UNDERSTANDING THE COORDINATE SYSTEMS

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ABSTRACT

This study deals with the challenge of making an educational innovation in teaching and learning coordinate system for the 10-12 year old elementary students. In order to help the students understand about the mathematical ideas of the coordinate systems we designed a six-step learning sequence (hypothetical learning trajectories/ HLT) based on Pendidikan Realistik Matematika Indonesia; an adaption of the Realistic Mathematics Education. In reaching the aim of this study which is to contribute to the local instructional theory of the coordinate system topic and in answering the proposed research question by examining how the designing activities can helps the students to learn and understand about the coordinate system, design research is employed in this study. The six designed activities and the HLTs are tested in the preliminary teaching (cycle 1, involved eight students of Grade 5), revised, and tested again in the classroom experiment (cycle 2, involved 34 students of Grade 5). These activities are implemented continuously in each cycle. Video and audio recordings, field notes, students' works, pre test and post test are the data collected in this study. This data then is analyzed. Our analysis is focused on how the designed activities help the students to understand the mathematical ideas behind the coordinate system. The findings suggest that the proposed contexts or activities (seat map of cinema, seat map of airplane, locate the sunken ship, modified rice field) which are combined with the teacher's questioning can make the students (participant of this study) think about the mathematical ideas of the coordinate systems. It can enhance students' understanding and brings out the students' different level of approaches and thinking of the ideas behind the coordinate system.

Keywords: *Coordinate systems, Realistic Mathematics Education, Pendidikan Matematika Realistik Indonesia, design research*

ABSTRAK

Penelitian ini menjawab tantangan dalam membuat sebuah inovasi dalam mengajarkan sistem coordinate di sekolah dasar. Kami mendesain enam aktifitas (hypothetical learning trajectories/ HLT) untuk membantu siswa kelas 5 dan 6 (10-12 tahun) memahami ide matematika dari sistem koordinat. Keenam aktifitas tersebut didesain berdasarkan karakteristik dari Pendidikan Realistik Matematika Indonesia (PMRI); adaptasi dari realistic mathematics education (RME). Penelitian ini bertujuan untuk memberikan kontribusi dalam local instructional theory mengenai koordinat sistem. Untuk itu, design research digunakan sebagai alat dan metodologi dalam penelitian ini. Desain aktifitas dan HLT diujicobakan dalam preliminary teaching (siklus 1, melibatkan 8 siswa kelas 5), direvisi, dan diimplementasikan dalam classroom teaching expereiment (siklus 2, melibatkan 34 siswa kelas 5). Keenam aktifitas tersebut diujicobakan secara berkelanjutan dalam setiap siklusnya. Video dan audio recording, field notes, dan dokumen hasil kerja siswa, hasil dari pre test dan post test adalah data yang dikumpulkan dalam penelitian ini yang kemudian dianalisis. Analisis difokuskan untuk menjawab pertanyaan penelitian yaitu, bagaimana aktifitas yang telah didesain dapat membantu siswa untuk memahami ide matematis dari koordinat sistem. Berdasrkan hasil analisis dapat disimpulkan bahwa siswa belajar mengenai koordinat sistem dari sesuatu yang riil ke abstrak. Mereka belajar megenai koordinat sistem melalui (masalah/aktifitas) konteks yang diberikan (bioskop, pesawat, melokasikan kapal tenggelam 1&2, sawah, dan Cartesian koordinat sistem). Konteks yang meaningful bagi siswa, dikombinasikan dengan pertanyaan yang diajukan oleh guru membuat siswa untuk berpikir mengenai ide matematika yang dibahas. Hal ini mendorong pemahaman siswa dan dapat memunculkan strategi siswa yang berbeda dalam menyelesaikan masalah yang diberikan.

Kata kunci: *Koordinat sistem, Realistic Mathematics Education, Pendidikan Realistik Matematika Indonesia, design research*

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CHAPTER I

INTRODUCTION

1.1 Background

Coordinate system is an important topic that should be learnt by students. Not only because it is an important component of graph and map reading (Blades & Spencer, 2001) or because it is an important factor in spatial development (Dickson, Brown, & Gibson, 1984) but also because it helps to organize the world. The coordinate systems also help us to communicate a location of an object easily and precisely. For instance, we can easily find a certain book in the library by looking at the code put on the books. That code is a coordinate system.

We cannot ignore the fact that we live in the coordinate system. We can find the coordinate systems everywhere. Our sitting arrangement in class, the houses in avenue, books in the library, and paragraphs in articles are some examples of the coordinate systems. So, coordinate system is everywhere and rich of sources/ context.

Although coordinate system is rich of contexts, it is rare to see the context used in teaching the coordinate system. In Indonesia, coordinate system is directly introduced to the students. Students only had given its form (how it looks like), its notation and its rule and they need to remember it. So in the first time the students learn about coordinate system, they directly and only learn the formal level of it. If there is context used in the learning the coordinate system then it is only used to introduce the topic or it is used to test the students' understanding. Yet, it does not help the students to understand the coordinate system. This is in line with what Gravemeijer and Doorman (1999) is pointed out. The role of context is limited to

the applications problem and be addressed in the end of learning sequence (Gravemeijer & Doorman, 1999).

Moreover, the common way of teaching of coordinate systems, which is only focus on its formal level and only require the students to remember the rules, leaves out the mathematical ideas of the coordinate systems. In other words, the mathematical ideas like the idea of origin and the idea that each point is unique are missed to be addressed. Hence, the students do not have a deep understanding of coordinate system. They might know that coordinate system is used to locate a point, but they do not understand how a good coordinate system should locate that point. Or why they have to start to locate from the certain point.

Without deep understanding of the coordinate system, students might face difficulties in learning it. Based on Sarama *et.al*. (2003) students tend to reverse x and y axis, especially when they have to plot point that have 0 in its coordinate (e.g. (0,6). In addition, some of students sometimes ignore the origin. They plot a new point started from the previous point, not the origin.

In order to help the students understand the coordinate system and the mathematical ideas behind it, we have to moves away from the current way of teaching coordinate system. We have to make innovation, so the learning can become meaningful for the students. This is in line with the current thinking of education in Indonesia starts to moves away from the transferring knowledge to teaching meaningfully (Sembirig, Hoogland, & Dolk, 2010).

To make the learning become meaningful for the students, rather than teach the coordinate system topic as ready-made product, we should give the students opportunity to experience and to understand the mathematical idea of

coordinate system. Rrealistic mathematics education (RME) and also Pendidikan Realistik Matematika Indonesia (PMRI) which is adaptation of RME suggest that instead of start the learning from the formal level, it is better if we start it from the informal level in which we used the 'real' meaningful context that near and familiar with the children. Rich and meaningful context which is combined with teacher's probing question can enhance students' thinking and understanding (Widjaja, Dolk, & Fauzan, 2010). This is in contrast with the common way of teaching of coordinate system in Indonesia which is lack of content and discussion.

So, this study deals with the challenge in making educational innovation in teaching and learning the coordinate systems. In order to help the students learn the coordinate system meaningfully. We design learning activities and the hypothetical learning trajectories (HLT) of coordinate system topic based on RME and so *Pendidikan Realistik Matematika Indonesia* (PMRI). In each activity, we use real context and combine it with teacher probing questioning to facilitate the students in understanding the mathematical ideas of coordinate system.

We mean to know "How can the students learn to understand a coordinate system?" by finding the answers of these two sub (research) questions: "What are the contexts which can be used to help or support students in understanding the coordinate systems?" and "How can the designed activities help the students to understand (the mathematical ideas) of coordinate systems?". And because the aim of this study is to contribute to the local instructional theory and to contribute to educational innovation of coordinate system topic, design research is chosen as the research tool and approach.

1.2 Research Aim

In order to make students aware of the importance of coordinate system (as organizing system and communicating) and to help students understand the big ideas of coordinate system and overcome their difficulties, I would like to conduct a study about understanding the coordinate systems. The aims of this research are to contribute to educational innovation and to an empirically grounded instructional theory for the coordinate system topic.

1.3 Research Question

In order to reach the research aims, in this research we design learning activities of coordinate system topic (based on RME approach) which can help and guide the students to understand the big ideas of coordinate system. So they will learn about coordinate system meaningfully instead of only remembering and memorizing the rule. The proposed research question is "How can the students learn to understand a coordinate system?". We will answer that research question by answering these following sub questions.

1. What are the contexts which can be used to help and/ or support students in understanding the coordinate systems?
2. How can the teaching materials (context problems) which designed by the researcher help the students

CHAPTER II

THEORETICAL BACKGROUND

2.1 The coordinate systems and its importance

The coordinate systems are important subject to be understood and learnt. Mathematicians and scientist need it to make a graph and equation. The sailor and pilot need it to find their ship and airplane location (in where they are) so they will not get lost. They also need it to find their way to their destination. The topologists and map makers need it to draw an island or country. It is also used to determine the time zones. It is also helps us to find the direction if we looking for a room in a building and etcetera. So, the coordinate systems not only used in locating a graph, but it is also used to locating a thing in a plane even in the three dimensional. It has important role in navigation and location.

Coordinate system is rich with context. We live in a world which is surrounded by coordinate system. We can easily find the coordinate systems around us. Our sitting arrangement in class, the houses in avenue, the seats of the plane and theater, books arrangement in the library, rooms in the building, paragraphs in articles etcetera are some examples of the coordinate systems.

Those coordinate systems are made by people to help them organizing the objects and help them to locate an object precisely. For instance, if we say a room number 126 it means the 26^{th} room on the first floor. So with the help of the coordinate system, people can easily find that object and can easily communicate its location precisely to other.

In geometry, coordinate system is a system which uses a set of numbers, direction, or angle to determine the fixed location of an (geometric) object in the

plane or in space (Woods, 1922). This means that the positions of an object or other geometric elements in the plane or in space is uniquely located by the coordinate. In addition, a coordinate only represent the location of a point or an object.

There are several sophisticated coordinate system which have been developed, such as, the rectangular coordinate system (so called Cartesian coordinate system), polar system, map coordinate system, etcetera. All of them is invented to give a precise and exact location of a point in the plane from a certain '*origin'* point/ location.

The idea that each point is uniquely located by a coordinate implies that it needs the unique origin. The starting point from where the objects are located or from where we should interpret the coordinate should be one (unique, same, and consistent). It is impossible for a system to have two different origins because it can leads to different locations. It will make the system inconsistent and imprecise in locating an object and make the user of that system get confused.

For the researcher, this implies that in helping the students to understand the coordinate system, we should help them to understand the mathematical ideas behind it. We have to facilitate the students so they can learn and understand the idea of origin and the idea of each point is unique. Furthermore, since coordinate system is rich of context, we can use the real context to make the learning more meaningful for the students.

2.2 Coordinate system in the Indonesian curriculum

In Indonesia, the topic of coordinate system is started to be given in second semester of the sixth grade of elementary school (for 11-12 year old students)

(Depdiknas, 2006). The standard and basic competences of the topic of coordinate system are described in the following table.

Table 2.1 *Standard competency and basic competencies of coordinate system*

Standard competence	**Basic competence**
Geometry and measurement 6. Use the coordinate system in solving a problem	6.2 Know the coordinate of an object 6.3 Determine the location/ coordinate of a point in the Cartesian coordinate system

In the common practice in Indonesia, the coordinate system topic is limited by only two coordinate systems, map coordinate system and (2-dimensional) Cartesian coordinate system. Those coordinate systems are taught to the students of Grade 6 directly. Teacher tells the definition of coordinate which is often defined as numbers which are used to locate a point in the plane or in space (Permana, Dadi, A., & Triyati). Teacher only tells that coordinate system is used to locate an object. Then teacher shows how the map and Cartesian coordinate systems look like and how to locate a point using that system. The students are not given a chance to find the system by themselves. They are only asked to remember the rule and the notion used.

Moreover, the common way of teaching coordinate system in Indonesia is lack of context. Even there is no context used in teaching the Cartesian coordinate system. If there is a context then that context is only used to introduce (as a preface) the word coordinate or used in the assessment questions. Yet it does not help the students to understand what coordinate (system) is and the mathematical ideas behind it.

2.3 Students' difficulties in learning coordinate system

The previous studies by Blades and Spencer (2001) about the 4 to 6 year old children's ability to use the coordinate references. Their study indicate that many children can use a coordinate reference system at the age of 4 and for 6 years old children success in doing tasks about understanding of grid. This finding support the Somerville's and Bryant's (1985) finding in which they found that by the age of 6 most children could use coordinates. Moreover Piaget *et. al.* (1960) argued that 4 year old children already know horizontal and vertical line, right-left, and distance.

Yet in Indonesia, the coordinate system topic is started to be taught in the Grade 6 (11-12 year old). Students directly introduced to Cartesian and map coordinate system. They only had given the shape (how it looks like) and its rule to use it. So in the first time the students learn about coordinate system, they directly learn the formal level of it.

For some people, the coordinate system is seen as an easy topic to be learnt. It is only an agreement which can directly to be told to the students as. Yet in fact students still face difficulties in learning the coordinate system.

Based on Sarama *et.al*. (2003) students tend reverse x and y, especially when they have to plot point that have 0 in its coordinate (e.g. (0,6)). In addition, some of students sometimes ignore the origin. They plot a new point started from the previous point, not the origin. Not only that, some students fail to relate coordinates and location of multiple points. For instance, they plot (20, 35) and (25, 35) each from the origin point. They cannot see that actually the second point is 5 point to the right from the first point. Moreover, based on Blades and Spencer

(2001), the children's struggle or mistake of determining the coordinate/location of an object is they often to see only one coordinate line rather than two coordinate lines.

Furthermore, although we can easily find the coordinate systems around us, some people and students do not recognize the other kind of coordinate systems beside the Cartesian coordinate system (or maybe map coordinate system). For example, the system of seat map in a plane or theater, teeth prefix name, rooms in a building and etcetera. They tend to not see it as a coordinate system. Those happen because students only learn the formal level of the coordinate system topic.

Moreover, directly tell the students the information and knowledge will make the learning become not meaningful for them. Only telling the result of the agreement is not enough. Students also need to know why and how that agreement and rule are needed and made. So, students need to experience the coordinate system by themselves.

2.4 Situate the Research

The common way of teaching the coordinate system which only asks the students to remember the rule, notion and the form of (Cartesian) coordinate system, leaves out the important mathematical ideas behind the coordinate system. The idea of origin and each point is unique are missed to be addressed.

In addition, the teaching which lack of content and only focus on remembering symbols and notions is not meaningful for the students. So, students might remember the notion and rules well, but it does not mean they understand it. Without a deeper understanding of coordinate system, students might make

mistakes in solving the problem related to the (Cartesian) coordinate system as explained in the 2.3.

Considering the fact above and noticing the current thinking in Indonesia which starts to move away from teaching mathematics as transferring knowledge to teaching meaningfully (Sembiring, Hoogland, & Dolk, 2010). We also need to moves away from the common way of teaching the coordinate system. We need to help the students to understand the coordinate system, not to remember. We need to make the learning process of coordinate system become meaningful for them.

However, there are only a few attentions on this topic. In Indonesia, there are only few researches on coordinate system. This is because of the opinions that coordinates system is an easy topic to be learnt and it is not a rich topic to be studied. Yet, coordinate system is a fundamental topic to be understood in mathematics. Not only because it helps us with our daily routine, but it is also needed for another topics in mathematics such as graph and function.

So this study deals with the challenge to help the students to understand the coordinate system. In other words, this study deals with the challenge to make a learning sequence which addresses the mathematical ideas of the coordinate system and involves the real contexts. In addition, it should be able to enhance the students thinking and give them opportunity to experience, or if possible, to reinvent the coordinate system. We also would like to know how the designed learning sequence can help the students to understand the mathematical ideas behind the coordinate system.

2.5 Reinventing and Understanding the Coordinate Systems

As I stated before that directly tell the students the information and knowledge will make the learning become not meaningful for them. Students may not understand the big ideas behind a coordinate system. And students may not realize the important of the coordinate system. So, we should give the students an opportunity to reinvent the coordinate systems.

That idea is walk along with Freudenthal's idea that 'mathematics as a human activity'. Freudenthal argued that students should be given the opportunity to reinvent or construct and develop their own mathematics and their mathematical thinking and reasoning (Gravemeijer, 2010). So mathematics should not be taught as 'ready-made product'. Teachers should facilitate, guide and foster that reinvention. So, the learning will be meaningful for them.

Let the students reinvent the coordinate system by themselves without any help is difficult. So, instead of let the students find the system by them without any helps, we should guide them to reinvent coordinate system. Based on Bakker (Bakker, 2004) there are three methods to support guided reinvention.

- The first method is "through experiment" (Freudenthal's idea). In this study, we engage students to explore and work on problems which give them opportunity to the students to reinvent the mathematics issue or idea themselves. For example, in the first and second activity, we give the students a chance to make a system for a airplane's seats and see and investigate how a system of teather's seats works. Furthermore, the students are also given a chance to investigate the importance of origin point through the corn field activity (finding the location of given coordinate).

- The second method is through study the history. if we look back to the history how the (Cartesian) coordinate system was invented by Descartes, we will know that the coordinate system was invented to locate a fly which landing on a ceiling. This situation is similar with the third activity designed in this study which asks the students to locate and communicate the location of a fire in the forest.
- The third method is using the students' informal strategies as source (Streefland, 1991). In this study, the students' informal strategy in locating a ship could be discussed and used as a consideration in finding another good system includes the Cartesian coordinate system.

Generally, in this study we guide the students to reinvent a coordinate system start by giving students contextual and real problem (didactical phenomenology) like finding a system of a plane's seats which is continued by looking how a system work then making a better coordinate system (Cartesian). In addition, manipulating the situation/ problem like making the field into squares (grids) and posing a question in about the mathematical ideas may help them to come up with a more structured and sophisticated coordinate system.

This is in a line with Blades and Spencer (2001). They argue that the task which did not provide or involve grid coordinate and asks the students to find out that system by themselves is difficult and need high order of thinking. Yet, when grids are provided, the young children are face less difficulty in using a coordinate reference. In addition, the task which is not presented in a concrete context make the task become harder for the younger children to understand.

However we realize that make the students to reinvent a coordinate system is need a long time and big effort. It will be not enough by only six meetings. So in this study, we try to be modest by only focusing our research in helping the students to understand the coordinate systems. Yet, we will not exclude the possibility that the students might reinvent or make up their own system through some provided activities.

So in this study, the researcher design a learning sequence (activities) which contains the familiar context, grid, questions that can help the students to understand the coordinate system. The learning sequence (design) will be developed based on the principles and five tenets of RME. We adjust the context with the Indonesian students' knowledge (PMRI) so it will meaningful for them.

2.6 (Indonesia) Realistic Mathematics Education Aspect in This Study

As we stated before that the activities in this study are designed based on the realistic mathematics education (RME) and so *Pendidikan Matematika Realistic Indonesia*/ PMRI; an adaptation of RME (Sembiring, Hoogland, & Dolk, 2010). In his research, Bakker (2004) stated that based on Treffers (1987), there are five characteristics/ tenets of RME. How those characteristics support the design of reinventing a coordinate system will be described as follow.

- Phenomenological exploration (the use of context)

In the previous study, Blades and Spencer (2001) argued that the task (of coordinate reference system) which is not presented in a concrete context make the task become harder for the younger children to understand. This is different with Somerville's and Bryant's (1985) task which is more realistic (meeting point

problem) for the younger children. So, this indicates that a context has important role in helping students to understand the coordinate system.

There are several contexts used in the design made like numbering the plane's seats and theater seats to understand how a system works and give an opportunity for students to make their system. The field context which is manipulated into squares is used to help the students to make up a grid system which can be brought into Cartesian system and aware of the origin point. Another context used is locating the sunken ship context which gives the students chance to explore other possible coordinate systems and give them experience in locating point in 2-dimesional plane.

In choosing the context, we also consider whether the context is "real" for the students. In other word, the students are familiar with those contexts, or at least it is imaginable. Furthermore, because we conduct the research in Indonesia (PMRI project), we adjust the contexts so it fit with the Indonesian students' especially the participants of this study.

- Using models and symbols

In this study, there is no particular model. However, the students may use/ produce a variety of models, schemes and symbols like grid, rectangles, column and row which help them to understand the proposed mathematical idea in each activity and understand the Cartesian coordinate system.

- Students' creation and contribution

Students create and contribute in making a system to number the plane's seats, finding how the seat map of the theater works, and making a system to locate something. Students' creation will be brought into class discussion. Furthermore,

students are expected to give active contribution in group activity or in class discussion.

- Students' activity and interactivity in the learning process

Rather than working individually, in the design made, students will work in pair or in small group. By working on the group, student can share their idea and thinking. So, they will learn more from each other.

- Intertwining

The contexts used are intertwined with other subject like knowledge about plane, theater, and navigation. So, students not only learn about mathematics but also another subject in a time. Moreover, by intertwined it with other subject or things/ context will make the memory of the learning easily to recall.

