DEVELOPING MULTIPLICATIVE STRUCTURE OF MEASURING AREA OF RECTANGLES AND SQUARES

MASTER THESIS

Anisa Fatwa Sari
NIM 107785050

THE STATE UNIVERSITY OF SURABAYA
POSTGRADUATE STUDIES
MATHEMATICS EDUCATION
2012

For the **endless love** *you gave me,*

For **lessons of courage** *and* **faith** *you taught me,*

For every single **pray** *through so many nights for me,*

For never ending **support** *you show me,*

I dedicate this thesis only for you,

My greatest **mother** *and my number one* **father** *in the world*

ABSTRACT

Sari, Anisa Fatwa. 2012. *Developing Multiplicative Structure of Measuring Area of Rectangles and Squares*. Master Thesis, Mathematics Education Study Program, Postgraduate Program of Surabaya State University.
Supervisor: Prof. Dr. Dwi Juniati, M. Si., and Dr. Agung Lukito, M.S.

Keywords*: Area measurement, multiplicative structure, rectangle, square, RME*

Area measurement is supported by four basic principles: complete covering, spatial structure, size relation, and multiplicative structure. Particularly in primary school, the two dimensional arrays for measuring area are usually given right away to students commonly in Indonesia without allowing them to construct it by themselves. It raises the need of constructing a conceptual basis to learn a formula of area which emphasizes the construction of spatial and multiplicative structure in measuring area.

This study used the design research as an approach. It was conducted to support students' in acquiring the initial concept of area measurement. It was focused on how a concrete model of two-dimensional arrays can support students' initial understanding of the formula for measuring area of rectangle and square. This study was conducted at one of the primary school in Surabaya during March – April 2012. A series of six activities was designed by using Realistic Mathematics Education (RME) approach. An instructional design contained those activities as the part of a hypothetical learning trajectory was implemented in the teaching and learning process. Six students were involved in the first cycle and 18 students became the subject of study in the second cycle. Data collections were generated from video recording during the teaching, collecting the students' work, giving pre-test and post-test, and interviewing the students.

After testing the hypothetical learning trajectory in the students' learning process, it has been found that the structuring array in the tasks play an important role in developing multiplicative structure. Complete covering, the use of multiplication, the arrangement of row by column structure, and the strip as handy tool for measuring area contribute to this development. The context of comparing two trays could raise the need of third object as unit measurement. Drying off crackers served as context to support the idea of complete covering. In the modeling level, the students can recognize the structure of arrays given in the figures. Although the students' are not yet able to make the grid perfectly, they already get the sense of partitioning shape into equal parts. It was indicated from their struggling to draw the grid of square in finding the area of two-dimensional shape.

ABSTRAK

Sari, Anisa Fatwa. 2012. *Developing Multiplicative Structure of Measuring Area of Rectangles and Squares*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Dwi Juniati, M. Si., dan (II) Dr. Agung Lukito, M.S.

Kata Kunci: *Pengukuran luas, struktur multiplikatif, persegi panjang, persegi*

Pengukuran luas tersusun dari empat prinsip dasar: *complete covering*, struktur spasial, hubungan antar ukuran, dan struktur multiplikatif. Di tingkat sekolah dasar di Indonesia, susunan petak dua dimensi dalam untuk mengukur luas diberikan langsung kepada siswa tanpa memberi mereka kesempatan untuk membangun sendiri struktur tersebut. Hal ini menumbuhkan kebutuhan siswa untuk mempelajari konsep dasar pengukuran luas sebagai landasan rumus mengukur luas. Konsep dasar tersebut menekankan pada pengembangan stuktur spasial dan multiplikatif saat mengukur luas.

Metode yang digunakan dalam penelitian ini adalah *design research*. Tujuan penelitian ini adalah mendukung proses belajar siswa dalam menguasai konsep dasar pengukuran luas. Fokus penelitian terletak pada bagaimana model kongkrit dari petak dua dimensi dapat mendukung pemahaman awal siswa untuk belajar tentang luas persegi panjang dan persegi. Penelitian ini dilaksanakan di salah satu sekolah dasar di Surabaya selama bulan Maret hingga April tahun 2012. Serangkaian aktivitas dikembangkan menggunakan pendekatan Pendidikan Realistik Matematika. Desain pembelajaran dengan 6 aktivitas yang merupakan bagian dari hipotesis trayektori pembelajaran (HLT) diuji dalam pembelejaran. Enam orang siswa dilibatkan pada siklus pertama dan 18 orang siswa turut menjadi partisipan pada siklus kedua. Proses pengumpulan data dilakukan dari rekaman video, pengumpulan hasil kerja tertulis siswa, pemberian pre-test dan post-test, serta wawancara dengan siswa.

Setelah pengujian HLT, ditemukan bahwa stuktur petak di dalam aktivitas siswa memainkan peranan penting untuk pengembangan struktur multiplikatif. C*omplete covering*, penggunaan perkalian, susunan struktur baris dikalikan kolom, dan strip persegi sebagai alat ukur luas berkontribusi dalam membantu siswa membangun struktus multiplikatif. Konteks membandingkan dua baki dapat menumbuhkan kebutuhan atas objek ketiga sebagai satuan pengukur luas. Proses penjemuran kerupuk di dalam baki berperan sebagai konteks untuk mempertajam ide *complete covering*. Pada level pemodelan, siswa dapat mengenali struktur petak persegi dalam gambar yang diberikan. Meskipun siswa belum mampu menggambar petak-petak persegi secara rapi, mereka telah mulai mendapatkan ide tentang membagi bangun datar menjadi beberapa bagian yang sama. Hal ini diindikasikan dari usaha mereka ketika mengambar petak persegi untuk menentukan luas dari sebuah bangun datar.

PREFACE

Accomplishing this thesis has been a great achievement for me. Through this hard times, I would say Alhamdulillah and grateful thanks to The Almighty Allah SWT for His blessing so that this thesis can be finished in due time. It is a great effort and the most challenging project during my study in the master program. I need a lot of motivations and supports from many people surrounding. That is why I want to express the deepest gratitude to:

1. Prof. Dr. Dwi Juniati, M.Si and Dr. Agung Lukito, M.S as supervisors who gave assistance and guidance during in writing this thesis.
2. Frans van Galen and Dr. Dolly van Eerde who give support strongly in designing activities and completing the methodological aspect of this study.
3. Jaap Den Hertog as the Coordinator of the IMPOME Program in Netherland.
4. All lecturers in Postgraduate Studies, UNESA and Freudenthal Institute, Utrecht University for all the supports.
5. Bu Anda and Bu Yuli who are willing to be partner in my research.
6. Riza agustiani, Intan Kemala Sari, and Devita kahardini for their kindness in helping me conducting the study.
7. All of my colleague in IMPOME program for their warmth and friendship during this program.

Finally, I consciously understand that this thesis is far from being perfect. Thus, any critics and constructive ideas will be gladly accepted.

Anisa Fatwa Sari

TABLE OF CONTENT

LIST OF TABLE

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.1 Background

Area measurement has already played an important role in human life since centuries ago when people dealt with Earth measurement. The area problem has also been studied since Babylonian civilization. It is also part of human culture, science, and technology and even can be found in our daily life (Hirstein, Lamb, & Osborn in Kordaki & Potari, 1998). Similar to the other units of measurement in mathematics, the measurement of area serves as a bridge between two critical aspects of mathematics: spatial relations and real numbers (Clements & Stephan, 2004). Area measurement is closely related to the number concepts in which it provides a model and an application for both numbers and its arithmetical operations (Skemp, 1986; Clements & Stephan, 2004). Moreover, Freudhental (1983) states that area models for the generalization of area measurement from discrete to continuous application can be natural means for teaching fractions and its multiplication.

Outhred & Mitchelmore (2000) states that the students' acquisition toward the formal way of measuring area is supported by four basic principles: complete covering, spatial structure, size relation, and multiplicative structure. Furthermore, spatial structuring is essential for the development of the notion of area and intimately related to multiplication

because two-dimensional arrays are a major model for and application of multiplicative thinking (Battista, Clements, Arnoff, Battista, & Van Auken Borrow, 1998). Consequently, students should learn covering a region with units and gradually develop their spatial ability to support them in constructing the multiplicative structure.

However, some research findings in mathematics education often reveal students' difficulties in learning area measurement. Zacharos (2006) states that these difficulties are attributed to the emphasis of using formulae while introducing this topic to students. In Indonesia, the first introduction of this topic is given at third grade of primary school. Fauzan (2002) argues that one of fundamental problem in Indonesia is that most of the learning objectives only focus on memorizing facts and concepts, and computational aspects (i.e. applying formula). Particularly, the two dimensional arrays for measuring area are usually given right away to students without allowing them to construct it by themselves. These findings raise the need of constructing a conceptual basis to learn a formula of area which emphasizes the construction of spatial and multiplicative structure in measuring area.

Realistic Mathematics Education gives a chance to change mathematics education in Indonesia. Applying the principles of Realistic Mathematics Education (RME), the students may get an opportunity to experience a concrete activity in covering a surface and enumerating the units in it. In turn, it enables them to construct the model of two dimensional arrays and the multiplicative structure in enumerating the units on the surface.

1.2 Research Questions

Based on the aim of this study, the research question is stated as follows.

How can multiplicative structure support students' initial understanding of area measurement for rectangles and squares?

1.3 Aim of the research

Considering the aforementioned issues, the aim of the present study is contributing to the development of a local instructional theory for area measurement. It will be conducted by designing and implementing classroom activities based on a RME approach that can support students' to understand the two- dimensional array and multiplicative structure in measuring area of rectangle and square. Specifically, this study is focused on the third grade of primary school in Indonesia.

1.4 Significance of the research

There are two significances that are expected to be achieved regarding this study. The first significance is to give an instructional theory on the area measurement. The second significance is providing clear view to mathematics teacher on how to develop learning activities especially for topic area measurement. This study is expected to give an overview of the design process of instructional activities and some considerations that must be taken into account in such a design process.

1.5 Definitions of key terms

In order to avoid misleading for the readers, the researcher defines some terms used in this study as listed below.

1. Array is a rectangular arrangement of units of area measurement in rows and columns.
2. Cracker is a traditional snack that usually made from flour with special seasoning. In Indonesian language, it is known as *kerupuk*.
3. Multiplicative structure is a row-by-column structure in the two-dimensional array of area measurement units. The characteristic of this structure is a spatial structuring that can be recognized visually. This structure can be also considered as the repetition of units iteration in a row or in a column.

CHAPTER II

THEORETICAL FRAMEWORK

2.1 Area measurement

Area is defined as an amount of surface inside the boundary of a two-dimensional shape (Baturo & Nason, 1996; Sarama & Clements, 2009). In general, measuring area can be defined as quantifying the amount of surface inside the boundary. Specifically, Clements & Stephan (2004) define an area measurement as tilling (or partitioning) a region with a two-dimensional unit of measure. Hence, measuring the area means enumerating how many two-dimensional units completely fit in the surface without gaps or overlapping.

There are some important aspects need to be addressed in learning area measurement. Outhred & Mitchelmore (2000) formulate four basic principles that constitute children's intuitive understanding of area measurement. These principles are complete covering, spatial structure, size relation, and multiplicative structure in which successively shows the children's acquisition in learning area measurement. In addition, Clements & Sarama (2009) describe some foundational concepts for learning area measurement as follows.

1. *Understanding the attribute of area*

   This concept concerns for giving a quantitative meaning to the amount of bounded two-dimensional surface. Prompting the need of quantitative statement for area usually start with comparing area of two shapes.

Intuitively, comparing area of two shapes can be stated by a relative statement such as “bigger (or larger) than” and “smaller than” (Yuberta, 2011). However, this relative statement cannot be applied in a case of comparing objects that cannot be directly compared. For instance, compare the surface of two tables in a different place will raise the need of the third object as a standard measure in order to quantify the amount of surface in those tables.

2. *Equal partitioning*

   Partitioning is defined as the mental act of splitting two-dimensional space into parts or units. In the mathematics textbook, this aspect is often given as a grid inside the shapes. However, Clement & Stephan (2004) argue that the construction of a two-dimensional array from linear units is nontrivial for children. In addition, it is crucial that students have opportunities to construct grids for themselves rather than simply observing ones that have been already prepared (Cavanagh, 2008).

   Therefore, students need to experience the activity of tiling a region with two-dimensional units and also discuss issues of leftover spaces, overlapping units, and precision. Discussions of these ideas lead students to mentally construct subregions of the whole shape that can be counted.

3. *Unit iteration*

   The issues related to this concept suggest the importance of completely covering space with units without overlapping and gaps. According to Cavanagh (2008), how a specified unit can be iterated until it completely

covers a flat surface without leaving gaps or overlaps is a basis of understanding of measuring area. Moreover, it is also important to address the extension of units beyond the boundaries if subdivisions of unit are needed.

According to Wijaya (2008), unit iteration is the process of finding how many units would fit to the attribute of the measured object. In case of measuring area, unit iteration is needed when a unit is not enough to cover up the measured space. In a more formal stage, unit iteration can involve a row (or a column) of square units for enumerating the area of shape. This argument is supported by Outhred & Mitchelmore (2000) that emphasize the significance of forming a row as unit iteration and the relation between the size of unit and the dimension of the shape. Furthermore, a concept of unit iteration supports the development of multiplicative structure to enumerate the units. According to Sarama and Clements (2009), tilling activity with incomplete units to cover rectangles encourage students to use multiplication principle along with the conceptual understanding behind it.

4. *Accumulation and additivity*

Similar to the concept of linear measurement, this concept concerns about composition and decomposition of shapes into sub regions with equivalent area.

5. *Structuring space*

Students need to comprehend the concept of two-dimensional array before they can construct the formula of measuring area meaningfully (Clements & Stephan, 2004). The structure of array in two-dimensional space related to the way a surface can be covered with unit squares that are arranged in row and column. Battista et al. (1998) stated that the structuring is significant for developing a concept of area measurement and closely related to multiplication because the two-dimensional array serves as model for and application of multiplicative thinking. Moreover, it is suggested that students must understand a row-by-column structure of the array in order to apply multiplication in determining the unit of measure. It will lead them to get conceptual understanding of the formula for measuring area.

The apprehension of students toward the structure of array differ one to another. Sarama & Clements (2009) classify the developmental levels in learning competences to understand and spatially structure namely (a) Area pre-recognizer; (b) Incomplete coverer; (c) Primitive coverer; (d) Primitive coverer and counter; (e) Partial row structurer; (f) Row and column structurer; (g) Array structurer. In order to comprehend the formula of area meaningfully, the students should apprehend the competence of array structurer.

6. *Conservation*

   The concept of area conservation is based on the understanding of getting exactly equal area if the parts of a given shape is rearranged and formed into another shape.

However, in order to focus on the aim of this study, the researcher only addresses some concepts from the aforementioned discussion. It includes area attribution, partitioning, units and its iteration, and also structuring space. These concepts are focused on the construction of multiplicative structure in enumerating unit covered the measured surface. Therefore, the instructional activities in this study are designed in such a way the students can build up their understanding by acquiring those concepts.

Clements & Sarama (2009) suggest that building upon students' initial spatial intuitions can lead to a more successful approach in learning area measurement. In addition, Clements & Stephan (2004) formulate a set of general instructional activities for area measurement that are described as follows:

1. *Investigation of covering region with a unit of measure*

   Through this activity, students need to realize that the entire shape must be covered and there must be no gaps or overlapping. The aim of this activity is constructing the idea of measurement units. It also includes a development of a measurement sense for standard units. In this study, the activity of ***covering a crackers tray*** promotes the emergence of unit

measurement. Starting from the use of rectangular cracker as a *"natural"* unit for measuring area, students then shift to the use of square crackers, and finally construct the idea of square units. It is in line with Zacharos (2006) who suggested that area can be measured using two-dimensional units such as squares and rectangles.

2. *Structuring array*

   This activity enables students to experience covering quantities with appropriate measurement units and enumerating the units. Through these experiences, students can make a link between the structure of rectangular arrays and counting by groups (i.e. row or column). It supports them for spatially structuring the shapes they are going to measure. Structuring the unit into an organized array can support students to achieve multiplicative thinking in measuring area.

   In this study, the activity of **covering surfaces using square unit** and **completing drawing of units contained in shape** can facilitate them to construct the structuring array. Although it is a long-term process for students to develop an idea of two-dimensional array of square, but Clements & Stephan (2004) confirm that even second grade students can make significant gains.

   Moreover, students can also construct the inverse relationship between the size of a unit and the number of units used in a particular measurement. The concept of this inversion is included in the measuring area of object using the different sizes of square unit strip. Though it may be not

explicitly addressed, students can still develop a sense of choosing appropriate size of unit to measure a certain surface.

3. *Linking the area measurement to the linear measurement*

   Outhred & Mitchelmore (2000) highlight the importance of a good understanding of linear measurement as prerequisite for learning area measurement. This understanding enables them to build a relation between unit size and rectangle dimensions. Therefore, there should be an activity in which students can learn that the length of the sides of a rectangle can determine the number of units in each row and the number of rows in the array. From this, they can construct two-dimensional space and correspond it to the multiplicative relations. However, the sequence of instructional activities in the present study is designed as a small part of a broader learning line in area measurement. Hence, it does not include this aspect in the design of lesson.

4. *Making a shortcut for measuring area*

   This activity is appropriate only in the advance phase of learning area measurement. It tries to facilitate them meaningfully learn to multiply the two dimensions of rectangle as a shortcut for determining the total number of units. The previous activity becomes a basis of the shortcut construction. It is reasonable since in a formal level, area measurement can be seen as the product of two linear measurements.

2.2 Realistic Mathematics Education

One of principles emphasized by Freudenthal (1991, p.56) in doing mathematics is the selection of learning situation within students' current reality which appropriate for horizontal mathematizing. It means that a problem situation involved in a mathematical learning must be experientially real or imaginable for students. In other word, by using their common sense students can be encouraged to expose their ideas and develop their own strategies to solve the problem.

As stated by Freudenthal, mathematics is as a human activity. Therefore, mathematics should be taught in such a way students can learn mathematics with experience-based instead of memorizing a ready-made algorithms or formulae. In line with this principle, this study attempts to develop an instructional design on teaching and learning area measurement. Through this instruction, it is expected that students could gain an insight about how to measure the area of a surface by engaging them in a series of meaningful activities instead of giving them a formula for measuring the area.

The process of designing a sequence of instructional activities in this study is guided by five tenets of realistic mathematics education defined by Treffers in Bakker (2004). The following description elaborates how these tenets fit in the present study.

1. Phenomenological exploration

A concrete contextual situation is used as a start of the instructional activity in this study. The mathematical activity involves students in

experiencing a real situation to be a basis of developing the mathematical concept. Moreover, this study starts with using a context of drying cracker as introduction area measurement. The aim of using this situation is prompting the students thinking in using the cracker as a natural unit to cover a cracker tray and determine the area.

2. *Using models and symbols for progressive mathematization*

This second principle characterizes a progression from a concrete level to the more formal level. The progression is bridged through the use of models and symbols. The initial understanding from the contextual activity will be brought into formal knowledge of area measurement. In this study, the students will be involved in several covering activities. Once they realize the efficiency of using unit square, they starting to use more formal objects for measuring area. They begin to develop their own model when they are asked to complete the drawing of the crackers tray. The drawing serves as a model to represent the arrangement of square crackers in two-dimensional array.

3. *Using students' own constructions and productions*

During the whole sequence of lesson, the students have opportunities to use and explore their strategy in producing solutions. The variety of responses in each activity will be discussed in the mathematical congress to facilitate them developing the area measurement concept embodied in it. The students can make their own construction during measuring different surface of shape using unit square strips. Through this activity they can

enhance their understanding of multiplicative principle in enumerating the area using unit squares.

4. *Interactivity*

The learning process of students involves an individual process as well as a social process. The latter process requires an existence of interaction among students and also between teacher and the students. Building this interaction can be done by designing a working group environment in which students communicate, compare, and justify their ideas. Almost all of the instructional activities in this study such as covering tray or measuring using strips are designed for group working that enables the students to interact each other.

5. *Intertwinement*

Intertwinement suggests integrating various mathematics topics in one activity. The instructional activities designed not only support learning for area measurement, but also support the development of spatial ability which is important in geometry domain. It also supported the students' comprehension toward multiplicative principle since the structure of array enables them to count the number of unit by using repeated addition.

2.3 Emergent Modeling

The second tenet of RME implies the use of models that support students' understanding of the area measurement concept. Gravemeijer (1994, p.100) described the changing process from *models-of* a certain situation

become *models-for* more formal reasoning. In the end of the lesson sequence in this study, it is aimed that the students can measure the area of surface of rectangle and square using standard unit (i.e. centimeter square). Though they are not expected to apply formal formula for measuring area, they are also expected to enumerate the number of units fit in the surface using multiplication. The implementation of the four levels of emergent modeling in the present study is described as follows:

1. *Situational level*

   Situational level is the basic level of emergent modeling where domain-specific, situational knowledge and strategies are used within the context of the situation. In this study, comparing two crackers tray is used as a contextual situation to promote the need of third object to measure the area of rectangular surface. This context can elicit the use of cracker as a natural unit measurement for covering the shape.

2. *Referential level*

   In this level, models and strategies refer to the situation which is contained in the problem. It means that *model-of* situation occur in this level. The use of paper as model referring the cracker that is be dried on the surface of the tray. Moreover, the completing drawing activity also encourages them to shift from the situational level to the referential level. This activity requires them to draw a square as a reference of square cracker in measuring the area.

3. *General level*

   In general level, *models-for* emerge where a mathematical focus on strategies dominates the reference to the context. In other word, in this level students are supported to develop a model which is applicable in different situations. The use of the unit square in this study serves as *model-for* area measurement when they become strips as tool for measuring different size of surface.

4. *Formal level*

   In the formal level, working with conventional procedures and notation is independent from the use of *model-for* mathematical activity. Though the end goals of this study do not reach the formal formula of area measurement, the use of multiplicative principle in enumerating units covered a surface and the standard unit for measuring area becomes the focus of the discussion.

2.4 Emergent perspective

Gravemeijer and Cobb (2006) state the use of *emergent perspective* as the framework for interpreting the classroom discourse and communication. In this study, the framework of learning area measurement can be viewed as a response to the issue of attempting to understand the concept of area measurement as it occurs in the social context of the classroom. This social context is divided into Social Perspective and Psychological Perspective which respectively concerns to the classroom community and the individual

students' reasoning. The social perspective includes the social norms, socio-mathematical norms, and classroom mathematical practices.

Social norms is defined as the ways of acting and explaining as the process of mutual negotiation between teacher and students. In this study, it is expected that the classroom will have different approach than the traditional ways of learning. It is important to put the norms of reform math class in to account such as students explain their reasoning and justify solution, indicate agreement and disagreement, try to make a sense the explanation given by other and question some possible alternative in such situation where a conflict in interpretation is apparent (Gravemeijer & Cobb, 2006).

Different from socio norms, Gravemeijer & Cobb (2006) define socio-mathematics norm as the ways of explicating and acting in whole class discussions that are specific to mathematics. In this study, it will be particularly about the concept of area measurement. This norms enable students to make independent judgments. For instance, each student might have their own ways of explaining what they understand about measuring area of a surface. It also enables the teacher and students to participate in a discussion to negotiate and redefine about the concept discussed in the learning process. These norms will also help teacher to facilitate students in their thinking process.

Moreover, Gravemeijer and Cobb (2006) describe the mathematical practices in this perspective as the normative ways of acting, communicating and symbolizing mathematically at a given moment in time. In the series of

instructional activities in the Hypothetical Learning Trajectory designed in this study, there will be some mathematical practices that are more specific to particular mathematical ideas. Comparing two trays of crackers will help the students to directly experience the mathematical practices of comparing the area. Students will also experience to use square units and strips of square units for measuring area of different surfaces. In the students' learning processes about the concept area measurement, their interpretation and their mathematical practices are related each other. It is implied that their mathematical development occurs as they contribute to the constitution of mathematical practices.

2.5 Area measurement in Indonesian curriculum

The area measurement in Indonesian curriculum is being introduced start from second semester of the third grade. The table below describes topic measurement for grade 3 in Indonesia curriculum.

Table 2.1: Topic measurement for third grader

The second Semester of Grade 3	
Standard Competence	**Basic Competence**
Geometry and Measurement **5.** Calculating perimeter and area of square and rectangular, and its application in problem solving	5.1 Calculating perimeter of square and rectangle 5.2 Calculating area of square and rectangle 5.3 Solving problems related to perimeter and area of square and rectangle

However, it is common that the focus in teaching area measurement in Indonesia is placed in understanding and applying the formula. It gives less emphasis to the reason behind the formula, the reason why it works. Indeed, the formal approach in measuring area is important but it should be based on a conceptual basis. Therefore, the present study would like to focus on the process of how students understand the use of the formula and its meaning.

According to the aforementioned discussion of the framework, the general research question in this study is elaborated into two specific sub questions as follows.

