

**LOOKING AT ANGLES: DEVELOPING A LOCAL INSTRUCTION
THEORY FOR LEARNING THE CONCEPT OF ANGLE BY
EXPLORING THE NOTION OF VISION LINES**

A THESIS

**Submitted in Partial Fulfillment of the Requirements for the Degree of
Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)**

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1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

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ABSTRACT

This study reports on a new approach of students' learning of the concept of angle in Indonesian primary schools. The study's context is employing the current education reform movement known as Pendidikan Matematika Realistik Indonesia (an Indonesian version of Realistic Mathematics Education) as the new approach in the teaching-learning process in the classroom. Using design research approach, a Hypothetical Learning Trajectory (HLT) was developed and a set of activities was performed to gain a better understanding of how the third grade students' (aged 8 – 9 years) understanding of the concept of angle may be fostered. Theoretical development is driven by an iterative process of designing instructional activities, performing teaching experiments and conducting retrospective analysis in order to contribute to local instruction theory on the concept of angle. The concept of angle emerged and evolved during a long term classroom activities involving vision lines and spatial representations. Data collections were generated from video recording of classroom events and group works, collecting student works, giving pre-test and post-test, and interviewing the students. The designed HLT was then compared with the students' actual learning trajectory during the teaching experiment in order to analyze whether the students learned or did not learn from what we had designed in the instructional sequence. Retrospective analysis of teaching experiment showed that by conducting the visual field activities involving vision lines and working with spatial representations, the students could develop their understanding of the concept of angle as well as their initial understanding of the notion of vision lines and blind spots. Based on these findings, it is recommended that PMRI as an adaptation of RME approach in Indonesia be implemented as an approach of teaching and learning the concept of angle in primary schools.

Keywords: *angle concept, PMRI, local instruction theory, visual field activities, spatial representations, hypothetical learning trajectory, design research*

ABSTRAK

Penelitian ini melaporkan suatu pendekatan baru dalam pembelajaran siswa pada materi konsep sudut di sekolah dasar, Indonesia. Konteks penelitian ini menggunakan gerakan reformasi pendidikan saat ini yang dikenal sebagai Pendidikan Matematika Realistik Indonesia (Realistic Mathematics Education versi Indonesia) sebagai pendekatan baru dalam proses belajar mengajar di kelas. Dengan menggunakan pendekatan design research, sebuah lintasan belajar dugaan (Hypothetical Learning Trajectory) kemudian dikembangkan dan seperangkat aktivitas diterapkan untuk memperoleh pemahaman yang lebih baik tentang bagaimana mengembangkan pemahaman siswa kelas tiga (berusia antara 8 – 9 tahun) tentang konsep sudut. Pengembangan secara teoritis dilaksanakan melalui proses interatif yang meliputi merancang aktivitas pembelajaran, melaksanakan pembelajaran (teaching experiment) dan melakukan analisis retrospektif (retrospective analysis) dalam rangka memberi kontribusi terhadap teori pembelajaran lokal (local instruction theory) pada konsep sudut. Konsep sudut muncul dan berkembang dalam aktivitas kelas yang cukup panjang yang melibatkan konsep garis pandang dan representasi spasial. Pengumpulan data dilakukan melalui beberapa hal meliputi membuat rekaman video tentang kejadian di kelas dan kerja kelompok, mengumpulkan hasil kerja siswa, memberikan tes awal dan tes akhir, dan mewawancarai siswa. Lintasan belajar dugaan yang telah dirancang kemudian dibandingkan dengan lintasan belajar siswa yang sebenarnya selama pelaksanaan pembelajaran (teaching experiment) untuk menganalisis apakah siswa belajar atau tidak belajar dari apa yang telah dirancang di rangkaian pembelajaran. Analisis retrospektif terhadap pelaksanaan pembelajaran menunjukkan bahwa dengan melaksanakan aktivitas visual field yang melibatkan garis pandang dan bekerja dengan representasi spasial, siswa mampu mengembangkan pemahaman mereka tentang konsep sudut serta pemahaman awal mereka tentang konsep garis pandang dan daerah tak terlihat (blind spots). Berdasarkan hasil penelitian ini, disarankan bahwa PMRI sebagai adaptasi pendekatan RME di Indonesia untuk digunakan sebagai suatu pendekatan dalam pembelajaran konsep sudut di sekolah dasar.

Kata kunci: *konsep sudut, PMRI, local instruction theory, aktivitas visual field, representasi spasial, lintasan belajar dugaan, design research*

SUMMARY

Angle is a complex concept defined in various contexts. There are three particular classes of angles definition occur repeatedly: an amount of turning about a point between two lines, a pair of rays with a common end-point, and the region formed by the intersection of two half-lines (Mitchelmore & White, 2000). Students quite often get confused to what an angle truly consist of, due to the many definitions of angles. This study aimed to develop an instructional sequence to learn the concept of angle in the primary school by exploring the notion of vision lines. This study also aimed to investigate how students develop their understanding of this essential topic through learning with the designed instruction.