2.7 The Role of the Teacher

Widjaja's, Dolk's, & Fauzan's (2010) find that meaningful context brings out mathematical thinking and discussion amongst the students. And if it is combined with the teacher's questioning related to the mathematical ideas, it will allow students to come up with different level mathematical approaches. This is in a line with the RME and so PMRI idea which is so called guided reinvention. The students are given freedom and opportunity to deal and experience the problem by themselves. Yet, it does not mean we do not provide any help for them. Teacher has the important role to play. How the teacher helps and guides the students, effects their thinking and understanding. In this study, teacher helps the students by asking the probing questions. Thus, the students start to think about the proposed mathematical ideas. In order to help the teacher, we provide the teacher with the clear teacher guides (Appendix C).

2.8 Emergent Perspective

Emergent perspective is use as the framework for interpreting the classroom discourse and communication (Gravemeijer, K., and Cobb, P., 2006). That framework is related to the social context (social perspective and psychological perspective) of the classroom. The social perspective itself consists of social norms, socio-mathematical norms, and mathematical practices.

- Social Norms

A social norm is defined as the ways of acting and explaining as the process of mutual negotiation between teacher and students. In this study, we expect to apply norms of reform mathematics class rather than to apply norms of traditional mathematics class. So, in this study, students are expected to be active, be able to explain and justify solutions, indicate agreement and disagreement, try to make a sense the explanation given by other and question some possible alternative solutions in the situation where a conflict of interpretation has become apparent (Gravemeijer, K., and Cobb, P., 2006). Moreover, because the students will work in a group, they should be able to communicate and negotiate each other. Yet, we realize it is a difficult task to do.

- Socio-mathematics norms

A Socio-mathematics norm is defined as the ways of explicating and acting in whole class discussion that is specific to mathematics (Gravemeijer, K., and Cobb, P., 2006). In this study, students are expected to be able in making independent judgments. They should be able to judge whether the statement stated by others is mathematically different, sophisticated, and acceptable. The designed activities in this study are made in such a way it has alternatives solutions and

leads to discussion. For example when the students have to make a system for the airplanes' seats, there will be more than one system occurs. So the students in this study needs to be able to make independent judgments.

- Mathematical practices

Mathematical practice is defined as the normative ways of acting, communicating, and symbolizing mathematically at a given moment in time (Gravemeijer, K., and Cobb, P., 2006). In this study, the students are given the opportunity to experience the mathematical practice in learning coordinate system through each activity in the learning series that have been made. For instance, the students experience to make a system for seats in the airplane, make a system to tell the location of a ship, etcetera.

CHAPTER III

METHODOLOGY

3.1 Participant

As we stated in the 2.2 “Coordinate system in the Indonesian curriculum”, the coordinate system is started to be taught in the 6th Grade (11-12 year old students). Ideally, this study should be conducted in the 6th Grade. However, due to the national examination issue, the designed learning sequence and activities are tested in the grade five (10-11 year old) which is still appropriate because they had learnt the needed prior knowledge. There are 42 students of Grade 5 which are involved in this study. Eight of them are involved in the first cycle (preliminary experiment) and the other 34 are participated in the classroom teaching experiment (cycle 2). What we mean by first and second cycle will be explained in the following part (3.2). This study is conducted in a primary school in Palembang. This is because this study is under the PMRI project.

3.2 Research approach

The purposes of the present study are to contribute to educational innovation and to an empirically grounded instructional theory for the coordinate system topic. Due of that, we design a learning sequence (hypothetical learning trajectory) completed with the students material as a help for teacher and students in learning and reinvent the coordinate system. That learning sequence is addressed for the students in the grade six of elementary school (11-12 year old). But we changed our target group as explained in the 3.1. Furthermore, the designed hypothetical learning trajectory is also used to investigate how the students learn (thinking and reasoning) and reinvent the coordinate system. So, designing and testing an

learning sequence (HLT) in the classroom setting is considered as the crucial part of this study.

In order to reach the aims of this study and finding the answer of the proposed research question, design research is employed in this study as the research approach and methodology. There are three phases in the design research (Gravemeijer and Cobb, 2006). They are the preparation and design phase, the design experiment (preliminary teaching and teaching experiment), and the retrospective analysis phase. Those three phases will be elaborated further as follow.

Phase 1: Preparation and design

The preparation and design phase is aimed to design and formulate a conjecture of local instructional theory known as hypothetical learning trajectory/ HLT). So in this phase, the researcher design the HLT which consists of the learning goals, a learning sequence activities/problems, the conjecture of students' learning and thinking of coordinate system, students materials, teacher guides, and other instrument needed for this study like pre test and post test, a list of classroom observation and a list of teacher interview. The HLT which is made in this phase is dynamic, which means that it can be elaborated, refined and adjusted to the actual learning experiment which will conduct in the second phase.

In addition, in this phase the researcher observes in the classroom which will be used in the second phase and interviews the teacher (which will participate in the second phase) related to his or her believe and teaching behavior. This is done to give an overview how is the socio norms and socio-mathematics norms in the class.

Phase 2: Design experiment

The second phase of design research is conducting the design experiment. In this study, the design experiment is conducted in two cycles. The first cycle called preliminary teaching is aimed to gather data and information needed to improve the HLT or design that have been made. That is done by testing it out in a small group of students (6 students) in the age of the target group (in this case is 5^{th} grader or 10-11 year old students). In this first cycle, the researcher plays a role as the teacher. The improved HLT then will be used in the second cycle (teaching experiment).

In the second cycle, the improved HLT is tested and implemented in the real classroom setting to see how the improved HLT works in the real classroom setting with a real teacher and to get some finding to refined or redesign the HLT. As a note, the students involves in the second cycle are different from the students in the first cycle. This is done to maintain the validation of the study (to maintain the condition of the students' prior knowledge which is expected to not understand yet about the topic issued in this study).

Phase 3: Retrospective analysis

The last phase of design research is retrospective analysis which is aimed to get the answer of the proposed research question. In other words, the researcher analyses and interprets the collected data to answer the proposed research question and to draw conclusion. The description of the data collection and analysis are elaborate in the next part (data collection and data analysis).

3.3 Data Collection

In order to answer the proposed research question, the data are collected during the pre-test, the preparation phase, the preliminary teaching experiment (first cycle), and the teaching experiment (second cycle), and the post-test. The data are gathered through interviews, observations, and collecting the written documents. The interviews and observations are recorded by the camera and video. Moreover, field notes also produced during the observations. The written documents which are collected are students' answer of pre-test and post-test, students' worksheet, and students' written work.

3.3.1 Preparation phase

In the preparation phase the researcher design the HLT and observes the classroom in where the second cycle will be conducted and interviews the teacher that will be involved in the second cycle.

Classroom observation

Classroom observation is done to get the impression of the situation of the classroom. In other words, the aim of the classroom observation is to gather the data about the classroom norms and socio mathematics norms of the class in where the second cycle will be conducted. This includes the culture of the classroom, the teacher's and the students' activity/behavior during the lesson, the interaction between the teacher and the students, and between the students, the teaching and learning process, and the class condition. In order to support the classroom observation, at first, the researcher made a list of the things to be observed (appendix A). Furthermore, a video registration is made and used to

helps the researcher to collect those data. In addition, the researcher makes field note of the things which are observed.

Interview with the teacher

Interviewing the teacher can gives us information/ data about teacher. The interview scheme is made (appendix B) and includes the questions related to the teacher's background and experience in teaching includes his/her knowledge about the realistic mathematics education (RME), the didactical knowledge (classroom management, teaching and learning approach and activities, and assessment), conceptual knowledge of the teacher, and the way she teach the coordinate system. In addition, it also helps the researcher in verifying what the researcher sees during the classroom observation and to know about the teacher. An audio registration of the interview is made.

3.3.2 Preliminary teaching experiment (first cycle)

Conducting the preliminary teaching experiment or also called pilot study as well as first cycle means trying out the design of activities/HLT in the small groups of students (6 students) who are in the same level/grade with the target group (5th grader/ 10-11 year old). The teaching and learning process is recorded by using video recorder and camera. Furthermore, the students' written works, students' note in the blackboard, poster, and any other written document are collected. Those data are collected to be used as basis in improving the design or hypothetical learning trajectory (HLT) that had been made.

3.3.3 Teaching experiment (second cycle)

In the second cycle, the revised design is tested out in the classroom with the teacher teaches during the lesson. The students of 5th grade (10-11 year old)

whose is involved in this cycle are different from the first cycle. The aim of this activity is to gather the data which is used to improve the HLT and to develop understanding of how it works. During this phase, the teacher implements the learning activities that had been designed. While the researcher do observation and mini interview the students (ask some question to the students during the working group or discussion) to know their thinking. The observation is focused and stick only on one group of students so we can follow their development in learning coordinate system. At least two video recorders are used to record the learning process and one video is focused on the focus group. Data collected from this activity are field note, video registrations, data of mini interview, and the students' written works (worksheet, poster, etc.).

3.3.4 Pre test and post test

The pre test and post test are both given to the students participate in the first cycle and second cycle. In the first cycle, the pre test and post test are done before and after the preliminary teaching experiment in order to whether the questions in the pre test and post test can be understood by students and to get some inputs that will be used to improve and refine the test items.

Different with the first cycle, in the second cycle pre test is given before the teaching experiment to know how far students know about the coordinate system and to know students prior knowledge. There are seven problems designed. The first problem is about completing the open number line which is aimed to see the students' prior knowledge about negative number and its location on the coordinate system. This is important because the designed activities cover the negative coordinate. The other six problems are aimed to know the students

knowledge about the coordinate systems such as, locating an object, locating a point, and plotting a point (see appendix D).

While post-test is given after the teaching experiment with purpose to know the development of the students' knowledge. Here we exclude the problem about completing the open number line. We still include the pre test problem but we change the number and we add some problems related to the coordinate systems (see appendix F).

As we explained, the problems written in the pre-test is slightly different with the one in the post-test but they have the same competencies. The problems used in both pre and posttest are have different level of difficulty. Students work individually on it. The process is not recorded. So the data collected from pre-and posttest is only students' written work.

3.3.5 Validity and reliability

Reliability and validity are important concerns of a research. Validity is about whether we really measure what we want to measure. And Reliability is about independence of the researcher. Based on Bakker and Vaneerde (2013) there are internal and external validity and reliability.

Internal validity refers to the quality of the data collections and credibility which is the soundness of the reasoning that has led to the conclusions (Bakker, A., and Vaneerde , 2013). In this study, the internal validity is improved by testing the conjectures with the other data material collected such as field notes, tests, and students' written work. Furthermore, a variation of the collected data gives the sources of triangulation which can improve the internal validity of the data. Moreover, in this study, the process of the data collection is described clearly so

the readers easily follow (track-ability) means improving the external reliability of this study.

3.4 Data Analysis

3.4.1 Pre-Test

As I explained in the previous part "Data Collection", the pre-test is given to the both students participates in the first and second cycle. The result of the pre-test from the first cycle is analyzed to investigate student preliminary knowledge and to get data and feedback which will used as basis to improve the pre-test before it is going to be used in the second cycle.

The result of the pre-test which is given in the second cycle is analyzed to investigate students' (the participants in the second cycle) preliminary knowledge in learning the coordinate system. The students' answers are looked carefully one by one and we see whether the students can answer the question correctly, how far they can solve the given problem, what strategy they used, do the students find difficulties or do mistake in answering the problem and so on. So, the test result is expected to reveal what students' already know about the coordinate system. This students' preliminary knowledge will affect the HLT. From this aspect the HLT will be adjusted in such a way so it will be appropriate with the students' prior knowledge.

3.4.2 Preliminary Teaching Experiment

The registered video, the field note, and the students' written works in the first cycle are analyzed and triangulate to investigate the learning process of the students participate in the first cycle. This is done to see whether the activities that have been made can really support students' learning of the coordinate system.

The analysis is done by checking the students' written work, video registration, field not and another collected data with focus on what actually happen in the learning process and students' thinking. In other words, the collected data is triangulated and compared to the HLT that have been made to see whether the students' thinking conjecture occurs as expected or if there is another conjecture besides the researcher prediction that surprisingly occur and whether the activity made can help the students to reach the settled learning goals. The HLT is assessed and will be refined based on the result of the data analysis of the preliminary teaching experiment phase. The activities that can help the student will be removed or improved, while the fine one (support the students learning) is kept.

3.4.3 Teaching Experiment

Similar to the preliminary teaching, in the teaching experiment (second cycle) the video recording (the video of the whole class and the video of a group that become the focus group), the field note, the observation data, the recording of the mini interview of the focus group are analyzed. This analysis is aimed to get the overview of the whole teaching and learning process in the real classroom and to get the overview of the working and discussion process of the students in the focus group. The analysis is done by checking all of the collected data thoroughly and triangulates them and compares them to the revised HLT. The researcher watches the registered videos and selected the important and relevant fragments which give data and evidence to answer the proposed research question. These fragments are then transcribed. And later together with the students' written works, it is analyzed by comparing it to the conjecture in the refined HLT to

investigate the students' learning and thinking of the coordinate system. The result of this analysis will be used to answer the research question, draw conclusion, and refine the improved HLT.

3.4.4 Post-Test

Same as the pre-test, the post-test are held in the first and second cycle. The result of the post-test from the first cycle is analyzed to see whether the items in the post-test really can give the expected data and whether it is understandable by the students. This data is also used as basis to improve the post-test. And the result of the post-test in the second cycle is compared to the result of the pre-test to see the students' development in understanding the concept of the coordinate system. This result is also used as evidence and support the researcher in answering the research question and drawing the conclusion. The analysis is done by looking/ checking the students' answer/ work to see whether there is a change or improvisation of the students' knowledge about the coordinate system and how they solve the problem.

3.4.5 Reliability

The (external) reliability of the data analysis can be improved by giving the clear description and transparent steps of the procedure followed and how the researcher do the study (each phase), collects the data, interprets the data, analyzes the data and draws the conclusion. The researcher also is being transparent in data analyzing and interpretation. Not only tell the success but also the failure. Giving the clear description in how the researcher work in the data analysis and being transparent in data interpretation and data analysis is the trackability aspect.

Moreover, the researcher also do a cross interpretation with colleagues and supervisors. In this study, all of the instruments (HLT, students' material, teacher guide, pre and post-test question) and the important segment of the teaching experiment are discussed by the researcher, supervisor and colleagues. Doing peer examination and discussion can improve the internal reliability of the data analysis and minimize the subjectivity of the researcher's point of view in interpreting the data.

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Hypothetical learning trajectory has an important role in design research. Not only as the product of the research but also as a tool in doing data analysis. Hypothetical learning trajectory (HLT) is offered the key aspects of planning mathematics lesson. It includes the mathematical goals of lesson/ activity, the description of the learning activity, and the hypothetical learning trajectory of students' thinking and learning/ conjecture of students' thinking (Simon, M. A., & Tzur, R., 2004).

In this chapter, I would like to present the designed HLT of learning the coordinate system for fifth graders (10-11 years old) consists of the mathematics learning goals, students' prior knowledge, description of activity, and conjecture of students' thinking. In this study, there are six activities designed which will be implemented in more or less six lesson. The HLT of each activity will be described as follow.

4.1 Activity 1: Seat map of a plane (What is a good system?)

Learning Goal:

Through this activity I hope that:

- Students can make a system to organize a thing, in this case is the plane's seats
- Students aware of row/line and column
- Students aware that to locate a thing precisely need at least two parameters (line/row and column). It is not enough if only consider one variable/parameter, for example just consider the row.
- Students understand what a good system is.

Starting point: There is no specific skill that needed to do this activity.

Mathematical activity:

Students and teacher discuss or talk about their experience in riding an airplane or their knowledge about airplane. If there is none that know about the sets of airplane then teacher can show them some picture of the airplane's seat map and start the discussion about the need of system.

Then the teacher In group of 4 or 5, the students are asked to solve this following problem and make a poster of their answer and thinking.

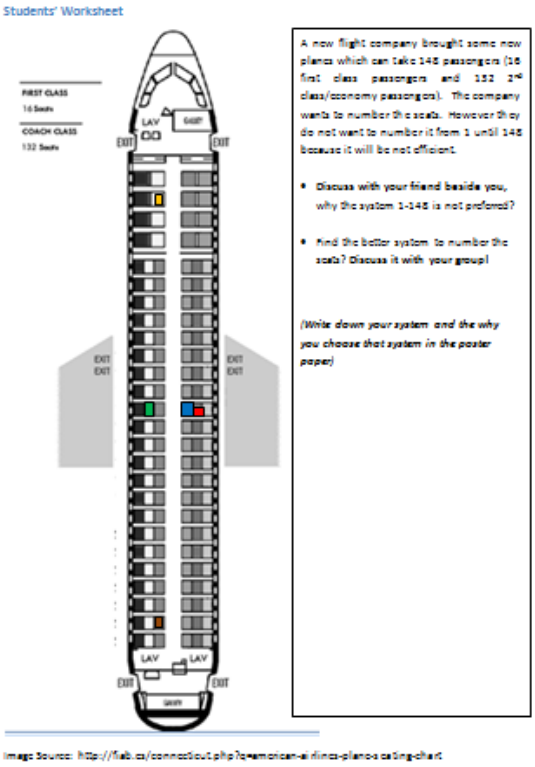


Image Source: http://flab.cs/connect/cut1.php?q=american-airlines-planes-seating-chart

A new flight company brought some new planes which can take 148 passengers (16 first class passengers and 132 2nd class/economy passengers). The company wants to number the seats. However they do not want to number it from 1 until 148 because it will be not efficient.

- Discuss with your friend beside you, why the system 1-148 is not preferred?

- In group, can you help the company to make a better system for the seat's number?

Should be noted that, the aim of this discussion is to make students aware that although there's a system 1-148 (they can numbers the seats from 1 to 148) but that system is not effective or cannot help much to the stewardess and passenger to find and tell the location of an object/seat effectively and easily. So there's better system that can help them to organize and locate a thing better than just count it one by one. In other word, this discussion is emphasized or focused on the importance of a system, why we need a system and what a good system is. Furthermore, orchestrate discussion about the criteria of a good system for the seats' number. Maybe it should not make the passengers and stewardesses confuse, easy to find, only for a passenger/ never happen become a seat for two different people, one seat for one person, etc.

Prediction of students' responses:

Question: "Why numbering the seats from 1 to 148 is not favorable?"

Students may answer:

- It is not favorable because it will difficult to find the seat
- It will take a long time to count the seat one by one or to see it one by one
- It is difficult to count because the number of the seats in the first class and second class is different. So we cannot do skip counting or multiply it easily.

The system that may occur:

- Students may make a system of the seat with only consider the number of line not the column. For example, seat in the row or line 3 (seats number 3).

- Students may differentiate the system within first class and second class. For the first class they may only number it from 1 to 16 but for the second class they may only use the number of line which is 1 to 22.
- Students may not differentiate the first and second class and start to number/ count the line from the first row till 26th rows.
- Students may also consider right and left seat (separate the group of seats as left and right group) and start to number the row. So they will make a system based on row and left right group.
- Maybe students symbolize the left and right as A and B
- Students may consider row and column. So they will number the seat as combination of the row and column like row 1 and column 2 as (1.2) or maybe just by telling "it is in the 1st row and 2nd column"
- Students may symbolize the row with number and column with alphabet (A to F) and differentiate the first class.
- Students may not differentiate the first and second class and symbolize the row as number 1 to 26, and column as A to F. While for the first class there will be only column A to D.
- Students may directly come up with the idea numbering the seat with "12A" (combine number and alphabet) because they ever ride the airplane.

Main Issue: the students may or may not find the good system. all of the students' answer is bring to the class discussion. The discussion is emphasized on why the systems is good system and why not. Here the good systems that may occur:

- Combining number and alphabet like 12A
- Reverse it so A1

- Combining number and number like 3.4
- Numbering it as 124 which mean first column seat number 24 (24^{th} row).
- We expect that students may answer that a system is considered as a good system if we can easily find a location of an object using that system and we can easily tell that location to other without any ambiguous meaning.

4.2 Activity 2: Seat map of a cinema (How a system works?)

Learning Goal:

Through this activity I hope that:

- Students can understand how a system work
- Students aware of another coordinate systems
- Students aware of row/line and column
- Students aware that to locate a thing precisely need at least two variables (line/row and column). It is not enough if only consider one variable, for example just consider the row.

Starting point: There is no specific skill that needed to do this activity.

