DESIGN RESEARCH ON ADDITION:
DEVELOPING MENTAL CALCULATION STRATEGIES ON ADDITION UP TO 20

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:

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GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

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STATEMENT PAGE

I hereby:

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State that:

1. All the data, information, analysis, and the statement in analysis and conclusion that presented in this thesis, except from reference sources in the result of my observation, research, analysis, and view with the guidance of my advisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

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ABSTRACT

Many prior studies revealed that most of young students tend to perform a counting strategy when they solve addition problems up to 10. This strategy is not longer useful when we want students to perform more abbreviated strategies to solve bigger addition problems. Meanwhile, many studies in Indonesia show that teachers directly teach students a standard algorithm without sufficient understanding of this algorithm. For this reason, we design a research in order to develop instructional activities on addition up to 20 that can support students to develop mental calculation strategies in learning addition up to 20 in grade 1 of primary school. In this design, we use structures that can support students' thinking process in developing mental calculation strategies such as making ten and using doubles. The students have to construct number relationships and big ideas such as doubles and combinations that make ten. Meanwhile, the research method which was used in this research is called design research, or development research, based on Realistic Mathematics Education (RME). The research conducted in SDN 179 Palembang, Indonesia on January – April 2011.

Keywords: mental calculation strategies, structure, number relationships, doubles, and combination that make ten.

ABSTRAK

Banyak penelitian sebelumnya menunjukkan bahwa sebagian besar siswa cenderung menyelesaikan masalah penjumlahan bilangan kurang dari 10 dengan cara menghitung satu-satu. Cara ini ini tidak lagi selamanya efektif apabila siswa dihadapkan dengan masalah penjumlahan bilangan yang lebih besar. Siswa diharapkan mengembangkan cara yang lebih efisien dan fleksibel. Sementara itu banyak penelitian di Indonesia menunjukkan bahwa guru secara langsung mengajarkan algoritma penjumlahan kepada siswa tanpa pemahaman yang cukup tentang algoritma tersebut. Dengan alasan ini, kami merancang sebuah penelitian untuk mengembangkan kegiatan pembelajaran tentang penjumlahan bilangan sampai 20 yang dapat mendukung siswa untuk mengembangkan strategi perhitungan secara mental di kelas 1 sekolah dasar. Dalam penelitian ini, kami menggunakan benda-benda yang terstruktur yang dapat mendukung proses berfikir siswa dalam mengembangkan strategi perhitungan secara mental seperti menjumlahkan ke bilangan 10 dan menggunakan bilangan kelipatan. Siswa diharapkan untuk mengkonstrak ide-ide tentang hubungan antar bilangan, bilangang kelipatan, dan pasangan bilangan berjumlah sepuluh. Sementara itu, metode penelitian yang digunakan dalam penelitian ini disebut penelitian desain, atau penelitian pengembangan, berdasarkan Pendidikan Matematika Realistik. Penelitian ini dilaksanakan di SDN 179 Palembang, Indonesia pada bulan Januari - April 2011.

Kata kunci*: strategi perhitungan secara mental, benda yang terstruktur, hubungan antar bilangan, bilangan kelipatan, dan pasangan bilangan sepuluh.*

SUMMARY

Several studies which analyze students' performances in addition focus on strategies students use to solve addition problems. One finding is that most of young students tend to perform a counting strategy when they solve addition problems. This strategy is not longer useful when the students have bigger addition problems. They have to develop more abbreviated strategies. Meanwhile, many studies in Indonesia show that teachers directly teach students a standard algorithm without sufficient understanding of this algorithm. For this reason, we design a research in order to develop instructional activities on addition up to 20 that can support students to develop mental calculation strategies in learning addition up to 20 in grade 1 of primary school. We use structures as that can support students' thinking process in developing mental calculation strategies such as making ten and using doubles. The students have to construct number relationships and big ideas such as doubles and combinations that make ten.

The approach used in this research was Realistic Mathematics Education (RME) that provides ideas on which mathematics should always be meaningful to students. The students are challenged to experience mathematics when they solve meaningful problems because mathematics is a human activity. In this research, we used *pempek Palembang* as a rich and meaningful contextual situation that can be the basis for developing understanding on mental calculation strategies on addition. From this situation, the students moved to use models and symbols for progressive mathematization. The circle representation of *pempek* and arithmetic rack serve as model of situation that later change into model for mathematical reasoning. The interactivity among students and also students and a teacher supports the learning process to shorter strategies in developing mental calculation strategies.

The research methodology used to get the data in this study was a design research. Actually there are three phases in a design research: preliminary design, teaching experiment, and retrospective analysis. In preliminary design, a hypothetical learning trajectory about developing mental calculation strategies on addition up to 20 was designed consisting three components: learning goals for students, mathematical activities, and hypothesis about the process of the students' learning. In teaching experiment, the hypothetical learning trajectory was tested and improved to the next teaching experiment. Then, the data were analyzed through retrospective analysis. Actually, the teaching experiment was conducted in SDN 179 Palembang, Indonesia that was divided into two parts. Part I was done with a small group, 5 students in the period of January to February 2011. Part II was done with 27 students in the period of March to April 2011.

In design the Hypothetical Learning Trajectory (HLT), some potential contextual situations were brought out in classroom activities. The first idea is to find combinations that make ten. The students started with a contextual situation that is making combinations of ten *pempek* which have two different kinds of *pempek*. There is a mini lesson I, parrot game, that still relate to develop understanding about combination that make ten. After that students work on a candy combination sheet to build knowledge about decomposing numbers up to ten. The next activity is flash card game. This game is used to support students' knowledge about number relationships up to ten. The learning trajectory is continued by exploring numbers up to 20. There are two main goals. The first one is that students are able to decompose numbers up to 20. There were two activities called "hiding monkey picture sheet" and "mini lesson II, parrot game". The second one is to support students' development of number relationships up to 20. The activity is called "exploring numbers using an arithmetic rack". In last activity students do some addition problems.

Those activities were tried during the preliminary experiment activity with a small group of students, 5 students, in the first grade. The result showed that the students needed to build the big ideas, doubles, compensation, part/whole relationship, and combinations that make a ten, in order to develop mental calculation strategies on addition up to 20. For the beginning of the lesson, many students still counted the objects, *pempek Palembang*, one-by-one. They synchronized one word for every object. They would change their strategy when they were asked other ideas to know the number of objects. The other finding was that the students gave reason using knowing facts not the structures of objects. This happened because the students were influenced by the previous activity that was decomposing number up to 10. In the lesson of hiding monkey picture sheet, the students decomposed numbers up to 20 in many pairs. This made students memorized some number pairs up to 20 and influenced them when they solved addition problems. Some students would not come to use making tens to solve addition problems up to 20 instead knowing the number facts. For the teaching classroom experiment, the hiding monkey picture sheet activity was changed into arranged beads in structuring ways and giving a worksheet .In the last lesson, the students worked with the arithmetic rack so that they showed it on structuring ways and come to some strategies such as using doubles, using the five-structure and making ten.

In the teaching experiment, the students started to find a combination of ten *pempek* by taking 5 *lenjer pempek* and 5 *bulet pempek.* This meant that they were familiar using the five-structure to find a combination that makes ten. Some groups of students used this combination to find other combinations of ten *pempek* without experiencing with real objects. Meanwhile the other groups of students always worked with real objects, the wax *pempek,* so they found some similar combinations and drew on the poster papers. In the parrot game, the students were able to find many combinations that make ten mentally. They were also able to represent combinations that make ten using the arithmetic rack. It seemed that they were able to recognize structures on the arithmetic rack. In the second lesson, some students gave reasons based on structures that they showed on flash cards, but the other students reasoned based on their knowledge on combinations that make a number. The third lesson was that the students were challenged to decompose numbers up to 10. The students were able to find many combinations for some numbers up to 10. They used fingers, arithmetic rack, and knowing mentally those combinations. In the mini lesson, parrot game, many students were able to find combinations of numbers up to 10 mentally. Meanwhile few students still needed fingers to represent that numbers.

The next lesson focused on number up to 20 and addition problems. In the fourth lesson, the students were challenged to build awareness of structures. They preferred to use five and ten-structures that were showed by their arrangement on making jewelry. During the fifth and sixth lesson, the students were able to recognize the structures of beads on the arithmetic rack. They knew that was five and ten-structures. Actually, not all students reasoned based on the structures on the arithmetic rack instead known number relationships. Throughout the last lesson, many students performed the big ideas of combinations that make ten. They also used doubles for near doubles and known facts to solve some addition problems. However, there were few students still used counting on strategy. Sometimes, they changed their strategies based on problems they had to solve. In generally, the students could use the big ideas such as doubles and combinations that make ten to perform mental calculation strategies such as making ten to solve addition problems up to 20.

RINGKASAN

Beberapa penelitian yang menganalisis kemampuan siswa dalam menyelesaikan masalah penjumlahan memfokuskan penelitianya terhadap cara siswa menyelesaikan masalah penjumlahan tersebut. Salah satu penemuan bahwa sebagian besar siswa kelas 1 SD cenderung menyelesaikan masalah penjumlahan dengan menghitung satu-satu. Cara ini tidak begitu mendukung ketika siswa menyelesaikan masalah penjumlahan bilangan yang lebih besar. Mereka harus mengembangkan carai yang lebih efektif. Sementara itu, banyak penelitian di Indonesia menunjukkan bahwa guru langsung mengajarkan algoritma penjumlahan kepada siswa tanpa pemahaman yang cukup oleh siswa tersebut. Untuk alasan ini, kami merancang penelitian dalam rangka mengembangkan kegiatan pembelajaran dalam penjumlahan bilangan sampai 20 yang dapat mendukung siswa untuk mengembangkan strategi perhitungan secara mental dalam belajar penjumlahan bilangan tersebut.

Adapun pendekatan yang digunakan dalam penelitian ini adalah Pendidikan Matematika Realistik (lebih dikenal dengan PMRI) yang menyatakan bahwa matematika adalah sesuatu kegiatan yang bermakna bagi siswa. Para siswa diberi kesempatan untuk mendapatkan pengalaman matematika ketika mereka memecahkan masalah karena matematika merupakan aktivitas manusia. Dalam penelitian ini, kami menggunakan pempek Palembang sebagai masalah matematika yang kaya dan bermakna yang dapat menjadi dasar untuk mengembangkan pemahaman mengenai strategi perhitungan secara mental dalam penjumlahan bilangan. Dari situasi ini, para siswa mengembangkan kemampuan memodelkan dan menggunakan simbol untuk mathematization yang progresif. Linkaran yang merepresentasikan pempek dan dekak-dekak berfungsi sebagai *model of* dari situasi yang kemudian berubah menjadi *model for* untuk penalaran matematika. Interaktivitas antara siswa dan juga siswa dan guru mendukung proses pembelajaran tentang cara yang lebih efektif dalam mengembangkan perhitungan secara mental.

Metodologi yang digunakan untuk mendapatkan data dalam penelitian ini adalah *design research*. Ada tiga tahapan dalam *design research* yaitu: *preliminary design, teaching experiment*, dan *retrospective analysis.* Pada tahap *preliminary design*, sebuah *Hypothetical Learning Trajectory* tentang cara perhitungan secara mental dalam menyelesaikan penjumlahan bilangann sampai 20 dirancang yang terdiri dari tiga komponen: tujuan pembelajaran bagi siswa, kegiatan matematika, dan hipotesis tentang proses belajar siswa. Kemudia ditahap *teaching experiment*, *Hypothetical Learning Trajectory* diuji cobakan dan kemudian diperbaiki untuk *teaching experiment* berikutnya. Setelah itu data yang diperoleh dianalisis pada tahap *retrospective analysis.* Untuk memperoleh data maka *teaching experiment* dilaksanakan di SDN 179 Palembang, Indonesia yang dibagi menjadi dua tahap. Tahap I dilakukan dengan 5 orang siswa siswa pada bulan Januari-Februari 2011. Tahap II dilakukan dengan 27 siswa pada bulan Maret-April 2011.

Dalam merancang *Hypothetical Learning Trajectory (HLT)* maka beberapa situasi kontekstual dibawa kedalam kegiatan pembelajaran. Ide pertama adalah menemukan pasangan bilangan berjumlah sepuluh. Para siswa mulai dengan situasi kontekstual yaitu menemukan susunan sepuluh pempek yang terdiri dari dua macam. Kemudian kegiatan permainan dengan burung beo yang masih berhubungan dengan mengembangkan pemahaman tentang pasangan bilangan yang berjumlah sepuluh. Setelah itu siswa bekerja pada lembar kerja siswa untuk mengembangkan pengetahuan tentang pasangan bilangan sampai sepuluh. Kegiatan selanjutnya adalah permainan kartu. Permainan ini masih digunakan untuk mendukung pengetahuan siswa tentang hubungan bilangan hingga sepuluh. *HLT* dilanjutkan dengan pemahaman bilangan sampai 20. Ada dua tujuan utama

yaitu: (1) siswa mampu menguraikan angka sampai 20. Ada dua kegiatan yaitu LKS menyembunyikan monyet" dan permainan burung beo. (2) mendukung pemahaman siswa tentang hubungan bilangan sampai 20. Kegiatan ini disebut "mengeksplorasi bilangan menggunakan dekak-dekak". Dikegiatan terakhir siswa menyelesaikan beberapa masalah yang berhubungan dengan penjumlahan.

Semua kegiatan yang sudah dirancang diuji cobakan terlebih dahulu dengan 5 orang siswa kelas 1 SD. Hasil penelitian menunjukkan bahwa siswa mmbutuhkan pemahaman tentang ide matematika mengenai bilangak kelipatan, hubungan antar bilangan, dan pasangan bilangan yang berjumlah sepuluh, dalam rangka mengembangkan cara perhitungan secara mental sampai 20. Pada awalnya, beberapa siswa masih menghitung pempek satu-satu. Kemudian mereka mengubah cara mereka ketika mereka diminta untuk menemukan cara yang lebih efektif untuk mengetahui jumlah pempek tersebut. Temuan lainnya adalah bahwa para siswa menggunakan pemahamannya tentang pasangan bilangan ketika memberikan jawaban dalam permainan kartu. Dalam mengerjakan LKS tentang menyembunyikan monyet, siswa menemukan beberapa pasangan bilangan sampai 20. Hal ini membuat siswa menghafal beberapa pasang bilangan hingga 20 dan mempengaruhi mereka ketika menyelesaikan penjumlahan bilangan. Akibatnya beberapa diantara mereka tidak menggunkan cara menjumlahkan ke bilangan sepuluhan dalam memecahkan masalah penjumlahan sampai 20. Maka pada *teaching experiment* berikutnya, kegiatan ini diubah menjadi menyusun manik-manik dan mengerjakan LKS yang berhubungan dengan dekak-dekak. Dalam pembelajaran terakhir, para siswa menggunakan susunan pada dekak-dekak sehingga memungkinkan mereka untuk menggunakan pasangan bilangan berjumlah sepuluh dan bilangan kelipatan.

Dalam *teaching experiment*, para siswa mulai menemukan susunan sepuluh pempek dengan mengambil 5 pempek lenjer dan 5 pempek telor. Ini berarti bahwa mereka telah terbiasa menggunakan bilangan berstruktur lima untuk menemukan pasangan bilangan berjumlah sepuluh. Dari susunan ini maka siswa menemukan pasangan bilangan yang berjumlah sepuluh lainya. Dalam permainan burung beo, siswa mampu menemukan pasangan bilangan berjumlah sepuluh secara mental. Dalam pembelajaran kedua, beberapa siswa memberi alasan berdasarkan pada susunan yang ditunjukkan pada kartu, tetapi siswa yang lain beralasan berdasarkan pengetahuan mereka tentang pasangan bilangan. Pelajaran ketiga adalah siswa menguraikan bilangan sampai dengan 10. Mereka mampu menemukan banyak pasangan bilangan yang jumlahnya sampai dengan 10. Mereka menggunakan jari, dekak-dekak, dan mengetahui secara mental.

Kegiatan selanjutnya memfokuskan pada pembelajaran bilangan sampai 20 dan penjumlahan bilangan. Dalam pembelajaran keempat dan kelima, siswa ditantang untuk membangun kesadaran akan susunan bilangan. Mereka lebih memilih untuk menggunakan susunan bilangan lima dan sepuluh yang ditunjukkan oleh pengaturan mereka pada penyusunan manik-manik. Selama pelajaran kelima dan keenam, para siswa mampu mengenali susunan manik-manik pada dekak-dekak. Mereka mengetahui itu sebagai susunan lima-lima dan sepuluh. Pada pembelajaran terakhir yaitu penjumlahan bilangan, banyak siswa menggunakan ide matematika tentang pasangan bilangan yang berjumlah sepuluh dan menggunakan bilangan kelipat. Namun, ada beberapa siswa masih menggunakan perhitungan satu-satu. Beberapa siswa menggunakan cara yang lebih fleksibel berdasarkan masalah penjumlahan yang mereka temui. Secara umum dapat disimpulkan bahwa siswa dapa menggunakan ide-ide matematika seperti bilangan kelipatan dan pasangan bilangan sepuluh untuk menyelesaikan masalah penjumlahan secara mental.

"Fainna talaba al-'ilm faridhat 'ala kulli muslim"

{Studying is obligatory upon every Muslim}

=Prophet Mohammed S.A.W=

I would like to dedicate this thesis with truly and great love to all people supporting me and staying beside me.

My special dedication is my mother, Alimah, and my father, Abdul Hamid. I wish they will stay in love forever.

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I wish this thesis, as the result of my research, will give contribution to improve the mathematics education in Indonesia, especially in the learning mental calculation strategies on addition up to 20 in grade 1 of primary school.

Palembang, Mei 2011

Zetra Hainul Putra

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CHAPTER I

INTRODUCTION

Several studies which analyze students' performances in addition focus on strategies or methods students use to solve addition problems (Adetula, 1996; Blote, Klein, & Beishuizen, 2000; Nwabueze, 2001; Saxton & Cakir, 2006; Torbeyns, Verschaffel, & Ghesquiere, 2004). Torbeyns et al (2004, 2009), for instance, found that students with the highest mathematical ability use more different strategies than students with the lowest mathematical ability. These last frequently use counting on strategies to solve addition problems. This strategy is not accurate and effective anymore when objects become larger because they need more time to synchronize between thinking and objects. Conobi et al (2002) also found that students who use advanced counting strategies such as decomposing one addend and then recombining the resulting numbers in a new order do not have a better understanding in solving addition problems. It means that they still have struggles to understand a principle that larger sets are made up of smaller sets. Thus, interventions are needed to be designed to help students to recognize patterns in the way in which objects can be combined progressing from a concrete to more abstract.

In traditional teaching-learning methods in Indonesia, teachers provide students a procedure or standard algorithm without students' understanding the underlying concepts (Armanto; 2002). Although students are able to solve addition problems by using algorithm, it is not meaningful practice for them. Meliasari (2008) found that Indonesian students in the first grade were not able to give reasons how the algorithm for addition works. They just said that the teacher taught algorithm to solve those problems. In other case, Sari (2008) also found that students at the second grade performed algorithm without sufficient understanding of numbers, so there are misconceptions of the students in doing the procedure. Some teachers argue that by learning algorithm students can solve problems easily. This indicates that mathematics is for these teachers a set of procedures which students should memorize and apply by rote whatever an operation.

However, there is a niche between students' development of understanding mathematical concepts and an algorithm which is taught formally at school. Teachers in Indonesia teach students the algorithm of addition directly after they learn addition up to 10 by counting. Students are taught to use the algorithm by adding ten and ones separately (*figure 1.1*). They do this procedure without understanding of place value.

$$\begin{array}{rl} 14 & \\ \underline{\;\;3} & + \\ 17 & \end{array}$$

Figure 1.1: An algorithm on addition

This algorithm does not support students' understanding of solving addition problems. Students need a bridge to move forward from counting to more flexible and effective strategies. Therefore, the realistic mathematics education offers an opportunity to change the traditional teaching-learning method in Indonesia. In this approach, students get opportunities to share their ideas in solving addition problems in a classroom discussion so that they can construct mathematical concepts on addition based on their understanding.

The aim of this research is to develop instructional activities on addition up to 20 by structures that can support students to develop mental calculation strategies in learning addition up to 20 in grade 1 of primary school. Consequently, the central issue of this research is formulated into the following main research question:

How can students develop mental calculation strategies on addition up to 20 in grade 1 of primary school in Indonesia?

We specify that research question into some sub research questions as follows:

1. What big ideas and strategies do students learn on addition up to 20?
2. What are the differences between the students in learning addition up to 20?
3. How does the lowest level of reasoning from students look like?
4. What are roles of the teacher to bring students to higher level strategies, mental calculation strategies on addition up to 20?
5. Which students can reach the highest level of strategies?

CHAPTER II

THEORETICAL FRAMEWORK

This chapter provides the theoretical framework that underlies the groundwork of this research. Some studies on addition were studied to identify the mathematical concepts that are required to develop mental calculation strategies in solving addition problems. Moreover, those studies were used in designing instructional activities about addition in which a learning process was taught starting from students' thinking and linking it to their daily life activities.

The real life contextual situations that are related to students' experiences were exploited as experience-based activities to build students' thinking and reach mathematical goals in learning addition. Therefore, some literature about realistic mathematics education was used to explain and investigate how mathematical thinking was build from the contextual situation starting and leading towards the more formal mathematics (Gravemeijer, 1994).

2.1 Mental calculation strategies on addition and big ideas

Mental calculation strategy is insightful calculation with mental rather than written representations of numbers (van den Heuvel- Panhuizen, 2001). This means that the students solve problems with a flexible strategy based on their abilities. Mental calculation strategies are different from a procedure or a standard algorithm because students construct their own strategies based on problems purposed.

Counting one by one is a basic of mental calculation strategies for students to solve addition problems. Firstly, they use counting-all procedure to solve problems such as five plus two (Sarama & Clements, 2009). They count out a set of five items, then count out two more items, and then count all those, and if they do not make a mistake than the answer is seven. After that they learn to perceive small amounts, such as two, three, or four, can often be seen as whole (subitize), so they do not need to count each quantity of objects. Hence, they construct a big idea, part/whole relationships, to develop more abbreviated strategies such as counting on strategies (Fosnot & Dolk, 2001; Fuson, 1988; Hughes, 1986).

Strategies, like counting on, and big ideas, like part/whole relationships, are important landmarks in the landscape of learning (Fosnot & Dolk, 2001). Without understanding these landmarks, students will use counting all when they solve addition problems. Counting on is a difficult strategy for students to construct because they almost have to negate their earlier strategy of counting from the beginning, but the construction of this strategy may bring about an understanding of part/whole relationship (Fosnot & Dolk, 2001).

When students have an understanding of part/whole relationship, they can develop other mathematical ideas. One of important mathematical ideas is doubles (4+4, 5+5, 6+6, etc) because it is a basis of other facts (Fosnot & Dolk, 2001). When doubling a number, students can count by two (i.e., two fives are equivalent to five twos). It underlies the relationship between odd and even numbers and also an important step in mathematical development.

The big idea of part/whole relationship also underlies the knowledge of all combinations that make ten and the subsequent strategy of making tens for addition (Fosnot & Dolk, 2001). Knowing the combinations that make ten is critical if we want students to be able to solve problems by making ten and then adding ones. Otherwise they will just use counting on strategy. For example, to solve problem like 7 + 4 by making a ten is 7 + 3 and adding one.

In the present study, we design a sequence of instructional activities to solve addition problems up to 20. The design focuses on the mental calculation strategies on working with doubles and making ten. In order to be able to use both strategies, students need to develop the big ideas of doubles, and combinations that make ten. To bring up these big ideas, we need to support students by structures that are built into a contextual situation.

2.2 Structures and calculation by structuring

The term of structuring is informed by Freudenthal, and his successors. He approved that doing mathematics consist of organizing phenomena into increasingly formal or abstract structures (Freudhental, 1991; Treffers, 1987). He proposed that students learn mathematics by structuring rather than forming concepts that get a grip on reality.

Ellemor-Collins et al (2009) argues that structuring is an activity that begins with content, experienced as realistic or common sense, and organizes it into more formal structures. In particularly, they argued that structuring numbers means organizing numbers more formally: establishing regularities in numbers, relating numbers to other numbers, and constructing symmetries and patterns in numbers. For example, consider a student adds 5 and 7 who first makes 10 from 5 + 5 and then uses a known fact that 10 and 2 more is 12. The student is structuring the numbers around 10 as a reference point: organizing the numbers and the operation by realizing that two fives make 10, and by using the regularities of numbers to add 2 to 10. Structuring numbers involves developing a rich network of number relations (Ellemor-Collins & Wright, 2009). Important structuring of numbers includes making doubles and combinations using 5 and 10 as referential. In this study, structuring is the operation of organizing, composing, and decomposing objects in a regular configuration to support mental calculation strategies.

Structures can be used to assist development of students’ ability of counting and arithmetic through conceptual subitizing (Clemment, 1999). In line with this idea, Steffee and Cobb (1988) suggested to use structures to support students develop abstract numbers and arithmetic strategies. For instance, the students often use fingers to solve addition problems because they are very familiar with structures on their fingers. The finger structures are able to support students to do more abbreviated mental calculation strategies.

In Realistic Mathematics Education (RME), a learning and teaching trajectory on addition up to 20 moves from calculation by counting, through calculation by structuring, to formal calculation (Treffers, 2001). In calculation by counting students can be supported where necessary by counting material such as blocks and fingers. To develop non-counting based calculation by structuring students can be helped by suitable models such as an arithmetic rack. In formal calculation students use numbers as mental objects for smart and flexible calculation without the need for structured materials.

In particularly, Treffers (2001) argued that numbers up to 20 are represented by means of three different structural models: a line model, such as establishing predecessor and successor numbers in the number sequences and recognizing 10 as a reference point in the sequence; a group model, such as grouping and splitting into doubles, fives, or ten. One example of the group models involves *tallying* – a skill that is closely linked with counting; and a combination model, a combined line and group model, such as the arithmetic rack – a variation on the traditional abacus.

In the present research, we use the combination model to support students' learning on structures up to 20. After that the structures are used to support mental calculation strategies on addition up to 20. Therefore, the instructional activities designe in the present study involved structural models to support students' thinking on number relationships up to 20.

2.3 Realistic Mathematics Education

Realistic Mathematics Education (RME) provides ideas on which mathematics should always be meaningful to students (Freudenthal, 1991). The term 'realistic' means that a problem situation must be experientially real for students. This does not indicate that the problem situations are always encountered in their daily life. An abstract mathematical problem can be a real for students when that problem is meaningful for them. Moreover, they are able to experience mathematics when they solve meaningful problems because mathematics is a human activity (Freudenthal, 1991). In Realistic Mathematics Education students get many opportunities to construct their own understanding. They are challenged to develop strategies in solving problems and discuss with other students. Therefore, the instructional activities on addition up to 20 are designed based on the tenets of Realistic Mathematics Education (RME) to guide students to develop their mathematical thinking from situational activities to formal mathematics.

2.3.1 Five tenets of realistic mathematics education

The five major learning and teaching principles that lie at the basis of Realistic Mathematics Education (RME) will be described in the context of the learning strand for addition. Those tenets defined by Treffers (1987, 1991) are described in the following way:

1. *Phenomenological exploration.*

   A rich and meaningful contextual situation should be used as the base of mathematical activities that can be the basis for developing understanding on addition. The contextual situation, making combinations of ten *pempek*, is used in this particular research as a starting point. Students have to find combinations of ten *pempek* which have two different kinds of *pempek*.

2. *Using models and symbols for progressive mathematization.*

   Models and symbols are used to bridge the gap between a concrete and abstract level. A variety of these can support students' thinking in the learning process, provided the models are meaningful for the students and have the potential to make generalizations. Therefore, making a drawing of combinations of ten *pempek* on a poster paper can serve as *model of* situation that can bridge from using the real objects as the concrete level to using a mathematical symbol as the formal level in solving addition problems. We also use the arithmetic rack (Treffers, 1991) as a *model for* students to develop their' thinking process in learning addition up to 20.

3. *Using students' own constructions and productions.*

   The learning of mathematics is promoted through students' own constructions and productions that are meaningful for them. Students are free to design their own strategies that can lead to the emergence of various solutions which can be used to develop the next learning process. The discussion among students which is guided by a teacher will support the process of reinventing mathematics in a shorter way than how it was invented in the history. Freudenthal (1991) used the term *guided reinvention* to name this process. During the activities and classroom discussions, students create their own constructions, for instance, make their own combinations of ten *pempek* and discuss their productions to figure out all combinations of ten *pempek*.

4. *Interactivity.*

   Interactions among students and also students and a teacher support the learning process to shorter strategies because they can express ideas and solutions of the given problems (Cobb & Yackel, 1996). They can learn from each other in small groups or in whole-class discussions. We assume a discussion in a group will build a natural situation for social interaction such as finding combinations of ten *pempek*, decomposing numbers up to 10 in a candy combination sheet, and exploring number up to 20 using an arithmetic rack. After that, the class discussion will provoke students to be able to negotiate to one another in an attempt to make sense of other's explanation.

5. *Interwinement.*

   An instructional sequence of learning process should be considered to relate one domain with other domains (Bakker, 2004). The integration domains will help students to learn mathematics in more effective way, for example learning addition and subtraction can be done simultaneously because subtraction inverses of addition.

In addition to these tenets, Realistic Mathematics Education (RME) also offers a principle for designing in mathematics education that is emergent modeling (Gravemeijer, 1994). We describe it in the following section.

