LOOKING AT ANGLES: DEVELOPING A LOCAL INSTRUCTION THEORY FOR LEARNING THE CONCEPT OF ANGLE BY EXPLORING THE NOTION OF VISION LINES

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

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FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
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APPROVAL PAGE

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State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

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ABSTRACT

This study reports on a new approach of students' learning of the concept of angle in Indonesian primary schools. The study's context is employing the current education reform movement known as Pendidikan Matematika Realistik Indonesia (an Indonesian version of Realistic Mathematics Education) as the new approach in the teaching-learning process in the classroom. Using design research approach, a Hypothetical Learning Trajectory (HLT) was developed and a set of activities was performed to gain a better understanding of how the third grade students' (aged 8 – 9 years) understanding of the concept of angle may be fostered. Theoretical development is driven by an iterative process of designing instructional activities, performing teaching experiments and conducting retrospective analysis in order to contribute to local instruction theory on the concept of angle. The concept of angle emerged and evolved during a long term classroom activities involving vision lines and spatial representations. Data collections were generated from video recording of classroom events and group works, collecting student works, giving pre-test and post-test, and interviewing the students. The designed HLT was then compared with the students' actual learning trajectory during the teaching experiment in order to analyze whether the students learned or did not learn from what we had designed in the instructional sequence. Retrospective analysis of teaching experiment showed that by conducting the visual field activities involving vision lines and working with spatial representations, the students could develop their understanding of the concept of angle as well as their initial understanding of the notion of vision lines and blind spots. Based on these findings, it is recommended that PMRI as an adaptation of RME approach in Indonesia be implemented as an approach of teaching and learning the concept of angle in primary schools.

Keywords: *angle concept, PMRI, local instruction theory, visual field activities, spatial representations, hypothetical learning trajectory, design research*

ABSTRAK

Penelitian ini melaporkan suatu pendekatan baru dalam pembelajaran siswa pada materi konsep sudut di sekolah dasar, Indonesia. Konteks penelitian ini menggunakan gerakan reformasi pendidikan saat ini yang dikenal sebagai Pendidikan Matematika Realistik Indonesia (Realistic Mathematics Education versi Indonesia) sebagai pendekatan baru dalam proses belajar mengajar di kelas. Dengan menggunakan pendekatan design research, sebuah lintasan belajar dugaan (Hypothetical Learning Trajectory) kemudian dikembangkan dan seperangkat aktivitas diterapkan untuk memperoleh pemahaman yang lebih baik tentang bagaimana mengembangkan pemahaman siswa kelas tiga (berusia antara 8 – 9 tahun) tentang konsep sudut. Pengembangan secara teoritis dilaksanakan melalui proses interatif yang meliputi merancang aktivitias pembelajaran, melaksanakan pembelajaran (teaching experiment) dan melakukan analisis retrospektif (retrospective analysis) dalam rangka memberi kontribusi terhadap teori pembelajaran lokal (local instruction theory) pada konsep sudut. Konsep sudut muncul dan berkembang dalam aktivitas kelas yang cukup panjang yang melibatkan konsep garis pandang dan representasi spasial. Pengumpulan data dilakukan melalui beberapa hal meliputi membuat rekaman video tentang kejadian di kelas dan kerja kelompok, mengumpulkan hasil kerja siswa, memberikan tes awal dan tes akhir, dan mewawancarai siswa. Lintasan belajar dugaan yang telah dirancang kemudian dibandingkan dengan lintasan belajar siswa yang sebenarnya selama pelaksanaan pembelajaran (teaching experiment) untuk menganalisis apakah siswa belajar atau tidak belajar dari apa yang telah dirancang di rangkaian pembelajaran. Analisis retrospektif terhadap pelaksanaan pembelajaran menunjukkan bahwa dengan melaksanakan aktivitas visual field yang melibatkan garis pandang dan bekerja dengan representasi spasial, siswa mampu mengembangkan pemahaman mereka tentang konsep sudut serta pemahaman awal mereka tentang konsep garis pandang dan daerah tak terlihat (blind spots). Berdasarkan hasil penelitian ini, disarakan bahwa PMRI sebagai adaptasi pendekatan RME di Indonesia untuk digunakan sebagai suatu pendekatan dalam pembelajaran konsep sudut di sekolah dasar.

Kata kunci: *konsep sudut, PMRI, local instruction theory, aktivitas visual field, representasi spasial, lintasan belajar dugaan, design research*

SUMMARY

Angle is a complex concept defined in various contexts. There are three particular classes of angles definition occur repeatedly: an amount of turning about a point between two lines, a pair of rays with a common end-point, and the region formed by the intersection of two half-lines (Mitchelmore & White, 2000). Students quite often get confused to what an angle truly consist of, due to the many definitions of angles. This study aimed to develop an instructional sequence to learn the concept of angle in the primary school by exploring the notion of vision lines. This study also aimed to investigate how students develop their understanding of this essential topic through learning with the designed instruction.

Design research was chosen to be a research approach in developing the instructional sequence for learning the concept of angles in the third grade of primary school. In this study, we designed what so-called Hypothetical Learning Trajectory (HLT) that consists of three components: the learning goal, the learning activities, and the hypothetical learning process – a prediction of how the students' thinking and understanding will evolve in the context of the learning activities (Simon, 1995). This HLT was then implemented to thirty-eight students of the third grade (i.e. SD Muhammadiyah 6 Palembang which had been involved in the Pendidikan Matematika Realistik Indonesia or Indonesian Realistic Mathematics Education project since 2010) through two cycles: preliminary teaching and teaching experiment.

Based on the findings of this study, it can be concluded that the students could develop more conceptual understanding of the concept of angle by exploring the notion of vision lines. This study also revealed that students started to grasp the concept of vision angles and developed their spatial visualization and spatial reasoning by learning through visual field activities and spatial representations. Through learning the classification of angles using paper fan, the students could grasp the angles equal and more than 180^{o}. Accordingly, the students significantly improved their understanding of many geometric terms such as straight angles and one circle angles.

RINGKASAN

Sudut adalah konsep yang kompleks didefinisikan dalam berbagai konteks. Ada tiga jenis definisi sudut yang sering muncul: jumlah putaran terhadap sebuah titik antara dua garis, sepasang sinar dengan satu titik temu, dan daerah yang dibentuk oleh perpotongan dua ruas garis (Mitchelmore & White, 2000). Siswa sering kali kebingungan tentang apa sebenarnya sudut tersebut, akibat banyaknya defines tentang sudut. Penelitian ini bertujuan untuk mengembangkan rngkaian pembelajaran tentang konsep sudut di sekolah dasar dengan mengeksplorasi ide garis pandang. Penelitian ini juga bertujuan untuk menginvestigasi bagaimana siswa mengembangkan pemahaman mereka tentang topik yang penting ini melalui pembelajaran yang telah didesain.

Design research dipilih sebagai pendekatan penelitian dalam mengembangkan rangkaian pembelajaran untuk mempelajari konsep sudut di kelas tiga sekolah dasar. Pada penelitian ini, kami merancang apa yang disebut lintasan belajar dugaan *(Hypothetical Learning Trajectory)* yang terdiri dari tiga komponen: tujuan pembelajaran, kegiatan pembelajaran, dan proses pembelajaran dugaan – prediksi tentang bagaimana pemikiran dan pemahaman siswa akan berkembang dalam konteks aktivitas pembelajaran (Simon, 1995). HLT tersebut kemudian di implementasikan kepada tiga puluh delapan siswa kelas tiga (i.e. SD Muhammadiyah 6 Palembang yang telah terlibat dalam proyek Pendidikan Matematika Realistik Indonesia sejak tahun 2010) melalui dua siklus:

pembelajaran awal dalam kelompok kecil and percobaan pembelajaran di satu kelas.

Berdasarkan hasil temuan dari penelitian ini, dapat disimpulkan bahwa siswa mampu mengembangkan pemahaman konseptual yang lebih baik tentang konsep sudut dengan mengeksplorasi ide garis pandang. Penelitian ini juga mengungkapkan bahwa siswa mulai memahami tentang konsep sudut pandang dan mengembangkan kemampuan visualisasi spasial dan penalaran spasial mereka dengan pembelajaran melalui aktivitas *visual field* dan representasi spasial. Dengan mempelajari klasifikasi sudut menggunakan kipas kertas, siswa mampu memahami sudut-sudut yang sama atau lebih besar dari 180^{o}. Sejalan dengan hal tersebut, siswa secara signifikan meningkatkan pemahaman mereka tentang banyak istilah-istilah geometri seperti sudut lurus dan sudut satu putaran.

"non-geometers do not enter"

"Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid."

Albert Einstein

"From the very beginning of his education, the child should experience the joy of discovery"

Alfred North Whitehead

I specially dedicated this thesis to:

My mother (Indotang) and my father (Buhari Mahmud) who have always showered me with unconditional love, uncountable prayers, encouragement, and support throughout my entire life. And also to my beloved sister (Rita Fitriani, S.Kep) and my beloved brother (Burhanuddin, SE) for their love, prayers, and support.

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I consciously understand that this thesis is far away from being perfect. Thus, any insightful critics and constructive ideas will be gladly accepted.

Palembang, June 2013

Bustang

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CHAPTER I

INTRODUCTION

1.1 Background

The origin of geometry cannot be separated from several key concepts that played a role in shaping its development from the early civilizations to the modern era such as the concept of angle (e.g. Ostermann, & Wanner, 2012; Hodgkin, 2005). However, the discussion about the nature of the concept of angle has been carried out for more than two thousand years and the debate is not over yet (Matos, 1990). Accordingly, there are many different definitions of angle during the course of history which vary significantly in their emphases (Keiser, 2004). There is no single definition of the concept of angle that all mathematicians and mathematics didacticians agree on. Mitchelmore and White (2000) called this peculiarity the multifaceted nature of the concept of angle in which each facet has different meanings depending on the physical and mathematical situations at hand. Researchers over the past decade or so have noted that there are various definitions of angle that originated from many different perspectives. Several authors found a wide variety of the definition of angle in the school mathematics textbooks (Strehl, 1983; Close, 1982; & Krainer, 1989; as cited in Mitchelmore & White, 1998). Matos (1990) has surveyed many different definitions of angle from the historical perspective and many mathematicians expressed their opinions about the definition of angle from a mathematical point of view (e.g. Euclid as cited in Heath, 1956; Freudenthal, 1983). Mitchelmore and White (2000) noted that there are three broad categories of definitions of angle that occur repeatedly:

viewing angle either as a measure of turning between two lines meeting at a point (angle as a rotation), as a pair of rays that extend from a common point (angle as inclination), or as a region bounded by the intersection of two half-lines with a common end-point (angle as a sector).

The research literature also indicates that students have difficulty in grasping the multifaceted nature of the concept of angle. Several studies pointed out that students often harbor many misconceptions, conceive an erroneous angle representation and have difficulty learning this concept in the school. For example, many students believe that the size of an angle depends on the length of its sides (Mitchelmore, 1998; Munier & Merle, 2009; Clements & Burns, 2000; Keiser, 2004), they also have difficulty when trying to understand about angles with measures of 0^o, 180^o, and 360^o (Keiser, 2004). They get confused recognizing right angles in different orientations and learning the use of the standard protractor (Mitchelmore, 1998). In Indonesia, several researches noted that geometry is one of the hardest topics for students to learn. Soedjadi (1991; as cited in Fauzan, 2002) found that many students have difficulties to determine whether an angle is a right angle or not.

Although several researches are conducted in order to deal with some of these misconceptions and confusions, the focus is still on the formal angle knowledge or on the abstract level (Mitchelmore, 1997; Cope & Simmons, 1991). Only a few studies investigated the informal knowledge students have about angles and the use of context to induce the students' understanding of the concept of angle (Mitchelmore, 2000; Fynn, 2010). Overall, in all of the above studies, the main

concern is still in the formal level and physical angle situations were not studied in its own right (Mitchelmore, 1997).

All of these numerous studies reveal three important things, namely the multifaceted concept of angle, the students' struggles in understanding the complexity of the concept of angle in their own development, and the lack of using realistic contexts as the means to support the students' learning of the concept of angle from the concrete level to the abstract level (e.g. Keiser, 2004; Mitchelmore, 1997). The misunderstandings that students hold about the concept of angle from a young age can be a crucial learning issue for them because these misconceptions will continue to exist until they learn geometry in the higher level. Furthermore, they will never get the real picture of the concept and they will lack understanding about how the concept of angle is represented and be used in the everyday life situations.

Keeping all of the aforementioned issues in mind, this study is paying close attention to the misconceptions primary students have about the concept of angle and how to encourage them to grasp a more conceptual understanding of this essential concept. This study hypothesizes that exploring the idea of vision lines by using visual field activities and spatial representations could be used as means to support students' learning development of the concept of angle. The use of visual field activities is inspired from the research done by Munier & Merle (2009) and is elicited from the notion of using realistic contexts as means to help students understand certain topics in mathematics. This idea has attracted many researchers as an appropriate starting point in the teaching and learning of

mathematics (e.g. Bakker, 2004; Doorman, 2005; Munier & Merle, 2009). Freudenthal (1991) emphasized this idea by stating that students should have the opportunity to reinvent the mathematics through progressive mathematization from the informal level to the formal level. The use of a spatial experiment as the starting point in learning the concept of angle is also relevant to the idea of Freudenthal (1991) that students should learn mathematics in a way that can enhance their common sense. This study conjectured and expected that exploring the idea of vision lines through visual field activities and spatial representations might result in meaningful insights for the students' in their understanding of the concept of angle and enabled them to invalidate misconceptions relating to the concept of angle.

1.2 Research Aims

The aims of this study are to contribute to mathematics education literature by providing ideas of using visual field activities and spatial representation tasks in learning the concept of angle and to contribute to an empirically grounded local instructional theory for learning the concept of angle in the primary school. In order to achieve these aims, an instructional sequence is designed based on the RME approach that can support students' understanding of the concept of vision lines and its relation to the concept of angle. The study focuses on the third grade of primary school in Indonesia.

1.3 Research Question

Based on the aims of the study stated above, the main research question is formulated as follows:

How can the concept of vision lines support the development of students' understanding of the concept of angle?

This main research question is split into two sub-research questions to investigate the whole process of students' learning of the concept of angle from exploring the idea of vision lines through the visual field activities and spatial representations to the formal concept of angle. The aim of the first sub-research question is to know how the visual field activities and spatial representations elicit and support the development of students' acquisition of vision lines. The sub-research question is formulated to achieve this aim is:

How do the visual field activities and spatial representations tasks elicit and support the development of students' acquisition of vision lines?

The aim of the second sub-research question is to investigate how to encourage students in developing the formal knowledge of the concept of angle that is elaborated from the concept of vision lines and vision angle. The sub-research question formulated to achieve this aim is:

How can pupils develop the mathematical concepts of angle elaborated and supported by their understanding of vision lines?

CHAPTER II

THEORETICAL BACKGROUND

This chapter provides the framework of thinking that was addressed to design instructional activities in this study. In this chapter, we begin with a general review of the main areas of conceptions in angle concept that need to be addressed in teaching the concept of angle, as documented in the previous and recent studies. The main focus in reviewing literatures about teaching and learning of the concept of angle is placed on identifying the multifaceted concept of angle and the common misconceptions related to conceptual understanding of the concept of angle. In this research, the visual field activities involving the notion of vision lines were used as experienced-based activities in order to support the development of students' learning of the concept of angle. To explain and investigate how the framework of learning the concept of angle by exploring the notion of vision lines in the visual field activities bring the students to grasp the more conceptual understanding, we employed the pedagogical and didactical philosophy of the domain-specific instruction theory of Realistic Mathematics Education (RME) that have been adapted into Indonesian context known as *Pendidikan Matematika Realistik Indonesia (PMRI)*. Because the research was conducted in Indonesia, this chapter also provides an overview about the concept of angle for elementary school in Indonesian curriculum.

2.1 The Multifaceted Concept of Angle

It has been established that angle is a multifaceted concept. Therefore, making a definition of angle becomes a difficult process because all definitions have limitations in describing the concept by emphasizing one facet more heavily than others (Keiser, 2004). Henderson and Taimina (2005; cited by Fyhn, 2007) define angle based on the three different perspectives: angle as geometric shape, angle as dynamic motion (angle as movement) and angle as measure.

In ancient history, some of the Greek geometers tried to define angles. In that time, most of the definitions of angle were included in one of these three categories: a relation, a quality, or a quantity (Keiser, 2004). One of the famous books in the history of geometry is the one which was written by Euclid. According to the definition of angle that Euclid presented in *book I* of the *Elements*, it seems that he thought of an angle as the space in between two lines. Freudenthal (1973), then explains Euclid's definition of angle:

> Euclid defines the angle as an inclination of lines (even curves were admitted); he meant half lines, because otherwise he would not be able to distinguish adjacent angles from each other… Euclid does not know zero angles, nor straight and bigger than straight lines… Euclid compares, adds and subtracts angles… Euclid takes the liberty of adding angles beyond two or even four right angles; the result cannot be angles according to the original definitions (p. 476-477).

Another mathematician, David Hilbert, was trying to define angles in his influential geometry books *Foundation of Geometry* as the follows:

> Let α be any arbitrary plane and $h,\ k$ any two distinct half-rays lying in α and emanating from the point O so as to form a part of two different straight lines. We call the system formed by these two half-rays $h,\ k$ an *angle* and represented it by the symbol $\angle(h,\ k)$ or $\angle(k,\ h)$ (Hilbert, 1950, p. 8).

From the point of view of a mathematics educator, Freudenthal (1973) in one of his famous books on mathematics education, *Mathematics as an Educational Task*, also shows his observation about the definition of angle:

Table 2.1. A Survey on Definitions of Angle Concept

Goniometry	**Elementary geometry**	**Analytic geometry**	**Space geometry**
Angle concept: the angle of			
An ordered	A non ordered pair of	An ordered	A non ordered
Half lines	Half lines in the	Lines	Lines
Oriented	Non oriented Plane, determined	Oriented	Non oriented
Mod 2π	Between 0^o and 180^o	Mod π	Between 0^o and 90^o

(Source: Freudenthal, 1973)

According to the explanation above, it can be concluded that even the mathematicians in the past were struggling in defining the concept of angle. This historical review of the concept of angle provides some important factors to consider when we think of the struggle students have in learning this concept in the classroom. Furthermore, considering the multifaceted concept and definition of angle from the historical perspective provide an insight for the researcher when designing a sequence of activities that seek to deal with the most common obstacles and conceptions of the students in learning the concept of angle.

2.2 Angle Comprehension and Misunderstandings

Many studies pointed out that children's conceptual understanding about the concept of angle needs to be investigated due to the several misconceptions and misunderstandings that exist in their thinking when dealing with that concept in the geometry classroom (e.g. Mitchelmore & White, 2000; Clements & Burns, 2000; Keiser, 2004; Munier&Merle, 2009). One of the possible reasons why students' learning of the concept of angle is complicated is that angle is indeed a difficult concept for children to understand. There are many different definitions of angles which vary in their emphases and take different meanings depending on the mathematical situation at hand (Keiser, 2004).

In the traditional geometry curriculum, the definition of angle is taken from the Euclid's definition. According to the definition presented in *Book I* of the *Elements*, Euclid stated that:

> A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line. And when the lines containing the angle are straight, the angle is called rectilinear (Health, 1956, p. 176).

It seems that Euclid's definition of angle excludes the zero angle and angles greater than or equal to a straight angle (Matos, 1990). Proclus and others (Keriser, 2004) also criticized this definition by arguing that if an angle is only one relationship between two rays, it seems to contradict with the fact that many angles exist for one inclination. For instance, a 90^0 angle can has the same form of lines with a 270^0, -270^0, -450^0, and 630^0 angle. So, it is quite reasonable that

students often harbor many misconceptions about angle and struggle to understand the angles with measures of 0^0, 180^0, and 360^0 when learning the concept of angle only from this kind of definition.

According to many studies conducted, it is also found that students who learn the concept of angle in the traditional teaching methods tend to think that the length of the sides affects the size of angle (Mitchelmore & White, 1998; Munier & Merle, 2009). Lehrer, Jenkins, and Osana (Munier & Merle, 2009) reported in their longitudinal studies that the misconception children have in making judgments about angles based on the effect of its length did not diminish during the three years of study. The study conducted by Berthelot and Salin (Munier & Merle, 2009) also revealed that three-quarters of all pupils cannot make sense out of the concept of angle unless it is presented in the primitive, schoolbook form, and that children have trouble recognizing an angle as a subfigure of another figure.

According to the aforementioned studies about angles, there are at least three important conclusions that can be drawn: angle as a complex concept defined in a variety of contexts, the struggle students have in understanding the concept of angle, and traditional teaching methods of the concept of angle with the lack of using realistic contexts as a means to support students' learning development. Furthermore, the multiple definitions of angle can also make students confused in their understanding about what an angle truly consists of. This study pays close attention to the misconceptions primary students have about the concept of angle and how to encourage them to grasp a more conceptual understanding of this

essential concept. Realistic contexts combined with visual field activities and spatial representations are employed as a starting point to encourage children to develop their own understanding and to reach the formal mathematical knowledge about the concept of angle. Accordingly, the notion of realistic mathematics education was studied from several literatures, as the framework for investigating and describing the mathematical reasoning students have in learning the concept of angle from the visual field activities shifted towards the formal mathematical knowledge.

2.3 The Notion of Realistic Mathematics Education

The concept of angle is closely related to the real world situation. Mitchelmore (1997) studied children's informal knowledge of physical angle situations and revealed a conclusion that children had an excellent informal knowledge of angle situations such as turn, slopes, crossings, bends, rebounds, and corners. Introducing the concept of angle from a physical angle situation such as visual field activities is one of the main concerns in this research and Realistic Mathematics Education (hereafter RME) becomes a domain specific design heuristic underlying the development of the instructional activities as well as a framework to understand the children's thinking in learning the concept of angle. RME is a new approach to mathematics education developed in the Netherlands. The influence of RME has been enormous around the world. Many countries such as USA, Indonesia, and South Africa have adopted and implemented RME theory in their education systems.

The development of RME and its ground educational theory still continues until recently. Freudenthal's view of mathematics as a human activity plays an important role in the development of RME. According to Freudenthal, mathematics must be connected to reality, stay close to children and be relevant to society in order to be of human value (Van den Heuvel-Panhuizen, 1996). The main activity in mathematics education, based upon Freudenthal's view of mathematics, is mathematizing. When setting 'mathematizing' as a goal for mathematics education, this can involve mathematizing mathematics and mathematizing reality (Gravemeijer, 1994). In Freudenthal's view, mathematizing is closely related to level-raising which is obtained when we do features that characterize mathematics such as generality, certainty, exactness, and brevity.

The idea for making mathematizing the key process in mathematics education is based upon at least two reasons. Firstly, mathematizing is not only a mathematicians' activity but also familiarizes the students with a mathematical approach to deal with everyday life situations. Secondly, mathematizing is closely related to the idea of reinvention. Freudenthal advocates that mathematics education should be organized as a process of guided reinvention, where students can experience a (to some extent) similar process as the process by which mathematics was invented (Gravemeijer, 1994). Later on, Adri Treffers's doctoral dissertation titled *Three Dimensions* (1987), supervised by Freudenthal, formulated the idea of two types of mathematization; he pronounced "horizontal" mathematization, related to the applied aspect of mathematics and "vertical" mathematization, related to the pure aspect of mathematics. Although this

distinction seems to be free from ambiguity, Freudenthal stated that it does not mean that the difference between these two forms of mathematization is clear cut and they are equal value. Treffers (1987) also described RME by means of the following five characteristics:

- The use of contexts.
- The use of models.
- The use of students' own productions and constructions.
- The interactive character of the teaching process.
- The intertwinement of various learning strands.

In answering the next step about how to proceed from visual field activities to the formal mathematics, the five tenets of RME offers a guideline and design heuristics.

2.3.1. Five Tenets of Realistic Mathematics Education

The process of designing a sequence of learning activities that starts from the physical angle situations such as visual field activities in this research was inspired by the five tenets of RME proposed by Treffers (1987) that are described below:

a) Phenomenological exploration

In the beginning of the activity, a real-world-context is used as the basis for concept formation. Instead of starting from a formal level, in this study the mathematical activity is embedded from a situation that is experientially real for the students. Accordingly, this study used the context of cat looking through mice hiding behind the jar as the contextual situation to be investigated by students.

b) Using models and symbols for progressive mathematization

The development from the context-bound notions towards more formal mathematical knowledge is gradually progressed by using models and symbols. Students' informal knowledge that formed from the context situation needs to be developed towards more formal knowledge of the concept of angle. Consequently, the "experiment with the screen" combined with the spatial representations activities in this study serve as bridging activities to elicit the students' formal level of understanding of the concept of angle.

c) Using students' own construction

Using students' own constructions and productions is considered an essential part of instruction. What students make on their own is assumed to be meaningful for them. Accordingly, the student-made drawing activities give students an opportunity to use their own construction and creativity.

d) Interactivity

The learning activities of students are a social process. Students can interact and learn from each other either in a small group or in a whole class discussion. The opportunity for students to make their own constructions can be used to elicit the interactivity among them by comparing and reflecting on the merits of different symbols and models. In this study, the experiment with the screen offers a room for students to discuss about the vision lines and make sense of the informal knowledge of the concept of angle.

e) Intertwinement

It is important to consider the integration of various domains in the instructional sequences. The contextual situations of the cat and mice as well as the experiment with the screen in the visual field activities do not only support the development of the concept of angle, but also the development of students' notion of spatial sense.

2.3.2. Emergent Modeling

As the second tenet of RME about models for progressive mathematization implies, we search for models that can support the process of mathematical growth of students from informal to more formal mathematical activity. Gravemeijer (1994, 1999) proposed the instructional design heuristic of emerging modelling as a mean of supporting the process of progressive mathematization of students. Ideally, models in RME perspective emerge from students' own activities and then these models gradually serve as a catalyst for a growth process to more formal mathematical knowledge (Gravemejier, 1998). Consequently, models play an important role as a basis for mathematical reasoning from context-specific *model of* situation to *model for* more formal reasoning on a formal level. The development from *model of* into *model for* is explicated by Gravemeijer into a four level-structure that represents levels of mathematical activity as shown in the following figure 2.1:

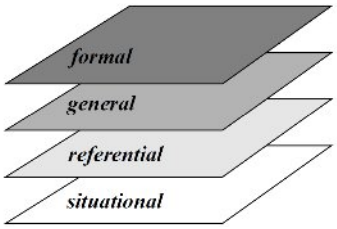


Figure 2.1 Four Levels of Emergent Modeling from Situational to Formal Reasoning (Gravemeijer, 1994, 1998, 1999)

These four levels of emergent modeling (figure 2.1) can be described as follows (Gravemeijer et al., 2000, p. 243):

1. Situational level: activity in the task setting, in which interpretations and solutions depend on understanding of how to act in the setting (often out-of-school settings).
2. Referential level: referential activity, in which models-*of* refer to activity in the setting described in instructional activities (mostly posed in school).
3. General level: general activity, in which model-*for* enable a focus on interpretations and solutions independently of situation-specific activity.
4. Formal level: reasoning with conventional symbolizations, which is no longer dependent on the support of model-for mathematical activity.

2.3.3. A Learning-Teaching Trajectory for the Concept of Angles

Within this study, we find out that it is very difficult to describe how the emergent modeling heuristic takes place in the teaching and learning of the concept of angles. It is because teaching and learning in geometry, includes the concept of angles, are less of linear structure. The various aspects of geometry

keep returning each time at a higher level, so that the teaching and learning for geometry concepts has more cyclic character (Van den Heuvel-Panhuizen et al., 2008). Furthermore, unlike other mathematics subjects such as numeracy (e.g. natural numbers, fractions, percentages, decimals) that are considered as a rich domain-specific didactical tradition employing RME theory, geometry in primary school, include angle concepts, is relatively unexplored. Consequently, within this study, we offer a teaching-learning trajectory for learning the concept of angle in the primary school.

In teaching and learning a (new) geometric concept or property, three phases are distinguished: experiencing, explaining, and connecting (Van den Heuvel-Panhuizen et al., 2008). These phases offer a direction in outlining the teaching-learning trajectory of geometric concepts and the accompanying didactic approach. In order to indicate what should be understood by these phases in this study, they are explained in the following:

1. Experiencing

The basic assumption is that the start of the teaching-learning process should always take place in a natural way and that, from there on, insight develops to a higher and higher level (Van den Heuvel-Panhuizen et al., 2008). In this study, the visual filed activities of constructing vision lines and blind spots of different observers are experience-based activities for students to experience how the vision of the cat looks like.

2. Explaining

Teaching geometry in RME is required to bring the students to the desire insight, through sketching a drawing or model. In this study, by making a schematic drawing "from above", the top view of situation, the students can explain why the blind spots are different for different position of observers. Here, the shift from in level is happening from the first experiencing phase to the explaining phase.

3. Connecting

The connecting phase means that the learned subject is connected to others concepts and phenomena, which should be lead to a deepening of insight (Van den Heuvel-Panhuizen et al., 2008). In this study, one can think of the use of angle concept in learning about polygon. In this process, students can discovers about polygon by making a sequence of turn of certain angle to one particular direction with the legs of the track that is formed are all equally long.

2.4 The concept of Angle in the Indonesian Curriculum

The concept of angle in the Indonesian curriculum is being introduced from the second semester of the third grade in primary schools. The table below describes the concept of angle as a topic under the umbrella of geometry and measurement for the third grades in the Indonesian curriculum.

Table 2.2 Angle Concept for Primary School Grade Three in the Second Semester in Indonesian Curriculum

The second semester of the third grade	
Geometry and Measurement	
4. Understanding the characteristics and the properties of simple plane figures.	4.1 Identifying many simple plane figures based on their characteristics and properties. 4.2 Identifying many kinds of angles and their measurement.

It can be seen that the focus of the teaching and learning the concept of angle in the third grade of the Indonesian curriculum is to guide students to identify many different kinds of angles and their measurement. However, what usually happens in the Indonesian mathematics classroom is that the students are taught in the traditional way of teaching by direct confrontation with the formal definition of angle from Euclid and using this definition to identify many different kinds of angles and their measurement. It gives less emphasis to the students to reinvent the definition of angle by themselves and build the mathematical reasoning about that definition. Instead of presenting the formal definition of angle to the students, the teacher should give them an opportunity to formulate and reinvent the definition by exploring the contextual situations and gradually developing their own understanding of the formal mathematical concept of angle.

CHAPTER III

METHODOLOGY

3.1. Research Approach

Theoretically, the aim of this research is to develop the local instructional theory to support students understanding of the concept of angle by exploring the idea of vision lines. Practically, the aim of this research is to investigate how the visual field activities and spatial representations could be used to support students' reasoning and reach the mathematical goals of the concept of angle. These two imply that there is a need for both the design of instructional means and the research about how these means support students' learning of the concept of angle. Therefore, design and research are highly intertwined for developing an instructional theory for learning the concept of angle.

The methodology of this research falls under the general heading of design research for at least two reasons. The research questions that were defined in the previous chapter start with " how can…" and "how do…". This describes that this study is interested not only to know whether the understanding of the concept of vision lines through visual field activities and spatial representations support students learning of the concept of angle, but also understanding about how. The second reason for choosing design research is that a ready-to-use local instructional theory for learning the concept of angle is lacking. In this sense, design research is chosen as the design heuristic in order to give a contribution in improving and making an innovation in education, especially in mathematics

education. The local instructional theory concerns both the process of learning and the means designed to support the learning. Therefore, in this research series of experienced-based activities is designed and developed as means to understand and improve of educational practices in the concept of angle for the third grade of primary school in Indonesia.

In the following, the definition of design research is given by Gravemeijer and Cobb (2006) by discussing the three phases of conducting a design experiment that are elaborated below:

1) Preparing for the experiment

In a design perspective, the goal of the preliminary phase of a design research experiment is to formulate a local instructional theory that can be elaborated and refined while conducting the experiment (Gravemeijer & Cobb, 2006). Accordingly, in this phase initial idea inspired and elaborated by studying literature such as journals, articles, and books which are related to the concept of angle. During this literature study, the researcher also started designing the learning activities. This sequence of learning activities including conjecture of students' thinking and strategies is developed and serves as the initial Hypothetical Learning Trajectory (HLT). This conjectured hypothetical learning trajectory is dynamic and can be changed and adjusted to students' actual learning process during the teaching experiments.

2) Experiment in the classroom

This second phase of design research is actually conducting the teaching experiment. According to Gravemeijer and Cobb (2006), the purpose of the

teaching experiment is both to test and improve the conjecture local instructional theory that was developed in the preliminary phase, and to develop an understanding of how it works. The aim of this teaching experiment is to collect the data in order to answer the research questions. So, in this phase the sequence of activities developed in the preliminary phase is implemented in the classroom within two cycles. The first cycle conducted as a pilot experiment taking only 5 or 6 students in the teaching process. The aim of pilot experiment is mainly to adjust both the content and the sequence of activities that have been developed and to improve it in order to get better design for the next cycle of teaching experiment. Then, the second cycle conducted as the actual teaching process in which the sequence of activities is conducted in the natural classroom environment. The mathematical content of the teaching experiment for these two cycles remains the same under the consideration that the second cycle is the revision of the first cycle. Before conducting a teaching experiment, the teacher and researcher have a preliminary discussion about the upcoming activity to adjust and make agreement about how the lesson might be delivered based on the teacher and researcher point of view. There is also a reflection about the whole learning process in the end of each lesson that concerns about the strong points and the weakest points of the lesson.

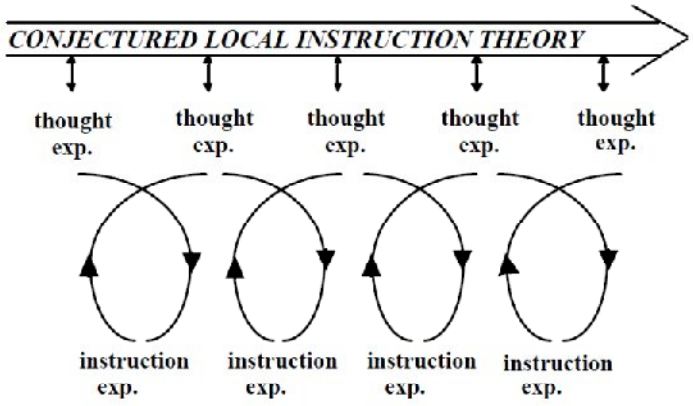


Figure 3.1 A Cumulative cyclic Process in Design Research (Gravemeijer, 2004)

At the heart of the design experiment lies a cyclic process of (re)designing and testing instructional activities and other aspects of the design (Gravemejier et al, 2006). A prototypical instructional sequence is developed in cyclic process of designing and revising instructional activities. In each cycle, the researcher conducts an anticipatory thought experiment by envisioning how the proposed instructional activities might be used in the classroom, and what students might learn as they participate in them. Actually, there is a reflexive relation between the local instructional theory and the thought and instruction experiments, as the theory gets revised and adjusted under influence of what is learned in the process (Gravemeijer, 2004).

3) Conducting retrospective analysis

The goal of the retrospective analysis depends on the theoretical intent of the design experiment. However, one of the primary aims is typically to contribute to the development of a local instruction theory (Akker et al, 2006). In this retrospective analysis, the hypothetical learning trajectory is used as a guideline and points of references in analyzing of the entire data set collected during the

teaching experiment. The HLT is compared with the actual teaching and learning process of students that take place in the classroom. The description of the analysis is not merely about the instances that support the conjecture, but also the ones that contradict with it. The conclusion of this analysis is used as the answers for the research questions.

In addition, the HLT continually develops through the different phases and has different functions depending on the phase of the design research (Bakker, 2004). In the preparation phase, the HLT functions as a guideline for designing the instructional materials that has to be developed. In the teaching experiment, the HLT functions as a guideline for both the teacher and researcher to determine the focus of the teaching, interviewing, and observing. In the retrospective analysis phase, the HLT serves as a guideline for the researcher to decide the focus of the analysis.

