DESIGN RESEARCH ON MATHEMATICS EDUCATION: DEVELOPING A MODEL AND USING CONTEXT TO SUPPORT STUDENTS' UNDERSTANDING IN THE TOPIC OF THE ADDITION INVOLVING NEGATIVE NUMBERS FOR THE FOURTH GRADE STUDENTS IN PRIMARY SCHOOL

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of
Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

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FACULTY OF TEACHER TRAINING AND EDUCATION

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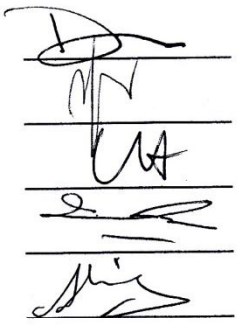

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ABSTRACT

Several researchers found that students have difficulty in understanding the concepts of either negative or signed numbers and in performing procedures with them when adding and subtracting the numbers. Moreover, for the fourth-grade Indonesian students who study the topic of negative numbers in general, they only study the rules and the procedures in studying the addition involving negative numbers. To overcome those problematic issues, several researchers suggest that the learning related to the negative numbers begins with problem posed in context and the number sense ability of the students must be improved. Therefore, a set of lessons is designed of which an appropriate context and a powerful model are elaborated aimed to make students learn the addition involving negative numbers meaningfully and improve their number sense so they can understand the concept of addition involving negative numbers. Design research was choosen as the approach or our reserach to contribute to empirically grounded instruction theory for the additon involving negative numbers. The equilibrium context which is used in this study is conjectured to lead the students to represent the context using signed number and use the model of number line or empty number line to manipulate the numbers. In addition, realistic mathematics education is chosen as the approach in the teaching-learning process in the classroom. Our finding suggests that in order to be able to understand the concept of addition involving negative numbers, the students need to understand the concept of negative numbers and to have model as a tool for counting. That can be helped by the equilibrium context and number line as the representation of signed numbers. Moreover, we recommend to the teachers in Indonesia to use RME or PMRI approach in their teaching. In our RME classroom, the use of context has stimulated the students to think informally and to represent a state using numbers and with the help of number line model, the students can solve a mathematical problem in formal level.

Keyword**:** *Number Sense, Number Line, Equilibrium Context, Empty Number Line, Realistic Mathematics Education.*

ABSTRAK

Beberapa peneliti terdahulu menemukan bahwa siswa mengalami kesulitan dalam memahami konsep bilangan negatif dan menerapkan prosedur ketika menambahkan atau mengurangkan bilangan tersebut. Lebih lanjut, pada umumnya, siswa kelas 4 sekolah dasar di Indonesia yang mempelajari topik bilangan negatif, hanya dibekali oleh aturan-aturan atau prosedur dalam mempelajari penjumlahan yang melibatkan bilangan negatif. Untuk mengatasi permasalahan tersebut, beberapa peneliti menyatakan bahwa pelajaran yang berkaitan dengan bilangan negatif sebaiknya dimulai dengan permasalahan kontekstual dan "*number sense*" siswa perlu ditingkatkan. Oleh karena itu, serangkaian pelajaran dan aktivitas yang memuat konteks dan penggunaan model untuk membuat siswa mempelajari penjumlahan yang melibatkan bilangan negatif secara bermakna dan meningkatkan "*number sense*" mereka sehingga mereka dapat memahami konsep penjumlahan yang melibatkan bilangan negatif. Penelitian desain dipilih sebagai acuan dalam penelitian ini untuk memberi kontribusi terhadap teori pengajaran pada penjumlahan yang melibatkan bilangan negatif. Konteks keseimbangan yang digunakan pada penelitian ini diduga mendukung siswa untuk berfikir dan merepresentasikan konteks tersebut menggunakan bilangan bertanda dan menggunakan garis bilangan atau garis bilangan kosong untuk memanipulasi bilangan tersebut. Juga, pendidikan matematika realistik dipilih sebagai pendekatan dalam proses belajar mengajar di kelas. Hasil penemuan dari penelitian ini menyarankan bahwa untuk membantu siswa dalam memahami konsep penjumlahan yang melibatkan bilangan negatif, siswa mesti memahami konsep bilangan negatif dan menggunakan model dalam menghitung. Hal tersebut dapat didukung oleh konteks keseimbangan dan garis bilangan sebagai representasi dari bilangan negatif. Lebih lanjut, kami menyarankan guru-guru di Indonesia untuk menggunakan pendekatan RME atau PMRI dalam pengajaran mereka. Pada kelas kami, penggunaan konteks menstimulasi siswa untuk berfikir secara informal dan mewakili suatu keadaan dengan menggunakan bilangan dan dengan bantuan model garis bilangan, siswa dapat memecahkan permasalahan matematika pada tingkat formal.

***Kata Kunci*:** *Number Sense, Garis Bilangan, Konteks Kesetimbangan, Garis Bilangan Kosong, Pendidikan Matematika Realistik.*

RINGKASAN

Fajar Arwadi. Design Research on Mathematics Education: Developing a Model and Using Context to Support Students' Understanding in the Topic of the Addition Involving Negative Numbers for the Fourth Grade Students in Primary School

Dalam pembelajaran matematika topik penjumlahan yang melibatkan bilangan negatif di kelas 4 sekolah dasar di Indonesia, tahap pertama dalam pembelajaran adalah pengenalan bilangan negatif yang biasanya menggunakan beberapa konteks seperti kedalaman laut, utang, dan sebagainya. Namun, ketika memasuki topik penjumlahan bilangan yang melibatkan bilangan negatif, guru tidak lagi menggunakan konteks, namun langsung mengajarkan siswa bagaimana melakukan penjumlahan yang melibatkan bilangan negatif dengan menggunakan model garis bilangan beserta cara-cara yang bersifat prosedural dalam menggunakan garis bilangan tersebut. Hal tersebut membuat pelajaran yang dilakukan siswa menjadi tidak bermakna dikarenakan siswa bergantung pada penghafalan bagaimana menjumlahkan bilangan bulat dalam garis bilangan yang bisa mengakibatkan kesalahan dalam melakukan perhitungan. Guna menghadapi hal tersebut, Kilpatrick, Swafford, & Bradford (2001) menyatakan bahwa siswa sebaiknya dihadapkan pada soal-soal kontekstual. Sebagai konsekuensi, diperlukan suatu model untuk mencapai jawaban pada tingkat yang lebih formal. Oleh karena itu, penelitian ini mengajukan pertanyaan penelitian, "Bagaimana penggunaan suatu konteks dan suatu model mendukung pemahaman siswa dalam mempelajari topik penjumlahan yang melibatkan bilangan negatif?

Hativa & Cohen (1995) menyatakan bahwa model yang pada umumnya digunakan dalam mempelajari bilangan negatif dapat dikategorikan pada dua model: model kesetimbangan, yakni model yang melibatkan dua entitas yang saling berlawanan yang dapat direpresentasikan oleh bilangan positif dan bilangan negatif, dan model garis bilagan. Beberapa peneliti terdahulu menyatakan bahwa, model garis bilangan lebih baik dibanding

model kesetimbangan. Namun, jika melihat dari konteks yang biasanya digunakan dalam mempelajari penjumlahan yang melibatkan bilangan negatif, konteks tersebut berhubungan erat dengan model kesetimbangan. Oleh karena itu, dalam penelitian ini, peneliti menggabungkan konteks kesetimbangan sebagai permasalahan kontekstual dan model garis bilangan. Oleh karena itu, penelitian ini mengelaborasikan pertanyaan penelitian menjadi tiga sub-pertanyaan penelitian:

a. *Bagaimana penggunaan konteks kesetimbangan mendukung siswa dalam melakukan penjumlahan yang melibatkan bilangan negatif*?
b. *Bagaimana kita mendukung siswa dalam mengembangkan model garis bilangan atau garis bilangan kosong?*
c. *Bagaimana model garis bilangan dan garis bilangan kosong mendukung siswa dalam melakukan penjumlahan yang melibatkan bilangan negatif pada tingkat formal?*

Dalam merancang rangkaian kegiatan instruksional, kami menerapkan ide pendidikan matematika realistik sebagai berikut. Kegiatan matematisasi diawali dengan situasi yang dapat dibayangkan siswa. Tujuan digunakannya situasi tersebut agar siswa dapat menggunakan pengetahuan informal mereka dari situasi tersebut. Dari kegiatan tersebut, siswa diberikan kegiatan untuk mengkonstruksi garis bilangan yang dapat memuat bilangan positif dan bilangan negatif. Tujuan dari kegiatan ini adalah untuk memberikan pengetahuan kepada siswa untuk menjembatani pengetahuan informal mereka dalam menjawab masalah kontekstual ke pengetahuan matematika secara formal. Dalam proses pembelajaran, siswa diperkenankan untuk menyatakan pendapat mereka dan memberi tanggapan atas jawaban dari siswa lainnya.

Penelitian ini bertujuan untuk mengembangkan kegiatan pembelajaran untuk memberikan kontribusi pada teori yang berlandaskan kegiatan empiris pada pembelajaran topik penjumlahan yang melibatkan bilangan negatif. Secara spesifik, penelitian ini

mengembangkan kegiatan pembelajran yang mengkombinasikan konteks dan model yang dapat digunakan oleh siswa untuk memahami penjumlahan yang melibatkan bilangan negatif. karena itu, *design research* dipilih sebagai jenis penelitian yang tepat untuk mencapai tujuan tersebut. Penelitian ini terdiri dari dua siklus, di mana tiap siklusnya terdiri dari tiga fase yang merupakan komponen utama dari penelitian desain, yaitu: *preliminary design, teaching experiment* dan *retrospective analysis.* Kami menganalisis data yang dikumpulkan selama fase eksperimental dalam *retrospective analysis* yaitu, hasil lembar kerja siswa, catatan lapangan, dan rekaman video. Kami membandingkan pembelajaran siswa yang terjadi di kelas dengan *hypothetical learning trajectory* (HLT) yang kami kembangkan. Hasil dari retrospective analysis pada siklus pertama yang melibatkan kelas kecil yang terdiri dari 6 orang siswa digunakan untuk menyempurnakan HLT yang akan diimplementasikan pada siklus kedua yang melibatkan kelas besar yang terdiri dari 26 orang siswa.

Terdapat enam aktivitas yang telah kami desain dalam HLT awal kami untuk diujicobakan , yaitu: kegiatan “menghitung kelereng dan menentukan posisi suatu bilangan pada garis bilangan”, kegiatan “menentukan posisi bilangan pada garis bilangan”, kegiatan mengurangkan bilangan sehingga hasilnya berupa bilangan negatif, kegiatan menjumlahkan bilangan negatif dan positif serta menjumlahkan dua bilangan negatif, kegiatan mengidentifikasi hubungan antara operasi yang berbeda, dan kegiatan menyelesaikan soal-soal non-kontekstual. Dari hasil siklus pertama kami, ditemukan bahwa kegiatan pembelajaran awal tidak mendukung siswa dalam menggunakan garis bilangan secara benar. Selain itu siswa juga kurang dapat memahami soal yang diberikan sehingga tidak mampu bekerja pada tingkat yang lebih formal.

Oleh karena itu, setelah melakukan analisis retrospektif, dalam *teaching experiment* siklus dua, terdapat enam aktivitas. Pada aktivitas pertama hingga ketiga, yakni aktivitas

menghitung kelereng, menghubungkan kuantitas dari banyaknya kelereng ke garis bilangan dan menentukan posisi suatu bilangan ke garis bilangan. kami menemukan bahwa siswa secara mental, menghitung kelereng dari kiri ke kanan, menghitung ke kanan ketika melakukan penjumlahan dan menghitung ke kiri ketika melakukan pengurangan, dan mereka dapat menggunakan berbagai macam strategi dalam menghitung. Dari aktivitas tersebut, siswa berkesimpulan bahwa bilangan yang lebih besar menempati sisi kanan garis bilangan, sebaliknya, bilangan yang lebih kecil menempati sisi kiri garis bilangan. Selain itu, ketika kuantitas bertambah, maka mereka melakukan perhitungan ke kanan dan ketika kuantitas berkurang, maka mereka melakukan perhitungan ke kiri.

Pada aktivitas keempat, siswa diberikan permasalahan kontekstual yang berkaitan dengan topik penjumlahan dua bilangan negatif. Siswa terlebih dahulu menjawab permasalahan secara informal, lalu dengan panduan dari guru, mereka diminta untuk menunjukkan bagaimana cara mereka mendapatkah hasil tersebut secara matematis. Dari hal tersebut, siswa mengilustrasikan jawaban informal mereka dengan merepresentasikan keadaan yang ada pada konteks tersebut dengan bilangan negatif dan secara matematis dengan menggunakan garis bilangan untuk memanipulasi operasi bilangan negatif tersebut. Lalu dari kegiatan tersebut, siswa dapat menjawab permasalahan kontekstual secara formal.

Pada aktivitas kelima, siswa diberikan permasalahan kontekstual yang berkaitan dengan topik penjumlahan bilangan negatif ditambahkan bilangan positif, serta soal non-kontekstual yang berkaitan dengan topik bilangan positif ditambahkan bilangan negatif. Proses kegiatan siswa dalam menjawab permasalahan kontekstual sama dengan proses di aktivitas keempat. Namun, ada pula siswa yang dapat menjawab permasalahan kontekstual tersebut secara matematis tanpa menggunakan model sekalipun. Pada aktivitas menjawab soal non-kontekstual, siswa langsung menggunakan garis bilangan dan merunut pada

pengetahuan informal mereka untuk bekerja dengan model garis bilangan tersebut: menentukan posisi awal, banyaknya lompatan, dan posisi akhir.

Pada aktivitas keenam, siswa diberikan soal non-kontesktual yang berkaitan dengan penjumlahan bilangan bulat secara umum. Dalam menjawab soal-soal tersebut, kami tidak hanya menjumpai siswa yang menggunakan garis bilangan, tapi terdapat pula siswa yang menjawab dengan menggunakan garis bilangan kosong.

Temuan penelitian kami menyarankan bahwa agar siswa mampu melakukan perhitungan yang melibatkan bilangan negatif tanpa mengandalkan penghafalan prosedur dan menghasilkan pembelajaran yang bermakna, siswa perlu dibekali pengetahuan tentang bilangan negatif beserta fungsinya, dapat mengkonstruksi model garis bilangan dengan menghubungkannya dengan situasi kontekstual, dan menggunakan garis bilangan dalam melakukan perhitungan.

Selebihnya, kami merekomendasikan kepada guru di Indonesia untuk menggunakan pendekatan pembelajaran PMRI sebagai pendekatan pembelajaran matematika di kelas. Pada kelas kami, penggunaan konteks menstimulasi siswa untuk berfikir secara informal dan mewakili suatu keadaan dengan menggunakan bilangan dan dengan bantuan model garis bilangan, siswa dapat memecahkan permasalahan matematika pada tingkat formal.

PREFACE

First of all, I am very thankful to Allah SWT. for the power given to me to finish this thesis. I'd like also to say many thanks to the people who gave me support during my study.

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I wish this thesis, as the result of my research, can contribute to the improvement of mathematics education in Indonesia, especially in the addition involving negative numbers domain in grade 4 elementary school.

Palembang, Mei 2012

Fajar Arwadi

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CHAPTER I

INTRODUCTION

The use of negative numbers in representing the magnitude value of things is mostly applied in daily life. Negative numbers are evenly used by children to denote the quantity of something. By understanding the concept and the addition and subtraction involving negative numbers, students are assumed to easily study the broader range of other mathematics lessons. However, most of them feel difficulty to understand the concept of integer and its operation particularly the operation involving negative numbers. Research showed that students have difficulties in understanding the concepts of negative or signed numbers and in performing procedures with them. Moreover, Gleaser (1981) identified about 20 different 'obstacles' for understanding negative numbers. Four of the highlighted obstacles can be described as follows:

1. Inability to manipulate isolated negative quantities.
2. Difficulty to make sense of isolated negative quantities.
3. Difficulty to unify the number line, that is to see it as one line, one axis, instead of two semi-lines opposite one another with different symbols, or understanding positive and negative quantities as having different quality.
4. Stagnation in the phase of concrete operations and not entering the phase of formal operations, i.e. seeing numbers as representing something substantial, concrete.

In addition, Hativa & Cohen (1995) stated that one of the difficulties of students in learning negative numbers is caused by the conflict between the practical meaning of magnitude or quantities associated with numbers in the early arithmetic teaching and the concept of negative numbers.

However, Kilpatrick, Swafford, & Bradford (2001) reveal that students generally perform better on problems posed in the context (debts and assets) of a story or through movements on a number line than on the same problems presented solely as formal equations. Therefore, giving the students contextual problem is important thing to do. Consequently, by working with contextual problems, models are needed to do progressive mathematization to reach formal level. As stated in Bakker (2004), models support the development from intuitive, informal, context-bound notions towards more formal mathematical concepts. However, in the other side, as stated in Glaser (1981) that one of difficulties in learning integer is that to make sense of isolated negative quantities. Therefore, it is necessary to enhance number sense of students. Since, according to Jordan et al (2009) number sense was most strongly related to the ability to solve applied mathematics problems presented in various contexts.

In learning negative numbers, students are given context to understand the meaning of negative numbers, e.g. the temperature, the sea level, and debt-possession context. However, while learning the addition involving negative numbers, Indonesian students are accustomed to directly work with mathematical symbol, i.e. numbers and to memorize several procedures or rules without knowing the meaning of what they

have learned. It is influenced by the conventional teaching method of their teachers which emphasizes on algorithm and procedures. This situation is relevant to what Liebeck (1990) said that the teacher only choose by-product course only, the teaching that provides the addition involving negative numbers in formal level only. Moreover, it is not easy to find contextually familiar problems for Indonesian children that can elicit the sense of negative numbers that can be brought to the concept of the addition involving negative numbers. Therefore, based on those issues, it is important to create a new approach in teaching students, particularly, the choosing of context, in order to make the learning of the addition involving negative numbers meaningful for them. Consequently, it is important to give young children experience-based activities that embody some basic concepts of negative numbers and the operation of them. Experience-based activities are crucial with Freudenthal's idea that stresses mathematics as a human activity, instead of subject matter that has to be transmitted (Freudenthal, 1983).

Based on those issues, it is necessary to design instructional activities in which appropriate context and powerful models are elaborated so they can support the development of students' own constructions, the sense of negative numbers elicited from contextual problems, and the number sense of the students. It is also a challenge to improve the mathematics education in Indonesian school. Realistic Mathematics Education (RME) is one of the approaches to change the mathematics education since it supports children to construct their knowledge starting from mathematical problem and to do progressive mathematization. Concerning the development of the number

sense of students, Berch (2005) states that one of the components of number sense that is relevant concerning integers is a mental number line on which representations of numerical quantities can be manipulated. The present study is aimed to develop classroom activities based on RME, the use of context, and the development and the use of mental number line to represent integers and the operation of them in learning the addition involving negative numbers. It was conjectured and expected that students' understanding on the addition involving negative numbers can be built upon the representation of integers in mental number line. I pose a research question: "*How can a context and a model support students' understanding in learning the addition involving negative numbers*?"

CHAPTER II

THEORETICAL FRAMEWORK

This chapter concerns the theoretical framework related to several key concepts of this research. Some literatures were studied to find out several aspects concerning the number sense of negative numbers, the development of students in learning the addition involving negative numbers, what kind of model which are commonly used in learning addition and subtraction involving negative numbers, and what understanding notion refers to. Furthermore, this theoretical framework is about Realistic Mathematics Education which becomes a basis to design a sequence of instructional activities of the addition involving negative numbers of which contextual situation can be shifted to more formal mathematics, the socio-norms and socio mathematical norms and the Indonesian curriculum of the addition involving negative numbers.

2.1. The Number Sense Concerning Integers

Negative numbers are numbers that are below zero. In addition to the definition of negative numbers, according to Rivard (1697-1778), negatives are certain magnitudes opposite to those which are regarded as positive (quoted in Schubring, 2005, p. 84). As stated in the introduction part of this study, number sense is really important to understand the concept of the addition involving negative numbers. Specifically, the students need to have the sense of negative

numbers in learning the addition involving negative numbers. Broadly speaking, number sense refers to awareness and understanding of the meaning and magnitude of number, the relationships among numbers, and the relative effect of number operations, including the use of mental computation and estimation. According to Bobis (1996) number sense refers to a well organized conceptual framework of number information that enables a person to understand numbers and number relationships and to solve mathematical problems that are not bound by traditional algorithms. Therefore, the sense of negative numbers can be defined as the understanding of the meaning of negative numbers, the relationship between one negative number and both other negative numbers and positive numbers, and the ability to operate the numbers using addition without relying on algorithm procedure. There are some findings that concern the sense of negative numbers. Berch (2005) formulated five components standing out as especially relevant concerning integers: elementary abilities or intuitions about numbers; ability to make numerical magnitude comparisons, ability to recognize benchmark numbers and number patterns, knowledge of the effects of the operation on numbers, and a mental number line on which representations of numerical quantities can be manipulated.

Moreover, some insights concerning the sense of negative numbers are also studied by several researchers. Stacey et al (2001b) state that the natural numbers are the primary elements from which concepts of other numbers, including negative numbers, are constructed. Therefore, it needs the presence of natural

numbers when students learn the addition involving negative numbers. Another insight that elicits the sense of negative numbers is that negative numbers are additive inverses and mirror the positive numbers with 0 as the reflection point. In addition, Gallardo & Hernandez (2005, 2006) formulate some conceptions of zero by children; one of them is total zero. Total zero is zero which is made up of opposite numbers i.e. positive numbers and negative numbers. Therefore, it can be suggested that to understand the addition involving negative numbers, it is necessary to also understand 0 as a benchmark number in which it becomes the mirror between positive numbers and negative numbers. Moreover, to make the sense of negative numbers, it is necessary to distinguish the quantities and the value of numbers that represent those quantities and also seeing number as relation as stated in Vergnaud (1982) that the quantity of something is not adequately represented by natural numbers but it can be represented as directed numbers. Directed numbers which are meant by Vergnaud here are the numbers that can be either positive or negative. Therefore, someone may put negative numbers for the value of certain quantities. Furthermore, as Berch (2005) stated, concerning the sense of negative numbers and the addition involving them, it is important to consider the presence of number line as the representation of positive numbers and negative numbers.

In learning the addition involving negative numbers, it is good to start with concrete level as Streefland (1993) suggests that the teaching includes learning strand with both concrete magnitudes or quantities and by-products. Furthermore,

to continue the phase of working with concrete level, several researchers recommend the use of models. Models are attributed as the role of bridging the gap between informal understanding connected to the 'real' and imagined reality on the one side, and the understanding of formal systems on the other side" (van den Heuvel-Panhuizen, 2003, p 13).

2.2. Number Line & Equilibrium Context

As stated in Hativa & Cohen (1995), the models which are commonly used in learning negative numbers can be categorized in two main models: equilibrium model and number-line models. In the equilibrium model, the numbers are elements of two opposite kinds such as black and white marbles, checks and bills, negative and positive charges, etc. In the number line model, numbers are represented either by positions on the number line or by displacements on this line.

Some studies emphasize the use of one model is better than the other model such as in Human and Murray (1987) suggesting that for the teaching signed numbers to elementary school children; the use of models based on embodiment of negative numbers in practical situations is not very beneficial. Thus it seems plausible to use a number line model rather than an equilibrium model. However, considering one of the causes of the occurrence of negative numbers in the history that is it started from two opposite things that is closely related to the equilibrium

model, so in this study, it uses the context containing two opposite things to elicit the sense of negative numbers for children which we call *equilibrium context*.

To summarize: The components of number sense related to the negative numbers can be covered by number line and contexts that contains two opposite things.

2.3. Empty Number Line

The finding of Fischer (2003) that people generally associate large magnitudes with right side of space and small magnitudes with left side of space has been taken as evidence for an existing mental number line where numbers are ordered from left to right, and increasing to the right. By understanding that negative numbers are less than zero and positive numbers, student can determine the position of them in the left of zero. According to Kilhamn (2011), it is perhaps surprising that a number line is not so easily understood. The reason for this could be that the number line is not treated as an explicit metaphor for numbers. It is a model that students learn to recognise, read and talk about, but they might not see it as a metaphor for numbers so that they may speak of numbers in terms of the number line. Therefore in this study, it designs a lesson in which the students can recognise that a number line is a representation of concrete objects by developing a mental number line derived from contextual situation. The generation of mental number line is empty number line. According to Treffers (1991) the empty number line consists of a horizontal line, on which one can mark numbers and also number operations. The empty number line is a power

tool that promotes powerful mathematical thinking. It helps children to show and explain their invented strategies, builds flexibility with numbers and scaffolds the mental representation of number and number operations to support mental arithmetic strategies (Fosnot, 2007, Beishuizen, 1993 & Gravemeijer, 1991). An empty number line can be used to describe the solution of two digit addition problems by making the numbers involved and drawing the jumps that correspond with the partial calculation (Gravemeijer, 1999). The use of empty number line also can create some strategies in counting numbers. Wright (2006) formulates some stages for early arithmetical learning, one of them is counting using counting on strategy to solve addition or may use counting back strategy. Also using non-counting-by-one strategies, such as doubles, add through ten, compensation, etc. In the present study, a sequence of instructional activities of the addition and of negative numbers is developed covering some crucial things as suggested by the aforementioned researchers. Based on those issues, in learning the addition involving negative numbers, the students will learn: (1) counting marbles using different kind of strategies, (2) making number line of integer with the help of marble sequence, (3) making mental number line, (4) adding two positive numbers and adding two negative numbers using the context of having and lack of marbles, 5) adding negative numbers by positive numbers, 6) adding positive numbers by negative numbers, and 7) solving several addition problems involving tens-numbers.

2.4. Understanding

Understanding is a crucial thing that need to be owned by students in learning. By having a good understanding, students can be considered successfull in learning something. According to Hiebert and Carpenter (1992), "…the mathematical idea is understood if its mental representation is part of a network of representations." (Hiebert & Carpenter, 1992 p. 67). Acoording to Kilhamn (2009a), an additional condition is of course that this network of representations will help to produce mathematically correct answers to mathematical problems. A very simplified operational definition of understanding negative numbers is to say that it is both having the ability to get correct answers when doing different kinds of calculations with negative numbers and being certain that the answers are correct. In addition, according to Kilhamn (2011), understanding in mathematics is a result of the process of making connections between ideas, facts, representations and procedures that are defined as mathematical. To assure that the students really understand, besides having correct answer for a problem given, in this study, the teacher needs to ask her students to explain their argumentation or reason why they have such an answer by explaining the ideas, concepts, and the procedures they have used. Specifically, the students need to be confident about the model representing a concept and how the use of the model can bring the students to formal mathematical answer.

2.5. Realistic Mathematics Education

The key idea of RME is that children should be given the opportunity to reinvent Mathematics under the guidance of an adult (teacher) (Fauzan, 2002). Meanwhile, Gravemeijer (1994) states that the main idea of Realistic Mathematics Education is students are given an opportunity to reinvent idea and concept of mathematics with a guide from adult or teacher through the exploration of some situations and real world problems. Therefore, this study develops design in which the students are able to construct knowledge of which they understand the addition involving negative numbers concept. According to Gravemeijer (1994), the term 'realistic' stresses that problem situations should be 'experientially real' for students. This does not necessarily mean that the problem situations are always encountered in daily life. Students can experience an abstract mathematical problem as real when the mathematics of that problem is meaningful to them. The instructional activities in this design are not absolutely encountered in daily life; however, they give meaningful problem and are imaginable for students.

(Zulkardi, 2002) states that RME has five tenets that are briefly explained below.

1. *The use of contexts in phenomenological exploration*

   In RME, the starting point of mathematics instruction should be experientially real to the student, allowing them to become immediately engaged in the contextual situation. The contextual situation in the instructional activity of this study is related to the opposite quantity which is the number of excess of marbles and the number of lack of marbles.

2. *The use of models or bridging by vertical instruments*

   The term *model* refers to situational models and mathematical models that are developed by the pupils themselves. First, the model is a *model of* a situation that is familiar to the pupils. By a process of generalizing and formalizing, the model eventually becomes an entity on its own. It then becomes possible to use this entity as a *model for* mathematical reasoning. The model which is primarily aimed in this study is number line.

3. *The use of pupils own creations and contributions*

   In each activity, the students are left to share and to discuss their ideas and strategies they use in solving problems. In the problem of "Having and lack of marbles" of which there is having certain number of marbles and another is lack of marbles, , the students can construct the value of the quantity into numbers and use some strategies to solve the problem.

4. *The interactive character of the teaching process or interactivity*

   Interaction among pupils and between pupils and teachers are essential part in RME instructional processes. Explicit negotiation, intervention, discussion, cooperation and evaluation are essential elements in a constructive learning process in which the students' informal methods are used as a vehicle to attain the formal ones. In this interactive instruction, pupils are engaged in explaining, justifying, agreeing and disagreeing, questioning alternatives and reflecting. In each instructional activity, the students present and communicate their ideas, strategies, and answers to the other students and teacher.

5. *The intertwining of various learning strands or units*

In RME, the integration of mathematical strands or units is essential. It is often called the holistic approach, which incorporates applications, and implies that learning strands should not be dealt with as separate and distinct entities.

2.6.Emergent Model

The implementation of the second tenet of RME produces the sequence of models which are used. Gravemeijer (1994) described how models-of a certain situation can become models-for more formal reasoning. The implementation of the four levels of emergent modeling in the present study as follows:

a. Situational level

The level of the situation; where domain-specific, situational knowledge and strategies are used within the context of the situation (Gravemeijer, 1994). To elicit the situation level in this study, the instructional activity uses a context in which someone is lack of certain number marbles and another has certain number of marbles. The students can represent those two states with different numbers: negative numbers and positive numbers.

b. Referential level

A referential level; where models and strategies refer to the situation which is sketched in the problem (Gravemeijer, 1994). The possible model which can come up in this level is the model in which students make visualization of what they know about the problems. In visualizing marbles, the students draw circles or small balls in which they resemble the form of marbles. However,

by considering that the students are not the first grade students who start to learn numbers, this kind of visualization is not likely to occur in the learning process.

c. *General level*

A general level; where a mathematical focus on strategies dominates the references to the context (Gravemeijer, 1994). In general model, the models-for emerge in which the students use either number line or empty number line as the representation of integers which can be used in every situation.

d. *Formal level*

The level of formal arithmetic; where one works with conventional procedures and notation (Gravemeijer, 1994). In this level, the students are able to work with numbers, i.e. adding negative numbers using numbers formally.

2.7.Social Norms and Social Mathematical Norms

In the process of learning, the students and the teacher have their own role. During the process of learning in this study, the teacher initiates and develops social norms that sustain classroom culture characterized by explanation and justification of solution, and argumentation: attempting to make sense of explanation given by others, indicating agreement and disagreement, and questioning alternatives in solutions in which a conflict in interpretation or solution has become apparent (Gravemeijer & Cobb, 2006). So, in this study we emphasize the importance of students' discussion with the other students and also

the teacher. Specifically, we use the term socio-mathematical norms described as normative understanding of what counts as mathematically different, mathematically sophisticated, and an acceptable mathematical explanation and justification. By this, students can contribute and share their mathematical ideas or thinking, whether they are the kind of different or alternative solutions. In addition, the acceptable or fix solution is not only determined by the teacher but also by the students.

2.8.The addition involving negative numbers in the Indonesian Curriculum

The topic of the addition involving negative numbers is taught in the fourth grade of elementary school. This topic is brought after the students already have learned whole numbers up to 1000, addition and subtraction of natural numbers, and also negative numbers in the previous topic. The Indonesian curriculum formulates some competencies that have to be had by the students as described in the table 2.1.:

#Table 2.1. Standar Competency and Basic Competences of Addition of Negative Numbers#

Standard Competence	Basic Competence
Adding Integers	• Adding two positive integers • Adding two negative integers • Adding positve integers and negative integers

As shown in the table, the basic competences of the curriculum relate to some indicators in this study. In the present study, some activities are created in order to let the students have more insight about integers, construct their understanding, and create some strategies.

2.9. Sub Research Question

Based on the introduction in the previous chapter and the theoretical framework, I pose sub-research questions:

a. *How can the use of a equilibrium context support students in doing the addition involving negative numbers*?
b. *How can we support students in developing number line or empty number line model*?
c. *How does either number line or empty number line support students in doing the addition involving negative numbers in formal level*?

CHAPTER III

METHODOLOGY

This chapter concerns the methodology of this study consisting of the explanation of the methodology of this study, data collection, data analysis, and the reliability and the validity of the data.

3.1. Methodology of the study

The main aim of this study is to contribute to an empirically grounded instruction theory for the addition involving negative numbers. In this study, we are interested in how we can support students' understanding in addition involving negative numbers. Therefore, a sequence of activities is developed as means to improve the quality of students' learning in that topic. The topic is brought to the students in grade 4 in elementary school. By that purpose, we choose design research as an approach of this study to answer the research question and achieve the goal of this research. Gravemeijer (2006) stated there are three phases of design research: the preparation for the experiment, the classroom experiment, and the restropective analyses.

3.1.1 Preparation for the Experiment

In the preparation phase, the researcher formulates a hypotetical learning trajectory (HLT) which consists of a mathematical goal, teaching and learning activities, and conjecture of student's thinking (Bakker, 2004). In this phase, the

HLT guides the design of instructional materials that have to be developed or adapted. This HLT can be elaborated and refined while conducting the experiment. The formulation of the conjecture of student's thinking can also be adjusted by conducting a pre-test and pilot experiment which is held before the teaching and learning activities phase, e.g. interviewing the teacher.

3.1.2. Experiment in the Classroom

In this phase, the researcher does an experiment in the classroom and tests the conjecture of student's thinking which is previously designed in preparation phase. The aim of this phase is to improve the conjecture learning trajectory and to develop an understanding how it works. In this phase, the HLT functions as a guideline for the teacher and researcher what to focus on in teaching, interviewing, and observing. Besides that, many data are collected like classroom observations, interviews, and student's work.

3.1.3. Retrospective Analysis

In this phase, the researcher does a retrospective analyses in which he analyzes all data, gets evaluation for hypotetical learning trajectory, and produces a new local instructional theory.

3.2. Data Collection

The data of this study will be obtained by conducting a sequence of activities in three phases. They are a preparation phase, a preliminary classroom experiment or the first cycle, and a classroom experiment or the second cycle. The data in this

research are gathered by doing an interview, observations, and collecting written documentation. The interview and observations are recorded by the camera and video. Documents which are collected are student's worksheets, materials, etc.

3.2.1. Preparation Phase

There are three main activities and data which will be collected in this phase:

Classroom Observation

To prepare classroom observation, the researcher firstly needs to make the draft of classroom observation in the preparation phase. A classroom observation is conducted to know the social norms and the socio-mathematical norms which are commonly occurred in the classroom, the management of classroom organization by the teacher, the way of teachers in teaching, ect. The data from this activity can be collected by using video camera, taking pictures during the learning process, and using written notes.

Interview with teachers

The interview with the teacher is useful to know the background, the didactical knowledge, the conceptual understanding of the teacher, and the way she teaches negative number addition and subtraction to the students so far. The data from this interview can be obtained by interviewing the teacher and using recording tool for transcription.

Pre-test

The pre-test is held to know the starting point of knowledge owned by the students. The students here are taken from two different classes. One class consists of six children and the other class consists of whole students in one class.

3.2.2. Preliminary Experiment

This phase consists of several activities such as, testing the HLT in which the instructional activities are brought for a small group consisting of six students, and conducting a post-test to know the progression of the students in learning. So the kinds of data collected after conducting this phase are written work of the students while learning in the classroom, what students write in the whiteboard, and the result of students' post-test. The data can be obtained by making recording using video camera, taking some pictures, and making field note.

3.2.3. Classroom Experiment

In this phase, the teacher conducts classroom experiment in a whole classroom and tests the conjecture of student's thinking which is previously designed and refined in the preparation phase and in the preliminary experiment. It should be noted that the students that participate in this experiment are different to those of who participate in preliminary experiment. The aim of this phase is to improve the conjecture learning trajectory and to develop an understanding how it works. In this phase, the

HLT functions as a guideline for the teacher and researcher what to focus on in teaching, interviewing, and observing. Besides that, many data are collected like classroom observations, mini interviews with the students, and student's work. In observing students while learning in the classroom, the observer focuses to one group of students consisting of two students. The choosing of one focus group aims to study the group in depth. The data can be obtained by making recording using video camera, taking some pictures, making field note, ect. In addition, the teacher also acts as observer to observe several groups.

3.2.4 Post Test

After the clasroom experiment has been conducted, the researcher conducts a post-test for students who are involved in the teaching experiment. This test is aimed to know the development of students' knowledge and specifically as a means to make assesment. The kind of data in this phase is written work of the students. The problems or the contents of the post-test are different to those of the pre-test since the students haven't studied yet about several competencies that are need to solve the problems in the post-test. However, some of them contain the same competency to those of the pre-test of which the problems in the pre-test are more informal than those of in the post-test. So, we can compare how the students solve the problems in the the two kinds of the test concerning to their strategies, models, and ideas. Specifically, the comparison is aimed to know whether the model which is learned in the

classroom experiment is useful for the students in answering the problems. For the detail of the data draft, see appendix A.

3.3. Validity and Reliability

According to Bakker (2004), internal validity refers to the quality of the data collection and the soundness of reasoning that has led to the conclusion. The internal validity of this study is gained by collecting the different types of data (data triangulation) such as video recording, audio recording, photographs, field notes, and written work of the students. This study will be conducted in a real classroom setting; therefore it also can guarantee the ecological validity.

Internal reliability refers to the reliability within a research project (Bakker, 2004). The internal reliability of this present study is improved by data registration itself. The data are collected using video recordings, audio recordings, and field note.

3.4. Data Analysis

3.4.1. Pre-test

The result of pre-test is analyzed by looking how the students answer the problems in the pre-test. Whether they are able to solve the problems, they use models and strategies, they use various ways to solve the problems are taken into the consideration of the analysis. The result of this analysis is used to refine the HLT which has been previously formulated.

