kuenya?

Kemudian berikan setiap kelompok lembar LKS (untuk setiap anggota) dan lembar bergambar kue beserta kacang medenya. **Situasikan siswa agar tidak melihat kerja kelompok lain agar tidak mempengaruhi susunan kacang mereka!**

<u>Beberapa kemungkinan kerja siswa</u>

1. Menumpukkan langsung kacangnya di atas outline kue dan membuyarkan lagi untuk dihitung. Kemungkinan ini berpotensi untuk ide kacang yang bertumpukkan (overlapping units)
2. Meletakkan satu-satu kacang medenya di atas kue sambil dihitung. Kemungkinan ini berpotensi untuk ide gap dan leftover (permukaan yang tak tertutupi)

Jumlah kacangnya ditulis dalam kotak yang tersedia. Minta semua kelompok beredar melihat hasil kerja kelompok lain. Kemudian lakukan diskusi kelas

3) Diskusi kelas *(20 menit)*

Diskusi berpusat pada investigasi mengapa jumlah kacang tiap kelompok berbeda.Beberapa sasaran amatan yaitu : *perbedaan ukuran kacang, kacang bertumpuk, dan bagian yang belum tertutup kacang (gap dan overlap)*

<u>Beberapa alternatif untuk memandu diskusi kelas</u>

- Jika kedua kemungkinan kerja siswa muncul, maka perbedaan jumlah kacang semakin besar, artinya perbedaan semakin dapat diamati.
  Perbedaan yang mencolok yang pertama adalah ukuran kacang, maka pandu siswa melihat efek beda ukuran ini terhadap jumlah kacangnya. Jika belum ada siswa yang berargumen, maka tanyakanlah *"bagaimana menurutmu ukuran kacang pada kue ini?", "bagaimana kira-kira akibatnya?"*
  Perbedaan mencolok kedua adalah susunan kacangnya. Mungkin ada siswa yang merespon kalau susunannya 'rapi', maka terima pendapat ini tapi jangan terlalu ditekankan. Susunan yang dimaksud adalah 'bertumpuk atau tidak', 'ada yang bercelah' atau tidak. Minta mereka menarik kesimpulan mengenai akibat susunan ini terhadap jumlah kacangnya. Salah satu pertanyaan *"Dapatkah kamu melihat perbedaan susunan kacang di kue-kue ini?"*
- Jika kemungkinan 1 tidak muncul, dengan kata lain hanya kemungkinan 2 saja yang muncul, maka perbedaan mungkin tidak terlalu dapat diamati. Mengenai ukuran kacang mungkin masih bisa di amati dampaknya. Namun, jika belum ada siswa yang mampu mengobservasi tentang susunan kacang (bertumpuk atau ada celah/bagian yang tak tertutupi) jangan dipaksakan karena mereka akan terbantu untuk melihatnya

di kegiatan 2. Tapi ini tidak tertutup kemungkinan jika ada siswa yang bisa berargumen, maka diskusikanlah.

Sasaran amatan siswa	Argumen siswa
Ukuran kacang	Jika ukuran kacangnya kecil-kecil, maka jumlah kacangnya jadi banyak. Jika ukuran kacangnya besar-besar, maka jumlah kacangnya sedikit. Kacang kue ini lebih banyak karena kacang kecilnya lebih banyak dll
Kacang bertumpuk	Kalau kacangnya bertumpuk, maka kacangnya lebih banyak, karena kacang lain diletakkan di atas kacang lain.
Celah atau bagian yang tak tertutupi kacang (gap dan leftover)	Makin banyak celah atau bagian yang tak tertutupi maka jumlah kacang di kue jadi sedikit.

Akhiri kegiatan 1 dengan pertanyaan *"Jadi bagaimana menurutmu jumlah kacang si pembuat kue?", "Apakah kamu menggunakan jumlah kacang kue yang kacangnya bertumpuk atau kue yang ada bagiannya tidak tertutup?"*. Arahkan siswa untuk melihat kue dengan kacang bertumpuk akan membuat kacang semakin banyak, si pembuat kue akan makin banyak membeli. Kalau kue dengan bagian yang tak tertutup jelas di luar kemauan si pembuat kue (permukaan kue penuh dengan kacang). Jadi arahkan siswa untuk menghindari tumpukan dan atau gap/leftover. Minta mereka memperbaiki susunan dan menghitung lagi.

4) Kegiatan 2 – *kue dengan 12 kacang mede (kelompok) (25 menit)*

Beri setiap kelompok lembar LKS kedua (bergambar dua kue berbeda ukuran, kue A kecil dan B besar) dan 12 kacang. Siswa melakukan observasi pada kegiatan 2.

Sasaran amatan : ***hubungan antara jumlah kacang dan ukuran kue; ide tumpukan dan gap / leftover***

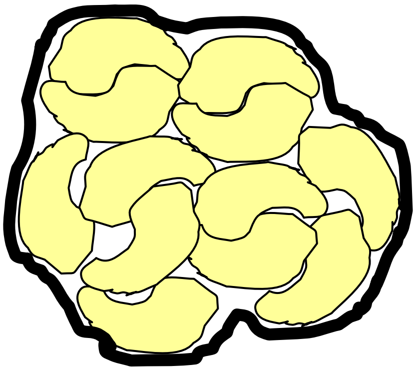
Kue	Argumen siswa
A (kecil)	Kacang menutupi permukaan kue dan bertumpuk
B (besar)	Kacang tidak seluruhnya menutupi permukaan kue (bisa jadi ada celah atau leftover) tidak ada yg bertumpuk

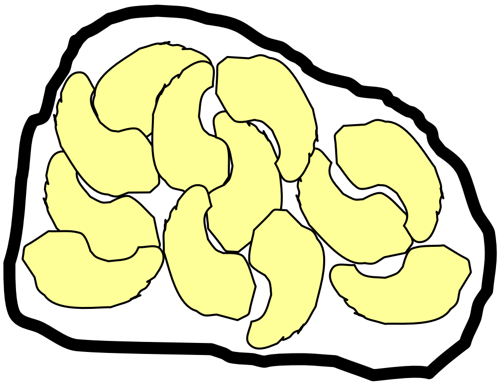
Arahkan siswa untuk melihat hubungan antara ukuran kue yang diberikan dengan jumlah kacang yang diberikan. Tanyakan *"bagaimana menurutmu ukuran kue untuk 12 kacang yang diberikan?"*

Kue	Argumen siswa
A (kecil)	Kue A kekecilan untuk 12 kacang yang diberikan, jadi kacang harus dikurang biar ga numpuk
B (besar)	Kue B kebesaran untuk 12 kacang yang diperlukan, jadi harus ditambah kacangnya

Lanjutkan dengan pertanyaan penting *"Jadi bagaimana seharusnya ukuran kue untuk 12 kacang tersebut?".* Kemudian minta siswa menggambar sendiri kuenya untuk 12 kacang yang diberikan. **Minta siswa untuk tidak menggunakan penggaris !!**

Sebelumnya tanyakan *"Apakah kacang bertumpukan dan bagian yang tak tertutupi diperbolehkan ketika kalian membuat kue dengan 12 kacangnya?".* Berikut beberapa kemungkinan gambar kue siswa





Minta setiap kelompok beredar melihat hasil kerja kelompok lain. Kemudian lakukan diskusi kelas.

Sasaran diskusi : *Membandingkan ukuran kue dengan bentuk berbeda-beda masing-masing kelompok dengan 12 kacang yang diberikan.*

Awali dengan pertanyaan *"Bagaimana bentuk kue yang dibuat oleh semua kelompok? Bedakah?".* Tanyakan dimana letak bedanya. Mungkin siswa akan langsung merujuk ke bentuknya. Arahkan siswa melihat perbedaan susunannya kemudian apakah masih ada celah atau bagian yang tak tertutupi yang muncul. Kemudian tanyakan *"Dengan 12 kacang ini apakah ukuran semua kue ini sama?".* Siswa mungkin berargumen kalau kue kelompok ini lebih besar karena banyak bagian yang tidak tertutupi. Catat ide penting ini.

Kemudian tanyakan *"Bagaimana caramu agar kue dengan 12 kacang ini ukurannya menjadi sama?".* Hal ini dimaksud agar siswa dapat meminimalisir bagian yang tidak tertutupi hingga ukuran kue yang tadinya agak besar jadi mengecil sedikit dan mendekati ukuran kue lainnya.Siswa mungkin berargumen kalau susunan kacangnya harus sama, dan bentuk kuenya juga sama. Arahkan siswa juga untuk meminimalisir bagian yang tidak ditutupi kacang walaupun susunan kacang/bentuk kue setiap kelompok beda.

5) Kegiatan 3 – *Ukur dan urutkan kuenya dengan kacang (individu) (10 menit)*

Beri lembar tugas individu dan kacang untuk setiap siswa. Minta siswa menemukan besar kue dengan menggunakan kacang. Kemudian minta mereka mengurutkannya berdasarkan jumlah kacangnya.

Kemudian akhiri kegiatan dengan diskusi singkat mengenai ukuran kue dengan kacang serta hubungan ukuran kue dengan banyaknya kacang yang diperlukan.

Argumen siswa *"kue ini besarnya 5 kacang"* misalnya atau "*kue ini butuh 5 kacang"* adalah argumen yang diharapakan kepada siswa untuk dapat disuarakan. Maka tanyakanlah *"berapa besarnya kue ini?"*. Kemudian diskusikan mengenai urutan kue berdasarkan jumlah kacang yang mereka gunakan.

Penutup

6) Minta siswa menyimpulkan inti materi pelajaran (2 menit)

Sasaran kesimpulan : Mengenai apakah boleh kacang bertumpuk atau bagian yang tidak tertutupi ketika mengukur besar kue; dan juga mengenai hubungan antara banyak kacang dan ukuran kue.

Rencana Pelaksanaan Pembelajaran

Subjek : Matematika
Kelas : III
Semester : 2
Alokasi waktu : 2 x 35 menit
Jumlah siswa : 15 orang
Pertemuan : 2

A. Standar Kompetensi :

Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

B. Kompetensi Dasar :

Memahami konsep pengukuran luas bidang datar melalui satuan unit luas

C. Tujuan Pembelajaran :

1) Siswa menginvestigasi bentuk unit yang dapat menutupi permukaan melalui konteks kue dan taburannya
2) Siswa mendiskusikan apakah bentuk petak atau non-petakan yang merupakan bentuk unit yang lebih baik untuk menutupi permukaan melalui konteks kue dan taburannya
3) Siswa memahami bahwa bentuk unit petak dapat menjadi bentuk unit terbaik untuk menutupi permukaan tanpa celah melalui konteks kue dan taburannya
4) Siswa menginvestigasi bentuk permukaan yang dapat ditutupi seluruhnya oleh unit petak melalui konteks kue dan taburannya

D. Indikator Pembelajaran :

1) Siswa dapat menyusun taburan tanpa bertumpukan
2) Siswa mendiskusikan bentuk taburan mana yang dapat menutupi permukaan kedua kue tanpa celah
3) Siswa dapat menemukan bahwa cip petak dapat menutupi permukaan kue oval namun meninggalkan bagian yang tidak tertutupi di pinggirannya sedangkan cip petak dapat menutupi keseluruhan permukaan kue petak
4) Siswa menyimpulkan bahwa kacang, cip bintang, dan M&M cokelat tidak menutupi keseluruhan permukaan kue karena terdapat celah diantaranya kecuali kalau taburan tersebut ditumpukkan.
5) Siswa menyimpulkan bahwa kue petak adalah bentuk kue yang dapat ditutupi dengan baik oleh cokelat petak. Kue oval tidak dapat ditutupi oleh keempat taburan dengan baik

E. Pengetahuan awal siswa :

Siswa sudah mempelajari ide tentang tumpukan antar unit, celah antar unit dan bagian yang tidak tertutupi pada sebuah permukaan dalam konteks menutupi permukaan kue dengan kacang mede. Siswa juga menyelidiki dampak dari susunan tersebut terhadap jumlah kacang yang dibutuhkan untuk menutupi permukaan kue. Siswa juga sudah menemukan besar kue dengan mengukur menggunakan kacang (tanpa overlap dan gap yang minim). Dalam pertemuan ini mereka akan menggunakan beberapa ide ini untuk menginvestigasi bentuk taburan/topping mana yang menutupi permukaan kue dengan baik (tanpa celah) dan bentuk kue mana yang tertutupi dengan baik pula (tanpa ada bagian yang belum tertutupi) oleh taburan tersebut.

F. Pendekatan : PMRI
G. Metode : Hands on activity, diskusi dan mengerjakan LKS
H. Alat dan bahan : Lembar Kerja Siswa (LKS), gambar kue dan taburan (kacang mede, tiruan cokelat cip bintang, tiruan M&M cokelat, dan tiruan cokelat cip petak), lem.

I. Kegiatan Pembelajaran

Pendahuluan

1) Pengenalan konteks – Kue dan taburannya

Mulai pelajaran dengan cerita tentang kue dan taburan. Untuk menarik perhatian mereka, tanyakan beberapa contoh taburan kue yang pernah mereka lihat. Bicarakan masalah susunan taburan tersebut. Ingatkan kembali mereka pada ide tumpukan antar unit dan celah atau bagian yang tak tertutupi. Kemudian, siswa diperkenalkan dengan empat macam bentuk taburan yang biasa digunakan untuk kue; yaitu kacang mede, M&M cokelat, cokelat cip petak dan cokelat cip bintang.Tanyakan apa mereka pernah melihatnya.

Kegiatan Inti

2) Menutupi permukaan kue dengan taburan

Berikan siswa masalah berikut

Pembuat kue akan membuat dua jenis kue, kue oval dan kue petak. Ia akan menutupi permukaan kue tersebut dengan empat taburan: kacang mede, cokelat cip bintang, M&M cokelat, dan cokelat cip petak tanpa bertumpuk. Dapatkah kamu membantunya untuk meletakkan taburan tersebut?

Kemudian temukan
1) Taburan mana yang dapat menutupi dengan baik permukaan kuenya?
2) Kue mana yang dapat tertutupi semuanya oleh taburan tersebut?
Setiap kelompok diberikan lembaran berisi gambar dua jenis kue, oval dan petak dan empat jenis taburan. Mereka juga diberi lem untuk menempelkan taburan tersebut

Catatan :

- Minta siswa untuk tidak langsung memberi lem lantas menempelkan ke kuepada taburannya. Mintalah mereka awalnya untuk menyusun taburan tersebut. Mendiskusikan dengan anggota kelompok mengenai susunan taburannya. jika mereka sudah yakin tentang susunannya, barulah diberi lem.
- Minta siswa untuk menghindari taburan yang menumpuk. Beri mereka pengertian bahwa dalam tugas ini, taburan tidak diperbolehkan menumpuk satu sama lain.
- Minta siswa untuk bekerja fokus agar tidak banyak waktu yang terlewati

Selama siswa bekerja dalam kelompoknya, beberapa kemungkinan susunan berikut dapat terjadi

- Siswa menyusun kacang mede dan celah serta bagian tak tertutupi di pinggiran ke dua kue muncul
- Siswa mungkin saja menyusun taburan sehingga keluar melewati outline kue. Minta siswa susun ulang dan didiskusikan lagi dengan anggota kelompoknya. Ingatkan siswa

untuk menghindari taburan yang menumpuk melalui pertanyaan *"apa boleh taburan ini menumpuk?*

Setelah mereka selesai menempelkan taburan di atas kue, minta mereka melakukan pengamatan kemudian mengerjakan LKS. LKS dikerjakan secara individual namun siswa diminta sebelumnya berdiskusi untuk menanggapi soal di LKS.