Mathematical activity:

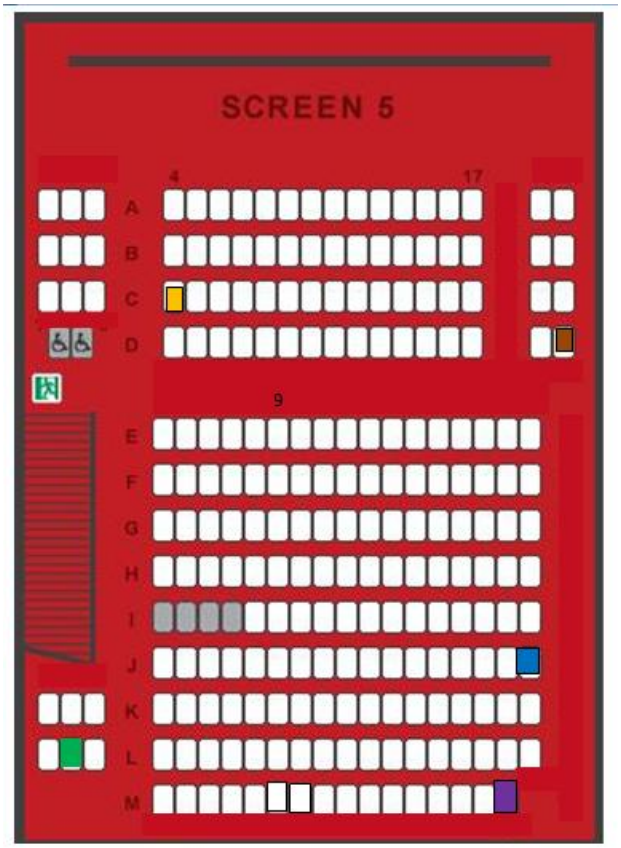
- Discussing the cinema's ticket

Image Source:
http://herli.web.id/wp content/uploads/2009/07/tiket_harry_potter_18_juli_2009-237x300.jpg

*Should be noted, t*he discussion is emphasized on the meaning of the number and alphabet on the ticket especially the seat number. Teacher may ask what students see in the ticket and ask more about the seat number. *"so what the number of the seat?" can you explain what is the meaning of F and 9? F and 10?"*

- The students will be asked to solve this following problem.

Students are given the map of the seats in a cinema. Teacher starts to tell a story about her friend namely Dona who is going to work on a cinema. Before she starts to work, her boss gives her the seat map of the theater and asks her to study and understand it. What can you tell about the map? Can you help her to understand the map especially about the number of the seats so if there is a person who ask her help to find his/her seat, they can tell him/her easily and precisely.



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Discuss with your group!

- What will you tell to my friend about the theater? What is important to be known by her? What can you tell about the seat map?
- What is the different between this seat map and the seat map of a plane that we made in the previous meeting?
- Can you make another different system to number the seat in this theater?

A prediction of students' responses

Students may answer they see: the name of the cinema, movie's title, date, time, price, number of the theater (4), and the seat number (F9 and F10 or F9 and F10).

For the next question,

- Students may respond that F9 means it is seat number F9. (*In this case teacher can ask how we can find it? do the number is written on the seat?)*
- 9 and 10 (because the seat numbers must be different)
- 9F and 10F (because he/she still thinks about the seat number on a plane) ➔ 9 or 10 is the row and F is the column.
- Students may answer it is row F and seat number 9; row F and 9th column; or row 9 and column F
- For the F10 students may answer similar like F9 or maybe there some students who will answer "it is next to F9".

Prediction of students' answer/respond of working group activity

- Students may start to describe the Theater. They may tell that the screen is in front of the chairs, or they will say it is in the north part near the exit. They may also tell that there are 231 chairs in total. It arranges in 13 rows and 20

column (some are only 19). There is also staircase in the exit and there are two places for the wheelchairs.

- Students may also come up with the detail explanation about the seats/ chairs. They may answer that the seats are arranged in row and columns. There are 20 columns (some only 19) and 13 rows. The seats are numbered with alphabet and number. The alphabet represents the row and number represents the column. It starts from the left to right.
- The difference between this seat map with the previous one is they have the different number of row and column. In the plane, the alphabet represents the column and the number represents the column. While in this system, it is reversed.
- Students may make another system by reverse the alphabet and number. Some students may use only number or alphabet to represent the row and column etcetera.

4.3 Activity 3: Sunken Ship 1 (Make a System to locate a sunken ship)

Learning Goal:

- Students can make a system to locate a thing in a plane from a certain point without any help of column and row or grid
- Students aware of the different types of coordinate system
- Students aware of the use of the coordinate system to locate an object in a plane
- Students understand the idea that a point is unique
- Students understand the idea that in making a system to locate a point at least need two aspects/ parameters or measurements.

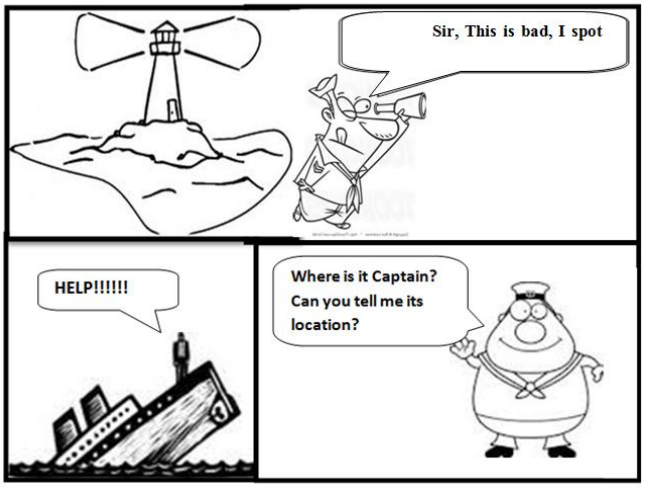
- Students aware of the origin point

Starting point:

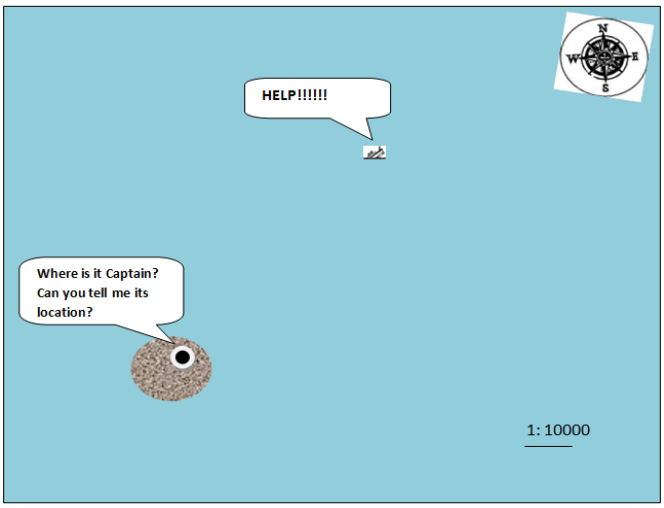
- Students already know about the direction like left-right and familiar with compass direction (north, east, south, and west, etc.)
- Students know about scale
- Students understand what a good system for locating an object is (first lesson).

Mathematical activity:

In this activity students are given this problem:



The position of the lighthouse and the ship can be seen as follow (satellite view).



How the captain supposed to tell/report the location of the sunken ship to the general? Remember, giving a report to the general must be precise and not ambiguous (lead to another or false location). **(Discuss it with your group!)**

Students' answer:

Students may locate the ship by considering only one aspect.

- Students may answer using the north, east, south west direction.
- They also may use right and left direction.
- Or they may thing about using clock as location like the position of two o'clock (student may ask considering the different point of view of the one that tell the information and the one that received it)
- They may use measurement. So the measure the distance of the sunken ship to the lighthouse.
- They may come up with the idea of angle.

Students may locate the ship by combining two aspects or two measurements.

- Students may locate the ship considering the angle and its distance from the lighthouse
- Instead of using the light house as the origin point, the students may use the corner of the map as origin point.
- Students may locate the ship by measure its distance to the lighthouse and to the one side of the map.

Main Issue: It should be emphasized that to locate a point or an object we need the origin point. In addition, locating the ship by only consider one parameter is not enough because it can lead to another location. Yet, we need a system which can locate an object precisely and uniquely. For example, if the students answer that

the location of the sunken ship is 10 km from the lighthouse, then it will not clear whether it is 10 km to the right, left, north or etcetera. So it is important that the system is made is a good system which have been learnt and discuss in the first meeting.

4.4 Activity 4: Playing in the Paddies fields (where is your origin point?)

Learning Goal:

- Students can tell a location of a place or an object precisely
- Students understand that each point is unique.
- Students understand about the origin point (they will not start to count/ name the first line as 1 but the second one and they have to consider from which view they see the map)
- Students are aware of vertical and horizontal grid lines and intersection of grids lines
- Students aware that to locate a thing precisely need at least two variables/parameters (vertical and horizontal axis/ grid lines). It is not enough if only consider one variable, for example just consider the vertical lines.
- Students know the positive coordinate system.

Starting point:

- Orientation, the students understand the direction like left-right, up-down, above-below.
- Students understand the term vertical and horizontal
- The students understand what is a good system to locate a thing

Mathematical activity:

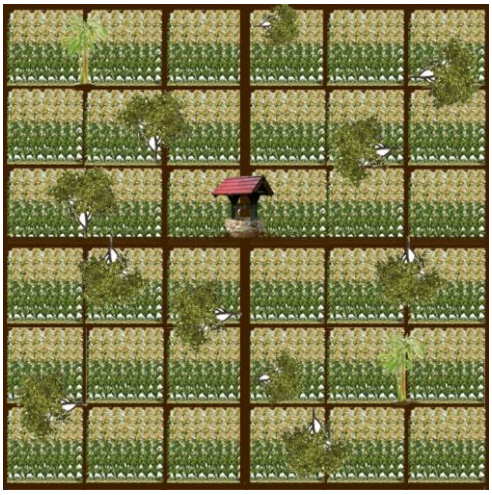
Children are given a map of paddies fields. These fields are specially built/ made for the recreational purpose. So that children from the city can experience the village environment.

Problem: Yesterday, class 6 of an elementary school "Pusri" went to the fields. They observed and helped the farmer to work on the field. In the break time, they played and walked around the fields. However, when a student namely "Dina" reached her home, she realizes that she lost her watch. She remembers that she take it off and put it in her pocket when she was playing in the fields. She is sure that it is dropped in the fields. She calls a farmer leader Mr. "Toni" for asking his help to find her watch.

Dina : "Hello.. "
Mr. Toni : yes hello
Dina : may I speak to Mr. Toni?
Mr. : yes I am Toni. With whom am i speak?
Dina : this is me, Dina, a students from the SD Pusri who came visit the fields yesterday.
Mr. : aah.. ya… i remember. What's matter Dina?
Dina : I lost my watch and I think it dropped in the fields.
Mr. : oh, i am sorry to hear that. What can i do for you?
Dina : would you mind to search it for me sir?
Mr. : sure.. but how?
Dina : I think I remember in where I dropped it. hmm… at that time I was stand in the corner of the field then I walked straight. When I meet first road I entered. I passed through on road and when I see the second road, I stopped and play in there.
Mr .: that's good.. wait i'll take a picture/ map of the fields then I will try to search it for you.
Dina : thank you so much sir.. Maybe in the weekend I'll go there and pick it up.

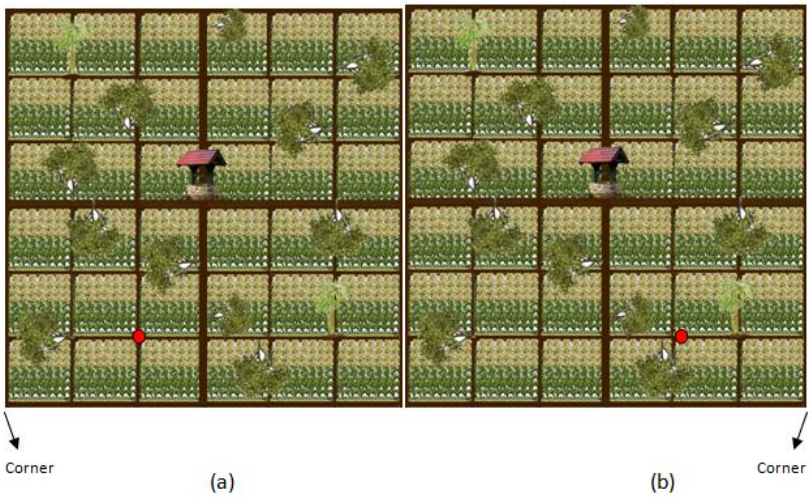
Discuss with your group! Could you help Mr. Toni to find the location of the watch based on the Dina statement? Why do you think that is the right location?

Map:



A list of students' answer

- Students might draw the map as grids (modeling it as grids)
- Different group may look at the map from different views. So they will have different corner to start with. This can cause the different location.

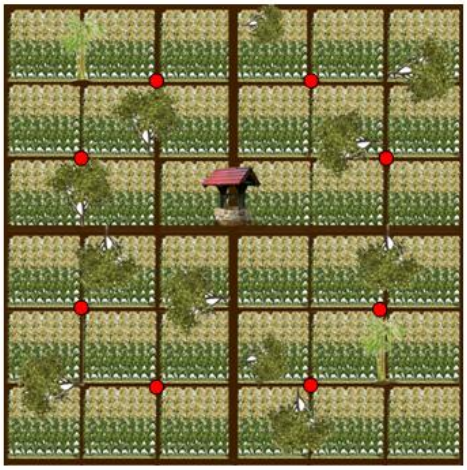


So if the students see the picture or map as above (picture a), they may use the left-below corner as starting point. And because Dina said that she walked straight and entered to the first road and stop when she met the second road, then the students will see the intersection of second and first road as the location of the watch (red dot in the picture). But if the students see the right-

below corner as starting or origin then they will end up in the position of red dot in the picture b.

- Students might start to count the first road from the main road/boarder.
- Students might not count the boarder. They skipped "0" zero.
- Students might count 1 to the right 2 up.
- Or maybe they will count 1 to the left and 2 up

The possible position of the watch that may occur can be seen as follow. The location is represented by red dot. These positions are resulted from the different point of view and origins



For the name of first, second road and so on, some students may start to count the corner as the first road. But other students may count the corner as zero which will give different locations.

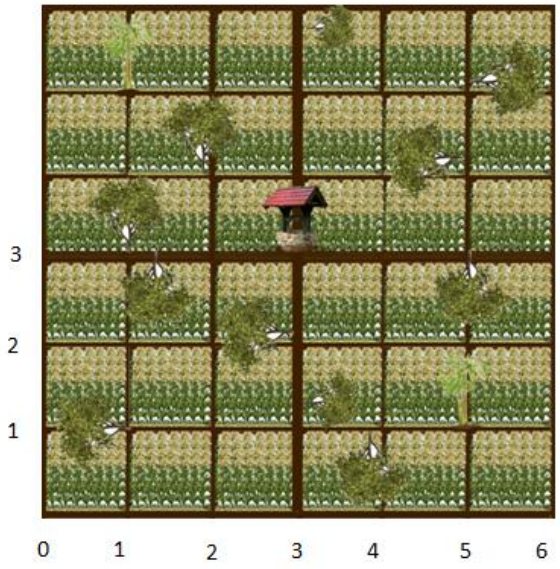
Main Issue: The discussion is emphasized on the need of an origin. The different locations that come up can be presented in the class. The teacher can ask the student how come they have the different location. Which is the correct one? What will you ask to Dina, so you can know the exact location? In making a

system to locate an object, we need the same origin so they can locate a thing precisely and uniquely.

After students agree about the origin, then the teacher can continue the discussion by proposing a question like: so where is the location of the watch? How do you supposed to tell Mr. Toni? In the intersection of which lines is it?

Those questions is aimed to help the student understand about the more formal system like coordinate (positive) which can help us to tell the location of an object easily and accurately/ efficiently. So rather than say walked straight, enter the first road and met the second road, we can just say that it is in the intersection of second and first road. We do not expect that the students already come up with the system like (1,2).

So the teacher can proposed the grids which represent the fields and roads and say that *"at first Dina stood her (corner). And this (point the first road) should be the first road, second and so on"* Like in the image below.



We predict that students may say that the location is in the intersection of first and second road. But some students may react that it is in the intersection of

second and first road. If the students say that it is the intersection of first and second road because she walked straight first and meet the first road then enter it. Then the teacher can ask "*How if Dina stands in the corner and walked to the right and meet the second road then enter it and stop when she meets the first road?"* These different answers can be discussed and we expected that they (students) make an agreement on it.

Later, the teacher poses several questions like guessing the location (similar to Dina's clue). This is done in order to give a chance for the students to practice their knowledge and to do a brief check whether the students understand about the concept that they just learnt.

In the end of the lesson, teacher can ask "*How if the location of the watch is in here (outside the grids: right, up, left, and down sides)".*

4.5 Activity 5: Sunken ship 2 (Locate the ships with the help of grids)

Learning Goal:

- Students can tell a location of a place or an object precisely
- Students can understand that although the point is not in the intersection of vertical and horizontal axis, it still can be located precisely (for example, (1, ½))
- Students can locate a point which involves negative number/ coordinate

Starting point:

- Students can produce the grid which is labeled by (0), 1, 2 and so on (exclude the boarder/ 1st line) for both horizontal and vertical lines (product from the paddies field activity).
- Students understand the positive grid system (previous lesson)

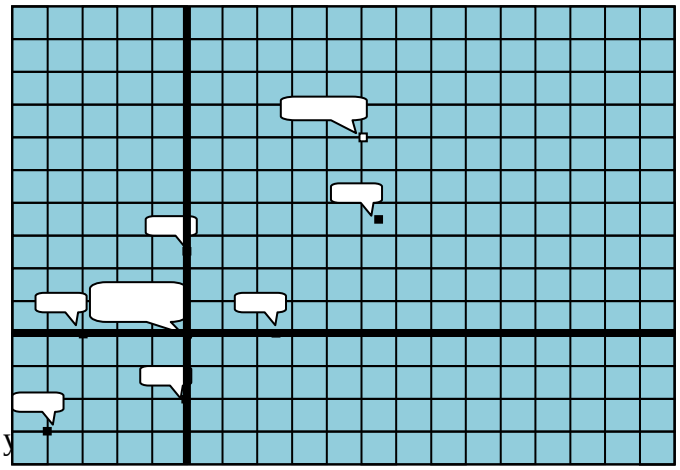
- Students understand negative number and fraction
- Students know how to measure a length

Mathematical activity:

This activity is continuation of the activity 3 (use the same context and problem). The difference is in this activity, the map is supported by grid lines.

Problem:

Do you still remember the story of sunken ship in the previous meeting? Following map is the satellite image of the sea, lighthouse and ships.



- Can y
- Can you locate the SAR teams from the lighthouse? (the light house as the origin)
- What did you find after locating the SAR teams?

Students' answer

- Instead to see the lighthouse as origin point, students may label the gridlines as they did in the previous activity (labeling it from 0 to the positive number). If this happen then the teacher can ask, how if I used the location of the lighthouse as the origin?
- Students may still use right-left, up and down. 5 line to the right and 4 line up

- Students may use the term vertical and horizontal lines to differentiate the axis
- For the location of the point which is not exact in the line intersection. The students may use ruler to measure it distance from the origin point.
- Students may count the squares in order to locate the point which is looks like in the square rather than measure its distance.
- For the negative location, the students may use 1 to the left (so using the positive number but in left direction rather than symbolize it as negative number)
- Students may recognize the negative coordinate because they learnt about number line
- Students may recognize that there is negative coordinate and coordinate involves fraction.

Main Issue: In this activity teacher help the students to understand the coordinate location that involves the negative number and fraction. Students make an agreement of how they will notate the location.

After the students understand about the negative and fraction coordinate, the teacher start to bring the grid into more formal form which is Cartesian coordinate system. Teacher draws the main axis and symbolizes the lighthouse as "0", and then the teacher asks the students to relocate the sunken ship and SAR teams in the given Cartesian coordinate system.

Furthermore, students are asks to discuss the position of the sunken ship from each team SAR and determine which team that is closer to the location of the sunken ship.

In the end, students are asked to discuss what the different between the cartesian system and another system to locate an object that they learnt like plane's and cinema's seat map etc.

4.6 Activity 6: Cartesian coordinate system

Learning Goal:

Through this activity I hope that:

- Students understand about the Cartesian system
- They know how to write the coordinate (notion)
- Students understand the agreement made (the rule) in the Cartesian coordinate system
- Students understand that Cartesian coordinate system is the one of ways to locate a point.

