DESIGN RESEARCH ON MULTIPLICATION: STRUCTURES SUPPORTING THE DEVELOPMENT OF SPLITTING LEVEL AT GRADE 3 IN INDONESIAN PRIMARY SCHOOL

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

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GRADUATE SCHOOL
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APPROVAL PAGE

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STATEMENT PAGE

I hereby:

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State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

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ABSTRACT

This research was conducted in MIN 2 Palembang and involved 28 students and one teacher in third grade. The purpose of this research is to design sequences of learning activities that support students to develop their mental calculation achieving splitting level on multiplication by numbers up to 20. The activities were designed by considering tenets of Realistic Mathematics Education (RME). In designing activities, we gave attention to what students' thinking process, how designed activities helped students in developing their mental calculation on multiplication until splitting level, how class discussion helped the low achieving students to learn, and how the role of the teacher in supporting students to learn. Giving structured objects, recognizing number relation in multiplication, and maintaining flexible calculation have important roles in supporting the development of mental calculation achieving splitting level.

Key words: design research, mental calculation, splitting level, structured objects

ABSTRAK

Penelitian ini telah dilaksanakan di MIN 2 Palembang dan melibatkan 28 siswa dan seorang guru di kelas tiga. Tujuan dari penelitian ini adalah merancang rangkaian aktivitas pembelajaran yang mendukung siswa untuk mengembangkan mental berhitung mereka hingga tahap splitting dalam perkalian bilangan dasar (tabel perkalian). Semua aktivitas dirancang dengan mempertimbangkan prinsip - prinsip dari *Realistic Mathematics Education (RME)*. Dalam merancang aktivitas, kita memperhatikan proses berpikir siswa, bagaimana aktivitas yang dirancang mampu membantu mereka dalam mengembangkan mental berhitung mereka dalam perkalian hingga tahap splitting, bagaimana diskusi kelas membantu siswa berkemampuan rendah untuk belajar, dan bagaimana peran guru dalam mendukung siswa untuk belajar. Pemberian objek – objek yang terstruktur, kemampuan mengenal hubungan antar bilangan dalam perkalian, dan pembiasaan berhitung secara fleksibel mempunyai peran penting dalam mendukung perkembangan mental berhitung siswa hingga tahap splitting.

Kata kunci: design research, mental berhitung, splitting level, objek - objek terstruktur

SUMMARY

This research purposed to develop classroom activities that support students in developing their mental calculation until they achieve splitting level on multiplication by numbers up to 20. We predicted that sequence of learning activities starting from structured objects could lead students to acquaint with splitting strategy. Therefore, we formulated a research question: “How can structures support the development of a splitting strategy in doing multiplication by numbers up to 20?”

Design research is used as a research methodology to attain research goals and discover the answer of research question. It was conducted in MIN 2 Palembang which has become PMRI partner school since 2006. It was done in two cycles: pilot experiment and teaching experiment. Five students in 3B were involved in pilot experiment (the first cycle) and 28 students of 3B became research subjects in teaching experiment (the second cycle). All data were collected including video, interview, observation, students’ worksheet, and poster.

We designed 5 activities based on tenets of RME in the first cycle. Because we found that some activities did not support to answer research question, we revised them. Finally, there were 6 improved activities which were tried out in the second cycle. The improved activities are: activity 1 (**two multiplications for one multiplication**) which purposed to provoke students to be able to calculate number of objects which were arranged into two parts; activity 2 (**Relation between multiplication by 10 and by 9**) which aimed to stimulate students to use the result of multiplication by 10 in determining the result of multiplication by 9; activity 3 (**Working with model for)** purposing to provoke students to able to draw rectangle model as a model for representing situation; activity 4 (**Multiplication one larger number and one smaller number)** where students can determine multiplication “before” and multiplication “after” of a multiplication; activity 5 (**From known multiplication to another multiplication)** where students are able to use multiplication that they have known in determining the result of a multiplication; and activity 6 (**Multiplication table activity)** where students can complete multiplication table, multiplication by 11 and multiplication by 12.

In this research, we gave attention to what students’ thinking process, how designed activities helped students in developing their mental calculation on multiplication until splitting level, how class discussion helped the low achieving students to learn, and how the role of the teacher in supporting students to learn.

To answer research question, we look at the sequence of learning activities which were improved and investigate if structures supports students to achieve splitting level. We conclude that giving structured objects, recognizing number relation in multiplication, and maintaining flexible calculation have important roles in supporting the development of mental calculation achieving splitting level. At the beginning, Students do not start to use splitting strategy if teacher do not give structured objects like spoon boxes which are arranged into two piles. We can help students to see structured objects and number relationship by relating problem into context situation. It is also important to give some kinds of situation like group situation and rectangle situation in order students are able to apply this strategy in any kind multiplication problem. At the end, they can solve an unknown multiplication if we stimulate them to use multiplication that they have already known.

The focus of this research is to help students to develop their mental calculation achieving splitting level on multiplication. This research is only a small part of learning trajectory in developing mental calculation on multiplication. However, they can apply this strategy in multiplication by number up to 20.

RINGKASAN

Penelitian ini bertujuan untuk mengembangkan aktivitas pembelajaran yang mendukung siswa untuk mengembangkan mental berhitung mereka hingga tahap splitting dalam perkalian dasar. Kita memprediksi bahwa rangkaian pembelajaran yang diawali dengan objek – objek yang terstruktur mampu mengarahkan siswa untuk mengenal strategy splitting. Oleh karena itu, kita merumuskan sebuah pertanyaan penelitian: "Bagaimana sruktur mampu mendukung perkembangan mental berhitung siswa hingga tahap splitting dalam perkalian dasar?"

Design research digunakan sebagai metodologi penelitian untuk mencapai tujuan penelitian dan menemukan jawaban atas pertanyaan penelitian. Penelitian ini telah dilaksanakan di MIN 2 Palembang yang telah menjadi sekolah partner PMRI sejak tahun 2006. Penelitian ini dilakukan dalam dua siklus: pilot eksperiment dan teaching eksperiment. Lima siswa di kelas 3B dilibatkan dalam pilot eksperimen (siklus pertama) dan 28 siswa di kelas 3B menjadi subjek penelitian di teaching eksperiment (siklus kedua). Semua data dikumpulkan seperti video, wawancara, observasi, lembar kerja siswa, dan poster.

Kita merancang 5 aktivitas di siklus pertama berdasarkan prinsip – prinsip RME. Beberapa aktivitas direvisi dikarena aktivitas tersebut tidak mencapai tujuan penelitian. Pada akhirnya, ada 6 aktivitas yang telah dikembangkan dan telah diujicobakan pada siklus 2. Aktivitas – aktivitas tersebut adalah: aktivitas 1 (*dua perkalian untuk satu perkalian*) yang bertujuan untuk memprovokasi siswa untuk mampu menghitung banyaknya objek yang disusun dalam 2 bagian; aktivitas 2 (*hubungan antara perkalian 10 dan perkalian 9*) yang bertujuan untuk menstimulasi siswa untuk menggunakan hasil dari perkalian 10 dalam menentukan hasil perkalian 9; aktivitas 3 (*bekerja dengan model for*) yang bertujuan untuk memprovokasi siswa untuk menggambarkan model persegi sebagai bentuk representasi situasi; aktivitas 4 (*perkalian satu bilangan yang lebih besar dan satu bilangan yang lebih kecil*) dimana siswa mampu menentukan perkalian "sebelum" dan perkalian "setelah" sebuah perkalian; aktivitas 5 (*dari perkalian yang diketahui ke perkalian yang belum diketahui*) dimana siswa mampu menggunakan perkalian yang telah mereka ketahui dalam menentukan hasil suatu perkalian; dan aktivitas 6 (*aktivitas tabel perkalian*) dimana siswa dapat melengkapi tabel perkalian, perkalian 11 dan perkalian 12.

Dalam penelitian ini, kita memperhatikan proses berpikir siswa, bagaimana aktivitas yang dirancang membantu siswa untuk mengembangkan mental berhitung mereka hingga tahap splitting, bagaimana diskusi kelas membantu siswa berkemampuan rendah untuk belajar, dan bagaiman peran guru dalam membantu siswa.