3) Diskusi kelas

Pajangkan hasil kerja kelompok di depan kelas dan mulailah diskusi. Beberapa aspek diskusi : *bentuk taburan mana yang dapat menutupi permukaan kue, dan bentuk kue yang mana yang dapat ditutupi dengan baik oleh taburan itu*

Diskusi kelas memegang peranan penting dalam pemebelajaran ini. Jadi, kelola diskusi dengan baik dan benar

<u>Beberapa alternatif untuk memandu diskusi kelas</u>

- Diskusikan per taburan. Jadi, diskusikan dulu apa kacang dapat menutupi permukaan kue oval dan petak dan seterusnya.
- Diskusikan per kue. Jadi, diskusikan apa empat taburan tersebut dapat menutupi permukaan kue oval. Kemudian baru dilanjutkan dengan kue petak.
- Minta siswa berargumentasi untuk setiap poin diskusi. Jangan berhenti jika siswa hanya berargumentasi *"Cip bintang tidak dapat menutupi permukaan kedua kue seluruhnya karena ada celah".*
  Gali ide ini lebih dalam dengan menanyakan "*Kenapa menurutmu susunan cip bintang menyebabkan celah?".* Mungkin siswa dapat berargumentasi karena bentuknya yang ada lengan-lengannya, sehingga bagian antar lengan tidak dapat ditutupi dengan baik oleh lengan cip bintang lainnya.

Argumentasi sasaran

Taburan tertentu (misal bintang) tidak dapat menutupi dengan baik tidak hanya karena antar cip ada celah, namun juga mendiskusikan mengapa celah itu bisa terbentuk. Dengan kata lain, bentuk atau struktur cip menjadi bahan pertimbangan.

<u>Beberapa kemungkinan argumentasi siswa</u>

Taburan	Argumen
Kacang mede	Selalu ada celah diantara kacang dan ada bagian tak tertutupi di pinggir kue oval. Karena bentuk kacang tidak memungkinkan untuk menutup erat satu sama lain
	Selalu ada celah diantara kacang dan ada bagian tak tertutupi di pinggir kue petak. Karena bentuk kacang tidak memungkinkan untuk menutup erat satu sama lain
Cokelat cip bintang	Selalu ada celah dan bagian yang tak tertutupi pada kue oval karena bintang punya lengan-lengan bagian di antara lengan tak dapat ditutupi baik oleh lengan binta lainya
	Selalu ada celah dan bagian yang tak tertutupi pada kue petak karena binta punya lengan-lengan bagian diantara lengan tak dapat ditutupi baik oleh lengan bintang lainya
Cokelat M&M	Bentuk bulat cip ini akan selalu ada celah di antaranya jadi tidak dapat menutupi permukaan kue oval secara keseluruhan

	Bentuk bulat cip ini akan selalu ada celah di antaranya sehingga tidak dapat menutupi permukaan kue petak secara keseluruhan
Cokelat cip petak	Bentuk petak dapat dirapatkan dan disusun erat satu sama lain sehingga tidak ada celah, namun tidak bisa menutupi bagian pinggir kue oval. Jadi tidak keseluruhan bagian kue yang tertutupi
	Bentuk petak dapat dirapatkan dan disusun erat satu sama lain sehingga tidak ada celah, semua bagian kue petak dapat tertutupi

Siswa mungkin juga akan berargumentasi kalau semua taburan dapat menutupi permukaan jika susunan bertumpuk diperbolehkan.
Argumen ini juga benar, namun segera ingatkan siswa kembali bahwa dalam tugas ini mereka menghindari taburan bertumpuk!
Minta siswa menyimpulkan dari hasil pengamatan kalau bentuk kue yang dapat ditutupi seluruhnya oleh cip petak (bentuk taburan yang menutupi dengan baik permukaan) adalah bentuk kue petak.

Penutup

4) kesimpulan
Minta siswa menyimpulkan pembelajaran mengenai kue dan taburannya
Sasaran kesimpulan : *bentuk (petak) yang dapat menutupi permukaan dengan baik, di lain pihak bentuk permukaan bagaimana yang dapat ditutupi dengan baik.*

Rencana Pelaksanaan Pembelajaran

Subjek : Matematika
Kelas : III
Semester : 2
Alokasi waktu : 4 x 35 menit (dua pertemuan)
Jumlah siswa : 15 orang
Pertemuan : 3 dan 4

A. Standar Kompetensi :

Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

B. Kompetensi Dasar :

Memahami konsep pengukuran luas bidang datar melalui satuan unit luas

C. Tujuan Pembelajaran :

1) Siswa memahami ide penggunaan unit seragam/identik untuk menutupi permukaan melalui konteks foto frame
2) Siswa menginvestigasi hubungan antara ukuran unit dan jumlah unit yang digunakan untuk menutupi permukaan melalui konteks foto frame
3) Siswa memahami penggunaan unit identik untuk megukur luas dan membandingkan luas beberapa permukaan melalui konteks foto frame
4) Siswa memahami dengan jumlah unit sama, bentuk permukaan dapat berbeda melalui konteks foto frame
5) Siswa menyimpulkan hubungan antara jumlah unit identik dengan ukuran luas permukaan melalui konteks foto frame
6) Siswa menemukan cara menghitung jumlah unit secara efisien dengan mengalikan

D. Indikator Pembelajaran :

1) Siswa dapat menyatakan jumlah masing-masing foto besar dan foto kecil yang digunakan pada foto frame sebagai ukuran framenya
2) Siswa dapat menyatakan jumlah foto kecil atau besar saja yang digunakan pada foto frame sebagai ukuran framenya
3) Siswa menyimpulkan bahwa dengan menggunakan foto kecil untuk menutupi permukaan maka jumlah fotonya akan lebih banyak dibandingkan jika foto yang digunakan adalah foto besar.
4) Siswa menggunakan foto besar saja atau foto kecil saja untuk mengukur luas foto frame dan membandingkan luas antar foto frame
5) Siswa menemukan bahwa dengan jumlah foto identik yang sama, bentuk frame dapat berbeda
6) Siswa menyimpulkan bahwa makin banyak jumlah foto identik yang digunakan, maka makin luas framenya, jika makin sedikit fotonya, makin kecil framenya
7) Siswa dapat mengalikan jumlah foto pada kolom dan jumlah foto pada baris untuk menemukan jumlah keseluruhan foto

E. Pengetahuan awal siswa :

Pada pertemuan pertama, siswa sudah memahami penggunaan kacang sebagai unit untuk mengukur luas kue. Mereka juga memahami bahwa semakin banyak kacang, makin luas kuenya. Pada pertemuan kedua, siswa sudah memahami bahwa bentuk unit petak adalah bentuk yang tepat untuk menutupi permukaan dengan baik tanpa celah. Mereka pun dapat melihat bahwa kacang selalu terdapat celah, sehingga lebih tepat memilih unit petak. Konsep pada dua pertemuan ini digunakan dalam konteks frame foto. Bentuk petak mewakili bentuk foto dan framenya. Sementara, ide unit dan luas digunakan pada foto dan framenya untuk menurunkan konsep luas melalui unit berikutnya.

F. Pendekatan : PMRI
G. Metode : Hands on activity, diskusi dan mengerjakan LKS
H. Alat dan bahan : Lembar Kerja Siswa (LKS), frame dan foto beda ukuran, lem, LCD dan powerpoint bergambar beberapa frame foto

I. Kegiatan Pembelajaran

Pertemuan 3

Pendahuluan

Coba minta siswa mengingat apa yang mereka pelajari pada pertemuan pertama tentang kue kacang mede.

<u>Beberapa poin penting yang diharapkan siswa dapat mengingatnya kembali</u>

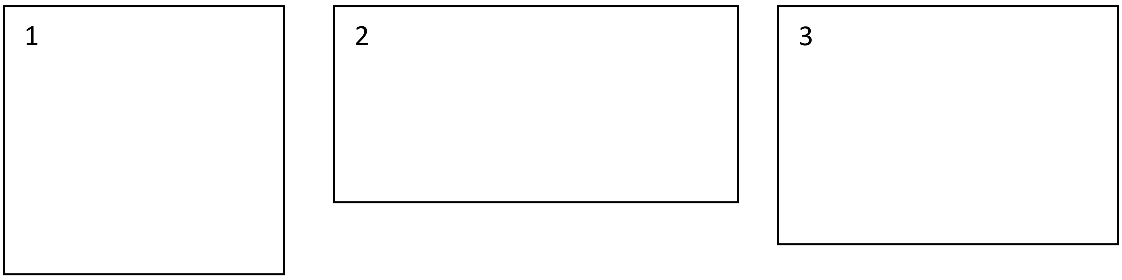
1. Bahwa untuk mengukur besarnya kue dapat digunakan kacang
2. Bahwa untuk mengukur kue dengan kacang, kacangnya tidak boleh bertumpuk dan tidak boleh ada bagian yang belum tertutupi kacang
3. Makin banyak kacang yang digunakan makin besar/luas kuenya dan sebaliknya.
4. Dengan jumlah kacang yang sama, bentuk kue bisa berbeda.

1) Pengenalan konteks – Membuat frame foto

Awali sedikit tanya jawab kepada siswa tentang frame foto, apa yang mereka ketahui, apakah mereka pernah melihat bentuk-bentuk frame dan susunan foto didalamnya dan sebagainya.

Kegiatan Inti

2) Kemudian tunjukkan tiga frame (berbeda bentuk dan ukuran) yang belum ditempeli foto (dalam keadaan kosong) di depan kelas

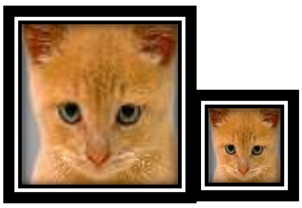


Ukuran frame (hanya untuk diketahui oleh guru)

1. 6x6 foto kecil
2. 4x9 foto kecil
3. 5x7 foto kecil

Ukuran foto (hanya untuk diketahui oleh guru)

Foto besar = 4 x foto kecil

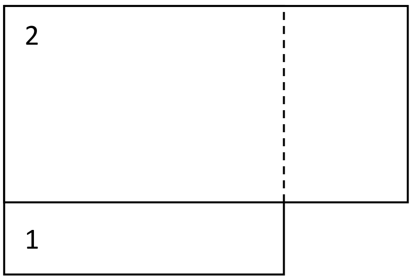


Kegiatan awal...
Minta siswa membandingkan ukuran framenya!

Kegiatan ini tidak dilakukan dalam kelompok. Lakukan seperti diskusi kelas. Dengan tiga frame minta siswa satu persatu atau beberapa orang secara langsung datang ke depan memegang dan menunjukkan di depan kelas ukuran dan urutan frame

<u>Kemungkinan cara siswa membandingkan</u>

- Langsung saja berargumentasi berdasarkan visual mereka kalau frame 1 lebih besar (misalnya) kemudian disusul dua frame berikutnya. Untuk strategi siswa yang seperti ini, beri pengertian kepada siswa kalau cara itu tidak cukup untuk melihatkan frame 1 memang lebih besar/luas. Minta siswa menunjukkan lagi mengapa menurut mereka frame 1 lebih besar. Mungkin sebagian mereka berargumentasi mengenai bentuknya saja. Misal frame 2 lebih besar karena lebih panjang. Tetap minta siswa menunjukkan mengapa.
- Siswa mungkin akan melakukan superposisi, membandingkan dengan cara menempelkan satu frame dengan frame berikutnya, kemudian membandingkan besarnya frame berdasarkan kelebihan bagian frame yang muncul.



Jika hanya strategi 1 yang muncul, minta siswa melakukan superposisi (jangan beri istilah superposisinya, tapi kegiatannya). Jika sudah ada yang melakukan strategi2, minta siswa lain melihat siswa yang melakukan superposisi tersebut!

Setelah siswa mendapatkan urutan frame berdasarkan ukuran dan mungkin akan berbeda-beda urutannya satu sama lain dikarenakan siswa hanya memprediksi kelebihan bagian pada saat superposisi, **kemudian minta siswa mencari ukuran setiap frame untuk melihatkan urutan mana yang benar**.

Lempar pertanyaan

Kegiatan berikutnya ....
"Bagaimana menurutmu mencari berapa luas/ukuran frame ini?"

Kemudian amati apa ada siswa yang berargumentasi ***"Gunakan fotonya".*** Jika belum ada, coba ingatkan lagi siswa dengan konteks kue kacang mede.

<u>Siswa diharapkan dapat berargumentasi sebagai berikut:</u>

Untuk menemukan besarnya kue dapat digunakan kacang. Maka dalam hal ini untuk mencari besar frame, dapat digunakan foto-fotonya.

Kemudian mintalah setiap kelompok memilih salah satu dari tiga frame tersebut. Kemudian, beri sepaket foto (didalamnya terdapat sejumlah foto beda ukuran, besar dan kecil, ukuran sudah dijelaskan sebelumnya di atas) dan lem/perekat. Kemudian minta siswa menutupi permukaan frame dengan fotonya. Untuk menghindari pertanyaan *"Apakah kedua foto yang digunakan, foto besar saja, atau foto kecil saja"*, instruksikan kepada mereka dengan jelas terlebih dahulu bahwa

- **manfaatkan foto yang diberikan untuk menemukan ukuran framenya atau**
- **mereka bebas menentukan foto yang akan mereka gunakan untuk menutupi permukaan frame**

Kemudian hitung berapa banyak foto yang kamu gunakan pada frame tersebut!

Minta siswa terlebih dahulu menyusun fotonya di atas frame. Jangan di lem langsung!

Perhatikan apa ada kelompok yang menyusun foto secara bertumpukan atau ada celah antar fotonya. Jika ada, lakukan sedikit tanya jawab dengan tujuan agar mereka dapat menyusun ulang foto agar tidak ada celah dan foto yang bertumpuk

- Siswa dalam kelompoknya memilih foto kecil saja atau besar saja, maka cari tahu alasan mengapa mereka menggunakan foto kecil saja atau besar saja sewaktu berjalan mengunjungi kelompok
- Siswa dalam kelompoknya memilih untuk menggunakan foto besar dan kecil di frame nya, maka cari tahu tahu alasan mengapa menggunakan kedua foto sewaktu berjalan mengunjungi kelompok

Minta siswa memajangkan hasil kerjanya di depan kelas. Berikut beberapa kemungkinan hasil kerja siswa berikut panduan melakukan diskusi kelas

1. Ada kelompok yang menggunakan foto kecil saja, foto besar saja (kelompok dengan frame 1) dan satu kelompok lagi menggunakan kedua foto pada framenya.

Minta siswa mengamati perbedaan foto yang digunakan di setiap frame. Minta argumentasi awal beberapa orang siswa. Kemungkinan argumen siswa

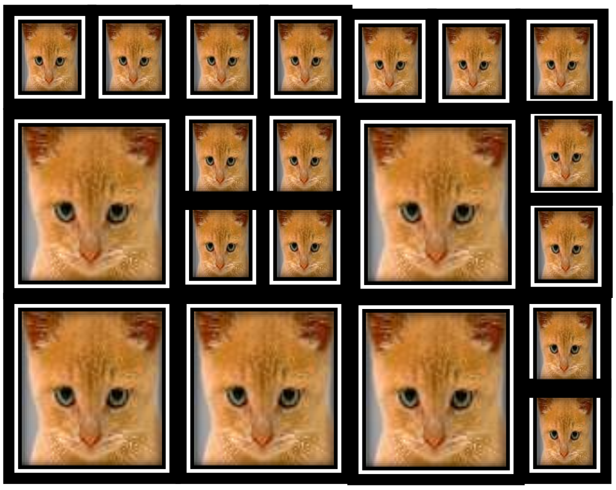
"Frame yang ini menggunakan kedua ukuran foto, sementara dua frame nya lagi menggunakan foto ukuran sama, baik besar atau kecil"

Kemudian minta siswa melaporkan berapa besar masing-masing frame dengan menggunakan foto.