3.2. Data Collection

3.5.1. Preparation Phase

The aim of data collection in the preparation phase is to study the relevant present knowledge of students who become the research subject in the teaching experiment. It is also aiming to have the insight about the learning environment of the classroom that is involved in this research. In the preparation phase, the researcher conducts a pre-test both in the preliminary teaching (cycle 1) and in the teaching experiment (cycle 2) aiming to reveal students' pre-knowledge relating to the concept of angle. In the pre-test, the data are collected from students' written work. Furthermore, following the pre-test, there is an interview conducted

with all students in the preliminary teaching and students from the focus group in the teaching experiment. The data from the interview are collected by making a video registration and collecting students' written work. The information from both the pre-test and interview are used to improve the initial HLT regarding the aspect of starting point of students and the activities. The students who are interviewed in the preparation phase are also observed during the teaching experiment.

It is also important to have an overview of how the learning process takes place in the classroom. Accordingly, in the preparation phase, the researcher also pays attention to the learning environment of the classroom. It concerns both social and socio-mathematical norms and other aspects that contribute to the mathematical learning process in the classroom. In this sense, the data is collected by observing the learning environment of the classroom and interviewing the teacher. The researcher makes field notes based on the list of observation points that has been developed and also makes a video recording during the interview session with the teacher.

3.5.2. Preliminary teaching (cycle 1)

Preliminary teaching is conducted both as the pilot experiment and as the first cycle of the research. The aim of preliminary teaching is to investigate students' thinking and reasoning about the problems in the HLT and to try out the conjecture about it. The preliminary teaching is conducted by trying out the initial HLT that has been developed with the small group of students who are in the middle level of understanding. Those students are also different with the students

who are going to work in the second cycle. The data is collected by collecting the students' written work and making a video recording of all the activities during the lessons. In this cycle, the researcher takes a role as a teacher and is assisted by his colleague to make a video registration during the lessons. The data collected during this preliminary teaching is used to revise and improve the initial HLT.

3.5.3. Teaching experiment (cycle 2)

The new improved HLT from the first cycle is used in this second cycle. The researcher also collects the data in this phase by making a video registration of the whole lessons and collecting students' written work. Furthermore, a short discussion with the focus group and the whole class discussion are also recorded in order to get the data about students' thinking and reasoning. The focus group in this case is the group that is explained in the section 3.2.1. The focus group consists of several students that are in the middle of level of understanding. The decision about students' level of understanding is based on the result of the pre-test and the interview with the teacher. Furthermore, in this phase the researcher uses two cameras, one as a standing camera recording the activities of students in the focus group and the other as dynamic camera that can be moved around the classroom to record all of activities during the whole lesson.

3.5.4. Post-test

In the end of the whole lessons, the researcher conducts a post-test with all students in both preliminary teaching and teaching experiment. This post-test is aiming to assess students' understanding about the concept of angle that they have learned. The data from the post-test are collected from the students' written work.

The students from the focus group in the teaching experiment are also interviewed about their works in the post-test. The goal of this interview is to know their thinking and reasoning in solving the problem given in the post-test. The data from this interview are collected by making a video registration.

3.3. Validity and Reliability

As mentioned before, this study involves different types of methods such as interviews, video observations, student written works and field notes to collect the data. In this research, it is very important to consider the aspects of validity and reliability regarding the data collection. It strengthens the quality of the research itself and also determines the impact of the research results. In brief, validity refers to whether the researchers really measure what they intend to measure and reliability refers to the independency of the researchers. The different types of methods are used to collect data such as observations related to the students' learning process, students' written work and a teacher interview. This different type of methods is used for doing method triangulation that can contribute to the internal validity of the research. Furthermore, collecting the data using video recording can improve the internal reliability of the research.

3.4. Data Analysis

3.4.1. Pre-Test

The result of the pre-test is analyzed in order to investigate the present relevant knowledge and to know the starting points of students about the concept of angle. The test result is expected to reveal the students' thinking about their informal knowledge, their understanding and their misconceptions about the

concept of angle. In this sense, the initial HLT is adjusted based on the result of the pre-test so that it is appropriate with the students' prior knowledge and their level of understanding.

3.4.2. Preliminary Teaching (cycle 1)

The data collected from the students' written work and video registration of all the activities during this first cycle is analyzed to investigate the learning process of students. The learning process of students is analyzed by testing the conjectures in the HLT. It means that the conjectures about students' learning process are compared and analyzed to the actual learning process of the students. In this sense, it can be seen which conjecture is really happened and which one is not. Moreover, it can also be seen whether the initial HLT support students in learning the concept of angle or not. Accordingly, the HLT is revised and improved based on the result of this first cycle and is implemented in the next second cycle.

3.4.3. Teaching Experiment (cycle 2)

The video recording of both the students' activities in the focus group and the whole class activities is analyzed to get the insight of the learning process of the students. Accordingly, the selected fragments or episodes are transcribed in order to make an interpretation of students' thinking and reasoning. These selected fragments together with the students' written work are compared and analyzed with the conjectures in the new improved HLT from the first cycle. The result of the analysis of this second cycle is used to answer the research questions, draw a conclusion, and also as a basis for redesigning and improving the HLT.

3.4.4. Post-Test

The result of post-test is analyzed by comparing the result of students from this post-test with the result of pre-test. The aim of this analysis is to investigate the development of students' learning and understanding of the concept of angle and also to reveal their strategies in solving the problems about angles. The result of analyzing the post-test also enhances the analysis of teaching experiment to draw a conclusion.

3.4.5. Validity and Reliability

It has been known that validity and reliability are important aspects to be concerned in all kinds of research, include design research. The validity and reliability of this research regarding the data analysis are strengthened as follows,

1) Validity

Validity of the research regarding the data analysis is elaborated in terms of internal and external validity. Internal validity refers to the quality of the data collections and the soundness of the reasoning that has led to the conclusions (also labeled as 'credibility') (Bakker et al, 2012). The data triangulation between the results of video registrations, students' written work, interviews and field notes contribute to the internal validity of the research. Testing and improving the conjectures in the HLT during the preliminary teaching and teaching experiment also contribute to the internal validity of the research. Moreover, in the first round of data analysis, the researcher ends up with a sequence of conjectures and refutations that are tied to specific episodes. In the second phase of retrospective analysis, these episode-specific conjectures and refutations were also generated

and tested at other episodes and other data materials such as field notes, tests, and other student work. The researcher also searches for counterexamples of these episode-specific conjectures. In this sense, it also contributes to the internal validity of the research. During the retrospective analysis, particular episodes reveal themselves to be pivotal. These pivotal episodes are analyzed with multiple theoretical instruments of analysis (theoretical triangulation) and contribute to the internal validity of the research.

External validity, often called generalizablity or transferability, concerns to which the results (instructional theory, educational activities and HLT) given by the research from the specific contexts are useful for the other contexts. By describing details of the participating students, framing the important issues or episodes, making a thick description of what happened in the teaching experiment and presenting the analysis of how these elements may have influenced the whole process, in such a way that others can find out can contribute to the external validity of the research. Furthermore, by showing the results of the research in such a way that it is helpful for outsiders (researchers and teachers) as the basis for them to make adjustment to their own situations or local setting can also contribute to the external validity of the research.

2) Reliability

Reliability of the research regarding the data analysis is also elaborated into internal and external reliability. Internal reliability refers to the reliability within the research itself. Internal reliability also refers to the reasonableness and argumentative power of inferences and assertions (Bakker, 2004). In the design

research, the internal reliability can be improved by discussing the critical fragments or episodes with the colleagues during the retrospective analysis about the interpretations and conclusions. It is called as inter-rater reliability. In this sense, the researcher needs to measure the inter-rater reliability and in this research there is no time to do it.

External reliability usually denotes as "trackability" or "traceability", meaning that the reader must be able to track or trace the learning process of the researchers and to reconstruct their study: failures and success, procedures followed, the conceptual framework used, and the reasons for certain choices must all be reported (Bakker et all, 2012). In this sense, by making a clear description of the way data were analyzed (transparency), the outsiders can easily follow of what the researcher has done and it contributes to the external reliability of the research. This norm of trackability fits well with Freudenthal's conception of developmental research or design research:

> *Developmental research* means:
> experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and this experience can be transmitted to others to become like their own experience.
>
> (Freudenthal 1991: 161)

3) Ecological validity

Gravemeijer and Cobb (2006) states that a central assumption that underpins our work is that instructional innovations developed in the course of a design research experiment can be used productively to support students' learning in the other classrooms. Accordingly, this research is carried out in the natural classroom

settings when the research enters the second cycle – teaching experiment. In this sense, it contributes to the ecological validity of the research. Furthermore, since the researcher and teacher are collaborated intensively in the research, feedback from the teacher on how the instructional activities are adjusted in the classroom can also strengthen the ecological validity of the research.

3.5. Research Subject and Timeline of the Research

The study was carried out within the International Master Program on Mathematics Education (IMPoME) program batch III. This program was launched since 2009 as a part of implementation and dissemination of PMRI approach in Indonesia (Zulkardi, 2009). Three universities: Freudenthal Institute for Science and Mathematics Education Institute Utrecht University, Sriwijaya University, and State University of Surabaya were collaborated to make this program happened. Several research have been conducted within IMPoME program exploring many mathematics topics in elementary schools such as decimal (Pramudiani, 2011), and volume measurement (Revina, 2011). The master thesis of IMPoME students can be downloaded online for free in this website (www.fisme.science.uu.nl/en/impome).

The study was conducted in the Primary School Muhammadiyah 6, Palembang. It involved third grade students from two different classes, III^B and III^C. The main consideration for choosing Primary School Muhammadiyah 6 Palembang in this study is because this school has been involved in the *Pendidikan Matematika Realistik Indonesia* project since 2010. The study was planned to have 2 cycles of HLT implementation. The first cycle involved 5

students from III^B and the second cycle involved 38 students from one class, III^C. The timeline of the study is summarized in the following table:

Table 3.1. Timeline of the Research

	Date	**Description**
Preparing for the Experiment		
Preparation	October – December 2012	Studying literatures and designing the initial HLT
Preliminary research to school (communicating with school and teacher)	6th – 7th of February 2013	Doing observation and communicating the plan of the research including HLT and research method with the teacher
Preliminary Teaching (The First Cycle)		
1st meeting	10th of February 2013	Pre-test and interview.
2nd meeting	14th of February 2013	Activity 1: The cat and the mice.
3rd meeting	20th of February 2013	Activity 2: Experiment with the screen.
4th meeting	25th of February 2013	Activity 3: Coloring the blind spot.
5th meeting	26th of February 2013	Activity 4: Understanding the vision angle.
6th meeting	27th of February 2013	Activity 5: Playing with paper fan.
Teaching Experiment (The Second Cycle)		
1st meeting	18th of March 2013	Pre-test.
2nd meeting	20th of March 2013	Activity 1: The cat and the mice.
3rd meeting	30th of March 2013	Activity 2: Experiment with the screen.
4th meeting	1th of April 2013	Activity 3: Coloring the blind spot.
5th meeting	2nd of April 2013	Activity 4: Understanding the vision angle.
6th meeting	3rd of April 2013	Activity 5: Playing with paper fan.

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

In all phases of the design research, the researcher develops so-called "Hypothetical Learning Trajectory" (HLT), which is regarded as an elaboration of Freudenthal' thought experiment. The notion of HLT was proposed by Simon (1995) entails that the teacher has to envision ways in which the students might engage as they participate in certain instructional activities, and then take into account about the students' potential lines of argumentation he or she wants to become taken-as-shared in the classroom community, related to the chosen learning goal. In this sense, HLT plays an important role as a way to explicate key aspects of planning mathematics lessons that promote students' development of new mathematical concepts and support the teaching mathematics for understanding. Simon introduces the description of HLT as follows:

> The hypothetical learning trajectory is made up of three components: the learning goal which defines the direction, the learning activities, and the hypothetical learning process – a prediction of how the students' thinking and understanding will evolve in the context of the learning activities. (Simon, 1995, p. 136)

As explained, an HLT is a vehicle for planning students' learning of particular mathematical concepts. Simon (1995) explained that the term "hypothetical" is based on the fact that the actual learning trajectory is not knowledge in advance. It means that the teacher can never be sure what the students will think and do or whether and how they will construct new interpretations, ideas and strategies until

they are really working on a problem (Fosnot & Dolk, 2001). In this sense, the designer has to envision the learning route in which students might engage with certain mathematical goal in mind. Furthermore, the term "trajectory" might also have a linear connotation (Bakker, 2004). Although we aim for a certain direction, the learning route is neither a rigid structure nor necessary linear. The students can go off in many directions as they explore, struggle to understand, and make sense of the world mathematically (Fosnot & Dolk, 2001).

As stated before, the aim of this study is to contribute to an empirically grounded instructional theory for understanding the concept of angle in the primary school level. In achieving this aim, the instructional activities are designed to encourage students to learn the concept of angle by exploring the notion of vision lines. The design of instructional activities in this study included the development of student worksheets, teacher guides, solution to the assignments, and tests. There are two main goals of the activities designed in the HLT. First, students can understand the notion of vision lines that embedded from the real life experience. Second, they can develop their understanding of the concept of angle by exploring the idea of vision lines when dealing with problems involving angle concept.

In the present chapter, we elaborate the HLT that is used and revised during the study. The HLT is implemented during the phase of teaching experiment in the third grade of primary school in Indonesia. It contains sequence of five lessons in the three weeks period of teaching that is designed to reach the aim of this study. In each lesson, we describe the starting point of students, the learning

goals, the mathematical activity, and the conjectures of students' thinking and reasoning. The instructional activities designed for learning the concept of angle that are embedded in the hypothetical learning trajectory are described as follows.

4.1. The Cat and the Mice

1. Starting points

The concept of angle is one of the mathematical topics that is taught in the second semester of the third grade of primary school in Indonesia. It is the first time for the primary students to learn the concept of angle. The starting points describe the relevant knowledge and skills students may already have and also to knowledge which may be necessary in order to support them in accessing and understanding this new topic. The starting points for the first lesson are as follows:

- Students know the concept of lines and points.
- Students are able to draw and visualize points and line segments.
- Students are able to reason why object can or cannot be seen. This knowledge is important because students need to develop their reasoning about the visible and invisible area.

2. The Learning Goals

Main goal:

Students investigate real situations involving vision lines and blind spots.

Sub goals:

- Students are aware of the different sides of viewing image.

- Students have a spatial perspective taking ability that is the ability to see the world from someone else's eyes.
- Students are able to reason why some mice can or cannot be seen by the cat.
- Students know that there are four mice that the cat can only see a half part of their body.

3. Description of Activity

To start with, the teacher brings the context of playing Hide and Seek. As this game is quite familiar with students, the teacher asks students about how they play the game. Specifically, how do students learn how to hide? What kind of object they want to be hiding of? Furthermore, in order to be a good hider, students need spatial perspective taking, that is the ability to see the world from someone else's eyes, to be able to find the best hiding place. For instance, a good hider must take into account where the area in which the seeker cannot see, where the seeker search first, and where to hide behind an object from the perspective of seeker? By exploring this game, we expect that students begin to talk about visible and invisible area and the reasons about it.

In order to help students make sense about the concept of vision lines, the teacher conducts a simple experiment with the students by using their arms. The teacher asks one of students to come in front of the classroom. The teacher asks him to sit down in a chair in front of the classroom facing his friends. The teacher also asks him that he is not allowed to move his eyes to the right or to the left. The teacher then stands a little far and in line with him, in his right or left side, and

asks him whether he can see the teacher without moving his eyes. After that, the teacher slowly moves to the front in a circle way and asks him again to stop the teacher as soon as he can see the teacher. In this case, the teacher asks the other students why this student can or cannot see the teacher. Why do you think so? Then, the teacher continues by hanging a book in front of this student and ask him whether he can see all the other students in the classroom or not. The teacher then asks the other students why he can only see some students. How if the teacher moves the book closer or far away from him? What happen with his view? This activity is also served as a bridging activity between the informal knowledge of students with Hide and Seek game and the problem about cat and mice.

Then, the teacher divides students into several groups. For instance, if there are 30 students in the classroom, then there will be 6 groups of five students. The teacher then introduces the problem about the cat and the mice. Before coming to the problem, the teacher encourages students to talk about the context and ask them whether they have ever seen a cat and mice in their house and their experience with them. Then, the teacher shares her or his problem by giving a worksheet to the students showing a picture of the cat and the mice. The picture is a panoramic drawing of situation and looks like below:

Figure 4.1. The First Problem (Panoramic Drawing)

Then, the teacher asks students to help him or her to decide how many mice the cat can see in this picture. Students start the activity by working in small groups and discussing about the problem. Each group also has one picture of the drawing. The teacher also suggests that students can draw lines in the drawing or anything they want to solve the problem. After a couple of minutes working and discussing about the first picture in groups, the teacher orchestrates a whole class discussion about it. The discussion focuses on the prediction of students about whether the cat can see the mice, and if so how many mice the students think that the cat can see in the picture. Then, the students are given the second worksheet showing a picture of the top view of the drawing. The picture looks like the one below:

Figure 4.2. The Second Problem (the Top View of the Drawing)

The teacher lets them work in groups. As in the first picture, the teacher also gives freedom to students to make a drawing or anything students want in this second picture that can help them to deal with the problem. After a couple of minutes working, the teacher conducts a classroom discussion. In this discussion, the teacher focuses on the students' strategies to solve the problem. In this sense, the teacher may ask students some questions such as: "how many mice that the cat can see? Why do you think so? Do you have different answer for the first and the

second picture? Why they are the same or different? Could you explain you reasoning to your friends? The teacher can also further the discussion by pointing the mouse hiding in the first picture but can be seen in the second picture. The teacher can ask students to think about it for a couple of minutes and then explain their answers to others. This discussion guides the students to understand about the different views of an object or situation better Furthermore, to give an insight about vision lines to the students, the teacher can ask them and have discussion whether the cat can really see the whole body of the mice that standing diagonally.

4. Conjecture of students' thinking

- When the students start to think about the first picture, they might start conjecturing about how many mice the cat can see in the picture such as 'I think the cat see all the mice' or 'I think the cat can only see the two cats beside the jar'.
- Some students may have a different answer or opinion between those two pictures. They might say that the cat in the first picture can see more mice than the cat in the second pictures or vice versa.
- Some students may realize that there is a mouse hiding in the first picture but can be seen in the second picture.
- There might be students that come up to the idea of drawing lines in the second picture. But, their lines may be different. Some students may just draw a parallel line while others draw diverging lines but do not meet in the eye of the cat. It is also possible that there are some students that can draw the vision lines of the cat touching the jar.

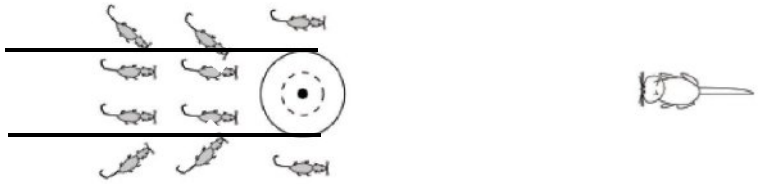


Figure 4.3. Parallel Line

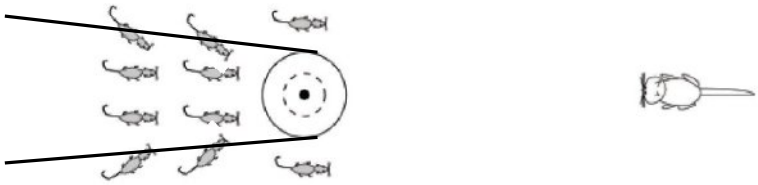


Figure 4.4. Diverging Lines but Do Not Meet Each Other

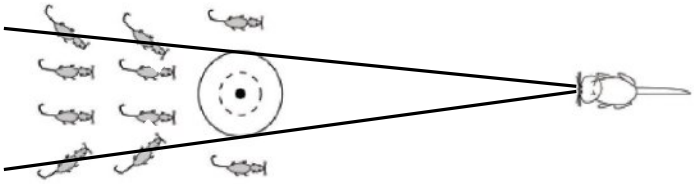


Figure 4.5. Vision Lines of the Cat (Correct Answer)

5. Discussion

The students are not expected to come up with a perfect answer or solution. But, it is expected that students informally use the idea of vision lines (i.e. imaginary straight lines from the eye to an object) as a tool to determine which mice can and cannot be seen by the cat and they use the top view to build their reasoning and thinking. In the first picture, the students may just make a prediction or conjecture about the number of mice that the cat can see. It is a good start because this activity is also aiming to develop the students' spatial sense in making a prediction or conjecture.

Since the third grade students already know about a ruler as a tool for measuring and drawing a line starting from the grade 2, so there might be some possibilities to use a ruler to draw a vision line in the picture. For those who draw a parallel line (figure 4.3) might argue to others that when you looking to the things in a real world you look it in a straight way. The teacher then can compare this opinion with other students who have drawing diverging lines (figure 4.4). The students who draw diverging lines might have a reason that when you see things in the real world, you see not only things exactly in front of you but also other things that are little bit on the left and right side of you. The teacher in this case emphasizes the discussion to the idea of vision lines. The teacher can continue the discussion by pointing out the four mice that can only be seen a part by the cat. The teacher can guide the students to discuss about it by showing his/her doubt about whether those mice can really be seen by the cat or not. The teacher can also post a further question such as what if the position of the cat far away or close to the jar? What happens with the number of the mice the cat can see? Is it less, more or remain the same? The teacher also furthers the discussion by pointing out the difference between those two pictures. The teacher might say that: “look to the first picture. There is a mouse hiding in the first picture but can be seen in the second picture. Is that true? Could you explain why it happens?

4.2. Experiment with the Screen

1. Starting points

From the previous lesson, students have learnt about the concept of vision lines informally. They have been worked in the activity of investigating situations

involving the concept of vision lines. Therefore, students are expected to have abilities as follows:

- Students are aware of the different sides of viewing image.
- Students have a spatial perspective taking ability that is the ability to see the world from someone else's eyes.
- Students can determine the number of mice that the cat can see and informally reason about the concept of vision lines.

2. Learning Goals

Main goal

Students are able to construct vision lines and blind spots in two and three dimensional representations.

Sub Goals

- Students aware that they can extend the vision lines endlessly.
- Students are able to realize that the vision lines are straight lines and they diverge.
- Students are able to explain that the vision lines go from the eye of the observer and touch the edge of the screen.
- Students are able to explain the idea of blind spots in the experiment.

3. Description of activity

To start with, the teacher reminds the students the context of the cat and the mice in the last meeting. The teacher also reminds students about the questions in the last meeting about what happen if the cat is further away or close to the jar.

To have a real life experience, the teacher brings the students outside the classroom to conduct an experiment. The teacher puts an opaque screen and a chair on the playground. In this activity, the teacher also prepares ropes and asks students to bring their school bags to be used. Students are divided into several small groups. For instance, if there are 30 students, then there will be 6 groups of 5 students. These groups are the same with the groups in the previous lesson. In order to use the time effectively, these 6 groups are divided over for each of the teacher and the researcher.

Before doing the experiment, the teacher asks one of students to be an observer and sit in a chair facing the screen. Then, the teacher puts some school bags in the visible (the left side or right side of the screen) and invisible area (behind the screen). In this case, the teacher promotes a discussion by asking some questions such as:

Can the observer see this bag? Why he can or why he cannot? Can you put the school bags where the observer can or cannot see it? What conclusion do you have about it?

After that, the teacher continues to do the experiment. The teacher explains the role of the experiment and reminds them that each group takes turn as an observer during the experiment. One student from the first group takes a position as an observer (the cat) and the other students from this group stand behind the observer.

Then, students from the other groups are carrying their school bags and standing in line close together behind the screen where they cannot be seen by the

observer. The teacher asks students behind the screen that they have to put their school bags where the observer can only see a part of it. One by one, the students behind the screen move sideways until coming out into the visible area and stop as soon as the observer can see them or give signal like 'I can see you now'. Then, the students put their school bags down at the place where they stand on and the observer can only see a part of the school bags. After doing this activity for each group, the teacher then asks students about whether they can predict the positions of the next school bags, why do you think so. The teacher also asks students about what the position of the school bags look like? Students validate their answer by putting a rope from the nearest school bag of the screen until the furthest one. Next, the teacher goes further by asking whether, for the same position of the observer, the students can farther apart or extend the rope (vision lines). If so, how long you can extend it? Why do you think so?

Then, the teacher guides the students to an understanding of the different positions of the observer and the influence on the position of the bags. In this sense, the teacher asks the students about how about the position of the bags may look like when the observer is further away or close to the screen. The teacher also promotes the discussion further by asking about how the position of the bags may look like when the observer is moving to the left or the right side facing the screen. Why do you think so? All the questions and discussions that happen in the experiment are related to the context of the mice and the cat from the first meeting by the teacher.

4. Conjecture of students' thinking

- Since the students put their school bags as soon as the observer can see them, some of the students might realize that the school bags are aligned.
- There might be students who do not believe that the two rows of school bags diverge. In that case, the teacher can propose students to use the rope to verify that the bags diverge.
- Answering the teacher question, there might be students who think that they can predict the next bags. They might validate their conjecture by doing the experiment again and using rope.
- There might be students that think they can extend the rope (vision lines) further, as far as they want. They might validate this conjecture by conducting the experiment again and using ropes.
- There might be students who realize the different direction of rows of the school bags when the observer moves in the different positions. The other students also validate this conjecture by doing the experiment again and using ropes.
- Some students may notice that the angle becomes smaller and smaller as soon as the position of the observer move sideways to the left or to the right of the screen.

5. Discussion

By conducting this experiment, it is expected that the students realize that the invisible area that cannot be seen by the observer is actually bordered by two half lines that go from the eyes of the observer and touch the screen edges. These lines

can be extended as far as desired. Before coming to that idea, some students may realize that there is an area where the observer can really see the school bags and there is an area where the observer cannot see the school bags. Students may also notice that there is a place where the observer can only see a half part of the school bags. Furthermore, some students may also notice that the school bags in which can only be seen a half part of it by the observer are in line and look like diverge. By putting the school bags into the visible area, the hidden area, and the middle are in which the observer can only see the half part of it, the teacher actually elicits the discussion to emphasize the idea of vision lines. In this sense, the teacher can guide the students to discuss about the lines the school bags formed and make sure that all students understand this idea. Then, following this discussion, the teacher can encourage students to talk about whether they can predict the next position of the bags and validate their conjecture by using ropes. The teacher should guide the students to understand that the vision lines in this problem is actually the imaginary straight lines from the eye of the observer to the screen that border the visible and invisible area (i.e. blind spots).

When students continue to do the experiment, there might be some of them think that the rope can be extended as far as desired. In that case, the teacher can ask the other students about this conjecture. Then, in order to validate and see whether it is true or not, the teacher can encourage students to do the experiment again with another rope. The teacher can also guide the students to realize that the ropes go from the eyes of the observer and touch the edge of the screen. Furthermore, the discussion about the vision lines goes deeper by guiding the

students to experience the different position of the observer (close, far away, right side and left side of the screen).

If the word of vision lines does not come up in the discussion, the teacher can introduce this word by explaining to the students that the rope is actually called the vision lines in geometry or mathematics. The teacher also guides the students to understand that the vision lines can be many, but in this activity the vision lines are the lines border the visible and invisible area. Even though the emphasis of this activity is on the notion of vision lines, there is possibility that students say something in the discussion about the idea of angle. React to this argument; the teacher can guide the students to discuss about the angle in terms of the experiment that they have done.

4.3. Coloring the Blind Spots

1. Starting points

Based on the previous lesson, students have already learned about constructing the vision lines and blind spots of the observer. Below is the list of the expected abilities and knowledge that students might have from the previous activities, such as:

- Students know that the vision lines can be extended endlessly.
- Students understand that the vision lines are straight lines and they diverge.
- Students are able to explain that the vision lines are the imaginary lines that go from the eye of the observer to the object (i.e. screen).
- Students are able to explain the idea of blind spots in the experiment.

2. The learning goals

Main goal

Students are able to imagine and visualize the vision lines and start to build their reasoning in terms of angle.

Sub goals

- Students are able to color the hidden area (blind spots) of different position of observer.
- Students are able to compare several different hidden areas.
- Students are able to realize that it is an angle that makes the drawing is different.
- Students are able to reason using their own words in terms of angle.
- Students are aware that the angle is the same for a given position of the observer; no matter how long the boundaries are (the boundaries are the vision lines).

3. Description of activity

In this lesson, students work on the problems in the student worksheet given by the teacher. Students are given several drawings in the problems. The drawings are the representation of the experiment done in the previous meeting showing several different positions of the observer and the screen in the different areas of the playground. Before asking the students to work on the problems, the teacher explain that the drawing is the top view of the experiment. In this sense, the teacher explains the meaning of top view to the students and make sure that everyone understand about it. The teacher also has asked students in the previous

meeting that they have to bring color pencils and a ruler to be used in this meeting. Then, the teacher asks students to be in their groups. The teacher also provides scissors and tracing paper for the students.

In the first activity, students are asked to color the area bounded by the vision lines of the observer when the observer's position varies (the observer is moving far away or close to the screen). In order to promote a discussion, the area of the playground is also different among the drawings. When the students have done with the activity of coloring the area of the drawings, they are asked to compare the drawings. In this sense, the teacher asks students to find their own way of comparing the drawings (using their fingers, superimposing, cutting, etc). In this activity, the teacher guides the students to discuss about whether the drawings are the same or different to each other? Can you explain to your friends, why do you think so?

After that, the teacher provides or hand out the second activity of coloring the area bounded by the vision lines of the observer in which the observer is moving to the left or to the right side of the screen. Like in the first activity, as soon as the students have done of coloring the area of the drawings, they are asked to compare those areas using their own methods or strategies. In this activity, the teacher can ask questions like: is the area the same for all of those drawings? Is it bigger or smaller? Can you explain to your friends why do you think so?

4. Conjecture of students' thinking

- Since the students have done the experiment in the previous meeting, there are some of them that are able to color the area bounded by the vision lines of the observer correctly.
- There is possibility that some students still struggle with the idea of vision lines and color the area incorrectly.
- Some students may compare the drawings by using their fingers, cutting or superimposing those four drawings.
- Some students may compare the drawings by using tracing paper.
- Some students may compare the drawings in terms of the area and say that the drawings are different.
- There are some students that may give their own reasoning in terms of angle when comparing the drawings and say those two drawings are the same in terms of angle but different in area.

5. Discussion

During their work, the teacher asks students to make a poster that depict their answer to the problems and the important things such as their reasoning and thinking that they have to share in the math congress (whole class discussion). The teacher explains to the students that the posters need to be clear for others to understand. The posters should not just be students' draft notes copied over. They should be concise and clear presentation of the important ideas or strategies that the students want to present. Making a poster is very important for students not only generates further reflection, but also challenges them to develop a convincing

and elegant argument or reasoning – an important part of mathematics. The teacher holds the math congress after the students finish in working for the two problems.

By doing these two activities of coloring and comparing the drawings, students are expected to feel the need of knowing the concept of angle and then use this concept in giving their reasoning to compare the drawings. There are some students that might color the area incorrectly because of their lack of understanding of the vision lines. In that case, the teacher orchestrates a whole class discussion in which the students are presenting their own different answer and having a discussion about it.

The teacher should emphasize the discussion to focus on reinventing the concept of angle when students are comparing the drawings. I this case, the teacher can guide students by asking some questions such as: do you have any idea how to compare these drawings? Are the drawings the same or different? What makes them the same or different? Can you explain to your friends why do you think so? In the first activity, if most of students say that those drawing are different to each other in terms of area, the teacher can guide the discussion into angles by doing superimposing of two drawings and ask questions such as: look at these two drawings, do you recognize something here? What is it? Can you explain to us about it? In the second activity, the teacher also guides the discussion in terms of angle by superimposing the drawings start from the biggest to the smallest one. The teacher can also further the discussion by putting the

drawings into one horizontal line and asks students about whether they recognize the pattern in it or not.

4.4. Understanding the Vision angle

1. Starting points

During the previous lesson, students have learned about visualizing the vision lines of different observers and compare them in terms of angle. Through those activities, students are expected to have abilities and knowledge such as:

- Students are able to draw the vision lines of observer in the different position.
- Students are able to compare the different vision lines of observer in terms of angle.
- Students know that the angle is getting smaller when the observer is moving to the left or to the right side of the screen.
- Students know that the angle is the same for a given position of the observer; no matter how long the boundaries are (the boundaries are the vision lines).

2. The learning goals

Main goal

Students are able to construct and draw the vision angle of the different observer's position.

Sub goals

- Students are able to construct the vision angle of the observer.

- Students know that the vision angle is different for different position of observer.
- Students are able to locate the vision angle of observer into a certain position.
- Students realize that one position can be placed by different piece of paper with the same angle.

3. Description of activity

In this lesson, the students do a small experiment. In the beginning, the teacher asks one of the students (as the observer) to go outside the classroom and stand in front of the door looking into the class. The teacher then asks this student to explain what he can see in the classroom in his position. Accordingly, the teacher then promotes a discussion by asking the other students questions such as: why he cannot see the others? Why do you think so? The teacher then asks the other two students to use rope in order to find the visible are of the observer. These two students pull the rope from the observer into inside the classroom. In this case, the teacher again promotes a discussion by asking the other students: what happens with the rope when the observer moves closer or far away to the door? Why do you think so?

Following this activity, the teacher then gives an exercise to be solved by the students. Students are asked to determine the vision angle of a child (observer). This child is standing in front of the door and looking into the classroom, like the experiment that students have done.

Then, the students are given a worksheet that shows a top view of the situation. The teacher explains to the students that it is a top view of the experiment that they have done before. In this top view, there are several different positions of the observer (a child) and students are asked to determine the visible area of the observer where s/he can see the situation of the classroom from the outside. The teacher asks the students to be in their groups for discussing and solving the problems. Each pair also has one student worksheet to work on. The teacher should tell the students in the last meeting that they have to bring a ruler and color pencils in this lesson. In doing the activity, students may use their ruler and color pencils.

For the first problem, the teacher explains to the students that in this problem they have to draw and color the visible area of the child (observer) from several different positions. In order to avoid confusion, the teacher asks the students that they may use different colors for different position of the observer. Students are also given a worksheet paper in which there is a picture of the classroom from the top view and different positions of observer looking into the classroom. In this sense, the teacher also asks students about the meaning of top view and gives an explanation when students do not understand about it. Then, students are asked to find the visible area of the child (observer) where s/he can see the situation of the classroom from outside. In this sense, students actually search for the vision angle of the observer in the worksheet that varies depending on the positions of the observer. The students have to color the visible area of each observer and explain their answer to others and make sure that they understand to each other. The

teacher orchestrates a discussion about the different answer of the students. The teacher can ask some questions such as: is there any position in the classroom that two different observers can see it? Why do you think so? Which position that the observer C cannot see it? Why do you think so?

In the second problem, the students also work in groups. In doing the activity, students are given several small papers by the teacher. Those small papers are actually an isosceles triangle with different length and angle. The teacher explains that this triangle is a drawing of the visible area of the observer from several different positions. In order to avoid confusion, the base of the isosceles triangle is not in a straight line. Students are also given a worksheet paper in which there is a picture of the classroom from the top view. Then, students are asked to find the position of the observer where he can see the situation of the classroom. There are several piece of small papers (the drawing of visible area of observer) and students are asked to find where the position of observer for each of these small papers. The students find the position of observer by placing the small papers into the picture on the worksheet correctly. After that, students have to explain their answer to others and make sure that they understand to each other. The teacher orchestrates a discussion about the different answer of the students.

4. Conjecture of students' thinking

The first problem

- Since the students have already learned about angle in the previous lesson, there might some students who can draw the vision angle of observer correctly and have a reason in terms of angle.