Untuk menjawab pertanyaan penelitian, kita melihat kembali aktivitas – aktivitas yang telah dikembangkan dan menginvestigasi jika struktur mampu mendukung siswa untuk mencapai tahap splitting. Kita menyimpulkan bahwa pemberian objek yang terstruktur, pengetahuan tentang hubungan bilangan dalam perkalian, dan pemantapan dalam menghitung secara fleksibel mempunyai peran yang penting dalam pengembangan mental berhitung siswa hingga pencapaian tahap splitting. Pada awalnya, siswa tidak akan memulai untuk menggunakan strategi splitting jika guru tidak memberikan objek – objek yang terstruktur seperti kotak – kotak sendok yang disusun ke dalam 2 bagian. Kita mampu membantu siswa untuk melihat objek – objek yang terstruktur dan hubungan dalam bilangan dengan menghubungkan masalah ke dalam konteks. Pemberian beberapa jenis struktur seperti struktur grup dan struktur persegi juga memiliki peran yang penting agar siswa mampu menerapkan strategy splitting dalam berbagai jenis masalah perkalian. Pada akhirnya, mereka mampu menyelesaikan perkalian yang tidak mereka ketahui dengan melibatkan perkalian yang telah mereka pahami.

Fokus penelitian ini adalah untuk membantu siswa dalam mengembangkan mental berhitung mereka hingga tahap splitting pada perkalian dasar (tabel perkalian). Penelitian ini hanya mencakup bagian kecil dari proses belajar dan pengembangan mental berhitung pada perkalian. Namun siswa mampu menerapkan strategi ini dalam perkalian oleh bilangan hingga 20.

PREFACE

I began to realize myself that period of doing my master on International Master Program on Mathematics Education (IMPoME) was very quickly coming to the end. I reflected that many things have been passed in this period. It became my learning trajectory in doing research on multiplication domain. Looking back on how I did this research made me aware that it was not an instant way in writing this thesis but it needed much effort. However, working with young children gave me insight how they are thinking and what we could do in helping them in learning multiplication.

This thesis is nothing without the supports from people surrounding me. Therefore, I would like to express my gratitude to:

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The last but not the least, I dedicated this thesis to my family: My beloved mother and father: Yusnateli and Cik Zen Anas who supported me to study abroad, allowed me to be far from them along one year, and always prayed for my health and my strength; and my lovely brothers: Pandi and Komas who always make me smile and take care our mother and father during I was not beside them.

Finally, I hope this thesis become useful for improvement the quality of mathematics education in Indonesia.

Writer,

Meryansumayeka

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CHAPTER 1 INTRODUCTION

Multiplication is a fundamental concept that supports concepts in mathematics like: division, fraction, percentage, etc. The knowledge of basic multiplication is also required in estimation and mental computation in multiplication (Heerge, 1985). Issac (Braddock, 2010) asked, “How can students use 80 x 40 to estimate 84 x 41 if they do not know 8 x 4?”

In Indonesia, students have learned multiplication since grade 2 and 3. However, most of the time, they are required to memorize multiplication table. Meanwhile, memorizing multiplication does not help students to solve extended multiplication because they do not know the meaning of it (Armanto, 2002).

Freudenthal (1991) argued that students need experience mathematics and achieve concept of mathematics including multiplication through sequence of learning activities since mathematics itself is ‘a human activity’. Gravemeijer (Foxman, and Beishuizen, 2002) suggested that learning process in multiplication should be started by informal strategies which are promoted both by realistic contextual problems and by mental calculation. Realistic context problems are not always connected with real world but is related to the emphasis that it puts on offering the students problem situations in which they can imagine (Van den Heuvel-Panhuizen, 2000). Using context problems help students to start constructing their knowledge because they can use their own experiences related to the problem. On the other hand, mental calculation is a way of approaching numbers and numerical information in which numbers are dealt with a handy and flexible way. It is not strictly related to a certain number area or operation (Van den Heuvel-Panhuizen, 2001). Thus, students also can learn multiplication using their own way based on their mental calculation without forcing them to memorize.

Dutch research has given a good deal of attention to mental calculation strategies under the influence of the 'realistic mathematics education' movement (Foxman and Beishuizen, 2002). In the increasing importance of mental arithmetic in multiplication, Van den Heuvel - Panhuizen (2001) categorizes three types of strategies. The first strategy is stringing strategy, where the multiplicand is seen as a whole; the second is splitting strategy, where the multiplicand is split into tens and ones; and the third one is varying strategy, where addition or subtraction concepts are also involved in splitting multiplicand. In another research which was done by Ambrose (2003), some important remarks in this study are made concerning the way the more basic strategies can evolve into more advanced strategies in a process of progressive abbreviation and level raising. Thus, students can involve their strategy which is meaningful for them in developing their mental calculation.

At the first, students learn multiplication as repeated addition. This strategy, where they do addition several times, is a kind of stringing strategy. In the process, they would have started learning multiplication tables. However, most of the time, students in Indonesia are required to memorize the multiplication table. Meanwhile, they can use their mental calculation when they face forgotten multiplication facts. They can relate multiplication that they know to find result of unknown multiplication. However, it is not enough to use multiplication as repeated addition in solving multiplication with larger numbers. They need to improve their mental calculation to higher level in multiplication. In this case, they start to acquaint splitting level where they can find result of unknown multiplication by splitting it into some multiplications that they have known.

Considering the need to develop students' mental calculation on multiplication and the need to connect mathematics to reality, we design instructional sequence of activities starting from structured context which is conjectured to be able to lead students to achieve splitting level on multiplication. Therefore, we formulate a research question:

"How can structures support the development of splitting strategy in doing multiplication by numbers up to 20?"

There are two sub research questions which are posed, namely:

1. *"How can group structures and rectangular structures lead students to apply splitting strategy?"*
2. *"How can splitting strategy be applied in multiplication by numbers up to 20?*

CHAPTER 2 THEORETICAL FRAMEWORK

This chapter describes the theoretical frameworks that are served as a basis for the research. It includes theories related to key concepts in this research such as Realistic Mathematics Education, emergent modeling, emergent perspective, mental calculation, splitting level, structures on multiplication, and learning multiplication.

A. Realistic Mathematics Education

Realistic Mathematics Education is a theory of teaching and learning in mathematics education that was firstly introduced and developed by Freudenthal Institute in Netherlands. One of central principles of Realistic Mathematics Education which is mostly determined by Freudenthal's idea in mathematics education is that mathematics is 'a human activity'. It means that mathematics should be taught in the way where students can experience and attain concepts. Therefore, this study design and develop a learning sequence in multiplication in which students can develop their mental calculation achieving splitting level on multiplication by numbers up to 20 through experiencing a sequence of learning activities rather than memorizing multiplication table. However, the sequence of learning activities in this study is only a part of longer series of learning trajectory in developing students' mental calculation on multiplication.

The sequence of learning activities is designed based on five tenets of Realistic Mathematics Education which are stated by Treffers (Bakker, 2004). We describe the five tenets of RME as following:

1. *Phenomenological exploration.*

   Contextual situation is used as a starting point in the first instructional activity. Beginning with contextual situation, students can use their informal knowledge since they experience the situation. *Kondangan*, a ceremony event in Indonesia that is held because of marriage or circumcision one or more sons, is a context that will be used in learning process about multiplication. It contains structured objects that can be used to lead student to achieve splitting levelon multiplication. Contexts such as spoon boxes arrangement and arrangement of guess chairs are considered as a starting point to stimulate student's informal multiplication strategies directing to splitting strategy.

2. *Using models and symbols for progressive mathematization.*

   The development from informal to formal mathematical concepts is a gradual process of progressive mathematization. From contextual situation, students start to mathematize by making model of situation and continue to generalize by making model for calculation. Rectangle models can be used to support their development of mental calculation achieving splitting level in multipication. These instruments are meaningful for the students in representing situation and have potentials for generalization and abstraction.

3. *Using students' own constructions and productions*.

   It is assumed that what students make on their own is meaningful for them. Hence, using students' constructions and productions is promoted as an essential part of instruction. Students use their own way in representing situation. They also work using their own strategies although we direct them to achieve splitting level.

4. *Interactivity.*

   Students' own contributions can be used to compare their works. Students can learn from each other in small groups or in whole-class discussions. Through discussion, they can evaluate and improve the way they achieve splitting level in multiplication. Discussion also can help low achieving students to learn.

5. *Intertwinement.*

It is important to consider an instructional sequence in its relation to other domains. In learning multiplication, students also can intertwint with another concepts like addition and subtraction in basic forms of mental calculations.

B. Emergent Modeling

To support progressive mathematization which is started from context situation, Grivemeijer (1991) argued that model can be used to bridge their informal and formal knowledge. He described how model of informal context situation become model for more mathematical reasoning. This following is the levels of emergent modeling from contextual situation to formal reasoning.