- Siswa akan melaporkan luas frame sebagai jumlah foto besar saja (frame dengan foto besar saja) dansebagai jumlah foto kecil saja (frame dengan foto kecil saja)

- Lihat apakah kelompok yang menggunakan kedua foto (besar dan kecil) dalam framenya melaporkan jumlah keseluruhan foto dalam satu bilangan saja.
  Contoh*: untuk frame (5x7) dengan 5 foto besar 15 foto kecil (seperti gambar di bawah ini) yang digunakan dilaporkan luasnya adalah 20 foto. Jika ada, jadikan prioritas pertama diskusi kelas.

Diskusi kelas mengenai frame yang memuat foto beda ukuran (besar dan kecil): bagaimana melaporkan luasnya?



Gambar di samping adalah **contoh** dari frame dengan menggunakan dua foto bersamaan berikut contoh susunannya.

Awali diskusi dengan menanyakan kembali berapa luas/besar frame ini.

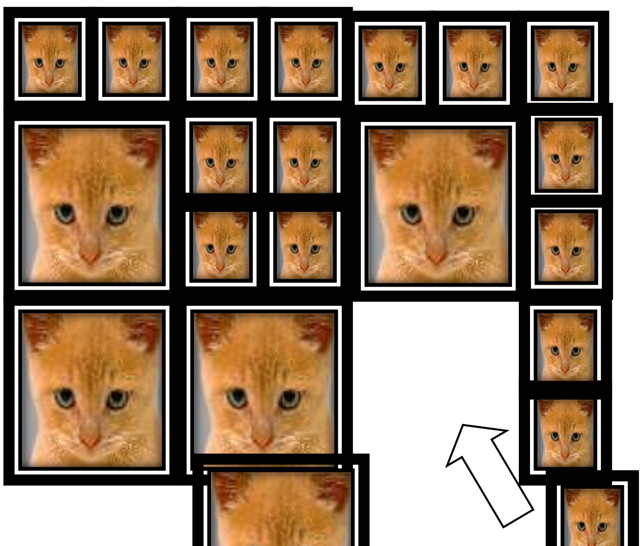
Jawaban yang diharapkan dari siswa :

"Luas/besarnya frame adalah 5 foto besar dan 15 foto kecil"

***Bisa saja frame lain, yaitu yang ukuran 1 dan 2 di atas**

Jika siswa masih menjawab 20 foto, beberapa tindakan berikut dapat dilakukan :

- Minta siswa melihat dua frame lainnya yang masing-masingnya dipasangi foto besar saja dan foto kecil saja. Minta mereka menyebutkan masing-masing besarnya. Siswa mungkin menjawab frame 2 besarnya 36 foto kecil, dan frame 1, 9 foto besar, lalu tanyakan kembali tentang luas frame 3 yang dijawab 20 foto oleh siswa *"menurut jawabanmu, 20 foto, kata foto disini merujuk ke foto ukuran yang mana?"*. Mungkin siswa menjawab kedua foto tersebut, besar dan kecil, namun mereka belum bisa melihat 20 foto itu merupakan jawaban yang kurang tepat. Mereka menyamaratakan foto besar dan foto kecil dalam penghitungan dan masih melaporkannya dalam satu bilangan. Jadi, 20 merujuk pada dua foto tersebut bersamaan dalam frame
- Jika siswa masih belum bisa menyatakan luas framenya 5 foto besar dan 15 foto kecil, mintalah siswa untuk berpikir ***bagaimana kalau seandainya salah satu foto besar yang ada di frame tersebut diganti dengan foto kecil.***



Lalu minta siswa menginvestigasi berapa jumlah foto kecil yang dapat digunakan untuk mengganti foto besar tersebut. Siswa mungkin akan menemukan bahwa foto besar adalah 4 kalinya foto kecil.

Tanpa menanggalkan foto besar, tempelah dengan selotip foto-foto kecil tersebut di atas foto besar tersebut (gambar disamping hanya untuk mengilustrasikan penggantian foto besar dengan foto kecil) dan perlihatkan kepada siswa bahwa foto besar dapat

Kemudian minta siswa menyebutkan ukuran frame setelah foto besar tersebut diganti dengan foto kecil. Siswa mungkin akan menjawab 23 foto. Minta siswa bandingkan dengan ukuran sebelumnya (20 foto) dan minta siswa memikirkan kenapa jumlah foto berubah. Siswa akan merespon karena jumlah foto kecilnya bertambah. Kemudian tanyakan beberapa pertanyaan berikut

- Berapa jumlah foto kecil semula/sebelum diganti?
- Berapa jumlah foto semula/sebelum diganti?
- Berapa jumlah foto kecil yang sekarang/setelah diganti?
- Berapa jumlah foto yang sekarang/setelah diganti?
- Apa yang dapat kamu simpulkan?

Siswa mungkin akan merespon, jumlah masing-masing foto sebelum dan sesudah, berubah. Kemudian tanyakan

"Untuk frame yang sama, bolehkah kita menyebutkan luas framenya 20 foto atau 23 foto? Bukankah artinya luasnya berbeda untuk frame yang sama?"

"Jika seseorang di luar sana bertanya berapa besar frame ini, akankah kamu menjawab 20 foto atau 23 foto saja? Apakah itu tidak membingungkan orang tersebut karena 20 foto dan 23 foto itu artinya ukuran framenya beda? Jadi bagaimana seharusnya melaporkan luas framenya? Cukupkah dengan menyebutkan 25 foto atau 28 foto saja? Apa yang kurang dalam hal ini?"

Siswa diharapkan dapat menyadari bahwa menyebutkan 20 foto atau 23 foto kurang tepat, mereka harus menyebutkan berapa jumlah foto besarnya, berapa jumlah foto kecilnya. Jadi, 5 foto besar dan 15 foto kecil untuk yang 20 foto tadi, atau 4 foto besar atau 19 foto kecil untuk 23 foto yang disebutkan tadi.

Jika tidak mudah bagi mereka menyimpulkannya, tanyakan *"karena jumlah foto besar dan foto kecil bisa berbeda-beda, maka haruskah kita mempertimbangkan masing-masing jumlahnya ketika ditanyakan luas framenya?".*

Untuk menguji pemahaman siswa, coba ganti lagi satu foto besar di frame tersebut dan tanyakan kepada siswa, bagaimana kamu menyebutkan luas frame ini? Siswa mungkin mengatakan 3 foto besar dan 23 foto kecil. Kemudian simpulkan, untuk frame yang sama dengan menggunakan foto beda ukuran, ada berapa cara menyebutkan luas framenya berdasarkan yang kita diskusikan. Minta siswa menyimpulkan

- 5 foto besar dan 15 foto kecil atau

- 4 foto besar dan 19 foto kecil atau
- 3 foto besar dan 23 foto kecil

Kemudian coba tanyakan siswa jika semua foto besar diganti foto kecil, apa yang terjadi? Siswa akan merespon bahwa ukuran frame menjadi 35 foto kecil.

Jika diskusi mengenai frame dengan foto beda ukuran ini selesai didiskusikan, bawa kembali siswa mengamati dua frame yang tersisa, frame dengan foto besar saja dan frame dengan foto kecil saja.

Diskusi mengenai frame dengan foto besar saja dan foto kecil saja

Frame 1 bisa saja memuat foto besar saja, sedangkan frame 2 tidak dapat hanya memuat foto besar saja, jadi dalam hal ini anggaplah siswa menggunakan semua foto kecil pada frame 2 tersebut. Minta siswa membandingkan kedua frame tersebut.

Siswa akan melaporkan bahwa frame 1 ukurannya 9 foto besar dan frame 2, 36 foto kecil. Kemudian tanyakan kepada siswa,

"Frame 1 luasnya dalam foto besar, frame 2 luasnya dalam foto kecil, dengan menggunakan foto yang berbeda ini, dapatkah kalian membandingkan frame 1 dan 2?"

Siswa mungkin berpikir dengan ukuran foto yang berbeda, mereka tidak dapat membandingkan framenya. Tanyakan *"Bagaimana seharusnya ukuran foto kedua frame? Apa yang akan kamu lakukan?"*

Siswa mungkin berpikiran untuk mengganti semua foto besar yang ada di frame 1 dengan foto kecil saja. Dari diskusi pertama, mereka menemukan 1 foto besar sama dengan 4 foto kecil. Dengan ide tersebut, minta salah satu siswa ke depan untuk mengganti foto besar dengan foto kecil. Jika memungkinkan, minta siswa tersebut menempel foto-foto kecil diatas foto besarnya. Jika tidak, lakukan untuk beberapa foto saja, misal memasang hanya 4 foto kecil, kemudian menghitung semua foto kecil pada frame tersebut. Siswa menemukan ada 36 foto kecil.

Kemudian tanyakan *"coba bandingkan jumlah foto yang digunakan jika kamu menggunakan foto kecil saja dibandingkan jika kamu menggunakan foto besar saja?"*

Siswa diharapakan dapat menyimpulkan bahwa jika ukuran foto yang digunakan makin kecil, maka jumlah foto kecil yang digunakan akan banyak, sebaliknya, jika foto yang digunakan besar, maka jumlah foto yang digunakan makin sedikit. Kemudian lanjutkan dengan pertanyaan *"Mengapa foto kecil jadi makin banyak digunakan?".* Siswa diharapkan dapat menjawab *"foto kecil hanya menutup sebagian kecil permukaan jadi dibutuhkan banyak foto kecilnya"*

Kemudian tanyakan ke siswa, *"coba sekarang bandingkan frame 1 dengan frame 2, apa yang dapat kamu simpulkan?"*

Siswa mungkin akan melihat secara langsung bahwa jumlah foto frame 1 dan 2 sama. Kemudian tanyakan kepada mereka *"Dengan jumlah foto sama, bagaimana menurutmu ukuran kedua frame tersebut?"*

Siswa diharapkan dapat menyimpulkan bahwa ukuran frame sama. Kemudian minta mereka bandingkan dengan frame 3 dengan 35 foto kecil tadi. Kemudian tanyakan *"Bandingkan frame 1 dan 2 dengan frame 3, apa yang dapat kamu simpulkan?"*

Siswa diharapkan dapat menyimpulkan bahwa frame 3 lebih kecil dari pada frame 1 dan 2.

Jika siswa sudah memahami semua frame dapat diukur dengan foto kecil saja dan dibandingkan ukuran frame nya dengan foto kecil tersebut. Coba tanyakan

"Mengapa frame 2 dan frame 3 tidak dapat memuat foto besar saja?". Perhatikan jawaban siswa, mereka mungkin berargumentasi kalau ada bagian frame yang tidak cukup ditutupi foto besar. Kemudian tanyakan *"Jadi, dapatkah kita membandingkan 3 frame tersebut dengan foto besar saja?"*. Siswa mungkin akan merespon tidak bisa.

Pelajaran mini

Kemudian beri kelompok tugas lanjutan dalam sebuah pelajaran mini. Beri siswa tugas berikut

Dengan foto yang tersedia, buatlah frame dengan ukuran

- 12 foto kecil
- 4 foto besar
- 2 foto besar dan 4 foto kecil

Kemudian diskusikan perbedaan bentuk framenya. Kemudian diskusikan ide penting bahwa dengan jumlah unit sama, bentuk frame bisa berbeda.

Jika kemungkinan 1 (setiap frame berbeda, memuat foto kecil saja, foto besar saja, dan campuran) tidak seutuhnya muncul, maka lihat kemungkinan kedua berikut panduan diskusi kelasnya.

2. Jika ketiga framenya masing-masing memuat foto beda ukuran (besar dan kecil)

Awali dengan pertanyaan *"Berapa luas masing-masing frame?"* kemudian *"Dapatkah kamu membandingkannya secara langsung?".* Jika siswa melaporkan ukuran frame dengan foto ukuran berbeda dalam satu bilangan, lakukan kegiatan berikut.

Pilih satu frame saja untuk didiskusikan, kemudian lakukan diskusi dengan cara yang sama seperti yang dicontohkan di atas (*Diskusi kelas mengenai frame yang memuat foto beda ukuran (besar dan kecil): bagaimana melaporkan luasnya?*).

Setelah diskusi tersebut, kemudian minta siswa melaporkan luas dua frame lainnya. Setelah itu, tanyakan kepada siswa *"Dapatkah kalian membandingkan luas frame secara* *langsung dengan ukuran yang sudah kalian dapatkan?", "Bagaimana seharusnya ukuran foto setiap frame agar dapat dibandingkan?", "Bagaimana kalau semua foto besar yang ada dalam frame diganti dengan foto kecil?"*

Kemudian minta siswa menghitung jumlahnya pada setiap frame dan tanyakan *"dapatkah kalian membandingkan ukuran framenya sekarang?"*

Kemudian tanyakan kepada siswa *"Apakah setiap frame dapat memuat foto besar saja?".* Minta mereka menginvestigasinya, jika perlu ganti foto-foto kecil yang ada dengan foto besar. Siswa akan menemukan bahwa frame 2 dan 3 tidak dapat memuat hanya foto besar saja. Jadi ketiga frame hanya dapat dibandingkan dengan menggunakan foto kecil saja.

Kemudian lakukan *pelajaran mini* seperti yang dijelaskan di atas. Jika kemungkinan 2 ini tidak muncul, antisipasi dengan kemungkinan berikutnya.

3. Jika frame 1 foto besar saja, dan frame 2 dan 3 foto kecil saja

Maka lakukan diskusi seperti yang diterangkan di atas (*Diskusi mengenai frame dengan foto besar saja dan foto kecil saja)* dengan sedikit penyesuaian (frame 1 juga bisa dibandingkang dengan frame 3, frame 3 sudah dalam kondisi disusun dengan 35 foto kecil).

Kemudian munculkan melalui slide, sebuah frame (contoh frame 3) yang memuat foto beda ukuran, kemudian minta siswa menyebutkan ukuran frame tersebut.

Ambil contoh gambar frame dengan foto beda ukuran di atas, jika siswa melaporkan ukuran framenya dalam satu bilangan yaitu 20 foto misalnya, maka lakukan diskusi seperti yang diterangkan di atas tersebut (*Diskusi kelas mengenai frame yang memuat foto beda ukuran (besar dan kecil): bagaimana melaporkan luasnya?*).

Setelah diskusi selesai, berikan *pelajaran mini* seperti yang dijelaskan di atas. Jika kemungkinan ini tidak muncul, antisipasi dengan kemungkinan 4

4. Jika setiap framenya memuat foto kecil saja

Awali dengan meminta siswa melaporkan ukuran setiap frame dalam foto kecil. Kemudian minta mereka membandingkannya.

Siswa mungkin akan menjawab *"Frame yang lebih banyak foto kecilnya, akan lebih besar ukurannya".*

Kemudian tanyakan, *"Bagaimana ukuran frame jika kalian menggunakan foto besar saja?".* Minta siswa mengganti foto-foto kecil pada frame dengan foto besar. Siswa akan menemukan bahwa frame 1 akan memuat 9 foto besar. Lalu tanyakan *"Apa yang dpat kamu simpulkan?", "Bagaimana jumlah fotonya setelah kamu ganti dengan foto besar?".* Siswa akan berargumen *"Jumlah fotonya menjadi sedikit karena menggunakan foto besar, kalau menggunakan foto kecil, jumlah foto kecilnya akan banyak"*

Argumen siswa ini menandakan bahwa siswa sudah dapat melihat hubungan inverse dari ukuran unit dan jumlah unit yang digunakan dalam perhitungan luas.

Sementara itu, siswa tidak dapat mengganti foto-foto kecil pada frame 2 dan 3 dengan foto-foto besar saja. Minta mereka menyimpulkan.

Siswa diharapkan dapat menyimpulkan bahwa frame 2 dan 3 tidak dapat ditempeli foto besar saja karena ada bagian yang tidak cukup ditutupi foto. Jadi siswa tidak dapat membandingkan frame-frame tersebut dengan menggunakan foto besar saja.

Kemudian munculkan melalui slide, sebuah frame (contoh frame 3) yang memuat foto beda ukuran, kemudian minta siswa menyebutkan ukuran frame tersebut.

