DESIGN RESEARCH ON MATHEMATICS EDUCATION:
CONSTRUCTING GEOMETRIC PROPERTIES OF RECTANGLE, SQUARE, AND TRIANGLE IN THE THIRD GRADE OF INDONESIAN PRIMARY SCHOOLS

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of
Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Faculty of Teacher Training and Education Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:
Ilham Rizkianto
NIM 20102812004

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
MAY 2012

APPROVAL PAGE

Research Title : Design Research on Mathematics Education: Constructing Geometric Properties of Rectangle, Square, and Triangle in the Third Grade of Indonesian Primary Schools

Student Name : Ilham Rizkianto

Student Number : 20102812004

Study Program : Mathematics Education

Approved by:

Supervisor I,

Prof. Dr. Zulkardi, M. I. Komp., M. Sc.

Supervisor II,

Dr. Darmawijoyo

Head of Mathematics Education Department,

Prof. Dr. Zulkardi, M. I. Komp., M. Sc.
NIP 19610420 198603 1 002

Dean of Faculty of Teacher Training and Education,

Prof. Dr. Tatang Suhery, M.A., Ph. D.
NIP 19590412 198403 1 002

Date of Approval: May 2012

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This Thesis had been examined in May 15th, 2012

Examination Committee	Signature
1. Prof. Dr. Zulkardi, M. I. Komp., M. Sc.	________________________
2. Dr. Darmawijoyo	________________________
3. Dr. Yusuf Hartono	________________________
4. Dr. Ratu Ilma Indra Putri, M. Si.	________________________
5. Dr. Somakim, M. Pd.	________________________

FACULTY OF TEACHER TRAINING AND EDUCATION
SRIWIJAYA UNIVERSITY
MAY 2012

STATEMENT PAGE

I hereby:

Name : Ilham Rizkianto

Place of Birth : Ponorogo

Date of birth : March 8th, 1987

Academic Major : Mathematics Education

State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

Palembang, May 16th, 2012

The one with the statement,

Ilham Rizkianto

NIM. 20102812004

ABSTRACT

Previous studies have provided that when learning shapes for the first time, young children tend to develop their own prototype for simple shapes. They use the prototype as the reference point for comparisons, but often fail when doing so since they do not yet think about the defining attributes or the geometric properties of the shapes. Most of the time, elementary students learn geometric properties of shapes only as empty verbal statements to be memorized, without any chance to experience the concepts meaningfully. In the light of it, a sequence of instructional activities along with computer manipulative was designed to support Indonesian third graders in constructing geometric properties of square, rectangle, and triangle. The aim of the present study is to develop a local instructional theory to support third graders in constructing geometric properties of rectangle, square and triangle. Consequently, design research was chosen as an appropriate means to achieve this research goal and realistic mathematics education was chosen as the approach in the teaching-learning process in the classroom. Thirty seven students of one third grade classes in SD Pupuk Sriwijaya Palembang, along with their class teacher, were involved in the study. Our findings suggest that the combination of computer and non computer activities supports third graders in constructing geometric properties of square, rectangle, and triangle in that it provides opportunities to the students to experience and to develop the concepts meaningfully while using their real experiences as the bases to attain a higher geometric thinking level.

Key concepts: Geometric properties, rectangle, square, triangle, design research, realistic mathematics education.

RINGKASAN

Ilham Rizkianto. Design Research on Mathematics Education: Constructing Geometric Properties of Rectangle, Square, and Triangle in the Third Grade of Indonesian Primary Schools.

Bangun datar merupakan konsep mendasar dalam perkembangan kognitif siswa dan juga matematika. Pada saat belajar mengenai bangun datar untuk pertama kalinya, siswa cenderung untuk mengembangkan prototipe mereka sendiri untuk bangun datar sederhana (Fox, 2000; Hannibal, 1999; Schifter, 1999; Clements et al., 1999). Clements dan Sarama (2009) menyatakan bahwa prototipe ini berupa benda-benda nyata di sekitar mereka, misalnya pintu untuk persegi panjang dan kotak untuk persegi,. Untuk segitiga, siswa menggunakan bentuk topi penyihir sebagai prototype mereka (Schifter,1999). Prototipe inilah yang mereka gunakan sebagai dasar pengkategorisasian bangun datar. Namun, mereka sering gagal melakukan kategorisasi jika bangun datar yang diberikan memiliki orientasi, rasio, kemiringan, atau ukuran yang berbeda. Hal ini dikarenakan mereka belum mempertimbangkan sifat-sefat geometri dari bangun-bangun tersebut.

Tujuan dari penelitian ini adalah untuk menginvestigasi bagaimana serangkaian aktivitas pembelajaran dan manipulatif komputer yang dikembangkan berdasarkan pendekatan Pendidikan Matematika Realistik Indonesia (PMRI) dapat membantu siswa kelas tiga sekolah dasar dalam mengkonstruksi sifat-sifat geometri persegi panjang dan persegi secara aktif bermakna yang mana akan membantu siswa untuk mengembangkan level berpikir geometri mereka, berdasarkan teori van Hiele (Fuys et al., 1984) dari level 0 (visualisasi) ke level 1 (analisis). Kegiatan pembalajaran akan dimulai dengan aktivitas baris berbaris di lapangan, di mana siswa menggunakan perintah maju sekian langkah ke depan dan hadap kanan untuk menyelesaikan tugas yang diberikan, dan dilanjutkan dengan beberapa aktivitas di kelas seperti baris berbaris di kertas, sebelum pada akhirnya menggunakan manipulatif komputer "jejak si kura" dan "geser segitiga" yang telah dikembangkan. Hal ini untuk menghindarkan siswa dari kegiatan menghapal sifat-sifat geometri sebagai pernyataan verbal kosong seperti yang terjadi di lapangan dan sesuai dengan apa yang dikemukakan oleh Battista (2001).

Metode penelitian yang digunakan adalah *design research* yang terdiri dari tiga fase: persiapan penelitian, penelitian di kelas, dan analisis retrospektif. *Design Research* digunakan dalam penelitian ini karena memberikan kesempatan untuk mengembangkan teori tentang proses pembelajaran dan segala hal yang mendukung proses pembelajaran tersebut pada saat yang bersamaan, termasuk di dalamnya manipulative komputer yang digunakan. Adapun subjek dari penelitian ini adalah 37 orang siswa kelas tiga Sekolah Dasar (SD) Pusri Palembang beserta seorang guru kelas mereka.

Berdasarkan investigasi yang dilakukan ditemukan bahwa kegiatan baris berbaris yang dikembangkan dapat membantu siswa dalam mengkonstruksi sifat-sifat geometri persegi, persegi panjang dengan memberikan kesempatan untuk merefleksi bangun datar yang telah

mereka buat berdasarkan perintah-perintah yang diperlukan. Mereka dapat mengungkapkan alasan kenapa sisi yang berhadapan pada persegi panjang dan persegi memiliki panjang yang sama. Penggunaan perintah untuk membentuk bangun datar secara bertahap memberikan ide kepada siswa bahwa suatu bangun terdiri dari bagian-bagian. Hal ini merupakan pengetahuan baru bagi mereka karena sebelumnya hanya memandang suatu bangun sebagai satu keutuhan. Penggunaan manipulatif komputer "jejak si kura" memberi kesempatan siswa membentuk bermacam-macam variasi persegi panjang dan persegi sehingga dapat melihat keteraturan yang mereka miliki. Berdasarkan sifat-sifat geometri persegi panjang dan persegi yang telah dikonstruksi, siswa dapat menyimpulkan bahwa persegi adalah juga merupakan persegi panjang. Terkait dengan sifat-sifat geometri segitiga, manipulatif komputer "geser segitiga" memberi kesempatan pada siswa untuk mengeksplorasi varian-varian berbeda dari segitiga. Sehingga siswa tidak hanya bertemu dengan segitiga-segitiga "umum" seperti segitiga sama sisi, segitiga sama kaki, atau segitiga dengan alas mendatar seperti yang biasa mereka temui di buku teks dan contoh yang diberikan guru, namun juga segala jenis segitiga. Pada awalnya, sebagian besar siswa mengatakan varian-varian berbeda dari segitiga ini bukan sebagai segitiga karena sangat berbeda jika dibandingkan dengan segitiga yang mereka ketahui. Namun setelah siswa mengkontruksi sifat-sifat geometri dari segitiga berdasarkan contoh-contoh yang mereka buat dan ketahui, mereka dapat melakukan kategorisasi segitiga dengan benar.

“Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid.”

Albert Einstein

“There is no royal road to Geometry.”

Euclid

I specially dedicated this thesis to:

My beloved parents and my special one, for their eternal loves and supports.

PREFACE

First of all, I am very thankful to Allah SWT for all the great things given to me. In this opportunity, I would also like to say thanks to all people who supported me in conducting my study, gave contribution and inspiration in writing this thesis and helped me in these two years.

1. I am so glad to be supervised by Dr. Frans van Galen, Prof. Zulkardi and Dr. Darmawijoyo because they gave supervision and assistance in writing this thesis. Also, they supported me by giving suggestions how to design the activities and how to support them in developing their geometrical thinking level. Therefore, I would like to express my gratitude to them.
2. The idea of this research was greatly influenced by my discussion with my beloved teachers, Dr. Jaap den Hertog, and Dr. Dolly van Eerde. Thank you very much to them.
3. I would never join this program if Prof. Robert Sembiring, Prof. Maarten Dolk and the PMRI Team did not launch this program in 2009.
4. It was Dr. Ratu Ilma, Dr. Yusuf Hartono, and all lecturers and staffs in Mathematics Education Department of Universitas Sriwijaya, who gave a lot of supports during my study in Palembang.
5. I would never go to Utrecht University without support from Nuffic – NESO and Dikti. Thanks to Pak Dede Herlan, , Bu Shita Wulandari and all NESO staffs, also to Pak Ridwan, Pak Muchlas, Bu Martha and all Dikti staffs for arranging the scholarship for us.
6. Special thanks also for Dr. Wayan Sadra and Dr. Sariasa, my lecturers in Universitas Pendidikan Ganesha who introduced me to PMRI and who gave me the recommendations to the PMRI Team as one of the candidates of the IMPoME 2010 scholarship awardees.
7. It was Bu Rima Yunidar, Bu Hafizah Pritiwi, and Bu Meldariani Roy SD Pusri tachers, who gave a lot of contributions during the teaching experiment.
8. Last but not least, I would like to thank to all my friends in IMPoME Utrecht University – Universitas Sriwijaya for the days we spent together. I would never forget all your kindness.

I wish this thesis, as the result of my study, can give contribution to improve the practice of mathematics education in Indonesia, especially in the geometry domain in grade 3 elementary school. And I hope other researchers can improve this study as I discussed and recommended in the last chapter of my work to give even better contributions.

Palembang, Mei 2011

Ilham Rizkianto

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CHAPTER I

INTRODUCTION

Shape is a fundamental concept in cognitive development (Clements & Sarama, 2009). For example, very young children use mainly shapes to learn the names of objects. Shape is also a fundamental idea in geometry and other areas of mathematics. Some previous studies (Fox, 2000; Hannibal, 1999; Schifter, 1999; Clements et al., 1999) have provided that when learning shapes for the first time, young children tend to develop their own prototype for simple shapes such as triangle, square, rectangle, and circle. According to Hanibal (1999), children's prototypes for those shapes are the "real" or "perfect" figures. Often, the prototypes are related to the real world objects, for example a triangle is a witches' hat (Schifter, 1999) and a rectangle is a door (Clements et al., 1999). Children use the prototypes as the reference points for comparisons (Hannibal, 1999). In making comparisons, children often fail if the shapes are in different orientation, ratio, skewness or size (Aslan & Arnas, 2007). They do not yet think about the defining attributes or the geometric properties of the shapes.

Most of the studies on children and shapes such as Clements et al. (1999) and Aslan & Arnas (2007) deal with children age 3-6 years old. There is no study about how third graders, age 8-9, construct geometric properties of simple geometric figures, such as square, rectangle, and triangle, that they can use to develop their geometric thinking (Fuys et al., 1984) from level 0 (visualization) to level 1 (analysis) and later level 2 (abstraction). Studies in this area are really needed since in Indonesia, third grade is the first time for the students to deal with this kind of concept and it is a good opportunity to encourage the students to think mathematically and logically. What is known from Battista (2001), elementary students learn geometric properties of shapes as empty verbal statements to be memorized, without any chance to experience it meaningfully.

Based on the idea of logo turtle of Pappert (1980), in which its combination with non computer activities designed to help students abstract the notion of path already put into practice by Clements et al. (1997) and empirically succeeds in providing a fertile environment for developing students' geometric thinking of simple two dimensional figures, a sequence of activities along with a computer manipulative will be designed to support Indonesian third graders in constructing geometric properties of square, rectangle, and triangle. In this study, it will be investigated how a sequence of instructional activities along with a computer manipulative designed for grade three Indonesian students can support them in constructing geometric properties of rectangle, square and triangle. The aim of this study is to develop a local instructional theory to support third graders in constructing geometric properties of rectangle, square and triangle with the research question: *How can a combination of computer and non computer activities support third graders in constructing geometric properties of square, rectangle, and triangle?*

CHAPTER II

THEORETICAL FRAMWORK

2.1 Geometric shapes

When learning shapes for the first time, young children tend to develop their own prototype for simple shapes, such as triangle, square, rectangle, and circle (Fox, 2000; Hannibal, 1999; Schifter, 1999; Clements et al., 1999). For those shapes, the prototype is the “real” or “perfect” figure (Hannibal, 1999). Often, the prototypes are related to the real world objects, for example a triangle is a witches’ hat (Schifter, 1999) and a rectangle is a door (Clements et al., 1999). Students use the prototype as the reference point for comparisons (Hannibal, 1999). In making comparisons, children often fail if the shapes are in different orientation, ratio, skewness or size (Aslan & Arnas, 2007). Furthermore, they do not yet think about the defining attributes or properties of shapes.

In a study reported by Clements et al. (1999) and also another study of Aslan & Arnas (2007), participants were children aged 3 to 6, the researchers found no gender differences in early geometric concept acquisition. During interviews, students were asked to identify shapes in collections of shapes. The focus of the study centered on four types of shapes, circle, square, triangle, and rectangle. It is found that students performed best with circles and squares than rectangle and triangle since the first two have less variance than the other two (Clements et al., 1999).

Some terms related to geometric shape will be defined. The first one is attributes. Attributes is used to mean any characteristics of a shape. Some attributes are defining attributes while others are non-defining. Example of defining attribute is straight sides, to be a rectangle a shape must have straight sides. A child might say that a shape is long or describe it as blue but neither of these attributes is relevant to whether a shape is rectangle or not. That is way such kinds of attributes are categorized as non-defining ones. Some defining attributes describe the parts of a shape, as a rectangle has four sides. Others are special attributes that will be defined as geometric properties. Geometric properties define a relationship between parts of a shape. For example in a rectangle, the opposite sides have to be in the same length and all the angles are right angle or 90^o in measure. These geometric properties are established by observing, measuring, drawing, and model making (Clements & Sarama, 2009). All these four aspects are used as the bases on designing instructional activities and computer manipulative in the present study. The geometric properties can be used to develop students' geometric thinking in that it supports students in understanding the classes of figures. Understanding the classes of figures can help students to not fall into common misconception that a square is not a rectangle (Battista, 2001; Erez & Yerushalmy, 2006; Clements & Sarama, 2009).

2.2 The van Hiele theory of the development of geometry thinking

The van Hiele theory of geometric thinking resulted from the doctoral work of Dina van Hiele-Geldof and her husband Pierre van Hiele at the University of Utrecht in the Netherlands (Fuys et al., 1984). The theory consists of five levels of understanding, numbered 0 through 4 by van Hiele. Recently, some studies such as Clements et al. (1999) which is also supported by Aslan & Arnas (2007) suggested that there exists another earlier level, called prerecognitive level. Children in this level pay attention to only some shapes' visual characteristics and are unable to identify many common shapes or distinguish among two dimensional figures in the same class. In table 1, adapted from Mistretta (2000), it will be given the 5 levels and characteristics of each.

Table 2.1. Characteristics of van Hiele levels of geometry thinking

Level	Characteristics
Level 0 Visualization	Students recognize figures by appearance alone, seen as a total entity, often by comparing them to a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.
Level 1 Descriptive/ Analysis	Students identify figures by their geometry properties, rather than by its appearance. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object. Therefore, definitions containing just pivotal information are not attained yet.
Level 2 Abstraction/ Informal deduction	Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and classes inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.
Level 3 Formal deduction	Logical reasoning ability is developed. Geometric proofs are constructed with meaning. Necessary and sufficient conditions are utilized with strong conceptual understanding.
Level 4 Rigor	Students can compare axiomatic systems. Theorems in different postulational system are established and analyzed.

In the relation with logo activities, Clements & Sarama (1995) found that logo activities might be used to encourage students to progress from visual level to descriptive/analysis level. For example with the concept of rectangle which is also part of this study, students at the beginning are able only to identify visually presented example (level 0). Using logo like manipulative designed, students will be asked to construct a sequence of commands to draw a rectangle. This activity allows or obliges the students to do what Papert (1980) calls externalize intuitive expectation. When student's intuition, in this case about rectangle, is translated into a sequence of commands, it will become more noticeable and accessible to reflection. In constructing procedure to make a rectangle, students will analyze the visual aspects of the rectangle and reflect on how its components are put together which encourages construction of geometric properties of the shape and level 1 (analysis) thinking. The geometric properties that are already constructed by the students will be used as the base of the discussion leading to the understanding of classes of figures which encourage level 2 (abstract) geometry thinking.

2.3 Realistic mathematics education

Mathematics has changed universally. There is shift from teaching as transmission of knowledge toward learning as construction of knowledge (Gravemeijer, 2010). Freudenthal (1991) also gives an emphasis on the construction of knowledge. He says that students should be allowed and encouraged to invent their own idea and use their own strategies. In other words, they have to learn mathematics in their own way. Furthermore, Freudenthal asserted that mathematics must be viewed as 'a human activity' instead of a ready-made product. That is why mathematics should be taught in such a way, so students by themselves can do and experience mathematics to grasp the concepts. In the light of it, this study designs a sequence of instructional activities and a computer manipulative, namely "Jejak Si Kura" (The turtle's path). This computer manipulative serves as an aid on teaching and learning two dimensional figures in which the students could gain more insight about paths, shapes and their geometrical properties through experiencing a sequence of meaningful activities instead of only memorizing empty verbal statements written on the text books or given by the teacher.

