DESIGN RESEARCH ON MATHEMATICS EDUCATION: DEVELOPING A MODEL TO SUPPORT STUDENTS IN SOLVING TWO DIGIT NUMBERS SUBTRACTION

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science (M. Sc)
in
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:
Nila Mareta Murdiyani
NIM. 20102812013

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
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APPROVAL PAGE

Research Title	:	Design Research on Mathematics Education: Developing a Model to Support Students in Solving Two Digit Numbers Subtraction
Student Name	:	Nila Mareta Murdiyani
Student Number	:	20102812013
Study Program	:	Mathematics Education

Approved by:

Supervisor I,

Prof. Dr. Zulkardi, M.I.Komp., M.Sc.

Supervisor II,

Dr. Ratu Ilma Indra Putri, M.Si

Head of
Mathematics Education Department,

Prof. Dr. Zulkardi, M.I.Komp., M.Sc.
NIP 19610420 198603 1 002

Dean of Faculty of Teacher Training
and Education,

Prof. Dr. Tatang Suhery, M.A., Ph.D.
NIP 19590412 198403 1 002

Date of Approval: May 2012

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NIM. 20102812013

This Thesis had been examined in May 15th , 2012

Examination Committee	**Signature**
1. Prof. Dr. Zulkardi, M.I.Komp.,M.Sc	__________________
2. Dr. Ratu Ilma Indra Putri, M.Si	__________________
3. Dr. Yusuf Hartono	__________________
4. Dr. Darmawijoyo	__________________
5. Dr. Somakim, M.Pd	__________________

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
MAY 2012

STATEMENT PAGE

I hereby,

Name	: Nila Mareta Murdiyani
Place of Birth	: Semarang
Date of Birth	: March 25^{th} , 1987
Academic Major	: Mathematics Education

State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that are presented in this thesis, except from reference source are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
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This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as cancelation of my degree that I have got through this thesis.

Palembang, May 16^{th} , 2012,

The one with the statement,

Nila Mareta Murdiyani

NIM. 20102812013

ABSTRACT

Subtraction has two meanings and each meaning leads to the different strategies. The meaning of "taking away something" suggests a direct subtraction, while the meaning of "determining the difference between two numbers" is more likely to be modeled as indirect addition. Many prior researches found that the second meaning and second strategy rarely appeared in the mathematical textbooks and teacher explanations, including in Indonesia. Therefore, this study was conducted to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate first grade of primary school students to develop a model in solving two digit numbers subtraction. Consequently, design research was chosen as an appropriate approach for achieving the research aim and Realistic Mathematics Education (RME) was used as a guide to design the lesson. This study involved 6 students in the pilot experiment, 31 students in the teaching experiment, and a first grade teacher of SDN 179 Palembang. The result of this study shows that the beads string could bridge students from the contextual problems (taking ginger candies and making grains bracelets) to the use of the empty number line. It also shows that the empty number line could promote students to use different strategies (direct subtraction, indirect addition, and indirect subtraction) in solving subtraction problems. Based on these findings, it is recommended to apply RME in the teaching learning process to make it more meaningful for students.

Key words: subtraction, design research, Realistic Mathematics Education, the beads string, the empty number line.

ABSTRAK

Pengurangan memiliki dua arti dan masing-masing arti mengarahkan ke strategi yang berbeda. Arti pengurangan sebagai “mengambil sesuatu” mendukung pengurangan langsung, sedangkan arti pengurangan sebagai “menentukan perbedaan dari dua bilangan” lebih mudah dimodelkan sebagai penjumlahan tidak langsung. Banyak penelitian sebelumnya menemukan bahwa arti pengurangan yang kedua dan strategi yang kedua jarang muncul di dalam buku matematika dan penjelasan guru, termasuk di Indonesia. Oleh karena itu, penelitian ini dilaksanakan dalam rangka memberikan kontribusi bagi pengembangan *local instruction theory* untuk pengurangan dengan mendesain aktivitas pembelajaran yang dapat memfasilitasi siswa kelas 1 sekolah dasar untuk mengembangan model dalam menyelesaikan pengurangan bilangan dua angka. Konsekuensinya, *design research* dipilih sebagai pendekatan yang sesuai untuk mencapai tujuan penelitian dan *Realistic Mathematics Education* (RME) digunakan sebagai panduan untuk mendesain pembelajaran. Penelitian ini melibatkan 6 siswa dalam *pilot experiment*, 31 siswa dalam *teaching experiment*, dan seorang guru kelas satu SDN 179 Palembang. Hasil dari penelitian ini menunjukkan bahwa manik-manik dapat menjembatani siswa dari masalah kontekstual (mengambil permen jahe dan membuat gelang biji-bijian) ke penggunaan garis bilangan kosong. Ini juga menunjukkan bahwa garis bilangan kosong dapat mendorong siswa untuk menggunakan strategi yang berbeda (pengurangan langsung, penjumlahan tidak langsung, dan pengurangan tidak langsung) dalam menyelesaikan masalah pengurangan. Berdasarkan penemuan ini, direkomendasikan untuk mengaplikasikan RME dalam proses belajar mengajar agar membuat pembelajaran semakin bermakna bagi siswa.

Kata kunci: pengurangan, design research, Realistic Mathematics Education, manik - manik, garis bilangan kosong.

SUMMARY

Subtraction is one of the basic number operations in mathematics which is familiar for students. Freudenthal (1983) said that subtraction results as the converse of addition and it often appears in students' daily life. Subtraction has two meanings and each meaning leads to the different strategies. According to Torbeyns, De Smedt, Stassens, Ghesquiere, & Verschaffel (2009), the meaning of "taking away something" suggests a direct subtraction, which means removing the subtrahend from the minuend. On the other hand, the meaning of "determining the difference between two numbers" is more likely to be modeled as indirect addition, which means counting on from the subtrahend until the minuend is reached. Many prior researches found that the second meaning and second strategy rarely appeared in the mathematical textbooks and teacher explanations, including in Indonesia. Therefore, this present study tried to provide a proper learning environment to support students in developing a model to construct their understanding of the meaning of subtraction and to choose the more efficient strategy to solve subtraction problems. This study is aimed to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate first grade of primary school students to develop a model in solving two digit numbers subtraction. Consequently, the central issue of this study is formulated into the following research question: *How can a model support students to solve subtraction problems up to two digit numbers in the first grade of primary school?*

Design research was chosen as an appropriate approach for achieving the research aim. Gravemeijer & Cobb (2006) stated that design research consists of three phases; those are preparing for the experiment, conducting the design experiment, and carrying out the retrospective analysis. This study contained two cycles of design experiment, namely pilot experiment and teaching experiment, and it took place on February until April 2012. The first cycle serves as a try out experiment in adjusting and improving the designed Hypothetical Learning Trajectory (HLT) to get the better design for the second cycle. Pre-test and post-test were conducted both in the pilot experiment and teaching experiment. This study involved 6 students in the pilot experiment who were different from the students in the teaching experiment, 31 students in the teaching experiment, and a first grade teacher of SDN 179 Palembang. The data were collected using video registration, photographs, students' written work, and field notes. In the retrospective analysis, the HLT and students' actual learning process during the teaching experiment were compared. The lessons were analyzed to observe what students and teacher do, how the activities work, and how the material contributed to the lesson. The development of students' strategies in solving subtraction problems can be seen by comparing the result of pre-test and post-test in the teaching experiment.

The process of designing a sequence of instructional activities was consulted by five tenets for Realistic Mathematics Education (RME) defined by Treffers (1987). In Indonesia, RME has been implemented for over last ten years, namely *Pendidikan Matematika Realistik Indonesia* (PMRI). First, the use of contexts in phenomenological exploration was facilitated by preparing ginger candies and grains bracelets to construct the meaning of subtraction. Second, the use of models for progressive mathematization was stimulated by applying the beads string as a "model of" the situation and the empty number line as a "model for" students' thinking in solving subtraction problems. Third, the use of students' own constructions and productions was promoted by giving students the opportunity to solve the problem in their own strategy. Fourth, the interactivity of the teaching learning process was encouraged by conducting the class discussion in which the

students can share their thinking and can receive the different ideas from their friends. Fifth, the intertwining of various mathematics strands or units could be seen by emphasizing the relation between addition and subtraction, not teaching subtraction separately.

The initial HLT was revised based on the pilot experiment and the discussion with the teacher. The adjustment and improvement of HLT were done by reversing the order of some activities, making the problems simpler both the mathematical content and the sentences used, changing the grains necklaces into the grains bracelets to make it more doable for students, and changing the numbers of grains from 30 - 23 into 28 - 21 to support students using various strategies in solving the problem. The revised HLT in this study consisted of six lessons, those are: working with ginger candies, working with grains bracelets, working with the beads string, working with the empty number line, working with the beads string and the empty number line, and solving subtraction problems.

In the teaching experiment, it could be seen that at the beginning the students used various strategies in solving subtraction problems, for example using fingers and algorithm. When facing the two digits numbers subtraction, they found difficulty to solve the problems with their previous strategies. They needed the ginger candies and the grains bracelets to help them in solving the contextual problems. Later on, they were facilitated to use a model. The beads string helped students as a “model of” the situation to solve the problems in subtraction. A string of beads functioned as a stepping stone that could bridge students from the contextual problems to the use of the empty number line. Then, the students were able to use the empty number line as a “model for” their thinking in solving subtraction. The empty number line served as flexible mental representation that can reflect students’ strategies to solve the problems and could help students to visualize the steps needed in counting to come to the result. The students were promoted to apply different strategies (direct subtraction, indirect addition, and indirect subtraction) that more make sense and more efficient for them. The students also were stimulated to make the solution simpler by applying “jumps of 10” in the empty number line.

This present study showed that RME (PMRI) approach could facilitate students in developing a model to support them in solving two digit numbers subtraction. Therefore, it is recommended for the mathematics teachers in Indonesia to apply RME (PMRI) in the other mathematical topics. This approach allows students to see mathematics as a “human activity” which makes the learning process more meaningful for them. The students are given the opportunity to "re-invent" mathematics with guide from the teacher. They will not see mathematics just as procedures to follow or rules to apply in solving the problems anymore.

RINGKASAN

Pengurangan adalah salah satu operasi bilangan dasar dalam matematika yang biasa dijumpai siswa. Freudenthal (1983) mengatakan bahwa pengurangan merupakan kebalikan dari penjumlahan dan sering muncul dalam kehidupan siswa sehari - hari. Pengurangan memiliki dua arti dan masing-masing arti mengarahkan ke strategi yang berbeda. Menurut Torbeyns, De Smedt, Stassens, Ghesquiere, & Verschaffel (2009), arti pengurangan sebagai "mengambil sesuatu" mendorong pengurangan langsung, yaitu mengambil bilangan pengurang dari bilangan yang dikurangi. Di sisi lain, arti pengurangan sebagai "menentukan perbedaan dari dua bilangan" lebih mudah dimodelkan sebagai penjumlahan tidak langsung, yaitu menghitung maju dari bilangan pengurang sampai mencapai bilangan yang dikurangi. Banyak penelitian sebelumnya menemukan bahwa arti pengurangan yang kedua dan strategi yang kedua jarang muncul di dalam buku matematika dan penjelasan guru, termasuk di Indonesia. Oleh karena itu, penelitian ini mencoba menyediakan lingkungan pembelajaran yang baik untuk mendorong siswa mengembangkan suatu model yang dapat membangun pemahaman mereka tentang arti pengurangan dan memilih strategi yang lebih efisien dalam menyelesaikan masalah pengurangan. Penelitian ini bertujuan memberikan kontribusi bagi pengembangan *local instruction theory* untuk pengurangan dengan mendesain aktivitas pembelajaran yang dapat memfasilitasi siswa kelas 1 sekolah dasar untuk mengembangan model dalam menyelesaikan pengurangan bilangan dua angka. Konsekuensinya, isu utama dari penelitian ini diformulasikan dalam rumusan masalah berikut: *Bagaimana suatu model dapat membantu siswa untuk menyelesaikan masalah pengurangan sampai dua angka di kelas 1 sekolah dasar?*

Design research dipilih sebagai pendekatan yang sesuai untuk mencapai tujuan penelitian. Gravemeijer & Cobb (2006) menyatakan bahwa *design research* terdiri dari tiga tahap yaitu persiapan penelitian, desain penelitian, dan *retrospective analysis*. Penelitian ini terdiri dari dua siklus desain penelitian, *pilot experiment* dan *teaching experiment*, yang berlangsung pada bulan Februari sampai April 2012. Siklus pertama berfungsi sebagai penelitian uji coba dalam menyesuaikan dan mengembangkan *Hypothetical Learning Trajectory* (HLT) yang telah didesain untuk memperoleh desain yang lebih baik di siklus kedua. Pre-test dan post-test dilaksanakan baik di *pilot experiment* maupun *teaching experiment*. Penelitian ini melibatkan 6 siswa dalam *pilot experiment* yang berbeda dengan siswa dalam *teaching experiment*, 31 siswa dalam *teaching experiment*, dan seorang guru kelas satu SDN 179 Palembang. Data-data dikumpulkan menggunakan video, foto, pekerjaan tertulis siswa, dan catatan lapangan. Dalam *retrospective analysis*, HLT dibandingkan dengan proses pembelajaran siswa yang sebenarnya. Pembelajaran dianalisis untuk mengamati apa yang dilakukan oleh siswa dan guru, bagaimana aktivitas pembelajaran berlangsung, dan bagaimana perlengkapan mengajar berkontribusi dalam pembelajaran. Perkembangan strategi siswa dalam menyelesaikan masalah pengurangan dapat dilihat dengan membandingkan hasil dari pre-test dan post-test.

Proses dalam mendesain serangkaian aktivitas pembelajaran didasarkan pada lima ciri *Realistic Mathematics Education* (RME) yang didefinisikan oleh Treffers (1987). Di Indonesia, RME sudah diimplementasikan selama lebih dari sepuluh tahun terakhir dengan nama Pendidikan Matematika Realistik Indonesia (PMRI). Pertama, penggunaan konteks dalam pengeksplorasian fenomena difasilitasi dengan menyiapkan permen jahe dan gelang biji-bijian untuk membangun arti dari pengurangan. Kedua, penggunaan model untuk peningkatan matematisasi distimulasi dengan mengaplikasikan manik-manik sebagai "*model of*" dari situasi dan garis bilangan kosong sebagai "*model for*" untuk pemikiran siswa dalam menyelesaikan masalah pengurangan. Ketiga, penggunaan produksi siswa

didorong dengan memberikan kesempatan kepada siswa untuk menyelesaikan masalah menggunakan strategi mereka sendiri. Keempat, interaksi dalam proses belajar mengajar didukung dengan melaksanakan diskusi kelas sehingga siswa dapat berbagi pemikiran mereka dan dapat menerima ide yang berbeda dari teman yang lain. Kelima, keterkaitan antara berbagai unit matematika dapat dilihat dari penekanan hubungan antara penjumlahan dan pengurangan, tidak mengajarkan pengurangan secara terpisah.

HLT awal direvisi berdasarkan *pilot experiment* dan diskusi dengan guru. Revisi HLT tersebut adalah dengan menukarkan urutan beberapa aktivitas, membuat permasalahan menjadi lebih sederhana baik dari isi matematika maupun dari bahasa yang digunakan, mengganti kalung biji-bijian menjadi gelang biji-bijian sehingga lebih mudah dibuat oleh siswa, dan mengganti angka yang digunakan dalam biji-bijian dari 30 - 23 menjadi 28 - 21 untuk mendorong siswa menggunakan strategi yang berbeda dalam menyelesaikannya. HLT yang telah direvisi dalam penelitian ini terdiri dari enam kegiatan pembelajaran, yaitu bekerja dengan permen jahe, bekerja dengan gelang biji-bijian, bekerja dengan manik-manik, bekerja dengan garis bilangan kosong, bekerja dengan manik-manik dan garis bilangan kosong, dan menyelesaikan berbagai masalah pengurangan.

Dalam *teaching experiment*, awalnya siswa menggunakan berbagai strategi dalam menyelesaikan masalah pengurangan, misalnya menggunakan jari dan algoritma. Ketika menghadapi masalah pengurangan dua angka, mereka mengalami kesulitan untuk menyelesaikannya dengan strategi mereka sebelumnya. Siswa membutuhkan permen jahe dan gelang biji-bijian untuk membantu mereka dalam menyelesaikan masalah kontekstual. Selanjutnya, siswa difasilitasi untuk menggunakan model. Manik-manik membantu mereka sebagai "*model of*" dari situasi untuk menyelesaikan masalah dalam pengurangan. Untaian manik-manik berfungsi sebagai batu loncatan yang dapat menjembatani siswa dalam berpindah dari masalah kontekstual ke penggunaan garis bilangan kosong. Kemudian, siswa dapat menggunakan garis bilangan kosong sebagai "*model for*" untuk pemikiran mereka dalam menyelesaikan masalah pengurangan. Garis bilangan kosong berfungsi sebagai representasi yang fleksibel yang dapat menggambarkan strategi siswa dalam menyelesaikan masalah dan dapat membantu siswa memvisualisasikan langkah-langkah yang mereka lakukan dalam mencapai hasil akhir. Siswa didorong untuk dapat mengaplikasikan strategi yang berbeda (pengurangan langsung, penjumlahan tidak langsung, dan pengurangan tidak langsung) yang lebih dapat mereka pahami dan lebih efisien bagi mereka. Siswa juga distimulasi untuk dapat menyederhanakan solusi dengan mengaplikasikan "lompat 10" pada garis bilangan kosong.

Penelitian ini menunjukkan bahwa pendekatan RME (PMRI) dapat memfasilitasi siswa dalam mengembangkan model untuk mendorong mereka menyelesaikan masalah pengurangan dua angka. Oleh karena itu, direkomendasikan bagi guru matematika di Indonesia agar mengaplikasikan RME (PMRI) untuk topik matematika yang lain. Pendekatan ini mengijinkan siswa untuk melihat matematika sebagai "aktivitas manusia" sehingga membuat proses pembelajaran lebih bermakna bagi mereka. Siswa diberikan kesempatan untuk "menemukan kembali" matematika dengan bimbingan dari guru. Mereka tidak akan lagi melihat matematika hanya sebagai prosedur yang harus diikuti atau aturan yang harus digunakan dalam menyelesaikan suatu masalah.

PREFACE

First of all, I am very thankful to Alloh SWT for all blessings given to me. The greeting is given to Rosulullah SAW as a role model for my life. In this opportunity, I would like to say thank you to all people who supported me during my study, in conducting my research and writing my thesis. My grateful thanks I give to:

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I also would like to say thank you to the readers of my thesis. I hope this research can give contribution for the development of mathematics education in Indonesia.

Palembang, May 2012

Nila Mareta Murdiyani

CURRICULUM VITAE

Nila Mareta Murdiyani was born on March 25th, 1987 in Semarang as a first daughter of Sudiyanto, S. Pd and Sri Murni Budi Astuti, B.A. She has one sister namely Riyana Aprilia Kurniawati. She lives with her parents at Perum Korpri Gang Kenanga 208 Jatimulyo Alian Kebumen, Central Java. She graduated from SDN Karangsari 2 Kebumen in 1999, SMPN 1 Kebumen in 2002, SMAN 1 Kebumen in 2005. She took Bachelor Degree in Mathematics Education from Semarang State University in 2005 and she graduated in 2009. She joined International Master Program on Mathematics Education (IMPoME) at Sriwijaya University and Utrecht University (2010 - 2012). Her email is nila_math@yahoo.co.id.

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CHAPTER I

INTRODUCTION

A. Background

Numbers and its operations are certainly the most important area of mathematics learning for students (Sarama & Clements, 2009). The studies of *Mathematical Thinking and Learning* reported the foundation that supports operating with numbers and make it clear that learning to calculate is not just a matter of learning a particular calculation procedure, but that it requires an understanding of number relationships and properties of operations. When using this understanding, calculating is not just a case of knowing the counting sequence and having a good memory but also one of thinking (van den Heuvel-Panhuizen & Treffers, 2009).

Subtraction is one of the basic number operations in mathematics which is familiar for students. Subtraction results as the converse of addition (Freudenthal, 1983) and it often appears in students' daily life. In solving subtraction problems, students have to think about the meaning of subtraction and the more efficient strategies to solve it. According to Fosnot and Dolk (2001), subtraction has two meanings; those are "taking away something" and "determining the difference between two numbers". Each meaning leads to the different strategies. The context of "taking away something" suggests a direct subtraction, which means removing the subtrahend from the minuend. On the other hand, the context of "determining the difference between two numbers" is more likely to be modeled as indirect addition, which means adding on from the subtrahend until the minuend is reached (Torbeyns, De Smedt, Stassens, Ghesquiere, & Verschaffel, 2009).

However, the indirect addition strategy, particularly with multi digit numbers, has received a little attention from researchers. The limited research interest for this complement strategy of direct subtraction is quite surprising because there are indications that indirect addition is not only computationally remarkably efficient but also very promising from a broader educational perspective (Torbeyns *et al*., 2009).

Moreover, in the Indonesian mathematical text books (see Djaelani & Haryono, 2008), the meaning of subtraction is explained only as “taking away something”. Teachers provide only removal contexts in teaching subtraction. In a traditional teaching learning method, teachers also teach students an algorithm of subtraction directly, subtracting tens and ones separately, after they learn subtraction up to 20 by doing physical activities or using drawing. It is meaningless for students because they do this procedure without understanding. It is also more difficult if students are confronted with borrowing and carrying procedures (Kamii & Lewis, 1993).

In this situation, teachers need to emphasize that subtraction also has a meaning of “determining the difference between two numbers” that will be more efficient to solve by indirect addition. Therefore, this present study tries to provide a proper learning environment by designing a sequence of meaningful mathematical activities to promote students in constructing their understanding of the meaning of subtraction and in choosing the more efficient strategy to solve subtraction problems up to 100.

In the present study, the instructional activities are started by providing contextual problems that have different meaning of subtraction. Then, the beads string is used as a “model of” the context situations. In the next step, students will work with an empty number line as a “model for” their thinking to solve two digit numbers subtraction in different situations. The empty number line is a flexible mental representation to support subtracting because it gives students a lot opportunity to apply different strategies in solving subtraction (van den Heuvel-Panhuizen, 2008).

This present study was based on the Realistic Mathematics Education (RME) or *Pendidikan Matematika Realistik Indonesia* (PMRI) approach in which facilitates different subtraction contexts that can be imagined by the students and meaningful for them. RME (PMRI) also serves as the framework to construct students conceptual knowledge of subtraction step by step from concrete to more abstract based on their level of understanding. Students are supported to give their own contribution in the teaching learning process by sharing their idea to others (Gravemeijer, 1994). Teachers play a role as a facilitator in providing guidance to help students reinvent their understanding in subtraction.

B. Research Aim

The aim of this study is to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate students to develop a model in solving two digit numbers subtraction.

C. Research Question

The central issue of this study is formulated into the following general research question: *How can a model support students to solve subtraction problems up to two digit numbers in the first grade of primary school?*

The general research question in this present study can be elaborated into two specific sub questions:

1) How can the beads string bridge students from the contextual problems to the use of the empty number line?
2) How can the empty number line promote students to apply different strategies in solving subtraction problems?

CHAPTER II

THEORETICAL FRAMEWORK

A. Subtraction

Subtraction is close to students' everyday life. Since they know numbers, they immediately learn the number operations including subtraction. When they realize the amount of something, they will recognize that the amount sometimes increase and decrease. If they add something, the amount will be increased; while if they subtract, the amount will be decreased. Addition and subtraction are related and cannot be separated each other. Before learning subtraction in the formal way, students already had informal knowledge about it. They already knew that when doing subtraction, the amount of something becomes less than the initial amount and the difference is less than the total. Students also already understood that they only can subtract the same item, for example book with book, money with money, etc.

Discussing subtraction after addition does not aim at a didactical separation and certainly not at a succession in the genetic and didactic process. In all contexts where addition is didactically offered, subtraction is implicitly present in order to be made equally explicit. Formally, subtraction results as the converse of addition, and in fact this aspect of subtraction should not be neglected (Freudenthal, 1983).

According to Fosnot and Dolk (2001), subtraction has two meanings; those are "taking away something" and "determining the difference between two numbers". The first meaning mostly appeared in the mathematical textbooks and teacher explanations. In the first meaning, the only matching action is that of removing. This interpretation of subtraction is too one sided. As Freudenthal (1983) already emphasized in his didactical phenomenological analysis of subtraction "explicit taken away suffices as little for the mental constitution of subtraction as uniting explicitly given sets suffices for addition".