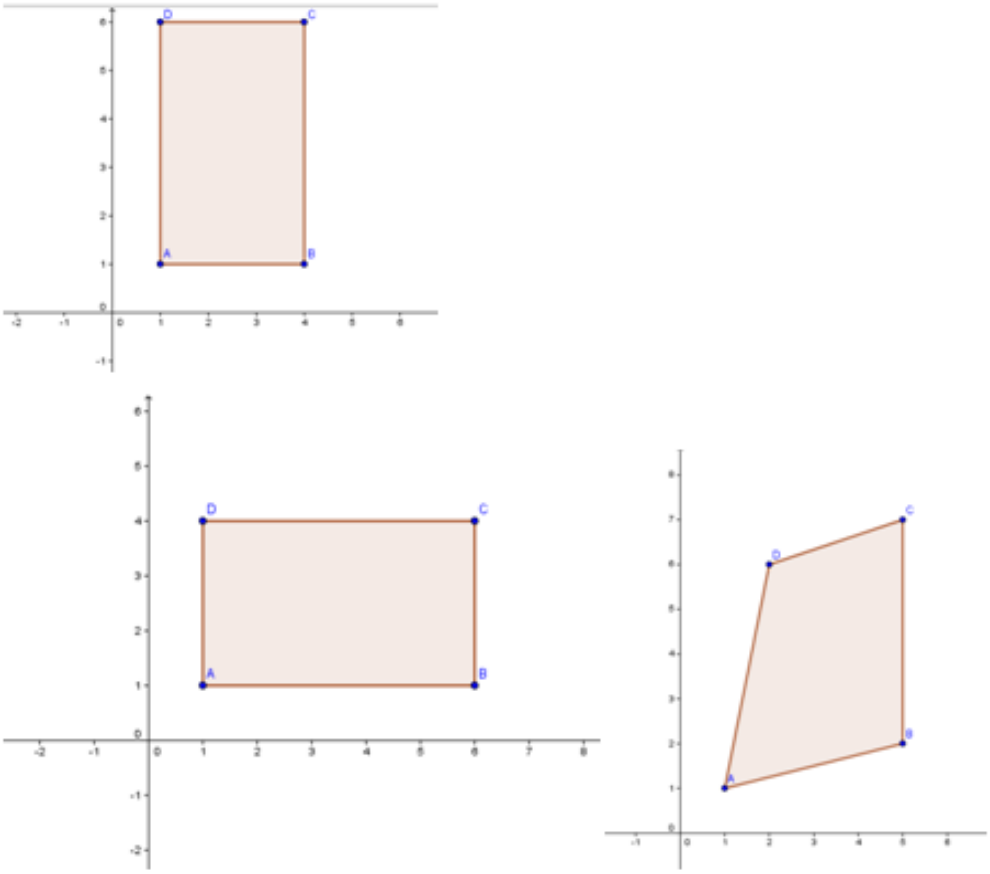
Starting point:

- Students aware of origin point
- Students know how to locate a point in a grid
- Students experienced the coordinate which involves negative number

Mathematical activity:

The students are given three different students' work in which those three students namely A, B and C were asked to locate the given coordinates and guess what figure can be made from the given coordinate. The given coordinates points are: (1,1); (4,1); (4, 6) and (1, 6) *(remember that the students still not know yet about this notation)*

And the students' A's, B's, C's answers are:



The students are asked to discuss in group and judge those three answers.

- Do the answer different? How?
- Which answer that you will choose? Why?

Follow up activities:

The next activity is guessing the figure. In this activity, the students are given some sets of coordinates and the students have to guess what figure that can be made from those points.

- (2,3); (4,3); and (2,5)
- (1,0); (4,0); (3,0); (1,3); (4,3); (5,3); (2,5) and (3,5)
- (0,2); (2,0); (4,2) and (2,4)

Later students are asked to make a sets of coordinates points then given to their friend and ask them to guess what figure can be made.

If there is still spare time, then the teacher gives the incomplete coordinates of a figure (parallelogram or others) and asks the students to determine the coordinate of the points so the figure will be completed. For example, I would like to make a parallelogram. I have some points: (2,-2); (6, -2) and (8,1) what point that I need to complete the parallelogram? How come you know it?

In order to check whether the students understand about the Cartesian system, the teachers engage the students in playing game. For example, *"So Now I am in the point (2,1) I walk 2 up, 3 left, 5 down, 2 right and 3 down. In which point my location now?"*.

Students' Answer

The most common students may answer maybe between A and B. Both of those answer is similar. The difference is only the way they interpret the coordinate notation. Some students may say that A's answer is better and some may say that B's answer is better. The A is interpretation of (x,y) while B is the reverse (y,x). Because the students still do not know yet about the Cartesian system notation. Teacher may like to bring these two answer into the whole class discussion. The discussion is to give a remark that the system that have been made include the Cartesian are a way or tool to locate an object precisely in which it involves the need of agreement. While for the C answer, the students may recognize that the mistake is because the location of some points is not from the

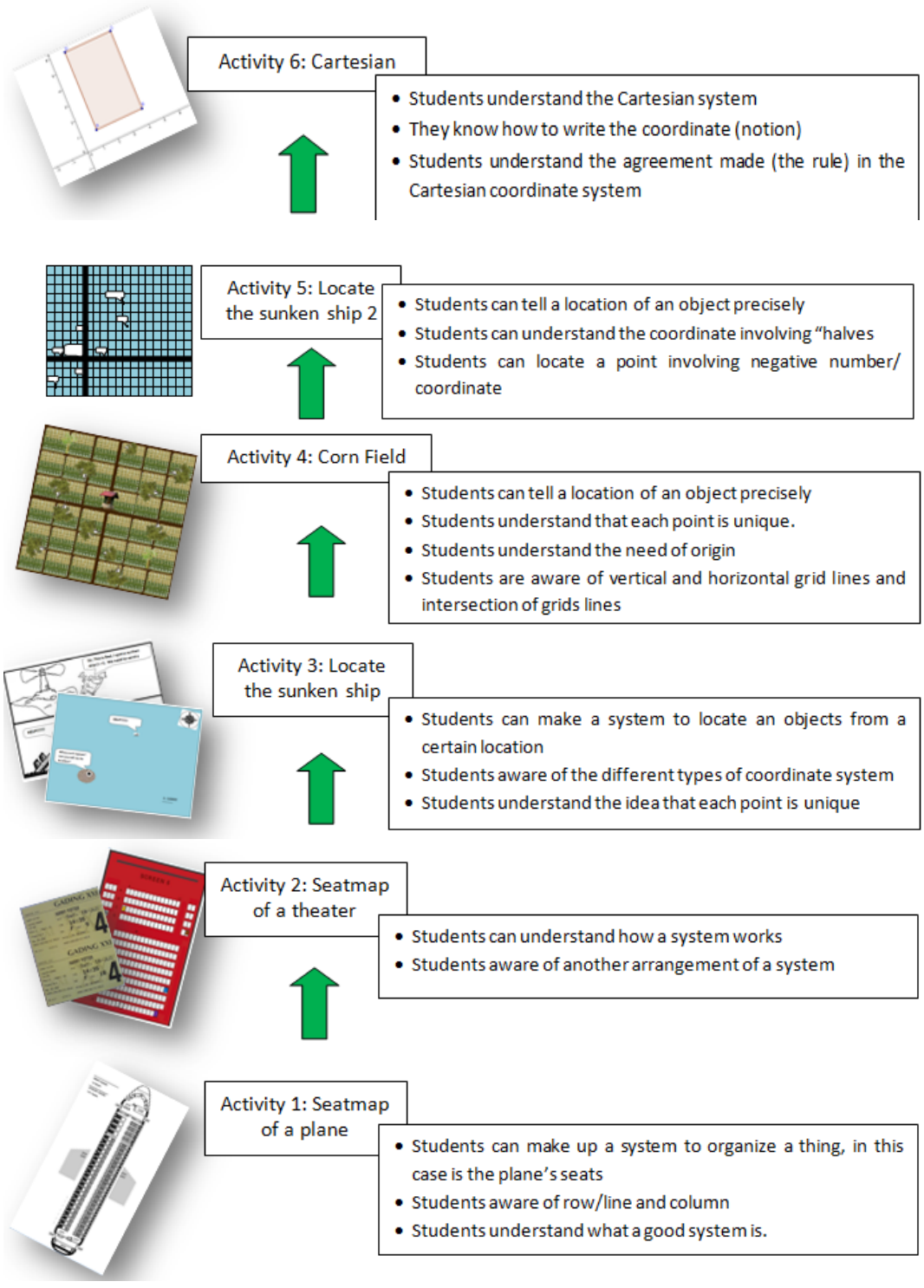
origin. From this answer they will sense the need of origin. For other questions, students may come up with the different answers.

Main Issue: In this activity, the term of Cartesian coordinate system and its notation is being introduced after the students discuss about the given problem. The teacher emphasize that the Cartesian coordinate system is one of a good systems which is made by mathematicians to locate an object precisely. So, it is how the mathematicians locate an object.

By giving the three different answer which are not all of them right is to make the students realize the common mistake that usually happen while locating the coordinate into Cartesian system. One of the is the reversal of x and y and ignoring the origin (point a location not from the origin). By giving a chance for the students to find out those mistakes by themselves we hope that they will not do the same mistake in the future.

The follow up questions/activities and game are given in order to make the students explore and understand more about the Cartesian coordinate system and to check fast the students understanding.

4.7 Learning Scheme: Understand the Coordinate Systems



Scheme 4.1 *The first learning sequence of "uderstanding the coordinate systems*

CHAPTER V

TESTING HYPOTHETICAL LEARNING TRAJECTORY

Chapter 5 describes the outcomes of the design experiment (preliminary teaching and classroom teaching experiment) and the retrospective analysis.

5.1 Design Experiment

After we designed the learning activities and HLT, we test that HLT into small group of students (cycle 1: preliminary teaching). From the cycle 1 we refine and improve the HLT and then it is tested out again in the real classroom situation (cycle 2: teaching experiment). In this part, first, we would like to give brief description on how the HLT is carried out in the preliminary teaching and teaching experiment. Then, we will give the retrospective analysis of the each cycle and the changes that we made on HLT.

5.1.1 Preliminary Teaching (Cycle 1)

In the preliminary teaching (cycle 1), we test our HLT into small group of students. There are eight students participated in this cycle who are separated into small group of 4. These eight students are from Grade 5 (5A). The HLT (includes pre test, activities, and post test) are tested in order to get the insight how the HLT work and get some finding or feedback which will be used to improve the HLT.

Pre Test

Before the learning activities are tested to the 8 participants, we give them the pre test first. There are four problems about the coordinate system given and to be solved for about 30 minutes. Students are working individually.

Activity 1 Seat map of an airplane

As explained in the HLT (chapter IV), this activity is given in a meeting (70 minutes). Students work on small group of 4-5 students. After the students telling their experience about airplane and discussing why numbering the seat from 1 to 148 is not preferable, they are asked to make up a system to locate a seat of a airplane precisely. This activity is not going smoothly. Students can not make up a system. Instead, they try to find good strategy to count the seat faster.

Activity 2 Understanding how a system used in a cinema works

This activity is should be given after the students solve the problem in the first activity. However, because the student can not make up a system for the first activity, we decided to give the second activity as a help for them. First, the context about cinema is introduced. Then, they are asked to find seats F9 and F10. In order to find that seats, they have to understand the used system works which is the purpose of this activity. After that, they are asked to make up a system to locate the seats of the airplane (first activity's problem).

Activity 3 sunken ship 1

As the other two activities, first, the context is introduced. Next, the students (in group) are asked to locate the ship precisely. The students come up with different answer as conjectured. The teacher helps them with asking some question so that they realize whether the the system they make is precise enough or not as written in the HLT. Students tend to use the system that similar as the system in the first and second activity.

Activity 4 Rice Field

After the context is introduced, the students in groups are asked to solve the given problem. Here the modified context can not bring thm into problematic situstion in where they should find some different possible origins. This is because of the drawing which blocks that possibility (explained in the retrospective analysis part). After they make an agreement about the origin, they are asked to locate some points. Here the students can do it well. They also make an agreement on how they should describe the location.

Activity 5 Sunken ship 2

Context is introduced, the students then are asked to locate some hips. Here they learnt about the negative and ½ coordinate. Students suppose to discuss in group, yet they work individually. The students use the similar system as used in cinema (second activity), so it kind of difficult to bring them to understand about the negative and "1/2" coordinates. We have to bring them to make a system similar as the fourth activity (using number- number). Then they can locate the ships.

Activity 6 Cartesian coordinate system

Here we introduce the Cartesian coordinate system. After that the students are asked to analyze and choose the right answer out of three proposed answer. Students supposed to work in grup. However, they work in pairs. There are two different answers. These answers then are discussed in class. After that, they try to solve the rest given problem.

5.1.2 Teaching Expereiment (Cycle 2)

In this cycle, we test the refined HLT in the real classroom. There are 34 students (different from the participant of the first cycle) and a teacher who participated. In

general, the activities are carried out for about 70 minutes each. Students work in small group of 4-5 students. These groups are maintained same for all six activities. The learning activities are, introduce the context, works on group/ group discussion, and the classroom discussion. Pre test and post test are given in the first and the end of meeting respectively for 30 minutes. How each activity is carried out will be described briefly as follow.

Activity 1 Cinema

The context about cinema is introduced. Teacher asks the students about their experience about the cinema. Teacher gives them a copy of cinema tickets. Teacher and the students discuss about the meaning of the numbers written on it. Then, they are asked to solve the given worksheet (see appendix F) in group. They have to write down their answer on the given poster paper. During the group discussion, teacher walks around and helps them by posing some problem questions. Teacher chooses some students to present their answer in front of the class. The teacher orchestrates the class discussion about how the system used in cinema works and the idea of each point is unique.

Activity 2 Airplane

The context about airplane is introduced. Teacher asks the students about their experience about the airplane. Teacher gives them s seat map of an airplane and asks them to make a system to locate the setas. Students write down their answer on the given poster paper (in the same group as first activity). Teacher chooses some students to present their answer in front of the class. During the group discussion, teacher walks around and helps them by posing some problem questions. All of the different anwer are presented in the class. Here the teacher

orchestrates the class discussion about “a good system” and how should they locate the seat. In the end, students are asked to determine which answer is the most effective system.

Activity 3 Sunken ship 1

Same as the othes activity, first, the context is introduced. Teacher gives the students an illustration of the problem and retelling the story on that illustration. Teacher gives the students a map of the sunken ship. Working on group, the students are asked to describe the location of the ship precisely. Tacher helps them by posing some problem questions related to the how precise the system they use can locate the sunken ship. Students write their system on the poster paper. All of the different anwers are presented in the class. Here the teacher orchestrates the class discussion about “a good system” and the possible systems to locate an object in a plane.

Activity 4 Rice Field

Teacher tell a story of Dina who losts her watch in the rice field. Teacher gives a map of the fields to the students. They work in group to find the location of the watch with given clues. Different from the first cycle, here the refined drawing/ context successes to bring the students into the problematic situation in where the students fin many possible origins. This then discussed. Students are asked to mark the location of the watch. Teacher asks the representative of the students with the different answer to explain their answer. The discussion are focused on the each point is unique and the need of the same unique origin. nezt, the students are asked to describe the location if the watch and some point includes “1/2”. Here

they make an agreement on how they should describe the location (horizontal first then vertical).

Activity 5 Sunken ship 2

In this activity, teacher reminds the students about the context used in the third meeting (sunken ship 1). But here, the map has grids and there are some rescue teams' ships. Teacher asks the students to describe the location of the ships. This activity takes a long time as expected. Students can locate the ships in the negative quadrant easily, but they find it difficult todescribe the location of the ship in the axis. Teacher guide the students who work in groups. In the end, teacher asks some students to describe the location of ships and orchestrate the class discussion.

Activity 6 Cartesian system

Teacher start the lesson by introduce the Cartesian coordinate system (form and its notion) by related it with the previous activity (fifth activity). Then, they are (still in group) given three answers of Ani, Budi and Erin about locationsome points in Cartesian. Students are asked to choose which answer is the right one and give the reason. Teacher helps them and checks their understanding by giving some probing question and act dumb as explained in teacher guide. Teacher asks their answers and brings it into class discussion. Next, because there's still time, the students are asked to solve the additional problem (worksheet F) in order to check and tested their knowledge about what they learnt.

5.2 Retrospective Analysis

In this phase, all the data collected are analysed. For the first cycle (preliminary teaching), we would like to focus our analysis on the how the HLT takes place in

the learning process. We would like to gather feedback and look for insight to improve the HLT. While in the second cycle (classroom teaching experiment), our analysis is focused on the how the HLT takes place in the class, how it supports students' thinking and on how the students thinking and understanding of the proposed mathematical ideas. Furthermore, we will look into the socio mathematical and socio norm in the classroom.

5.2.1 Preliminary Teaching (Cycle 1)

Pre Test

The pretest is given to the eight students from class 5A which will participate in the first cycle a few days before the activity 1 is implemented. These students are from different levels. There are low, middle and high achievement students. And the high achievement students it is known from the homeroom teacher that she is already learnt about coordinate system in the extracurricular. However, her pretest result shows that although she knows and learnt about Cartesian coordinate system, she cannot tell the location of an object precisely.

In addition, she also cannot tell the coordinate of the give points in the Cartesian coordinate system. Instead of locate it using the Cartesian system, she locate the point by only consider the (compass) direction. Furthermore, from the mini interview, it is known that she can locate/ plot a point with given coordinate but she cannot locate the coordinate of a given point in the Cartesian system.

The pretest itself consists of four problems. The first problem is aimed to know whether the students already learnt and know about negative, 0, and positive number and its position in the (vertical and horizontal) number line. The second problem is aimed to know whether the students already understand about

coordinate position. We would like to know how precise the students locate an object. It is also aimed to get a brief view how the students will solve those kind of problem. The third question is about position of points in the Cartesian coordinate system. And the last question is determining the location of a point in which if the given point and missing point are connected it can make a rectangle (2D figure). So in the last question we would like to know how far the students can determine a thing and tell its location. We do not include the problem about plotting a point because the students still do not know yet about the notation of coordinate itself.

Almost all of the students are able to complete the open number line. Only 1 out of 8 students missed the zero. So from this result we conclude that the student know negative, zero, and positive number and its position in number line. While for the missing zero can be a constraint and will be discussed while they learnt about origin point.

Student's answers are different. There are students who used compass direction and left-right direction to tell the location of an object. Some of them use other object as origin point (door, teacher's desk, left desk, right desk etc.) in other words, they use different origin. There are also some students who count the desk, but they do not tell from where they count it.

Activity 1

There are some remarks and finding that become the consideration of the researcher. First is about the use of airplane as the context. Some supervisors and colleagues doubted if the students are familiar with airplane. They argued that students may be more familiar with train or bus. So instead of using airplane

maybe it is better to use train or bus as the context. Surprisingly, when the teacher, in this case is the researcher herself, asks if they know about airplane, almost all of the students ever rode the plane and have experience about it. This means that the change of the context is not needed. We can keep the airplane context and do not need to change it into train or bus.

Second is about the group, it seems that the students do not use to work in group consists of boys and girls. They prefer to work with the same gender, the girls and the boys. The girls claimed that it is not comfortable to work with the boys and so do the boys. Thus, the discussion does not going smoothly. So for the next meeting the researcher decides to separate them into boys and girls. Regarding this issue in the cycle 2, the researcher discusses it with the teacher in charge of second meeting. The teacher said that for the students that will be participated in the second meeting, they already use to work in random group. So it will be no problem.

Regarding of how the HLT works and the students' thinking, we find out that the students face the difficulty in making a system to locate the seats in the airplane. Although they notice that the seats are arranged in rows and columns, but they do not know how to make the system. The only system that they know is numbering it from 1 to 148. Moreover, although the students know that numbering the seats from 1 to 148 is cost time and does not effective, they still stick with the 1-148 system (numbering the seat from 1 to 148) ad cannot come up with other system.

Furthermore, when the researcher asks the students to locate some other seat to make them think about a system that can locate the seats easily, they tend

to find out a way find out the number of the seat easily if the seats are located with 1-148 system. For example, when researcher asks them to locate the 4^{th} seat in 12^{th} row, they will do skip counting of 4 and 6 and to find out that it is a seat with number 62 instead of say that it is in the 12^{th} row and the fourth seat from left. The students' thinking might be limited to the number and 1-148 system since we proposed that system in the beginning. It might be different if we do not tell any system at all.

Because the students get confused and do not come up with an idea of system at all, we decide to give them the second activity first which is about understanding how a system works (cinema problem). We give them the second problem in order to make the students aware of other system beside 1-148 system.

Activity 2

The second activity is about understanding the system of locating the seats which is used in the cinema. This activity is designed in order to help the students aware of the use of system in locating an object and understand how a system works. Here they also will learn that a good system can help them to locate an object precisely and easily. So every object has a unique location. In a system there is no an object that have two or more coordinate/location and there will be no two or more same objects with have the same location/ coordinate

As written in HLT and teacher guide, first the teacher introduces the context. All of the students are familiar with the context. All of them ever watched movie in cinema. They also know that F9 and F10 (number that written in the tickets) represent the number of the seats.

Because there is a sudden change in the order of the activity, then there is a question that should be given but cannot be asked. For instance, the question "what is the different between this seat map and the seat map of the airplane that we made in the previous meeting?". That question requires the knowledge and the students' answer of the problem in the first activity. Yet, they still not solve the first problem yet.

So for the next, teacher asks them to determine the location of seats F9 and F10 in the given seat map. Students work in the same group as activity 1. Both groups find the same seats. However, the seats that they locate as F9 and F10 are actually seats number F12 and F13.

This is happen because they count the first seat in the row F as 1. They do not notice that it should be counted as 4 although they know that the seats placed as long as the line/column 4 is all number 4. So when the teacher asks them "Where is the seat with number F4? What is the meaning of the "9" here? And did not you said before that all of the seats in this line should have number 4?" then they realize their mistake and they find the right seats.

That issue should be emphasized in the discussion, that a system should be able to locate an object precisely and there will be never two or more seats with the same number and reverse. The discussion also focused on the effectiveness of the system. After the students understood how the system used in the cinema works, the teacher asks them if it is easy to find the location of the seat using that system. The students agree on it. They also said that the system used in the cinema which involves the use of row and lines/columns is more helpful than numbering the seats with 1 to 148 for example.