In designing the computer manipulative and sequence of following instructional activities, it is consulted to five tenets of realistic mathematics education (de Lange, 1987; Gravemijer, 1994), which are described as following:

a. Phenomenological exploration

In designing the activities and the computer manipulative, a contextual situation, parade activity, serves as the preliminary activity. Students who have roles as the commanders, with the help of other students, are faced to the situation where they need to give a sequence of orders, consisted of going forward certain number of steps and right face (turn right 90^o), to their soldier students such that the soldiers can go and back to the initial point. They will investigate two dimensional figures formed by the straight lines that are the paths of the soldiers' movements. The mathematical activity is really started from a situation that is experientially real for the students.

b. Bridging by vertical instrument

The second tenet of RME is bridging from a concrete level to a more formal level by using models and symbols. In the manipulative and the following activities, the students can develop their own model or symbol to represent the paths and two dimensional figures formed. Later on they can use the paths of the turtle as model for thinking about rectangle, square, and triangle along with their geometry properties. In the present study, the development from concrete to more abstract level can be seen as the shift in students' reasoning related to their geometric thinking level.

c. Students' own constructions and productions

In each activity, Students are free to discuss what strategies they are going to use and why a certain shape can or cannot be formed by a sequence of orders. In doing so, they will develop the strategies or procedures that work for them in order to solve a certain problem, for example finding procedures to make a rectangle. While experiencing the activities with their own ways, they will construct the geometrical properties of the shapes they form.

d. Interactivity

The learning process of students has to be seen as both individual and social process. Therefore, social interaction emerging in the classroom is important part of the whole class performance. Working in groups will build a natural situation for social interaction such as developing their strategies to go and back to the initial position. They will experience the activities and discuss together to come to the geometric properties of rectangle, square, and triangle.

e. Intertwinement

Intertwinement suggests integrating various mathematics topics in one activity. The computer manipulative and instructional activities designed not only support learning for simple geometric figures: paths, rectangles, squares, triangles, and the geometry properties of certain shapes, but also support geometric processes: measuring, turning, visualizing and arithmetic: computation and estimation.

2.4 Computer manipulative in mathematics education

ICT based products such as game or computer manipulative can promote learning material in such a more intriguing way so the students can easily get engaged to the learning activities. It also avoids teachers from spending too much time in boring-repeating activities in order to simulate mathematical phenomena. A computer manipulative lesson requires the same amount and quality of planning as any others mathematics lessons (Bell, 1978), otherwise we will only modernize the lesson without making any improvement in learning (Batista, 2001).

That is why it needs a sequence of instructional activities that makes sure it can be used properly in order to foster students' understanding in mathematics concepts. In this case, design research is the best approach in doing research in ICT because it provides chance in developing both the manipulative and activities following it.

Figure 2.1. Computer manipulative involved in the study

The design of instructional computing environments in geometry should be based not only on the general constructivist theory of learning, but also on research dealing with how students construct understanding on certain geometric ideas. Papert (1980) discusses turtle geometry as ego-syntonic or fitting the ways of thinking of the child as a geometric knowledge builder, while Clements et al. (1997) provide evidence that a combination of turtle based manipulative and non computer activities could provide a fertile environment to develop students' geometric thinking of two dimensional figures. Some other studies (Battista, 2001; Gurevich et al., 2005; Erez & Yerushalmy, 2006; Chang et al., 2007) that used computer manipulative to support students in learning geometry have also provided that the use of computer manipulative produce significant learning effect on students' geometric thinking. However, Gurevich et al. (2005) emphasize that the positive effect of the computer manipulative will be achieved when it us used in 'real time', when it is found appropriate by the teacher during a geometry lesson.

To be consistent with the general constructivist theory, computer environments should encourage student problem solving and inquiry (Battista, 2001). Students should have chance to make and test conjectures for posed problems, what they will do in the designed learning activities. Using computer manipulative designed, they will also be encouraged to explore their own ideas related to the concept learned, in this case geometry properties of rectangle, square, and triangle. The designed computer manipulative will also support students' development and use of appropriate mental models for dealing with physical, conceptual, and symbolic mathematical phenomena as suggested by Battista (2001). Furthermore, Paths created by the turtle will serve as student's model for thinking geometry properties of rectangle, square, and triangle as found by Clements & Sarama (1995). The instructional activities accompanying the use of computer manipulative designed, for example activity 'paths with the same length' requiring students produce as many as possible procedure to make rectangle with a certain number of turtle steps, support reflection on and abstraction of the mental operations necessary for properly conceptualizing and reasoning about mathematizations of the problem.

Since mathematical conceptualizations and associated mental models result from reflection on and abstraction of student's own mental actions, computer manipulative must make those actions and their consequences more accessible to reflection. When student's intuition, in this case about rectangle, is translated into a sequence of commands, it will become more noticeable and accessible to reflection (Papert, 1980). In constructing procedure to make a rectangle, students will analyze the visual aspects of the rectangle and reflect on how its components are put together which encourages level 1 (analysis) thinking. The gap between student's predictions and what actually happens provide a constant source of perturbations requiring accommodations that lead to increasingly sophisticated conceptions (Battista, 2001).

2.5 Emergent Perspective

Gravemeijer and Cobb (2006) state that emergent perspective is a framework that is used for interpreting classroom discourse and communication. The framework can be viewed as a response to the issue of attempting to understand mathematical learning as it occurs in the social context of the classroom. There are three aspects of emergent perspective that will be elaborated as follows.

- Social norms

Gravemeijer and Cobb (2006) confirm that social norms refer to expected ways of acting and explaining that become established through a process of mutual negotiation between the teacher and the students. The social norms that have been discussed in this study is the social norms of the classroom that engage in mathematics reform. For whole class discussions in reform math class, some examples of social norms include the obligations for the students to explain their reasoning and justify solution, indicate agreement and disagreement, try to make a sense the explanation given by other, and question some possible alternative in such situation where a conflict in interpretation is apparent (Gravemeijer & Cobb, 2006). The psychological perspective that correlate to social norms concerns the teacher's and students' individual belief about their own and others' roles.

- Socio-mathematical norms

Socio-mathematical norms refer to the expected ways of explicating and acting in the whole class discussion that are specific to mathematics. The students' personal beliefs about what makes a contribution acceptable, different, sophisticated or efficient encompass the psychological correlate of the socio-mathematical norms (Gravemeijer & Cobb, 2006). In the present study, the norms will be particularly about the geometric properties of rectangles, squares, and triangles. The norms encourage students to make independent judgements, for example every student might have their own ways of explaining what they understand about geometric properties of rectangle. It means that the teacher cannot merely state specific guidelines for what types of solutions are acceptable and expect the guidelines to be understood and enacted by students. Instead, the socio-mathematical norms are continually negotiated and redefined as the teacher and students participate in discussions.

- Classroom mathematical practices

Classroom mathematical practice refers to the normative ways of acting, communicating, and symbolizing mathematically at a given moment in time. The focus on mathematical practice are specific to particular mathematical ideas or concepts. Individual student' mathematical interpretations and actions constitute the psychological correlates of classroom mathematical practices.

2.6 Third graders' geometry in Indonesian curriculum

The geometry lesson concerning on shape is already given from the first grade in Indonesia, but it is not until in the third grade the students deal with the geometric properties of the shapes. The geometry and measurement curriculum for the third grade in Indonesia which is developed by National Bureau for Educational Standardization (BNSP) is given in Table 2. Most of the times, in their classroom or textbook, students tend to only deal with typical forms of each shape. For example squares with horizontal bases, equilateral or isosceles triangles with horizontal base, and rectangles with horizontal base and ratio length and width 2:1. They do not really have chances to see and discuss other examples of the shapes such as rotated squares or thin triangles.

Related to the geometric properties of the shapes, students never have opportunities to experience the concepts meaningfully. What they do is only memorizing what is already written in their textbook as empty verbal statements without any meaning. It is really not in line with Clements and Sarama (2009) suggest for the construction of geometric properties, which should include observing, measuring, drawing, and model-making. In addition, the use of computer in learning geometry in primary school is also still very rare in Indonesia.

Table 2.2. Geometry and measurement curriculum for third grade in Indonesia

Competency standard	Basic competencies
Geometry and Measurement 4. Understanding elements and properties of simple two-dimensional figures	4.1 Identifying various simple two-dimensional figures based on their properties and elements 4.2 Identifying various angles based on their types and magnitudes

CHAPTER III

METHODOLOGY

In this chapter the methodological aspects of the present study will be discussed thoroughly. It concerns the research approach, the data collection, and the data analysis.

3.1 Research Approach

Design research as an approach of a study is aiming at the development of theories about both the processes of learning and the means that are designed to support that learning (Gravemeijer & Cobb, 2006). The aim of the present study is to develop a local instructional theory to support third graders in constructing geometric properties of rectangles, squares, and triangles. Therefore, a sequence of instructional activities and a computer manipulative were developed as means to improve educational practices in the construction of geometric properties of squares, rectangles, and triangles for third graders of elementary school in Indonesia. Considering the aim of the present study, design research is suitable to answer the research question and to contribute to the research aim.

Gravemeijer & Cobb (2006) define design research by discussing the three phases of conducting a design experiment which are preparing the experiment, experimenting in the classroom, and retrospective analysis. We will now describe these three phases of a design research.

a. Preparing for the experiment

Gravemeijer & Cobb (2006) affirm that the goal of the preliminary phase of a design research experiment is to formulate a conjectured local instructional theory that can be elaborated and refined while conducting the experiment. In this phase, a sequence of instructional activities containing conjectures of students' strategies and thinking was developed. The conjectured hypothetical learning trajectory is dynamic and could be adjusted to the students' actual learning during the teaching experiment. In this phase, the pre-knowledge of the students who became the subjects in the teaching experiment period was useful to be investigated. The pre-knowledge of the students is important for determining the starting points of the instructional activities and adjusting the initial hypothetical learning trajectory.

b. The teaching experiment

The purpose of the design experiment is both to test and improve the conjectured local instructional theory that was developed in the preliminary phase (Gravemeijer & Cobb, 2006). In the present study, the teaching experiment was conducted in six seventy-minute lessons. The teaching experiment emphasizes that both ideas and conjectures could be adjusted while interpreting students' learning process. Before doing the teaching experiment, the teacher and the researcher discussed the upcoming activity, and after each lesson, the researcher and the teacher reflected on the whole class performance. There were two cycles in this present study. The first cycle was a pilot experiment, taking 4 students to participating in the teaching experiment, while the second cycle was conducted in a classroom with 40 students. The goal of the pilot experiment was to adjust and to improve the initial HLT in order to get a better design for the second cycle of the teaching experiment.

c. Retrospective analysis

Data that had been gathered from the teaching experiments were analyzed in this phase. The results of the retrospective analysis were used to answer the research questions, to draw a conclusion, and to redesign the HLT. In general, the purpose of the retrospective analysis was to develop a well considered and empirically grounded local instruction theory.

3.2 Data Collection

3.2.1 Preparation Phase

Data collection in this phase was aiming at getting general information related to the students that were the subject of the study, the teacher who collaborated with the researcher in conducting the study, and the learning environment of the class in which the study was conducted. In the light of it, the following types of data will be collected.

- Classroom observation

The classroom observation data were collected by the researcher with the help of his colleague. While observing a seventy-minute mathematics lesson in the classroom where the teaching experiment was conducted, the researcher and his colleague took written field notes of important observation points, such as social and socio-mathematics norms, according to the list of observation points already made beforehand (see appendix 3). Some pictures from the lesson were captured by the researcher using a digital camera.

- Study material

In order to get more information about how the students learned in the classroom, the researcher asked the teacher about the study material used by the students. It included textbook, worksheet, and other sources student used in the learning activities. In addition, the researcher also found some other mathematics textbooks for third grade primary school.

- Pre-test

A written pre-test (see appendix 4) was given to 4 students who participated in the preliminary teaching experiment. In giving the written pre-test to the students, which was later on followed by interview with some students related to their answers and thinking processes, the researcher was assisted by his colleague.

- Interview

There were 2 kinds of interview in this preparation phase, namely interview with the teacher and interview with the students. An interview with the teacher was conducted just after the classroom observation. It focused on the teacher's education background, her experience with PMRI, the subject would be taught, and some information related to the students that could not be obtained from the observation. Although an interview scheme was already prepared for the interview (see appendix 2), there was also a possibility to add some questions according to the findings from the classroom observation. Another interview was conducted with some students after each pre-test, in order to get more information about their answers to the questions on tests. All four students who participated in the preliminary teaching experiment were interviewed, but only four students from the teaching experiment class were interviewed. These 4 students were deliberately chosen, based on the teacher's suggestion, to be members of the focused group on the teaching experiment phase. In conducting the interview, the researcher, with the help of his colleague, recorded the entire interview using an audio recorder.

3.2.2 Preliminary teaching experiment

Data collection in this phase was aiming at trying out the initial HLT designed. For these purposes, the following types of data were collected.

- Classroom observation

Six instructional activities already designed were tried out with 4 students in this phase. These 4 students were selected by considering the teacher's suggestion and all of them were not members of the class in which teaching experiment (second cycle) phase was conducted. Since the researcher had role as a teacher in this phase, one of his colleagues helped him record the entire learning activities, using a video recorder, and taking written field notes. Some important moments were captured using a digital camera by the researcher's colleague.

- Students' work

Students' written work from all six instructional activities was collected during this phase. With the help from his colleague, the researcher collected posters and worksheet made by the students and also the screenshot of the computer as the students work with the computer manipulative.

3.2.3 Teaching Experiment

Data collection in this phase was aiming at trying out the HLT that was already improved based on the result of the preliminary teaching experiment. Therefore, the following types of data were collected.

- Pre-test

A written pre-test that was an improved version of that used in the preparation phase was given to one class of students who participated in the teaching experiment. In giving the written pre-test to the students, which was later on followed by interview with some students related to their answers and thinking processes, the researcher was assisted by his colleague.

- Classroom observation

One class of third graders comprised the subject of the study in this phase. Six instructional activities that were already improved were tried out with the whole class. Although there were observations of whole class learning, not all students were observed by the researcher.

The focuses of the observation were 4 students already chosen beforehand. These 4 students were chosen based on the teacher's suggestion so that it was obtained 1 heterogeneous group of 4 students. Since the teacher of the class did the teaching, the researcher focused on doing the observation. With the help of his colleague, the researcher recorded the entire learning activities using a video recorder located near the focus group. Witten field notes were also taken during the learning activities by both the researcher and his colleague. The timing of interesting moments was written down on the notes based on the time showed by the video recorder, so it was helpful when doing the video analysis later on. In addition, a digital camera was also used to capture some interesting pictures from the learning activities.

- Group observation

Besides observing the whole class learning, the interaction within the focus group was also observed. With the position of the video camera near the focus group, it was easy to make adjustment so that interactions within each group can be recorded clearly. There were also opportunities to pose some questions to the students in the focus group in order to get more information about their learning. Written field notes were also taken during this observation.

- Students' work

Written work of the 4 students in the focus groups from all six instructional activities was also collected during this phase. With the help from his colleague, the researcher collected posters and worksheet made by the students and also the screenshot of the computer screen as they worked with the computer manipulative.

3.2.4 Post-test

The post-test administered was aiming at getting information about what the students had learned during the preliminary teaching experiment or the real teaching experiment. A written post-test (see appendix 5) was given to the 4 students participating in the preliminary teaching experiment and also to the 4 members of the focus group from the teaching experiment. In giving the written post-test to the students, which was later on followed by an interview with all students related to their answers and thinking processes, the researcher was helped by his colleague.

3.2.5 Validity and Reliability

In the present study, various kinds of data had been gathered. They were video recording of classroom observations and students' activities, students' work, students' written test, interview, and field notes. The entire learning activities were recorded using a video camera and all written work from the focused students were also collected. These various kinds of data related to the students' learning process were used as a base of data triangulation that in turn contributed to the internal validity of the present study. The HLT that was compared with the actual learning taking place in the teaching experiment also contributed to the internal validity of the present study. Related to the internal reliability of the study, it was contributed by registering the data using a video camera.

3.3 Data Analysis

3.3.1 Pre-test

The written pre-test answers and strategies from the students were examined by the researcher and one of his colleagues. With additional information provided by the transcripts of the interview related to the students' answer and thinking processes, starting points of the students participating in the preliminary teaching experiment and the teaching experiment (second cycle) were determined. These starting points were used to adjust the HLT already designed so that the learning activities were suitable to the students' levels of understanding.

3.3.2 Preliminary Teaching Experiment

Data of classroom observation and the students' work gathered in this phase were analyzed by the researcher and one of his colleagues. Related to the video analysis, the video was watched and was cut based on the timing of interesting moments already written on the field notes, so it leaved out the irrelevant parts. Transcript of the video was also provided. The analysis of the preliminary teaching experiment was done by comparing the HLT with the actual learning taking place. The purpose of the analysis was to know to what extent the actual learning taking place matches with what was conjectured in the HLT and to found new insights that were used to refine the HLT and to improve the practice in the next cycle.

3.3.2 Teaching Experiment

With the help of his colleague, data of classroom observation, group observation and the students' work collected in this phase were analyzed by the researcher. The video recordings were treated in the same way as it is in the preliminary teaching phase. By considering also the field notes and the students' work, the actual learning taking place was compared with what was conjectured in the improved HLT. The analysis was done in order to get information that was used to answer the research question, to draw a conclusion, and to redesign the HLT.

3.3.3 Post-test

The post-test was analyzed by examining the result of students' written post-test and the interview results related to the post-test. In the analysis, which was done by the researcher and his colleague, the students' answers in this post-test were compared with the result of their pre-test. The intention of the comparison was to see what the students had learned during the teaching experiment. The result of the analyses contributed to the conclusions of the present study.

3.4 Research subject and the timeline of the research

The present study was conducted in class 3B SD Pusri Palembang with 37 students in it, including the 4 focus group students, and Bu Melda as the class teacher. In total there were 8 meetings: 6 for the lessons, 1 for pre-test, and 1 for post-test. There was also a pilot study conducted before it. The pilot study involved 4 students from class 3C SD Pusri Palembang and also had 8 meetings in total. In the learning process, the students were given a combination of computer and non computer activities. In the non computer activity, the students mainly worked with a parade context while in the computer activity they worked with the turtle path and the triangle manipulative.