2.3.2 Emergent modeling

Based on the second tenet of Realistic Mathematics Education (RME) about models and symbols for progressive mathematization, a sequence of models needs to be developed to help students from formal to informal mathematical activity. Students should be given an opportunity to reinvent mathematics by experientially real for the students (Gravemeijer & Stephan, 2002). In the case of learning addition, the students start from an activity that involves real objects such as *pempek Palembang*. After that they make a representation that serves as model of situation. This process can be characterized as emergent modeling.

Emergent modeling is one of the core heuristics for instructional development in Realistic Mathematics Education (RME). Gravemeijer (1994, 1997) describes how *models-of* a certain situation can become *model-for* more formal reasoning. The following figure describes the levels of emerging modeling.

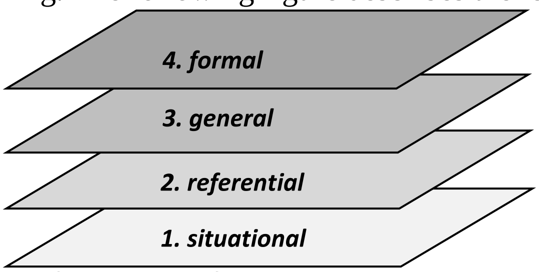


Figure 2.1: The four levels of mathematical activities in emergent modeling

These levels of emergent modeling in this research can be described as follows:

1. Situational level

   In this level domain specific, situational knowledge and strategies are used within the context of the situation (mainly out of school situation). In this study, making combinations of ten *pempek* is the real contextual situation which is taken into classroom activities that relates to students' daily life activities. This activity aims to find combinations that make ten.

2. Referential level

   Referential level is the level of *model-of* where models and strategies refer to the situational which is sketched in the problem (mostly post in a school setting). The making representations of combinations of ten *pempek* encourage students to move from situational level to referential level. In this research, *the mpek-mpek representations* will serve as *model-of* situations.

3. General level

   General activity, in which *model-for* enable a focus on interpretations and solutions independently of situation-specific imagery. Students make general representations such as *circles* and an *arithmetic rack* becomes *model-for* situation. In this level, *the circles and the arithmetic rack* are independent from students' thinking in a real contextual situation.

4. Formal level

   Reasoning with conventional symbolizations, which is no longer dependent on the support of *model-for* mathematical activity. Students move from general representation into formal mathematics notation. In this level, they use *numbers* as mental objects for flexible calculation without the need for structured materials.

CHAPTER III

METHODOLOGY

This chapter describes the methodology which was used to reach the goals and answer the research question. There are four issues discussing in this chapter: (a) design research, (b) data collection, (c) data analysis, and (d) validity and reliability.

A. Design research

The research methodology that we used in this study was a design research. Design research or also known as developmental research is aimed to develop theories, instructional materials and empirical grounded understanding of how the learning process works (Bakker, 2004; Drijvers, 2003; Gravemeijer, 1994). Bruner (in Drijvers, 2003) said that the main objective of design research was understanding and not explaining. This objective implies that understanding how the learning process is done in a classroom activity is a core of design research. Therefore, we used the design research to design instructional activities to develop a local instructional theory and to know students' thinking process about addition up to 20. In the following, we describe three phases of conducting a design research. (Gravemeijer & Cobb, 2006).

1. Preliminary Design

   In this phase, we studied some literatures about addition, realistic mathematics education, and designed research to support in designing a learning trajectories. After getting some knowledge, we tried to formulate a hypothetical learning trajectory consisting three components: learning goals for students, mathematical activities, and hypothesis about the process of the students' learning (Simon, 1995; Simon & Tzur, 2004). We tried to find contextual situations that could be meaningful for Indonesian students and discussed these with supervisors who are experienced in designing for mathematics education. We also adjusted some activities that had used in a previous research, and made conjectures about the learning process that could happen in a classroom.

2. Teaching Experiment

   The aim of the teaching experiment was to test the hypothetical learning trajectory and improved the conjectured learning trajectory. During this phase, we collected data such as classroom observation, teacher and students' interview, field note, and students' work. In this design research, the teaching experiment was done in two cycles, so we revised the hypothetical learning trajectory after the first circle. We got more information about students' learning process and improved the local instructional theory. Hence, the commutative cyclic process of this research to improve a local instructional theory describe by figure below.

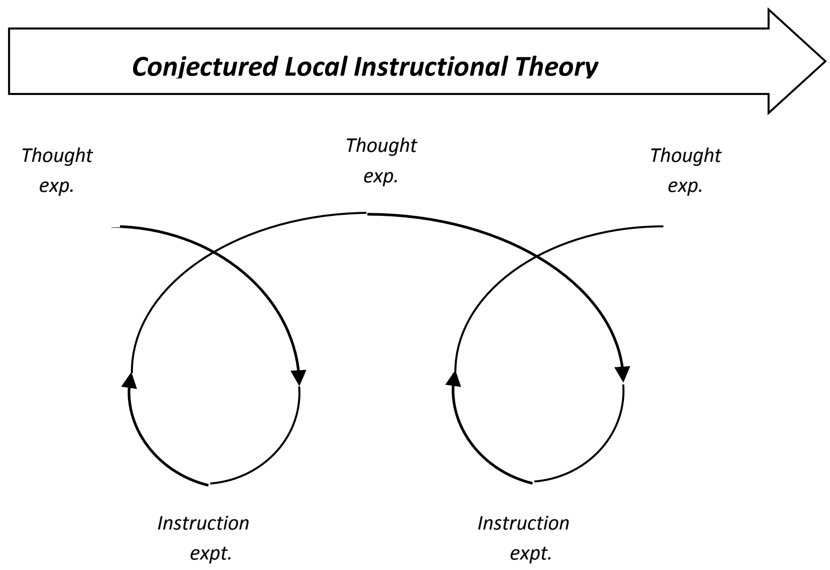


   Figure 3.1: A cumulative cyclic process

3. Retrospective Analysis

   In this phase, we analyzed the data that we got during the teaching experiment. The hypothetical learning trajectory from the first cycle was used to compare with students' actual learning. As a result, we revised the next hypothetical learning trajectory and redesigned instructional activities. After that, the second cyclic was done and analyzed the data from the teaching experiment. The result of the retrospective analysis contributed to the local instructional theory and gave an evaluation to improve the initial hypothetical learning trajectory.

B. Data collection

The research had been conducted in SDN 179 Palembang, Indonesia. We took data from a class in the first grade. The experimental class consisted of 27 students at the age 7 to 8 years old. We divided the experiment into two parts. Part I had been done with a small group, 5 students that were different from the whole class students, in the period of February 2011. We investigated students' knowledge and tried out the activities of the hypothetical learning trajectory. The second part of this research had been done with the whole class in the period of March to April 2011. At this time, we revised the hypothetical learning trajectory and test the improved HLT.

The data had been collected in both periods trough observing the classroom activities, interviewing the teacher and students, collecting students' work, and making field notes. We used two cameras to record students' activities during the lesson. A camera was a static camera that recorded the whole class activity, and the other one was a dynamic camera that recorded a specific activity such a group discussion. Photos had been taken during the classroom activities. We represent the outline of data collection in the following timeline:

Table 3.1: The outline of data collection

Date	Activities	Data Collection	Goals
Preliminary Design			
October – December 2010	Studying literatures and designing initial HLT		
January 2011	Discussion with teachers	Interviewing and field notes	Communicating the designed HLT
	Classroom observation	Pre-Assessment,	Observing students' current

		interview, and video recording	knowledge of addition up to 20, finding socio norms, and socio-mathematical norms.
Teaching Experiment I			
01 February 2011	Lesson I (Making combinations of ten *pempek*)	Video recording and students' work	Finding combinations that make ten
02 February 2011	Mini Lesson I (Parrot game) and Lesson II (Candy combination sheet)	Video recording and students' work	Decomposing numbers up to 10
05 February 2011	Lessons III (Flash card game)	Video recording	Building knowledge about number relationships
08 February 2011	Lesson IV (Hiding monkey picture sheet)	Video recording and Students' work	Decomposing numbers up to 20
10 February 2011	Mini Lesson II (Parrot game) and Lesson V (Exploring numbers up to 20 using the arithmetic rack)	Video recording	Understanding about number relationships up to 20
12 February 2011	Lesson VI (Addition up to 20)	Video recording and students' work	Solving addition problems up to 20
Revising HLT			
February – March 2011	Redesigning HLT		A new HLT that called HLT II
Teaching Experiment II			
29 March 2011	Lesson I (Making combinations of ten *pempek*)	Video recording and students' work	Finding combinations that make ten
30 March 2011	Mini Lesson I (Parrot game) and Lessons II (Flash card game)	Video recording and students' work	Building knowledge about number relationships
02 April 2011	Lesson III (Candy combination sheet) and Mini Lesson II (Parrot game)	Video recording and students' work	Decomposing numbers up to 10
04 April 2011	Lesson IV (Making Jewelry)	Video recording and students' work	Awareness of structures
06 April 2011	Lesson V ((Exploring numbers up to 20 using the arithmetic rack)	Video recording and students' work	Understanding about number relationships up to 20
12 April 2011	Lesson VI (Worksheet based on the arithmetic rack)	Video recording and students' work	Using combinations that make ten on solving some addition problems on the arithmetic rack
12 April 2011	Lesson VIII (Addition up to 20 with contextual problems)	Video recording and students' work	Solving addition problems up to 20
13 April 2011	End-Assessment	Video recording and students' work	To know students' performances on mental calculation strategies

C. Data analysis

Data which were collected during teaching experiments had been analyzed in a retrospective analysis. In the analysis, the Hypothetical Learning Trajectory (HLT) were compared to students' actual learning based on video recording, field note, and students' work. The data from video recording were selected into some fragments in which showed students' learning processes. The fragments were registered for a better organization of the analysis. Actually not all video recording were analyzed but a part relevant to students' learning.

After getting some videos, we transcribed conversations between a teacher and students and among students during group discussions. Then we analyzed and gave interpretations of students' thinking process. We also interviewed some students to get more insight about their mathematical thinking. The other data such as teacher's interview and students' work (data triangulation) were used to improve a validity of this research. Discussions with supervisors also improved a quality of this research.

We decided to analyze the lessons in two ways that were analysis on a daily lesson and on all the lessons. Analysis on a daily lesson focused on the intended students' thinking process on that activity, and analysis of whole lessons focused on intertwining among one lesson to others to find out the succession of students' learning process. At the end, a conclusion was drawn based on a retrospective analysis and answers the research question. We also gave a recommendation to improve the hypothetical learning trajectory on addition.

D. Validity and reliability

Validity and reliability of data are important issues in doing a design research. In validity, we concerned on a quality of a data collection and conclusion that was drawn based on the data, and we used reliability to preserve the consistency of data analysis. In the following, we described more about validity and reliability (Bakker, 2004).

Internal validity refers to a quality of data collections and soundness of reasoning that has led to a conclusion. We used many sources of data to guarantee an internal validity, namely video recording of classroom observations, students' work, field note and teachers' interviews. We also tested conjectures during retrospective analysis.

External validity can be interpreted as a generalizability of results. It was not easy to generalize the results from specific contexts as to be useful for other contexts, but we challenged it by presenting the results in such a way that others could adjust them. Since we tried the hypothetical learning trajectory in a real classroom setting, we found the results than could be generalized.

Internal reliability refers to reasonableness and argumentative power of inferences and assertions. To improve the internal reliability on this research, the data on the video recording was transcribed in some episodes, and discussed the critical learning process with the supervisors and colleagues.

External reliability means that a conclusion of study should depend on subjects and conditions, and not on the researcher. To keep the external reliability of this research, we recorded teaching experiments using video recording and students' worksheet. In fact readers were able to track the learning process of students and reconstruct the study (trackability of the research).

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

The purpose of this research was to develop instructional activities that support students' learning process and to know students' thinking in learning addition up to 20. We design a Hypothetical Learning Trajectory (HLT) that consists of the goals for students' learning, the mathematical activities that are used to promote students' learning, and hypotheses about a process of students' learning (Simon, 1995; Simon & Tzur; 2004). In this chapter, we describe a set of activities which contain some mathematical goals for students. We make hypotheses about the process of the students' learning in each activity.

In design the Hypothetical Learning Trajectory (HLT), we start to find some potential contextual situations to be brought out in classroom activities. The first idea is to find combinations that make ten. We choose a contextual situation that is making combinations of ten *pempek* which have two different kinds of *pempek*. There is a mini lesson I, parrot game, that still relate to develop understanding about combination that make ten. After that students work on a candy combination sheet to build knowledge about decomposing numbers up to ten. The next activity is flash card game. We use medicine tablet as an idea to design the flash card game. This game is used to support students' knowledge about number relationship up to ten.

The learning trajectory is continued by exploring numbers up to 20. There are two main goals. The first one is that students are able to decompose numbers up to 20. We design activities called "hiding monkey picture sheet" and "mini lesson II, parrot game". The second one is to support students' development of number relationships up to 20. The activity is called "exploring numbers using arithmetic rack".

In last activity students do some addition problems. We design two different addition problems. The first one is that students solve one digit addition problem which challenges them to perform decomposing number up to ten and adding through ten. In second one is students add two digit numbers with one digit number where students can use their knowledge about number relations and decomposition numbers up to 20 such as ten and ones. The intended activities will be explained next in more detail.

4.1 Lesson I (Making combinations of ten *pempek Palembang*)

Goal

The goal of this activity is that students are able to find combinations that make ten.

Description of Activities:

In this activity, students are challenged to make combinations of ten *pempek*. They have to find many different combinations of ten *pempek* that can be put on plates. Since they often find this situation in the daily life such as buying *pempek* in canteens or traditional markets, or getting *pempek* in parties, they can figure out what combinations of ten *pempek* can be made.

At the beginning of the activity, teacher shows some *pempek* to the students and tells a problem about a host wants to put ten *pempek* on a plate. The following problem is told to the students:

"Yesterday I went to my sister house. She told me that she wanted to make a birthday party for his son. She wanted to serve guests with pempek. She has two kinds of pempek that are egg pempek and beef pempek. She wanted to arrange ten pempek on each plate, so what different combinations are there?

Figure 4.1: A combination of ten pempek

First, students are asked to talk in a group (4 to 5 students) for 2 minutes, after that they have to share their ideas about it. This is a short discussion for warming up their ideas about combination of ten *pempek.* After that they have to draw combinations of ten *pempek* on a poster paper.

When the students do not understand this problem, the teacher can ask a student to take ten artificial *pempek* from a plastic bag and put on a plate. The other students have to observe what a combination of ten *pempek* he or she makes. After that the teacher asks students:

"*Can you figure out other combinations of ten pempek? Let's work in your groups and write your combinations on a poster paper."*

Conjectures of students' representations and thinking

Students' representation

- Some students draw *pempek* precisely. This representation will serve as a *model of* situation (Gravemeijer, 2006).
- Some students draw circle to represent egg *pempek* and rectangle for *lenjer pempek.* This representation will serve a bridge to move from *model of* to *model for* situation.
- Some students draw circle to represent *pempek.* They differentiate both *pempek* by using two different colors such as red for egg *pempek* and blue for *lenjer pempek*. This representation will be used into a classroom discussion and become *a model for* situation.
- Some students do not make draws but write numbers to represent quantity of *pempek.* These students have been able to relate between quantity objects and numbers representing objects.

Students' thinking

- Some students think that they need to represent ten *pempek* using fingers. They will fold some fingers to represent egg *pempek* and the other fingers to represent *lenjer pempek*. Some of them will use counting strategies and others can recognize the finger structure.
- Some students will start to think when they have ten egg *pempek,* so there is no *lenjer pempek,* and when they have nine egg *pempek,* so they have to have one *lenjer pempek,* and so on.
- Some students will start from five egg *pempek* and five *lenjer pempek* (doubles).
- Some students will find that if there are, for instance, four egg *pempek* and six *lenjer pempek,* and six egg *pempek* and four *lenjer pempek* (commutative property of addition).

Discussion

After students discuss and work in groups to find combinations of ten *pempek* and write their answers on poster papers, the teacher leads a classroom discussion. The teacher starts by choosing a group to present their work. The first presentation is a group who just finds few combinations of ten *pempek*. This group is chosen because it can challenge other students to think about other combinations of ten. After the first group gives the presentation, the teacher asks the students:

What are other possibilities combinations of ten pempek you get?

Then the teacher chooses a group to share their work. If students can follow the classroom discussion and understand what they discuss about, the teacher can choose a group who finds many combinations of ten *pempek*. Otherwise, the teacher can select a group who just get few combinations but different from previous one, and give opportunities to all groups to share their work.

After they finish doing presentations, the teacher can ask students:

What do all combinations of ten pempek you get?

The teacher can make a table and ask students to fill in.

Table 4.1: Combinations of ten pempek

Combinations of ten pempek	
Egg *pempek(s)*	*Lenjer pempek(s)*

Mini lesson I (Parrot game)

Goals

The goal of this activity is that students develop understanding about combinations that make ten.

Description of Activities

A Parrot game (*Figure 4.2*) is adopted from waku-waku that is developed in the Netherlands (Menne, 2001; Treffers, 2001). In this game, students are challenged for reproductive practices because they have to think about combinations that make ten. The teacher can start the activity by showing a colorful parrot puppet on her hand and tell to the students that:

This is a Parrot and he can say 10. What a problem that can give the answer 10?

Figure 4.2: A parrot game picture

Conjecture of students' thinking

Students will come up with many own productions. Since they have learnt about finding combinations of ten *pempek* from the previous activity, they will use their knowledge about it. Our conjectures are that some students still need to think about the concrete objects, but others can figure out the combinations that make ten. When a student gives a wrong combination, the teacher takes it into a discussion. The teacher can use an arithmetic rack to make representations.

4.2 Lesson II (Candy combination sheet)

Goal

The goal of this activity is that students are able to decompose numbers up to ten.

Description of the activity

The students will work on a candy combination sheet. Before they work in the worksheet, the teacher tells a contextual situation to the students.

In a birthday party, there is a can which contains two different candies that are chocolate and peanut candies. Your friend asks you to take 5 candies without see into the can, what combinations of candies can you get?

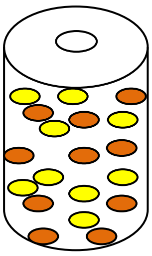


Figure 4.3: A can contains chocolate and peanut candies

First students will work in group around 5 to 10 minutes to figure out combinations of 5 candies. They can use the arithmetic rack to represent the candies. They have to record what combinations of five candies they get on a paper. If the students do not understand this activity, the teacher can show a representation by using the arithmetic rack. For instance, the teacher shows a bead on the first rack, and says "*if I have one, what is others?*"

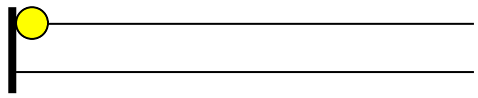


Figure 4.4: Representing a candy using an arithmetic rack

Conjectures of students' thinking and strategies

- Some students represent chocolate candies with beads on the top and peanut candies with beads on the bottom. They first think that if they take 1 bead on the top, they have to take 1 bead on the bottom, 1 more bead on the top, 1 more bead on the bottom, and the last bead on the top, so they get 3 beads on the top and 2 beads on the bottom. Other students can directly think that they can take 2 beads on the top, 2 beads on the bottom, and 1 bead more on the top or bottom. They continue this strategy until they get combination 5 and 0.
- Some students represent candies with different color of beads, for instance red for chocolate candies, and blue for peanut candies. They first think that if they take 1 white bead, they have to take 1 blue bead, 1 more white bead, 1 more blue bead, and the last white bead, so they get 3 white beads and 2 blue beads. Other students can directly think that they can take 2 white beads, 2 blue beads, and 1 white or blue bead more. They continue this strategy until finding all combinations.

- Some students think that they can start by taking 5 beads on the top, so there is no bead on the bottom, and then they move 1 bead on the top and substitute with 1 bead on the bottom, and so forth.
- Some students do not need to use the arithmetic rack, but they use fingers as representations. They fold some fingers as representation of chocolate candies and other fingers for peanut candies.
- Some students can find the combination of 5 candies mentally.

Discussion

After students work in group, they have to discuss their ideas in a classroom discussion. They have to tell what combinations of candies they get, and how they get those combinations. At the end, the teacher makes a table and asks students to fill in (*Table 4.2*). The students continue working on a candy combination sheet.

Table 4.2: Candy combination sheet

Chocolate candies	Peanut candies

4.3 Lesson III (Flash Card Game)

Goals

The goal of this activity is that students build knowledge about number relationships such as doubles, almost doubles, and five- and ten- structures.

Description of Activities

In this activity, students will play flash card. In this game, the teacher will show some cards and ask students to tell what they see. They only have a few seconds to see a card thus they challenge to not count one-by-one.

At the beginning of the activity, the teacher asks students about their experiences when they are sick. The teacher gives the following question.

"*Can you tell me what your mother gives when you are sick?"*

After that the teacher can ask students other questions, for instance *"How many medicines do you take for a day?* Or *do you know how many medicines are there on a box?"*

The teacher continues the lesson by showing a medicine tablet (*figure 4.5*) to the students and asks them:

"How many medicines do you see?"

Figure 4.5: Medicines on a medicine tablets

Conjectures of students thinking

- Some students see there are three medicines on the top and four medicines on the bottom, so altogether are seven.
- Some students use combination of ten. They see that there are three medicines have been used, so seven medicines remain.
- Some students have an idea by seeing the medicines as doubles, so they count 2, 4, 6, and add 1 more become 7.

Discussion

After the students think for a few second, they have to share their ideas. The teacher has to provoke students to give reasoning, for instance, when they just tell the number of medicines. The teacher can give a question: "*how do you know it?*" This question will lead them to come to the idea of number relationships. They will say that there are three on the top and four on the bottom, so altogether are seven medicines. The teacher continues the activity by showing other flash cards. We design ten flash cards, and the following figure is one example of those.

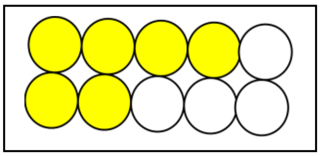


Figure 4.6: A flash card

4.4 Lesson IV (Hiding monkey picture sheet)

Goal

The goal of this activity is that students are able to decompose numbers up to twenty.

Description of Activity

At the beginning of the activity, the teacher tells a story about monkeys and a Sumatera tiger in the jungle.

In a jungle, there are 12 monkeys and a Sumatera tiger. The tiger wants to eat the monkeys, so the monkeys have to hide on the trees. One day, the tiger goes to a place where the monkeys often play there. When the tiger arrives, the monkeys hide in two trees. How many in each tree? What are the possibilities?

While the teacher tells the story, she has to show the following figures to the students respectively.

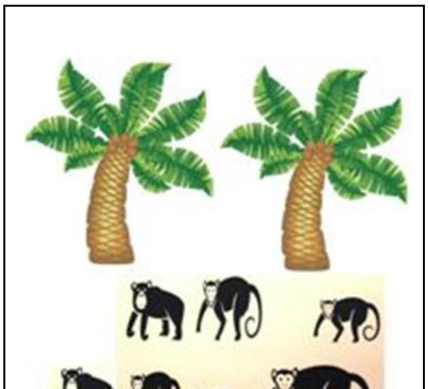


Figure 4.7.a: Monkeys and trees *Figure 4.7.b: A Sumatera tiger and trees*

The students will work in a group of 3 to 4 for 10 to 15 minutes. They will get the pictures above and discuss where the monkeys have gone. After discussing in a group they will do a classroom discussion. The students can use the arithmetic rack to support their thinking. When they still do not understand what they have to work, the teacher can tell to the students; *"if there are two monkeys in a tree, where do other monkeys hide?"* The teacher can support students by making a representation on the arithmetic racks.

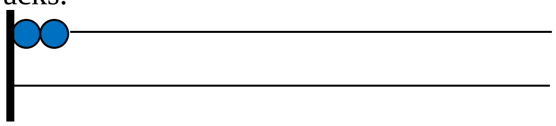


Figure 4.8: A representation of two monkeys on the arithmetic rack

Conjectures of students thinking

- Some students think that they can start by putting one monkey in the first tree, the second monkey in the other tree, and so forth until they finish moving all monkeys to the trees. They will get 6 monkeys in each tree.
- Some students think that they can start by putting all monkeys to the first tree, no monkey to the other tree, and then find other combinations such as 11 monkeys in the first tree, and 1 monkey in the other tree, so forth.
- Some students think that they can use the arithmetic rack to support their thinking. They make representations of 12 monkeys by taking 12 beads, and make combinations of 12 beads.

The students share their ideas how they get the answers. After students understand how to work with hiding monkeys, they will work on the hiding monkey picture sheet. The following figure shows an example of the worksheet.

Table 4.3: A hiding monkey picture sheet

$\boxed{15}$ Monkeys

Tree 1	Tree 2

Mini Lesson II (Parrot game)

Goal

The goal of this activity is that students develop understanding of decomposing numbers up to 20.

Description of Activities

The students will play the parrot game. In this game, they have to figure out some numbers that make combinations of a number. The teacher starts the activity by showing a colorful parrot puppet on her hand and tells to the students that the Parrot now can say numbers up to 20. They want to make the parrot look clever, so they have to think of a problem that gives the answer. For example teacher says that:

This is a Parrot and he can say 16. What a problem that can give the answer 16?

Figure 4.9: A parrot game picture

Conjecture of students' thinking

Our conjectures that students can give many problems that have result 16 because this problem is an opened problem. Some students still need to think concrete objects such as beads on the arithmetic rack, and others can figure out number relationships up to 20 mentally. When a student gives a wrong problem, the teacher takes it into a discussion, and use the arithmetic rack to help students.

4.5 Lesson V (Exploring numbers up to 20 using the arithmetic rack)

Goals

The goal of this activity is that students are able to develop their understanding about number relationships up to 20.

Description of Activities

The students will play flash card game using the arithmetic rack. This game is adopted from *Rekenweb* game that is developed in the Netherlands. In this game, the teacher shows a card (the arithmetic rack) to the students in a few seconds. In fact the students do not have a lot of time to count one-by-one how many beads on that card, so they need to do a fast counting such as structuring by five or ten. We hope this game can stimulate students to count by structuring. For example, the teacher shows the following figure and asks students:

How many beads do you see?

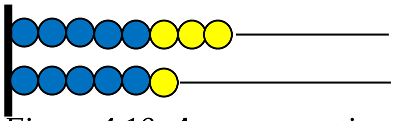


Figure 4.10: A representation of a number using an arithmetic rack

Conjectures of students' thinking

- Some students will reason that there are 5 red beads on the top, 5 red bead on the bottom, 3 blue beads on the top and 1 blue bead on the bottom. After that they add 5 and 5 is 10, add 3 more is 13, and 1 more is 14.
- Some students reason that there are 8 beads on the top because 5 red and 3 blue beads, and 6 beads on the bottom because 5 red and 1 blue beads. They add 8 and 6 and by counting on or adding by ten and get 14.
- Some students reason that there are 10 red beads, 3 blue beads on the top and 1 blue bead, so 10+3+1=14.
- Some students reason that they move one blue bead from top to the bottom so they get 7 beads on the top, 7 bead on the bottom, and altogether is 14 (using doubles).

4.6 Lesson VI (Addition up to 20 with contextual problems)

Goals

The goal of this activity is that students are able to perform combinations that make ten and decomposition of other numbers in solving addition problems up to twenty.

Description of Activities

Students will be given two addition problems up to 20 respectively.

The first problem is:

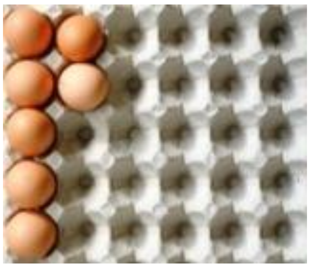


Figure 4.11: Seven eggs on a box

Bayu sees there are 7 eggs on the box in the kitchen. After that his mother comes from market and buys 8 eggs more. How many eggs they have now?

Conjectures of students' thinking

- Some students will solve the problem by using counting on strategies. They start from 7 and use their fingers to do counting.
- Some students solve problem by decomposing numbers. 7=5+2 and 8=5+3, so 5+5=10 and 2+3=5. After that they get 10+5=15.
- Some students use combination of ten to solve this problem. 7+8=7+3+5=10+5=15.
- Some students need models to support their thinking that can be fingers, arithmetic rack, and making a representation of eggs.
- To find the answer using the arithmetic rack, some students will use the strategies of composition by ten. First, they take 8 beads from the first rack, and 7 beads from the second rack. They move 2 beads from the second one and substitute by taking 2 beads from the first one, so the answer is 15 beads.
- Some students take 8 beads from the first rack, and 7 beads from the second racks. They decompose 8 into 5 and 3 beads, and 7 into 5 and 2 beads, so 5 and 5 altogether is 10, and 2 and 3 altogether is 5, so the result of 15 beads. In finding the answer, students probably use structuring strategy combine with counting strategy.

- Some students take 8 beads from the first rack and continue to take 7 beads from the remainder of first rack, and the end they count all beads or find the structure such as 5, 5, and 5, so the result is 15 beads.

After students work around 10 to 15 minutes, the teacher asks students to share their ideas. The teacher asks students:

How do you get the answer?

The students will tell the answer and also how they get the answers. The teacher has to give opportunities to the students to share their ideas. We hope students can realize that solving the problem using decomposition is more flexible that counting. After that the teacher challenges student by giving the other problem.

The second problem is:

Figure 4.12: Thirteen candies

Ani has 13 candies in her pocket, and then she get 4 more candies, so how many candies does Ani have now?

Conjectures of students' thinking

- Some students still solve the problem by counting on strategy starting from 13.
- Some students decompose 13 into 10 and 3, add 4 to 3 that equals to 7. In the end 10+7=17.
- Some students decompose 4 into 2 and 2, add 2 to 13 that equals to 15, and finally 15+2=17.
- Some students decompose 13 into 10 and 3. Since they know number relations that 3 and 4 become 7, 10+7=17.