Ambil contoh gambar frame dengan foto beda ukuran di atas, jika siswa melaporkan ukuran framenya dalam satu bilangan yaitu 20 foto, maka lakukan diskusi seperti yang diterangkan di atas tersebut (*Diskusi kelas mengenai frame yang memuat foto beda ukuran (besar dan kecil): bagaimana melaporkan luasnya?*).

Setelah diskusi selesai, berikan *pelajaran mini* seperti yang dijelaskan di atas.

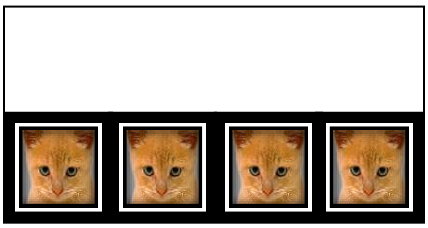
Sampai di sini adalah batas **pertemuan ke tiga**, jika waktu tidak cukup, maka sebagian kegiatan dapat dilakukan di pertemuan keempat.

Pertemuan 4

3) Pelajaran mini 1 – Gambar cepat frame yang belum selesai ditempeli foto

Lakukan diskusi kelas kemudian tampilkan secara cepat frame yang belum selesai ditempeli foto satu persatu melalui powerpoint dan LCD. **Setiap frame tersebut tanyakan luas framenya.**

1.



Kemungkinan strategi siswa

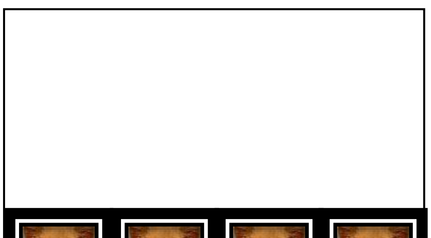
1. Hitung satu satu

2. Doubling (4x2)

Luas frame = jumlah foto = 8 foto

Jika ada siswa yang hitung satu-satu segera hilangkan dengan cepat slide framenya. Jika ada siswa yang doubling, minta siswa tersebut menyatakan alasannya mengapa dan minta siswa lain memperhatikan. Argumen sasaran : baris fotonya dikali dua

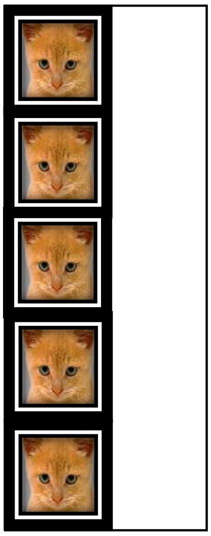
2.



Perlihatkan frame ini agak lama dibandingkan frame 1, tujuannya agar siswa dapat mengira jumlah foto perkolom

Kemungkinan strategi siswa

3.



Kemungkinan strategi siswa

Doubling (5x2)

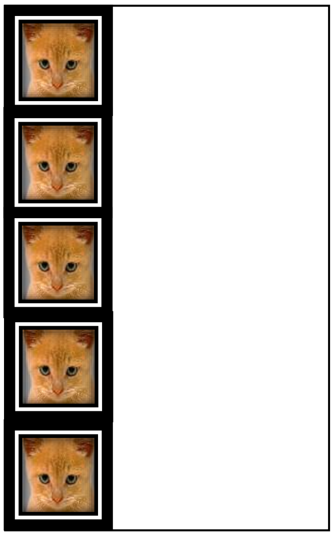
Luas frame = jumlah foto = 10 foto kecil

4. perlihatkan agak lama frame ini, agar siswa dapat memperkirakan jumlah foto perbaris

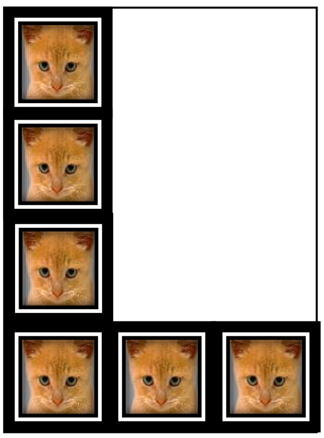
Kemungkinan strategi siswa

Mengalikan dengan 3 jumlah foto perkolom

Luas frame = jumlah foto = 15 foto



5.



Siswa sudah dapat menghitung cepat dengan mengalikan jumlah foto dalam satu baris/kolom dengan banyaknya baris yang ada.

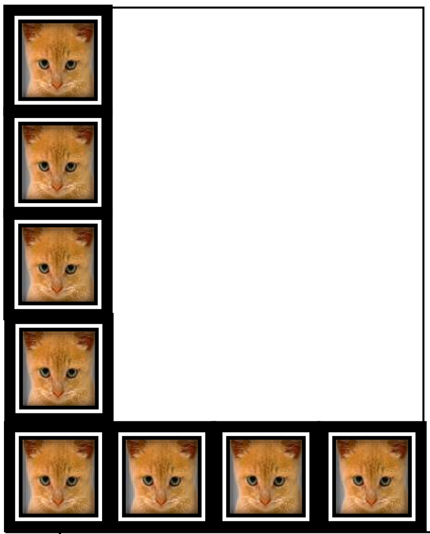
Maka dalam foto 5, siswa dapat melihat bahwa jumlah fotonya adalah banyak foto dalam satu baris (3) dikalikan banyaknya baris (4) atau jumlah foto dalam satu kolom (4) dikalikan banyaknya kolom (3)

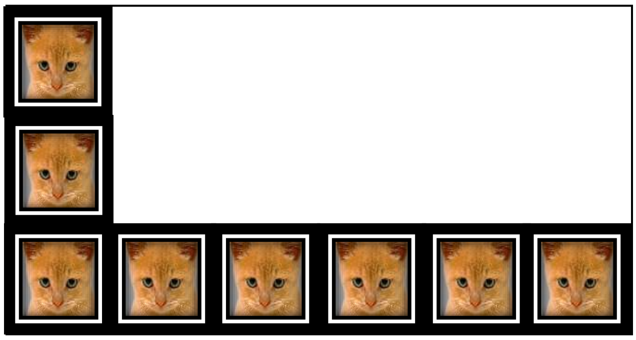
Maka siswa sudah bisa menghitung cepat dengan mengalikan, 3x4 atau 4x3, beri satu gambar lagi

Luas frame = jumlah foto = 12 foto

Untuk menguji pemahaman siswa tentang penggunaan strategi mengalikan untuk menghitung jumlah unit secara cepat, tunjukan slide dengan dua frame berikut

Manakah frame yang lebih luas ? (setelah beberapa saat hilangkan slidenya, untuk mencegah siswa yang masih menghitung satu-satu fotonya)





Siswa menjawab banyaknya foto

3x6 = 18 foto, sehingga luas framenya 18 foto

Siswa menjawab banyaknya foto

4x5 = 20 foto, sehingga luas framenya 20 foto

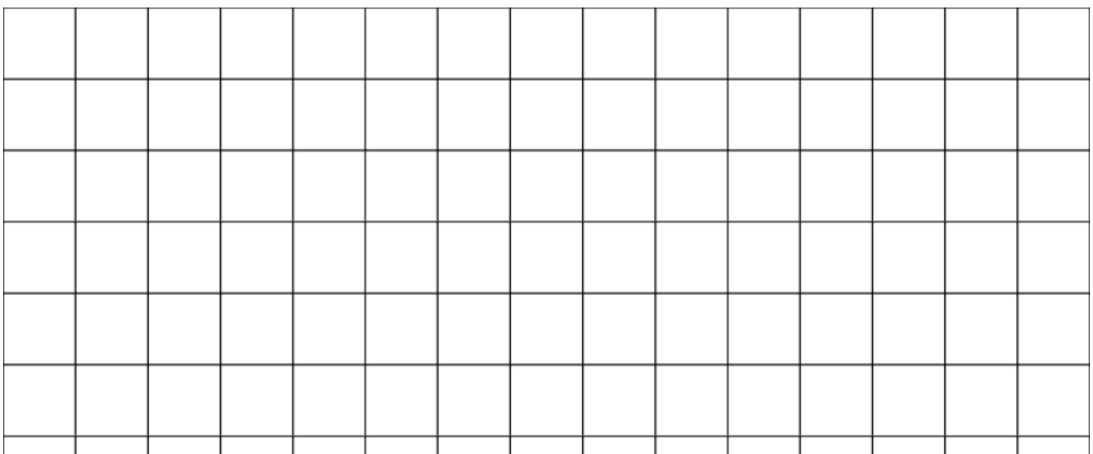
Siswa kemudian dapat menentukan frame yang lebih luas berdasarkan jumlah fotonya.

4) Pelajaran mini 2 – Manakah karpet yang lebih luas?

Awali dengan tanya jawab tentang lantai keramik, apa persamaannya dengan frame foto. Kemudian daptkah keramik juga digunakan untuk mengukur luas benda? Coba tanyakan contoh bendanya kepada siswa!

Kemudian berikan gambar tampak atas karpet di atas lantai melalui slide dan LCD.

Manakah karpet yang lebih luas ? Bagaimana caramu mengetahuinya ?



Quick show frame yang uncomplete >> efisien counting

Membandingkan besar frame

Mini lesson >> membandingkan luasan karpet

Kemungkinan jawaban siswa :

- Siswa secara langsung memilih karpet kuning karena berbentuk persegi, jadi kelihatan besar. Untuk jawaban seperti ini, minta siswa tersebut menunjukkan mengapa karpet kuning lebih besar.
- Siswa menghitung jumlah keramik yang tidak kelihatan dibawah karpet. Ingatkan siswa tentang cara menghitung yang cepat dan efisien tanpa harus menghitung satu-satu lagi.
- Siswa menerapkan strategi perkalian untuk mendapatkan luas masing-masing karpet. Siswa pertama menghitung jumlah karpet pada baris dan kolom setiap karpet kemudian mengalikannya. Dari hasilnya mereka dapat menemukan karpet mana yang lebih luas.

4) Kesimpulan

Minta siswa menyimpulkan hal penting yang mereka pelajarai selama pertemuan 3 dan 4

Sasaran kesimpulan siswa

- Bahwa foto dapat digunakan untuk mengukur frame
- Bahwa dengan foto beda ukuran yang digunakan dalam sebuah frame, maka luasnya dapat dinyatakan dalam sekian foto besar sekian foto kecil, atau sekian foto kecil saja, atau sekian foto besar saja
- Membandingkan ukuran beberapa frame, dipergunakan foto denagn ukuran yang sama
- Makin banyak foto identik diguankan, makin besar framenya
- Menghitung cepat dengan mengalikan dapat dilakukan untuk menghitung banyaknya foto dalam frame.

Rencana Pelaksanaan Pembelajaran

Subjek : Matematika
Kelas : III
Semester : 2
Alokasi waktu : 2 x 35 menit
Jumlah siswa : 15 orang
Pertemuan : 5

A. Standar Kompetensi :

Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

B. Kompetensi Dasar :

Memahami konsep pengukuran luas bidang datar melalui satuan unit luas

C. Tujuan Pembelajaran :

1) Siswa mengukur luas bentuk non petak dengan menggunakan unit petak melalui konteks kue dan meja oval di atas keramik
2) Siswa menggunakan strategi perkalian sebagai cara efisien menemukan jumlah unit
3) Siswa dapat melakukan perkiraan jumlah unit dalam menentukan luas permukaan berbentuk non petak.

D. Indikator Pembelajaran :

1) Siswa dapat menggunakan perkalian untuk menemukan jumlah cokelat cip petak pada kue petak
2) Siswa dapat menemukan cara untuk menutupi permukaan kue oval dengan cokelat cip petak denganc ara memecah cip petaknya
3) Siswa dapat melakukan perkalian untuk hitung keramik efisien di dalam outline meja oval pada saat menentukan luas meja oval
4) Siswa dapat melakukan estimasi/perkiraan jumlah keramik yang digunakan pada bagian pinggir meja oval dan menemukan jumlah perkiraan total keramik sebagia luas permukaan meja oval

E. Pengetahuan awal siswa :

Pada pertemuan kedua, siswa menemukan bahwa bentuk petak adalah bentuk yang tepat untuk menutupi permukaan tanpa celah dan tumpukan, namun saat itu mereka belum mengetahui secara langsung bahwa jumlah unit adalah representasi luas permukaannya. Mereka juga mengetahui bahwa bentuk non petak tidak dapat ditutupi seluruhnya oleh unit petak. Pada pertemuan ini mereka akan menemukan strategi untuk menutupi permukaan yang non-petak dengan memecah unit. Siswa sudah memahami pada pertemuan 3 dan 4 bahwa jumlah unit merepresentasikan luas permukaan melalui konteks frame foto dan karpet. Dalam pembelajaran ini mereka akan mengetahui luas permukaan kue dengan cokelat cip petak dan luas permukaan meja oval dengan menggunakan keramik. Dengan pengetahuan ini, siswa akan membangun pemahaman mengenai mengukur luas bentuk non petak dengan unit petak dan melakukan estimasi terhadap hasil pengukurannya.

F. Pendekatan : PMRI

G. Metode : Hands on activity, diskusi dan mengerjakan LKS

H. Alat dan bahan : Lembar Kerja Siswa (LKS), gambar kue dan tiruan cokelat cip petak), lem, gambar tampak atas meja oval di atas lantai keramik.

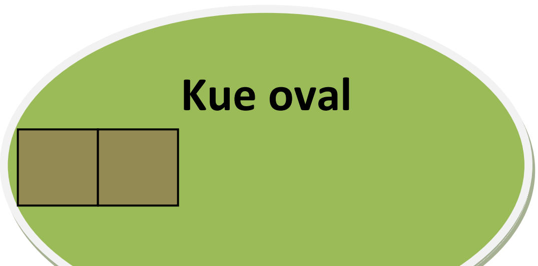
I. Kegiatan Pembelajaran

Pendahuluan

Awali pertemuan dengan meminta siswa mengingat pelajaran ke dua, tentang mencari bentuk taburan yang tepat untuk menutupi permukaan kue dan bentuk kue yang seperti apa yang dapat tertutupi keseluruhannya dengan taburan tersebut. Kemudian minta siswa mengingat pelajaran 3 dan 4 tentang penggunaan foto untuk mengukur besar frame. Kemudian beri tahu siswa bahwa pada kegiatan pertama ini mereka akan bekerja dengan kue dan cokelat cip petak. Kemudian tanyakan

"Dapatkah kalian menemukan besarnya kue? Bagaimana caramu?"

Siswa mungkin akan mengatakan bahwa cokelat cip dapat digunakan untuk menemukan besarnya kue.Kemudian berikan gambar dua buah kue, petak dan oval beserta tiruan cokelat cip petak untuk setiap kelompok.



Kegiatan Inti

1) Kegiatan 1 – ukur luas permukaan atas kuenya

Siswa menyusun cip petaknya di atas masing-masing kue.

Minta siswa menggunakan cara cepat meghitung dengan perkalian, agar siswa tidak menghitung satu-satu lagi

Siswa dengan mudah menemukan banyaknya cip petak pada kue petak. Ketika ada kelompok yang mencoba menempel semua cip petak pada kue petak, tanyakan mereka, apa perlu mereka melakukannya.

Ingatkan mereka pada perkalian, mereka diharapkan cukup menyusun cip pada baris dan kolom kemudian mengalikannya, tapi janagn mendemonstrasikan caranya! Biar mereka sendiri yang melakukannya.

Untuk kue oval, siswa akan menemukan bahwa di pinggiran kue tidak dapat ditutupi dengan cip. Minta mereka menghitung cip yang terpakai. Kemudian pajang hasil kerja siswa dan lakuakn diskusi kelas.

2) Diskusi kelas – Berapa besar kue

Siswa akan dengan mudah menemukan besar/luas kue petak dengan menemukan jumlah cip petak melalui perkalian. Namun, siswa sedikit mengalami kesulitan menemukan luas/besar kue oval karena dipinggiran kue ada yang belum tertutup cip petak.