3.4.2. Preliminary teaching experiment

The analysis of data obtained in the preliminary teaching experiment is done by checking and analyzing the student's work and observing the video recording and reviewing some notes taken from the observation. The main focus of this observation is to compare the hypotheses of students' learning and thinking to the actual students' learning process. The analysis gives data which can be used to refine the HLT which has been previously formulated.

3.4.3. Teaching experiment

The analysis of data obtained in the teaching experiment is quite similar to that of in the preliminary teaching experiment. However, in the analysis of student's work, we only focus on several students including the focus group who have the learning process and development representing the whole class. The data are used to make the conclusion concerning the HLT and to answer the research question.

3.4.4. Post test

The analysis of the post test is done by checking the students' work of the students. Specifically, the focus is to see the change of how students answer the problems given compared to their work in the pre-test. The other focus also is to see how they work and think in solving the problems. This analysis gives data to answer the research question.

3.5. Validity & Reliability of Data Analysis

To improve the internal validity of this study, during the retrospective analysis, the conjectures that are generated in each activity will be tested and other data materials such as video registration, field notes, and written work of the students will be analyzed. Having these data, data triangulation is conserved so that the quality of the conclusions can be controlled.

To improve the internal reliability, the important segments of the teaching experiment are discussed by the researcher, the supervisors and colleagues. This cross interpretation (inter-subjectivity) will minimize the subjectivity of the researcher's point of view. External reliability is obtained if the reader can follow the track of the learning process in this study and to reconstruct their study (trackability). In order to do so, two dynamic cameras are used to record every important moment in the teaching learning process. Besides that, the field notes during the implementation of classroom experiment are also made to describe in detail the crucial event in the classroom activities.

3.6. Research Subject and the Timeline of the Research

Students and a teacher of grade 4 in an Indonesian elementary school in Palembang-Indonesia, that is SD 119 Palembang, were involved in this research. The students were about 9 to 10 years old. SD 119 Palembang has been involved in the *Pendidikan Matematika Realistik Indonesia* or Indonesian Realistic Mathematics Education project since 2010.

The experiment of this research was divided into two parts, namely preliminary experiment and teaching experiment based on improved HLT. In the first part, there were 6 students involved. We tested some of the activities in our initial HLT to these students. We wanted to investigate the students' thinking of the tasks and problems in the HLT and tested our conjectures about it.

In the second part, we first improved the initial HLT and then test the improved HLT to the whole class. Here, we had the "real" teaching experiment. Those students who were involved in the second phase were not the students in the first phase.

As we mentioned that in the second phase, students worked in groups to discuss their ideas in solving tasks and problems in the activities. From 26 students, there were 12 groups of 2-3 students.

Based on our explanation above about the planning of our research, we can summarize the timeline of this research on the table 3.1. as follows:

#Table 3.1. Timeline of The Research#

DESCRIPTIONS	DATE
Preliminary Design	
Studying literature and designing initial HLT	August 2011 – 13 February 2012
Discussion with teacher	14 February 2012

Preliminary Experiment	
Classroom Observation	15 February 2012
Pre-Test	18 February 2012
Lesson 1	21 January 2012
Lesson 2	23 January 2012
Lesson 3	25 February 2012
Lesson 4	28 February 2012
Lesson 5	1 March 2012
Lesson 6	4 March 2012
Post-test	6 March 2012
Analyzing the Preliminary Experiment and Improving the HLT	
Discussion with Teacher	15 March2012
Preparation for Teaching Experiment	15 – 20 March 2012
Teaching Experiment	
Lesson 1	24 March 2012
Lesson 2	26 March 2012
Lesson 3	27 March 2012
Lesson 4	28 March 2012
Lesson 5	29 March 2012
Lesson 6	30 March - 31 March 2012
Final Assessment	6 April 2012

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Hypothetical Learning Trajectory (HLT) is used as a term to describe key aspect of planning mathematics lesson (Simon, 1995) as cited in Simon and Tzur (2004). It includes the mathematical goals of lesson for the students, the learning activities, and the hypothetical learning trajectory concerning the process of students' thinking and learning.

Students' current knowledge is the starting point of this study. The students in grade four have already studied several topics related to integer addition and subtraction namely positive numbers up to 1000, addition, subtraction, etc. Before they studied about integer addition and subtraction, they have already been introduced by their teacher about negative numbers. By that, the students already know that negative numbers are less than 0 and positive numbers.

In this HLT, there are several learning goals expected to be reached by the students during the series of lessons in three weeks period. To reach the formulated goals, we designed a sequence of instructional learning for integer addition and subtraction which consists of five lessons, which are elaborated as follows:

4.1.Lesson 1: Making Number Line from Marbles activity

a. Activity 1. : Counting Marbles

The starting point and the goal of this activity are described in the table 4.1.:

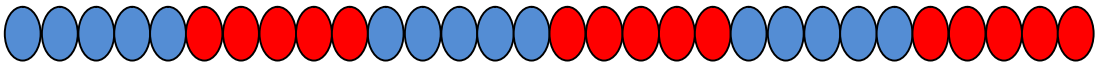
#Table 4.1. The starting point and the goal of the activity 1 in Lesson 1#

Indicator	**Starting Point**	**Goal**
Knowledge	• Students know some ideas about numbers • Students are familiar with grouping	• Students know about counting forward and backward
Skill	• Students have already known how to count something by ones • Students have already known how to count on	• Students can use model of marbles in counting numbers and use different strategies. • Students can calculate the marbles and use some strategies. • Students can calculate the marble either forward or backward.
Attitude	• Students have been accustomed in the situation when they have different answers to the answers of their friends	• Students express their point of view toward their friends who solve counting marbles problem using more elegant way i.e. jumping by tens, moving to friendly number, etc.

Activity description:

Teacher will show a picture of 60 marbles with two different colors, e.g. white and blue to the students in front of the class. Then teacher holds discussion with students by asking some students to show how they answer and let the other students respond.

These are the marbles of Budi:



a. *How many Budi's marbles? How do you count them?*
b. *Could you count backward?*
c. *Where is the number 16 on the marbles? How do you get that?*
d. *Where is the number 24 on the marbles? How do you get that?*
e. *How do you count the marbles from 6 to 25? How many marbles are they?*
f. *How do you count the marbles from 21 to 7? How many marbles are they?*

Hypotheses of students' thinking and learning:

a. Most of the students think that the marbles are arranged by group of five and there are four group of marbles, so they will answer 30 by counting the marbles from the left to the right by 5 as many as 6 times and some other students just count the marbles by ones.

b. Most of the students think the same as in the part a, they count by five from the right to the left by 5 as many as 5 times and some other students just count the marbles by ones.

c. Students determine the position of 16 by counting by five three times and count next by ones as many as 1 time since they think that 16 is 1 next from 15 and 15 is derived from counting the marbles from the left by fives three times.

d. Some students determine the position of 24 by counting the marbles by fives four times and next counting by ones as many as 4 times. Since they think that 24 is 4 next from 20 and 20 is derived from counting the marble from the left by fives four times.

e. Students count the marbles by starting from the marbles number 6 and jumping by ones four times and next jumping by fives as many as three times to get 25. They count the marbles as many as 19 marbles.

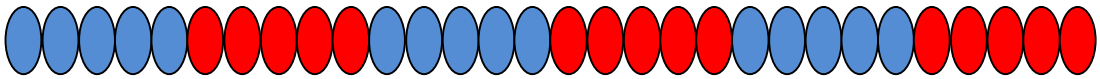
f. Students count the marbles by starting from the marbles number 21 and jumping backward by ones to 20, and next jumping by fives twice, and next jumping by ones three times to get 7. Then they count the marbles as many as 14 marbles.

Teacher's follow up reaction/Discussion:

After students solve a problem in the front of the class, she asks the other students to respond and compare their answers. Once, she asks students to answer the

problems using another way. After the whole problems have been solved and discussed, the teacher gives all students paper contenting problem and the picture of marble sequence like the picture that she has presented in the previous activity.

The problems:



1. *Solve 13 marbles + 16 marbles*
2. *Solve 29 marbles – 12 marbles*

Hypothesis of students' thinking and work:

1. Some students just answer the operation by using algorithmic way since they know already them. Other students use marbles as model to show their works by counting the marbles by using some jumps. They count by starting from 13 and jumping by ones twice then jumping by five twice, then jumping by ones four times to get 29 marbles.
2. Some students just answer the operation by using algorithmic way. Other students work with marbles. They start by deciding first 29 marbles and removing 12 marbles, finally the marbles remain 17. Some students start at marble number 29 and jumps by ones four times to 25. Next they jump by fives one time and finally jump by ones three times.

b. Activity 2: determining the position of numbers in number line

The starting point and the goals of this activity are described in the table 4.2:

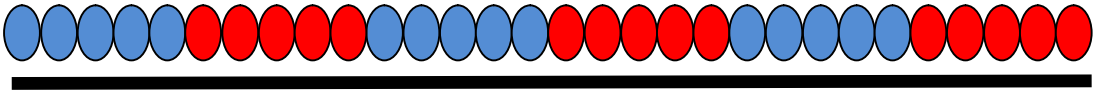
#Table 4.2. The starting point and the goal of activity 2 in lesson 1#

Indicator	**Starting Point**	**Goal**
Knowledge	• Students already know number line • Students already know about jumping by tens.	• Students can determine the position of numbers in number line by corresponding the position of the marbles to the number line using some strategies. • Students can know that the number line can flexibly represent the numbers. • Students can know that the order of the numbers in number line, i.e. small numbers in the left and big numbers in the right.
Skill	• Students already know determining the position of number in the marbles	• Students are able to correspond the marbles to the line.
Attitude	• Students have been accustomed in the situation when they have different answers to the that of their friends	• Students express their point of view toward their friends who find the position of numbers in the line using different strategies.

Activity description:

Students are divided into group of two then they are given paper by teachers consisting of two problems. Each of the problems consists of some sub-problems in which each sub-problem is discussed by teacher and students before students are asked to do the next part. Here, the following picture of sequence of marbles is the same as the picture provided in the previous activity to show students that number line can flexibly represent numbers. The picture also contains a horizontal line put below the sequence of marbles.

1. **Problem 1**



   a. *Where is the position of 1 and 0 on the marbles sequence? Where is the position of 1 and 0 in the line? How do you determine that?*
   b. *Where is 10 on the marbles? Where is the position of 10 in the line? How you determine that?*
   c. *Where is 15 in the line? How do you determine that?*
   d. *Where is 30 in the line? How do you determine that?*
   e. *Complete the number line with other numbers!*

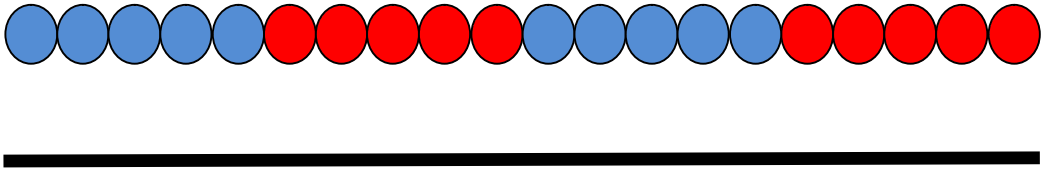
f. *What conclusion do you get about the numbers in the number line? Whether the righter position on the numbers in the number line, the greater or the lesser they are?*

Hypothesis of students' thinking and work:

Students choose the end part of the marble number 1 and draw a line from it to the number line. Then they have corresponding position of number 1 in the number line. In determining the position of 0, the students choose the left part of the left-most marble and draw the line from it to the line since they think that 0 represents nothing marble.

The students will do the same procedure in answering part b as well as in answering part c, d, and e. For other students, they just mentally put numbers in the line without drawing connecting lines. In answering question f, students will take conclusion that numbers get higher when their position in the right and get less when their position in the left because they think and see that the numbers in the right side are bigger than those of in the left side.

2. Problem 2



a. *Where is the position of the number 1 and 0? Where is the position of them in number line?*

b. *Where is the position of the number 10? Where is the position of it in number line?*

c. *Where is the position of 20 in number line?*

d. *Could you determine the position of 21 and 25?*

e. *What conclusions do you get about the numbers in the number line? Whether the righter position on the numbers in the number line, the greater or the lesser they are?*

Hypotheses of students' thinking

In part a, b, and c, the students will do the similar process like solving the problem number 1. The crisis will occur in solving part d. Some students will answer that there is no position of 21 and 25 in the number line since the line is over. Other students will draw five more marbles and they will be able to determine the position of those numbers. Some students think that the position of the numbers 21 and 25 in this problem is the same as that of in the line of the previous problem. They will have the same thinking and conclusion as in the problem number 1 in answering the question part e.

Discussion:

When the crisis occurs in the part d, the teacher asks the students who works with 5 more marbles to present their answers in the front of the class. So by seeing the work and explanation of them, it is expected for students who couldn't solve the problem correctly to realize their mistake. Especially for the conclusion about the numbers in number line, the teacher needs to trigger the students who couldn't make the precise conclusion by asking: *are the numbers in the right bigger or less than the numbers in the left*?.

4.2.Lesson 2: Determining the position of numbers including negative numbers in number line.

The starting point and the goal of this activity are described in the table 4.3.:

#Table 4.3. The starting point and the goal of lesson 2#

Indicator	**Starting Point**	**Goal**
Knowledge	• Students already know number line. • Students already know negative numbers. • Students already know that negative numbers are less than positive numbers.	• Students are able to recognize the position of negative numbers on number line. • Students are able to know the idea of zero as the mirror number.

Indicator	Starting Point	Goal
Skill	• Students already know how to determine the position of numbers in number line.	• Students are able to determine the position of negative numbers in number line. • Students are able to determine the position of numbers by identifying the proper distance of those numbers.
Attitude	• Students have been accustomed in the situation when they have different answers to the answers of their friends	• Students express their point of view toward their friends who extend the line to mark the negative numbers in number line.

Activity description:

This kind of activity is almost similar to the sub-activity 1.2. However, to determine the position of numbers, the students are not given the help of marble picture. The students are divided into group of two and given four papers by teachers of which each paper consists of a problem. Each of the problems is discussed by teacher and students before students are asked to do the next part.

Problem:

Put certain numbers on the following lines:

a. 15, 30, and 45

b. 5, 25, and 75

c. 10, 5, 1, -1, -2, -3, -4, -5, -10 and -20

d. 10, 20, 30, 40, -10, -15, -20, -25, -30, -40, and -50

Hypotheses of students' thinking and work

a. In determining the position of 15, 30 and 45, students will take the middle part of the line which is 30. Then they continue to find 15 which is in between 0 and 30 of which 15 is the centre of the range, as well as 45 which is in between 30 and 60. Other students don't consider the distance among numbers and just directly put 15, 30, and 45 in arbitrary position.

b. In determining the position of 5, 25, and 75, students will take the middle part of the line which is 50. Then they are able to determine the position of 25 and 75 like doing the process in the part a. in determining the position of 5, some students split the range between 0 and 50 by 5 splits, and determine the position of 10 in the first split mark, then continue to determine the position of 5 in

between. Other students don't consider the distance among numbers and just directly put5, 25, and 75 in arbitrary position.

c. Students are easily able to determine the position of 10, and 5. In determining the position of 1, they put in the right of 0. In determining the position of -1, some students answer that there is no position of -1 in the number line. Other students directly extend the line to the left and putting -1 in the left of 0. They continue working by respectively putting -2, -3, -4, and -5 in the left. In determining the position of -10, and -20, they extend the line as long as the previously given line to the left. They think that the length from 0 to -10 is the same as that of from 0 to 10, so they can determine the position of -10. They do the same process to determine the position of -20.

d. Students determine the position of 25 first, and then they split the range from 0 to 25 by 5. Then like in part d, they are able to find the position of 5. Next, they are able to find the position of 10 of which the length from 0 to 10 is twice of the length from 0 to 5. Next they are able to find the position 0f 20, 30, and 40. As well as the process of finding the position of negative numbers, students extend the line and measure length from 0 to -10, -20, -30, -40, and -50. Especially in determining the position of -15, and -25, they put respectively those numbers in between -10 and -20 and -20 and -30.

Teacher's follow up reaction/Discussion:

a. In the matter of the arbitrary position of students' work, teacher asks the students to illustrate the giving of marbles in certain amount depend on the maximum number on the line.
b. In the matter of the answer of students that there is no negative numbers in the line given, teacher asks the students to try to extend the line to the left and asks them again where the position of those numbers.

4.3.Lesson 3 : Positive numbers subtracted by larger positive numbers which results negative numbers

The starting point and the goal of this lesson are described in the table 4.4.:

#Table 4.4. The starting point and the goal of lesson 3#

Indicator	Starting Point	Goal
Knowledge	• Student already know about negative numbers. • Students already know about the position of negative numbers in number line. • Students already know that having something or having excess of something is opposite to lack of something.	• Students are able to know that subtracting numbers by larger numbers result negative numbers. • Students are able to know the idea about directed numbers that is positive numbers and negative numbers obtained their representations of which a thing represented by positive numbers is opposite to that of represented by negative numbers. In this case lack of certain numbers of something is represented by negative numbers and having or excess of certain numbers of something is represented by positive numbers.
Skill	• Students already know to	• Students can use the model of

Indicator	Starting Point	Goal
	determine the position of numbers in number line. • Students already know how to subtract numbers in number line.	number line in showing that subtracting large numbers from small numbers results negative numbers.
Attitude	• Students have been accustomed in the situation when they have different answers to the answers of their friends	• Students express their point of view toward their friends who determine that it will result negative numbers when a number is subtracted by a larger number.

Problem:

Budi has 6 marbles. He plans to give them to his friends of which each friend gets one marble. What happen to the stock of Budi's marbles if the number of his friends is:

a. 2

b. 4

c. 6

d. 8

e. 10

Hypothesis of the students' thinking and working:

a. The students think that the problem concerns the subtraction of two numbers. They subtract 6 by 2 to get 4. In visualizing their answers, the students draw number line. Starting with 6, they move to the left as many as 2 steps to get 4. They then get the conclusion that Budi still have 4 marbles.
b. The students think that the problem concerns the subtraction of two numbers. They subtract 6 by 4 to get 2. In visualizing their answers, the students draw number line. Starting with 6, they move to the left as many as 4 steps to get 2. They then get the conclusion that Budi still has 2 marbles.
c. The students think that the problem concerns the subtraction of two numbers. They subtract 6 by 6 to get 0. In visualizing their answers, the students draw number line. Starting with 6, they move to the left as many as 6 steps to get 0. They then get the conclusion that Budi has no stock of marbles anymore.
d. The students think that the problem concerns the subtraction of two numbers. They subtract 6 by 8. Several students answer the problem: 2 and they get the conclusion that Budi has 2 marbles. The other students answer the problem -2 and they get the conclusion that Budi has lack of 2 marbles.
e. The students think that the problem concerns the subtraction of two numbers. They subtract 6 by 10. Several students answer the problem: 4 and they get the conclusion that Budi has 4 marbles still. The other students answer the problem: -4 and they get the conclusion that Budi has lack of 4 marbles.

Teacher’s following direction:

The teacher asks the students to visualize their answers when they are asked to present their answers in front of the class. When, the students answer 2 and 4 in part d and e respectively obtained by doing subtraction, the teacher directs or asks them to answer all problems using number line.

4.4. Lesson 4:

Activity 1: The addition of negative numbers and positive numbers

The starting point and the goal of this lesson are described in the table 4.5.:

#Table 4.5. The starting point and the goal of the activity 1 in lesson 4#

Indicator	Starting Point	Goal
Knowledge	• Students already know that lack of certain number of something is represented by negative numbers and excess of or having number of something is represented by positive numbers. • Students already know about the position of negative numbers in number line. • Students already know that having something or having excess of something is opposite to lack of something	• Students are able to know that numbers become increase when they are added by positive numbers.
Skill	• Students already know to determine the position of numbers in number line. • Students already know how to add numbers by positive	• Students can use the model of empty number line or number line in showing that adding negative numbers by positive numbers has

Indicator	Starting Point	Goal
	numbers using number line.	opposite direction to adding them by negative numbers.
Attitude	• Students have been accustomed in the situation when they have different answers to the answers of their friends	• Students express their point of view toward their friends who determine that the numbers become increase when they are added by positive numbers.

Yesterday, Ilham has lack of 5 marbles and today he gets some new marbles from his friend. Does he still has lack of marbles if he gets new marbles as many as:

a. 3?
b. 5?
c. 8?
d. 10?

If he doesn't have lack of marbles anymore, how many marble does he has?

Hypothesis of the students' thinking and working:

a. Students answer that Ilham still has lack of 2 marbles. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 3 steps to get -2.

b. Students answer that Ilham has no marble. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 5 steps to get 0.

c. Students answer that Ilham doesn't have lack of certain marbles, instead, Ilham now has 3 marbles. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 8 steps to get 3.

d. Students answer that Ilham doesn't have lack of certain marbles, instead, Ilham now has 5 marbles. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 10 steps to get 5.

Activity 2: The addition of two negative numbers

The starting point and the goal of this lesson are described in the table 4.6.:

#Table 4.6. The starting point and the goal of the activity 2 in lesson 4#

Indicator	Starting Point	Goal
Knowledge	• Students already know that lack of certain number of something is represented by negative numbers and excess of or having number of something is represented by positive numbers.	• Students are able to know that the addition of two negative numbers results negative numbers

Indicator	Starting Point	Goal
	• Students already know about the position of negative numbers in number line. • Students already know that having something or having excess of something is opposite to lack of something	
Skill	• Students already know to determine the position of numbers in number line. • Students already know how to add numbers by positive numbers in number line of which the direction is to the right.	• Students can use the model of empty number line or number line in showing that adding negative numbers by negative numbers results negative numbers.
Attitude	• Students have been accustomed in the situation when they have different answers to the answers of their friends	• Students express their point of view toward their friends who determine that it will result negative numbers too when a negative number added by negative number.

Problems

Yesterday, Eka had excess of four marbles. Today, he has excess of two marbles. What is the excess total of Eka's marbles?

Yesterday, Anton had lack of three marbles. Today, he has lack of four marbles. What is the shortage total of Anton's marbles?

What is the difference of the marble possessions between Eka & Anton?

Hypothesis of the students' thinking and working:

Some students consider the lack of numbers of marbles means negative and give the value of Anton's possession that is -7. They also consider in Eka's possession that having excess of marbles means positive and give the value of Anton's possession that is 6. So, they get the difference of the marble possessions between Anton & Eka that is 13. Other students only answer for the case Anton's possession that he lacks 7 marbles and in the case of Eka's possession that he has 6 marbles. They count the difference that is 13. The other students answer that the difference between Eka's marble and Anton's marble is 1.

4.5. Lesson 5: Finding the relationship among different operations

The starting point and the goal of this lesson are described in the table 4.7.:

#Table 4.7. The starting point and the goal of lesson 5#

Indicator	**Starting Point**	**Goal**
Knowledge	• Students already know about the addition of two positive numbers. • Students already know about the position of negative numbers in number line.	• Students are able to know that positive numbers, when added by negative numbers, are the same as subtracting them. • Students are able to know that subtracting numbers by negative numbers is the same as adding the numbers by positive numbers.
Skill	• Students already know to determine the position of numbers in number line. • Students already know how to add positive numbers by positive numbers in number line of which it moves to the right from the starting point.	• Students can use the model of empty number line or number line in showing that adding positive numbers by negative numbers has opposite direction to adding them by positive numbers. • Students can use the model of empty number line or number line in showing that subtracting numbers by negative numbers has opposite direction when subtracting them by positive numbers and has the same direction when adding the numbers by positive numbers.

Indicator	Starting Point	Goal
Attitude	• Students have been accustomed in the situation when they have different answers to the answers of their friends	• Students express their point of view toward their friends who have different answers.

Problem:

1. Solve:

- 5 + 2 = ….
- 5 + -2 = ….

2. Solve:

- 5 – 2 = …
- 5 – (-2) = …

Hypothesis of the students' thinking and working:

1.

- 5 + 2, students mentally answer this problem by writing down 7.
- 5 + (-2), several students mentally answer 7. The other students use number line to answer the problem. Starting from 5, they move to the left

as many as 2 steps to get 3. They move to left because 5 + (-2) has opposite direction in number line to that of 5 + 2.

2.

- 5 – 2, students mentally answer this problem by writing 3.
- 5 – (-2) , several students mentally consider that the answer is 3. The other students use number line to answer this problem. Starting from 5, they think that because 5 - (-2) is not the same as 5 + (-2) they move to the right in number line as many as 2 steps to get 3.

4.6.Lesson 6: Solving several problems with greater numbers using number line

The starting point and the goal of this lesson are described in the table 4.8.:

#Table 4.8. The starting point and the goal of lesson 6#

Indicator	**Starting Point**	**Goal**
Knowledge	• Students already know about the flexibility of line in representing integers. • Students already know some concepts of the addition and subtraction involving negative numbers. • Students already know about the position of negative numbers in number line.	Students are able to add and to subtract negative numbers in the problems involving greater numbers.
Skill	• Students already know to determine the position of numbers in number line.	• Students can use the model of empty number line or number line in solving the addition and subtraction involving negative numbers
Attitude	• Students have been accustomed in the situation when they have different answers to the answers of their friends	• Students express their point of view toward their friends who have different answers.

Problems:

a. 14 – 31 = ... ?

b. 25 + (-15)

c. -23 + 40

d. -15 + -25

e. 12 – (-16)

Hypothesis of the students' thinking and working:

a. Several students use number line to visualize the process of the work. They start with 14 in number line and make jump by ones as many as 31 steps to the left to get -17. The other students use the combination of jumping by five-strategy and jumping by ones to move to the left in number line.

b. Several students use number line to visualize the process of work. They start with 25 in number line and make jump by ones as many as 15 to the left, to get 10. The other students use jump by five-strategy to move to the left to get 10.

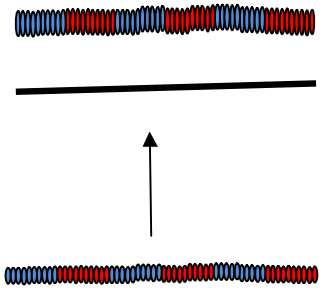
c. Several students use number line to visualize the process of work. They start with -23 in number line and make jump by ones as many as 40 to the right, to get 17. The other students use the combination of jumping by five-strategy and jumping by ones to move to the right in number line to get 17.

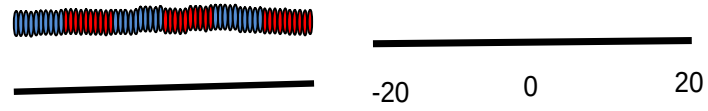
d. Several students mentally answer -40 for this problem. Other students use number line by starting from -15 and move to the left to as many as 25 steps

in number line. The other students also use number line and apply jumping by five strategyto get 17 as the answer.

e. Students use number line by starting from -12 and next jump by ones to the right as many as 16 times to get 4.

Visualization of Learning Trajectory





Counting Marbles arranged in the sequence and positioning numbers in number line with the help of marbles.

Positioning numbers including both positive numbers and negative numbers in number line without the help of marbles

Finding negative numbers by subtracting positive numbers by larger positive numbers

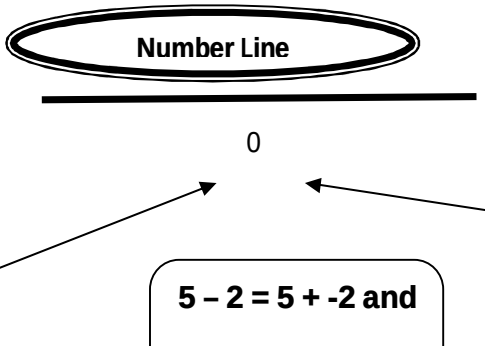
Adding negative numbers with positive numbers and negative numbers

Finding the relationship among several different operations

Solving several problems related to the addition and subtraction of negative numbers involving tens-numbers

5 – 2 = 5 + -2 and

5 + 2 = 5 – (-2)



a. 14 – 31
b. 25 + (-15)
c. -23 + 40
d. -15 + -25

CHAPTER V

RETROSPECTIVE ANALYSIS

In this chapter, the retrospective analysis of data collected from pre-test, the preliminary experiment, and the post-test were described. The result of this research is the underlying principles explaining how and why our design works. The hypothetical learning trajectory served as a guideline in the retrospective analysis to investigate and explain students' thinking in learning the addition and subtraction involving negative numbers.

5.1. Pre-Assessment

The pre-test was aimed to know students' current knowledge and ability. In the previous grades and lessons, the students have already learned about natural numbers, zero, negative numbers, and the function of negative numbers as a number that represents the quantity of something that is opposite to the thing that is considered as positive.

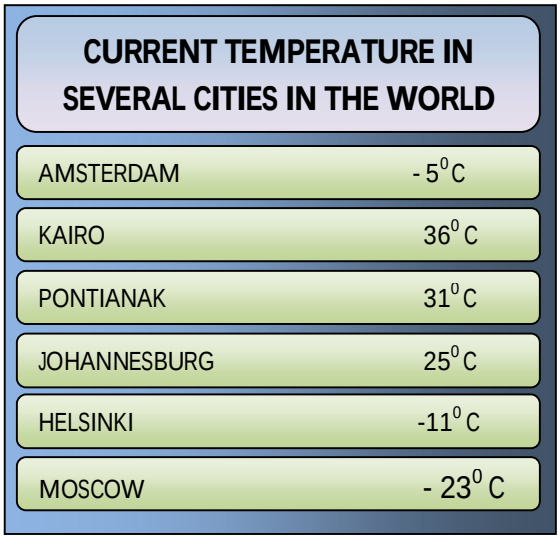
By giving this pre-test, we wanted to know how deep the understanding of the students in the aforementioned topics as their starting point and to know where our departure point should be. We also wanted to detect what are the difficulties of the students. Specifically, we want to know whether students recognize the order of integers given from smallest to the largest or vice versa, the function of

negative numbers, and how students subtract numbers. The students worked individually, and the pre-test consists of five problems as follows:

Problem 1

Problem 1 is about ordering the integers given in the form of the temperature of several cities. This problem is aimed to know the understanding of the students in recognizing that negative numbers are less than positive numbers. In addition, they also know the order of some negative numbers, e.g. -10 < -5.

The data of a weather station show the temperature of some cities around the world as shown in the diagram besides:



The students were asked to order the cities whose temperature from the lowest to the greatest.

Of six students, only three arranged the cities whose temperatures or the temperatures from the smallest to the largest correctly. However, two of the three students firstly

thought that -5 was less than -11 and -11 was less than -23. After they had been given by the researcher a stimulation in the form of question: is -1 less than -2?, they changed their mind to consider that -23 is less than -11 and -11 is less than -5. Meanwhile, the other two students considered that -5 is less than -11, and -11 is less than -23 although had been given the question which is the same as that of the two students who finally had correct answer because of the question. Nevertheless, the other student arranged the numbers in arbitrary position.

Problem 2

Problem 2 is about ordering the integers given in the form of numbers. The difference between this problem and the previous problem is that, in this problem, the students are asked to order the integers from the largest to the smallest. This problem is aimed to know the understanding of the students in recognizing that negative numbers are less than positive numbers. In addition, they also know the order of some negative numbers, e.g. -6 is bigger than -31.

Order these numbers from the smallest to the largest:
20, -31, -6, 3

In this problem, there are two students who give correct answer. Two of them are the students who have correct answer in problem 1. Meanwhile, the other students have various answers: considering that -31 is greater than -6 and arranging the numbers in arbitrary position.

Problem 3

This problem is asked to know the ability of the students in determining the quantitative state that is represented by certain numbers providing that the magnitudes opposite to that state is known, by concerning the definition of Rivard about negative numbers that negatives are certain magnitudes opposite to those which are regarded as positive.

- If number 5 represents the excess of 5 marbles, then what state does -5 represent?
  Your answer:
  Your reason: .....

- If number 3 represents the excess of 3 marbles, then what state does 0 represent?
  Your answer:
  Your reason:

There are four students who have correct answers in the two questions given. Meanwhile, there is one student who has correct answer in the first question and the other student has no correct answer at all.

Problem 4

This problem is asked to know the starting point of the students in representing the quantitative state by integers and to operate the integers and identifying two opposite states that is lack/shortage and excess that can elicit the sense of negative numbers. Here the following problem:

Mr. Joko is the owner of food stall Sumber Sedap that provides Fried Chicken. Every day he prepares 60 portions of foods. Explain what happen to the portions of the food in Mr. Joko' stall if in a day, the number of costumers coming is:

a. 45
b. 53
c. 60
d. 69
e. 79

Most of the students used subtraction in solving this problem and successfully identified that Mr. Joko has excess, has enough, or has lack of fried chicken. However, three of them did wrong calculation in problem a, of which they answered that Mr. Joko has excess of 25 fried chickens. One student totally gave wrong answer in all questions and seemed not to understand the problem.

Problem 5

Problem 5 is about the subtraction of two numbers. This problem is asked to know the starting point of the students in operating integers, the strategy used by the students, and the model that is used.

Calculate:

23 – 13 = ...
23 – 32 = ...

In this problem, there are three students who give correct answer for both questions. Two of them used algorithm way in answering the first question and the other student directly answered the problem. In the second question, they seem to be clueless and had difficulty in answering the problem. However, when being guided by the researcher about the order of integers, several of them eventually determine the answer correctly. For the other three students, they correctly answered the first problem. Two of them used algorithm way and the other direclty solve the problem without showing any process.

Problem 6

This problem is about the difference of two numbers. This problem is asked aimed to know the starting point of the students in understanding the word: difference, the strategy, and the model used by the students in answering the problem.

a. 21 and 12
b. 21 and 7

In this problem, only one of them had wrong answer. For the other five students, they considered the problem as the subtraction of two numbers and most of them used algorithm way. The student who gave wrong answer showed no strategy.

From the result of the pretest, several students still don't know the right order in negative numbers, for instance considering that -5 is less than -23 and -31 is greater than -6. It is conjectured that they need to have a model. In addition, some of them still don't know the function of negative numbers in representing the magnitude of something which is opposite to the thing that is considered as positive. Moreover, most of them can identify the states which are opposite to each other based on their values i.e. excess and lack. All of them can subtract two positive numbers; however, most of them cannot identify that positive numbers subtracted by larger positive numbers result negative numbers. Furthermore, they already know about the

difference of two numbers. Another thing that can be observed by the students is that they can think in abstract way by seeing that they really understand the problem provided in the form of story. To overcome the difficulty of several students who didn't do well in this pre-test and to make them more ready to follow the lesson in the preliminary experiment, after the pre-test had been conducted, we scaffold the students by teaching them several essential lessons related to the pre-test that an integer before 0 is -1, -2 is before -1, ect. Therefore they, who don't know that -31 is less than -6, can realize the right order of integers. In addition, to make students more aware of the right order of integers, we decide to introduce number line to the students as a tool to represent integers. Table 5.1. shows the items of each student in the pre-test.

Table 5.1. Table of students' answer result in the pre-test

Problem / Students	**1**	**2**	**3**		**4**					**5**		**6**	
			a	**b**	**a**	**b**	**c**	**d**	**e**	**a**	**b**	**a**	**b**
Rizki	√	×	√	√	×	√	√	√	√	√	×	√	√
Melisa	×	×	√	√	×	×	√	√	√	√	×	√	√
April	×	√	√	×	×	√	√	√	√	√	×	√	√
Vinna	√	√	√	√	√	√	√	√	×	√	√	√	√

Tia	√	√	√	√	√	√	√	√	√	√	√	√	√
Sela	×	×	×	×	×	×	×	×	×	√	×	×	×

5.2. Preliminary Experiment

Based on the pre-test results, we decided to start with a task in which it can visualize integers in right order before they study the addition and subtraction of negative numbers. We worked with 6 students and our investigation was focused on finding out how students analyze the context given and use number line as a model and a tool to do the addition and subtraction of negative numbers. The students who worked with us were Vinna, Tia, Melisa, Sela, Rizki, and April. Based on their teachers, they have different academic ability: Tia and Vinna represent the high achiever students; Rizki and April represent the average students; and Sela and Melisa represent the low achiever students. The result of this pilot experiment would give us feedback for the improvement of our hypothetical learning trajectory.

5.2.1. Lesson 1: "Making Number Line from Marbles" Activity

In the first activity of the preliminary experiment, we tried out "Making Number Line from Marbles" activity. In this activity, the students were asked to determine the position of numbers in number line. First, we showed them a picture of 30 marbles in sequence and asked them to determine the number of marbles. In this lesson, the students mentally counted from the left to the right. Specifically in counting, some students could count using more effective ways i.e. counting by twos, counting by fives, and counting by tens, meanwhile the other students used counting by ones. Also, in the activity of finding certain numbers in the marble sequence, most

of the students found them using counting by ones strategy from the left to the right. In the activity of counting marbles from one marble to another marble, the students only used counting by ones strategy. The group of marbles based on their colors didn't stimulate the students in counting using more effective strategy. In the activity of adding and subtracting marbles, the picture which is available in the instructional activity helped several students who didn't use algorithm way. In determining particular number of marbles and the number of marbles either added or subtracted, the students counted from the left to the right, counted to the right when adding, and counted to the left when subtracting. However, most of the students keep using their previous knowledge for this exercise, i.e. algorithm way.

It can be concluded that the grouping of marbles based on the color helps the students how to count marbles using several different strategies. The further analysis of this lesson is that students count from the left to the right. They start counting from 1 next to the right to 2, next to the right to 3, and so on. Consequently, they could identify the position of numbers in the marble sequence by taking the advantage of counting from the left to the right. This insight helped the students to find the position of certain numbers in the second activity. However, in the preliminary part of the second activity, several students couldn't identify the correct position of 0. The students put it in the right side of 1. It shows that the students didn't grasp understanding and couldn't keep in mind the conclusion that the position of smaller numbers is in the left side of bigger number. However, all students correctly could

determine the position of 21 and 25. Here, the students were quite helped by the question asking the position of 20 in the line, and by knowing the order of integers, so the students could successfully identify where the position of 21 and 25 are.