- Some students still need models to support their thinking so that they use fingers or arithmetic rack.
- Some students solve the problem by counting on using fingers.
- Some students who use the arithmetic rack take 10 beads by counting, grouping of five, or knowing the structure of arithmetic rack that is 10 beads in the first row. After that they take 3 more to get 13, and they take 4 by counting on or combination 2 and 2, so they get the result is 13+2=15+2=17.

After students work around 10 to 15 minutes, they have to share their ideas how to get the answer. We hope that the discussion will lead them to realize that using decomposition numbers such as ten and ones, after that adding ones to other numbers will give them opportunities to do more flexible strategies in solving addition problems.

CHAPTER V

RESTROSPECTIVE ANALYSIS

In this chapter, we describe the retrospective analysis of data from pre-assessment, preliminary experiment activities, teaching experiment activities, and post-assessment. The result of this research was core principle that explains how and why this design works. We used the hypothetical learning trajectory as a guideline in the retrospective analysis to investigate and explain students' thinking in learning and developing mental calculation strategies on addition up to 20.

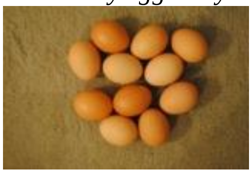
5.1 Pre-Assessment

The aim of pre-assessment was to know students' pre-knowledge not to test students' ability on addition up to 20. First, we designed eight problems, and then we tried with some first grade students in 1.A in SDN 179 Palembang. After we tried those problems to the students, we found that those problems were too much for students because students did not have enough time to solve those problems, and some problems are similar ideas. Then, we revised those problems became two problems, and we tried with first grade students from 1.A, 1.D, and 1.E. We described the students' thinking in solving both problems.

In the first problem, we gave students a figure (*figure 5.1*), asked them to tell the number of eggs on that figure, and described strategies to know the answer. Many students from those classes knew the number of eggs that is eleven, and they knew the number of eggs by counting one-by-one (*dihitung*). Since we wanted to know more about students' ideas about the number of eggs on the figure, we interviewed some students. We described the result on the following segment.

Figure 5.1: Un-structuring eggs

Researcher : How many eggs do you see on the figure?



Atha : These (Pointing the figure and then using his fingers) are 3, plus 3, plus 3, and plus 2 equa 11.
Researcher : Which one are three?
Atha : These are 3, 3, 3, and plus 2 equal 11. (Pointing 3 eggs, 3 eggs, 3 eggs, and 2 eggs from the top to the bottom)

When we asked the number of eggs on the figure, Atha did not give directly the number of eggs instead told the strategy how he knew the number of eggs. This is showed by the phrase "*These are 3, plus 3, plus 3, and plus 2 equal 11".* He did not count the eggs one by one, but he could see the eggs arranged in some small groups. Atha also used a mathematical language that was plus indicating addition of some small groups of eggs. By pointing the objects on the figure showed that Atha saw the eggs arranged in structuring way. After that we asked other students how they knew the eggs on the figure. We describe the interview on the following segment.

Researcher : How do you know 11?
Rista : Counting
Researcher : Explain your answer!
Rista : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11. (Pointing each egg using a ruler)

Rista used *counting* to know the number of eggs on the figure. Counting in her answer means that she counted the object one-by-one. This was showed by the phrase '*1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11*'. She could synchronize one word for every object and had a big idea that was one-to-one correspondence. Rista could not see the eggs on the structuring way. Many students had the similar strategy, counting one-by-one, and big idea, one-to-one correspondence, with Rista. These were showed by some students' answers on the worksheet (*figure 5.2*).

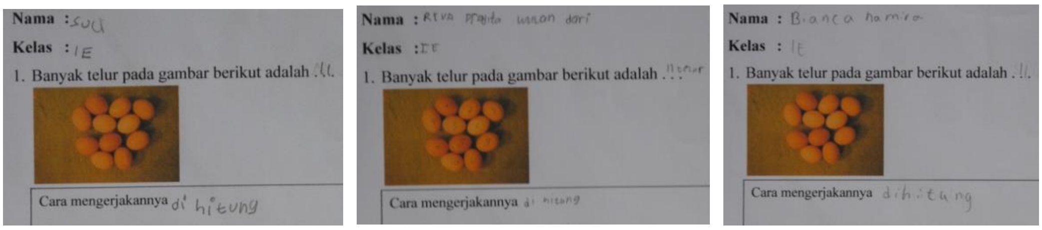


Figure 5.2: Some students' work (Counting-one-by-one)

The second problem is different from the first one because the students are given a figure of medicines in which the medicines are arranged using the five-structure. We expect that students could recognize the structure on the figure (*figure 5.3*), so they could have different ideas in knowing the number of medicines.

Figure 5.3: Ten medicines on a medicine tablet

Based on students' answers on their worksheet, all students knew the number of medicines that were ten, and many students knew the number of medicines by counting one-by-one (*dihitung*). A student, Atha, wrote on his worksheet that there were 10 because 5 + 5 = 10 (*figure 5.4*). He could recognize the structure of medicines that were arranged using the five-structure. To know more about students' strategies and big ideas to solve this problem, we interviewed two students described on the following segment:

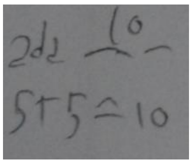


Figure 5.4: A student's work using the five-structure

Researcher : How many medicines do you see on this picture?

Agung : *Medicines*
Researcher : *Yes*
Agung : *1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. (Pointing each medicine for each number)*
Researcher : *And you Rizki, how many medicines do you see on this picture?*
Rizki : *10*
Researcher : *How do you know?*
Rizki : *Counting*
Researcher : *How do you count?*
Rizki : *5 plus 5*

Agung did not give the number of medicines on the figure instead showed his strategy to know the number of medicines. This was showed by the phrase '*1, 2, 3, 4, 5, 6, 7, 8, 9, and* 10'. He counted the object one-by-one and synchronized one work for one object. He used a big idea of one to one correspondence. He could not see the medicines on the five-structure. This was different from Rizki's thinking. He could see the number of medicines as 5 plus 5. This was showed by the phrase '*5 plus 5*'. The word 'counting' that Rizki used did not indicate that he counted the medicines one-by-one, but he used the five-structure. He also used the mathematical language that was plus that means adding 5 and 5.

In general, many students still used counting one-by-one to know the number of objects. They could synchronize one work for one object and had a big idea that was one-to-one correspondence. Although we showed the students a problem that was arranged using the five-structure, many students still used counting one-by-one. Some students were able to see the structures of objects, so they knew the number of objects quickly. We think that by discussion among the students, it will provoke other students to see the structures of the objects in case they could have an idea in knowing the number of objects quickly. This situation is appropriate for us to try the hypothetical learning trajectory.

5.2 Preliminary Experiment Activities

The designed hypothetical learning trajectory was tried out with five students in the first grade in SDN 179 Palembang. The students that we chose were different from students that we will conduct our research for whole class activities. We take those students from I.D class randomly. We tried all activities that we had designed to find out how this design works and to test our conjectures about students' thinking and learning processes. The result of this preliminary experiment will give us feedback to improve our hypothetical learning trajectory.

Lesson I (Making combinations of ten *pempek Palembang*)

The activity in the lesson I of preliminary experiment is that students are challenged to make combinations of ten *pempek.* First, we showed them a picture of *pempek* (*Figure 5.5a*) to get willing that they knew about this contextual situation. After they saw the picture, they directly recognized it and also the name of two kinds of *pempek,* egg *pempek* and *lenjer pempek.* After that we gave them a story where a mother wants to serve guesses with *pempek* in her son birthday party. She wants to know what combinations of ten *pempek* she can make. Since first grade students could not visualize those combinations, we initiated to make *pempek* from wax (*figure 5.5b)* and asked them to experience with it.

Figure 5.5a: Pempek

Figure 5.5b: wax pempek as a model of situation

The lesson was continued by asking the students to experience by putting ten *pempek* on a plate. The following is a segment from the video and audio recording.

Researcher : How do you put 10 pempek on the plate?
All students raise their hand
Riko : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. (Taking eggs pempek one-by-one from a box to a plate)
Researcher : Let we see. Riko has taken 10 pempek. What kinds of mpek- mpeks are those?
Students : egg pempek.
Researcher : Do you have another idea?
Students : Yes
Researcher : What is your idea?
Nada : I will count. I count 5 and 5.
Researcher : What do you mean 5 and 5?
Students : 10
Researcher : So, It is different from Riko's. How did Riko take them?
Students : One-by-one.
Researcher : Nada, How do you take it? Show us!
Nada took pempek and put on the plate
Riko : Nada will count quicker, 5 and 5.
Siti : 1, 2, 3, 4, and 5 (Counting pempek on the plate)
Nada took 5 pempek more and put on the plate.

Two students, Riko and Nada, showed different ideas in putting ten *pempek* on a plate. The phrase '*1, 2, 3, 4, 5, 6, 7, 8, 9, 10*' indicates how Riko put ten *pempek* on a plate. He used one-to-one tagging and synchronized one word for every object. He had a big idea of one-to-one correspondence. Hence, Nada used the five-structure to put ten *pempek* on a plate. The phrase '*I count 5 and 5*' describes her big idea of doubles. The word '*quicker*' said by Riko indicates that he realized that using the five-structure was better that counting one-by-one. Siti, the other student, still counted the *pempek* one-by-one to check Nada's. This indicated that she still was not influenced by Nada's strategy, using the five-structure. Since the students just put same *pempek* on the plate, and we expected that students put combinations of ten *pempek*, we tried to provoke students by asking questions.

Researcher : If we mix the pempek, what combinations of pempek will we get?
Nada : Lenjer pempek and egg pempek
Researcher : How many are those to make 10?
All students : 5 and 5
Researcher : Do you have other ideas?
Riko : 6 and 4
Inaya : Aaaa…
Reseacher : 6 and 4, are you sure? Why do you say 6 and 4, Riko? Explain to us!

Riko : 10 (folding 4 fingers).

The phrase '*5 and 5*' showed that the students are familiar with the five-structure. They had an idea that was double of five making ten. Starting from 5 and 5, the students could find other combinations that make ten. Such as Riko said '*6 and 4*' is the other combination that make ten. He came to a big idea that is compensation because 5 and 5 is similar to 6 and 4.

After students figured out some combinations of ten *pempek,* we gave them a paper to draw combinations of ten *pempek.* The result is that the students come up with some different drawing as follows:

Inaya started by drawing an egg *pempek* and then a *lenjer pempek.* She continued it until she got 10 *pempek* (*Figure 5.6a*). She wrote 10 in the right side of her paper. She explained that there were 5 egg *pempek* and 5 *lenjer pempek.* In her drawing, we can see that she drew the *pempek* in pairs, so there were five pairs of *pempek.* She came up with a big idea of doubles. In other case, Bimo drew ten *lenjer pempek* in a plate and ten egg *pempek* in the other plate (*Figure 5.6b*). He did not make a combination of ten *pempek* instead of each kind of *pempek* in each plate.

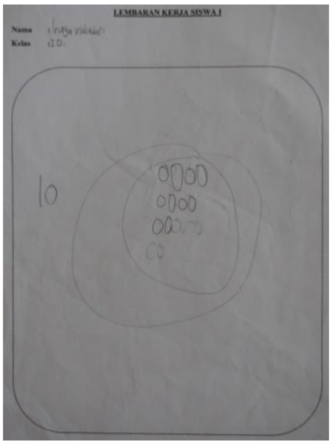


Figure 5.6a: Inaya's drawing

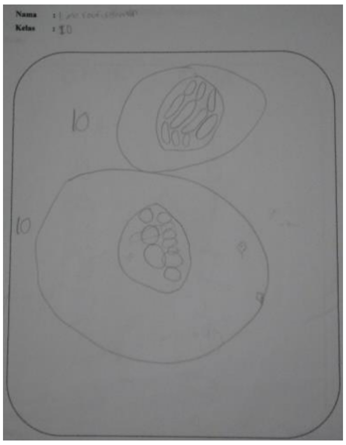
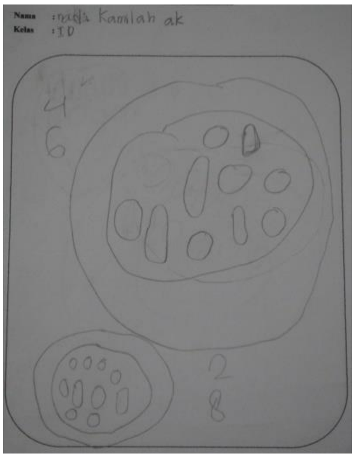
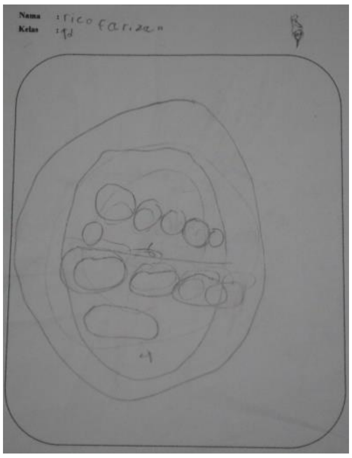


Figure 5.6b: Bimo's drawing

Riko and Nada drew a similar combination of ten *pempek*, 6 egg *pempek* and 4 *lenjer pempek*, but different way of drawing. Nada drew *pempek* randomly (*Figure*

5.7a). Otherwise, Riko drew *pempek* more structuring in which he drew 6 egg *pempek* on the top and 4 *lenjer pempek* on the bottom (*Figure 5.7b*). He decomposed 6 egg *pempek* into 5 and 1 and 4 *lenjer pempek* into 3 and 1. Nada also drew the other combination of ten *pempek* that was 8 egg *pempek* and 2 *lenjer pempek.* She drew this combination more structuring in which 4, 4, and 2 *pempek*, but she still did not drew some *pempek* in a group. Different from Riko and Nada, Siti drew combinations of ten *pempek* using the five-structure (*Figure 5.7c*). In the biggest plate, she drew 5 egg *pempek* and 5 *lenjer pempek.* She came up with a big idea that is doubles of 5 making ten. She also made the other drawing that was 6 *lenjer empek-empeks* and 4 egg *empek-empeks*, but she still arranged using the five-structure. In the second line of her drawing, she drew 1 *lenjer pempek* and 4 egg *pempek.* She came up to a big idea of compensation that was 5 and 5 similar to 6 and 4.





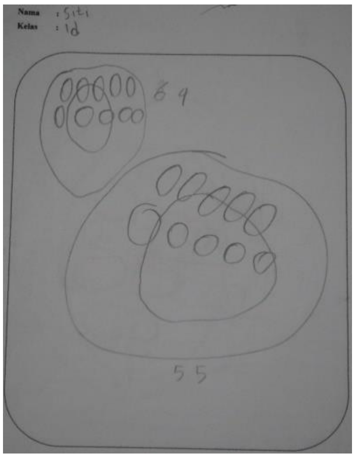


Figure 5.7a: Nada's drawing *Figure 5.7b: Riko's drawing* *Figure 5.7c: Siti's drawing*

In the activity on this lesson, we compared our conjectures of the hypothetical learning trajectory with the actual learning process in preliminary first activity. We found that the students used fingers to find combinations of ten *pempek,* and started finding a combination of ten *pempek,* 5 egg *pempek* and 5 *lenjer pempek.* From this combination, they could figure out other combinations such as 6 egg *pempek* and 4 *lenjer pempek.* Two big ideas, doubles and compensation, emerged from this activity, and they also were familiar using the five-structure. Hence, some students still counted the object one-by-one and they did not find all combinations of ten *pempek,* but they could model of situation by making drawing of *pempek.* As our conjecture, the students drew circle to represent egg *pempek* and rectangle to represent *lenjer pempek.* This representation served as model of situation.

Mini lesson (Parrot game)

Before the students continued the activity in the lesson II, they first played the parrot game (*figure 5.8*) in a mini lesson. The goal of the activity in this mini lesson was that students develop understanding about combinations that make ten. They were given a question that was to find combinations that make ten. They did not use pencil and paper, so they had to think mentally. The activity was described on the following segment.

Figure 5.8: A parrot game activity

Researcher : *This is a Parrot. Today he can say 10. Ten, ten. (Shaking Parrot head)*

Students : haha… (Laughing)
Researcher : So if you can say combinations that make ten, please raise your hand, so the Parrot can speak!
All students raised their hands
Researcher : Ok, Inaya!
Inaya : 2 plus 8.
Researcher : 10, who is next?
Sari : 5 and 5 (Showing by her fingers)
Researcher : 10, Lets Bimo!
Bimo : 2 and 6
Researcher : Mmm…(Shaking Parrot head)
Researcher : What number did Bimo say?
Nada : 2 and 6
The researcher shows an arithmetic rack and asks Bimo to check it
Researcher : Please take 6 Bimo!
Bimo : 1, 2, 3, 4, 5, and 6. pointing each bead on the top)
Researcher : What is the next number?
Riko : 2
Bimo : 1, 2 (pointing each bead on the bottom)
Researcher : How many beads are those?
Students : 1, 2, 3, 4, 5, 6, 7, and 8 (Saying 8 loudly)
Researcher : Do you have any idea to know it 8?
Nada and Inaya : I know
Siti : Minus 2
After that the researcher asks other students
Researcher : Nada!
Nada : 4 plus 6
Researcher : 10, Riko!
Riko : 7 plus 3
Researcher : 10, Bimo!
Bimo : 9 plus 1
Researcher : 10.

The students were able to find many combinations that make ten mentally. Some students said the combinations that make ten as number pairs, and the others said on addition that was indicated by word 'plus'. Starting from Inaya, She argued that 2 plus 8 to make ten and Siti used the five-structure that was 5 and 5. She still preferred to work on doubling of 5. When we asked Bimo, he gave an incorrect combination that was 6 and 2. He did not realized on Inaya's answer that was 2 plus 8 become 10. We asked Bimo to check his answer using the arithmetic rack. Since he did not have many experiences to use the arithmetic rack, he was not familiar with the structure of the arithmetic rack. He still used counting one-by-one when he took 6 and 2 beads on the arithmetic rack and counted three times when adding. To provoke the other students to know the number of beads taking by Bimo in more effective way, we asked ideas of the other students. The phrase '*minus 2*' indicated that Siti had a big idea that was part/whole relations. She showed a relationship between addition and subtraction. The other students, Nada said 4 plus 6 and Riko said 7 plus 3. There was a relation between combinations telling by Nada to Riko. There was a big idea of compensation, 4 plus 6 equal to 7 plus 3. Finally, Bimo could figure out a combination of ten that is 9 and 1.

We concluded that the students were able to find combinations of ten mentally. They directly gave answers when we asked to figure out some combinations of ten. Each student could give different combinations of ten. These were based on our conjectures that they would give variety of combinations of ten, but these were still on addition.

Lesson II (Candy combination sheet)

In the activity on this lesson, the students were asked to figure out what combinations of candies they would get if they had to take some candies on a can containing two kinds of candies, chocolate and peanut candies. Since they could not see inside the can, they challenged to figure out what combinations they would get. We first showed them a picture (*Figure 5.9*) and told that there were some candies on that box and you were asked to take 5 candies. When we asked them to think what combinations they could get, they said that they could not figure out those combinations, so we offered a plastic bag contained 10 blue beads and 10 white beads as representations two kinds of candies. We asked some students to take some beads inside the plastic bag. The activity is described on the following segment.

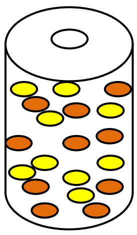


Figure 5.9: A can contains chocolate and peanut candies

Researcher : Let's take 5 on this plastic bag, Nada!
Nada takes 5 beads on the plastic bag
Researcher : What did Nada get?
Inaya : 5 (saying loudly)
Siti : 1, 2, 3, 4, and 5
Researcher : What combination does she get?
Inaya and Siti : Chocolate and peanut.
Researcher : How many chocolates does she get?
Students : 2
Researcher : How many peanuts does she get?
Students : 3

Inaya was able to recognize the number of beads taking by Nada. The word '*5'* indicated that she could subitize the five objects. She did not need to count the beads one-by-one. Meanwhile, Siti still needed to count the beads one-by-one and synchrony: one word for every object. Since the students did not tell about combination that Nada took, we had to provoke them by giving a question '*what combination does she get?*'. The students could perceive there were 2 chocolate and 3 peanut candies. They could subitize for the small amount of objects. After that we gave opportunity to Siti to take 5 beads on the plastic bag. Siti put her hand inside the plastic bag and then showed the beads she got to all students.

Riko : This is similar from the previous one.
Riko and Inaya : But the peanuts are 2 and the chocolate are 3.

The word '*similar´* said by Riko indicated that the number of beads were similar to the previous one, but Riko together with Inaya realized that there were different in combination. They had a big idea that was commutativity.

After students knew how to find combinations of two candies, we gave them a worksheet in which they had to find as many as possible combinations of 5 candies. To support their thinking, we allowed them to used beads on the plastic bag, fingers and the arithmetic rack as model of situation.

Siti and Riko modeled the situation with circle representations. Siti could find three different combinations of 5 candies that are 3 and 2, 2 and 3, and 4 and 1 (*Figure 5.10a*). There are three same combinations of 2 and 3 there. We observed that Siti took five beads, representing of candies, on the plastic bag and found those combinations. She got difficulties to represent when she got 5 beads in the same color. She could not be able to connect between 5 and 0. Otherwise, Riko drew two combinations of 6 candies (*Figure 5.10b*). When we asked him, he said that he took 6 beads on the plastic bag, so he wrote those combinations. This was a mistake that a student made, so we have to give instruction to the students clearly for the real classroom teaching experiment.

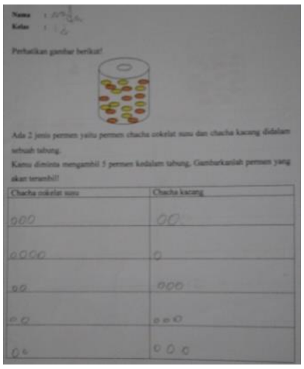


Figure 5.10a: Siti's work

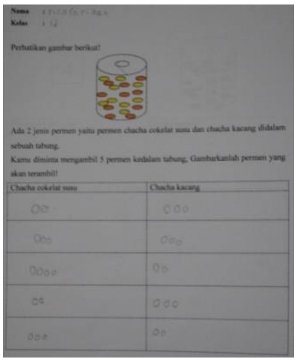


Figure 5.10b: Rico's work

Nada did not only model of situation with circle representations but also with numbers to represent the number of candies. She found four different combinations of 5 candies that were 4 and 1, 1 and 4, 2 and 3, and 3 and 2 (*Figure 5.11*). Based on our observation, Nada just used beads to find first and second combinations after that she was able to figure out other combinations. This indicated that she only need the real objects to figure out some combinations of 5 candies, and used the known fact to find other combinations of 5 candies.

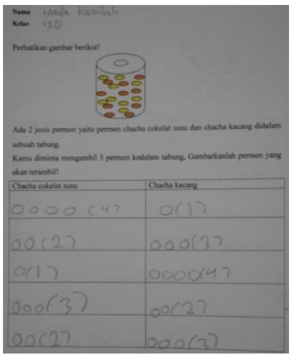


Figure 5.11: Nada's work on candies combination sheet

Inaya and Bimo did not model of situation with circle representation, but they directly wrote number representations to represent the number of candy combinations. Based on our observation Inaya took 5 beads on a plastic bag for the first and second activity. After that she used her fingers to find other combinations, but she just wrote two different combinations of 5 candies, 2 and 3, and 3 and 2 (*Figure 5.12a*). Actually we saw that she found 5 beads in the same color but she did not sure with her finding, so she did not write it on her worksheet. She got difficulties to connect 0 with 5 as a combination of 5 candies. In other case, Bimo wrote 4 and 1 in the first row because when we asked him to take 5 beads on the plastic bag, he got that combination (*Figure 5.12b*). He wrote wrong combinations for second, fourth, and fifth row since he took the numbers of beads that are different from what we asked him. Although he found 4 and 1 became 5, he did not realize that it should not be possible 4 and 2 become 5.

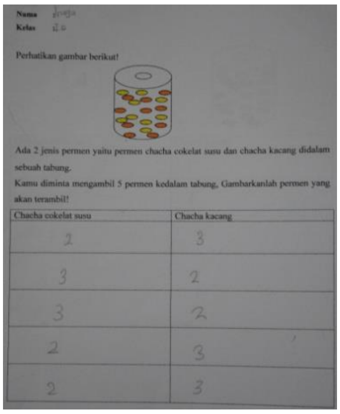


Figure 5.12a: Inaya's work

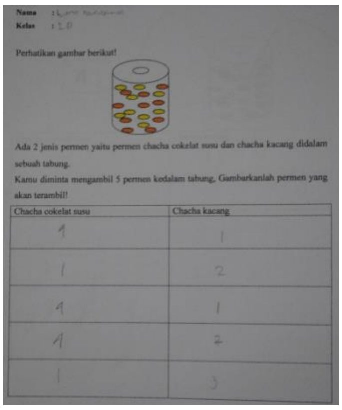


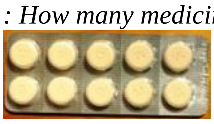
Figure 5.12b: Bimo's work

From this activity, we make some conclusions. Some students needed modeling of situation. They needed real objects such as blue and white beads as model of chocolate and peanut candies. A student, Inaya, used fingers to find combinations of 5 candies, and Nada could figure out some combinations of 5 candies mentally. Bimo and Riko made a mistake in finding combinations of 5 candies. This happened because they took candies less or more than the instruction on the worksheet. We have to give a clear instruction to the students in a real classroom teaching experiments. The students also got difficulties to represent when they found some colorful beads on the plastic bag. They need to connect 0 and 5 as a combination of 5 candies. The big idea, commutativity, emerged in this activity. Hence, some students still count one-by-one and had a big idea that was one-to-one correspondence. The students also did not use the arithmetic rack since they still was not familiar with it yet.

Lesson III (Flash card game)

We tried the flash card game to build students' knowledge about number relationships such as doubles, almost doubles, and five structures. We first showed the students two medicine tablets to make them recognize a situation. After that we showed them some flash card consecutively. The flash card served as model of situation that represented the number of medicines. The following segment described the learning process on this lesson.

Researcher *: How many medicines do you see on it? (Showing a medicine tablet)*



Students *: I know, 10 (Raising their hands)*
Researcher *: Lets Bimo answers!*
Bimo *: 10*
Researcher *: How do you know?*
Bimo *: 5 plus 5*

The students knew the number of medicines on the tablet although they did not have enough time to count one-by-one. This meant that they were familiar with this contextual situation. The phrase '*5 plus 5*' showed that Bimo knew the structure of the medical tablet that was the five-structure. Then, we showed the other medicine tablet to the students as follows.

Researcher *: Let's guess how many medicines on it? (Showing a medicine tablet)*

Please raise your hand if you know!

Students	*: I know (Raising their hands)*
Inaya	*: 9*
Researcher	*: How do you know 9?*
Siti interrupts	
Siti	*: I know Sir*
Inaya	*: I know, I know, 8 . . ., 7 plus 1.*
Researcher	*: Please see again from this medicine tablet (Showing the medicine tablet)*
Riko	*: Me, Sir!*
Researcher	*: Riko!*
Siti interrupts	
Siti	*: Minus 1 (Pointing a hole in the tablet)*
Inaya	*: Minus 1 (Also pointing a hole in the tablet)*
Researcher	*: Riko, how do you know it 9?*
Riko	*: 8 plus 1*

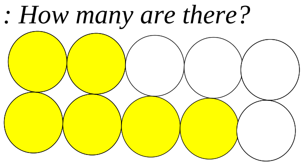
Researcher	*: Which one are 8?*
Siti	*: These (Pointing 8 medicines and then a medicine)*

The phrase '*I know*' indicated that the students knew the number of medicines that was 9. Some students need more time to reason how they knew the number of medicines. Such as Inaya said first '*8…*' and then changed '*7 plus 1*'. She first started with the correct number that was 8, but she then changed to 7 and 1. This was influenced by a situation in which she needed more time to concentrate and the classroom norm in which some students interrupted while other students gave the reason. The phrase '*minus 1*' showed that Siti seemed to connect the number of medicines was ten, and there was one loss. She seemed to have big ideas of combinations that make ten and part/whole relations: relationship between addition and subtraction. Inaya also had similar argument with Siti. Meanwhile, the phrase '*8 plus 1'* showed that Riko had different perspective in knowing the number of medicines. Although he could not point which medicines he meant 8 and 1, he agreed with Siti that pointed 8 medicines on the right side. The medicines were arranged by 4 on the top and 5 on the bottom, so this seemed that the students had a big idea that was doubles. Then we asked the other students to give their reason on that medicine table figure.

Researcher : and Bimo, How do you know it?
Bimo : 6 plus 3
Researcher : Ok, and you Nada?
Riko interrupts
Riko : 5 and 4
Researcher : Which one are 5?
Riko : On the bottom
Researcher : And on the top?
Riko : 4
Nada : I know 8 plus 1.

The phrase '*6 plus* 3' indicated that Bimo did not give a reason based on the figure he showed but his knowledge of a combination that makes 9. This was influenced by previous activity in which the students had to decompose some numbers up to ten. Riko showed there were 5 medicines on the top and 4 medicines on the bottom. He showed the structure on the medicine tablet, so he came to an idea that was almost doubles. Meanwhile Nada had similar answer with Riko's in the previous one that was 8 plus 1. Then we showed the students a following flash card.

Researcher : How many are there?