Design research was chosen to be a research approach in developing the instructional sequence for learning the concept of angles in the third grade of primary school. In this study, we designed what so-called Hypothetical Learning Trajectory (HLT) that consists of three components: the learning goal, the learning activities, and the hypothetical learning process – a prediction of how the students' thinking and understanding will evolve in the context of the learning activities (Simon, 1995). This HLT was then implemented to thirty-eight students of the third grade (i.e. SD Muhammadiyah 6 Palembang which had been involved in the Pendidikan Matematika Realistik Indonesia or Indonesian Realistic Mathematics Education project since 2010) through two cycles: preliminary teaching and teaching experiment.

Based on the findings of this study, it can be concluded that the students could develop more conceptual understanding of the concept of angle by exploring the notion of vision lines. This study also revealed that students started to grasp the concept of vision angles and developed their spatial visualization and spatial reasoning by learning through visual field activities and spatial representations. Through learning the classification of angles using paper fan, the students could grasp the angles equal and more than 180° . Accordingly, the students significantly improved their understanding of many geometric terms such as straight angles and one circle angles.

RINGKASAN

Sudut adalah konsep yang kompleks didefinisikan dalam berbagai konteks. Ada tiga jenis definisi sudut yang sering muncul: jumlah putaran terhadap sebuah titik antara dua garis, sepasang sinar dengan satu titik temu, dan daerah yang dibentuk oleh perpotongan dua ruas garis (Mitchelmore & White, 2000). Siswa sering kali kebingungan tentang apa sebenarnya sudut tersebut, akibat banyaknya definisi tentang sudut. Penelitian ini bertujuan untuk mengembangkan rangkaian pembelajaran tentang konsep sudut di sekolah dasar dengan mengeksplorasi ide garis pandang. Penelitian ini juga bertujuan untuk menginvestigasi bagaimana siswa mengembangkan pemahaman mereka tentang topik yang penting ini melalui pembelajaran yang telah didesain.

Design research dipilih sebagai pendekatan penelitian dalam mengembangkan rangkaian pembelajaran untuk mempelajari konsep sudut di kelas tiga sekolah dasar. Pada penelitian ini, kami merancang apa yang disebut lintasan belajar dugaan (*Hypothetical Learning Trajectory*) yang terdiri dari tiga komponen: tujuan pembelajaran, kegiatan pembelajaran, dan proses pembelajaran dugaan – prediksi tentang bagaimana pemikiran dan pemahaman siswa akan berkembang dalam konteks aktivitas pembelajaran (Simon, 1995). HLT tersebut kemudian di implementasikan kepada tiga puluh delapan siswa kelas tiga (i.e. SD Muhammadiyah 6 Palembang yang telah terlibat dalam proyek Pendidikan Matematika Realistik Indonesia sejak tahun 2010) melalui dua siklus:

pembelajaran awal dalam kelompok kecil and percobaan pembelajaran di satu kelas.

Berdasarkan hasil temuan dari penelitian ini, dapat disimpulkan bahwa siswa mampu mengembangkan pemahaman konseptual yang lebih baik tentang konsep sudut dengan mengeksplorasi ide garis pandang. Penelitian ini juga mengungkapkan bahwa siswa mulai memahami tentang konsep sudut pandang dan mengembangkan kemampuan visualisasi spasial dan penalaran spasial mereka dengan pembelajaran melalui aktivitas *visual field* dan representasi spasial. Dengan mempelajari klasifikasi sudut menggunakan kipas kertas, siswa mampu memahami sudut-sudut yang sama atau lebih besar dari 180° . Sejalan dengan hal tersebut, siswa secara signifikan meningkatkan pemahaman mereka tentang banyak istilah-istilah geometri seperti sudut lurus dan sudut satu putaran.

“non-geometers do not enter”

“Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid.”

Albert Einstein

“From the very beginning of his education, the child should experience the joy of discovery”

Alfred North Whitehead

I specially dedicated this thesis to:

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I consciously understand that this thesis is far away from being perfect. Thus, any insightful critics and constructive ideas will be gladly accepted.

Palembang, June 2013

Bustang

TABLE OF CONTENT

ABSTRACT	v
ABSTRAK	vi
SUMMARY	vii
RINGKASAN	ix
PREFACE	xii
TABLE OF CONTENT	xiv
LIST OF TABLES	xvii
LIST OF FIGURES	xviii
LIST OF APPENDICES	xix
CHAPTER I: INTRODUCTION	1
1.1 Background	1
1.2 Research Aims	4
1.3 Research Question	4
CHAPTER II: THEORETICAL BACKGROUND	6
2.1 The Multifaceted Concept of Angle	6
2.2 Angle Comprehension and Misunderstanding	9
2.3 The Notion of Realistic Mathematics Education	11
2.3.1 Five Tenets of Realistic Mathematics Education	13
2.3.2 Emergent Modeling	15
2.3.3 A Learning-Teaching Trajectory for the Concept of Angle	16
2.4 The Concept of Angle in the Indonesian Curriculum	18
CHAPTER III: METHODOLOGY	20
3.1 Research Approach	20
3.2 Data Collection	24
3.2.1. Preparation Phase	24
3.2.2. Preliminary Teaching (cycle 1)	25
3.2.3. Teaching Experiment (cycle 2)	26
3.2.4. Post-Test	26
3.3 Validity and Reliability	27