Activity 3

In this third activity, the students are asked to locate the sunken ship. At first the students answers that occur is the same like we conjectured, only locate it using the compass direction or using the distance. Surprisingly, after they realized that locate the ship only using compass direction is not precise enough, they come up with the idea of using the similar system like they used in "airplane" activity (Figure 5.1). This is beyond the prediction.

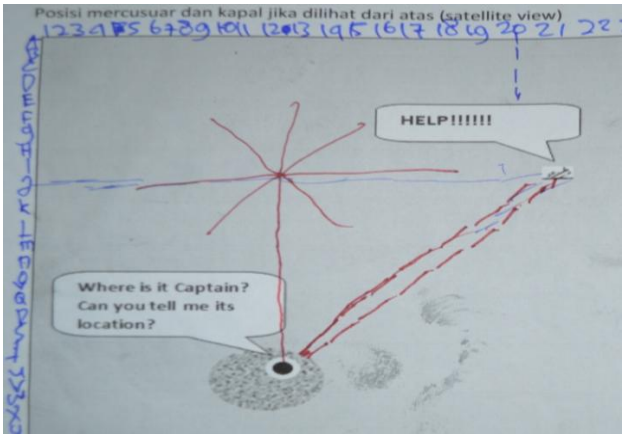


Figure 5.1 System made by the student to locate the sunken ship

Furthermore, although there are some students who used the similar system, they got the different location. These answers are compared, and then the students notice that the location is different because they use the different starting point (up-corner, bottom-corner, and lighthouse). In addition, the also notice that the different distance of the point in the axis makes them to get the different location. Those noticing will be added in HLT.

Activity 4

For the 4th activity which is about looking for a loss watch in the field. We notice that the sentence used to give the direction of the loss watch is ambiguous for the

students. Not only that, the drawing (in where there are some trees which looks like it covers the road) makes the students thinking that those roads cannot be passed through. Hence, they find that only one corner possible although at first they feel confused and unsure which corner it is. Furthermore, the discussion is spent more in making the same perception of the clues/ direction and navigation.

Regardless to those facts, in the end the students are able to determine the location of some points. Even when the researcher asks them to locate a point which the location is not explicitly in the picture, they made it. For example, when the researcher asks Renata to plot or locate point (7, 2) she can do it. To be noted, the grids presented in the whiteboard is only consists of 6 vertical and horizontal lines. She estimates the position of the 7^{th} line/ road of horizontal position and then goes up 2 steps. Students also have no difficulty to determine the location of the point in the axis (involve zero). They can see the "0" distance and can differentiate between, for example, (0, 3) and (3, 0). They keep the agreement that they make.

Activity 5

The 5^{th} activity is about locate the sunken ship and the rescue teams. The context used is similar with the context in the 3^{rd} activity. The difference is the map used in this activity is completed with grids. Moreover, we add some rescue team's ships which is placed in the negative coordinate and "halve". This activity is aimed to help the students learn and understand about negative coordinate and halves coordinate. It should be noted that in the previous meeting, the students already made an agreement in locating a point which is they have to look at the horizontal axis first then the vertical one.

From the observation and video recording, we can know that the students try to use the similar system like in cinema and plane instead of using the system that they learnt in the fourth meeting (system that similar to Cartesian) as they did in the third meeting (see Figure 5.1). So they see the grid as square and see the location as an area instead of the intersection of the grid lines. Hence they find it difficult to understand about halves coordinate. I think we should start to introduce the location involves halves-positive number/coordinate in the 4th activity in where they use the similar system as Cartesian instead of system used in the airplane or cinema. So, they can see the halves.

Furthermore, the students start their system in the different origin. Although the problem asks them to locate the ships from the lighthouse, most of them still locate it from the corner of the map. Thus, it is take times to bring them into negative coordinate.

Activity 6

In the last activity we introduce the formal coordinate system to the students. In here the students will learn about Cartesian system and its notion. It should be noticed that the students actually already learnt about the origin and the agreement needed to locate a point in the Cartesian. So in this activity we only need to (more) elaborate what students already know.

After the students learn about the term Cartesian and its notion, they are asked to analyze and choose the answer of three students called A, B, and C related to the Cartesian problem (see HLT of the 6th activity in the 4.6). Those three answers shows the (student A, B, and C) different ways of plotting points in the Cartesian coordinate system. Student A plots the point in the right way (x, y).

Yet student B reverse y and x, so instead of locating it as (x, y) B plot the point as (y, x). While for C, He plots the second point from the first point instead of from the origin. B's and C's answers are common mistakes that students do when they are plotting points in the Cartesian system.

From the students answer, observation and video registration, we know that some students choose A's answer as the right answer, and some choose B's. But after the students who choose A explain their reason why they choose A and said that based on the agreement they have to locate a point start from the horizontal axis, the students who chose B, realize their mistake and return to choose A indeed. There is no student who chooses C's answer. After the students discuss that problem, they are given some additional problem related to the Cartesian coordinate system. Here they practice what they have learnt.

Post Test

From the pos test we can see that there is improvement in students understanding of coordinate system. Students can locate a thing more precisely. There no more students who locate Andi's desk by say "*it is near the teacher's table*" (question 1 in post test and question 2 in the pre test; see appendix E). They can make their own way to locate an object.

Moreover, the students also able to determine the coordinates of points in the Cartesian coordinate system. Furthermore, students also able to plot points with given coordinates, although there is a student who reverse x and y. They locate the first number in the given coordinate as in y-axis. But when researcher ask her explanation she show that she understand about the agreement and she also realize that her answer is wrong. There is no student who makes mistake in

locating or plotting a point related to the origin. They know the origin and they locate and plot each point from the origin.

5.2.2 Conclusion of the first cycle

From the first cycle, we can conclude that the activity can help the students to understand the coordinate system. Students are familiar with the contexts. It can engage and motivate them to learn about the proposed topic. Moreover, the context with the probing question can make the students to think through the mathematical idea of coordinate system.

Social norm is become the issue. Although students enthusiast to present their answer, but they do not want to hear their friend explanation. Rule for questioning, giving opinion and any other classroom rule are needed to coordinate the class and discussions.

We experience that the role of the teacher is really important in enhance the students thinking. So, we have to make our intension towards the learning become clear to the teacher in cycle 2. We need to help the teacher by providing the clear teacher guide and discuss each activity together.

We found that it is too difficult for students to make up a system in the first activity. This is because they stuck with number. They find strategies to count the seat easier rather than find a system to locate it easily and precisely. So, we decide to change the order of the first and second activity. We also improve some drawings and language used, so it can be understood and lead to the right action as we conjectured. We use these feedbacks to improve our HLT. The details explanation about the refine HLT are described in the next part (5.2)

5.2.3 Refined HLT

Based on the remarks and findings of the first cycle and teacher's suggestion, there are some changes made in the hypothetical learning trajectory. The changes are:

- We change some questions/pictures in the pretests and posttest because it is not clear (there are some points that not in it right position/ moves away). We also add some questions (in pre test) related to locating and plotting a point in the Cartesian system. We decide to give that question even though we know that most of the students still do not know the Cartesian and its notion. We would like to know how far they know about it and then compare it with the post test result. In addition, we add the slanted coordinate system problem in the post test to know whether students can understand about the new system similar to Cartesian. The new pre test and post test are attached as appendix E and appendix G).
- We change the learning sequence (appendix C). We reverse the order of the first and second activity. The cinema activity which is given in the second lesson is given in the first activity. While the plane activity is become the second activity. This is done because in the first cycle, we found that students face difficulty in make up a system to locate a seat in airplane.

  Thus, there are also changes in the given problems. We erase the question "what is the different between this seat map and the seat map of the airplane that we made in the previous meeting?" in the cinema problem. That question cannot be asked since the students do not learnt yet about the airplane problem. In the airplane problem, we do not reveal yet that the system

numbers the seat from 1 to 148 is not effective. Yet, we discuss it in the classroom discussion, and ask the students to find and feel it by themselves. There is no change in the way teacher orchestrate and guide the students. The new activity can be seen in the teacher guide (appendix D).

- We change the drawing of the map on the third activity (see appendix F). We adjust the distance of the sunken ship as 5 kilometers. However, the students tend to measure the distance by a ruler, so the distance becomes different. There is no change in the activity and the conjecture of the students thinking.
- We revised the drawing of the field. Instead place the tree in the road; we make it in the middle of the field so that it will not cover the road. We also paraphrase the conversation between Dina and Pak Toni. We change it into narration instead of conversation. It is done because if we presents it in the conversation from, the each students would like to try that conversation as drama and it will takes time. Moreover, students will only focus on play the role in the conversation instead on the conversation itself. There is no particular change in the students' thinking conjecture and the way teacher should guide and help the students. We start to introduce the halves coordinate in the fourth meeting (see appendix F). This can be done by asking the students to locate a point which is in the "halves"/middle of road.
- For the fifth activity, we change the text in the callouts. So instead of "HELP" we will write "Sunken ship". This is done to make the picture clear. The students will easy to notice which one is the sunken ship (see appendix F: worksheet).

- For the last activity, as the suggestion from the teacher, we change the A, B, C with real names Ani, Budi and Erin. So the students will not confuse. We also change the Erin's answer. At first, we make Erin to plot the coordinates from other coordinates instead of origin (see Chapter 4). In the new HLT, we make Erin locate the other three coordinates from the point (1,1) (see students' worksheet).

This refined HLT then is tested in the real classroom situation (cycle 2). We mean to know how thw designed activites work in the real classroom situation. In addition, we mean to gather data and examine how the students' thinking and learning of the coordinate systems through the given activities. Before that, we would like to give the brief overview of the students and the teacher.

5.2.4 Overview of the students

From the first class observation, we find out that the students participant of cycle 2 are active students. Some of them show that they are independent learner. Before they ask for the teacher helps, they try to solve the problem by themselves first. Of course, there are also students who are passive and quiet. Some students are brave enough to volunteer themselves to solve the problem in the board.

However, they are still seemed unfamiliar with class presentation. When the teacher asks them to explain their answer, they are still shy and cannot speak their thinking well. This is supported with the teacher' information, the teacher says that they are still rare to have a class presentation and discussion. Yet the teacher is enthusiast to do the changes and eager to make her students more active and can argue well of a problem.

In addition, based on the teacher explanation, the students are able to work in group. They are familiar to work with boys or girl. So we do not need to separate girls and boys. Regarding the classroom, the tables are chair are moveable, so it is easy to make working tables for the groups.

For the researcher, this information implies that we can just separate the students into small group consists of 4-5 students mixed boys and girls. We do not need to separate them to avoid the working group might not work as in the first cycle. Because there are 34 students, so there are 8 groups. In addition, that information gives a '*green lamp*' for the researcher to implement the activities that rich of working group and discussion. Although the students are still shy to explain their answer, they still want to try. We hoped that we can improve it and engage them to argue and make them able to speak their mind.

5.2.5 Overview of the teacher

The teacher is a bachelor majoring mathematics education and the homeroom teacher of the students participates in cycle 2. However she never teaches coordinate system before. Yet it is not a big problem remembering the teacher educational background. We know and ever work with the teacher before, in implementing a small activity which is designed based on Realistic Mathematics Education (RME). So even though the teacher never learnt about RME before, she experienced it.

Based on experience of working together, we also know that this teacher is a patient teacher. She is modest in helping the students (not directly reveal the answer, but guide them to find the answer). And fro the observation, we know that she can manage the class well. The students are feel comfort with her

existence, yet they still respect her. We also know that this teacher is cooperative enough. She is the type of teacher who want to improve her teaching and eager to do the changes and to make her students more active and can argue well of a problem.

For the researcher, that information is important to be known because the teacher will play an important role in the learning process. The activities used in this study are designed to help the students by combining real context and teacher's probing question. If the teacher is a traditional teacher, then it will be difficult to implement the activities and the result might not as expected. Although the teacher is promising enough and we provide her with the teacher guide, but we still discuss and reflect on each activity. It is still important to make the teacher clear with her role in the teaching and learning process.

5.2.6 Classroom Teaching Experiment (Cycle 2)

In the cycle 2, the refined HLT is tested in the real classroom which involves 34 students of 5^{th} grade who are different from the participants of the first cycle. The students are separated into small groups consists of 4-5 students. We choose group 3 and 4 as the focus group in this study.

Same as the first cycle, there are six activities tested in the cycle 2. In this cycle, we ask a teacher orchestrate the teaching and learning process. The researcher only act as observer yet intervene the teaching and learning process occasionally. Before we describe the teaching experiment of each activity, we would like to give an overview of the students and the teacher involves in this cycle.

Pre Test

Pre test is given, for about 30 minutes, to the students who participate in the second cycle. There are should be 34 students. However, two of them do not attend the class at that time. So, there are 32 students in total who get the pre test. The pre test is aimed to know the students' knowledge about the needed prior knowledge and about the coordinate systems itself. The questions of the pre test can be seen in appendix E.

From the result of the pre test we can see that all of the students already know about the negative integers and its position on the number line. Regarding students ability in locating an object precisely, there are some students who are able to do that. They locate the Andi's desk as in the second row and second column. However there are still a lot of the students who only locate it as in the second row or near the teacher's desk. This means that most of the students still do not understand that in each object should be uniquely located.

Furthermore, most of the students do not aware of the system around themselves. Although the students already learnt about map and longitude in Geography, they do not notice the map system. There are only 2 out of 32 students who notice it. Hence, most of the students cannot locate the given city using the provided systems.

Regarding the students' ability in locate a point in the Cartesian coordinate system, most of the do not know yet about the system. However there are students who are able to locate the point even write it in Cartesian notion. However, they do not understand what they write (about the notion). They just copy and guess the meaning of the given coordinate/ notion in the last question (about plotting a

point). Hence, there are many students who reverse x and y axis. There are also many students who only locate the point by saying, for example, "*the location of A is near 2*". Furthermore, Most of the students also cannot plot the point with given coordinate correctly. This is because they do not know yet the meaning of the notion.

Activity 1: Understands how the system works (Cinema Problem)

The first activity in the cycle 2 is "understands how the system works (cinema problem)". It is different from the first cycle in where the first activity is about making a system to number the seats of an airplane. As we explained on the analysis of cycle 1, considering the students' difficulty, we decided to change the order of the activity. So, in the cycle 2, the first activity is about cinema problem and the second one is about numbering seats of an airplane.

From this activity we hope that the students will aware of the coordinate (location) system used in cinema and know how it works. We also hope that through this activity, the students will be aware of rows and columns related to the coordinate (location) system used.

The learning process is started with introducing the context to the students. The teacher introduces the context by asking the students about their experience to be in the cinema. As expected, students familiar with the context. Most of them have been in cinema.

Then the teacher shows them the tickets (Figure 5.2) and asks them about the meaning of the numbers written on it.

Figure 5.2 Ticket of a cinema

Students know the meaning of the numbers well. They know that 18 is the date, 14: 35 is the time, 4 is the number of the room, and F9 and F10 are the number of the seats.

Later, the teacher gives the seat map of the cinema (Figure 5.3).

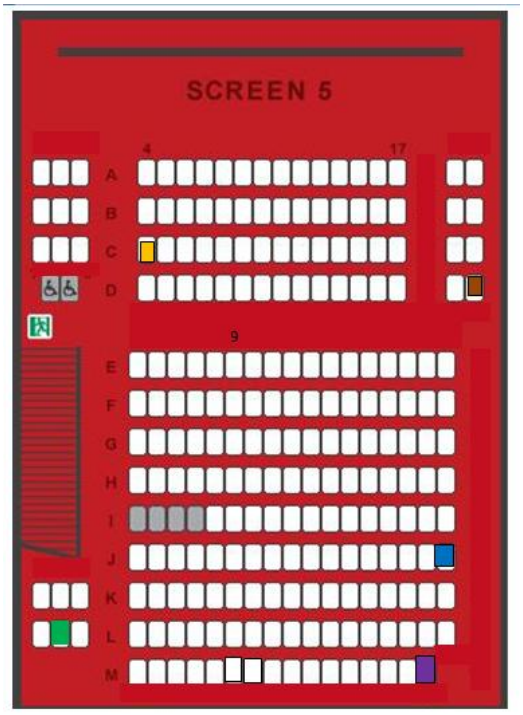


Figure 5.3 Seat map of a cinema

In group of 4-5, the students are asked to look the seat map and tell what they know about the seat map especially the arrangement of the seats. Here the students notice that the cinema has many seats which are arranged in rows and columns. Group 2 write that there are 17 chairs in a row and there are 13 rows (Figure 5.4). But students in Group 8 notice that the number of the seats in each row is different. This group also notice that each rows has name A, B, … and for the first four rows A, B, C, and D have 19 seats while the others rows have the different numbers of seat (Figure 5.5).

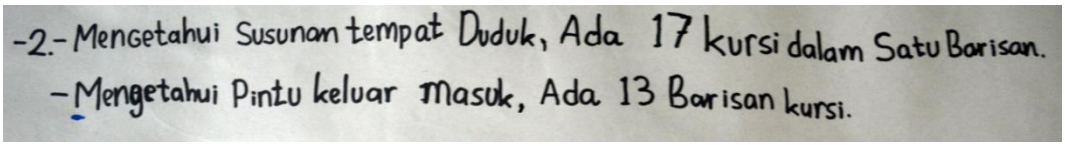


Figure 5.4 The answer of Group 2, there are 17 seats in a row

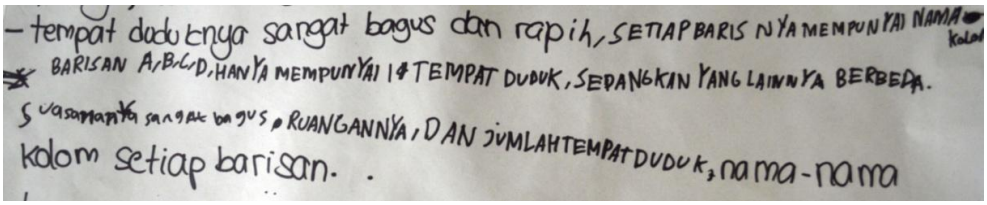


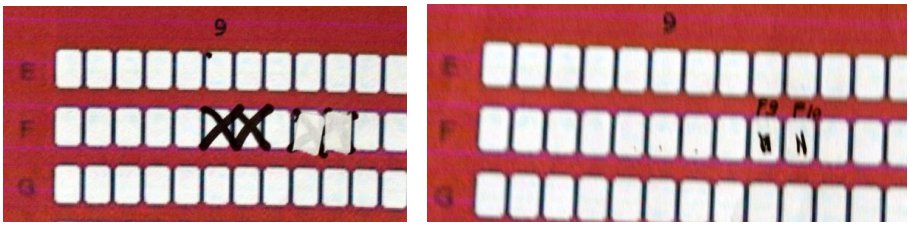
Figure 5.5 The answer of Group 8 that state that the number of the seats in each row is different

Later, the teacher asks the students to determine the seats with number F9 and F10 on the seat map. The given problem and seat map give both constraints and helps for the students to understand how the system used works and understand the idea that each point is uniquely located in a system. Furthermore, the problem given makes the students aware that there are systems to locate an object

What we means by helps here are the seats arrangement on the seat map which are arranged in rows and columns and some signs that show the name of the row and the number of the seats. While the constraint is the number of seat on

each rows is different, so, there are unavailable seats like E1, E2, E3, .. etc. If the students really understand the system then they can reason about the constraint.

As we predict on the HLT, there are two different answers occurs. There are some students who mark the seat below the E9 as seat F9 (Figure 5.6.a). But, there are also some students who claim seat which should be number F12 as F9 (Figure 5.6.b).



(a) (b)

Figure 5.6 Students' different answers of the location of F9 and F10

Most of the students tend to claim seat which should be number F12 as seat F9. This is because they count the seat in row F from one instead of four (Figure 5.7). They see seat with number F4 as seat F1.

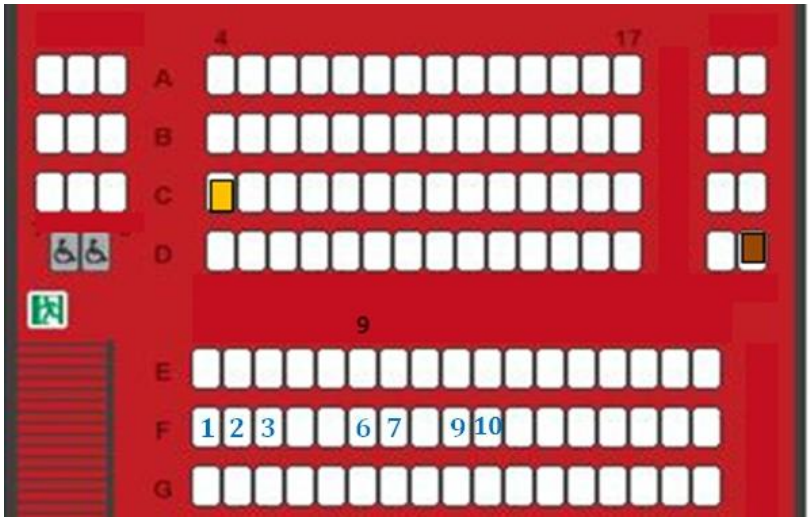


Figure 5.7 The way students' count the seats in row F

The reason why the students see the seat F12 as seat F9 is because they do not notice the signs “4”, “17”, and “9” which represent the number (column) or do not understand its meaning (Transcript 1).