Maka tanyakanlah *"Lalu, bagaimana caramu agar pinggiran tersebut dapat tertutup?", "Apa yang dapat kamu lakukan untuk menutupinya?"*

Siswa mungkin akan menjawab, potong/sobek/belah/pecah cip nya.

Catat bahwa ini adalah ide pentingnya! Karena ini akan membawa ke konsep unit kompensasi (beberapa pecahan unit yang dapat digabung menjadi 1 unit). Dengan strategi ini, siswa dapat melakukan estimasi hasil pengukuran dengan menggunakan unit!

Kembalikan hasil kerja mereka dan minta mereka menyelesaikan pengukuran besar kue oval dengan cip tersebut. Kemudian perhatikan bagaimana mereka melakukannya! Hal penting berikut harus di amati

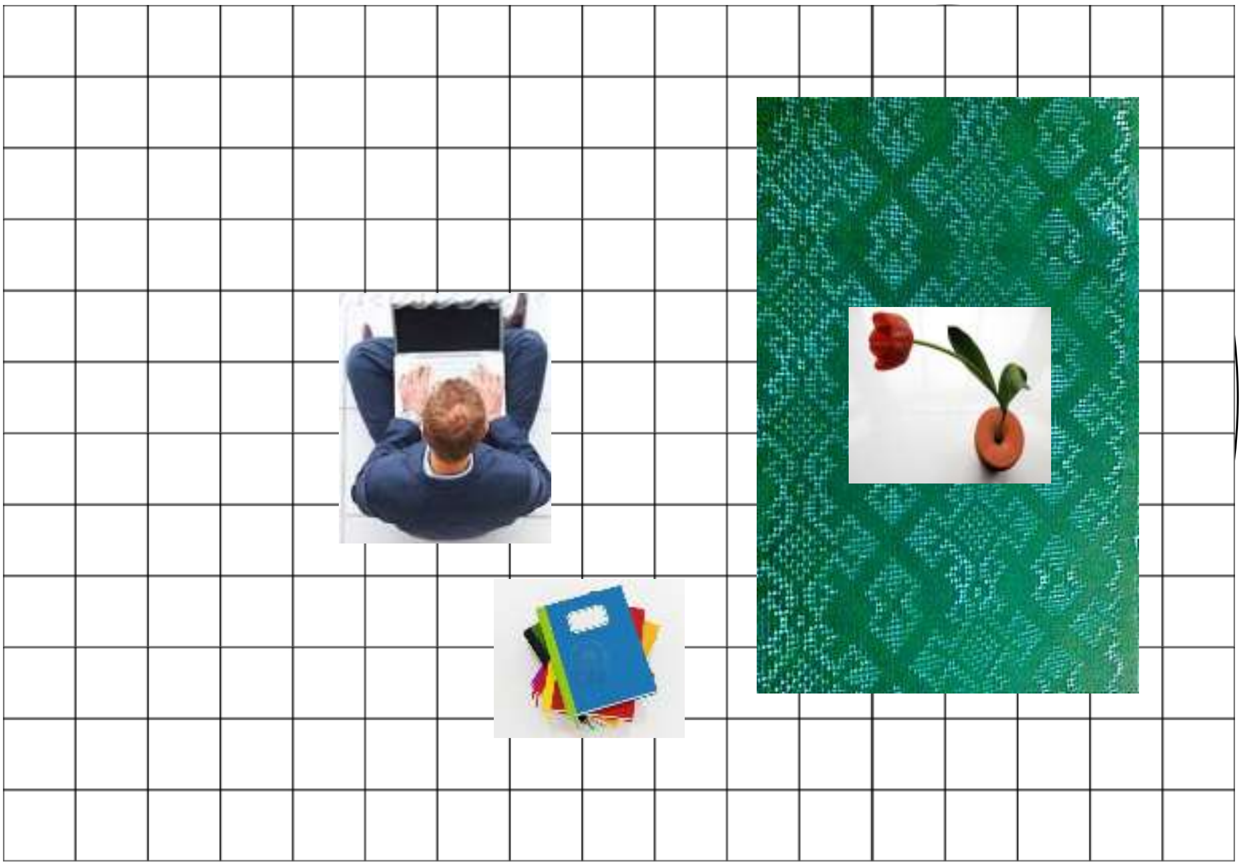
- Apa mereka melakukan pemecahan cip dan menempel pecahan tersebut sehingga dapt menutupi bagian pinggirnya. Amati, apa mereka menghitung pecahan tersebut sebagai 1 cip. Misal pecahan ini dan itu dihitung satu cip karena berasal dari sebuah cip yang dipecah.

Kemudian minta siswa melaporkan hasil pengukuran mereka. Lihat apakah siswa datang dengan jawaban atau estimasi luas kue oval yang berbeda-beda. Jika ada perbedaan, lakukan diskusi dan cari tahu bersama siswa apa yang menyebabkan perbedaan.

Langkah selanjutnya, minta siswa membandingkan kue manakah yang permukaan atasnya lebih luas/besar?

3) Tugas – Luas permukaan meja oval

Berikan setiap kelompok gambar tampak atas meja oval di atas lantai keramik dan minta siswa menemukan luas meja ovalnya.



Berikut adalah kemungkinan strategi siswa:

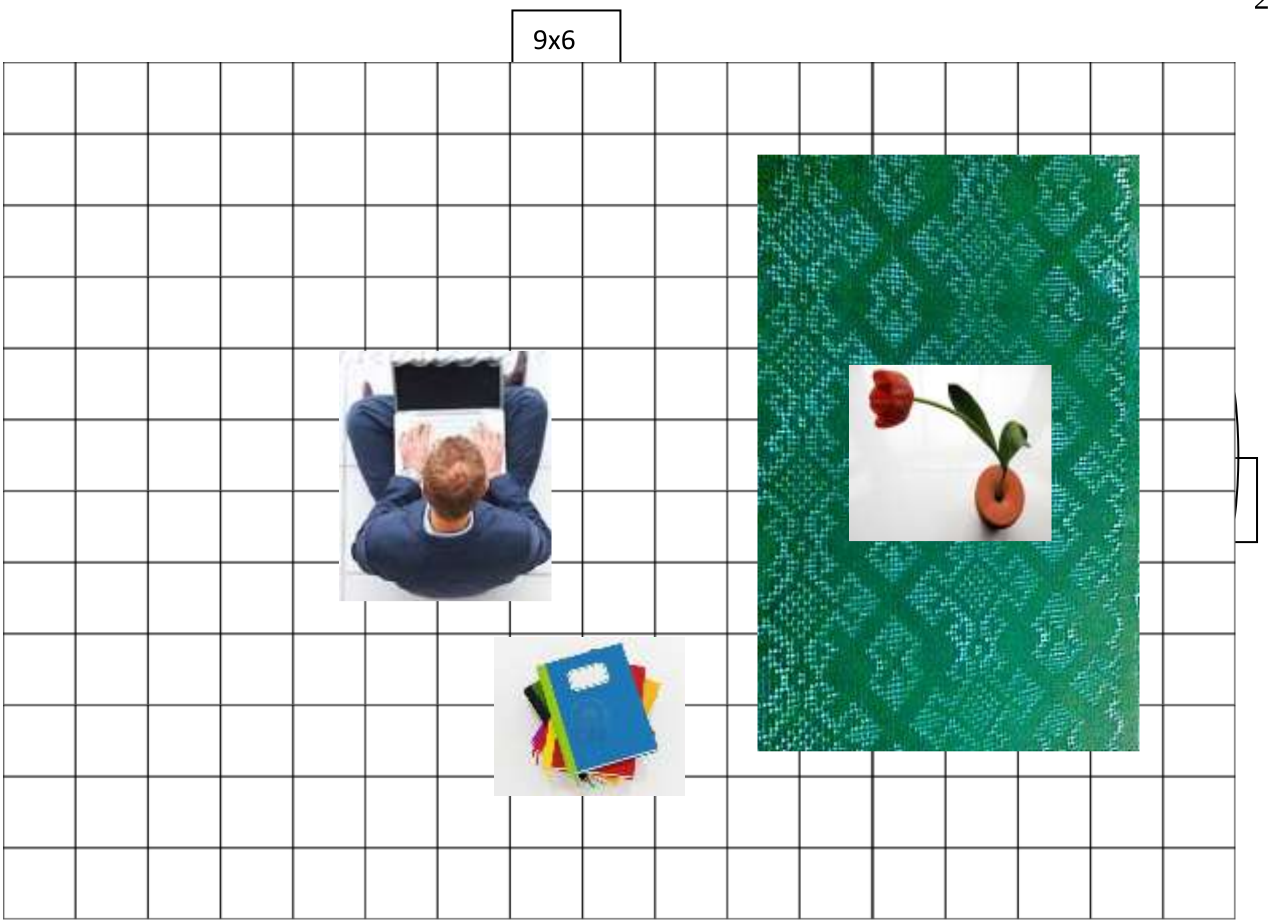
- Siswa menghitung jumlah jumlah keramik yang tertutupi oleh taplak meja (taplak meja diguankan untuk siswa dapat menghindari menhitung satu, dan juga untuk membangkitkan ide penggunaan perkalian dalam menghitung luas meja oval ini). Beberapa strategi perkalian dapat diterapkan: 6x9, 4x6, 6x7 kemudian siswa menghitung sisa keramiknya dengan cara mencari potongan-potongan keramik yang dapat digabung dan dihitung menjadi 1 keramik, kemudian melakukan estimasi
- Siswa menemukan estimasi luas meja oval dengan menggunakan keramik

Minta kelompok siswa melaporkan luas meja dan bagimana mereka menemukannya secara singkat kemudian lakukan diskusi kelas.

4) Diskusi kelas – berapa luas meja ovalnya?

Poin diskusi : Penggunaan perkalian untuk menemukan luas, dan estimasi luas meja

- Diskusikan dengan siswa apa saja kemungkinan perkalian yang digunakan untuk menemukan sebagian jumlah keramik pada meja oval tersebut.



Siswa mungkin jawab dengan beberapa perkalian seperti yang ditunjukkan pada gambar di atas. Diskusikan apa kekurangan dan kelebihannya menggunakan perkalian tersebut. **Tanya jawab dengan siswa perkalian lain apa saja yang mungkin digunakan untuk menemukan sebagian jumlah keramik pada meja oval. Minta mereka menunjukkan dimana mereka menggunakan perkalian itu.**

Kemudian siswa diminta menyimpulkan kenapa mempergunakan perkalian dalam menemukan luas meja oval ini. Siswa diharapkan dapat menyimpulkan bahwa mereka dapat membentuk perkalian apa saja sesuai dengan wilayah persegi yang mereka buat pada outline permukaan non petak untuk menghitung cepat dan efisien sebagian jumlah keramik pada meja oval.

- Apabila ada perbedaan hasil estimasi setiap kelompok, cari tahu bersama siswa dimana letak perbedaannya, kemudian bandingkan hasil estimasi pengukuran kelompok manakah yang lebih teliti.

Penutup

Minta siswa menyimpulkan pelajaran hari ini. Beberapa sasaran kesimpulan siswa

- Permukaan nonpetak dapat ditemukaan dengan menggunakan unit petak
- Perkalian dapat digunakan untuk emnemukan sebagian jumlah unit pada permukaan non petak
- Estimasi/perkiraan dapat dilakukan untuk menemukan luas permukaan non petak dengan ide memecah unit dan mnghitung bagian pecahan tersebut.

Rencana Pelaksanaan Pembelajaran

Subjek : Matematika
Kelas : III
Semester : 2
Alokasi waktu : 2 x 35 menit
Jumlah siswa : 15 orang
Pertemuan : 6

A. Standar Kompetensi :

Menghitung keliling, luas persegi dan persegi panjang, serta penggunaannya dalam pemecahan masalah

B. Kompetensi Dasar :

Memahami konsep pengukuran luas bidang datar melalui satuan unit luas

C. Tujuan Pembelajaran :

1) Siswa menggunakan petak-petak/grid untuk menemukan luas permukaan berbentuk petak atau non petak
2) Siswa melakukan perkiraan luas permukaan non petak menggunakan petak-petak/grid

D. Indikator Pembelajaran :

1) Siswa menggunakan kertas petak-petak transparan untuk menemukan luas permukaan yang petak dan melakukan perkiraan luas untuk permukaan yang non petak
2) Siswa memahami bahwa luas yang diperoleh melalui petak-petak/grid bergantung pada ukuran petak yang digunakan
3) Siswa dapat membandingkan mencari luas dengan menggunakan kacang dan dengan menggunakan peta-petak dari segi ketelitian.
4) Siswa mampu membuat objek dan petak transparan dengan luas yang mereka tentukan sendiri

E. Pengetahuan awal siswa :

Pada pembelajaran sebelumnya, siswa sudah bisa melakukan estimasi luas permukaan non petak dengan menggunakan unit petak. Dengan pengetahuan ini siswa akan diberi model unit grid melalui petak-petak transparan dan menemukan luas objek/permukaan/bidang yang diukur.

F. Pendekatan : PMRI

G. Metode : Hands on activity, diskusi dan mengerjakan LKS

H. Alat dan bahan : Lembar Kerja Siswa (LKS), kertas petak transparan, tampak atas kue, spidol warna, penjepit kertas atau double tip

I. Kegiatan Pembelajaran

Pendahuluan

Minta siswa mengingat kembali pelajaran sebelumnya melalui tanya jawab. Kemudian awali pelajaran dengan meminta siswa membandingkan besar permukaan telapak tangan mereka. Lihatlah apakah siswa dapat membandingkan dengan melekatkan telapak tangan satu sama lain. Kemudian tanyakan mana yang lebih besar. Kemudian minta siswa berkelompok dan memilih satu utusan mereka. Kemudian setiap utusan kelompok dibandingkan besar telapak tangannya. Siswa mungkin bisa melihat mana yang lebih besar serta membuat urutannya.

Bagaimanapun tanyakan kepada siswa bagaimana kamu mengetahui berapa besar atau luas telapak tangannya. Kemudian perkenalkan salah satu alat yang digunakan untuk mengukur luas, yaitu petak-petak/grid transparan. Demonstarsikan cara menjiplak telapak tangan di atas kertas. Buat outline tertutup telapak tangannya sehinga dapat dicari luasnya. Kemudian minta salah satu siswa dalam kelompok untuk membuat outline telapak tangannya/jiplak telapak tangannya. Berikan petak transaparan ukuran pertama untuk setiap kelompok. Beserta spidol warna.

Ingatkan siswa mengenai cara menghitung cepat sebagian jumlah unit dengan perkalian.

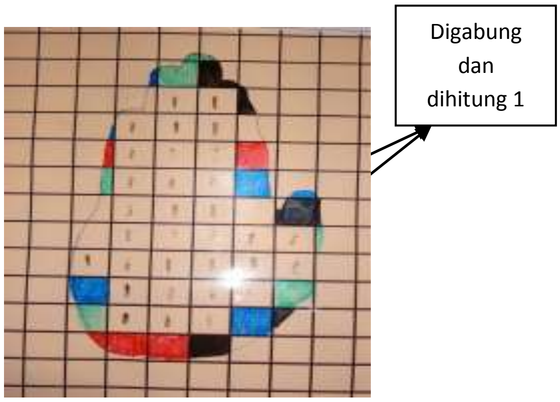
Kegiatan Inti

1) Berapa luas telapak tangannya?

Siswa membuat outline telapak tangan teman sekelompoknya, kemudian meletakkan petak transaparan di atas outline telapak tangannya. Spidol warna digunakan untuk menandakan bagian-bagian unit yang bisa digabung dan dihitung menjadi satu unit petak.