1) *How do the third grade students structure two-dimensional array in covering a surface to measure area?*
2) *How can the structure of two-dimensional array support students' development of the multiplicative structure?*

CHAPTER III

RESEARCH METHOD

3.1 Research approach

Design research is used as an approach in this study since it is appropriate to the aim of this study. This study is aimed to provide an empirically grounded theory about how mathematical instruction works in the domain of area measurement. The theory concerns both the process of learning and the means designed to support that learning. Therefore, during this study series of activities are designed and developed as means to contribute the improvement of educational practices in area measurement for the third grade of primary school in Indonesia. Specifically, this study focuses on how a concrete model of two-dimensional array can support students' initial understanding of the formula for measuring area of rectangle and square.

The definition of design research is given by Gravemeijer & Cobb (2006) in the discussion of the three phases of conducting this type of research. These phases are described as follows.

1. Preparing for design study

   The goal of the preliminary phase of a design research experiment is to formulate a conjectured local instructional theory that can be elaborated and refined while conducting the experiment (Gravemeijer & Cobb, 2006). The sequence of instructional activities including conjectures of students' thinking and students' strategies is developed in this phase. This sequence

serves as initial Hypothetical Learning Trajectory (HLT). However, this initial HLT is dynamic in which it can be adjusted to the students' actual learning during the teaching experiment phase.

2. Conducting the design study

   According to Gravemeijer & Cobb (2006), the purpose of this phase is both to test and improve the conjectured local instruction theory that was developed in the preliminary phase, and to develop an understanding of how it works. In this phase, the sequence of activities developed in the previous phase is implemented in the classroom. In this study, the teaching implementation of the design will be conducted in six lessons. During the teaching, the conjectures in the HLT will be tested and adjusted while reacting to students' learning process. There will be two cycles for this implementation. The first cycle will be conducted as a pilot experiment. The content of the teaching of both cycles will be the same based on the consideration that second cycle is the revision of the first cycle. Before conducting each lesson, researcher and teacher hold a preliminary discussion about the upcoming activity to adjust and make agreement about how the lesson will run based on teacher and researcher point of view. There will be also a reflection of the whole learning process after the lesson that concerns about the strong points and the weak points of the lesson.

3. Retrospective Analysis

The goal of the retrospective analyses depend on the theoretical intent of the design implementation. However, one of the primary aims is typically to contribute to the development of a local instruction theory (Gravemeijer & Cobb, 2006). In this retrospective analysis, researcher will make an analysis of the entire data set collected in the teaching. The HLT is compared to the actual learning process of the students. The explanation is not merely about the instances that support the conjectures, but also the examples that contradict the conjectures. Based on this analysis, the research question will be answered. The recommendations of how the next HLT should be improved for further study are underpinned by the analysis. In addition, the HLT has different three functions in each phase of the design research (Bakker, 2004). In the phase of preparing the design study, the HLT guides the design of instructional activities that have to be developed. In the implementation phase, it functions as a guideline the focus of teaching, interviewing, and observing. During the retrospective analysis, it serves as a guideline in determining what to focus on in the analysis.

3.2 Subjects of the study

This design study was conducted in the Elementary School Laboratory of Unesa (SD Laboratorium Unesa). It involves third grade students from two

different classes, 3C and 3D. The students who involved in each cycle were different.

For the first cycle, the researcher involves only 6 students from 3C. It was because the goal of the implementation of design in this cycle is mainly to adjust with the initial HLT and improve it to get a better design for the next cycle. The selection of the students was based on the recommendation from their mathematical teacher. These students were those who have middle level of achievement. During this cycle, the researcher itself took a role as the facilitator.

Different from the previous cycle, the implementation of the design in the second cycle involved a class of students from 3D. The number of students in this class was 18. In addition, there were four out of these students chosen as the focus of observation during the teaching. These four students were selected based on the recommendation of teacher who knew which students were in the middle level of achievement. Later, these students would be called the focus group. The facilitator of learning during this cycle was the teacher.

3.3 Data collection

3.3.1 Preparation phase

The data collection in the preparation phase is aimed to investigate pre-knowledge of students in class 3D. It is also aimed to do an orientation to the learning atmosphere of the class that will involve in the second cycle.

The information about students' pre-knowledge will be used to do first adjustment of the initial HLT regarding the aspect of starting point of the instructional activities.

Moreover, the classroom orientation is important to get an overview of how the learning process works in that class. It concerns both social and socio-mathematical norms in the classroom and other aspects that constitute the mathematical learning. This kind of data is collected by observing the learning environment in the classroom and interviewing teacher. The researchers write notes based on the list of observation points during the observation and make an audio recording during the interview. Both observation points and the interview scheme with teacher are provided in the appendix section.

3.3.2 Pre-test

Pre-test is used to assess students' pre-knowledge regarding the concept of area measurement. This test is given to all students involved in the beginning of the series of lessons. An interview is also conducted to know the students' way of structuring two-dimensional arrays. In the first cycle, the interview is conducted for all students. However, in the second cycle only focus group is interviewed about their pre-test. It is aimed to get clarification of their thinking and reasoning. The test item for pre-test and the question of the interview can be seen in Appendix E.

3.3.3 Preliminary teaching

The first cycle of the design study served as preliminary teaching. In this cycle, the instructional activities are tried out with six students (See 3.2). The goal of this phase is to investigate students' thinking of the problems in the HLT and to test the conjectures about it. In turn, the data collected from this phase is used to improve the HLT. The data is collected by making a video recording of the activities during the lessons and collecting students' written work. The video is recorded by using one camera while the researcher takes the role as a teacher in the lessons. One colleague assists the researcher in making a video registration.

3.3.4 The implementation of the improved-HLT

The improved HLT from the first cycle will be tested in this phase. The students' work and strategies in solving the problems during the lesson is observed by video. Short discussions with focus group (See 3.2) and class discussions are also recorded to investigate students' reasoning. Moreover, the videotaping during the teaching experiments is recorded by using one static camera focusing on one group of students and a dynamic camera to record the activities in classroom. In this phase, the researcher takes role as observer and one colleague of the researcher will makes video registration of the dynamic camera. In some extent, the researcher makes an intervention during the learning process by posing question to students in the group.

3.3.5 Post-test

Post-test is used to assess students' development in understanding the concept of area measurement. This test is given to all students at the end of the whole lessons in each cycle. There is an interview with the focus group about their post-test. It is aimed to get clarification of their thinking and reasoning. The questions in the post-test are slightly similar to the pre-test, but there is an additional question. These items can be seen in Appendix G.

3.3.6 Validity and reliability

This study involves different types of data such as interview data, video observations, and students' written works. The interview is recorded by using an audio device, while the observation process is recorded by using video-tape tool. The different types of data that relate to the students' learning process will be used as source triangulation to contribute the internal validity of the data. Testing conjectures in the HLT during the teaching also contributes to the internal validity. The data registration from different methods in collecting the data will contribute to the reliability of the data.

3.4 Data analysis

3.4.1 Pre-test

The result of the pre-test is analyzed to investigate the starting point of students in learning area measurement. The test result is expected to reveal

students' thinking about a unit for measuring area and their strategies in enumerating the units covered the surface. From this aspect, the HLT will be adjusted in such a way that it is appropriate to students' pre-knowledge.

3.4.2 Preliminary teaching

The video and the students' written works in the first cycle are analyzed to investigate the learning processes of the students. The learning process of students is assessed by testing the conjectures in the HLT. It means the assumptions about students' learning will be compared to the students' actual learning. From this analysis, it can be seen which part of the HLT supports students in learning and which part is not. In turn, an improved HLT will be designed and will be implemented in the next cycle based on the analysis of this phase.

3.4.3 The implementation of the improved-HLT

The video recording will be observed to get the overview of the whole teaching and learning process in the classroom. During watching the video, the researcher makes field note. This note is used to make general description of the lesson including notes of important things such as students' ideas and strategies. The researcher then selects interesting fragments and makes transcription of it. Then the selected fragments will be transcribed to make an interpretation of students' thinking. However, the irrelevant part of the students' learning process will be neglected. The selected fragments and the students' written works in the second cycle the conjectures will be tested by comparing it to the conjectures in the

improved HLT. The result of this analysis will be used to answer the research question, draw the conclusion, and also become a basis to redesign the improved HLT.

3.4.4 Post-test

The result of post-test is analyzed by comparing it to the result of pre-test. It is aimed to investigate students' development in understanding the concept of area measurement and their strategies in solving problems in measuring area. It will enrich the analysis of the teaching experiment to draw a conclusion.

3.4.5 Reliability

The reliability of the data analysis is contributed by two aspects namely the track ability and the inter subjectivity. The clear description of how the researcher works on this study so that people can easily follow the process constitutes the track ability aspect. This description contains the explanation of the process of doing each phase of this study. It includes the process of how the preparation is done, how the teaching experiment was happening, and how the researcher analyze, interpret the data, and give conclusion. Moreover, a cross interpretation with colleagues and supervisors is needed to fulfill the inter subjectivity aspect. It is done to avoid the researcher's own point view or subjectivity toward the interpretation of the data collected.

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

According to Simon (1995) as cited in Simon and Tzur (2004), Hypothetical Learning Trajectory (HLT) is used as a term to describe key aspects of planning mathematics lesson. It includes the mathematical goals of students, the mathematical learning activities, and the hypotheses about the process of students' thinking and learning.

The aim of this study is to contribute to the development of a local instruction theory for area measurement. In achieving this aim, the instructional activities are designed to facilitate students to learn the initial understanding of the formula for measuring area of rectangles and squares. There are two main goals of the activities designed in the HLT. First, students can structure two dimensional arrays in covering the surface when measuring area. Second, they can develop multiplication structure in the array and then apply multiplication as the most efficient way in enumerating unit squares.

In this chapter, we elaborate the HLT that will be used in the present study. It contains sequence of six activities in three weeks period of teaching that is designed to reach the aim of this study. This HLT will be implemented in the third grade of primary school in Indonesia. In each lesson, we will describe the starting point of the students, the learning goals, the mathematical activity, and the conjectures of students' thinking.

4.1 Comparing two cracker's trays

A. Starting points

Area measurement is one of mathematical topic that will be taught in the second semester of third grade of primary school. It is the first time for them in learning about the concept of area measurement. The knowledge and the skills that students already have as prerequisite are as follows.

- Students know rectangle and its visual representation
- Students know square and its visual representation
- Students can draw square and rectangle
- Students can determine the number of objects. This skill is important because students need to enumerate units covered the surface. It is adequate that they can do it either use counting one by one or do skip counting.
- Students can do multiplication of whole numbers up to hundred. They already learnt this skill in the second semester of the second grade level.

B. The Learning goals

Main goal:

- Students understand the use of identical unit as a unit for measuring area (the attribution of area)

Sub goals:

- Students know that measuring area of a shape can be done by covering it with identical units and enumerating the units

- Students can cover the rectangle using units without overlap and leave a gap
- Students can count the number of units covered the shape by using their own strategies
- Students can compare the area of two rectangles by comparing the number of units covered each rectangle

C. Description of Activity

In this activity, teacher tells students about drying cracker under the sun. The teacher then shows two rectangular bamboo trays in different size. One is wider while another is longer. Teacher also shows some raw crackers to the students.

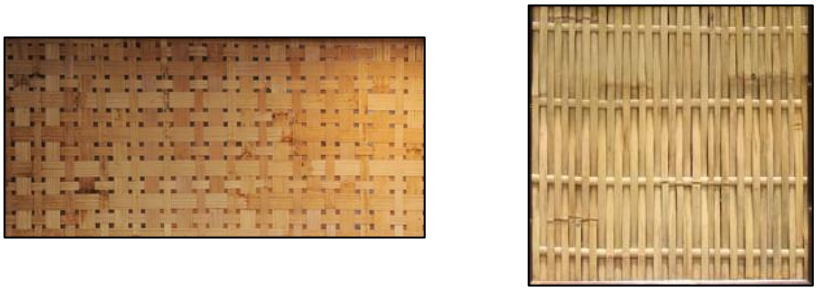


Figure 4.1: Two bamboo trays for drying the crackers

Teacher tells the students that he/she needs the largest tray to be used in drying crackers. The students are asked what the meaning of the largest tray is. Teacher then hold a class discussion about it. There may be some students who say that the largest tray contains more crackers. After students realize that the number of crackers contained in the tray can indicate the area of trays, they will work in a small group to compare the

area of trays. Each group will be given two pieces of cardboard as a representation of the trays. They are also given several rectangular papers with two different sizes as a substitution of using real crackers. It uses papers as a substitution because the shape of the real crackers is not neat. The task are describing the method to compare the trays and telling how large each cracker tray is in order to be compared during the class discussion.

Figure 4.2: Two rectangular papers as substitution for crackers

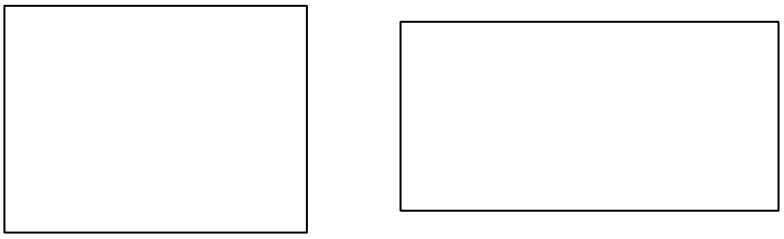


Figure 4.3: Two cardboards as substitution of two different trays

D. Conjectures of students' thinking

In determining the number of crackers contained in the tray, the students may put the rectangular paper in the cardboard. Some students may put the papers but overlapping each other or even leaving gap among it. Some other students may already cover the cardboard completely without gap and overlap.

In the class discussion, the teacher compares the result of students' work and their method. Most of students may cover each tray with different type of crackers. They may tell the teacher that the tray with the most

crackers is bigger. These students do not aware of the difference size of the rectangular papers. This type of strategy can be seen in Figure 4.4 and Figure 4.5.

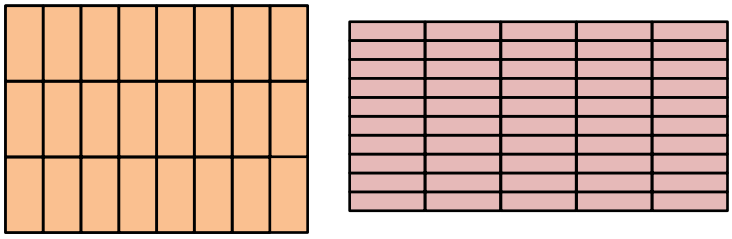


Figure 4.4: Students cover different trays with different types of crackers

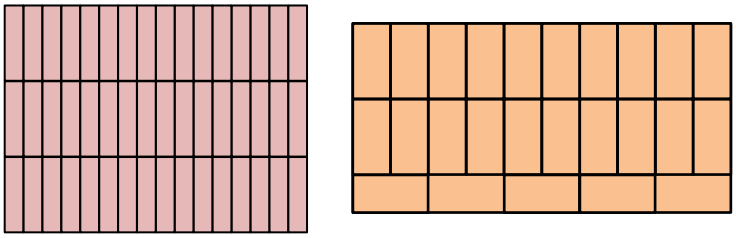


Figure 4.5: The same strategy with Figure 4.4 but the crackers are swapped

However, it is expected that some students give a response that those trays should be measured using the same crackers. By giving such a response, it can be inferred that these students already realize the identical size of unit is required to compare area of two shapes requires. Their strategies are covering one tray with one size of crackers, and then use the same size of crackers to cover another tray (see Figure 4.6 as an example). Teacher should emphasize this strategy and invite the students to compare two trays using another size of crackers to check whether they can get the same result.

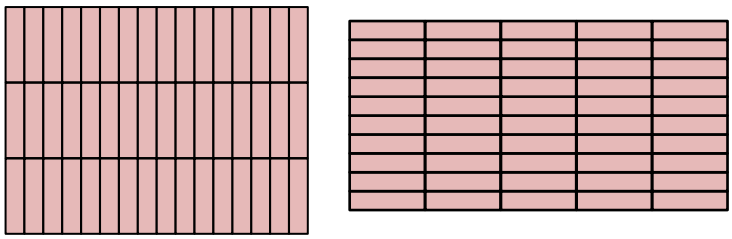


Figure 4.6: Covering trays with the same crackers

If there is no student answer in this way, that is because they ignore the size of the crackers, teacher can pose prompting question.

"Look at the crackers, what do you think about the size of them. Do those crackers have same size?"

The next discussion will be focused on students' strategies in enumerating the crackers. They may use different ways in enumerating the crackers. Some of them may just count them one by one, the others may do skip counting by considering the crackers in one row or column and then add it as many as the number of cracker in the different direction. It is also possible that some students already come up with the idea of multiplication. All possible way of counting is allowed, and the most efficient way to count the crackers will be discussed in the next lesson.

4.2 How large is the tray? (Measuring area of tray by using square crackers)

A. Starting point

From the previous activity, students have learnt that comparing area of two rectangles can be done by covering the shapes with units. They have been involved in the activity of complete covering (covering without gap or overlap). Therefore, students are expected to have abilities as follows.

- Students can cover the rectangle using units without overlap and leave a gap
- Students can count the number of units covered the shape by using their own strategies
- Students can state the area of rectangle by referring to the number of units that cover the rectangle

B. Learning Goals

Main Goal:

- Students can use multiplication as the efficient way in enumerating the number of unit covered the shape

Sub goals:

- Students can determine the number of units in a row and in a column
- Students can determine total number of units by multiplying the number of units in a row with the number of rows
- Students can determine total number of units by multiplying the number of units in a column with the number of column
- Students can determine total number of units by multiplying the number of units in a row with the number of units in a column

C. Description of Activity

Teacher shows a bamboo tray that is covered by some square crackers to the students. Teacher tells them that the raw crackers will be fried for a

special moment. However, teacher needs more tray as a place for more crackers. The carpenter who made the tray asks the teacher how large the tray is in order to make another one. Teacher then asks them the way they can inform the others about how large the tray is. It is sufficient if students state that the area of the tray can be indicated by the number of crackers on the tray. Teacher can remind them about determining the largest tray in the previous activity if the students do not have an idea how to it.

The teacher then gives each group of students a cardboard as substitution of tray. The first cardboard given to students is the rectangular cardboard. Teacher also distributes some square papers as substitution of crackers. The students are asked to determine how many crackers that can be put in that size of tray. They are also asked to give explanation about their method in counting the squares. The class discussion will be held after the students finish their work. The second cardboard will be distributed to students after the class discussion as an exercise for students.

D. Conjectures of students' thinking

- As the students put all squares on the tray, they may count the cracker one by one. The illustration of this strategy can be seen in Figure 4.7.

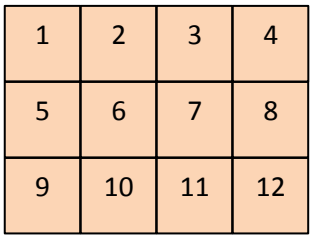


Figure 4.7: Counting one by one strategy

- Students may also count the papers in a row and then add it up so many times the number of rows (skip counting).

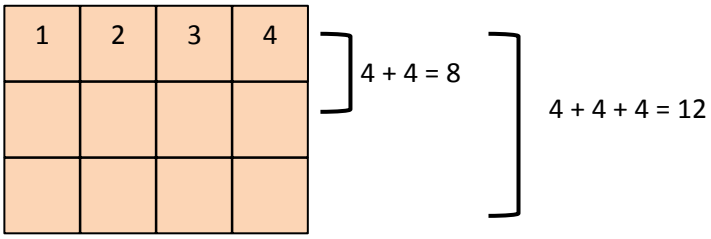


Figure 4.8: Skip counting by rows

- In the similar way of counting by rows, students may count the papers in a column and then add it up so many times the number of columns.

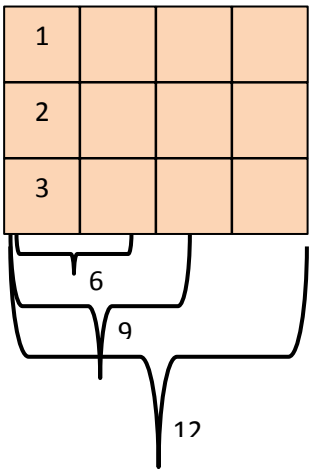


Figure 4.9: Skip counting by column

- Students who already have an idea of multiplication may figure out the total number of squares by multiplying the number of units in a row.

4.3 Square or rectangular crackers? (Determining the flexible unit for measuring area)

A. Starting points

Based on students' previous activities, it is expected that they already acquired abilities as follows.

- Students understand that area of rectangle and square can be measured by covering those shapes with identical units
- Students are able to cover the shape without gap and overlap
- Students can use multiplication to determine the total number of units that cover the shapes

B. Learning goals

The aim of this activity is supporting students in realizing the efficiency of using square as a unit measurement.

Main goal:

- Students can determine the most efficient unit for area measurement

Sub goals:

- Students can measure the area of rectangle and square using rectangular units

- Students can measure the area of rectangle and square using square units
- Students know that multiplication can be easily applied if the unit used for area measurement is square

C. Description of Activity

In the previous activity, the students already discuss the use of multiplication in counting the number of units covered a shape. Now, they will measure one tray using two different kind of unit. Each group of students will get a cardboard, rectangular papers with size 10 cm x 5 cm, and 10 cm square papers. They are asked to determine the area of the tray using those units. The main theme of the class discussion following the activity is which shape is more efficient if we want to apply a multiplication in enumerating the units.

D. Conjectures of students' thinking

- The students may put the rectangular crackers in the different directions, but it will make them difficult to apply multiplication. The rectangle can be also a unit measurement, but it requires one to be aware of the direction when applying multiplication.

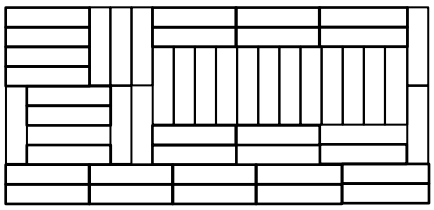


Figure 4.10: Putting the rectangular units in the different direction

- It is expected that some students realize the flexibility of square. They are expected to reason that no matter how to put the square it is always in the same direction. Therefore, using square makes people easier to apply multiplication because the direction of arranging them will always be the same.

4.4 Measuring area with limited number of units

A. Starting points

Based on the previous activities, students have already learnt area measurement as covering shape with units and learnt the multiplication as the efficient method to enumerate the units. The following list shows the abilities that are expected to be acquired by students from the activities.

- Students know area can be measured by covering a shape with identical unit
- Students know area can be referred to the number of units covering the shape
- Students can apply multiplication to find the total number of units that cover the shape

B. Learning Goals

In the previous activities (see 4.2 – 4.3), students started to recognize row-by-column structure by applying the multiplication and arguing the most efficient shape of unit in such a way multiplication can be hold. This activity is aimed to supporting students more in developing the structure of array in measuring area using a unit square. This aim is formulated as learning goals as follows.

Main goal:

- Students are able to develop row-by-column structure of the units

Sub goals:

- Students are able to determine the total number of units needed to cover the whole shape if they area only provided by limited number of square
- Students can understand that multiplication can be used to predict the total number of units

C. Description of Activity

Teacher tells the students that he/she needs to know the area of a tray, but she only has limited square units to cover it. Teacher gives each group of students a cardboard and limited number of square units. The task is predicting the area of the cardboard (as substitution of the tray) using some square unit. The units given are only sufficient to cover a row and a

column of the shape. The main theme of a class discussion is finding the easy way of predicting the area of the tray.

As additional activities, the students are also given a worksheet which contain incomplete square covering a shape. The task remains the same; determine the number of square needed to cover the whole shape.

D. Conjectures of students' thinking

- Some students may put the given square one by one in a row and counting them continuously for other rows.

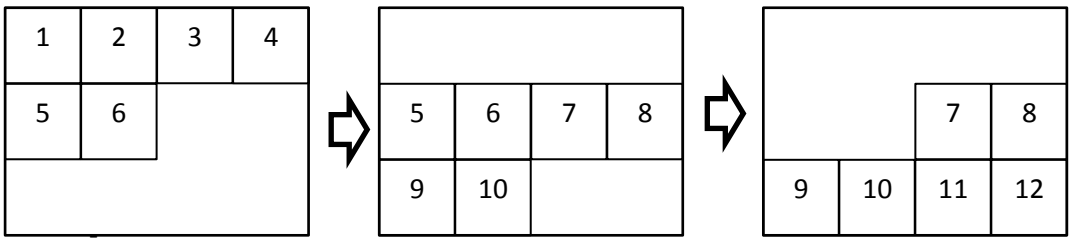


Figure 4.11: Iterating the units one by one

- Some students may still do skip counting by considering the number of papers in one row (or column) and then iterating the units.