Torbeyns, *et al.* (2009) described the strategies to solve subtraction problems in three different ways. They distinguished (1) direct subtraction, which means taking away the subtrahend from the minuend; (2) indirect addition, which means adding on from the subtrahend until the minuend is reached; and (3) indirect subtraction, which means subtracting from the minuend until the subtrahend is reached. Subtrahend is the number being taken away/ the smaller number, while minuend is the number that has something taken away from it/ the bigger number. According to them, splitting, stringing, and varying belong to the class of direct subtraction strategies, whereas indirect addition is considered as a separate class of strategies which do not fit the three of them.

The indirect addition strategy, particularly with multi digit numbers, has received little attention from researchers so far. The limited research interest for this strategy as a complementary strategy for the conventional direct subtraction is quite surprising. Indirect addition seems to have some computational advantages over direct subtraction, at least for a particular kind of subtraction problem with a relatively small difference between the subtrahend and the minuend. Besides that, indirect addition is a valuable strategy from a broader educational perspective because it clearly expresses the relation between addition and subtraction (Torbeyns *et al.*, 2009).

The strategies that will be used in solving subtractions are influenced by the numbers involved. If minuend and subtrahend are far away, students are expected to use direct subtraction; while if two numbers are close together, they should use indirect addition strategy. Indirect addition also can be a good alternative for problems which require crossing the ten (Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2011). It can be assumed that people decide to solve two digit subtraction problems either by means of regular subtraction or by means of an addition depending on which of both processes requires the fewest and easiest steps (Peters, De Smedt, Torbeyns, Ghesquiere, & Verschaffel, 2010).

Besides the numbers involved, the way subtraction problems are presented also influences the strategies that students will use. Several studies revealed that bare number problems hardly evoke the use of indirect addition, which can be explained by the presence of the minus sign that emphasizes the “taking away” action, which is equally true for minus words (like lost or gave away or fewer) in words problems (van den Heuvel-Panhuizen, 1996). Contextual problems, on the contrary, lack this operation symbol and therefore open up both interpretations of subtraction (van den Heuvel-Panhuizen, 2005). Moreover, the action described in the context of a problem may prompt the use of a particular strategy.

There is a need for providing a learning environment that can support students to apply the direct subtraction and indirect addition strategy properly. Based on available research on the didactical insights, the following design principles might be recommended: (1) integrating the development of both conceptual knowledge and procedural solution of subtraction (Baroody, 2003); (2) including both bare number problems and context problems in subtraction (van den Heuvel-Panhuizen & Treffers, 2009); (3) creating a classroom culture that is supportive to the development of adequate beliefs about and attitudes towards strategy flexibility (Baroody, 2003; Verschaffel, Torbeyns, De Smedt, Luwel, & Van Dooren, 2007); and (4) applying the empty number line as a model for solving different subtraction problems (van den Heuvel-Panhuizen & Treffers, 2009).

Therefore, this present study tries to provide instructional activities to construct students’ understanding of the meaning of subtraction and the more efficient strategy to solve subtraction problems up to two digit numbers based on the design principles above.

B. Subtraction in the Indonesian Curriculum

Subtraction is taught in Elementary School from grade 1 until grade 6. But, the basic of subtraction (subtraction in whole numbers) is taught from grade 1 until grade 3. There are the standard competence and basic competence of subtraction for first grade in the Indonesian Curriculum (Depdiknas, 2006).

Table 1.Standard competence and basic competence of subtraction in grade 1 semester 1

Standard Competence	Basic Competence
Numbers 1. Doing addition and subtraction up to 20	1.3 Doing addition and subtraction up to 20 1.4 Solving problems related to addition and subtraction up to 20

Table 2.Standard competence and basic competence of subtraction in grade 1 semester 2

Standard Competence	Basic Competence
Numbers *4. Doing addition and subtraction up to two digit numbers in problem solving*	*4.4 Doing addition and subtraction up to two digit numbers* *4.6 Solving problems related to addition and subtraction up to two digit numbers*

This present study will focus on subtraction in the second semester of grade 1. In this grade, students already had the sense of subtraction up to 20 by doing physical activities or using drawing in the first semester. At the other hand, in the beginning of second semester, they have not been taught the algorithm of subtracting tens and ones separately; and also borrowing and carrying procedures. It makes easier to develop a model and to build students number sense of subtraction. The algorithms are harmful for young children because they actually work against the development of children's understanding of place value and number sense (Kamii & Lewis, 1993).

The standard competence and basic competences of subtraction above are elaborated in the mathematical textbooks. However, in Indonesian mathematical textbook (see Djaelani & Haryono, 2008), the meaning of subtraction is explained only as "taking away something". The book only provides removal contexts in teaching subtraction. It only uses the words that have the meaning of removed something such as dead, leave, disappeared, damaged, taken, and broken.

C. The Empty Number Line in Solving Subtraction Problems

The powerful tool to support the "two ways traffic" of subtraction (taking away and adding on) is the empty number line. It was Freudenthal (1983) who pleaded for using what he called "geometrical concreteness of the number line" in which the two methods connected to the two interpretations of subtraction can be observed, namely "taking away at the start" and "taking away at the end".

The empty number line began to make progress as a didactic model during the 1990s in mathematics education in the Netherlands because of the ideas of Weill (1978), Whitney (1988), and Treffers (1989). Weill demonstrated how abbreviated calculations can be performed by placing numbers below and above an empty number line. Whitney supplemented this concept with the idea that a vertical mark before a number on the empty number line represents the location of a toothpick on the string of beads. Treffers saw possibilities for learning to do abbreviated and flexible arithmetic with this model by assigning clear meaning to this mark while also using the "jump approach" (Menne, 2001).

Studies conducted by Veltman (1993) and Klein (1998) then demonstrated that the empty number line is a useful scheme for adding and subtracting up to 100. Veltman showed how helpful the empty number line is for making students aware of the two strategies and for choosing the more efficient strategy for doing subtraction problems.

According to Treffers (1989), before students can operate numbers on an empty number line, they must be able to count with tens and ones, to locate numbers on a string of beads and on an empty number line, and to take a jump of ten from any number (Menne, 2001).

Empty number line's attended use is as a flexible mental representation to support adding and subtracting, rather than measuring line from which the exact results of operations can be read. It is intended as a flexible model that should give students a lot of freedom, and this includes both flexibility in the ways of recording results and flexibility in the jumps students make to solve the problems (van den Heuvel-Panhuizen, 2008). Therefore, this present study emphasizes the importance of applying empty number line to solve two digit subtraction problems either using direct subtraction or using indirect addition strategy.

However, there is a possibility of confusion when the empty number line that is meant as a counting line (referring to discrete quantities) is used as a measuring line (referring to continuous quantities). Doing a calculation based on such a line means "reading off" the number at which it arrives after carrying out the operation, while the empty number line is meant for structuring the consecutive calculation steps and recording them (van den Heuvel-Panhuizen, 2008).

That the empty number line refers to discrete quantities was clearly expressed by Whitney (1985) when he used toothpicks to indicate the numbers or, more correctly, the amount of beads. By using toothpicks, Whitney combined the two types of numbers (quantity numbers and measuring numbers) in one model. More importantly, this model clarified the difference between these two types of numbers. The "measuring" eight (at the end of the first toothpick) indicates that there are eight beads to the left of it. However, at the same time the model makes clear that this measuring eight does not coincide with the “quantity” eight, the interval after the eighth bead. This could solve the difficulties that up to that point had obstructed the use of number lines (Treffers, 1991).

It was often unclear for both students and teachers what should be counted: the beads or the intervals. Whitney’s toothpicks clarified the difference between the two, and at the same time indicated their connection. By introducing the children to the string of beads that used toothpicks to mark certain amounts, the foundation was created for the empty number line as a didactical model to support adding and subtracting with whole numbers, and should not to treat the empty number line as a measuring line (van den Heuvel-Panhuizen, 2008).

Therefore, in this present study, a string of beads is used as a stepping stone in moving from contextual problems to the use of empty number line as a powerful model in solving subtraction problems up to two digit numbers.

D. Realistic Mathematics Education

Realistic Mathematics Education (RME) is an answer to reform the teaching and learning in mathematics. The present form of RME is mostly determined by Freudenthal’s view about mathematics. According to him, mathematics must be connected to reality, stay close to children and be relevant to society, in order to be of human value. Instead of seeing mathematics as subject matter that has to be transmitted, Freudenthal stressed the idea of mathematics as a “human activity”. Education should give students the "guided" opportunity to "re-invent" mathematics by doing it. This means that in mathematics education, the focal point should not be on mathematics as a closed system but on the activity, on the process of mathematization (Freudenthal, 1968).

Later on, Treffers (1987) formulated the idea of two types of mathematization explicitly in an educational context and distinguished the horizontal and vertical mathematization. In horizontal mathematization, the students come up with mathematical tools which can help them to organize and solve a problem located in a real-life situation. On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. In short, horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols (Freudenthal, 1991).

Therefore, a sequence of meaningful activities are provided in this present study to construct students' understanding of the meaning of subtraction and the more efficient strategy to solve two digit numbers subtraction problems, instead of only using the algorithm. The horizontal mathematization is happened when the students can translate the real world problems (the contexts of taking candies and making necklaces) into the mathematical problems and can visualize the solutions of the problems in the different ways. The vertical mathematization is occurred when the students can develop models (the beads string and the empty number line) and can apply those models in a general situation to solve subtraction problems.

The process of designing a sequence of instructional activities was consulted by five tenets for RME defined by Treffers (1987). Those tenets in this study are described as following.

1. The use of contexts in phenomenological exploration

   The mathematical activity is started from local contexts situation that are experientially real for students. The context of taking ginger candies is used to construct the meaning of subtraction as "taking away something". For constructing the meaning of subtraction as "determining the difference between two numbers", the context of making necklaces from grains is used. Those contexts are familiar for students and close to their everyday life.

2. The use of models for progressive mathematization

   Models are used as a bridge from concrete level to more formal level. Typical of progressive mathematization is that students in every phase can refer to the concrete level of the previous step and infer meaning from that. Firstly, students will explore the different contexts of taking candies and making necklaces. Then, they will make a visualization of the solution in their own way. Later on, a string of beads can serve as a powerful model to represent the situation of those contexts. Students can see the meaning of subtraction given in the beads string. In the next level, an empty number line can represent the general situation and can reflect the students' thinking in solving subtraction problems.

3. The use of students' own constructions and productions

   In the activity of taking candies and making necklaces, students are given the opportunity to solve the problem in their own strategy. Students are also asked to make their own productions in finding as many as possible the combinations of numbers from the given numbers using addition and subtraction operations. Class discussion is conducted in every meeting to discuss different ways to solve subtraction problems so that every student can get new insight from their friends and can choose more efficient strategy that makes sense for them.

4. The interactivity of the teaching process

   The teaching learning process can be interactive if there are occurred a vertical interaction between teacher and students and a horizontal interaction among students. Teacher plays a role as a facilitator to support students' understanding by providing social interaction in the classroom. In solving subtraction problems, students always work in group and will share their idea to others. By this interaction, students can develop their thoughts and can learn to respect each other.

5. The intertwining of various mathematics strands or units

   The sequence of instructional activities in this study not only emphasizes the meaning of subtraction and the strategy to solve subtraction problems, but also stresses the relation between addition and subtraction. Moreover, students are not taught the algorithm of subtraction directly in order to build their number sense: the relation among numbers.

RME continually works toward the progress of students. In this process, models which originate from context situations and which function as bridges to higher levels of understanding play a key role. The switch from informal to more formal level can be characterized as emergent modeling. Gravemeijer (1994) described how "model of" a certain situation can become "model for" more formal reasoning. The levels of emergent modeling in this present study are shown in the following.

1. Situational level

   The interpretations and solutions of the activity in this level depend on the understanding of how to act in the setting. This basic level of emergent modeling uses situational knowledge and strategy within the context of the situation. The contexts of taking ginger candies and making necklaces from grains are provided to emphasize two different meanings of subtraction.

2. Referential level

   The referential level is the level of "model of" in which the use of model and strategy refers to the situation described in the instructional activities. Students are promoted to shift from situational level to referential level when they have to make representation as the "model of" their strategies in solving subtraction. The use of beads string leads them to move from the contexts level to the next level in using the empty number line.

3. General level

   In the general level, students need to develop a “model for” their thinking that can be used in different situations. This model makes possible a focus on interpretations and solutions independently from specific situation, in this case the situations of taking candies and making necklaces. The empty number line serves as the “model for” students thinking in representing any kinds of subtraction problems.

4. Formal level

   Students use reasoning with conventional symbolizations which is no longer dependent on the support of “model for” mathematical activity in the formal level. However, this study is not going further to the formal level. In the final assessment, students are expected to have a good number sense in solving different subtraction problems. They are requested to look at the number first to decide on a strategy. But, they are still allowed to apply the empty number line to make them easier to come to the solution.

RME approach is not familiar for students. Therefore, it is a need for adjusting students’ belief about their own roles, the others’ roles, and the teacher’s roles in the classroom. Emergent perspective is used for interpreting the classroom discourse and communication (Gravemeijer & Cobb, 2006). The emergent perspectives adapted in this present study consist of social norms, socio-mathematical norms, and classroom mathematical practices.

E. *Pendidikan Matematika Realistik Indonesia*

Inspired by the philosophy of RME, a team of Indonesian Educators developed an approach to improve mathematics learning in Indonesian schools. It is known as *Pendidikan Matematika Realistik Indonesia* (PMRI), an Indonesian adaptation of RME. It was developed through design studies in Indonesian classrooms, later becoming a movement to reform mathematics education in Indonesia. According to Sembiring, Hadi, & Dolk (2008), the approach to reform adopted by PMRI involves:

1. Bottom-up implementation.
2. Materials and frameworks based on and developed through classroom research.
3. Teachers being actively involved in designing investigations and developing associated materials.
4. Day-by-day implementation strategies that enable students to become more active thinkers.
5. The development of contexts and teaching materials that are directly linked to school environment and the interests of students.

After ten years of PMRI development and pilots, a vast body of knowledge has been acquired on PMRI and on what is considered good PMRI education in Indonesia. Many experiences contributed to the slowly developed ideas of good standards for various aspects of PMRI, including PMRI lesson (Hadi, Zulkardi, & Hoogland, 2010). Standards for PMRI lesson used in this present study are explained as following.

1. A PMRI lesson fulfills the accomplishment of competences as mentioned in the curriculum.

   This study is in line with the standard competence and basic competence on subtraction that should been reached for first grader in Indonesia.
2. A PMRI lesson starts with a realistic problem to motivate and help students learn mathematics.

   This study starts with contextual problems about taking ginger candies and making grains necklaces which are familiar for Indonesian students.
3. A PMRI lesson gives students opportunities to explore and discuss given problems so that they can learn from each other and to promote mathematics concept construction.

In every meeting, the students work in group to share their thinking about the problems given. Later on, some of them are asked to present their solution in the class discussion.

4. A PMRI lesson interconnects mathematics concepts to make a meaningful lesson and intertwine knowledge.

   In this study, the concept of subtraction is related with the concept of addition. They cannot be separated from each other.

5. A PMRI lesson ends with a confirmation and reflection to summarize learned mathematical facts, concepts, and principles, and is followed by exercises to strengthen students' understanding.

   In every meeting, the teacher gives summary about the learning process and asks the students if they have any questions about the lesson. This study ends with post-test that will examine students' understanding on subtraction and students' strategies in solving subtraction problems.

CHAPTER III

METHODOLOGY

A. Research Approach

This present study is aimed to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate students to develop a model in solving two digit numbers subtraction. Consequently, this study will be based on the design research approach as an appropriate methodology for achieving the research aim since the purpose of design research is to develop theories about both the process of learning and the means designed to support that learning. Design research consists of three phases; those are preparing for the experiment, conducting the design experiment, and carrying out the retrospective analysis (Gravemeijer & Cobb, 2006). The characteristic of these phases is the cycles always refined to form a new cycle in the emergence of a local instruction theory, as the figure below shows:

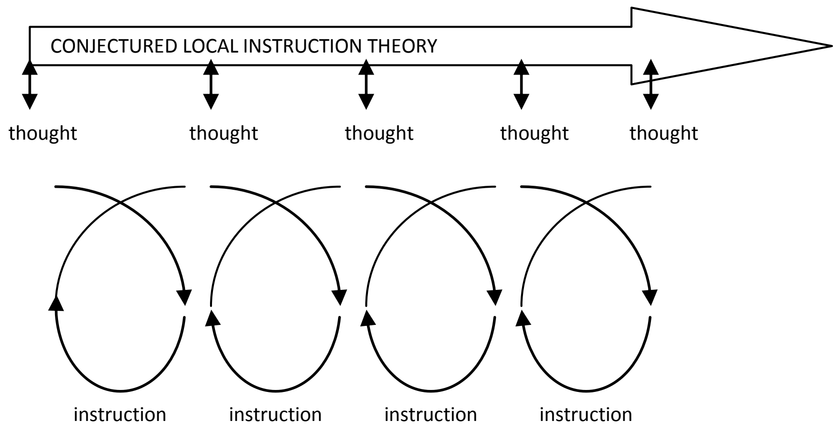


Figure 1. The cycles of design research

In this study, a sequence of instructional activities is designed as a flexible approach to improve educational practices for subtraction in the first grade of primary school in Indonesia. The three phases in this design research are described as following.

1. **Preparing for the experiment**

   The first step in this phase is studying literature about subtraction, realistic mathematics education approach, and design research approach as the bases for designing the instructional activities. Later on, an Hypothetical Learning Trajectory (HLT) is designed containing three components: the learning goals that define the direction; the planning of mathematical activities and the instruments that will be used; and the conjectures of learning process in which teacher anticipates how students' thinking and action could evolve when the instructional activities are used in the classroom (Simon & Tzur, 2004). During the preparation, the HLT guides the design of instructional activities that have to be developed. The initial HLT can be adjusted to students' actual learning during the teaching experiment. The next steps are conducting a classroom observation, an interview with teacher, and pre-test to investigate the starting positions of the students and to support the elaboration of the initial HLT.

2. **The design experiment**

   In this phase, the instructional activities are enacted and modified on a daily basis during the experiment. Before conducting the teaching experiment, the researcher and the teacher discuss the upcoming activity. And after conducting the teaching experiment, the researcher and the teacher make a reflection of whole learning process in the classroom, what are the strong points and the weak points. During the teaching experiment, the HLT functions as a guideline what to focus on in teaching, interviewing, and observing. This present study contains two cycles of design experiment. The first cycle serves as a pilot experiment in adjusting and improving the designed HLT to get the better design for the second cycle. In this study, the teaching experiments are conducted in six lessons in which each lesson needs 70 minutes.

3. The retrospective analysis

In this phase, all data in the experiment are analyzed. During the retrospective analysis, the HLT functions as a guideline in determining what to focus on in the analysis. The form of analysis of the data involves an iterative process. The HLT is compared with the students' actual learning. There should be explained not only the instances that support the conjectures, but also the examples that contradict the conjectures. Underpinned by the analysis, the research questions can be answered and the recommendation of how the next HLT should be improved for further studies can be made. In general, the purpose of this retrospective analysis is to develop a well considered and empirically grounded local instruction theory.

B. Research Subject and Timeline of the Research

This present study was conducted in the SDN 179 Palembang which is the partner school of PMRI. The experiment of this study consisted of two cycles, pilot experiment and teaching experiment. The participants of the pilot experiment were 6 first grade students from class 1E of SDN 179 Palembang. In the teaching experiment, there were 31 first grade students from class 1D and a first grade teacher of SDN 179 Palembang involved. The students were about 6 or 7 years old.

The timeline of the research can be seen in the table below.

Table 3. The timeline of the research

Descriptions	**Date**
Preparing for the Experiment	
Studying the literature	September - November 2011
Designing the initial HLT	November 2011 - January 2012
Discussion with the teacher	10 February 2012
Pilot Experiment (First Cycle)	

Pre-test	13 February 2012
Lesson 1 (Working with ginger candies)	14 February 2012
Lesson 2 (Working with grain bracelets)	17 February 2012
Lesson 3 (Working with the beads string)	20 February 2012
Lesson 4 (Working with the empty number line)	21 February 2012
Lesson 5 (Working with the beads string and the empty number line)	24 February 2012
Lesson 6 (Solving subtraction problems)	27 February 2012
Post-test	2 March 2012
Retrospective Analysis for the Pilot Experiment	
Discussion with the teacher	3 March 2012
Analyzing the pilot experiment	5 March 2012 - 10 March 2012
Improving the HLT	12 March 2012 - 17 March 2012
Teaching Experiment (Second Cycle)	
Classroom observation	14 February 2012
Interview with the teacher	21 February 2012
Pre-test	1 March 2012
Lesson 1 (Working with ginger candies)	19 March 2012
Lesson 2 (Working with grain bracelets)	20 March 2012
Lesson 3 (Working with the beads string)	26 March 2012
Lesson 4 (Working with the empty number line)	27 March 2012
Lesson 5 (Working with the beads string and the empty number line)	29 March 2012
Lesson 6 (Solving subtraction problems)	2 April 2012
Post-test	3 April 2012
Retrospective Analysis for the Teaching Experiment	
Discussion with the teacher	5 April 2012
Analyzing the teaching experiment	April 2012
Answering the research questions and drawing the conclusions	May 2012

C. Data Collection

1. Preparation Phase

a. Classroom observation

A classroom observation was conducted to get the overview of the social norms, the socio- mathematical norms, the teaching methods, the classroom organizations, the rules of the class, the students' work, and the time management from the teaching learning process of the students who will become the subjects in the teaching experiment. To collect the data, a video registration by one dynamic camera, photographs, and written notes were used.

b. Interview with teacher

An interview with the teacher who conducted the teaching experiment was held after the classroom observation to get more information about classroom interactions which cannot be observed directly, for example about the teacher's difficulties to teach the subject, the level of students' understanding, teacher's experience with RME approach, and students' experience with RME approach. Interview with teacher was conducted also to communicate and to discuss the designed HLT. The data were gathered by audio registration and field notes.

c. Pre-test

Pre-test was conducted to know the starting points of the students who became the subjects in the teaching experiment and what they should learn. It was held both in the first cycle and second cycle of the teaching experiment. Students' worksheets and an interview with four students were used to gather the data.

2. Pilot Experiment (First Cycle)

The pilot experiment was conducted to know the prior knowledge of the students and to try out the initial HLT. The participants were 6 first grade students who were not the same as the students involved in the second cycle. They were chosen by recommendation of their teacher. The researcher played a role as a teacher, while the teacher joined this pilot experiment to get more sense about how the teaching learning process will come along in the classroom. It is an important phase as a base to revise the HLT before conducting the teaching experiment. The data of pilot experiment were collected by video registration with one dynamic camera, by photographs, and by written notes.

3. Teaching Experiment (Second Cycle)

a. Classroom observation

A classroom observation was conducted to gain the data about whole class activities during the teaching experiment from the research subjects, the first grade students of SDN 179 Palembang. In collecting the data about classroom observation, a video registration by one dynamic camera, photographs, and written notes were used.

b. Group observation

A group observation was held to gather the data about group work discussions during the teaching experiment. This present study will follow the interesting discussions of one focus group that consists of four students with the different abilities to get more detail information about the process of students' understanding on the subject. The researcher sat down next to the focus group and sometimes conducted a short discussion to investigate students' reasoning. To collect the data about group observation, a video registration by one dynamic camera, photographs, and written notes were used.

4. Post-test

Post-test was conducted to know the end points of the students after the teaching experiment and what they have learned. It was held both in the first cycle and second cycle of the teaching experiment. The data were gained from students' worksheets that consist of students' final answer and students' strategies to solve the problems. The interview with four focus students was conducted to find out the reason behind their solutions.

5. Validity and Reliability

Internal validity refers to the quality of the data collection and the soundness of reasoning that has led to the conclusion (Bakker, 2004). The internal validity of this study was gained by collecting the different types of data (data triangulation) such as video recording, audio recording, photographs, field notes, and written work of students. This study was conducted in a real classroom setting; therefore it also can guarantee the ecological validity.