- There might students who color incorrect vision angle of the observer because of misunderstanding of vision lines.
- Some students might notice that there is a position in the classroom where the observer cannot see it.

The second problem

- Since the students have already learned about angle in the previous lesson, there might some students who can put the small papers correctly and have a reason in terms of angle.
- There might students who realize that one position can be placed by two or more small papers with the same angle but different length of sides.
- Some students might noticed that the small papers are different depend on the position of the observer.

5. Discussion

It is expected that the students discusses about the small experiment that they do in the beginning of the lesson. In this case, the teacher encourages them to talk about the vision angle of the observer. Furthermore, the students also discuss about where the place that the observer can or cannot see in the classroom. The students have to explain their reasoning so that the others can understand. They might validate their answer by conducting the experiment again and using rope.

The first problem

The activity of coloring the visible area of the observer is aiming to develop students understanding of the concept of vision angle through comparing the

different position of the observer. We expect that students realize that the coloring area is different for the different position of the observer. In this sense, the activity gives a chance for students to realize that for the same drawing of the classroom, the vision angle can be different if the position of the observer is different. The teacher can promote this idea by discussing the answer of students.

The second problem

The activity of finding the right small paper (i.e. isosceles triangle) is also aiming to develop students understanding of the concept of angle through comparing the different position of the observer. We expect that students realize that the small paper (isosceles triangle) is different for the different position of the observer. In this sense, the activity gives a chance for students to realize that for the same drawing of the classroom, the angle can be different if the position of the observer is different. The teacher can promote this idea by discussing the answer of students. The teacher also can further the discussion by asking whether for the same position of the observer, we can put the different small paper. Why do you think so? The teacher can also further the discussion by pointing out a certain position inside the classroom and asks students whether the observer can see it or not? Where the position of the observer that s/he can or cannot see it? Why do you think so?

4.5. Playing with Paper Fan

1. Starting points

Based on the previous lesson, students have learned about construct and draw the vision angle of the different observer's position. Accordingly, students are expected to have abilities and knowledge such as:

- Students know and are able to construct the vision angle.
- Students know that the vision angle is different for different position of observer.
- Students know that the vision angle is the same for given observer position no matter how long the boundaries are.

2. The learning goals

Main goal

Students understand about acute, right, straight, obtuse, and reflex angle and its different orientation.

Sub goals

- Students are able to realize the acute, right, obtuse, and straight angles.
- Students aware that the same angle can be placed in different orientation.
- Students realize the existence of reflex angles (inside and outside angles) for a pair of interesting half lines.

3. Description of activity

In this lesson, the teacher introduces a paper fan to the students as a tool for understanding the concept of acute, right, obtuse, and straight angle. This paper fan has a piece of color paper in between its two arms. This instrument is also used to introduce the students the idea of reflex (inside and outside) angles.

The teacher put the paper fan in front of his/her eyes horizontally, as the paper fan represent the vision angle of the teacher. Then, the teacher starts the activity by opening up one of the branches of the paper fan little by little and then asking the students about how the angle would change. The teacher then continues to open up the branches until they form a right angle. In this case, the teacher asks students whether they recognize the angle or not because right angle is considered as a common angle. Some of the students may realize that it is a right angle because it is quite often seen by students in their daily life. In this case, the teacher goes further by asking the students to point right angles in their surroundings. Then, the teacher continues to spread the branches apart. When the branches are flat, the teacher also asks some questions to students in order to promote a discussion. The questions such as: is there any angle here? Why or why not? When the students are in doubt, the debate set in. The teacher can facilitate the discussion by asking the students who think that there is an angle in that paper fan and explain their reasoning to others. The teacher then goes on to move the paper fan branches even farther apart. When the branches form a right angle in different position, the teacher, again asks some questions to students. In this case, the

teacher asks students about what happens with the angle? Is that right angle? Is that different or same with the previous right angle? Why do you think so?

Students might also realize that there is a reflex angle (inside and outside angle) when opening up the branches. In case the students do not realize about the reflex angle, the teacher can ask them questions like how many angles do you see here? Why do you think so? The teacher then slowly moves the paper fan branches until they are superimposing to each other and asks the students about the angle in this case. Do you still see the angle? Why you think so? In the end, the teacher gives exercises for students to be solved in groups. The exercise contains the pictures of the paper fan in several different positions that representing those four special angles.

4. Conjecture of students' thinking

- Since the students have already been introduced the concept of angle in the last activity, some of them may notice the right angle formed by the paper fan. It is also because the right angle is a common angle for them.
- Students may give a response for the activity differently. For example, some students may say that there is no angle when the paper fan branches are flat, while others say there is still angle in it.
- Some students may realize the reflex angle (inside and outside angle) by noticing the paper in between the branches.
- Some students may say that there is an angle when the branches are superimposing to each other by noticing the paper in between.

5. Discussion

It is expected that students discuss about whether there is angle or not when the paper fan is in a certain position. Some students might say that there is an angle when the paper fan is flat or superimposing to each other by recognizing the paper between the two branches. The teacher then asks students to explain their reasoning why they think there is an angle or no angle in that position. Why do you think so? What is the form of the paper? The teacher also emphasizes the discussion about the reflex angle. The teacher asks students whether they recognize the two angles when opening up the paper fan. The teacher can further the question about it by asking whether the two angles are the same or different? Is that possible to have the same inside and outside angle? When? Why do you think so? In case that the students do not realize these angles, the teacher can guide students to pay attention to the paper of the paper fan and propose a question such as do you see any difference with the angle? What is it? Why do you think so?

CHAPTER V

TESTING THE HYPOTHETICAL LEARNING TRAJECTORY

In the previous chapter we described about how we designed a Hypothetical Learning Trajectory (HLT) in the concept of angle for grade 3. The notion of vision lines had turned out to be difficult to be developed, though students in the small group were able to reason with drawing. In the present chapter we provide the retrospective analysis of data collected from both the first and the second cycle. We present the results of the first cycle in which we test the HLT that was developed for small group, the third grade of primary school with 5 students. We first describe the result of the pre-test to give an impression of the starting point of students' learning process about the concept of angle and the notion of vision lines. The remarks from the pre-test are used as consideration to adjust the initial HLT into HLT1. Next, we compare the HLT1 specified for particular activities with students' actual learning in the small group. In doing so, we analyze how the activities support the development of students' notions of vision lines, and then the concept of angle. Afterwards, we present the results of the post-test. All the findings and remarks from the first cycle are used to make some refinements into HLT2. The analysis of the HLT implementation in the second cycle is presented in the same order with the first cycle. In the conclusion, a summary of the results of testing the HLT gives an answer to the first research question of how the concept of vision lines can support the development of students' understanding of the concept of angle.

5.1. Teaching Experiment

The term of "teaching experiment" in this study does not refer to the format experiment in which a limited number of variables is manipulated and effects on other variables are measured. In this study, the researcher and the teacher during the teaching experiment phase, used the designed instructional activities and certain type of instruction that seem most appropriate at that moment according to the HLT. The teaching experiment phase was conducted within two cycles, namely preliminary teaching (first cycle) and teaching experiment (second cycle). The improvement was made into the HLT and the designed activities from the findings and remarks of the first cycle and then implemented into the second cycle. In this part, we will briefly explain about the process of teaching experiment both the first and the second cycles and then going deeply into the retrospective analysis of each cycle.

5.1.1. Preliminary Teaching (First cycle)

During the preliminary teaching (first cycle) we tested out the designed instructional activities and type of instruction in the intial HLT into one small group of five students. These students were from the third grade of SD Muhammadiyah 6, Palembang. The researcher himself acted as a teacher in this first cycle. Observations and analysis about what is going on during the preliminary teaching was made and conjectures that are embodied in the instructional activities were evaluated based on the remarks and findings to improve the initial HLT.

Pre-test

Before going the first lesson in the first cycle, the students took a 30-minute test. This test was aimed to get an impression of their prior knowledge about the notion of vision lines and the concept of angle, which was important information for conducting the HLT. In this test, the students worked individually.

Lesson 1: the cat and the mice

In this lesson, the students were given the worksheet with three different drawings depicting the situation of the cat looking through the mice behind the jar. The students then were asked to determine the number of the mice that the cat can see in those three different drawings. There were several different answers of the students in this problem. The students were also able to draw the connecting lines depicting the vision of the cat. In order to find out how the vision of the cat look like and the number of the mice that the cat can see, the students then conducted the experiment in the next day.

Lesson 2: experiment with the screen

In the second day, the students were eoncuraged to conduct an experiment with the screen. The students actually constructed the vision lines of different observers. Although, the students were able to do the experiment, the activity did not go smoothly. Intead of doing in the playground, the students conducted the experiment in the classroom due to the technical problem. The playground was used by other students for doing sport activities. The students also did not

use their school bags because there were not enough. So, the researcher proposed to use the boxes instead of school bags.

Lesson 3: coloring the blind spots

As the follow up activity of the experiment, the students were given a worksheet showing several pictures of the top view of observers looking through the screen. The students then were asked to color or shade the hidden area in which the observer cannot see. Some students were really struggling in doing this activity. At first, none of them were able to draw the vision lines of the observer in the drawing. They only made some point representing the hidden area. Only after the guidance of the researcher by proposing several questions, the students could come up with several answers such as drawing two perpendicular lines in both end sides of the screen.

Lesson 4: understanding the vision angle

In this lesson, the students were encouraged to construct the vision angle of the observer who stood in front of the classroom. The teacher asked one of the students to be the observer and the other two students pulled the rope from the observer into the classroom representing the vision angle. After discussion this activity, the students then were given a worksheet depicting the top view of the classroom and some observers in front of the door. They students had to draw the vision angle of those different observers. Although, the students were able to draw the vision angle, they still made a mistake in their drawing in which the vision lines that they drew did not touch both the two sides of the door.

Lesson 5: playing with paper fan

In the last meeting, the students were given an activity of finding some types of angle by using the paper fan. As the teacher in the classroom, the researcher opened the braches of paper fan little by little until it showed certain types of angle. The students then discussed whether there is an angle in those paper fans and their reason about it. Although the students were struggling to understand about the straight and one circle angles, at the end they could realize those angles with the guidance of the researcher.

5.1.2. Teaching Experiment (Second cycle)

After testing the HLT with the small group of students, the researcher made some refinements based on the remarks and findings from the first cycle. Then these improved HLT and the instructional activities were used in the classroom with 38 students. In this second cycle, the mathematics teacher of the school was involved and acted as a teacher for the whole lessons. The teacher divided the students into several small groups consisted of 4 or 5 students. The researcher and the teacher also chosed one group of students as a focus group by considering their mathematics grade and their eagerness to speak in the discussion.

Lesson 1: the cat and the mice activity

As it has been explained in the first cycle above, there were not so many differences made in this lesson for the second cycle. The teacher introduced the problem about the cat looking through the mice hiding behind the jar by asking their experience with cat and mice. The students then were asked to answer the problem in the worksheet (see appendix v) with their own group. As in the first

cycle, some students were also able to draw the connecting lines that went from the eye of the cat to the mice in the second cycle. The students also came up with different answer about the number of mice that the cat can see in the three drawings. The teacher then orchestrated a whole class discussion about this issue and asked some students with different solution to exlain their answers to the others.

Lesson 2: experiment with the screen

In the next day, the students conducted visual field activities on the playground. They constructed the vision lines of several different observers with the guidance of the teacher and the researcher. At this time, the experiment took place on the playground and the students used their school bags instead of boxes. In this activity, the concept of angle emerged when the students discussed about the different forms of vision lines and blind spots when the observers moved closer to or far away from the screen. The students noticed that it was an angle that delineated the change of the vision lines.

Lesson 3: coloring the blind spots

As it has been done in the first cycle, the students in the second cycle were also given a worksheet depicting the ground plan of the experiment in the previous lesson. It was quite different with the first cycle in which the students in the second cycle came up with several different drawings of vision lines and blind spots. At least there were four different answers among the students, ranging from the one that was closely related to the context of experiment to the correct drawing of vision lines and blind spots of the observer.

Lesson 4: understanding the vision angle

In this lesson, the students were encouraged to construct the vision angle of the observer as it was done in the first cycle. As there were many students in the classroom, the teacher had a discussion about the activity with some of the students both in the visible and invisible area of the observer. From this activity, the students noted that as the observer moved closer to or far away from the door, the vision angle was also changed, bigger or smaller. Following the activity of constructing vision angle of the observer, the teacher gave a worksheet for the students to work on with their own groups. Although the students had constructed the vision angle of the observer, they still made some mistakes in their drawing. It shows that making a drawing of three dimensional situations was not easy for the students.

Lesson 5: playing with the paper fan

In the last lesson, the teacher encouraged students to discover the classification of angles and find out several types of angle by using paper fan. The teacher introduced several types of angle by opening the branches of the paper fan little by little until it formed certain type of angle. For each type of angle that the paper fan formed, the teacher had a whole-class discussion with the students to discuss whether there is an angle in that paper fan or not. Later on, the teacher introduced the name of angle to the students. Following this activity, the students were given a worksheet depicting several drawings of paper fan and the name tag of several types of angle. The students then were asked to name the angle formed by the paper fan and to find out other types of angle by combining two or more

drawings of paper fan. Generally, the students were able to recognize the types of angle quite easily, except for straight and one circle angles. Some of the students still perceived that those two angles did not exist because they could find out the two branches or lines.

5.2. Retrospective Analysis

The data collected during the teaching experiment phase was analyzed in the retrospective analysis. In this phase, the HLT was compared with the actual learning process of the students. On the basis of such anlyses we answered the research question proposed in this study and contribute to the local instruction theory for learning the concept of angle. We divided our analysis into two parts, the first cycle and the second cycle. We first transcribed the fragments that interested to be analyzed and could inform us about the thinking and learning of the students. We read all the transcripts and watched the videotapes chronologically episode-by-episode (Bakker, 2004). The HLT and research question became a guideline to analyze and made interpretation about students' learning. These conjectures were tested by looking for confirmation or couter-examples. We also looked for the social norm and soci-mathematical norms of the students. The results of retrospective analysis were used as the basis for adjusting and improving the HLT and for answering the research question.

5.2.1. Remarks on Students' Pre-Knowledge in the First Cycle

Before doing the first lesson, students took a 30-minutes test. This test was followed by an interview aiming to reveal the students' thinking and reasoning in answering the problems in the test. The test and interview were done to get an

impression of students' prior knowledge of the concept of angle, particularly about the notion of vision lines, which was important information for adjusting the HLT. In particular we wanted to find out how students would solve the problem involving the notion of vision lines without preparatory instruction and how well they understood the concept of angle. During the investigation through pre-test, the researcher found several critical issues of students' present relevant knowledge that corresponds to the learning design in HLT as follows:

a) The informal knowledge of vision lines and vision angle

Generally, the students were able to reason in terms of vision lines and vision angle informally. Three out of five students said that your view becomes bigger as you get closer to the window, and vice versa. It shows that those students were able to imagine their views to answer the problem. They were also able to do spatial visualization and spatial thinking involving the notion of vision lines and vision angle. The other two students misunderstood about the problem.

b) Difficulties with defining angles and conceiving of 180^o and 360^o angles

As it is usually done in the school, thinking of angles as distances between to rays excludes any angle greater than or equal to 180^o. Those five students in this first cycle also excluded a straight angle, and they had a common reason to support this exclusion. They argued that it was not angle because it is just a straight line. They could not find the two lines that meet in a common point as they understood about angles. Likewise, the 360^o angle did not fit into their concept of image about angle. Their comments show how they rely heavily on their everyday experience with the physical word as they attempt to make sense of

mathematical concepts of angle. They only accept the certain angles involving two rays meeting in a common point. Their concept image of angles excludes the turning aspect of angle (the dynamic sense of angle) and therefore it was difficult for them to conceptualize 180^o and 360^o angles. Furthermore, students also struggled to define angles. Four out of five students said the definition of angle by listing the types of angle: acute, right, and obtuse angles. Only one of them tried to define angle. She said that angle is (something) between two joined lines.

c) The ability to know the angle classifications and the example from their daily life

All of students in the first cycle were able to mentioned the three types of angles namely, acute angles, right angles, and obtuse angles. They were also able to explain the definition and difference of them. Furthermore, the students also could mention several kinds of angle in their life such as angles in the doors, white board, table, etc. But, all of them only mentioned examples of right angle. None of them were able to give an example of obtuse or acute angles. It shows that most of children see right angles as special angles and it is easy for them to recognize this angle in their daily life.

5.2.2. Preliminary Teaching (first cycle)

The five students involved in the preliminary teaching were Radit, Adinda, Ghazy, Dewa and Haura. They worked with the activities in an improved-HLT based on the results of pre-test called HLT 1. The initial HLT was adjusted to the present knowledge of the students by considering the pre-test results. The feedback and remarks from this cycle were used as consideration for improving

the hypothetical learning trajectory. The analysis of how the HLT 1 works during the first cycle is explained as follows:

a) Investigating real situations involving vision lines and blind spots

Activity 1: Determining which mice can and cannot be seen

This activity was aimed to encourage students to investigate and explore real situations that involve the idea of vision lines and blind spots. It was expected that students started to talk about vision lines and blind spots informally.

The context is about the cat looking to the mice behind the jar. The idea is that there is an object (the jar) blocking the view of the cat. Then, the students have to determine which mice can and cannot be seen by the cat. In the first problem, students were given the panoramic drawing of the situation. The students, after reading and paying attention to the picture in the problem, directly said that the cat can only see one mouse that is on the right side of the jar.

Researcher	: How many mice that the cat can see?
Ghazy and Haura	: one (raising their finger together)
Researcher	: raditya?
Radit	: (really sure) one.
Researcher	: why is it one? Which mouse do you mean?
All	: (read the problem again and silent).
Researcher	: is there any other answer? Dewa said it is only one, the others cannot be seen.
Radit	: yes, it is. The mouse besides the jar.
Researcher	: is it the only one?
Radit	: So, it is seen from the cat to that mouse (pointing his pen from the eye of the cat to the mouse beside the jar).
Researcher	: where it can be seen?
Radit	: from here (pointing the cat) to here (pointing the mouse beside the jar).
Researcher	: how about the others, then?
Radit	: They cannot be seen because they are behind the jar.

At this point, students directly guessed that the cat can only see one mouse besides the jar. The phrase “they cannot be seen because they are behind the jar” by Radit illustrates that he had come to understand about the idea of visible and invisible area. He perceived that the other mice were in the visible area behind the jar since the view of the cat was blocked by the jar. It is also found that some of the students were able to make a drawing of connecting lines that goes from the eye of the cat to the mouse. These lines acted as a visual proof of which mice that can be seen by the cat.

Later on, as expected in the HLT, there is one student who recognized the existence of the mice on the other side of the jar. He explained to the others that there is another mouse on the left side of the jar and the cat can also see it.

Dewa	: I know. Actually, in this side, there are two mice that can be seen because this mouse (pointing the hidden mouse on the left side) is covered by this (pointing the jar). So, it looks like only one.
Researcher	: So, there are two that can be seen?
Dewa	: Yes, this one (pointing the hidden mouse on the left side) is covered.
Researcher	: can it be seen by the cat? I mean the hidden one?
Dewa	: yes, because if this is opened (the jar) it can be seen because it (the mouse on the left side) is here.
Researcher	: How is it look like in the drawing?
Dewa	: (showing the top view of the drawing) like this.

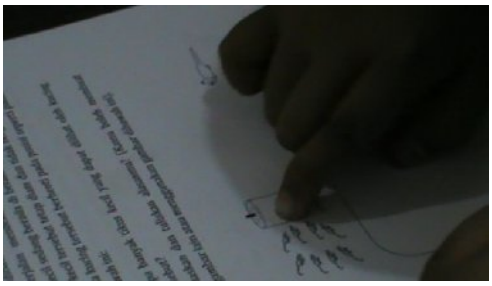


Figure 5.1 Pointing the Hidden Mouse

Based on the selected fragment above, we know that this student recognized the hidden mouse by comparing the first and the second picture. Accordingly, the students then continue to work on the second problem. They were asked to compare whether there is a difference in terms of the number of mice that the cat can see between the first and the second pictures. Researcher tried to provoke students' spatial thinking with the problem of comparing the front view and top view of the same situation.

Researcher	: so, in terms of number, is the number (of mice) that the cat can see in the first and second picture the same?
All students	: no.
Researcher	: it is not the same.
Radit	: in that picture is six (pointing the second picture).
Researcher	: how about this? (showing the first problem)
Radit, Haura, Gahzy	: one.
Dewa	: for this is one (pointing the first picture) but this one is covered (pointing the hidden mouse) so, it is one.
Researcher	: how about the second picture? How many mice that the cat can see?
Radit	: my answer is six.
Haura	: (counting the mice) six.
Dewa	: two.
Researcher	: Adinda? (showing the first picture)
Adinda	: one.
Researcher	: how about the second picture? Also one?
Adinda	: (shaking her head indicating yes)

Based on the discussion quote above, it can be seen that most of students perceived that those two pictures are different in terms of the number of mice that the cat can see. One of students, Dewa, was able to realize that those two pictures are the same. But, because the mouse in the first picture is hidden, it makes him

think that the first and the second pictures were different in terms of the number of mice that the cat can see.

Furthermore, most of students in this problem were confused with the picture of spider in the second problem. This was put in the problem to emphasize that the second drawing, the top view of situation, was seeing from above by the spider. It was intended to help the students in imagining the situation. But, it made some students confused by comparing the cat and the spider. They wrote that the cat can see two or six mice but the spider can see all the mice. Because of this confusion, the extensive discussion during this activity was conducted to encourage students to think and compare the panoramic drawing of situation in the first problem and the top view of situation in the second problem. Based on this remark, the researcher decided to improve the student worksheet of the first meeting. The worksheet would be more explicit by showing a clear statement to compare the panoramic drawing and the top view of the situation.

Next, students were asked to solve the third problem about what happen with the number of mice that the cat can see if the cat gets closer to the jar. As in the previous problem, in the third problem students were also given a picture of situation showing the cat's position closer to the jar. Then, there also a statement of Andy in the problem said that as the cat gets closer to the jar, the cat can see fewer mice. Students then were asked a question about whether they agree with Andy's statement or not and explain their reasoning. Four out of five students said that as the cat gets closer to the jar, the cat see more and more mice. Unfortunately, most of students did not read the question in the problem carefully.

Although they agree with Andy, their explanations contradict with Andy's statement, as describe in the following transcript:

Researcher	: we start from Ghazy? What is your answer Ghazy?
Ghazy	: (read his answer) agree, because the closer the cat, the more mice that can be seen and the cat is getting closer in front of the mice.
Researcher	: so, Ghazy's opinion is the closer the cat, the more mice that can be seen. How about you Haura?
Haura	: I agree with Andy's opinion because only two mice that can be seen because the others are covered by the jar.
Researcher	: Adinda, read your answer loudly.
Adinda	: agree. My reason is because as the cat get closer to the mice, the cat sees less and less mice and there are two mice that can be seen by the cat.

Based on the discussion in the fragment above, it can be seen that Ghazy and Haura had different opinion with Andy about the problem, although they said that they agreed with Andy. Those students seemed to have difficulty in imagining how the vision of the cat may change as the cat gets closer to the jar. Only one of them, Adinda, said that as the cat gets closer to the mice, the cat sees less and less mice. This opinion stated by Adinda illustrates that she had come to understand that the vision of the cat may change as the cat gets closer to the jar and it does affect the number of mice that the cat can see.

The other important remark from the first lesson is that some of the students tried to draw connecting lines that go from the eye of the cat to the mice. But, it was curved because they did not use any ruler.

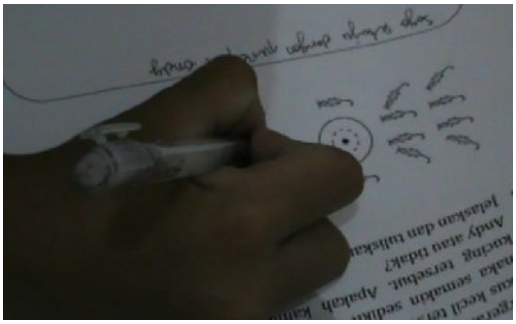


Figure 5.2. Drawing a Line without Using a Ruler

All of the students' drawings were the same in which they drew lines go from the eye of the cat to the mice but did not touch the two sides of the jar. It shows that the students grasped the idea of line of sight or line of vision that is any line that goes from the eye to the object. But, they hardly understood about the vision lines (e.g. the lines from the eye of the cat to the mice touching the jar on the both sides) that is bounded the visible and invisible area. As in the previous problem, it may also be interpreted that the connecting lines that students drew in the third problem is also a visual proof showing which mice that the cat can see.

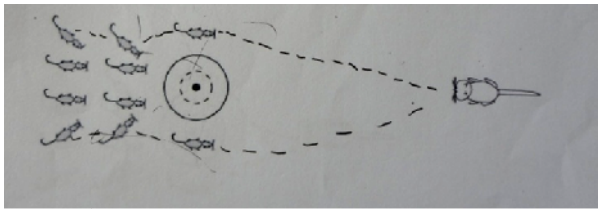


Figure 5.3. Student's Drawing of Connecting Lines

Generally, those three problems show some remarks on students' understanding toward vision lines and blind spots. Specifically, students perceived that there were some mice that could be seen by the cat and the vision of the cat may change as the cat gets closer or further away from the jar. Therefore, the idea

of vision, vision lines and blind spots should become the main issue in the next cycle.

Furthermore, through these activities that involve taking a point of view of situation, the students have experienced that certain situations such as the situation of cat looking to the mice hiding behind the jar can look quite differently when they are viewed from another side. Another step further in the students' development of taking a point of view of the situation is thinking up different views themselves. By working with those three problems, the students were trying to make a visual model of situation in their mind and manipulate it mentally.

b) Constructing the vision lines and blind spots

Activity 2: Constructing the vision lines and blind spots of different position of observer

This activity is aimed to support students in constructing and understanding the vision lines and blind spots from different position of observer. Furthermore, by constructing the vision lines and blind spots of different observer, students were able to validate their answers of the cat and mice problem in the previous lesson. Figuring the different forms of vision lines from the different position of observer is also important for student to understand the relationship between the vision lines and the concept of angle.

Each of students in this experiment took their turn to be the observer sitting in front of the screen. The researcher then put the school bag in the area where it can be seen and cannot be seen by the observer. The researcher then asked about students' comments of this activity by proposing question why they cannot see the

bag in all area. Most of students said that they cannot see the bag because it is covered by the screen. The important thing of this initial activity is that students realized that there is an area in which they can see the bag and there is also an area in which they cannot see the bag.

Following this activity, students then started to do the experiment by having one observer in front of the screen and the other students stand behind the screen. Instead of using their school bags, students used boxes when they did the experiment. The reason was that there were only five students in this lesson, and they needed more bags in doing the experiment. So, the researcher decided to use boxes instead of school bags.

One by one, students behind the screen moved sideways to the left and right side of the screen and stopped when the observer could see a half part of their body. Next, the researcher put the boxes as the representation of the students' position. After doing this activity for two pairs of students, the researcher then asked them whether we can predict the next position of the students behind the screen or not. Dewa said that we can predict the next position of the boxes and it is in line. The following fragment shows a discussion about the position of the bag is in line.

Researcher	: where is the position of the third?
Dewa	: here (pointing the position behind the second boxes).
Researcher	: why it should be here Dewa?
Dewa	: (pointing from the screen to the third position of the boxes and whispering).
Researcher	: why is it here? Say it loudly.
Dewa	: Because of this (pointing that the position of boxes is in line).
Researcher	: how about here? How about the other side?
Dewa	: (Pointing the position behind the second boxes).

Researcher : do you understand what Dewa said?
Ananda : no.
Researcher : Dewa, could you explain to your friends about your answer. Is this in line?
Dewa : So, this is in line to here (pointing the screen to the third boxes) and the third box is here. If the room is bigger, the line from here to here (pointing from the screen to the second boxes) diverges. If the room is bigger it will be over here (pointing the wall).
Researcher : it is also for the other side, isn't it?
Dewa : ya (pointing the line from the screen to the wall).

It shows that Dewa knows that we can determine the next position of the box by placing it in line with the other boxes. He also knows that they could have placed the boxes further in line behind the screen beyond the wall of the classroom. His explanation illustrates that Dewa had come to understand that those boxes were in line and could be extended as far as we want, even beyond the wall of the classroom. Unfortunately, the researcher forgot to ask him whether he could realize that those boxes were actually the vision of the observer.

Next, the researcher put the position of observer a little bit further away from the screen. The other student then became an observer and the experiment was repeated again as in the first activity. After doing the experiment, the researcher posed a question for students to compare the first and the second experiment. At first, students said that the boxes in the second experiment, the observer was further away from the screen, was larger than the first experiment. They said so because the boxes of the first experiment were omitted from the field. So, the students only made a guess without realizing the first and the second situation. Then, the researcher asked students again to compare the first and the second experiment by representing the form of boxes in those two experiments with his

hand. In this moment, students realized that when the observer farther away from the screen in the second experiment, the area between two in line boxes became smaller.

Generally, students realized that boxes representing the vision lines of the observer would become smaller as the observer moves further away from the screen. They also noticed that as the observer moves to the right of the screen, the form of the boxes also becomes smaller. However, the idea about angle did not come up during the discussion. The students realized the change of the form of the boxes as the observer moved further away from the screen, but did come up with the idea that it is an angle that makes them different. This issue should be one of the main topics in the discussion in the next cycle.

c) Coloring blind spots

Activity 3: Delimiting the hidden area of the screen

This main goal of this activity is to support students to be able to emerge the concept of angle as a delineator of the hidden area behind the screen. The teacher put up the four different diagrams, in which students had to draw the hidden area for various positions of observer, as in the playground experience. At first, all students only made some points in the hidden area behind the screen for those four different positions of observer.

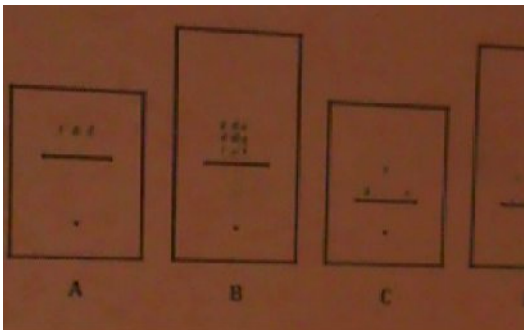


Figure 5.4. Some Points Representing the Hidden Area

This fact shows that students still think about placing objects such as the boxes in the experiment (in this activity is the dots) in the hidden area where they cannot be seen by the observer. When the researcher asked students about their answers, they misunderstand about how the hidden area (blind spots) and the vision lines may look like in the drawing. In fact, no one student can draw the vision lines of the observer. Students also misunderstand about the direction of the problem. They perceived that the term 'coloring the hidden area' in the problem is to place the dots in the hidden area. Only after asking students to color or shaded all the hidden area and explaining with the area of figure, they are able to draw the hidden area of the observer.

Figure 5.5. Students' Different Answers of Coloring the Hidden Area

In the figure above, it shows that students think the observer cannot see the strip perpendicular to the screen. The solution obtained show how difficult it is for

students to understand the concept of visual field such as vision lines and blind spots. Those students were not aware that what could not be seen was more than just the ‘strip’ which means that the concept of vision lines had not yet been acquired or at the very least was not operational here.

In response to this fact, the researcher reminds students about the experiment that they have done in the previous meeting. The researcher reminded students about the position of the boxes in the experiment and asked them to color the hidden area that they figured out during the experiment. The researcher also asked students about whether the two lines bounded the visible and invisible area were met somewhere or not. Students then answered that those two lines met in the observer and they colored the area within those two lines. This obstacle probably came up in the classroom because the students did not understand that the problem is about the top view or ground plan of the situation. In the next cycle, the understanding of top view or ground plan of situation needs to be concerned by the teacher during the lesson.

d) Understanding the vision angle

Activity 4: Discovering and drawing the vision angle of observer

The main goal of this activity is to encourage students to be able to construct and draw the vision angle of the different observer’s position. In the beginning, the students had a short experiment constructing the vision angle of observer standing in front of the classroom. There were two other students pulled the rope from the observer to the classroom representing the vision angle of the observer.

Following this experiment, students worked in groups to solve the problem on the worksheet. The problem asked students to determine the vision angle of several different observers standing in front of the classroom. The first question is about whether the observers in the position A and C can see the teacher's table or not. In response to this question, students had different answers and a discussion was organized.

The researcher: Haura, please read your answer? What is your answer?
Haura : (reading her answer in the worksheet)
The researcher: Haura, please read it loudly.
Haura : cannot see the teacher's table because the door covers our view.
The researcher: cannot see. Haura's answer is cannot see. Because the question is can the child standing on A and C see the teacher's table? Haura's answer is both of them cannot see the teacher's table. Anyone agree with Haura that both of them cannot see the teacher's table?
Radit : No.
Dewa : No.
The researcher: How about you Adinda? Please, read your answer.
Adinda : A cannot see because there is a door besides A and C is able to see because its door is on the right side but for A is on the left side. Because the teacher's table is placed on the left side.
The researcher: Ok. So, according to Adinda, position A cannot see it because, because of what? What your reason again Adinda? Read it loudly.
Adinda : because there is a door besides A.
The researcher: Ok. Who is agreed with Adinda that position A cannot see it but position C can see it?
Radit : Me.
The researcher: Radit, what is your answer? Read your answer.
Radit : A cannot see the teacher's table because A can only see the table 1. While C can be seen from the teacher's table and C can see the table 5 and 2.

From the discussion above, it seems that most of the students know that the observer in the position C can see the teacher's table while the observer in the position A cannot. This fact shows that students are able to imagine the situation

and know that the vision angle of is different for different position of observer. Although their answers are correct for the first question in the worksheet, students still made a mistake in drawing the vision angle of the observer. Most of the students drew the line from the observer's position into the classroom without touching the right and the left side of the door. It reflects that students did aware that the vision angle of the observer was bounded by the two sides of the door.

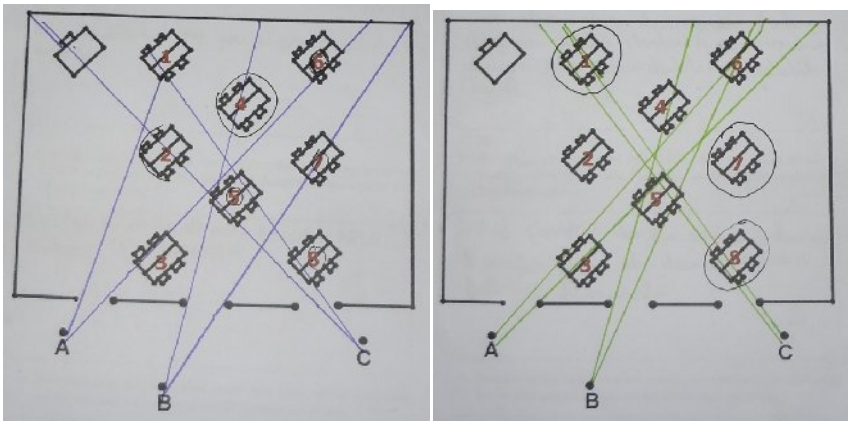


Figure 5.6. Students' Drawing of Vision angle for Different Observers

In the next question, students were asked about whether there is a table that cannot be seen by the observer in the position A or not. Most of the students said that the table 5, 7, and 8 cannot be seen by the observer because they are covered by the door. Although their answers are right, the students still make a mistake by saying that the table 1 and 4 cannot be seen by the observer in the position A. It shows that students did not pay attention to the drawing that they made.

Furthermore, the students also did not discuss about the different positions of the observer in front of the classroom. The researcher forgot to encourage the students to realize the different position of the observer and what the students can say about it relating to the vision angle. For example, they did not compare the

observer's position in A and B and its relation to the vision angle. They did come up to an understanding that as the observer gets closer to the door, the vision angle becomes bigger and vice versa. This important issue should be discussed in the next cycle.

e) Playing with Paper Fan

Activity 5: Discovering the zero, acute, right, straight, obtuse, and reflex angle and its different orientation.