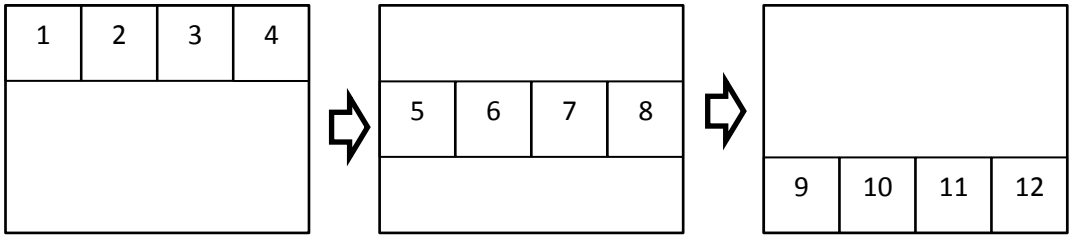


Figure 4.12: Iterating the units by row

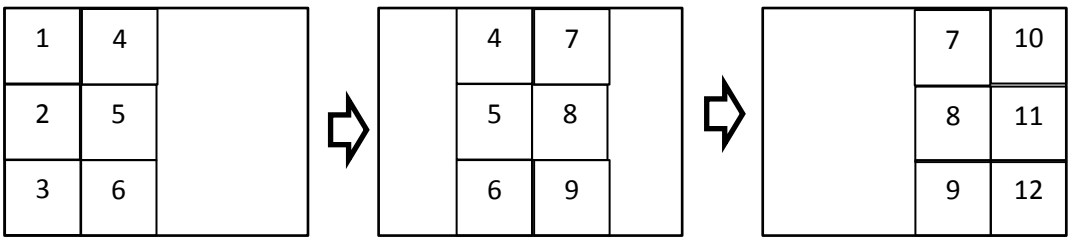


Figure 4.13: Iterating the units by column

- Some students may put the given square in the edge of tray for different direction and then multiply the number of units in each edge.

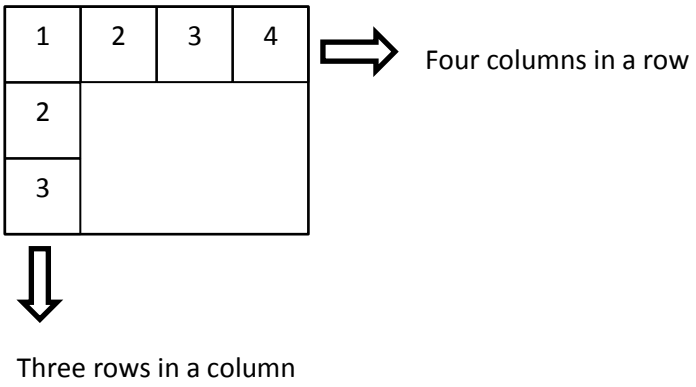


Figure 4.14: Row by column structure

4.5 Measuring area using strip of square units

A. Starting point

During the previous activities, students have already learnt the way of measuring area and the method for enumerating units. They have also learnt to construct the structure array by themselves (see sub section 4.4). Through those activities, students are expected to acquire the following abilities as the starting point of this lesson.

- Students know area can be measured by covering a shape with identical unit
- Students know area can be referred to the number of units covering the shape
- Students can apply multiplication to find the total number of units that cover the shape

- Students can understand that multiplication can be used to predict the total number of units if they are given a limited amount of units

B. Learning Goals

Main goal:

- Students are able to measure area by using strip of unit squares

Sub goals:

- Students can choose an appropriate size of unit square to measure different sizes of shape
- Students can apply multiplication in predicting the total number of units

C. Description of Activity

Teacher gives three different strips of square paper to each group of students. Students are asked to measure the area of different shapes in their classroom. They must measure one surface of object using one strip. They also should write the list of three objects and its area. The main theme of the class discussion is the method of how each group measure the area of objects in their list.

D. Conjectures of students' thinking

The students may get different list of surfaces of object in the classroom. In measuring the small surface some students may use the small strip, while they use the big strip for measuring the big region.

In counting of units, some students may use skip counting by rows (or column) because the strips may help them to structure the units. Some other students may use multiplication after they find the number of units in the vertical direction as well as in the horizontal direction.

4.6 Measuring area using strip of standard units

A. Starting points

Since students are involved in the previous activities, it is expected that they already acquired the following abilities.

- Students know area can be measured by covering a shape with identical unit
- Students know area can be referred to the number of units covering the shape
- Students can apply multiplication to find the total number of units that cover the shape
- Students can understand that multiplication can be used to predict the total number of units if they are given a limited amount of units
- Students can measure the surface of object by using strips of unit squares

B. Learning Goals

Main goal:

- Students are able to measure area using standardize unit

Sub goals:

- Students can measure the area of surface by using a strips of centimeter square
- Students can measure the area of surface by using a strips of decimeter square

C. Description of Activity

Teacher gives each group of students a cardboard and two different strips of standardize unit square (i.e. in decimeter square and centimeter square). Teacher informs students that these two standard units of measurement are commonly used by people to communicate about the area achievement other.

Teacher asks students to measure the area of the cardboard by using those different strips. The main theme of the class discussion is about quantifying the area of the shape in standard units of measurement. Another important aspect that can be addressed during the discussion is determining which unit is more appropriate for measuring the cardboard (decimeter square or centimeter square).

D. Conjectures of students' thinking

- Some students may keep telling the number of units as a square unit.
- Some students may already aware of the standard unit measurement and state the area of the shape by using this unit.

CHAPTER V

RETROSPECTIVE ANALYSIS

This chapter provides the retrospective analysis of data collected from both the first and the second cycle. The analysis will be described chronologically starting from the remarks of students' pre-knowledge in the first cycle. These remarks are used as consideration to adjust the initial HLT into HLT 1. The students' learning process and the remarks from the post-test are described afterwards. These descriptions lead to general conclusion of students' learning process of the first cycle. Based on the finding and explanation in each activity in the first cycle, some refinements were made into HLT 2. The analysis of implementation of HLT in the second cycle is presented in the same order with the previous cycle. In addition, the mathematical ideas occurred in the experiment became the main issue of the analysis. It can be projected either from focus group or the rest of students in the classroom.

5.1 The research timeline

Table 5.1: Research Timeline

DATE	**DESCRIPTION**
Preparation phase	
November 2011 – Feebruary 2012	Studying literature and designing initial HLT
5^{th} – 6^{th} March 2012	Discussion with teachers
Preliminary teaching (the first cycle)	
7^{th} March 2012	Pre-test
8^{th} March 2012	Activity 1: Comparing two trays using crackers

12th March 2012	Activity 2: Measuring area of tray by using square crackers Activity 3: Determining flexible unit for measuring area
13th March 2012	Activity 4: Measuring area without enough number of units
14th March 2012	Activity 5: Measuring area using strip of square units Activity 6: Measuring area using strip of standard units
15th March 2012	Post-test
Trying out the design in the second cycle	
14th March 2012	Pre-test
15th March 2012	Interview with focus group regarding the result of pre-test.
19th March 2012	Activity 1: Comparing two trays using crackers
21st March 2012	Activity 2: Measuring area of tray by using square crackers
22nd March 2012	Activity 3: Determining flexible unit for measuring area
2nd April 2012	Activity 4: Measuring area without enough number of units
4th April 2012	Activity 5: Measuring area using strip of square units
5th April 2012	Activity 6: Measuring area using strip of standard units Post-test
9th April 2012	Interview for clarifying the result of post-test.

5.2 Remarks on the students' pre-knowledge in the first cycle

Topic area measurement is closely related to introduction of two-dimensional shape. In grade three, the students have already learnt about it. In addition, they also learnt how to determine the perimeter of a two-dimensional shape. This fact is also signified by teacher during the interview.

Six students who are involved in the first cycle were given two items of written test and one oral problem. The aim of pre-test was to know students' current knowledge and ability. Specifically, the result of pre-test was used to check the correspondence between the starting point of initial HLT and the students' pre-knowledge. During the investigation through pre-test, the researcher found several critical issues of students' pre-knowledge that correspond to the learning design in HLT as follows:

5.2.1 The lack of awareness toward attribution of area during comparing the area of two shapes

Generally, the students built on their statement of a larger area to the dimension of the shapes. One student stated the longer rectangle has larger area. Instead of using square units around the shapes to quantify the area of each shape, they used the units outside the shapes as the reference to its size. Indeed, the dimension could lead to the area, but this case tends to refer to the perimeter. (See Appendix E)

5.2.2 The relation between the structuring arrays and the way of counting the units

This finding is based on the students' responses to the second item of the pre-test. The students are given rectangles with incomplete square units in each rectangle. They must estimate the total number of units needed to cover the rectangle. The students' responses showed three different ways of counting. The first way is that students can directly apply multiplication with or without completing the drawing of the arrays in the given

rectangle. The second way is that students use repeated addition based on the number of units in a row or in a column. The third way of students is using complete the drawing and then count the units one by one. However, there is a student who uses a unique method. This student estimates the missing units then add it to the units that are already given. It can be considered as the fourth type of structuring arrays.

From these different ways of counting, it can be inferred that the students already build the relation between the structuring arrays in the figures and the way they count. Some of them could build the multiplicative way from the structure while the others were still in the process of it (repeated addition or counting one by one).

5.2.3 The various iteration process among students

Interview in the pre-test is aimed to know how the students deal with unit iteration process. Individually, the students are given two square units and a figure of rectangle. They are challenged to estimate the total number of units needed to cover the rectangle. There are two students who are unaware of overlapping during doing the iteration of units. In other case, three students already apply multiplication by finding out the number of units in a row and in the column. One student can correctly predict the number of units needed to cover the rectangle by using unit iteration.

5.3 Preliminary teaching

The six students who were involved in this cycle were Irsyad, Vitto, Vio, Tina, Ayin, and Rachel. They worked with the activities in an improved-HLT

called HLT 1. Design of initial HLT is adjusted to students' current knowledge by considering pre-test results (See Appendix F). The feedback from the result of this experiment will become consideration for the improvement of the hypothetical learning trajectory. The analysis of how the HLT 1 works during the first cycle is explained as follows.

5.3.1 Developing the idea of identical units to measure area

Activity 1: Comparing two trays using crackers.

This activity was aimed to facilitate students in developing the idea of identical units to measure area. In addition, covering the shape without gap and overlap with units was also important ideas that were addressed in this activity.

Using the context of comparing two bamboo trays, students were asked to determine the largest tray. One tray was wider, while another was longer. The students directly gave a response that the longer one was the biggest one.

Researcher	: Can you show me how do you compare them?
Irsyad	: (*Use hand span to measure the length of each rectangle*). This is four (*pointing out the wider tray*) and that is three (*pointing out the longer tray*).
Vitto	: What do you mean with that?
Researcher	: Okay, we will use the tray as a place to dry off the crackers. What do you think about the number of crackers in the biggest tray?
Students	: *silent*
Researcher	: Do you think there are more crackers or less crackers in the biggest tray?
Students	: More crackers.

At this point, students can refer to the number of crackers as indication of the biggest tray. Though the idea of reference to compare two trays did not

come from the students, the anticipated question could give hint for them. They then got two cardboards as substitution of tray, rectangular and circular paper as representation of crackers. They were asked to determine the biggest tray by using those crackers.

As expected in the HLT, there were gaps among the units put in the cardboard. The discussion about this idea is captured in the following quotation.

Researcher : How can we get maximum number of crackers in the tray?
Tina : Put them closer to each other!
Researcher : How about another group?
Irsyad : (*talk to his groupmate*) we should also put them closer.
Ayin : It will not dry well.
Vitto : (*talk to Tina and Rachel*) don't put it too close.
Researcher : Do these two crackers overlap each other?
Tina and Ayin : No.
Researcher : Do you think all crackers will get enough sunlight to get dry?
All students : Yes.
Researcher : So, is it allowed to put the crackers close enough without overlap?
All students : Yes, it is allowed.
Researcher : Don't forget to put the crackers so that we get the maximum number.

Figure 5.1: Covering with gaps

Based on the discussion quoted above, the students then continued to work on covering the trays. However, both groups covered each tray with different crackers. Researcher tried to provoke students with the idea of identical units to compare area.

Researcher : You put yellow crackers (*circular*) in this tray and green crackers (*rectangular*) in that tray. Can we know which tray is the biggest one?
Tina : This one (*pointing out the longer tray covered by rectangular crackers*).
Researcher : Why?
Tina : Because it contains more crackers.
Vitto : No, the size of crackers is different!
Researcher : How about another group? Do you think we can compare the trays if we cover each tray with different crackers?
Ayin : No, because the size of crackers is different.

Considering students' pre-knowledge that they found it was difficult to recognize the need of identical units as reference to compare area. Through this activity they used the different size of crackers as consideration in comparing the area. From this stage, students needed to be guided to select the appropriate unit to measure area.

Researcher : Our goal is comparing the tray so that we know which one is the biggest. Now, see this tray (the one that covered by circular crakcers), it still leaves gap. How about the other tray?
Ayin : It is fully covered.
Vio : It is covered.
Researcher : We have spaces left in this tray and we do not know how large the spaces are. So, which cracker do you choose to determine the biggest tray?
Students : (*pointing out the tray covered with rectangular tray*)
Researcher : Now, let us determine how many rectangular crackers needed to cover the other tray.

The decision to choose the rectangular cracker as units already emerged. The students then started to measure the wider tray using rectangular

crackers because they already measured the longer tray using the same crackers. Both groups found that the longer tray could be covered by 25 rectangular crackers. All of them covered the tray with crackers in the same direction as shown in Figure 5.2. They also used the similar method to count the number of crackers. They counted the crackers on the first row, added it twice since the second row contains the same number of crackers, and then added to the remaining crackers.

Figure 5.2: Complete covering in the same direction

During the covering activity of the wider tray each group did different method. The first group of students arranged the first two rows in the same direction, but they put the rectangle in the last row in the different direction. They counted the crackers in the first row and multiplied it by two because the second row contains the same number of crackers with the same direction. In addition, they add it to the remaining number of crackers by counting on one by one. However, the other group used

different method. They arranged the crackers in the same direction. The crackers were put vertically in three rows. Instead of counting one by one or using repeated addition, they already applied multiplication. They said that there were three rows of eight crackers so that there were three times eight crackers.

The problem occurred when the students must write down their covering activity and drawing conclusion about the largest tray. The extensive discussion during this activity was also overwhelming for them. It would be a challenge when it would be implemented in the real class. Therefore, the researcher decided to improve the worksheet. The worksheet would be more explicit by providing statement that should be filled by students.

In addition, the organization of the discussion would also be changed. During students were covering the tray, teacher would walk around and ensure that they do a complete covering. However, discussion about the reason why one tray is bigger than another, the way of counting the units, and choosing the appropriate crackers for covering the tray, the way of counting can be done in the whole class discussion.

5.3.2 Developing idea of multiplication in counting units

Activity 2: Measuring area of tray using square crackers

This activity is aimed to support students in developing the idea of multiplication as an efficient way to count the units. It was divided into two parts, measuring the area of rectangle and square. Either the rectangular or square tray was replaced by cardboard.

Vitto, Tina, and Rachel as a group started to cover the rectangular cardboard. They covered a row and two consecutive columns with square units. After counted the number of units in a row and a column, Vitto applied multiplication. However, this group kept covering the cardboard until the whole surface was covered with units.

Another group covered the tray by putting units in a row and the left column of the rectangular cardboard. They also applied multiplication to determine the number of units that should cover the cardboard.

The researcher explored the reason of using multiplication by asking them question why they could use it. Vitto said that he used it because his mother said so. The following transcript shows a fragment of discussion that highlight the idea of multiplication.

Ayin	: How if we know the result without covering the whole surface?
Researcher	: What is the result?
Ayin	: (*counting the units in a column and in a row*) So, we only need to multiply six by eight.
Researcher	: Why has to be six times eight?
Ayin	: (*counting the units in a column*) This is six, (*counting the units in a row*) and this is eight. So, six times eight equals forty eight.
Researcher	: Why can we multiply? How do you learn multiplication?
Irsyad	: Like this (*showing 'finger calculation' for six times eight*)
Researcher	: Okay, that is the way we calculate. Now we have six times eight. How many eight are there?
Ayin	: I know. We add six plus six eight times.
Researcher	: What does it means here? (*pointing out the structure of units*).
Ayin	: Six plus six plus six….. (*pointing out the columns while counting*).
Researcher	: Where are six?
Ayin	: (*pointing out a column*)
Researcher	: How many sixes are there?
Students	: Eight.

Researcher : How many units here? (*pointing out the second column that only have one unit*).
Students : Six.

In the second part of this activity, the students dealt with figuring out the number of units that could cover a square cardboard. Both groups covered the left column and the bottom row and applied multiplication to determine the total number of square units.

Figure 5.3: Row by column structure

According to students' responses in this activity, they basically know about using multiplication as the efficient way in counting units in such a structure. They did not even need to cover the whole. They just arranged the units in a row by column structure to know the numbers that will be multiplied. However, they did not comprehend the reason why they can use multiplication. By posing question like fragmented above, it is expected that students can make relation between repeated addition and the whole structure units covered the surface. This activity also shows remark on students' understanding toward multiplication. Specifically, they

perceived six times eight as repeated addition of six until eight times. Therefore, the idea of repeated addition and its relation to the structure should become the main issue in this activity for the next cycle.

5.3.3 Determining the flexible unit for measuring area

Activity 3: Measuring area of tray using different types of units (square and rectangle)

The main goal of this activity is to facilitate students realize the reason of using square as standard units for area measurement. They measured square cardboard by using square units and rectangular units, respectively. They were confronted with the idea of flexible units in order to apply multiplication.

In this activity, all students work together. The researcher explained to them the term ‘square units’ instead of keeping use ‘crackers’ as reference. The students put square units along the edge of a cardboard. However, the students assumed the total number of units was 40. After the researcher reminded the students about the question, Vitto came with the answer 100. He counted units in a column and in a row and then applied multiplication. The researcher then reviewed the reason why they could use multiplication (similar discussion with the previous activity).

The students then worked with covering the cardboard by using the rectangular units. The researcher also informed them about the term ‘rectangular unit’ instead of using ‘rectangular crackers’. They put the units vertically and tried to cover the whole. In the middle of their

covering activity, Ayin suggested to use multiplication. At that time, they already covered two rows from top, column, and the right column. They then counted units in a row and in a column, multiplied the numbers, and got 50. However, they still kept covering the surface until it was fully covered. The researcher then held a discussion about their work like quoted in the transcript below.

Researcher : What is the area of the cardboard?
Students : 50 rectangular units.
Researcher : How do you count the units?
Students : Use multiplication.
Researcher : Which [number of] units that you use for multiplication?
Vitto and Tina : In the top row and the side [a column].
Researcher : How if we use the number of units in the column and bottom row?
Students : That's okay.
Researcher : Why?
Rachel : Because [the number of units is] same.
Researcher : How if I change the direction? (*changing the direction of some units into horizontal*)
Vitto : It is not allowed.
Researcher : Why?
Vitto : One, two, three, four, five (*count the units in a column*), six, seven, eight, nine, ten (*count the units in a top row*). Eh, one, two three (*count units in another column*). I don't know, but it's just not allowed.
Researcher : Okay, anybody know the reason why we cannot use multiplication?
Students : (*silent*)
Researcher : How many units are in this direction? (*pointing out units in the first row*)
Students : Six.
Researcher : Here? (*pointing out units in the column*).
Students : Five.
Researcher : We already counted all units. What is the total?
Students : Fifty.
Researcher : So, what will happen if we apply multiplication now?
Students : We get wrong answer.
Vitto : We get thirty.
Researcher : **What should we do if we still want to use multiplication for counting rectangular units?**

Vio : It must be in ‘stand up’ [vertical] direction.
Irsyad : We must put them closer.
Vitto : It must be same.
Researcher : What you mean by ‘same’?
Vitto : The position.
Researcher : How about the [position of] square unit?
Students : It is all the same.
Researcher : **Suppose we want to use multiplication in counting the units, which unit do you choose to be used?**
Students : **Square.**
Researcher : Why?
Vitto : **Because each side has the same length, so it will remain the same although we rotate it.**
Researcher : What about rectangular unit?
Students : It is not the same.

The students did not put the rectangular crackers in the different direction as conjectured in the HLT. That is why the researcher challenges them by changing some units into different direction. The discussion afterward shows that this conflict can trigger students’ thinking about multiplication issue. The different direction of some rectangular units will lead them to a result that completely different from what they think. The change of the direction also intrigues them to think about position of units and the difference between square and rectangle. However, the issue of using rectangle as unit for measuring area did not occur in this activity. It is allowed to use rectangle as unit for area measurement as long as people aware of the direction if they want to apply multiplication. This issue will be discussed in the next cycle.

5.3.4 Constructing the shortcut to determine area

Activity 4: Measuring area by using limited number of units

Before this activity started, the researcher reviewed the discussion from the last activity. This review was about the reason why square was more flexible.

The idea behind the fourth activity is row-by-column structure. Given limited number of square units, students were challenged to estimate the area of a rectangle and a square.

In the first part of this activity, students worked with square cardboard. This cardboard could be covered with 64 unit squares. However, students only got 10 units. The first group put 8 square units in a row and 2 other units in the right column. They then did iteration along the column and got eight. Using multiplication they found that the area of the cardboard was 64. However, another group used a different method. They put 8 units in a row, one unit in the left column, and another unit in the right. Tina said that there were 10 units and they should remember it first. They then move some units from the row to the left column until it has fully covered. Tina told that the area was eight times ten, but Vitto said that it was 96 instead of 80. Vio then reminded them about the number of units in a row and in a column. Tina then said that it was eight times eight equals sixty four.

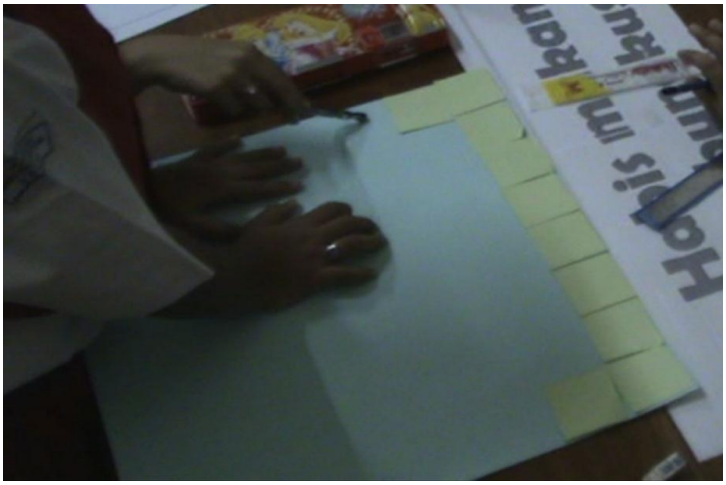


Figure 5.4: Work of second group

The researcher tried to intrigue students' thinking about the structure of arrays. The position of units moved either from the left column to another or from top row to another row, the students confirmed that the area of cardboard were same.

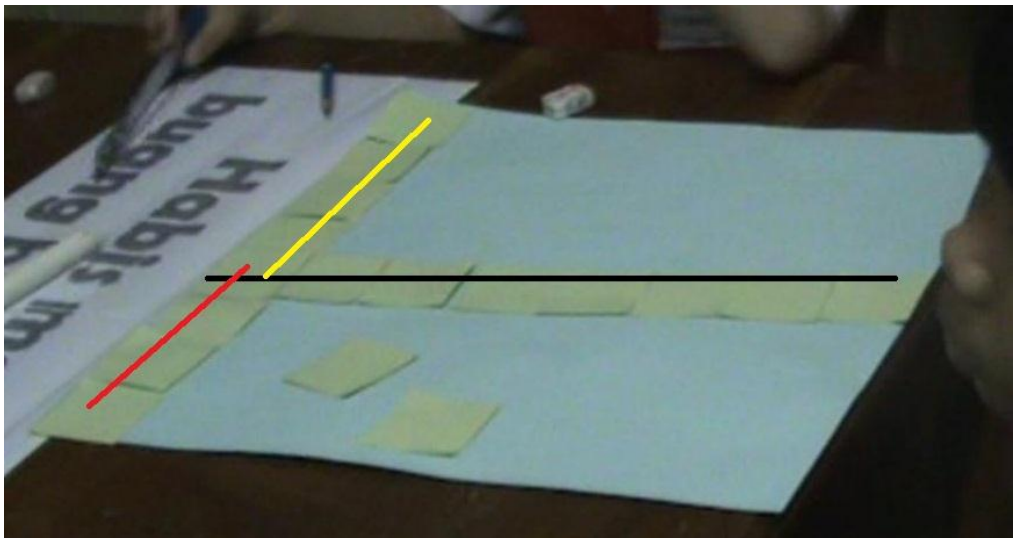


Figure 5.5: Additivity of area

An interesting moment occurred when the researcher put eight units in the top row and arranged 7 units in the fourth column. This event is depicted in the following transcript (reference: Figure 5.5).

Researcher : Is there any other way to know the area of this cardboard?
Vitto : I know. One, two, three, four, six, seven, eight (*counting units along the black line*). One, two, three, four, five (*counting units along the yellow line*). Eight times five is forty.
Researcher : How about this part? (*pointing out the area around black and red line*)
Vitto : Wait, we keep forty first. One, two, three, four, five, six, seven, eight (*counting units along the black line*). One, two, three, four (*counting units along red line*). Eight times four equals to thirty six. Then forty plus thirty six….
Researcher : How many is that?
Vitto : Seventy six.
Researcher : Why is the result different from what we got before?
Tina : **Because it was counted twice (*pointing out units along black line*).**
Researcher : So, we must subtract these units. How many units are here?
Tina and Vio : Eight
Vitto : 76 – 8 = 68.
Tina : It is still greater than before.
Researcher : Okay, we will ask to another group. Vitto has an idea to determine the area of this cardboard. First, he multiplied eight (black line) by five (yellow line), so the area of this part is…
Students : Forty
Researcher : And then he multiplied eight by four. So, what is the area of this part?
Students : Thirty two
Vitto : Oh, thirty two
Researcher : So, the area of the whole cardboard is …
Students : 40 + 32 = 72
Researcher : Why is the result different from what we got before?
Irsyad : Because we use addition.
Tina : Because we count this [units in the black] twice.
Researcher : So?
Vitto : 72 – 8 = 64

In the second part of this activity, the students were given a rectangular cardboard. Both groups could easily arrange the units in a row-by-column structure because the units were sufficient to do it. One row contained six units, while one column contained four units. The students use

multiplication to estimate the area of the cardboard. In addition, the researcher also explored Vitto's idea in this activity. The students confirmed that the area were unchanged either they estimate it as whole or splitting the estimation into two parts.