Internal reliability refers to the reliability within a research project (Bakker, 2004). The internal reliability of this present study was improved by data registration itself. The data were collected using video and audio recordings, not only observations and notes by researcher.

D. Data Analysis

1. Pre-test

The written work of students in the pre-test was analyzed by looking at the students' final answer and students' strategies to solve the problems. The result of this analysis was used as a base to determine the starting points of the students and what they should learn.

2. Pilot Experiment (First Cycle)

All data from first cycle experiment which were gathered by selected video registrations, photographs, field notes, and students' work were analyzed to compare the assumption about students' learning in the HLT I with the students' actual learning. From this analysis, it can be seen which part of the HLT supported students' learning and which part was not. This analysis was used to revise and to improve the HLT for second cycle (HLT II).

3. Teaching Experiment (Second Cycle)

The whole video recording was watched to get the overview of the teaching learning process in the classroom. During watching the video, the field notes also were written to make a general description of the activity. The researcher made the timing of the important moment especially in the focus group discussions. The selected fragment was transcribed to make the interpretation of students' thinking. The parts that were not relevant with the students' learning process were ignored. The researcher also selected the interesting written work from the students. Students' actual learning was compared with the conjectures in the HLT II. The field notes from the observer and the related photographs also were used in the analysis. The final analysis was accomplished by the researcher with cooperation and review from supervisors and colleagues to increase the validity and reliability of this present study. This analysis was used to answer the research questions, to draw the conclusions, and to redesign the HLT for further studies.

4. Post-test

The students' final answer in the post-test was compared with the answer in the pre-test. All final answers were checked to get the quantitative data of all students in the classroom. We could see the development of students' strategies in solving subtraction problems by this comparison. Some strategies from focus group students were examined to investigate the development of their understanding. The result of this analysis was used to know the end points of the students and what they have learned after the teaching experiment.

5. Validity and Reliability

To improve the internal validity in this study, during the retrospective analysis, the conjectures that were generated in each activity were tested and other data materials such as video registration, audio registration, field notes, and written work of students were analyzed. Having these data, data triangulation was conserved so that the quality of the conclusions can be controlled.

External validity is mostly interpreted as the generalizability of the result. If lessons learned in one experiment are successfully applied in other experiments, this is a sign of successful generalization. The challenge is to present the result of this study in such a way that others can adjust them in their local contingencies.

In this study, the internal reliability was gained by discussing the critical protocol segments in the teaching experiments with the supervisors and colleagues. This cross interpretation (inter-subjectivity) reduced the subjectivity of the researcher's point of view.

External reliability is obtained if the reader can follow the track of the learning process in this study and to reconstruct their study (trackability). In order to do so, two dynamic cameras were used to record every important moment in the teaching learning process. Besides that, the field notes in the observation sheets were also used to describe in detail the crucial event in the classroom activities.

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

This chapter provided the Hypothetical Learning Trajectory (HLT) for our study in subtraction. The goal of this present study is to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate students to develop a model in solving two digit numbers subtraction. Consequently, this HLT contains six sequences of activities in three weeks aimed to reach the goal of the study. This study was conducted in the first grade of primary school in Indonesia.

For each instructional activity, we will describe the goals, the starting positions, the description of activity, the conjectures of students' thinking and teacher's reactions. The intentions of these activities are students can understand the two meaning of subtraction ("taking away something" and "determining the difference between two numbers") and students can apply the more efficient strategies in solving subtraction problems (direct subtraction or indirect addition). The activities are elaborated as follows:

A. Lesson 1 (Working with Ginger Candies)

1. Goals

- **Knowledge**

- Students know that subtraction is the converse of addition
- Students know the meaning of subtraction as "taking away something"

- **Skills**

- Students can subtract two numbers up to 100
- Students can use counting back
- Students can make a representation from the solutions

- **Attitude**

- Students are willing to find the strategies in solving subtraction problems
- Students are willing to verbalize the solutions
- Students are willing to express their point of view about the meaning of subtraction

2. **Starting Positions**

- **Knowledge**

- Students know the names of the numbers up to 100
- Students know the number symbols
- Students know the relation between two numbers
- Students know the addition operation
- Students know the subtraction operation

- **Skills**

- Students can add two numbers up to 20
- Students can subtract two numbers up to 20
- Students can count the real objects
- Students can count from any numbers
- Students can locate the numbers

- **Attitude**

- Students are willing to solve the contextual problems
- Students are willing to work with real objects

3. **Description of Activity**

- Teacher poses a worksheet that consists of two different contexts of subtraction using the ginger candies. The contexts are the following:
  a. Donna has 20 ginger candies. Ryan gives to her 6 more candies. How many candies that Donna has right now?

b. Donna has 26 ginger candies. She gives some candies to Andre. She still has 20 ginger candies right now. How many candies that Donna gives to Andre?

- Students solve the problems in the group of four, so they have opportunities to share their thinking to others.
- Teacher provides the ginger candies to each group, so they can do hands on activity.
- Two groups of students present their solution in front of the class. Teacher discusses the students' solution. She guides students to find the relation between the first and the second context, which is the relation between addition and subtraction.
- Teacher poses the third context as following:

c. Donna has 26 ginger candies. She gives 6 of those candies to Andre. How many candies that Donna still has?

- Two groups of students present their solution in front of the class. Teacher discusses the students' solution. She guides students to understand that the meaning of subtraction is "taking away something" in this context.

4. Conjectures of Students' Thinking and Teacher's Reactions

Table 4. Conjectures of students' thinking and teacher's reactions for lesson 1

No	Students' Thinking	Teacher's Reactions
1.	For the first context: Some students will use the ginger candies to represent the situation. They will add the candies one by one, or two by two, or three by three, or directly six candies, etc. Some students will draw the ginger candies to represent the situation. They will draw the additional candies one by one, or two by two, or three by three, or directly six candies, etc. Some students only write the numbers. They will use their fingers to count forward the number one	Teacher supports students to make visualization for their solution.

	by one, or two by two, or three by three, or directly six, etc.	
2.	For the second context: Some students will use the ginger candies to represent the situation. They will take the candies one by one, or two by two, or three by three, or directly 20 candies, etc. Some students will draw the ginger candies to represent the situation. They will remove the candies one by one, or two by two, or three by three, or directly 20 candies, etc. Some students only write the numbers. They will use their fingers to count backward the number. It is also possible if some students will count forward the number from 20.	Teacher supports students to make visualization for their solution.
3.	For the third context: Some students will use the ginger candies to represent the situation. They will take the candies one by one, or two by two, or three by three, or directly six candies, etc. Some students will draw the ginger candies to represent the situation. They will remove the candies one by one, or two by two, or three by three, or directly six candies, etc. Some students only write the numbers. They will use their fingers to count backward the number one by one, or two by two, or three by three, or directly six, etc.	Teacher supports students to make visualization for their solution.

B. Lesson 2 (Working with Grain Necklaces)

1. Goals

- **Knowledge**

- Students know that subtraction is the converse of addition
- Students know the meaning of subtraction as "determining the difference between two numbers"

- **Skills**

- Students can subtract two numbers up to 100
- Students can use counting on
- Students can make a representation from the solutions

- **Attitude**

- Students are willing to find the strategies in solving subtraction problems
- Students are willing to verbalize the solutions
- Students are willing to express their point of view about the meaning of subtraction

2. Starting Positions

- **Knowledge**

- Students know the names of the numbers up to 100
- Students know the number symbols
- Students know the relation between two numbers
- Students know the addition operation
- Students know the subtraction operation

- **Skills**

- Students can add two numbers up to 20
- Students can subtract two numbers up to 20
- Students can count the real objects
- Students can count from any numbers
- Students can locate the numbers

- **Attitude**

- Students are willing to solve the contextual problems
- Students are willing to work with real objects

3. **Description of Activity**

- Teacher poses a worksheet that consists of two different contexts of subtraction using the grain necklaces. The contexts are the following:
  a. Farah should make two necklaces from grains for art class. She is stringing until 33 grains for the first necklace right now. She still needs 7 more grains to finish her first necklace. How many grains in total which are needed to make the first necklace?
  b. Right now, Farah is stringing until 33 grains for the second necklace. How many more grains she needs to finish her second necklace if the second necklace consists of 40 grains?
- Students solve the problems in the group of four, so they have opportunities to share their thinking to others.
- Teacher provides the grain necklaces to each group, so they can do hands on activity.
- Two groups of students present their solution in front of the class. Teacher discusses the students' solution. She guides students to find the relation between the first and the second context, which is the relation between addition and subtraction.
- Teacher poses the third context as following:
  c. Farah already finished the first necklace that consists of 40 grains. She is stringing until 33 grains for the second necklace. How many grains the differences between the first and the second necklace right now?
- Two groups of students present their solution in front of the class. Teacher discusses the students' solution. She guides students to understand that "taking away something"

does not make sense in this context. Students will get a new understanding that the meaning of subtraction is also “determining the difference between two numbers”.

4. Conjectures of Students’ Thinking and Teacher’s Reactions

Table 5. Conjectures of students’ thinking and teacher’s reactions for lesson 2

No	Students’ Thinking	Teacher’s Reactions
1.	For the first context: Some students will use the grain necklaces to represent the situation. They will add the grains one by one, or two by two, or three by three, or directly seven grains, etc. Some students will draw the grain necklaces to represent the situation. They will draw the additional grains one by one, or two by two, or three by three, or directly seven grains, etc. Some students only write the numbers. They will use their fingers to count forward the number one by one, or two by two, or three by three, or directly seven, etc.	Teacher supports students to make visualization for their solution.
2.	For the second context: Some students will use the grain necklaces to represent the situation. They will add the grains one by one, or two by two, or three by three, etc, until 40 is reached. Some students will draw the grain necklaces to represent the situation. They will draw the additional grains one by one, or two by two, or three by three, etc, until 40 is reached. Some students only write the numbers. They will use their fingers to count forward the number until 40 is reached. Maybe some students will realize that this context not only can be seen as addition problem but also	Teacher supports students to make visualization for their solution.

	can be seen as subtraction problem.	
3.	For the third context: Some students will use two grain necklaces that consist of 40 and 33 grains to represent the situation. They will compare those two necklaces and see the difference. Some students will only use one grain necklace which is consisted of 40 grains. They will take the grains one by one, or two by two, or three by three, etc, until that necklace consists of 33 grains. Some students will only use one grain necklace which is consisted of 33 grains. They will add the grains one by one, or two by two, or three by three, etc, until that necklace consists of 40 grains.	Teacher supports students to make visualization for their solution.

C. Lesson 3 (Introducing the Beads String)

1. Goals

- **Knowledge**

- Students know how to use the beads string
- Students know how to use "ten catcher"
- Students know the rule to draw the beads string

- **Skills**

- Students can use the beads string to represent the real objects
- Students can use the beads string as a "model of" the situation
- Students can solve addition and subtraction problems in the beads string
- Students can count with tens and ones up to 100 in the beads string

- **Attitude**

- Students are willing to use the beads string as a representation for the contexts
- Students are willing to share their thinking

2. Starting Positions

- **Knowledge**

- Students know the relation between two numbers in the beads string

- **Skills**

- Students can count from any numbers in the beads string
- Students can locate the numbers in the beads string

- **Attitude**

- Students are willing to work with the beads string

3. Description of Activity

- Teacher poses two contextual problems from the previous activities. The contexts are the following:
  a. Donna has 26 ginger candies. She gives 6 of those candies to Andre. How many candies that Donna still has?
  b. Farah already finished the first necklace that consists of 40 grains. She is stringing until 33 grains for the second necklace. How many grains the differences between the first and the second necklace right now?
- Teacher lays down a string of beads consists of 100 beads in the whiteboard. Each ten beads have different color. Teacher also provides a tool, namely "ten catcher", to catch ten beads.
- One student presents the solution for the first context that he/ she had done before using the beads string. Teacher draws the representation of the solution.
- One student presents the solution for the second context that he/ she had done before using the beads string. Teacher draws the representation of the solution.
- Teacher discusses the difference of students' solution. She also emphasizes the rule to draw beads string (to write the number in the interval of the beads).

- Teacher gives a string of problems consists of five addition problems and five subtraction problems. This string of problems can lead students in using jumps and hops in the beads string. The structure of beads can help students to apply jumps and hops.

  The string of problems is the following:

Addition problems	Subtraction problems
a. 12 + 3 = …	a. 76 – 4 = …
b. 12 + 10 = …	b. 76 – 10 = …
c. 12 + 13 = …	c. 76 – 14 = …
d. 12 + 30 = …	d. 76 – 30 = …
e. 12 + 33 = …	e. 76 – 34 = …

- Students work in pair. Each pair gets the drawing of beads string and a manipulative of “ten catcher”.
- Teacher conducts class discussion to discuss students’ solution.

4. Conjectures of Students’ Thinking and Teacher’s Reactions

Table 6. Conjectures of students’ thinking and teacher’s reactions for lesson 3

No	Students’ Thinking	Teacher’s Reactions
1.	Some students still find difficulty to translate the context into the use of beads string.	Teacher guides students using a real beads string.
2.	Some students can translate the context into the use of beads string correctly.	Teacher emphasizes the rule to draw beads string.
3.	Some students only count one by one in the beads string.	Teacher promotes students to use “ten catcher”.
4.	Some students already used “ten catcher” in the beads string.	Teacher promotes students to apply jumps and hops.

D. Lesson 4 (Applying the Beads String)

1. Goals

- **Knowledge**

- Students know how to make jumps in the beads string
- Students know how to make hops in the beads string

- **Skills**

- Students can make jumps of ten or jumps via ten in the beads string
- Students can find as many as possible the ways (the combinations of numbers) to come to the result

- **Attitude**

- Students are willing to make their own productions

2. Starting Positions

- **Knowledge**

- Students know how to use the beads string
- Students know how to use "ten catcher"
- Students know the rule to draw the beads string

- **Skills**

- Students can use the beads string as a "model of" the situation
- Students can solve addition and subtraction problems in the beads string
- Students can count with tens and ones up to 100 in the beads string

- **Attitude**

- Students are willing to work with the beads string

3. Description of Activity

- Students are asked to make their own productions to explore jumps and hops in the beads string.

- Students work in the group of four. Each group gets the drawing of beads string and a manipulative of “ten catcher”.
- Each group should find as many as possible the ways (the combinations of numbers) to come to the result using addition and subtraction operations.

  The result of addition operations must be 27, 44, and 85.

  The result of subtraction operations must be 9, 31, and 60.
- Teacher conducts class discussion to discuss students’ productions.

4. Conjectures of Students’ Thinking and Teacher’s Reactions

Table 7. Conjectures of students’ thinking and teacher’s reactions for lesson 4

No	Students’ Thinking	Teacher’s Reactions
1.	Some students only count one by one in the beads string.	Teacher promotes students to use “ten catcher”.
2.	Some students already used “ten catcher” in the beads string.	Teacher promotes students to apply jumps and hops.
3.	Some students can use jumps and hops in the bead strings.	Teacher supports students to make jumps of ten.
4.	Some students only find one solution (one combination of numbers).	Teacher supports students to find other possibilities.
5.	Some students can find more than one combination of numbers from the given number.	Teacher asks students to share their thinking in the discussion.

E. Lesson 5 (Introducing the Empty Number Line)

1. Goals

- **Knowledge**

- Students know how to use the empty number line
- Students know the rule to draw the empty number line

- Students know the relation between the drawing of beads string and the drawing of empty number line

- **Skills**

- Students can use the empty number line to represent the beads string
- Students can use the empty number line as a "model for" their thinking
- Students can solve addition and subtraction problems in the empty number line
- Students can count with tens and ones up to 100 in the empty number line

- **Attitude**

- Students are willing to use the empty number line as a representation for the beads string
- Students are willing to share their thinking

2. **Starting Positions**

- **Knowledge**

- Students know the relation between two numbers in the empty number line

- **Skills**

- Students can count from any numbers in the empty number line
- Students can locate the numbers in the empty number line

- **Attitude**

- Students are willing to work with the empty number line

3. **Description of Activity**

- Teacher poses two contextual problems from the previous activities. The contexts are the following:
    a. Donna has 26 ginger candies. She gives 6 of those candies to Andre. How many candies that Donna still has?

b. Farah already finished the first necklace that consists of 40 grains. She is stringing until 33 grains for the second necklace. How many grains the differences between the first and the second necklace right now?

- One student draws a string of beads that represents the solution for the first context and the teacher demonstrates the shift from the drawing of beads string to the drawing of empty number line.
- One student draws a string of beads that represents the solution for the second context and the teacher demonstrates the shift from the drawing of beads string to the drawing of empty number line.
- Teacher discusses the shift from the drawing of beads string to the drawing of empty number line to make students aware of the flexibility of empty number line (the interval of numbers does not depend on the length of line).
- Teacher poses a worksheet that consists of the string of problems in the activity 3. Below each problem is provided the drawing of beads string and the drawing of empty number line. The string of problems is the following:

Addition problems	Subtraction problems
a. 12 + 3 = …	a. 76 – 4 = …
b. 12 + 10 = …	b. 76 – 10 = …
c. 12 + 13 = …	c. 76 – 14 = …
d. 12 + 30 = …	d. 76 – 30 = …
e. 12 + 33 = …	e. 76 – 34 = …

- Students in pair are asked to represent the solution of the problems using the drawing of beads string and the drawing of empty number line. This activity can promote students to apply jumps and hops in the empty number line.
- Teacher conducts class discussion to discuss students' solution.

4. Conjectures of Students' Thinking and Teacher's Reactions

Table 8. Conjectures of students' thinking and teacher's reactions for lesson 5

No	Students' Thinking	Teacher's Reactions
1.	Some students still find difficulty to transform the drawing of beads string into the drawing of empty number line.	Teacher guides students in person.
2.	Some students can transform the drawing of beads string into the drawing of empty number line correctly.	Teacher asks students to help the others.
3.	Some students only count one by one in the empty number line.	Teacher promotes students to count with tens and ones.
4.	Some students already counted with tens and ones in the empty number line.	Teacher supports students to apply jumps and hops.

F. Lesson 6 (Applying the Empty Number Line)

1. Goals

- **Knowledge**

- Students know the meaning of subtraction
- Students know the relation of the numbers involved
- Students know how to make jumps and hops in the empty number line

- **Skills**

- Students can find the solution to solve the contextual problems and the bare number problems in subtraction using the empty number line
- Students can make jumps of ten or jumps via ten in the empty number line
- Students can apply direct subtraction and indirect addition in the empty number line

- **Attitude**

- Students are willing to solve the problems in their own way
- Students are willing to present their solutions

2. Starting Positions

- **Knowledge**

- Students know how to use the empty number line
- Students know the rule to draw the empty number line
- Students know the relation between the drawing of beads string and the drawing of empty number line
- Students know the contextual problems and the bare number problems

- **Skills**

- Students can use the empty number line as a "model for" their thinking
- Students can count with tens and ones up to 100 in the empty number line

- **Attitude**

- Students are willing to work with the empty number line
- Students are willing to work with the contextual problems and the bare number problems

3. Description of Activity

- Teacher poses a worksheet that consists of four problems of subtraction as "taking away something" and "determining the difference between two numbers" in the context format and bare number format. The problems are the following:
    a. Father has 47 fish in the pond. Because of the flood, 13 of his fish disappeared and 9 of the remained fish were death. How many fish that father still has now?
    b. Anna loves to read Indonesian folktale book. She read "The legend of Lake Toba". That book has 90 pages. She has finished page 52. How many more pages does Anna need to finish her reading? Her friend, Liza, also read the same book. She is finishing page 38 right now. How many more pages that Liza should read to finish the book?

c. 62 – 59 = …

d. 54 – 15 = …

- Students solve the problems in the group of four, so they have opportunities to share their thinking to others. Teacher walks around the class and facilitates help for students.
- Teacher conducts class discussion to discuss different students' solution. She leads students to find the meaning of subtraction in each problem and the more efficient strategies to solve it. The discussion focuses on direct subtraction (removing the subtrahend from the minuend) and indirect addition (adding on from the subtrahend until the minuend is reached) in solving subtraction problems.

4. Conjectures of Students' Thinking and Teacher's Reactions

Table 9. Conjectures of students' thinking and teacher's reactions for lesson 6

No	Students' Thinking	Teacher's Reactions
1.	Students will use different strategies to solve the problems. Some of them still use drawing, some of them will use the beads string, and some of them will use the empty number line.	Teacher promotes students to use more efficient strategy in solving the problems.
2.	Some students will apply the empty number line as a model to solve the problems, but they still use same reasoning in each problem (using all direct subtraction or all indirect addition).	Teacher supports students to look back at the context and the numbers involved.
3.	Some students will apply the empty number line and they are able to use counting back or adding on one by one.	Teacher supports students to make jumps and hops.
4.	Some students are able to use more advance strategies in the empty number line, not only counting back or adding on one by one. They might think to make jumps of ten or jumps via ten.	Teacher asks students to share their thinking with the others.
5.	In the problem a, most students will apply counting back strategy using the empty number line. They	Teacher asks students to share their thinking in the

	will take 13 from 47 first, and then they will take 9 from the rest. However, it is possible that some students will add 13 and 9 first, and then they will remove the sum of 13 + 9 from 47.	discussion.
6.	In the first question on problem b, students will apply adding on strategy using the empty number line. Most of them can solve correctly and get 38 as a result. In the second question, some of them also apply adding on strategy using the empty number line. However, it is possible that some students realize the relationships between first and second questions. They will directly come up with an answer 52 pages.	Teacher asks students to share their thinking in the discussion.
7.	Some students already have a good number sense to solve problem c and d. They look at the number first to decide on a strategy. If minuend and subtrahend are far away, they will use counting back (direct subtraction); while if two numbers are close together, they will use adding on (indirect addition).	Teacher asks students to share their thinking with the others.

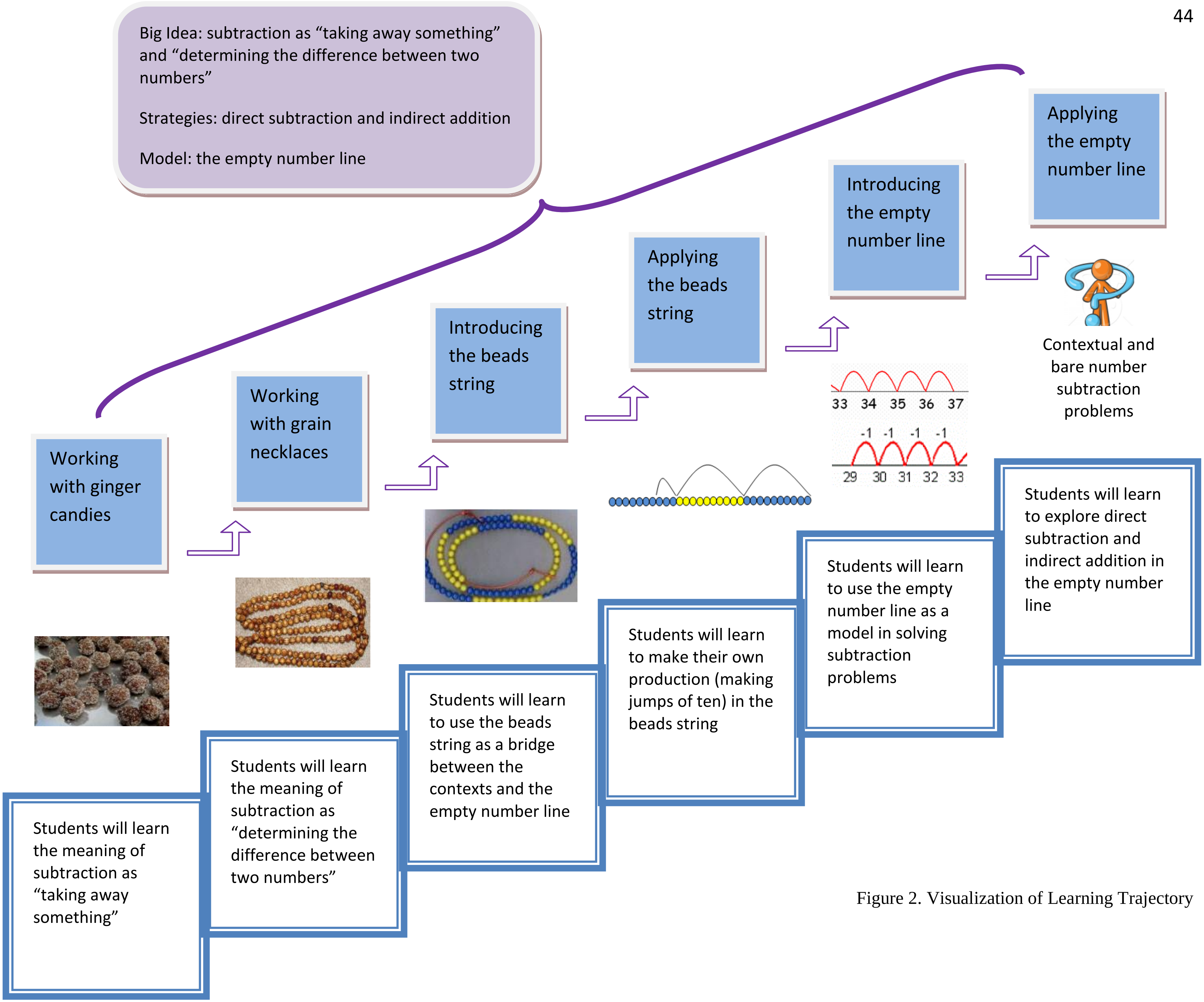


Figure 2. Visualization of Learning Trajectory

CHAPTER V

RETROSPECTIVE ANALYSIS

A. First Cycle

The first cycle was conducted in three steps: pre-test, pilot experiment, and post-test. The participants were 6 first grade students from class 1E of SDN 179 Palembang. Those students represent the whole class because they have different abilities (1 high achiever student, 3 average students, and 2 low level students). The results of the first cycle are used to make adjustment and improvement for the teaching experiment in the second cycle.