In the last activity, students already started to develop their understanding of the concept of angle. As a follow up activity, they were going to find out the types of angle such as the zero, acute, right, straight, obtuse, and reflex angle as well as its different orientation. The students were facilitated to find out those types of angle and its different orientation by playing paper fan.

Starting this activity, the researcher introduced the paper fan to the students. Next, the paper fan was used to introduce some angle terminologies and to raise a number of questions, particularly about reflex angle (which had not yet been mentioned because they were not in the physical situation). The researcher opened the paper fan little by little and asked the students how the angle would change. The researcher continued to open up the branches of the paper fan until they formed a right angle. Although it is a common angle for students, not all of them were quickly noticed that it is a right angle. Some of them said that it was an obtuse angle.

Then, the researcher continued to spread the branches apart and asked the students whether there is an angle or not. When the branches formed an obtuse

angle, most of the students could recognize it. As the researcher open the branches even further apart and formed a straight angle, the students came up with different opinions and the discussion set in.

Researcher	: Now, I open it again (spread the branches until formed a straight angle).
Students	: Obtuse (angle).
Dewa	: a half of circle.
Researcher	: (take a stick and showing to the students) how many angles for this?
Students	: no, nothing.
Researcher	: how about this, then (showing the paper fan again)?
Students	: there are two (pointing both the end side of the branches).
Researcher	: (showing the right angle with paper fan) how about this? Right angle?
Students	: yes.
Researcher	: and if I open it (open the branches until form a straight angle), what is this? Is there any angle for this?
Students	: nothing.
Researcher	: why? You said this (paper fan form an obtuse angle) has an angle, but this (paper fan form a straight angle) is not, why?
Radit	: because this one (pointing the paper fan form an obtuse angle) is a little bit slanted, whereas this one (pointing the paper fan form an obtuse angle) is straight.

This fragment above illustrates how the students were struggling in grasping the notion of straight angle. They realized that the two branches were in a straight line but they could not see that there is an angle in between those branches. Perhaps, it was due to the misconception students have regarding the concept of angle that there is angle if the two lines meeting in one point are slanted to one another. They also did not know about what is the angle exactly, is it the vertex, the area in between two lines or all together? Only after the researcher pointed out the paper in between branches the students could see that there is an angle when

the paper fan is flat and it is called straight angle. This important issue should be one of the main concerns in the next cycle.

5.2.3. Remarks on Students Knowledge in the Post-Test (1st cycle)

The researcher gave a post-test to the students in the end of the learning process in the first cycle. This test served as clarification of the students' knowledge development that had been observed and analyzed in the learning process of the first cycle. This test was not intended to be compared with students' pre-knowledge before coming to the first cycle. Most of the questions in the post-test were similar with questions in the pre-test. There is an additional problem in the pre-test asking students' reasoning about whether there is any different between what they see if they move closer of further away from the window. The purpose of this change is to reveal the students' reasoning and thinking about their understanding of vision lines and blind spots. The picture in the question 5 was changed from rectangle to triangle. There is also an additional problem for the last question asking about the types of angle that the drawing showed in the worksheet and about the students' reasoning about it. It is aimed to know whether students are able to know some types of angle or not. According to the results of post-test, the researcher noted several important remarks as the following:

a) The initial understanding of vision lines

Given tow pictures of someone looking out of the window (close to and far away from the window), students determine which picture that they can have a bigger view than the other. By answering this problem, it shows that students start

to realize the important of line of sight in determining which picture that have a bigger view.

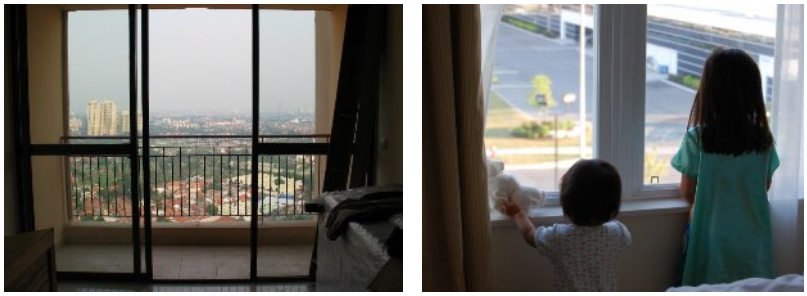


Figure 5.7. Two Different Views of Looking Out of the Window

The students could mentally imagine how their view would be look like if they were at those two positions. Comparing those two pictures could promote students' visualization ability toward different situations. Starting to discuss about why in these two situations of looking out of the window can have different view can raise the need of knowing about angles, particularly about vision angle.

b) Recognizing straight, right and one circle angles

The students already developed their initial understanding of classifying angles from the activity with paper fan. In the post-test, they were given three objects and asked to determine whether there is an angle in those objects or not. Although students have learned about one circle angle, most of them said that there is no angle in the circle and straight objects. It shows that students still perceived that angle has to have two lines in which one line is slanted to another. But, it is quite easy for students to recognize right angle. All of the students were able to see that the square object has four right angles. We assumed that it is because right angle is a common angle for students.

c) Finding the examples of angle in their surroundings

The other problem in the post-test was asking students to list some of examples of angle that students recognize in their surroundings. Unfortunately, since students quite familiar with the right angle, almost all the examples mentioned by the students were the examples of right angle. Only one student mentioned the example of acute angle. The students might also be confused in finding the example of angle besides right angle because there are not so many things in the classroom that have acute, straight, obtuse or one circle angles.

5.2.4. Conclusion of Students' Learning Process in the First Cycle

Based on the observation in the preliminary teaching, we can draw conclusion that learning the concept of angles by exploring the notion of vision lines had several obstacles and mistakes. In the first activity, the students worked with the realistic problem about the cat looking through the mice hiding behind the jar. In that activity, the students showed their spatial visualization and spatial reasoning ability of grasping the notion of vision lines informally by drawing connecting lines that went from the eye of the cat to the mice. They also were able to recognize the difference between the panoramic drawing and the top view of the situation. But, they did some mistakes in understanding how the vision lines could change as the cat moved closer to or far away from the jar. Their drawing also did not represent the intended vision lines that bounded the visible and invisible area of the cat. It shows how difficult for the students to imagine the situation and draw it into the paper.

In the other activity, the students were able to construct the vision lines of the observer in the visual field activities. They knew that the vision lines were in the straight line and could be extended as far as desired. However, this activity did not elicit the concept of angle among the students. It means that the concept of angle did not emerge in this visual field activity. The students were also struggling in drawing the hidden area (blind spots) for different positions of observer.

In the activity of finding the classification of angles through paper fan, the students were struggling in conceiving the straight angles. They tended to perceive that there was no angle when the paper fan was flat because they could not see the two branches or there were no bend or curve. This issue became one of the main concerns in the teaching experiment. In general, although the students were struggling in learning the concept of angle during the preliminary teaching, they showed greater progress from the first time they worked with the problems.

5.2.5. Improvement of the Hypothetical Learning Trajectory

There were some revisions based on the remarks and insight from the first cycle that was made to improve the HLT. These improvements were made either for practical or didactical reasons. This improved HLT was implemented in the second cycle.

For the pre-test and post-test, we made some revisions. We changed the problem involving the notion of vision lines from asking about the view of looking out of the window into asking about the view of looking to the ship from the small rowboat. We reformulate the questions in the pre-test and post-test as well. The results of post-test in the cycle one showed that the given problems did

not assess the whole learning series. It can be said that the problems were quite general and need to be more specific and the given pictures were quite abstract.

For the problem about the cat looking to the mice hiding behind the jar in the worksheet 1, we changed the question 2. We omitted the picture of spider and changed with the panoramic drawing of the cat looking to the mice. This revision was based on the remark in the first cycle that some students misunderstood the presence of spider in the worksheet.

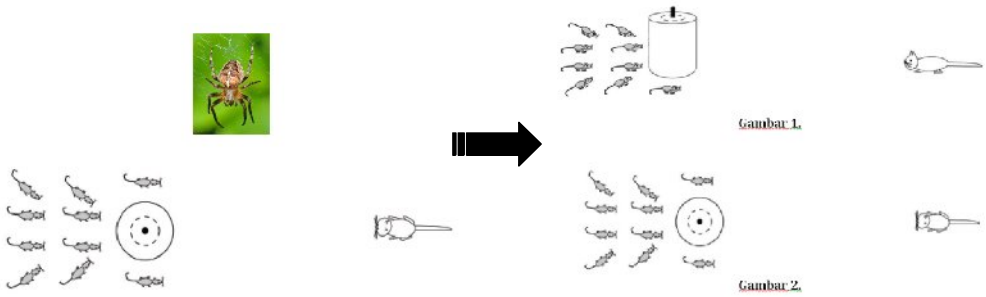


Figure 5.8. From Left to Right: the Change of Worksheet in the Problem 2

In the last activity of determining the types of angle from the pictures of paper fan, we added a space for students to write down their answer that was missed in the worksheet of the first cycle. Since the students had some initial understanding about the degree of angle, we added one activity of measuring angles by using unit of 10 and 50 degree angles. We gave the students some small papers with 10 and 50 degree angles and asked them to use it in order to measure the degree of several types of angle.

Considering the different number of students from the 1st cycle to the 2nd cycle, in which the 2nd cycle consisted of 38 students, we decided to rearrange the group activities so that all the group members could be involved in the learning

process. We also changed the social norms of the classroom from asking students' answer one by one to conducting classroom discussion.

5.2.6. Remarks on Students Knowledge in the Pre-Test (2nd cycle)

There were 38 students who were given the pre-test in the second cycle. There were some changes made in this pre-test as an improvement of the pre-test from the first cycle. The result of the pre-test is analyzed in order to investigate the present relevant knowledge and to know the starting points of students about the concept of angle.

Generally, the students' pre-knowledge in this second cycle was more or less the same with the students' pre-knowledge in the first cycle. Some students were having a sense of vision lines and vision angle. They worked with the problem depicting a small rowboat rowing toward a ship and asked why the captain of the ship cannot see you in the rowboat. With this problem, students realized that the shape of the ship affect their field of vision. So, in some spots, the captain was not able to see the small rowboat. The pre-test also reveals that most of the students did not recognize the 0^o, 180^o, and 360^o angles. Their concept image of angles excludes the turning aspect of angle and therefore it was difficult for them to conceptualize 180^o and 360^o angles. Most of students were also able to recognize things in their life that have an angle such as a door and a table. But, mostly the examples that they mentioned is for a right angle as it is a common angle for them. These findings show that students have difficulties in dealing with the concept of angle.

5.2.7. Teaching Experiment (second cycle)

a) Investigating real situations involving vision lines and blind spots

Activity 1: Determining which mice can and cannot be seen

For the problem 1, all of students were trying to answer the problem by determining the number of mice that the cat can see. Some students argued that the cat can only see the three mice beside the jar because the others are hiding behind the jar. But, there are also some of them said that the cat can see five mice, three on the right, and two on the left side of the jar. It shows that students were able to positioning themselves in the eye of the cat and recognizing that some mice can be directly seen by the cat. The fragment below depict the moment when the researcher asked one of students about their answer and drawing of this problem.

Researcher	: what does the line mean, Ikhsan? (pointing the line drawn by the student).
Ikhsan	: Its vision, sir. Its vision is impenetrable by only looking to this side (pointing the line that goes to the jar). So, it can only see three (pointing the three mice besides the jar).
Researcher	: Others? (pointing the other mice).
Ikhsan	: Others are invisible because they are behind this (pointing the jar), behind the jar.
Researcher	: Behind the jar.
Students	: (laughing)
Reseacher	: How many lines from the cat? How many lines?
Ihksan	: I do not know.
Researcher	: How many lines did you draw?
Ikhsan	: Three.
Researcher	: How many mice touched?
Ikhsan	: Three.
Reseacher	: How about the other mice? (pointing the other mice).
Ikhsan	: Untouched (by the line), because of this (pointing the jar).
Reseacher	: So?

Ikhsan : they cannot be seen by the cat. Because the cat stops here (pointing the cat).

Furthermore, most of the students were able to draw lines with different form that represent the vision of the cat. Although they were able to make an inferential reasoning for answering the problem, they still make a mistake in their drawing. Probably, their drawing is not the vision lines of the cat, but just a line that proves the cat can see certain mice as they drew those lines for each of the mice.

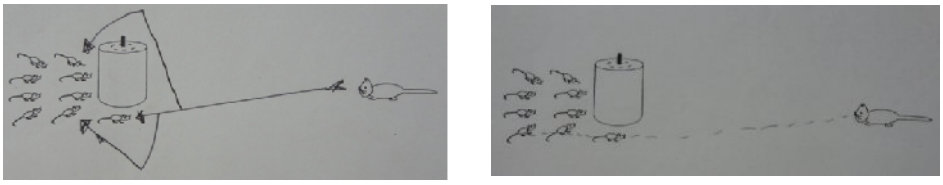


Figure 5.9 Students' Drawing of the Vision Lines of the Cat

In the problem 2, students tried to compare the side view (picture 1) and the top view (picture 2) of the situation. Some of the students were able to recognize that those two pictures are different in terms of the number of the mice that the cat can see. In the first drawing, the cat can only see three mice, but in the second drawing the cat can see six mice. There were interesting discussion between the researcher and the focus group regarding their opinions about comparing the pictures. They were struggling in understanding the second picture showing the top view of situation. The transcript below quoted from it.

Researcher : how many mice that the cat can see in this picture (pointing the second picture)?
Rahma & Diendha : six.
Researcher : you all agree that there are six? How many mice that the second cat (the can in the second picture) can see?
Aafiyah : many. (then change her answer) only two.
Suci : (pointing the mice beside the jar) these (two) are in the

outside (of the jar). The jar is in the front (of the mice). So, these (mice) cannot be seen (by the cat).

Aafiyah : in this (second picture), the spider is in above, so it can see six mice.

Suci : So, it means all (mice) can be seen, if the spider is in above, all (mice) can be seen (by the spider).

In this focus group, the students have different answers among them. Rahma and Diendha said that there are six mice that the cat can see in the picture 2. But, Suci said that there are only two mice since the others are covered by the jar and the two mice in the front. Aafiyah seemed to misunderstand the existence of the spider in the problem. It reflects that understanding the top view of situation is quite difficult for students. Although, Rahma and Diendha said there are six mice that can be seen by the cat in the picture 2, they cannot give any reason why they said so. Suci tends to think that the two mice in the front blocking the view of the cat so that the other four mice behind cannot be seen by the cat. It is because Suci did not draw the vision lines of the cat, as she did in the first drawing, that can show her whether the two mice in the front blocking the view of the cat or not.

However, the students' work in the worksheet reveals another fact that was not recorded in the video. The two students in this focus group (Rahma and Diendha) realized that the picture 1 and picture 2 were different because there were nine mice in the picture 1 and ten mice in the picture 2. So, they actually noticed that there was one mouse missing in the picture 1. But, they did not come up with any reason why this thing happened. It seems that students treated these two pictures to be different by only counting the number of the mice for each picture.

In general, some students in the focus group started to develop their spatial visualization sense in comparing the two pictures but they were struggling in finding an explanation about what they have noticed from the problem. The students seemed to think that those two pictures were different because there was no a statement in the problem saying that those pictures were taken from the same situation. It raises the need to revise the problem.

In the problem 3, some students said that they agreed with the opinion stated in the problem. They argued that as the cat gets closer to the jar, the cat see less mice. Even, if the cat moves very close to the jar, the cat cannot see the mice. But, other students have different opinion and disagree with the opinion stated in the problem. Some of them argued that if the cat gets closer to the jar, the cat should be able to see more mice. Although students' answer were different, it seems that they realized that when the movement of the cat can affect the number of the mice that the cat can see.

According to this activity, we can realize that most of the students had a sense about the notion of vision lines and blind spots. They know that the vision of the cat is different if the cat move closer or farther away from the jar. They also tried to make a drawing of the vision of the cat. But, since vision lines cannot be seen in a reality, it is understandable that students were struggling to put it in a drawing. By doing visual field activities on the playground, constructing the vision lines of the observer, the students might see that the vision lines intended to be discussed are the ones that bounded the visible and invisible area. Furthermore, when students construct the vision lines, the notion of angle might emerge since

students can see that the vision lines change as the observer moves into different position.

b) Constructing the vision lines and blind spots

Activity 2: Constructing the vision lines and blind spots of different position of observer

In this activity, the teacher encouraged students to conduct visual field activities of constructing vision lines and blind spots of observer. The teacher explained to the students that they were going to construct the situation of the cat and mice problem in the last meeting. The teacher put a screen and a chair on the school playground and asked one group of students to be an observer. A student-observer sat in the chair facing the screen and the other students took their school bags lined up behind the screen where they could not be seen, approximately in the middle of the screen.

Figure 5.10. Conducting the Visual Field Activities

One by one student in pairs moved sideways from behind the screen until the observer could see them and then they put their school bag down on the location where they stopped. After doing this activity, the teacher posed questions about whether they could predict the next position of school bags. This question is

important to be asked in order to see whether the students could make a prediction or conjecture about what they just have done. Furthermore, this question might reveals whether students realize the pattern of the bags and their reasoning of why they put it on the certain place instead of others. Most of the students said that they could do that and then put their school bags in line with the others. One of the students used her hand to see where she should put the school bags so that it is in line with the others.

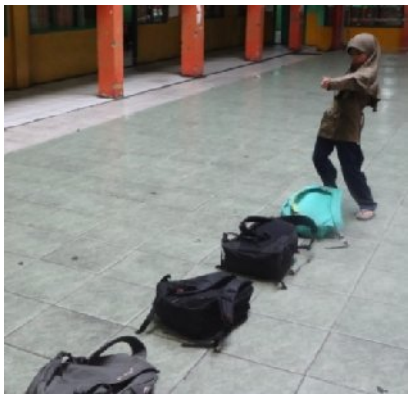


Figure 5.11. Student's Strategy in Predicting the Next Position of the School Bag

Students then observed the position of the bags and realized that they were "diverging" or "in line". The teacher proposed to use a rope to check and verify their answer by laying out two ropes along the bags. Following this activity, the teacher propose another question to the student asking about whether there is any difference if the number of school bags are different for the same observer position. The episode below captured the discussion between the teacher and students about this issue.

Teacher : now, I asked you. If those bags are (only) three and I put ten (bags), is there any difference (between them)?
Students : there is.
Teacher : where is the difference?

Zelvan : in the left and the right side.
Teacher : what is the difference?
Suci : big (showing her hands representing the form of the experiment).
Students : big and small.
Teacher : If they are a few, how is the position?
Students : small.
Teacher : If they are many?
Students : big.
Teacher : going where? On the back?
Students : yeah (nodding their head indicating agreement)
Teacher : If they are a few it is small, if they are many it is bigger. So, what is that? (showing her hands representing the form of the experiment) what is it representing?
Suci : vertex.
Teacher : where is the vertex?
Students : here (pointing with different direction).
Teacher : where?
Students : here (pointing the observer's position).
Rahma : the chair.
Teacher : in the chair, are you sure?
Suci : (nodding their head indicating agreement)

It seemed that students tended to think about the area in between the two bags so that they perceived that three bags and ten bags were different for the same observer position. They noticed that as there were more bags, the arrangement of the bags became longer and the area in between became wider, as well. It is actually the common misconception about angle that most of students generally have in mind by perceiving that the length of the arms (arrangement of bags in both sides) affect the size of angle.

In the discussion, the students also came up with the notion of vertex. In Indonesian language, vertex literally means angle point. They said "angle point" to describe that the two arrangements of school bags were met in one point that is

the observer position. So, basically, the notion of angle was emerged during this discussion in which students already noticed that the hidden area bounded by the school bags was delineated by angle. Although, students could come up with angle during this activity, they still hold the misconception about it perceiving that the size of angle depends on the length of its sides. Perhaps, it is due to the fact that the angle that students built in this visual field activity is a sector angle (static sense of angle). They were only paying attention to the area of the blind spots in between two ropes. These observations raised the need for adding another activity of working with turning angle (dynamic sense of angle) that may facilitate students to deal with this misconception.

Next, the teacher asked students to use the rope as a tool to validate their answer that the school bags were in the straight or oblique line. Following this activity, the teacher then encouraged students to conduct the same experiment of constructing the vision lines of the observer, but in the different positions (closer to and far away from the screen). The following transcript shows the discussion between the students, teacher and researcher about what the students noticed from the experiment.

Teacher : the comparison, between the form of school bags now and in the first (experiment). Where is the comparison? If in the earlier (the first experiment) the position (of the observer) is closer to (the screen), how is the form of school bags? But, now the position (of the observer) is farther away from the jar (the screen), how is the form of the school bag?
Suci : smaller.
Teacher : smaller. Where is smaller? Which one?
Students : (whispering) its form (the school bags).
Teacher : the form of the position (of the school bags).

Researcher	: How is the comparison between the form of school bags in the earlier (experiment) and now?
Nazir	: smaller.
Teacher	: the position of school bags is smaller (than before). But, in the earlier is bigger.
Suci	: (nodding her head indicating agreement)
Students	: yes.
Teacher	: because of? Why?
Suci	: the jar (pointing the screen) is far away (from the observer).
Teacher	: because the jar is far away. But, in the earlier the jar was closer.
Suci	: (nodding her head indicating agreement)
Students	: yes, mom.
Teacher	: if the jar is closer to (the observer), the position of the school bags is more?
Students	: farther away.
Teacher	: farther away means?
Students	: bigger.
Researcher	: how is the form of school bags, if this (pointing the screen) is far away from the chair (observer)?
Students	: smaller.
Researcher	: if it (the screen) is closer to the chair?
Students	: bigger.
Researcher	: what does bigger and smaller (mean)?
Rahma	: angle point (vertex).

It seems that students already have noticed that as the observer moves (closer to or far away) from the screen, the form of the school bags is also changed. It shows that the students, after doing this experiment of constructing the vision lines for different positions of observer, started to develop their spatial sense of vision. Furthermore, they noticed that it is the angle point (vertex) that delineated the smaller and bigger form of the school bags. However, this visual field activity still did not help students in dealing with the most common misconceptions about angle namely that the size of angle is determined by the length of its arms.

c) Coloring blind spots

Activity 3: Delimiting the hidden area of the screen

In the third meeting, the teacher put up two problems with different diagrams depicting the ground plan of the situation in the visual field activities. Then, the students were asked to color or shade the blind spots (hidden area) for various positions of observer, as in the visual field activities.

In the first problem asking to color four different diagrams, students in the focus group were struggling to color or shade the hidden area bounded by the vision lines. It was due to the fact that the students did not use or draw the vision lines. So, they directly shaded some areas behind the horizontal line (representing the screen). They did not really understand that they had to have color or shade all the hidden area, not only the one that close to the horizontal line. The fragment below shows the discussion between researcher and the students in encouraging them to understand the problem.

Researcher : yesterday, you had done the experiment. Yesterday, you sat down here (pointing the points representing the observer positions). Now, which areas you cannot see? If you are here (pointing the first position of observer), which areas you cannot see?
Rahma : here (pointing the area near the horizontal line)
Researcher : if here (pointing the area a little bit far away from the horizontal line), can you see it?
Rahma : no.
Researcher : so, it is also (included). Now, shade all the areas you cannot see?
Rahma : until here (pointing the area far away from the horizontal line)?
Researcher : ya.

The students in this focus group did not draw the vision lines that bounded the visible and invisible area. They directly made small boxes for area that could not

be seen by the observer. Perhaps, those students imagined the visual filed activities in which they used school bags to represent the vision lines of the observer. So, their drawing might represent the school bags placed on the hidden area (blind spots).

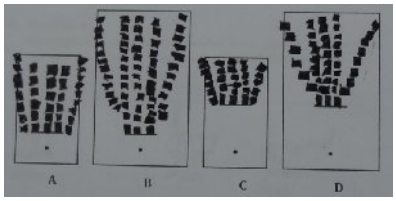


Figure 5.12. Student's Drawing of Hidden Area (Blind Spots) in the Focus Group

The students in this group also noticed the similarities and differences among those four diagrams. They notice, for example, that the distance between the observer and the screen is different among the diagrams. The transcript below shows the discussion between the researcher and one of the students in the focus group about her understanding of the diagrams.

Researcher : are the picture A and B the same? Or different?
Yulianti : same.
Researcher : what makes them the same?
Yulianti : the screen and the observer are closer (to each other).
Researcher : is there any difference or not?
Yulianti : yes, there is.
Researcher : what is the difference?
Yulianti : this (pointing the different lengths of its sides). This one is high (pointing the sides of the picture B) and this one is not (pointing the sides of the picture A).
Researcher : what about picture B and C? Do they different?
Yulianti : yes.
Researcher : what is the difference?
Students : big and small.
Yulianti : this is one is farther (pointing the distance between the screen and the observer in the picture C). This is one is closer (pointing the distance between the screen and the observer in the picture B).

Although, the students were able to see the difference between the diagrams, they did not make any relation between the differences and its implication to the vision of the observer. They could not arrive to the conclusion that as the observers get closer to the screen, the hidden area becomes wider.

However, we also saw other drawings that made by the students. Even some of them were able to draw the vision lines and shade the blind spots of different observers. So, together there are at least four types of drawings from the students.

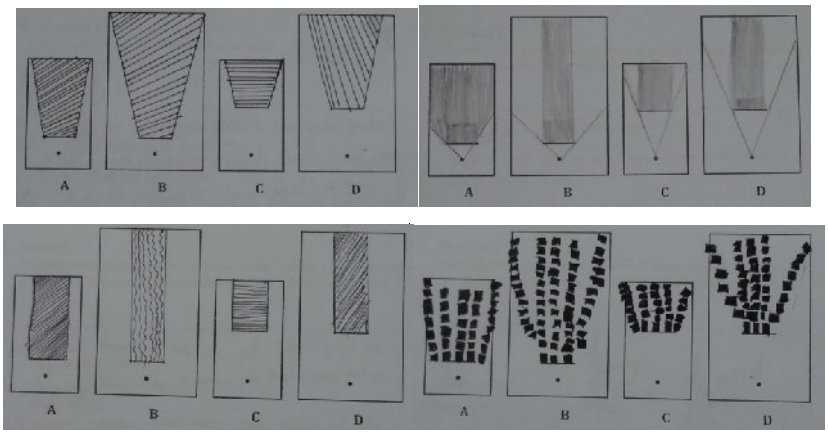


Figure 5.13. Student's Drawing of Hidden Area (Blind Spots)

Some students answered correctly by drawing the vision lines that went through the end sides of the screen and met in the observer's position. There were also some of them drew the vertical lines perpendicular to the screen (strip solution). The other students drew oblique lines that went through the end points of the screen but did not go through the observer and a few students drew boxes (the representation of school bags) as the visualization of the visual field activities that they did in the previous meeting. The variety of answers among students shows how difficult it is for them to grasp the concept of vision lines and blind

spots. It also describes their struggle in making a shift from the experiencing activities (visual field) to the imagining activities (make a drawing).

In the second problem, the students were asked to color three different diagrams. The diagrams were different in which the observers have different position from the left, center and right side of the screen as shown below:

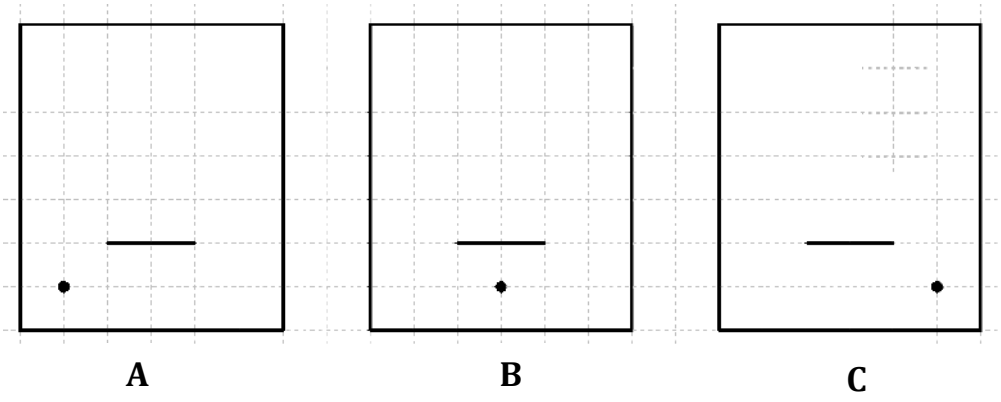


Figure 5.14. The Top View of Different Positions of Observer

In this problem, students were struggling to color the hidden area of the observers who sit down on the left and the right side of the screen. Although, they understood the problem that they had to color or shade the hidden area, they were struggling to draw the vision lines that bounded the visible and invisible area. The episode below captures the discussion between the researcher and one of the students from the group seven. It shows how the researcher was trying to guide the student to draw the intended vision lines.

Researcher	: what are these lines (mean), Nanda?
Nanda	: these ones (pointing the vision lines from the observer to the screen)?
Researcher	: ya.
Nanda	: the ones that can be seen, sir.
Researcher	: here (pointing the area outside of the vision lines), can it be seen?

Nanda : ya.
Researcher : what about here (pointing the area inside of the vision lines a little bit far away from the horizontal line)?
Nanda and her friend : no.
Researcher : why you only shade this area (pointing the area near the horizontal line shaded by her)
Nanda : oh, ya (realizing her mistake and directly shade all the area in between the vision lines). All of them.
Researcher : ya. Nanda, in the (picture) A, the (vision) line that goes from here (pointing the observer) is this? Is that the only one (pointing the vision line from the observer to the screen)?
Nanda : (starting to think about it).
Researcher : Here, (pointing the picture B) there are two (vision lines) that you made.
Nanda : ya. So, here (picture A) as well, sir.
Researcher : ya. Where the other line for (picture) A?
Nanda : this one (pointing any line). This one (pointing line from the observer to the screen).
Researcher : ya.
Nanda : (want to make a horizontal line that goes from the observer).
Researcher : (directly reply) no. here (pointing the vision lines in the picture B) there are only two (lines) that go to the screen. In that (picture), how many (lines) that go to the screen?
Nanda : (thinking for a while) three.
Researcher : here (pointing the picture B) there are only two (lines) that go to the screen. Here (pointing the picture A), how many (lines) that go to the screen?
Nanda : one.
Researcher : where is the other, then?
Nanda : oh, ya (realizing her mistake). (after thinking for a while), here (pointing horizontal line from the observer) sir?
Researcher : the one that touch the other side of the screen. Here (pointing the picture B), there are (lines) that touch the left and the right side of the screen. Here (pointing picture A), (the line) only touches the left side, where is (the line) that touches the right side (of the screen)?
Nanda : this one, sir (making horizontal line from the observer).
Researcher : but, does it touch the screen?

Nanda : no.
Researcher : the one that touches the end side of the screen.
Nanda : like this (putting the ruler from the observer to the end side of the screen)?
Researcher : ya.

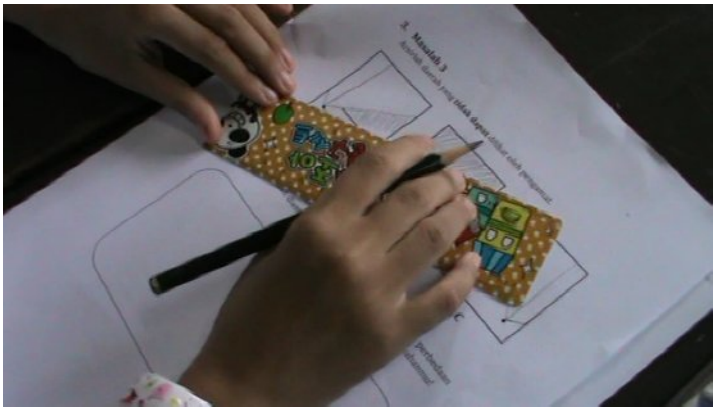


Figure 5.15. Student Draws the Vision Lines in the Second Problem

Although, the students in this group were able to draw the vision lines for the second drawing (observer sit in the center in front of the screen), they could not draw the intended vision lines for the observers who sit on the left and the right side of the screen. Only after the researcher asked them several questions to guide them, they were able to draw the intended vision lines that bounded the visible and invisible area in the problem.

In general, it can be concluded that in the third meeting, students were struggling to draw the vision lines and blind spots for different positions of observer. Their drawings were different to each other depending on their understanding of vision lines and blind spots from the previous meeting. For some students, their thoughts were much more related to the visual field activities by drawing small boxes representing the school bags. Whereas, for others, they could

draw lines and color the hidden area (blind spots) but the lines did not meet in the observer.

d) Understanding the vision angle

Activity 4: Discovering and drawing the vision angle of observer

In the fourth meeting, teacher encouraged students to construct and draw the vision angle of different position of observer. The teacher asked one of students to be an observer standing in front of the door looking into the classroom. Then, two other students constructed the vision angle of the observer by pulling a rope from the student-observer into the classroom where this observer could see a half part of their bodies. After doing this kind of activity for different position of the observer (closer to or far away from the door), the students recognized that as the observer gets closer to the door, the area in between the ropes (i.e. vision angle) is getting bigger and vice versa.

Figure 5.16. Constructing the Vision angle of the Observer

Following this activity, the teacher then gave a worksheet for students depicting the ground plan of the classroom. Then, the students were asked to draw the vision angle of the different observer's position in front of the classroom. Some of students were able to draw the vision angle correctly. The fragment

below shows a discussion between the researcher and student from the group one about his answer for the first problem.

Ikhsan	: (explaining to his friend) (touches) the end side of the door can see until here (pointing the end side of the vision lines of the observer). Teacher, he can see. Here (pointing the table 1), here (pointing the table 2).
Researcher	: can the table 2 be seen (by the observer)?
Ikhsan	: it can be seen.
Researcher	: it can be seen. How about table 5?
Ikhsan	: no.
Researcher	: why not?
Student	: because it is covered.
Ikhsan	: because it is covered by this (pointing the wall of the classroom). It is blocked.
Researcher	: how about the teacher's table? Can it be seen?
Ikhsan	: yes, it can be seen. Straight (drawing a straight line from the observer to the teacher's table).

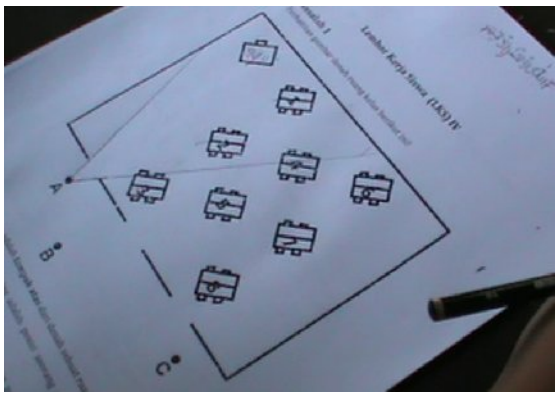


Figure 5.17. Student's Drawing of Vision Angle

Form the discussion above, it can be seen that the students in this group were able to draw the vision angle of observer correctly. They also recognized the visible and invisible area from the drawing. It implies that the students had the ability to visualize and manipulate mental images involving the concept of vision angle.

Although, the students had the activity of constructing the vision angle in the classroom, most of them made an incorrect drawing. It shows that drawing an abstract geometrical concept such as vision angle is quite hard for students. They also made a mistake in answering the question about which desks that can be or cannot be seen by the observer. The fragment below shows how the student in the focus group was struggling to determine which desk that can be seen by the observer.