The timeline of the present study can be summarized in a table as follows.

Table 3.1. The Research time line

Descriptions	Date
Preliminary Design	
Studying literature and designing initial HLT	September 2011 – January 2012
Discussion with the teacher	20 February 2012
Preliminary Teaching Experiment	
Classroom observation	24 February 2012
Pre-test & Interview	27 February 2012
Trying out the parade activity	1 March 2012
Trying out paper parade activity	2 March 2012
Trying out working with triangle activity	5 March 2012
Trying out the relation between rectangle And square	6 March 2012
Trying out missing measure activity	7 March 2012
Trying out paths with the same length activity	8 March 2012
Post-test & Interview	9 March 2012
Analyzing the preliminary experiment and improving the HLT	
Discussion with the teacher	10 March 2012
Preparation for teaching experiment	11 March 2012 – 20 March 2012
Teaching experiment	
Pre-test & Interview	22 March 2012

Lesson 1	26 March 2012
Lesson 2	27 March 2012
Lesson 3	28 March 2012
Lesson 4	29 March 2012
Lesson 5	2 April 2012
Lesson 6	3 April 2012
Post-test & Interview	4 April 2012

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Hypothetical learning trajectory is used as an approach to identify and describe relevant aspects associated with a mathematics lesson plan. In the hypothetical learning trajectory, we describe students' starting points related to the learning activity, the goal for students' learning, the mathematical task that will be used to promote student learning, and hypotheses about the process of students' learning.

A sequence of instructional activities with a computer manipulative for students' constructions of simple geometric shapes in this study was designed based on the hypothesized students' learning trajectory. In this study, it was investigated how a sequence of instructional activities along with a computer manipulative designed for grade three Indonesian students supported them in constructing geometric properties of rectangle, square and triangle. The instructional activities consisted of six activities that were conducted in six meetings. The hypothetical learning trajectory is elaborated in the instructional activities as follows.

A.Parade Activity (2x35 minutes)

a. Starting point:

Knowledge: Students already know the shapes of rectangle and square, but they base their categorizations on the shapes' overall appearances.

Skills: Students can draw rectangle and square.

Attitudes: Students do not show consideration on shapes' geometric properties

b. Goal:

Knowledge: Students define straightness; students recognize that shape consists of parts

Skill: Students use paths to form geometric shapes

Attitude: Students show consideration on commands that can or cannot form a rectangle or a square

c. Activity:

Working in a group of four, one student becomes a commander, another one becomes a soldier, and the other two become the secretary and the drawer. The commander can give order to the soldier to either move certain number of steps forward or turn right. The commander should give a sequence of commands such that the soldier at the end of the orders comes back to the initial point. Meanwhile, the secretary and the drawer will record paths formed by the soldier movement along with the commands given in drawings on a poster, which will be the source of class discussion, and also make relation between the paths and the commands. They will do it for several different geometric figures. By doing this activity, students will make their own paths and identify what kind of two dimensional figures can be formed by their paths. They will be suggested to pay attention to the straightness and the idea that shape consist of parts. Students walk, describe, discuss and create paths. In addition they will also identify why some sequences of commands works and why others not. This activity introduces a formal geometric symbolization. Students will experience learning mathematics outside the classroom, what they never do before. This outdoor activity is continued by whole class discussion.

d. Conjectures about students' actions and thinking

Students will try to make their first path by moving certain number of steps forward then they will try to do the turn right command. For the second path, there are two possibilities. The first one, they will move forward with the same number of steps as they do for the first path. Students doing this are either aware that they could make a square or just doing it without purpose, just trying. Students who are aware that they could make a square or a rectangle know that the right turn they do will make a kind of right angle, angles they see on rectangle or square, even they do not know yet the term "right angle". The second possibility, they will move different number of steps forward, with no purpose, just trying. The next command will be turn right again before moving certain number of steps forward. In choosing the number of steps, they will consider the length of the first path or just try until realizing that in order to be able to back to the initial point effectively, the third path have to be in same length with the first one. They will do the last turn right before finally choosing the number of steps needed to get back to the initial point. They will record paths they make and do it for several times to make different figures. There will be a possibility that students make figures other than rectangle or square. The figure will be composition of several rectangles and squares. They are also required to record all geometric figures and commands related that they make, even they not succeed to back to the initial point in that figure. It will be a source of discussion, why certain commands work and why others not.

In the class discussion, the teacher could ask students about what kind of figure they make and how they could make that. When making connection between commands they use and the shape formed, they will recognize that shape consist of parts that relates one another. The students are also asked to identify why certain commands work and why others not which implicitly suggests the geometric properties of square and rectangle. The teacher also could ask the students whether some groups make rectangle or square. Since students will record their paths as a poster on a plain big paper, there is possibility that figures they make do not exactly look like square or rectangle, but they can argue that a close shape can be formed only if it satisfies a certain condition, for example opposite sides must be in the same length. Based on the paths that students form, the discussion will also address the needs of straightness of the shape.

B. Paper Parade (2x35 minutes)

a. Starting point:

Knowledge: Students define straightness; students recognize that shape consists of parts

Skill: Students use paths to form geometric shapes

Attitude: Students show consideration on commands that can or cannot form a rectangle or a square

b. Goal:

Knowledge: (i) Students recognize length structure of rectangle and square; (ii) Students analyze that shape is invariant under rotation.

Skill: Students use paths to form square and rectangle

Attitude: Students express point of view on some geometric properties of rectangle and square

c. Activity:

Using commands from the previous activity: moving forward certain number of steps and right face, students have to draw different figures on the grid paper. They are also required to write down all the commands needed for each figures. Students give imaginary commands to his/her pencil and make several different two dimensional figures on the grid paper. What they can make using forward and right face command are rectangles and squares or figures consisting of those two shapes. Students will identify geometric properties of rectangle and square, related to their sides' lengths. By obligating the students mention commands needed to make the paths, it keeps them to make connections between mathematical symbols and geometric representation. In the classroom discussion, the teacher who has a big grid paper attached on the blackboard will deliberately remove the tape on one upper corner of the grid paper such that the grid paper not in horizontal position anymore. Students will have discussion whether rectangle or square already drawn on it is still rectangle or square or not.

d. Conjectures about students' actions and thinking:

   Since this activity is almost similar with the first one, most students will not find any difficulty. The use of grid paper will structure their figures, so they will see clearly rectangle or square. The teacher could start the discussion by asking how to make a rectangle or square. Students will start to saying the procedures related to the number of steps and right face. The teacher will ask the students to see whether there are regularities in rectangle or square by asking them to see the relation between the shape's parts. Students will make conclusion related to the lengths of shape's sides. They will be able to make conclusion that rectangle and square have four sides, opposite sides always in the same length, and special for square, all sides are in the same length, but they do not say anything about the angles' measures, what they will deal in the next activity. Related to the one corner hanging grid paper in front of the class, some students would say rectangle or square already drawn there is not rectangle or square anymore since the figure has no horizontal base anymore. The teacher will invite groups of students that still say the drawn figures are rectangle or square to explain their opinion. At the end, all students will come to conclusion that rotation does not change shape and kind of shape is determined by its geometric properties.

C. Working with triangles (2x35 minutes)

a. Starting Point:

Knowledge: students analyze that shape is invariant under rotation.

Skill: Students use paths to form square and rectangle

Attitude: Students express point of view on some geometric properties of rectangle and square

b. Goal:

Knowledge: (i) Students define angle, (ii) Students recognize angle measures, (iii) Students define right angle, (iv) Students construct geometric properties of triangle

Skill: Students use paths to form equilateral triangle

Attitude: Students show consideration on variant of triangles

c. Activity:

Students are introduced to a logo like computer manipulative. They will be asked to do almost the same things as they do in the two previous activities. They can give command to the turtle to form a geometric figure. The difference is with this computer manipulative students have to give magnitude (multiplications of 30) to the turn they try to make. They also have two different turn, left and right. Asked to form an equilateral triangle using the manipulative, students will define angle, recognize angle measures, and define right angle. Given the second computer manipulative, in which the students can click three different points to make a triangle and later on can freely drag all three vertices, students will record and identify not only exemplar (typical form) but also variants (other examples) of triangles. Class discussion will also be held to address all involved concepts.

d. Conjectures about students' actions and thinking:

Students will try to do some commands. They will try move forward certain steps first which they already familiar with. They will find difficulty to use the turn with magnitude and direction command. They do not have idea what the magnitude represents. Teacher could give help by asking what magnitude needed to perform their usual right face command. They will try and find that turn right 90 is exactly like their usual right face. By asking to consider other magnitude such as 0, 30, 60, and 120, together with the teacher students will discuss and define angle, in this case also interior and exterior angle. Students could define angle as two lines that meet to make a corner. They will get the idea that angel measures is a magnitude that determines the relation between two lines that meet and make a corner. With the help of the teacher, they will define right angle which equals to their usual right face command. In addition, the teacher could ask whether they have tool to determine an angle is a right angle or not. Students could start mentioning the corner of their books or the configuration formed when they stretch their thumbnail and point finger. Based on this knowledge, student will find that turn right 120, which is the measure of the exterior angle, is needed to form an equilateral triangle and all its (interior) angles are 60^o. By reflecting to commands they use and shape they make, students will construct geometric properties of an equilateral triangle. The use of the second manipulative will provide opportunity to the students to make and see variants of triangles. There is possibility that students will say one triangle is not triangle since its base is not horizontal or it is so thin, the class discussion will be held to find what properties are needed by a shape to be triangle. The students will come to the conclusion that a shape can be identified as triangle if it has three sides and three angles.

D. The relation between rectangle and square (2x35 minutes)

a. Starting Point:

Knowledge: (i) Students define angle, (ii) Students recognize angle measures, (iii) Students define right angle, (iv) Students recognize length structure of rectangle and square;

Skill: Students use paths to form rectangle and square

Attitude: Students express point of view on some geometric properties of rectangle and square

b. Goal

Knowledge: Students construct geometric properties of rectangle and square,

Skill: Students determine the relation between rectangle and square

Attitude: Students express point of view on the relation between rectangle and square

c. Activity:

Students will still work with the first computer manipulative. They now will make rectangle and square. They will be asked to identify geometric properties of those shapes including their angles. They will also analyze the relationship between rectangle and square.

d. Conjectures about students' actions and thinking:

Students who already have idea about angle will construct the geometric properties of rectangle and square. In this activity, they will also pay attention to angles on those shapes, not only to the sides' lengths as they do on the paper parade. By asking question what properties of rectangle held by square and vice versa after students make list all geometric properties of those shapes, the teacher can guide the students to come to the conclusion that square is a rectangle. It is not easy to come to the conclusion that a square is a rectangle since the language (Bahasa Indonesia) suggests the misconception. Rectangle is translated as *persegi panjang* which literally means long square. Students will hardly accept that a square is a rectangle. By making list and discussing the geometric properties of rectangle and square, it will be seen that all geometric properties of rectangles are also held by square. Then the students are expected to draw a conclusion that square is a special kind of rectangle. Square is one class of rectangle.

E. Missing Measures (2x35 minutes)

a. Starting point:

Knowledge: Students construct geometric properties of rectangle, square, and triangle

Skill: Students determine the relation between rectangle and square

Attitude: Students express point of view on the relation between rectangle and square

b. Goal:

Knowledge: Students apply geometric properties of rectangle, square, and triangle to find missing measures of two dimensional figures

Skill: Students determine the missing measures of two dimensional figures

Attitude: Students show consideration on the reasoning in finding the missing measures

c. Activity:

Students should find the missing measures including lengths and angles to complete partially drawn paths. Students have to figure out what commands should they give so the turtle can complete the side of the figure. They have to analyze geometric properties of the figures and use mental computation or estimation to find out the missing measures. Class discussion will be held after all students finishing their tasks.

d. Conjectures about students' actions and thinking:

Some students will do trial and error at the beginning, but after asked to consider the geometric properties held by the shapes, they will find it easier to find the missing measures. In the class discussion, the teacher could ask "why are you sure that your answer is correct?" to the students in order to emphasize the use geometric properties of the shapes in reasoning.

F. Paths with same length (2x35 minutes)

a. Starting point: Knowledge: (i) Students apply geometric properties of rectangle and square to find missing measures of two dimensional figures, (ii) Students construct geometric properties of rectangle and square

Skill: (i) Students determine the missing measures of two dimensional figures, (ii) Students use path to form rectangle and square

Attitude: Students show consideration on the reasoning in finding the missing measures

b. Goal:

Knowledge: Students apply geometric properties of rectangle to form several rectangles with the same perimeters

Skill: Students determine the dimensions of rectangles that have the same perimeter

Attitude: (i) Students show consideration on the reasoning in finding the procedures, (ii) Students express point of view in finding the procedures

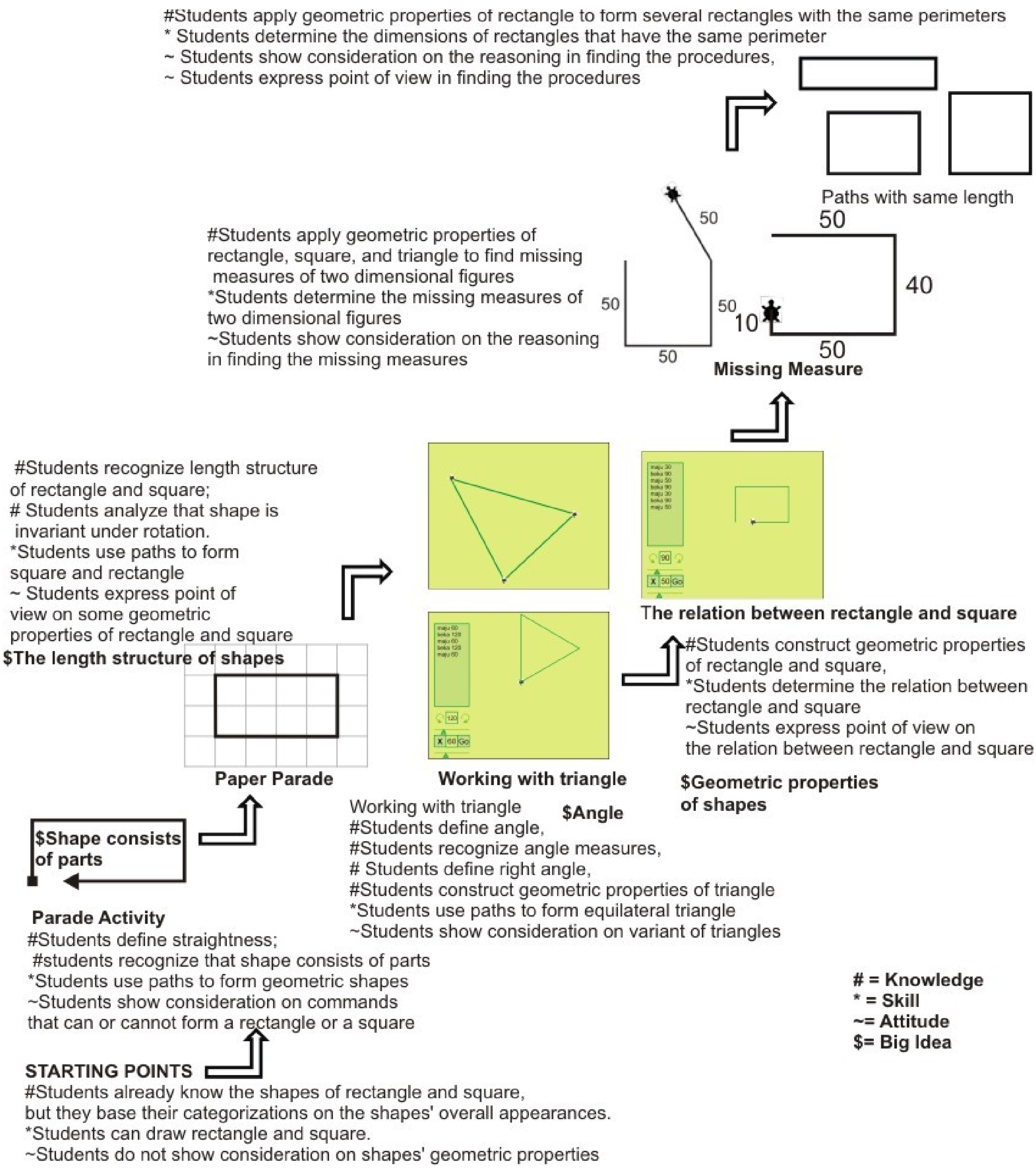
c. Activity:

In this activity, students are supposed to write down as many as possible procedures, set of commands, to draw different rectangles that have paths with same lengths in total, for example 200 steps. Students will not work with the computer at the first. They start by planning one procedure and check it using the logo turtle to get feedback whether they have done it correctly or not. Then they revise it or start planning new procedures and check again with the turtle. Similar with missing measures, students have to analyze the geometric situation involved and use mental calculation or estimation to get the procedures. Classroom discussion will be held to find the best strategy to do the task.

d. Conjectures about students' actions and thinking:

Some students will start to make drawings and do trial and error to complete this task, but after suggested to consider the geometric properties of rectangle, they will find it easier. Some students will also find another property of rectangle that could help them solving this task, namely the sum adjacent sides is always one half of the perimeter. Some students can do it mentally and easily find many procedures while other students make a procedure and try it in the computer manipulative before revising it or making another one. The discussion can be started by asking why students choose certain numbers in their procedures and why certain procedures do not work.

G. The visualization of learning trajectory



CHAPTER V

RETROSPECTIVE ANALYSIS

In this chapter, the retrospective analysis of all the data already collected from the preliminary phase and the teaching experiment phase, including also the result of pre-test and post-test in each phase, are described. The result of this study is underlying principles explicating how and why the present design works. The hypothetical learning trajectory serves as a guideline in the analysis to investigate how a combination of computer and non computer activities can support third graders in constructing geometric properties of square, rectangle, and triangle.