1. Pre-Test

Most of the students could answer part A question number 1, 2, 3, 5, 6 correctly. Number 4 is the most difficult question for them. Only one student could fill the blank boxes with the desired numbers. Her strategy is she counted the differences between the first box and the second box. She found that she have to count 10 numbers from the one box to the next box.

For part B, all students could give the right answer for question number 1, 3, and 4. They used their fingers to solve those problems with counting back strategy. For question number 5 up to 8, most of the students had difficulties to answer correctly. The interesting case was one of the students could answer question number 7 and 8 using the concept of place value, the algorithm of subtracting tens and ones separately. She started to subtract tens with tens and ones with ones. In the question number 8, she got the correct answer. Her answer is: 85 - 34 = 51 because 8 - 3 = 5 and 5 - 4 = 1. For question number 7, she used the same strategy. She wrote: 51 - 49 = 18 because 5 - 4 = 1 and 1 - 9 (she might think 9 - 1) = 8.

From the result of pre-test in the first cycle, we make the adjustment and the improvement for the pre-test in the second cycle. Because of the limited time, we make the question fewer. We make the font bigger and the sentence simpler. We add the picture in the question about the age differences to promote students' idea in solving the problem. We change the context about travelling to the context about reading a book and we add the picture of it. This context is more familiar for students. The last, we write "the strategy that I use: " in the answer box to guide students to write not only the final answer but also the strategy behind their solution.

2. Pilot Experiment

Lesson 1 and lesson 2 are still same with the initial HLT. Based on the students' reaction and the teacher's advice, we changed the activities in the lesson 3, 4, and 5. We reversed the order of activities, we removed some activities and we added some other activities. In the lesson 6, we only changed the questions in the worksheet.

a. Lesson 1 (Working with Ginger Candies)

All students could answer the first question correctly (20 + 6 = 26) and all of them used ginger candies. They still used ginger candies to solve the second question. One of them used different strategy (26 - … = 20) while the others wrote 26 - 20 = … In the discussion, all of them could understand that two strategies above have the same meaning.

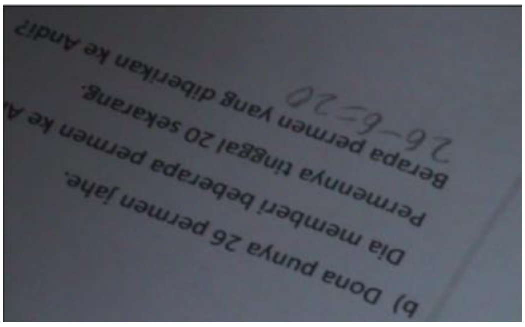


Figure 3. Student's strategy to solve ginger candies' problem

Students seemed know the relation between addition and subtraction in the first and second question, but they could not say that "subtraction is the converse of addition". The teacher said that it is impossible for first grade students to say "subtraction is the converse of addition". We will pose more questions in the second cycle to make sure that the students understand the relation between addition and subtraction. For example: if 21 + 7 = 28, then 28 - 7 = … and 28 - 21 = … The students have to answer those questions as fast as possible without counting.

All students did not find difficulties to answer the third question. They used ginger candies to solve 26 - 6 = 20. They also could understand that the meaning of subtraction is "taking away something" in this context. It is same with the meaning of subtraction in other contexts they usually faced.

b. Lesson 2 (Working with Grain Bracelets)

We made the activity more doable for students by changing the activity of making necklaces into making bracelets and changing the number of grains. It was time consuming when students are asked to string two bracelets. So, in the second cycle we will already prepare the second bracelet for each group.

The first question could be answered correctly by all students (23 + 7 = 30) and they used grain bracelets. In the second question, all of them used the same strategy (23 + … = 30). No one could come up with the idea of 30 - 23 = … For the second cycle, we will change the second question with the context that can stimulate students to use subtraction idea so that they can understand the relation between addition and subtraction in the first and second question.

For the third question, the students knew the differences between the grains in the first and second bracelet by comparing those bracelets. When we asked them to write the mathematical notation, they look confused. We guided them to come to the notation 30 - 23 by drawing the picture of those bracelets. They look surprised after knowing that subtraction also has the meaning “determining the difference between two numbers”. We asked more questions with different numbers to make students more familiar with this other meaning of subtraction.

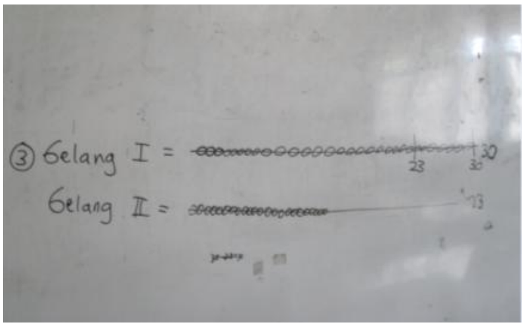


Figure 4. Student’s strategy to compare two grains bracelets

c. Lesson 3 (Working with the Beads String)

For question number 1 (26 - 6), as we predicted before, there are some students who took away the beads string from 26 and some students who took away from 1. We compared their strategies and we agreed that it is easier to solve this question by taking away from 26. For question number 2 (30 - 23), students also used two different strategies. Because the minuend is 30, it is quite similar to take 23 from 30 and from 1. We could see clearly that the differences are 7 using both strategies. In the second cycle, we will change the number become 28 - 21 so the students can see that this type of question can be solved easily using counting on strategy. Some students still confused what is the answer. We need to make a circle for the final answer because the answer for counting back strategy is the number in the beads string and the answer for counting on strategy is the difference between two numbers.

Figure 5. Student's strategy using the beads string

Most of the students had difficulties to solve worksheet 3. Only one of them could apply jumps and hops in the drawing of beads string. We skipped the remained questions and we asked students to make "jumps of 10" in the beads string using "ten catchers". We gave them example by jumping from 0 – 10 – 20 – 30 etc and 1 – 11 – 21 – 31 etc. The students tried to jump from different numbers, not only jumping forward but also jumping backward. We also wrote the jumping notation in the whiteboard. For the second cycle, we will use jumping activity in the beads string first and we will give the worksheet later in the lesson 5. We will mix this worksheet with the worksheet about jumps and hops in the empty number line.

d. Lesson 4 (Working with the Empty Number Line)

We removed the activity in the initial HLT about students' own production. Based on students' understanding so far, it is difficult for them to do the initial activity. They also did not familiar with this kind of activity. The teacher suggested us to conduct the same activity in the lesson 3 using the empty number line. By doing the activities in the lesson 3 and lesson 4 consecutively, the students will see the connection among the beads string, the drawing of beads string, and the empty number line in solving subtraction problems.

At first, in the drawing of beads string, one student still took away from 1 when solving 26 - 6. He realized by himself that it is easier to take away from 26, as we discussed in the previous lesson. Using two strategies, taking away at the start and taking away at the end, we demonstrated the shift from the drawing of beads string to the empty number line. By seeing these drawing and conducting class discussions, some students could realize that the first question (taking away context and small subtrahend) is solved easily using counting back (direct subtraction strategy). On the other hand, the second question (difference context and large subtrahend) is solved easily using counting on (indirect addition strategy).

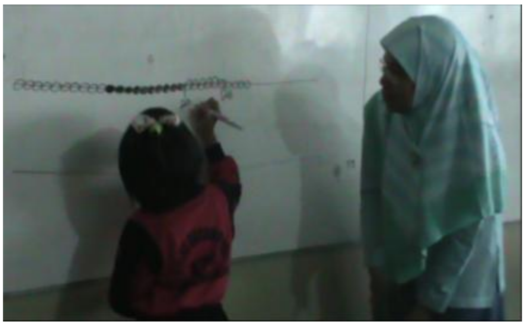


Figure 6. The shift from the beads string to the empty number line

We gave the example of jumping forward and jumping backward in the empty number line. Most of the students could make jumping forward correctly. Even, there is a student who could draw jumps of ten from 6 until 196. Some students still faced difficulties in doing jumping backward. We will ask the students to write the notation of jumping forward and backward in their paper for the second cycle to make them can see clearly the pattern of “jumps of 10”.

e. Lesson 5 (Working with the Beads String and the Empty Number Line)

We gave worksheet 5 to the students and we asked them to represent the string of problems into the drawing of beads string and the empty number line. Some students could apply “jumps of 10”, but there are two students who still counted one by one. One student used interesting strategy when solving 12 + 13. Instead of using 12 + 10 + 1 + 1 + 1, she used 12 + 8 + 2 + 3. In solving 12 + 30, she was able to reverse the question become 30 + 12 = 30 + 10 + 2. Another student found difficulties to answer the bigger number. She used arithmetic rack (100 beads) to solve 12 + 30 and 12 + 33.

After solving the worksheet, we asked students to solve the problems in the whiteboard. Two students came up in front of the class and solved two problems: 31 - 3 and 31 - 29. We prepared the drawing of 40 beads in the whiteboard. The student could answer correctly the first problem. The other student made a mistake in solving the second problem. First, she gave the mark for beads number 31. Then, she counted 29 by making the circle from beads number 30 to 20, 20 to 10, and 10 to 1. At the beginning, she thought that the answer is 1. Later on, she saw the rest of the beads and she counted all of them. She got that the answer is 11. We helped her to come back to the question by only drawing 31 beads. She realized her mistake and she got 2 as the result.

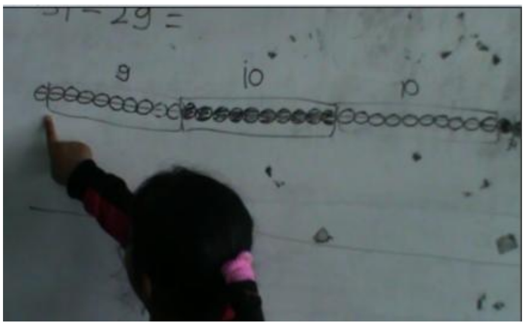


Figure 7. Student’s strategy to solve 31 - 29

We will make improvement for the second cycle by paying more attention to the choice of numbers that can stimulate students to use different strategies in solving subtraction (direct subtraction and indirect addition). We will make the question fewer (6 questions) and more focus on subtraction problems.

f. Lesson 6 (Solving Subtraction Problems)

We posed worksheet 6 with some modifications based on the advice from the teacher. We made the problem number 1 and number 2 simpler and not too complicated. We also added two questions to give more exercises for students in solving the different types of subtraction problems. We promoted students to use the empty number line in solving the problems by drawing the picture of it in the answer box.

In the first question (26 - 10), most of the students tried to use the empty number line, but some of them did not sure how to start and asked for help. One student drew 26 beads in the empty number line and colored the beads number 11 until 20 so that each ten beads have different color. She took away first 10 beads and got 10 + 6 = 16 beads as the result. For the second question (reading a newspaper) and the third question (age differences), all students could recognize that those questions are the subtraction problems.

Two students solved the next two questions (56 - 4 and 65 - 61) in the whiteboard. The student got the right answer for the fourth question using direct subtraction strategy. For the fifth question, the student firstly used direct subtraction. She felt difficulty to continue her counting. We asked her to use other strategy. She remembered indirect addition strategy and she got 4 as the answer.

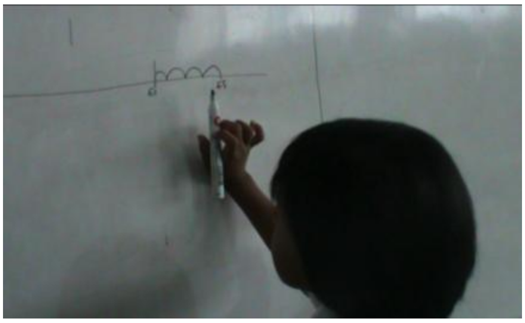


Figure 8. Student’s strategy to solve 65 - 61

The other interesting finding was there is a student who asked us whether she can use her previous strategy (subtracting tens and ones separately) to solve 65 - 61. We wanted her to present her strategy in front of the class. She said that the answer is 4 because 6 - 6 = 0 and 5 - 1 = 4. We posed the question number 6 (61 - 59) and let her to solve it using her own way.

Researcher: What is the answer?
Student : 18
Researcher: Why?
Student : Because 6 - 5 = 1 and 9 - 1 = 8.
Researcher: 6 - 5 = 1, but 9 - 1 or 1 - 9?
Student : Oh, I make a mistake, it is 1 - 9.
Researcher: What is the result of 1 - 9?
Student : (thinking) We cannot subtract 9 from 1.
Researcher: So?
Student : The problem cannot be solved using this strategy.

Later on, we asked her to solve both questions (65 - 61 and 61 - 59) using the empty number line. She could use counting on strategy properly. She found that the answer for the first question is 4, same with the answer using her strategy. She also could get 2 as the answer for the second question. We brought this problem into class discussion and most of the students could realize that the empty number line is an appropriate model to solve subtraction problems. We will allocate more time to discuss different students' solution in the second cycle.

3. Post-Test

We used 8 questions which are same with the questions in the part B of pre-test. We made the font bigger and the sentence simpler. We added the picture in the some questions. The students could understand the problems and they recognized that all of the problems are the subtraction problems, but not all of them used the empty number line in solving those problems.

For question number 1 up to number 4, only one student directly drew and used the empty number line. The others used their fingers to get the answer. Even, two students still used their fingers until the last question like what they did in the pre-test. However, one of them could make a switch between direct subtraction and indirect addition strategy based on the numbers involved. One student also still used the arithmetic rack to solve the question number 5 until number 8. We let them to solve the problems in their own way.

The student who has the place value strategy still used her strategy in solving question number 5 (68 - 13) and question number 6 (45 - 32). However, she already understood that her strategy cannot be used anymore to solve question number 7 and 8 (51 - 49 and 75 - 26). She was able to use the empty number line to solve those questions. Firstly, she used indirect addition strategy for both questions. Then, she realized that 26 is too far away from 75. She changed her strategy and she found that direct subtraction is easier to solve the last question.

For the second cycle, we will compare the strategy using fingers and using the empty number line like we compared the empty number line with the place value strategy in the sixth lesson. We hope the students can realize by themselves that in solving subtraction problems with big numbers (more than 20), using fingers is difficult and leads to wrong answer. We also will not allow students to use arithmetic rack or other tools to solve subtraction problems in the post-test.

B. The Improved HLT

We made the improvement of the HLT with reversing the order of some activities, making the problems simpler both the mathematical content and the sentences used, changing the grains necklaces into the grains bracelets to make it more doable for students, and changing the numbers of grains from 30 - 23 into 28 - 21 to support students using various strategies in solving the problem.

Briefly, the changes from the initial HLT into the revised HLT based on the pilot experiment and the discussion with the teacher can be described in the table below.

Table 10. The revised HLT

Lesson	Activity	Goals
1	Working with Ginger Candies	- Students are able to understand that subtraction is the converse of addition - Students are able to understand the meaning of subtraction as "taking away something"
2	Working with Grain Bracelets	- Students are able to understand that subtraction is the converse of addition - Students are able to understand the meaning of subtraction as "determining the difference between two numbers"
3	Working with the Beads String	- Students are able to use the beads string as a "model of" the situation in solving subtraction problems - Students are able to make "jumps of 10" in the beads string
4	Working with the Empty Number Line	- Students are able to use the empty number line as a "model for" their thinking in solving subtraction problems - Students are able to make "jumps of 10" in the empty number line
5	Working with the Beads String and the Empty Number Line	- Students are able to make a shift from the drawing of beads string into the empty number line - Students are able to apply counting back strategy (direct subtraction) and counting on strategy (indirect addition) in solving subtraction problems - Students are able to make "jumps of 10" in the beads string and in the empty number line

6	Solving Subtraction Problems	- Students are able to use the empty number line in solving subtraction problems - Students are able to apply counting back strategy (direct subtraction) and counting on strategy (indirect addition) in solving subtraction problems - Students are able to make “jumps of 10” in the empty number line

C. Second Cycle

The second cycle was conducted also in three steps: pre-test, teaching experiment, and post-test. The participants were 31 first grade students from class 1D of SDN 179 Palembang. We had a focus group consists of 4 students who have different abilities (1 high achiever student, 2 average students, and 1 low level students). We observed in detail the learning process of the focus group.

In this section, we compared our improved HLT and students’ actual learning process during the teaching experiment. We looked to the video recordings and selected some critical moments. We also collected the written works of the students. We analyzed the lesson to observe what students and teacher do, how the activities work, and how the material contributed to the lesson. We investigated whether the HLT supported students’ learning. The result of the retrospective analysis in this teaching experiment would be used to answer the research questions.

1. Pre-Test

Pre-test consisted of two parts. Part A is given to know the prior knowledge of the students; those are counting from any number (from question number 1 until number 3) and locating the numbers (from question number 4) as a basic skill to work with the beads string and the empty number line. It is expected if the students can answer question number 2 and number 3 correctly; they already have the sense of "jumps of 10" towards a number.

Part B is given to know the starting points of the students and what they should learn in subtraction. It will give the overview of students' strategies in solving the subtraction problems. They already learnt the subtraction up to 20 (question number 1 and 2), but they do not learn yet the subtraction up to 100 (question number 3 until 6). The problems consisted of contextual problems (taking away and adding on contexts) and bare number problems (large difference and small difference numbers).

The question number 1 from part A could be answered correctly by all students. For question number 2, only 2 students got the wrong result; and for question number 4, only 5 students made a mistake. Twelve students found difficulty to answer the question number 3.

There are several strategies that students applied to solve the questions from part B. We could find the students who used their fingers, the students who used the arithmetic rack, the students who already used the algorithm of subtracting tens and ones separately, and also some students who made a drawing to come to the solution.

The students' answer for the questions from part B can be shown in the table.

Table 11. The students' answer for the questions from pre-test part B

No	**Correct Answer**				**Wrong Answer**	**No Answer**
	Using Fingers	**Using Rack**	**Using Algorithm**	**Using Drawing**		
1.	20 students	6 students	-	2 students	3 students	-
2.	16 students	4 students	-	-	10 students	1 student
3.	6 students	4 students	12 students	-	6 students	3 students
4.	2 students	4 students	10 students	-	12 students	3 students
5.	2 students	6 students	-	-	17 students	6 students
6.	-	6 students	-	-	18 students	7 students

2. Teaching Experiment

a. Lesson 1 (Working with Ginger Candies)

In lesson 1, we expected the students are able to understand that subtraction is the converse of addition and the students are able to understand the meaning of subtraction as “taking away something”.

The teacher started the lesson by reminding students about the subtraction up to 20 that they have learned before in the first semester. She posed an example “There are 18 girls in this class and 13 of them are wearing pigtail. How many girls in this class did not wear a pigtail?” Most of the students used their finger to count the result and most of them got the correct answer.

After that, the teacher gave worksheet 1.1 that consists of two contexts about addition and subtraction. The first context is “Dona has 20 ginger candies. Rani gives to her 6 more candies. How many candies that Dona has right now?” The second context is “Dona has 26 ginger candies. She gives some candies to Andi. She still has 20 ginger candies right now. How many candies that Dona gives to Andi?” The teacher also provided the ginger candies for each group as a manipulative to help them in counting.

Like in the first cycle, all groups could answer the first question correctly (20 + 6 = 26), but they used different strategies. Group Jambu still used their fingers to count the result. They opened 6 fingers and started to fold them one by one from 21 until 26. While Group Mangga used the ginger candies to find the answer. They arranged the candies in two groups of 10 and they added it with 6 candies.

Figure 9. Group Jambu solved 20+6 Figure 10. Group Mangga solved 20+6

Students in the focus group (Group Apel) also had different strategies. Syauqi and Vera directly got the correct answer; they could do the mental calculation. Febi still did not sure with her friends' answer and she used the ginger candies. She counted the candies one by one. Vera helped Febi in counting the candies. Fakhri draws the candies when Febi is counting.

For the second question, there are two strategies which are used by the students, 26 - 20 = … and 26 - … = 20. The students in the focus group also had different strategies. Fakhri, the one who holds the paper and pencil, started to draw the candies. He drew 26 candies, and then he made a square for 6 candies. He drew 20 candies again after the equal sign. Later on, he wrote the mathematical notation above the drawing, 26 - 6 = 20. The researcher asked him to explain the answer.

Researcher: What is your answer?
Fakhri : This is our answer, 26 - 6 = 20 (pointing at their worksheet).
Researcher: So, how many candies that Dona gives to Andi?
Fakhri : 20 candies
Syauqi : Wait…wait…the answer is 6, isn't it?
Fakhri : hmm…(look confused)
Researcher: How about you, Vera and Febi?
Vera : I think the answer is 6.
Febi : I do not count it yet.
Researcher: Ok, let's back to the question. How many ginger candies that Dona has before?
Students : 26 ginger candies
Researcher: She gives some candies to Andi, right?
Students : yes
Researcher: How many candies that she still has right now?
Students : 20
Researcher: The question is how many candies that Dona gives to Andi?

Fakhri : Oh, I know. The answer is 6. I mean, these are Dona's candies (pointing at 26), she gives these candies to Andi (pointing at 6), and these are the rest of the candies (pointing at 20).
Researcher : All agree that the answer is 6?
Students : yes
Researcher: Fakhri, please make a circle for the final answer so that you did not get confused.
Fakhri : Ok

From the fragment above, we could see that Fakhri could understand the question and could find the correct answer, but he got confused to determine the final answer. When looking back at the question, he could explain his reasoning well. Syauqi and Vera could get the result using the mental calculation, while Febi still need the real thing to help her in counting. After the conversation, instead of making a circle for the correct answer, Fakhri changed the strategy become 26 - 20 = 6. He also changed the drawing. He made a square for 20 candies and he drew 6 candies after the equal sign.

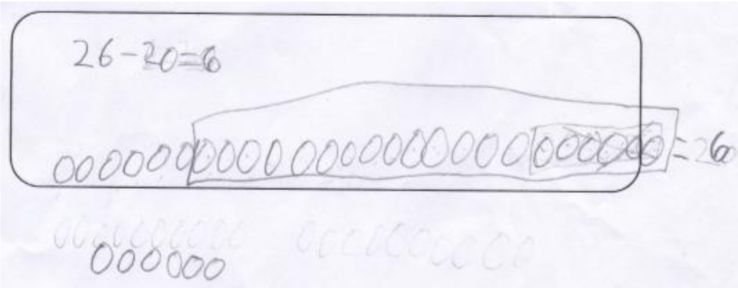


Figure 11. Focus group's strategy to solve 26-20

The teacher conducted the class discussion to discuss the students' solutions for the first and second question. She also asked "What is the relation between addition and subtraction in the first and second context?" As we predicted before, all students are silent, they could not say that "subtraction is the converse of addition." Then, she guided the students to pay more attention for the three numbers involved.

Teacher : What is the mathematical notation we get from the first question?
Students : 20 + 6 = 26
Teacher : From the second question?