Transcript 1

1. …
2. *Teacher : where is F9?*
3. *Renita : (counting the seats on row F from 1 to 9. See figure 8) here*
4. *Teacher : how about A4*
5. *Renita : (count the 4 seats on row A, yet she counts the fourth seat as the first) here..*
6. *Teacher : what is the meaning of this number (point the number sign “4”)*
7. *Renita : number (of the seat) … (thinking)*
8. *Teacher : if I ask where is A4?*
9. *Renita : Here (point the right location)*
10. *Teacher : if D4?*
11. *Renita : here (easily found the right seat)*
12. *Teacher : how about D9?*
13. *Kaisar : here (point the seat of D9)*
14. *Renita : (check it by counting the seat from 4 to 9 and agree with Kaisar’s answer)*
15. *Teacher : so where is F9?*
16. *Renita & Kaisar : (points the right seat)*

From that transcript we can see that at first they claim that seat F12 is seat with number F9. But after the teacher asks them the meaning of the numbers (4, 9, 17) written on the seat map they realize their mistake. So make the meaning of the written alphabets and numbers clear help the students to understand the system.

Another reason why the students see the seat F12 as seat F9 is because although they notice the signs (written numbers and alphabets) and understand its meaning, they cannot accept that there are no seat with number F1, F2, and F3 like happen in Group 4. This group now that seat E1, E2, and E3 are not exists in that seat map. But they are not sure with their answer. To see something that is not available (cannot be seen) is abstract and difficult for those students. Moreover,

the children are seems not used to be asked with question which have "no" (abstract) answer. They used to the questions which answer can be seen and be shown (not abstract).

However, not all students face that problem (difficult to accept the idea that there are some seats which are not in the seat map). There are also students who can understand the system and know that there are some seats which are not in there. They even can explain well why the first seat in row F has number F4 and not F1 (Transcript 2).

Transcript 2

…

1. *Teacher : where is the seat F9?*
2. *Naufal : (count the seat on row F backwards from 17 till 9) here..*
3. *Fadya & Hendra : (check the Naufal's answer by counting onwards the seat on row F from 4 to 9) yes there (point the same location as Naufal)*
4. *Teacher : How about E4?*
5. *Naufal : Here (points the right position of E4 which is the first seat on row E)*
6. *Teacher : how come you know?*
7. *Naufal : because this is 4 (point the A4 and then he make a line with his finger from A4 to E4, as long as the seats in line/ column 4) so here it is (E4).*
8. *Other students : nod (agree with Naufal's answer)*
9. *Teacher : How about E2?*
10. *Naufal : (confused) here maybe (points location besides the last seat on row E)*
11. *Other students : (thinking)*
12. *Teacher : where is A2?*
13. *Students : here (point the right location of seat A2)*
14. *Teacher : and so E4? Where is it?*
15. *Naufal : (points the right location of E2) here … (laughing)*
16. *Other students : (laughing)*
17. *(Here the students laugh because they found out that there's no seat in location of E2. Yet it is a stairway)*
18. *Teacher : so where is E2?*

19. Fadya : (shake her head which means there is none E2)
20. Other students : there is none …

Furthermore, those both answers are presented in the class and discussed. The discussion is focused on the meaning of alphabet and numbers written on the seat map, and how the system used in cinema works. After the students understand how the system works, teacher asks them about the seat E1, E2, etc. Most of the students' answers that those seats are not exist. Here, Group 4 finally feeling sure and can accept that there are some seats which are not exist.

Another important issue that addressed is the idea that each point/ object is unique (uniquely located by a system). Placing the students in the problematic situation in where they find the different seats for a location/ number (Transcript 3) makes them aware and makes them senses that it is impossible for a number to have to different seat.

Transcript 3

…
1. Teacher : so where is F9?
2. Renita & Kaisar : (points the right seat)
3. Teacher : how about F3?
4. Renita : (thinking and then counting backward from the position of F9 but she cannot find F3 because it does not exist)
5. Kaisar : (counting onward from F4 but he count it as F1) here (points the seat F6)
6. Teacher : how about F6 then? Kaisar said that here (F6) is F3. Now where is F6?
7. Kaisar : (he seems want to point the seat F9 as F6 but he does not do it)
…

From that Transcript 3, we can see that Kaisar senses that it is impossible for seat in the location of F9 to be seat F6. When the teacher asks him where is F6

he seems want to point seat F9 but because he know that it should be F9, he does not do it.

Furthermore, based on the video registration we can know that the students understand about the idea that each point is unique. They argue that having two different seats with the same number is impossible. They said that each ticket/ seat number on ticket is for a seat (Transcript 4).

Transcript 4

...

1. *Teacher : have you ever seat on the wrong seat in the cinema? Or maybe is there someone who seats on your seat?*
2. *Students : Yes..*
3. *Teacher : Why ca it happen?*
4. *Hendra : They take the wrong seat*
5. *Teacher : Is it possible for a ticket to has two different seats?*
6. *Students : No.. (shake their heads)*
7. *Teacher : So do you mean that a ticket is for a seat?*
8. *Students : yes*

Another finding from this activity is that some students can relate the location of an object with location of another object. They can determine the location of a seat by knowing the location of other seat. For example, students in group 5 write that seat with number F9 will be below E9 (Figure 5.8). They know that the position of seats on row F is behind the seats on row E, and the seat with number 9 will be placed in a line/ column. So the location of F9 will be below of E9.

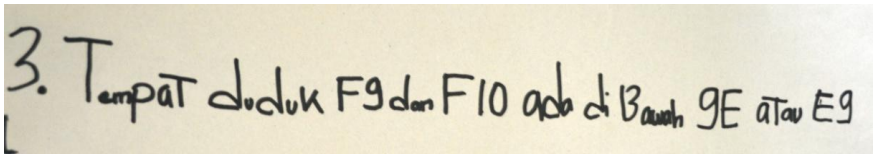


Figure 5.8 Students' answer which states that the position of F9 is behind E9

Moreover, from the video registration, we also can see that the similar answer is give by Group 7. When the teacher ask them how they can find the location of seat F9 and F10, they say that they simply find F9 by looking at seat on row F and the line of seat with number 9 and beside it is seat F10 (Transcript 5).

Transcript 5

1. *Teacher* : *ok lets listen to Renita explanation?*
2. *Renita* : *the location of F9 is here (point the seat F9 on the map). We can know by looking the column/line of seat with number 9(point the "sign 9" in the seat map) and row F. show it will be here (point the location of F9). And F10 is beside F9.*

…

While students in group 6 know that the location of seat F10 is beside F9. They write that F9 is in the row F and the 9th line, while F10 is in the row F and the 10th line which is beside F9 (Figure 5.9).

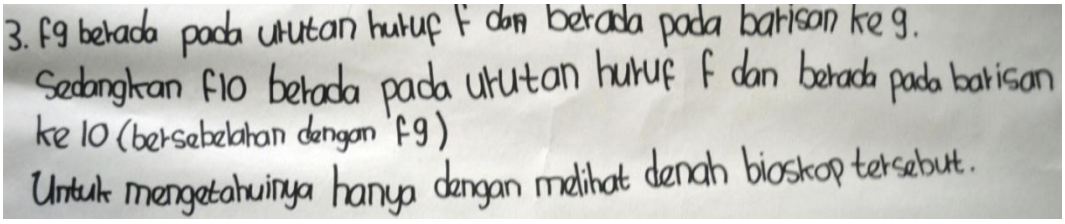


Figure 5.9. Students' answer which shows the relationship of F9 and F10

Activity 2: Making a system to number the seats on an airplane

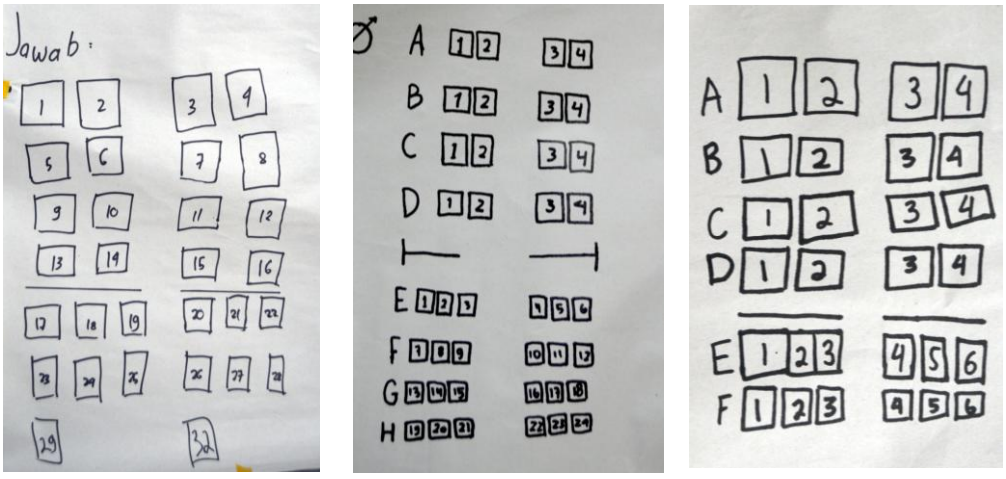
Differ from the first cycle, the activity "making a system to number the seats on an airplane" is given as the second activity in this cycle (cycle 2). And in this cycle, the students are given more freedom to make their own system. If in the first cycle the students can make their own system exclude numbering the seats from 1 to 48, here (in the cycle 2) it is included.

Through this activity we hope that the students able to make a system to locate a thing (seat on airplane), aware of the row and column, understand what a good system is, and they will aware that to locate a thing precisely they need at least two parameters (in this case is row and column).

The learning process is started by introducing the context. The students are familiar with this context. Almost all of them ever rode it. The students eagerly tell their experience with airplane. Next, the teacher introduces the problem.

The students are given the seat map of an airplane (appendix F). They have to make a system to number the seat map. So, the passengers and stewardess can find the location of a seat easily. They discuss this problem with the same group as in the first meeting. The system made should give a unique location for a seat.

There are different systems that occur. Here the three different system that made by the students to locate/ to number the seats of a plane (Figure 5.10).



(a) (b) (c)

Figure 5.10 Systems made by the students to locate the seats of an airplane

The first group (Figure 5.10 a) numbers the seats using number from 1 to 148, from left to right. They do not separate between business class and economy. This means that they still not aware of the row-column arrangement. Numbering

the seats from 1 to 148 is not wrong. By numbering it from 1 to 148 we still can locate the seat precisely. However it less effective and needs time to find a seat.

While the other group (group 4) numbers the seats using alphabet and number (Figure 5.10b). If we look at the answer in detail, we can see that they notice that the seats are arranged in rows and columns. They number the rows using alphabet while the column is represented by number. But because the number of the columns is different between the business and economy class, they decided to numbering the column of the business seat and numbering one by one for the economy class. So, they separate the seats become two groups and differentiate the system for each group. For the business seats, they number it as A1 to D4. Yet for the economy class, they number it from E1, E2, …, F7, … G12 .., etc.

The system like in Figure 5.10c is the most popular system. Most of the students/ groups come up with the idea to number the seats using alphabet and number. They make the similar system like used in the cinema (they learnt it in meeting 1). Furthermore, this system is the sophisticated system.

The students who come up with this system notice that the seats are arranged in rows and columns. They represent the row with alphabet and the column with number 1 to 4 (for business seat) and 1 to 6 (for the seats in the economy class). So for the first four rows, they number it from A1 to D4, and for the rest they number it from E1, E2, E3, E4, E5, and E6 continue with F1, …, F6, and so on. Here they make up the system by consider two parameters, row and column.

From these three answers, we know that the students already know how to make a system to number the seats of an airplane. Their system can locate the seat precisely. Just, whether the system made are good or not and which one is the most effective are become the next issue to be discussed.

The idea that a good system to locate an object should be easy to understand, effective (can be used to a seat easily and fast), and can locate a point/ object uniquely is what we want to be understood by the students. So in the classroom discussion, the children will decide by themselves whether this system is a good system or not. Yet, still, the teacher helps them by asking several questions, like ask them to locate a certain seat. By asking them to locate a certain seat using those different systems, they will experience which system is most effective and better to used. Here the students claim that the first system (numbering the seat from 1 to 148) is not a good system. Although it can locate the seats precisely, it consumes much time. It is also not effective because we need to see the number one by one. They prefer to use the third system (Figure 5.10c).

Actually there is still another interesting system that comes up beyond the conjecture. There is a student who comes up with the idea to separate the seats in two groups A (left group) and B (right group). Then he represents the row with number (1 to 26) and the column with 1 and 2 (for the first four rows) and 1, 2, and 3 (for the rest rows). So the first seat is A11 which means seat in group A (left), first row and the first seat from left, B12 means seat in the group B (right), first row and the second seat. This answer can be seen in this following picture (Figure 5.11).

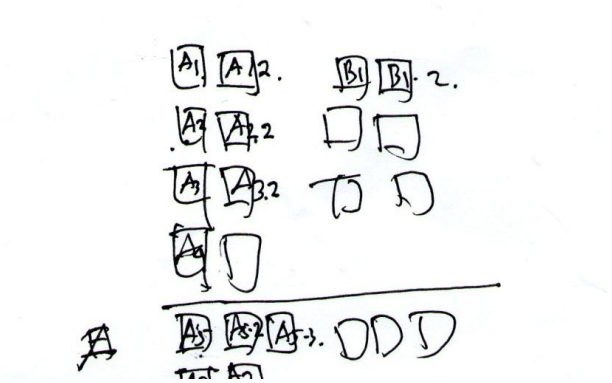


Figure 5.11 System to number plane's seats made by Group 6

However in the end group 6 do not use this system, rather they used the system like Figure 5.10c. They argue that system like in figure c is easier to understand and more effective to be used rather than the system in figure d.

Through this activity, students not only learn to make a system to locate the seat on an airplane, but they also learn that a good system should be able to locate a point/ an object uniquely. And to locate a seat precisely, it is not enough to only consider only row or column.

When the teacher asks the students in Group 4 to find the seat with number 5 they said that they cannot find the exact seat. The information that teacher has is not enough, something is missing which is the number of the row. The similar answer also occurs in group 3. When the teacher asks them to find the seat with number F they said that they cannot find the exact seat. They also argue that we also cannot seat in any seats with number F, because it may be other people seats.

Activity 3: Locate the sunken ship 1

After the students understand how a system worked (e.g. cinema problem) and able make their system to locate objects which are arranged in rows and columns (airplane problem), in the third activity, the students are challenged to make their own system to locate an object in a plane. So they will aware of another kind of

coordinate (location) systems and aware of the origin, and understand the idea that each point is unique. A point or an object is uniquely located in a system. There will be no two different points with same location/ coordinate and there will be no a point which have two different locations/ coordinate.

As usual, the learning process is started with introduction of context. Teacher gives a map of the sunken ship to the students (appendix F). In the same groups from the previous meetings, students are asked to locate the sunken ship from the lighthouse. The proposed context is open enough to offer freedom for the students to make their own (good) system to locate the ship which is often seen as difficult problem. So we give helps for the students like providing the compass direction and a line that represents 1km distance.

Various answers occur in this activity. From the students answer we can know that all of the groups can locate the sunken ship precisely. Even there are groups who manage to make the system which involve horizontal and vertical distance (Group 2, 3, 4, and 8) and a system which involve angle (Group 1).

Yet, at first, all groups locate the sunken ship by considering only one parameter, compass direction or direct distance (lighthouse-ship). The students who locate the sunken ship using the compass direction say that the location of the sunken ship is in the north-east of the light house (Figure 5.12). The students who locate the ship by measuring the distance of the ship from the lighthouse claim that the location of the ship is 8 kilometers from the lighthouse.

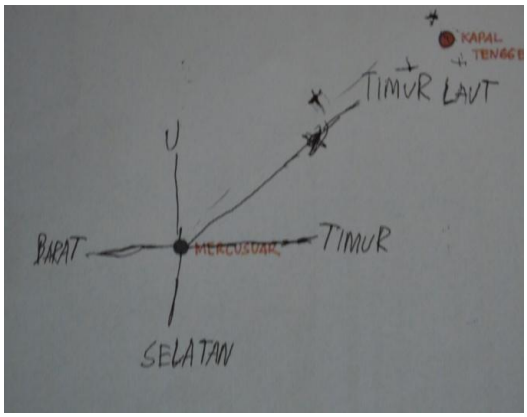


Figure 5.12 Students locate the sunken ship by only consider the direction

Locating the ship by only considering the direction or distance is not precise enough. There are many possible locations which can be represented by "north east" or "8 kilometers from the lighthouse". This mean that the system used to locate the sunken ship is not good enough.

To make the students aware of that fact and challenge them to think of a good system, teacher shows them some possible locations that can be lead by their answer. For the students who claim that the location of the ship is in north east, teacher points some locations which are also in the same direction (north-east). And for the students who locate the sunken ship as 8 kilometers from the light house. Teacher shows them that 8 kilometers can be 8 kilometers to the north, east, or other direction.

Showing the fact that their system leads to unwanted location can make the students aware that they have to make up a way to locate the ship precisely. They start to think that either only locating the ship by using compass direction or measure the distance is not enough to locate the ship precisely. That system is not a good system because it can locate the ship/ point uniquely.

Later, some of them then try to locate the ship by considering 2 parameters. They combine compass direction and distance. They say that the location of the sunken ship is 8 km to the north-east from the light house (Figure 5.13). By combining both compass direction and distance they get a more precise location/ system.

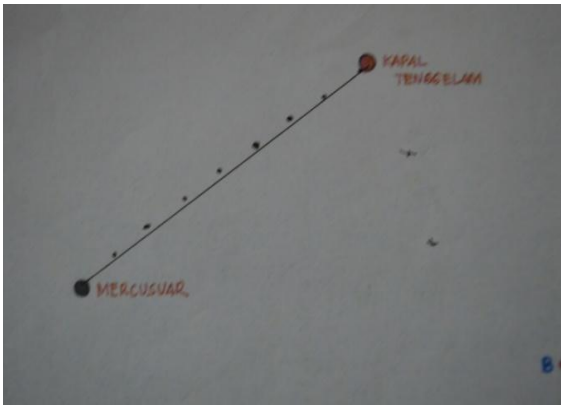


Figure 5.13 A system made by the students which consider distance and direction

Actually, locating the sunken ship by considering the direction and distance is precise enough. But when the teacher asks the students to tell the location of some points which are 8 km from the lighthouse but not exactly in the north-east (near the north east) they start to think about the more sophisticated systems like what happen with Group 4 (Transcript 6).

Transcript 6

...

1. *(The teacher asks the location of the sunken ship)*
2. *Leony : 5 km from the lighthouse*
3. *Teacher : How if I said 5 km from the lighthouse is in here? (points another locations which are also can be represented as 5km from the lighthouse)*
4. *Leony : To the north-east*
5. *Teacher : North-east, so it is in this direction (point the direction of north-east). But how if I go to 5 km to here (near north east)?isn't it still in north east?*
6. *Students : (nod)*

7. Leony : but it should be 5 kilometers
8. Teacher : here is 1, 2, 3, 4, 5 km from the lighthouse (the teacher shows that the point/ location pointed by her is 5 km from the lighthouse)
9. Students : (seems they are thinking)
10. Teacher : (leave them and give them time to discuss with their group)
…

The students (group 4) discuss it with their group. From the audio recording we know that they try several ways. One of the students proposed the idea to locate the ship by going straight to the north and then turn to the 3 o'clock direction. And when the teacher backs and asks them again about the location of the sunken ship, the students say that they can locate it by going 4km sideways/ *miring (*Transcript 7*).*

Transcript 7

…
1. Teacher : so how is it?
2. Leony : it is 4km sideways from the lighthouse
3. Teacher : sideways? … Which side? Here also sideways, here, here (teacher points some location which can be represented by 4km sideways)
4. S2 : 4km
5. Teacher : here is 4km sideways
6. Cowok : 360 sideways
7. Leony : how if like this, we go straight … how was it before… here, 4 kilometers straight (to the north) and then turn to the east 5 kilometers.
8. Teacher : is it 5 km?
9. Leony : (measure the distance with the ruler, in which 1 cm represents 1 km)

This group finally find that the location of the sunken ship is 5 km to the north and 6.5 km to the right, or go straight to the north as far as 5 km then turn to the east as long as 6.5 km (Figure 5.14).