Researcher : the desk 5, who (observer) can see it?
Rahma : B and C.
Researcher : why?
Rahma : I do not know.
Researcher : let's see the picture. Where is the desk 5?
Rahma : here (pointing the desk 5 in the picture).
Researcher : who can see it?
Rahma : here B (pointing the straight line from observer B) and here C (pointing the straight line from observer C).
Researcher : why it (desk 5) can be seen?
Rahma : I do not know.
Researcher : is it inside or outside of the lines (rope)?
Rahma : inside.
Researcher : if it is inside, can it be seen or not?
Rahma : can be seen.
Researcher : how about outside?
Rahma : no.
Researcher : so, it can be seen because?
Rahma : I do not know.
Researcher : is it inside or outside?
Rahma : inside.
Researcher : so, it can be seen if?
Rahma : inside the rope (line).

So, we may conclude that the students know that the vision angle is different for different position of observer and it is getting bigger as the observer comes closer to the door or vice versa but they are struggling in making a drawing. Some

students were also struggling in making their reasoning about which desk that can be seen by the observer, although they made a correct drawing of vision angle. This fact implies that making a correct drawing does not automatically shows that the students also understand their drawing.

e) Playing with Paper Fan

Activity 5: Discovering the zero, acute, right, straight, obtuse, and reflex angle and its different orientation.

The last activities in the previous lessons about constructing vision lines and understanding vision angle emphasize the idea of sector angle. This way of thinking involves a static sense of angle. If only introduced a static sense of angle to the students, they run the risk of grasping only this conception when dealing with angles. Accordingly, the teacher introduced the paper fan that emphasises a dynamic sense of angle. Using this paper fan, students might view angles as inclination of one direction with respect to another, or space between two directions. As students manipulate this instrument, open and close the branches of the paper fan, this instrument might promote a conception of angle in terms of openness.

Starting this activity, the teacher introduced the paper fan to the students. Next, the paper fan was used to introduce some angle terminologies and to raise a number of questions, particularly about straight and one circle angles (which had not yet been mentioned because they were not in the physical situation). The teacher opened the paper fan little by little and asked the students how the angle would change. As the teacher opened the paper fan and formed acute angle, most

of the students were able to recognize it and pronounced that it was an acute angle. The fragment below shows how students directly recognized the actue angle in the paper fan:

Teacher : (showing the paper fan) what angle for this?
Students : acute (angle).
Teacher : who said it is acute (angle)?
Students : (raise their hands)
Teacher : now, I ask you. Is there any thing or object in this classroom that form acute angle?
Students : there is. Ruler.
Teacher : who knows?
Students : (pointing one side of the ruler) this.

Figure 5.18. Students are pointing the acute angle in the ruler

Following this discussion, the teacher introduced acute angles in different orientation and then asked some students to draw the different forms of acute angles in the blackboard.

Figure 5.19. Students draw acute angles in different orientations

The teacher continued to open up the branches of the paper fan until they formed a right angle, which was quickly noticed by the students. Some of them pointed out the examples of right angle in their classroom such as the side of the door and whiteboard. The teacher continued to spread the branches apart, and the students directly commented that it was straight. When the teacher asked students about the form of paper fan, students came up with different answer and the discussion set in.

Teacher : Now, if I open it again?
Students : straight.
Teacher : angle?
Students : straight.
Teacher : (pointing along the paper between two branches) what we called this?
Diendha : a half of turn.
Teacher : a half of turn, right?
Diendha : (nodding her head indicating agreement)
Teacher : (asking) students in the back, what is the name of this (angle)? (showing the paper fan)
Some students : straight. A half of turn. A half of circle.

It seems that most of the students recognized that the two branches of the paper fan were in a straight line. The teacher, instead of asking whether the students really saw the existence of angle in between those branches, asked the students about the degree of the angle. Here we can say that although some of the students pronounced the word such as straight or a half of turn, they probably did not see the existence of straight angle in that paper fan. The teacher then went on to move the branches of the paper fan even farther apart. As the teacher already said about degree of angle, the students commented the form of paper fan in terms

of the degree of angle. When the paper fan formed one full circle, some of the students directly said that it was 360^{o}. Perhaps, it is because the students know that one circle is equal to 360^{o} without knowing the meaning of it.

Following this activity, the teacher gave a worksheet to the students depicting several pictures of paper fan in different forms and orientations. Students then were asked to determine what kind of angle that the paper fan formed in those pictures. The students' work in the worksheet shows that most of them were able to recognize acute, right, straight and one circle angles. But, there were some students who struggled in recognizing those angles in different orientations.

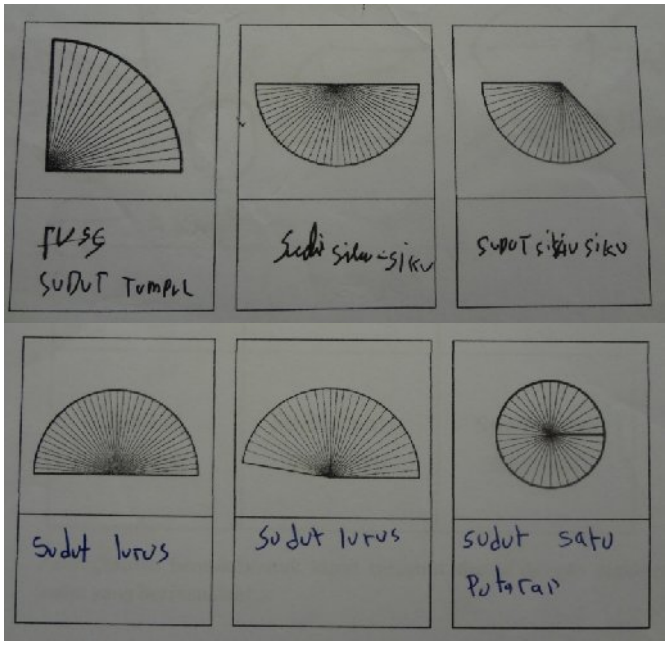


Figure 5.20. Students' mistake in recognizing the angles in the paper fan

There was also a problem in the worksheet asking the students to combine the pictures of paper fan, two or more, to form another type of angle. In this problem, most of the students came up with an idea of combining two straight angles to become one circle angles. There was also student who combined four right angles

to become one circle angle. Some of the works of students in this problem are shown in the figure 5.20 below:

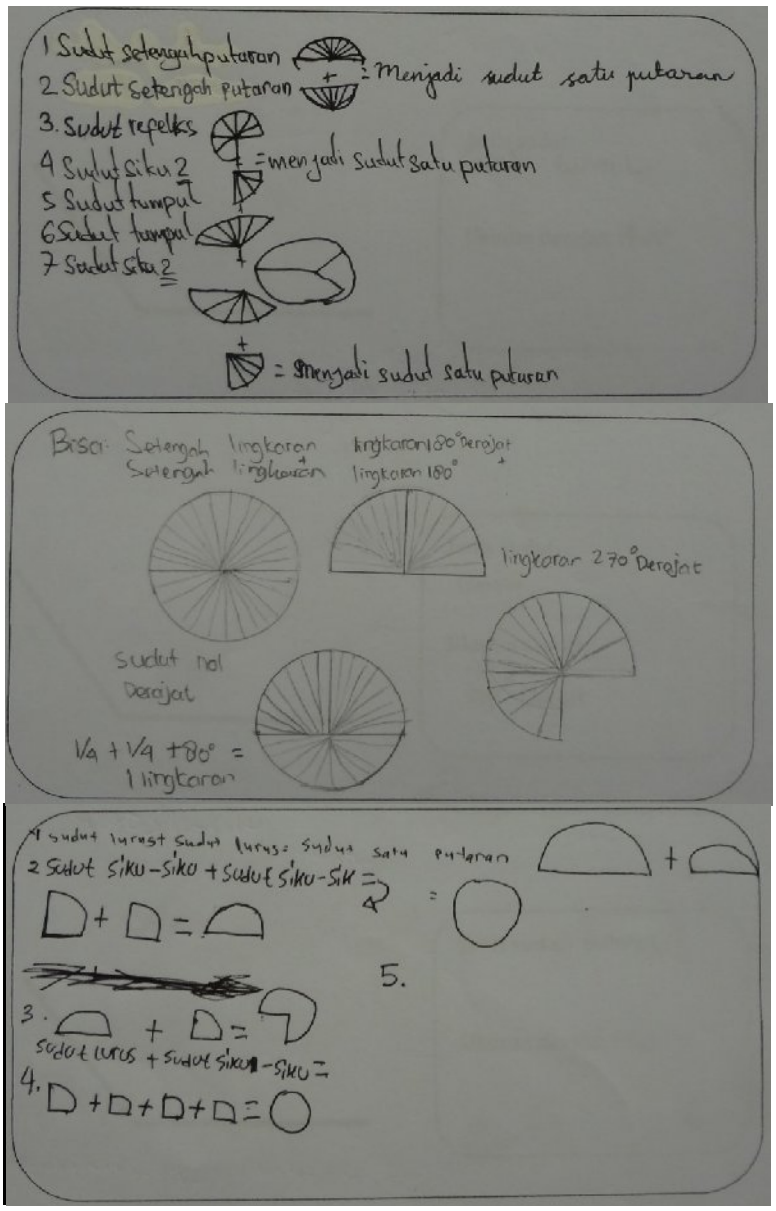


Figure 5.21. Students' answer of combining two or more paper fans

5.2.8. Remarks on Students Knowledge in the Post-Test (2nd cycle)

The researcher gave a written post-test to the students after completing the whole learning series in the second cycle. The post-test was aimed to assess students' conceptual understanding of angle due to the instructional activities given. The findings of post-test were also used to support data for drawing a conclusion of the whole learning process during the second cycle. The questions in the post-test were similar with questions in the pre-test because it was aimed to investigate the development of students' understanding after following the learning process.

There were 28 students attended the post-test, including the four students from the focus group. According to the results of post-test the researcher noted several important points as the following:

a) The initial understanding of vision lines

In the post-test, the students were given several pictures depicting a small rowboat rowing toward the ship. The students were positioned in the small rowboat. They then were asked to explain why in the certain position he/she could not see the captain of the ship. Some of the students were able to explain that he or she could not see the captain because the bow of the ship was blocking the view and covered the captain. In this sense, the students were able to imagine and positioned themselves in the situation given in the problem. Although the students could reason about vision lines in formally, no one of them was able to mention about vision lines in this problem.

By comparing several pictures of a small rowboat rowing toward the ship, the students developed their spatial visualization and spatial reasoning in terms of vision lines. They realized that their view is changed as the rowboat gets closer to the ship.

b) Conceiving the straight, right and one circle angles

As the students had learned about the classification of angles using paper fan, they were given several questions asking about their understanding of several types of angles in both paper fan pictures and concrete examples.

In the problems using paper fan pictures, some of the students were able to recognize several types of angles include: acute, right, obtuse, straight and one circle angles. Although, the students were able to recognize the straight and one circle angles, their understanding seems to be related to the paper fan context. They could not recognize those two types of angles when they were given another context differ from the paper fan. For example, it was quite often the students recognized the straight angle in the paper fan pictures, but they could see the straight angles in the stick. Perhaps, it was due to the fact that in the paper fan pictures, the students saw the area in between its two branches.

5.2.9. Conclusion of Students' Learning Process in the Second Cycle

According to the whole learning series, we can conclude that introducing the concept of angle by exploring the notion of vision lines had several obstacles and mistakes. Perhaps, it is due to the fact that linking the geometric concepts such the concept of angle with spatial visualization involving vision is not a common way of learning for the students. Although, there were some confusion at the beginning

and mistakes during the teaching and learning process, students can gain more conceptual understanding about the concept of angles and the notion of vision lines and how they were related.

Working with the realistic problem about the cat looking through mice hiding behind the jar, the students started to grasp the notion of vision lines by drawing connecting lines that went from the eye of the cat to the mice. But, their drawing of connecting lines was directly went from the eye of the cat to the mice which shows that the students could grasp the concept of vision but did not aware of the intended vision lines bounded the visible and invisible area. The different drawings presented in the problem (panoramic and top view drawings) could also facilitate students to gain experience with taking point of view. They were able to say what can be seen from a certain point or position, and what cannot. They were able to indicate whether an object (the mice) can be seen by (the cat) at a certain point or position. It means that they could imagine and describe how something is seen from a certain position.

Some of the students were also able to imagine how the vision of the cat would change as the cat moves closer to or far away from the jar. It shows that the students have developed their spatial visualization and spatial reasoning by being able to imagine how the cat's point of view might change.

When the students constructed the vision lines of the cat in the visual field activities, they recognized that the vision lines were straight lines and they were slated or diverged. They also figured out that the vision lines were changed as the observer position moved closer to or far away from the screen. Following this

discovery, they could grasp the idea that it was an angle that makes the vision lines were different by the guidance of the teacher. In this sense, the students could make a shift from experience phase to explaining shape.

Supporting the development of students' understanding toward the concept of angle, they were given a ground plan picture of visual field activities and asked to draw the vision lines and shade the blind spots. There were at least four different drawings made by the students depicting the vision lines and hidden area (blind spots) for different positions of observer. Their drawings were varying ranging from the situation-related drawing (drawing the school bags representing the hidden area) to the formal drawing of vision lines and blind spots.

Sharpening the students' comprehension of the concept of angle and the relation with the vision lines, students were given the problem involving vision angle. As in the visual field activities, the students in this lesson could grasp the idea that the vision of the observer was changed (bigger or smaller) as the observer moved closer to or far away from the door of the classroom.

Using the paper fan to introduce the classification of angles, the category of angles was composed of basic-level entities such as acute, right, and obtuse angles for most of the students. Furthermore, acute, right and obtuse angles were central elements in the classification of angles for the students. Some of the students were also able to recognize the straight and one circle angles. However, their understanding toward these two types of angle was closely related to the context of paper fan. Perhaps, it was due to the fact that in the paper fan, the students were able to see the area in between its two branches.

CHAPTER VI

CONCLUSION AND SUGGESTION

The purpose of the present study is to contribute to an empirically grounded local instruction theory for learning the concept of angles in the primary school. An instruction theory, in short, is a theory of how students can be supported in learning a particular topic, in our case the concept of angles. The contribution of the present study to such a local instruction theory is summarized in this final chapter, which consists of a reflective and prospective component. In the reflective component, we explain about the conclusion of the study drawn from the whole learning series to answer the research question and more general results relevant to a local instruction theory on the learning of the concept of angle. In the prospective component, we give suggestions and recommendations for teaching, instructional design, PMRI team and future research within the domain-specific of the concept of angles.

6.1 Conclusion

6.1.1. Answer to the research question

The main research question of this study: *"How can the concept of vision lines support the development of students' understanding of the concept of angle?*

To answer the main research question, the two sub research questions as elaborated in the end of the second chapter are answered beforehand. Both sub research questions are answered by summarizing the analysis of all activities in this study.

The answer of the first sub question

How do the visual field activities and spatial representations tasks elicit and support the development of students' acquisition of vision lines?

The first sub question is answered by summarizing a reconstruction of the hypothetical learning trajectory (HLT) on the basis of what has been learned from this study. As explained in the Chapter V of this study, the visual field activities and spatial representations could be used to encourage students to grasp the concept of vision lines and blind spots. The students' acquisition of the concept of vision lines was elicited in the context of the cat looking through the mice and was elaborated into three phases. These phases are described as follows:

1. Exploring the context of the problems

The concept of vision lines was elicited by encouraging the students to work on the problems depicting the situation of the cat looking through the mice behind the jar. The context of the problem was quite naturally introduced by the teacher by asking the students about their experience with the cat and mice. As the students work with the three problems showing the situation of the cat and the mice from different point of view (panoramic drawing and top view), they not only know what can be seen by the cat from certain position, but they also more and more capable of explaining the situation by showing their reasoning involving the concept of vision lines and blind spots. The students could use their understanding of vision lines in the situation to reason that some mice were invisible form certain point of view. In this phase, being able to explain the

geometric situation is not important. At most, the students were expected to be able to give a situation-based explanation.

Through working with these meaningful geometric problems, the students also developed their ability related to spatial visualization and reasoning. Some students were trying to draw the vision lines of the cat by making a connecting line that goes from the eye of the cat to the mice. Even though their insight is still very local and based on the specific situation, there are still some significant signs of progress in the students' development both their understanding of the concept of vision lines and their ability of spatial visualization and reasoning. The problems of the cat looking through the mice allowed the students to positioning themselves in the eye of the cat and imagine how the vision of the cat might look like. In this sense, the students learn how to take a point of view mentally and build their own spatial reasoning.

2. Experiencing the visual field activities

In order to figure out how the vision of the cat may look like when looking through the mice hiding behind the jar, the teacher encouraged students to conduct visual field activities of constructing vision lines and blind spots. These activities play an important role in supporting the students' understanding of vision lines and blind spots. These activities also allowed students to experience how the vision of the cat changed as the cat moved closer to or far away from the jar. With the help from the teacher, the students could find out that the change of vision of the cat is related to the angle concept. Accordingly, the concept of angle emerged

as the students worked on constructing vision lines and blind spots for different positions of observer.

Through these visual field activities the students also had an opportunity to experience the situation, to be "a cat", and it could elicit their skills in spatial visualization and spatial reasoning to link these visual field activities with the experience they have had in working with the contextual problems. Making constructions in the visual field activities also supported the students to go from working with two-dimensional object on the paper to three-dimensional object in the playground. In this sense, the activity offered a learning environment for them to shift from imagining with the mental object (panoramic drawing and top view) to constructing the real situation. That the students really carried out the activities causes that the understanding of the vision lines and the concept of angle sink in.

3. Making a drawing

To emphasize the students' understanding of the vision lines and blind spots, the teacher encouraged them to construct the vision lines and blind spots of different positions of observer in the two-dimensional shape (paper). It is the students themselves that had to think of ways to draw geometric shapes such as vision lines and blind spots. By working on the problems of colouring the hidden area (blind spots), the students also pointed out that vision lines were different for different positions of observer. By comparing the different drawings for different position of observer, the students also recognized that it was an angle that made those drawings different.

Furthermore, the students made a drawing of vision lines and blind spots in the ground plan or top view of the situation. In this way, the students moved toward more schematic drawing of the real situation. By allowing the students to make a drawing as the spatial representations of the visual field activities, they could develop their spatial ability and spatial reasoning involving vision lines and blind spots.

The answer of the second sub question

How can students develop the mathematical concepts of angle elaborated and supported by the understanding of vision lines?

The relation between the vision lines and the concept of angle emerge in the phase of visual field activities. The students came up with the notion of angle as they recognized that it was an angle that delimited the area in between the two ropes representing the vision lines of the observer. Furthermore, the students also, with the guidance of the teacher, came up with the conclusion that the vision of the observer changes as he or she gets closer to or far away from the screen. At this point, the teacher should guide the students to relate this conclusion with the concept of angle that they have mentioned beforehand. The teacher should led the students to build their reasoning, employing the term angle, to explain what happen with the vision of the observer when he or she is closer to or farther away from the screen. Unfortunately, the teacher did not do it in this study.

The students also developed their understanding about the notion of vision angle. By working with the physical activities of constructing the vision angle of observer standing in front of the classroom, the students started to grasp the

concept of vision angle. They understand that as the observer gets closer to or farther away from the classroom, the vision angle will also change bigger or smaller. They were also able to draw the vision angle of observer on the paper, although some of them still struggled with it.

Although the students were able to come up with the concept of angle by themselves through visual field activities and spatial representation tasks, they still could validate the common misconception of angle perceiving that the size of angle are determined by the length of its arms. Probably, the reason is that the teacher did not emphasize this issue during the visual field activities. It raises the challenge to explain to the teacher about this issue in the next research employing the visual field activities.

The students were also able to recognize some types of angle such as acute, right, and obtuse angles during the activity of playing with paper fan. Although, the teacher did not relate the paper fan activity with the notion of vision lines, most of the students were able to recognize 180o (straight angle) and 360o (one circle angle) degree angles by playing with the paper fan. However, their understanding of these two difficult angles is much closely related to the context of paper fan. When the students were dealing with the drawing of straight angle without the context of paper fan, the students tended to say it is not an angle anymore. It raises the need to add another activity that offers the students to work on these two types of angle intensively.

The answer of the research question

How can the concept of vision lines support the development of students' understanding of the concept of angles?

Learning the concept of angles by exploring the notion of vision lines could bring the students to grasp a more conceptual understanding about the angles concept. Through experiencing different activities of imagining and constructing, the concept of angle could be emerged among the students. By doing the visual field activities of constructing the vision lines, the students realized that the hidden area (blind spots) were determined by an angle whose vertex was in the observer and whose sides went from the eye of the observer to the end sides of the screen. Furthermore, the students also revealed that as the observer moved closer to or far away from the screen, the vision lines and the angle were also changed. Although it was not happened in our study, we hypothesized that these visual field activities could also help the students in invalidate the common misconception about angles that the length of the side determines the size of angle. It was the teacher that has the important role in guiding the students to invalidate this common misconception by asking right questions related to the visual field activities.

Introducing the concept of angles into the third grade students of primary school by exploring the notion of vision lines is quite different from the traditional teaching method that mostly done in schoolbook type of situation. These kinds of teaching make the concept of angles meaningful for students and break away from

the conventional teaching method that separates students from their experience with the physical situations.

6.1.2. Local instruction theory on learning the concept of angle

The aim of the present study is to contribute to the development of local instruction theory for the concept of angle in the grade three of primary school. According to Gravemeijer (2004) local instruction theory refers to the description of, and rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topic. The tool and the activities proposed in the instructional design are summarized in the following table.

Table 6.1. Local Instruction Theory for Learning the Concept of Angle

Activity	Tool	Main Goals	Description of Activity
The cat and the mice	The panoramic drawing and the top view of the situation, ruler	Students investigate real situations involving vision lines and blind spots.	- Teacher poses a worksheet that consists of three problems depicting the situation of the cat looking through the mice where some mice are hiding behind the jar. - Students with their groups discuss which mice that can and cannot be seen by the cat and build their reasoning about it.
Experiment with the screen	A chair, opaque screen, school bags, ropes,	Students are able to construct vision lines and blind spots in three dimensional representations.	- Teacher encourages students to conduct the visual field activities of constructing the vision lines of observer by using a screen and school bags. - Each group of students takes their turn to be an observer and discuss the activity that they have done.
Colouring the blind spots	Color pencils, ruler	Students are able to imagine and visualize the vision lines	- Teacher poses a worksheet that consists of several drawings depicting the situation of the observer

		and build their reasoning in terms of angle.	looking through the screen. - Students with their groups discuss about the problems and colour the hidden area (i.e blind spots) behind the screen where it cannot be seen by the observer.
Understanding the vision angle	Ropes, ruler, small paper	Students are able to construct and draw the vision angle for different positions of observer	- Teacher encourages students to construct the vision angle of observer who stands in front of the door of the classroom. - Students with their group work on the worksheet depicting the ground plan of the classroom in which they have to draw the vision angle of different observers.
Playing with the paper fan	Paper fan, some small triangles with certain degree angles.	Students understand about acute, right, straight, obtuse, reflex and one circle angle in different orientations.	- Teacher introduces the types of angle by using paper fan and promotes the students' understanding by asking them questions related to the types of angle. - Students work with their group on the worksheet about determining the types of angle for several different drawings of paper fan.

6.2 Reflection

6.2.1. The weakness point of the study

The researcher realized several weakness points during conducting this study. First, there was lack of communication between the researcher and the teacher. It was very hard to find enough time to discuss the design together with the teacher and make a consensus. It leaded to the fact that there were some significance differences between the teacher's interpretation toward the design and the intention of the researcher. There were also not so many discussions with the

teacher about the social norms and socio-mathematical norms that have to be established in the classroom. These factors seem to affect the teaching and learning process in the second cycle sometimes went a bit difference with the design in the HLT.

Second, the researcher found that it was very hard to implement the visual field activities in the classroom that have many students, for example more than 20 students. Although it could be done in this study, there were some challenges facing by the teacher and the researcher in managing the students to be focused on the activities. For classroom with more than 20 students, we suggest to divide the students into two big groups and do the visual field activities in two meetings, each meeting for each group.

Third, the students were not familiar to work on the group. Based on the interview with the teacher and the classroom observation, most of the time they worked individually or in a pair with another student. It implied that the social norm of working in a group was not yet built in this study. Consequently, it was revealed that the students seemed not accustomed to work with their friends in the group. It was quite often they work individually or in pairs with another student.

6.2.2. Reflection on the important issues

Understanding the students' thinking and their learning process were not merely the focus of this study. Several important issues were also noted during the implementation of the design. The reflection these important issues are described as follows:

Heuristic of (Indonesian) Realistic Mathematics Education

In designing and conducting the study, we employed the notion of RME theory that was adapted into Indonesian local situations. However, the instructional sequence designed in this study was only a part of long series of learning trajectories for learning the concept of angles. The proposed instructional sequence for learning the concept of angles is not a final design. It is a part of developing a local instruction theory for the concept of angles and also an early stage of adapting RME theory in the Indonesian context in the topic of angle concepts through the PMRI project. The adaptation of principles and tenets of RME in the concept of angles will take longer time than this limited study. Although, this study was limited, the proposed idea of employing phenomenological exploration such as vision lines could be used as a starting point in developing the students' understanding of the concept of angles.

Social Norms and Socio-Mathematical Norms

The study was expected to develop certain social norms or didactical contract and socio-mathematical norms that used to be established in the RME classroom. Examples of these social norms in reform math classrooms include obligations for the students to explain and justify solutions, attempt to make sense of explanations given by others, indicate agreement and disagreement, and question alternatives in situations where a conflict in interpretations or solutions is apparent (Gravemeijer, K., & Cobb, P., 2006). A socio-mathematical norm that was important in this study was that of what counted as acceptable explanation. However, these two norms were not well established in this study.

In the present study, we involved the teacher who has followed several PMRI activities such as workshop and seminar. However, her understanding of PMRI is limited to the use of model. She is also not accustomed to orchestrate a whole class discussion or proposing a guiding question to the students. Accordingly, several norms that used to be in the traditional teaching method were still exist in our classroom such as teacher gave instruction, asked closed questions and the students tried to understand the teacher and acted according to the teacher's expectation.

The establishment of social norms and socio-mathematical norms in the certain classroom takes time. Consequently, the teacher plays an important role in facilitating the development of these norms in the PMRI classroom through a process of negotiation. However, because the teacher in this study already used to the traditional teaching method of explaining all the materials to the students for many years, it was quite hard for her to develop the social and socio-mathematical norms of PMRI in her classroom. In general, it raises the need of helping teachers in Indonesia to develop these two norms in PMRI classroom in order to encourage them to move from perceiving teacher as the provider of knowledge to teacher as a knowledgeable orchestrator.

6.3 Suggestion

This study has been in search of ways to engage students in learning the concept of angles and offered them opportunities to grasp the concepts by themselves. However, as this study only focused on the concept of angles by exploring the notion of vision lines, we recommended for further study to search

and explore another context or didactical phenomenology to learn the concept of angles. This is in line with what has been proposed by Freudenthal (1983, 1973) that teaching and learning of the concept of angles should be done by exploring all the angle concepts with various phenomenological approaches. If one concept is forbidden, students will never learn to distinguish all the concepts of angles.

This study can be used as an initial study for developing a local instruction theory for the concept of angles. We recommend to the PMRI team to conduct several studies in searching the Indonesian contexts for learning the various concepts of angles. We also proposed to include the visual field activities and spatial representations tasks for learning the concept of angle in the PMRI textbook.

REFERENCES

Bakker, A. (2004). *Design research in statistics education: on symbolizing and computer tools*. (Doctoral dissertation). Utrecht: CD-β Press, Center for Science and Mathematics Education.

Bakker, A., & va Eerde, D. (2012). An introduction to design-based researcher for master and PhD students. In Biker-Ahsbahs, A., Knipping, C., & Presmeg, N. (Eds.), *Doing (qualitative) research: methodology and methods in mathematics education*. Paper submitted for publication March 15, 2012.

Cope, P. & Simmons, M. (1991). Children's exploration of rotation and angle in limited logo microworlds. *Computer Education, 16*(2), 133-141. doi:10.1016/0360-1315(91)90019-N

Doorman, M. L. *Modelling motion: from trace graphs to instantaneous change*. (Doctoral dissertation). Utrecht: CD-β Press, Center for Science and Mathematics Education.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: constructing number sense, addition, and subtraction*. Portsmouth, NH: Heinemann.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht: Kluwer Academic Publishers.

Freudenthal, H. (1991). *Revisiting mathematics education-china lectures.* Dordrecht: Kluwer Academic Publishers.

Fynn, A. B. (2010). Climbing and angles: a study of how two teachers internalize and implement the intentions of a teaching experiment. *The Montana Mathematics Enthusiast, 7(2-3)*, 275-294.

Gravemeijer, K. (1994). *Developing realistic mathematics education.* Utrecht: CD-ß Press / Freudenthal Institute.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 1,* 155-177. doi:10.1207/s15327833mtl0102_4

Gravemeijer, K.P.E., Cobb, P., Bowers, J. & Whitenack, J. (2000). Symbolizing, modeling, and instructional design. In P. Cobb, E. Yackel & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms:*

perspectives on discourse, tools, and instructional design (pp. 225-273). Hillsdale, NJ: Lawrence Erlbaum Associates.

Gravemeijer, K. (2004a). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6(2)*, 105-128. doi:10.1207/s15327833mtl0602_3

Gravemeijer, K. (2004b). *Creating opportunities for students to reinvent mathematics*. Paper presented as regular lecturer at the 10th international congress on mathematics education, Lyngby-Taarbæk, Denmark.

Gravemeijer, K., & Cobb, P. (2006). Design research from the learning design perspective. In Van den Akker, J., Gravemerijer, K., McKenney, S., & Nieveen, N (Eds.), *Educational design research*. London: Routledge.

Health, T. L. (1956). *The thirteen books of euclid's elements (vol 1)*. New York, NY: Dover Publications.

Hodgkin, L. (2005). *A history of mathematics: from mesopotamia to modernity*. Oxford: Oxford University Press.

Keiser, J. M. (2004). Struggles with developing the concept of angle: comparing sixth-grade students' discourse to the history of the angle concept. *Mathematical Thinking and Learning, 6(3)*, 285-306. doi:10.1207/s15327833mtl0603_2

Matos, J.(1990). The historical development of the concept of angle. *The Mathematics Educator, 1(1)*, 4-11. Retrieved from http://math.coe.uga.edu/TME/Issues/v01n1/2matos.pdf

Mitchelmore, M. (1997). Children's informal knowledge of physical angle situations. *Learning and Instruction, 7(1)*, 1-19. doi:10.1016/S0959-4752(96)00007-2

Mitchelmore, M. (1998). Young students' concepts of turning and angle. *Cognition and Instruction, 16(3)*, 265-284. doi:10.1207/s1532690xci1603_2

Mitchelmore, M. & White, P. (1998). Development of angle concepts: a framework for research. M*athematics Educational Research Journal, 10(3)*, 4-27. doi:10.1007/BF03217055

Mitchelmore, M. & White, P. (2000). Development of angle concepts by progressive abstraction and generalization. *Educational Studies in Mathematics, 41*, 209-238.doi:10.1023/A:1003927811079

Munier, V., Devichi, C., & Merle, H. (2008). A physical situation as a way to teach angle. *Teaching Children Mathematics, 3,* 402-407.

Munier, V. & Merle, H. (2009). Interdisciplinary mathematics-physics approaches to teaching the concept of angle in elementary school. *International Journal of Science Education, 31(14)*, 1857-1895. doi: 10.1080/09500690802272082

Ostermann, A., & Wanner, G. (2012). *Geometry by Its History*. Heidelberg: Springer.

Pramudiani, P., Zulkardi, Hartono, Y., van Amerom, B. (2011). A Concrete Situation for Learning Decimals. *Indonesian Mathematical Society Journal on Mathematics Education, 2(2)*, 215-230.

Revina, S., Zulkardi, Darmawijoyo, van Galen, F. (2011). Spatial Visualization Tasks To Support Students' Spatial Structuring In Learning Volume Measurement. *Indonesian Mathematical Society Journal on Mathematics Education, 2(2)*, 127-146.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26,* 114-145.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction the wiskobas project*. Dordrecht: Reidel Publishing Company.

Van den Akker, J., Gravemerijer, K., McKenney, S., & Nieveen, N (Eds.). (2006). *Educational design research*. London: Routledge.

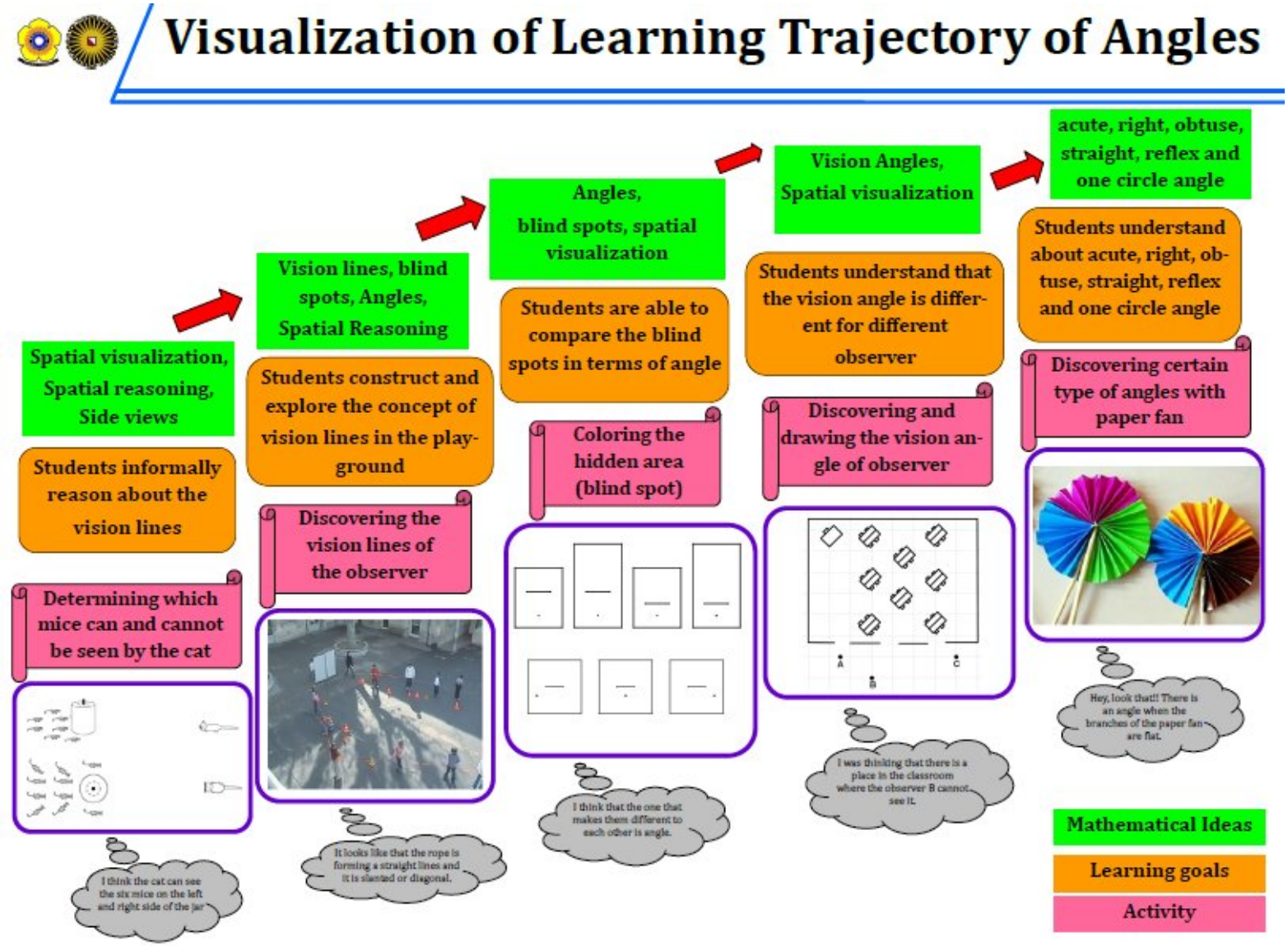
Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. Utrecht: CD-ß Press / Freudenthal Institute, Utrecht University.