Students : 26 - 20 = 6
Teacher : We have one more notation, right?
Ravli : We also can write 26 - 6 = 20
Teacher : It is correct. So, what can you see from these three notations?
Allya : If we add 20 and 6, we get 26. So, if we subtract 20 from 26 we will get 6, and if we subtract 6 from 26 we will get 20.
Teacher : Any other opinion? (all students are silent) Do you agree with Allya's answer?
Students : Yes

The teacher posed three questions to each group. They have to answer those questions as fast as possible. The examples of the question for Group Apel are: 20 + 7 = …, then 27 - 7 = … and 27 - 20 = … Five groups could answer all of the questions correctly without counting, they were able to understand the relation between addition and subtraction. Three other groups also could give the correct answer, but they still counted the number one by one. One group got wrong result when counting. The teacher asked the students to check the other groups' answer.

Teacher : Let's check this answer (pointing at the whiteboard). Where is the mistake?
Students : 27 - 5 = 21
Teacher : It is, right? (pointing at 27 - 5 = 21)
Students : yes
Teacher : Now, there is 27, there is 5. There 22, but here 21. So, what number should be here?
Students : 22
Teacher : That's right, 22. Do you understand?
Students : yes

Later on, the teacher gave worksheet 1.2 that consists of a context about subtraction as "taking away something". The context is "Dona has 26 ginger candies. She gives 6 of those candies to Budi. How many candies that Dona still has?" All groups did not find difficulties to answer this question. They used different strategies like they did to solve worksheet 1.1. They were able to write 26 - 6 = 20. They also could understand that the meaning of subtraction is "taking away something" in this context.

From lesson 1, we concluded that most of the students could solve the addition and subtraction problems. They used various strategies in solving the problems. Some students used their fingers, some of them used the ginger candies, other students drew the picture of candies, and even there are some students who were able to do the mental calculation for easy number. Most students also were able to understand that subtraction is the converse of addition implicitly. It is indicated by the students could solve the subtraction problems without counting, only by seeing their relation with the addition problem before. All students did not find difficulties to understand the meaning of subtraction as “taking away something” because the already familiar with it.

b. Lesson 2 (Working with Grain Bracelets)

In lesson 2, we expected the students are able to understand that subtraction is the converse of addition and the students are able to understand the meaning of subtraction as “determining the difference between two numbers”.

The teacher started the lesson by asking students about the meaning of subtraction. There are various answers from them such as given, taken, broken, damaged, dead, disappeared, and leave. All of the answers refer to the meaning of subtraction as “taking away something”.

Then, the teacher posed worksheet 2.1 that consists of two contexts about addition and subtraction. The first context is “Farah is stringing 21 grains to make a bracelet. She needs 7 more grains. How many grains which are needed to make Farah’s bracelet?” The second context is “Farah’s bracelet consists of 28 grains. Those 7 grains are lost. How many grains the remaining?” The teacher also gave the grains bracelet to each group, so they can do hands on activity.

The students did not find difficulties to answer the addition problem like in the lesson 1. All of them could give the correct answer for the first question (21 + 7 = 28). Most students also could understand that the second question is the subtraction problem (28 - 7 = 21). Each group had different strategies to solve the problems. Some groups needed the grains as a manipulative in counting. Some groups only used their fingers. The others made a drawing of the grains bracelet in their worksheet.

Both Group Durian and Group Melon used the grains to find the answer of the problems, but they applied different ways. Group Durian used the grains one by one. Sometimes they lost of track when counting and repeated the counting from one. On the other hand, Group Melon used the grains in the form of bracelet as a representation of the situation in the context. They counted the grains two by two.

Group Mangga changed their strategy in solving the problems. In the previous lesson, they used the ginger candies to find the answer. In this lesson, they used their fingers to come to the solution. They applied counting on strategy for the first question and counting back strategy for the second question.

There is a group, Group Nanas, who could write three formal mathematical solutions. They were able to count the tens and ones separately. The teacher asked them to explain their solution. They said “Farah has 21 grains and she needs 7 more, so it is 21 + 7.” Instead of explaining what they already wrote in the worksheet, they practiced their saying using the grains bracelet. They got 28 as the answer. By looking at the bracelet, they could find the relation between the first and the second question. They said that the second question is the converse of the first, “From the first question we get 21 + 7 = 28. So if Farah has 28 grains and she lost 7 grains, she still has 21 grains in her bracelet.”

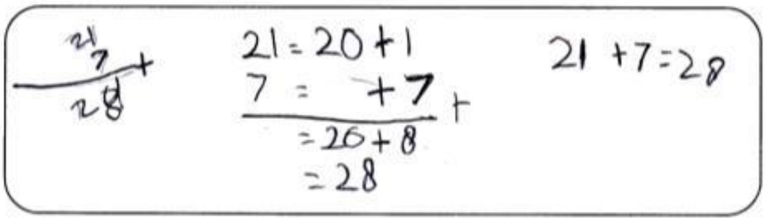


Figure 12. Group Nanas' strategy to solve 21+7

Most students in the focus group could answer the first question quickly. Vera, Syauqi, and Fakhri used their fingers with counting on strategy. Febi preferred to use the grains bracelet to solve the problems. She did not count the grains one by one. She already knew that each different color consists of 10 grains, so that she counted from 10, then 10, and 1. She put 7 grains more and she got 28 as the answer for the first question.

For the second question, Vera and Syauqi still used their fingers. Both of them switched the strategy into counting back. Febi also still used the grains bracelet and she counted by 10. Fakhri used the more advanced strategy. He did not need to count anything. He could see the relation of this question with the previous question. "If we have 21 + 7 = 28, then 28 - 7 must be 21," he said.

After all groups finished their work, the teacher asked them to present their solution voluntarily. Group Jeruk explained their answer for the first problem. They drew the grains in the whiteboard and they wrote the mathematical notation below the drawing. The solution for the second problem is presented by Group Mangga. They used their fingers, like they did in the group discussion.

The teacher asked whether there are other students who have different strategy or different answer. Fakhri came up in front of the class and presented his idea in solving the second problem.

Teacher : What is your strategy?
(Fakhri writes 21 + 7 = 28, and then he writes 28 - 7 = 21 in the below)
Teacher : Please explain your strategy to your friends.

Fakhri : Uhm…(look confused how to start)
Teacher : What is it? (pointing at 21 + 7 = 28)
Fakhri : This is the answer for the first question.
Teacher : And what is it? (pointing at 28 - 7 = 21)
Fakhri : This is the answer for the second question.
Teacher : How do you get this? (pointing at 21 + 7 = 28)
Fakhri : By counting with my fingers.
Teacher : And how do you get this? (pointing at 28 - 7 = 21)
Fakhri : I do not need to count anymore.
Teacher : Why?
Fakhri : If we know 21 + 7 = 28, then 28 - 7 is 21.
Teacher : Students, do you understand Fakhri's strategy?
(Some students nodded his head)
Vera : It is like what we did yesterday about addition and subtraction.
Teacher : Yes, it is right. Now, please answer this question quickly, without counting. What is the result of 28 - 21?
Students : 7
Teacher : Good

Later on, the teacher posed worksheet 2.2 that consists of a context about subtraction as "determining the difference between two numbers". The context is "Farah's first bracelet consists of 28 grains. Farah's second bracelet consists of 21 grains. How many grains the differences between the first and the second bracelet?" The teacher also provided the 28 grains' bracelet and the students can use the grains that they got before as the second bracelet.

Most students found difficulties to understand the meaning of "differences". The teacher guided them to put in a row and to compare the first and the second bracelet. They got 7 as the differences between those bracelets, but no one could write the mathematical notation from this context.

Figure 13. Comparing two grains bracelets

The focus group preferred to use the grains bracelets because they did not know yet the meaning of the context.

Researcher: Where is the first bracelet?
Vera : This (pointing at the 28 grains' bracelet)
Researcher: How many grains it consists of?
Vera : 28 grains
Researcher: How do you know?
Vera : By counting
Researcher: You count it one by one?
Syauqi : Yes
Vera and Febi : No
Febi : This is 10 black grains; this is 10 chocolate grains; and this is 8 black grains. So, the total is 28 grains.
Syauqi : Oh, I see
Researcher: Syauqi, can you count the second bracelet?
Syauqi : Of course. It consists of 21 grains because this is 10 chocolate grains; this is 10 black grains; and this is 1 chocolate grain.
Researcher: What will you do to find the differences between the first and the second bracelet?
Febi : I will add it.
Syauqi : I will subtract it.
Researcher: How about you, Vera and Fakhri?
(Vera and Fakhri are silent)

The researcher helped them to determine the difference between the grains in the first and the second bracelet by comparing those bracelets.

Researcher: Let's we compare the bracelets. How many grains in the first bracelet?
Students : 28
Researcher: How many grains in the second bracelet?
Students : 21
Researcher: Let's we put those bracelets in a row.
(Students put the first and the second bracelet in a row and compare them)
Researcher: Where is the difference?
Students : This (pointing at the more grains from the first bracelet)
Researcher: How many grains is it?
(Students count the grains one by one)
Students : 7 grains
Researcher: So, from which we get 7?
Vera : Subtraction
Researcher: Can you write down in the worksheet?
(Vera writes down 28 in the worksheet)
Researcher: What is the subtrahend? The second bracelet?
Vera : 21
Researcher: What is the result?
Vera : 7

Researcher: Everyone is clear? This context is also a kind of subtraction problems. If we have 28 grains in the first bracelet and we have 21 grains in the second bracelet, then the difference between those bracelets is 28 - 21 = 7.

Figure 14. The difference between two grains bracelets

From the dialog above, we knew that Vera and Febi could find the pattern of the grains bracelets (grouping by 10). At first, Syauqi counted the grains one by one. After hearing Febi's explanation, he also could count the bracelets using the group of 10. Vera and Fakhri had no idea to find the differences between the first and the second bracelet without help. Febi said that it is an addition problem, while Syauqi thought that it is a subtraction problem. With help from us by comparing the two bracelets, the students could recognize the differences between those bracelets. After getting the differences, they realized that the context is also a type of subtraction problems.

After writing the mathematical notation, we asked the focus group to make a drawing that represents the situation in the context. Vera started to draw the first bracelet. She made 10 empty circles, then 10 shaded circles, and 8 empty circles. Febi continued to draw the second bracelet below the first bracelet. She drew 10 empty circles, then 10 shaded circles, and 1 empty circle. Fakhri was asked to determine the differences between those bracelets. He surrounded 7 empty circles from the first bracelet as the differences.

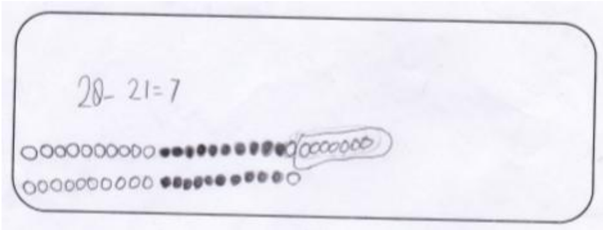


Figure 15. Focus group's strategy to solve 28-21

The teacher gave the opportunity to the group who wants to explain their solution in front of the class. Allya from Group Nanas presented her strategy in solving the problem. She made a drawing of the grains for the first and second bracelet consecutively and she made a line as a border for each 10 grains. She also made a line after 21st grains. She wrote the number of the grains next to the drawing and she wrote that the difference is 7. She made a square surrounding the 7 remaining grains from the first bracelet. Unfortunately, she forgot to write the mathematical notation from that problem.

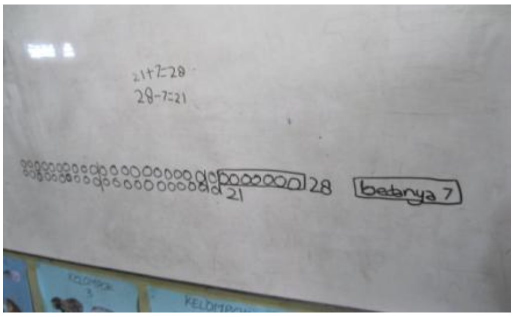


Figure 16. Allya's strategy to solve 28-21

In the end of the lesson, the teacher emphasized that the meaning of subtraction are not only "taking away something", but also "determining the difference between two numbers". She also said that there are various ways in solving subtraction problems by giving an analogue with there are various ways to come to the Ampera Bridge.

The teacher asked one more question to check students' understanding about the meaning of subtraction as "determining the difference between two numbers".

She gave an example of age difference. The problem is "Mrs. Mona (the other teacher in the class) is 28 years old and Fakhri is 7 years old. How is the age difference between Mrs. Mona and Fakhri?" Most students could answer correctly that the age difference is 28 - 7 = 21 year.

From lesson 2, we could conclude that most of the students were able to solve the addition and subtraction problems; even they could see the relation between those problems. They used different strategies that more make sense for them. Some students could count not only one by one, but also by group of other numbers (group of 2 or group of 10). It is quite difficult for the students to construct the other meaning of subtraction. Most of them took a long time and needed a guidance to understand that subtraction also has the meaning "determining the difference between two numbers". With help of the manipulative, in this case comparing two grains bracelets, the students could see the meaning of "differences" in the subtraction contexts. They also could make the mathematical notation of those contexts.

c. Lesson 3 (Working with the Beads String)

In lesson 3, we expected the students are able to use the beads string as a "model of" the situation in solving subtraction problems and the students are able to make "jumps of 10" in the beads string.

The teacher started the lesson by asking students about the meaning of subtraction. Most of them could answer that subtraction has the meaning "taking away something" and "determining the difference between two numbers".

Later on, the teacher gave worksheet 3 that consists of two contexts about two different meanings of subtraction. The contexts are still the same with the context in the lesson 1 and lesson 2. The first problem is "Dona has 26 ginger candies. She gives 6 of those candies to Budi. How many candies that Dona still has?" And the second is "Farah's first bracelet consists of 28 grains. Farah's second bracelet consists of 21 grains. How many grains the differences between the first and the second bracelet?" The teacher prepared small beads string which is contained of 50 beads for each group and one big beads string contained of 100 beads in front of the class. She stressed that every different color consists of 10 beads.

As we predicted before and like in the first cycle, two different strategies appeared in solving two problems above. There are some groups who always doing subtraction from the back, a group who always doing subtraction from the front, and also two groups who were able to switch their strategy based on the context and the number.

Doing subtraction from the back is the most often strategy used by the students. Group Pisang is the example of the group who used this strategy in solving the problems. In the first question, they could easily take 6 from 26 and they got 20 as the result. For the second question, they faced difficulty to take away 21 from 28. After a couple of time, they finally found that the answer is 7.

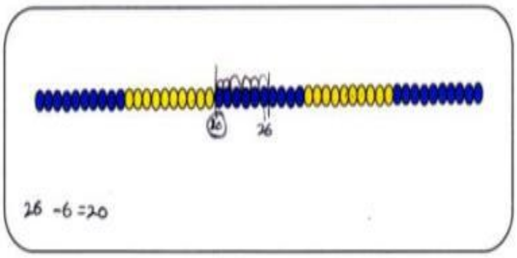


Figure 17. Group Pisang solved 26-6

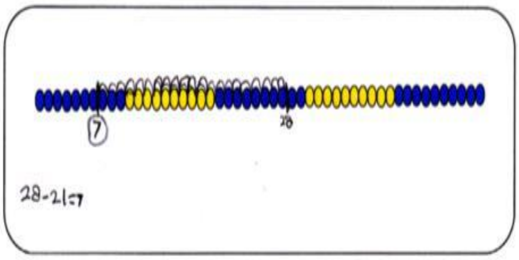


Figure 18. Group Pisang solved 28-21

Doing subtraction from the front is not a common strategy, but Group Duku used it. For the first question, they picked up 26 beads and they took away 6 beads from the front. They needed a long time to count the remaining beads one by one. In the second question, they picked up 28 beads by using group of 10 (10 + 10 + 8), but they took away 21 beads from the front one by one. They got 7 as the differences between 28 and 21.

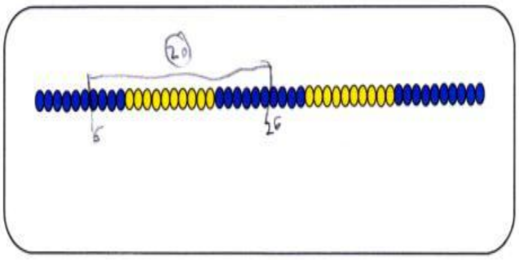


Figure 19. Group Duku solved 26-6

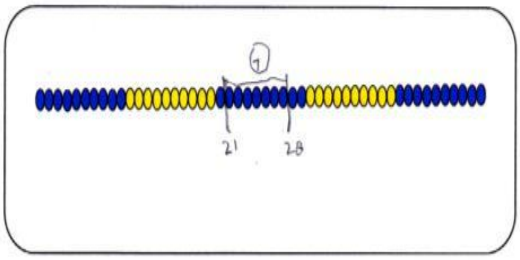


Figure 20. Group Duku solved 28-21

Two groups who could switch their strategy are Group Apel (the focus group) and Group Nanas. They did subtraction from the back for the first question and subtraction from the front for the second question. They also could count the beads using group of 10.

For question number 1, all students in the focus group had same answer. They started to work using the beads string.

Researcher: Where is 26?
Vera : This (pointing at 26th beads)
Researcher: How do you count it? One by one…or…
Vera : This is 10 (pointing at the chocolate beads), this is 10 (pointing at the black beads), and this is 6 (pointing at the chocolate beads)
Researcher: What is the question?
Fakhri : Subtraction
Researcher: What subtraction?
Febi : 26 minus 6
Researcher: What will you do next?
Vera : I take it (take away 6 beads from the back)
Researcher: What is the result?
Students : 20
Researcher: How do you know it is 20?
Vera : This is 10 (pointing at the chocolate beads) and this is 10 (pointing at the black beads)
Researcher: Ok. Good

After worked with the beads string, the students represented what they did in the worksheet. First, they wrote the mathematical notation from the question.

Researcher: Just now, we used the beads string. Can you draw it here? (pointing at the worksheet). Where is 26?
Febi : This (pointing at 26th beads). Then?

Researcher: Can you make a border line after 26th beads?
(Febi makes a border line after 26th beads)
Researcher: How many beads should we subtract?
Students : 6
Researcher: So, we take away 6.
(Febi counts 6 from 26th beads to the right, while Vera counts 6 from 26th beads to the left)
Researcher: If we take away something, is it to the right or to the left?
Vera : It must be to the left.
(Febi realizes her mistake and she also counts to the left)
Researcher: What is the result?
Students : 20
(Febi makes a border line after 20th beads)

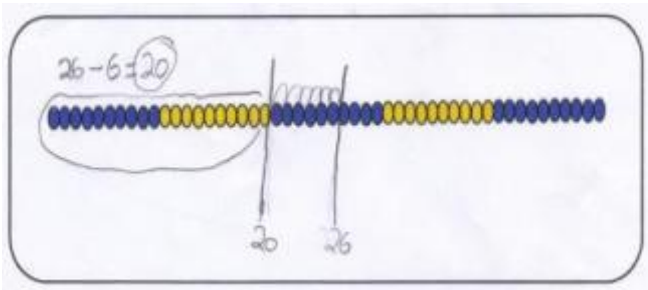


Figure 21. Focus group solved 26-6

For question number 2, the focus group's students did the same thing. Their drawing in the worksheet is correct. They used subtraction from the front. But in the mathematical notation, they wrote 9, instead of 7, as the final answer. The students said that they had different answer. Febi's answer is 9, Vera's answer is 7, Syauqi's answer is 8 (he did not count yet), and Fakhri's answer is 7. The researcher asked them to check their work using the beads string.

Researcher: Let's check your work using the beads string. What will be the result? Where is 28?
Febi : This is 20 (counting one by one to the right until 28 is reached)
Vera : This (directly pointing at 28th beads)
Researcher: How many beads should we subtract?
Students : 21
Researcher: Where is 21?
(Febi and Vera directly pointing at 21st beads and make a border with their pencil and their finger)
Researcher: So, where is the result? How many beads?
Vera : 1, 2, 3, 4, 5, 6, 7
(Febi still continues her counting until 30th beads)
Febi : 9
Researcher: Where is 9?

Febi : This (pointing until 30th beads)
Researcher: Where is the result? It is, right? (pointing at 21st beads until 28th beads)
Vera : We do not use it (pointing at 29th and 30th beads)
Researcher: Where is the result? Febi, do we use these two beads? (pointing at 29th and 30th beads)
Febi : Hmm…No…
Researcher: Good. We do not need to use these two beads. So, where are the beads that we have to count?
Febi : This (pointing at 21st beads until 28th beads)
Researcher: Please count it. What is the result?
Febi : 7
Researcher: All of you agree with the answer?
Students : Yes

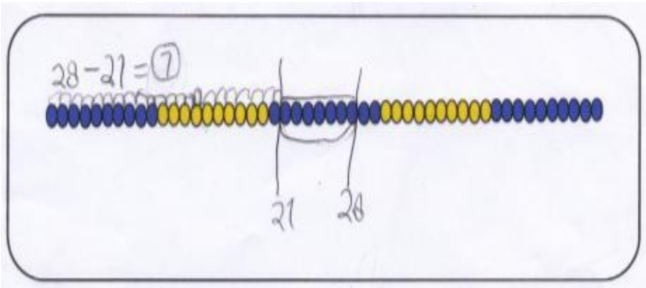


Figure 22. Focus group solved 28-21

From the moment above, we knew that Vera was able to understand the questions and she could find the easier strategy to come to the solution. She did not face difficulty to work with the beads string and to represent her work in the drawing. Febi also could understand the questions, but she was careless in solving it. In the first question, she was wrong in determining the direction of doing subtraction. In the second question, she made a mistake to find the answer because she also counted the 29th and 30th beads. By sharing with her friends, Febi realized her mistake soon and she could revise it. In this fragment, we also could see that the boys did not participate much in the group discussion. Fakhri was still able to follow the learning process, but Syauqi seemed lost his concentration.

The teacher held the class discussion to discuss the students' strategies. She asked the students to use the big beads string as a "model of" the situation in solving subtraction problems. Allya, from Group Nanas, came up in front of the class. The teacher asked her to count the total beads in the beads string first. She only counted one color and she said that each color contains 10 beads. She found that the beads string consists of ten groups of 10, so the total is 100 beads. Then, she presented her solution in subtracting 26 with 6 from the back.

The second problem was solved by Keisya, from Group Pisang. She picked up 28 beads and she took away 21 beads from the back one by one like what she did in her group. Because it needed a long time, the teacher asked the students who have the easier way to share their strategy. Febi tried to explain her answer using the beads string in front of the class. She picked up 28 beads and she directly took away 21 beads from the front. She easily got 7 beads as the result of 28 - 21.

Figure 23. Keisya solved 28-21

Figure 24. Febi solved 28-21

After solving the problems using the beads string, the teacher asked the students to draw the representation of the solutions in the whiteboard. Ravli, from Group Melon, made a drawing for the first problem. He made 10 shaded circles, 10 empty circles, and 6 shaded circles again. He made a line border after 20th circles and he got 20 as the answer. He also wrote the mathematical notation for it.

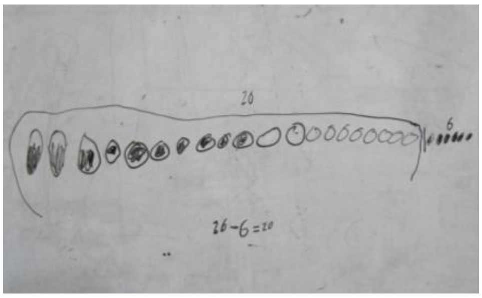


Figure 25. Ravli’s strategy to solve 26-6

The solution for the second problem was drawn by Vera.

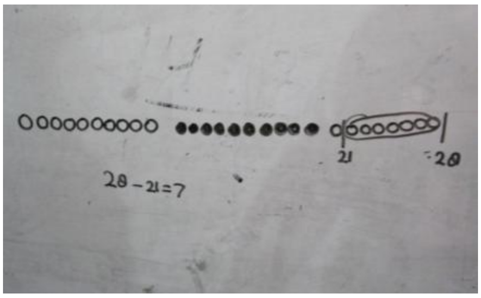


Figure 26. Vera's strategy to solve 28-21

Before continuing the lesson, the teacher summarized what they already did in solving two problems from worksheet 3. The first problem (26 - 6) can be solved easier by doing subtraction from the back. On the other hand, the second problem (28 - 21) can be solved easier by doing subtraction from the front. The students can choose the strategy that more makes sense for them. The teacher also emphasized students to make a circle for the final answer.