5.1 Preliminary teaching experiment

Pre-test

The pre-test given to the four students involved in the first cycle (see appendix 4) exactly gave result matching what is already conjectured before. The students based their categorization of shapes on the shapes' overall appearance. No attention was paid to the shapes geometrical properties. They could not explain why a certain shape is a rectangle or a square. Two students mentioned about sides and angles but really did not mean it. They could categorize squares and rectangles with a horizontal base, but when it was slightly rotated, they did not see it as a square or a rectangle anymore. Similar thing happened to the categorization of the triangle task. They could say that 'normal' triangles like equilateral triangle, right triangle, and isosceles triangle whose bases are horizontal lines as triangle, but failed to categorize non exemplar triangles like ones with non horizontal base or irregular lengths as triangle. Related to the van Hiele level of geometrical thinking, it could be inferred that these students were in the level 0, visualization.

Parade activity

In this activity, students were doing parade activity in the school yard. In a group of four, one student became the commander, another became the soldier, and the other two became the secretary and the drawer. With 2 different types of commands, forward certain steps and right face, the commander had to give the soldier order to move and go back to the initial position at the end. They had to determine what kind of shape formed by the paths the soldier makes.

As conjectured, by taking some time in doing the activity, the students could realize that they could make rectangle and square. By reflecting to the commands needed to form those shapes. They also realized that shape consists of parts, sides and vertices. In addition they could argue that the sides of rectangle or square have to be straight lines.

Paper parade

After doing parade activity, the students were asked to do almost the same activity, but on the grid paper. Each student has to be a commander and treat his or her pencil as the soldier. Doing this activity, the students were helped by the grid to structure square and rectangle they made. They also could perceive the length structure of rectangle and square as conjectured and got insight that the opposite sides have to be in the same length.

Working with triangle

In this activity students started working with the computer manipulative the turtle path. This manipulative has similar idea with the parade activity. The differences are the soldier becomes a turtle and right face command is replaced with turn command which has two directions, left and right, and also magnitudes. By using this manipulative the students have opportunities to create more shape such as an equilateral triangle.

Students could define angle using their own word. For example one student said that angle is two lines that meet in one point. They could also define right angle and give example how to determine whether an angle is a right one by using one corner of the paper. Related to the geometric properties of triangle, the students could say that a shape is a triangle if it has three sides and three vertices as already conjectured before.

In this activity, the students also worked with the second manipulative, namely triangles drag. The students chose any three points on the screen. These three points will be connected to one another to form a triangle. After that, the students were asked to drag the triangle's vertices in order to form an interesting figure. They had to determine whether the new figure they made is a triangle or not. Since the students never dealt with non exemplars triangles, they said that some figures they made not triangles. Their reasons are the shape is too pointy, it is very thin, and it does not look like a triangle. All this reactions are already conjectured before. After having discussion about what is needed by a figure in order to be a triangle, they could come to the conclusion that as long as one shape has three sides and three vertices, it is a triangle.

The relation between rectangle and square

Students still worked with the computer manipulative in this activity, they had to make rectangles and squares before finding the regularities of those two and investigating their relation based on their geometric properties. Students gave respond as conjectured in the HLT, they could conclude that a square is also a rectangle.

Missing measures

In this activity the students were given non-finish figures, drawn using turtle path manipulative, and had to give commands to the turtle so it could complete the figures. At the beginning, the students started by doing trial and error, as conjectured, but after suggested to consider the geometric properties of the shapes, they could easily find the answers and were able to give reasons why they used certain commands to complete the given shapes.

Paths with the same length

Students, in this activity, were challenged to write down as many as possible commands needed to form a rectangle whose total length of it sides is 200. As they did in the previous activity, they started to do trial and error. They found problem since the number is a big one for them. After the number was changed to 20, they could do the task well. Starting with trial and error, they could utilize the geometric properties of rectangle and square to find many possible commands, but they could not yet conclude that for rectangle the sum of adjacent sides is always one half of the total length of all sides.

Post-test

The result of the post-test in the pilot study informed that 3 out of 4 students already grasped the geometric properties of rectangle, square, and triangle. They could determine whether one shape is a triangle, a rectangle, or a square base on its geometric properties. They could also give reasoning why a certain shape, for example, is a rectangle by mentioning its geometric properties. They could also solve problems related to the shapes' geometric properties. Compared to the result of the pre-test, it could be said that they already construct the geometric properties of rectangle, square, and triangle and already in a higher level of geometrical thinking, in this case level 1 analysis.

5.2 The Improved Hypothetical Learning Trajectory

Based on findings in the teaching experiment phase, some activities are revised in order to get optimal results. For the parade activity, the students will be given symbols to record the commands they use since it costs a lot of time to write down the commands completely. From the pilot study, the students wrote down all the commands they said. For example when they said go forward 5 steps, they also wrote it down on their papers. From the beginning of the activity, the students will be suggested to use M, stands for maju (Indonesian word for forward), to record go forward commands and HK, stands for hadap kanan (right face), to record right face command. For example, when the students give commands go forward 5 steps and right face, they only need to write M5 and HK on their paper, so they could do the activity effectively. Another improvement for the first activity is the soldier students should give mark to the point where they stand, using small paper, before doing a right face command, so the students could explicitly see vertices of the shapes they make and will not loss track.

For the second activity, paper parade, improvement is addressed to the size of the square on the grid paper. Since the square is very small, it makes hard for the students to make good drawings and establish a relation between what they have done in the previous activity with the paper parade activity. For example, in the parade activity, they make a big square with only 4 steps forward, but on the grid paper, go forward 4 steps will form a very short line. Another reason to make the square on the grid paper bigger is expectation that the students will be really helped to structure shapes they make and perceive the length structure of rectangle and square.

In working with triangle activity, there are also some improvements made. The first one is related to the turtle path manipulative. The use of the number of pixels to represent the number of steps with the ratio 1 pixel for 1 step seems inappropriate since 1 pixel is very small. It will make some errors that disturb the students in constructing geometric properties of the shapes. As an illustration, commands M100, BK90, M100, BK90, M100, BK90, M100 and commands M100, BK90, M100, BK90, M100, BK90, M98 will make the same square in the screen since 2 steps are very small. That is why a scale is needed. In this case, the improvement is to make 100 pixels equal to 1 step. Another reason to use this scale is to give opportunities to the students to make relations between what they have already done in the parade activity and what they do with the turtle path manipulative. For example, after the improvement, they will find in the turtle path manipulative that a square with sides 3 steps are big enough, just like what they get when doing it in the parade activity.

For the fourth activity, the relation between rectangle and square, the improvement is addressed to the students' worksheet. A two-column table is provided. The first column is used to list all the regularities of rectangle resulted from the students' observation while the other one is used for those of square, so the students can easily compare and establish a relation between those two shapes.

In missing measure activity, the numbers included in the problems are changed. Since some students were having problems with calculation including big numbers, in this case more than 100, they struggled more with the numbers not the geometric properties of the shapes. Hence it is decided to change the numbers with small numbers, with 20 as the biggest numbers. Our expectation is the students not struggle with the calculation, so can more focus on the application of the geometric properties of the shapes.

For the last activity, paths with the same length, the improvement is almost similar with the previous one. The total length of all sides is reduced from 200 to 20, so the students can more focus on the geometric properties of the rectangle. Since the students in the preliminary teaching could not attain the property that the sum of two adjacent sides of rectangle is always one half of the total of the all its sides length, one task about it, with a conclusion box, is provided in the worksheets. Three variants of rectangle with perimeter 10, 12, and 16 are also provided as exercises to see whether the students already grasp the property or not.

5.3 Teaching Experiment

Pre-test

The result of pre-test given to all 37 students participating the teaching experiment phase (see appendix 4) informed more or less the same thing as what was acquired from the pre-test for the four students in the preliminary teaching phase. All the students failed to identify rotated squares as squares. It also happened for rectangle and triangle. They could not make good categorization of shapes. They based their categorization solely on the shapes' whole appearances. Some of them mention about side and angle but really did not mean it. Hence, according to van Hiele level of geometrical thinking, it could be said that they were still on the level 0, visualization.

Since they were still on the visualization reasoning level, which means they had not grasped the geometrical properties of shapes, it was not surprising that they could not determine the relation between rectangle and square in the question number 4 in the pre-test, which needs a higher level of geometrical reasoning. Asked whether a square is also a rectangle, all the students said no and most of them gave reason that indicated they were on level 0, reasoning. They said that square looks like box and rectangle has a long shape. No attention was paid to the shapes' geometrical properties.

Lesson 1:

There was only one activity in the first lesson, namely the parade activity. In this activity, the students were working in groups of four. One student became a commander, another one became a soldier, and the other two became a secretary and a tracker. Having only two kinds of commands namely move forward certain steps and right face, the commander had to give order to the soldier to move and go back to the initial position at the end. What the secretary and the tracker had to do was to record all the commands the commander gives and to draw all the paths formed by the soldiers' movement.

All the students including four focus group students were enjoying the activity very much since they did the activity outdoor, in the schoolyard. It was the first time for them learning mathematics outside the classroom. They gave commands, argued one another, and laughed together, but did not forget what they had to do there. As conjectured, they were able to form rectangle and square. Here will be described one interesting fragment from what the focus group students did in the parade activity.

Figure 5.1 Students doing the parade activity

Rafi, the commander, gave command move forward 2 steps to the soldier, Alfaris. After he moved two steps forward and put a sign on the spot he stood, the commander gave him order right face. Again, he was ordered to go forward 2 steps. Although the order was the same one as before, he did something different. He moved two steps but with smaller steps than before. The commands forward 2 and right face were repeated until Alfaris got back to the initial position. Since the size of his steps did not constant, he formed a rectangle on the field, instead of a square. Recognizing that there was something wrong, Aslam, the secretary, said something to Rafi. As pointing to commands he had recorded on the paper, he said that it should be a square since all the sides were formed by forward 2 commands. He also mentioned that the steps of Alfaris that were not consistent as the reason. After he gave the correction by doing forward 2 commands with constant steps, Rafi erased one line and replaced it with one ended at the point Aslam stopped. It made they had a figure of square on the yard. All of them agreed with that figure.

Based on the fragment described above, we could infer that the students could make connection between the shapes and commands given. They knew that each command will affect the resulted figure and the same commands will also yield the same figures. Reflecting to the commands they gave and the resulted figures, they could decide whether the figure is the right one or not. They knew that with the right face command if all the sides are in the same length, they would have a square.

In the classroom discussion, the teacher, Bu Melda, asked the students to use commands they recorded on the paper to reflect figures they made. With this instruction, the students could conclude that shapes, in this case rectangle and square, consist of parts. They could also mention that those shapes have four sides, that have to be straight, and four vertices.

The parade activity gives opportunity to the students to make connection between shapes they make and the given commands. By obligating the students to record all the commands needed to form a shape, it can be a source of reflection about how a shape is formed. As the result, the students will perceive a shape not only as a whole but also a combination of parts, as conjectured in the HLT. A classroom discussion about the need of straight lines in rectangle and square gives the students opportunities to reason explicitly that a shape is not a square nor a rectangle if the sides are not straight.

Lesson 2:

In the second lesson, there was only one activity which is the paper parade. Considering the name of the activity solely, it was easy to guess that this activity must have connection with the first one. Indeed, this activity was almost similar with the first one except it was done on grid papers, not in the school yard anymore. Students working in pair were supposed to give commands as they did in the first activity, but their soldier now was their pencil that walks on the grid paper.

Four focus students, who were split into two pairs, and other students were doing this activity before having class discussion about the length structure of rectangle and square and shape that is invariant under rotation. While doing the activity, one student Nadia, who already made two rectangles, was interviewed by the researcher. Here is given the transcript (I stands for the researcher while N for nadia).

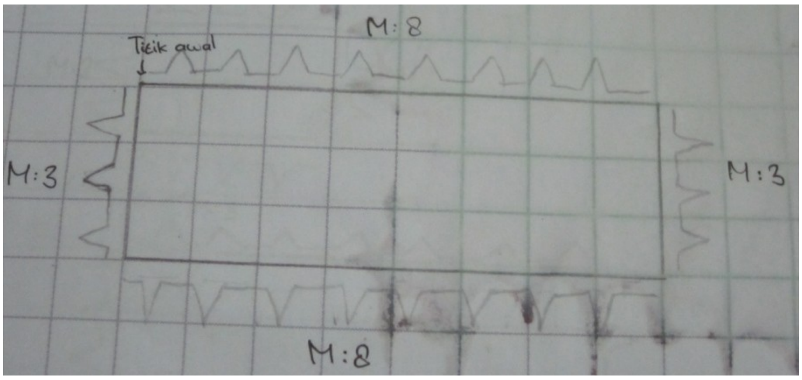


Figure 5.2. Nadia's drawing

I : what are you doing?
N : making a rectangle
I : How do you make it? Can you explain?
N : What?
I : Where is the rectangle?

N : these two (pointing to two rectangles she already made)

I : Are both of these rectangles?

N : yes, later I will also make a square

I : How do you know that is a rectangle?

N : Because, these two sides are equal

I : Which? why?

N : Because the sides, this and this are equal (pointing to the bottom and the up side of the rectangle)

I : why?

N : These sides are equal. This is 3. (pointing to the right side) This is 8. (pointing to the bottom side)

I : Then?

N : This is also 3 (pointing to the left side) and this is also 8 (pointing to the up side)

I : So, because these two sides are equal, the shape is a rectangle?

N : Yes

Based on the transcript, it could be inferred that the paper parade activity with the use of grid paper could support Nadia geometrical reasoning. Nadia could use the length structure of the figure she made on the grid paper to reason why that figure is a rectangle. This is already conjectured in the HLT. The use of the grid paper really helps the students to structure figures they make, so they can recognize the length structure of the shapes and clearly see whether shapes they make rectangle or square. The pattern on the grid also helps the students to recognize the regularity of rectangle's and square's corners. Even though the students have not known the term 'right angle' or 'angle', they will perceive that the corners of rectangle and square have a special form.

Although the use of the grid paper helps most students, including all focus students, there were still some students who did not get support from the grid. Instead of starting their drawing from the intersection of the grid, they started within the pattern square. It made they could not structure their figures and could not construct the length structure of the square and rectangle. It was a pity that the teacher and the researcher did not see this phenomenon earlier, so they could not find the reason why they did not get help from the grid.

Lesson 3:

In the third lesson, there was one activity namely working with triangle. Using computer manipulative the turtle path the students were asked to form an equilateral triangle. While doing so, they had also to establish their own definition about angle and perceive that there was a magnitude determining the relation between two segments that meet at one end. They also worked with computer manipulative, triangle drag. With this manipulative, they could choose any three points on the screen. These three points will be automatically connected one another to form a triangle. Then they could drag every vertex to form other shapes. The students in groups of four were asked to make nine variants of the shapes using this manipulative. Then they have to decide which shapes are triangles. It probably looks strange for someone reading this since it is obvious that all the shapes made will be triangle, but it was not obvious for the students. According to what was conjectured and also supported by the result of the pre-test, the students were not used to deal with non variant triangle. They failed to identify non exemplar triangles such as triangle with non horizontal base or a thin triangle as a triangle. Here will be given one transcript from one focus group student Seli while working with the triangle drag manipulative. She and her group already made 9 figures and recorded them on the worksheet. The researcher asked her whether one non exemplar triangle, number 9 in her worksheet, is triangle or not. (I stands for researcher, S for Seli, A for Aslam, and F for Alfaris)

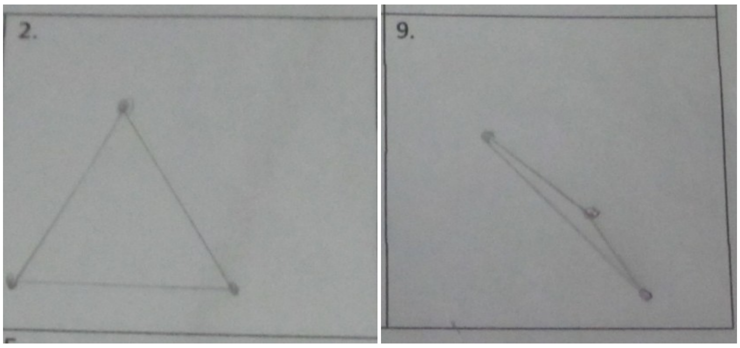


Figure 5.3. Seli's triangles number 2 and 9.

I : Seli, are all the figures your group made triangle?

S : Mmm..

I : What about figure 9? Is it a triangle?

S : I think it is not.

I : Why? Do you have a reason for that?

S : I just feel it is not a triangle. It is too thin and pointy. It is strange.

I : What do you think Aslam?

A : That is a triangle since it has three sides and three vertices.

F : Ya. That is a triangle. Having three sides and three vertices.

I : So, Seli. What do you think now?

S : Mmm. I don't know.

I : What about figure 2. Is it a triangle?

S : Yes.

I : How do you know that?

S : It looks like a triangle.

I : How many sides it has?

S : Three

I : What else can you say about figure 2?

S : It has three… vertices.

I : look again at figure 9, what about its sides and vertices?

S : Three sides and vertices

I : So? Is it a triangle?

S : Probably yes, but it is strange.

From this fragment, it is evident that Seli had difficulty in identifying non exemplar triangle. Seli also still based her categorization on the prototype of triangle in her head. The triangle drag manipulative gave opportunities to Seli and other students to deal with variants of triangles that they rarely found in the textbooks and classroom. By asking Seli to reflect to kind of triangle she was already familiar with, she could say about the regularities of a triangle. Suggested to apply it to the non familiar triangle, she into some extent agreed that figure 9 was triangle. She still had doubt since not used to dealing with that kind of triangle. Students need to get more often non exemplars kind of shapes, so they will have variety prototypes of those shapes.

Reflecting to the HLT already conjectured for this activity, almost all the students at the end of the activity succeeded in determining that all the figures are triangle. They also could make a conclusion that a shape is a triangle as long as it has three sides and three vertices. For the focus group students, although during the activity one student Seli struggled in determining whether triangle 9 is a triangle or not, at the end she could also make the conclusion.

Lesson 4:

In this lesson, the main activity was to find the relation between rectangle and square. Students had to construct several different rectangles and squares. Then they had to find regularities that were also the properties of each shape and recorded it on the given table. Next using the list of properties of square they already made, the students had to check whether a rectangle is a square. The same thing also happened for a square. Using the list of properties of rectangle they made, they had to check whether a square is also a rectangle.