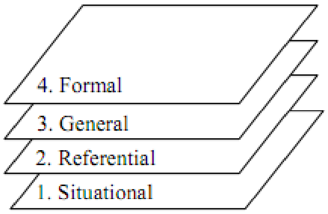


Figure 2. 1 Level of Emergent Modeling

In designing instructional sequence that support student's acquisition of mental multiplication skill, we start from "Kondangan" situation as a context to learn more about multiplication. The representation of the arrangement of guess chairs, plates, and glass that students make by themselves can be used as model of situation. In development of their understanding, rectangle model emerge as model for more formal way in calculation. In final level, they are mentally able to do calculation in numerical way without using rectangle model anymore. However, this research is only a part of learning trajectory in developing student's mental calculation in multiplication grade 3. Therefore, we only design activities to the stage that students are able to create model of situation and they are able to use number relation in solving multiplication problem in one digit numbers.

C. Emergent Perspective

According Gravemeijer and Cobb (2006), emergent perspective is a framework that is used in interpreting classroom discourse and communication. It is viewed as a response to understand mathematical learning when it occurs in the social context of the classroom. There are two kinds of norms that are carried out in this research, social norms and socio mathematical norms.

Social norms are ways of interaction between teacher and students, these norms include ways of acting and explaining. In this research, socio norms that are expected include obligation for students to explain and justify solution in multiplication problem, attempt to make sense of explanation which is given by other students, indicate agreement and disagreement with multiplication solution that is presented by other students, and ask if they do not understand.

Socio mathematical norms are ways of explicating and acting in whole class discussion that are specific to mathematics. They include different mathematical solutions, sophisticated mathematical solutions, efficient mathematical solutions, and acceptable mathematical explanation and justification. In the area of multiplication, students may use different ways in solving multiplication problem whether they use repeated addition, doubling, or splitting – use number relation in their solution. Among those ways, they will see that using number relation is an efficient and acceptable way especially when they have larger number. Of course, in the development of their mental calculation, they need to know the reason of their action.

D. Mental Calculation

Van den Heuvel - Panhuizen (2001) defined that mental calculation is skillful and flexible calculation based on known number relationships and number characteristics. In line with Van den Heuvel – Panhuizen, Buijs (2008) stated that mental computation refers to a fluent and flexible way of computation that is mainly done by heart and in which appropriate number relations and mathematical properties are used. Maclellan (2001) state that Sowder mentioned mental calculation as the process of carrying out arithmetical operations without the aid of external devices. Particularly, mental calculation necessarily uses strategies which are very different from the algorithms associated with pencil-and-paper procedures. Threlfall (2002) state that Thompson suggested mental calculation 'strategies' as the application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known. Thus, mental calculation is a flexible way that students can use to calculate based on number relationships and they have the freedom to use their own ways.

According to Van den Heuvel (2001), there are three elementary forms of mental calculation in multiplication which continue on from each other. They are stringing level, splitting level, and varying level. Since we focus in supporting students to achieve splitting level, we describe the explanation of the splitting level in the following sub chapter.

E. Splitting Level on Multiplication

Splitting strategy involves "breaking down" numbers where multiplicand or multiplier is no longer seen as a whole but it is separated into other numbers – it is usually split into tens and ones in multiplication with larger numbers. In the area of basic multiplication, this is characterized as structuring based multiplication where a problem, for instance, a 6 x 8 structure, is no longer solved by step-by-step counting but is reached with only one intermediate step using the known table product of 5 x 8.

According to Ambrose (2003), splitting is acquainted alongside when they have extended repeated addition or doubling which is categorized as stringing strategies. Using multiplication situations based on different structure will lead to the splitting strategy becoming independent of the concrete examples. In the area of multiplication, three kinds of context situation like a line situation, group, and rectangular structure support the insight that 6 x 8 is 5 x 8 plus 1 x 8 (Van den Heuvel, 2002)

Ambrose (Buijs, 2008) mentioned strategies in multi-digit multiplication problems - which are used by students that have not had any formal instruction on the subject - vary from elementary solution strategies such as repeated addition and various form of doubling to more sophisticated strategies based on decimal splitting. It is a process for students to develop their strategy in multiplication where the basic strategies can be involved into more advanced strategies. In this research, we want to support students to develop their strategy until splitting level through designing a learning trajectory for the domain of multiplication based on the realistic approach of mathematics education. We try with basic multiplication because students have to be advanced with multiplication table before they acquaint with splitting level in multiplication with larger numbers.

F. Structures on Multiplication

Freudental (1991) suggested that context situation should be used to promote students to learn mathematics because they can use their experience to construct new knowledge. However, the chosen context should be appropriate with attained mathematical idea. Using structured objects are believed to be able to make students aware of splitting strategy on multiplication. In this study, structured objects mean objects that are arranged in certain condition such that students recognize parts of arrangement and can use multiplication in each part.

Freudenthal (1991) believed that structuring is a mean of organizing physical and mathematical phenomena, and even mathematics as a whole. In this study, structures which are used are structured objects in group situation and rectangular situation. There are group situation such spoon boxes arrangement and rectangular situation such guest chair arrangement which are used at the beginning of instructional sequence of learning activities. In group situation, some spoon boxes are arranged into some parts purposing students can calculate them in the partition. Guest chair arrangement in rectangular situation provides chair arrangement based on rows and columns.

G. Learning Basic Multiplication

In Indonesia, basic multiplication facts generally have been taught in second and third grade. They have learned it as repeated addition in the first time (Armanto, 2002). However, most of the time, they are required to memorize it. Whereas, Issac (Braddrock, 2010) argue that memorizing can cause anxiety in students which can lead to a lack of motivation and a bad attitude toward mathematics. Otherwise, children with deep conceptual understanding of multiplication will have an advantage when they have faced with a forgotten multiplication fact. Besides that, Hergee (1985) believes that knowledge of basic multiplication support flexible mental calculation. Since they only know multiplication as repeated addition, Clark and Kamil (Braddrock, 2010) argue it requires higher order multiplicative thinking which the child develop out of addition.

In learning basic multiplication, Issac & Carrols (Braddrock, 2010) argue that basic multiplication facts should begin with real world multiplication situation that students can model with manipulative and count all the objects. It is in line with some tenets of RME that instruction activity is started with contextual situation and let students to make representation of situation. Drawing pictures such as equal groups and arrays should be a part of the early instruction of basic facts. Using multiplication situation like group or rectangular situation will lead them to recognize multiplication structure. Because of structure context, it promotes students to use splitting strategy.

Sherin and Foson (Braddrock, 2010) believe that students will learn some basic facts on their own like multiplication by 5 and multiplication by 10 since they have ability to count by 5 and 10, but other higher factors like 6, 7, 8, and 9 require explicit instruction. However, Angileri argues that they can use lower factors (from 1 until 5) to solve problems with higher factors.

CHAPTER 3 METHODOLOGY

This chapter describes methode which is used in this research including research methodology, research subject, timeline of research, data collection, data analysis, validity and reliability.

A. Research Methodology

The topic of this study is the development of mental calculation achieving splitting level on multiplication. More specifically, we want to know if our designed instructions will help students to develop their mental calculation until they reach splitting level on basic multiplication. Therefore, instructional sequences are designed. For that purpose, design research is used as a research methodology to achive research goals and discover the answer of research question.

According to Gravemeijer (2006), there are three phases of design research: preparation for the experiment, experiment in the classroom, and restropective analyses. These phases are descibed as following:

1. Preparation for the experiment

This phase contains preparations before experiment is conducted. It includes designing hypothetical learning trajectory, determining research subjetcs, and doing pre assessment. A hypotetical learning trajectory which is designed consists of mathematical goal, teaching and learning activities, and conjecture of student's thinking. The designed hypothetical learning trajectory will be implemented in a experiment phase. Before any instruction begin, we also do observation in school where we will conduct the research. Observation includes contact with teacher, condition of school, and students. It is also a phase to choose a number of students which are involved as research subjects. However, pre assessment is also useful to be done. The purpose is to investigate students' pre – knowledge about the way they learn multiplication and how far they have learned it. It is used as starting point before we do insructional activities. It includes interview with teacher and written test for students in whole class. In written test, students had to answer some open questions. For further need, it is possible to do interview for some students to know more specific information. After hypothetical learning trajectory have been designed, research subjects have been chosen, and pre assessment have been done; designed hypotetical learning trajectory is conducted in classroom experiment.