5.2.2. Lesson 2: Determining the position of numbers including the position of negative numbers in number line.

We conducted this lesson as the continuance of the first lesson. We want to make the students learn how to make number line involving negative integers. We gave a paper containing problem to the students as shown in the following:

Problem:

Put certain numbers on the following lines:

a. 15, 30, and 45

b. 5, 25, and 75

c. 10, 5, 1, -1, -2, -3, -4, -5, -10 and -20

d. 10, 20, 30, 40, -10, -15, -20, -25, -30, -40, and -50

5.2.3. Lesson 3: Positive numbers subtracted by larger positive numbers which results negative numbers

We conducted this lesson as the continuance of the second lesson. We want to make the students construct and know the knowledge of negative numbers derived from the subtraction of two positive numbers. We divided the students in three groups of two, where Sela is with Melisa, Tia is with April, and Vinna who is with Rizki. The following activity in this lesson is shown in the following:

Budi has 6 marbles. He plans to give them to his friends of which each friend gets one marble. What happen to the stock of Budi's marbles if the number of his friends is:.

a. 2
b. 4
c. 6
d. 8
e. 10

In this lesson, it looked that the students understood what operation and symbol representing the context given. It can be seen by their written works that started at mathematical symbol of which the students used subtraction. However, to stimulate the students using number line, the researcher asked them to show their work using number line. The group of Tia started at 6 and moved to 2. It seemed that the group didn't know how to work with number line. It considered that the stop position represents the number subtracting the other number. However, the group of Vinna used number line and worked correctly. In the question having negative number as

the answer, most students only answered contextually that is Budi lack of certain number of marbles. Then the researcher asked them about the operation involved to use number line to show the process of their work. The students agreed the operation meant in this context is 6 – 8. The group of Tia considered that the result is 2. Therefore, the researcher guided them so the students by themselves realized that it is not correct. In showing the work using number line, the students hesitantly started at 8. Here it showed that the students inconsistently worked with number line. Meanwhile, Vinna could work with number line correctly for this problem. By seeing the result from this lesson, it can be concluded that the previous lessons don't quite help most students in working with this lesson. Most students don't know how to work with number line. Specifically, they didn't know how to illustrate the operation which is used: where it starts, how many jump needed, what does it mean if it stops at certain number, ect. Some abilities that were not sustained by the previous lessons.

5.2.4. Lesson 4

Activity 1: The addition of negative numbers and positive numbers

This lesson was conducted as the continuance of lesson 3. The aim of the activity is to make the students able to find the correct result of negative numbers added by positive numbers. The problems given to the students are shown in the following:

> Yesterday, Ilham has lack of 5 marbles and today he gets some new marbles from his friend. Does he still have lack of marbles if he gets new marbles as many as:
>
> a. 3?
> b. 5?
> c. 8?
> d. 10?

In the fourth lesson, in working on the first question, all students had incorrect solution. They also didn't know how to work with number line from this context. There is a group which successfully represented the states in the context using numbers appropriately and used the addition as the operation correctly. It represented the state of lack of marble by negative numbers and the addition of marble is represented by plus sign. However, it then answered 8 as the result of the operation. In working with number line, the students correctly determine the direction of move which is to the right. They recognized the contextual knowledge that getting marbles means increase and it should move to the right in number line. Again the same as the problem of students in the third lesson, it didn't know how to work with it using

number line procedurally, i.e. the mistake in the number of jump needed and the stop position. The students didn't correspond the number of marbles got by Ilham and the number of jump in the number line. The group of Tia which wrote -5 + 3 = 8 considered that 8 is the number of jump, and 3 is the stop position in the number line. The group of April, firstly assumed that the answer is 3 since Ilham gets 3 marbles. Then it adjusted the operation: -5 + 3 = 3 to the number line of which it started at -5 and directly move to 3. The conclusion from this lesson is that, the students were helped by the context in which they can isolate negative quantity into negative number and determine the operation which should be used. However, in working with number line, they seemed not to understand about the procedure. It can be concluded that the mistake of Tia's group is caused by the misconception of what it had in the previous lesson. The lack knowledge of using number line also caused the the group of April to the wrong understanding. However, when being told by the researcher the right procedure to work on these problems, the students had good progress. By seeing this fact, the inability of the students to work with the contextual problem might be influenced by the typical learning process they always experience of which the procedure and the facts in the form of ready-made mathematics need to be memorized rather than constructing their mathematical knowledge by themselves.

Activity 2: The addition of two negative numbers

This activity is the continuance from the previous activity to cover the competency, the addition of two negative numbers. The instructional activity is shown in the

following:

a. Yesterday, Eka had excess of four marbles. Today, he has excess of two marbles. What is the excess total of Eka's marbles?
b. Yesterday, Anton had lack of three marbles. Today, he has lack of four marbles. What is the shortage total of Anton's marbles?
c. What is the difference of the marble possessions between Eka and Anton?

In the problem covering the addition of two negative numbers, all students successfully identified the numbers represented the quantity in the context. Consequently, they could answer the problem correctly. Here, the role of context quite helped the students in determining the numbers as the representation of certain quantity.

5.2.5. Lesson 5: Finding the relationship among different operations

The instructional activity of this lesson is shown in the following:

Problem:

1. Solve:
   - 5 + 2 = ….
   - 5 + -2 = ….

2. Solve:
   - 5 – 2 = …
   - 5 – (-2) = …

All students correctly answered the first question in the question number 1. All of them, except Tia, used usual addition: 5 + 2 = 7. Whereas Tia used number line of which she started at 5 then jumped to 7. In the discussion session, Tia came forward and used number line as the model in solving the problem. She started with 5 and jumped to the right by two to 7.

Then we continued to work on the second question. Only Tia did have correct answer for this question. Melisa and Sela wrote down 7 for the answer. Meanwhile, Rizki used number line of which he started at 5 and moved to 2. An awkward thing occurred when Vinna used number line. She started at -2 and jumped to 5. However, when Vinna presented her work in the front of the class, Vinna used number line and put a little bit strange numbers in it. She wrote -2 in the right of -1. Then, the researcher interrupted and asked the other students to respond. Then Tia gave respond by saying that -1 is in the wrong position. Vinna then made a new number line with

numbers in it. She now positioned the numbers correctly. She started with -2 and directly jumped to the right to 3. Melisa also presented its answer in the whiteboard; she answered -7 as the result. However, when she was asked the reason why she answered -7, she couldn't give an argumentation. Meanwhile, Rizki also came forward to present his answer. He drew number line and started with 5. Then, he directly made a jump from 5 to 2. When, the researcher asked him why he jumped by threes, he couldn't give an argumentation. Tia then came forward and also used number line. She started with 5 and directly jumped to 3. When she was asked how many jump of ones you made, she answered 2 by drawing two jumps of ones in the number line.

Then, we continued to work on the question number 2: 5 – 2 = …. ?. The researcher tried to see how Rizki work. This following extract of our video shows the conversation between the researcher and Rizki.

1.	**Researcher**	: (*seeing Rizki draw number line*) where do you start Rizki?
2.	**Rizki**	: 5
3.	**Researcher**	: what direction you will jump?
4.	**Rizki**	: the left
5.	**Researcher**	: how many jump of ones?
6.	**Rizki**	: three
7.	**Researcher**	: why three?
8.	**Rizki**	: because 5 – 2 = 3.
9.	**Researcher**	: because the result is 3 so you make jump by threes?
10.	**Rizki**	: (*nodding*)

In the discussion session, Sela was the first student who came forward. She used her fingers in solving the problem. Firstly, she showed her five fingers in her right hand while saying “five”. Surprisingly, she then showed her two fingers in her left hand while saying “subtracted by two”.

Then, Melisa came forward and used the usual subtraction: 5 – 2 = 3. She seemed to memorize that five subtracted by two results three. Next, Rizki came forward to show his work. Surprisingly, his work in the whiteboard was different to his previous work. He just made jump of two once.

Then, we continued to work on the question number 2: 5 – 2 = ….?. Melisa came forward and used the usual subtraction: 5 – 2 = 3. She seemed to memorize that five subtracted by two results three. Next, Rizki came forward to show his work. Surprisingly, his work in the whiteboard was different to his previous work. He just made jump of two once. Next, we continued to work on the second question in question number 2: 5 – (-2) = …?. In the discussion session, Rizki was the first student to come forward to present his work. He drew number line and started at 5. He then made a jump of 8 to get -3. After that, Sela and Melisa in turn came forward and wrote down 5 – (-2) = -3. The researcher asked them to explain why she got -3, but they couldn’t give a clear argumentation. The last two students who came forward were Vinna and Tia. She precisely had the same answer and strategy as that of Rizki.

Here, it can be seen that most of the students couldn't think the relationship between the operations of which the addition is the inverse of the subtraction that leads to the difference direction of move. It can be concluded that, most of the students seemed not to be ready to work with the problems since the relation between the previous lessons and this lesson is weak. It can be also remarked that students felt difficulty in working with "naked task".

By seeing the result occurred in this lesson, the competency of subtracting numbers by negative numbers is quite too difficult for children considering the limited time of the first cycle implementation. The plan of us in including the exercise covering that competency in the next lesson is cancelled.

5.2.6. Lesson 6: Solving several problems with greater numbers using number line

This lesson is the continuance from the previous lesson. The researcher expected the students to use empty number line which is more flexible and efficient instead of using usual number line involving too many numbers in it. The instructional task is shown in the following:

Solve:

a. 14 – 31
b. 25 + (-15)
c. -23 + 40
d. -15 + -25

The students started the activity by solving the question part a. The researcher saw how the students solve the problem. At first, he checked how Rizki worked with the problem. Since he saw Rizki drew number line, then he asked what number you start. However, Rizki didn't answer and he seemed not to know how to start. Then, the researcher came to Sela and he saw her using algorithm way to solve the problem of which she subtracted 4 by 1 to get 3 and subtracted 1 by 3 to get 2. So, she got 23 as the result. In the other side, like Rizki, Vinna also used number line. She started at 14 and then jumped to the left.

In the discussion session, Melisa came forward at first. She used algorithm way to solve the problem. Her process in solving the problem is precisely the same as that of

Sela and got 23 as the result. Then, Rizki came forward to present his answer. He used number line as a model to solve the problem. He started at 14 and surprisingly he made a jump to the right to 23. It contradicted to his idea just now that he jumped to the left. When, the researcher asked him to explain why he made a jump to the right, he couldn't give argumentation. The next student who came forward is Vinna. Astonishingly, she also used algorithm way to solve the problem. The different thing compared to Sela and Melisa, she got -23 as the answer. -2 is derived from 1 – 3. Finally, Tia came forward and used an interesting way to solve the problem. She didn't draw a line, instead, she just drew the jump as many as 31. After that, she wrote down the numbers in each jump starting from 14 and finally she got -19. The answer was almost true.

Then, we continued to work on the next question: 25 + (-15). The researcher firstly came to Rizki to check how he worked. Because he saw Rizki drawing a line, he asked him where he started and what direction he would jump. Rizki replied that he started at 25 and he would jump to the left. The researcher then checked how Vinna worked, again Vinna assumed that if we add something, then we should make a jump to the right.

In the discussion session, Sela again used algorithm way to solve this problem. She added 5 by 5 to get 10. However, she only wrote 0 and kept 1 to the next column and adding it by 2 and 1 to get 4. So the final result according to her is 40. Astonishingly, Vinna also used the same way as Sela did and got 40. In the other side, Rizki used

number line to solve the problem. He started at 25 then directly moved to the left beyond 0 to get -40. Whereas Tia also used number line and only wrote down several numbers that are 25, 10, and 0. She directly made a jump from the number 25 to the left to the number 10.

Then we continued to work on the next problem that is -23 + 40, again, the researcher firstly checked how Rizki thought and worked. Surprisingly, he saw Rizki using algorithm way to solve the problem. Rizki got 23 as the answer derived from adding 0 by 3 which results 3 in one column and -2 by 4 which results 2 in the other column.

However, in the discussion session, she used algorithm way to solve the problem and got 63 as the result.

In the other side, Tia also used algorithm way to solve the problem. She subtracted 0 by 3 in the left side and wrote down since she has taken 10 from the 40. Then it remained 3 in the right side. Therefore, Tia wrote down 1 as the result in the right side obtained by subtracting 3 by 2. Therefore, the final result was 17.

Seeing too many unpredictable situation from the students, the researcher scaffold the students to bear in mind that when they find such a problem, a number added by another number, they must consider where they start at the number line which is determined by the number written before the operation sign. Then they should determine what direction to move make a move as many as certain number determined by the number written after the operation sign.

Then, we continued to work on the last problem that is (-25) + (-15). Melisa did algorithm way and got result -40. In the other side, surprisingly, Rizki used empty number line to solve the problem. He started at -15 then made a jump by fives as many as five times to get -40. Meanwhile, Tia and Vinna also used empty number line. They started at -15 and made a jump by ten once and made a jump by fifteen next to get -40. So, in this lesson, all of the students, except Tia, couldn't do well in the first three problems. Since several of them applied algorithm way to the addition involving negative numbers and the other student, Rizki, couldn't find the incorrect answer since he made a wrong direction in jumping in number line. He answered the problem based on his belief and next illustrated it using number line with various strategies.

5.3. Post-Test

The post-test was conducted after the implementation of the pilot experiment. There are six problems that are tested for the students. The first problem is about the subtraction of two numbers. There are three sub-problems of which one of the sub-problems results negative numbers. Tia and April made correct answer of this sub-problem. Tia used number line by only made two jumps: from 12 to 0 and then from 0 to -4. Meanwhile, April used algorithm way to solve the problem. She seemed mentally to start analyzing the context first, that is Rizki has lack of 4 marbles when he had friends as many as 16 and corresponded that state to the number -4. Vinna, in the other side, didn't use both algorithm way and number line, instead, she used tally way in which she drew 16 vertical line for the Rizki's friend and 12 vertical line for the marbles of Rizki. She concluded that Rizki's marble is lack 4, however, she wrote down 12 – 16 = 4. Melisa herself, only described the stock of Budi's marble using statements.

In the second problem, surprisingly, only Vinna did wrong in solving the problem of which she used algorithm way. Actually, she got -11 at first by subtracting 5 by 6 and 2 by 3, however, she changed her answer to 12. Tia, again, used simple way to solve the problem. She used number line and started at 36, then jumped to 0, then jumped to -11. April and Melisa used algorithm way and got -11. Meanwhile, Rizki used number line and used counting by ones strategy.

In the third problem, only Melisa had wrong answer since she used algorithm way and considered that 0 subtracted by 8 is 8. Meanwhile, the others used number line. Especially, for Tia, she used empty number line by jumping from 10 to 0 and then from 0 to -8. Whereas the others used counting by ones strategy.

In the fourth problem, all of the students made correct answer. April and Melisa used algorithm way. They added 8 by 2 to get 0 in the right column and kept 1 for the left column and added it by 1 to get 2. Therefore, the result is 20. Because they assumed that negative numbers added by negative numbers are negative so they put negative sign to the number. Whereas, the others used number line of which Rizki and Vinna used counting by ones strategy and Tia used empty number line by simply wrote down -8, -12, and -20. She made two jumps that are from -8 to -12 and -12 to -20.

In the fifth problem, Tia and Melisa made a correct answer. Melisa consistently used algorithm way to solve the problem. She did the same way as that of in number 4. Meanwhile, Tia again used empty number line. The others also used number line, however, made a wrong process. There is student who jumped to -12 to -16, there is student who jumped from -12 to -16 since she thought that when we added number, we must jump to the right in number line, and there is also, Rizki, who made a wrong calculation in using counting by ones strategy. He moved from -12 to -29.

In the sixth problem, in part a, April and Melisa were the students who made incorrect answer. She moved from -6 to 9 using number line. Meanwhile, Melisa didn't show any process in solving the problem. Vinna used number line and started at -6 then jumped to 9. Rizki also used number line and used counting by ones strategy. Meanwhile, Tia used empty number line from -6 to 0 and then from 0 to 3.

In part b, April again did the same process like that of in part a by moving from -15 to 11. The same process was also done by Rizki. Melisa made correct answer but she did show no process. Whereas Vinna used number line and Tia used empty number line.

In part c, April and Melisa were the students who made incorrect answer. She used number line and again she did the same process like that of in part b and c by moving from 8 to -5. Unpredictably, she continued to move to -13. Rizki and Vinna used number line and Tia used open number line.

In part d, only did Vinna, Tia, and Melisa made correct answer. Vinna used counting by ones-strategy while Tia used empty number line by using counting by twelve and counting by three. While the others made incorrect answer in which Rizki jumped from 12 to the right to get 26, and April made a jump from -12 to -27.

In part e and f, there is no student who made correct answer. There is no one of them who considered that the numbers subtracted by negative numbers is the same as added by positive numbers. It seems that the problems are difficult for the students.

The result of post-test of each student is shown in the following table.

Table 5.2. Table of students' answer result in the post-test

Problem / Students	1			2	3	4	5	6					
	a	**b**	**c**					**a**	**b**	**c**	**d**	**e**	**f**
Rizki	√	√	×	√	√	√	×	√	×	√	×	×	×
Melisa	√	√	×	√	×	√	√	×	√	√	×	×	×
April	√	√	√	√	√	√	×	×	×	×	×	×	×
Vinna	√	√	×	×	√	√	×	√	√	√	√	×	×
Tia	√	√	√	√	√	√	√	√	√	√	√	×	×

5.4.Concluding Remarks of the Preliminary Experiment

Although the context of excess and lack of marbles is useful to elicit the concept of negative numbers as the numbers that represent quantity that is opposite to the quantity that is considered positive, our learning is failed to connect the context to the number line model. Number line is useful model for the students in counting the numbers either involving or resulting negative numbers providing that it is understood how to use it. However, most of our students didn't know how to use number line. For several students, they had

problems in ordering numbers in number line. Therefore, we can judge that the learning, especially the lesson 1 and lesson 2 don't seem quite meaningful for children to help them using number line correctly. Consequently, the students often show inconsistency of which they forget the knowledge they got in previous lessons especially about the using of number line.

Another remark that can be seen is that several students tend to use algorithm way since it seems simple for them in counting larger magnitude numbers. However, they didn't know that that common strategy is not suitably applied when working with the addition involving negative numbers. Furthermore, the problems of solving larger magnitude numbers stimulate student to use empty number line and to create several strategies. It should be notified also that the competencies covered by the HLT and the post-test are too much for the students especially for the low achiever and the middle achiever students since they are brought in a short time. Consequently, we would only provide the topic of the addition involving negative numbers in our second cycle.

5.5. The Improvement of Hypothetical Learning Trajectory

Based on the result of our preliminary experiment and the post-test, and also from the discussion with our supervisors, we would like to improve and adjust the activities in our initial HLT. The plans of the HLT adjustment and improvement are formulated in the following:

a. We'd like to reduce the number of competencies of the lesson by only providing the addition involving negative numbers. Consequently, several lessons related to the subtraction involving negative numbers will be deleted.
b. Concerning the emphasizing of students' understanding especially for low achiever students in the second cycle, in the first lesson, we will only provide the students with counting marbles activity and make the learning of determining the position of numbers with the help of marbles become our second lesson. In counting marbles activity, we will modify the questions to be more realistic so it becomes more meaningful for them and support the students to work using number line properly.
c. In the part of determining the position of numbers with the help of marbles, the teacher should emphasize the students about the length of the line represented by the number of marbles. The teacher can teach that topic in the end of the lesson or before the third lesson is conducted. We also would like to add problem in this lesson in which the students are asked where the position of the lack of certain numbers of marbles in the line.
d. Determining the position of numbers in number line will become the third lesson. We'd like to reduce the level of difficulty by choosing small numbers.
e. The competency of adding two negative numbers will become our fourth lesson.

f. The competency of adding negative numbers by positive numbers and vice versa will become our fifth lesson.

g. Finally, we will provide several problems to the students covering all the competencies of the addition involving negative numbers in the seventh lesson of which there are some problems involving large numbers to stimulate the students using empty number line.

5.6. The Refined Hypothetical Learning Trajectory

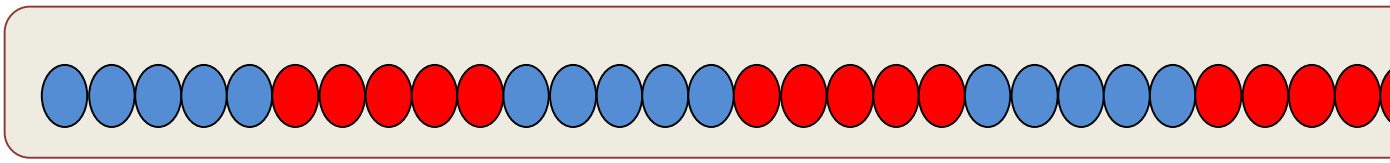
5.6.1. Lesson 1 : *Counting Marbles*

Goal: The students can count marbles using different strategies and can take conclusion that small numbers are in the left and large numbers are in the right. In addition, the students recognize that when doing addition, the counting goes to the right and when doing subtraction, the counting goes to the left.

Activity 1:

Teacher puts the picture of the sequence of marbles and next she orally asks the students several questions in the following:

These are Budi's marbles:



a. *How many marbles that Budi has in the picture? How do you count them?*
b. *Could you count the marbles backward? (counting the marbles from the right to the left)*
c. *If Budi only has 16 marbles, determine to what extent his marble in the picture?*
d. *If Budi only has 24 marbles, determine to what extent his marble in the picture?*

e. *Count as many as 16 marbles to the right by starting from the fifth marble! In what marble you stop?*

f. *Count as many as 12 marbles to the left by starting from the twentieth marble! In what marble you stop?*

Hypothesis of students' learning:

a. The students will answer 30 of which there are students, from the left to the right, counting the marbles by ones, some others count by twos, others count by fives, and the others count by tens.

b. The students count backward, i.e. from the right to the left, of which there are students counting by ones, counting by twos, count by five, and the others count by tens.

c. The students determine the answer by counting the marbles from the left by ones sixteen times, and the others count by fives, specifically, they count by fives three times and next count by ones one time.

d. The students determine the answer by counting the marbles from the left by ones as many as twenty four times, and the others count by fives, specifically, they count by fives four times and next count by ones four times.

e. Students count the marbles starting from the fifth marble and use count by ones strategy to count the marble as many as sixteen times and stop in the twenty first marble.

f. Students count the marbles starting by the twentieth marble and use count-by ones strategy in counting the marble and stop in the eight marble.

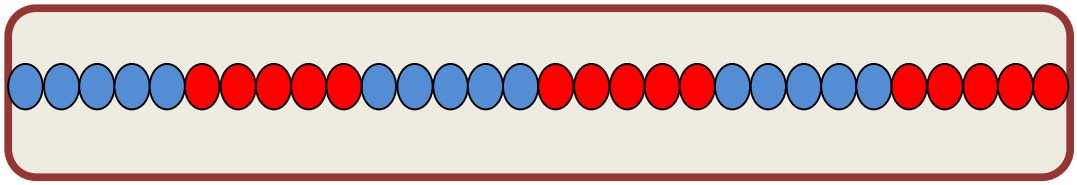
After these questions are asked, the teacher then gives a paper containing two questions that are shown in the following:

Activity 2: Solve the following problems and show how you solve them:

1. Count :

   13 marbles + 16 marbles : ….

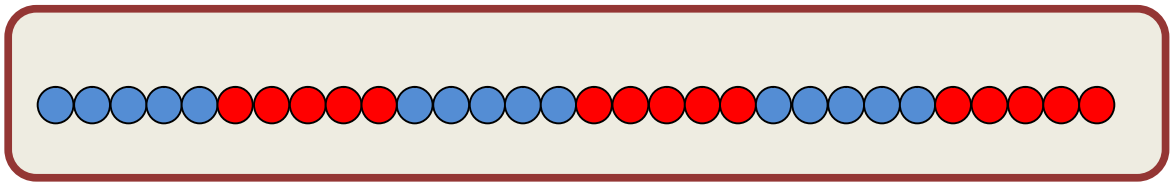
Your answer:



2. Count:

   29 marbles – 12 marbles = ….

   Your answer:



Hypothetical of students' learning:

Some students will solve it using algorithm way, meanwhile, the other students will solve it by utilzing the picture of marbles that is available in the problem and counting them using counting by ones-strategy:

a. Students firstly look for the thirteenth marble and do counting to the right as many as sixteen times and they stop at the twenty nineth marble.
b. Students firstly look for the twenty nineth marble and do counting to the left as many as twelve times and they stop at the seventeenth marble.

5.6.2. Lesson 2: Making Number Line

Goal:

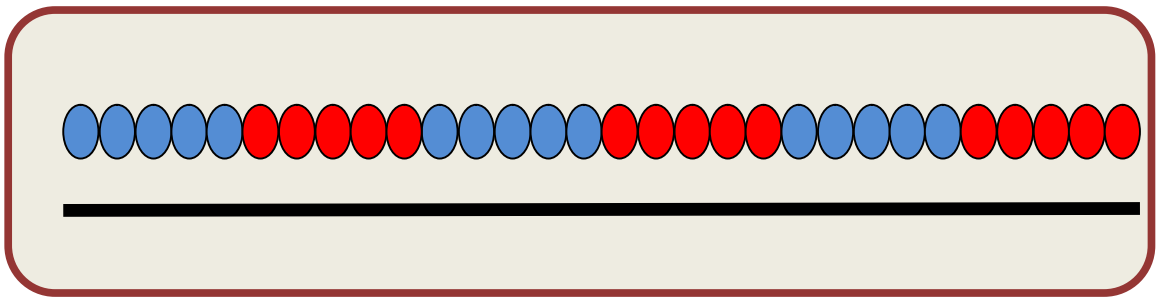
- Students can determine the position of numbers in number line by corresponding the position of the marbles to the number line using some strategies.
- Students can know that the number line can flexibly represent the numbers.
- Students can know that the order of the numbers in number line, i.e. small numbers in the left and big numbers in the right.

Activity description:

The students are given by the teacher tasks containing two problems shown in the following:

Problem 1:

The picture of marble sequence below can show the number of Budi's marbles:



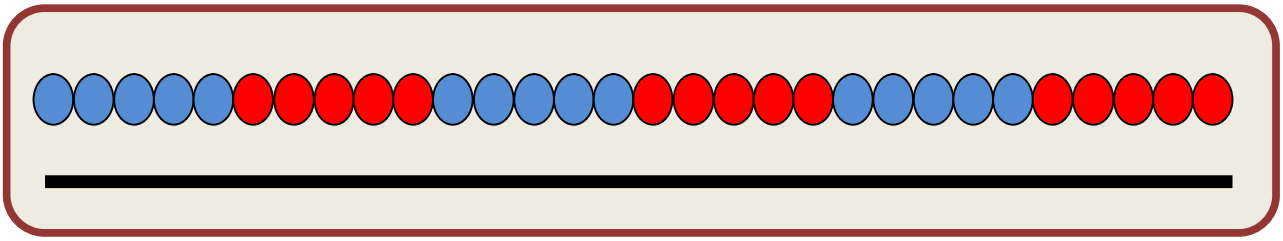
a. If Budi only has 1 marble, put the mark in the picture of marble sequence! Where is the position of the number in the line?

b. If Budi only has 10 marbles, put the mark in the picture of marble sequence! Where is the position of 10 in the line?

c. If Budi only has 15 marbles, gives the mark of the number on the line!

d. If Budi has 30 marbles, gives the mark of the number on the line!

e. Complete the following number line which is below the marble sequence with numbers from 1 to 30!



f. What conclusion you get about the numbers in the number line?

Conclusion:

.....

.....

.....

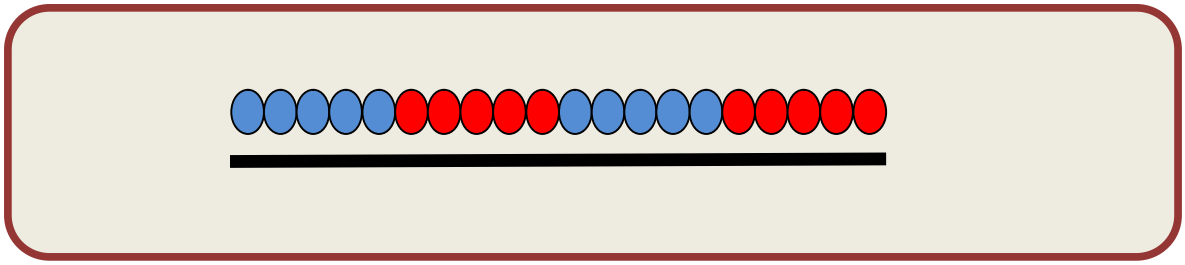
.....

Hypothesis of students' thinking and learning:

a. Students will put the mark in the right side of the first marble and put the mark for the position of 1 by corresponding the position of the first marble in number line.
b. Students will put the mark in the right side of the tenth marble and put the mark for the position of 10 by correspnding the position of the tenth marble in number line.
c. Students will put the mark for the position of 15 in number line by corresponding it to the fifteenth marble.
d. Students will put the mark of the position of 30 in number line by corresponding it to the right side the thirtieth marble.
e. Students will complete the number line with numbers from 1 to 30 by applying their procedures of part a, b, c, and d.
f. Several students take the conclusion that number become larger if they are in the right and become less if they are in the left. However, there are students who take unclear conclusion.

Problem 2

The pictures below can show the number of Budi's marbles



a. If Budi only has 1 marble, put the mark in the picture of marble sequence! Where is the position of the number in the line?
b. If Budi doesn't have any marble, mark its position in the marble sequence!
   What number that represents Budi has no marble?
   answer: ….
   Where is the position of the number in number line?
c. If Budi only has 10 marbles, mark its position in the marble sequence! Where is the position of 10 in the line?
d. If Budi has 20 marbles, mark its position in marble sequence! Where is the position of 20 in the line?
e. If Budi has 21 marbles, could you give mark to its position in the marble sequence? Where is the position of 21 in the line?
f. If Budi has 25 marbles, mark its position in the line?

g. If Budi is lack of 1 marble, could you give mark in the picture of marble sequence?

What number that represents that state?

answer: ……..

where is the position of the number in the line?

h. If Budi is lack of 2 marbles, could you give mark in the picture of marble sequence?

What number that represents that state?

answer: ……..

where is the position of the number in the line?

i. If Budi is lack of 5 marbles, could you give mark in the picture of marble sequence?

What number that represents that state?

answer: ……..

where is the position of the number in the line?

j. What conclusion you get about the numbers in the number line?

Conclusion:

……
…….

……

Hypothesis of the students' thinking and learning:

a. Students will put the mark in the right side of the first marble and put the mark for the position of 1 by corresponding the position of the first marble in number line.
b. Students will put the mark in the left side of the first marble. They consider that the state is represented by zero and and put the mark for the position of the number by corresponding the left part of the first marble in marble sequence to the line.
c. Students will put the mark in the right side of the tenth marble and put the mark for the position of 10 by corresponding the position of the tenth marble in number line.
d. Students will put the mark in the right side of the twentieth marble and put the mark for the position of 20 by correspnding the position of the twentieth marble in number line.
e. Several students have no idea in determining the position of having 21 marbles in marble sequence. The other students add one marble to the right and are also be able to mark the position of it in the line.
f. Several students have no idea in determining the position of having 25 marbles in the line. The other students add five more marbles to the right and are also be able to mark the position of the number in the line.

g. Several students have no idea in determining the position of lacking of 1 marble. The other student draw one marble in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -1.
h. Several students have no idea in determining the position of lacking of 2 marble. The other student draw two marbles in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -2.
i. Several students have no idea in determining the position of lacking of 5 marble. The other student draw two marbles in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -5.
j. Several students take conclusion that number become larger when they are in the right and become less when they are in the left. The other students also take conclusion that the position of negative numbers are in the left of positive numbers.

5.6.3. Lesson 3: Positioning Integers in Number Line

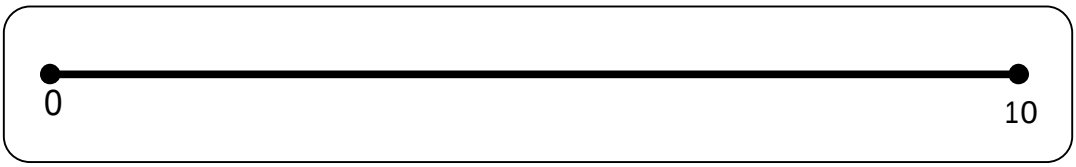
Goal: Students are able to recognize the position of negative numbers on number line.

The activity:

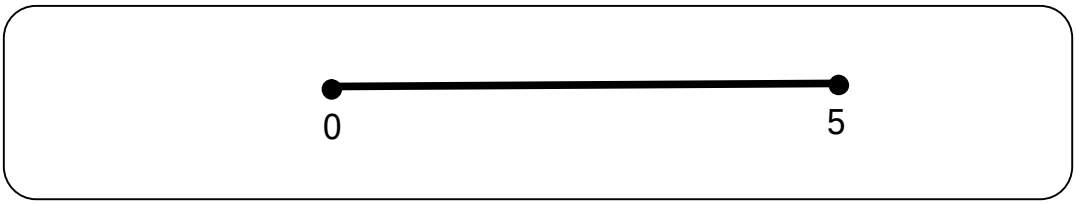
The students are asked to work with the tasks containing three problems shown in the following:

Put the numbers in the available number line!

a. 5



b. 1, 2, 3, and 4

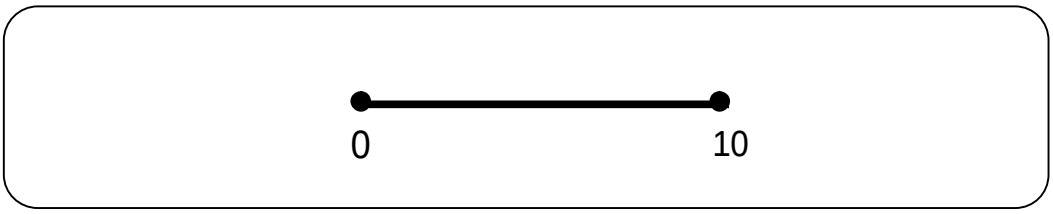


Could you determine the position of these following numbers to the line above:

- 6
- 7
- 10

- -1
- -2
- -3
- -4
- -5

c. 5, -5, and -10



What conclusion you get from those number lines?

.....

Hypothesis of students' thinking and learning:

a. In determining the position of 5, several groups take the arbitrary position of 1, 2, 3, 4, and next 5. Some other groups take the middle position of the line and put mark of it as the position of 5. Since they think that 0 is added by 5 is 5 and 5 is added also by 5 is 10 so 5 is exactly in the middle of the line.

b. The groups consecutively put 1, 2, 3, and 4 between 0 and 5. Next, several groups put 6, 7, 10, -1, -2, and -5 in the range between 0 and 5 in arbitrary position. The other groups then extend the line to the right to put 6, 7, and 10. In determining

the position of the negative numbers, -1, -2, and -5, they extend the line to the left.

c. Several groups put all the numbers in the range between 0 and 10. Meanwhile, the other groups put 5 exactly in the middle of the line and extend the line to the left. These groups put -5 in the position of which its distance from zero is the same as the distance between 0 and 5. As well as the process of determining the position of -5, these groups also determine the position of -10 based on the distance from 0 to 10. There are also groups which need to write down, for instance 1, 2, 3, 4 to determine the position of 5 and write down -1, -2, -3, and -4 to determine the position of -5.

5.6.4. Lesson 4: The Addition of Two Negative Numbers

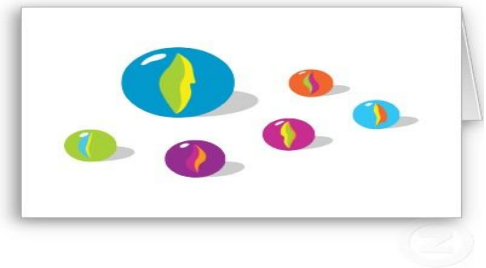
Goal:

Students are able to know that the addition of two negative numbers results negative numbers

The activity:

The students are asked to work with the tasks containing three problems shown in the following:

1. Yesterday, Eka had four marbles. Today, he has two marbles. What is the total number of Eka's marbles?



Your answer: reason/your work:

2. *Yesterday, Anton was lack of 3 marbles. Today, again he was lack of 4 marbles. How many total marble shortage of Anton?*

Your answer: reason/your work:

3. Is the difference of Eka's marble and Anton's marble is 1?

answer : reason/your work:

Hypothesis of the students' thinking and learning:

a. The students firstly answer that Eka totally has 6 marbles and they think that 6 marbles is obtained by adding four by two which gets six.

b.

- Several students answered that Anton is totally lack of seven marbles since they think that the problem is about adding two positive numbers that is 3 + 4 = 7.
- The other students also get that Anton is totally lack of seven numbers and then they use number line and consider that lack means negative, so they start at -3 and move to the left as many as 4 steps to get -7.

c.

- Students consider that the difference is 1 since they see 6 and 7 as the answers in respectively part a and part b.

- However, some of those students will think critically by considering that lack of seven mables can be represented by the number negative seven.
- The other students consider that the difference is 13 by seeing the number line of which they count from 6 to -7 or vice versa.

5.6.5. Lesson 5: *The Addition of Positive Numbers and Negative Numbers*

Goal: students are able to work with the addition of negative numbers and positive numbers problem.

The activity:

This lesson consists of two main activities. In the first activity, the students are asked to work with the tasks containing four problems shown in the following:

1. Yesterday, Ilham was lack of 5 marbles and today he gets some new marbles from his friend. Is still he lack of marbles if he gets new marbles as many as ***3 marbles?***

answer: reason: your work :

2. Yesterday, Ilham was lack of 5 marbles and today he gets some new marbles from his friend. Is he still lack of marbles if he gets new marbles as many as ***5 marbles?***

answer:

reason:

your work :

3. Yesterday, Ilham was lack of 5 marbles and today he gets some new marbles from his friend. Is he still lack of marbles if he gets new marbles as many as ***8 marbles?***

answer:

reason:

your work :

4. Yesterday, Ilham was lack of 5 marbles and today he gets some new marbles from his friend. Is he still lack of marbles if he gets new marbles as many as ***10 marbles***?

answer:

reason:

your work :

Hypothesis of the students' thinking and working:

1.

- Several Students answer that Ilham still has lack of 2 marbles since in their minds they think that the addition of 3 marbles doesn't completely fulfill the lack of Ilham's marble. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 3 steps to get -2.
- The other students consider that Ilham has 3 marbles since Ilham gets 3 new marbles.

2.