All the students including focus group were working in groups of four. They were allowed to work with the turtle path manipulative to construct variants of rectangle and square before constructing their geometric properties. The focus group did not find any difficulty in finding the properties task, neither did the other groups. Some groups found some difficulties in the checking task. They did not rely only on the geometric properties of the shapes they already established but on their pre-knowledge of rectangle and square. Rectangle has to be long and square has to look like a box. Here will be given one students' answer for checking task, whether a rectangle is a square or not.

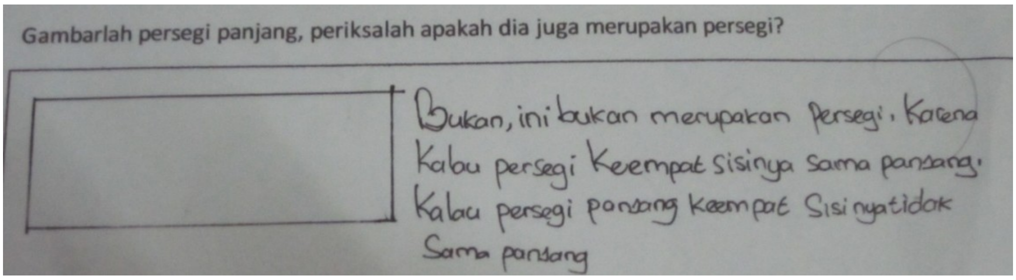


Figure 5.4. Exampel of students answer in lesson 4

The instruction is draw a rectangle, then check whether it is also a square. This group wrote down as follows. *No, it is not a square, because square has four sides in the same length. If rectangle, not all four sides are in the same length*. This group also could make a good conclusion at the end. They wrote rectangle is not square since there is one property that is not common. Square is rectangle since all the properties of rectangle are similar with those of square.

Encouraged to make use the geometric properties of rectangle and square, this group succeeded in determining relations between rectangle and square. They started to get used to use shapes' geometric properties to categorize shape. As conjectured, in making list of properties of the shapes, they listed all that they could see, not only the defining properties. Hence it can be said that they were already in geometrical reasoning level 1, descriptive/analysis. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object. Therefore, definitions containing just pivotal information are not attained yet. Considering all the students on the class in the end of lesson, after having discussion led by the teacher, all the students could attain that a square is also a rectangle.

Lesson 5:

In lesson five, there was only one activity, namely missing measures. In this activity, the students, in group of four, had to give commands to the turtle in order to finish the half drawn figures given. There were six questions for the students including square, rectangle, and triangle (see appendix 6). The students had to predict the needed commands by themselves before checking it using the turtle path manipulative. The students were expected to use the geometric properties of shapes that they have constructed in solving the problem. The focus group students were able to solve the first three problems that are not including triangle or rotated figures, so were the other groups. For the other three problems, no group could solve them correctly. It is probably caused by the presence of triangles or rotated figure in the last three problems. Here is given an example of students' work for question 2 from the focus group.

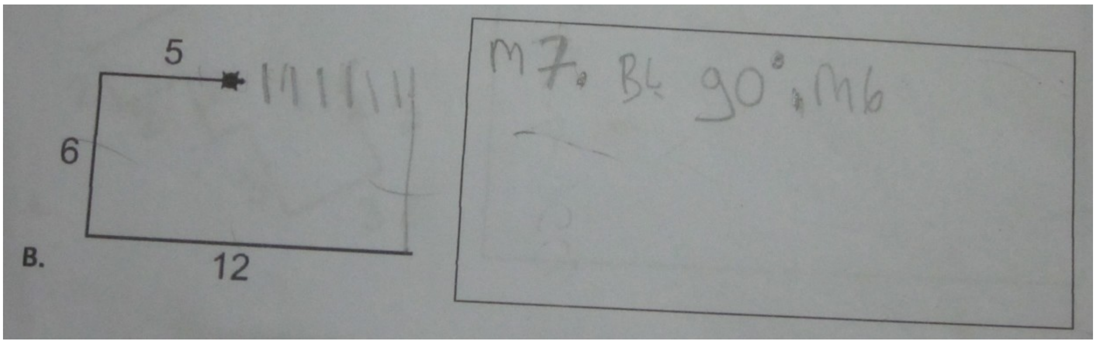


Figure 5.5. Example of students answers in lesson 5

In the image above, we could see that these students gave the correct answer, M7, BK 90, M6. Looking at the drawing they made, they clearly understood that the completed figure is a rectangle. It seemed that they counted on the number 6, 7, 8, 9, 10, 11, 12 as drew a tally for each number. They had no problem with the property of rectangle stating that the opposite sides have to be in the same length and used it correctly to solve this problem.

Related to the questions number 4 and 6 that include triangles on the figure in which no group could solve it correctly, it indicated that they still could not utilize the geometric properties of equilateral triangle to solve problems as expected. It is probably caused by the fact that they did not really construct the geometric properties of equilateral triangle in the lesson 3. Even they did make equilateral triangles in that lesson, they did it by trial and error, so they did not grasp the concept.

Lesson 6:

In this last lesson, there was also one activity, paths with the same lengths. In this activity the students had to make several different rectangles using the commands that have the same total length of sides, 20. The students also had to make a conclusion related to the sum of the lengths of two adjacent sides of the rectangle and use it to solve 3 problems given. In doing so, all the students did trial and error at the beginning. For example, students from the focus group at the first chose number 7 as the length of the first side. Using the geometrical properties of rectangle that opposite sides have to be in the same length, they added up 7 by 7 to get 14. Since the total is 20, they subtracted 14 from 20 to get 6 and divided it by to 2 to get 3 as the length of the other two sides. Hence, he got a rectangle with length 7 and width 3. Based on this rectangle, they generated the second and the third rectangle. For the second one, they distributed 1 from the left side to the bottom side and 1 from the right side to the above one to get a rectangle with length 8 and width 2. The same procedure was carried out by them to get the third rectangle whose width and length are 9 and 1 respectively. All the three rectangles are in the Figure 5.6 as follows.

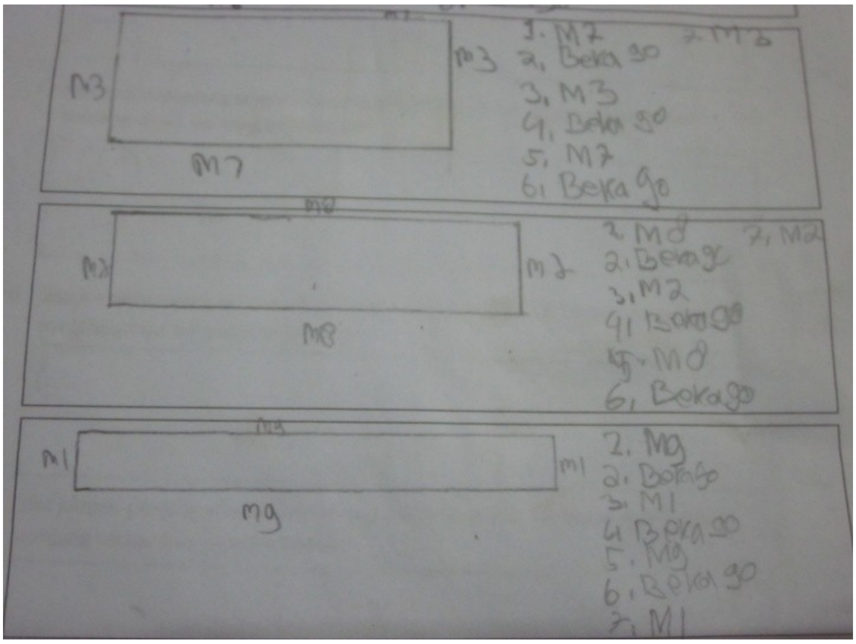


Figure 5.6. Three rectangles made by the focus group students

These students had correctly applied the geometric properties of rectangle, the equality of the opposite sides, to solve the problem but they had not attained another geometrical property namely the sum of two adjacent sides is equal to one half of the total length of the all sides. Trying to answer question number 5, asking them to count the total length of the adjacent sides of every rectangle they made, and with the help of the teacher, the focus group students and some other students could reach to the conclusion that the sum of two adjacent sides is equal to one half of the total length of the all sides and used this conclusion to easily answer the last three questions. The other students who could not yet attain this property were also able to answer those three questions by drawing or using the equal opposite sides property.

Post-test

The result of the post-test in the teaching experiment gave insight that some students already grasped the geometric properties of rectangle, square, and triangle. They could determine whether one shape is a triangle, a rectangle, or a square base on its geometric properties. They could also give reasoning why a certain shape, for example, is a rectangle by mentioning its geometric properties not solely by its whole appearance anymore. The relation between rectangle and square were also established. They could also solve problems related to the shapes' geometric properties. Compared to the result of the pre-test, it could be said that they already constructed the geometric properties of rectangle, square, and triangle and already in a higher level of geometrical thinking, in this case level 1 analysis.

However, not all the results fulfill the expectation. One interesting finding comes from question number 3, the triangle categorization. Almost all the students categorized 'triangles' with non straight lines as triangle. One example of focus group student answer is given in the Figure 5.7. It is interesting since their problems that is known from the pre test is fail to identify triangle number 11 as a triangle. Now, after the post-test, they succeeded to identify triangle number 11 as a triangle but they also categorized figures number 3, 5, 7, and 14 as triangle. Asked about their reason, they said that since these figures are having 3 sides and 3 vertices, there are triangles. Suggested to consider the straightness of the sides, they argued that it is not necessary to have straight sides in triangle. It is only necessary for square and rectangle. Hence, more discussion about straightness is really needed in the future.

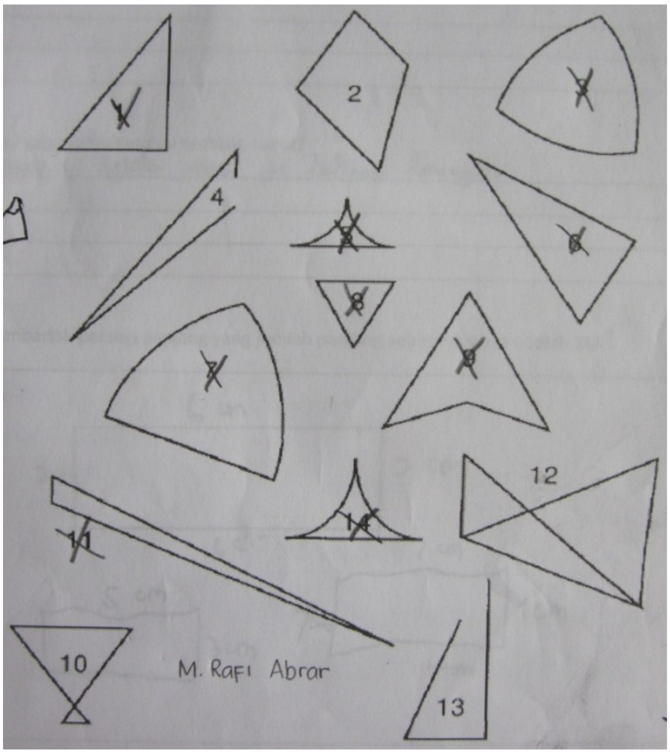


Figure 5.7. Example of focus group students answer for triangle categorization task

CHAPTER VI

CONCLUSION AND DISCUSSION

In this chapter, the research question will be answered based on the retrospective analysis. Then we reflect on some issues in this study and elaborate recommendation for further studies to improve the mathematics education in Indonesia.

6.1 Conclusion

Conclusion in this section is addressed to answer the research question leading the present study: *How can a combination of computer and non computer activities support third graders in constructing geometric properties of square, rectangle, and triangle?*

The parade activity supports the students in that it provides opportunities to the students to reflect shapes they already make using the commands needed to construct those shapes. They could perceive the reasons why the opposite sides of rectangle and square have to be in the same length. The use of the commands to form the shapes step by step gives the idea to the students that a shape consists of parts. It is a new insight for them since before doing the activity they only see a shape as a whole. The paths made by the soldier using the forward command which is straight lines lead to the discussion that it is necessary to have straight sides in order to form shapes such as rectangle or square. In addition, using what they get from the parade activity, they can argue that one shape they make will not be a rectangle or a square if the opposite sides are not in the same length.

The paper parade activity, which is the replication of the first one but on a paper grid, helps the students to structure rectangle and squares they construct in drawings. The grids on the paper also encourage them to grasp the length structure of the shapes in that two opposite sides of rectangle or square are always in the same length. Small squares pattern made by the grids also helps the students to perceive the regularities in the rectangle and square corner even they have not known the notion of right angle or the angle itself. The structure of the shapes provided by the grids paper that is already grasped by the students could lead them to one conclusion that shape is invariant under rotation.

The first computer manipulative, the turtle path, provides opportunities to the students to construct more shapes than they could do using the commands in the parade activity since they could input numbers indicating the degree of the turn the turtle do. Two directions of turn, left and right, are also provided. The activity following the use of this manipulative provides the students opportunities to perceive that there is a magnitude determining the relation between two segments having one end in common and define angle for themselves. With this manipulative, the students could construct an equilateral triangle. The given commands that are recorded on the screen could be the source of reflection for the students to construct the geometric properties of an equilateral triangle.

The second computer manipulative, the triangle drag, really supports the students in constructing the properties of triangles in that it gives chances to the students to construct more variants of triangles and reflect whether they could identify those triangles as triangle or not. In the textbooks or in the classroom, the students are only used to deal with exemplars of triangles such as right triangles and triangles with horizontal bases. They need to see more variants of triangle in order to grasp the idea about what triangle is.

The fourth activity that asks the students to establish the relation between rectangle and square encourages the students to construct the geometric properties of those shapes. The properties of the rectangle and square already established and recorded in a table will be used to check whether a square is a rectangle and vice versa. Doing this activity, the students will draw a conclusion based on the two shapes geometrical properties that a square is also a rectangle. Succeeding in attaining the geometrical properties of rectangle and square and also the conclusion that a square is a rectangle could develop students' geometrical thinking into a higher level than visualization.

The fifth activity, missing measures, supports the students by providing opportunity to apply all the geometrical properties of rectangle, square, and triangle they have already constructed in solving the geometrical problems, so they could evaluate their knowledge and feel it meaningful. The use of the turtle path manipulative in this activity also provides them chances to make conjectures and reflection of the problems they have to solve.

The last activity, paths with the same lengths, gives the students problems that encourage students to construct another geometric property of rectangle, namely the sum of two adjacent sides is one half of the total length of all the sides. This property could lead the students to the reinvention of the formula to find perimeter of one rectangle.

6.2 Reflection on the important issues

RME

In designing the sequence of instructional activities and the computer manipulative, we consult to some ideas of realistic mathematics education. The sequence of activities designed in this study is only a part of longer series of learning trajectories in constructing geometrical properties of shapes. The descriptions of the principals of RME applying to mathematics learning as a process will take months or years and perhaps could not be applicable to a short series of activities as in the present study that is limited by the time.

In the present study, the use of the parade activity as the contextual situation in which the students using the two commands, forward and right face, to move and go back to the initial position could support them in grasping some issues such as shape consists of parts and the needs of straight lines in rectangle and square. This activity which relates their prototypes of shapes with commands given and drawn figures could develop their geometrical reasoning level. After doing this activity, the students who did not know why the opposite sides of a rectangle have to be in the same length could argue that the shape will not be a rectangle if the opposite sides are in different length. They base their reasoning on what they experience in the parade activity.

Classroom discussion

In the 3B class, with 37 students, working in groups or classroom discussion is not a common thing. From the observation, it was acquired that there were only small numbers of the students participating discussions started by the teacher. Other students were busy with their own activities. Related to working on groups, the teacher almost never organized the students in that way. She believes that working on group will give more opportunity to the students to talk one another, that is not related to mathematics, and make a noise. It could be said that it was new for the students and also for the teacher to have this kind of socio mathematics norm.

During the six lessons, the students worked in small groups of students, but sometimes also in pairs. They first had discussions in their group to solve such a problem in each activity and then in the class discussion they shared their idea to the rest of the class. The students were free to express their ideas and use their own representations and strategies. The students compared their solutions and reflected to others. They also questioned one another. The students with low levels of understanding could be supported by involving them in the discussion.

The role of the teacher

In the present study, the teacher who was involved, Bu Melda is a brand new teacher. This year is her first teaching year, but she was ever involved in one study conducted by another master student. According to the interview, she supported the idea that class discussion could support students in learning mathematics, but she said it is not easy to have discussion with third grade students. Related to the use of computer, she has experience on that since ever conducting a study about computer manipulative. During the present study, she showed that she would like to promote the development of socio-mathematical norms in her class.

6.3 Discussion

One issue that is worth to be discussed in this section is the phenomenon that almost all the students in the second cycle, the teaching experiment phase, identify 'triangles' with non straight lines as triangles in the post-test. This becomes most interesting since at the pre-test they did not choose these 'triangles' as triangle. At that time they also did not choose the 'thin' triangle with non horizontal base, but it is already predicted beforehand since they laid their categorization on the shapes whole appearances based on the prototypes of the shapes they have on their mind. In the post-test, they did choose the 'thin' triangle with reasoning that figure has three sides and three vertices, what they grasped from the third lesson, working with triangles activity. Using the same reasoning almost all the students also identified all the 'triangles' with non straight sides as triangle.

Suggested to consider the straightness of the sides they learnt in the first activity, they argued that the straight side is only necessary for rectangle and square, not for triangle. The focus group students, as the representation of all the students in the class, explained it for sure with no doubt. Hence, more discussion about straightness is really needed in the future. A context that is solely supporting the students in constructing geometric properties of triangle is probably needed since in the context of the parade activity in the present study concerns only to rectangle and square.

6.4 Further studies

In the present study, the main context, the parade activity could only deal with rectangle and square. The results of the second cycle post-test show that the students identify ‘triangle’ with non straight sides as triangle could be caused by the absence of a contextual situation that the students could use as the reference in identifying triangle, like what the parade context does for rectangle and square. A study focusing on finding the appropriate context to support the students in perceiving the properties of triangle is needed as the follow up of the present study. A question about how the students perceive the concept of straightness and use it to identify shapes could also be raised in the further study.

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Appendix 1

Teacher Guide

A. General Introduction about the role of the teacher

In conducting a study especially when the teacher herself is the one conducting the lessons, the role of the teacher will be the main part of discussion. The role that the teacher holds here cannot be separated from the teaching qualities that she has. There are some aspects of teaching qualities that should be acquired by teacher to facilitate children in mathematizing during learning process (Fosnot et al., 2006).