2. Experiment in the classrrom

In this phase, we do experiment in classroom and test the conjecture of student's thinking which is designed in preparation phase. The aim of this phase is to improve learning trajectory in hopes of helping students to aquaire multiplication basic facts through developing their mental calculation. The experiment is conducted in 6 lesson. Each lesson emphasizes mathematical idea and students' learning process. Before doing each lesson, researcher and teacher discuss about upcoming lesson and make reflection after each lesson is done. During experiment, all data including classroom observations, interviews, poster, and students' worksheet are collected. These gathered data will be analyzed in restropective analyses phase.

3. Restropective analyses

After data are gathered, we analyze all data and get evaluation for hypotetical learning trajectory. According to restropective analysis result, the hypotetical learning trajectory is developed and is tried out in the next cycle. At the end of the last cycle, we discover the answer of research question about what kind of structures that support students to develop splitting strategy in multiplication.

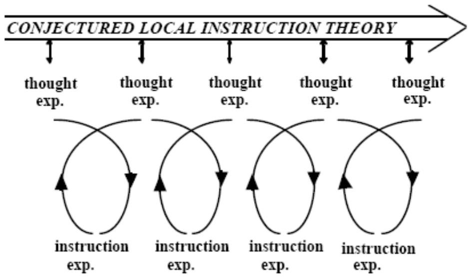


figure 3. 1 The cycle process of design research (Gravemeijer, 2004)

B. Research Subjects

This research was conducted in MIN 2 Palembang which has become school partner of PMRI since 2006. Twenty eight students of 3A, one group which consist of 5 students in 3B, and a teacher of grade 3 were involved in this research. The students were about 8 to 9 years old and they had learned about multiplication as repeated addition in grade 2.

C. Time Line of Research

The organization of this research is described in the following table:

Date	Activity	Purpose
Preliminary Experiment		
25 January 2011	The first meeting with teacher and headmaster	Asking permission to conduct research in MIN 2
28 January 2011	Observation and pre test in 3B	Observing students and investigating their pre-knowledge about multiplication in 3B.
29 January 2011	Consultation with supervisor and peer validation	Validating research instrument
1 February 2011	Interview and consultation with teacher.	Discussing with teacher about designed learning instruction that will be conducted
4 February 2011	Discussion with teacher	Discussing the coming up activity that is described in “teacher guide”

8 February 2011	Pretest in 3A and determining research subject	Investigating pre-knowledge of students in 3A and determining research subject for the first cycle and the second cycle.
Pilot Experiment (cycle 1)		
9 February 2011	Activity 1: calculating the number of pans in *Kondangan* event	Students are able to solve multiplication using repeated addition
10 February 2011	Activity 2: Calculating the number of spoons in boxes	Students can use "Number Relation" in solving multiplication problem.
11 February 2011	Activity 3: Arranging guests chairs	Students are able to make structure in multiplication
12 February 2011	Consultation with supervisor	Discussing some findings in pilot experiment for the first three meetings
14 February 2011	Activity 4: calculating the number of guest chairs and attending guests	Students can solve problem of multiplication by 10 and multiplication by 9
16 February 2011	Activity 5: Solving multiplication problem with larger numbers	Students are able to solve multiplication problem with larger numbers using number relation between tens and ones
Analyzing the Preliminary Experiment and Improved HLT		
17 – 20 February 2011	Revision according to the result of the first cycle and discussion with teacher and supervisor	Producing improved activities that will be tried out in the second cycle.
Teaching Eksperimen (cycle 2)		
16 March 2011	Activity 1: Using two multiplications for one multiplication	Students are able to use the result of two multiplications in determining the result of a multiplication.
19 March 2011	Activity 2: Relation between multiplicand by 10 and by 9	Students can use the result of multiplication by 10 in determining the result of multiplication by 9
22 March 2011	Activity 3: Working with model for	Students are able to draw rectangle model as a model for representing situation
23 March 2011	Activity 4: Multiplication One larger number and one smaller number	Students can determine multiplication "before" and multiplication "after" of a multiplication
12 April 2011	Activity 5: From known multiplication to another multiplication	Students are able to use multiplication that they have known in determining the result of a multiplication
14 April 2011	Activity 6: Multiplication table	Students can complete multiplication table, multiplication by 11 and by 12 by using number relation.
16 April 2011	Final Assessment	

D. Data Collection

The data in this research are gathered by interview, observation, and documents. These instruments are used to provide triangulation in the data. Interview and observations are recorded by videos where there are two video cameras which are provided; static camera and dinamic camera. Static camera is a camera which is put in a corner of classs and is used to record all of situation in class meanwhile dinamic camera is used to record a unique cases which are found in class when experiment is conducted. Documents which are collected include students' worksheets, posters, materials, etc

E. Data Analysis

After data are collected in experiment phase, we extensively analyze them during retrospective analysis phase. In the analysis, we compare the Hypothetical Learning Trajectory and the actual student's learning that happened during experiment based on the video recording. At the beginning, we watch the whole video and look for segment in which students did or did not do what we expected. We also figure out some fragments in which unexpected cases take place. Then, we registered these selected fragments for a better orgnization of the analysis and leave out unrelevant videos.

After that, we transcribe the conversation between teacher and students in the selected fragments. Then we analysis by looking at short conversations and student's gestures in order to make interpretation of student's thinking procesess. In doing this, we also discuss the interpretation of student's learning with other friends.

To improve validity of research, we do not only analyze video record but we also use other sources like teacher's interview and students's work. we also ask opinions from supervisors as the experts, discuss the analysis and improve them. At the end, we make conclusion based on restropective analysis and answer the research question.

F. Validity and Reliability

To support validity in this research, we use many sources like teacher interview, video recording of classroom observations, and students's work. Having these data allows us to conserve the triangulation so that we can control the quality of conclusion.

According Arthur (2004), to improve internal reability of the research, we discuss the critical episodes with colleagues (peer examination). We register and record the data in such way that it is clear where the conclusion come from. In this way, we take care the external reliability and the trackability.

CHAPTER 4 HYPOTHETICAL LEARNING TRAJECTORY

According Simon (2004), a Hypothetical Learning Trajectory consists of goals for student's learning, teaching and learning activity, and hypoteses about processes of the student's learning. The mathematical goals which want to be reached are as directions for teaching and learning process and hypotesis of student's learning. The selection of learning tasks and the hypoteses of student's learning are interdependent where the tasks are selected based on hypothesis about the learning process. The hypothesis of the learning process is based on the tasks involved. Grameveijer (2004) added that the activity of designing instructional activities is guided by a conjectured local instruction theory, which is developed in advance, and which is refined and adjusted in the process.

This table below contains learning trajectory which consists of mathematical goals, activities, and conjectured of students' learning process in achieving splitting level on basic multiplication facts.

Table 1 Hypothetical Learning Trajectory

Goal	**Activity**	**Mathematical idea**	**Conjecture**
Students can solve multiplication problem by repeated addition	**Calculating *Kondangan* Warmer Square Pans**	Multiplication as a repeated addition	- They make drawing like rectangles, lines or numbers to represent pans. - They use repeated addition in their calculation.
Students are able to use number relation to solve the problem	**Calculating Spoons**	Number Relation	Students may use repeated addition in determining the number of spoons students use number relation when they know that number of spoons in 8 boxes can be known when they add number of spoons in 3 boxes and number of spoons in 5 boxes.
students are able to make structure	**Arranging Guest Chairs**	Multiplication Structure	they may make drawing like the actual chairs, rectangles, circles, or lines as representation of chair

			arrangements. In determining the number of chairs, they may calculate how many chairs in the left side and in the right side then they sum up to get the total. They may use repeated addition in calculating the number of chairs in each side.
Students can solve multiplication by ten problem.	**Chair for Guests**	Multiplication by ten	Some students solve by repeated adding or doubling, to compute all chairs. When they are able to see their drawing in different side they can calculate in different way, they exchange the numbers.
Students are able to use subtraction from the result of multiplication by tens to determine the result of multiplication by nine	**Attending Guests**	Multiplication by nine	Some students may still use repeated addition. The others may use subtraction from the result of multiplication by ten.
Students can solve multiplication problem with larger number than 10 by using number relation between tens and ones.	**Solving Multiplication with Larger Number by Number Relation.**	Number Relation in Multiplication with larger numbers	In solving multiplication larger than ten, they may solve via multiplication by ten plus multiplication by ones.