The last part of this activity was reinforcement for students about multiplication and the structure of arrays. They were challenged to estimate the area of two given figures individually. Three students directly used multiplication without drawing anything. One student completed the drawing by making the grid, another student only drew line to get the grid along top row and a column, and the last student only drew some dots to signify the units. Some of students' work can be seen in the figure below.

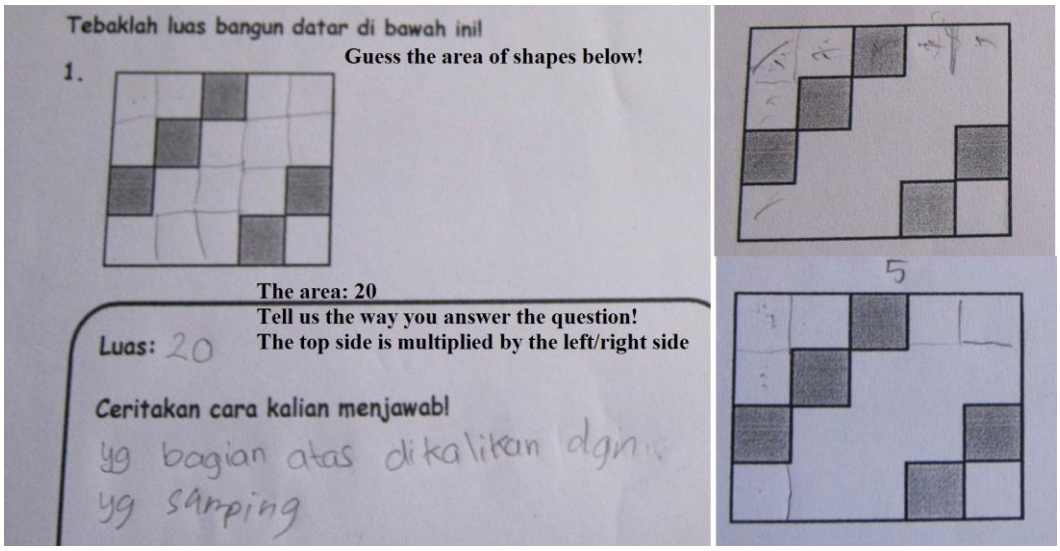


Figure 5.6: Students' work in estimating the area of rectangle

Students' responses of this activity were quite surprising. Firstly, there were various ways of iteration shown by students in the first part of this activity. The first group of students did iteration that was not expected before, while another group used iteration to get row-by-column structure.

It was possible to happen since the number of units given to students was insufficient to directly apply row-by-column structure. Supporting this argument, the students easily formed row-by-column structure in the second part of this activity. That is why the order of giving the cardboard will be the same in the next cycle. It is expected that students will come up with different ways of iteration for the first cardboard, while they are facilitated explicitly to form row-by-column structure in the second cardboard.

Second, it was very interesting when the idea of additivity for area measurement emerged from students. Actually, this idea was not included as the main idea of this study. However, the researcher decided to discuss it with students because it was closely related to the structure of arrays. Although the students did a mistake by counting a column twice, they already raised the idea of splitting the whole surface into two parts.

Third, students' achievement was projected in the result of their exercises. Although it showed different achievement, all students already got the idea of multiplication by considering the number of units in a row and in a column.

Considering these analyses, this activity will not be changed in the next cycle. If possible, the researcher is also going to discuss the idea of additivity of area measurement with the focus group.

5.3.5 Constructing the row-by-column structure

*Activity 5: Measuring area of rectangle and square by using strip of square unit*s

In the last activity, students already started to develop row-by-column structure. As a follow up, they were going to measure area of surfaces by using strip of square units. Through this activity, students were facilitated to measure different sizes of surfaces by using different sizes of units.

Starting this activity, the researcher introduced three sizes of strips. There were big, medium, and small strips of square units. The big strip was a strip of 1 dm^2 squares. The medium strip was 25 cm^2 squares strip; while the small strip contains 1 cm^2 squares. Students were asked to find anything around them to be measured by using those strips. The surface of things they measured must be rectangular or square.

The students measured the surface of a table, a chair, a paper, and many more. However, some of surfaces of those things were not rectangular or square. Some of them just resemble rectangle or square but has oval edge instead of right angle. It was also hard to help students focused on their work. Therefore, the researcher decided to do anticipation activity. Students were given two cardboards. One was rectangular, while another was square. They were asked to measure its area.

Both groups could use multiplication as the shortcut for measuring the area of the cardboard. The discussion in this activity served as the way to check

the result of area measurement that came from both groups. The following transcript shows the discussion when students were working.

Vitto	: This is ten squares (*put small strip in the edge of square cardboard*).
Researcher	: How about this part? (*pointing out the left over space of the edge*)
Tina	: We mark here first.
Vitto	: (*put small strip after the mark*) this is also ten, so the total is twenty.
Tina	: This edge is also twenty.
Vitto	: So, the area is twenty times four.
Researcher	: **How do we measure the area of surface in the previous activity?**
Vitto	: Ah, it must be twenty times twenty.
Researcher	: Why?
Students	: (*silent*)
Researcher	: How many square units in this direction? (*pointing out the left column of cardboard*)
Students	: Twenty.
Researcher	: How many twenties needed to cover the whole?
Students	: Twenty.

Although students finally got the correct result for their measurement, they needed to struggle in the beginning. As quoted in the transcript above, students perceived that the area of the square cardboard was four times the number of units covering the edge of the shape. It seemed that they mixed up the concept of area with perimeter. However, posing a question that reminded the students about the previous activities made them realize the way of measuring the area.

Considering the improvisation in the middle of this activity, the researcher will change it for the next cycle. The number of students in the next cycle is greater than in this cycle. It may raise the possibility that students will get confused or even do not focus on the goal of the activity. The plan is

providing some objects that have rectangular or square surface. The students in a group will measure each object in turn with other groups. By doing this arrangement, it is expected that students can be interactive and still can keep focused on their activity. In addition, the researcher will also make the small strip ten squares longer so that it will be easier for students to measure larger surface.

5.3.6 Developing the use of standard unit for area measurement

Activity 6: Measuring area of rectangle and square by using strip of standard units

From previous activity, students got an experience in measuring the area by using strip of square units. Through the last activity, the researcher tried to introduce standard units of area measurement to them. Indeed, they already used square units –the shape of standard unit of area measurement– in previous activities with awareness that it was more flexible than rectangular unit. However, they main idea was introducing the terms cm^2 and dm^2 as reference of unit. It served as the more formal way to state the area. It was expected that students would begin to use those terms as replacement of crackers and square units.

Starting the activity, the researcher posed a question about the unit of length that is familiar to students. They could mention several units such as meter, centimeter, kilometer, etc. The researcher then introduced two standard units that could signify the area namely cm^2 and dm^2. Those two

units were represented by a square in a small strip and a big strip respectively.

Students worked in a group to measure the area of two cardboards –one rectangular and another was square. These cardboard were exactly the same with the previous activities. Both groups could measure the area of square cardboard and then state it in standard units. However, one group did not measure the rectangular cardboard correctly using a strip of cm^2. They measured a column of cardboard as thirteen while in fact it was fifteen. Helping them correct their answer, the researcher asked them to demonstrate their measurement. They finally got the correct result.

This activity was merely a "brief" introduction for students to know two standard units of area measurement. Therefore, there was no intensive discussion but checking the result of the measurement. The idea of relation or conversion between cm^2 and dm^2 was not also covered because it was the first time they work with these units. Trying out this activity was aimed to check whether it could be handled by students after doing the previous activities.

The conflict occurred when all students found it difficult to measure the rectangular cardboard by using a strip of dm^2. The size of cardboard was 20 x 15 cm^2. There was space left over if they put the strip horizontally and a half unit over if they put the strip vertically. Since the activity was based on covering, those case was considered as "something that cannot be measured" using a dm^2 unit. In fact, people could still state the area by

using dm^2 –two times one and half. Considering this fact, the researcher will provide rectangular cardboard that can be measured by using a dm^2 in the next cycle.

5.4 Remarks on the students' knowledge in the post-test (1st cycle)

The researcher posed a post-test to students in the end of learning process in the first cycle. This test served as clarification of for the students' knowledge development that had been observed in the learning process of the first cycle. It was not meant to compare it to the students' pre-knowledge before doing the first cycle. The questions were similar with questions in the pre-test. The term *"larger"* in the first question was changed into *"bigger area"*. The estimation of total number of tiles in the second question was replaced with *"estimate the area"*. The purpose of these changes was to shift common term into more mathematical way. However, there was a question added in the post test. Given a figure of rectangle and square, the students were asked to determine the area of each shape. The length and the width of each shape were given in the picture. According to the results of post-test, the researcher noted some important points as the following. (See appendix G)

5.4.1 The awareness toward identical units

Given two different sizes of pools in the pre-test, students determine the larger pool by referring to the shapes or dimension of the shapes. In the post-test, they start to realize the use of identical units as reference to determine the pool that has bigger area (Figure 5.7)

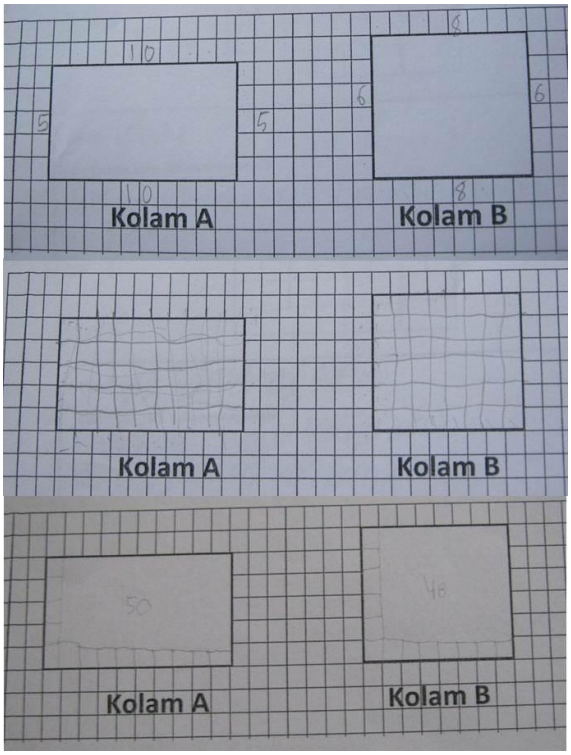


Figure 5.7: Identical units as reference

The different reference used by students highlight the importance of promoting indirect comparison. Comparing the area of shapes that cannot be directly compared can raise the need of identical units. The need of units to compare the area can be the starting point of using the units as reference to state the area of one shape.

5.4.2 Structuring arrays and its support to the idea of multiplication

The students already developed the idea of structuring arrays in the different levels in the beginning of this study (see 5.2.2). In the post-test, they were given two figures and asked to determine the area of each shapes. These figures were completely different from figures in the pre-test. After working on six activities, most of students' responses of questions can be projected into one strategy. These students applied multiplication in estimating the

area either they completed the drawing or they drew row-by-column structure.

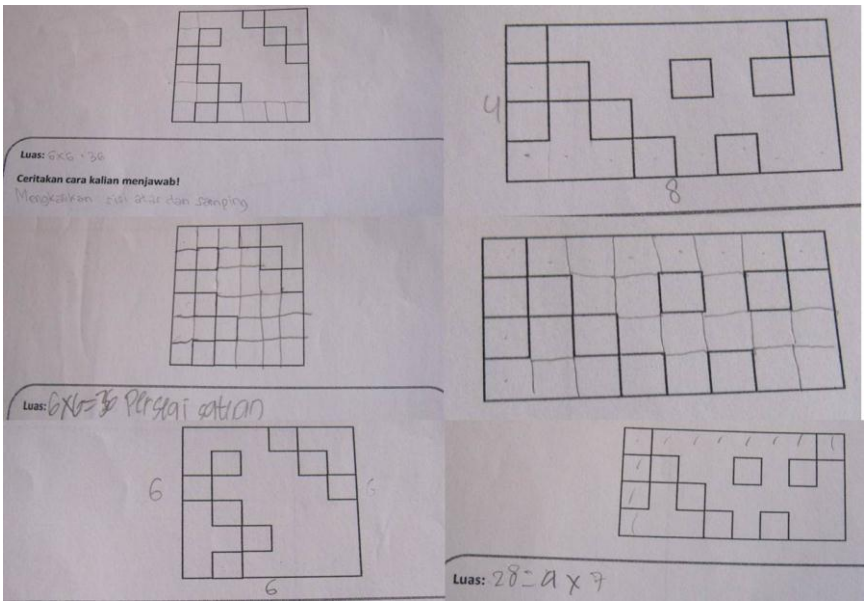


Figure 5.8: Estimating the area of given figures

The shift of students' method in estimating the area underlined the idea behind the activities in this cycle. Complete covering, decreasing the number of units and the use of strip of square units may contribute to the development of row-by-column structure. In fact, such a structure is closely related to multiplication in finding the area of shape. Though some students still needed to complete the square units inside the shape, they only counted unit in one row and one column to apply multiplication. It was indicated from dots inside units in such a row and a column. The researcher assumes that these students already start to develop the idea of row-by-column structure at some extent.

5.4.3 Visualization of measuring process

The rationale behind the last question in the post-test was to challenge students to make a visualization of the measurement. However, only one student could draw square units as an aid of area measurement. The rest of the students found it difficult to do so and then prefer to use a cm^2 strip. It was remarkable that one student could make a relation between covering activities –which was hands on activities– and visualization of the area measurement. Since the students did not explore the relation between unit of length and unit of area measurement, this result showed that covering activities served as good start for making such a relation.

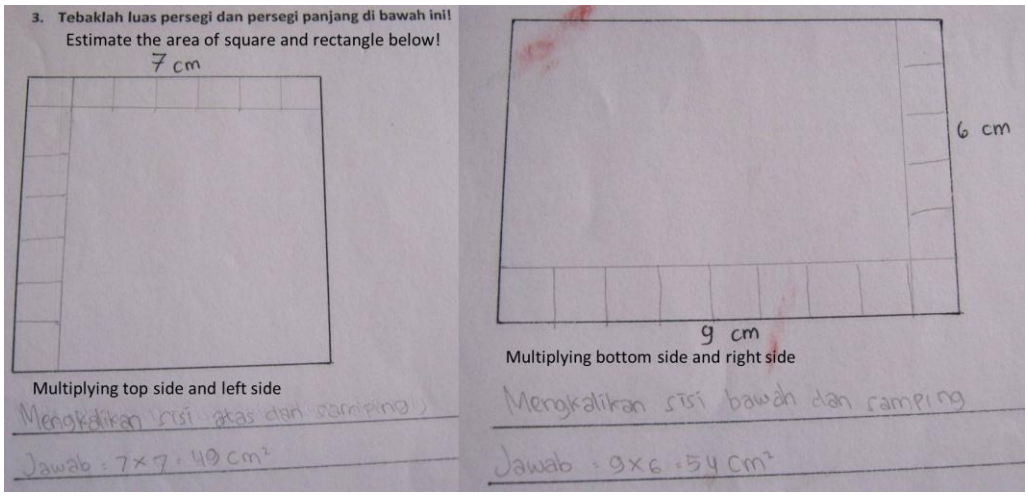


Figure 5.9: Student's visualization of area measurement

5.5 Conclusion of students' learning process in the first cycle

As shown in the remark on the pre-test in the beginning of this cycle, the students were lack of awareness toward identical units to compare area indirectly. Experiencing the comparison of two trays by using circular and rectangular crackers, they started to realize the need of using 'third object' as

aid to compare area of two surfaces that was difficult to be compared directly. They finally utilized the square units around the given figures as reference to determine the bigger pool.

The context of drying off crackers could help the students to do complete covering –without gaps or overlap. By discussing whether the crackers will get enough sunlight or not, students could cover the whole surface without gap and overlap.

The students already recognized the structure of arrays in the different level. Their starting point showed various ways in estimating total number of units covered the given figure. Counting one by one, repeated addition, and multiplication were their strategies in their post-test. Working with complete covering helped them comprehend area measurement as partitioning. Structuring the space by arranging rectangular and square units also contributed to the idea of using unit iteration. The students could apply multiplication by enumerating units in a row and so many rows in a column. When number of units was limited, some students put the units in a row and used single unit iteration along a column. When the units were enough to form row-by-column structure, the students could easily make it and then apply multiplication.

Sharpening the development of students' comprehension toward row-by-column structure, they were asked to measure area by using strip of square units. They were not merely measure area by using one size of strip, but they

could use different sizes of strips. They were also introduced to two standard unit of area measurement, namely cm^2 and dm^2.

There was an interesting finding when the students worked on covering with insufficient number of units. One student raised the idea of additivity of area measurement. This student told the idea of estimating the area by splitting the whole surface into two parts, estimating the area of each part, and then adding the estimation to get the area of the whole surface.

5.6 HLT 2 as the refinement of the HLT 1

Considering the analysis of the first cycle, the design of the series of activities basically can support the development of multiplicative structure quite well. However, some refinements are still needed to improve the HLT before the second cycle. These improvements were actually made for practical reason.

First, the students found it difficult to write their work and conclusion in the first activity. As the adjustment, the worksheet in the first activity will be more explicit to guide the students in comparing the area.

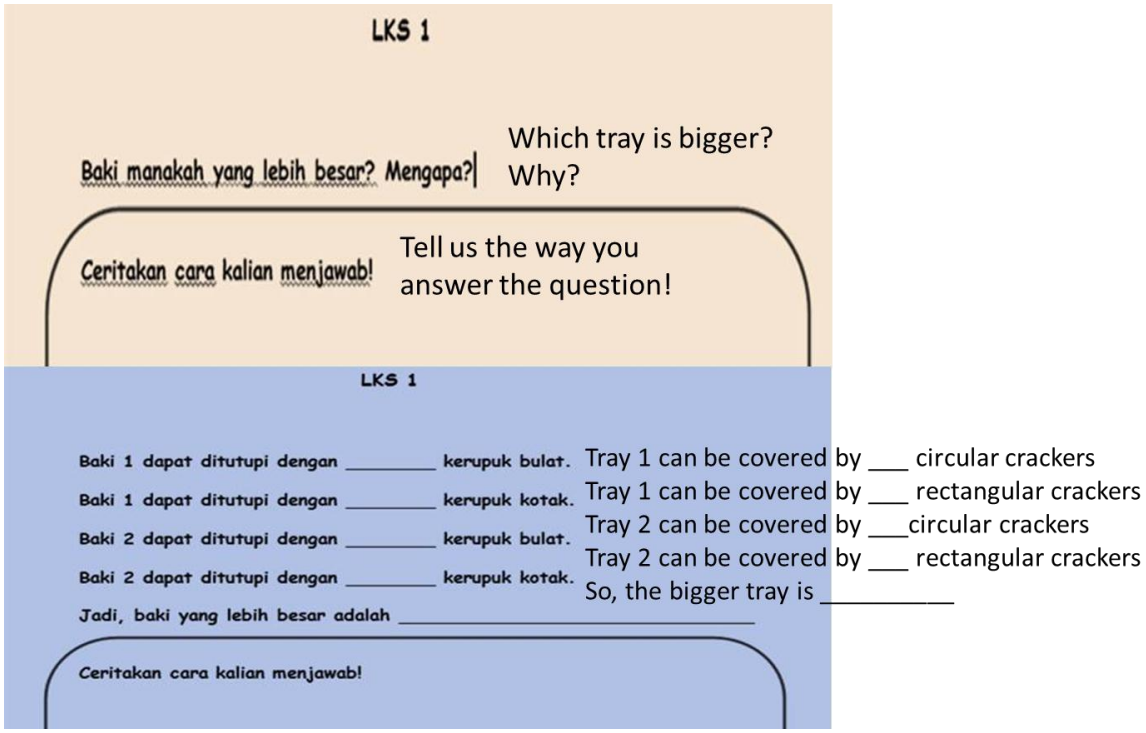


Figure 5.10: From top to bottom: the change of worksheet in activity 1

Second, in the second activity students were not familiar with the term 'conclusion'. The researcher will discard the question about that and discuss with teacher to hold class discussion instead.

Third, as the conclusion in the end of the fifth activity (see 5.3.5), the researcher will provide five objects whose surfaces are rectangle or square. It is also because the students in the second cycle will be worked in groups. They will measure the area of each object in turn with the other group. The objects and the size of each shape are described in the following table. The rationale behind the selection of the size is that some of these objects can be easily measured by using one size of strip but may be difficult to do so using another size of strip.

Table 5.2: The object for area measurement in the fifth activity

No.	Object	Size
1.	Piece of newspaper	10 cm x 15 cm
2.	Cardboard	10 cm x 10 cm
3.	Photograph	20 cm x 25 cm
4.	Calendar	50 cm x 40 cm
5.	Styrofoam	30 cm x 40 cm

For instance, the calendar can be measured by using a big strip (10 cm^2 strip) and a medium strip (5 cm^2 strip) but it will be too cumbersome for the students to measure it by using a small strip (1 cm^2 strip).

Fourth, as the conclusion in the last activity the researcher will provide several cardboards that can be measured by using 1 cm^2 strip and 1 dm^2 strip. The dimension of cardboard is 20 cm x 20 cm, 20 cm x 30 cm, and 20 cm x 40 cm.

5.7 Remarks of students' pre-knowledge in the pre-test (2^{nd} cycle)

There were 18 students who were given the pre-test in the second cycle. The question of the pre-test in this cycle was exactly the same with that of the first cycle. In addition, four students chosen as focus group were interviewed afterward. The same question with the interview in the first cycle was posed to them. Moreover, this interview also served as the clarification of students' answer in the written test.

Generally, the students' pre-knowledge in this cycle was not quite different from students' pre-knowledge in the first cycle. The detail analysis of pre-test in the second cycle can be seen in Appendix H.

Similar to the first cycle, students in the second cycle was **lack of awareness toward attribution of area during comparing area of two shapes**. The written test showed that students used perimeter and the shape of figures as reference to determine the largest pool. This result was clarified in the interview with focus group. Their explanations during interview were in line with it. However, one student seemed to start developing the idea of identical units to compare area. Although this student did not count the total squares in the first figure correctly (it must be 5 x 10, not 10 x 20 instead), the researcher assumed that he developed the idea of identical units at some extent.

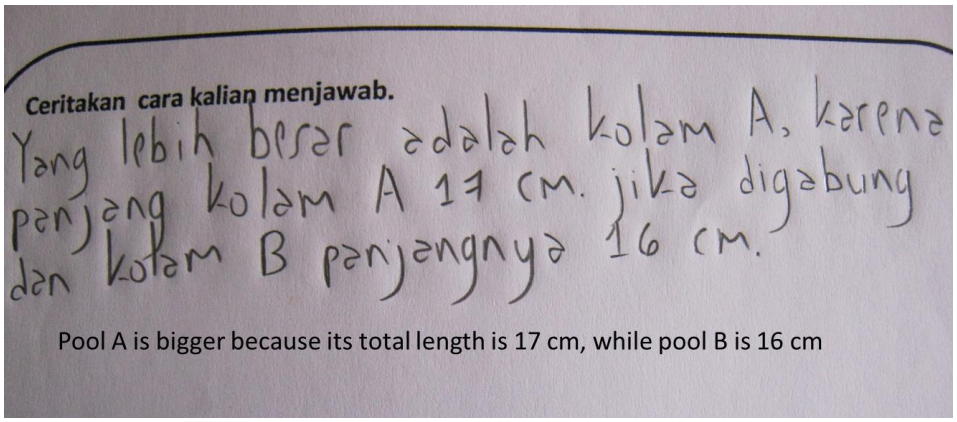


Figure 5.11: Students' response that using 'perimeter' as reference of the bigger pool

Unlike in the first cycle, the students in this cycle **did not make relation between the structure of arrays in the figure and the way of counting the units** in the figure. Most of them seemed to misunderstand the question. They thought that they should determine the remaining tiles needed to cover the whole instead of the total number of tiles. Only three out of eighteen understood the question correctly. These students completed the drawing,

count the squares given, and then added it to the remaining squares (the fourth way, see 5.2.2). However, unclear reasoning shown by students might be clarified if they had an opportunity to explain it. One student who gave unclear reasoning happened to be in the focus group. So, the researcher asked clarification about his response in written test (see Figure 5.11) during interview.