Van den Heuvel-Panhuizen, M., Buys, K. (2008). *Young children learn measurement and geometry. A learning-teaching trajectory with intermediate attainment targets for the lower grades in primary school*. Rotterdam: Sense Publishers.

Wijaya, A. (2008). *Design research in mathematics education: indonesian traditional games as preliminaries in learning measurement in length*. (Master Thesis). Utrecht University.

Zulkardi. (2002). *Developing a learning environment on realistic mathematics education for indonesian students teachers*. University of Twente.

HLT VISUALIZATION



Appendix I
The teacher's Interview Scheme

Background:

- How long have you been teaching in primary school?
- Do you teach only mathematics or all the entire subjects?
- How long have you taught mathematics for the third graders?

Teaching process:

- What is your experience in teaching mathematics in the third grade?
- Do you have experience in teaching the concept of angle for the third graders?
- What kind of textbook do you use to teach mathematics in the third grade?
- Do you always follow the textbook in teaching mathematics?
- What kind of difficulty do you have in teaching the concept of angle in the third grade?
- What kind of difficulty students have in understanding the concept of angle?
- How many meetings do you need to teach the concept of angle?
- Do the third graders have learned about the concept of vision lines?
- Are you get used to orchestrate a classroom discussion with students?
- How do you manage the discussion? What is your consideration?
- Do you always use real world contexts to introduce a new mathematics topic for students?
- Do you have any specific rule in the class? (reward or punishment, raising finger, giving turn)

About PMRI:

- Have you ever heard about PMRI?
- Do you have any experience about PMRI or teaching with this approach?
- Do you think it is possible to teach mathematics by using PMRI approach?
- What do you think about using real contexts in teaching mathematics?

About students:

- What the age of students in the third grade?
- Do you know who are the high achievers and low achievers in your classroom?
- Do the students get used to work in small group?
- How do you group them?

Appendix II
The classroom observation Scheme

Classroom environment:

- How do teacher open the lesson?
- How students sit in the classroom?
- How do teacher interact with students? How friendly the teacher to the students?
- Where does the teacher position during the lesson? (mostly stand in front of the class or moving around)
- How do students interact to each other?
- Are students mostly silent or actively talking during the lesson?
- How do students participate in the lesson?
- Do the students work individually, in pairs, or groups? (If in groups, how does the teacher group the students?)
- Does the teacher give an equal chance to the students to express their opinion?
- Is there any irrelevant behavior of the students during the lesson? How teacher deal with it?
- Do the students pay attention when the teacher gives an explanation?
- Is there any students acting bossy or too silent in the classroom?
- How the teacher ends the lesson?

Teaching and learning process:

- How do teacher teach mathematics in the classroom? (demonstrating/explaining, or promoting a discussion)
- Does the teacher give a chance for students to think for a while before giving a response?
- How the teacher responses to the students opinion or answer?
- Does the teacher always follow the textbook during the lesson?
- Does the teacher give student worksheet to the students?
- How do teacher deal with the time management?
- Is there any discussion about students' thinking and reasoning?
- Is there any mathematical model used in the lesson?
- How do teacher guide the students to understand the topic?
- Does the teacher appreciate the different opinion of the students?
- How the teacher facilitates different solutions of the students?
- Does the teacher discuss with the students about elegant, efficient, sophisticated solution?

Appendix III
Petunjuk Guru
Pertemuan 1

Aktivitas
Kucing dan Tikus

Alokasi waktu
2 x 35 menit

Materi
- Kursi
- Meja
- Tali
- Kotak kecil
- Lembar kerja siswa

Tujuan Pembelajaran
Tujuan utama
Siswa menginvestigasi situasi nyata yang melibatkan garis pandang dan daerah yang tak terlihat.
Penjabaran Tujuan
- Siswa menyadari tentang berbagai sudut pandang dalam melihat suatu objek.
- Siswa mengembangkan kemampuan perspektif spasial yakni kemampuan untuk melihat objek dari sudut pandang orang lain.
- Siswa mampu berargumen mengapa beberapa tikus dapat atau tidak dapat dilihat oleh kucing pada masalah tersebut.
- Siswa mengetahui bahwa ada empat tikus yang hanya dapat dilihat setengah badannya oleh kucing pada masalah tersebut.

Gambaran aktivitas:
- Guru membentuk kelompok yang terdiri dari empat orang siswa dan menyuruh mereka berkumpul dengan kelompoknya. Setiap kelompok juga mempunyai satu lembar LKS. Guru menyampaikan kepada siswa bahwa kelompoknya akan tetap sama selama pembelajaran matematika. Guru juga meminta siswa untuk menyiapkan penggaris dan pensil untuk digunakan dalam mengerjakan LKS. Guru menekankan kepada siswa bahwa semua gambar pada LKS 1, LKS 2, dan LKS 3, adalah merujuk pada situasi yang sama namun dilihat dari posisi yang berbeda.

- Guru memperkenalkan konteks kucing dan tikus kepada siswa. Guru mengelaborasi konteks tersebut lebih jauh dengan menanyakan pengalaman siswa tentang tikus dan kucing. Guru memberikan beberapa pertanyaan seperti: *apakah kalian sering melihat kucing dan tikus? Apakah kalian mempunyai kucing atau tikus di rumah?*
- Guru kemudian membagikan kepada siswa Lembar Kerja Siswa (LKS) 1 dan meminta mereka untuk membaca dan memahami masalah dalam LKS 1 tersebut. Kemudian, jika tidak ada pertanyaan tentang LKS 1 tersebut, guru meminta siswa untuk berdiskusi dan menjawab masalah bersama-sama dengan kelompok mereka. Guru menyampaikan kepada siswa bahwa mereka harus mampu meyakinkan setiap anggota kelompok tentang jawaban mereka. Guru melakukan kegiatan tersebut untuk LKS 2 dan LKS 3.
- Masalah pada LKS tersebut adalah tentang menentukan jumlah tikus yang dapat dilihat oleh kucing. Ada beberapa gambar situasi kucing dan tikus yang diperlihatkan pada LKS tersebut dan siswa diminta untuk menentukan tentang jumlah tikus yang dapat dilihat oleh kucing tersebut dan apakah jumlah tersebut sama untuk semua gambar atau tidak.

Peran Guru

- Selama siswa bekerja

  Guru memberikan waktu kepada siswa untuk bekerja dan berdiskusi di dalam kelompok mereka masing-masing. Guru juga memberikan kebebasan kepada siswa untuk membuat gambar atau apapun yang mereka inginkan pada gambar di LKS yang dapat membantu mereka menyelesaikan masalah tersebut. Setelah beberapa menit bekerja, guru akan mengajak siswa untuk melaksanakan diskusi kelas. Guru menekankan pada penalaran siswa tentang jawaban mereka. Jika siswa hanya mampu menjawab soal tersebut dengan cara menerka atau membuat dugaan sementara, guru dapat membimbing siswa tersebut dengan mengajukan pertanyaan seperti:

  Apa yang dapat kamu lakukan pada gambar tersebut untuk mendukung jawaban kamu?

  Bagaimana caranya kucing tersebut bisa melihat tikus itu?

  Hai, coba perhatikan gambar kedua! Pada gambar kedua ini saya bisa melihat tikus ini, tapi pada gambar pertama tidak? Kenapa ya kira-kira? Ada yang bisa bantu saya mencari tahu alasannya?

  Saya tidak yakin kalau kucing tersebut bisa melihat keempat tikus tersebut secara keseluruhan, kalian setuju atau tidak? Mengapa tidak?

Sebagai tambahan, mungkin akan ada siswa yang berpendapat bahwa kucing tersebut dapat melihat semakin banyak tikus saat dia bergerak semakin mendekati tempat makanan tersebut. Jika ada siswa lain yang berpedapat berbeda, guru dapat melaksanakan diskusi untuk dua pendapat yang berbeda tersebut. Jika tidak, pendapat siswa yang salah tersebut, akan menjadi salah satu alasan untuk melakukan kegiatan eksperimen di pertemuan kedua.

- Selama diskusi

  Point diskusi pertama adalah tentang masalah pertama. Guru akan bertanya kepada siswa tentang berapa banyak tikus yang dapat dilihat oleh kucing pada gambar pertama. Kebanyakan siswa mungkin akan menjawab bahwa kucing terebut hanya bisa melihat satu tikus di sebelah kiri tempat makanan tersebut karena yang lainnya tertutup oleh tempat makanan tersebut. Jawaban lain yang mungkin dari siswa adalah kucing tersebut dapat melihat tiga tikus di sebelah kiri dengan menggambar garis dari kucing ke tikus tersebut. Menanggapi jawaban siswa yang beragam, guru mengorkestrasi jalannya diskusi dengan menekankan pada alasan siswa dari jawaban yang mereka ajukan.

Gambar 1. Masalah pertama

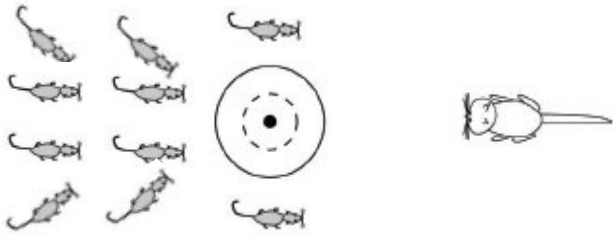
Point diskusi kedua adalah tentang masalah kedua pada Lembar Kerja Siswa 2. Pada diskusi ini, guru fokus pada strategi siswa untuk menyelesaikan masalah tersebut. Guru bertanya kepada siswa perbedaan apa yang dapat dilihat oleh siswa dari kedua gambar tersebut? Guru juga menekankan kepada siswa bahwa kedua gambar tersebut diambil dari situasi yang sama, hanya saja gambar pertama adalah tampak dari depan sedangkan gambar kedua adalah tampak atas.

Guru juga dapat melanjutkan diskusi lebih jauh dengan menunjukkan tikus yang tidak terlihat pada gambar pertama tetapi dapat dilihat pada gambar kedua. Guru memberikan siswa kesempatan untuk berpikir beberapa saat tentang hal tersebut dan kemudian mempersilahkan siswa untuk

menjelaskan jawaban mereka kepada siswa lain. Guru juga mengecek pemahaman siswa tentang jawaban yang telah disampaikan dengan cara meminta siswa yang lain untuk mengulangi jawaban tersebut. Diskusi tersebut akan membawa siswa untuk memahami tentang perbedaan pandangan dalam melihat objek atau situasi dengan lebih baik.

Gambar 2. Masalah kedua

Point diskusi ketiga adalah tentang masalah ketiga pada LKS 3. Guru bertanya kepada apa yang terjadi pada jumlah tikus yang dapat dilihat oleh kucing tersebut jika dia bergerak semakin mendekati tempat makanan tersebut? Kemungkinan siswa akan muncul dengan jawaban yang berbeda-beda. Sebagian besar dari mereka mungkin menjawab semakin dekat kucing dengan tempat makanan maka semakin banyak tikus yang dapat dilihatnya. Namun, mungkin ada sebgian siswa yang menjawab sebaliknya. Dalam hal ini, guru tidak menyalahkan atau membenarkan jawaban salah satu siswa tetapi mengorkestrasi diskusi yang menekankan pada penalaran siswa, alasan yang mereka miliki sehingga menjawab demikian.



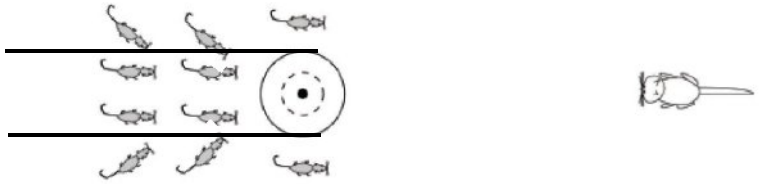
Gambar 3. Masalah ketiga

Dugaan pemikiran siswa:

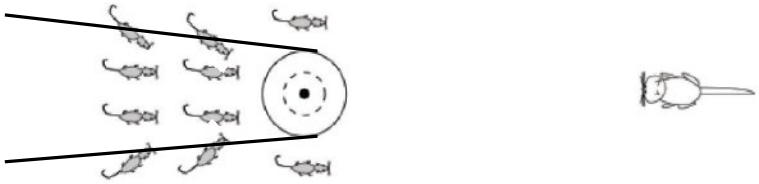
- Ketika siswa pada awalnya mulai memikirkan masalah tentang gambar pada LKS 1, mereka mungkin mulai menduga tentang berapa banyak tikus yang dapat dilihat oleh kucing pada gambar tersebut. Siwa mungkin akan

menjawab seperti ‘menurut saya kucing tersebut dapat melihat semua tikus’ atau ‘menurut saya kucing tersebut hanya dapat melihat dua tikus yang berada di samping tempat makanan tersebut’. Guru meminta siwa untuk menjelaskan lebih jauh alasan mereka tentang jawaban tersebut.

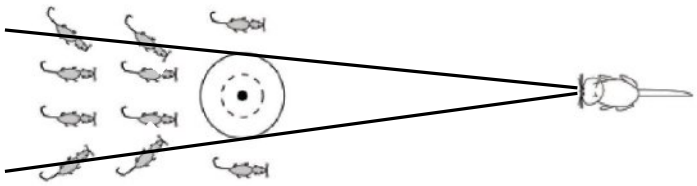
- Beberapa siswa mungkin mempunyai jawaban atau pendapat yang berbeda untuk kedua gambar pada LKS 1 dan LKS 2 tersebut. Siswa mungkin mengatakan bahwa kucing pada gambar pertama LKS 1 dapat melihat lebih banyak tikus dari pada kucing pada gambar kedua LKS 2 ataupun sebaliknya. Guru meminta siwa untuk menjelaskan lebih jauh alasan mereka tentang jawaban tersebut karena gambar pada LKS 1 dan LKS 2 sama-sama dari kondisi yang sama namun mengapa berbeda jumlah tikus yang dapat dilihat oleh kucing tersebut.
- Beberapa siswa mungkin sadar bahwa ada tikus yang tidak terlihat pada gambar pertama LKS 1 tapi dapat dilihat pada gambar kedua LKS 2. Guru kemudian meminta siswa menjelaskn pendapat mereka mengapa terjadi hal yang demikian untuk kondisi yang sama?
- Mungkin ada siswa yang muncul dengan ide menggambar garis pada gambar kedua LKS 2. Tapi, garis yang mereka gambar mungkin berbeda-beda. Beberapa siswa mungkin hanya menggambar garis sejajar (gambar 4) sedangkan yang lain menggambar garis miring namun tidak bertemu pada mata kucing tersebut (gambar 5). Kemungkinan juga beberapa siswa yang dapat menggambar garis pandang kucing tersebut dengan benar yakni garis pandang tersebut menyinggung kedua ujung sisi tempat makanan tersebut (gambar 6).



Gambar 4. Siswa menggambar garis sejajar



Gambar 5. Siswa menggambar dua garis miring tapi tidak saling bertemu



Gambar 6. siswa menggambar garis pandang kucing tersebut dengan benar

Menutup pembelajaran

Guru merefleksi kegiatan yang telah dilakukan dan meminta beberapa orang siswa untuk menjelaskan kepada teman-temannya yang lain tentang apa yang telah mereka pelajari pada pertemuan hari ini. Guru juga dapat membantu siswa untuk menyimpulkan pembelajaran pada pertemuan tersebut dengan mengajukan beberapa pertanyaan seperti:

Bagaimana caranya kita tahu jumlah tikus yang dapat dilihat oleh kucing tersebut? Bagaimana gambar itu bisa membantu kita?

Adakah perbedaan jumlah tikus yang dapat dilihat oleh kucing dari ketiga gambar tersebut? mengapa demikian?

Petunjuk Guru
Pertemuan 2

Aktivitas
Eksperimen dengan Layar

Alokasi waktu
2 x 35 menit

Materi
- Layar
- Kursi
- Rope/lakban hitam
- Tas sekolah
- Lembar kerja siswa

Tujuan Pembelajaran
Tujuan utama
Siswa mampu merekonstruksi garis pandang dan daerah yang tidak terlihat dalam bentuk dua dan tiga dimensi.
Penjabaran Tujuan
- Siswa akan menyadari bahwa mereka dapat memperpanjang garis pandang sampai tak berhingga.
- Siswa mampu menyadari bahwa garis pandang berbetuk garis lurus dan miring.
- Siswa mampu menjelaskan bahwa garis pandang berasal dari mata pengamat dan menyentuh pinggir layar.
- Siswa mampu menjelaskan tentang daerah yang tidak terlihat pada kegiatan eksperimen.

Gambaran aktivitas:
- Guru meminta siswa untuk menempatkan kursi di depan layar. Guru meminta siswa untuk menyiapkan catatan selama kegiatan eksperimen berlangsung karena setelah kegiatan eksperimen tersebut siswa akan diminta menjelaskan apa yang telah mereka pelajari dari kegiatan tersebut.
- Guru meminta salah seorang siswa untuk duduk di kursi menghadap layar sebagai pengamat.
- Guru menempatkan tas pada daerah yang terlihat (di sebelah kiri atau kanan layar) dan daerah yang tidak terlihat (di belakang layar) oleh pengamat.

- Guru bertanya kepada siswa apakah pengamat bisa melihat tas tersebut atau tidak dan alasan atau argument tentang itu. Diharapkan siswa akan mengatakan bahwa tas tersebut tidak dapat dilihat oleh pengamat karena tertutup oleh layar tersebut. Yang penting adalah siswa mengetahui bahwa ada daerah yang tidak terlihat dan ada daerah yang terlihat oleh pengamat.
- Guru memulai kegiatan eksperimen dengan semua kelompok. Guru mengingatkan siswa bahwa mereka tetap berada pada kelompok mereka seperti pada pertemuan sebelumnya. Guru juga menyampaikan bahwa setiap kelompok mempunyai giliran untuk menjadi pengamat.
- Guru meminta satu kelompok untuk menjadi pengamat (setiap anggota kelompok akan mengambil giliran sebagai pengamat) dan siswa lain akan berdiri di belakang layar sambil membawa tas sekolah mereka.
- Guru menjelaskan aturan kegiatan eksperimen tersebut bahwa siswa yang berada di belakang layar harus meletakkan tas sekolah yang mereka bawa pada tempat dimana pengamat hanya dapat melihat sebagian badan siswa tersebut.
- Guru juga menyampaikan kepada siswa bahwa mereka bisa menggunakan tali atau lakban selama proses eksperimen.

Peran Guru

- Guru membimbing siswa untuk melakukan kegiatan eksperimen dan memastikan bahwa setiap kelompok mempunyai kesempatan untuk menjadi pengamat. Sangat penting bagi siswa untuk menjadi pengamat karena mereka akan melihat secara langsung tentang apa yang dimaksud dengan garis pandang dan daerah yang tidak terlihat dalam kegiatan eksperimen tersebut. Guru juga membimbing siswa untuk menggunakan tali atau lakban selama proses eksperimen. Tali atau lakban tersebut sangat penting untuk digunakan dalam proses eksperimen sebagai alat untuk memvalidasi jawaban siswa tentang apakah tas tersebut membentuk garis lurus dan apakah tas tersebut akan bertemu pada satu titik atau tidak. Jika siswa tidak sampai pada ide tersebut, guru bisa membantunya dengan memberikan pertanyaan seperti:
  Menurut kamu bagaimana bentuk tas-tas tersebut?
  Bagaimana kita bisa yakin bahwa bahwa tas tersebut berada pada satu garis?
  Bisakah kita menggunakan tali atau lakban tersebut?
- Selama diskusi
  Guru harus menekankan bahwa tas sekolah tersebut berada pada satu garis. Guru juga harus memastikan jika siswa mengerti bahwa tas sekolah tersebut dapat diletakkan sejauh yang kita inginkan sampai tak berhingga

mewakili garis pandang pengamat. Jika hal ini tidak muncul selama kegiatan eksperimen, guru dapat membimbing siswa dengan mengajukan pertanyaan seperti:
dapatkah kita memprediksi posisi berikutnya dari tas tersebut? Seberapa jauh kita bisa memperpanjang posisi tas atau tali tersebut? Apakah tali atau lakban tersebut bertemu pada satu titik?

(source: Munier, et al., 2008)

Susunan tas sekolah tersebut akan terlihat seperti pada susunan *traffic cone*s pada gambar diatas.

Guru juga harus membimbing siswa untuk menjadi pengamat dimana posisi kursi pengamat tersebut berpindah-pindah ke sebelah kiri atau kanan layar. Dalam hal ini, guru harus membimbing siswa untuk berdiskusi tentang perbedaan bentuk dari tas sekolah tersebut. Guru bisa memberikan beberapa pertanyaan yang dapat membimbing siswa seperti:
Apa yang terjadi pada bentuk tas sekolah tersebut jika pengamat bergerak mendekat atau menjauh dari layar?
Bagaimana jika pengamat bergerak ke sebelah kiri atau kanan layar?
Dapatkah kamu memberikan penjelasan mengapa hal itu terjadi?
Guru juga mengingatkan siswa untuk membuat catatan tentang kegiatan eksperimen dan diskusi yang mereka lakukan.

Menutup pembelajaran
Guru merefleksi kegiatan yang telah dilakukan dan meminta beberapa orang siswa untuk menjelaskan kepada teman-temannya yang lain tentang apa yang telah mereka pelajari pada pertemuan hari ini. Guru juga dapat membantu siswa untuk menyimpulkan pembelajaran pada pertemuan tersebut dengan mengajukan beberapa pertanyaan seperti:
Apa yang dapat kalian sampaikan tentang tas sekolah atau tali tersebut?

Apa yang terjadi jika pengamat bergerak mendekat, menjauh, ke sebelah kiri, atau ke sebelah kanan layar?
Dapatkah kalian menyimpulkan apa yang membuatnya berbeda?

Petunjuk Guru
Pertemuan 3

Aktivitas
Mewarnai Daerah yang Tertutup

Alokasi waktu
2 x 35 menit

Materi

- Pensil berwarna
- Penggaris
- Kertas A3

Tujuan Pembelajaran
Tujuan Utama
Siswa mampu membayangkan dan memvisualisasi garis pandang serta mulai membangun penalaran mereka tentang sudut.
Penjabaran Tujuan

- Siswa mampu mewarnai daerah yang tidak terlihat (*blind spots*) untuk posisi pengamat yang berbeda.
- Siswa mampu membandingkan beberapa daerah yang tidak terlihat yang berbeda.
- Siswa mampu menyadari bahwa sudut yang membuat gambar tersebut berbeda.
- Siswa mampu bernalar dengan menggunakan kata-kata mereka sendiri dalam hal konsep sudut.
- Siswa akan menyadari bahwa sudut tetap sama untuk satu posisi pengamat, tidak perduli bagaimanapun panjang garis pembatasnya (garis pembatasnya adalah garis pandang).

Gambaran Aktivitas

- Guru Teacher menuntun siswa untuk berada dalam kelompok mereka dan meminta mereka untuk mempersiapkan pensil warna dan penggaris.
- Guru menjelaskan kepada siswa tentang tugas yang akan mereka kerjakan bersama dalam kelompok mereka masing-masing. Guru menjelaskan bahwa gambar tersebut adalah representasi dari tampak atas kegiatan eksperimen yang telah dilakukan pada pertemuan sebelumnya.

- Guru meminta siswa untuk membuat poster dari jawaban mereka terhadap permasalahan tersebut. Guru juga menjelaskan bahwa poster harus memuat strategi yang digunakan siswa dalam menjawab masalah tersebut dan dijelaskan dengan jelas sehingga dapat dipahami oleh siswa yang lain.
- Guru membagikan LKS 1 kepada siswa dan meminta mereka untuk berdiskusi dan menyelesaikan masalah tersebut. Kemudian, guru memberikan LKS 2 kepada mereka dan memberikan kembali kesempatan kepada mereka untuk bekerja dalam kelompok mereka masing-masing.
- Guru meminta siswa untuk meletakkan poster mereka ditempat dimana poster tersebut mudah dilihat oleh siswa yang lain.
- Guru memberikan siswa waktu yang cukup untuk melihat poster dari siswa lain dan memberikan catatan kecil yang berisi tentang komentar atau pertanyaan mereka terhadap poster tersebut.
- Guru mengatur kongres matematika (diskusi kelas) dengan meminta beberapa orang siswa dari kelompok yang berbeda untuk menjelaskan strategi atau jawaban mereka yang berbeda-beda.

Peran Guru

- Selama siswa bekerja

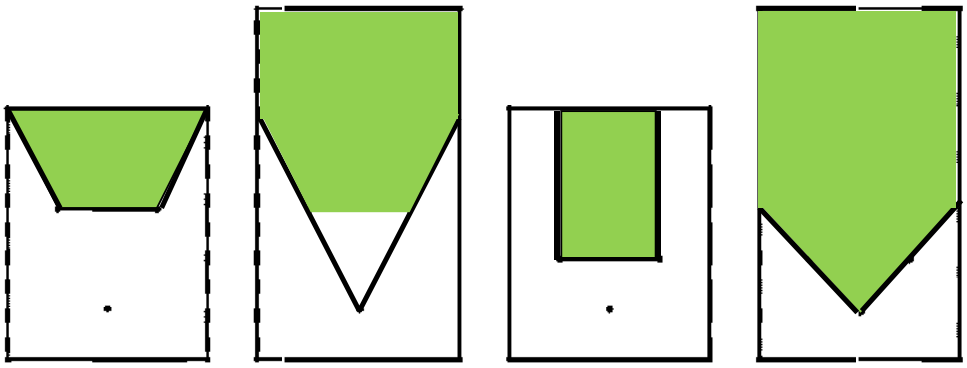
  Guru harus membimbing siswa untuk mewarnai daerah yang tidak terlihat (*blind spots*). Mungkin akan ada beberapa orang siswa yang mewarnai daerah yang tidak terlihat dengan cara yang salah. Dalam hal ini, guru dapat mengajukan pertanyaan yang membangun pemahaman siswa, seperti:

  Apakah kamu menyadari bahwa gambar tersebut kelihatan seperti kegiatan eksperimen pada pertemuan sebelumnya?

  Apakah kamu masih ingat bentuk tali pada eksperimen tersebut?

  Guru juga membimbing siswa untuk berdiskusi bersama dengan kelompok mereka tentang perbedaan diantara gambar-gambar tersebut. Guru meminta mereka untuk menemukan strategi untuk membandingkan gambar tersebut dan alasan dari jawaban mereka tersebut. diharapkan siswa mampu sampai pada berbagai macam strategi seperti menumpukkan satu sama lain (*superimposing*), menggunakan kertas perekat, atau menggunakan jari-jari mereka.
- Selama diskusi

  Point diskusi pertama adalah tentang bagaimana siswa mewarnai daerah yang tidak terlihat tersebut. Guru membimbing siswa untuk mendiskusikan perbedaan jawaban dan alasan mereka. Beberapa jawaban siswa yang mungkin dari masalah tersebut adalah sebagai berikut:



Guru juga harus yakin bahwa siswa yang mempunyai jawaban yang salah menyadari kesalahan mereka.

Kemudian, point diskusi yang kedua adalah tentang bagaimana kita dapat membandingkan ke empat gambar yang pertama tersebut. dalam hal ini, guru mengatur strategi siswa yang berbeda-beda dalam membandingkan gambar-gambar tersebut. Penekanannya ada pada apakah siswa membandingkan gambar tersebut dengan benar atau tidak dan apakah siswa yang lain memahamai tentang jawaban siswa tersebut.

Point diskusi ketiga adalah tentang apa yang terjadi jika pengamat bergerak ke sebelah kanan atau sebelah kiri layar, seperti pada LKS 2. Guru harus membimbing siswa untuk menemukan bahwa sudutnya menjadi semakin kecil ketika pengamat bergerak ke sebelah kiri atau sebelah kanan dari sisi layar tersebut.

Menutup pelajaran

Guru mereview aktivitas dan meminta beberapa orang siswa untuk menjelaskan kepada yang lain tentang apa yang telah mereka pelajari dari pertemuan ini. Guru juga bisa mereview aktivitas pembelajaran dengan mengajukan pertanyaan kepada siswa seperti:

Apa yang membuat gambar-gambar tersebut berbeda?

Apa yang terjadi dengan gambar tersebut ketika pengamat bergerak ke sebelah kiri atau sebelah kanan dari sisi layar tersebut?

Petunjuk Guru
Pertemuan 4

Aktivitas
Memahami konsep sudut pandang

Alokasi Waktu
2 x 35 menit

Materi
- Tali
- Penggaris
- Pensil warna

Tujuan Pembelajaran
Tujuan Utama
Siswa mampu mengkonstruksi dan menggambar sudur pandang dari posisi pengamat yang berbeda-beda.
Penjabaran Tujuan
- Siswa mampu mengkonstruksi sudut pandang dari pengamat.
- Siswa mengetahui bahwa sudut pandang berbeda-beda untuk posisi observer yang berbeda.
- Siswa mampu menempatkan sudut pandang dari suatu pengamat pada posisi tertentu.
- Siswa menyadari bahwa suatu posisi dapat ditempati oleh berbeda-beda potong kertas yang mempunyai sudut yang sama.
- Siswa mampu membedakan mana potongan kertas yang mempunyai sudut yang besar dan potongan kertas yang mempunyai sudut yang kecil.
- Siswa menyadari bahwa panjang sisi potongan kertas tersebut tidak mempengaruhi ukuran sudut dari potongan kertas terebut.

Gambaran Aktivitas
- Guru meminta salah satu siswa untuk menjadi pengamat. Siswa tersebut keluar kelas dan berdiri di depan pintu menghadap ke dalam kelas tersebut. Guru bertanya kepada siswa tersebut apa saja yang dapat ia lihat didalam kelas dan apakah ia dapat melihat semua siswa di dalam kelas atau tidak.
- Guru kemudian berdiskusi dengan siswa tentang mengapa ada yang tidak dapat dilihat dan ada yang dapat dilihat oleh pengamat tersebut? Guru juga

bertanya kepada siswa mengapa ada siswa di dalam kelas yang tidak dapat melihat pengamat di luar kelas tersebut?

- Guru memverifikasi jawaban siswa yang mungkin berbeda-beda dengan cara meminta dua orang siswa untuk menarik tali dari pengamat tersebut ke dalam kelas yang menjadi pemisah antara daerah yang terlihat dan daerah yang tidak terlihat oleh pengamat tersebut. Dalam hal ini, guru membantu siswa untuk mengkonstruksi sudut pandang dari pengamat tersebut.
- Guru menekankan kepada siswa bahwa daerah yang berada diantara dua tali tersebut adalah daerah yang terlihat oleh pengamat. Sedangkan, daerah yang berada diluar tali tersebut adalah daerah yang tidak dapat dilihat oleh pengamat. Guru juga menyampaikan istilah baru kepada siswa bahwa yang dibentuk oleh dua tali tersebut adalah **sudut pandang** dan tali tersebut adalah **garis pandang**.
- Guru kemudian berdiskusi lebih jauh dengan siswa dengan mengajukan beberapa pertanyaan, seperti:
  Apa yang terjadi dengan tali tersebut jika pengamat bergerak maju mendekati pintu kelas atau bergerak mundur menjauhi kelas
  Bagaimana jika pengamat bergerak ke sisi sebelah kiri atau ke sebelah kanan dari pintu tersebut?
  Guru membimbing siswa untuk memverifikasi jawaban mereka dengan melaksanakan kegiatan eksperimen kembali dengan tali tersebut untuk posisi pengamat yang berbeda-beda (jauh dan dekat dari pintu kelas).
- Guru meminta siswa untuk berkumpul dengan kelompok mereka masing-masing dan membagikan LKS 1 kepada mereka yang menunjukkan tampak atas dari kegiatan eksperimen yang telah mereka lakukan sebelumnya. Guru menjelaskan kepada mereka bahwa mereka harus menggambar dan mewarnai **daerah yang terlihat** oleh pengamat yakni daerah dimana pengamat yang berdiri di depan kelas tersebut dapat melihat situasi di dalam kelas.
- Guru memberikan LKS 2 dan juga potongan kertas berbentuk segitiga dengan ukuran yang berbeda-beda (besar- kecil) untuk setiap kelompok dan meminta siswa untuk berdiskusi dan menyelesaikan masalah di KLS 2 tersebut. Guru menjelaskan kepada mereka bahwa potongan kertas tersebut adalah daerah yang dapat dilihat oleh pengamat dan mereka harus menentukan dimana posisi pengamat di denah kelas yang ada di LKS 2 tersebut untuk setiap potong kertas yang mereka miliki.
- Guru kemudian berdiskusi dengan siswa tentang jawaban siswa pada LKS 2 tersebut. Guru memfokuskan diskusi pada pemahaman siswa bahwa

potongan kertas yang berbeda ukuran dapat menempati sebuah posisi pengamat yang sama karena mempunyai sudut yang sama.

- Guru juga meminta siswa untuk membandingkan potongan-potongan kertas tersebut kemudian menanyakan kesimpulan apa yang mereka peroleh dari kegiatan tersebut. Guru juga dapat mengajukan pertanyaan seperti:
  Potongan kertas manakah yang mempunyai sudut paling besar? Jelaskan alasanmu!
  Potongan kertas manakah yang mempunyai sudut paling kecil? Jelaskan alasanmu!
  Manasajakah potongan kertas tersebut yang mempunyai sudut yang sama?Jelaskan alasanmu?

Peran Guru

- Selama siswa bekerja
  Guru membantu siswa untuk melakukan kegiatan eksperimen dan membimbing mereka memvisualisasikan sudut pandang dengan menggunakan tali. Guru juga membimbing siswa untuk berdiskusi dengan kelompok mereka tentang gambar daerah yang terlihat yang berbeda-beda untuk posisi pengamat yang berbeda. Pada masalah kedua, beberapa siswa mungkin kebingungan tentang posisi potongan kertas tersebut, potongan kertas yang berbeda dapat menggambarkan posisi pengamat yang sama. Dalam hal ini, guru membimbing mereka untuk menemukan jawabannya sendiri melalui diskusi bersama dengan kelompok mereka.
- Selama diskusi
  Point diskusi pertama adalah tentang tentang kegiatan eksperimen yang dilaksanakan di kelas. Guru membimbing siswa untuk menyadari bahwa tali tersebut merepresentasikan garis pandang dari pengamat dan menjelaskan kepada mereka bahwa daerah yang terlihat bersama dengan garis pandang tersebut adalah sudut pandang dari pengamat tersebut.
  Point diskusi kedua adalah tentang menggambar garis pandang untuk posisi pengamat yang berbeda-beda. Guru membimbing mereka menyadari bahwa ada daerah dimana semua pengamat di LKS tersebut dapat melihatnya secaa bersama-sama. Guru juga membimbing siswa untuk membandingkan gamabr yang berbeda-beda tersebut (sudut pandang pengamat) dengan menggunakan istilah sudut.
  Point ketiga diskusi adalah tentang potongan kertas yang merepresentasikan sudut pandang pengamat. Guru menekankan diskusi tentang mengapa potongan kertas yang berbeda-beda dapat mempunyai posisi pengamat yang sama.

Menutup pelajaran

Guru mereview aktivitas yang telah dilakukan dan meminta beberapa orang siswa untuk menjelaskan kepada temannya yang lain tentang apa yang telah dipelajari hari ini. Guru juga dapat mereview aktivitas yang telah dilakukan dengan mengajukan pertanyaan seperti:

Apa yang membedakan sudut pandang dari pengamat-pengamat tersebut?

Mengapa untuk posisi pengamat yang sama dapat mempunyai potongan kertas yang berbeda?

Manasajakah potongan kertas yang mempunyai sudut yang besar dan yang kecil? Mengapa demikian?