Preknowledge

We expect that students are fully confident in solving multiplication problem by repeated addition since they have already learned multiplication in grade 2. It is good as starting point to develop splitting strategy in multiplication.

If students have not been advanced with multiplication table, the design of learning trajectories is purposed to help them in understanding multiplication table by using splitting strategy not memorizing it. In the following, we describe the instructional activities in more detail:

A. Activity 1 : Calculating *Kondangan* Warmer Square Pans

Goal : Students can solve multiplication problem by repeated addition

Problem :

In *Kondangan* event, there are 4 tables as places to put food for guests.

In each table, there exists 6 warmer square pans.

a. Draw the situation of the tables in that *Kondangan* event!

b. How many pans are there on all tables?

Description of Activity :

Students are asked what the situation is about. Futhermore, they are asked to make representation of situation and calculate how many all warmer square pans which are provided. Question a) is purposed to know the way they represent the situation. Question b) is purposed to know the way they solve the problem. After students work in their group, there is discussion which is aimed to present their answers. Through discussion, students learn from their friends. They tell the result and how they get the result. We expect that students have been well advanced with repeated adition in solving this kind of problem.

Conjectures:

Based on the problem, in answering question a) students may draw 4 rectangles as representation of tables. Then, they give drawing of warmer square pans like 6 rectangles, 6 lines or number 6 to represent square pans in that tables. In answering question b), they might calculate them by repeated addition or doubling. They add 6 as many 4 times to find the result. The others might add two first 6 and two last 6 then sum them up together. There might also some students who know the result of 4 x 6 since they learn it by heart.

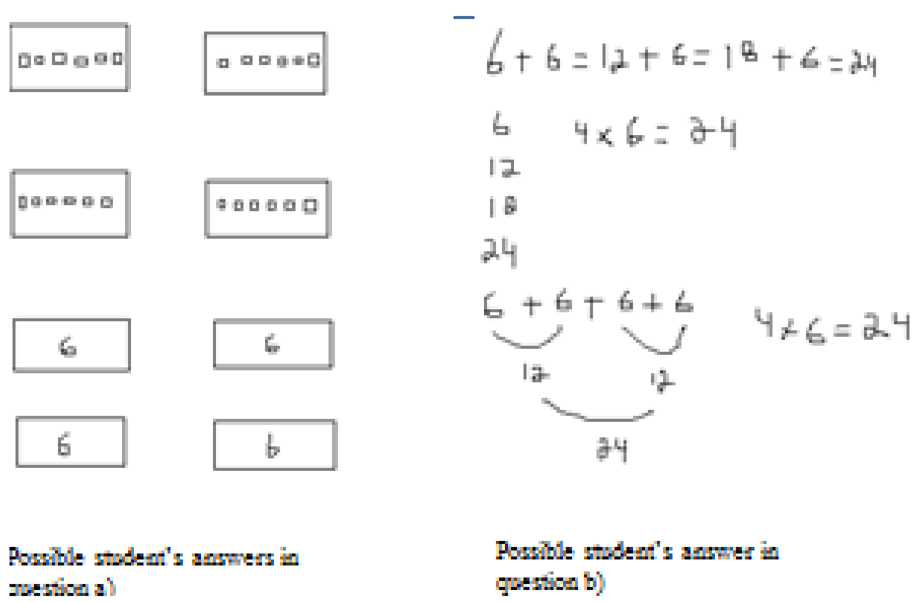


figure 4. 1 Possible students' anwers in question a) and b)

Discussion:

In discussion, For students who just write down 4 x 6 = 24 since they learn it by heart, teacher may ask questions: what is the meaning of 4 x 6? What do you do to find the result of 4 x 6? From the problem 4 x 6, which number that is repeated? These questions are purposed to make them sure with their answer not because they just memorize it but they really know the meaning of the numbers. We expect that students have been advanced with repeated addition in solving this kind of problem. Although they know the result by heart, they know the reason and how it works.

B. Activity 2: Calculating Spoons

Goal : students are able to use number relation to solve the problem

Problem:

Mother bought spoons in some boxes in which there are 6 spoons in each box.

a) If mother bought 5 boxes, how many spoons are there ?

b) Evidently, mother need more spoons so that she buy 2 more boxes, how many boxes does mother have now?
c) How many spoon does mother has? How do you calculate them?

Description:

Using real boxes, students are asked how many boxes of spoon that mother has at the first time. They are asked to make representation of spoon boxes before and after mother add more boxes. Futhermore, they are also asked to calculate how many spoons mother has at the end. They work in group and use boxes such that they can determine the numbers of spoons. Then, there is class discussion which is purposed to present some student's works. Beside telling the result, they also tell how they find it. We expect students are able to use number relation. In this kind of problem, they can add the number of spoon in five boxes plus that of two boxes to find the number of spoons in seven boxes. Through discussion, students, who still use repeated addition in finding the number of spoon in those three stacks, can learn that they can use number relation to answer multiplication problem like that. It is also as a way to understand multiplication table where they do not need to memorize it but they can use number relation to find the result.

Conjecture:

Based on the problem, students may use repeated addition in determining the number of spoons. They know that 5 x 6 means adding 6 as many as 5 times. Some students might know the result of 5 x 6 since they learn it by heart. In determining the number of spoons altogether, students might add the result of 5 x 6 and result of 3 x 6. When they know the result of 5 x 6, they use it in determining the result of 8 x 6. They just add three more 6 since they know that 8 boxes of spoons which contain 6 spoons mean 5 boxes plus 3 boxes. Therefore, they add them and get the result. Here, students use number relation where they know that number of spoons in 8 boxes can be known when they add number of spoons in 5 boxes and that of 3 boxes.

Discussion:

In counting how many spoons in each pile, most of the time students use repeated addition. In this situation, teacher can provoke question: what is the possible way to calculate them without repeatedly add them from beginning to find the number of spoons in second and third pile? With this question, they may consider and think another way. The critical situation here is when they are not able to use number relation the way that they can use the result of the previous pile (first pile and second pile) to find the number of spoons in third pile. If this happen, they still back to use repeated addition as their favorite way. Through discussion, we expect that they also learn from their friends who are able to use number relation in their calculation so that they rethink that they can use the way like the other friends do, using number relation. Teacher may give another similar problems – multiplication by another number- to explore their understanding.

C. Activity 3: Arranging Guest Chairs

Goal: Students are able to make structure

Problem:

In preparation a Kondangan event, commitees want to arrange chairs for guest.

a) Using small artificial guest chairs which are given, how do you make the chair arrangement?
b) Draw their representation in your paper.

c) How many chairs are there?

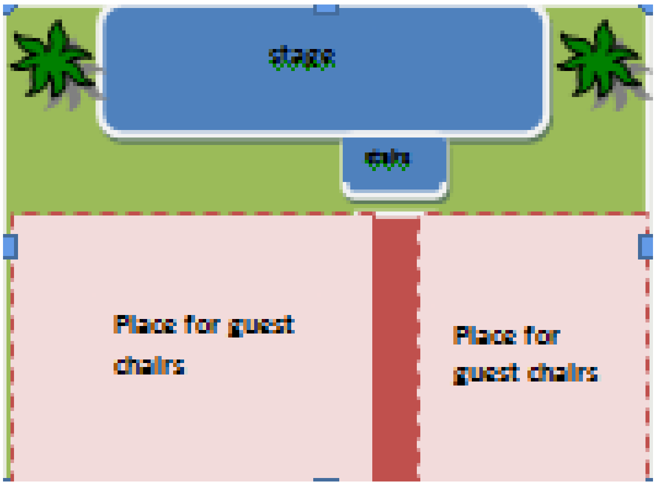


figure 4. 2: place of mini chairs

Description:

After telling the condition of places that are used to put guest chairs, students are asked to arrange guest chairs by using small artificial chairs which are provided in certain numbers. There are 56 chairs which are used; spaces of places for guest chairs that are provided are enough for 5 colums in left side and 3 colums in right side and there are 7 rows of chairs (students do not know this condition at the first time); but students are let to arrange by themselves and find out the condition. We expect that they like to arrange chairs. Then, they are asked to make representation of them in their paper and calculate how many chairs there are. Futhermore, they are engaged to calculate the number of chairs using representation that they make in their paper. At the end, there is class discussion after they work in their group. In this discussion, they present the result and tell the way they find it. If there is different way that they find from their friends, they tell their own way. We expect that they are able to find the result through number relation.