In the next session, the teacher guided students to make "jumps of 10" in the beads string, both jumping forward and jumping backward. She used the help of "ten catchers" to catch 10 beads. At the beginning, the teacher jumped from 0 to 10, from 10 to 20, etc, until 90 to 100. She also gave an example by jumping from 1 to 11, from 11 to 21, etc, until 81 to 91.

Figure 27. Jumping forward in the beads string

The teacher challenged students to do jumping forward from different numbers. Shadiqa tried to jump from 2. He caught 10 beads using "ten catchers", but he counted the result. The teacher asked him not to count the result anymore.

Teacher : It is 2, right?
Shadiqa : Yes
Teacher : We already made "jumps of 10" using this "ten catchers". Do you know how many beads is it? (pointing at 10 chocolate beads)
Shadiqa : 10
Teacher : How many more these beads? (pointing at 2 white beads)
Shadiqa : 2
Teacher : So, how many beads we got after jumping by 10?
Shadiqa : 12
Teacher : How if we jump again? (making "jumps of 10" using "ten catchers")
Shadiqa : Uhm…22

After getting the pattern of "jumps of 10", Shadiqa could continue his jumping until 92 without counting the beads.

The next turn to do "jumps of 10" was given to Gisya. She made jumping forward from 3. She did not find difficulty to use "ten catchers" and to know the result of her jumping without counting.

The teacher wrote the notation of jumping forward by 10 in the whiteboard. She made some questions and she asked the students to answer those questions together. Most of the students could give the correct answer smoothly.

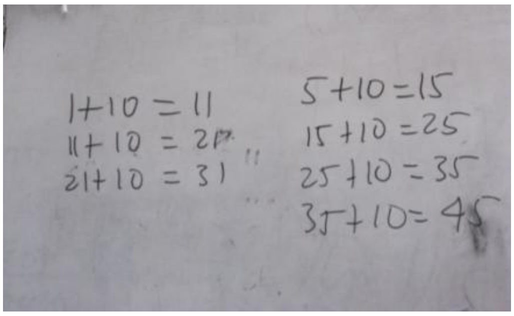


Figure 28. The notation of jumping forward

The teacher continued the lesson by giving the examples of jumping backward. The students did not face difficulty to determine the result of "jumps of 10" in the beads string from 100 to 90, from 90 to 80, etc, until 10 to 0 using "ten catchers". However, they look confused when determining the result of "jumps of 10" from 99. Most students answered 11. The teacher said that the number of the beads should be counted from the left. The students counted the beads from the left and they got 89 as the result. Then, the teacher promoted them to find the pattern of jumping backward that is the reverse of jumping forward. Finally, some students could jump until 9 without counting the beads anymore.

The teacher gave the chance for the students to do “jumps of 10” from 92. Mercy tried to jump backward in the beads string. In the jumping from 92, she counted the result from the left. After getting 82, she still counted the result of her jumping. Gradually, she could continue the next jumping from 72 to 62, from 62 to 52, and from 52 to 42 without counting.

Figure 29. Jumping backward in the beads string

The notation of jumping backward from 92 was written by the teacher in the whiteboard. She asked the students to answer the result of Mercy’s next jumping. Most students were no need to count in getting the correct answer.

From lesson 3, we concluded that most students could recognize the meaning of subtraction as “taking away something” and “determining the difference between two numbers”. The beads string could help the students as a “model of” the situation in solving subtraction problems. When they got confused to solve the word problems, they could represent the situation of the problems in the beads string first. Most of the students could explain with the drawing what they already did in the beads string. There are two strategies which are used by the students in solving subtraction problems, doing subtraction from the back and doing subtraction from the front. Even, some students were able to use those strategies alternately. They could share their strategies in the class discussion so that their friends could choose the easier strategy for the different problems. The beads string and the "ten catchers" were helpful to guide the students in finding the pattern of "jumps of 10", either jumping forward or jumping backward. Most students were able to do "jumps of 10" without counting anymore.

d. Lesson 4 (Working with the Empty Number Line)

In lesson 4, we expected the students are able to use the empty number line as a "model for" their thinking in solving subtraction problems and the students are able to make "jumps of 10" in the empty number line.

The teacher started the lesson by reminding students that they can use different strategies to solve subtraction problems, for example doing subtraction from the back and doing subtraction from the front. The teacher also said that if in the previous lesson they worked with the beads string, then in this lesson they will work with the empty number line.

After that, the teacher posed worksheet 4 that consists of the same contexts with worksheet 3. The difference is in the worksheet 3 she only provided the drawing of the beads string, while in the worksheet 4 she provided the drawing of the beads string and the empty number line in the answer box.

Because the students already discussed the solution of the problems, most of the groups applied subtraction from the back for the first problem and subtraction from the front for the second problem. However, there is a group, Group Pisang, who still used the same strategy like they did in the lesson 3, doing subtraction from the back for both first and second problem. This strategy made sense for them because they could get the correct answer in the drawing of the beads string and in the empty number line.

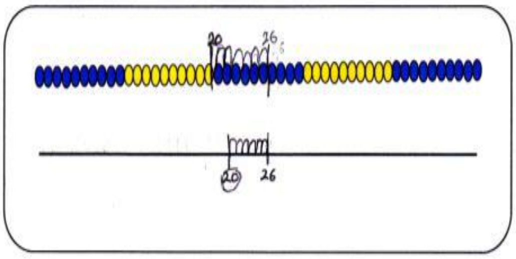


Figure 30. Group Pisang solved 26-6

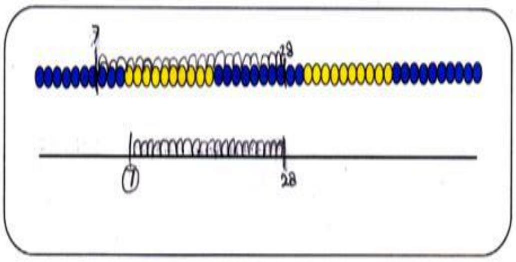


Figure 31. Group Pisang solved 28-21

There are several ways from the students to make the shift from the drawing of the beads string to the empty number line. Grup Nanas started to write number 26 in the empty number line. Then, they made 6 beads from number 26 to the left. They also made the small jumps above the 6 beads. They wrote 20 as the result. They did the same thing for the second question. They started from number 21 to number 28 with counting on.

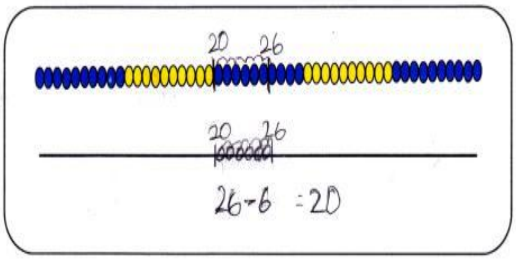


Figure 32. Group Nanas solved 26-6

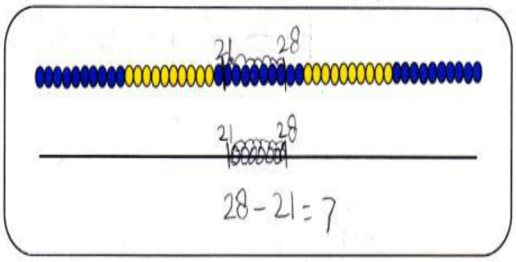


Figure 33. Group Nanas solved 28-21

Grup Jeruk only wrote number 26 in the empty number line below the 26th beads in the beads string and they wrote number 20 in the empty number line below the 20th beads in the beads string. They did not make the small jumps between number 26 and 20. For the second question, they also only wrote number 21 and 28.

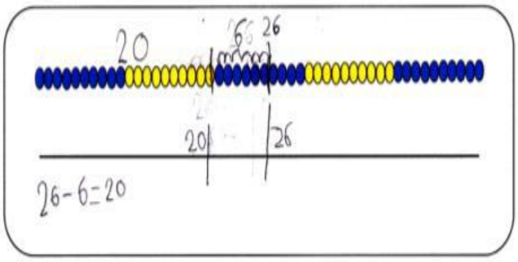


Figure 34. Group Jeruk solved 26-6

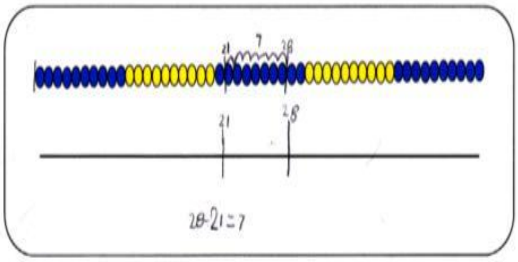


Figure 35. Group Jeruk solved 28-21

The focus group was able to make the shift from the drawing of the beads string to the empty number line perfectly for the first question. The teacher only guided them to change the beads with the line. In the second question, they started to work from 28. Then, they made 7 small jumps to the left until 21 was reached.

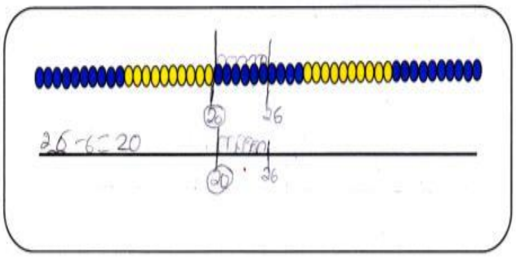


Figure 36. Focus group solved 26-6

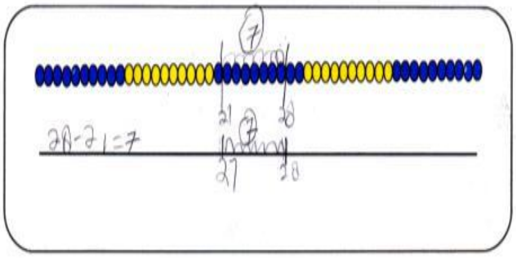


Figure 37. Focus group solved 28-21

From the students' worksheet above, we could see that the similarity among students' ways in shifting from the drawing of the beads string to the empty number line was they tended to write the number in the empty number line exactly below the position of the bead in the beads string. The teacher emphasized that in solving subtraction problems, the students can start from any number and any position in the empty number line.

Later on, the class discussion was conducted so that the students could share their ideas in demonstrating the shift from the drawing of the beads string to the empty number line. Vania, from Group Duku, presented the solution for the question number 1. Actually, the teacher asked her to write the solution only using the empty number line. The teacher already drew an empty number line in the whiteboard. Instead of using the empty number line, she made the beads in that line and she solved the question like what she did in the beads string.

Figure 38. Vania’s strategy using the beads string

The teacher drew an empty number line below Vania’s beads string and asked her to use it. Vania was confused how to start in writing the solution. She made the short vertical line to represent each number from number 1. The teacher said that she does not need to do that and she can choose any position for number 26 in the empty number line. Finally, Vania could realize that the empty number line is simpler model than the beads string.

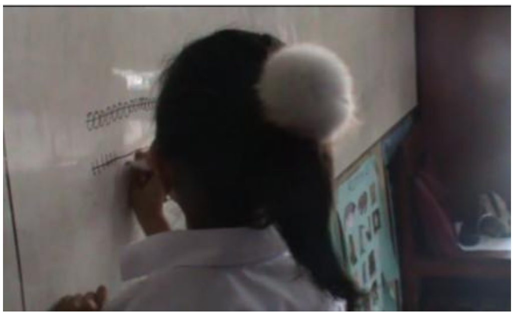


Figure 39. Vania’s strategy using the empty number line

The question number 2 was presented by Fakhri. The teacher guided him to start from 21 and to use counting on strategy. Fakhri made the small jumps to the right until 28 was reached. At first, he wrote 7 above the last jump. Then, he recognized that the last jump should be 28 and the final answer is the total of jumps from 21 to 28 (7 jumps).

The teacher gave two more exercises (26 - 3 and 26 - 19) to make students familiar with the different strategies in solving subtraction problems, in this case counting back (direct subtraction) and counting on (indirect addition), in the empty number line. Without guidance, the students automatically used counting back strategy for two exercises above. All groups, except Group Durian, got the correct answer for the first exercise. Group Durian made 3 jumps from 26 to the left, but they made a space between the last jump and the result. They got 22 as the final answer.

The example of the group who used counting back strategy for both the first and the second exercise is Group Jambu. They did not face difficulty to solve 26 - 3, but they lost the track when counting 26 - 19. They doubted with the result, whether 7 or 6. The researcher helped them to find the right answer.

Researcher: What is the result here? (pointing at the last jump)
(The students look confused)
Researcher: How many jumps from 26?
Gisya : 19
Researcher: Ok, let's count it together.
Gisya : 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7
Researcher: So, what is the result?
Gisya : 7
Researcher: How do you feel? It takes a long time, right?
Gisya : yes
Researcher: We have 26 - 19. Right now, we did not count from 26, but we count from 19. Please write 19.
(Gisya writes 19 in the new line)
Researcher: Where should we go from 19 to reach 26? Is it to the right or to the left?
Gisya : To the right
Researcher: How many jumps do you need from 19 to reach 26?
Gisya : 20, 21, 22, 23, 24, 25, 26 (drawing the jumps from 19 to 26)
Researcher: Let's count your jumps.
Gisya : 1, 2, 3, 4, 5, 6, 7 (counting the jumps from 19 to 26)
Researcher: So, 26 - 19 also has the meaning that how many jumps we need from 19 to 26. What is the answer?
Gisya : 7
Researcher: Which one do you think easier? This (pointing at counting back strategy) or this one (pointing at counting on strategy)?
Students : This one (pointing at counting on strategy)
Researcher: Yes. It needs fewer jumps.
(The students nod their head)

Figure 40. Group Jambu's strategies to solve 26-19

The focus group (Group Apel) also applied counting back strategy for exercise number 1 and number 2. Before using the empty number line, Syauqi already used his fingers to count the answer. He got 23 for 26 - 3, but he got 13 for 26 - 19. The researcher asked him and the group to use the empty number line in solving the questions.

Researcher: What is the result for the second question?
Syauqi : 13
Researcher: Where is 13 come from?
Syauqi : 9 minus 6 equals 3 and 2 minus 1 equals 1, so the result is 13.
Researcher: It is 26 - 19, isn't it? So, it is not 9 minus 6 but 6 minus 9.
(Syauqi look confused)
Researcher: Let's find the result with the empty number line.
(Syauqi draws the empty number line for the second question)
Researcher: What will you do?
(Syauqi makes 19 small jumps from 26 to the left)
Researcher: What is the answer?
(The students count it together and they get 7 as an answer)
Researcher: Do you want to know the easier strategy?
Students : Yes
Researcher: Right now, we will count from 19.
(Syauqi draws a new line and writes 19)
Researcher: Where should we go to reach 26?
Students : Here (pointing at the right side)
Researcher: How many jumps do you need from 19?
(Syauqi makes the small jumps from 19 to 26)
Researcher: What is the result?
Students : 7
Researcher: Using the empty number line, we got the same result, right?
Students : Yes
Researcher: So, 26 - 19 also has the meaning that how many jumps we should make from 19 to reach 26.
(Students nod their head)
Researcher: From these two strategies, which one do you prefer?
Students : This one (pointing at counting on strategy)

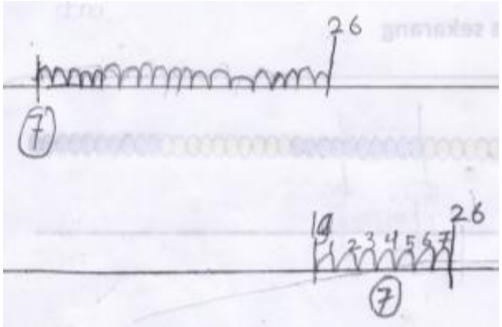


Figure 41. Focus group's strategies to solve 26-19

The two fragments from Group Jambu and Group Apel above proved that the students tended to use counting back strategy in solving subtraction problems.

The students were given the opportunity to present their solution to their friends. Anisa, from Group Melon, wrote her group's solution for the first exercise in the whiteboard. She started from 26, and then she made 3 small jumps to the left. She counted back from 26 and she got 23 as the answer.

The teacher asked other groups who did not solve the second exercise by counting back 19 times from 26 to share their strategy. Gisya, from Group Jambu, tried to present her group's strategy. The researcher already guided her to use counting on in solving the second exercise. Surprisingly, Gisya changed her strategy; she did not use counting on anymore. Instead of starting from 19, she started from 26. She counted back until 19 was reached. Later, she counted her small jumps from 26 to 19 and she got 7 jumps. She was able to re-invent the new strategy (indirect subtraction) in the empty number line.

Figure 42. Gisya's strategy to solve 26-19

The teacher emphasized that there are several strategies to solve subtraction problems in the empty number line. The students are allowed to use any strategy that can help them to solve the problems in the easiest way. In the case of two exercises above, the students could see that the first exercise (small subtrahend) was solved easily using direct subtraction; whereas the second exercise (large subtrahend) was solved easily using both indirect addition and indirect subtraction.

Then, the teacher continued the lesson by posing worksheet 4.2 that consists of the questions about “jumps of 10” from different numbers, 3 questions of jumping forward and 3 questions of jumping backward in the empty number line. She guided the students to make the larger jump for “jumps of 10” to differentiate it with jumps one by one.

Most students did not find difficulty to make “jumps of 10” in the empty number line. They also already knew that the pattern of jumping backward is the reverse of jumping forward. However, there are some groups who had a misinterpretation in doing so. For the question number 1, the students were asked to make “jumps of 10” forward started from 3. At first, Group Durian did it wrong by making jumps of 3. For the question number 2, Group Duku made the same mistake. Instead of making “jumps of 10” started from 5, they made jumps of 5.

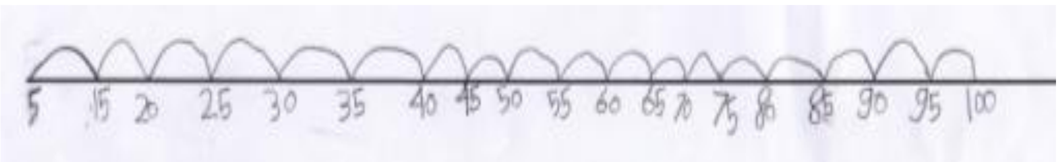


Figure 43. Jumps of 5

Not all groups could get the pattern of “jumps of 10” in the empty number line. Group Mangga is the example of the group who used their fingers to count the result of the jumping. Although after making some jumps they could find the pattern of “jumps of 10”, they still checked their result by counting.

All students in each group participated in doing “jumps of 10”. Some of them did it together and some others did it alternately. They corrected each other because some students still tended to make jumps one by one, for example Febi from the focus group. Some other students also often made wrong jump because they did not pay attention to their jumping.

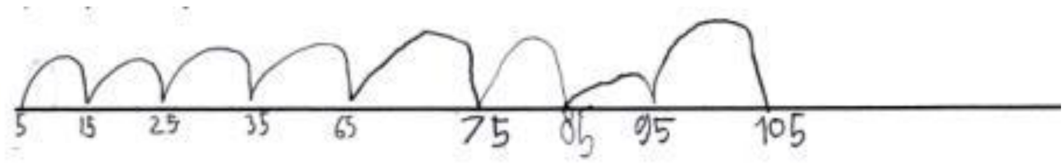


Figure 44. Jumps of 10 mistake

There are some groups who were able to continue their jumping forward more than 100. Group Jambu, Group Jeruk, Group Nanas, and Group Apel could find the pattern of “jumps of 10” after 100 correctly. Contrarily, Group Duku made a mistake in continuing their jumping. After jumping up to 93, they wrote 103, 203, 303, etc.

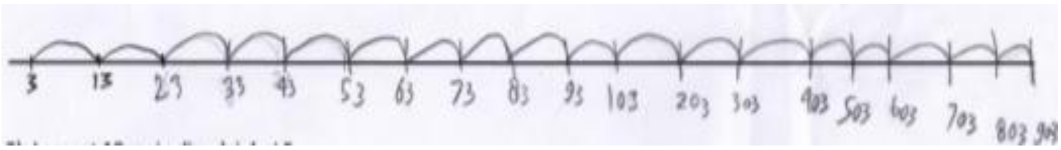


Figure 45. Jumps of 10 more than 100

Before ending the lesson, the teacher asked the students to make “jumps of 10” forward and backward together in the whiteboard. Most of the students could make those jumping correctly without counting.

From lesson 4, we could conclude that most students did not face difficulty in making the shift from the drawing of beads string to the empty number line. The teacher only guided them to change the beads with the line to make the model simpler. Most of them tended to write the number in the empty number line exactly below the position of the bead in the beads string. The students were able to use the empty number line as a “model for” their thinking in solving subtraction problems.

They could use the empty number line to solve the different situations with different numbers. The teacher emphasized that to solve subtraction problems, the students can start from any number and any position in the empty number line. The class discussion was very important to make the students realized the possibility in solving subtraction problems with more than one strategy (counting back). Some students could distinguish in applying counting back strategy (direct subtraction) or counting on strategy (indirect addition) based on the problems. Even, there is a student who could re-invent the other strategy by herself, namely indirect subtraction. Almost all students did not have a problem to make “jumps of 10” in the empty number line both jumping forward and jumping backward.

e. Lesson 5 (Working with the Beads String and the Empty Number Line)

In lesson 5, we expected the students are able to make a shift from the drawing of beads string into the empty number line; the students are able to apply counting back strategy (direct subtraction) and counting on strategy (indirect addition) in solving subtraction problems; and the students are able to make “jumps of 10” in the beads string and in the empty number line.

At the beginning, the teacher reminded students that they can use the beads string and the empty number line as a model in solving subtraction problems. In this lesson, they will practice more to work with both the beads string and the empty number line.

The teacher gave worksheet 5 that consists of 6 bare number problems in subtraction, 3 problems with large difference between minuend and subtrahend; and 3 problems with small difference between minuend and subtrahend. Those problems will promote students to use different strategies (direct subtraction and indirect addition). The students were asked to represent the problems into the drawing of beads string and the empty number line.

For the first problem (56 - 4), most students used direct subtraction in the beads string and the empty number line. They started from 56, and then they counted back 4 times. They got 52 as the answer. However, not every group got the result by counting her jumping, Group Mangga for example. Although they made a correct drawing, they still counted their fingers to come to the result.

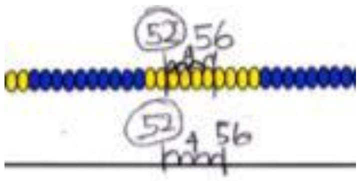


Figure 46. Students' strategy to solve 56-4

The focus group also used direct subtraction to solve the first problem. Vera directly said that the answer is 52. She got it by counting back with her fingers. Syauqi drew the solution in the beads string and the empty number line. He made the similar solution with the other groups. He only added the number of jumps above his jumps.

Most of the students applied direct subtraction for the second problem (56 - 10). They used counting back to the left from 56 one by one. There is a group, Group Pisang, who were able to apply what they already got in the previous lesson about "jumps of 10". They directly made a jump from 56 to 46 in the beads string. They also made a large jump in the empty number line to represent "jumps of 10".

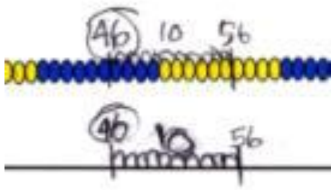


Figure 47. Students' strategy to solve 56-10

For the second problem, the focus group also applied direct subtraction to solve it. Febi made 10 jumps backward from 56. Instead of counting it one by one, she counted it from the front. She got that the position of the result is 46. The researcher asked whether anyone knew the easier way in solving this problem. She stimulated students by asking “How is the result of 56 - 20?” Fakhri remembered about jumping backward and he could answer that the result is 36 because 56 - 10 = 46 and 46 - 10 = 36. When returning to the worksheet, he could draw a “jumps of 10” in the empty number line.

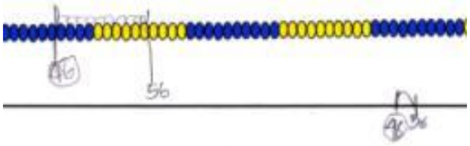


Figure 48. Focus group’s strategy to solve 56-10

We also found a group who used the different strategy. Group Jambu took away 10 beads from the front in the beads string. They counted the rest of the beads from 11 to 56 one by one. At first, they got 45 as the result. Then, the researcher asked them to count again by using group of 10. Finally, they could get the correct answer by counting 10 + 10 + 10 + 10 + 6.