- Several students answer Ilham is not lack of marble anymore since they think that the addition of 5 marbles completely fulfill the lack of Ilham's marble. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 5 steps to get 0.

- The other students consider that Ilham has 5 marbles since Ilham gets 5 new marbles.

3.

- Several students answer that Ilham doesn't have lack of certain number of marbles; instead, Ilham now has 3 marbles. Some other students don't have idea at first. Therefore, they visualize the problem using number line. In visualizing, several students draw number line and start with -5 and move to the right as many as 8 steps to get 3.
- The other students consider that Ilham has 8 marbles since Ilham gets 8 new marbles.

4.

- Students answer that Ilham doesn't have lack of certain marbles; instead, Ilham now has 5 marbles. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 10 steps to get 5.
- The other students consider that Ilham has 10 marbles since Ilham gets 10 new marbles.

In the second activity, the students were asked to solve the following problems.

Solve:

- Is the result of 3 + (-5) is the same as that of -5 + 3?

Answer: your reason or your work:

- Is the result of 5 + (-5) is the same as that of -5 + 5?

Answer: your reason or your work:

- Is the result of 8 + (-5) is the same as that of -5 + 8?

Answer: your reason or your work:

- Is the result of 10 + (-5) is the same as that of -5 + 10?

Answer: your reason or your work:

Conjecture of student's thinking and learning:

1.

- Several students will answer yes and they used number line to prove it. In the problem -5 + 3, they start at -5 and move to the right to -2. Meanwhile, in the problem 3 + -5, they start at 3 and move to the left to -2.
- Several students answer no since they think that -5 + 3 equals -2 but 3 + -5 equals 8.

2.

- Students will answer yes and they used number line to prove it. In the problem -5 + 5, they start at -5 and move to the right to 0. Meanwhile, in the problem 5 + -5, they start at 5 and move to the left to 0.
- Several students answer no since they think that -5 + 5 equals 0 but 5 + -5 equals 10.

3.

- Students will answer yes and they used number line to prove it. In the problem -5 + 8, they start at -5 and move to the right to 3.
- Several students answer no since they think that -8 + 5 equals -3 but 5 + -8 equals 13.

4.

- Students will answer yes and they used number line to prove it. In the problem -5 + 10, they start at -5 and move to the right to 5. Meanwhile, in the problem 10 + -5, they start at 10 and move to the left to 5.
- Several students answer no since they think that -10 + 5 equals -5 but 5 + -10 equals 15.

5.6.6. Lesson 6: *Excercise on "Naked Task"*

Goal:

Students are able to add negative numbers in the problems involving great numbers.

The activity:

Solve:

1.

a. 5 + 6 = …

b. 11 + 15 = …

c. 14 + 31 = …

2.

a. 8 + (-5) = …

b. 12 + (-16) = …

c. 20 + (-25) = …

3.

a. -6 + 7 = …

b. -14 + 18 = …

c. -23 + 40 = …

4.

a. -5 + -3 = …

b. -11 + -13 = …

c. -15 + -25 = …

Hypothetical of students' thinking and learning:

1.
    a. The students will directly answer the problem that is 11. In showing their work, they solve using number line by starting from 5 and jump by ones as many as 6 times to get 11.
    b. Several students work with number line by starting from 11 and make jump by ones as many as 15 times to the right. The other students make jump by five strategies.
    c. Several students work with number line by starting from 14 and make jump by ones as many as 31 times to the right. The other students make jump by five strategies.
2. **.**
    a. The students answer the problem by using number line. They start at 8 and jump to the left using jump by ones strategy : jumping by ones as many as five times. The other students make jump by fives strategy. However, there are also students who make jump to the right.
    b. The students answer the problem by using number line. They start at 12 and jump to the left using jump by ones strategy : jumping by ones as many as sixteen times. The other students make jump by fives strategy. However, there are also students who make jump to the right.

c. The students answer the problem by using number line. They start at 20 and jump to the left using jump by ones strategy : jumping by ones as many as twenty five times. The other students make jump by fives strategy. However, there are also students who make jump to the right.

3. .

a. The students use number line by starting at -6 and count by ones to the right as many as seven times to get 1. The other students use count by fives strategy.

b. The students use number line by starting at -14 and count by ones to the right as many as seven times to get eighteen times. The other students use count by fives strategy.

c. The students use number line by starting at -23 and count by ones to the right as many as seven times to get forty times. The other students use count by fives strategy.

4. .

a. The students use number line by starting at -5 and count by ones to the left as many as three times to get -8.

b. The students use number line by starting at -11 and count by ones to the left as many as thirteen times to get -24. The other students use count by fives strategy.

c. The students use number line by starting at -15 and count by ones to the left as many as twenty five times to get -40. The other students use count by fives strategy.

5.7.Teaching Experiment

In this section, we compared our improved HLT and students' actual learning process during the experimental phase. We investigated how the HLT supported students' learning. We looked to the video recordings and selected some critical moments. We also analyzed their written works such as worksheet as another source. We analyzed every day lesson to investigate what students and teacher do, how the activities work, and how the material contributed to the lesson. We also looked with broader view and searched for connections between the lessons and tried to find out how earlier lessons supports the following ones. The result of the retrospective analysis in this teaching experiment would be used to answer our research questions.

5.7.1. Lesson 1: Counting marbles

This lesson consists of two activities, i.e. the first is counting marbles and the second is solving addition and subtraction problem. In the first activity, the teacher told the students that the picture that had been put in the whiteboard showed the marbels of Budi. Then the teacher asks the first question to the students that is how many marbles in the picture?. The students in their chairs answered that the number of marbles are 30. The teacher then asked the students how they count them. Imelda was the first to come forward, she used counting by ones strategy from one to thirty in which she counted from the left to the right. Next, Indira came forward and used counting by twos strategy, from two, four, six and so on till thirty.

Then the teacher asked the students to count the marbles from the right to the left in the marble sequence. Tri came forward and used count by ones strategy from one to thirty. The next student who came forward was Icha. She used counting by tens strategy: ten, twenty, thirty. Then, Attiya came forward and used counting by fives strategy: five, ten, fifteen, twenty, twenty five, and thirty.

After the second question was explained, the teacher asked continued to the third question that is if Budi only has 16 marbles, determine to what extent his marble in the picture?. However, the teacher paraphrased the question by asking where the position of 16 in the marble sequence is. Edi then came forward and counted the marbles using counting by ones strategy till getting the sixteenth marble from the left to the right. The next student who came forward is Dini that used counting by twos strategy from the left to the right. Then, Indira also counted the marble in the whiteboard and used counting by fours strategy from the left to the right.

Next, teacher asked the fourth question that is if Budi only has 24 marbles, determine to what extent his marble in the picture?. However, the teacher paraphrased the question by asking where the position of 24 in the marble sequence is. The first student who came forward was Suci that used counting by ones strategy from the left to the right. Tegar was the second student and also used counting by ones strategy from the left to the right. Next, Attiya came forward and used counting by sixes strategies from the left to the right. Afterward, Icha and Gina came forward and

respectively used counting by eights strategy and counting by ones strategy from the left to the right

The next occasion is the teacher asked the students the fifth question that is counting 16 marbles to the right starting from the fifth marble and in what marble it stops. The students who came forward were Theo and Indira. They started from the fifth marble from the left and counted using counting by ones strategy as many as sixteen times. They stopped at the twenty first marble. Then, the last question was asked that is counting 12 marbles to the left starting from the twentieth marble. The student who came forward was Fitrah who solved the problem using counting by ones strategy. Finally, he said that he stopped at 8.

After the session of this problem had ended, the teacher asked all students about the conclusion of their. She directed the students to think that when counting from the left to the right, the number increase or the number of things become more; and when counting from the right to the left, the number decrease.

By observing those activities, we can see that the students' strategies in counting marbles using more effective strategies i.e. counting by twos, counting by fives, ect in the starting point of the lesson don't affect the students using the same strategies when working on the next problems i,e, in counting the marbles from the predetermined marble to another marble, adding marbles and subtracting marbles. However, the useful contribution of the problems in the starting point of this lesson

thing seen in this lesson is, when the students will add the marbles, they move from the left to the right and when they subtract the marble, they move from the right to the left in the marble sequence. Here, the students mentally count from the left to the right that lead them to the concept that smaller numbers are in the left and larger numbers are in the right. Here is the table 5.20.showing the comparison between our conjectures in the activity 1 and the actual work of students:

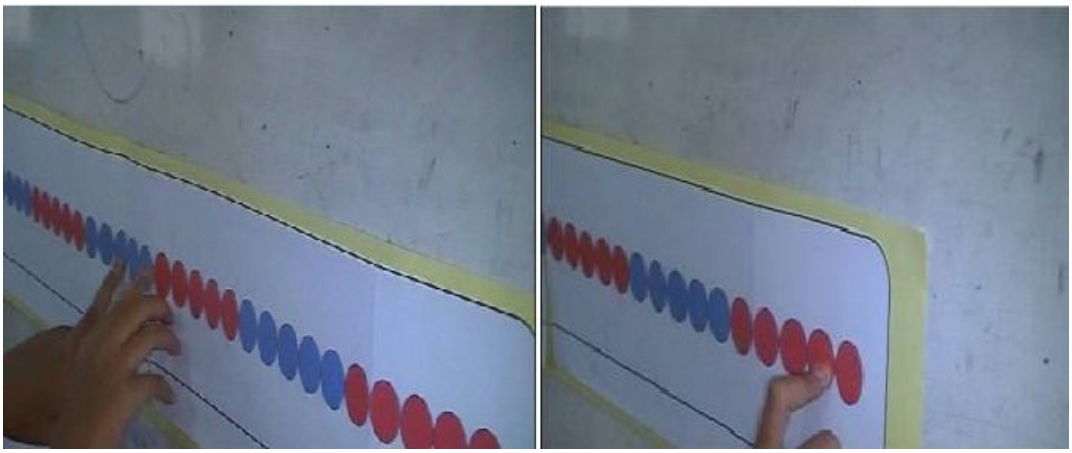
#Table 5.3. The comparison between the conjecture of students thinking and learning and the students' actual work of first activity of lesson 1 #

No		Conjecture of Students' thinking and learning	Actual learning of students
1	A	The students will answer 30 of which there are students, from the left to the right, counting the marbles by ones, some others count by twos, others count by fives, and the others count by tens.	Students answer 30 by using counting by ones strategy and counting by twos strategy
	B	The students count backward, i.e. from the right to the left of which there are students counting by ones, counting by twos, counting by fives, and the others count by tens.	Students counted using counted by ones strategy, counted by fives strategy, and counting by tens strategy.
	C	The students determine the answer by counting the marbles from the left by ones sixteen times, and the others count by fives, specifically, they count by fives three times and next count by ones one time.	Students counted till getting the sixteen marbles using counting by ones strategy, counting by twos strategy, and counting by fours.
	D	The students determine the answer by counting the marbles from the left by ones as many as twenty four times, and the others count by fives, specifically, they count by fives four times and next count by ones four times.	Students tried to get the twenty fourth marble using counting by ones strategy, counting by twos strategy, counting by fours strategy.
	E	Students count the marbles starting from the fifth marble and use count by ones strategy to count the marble as many as sixteen times and stop in the twenty first marble.	They started from the fifth marble and used counting by ones strategy to count the marbles till sixteen times. They said that, they stopped at the twenty first marble.
	F	Students count the marbles starting by the twentieth marble and use count-by ones strategy in counting the marble and stop in the eight marble.	The student who came forward was Fitrah who solved the problem using counting by ones strategy. Finally, he said that he stopped at 8.

In the table 5.3., we can conclude that some of our conjectures are fulfilled that is students counted the marbles using several strategies. Students always counted the marbles from the left to the right except when working on the problem counting marble from the right to the left.In doing counting from certain number to another number, the conjecture in which the students jump to the nearest five and doing counting by fives is not fulfilled because the students always use constant jump when counting. When counting marbles from certain number to the right, the students recognized that the stop position is larger number compared to the number in start position and when counting marbles from certain number to the left, the students recognized that the stop position is smaller number compared to the number in start position.

The next activity done by the students was solving the problems available in the worksheet given. As set before, the students were divided into group of two. The first problem that is 13 marbles + 16 marbles was solved first and several groups came forward to present their answers in the discussion session. The groups who came forward were the group of Attiya and Dwiki and also the group of Imelda and Aisyah who answered 29 and used counting by ones strategy which started from the thirteenth marble and to the twenty ninth marble. The following picture shows the process of Attiya in adding in which he stopped at the thirteenth marble first shown in the left picture and next he stopped at the twenty nineth marble shown in the right picture.

#figure 5.1. The process done by Attiya in adding marbles#



The second problem that is 29 marbles – 12 marbles was discussed. The group that came forward was the the group of Rahmat and Rizki who answered 17 marbles by counting from 29 to the left to 17 using counting by ones strategy.

In one of the closing part of this lesson, the teacher asked the students question related to the increase and the decrease of the number when counting to certain direction, i.e. counting to the right makes the number increase. All of the students agree that when counting to the right the number become increase and when counting to the left, the number become decrease.

By seeing the description of this activity, we can see that when adding marbles, the students count from the left to the right and when subtracting marbles, the students count from the right to the left. The following table shows the comparison between our conjectures in the second activity and the actual work of students:

#Table 5.4. The comparison between the conjecture of students thinking and learning and the students' actual work in the second activity of lesson 2#

Questions	**Conjecture of Students' thinking and learning**	**Actual learning of students**
1	Some students will solve it using algorithm way, meanwhile, the other students will solve it by utilzing the picture of marbles that is available in the problem and counting them using counting by ones-strategy	Students utilized the marble pictures. They started to find the thirtheenth marble and next counted by ones from til sixteen times to reach the twenty nineth marble.
2	Some students will solve it using algorithm way, meanwhile, the other students will solve it by utilzing the picture of marbles that is available in the problem and counting them using counting by ones-strategy	Students utilized the marble pictures. They started to find the twenty nineth marble and counted by ones till seventeen to get the twelfth marble.

Conclusion

In the table above, we can see that the conjecture in which the students used algorithm way is not fulfilled. It is caused by the sequence of marble which is available in the instructional activity which leads the students to the usage of marbles in counting. However, the conjecture in which the students utilized the marbles in counting, is completely fulfilled since the students have conception that is smaller numbers are in the left and larger number are in the right that is derived from the previous activity. Consequently, the students counted to the right when adding and counted to the left when subtracting. Moreover, the students just used counting by ones strategy because the starting number of each question is not 1 and in that case the students used constant jump. Here, we can also conclude that the activity supports the students to come up with the idea of several mathematical concepts: considering smaller numbers are in the left, larger numbers are in the right, counting to the right when adding, counting to the left when subtracting, and counting marbles with several different strategies.

5.7.2. Lesson 2: Making Number Line

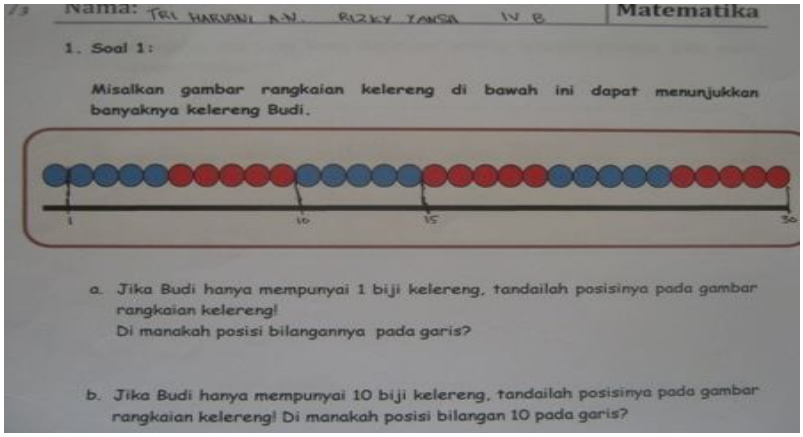
This lesson was conducted one day after the implementation of the first lesson. Before, the students solved the problems given, the teacher reminded the students that the position of numbers is in the right part of the marble. After that, the teacher gave the students worksheet containing several questions. In the first problem, without being guided, most of the students, including our focus group, Tri & Rizki, did well in determining the position of the numbers asked. They put mark in the marble sequence when Budi has certain number of marbles and corresponding the position of the mark to the line to determine the position of the number in the

line. In the question about the conclusion about the numbers in the line, nearly all the students take their own conclusions which were not expected by the researcher before, for instance, there are groups that wrote down completing the line with numbers from 1 to 30 as the conclusion, and there are groups also concluding that the position of number lay in the right of part of the marble. Except the group of Gina and Rahma which concluded that the numbers become larger to the right and become less to the left in the line. After the students had finished their work on the first problem, one of the session was that the teacher asked the group of Attiya and Dwiki (AD) to read the conclusion in front of the class. The discussion was shown by the following fragment of our video:

1. ***AD*** : to the left, the number increase and to the right, the number become
2. smaller
3. ***Teacher*** : is it right?
4. ***All students*** : wrong
5. ***Teacher*** : what about your respond, Gina, try to read the conclusion in the front of
6. the class
7. ***Gina*** : to the right become increase, and, in contrast, to the left more decrease.
8. ***Teacher*** : yes, right.
9. ***Teacher*** : so, if the position is in the right then it become.... ?
10. ***All students*** : large
11. ***Teacher*** : yes, or become increase. If the position is in the left then it become... ?
12. ***All students*** : small.

The figure 5.2. and the figure 5.3. shows the work of our focus group and the conclusion about numbers in number line made by the group of Gina respectively

#Figure 5.2. The work of the focus group in lesson 2#


Nama: TRI HARYANI A.N. RIZKY YANSA IV B

Matematika

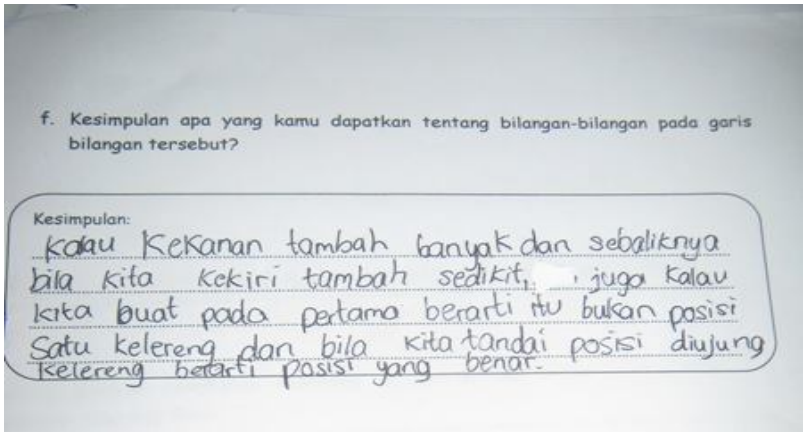
1. Soal 1:

Misalkan gambar rangkaian kelereng di bawah ini dapat menunjukkan banyaknya kelereng Budi.

a. Jika Budi hanya mempunyai 1 biji kelereng, tandailah posisinya pada gambar rangkaian kelereng!
Di manakah posisi bilangannya pada garis?

b. Jika Budi hanya mempunyai 10 biji kelereng, tandailah posisinya pada gambar rangkaian kelereng! Di manakah posisi bilangan 10 pada garis?

#Figure 5.3. The conclusion of Gina's group#


f. Kesimpulan apa yang kamu dapatkan tentang bilangan-bilangan pada garis bilangan tersebut?

Kesimpulan:
Kalau kekanan tambah banyak dan sebaliknya bila kita kekiri tambah sedikit, juga kalau kita buat pada pertama berarti itu bukan posisi satu kelereng dan bila kita tandai posisi diujung kelereng berarti posisi yang benar.

By observing these activities, we can interpret that most of the groups successfully corresponded the marble to the line in which they see the line as the representation of numbers. They mentally know that the larger numbers occupy the right space and the

smaller numbers occupy the left space of the line. The following table shows the comparison between our conjectures in this activity and the actual work of students:

#Table 5.5. The comparison between the conjecture of students thinking and learning and the students' actual work of first activity in lesson 2#

No		Conjecture of Students' thinking and learning	Actual learning of students
1	a	Students will put the mark in the right side of the first marble and put the mark for the position of 1 by corresponding the position of the first marble in number line.	Students put mark in the marble sequence when Budi has certain number of marbles and corresponding the position of the mark to the line to determine the position of the number in the line.
	b	Students will put the mark in the right side of the tenth marble and put the mark for the position of 10 by correspnding the position of the tenth marble in number line.	
	c	Students will put the mark for the position of 15 in number line by corresponding it to the fifteenth marble.	
	d	Students will put the mark of the position of 30 in number line by corresponding it to the right side of the thirtieth marble.	
	e	Students will complete the number line with numbers from 1 to 30 by applying their procedures of part a, b, c, and d.	
	f	Several students take the conclusion that number become larger if they are in the right and become less if they are in the left. However, there are students who take unclear conclusion.	Nearly all the students take their own conclusions which were not expected by the researcher before, for instance, there are groups that wrote down completing the line with numbers from 1 to 30 as the conclusion, and there are groups also concluding that the position of number lay in the right of part of the marble. Except the group of Gina and Rahma which concluded that the numbers become larger to the right and become less to the left in the line

By seeing the comparison between our conjectures and the actual learning of the students in the table 5.5., the conjectures are fulfilled since the students mentally know how to correspond the quantity of marbles to the number line. By seeing the numbers in number line, they concluded that the larger numbers occupied the right space and the smaller numbers occupied the left space of the number line.

After the first problem had been discussed, the students continued to work on the second problem. All of the students found no difficulty in answering the first four problems of which they determine the position of 1, 10, and 20 in the marble sequence and in the line and also considered that 0 represents the state in which Budi has no marble. Most of the groups also found no difficulty in answering the position of 21, 22, and 25 in the line. They drew five more marbles to the right and also extended the line to the right. The problem occurred when they faced the question g, that is determining the position in the marble sequence the state of lack of 1 marble, the number representing that state, and also the position of the number in the line. Since there are groups considering that the state of question g was the continuance of the state occurring in the question f, i.e. when Budi has 25 marbles and now he is lack of 1 marble, then his marble is as many as 24. This type of answer was had by the group of Indira & Fariz. Consequently, they didn't know how to determine the position of the number in the line. Moreover, there were also several groups, e.g. Rendi's group which didn't have idea how to work with the negative quantity. In the other hand, some groups did well in these problems, for instance, the group of Teo &

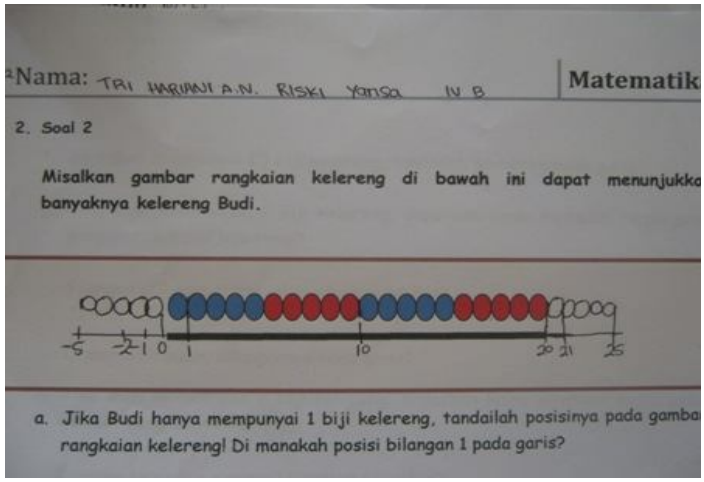
Dini and also the group of Tri and Rizki successfully drew the marble to the left and, considered that -1 represented the state of lack of 1 marble and determine the position of -1 in the line as well as they did the same procedure to the question h and question i in determining the position of -2, and -5.

By considering that several groups didn't have idea of certain problems, to explore their idea and to assure that the groups actually understood the concept of number line, then the researcher conducted interview to ask several questions to the groups. The interview with Indira's group is shown by the following fragment of our video and audio:

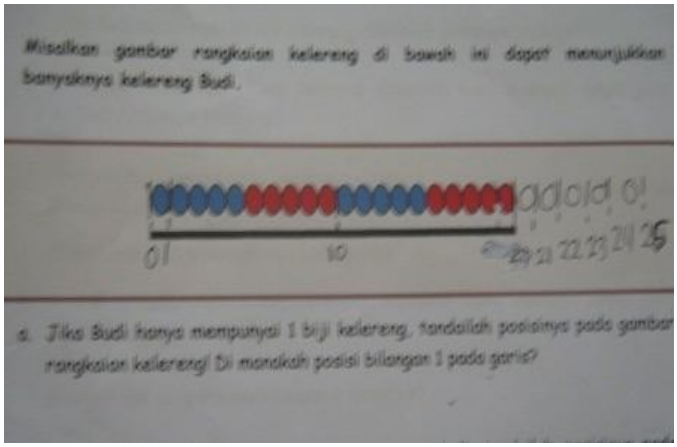
1. ***Researcher*** : If Budi doesn't have any marble, where is the position of it in the line?
2. ***Indira*** : here (*pointing out the most-left part of the line*)
3. ***Researcher*** : where is the position if Budi is lack of 1 marble in the picture?
4. ***Indira*** : here (*pointing out the left part of the marble sequence*)
5. ***Researcher*** : give sign!
6. ***Indira*** : (*drawing one circle in the left*)
7. ***Researcher*** : what is the number?
8. ***Indira*** : negative one

By seeing the description of this activity, we can interpret that all groups successfully corresponded the marbles to the line as they see that line can represent numbers. Most of the groups, including our focus group, also were aware of the need to draw more marbles to the right based on the conception that the more marbles Budi has, the more marbles they need to draw in the right. In determining the position of lack of certain marbles, our focus group successfully identified that the position is in the left and recognized that the state is represented by negative numbers. However, for some other groups which identified that, e.g. the lack of one marble was represented by 24, the students thought that the state was related and as the continuance to that of state in the previous question, i.e. Budi has 25 marbles. However, they actually knew that lack of marble without connecting that state to another state was represented by negative number. The figure 5.4 and figure 5.5. show the focus group's and Rendis' group work in making number line respectively.

#Figure 5.4. The group of Tri's work in making number line#



#Figure 5.5. The group of Rendi's work in making number line#



The comparison between our conjectures and the actual learning of the students shown in the following table:

#Table 5.6. The comparison between the conjecture of students thinking and learning and the students' actual work of the second activity in the lesson 2 #

No	Conjecture of Students' thinking and learning	Actual learning of students
a	Students will put the mark in the right side of the first marble and put the mark for the position of 1 by corresponding the position of the first marble in number line.	Students put mark in the right side of the first marble and connect to the mark straightly to the line below it.
b	Students will put the mark in the left side of the first marble. They consider that the state is represented by zero and and put the mark for the position of the number by corresponding the left part of the first marble in marble sequence to the line.	Students put mark in the left side of the first marble and connect to the mark straightly to the line below it. They also know that the state is represented by zero.
c	Students will put the mark in the right side of the twentieth marble and put the mark for the position of 20 by correspnding the position of the twentieth marble in number line.	Students will put the mark in the right side of the twentieth marble and put the mark for the position of 20 by correspnding the position of the twentieth marble in number line.
d	Several students have no idea in determining the position of having 21 marbles in marble sequence. The other students add one marble to the right and are also be able to mark the position of it in the line.	Students draw one more marble to the right and put mark in the twenty first marble, and extend the line to the right as well and correspond the mark to the line.
e	Several students have no idea in determining the position of having 25 marbles in the line. The other students add five more marbles to the right and are also be able to mark the position of the number in the line.	Students draw five more marbles to the right and put mark in the twenty fifth marble, and extend the line to the right as well and correspond the mark to the line.
f	Several students have no idea in determining the position of lacking of 1 marble. The other student draw one marble in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -1.	Several students considered that lack of one marble means 24 marble. The other students draw one marble in the left and determine the position of the state in the line by considering that the number representing that state is -1.

g	Several students have no idea in determining the position of lacking of 2 marble. The other student draw two marbles in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -2.	Several students have no idea in determining the position of lacking of 2 marble. The other student draw two marbles in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -2.
h	Several students have no idea in determining the position of lacking of 2 marble. The other student draw two marbles in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -2.	Several students have no idea in determining the position of lacking of 2 marble. The other student draw two marbles in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -2.
i	Several students have no idea in determining the position of lacking of 5 marble. The other student draw two marbles in the left and are also be able to determine the position of the state in the line by considering that the number representing that state is -5.	

Conclusion

By seeing the table 5.6., our conjectures in which the students are able to correspond the magnitude of number of marbles to the line representing positive numbers are fulfilled. Because the students are aware to the concept: “larger numbers are in the right”. Besides that our conjectures in which the students draw some marbles to the left to determine the position of lack of certain marbles, the number representing that state, and to correspond the numbers to the number line are fulfilled by our focus group and several other groups since they had conception that lack of marbles are less in quality than having no marble. However, they are not fulfilled by several groups since the students considered that the state of lack is the continual event from the previous problem. Therefore, we can conclude that this lesson supports the development of students’ understanding of the concept of number line which can represents both positive numbers and negative numbers.

5.7.3. Lesson 3: Positioning integers in number line

This lesson was conducted as the continuance of the lesson 2. To avoid the students having no idea of the problems in this lesson, the teacher explained several things related to the lesson 2 before the students work on the problems. In that session, by utilizing the picture of marble sequence and line, teacher asked several useful questions so all of the students understand that the line can represent numbers, the position of numbers, both positive numbers and negative numbers, in the line, and the fact that the numbers increase to the right and decrease to the right.

After that, the students were pleased to work on the problems given. Here, we observed some groups including our focus groups: group of Teo & Dini, Tri & Rizki, Gina & Rahma, and Indira & Fariz. Meanwhile, the teacher controlled the other groups. In the first problem, all the observed groups put 5 in the middle of the line without writing and putting down numbers 1, 2, 3, and 4. Here, the following transcription taken from our video segment of which Theo & Dini presented their work.

1. ***Teacher*** : what is the reason why you put 5 in the middle?
2. ***Theo*** : (*pointing out the line from the left and counting till 5*) because 1, 2, 3, 4,
3. 5 and 1, 2, 3, 4, 5 (*pointing out the line from 5 and counting till 10*)

It can be suggested that, Theo's group considered that it jumped 5 from 0 to 5 and from 5 to 10.

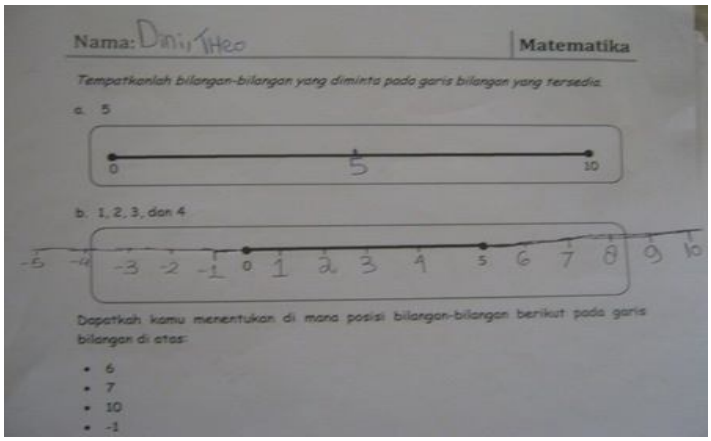
In the second problem, the groups put 1, 2, 3, and 4 in the available line. In putting the numbers 6, 7, and 10, the groups extended the line to the right and in putting the numbers from -1 to -5, the groups extended the line to the left. Here, the transcription taken from our video segment of which researcher interviewed Theo & Dini aiming to know their idea when Theo tried to solve the problem putting the numbers 6, 7, and 10 and Dini tried to solve the problem putting the numbers -1, -2, -3, -4, and -5.

1. ***Researcher*** : (*seeing Theo extended the line to the right*) why do you extend the line to
2. the right?
3. ***Theo*** : because it is not enough (*referring the line which is not enough*)
4. ***Researcher*** : what about the position of 7?
5. ***Theo*** : in the right.
6. ***Researcher*** : where is -1?
7. ***Dini*** : in the left (*referring the left part of 0*)
8. ***Researcher*** : why is -1 in the left ?
9. ***Dini*** : because to the right, increase, to the left, decrease.
10. ***Researcher*** : so, -1 is less than 0?

11. ***Dini*** : yes

The following pictures show the work of Dini's group in lesson 3

#Figure 5. 6. The group of Dini's work of problem a and b#



In working on the last problem, Tri's group and Indira's group directly put -5, and -10 in the proper position without writing down the other numbers in between such as -1, -2, ect. Meanwhile, Theo's group and Gina's group firstly wrote down numbers -1,- 2, -3, and -4 before putting -5 and wrote down -6, -7, -8, and -9 befor putting -10 in the extended line. This following conversation is our transcription taken from the fragment of our video in which the researcher interviewed Indira's group to explore more the idea of Indira's group.

1. ***Researcher*** : is it the same from 0 to 5 and 0 to -5?
2. ***The group*** : (*showing no response*)
3. ***Researcher*** : how many jump from 0 to 5?

4. ***Indira*** : five

5. ***Researcher*** : how many jump from 0 to -5?

6. ***Indira*** : five also

7. ***Researcher*** : so, is it the same from 0 to 5 and 0 to -5?

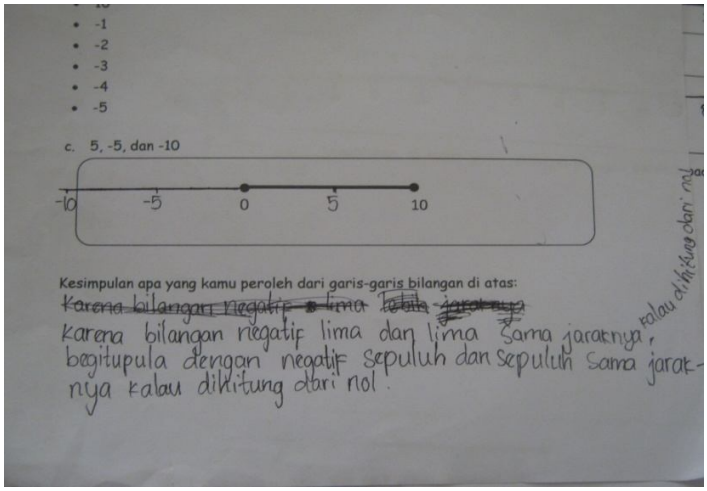
8. ***Indira*** : same

9. ***Researcher*** : is it the same from 0 to 10 and 0 to -10?

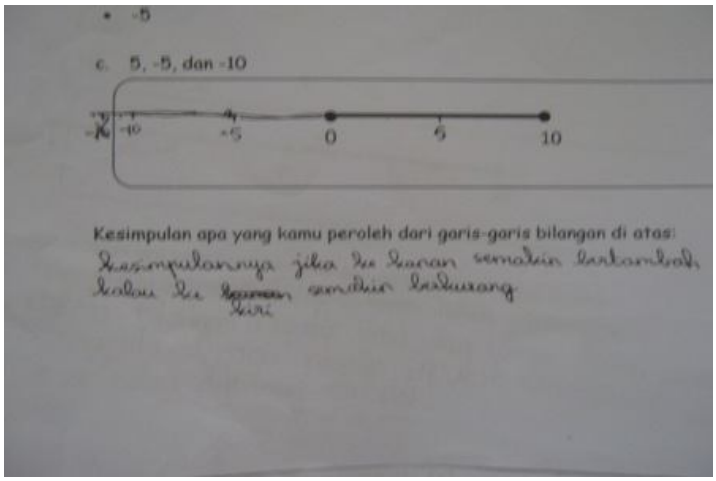
10. ***Indira*** :same.

The following pictures, figure 5.7., figure 5.8., and figure 5.9. show the work of several groups on problem c.

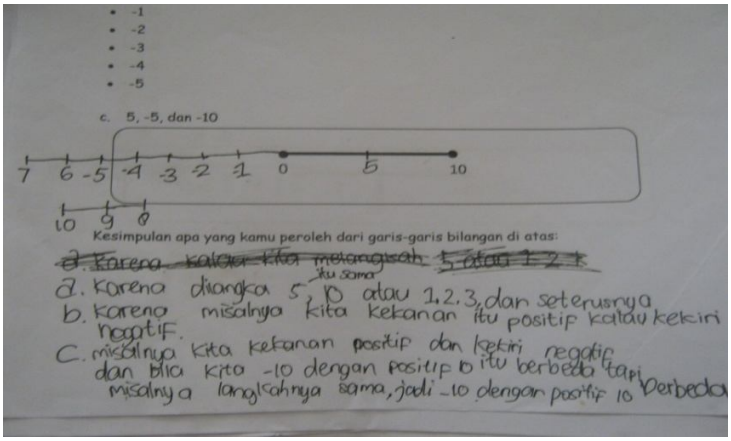
#Figure 5.7. The Indira's group work in problem c of lesson 3#



#Figure 5. 8. The focus group's work in problem c of lesson 3#



#Figure 5. 9. The focus group's work in problem c of lesson 3#



By observing this lesson, it can be seen that students have no difficulty in determining the position of numbers without the help of marble sequence. The students' knowledge of numbers in number line is really useful for the students in determining the position of positive numbers and negative numbers in line.

The following table shows the comparison between our conjectures of this lesson and the actual work of the students.