Gambar di samping adalah salah satu contoh kerja siswa. Minta mereka melakukan perhitungan yang teliti, kemudian menulis luas nya dalam outline tangan yang bersangkutan serta melaporkan perkiraan jumlah petaknya sebagai luas telapak tangannya. Agar transaparan tidak lepas, beri penjempit/double tip.

Kemudian bandingkan hasil pengukurannya, apakah sesuai dengan urutan yang dibuat siswa dengan mengukur telapak tangan secara langsung sesaat sebelum siswa bekerja dengan petak transparan. Kemudian lakukan diskusi.



2) Diskusi – grid lain yang beda ukuran petak-petaknya

<u>Beberapa poin diskusi</u>

- Beberapa perkalian yang bisa digunakan untuk menemukan jumlah beberapa petak dalam outline telapak tangan. Misal, perhatikan gambar telapak tangan di atas, maka untuk menghitung jumlah petak dalam telapak tangannya, dapat digunakan perkalian 3x8 atau 5x2, atau mungkin 4x4 sesuai wilayah yang ingin dicari jumlah petaknya

- Mengenai estimasi/perkiraan luasnya. Diharapkaan siswa dapat menyadari bahwa mengukur luas permukaan non petak dengan grid hasilnya bisa berupa perkiraan, tidak tepat pas, karena mereka menggunakan perkiraan jumlah unit. Tapi, perkiraan akan mendekati luas sebenarnya jika mereka memperkirakan dengan teliti.
- Bagaimana hasil pengukuran luasnya jika menggunakan petak transparan lain dengan ukuran petak-petaknya lebih kecil. Tunjukkan kepada siswa transparan kedua. Kemudian minta siswa mendiskusikan hasil pengukurannya. Jika memingkinkan lakukan dulu pengukurannya secara cepat.
  Kemungkinan reaksi siswa: *"Maka jumlah petaknya makin banyak karena ukurannya petaknya lebih kecil"*. Reaksi ini berpotensi sekali untuk muncul.
  Kemudian tanyakan: *"Jadi apa yang dapat kalian simpulkan?"*
  Arahkan siswa untuk dapat menyimpulkan bahwa luas telapak tangan atau permukaan lainnya yang diperoleh tergantung ukuran petak yang digunakan.

  Kemudian tanyakan kepada siswa : *"Dapatkah kamu membayangkan petak lainnya yang beda ukuran pula dengan kedua petak ini?"*

  Kemudian minta siswa untuk membuat petak-petak mereka sendiri untuk mengukur telapak tangan teman kelompoknya.

3) Mengukur luas telapak tangan dengan petak-petak buatan sendiri

Minta siswa yang sama setiap kelompok untuk dijiplak telapak tangannya kembali di kertas yang lain. Kemudian pada kertas yang sama minta kelompok merundingkan ukuran petak yang akan mereka buat untuk mencari luas telapak tangan mereka sendiri.

Minta mereka penggunakan penggaris. Perhatikan kerja kelompok, apakah ada ukuran setiap petak yang tidak sama/tidak konsisten. Jika ada bersiaplah untuk membuat hal ini jadi bahan diskusi. Setelah selesai minta mereka mencari luas telapak tangannya.

4) Diskusi – Hasil pengukuran dengan petak-petak buatan sendiri

Poin diskusi

- Ada ukuran petak-petak yang tidak sama dalam petak-petak yang mereka buat? Diskusikan apa akibatnya jika berbeda, minta mereka mengingat pelajaran tentang frame foto dengan menggunakan dua foto beda ukuran. Kemudian coba bandingkan petak-petak ini dengan petak-petak kelompok lain yang setiap ukuran petaknya sama. Diskusikan apa akibatnya jika ada petak yang berbeda ukuran.

5) Refleksi – menggunakan kacang VS menggunakan petak-petak

Minta siswa dalam kelompoknya mengukur luas kue dengan petak-petak. Kemudian minta mereka membandingkan mencari luas kue dengan kacang dan dengan petak-petak.

Sasaran jawaban

Bahwa dengan menggunakan petak-petak hasil pengukuran lebih teliti karena petak-petak itu rapat sedangkan kacang ada celah diantaranya sehingga ada bagian yang belum tertutup.

Penutup

Minta siswa menyimpulkan pelajaran hari ini.
Beberapa kesimpulan yang bisa ditarik

- Bahwa pengukuran luas dengan grid/petak-petak hasilnya bergantung pada ukuran petak-petak pada grid tersebut
- Estimasi dapat dilakukan utnuk emnemukan luas permukaan dengan menggunakan grid/petak-petak

Appendix E
Pupil Worksheet and Materials

Dikerjakan bersama kelompok

Lembar Kerja Siswa 1

Nama :

Kelas/Kelompok :

Kegiatan 1 – *Pembuat kue kacang mede*

Bersama kelompokmu, letakkan kacang mede di atas cookie!

Berapa jumlah kacang mede di kue mu?

Biarkan kacangnya di atas cookie tersebut, kelompok lain akan melihat hasil kerjamu

Bandingkan hasil kerjamu dengan kerja kelompok lain.

Apakah banyak kacang setiap kelompok sama?

....

Apa menurutmu yang menyebabkan jumlah kacang setiap kelompok berbeda?

Tulis dan jelaskan jawaban pertanyaan di atas dalam awan berpikir berikut !

____

Untuk kelompok
(kerjakan bersama)

Kegiatan 2 – *Kue dan 12 kacang mede*

Bekerjalah di setiap kue. Tutupi kue dengan kacang, kerjalah di dalam kelompok! Diskusikan hasil temuanmu!

Banyak kacang yang tersedia. : ....

Kue A

Kue B

Tulis apa yang kamu temukan berdasarkan pengamatanmu!

Kue A

....

Kue B

....

Bagaimana seharusnya bentuk dan ukuran kue dengan 12 kacang?. Buat kuenya dengan menggunakan pensil. Kerjakan di bagian kosong di bawah ini!

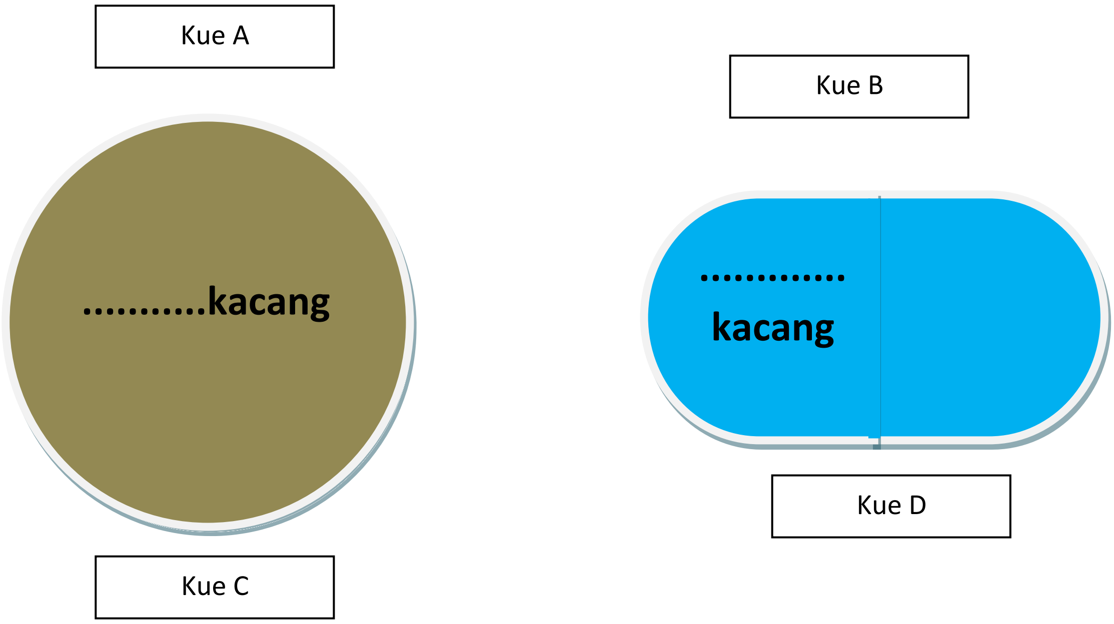
Setelah selesai mengerjakan, simpan pekerjaanmu dan kunjungi dan lihat hasil kerja kelompok lainnya

Kegiatan 3 – *Ukur dan urutkan kue kacang medenya*

Bekerjalah sendiri dan temukan berapa besarnya kue dengan menggunakan kacang mede!

.... kacang

......................kacang



Dengan jawabanmu tersebut, coba urutkan kue berdasarkan banyak kacang medenya.

1............ 2............ 3............... 4................

Kue mana yang paling besar? ....

Berapa besarnya?.................kacang

Bagaimana ukuran kue jika kacangnya semakin banyak?

Bagaimana ukuran kue jika kacangnya semakin sedikit?

Pembuat kue dan kue kacang medenya

Untuk guru dan diceritakan ke siswa

Seorang pembuat kue akan membuat kue penuh dengan kacang mede di atas permukaan kuenya (tunjukkan bentuk outline kuenya). Karena kacang mede tergolong mahal di pasaran, si pembuat kue tidak ingin untuk sejumlah kuenya, banyak kacang mede yang berlebih. Dengan demikian ia coba mengira-ngira

Gambar untuk kegiatan 1

Untuk kelompok

Di bawah ini adalah gambar tampak kue

Gunakan gambar ini untuk aktivitas bagian 1

Dan tuliskan banyak kacang yang digunakan di kotak yang disediakan di bawah cookie!

Biarkan kacangnya tetap di atas cookie dan biarkan gurumu dan kelompok lainnya melihat hasil pekerjaanmu!

Dikerjakan bersama kelompok

Lembar Kerja Siswa 2

Nama :
Kelas/Kelompok :

Petunjuk

Jawablah pertanyaan di bawah ini secara perorangan setelah berdiskusi dengan kelompokmu

Cookie A (Tulis hasil pemeriksaanmu terhadap tiap-tiap taburan kue)

1. Kacang mede

- *Dapatkah taburan ini menutupi seluruh permukaan kuenya?* ....
- *Mengapa?*...
  ....

2. Cokelat M&M

Kue B (Tulis hasil pemeriksaanmu terhadap tiap-tiap taburan kue)

Cookie A (Tulis hasil pemeriksaanmu terhadap tiap-tiap taburan kue)

1. Kacang mede

- *Dapatkah taburan ini menutupi seluruh permukaan kuenya?* ....
- *Mengapa?*..
  ....

2. Cokelat M&M

- *Dapatkah taburan ini menutupi seluruh permukaan kuenya?* ....
- *Mengapa?*..
  ....

3. Cokelat cip bintang

- *Dapatkah taburan ini menutupi seluruh permukaan kuenya?* ....
- *Mengapa?*..
  ....

4. Cokelat cip petak

Petunjuk

Jawablah pertanyaan di bawah ini seara perorangan setelah diskusi kelas

Taburan mana yang menutupi seluruh permukaan kue? .... *Mengapa?..............*

Kue bentuk manakah yang permukaannya tertutupi seluruhnya? .... *Mengapa?.........................*

Cerita pembuat kue dan taburan cookienya

Untuk diceritakan ke siswa

Pembuat kue akan membuat dua jenis kue, kue oval dan kue petak. Ia akan menutupi permukaan kue tersebut dengan empat taburan: kacang mede, cokelat cip bintang, M&M cokelat, dan cokelat cip petak tanpa bertumpuk. Dapatkah kamu membantunya untuk meletakkan taburan tersebut?

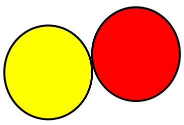
Kemudian temukan
1) Taburan mana yang dapat menutupi seluruh permukaan kuenya?
2) Kue mana yang dapat tertutupi semuanya oleh taburan tersebut?

Dikerjakan bersama kelompok

Gunakan tiruan taburan yang terbuat dari kertas berikut ini dan juga kacang mede asli. Tutupi permukaan cookienya, kemudian jawablah pertanyaan pada LKS!

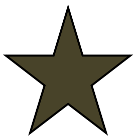
Kacang mede

Cookie A



Cookie A

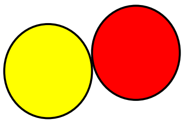
Cokelat M&M



Cookie A

Cookie B

Kacang mede



Cookie B

Cokelat M&M

Cookie B

Bekerja dalam kelompokmu, isi LKS ini perorangan

Lembar Kerja Siswa 3

Nama :
Kelas/kelompok :

Menemukan berapa besar frame nya

Frame nomor berapa yang kamu pilih?................

Foto ukuran mana yang kamu gunakan untuk menutupi frame?..............................

Berapa jumlah foto yang digunakan untuk menutupi frame?..........................

Berapa besar framenya? ....

STOP !! Jangan diisi bagian berikut sebelum diminta gurumu!

Membandingkan ukuran frame foto

Bagaimana seharusnya ukuran foto di setiap frame agar dapat dibandingkan ukuran ketiga frame tersebut?

Tulis jawaban dan alasanmu dalam kotak di bawah ini!

Tuliskan urutan frame foto berdasarkan besar atau luasnya

1. 2.3.

Pelajaran Mini – Buatlah frame mu dengan foto yang diberikan

1) 12 foto kecil , 2) 4 foto besar , 3) 2 foto besar dan 4 foto kecil

Bekerja dalam kelompokmu, isi LKS ini perorangan

Lembar Kerja Siswa 4

Nama :

Pelajaran mini – Menemukan Luas Frame dengan Gambar Cepat LCD

Tulis berapa luas setiap frame yang dilihatkan secara cepat melalui LCD dan jelaskan caramu menemukan luasnya!

Frame 1

Frame 2

Frame 3

Frame 4

Frame 5

Berapa luas frame A ? ....

Tuliskan caramu menemukan luasnya

Berapa luas frame B ? ....

Tuliskan caramu menemukan luasnya

Mana frame yang lebih luas? Tulis jawabanmu dan alasanmu di kotak di bawah ini!

Berapa luas karpet A ? ....

Tuliskan caramu menemukan luasnya

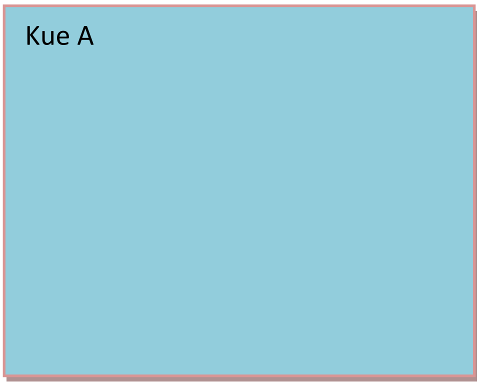
Berapa luas karpet B ? ....

Tuliskan caramu menemukan luasnya

Mana karpet yang lebih luas? Tulis jawaban dan alasanmu di kotak di bawah ini!

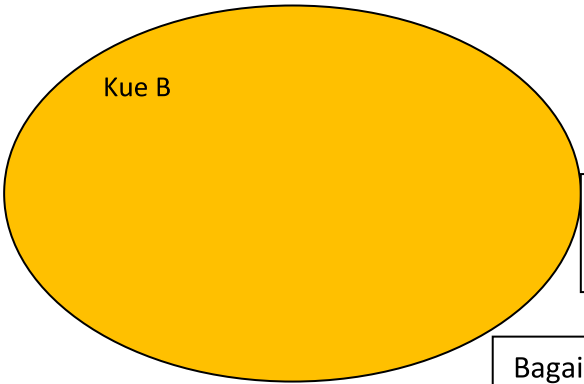
Diberikan kepada kelompok

Mini lesson



Luas kuepetak = ... cip petak

Bagaimana caramu menemukannya?