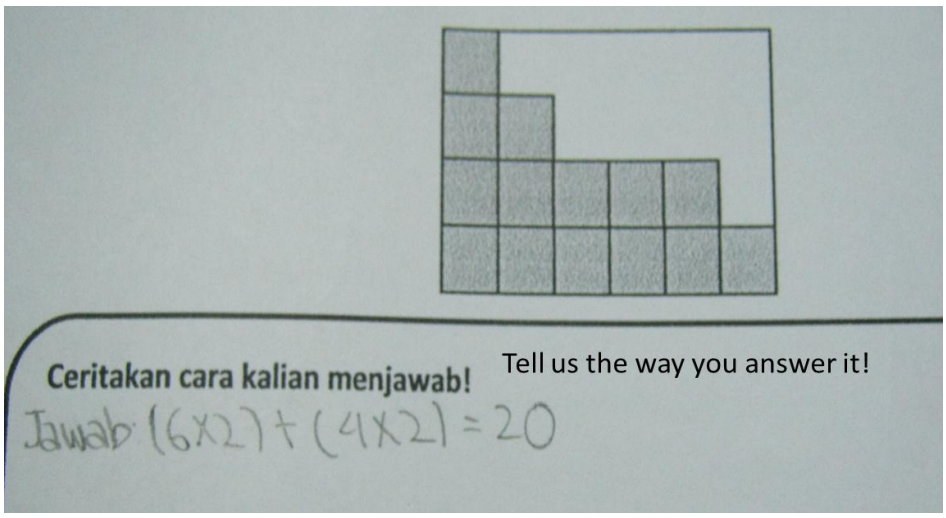


Figure 5.12: An example of unclear reasoning

The student explained that both the bottom row and the top row contained six. It was the reason of six times two. In addition, four times two was obtained from squares in the left and the right column. He only considered the squares in the edge of the shape without aware of the other squares covered the whole shape. At this point, the researcher concluded that this student did not comprehend the structure of arrays yet.

As additional investigation of students' pre-knowledge in this cycle, the researcher held an interview about the way of iteration process in covering a surface. This interview was only for students in the focus group. Without

making generalization, the students' response in this interview was expected to depict the general ability of students. It was because students in the focus group represented those with middle level of achievement. They were asked to estimate the number of square units needed to cover a given rectangle.

Similar to the response of the interview of the first cycle, the students in this cycle also showed **the various ways of iteration in estimating total units**. The variety of iteration ways was also very similar. Two students performed row-by column iteration to apply multiplication, one student did one-by-one iteration, and another student still got confused in doing iteration.

Considering these remarks on students' pre-knowledge of the second cycle, the researcher did not make any more adjustment for HLT 2. It will be implemented in the second cycle. The report of activities in this cycle will be described in the following sub chapter.

5.8 Trying out the HLT in the second cycle

The implementation of the design in the second cycle will be explained in this section. This experiment was based on HLT 2. It was conducted in a class consist of 20 students. However, it sometimes changed because one or two students was absent during a lesson. In this cycle, four students become a focus group that will be a main focus of observation. These students were chosen according to the consideration from the mathematics teacher. They were students with middle level of achievement in mathematics. They were Dimas, Aura, Aryo, and Putu.

The description of this experiment was elaborated based on each lesson since there was only one activity in one lesson. The chronology of description was explaining general description and remark, the fragment of interesting conversation, and then the analysis of the lesson.

5.8.1 Developing the idea of identical unit to measure area

Activity 1: Comparing two trays using two different crackers

The first activity in this cycle was built on the idea of using identical unit as reference in comparing area. In addition, complete covering without gaps and overlaps also the main idea of this activity.

Teacher started the lesson by reviewing the perimeter which was a topic of the last meeting. The teacher then showed a bamboo tray to students and held brief conversation to check whether students familiar with it. Showing two bamboo trays –one was longer, another was wider–, teacher asked them the bigger one. Students directly said that the longer was the larger. The teacher put a tray one another, and then repeat the same question. One student told that it was difficult to compare the trays because one was big and another one was longer. Following that response, the teacher showed two different crackers. The students were asked to investigate the tray that contained more crackers. Two cardboards, rectangular papers and circular paper were given to each group of students. Those objects were substitution for trays, rectangular crackers and circular crackers respectively.

In covering longer tray, the focus group put two rows of rectangle vertically and one row horizontally so that the whole surface of tray were fully covered (see Figure 5.13 A). However, some other students put the rectangular paper in the same direction (Figure 5.13 B). Moreover, all students put the rectangular papers in the same direction –vertically or horizontally– when covering the wider tray (Figure 5.13 C).

Enumerating the papers, the students did not show various ways. All of students count them one by one. Apart from that, the focus group could perform skip counting such as 10 + 10 + 5 and 5 + 5 + 5 when the researcher asked them to count in another way.

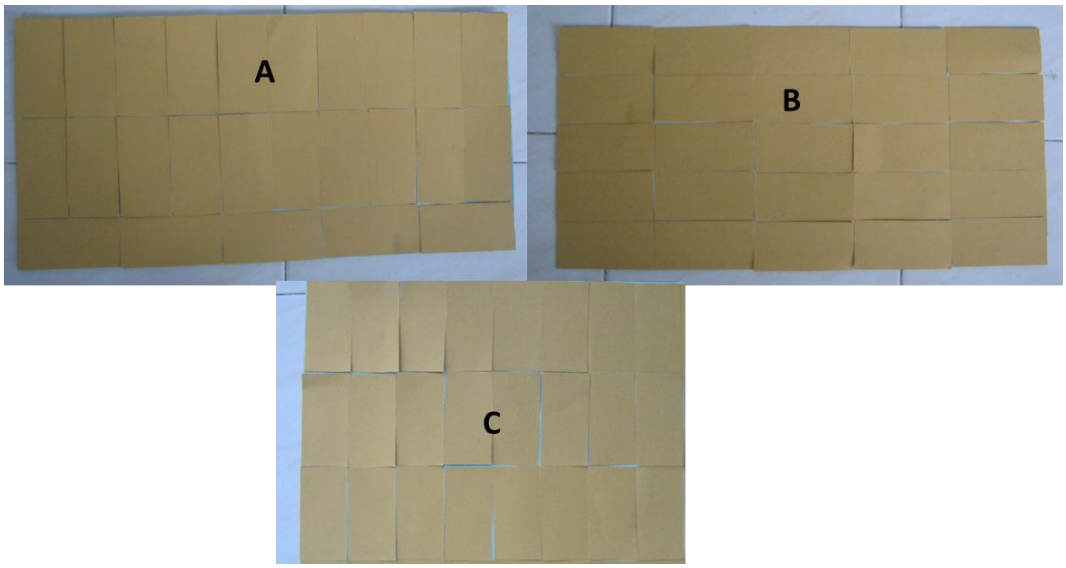


Figure 5.13: Students' strategy in covering the cardboard

The class discussion was held to check the result of students' work. It was about the number of rectangular crackers and circular crackers that covered each tray. However, some covering activities were repeated by demonstrating it once more because some students got the different result.

It happened on covering tray using circular crackers. In this demonstration, it was revealed that some students did not aware of gaps and overlap. They prefer to put circular paper overlap each other than to leave some spaces uncovered.

Figure 5.14: Demonstration of covering the wider tray by using circular crackers

Considering the process of drying off the crackers, there was a student could explain why they could not put the cracker overlap each other. This student said that the crackers would not dry well. The teacher then raised the fact of cracker might be fall if they kept putting the same number of cracker without overlap. Therefore, the conclusion was to find out the maximum number of circular paper and leave some part uncovered if it could not be covered by those papers. However, this conclusion implied that the maximum number of crackers might be obtained though some parts of surface were not covered.

After all the result of covering activity was checked, the next discussion was to conclude which tray was bigger than another. All students answered

that tray 1 (the longer one) was bigger than tray 2 (the wider) because it contained more crackers. The following statement was quoted from a student in the focus group who explained the reason of the answer.

"*Tray 1 is bigger because it [contains] more crackers. For circular crackers, it contains 18 [crackers] while tray 2 contains 15 [crackers]. For the rectangular crackers, tray 1 contains 25 [crackers] while tray 2 contains 24 [crackers]*."

However, some of the students stated that the second tray 1 was the biggest tray. Their statement seemed to be based on the result of their covering activity. It showed that the number of rectangular crackers covered the tray 1 was more than the number of circular crackers covered the same tray.

There was interesting discussion between researcher and focus group about how they conclude the bigger than another. It was held before the class discussion was started. The transcript below quoted from it.

Researcher	: What do you think about the bigger tray?
Students	: (*silent*)
Researcher	: Does it contain more crackers or fewer crackers?
Dimas	: More crackers.
Researcher	: So, which one is bigger?
Dimas	: Tray 1 contains 25 [rectangular crackers] and 18 [circular crackers]. Tray 2 contains 24 [rectangular crackers] and 20 [circular crackers]. (*pointing out the answer on the worksheet*)
Researcher	: Can we compare the tray by using rectangular and circular crackers?
Students	: No.
Researcher	: So, how can we compare?
Dimas	: **Rectangular to rectangular, circular to circular**.
Researcher	: Then, which one is bigger?
Students	: Tray 2.

Researcher : Look at your answer once more.
Students : Tray 2.
Researcher : Why?
Dimas : Because this is 18 [tray 1, rectangular crackers] and that is 20 [tray 2, rectangular crackers]. This is 25 [tray 1, circular crackers] and that is 24 [tray 2, circular crackers].
Researcher : If we consider the circular crackers, which one is bigger?
Students : Tray 2.
Researcher : If we consider the rectangular crackers, which one is bigger?
Dimas : Tray 1. There must be something wrong.
Researcher : Show me the way you cover the trays using circular crackers!

……

Researcher : How many circular crackers covered tray 2?
Students : Fifteen.
Researcher : So, which one is bigger?
Students : Tray 1.
Researcher : Why?
Aryo : This [18] is greater than that [15], and this [25] is greater than [24].

The transcript above shows that students in the focus group start to develop the idea of identical units to compare area. They used one type of crackers as reference to compare the trays. However, their mistake during covering tray 2 with circular papers also applied for the rest of students. Fortunately, the teacher checked the result of each covering and asked for demonstration if the different results among students exist.

Generally, students started to get an idea of complete covering. Through class discussion, the teacher facilitated students to sharpen their awareness toward complete covering without gaps and overlaps. The context of drying off crackers gave a hint for these ideas. However, the idea of identical units to compare the area was not explicitly occurred in the beginning of this activity. The students were just asked to do an

investigation to find out the tray that contains more crackers. They were not asked how they could determine the bigger tray respect to the number of crackers contained in the tray. They only explored this idea through the class discussion. Moreover, the students did not discuss about the idea that the comparison of the tray must be respect to the same cracker in the class discussion. They only referred the biggest tray from the biggest number of crackers that was contained in the tray. As the implication, some of them gave statement about "the second tray 1". It is inferred that they still compared the number of rectangular crackers to that of circular crackers which covered tray 1.

In addition, the comparison between the use of rectangular and circular units to cover the surface did not discussed yet. As bridge for the next activity, the use of square crackers will be based on the idea of covering surface by using another variant of crackers.

Looking back to the way students' structure two-dimensional arrays in this activity, all of their strategies was predicted in the conjectures. Although some of them arranged the units in the same direction, none of them used multiplication or even skip counting to enumerate the units. The researcher inferred that the students did not yet build the relation between the structure of arrays and the way of counting the units. Nevertheless, it was a good start for them because the students would explore further about this in the next activities.

Another remark of this activity was the way students' work on their task. They seemed to found it was difficult to organize their task. The worksheet that has been improved from the last cycle was not directly simplifying this problem. In this activity, they must cover two cardboards by using two different types of units. They needed to be aware of which tray they covered using a type of units so that they can fill the worksheet correctly.

5.8.2 Developing idea of multiplication in counting units

Activity 2: Measuring area of tray by using square crackers

Starting the idea of attribution of area in the previous activity, this activity was aimed to facilitate students in constructing the idea of multiplication in enumerating units of area measurement.

After reviewing the last mathematical lesson, the teacher introduced to students another shape of crackers that was different from the last meeting, a square cracker. The teacher then explicitly asked them to measure the cardboards –as substitution of tray– by using square crackers. Instead of giving the cardboards one by one, the teacher distributed two different cardboards –one was square and another was rectangular– and square papers to each group of students at the same time.

Almost all students including the focus group did cover the whole surface with square papers. These students showed various ways during the covering process (see Figure 5.15). Some of them started with cover the most left column and the top row. Some other students covered the top row first and then filled in the space in the next row. Another ways of covering

shown by students was starting to cover the cardboard from the most left column and the most right column at the same time and then filled in the space between them. However, there were some students who only put the papers in one row and one column.

As predicted in the HLT, all students used multiplication to enumerate the squares. However, the students was not merely used this as their strategy. Students in a group might have different idea on enumerating the squares. After one student in the focus group suggested to use multiplication and got the result, another member still checked it by using counting one by one. Another case in another group showed that when a student started to count one by one his friend proposed to use multiplication. In addition, none of these students used skip counting.

Figure 5.15: Various ways of iteration process of covering the cardboard

During covering and counting process, the researcher walked around to observe the students' work. Anytime students used multiplication, the researcher posed question to clarify why they could use it as their strategy in enumerating the squares. Most of them responded that it was easier and faster. The researcher then discussed the relation between the number of squares in a row and in a column. It was done by posing question such as "*how many squares in this row?*', "*how many rows we have here?*".

The researcher together with the teacher made an improvisation in this activity. It was because the whiteboard in the classroom has the grid on it. Luckily, the size of square that compiled the grid was congruent to the square papers used in the students' activity. Therefore, before the class discussion was started the teacher drew square and rectangle on the board. The dimension of the figures was exactly the same with the cardboard. These figures would be used as tool for clarifying students' strategy in enumerating the units.

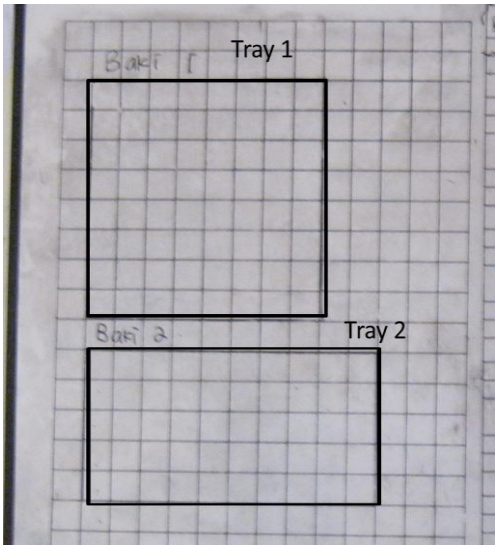


Figure 5.16: The drawing on the board

Each group of students was asked to write their result of covering activity in the other part of the board. All students found the same result for the rectangular cardboard (50 crackers). However, unlike most of students whose result was 64 crackers the focus group wrote the square cardboard needed 49 crackers only. The interesting discussion about this rose up. The following transcript showed the conversation during the class discussion.

Teacher	: Please the representative of group five (the focus group) explain to us how you get 49!
Dimas	: (*come forward*)
Teacher	: (*giving the square cardboard to Dimas*) Now, everybody looks at Dimas!
Dimas	: (*putting the cardboard on the drawing in the board*)
Teacher	: Does tray 1 brought by Dimas has the same size with the drawing on the board?
Students	: Yes.
Teacher	: (*giving a square unit to Dimas to be put on the board*) Does the square cracker has the same size with the square on the board?
Students	: Yes.
Teacher	: Dimas, show us how many crackers can be contained in tray 1!
Dimas	: (get confused)

Teacher : Okay, another member of the group may join Dimas to help him.
Aryo : (*come forward and then counting the squares together with Dimas*). It is 64.
Dimas : (*whispering to Aryo*) It was because we counted from here (*pointing out the second column*), not here (*pointing out the first row*).
Teacher : If you got the answer, explain to us!
Dimas : I counted from here (second row), not from here (the first row). It was because the units cover the outside of cardboard and do not overlap.

Figure 5.17: Checking the size of cardboard and the square units

Based on the transcript above, the researcher inferred that some students in focus group still needed more time to perceive the idea of complete covering. They could do complete covering on the rectangular cardboard (Figure 5.18 A) while they actually missed one row and one column when covering the square cardboard (Figure 5.18 B). It was also presumed that they considered the demonstration of covering with circular units in the last meeting. At that time, the left over space were allowed if the tray could contain no more circular crackers. Therefore, in this activity the students also let some parts of cardboard were uncovered because they thought there was no space for more square crackers. Apart from that, having an opportunity to clarify their answer on the whiteboard helped them realize the reason why they got different result from the other.

Hopefully it could support them to comprehend the idea of complete covering.

Figure 5.18: The work of the focus group

In the HLT, it was planned that the class discussion was also addressed the relation between the whole structure of two-dimensional arrays. However, the teacher explored repeated additions as another strategy to enumerate the squares. The teacher highlighted multiplication as easier, faster, and more effective instead of exploring the relation between repeated addition and the multiplication performed by students. The teacher merely discussed the comparison. Nevertheless, the discussion between the researcher and each group when students were working might facilitate them at some extent. Although it did not happen in the whole class, some

students explicitly showed their comprehension toward the relation between the structure of arrays and multiplication (Figure 5.19). Regarding this achievement, the learning will be continued to next activity in the next lesson.

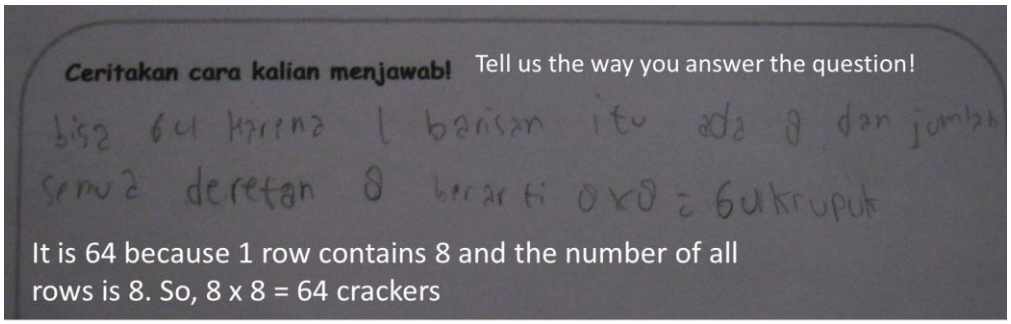


Figure 5.19: Students' reasoning in using multiplication

In the end of this activity, the teacher introduced the term area to students. It was explained to them that covering the surface was area measurement. The students were asked to differentiate area and perimeter. Some students stated that the area was something inside the boundaries of the shape.

5.8.3 Determining the flexible unit for measuring area

Activity 3: Measuring area of tray using different types of units (square and rectangle)

As the students started to use multiplication to enumerate the units covered a surface, they will be confronted with the idea of flexible units in applying multiplication. Although people in general can measure the area of two-dimensional shape by using rectangular units, they need to put them in the same direction to apply multiplication. In this activity, the students measured a square cardboard by using square units and rectangular units, respectively.

Starting the lesson, the teacher reviewed the last meeting. The students were asked to show the area of their mathematics book cover. Almost all students pointing out the surface of the book cover. Then, they were introduced to term square units and rectangular units. It was intended to support them using those terms instead of keeping use ‘crackers’. Each group received the cardboard and two types of units.

The students showed various strategies in finding out the area of cardboard by using square units. The students in the focus group as well as some other students did complete covering. These students did it by starting to cover the cardboard with square units from any direction. However, the rest of students only covered a column and a row. In other word, they formed row by column structure. Although some students performed complete covering, they only counted the units in a row and in a column. They then applied multiplication like the other students who used row by column structure.

Figure 5.20: Students' strategies in covering the cardboard

Figuring out the area of cardboard by using rectangular units, the students also used the various ways of covering. The focus group covered the cardboard with units starting from bottom and top row at the same time. They put the units in the same direction and then covered a column with the units before applying multiplication. Some students fully covered the cardboard with rectangular units and put them either in the different or the same direction. Some other students formed row by column structure of the rectangular units in the same direction. Finding out the area of the cardboard, the students used multiplication if the direction of rectangular units was the same. In the other hand, some students used counting one by one if the direction of units was different.

The researcher talked with the focus group after they finished figuring out the area of cardboard using rectangular units. The following transcript described the discussion with focus group.

Figure 5.21: Discussion with the focus group

Researcher	: Can we know the area if we only put the rectangular units like this (Figure 5.21)?
Students	: We can.
Aryo	: 1, 2, 3, 4, 5 (*counting along the column*). 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (*counting along a row, while Aura also counting another one*).
Aryo	: It is fifty.
Researcher	: How come?
Aryo	: From five times ten.
Researcher	: (*rotating some units in the column into different direction*) If the units are in the different direction, can we still use multiplication?
Aryo	: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (*counting along the column*), 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (*counting along the row*).
Aryo	: It is hundred.
Dimas	: It is eighty (*considering the column only contained 8 units because the other two were in the different direction*).
Researcher	: Okay, now let's cover the whole surface to prove how many rectangular units on it.
Students	: (*completing the covering process*).
Researcher	: So, what is the total number of units?
Students	: Fifty.

Researcher : Hm, it is the same with our previous result, five times ten. So, can we use multiplication if the direction of the units is different?
Dimas : We can't.
Researcher : **How if we still want to use multiplication?**
Dimas : **All should be like this (*horizontal*) or like this (*vertical*).**
Researcher : **Which one do you prefer if we want to apply multiplication –the rectangular or the square?**
Aryo : **The rectangular one.**
Dimas : Rectangular.
Researcher : Why?
Aryo : **It is longer than the square.**
Dimas : **We need less number of units to cover the whole.**
Researcher : (*removing some units and only keeping a column and a row of units*) Can we use multiplication if we put the units like this?
Aryo : The result is hundred.
Researcher : What is the number of units covered the whole when we count them one by one?
Students : Fifty.
Researcher : So, how if we want to use multiplication without considering the direction of units?
Dimas : the square is better.
Aryo : Yah, the square (taking one square unit and rotating it) because it is still the same if we rotate it.

This kind of conversation also occurred in the dialogue between the researcher and the rest of students in the other groups. In the beginning, the students chose the rectangular units because it was longer or they only need less number of units to cover the whole. By challenging them to consider the total number of units by counting one by one, they could directly compare the result.

In addition, the teacher held a class discussion to highlight the idea of flexible units to measure the area. The teacher asked the students to choose the more flexible unit between rectangle and square if they want to apply

multiplication. Dimas is asked to explain his reasoning in choosing square unit. This discussion is depicted in the following transcript.

Dimas	: Because when it is being rotated (*showing square unit and rotating it*), the shape will remain the same.
Teacher	: Show it to your friend.
Dimas	: When it (*square unit*) is being rotated, the shape will be same. But when it (*rectangular unit*) is being rotated, it (*the direction*) will not same anymore.
Teacher	: **Can we count the rectangular units [that cover a surface] if the directions of the units are different?**
Dimas	: **We cannot apply multiplication, but we can still count them one by one.**
Teacher	: Do you understand, Fathan? Please repeat Dimas's explanation!
Fathan	: Because it will be the same although it is being rotated.
Teacher	: Which one?
Fathan	: The square unit.
Dimas	: (*come to whiteboard to demonstrate the rotation*)
Dimas	: If it is being rotated, it will be the same (*demonstrating the rotation of square unit*). If it is being rotated, it will be not the same. There will be two directions.
Teacher	: **Can we count the units if the directions are different?**
Dimas	: Yes, we can.
Teacher	: How?
Dimas	: **Counting the units one by one.**
Teacher	: **How if we want to apply multiplication?**
Dimas	: **We use this (*showing a square unit*).**
Teacher	: Why can we say that the square unit is better?
Joubu	: It is smaller.
Teacher	: Joubu said [because] it is smaller.
Dimas	: We can rotate it without changing the direction.
Teacher	: What do you think, Rifqy?
Rifqy	: Because it (*each side*) has the same length.
Teacher	: Good. The length of each side has the same length.
Teacher	: How about it [showing the rectangular unit]?
Students	: It (*the length of each side*) is different.
Rifqy	: Two [parallel] sides have the same length.
Teacher	: The length of this side (*pointing out the length*) is same with this (*pointing out the parallel length*). The length of this side (*pointing out the width*) is same with this (*pointing out the parallel width*).

From those two transcripts of discussion, it can be inferred that the students in the focus group could chose square unit as flexible unit in area measurement. It was also applied to the rest of students because similar discussion occurred in each group of students. However, there was no explicit statement or question about the use of rectangular unit in area measurement. There was also no exploration about how if the rectangular units were placed in the same direction.

Looking back to the way students covered the cardboard by using rectangular units, some of them did it as predicted in the HLT. They put the units in the different direction and found it was difficult to apply multiplication. In the contrary, the other students who put the units in the same direction could easily do it. Moreover, the students only considered the units in a row and a column although they did complete covering with square units. Therefore, the structure of two-dimensional arrays formed by students has something to do with multiplicative structure. It could help students recognize the number of units in a column (or a row) and so many column (or a row) there.

5.8.4 Constructing the shortcut to determine area

Activity 4: Measuring area without enough number of units

In the last activity, the students already took consideration of the direction of units to apply multiplication. They chose square units as the better units if they want to use multiplication in finding the area of cardboard.

Following this achievement, this activity attempted to give more support to students in developing the idea of multiplication.