Inaya : 6
Siti : I know 6
Nada : 6
Researcher : How do you know 6, Siti?
Siti : Because there are 2 on the top and 5 on the bottom.
Inaya : 3 on the top (Showing three by her right fingers)
Riko interrupts
Riko : 2 on the top
Bimo : On the top.., on the top are 2, on the bottom are…
Siti interrupts
Siti : 4 (Showing 4 fingers on the right hand)
Researcher : Let's see! What is the correct one? (Showing a flash card game)
Siti : 2 plus 4.

The students could figure out the colorful circle on the flash card that was 6, but they needed to negotiate when we asked their reasoning. The students saw the yellow circles as two parts that were top and bottom. Siti said '*2 on the* top' was correct but '5 *on the bottom*' was incorrect number to make 6. Inaya also gave an incorrect answer by saying '*3 on the top*'. Meanwhile, Riko and Bimo corrected the number of the colorful circle on the top that was 2. The word '*4'* said by Siti showed that she tried to find the correct reasoning that was 2 on the top and 4 on the bottom. At this time, Interruptions among students often happened when a student knew the answer while the other students gave an incorrect one. This is a mathematics socio norm that we found in this teaching experiment in the first grade students.

In this activity, we found that the students developed their strategies to know the number of medicines that we showed. They did not use counting one-by-one anymore, but they move to more abbreviated strategies such as doubling, using doubles for near doubles, and using the five-structure. By giving this activity, students developed some big ideas, part/whole relations, doubles, and combinations that make ten. Flash card game using a ten-frame served as model of situation, and later on could serve as model for any situation.

Lesson IV (Hiding monkey picture sheet)

We started the lesson by giving a story that there was a jungle where some monkeys live there. The monkeys like to play in a place where there are two coconut trees there. While the monkeys play, a tiger goes there to eat them. Unluckily, the tiger does not find any monkey there, so the students are asked to tell where the monkeys hide. By giving this situation, we hope that students are able to figure out how to help the monkeys to hide in both trees.

Researcher : Where do the monkeys go?
Siti : I do not know, I think they go to the trees. Yes to the trees.
Researcher : Trees, how many monkeys are there?
Students : 12 monkeys
Researcher : Where do they go?
Nada : In the trees
Researcher : In the trees, who knows how many monkeys go to this tree (pointing a tree) and to this tree (Pointing the other tree).
Nada : These are . . .
Riko interrupts
Riko : 6 and 6
Researcher : Ok, Riko has an idea, 6 in this tree (Pointing a tree) and 6 in the other tree (Pointing the other one).
Researcher : Inaya! Riko said that 6 in this tree and 6 in the other one.
Inaya : I am . . .

Riko interrupts

Riko	*: 10 plus 2. 10 go to this tree (pointing the tree on the left side) and 2 go to the other tree. Hahaha…*
Researcher	*: 10 go here (Pointing the tree on the left side)*
Riko	*: equal 12.*
Siti	*: I am Sir, 10 plus 2.*
Inaya	*: I am also 10 plus 2.*
Bimo	*: 9 plus 3.*
Researcher	*: 9 and 3. 9 are in this tree (Pointing the tree on the left) and 3 are in this tree (Pointing the other one)*
Researcher	*: and Nada?*
Nada	*: 9 plus 3*

The first idea to hide those monkeys to both trees was come from Riko. He separated 12 monkeys in equal numbers that was showed by phrase *‘6 and 6’.* At this time, Riko had a big idea of doubles. The doubles is one of big ideas that students often used when they had to decompose even numbers. Riko also did not get difficulties to find other combinations of 12 monkeys. He, together with Siti and Inaya, decomposed 12 into 10 and 2. They used the ten-structure to decompose number more than ten. Meanwhile, Bimo and Nada decomposed 12 into 9 and 3. This seemed that they had a big idea of compensation, 10 plus 2 equal to 9 plus 3. After that we also asked the students to give other ideas.

Researcher	*: Does anyone have other ideas?*
Inaya	*: 5 plus 10.*
Researcher	*: yeah?*
Students	*: Aaa???*
Researcher	*: 5 plus 10?*
Nada	*: 15*
Researcher	*: how many monkeys do we have?*
Students	*: 12*
Researcher	*: 5 plus 10, lets we see!*
Inaya	*: eh 9 plus 3.*
Researcher	*: 5 and 10, how many are those? (Showing an arithmetic rack)* *Which one are 5?* *How many white beads are on this? (Pointing 5 white beads on the top)*
Students	*: 5*
Researcher	*: Lets we take 10 on the bottom!* *How many are these? (Pointing 5 beads on the bottom)*
Students	*: 5*
Researcher	*: And these (Pointing 5 blue beads on the bottom)*
Students	*: 5*
Researcher	*: Altogether? (Pointing beads on the bottom)*
Riko	*: 25*
Researcher	*: Are there 25?*
Nada	*: No, Those are 15.*

Inaya gave an answer that was 5 plus 10 that made 12. We found that she just gave an answer respectively, so we asked her to explain about it, but Nada directly gave an answer that was 15. She could connect between 10 and 5 that made 15. The phrase '*9 plus 3*' showed that Inaya realized that her first answer was not correct and tried to find the correct combination that makes 12. To check the total of 5 plus 10, we offered the arithmetic rack to the students. The students count the number of beads one-by-one since they recognized the structure of beads on the arithmetic rack. A student, Riko, struggled to know the total of 5 beads on the top and 10 beads on the bottom. He said there were 25 altogether, but Nada said '*those are 15'* surely. We found that finally Riko shacked his had as an agreement about Nada's answer.

To know more students' ideas in decomposing numbers up to 20, we gave them worksheets (*Figure 5.13*) in which they have to determine how many monkeys there are. After that they have to hide the monkeys on the tree.

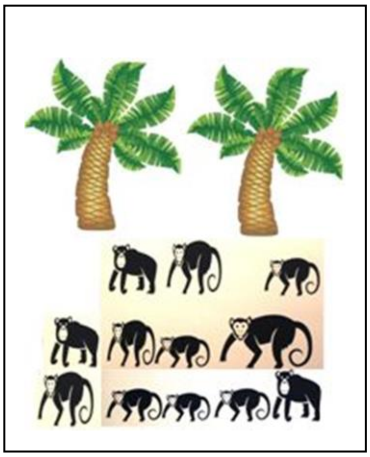


Figure 5.13: Monkeys and trees

The students knew that there were 12 monkeys on that figure (*figure 5.13*). They wrote their strategies how they knew the number of monkeys. Nada and Bimo represented the monkeys with circle representations. These representations served as model of situation. Nada represented that there were 5+7(*Figure 5.14a*). She explained that there were 5 monkeys on the bottom and 7 monkeys from 3 on the top and 4 on the second row. She could subitized the small objects and saw the structure of monkeys. And the other hand, Bimo represented the number of monkeys as 10+2 (*Figure 5.14b*). Bimo decomposed the number of monkeys into ten and ones. This was similar to Inaya's answer. She also wrote 10+2 (*Figure 5.14c*). She explained that she saw that there were 2 monkeys on the left side. Inaya saw the figure in structuring way. Siti wrote 6+6 = 12 (*Figure 5.14d*). This seemed that she had a big idea of doubles. Meanwhile, Riko first wrote 7+5, and then he changed into *seven + five* (*Figure 5.14e*). His answer was similar to Nada's but different representation. He counted the number of monkeys from top to the bottom because the number of monkeys on the first and second row is 7 and the bottom is 5.

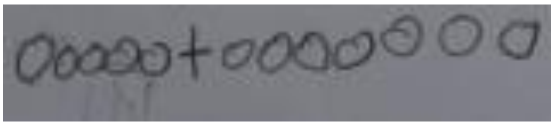


Figure 5.14a: Nada's solution

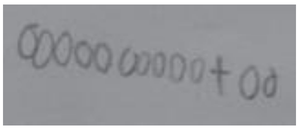


Figure 5.14b: Bimo's solution

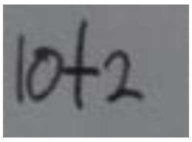


Figure 5.14c: Inaya's solution

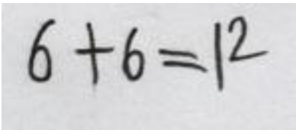


Figure 5.14d: Siti's solution

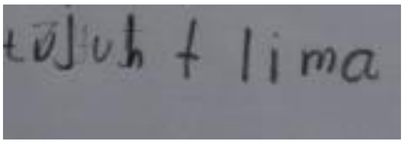


Figure 5.14e: Riko's solution

When we gave the students the next problem that was to hide those monkeys into two coconuts trees, three students, Nada, Siti, and Riko, had similar ideas to their solution in finding the number of monkeys. Nada put 5 monkeys in the first tree and 7 monkeys in the second tree, Siti put 6 monkeys in each tree, and Riko put 5 monkeys in the first tree and 7 monkeys in the second tree. Bimo changed his idea into 8 monkeys on the first tree and 4 monkeys in the second tree. Bimo was able to find the other combination that makes 12. Inaya also changed her idea into 6 monkeys in each tree. She had similar answer to Siti that was using doubles.

From hiding monkey picture sheet, we concluded that the students were able to decompose number up to 20. In this case, they were familiar to decompose 12 into 10 and 2, 6 and 6, and 5 and 7. They students built some big ideas such as doubles and compensation. They also developed some strategies, using doubles, the five-structure, and compensation. However, we still found that some students still used counting one-by-one when they had to tell the number of monkeys on the figure.

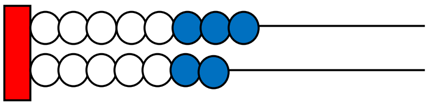
Lesson V (Exploring numbers up to 20 using the arithmetic rack)

In this lesson, the students explored number relations up to 20 using the arithmetic rack. We showed some representations of numbers using the arithmetic rack in a few second to the students. Since they did not have enough time to count the beads one-by-one, they had to count in structuring way. Based on our conjecture, they would count by five, ten, or doubles. We started the activity by representing a number using the arithmetic rack, asked them to tell about the number of beads on that arithmetic rack and told about how they knew the answer.

When we showed to the students an arithmetic rack, they were able to know the number and the structures of beads on it. The students knew that the total beads on the arithmetic rack are 20 containing 10 beads in each line. They also knew that in each line contains 5 white beads and 5 blue beads. They knew it since they had experienced to represent some numbers on the parrot game.

We continued the activity by showing a representation of numbers using the arithmetic rack and asked them to give the answer. We describe the discussion on the following segment:

Researcher moves and shows 15 beads, 8 on the top and 7 on the bottom, to the left side in a few seconds. The students raised their hands to give the answers.



Researcher	*: Ok, Inaya, How many are these?*
Inaya	*: 17 (pause), 15.*
Researcher	*: Which one is your answer, 15 or 17?*
Inaya	*: 15*
Researcher	*: Do you have other ideas?*

Siti shaked her hand to indicate that she did not have an idea.

Bimo	*: 14*
Researcher	*: Bimo's answer is 14.*
Nada	*: My answer's is 15.*
Riko	*: 17*
Researcher	*: Inaya, How did you know 15?*
Riko	*: Sir, my answer is also 15.*
Inaya	*: 10 plus 5.*
Researcher	*: and Nada?*
Bimo	*: 15*
Nada	*: 12 plus 3.*
Researcher	*: Let's we see! A few minutes ago, I moved these (Moving 8 beads to the left) and these (Moving 7 beads to the left)*
Nada	*: hahahah…*
Researcher	*: How many are these?*
Bimo	*: 15*
Researcher	*: Inaya, How do you know 10? (since Inaya did not react, the researcher asked) What did Inaya say before?*
Nada	*: 10 plus 5*
Researcher	*: Which one are 10?*
Inaya	*: These (Pointing 10 white beads and Nada also points white beads)*
Researcher	*: And these? (Pointing 5 blue beads)*
Nada	*: 5*
Nada	*: These are 10 (Pointing white beads), these are 2 (point 2 blue beads on the bottom, and these are 3 (Pointing 3 beads on the top)*

Some students struggled to know the number of beads when we showed the arithmetic rack in few seconds. They needed to count the objects more flexible way such as grouping by 5 or 10. The phrase '*17 (pause) 15*' showed that Inaya was doubt with her answer, but she could correct it while we asked her again. Bimo and Riko also struggled to know the number of beads, but they could correct their answers after they listened the other students' explanation. Inaya saw the structure of beads as 10 plus 5. She showed the beads based on those colors in which there were 10 white colorful beads (5 and 5 beads) and 5 blue colorful beads (3 beads on the top and 2 beads on the bottom). This showed that she used the ten-structure combined with the five-structure. In other hand, Nada had different strategies to know the number of beads on that arithmetic rack. She showed the beads as 12 + 3. She explained that there was 10 + 2 + 3. She used decomposing strategy to know the number of beads. She decomposed 8 into 5 + 3 and 7 into 5 +2.

We gave students the next problem in which we represented 17 using 10 beads on the top and 7 beads on the bottom. Since we just showed students in a few second, a student, Siti, was still not able to know the number of beads on the arithmetic rack. When we asked Bimo the number of beads on that arithmetic rack, he still doubted between 17 and 19 beads. After that we asked Riko to give his answer and reason. The following segment describes the discussion with the students.

Researcher : Riko!
Riko : 17
Researcher : How do you know 17?
Riko : Ooo…
Inaya interrupts
Inaya : 10 plus 7
Riko : 15 plus 2

The phrase '*10 plus 7*' indicated that Inaya recognized the structure of beads in which 10 beads on the top and 7 beads on the bottom. Meanwhile, Riko said *'15 plus* 2'. This meant that he combined 10 beads on the top together with 5 white beads on the bottom, and by adding 2 blue beads on the bottom become 17. He counted the beads using the ten-structure combining the five-structure. Both students had different strategies to reason 17 on the arithmetic rack, but they could see the number of beads in more structuring way.

We gave the next representation of a numbers using the arithmetic rack to Siti since she was still not able to tell her idea on the previous two activities. We showed a representation of 13 using the arithmetic rack, 8 on the top and 5 on the bottom. We describe on the following segment.

Researcher : How many beads are these?
Siti : I do not know
Researcher : Ok, How many beads are these? (Moving 3 white beads to the right and pointing 10 blue beads on the left)

Siti : 10 plus 3, so 13.

For the first time, Siti did not have an idea how many beads that we showed to her. We tried to ask her first the blue beads on the left. She could give her answer that was 10 directly without counting the object. She actually knew the structure of number that was represented by beads on the arithmetic rack. She also could give her answer that 10 plus 3 equals 13. Her struggling was that she needed more time to see the number of beads comparing to the other students.

From exploring numbers up to 20 using the arithmetic rack, we concluded that the students needed times to see the structures of beads on the arithmetic rack, but we had to not give too much time because they would count the beads one-by-one. The students had an idea that numbers can be related one each other. They preferred to see the structured of beads as ten and ones. They sometimes showed the number of beads based on the color such as seeing the white beads first that adding to the blue beads. By discussion and giving some opportunities to those students, they could engage on this activity.

Lesson V (Addition up to 20 with contextual problems)

In this lesson, we gave students two contextual problems that involved addition up to 20. The different of the first problem from the second one is that the students have to add two digit numbers to one digit number in the second problem. Actually, the goal of this activity was that students were able to perform combinations that make ten and decompose of other numbers in solving addition problems up to 20.

The students first worked few minutes to solve a problem as follows:

Bayu sees there are 7 eggs on the box in the kitchen. After that his mother comes from market and buys 8 eggs more. How many eggs they have now?

Based on our observation, the students had different strategies to solve this problem. We described students' answers as follows.

Inaya gave an answer that was 15 eggs. She wrote her strategy that was counting (*dihitung*) (*Figure 5.15*). Based on our observation, she used counting on strategy because she counted from 7 by using her fingers. She synchronized one word for every object. She struggled to move to other strategies to solve this problem.

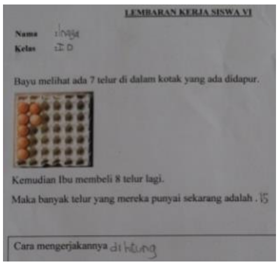
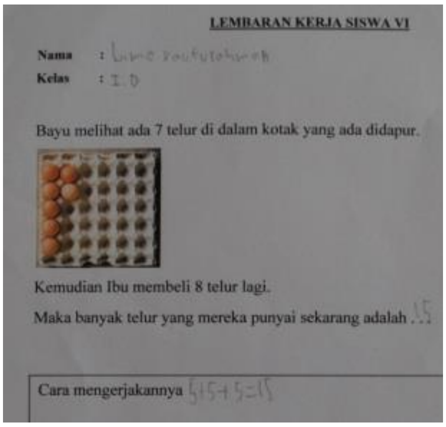


Figure 5.15: Inaya's works on eggs on a box

Bimo wrote his strategy as 5+5+5=15 (*Figure 5.16a*). Based on our observation, Bimo used an arithmetic rack as model of situation. He first took 7 beads on the top and 8 beads on the bottom. He decomposed 7 beads into 5 white beads and 2 blue beads and 8 beads into 5 white beads and 3 blue beads. Then he made five from 2 blue beads and 3 blue beads. As a result, he used the five-structures, so 3 of 5 made 15. Meanwhile, Siti wrote 10 + 5 = 15 (*Figure 5.16b*). She explained that 2 of 5 white beads made 10 and 2 blue beads and 3 blue beads made 5. She worked using the five-structure, making ten, and using doubles. He built big ideas that were doubles and combinations that make ten.



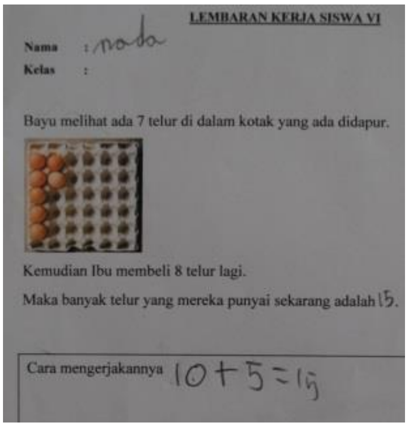
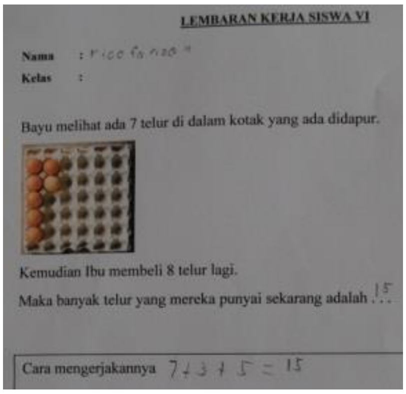


Figure 5.16a: Bimo's work *Figure 5.16b: Siti's work*

Riko and Nada have similar idea in solving this problem. They wrote 7+3+5=15 (*Figure 5.17a and 5.17b*). They used the big idea of combinations that make ten. They decomposed 8 into 3 and 5 then added 3 to 7 became 10. In Nada's worksheet, we saw that she added 10 to 5 equal 15. Both students used making tens strategies to solve this problem.



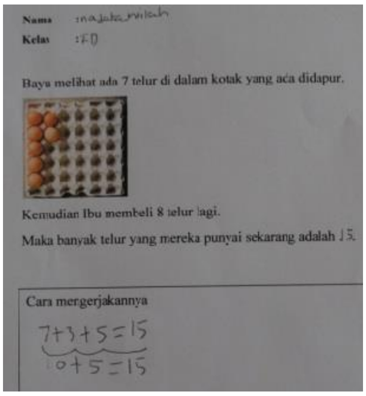


Figure 5.17a: Riko's work *Figure 5.17b: Nada's work*

After discussion how they solved first problem, we gave them the second problem that is described as follows:

Ani has 13 candies in her pocket, and then she gets 4 more candies, so how many candies does Ani have now?

Inaya, Siti, and Bimo's answer was 17. Inaya wrote her strategy in the worksheet that 16+1=17. She explained that she used counting on. She counted from 13 to 16 and then added 1 more from 16 equals 17. She preferred to use counting on strategy using her fingers than the arithmetic rack. Meanwhile, Siti wrote 2 strategies on her worksheet. The first one, 16+1=17 (*Figure 5.18a*), was similar to Inaya's strategies. Her explanation was similar to Inaya that she counted from 13. When we provoked her to show her answer using the arithmetic rack, she found the second solution that is 10+7=17 (*Figure 5.18b*). She saw there were 10 beads on the top and 7 beads on the bottom. She built the idea of decomposing ten and one. In other hand, Bimo wrote in his worksheet that 10+6+1=17 (*Figure 5.18c)*. He used the arithmetic rack as a model for situation. He showed his strategy to solve this problem using the arithmetic rack that he first took 10 beads on the top, took 6 beads on the bottom and added 1 equal 17. He had strategies that were decomposing ten and ones, using doubles of three to get 6, and adding 1 to get 17. This seemed that he develop the big ideas of doubles and combinations that make ten.

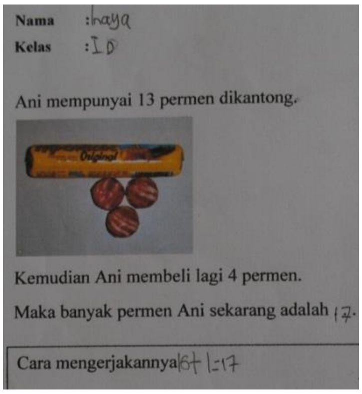


Figure 5.18a: Inaya's work

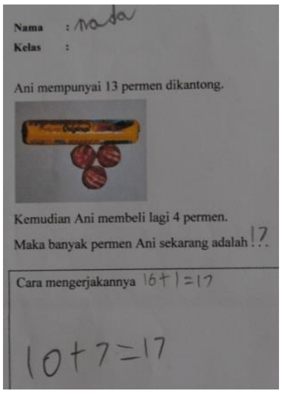


Figure 5.18b:Siti's work

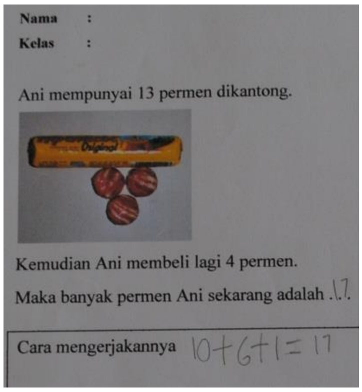


Figure 5.18c: Bimo's answer

Riko's answer was also 17. In his worksheet, he wrote 5+5+ as his strategy (*Figure 5.19a*). Based on our observation, he used the arithmetic rack as a model of situation. He took ten beads on the top, three beads on the bottom to represent 13. After that he added 4 more beads on the bottom. He represented the beads on the arithmetic rack into numbers on the beads. Since he showed there were doubles of 5 on the first rack, he wrote 5+5. He just wrote numbers to represent the beads he saw on the arithmetic rack.

Nada's answer was 16. When we asked how she got 16, she explained that 13 plus 3 is 16. We asked her how she got 3, she reread the problem that we gave and realized that was 4. Her struggle was that she did not read carefully the problem. After that she revised her answer became 10+3+4=17 (*Figure 5.19b*). She decomposed 13 into 10 and 3. She could perform number relationships and decompositions ten and ones. After that she wrote 13+4=17. First, we thought that she would add 3+4 become

7, but she added 10+3=13, and 13+4=17. Based on our observation, she did not count on 13, but she directly wrote 17 that meant she knew facts.

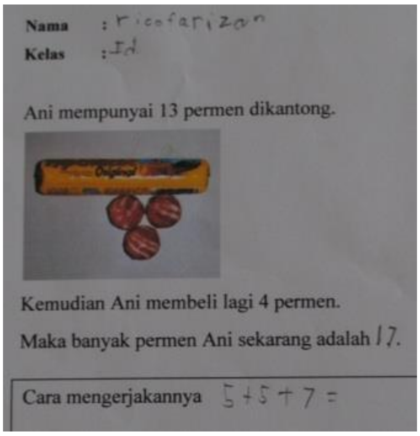


Figure 5.19a: Riko's work

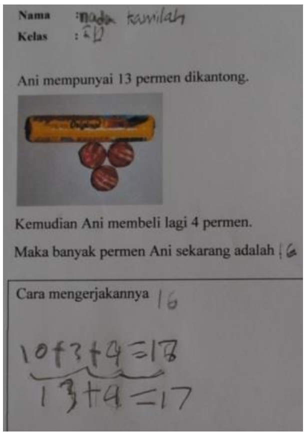


Figure 5.19b: Nada's work

Based on students' worksheet, we drew some conclusion. The students used different strategies and big ideas to solve those problems. They had the big ideas of doubles and combinations that make ten, and they used some strategies such using the five-structure, making tens, using the know facts, and counting. The students were also still need to use the arithmetic rack as model for situation. Although they had those strategies, some students still used counting on strategy. We think that the students need more activities to improve their big ideas and strategies.

5.3 Conclusion of the Preliminary Experiment Activity

The preliminary experiment activity with a small group of students (5 students) in the first grade showed that the students needed to build the big ideas, doubles, compensation, part/whole relationship, and combinations that make a ten, in order to develop mental calculation strategies on addition up to 20. However, to develop students' understanding about some mental calculation strategies such as using the five-structure, making tens, using doubles, and using doubles for near doubles, it was needed some activities that build students' knowledge on number relationships up to 20 for the teaching experiment.

Based on our observation in the preliminary experiment activity we can draw some conclusion. For the beginning of the lesson, many students still counted the objects (*pempek Palembang*) one-by-one. They synchrony one word for every object, but a student, Nada, could count the objects by group of five. She could subitize the small objects. We found that two students kept counting one-by-one and counting on when we did not provoke them to do more abbreviated strategies. Both students struggled to change their strategy although they worked on structuring objects. They would change their strategy when we asked whether they had other ideas to know the number of objects. Meanwhile, the other three students could develop the big ideas, doubles and combinations that make 10, in solving addition problems.

The other finding was that the students gave reason using knowing facts not the structure of objects that we saw to them. We could see on the flash card game in which Bimo gave a reason for 9 medicines not based on the medicine he saw, but he reasoned based on what combinations that make 9. This happened because the students were influenced by the previous activity that was decomposing number up to 10. We will change the order of those activities for the teaching experiment with whole class students.

In the lesson of hiding monkey picture sheet, some students still counted the monkeys one-by-one. We realized that the monkeys were arranged in not good structures. Since the students decomposed numbers up to 20 in many numbers, this made students memorize some number pairs up to 20 and influenced them when we gave the students addition problems at the end. They would not come to use making tens to solve addition problems up to 20 instead knowing the number pairs. For the teaching classroom experiment, we will not use the activity in this lesson more instead giving students a worksheet.

In the last lesson, we found that two students, Inaya and Siti, still used counting on strategy. Although they used the arithmetic rack to solve the addition problems, but they still counted the object one-by-one. They did not use the structure of beads on that arithmetic rack. The other three students also used the arithmetic rack to solve the addition problems. They used the structure on the arithmetic rack so that they come to some strategies such as using the five-structure and making ten. We think that the students need to share their idea so other students can develop their strategies in solving addition problem up to 20.

In general, the main purpose of the preliminary experiment activity was improving the designed Hypothetical Learning Trajectory. From the weaknesses that we found in the preliminary experiment activity, we will adjust our HLT. We name our revised HLT with “Hypothetical Learning Trajectory II (HLT II). Later, we will see how HLT II works in the teaching experiment.

5.4 The Hypothetical Learning Trajectory II

The lesson using the concrete objects such as *pempek Palembang* given in the beginning of the lessons are very useful to support students’ big ideas to develop mental calculation strategy. However, we still need to improve and adjust the activities in our initial Hypothetical Learning Trajectory.

In the first lesson, we first ask one or two students to show their ideas in making combinations of ten *pempek* in front of class and other students observe what combinations there are. Since we want the students to be able to find many combinations of ten *pempek* and have experience of this activity, we will prepare some *pempek* that are made from wax to each group. The teacher has to provoke the students to make representations that they find.

For the mini lesson, the Parrot game, we will do in the beginning of second meeting and at the end of third meeting. As we found in the preliminary experiment activity, the students were really interested to do this activity. They wrote in their reflection. The goal of this activity is to develop students' understanding of combinations that make ten and other numbers up to 10.

In the second lesson, flash card game, we will give students a worksheet in the end of activity. The goal of the worksheet is that students are able to represent numbers into circle representations. The students will give a numbers, and they have to color circles based on that number. In this activity, we can observe what big ideas students use when they color those circle.

In the third lesson, candy combination sheet, we found that the students had an idea about distributive properties on addition. We think that it is interesting topics to bring into discussion, but we will not give more attention on it because our research just focuses on decomposing number up to 10. In this activity, we first design an activity in which students will work in group to find a combination of numbers up to 10. In the preliminary experiment activity, we asked students to find combinations that make 5, but we change into 7 in this activity because they will work in a group and they have to find many combinations of that number. To find combinations up to 10, the students are allowed to use the arithmetic rack. Each group of students, 4 to 5 students, will get an arithmetic rack, so they can discuss each other.

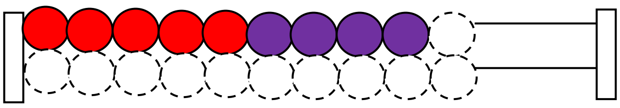
We design an activity on the fourth lesson that is making jewelry. The goal of this lesson is that students build awareness of structures. In this lesson, students are given 20 beads containing two colorful beads, blue and pink beads. They have to arrange those beads in order other students can recognize the number of beads easily. We expect that students will arrange the beads into doubles, the five-structure, and ten-structures.

In the fifth lesson, we found in the preliminary experiment activity that when we showed to the students some beads on the arithmetic rack, they had many ideas to know the number of beads. Some of their ideas is doubles, combinations that make ten, and compensation. Since we want them to use some big ideas when we give addition problem, they have to discuss which approach they prefer to use. In the end in this activity, we give students a worksheet that is similar to the third activity but different from numbers.