#Table 5.7. The comparison between the conjecture of students thinking and learning and the students' actual work of the lesson 3 #

No	Conjecture of Students' thinking and learning	Actual learning of students
a	In determining the position of 5, several groups take the arbitrary position of 1, 2, 3, 4, and next 5. Some other groups take the middle position of the line and put mark of it as the position of 5. Since they think that 0 is added by 5 is 5 and 5 is added also by 5 is 10 so 5 is exactly in the middle of the line.	Students put 5 in the middle of the line since the students think that there are five numbers in between 0 and 5 and there are also five numbers between 6 and 10.
b	The groups consecutively put 1, 2, 3, and 4 between 0 and 5. Next, several groups put 6, 7, 10, -1, -2, and -5 in the range between 0 and 5 in arbitrary position. The other groups then extend the line to the right to put 6, 7, and 10. In determining the position of the negative numbers, -1, -2, and -5, they extend the line to the left	The groups consecutively put 1, 2, 3, and 4 between 0 and 5. Next, several groups put 6, 7, 10, -1, -2, and -5 in the range between 0 and 5 in arbitrary position. In putting the numbers 6, 7, and 10, the groups extended the line to the right. In putting the numbers from -1 to -5, the groups extended the line to the left.
c	Several groups put all the numbers in the range between 0 and 10. Meanwhile, the other groups put 5 exactly in the middle of the line and extend the line to the left. These groups put -5 in the position of which its distance from zero is the same as the distance between 0 and 5. As well as the process of determining the position of -5, these groups also determine the position of -10 based on the distance from 0 to 10. There are also groups which need to write down, for instance 1, 2, 3, 4 to determine the position of 5 and write down -1, -2, -3, and -4 to determine the position of -5.	There is a group who directly put 5, -5, and -10 without writing down other numbers. There are also the groups, for instance, that write down -1, -2, -3, and -4 before writing down -5.

Conclusion

By seeing the table above, we can conclude that our conjectures to which the students did well of this lesson are fulfilled. It is caused by the fact that the students have already had conception that small numbers occupy left space of the line and large numbers occupy right space of the line which are derived from the previous lesson. Concerning the conjectures to which the students put the numbers in arbitrary position which are not fulfilled, because some students were given supporting questions from the teacher, i.e. where is the position of negative numbers?, where is the position of larger numbers?, are the numbers in the left or in the right? which led them to think and to answer the problems correctly. Therefore, this lesson supports students to develop a number line representing both positive numbers and negative numbers and to have conception that negative numbers are in the left space and positive numbers are in the right space.

5.7.4. Lesson 4: The addition of two negative numbers.

In the implementation of the lesson, as usual, the students worked in pairs. The teacher controlled the students when they are working. In the first problem, all focus groups did well in answering the question and most of them were able to use model of number line as the visualization of their answers.

In guiding the students, the teacher asked several useful questions aimed at helping the students to think and to work more mathematically and exploring their ideas. One of the processes of guiding the students is shown by the following transcription of our video segment of which Dona's group have answered six marbles:

1. ***Teacher*** : what is the process in getting the six marbles?
2. ***Chaca*** : (*telling Dona*: 4 + 2, because Dona was the writer)

   (Dona then wrote down 4 + 2 that results 6).

Moreover, to know the idea of one of the groups, the researcher did interview to Tri's group. The conversation is shown by the following transcription taken from the segment of our video:

1. ***Researcher*** : how do you get 6? There must be a reason. What did you do so you get
2. 6? What operation you used?
3. ***Tri*** : plus
4. ***Researcher*** : what plus what?

5. ***Tri*** : four plus 2
6. ***Researcher*** : could you use number line?

   Tri directly drew number line.
7. ***Researcher*** : if Eka has 4 marbles, where is its position in the line?

   (Tri wrote down some numbers in the line from 1 to 4).
8. ***Researcher*** : sign it.

   (Tri then made a circle to circle 4 as the sign)
9. ***Researcher*** : now, if he has 2 more marbles, is it to the right or to the left?
10. ***Tri*** : to the right.
11. ***Researcher*** : how many jump?
12. ***Tri*** : two times

In the second problem, the researcher interviewed Tri's group again to know the idea of the students.

1. ***Researcher*** : Anton was lack of 3 marbles and today he is lack of 4 marbles. What is

2. the marble shortage total of Anton?

(Tri's group didn't show any response).

3. ***Researcher*** : could you work using number line?

(Tri then drew a line and the researcher asked the group).

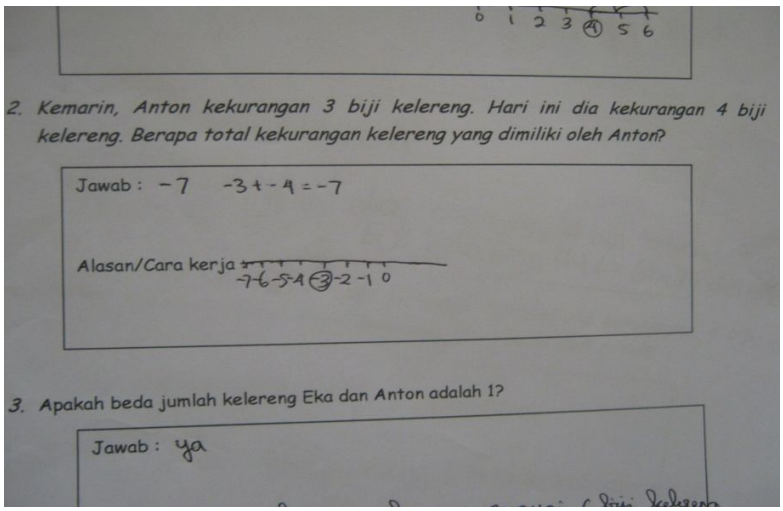
4. ***Researcher*** : where is the position of lack of 3 marbles in the line?

(Tri then wrote down the numbers in the line from 0 to the left to -3)

5. ***Researcher*** : if we lack of marble again, we jump to what direction in number line? To
6. the right or to the left?
7. ***Tri*** : to the left
8. ***Research*** : how many jump?
9. ***Tri*** : as many as four
10. ***Researher*** : ok, do you know the word “total”, it means the whole, isn’t it?.
11. Therefore, it is added, isn’t it? It means negative three plus negative four
12. equals ...?
13. ***Tri*** : negative seven.

However, there is a group firstly answering that Anton is lack of 7 marbles and it illustrates it using number line by starting from -3 to -7. The figure 5.10 shows the focus group’s work of problem number 2.

#Figure 5.10. The focus group's work on the second problem of lesson 4#



In the third problem, all groups considered that the difference is 1 because most of them saw the number 6 in the case of Eka and the digit 7 in the -7 in the case of Anton. However, when being guided by the teacher and the researcher, they eventually understood that the difference is not 1.

In this lesson, we can see that the role of the teacher is to make the students think mathematically and make the students can jump to the conclusion gained by the problem. The students' knowledge of number line in the previous lessons help the students to determine what direction to move. The context also quite supports the students in understanding the addition of two negative numbers and the model helps the students to show that negative numbers added by negative numbers is also negative numbers. The following picture shows the work of our focus group:

The following table shows the comparison between our conjectures of students' thinking and learning of this lesson and the actual work of the students.

#Table 5.8. The comparison between the conjecture of students thinking and learning and the students' actual work of the lesson 4 #

No	Conjecture of Students' thinking and learning	Actual learning of students
a	The students firstly answer that Eka totally has 6 marbles and they think that 6 marbles is obtained by adding four by two which gets six.	The students firstly answer that Eka totally has 6 marbles and they think that 6 marbles is obtained by adding four by two which gets six.
b	• Several students answered that Anton is totally lack of seven marbles since they think that the problem is about adding two positive numbers that is 3 + 4 = 7. • The other students also get that Anton is totally lack of seven numbers and then they use number line and consider that lack means negative, so they start at -3 and move to the left as many as 4 steps to get -7.	Most of the students answering that Anton totally lacks of seven marbles derived from 3 plus 4. However, several of them used number line and represented the state by negative numbers and got negative 7 since they know that lack means negative.
c	• Students consider that the difference is 1 since they see 6 and 7 as the answers in respectively part a and part b. • However, some of those students will think critically by considering that lack of seven mables can be represented by the number negative seven. • The other students consider that the difference is 13 by seeing the number line of which they count from 6 to -7 or vice versa.	Nearly all students agreed that the answer is 1, however, there are several students that considered the difference is 13.

By seeing the comparison in the table 5.8., the conjectures to which the students answered six marbles and explained the reason how to get it is fulfilled because the concept of addition of positive quantities is easy for the students. The conjecture to which the students visualize the addition of positive numbers is fulfilled because the students can recognize what direction they should move in number line.

Conclusion

The conjecture to which the students will answer 7 for the lack total of Anton's marble is fulfilled since the students recognized that the problems concerned the addition of two numbers and were able to solve the problems contextually. The conjecture to which the students visualize the addition of two negative numbers representing the state of lack of marbles and get -7 derived from -3 and -4 is fulfilled because the students represented the lack of marbles by negative numbers. They also knew that the state of lack again meant that Anton is more lack and caused the move in number line was directed to the left. The role of the teacher that is asking too much questions that can stimulate the thinking of several students led the students to come up to the concept of the addition of negative numbers.

Therefore, this lesson in which there are equilibrium context and the use of number line to count, support the students' thinking in answering the addition of two negative numbers formally.

5.7.5. Lesson 5: The addition of negative numbers and positive numbers.

This lesson was conducted after the implementation of the lesson 4. It consists of two activities, i.e. one activity concerns negative numbers added by positive numbers and the other is about positive numbers added by negative numbers. As usual, the students worked in their own pair. The researcher and the teacher equally shared the groups to observe. The teacher came to see how the group of Imelda worked because she saw that Imelda answered yes that is Ilham is still lack of marbles. The following conversation between the teacher and Imelda's group shows the transcription taken from our video segment.

1. ***Teacher*** : where is the position of lack of 5 in the line
   (Imelda then wrote numbers from -1 to the left to -5)
2. ***Teacher*** : today, he got several new marbles from his friend. Is he still lack of marbles if
3. he gets new marbles as many as 3 marbles? How many marbles of which he is
4. lack?
5. ***Imelda*** : (*nodding*) 2 marbles
6. ***Teacher*** : if he gets new marbles, is it to the right or to the left?
7. ***Imelda*** : to the right

Then, the teacher wanted to know how they got lack of 2 marbles using number line. Imelda started from -5 and she knew that it goes to the right since Ilham gets new marbles. However, she didn't know how to work with it. She made a jump of two from -5 to -3. However, after receiving too many questions from the teacher that stimulates the thinking of the students, the group eventually did correct procedure.

Meanwhile, in our focus group, it firstly answered yes for the problem. When being asked to explain the reason, it showed a convincable reason. However, when trying to visualize using number line, it felt hesitant how to move from -5. The following transcription taken from our video segment showed the conversation between the researcher and the focus group of which Tri has drawn number line with number -1 to the left to -5.

1. ***Researcher*** : is it to the right or to the left if he got new marbles?
2. ***Tri*** : to the left
3. ***Researcher*** : why is to the left if getting new marbles?

   (The group showed no response).
4. ***Researcher*** : does the number of marble increase?
5. ***Tri*** : increase
6. ***Researcher*** : so, is it to the left or to the right?

   (The group showed no response).
7. ***Researcher*** : Rizki, to the left or to the right?
8. ***Rizki*** : to the right

9. ***Researcher*** : why is it to the right?

   (Rizki didn't respond)

10. ***Researcher*** : Tri, do you agree to the Rizki's answer?

    (Tri looked hesitant)

11. ***Researcher*** : if the marble become increase, is it to the right or to the left?

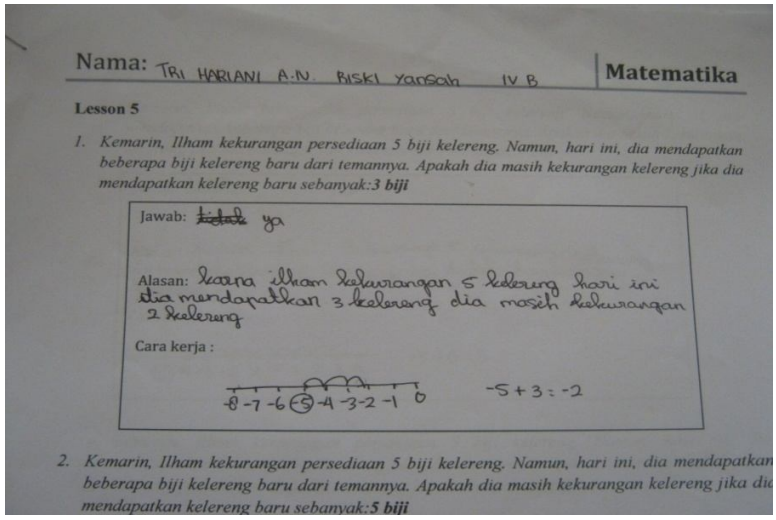
12. ***Tri*** : to the right.

Then, Tri drew three more jumps and stopped at -2. In the group of Gina & Rahma, it answers the lack of 2 by using number line starting from -5 representing the state of lack of 5 and moved to the right as many as three jumps and stopped at -2. In determining the operation of the problem, the group correctly formulated the mathematical notation of it which is -5 + 3 = -2.

However, there was also a group that is group of Theo and Dini which directly answered -5 + 3 = -2 after answering yes to state that Ilham is lack of marble.

In the next three problems, the focus group followed the procedure of problem 1 and did quite well in answering the problems. The figure 5.11. shows how the focus group worked.

#Figure 5.11. The work of the focus group on problem 1 lesson 5#



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Lesson 5

1. *Kemarin, Ilham kekurangan persediaan 5 biji kelereng. Namun, hari ini, dia mendapatkan beberapa biji kelereng baru dari temannya. Apakah dia masih kekurangan kelereng jika dia mendapatkan kelereng baru sebanyak:* ***3 biji***

Jawab: ~~tidak~~ ya

Alasan: karna ilham kekurangan 5 kelereng hari ini dia mendapatkan 3 kelereng dia masih kekurangan 2 kelereng

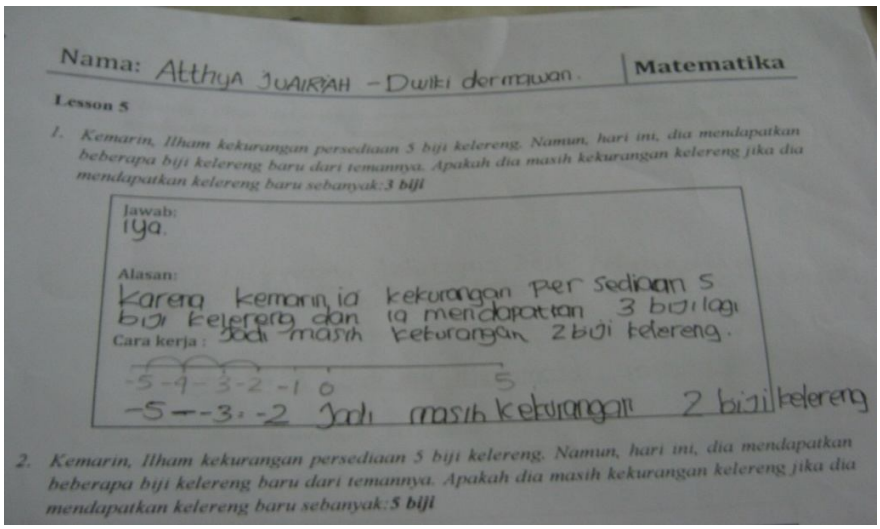
Cara kerja :

-8 -7 -6 -5 -4 -3 -2 -1 0 -5 + 3 = -2

2. *Kemarin, Ilham kekurangan persediaan 5 biji kelereng. Namun, hari ini, dia mendapatkan beberapa biji kelereng baru dari temannya. Apakah dia masih kekurangan kelereng jika dia mendapatkan kelereng baru sebanyak:* ***5 biji***

However, there was also a group which didn't write down the mathematical notation for the formal operation used in the problem 1. The group of Attiya wrote down -5 – (-3) = -2. The figure 5.12. below shows the work of Attiya's group

#Figure 5.12. Attiya's work on problem 1 of lesson 5#



Nama: Atthya JUAIRIAH - Dwiki dermawan. | Matematika

Lesson 5

1. *Kemarin, Ilham kekurangan persediaan 5 biji kelereng. Namun, hari ini, dia mendapatkan beberapa biji kelereng baru dari temannya. Apakah dia masih kekurangan kelereng jika dia mendapatkan kelereng baru sebanyak:* ***3 biji***

Jawab: iya.

Alasan: Karena kemarin ia kekurangan persediaan 5 biji kelereng dan ia mendapatkan 3 biji lagi jadi masih kekurangan 2 biji kelereng.

Cara kerja :

-5 -4 -3 -2 -1 0 5

-5 - -3 = -2 Jadi masih kekurangan 2 biji kelereng

2. *Kemarin, Ilham kekurangan persediaan 5 biji kelereng. Namun, hari ini, dia mendapatkan beberapa biji kelereng baru dari temannya. Apakah dia masih kekurangan kelereng jika dia mendapatkan kelereng baru sebanyak:* ***5 biji***

By seeing the description above, we can see that several students still don't understand technically in using number line. Since they need much support from the teacher in answering the problem mathematically in which they used number line and to write it down in formal operation. In our focus group, the students need to think contextually before working with number line, i.e. identifying that getting marbles causes the addition of marbles and consequently it causes that the direction of move in number line goes to the right. Meanwhile, the group of Theo & Dini was supported by the context to determine the formally mathematical answer. Here, the context can support students in understanding the concept of negative numbers added by positive numbers and the model can help students to show the process and the result of the question. In the case of Attiya's group, the interpretation of context and the use of number line don't assure that the students can write the correct mathematical symbol. Because Attiya's group didn't refer to the story in the context of which getting means adding. The following table shows the comparison between our conjectures and the actual work of students in this activity.

#Table 5.9. The comparison between the conjecture of students thinking and learning and the students' actual work of the first activity in lesson 5 #

No	Conjecture of students' thinking and learning	Actual work of the students
a	• Several Students answer that Ilham still has lack of 2 marbles since in their minds they think that the addition of 3 marbles doesn't completely fulfill the lack of Ilham's marble. In visualizing their answers, several students draw	Students firstly answered that Ilham is lack of 2 marbles. When they were asked to illustrate

	number line and start with -5 and move to the right as many as 3 steps to get -2. They then conclude that -5 + 3 = -2. • The other students consider that Ilham has 3 marbles since Ilham gets 3 new marbles.	using number line, they started at -5 and moved to the right to -2. They then concluded that -5 + 3 = -2
b	• Several students answer Ilham is not lack of marble anymore since they think that the addition of 5 marbles completely fulfill the lack of Ilham's marble. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 5 steps to get 0. They then conclude that -5 + 5 = 0. • The other students consider that Ilham has 5 marbles since Ilham gets 5 new marbles.	Quite the same as the actual work of the students in part a
c	• Several students answer that Ilham doesn't have lack of certain number of marbles; instead, Ilham now has 3 marbles. Some other students don't have idea at first. Therefore, they visualize the problem using number line. In visualizing, several students draw number line and start with -5 and move to the right as many as 8 steps to get 3. They then conclude that -5 + 8 = 3 • The other students consider that Ilham has 8 marbles since Ilham gets 8 new marbles.	Quite the same as the actual work of the students in part a
d	• Students answer that Ilham doesn't have lack of certain marbles; instead, Ilham now has 5 marbles. In visualizing their answers, several students draw number line and start with -5 and move to the right as many as 10 steps to get 5. They then conclude that -5 + 10 = 5. • The other students consider that Ilham has 10 marbles since Ilham gets 10 new marbles.	Quite the same as the actual work of the students in part a

Conclusion

By seeing the table above, we can see that our conjectures to which the students answer yes or no about whether Ilham is still lack, are fulfilled because the students really understand the problem and can answer it contextually with convincable reason. In visualizing the contextual answer using number line, some students felt difficulty and needed help from teachers or researchers as well as in determining the formal notation of the operation elicited from the problem. However, our focus group and the group of Theo and Dini directly determined the formal notation of the operation. Here, the context can support students in understanding the concept of negative numbers added by positive numbers and the model can help students to show the process and the result of the question.

The next activity was working on the second worksheet that covers the competency of positive numbers added by negative numbers. Considering the limited time, this activity was conducted in the next day after the first students' worksheet had been done. In working the problems, most of them answered yes for all the problems. Here, the following transcription taken from the segment of our video showing when the researcher interviewed the focus group:

1. ***Observer*** **:** so, the answer of number 1 is the same? What question that
2. upper number line stand for?

3. ***Tri*** : three added by negative five.
4. ***Observer*** : three added by negative five. Where does it start?
5. ***Tri*** : three
6. ***Observer*** : why is it to the left?
7. ***Tri*** : because it is lack of five
8. ***Observer*** : and the second one?
9. ***Tri*** : negative five
10. ***Observer*** : added by three equals negative two. Why is it to the right?
11. ***Tri*** : because it increases

In another case, the teacher observed several students and took care the students who couldn't determine whether the result of -5 + 3 is the same as that of 3 + (-5). For instance, in the group of Rendi and Rizki which could determine that -5 + 3 equals -2 using number line, however, it has no idea how to work with 3 + (-5). The teacher then came to the group and asked questions that can stimulate them to think and to work. Here, the following transcription taken from our video fragment showing the conversation between the teacher and the group.

1. ***Teacher*** : what is the number? added by what negative number?
2. ***Rendy*** : three
3. ***Teacher*** : no, this is 3, isn't it? Is 3 positive or negative? Positive or
4. negative? 3 is positive number, isn't it?
5. ***Rizki*** : (*nodding*)

6. ***Teacher*** : added by what negative number? negative .... (giving clue)

7. ***Rizki*** & ***Rendy*** : five

8. ***Teacher*** : so, how many jump for that?

9. ***Rendy*** : eight

10. ***Teacher*** : just count. What direction if it is added by negative? To the

11. left or to the right?

12. ***Rizki*** : left

13. ***Teacher*** : ok, left, go on

14. ***Rizki*** : two more (*telling Rendy*) two more

15. ***Teacher*** : where does it start kids? That negative five. Added, added by

16. negative five. Added by negative five, where is it (*the stop position*) to

17. what extent you count?

18. ***Rizki*** : from here (*pointing at 0*)

19. ***Teacher*** : Ramadhan, count it! Count.

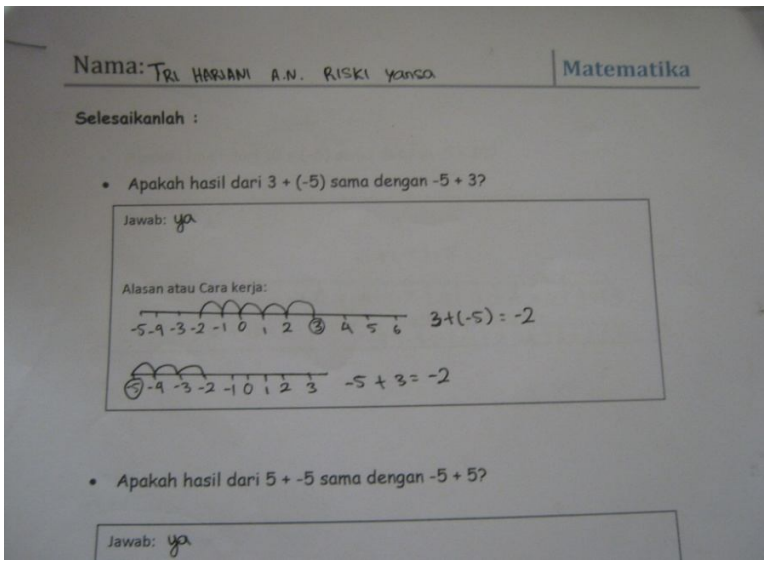
(*Rizki made two more jumps*).

20. ***Teacher*** : where does it stop?

21. ***Rizki*** : negative two.

The figure 5.13. shows the focus groups' work on the first problem:

#Figure 5. 13. The work of the focus group on problem 1#



By seeing the description above, we can interpret that in doing the operation, our focus group refers to the equilibrium context to work on number line, i.e. negative means lack, and lack means decreasing and moving to the left in number line. Conversely, adding positive numbers means increasing and moving to the right in number line. In the group of Rendi and Rizki, it felt difficulty to determine the result of 3 + (-5) since it didn't refer back to the concept of what the addition involving negative number means in the term of context, i.e. lack of something and what it means to the direction of move.

Here is the comparison between our conjectures and the actual learning of the students shown in the following table.

#Table 5.10. The comparison between the conjecture of students thinking and learning and the students' actual work of the second activity in lesson 5 #

No		Conjecture of students' thinking and learning	Actual work of the students
2	a	• Several students will answer yes and they used number line to prove it. In the problem -5 + 3, they start at -5 and move to the right to -2. Meanwhile, in the problem 3 + -5, they start at 3 and move to the left to -2. • Several students answer no since they think that -5 + 3 equals -2 but 3 + -5 equals 8.	Students considered that the answer is the same since they similarly result -2. In the problem -5 + 3, they start at -5 and move to the right to -2. Meanwhile, in the problem 3 + -5, they start at 3 and move to the left to -2.
	b	• Students will answer yes and they used number line to prove it. In the problem -5 + 5, they start at -5 and move to the right to 0. Meanwhile, in the problem 5 + -5, they start at 5 and move to the left to 0. • Several students answer no since they think that -5 + 5 equals 0 but 5 + -5 equals 10.	Quite the same as the actual learning of students in part a
	c	• Students will answer yes and they used number line to prove it. In the problem -5 + 8, they start at -5 and move to the right to 3. Meanwhile, in the problem 8 + -5, they start at 8 and move to the left to 3. • Several students answer no since they think that -8 + 5 equals -3 but 5 + -8 equals 13.	Quite the same as the actual learning of students in part a
	d	• Students will answer yes and they used number line to prove it. In the problem -5 + 10, they start at -5 and move to the right to 5. Meanwhile, in the problem 10 + -5, they start at 10 and move to the left to 5. • Several students answer no since they think that -10 + 5 equals -5 but 5 + -10 equals 15.	Quite the same as the actual learning of students in part a

Conclusion

By seeing the table 5.10., we can see that our conjectures in which the students answer yes with correct procedures in using number line are fulfilled since the students understand that the movement in number line in the problem can be referred back to the equilibrium context and the use of number line to help them to count. Although, there are students who couldn't directly think how they work, but when being guided by the teacher to refer back to the concept of numbers in number line, they eventually know how to work with the problems. Therefore, we can conclude that the equilibrium context is useful for the students to work using number line and next to find the final result of the question using number line.

5.7.6. Lesson 6: *Excercise on "Naked Task"*

This lesson was conducted in the next day after the fifth lesson had been conducted. This lesson is in the form of activity in which the students solved the tasks related to the addition of integers covering all competencies: the addition of positive numbers, addition of negative numbers and positive numbers, and also the addition of negative numbers. In this lesson, considering the time constraint and the number of questions which are so many, the students were divided into several groups of which each group consists of either four or five students. In the first problem which is about the addition of two positive numbers, all groups had correct answers and used number line to solve the questions. Here, most of the groups firstly answered the result and then used number line to prove whether it is correct. In using number line, the groups have various strategy to solve the problem. Here, the following transcription taken from the segment of our video showing the teacher involved the conversation with Imelda's group when the group worked on the question 1b:

1. ***Teacher*** : what is 11 added by 15?
2. ***Imelda*** : 26
3. ***Teacher*** : at what number it stops in the number line?
4. ***Imelda*** : twenty six
5. ***Teacher*** : (*seeing the jump from 11 to 20*) what jump you used in here? I mean 22
6. here, ok, what jump from 11 to 22?

7. ***Imelda*** : eleven
8. ***Teacher*** : from 22 to 25?
9. ***Aisyah*** : four.

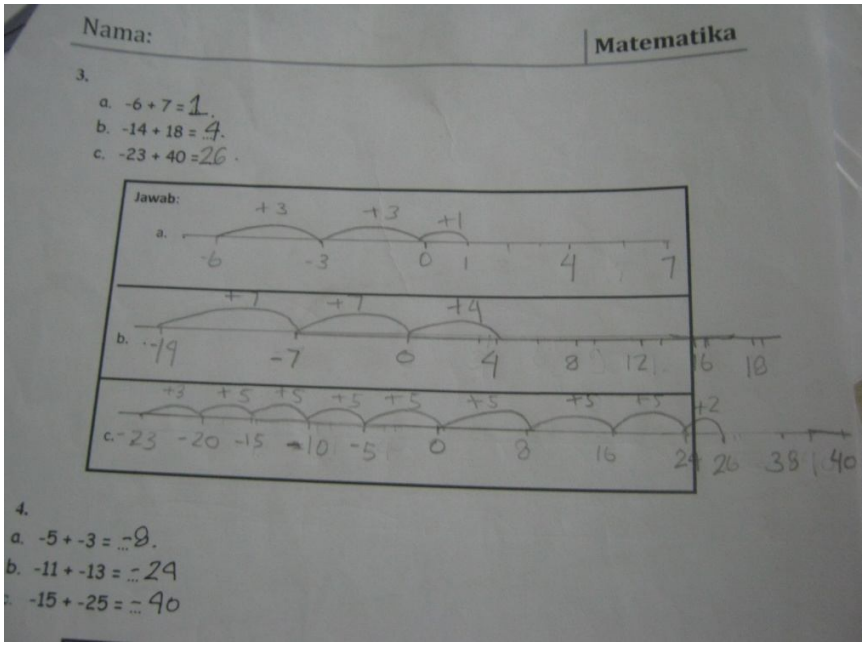
Although, they seemed to use various strategy, it seemed that they still used strategy of jumping by ones since they keep writing down the numbers one by one.

In Atthya's group, the group mentally answered the questions first and used number line to show their work. Strangely, for instance in answering the question 11 + 15 that equals 26, it started with 0 and made jump of 4 and jump of 2 to get 26 without starting at 11. Similarly, it made jump of nine by starting at 0 and stopped at 45 in answering the question c. However, in working on the problem 2, they started at the first number at the question, i.e. starting at 8 in working on question 2a, and made a jump of five to the left to get 3. In addition about the work of Atthya's group, it didnt' write the whole numbers one by one, instead, they just wrote down the number of which they stopped after making a certain jump. It is different to the work of Gina's group that wrote down the numbers one by one although it used various jumps in working with number line.

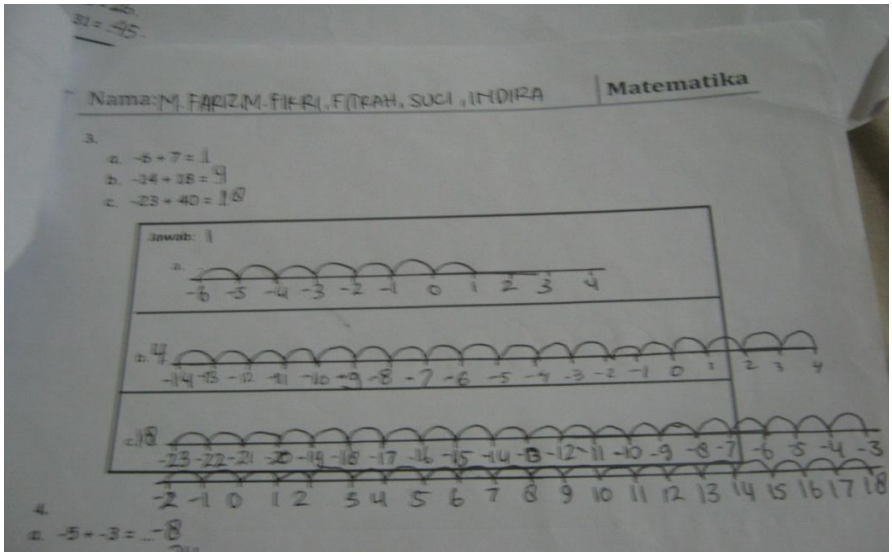
Meanwhile, in the group of Indira, it completely used the strategy of jumping by ones to solve all the problems. And in our focus group, it also used the jumping by ones-strategy at first, however when worked on the problem involving large number, it

didn't use jumping by ones-strategy anymore, instead, it directly used jumping by tens-strategy. The figure 5.14 and the figure 5.15 show the works of several groups:

#Figure 5.14 The work of Attiya's group of lesson 6#



#Figure 5.15. The work of Indira's group of lesson 6#



By seeing the description above, we can interpret that, the students are able to work with "*naked task*" the task without contextual problems. We can see that the knowledge of students gained in the previous lessons help the students to work on the problems in this lesson. The choose of questions also stimulate the students using several different strategies. Moreover, the model of number line holds important role to help the students work on the problems.

The comparison between our conjectures of students' learning and the actual learning of the students in this lesson is shown in the table 5.11.

Table 5.11. The comparison between the conjecture of students thinking and learning and the students' actual work of the second activity in lesson 5

No		Conjecture of Students' thinking and learning	Students' actual learning
1	a	The students will directly answer the problem that is 11. In showing their work, they solve using number line by starting from 5 and jump by ones as many as 6 times to get 11.	Students direcly answer the problem that is 11. The students will directly answer the problem that is 11. In showing their work, they solve using number line by starting from 5 and jump by ones as many as 6 times to get 11.
	b	Several students work with number line by starting from 11 and make jump by ones as many as 15 times to the right. The other students make jump by five strategies.	Students directly answered the problem that is 26. There is a group using number line and jumping by ones strategy by starting at 11 to 26. There is also the students using number line starting from 0 and using counting by using jumping by fours and jumping by twos.
	c	Several students work with number line by starting from 14 and make jump by ones as many as 31 times to the right. The other students make jump by five strategies.	Students directly answered the problem that is 45. There is a group using jumping by fourteens and jumping by threes. There is also group using the combination of jumping by ones and there is also using the combination of jumping by fours and jumping by ones.
2	a	The students answer the problem by using number line. They start at 8 and jump to the left using jump by ones strategy : jumping by ones as many as five times. The other students make jump by fives strategy. However, there are also students who make jump to the right.	Students used number line firs and started from 8. Most of them jumped to 3 using jumping by ones. There is also group using the combination of jumping by fives and jumping by threes.
	b	The students answer the problem by using number line. They start at 12 and jump to the left using jump by ones strategy : jumping by ones as many as sixteen times. The other students make jump by	Quite the same to the actual work of students in 2a, however, the group who used the combination of jumping by fives and jumping by threes using jumping by elevens and jumping by thirteens.

		fives strategy. However, there are also students who make jump to the right.	
	c	The students answer the problem by using number line. They start at 20 and jump to the left using jump by ones strategy : jumping by ones as many as twenty five times. The other students make jump by fives strategy. However, there are also students who make jump to the right.	Quite the same to the actual work of students in 2a, however, the group who used the combination of jumping by fives and jumping by threes using jumping by fifteens and jumping by twenty fives.
3	a	The students use number line by starting at -6 and count by ones to the right as many as seven times to get 1. The other students use count by fives strategy.	Students used number line first and moved to the right from the starting number. Most of them used counting by ones strategy and there is a group using counting by threes strategy.
	b	The students use number line by starting at -14 and count by ones to the right as many as seven times to get eighteen times. The other students use count by fives strategy.	Quite the same as the actual learning of the students in 3a. However the group using jumping by threes strategy used jumping by seven strategy in this problem.
	c	The students use number line by starting at -23 and count by ones to the right as many as seven times to get forty times. The other students use count by fives strategy.	Quite the same as the actual learning of the students in 3a. However the group using jumping by threes strategy used jumping by five strategy in this problem.
4	a	The students use number line by starting at -5 and count by ones to the left as many as three times to get -8.	Students used number line first and moved to the left from the starting number. Most of them using jumping by ones strategy. there is also a group using the combination of jumping by fives and jumping by threes strategy
	b	The students use number line by starting at -11 and count by ones to the left as many as thirteen times to get -24. The other	Quite the same as the actual learning of students in part a, however, the group using the combination of jumping by fives

		students use count by fives strategy.	and jumping by threes using jumping by elevens and jumping by thirteens strategy in this problem.
	c	The students use number line by starting at -15 and count by ones to the left as many as twenty five times to get -40. The other students use count by fives strategy.	Quite the same as the actual learning of students in part a, however, the group using the combination of jumping by fives and jumping by threes using jumping by twenty fives and jumping by fifteens strategy in this problem.

Conclusion

By seeing the table 5.11., our conjectures concerning the various strategies used by the students in working using number line to solve the problems are fulfilled. Because the students have a good conception about how to work with these problems, memorizing some facts, and several students don't rely only on counting by ones-strategy. Therefore, we also can conclude that both number line and empty number line also supports student's development and to help them in adding negative numbers.

5.8. The Analysis of Pre-test & Post-test

Pre-test

This pre-test is quite different to that of in the first cycle. Since it consists of questions related to the topic of addition of negative numbers only. There were 25 students participating in this pre-test. The problems in the pre-test are shown in the following:

Giving explanation for your answer:

1. Order the following numbers from the largest to the smallest:

   -3, 5, 2, -6

Answer:

Explanation/reason of why you order the numbers in that order:

Problem 1 is about ordering the integers given from the largest to the smallest. This problem is aimed to know the understanding of the students in recognizing that negative numbers are less than positive numbers. In addition, they also know the order of some negative numbers, e.g. -6 < -3. Of 25 students, there are 13 students doing correct in this problem. There ore two other students who actually understand

the order of the integer but their answers are not relevant to the problems given, i.e. one students didn't see the negative sign in -6, one student wrote down 2 before 5.

2.

- If 5 represents having 5 marbles, then what does -5 show?
  Reason: .....

- If 3 represents having 3 marbles, then what does 0 show?

  Reason: ....

This problem is asked to know the ability of the students in determining the quantitative state that is represented by certain numbers providing that the magnitudes opposite to that state is known, by concerning the definition of Rivard about negative numbers that negatives are certain magnitudes opposite to those which are regarded as positive. There were 10 students who did correct for this problem because they understand the problem. The other students didn't answer correctly since they seemed not to understand the problem very well.

3. Yesterday, Reza was still lack of 5 marbles. Today, he is lack again 8 marbles. How many lack total of marbles that Reza has?

Answer:

This problem was given aimed to know how students solve the addition problem in context. In this problem, there were 14 students who correctly answer the problems. Even, three of them used notation negative for 13. The other students seemed not to understand and didn't identify the problem as the addition.

4. Hadi is lack of 5 marbles. If he gets 3 new marbles from his friend, is he still lack of marbles?

answer:

If he is still, how many marbles he is still lack?

answer:

if not, how many marbles he has now?

answer:

This problem was given to know the student's reasoning in solving problem in context and how they explain the way he got the answer like that. There were 12 students correctly answered this problem. However, they just answered the problem contextually and didn't show any strategy or model, i.e. representing the quantitative state into numbers.

5. Determine the result of:

a. (-4) + (-6) = ….

Answer:

b. (-4) + 6 = ….

Answer:

c. 6 + (-4) = ….