Conjecture:

Using small artificial guest chairs, students arrange them in the right and the left side. They may arrange the chairs in the right side first or in the left side first. They may arrange the chairs based on rows or coloms. After they full one side they move to the other side. In representing the arrangement of chairs, they may make drawing like the actual chairs, rectangles, circles, or lines as representation of chair arrangements. Since the place for chairs are limited, they will not arrange more than 5 colom in the left side and not more than 3 colom in the right side. In determining the number of chairs, they may calculate how many chairs in the left side and in the right side then they sum up to get the total. They may use repeated addition in calculating the number of chairs in each side. Some students may know the number of chairs is same with the result of 8 x 4. There are also students who use multiplication in each side. They determine the result of 5 x 7 and 3 x 7. They add the result of those multiplications to get the number of chairs altogether.

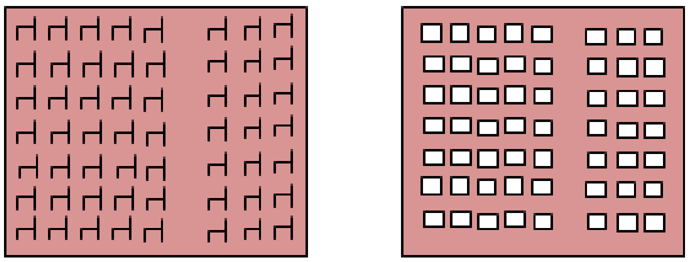


figure 4. 3: some student's representations

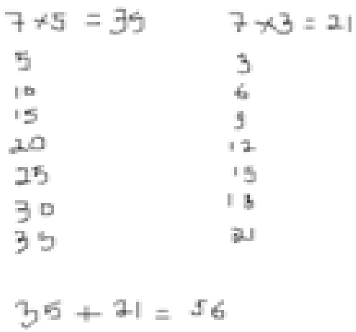


figure 4. 4: One of student's possible way

Discussion

In discussion, after they tell that they can find how many chairs there are through calculating the number of chairs in the left side - that is 5 x 7 - and the right side – 3 x 7, teacher poses questions: if you ignore the partition between right and left side, how many coloms are they? What can you say about their relation when they are with or without the partition? What can you conclude? By asking them those question, they can relate that (5 x 7) + (3 x 7) = 8 x 7. We expect that through this activity they are able to use number relation. They learn that, they can solve via 5 x 7 plus 3 x 7 in finding the result of 8 x 7. Teacher can give another similar multiplication problem by another different numbers.

D. Activity 4: Chairs for Guests

Goal : Students can solve problems of multiplication by ten.

Problem:

In a kondangan event, committees provide 10 rows of chairs which each consist of 6 chairs. How many guests can sit on that chairs?

Description of activity:

Students are asked to show their ways in calculating the numbers of chairs. The question is purposed in order they get used to solve kind of problem using multiplication. Furthermore, the question is posed to develop their understanding of multiplication by ten. Through discussion, we expect that students not only can solve this multiplication problem but they also are able to calculate when they see in different side of their drawing that they make as the representation of situation.

Conjecture:

Students may make drawing to represent the situation. Some of them may use multiplication by ten and know the result of this multiplication. For students who do not know the result of this multiplication, they use repeated addition or doubling to compute all chairs. They add 6 as many as 10 times. When they see their drawing in different side, they may think that they can do 10 x 6 because they see there are 6 rows and 10 coloms. Then they add 10 as many as 6 times.

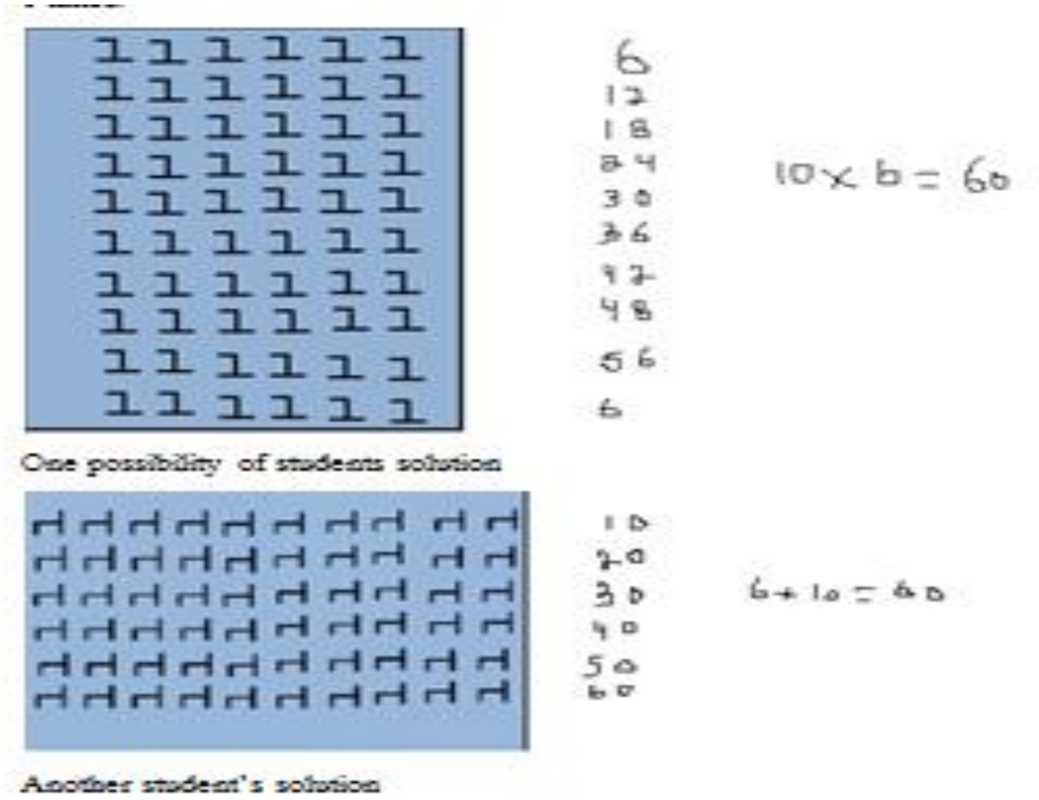


figure 4. 5: some possibilities of students' solution of problem "chairs arrangement"

Discussion:

In discussion, they present the way to determine quantity of the chairs. Teacher may ask question: How do you find the result? Students may have different way in calculation. Some students may calculate from 10 x 6. The other may use 6 x 10. Teacher can ask: based on the problem, what is multiplication that suitable with that problem? They will say 10 x 6. Then, teacher asks: what is the meaning of 10 x 6? They explain the reason that there are 10 rows consisting 6 chairs in each row. For students that solve via 6 x 10. Teacher may ask: what is the meaning of 6 x 10? Students may explain that because they see from different side that there are 6 rows consisting 10 chairs in each row. No matter what side that they see, as long as they know the reason of their way. Through discussion, students presents their way whether they solve via 10 x 6 or 6 x 10. At the end, they know that the result is same.

E. Activity 5: Attending Guests

Goal : Students are able to use subtraction from the result of multiplication by ten to determine the result of multiplication by nine

Problem:

From 10 rows of chairs which consist of 6 coloms of chairs, only 9 rows are fulled. How many attending guests are there?

Description of Activity:

Relating to the previous activity that commitees provide 10 rows of chairs which consist of 6 coloms, students are asked how many guests that attend when they only fulfill 9 rows of chairs. Students are provoked in order they do not use repeated addition. They are asked whether they can relate the problem with the problem in the previous activity. There is class discussion where they present how they find the result.

Conjecture:

Some students may still use repeated addition, they add 6 as many as 9 times since they know that there are 9 rows that are fulfilled. The other students may start from the result of 5 x 6 and add 6 as many as 4 times. Another students just take away 6 from the result of 10 x 6 because they know that only one row that is not fulfilled.

Discussion:

In discussion, teacher may ask: how do find the result? The answer may vary. The different students' answers can be used in discussion. Teacher may provoke students to see relation between the situation of attending guest and all chairs by posing question: what can you say about the situation of guests and all cairs. We expect that students involve subtraction in finding the number of guests. At the end, students learn that they can take away a numbers from the result of multiplication by ten in finding the result of multiplication by nine

F. Activity 6: Solving Multiplication with Larger Number by Number Relation

Goal: Students can solve multiplication problem with larger number than 10 by splitting the multiplication into multiplication by ten and multiplication by ones.