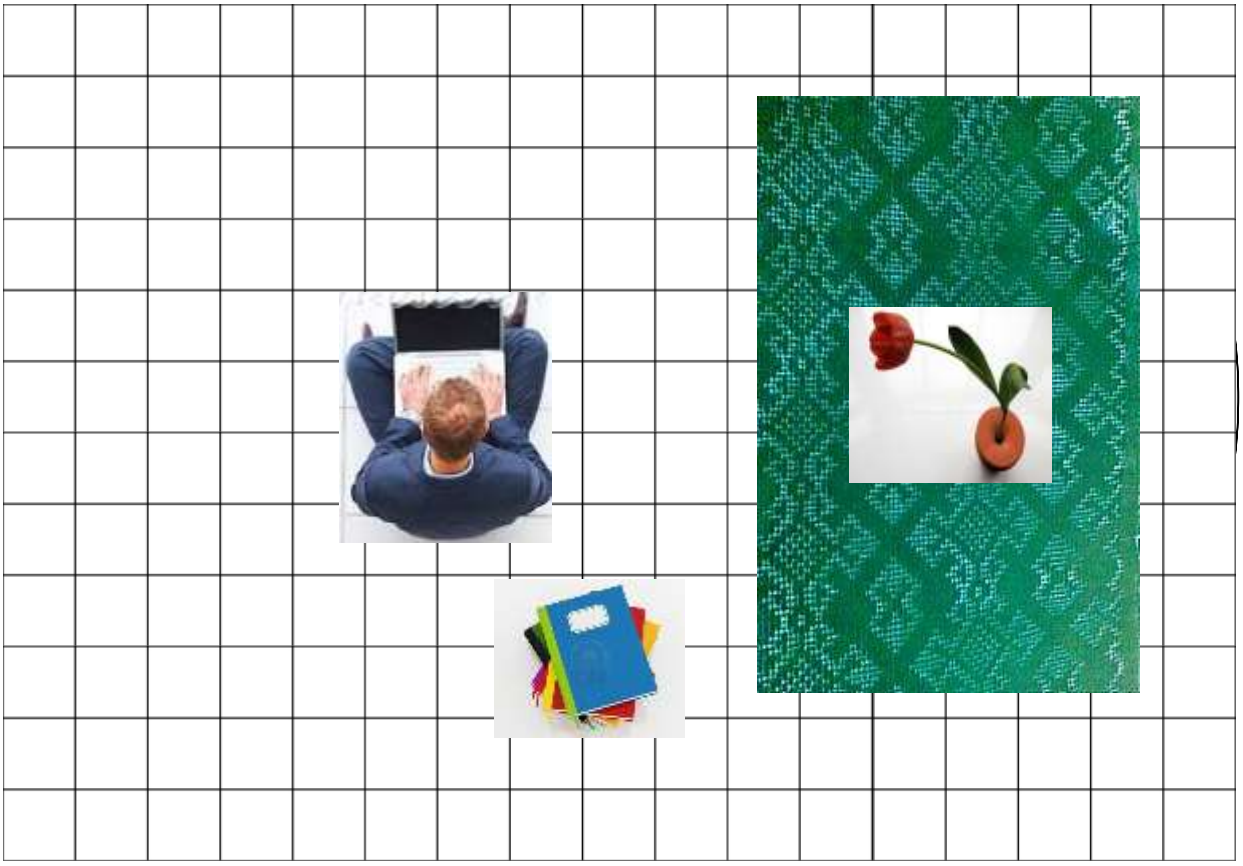


Luas kue oval= ..... cip petak

Bagaimana caramu menemukannya?

Diberikan kepada setiap siswa

Luas meja oval



Luas meja oval = ...keramik

Tuliskan bagaimana caramu menemukannya!

Kerjakan bersama kelompokmu

Lembar Kerja Siswa 6

Kelas/Kelompok :

Seberapa luas telapak tangannya? Ukur dengan transparan

1. Jiplaklah telapak tangan teman kelompokmu!
2. Kemudian gunakan petak-petak transparan dan spidol untuk mengukur luasnya.
3. Hasil pengukuran, tuliskan dalam jiplakan tangan!

Kerjakan sendiri

Nama :
Kelas/kelompok :

Pengukuran dengan transparan kedua

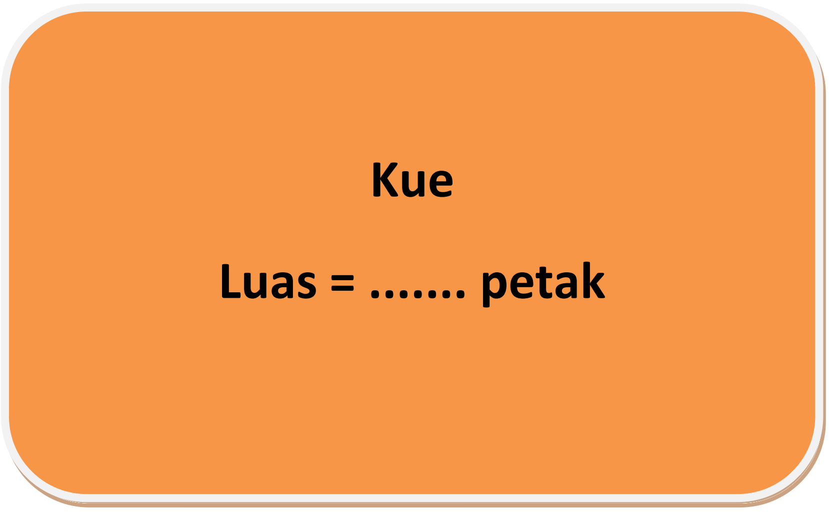
Bagaimana hasil pengukuran luas telapak tangan dengan menggunakan petak transparan kedua dibandingkan dengan menggunakan petak transaparan pertama?

Buat petak-petakmu sendiri

Jiplaklah dulu telapak tangan temanmu, kemudian buat petak-petakmu sendiri dengan penggaris dan temukan luasnya! Buat pada bagian di bawah ini!

Carilah luas kue dengan petak-petak transparan

Pilih petak transparanmu, kemudian ukur luas kue berikut!



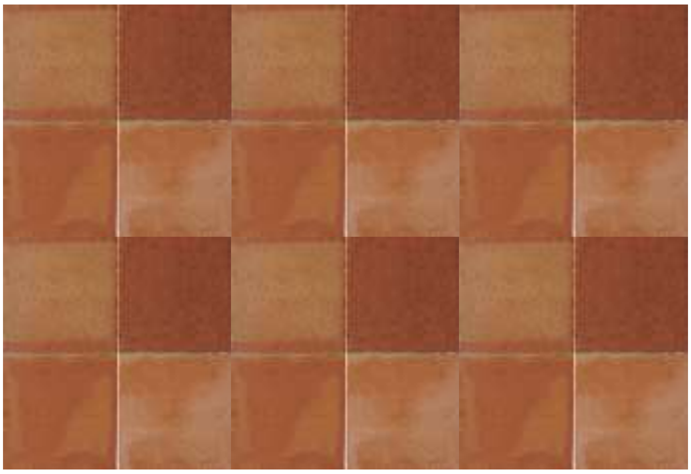
Bandingkan

Apa perbedaan mengukur luas dengan menggunakan kacang dan dengan menggunakan petak-petak? Yang mana menurutmu lebih baik digunakan untuk mencari luas? Tulis jawaban dan alasanmu di bawah ini!

Appendix F
Pretest for Cycle 1

1. This following is the picture of strawberry tart. Do you know how large is the top surface of tart? How do you find it?

Write your answer and your way to find the answer in this box



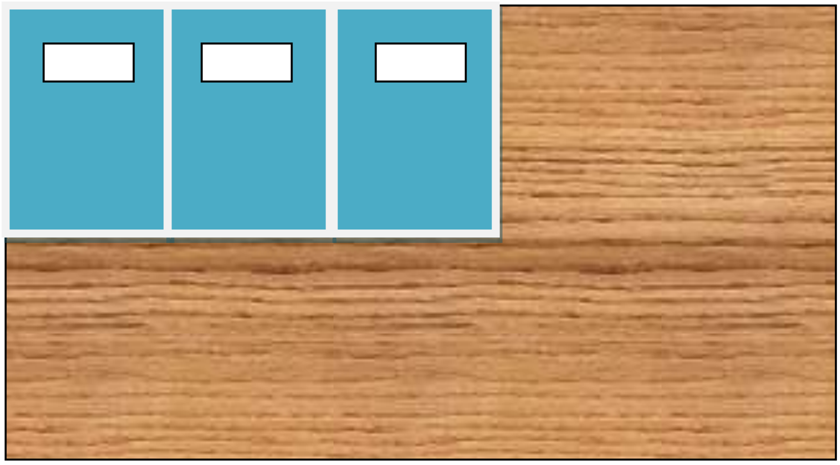
2. What is the area of the following tiled floor? How do you know? Write your answer on the following box!

Write your answer and your way to find the answer in this box

Paper quilt 1

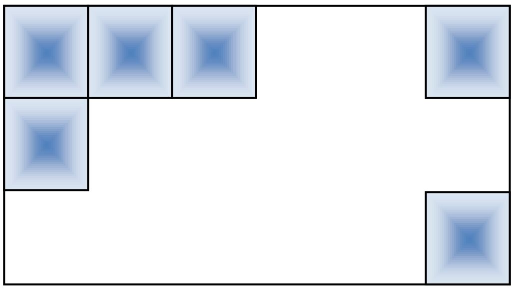
Paper quilt 2

Write your answer and your way to find the answer in this box

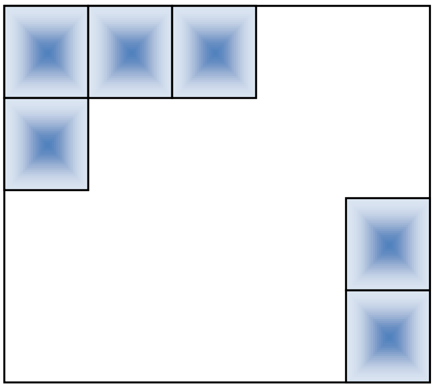


4. Ani wants to find the size of her table surface. She arranges the books on top of it. Do you know how large her table is? How do you know it? Write your answer!

Write your answer and your way to find the answer in this box



Floor A



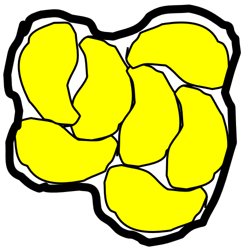
Floor B

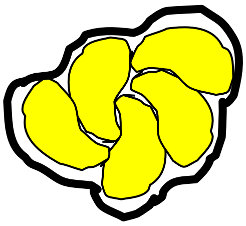
5. Which floor is the larger? Why is it so? Explain your answer in the following box!

Write your answer and your way to find the answer in this box

AppendixG
Posttest (Cycle 1)

1. Look at the following cashewnut cookies! Which cookie do you think is larger?



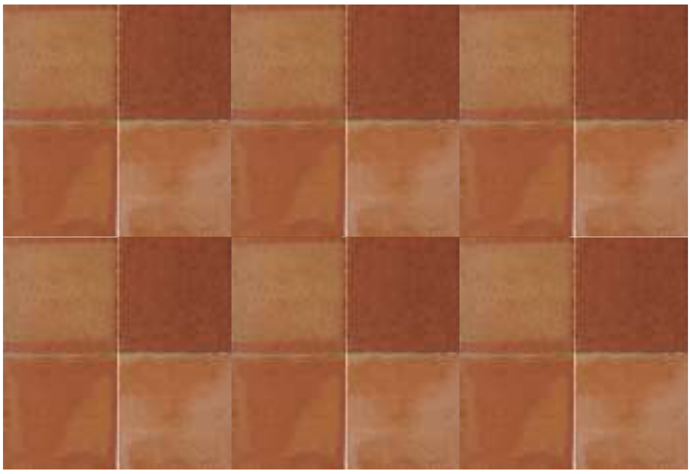


Cookie A

Cookie B

Write your answer and tell how you find it!

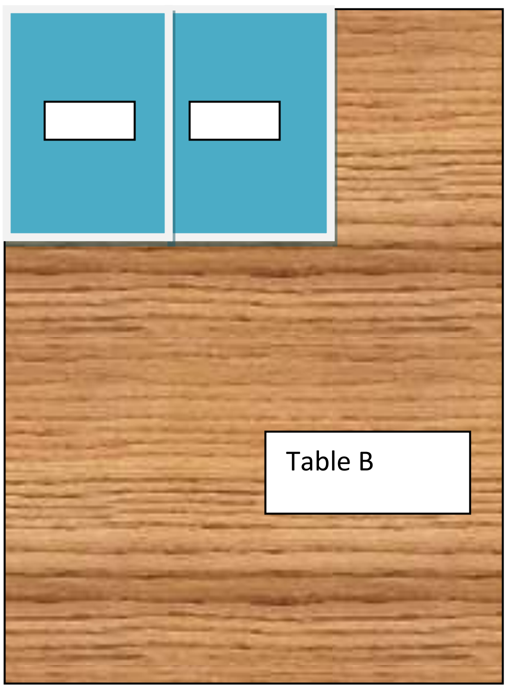
2. What is the area of the following tiled floor?



Write your answer and tell how you find it!

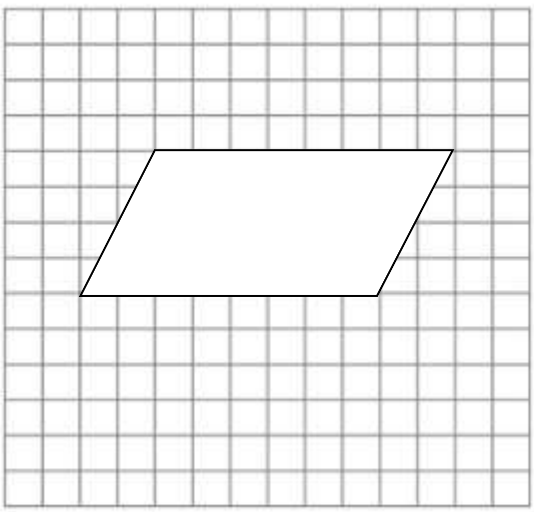
3. The following pictures are the top view of table and the books. Can you determine which table surface is larger?

Table A



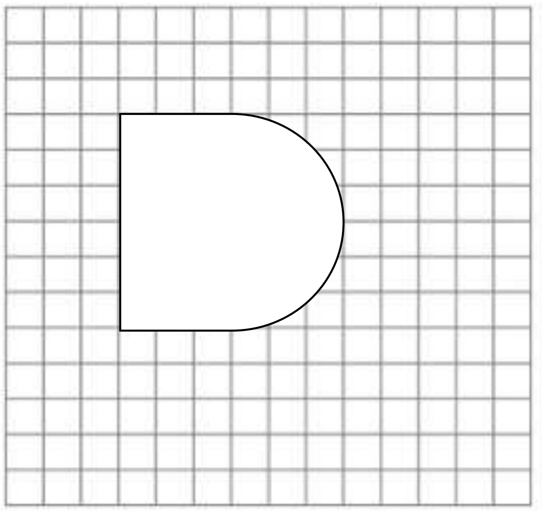
Write your answer and tell how you find it!

4. Can you find the area of the following plane?



Write your answer and tell how you find it!

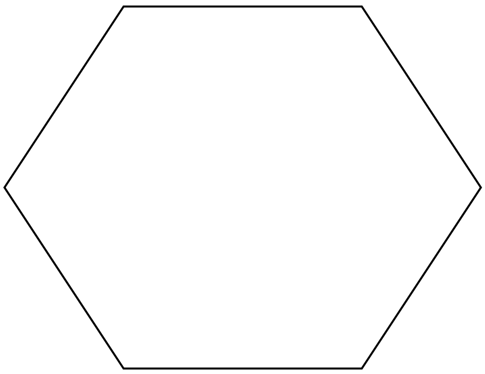
5. Can you find the area of the following plane?



Write your answer and tell how you find it!

6. Based on the answer of the questions 4 and 5, can you determine which plane is larger?Write your answer and tell how you find it!