Answer:

This problem was given to know how students work on the problems without context, and what strategy or model they used. There were 13 students correctly answered the problem part a. However, they didn't use specific reason for determining the answer instead they only relied on the fact that 6 + 4 is 10 and because both numbers in the left are negative, so they put negative sign for 10. For the next two problems, there was only one student who correctly answered the problems. However, he didn't use any representation how he answered the problems.

The result of the pretest is given in the table 5.12.:

Table 5.12. The result of the students' pre-test.

PROBLEM		C	IC
1		13	12
2		14	11
3		10	15
4		12	13
5	a	13	12
	b	1	24
	c	1	24

The frequency of the type of students' answer who did correctly in solving the contextual problem containing the concept of negative numbers added by positive numbers is shown in the following table.

#Table 5.13. Number of students who used certain type of answers#

CX	M	F
12	0	0

Nb:

- C: number of students doing correct
- IC: number of students doing incorrect
- CX: number of students who solved the problem contextually
- M: number of students using a model
- F: number of students answering in formal mathematical level.

It can be seen from the table that several students didn't know the order of integers as well as didn't know also what the function of negative numbers in representing a state. In solving mathematical contextual problem, the students used their informal knowledge in answering the problem. However, surprisingly, most of the students didn't know how to work with contextual problem related to the addition of two quantities. Furthermore, nearly all the students still had no idea how to work with negative numbers "naked-task" problem.

Post-Test

The post-test of this study is somewhat different to the pre-test. However, it still contains the same competency as pre-test has. The content of the test is in the form of exercises related to the lessons that the students has solved as shown in the following.

1. Achmad has 6 marbles, then he gets 4 marbles from his friend. Meanwhile, Banu is lack of 5 marbles and then he is lack of 3 marbles. Count the difference between the marbles Achmad has and Banu has!

   Answer:

2. Rudi is lack of 6 marbles, however, later he gets 7 marbles from his friend. Is he still lack of marbles? If he is not, how many the excess of marble he has now?

   Answer

3. Solve
   -12 + -16 = ... ?

   Answer:

4. Determine the result of:
   a. -5 + 4 = ... ?
   b. -6 + 15 = ... ?

Answer:

a. .

b. .

5. Determine the result of:
   a. 5 + -3 = ...
   b. 10 + -19 = ...

Answer:

a..

b.

In general, the students who had shown good performance during learning process, did good in this post-test. Number line model was used in each problem by them. The following shows the comparison of the quantitative overview between the pre-test and the post-test.

The following table shows the frequency comparison of type of students' answer between pre-test and post-test containing the concept of negative numbers added by positive numbers.

Nb:

- CX: number of students who solved the problem contextually
- M: number of students using a model
- F: number of students answering in formal mathematical level.

This following table shows the comparison between the result of pretest and that of post test in the term of the number of students doing correct of the problems containing three concepts of addition of negative numbers.

#Table 5.14.The comparison between the result of pre-test and post-test#

COMPETENCY	PRE-TEST	POST-TEST
ADDITION OF TWO NEGATIVE NUMBERS	13	9
NEGATIVE NUMBERS ADDED BY POSITIVE NUMBERS	1	13
POSITIVE NUMBERS ADDED BY NEGATIVE NUMBERS	1	9

CHAPTER VI

CONCLUSION AND DISCUSSION

The aim of this research is to develop classroom activities that support students in learning addition of negative numbers. In this research, we posed a research question: *"How can a context and a model support students in adding negative numbers?"* In this chapter, we firstly answer the research question based on our retrospective analysis, and then we reflect on some issues in this study and elaborate recommendation for further studies to improve the mathematics education in Indonesia.

6.1.Answer to Research Question

"How can a context and a model support students in adding negative numbers?"

To answer this question, we elaborate the research question to be more specific and result sub-research questions that have been formulated as listed in the following:

a. *How can the use of a equilibrium context support students in doing the addition involving negative numbers*?
b. *How can we support students in developing number line or empty number line model*?
c. *How does either number line or empty number line support students in doing the addition involving negative numbers in formal level*?

Then, we looked at the sequence of learning activities whether it help the students in developing a model and investigate the role of equilibrium context as well as number line or empty number line in the students' learning. Then, we can conclude how their role evolves during the activities.

6.1.1. Specific Conclusion in each sequence of the students' learning.

The equilibrium context used in this study is having marbles and lack of marbles. Moreover, to connect it to the model of number line as the supporting tool to develop the student's thinking from informal, intuitive, to formal mathematical concept, the marbles are horizontally arranged in sequence such that they can be connected to a space as symbolical representation including some mathematical concepts, i.e. number line or empty number line. Therefore, in lesson 1 we conducted an activity to which the students count the marbles thoroughly, to determine to what extent the marble someone has in marble sequence when the students are given certain quantity, to count the marbles from one marble to another marble both from the right to the left and from the left to the right. Eventually, after conducting the activities of this lesson and seeing the result, we conclude that the activity of which, the arrangement of marbles in horizontal form supports the students to come up with the idea of several mathematical concepts: considering small numbers are in the left, large numbers are in the right, counting to the right when adding, counting to the left when subtracting, and counting marbles with several different strategies.

In lesson 2, the students were asked to correspond the quantity of marbles someone has to the line below it. In the first activity in which someone is only in the state of having marbles, the students have no difficulty in corresponding the quantity of marbles to the number line below by putting a number representing the quantity of certain number of marbles in the line. After implementing this activity and seeing the result of the students, we conclude that this lesson elicits the mental thinking of the students in corresponding the quantity of marble to the number line. By seeing the numbers in number line, they concluded that the larger numbers occupied the right space and the smaller numbers occupied the lef space. Therefore, this lesson of which the students are asked to tacitly correspond the quantity of marbles to the line below supports the students' development in understanding the concept of number line as a representation of numbers. In the second activity, the students were asked to correspond the quantity of marbles someone has to the line below it as well as to think to determine the position of marbles that someone is lack of in marble sequence, the number that represents that state, and the position of the number in number line. After implementing the activity and seeing the result of the students' work, we conclude that our conjectures in which the students are able to correspond the magnitude of number of marbles to the line representing positive numbers are fulfilled. Because the students are aware to the concept: "larger numbers are in the right". Besides that our conjectures in which the students draw some marbles to the left to determine the position of lack of certain marbles, the number representing that state, and to

correspond the numbers to the number line are fulfilled by our focus group and several other groups since they had conception that lack of marbles are less in quality than having no marble. However, they are not fulfilled by several groups since the students considered that the state of lack is the continual event from the previous problem. Therefore, we can conclude that this lesson supports the development of students' understanding of the concept of number line which can represents both positive numbers and negative numbers.

In lesson 3, the students were asked to position numbers in a line. Here, the students found no difficulty in positioning the numbers in number line. They even extended the line to the left to represent the negative numbers in the problem using number line. By seeing the actual work of the students, we can conclude that our conjectures to which the students did well of this lesson are fulfilled. It is caused by the fact that the students have already had conception that smaller numbers occupy left space of the line and larger numbers occupy right space of the line which are derived from the previous lesson. The choosing of numbers in the problems also supports several groups to create empty number line. Concerning the conjectures to which the students put the numbers in arbitrary position which are not fulfilled, is caused by the fact that the students who couldn't work at first were given supporting questions from the teacher, i.e. where is the position of negative numbers?, where is the position of larger numbers?, are the numbers in the left or in the right? which led them to think and to answer the

problems correctly. Therefore, this lesson supports students to develop a number line or empty number line representing both positive numbers and negative numbers and to have conception that negative numbers are in the left space and positive numbers are in the right space of the line.

In lesson 4 we conducted activities in which the students were asked to solve contextual problem related to the addition of two negative numbers. By seeing the result of students' work, we conclude that the conjecture to which the students will answer 7 for the lack total of Anton's marble is fulfilled since the students recognized that the problems concerned the addition of two numbers and they were able to solve the problems contextually. The conjecture to which the students visualize the addition of two negative numbers representing the state of lack of marbles and get -7 derived from -3 and -4 is fulfilled because the students represented the lack of marbles by negative numbers. They also knew that the state of lack again meant that Anton is more lack and caused the move in number line was directed to the left. The role of the teacher that is asking too much questions that can stimulate the thinking of several students led the students to come up to the concept of the addition of negative numbers. Therefore, this lesson in which there are equilibrium context which is used as a base to represent the states within the context using signed numbers and the use of number line which is used to count, support the students' thinking in answering the addition of two negative numbers formally.

In lesson 5, we conducted lesson consisting of two activities in which in the first activity, the students were asked to solve contextual problem containing the concept of negative numbers added by positive numbers. By seeing the result of students' work, we conclude that our conjectures, to which the students answer yes or no about whether Ilham is still lack, are fulfilled because the students really understand the problem and can answer it contextually with convincable reason. In visualizing the contextual answer using number line, some students felt difficulty and needed help from teachers or researchers as well as in determining the formal notation of the operation elicited from the problem. However, some groups including our focus group directly determined the formal notation of the operation without the help of teacher. Here, the context can support some students in understanding the concept of negative numbers added by positive numbers and the model can help students to show the process and the result of the question in formal mathematical notation.

In the second activity, the students were asked to solve "*naked task*" containing the concept of addition of negative numbers and positive numbers. By seeing the result of students' work, we can conclude that our conjectures in which the students answer yes with correct procedures in using number line are fulfilled since the students understand that the movement in number line in the problem can be referred back to the equilibrium context and the use of number line to help them to count. Although, there are students who couldn't directly think how they work, but when being guided by the teacher to refer back to the concept of

numbers in number line, they eventually know how to work with the problems. Therefore, we can further conclude that the equilibrium context is useful for the students to work using number line in which they can determine what direction they move in number line and next to find the final result of the question using number line.

In the last lesson, i.e. lesson 6, the students were asked to solve several exercises containing all kinds of addition of integers competency, i.e. the addition of two positive numbers, addition of negative numbers and positive numbers, and the addition of two negative numbers. By seeing the result of the students' work, we can conclude that our conjectures concerning the various strategies used by the students in working using number line to solve the problems are fulfilled because the students have a good conception about how to work with these problems and memorizing some facts. Moreover, several students didn't rely only on counting by ones-strategy but also used different strategies. Therefore, we also can conclude that both number line and empty number line also supports student's development and to help them in adding negative numbers.

6.1.2. Conclusion

In short, this research has shown that equilibrium context and number line model or the combination of the two play an important role in supporting the students in doing the addition involving negative numbers correctly. Specifically, the state of having something and lack of something support the students' thinking to answer

the contextual problem informally and to represent both states as two opposite things using positive numbers and negative numbers respectively. The arrangement of collection of marbles in horizontal form and the understanding of having something is better than lack of something leads the students to develop number line model containing integers in which students represent the numbers representing those states in a line in which positive numbers or larger numbers occupy the right space and negative numbers or smaller numbers occupy the left space in number line or empty number line. Furthermore, as the number line or empty number line provides numbers that can be connected to the numbers representing certain quantity, it supports the students to think from informal knowledge to more abstract level of thinking to be able to work with and to manipulate the numbers and to get the result of certain operation correctly.

6.2. **Reflection on the Important Issues**

In this research we not only focused on students' thinking process but also we observed how activities helped the students in building their mental model of the situation, how the discussion help the low achieving students to learn, and how the role of the teacher in supporting the students learning.

6.2.3. **Realistic Mathematics Education**

In designing the sequence of instructional activities, we consult to some ideas of realistic mathematics education (RME). In RME, the starting point of mathematics instruction should be experientially real to the student, allowing them to become immediately engaged in the contextual situation.

The contextual situation in the instructional activity of this study is related to the opposite quantity which is having number of marbles and the number of lack of marbles. In the activity of counting marbles and corresponding the quantity of marbles to the line, it leads to the development of model in which the students used the model to make counting.

6.2.4. **Classroom Discussion**

In this classroom, the teacher and the students were struggling to develop a constructive discussion. There were 26 students in the class which were distributed to 12 groups of which each group consists of 2 till 3 students. We observed that while doing some activities, only high achiever students who actively worked and the other just saw his or her friend without giving significant contribution. The teacher accepted that the culture of working in group of which there is an intension discussion within to share idea to each other was not familiar with the students. Since when we interviewed the teacher before conducting the teaching experiment, the teacher said that, only the high achiever students who could follow the learning and understand the topic being studied while the other students tend to be silent during learning process in the classroom. We could say that it was new for the students to have these socio mathematical norms.

6.2.4 **The role of the teacher**

In this research, the teacher has been teaching for 30 years and she ever teaches in all grades in primary school. She had a lot of experiences in

teaching but it was really new for her to conduct a classroom environment as in this research, open and students center. Based on the interview with her, she told that it was not usual to have discussion in mathematics lessons. However, she is an open-minded person who would like to apply the innovation of the new approach in teaching mathematics in her classroom.

During the experiment, she was rather awkward since it was the first time for her to be observed using video while teaching her students. However, she has shown a good performance in keeping promoting the equilibrium context and the use of model of number line while guiding the students in counting. Sometimes, she asked the students to compare their answers and find the correct one. It was not easy for the teacher to manage the students' discussion. However, she has been trying to promote the development of socio-mathematical norms in her classroom.

6.3. Discussion

In this section, we would like to discuss the most important issue in this research that is addition of negative numbers. As we explained before in chapter 2, in this research, we would like to support students in adding negative numbers using equilibrium context and number line model. Then, there are advices from some other researchers, first: We need to promote another model to the students so that the students who couldn't work with number line can refer to another model to solve problems related to addition of negative numbers.

The second advice is related to the context used. For instance, in the context of Ilham lacks of 5 marbles. After Ilham gets 5 new marbles, he should give them to the others as the process of "*paying debt*". Therefore, the students don't think that Ilham has 5 marbles, instead the number of marble in his hand is 0.

In the first advice, it is good to have another kind of model. Based on the literatures the researcher has read, there are two kinds of models commonly used, besides number line model that is used in this study, there is also equilibrium model to which we use two kinds of opposite concrete things. Therefore, the notion of equilibrium model here is also used in this study as a context. Concerning the second advice, it is very useful to provide a story like that, however, in the other side, the researcher is afraid to the students' interpretation in modelling such a long story to mathematical model which might lead to the wrong mathematical notation, i.e. lack of certain marble is represented as negative numbers and getting new marbles means adding. If the story is continued to giving the marble to others, some students might think that the story involves two kinds of operations, that are addition and subtraction, since giving marbles to others means taking away that leads to subtraction operation.

6.4. Recommendations

In this section, we would like to give our recommendation related to apply the RME approach in the classroom, to teach the addition involving negative numbers to grade 4, and suggestion for further studies.

6.4.1. **Realistic Mathematics Education**

In our RME classroom, the use of context stimulates the students to think of a way to solve problems given. The students could bring their informal knowledge to get ideas to work with mathematical model and to get the final result in the end.

Therefore, we recommend to the teachers in Indonesia to use RME or PMRI approach in their teaching. This approach allows the students to construct their own knowledge, leads the teachers to know how to guide the students so they can be active learners. Furthermore, the teacher and the students will see mathematics as human activity to which they find mathematics not just as procedures and rules that need to be memorized without meaningful process.

6.4.2. **Addition of Negative Numbers**

In Indonesian primary school, the typical problems faced by the students are in the form of "*naked task*" that is problem without contextual situation. After that, they are asked to solve the problem using number line which consists of several procedures and rules to work with. This can lead students to be passive learners. In another instance, the use of object collection that can lead to the equilibrium model and the use of number line is seen as separated things. However, in this study, the use of equilibrium context and the connection of it to develop number line can combine the two kinds of notions as the important elements in learning negative numbers.

6.4.3. **Further studies**

In our study, we only focus on the addition involving negative numbers. There are several topics that need to be taught in primary school, i.e. subtraction involving negative numbers. The new challenge is how to support the students' understanding to study subtraction involving negative numbers using a context and a model.

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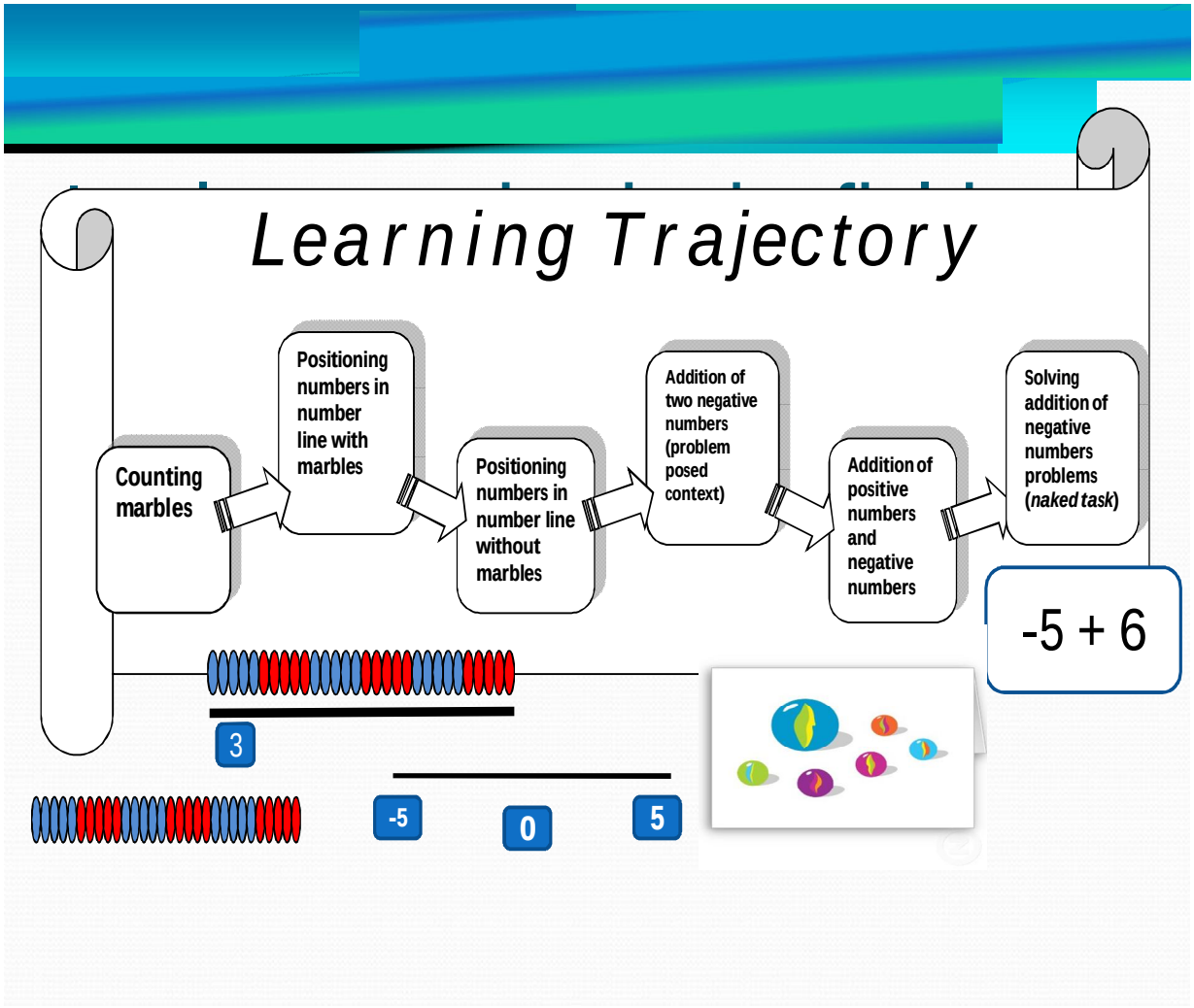
Appendix

Appendix A


Learning Trajectory
Counting marbles
Positioning numbers in number line with marbles
Positioning numbers in number line without marbles
Addition of two negative numbers (problem posed context)
Addition of positive numbers and negative numbers
Solving addition of negative numbers problems (naked task)
-5 + 6
3
-5
0
5

Appendix B

1. Draft of Interview with teacher:

Questions:

1.1. Background of teachers:

a. *What is your name?*
b. *How long you have been teaching mathematics in elementary school?*
c. *In which grades you ever teach in the elementary school?*
d. *What grade you currently teach now? How many years have you been teaching in that grade?*

1.2. Teaching-Learning Process

a. *How do you usually teach negative numbers and addition and subtraction of integers in classroom?*
b. *Do you usually use context in teaching addition and subtraction of integers in classroom?*
c. *Do you usually use model in teaching addition and subtraction of integers in classroom?*
d. *How do you usually manage your students in the teaching-learning process? Do they work individually or do they work in group? Why?*
e. *When you divide the students learning in groups, do you consistently match the students to their friends? Why?*
f. *How long do you usually use in teaching addition and subtraction of integers?*
g. *Do you usually hold discussion after the students solve the problems given?*
h. *Do you usually face difficulty in teaching this topic? If yes, what kind of difficulty is?*

i. *What criteria you usually use to reach your goals in teaching this kind of topic?*
j. *Could you describe your about your students (what age, how many boys and girls, ect.)?*
k. *What kind of difficulty they often find while studying mathematics?*
l. *Who are the leading students in your class?*
m. *Could you describe why those students are capable in studying math?*
n. *Who are the students who are rather low in your class?*
o. *Why those students find difficulty in studying math?*
p. *What kind of situation the students like during learning process in classroom? Do they like studying in group or individually?*
q. *Have you ever heard about PMRI (Pendidikan Matematika Realistik Indonesia)?*
r. *What do you hear about it?*
s. *Have you ever apply PMRI teaching in your classroom?*

2. Classroom Observation

2.1.Before Experiment

2.1.1. The observation of classroom

- The feasibility of the classroom, the size of the classroom compared to the number of students. This can be observed by making some pictures in the classroom.
- There are some additional tools or pictures in the classroom that help students to learn. This can be observed by making some pictures in the classroom
- The way the tables and the chairs are managed. This can be observed by making some pictures in the classroom.

2.1.2. The observation to the students

- The number of students in classroom, the number of boys, and the number of girls. This can be observed by seeing the attendance list had by the teacher.
- The way they are organized by the teacher while the learning process is occurring. This can be observed by making some pictures in the classroom.
- The students who are actively participating in the learning process especially in the discussion session. This can be observed by observing video and making notes.
- The table-mate of the students in several times of learning. This can be observed by being there several times and making some pictures in the classroom.

2.1.3. The observation to the teacher

- How teacher begins the class, whether she motivates her students or she directly give students problems and the reason behind this. This can be observed by seeing the video.
- How long the time teacher provides to let students work and to lead the discussion and the reason of it. This can be observed by observing video and making notes.
- Whether the teacher really follows the guide of lesson provided by the school curriculum or the teacher has applied her own rule. This can be observed by observing video and making notes.

2.1.4. The observation of the interaction between students and teacher

There are some kinds of data concerning the impression of teacher:

- How teacher determines the students who come forward to explain their answers in the front of the class, e.g. whether she appoints the student s who is smart or not. This can be observed by making note.
- How students answer the question of the teacher, whether they raise their hand or wait the instruction of the teacher. This can be observed by making note.

2.2. During the Experiment

2.2.1. First Cycle:

This first cycle involves around 5 or 6 students and a teacher.

a. Observation to the students

- Whether the students find difficulty in working with problems. This can be observed by observing video, doing mini interview, and making notes.
- Whether several students only who actively participate in learning. This can be observed by observing video, doing mini interview, and making notes.

- Whether students wait for the instruction of teacher to share their ideas or directly raise their hand to share. This can be observed by observing video, and making notes.
- Whether students apply the models and the strategies as conjectured by the researcher. This can be observed by observing video, doing mini interview, making notes, and reviewing the work of the students.
- Whether the students only sit nearby to the same student in each lesson. This can be observed by observing video and making notes.

b. Observation to the teacher

- How teacher begins the class, whether she motivates her students or she directly give students problems. This can be observed by seeing the video.
- How long the time teacher provides to let students work and to lead the discussion and the reason of it. This can be observed by observing video and making notes.
- Whether the teacher really follows the guide of lesson provided by the researcher or the teacher has applied her own rule. This can be observed by observing video and making notes.

2.2.2. The second cycle

This cycle involves whole class and a teacher

a. Observation to the students

- Whether the students find difficulty in working with problems. This can be observed by observing video, doing mini interview, and making notes.
- Whether several students only who actively participate in learning. This can be observed by observing video, doing mini interview, and making notes.

- Whether students wait for the instruction of teacher to share their ideas or directly raise their hand to share. This can be observed by observing video, and making notes.
- Whether students apply the models and the strategies as conjectured by the researcher. This can be observed by observing video, doing mini interview, making notes, and reviewing the work of some of the students.
- Whether the students only sit nearby to the same student in each lesson.

b. Observation to the teacher

- How teacher begins the class, whether she motivates her students or she directly give students problems. This can be observed by seeing the video.
- How long the time teacher provides to let students work and to lead the discussion and the reason of it. This can be observed by observing video and making notes.
- Whether the teacher really follows the guide of lesson provided by the researcher or the teacher has applied her own rule. This can be observed by observing video and making notes.

3. Written works of pupils

3.1. Pretest

- Whether students solve the problems correctly.
- Whether students use model in solving problems or directly use formally mathematical formula.
- Whether students use various strategies in solving problems.

3.2. Written works during lessons

- Whether students solve the problems correctly.

- Whether students use model in solving problems or directly use formally mathematical formula as conjectured by the HLT.
- Whether students use various strategies in solving problems as conjectured by the HLT.
- Whether there is a change in the way of students in answering the problems compared to their pretest.

Appendix C

Panduan Guru

Panduan Guru di Pertemuan Pertama

Pelajaran : Menghitung Kelereng dan Membuat Garis Bilangan

Kelas : IV

Semester : II

Alokasi Waktu : 70 Menit

Peran dari Guru:

Peran dari guru adalah sebagai fasilitator pembelajaran. Peranan ini dijelaskan secara rinci sebagai berikut:

a. **Memimpin diskusi**
   Di tiap pembelajaran, terdapat diskusi antara guru dan siswa dan juga antara siswa dan siswa lainnya.
b. **Mengarahkan proses dan aktivitas pembelajaran**
   Guru menginformasikan ke siswa tentang proses pembelajaran dan aktivitas yang akan dikerjakan oleh mereka. Hal ini bertujuan agar proses belajar mengajar berjalan dengan lancar.
c. **Mendorong partisipasi aktif dari siswa untuk menyalurkan gagasan mereka**
   Sesering mungkin, guru mesti membuat siswa aktif berpartisipasi dalam proses pembelajaran dan penyaluran idea tau gagasan ke siswa lainnya.
d. **Devil's advocate**
   Di samping memimpin diskusi, guru terkadang menguji argumentasi siswa dengan cara tidak menyetujui gagasan atau jawaban dari siswa tersebut.
e. **Menyediakan aktivitas pembelajaran**
   Pada pembelajaran di kelas, guru memulai pembelajaran dengan cara memberi permasalahan/soal matematika kepada siswa

Eksplorasi Fenomenologis

Konteks yang digunakan dalam pelajaran ini adalah menghitung biji kelereng yang disusun dalam bentuk barisan. Konteks ini digunakan karena dapat membawa siswa ke dalam konsep garis bilangan sebagai alat representasi bilangan.

Level Kongkrit

Penggunaan material pembelajaran digunakan untuk memunculkan level kongkrit pada pelajaran ini. Material yang digunakan adalah kertas besar yang berisikan gambar barisan kelereng yang terdiri atas dua warna berbeda. Material ini dapat melibatkan siswa untuk menggunakan beberapa strategi dalam menghitung dengan menggunakan garis bilangan. Strategi yang dapat muncul adalah melompat sepuluh, beralih ke puluhan yang familiar, dll.

Keragaman level pembelajaran

Level pembelajaran yang berbeda yang dapat muncul ketika proses pembelajaran berjalan adalah menghitung kelereng dengan gambar dan menghitung kelereng tanpa bantuan gambar.

Interaksi Sosial

Ada dua macam interaksi yang dapat terjadi selama pembelajaran di kelas yakni interaksi vertical dan interaksi horizontal. Interaksi vertical terjadi ketika guru bertanya ke siswa untuk maju menghitung kelereng dan memberitahukan strategi yang mereka gunakan. Sedangkan, interaksi horizontal terjadi ketika siswa merespon strategi dan jawaban siswa lainnya.

Refleksi

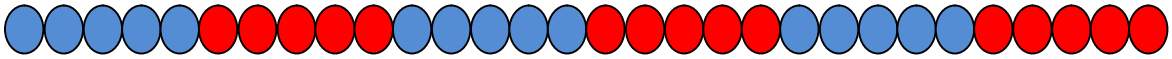
Untuk meyakinkan bahwa siswa memperoleh pemahaman mendalam tentang pelajaran ini, guru senantiasa melakukan interaksi yang intensif kepada siswa ketika mereka bekerja dengan soal-soal yang diberikan dan menanyakan ke siswa apa yang mereka dapatkan dari pelajaran ini.

Aktivitas 1: Menghitung Kelereng (35 menit)

Deskripsi aktivitas:

Di depan kelas, guru akan menunjukkan gambar 30 biji kelereng ke siswa yang terdiri dari dua warna berbeda yakni biru dan merah di mana kelereng-kelereng tersebut disusun berdasarkan warnanya di mana lima kelereng biru berjejer lalu diikuti oleh lima kelereng merah lalu lima kelereng biru lagi dan seterusnya. Kemudian guru mengajukan beberapa pertanyaan dan menjalankan diskusi dengan cara mengajukan beberapa pertanyaan dan meminta siswa untuk menunjukkan jawabannya serta memberi kesempatan kepada siswa yang lainnya untuk menanggapi.

Gambar yang ditunjukkan ke siswa:



#Gambar 1. Rangkaian biji kelereng #

a. Ada berapa banyaknya biji kelereng yang terdapat dalam gambar? Bagaimana cara kamu menghitungnya?

1. Guru meminta salah satu siswa untuk maju dan bertanya ke siswa tersebut, misal A:

   "ada berapa biji kelereng yang terdapat pada gambar?"

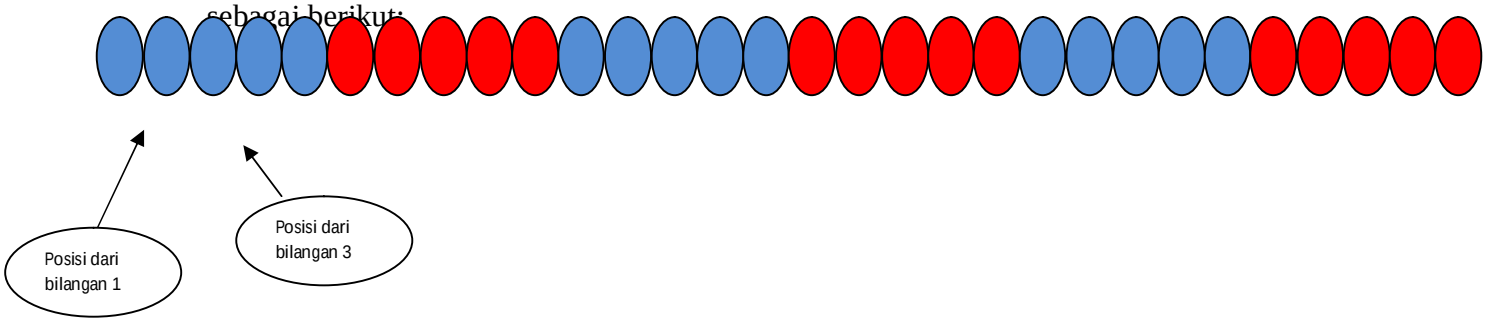
2. Guru memberi kesempatan kepada A untuk berfikir dan menghitung
3. Guru bertanya ke A:
   "*dapatkah kamu menunjukkan kepada kami bagaimana kamu menghitung kelereng-kelereng tersebut"*!
4. Ketika siswa tersebut telah menjawab pertanyaan tersebut, tanpa bergantung dari benar atau tidaknya, guru mengajukan pertanyaan ke siswa lainnya:

   "apakah kamu setuju bahwa banyaknya biji kelereng adalah ….?" (bergantung dari jawaban A)
5. Guru lalu menanyakan ke beberapa orang siswa (satu atau dua orang) untuk maju ke depan kelas dengan menanyakan:
   "*dapatkah kamu mempunyai cara yang lain dalam menghitung biji kelereng yang ada pada gambar*?"

b. Dapatkah kamu menghitung dari belakang (dari kanan ke kiri)?

1. Guru bertanya kepada salah seorang siswa, misalkan B, untuk maju ke depan dan menanyakan ke B:
   "*Dapatkah kamu menghitung dari belakang?*"
2. Guru lalu bertanya ke beberapa orang siswa (satu atau dua siswa yang lain) untuk maju dan menanyakan mereka:
   "*apakah kamu mempunyai cara yang lain?*"
3. Guru bertanya ke siswa lainnya yang mempunyai cara berbeda untuk menjelaskan strateginya di papan tulis.

Setelah sesi diskusi pada pertanyaan bagian b selesai, guru memberitahukan siswa bahwa posisi bilangan pada gambar rangkaian kelereng terletak di sisi kanan pada kelereng dengan cara mengilustrasikan posisi dari 1 dan 3 pada rangkaian kelereng sebagai berikut:



c. *Jika Budi hanya mempunyai 16 kelereng, tentukan sampai di manakah posisi kelerengnya berada?*

1. Guru menyuruh salah seorang siswa, misal C, untuk maju dan bertanya ke C:
   "*Jika Budi hanya mempunyai 16 kelereng, tentukan sampai di manakah posisi kelerengnya berada?"*
2. Guru kemudian bertanya ke C jika C telah menunjukkan posisi dari bilangan 16.
   "*bagaimana kamu mengetahui bahwa di situ adalah posisi dari bilangan 16*?"
3. Guru lalu bertanya ke siswa lainnya:
   "*apakah kalian setuju dengan jawaban dari C*?"
4. Guru lalu bertanya lagi ke siswa yang lainnya:
   "*apakah kamu mempunyai cara yang lain untuk menunjukkan ke kami sampai di manakah posisi kelerengnya berada jika ia mempunyai 16 biji kelereng?*"

5. Guru lalu bertanya ke siswa yang mempunyai cara yang lain untuk menjelaskan strateginya di papan tulis.

d. Jika Budi hanya mempunyai 24 kelereng, tentukan sampai di manakah posisi kelerengnya berada?

1. Guru menyuruh salah seorang siswa, misal D, untuk maju dan bertanya ke D:
   "*Jika Budi hanya mempunyai 24 kelereng, tentukan sampai di manakah posisi kelerengnya berada?"*
2. Guru kemudian bertanya ke D jika D telah menunukkan posisi dari bilangan 24.
   "*bagaimana kamu mengetahui bahwa di situ adalah posisi dari bilangan 24*?"
3. Guru lalu bertanya ke siswa lainnya:
   "*apakah kalian setuju dengan jawaban dari C*?"
4. Guru lalu bertanya lagi ke siswa yang lainnya:
   "*apakah kamu mempunyai cara yang lain untuk menunjukkan ke kami sampai di manakah posisi kelerengnya berada jika ia mempunyai 24 biji kelereng?*"
5. Guru lalu bertanya ke siswa yang mempunyai cara yang lain untuk menjelaskan strateginya di papan tulis.

e. ***Hitunglah sebanyak 16 kelereng yang ada pada gambar dengan memulai menghitung ke kanan dari bilangan 5. Di bilangan apa kamu berhenti?***

1. Guru bertanya ke salah seorang siswa, misalkan E, untuk maju dan bertanya ke E:

"*bagaimana caramu menghitung sebanyak 16 kelereng dari bilangan 5 ke kanan?"*

2. Guru lalu bertanya ke E jika E telah selesai menghitung.
"*di bilangan apa kamu berhenti?*"
3. Guru lalu bertanya ke siswa lainnya:
"*apakah kamu setuju dengan jawaban dari E*?"
4. Guru lalu bertanya ke siswa lainnya lagi:
"*apakah kamu mempunyai cara yang lain?*"
5. Guru bertanya ke siswa yang lain yang mempunyai cara yang berbeda untuk menjelaskan jawaban mereka di papan tulis.

f. ***Hitunglah sebanyak 12 kelereng yang ada pada gambar dengan memulai menghitung ke kanan dari bilangan 5. Di bilangan apa kamu berhenti?***

1. Guru bertanya ke salah seorang siswa, misalkan F, untuk maju dan bertanya ke F:

"*bagaimana caramu menghitung sebanyak 12 kelereng dari bilangan 20 ke kiri?*"

2. Guru lalu bertanya ke F jika F telah selesai menghitung.
   "*di bilangan apa kamu berhenti?*"
3. Guru lalu bertanya ke siswa lainnya:
   "*apakah kamu setuju dengan jawaban dari F*?"
4. Guru lalu bertanya ke siswa lainnya lagi:
   "*apakah kamu mempunyai cara yang lain?*"
5. Guru bertanya ke siswa yang lain yang mempunyai cara yang berbeda untuk menjelaskan jawaban mereka di papan tulis.

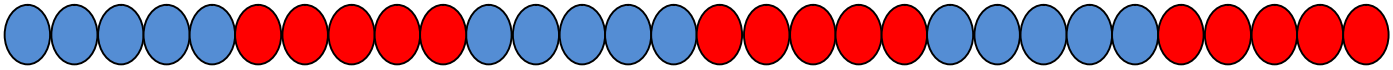
Pertanyaan-pertanyaan Intervensi:

Pertanyaan-pertanyaan intervensi diajukan ketika terjadi hal-hal berikut:

a. Siswa menggunakan strategi:
   - Menghitung dengan lompatan lima -> guru bertanya ke siswa: *bagaimana kamu bisa mengetahui bahwa kamu menghitung dengan lompatan lima*? (pertanyaan ini ditujukan untuk mengetahui penalaran siswa ketika menghitung dengan lompatan lima
b. Ketika siswa menghitung bilangan dengan cara menunjuk bagian tengah pada kelereng. Keadaan ini dapat terjadi ketika siswa memecahkan masalah seperti pertanyaan c, d, e, dan f.
   - Guru memberitahu siswa dan bertanya: *seperti yang saya beritahukan kepada kalian, di mana semestinya kalian menunjuk bagian pada kelereng dalam menghitung atau menentukan posisi bilangan?*

   Setelah semua pertanyaan dijawab dan didiskusikan, guru memberi siswa lembaran yang berisikan gambar rangkaian kelereng seperti gambar yang telah ia tunjukkan di kegiatan sebelumnya.