Each group of students got two cardboards –a yellow rectangular cardboard and a blue square cardboard. They only received ten square units to estimate the area of each cardboard. These units were only enough to cover one row and one column of the rectangular cardboard. It was not applied for the square cardboard, because these units could only cover one row (or one column) and two units as remainder. In the worksheet, the students were asked to estimate the square cardboard first. In fact, some students just started to work on estimating the area of rectangular cardboard and then filled in the question in the worksheet.

Figure 5.22: Students' strategies in estimating the area

In estimating the square cardboard, some students –including focus group– filled in a row with units, removed the units, and then covered a column. Some other students put eight units in a column and two units in a row. They then moved six units from the column to the row. All students found out that the area of the square cardboard was 64 square units. They used multiplication as their way to estimate the area.

Estimating the area of rectangular cardboard, all students used the same method. They put square units in one row and one column and then applied multiplication. There was interesting discussion among some students about their estimation of the area of the rectangular cardboard. This discussion is quoted in the following transcript.

Lala	: It is six time four.
Rifqy	: It is twenty four.
Rayhan	: No, it is six times two.
Rifqy	: **No, it is 6 + 6 + 6 + 6 (*pointing out the units on their cardboard*)**. There are four sixes.
Rayhan	: Four times six, sixty four.

After finishing their work, the students discussed the result of their work in the class discussion. The teacher clarifying students' work by asking in which part of cardboard they put the units. The teacher also clarified how the students estimate the total number of units.

Comparing the students' responses to the HLT, all students formed row by column structure in estimating the area of the cardboard. None of them used row or column iteration to get the total number of units that could cover the whole surface of the cardboard. Supporting by the transcript of the conversation above, it can be inferred that the students did not need to

cover the whole cardboard for recognizing the relation between the structure of arrays and multiplication. The row by column structure may contribute to the development of the multiplicative structure.

The trying of this activity in the previous cycle revealed that the order of distributing the cardboard could support them in forming row by column structure with insufficient units (See 5.3.4). However, it was different from this cycle. In this activity, the cardboards were given to students at the same time. Some of them worked on the rectangular cardboard first which enabled them to form row by column structure easily. They then tried to form the same structure to estimate the area of the square cardboard. It shows that no matter which cardboard that is chosen by students as their first work, it may support them on their next work.

According to the previous cycle, the idea of additivity of area measurement occurred in this activity. However, none of students came up with this idea. The researcher only tried to discuss it with focus group. The students in the focus group could easily estimate the area of each part of the cardboard when the researcher split it into two parts.

Like in the previous cycle, after this activity the students got an individual exercise in estimating the area of given figure. This task was exactly the same with the individual task in the previous cycle. From 18 students, only one student did not finish his work. He only completed the drawing. Sixteen out of seventeen students used multiplication to estimate the area of the figures. Seven students completed the drawing, three students

completing one row and one column, and six students did not draw anything. One last student completed the drawing, counted the squares on the given drawing, and then added it to the additional squares formed by the drawing.

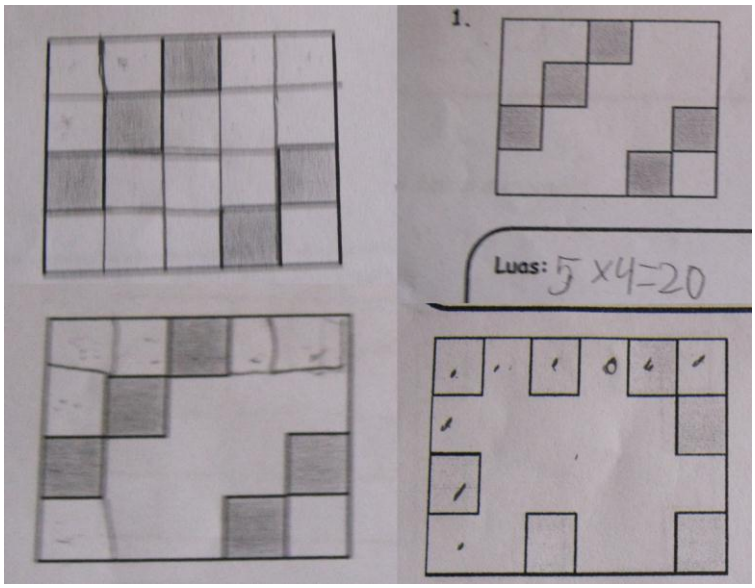


Figure 5.23: The individual task on estimating area

The students' responses on the task support the conjectures of the researcher. Although some students still needed to complete the drawing, they only consider the number of square units in a row and in a column. All students started to develop row by column structure and made relation with multiplication.

5.8.5 Constructing the row-by-column structure

Activity 5: Measuring area using strip of square units

Covering cardboards with insufficient number of square units, students started to develop row by column structure. In this activity, they would measure five objects by using strips of squares units. There were three

strips with different sizes. The big strip was 10 units of 1 dm^2 squares. The medium strip was 10 units of 25 cm^2 squares; while the small strip contains 20 units of 1 cm^2 squares.

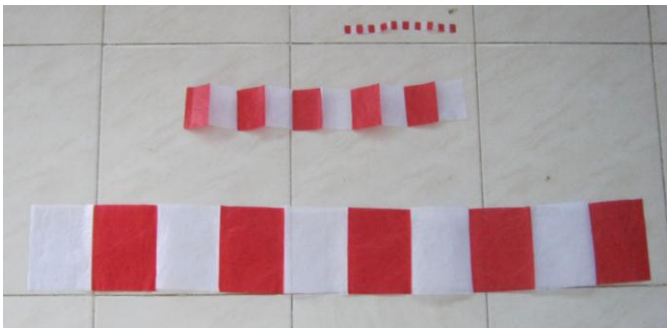


Figure 5.24: Three different sizes of strip

Starting this activity, the teacher introduces the strips and the notation for each of the strip. As written in the worksheet, the notation for big, medium, and small strip were B, S, K, respectively. The teacher then explained the rule in this activity. Each group of students would measure each object in turn with the other groups until all objects were completely measured. Before they started, the teacher reviewed the way they measure area in the previous activity.

All students have the same method in finding the area of each object. They put the strip in the vertical direction and then lay it in the horizontal direction. Using the number of units in those directions, they applied multiplication. Some students could measure the area of an object with large surface by using small strip while the other could not do it. If the small strip could not cover the whole edge, they marked the last position of the strip and then used that last position as new start point to get the number of units in one direction. Moreover, some students have an idea to

measure the table. Although it was not included in the worksheet, all students could measure the four tables of their group by using big strip correctly.

Figure 5.25: Measuring the surface of object

The students' work on measuring the area of all objects was reviewed in the class discussion. The students were asked to demonstrate the way they measure one object. The demonstration showed the strip used by students, the way students put the strip, and how they got the area of the object. Another group was invited to show their work if they used different type of strip. It was done for five objects. The following transcript shows the review of area measurement on one object.

Teacher : The group with Styrofoam on their table please come forward and explains your answer!

(*Rifqy, Lala, and Rayhan come forward bringing big strip, medium strip, and the Styrofoam*).

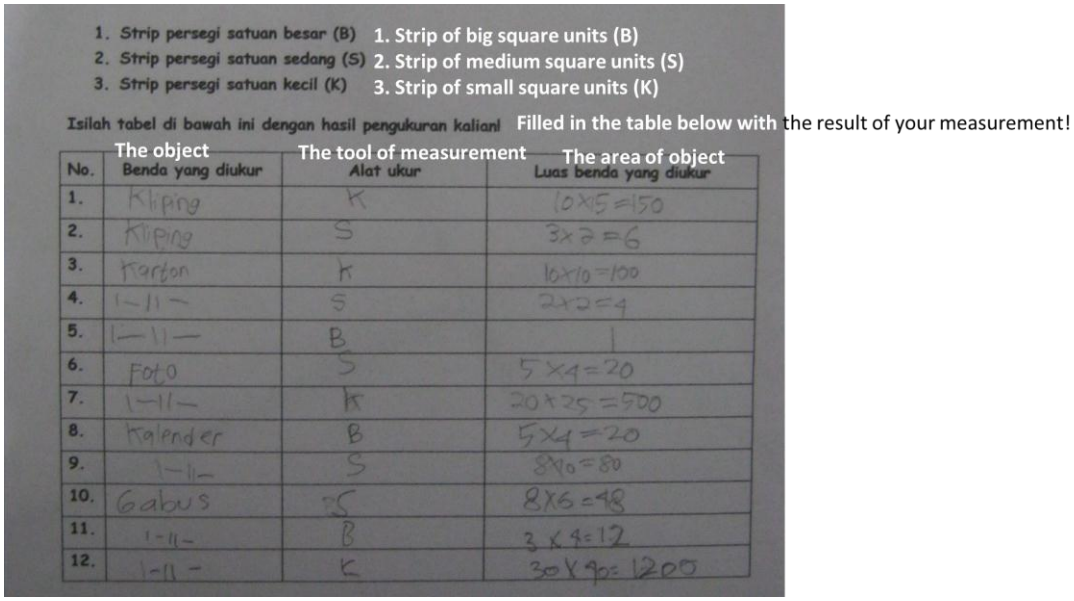
Rayhan : The object is Styrofoam. The strip used is big strip. (*reading the table in the worksheet*).

Rifqy : (*putting the strip in the horizontal direction*).

Lala : 1, 2, 3, 4 (*counting the square units*).
Rifqy : (*putting the strip in the vertical direction*)
Lala : 1, 2, 3 (*counting the units*). It is four times three, twenty four.
Teacher : (*asking to the rest of students*) Do you find the same result by using the big strip?
Students : Yeah.
Teacher : Now, try using the medium strip.
Rifqy : (*putting the medium strip in horizontal direction*)
Lala : 1, 2, 3, 4, 5, 6, 7, 8.
Rifqy : (*putting the medium strip in vertical direction*)
Lala : 1, 2, 3, 4, 5, 6.
Teacher : So?
Rayhan : (*reading the worksheet*) six times eight is forty eight.
Teacher : Is there anyone who gets different result using this strip?
Students : No.
Teacher : How about the small strip?
Rifqy : We do not use it.
Lala : It was not enough.
Dimas : (*raising his hand*) We use the small strip.
Teacher : What is the result?
Dimas : 1200.
Teacher : Show us then.
Dimas : (*putting the small strip in the horizontal direction*) It is twenty, (*marking the last point and using it as the new starting point*) it is also twenty.
Dimas : It is forty.
Teacher : Okay, it is from twenty plus twenty.
Dimas : (*putting the strip in the vertical direction*) It is twenty (*do the same thing with the last measurement*) and ten.
Teacher : Dimas's group measures this object by using the small strip. This direction contains forty and this contains thirty. So, what is the result of forty times thirty?
Dimas : 1200.

Figure 5.26: The demonstration of measuring area

In the class discussion, the idea of complete covering was also addressed. It was not allowed to use a type of strip if there one square unit was not intact cover the surface. It was because the students were still at the beginning of learning area measurement. A half or any part of units was not considered as one unit.



1. Strip persegi satuan besar (B) 1. Strip of big square units (B)
2. Strip persegi satuan sedang (S) 2. Strip of medium square units (S)
3. Strip persegi satuan kecil (K) 3. Strip of small square units (K)

Isilah tabel di bawah ini dengan hasil pengukuran kalian! Filled in the table below with the result of your measurement!

No.	The object Benda yang diukur	The tool of measurement Alat ukur	The area of object Luas benda yang diukur
1.	Kliping	K	10x5=150
2.	Kliping	S	3x2=6
3.	Karton	K	10x10=100
4.	1-11-	S	2x2=4
5.	1-11-	B	1
6.	Foto	S	5x4=20
7.	1-11-	K	20x25=500
8.	Kalender	B	5x4=20
9.	1-11-	S	8x10=80
10.	Gabus	S	8x6=48
11.	1-11-	B	3x4=12
12.	1-11-	K	30x40=1200

Figure 5.27: The students' work on area measurement of five objects

Looking back to the HLT, the students' strategy in measuring the area of objects was forming row by column structure. None of them used row iteration or row iteration. It can be inferred that the use of multiplication – especially in the two previous activities– may contribute to the development of students' thinking of multiplicative structure. The structure of strip itself may also give significance to tendency of students in forming row by column structure. The nature of the strip may facilitate students to easily see the number of units either in vertical or horizontal direction.

Clarifying students' work by using demonstration could also facilitate students in choosing the unit of area measurement. They could see that one object can be measured by using different size of units and lead to the different result. It supported the measuring activity that they have already done in a group. All students could use at least two types of strip for measuring area.

5.8.6 Developing the use of standard unit for area measurement

Activity 6: Measuring area using strip of standard units

This last activity was actually an additional activity in this series of activities. It was intended to give the students brief introduction to several standard units of area measurement, cm^2 and dm^2. As the refinement from the previous cycle, the students were given three cardboards that could be measured by using 1 cm^2 strip and 1 dm^2 strip. The dimension of each cardboard was 20 cm x 20 cm, 20 cm x 30 cm, and 20 cm x 40 cm.

Before the students started to measure the cardboards, the teacher asked them to mention standard unit for length. The students could mention several standard units such as millimeter, meter, centimeter. The teacher then introduced two standard units for area measurement by showing two strips –strip of 1 cm^2 and strip of 1 dm^2. The teacher explained that one square in each strip stands for 1 cm2 and 1 dm^2 respectively. The students were also introduced to the notation of the units. In addition, the teacher reviewed how they could measure the area of surface of object. The students stated that they could lie the strip in vertical direction (left or right

column), lie it in the horizontal direction (top or bottom row), and then apply multiplication.

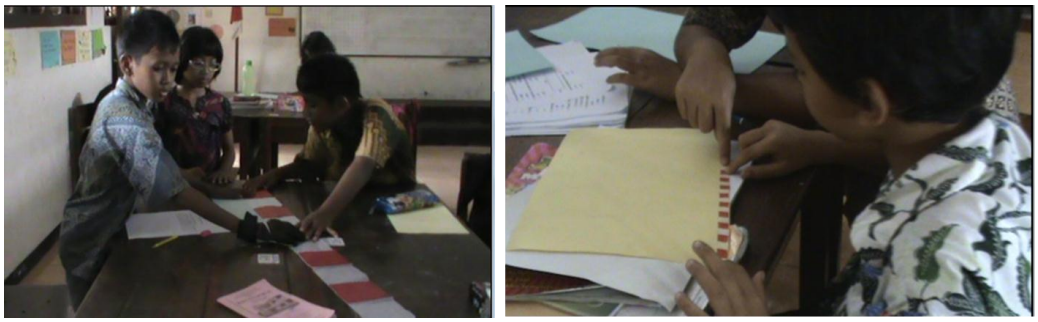
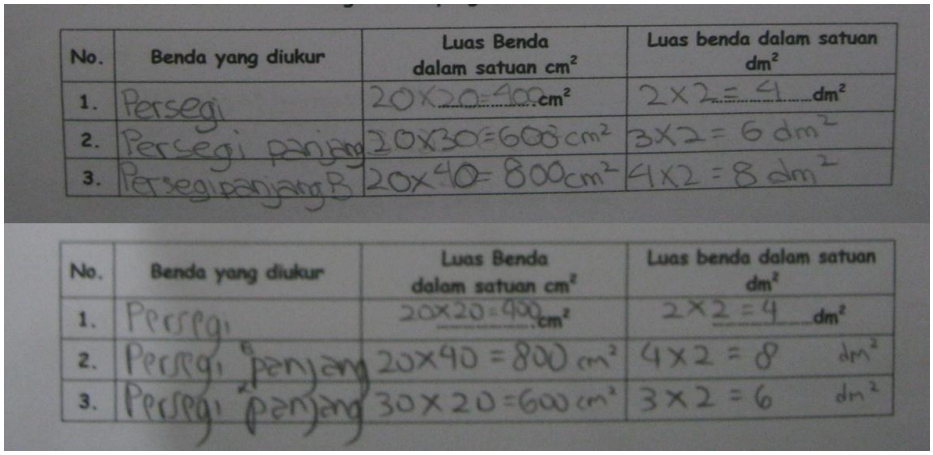


Figure 5.28: The students measured the area of given cardboard

During the measuring activity, all students could measure the area of cardboards by using the strips. They did area measurement just like what they said in the beginning of this activity. They could even measure one row (or column) although the strip of 1 cm^2 could not cover the whole row (or column). All students marked the last position of the strip and then used it as the next start point. They then added the result of those measurements to get the number of 1 cm^2 in one direction.

However, they sometimes confused in filling the table in the worksheet. They could not easily record the result of their measurement because they lost track to collect the data of which cardboard they measured, which strip they used, and what the result of their measurement was. The teacher walked around to help them to do it. Nevertheless, all students finally finished their work and completed the given table with the result of their measurement (Figure 5.29).



No.	Benda yang diukur	Luas Benda dalam satuan cm^2	Luas benda dalam satuan dm^2
1.	Persegi	20X20=400 cm^2	2X2=4 dm^2
2.	Persegi panjang	20X30=600 cm^2	3X2=6 dm^2
3.	Persegi panjang B	20X40=800 cm^2	4X2=8 dm^2

No.	Benda yang diukur	Luas Benda dalam satuan cm^2	Luas benda dalam satuan dm^2
1.	Persegi	20X20=400 cm^2	2X2=4 dm^2
2.	Persegi panjang	20X40=800 cm^2	4X2=8 dm^2
3.	Persegi panjang	30X20=600 cm^2	3X2=6 dm^2

Figure 5.29: The result of area measurement performed by students

Challenging the students, the researcher came to each group and posed a question. The students were asked to look at the ceiling of their classroom. They were then introduced that one square at the ceiling was 1 m^2 and asked to determine the area of the ceiling. All students could give correct response to the question.

According to work of students during this activity, it can be inferred that this introduction fit to the students' achievement so far. The challenge gave by the researcher also proved that the students can be introduced to another units of area measurement. In the next stage, they need to learn the relation between the standard units of measurement and the length units in order to comprehend the shortcut of measuring area (the formula).

5.9 Remarks of the students' knowledge in post-test (2^{nd} cycle)

The researcher gave a written post-test to students after trying out activities in the second cycle were done. The findings of the post-test were also used to support data for drawing the conclusion of the whole learning process during the second cycle. The questions were similar with questions in the pre-test

because it was aimed to investigate the students' achievement after the learning process. The test items in this cycle were similar to the written post-test in the first cycle. The difference was in the last question where the size of the rectangle and the square was not given in the picture.

There were 19 students attended the written test, including three students in the focus group. One student of focus group was absent during the post-test. Therefore, only three students of the focus group were interviewed afterward. According to the results of post-test, the researcher noted some important points as the following. (See appendix I for complete report of the post-test).

5.9.1 The lack of awareness toward identical units in comparing area

Considering the number of students who used the squares around the figures to compare the area, it seemed they were lack of awareness toward identical units. Even in the focus group, only one student who could use the squares, find the area, and then determine the figure with the largest area. Looking through the first activity in this cycle, students' responses of this question may be related to it. In finding the largest tray, they only referred to the larger number of crackers without being aware that the crackers should be the same (see 5.3.1).

In addition, this result may be caused by lack of students' motivation to do this test. In the beginning of the test, most of students were reluctant to do it. They said they already did the same test before. Although the question in the post-test used term *"bigger area"*, the students seemed do not

realize it. It was quite surprising since they were already able to find the area of a given surface.

However, the interview with focus group showed different things. In the written test, two out of three students in the focus group gave incorrect reasoning. One of them said that the longer pool has bigger area, while another gave unclear reasoning. But once they realize the question, they could find the area of each pool and then determine which one was the biggest. Therefore, although students may lack of the awareness toward identical units they may be able to determine the area of each shape and then find the largest one.

5.9.2 The use of multiplicative structure in estimating area

The students' responses for the second question in the post-test showed an improvement from the pre-test. In the post-test they showed that they already built the relation between the structure of arrays and the way of counting the units. Most of them could use multiplication although they showed different ways in perceiving the given structure of arrays. Some of these students completed the drawing while the others completed row by column structure or even drew nothing on the figure.

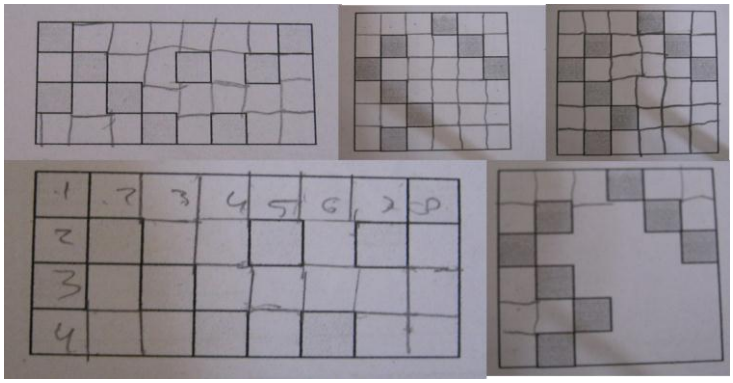


Figure 5.30: The drawing added by students on the given figure

During the interview with focus group, the researcher asked a student who did not draw anything to explain the reasoning. Using a ruler, this student showed how he used the structure of arrays in the figure to find out the number of square units in a row and in a column (See Figure 5.31).

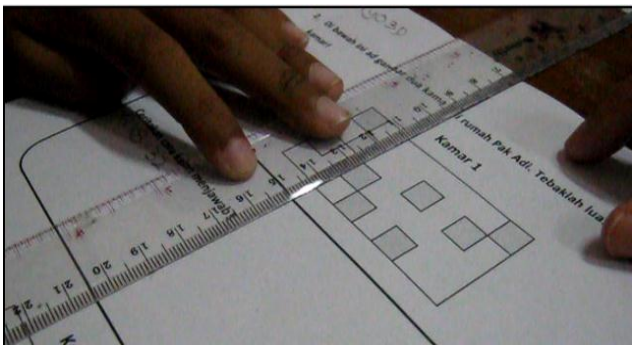


Figure 31: The way students use the structure of arrays to estimate area

This result showed that students' already develop the idea of multiplicative structure. It is inferred from the fact that they used the structure to find out the numbers they should multiply to find the area. Although some students still needed to complete the drawing, they only consider the units in a row and in a column before applying multiplication.

The students' achievement in this test also implies that the series of activities may support students in comprehending the idea of row by column structure. The series of activities was starting from complete covering, decreasing the number of units and then using of strip of square units. This order of the activities may contribute to students' learning process in developing multiplicative structure.

5.9.3 The visualization of area measurement

Eight out of nineteen students could use the strip of 1 cm^2 to measure the area of each given figure. They could get the correct answer by using that tool. However, most of students attempted to draw grid of squares or row by column structure of squares as an aid to measure area. Their struggle led to different results. Some students who drew the square closely congruent to 1 cm^2 got the correct answer, while some other students could not do it. Most of them did not even aware about the congruency of units used in the area measurement. Nevertheless, it showed that they started to develop the idea of partitioning to determine the area of the shape.

In the interview of focus group, it was revealed that the students realize about the incongruence of the square in the grid they made. When they were asked to determine the area by using 1 cm^2 strip, they then realized that the squares used as units of area measurement must be congruent. It implies that they need more support to comprehend this idea.

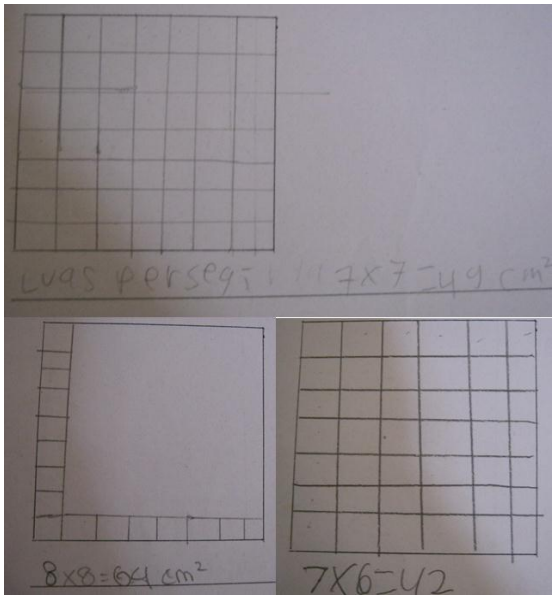


Figure 5.32: Students' effort in drawing the grid

This result was quite remarkable since most of students tried to visualize the process of area measurement. Though their activities were hands on activities, they attempted to make relation between covering activities and the drawing they wanted to make. Since the students did not explore the relation between unit of length and unit of area measurement, this result showed that covering activities served as good start for making such a relation. It also shows that covering activities can be a good start for students in measuring area. Following this start, the students need to learn how to model the covering activity into making grid and then learn the relation between unit of length and unit for measuring area.

CHAPTER VI

CONCLUSION AND DISCUSSION

This chapter contains three main parts; namely, conclusion, discussion, and recommendation. First, it will explain the conclusions that consist of the answer to the research question and a local instructional theory on the learning of area measurement. Second, the discussion will provide information about important issues in this study. The last part of this chapter will elaborate further recommendations for further educational research especially in domain of area measurement.