The first aspect is the pedagogy. Based on pedagogy aspect, teacher has to facilitate students' construction of knowledge. Teacher gives time for students to think after asking a question and encourages students to share their opinion to others. In the classroom discussion, teacher facilitates students in explaining their mathematical ideas and strategies in solving problems.

Another aspect that has to be acquired is about the use of context. Teacher is able to adapt and modify the context as she works with different students who have different levels of understanding. The instructional activities containing some contexts that have been designed beforehand will serve as guideline in the teaching learning process so that she can adjust or modify them when the unexpected event happens.

The mathematical content is also the main part of the teaching qualities. It consists of mathematical ideas, strategies, model, and etc that have to be attained by the students. It can be done by promoting the discussion amongst students. For example in constructing geometrical properties of rectangles, squares, and triangles, teacher promotes discussion whether rotation changes the shape or not.

B. Practical information for each lesson

1. **Lesson 1 – Parade activity (2x35 minutes)**
   a. Goal
   Knowledge: Students define straightness; students recognize that shape consists of parts
   Skill: Students use paths to form geometric shapes
   Attitude: Students show consideration on commands that can or cannot form a rectangle or a square
   b. Materials
      Poster papers, markers, glue
   c. Classroom organization
      Students will work on groups of 4. The first part of lesson will be done outside the classroom. Presentation and discussion will be done in the classroom.
   d. Learning activities
      - Introduction (5 minutes)
        The teacher introduces the parade context. She will ask some students that already know it to give example how the parade works. She asks the students to form groups of 4 and then explains what students should do. Working in a group of four, one student becomes a commander, another one becomes a soldier, and the other two become the secretary and the drawer. The secretary and the drawer will record

paths formed by the soldier along with commands needed in a poster. The commander can give order to the soldier to either move certain number of steps forward or right face (turn right 90^o). The commander should give a sequence of commands such that the soldier at the end of the orders comes back to the initial point.

- Parade Activity (20 minutes)
  The activity is done outside the classroom, so the students will have more space. While the students are doing the activity, the teacher pays attention to all students, making sure that they do it in the right way and record all the commands and paths made on paper. The teacher should allow the students making mistakes, in this case the soldier students cannot go back to the initial position. She will suggest the students to keep recording the paths and commands so it can be the source of discussion later on. If there are groups that make only a line, which is possible by doing two 'right face' commands in a row, the teacher can ask them to make more interesting paths. She also has to make sure that there are groups make rectangle or square. All posters will be hanged in the class.
- Making posters (10 minutes)
  The teacher gives the students time to choose their best three paths and draw them along with the related commands in a big poster. The teacher should make sure that there are rectangle, square and other figures in students posters and all related commands written down besides the figures.
- Presentation & discussion (30 minutes)
  The teacher should give chance to one group that want to present their poster. The presenting group should name every figure they make and explains the relation between the commands and the figures. The intention here is the students recognize that each figure they make consists of parts. The teacher can ask them about the shape of the figure whether it is one of rectangle or square or others. Students should explain for example why they categorize one figure they make as a rectangle. Making relation to the commands needed, the teacher will encourage the students to discuss some properties such as the lines have to be straight lines and opposite sides have to be in the same length. Examples of questions that can be asked by the teacher are "what if the line is not straight as you make using the path of your parade? Is it still a rectangle?", "what make you sure you have already formed a rectangle?", and "why some commands success and others do not success to bring back the soldier to the initial position?"
- Reflection (5 minutes)
  At the end of session, together with the students, the teacher will make conclusion about have already been learned today. The main issues are about the straightness, shape consists of parts, and conditions needed to make the commands success.

e. HLT

a. Starting point:

Knowledge: Students already know the shapes of rectangle and square, but they base their categorizations on the shapes' overall appearances.

Skills: Students can draw rectangle and square.

Attitudes: Students do not show consideration on shapes' geometric properties

b. Goal:

Knowledge: Students define straightness; students recognize that shape consists of parts

Skill: Students use paths to form geometric shapes

Attitude: Students show consideration on commands that can or cannot form a rectangle or a square

e. Activity:

Working in a group of four, one student becomes a commander and another becomes a soldier. Two other students will record paths formed by the soldier in a poster. The commander can give order to the soldier to either move certain number of steps forward or turn right. The commander should give a sequence of commands such that the soldier at the end of the orders comes back to the initial point. Meanwhile, the other two will record paths formed by the soldier movement in drawings on a poster, which will be the source of class discussion, and also make relation between the paths and the commands. They will do it for several different geometric figures. By doing this activity, students will make their own paths and identify what kind of two dimensional figures can be formed by their paths. They will be suggested to pay attention to the straightness and the idea that shape consist of parts. Students walk, describe, discuss and create paths. In addition they will also identify why some sequences of commands works and why others not. This activity introduces a formal geometric symbolization. Students will experience learning mathematics outside the classroom, what they never do before. This outdoor activity is continued by whole class discussion.

f. Conjectures about students' actions and thinking

Students will try to make their first path by moving certain number of steps forward then they will try to do the turn right command. For the second path, there are two possibilities. The first one, they will move forward with the same number of steps as they do for the first path. Students doing this are either aware that they could make a square or just doing it without purpose, just trying. Students who are aware that they could make a square or a rectangle know that the right turn they do will make a kind of right angle, angles they see on rectangle or square, even they do not know yet the term "right angle". The second possibility, they will move different number of steps forward, with no purpose, just trying. The next command will be turn right again before moving certain number of steps forward. In choosing the number of steps, they will consider the length of the first path or just try until realizing that in order to be able to back to the initial point effectively, the third path have to be in same length with the first one. They will do the last turn right before finally choosing the number of steps needed to get back to the initial point. They will record paths they make and do it for several times to make different figures. There will be a possibility that students make figures other than rectangle or square. The figure will be composition of several rectangles and squares. They are also required to record all geometric figures and commands related that they make, even they not succeed to back to the initial point in that figure. It will be a source of discussion, why certain commands work and why others not.

In the class discussion, the teacher could ask students about what kind of figure they make and how they could make that. When making connection between commands they use and the shape formed, they will recognize that shape consist of parts that relates one another. The students are also asked to identify why certain commands work and why others not which implicitly suggests the geometric properties of square and rectangle. The teacher also could ask the students whether some groups make rectangle or square. Since students will record their paths as a poster on a plain big paper, there is possibility that figures they make do not exactly look like square or rectangle, but they can argue that a close shape can be formed only if it satisfies a certain condition, for example opposite sides must be in the same length. Based on the paths that students form, the discussion will also address the needs of straightness of the shape.

2. Lesson 2 – Paper parade (2x35 minutes)

a. Goal

Knowledge: (i) Students recognize length structure of rectangle and square; (ii) Students analyze that shape is invariant under rotation.

Skill: Students use paths to form square and rectangle

Attitude: Students express point of view on some geometric properties of rectangle and square

b. Materials

Big grid paper, grid papers

c. Classroom organization

Students will work in pairs, followed by classroom discussion

d. Learning activities

- Introduction (5 minutes)

  The teacher asks some students to tell what they learned in the previous class. Then, she explains what they will do today, similar with the previous one but on a grid paper.

- Paper Parade (20 minutes)

  The teacher allows students in pair to discuss and make some drawings, using commands they have from the previous activity, on grid papers. They have to make at least 10 different figures and write down all commands needed. The teacher will suggest the students to really pay attention to what shape they make and how they can end up with that shape.

- Presentation & discussion (25 minutes)

  Teacher will ask two pairs of students to draw their figures on the big grid paper attached on the black board. She has to make sure that there are square, rectangle, and other figures drawn on the big grid paper. The chosen pair will explain what kind of shapes they make, how they make it, and why they are sure that shape they make is a rectangle or square. The teacher will emphasize the length structure of the shapes that students can recognize by the structure of the grid paper. She can ask question such as what will happen if the opposite sides are not in the same length and why it is necessary to have opposite sides in the same length.

- Another discussion (15 minutes)
  The teacher who has a big grid paper attached on the blackboard will deliberately remove the tape on one upper corner of the grid paper such that the grid paper not in horizontal position anymore. Students will have discussion whether rectangle or square already drawn on it is still rectangle or square or not. The teacher can ask question such as "well, after what accidentally happened to our big paper, are the shapes still the same?". By pointing one rectangle drawn, which is now in a rotated position, she can ask whether it is still a rectangle or not. The students should use their argument on their decision which leads to the use of geometric properties of the shapes. After some discussion related to the form and length structure of the shapes, they will come to conclusion that rotation does not change shape.
- Reflection (5 minutes)
  At the end of session, together with the students, the teacher will make conclusion about have already been learned today. The main issues are about the length structure of rectangle and square and shape that is invariant under rotation.

e. HLT

e. Starting point:
   Knowledge: Students define straightness; students recognize that shape consists of parts
   Skill: Students use paths to form geometric shapes
   Attitude: Students show consideration on commands that can or cannot form a rectangle or a square
f. Goal:
   Knowledge: (i) Students recognize length structure of rectangle and square; (ii) Students analyze that shape is invariant under rotation.
   Skill: Students use paths to form square and rectangle
   Attitude: Students express point of view on some geometric properties of rectangle and square
g. Activity:
   Using commands from the previous activity: moving forward certain number of steps and right face, students have to draw different figures on the grid paper. They are also required to write down all the commands needed for each figures. Students give imaginary commands to his/her pencil and make several different two dimensional figures on the grid paper. What they can make using forward and right face command are rectangles and squares or figures consisting of those two shapes. Students will identify geometric properties of rectangle and square, related to their sides' lengths. By obligating the students mention commands needed to make the paths, it keeps them to make connections between mathematical symbols and geometric representation. In the classroom discussion, the teacher who has a big grid paper attached on the blackboard will deliberately remove the tape on one

upper corner of the grid paper such that the grid paper not in horizontal position anymore. Students will have discussion whether rectangle or square already drawn on it is still rectangle or square or not.

h. Conjectures about students' actions and thinking:
Since this activity is almost similar with the first one, most students will not find any difficulty. The use of grid paper will structure their figures, so they will see clearly rectangle or square. The teacher could start the discussion by asking how to make a rectangle or square. Students will start to saying the procedures related to the number of steps and right face. The teacher will ask the students to see whether there are regularities in rectangle or square by asking them to see the relation between the shape's parts. Students will make conclusion related to the lengths of shape's sides. They will be able to make conclusion that rectangle and square have four sides, opposite sides always in the same length, and special for square, all sides are in the same length, but they do not say anything about the angles' measures, what they will deal in the next activity. Related to the one corner hanging grid paper in front of the class, some students would say rectangle or square already drawn there is not rectangle or square anymore since the figure has no horizontal base anymore. The teacher will invite groups of students that still say the drawn figures are rectangle or square to explain their opinion. At the end, all students will come to conclusion that rotation does not change shape and kind of shape is determined by its geometric properties

3. Lesson 3 – Working with triangles (2x35 minutes)

a. Goal

Knowledge: (i) Students define angle, (ii) Students recognize angle measures, (iii) Students define right angle, (iv) Students construct geometric properties of triangle

Skill: Students use paths to form equilateral triangle

Attitude: Students show consideration on variant of triangles

b. Materials
Computer manipulative software, PC

c. Classroom organization
Students will work in pairs in the computer room and have discussion there.

d. Learning activities

- Introduction (10 minutes)
The teacher asks some students to tell what they learned in the previous class. Then, she explains what they will do today. They will work with computer manipulative and have some fun with it. The teacher introduces the manipulative to the students and tells them how to use it.
- Defining angle (20 minutes)
At the beginning with the manipulative, the teacher can ask each pair to make a certain number of steps forward. Then she asks them to find the turn that is similar with their usual right face command. The will investigate and get turn right 90 as

the answer. If after making turn they move certain steps forward again, they will have two segments forming an angle. Teacher will have discussion where they ever see this kind of figures before, for example in the corner of a book or rectangle. Students could define angle as two lines that meet to make a corner. Considering the number they insert when making turn they will be aware that angle has measures. The number they insert is the measure of the exterior angle. The teacher can ask the relation between the interior and the exterior angle, which their sum equals 180. With the help of the teacher, they will define right angle which equals to their usual right face command and has 90^o as the measure. In addition, the teacher could ask whether they have tool to determine an angle is a right angle or not. Students could start mentioning the corner of their books or the configuration formed when they stretch their thumbnail and point finger.

- Forming equilateral triangle (15 minutes)
  Using the knowledge about angle, the students will be asked to form an equilateral triangle. They will investigate how to form an equilateral triangle. By reflecting to commands they use and shape they make in the discussion, students will construct geometric properties of an equilateral triangle with emphasize on all sides are in the same length and all the angles are 60^o. Teacher can ask questions such us how the sides of an equilateral triangle is connected one another and what regularities you find in this kind of shape.
- Variants of triangle (20 minutes)
  Students will work with the second manipulative. The manipulative will provide opportunity to the students to make and see variants of triangles. Teacher can start from a regular triangle such as an isosceles or an equilateral one and drag the vertices so it becomes non familiar shape of triangle. She can ask whether it is still triangle or not. There is possibility that students will say one triangle is not triangle since its base is not horizontal or it is so thin, students will try to drag the vertices of the triangle. The class discussion will be held to find what properties are needed by a shape to be triangle. The students will come to the conclusion that a shape can be identified as triangle if it has three sides and three angles
- Reflection (5 minutes)
  At the end of session, together with the students, the teacher will make conclusion about what have already been learned today. The main issues are about the angle, right angle, equilateral triangle and geometric properties of triangle.

e. HLT

a. Starting Point:

Knowledge: students analyze that shape is invariant under rotation.

Skill: Students use paths to form square and rectangle

Attitude: Students express point of view on some geometric properties of rectangle and square

b. Goal:

Knowledge: (i) Students define angle, (ii) Students recognize angle measures, (iii) Students define right angle, (iv) Students construct geometric properties of triangle

Skill: Students use paths to form equilateral triangle

Attitude: Students show consideration on variant of triangles

c. Activity:

Students are introduced to a logo like computer manipulative. They will be asked to do almost the same things as the do in the two previous activities. They can give command to the turtle to form a geometric figure. The difference is with this computer manipulative students have to give magnitude (multiplications of 30) to the turn they try to make. They also have two different turn, left and right. Asked to form an equilateral triangle using the manipulative, students will define angle, recognize angle measures, and define right angle. Given the second computer manipulative, in which the students can click three different points to make a triangle and later on can freely drag all three vertices, students will record and identify not only exemplar (typical form) but also variants (other examples) of triangles. Class discussion will also be held to address all involved concepts.

d. Conjectures about students' actions and thinking:

Students will try to do some commands. They will try move forward certain steps first which they already familiar with. They will find difficulty to use the turn with magnitude and direction command. They do not have idea what the magnitude represents. Teacher could give help by asking what magnitude needed to perform their usual right face command. They will try and find that turn right 90 is exactly like their usual right face. By asking to consider other magnitude such as 0, 30, 60, and 120, together with the teacher students will discuss and define angle, in this case also interior and exterior angle. Students could define angle as two lines that meet to make a corner. They will get the idea that angel measures is a magnitude that determines the relation between two lines that meet and make a corner. With the help of the teacher, they will define right angle which equals to their usual right face command. In addition, the teacher could ask whether they have tool to determine an angle is a right angle or not. Students could start mentioning the corner of their books or the configuration formed when they stretch their thumbnail and point finger. Based on this knowledge, student will find that turn right 120, which is the measure of the exterior angle, is needed to form an equilateral triangle and all its (interior) angles are 60o. By reflecting to commands they use and shape they make, students will construct geometric properties of an equilateral triangle. The use of the second manipulative will provide opportunity to the students to make and see variants of triangles. There is possibility that students will say one triangle is not triangle since its base is not horizontal or it is so thin, the class discussion will be held to find what properties are needed by a shape to be triangle. The students will come to the conclusion that a shape can be identified as triangle if it has three sides and three angles

4. **Lesson 4 – The relation between rectangle and square (2x35 minutes)**

a. Goal

Knowledge: Students construct geometric properties of rectangle and square

Skill: Students determine the relation between rectangle and square

Attitude: Students express point of view on the relation between rectangle and square

b. Materials

Computer manipulative, PC

c. Classroom organization

Students will work in pair in the computer room and also will have discussion there.

d. Learning activities

- Introduction (5 minutes)

  The teacher asks some students to tell what they learned in the previous class. Then, she explains what they will do today. They will investigate the relation between rectangle and square.
- Forming rectangle & square (20 minutes)

  The teacher asks students to form several rectangles and squares using the first computer manipulative. While they are doing so, the teacher asks them to pay attention to the commands needed. Students are also asked to make a list of regularities that they can see amongst the rectangles and also amongst the squares. If there was no students writing about the right angle, the teacher could suggest them to consider what they learn in the previous meeting and ask about the relation between the adjacent sides of rectangle and square. The lists students make will become the source of the class discussion.
- Presentation and discussion (40 minutes)

  The teacher asks one pair to write their list of geometric properties of rectangle and another pair to do so for those of square. The discussion will be started by asking other students whether they have other properties that should be included in the lists or whether they agree with already written list. After getting the fix lists, the discussion moves toward the relation between rectangle and square. Teacher asks which properties of rectangles are held by square and vice versa. Teacher will lead the discussion until students come to the first conclusion that all geometric properties of rectangle are held by square but not the other way around. In doing so, teacher could ask questions such as "what properties of rectangle that square also has?" or "what regularities of square that you don't find in rectangle?". Next, the teacher will lead the discussion to determine the relation between square and rectangle. Teacher will ask a question, "Do you agree if one says that a square is a rectangle?" Start from this question the discussion will be continued by the students before finally they come to the conclusion that square is a special kind of rectangle.
- Reflection (5 minutes)

  At the end of session, together with the students, the teacher will make conclusion about have already been learned today. The main issues are about the geometric properties of rectangle and square and the relation between those shapes.

e. HLT

a. Starting Point:

Knowledge: (i) Students define angle, (ii) Students recognize angle measures, (iii) Students define right angle, (iv) Students recognize length structure of rectangle and square;

Skill: Students use paths to form rectangle and square

Attitude: Students express point of view on some geometric properties of rectangle and square

b. Goal

Knowledge: Students construct geometric properties of rectangle and square,

Skill: Students determine the relation between rectangle and square

Attitude: Students express point of view on the relation between rectangle and square

c. Activity:

Students will still work with the first computer manipulative. They now will make rectangle and square. They will be asked to identify geometric properties of those shapes including their angles. They will also analyze the relationship between rectangle and square.

d. Conjectures about students' actions and thinking:

Students who already have idea about angle will construct the geometric properties of rectangle and square. In this activity, they will also pay attention to angles on those shapes, not only to the sides' lengths as they do on the paper parade. By asking question what properties of rectangle held by square and vice versa after students make list all geometric properties of those shapes, the teacher can guide the students to come to the conclusion that square is a rectangle. It is not easy to come to the conclusion that a square is a rectangle since the language (Bahasa Indonesia) suggests the misconception. Rectangle is translated as persegi panjang which literally means long square. Students will hardly accept that a square is a rectangle. By making list and discussing the geometric properties of rectangle and square, it will be seen that all geometric properties of rectangles are also held by square. Then the students are expected to draw a conclusion that square is a special kind of rectangle. Square is one class of rectangle.