   Permasalahan:



1. *Hitunglah 13 kelereng + 16 kelereng*
2. *Carilah hasil dari 29 kelereng – 12 kelereng*

Setelah siswa selesai, guru bertanya ke salah seorang siswa untuk mempresentasikan jawaban pada soal nomor 1dan 2. Ketika siswa tersebut menggunakan cara algoritma dalam menjawab, guru meminta ke siswa tersebut untuk mengilustrasikan jawabannya dengan cara menyediakan gambar rangkaian kelereng. Lalu guru bertanya ke siswa tersebut dengan mengajukan pertanyaan seperti berikut:

a. *Di mana kamu memulai menghitung pada gambar*?
b. *Lalu bagaimana cara kamu menghitung*?

Ketika siswa telah menyelesaikan pertanyaan dengan cara mengilustrasikannya dengan gambar rangkaian kelereng, guru bertanya ke siswa lainnya (satu atau dua orang siswa) untuk menjawab dan mengilustrasikan jawaban mereka di papan tulis dengan menanyakan:

apakah kamu mempunyai cara lain untuk memecahkan masalah tersebut?

Pertanyaan-pertanyaan intervensi:

Pertanyaan-pertanyaan intervensi diajukan ketika terjadi keadaan-keadaan sebagai berikut:

a. Ketika siswa mengggunakan strategi:
   - Menghitung dengan lompatan 5 ->guru bertanya ke siswa: *bagaimana kamu mengetahui bahwa kamu menghitung dengan lompatan 5*?
   - Menghitung dengan lompatan 5 ->guru bertanya ke siswa: *bagaimana kamu mengetahui bahwa kamu menghitung dengan lompatan 5*?
   - Menghitung dari belakang -> guru bertanya ke siswa: *mengapa kamu menghitung dari belakang*?

b. Ketika siswa menghitung bilangan dengan cara menunjuk bagian tengah pada kelereng.
   Guru memberitahu siswa dan bertanya: *seperti yang saya beritahukan kepada kalian, di mana semestinya kalian menunjuk bagian pada kelereng dalam menghitung atau menentukan posisi bilangan?*

Ketika seluruh aktivitas telah dilaksanakan, guru meminta tiap-tiap kelompok untuk menyampaikan kesimpulan dari apa yang telah mereka pelajari. Adapun kesimpulan yang diharapkan dapat ditangkap dan disampaikan oleh siswa adalah:

1. Menghitung dengan lompatan 5 lebih cepat dibanding dengan melompat satu-satu.
2. Ketika menghitung ke kanan, maka bilangan akan semakin besar.
3. Ketika menghitung ke kiri, maka bilangan akan semakin kecil.

Panduan Guru di Pertemuan Kedua

Pelajaran **:Membuat Garis Bilangan**

Kelas **: IV**

Semester **: II**

Alokasi Waktu **: 70 Menit**

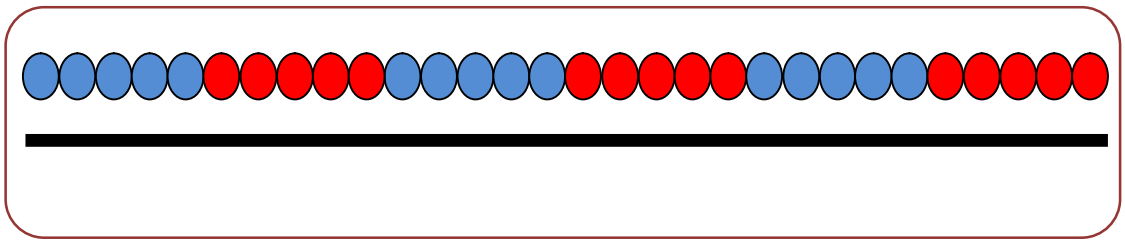
Peran dari Guru:

Menentukan posisi bilangan pada garis bilangan

Deskripsi Kegiatan Guru:

1. Guru membagi siswa menjadi beberapa kelompok di mana tiap kelompok terdiri dari 4orang siswa. Kemudian, mereka diberi lembaran soal oleh guru yang berisikan dua buah soal di mana tiap soal dikerjakan selama 15 menit dan tiap-tiap soal didiskusikan sebelum siswa diminta mengerjakan soal berikutnya. Adapun soalnya sebagai berikut.

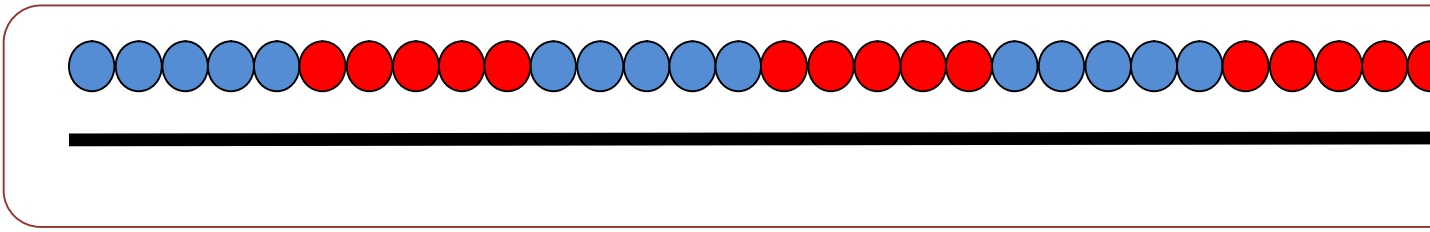
Soal 1



a. Jika Budi hanya mempunyai 1 biji kelereng, tentukan di manakah posisinya pada gambar rangkaian kelereng? Di manakah posisi bilangan 1 pada garis?

b. Jika Budi hanya mempunyai 10 biji kelereng, tentukan di manakah posisinya pada gambar rangkaian kelereng? Di manakah posisi bilangan 10 pada garis?

c. *Tentukan di manakah posisi pada garis, jika Budi hanya mempunyai 15 biji kelereng?*
d. *Tentukan di manakah posisi pada garis, jika Budi mempunyai 30 biji kelereng?*
e. *Lengkapilah garis bilangan berikut dengan bilangan-bilangan yang lain!*



f. *Kesimpulan apa yang kamu dapatkan tentang bilangan-bilangan pada garis bilangan tersebut?apakah bilangan-bilangan tersebut semakin besar ataukah semakin kecil jika posisinya makin ke kanan?*

3. Guru menyuruh tiap-tiap kelompok memasang pekerjaan mereka di papan tulis dan tiap-tiap siswa diminta untuk melihat pekerjaan kelompok yang lainnya.
4. Guru lalu meminta 2 kelompok untuk maju ke depan merepresentasikan jawaban mereka. Diharapkan, kelompok yang maju terlebih dahulu adalah kelompok yang mempunyai pekerjaan yang kurang tepat, dan selanjutnya kelompok yang mempunyai jawaban yang benar.

Hipotesis jawaban siswa dan reaksi guru:

a. .

- Siswa memilih bagian kanan dari kelereng pertama dan menarik garis dari kelereng tersebut ke garis bilangan. Sehingga mereka mengkorespondensikan posisi kelereng pertama ke garis bilangan..
- Siswa lainnya menaruh tanda posisi bilangan 1 tanpa mengkorespondensikannya dengan kelereng pertama.

b.

- Siswa memilih bagian kanan dari kelereng kesepuluh dan menarik garis dari kelereng tersebut ke garis bilangan. Sehingga mereka mengkorespondensikan posisi kelereng kesepuluh ke garis bilangan..

- Siswa lainnya menaruh tanda posisi bilangan 10 tanpa mengkorespondensikannya dengan kelereng kesepuluh.

c.

- Siswa menentukan posisi bilangan 15 di garis bilangan dengan menghubungkannya dengan kelereng ke-lima belas.
- Siswa lainnya menaruh tanda posisi bilangan 15 tanpa mengkorespondensikannya dengan kelereng kelimabelas.

d.

- Siswa menentukan posisi bilangan 30 di garis bilangan dengan menghubungkannya dengan kelereng ke-tiga puluh.
- Siswa lainnya menaruh tanda posisi bilangan 30 tanpa mengkorespondensikannya dengan kelereng ketigapuluh.

e. .

- Siswa menentukan posisi bilangan-bilangan yang lain dengan menghubungkannya ke kelereng yang sepadan.
- Siswa lainnya menaruh tanda posisi bilangan-bilangan yang lain secara sebarang

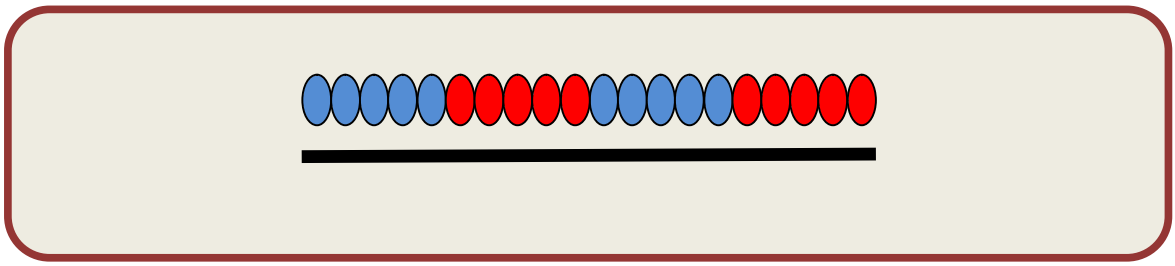
f. Siswa akan mengambil kesimpulan bahwa semakin ke kanan maka bilangan akan semakin membesar dan jika semakin ke kiri maka bilangan akan semakin kecil.

Pertanyaan-pertanyaan yang dapat diajukan guru selama proses pembelajaran dengan tujuan membimbing siswa:

a. "*Di mana posisi suatu bilangan pada kelereng?*". Pertanyaan ini diajukan ketika siswa menghubungkan bagian tengah kelereng dengan garis dalam menentukan posisi suatu bilangan.
b. *"Apakah panjang garis bilangan yang memuat bilangan dari 1 sampai 10 hanya setengah dari panjang garis bilangan yang memuat bilangan dari 1 sampai 20*?".

Soal 2

Gambar dari soal 2 ini berbeda dengan gambar di soal 1. Gambar tersebut diilustrasikan sebagai berikut:



a. Jika Budi hanya mempunyai 1 biji kelereng, tandailah posisinya pada gambar rangkaian kelereng! Di manakah posisi bilangan 1 pada garis?

   Hipotesis jawaban siswa:
   Siswa akan memberi tanda pada sisi kanan kelereng pertama dan memberi tanda untuk posisi bilangan 1 di garis bilangan dengan menghubungkan tanda posisi mempunyai 1 kelereng ke garis bilangan di bawahnya.

b. Jika Budi tidak mempunyai kelereng sama sekali, tandailah posisinya pada gambar rangkaian kelereng!

   Bilangan apa yang mewakili jika Budi tidak mempunyai kelereng sama sekali?
   Jawab: ….

   Di manakah posisi bilangannya pada garis?
   Hipotesis jawaban siswa:
   Siswa akan menaruh tanda di sisi kiri kelereng pertama. Mereka menganggap bahwa keadaan tersebut diwakili oleh bilangan 0. Dan memberi tanda untuk posisi bilangan 0 dengan menghubungkan tanda di sebelah kiri bilangan pertama ke garis bilangan di bawahnya.

c. Jika Budi hanya mempunyai 10 biji kelereng, tandailah posisinya pada gambar rangkaian kelereng! Di manakah posisi bilangan 10 pada garis?
   Hipotesis jawaban siswa:

Siswa akan menaruh tanda di sebelah kanan pada kelereng ke sepuluh dan memberi tanda untuk bilangan 10 dengan menghubungkan tanda posisi mempunyai sepuluh kelereng ke garis bilangan di bawahnya.

d. Jika Budi mempunyai 20 biji kelereng, tandailah posisinya pada gambar rangkaian kelereng! Di manakah posisi bilangan 20 pada garis?
   Hipotesis jawaban siswa:
   Siswa akan menaruh tanda di sebelah kanan pada kelereng ke kedua puluh dan memberi tanda untuk bilangan 20 dengan menghubungkan tanda posisi mempunyai dua puluh kelereng ke garis bilangan di bawahnya.

e. Jika Budi mempunyai 21 biji kelereng, dapatkan kamu memberi tanda pada gambar rangkaian kelereng! Di manakah posisi bilangan 21 pada garis?
   Hipotesis jawaban siswa:
   Beberapa siswa tidak dapat mengetahui bagaimana menentukan posisi mempunyai 21 kelereng pada rangkaian kelereng. Siswa lainnya, menggambar satu biji kelereng di sebelah kanan dan akhirnya mereka dapat menentukan posisi bilangan 21 pada garis bilangan dengan memperpanjang garis ke kanan.

f. Jika Budi mempunyai 25 biji kelereng, tandailah posisinya pada garis?
   Hipotesis jawaban siswa:
   Beberapa siswa tidak mengetahui bagaimana cara menentukan posisi mempunyai 25 biji kelereng pada garis. Siswa lainnya menambah 5 kelereng ke kanan dan akhirnya mampu untuk menentukan posisi bilangan 25 pada garis.

g. Jika Budi kekurangan 1 biji kelereng, dapatkah kamu memberi tanda pada gambar rangkaian kelereng!?
   Bilangan apa yang mewakili keadaan tersebut?
   Jawab: ……..

   Di manakah posisi bilangannya pada garis?

   Hipotesis Jawaban siswa:
   Beberapa siswa tidak mengetahui bagaimana di mana posisi kekurangan 1 biji kelereng. Siswa lainnya menggambar satu kelereng ke kiri dan akhirnya mampu menentukan posisi bilangan yang merepresentasikan kekurangan 1 biji kelereng yakni -1 pada garis bilangan.

h. Jika Budi kekurangan 2 biji kelereng, dapatkah kamu memberi tanda pada gambar rangkaian kelereng?

Bilangan apa yang mewakili keadaan tersebut?
Jawab: ……

Di manakah posisi bilangannya pada garis?

Hipotesis jawaban siswa:

Beberapa siswa tidak mengetahui bagaimana di mana posisi kekurangan 2 biji kelereng. Siswa lainnya menggambar dua kelereng ke kiri dan akhirnya mampu menentukan posisi bilangan yang merepresentasikan kekurangan 2 biji kelereng yakni -2 pada garis bilangan.

i. Jika Budi kekurangan 5 biji kelereng, dapatkah kamu memberi tanda pada gambar rangkaian kelereng!

Bilangan apa yang mewakili keadaan tersebut?
Jawab: …..
Di manakah posisi bilangannya pada garis?

Hipotesis jawaban siswa:

Beberapa siswa tidak mengetahui bagaimana di mana posisi kekurangan 5 biji kelereng. Siswa lainnya menggambar lima kelereng ke kiri dan akhirnya mampu menentukan posisi bilangan yang merepresentasikan kekurangan 5 biji kelereng yakni -5 pada garis bilangan.

j. Kesimpulan apa yang kamu dapatkan tentang bilangan-bilangan pada garis bilangan tersebut?

Kesimpulan:

……
…….

……
…….

Hipotesis jawaban siswa:
Beberapa siswa berkesimpulan bahwa bilangan akan semakin besar ketika mereka berada di kanan dan semakin kecil ketika mereka berada di kiri. Siswa lainnya mengambil kesimpulan bahwa posisi bilangan negatif berada di sebelah kiri bilangan positif.

Aktivitas guru:

1. Guru menyuruh tiap-tiap kelompok memasang pekerjaan mereka di papan tulis dan tiap-tiap siswa diminta untuk melihat pekerjaan kelompok yang lainnya.
2. Guru lalu meminta 2 kelompok untuk maju ke depan merepresentasikan jawaban mereka. Diharapkan, kelompok yang maju terlebih dahulu adalah kelompok yang mempunyai pekerjaan yang kurang tepat, dan selanjutnya kelompok yang mempunyai jawaban yang benar.

Pertanyaan-pertanyaan yang dapat diajukan guru selama proses pembelajaran dengan tujuan membimbing siswa:

a. "*Di mana posisi suatu bilangan pada gambar rangkaian kelereng?*". Pertanyaan ini diajukan ketika siswa menghubungkan bagian tengah kelereng dengan garis dalam menentukan posisi suatu bilangan.
b. *"Apakah panjang garis bilangan yang memuat bilangan dari 1 sampai 5 hanya setengah dari panjang garis bilangan yang memuat bilangan dari 1 sampai 10*?".
c. Untuk siswa yang tidak mempunyai ide untuk menentukan posisi, misalnya, mempunyai 21 kelereng, guru dapat bertanya ke siswa tersebut: "*kalau mempunyai 20 biji kelereng kan di ujung kanan kelereng terakhir, nah kalau kelerengnya menjadi 21, kelerengnya bertambah berapa*?"
d. Untuk siswa yang tidak mempunyai ide untuk menentukan posisi, misalnya, kekurangan 1 biji kelereng, guru dapat bertanya ke siswa tersebut dengan menstimulasi fikiran mereka tentang posisi mempunyai sejumlah kelereng dimulai dengan mempunyai 3 kelereng, mempunyai 2 kelereng, lalu mempunyai 1 kelereng.

Panduan Guru Pertemuan Ketiga.

Pelajaran **:Membuat Garis Bilangan**

Kelas **: IV**

Semester **: II**

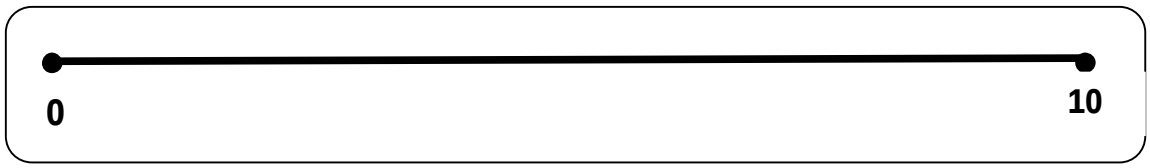
Alokasi Waktu **: 70 Menit**

Deskripsi Kegiatan Guru:

1. Siswa dibagi menjadi beberapa kelompok di mana tiap kelompok terdiri dari 2 orang siswa. Kemudian, mereka diberi lembaran soal oleh guru yang berisikan tiga buah soal di mana tiap soal dikerjakan selama 15 menit dan tiap-tiap soal didiskusikan sebelum siswa diminta mengerjakan soal berikutnya. Adapun soalnya sebagai berikut.

Tempatkanlah bilangan-bilangan yang diminta pada garis bilangan yang tersedia.

a. 5



Dugaan jawaban siswa:

- Dalam menentukan posisi bilangan 5, beberapa kelompok memulai menandai posisi dari bilangan 1 terlebih dahulu dengan menandai posisi bilangan 1 di sebelah kanan 0. Lalu mereka secara berturut-turut menandai posisi bilangan 2, 3, dan 4 di sebelah kanan bilangan 1. Lalu mereka menentukan dan menandai posisi bilangan 5 di sebelah kanan 4.

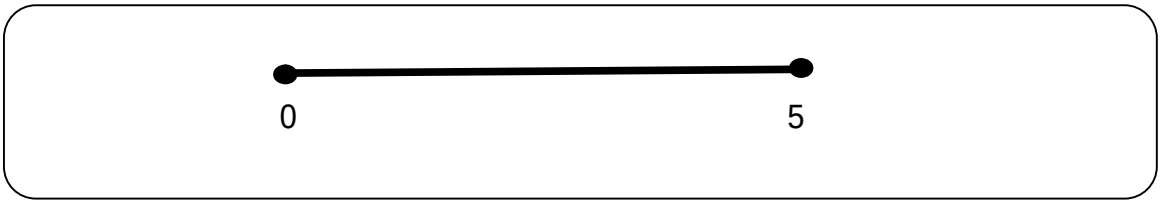
Pertanyaan guru:

1. Bisa gak menentukan posisi 5 tanpa menulis -1, -2, -3, dan -4 terlebih dahulu?

Sama gak jarak dari 0 ke 5 dan jarak dari 5 ke 10?

- Kelompok yang lainnya hanya menaruh bilangan 5 di tepat tengah-tengah garis karena mereka melihat bahwa pada garis bilangan yang tersedia, tersedia bilangan 0 dan 10. Dan mereka menganggap bahwa 5 berada di tengah-tengah antara 0 dan 10.
  Pertanyaan guru:
  Kenapa kalian menaruh bilangan 5 di tengah-tengah garis?

b. 1, 2, 3, dan 4



Dapatkah kamu menentukan posisi bilangan-bilangan berikut:

- 6
- 7
- 10
- -1
- -2
- -5

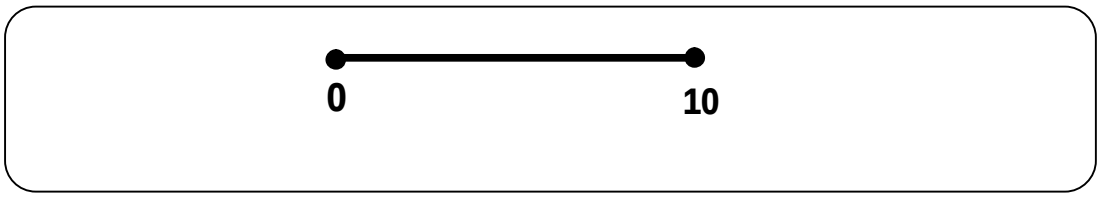
Dugaan atas jawaban siswa:

- Pada soal menaruh bilangan 1, 2, 3, dan 4, tiap-tiap kelompok menaruh bilangan-bilangan tersebut secara berurutan antara interval 1 sampai dengan 5 pada garis.
- Pada soal menaruh bilangan 6, 7, dan 10, beberapa kelompok memperpanjang garis ke kanan lalu menaruh bilangan 6 di sebelah kanan 5, dan 7 di sebelah kanan 6. Lalu mereka menempatkan bilangan 8 dan 9 Lalu mereka menaruh 10 di sebelah kanan 9.
- Pada soal menaruh bilangan -1, -2, dan -5, beberapa kelompok tidak mengetahui cara menempatkan bilangan-bilangan tersebut. Ada yang tidak menempatkan bilangannya sama sekali dan ada pula yang menaruh bilangan-bilangan tersebut pada garis interval 0 sampai dengan 5.
  Pertanyaan guru:

Kalau bilangan semakin ke kiri pada garis semakin kecil atau semakin besar?
Terus, bilangan negatif lebih kecil gak dari pada bilangan positif?
Jadi, di mana posisinya kalau bilangan negatif lebih kecil dari pada bilangan positif?

- Kelompok lainnya, mereka memperpanjang garis ke kiri, lalu menempatkan bilangan -1, -2, secara berturut-turut di sebelah kiri 0. Lalu, mereka menaruh tanda untuk posisi bilangan -3 dan -4 secara berturut-turut di sebelah kiri -2, setelah itu barulah mereka menempatkan -5 di sebelah kiri -4.

c. 5, -5, dan -10



Dugaan atas jawaban siswa:

1. Dalam menentukan posisi bilangan 5.

- Beberapa kelompok menaruh bilangan 5 tepat di tengah-tengah garis yang disediakan.
- Beberapa kelompok menaruh bilangan 5 setelah mereka menandai posisi bilangan 1, 2, 3, dan 4 secara berturut-turut di sebelah kanan 0.

2. Dalam menentukan posisi bilangan -5

- Beberapa kelompok menaruh tanda posisi -5 di interval antara 0 sampai 10 pada garis.

Pertanyaan guru:

-5 termasuk bilangan negatif tidak? Terus bilangan negatif itu posisinya di sebelah kiri 0 atau kanan 0?

- Beberapa kelompok lainnya memperpanjang garis ke kiri lalu menaruh bilangan -5 setelah mereka memberi tanda untuk posisi -1, -2, -3, dan -4 secara berturut-turut di sebelah kiri 0.
- Beberapa kelompok lainnya, langsung memanjangkan garis yang tersedia ke kiri sepanjang jarak antara 0 sampai dengan 5, lalu mereka memberi tanda pada ujung garis tersebut dengan bilangan -5.

3. Dalam menentukan posisi bilangan -10
   - Beberapa kelompok menaruh tanda posisi -10 di interval antara 0 sampai 10 pada garis.

     Pertanyaan guru:

     -10 termasuk bilangan negatif tidak? Terus bilangan negatif itu posisinya di sebelah kiri 0 atau kanan 0?

   - Beberapa kelompok lainnya memperpanjang garis ke kiri lalu menaruh bilangan -10 setelah mereka memberi tanda untuk posisi -6, -7, -8, dan -9 secara berturut-turut di sebelah kiri -5.
   - Beberapa kelompok lainnya, langsung memanjangkan garis yang tersedia ke kiri sepanjang jarak antara 0 sampai dengan 10, lalu mereka memberi tanda pada ujung garis tersebut dengan bilangan -10.
2. Guru lalu menyuruh tiap-tiap kelompok memasang pekerjaan mereka di papan tulis dan tiap-tiap siswa diminta untuk melihat pekerjaan kelompok yang lainnya.
3. Guru lalu meminta 2 kelompok untuk maju ke depan merepresentasikan jawaban mereka. Diharapkan, kelompok yang maju terlebih dahulu adalah kelompok yang mempunyai pekerjaan yang kurang tepat, dan selanjutnya kelompok yang mempunyai jawaban yang benar.

Panduan Guru Pertemuan Keempat.

Pelajaran : Menjumlahkan dua bilangan positif dan dua bilangan negatif

Kelas : IV

Semester : II

Alokasi Waktu : 70 Menit

Deskripsi Kegiatan Guru:

1. Guru mengulang secara singkat pelajaran pada pertemuan sebelumnya yakni materi yang berkenaan dengan garis bilangan.
2. Siswa dibagi menjadi beberapa kelompok di mana tiap kelompok terdiri dari 2 orang siswa. Kemudian, mereka diberi lembaran soal oleh guru yang berisikan dua buah soal di mana tiap soal dikerjakan selama 15 menit dan tiap-tiap soal didiskusikan sebelum siswa diminta mengerjakan soal berikutnya. Adapun soalnya sebagai berikut.

1. *Kemarin, Eka masih memiliki 4 biji kelereng. Hari ini, dia memiliki 2 biji kelereng lagi. Berapa total kelereng yang dimiliki oleh Eka*.?

Dugaan jawaban siswa:

- **6 / banyaknya kelereng yang dimiliki oleh Eka adalah 6.**
  Alasan/cara kerja:
  4 + 2 = 6.

2. *Kemarin, Anton kekurangan 3 biji kelereng. Hari ini dia kekurangan 4 biji kelereng. Berapa total kekurangan kelereng yang dimiliki oleh Anton*?

- **7 / Anton kekurangan 7 biji kelereng.**
  Alasan/cara kerja:

3 + 4 = 7.

Pertanyaan yang diajukan guru:

Apa arti dari bilangan 7 yang kamu tulis? (pertanyaan ini diajukan kalau siswa hanya menulis bilangan 7 saja)

Kalau kekurangan kelereng diwakili bilangan positif atau negatif?

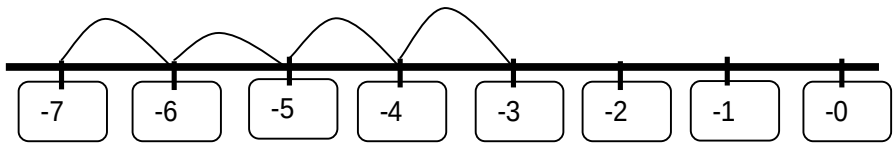
- **-7/Anton kekurangan 7 biji kelereng.**

  Alasan/cara kerja:

  -3 + -4 = -7.

  Pertanyaan guru:

- Kenapa negatif 3 ditambah negatif 4 sama dengan negatif 7? Apa alasannya?
- Coba pakai garis bilangan kerjanya.
- Di mana posisi kekurangan 3 biji kelereng pada garis bilangan?
- Kalau ke kekurangan lagi, dari bilangan 3 kita lompat atau pindah ke mana pada garis bilangan? Ke kiri atau ke kanan?
- Kenapa ke kiri? (kalau siswa jawab ke kiri) Kenapa ke kanan? (kalau siswa jawab ke kanan)
- Sebanyak berapa kali pindahnya/lompatnya?



3. *Apakah jumlah kelereng Eka dan Anton bedanya sebanyak 1?*

   Dugaan jawaban siswa:

   - **Ya / Bedanya 1**

     Alasan: karena Eka punya 6 kelereng dan Anton punya 7 kelereng.

     Pertanyaan guru:

     Bagaimana cara kerja kalian? (siswa menjawab: 7 – 6)

     Jadi, Anton juga punya 7 kelereng gitu?

     Coba gambar pakai garis bilangan, di mana posisi kelereng Eka dan kelereng Anton?

   - **Tidak/ bedanya 13**

     Alasan: karena Eka punya 6 dan Anton kekurangan 7.

3. Guru lalu menyuruh tiap-tiap kelompok memasang pekerjaan mereka di papan tulis dan tiap-tiap siswa diminta untuk melihat pekerjaan kelompok yang lainnya.
4. Guru lalu meminta 2 kelompok untuk maju ke depan merepresentasikan jawaban mereka. Diharapkan, kelompok yang maju terlebih dahulu adalah kelompok yang mempunyai pekerjaan yang kurang tepat, dan selanjutnya kelompok yang mempunyai jawaban yang benar.
5. Pada saat terjadinya diskusi, kemungkinan siswa menjawab secara kontekstual. Oleh karena itu, guru menanyakan ke mereka bagaimana mereka mendapatkan hasil seperti itu. Misalnya: *bagaimana kamu bisa mendapatkan bilangan tersebut*?
6. Guru menanyakan ke siswa untuk mengilustrasikan pekerjaan mereka dengan garis bilangan.
7. Guru menanyakan kesimpulan tentang apa yang siswa telah pelajari khususnya tentang konsep matematika yang mereka telah terima.

Kesimpulan:

Kalau bilangan negatif ditambah bilangan negatif hasilnya bilangan apa?

Panduan Guru Pertemuan Kelima.

Pelajaran : Menjumlahkan bilangan negatif dengan bilangan positif

Kelas : IV

Semester : II

Alokasi Waktu : 70 Menit

Deskripsi Kegiatan Guru:

1. Siswa dibagi menjadi beberapa kelompok di mana tiap kelompok terdiri dari 2 orang siswa. Kemudian, mereka diberi lembaran soal oleh guru yang berisikan empat buah soal di mana tiap soal dikerjakan selama 5 menit dan tiap-tiap soal didiskusikan sebelum siswa diminta mengerjakan soal berikutnya. Adapun soalnya adalah sebagai berikut.

Kemarin, Ilham kekurangan persediaan 5 biji kelereng. Namun, hari ini, dia mendapatkan kelereng baru dari temannya. Apakah dia masih kekurangan kelereng jika dia mendapatkan kelereng baru sebanyak:

a. 3 biji

Jawab:

Jika dia tidak lagi kekurangan kelereng, berapa banyak biji kelereng yang ia miliki?

Dugaan jawaban siswa:

- Beberapa kelompok menjawab: Ilham masih kekurangan 2 kelereng karena mereka berfikir bahwa penambahan 3 kelereng tidaklah cukup bagi Ilham agar ia tidak kekurangan kelereng lagi.
  Pertanyaan guru:
  Bagaimana cara kamu mendapatkan jawaban bahwa Ilham kekurangan 2 kelereng?

Operasi apa yang kamu pakai?

- Kelompok lainnya menganggap bahwa Ilham mempunyai 3 biji kelereng, karena ia mendapatkan 3 kelereng baru.

  Pertanyaan guru:

  Coba kerja pakai garis bilangan, gambar garis bilangan terus di mana posisi kekurangan 5 biji kelereng pada garis bilangan. Nah kalau Ilham dapat 3 kelereng baru, terjadi penambahan atau pengurangan?
  Kalau terjadi penambahan, ke kanan atau ke kiri?
  Sebanyak berapa kali ke kanan?
  Di bilangan mana berhentinya?
  Jadi dari garis bilangan tersebut, bilangannya ditambah atau dikurangi?
  Bilangan apa tambah bilangan apa?
  Hasilnya?

b. 5 biji

Jawab:

Jika dia tidak lagi kekurangan kelereng, berapa banyak biji kelereng yang ia punya?

Dugaan jawaban siswa:

- Beberapa kelompok menjawab Ilham tidak lagi kekurangan kelereng, karena kekurangannya tertutupi oleh 5 kelereng baru.

  Pertanyaan guru:

  Bagaimana cara kamu mendapatkan jawaban Ilham tidak lagi kekurangan kelereng?
  Operasi apa yang kamu pakai?
- Kelompok lainnya menganggap bahwa Ilham mempunyai 5 biji kelereng, karena ia mendapatkan 5 kelereng baru.

  Pertanyaan guru:

  Coba kerja pakai garis bilangan, gambar garis bilangan terus di mana posisi kekurangan 5 biji kelereng pada garis bilangan. Nah kalau Ilham dapat 5 kelereng baru, terjadi penambahan atau pengurangan?

Kalau terjadi penambahan, ke kanan atau ke kiri?
Sebanyak berapa kali ke kanan?
Di bilangan mana berhentinya?
Jadi dari garis bilangan tersebut, bilangannya ditambah atau dikurangi?
Bilangan apa tambah bilangan apa?
Hasilnya?

c. 8 biji

Jawab:

Jika dia tidak lagi kekurangan kelereng, berapa banyak biji kelereng yang ia punya?

Dugaan jawaban siswa:

- Beberapa kelompok menjawab Ilham tidak lagi kekurangan kelereng tapi ia telah mempunyai 3 kelereng, karena kekurangannya tertutupi oleh 8 kelereng baru.

  Pertanyaan guru:
  Bagaimana cara kamu mendapatkan jawaban bahwa Ilham tidak lagi kekurangan kelereng dan ia sekarang memiliki 3 kelereng?
  Operasi apa yang kamu pakai?

- Kelompok lainnya menganggap bahwa Ilham mempunyai 8 biji kelereng, karena ia mendapatkan 8 kelereng baru.

  Pertanyaan guru:
  Coba kerja pakai garis bilangan, gambar garis bilangan terus di mana posisi kekurangan 5 biji kelereng pada garis bilangan. Nah kalau Ilham dapat 8 kelereng baru, terjadi penambahan atau pengurangan?
  Kalau terjadi penambahan, ke kanan atau ke kiri?
  Sebanyak berapa kali ke kanan?
  Di bilangan mana berhentinya?
  Jadi dari garis bilangan tersebut, bilangannya ditambah atau dikurangi?
  Bilangan apa tambah bilangan apa?
  Hasilnya?

d. 10 biji

Jawab:

Jika dia tidak lagi kekurangan kelereng, berapa banyak biji kelereng yang ia punya?

Dugaan jawaban siswa:

Beberapa kelompok menjawab Ilham telah mempunyai 5 kelereng, karena kekurangannya tertutupi oleh 10 kelereng baru.

Pertanyaan guru:

Bagaimana cara kamu mendapatkan jawaban bahwa Ilham tidak lagi kekurangan kelereng dan ia sekarang memiliki 5 kelereng?

- Kelompok lainnya menganggap bahwa Ilham mempunyai 10 biji kelereng, karena ia mendapatkan 10 kelereng baru.

Pertanyaan guru:

- Coba kerja pakai garis bilangan, gambar garis bilangan terus di mana posisi kekurangan 5 biji kelereng pada garis bilangan. Nah kalau Ilham dapat 8 kelereng baru, terjadi penambahan atau pengurangan?
  Kalau terjadi penambahan, ke kanan atau ke kiri?
  Sebanyak berapa kali ke kanan?
  Di bilangan mana berhentinya?
  Jadi dari garis bilangan tersebut, bilangannya ditambah atau dikurangi?
  Bilangan apa tambah bilangan apa?
  Hasilnya?

2. Guru lalu meminta 2 kelompok untuk maju ke depan merepresentasikan jawaban mereka. Diharapkan, kelompok yang maju terlebih dahulu adalah kelompok yang

mempunyai pekerjaan yang kurang tepat, dan selanjutnya kelompok yang mempunyai jawaban yang benar.

3. Pada saat terjadinya diskusi, kemungkinan siswa menjawab secara kontekstual. Oleh karena itu, guru menanyakan ke mereka bagaimana mereka mendapatkan hasil seperti itu. Misalnya: *bagaimana kamu bisa mendapatkan bilangan tersebut*?

4. Guru menanyakan kesimpulan tentang apa yang siswa telah pelajari khususnya tentang konsep matematika yang mereka telah terima.

Panduan Guru Pertemuan Keenam.

Pelajaran : Menjumlahkan bilangan positif dengan bilangan negatif

Kelas : IV

Semester : II

Alokasi Waktu : 70 Menit

Deskripsi Kegiatan Guru:

1. Siswa dibagi menjadi beberapa kelompok di mana tiap kelompok terdiri dari 2 orang siswa. Kemudian, mereka diberi lembaran soal oleh guru yang berisikan empat buah soal di mana tiap soal dikerjakan selama 5 menit dan tiap-tiap soal didiskusikan sebelum siswa diminta mengerjakan soal berikutnya. Adapun soalnya adalah sebagai berikut.

Selesaikanlah

a. Apakah hasil dari -5 + 3 sama dengan 3 + (- 5)?

b. Apakah hasil dari -5 + 5 sama dengan 5 + (-5)?

c. Apakah hasil dari -5 + 8 sama dengan 8 + (-5)?

d. Apakah hasil dari -5 + 10 sama dengan 10 + (-5)?

Dugaan tipe jawaban siswa:

- Beberapa siswa memvisualisasikan permasalahan dengan menggunakan garis bilangan. Mereka mulai dengan 3 lalu berpindah ke kiri sebanyak 5 langkah menuju -2.

  Pertanyaan guru:
  "*Kenapa pindahnya ke kiri*?"

- Beberapa siswa memvisualisasikan permasalahan dengan menggunakan garis bilangan. Mereka memulai dengan 3 ke kanan sebanyak 5 langkah menuju 8.

  Pertanyaang guru:

  "*Kenapa pindahnya ke kanan*?"