6.1 Conclusion

6.1.1. Answer to the research question

The main research question of this study is "*How can structuring arrays support students' initial understanding of area measurement for rectangles and squares?"* There are two specific sub questions elaborated in the end of second chapter of this study, namely:

1) *How do third grade students structure two dimensional arrays in covering a surface to measure area?*
2) *How can the structure of arrays support students' development of the multiplicative structure?*

In order to answer the general research question, the sub questions will be answered beforehand. Both sub questions will be answered by summarizing the analysis of all activities in this study as explained in

section 5.8. The first sub question will be more focused on the way of covering; meanwhile the second sub question will elaborate the relation between the way of covering and the idea of multiplication.

The answer of the first sub question

How do third grade students structure two-dimensional array in covering a surface to measure area?

The foundation of structuring the two-dimensional arrays is complete covering without gap and overlap. The context of drying off the crackers can raise these ideas (See 5.8.1). In the first activity, the students put rectangular units in the same direction when covering the surface. They started put the units either from the first column or the first row and so on until all units cover the surface. If the spaces left are not enough to be covered by the units in the same direction, they will put them in another direction. When they are challenged to choose the flexible unit to measure area, they start from any direction and put the rectangular units in different directions.

It was not applied for covering with the square units. Since the direction will remain the same when putting the square units, the students tend to structure the units from any starting point. Some of them –including students in the focus group– put the units starting from both left and right column (or from top row and bottom row) at the same time until the whole surface is fully covered by the units. Some other students structure the

units by row iteration. When students are asked to predict the area of rectangle and square by using limited number of units, they find out the number of square units needed to cover a row and a column before multiply it.

In first cycle, students came up with the idea that the row or the column do not always in the edge of the surface. Surprisingly, they also stated the idea of additivity of area. Since the students in the second cycle did not come with this idea, the researcher tried to discuss it with the focus group. It is confirmed that the area of shape will remain the same either they directly measure the whole or splitting it into two parts and measure each part separately.

The answer of the second sub question

How can the structure of two-dimensional array support students' development of the multiplicative structure?

The relation between structure of arrays and multiplicative structure emerge in a stage of enumerating units. Students come up with different ways of counting. Unlike the conjecture in the HLT, the students only use counting one by one and multiplication. The skip counting is rarely used by the students.

The use of multiplication is started in the second activity when students measure the area of cardboard by using square units. They just know they can multiply but have no reason why they can do it. By using the structure

of arrays, they are supported to recognize the repetition of number of units in rows or columns. Although it is not explicitly discussed in the class discussion, each group of students is facilitated to comprehend the reason behind the use of multiplication. The fragment in 5.8.4 shows students' comprehension toward the relation between the structure of arrays, repeated addition, and multiplication.

Choosing the flexible unit for measuring area, the students are supported by the nature of rectangular units. The students find it difficult to apply multiplication when they put the units in the different direction. However, the use of rectangular units for measuring area and its condition for applying multiplication (the direction must be the same) are not explicitly elicited in the class discussion. Nevertheless, the students are still aware that different direction of rectangular units makes it impossible to use multiplication and they still can use counting one by one to find the area (*see the fragment in 5.8.3*).

The number of units provided for students are extremely decreased so that they need to estimate the total number of units covered the whole surface in finding the area. The students form row by column structure and then use multiplication to predict the area. Such a structure is also being emphasized when the students use strips to measure the area. They tend to put the strips at the edge of the surface in both vertical and horizontal direction. Once they can form row by column structure, they can easily see

the number of units in the different direction to apply multiplication. This structure becomes a kind of shortcut for them in finding the area.

The answer of the research question

How can multiplicative structure support students' initial understanding of area measurement for rectangles and squares?

The structure of two dimensional arrays is based on the idea of complete covering using square units. Experiencing the activity of covering surface with identical units, the students start to develop the idea of partitioning two-dimensional space into equal parts and enumerate the parts as the attribution of area. By using the context of drying off the crackers, complete covering without gaps or overlap is elicited. Although the size of circular units used in the first activity makes the students lack of awareness toward complete covering, the incomplete structure of square units help them to grasp this idea (*See 5.8.2* – the fault of focus group in finding the area of square cardboard).

In activity 2, the complete structure of square units raises the use of multiplication. The students only need to be facilitated in comprehending the reason why they can use multiplication. It can be done by making relation between the whole structure and the repetition of the number of units either in a row or in a column.

Through activity 3, the students experience the comparison between using rectangular units and square units in measuring area. The structure of

rectangular arrays enables the students realize that multiplication can only be done if the units are in the same direction. They choose square unit as the flexible unit for area measurement because they can use multiplication no matter the direction of it.

Providing the limited number of units can support students to form row by column structure in estimating the area. The individual task on estimating the area serves as reinforcement for students in measuring area of two-dimensional shape. They recognize the drawing of two-dimensional arrays. It is shown by their responses namely, completing the drawing, drawing row by column structure, or even mentally imagine the row by column structure (*see 5.8.4* – the exercise). Moreover, once they realize this structure, the strips can reinforce students' thinking in using the shortcut for measuring area. They do not even need to do row or column iteration with the strip (*see 5.8.5 and 5.8.6*). The comprehension of the shortcut in measuring area is the initial understanding for the next stage in area measurement.

From these summaries of the activities, this study has shown that the structuring arrays in the tasks play an important role in developing multiplicative structure. The ability in complete covering, the use of multiplication, the arrangement of row by column structure, and the strip as handy tool for measuring area contribute to this development. In the modeling level, the students can recognize the structure of arrays given in the figures. Although the students are not yet able to make the grid neatly,

they already get the sense of partitioning shape by struggling to draw grid of square in finding the area of two-dimensional shape (*see 5.9.3*).

6.1.2. Local Instructional Theory on learning area measurement

The aim of the present study is contributing to the development of a local instructional theory for area measurement in grade 3 of primary school. According to Gravemeijer and Cobb (2006), a local instructional theory consists of conjectures about a possible learning process and possible means of supporting that learning process. The tool and the contextual activity proposed in the instructional design are summarized in the following table.

Table 6.1: Local Instructional Theory in learning area measurement

Activity	Tool	Imagery	Practice	Concept
Comparing two trays using crackers	Cardboards, circular units, and rectangular unit	Signifies the need of 'third object' to compare area becomes the measuring unit	Indirect comparison, complete covering	The attribution of area
Measuring area of tray by using square crackers	Cardboards and square unit	Signifies the attribution of area by using structure of identical units	Complete covering	The use of multiplication in enumerating the units

Activity	Tool	Imagery	Practice	Concept
Determining flexible unit for measuring area	Square cardboard, rectangular units and square units	Signifies the difference between using rectangular and square units	Determine flexible unit for measuring area	The awareness of the direction of units in applying multiplication
Measuring area without enough number of units	Cardboards, limited number of square units	Signifies the arrangement of units	Estimate the area	The shortcut to determine area
Measuring area using strip of square units	Objects that its surface are rectangle or square, three different sizes of strip	Signifies the structure of arrays in row by column structure	Measure different size of surfaces	The row by column structure
Measuring area using strip of standard units	Cardboards, 1 cm^2 strip, and 1 dm^2	Signifies the introduction of standard units for area measurement	Use standard unit for area measurement	The standard units of area measurement

6.2 Discussion

6.2.1. The weakness points of study

The researcher realized that there are some weaknesses during conducting this study. First, it is related to the preparation of teaching experiment in the second cycle. The interval between the first and the second cycle was very short. As the implication, the researcher could not reflect the

students' learning process in the first cycle optimally for the improvement in the next cycle.

Second, there was lack of communication between the teacher and the researcher. It was very hard to find enough time to elaborate the design together and make a consensus. These factors seem to have significance to the difference between the teacher's interpretations toward the design with the intention of the researcher. As the consequence, the learning processes sometimes a bit different from the design in the HLT.

Third, the use of context in drying off crackers might be not familiar to the students who lived in city generally. It might be recognized by the students in rural area. However, the students involved in this study showed that they could understand the situation by telling them the idea of drying off crackers as one step in making the crackers and providing them the real bamboo tray and also the real crackers. Without neglecting this fact, the researcher suggests the next study should give more preliminary introduction about it. It can be done by providing a video about the cooking process of crackers so that students can really imagine the situation.

Fourth, this research did not elaborate further about the learning styles of the students. The students' learning styles including visual, auditory and kinesthetic might affect the learning process of the students. It might also affect the way of organizing the classroom. Based on the information from interview with the teacher, the students rarely worked in a big group. Most

of the time they learn individually or in a pair with another student. The researcher did not take this as consideration in organizing the class. In the other hand, the tool and learning material were prepared for students to work in groups. The socio-mathematical norm of working in a group was not yet built in this study. As the result, in the teaching experiment it was revealed that students seemed not accustomed to cooperate with their friend in a group. They often worked as two pairs of students.

6.2.2. Reflection on the important issues

The students' thinking and their learning process were not merely the focus of observation in this study. Some important issues were also noted during the implementation of the design. The reflection of these points is described as follows.

- **The concept of multiplicative structure**

  As noted in the theoretical framework, multiplicative structure was an important concept of initial understanding of area measurement. It included the concept of estimating area by using an iteration of a single unit measurement. Unfortunately it was not addressed intensively in this study. It would be better if students were involved in an activity in which they can do such iteration. Teacher may also address this concept by reminded the students how to do iteration in a row, in a column, and in a whole structure that consist of repetition of a column (or a row).

  However, the researcher kept implementing the series of activities that was designed beforehand because the interview in the pre-test showed

that students' were able to do iteration by using two units measurement in estimating area.

- **The use of context**

  In the start of the activities, the context of drying of the crackers can raise the idea of complete covering. Following this context, the context of telling people how large the tray should be highlighted as the support for acquiring the attribution of area. However, this context was not elicited explicitly in the class discussion. The discussion was only about the number of crackers that could cover the whole surface of the tray.

  Nevertheless, although the next activities shifted the term crackers into the square units, some students sometimes still used the term crackers in their written worksheet. It means that some students refer the crackers as the units. It also indicated that some of them were still at the concrete level in learning area measurement.

- **The size of unit measurement**

  During determining the dimension of cardboard, the size of the unit was also considered. The size of rectangular and the square unit fit well on the design. However, the size of circular units in the first activity seems to trigger a problem. The units could not cover the whole surface and led the students to be unaware of complete covering. It was because the circular units were made as similar as the real circular crackers. The

researcher was unaware about the fact that the units would not be fit in the cardboard.

- **The role of teacher and the intervention of the researcher**

  According to the information from interview with the teacher, she had been teaching for 2 years. During the period of this study, it was the first time for her in teaching third graders. She had an anxiety in teaching mathematics because she graduated from the degree of social science. That was why during the teaching experiment, the teacher and the researcher agreed to work together in guiding the students.

  Although it seemed that the researcher did intervention in the experiment, it was not the case. The teacher actually played the main role in the learning process, while the researcher observed and clarified students' strategies or their thinking during the activity. In addition, the teacher was so eager to discuss with the researcher and then made some improvisation during the learning.

6.3 Recommendation for further research

In this study, the researcher only focus on the initial understanding in area measurement by exploring complete covering and structure of arrays. After the series of activities, the students get the sense of visualizing the process of area measurement in the two-dimensional figure. Further research is needed to elaborate the shift from hands-on activity –like in this study– to the more formal way in measuring area of two-dimensional shape. The follow up study

is expected to answer how the students are facilitated to model the partitioning and covering activity into two-dimensional figure. From this, the students can learn how to build the grid neatly.

This study supported students to know two kind of standard unit for area measurement. However, it did not explore the relation between standard unit of area measurement and that of length measurement. It raises the possibility of further study in developing such a relation so that students can use the length as the shortcut to measure area of two-dimensional shape including rectangle and square.

References

Bakker, A. (2004). *Design Research in Statistics Education. On Symbolizing and ComputerTools*. Amersfoort: Wilco Press.

Battista, M. T. & Clements, D.H., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education*, *29* (5), 503 – 532.

Battista, M.T. (2004). Applying cognition-based assessment to elementary school students' development of understanding of area and volume measurement. *Mathematical Thinking and Learning, 6 (2),* 185-204.

Baturo, A. & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, *31*, 235-268.

Cavanagh, M. (2008). Area measurement in year 7. *Reflection, 33* (1), 55 – 57.

Clements, D. H., and Sarama, J. (2009). *Learning and teaching early mathematics: the learning trajectories approach.* New York: Routledge.

Clements, D. H., and Stephan, M. (2004). Measurement in Pre-K to grade 2 mathematics. In Clement, D. H., Sarama, J., & DiBiase, Ann-Marie (Eds.), *Engaging young children in mathematics: standards for pre-school and kindergarten mathematics education, 299 – 317.* Mahwah, NJ: Larence Erlbaum Assiciates, Inc.

Depdiknas. (2006). *Kurikulum Tingkat Satuan Pendidikan Sekolah Dasar*. Jakarta: Depdiknas.

Fauzan, A. (2002). *Applying Realistic Mathematics Education (RME) in Teaching Geometry in Indonesian Primary Schools.* Enschede: PrintPartners Ipskamp.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structure*. Dordrecht: Reidel

Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures.* Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD Beta Press.

Gravemeijer, K., & Cobb, P. (2006) Design research from the learning design perspective. In Van den Akker, J., Gravemerijer, K., McKenney, S., & Nieveen, N (Eds.), *Educational Design Research*. London: Routledge.

Kordaki, M. & Potari, D. (1998). Children's approaches to area measurement through different contexts. *Journal of Mathematical Behavior*. *17* (3), 303–316.

Outhred, L. N., & Mitchelmore, M. C. 2000. Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education*, *31* (2), 144 – 167.

Sarama, J., and Clements, D. H. (2009). *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children*. New York, NY: Routledge.

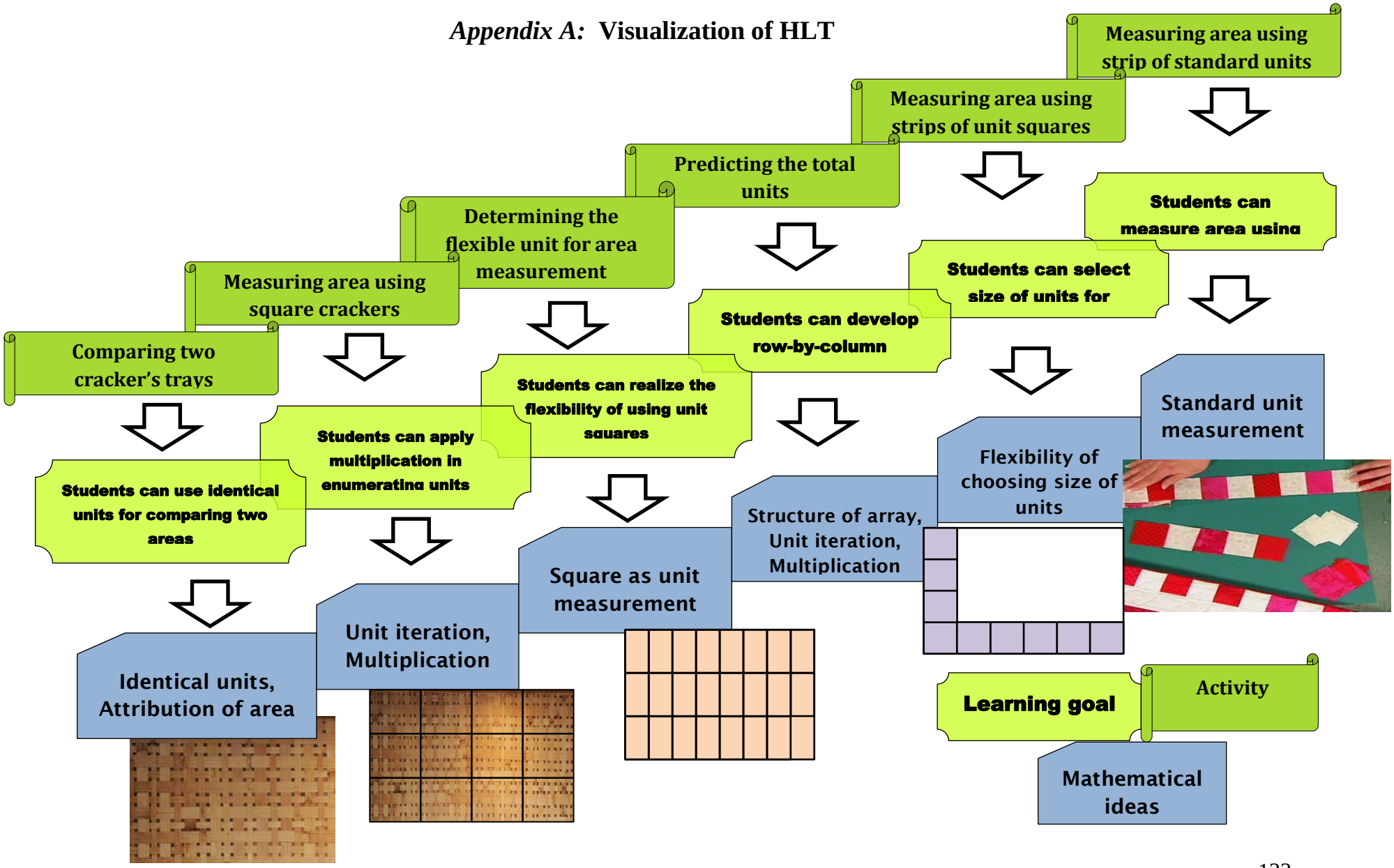
Skemp, R. R. (1987). *The Psychology of Learning Mathematics*. New Jersey: Lawrence Erlbaum Associates.

Wijaya, A. (2008). *Design Research in Mathematics Education Indonesian Traditional Games as Preliminaries in Learning Measurement of Length*. Utrecht University.

Yuberta, Kurnia R. (2011). *Design Research on Developing Unit in Area Measurement For Grade 3 in Indonesian Primary School*. Sriwijaya University.

Zacharos, K. (2006). Prevailing Educational Practices for Area Measurement and Students' Failure in Measuring Areas. *Journal of Mathematical Behavior*. *25*, 224 – 239.

Appendix A: Visualization of HLT



Appendix B

The teacher's interview scheme

Background:

- When do you start to teach in elementary schools?
- In which grade(s) you have experienced in teaching mathematics?
- How long have you been taught the third grade pupils?

Teaching process:

- What is your experience in teaching area measurement for rectangle and square?
- What is the textbook that you use in the third grade?
- What is the difficulty in facilitating the pupils to learn the measurement of area?
- What is the difficulty of the pupils in understanding the concept of area measurement?
- How many meeting do you need to teach area measurement?
- Do the third grade students can draw rectangle and square?
- Have you ever heard about PMRI?
- Do you have any experience about PMRI or teaching with this approach?
- Do you think it is possible to use PMRI as approach in your classroom?

Class organization:

- How do you usually organize the pupils during the lesson?
- (If the teacher organize students in group) How do you organize students to work in pair/groups? What is your consideration to do it?
- Is there any specific rule in the class? (reward or punishment, raising finger, giving turn)
- How do students interact among others?
- What is the consideration of pointing a pupil during the discussion?

Appendix C

The classroom observation scheme

Teaching process

- How does the teacher teach? (mostly explaining or promote the discussion)
- Where does the teacher position? (stand in front of the class or moving around the classroom)
- What is the textbook used by teacher?
- Does the teacher use whiteboard?
- Does the teacher use worksheet other than textbook?
- Is there the discussion of pupils' thinking?
  (Asking the correct answer or explanation)
- Is there any mathematical model used in the lesson?
- Does the teacher give opportunity to students for thinking for a while before giving response?

Classroom organization:

- How do teacher interact with students?
- How do students interact among others?
- Time management of the lesson
- How do the pupils sit in the classroom?
- How do the pupils participate in the lesson?
- Do the pupils work individually or in group?
  (If it is in the group, how many pupils are in the group?)
  (How does the teacher group the pupils?)
- Does the teacher pointing the same pupil all the time?
- Is there any irrelevant behavior during the lesson?

Appendix D

TEACHER GUIDE

LESSON 1

COMPARING TWO CRACKER'S TRAYS

Duration of lesson

2 X 35 minutes

Material

- Cardboard representing two trays with different dimension:
  Tray A (50 cm x 25 cm) and Tray B (40 cm x 30 cm)
- Rectangular paper representing different size of rice crackers:
  Crackers 1 (10 cm x 5 cm) and Crackers 2 (10 cm x 2,5 cm)

Learning goals

Main goal:

- Students understand the use of identical unit as a unit for measuring area (the attribution of area)

Sub goals:

- Students know that measuring area of a shape can be done by covering it with identical units and enumerating the units
- Students can cover the rectangle using units without overlap and leave a gap
- Students can count the number of units covered the shape by using their own strategies
- Students can compare the area of two rectangles by comparing the number of units covered each rectangle

Description of Activity

- Teacher makes group of four students and asks them to sit with their group. Teacher informs students that the groups remain the same for mathematics lesson.

- Teacher tells students about drying cracker under the sun. The teacher then shows two rectangular bamboo trays in different size. One is wider while another is longer. Teacher also shows some raw crackers to the students.
- Teacher tells the students that she needs the largest tray to be used in drying crackers.
- Teacher asks students what the meaning of the largest tray is and holds a class discussion about it. There may be some students who say that the largest tray contains more crackers. If there is no students come with idea of indicating the number of crackers, teacher can pose question as follows.

  "What do you think about the number of crackers that is put on the tray when it is being dried?"
- After students realize that the number of crackers contained in the tray can indicate the area of trays, they will work in a small group to compare the area of trays.
- Teacher gives each group two pieces of cardboard as a representation of the trays. They are also given rectangular papers with two different sizes as a substitution of using real crackers.
- The task are describing the method to compare the trays and telling how large each cracker tray is in order to be compared during the class discussion. It is possible that students will mix the different crackers in one tray; therefore they are not allowed to put different crackers in one tray.

Role of teacher

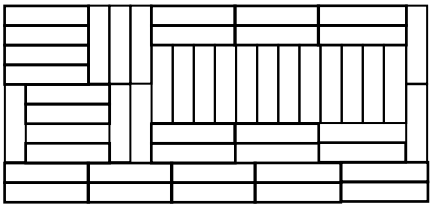
- *During* students' *working time*

  It is important to assure that each group of students does complete covering before they count the units. Some students may put the papers but overlapping each other or even leaving gap among it. If it happens, the teacher can give stimulating question.

  "How can we get the maximum number of cracker contained in the tray?"

  "How can we sure that all crackers are dry enough before it is fried?"

In addition, teacher may found that some students cover the tray with units in a different direction as in the following figure. This kind of covering is acceptable as long as there is no gap or overlap.



- *During class discussion*

  The first point of the discussion is the units used by the students to cover the cardboard. Teacher can asks a group that compares two trays using identical unit and asks them to explain their strategy. Suppose this group of students compares the trays using crackers 1, teacher can asks all students to compare the trays using crackers 2.

  If there is no student answer in this way, it means all group compare the area of trays using different size of units. The possibilities of the area of trays are described in the following table. From the table, there will be two possible answer, either tray A is bigger than tray B (50 is larger than 24) or the inverse (48 is larger than 25). Teacher can ask two groups with different answer to explain their answer.

	Tray A	Tray B
Crackers 1	25 units of crackers 1	24 units of crackers 1
Crackers 2	50 units of crackers 2	48 units of crackers 2

  Teacher can asks the following question to prompt students' thinking.

  "Look at the crackers, what do you think about the size of them? Do those crackers have same size?"

  "Do you have an idea how can we determine which tray is bigger so that we can get the same answer?"

The next discussion will be focused on students' strategies in enumerating the crackers. Teacher can give appreciation for all strategies of counting but he/she should emphasize the efficiency of the strategy. For instance, teacher tells students that skip counting is faster than counting one by one.

Closing the discussion, teacher should refer to the "real tray". After the discussion come to a conclusion which tray is larger, teacher must give a statement which bamboo tray is larger than another. Teacher need to emphasize the reason by asking the students why it is larger than another. It is expected that students answer because it contains more crackers.

- *Closing the lesson*

  Teacher review the activity and asks students about what they already done today. Teacher can also refer to the activity by posing the following question.

  "How can we find out the larger tray?"

  "What should we do to compare it?"

  Teacher can reformulate students response by emphasizing that determining the larger tray is the same with comparing area.

TEACHER GUIDE

LESSON 2

HOW LARGE IS THE TRAY?

(Measuring area of tray by using square crackers)

Duration of lesson

2 X 35 minutes

Material

- Cardboard representing two shapes of trays
  Rectangular tray (40 cm x 30 cm) and Square tray (40 cm x 40 cm)
- Square paper representing the crackers: 10 cm x 10 cm

Learning goals

Main Goal:

- Students can use multiplication as the efficient way in enumerating the number of unit covered the shape

Sub goals:

- Students can determine the number of units in a row and in a column
- Students can determine total number of units by multiplying the number of units in a row with the number of rows
- Students can determine total number of units by multiplying the number of units in a column with the number of column
- Students can determine total number of units by multiplying the number of units in a row with the number of units in a column

Description of Activity

- Teacher shows a bamboo tray that is covered by some square crackers to the students. Teacher tells them that the raw crackers will be fried for a special moment. However, teacher needs more tray as a place for more crackers. The carpenter who made the tray asks the teacher how large the tray is in order to make another one.