Problem:

For the need of *Kondangan* event, mother bought 10 boxes of spoons where there are 6 spoons in each box. Evidently, she thinks that it is not enough so that she buys 3 more boxes.

a) How many boxes of spoons that she buys?

b) How many spoons are there? How do you find the result?

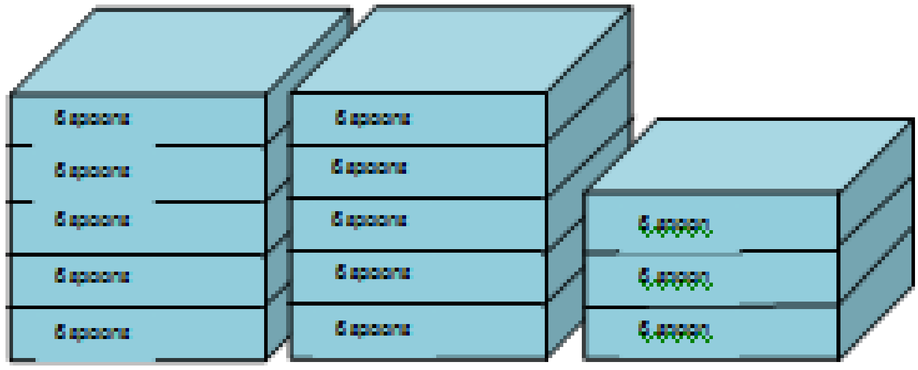


figure 4. 6: spoon boxes

Description:

This activity is purposed to ask student the result of multiplication and also to know the way they solve the problem whether they can use number relation or not, especially when we use the larger numbers. Picture of boxes are provided in order when they have difficult with the number they can imagine the situation. Question a) is purposed to ask the number of boxes altogethers. Question b) is aimed to know the way they find the result. There is class discussion where students present their works. We expect thet they are able to involve multiplication by ten to find the result of multiplication by number up to 20.

Conjecture:

Based on the problem, students may use multiplication 13 x 6 since they see that there are 13 boxes. They use repeated addition to solve the problem where they add 6 as many as 13 times. Some students may calculate 10 x 6 at the first time then calculate 3 x 6 and sum up their result. These students know that 13 boxes means the first 10 boxes plus 3 more boxes. Most students know 10 x 6 but they have a litlle work to find the result of 3 x 6. They may use repeated addition to find it or some students may know it by heart.

Discussion:

In discussion, students present their works. For students that write the problem 13 x 6, teacher ask: what is the meaning of that number? They explain that there are 13 boxes of 6 spoons. Teacher may ask: how do you find the result of that multiplication? We expect that there are students who can explain that they can find the result via 10 x 6 plus 3 x 6 to calculate the number of spoons in 13 boxes. Through discussion, we expect that other students can learn from their friends. The other students can be influenced that they can use splitting strategy in finding the result of multiplication with larger number.

CHAPTER 5 RESTROPECTIVE ANALYSIS

This chapter contains analysis of data collections in pilot experiment - it includes analyses of pre test and activities which have been done in the first cycle; and analysis of teaching experiment. The purpose is to compare hypothetical learning trajectory with students' actual learning. The result of analyses will be used to revise activities which have been designed. Furthermore, the improved activities will be tried out in teaching experiment as the second cycle.

E. Pre - Test

Pretest which was given is purposed to know what students have known about multiplication. Generally, we expected that students could explain what in their mind related to situation which was given in pictures. They said what they saw, what they can explain, and how to calculate objects. We wanted to see if they count objects or they used multiplication. Furthermore, we wanted to know whether they have been advanced in repeated addition or not. This is a basic condition before they are supported to develop their strategy into splitting and varying strategy– the next level of mental calculation after they have been advanced in repeated addition which is categorized as stringing strategy. To know what level they are, it is useful to give them written pretest. The pretest contained some open questions in order we get information as much as we can about what pre knowledge that they have. It was given for 28 students in 3B and 3A but later only 5 students in 3B were involved as subjects in the first cycle. It consisted of 3 problems in which each contained some questions (see appendix A).

First problem which consisted of 3 questions is purposed to find information about what students can explain about "Kondangan situation" especially question a and b. More specific, question c was asking how they determined the number of plates in the picture which was given.

We expected that they told what objects that they saw including the number of objects in the picture. To determine the number of plates, we predicted that they did not count them one by one but they can use multiplication at least repeated addition. In fact, when they had this pretest, they told what they have ever seen related to Kondangan situation. They told almost all things in the picture and explained what kind of food that they usually find in the Kondangan situation like Rendang, Pindang, Chicken Curry, etc. Since they knew this weekly event, they told what they experience. Some students also made drawing to represent the objects in that situation. The others explained in sentences. In answering questions c, some students tried to find out via repeated addition in determining the number of plates. For instance, Barza and Agim added 15 as many 5 times. However, they are not good enough in translating the problem into multiplication. It can be seen in their worksheet that they wrote 15 x 5 but they added 15 as many 5 times. Few students were able to use algorithm in finding the result of multiplication like Fadhil did. The other students only wrote the number without writing how they found the number. Through interview, Ade said that she got it by adding 15 as many 5 times.

Like problem 1, problem 2 also consisted of some questions, there were question a and b. Question a was purposed to know what they could do if they have given a picture of some piles of spoon boxes like picture below. We expected that students could see spoon boxes in each pile. They explained that they saw 3 piles of boxes. There were 2 boxes, 5 boxes, and 7 boxes in pile (a), (b), and (c). They could determine the number of spoons in each pile even they could relate the number of spoons in pile (a) and (b) to get the number of spoons in pile (c). In fact, they saw overall boxes. They knew that there were 14 boxes altogether. Some students also told that those were very nice boxes which were used to keep spoons. They talked about the condition and function of boxes rather than quantity of boxes since they were asked about what their opinion about those boxes.

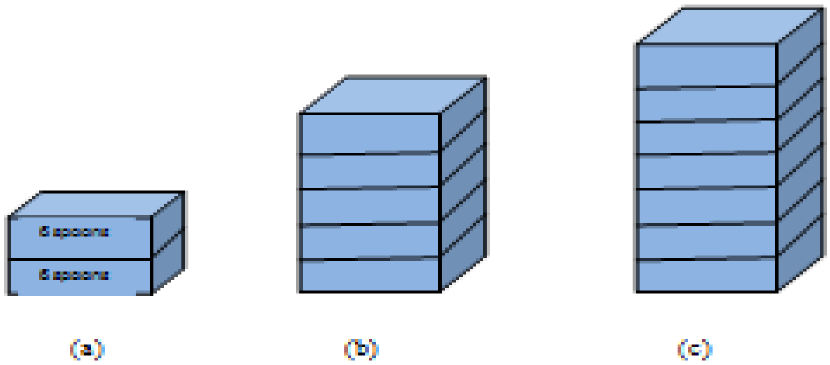


figure 5. 1 spoon boxes in some piles

Question b was aimed to know how they determined the number of spoons. We expected that they calculate the number of spoon in each pile where they get 12 spoons by adding 6 as many 2 times in determining the number of spoons in the first file. They add 6 as many 5 times in determining the number of spoons in the second pile and add 6 as many 7 times in determining the number of spoons in the third pile. In fact, many students saw boxes altogether. They stated that there were 14 boxes. Only some students were able to see the number of boxes in each pile like Fadhil Siddiq. He stated that there were 2 boxes in first pile, 5 boxes in second pile, and 7 boxes in third pile. However, most of them including Fadhil added 6 as many 14 times to find the number of spoons and there were also students who miscount on it.

Problem 3 was given to know the way they count chairs that were arranged based on columns and rows. It consisted of 3 problems (see problem 3 in appendix A). We expected that students could see that there were many chairs and thought how to calculate all chairs. When they saw that the chairs were arranged in rows and columns, they are expected to use repeated addition even multiplication since they know that there are some rows, for example, 13 rows with 7 chairs in each row. In this case, we predicted that they add 7 as many 13 times in determining the number of chairs or use multiplication 13 x 7 or 7 x 13. In fact, they said that it was hard when they saw many chairs. Since all chairs were shown in pictures, most of students counted the chair one by one until the last chair to find how many they are. However, there were also students who could determine the result through multiplication although not all problems could be answered in correct result. For example problem b, they knew the number of chairs in 10 rows by multiplying 10 x 7 but they had difficulty in determining the result of 3 x 7 so that they did not get exact result.