5. **Lesson 5 – Missing measures (2x35 minutes)**

a. Goal

Knowledge: Students apply geometric properties of rectangle, square, and triangle to find missing measures of two dimensional figures

Skill: Students determine the missing measures of two dimensional figures

Attitude: Students show consideration on the reasoning in finding the missing measures

b. Materials

Computer manipulative, worksheet

c. Classroom organization

Students will work in pair in the computer room and also will have discussion there.

d. Learning activities

- Introduction (5 minutes)

  The teacher asks some students to tell what they learned in the previous class, about geometric properties of rectangle, square, and triangle. Then, she explains what they will do today. They will try to write down commands needed to complete figures drawn using paths made by the turtle.

- Finding missing measures (30 minutes)

The teacher will give the students worksheets containing 8 problems related to the missing measures. In pair, students will try to solve the problem. The teacher will ask the students to solve the problem on the paper before justifying their solution using the first computer manipulative. If some pairs found difficulty, the teacher would suggest them to consider what they have learned before about the geometric properties of rectangle, square, and triangle. When the students already make their solution on the paper, the teacher will allow them to check their solution using the computer manipulative. Teacher will also encourage them to pay attention to the commands needed and to find another possible solution.

- Presentation & discussion (30 minutes)

  After all pairs ready, class discussion is held. Some pairs will be chosen to present their solution for one problem they choose. Other students have chance to ask question and express their opinion about solution given by the presenting pair. The teacher will lead the discussion to make sure that the students use the geometric properties of the shapes as the bases of their reasoning. The teacher could ask questions such as "why are you sure that your answer is correct?" and "if there is no computer manipulative, how will you make your friends sure that answer you give is the right one?". In addition, the teacher could also ask for another possible solution.
- Reflection (5 minutes)

  At the end of session, together with the students, the teacher will make conclusion about have already been learned today. The main issues are about the geometric properties of rectangle, square, and triangle and how they are used to solve the given problems.

e. HLT

a. Starting point:

Knowledge: Students construct geometric properties of rectangle, square, and triangle

Skill: Students determine the relation between rectangle and square

Attitude: Students express point of view on the relation between rectangle and square

b. Goal:

Knowledge: Students apply geometric properties of rectangle, square, and triangle to find missing measures of two dimensional figures

Skill: Students determine the missing measures of two dimensional figures

Attitude: Students show consideration on the reasoning in finding the missing measures

c. Activity:

Students should find the missing measures including lengths and angles to complete partially drawn paths. Students have to figure out what commands should they give so the turtle can complete the side of the figure. They have to analyze geometric properties of the figures and use mental computation or estimation to find out the missing measures. Class discussion will be held after all students finishing their tasks.

d. Conjectures about students' actions and thinking:

Some students will do trial and error at the beginning, but after asked to consider the geometric properties held by the shapes, they will find it easier to find the missing measures. In the class discussion, the teacher could ask "why are you sure that your answer is correct?" to the students in order to emphasize the use geometric properties of the shapes in reasoning.

6. **Lesson 6 – Paths with same length (2x35 minutes)**

a. Goal

Knowledge: Students apply geometric properties of rectangle to form several rectangles with the same perimeters

Skill: Students determine the dimensions of rectangles that have the same perimeter

Attitude: (i) Students show consideration on the reasoning in finding the procedures, (ii) Students express point of view in finding the procedures

b. Materials

Computer manipulative, worksheet

c. Classroom organization

Students will work in pair in the computer room and also will have discussion there.

d. Learning activities

- Introduction (5 minutes)

  The teacher asks some students to tell what they learned in the previous class, about geometric properties of rectangle and square. Then, she explains what they will do today. They will try to write down as many as procedures to form rectangle with 200 steps.

- Forming rectangles with 200 steps (30 minutes)

  The teacher will give every pairs A4 papers, they should make as many as possible procedures to form rectangle with 200 steps. Besides writing down the procedures, they also have to draw the rectangle corresponded to every procedures. Every time they finish one procedure, they can check whether the procedure works or not using the first computer manipulative. While they are doing so, the teacher could come to some pairs and ask why they choose certain numbers in their procedures. For pairs that are still having problem, the teacher could also suggest to consider about geometric properties of rectangles that they have learned before.

- Presentation & discussion (30 minutes)

  After all pairs ready, class discussion is held. Some pair will be chosen to present their figures and procedure. Other students have chance to ask question and express their opinion about solution given by the presenting pair. The teacher will lead the discussion to make sure that the students use the geometric properties of the shapes as the bases of reasoning. The teacher could ask questions such as "why are you sure that your answer is correct?", "if there is no computer manipulative, how will you make your friends sure that answer you give is the right one?" and "why some procedures do not work correctly?". In addition, the teacher could also ask for another possible solution. In this occasion, the teacher will introduce the term perimeter as the total length of all sides of the shapes. Using the term 'perimeter', the students will be encouraged to construct one 'new' geometric property of rectangle, namely the sum of every adjacent sides of a rectangle equals to one half of its perimeter. In the light of it, the teacher could ask questions such as what is the sum of rectangle's adjacent side that they make and what is its relation with the perimeter. Other numbers of steps for example 100 can be used to make this property obvious.

- Reflection (5 minutes)

  At the end of session, together with the students, the teacher will make conclusion about have already been learned today. The main issues are about the geometric properties of rectangle, perimeter and one 'new' property that sum of adjacent sides is one half of the perimeter.

e. HLT

a. Starting point:

Knowledge: (i) Students apply geometric properties of rectangle and square to find missing measures of two dimensional figures, (ii) Students construct geometric properties of rectangle and square

Skill: (i) Students determine the missing measures of two dimensional figures, (ii) Students use path to form rectangle and square

Attitude: Students show consideration on the reasoning in finding the missing measures

b. Goal:

Knowledge: Students apply geometric properties of rectangle to form several rectangles with the same perimeters

Skill: Students determine the dimensions of rectangles that have the same perimeter

Attitude: (i) Students show consideration on the reasoning in finding the procedures, (ii) Students express point of view in finding the procedures

c. Activity:

In this activity, students are supposed to write down as many as possible procedures, set of commands, to draw different rectangles that have paths with same lengths in total, for example 200 steps. Students will not work with the computer at the first. They start by planning one procedure and check it using the logo turtle to get feedback whether they have done it correctly or not. Then they revise it or start planning new procedures and check again with the turtle. Similar with missing measures, students have to analyze the geometric situation involved and use mental calculation or estimation to get the procedures. Classroom discussion will be held to find the best strategy to do the task.

d. Conjectures about students' actions and thinking:

Some students will start to make drawings and do trial and error to complete this task, but after suggested to consider the geometric properties of rectangle, they will find it easier. Some students will also find another property of rectangle that could help them solving this task, namely the sum adjacent sides is always one half of the perimeter. Some students can do it mentally and easily find many procedures while other students make a procedure and try it in the computer manipulative before revising it or making another one. The discussion can be started by asking why students choose certain numbers in their procedures and why certain procedures do not work.

Appendix 2

Interview scheme for teacher

a. Background
   - Would you mind telling me your teaching experience?
   - How long have you been teaching in primary school?
   - Are enjoying teaching in primary school?
   - What is your educational background?
b. Social Norm
   - What are the students allowing to do during the lesson?
   - What are the students not allowing to do during the lesson?
c. Socio-mathematics Norm
   - How important is giving the students opportunity to ask question during the lesson?
   - What is your reaction to the students' questions?
   - What is your reaction to the student's answer if it is already the right answer? If it is the wrong one?
d. PMRI
   - Have you ever heard about PMRI?
   - What is your experience with PMRI?
   - What do you think about PMRI?
   - What is the difficulty to put PMRI approach into practice?
e. The subject
   - What do you know about geometric properties of simple shapes such as rectangle, square, and triangle?
   - Have you ever taught this subject before?
   - What your students (third graders) already know about this subject?
   - How do you teach this subject to the students?
   - What textbook do you use?
   - How do they learn this subject?
   - What are their difficulties in this subject?
f. The learning activities
   - How do you do the lesson?
   - Do you use any models?
   - Do the students have any experience working in group?
   - How do you group the students?
   - What is your experience in using computer for the lesson?
g. The students
   - How many students are in your class?
   - How are their participations in the math class?
   - How are their levels of understanding?
   - Are they motivated in learning math?

Appendix 3

Classroom observation scheme

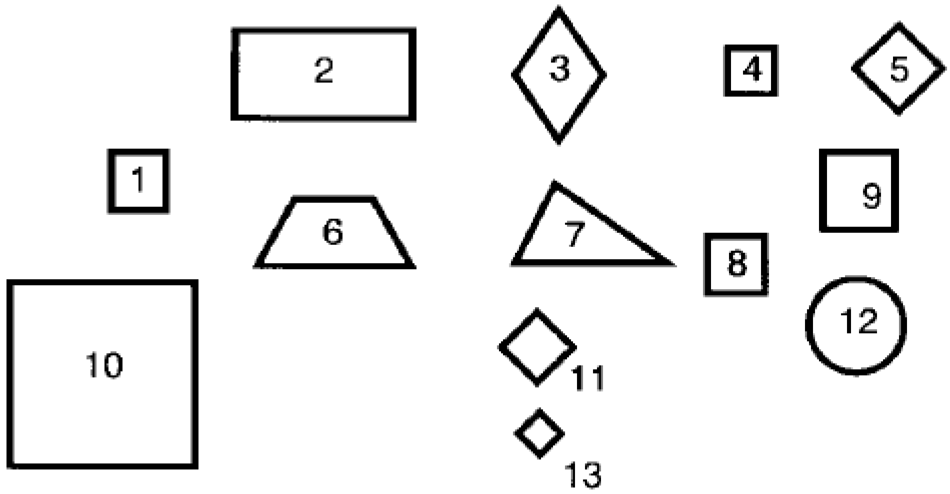
a. The number of the students on the class
b. How the teacher starts the lesson
c. How the teacher teaches the students
d. How the teacher interacts with the students
e. How the students interact with one another
f. How the students interact with the teacher
g. How the class is organized (students' sitting positions, students work individually, in pair, or in group)
h. The social norm
i. The socio-mathematics norm
j. The learning environment
k. How the teacher starts the discussion
l. How many minutes are the effective learning time
m. Students' participations in the learning activities
n. What kind of material is used by the teacher

Appendix 4

Pre-test

Nama:

1. Silanglah (x) setiap bangun yang merupakan persegi!



a. Apakah bangun nomor 4 adalah persegi? Kenapa?

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b. Apa yang kamu ketahui tentang persegi?

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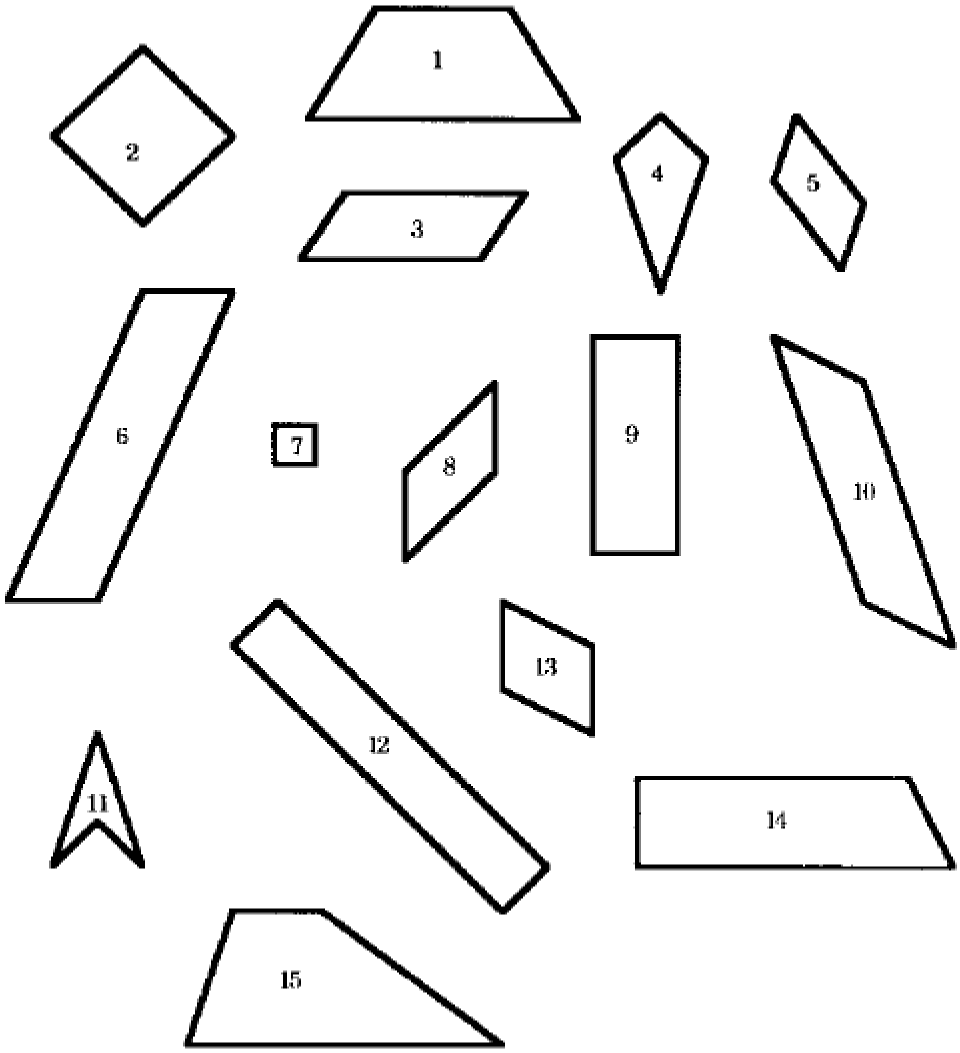
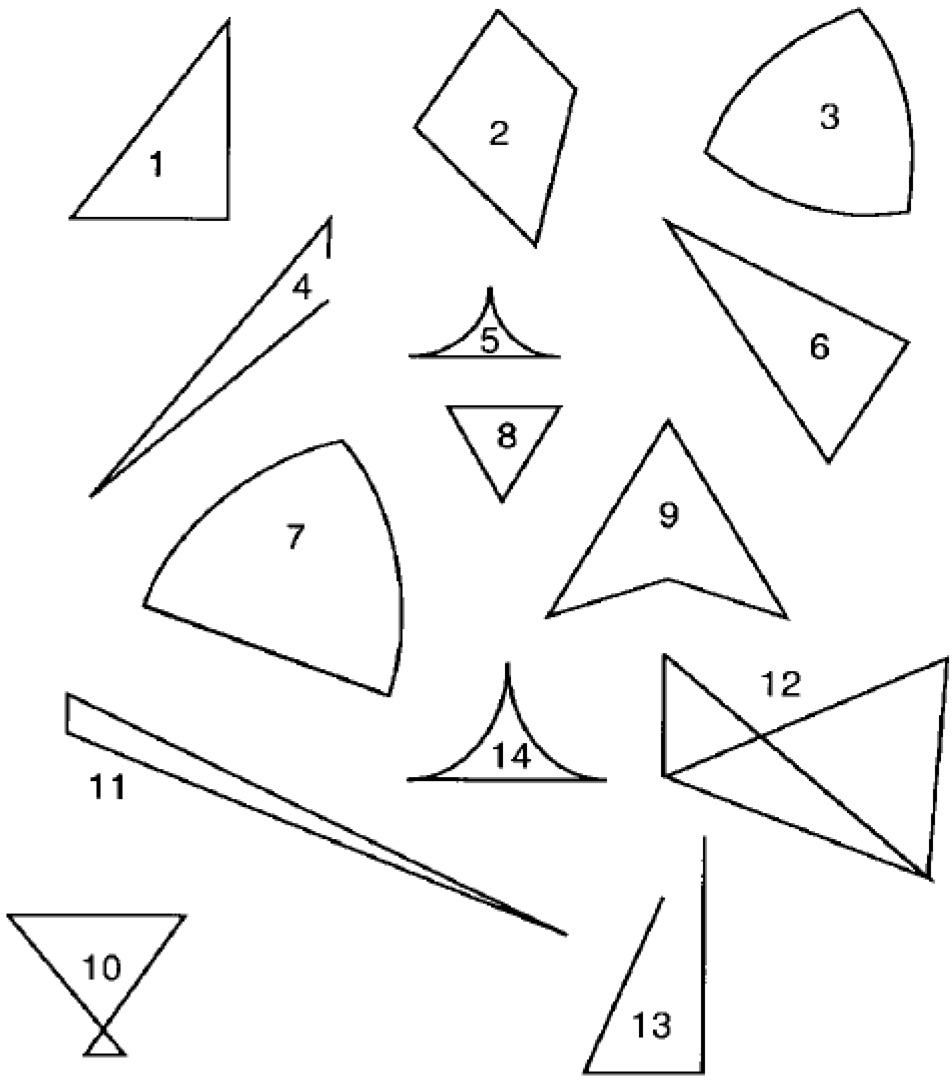
2. Silanglah (x) setiap bangun yang merupakan persegi panjang!



a. Apakah bangun nomor 9 adalah persegi panjang? Kenapa?

b. Apa yang kamu ketahui tentang persegi panjang?

3. Silanglah (x) setiap bangun yang merupakan segitiga!



a. Apakah bangun nomor 11 adalah segitiga? Kenapa?

b. Apa yang kamu ketahui tentang segitiga?