7. Make your own grid to find the area of following plane figure!

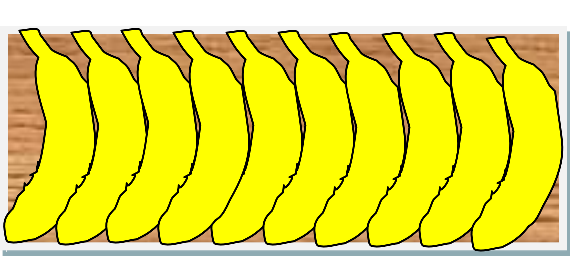


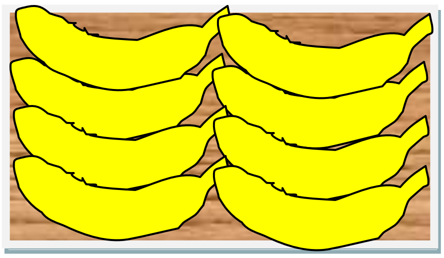
Write your answer and tell how you find it!

Appendix H
Pretest (Cycle 2)

Name :

1. The following pictures are two banana racks, A and B. Which rack has the larger surface?

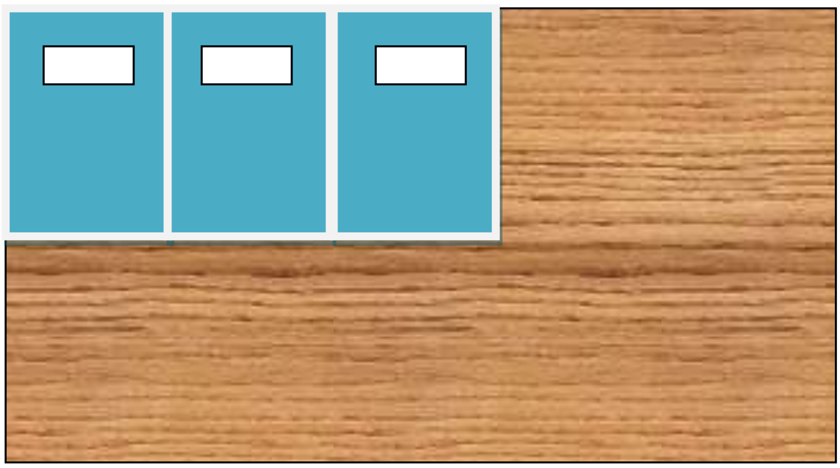




Rack A Rack B

Write your answer down here and describe how you find it!

2. Take a look at the following picture!



Ani wants to find how large is her table surface. She put her books on her table.Do you know how large is her table? Write your answer down here and describe how you find it!

3. The following picture are two paperquilts.

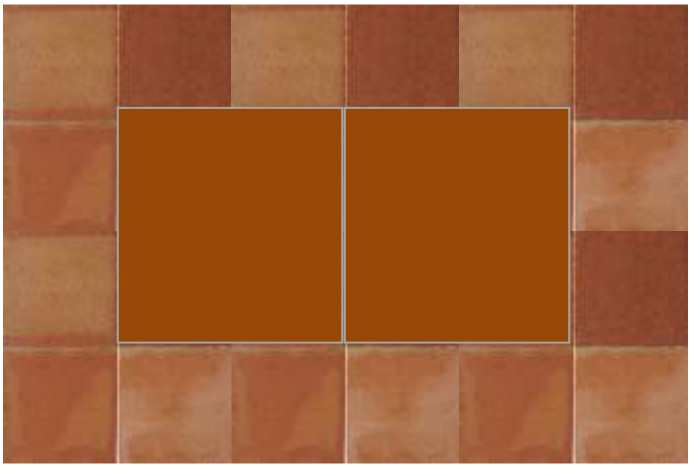
Paperquilt 1

Paperquilt 2

Which one is the larger one?

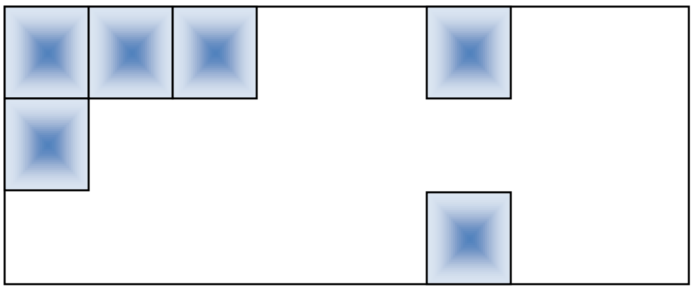
Write your answer down here and describe how you find it!

4. How large is the following tiled floor?

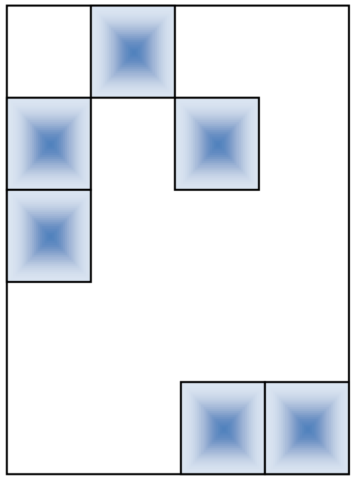


Write your answer down here and describe how you find it!

5. The following pictures are two unfinished tiled floor.



Floor A



Floor B

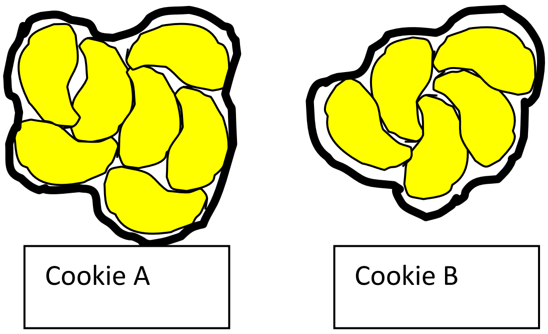
Which floor is the larger one?

Write your answer down here and describe how you find it!

Appendix I

Posttest (Cycle 2)

1. Look at these two cashewnut cookies! Which cookie is larger?Write your answer down and describe how you find it!

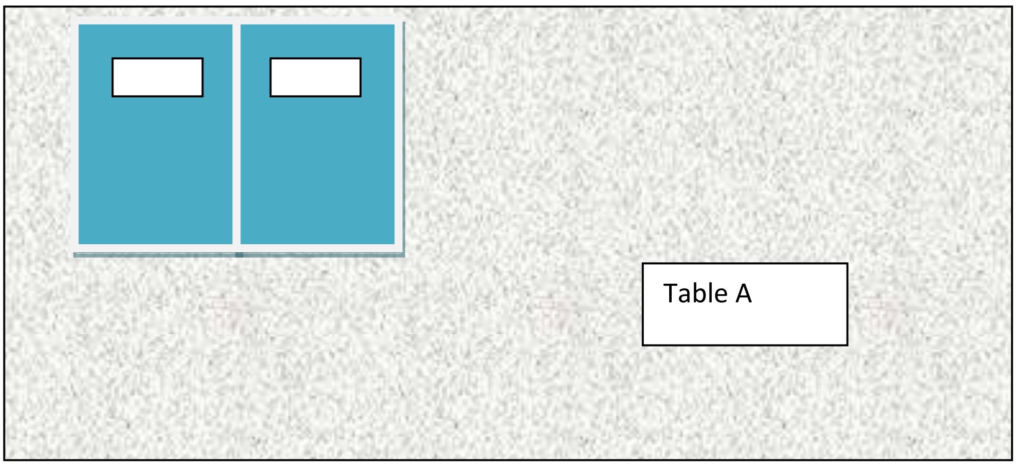


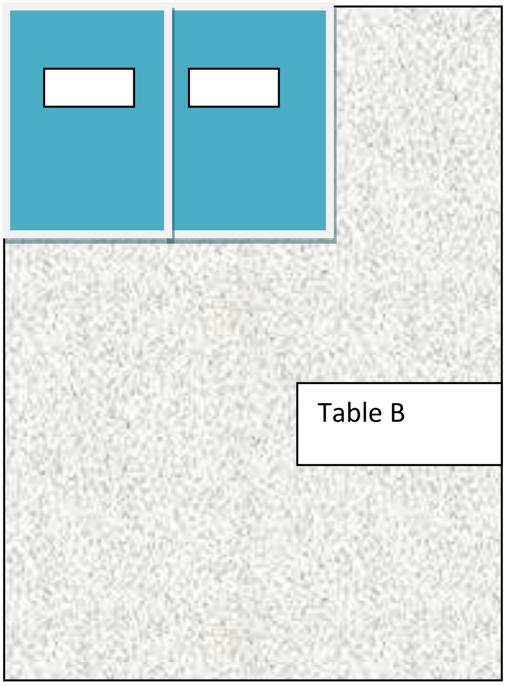
Write your answer down here and describe how you find it!

2. What is the area of following tiled floor?Write your answer down and describe how you find it!

Write your answer down here and describe how you find it!

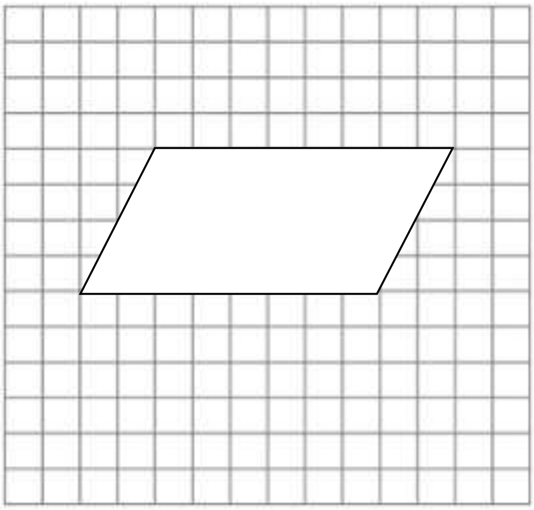
3. These following pictures are the top view of table and books. Can you figure out which table surface is the larger one?





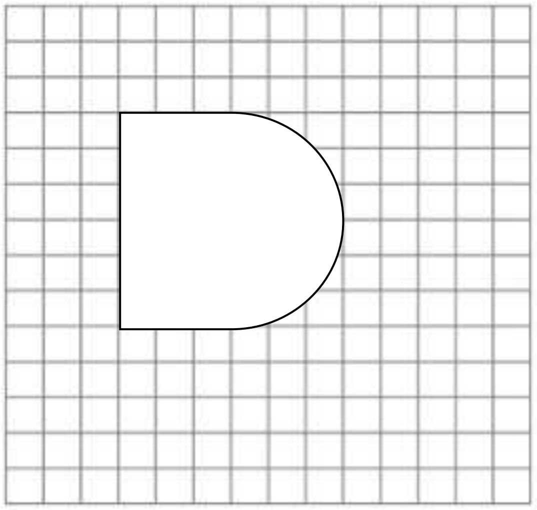
Write your answer down here and describe how you find it!

4. Can you find out the area of following plane figure?



Write your answer down here and describe how you find it!

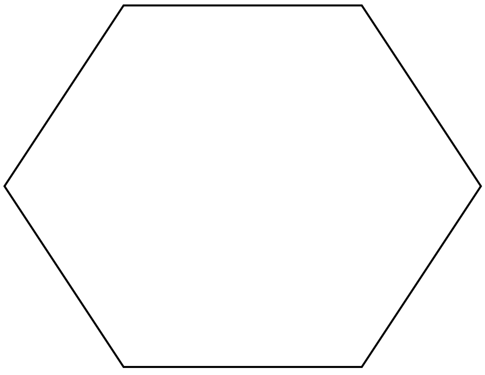
5. Can you find out the area of following plane figure?



Write your answer down here
and describe how you find it!

6. Based on the question number 4 and 5, which plane is larger? Write your answer down here and describe how you find it!

7. Make your own grid to find the area of following plane figure!



Write your answer down here
and describe how you find it!

Appendix J

The Overview of Students Response on Pretest Result (Cycle 2)

School : UNESA Laboratory Elementary School
Class : IIIA
Students : 15 (14 attended the test)

*See Appendix H Comparing area Measuring area

No	Problems*	Students' response
1	Two banana racks	8 students responded rack A as the larger one for it contains more bananas. 1 pupil responded rack A as the larger one for it looks longer 1 other pupil chose rack B for it looks larger 1 pupil chose rack B for the arrangement of banana that makes it larger. 3 other students chose rack A for the arrangement which makes it larger.
2	Tables and books	5 students used the books to express how large the table surface was The remaining students used imaginary ruler (since using ruler was not allowed) and explore in their mind the ruler on the table (clarified when interviewed). Some only referred to lenght of table, some referred to the width of table, while some other referred to the perimeter.
3	Two paperquilts	8 students chose paperquilt 1 for it contains more square papers in it 6 other students chose paperquilt 1 for it looks longer than paperquilt 2
4	Tiled floor	1 students answered correctly by the use of tile. 2 other students already regarded the tiles, but miscalculation 1 students express the area in the way : lenght is 6 tiles, width 4 tiles Other 10 students created their imaginary ruler and did some exploration on the tiled floor by that imaginary ruler. Some measured length, some emasured the width.
5	Two unfinished tiled floor	5 students answered floor A as the larger one since it contains more tiles 7 students answered floor A for it looks longer 2 students answered both floor were the same

Suggested groups discussed with teacher

Focus group 1

1. Rama
2. Sasikirana (Sasi)
3. Raffa
4. Nastity Ivory (Asti)
5. Franscisca Ranti (Ranti)

Group 2

1. Hammada Nadhif (Adhif)
2. Talitha Rizkina (Tita)
3. Bunga
4. Aisyah Dyah M (Aisyah)
5. Adit

Group 3

1. Munawar
2. Hiu
3. Thalia Olga (Olga)
4. Ersa Setyafirliani (Ersa)
5. Desyanti M. Putri (Dinda)

Appendix K

The Overview of Students Response on Posttest Result (Cycle 2)

School : UNESA Laboratory Elementary School
Class : IIIA
Students : 15

*See Appendix I — Comparing area (yellow) — Measuring area (grey)

No	Problems*	Students' response
1	Two cashewnut cookies	14 (inlcuding our focus group)students responded that cookie A was the larger one for it contains more nuts 1 pupil did not yet regard the nut at all, this pupil reasoned by visual reasoning
2	Tiled floor	10 students (including four of our focus group students)used the small tiles to derive the area of tiled floor (30) 2 students (including one of our focus group students) used the larger tiles to derive the area of tiled floor (7,5) 1 pupil used smaller tiles but did miscalculation 1 pupil got wrong answer and provided unclear reasoning 1 pupil used imaginary ruler and did exploraion on the perimeter
3	Two tables and books	13 students (including four of our focus group students) used the book to derive each area of the table then compared them to see which one was larger 2 (including one of our focus group students) students used visual reasoning, not used the books
4	Plane figure with grid	3 students (including two of our focus group students) drew the grid inside the outline precisely and did the best aprroximation of the number of squares 9 students (including three of our focus group students) generally drew the grids inisde the outline, but did poor counting and estimation 3 students drew the wrong structure of square grids inside the outline and did wrong calculation
5	Plane figure with grid	7 students (including four of our focus group students) drew the grid inside the outline precisely and did the best approximation of the number of squares 5 students (including one of our focus group students) generally drew the grids inisde the outline, but did poor counting and estimation 3 students drew the wrong structure of square grids inside the outline and did wrong calculation
6	Compare plane question no. 4 and 5	Students regarded the number of squares on each plane figure when comparing the area

7	Make own grid and measure	6 students (including two of our focus group students) could make the square grid almost precisely and performed counting and estimating the number of tiles as the area of figure correctly 3 students (including two of our focus groups students) drew the square grid not really well (there were some different sized grid squares) and they performed an inaccurate estimation of the number of squares 6 students (including one of our focus group students) drew unstructured square grids and some of them performed the incorrect way of counting.