In the sixth lesson, Students will work on a worksheet. The goal of this activity is that students perform combinations that make a ten. On that worksheet, there are some beads have been colored. The students are given a problem in which they have to color some more beads. After that they have to write how they do it. For instance: we give the students a problem as follows:

Look at the picture below:

Please colored 4 beads more!



How do you do that?

From this problem, our conjecture that some students will color one more beads on the first row then 3 more beads on the second row. Those students have a big idea that was combinations that make ten. The other students will color 4 beads on the second row, and use counting on from 9. By a classroom discussion, the students can develop their strategies in solving addition up to 20 for the next activity.

For the last lesson, we think that two problems that we will give to the students are enough to explore students' big ideas and strategies in solving addition problems. We just need to provoke students to not use counting one-by-one strategy by offering the arithmetic rack. We will design that the students will work in pair so they can share their ideas to find the strategies in solving those problems.

We discussed with the teacher about all changes we made. The teacher also gave some suggestion to improve this Hypothetical Learning Trajectory, and we try to adjust this initial HLT together. We describe the change of the Hypothetical Learning Trajectory on the following table.

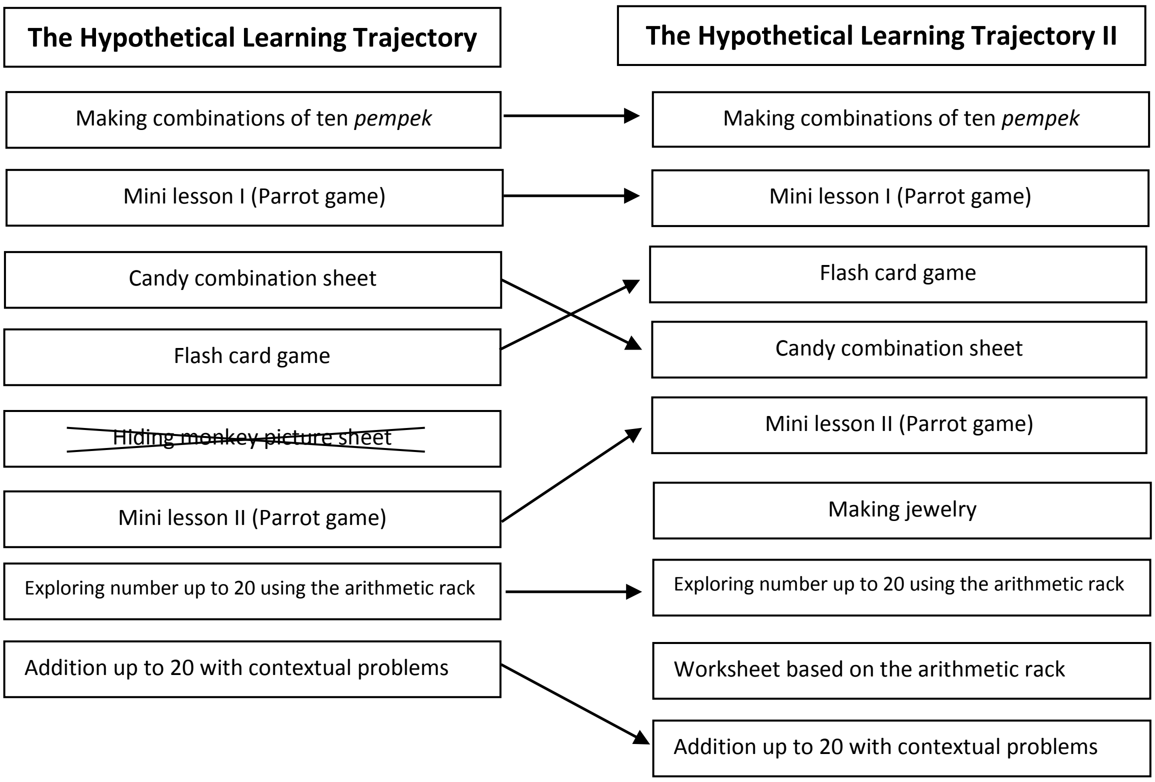


Figure 5.20: A diagram of the improved Hypothetical Learning Trajectory

5.5 Teaching Experiment Activity

In this section, the Hypothetical Learning Trajectory II was compared to students' actual learning process during the teaching experiment. We did an investigation whether or not the HLT II supported students' learning. We observed the classroom learning process by looking the video recordings and selecting some critical moments. We also used field notes and analyzed students' written works such as posters and worksheets as other sources. We conducted everyday analysis of the lesson and investigated what students and teacher did, how the activities worked, and how the materials contributed to the lesson. We also looked at connections among the lessons, and we tried to investigate how earlier lessons supported the following ones. We used the result of the retrospective analysis in this teaching experiment activity to answer our research question.

Lesson I (Making combinations of ten *pempek Palembang*)

In the first lesson, we designed an activity in which the students worked using a concrete contextual situation as the base of mathematical activities. We used the activity of making combinations of ten *pempek* as the experience-based activity. The goal of this activity was that the students were able to find many combinations of ten *pempek* containing *lenjer pempek* and egg *pempek.* At this time, the students still worked on non-formal situation in which they could experience by using wax *pempek.*

The teacher started the lesson by giving a contextual situation to the students. The teacher went to her sister's house, Bu Ani's house, yesterday. Bu Ani served the teacher with special food from Palembang, *pempek.* There were many kinds of *pempek*, but Bu Ani just made two kinds of *pempek*, egg *pempek* and *lenjer pempek.* Then, the teacher showed egg *pempek* and *lenjer pempek* that were made from wax to the students and asked them how many *pempek* actually put on a plate. Some students answered there were 5 *pempek*, and the other students answered there were 10 *pempek*. This showed that the students were familiar with this situation and recognized the number of *pempek* served on a plate. After that the teacher asked some students to put ten wax *pempek* on a plate.

The first student, Atha, put ten *bulet pempek* on a plate (*Figure 5.21a*). He did not make a combination between *lenjer pempek* and egg *pempek*. Based on our observation, Atha just took those *pempek* one-by-one. He synchronized one word for every object, so he still worked with a big idea that was one-to-one correspondence. What Atha did was not so different from Zaidan. He also put same *pempek* on a plate (*Figure 5.21b*). He put ten *lenjer pempek* and counted the number of *pempek* one-by-one. Otherwise, Firza made a different combination of ten *pempek* (*Figure 5.21c*). He first put 5 *lenjer pempek* on a plate. He did not put those *pempek* one-by-one instead put by 2, 2, and 1 *lenjer pempek*. He could subitize small amount of objects. Then he put 5 egg *pempek* on that plate, so there were 5 *lenjer pempek* and 5 egg *pempek* that made a combination of ten *pempek*. At this time, Firza used the five-structure in which doubles of five made ten. After that the students worked in group to find many combinations of ten *pempek* that they could put on each plate, and they had to record their work on a poster paper.

Figure 5.21a: Atha's work

Figure 5.21b: Zaidan's work

Figure 5.21c: Firza's work

In our observation, we found that all groups of students started making drawings of 5 *lenjer pempek* and 5 egg *pempek*. Three groups, grape, strawberry, and mango groups, made more than one drawings of 5 *lenjer pempek* and *5* egg *pempek* on their poster papers. When we interviewed a student from grape group, Farhan, he said that all combinations were similar in numbers that were 5 *lenjer pempek* and *5* egg *pempek,* but he did not reason why he made more than one drawing. From this situation, we concluded that the students were familiar to use the five-structure, and doubles of five making ten. It was one of our conjectures that the students started from 5 egg *pempek* and 5 *lenjer pempek*.

Based on the following poster papers, Strawberry group made 7 different drawings (*Figure 5.22a*), but they just made 4 different combinations of ten *pempek* that were 5 *lenjer pempek* and 5 egg *pempek,* 6 *lenjer pempek* and 4 egg *pempek,* 10 *lenjer pempek* with no egg *pempek,* and 10 egg *pempek* with no *lenjer pempek*. They did not draw those combinations on structuring ways. We also observed that they drew those combinations based on their experienced with real objects.

In line with strawberry group, mango group made 10 different drawing (*Figure 5.22b*), but they also made 4 different combinations of ten *pempek* that were 5 *lenjer pempek* and 5 egg *pempek*, 4 *lenjer pempek* and 6 egg *pempek,* 8 *lenjer pempek* and 2 egg *pempek,* and 10 *lenjer pempek* with no egg *pempek.* They drew those *pempek* in structuring ways such as 5 *lenjer pempek* on the first line and 5 egg *pempek* on the second line. It seemed that they realized drawing objects on structuring ways was recognizable that un-structuring one. This group also worked with real objects to find all combinations of ten *pempek.*

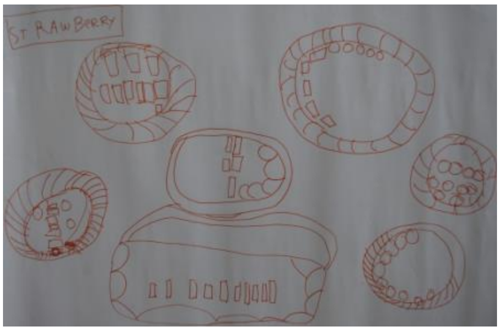


Figure 5.22a: Strawberry group's work

Figure 5.22b: Manggo group's work

Orange group made 6 different drawings (*Figure 5.23a*), and they found 5 different combinations of ten *pempek* that were 5 *lenjer pempek* and 5 egg *pempek*, 6 *lenjer pempek* and 4 egg *pempek,* 2 *lenjer pempek* and 8 egg *pempek*, 7 *lenjer pempek* and 3 egg *pempek*, and 1 *lenjer pempek* and 9 egg *pempek.* Actually they first made a wrong combination for 1 *lenjer pempek* and 9 egg *pempek.* They first drew 2 *lenjer pempek* and 9 egg *pempek.* When we interviewed a student from that group, Raka, about that combination, he said that it was 11 *pempek* in total, so it should be erased a *pempek* to make 10.

Similar situation was also found on the grape group in which they erased two *lenjer pempek* since they made a combination of 10 *lenjer pempek* and 2 egg *pempek.* Farhan, a student on the grape group, explained that there were 12 altogheter, so they had to take away two *lenjer pempek* to make a combination of 8 *lenjer pempek* and 2 egg *pempek.* This group made 4 similar combinations of ten *pempek* that were 5 *lenjer pempek* and 5 egg *pempek.* From ten different drawings (*Figure 5.23b*), they found 7 different combinations of ten *pempek* that were 5 *lenjer pempek* and 5 egg *pempek*, 6 *lenjer pempek* and 4 egg *pempek,* 7 *lenjer pempek* and 3 egg *pempek*, 8 *lenjer pempek* and 2 egg *pempek,* 1 *lenjer pempek* and 9 egg *pempek,* 3 *lenjer pempek* and 7 egg *pempek,* 10 *lenjer pempek* with no egg *pempek,* and 10 egg *pempek* with no *lenjer pempek.* Based on our observation, this group just worked with real objects to find some combinations of ten *pempek*, and then they think mentally by looking the relations among combinations they had made.

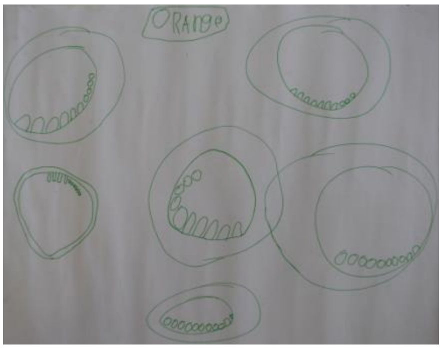
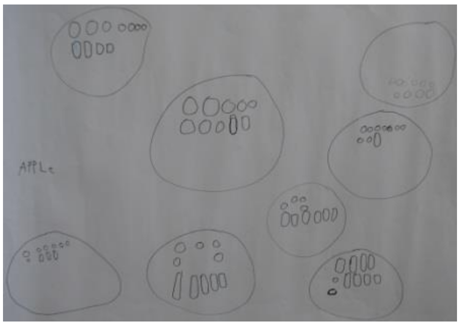


Figure 5.23a: Orange group's work *Figure 5.23b: Grape group's work*

The other groups were apple group. This group made 8 different drawings and found 7 different combinations of ten *pempek.* Apple group found combinations of ten *pempek* that were 1 *lenjer pempek* and 9 egg *pempek*, 2 *lenjer pempek* and 8 egg *pempek*, 3 *lenjer pempek* and 7 egg *pempek*, 5 *lenjer pempek* and 5 egg *pempek,* 6 *lenjer pempek* and 4 egg *pempek,* 8 *lenjer pempek* and 2 egg *pempek,* and 10 egg *pempek* with no *lenjer pempek* (*Figure 5.24a*). They drew some *pempek* in structuring ways such as decomposing *lenjer pempek* with egg *pempek*, or 5 *pempek* on the first line and 5 *pempek* in the second line. Actually, this group made an incorrect combination of ten *pempek*, but they fixed it during a classroom discussion.

Melon group made 7 different combinations of ten *pempek* that were 2 *lenjer pempek* and 8 egg *pempek,* 3 *lenjer pempek* and 7 egg *pempek*, 4 *lenjer pempek* and 6 egg *pempek,* 5 *lenjer pempek* and 5 egg *pempek,* 6 *lenjer pempek* and 4 egg *pempek*, 9 *lenjer pempek* and 1 egg *pempek,* and 10 egg *pempek* with no *lenjer pempek* (*Figure 5.24b*). They also drew in structuring ways by decomposing *lenjer pempek* with egg *pempek.* Both groups also worked with real objects for finding some combinations of ten *pempek*, and then they thought mentally by looking the relations among combinations of ten *pempek.* It seemed that the students build knowledge of number relationships. They also used the idea of commutativity such as by finding 4 *lenjer pempek* and 6 egg *pempek*, and then they also found 6 *lenjer pempek* and 4 egg *pempek.*



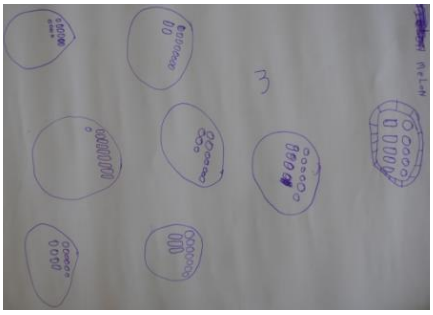


Figure 5.24a: Apple group's work *Figure 5.24b: Melon group's work*

The group worked was continued by group presentation. We chose the following fragment from Apple group's presentation because the other groups could react with their presentation. They were able to clarify whether a group made a wrong combination of ten *pempek.*

Atha : There are 8 lenjer pempek and 1 egg pempek. (Pointing the drawing on the poster paper)
Teacher : So… There is 1 egg pempek. Listen! (Asking students to focus on the discussion) And there are 8 lenjer pempek.
Firza : Wrong.
Farhan : Wrong.
Teacher : Is it wrong?
Students : Wrong. (Saying enthusiastic)
Teacher : Why is it wrong?
Students : There are 9 altogether.
Teacher : Why is it wrong?
A student : Yes, because there is less 1 egg pempek.
Teacher : Oh, it is less 1. What should be there?
Student : 10.
Teacher : So, how to make it 10?
Atha : My friend did it. (Pointing his friend)
Teacher : Oh, she forgot to write it. Please repair! Which one do you want to draw?
Atha pointed a drawing on the poster paper.
Atha : Drawing one more. (Saying weakly)
Teacher : Which one is less?
Student : 1
Teacher : Which pempek do you mean?
Student : An egg pempek. (Atha also said it)
Atha drew an egg pempek on his poster paper.
Teacher : So, there is a discussion to repair a mistake.

Atha were able to present his group's work to other students. He explained the number of *pempek* his group drew to other friends. Since his voice too weak, so the teacher repeated his work to other students. It was a socio norm that we found in the classroom teaching experiment. The other socio norm that we found was that the students said something together. When the teacher repeated what Atha said '*there is 1 lenjer pempek and 9* egg *pempek',* the students directly judged that was not correct. It showed by the phrase '*wrong*'. The students recognized that 8 and 1 make 9. Then the phrase '*because it is less one'* indicated that the students knew that they neede 1 more to make ten, but it was also possible that they had a big idea that was part/whole relationship: relationship between addition and subtraction.

Summary

From the activity in this lesson, we make some conclusions. The students started making a drawing of 5 *lenjer pempek* and 5 egg *pempek.* It seemed that they were familiar using the five-structure and doubles of five making ten. They had an idea of doubles to make a combination of ten. Some groups were able to find many combinations of ten *pempek* and the other group drew a similar combination, *5 lenjer pempek* and 5 egg *pempek,* up to 5 drawings. This was influenced by their experience putting *5 lenjer pempek* and 5 egg *pempek* on a plate in many times without checking their previous drawing. In the discussion, the students were able to clarify when they found an incorrect combination of ten *pempek.* From a discussion, the students built the big idea of part whole relationships since they connected between addition and subtraction.

Mini Lesson I (Parrot Game)

In the mini lesson I, the students played a game that was a Parrot game. The goal of this game was to develop students' understanding about combinations that make ten. The teacher started the lesson by showing a Parrot to the students and told that the Parrot was able to know combinations that make ten. The students have to tell combinations that make ten.

Teacher : Please raise your hand if you know combinations that make 10!
The students raised their hand

Teacher : Rafi.
Rafi : 8 plus 2.
Teacher : yes, 8 and 2. True, true, and true (Parrot said)
Teacher : Who is next? (The students raised their hands) Fabella!
Fabella : 7 plus 3.
Teacher : Do not say addition, but combinations!
Students : 7 and 3.
Fabella : 7 and 3.
Teacher : True, true, and true (as a Parrot)
Teacher : Please raise your hand if you know! Agung! (The students raised their hands)
Agung : 6 and 4 (Agung said weakly)
Teacher : True, true, and true (as a Parrot)
Teacher : Please raise your hand if you can say a combination! (The students raised their hands) Firza!
Firza : 5 and 5
Teacher : True, true, and true (as a Parrot)

The phrase '*8 plus 2*' and '7 *plus 3*' showed that Rafi and Fabella thought combination that make 10 as addition two numbers that equal ten. Both students used the word '*plus'* to reason combinations that make ten, but the phrase '*do not say addition, but combinations*' said by the teacher influenced students to change '*plus'* became *'and'*. When we saw the phrase '*8 plus 2'*, '*7 plus 3*', *6 and 4,* and *'5 amd 5'* that were said by four students respectively, it seemed that the next student said a combination was inspired by the previous student, for instance: Agung said *'6 and 4',* then Firza said *'5 and 5'*. Those students develop a big idea of compensation that was moving 1 from 6 to 4 to make 5 and 5.

Teacher : Ok, Please talk directly if you know combinations that make 10!
Firza : 9 and 1
Agung : 4 and 5
Teacher : Ah!
Chantika : 5 and 5
Teacher : Mmmm (as a Parrot)
Rista : 5 and 5
Tata : 4 and 6

Teacher : *Is it correct 4 and 5?*
Students : *Incorrect*
Teacher : *So, what is the correct one?*
Students : *5 and 5*
Teacher : *What is a pair of 4?*
Students : *6*

The other combination that makes ten the students said was 9 and 1, but a student, Agung, said an incorrect combination that was showed by the phrase *'4 and 5'*. The other students directly corrected it. Chantika and Rista corrected it to be 5 and 5. It seemed that they added 1 to 4 became 5. Otherwise, the phrase *'4 and 6'* showed that Tata had a different idea with her friends. It seemed that she added 1 to 5 became 6.

After playing the Parrot game, the teacher showed an arithmetic rack and asked some students to experience on it to represent combinations that make 10. Based on our observation, the students were able to recognize the number of beads on the arithmetic rack. It seemed that they were familiar with the structures of beads on the arithmetic rack. In fact, they did not count the beads one-by-one when the teacher asked them to represent combinations that make ten. Raka represented a combination of ten as 4 and 6, Zaidan represented 5 and 5, Rafi represented 7 and 3, and Firza represented 8 and 2. Those representations were shown by the following figures.

Figure 5.25 Raka, Zaidan, Rafi, and Firza's representations of combinations that make ten

Summary

From mini lesson I (Parrot game), it can be concluded that the students first thought that combinations that make ten as an addition of two numbers. It seemed there was a problem between students and teacher’s language, but the students were able to find many combinations that make ten. Some students used the big idea of compensation to find other combinations that make ten. A mathematical socio norm happened in a discussion on a classroom that other students directly corrected when they found an incorrect combination made by their friends. The students were also able to represented combinations that make ten using the arithmetic rack. It seemed that they were able to recognize the structure on the arithmetic rack.

Lesson II (Flash card game)

In this lesson, we designed an activity, flash card game, in order to build students’ knowledge about number relationships such as doubles, almost doubles, and five- and ten- structures. We used medicine tables as a contextual situation, and then we modified it became flash cards. We designed some flash cards that represented the number of medicines.

The teacher started the lesson by asking students’ experiences when they were sick. The teacher asked them what a doctor or mother gave to help them. The students told that they used medicines such as syrup and tablets. After that the teacher showed a medicine tablet and asked their comments on it as follows.

Teacher : Who can give comments about this? (Showing a medicine tablet to the students)

The students raised their hands
Teacher : Gibran!
Gibran : Medicines
Hafiz : Candies
Students : Syrup
Teacher : Is it syrup?
Raka : 10
Teacher : How many medicines are these? (Asking Raka)
Raka : 10

Teacher : Raka said it was 10.
Teacher : Do you have other ideas?
Firza : 8 and 2 (Saying weakly)
Teacher : What? 5 and 2? Do you mean there were 2 of 5?(The teacher did not hear Firza's answer clearly)
Firza : no, 8 and 2.
Teacher : Oh, 8 and 2. How do you know it?
Firza pointed the medicine tablets as follows:

Teacher : So these are 8 plus 2. (Pointing the medicine tablet)
Teacher : Do you have other ideas?
Aqila : 3 and 7
Teacher : Which one do you mean?
Aqila pointed the medicine tablets as follows:

Teacher : Do you have other ideas?
Atha : 6 and 4
Teacher : Which one do you mean 6?
Atha pointed 6 medicines first and then 4 medicines as follows:

Teacher : Who is next?
Chantika : I have an idea that was 5 and 5.
Teacher : Which one do you mean 5 and 5?
Chantika pointed the medicines as follows:

For the first time, the students did not come to a mathematical idea. This showed by the word '*medicine', 'candies',* and '*syrup*'. Since the teacher asked the students more about their opinion on that medicine tablet, a student, Raka, had an idea that was 10 medicines. This showed by the word *'10'*. Since there was no more interaction between the teacher and Raka, we could not know how Raka knew the number of medicines on that tablet. The phrase *'8 and 2'* said by Firza was a combination that makes a ten. It seemed that he did not give a reason based on structuring on the medicine tablet instead saying a combination that makes ten. This was followed by other students. The other students said some combinations that make ten. It seemed that they had a big idea that was compensation. This could be seen in which Aqila said *'3 and 7'* after Firza said '*8 and 2*'. However, the phrase *'5 and 5'* seemed that Chantika used the five-structure to know that doubles of five made ten. After that the teacher showed another medicine tablet to the students. Then the teacher continued by showing a flash card game.

Teacher : *You have to tell about the number of medicines on this tablet. Ok are you ready? (Showing a medicine tablet)*

Students : *9 medicines*
Zaidan : *5 and 4*
Teacher : *yes, 5 and 4. Do you have other ideas?*
Firza : *7 and 3.*
Teacher : *Mmm. 5 and 4. (The teacher did not react to Firza's answer)*
Harnita : *7 and 2.*
Chantika : *4 and 5*
Niken : *3 and 6.*
Fabella : *3 and 5.*
Teacher : *Please see the medicine tablet first!*
Chantika : *4 and 5.*
Gibran : *8 and 1.*

The students were able to know quickly that the number of medicines was 9. A student, Zaidan, said *'5 and 4'*. It seemed that he showed the structure of medicines on that tablet that was 5 medicines on the bottom and 4 medicines on the top. Meanwhile, Firza still thought on a combination that makes ten. It was showed by the phrase '*7 and 3*'. We found that the teacher did not react to firza's answer directly instead repeating 5 and 4. The phrase '*7 and 2*' seemed that Harnita tried to find a combination that makes 9 medicines using Firza's answer. Actually, it seemed that many students gave some reasons based on combinations that make a number instead reasoning based on structures on that medicine tablets. This showed by the phrase *'3 and 6*' and *'8 and 1*'. Since the teacher did not ask Fabella's idea why she said 3 and 4, we did not know her thinking on that medicine tablet.

Teacher : *Raise your hands first and then you can give an answer!*

The students raised their hands, but some students told the answers.

Some students : 8
Teacher : Keep silent and just raise your hands! Rizki!
Rizki : 8
Teacher : How do you know it?
Rizki : Thinking
Fabella : 5 and 3.
Teacher : yeah 5 and 3.
Teacher : Ok, can you tell other ideas?
Rafi : 4 and 4.
Teacher : Let's talk Niken!
Niken : 3 and 5. (Niken used her fingers, 3 fingers on her right hand and 5 fingers in her left hand)
Firza : 4 and 4.

Some students said 7 and 1 and the other students said 2 and 6. Raka, one of them, said 2 and 6 and the teacher asked him to show it. He showed as follows:

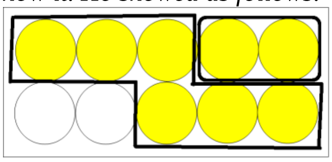


Figure 5.26: Raka's idea 2 and 6 on flash card

The word '*thinking'* showed that Rizki thought mentally to know the number of colorful circles on that flash card game. He knew there were 8 colorful circles on that figure. In other hand, Fabela and Niken gave a reason based on the structures of colorful circles on that figure. It was showed by the phrase *'5 and 3'* and *'3 and 5'*. Niken used her fingers as concrete objects to represent the number of circles on that flash card. It seemed that she connected 5 circles on the bottom with her left fingers, and 3 circles with her right fingers. The other students, Rafi and Firza used known facts to reason on the number of colorful circles on that flash card game. The phrase *'4 and 4'* showed that they tried to find number pairs that make 8. It seemed that they came to the idea of doubles. Meanwhile, Raka also gave a reason on number pairs that make 8 that were 2 and 6. His reason that was showed by the figure 5.26 was unpredictable. He pointed 2 colorful circles on the first bottom, and said that there were six more circles that were 3 on the top and 3 on the bottom. He constructed his own structure.

After the teacher gave some flash card game, the students continued to work on a worksheet. They worked in pair to color some circles to represent the number of medicines that were still on that tablet. In general, we found that many students used the five-structures. They colored 5 circles on the top first then continued to the bottom, but we found three students' answers on their worksheets that they also used doubles and near doubles. They had a big idea that was doubles. This showed by the following students' answer on those worksheets (*Figure 5.27).*

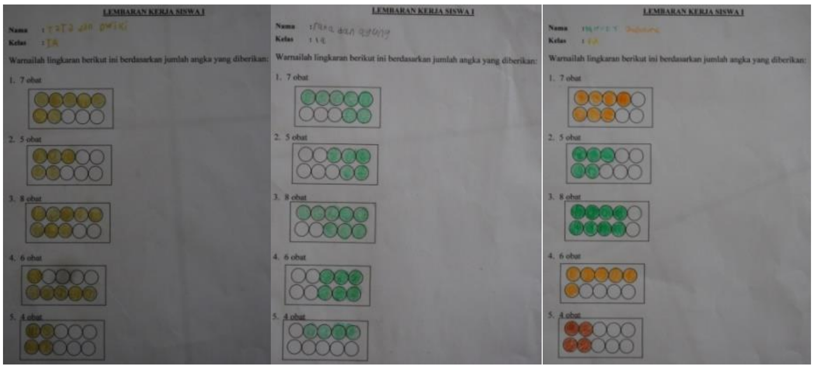


Figure 5.27: Student's work on flash card worksheets.

Summary

From this lesson, we concluded that some students gave a reason based on structuring that they showed on flash cards, but the other students gave reason based on their knowledge on combinations that make a number. Since they knew combinations that make ten, they reasoned on it while the teacher showed ten medicines on a medicine tablet. Although they did not totally reason on structuring numbers, we still found some students had big ideas that were doubles, and compensations. The students also used some strategies to reason how they know the number of circles on the flash card. They used known facts, the five-structure, compensations, and doubles for near doubles. We also found that two mathematical socio norm that the students gave an answer together, and the teacher did not react directly to the students when the students gave an incorrect answer.

Lesson III (Candy combination sheet)

In this lesson, the students were challenged to decompose number up to ten. They started by experiencing on finding combinations two candies that were taken on a can without seeing inside the can. Before the students did this activity in a group, the teacher started the lesson by giving a contextual situation in which there was a birthday party. In that birthday party, a host asked the teacher to take 7 candies on a can containing grape candies and milky candies. They have to figure out as many as possible combinations of those candies. The candies on this research were substituted by beads that put on a can.

For the first time, the teacher asked the students to figure out how many milky candies she got, if she had 5 grape candies. We observed that the students could figure out that there were 2 milky candies. A student, Farhan showed by his fingers, 5 fingers in his right hand and 2 fingers in his left hand. This meant that the students were able to figure out what combinations they could make from 7 candies. After telling the contextual situation to the students, and two students tried to take 7 beads on a can in front of class, the students worked in group to find as many as combinations of 7 candies.

Based on our observation, three groups, orange, melon, and strawberry missed understanding on the task that they had to do. They did not try to find combinations of 7 candies, but they count the number of candies on the can. Since there were many groups still did not understand the task, the teacher repeated the information on that task.