- Teacher then asks them the way they can inform the others about how large the tray is. It is sufficient if students state that the area of the tray can be indicated by the number of crackers on the tray. Teacher can remind them about determining the largest tray in the previous activity if the students do not have an idea how to it.
- The teacher then gives each group of students a cardboard as substitution of tray. The first cardboard given to students is the rectangular cardboard. Teacher also distributes some square papers as substitution of crackers.
- The students are asked to determine how many crackers that can be put in that size of tray. They are also asked to give explanation about their method in counting the squares.
- The second cardboard will be distributed to students after the class discussion as an exercise for students.

Roles of teacher

- *During students' working in a group*

  While students working, teacher needs to make sure that all group do complete covering. If students count the unit one by one, teacher can also motivate them to find the easier (or faster) way to determine the number of units.
- *During class discussion*

  If students come with different ways of enumerating units, teacher can asks groups with different method to explain their solution. Teacher then ask the students which method is easier than the other.

  If there is no students apply multiplication but use skip counting, teacher can promote students' thinking by asking the following question.

  "How many crackers are in one row?"

  "How many rows do we have?"

  "Do you have an idea how to count the crackers quickly?"

  Teacher can also promote students' thinking about the structure of units in a column.

In the second discussion after students work with square cardboard, teacher can give more emphasize to multiplication as the easier way to enumerate the units. Teacher can reformulate students' statements about their strategy in an appropriate reasoning as follows.

"There are … crackers in a row and … rows in the tray, therefore there are … crackers on the tray"

"There are … crackers in a column and … column in the tray, therefore there are … crackers on the tray"

"There are … crackers in a row and … crackers in a column, therefore there are … crackers on the tray"

Each time the discussion come to the conclusion about the total number of crackers, teacher needs to refer to the context. Teacher can ask students how large the tray is. Teacher should reformulate students' statement in an appropriate way such as follows.

"The tray can be covered by … crackers"

- *Closing the lesson*

Teacher informs students that determining how large the tray using means measuring the area of tray using crackers. Teacher review the activity today by asking students about what they have done. Teacher emphasizes students' statement by reformulating students' response like in the following statement.

"The area of rectangular tray is …"

"The area of square tray is …"

TEACHER GUIDE
LESSON 3
SQUARE OR RECTANGULAR CRACKERS?
(Determining the flexible unit for measuring area)

Duration of lesson

2 X 35 minutes

Material

- A cardboard with size 50 cm x 50 cm
- Square papers 10 cm x 10 cm and rectangular papers 10 cm x 5 cm

Learning goals

The aim of this activity is supporting students in realizing the efficiency of using square as a unit measurement.

Main goal:

- Students can determine the most efficient unit for area measurement

Sub goals:

- Students can measure the area of rectangle and square using rectangular units
- Students can measure the area of rectangle and square using square units
- Students know that multiplication can be easily applied if the unit used for area measurement is square

Description of Activity

- Teacher reminds students about the previous activities by asking them to explain what they have done.
- Teacher distributes a cardboard and the units (both rectangular and square units).
- Teacher asks students to measure the area by using those different units.

Role of teacher

- *During students work in a group*

  While students working, teacher needs to make sure that all group do complete covering. If students count the unit one by one, teacher can also motivate them to use the easier (or faster) way to determine the number of units like they did in the previous activities.

- *During the class discussion*

  Before teacher start the discussion about the shape of units, teacher review how students measure the area. It is expected that students can explain that area measurement can be done by doing complete covering of shape with units.

  Teacher can ask students who put the rectangular crackers in the different directions to explain their method in counting the rectangular units. Teacher can promote their thinking by asking, *"What should we do if we want use multiplication in finding the total number of units?"*

  "How should we arrange the rectangles if we want use multiplication?"

  If all students already put the rectangle in the same direction, teacher can intrigue them by rearranging the rectangles in the different direction. Teacher then asks them, *"Can we still apply multiplication if we put the rectangles in the different direction? Why do or why not?"*

  Teacher then asks a group of students to explain their method while measuring the cardboard using square unit. Teacher can asks the whole class, *"Can we still apply multiplication if we put the square in the different direction?"*

  After the discussion come to a conclusion, teacher can tell students that square unit is more flexible to be used in measuring area. Teacher then recheck students' understanding by posing the following question.

  "Can we measure area by using rectangular unit?"

  "Why do we agree that using square is easier than using rectangle in covering the shape?"

- *Closing the lesson*

  Teacher review the activity today by emphasizing the flexibility of using square units.

TEACHER GUIDE
LESSON 4
MEASURING AREA WITHOUT ENOUGH AMOUNT OF UNITS

Duration of lesson

2 X 35 minutes

Material

- A cardboard with size 50 cm x 40 cm
- Square papers 10 cm x 10 cm

Learning goals

Main goal:

- Students are able to develop row-by-column structure of the units

Sub goals:

- Students are able to determine the total number of units needed to cover the whole shape if they area only provided by limited number of square
- Students can understand that multiplication can be used to predict the total number of units

Description of Activity

- Teacher tells the students that he/she needs to know the area of a tray, but she only has limited square units to cover it.
- Teacher gives each group of students a cardboard and limited number of square units.
- The task is predicting the area of the cardboard (as substitution of the tray) using some square unit. The units given are only sufficient to cover a row and a column of the shape.
- As additional activity after class discussion, the students are also given a worksheet which contain incomplete square covering a shape. The task remains

the same; determine the number of square needed to cover the whole shape. This task is individually.

Roles of teacher

- *During students work in a group*

  Teacher should emphasize the concept of area measurement by asking students what the area means in a term of covering shape with unit. Teacher should also highlight the meaning of prediction of the area. Students must understand that they need to find total number of units, instead of the remaining units needed to cover the shape. Teacher can promote this thinking by asking students to imagine how many units can the shape.

- *During class discussion*

  Teacher can appreciate any students' strategy in iterating the units. However, teacher should choose a group with the lowest level of strategy occurred in the classroom (either iterating one by one or iterating by row/column). Teacher then promote their thinking to find the easier way of doing it. Teacher can also directly promote the use of multiplication by asking students, *"Can we use a multiplication to predict the total number of units?"*

  If all students already come up with using multiplication, teacher can emphasize the structure of array. Teacher can ask students the possible arrangement of units so that they can easily find the numbers to be multiplied.

  After the discussion come to the conclusion, teacher should review the result of the measurement. Teacher need to emphasize that the total number of square units indicates the area of the shapes. Teacher can pose a question like, *"What do the total number of units tell us?"*

  In discussing the individual work, teacher can do the same method.

- *Closing the lesson*

  Teacher review the activity today by asking the students what they have done during the lesson.

  "How can we measure area if we don't have enough unit squares?"

"How can we arrange the units (or draw it) if we want to apply multiplication?"

Teacher can reformulate students' response to draw conclusions of the lesson.

TEACHER GUIDE
LESSON 5
MEASURING AREA USING STRIP OF SQUARE UNITS

Duration of lesson

2 X 35 minutes

Material

- Three different strips of square papers: 1-cm-square strip, 5-cm-square strip, 10-cm-square strip

Learning goals

Main goal:

- Students are able to measure area by using strip of unit squares

Sub goals:

- Students can choose an appropriate size of unit square to measure different sizes of shape
- Students can apply multiplication in predicting the total number of units

Description of Activity

- Teacher gives three different strips of square paper to each group of students. One is called small strip, another is medium strip, and the other one is big strip.
- Students are asked to measure the area of three different shapes in their classroom. The surface of object must be rectangle or square. Each shape must be measured by using one type of strip.

The Role of teacher

- *During students work in group*

  Teacher should make sure that students measure the area and do not measure length. Teacher can promote students to think about the previous activities.

Teacher can also ask students to imagine how many strips need to cover the shape, and draw a conclusion how many square units on the shape.

- *During the class discussion*

  Teacher can choose one group of students for each measurement that use each type of strip. Teacher need to emphasize students' method in measure the area. Teacher should also pay attention to the use of multiplication. If needed, teacher can ask students to draw a picture that represent their measurement. For instance, teacher asks students to draw the direction they lay down the strip and asks them to explain how they can apply multiplication. Each time students explain their solution, teacher can reformulate the explanation in order to highlight the important aspects. These aspects include the way of putting the strips in such a way the multiplication can be applied. It also include that measuring using strip means "imagining" a row (or a column) of unit squares.

  Teacher also needs to discuss how to choose the strip. Teacher can ask students with a question like, *"Which strip that we should use if we want to measure the surface of blackboard?"*

  Teacher should also promote students' thinking that they still can measure the larger surface with the small strip, but it will be difficult.

  After the discussion come to the conclusion, teacher need to refer back to the problem. For instance, teacher can make a list of the object that is measured by students. It is important to note that objects in classroom mostly three dimensional shapes, therefore teacher must explain that in fact students measure the surface of objects.

- *Closing the lesson*

  Teacher review the activity today by asking students what they have already done. Teacher should emphasize to the meaning of measuring area. It is expected that students can explain area measurement using strip as predicting how many strip can cover the shape. In turn, it leads to the finding of the number of square units that can cover the shape.

TEACHER GUIDE

LESSON 6

MEASURING USING STRIP OF STANDARD UNITS

Duration of lesson

2 X 35 minutes

Material

- A cardboard with size 30 cm x 30 cm
- 1 cm^2 strip and 1 dm^2 strip

Learning goals

Main goal:

- Students are able to measure area using standardize unit

Sub goals:

- Students can measure the area of surface by using a strips of centimeter square
- Students can measure the area of surface by using a strips of decimeter square

Description of Activity

- Teacher gives each group of students a cardboard and two different strips of standardize unit square (i.e. in decimeter square and centimeter square). Teacher informs students that these two standard units of measurement are commonly used by people to communicate about the area of shapes to others.
- Teacher asks students to measure the area of the cardboard by using those different strips.

The Role of teacher

- *During students work in a group*

  Teacher must be aware of students' confusion between a standard for length measurement and for area measurement (cm and cm^2 as well as dm and dm^2). Teacher should emphasize that the standard unit for measuring area is square unit. If it is needed, teacher can give brief explanation about the difference

between those standard units. Teacher can refer the standard units of area measurement to the unit square from the previous activity.

Teacher needs to always keep students to think about the concept of area measurement. Teacher can promote this thinking by asking the following question in a group of students.

"How do you measure the cardboard?"

"How do you determine the number of units?"

Teacher can ask students to visualize their solution in a drawing to make clear how they put they strip on the cardboard. Teacher should remind students to state the result of their measurement in standard units (either cm^2 or dm^2).

- *During class discussion*

  Teacher can ask a group of students to present their solution. The focus of the discussion is explaining the way of measuring the cardboard. Teacher should emphasize that measuring area is covering the whole. Teacher should also promote students' thinking that strip of unit square is an aid to make the measurement easier.

  Teacher should also aware of students' understanding toward the measurement. Since the strip is an aid, teacher can refer the area measurement to the covering activity. Students can be asked to explain how the strip can cover the entire shape (keyword: imagine). It can be done by asking students to draw their solution. The drawing can be include the way of putting strips on the shapes as well as completing the drawing with unit square.

  Students should be aware of the use of standard units. In this case, the must state the result of their measurement in a standard unit. Teacher can also promote students' thinking in finding the relation between cm^2 and dm^2. In the case of the cardboard, teacher can ask students which standard unit is easier to be used to measure the area.

  Teacher can also give additional problems about finding the area of rectangle and square using the strip in a worksheet. Students' answer may vary as they can measure the shape using both standard units. In discussing students'

individual work, teacher can still emphasize the concept of area measurement, the use of multiplication, and the statement of the result of the measurement.

- *Closing the lesson*

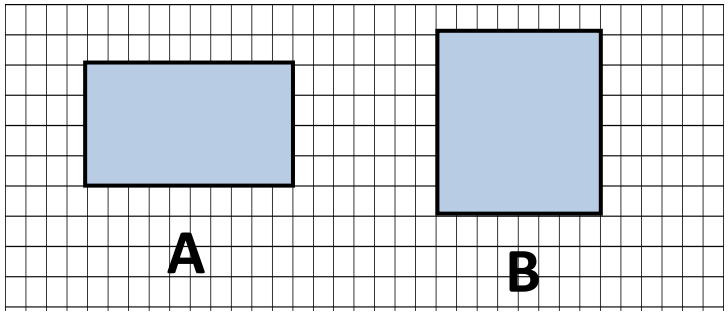
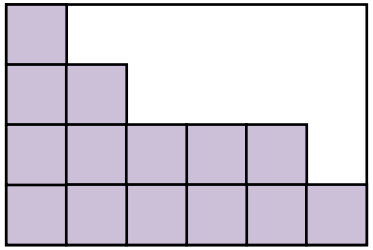
  Teacher review the lesson by asking the following the question.

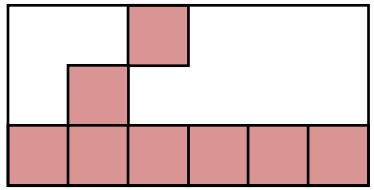
  "How can we measure the area of the cardboard?"

  "How can we determine the number of units that cover the cardboard if we only have the strip?"

  Teacher can also reformulate students' statement to highlight the important aspect in the lesson.

Appendix E: The analysis of pre-test in the first cycle

No.	Question	Response
1.	Given the following picture, students are asked to determine which pool is bigger. 	All students said that pool A is bigger. Five of them build their comparison on the number of squares outside the pool. Among these five students, there is one student who refers the square outsides the shapes as length and width. Another student said that pool A is bigger because it is a rectangle (longer than the other pool.)
2.	Given the picture of tiling process in the room of pak Adi's house, students are asked to estimate the total number of tiles that can cover the whole room. 	Response to the first picture: All students give the correct answer, 24 tiles. Two students apply multiplication. One of these students needs to draw some lines, while the other does not draw anything to the picture. The other two students draw lines to get the view of whole room and then count the squares one by one. One student uses repeated addition (4 + 4 + 4 + 4 + 4 + 4 = 24) without draw anything. Without the aid of additional lines, the last student counts the tiles that are already there, and then add it to the remaining tiles (14 + 10 = 24).

		Response to the second picture: All students give the correct answer, 18 tiles. The first two students use multiplication. However, both of them draw two squares in the left side of the room in order to know how many rows in the room. The other two students keep completing the room with square tiles, and then count it one by one. One student uses repeated addition with drawing nothing (6 + 6 + 6 =18). It is quite different from what she does in the first picture. In this picture, she counts the squares horizontally. The last student also keep doing her way by counting the tiles in the room and add it to the remaining tiles needed to cover it.
Interview		
3.	Given a figure of rectangle and two square units. Students are asked to estimate the total number of units that can cover the rectangle.	*First student:* She perceives one row contains three squares after she put two squares side by side. She then moves the squares down to get the next row. However, she stops and then assumes that there are three rows more below by moving his hand. She says the units needed are 15. *Second student:* She finds out the number of units in a row, figure out the number of units in a right column, and then applies multiplication to get twelve.

		Third student: In the beginning, she moves each square in left and right column to do iteration. After the researcher asks her to explain her method, she does iteration one by one. Her final response is 12. *Fourth student:* At first, she does iteration by using two squares along the left column, bottom row, right column, and then comes back to the first row. She says the total units needed are 20 with doubt. When the researcher asks clarification, she does iteration in the left column and bottom row and then applies multiplication. *Fifth student:* He does iteration along the left column and bottom row. He then answers surely 12. The researcher asks how he gets 12, he then responses that it comes from three times four. *Sixth student:* He cannot do the iteration correctly because he keeps moving the unit without being aware that there is overlapping. He says the total units needed are 10.

Appendix F

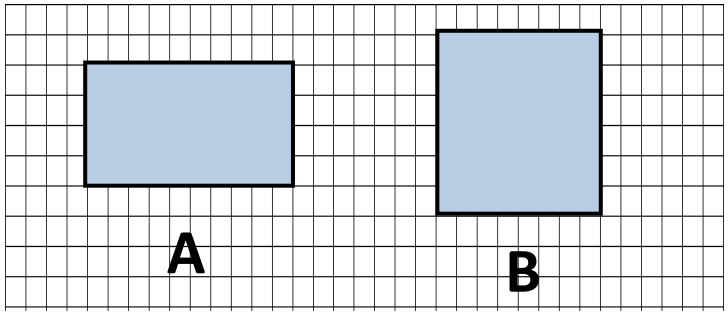
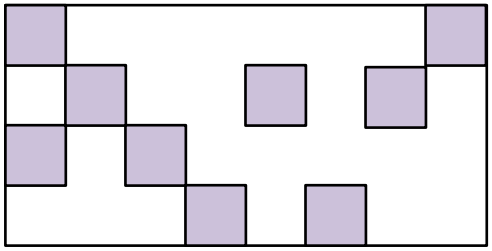
HLT 1 as refinement of the initial HLT

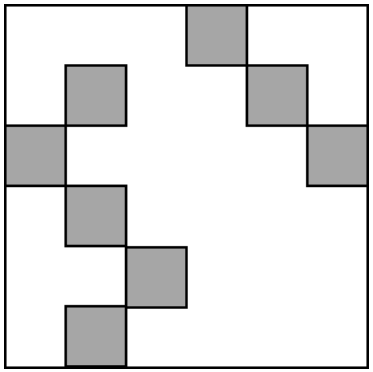
Lesson	Activity of initial HLT	Refinement of activity (HLT 1)	Rationale behind the refinement
1.	**Comparing two trays using rectangular units (crackers) with different sizes.** Tray A: 50 cm x 25 cm Tray B: 40 cm x 30 cm Crackers 1: 10 cm x 5 cm Crackers 2: 10 cm x 2,5 cm	Comparing two trays if the units given are rectangular and circular. The dimension of the tray is same. The rectangular crackers: 10 cm x 5 cm The diameter of circular crackers: 7,5 cm	Due to the pre-test results, students are still unaware of the need of third object to compare the area. It is expected that the idea of identical unit can be emerged from the use of different types of crackers.
2.	**Measuring area of tray by using square crackers.** Students are given two trays; one is rectangular while the other is square. The task is to measure the area of each shape by using square crackers. Rectangular tray: 40 cm x 30 cm Square tray: 40 cm x 40 cm Square crackers: 10 cm x 10 cm	The activity is not change. However, the dimension of square units used in the measurement is changed into 5 cm x 5 cm.	Based on the students' ability in doing iteration of units, the use of large square will give less challenge to them. Since the aim of this activity is to develop multiplicative structure, the number of units needs to be added. If the original number of units remains the same, students can use addition and probably cannot see the need of using multiplication. In addition, the researcher find a real (almost) square cracker which has length approximately 5 cm.

3.	**Determine the flexible unit for measuring area.** Students are given square cardboard which has length 50 cm. They are also given two types of unit measurement. One is rectangular (10 cm x 5 cm), and the other is square (10 cm x 10 cm).	The activity remains the same, but the dimension of the square unit becomes 5 cm x 5 cm.	As the follow up of second activity and for practical reason, the dimension of unit square in this activity remains the same with the previous activity.
4.	**Measuring area with insufficient number of units.** Students are given a cardboard with size 50 cm x 40 cm and square for unit measurement (10 cm x 10 cm). The task is estimating the area of the cardboard if they only have limited number of units. The unit measurement given to the students can be used to cover a column and a row.	There will be to cardboard given to students. The first one is a square with size 40 cm x 40 cm, while the other is a rectangle with dimension 30 cm x 40 cm. The unit measurement given to the students is a square with size 5 cm x 5 cm. The number of units given is ten units. It can cover a row and a column of the rectangle, but it does not apply the same for the square cardboard.	Two different shapes of cardboard can sharpen students' understanding of measuring different shapes. In the worksheet, measuring the square cardboard is in the first order because it is more challenging. The second measurement serves as help if students stuck in their first work. It is easier since the units are sufficient to cover a column and a row.
5.	**Measuring area using strip of square units.** Students are given strips of unit square with three different sizes. They are asked to measure the area of rectangular and square objects around	The researcher prepares anticipation for this activity. The anticipation is preparing two cardboard, a rectangle with size 20 cm x 20 cm and a square 20 cm x 15 cm. These cardboard will be used if the students	The anticipation is based on the fact that students in this grade are active.

	them.	could not find appropriate object to be measured.	
6.	**Measuring area using strip of standard units.** Given strip with two standard units (strip of cm^2 and dm^2), students are asked to measure a cardboard. The size of the cardboard is 30 cm x 30 cm.	The size of cardboard will be adjusted based on the previous activity.	Students can connect this activity to the previous one since formalize the standard unit is not easy for children.

Appendix G: The analysis of post-test in the first cycle

No.	Question	Response
1.	Given the following picture, students are asked to determine which pool has **bigger area**. 	All students said that pool A is bigger. One student draws grids inside two shapes and states the number of square units in each shape. Four students draw row-by-column structure of square units inside each shape. One student draws nothing but counting squares beside each side. This student signifies the "length" of each side by writing a number around it.
2.	Given the picture of tiling process in the room of pak Adi's house, students are asked to estimate the area of each room. 	**Response to the first figure:** Two students signify the square unit in the top row and left column with dots. However, one of them miscalculated the units as 4 x 7 while another can get correct answer. The other two students complete the square units inside the rectangle and then apply multiplication correctly. One student also uses multiplication but she only need to complete the squares in the bottom row. The last students only signify the number of units in a row and in a column by writing a number beside each side. Like the others, this student uses multiplication to determine the area.

		**Response to the second figure:** All students get the correct answer for this figure. One student signifies units in a row and a column by using dots and then applies multiplication. Three students complete the drawing with square units. Two of these students use multiplication while another one counts the units one by one. There is one student draw row-by-column structure before use multiplication. The last student draws nothing but write numbers to signify the number of units in a row and a column. This last student also applies multiplication.
3.	Given a figure of rectangle and square. Students are asked to determine the area of each shape. The length and the width of each shape were given in the picture. The size of rectangle is 9 cm x 6 cm, while the length side of square is 7 cm.	All students use multiplication to determine the area of each shape. However, five of them prefer to use a cm^2 strip as tool for measurement. Only one student who can draw cm^2 units in a row-by-column structure to determine the area of shapes.

Appendix H

The analysis of pre-test in the second cycle

Question No.	Number of students	Responses
1.	1	This student wrote pool A was bigger than pool B. The reasoning was because pool A= 20 x 10 (it should be 10 x 5) and pool B = 8 x 6 = 48. This student seemed to get an idea of using identical units.
	4	The students give a correct answer, pool A. However, their reasoning is based on **perimeter as the reference** of bigger pool. One of these students measured the perimeter of each shape by using ruler and stated it in cm. Another student also clearly stated the length of each side in cm although did not state the total perimeter. The other students only gave a statement that the length of side in pool A was longer than pool B.
	3	These three students answered pool A is the bigger one because **its shape was a rectangle**.
	8	All of these students answered pool A is bigger than pool B. However, **their reasoning was quite unclear**. Some of them seemed to count the squares around the shape but did not give clear explanation.
	2	These students answered that pool B than pool A. They did not give clear explanation about their answer.
2.	3	These students gave the correct answer for the figures, 24 and 18 tiles respectively. They completed the drawing, counted the squares in the original drawing, and then added it to the remaining tiles they drew.

	12	These students interpreted the question as “the remaining tiles needed to cover the whole”. Their answer was ten tiles more for both figures. Five of these students completed the squares inside each figure, while the other did not do that.
	4	These four students gave unclear answer.
3.	**Interview with focus group**	
		Two out of four students used two squares to do iteration along a column and a row. They then applied multiplication. Another student did one-by-one iteration and still got the correct answer. Although the last student got the correct answer, she got confused in doing iteration. This student just moved around two square randomly.

Appendix I

The analysis of post-test in the second cycle

Question No.	Number of students	Responses
1.	3	The students can **use the square units around the figure** as reference to compare the area of pools. One of these students is from the focus group. He clearly stated the area of each pool and then wrote that pool A has bigger area.
	3	The students give a correct answer, pool A. However, their reasoning is based on **perimeter as the reference** of pool that has bigger area. One of these students measured the perimeter of each shape by using ruler and stated it in cm. Another student also clearly stated the length of each side in cm although did not state the total perimeter. The other students use the squares around each figure to determine the perimeter instead of using it to find the area of each figure.
	2	The students answered pool A is because **its shape was a rectangle**.
	3	The students answered pool A is bigger than pool B because pool A is **longer** than pool B
	8	All of these students gave correct answer. However, **their reasoning was quite unclear**. Some of them wrote multiplication like 7 x 10 or 6 x 6 without clear information how they got the numbers. Some other students just gave statements like pool A is bigger because B is not big, etc.
2.	16	The students gave the correct answer for the figures, 32 and 36 tiles respectively. Two out of these students

		completed the drawing and used counting one by one to find the area of each figure. Six students who also completed the drawing used multiplication in figuring out the area. The other two students only completed the drawing for a row and a column before they applied multiplication. The last six students did not draw anything and directly applied multiplication.
	3	The students did not finish their work. However, two of them already completed the drawing.
3.	4	The students drew a grid-like on each given figure. The grid contained square-like which size was close to 1 cm^2. One student even only drew a row and a column of square unit.
	8	The students used 1 cm^2 to measure the area of each figure.
	4	The students drew line or seemed in the process of drawing grid. However, they gave clear reasoning or explanation.
	3	The students draw grid-like inside each figure, but since the square-like in the grid was not close to the size of 1 cm^2 they got wrong answer.