3.4 Data Analysis	27
3.4.1. Pre-Test	27
3.4.2. Preliminary Teaching (cycle 1)	28
3.4.3. Teaching Experiment (cycle 2)	28
3.4.4. Post-Test	29
3.4.5. Validity and Reliability	29
3.5 Research Subject and Timeline of the Research	32
CHAPTER IV: HYPOTHETICAL LEARNING TRAJECTORY	34
4.1 The Cat and the Mice Activity	36
4.2 Experiment with the Screen Activity	42
4.3 Coloring the Blind Spots Activity	48
4.4 Understanding the Vision Angle Activity	53
4.5 Playing with Paper Fan Activity	59
CHAPTER V: TESTING THE HYPOTHETICAL LEARNING	
TRAJECTORY	63
5.1 Teaching Experiment	64
5.1.1. Preliminary Teaching (first cycle)	64
5.1.2. Teaching Experiment (second cycle)	67
5.2 Retrospective Analysis	70
5.2.1. Remarks on Students' Pre-Knowledge in the First Cycle	70
5.2.2. Preliminary Teaching (first cycle)	72
5.2.3. Remarks on Students' Knowledge in the Post-Test (1 st cycle)	89
5.2.4. Conclusion of Students' Learning Process in the First Cycle	91
5.2.5. Improvement of Hypothetical Learning Trajectory	92
5.2.6. Remarks of Students' Pre-Knowledge in the Second Cycle	94
5.2.7. Teaching Experiment (second cycle)	95
5.2.8. Remarks on Students' Knowledge in the Post-Test (2 nd cycle)	118
5.2.9. Conclusion of Students' Learning Process in the Second Cycle	119
CHAPTER VI: CONCLUSION AND SUGGESTION	122
6.1 Conclusion	122

5.1.1. Answer to the Research Question	122
5.1.2. Local Instruction Theory for Learning The Concept of Angle	129
6.2 Reflection	130
6.2.1. The weakness point of the study	130
6.2.2. Reflection on the important issues	131
6.3 Suggestion	133
REFERENCES	135
APPENDICES	139
CURRICULUM VITAE	210

LIST OF TABLES

Table 2.1. A Survey on Definitions of Angle Concept	7
Table 2.2. Angle Concept for Primary School Grade Three in the Second Semester in Indonesian Curriculum	18
Table 3.1. Timeline of the Research	33
Table 6.1. Local Instruction Theory for Learning the Concept of Angle	129

LIST OF FIGURES

Figure 2.1. Four Levels of Emergent Modeling from Situational to Formal Reasoning (Gravemeijer, 1994, 1998, 1999)	16
Figure 3.1. A Cumulative cyclic Process in Design Research (Gravemeijer, 2004)	23
Figure 4.1. The First Problem (Panoramic Drawing)	38
Figure 4.2. The Second Problem (the Top View of the Drawing)	39
Figure 4.3. Parallel line	41
Figure 4.4. Diverging Lines but Do Not Meet Each Other	41
Figure 4.5. Vision Lines of the Cat (Correct Answer)	41
Figure 5.1. Pointing the Hidden Mouse	74
Figure 5.2. Drawing a Line without Using a Ruler	78
Figure 5.3. Student's Drawing of Connecting Lines	78
Figure 5.4. Some Points Representing the Hidden Area	83
Figure 5.5. Students' Different Answers of Coloring the Hidden Area	83
Figure 5.6. Students' Drawing of Angle of Vision for Different Observers ...	86
Figure 5.7. Two Different Views of Looking Out of the Window	90
Figure 5.8. From Left to Right: the Change of Worksheet in the Problem 2	93
Figure 5.9 Students' Drawing of the Vision Lines of the Cat	96
Figure 5.10. Conducting the Visual Field Activities	99
Figure 5.11. Student's Strategy in Predicting the Next Position of the School Bag	100
Figure 5.12. Student's Drawing of Hidden Area (Blind Spots) in the Focus Group	105
Figure 5.13. Student's Drawing of Hidden Area (Blind Spots)	106
Figure 5.14. The Top View of Different Positions of Observer	107
Figure 5.15. Student Draws the Vision Lines in the Second Problem	109
Figure 5.16. Constructing the Angle of Vision of Observer	110
Figure 5.17. Student's Drawing of Angle of Vision	111

Figure 5.18. Students are pointing the acute angle in the ruler	114
Figure 5.19. Students draw acute angles in different orientations	114
Figure 5.20. Students' mistake in recognizing the angles in the paper fan	116
Figure 5.21. Students' answer of combining two or more paper fans	117

LIST OF APPENDICES

Appendix I. The Teacher Interview Scheme	131
Appendix II. The Classroom Observation Scheme	132
Appendix III. Teacher Guide	133
Appendix IV. Pre-Test and Post-test Indonesian version	154
Appendix V. Lembar Kerja Siswa	158
Appendix VI. Rencana Pelaksanaan Pembelajaran	170