Teacher	*: What’s number is it? (Pointing the number on the poster paper)*
Students	*: 7.*
Teacher	*: Let’s give an example! How many grape candies could we get?*
Atha	*: 0.*
Chantika	*: 10.*

Firza : *4*
Teacher : *It is 7. (Pointing 7 on the poster paper)*
Teacher : *How many is it? (Pointing grape)*
Students : *4.*
Teacher : *How many should be the milky candies?*
Students : *3.*

The word *'10'* showed that Chantika counted the number of grape candies on the can. There were 10 grape and 9 milky candies (*Figure 5.28)*. Meanwhile, the phrase '*it is 7'* indicated that the teacher tried to emphasize there should be 7 altogether, so it was not so possible there were 10 grape candies.

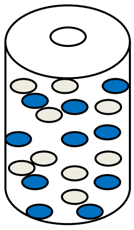


Figure 5.28: A can contains grape and milky candies.

To find some combinations of 7 candies, the students used different ideas, strategies, and models. Since each group was facilitated with two models, beads on a can and arithmetic rack, they could experience with those and chose models that they prepared. We also found some students used their fingers and knew mentally. We described students' works as follows.

The students from the grape's group used beads on a can and also the arithmetic rack. Atha made a combination of 7 candies by taking 3 green beads on the top and 4 green beads on the bottom (*Figure 5.29*). Meanwhile, Aqila showed there were a new combination of 7 candies from 2 green beads and 5 red beads. In fact they found two different combinations of 7 candies that they wrote in the second and third column (*Figure 5.29)*. From grape's worksheet, there was a big idea that they found that was a commutative property on addition. The students in this group struggled to find the fifth combinations. They tried some combinations but it had been found in the previous one. Finally, they found 7 grape candies, and there was no milky candy.

Figure 5.29: Apple group's work and worksheet on a candy combination sheet

The students in strawberry group used their fingers to find a combination of 7 candies. Zaidan showed to his friends 5 left fingers and 2 right fingers to represent grape and milky candies. They found a combination of 7 candies that was written in the first column. Based on our observation, they moved from using fingers to the arithmetic rack (*Figure 5.30).* They found a combination of 7 candies that was 1 grape candies and 6 milky candies. This was a learning process to find a flexible model to solve a mathematical problem. This group also had a big idea that was commutativity.

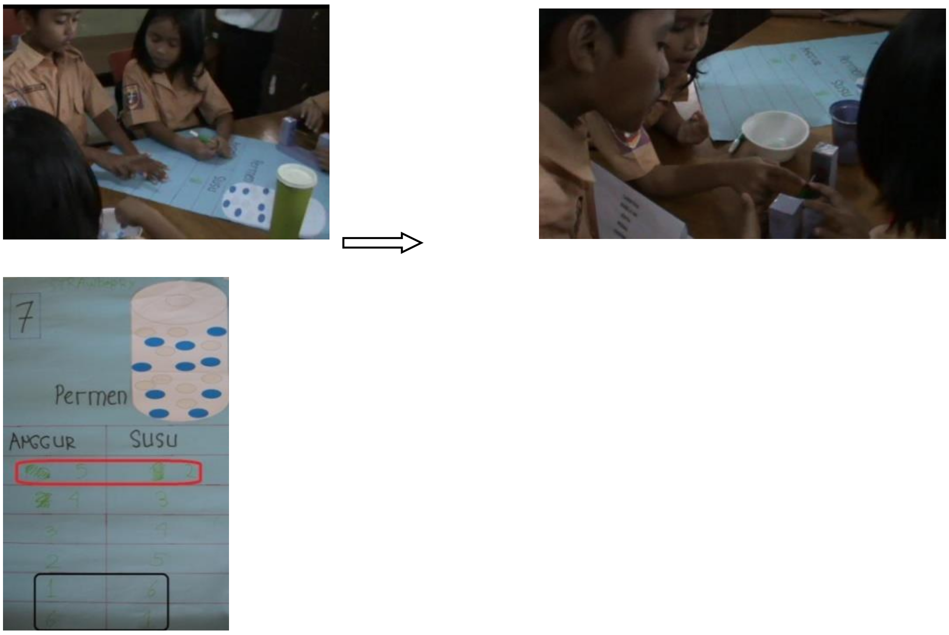


Figure 5.30: Strawberry group's work (From fingers to an arithmetic rack).

The big idea of commutativity also emerged from mengo and melon's worksheet. The students in melon's group found that if there were 6 grape candies and

1 milky candy, there was also possible to have 1 grape candy and 6 milky candies. The students in mengo group found combinations of 4 and 3 became 3 and 4 in another way. Meanwhile, the students in grape group had a different strategy. Based on the worksheet (*Figure 5.31),* they used known facts to know other combinations of 7 candies such as 4 and 3 became 5 and 2. Those students were emerged a big idea of compensation.

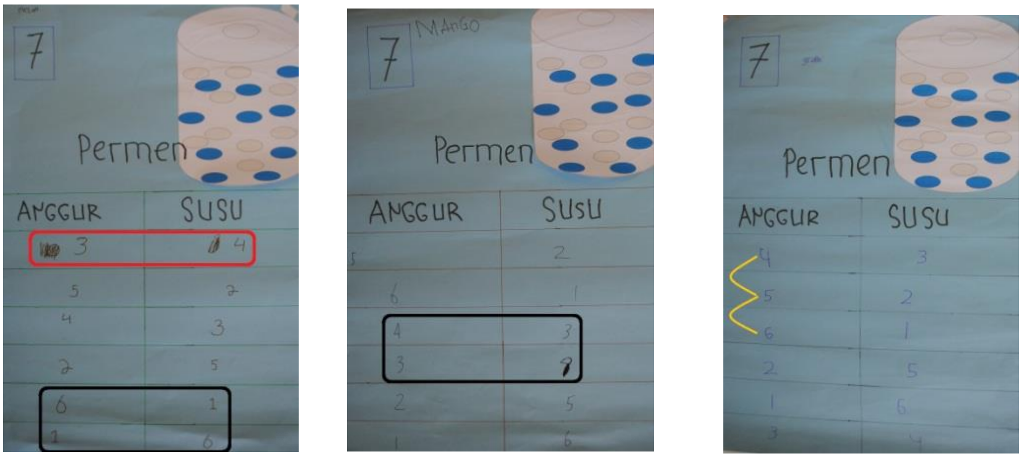
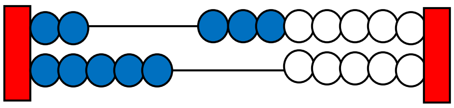


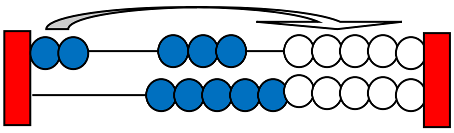
Figure 5.31: Melon, Manggo, and Grape groups' work on candy combination sheets

After the students worked in group to find many combinations of 7 candies, they had to present their work in a classroom discussion. Firza, a student from grape's group presented their worked. He told how they found combinations of 7 candies.

Firza : These are 2. (Moving 2 white beads directly to his left side) and plus 5.
Firza : 1, 2, 3, 4, and 5. (Moving the next beads to the left side one-by-one).
Teacher :Firza moved 2 (Moving 2 blue beads to the left side as Firza's did). What is a number pair of that?
Students : 5.
Teacher : Who can move 5 directly? Please Raka!
Raka went in front of class and move 5 beads on the bottom directly.



Teacher : Do you have another idea?
The students raised their hands.
Teacher : Let's go!
Nike went in front of class moved three blue beads next to two blue beads and pointed 5 white beads.



The students showed different ideas to represent a combination that makes 7. Firza showed his strategy that was to take 5 beads more next to two first beads, but he still used counting on. Meanwhile, Raka and Nike were able to perceive small amount such as five. Raka represented 2 and 5 by taking 5 blue beads on the bottom. He recognized that there were 5 blue beads on the bottom. Meanwhile, Niken represented as different color. She represented 5 by white beads. She was also able to recognize the structure of beads on the arithmetic rack.

The students continued the lesson by working in pairs on a worksheet. There were some big ideas showed on their work (*Figure 5.32*). The big ideas that we found on those worksheet were doubles, compensation, and commutativity. The students used doubles for near doubles and compensation.

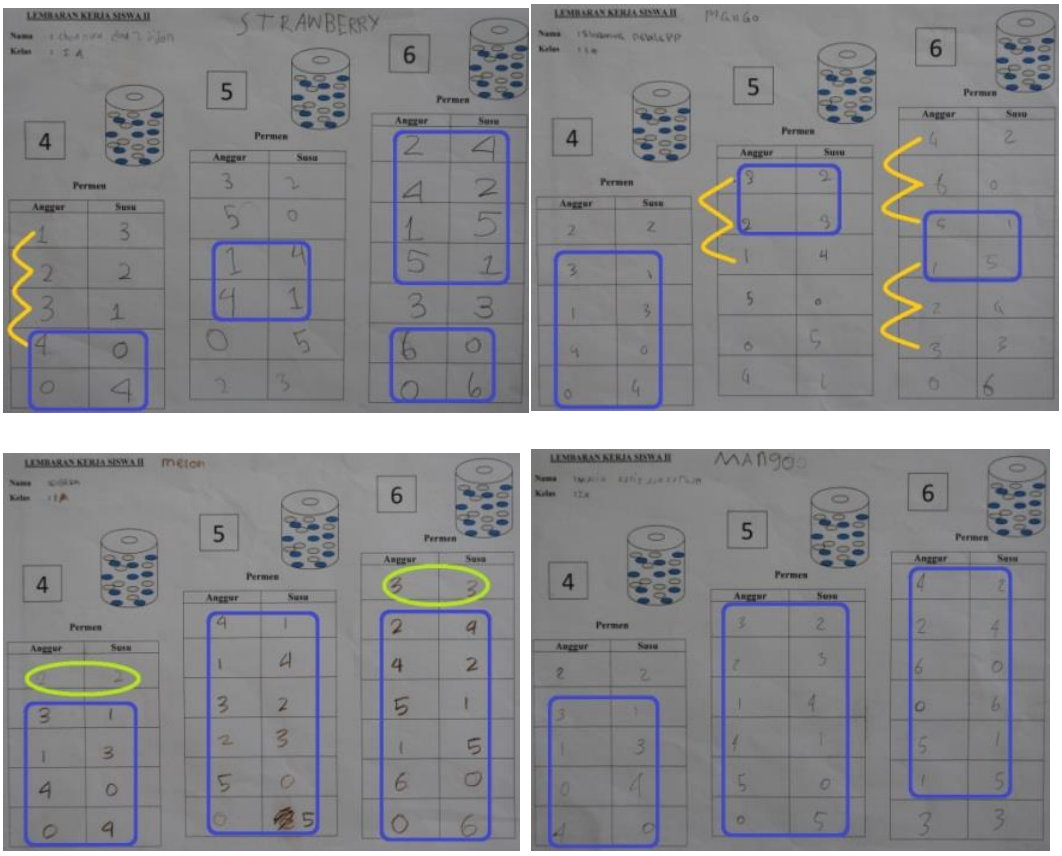


Figure 5.32: Some students' work on candy combination sheet

Summary

From this lesson we could make a conclusion. The students were able to find many combinations for some numbers up to 10. To find those combinations, they used fingers, arithmetic racks, and knowing mentally. A group of students was able to move from using fingers to an arithmetic rack. The students were able to develop some big ideas, doubles, compensation, and commutativity. The idea of commutativity is often used by students to find other combinations of numbers up to 10.

Mini Lesson II (Parrot Game)

The activity in the mini lesson II was similar to the mini lesson I, but the difference was the numbers that was used. At this time, the students were challenged to find combinations that make a number up to 10, for instance 7. The goal in this lesson was that students developed understanding about decomposing numbers up to 10. They had to give combinations of a number mentally.

Teacher : Last time, we had learnt combinations that make 7.

The teacher asked students to raise their hands if they wanted to talk.
Teacher : Ok. Please tell combinations that make 7! Ok Agung.
Agung : 5 plus 2.
Teacher : 5 and 2, true, true, and true. (Shaking the Parrot head). Who can tell more?
Some students raised their hands.
Rafi : 4 and 3.
Teacher : 4 and 3, true, true, and true. (Shacking the Parrot head).Ok, who is next?
Fabella raised her hand.
Teacher : yes, Fabella.
Fabella : 3 and 4.
Teacher : 3 and 4, true, true, and true. (Shaking the Parrot head). Who is next?
Maudy raised her hand.
Teacher : Maudy!
Maudy : 5 and 6. (Saying weakly)
Teacher : The Parrot cannot hear.
A student : 2 and 5.
Maudy : 5 and 6.
Teacher : 5 and 6, Mmmm…(The parrot said). What does it mean?
Chantika : It was incorrect.

Teacher : *What is the correct one?*
Chantika : *1 and 6.*
Teacher : *Yes, Harnita!*
Harnita : *2 and 5.*

The phrase ‘*5 plus 2*’ showed that Agung was able to find a combination that makes 7. Based on our observation, he used his fingers and represented 5 using left hand and 2 using right one. Rafi, the student sat next to Agung, said a combination that was 4 and 3. It seemed that he had a big idea that was compensation by knowing facts. Meanwhile, the phrase ‘*3 and 4*’ showed that Fabella came to the idea of commutativity. A student, Maudy, gave an incorrect combination that makes 7. When the teacher asked her to show her answer using the arithmetic rack, she took 5 white beads first by one-by-one tagging, then took a blue bead next to five white beads to make 6. She said that she added 5 and 1 to make 6. Her reasoning was different from what we conjected that she would take 5 beads and 6 other beads. It seemed that Maudy wanted to make 7 by counting on. Two students corrected Maudy’s work by finding other combinations by connecting to 6 and 5.

Summary

From this Mini Lesson, we concluded that the students were able to find many combinations that make numbers up to 10. In this case, we focused on a number that was 7. Some students still used fingers to represent 7 and found 5 and 2 as a combination that make 7. Meanwhile, the other students were able to know fact mentally. When a student gave an incorrect combination, the other students were able to correct it. From this activity, the students used the big ideas such as compensation and commutativity.

Lesson IV (Making Jewelry)

To build students' awareness of structures, the teacher asked students to arranged 20 beads containing blue and pink beads. The students had to arrange those beads in order other people were able to know the number of beads quickly. They also had to make a representation of their work on poster papers.

Based on our observation, there were three different arrangements that the students made. They arranged beads into 10 pink beads and 10 blue beads, a blue bead and a pink bead up to 20 beads, and combined 2 of 5 blue beads with 10 pink beads (*Figure 5.33)*. From those arrangements the students could develop their awareness of structures.

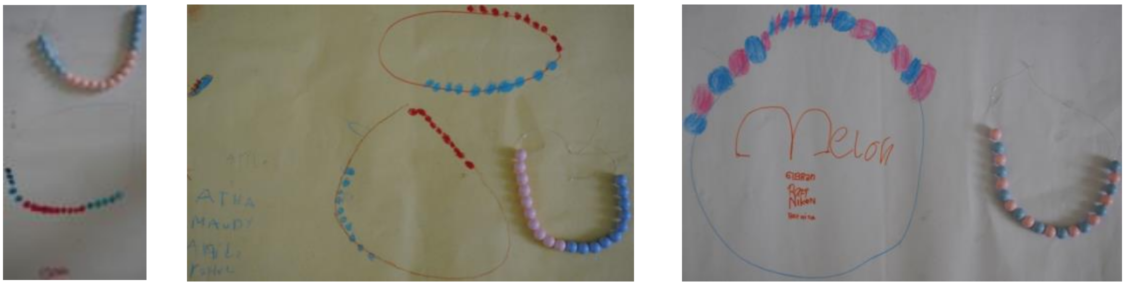


Figure 5.33: Some groups' work on arranging 20 beads.

From students' poster papers, we found that three groups, apple, orange, and mango, made a similar arrangement that was 10 pink beads and 10 blue beads. Two groups, strawberry and mellon's groups, also made a similar arrangement that was a blue bead and a pink bead up to 20 beads. Meanwhile, a group, grape's group, made a different arrangement that was 2 of 5 blue beads with 10 pink beads. After making the poster paper, they explained why they chose those arrangements.

Teacher : *Tell to your friends how do you know the numbers of beads on this figure quickly?*

Rafli : *The pink beads are 10.*
Teacher : *So you first looked at the pink beads! How many are they?*
Rafli : *10.*
Teacher : *These? (Pointing blue beads on the left side)*
Rafli : *5.*
Teacher : *and these?*
Rafli : *5.*

The phrase *'the pink beads are ten'* showed that he knew the number of pink beads on his work, but he together with his friends still counted the pink beads one-by-one during arranging the beads except Firza. He counted the pink beads by two up to 4 and then used counting on. The idea of dividing blue beads into 2 of 5 indicated that the students in this group used the five-structure and double of five to make ten. When we interviewed Firza, he explained that he showed the blue beads first. Since there were two groups of blue beads making ten, the pink beads were also ten. It seemed that knowing the number of blue beads was able to know the number of pink beads because it should be 20 beads. Then the teacher asked other groups' work.

Teacher : *How many beads are these? (Pointing melon's work)*

Rafi and Niken : *20.*
Teacher : *How do you know it quickly?*
Rafi : *10 and 10.*
Teacher : *How did you arrange it?*
Niken : *one-by-one.*
The teacher continued to ask strawberry's group because they made a similar arrangement with melon's group.
Teacher : *How do you know it quickly? (Pointing strawberry's work)*
Chantika : *10 blue beads and 10 pink beads.*

The phrase *'10 and 10'* showed that Rafi connected between the number of pink and blue beads to the number of beads altogether. Meanwhile, they arranged the beads one-by-one that meant they had a big idea that was one-to-one correspondence. It meant that if they had one blue bead, so there was also one pink bead. In fact when they had ten blue beads, they also had ten pink beads. This idea was told by Zaidan and Chantika, strawberry's group, when we interviewed them. They said that they just needed to count pink beads to know the number of blue beads. On making a representation of the arrangements, melon's group made 22 beads. They said that they drew two more beads than original jewelry. It showed that they knew numbers more than 20. Then the next group told their work.

Teacher : *Why do you choose this arrangement?*

Atha : *because these are 10 (Pointed blue beads), and these are 10 (Pointing pink beads)*

The phrase '*because these are 10, and these are 10*', showed that Atha, together with their friends, preferred to arrange the beads in similar color. It seemed that apple's group used the ten-structure as a basis to arrange the beads.

Summary

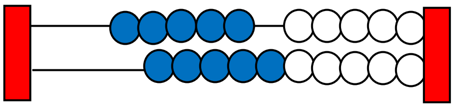
As our conclusion from this lesson that the students preferred to arranged the beads into five and ten-structures, but we also found some groups explained that they arranged the beads based on beauty value. This meant that some students did not followed the instruction by the teacher that they had to arrange the beads in order other people could recognize the number of beads quickly. Probably we had to give students the number of pink beads did not equal to blue beads, so the students could think what arrangement they had to make.

Lesson V (Exploring numbers up to 20 using the arithmetic rack)

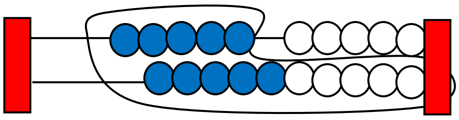
The activity using the arithmetic rack in this lesson was aimed to develop students' understanding about number relationships up to 20. In this activity, the students were challenged to give some reasons based on a structure on the arithmetic rack. The teacher started the lesson by asking students about their awareness on the structures of the arithmetic rack.

Teacher : *How many beads do you see on it? (Showing the arithmetic rack)*

Students : *20.*
Teacher : *How do you know?*
Students : *Thinking.*
Teacher : *How do you know? (Asking Rahul)*
Rahul : *10 and 10.*
Teacher : *Do you have other ideas?(Asking other students)*
Rizki : *5, 5, 5, and 5.*
Teacher : *So, Rizki showed there were 5, 5, 5, and 5, did not it Rizki?(Moving each group of five beads to the left sode)*
Rizki : *yes*
Teacher : *And Rahul? Let's showed it!*
Rahul went next to the teacher and showed his answer as 10 beads on the bottom and 10 beads on the top.
Firza : *I have an idea!*
Teacher : *yes, show it!*
Firza went closed to the teacher, and used the arithmetic rack to show it.
Firza : *5, 5, and 10.*

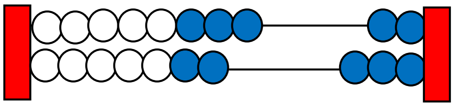


Teacher : *Do you still have other ideas?*
Niken raised her hand and went closed to the teacher to show her answer using the arithmetic rack.
Niken : *5 and 15.*



The word '*20*' showed that the students recognized the number of beads on the arithmetic rack. They also were able to reason how they knew the number of beads on that arithmetic rack. The work *'thinking'* indicated that the students did not need to count the beads one-by-one. A student, Rahul, saw the structures of beads as 10 and 10. He showed as 10 beads on the top and 10 beads on the bottom. Meanwhile, Rizki showed the structuring of beads as the five-structure. The other two students, Firza and Niken, showed the relations among those numbers, so they were able to see the beads on the arithmetic rack arranging in many ways but still relating to the five and ten-structure. After that the teacher asked students to tell what they saw on a number representation using the arithmetic racks.

Teacher : How many beads do you see?



The students raised their hands and answer:
Students : 15.
Teacher : Malik!
Malik : 15.
Teacher : How do you know it?
Malik : White beads are 5.
Teacher : How many white beads do you see?
Malik : White beads are 10.
Teacher : and then?
Malik : plus 5.
Teacher : Do you have other ideas?
Firza : 5 plus 10.

For the first time, Malik just said there were 5 white beads. It probably that he just showed the white beads on the top or bottom of the arithmetic rack. Then he changed his answer to be 10 white beads. It seemed that he realized himself that there were 2 groups of 5 white beads, and then recognized there were 5 blue beads. It seemed that he was able to perceive the small objects such as 2 blue beads on the top and 3 blue beads on the bottom make 5 blue beads. Meanwhile, the phrase '*5 plus 10'* indicated that Firza had a big idea of commutativity. He used this idea to know the number of beads on the arithmetic rack.

Based on our observation, the teacher asked Zaidan which one he meant 10 and 5. We first conjectured that Zaidan saw there were 10 white beads, but he had different idea on it. He had a big idea that was combinations that make ten. He moved two blue beads on the bottom next to 8 beads to make 10 (*Figure 5.34*). Then He decomposed 7 beads into 2 and 5 beads. He finally explained that there were 15 altogether. From Zaidan's work, we concluded that he used making tens instead the five-structure.

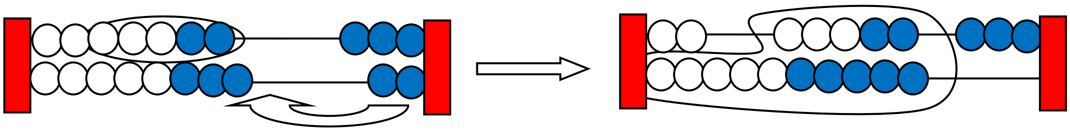


Figure 5.34: Zaidan's idea to know the number of beads on the arithmetic rack.

On the students' worksheet, we found there were some students colored the frame using the five-structure to make 10 beads. We could see, for instance: on Harnita and Najwa's work (*Figure 5.35),* that they colored 10 circles on the left side with yellow and the other beads with dark blue. It seemed that they decomposed a number into ten and ones and colored the frame based on the structuring on the arithmetic rack. Meanwhile, other students first colored 10 circles on the top and continued coloring the circle on the bottom. Those students colored the frame without connecting with the structuring on the arithmetic rack. We also found on Farhan and Aria's worksheet that they colored whole circles for problem 1 and 3 (*Figure 5.35*), then he crossed some circles. They explained that they colored too many circles, so they had to subtract it. This indicated that the students had the idea of part/whole relationships; relations between addition and subtraction.

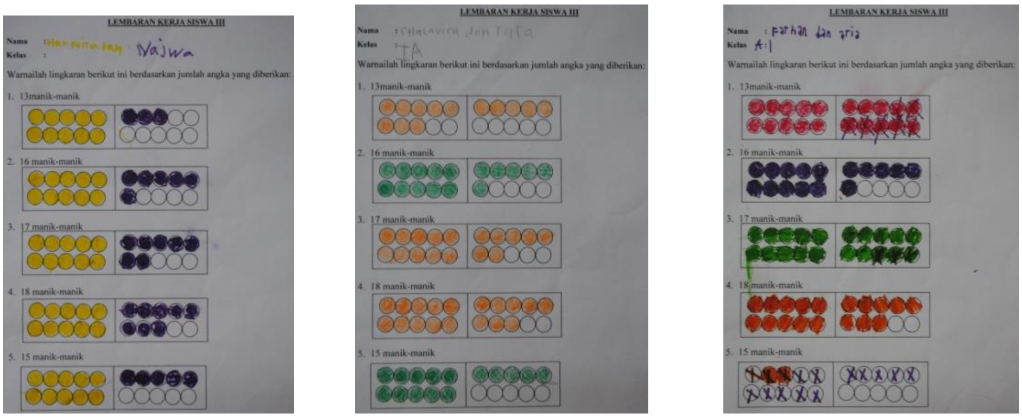


Figure 5.35: Students' work on exploring beads on an arithmetic rack

Summary

During this lesson, we found that the students were able to recognize the structure of beads on the arithmetic rack. They recognized it as the five- and ten-structure. When the teacher represented a number using the arithmetic rack, most students knew the number of beads quickly because they first saw a group of beads, for instance white beads, that make ten, then they showed the other one and used known facts such as 2 and 3 equal 5. However, a student, Zaidan, had a big idea that was combinations that make ten. For the next lesson, the teacher could provoke students by trying Zaidan's idea to know the number of beads on the arithmetic rack.

Lesson VI (Worksheet based on the arithmetic rack)

In this lesson, we gave the students four problems in which they were challenged to perform combinations that make ten. They had to colored some more beads then wrote how they knew the number of beads. Since the students had some experiences in the previous lesson, they probably could use their previous knowledge on the structures of an arithmetic rack.

The first problem was that there were 8 beads had been colored, and then the students had to color 4 more beads. For this problem, there were two different colors that students made. Some students colored four beads on the second line. Those students wrote their strategies to know the number of beads altogether as 8 + 4 = 12 (*Figure 5.36a).* Based on our observation those students used counting on. They counted from 8, 9, 10, 11, and 12, so they gave the answer as 12. The other students colored 2 more beads on the first line to make 10, and then they colored 2 more beads on the second line (*Figure 5.36b).* They wrote their strategies to know the number of beads as 10 + 2 = 12. Those students used a strategy that was making ten. They used a big idea of combinations that make ten. In more detail, Firza and Renald described how they found 10 + 2. They wrote on their worksheet as 5 + 3 + 2 + 2. Those numbers represented the colorful beads on the arithmetic rack. They were influenced by the structure on the arithmetic rack.

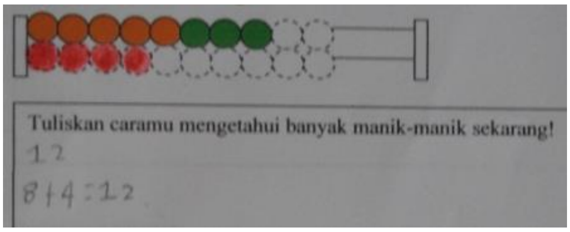


Figure 5.36a: Rista and Nabila's work

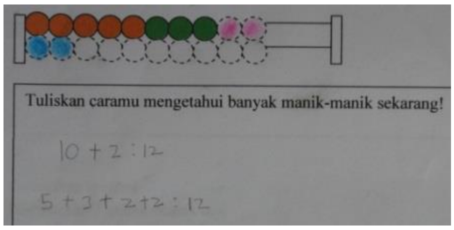


Figure 5.36b: Firza and Renald's work

The second problem is that there were 12 beads had been colored, and then the students had to color 5 beads more. All students colored 5 more beads on the second line with a color. Many students wrote their strategies to know the number of beads as 10 + 7 = 17 (*Figure 5.37a*). Some students wrote 5 + 5 + 7 = 17 in which they looked the structures of beads on the first line. There were 2 of 5 beads. Meanwhile, we found a group, Rafi and Rizki, had different idea to know the number of beads. They had a big idea that was doubles (*Figure 5.37b*). They counted the number of beads by two up to 14, and then they used counting on from 14 to 17. It meant that some students preferred to use doubles to know number of beads.

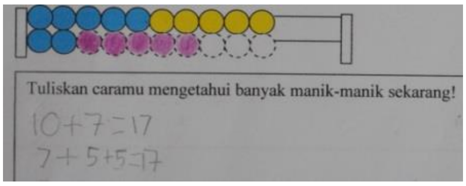


Figure 5.37a: Gibran and Dwiki's work

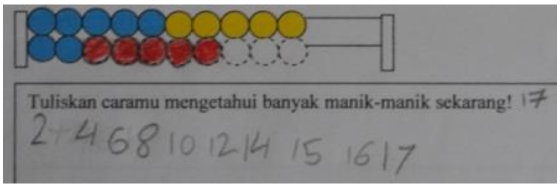


Figure 5.37b: Rafi and Rizki's work.