Petunjuk Guru
Pertemuan 5

Aktivitas
Bermain dengan kipas kertas

Alokasi Waktu
2 x 35 menit

Materi
- Kipas kertas
- Penggaris
- gunting

Tujuan Pembelajaran
Tujuan Utama
Siswa memahami tentang sudut lancip, siku-siku, tumpul, lurus, satu putaran dan refleksi dalam berbagai bentuk dan orientasi.
Penjabaran Tujuan
- Siswa mampu menyadari tentang sudut lancip, siku-siku, tumpul, lurus, dan satu putaran.
- Siswa menyadari bahwa sudut yang sama dapat di letakkan atau dibentuk dengan berbagai macam orientasi.
- Siswa menyadari tentang sudut refleksi (sudut dalam dan sudut luar) untuk sepasang garis yang berpotongan di tengah.

Gambaran Aktivitias
- Guru memperkenalkan kipas kertas kepada siswa dan bertanya kepada mereka apakah mereka familiar dengan kipas kertas tersebut.
- Guru kemudian meletakkan kipas kertas tersebut didepan matanya dan mulai membuka kipas kertas tersebut sedikit demi sedikit. Dalam hal ini, Guru bertanya kepada siswa tentang apa yang mereka lihat dan apa yang terjadi dengan kertas diantara dua lengan kipas kertas tersebut. sudut apa yang dibentuk oleh kipas kertas tersebut?
- Guru kemudian melanjutkan membuka kipas kertas tersebut sampai membentuk sudut siku-siku. Kemudian, guru bertanya kepada siswa apakah mereka mengenali sudut siku-siku tersebut atau tidak. Sebagai tindak lanjut, guru kemudian bertanya kepada siswa untuk menyebutkan contoh-contoh sudut siku-siku yang ada di sekitar mereka.

- Guru kemudian melanjutkan untuk membuka kipas kertas tersebut lebih lebar lagi sampai membentuk sudut tumpul dan kemudian sudut lurus. Guru kemudian bertanya kepada siswa apakah ada sudut pada kipas kertas tersebut atau tidak. Ada kemungkinan bahwa siswa tidak menyadari adanya sudut pada kipas kertas tersebut. dalam hal ini, guru dapat mengajukan pertanyaan seperti:
  Apakah kalian melihat kertas yang berada diantara dua lengan kipas kertas tersebut?
  Apakah kertas tersebut mengindikasikan sudut seperti pada posisi kipas kertas sebelumnya?

  Jika siswa sudah menyadari bahwa ada sudut pada posisi kipas kertas tersebut, guru memperkenalkan istilah sudut tersebut kepada siswa bahwa sudut tersebut dikenal dengan nama sudut lurus. Untuk mengecek pemahamam siswa tentang sudut tersebut, guru kemudian bertanya kepada siswa apakah ada sudut pada benda-benda seperti penggaris, stik, pensil, dll.
- Guru kemudian melanjutkan membuka kipas kertas tersebut lebih lebar lagi, lebih dari sudut lurus. Guru kemudian bertanya kepada siswa ada berapa sudut yang terbentuk pada kipas kertas sekarang. Jika siswa tidak menyadari tentang sudut refleksi, guru membimbing siswa untuk melihat bahwa ada daerah lain yang dibentuk oleh dua lengan kipas kertas tersebut selain yang di representasikan oleh kertas tersebut. Guru kemudian berdiskusi dengan semua siswa di kelas tentang hal ini.
- Guru kemudian membuka kipas kertas tersebut sampai terbuka penuh dan membentuk sudut satu putaran. Guru bertanya kepada siswa apakah mereka melihat ada sudut pada kipas kertas tersebut atau tidak. Mungkin ada siswa yang berpendapat ada sudut dan mungkin juga sebagian besar siswa mengatakan tidak ada. Dalam hal ini, guru kemudian mendiskusikan dengan siswa kemudian pada akhirnya guru memperkenalkan kepada siswa tentang nama sudut tersebut yang disebut dengan sudut satu putaran.
- Guru meminta siswa untuk tetap berada pada kelompok mereka masing-masing kemudian guru membagikan LKS kepada siswa. Pada LKS tersebut, siswa diminta untuk memasangkan potongan gambar-gambar kipas kertas dan nama-nama sudut yang ada di LKS tersebut. Siswa kemudian diminta menempelkannya pada poster yang telah disediakan. Siswa juga dapat mengkombinasikan dan menggabungkan gambar kipas kertas tersebut untuk membentuk sudut yang berbeda.

Peran Guru

- Selama siswa bekerja

  Guru harus membimbing siswa untuk berdiskusi apakah terdapat sudut pada kipas kertas dalam posisi tertentu atau tidak. Guru juga menekankan pentingnya siswa mengenali sudut yang sama dalam orientasi dan bentuk yang berbeda dengan menggunakan kipas kertas. Dalam hal ini, guru dapat mengajukan pertanyaan seperti:

  Bagaimana jika saya menggerakkan kipas kertas pada posisi seperti ini? Apakah masih sama sudutnya atau tidak?

  Bagaimana pendapat kamu tentang sudut pada posisi kipas kertas seperti ini (terbuka setengah lingkaran)? Apakah ada sudut atau tidak?

  Guru juga harus mengingat bahwa ide tentang berbagai bentuk sudut termasuk hal baru bagi siswa pada kelas tiga. Jadi, guru dapat memperkenalkan nama dari sudut-sudut tersebut setelah siswa mengenali bahwa ada sudut pada posisi tersebut.

  Mungkin juga beberapa siswa muncul dengan berbagai macam jawaban, seperti halnya dengan perbedaan bentuk atau orientasi dari sudut tersebut, misalnya sudut siku-siku. Dalam hal ini, guru membimbing siswa untuk berdikusi dalam kelompok mereka masing-masing dan menyadari bahwa sudutnya tetap sama namun berbeda bentuk dan orientasi. Mungkin siswa akan mengalami kesulitan dalam menyebutkan nama sudut untuk dua atau lebih gambar kipas kertas yang digabung. Dalam hal ini, guru menggunakan dua atau lebih kipas kertas untuk membimbing siswa memahami dan menemukan nama dari sudut yang dibentuk oleh kipas kertas tersebut.

- Selama diskusi

  Guru mengorkestrasi diskusi kelas meminta beberapa orang siswa untuk menjelaskan jawaban dan alasan mereka tentang masalah yang ada pada LKS. Focus diskusi ini adalah pada strategi dan penalaran siswa dalam menjawab masalah tersebut. guru harus memastikan bahwa satu jenis sudut dapat diletakkan dalam berbagai bentuk orientasi (berbagai bentuk kpas kertas) dan dapat juga mempunyai panjang lengan yang berbeda-beda (panjang lengan kipas kertas bisa berbeda-beda untuk satu sudut yang sama).

  Guru dapat meminta siswa untuk merepresentasikan beberapa jenis sudut dengan menggunakan kipas kertas dan mendiskusikan tentang hal tersebut. Khususnya, mendiskusikan apa yang membuat suatu jenis sudut berbeda dengan sudut yang lain.

Menutup pelajaran

Guru mereview aktivitas yang telah dilakukan dan meminta beberapa orang siswa untuk menjelaskan kepada temannya yang lain tentang apa yang telah ia pelajari dari kegiatan hari ini. Guru dapat juga mereview pembelajaran dengan cara mengajukan beberapa pertanyaan kepada siswa seperti:

Berapa banyak jenis sudut yang kalian ketahui dari kegiatan ini? Sudut apasajakah itu?

Apa yang membedakan satu jenis sudut dengan sudut yang lain?

Apa itu sudut lurus, sudut refleksi dan sudut satu putaran?

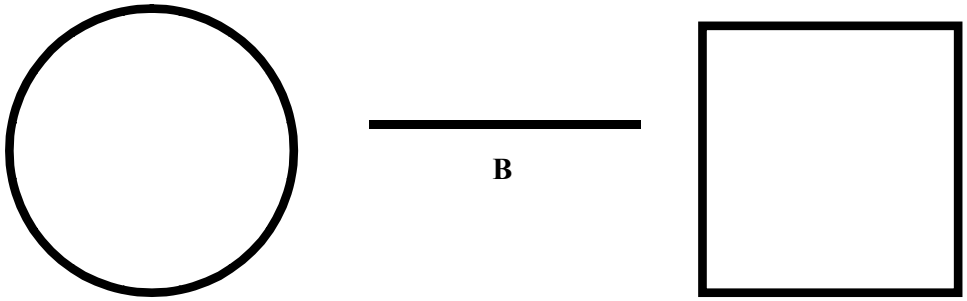
Appendix IV
Pre-test and Post-test

1. Bayangkan kamu sedang berdiri di depan sebuah jendela seperti pada gambar dibawah ini.

Apakah ada perbedaan yang kamu lihat ketika kamu bergerak semakin dekat dengan kamu bergerak semakin jauh dari sebuah jendela? Jelaskan alasanmu?

2. Jelaskan apa yang kamu ketahui tentang sudut!

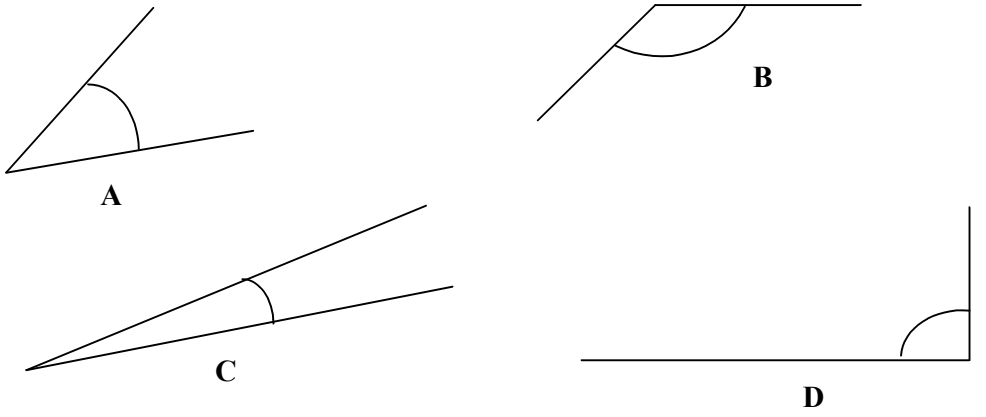
3. Tentukan apakah terdapat sudut pada masing-masing gambar dibawah ini atau tidak! Jelaskan alasanmu!



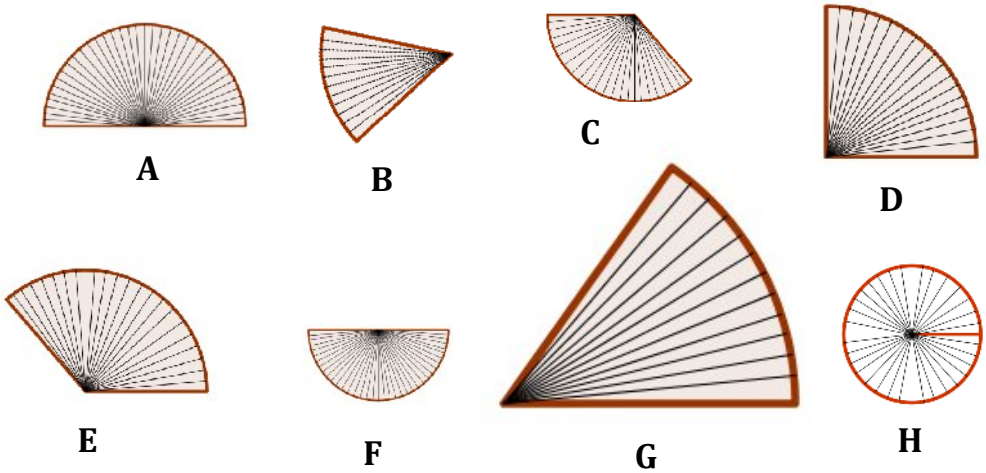
A **B** **C**

4. Sebutkan contoh benda-benda disekitarmu yang mempunyai sudut? Sudut apa sajakah itu?

5. Perhatikan gambar dibawah ini! Gambar yang manakah yang mempunyai sudut paling besar? Gambar manakah yang mempunyai sudut paling kecil? Jelaskan alasanmu!



6. Dibawah ini adalah beberapa gambar dari kipas kertas.
   a) Tentukan apakah terdapat sudut pada gambar tersebut atau tidak! Jelaskan alasanmu!



b) Tentukan gambar manakah yang mempunyai sudut paling besar dan gambar manakah yang mempunyai sudut paling kecil! Jelaskan alasanmu!

Appendix V

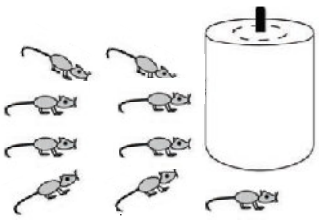
Lembar Kerja Siswa I

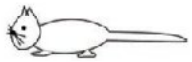
1. Masalah 1

Perhatikan gambar dibawah ini:

Seekor kucing berjalan mendekati sebuah tempat makanan dengan beberapa tikus kecil yang berada di belakangnya. Jika kucing tersebut berhenti pada posisi seperti pada gambar dibawah ini:

Berapa banyak tikus kecil yang **dapat dilihat** oleh kucing tersebut? Jelaskan alasanmu! (kamu boleh menggambar/mencoret gambar dibawah ini untuk mendukung jawabanmu).





2. Masalah 2

Gambar 2 berikut ini adalah **tampak atas** dari posisi kucing dan tikus kecil tersebut.

Apakah ada perbedaan antara jumlah tikus kecil yang dapat dilihat oleh kucing pada **gambar 1** dengan **gambar 2**? Jelaskan alasanmu! (kamu boleh menggambar/mencoret gambar dibawah ini untuk mendukung jawabanmu).

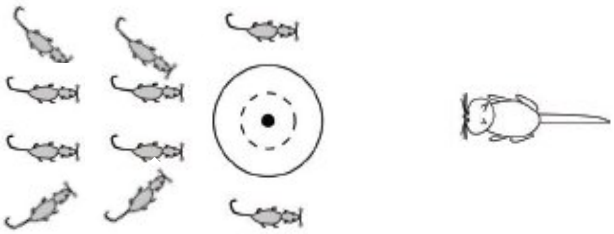
Gambar 1.

Gambar 2.

3. Masalah 3

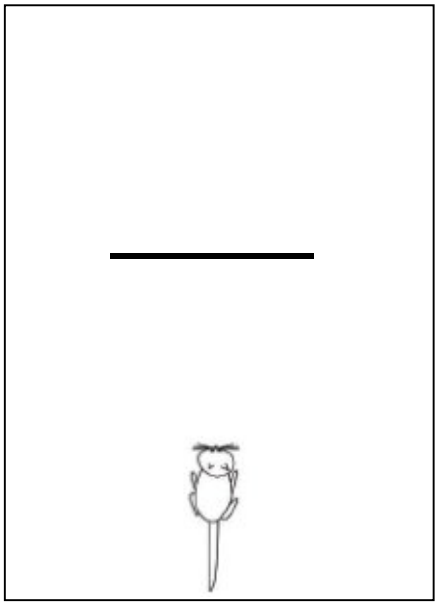
Kemudian, kucing tersebut bergerak mendekati tempat makanan tersebut. Tikus kecil tersebut tetap diam dan tidak bergerak dari tempatnya.

Bagaimana pendapat kamu tentang jumlah tikus yang dapat dilihat oleh kucing tersebut? Apakah semakin banyak, semakin sedikit, atau sama saja? Jelaskan alasanmu! (kamu boleh menggambar/mencoret gambar dibawah ini untuk mendukung jawabanmu).



4. Masalah 4

Gambar berikut adalah tampak atas dari seekor kucing yang melihat ke sebuah dinding. Arsirlah daerah yang tida dapat dilihat oleh kucing tersebut!



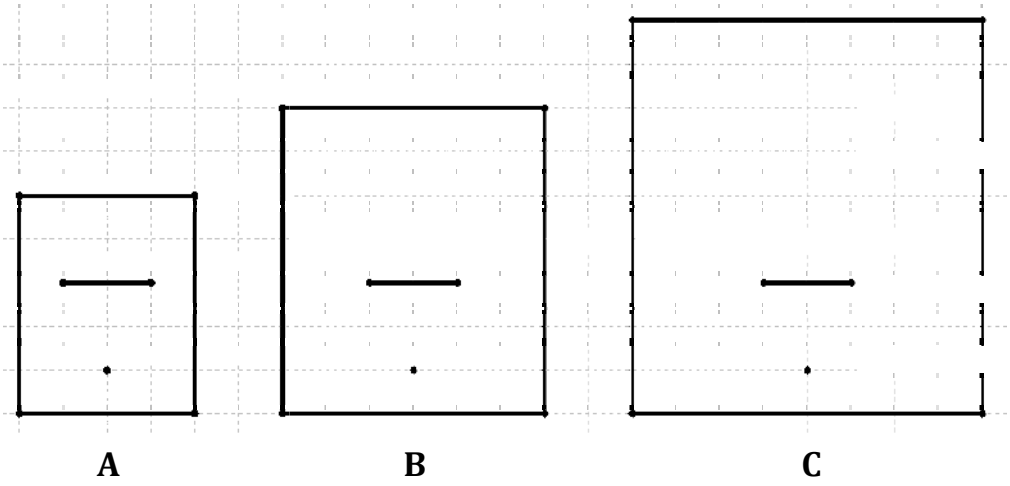
Lembar Kerja Siswa III

Petunjuk

Perhatikan beberapa gambar dibawah ini. Gambar tersebut adalah gambar **tampak atas** dari kegiatan eksperimen yang telah kita lakukan kemarin. Titik pada gambar tersebut adalah sang pengamat (siswa yang duduk di kursi menghadap layar), dan garis tersebut adalah layar yang berada di hadapan pengamat tersebut.

1. Masalah 1

Arsirlah daerah yang **tidak dapat** dilihat oleh pengamat.

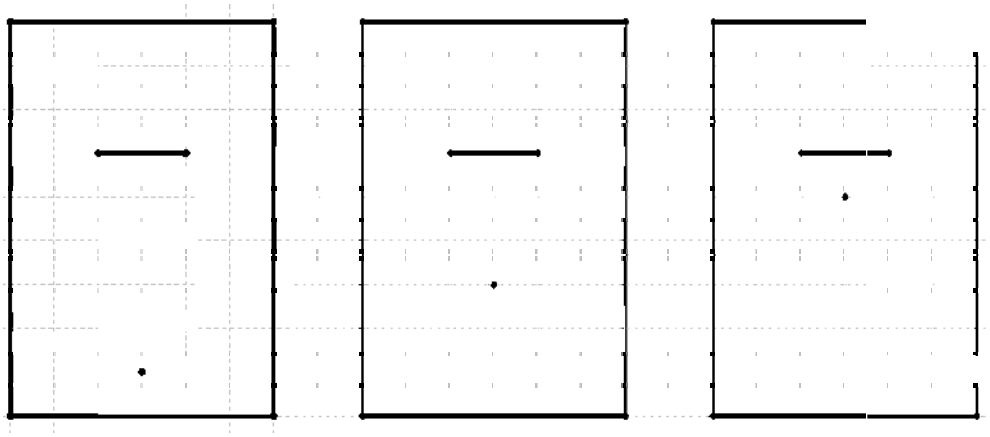


2. Masalah 2

Bandingkanlah gambar-gambar tersebut diatas. Apakah ada perbedaan daerah yang tidak dapat dilihat oleh pengamat diantara ketiga gambar tersebut? Jelaskan jawabanmu!

3. Masalah 3

Arsirlah daerah yang **tidak dapat** dilihat oleh pengamat.



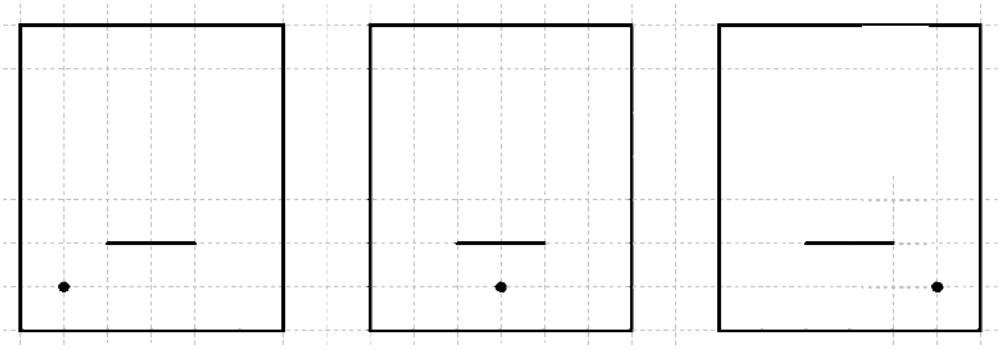
A **B** **C**

4. Masalah 4

Bandingkanlah gambar-gambar tersebut diatas. Apakah ada perbedaan daerah yang tidak dapat dilihat oleh pengamat diantara ketiga gambar tersebut? Jelaskan jawabanmu!

5. Masalah 5

Arsirlah daerah yang tidak dapat dilihat oleh pengamat.



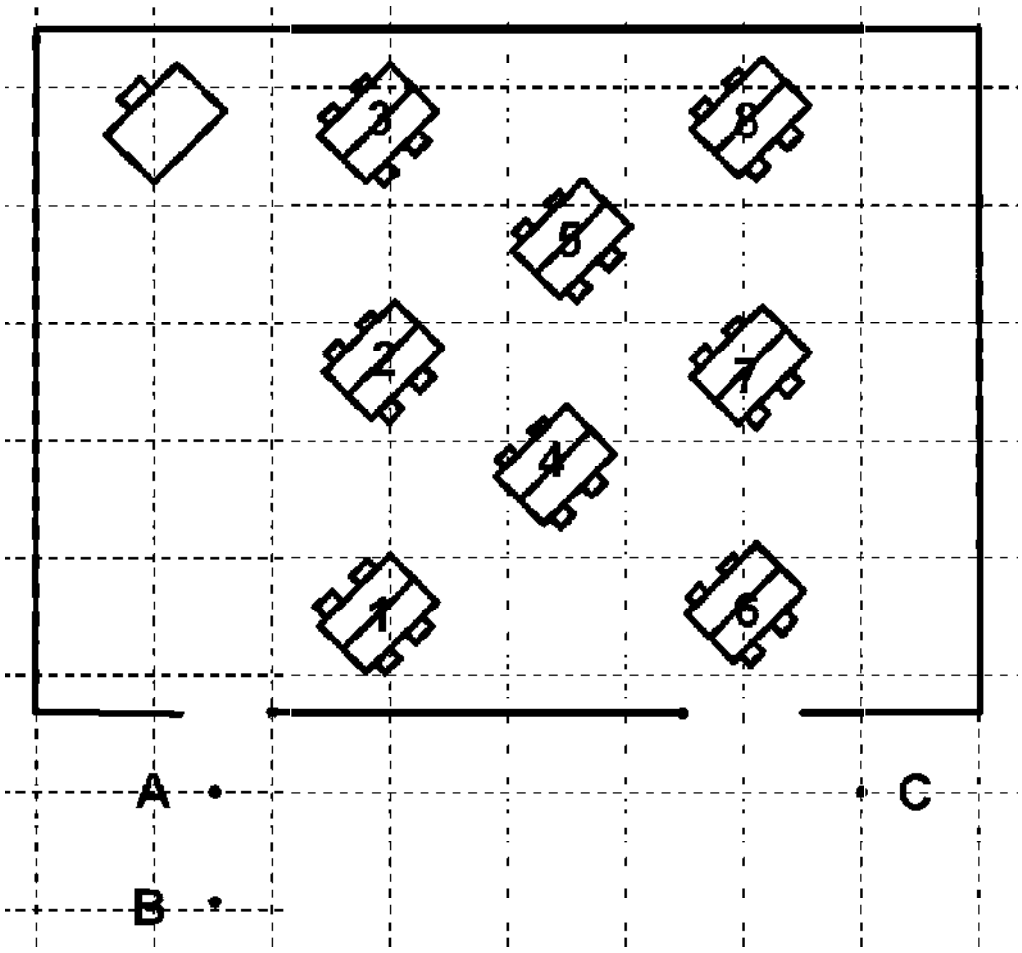
6. Masalah 6

Bandingkanlah gambar-gambar tersebut diatas. Apakah ada perbedaan daerah yang tidak dapat dilihat oleh pengamat diantara ketiga gambar tersebut? Jelaskan jawabanmu!

Lembar Kerja Siswa IV

1. Masalah 1

Perhatikan gambar denah ruang kelas berikut ini!



Gambar diatas adalah **tampak atas** dari denah sebuat ruang kelas.

A, B, dan C masing-masing adalah posisi seorang anak yang berdiri melihat ke dalam kelas.

a) Gambarlah **sudut pandang** untuk setiap posisi anak yang berdiri pada denah tersebut.

b) Dapatkah anak yang berdiri di posisi C melihat meja guru? Tuliskan alasanmu!

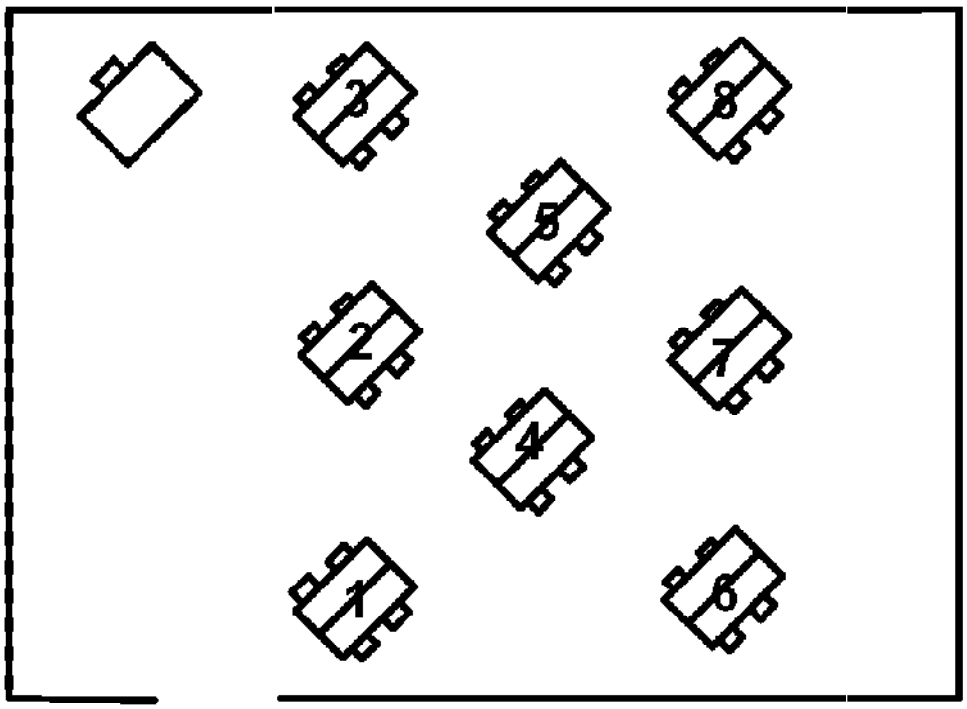
c) Adakah bangku yang tidak dapat dilihat oleh anak yang berdiri di posisi A tersebut? Jika ada, bangku berapa sajakah itu? Tuliskan alasanmu!

d) Diantara ke tiga posisi tersebut, posisi yang mana sajakah anak yang berdiri bisa melihat bangku nomor 5? Tuliskan alasanmu!

e) Diantara posisi A dan B, posisi manakah yang melihat lebih banyak daerah di dalam kelas? Jelaskan alasanmu!

2. Masalah 2

Perhatikan gambar denah ruang kelas berikut ini!



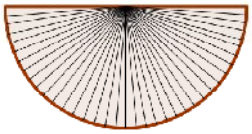
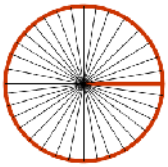
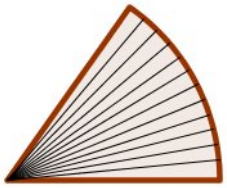
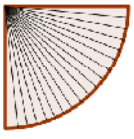
a) Gambar diatas adalah **tampak atas** dari denah sebuat ruang kelas. Potongan kertas yang diberikan oleh guru adalah representasi dari daerah yang dapat dilihat oleh anak yang berdiri memandang ke dalam kelas. Tentukan posisi anak tersebut berdasarkan potongan kertas yang telah dibagikan!

b) Bandingkanlah potongan kertas yang telah dibagikan tersebut. Tentukan potongan kertas manakah yang mempunyai sudut paling besar dan potongan kertas manakah yang mempunyai sudut paling kecil? Jelaskan Alasanmu!

Lembar Kerja Siswa V

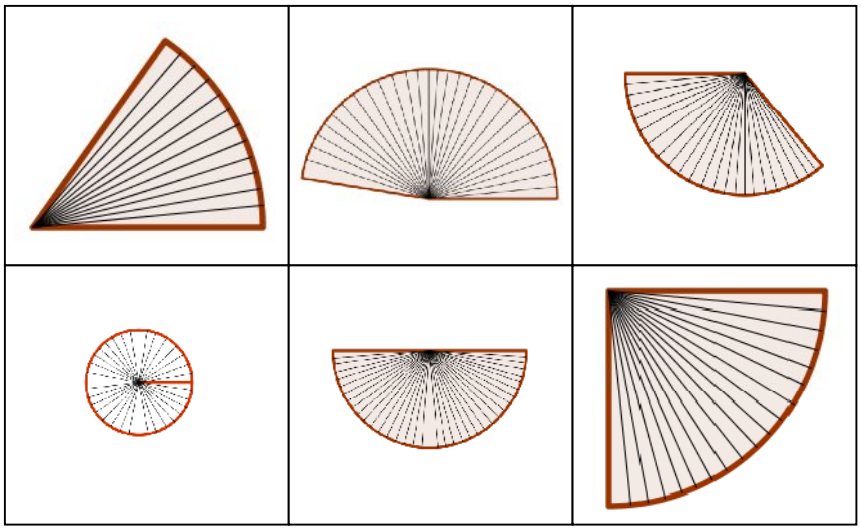
1. Masalah 1

Dibawah ini adalah beberapa bentuk kipas kertas. Tentukanlah jenis sudut yang dibentuk oleh kipas kertas tersebut.

		
		
Sudut lancip	Sudut lancip	Sudut tumpul
		
Sudut siku-siku	Sudut siku-siku	Sudut tumpul

2. Masalah 2

Urutkanlah gambar kipas kertas dibawah ini dari yang mempunyai sudut paling kecil sampai yang mempunyai sudut paling besar!



3. Masalah 3

Perhatikan bentuk-bentuk kipas tersebut. Dapatkah kalian menggabung (dua atau lebih) bentuk-bentuk kipas tersebut sehingga membentuk jenis sudut lain! Ceritakan cara kalian menjawab!

Appendix VI

Rencana Pelaksanaan Pembelajaran 1

(RPP)

Sekolah : SD Muhammadiyah 6 Palembang

Mata Pelajaran : Matematika

Kelas/Semester : II/Genap

Pertemuan : 1 (Pertama)

Alokasi Waktu : 2 x 35 menit

Standar Kompetensi : 4. Memahami unsur dan sifat-sifat bangun datar sederhana

Kompetensi Dasar **:** Mengidentifikasi berbagai jenis sudut

A. Tujuan Pembelajaran:

- Siswa menyadari tentang berbagai sudut pandang dalam melihat suatu objek.
- Siswa mengembangkan kemampuan perspektif spasial mereka yakni kemampuan untuk melihat objek dari sudut pandang orang lain.
- Siswa mengembangkan kemampuan visualisasi dan penalaran spasial mereka.
- Siswa mulai mengetahui tentang konsep garis pandang dan daerah yang tidak terlihat (*blind spots*) secara informal.

B. Indikator :

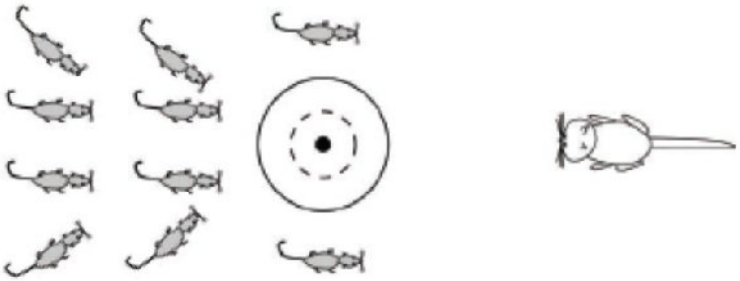
- Siswa dapat menjelaskan situasi kucing dan tikus yang dilihat dari tampak depan dan tampak atas.
- Siswa dapat menjelaskan tikus yang dapat dilihat dari sudut pandang kucing.
- Siswa dapat menyebutkan alasan mereka tentang mengapa ada beberapa tikus yang dapat dan tidak dapat dilihat oleh kucing.
- Siswa dapat menjelaskan perubahan garis pandang kucing pada saat berada dekat dan jauh dari tempat makanan.
- Siswa dapat menggambar garis pandang yang berasal dari mata kucing ke tikus yang berada di dekat tempat makanan.

C. Materi Pembelajaran

Konsep sudut adalah salah satu konsep geometri yang sulit untuk dipahami oleh siswa karena konsep sudut tidak mempunyai satu definisi tunggal. Untuk memepelajari konsep sudut tersebut, siswa di bimbing untuk menemukan sendiri konsep tersebut melalui beberapa kegiatan dan aktivitas yang melibatkan konsep garis pandang. Pada pembelajaran di pertemuan pertama ini, guru memberikan masalah tentang menentukan jumlah tikus yang dapat dilihat oleh kucing pada posisi tertentu kepada siswa untuk di kerjakan. Ada tiga gambar berbeda yang dapat di analisis oleh siswa dan ditentukan berapa jumlah tikus yang dapat dilihat oleh kucing pada setiap gambar tersebut.