For the problem number 3 (56 - 14), most groups still used direct subtraction one by one. However, there is a group who already realized that 14 consist of 10 and 4. Group Jeruk made 14 jumps in the drawing, but they did not count it back one by one. They made a line after 10 jumps and they knew 46 as the result of 10 jumps. They continued their jumps 4 times and they started to do counting back. They got the right answer that is 42.

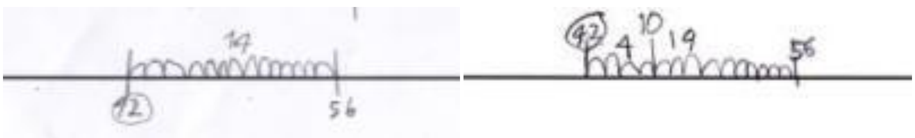


Figure 49. Students’ strategy to solve 56-14

The teacher brought the solution from Group Jeruk in front of the class because it is easier than others. Jumping 10 backward first like they did in the lesson 4 was faster than counting back 14 one by one. The teacher gave another example with this kind of solution in the whiteboard (58 - 12). She also asked the students to solve an exercise (69 - 14). Ravli could solve the exercise correctly in the empty number line. He jumped back 10 times from 69 and he wrote 59. He made 4 more jumps and he started to count back. He got 55 as the result.

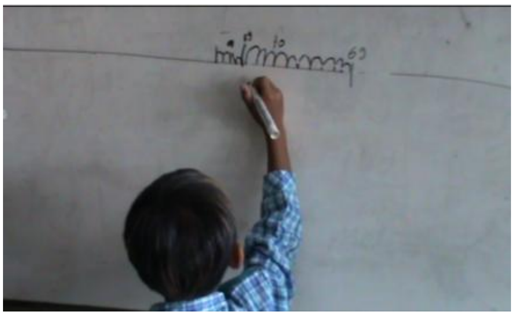


Figure 50. Ravli’s strategy to solve 69-14

The focus group was able to apply “jumps of 10” in the empty number line. Firstly, Fakhri only made 2 jumps, a jump from 56 to 46 and a jump from 46 to 42. The researcher said that the jump from 46 to 42 will make other people confused. We only use a large jump to represent “jumps of 10” and a small jump to represent jumps one by one. Fakhri revised his drawing. He made 4 jumps from 46 to 42.

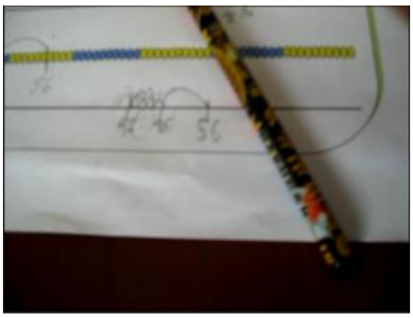


Figure 51. Focus group’s strategy to solve 56-14

Syauqi said that he had other strategy to answer 56 - 14 without using the empty number line. The researcher allowed him to show his strategy. He got that the result is 42 because 6 - 4 = 2 and 5 - 1 = 4. Vera said that she also knew that strategy. She got 42 from 5 - 1 = 4 and 6 - 4 = 2. Both of them could solve the problem by subtracting tens and ones separately. The difference is Syauqi subtracted the ones first, while Vera subtracted the tens first.

The researcher challenged them to solve another problem, 21 - 13, using their strategy.

Researcher: Can you solve 21 - 13 using your strategy?
Syauqi : 3 - 1 = 2 and 2 - 1 = 1, so the answer is 12.
Vera : I also get 12 because 2 - 1 = 1 and 1 - 3 = 2.
Researcher: Syauqi said that 3 - 1 = 2 and 2 - 1 = 1. Is it 3 - 1 or 1 - 3?
Vera : It is 1 - 3.
Researcher: So, what is the result of 1 - 3?
Vera : 2
Researcher: Are you sure?
Fakhri : I think we cannot subtract 3 from 1.
Syauqi : Hmm…because we cannot do 1 - 3, so we solve 3 - 1.
Researcher: Is it allowed?
(The students shook his head)
Researcher: We did not allow to change 3 - 1 because the problem is 21 - 13. But, we also cannot subtract 1 with 3. What will we do?
Febi : We cannot use this strategy to solve 21 - 13.
Researcher: Right now, can you solve using your fingers?
(The students use their fingers, but they get different answer)
Researcher: Ok, let's count it together.
(Fakhri opens his 10 fingers, Syauqi opens his 10 fingers, and Vera opens 1 finger that makes 21. Febi folds 13 of their fingers and counts the fingers which still open)
Researcher: So, what is the answer?
Students : 8
Researcher: Let's solve this problem using the empty number line.
(Syauqi draws a line in the paper and he writes the number 21)
Researcher: What is next?
Fakhri : We make a "jumps of 10" to the left and 3 more jumps.
Researcher: What is the answer?
Students : 8

From the fragment above, we saw that Syauqi and Vera already knew the algorithm of subtracting tens and ones separately. However, they used it differently. Syauqi subtracted the ones first and Vera subtracted the tens first. They look confused when facing a problem that needs borrowing and carrying procedures. Syauqi tried to reverse the numbers. On the other hand, Vera calculated the opposite numbers. The students gave up in solving 21 - 13. They thought that not every problem can be solved by subtracting tens and ones separately. Using fingers, the one who could count correctly is Syauqi. The others got the correct answer when solving the problem together. The researcher asked the students to solve 21 - 13 using the empty number line. They could work well and they got 8 as a result. They recognized that the empty number line is helpful in solving subtraction problems.

For the fourth problem (56 - 54), all groups used direct subtraction in the beads string. They often complained that they are tired when counting. Most of them got the wrong result because they lost of track in counting the beads. Group Durian did counting back 54 times one by one and they finished in 1. Although Group Pisang could come to the right answer, they needed very long time to do it.

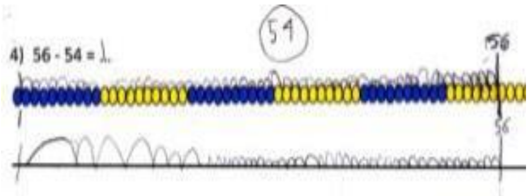


Figure 52. Group Durian's strategy to solve 56-54

The teacher guided students to use other strategy. She supported students to come to the indirect addition strategy by posing some questions and some examples.

Teacher : What is the result of 6 - 4?
Students : 2
Teacher : How many steps do you need from 4 to reach 6?
Students : 2
Teacher : I have a question, how many steps do you need from 52 to reach 57?
Dimas : 5
Teacher : Let's check Dimas' answer.
(The teacher draws a line and she writes 52)
Teacher : Let's make jumps from 52 to 57.
(The teacher makes jumps from 52 to 57 and asks the students to count the jumps)
Teacher : How many jumps from 52 to 57?
Students : 5

Teacher : I can represent this drawing with 57 - 52. So, what is the result of 57 - 52?
Students : 5
Teacher : What about 59 - 56? From what number we should start?
Allya : We start from 56.
Teacher : That is right. We will count how many jumps we need from 56 to 59.
(The teacher makes jumps from 56 to 59 and she asks the students to count the jumps again)
Teacher : What is the result of 59 - 56?
Students : 3

Figure 53. Teacher guided the indirect addition strategy

The teacher said that the students can use this strategy to solve the question number 4, 5, and 6. It is easier because they do not need to take a long counting. Most students could apply indirect addition strategy in solving the question number 4. They were able to get the correct answer without time consuming.

Before the teacher's explanation, the focus group also used direct subtraction to solve the fourth problem. Fakhri started to solve the problem in the empty number line. He wrote 56 first and he made 5 large jumps to the left. He counted the result of the large jumps one by one and he got 6 in the last jump. He continued to make 4 small jumps and he directly wrote 2 for the final answer. Fakhri already knew that the answer must be 6 minus 4. After knowing indirect addition strategy, the focus group could apply this strategy easily both in the beads string and the empty number line.

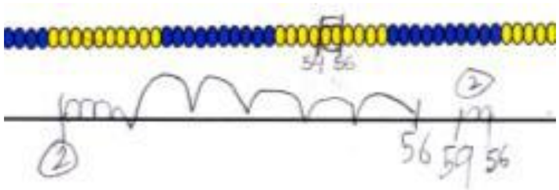


Figure 54. Focus group's strategy to solve 56-54

Most of the students used indirect addition again for the fifth problem (56 - 46). They applied counting on from 46 one by one until 56 was reached. They got 10 as the answer because they needed 10 jumps from 46 to 56. Firstly, the focus group also made the jumps from 46 to 56 one by one. Seeing the result was 10, Syauqi said that they also could make 1 large jump which represents "jumps of 10".

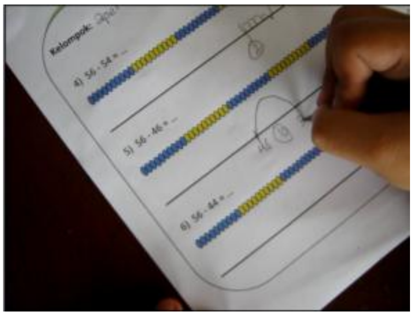


Figure 55. Focus group's strategy to solve 56-46

For the problem number 6 (56 - 44), most groups still applied indirect addition strategy one by one. There are two groups, Group Jeruk and Group Nanas, who started with counting on by 10, and then they continued the jumps until the desired number was reached. They wrote 54 as the result of 10 jumps. They got that the final answer is 12 because they needed 2 more jumps to reach 56.

The focus group directly solved the sixth problem in the empty number line using indirect addition. Vera made a large jump from 44 to 54. Then, she made 2 small jumps from 54 to 56. She wrote 12 as the result of 56 - 44. Syauqi said that they also could solve the problem starting from 56. He is asked to present his strategy in front of the class later.

Tiara, from Group Jeruk, shared the solution from her group for the problem 6 to her friends. She drew 10 jumps from 44 and she made a line as a border. She continued to draw 2 jumps until 56 was reached. The answer is 10 + 2 = 12.

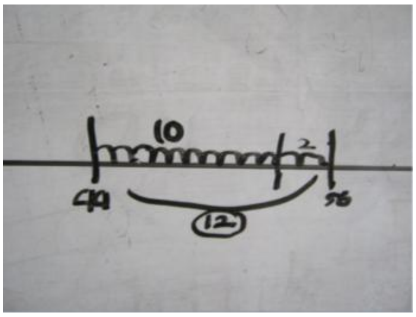


Figure 56. Tiara’s strategy to solve 56-44

Syauqi presented the other strategy to solve 56 - 44 in the whiteboard. He started to write 56 in the empty number line. Later on, he made a large jump backward from 56. He wrote 46 as the result of this “jumps of 10”. He continued in making the jumps from 46 to reach 44. He got 12 as the result for 56 - 44. Syauqi’s strategy is same with Gisya’s strategy in the previous lesson, namely indirect subtraction.

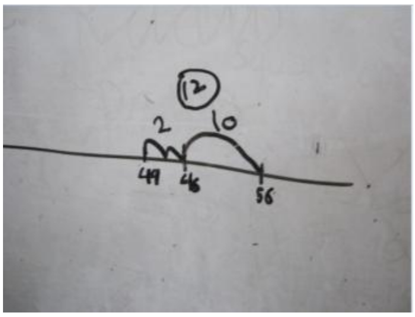


Figure 57. Syauqi’s strategy to solve 56-44

The teacher repeated Tiara’s and Syauqi’s solution before ending the lesson. Tiara made the jumps from 44 to 56 using counting on strategy. This strategy is called indirect addition. On the other hand, Syauqi applied counting back strategy to count the jumps from 56 to 44. It is called indirect subtraction. To make the counting easier and faster, they can use “jumps of 10” in those two strategies.

From lesson 5, we concluded that most students were able to represent the solution of the problems in the drawing of beads string and in the empty number line. The students tended to apply direct subtraction (counting back strategy) in solving all of the problems given. By guidance from the teacher, they also could use indirect addition (counting on strategy) to solve certain problems. Like in the previous lesson, there is a student who preferred to use indirect subtraction. In the class discussion, the students recognized that both indirect addition and indirect subtraction are more efficient to solve the problems which have small difference between minuend and subtrahend. Although most of the students did not find difficulty to make "jumps of 10" in the beads string and in the empty number line, they still could not apply it in solving subtraction problems. They mostly used jumping forward (counting on) or jumping backward (counting back) one by one.

f. Lesson 6 (Solving Subtraction Problems)

In lesson 6, we expected the students are able to use the empty number line in solving subtraction problems; the students are able to apply counting back strategy (direct subtraction) and counting on strategy (indirect addition) in solving subtraction problems; and the students are able to make "jumps of 10" in the empty number line.

First of all, the students are reminded that they can apply both counting back and counting on strategy to solve subtraction problems. They also can use the empty number line as a model to represent their idea about the problems and their solution in solving the problems. They also are reminded about "jumps of 10" in the empty number line.

The teacher posed worksheet 6 that consists of 6 subtraction problems. Three of them are contextual problems of subtraction as "taking away something" and "determining the difference between two numbers". Three others are bare number problems with large difference and small difference between minuend and subtrahend. There is an empty number line in the answer box to stimulate students to use it in solving the problems.

The first problem is "Sari has 26 fish and 10 of her fish died. How many fish the remaining?" For this problem, all groups used direct subtraction strategy and they got 16 as the answer. Some groups, for instance Group Mangga, still made jumping backward one by one from 26. Group Apel (focus group) is the example of the groups who could make "jumps of 10" in the empty number line. Other group, Group Duku, made 10 small jumps from 26 to the left at the beginning. Then, they made a large jump above the small jumps as the sign of "jumps of 10".

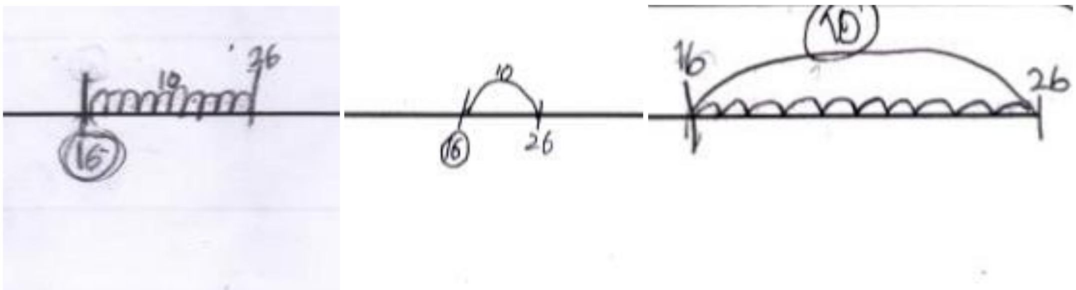


Figure 58. Students' strategies to solve 26-10

The second problem is "Father is reading a newspaper. The newspaper has 60 pages. Father already read 52 pages. How many more pages that father should read to finish the newspaper?" Most of the students could understand the meaning of the problem. They applied indirect addition to solve the problem. They counted on from 52 one by one until 60 was reach. There is a group, Group Duku, who made "jumps of 10" from 52. They wrote 62 as the result of their jump. They realized that their jump exceed the desired number. They revised their strategy with making one by one jump. The answer is 8 because they needed 8 jumps from 52 to 60.

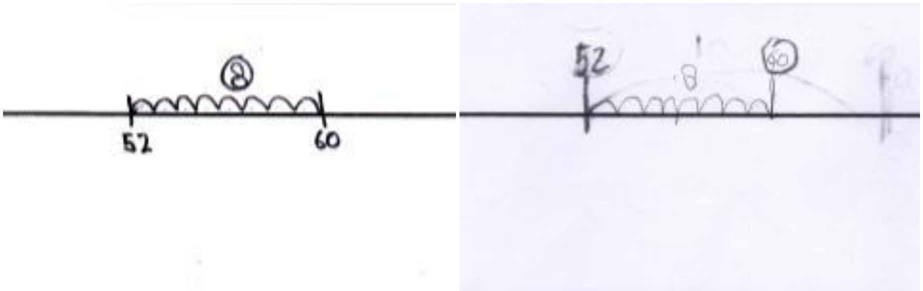


Figure 59. Students' strategies to solve 60-52

The focus group also used indirect addition strategy in solving the second problem. They were able to understand the meaning of the problem, but they had difficulty to write the mathematical notation from the problem in the subtraction format.

Researcher: What kind of problem is it?
Syauqi : Subtraction
Vera : Addition
Researcher: What will you write first? 60 or 52?
Febi : 52 (writing 52 in the empty number line)
Researcher: Next?
Vera : Jumping forward until 60.
(Febi makes jumping forward from 52 to 60)
Researcher: How is the result? Let's count it together.
Students : 8
Researcher: Can you write the mathematical notation from this?
Febi : 52 + 8 = 60 (writing 52 + 8 = 60 in the worksheet)
Researcher: Syauqi, just now you said that it is a subtraction problem, right? Can you write the mathematical notation for subtraction?
Syauqi : 60 - 8 = 52
Researcher: Please, look at the problem again. What is the question?
Syauqi : How many more pages that father should read to finish the newspaper?
Researcher: So, the answer is…
Syauqi : 8
Researcher: How is the mathematical notation?
Fakhri : 60 - 52 = 8
Researcher: Do you agree with Fakhri's answer?
Students : Yes
(Fakhri writes 60 - 52 = 8 in the worksheet)

From the moment above, we could see that the focus group's students could understand the problem although they had different point of view. Vera saw the problem as addition problem and Syauqi saw it as subtraction problem. They solved the problem using indirect addition strategy and they got the correct answer. Febi wrote 52 + 8 = 60 as the mathematical notation from the problem based on what she already did in the empty number line. When being asked the mathematical notation as subtraction problem, Syauqi answered 60 - 8 = 52. He only saw Febi's notation without looking back at the question. Fakhri could revise Syauqi's answer. He said that the notation for subtraction is 60 - 52 = 8.

The third problem is "Toni is 33 years old. His brother is 23 years old. How many years the age differences between Toni and his brother?" Most students recognized that this problem is a kind of subtraction problem which has the meaning "determining the difference between two numbers". This problem led to various strategies from the students. Group Jeruk used indirect addition one bye one. They started from 23 and they made small jumps until 33. Group Melon also used indirect addition strategy, but they were able to make "jumps of 10" forward. They drew an arrow in their large jumps to represent the jumps from 23 to 33.

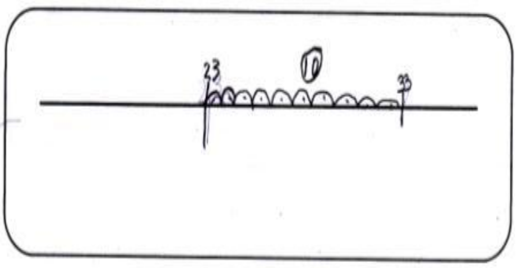


Figure 60. Group Jeruk solved 33-23

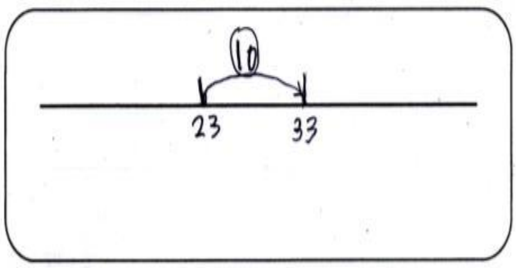


Figure 61. Group Melon solved 33-23

Group Pisang applied direct subtraction to solve the third problem. They wrote 33 in the empty number line and they made 23 jumps to the left one by one. They could get the right answer although they took a long time to count it. Group Mangga also used direct subtraction, but they got the wrong result. At first, Group Nanas also would apply direct subtraction strategy. They made 2 large jumps and 3 small jumps. They got 23 as the result of the first large jump and they immediately realized that the answer of the problem is 10. Accidentally, Group Nanas changed their strategy become indirect subtraction.

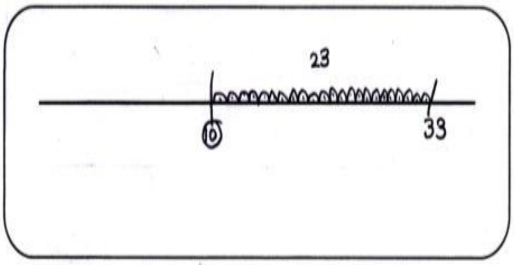


Figure 62. Group Pisang solved 33-23

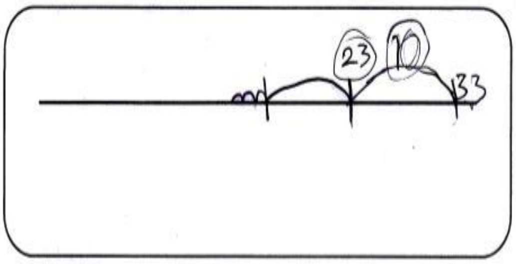


Figure 63. Group Nanas solved 33-23

The indirect subtraction strategy also was used by the focus group. Fakhri started to write 33 and he directly made a large jump to the left. He got 10 as the difference between 33 and 23. Febi wrote 33 - 10 = 23 as the mathematical notation from the third problem. Syauqi disagreed with Febi's answer. He said that the notation must be 33 - 23 = 10 because the known numbers are 33 and 23 and the unknown number is 10. The others agreed with Syauqi's answer. Then, Febi revised the notation in the worksheet.

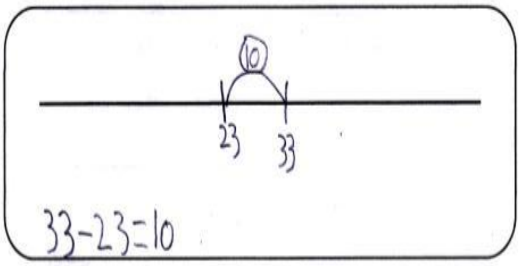
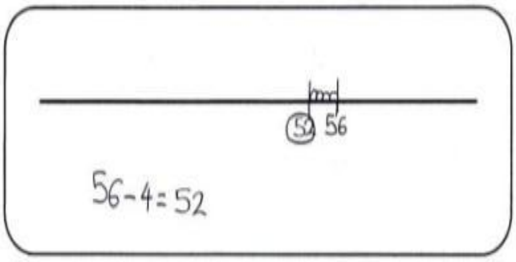


Figure 64. Focus group solved 33-23

The problem number 4 (56 - 4) is different from the previous problem. It did not support students to come with different solutions. The students applied direct subtraction strategy to solve this problem. They made 4 small jumps backward from 56 and they got that the result is 52. The focus group also used the same strategy with the other groups. Group Jeruk made a mistake by jumping forward instead of jumping backward.



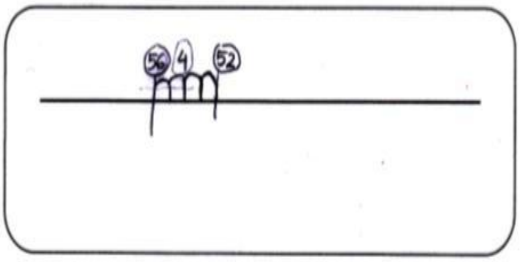


Figure 65. Focus group solved 56-4

Figure 66. Group Jeruk solved 56-4

Most of the students were already familiar with the bare number problems in subtraction which can be solved easily using indirect addition, for example the problem number 5 (65 - 61). Most of them started to make the jumps from 61 to 65 and they found 4 as the answer. Group Melon also got 4 as the result, but they only made 1 jump from 61 to 65. Group Jambu made the jumps in the opposite direction, from 65 to 61. It means that they used indirect subtraction strategy in solving the problem, like they ever did in the previous lesson.

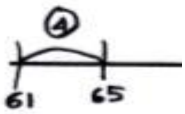


Figure 67. Group Melon solved 65-61

The students in the focus group had different strategies to solve the fifth problem. Fakhri applied direct subtraction strategy. He wrote 65 in the empty number line. And then, he made “jumps of 10” backward 6 times and 1 small jump. He wrote the result of each large jump and he got 5 as result of the last large jump. So, the final answer that he got is 5 minus 1 equal to 4. Syauqi said that Fakhri’s solution was time consuming. He would use the other strategy, indirect addition. The researcher asked him to draw an empty number line below Fakhri’s solution. Syauqi started from 61 and he made jumping forward. He wrote the result of each jump. He only needed 4 jumps to reach 65. He already proved that indirect addition is more efficient in solving this problem.