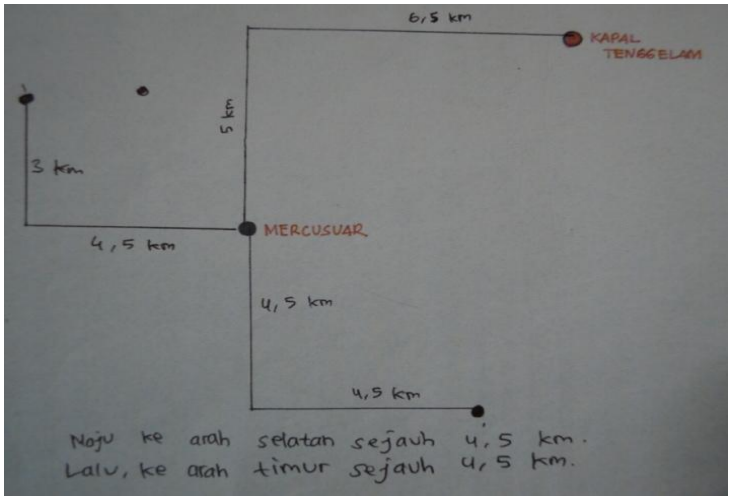


Figure 5.14 A system made by the which consider the distance and two dimensions measurement (north and east)

From the transcripts, we can see that with the helps and guidance from the teacher, Group 4 finally can locate the sunken ship precisely with considering two parameters: horizontal distance and vertical distance. They also understand that the good system can locate an object precisely without mislead to any other location. In other words, they know that each point is unique.

If we look into details to the transcript, we can see that the students keep thinking about more sophisticated system because the teacher shows them that their proposed system "5km to the north east" cannot locate the ship precisely. Teacher says that points near the north east are considered as in the north east area (Transcript 6 line 5) which is agreed by the students. So, by only saying that the location of the ship is 5km to the north east can lead to other location.

However, there is a mistake in how the teacher and students define the north east direction. North east direction does not have area. It is exactly 45^0 from the north and east. So the points near the north east cannot be considered to be in north east. In addition, the "5km to the north east" system is a good system which can locate the ship precisely.

Yet, that misconception leads to the further discussion about the more sophisticated system to locate the sunken ship (measuring the horizontal and vertical distance). By showing that there are points which can be located with the same direction/ coordinate, we challenge the students to think about the more sophisticated system. This is also the same with showing them that there is point which cannot be located by the proposed system.

So, in the next implementation, the way we challenge the students to think about more sophisticated system can be done by asking them to locate the another points which are not exactly in the direction of north east, shout east, north north east and so on (not in the direction of 45^0 and its multiplication).

Regardless the misconception happened, the fact that students can locate the sunken ship precisely is still beyond the expectation of the researcher and supervisors, although it is already conjectured as written in HLT.

Another surprising thing occurs is that Group 3 locates the sunken ship considering the distance of the ship from the lighthouse, the direction and the angle. They claim that the location of the sunken ship is 135^0 to the north-east and around 8 km from the lighthouse (Figure 5.15).

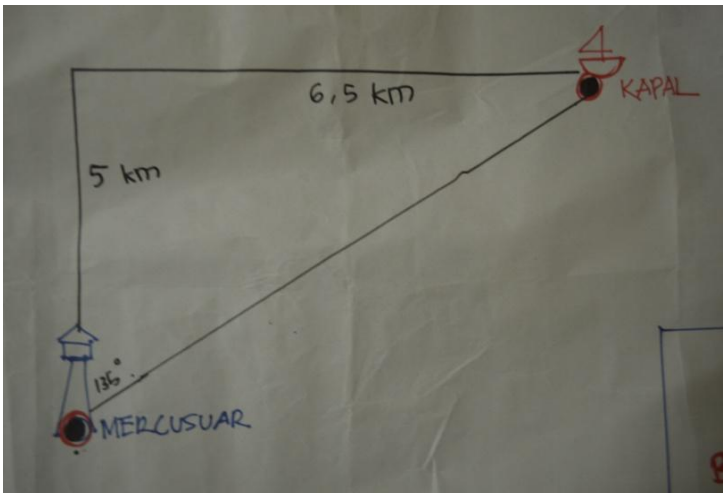
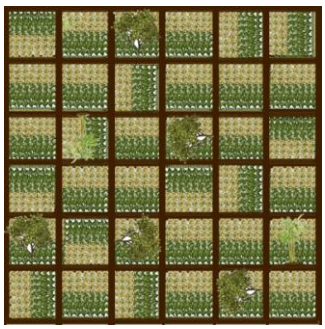


Figure 5.15 Students make up a system to locate a sunken ship by involving angle

From the analysis above, we can see the improvement/ development of the students' thinking in locating a point. From only considering one parameter, two parameters not perpendicular, till considering two perpendicular parameters. In other words, given the designed activity/ context and with the guidance from the teacher, the students show a lot improvement in making a sophisticated system to locate and object precisely.

Activity 4: Rice field (finding the lost watch)

The fourth activity is about finding a lost watch in the rice field. Same as the previous activity, this activity is started by introducing the context. The teacher tells a story about Dina who lost her watch when she is playing in the rice fields. Teacher also gives the map/ drawing of the field and the clues of the location of the watch as follow.



Clues: Dina only remembers that at that time she stood at the corner of the field. She walked straight/ berjalan lurus till she met the roads intersection. Then she turned/ belok and walked straight. Later she met intersection of roads but she still keep walk straight. She then met another intersection and she stopped. She thought that she dropped her watch there.

The aim of this activity is to make the students aware of the need of origin point and that origin point in a system should be unique. In addition, we hope that through this activity students also can tell location of an object precisely,

understand that each point is unique, aware of the vertical and horizontal grid lines, and know about the positive coordinate.

From the video registration we can see that during the group discussion, almost all of the students get confused to determine the corner in where Dina stood as the origin/ starting point. They argue that there are many corners that can be seen in the map.

Naufal (group 3) argues that there are many corners. And each corners can produces different final locations. This statement also supported by his friend (Hendra). They point some possible locations from the different corner/ origin (Figure 5.16.).

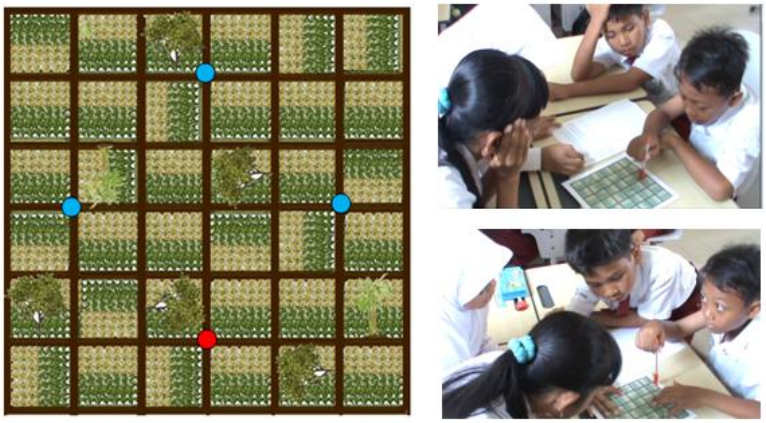


Figure 5.16 Possible locations of the lost watch pointed by group 3

From Figure 5.16 above, we can see that Naufal points one of the possible location of the watch (red point), and Hendra show another possible locations (blue points).

Naufal also shows another possible location if the origin is in the corner of small square/ field (Figure 5.17). So if the origin is in the location marked by yellow triangle, then the location of the watch is in the red point.

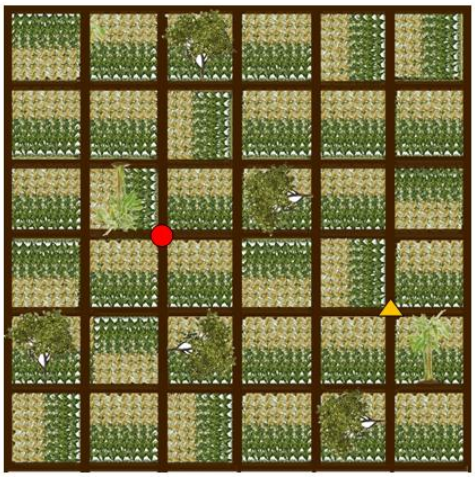
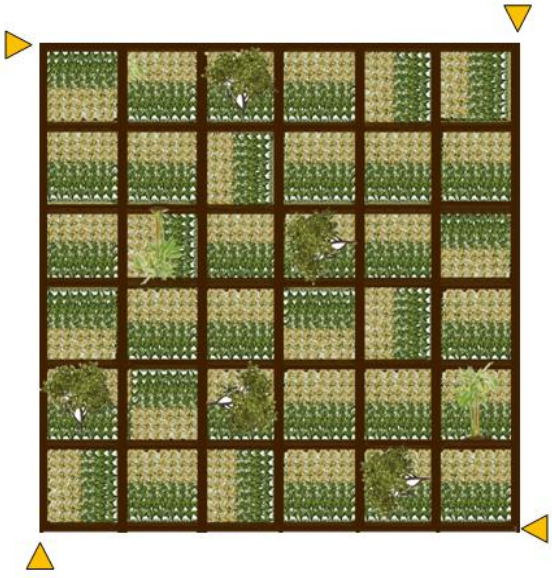


Figure 5.17 Naufal's answer of another possible location of the watch

When teacher gives additional clue that the starting point is “left bellow corner”, Naufal still can not sure which one it is. He knows and aware that left bellow corner is not unique. It may be different as each person point of view. He claims that there are four possible left bellow corners in the map. He gets different corners if he change his direction to see the map by rotating it 90^0. The similar thinking also occurs in other groups. They say that left bellow corner is not only one. As the map is rotated the location of “left below corner” will be different



(

Figure 5.18).

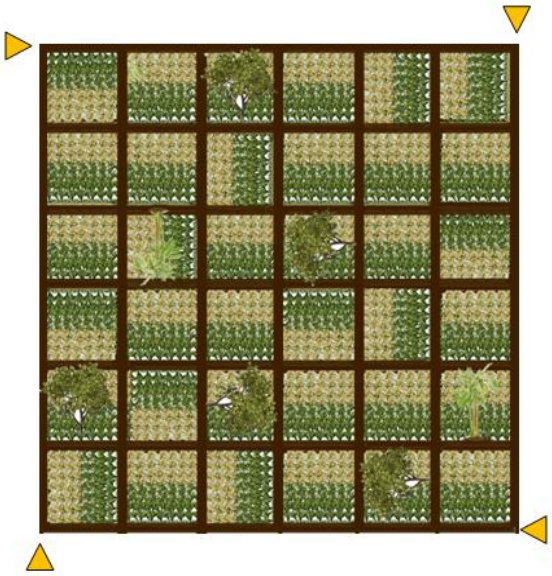
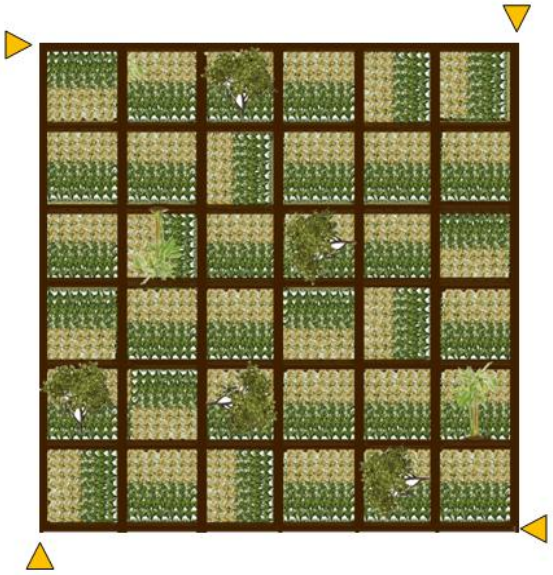


Figure 5.18 The possible origins



From ***Figure 5.18*** we can see that there are four (left-below) corners/origins from the different view. The triangles represent the four corners, and its direction represents from where we should see the map.

Students' confusion is what we expected. By placing the students in that situation, students will learn and aware that they need the same unique origin in a system. If a system has different origins then it will give different locations for a

point and show different points for one location/direction. Later, they make an agreement about which corner should be used as origin.

Regarding find the location of the watch, only 2 groups out of 8 groups who can find the right location, they are Group 4 and 7 (Figure 5.19).

Figure 5.19 Students' aswer of the watch's location

From the video recording and the students' answers (Figure 5.19), we can know that the cause of the wrong answer is the wrong interpretation of word "turn/ *belok*". We define the word "turn/ *belok"* as only change our direction. So if we walk to the north, "turn right" means that we change our direction to the east and as consequence we face the east (Figure 5.20 left). While group 6, 3 and 8 interpret the word "turn right" as change direction to the east and go/walk straight until they meet another intersection (Figure 5.20 right).

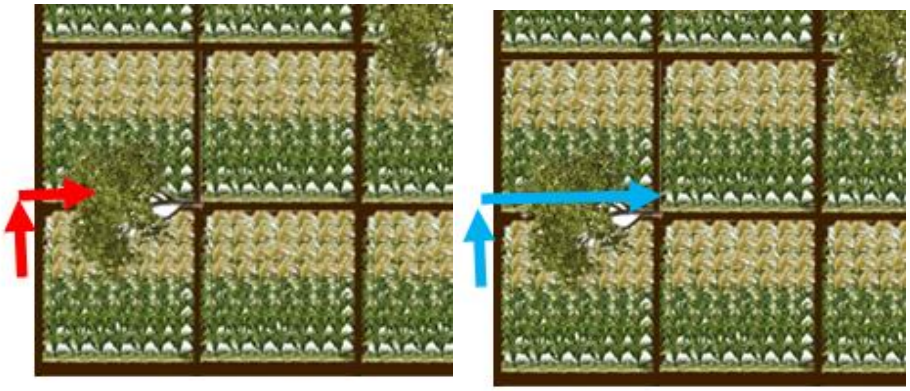


Figure 5.20 Students' misinterpretation of the word "turn/ belok"

Another mistake in navigation that leads to the wrong location is the misinterpretation of word “go straight/ (*berjalan) lurus*”. There are some groups (Group 1 and 2) who interpret the word “go straight” as go/ walk to the north. This misinterpretation is caused by the additional clue. When the students get confused to which direction they have to walk from the corner (there are two possible direction: north and east), teacher and researcher decide to limit the possible answer by giving additional clue *“stand in the corner ad go straight to the north”*. Due of that, students in group 1 and 2 always interpret “go straight” as walk toward north. So, the clue that says *“... Dina turns right and then walks straight”* is interpreted as *“... Dina turns right and then walks to the north”* like showed in Figure 5.21.

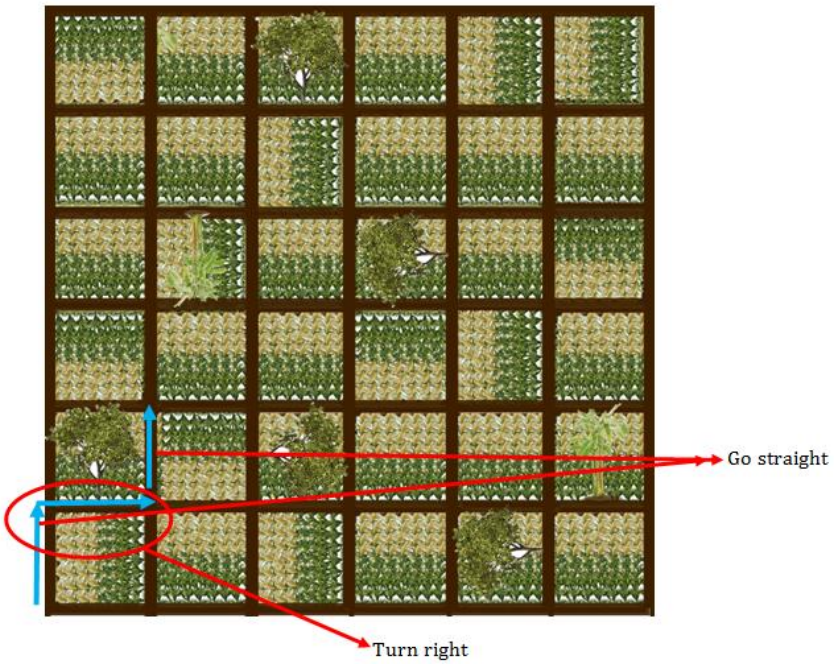


Figure 5.21 Students' misinterpretation on the word "go straight/ lurus"

So if we see the students's answer once again (Figure 5.19), we can see that group 1 and 2 misinterpret the word "turn/ belok" and the word "staright/ lurus". They walk to the north from the corner, then turn and walk through till meet the intersection, and then walk straight to the north (actually here they turn left) (Figure 5.22).

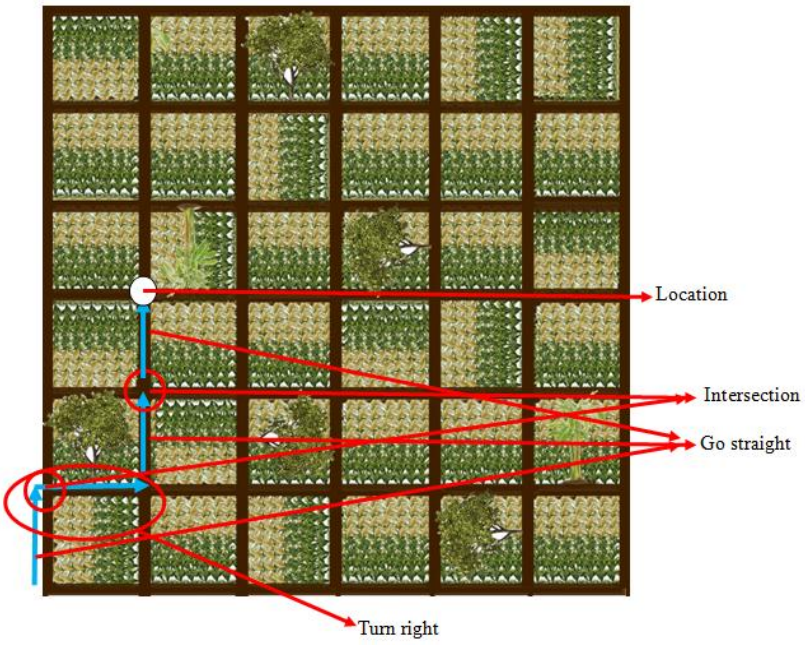


Figure 5.22 Walkpath made by Group 2

While for group 3, 6, and 8, they walk straight to the north from the corner, then turn right and walk till they meet the intersection (should be only turn right/ change the direction). Then they walk straight passing the intersection and stop at the next intersection (Figure 5.23).

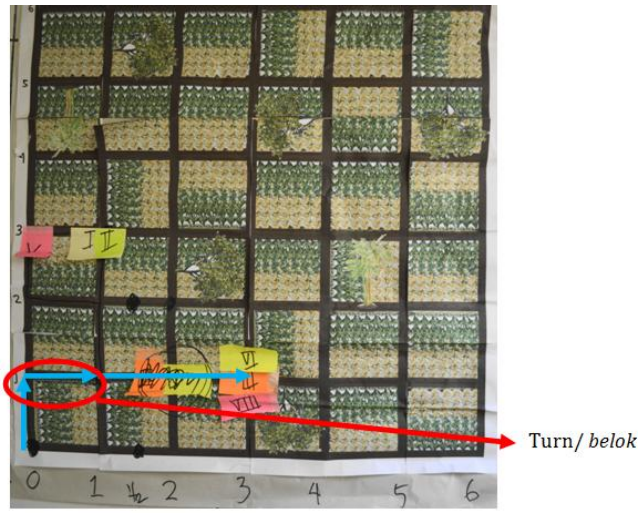


Figure 5.23 Walkpath made by Group 2, 6, and 8

These different answers and misinterpretation are discussed in class. After making the misinterpretation clear, teacher continues the classroom discussion. The issue addressed is how to tell the location of the watch (making a system to locate the location of the lost watch).

The one who proposed to numbers the roads is the teacher. But the students come up with idea of "0" by themselves. From the observation and field notes, we know that the students can tell the location of the point (location of the lost watch). Even some of them come up with their own notation like "2 and 3" and "2,3" which mean (2,3) or the point is located in two steps to the right and 3 steps vertical/ to the north. When the teacher plays dumb and say this (points (3,2)) is also 2 and 3, the students start to think and say that they are different

point. They say that the location asked by the teacher is 3 to the right and 2 up/ to the north. So it is different. This means that they understand that each point is unique. One location should only for one point and vice versa.

Moreover, students also understand that they need to make an agreement from where they should see the location first. So they can differentiate between (2,3) and (3,2). They agree to see the horizontal distance first then the vertical distance.

The further discussion is about locating a point which is not exactly in the intersection of the roads, rather than in between. In other words, they discuss about the coordinate which involves halves like (2 ½, 1). Most of the students can understand it easily. They say that to determine the location of point located in (2 ½, 1) they have to walk 2 and a half step to the right/ horizontally and then walk 1 step to the north/ up.

Yet, there is a student who still gets confused. Kaisar says that to reach location of (2 ½, 1) we should walk 2 steps to the right, 1 up, then walk a half to the right again. He argues that we should follow the road, and it is impossible for us to walk pass the paddies. This shows that Kaisar still think in the real situation. He still sees the field and not yet sees the map of the fields as squares or grids or intersections of lines. The teacher then says that we are allowed to pass through the paddies, so we can directly walk 2 and a half step to the right and 1 up instead of following the road and he seems to understand it.