Gambar 1. Masalah pertama

Gambar 2. Masalah kedua



Gambar 3. Masalah ketiga

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	• Berdoa *Apersepsi* • Siswa ditanya tentang pengetahuan awalnya mengenai tikus dan kucing, misalnya pengalaman mereka tentang kucing dan tikus, pernah melihat kucing atau tikus di rumah mereka. • Guru memotivasi siswa dengan menyampaikan kegiatan yang akan dilakukan oleh siswa. • Guru membagi siswa kedalam beberapa kelompok kecil yang terdiri dari 4 atau 5 orang siswa.	20 menit
Kegiatan Inti	1. Guru membagikan Lembar Kerja Siswa (LKS) kepada siswa. 2. Guru menjelaskan masalah 1, 2 dan 3 yang menunjukkan gambar pertama, kedua dan ketiga secara berurutan kepada siswa yaitu: *Seekor kucing berjalan mendekati sebuah tempat makanan dengan beberapa tikus kecil yang berada di belakangnya. Jika kucing tersebut berhenti pada posisi seperti pada gambar, berapa banyak tikus kecil yang* ***dapat dilihat*** *oleh kucing*	40 menit

	tersebut? Jelaskan alasanmu! kalian boleh menggambar/mencoret gambar tersebut untuk mendukung jawabanmu. 3. Siswa diberi waktu 15 menit per masalah untuk berdiskusi dengan teman kelompoknya tentang berapa banyak tikus yang dapat dilihat oleh kucing tersebut beserta dengan alasannya. 4. Guru meminta siswa untuk mendiskusikan masalah kedua dan ketiga bersama dengan teman kelompoknya. 5. Guru memantau diskusi siswa dan mencatat jawaban siswa yang berbeda. 6. Guru meminta siswa yang mempunyai jawaban yang berbeda-beda untuk menjelaskan jawaban dan alasan mereka kepada siswa yang lain. 7. Guru membimbing siswa untuk melakukan diskusi kelas. Beberapa topik diskusi yaitu: *Apa yang dapat kamu lakukan pada gambar tersebut untuk mendukung jawaban kamu?* *Bagaimana caranya kucing tersebut bisa melihat tikus itu?* *Coba perhatikan gambar kedua! Pada gambar kedua ini saya bisa melihat tikus ini, tapi pada gambar pertama tidak? Kenapa ya kira-kira? Ada yang bisa bantu saya mencari tahu alasannya?*	

	Saya tidak yakin kalau kucing tersebut bisa melihat keempat tikus tersebut secara keseluruhan, kalian setuju atau tidak? Mengapa tidak? *Adakah perbedaan jumlah tikus yang dapat dilihat oleh kucing dari ketiga gambar tersebut? mengapa demikian?*	
Kegiatan akhir	Siswa dan guru melakukan refleksi pembelajaran dengan memberikan beberapa pertanyaan seperti: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? • Adakah perbedaan jumlah tikus yang dapat dilihat oleh kucing ketika dekat atau jauh dari tempat makanan?	10 menit

F. Media Pembelajaran

- Lembar Kerja Siswa 1
- Penggaris

G. Penilaian

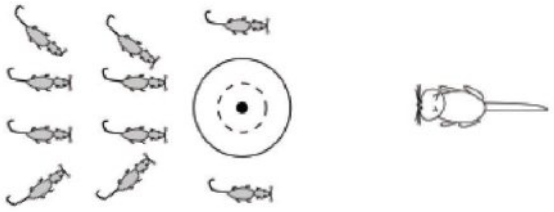
Bentuk test : LKS 1

Bentuk soal : Tertulis isian

Indikator dan soal

Indikator	Soal
1. Siswa dapat menetukan jumlah tikus yang dapat dilihat oleh kucing berdasarkan gambar	Berapa banyak tikus kecil yang **dapat dilihat** oleh kucing tersebut? Jelaskan alasanmu! (anda boleh menggunakan gambar dibawah ini).

tampak depan yang ada di LKS 1.	 Gambar 1.
2. Siswa dapat membandingkan antara jumlah tikus yang dapat dilihat kucing pada gambar tampak depan dan gambar tampak atas.	Apakah ada perbedaan antara jumlah tikus kecil yang dapat dilihat oleh kucing pada **gambar 1** dengan **gambar 2**? Jelaskan alasanmu! (anda boleh menggunakan gambar dibawah ini).  Gambar 1  Gambar 2
3. Siswa mengetahui bahwa garis pandang kucing berubah ketika kucing tersebut bergerak mendekati tempat makanan tersebut.	Kemudian, kucing tersebut bergerak mendekati tempat makanan tersebut. Tikus kecil tersebut tetap diam dan tidak bergerak dari tempatnya. Seorang siswa, Andy berpendapat bahwa jika kucing bergerak semakin mendekati tempat makanan tersebut, dan tikus kecil tersebut tidak berpindah tempat, maka semakin sedikit tikus kecil yang dapat dilihat oleh kucing tersebut. Apakah kamu setuju dengan pendapat Andy atau tidak? Jelaskan alasanmu! (kamu boleh menggunakan gambar dibawah ini).

	 Gambar 3.

Rubrik Penilaian

No	Kunci jawaban	Skor
1.	Jumlah tikus yang dapat dilihat adalah 6, karena ada satu tikus yang tidak terlihat yang berada di samping tempat makanan.	4
2.	Tidak ada perbedaan dalam hal jumlah tikus yang dapat dilihat oleh kucing pada kedua gambar tersebut karena kedua gambar tersebut diambil dari situasi yang sama.	5
3.	Setuju. Karena ketika kucing semakin mendekat ke tempat makanan, pandangan kucing tersebut semakin terhalangi oleh tempat makanan sehingga semakin sedikit tikus yang dapat dilihatnya.	4
	Jawaban salah	0
	Total skor	13

$$\text{Nilai} = \frac{\qquad}{\qquad} \times 100$$

Palembang, Maret 2013

Guru Kelas IIIE, Peneliti

Humaro, S.Pd. Bustang

Menyetujui,
Kepala SD Muhammadiyah 6 Palembang

Mardiana Sari, S.Pd.

Rencana Pelaksanaan Pembelajaran 2

(RPP)

Sekolah : SD Muhammadiyah 6 Palembang
Mata Pelajaran : Matematika
Kelas/Semester : II/Genap
Pertemuan : 2 (kedua)
Alokasi Waktu : 2 x 35 menit
Standar Kompetensi : 4. Memahami unsur dan sifat-sifat bangun datar sederhana
Kompetensi Dasar **:** Mengidentifikasi berbagai jenis sudut

A. Tujuan Pembelajaran:

- Siswa mampu mengkonstruksi garis pandang dan daerah yang tidak terlihat.
- Siswa menyadari bahwa garis pandang dapat diperpanjang sampai tak berhingga.
- Siswa menyadari bahwa garis pandang berbentuk garis lurus dan miring.
- Siswa mengetahui bahwa garis pandang adalah garis yang berasal dari mata pengamat ke objek yang dilihat.
- Siswa mampu menjelaskan perubahan garis pandang yang terjadi pada saat pengamat mendekat atau menjauh dari layar.

B. Indikator :

- Siswa mampu mengkonstruksi garis pandang dan daerah yang tidak terlihat untuk beberapa pengamat yang berbeda.
- Siswa dapat menjelaskan bahwa garis pandang dapat diperpanjang sampai tak berhingga.
- Siswa dapat menjelaskan bentuk garis pandang yang terbentuk.
- Siswa dapat menjelaskan tentang garis pandang dari kegiatan yang mereka lakukan.
- Siswa mampu menjelaskan tentang perubahan garis pandang yang terjadi pada saat pengamat mendekat atau menjauh dari layar.

C. Materi Pembelajaran

Garis pandang adalah garis-garis lurus yang berasal dari mata seorang pengamat ke objek atau benda yang dilihatnya. Konsep garis pandang merupakan konsep yang abstrak. Oleh karena itu, siswa dituntun untuk mengkonstruksi garis pandang tersebut dalam bentuk tiga dimensi. Siswa akan mengonstruksi garis pandang dan daerah yang tidak terlihat dari pengamat yang berbeda-beda posisinya.

(source: Munier, et al., 2008)

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	• Berdoa *Apersepsi* • Siswa diingatkan kembali tentang masalah kucing dan tikus pada pertemuan sebelumnya dan mengaitkan masalah tersebut dengan kegiatan yang akan dilakukan. *Guru:kemarin kita telah menyelesaikan masalah tentang berapa jumlah tikus yang dapat dilihat oleh kucing. Dari diskusi kemarin, ada beberapa jawaban yang*	20 menit

	berbeda yang kita dapatkan. Untuk mengetahui berapa jumlah tikus yang dapat dilihat oleh kucing tersebut yang sebenarnya, maka hari ini kita akan melakukan kegiatan eskperimen. • Guru memotivasi siswa dengan menyampaikan kegiatan yang akan dilakukan oleh siswa. *Guru: Pada pertemuan ini, kita akan melakukan kegiatan eksperimen di lapangan sekolah. Pada kegiatan tersebut, ada kelompok yang bertindak sebagai kucing (pengamat) yang duduk di kursi menghadap layar (tempat makanan). Siswa yang lain berdiri di belakang layar berpasang-pasangan dengan membawa tas masing-masing.* • Guru membagi siswa kedalam beberapa kelompok kecil yang terdiri dari 4 atau 5 orang siswa.	
Kegiatan Inti	1. Guru meminta siswa kelompok 1 untuk menjadi pengamat dan siswa kelompok lain berdiri di belakang layar berpasang-pasangan. *2.* Guru menjelaskan aturan kegiatan eksperimen kepada siswa: *Guru: setiap kelompok akan mempunyai kesempatan menjadi pengamat. Siswa yang berdiri di belakang layar bergerak ke*	40 menit

	samping kiri atau kanan layar sampai pengamat dapat melihat sebagian badan mereka kemudian berhenti dan meletakkan tas mereka di tempat berhenti tersebut. 3. Guru membimbing siswa untuk melakukan kegiatan eksperimen dan memastikan bahwa setiap kelompok mempunyai kesempatan untuk menjadi pengamat. 4. Guru mengajukan beberapa pertanyaan terkait dengan kegiatan eksperimen yang dilakukan oleh siswa, seperti: *Menurut kamu bagaimana bentuk tas-tas tersebut?* *Bagaimana kita bisa yakin bahwa bahwa tas tersebut berada pada satu garis?* *Bisakah kita menggunakan tali atau lakban tersebut?* 5. Guru membimbing siswa melanjutkan kegiatan eksperimen untuk posisi pengamat yang berbeda (menjauh, mendekat, kiri dan kanan dari layar) dengan siswa dari kelompok yang berbeda yang menjadi pengamatnya secara bergantian. 6. Guru membimbing siswa untuk melakukan diskusi bersama-sama. Beberapa topik diskusi yaitu: *Apa yang terjadi pada bentuk tas sekolah tersebut jika pengamat bergerak mendekat atau menjauh dari layar?* *Bagaimana jika pengamat bergerak ke*	

	sebelah kiri atau kanan layar? *Dapatkah kamu memberikan penjelasan mengapa hal itu terjadi?*	
Kegiatan akhir	Siswa dan guru melakukan refleksi pembelajaran dengan memberikan beberapa pertanyaan seperti: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? • Apa yang terjadi jika pengamat bergerak mendekat, menjauh, ke sebelah kiri, atau ke sebelah kanan layar? • Dapatkah kalian menyimpulkan apa yang membuatnya berbeda?	10 menit

F. Media Pembelajaran

- Layar
- Kursi
- Tali
- Lakban hitam
- Tas sekolah

G. Penilaian

Bentuk test : Diskusi dan Tanya jawab

Bentuk soal : lisan dan observasi

Rubrik observasi

Kriteria pengamatan	Ya	Tidak
1. Siswa mampu mengonstruksi garis pandang pengamat yang berbeda-beda dengan benar.		
2. Siswa mampu menjelaskan bahwa tali (garis pandang) tersebut berbentuk lurus dan dapat		

diperpanjang sampai tak berhingga.		
3. Siswa mampu menjelaskan bahwa tali (garis pandang) tersebut berbeda-beda untuk setiap posisi pengamat yang berbeda.		
4. Siswa mampu menjelaskan bahwa yang membedakan garis pandang pengamat-pengamat tersebut adalah sudut atau sudut pandangnya.		
5. Siswa mampu menjelaskan bahwa garis pandang semakin mengecil ketika pengamat bergerak ke samping kiri atau kanan layar.		

Palembang, Maret 2013

Guru Kelas IIIE, Peneliti

Humaro, S.Pd. Bustang

Menyetujui,

Kepala SD Muhammadiyah 6 Palembang

Mardiana Sari, S.Pd.

Rencana Pelaksanaan Pembelajaran 3

(RPP)

Sekolah : SD Muhammadiyah 6 Palembang

Mata Pelajaran : Matematika

Kelas/Semester : II/Genap

Pertemuan : 3 (ketiga)

Alokasi Waktu : 2 x 35 menit

Standar Kompetensi : 4. Memahami unsur dan sifat-sifat bangun datar sederhana

Kompetensi Dasar **:** Mengidentifikasi berbagai jenis sudut

A. Tujuan Pembelajaran:

- Siswa mampu membayangkan dan memvisualisasikan garis pandang untuk pengamat yang berbeda-beda.
- Siswa mampu menjelaskan dan mengaitkan perbedaan garis pandang tersebut dengan konsep sudut.

B. Indikator :

- Siswa mampu mewarnai daerah yang tidak terlihat (*blind spots*) untuk posisi pengamat yang berbeda.
- Siswa mampu membandingkan beberapa daerah yang tidak terlihat yang berbeda.
- Siswa mampu menyadari bahwa sudut yang membuat gambar tersebut berbeda.
- Siswa mampu bernalar dengan menggunakan kata-kata mereka sendiri dalam hal konsep sudut.
- Siswa akan menyadari bahwa sudut tetap sama untuk satu posisi pengamat, tidak perduli bagaimanapun panjang garis pembatasnya (garis pembatasnya adalah garis pandang).

C. Materi Pembelajaran

Mengembangkan kemampual visualisasi and penalaran spasial siswa merupakan hal yang sangat penting. Hal ini dapat dilakukan dengan membimbing siswa untuk menggambar garis pandang dan daerah yang tidak terlihat pada kertas (2 dimensi). Selain itu, dengan menggambar garis pandang

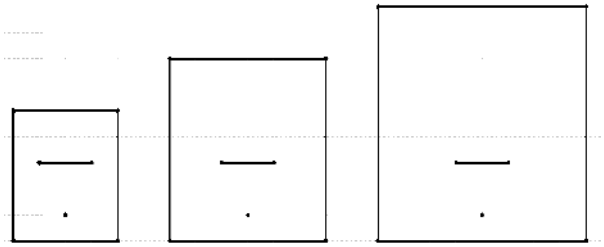
dan daerah yang tidak terlihat untuk posisi pengamat yang berbeda-beda, siswa dapat sampai pada konsep sudut.

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	• Berdoa *Apersepsi* • Siswa diingatkan kembali tentang kegiatan eksperimen yang telah mereka lakukan pada pertemuan sebelumnya. • Guru memotivasi siswa dengan menyampaikan kegiatan yang akan dilakukan oleh siswa. *Guru: Pada pertemuan ini, kita akan menggambar garis pandang dan daearah yang tidak terlihat untuk posisi pengamat yang berbeda-beda pada LKS yang telah disediakan. Pada LKS tersebut, terdapat gambar pengamat dan layar yang dilihat dari atas.* • Guru membagi siswa kedalam beberapa kelompok kecil yang terdiri dari 4 atau 5 orang siswa.	20 menit
Kegiatan Inti	1. Guru membagikan LKS kepada siswa dan menjelaskan tentang masalah yang ada pada LKS tersebut.	40 menit

	2. Guru menjelaskan petunjuk mengerjakan masalah yang ada pada LKS tersebut kepada siswa: *Perhatikan beberapa gambar dibawah ini. Gambar tersebut adalah gambar **tampak atas** dari kegiatan eksperimen yang kita lakukan kemarin. Titik pada gambar tersebut adalah sang pengamat (siswa yang duduk di kursi menghadap layar), dan garis tersebut adalah layar yang berada di hadapan pengamat tersebut.* *Arsirlah daerah yang tidak dapat dilihat oleh pengamat.*  3. Guru membimbing siswa untuk melakukan kegiatan diskusi dengan teman kelompok mereka masing-masing. 4. Guru meminta siswa untuk menjawab masalah selanjutnya yang ada pada LKS tersebut. 5. Guru membandingkan jawaban siswa yang berbeda-beda sambil memeriksa jalannya diskusi siswa dengan teman kelompoknya. 6. Guru membimbing siswa untuk melakukan diskusi bersama-sama. Beberapa topik diskusi yaitu:	

	Apa yang membuat gambar tersebut berbeda-beda? *Konsep matematika apa yang dapat kalian simpulkan dari kegiatan membandingkan gambar-gambar tersebut?* *Apa yang terjadi jika pengamat bergerak mendekat atau menjauh dari layar? Mengapa demikian?*	
Kegiatan akhir	Siswa dan guru melakukan refleksi pembelajaran dengan memberikan beberapa pertanyaan seperti: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari?	10 menit

F. Media Pembelajaran

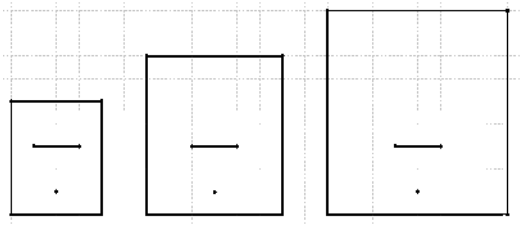
- Pensil warna
- penggaris

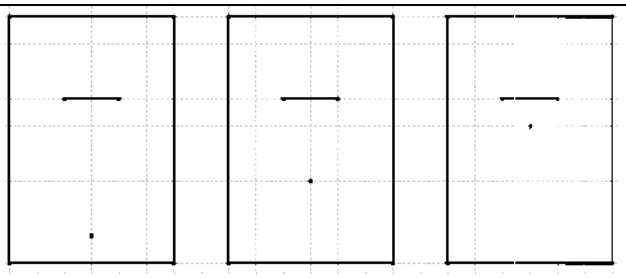
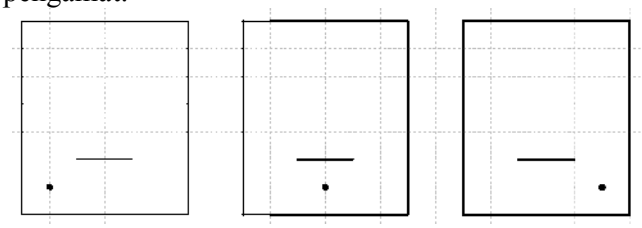
G. Penilaian

Bentuk test : LKS III

Bentuk soal : tertulis dan isian

Indikator dan soal

Indikator	Soal
1. Siswa mampu menjelaskan bahwa panjang garis pandang tidak mempengaruhi ukuran sudut pandang pengamat.	Arsirlah daerah yang **tidak dapat** dilihat oleh pengamat. 
2. Siswa mampu	Arsirlah daerah yang **tidak dapat** dilihat oleh pengamat.

menjelaskan bahwa semakin dekat pengamat maka semakin besar pula sudut pandangnya, begitupun sebaliknya.	
3. Siswa mampu menjelaskan bahwa daearah yang tidak terlihat semakin mengecil ketika pengamat bergerak kesamping kiri dan kanan layar.	Arsirlah daerah yang **tidak dapat** dilihat oleh pengamat. 

Palembang, Maret 2013

Guru Kelas IIIE, Peneliti

Humaro, S.Pd. Bustang

Menyetujui,

Kepala SD Muhammadiyah 6 Palembang

Mardiana Sari, S.Pd.

Rencana Pelaksanaan Pembelajaran 4

(RPP)

Sekolah : SD Muhammadiyah 6 Palembang

Mata Pelajaran : Matematika

Kelas/Semester : II/Genap

Pertemuan : 4 (keempat)

Alokasi Waktu : 2 x 35 menit

Standar Kompetensi : 4. Memahami unsur dan sifat-sifat bangun datar sederhana

Kompetensi Dasar **:** Mengidentifikasi berbagai jenis sudut

A. Tujuan Pembelajaran:

- Siswa mampu mengkonstruksi dan menggambar sudur pandang dari posisi pengamat yang berbeda-beda.

B. Indikator :

- Siswa mampu mengkonstruksi sudut pandang dari pengamat.
- Siswa mengetahui bahwa sudut pandang berbeda-beda untuk posisi observer yang berbeda.
- Siswa mampu menempatkan sudut pandang dari suatu pengamat pada posisi tertentu.
- Siswa menyadari bahwa suatu posisi dapat ditempati oleh berbeda-beda potong kertas yang mempunyai sudut yang sama.
- Siswa mampu membedakan mana potongan kertas yang mempunyai sudut yang besar dan potongan kertas yang mempunyai sudut yang kecil.
- Siswa menyadari bahwa panjang sisi potongan kertas tersebut tidak mempengaruhi ukuran sudut dari potongan kertas terebut.

C. Materi Pembelajaran

Sudut pandang adalah sudut yang dibentuk oleh dua buah garis pandang yang bertemu pada satu titik. Sudut pandang adalah konsep matematika yang abstrak. Oleh karena itu, dengan membimbing siswa mengkonstruksi sudut pandang pengamat yang berbeda-beda, mereka akan memiliki pemahaman tentang konsep sudut pandang tersebut.

Pada pembelajaran ini, guru menuntun siswa untuk mengkonstruksi sudut pandang pengamat yang berbeda-beda kemudian menggambarnya pada LKS yang telah disediakan.

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	• Berdoa *Apersepsi* • Siswa diingatkan kembali tentang konsep sudut pada pertemuan sebelumnya. • Guru memotivasi siswa dengan menyampaikan kegiatan yang akan dilakukan oleh siswa. *Guru: Pada pertemuan ini, kita akan mengonstruksi dan menggambar sudut pandang untuk posisi pengamat yang berbeda-beda. Setelah mengonstruksi sudut pandang tersebut, kita akan mengerjakan masalah yang ada pada LKS yang berkaitan dengan sudut pandang. Pada LKS tersebut, terdapat gambar denah kelas dan pengamat yang dilihat dari atas.* • Guru membagi siswa kedalam beberapa kelompok kecil yang terdiri dari 4 atau 5 orang siswa.	20 menit

Kegiatan Inti	1. Guru meminta salah satu siswa untuk menjadi pengamat. Siswa tersebut keluar kelas dan berdiri di depan pintu menghadap ke dalam kelas tersebut. Guru bertanya kepada siswa tersebut apa saja yang dapat ia lihat didalam kelas dan apakah ia dapat melihat semua siswa di dalam kelas atau tidak. 2. Guru kemudian berdiskusi dengan siswa tentang mengapa ada yang tidak dapat dilihat dan ada yang dapat dilihat oleh pengamat tersebut? Guru juga bertanya kepada siswa mengapa ada siswa di dalam kelas yang tidak dapat melihat pengamat di luar kelas tersebut? 3. Guru kemudian berdiskusi lebih jauh dengan siswa dengan mengajukan beberapa pertanyaan, seperti: *Apa yang terjadi dengan tali tersebut jika pengamat bergerak maju mendekati pintu kelas atau bergerak mundur menjauhi kelas* *Bagaimana jika pengamat bergerak ke sisi sebelah kiri atau ke sebelah kanan dari pintu tersebut?* 4. Guru meminta siswa untuk berkumpul dengan kelompok mereka masing-masing dan membagikan LKS 1 kepada mereka yang menunjukkan tampak atas dari kegiatan eksperimen yang telah mereka lakukan sebelumnya. Guru menjelaskan kepada mereka bahwa mereka harus	40 menit

	menggambar dan mewarnai **daerah yang terlihat** oleh pengamat yakni daerah dimana pengamat yang berdiri di depan kelas tersebut dapat melihat situasi di dalam kelas. 5. Guru membimbing siswa untuk melakukan kegiatan diskusi dengan teman kelompok mereka masing-masing. 6. Guru meminta siswa untuk menjawab masalah selanjutnya yang ada pada LKS tersebut. 7. Guru membandingkan jawaban siswa yang berbeda-beda sambil memeriksa jalannya diskusi siswa dengan teman kelompoknya. 8. Guru membimbing siswa untuk melakukan diskusi bersama-sama. Beberapa topik diskusi yaitu: *Potongan kertas manakah yang mempunyai sudut paling besar? Jelaskan alasanmu!* *Potongan kertas manakah yang mempunyai sudut paling kecil? Jelaskan alasanmu!* *Manasajakah potongan kertas tersebut yang mempunyai sudut yang sama?Jelaskan alasanmu?*	
Kegiatan akhir	Siswa dan guru melakukan refleksi pembelajaran dengan memberikan beberapa pertanyaan seperti: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? • *Apa yang membedakan sudut pandang dari pengamat-pengamat tersebut?*	10 menit

	• *Mengapa untuk satu posisi pengamat yang sama bisa mempunyai potongan kertas yang berbeda?* • *Apakah ada pengaruh panjang garis pandang terhadap sudut pandang pengamat yang memiliki posisi yang sama?*	

F. Media Pembelajaran

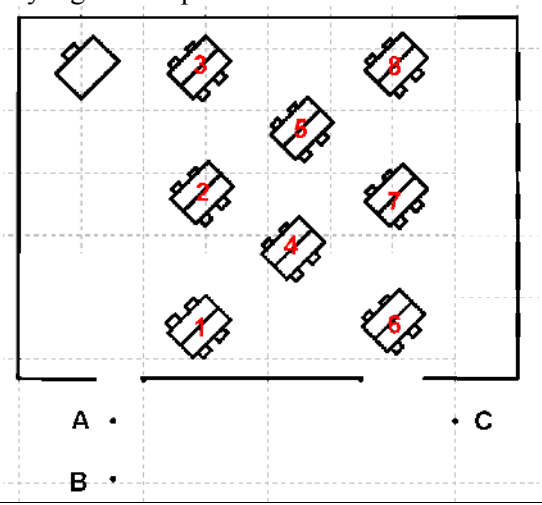
- LKS IV
- Pensil warna
- penggaris

G. Penilaian

Bentuk test : LKS IV

Bentuk soal : tertulis dan isian

Indikator dan soal

Indikator	Soal
1. Siswa mampu menggambar sudut pandang untuk posisi pengamat yang berbeda-beda. 2. Siswa mampu menjelaskan bahwa panjang garis pandang tidak mempengaruhi ukuran sudut pandang pengamat.	Gambarlah **sudut pandang** untuk setiap posisi anak yang berdiri pada denah tersebut. 
	Dapatkah anak yang berdiri di posisi C melihat meja guru? Tuliskan alasanmu!
	Adakah bangku yang tidak dapat dilihat oleh anak

	yang berdiri di posisi A tersebut? Jika ada, bangku berapa sajakah itu? Tuliskan alasanmu!
	Diantara ke tiga posisi tersebut, posisi yang mana sajakah anak yang berdiri bisa melihat bangku nomor 5? Tuliskan alasanmu!
	Diantara posisi A dan B, posisi manakah yang melihat lebih banyak daerah di dalam kelas? Jelaskan alasanmu!

Palembang, Maret 2013

Guru Kelas IIIE, Peneliti

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Rencana Pelaksanaan Pembelajaran 5

(RPP)

Sekolah	: SD Muhammadiyah 6 Palembang
Mata Pelajaran	: Matematika
Kelas/Semester	: II/Genap
Pertemuan	: 5 (kelima)
Alokasi Waktu	: 2 x 35 menit
Standar Kompetensi	: 4. Memahami unsur dan sifat-sifat bangun datar sederhana
Kompetensi Dasar	**:** Mengidentifikasi berbagai jenis sudut

A. Tujuan Pembelajaran:

- Siswa memahami tentang sudut lancip, siku-siku, tumpul, lurus, satu putaran dan refleksi dalam berbagai bentuk dan orientasi.

B. Indikator :

- Siswa mampu menyadari tentang sudut lancip, siku-siku, tumpul, lurus, refleksi, dan satu putaran.
- Siswa menyadari bahwa sudut yang sama dapat di letakkan atau dibentuk dengan berbagai macam orientasi.
- Siswa menyadari tentang sudut refleksi (sudut dalam dan sudut luar) untuk sepasang garis yang berpotongan di tengah.

C. Materi Pembelajaran

Mengidentifikasi berbagai jenis sudut merupakan materi yang cukup sulit bagi siswa. Dalam pembelajaran ini, siswa dibimbing untuk mengenal berbagai jenis sudut, diantaranya sudut lancip, siku-siku, tumpul, lurus, refleksi dan satu putaran. Sudut lurus, refleksi dan satu putaran adalah jenis sudut yang sangat sulit diindentifikasi oleh sebagian besar siswa. Dengan menggunakan kipas kertas, siswa dibimbing untuk melihat daerah yang terbentuk diantara dua buah lengan kipas kertas tersebut sehingga siswa mengetahui tentang sudut lurus, refleksi dan satu putaran.

D. Pendekatan Pembelajaran

Pendekatan PMRI (Pendidikan Matematika Realistik Indonesia)

E. Kegiatan Pembelajaran

Kegiatan	Uraian	Waktu
Kegiatan Awal	• Berdoa *Apersepsi* • Siswa diingatkan kembali tentang konsep sudut pada pertemuan sebelumnya. • Guru memperkenalkan kipas kertas kepada siswa dan bertanya kepada mereka apakah mereka familiar dengan kipas kertas tersebut. • Guru memotivasi siswa dengan menyampaikan kegiatan yang akan dilakukan oleh siswa. *Guru: Pada pertemuan ini, kita akan memepelajari berbagai macam sudut dengan menggunakan kipas kertas seperti yang kita lihat sekarang. Kipas kertas tersebut akan saya bukan pelan-pelan dan kalian menyebutkan apakah ada sudut atau tidak pada posisi tertentu dari kipas kertas tersebut.*	20 menit
Kegiatan Inti	1. Guru meletakkan kipas kertas tersebut didepan matanya dan mulai membuka kipas kertas tersebut sedikit demi sedikit. Dalam hal ini, Guru bertanya kepada siswa tentang apa yang mereka lihat dan apa yang terjadi dengan kertas diantara	40 menit

	dua lengan kipas kertas tersebut. *Apakah ada sudut pada posisi kipas kertas saat ini?* *Sudut apa yang dibentuk oleh kipas kertas tersebut?* 2. Guru kemudian melanjutkan membuka kipas kertas tersebut sampai membentuk sudut siku-siku. Kemudian, guru bertanya kepada siswa apakah mereka mengenali sudut siku-siku tersebut atau tidak. Sebagai tindak lanjut, guru kemudian bertanya kepada siswa untuk menyebutkan contoh-contoh sudut siku-siku yang ada di sekitar mereka. 3. Guru kemudian melanjutkan untuk membuka kipas kertas tersebut lebih lebar lagi sampai membentuk sudut tumpul dan kemudian sudut lurus. Guru kemudian bertanya kepada siswa apakah ada sudut pada kipas kertas tersebut atau tidak. Ada kemungkinan bahwa siswa tidak menyadari adanya sudut pada kipas kertas tersebut. dalam hal ini, guru dapat mengajukan pertanyaan seperti: *Apakah kalian melihat kertas yang berada diantara dua lengan kipas kertas tersebut?* *Apakah kertas tersebut mengindikasikan sudut seperti pada posisi kipas kertas sebelumnya* 4. Guru kemudian melanjutkan membuka kipas kertas tersebut lebih lebar lagi, lebih	

	dari sudut lurus. Guru kemudian bertanya kepada siswa ada berapa sudut yang terbentuk pada kipas kertas sekarang. Jika siswa tidak menyadari tentang sudut refleksi, guru membimbing siswa untuk melihat bahwa ada daerah lain yang dibentuk oleh dua lengan kipas kertas tersebut selain yang di representasikan oleh kertas tersebut. Guru kemudian berdiskusi dengan semua siswa di kelas tentang hal ini. 5. Guru kemudian membuka kipas kertas tersebut sampai terbuka penuh dan membentuk sudut satu putaran. Guru bertanya kepada siswa apakah mereka melihat ada sudut pada kipas kertas tersebut atau tidak. Mungkin ada siswa yang berpendapat ada sudut dan mungkin juga sebagian besar siswa mengatakan tidak ada. Dalam hal ini, guru kemudian mendiskusikan dengan siswa kemudian pada akhirnya guru memperkenalkan kepada siswa tentang nama sudut tersebut yang disebut dengan sudut satu putaran. 6. Guru meminta siswa untuk tetap berada pada kelompok mereka masing-masing seperti pada pertemuan sebelumnya kemudian guru membagikan LKS kepada siswa. Pada LKS tersebut, siswa diminta untuk memasangkan potongan gambar-gambar kipas kertas dan nama-nama sudut	

	yang sudah di gunting pada LKS tersebut dan menempelkannya pada poster yang telah disediakan. Siswa juga dapat mengkombinasikan dan menggabungkan gambar kipas kertas tersebut untuk membentuk sudut yang berbeda. 7. Guru membimbing siswa untuk melakukan kegiatan diskusi dengan teman kelompok mereka masing-masing. 8. Guru membandingkan jawaban siswa yang berbeda-beda sambil memeriksa jalannya diskusi siswa dengan teman kelompoknya. 9. Guru membimbing siswa untuk melakukan diskusi bersama-sama. Beberapa topik diskusi yaitu: *Bagaimana jika saya menggerakkan kipas kertas pada posisi seperti ini? Apakah masih sama sudutnya atau tidak?* *Bagaimana pendapat kamu tentang sudut pada posisi kipas kertas seperti ini (terbuka setengah lingkaran)? Apakah ada sudut atau tidak?* *Suut apasajakah yang dapat digabung sehingga membentuk jenis sudut baru?*	
Kegiatan akhir	Siswa dan guru melakukan refleksi pembelajaran dengan memberikan beberapa pertanyaan seperti: • Apa yang kita pelajari hari ini? • Hal penting apa saja yang kita pelajari? • *Jelaskan jenis-jenis sudut yang kalian pelajari hari ini?* • *Apa itu sudut lurus, refleksi dan satu putaran?*	10 menit

F. Media Pembelajaran

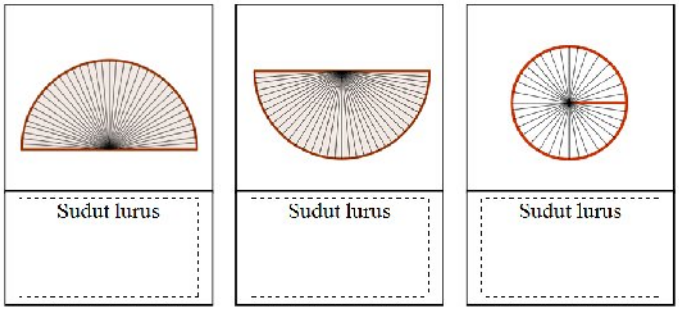
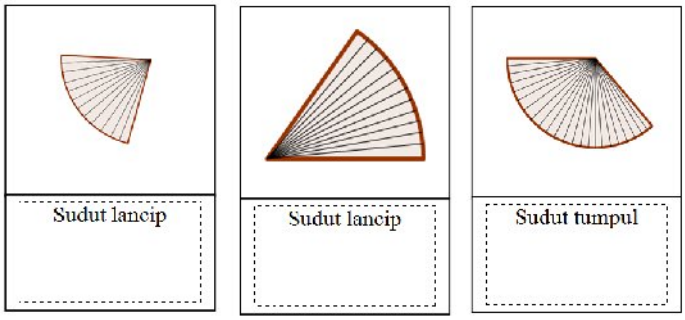
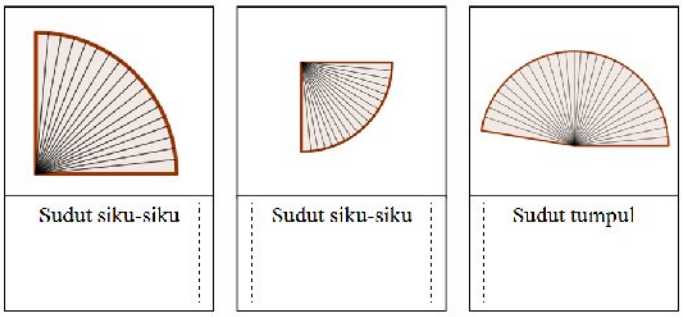
- LKS V
- Kertas poster
- Lem

G. Penilaian

Bentuk test : LKS V

Bentuk soal : tertulis dan isian

Indikator dan soal

No	Kunci Jawaban	Skor
1.		10
2.		10
3.		10
4.	Sudut lancip, sudut siku-siku, sudut tumpul, sudut tumpul, sudut lurus, sudut satu putaran	10

Nilai: (skor : 4) x 10

Palembang, Maret 2013

Guru Kelas IIIE,

Humaro, S.Pd.

Peneliti

Bustang

Menyetujui,

Kepala SD Muhammadiyah 6 Palembang

Mardiana Sari, S.Pd.

CURRICULUM VITAE

Bustang was born in Tacipi, Bone regency, South Sulawesi, Indonesia, on December 12, 1988. He took his first formal education in kindergarten TK Tomporeng Kesso, Tacipi and graduated in 1994. He continued his study into elementary school SD Inpres 10/73 Tacipi graduated in 2000. After that he went to junior high school SMPN 1 Ulaweng graduated in 2003. He completed his secondary school at the senior high school SMUN 1 Watampone graduated in 2006. In the same year, he studied mathematics education at the Mathematics Department, Faculty of Mathematics and Science, State University of Makassar and awarded him a bachelor degree, *sarjana pendidikan*, S.Pd in 2010. During the academic year of 2009 – 2011, he was devoted to be a lecturer assistant in the Mathematics Department, State University of Makassar. In 2011, he then followed the International Master Program on Mathematics Education in collaboration between Sriwijaya University, Indonesia and the Freudenthal Institute for Science and Mathematics Education, Utrecht University, The Netherlands.