- Beberapa siswa menganggap bahwa jawabannya adalah 8.

  Pertanyaang guru:

  "*bagaimana caranya kamu mendapatkan jawaban tersebut*?"

2. Guru lalu menyuruh tiap-tiap kelompok memasang pekerjaan mereka di papan tulis dan tiap-tiap siswa diminta untuk melihat pekerjaan kelompok yang lainnya.
3. Guru lalu meminta 2 kelompok untuk maju ke depan merepresentasikan jawaban mereka. Diharapkan, kelompok yang maju terlebih dahulu adalah kelompok yang mempunyai pekerjaan yang kurang tepat, dan selanjutnya kelompok yang mempunyai jawaban yang benar.

Panduan Guru Pertemuan Ketujuh.

Pelajaran : Menjumlahkan bilangan bulat

Kelas : IV

Semester : II

Alokasi Waktu : 70 Menit

Deskripsi Kegiatan Guru:

1. Siswa dibagi menjadi beberapa kelompok di mana tiap kelompok terdiri dari 4 orang siswa. Kemudian, mereka diberi lembaran soal oleh guru yang berisikan empat buah soal di mana tiap soal dikerjakan selama 5 menit dan tiap-tiap soal didiskusikan sebelum siswa diminta mengerjakan soal berikutnya. Adapun soalnya adalah sebagai berikut.

Selesaikanlah

1.

a. 5 + 6 = …
b. 11 + 15 = …
c. 14 + 31 = …

Jawaban: a. .
b. .
c. .

Dugaan jawaban siswa dan pertanyaan/reaksi dari guru:

a. Ketika siswa menjawab pertanyaan secara langsung tanpa melalui proses, guru bertanya ke siswa untuk menunjukkan cara kerja mereka.
b. Siswa menjawab dengan langsung pertanyaan dan menggunakan garis bilangan untuk menunjukkan cara kerja mereka.

2.

a. 8 + (-5) = …
b. 12 + (-16) = …
c. 20 + (-25) = …

jawaban: a. .
b. .
c. .

Dugaan jawaban siswa dan reaksi/pertanyaan guru:

a. Siswa menggunakan garis bilangan dengan memulai dari bilangan positif lalu melakukan lompatan ke arah kiri
b. Siswa menggunakan garis bilangan dengan memulai dari bilangan positif lalu melakukan lompatan ke arah kanan dan guru bertanya ke siswa tersebut mengapa siswa tersebut melakukan lompatan ke arah kanan.

3.

a. -6 + 7 = …
b. -14 + 18 = …
c. -23 + 40 = …

jawaban: a. .
b. .
c. .

Dugaan jawaban siswa:

Siswa menggunakan garis bilangan dengan memulai dari bilangan negatif dan membuat lompatan ke kanan

4.

a. -5 + -3 = …
b. -11 + -13 = …
c. -15 + -25 = …

jawaban: a.
b. .
c. .

Dugaan jawaban siswa:

Siswa menggunakan garis bilangan dengan memulai dari bilangan negatif dan membuat lompatan ke arah kiri.

Dugaan tipe jawaban siswa:

- Beberapa siswa memvisualisasikan permasalahan dengan menggunakan garis bilangan. Mereka mulai dengan 3 lalu berpindah ke kiri sebanyak 5 langkah menuju -2.

  Pertanyaan guru:
  "*Kenapa pindahnya ke kiri*?"

- Beberapa siswa memvisualisasikan permasalahan dengan menggunakan garis bilangan. Mereka memulai dengan 3 ke kanan sebanyak 5 langkah menuju 8.

  Pertanyaang guru:

  "*Kenapa pindahnya ke kanan*?"

- Beberapa siswa menganggap bahwa jawabannya adalah 8.

  Pertanyaang guru:

  "*bagaimana caranya kamu mendapatkan jawaban tersebut*?"

2. Guru lalu menyuruh tiap-tiap kelompok memasang pekerjaan mereka di papan tulis dan tiap-tiap siswa diminta untuk melihat pekerjaan kelompok yang lainnya.
3. Guru lalu meminta 2 kelompok untuk maju ke depan merepresentasikan jawaban mereka. Diharapkan, kelompok yang maju terlebih dahulu adalah kelompok yang mempunyai pekerjaan yang kurang tepat, dan selanjutnya kelompok yang mempunyai jawaban yang benar.

Appendix D

Rencana Pelaksanaan Pembelajaran (RPP)

Nama Sekolah : SDN 119 Palembang
Mata Pelajaran : Matematika
Kelas/Semester : IV (Empat) / 2
Jumlah Pertemuan : Satu kali pertemuan
Pertemuan ke- : 1

A. Standar Kompetensi
Mengurutkan Bilangan Bulat
B. Kompetensi Dasar
Menentukan Letak Bilangan Bulat Pada Garis Bilangan
C. Indikator Pencapaian Kompetensi
Menentukan Letak Bilangan Bulat Pada Garis Bilangan
D. Tujuan Pembelajaran
Siswa dapat menentukan letak bilangan bulat pada garis bilangan
E. Materi Ajar : Bilangan Bulat
F. Alokasi Waktu : 2 jam pelajaran (@ 35 menit)
G. Pendekatan Pembelajaran : Pendidikan Matematika Realistik Indonesia (PMRI)
Metode Pembelajaran : Pemecahan masalah, diskusi, dan tanya jawab
H. Kegiatan Pembelajaran

Aktivitas 1

1. **Pendahuluan (5 menit)**
   a. Apersepsi/Motivasi.
   b. Guru membagi seluruh siswa ke dalam beberapa kelompok, di mana masing-masing kelompok terdiri dari 4 orang.

2. **Inti (20 menit)**
   1. ***Eksplorasi***
      a. Guru menanyakan secara lisan pertanyaan-pertanyaan yang terdapat pada panduan guru ke siswa dan meminta mereka menunjukkan cara kerjanya di depan kelas.
      b. Guru berperan sebagai fasilitator selama proses pembelajaran berlangsung untuk membimbing siswa menyelesaikan permasalahan yang diberikan agar sesuai dengan tujuan yang diharapkan.
   2. ***Elaborasi***
      a. Ketika siswa telah menunjukkan cara kerjanya di depan kelas, guru mengarahkan siswa lainnya untuk menanggapi jawaban temannya dan melakukan diskusi.

b. Guru memberikan kesempatan ke siswa lainnya untuk maju ke depan kelas apabila mereka memiliki cara yang lain dalam menyelesaikan pertanyaan guru.
c. Untuk siswa yang maju ke depan kelas, guru mengarahkan agar siswa dalam kelompok tersebut menjelaskan bagaimana strategi mereka dalam menyelesaikan permasalahan yang telah diberikan.

3. Konfirmasi

a. Guru membimbing jalannya diskusi kelas pada saat siswa menjelaskan strategi mereka. Dalam hal ini guru mengarahkan agar setiap siswa mendengarkan siswa yang maju mengerjakan pertanyaan di depan kelas. Guru juga menfasilitasi proses pembelajaran dengan mengajak siswa berani bertanya kepada temannya dan menyampaikan ide yang mereka miliki.
b. Saat siswa mengemukakan cara yang menarik, guru dapat memberikan sejumlah pertanyaan untuk memperoleh pengetahuan yang mendalam (*insight*) dari siswa tentang cara yang ia gunakan.
c. Pada akhir dari diskusi, guru mengarahkan agar siswa mendapatkan pemahaman tentang materi pelajaran sesuai tujuan yang diharapkan.

Aktivitas 2

1. Inti (30 menit)

1. Eksplorasi

a. Guru membagikan LKS yang di dalamnya berisi sejumlah pertanyaan terkait penjumlahan dan pengurangan bilangan bulat.
b. Ketika siswa bekerja dalam kelompoknya, guru mengarahkan mereka agar saling berdiskusi dalam kelompoknya untuk menjawab seluruh pertanyaan yang ada di LKS.
c. Guru berperan sebagai fasilitator selama proses pembelajaran berlangsung untuk membimbing siswa menyelesaikan permasalahan yang diberikan agar sesuai dengan tujuan yang diharapkan.

2. Elaborasi

a. Siswa bekerja dalam kelompoknya dan diberikan kesempatan untuk menjawab seluruh pertanyaan yang ada di LKS dengan menggunakan berbagai strategi yang dimilikinya.
b. Setiap kelompok menempelkan hasil kerjanya di dinding dan Guru memberi kesempatan kepada siswa untuk melihat hasil kerja kelompok lain.
c. Guru menanyakan apa yang siswa dapatkan ketika mereka membandingkan jawaban kelompok mereka dengan jawaban kelompok lain. Setelah terjadi diskusi beberapa saat, dua kelompok diminta mempresentasikan hasil kerja mereka di depan kelas. Dalam hal ini, siswa diharapkan dapat maju dengan sendirinya dan guru memotivasi siswa agar berani tampil di depan kelas.
d. Dua kelompok yang melakukan presentasi diharapkan adalah kelompok yang jawabannya menarik (jawabannya tepat, strateginya unik dan sesuai dengan tujuan pembelajaran) dan kelompok yang jawabannya kurang tepat.

e. Untuk kelompok yang akan berpresentasi, guru mengarahkan agar siswa dalam kelompok tersebut menjelaskan bagaimana strategi mereka dalam menyelesaikan permasalahan yang telah diberikan.

3. Konfirmasi

a. Guru membimbing jalannya diskusi kelas pada saat tahap presentasi. Dalam hal ini guru mengarahkan agar setiap kelompok mendengarkan kelompok yang presentasi dan memberikan pertanyaan kepada kelompok yang presentasi tersebut. Guru juga menfasilitasi proses pembelajaran dengan mengajak siswa berani bertanya kepada temannya dan menyampaikan ide yang mereka miliki.
b. Kelompok yang jawabannya kurang tepat dapat memprentasikan hasil kerjanya terlebih dahulu, kemudian kelompok lain diminta untuk membandingkan dengan jawaban kelompoknya masing-masing dan melakukan koreksi sehingga terbentuk kesadaran pada siswa mana jawaban yang tepat.
c. Saat kelompok yang jawabannya menarik berpresentasi, guru dapat memberikan sejumlah pertanyaan untuk memperoleh pengetahuan yang mendalam (*insight*) dari siswa tentang jawaban yang dituliskannya pada lembar jawaban dan strategi yang digunakannya.
d. Pada akhir dari diskusi, guru mengarahkan agar siswa mendapatkan pemahaman tentang materi pelajaran sesuai tujuan yang diharapkan.

2. Penutup (10 menit)

a. Siswa diarahkan untuk merangkum materi yang telah didiskusikan dan menarik kesimpulan dari proses pembelajaran yang telah dilakukan.
b. Guru memberikan informasi tentang pembelajaran yang akan datang.

I. Media/Sumber Belajar

1. Kertas *plano*, spidol, *double tip*
2. Kertas karton
3. Lembar Kerja Siswa (LKS)
4. Buku Matematika Kelas IV
5. *Teacher Guide* / Panduan Guru

J. Penilaian Hasil Belajar

Penilaian dilakukan melalui penilaian proses belajar siswa. Penilaian proses dilakukan dengan mengamati aktivitas siswa selama proses belajar mengajar berlangsung, yang meliputi strategi yang digunakan siswa dalam menyelesaikan permasalahan yang diberikan, kemampuan siswa dalam memberikan penjelasan dan argumentasi atas penyelesaian masalah ketika diskusi di kelompoknya atau di kelas, serta kemampuan siswa untuk mempertahankan argumentasinya atau memberikan tanggapan terhadap ide dari temannya yang lain.

Rencana Pelaksanaan Pembelajaran (RPP)

Nama Sekolah : SDN 119 Palembang
Mata Pelajaran : Matematika
Kelas/Semester : IV (Empat) / 2
Jumlah Pertemuan : Satu kali pertemuan
Pertemuan ke- : 2

A. Standar Kompetensi
Mengurutkan Bilangan Bulat
B. Kompetensi Dasar
Menentukan Letak Bilangan Bulat Pada Garis Bilangan
C. Indikator Pencapaian Kompetensi
Menentukan Letak Bilangan Bulat Pada Garis Bilangan
D. Tujuan Pembelajaran
Siswa dapat menentukan letak bilangan bulat pada garis bilangan
E. Materi Ajar : Bilangan Bulat
F. Alokasi Waktu : 2 jam pelajaran (@ 35 menit)
G. Pendekatan Pembelajaran : Pendidikan Matematika Realistik Indonesia (PMRI)
Metode Pembelajaran : Pemecahan masalah, diskusi, dan tanya jawab
H. Kegiatan Pembelajaran

1. **Pendahuluan (5 menit)**
   a. Apersepsi/Motivasi.
   b. Guru mengarahkan siswa untuk tetap duduk secara berkelompok sebagaimana pada pertemuan sebelumnya.
   c. Guru mengingatkan siswa kembali tentang pelajaran sebelumnya, yaitu tentang posisi pada rangkaian kelereng apabila Budi memiliki kelereng dalam jumlah tertentu.

2. **Inti (55 menit)**
   1. ***Eksplorasi***
      a. Guru membagikan LKS yang di dalamnya berisi sejumlah pertanyaan terkait penentuan posisi bilangan bulat pada suatu garis bilangan.
      b. Ketika siswa bekerja dalam kelompoknya, guru mengarahkan mereka agar saling berdiskusi dalam kelompoknya untuk menjawab seluruh pertanyaan yang ada di LKS.
      c. Guru berperan sebagai fasilitator selama proses pembelajaran berlangsung untuk membimbing siswa menyelesaikan permasalahan yang diberikan agar sesuai dengan tujuan yang diharapkan.

   2. ***Elaborasi***
      a. Siswa bekerja dalam kelompoknya dan diberikan kesempatan untuk menjawab seluruh pertanyaan yang ada di LKS dengan menggunakan berbagai strategi yang dimilikinya.

b. Setiap kelompok menempelkan hasil kerjanya di dinding dan Guru memberi kesempatan kepada siswa untuk melihat hasil kerja kelompok lain.
c. Guru menanyakan apa yang siswa dapatkan ketika mereka membandingkan jawaban kelompok mereka dengan jawaban kelompok lain. Setelah terjadi diskusi beberapa saat, dua kelompok diminta mempresentasikan hasil kerja mereka di depan kelas. Dalam hal ini, siswa diharapkan dapat maju dengan sendirinya dan guru memotivasi siswa agar berani tampil di depan kelas.
d. Dua kelompok yang melakukan presentasi diharapkan adalah kelompok yang jawabannya menarik (jawabannya tepat, strateginya unik dan sesuai dengan tujuan pembelajaran) dan kelompok yang jawabannya kurang tepat.
e. Untuk kelompok yang akan berpresentasi, guru mengarahkan agar siswa dalam kelompok tersebut menjelaskan bagaimana strategi mereka dalam menyelesaikan permasalahan yang telah diberikan.

3. Konfirmasi

a. Guru membimbing jalannya diskusi kelas pada saat tahap presentasi. Dalam hal ini guru mengarahkan agar setiap kelompok mendengarkan kelompok yang presentasi dan memberikan pertanyaan kepada kelompok yang presentasi tersebut. Guru juga menfasilitasi proses pembelajaran dengan mengajak siswa berani bertanya kepada temannya dan menyampaikan ide yang mereka miliki.
b. Kelompok yang jawabannya kurang tepat dapat memprentasikan hasil kerjanya terlebih dahulu, kemudian kelompok lain diminta untuk membandingkan dengan jawaban kelompoknya masing-masing dan melakukan koreksi sehingga terbentuk kesadaran pada siswa mana jawaban yang tepat.
c. Saat kelompok yang jawabannya menarik berpresentasi, guru dapat memberikan sejumlah pertanyaan untuk memperoleh pengetahuan yang mendalam (*insight*) dari siswa tentang jawaban yang dituliskannya pada lembar jawaban dan strategi yang digunakannya.
d. Pada akhir dari diskusi, guru mengarahkan agar siswa mendapatkan pemahaman tentang materi pelajaran sesuai tujuan yang diharapkan.

4. Penutup (10 menit)

a. Siswa diarahkan untuk merangkum materi yang telah didiskusikan dan menarik kesimpulan dari proses pembelajaran yang telah dilakukan.
b. Guru memberikan informasi tentang pembelajaran yang akan datang.

I. Media/Sumber Belajar

1. Kertas *plano*, spidol, *double tip*
2. Kertas karton
3. Lembar Kerja Siswa (LKS)
4. Buku Matematika Kelas IV
5. *Teacher Guide*

J. Penilaian Hasil Belajar

Penilaian dilakukan melalui penilaian proses belajar siswa. Penilaian proses dilakukan dengan mengamati aktivitas siswa selama proses belajar mengajar berlangsung, yang

meliputi strategi yang digunakan siswa dalam menyelesaikan permasalahan yang diberikan, kemampuan siswa dalam memberikan penjelasan dan argumentasi atas penyelesaian masalah ketika diskusi di kelompoknya atau di kelas, serta kemampuan siswa untuk mempertahankan argumentasinya atau memberikan tanggapan terhadap ide dari temannya yang lain.

Rencana Pelaksanaan Pembelajaran (RPP)

Nama Sekolah : SDN 119 Palembang
Mata Pelajaran : Matematika
Kelas/Semester : IV (Empat) / 2
Jumlah Pertemuan : Satu kali pertemuan
Pertemuan ke- : 3

A. Standar Kompetensi
Mengurutkan Bilangan Bulat
B. Kompetensi Dasar
Menentukan Letak Bilangan Bulat Pada Garis Bilangan
C. Indikator Pencapaian Kompetensi
Menentukan Letak Bilangan Bulat Pada Garis Bilangan
D. Tujuan Pembelajaran
Siswa dapat menentukan letak bilangan bulat pada garis bilangan
E. Materi Ajar : Bilangan Bulat
F. Alokasi Waktu : 2 jam pelajaran (@ 35 menit)
G. Pendekatan Pembelajaran : Pendidikan Matematika Realistik Indonesia (PMRI)
Metode Pembelajaran : Pemecahan masalah, diskusi, dan tanya jawab
H. Kegiatan Pembelajaran

Aktivitas 1

1. Pendahuluan (5 menit)

a. Apersepsi/Motivasi.
b. Guru mengarahkan siswa untuk tetap duduk secara berkelompok sebagaimana pada pertemuan sebelumnya.
a. Guru mengingatkan siswa kembali tentang pelajaran sebelumnya, yaitu tentang posisi pada rangkaian kelereng apabila Budi memiliki kelereng dalam jumlah tertentu serta karakteristik garis bilangan.

2. Inti (30 menit)

1. Eksplorasi

a. Guru membagikan LKS yang di dalamnya berisi sejumlah pertanyaan terkait penentuan posisi bilangan bulat pada suatu garis bilangan.
b. Ketika siswa bekerja dalam kelompoknya, guru mengarahkan mereka agar saling berdiskusi dalam kelompoknya untuk menjawab seluruh pertanyaan yang ada di LKS.
c. Guru berperan sebagai fasilitator selama proses pembelajaran berlangsung untuk membimbing siswa menyelesaikan permasalahan yang diberikan agar sesuai dengan tujuan yang diharapkan.

2. Elaborasi

a. Siswa bekerja dalam kelompoknya dan diberikan kesempatan untuk menjawab seluruh pertanyaan yang ada di LKS dengan menggunakan berbagai strategi yang dimilikinya.
b. Setiap kelompok menempelkan hasil kerjanya di dinding dan Guru memberi kesempatan kepada siswa untuk melihat hasil kerja kelompok lain.
c. Guru menanyakan apa yang siswa dapatkan ketika mereka membandingkan jawaban kelompok mereka dengan jawaban kelompok lain. Setelah terjadi diskusi beberapa saat, dua kelompok diminta mempresentasikan hasil kerja mereka di depan kelas. Dalam hal ini, siswa diharapkan dapat maju dengan sendirinya dan guru memotivasi siswa agar berani tampil di depan kelas.
d. Dua kelompok yang melakukan presentasi diharapkan adalah kelompok yang jawabannya menarik (jawabannya tepat, strateginya unik dan sesuai dengan tujuan pembelajaran) dan kelompok yang jawabannya kurang tepat.
e. Untuk kelompok yang akan berpresentasi, guru mengarahkan agar siswa dalam kelompok tersebut menjelaskan bagaimana strategi mereka dalam menyelesaikan permasalahan yang telah diberikan.

3. Konfirmasi

a. Guru membimbing jalannya diskusi kelas pada saat tahap presentasi. Dalam hal ini guru mengarahkan agar setiap kelompok mendengarkan kelompok yang presentasi dan memberikan pertanyaan kepada kelompok yang presentasi tersebut. Guru juga menfasilitasi proses pembelajaran dengan mengajak siswa berani bertanya kepada temannya dan menyampaikan ide yang mereka miliki.
b. Kelompok yang jawabannya kurang tepat dapat memprentasikan hasil kerjanya terlebih dahulu, kemudian kelompok lain diminta untuk membandingkan dengan jawaban kelompoknya masing-masing dan melakukan koreksi sehingga terbentuk kesadaran pada siswa mana jawaban yang tepat.
c. Saat kelompok yang jawabannya menarik berpresentasi, guru dapat memberikan sejumlah pertanyaan untuk memperoleh pengetahuan yang mendalam (*insight*) dari siswa tentang jawaban yang dituliskannya pada lembar jawaban dan strategi yang digunakannya.
d. Pada akhir dari diskusi, guru mengarahkan agar siswa mendapatkan pemahaman tentang materi pelajaran sesuai tujuan yang diharapkan.

3. Penutup (10 menit)

a. Siswa diarahkan untuk merangkum materi yang telah didiskusikan dan menarik kesimpulan dari proses pembelajaran yang telah dilakukan.
b. Guru memberikan informasi tentang pembelajaran yang akan datang.

I. Media/Sumber Belajar

1. Kertas *plano*, spidol, *double tip*
2. Kertas karton
3. Lembar Kerja Siswa (LKS)
4. Buku Matematika Kelas IV
5. *Teacher Guide*

J. Penilaian Hasil Belajar

Penilaian dilakukan melalui penilaian proses belajar siswa. Penilaian proses dilakukan dengan mengamati aktivitas siswa selama proses belajar mengajar berlangsung, yang meliputi strategi yang digunakan siswa dalam menyelesaikan permasalahan yang diberikan, kemampuan siswa dalam memberikan penjelasan dan argumentasi atas penyelesaian masalah ketika diskusi di kelompoknya atau di kelas, serta kemampuan siswa untuk mempertahankan argumentasinya atau memberikan tanggapan terhadap ide dari temannya yang lain.

Rencana Pelaksanaan Pembelajaran (RPP)

Nama Sekolah : SDN 119 Palembang
Mata Pelajaran : Matematika
Kelas/Semester : IV (Empat) / 2
Jumlah Pertemuan : Satu kali pertemuan
Pertemuan ke- : 4

A. Standar Kompetensi
Menjumlahkan Bilangan Bulat
B. Kompetensi Dasar
Menjumlahkan dua bilangan positif dan dua bilangan negatif
C. Indikator Pencapaian Kompetensi
Menjumlahkan dua bilangan positif dan dua bilangan negatif
D. Tujuan Pembelajaran
Siswa dapat menjumlahkan dua bilangan negatif
E. Materi Ajar : Bilangan Bulat
F. Alokasi Waktu : 2 jam pelajaran (@ 35 menit)
G. Pendekatan Pembelajaran : Pendidikan Matematika Realistik Indonesia (PMRI)
Metode Pembelajaran : Pemecahan masalah, diskusi, dan tanya jawab
H. Kegiatan Pembelajaran

Aktivitas 1

1. **Pendahuluan (5 menit)**
   a. Apersepsi/Motivasi.
   b. Guru mengarahkan siswa untuk tetap duduk secara berkelompok sebagaimana pada pertemuan sebelumnya.
   c. Guru mengingatkan siswa kembali tentang pelajaran sebelumnya, yaitu tentang posisi bilangan positif, 0, dan bilangan negatif pada garis bilangan.

2. Inti (30 menit)

1. Eksplorasi

a. Guru membagikan LKS yang di dalamnya berisi sejumlah pertanyaan terkait dengan penjumlahan dua bilangan bulat.
b. Ketika siswa bekerja dalam kelompoknya, guru mengarahkan mereka agar saling berdiskusi dalam kelompoknya untuk menjawab seluruh pertanyaan yang ada di LKS.
c. Guru berperan sebagai fasilitator selama proses pembelajaran berlangsung untuk membimbing siswa menyelesaikan permasalahan yang diberikan agar sesuai dengan tujuan yang diharapkan.

2. Elaborasi

a. Siswa bekerja dalam kelompoknya dan diberikan kesempatan untuk menjawab seluruh pertanyaan yang ada di LKS dengan menggunakan berbagai strategi yang dimilikinya.

b. Setiap kelompok menempelkan hasil kerjanya di dinding dan Guru memberi kesempatan kepada siswa untuk melihat hasil kerja kelompok lain.
c. Guru menanyakan apa yang siswa dapatkan ketika mereka membandingkan jawaban kelompok mereka dengan jawaban kelompok lain. Setelah terjadi diskusi beberapa saat, dua kelompok diminta mempresentasikan hasil kerja mereka di depan kelas. Dalam hal ini, siswa diharapkan dapat maju dengan sendirinya dan guru memotivasi siswa agar berani tampil di depan kelas.
d. Dua kelompok yang melakukan presentasi diharapkan adalah kelompok yang jawabannya menarik (jawabannya tepat, strateginya unik dan sesuai dengan tujuan pembelajaran) dan kelompok yang jawabannya kurang tepat.
e. Untuk kelompok yang akan berpresentasi, guru mengarahkan agar siswa dalam kelompok tersebut menjelaskan bagaimana strategi mereka dalam menyelesaikan permasalahan yang telah diberikan.

3. Konfirmasi

a. Guru membimbing jalannya diskusi kelas pada saat tahap presentasi. Dalam hal ini guru mengarahkan agar setiap kelompok mendengarkan kelompok yang presentasi dan memberikan pertanyaan kepada kelompok yang presentasi tersebut. Guru juga menfasilitasi proses pembelajaran dengan mengajak siswa berani bertanya kepada temannya dan menyampaikan ide yang mereka miliki.
b. Kelompok yang jawabannya kurang tepat dapat memprentasikan hasil kerjanya terlebih dahulu, kemudian kelompok lain diminta untuk membandingkan dengan jawaban kelompoknya masing-masing dan melakukan koreksi sehingga terbentuk kesadaran pada siswa mana jawaban yang tepat.
c. Saat kelompok yang jawabannya menarik berpresentasi, guru dapat memberikan sejumlah pertanyaan untuk memperoleh pengetahuan yang mendalam (*insight*) dari siswa tentang jawaban yang dituliskannya pada lembar jawaban dan strategi yang digunakannya.
d. Pada akhir dari diskusi, guru mengarahkan agar siswa mendapatkan pemahaman tentang materi pelajaran sesuai tujuan yang diharapkan.

4. Penutup (10 menit)

a. Siswa diarahkan untuk merangkum materi yang telah didiskusikan dan menarik kesimpulan dari proses pembelajaran yang telah dilakukan.
b. Guru memberikan informasi tentang pembelajaran yang akan datang.

I. Media/Sumber Belajar

1. Kertas *plano*, spidol, *double tip*
2. Lembar Kerja Siswa (LKS)
3. Buku Matematika Kelas IV
4. *Teacher Guide*

J. Penilaian Hasil Belajar

Penilaian dilakukan melalui penilaian proses belajar siswa. Penilaian proses dilakukan dengan mengamati aktivitas siswa selama proses belajar mengajar berlangsung, yang

meliputi strategi yang digunakan siswa dalam menyelesaikan permasalahan yang diberikan, kemampuan siswa dalam memberikan penjelasan dan argumentasi atas penyelesaian masalah ketika diskusi di kelompoknya atau di kelas, serta kemampuan siswa untuk mempertahankan argumentasinya atau memberikan tanggapan terhadap ide dari temannya yang lain.

Rencana Pelaksanaan Pembelajaran (RPP)

Nama Sekolah : SDN 119 Palembang
Mata Pelajaran : Matematika
Kelas/Semester : IV (Empat) / 2
Jumlah Pertemuan : Satu kali pertemuan
Pertemuan ke- : 5

A. Standar Kompetensi
Menjumlahkan Bilangan Bulat
B. Kompetensi Dasar
Menjumlahkan bilangan positif dan bilangan negatif
C. Indikator Pencapaian Kompetensi
Menjumlahkan bilangan positif dan bilangan negatif
D. Tujuan Pembelajaran
Siswa dapat menjumlahkan bilangan positif dan bilangan negatif
E. Materi Ajar : Bilangan Bulat
F. Alokasi Waktu : 2 jam pelajaran (@ 35 menit)
G. Pendekatan Pembelajaran : Pendidikan Matematika Realistik Indonesia (PMRI)
Metode Pembelajaran : Pemecahan masalah, diskusi, dan tanya jawab
H. Kegiatan Pembelajaran

Aktivitas 1

1. **Pendahuluan (5 menit)**

a. Apersepsi/Motivasi.
b. Guru mengarahkan siswa untuk tetap duduk secara berkelompok sebagaimana pada pertemuan sebelumnya.
c. Guru mengingatkan siswa kembali tentang pelajaran sebelumnya, yaitu tentang penjumlahan dua bilangan negatif.

2. Inti (30 menit)

1. Eksplorasi

a. Guru membagikan LKS yang di dalamnya berisi sejumlah pertanyaan terkait dengan penjumlahan bilangan positif dan negatif.
b. Ketika siswa bekerja dalam kelompoknya, guru mengarahkan mereka agar saling berdiskusi dalam kelompoknya untuk menjawab seluruh pertanyaan yang ada di LKS.
c. Guru berperan sebagai fasilitator selama proses pembelajaran berlangsung untuk membimbing siswa menyelesaikan permasalahan yang diberikan agar sesuai dengan tujuan yang diharapkan.

2. Elaborasi

a. Siswa bekerja dalam kelompoknya dan diberikan kesempatan untuk menjawab seluruh pertanyaan yang ada di LKS dengan menggunakan berbagai strategi yang dimilikinya.
b. Setiap kelompok menempelkan hasil kerjanya di dinding dan Guru memberi kesempatan kepada siswa untuk melihat hasil kerja kelompok lain.
c. Guru menanyakan apa yang siswa dapatkan ketika mereka membandingkan jawaban kelompok mereka dengan jawaban kelompok lain. Setelah terjadi diskusi beberapa saat, dua kelompok diminta mempresentasikan hasil kerja mereka di depan kelas. Dalam hal ini, siswa diharapkan dapat maju dengan sendirinya dan guru memotivasi siswa agar berani tampil di depan kelas.
d. Dua kelompok yang melakukan presentasi diharapkan adalah kelompok yang jawabannya menarik (jawabannya tepat, strateginya unik dan sesuai dengan tujuan pembelajaran) dan kelompok yang jawabannya kurang tepat.
e. Untuk kelompok yang akan berpresentasi, guru mengarahkan agar siswa dalam kelompok tersebut menjelaskan bagaimana strategi mereka dalam menyelesaikan permasalahan yang telah diberikan.

3. Konfirmasi

a. Guru membimbing jalannya diskusi kelas pada saat tahap presentasi. Dalam hal ini guru mengarahkan agar setiap kelompok mendengarkan kelompok yang presentasi dan memberikan pertanyaan kepada kelompok yang presentasi tersebut. Guru juga menfasilitasi proses pembelajaran dengan mengajak siswa berani bertanya kepada temannya dan menyampaikan ide yang mereka miliki.
b. Kelompok yang jawabannya kurang tepat dapat memprentasikan hasil kerjanya terlebih dahulu, kemudian kelompok lain diminta untuk membandingkan dengan jawaban kelompoknya masing-masing dan melakukan koreksi sehingga terbentuk kesadaran pada siswa mana jawaban yang tepat.
c. Saat kelompok yang jawabannya menarik berpresentasi, guru dapat memberikan sejumlah pertanyaan untuk memperoleh pengetahuan yang mendalam (*insight*) dari siswa tentang jawaban yang dituliskannya pada lembar jawaban dan strategi yang digunakannya.
d. Pada akhir dari diskusi, guru mengarahkan agar siswa mendapatkan pemahaman tentang materi pelajaran sesuai tujuan yang diharapkan.

5. Penutup (10 menit)

a. Siswa diarahkan untuk merangkum materi yang telah didiskusikan dan menarik kesimpulan dari proses pembelajaran yang telah dilakukan.
b. Guru memberikan informasi tentang pembelajaran yang akan datang.

I. Media/Sumber Belajar

1. Kertas *plano*, spidol, *double tip*
2. Lembar Kerja Siswa (LKS)
3. Buku Matematika Kelas IV
4. *Teacher Guide*

J. Penilaian Hasil Belajar

Penilaian dilakukan melalui penilaian proses belajar siswa. Penilaian proses dilakukan dengan mengamati aktivitas siswa selama proses belajar mengajar berlangsung, yang meliputi strategi yang digunakan siswa dalam menyelesaikan permasalahan yang diberikan, kemampuan siswa dalam memberikan penjelasan dan argumentasi atas penyelesaian masalah ketika diskusi di kelompoknya atau di kelas, serta kemampuan siswa untuk mempertahankan argumentasinya atau memberikan tanggapan terhadap ide dari temannya yang lain.

Rencana Pelaksanaan Pembelajaran (RPP)

Nama Sekolah : SDN 119 Palembang
Mata Pelajaran : Matematika
Kelas/Semester : IV (Empat) / 2
Jumlah Pertemuan : Satu kali pertemuan
Pertemuan ke- : 6

A. Standar Kompetensi
Menjumlahkan Bilangan Bulat
B. Kompetensi Dasar
Menjumlahkan 2 bilangan bulat.
C. Indikator Pencapaian Kompetensi
Menjumlahkan dua bilangan positif , bilangan positif dan bilangan negatif, bilangan negatif dan positif, dan dua bilangan negatif.
D. Tujuan Pembelajaran
Siswa dapat menjumlahkan bilangan positif dan bilangan negatif
E. Materi Ajar : Bilangan Bulat
F. Alokasi Waktu : 2 jam pelajaran (@ 35 menit)
G. Pendekatan Pembelajaran : Pendidikan Matematika Realistik Indonesia (PMRI)
Metode Pembelajaran : Pemecahan masalah, diskusi, dan tanya jawab
H. Kegiatan Pembelajaran

Aktivitas 1

1. Pendahuluan (5 menit)

a. Apersepsi/Motivasi.
b. Guru mengarahkan siswa untuk tetap duduk secara berkelompok sebagaimana pada pertemuan sebelumnya.
c. Guru mengingatkan siswa kembali tentang pelajaran sebelumnya, yaitu tentang penjumlahan dua bilangan negatif dan penjumlahan bilangan positif dan negatif.

2. Inti (30 menit)

1. Eksplorasi

a. Guru membagikan LKS yang di dalamnya berisi sejumlah pertanyaan terkait dengan penjumlahan dua bilangan bulat.
b. Ketika siswa bekerja dalam kelompoknya, guru mengarahkan mereka agar saling berdiskusi dalam kelompoknya untuk menjawab seluruh pertanyaan yang ada di LKS.
c. Guru berperan sebagai fasilitator selama proses pembelajaran berlangsung untuk membimbing siswa menyelesaikan permasalahan yang diberikan agar sesuai dengan tujuan yang diharapkan.

2. Elaborasi

a. Siswa bekerja dalam kelompoknya dan diberikan kesempatan untuk menjawab seluruh pertanyaan yang ada di LKS dengan menggunakan berbagai strategi yang dimilikinya.
b. Setiap kelompok menempelkan hasil kerjanya di dinding dan Guru memberi kesempatan kepada siswa untuk melihat hasil kerja kelompok lain.
c. Guru menanyakan apa yang siswa dapatkan ketika mereka membandingkan jawaban kelompok mereka dengan jawaban kelompok lain. Setelah terjadi diskusi beberapa saat, dua kelompok diminta mempresentasikan hasil kerja mereka di depan kelas. Dalam hal ini, siswa diharapkan dapat maju dengan sendirinya dan guru memotivasi siswa agar berani tampil di depan kelas.
d. Dua kelompok yang melakukan presentasi diharapkan adalah kelompok yang jawabannya menarik (jawabannya tepat, strateginya unik dan sesuai dengan tujuan pembelajaran) dan kelompok yang jawabannya kurang tepat.
e. Untuk kelompok yang akan berpresentasi, guru mengarahkan agar siswa dalam kelompok tersebut menjelaskan bagaimana strategi mereka dalam menyelesaikan permasalahan yang telah diberikan.

3. Konfirmasi

a. Guru membimbing jalannya diskusi kelas pada saat tahap presentasi. Dalam hal ini guru mengarahkan agar setiap kelompok mendengarkan kelompok yang presentasi dan memberikan pertanyaan kepada kelompok yang presentasi tersebut. Guru juga menfasilitasi proses pembelajaran dengan mengajak siswa berani bertanya kepada temannya dan menyampaikan ide yang mereka miliki.
b. Kelompok yang jawabannya kurang tepat dapat memprentasikan hasil kerjanya terlebih dahulu, kemudian kelompok lain diminta untuk membandingkan dengan jawaban kelompoknya masing-masing dan melakukan koreksi sehingga terbentuk kesadaran pada siswa mana jawaban yang tepat.
c. Saat kelompok yang jawabannya menarik berpresentasi, guru dapat memberikan sejumlah pertanyaan untuk memperoleh pengetahuan yang mendalam (*insight*) dari siswa tentang jawaban yang dituliskannya pada lembar jawaban dan strategi yang digunakannya.
d. Pada akhir dari diskusi, guru mengarahkan agar siswa mendapatkan pemahaman tentang materi pelajaran sesuai tujuan yang diharapkan.

6. Penutup (10 menit)

a. Siswa diarahkan untuk merangkum materi yang telah didiskusikan dan menarik kesimpulan dari proses pembelajaran yang telah dilakukan.
b. Guru memberikan informasi tentang pembelajaran yang akan datang.

I. Media/Sumber Belajar

1. Kertas *plano*, spidol, *double tip*
2. Lembar Kerja Siswa (LKS)
3. Buku Matematika Kelas IV
4. *Teacher Guide*

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