Activity 5: Locate the sunken ship 2

The context used in this activity is similar with the one used in the third meeting. The difference is in this activity, the given map is completed with grids. In addition, we placed some rescue teams' ships.

Students work in the same group as the previous meetings. They are asked to locate the sunken ship and the rescue teams' ships. The rescue team's ship are placed in several locations so it will involves positive coordinate, negative coordinate, halves, and coordinate that involves zero "0" (in axis).

To be note, the students already learn about origin and make an agreement in locating a point (see the horizontal distance first then vertical). From the previous meetings they show understanding on it. They also learnt about positive coordinates and (involves) halves coordinates.

During the group discussion, teacher and researcher walk around. From the observation we know that the students can locate the ships (sunken ship and rescue teams') which are placed in the positive and negative coordinates easily. Some of the students (Group 3, 4, 2, and 1) are also able to locate the ships which are in the axis and halves. They can differentiate between the point in the vertical axis and horizontal axis (0, 2 ½) and (2 ½, 0). They understand that points in the horizontal axis will have "0" vertical distance and vice versa.

However, from the transcript below (Transcript 8), we can see that there are students (group 7) who struggle to understand how to locate a point which is in the axis/ involves "0" like (0, 2 ½). They say that the location of the ship A is (1, 2 ½). And when they know that the coordinate that they proposed is not the location of the ship, Kaisar come up with the idea of walk path (go to the (1,2) then turn to

(0,2) and then continue to the destination (0, 2 ½). In other words, he locate the ship as "*1 to the right, 2 up then 1 to the left then ½ up"*

Transcript 8

(Here the students already find that the location of Team A is 2 ½ up/ to the north from light house. And now researcher try o ask them about it horizontal distance from the light house. And they answer that it is in the coordinate (1, 2 ½))

…

Renita : 1, 2 ½

1. *Researcher : 1, 2 ½ ? that means go to the right 1 and*
2. *Students : 2 ½ up. Ok, 1 to the right and 2 ½ up (point the walk path) 1, 2 and ½ here the position (point the coordinate (1, 2 ½)). It should be here (the location of A: (0, 2 ½)*
3. *Kaisar : Oh go here first (from the (1, 2) to the (0, 2))*
4. *…*
5. *Researcher : this (the sunken ship) is 5 to right and 6 up. Now this (A), how far it goes to the right?*
6. *Kaisar : 1 to the right*
7. *Researcher: if 1 then it should be in this line (point the line passes through "1") while it should be in this line (point the vertical axis)*
8. *Kaisar : how if it like this, 1 horizontal, 2 up,*
9. *Renita : 2 ½*
10. *Kaisar : 2 up then 1 to the horizontal then ½ up.*
11. *Researcher : which side of horizontal direction?*
12. *Researcher : which side of horizontal direction?*
13. *Kaisar : 1 to the right then 2 up then 1 to the left..*

...

From the continuation of the discussion (Transcript 9) we can see that although the students in the group 7 finally realize that team A is in the "0" lines (horizontal axis) or in one line with the origin and the vertical distance is 2 ½, they still avoid to say that the coordinate is (0, 2 ½). They avoid "0". They say that its location is in "*line of 2 ½"* (Transcript 9 line 18).

Transcript 9

…

1. *Researcher : points lay in here (line that passes '5') are points in what line?*

2. *Kaisar : 5 (he see the previous system that they made before with the corner as the origin)*
3. *Researcher : don't see it again*
4. *Kaisar : "0"*
5. *Researcher : how about the points here?*
6. *Kaisar : 5*
7. *Researcher : and here is 1 isn't it? How about the horizontal distance of this point (A)?*
8. *Renita : "0"*
9. *Researcher : and vertical?*
10. *Kaisar : 2….. 3*
11. *Researcher : 2 is here isn't it? while it (A) is in this line (point the line pass through 2 ½ of vertical axis)*
12. *Kaisar : 2 ½*
13. *Researcher : you said that the horizontal is*
14. *Renita : "0"*
15. *Researcher : and vertical*
16. *Renita : 2 ½*
17. *Researcher : so?*
18. *Renita : line of 2 ½*
19. *Researcher : horizontal?*
20. *Kaisar : horizontal to the right 1*
21. *Researcher : horizontal, to the right 1 is in this line isn't it? (points the line pass though 1 of horizontal axis), it should be in this line isn't it? (vertical axis)..*
22. *Kaisar : here (from the 1 of the horizontal axis) vertical 2.. horizontal… horizontal … to the left .. (confuse) hehehe*

…

And for Kaisar, although he knows that the ship A is in the vertical axis (line "0") he still stuck with his idea "*1 to the right, 2 up then 1 to the left then ½ up"*. Kaisar avoid the "0" distance. This is because "0" is abstract for him.

Some of the students also determine the location of the point from the different origins. They used light house are origin and then they switch it into sunken ship when they have to determine the location of SAR ship which is in the axis. This is because if they use the sunken ship as origin then the points which are in the axis become not in the axis anymore. So they can clearly see its horizontal

and vertical distance. From that case and the group 7 case, we can see that "0" is an abstract thing for the students. They tend to avoid it. It is a difficult concept for them.

It might be better to emphasize the discussion about the coordinate of a point in the axis in the previous meeting (rice field). We can start the discussion by asking them to locate point in (3,0) and compare it with (0,3). If the students understand that (3,0) means that they only need to walk to the right 3 steps and (0,3) means that they only need to walk to the north 3 steps, then the concept of coordinate in the axis is easier to be imagined and understood. This is because in the previous meeting they only see the positive axis and it is more real for them.

However, there are also some groups who face no difficulty in seeing the zero distance. They can easily locate the ship A. furthermore, students have no difficulty in understanding the coordinate that involves halves and negative.

Activity 6: Cartesian system

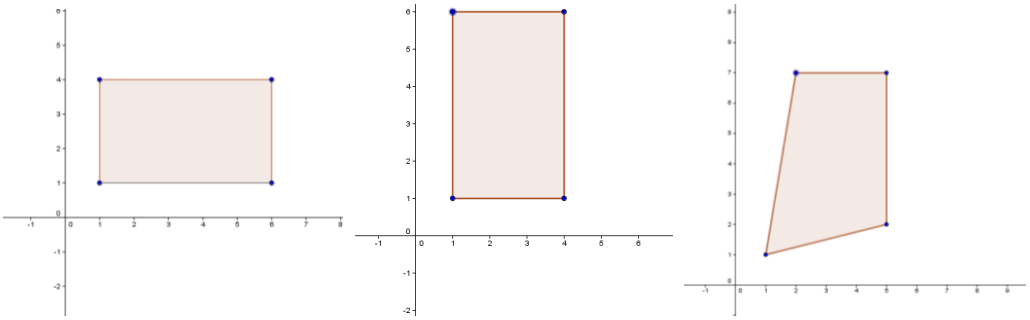
The last activity is about Cartesian coordinate system. This activity is the formal level of the coordinate (location) system. Here the term Cartesian and its form, x-y axis, origin, and coordinate notion are introduced. Teacher relates it with the previous activity (sunken ship 2) which actually has the similar form with Cartesian system. The rule to locate points and the idea behind the Cartesian system are actually had been learnt in the previous meetings. Students just need to adjust it a bit. They only need to adjust the way of notate the coordinate, and adjust the term horizontal as x-axis and vertical as y-axis.

After the students know about Cartesian system and its notation, teacher gives them some problems related to the Cartesian system. The first problem is

given to make them aware of some common mistakes in locating a point in Cartesian system which are reversing (x, y) become (y, x), and ignoring the origin (locating point from the other point instead of origin).

The students are showed three students' answer (Budi's, Ani's, and Erin's) of this following problem.

<u>Problem:</u> Locate these points in the Cartesian coordinate system, connect it and see what figure is formed.
Points: (1, 1); (4, 1); (4, 6); and (1, 6)
The following figures are Budi's, Ani's, and Erin's answer respectively (from left to right).



6 out of 8 groups are able to find the correct answer of the problem. And when asked, they can argue well. From the field note we can know that the students stated that Ani's answer is the correct one because all of the points are there. Budi's answer and Erin's answer do not have some points. And when the teacher play dumb by asking that "*who say Budi's have no some points, all of the points are there, teacher points (6, 1) in the point (1,6)"*, they know that what the researcher said is wrong. They argue that what the researcher said is not like the agreement that be made (notate the horizontal distance/axis first then vertical), (x,y) not (y,x).

However there are some students who choose Budi's although they know and understand about the agreement. Group 2 argues that Budi's answer is the

right one because in Budi's answer there are points which have the same coordinates as the given coordinates. When the researcher asks them to show which point who has the coordinate (1, 6), they point the point (6, 1). Then the researcher asks their opinion about Ani's answer, they say that it does not have some points like point (1, 6). And when the researcher asks them about the agreement, they know that it should be horizontal axis first. Not long after the researcher left them, they realize their mistake and Umu says that the right answer is Ani's.

There is no one who chooses Erin's answer. All of them claim that Erin's answer is wrong because the points are not fit with the given coordinates. Based on the mini interview with group 3, they do not know what the mistake in Erin's answer beside that the points in there are not representation of the given coordinates. They do not know how Erin can come up with her answer. Then researcher tell them that Erin said that first she locate the point (1, 1). After that she would like to locate the point (4, 1). From (1, 1) she walks 4 steps to the right and then 1 up. Reacts to that, Fadya says that we cannot do like that. We have to start from (0, 0) which is agreeing by her friends. From that mini interview we know that the students in group 4 aware and understand about the origin.

From the students' answer of other problem given in this activity, we can see that the students are able to locate points with given coordinates (Figure 5.24 students' answer of the follow up problem (Figure 5.24). Students are asked to locate and connect points (2, -2); (6, -2); (8, 1) and then find the next point so it can be a parallelogram.

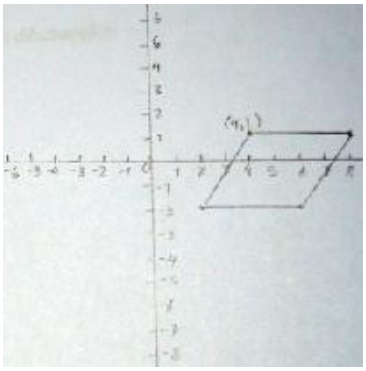


Figure 5.24 students' answer of the follow up problem

However, we not sure if the point has coordinate which involves halves or zero “0” (in the axis). This is because the coordinates given in the problem only consists of negative and positive integer/ coordinates.

Post Test

Post test is given to the students in the end of the learning sequence. The questions asked in the post test are similar to the pre test. We only change the number and coordinates of the point. We also exclude the first problem of pre test which is aimed to know the students’ prior knowledge of negative number. Yet, we add a slanted coordinate in order to check whether students able to understand the system although it is slanted Cartesian coordinate system.

From the pre test we can see that students show an improvement of their knowledge about coordinate systems. Most of them locate the Andi’s desk precisely. They also notice the map systems and able to locate the pointed city correctly. Furthermore, the students are able to locate the points in the Cartesian coordinate system without guessing its mean. They are also able to plot points on the Cartesian system.

Regarding the coordinate which involves “0” zero and halves, most of the students are understand it. They can locate and plot points with that coordinate, although, there is also some of them who still cannot do it.

5.2.7 Conclusion of the second cycle

From the cycle 2, we can conclude that the learning process and the discussion is going well and better than in the first activity. The teacher can manage the classroom well. And the students have no problem in working on a group.

The students react to each activity as conjectured in HLT. The use of context is proven to be helpful in engage the students in the learning process. Combined with the teacher’s probing question, it can enhance the students to think about the proposed mathematical idea and enhance the students’ different strategies. In other words, the designed activities can help the students to understand the coordinate system.

The learning sequence is going smoothly. Although, there are still some things that should be more considered, such as the activity of locating a point in the axis and students prior knowledge about navigation (interpretation of ' turn and go straight).

CHAPTER VI

CONCLUSION AND SUGGESTION

6.1 Conclusion

In this part we would like to answer the proposed research questions. It should be noted that the following answer is the outcomes of this study. So, the outcomes are limited by the target group and participants as described in the Chapter III.

We mean to know how the students can learn to understand the coordinate system. We would try to elaborate the answer to that question by answering the proposed sub questions, 1. What context that can be used to help the students to understand the coordinate system; and 2. How that context can support the students to learn and understand about the mathematical idea of coordinate system. In order to avoid redundant of information, we would like to address both questions in once.

The findings suggest that the "experientially real" context problems which are proposed in this study are able to support students in learning and understanding the big ideas of the coordinate systems. We can use the cinema problem to help the students understand how a system works. From this activity the students understand that the system used in the cinema locates the seat uniquely and precisely. They found out that it is impossible for a seat to have two different locations and vice versa. In addition, the students notice the role of rows and columns in a system. They can locate or tell the location of seat by considering the row and column. They also understand that locating a system only by considering one parameter (row or column) is not enough.

The next problem, airplane problem, gives chance to the students to make their own system. It also supports the students to understand the good system and the idea that each point should be uni 109 ıted. The students come up with different system; numbering the seat from one to 148 and using the similar system as in the cinema (involving alphabet and number). From this activity the students senses that numbering the seat from 1 to 148 is not as effective as numbering the rows with alphabet and the columns with number. They know that using 1-148

system consumes time and need effort to find the seat. So, besides learn about the idea of each point is unique, the students also make a judgment of the most effective and better system.

The students broaden their understanding about coordinate system by making a system to locate an object in the plane. “the sunken ship 1” context problem offer an opportunity for the students to experience another kind of system which is different from the system used in cinema and airplane. This context, again, makes the students understand of an idea that each point is unique. Furthermore, here they show an improvement of the development level in locate a point. With the teacher’s guidance, the student’ thinking move away from locating a point by only consider one parameter to the sophisticated system. For instance, the system involves angle and the system considers two perpendicular parameters.

The students learn about the origin through the rice field problem. The modified drawing of the rice fields bring the students into the problematic situation. This problematic situation enhances them to think about an unique origin. The students sense that to locate an object precisely, they need to know in where the origin is. They found out that different origins (in a system) can produce different locations with the same clues/ coordinate. In addition, this context can bring the students to the more abstract level. Through this context, the students can see the grids, vertical and horizontal axis which is used to introduce the positive coordinate to the students. From this activity, the students made an agreement on how they should locate an object (horizontal, vertical).

Students’ understanding about the positive and halves coordinate system is challenged by giving them the context problem that force them to think about

negative coordinate. The sunken ship 2 context offers that opportunity. Making the students locate the rescue team's ships which are placed in the left and below the light house make the students think about the negative coordinate and so the halves coordinate.

In the end, the students are introduced to Cartesian coordinate system. Here their knowledge of origin, idea of each point is unique, and the agreement that they made to locate the point help them to understand the Cartesian coordinate system and its rule. The last proposed context is an abstract context which can make the students aware of some common mistakes happen when locating or plotting a point in the Cartesian system. By asking the students to analyze and choose a right answer out of three options can make the students to keep aware of the agreement on locating point which had been made.

So, in general we can conclude that students learn to understanding the coordinate system from the concrete to the more formal level. They learn the mathematical ideas of the coordinate system in each activity through the context. In other words, the real context and the challenging activities proposed in this study can enhance the students' thinking and understanding about the coordinate systems. Moreover the familiar and meaningful context motivates the students to learn about the coordinate system. This is also supported by the post test result which shows improvement than the pre test. This implies that the learning activities which are designed in this study play an important role in supporting the students (in this study) to understand the mathematical ideas of the coordinate systems.

Beside the contexts, the finding suggests that the teacher also plays an important role in supporting the students to understand the coordinate system. Probing question asked by the teacher makes the students think through and critically about the mathematical contexts which have been addressed. Moreover, how much the teacher gives helps to the students affects the students' thinking and learning.

6.2 Reflection

In this part we would like to reflect on some elements related to this study such as the social norm and socio mathematics norm, role of the teacher, and RME. In addition, we would like to discuss the students' development level of locate a point related to the Piaget's level (Piaget et al., 1960).

6.2.1 Social Norm and Socio Mathematics Norm

Ideally, in this study we expect the students to be active, be able to explain and justify solutions, indicate agreement and disagreement, try to make a sense the explanation given by other and question some possible alternative solutions (Gravemeijer, K., and Cobb, P., 2006). Yet, it is a difficult task. It does not emerge automatically.

However, we satisfied with the students' improvement in their behavior. From the first classroom observation, we know that the students are active students but they are still not able to speak their mind and explain their answer in front of the class. And although at first they still shy and feel nervous when they are asked to present and explain their answer in front of the class, but after several time, they are not nervous anymore. Even they volunteer themselves to explain the answer.

In addition, they learn to listen to other students' answer and explanation. At first, when they see their friend explains her/his answer, they will tease him/her and sometimes laugh at him/her mistakes. This habit is fade away. Even if the teacher forgets to give reinforcement (applause) to her/him, the students take the initiative.

Furthermore, the students are able to speak their agreement and disagreement of other's answer. At first, they only whisper their disagreement (mumbling to themselves). But in the fourth meeting, they speak it out to the class. They speak their reason why they do not agree on the answer and show their answer. They learn to make a judgment on the other's answer and their own answer.

This improvement might be caused by the learning situation that forces them to argue, explain, and present. In the six designed activities, there are always a discussion and teacher' probing questions. This is also because the teacher always tries to engage them in the discussion by asking their opinion, If this learning environment is continued then the ideal good norm might happen.

6.2.2 (Indonesian) Realistic Mathematics Education

The use of context is the crucial element in this study. It has been showed that the context help the students to understand the mathematical idea behind the coordinate system. Contexts used in this study are real for the students. They know about airplane, cinema, rice field, and ships. It can engage and motivate them. In addition, the students' contribution and various different creations make the discussion of the mathematical ideas is possible to be done.

6.2.3 Role of the teacher

As we stated that teacher plays a crucial role in orchestrate discussion and enhance students' thinking by giving the probing questions. The teacher should be really modest and patient in guide and help the students. Instead of shows the answer, teacher should scaffold them. Teacher in this study is a good teacher. She is really patient in help the students to understand the mathematical ideas. She does not directly tell the students which answer is the better answer, rather she let the students to decide it by themselves. She is also eager to know what students' think.

6.2.4 Students' developmental level of locate a point

The outcomes of the third activity which is about locating the sunken ship shows that students come up with many systems. With the help of the teacher, there are some students who are able to make a sophisticated system to locate the ship precisely. Yet, there are also some students who are not. This suggests that the students have the different level of developmental thinking in locate a point.

Relate to the Piaget's level of child's developmental thinking of locate a point (Piaget et al., 1960), the finding shows that the students' level of developmental thinking of location and object are different. At first, most of the students are in the first or second level. The students only locate the sunken ship by considering the compass direction. They say that the location of the ship is in the north-east. This means that they only use visual estimation in locate the ship. There are also students who locate the ship by measuring the ship's distance from the lighthouse. They use the measurement tool, but they only consider one parameter/ measurement.

Furthermore, with the guidance from the teacher, they move away from the first and second level to the third level. Here, the students start to recognize that they need two measurements to locate the ship precisely. They locate the ship by combine the distance of the ship from the lighthouse and its direction. They find out that the location of the ship is "*8 km to the north-east".* Here, although the students recognize two measurement, but they are not perpendicular yet.

In the end, there are some students who can find the sophisticated systems. A group finds out a polar system (involves angle) and four groups who finds out the perpendicular system. This implies that they can construct an interiorized mathematics coordinate system which is the fifth level.

6.3 Suggestion for teaching, design, and research

6.3.1 Suggestion for teaching and Instructional design

As we suggest in the chapter 2, that we should moves away from the common way of teaching the coordinate system. The result of this study suggest that the in teaching the coordinate system, it should be started with the real context. But not as a preface or intro but as the crucial element that can force the student to think about the mathematical ideas. So it should be a powerful meaningful real context.

Regarding the time allocation, the students needs time to discuss and solve the given problem. In the real teaching practice we can avoid it by giving the problem as homework, so in the class they only have to discuss it.

From the retrospective analysis, we find that there are students that face a difficulty in locating the points which are in the axis. The finding suggests that those students still think in the concrete/ real level. They have difficulty in seeing something abstract like zero. This is also happening when they have to find the

not-exist seat. Although they cannot find it and know that it is not exist, they are uncertain and doubt their thinking. This is needed to be considered and more addressed in the future learning.

6.3.2 Suggestion for the future research

Realizing the fact that coordinate system is rich of context, we still can explore the possible context to be used in teaching coordinate system. There is the possibility to teach the coordinate system from the really real context around us like the way we organize files in our computer. Or we can use technology and game as suggested by some studies, one of them is a study by Sarama et al (2003).

Furthermore, the mathematical ideas related to coordinate system addressed in this study are each point is unique, origin, a good system and plotting and locating a point. Yet, we do not address and explore about the distance of two points and the relationship of two points. For example we can find the location of a point by knowing the location of the other point. So, we can extend this study to that extend. Furthermore, this study is limited in the two dimensional coordinate system. We do not address the three dimensional coordinate system yet.

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