For the third problem, the students were showed that there were 7 beads had been colored, and then they were free to color 6 more beads. In this problem, we could observe whether they used their knowledge on the arithmetic rack or not. To solve this problem, we found four different students representations on the number of beads. First, Arya and dini drew 6 more beads on the first line (*Figure 5.38a*). It meant that they did not use their experience on the arithmetic rack. They wrote 7 + 6 = 13. Based on our observation, they used counting on strategy that they counted from 7 to 13. The second one, we found some students drew 5 more beads on the first line, and then one bead on the second line (*Figure 5.38b*). We took a group of students' work on it, Firza and Renald's work. Based on our observation, they struggled to know the number of beads altogether. First, Firza thought that there were 10 beads on the first line, but he was doubt. Then he counted the beads one-by-one two times to make sure their answer. They wrote 10 + 3 = 13 based on known fact, and then 5 + 7 +1 = 13 indicated that there were 5 red beads, 7 more beads on the first line, and 1 bead on the second line. From these two representations, we found that students struggled to know the number of beads quickly, so they used one-by-one tagging.

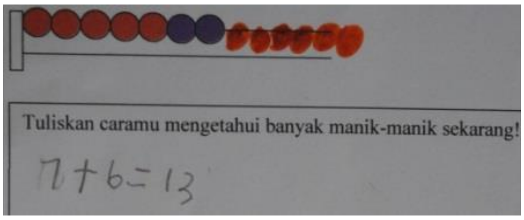


Figure 5.38a: Arya and Dini's work

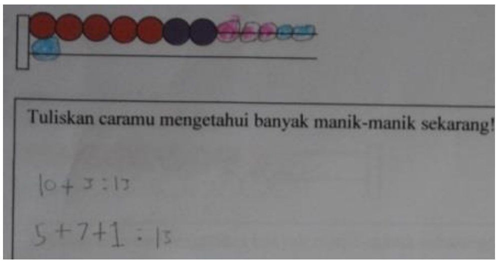


Figure 5.38b: Firza and renald's work

The other students' work was that they drew 3 more beads on the first line to make 10 beads then drew 3 more beads on the second line (*Figure 5.39c*). A group of students' work on it was Rahul and Zaidan. They had a big idea that was combinations that make ten and related to the structures of the arithmetic rack. 10 + 3 = 13 indicated the number of beads on the first line adding to second line. To get 10, they applied their knowledge on combinations that make ten that was 7 + 3 = 10. From 10 + 3 = 13, they found 11 + 2 = 13. They had a big idea that was compensation and used known fact. The last representation that we found was that some students drew 6 more beads on the second line, for instance, Rista and Nabila's work. They also wrote 10 + 3 = 13 (*Figure 5.39b*). It seemed that they showed the structure of the beads on the arithmetic rack in which 5 beads on the top together with 5 beads on the bottom make 10. They worked on the five-structure and doubles of five.

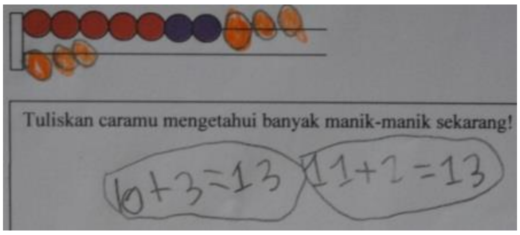


Figure 5.39a: Rahul and Zaidan's work

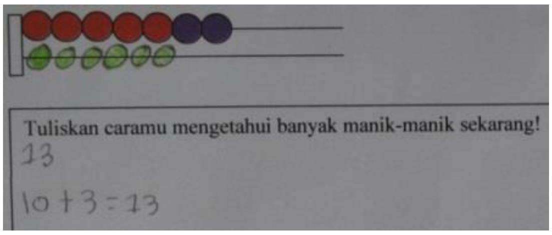


Figure 5.39b: Rista and Nabila's work

The last problem, the students were given that there were 14 beads had been colored, and then they had to color 3 more beads. There were two different students' representations. Some students, for instance: Malik and Rafli, drew 3 more beads next to 10 beads on the first line. It seemed that they did not realize the structure of an arithmetic rack. Based on our observation, they used counting on strategy that they counted from 14 up to 17 and then wrote 14 + 3 = 17 (*Figure 5.40a*). The second representation was that the other students drew 3 beads on the second line. It seemed that they knew the structure of beads on the arithmetic rack. In general, we found two different students' answers. First, Gibran and Dwiki performed making ten and decomposing ten and ones. They showed the structure of beads on that figure (*Figure 5.40b*). They also were able to use known fact that was 4 + 3 equal 7, and adding 10 and 7 become 17. The second one is that Farhan and Raka's work. The word '*9 + 8 = 17'* showed that they seemed using doubles for near doubles (*Figure 5.40c*). They first counted beads by 2 up to 16 then adding 1 to get 17. Meanwhile, 9 + 5 + 3 = 17 indicated that they decomposed 8 into 5 and 3. From these three different students' answers, we concluded that the students used counting on, making ten, using doubles for near double, and decomposing ten and ones. They had big ideas of doubles and combinations that make 10.

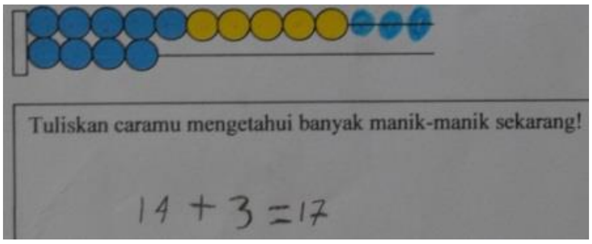
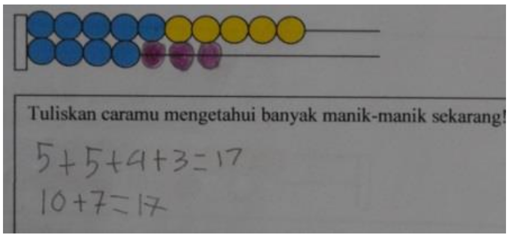


Figure 5.40a: Malik and Rafli's work



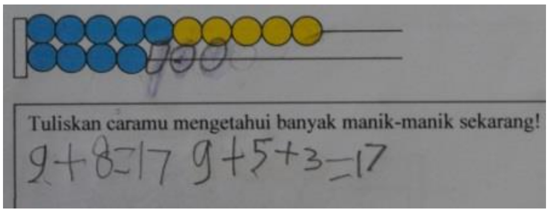
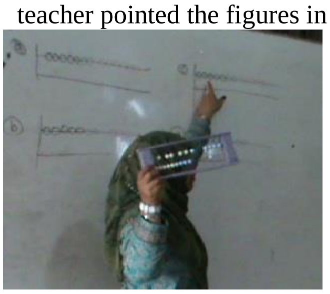


Figure 5.40b: Gibran and Dwiki's work *Figure 5.40c: Farhan and Raka's work*

Since there were some students made a drawing of beads that were not similar to the arithmetic rack, the third and fourth problem, the teacher took the third problems into a discussion. The teacher first made four different drawings based on students' answers on a white board and asked students to think which drawings were suitable with the arithmetic rack.

Teacher (The : Who drew this figure? These are 7, these are 5, and one beads on the second line. teacher pointed the figures in order)



Firza raised his hand.

Teacher : Yes Firza. These are 7, then you added 5, but these are only 3, how do you think on it?

Firza thought for a while, and then he gave a code that the teacher had to move 3 beads to next to seven beads. After that he told.

Firza : It should be on the second line.

Teacher : So, it had to be on the second line. These mean three on the first line, then three on the second line, don't these?

Student : yes

The phrase '*It should be on the second line*' showed that Firza realized himself that he made a wrong drawing. He had not to draw 5 beads next to the seven beads on the first line instead to draw three beads on the second line. The other students also agreed on it that they had not to draw more than 10 beads on the first line. Finally, the teacher asked students to comment which drawings could be similar to the structures on the arithmetic rack and emphasized that the maximum beads on each line were 10 beads. It seemed a guideline that the teacher gave to the students.

Summary

From this lesson, we concluded that the students used different strategies to solve those four problems. Some students still used basic strategies that were counting one-by-one and counting on. Meanwhile, the other students were able to perform strategies such as the five- and ten-structure, doubles for near doubles, making tens, compensation, and known fact. From those strategies, they develop the big ideas of doubles, compensation, and combinations that make ten. In this lesson, the students still worked on models with arithmetic rack, and some students sometimes still used fingers to represent beads on those problems. It meant that the students still needed model to solve some addition problems. We also found that some students drew beads not based on the arithmetic rack.

Lesson VII (Addition up to 20 with contextual problems)

In the last lesson, we gave two contextual problems to the students. We expected that the students were able to perform combinations that make ten and decomposition of other numbers in solving those problems. Eggs on the box were a contextual problem that we chose because the students were familiar enough on this situation. First problem was that there were 7 eggs on the box, and then Mrs. Ayu bought 8 eggs more and put on that box. How do you know the number of eggs now?

There were two different representations that students made on the box. First, many students drew 3 more eggs on the second row, and then 5 more eggs on the third row. It seemed that the students made a combination of ten that was 7 + 3 = 10 (*Figure 5.41a*). The second one was that a group of students, Raka and Farhan, drew 8 eggs on the first and second columns. It seemed that he made a combination of ten that was 8+2=10 (*Figure 5.41b*).

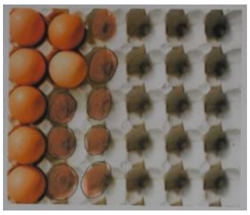


Figure 5.41a: Eggs on the box (7+3+5)

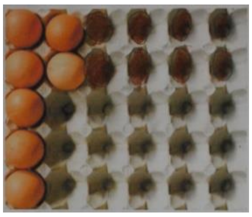
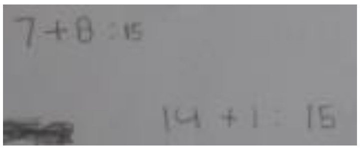


Figure 5.41a: Eggs on the box (5+2+8)

Based on students' worksheet, we found some students strategies. Some students used doubles for near doubles, for instance: Firza and Renalds (*Figure 5.42a*). They showed to us that they counted by two up to 14 then adding 1 to get 15. Those students had a big idea that was doubles. Other students used the strategy that was making ten. Chantika and Maudy (*Figure 5.42b*) wrote in their worksheet as 5+2+3+5=15. When we asked them, they explained that 5+2+3=10 and then adding 5 to make 15. We also saw on other students' work. Gibran and Dwiki did not only use making ten, but also the five-structure (*Figure 5.42c*). They also found relationships among numbers and had big ideas of compensation, and combinations that make ten. More about number relationships, Rafi and Rizki found many number relationships (*Figure 5.42d*). From 10+5=15, they found the other 5 problems that also gave the same results. They used the big idea of compensations. They were influenced by first and third lesson in which the students were challenged to find many combinations that make ten and other numbers up to ten. However, we still found some students used counting one-by-one to know the number of eggs on the figures.



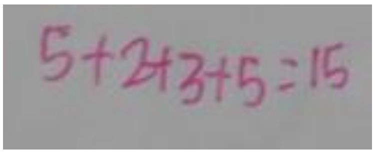
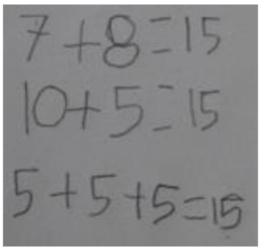


Figure 5.42a: Firza and Renald's work *Figure 5.42b: Chantika and Maudy's work*



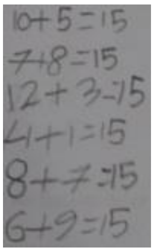


Figure 5.42: Gibran and Dwiki's work *Figure 5.42: Rafi and Rizki's work*

After the students solved the first problem, they had to solve the second one. In this problem, the students were challenged to decompose a number into ten and ones to solve an addition problem. We gave them the problem as follows: There were 13 eggs on the box. Mrs. Ani bought 3 eggs more and put on that box. How do you know the number of eggs now? Since there was a picture of 13 eggs on the box, the students were able to figure it out and made a drawing if necessary.

Based on students' answers on their worksheets we generally found two different students' answers. Some students still used counting on strategy that they counted from 13 up to 16 (*Figure 5.43a)*. However, some students could perform decomposing ten and ones *(Figure 5.43b)*. Those students performed number relationships to solve this problem. It seemed that they were able to see the structure of eggs arranging on the box. More about it, Gibran and Dwiki wrote 10 + 6 = 16 were similar to 5 + 5 + 6 = 16. They could relation between 10 and 5 + 5. They had a big idea that was combinations that make ten. They also performed the idea of compensation such as 5 + 5 was similar to 6 + 4.

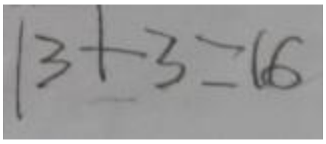


Figure 5.43a: Raka and Farhan's work

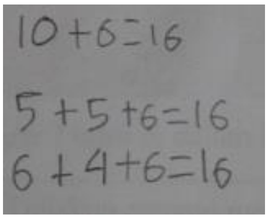


Figure 5.43b: Gibran and Dwiki's work

Summary

Throughout this lesson, we concluded that most of students solved first problems using making tens. They were able to see the structure of eggs on the box. Some students also used doubles for near doubles, and known facts to know the eggs altogether. However, we still found the students that used counting on strategy. Sometimes, some students changed their strategies when they worked on different problems and contextual situations; such as Firza and Renald using doubles in this problem that different from using making tens in previous lesson.

5.6 Analysis throughout All Lessons

In the analysis throughout all lessons, we first looked at learning process on each lesson and searched for connections between those lessons. We focused on the students' learning trajectory throughout those lessons. We wanted to know whether the activities had supported students in learning additions up to 20.

In the first lesson, we found that most of students started to find a combination of ten *pempek* by taking 5 *lenjer pempek* and 5 *bulet pempek.* This meant that they were familiar using the five-structure to find a combination that make ten. Some groups of students used this combination to find other combinations of ten *pempek* without experiencing with real objects. They just used the ideas of compensation and commutativity. Meanwhile the other groups of students always worked with real objects, the wax *pempek*, found some similar combinations and drew on the poster papers. By classroom discussion, the students were able to clarify when they found incorrect combinations of ten *pempek* and made a conclusion of all combinations of ten *pempek* they could make. Finally, the students were able to think those combinations that make ten mentally without thinking on combinations of ten *pempek.*

The next lesson is that mini lesson I (Parrot game). The goal of this lesson still related to the first one that was to develop students' understanding about combinations that make ten. In this lesson, the students were able to find many combinations that make ten when the Parrot asked them. Some students could think mentally, but the other used their fingers to find combinations that make ten. The students also used the big idea of compensations to find combinations that make ten, for instance; 4 and 6 changing into 3 and 7. They were also able to represent combinations that make ten using the arithmetic rack. It seemed that they were able to recognize the structure on the arithmetic rack.

In the second lesson, we found that some students gave reasons based on structuring that they showed on flash cards, but the other students reasoned based on their knowledge on combinations that make a number such as reasoning on combinations that make ten when they showed ten medicines on a medicine tablet. Although some students did not reason on structuring numbers, they still developed big ideas of doubles and compensation. They could perform the known fact, five-structure, compensation, and doubles for near doubles to know the numbers of colorful circles on flash card. We found two mathematical socio norms that the students sometimes gave an answer together, and the teacher did not react directly when the students gave an incorrect answer.

The third lesson was that the students were challenged to decompose numbers up to 10. The students were able to find many combinations for some numbers up to

10. They used fingers, arithmetic rack, and knowing mentally those combinations. A group of students was able to move from using fingers to an arithmetic rack. Finally, they just used the idea of commutativity to find other combinations. The students developed some big ideas, doubles, compensations, and commutativity. The last big idea, commutativity, was often used by students to find other combinations of numbers up to 10.

To develop students' understanding on decomposing numbers up to 10, the students played a parrot game, mini lesson II. In this lesson, many students were able to find combinations of numbers up to 10, for instance: 7, mentally. Meanwhile few students still needed fingers to represent that numbers. When a student gave an incorrect combination, the other students directly corrected it. In this activity, the students still preferred to use compensation and commutativity.

In the fourth lesson, the students were challenged to build awareness of structures. They preferred to use five and ten-structures that were showed by their arrangement on making jewelry. However, we also found that some students explained that they arranged beads based on beauty value. It seemed that they really did not followed an instruction that they had to arranged the beads in order other people could recognize the number of beads quickly.

During the fifth lesson, we found that the students were able to recognize the structure of beads on the arithmetic rack. They knew it as five and ten-structures. Actually, not all students reasoned based on the structure on the arithmetic rack instead known number relationships. As an example, when we represented 8 beads on the first line and 7 beads on the second line, the students knew there were 15, but they reasoned on combinations that make 15. However, we found few students could perform on combinations that make ten such as 8 and 7 beads representing on the arithmetic rack.

They knew that by moving 2 beads to the first line making ten. The big idea of combinations that make ten were emerged in this lesson.

The sixth lesson still connected to the fifth one because the students still worked on an arithmetic rack. In solving four problems in this lesson, the students used various strategies, counting on, five- and ten-structure, doubles for near doubles, making tens, and compensations, and known fact. From those strategies, the students developed the big ideas of doubles, compensations, and combinations that make ten. They still worked on models with the arithmetic rack, and some students used fingers to represent beads on those problems. It seemed that they still needed models to solve addition problems.

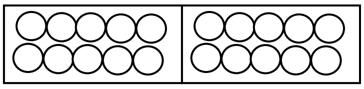
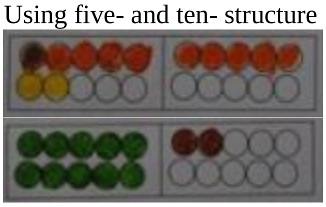
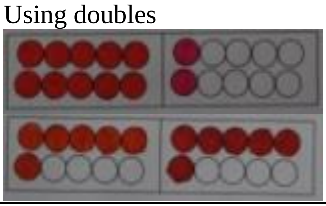
Throughout the last lesson, we found that many students performed the big ideas of combinations that make ten. They were able to see the structure of arranging eggs on the box. They also used doubles for near doubles and known facts to know the eggs altogether. However, we still found that some students used counting on strategy. Sometimes, few students changed their strategies based on problems they had to solve. In generally, the students could use the big ideas such as doubles and combinations that make ten to solve the problems on this lesson.

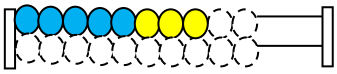
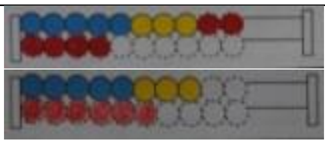
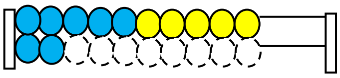
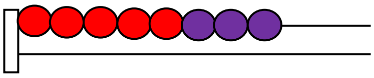
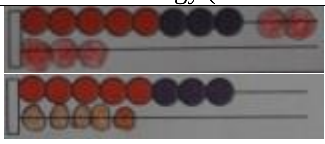
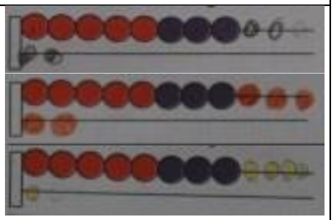
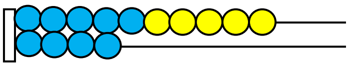
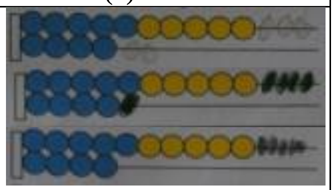
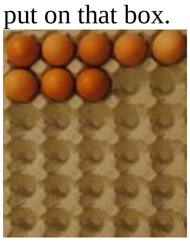
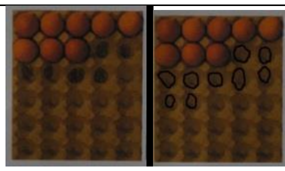
5.7 Final Assessment

At the end of the series of lessons in the teaching experiment activities, we conducted an assessment to see whether the activities supported the students in learning additions up to 20. We designed 8 problems on this assessment. The problems were about number relationships, structuring numbers, and addition problems up to 20 involving arithmetic rack representations and eggs on the box. At this time, the students worked individually, so we could know if the students learnt from previous lesson in groups or pairs. There were 27 students, so there were 27 different students' answers.

To analyze this end assessment, we made an analysis table (*Figure 2.26*). We described the students' strategies to solve those problems. Those students' answers represented all students' strategies and drawings on their worksheets. We also grouped the answers into correct and incorrect answers to know students' big ideas and struggles. Then, we described their tendency in solving those problems.

Table 5.1: Analysis of students' strategies on the end assessment

Problem	Correct students' solutions	Incorrect students' solutions
1. How do you know the number of beads? 	- Using the ten-structure (10 + 4 = 14, 10 + 3 + 1 = 14) - Known fact, counting on (8 + 6 = 14) - Decomposing, using the five-structure, and making ten (5+3 and 5+1, 5 + 5 = 10 and 3 + 1 = 4, 10 + 4 = 14) - Decomposing (5 + 3 + 5 + 1 = 14, 3 + 2 + 3 +5 + 1 = 14) - Unknown strategy (5+3+6 = 14)	- 10 + 6 (4) = 14
2. How do you know the number of medicines? 	- Decomposing, Using the-five structure, and making ten (5 + 5 + 3 + 4 = 10 + 7 = 17) - Using the ten-structure (10 + 7 = 17) - Using doubles for near doubles (8 + 9 = 17) - Using the five-structure (5 + 5 + 7 = 17, 5 + 5 + 3 + 3 + 1 = 17) - Unknown strategy (5 + 12 = 17)	
3. Please color the following figure based on the number of beads! 12beads 	- Using five- and ten- structure  - Using doubles 	

4. Please color 6 beads more!  How do you know the number of beads?	 - Making ten (5 + 3 + 2 + 4 = 10 + 4 = 14, 8 + 2 + 4 = 14) - Using the five- and ten-structure (5 + 5 + 4 = 14) - Using the ten-structure (10+4 =14) - Decomposing, and almost ten (5 + 3 + 6 = 5 + 9 = 14) - Using known fact, counting on (8 + 6 = 14) - Using doubles (7 + 7 = 14) - Unknown strategy (11 + 3 = 14)	- 10 + 4 = 54 (14)
5. Please color 4 beads more!  How do you know the number of beads?	 - Using the five-structure, making ten, and decomposing (5 + 5 + 2 + 4 = 10 + 6 = 16) - Using the five- and ten-structure (5 + 5 + 6 = 16) - Using the ten-structure (10+6 = 16) - Using counting on (12 + 4 = 16) - Using doubles for near doubles (7 + 9 = 16) - Unknown strategy (11 + 5 = 16)	- 5 + 2 + 5 (+3) - 10 (+2) + 4 = 14 - 10 + 16 (6) = 16 - 14 (12) + 5 = 6
6. Please color 5 beads more!  How do you know the number of beads?	 - Using the five-structure, Making tens (5 + 5 + 3 = 13) - Using the ten-structure (10 + 3 = 13) - Using the known fact, using counting on (8 + 5 = 13) - Unknown strategy (11 + 2 = 13)	 - 8 + 4 (5) = 11 - 5 (+ 5) + 3 = 13 - 10 + 3 = 43(13) - 5 + 3 + 2 + 2 (3) 10 + 2 (3) = 12
7. Please color 4 beads more!  How do you know the number of beads?	 - Using the five-structure (5 + 5 + 5 + 4 = 19, 5 + 5 + 9 = 19) - Using the ten structure, using doubles for near doubles (10 + 9= 19) - Counting on (14 + 5 = 19)	 - 14 + 15 (5) = 29
9. There are 8 eggs on the box. Andy buys 5 eggs more and put on that box.  How do you know the	 - Making tens (8 + 2 + 3 = 13, 4 + 6 + 3 = 13) - Using the ten-structure(10 + 3 = 13) - Using the five-structure (5 + 5 +	 - 8 + 9 (6) = 17 - 5 + 3 + 6 (5) = 14 - 5 + 5 (+3) = 13

number of eggs now?	3 = 13) - Known fact (8 + 5 = 13)	

The aim of the first and second problems was similar that was to know whether students could perform their knowledge on number relationships and how they saw the structures of the beads and medicines that were arranged. Based on students' answers on their worksheet (*Table 5.1*), we found that the students were able to see the relations among numbers, for instance 5 plus 5 equals 10 and 8 could be decomposed into 5 and 3. They were able to perform on number relationships since they saw the structures of beads and medicines that were arranged. From this situation, the students used different strategies to tell how they knew the number of beads and medicines. They used decomposing numbers, five- and ten- structures, making tens, and doubles for near doubles. From those strategies, we concluded that the students had big ideas of doubles and combinations that make ten.

In the third problems, we wanted to know how the students represented a number on circle representations. From their representations on the table 5.1, we found that the students worked on five- and ten- structures and doubles. The students worked on five- and ten- structures seemed that they represented 12 beads as 5 + 5 + 2 and 10 + 2. Meanwhile the students worked on doubles seemed that they represented 12 beads as 6 + 6 and (5 + 1) + (5 + 1). However, we found that a student colored all beads. We thought that he did not read the problem carefully and directly colored all beads on that figure.

Problem 4 and 6 had similar aim that was to know what strategies and big ideas students used to solve the problems that involved addition of one digit number up to 20 and to know how students represented some more beads on the arithmetic rack drawing. On the table 5.1, we found that the students used various strategies that were counting on, using the five- and ten-structures, making ten, using known fact, and using doubles. Based on those strategies, the students had big ideas of doubles and combinations that make ten. We also found some students' struggles. In problem 6, the students drew beads on the first line that the total of beads more than ten. It seemed that those students did not see the structure of beads related to the arithmetic rack. Some students also struggled on writing numbers such as 10 + 4 = 14 that a student wrote 14 as 54.

Problem 5 and 7 also had similar aim that was to know what strategies and big ideas students used to solve problems that involved addition of two digit numbers with one digit number up to 20 and to know how students represented some more beads on the arithmetic rack drawing. To solve both problems, the students used various strategies that were decomposing ten and ones, using five- and ten- structure, doubles for near doubles, and known fact. However, we still found that some students still used counting on strategies. From those students' strategies, we showed that the students had big ideas of doubles and combinations that make ten. It seemed that they preferred to use both big ideas to solve various addition problems. Although the students wrote correct answers on their worksheet, some of them still struggled in drawing. They drew more beads next to 10 beads on the first line. It seemed that they still did not see the structure of beads related to the arithmetic rack. Some students made some mistakes such as making wrong number representations, counting the same numbers in two times, and forgetting to write a number to represent some beads.

In the last problems, we gave students a contextual problem in order to know students' strategies and big ideas. Based on their answers that were described on the table 5.1, we found that the students really preferred to use making ten, using five- and ten- structures, and know fact. It seemed that the students performed their big ideas of combinations that make ten. It meant that combinations that make ten were crucial if we wanted students to solve addition problems in more abbreviated strategies.

CHAPTER VI

CONCLUSION AND RECOMMENDATION

In this chapter, we conclude our research and try to answer the research questions. We reflect some information about important issues and also give some recommendations for further research especially on addition up to 20. We elaborate those two components on the following subchapters.

A. Conclusion

Many students for the first time just knew some strategies, counting, one-to-one tagging, and synchrony: one word for every object, counting three times when adding and counting on, and the big idea of one-to-one correspondence. During the learning process, they learnt some new strategies on addition up to 20. They learnt to subitize small objects, skip counting, use the five- and ten-structure, use know facts, use compensation when decomposing numbers up to 10, and use doubles and making ten to solve some addition problems up to 20. They also learnt the new big ideas to support those strategies, compensation, part/whole relationship, commutativity, doubles, and combinations that make ten to support their thinking process.

However, we found some differences between the students in learning addition up to 20. Some students, the lowest level of thinking, used counting one-by-one strategy in learning addition up to 20. Other students, middle level of thinking, used counting on strategy, and the other students, the highest level of thinking, were able to use more abbreviated strategies such as using the five- and ten-structure, making ten, and using doubles. In line with Gravemeijer (1994), we found that some students worked with concrete objects such as fingers, beads, and the arithmetic rack in learning addition up to 20. Other students made drawing as a *model of* situation, and the other students used numbers as mental objects for smart and flexible calculation without the need for structured materials.

For the lowest level of reasoning from students, they reasoned based on a concrete objects. It meant that they often worked with real objects as a mental object in learning addition up to 20. The drawing representations and using models, such as the arithmetic rack really gave support to the students to reason of their answers. For some students, they often gave reason using counting one-by-one although the objects was arranged on structuring ways. They did not realize the important of structures in doing more abbreviated counting strategies.

To bring the students from counting one-by-one, the lowest level strategies, to more abbreviated mental calculation strategies such as making ten and using doubles in learning addition up to 20, the teacher had some important roles. The first role was that given a rich and meaningful contextual situation, *pempek Palembang,* as the base of mathematical activities. By giving this contextual situation, the students were able to develop their strategies from situational level to referential level by making drawing representation. The guide the teacher gave to the students in a classroom discussion helped students to develop their strategies and big ideas from counting one-by-one to more abbreviated mental calculation strategies using the five- and ten-structures. Models, such as arithmetic rack, used by the teacher gave a support to the students to not count the objects one-by-one instead by group of five and ten.

Actually, not all students were able to reach the highest level strategies because they were different abilities in learning additions up to 20. The students who were able to subitize small objects, and showed the important of doing calculations by structuring were able to develop their strategies from counting one-by-one to more abbreviated mental calculation strategies. The students who were able to know number facts up to

10 mentally and number relationships were also able to reach the highest level strategies.

The explanations above describe students learning process on mental calculation strategies on addition up to 20 in grade 1 of primary school in Indonesia. Those also gave answers for five our sub research questions that were stated in the first chapter.

B. Reflection

There are three important issues that we would like to reflect for the future research on addition up to 20. The first one is the implementation of Realistic Mathematics Education (RME). Based on the first tenet of RME, we found that a rich and meaningful contextual situation, such as *pempek Palembang,* really gave support for developing students' mental calculation strategies on addition up to 20. The use of meaningful contextual situations for teaching could give important implications for understanding how informal and formal learning supported students' learning process and gave motivation in learning.