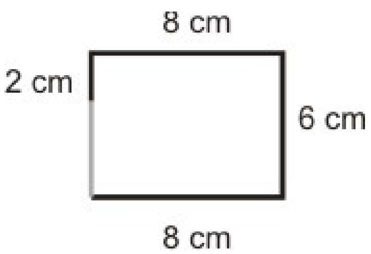
4. Apakah setiap persegi adalah persegi panjang? Jelaskan jawabanmu!

5. Apa yang kamu ketahui tentang sudut?

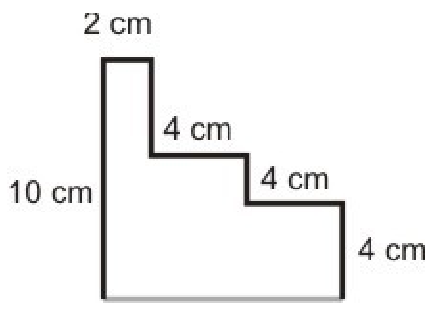
6. Gambarlah persegi panjang yang jumlah panjang seluruh sisinya adalah 12

7. Berapakah panjang garis abu-abu di setiap bangun di bawah? Berikan penjelasamu!

a.



b.

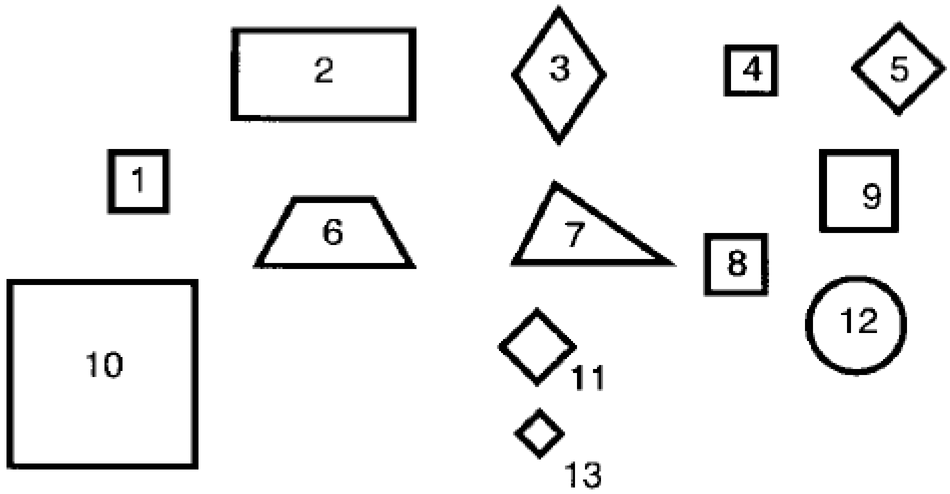


Appendix 5

Post-test

Nama:

8. Silanglah (x) setiap bangun yang merupakan persegi!



c. Apakah bangun nomor 11 adalah persegi? Kenapa?

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d. Sebutkan sifat-sifat persegi yang kamu ketahui!

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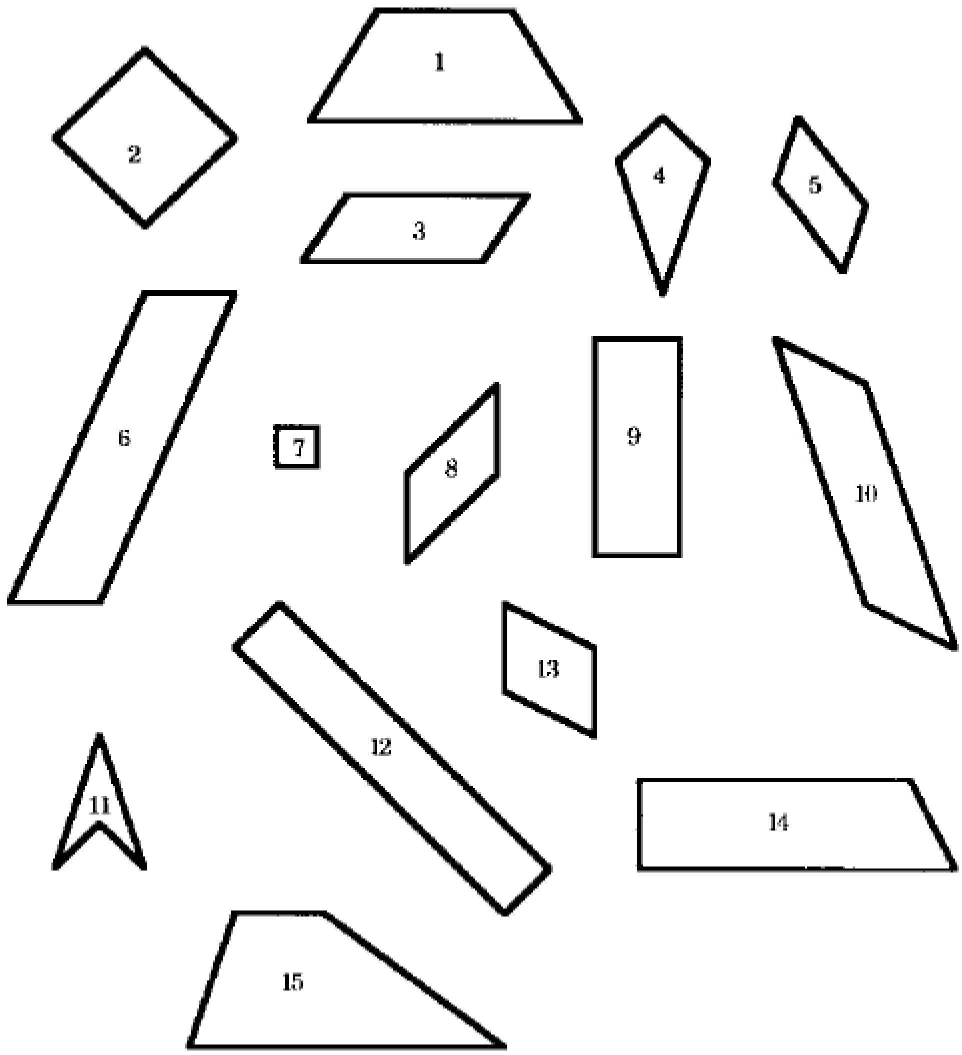
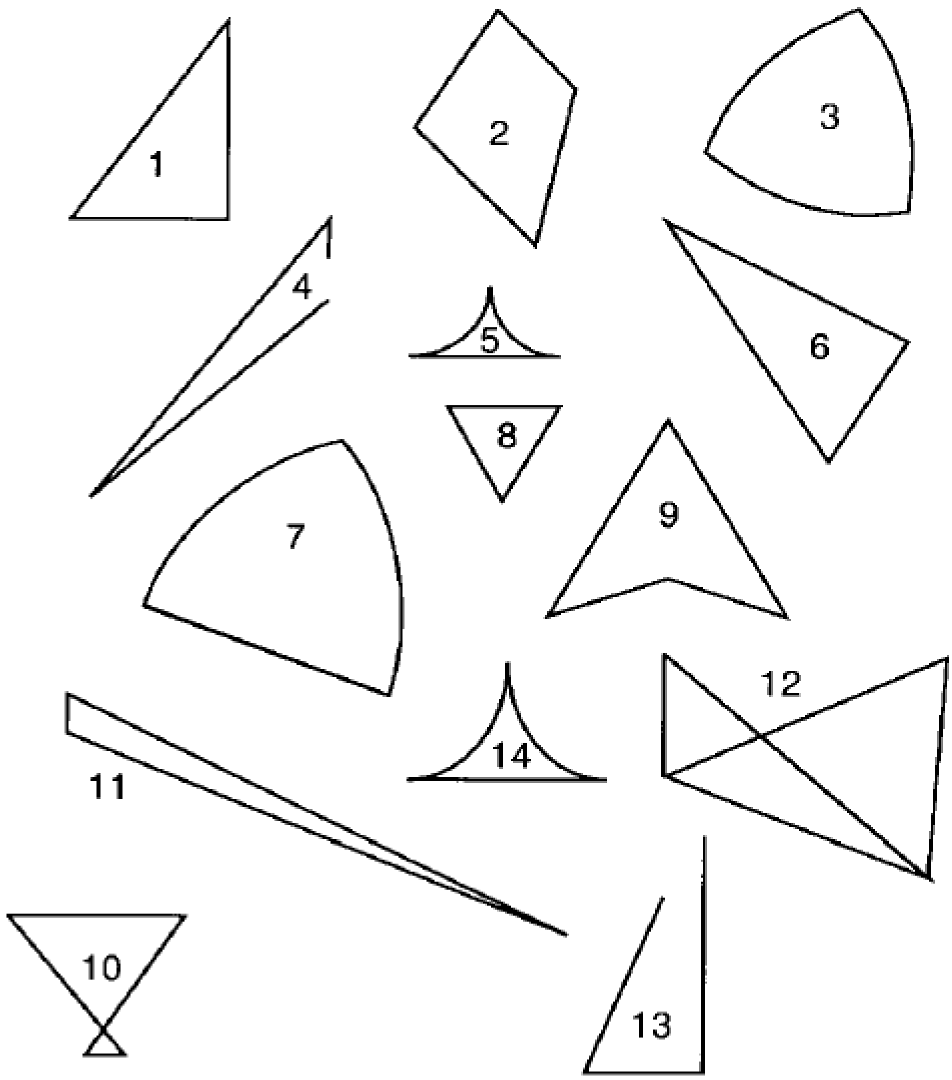
9. Silanglah (x) setiap bangun yang merupakan persegi panjang!



c. Apakah bangun nomor 6 adalah persegi panjang? Kenapa?

d. Sebutkan sifat-sifat persegi panjang yang kamu ketahui!

10. Silanglah (x) setiap bangun yang merupakan segitiga!



c. Apakah bangun nomor 7 adalah segitiga? Kenapa?

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d. Sebutkan sifat-sifat segitiga yang kamu ketahui!

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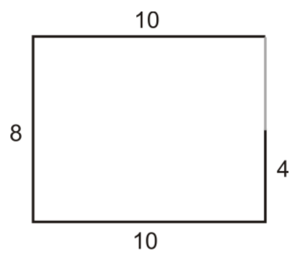
11. Apakah setiap persegi adalah persegi panjang? Jelaskan jawabanmu!

12. Apa yang kamu ketahui tentang sudut?

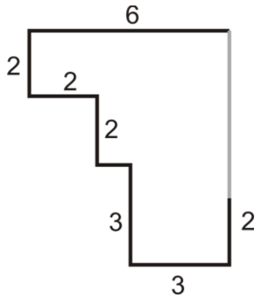
13. Gambarlah persegi panjang yang jumlah panjang seluruh sisinya adalah 16!

14. Berapakah panjang garis abu-abu di setiap bangun di bawah? Berikan penjelasamu!

a.

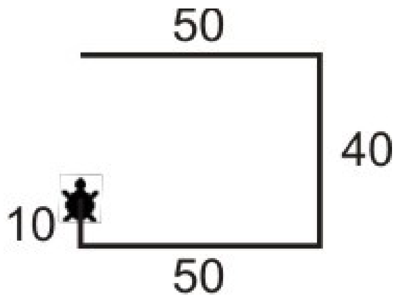


b.

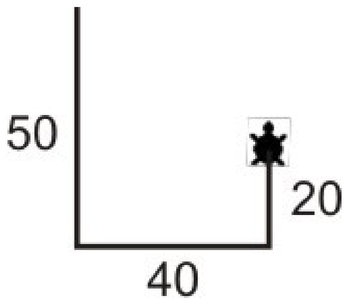


8. Tuliskan perintah untuk melengkapi gambar berikut dan berikan penjelasanmu!

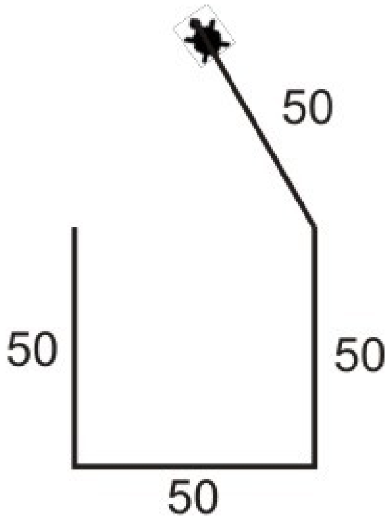
a.



b.



c.



Appendix 6

Lesson 3-1

Students' worksheets

1. Coba berikan perintah **Maju 10, Belok Kanan (BeKa) 30, Maju 10** kepada si kura-kura! Gambar apa yang terbentuk?

2. Cobalah maju dengan beberapa Belok kanan dengan besar yang berbeda, misalnya Beka 60 atau Beka 120. Kemudia maju lagi. Apa yang terjadi?

3. BeKa berapa kah yang sama seperti perintah Hadap Kanan yang diberikan kepada prajurit di pertemuan sebelumnya? Bentuk apakah yang terjadi?

4. Tulislah perintah yang harus diberikan kepada kura-kura sehingga dia bisa membentuk segitiga sama sisi.

   Tulislah di sini jika kamu punya perintah yang lain.

Lesson 3-2

Geserlah ketiga titik yang berada di layar. Kemudian gambarlah gambar yang terbentuk.

1.	2.	3.
4.	5.	6.
7.	8.	9.

Gambar nomor berapa sajakah yang merupakan segitiga? Berikan juga alasanmu!

Gambar nomor berapa sajakah yang bukan merupakan segitiga? Berikan juga alasanmu!

Lesson 4

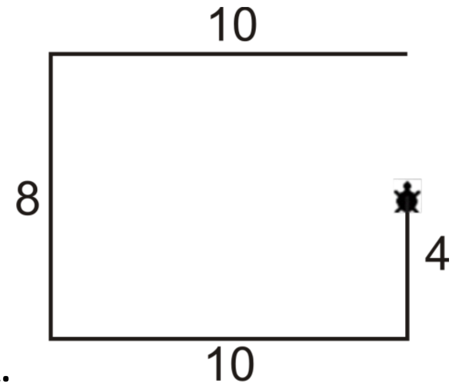
SIFAT-SIFAT PERSEGI	SIFAT-SIFAT PERSEGI PANJANG

Gambarlah persegi panjang, periksalah apakah dia juga merupakan persegi?

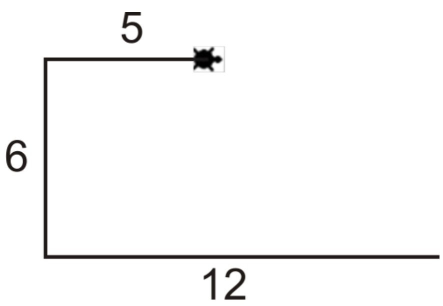
Gambar lah persegi di bawah, periksalaha apakah dia juga merupakan persegi panjang?

Apa yang dapat kalian simpulkan?

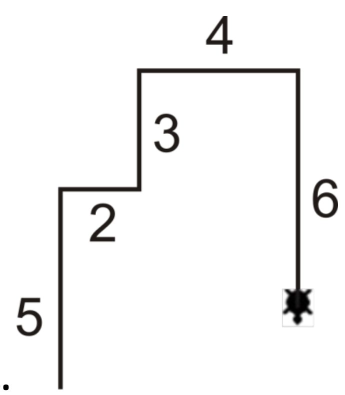
Tuliskanlah perintah yang diperlukan untuk melengkapi setiap gambar berikut!
Cobalah dulu diskusikan dengan teman sekelompokmu, sebelum mencobanya dengan komputer!



A.

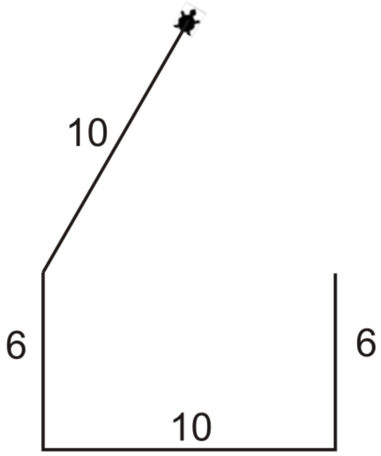


B.

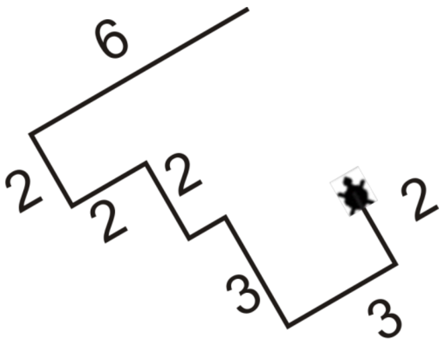


C.

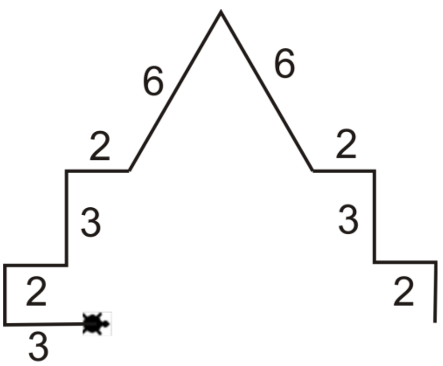
D.



E.



F.



1. Berapakah jumlah panjang seluruh sisi persegi panjang yang terbentuk dari perintah berikut:
   Maju5, BeKa90, Maju4, BeKa 90, Maju5, BeKa 90, Maju 4?

2. Tulislah 4 perintah berbeda yang dapat membentuk persegi panjang yang jumlah panjang seluruh sisinya adalah 20!

3. Sebelum memeriksanya dengan komputer, bagaimana kamu bisa yakin bahwa seluruh perintah yang kamu tuliskan itu benar?

4. Hitunglah, berapa jumlah panjang setiap sisi yang berdekatan dari kelima persegi panjang yang telah kamu buat! Apa yang dapat kamu simpulkan?

5. Jika jumlah panjang seluruh sisi persegi panjang adalah 10, berapakah jumlah panjang setiap sisi yang berdekatan?

6. Jika jumlah panjang seluruh sisi persegi panjang adalah 12, berapakah jumlah panjang setiap sisi yang berdekatan?

7. Jika jumlah panjang seluruh sisi persegi panjang adalah 16, berapakah jumlah panjang setiap sisi yang berdekatan?