For the last problem (61 - 59), most students also used indirect addition. They made 2 jumps from 59 to 61. Group Durian also made 2 jumping forward, but they wrote 61 in the left side and 59 in the right side. Group Jambu still preferred to apply indirect subtraction by making jumping backward from 61 to 59 and they got 2 as the answer.

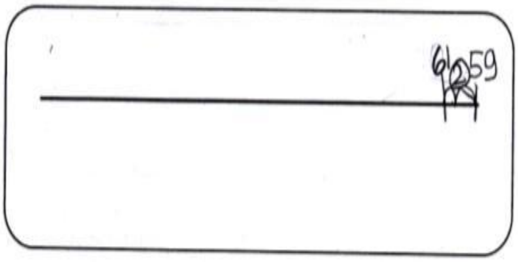


Figure 68. Group Durian solved 61-59

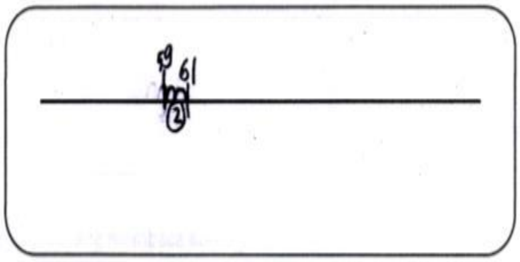


Figure 69. Group Jambu solved 61-59

A student in the focus group, Febi, wanted to use the direct subtraction strategy. She started to write 61 in the empty number line. She made small jumps from 61 to the left. Vera said that Febi's strategy was not efficient, like Fakhri's strategy in the previous problem. Then, Febi realized her ineffectiveness. She switched her strategy become indirect addition. She could get the answer easily.

The mathematical congress will be started if all of the groups already finished their work. Before the mathematical congress begins, the focus group already finished all problems in the worksheet. Therefore, the researcher posed additional question for them. In the lesson 5, Syauqi and Vera were able to use the algorithm of subtracting tens and ones separately in solving subtraction problems. The researcher asked Syauqi to solve the problem number 5 and 6 in the worksheet using this strategy.

Researcher: Can you solve the fifth problem using your previous strategy? (writing 65 - 61 in the paper)
Syauqi : The answer is 4 (writing 4 in the paper)
Researcher: Where 4 come from?
Syauqi : 5 - 1 = 4 and 6 - 6 = 0. So, the result is 4.

Researcher: Ok, the result is same with the result using the empty number line, right? Now, please solve the sixth problem.
(Syauqi writes 61 - 59 in the paper)
Researcher: What is the result?
Syauqi : 18 (writing 18 in the paper) because 9 - 1 and 6 - 5.
Researcher: Why 9 - 1? It is 1 - 9, isn't it?
Syauqi : That's right (thinking) So, we have to borrow.
We cross out 1 and 6. Uhm… and then… we change 6 with 5 and 1 with 11 (writing in the paper)
Researcher: Then?
Syauqi : 11 - 9… 10 - 9 = 1… 1 + 1 = 2 (counting with his fingers)
5 - 5 = 0. So, it is 2.
Researcher: What is the answer?
Syauqi : 2 (writing 2 in the paper)
Researcher: It is also same with the result using the empty number line, right? Which one easier for this problem? Using the empty number line or using your strategy?
Syauqi : Using the empty number line (pointing at the empty number line)
Researcher: Yes. Using your strategy needs borrowing procedures. If we forget to do that, we will come to the wrong result.
(Syauqi nods his head)
Researcher: Using the empty number line, we only need to jump from 59 to 61.

From the dialog above, we knew that Syauqi could use the algorithm of subtracting tens and ones separately starting from subtracting the ones first, like what he did in the previous lesson. He made a progress in using this strategy although he made a mistake in the beginning. In lesson 5, he was not able to solve the problem which needs borrowing and carrying procedures. Surprisingly, he could apply the procedures well in this lesson. Even, he could get the correct answer. However, he realized that using the empty number line is easier than using the algorithm of subtracting tens and ones separately. He also preferred to use the empty number line in solving subtraction problems.

After that, it was Vera's chance to explain the algorithm of subtracting tens and ones separately in solving the problem given by the researcher.

Researcher: Please solve this problem using your yesterday's strategy (writing 57 - 52 in the paper)
Vera : 5 - 5 = 0 and 7 - 2 = 5.
Researcher: What is the result?
Vera : 5 (writing 5 in the paper)

Researcher: Now, please solve this problem (writing 52 - 48 in the paper)
Vera : 5 - 4 = 1 and 2 - 8 .... Uhm ... (thinking) We cannot subtract 2 - 8. We cannot use this strategy to solve the problem.
Researcher: What will you do?
Vera : I will use the empty number line (drawing a line in the paper). We can use it for any subtraction problems.
Researcher: From what number you will start?
Vera : 48
(Vera writes 48 in the empty number line. Then, she makes a jumping forward and she writes 49, etc, until 52 was reached)
Researcher: What is the answer?
Vera : 4 (writing 4 above the jumps)

From the conversation above, we could see that Vera was able to apply the algorithm of subtracting tens and ones separately. She started to subtract the tens first, like what she did in lesson 5. She said that this strategy is not applicable for the problems which need borrowing and carrying procedures. She preferred to use the empty number line. She thought that the empty number line is a flexible model to solve any different subtraction problems.

During the group discussion, the teacher walked around the class to see what the students do. She found that there are two strategies (indirect addition and indirect subtraction) which are used by the students to solve the subtraction problems with small difference between minuend and subtrahend. Therefore, the teacher repeated those strategies in front of the class. She wrote 58 - 53 as an example and she solved it using indirect addition (counting on). She started from 53 and she made jumping forward to reach 58. Later on, Tiara was asked to solve 58 - 53 using indirect subtraction (counting back). She made 5 jumping backward from 58 to 53. Both indirect addition and indirect subtraction had the same result.

Figure 70. The teacher solved 58-53 Figure 71. Tiara solved 58-53

After all groups finished the worksheet, the teacher conducted the mathematical congress to discuss the students' solution. Syauqi, from Group Apel, presented his group's strategy in solving the first problem. He used direct subtraction strategy. He wrote 26 in the empty number line and he made a "jumps of 10". He got 16 as the answer.

The second problem was solved by Dike (Group Pisang) and Fitriah (Group Melon). Dike applied indirect subtraction by jumping backward one by one from 60 to 52. On the other hand, Fitriah applied indirect addition by jumping forward one by one from 52 to 60. She wrote the result of each jump. Both of them got 8 as the answer of the problem.

Dimas (Group Jeruk) and Allya (Group Nanas) came up in front of the class to write the solution for the problem number 3. Dimas used indirect addition strategy by making the jumps from 23 to 33 one by one. On the contrary, Allya used indirect subtraction strategy by making a "jumps of 10" from 33 to 23.

Figure 72. Dimas solved 33-23 Figure 73. Allya solved 33-23

The problem number 4 was presented by Athayah, from Group Mangga. She applied direct subtraction by jumping backward 4 times from 56. She got 52 as the answer. The teacher asked for the other possible strategies because all groups used the same strategy. Ravli, from Group Melon, tried to use indirect addition in solving the problem. He started from 4 and he made 5 large jumps. The result of his large jumps is 54. He made 2 small jumps to reach 56. So, the answer is 52 because he needed 52 jumps from 4 to 56. Although Ravli could get the correct answer, he preferred to use direct subtraction in solving this kind of subtraction problem.

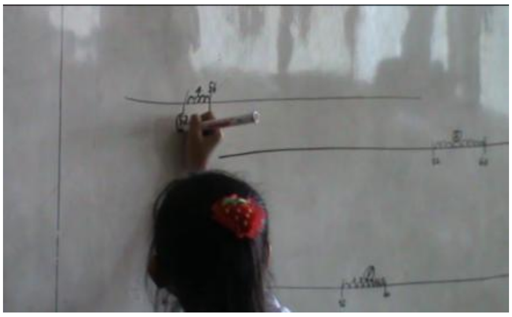


Figure 74. Athayah solved 56-4

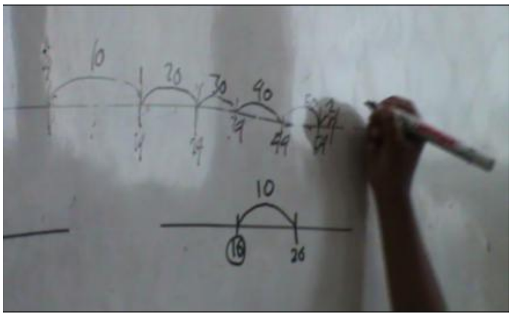


Figure 75. Ravli solved 56-4

Adisa, from Group Durian, solved the fifth problem in the whiteboard. She used indirect addition strategy by making the jumps forward from 61 to 65. She got 4 as the result. In solving the sixth problem, Mercy (Group Duku) also used indirect addition strategy. She made the jumps forward from 59 to 61. She could get 2 as the answer for the last problem easily.

In the end of the lesson, the teacher made reflections about the lesson and the whole teaching learning process. She said that subtraction has two meanings, those are “taking away something” and “determining the difference between two numbers”. The students were suggested to use the empty number line as a model in solving subtraction problems. They also were suggested to choose the easiest strategy (direct subtraction, indirect addition, and indirect subtraction) based on the problems. It is better if they can apply “jumps of 10” to simplify the solution.

From lesson 6, we could conclude that most of the students were able to use the empty number line as a model in describing their idea about the problems and in representing their solution to solve the problems. By representing students' strategies in the empty number line, each step in students' thinking could be recorded. Therefore, it allowed them to track errors. The students did not face difficulty to solve the contextual problem in subtraction and to find the meaning of subtraction on it. Some students could apply not only direct subtraction, but also indirect addition and indirect subtraction in solving different subtraction problems. Some others still tended to use only one strategy, mostly direct subtraction strategy first, in solving all problems. The empty number line could help the students to visualize the steps needed in counting to come to the result. After finding the difficulty in counting, the students would realize to switch their strategy into indirect addition or indirect subtraction. Some of the students also were able to make the solution simpler by applying "jumps of 10" in the empty number line.

3. Post-Test

Post-test is given to know the end points of the students after the teaching experiment and what they have learned. We used 8 questions which 6 of them are same with the questions of part B in the pre-test. By comparing the result from pre-test and post-test, it can be seen the development of students' strategies; whether they are able to apply direct subtraction, indirect addition, and indirect subtraction in solving subtraction problems up to 100 using the empty number line.

There are several strategies that students applied to solve the questions in the post-test. Most students could apply the empty number line with different strategies (direct subtraction, indirect addition, and indirect subtraction). It was easy to determine the students who used direct subtraction strategy. On the other hand, it was difficult to differentiate the students who used indirect addition/ indirect subtraction because the representations of the solutions in the drawing are same. We only could determine the students who used indirect addition/ indirect subtraction if we saw the process of their solutions. We also could determine them if they applied “jumps of 10” in their drawing.

We still found the students who used the mental calculation for easy numbers, the students who used their fingers, and the students who used the algorithm of subtracting tens and ones separately. We did not allow students to use the arithmetic rack and other tools in helping them to answer the questions. Only some students could apply “jumps of 10” to simplify the solution in the empty number line.

The students' answer for the questions from post-test can be shown in the table below.

Table 12. The students' answer for the questions from post-test

No.	**Correct Answer**							**Jumping**	**Wrong Answer**
	Using Mental Calculation	**Using Fingers**	**Using Algorithm**	**Using Direct Subtraction (DS)**	**Using Indirect Addition (IA)**	**Using Indirect Subtraction (IS)**	**Using IA or IS**		
1.	2 students	3 students	-	15 students	3 students	2 students	6 students	4 students	-
2. (2 pre-test)	-	3 students	-	9 students	2 students	1 student	13 students	-	3 students
3.	-	3 students	-	15 students	2 students	1 student	9 students	2 students	1 Student
4. (1 pre-test)	-	5 students	-	10 students	1 student	1 student	12 students	1 student	2 students
5. (3 pre-test)	-	-	4 students	19 students	1 student	-	1 student	12 students	6 students
6. (4 pre-test)	-	-	2 students	6 students	5 students	1 student	11 students	11 students	6 students
7. (5 pre-test)	-	-	-	-	2 students	2 students	18 students	-	9 students
8. (6 pre-test)	-	-	-	13 students	4 students	1 student	-	12 students	13 students

We could see the development of the students in solving two digit numbers subtraction by comparing the result from pre-test and post-test. We also could see the difference of students' strategies before and after the learning process in the table.

Table 13. The students' strategies in the pre-test and post-test

Problems	**Correct Answer**		**Strategies**	
	Pre-test	**Post-test**	**Pre-test**	**Post-test**
12 - 8	28 students	29 students	20 students use fingers 6 students use rack 2 students use drawing	5 students use fingers 10 students use DS 1 student use IA 1 student use IS 12 students use IA or IS
15 - 9	20 students	28 students	16 students use fingers 4 students use rack	3 students use fingers 9 students use DS 2 students use IA 1 student use IS 13 students use IA or IS
68 - 13	22 students	25 students	6 students use fingers 4 students use rack 12 students use algorithm	4 students use algorithm 19 students use DS 1 student use IA 1 student use IA or IS
45 - 32	16 students	25 students	2 students use fingers 4 students use rack 10 students use algorithm	2 students use algorithm 6 students use DS 5 students use IA 1 student use IS 11 students use IA or IS
51 - 49	8 students	22 students	2 students use fingers 6 students use rack	2 students use IA 2 students use IS 18 students use IA or IS
75 - 26	6 students	18 students	6 students use rack	13 students use DS 4 students use IA 1 student use IS

There are some strategies that focus group students used in the pre-test and post-test. Febi showed the development of her understanding of the meaning of subtraction. In the pre-test she thought that the problem about age differences is a kind of addition problem, but she already knew that age differences problem is a kind of subtraction problem in the post-test. She also was able to apply indirect addition strategy in the empty number line.

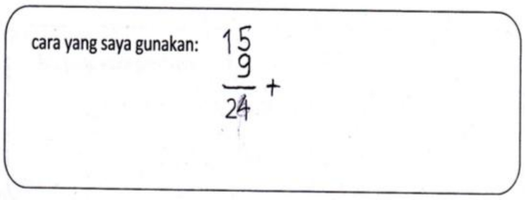


Figure 76. Febi solved 15-9 in the pre-test

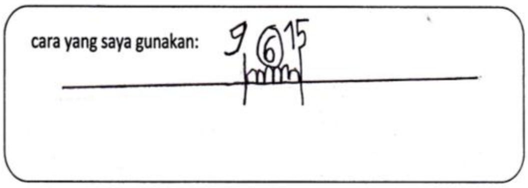


Figure 77. Febi solved 15-9 in the post-test

Vera tended to solve all subtraction problems using her fingers in the pre-test. For the small number problems, she could get the correct answer. When facing the large number problems, such as 68 - 13, she often lost of track in counting and got the wrong result. In the post-test, she was able to use the empty number line to solve two digit numbers subtraction; even she could apply “jumps of 10”.

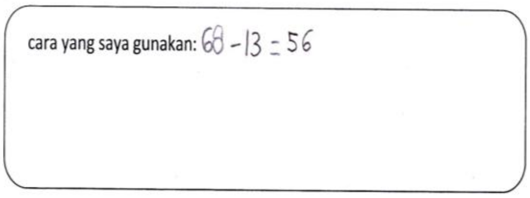


Figure 78. Vera solved 68-13 in the pre-test

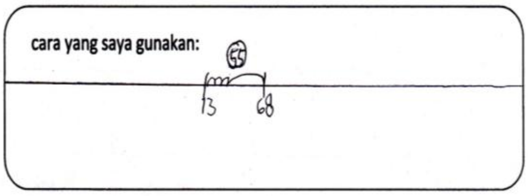
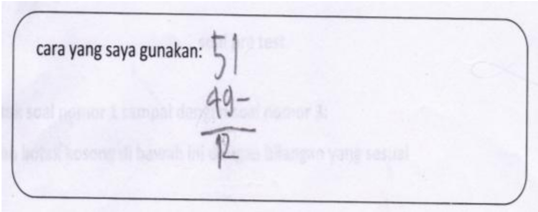


Figure 79. Vera solved 68-13 in the post-test

Fakhri already used algorithm to solve some problems in the pre-test that he cannot count with his fingers. But, he made a mistake when solving the problem with borrowing and carrying procedures. He forgot that he already borrowed the tens from the minuend. In the post-test, he changed his strategy using the empty number line. He could get the answer easily by applying indirect subtraction strategy.



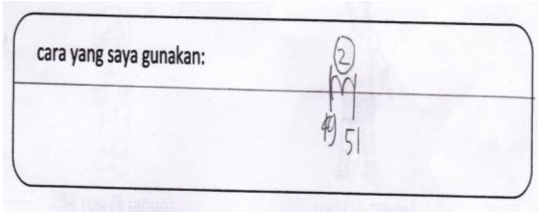
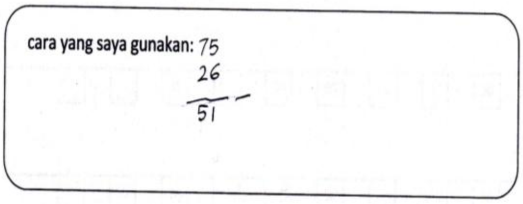


Figure 80. Fakhri solved 51-49 in the pre-test Figure 81. Fakhri solved 51-49 in the post-test

Syauqi also used algorithm in the pre-test. Similar with Fakhri, he faced difficulty to apply borrowing and carrying procedures. He subtracted the ones first, and then he subtracted the tens. Unfortunately, he did not pay attention for the minuend and subtrahend. For example, if he found 5 - 6, he though that the result is same with 6 - 5. In the post-test, he felt that using the empty number line is easier than using the algorithm. He was able to apply direct subtraction and “jumps of 10”.



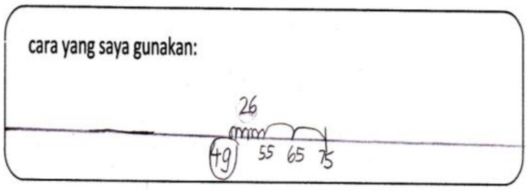


Figure 82. Syauqi solved 75-26 in the pre-test Figure 83. Syauqi solved 75-26 in the post-test

CHAPTER VI

CONCLUSION AND RECOMMENDATION

A. Conclusion

The aim of this study is to contribute to the development of a local instruction theory for subtraction by designing instructional activities that can facilitate students to develop a model in solving two digit numbers subtraction. Consequently, the central issue of this study is formulated into the following general research question: *How can a model support students to solve subtraction problems up to two digit numbers in the first grade of primary school?*

The general research question in this present study can be elaborated into two specific sub questions:

1) How can the beads string bridge students from the contextual problems to the use of the empty number line?
2) How can the empty number line promote students to apply different strategies in solving subtraction problems?

1. The Answer of First Sub Question

The instructional activities were started from the contextual situations that are experientially real for students. The use of real objects, ginger candies and grains bracelets, could help students to connect the problems given with their everyday life. The context of taking ginger candies was used to construct the meaning of subtraction as “taking away something” and the context of making grains bracelets was used for constructing the meaning of subtraction as “determining the difference between two numbers”.

In the next activity, the ginger candies and the grains bracelets were not exist anymore. The use of beads string referred to the situation described in the contexts. The students were stimulated to shift from situational level to referential level when they have to make representation as the "model of" the situation in solving subtraction problems. A string of beads could serve as a powerful model to represent the situation of those contexts because the students would see the two meanings of subtraction on it.

In the further activity, the beads string was changed with the model which is simpler and can be applied in the general level. The empty number line served as a "model for" students' thinking in solving the different situation of subtraction independently from a specific situation. The beads string bridged students from the contextual problems to the use of the empty number line. By using a string of beads first, the students could clearly see the empty number line as a counting line that refers to the discrete quantities. To solve subtraction problems, the students could start from any number and any position in the empty number line.

2. The Answer of Second Sub Question

Based on the activity in lesson 5 and 6, the empty number line was a useful scheme for subtracting up to 100. It served as flexible mental representation that can reflect the students' thinking in solving subtraction problems. The students could describe their idea about the problems and could represent their solution to solve the problems in the empty number line.

Most students still tended to use only one strategy, mostly direct subtraction strategy first, in solving all subtraction problems. The empty number line was helpful to make the students recognized the possibility in solving subtraction with more than one strategy. It could visualize the steps needed in counting to come to the result. After finding the difficulty in counting using direct subtraction, the students would realize to switch their strategy into indirect addition or indirect subtraction.

The empty number line promoted students to apply different strategies in solving subtraction by making students aware of the three strategies and the more efficient strategy based on the problems. It also showed the flexibility in making the jumps. The students could make the solution simpler by applying “jumps of 10” in the empty number line.

3. The Answer of General Research Question

At first, the students used various strategies to solve subtraction problems. Some students used their fingers, some of them used the drawing, other students used the algorithm, and some students could do the mental calculation for easy number. There were also the students who used the arithmetic rack to calculate the result.

When facing the two digit numbers subtraction problems, the students found difficulty to solve the problems with their previous strategies. They needed the ginger candies and the grains bracelets to help them in solving the contextual problems. Later on, those real objects did not longer exist. The students could not rely on them every time in solving the problems.

The students were facilitated to use a model. The beads string helped students as a “model of” the situation to solve the problems in subtraction. A string of beads functioned as a stepping stone in moving from the contextual problems to the use of the empty number line. Then, the students were able to use the empty number line as a “model for” their thinking in solving subtraction problems. They could apply different strategies (direct subtraction, indirect addition, and indirect subtraction) that more made sense and more efficient for them. As a conclusion, the model could support students to solve subtraction problems up to two digit numbers in the first grade of primary school.

B. Recommendation

1. Recommendation for the Teachers in Indonesia

This present study was based on the RME (PMRI) approach in which it provided the meaningful learning process for the students. We facilitated the use of contexts in phenomenological exploration by preparing ginger candies and grains bracelets to construct the meaning of subtraction. Then, we stimulated the use of models for progressive mathematization. The students in every phase could refer to the concrete level of the previous step and they could infer the meaning from that. We applied the beads string as a “model of” the situation” and we used the empty number line as a “model for” students’ thinking in solving subtraction problems.

In this study, we also promoted the use of students’ own constructions and productions. In each lesson, the students were given the opportunity to solve the problem in their own strategy first. After that, we encouraged the interactivity of the teaching process by conducting the class discussion. The students could share their thinking and they could receive the different ideas from their friends. The intertwining of various mathematics strands or units in this study could be seen by not only teaching subtraction as a separated concept but also stressing the relation between addition and subtraction.

Therefore, we recommend for the teachers in Indonesia to apply RME (PMRI) approach in the other mathematical topics. This approach allows the students to see mathematics as a “human activity” which makes the learning process more meaningful for them. The students are given the opportunity to "re-invent" mathematics with guide from the teacher. They will not see mathematics just as procedures to follow or rules to apply in solving the problems anymore.

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2. Recommendation for the Other Researchers

We have two suggestions for the other researchers to enhance the HLT in this present study. First of all, from the beginning the researcher should recognize the possibility that the students will find three strategies, not only direct subtraction and indirect addition, but also indirect subtraction strategy. It is normally to be happened because the three strategies are related each other. By recognizing this possibility, the researcher could make a better learning trajectory and its anticipation in each lesson to support the use of those strategies.

Second, it was difficult to differentiate the students who used indirect addition or indirect subtraction. The representations of the solutions in the drawing for those two strategies were same when they made the jumps one by one. It is impossible to see the process of all students to come to the solution. Consequently, we should guide students to give a sign for the direction of their drawing from the start to the end. If they started from the subtrahend (the smaller number), then they used indirect addition. On the other hand, if they started from the minuend (the larger number), then they used indirect subtraction.

This present study proved that RME (PMRI) approach could help students in developing a model to support them in solving two digit numbers subtraction. The use of the beads string could bridge students from the contextual problems to the use of the empty number line. Later on, the use of the empty number line could promote students to apply different strategies (direct subtraction, indirect addition, and indirect subtraction) in solving subtraction problems. Therefore, we also recommend for the other researchers to continue this study by conducting a further research using the empty number line to support students in solving multi digit numbers subtraction.