The second important issue is classroom discussion: teacher's role and students' social interaction. Base on the fourth tenet of RME, the *interactivity,* emphasizes on students' social interaction to support students' learning process. The students learnt from each other in small groups or in a whole-class discussion. A group discussion built a natural situation for social interaction, and the class discussion provoked students to be able to negotiate to one another in attempt to make sense other' explanation. As our finding, some students were able to discuss in their groups to negotiate their ideas. However, we also found that some students got difficulties to discuss in a group, so when a student made a mistake, the other students directly gave a judgment. It meant that they did not used to discuss in their daily classroom activities. In fact, the teacher has an important role in orchestrating social interaction to reach the goals in the class discussion.

There are some roles of the teacher during the teaching experiment activities. The first one is that the teacher made a role for the students in order to communicate their ideas in classroom discussion. The second role of the teacher is that providing students an opportunity to present their works. Stimulating social interaction among students in classroom discussion is the third role of the teacher. The fourth role of the students is that to emphasize students' ideas. The last teacher one is that to ask for clarification.

The last issue is about methodology used in this research. We used design research. In this methodology, we were challenged to design instructional activities by using a Hypothetical Learning Trajectory (HLT) as a guide line. We got some opportunities to revise the design for the future research. The design is always developed to build for better instructional activities.

C. Recommendation

In learning mental calculation strategies on addition up to 20 in grade 1 of primary school in Indonesia, the students had different level of counting. Many students for the first time still used counting one-by-one and counting on strategies. During the learning process, some students changed their strategies to smart and more flexible strategies. The change of students' strategies based on supports given to them. One of supports is a rich and meaningful contextual situation. Then for the next study, the research has to use contextual situations so that all students at their own level can learn.

In this research, we sometimes did not look to all students' different strategies and big ideas because our research just focused on some mental calculation strategies such making ten and using doubles. Our suggestion for the future study that is to look to the different strategies and bring them into classroom discussion, so every student can learn based on their own level. The appropriate model, the arithmetic rack, also gave support in learning addition up to 20, but in this research we did not use it optimally. For the future study, there is a need to use an appropriate model consistently so that model can support students to move to using numbers as a mental object.

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APPENDICES

Appendix 1: Pre-Assessment

PRE-ASSESSMENT

Name :

Date :

1. Look at the picture!

How many eggs do you see on the picture? How do you know?

2. Look at the picture!

How many medicines on the following figure? How do you know?

Appendix 2: Students' worksheet

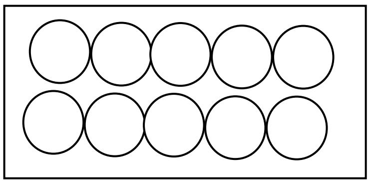
Worksheet I (flash card game)

Name :

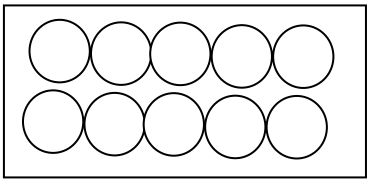
Class :

Please color the following figures based on the number of medicines!

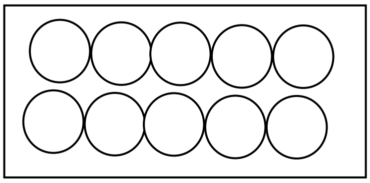
1. 7 medicines



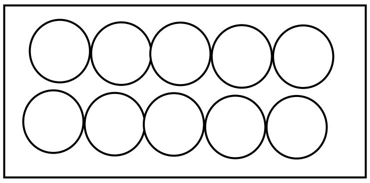
2. 5 medicines



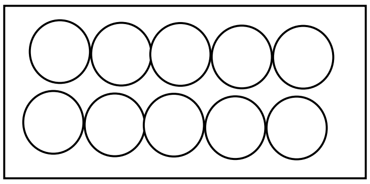
3. 8 medicines



4. 6 medicines



5. 4 medicines

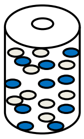


Worksheet II (Candy combination)

Name :

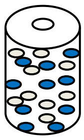
Class :

1. $\boxed{4}$

Candies 

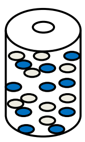
Grape	Milky

2. $\boxed{5}$

Candies 

Grape	Milky

3. $\boxed{6}$



Candies

Grape	Milky

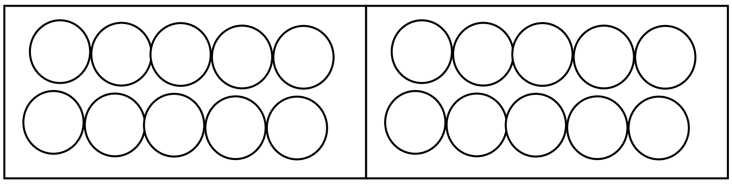
<u>Worksheet III (Exploring the Arithmetic rack)</u>

Name :

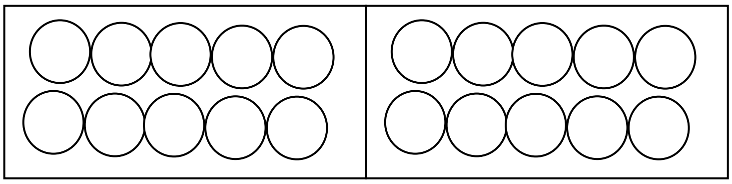
Class :

Please color the following figures based on the number of beads!

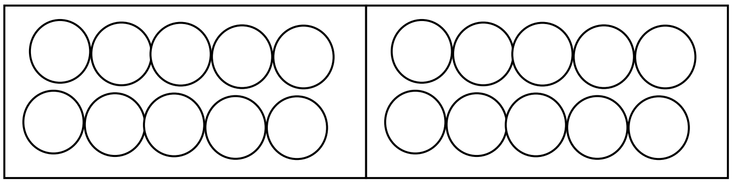
1. 13 beads



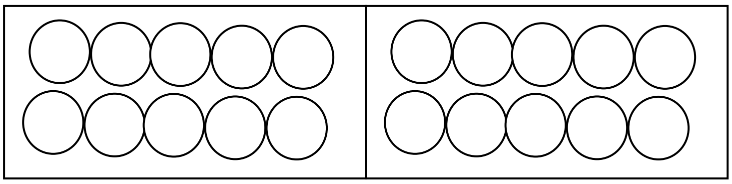
2. 16 beads



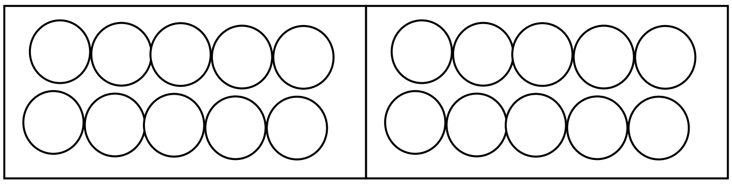
3. 17 beads



4. 18 beads



5. 15 beads

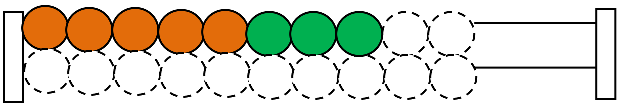


Worksheet IV

Nama :

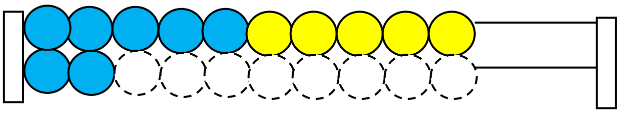
Kelas :

1. Look at the following figure.
   Please color 4 beads more!



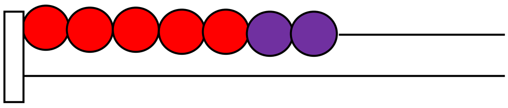
How do you know the number of beads now?

2. Look at the following figure.
   Please color 5 beads more!



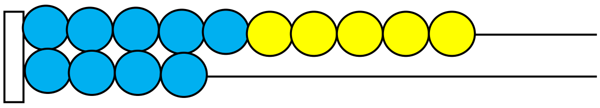
How do you know the number of beads now?

3. Look at the following figure.
   Please color 6 beads more!



How do you know the number of beads now?

4. Look at the following figure.
   Please color 3 beads more!



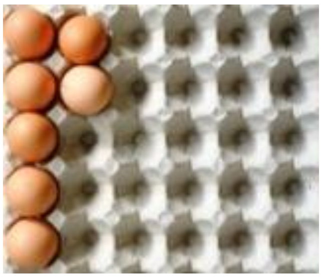
How do you know the number of beads now?

Worksheet V (Addition problems)

Name :

Class :

1. Look at the following figure!



There are 7 eggs on the box. Mrs. Ayu buys 8 eggs more and put on that box.

How do you know the number of eggs now?
So the numbers of eggs are . . .

2. Look at the following figure!

There are 13 eggs on the box. Mrs. Ani buys 3 eggs more and put on that box.

How do you know the number of eggs now?

Apendix 3: End-Assessment

END-ASSESSMENT

Name :

Class :

1. Look at the following figure!

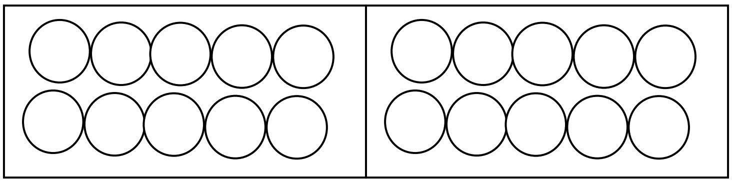
How do you know the number of beads?   So the numbers of beads are . . .

2. Look at the following figure!

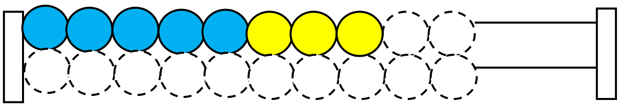
How do you know the number of medicines?   So the numbers of beads are . . .

3. Please color the following figure based on the number of beads!

12 beads



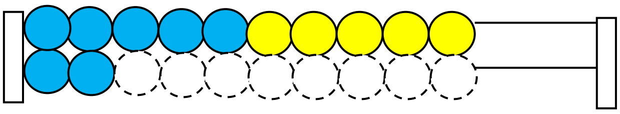
4. Look at the following figure.
   Please color 6 beads more!



How do you know the number of beads?

So the numbers of beads are . . .

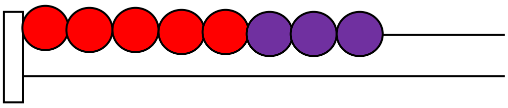
5. Look at the following figure.
   Please color 4 beads more!



How do you know the number of beads?

So the numbers of beads are . . .

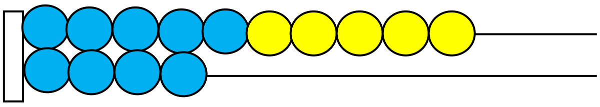
6. Look at the following figure.
   Please color 5 beads more!



How do you know the number of beads?

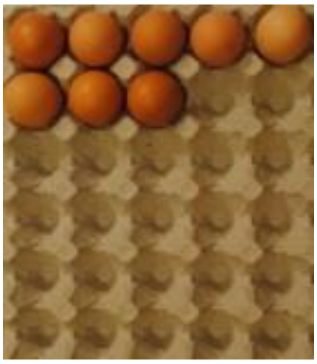
So the numbers of beads are . . .

7. Look at the following figure.
   Please color 3 beads more!



How do you know the number of beads?
So the numbers of beads are . . .

8. There are 8 eggs on the box. Andy buys 5 eggs more and put on that box.



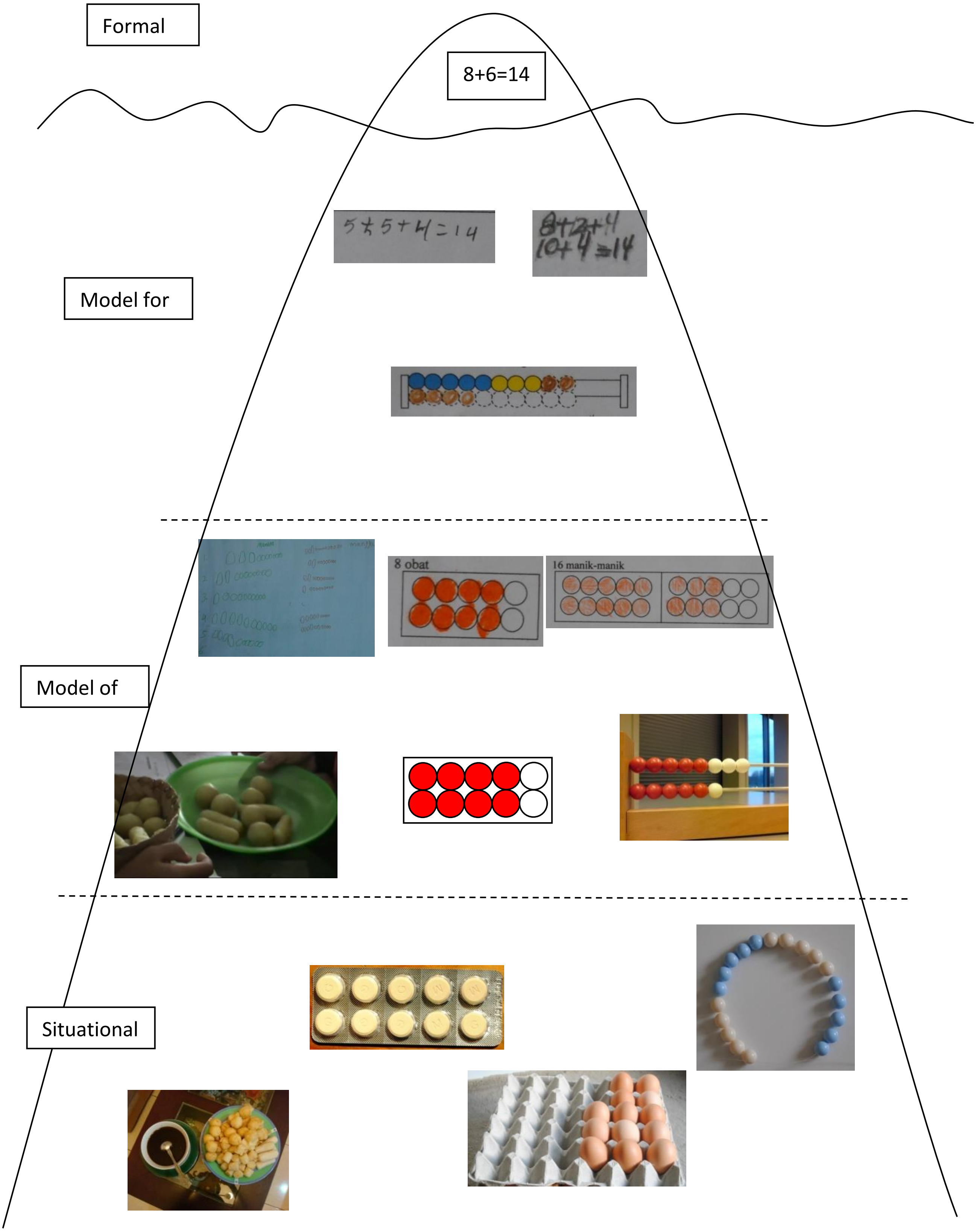
How do you know the number of eggs now?

So the numbers of beads are . . .

Appendix 4: Frequency analysis of final assessment

Question	Students' strategies		Percentage
1.	Ten-structure	17	74 %
	Five-structure	3	
	Unstructured	7	26 %
2	Ten-structure	24	93 %
	Five-structure	1	
	Unstructured	2	7 %
3	Doubles	2	96 %
	Ten-structure	22	
	Five-structure	2	
	Others	1	4 %
4	Making ten	14	56 %
	Doubles	1	
	Others	12	44 %
5	Decomposition to 10	16	70 %
	Decomposition to 5	3	
	Others	8	30 %
6	Making ten	15	59 %
	Using five-structure	1	
	Others	11	41 %
7	Decomposition to 10	17	67 %
	Decomposition to 5	1	
	Others	9	33 %
8	Making ten	7	37 %
	Using five-structure	2	
	Using doubles for near doubles	1	
	Others	17	63 %

Appendix 5: Ice berg on addition up to 20



Appendix 6: Lesson plan

Rencana Pelaksanaan Pembelajaran

Subjek **:** Matematika

Kelas **:** I

Semester **:** -

Alokasi waktu **:** 2 x 35 menit

Jumlah siswa **:** 27 orang

Aktivitas **: I**

Standar Kompetensi :

Melakukan penjumlahan bilangang sampai 20

Kompetensi Dasar :

Melakukan penjumlahan bilangan sampai 20

Indikator Pembelajaran

Siswa dapat menemukan pasangan bilangan yang hasilnya 10

Sumber belajar **:** Alat Peraga (Replika Empek-empek dari lilin)

Pendekatan **:** PMRI

Metode **:** Kerja Kelompok, Presentasi Poster dan Diskusi

Kegiatan pembelajaran :

a. Pendahuluan (10 menit)

- Pembukaan
- Guru melakukan apersepsi
- Guru menunjukkan konteks pembelajaran (*Empek-empek Palembang*) yang akan digunakan dalam aktivitas pembelajaran.
- Guru menceritakan sebuah permasalahan kepada siswa yaitu :
  Kemaren Ibu pergi kerumah saudara Ibu yaitu bu Ani. Bu Ani bercerita bahwa dia ingin mengadakan pesta buat anaknya. Bu Ani ingin menghidangkan empek-empek untuk tamu-tamu yang akan datang. Dia memutuskan untuk membuat dua jenis empek-empek yaitu empek-empek lenjer dan empek-empek adaan (Sambil memperlihatkannya kepada siswa). *Bu Ani ingin menyusun 10 empek-empek disetiap piring yang terdiri dari 2 jenis empek-empek tersebut. Jadi apa saja susunan yang bisa dibuat oleh Ibu Ani?*
- Siswa kemudian diberikan 1 kantong empek-empek yang terbuat dari lilin dan melakukan percobaan didalam kelompoknya.

b. Kegiatan inti (50 menit)

- Siswa mengerjakan masalah yang diberikan guru dalam kelompoknya.
- Siswa mendiskusikan dalam kelompok penyelesaian masalah yang diberikan.
- Siswa menyajikan pekerjaan mereka didepan kelas.

- Guru memfasilitasi siswa untuk mendiskusikan jawaban mereka dengan siswa lainya.
- Siswa menyimpulkan hasil kerja yang dilakukan dalam aktivitas pembelajarannya dengan bimbingan guru.

c. **Penutup (10 menit)**
- Siswa dengan bantuan guru menyimpulkan hasil kegiatan pembelajaran tersebut.
- Salam

Rencana Pelaksanaan Pembelajaran

Subjek **:** Matematika

Kelas **:** I

Semester **:** -

Alokasi waktu **:** 2 x 35 menit

Jumlah siswa **:** 27 orang

Aktivitas **: II**

Standar Kompetensi :

Melakukan penjumlahan bilangang sampai 20

Kompetensi Dasar :

Melakukan penjumlahan bilangan sampai 20

Indikator Pembelajaran

Siswa dapat menemukan pasangan bilangan yang kurang dari 10

Sumber belajar **:** Lembaran Kerja Siswa, Alat Peraga (Boneka tangan burung Beo, Obat tablet, dan flash card)

Pendekatan **:** PMRI

Metode **:** Kerja Kelompok, Diskusi dan LKS

Kegiatan pembelajaran :

a. Pendahuluan (15 menit)
- Pembukaan
- Guru mengingatkan kembali siswa tentang pasangan bilangan yang menghasilkan 10 dengan *mini lesson* burung Beo.
- Guru memperkenalkan situasi baru kepada siswa yaitu mengenai obat tablet.

b. Kegiatan inti (45 menit)
- Guru memperlihatkan obat tablet dan kartu yang merepresentasikan obat tablet dalam waktu yang singkat (ada sekitar 8 kartu)
- Siswa diminta untuk menceritakan tentang banyak obat yang mereka lihat.
- Siswa diminta alasan untuk setiap jawaban yang diberikanya dan siswa lain diminta untuk berkomentar tentang jawaban tersebut.
- Siswa selanjutnya mengerjakan permasalahan pada Lembaran Kerja Siswa secara berpasanga.

c. Penutup (10 menit)
- Siswa dengan bantuan guru menyimpulkan hasil kegiatan pembelajaran tersebut.
- Salam

Rencana Pelaksanaan Pembelajaran

Subjek **:** Matematika

Kelas **:** I

Semester **:** -

Alokasi waktu **:** 2 x 35 menit

Jumlah siswa **:** 27 orang

Aktivitas **: III**

Standar Kompetensi :

Melakukan penjumlahan bilangang sampai 20

Kompetensi Dasar :

Melakukan penjumlahan bilangan sampai 20

Indikator Pembelajara

Siswa dapat memahami hubungan antar bilangan seperti bilangan kelipatan

Sumber belajar **:** Lembaran Kerja Siswa, Alat Peraga (Manik-manik, gelas mineral, dan dekak-dekak)

Pendekatan **:** PMRI

Metode **:** Diskusi, Presentasi dan LKS.

Kegiatan pembelajaran :

a. Pendahuluan (10 menit)
- Pembukaan
- Guru menceritakan permasalahan baru kepada siswa yaitu:
  Disebuah pesta ulang tahun, Yang berulang tahun menyediakan sebuah tabung yang berisi dua jenis permen yaitu permen anggur dan permen coklat. Lalu kamu disuruh untuk mengambil 7 permen tampa melihat kedalam tabung tersebut, kira-kira permen apa yang akan kamu dapatkan?

b. Kegiatan inti (45 menit)
- Siswa mengerjakan masalah yang diberikan guru secara berkelompok 4-5 orang.
- Siswa mendiskusikan selama 10 menit untuk menemukan penyelesaian masalah yang diberikan didalam kelompoknya.
- Siswa mendiskusikan jawaban mereka secara bersama-sama dengan panduan guru.
- Siswa menyimpulkan hasil kerja yang dilakukan dalam aktivitas pembelajarannya dengan bimbingan guru.
- Siswa bekerja di Lembaran Kerja Siswa secara berpasangan.

c. Penutup (15 menit)
- Siswa degan bantuan guru menyimpulkan hasil kegiatan pembelajaran tersebut.
- Guru mengajak siswa untuk menyebut pasangan bilangan kecil dari 10 dengan *mini lesson* burung Beo.
- Salam

Rencana Pelaksanaan Pembelajaran

Subjek **:** Matematika

Kelas **:** I

Semester **: -**

Alokasi waktu **:** 2 x 35 menit

Jumlah siswa **:** 27 orang

Aktivitas **: IV**

Standar Kompetensi :

Melakukan penjumlahan bilangang sampai 20

Kompetensi Dasar :

Melakukan penjumlahan bilangan sampai 20

Indikator Pembelajaran

Siswa memahami akan pentingnya susunan dalam membantu melakukan perhitungan secara cepat.

Sumber belajar **:** Lembaran Kerja Siswa, Alat Peraga (Manik-manik dan nilon)

Pendekatan **:** PMRI

Metode **:** Diskusi dan Presentasi

Kegiatan pembelajaran :

a. Pendahuluan (10 menit)
- Pembukaan
- Guru mengenalakan situasi kepada siswa dimana biasanya orang-orang suka menggunakan asesoris seperti kalung ke pesta.

b. Kegiatan inti (50 menit)
- Guru Meminta siswa untuk menyusun 20 manik-manik yang terdiri dari 20 warna sedemikian hingga orang yang melihatnya dapat mengetahui banyak manik-manik tersebut dengan cepat.
- Siswa bekerja dikelompoknya untuk menyusun manik-manik tersebut dan kemudian menggambar susunan tersebut dikarton.
- Siswa kemudian merepresentasikan hasi kerjanya dan menjelaskan mengapa susunan yang mereka buat mudah untuk dikenali banyak manik-maniknya.
- Siswa yang lain member komentari hasil kerja kelompok lain.

d. Penutup (10 menit)

e. Siswa dengan panduaan guru membuat kesimpulan tentang kegiatan menyusun benda-benda dalam memudahkan perhitungan.

- Salam

Rencana Pelaksanaan Pembelajaran

Subjek **:** Matematika
Kelas **:** I
Semester **:** -
Alokasi waktu **:** 2 x 35 menit
Jumlah siswa **:** 27 orang
Aktivitas **: V**

Standar Kompetensi :

Melakukan penjumlahan bilangang sampai 20

Kompetensi Dasar :

Melakukan penjumlahan bilangan sampai 20

Indikator Pembelajaran

Siswa dapat mengenal hubungan antar bilangan yang lebih besar dari 10 dan kurang dari 20.

Sumber belajar **:** Lembaran Kerja Siswa, Alat Peraga (Dekak-dekak)
Pendekatan **:** PMRI
Metode **:** Diskusi dan Lembaran Kerja Siswa

Kegiatan pembelajaran :

a. Pendahuluan (10 menit)
- Pembukaan
- Guru mengenalakan susunan manik-manik yang ada pada dekak-dekak.

b. Kegiatan inti (50 menit)
- Guru merepresentasikan berbagai bilangan dengan dekak-dekak dalam waktu yang singkat.
- Siswa diminta untuk menebak banyaknya manik-manik yang ditampilkan.
- Siswa diminta alasan untuk setiap jawaban yang diberikanya dan siswa yang lain diminta untuk berkomentar tentang jawaban tersebut.
- Siswa mengerjakan beberapa latihan yang berhubungan dengan susunan dekak-dekak.

c. Penutup (10 menit)
- Siswa dengan bantuan guru menyimpulkan hasil kegiatan pembelajaran tersebut.
- Salam

Rencana Pelaksanaan Pembelajaran

Subjek **:** Matematika

Kelas **:** I

Semester **:** -

Alokasi waktu **:** 2 x 35 menit

Jumlah siswa **:** 27 orang

Aktivitas **: VI**

Standar Kompetensi :

Melakukan penjumlahan bilangang sampai 20

Kompetensi Dasar :

Melakukan penjumlahan bilangan sampai 20

Indikator Pembelajaran

Siswa dapat melakukan penjumlahan bilangan sampai 20 dengan menggunakan bilangan 10.

Sumber belajar **:** Lembaran Kerja Siswa

Pendekatan **:** PMRI

Metode **:** Diskusi dan Lembaran Kerja Siswa

Kegiatan pembelajaran :

a. Pendahuluan (10 menit)
- Pembukaan
- Guru melakukan apersepsi

b. Kegiatan inti (50 menit)
- Guru memberikan soal latihan kepada siswa dan memintanya mengerjakan dalam waktu 5 menit secara berpasangan.
- Siswa melakukan diskusi dengan bimbingan guru.
- Guru memberikan masalah kedua, ketiga sampai keempat secara bergantian dan mendiskusikannya secara bersama-sama.

c. Penutup (10 menit)
- Siswa dengan bantuan guru menyimpulkan hasil kegiatan pembelajaran tersebut.
- Salam

Rencana Pelaksanaan Pembelajaran

Subjek **:** Matematika
Kelas **:** I
Semester **:** -
Alokasi waktu **:** 2 x 35 menit
Jumlah siswa **:** 27 orang
Aktivitas **: VII**

Standar Kompetensi :
Melakukan penjumlahan bilangang sampai 20

Kompetensi Dasar :
Melakukan penjumlahan bilangan sampai 20

Indikator Pembelajaran
Siswa dapat melakukan penjumlahan bilangan sampai 20 dengan bermacam cara.

Sumber belajar **:** Lembaran Kerja Siswa.
Pendekatan **:** PMRI
Metode **:** Diskusi dan Lembaran Kerja Siswa

Kegiatan pembelajaran :

c. **Pendahuluan (10 menit)**
   - Pembukaan
   - Guru melakukan apersepsi

d. **Kegiatan inti (50 menit)**
   - Guru memberikan sebuah masalah yang berhubungan dengan penjumlahan.
   - Siswa mengerjakan masalah tersebut secara berpasangan.
   - Siswa melakukan diskusi dengan bimbingan guru.
   - Guru memberikan masalah kedua yang masih berhubungan dengan penjumlahan.
   - Siswa mengerjakan masalah tersebut secara berpasangan.
   - Siswa melakukan diskusi dengan bimbingan guru.

d. **Penutup (10 menit)**

Curriculum Vitae

Author is the youngest son of 5 siblings. He was born in Cimpago, Pariaman-West Sumatera, Indonesia at June 22nd, 1985. His parents are Abdul Hamid and Alimah.

The author first completed his study in primary school in SDN 01 Kabun Cimpago, Pariaman in 1997. In the same year, he directly continued his study to junior hogh school at SLTP N 05 Pariaman and completed in 2000. Then, He continued his study to senior high school in SMU N 2

Pariaman and completed at 2003. After that, he was accepted as a college student at Mathematics, Science and mathematics Faculty, in University of Riau and completed at 2007.

After finished the undergraduate program at University of Riau until 2009, the author worked at Rab University as an assistance of lecturer. He taught statistics, mathematics logic, and linier and integer programming. At the same time, he also worked at Islamic Junior High School As- Shofa Pekanbaru until 2009.

In 2009, the author received an International Master Program on Mathematics Education (IMPoME) scholarship from the cooperation among IP-PMRI (*Institut Pengembangan-Pendidikan Matematika Realistik Indonesia*), DITJEN DIKTI, and NUFFIC-NESO. Three universities are involved in this program, Sriwijaya University and Surabaya State University, Indonesia, and Utrecht University, the Netherlands.

The author accepted as an IMPoME student at Mathematics Education Department at Sriwijaya University. Then He also studied for a year in Freudenthal Institute, Utrecht University about Realistic Mathematics Education.