According to result of pretest, most of students are able to do repeated addition in finding the number of objects which are arranged in group situation like problem 1 and 2 but when they have rectangle situation like problem 3, most of the time they counted one by one. We concluded that the level of students who are able to do repeated addition is categorized in stringing level – the first level of mental calculation in multiplication.

Based on interview and the result of pretest, multiplication by 6, 7, 8, and 9 are categorized as uneasy multiplications for students to learn. Some students admit that they have known multiplication by 1 until 5 and multiplication by 10 but they are struggled in multiplication by 6, 7, 8, and 9.

F. Pilot Experiment

This experiment was done to prove hypothetical learning trajectory that was designed and to compare with the real situation. Five students in 3B, which were selected randomly based on different ranks in their class, were involved as subjects in this first cycle. They were Agim, Barza, Hasbi, Ade, and Arif. Meanwhile, a whole class of 3A will be the subject in the second cycle. All activities which were done in the first cycle are described as follow:

Activity 1

The purpose of this activity was that students could solve multiplication problem by repeated addition. For this purpose, teacher gave problem like: “if there are 6 pans on a table and there exist 4 tables which are provided, how many pans are there?” Students were asked to imagine and calculate how many pans that were provided if there were 4 tables.

We expected students could made drawing at the first time to describe the situation. In fact, all students were able to work in formal way since they used number in their calculation. After they knew the problem, some students even are able to state multiplication. For example: Agim stated that it was 6 x 4. When he thought that it was 6 x 4, he tried to find the result by adding 4 as many 6 times. The other students also did like what Agim and Hasbi did.

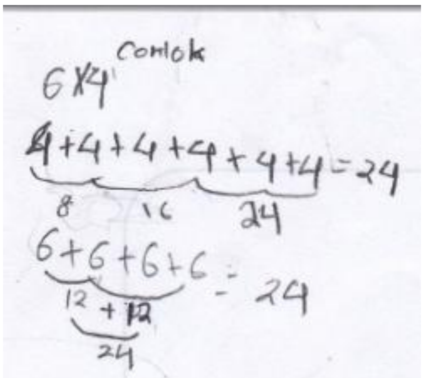


figure 5. 2 (a): Agim’s work

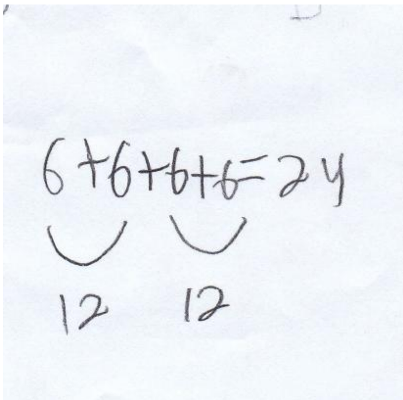


figure 5. 3 (b): Hasbi’s work

During discussion, researcher asked Agim about the result and the way he get the result.

Researcher : Now, tell us Agim. How many do you get?

Agim : twenty four
Researcher : twenty four? How do you find twenty four?
Agim : there are 4 as many 6 times
Researcher : 4 as many 6 times?
Agim : there are also 6 as many 4 times
Researcher : Emm.. 4 as many 6 times? 6 as many 4 times? Which one do you mean?
Agim :There are 4 as many 6 times and 6 as many 4 times. Both are 24.
Researcher : oke

Agim was inconsistent in his way since he made two ways in finding the result. He added both 4 as many 6 times and 6 as many 4 times. He did it because he knew the results were same. Therefore, he did not care what number he added whether it was 4 or 6, he found their results same. At the beginning, Hasbi stated 6 x 4 since he was influenced by Agim and Arif that said 6 x 4 before. However, because there were 6 spoons in one box, Hasbi used number 6. He added 6 as many 4 times because he knew that there were 4 boxes. He calculated the number of spoon based on the situation of 4 boxes of spoons and he knew that each box consisted of six spoons. Although he was not able to state into multiplication, he found the result through repeated addition based on spoons situation.

Agim stated 6 x 4 because he might think that the first number in the problem was 6 and the second number was 4. To find the result, he was doing suitable repeated addition with the multiplication that he used. At the beginning, he knew that 6 x 4 equals to add 4 as many 6 times. At the end, he also added 6 as many 4 times since he knew that Hasbi had same result with him. He wrote those ways because he was not sure which one the correct way.

In finding the result of repeated addition, most of students counted the number up. They used their fingers in counting the number. Some students mixed their ways. They doubled two consecutive numbers and counted the last number up like what Agim did.

figure 5. 3: Students use their fingers

Agim : eight, sixteen..
sixteen
(use his eight fingers)
yes, twenty four

Hasbi : I said twenty four. I don't need to write..
just count by my fingers
(propel his fingers) twenty four

Agim added 4 as many 6 times. He seems doubling 4, doubling 8, and added the other 8 to find the result. Hasbi was sure that he found the right result because Agim had same result with him. Although Hasbi use different addition, he also got the result, 24.

What they did is similar with my conjecture that they use repeated addition and doubling in solving the problem. The difference was that they were not able to state multiplication that is suitable with the problem. According to the result of pretest, they could solve multiplication problem via repeated addition. However, they were not good enough in translating multiplication related to the problem. In 4 tables' problem, they saw it as 6 x 4 not 4 x 6. They tried adding 4 as many 6 times. Some students wrote 6 x 4 and 4 x 6. They argued that they are similar since they have same result.

Since students were not really good in maintaining the concept of multiplication where they have not been advanced in using right repeated addition, there might be needed one activity that purposes in order students can translate problem into multiplication and do the right repeated addition based on the problem. It must be done before second cycle since we assume that students have already been advanced in repeated addition.

To support them to be able to do right repeated addition, they need to involve situation in the problem. Using situation can help them to understand the number so that they know whether the repeated addition that they write makes sense with the problem or not. Teacher can pose questions like: "what is the meaning of 6 x 4?" we expect that they know the meaning of 6 x 4 that there are 6 boxes which consist of 4 objects in each box. This case is different with the situation which is described in the problem. In fact the problem is about 4 x 6 where there are 4 boxes which each box consists of 6 spoons. To convince students, students are asked to draw the situation of those boxes. Then, teacher can pose more questions like: *"How many spoons are in one box? How many spoons in two boxes? How do you know? What do you think about addition that you made?"* Through those questions, students are provoked to think the multiplication and repeated addition based on the problem. Although they get through different way, they are expected to be able to explain.

After they are able to translate problem into multiplication and they are able to do right repeated addition based on multiplication, they are given a similar problem using one larger number than the previous number. For instance, the previous problem is about 7 x 6 then they are asked the result of 8 x 6. This problem is purposed in order students can use the result of 7 x 6 to find the result of 8 x 6 - they just need to add one more 6. Giving this problem is also a transition from they use repeated addition to they use number relation where they do not need to do repeated addition in finding the result of 8 x 6 but they just add 6 to the result of 7 x 6.

This activity will not be included in the second cycle but it will be done before the second cycle. It means that when we begin the second cycle, students have already been advanced in repeated addition. Since we focus to develop their mental calculation into acquainting with splitting and varying strategy, the first activity in the second cycle must be focused in order students start to acquaint with splitting strategy. In the second cycle, students are guided to know number relation in multiplication where they can use multiplication which they have known in previous problem to find the result of a multiplication. According to definition of mental calculation that students can use flexible and skillful way based on number relation and its characteristics in their calculation, students can start from acquiring number relation and its characteristics to develop their mental calculation. To reach this purpose, the first activity in the second cycle is an activity which is similar with the second activity in the first cycle where we focus the ways how to support them to use number relation in multiplication.

Activity 2

This activity purposed in order students was able to use number relation to solve the problem. For this purpose, students were given some real spoon boxes and problem as following:

a. "Mother bought 5 boxes of spoons in which each box contain 6 spoons. How many spoons did she buy?
b. However, she felt that she needed to buy more spoons so that she bought 3 more boxes. How many spoons altogether did mother have?"

At the beginning, students were asked to determine the number of spoons that mother bought in the first time. Then, they also calculated the number of spoons that mother bought in the second time. By giving problem like this, they were stimulated to calculate all spoons by adding the number of spoons when mother bought in the first time and in the second time.

In this activity, we expect that students add the numbers of spoons in problem a with the number of spoons in problem b. In this case, they can involve the result of multiplication in problem a to find result of multiplication in problem b. In fact, when they were asked to work in their worksheet, some of them seemed knowing the numbers of all boxes from adding the first 5 boxes with 3 boxes like what Barza did.

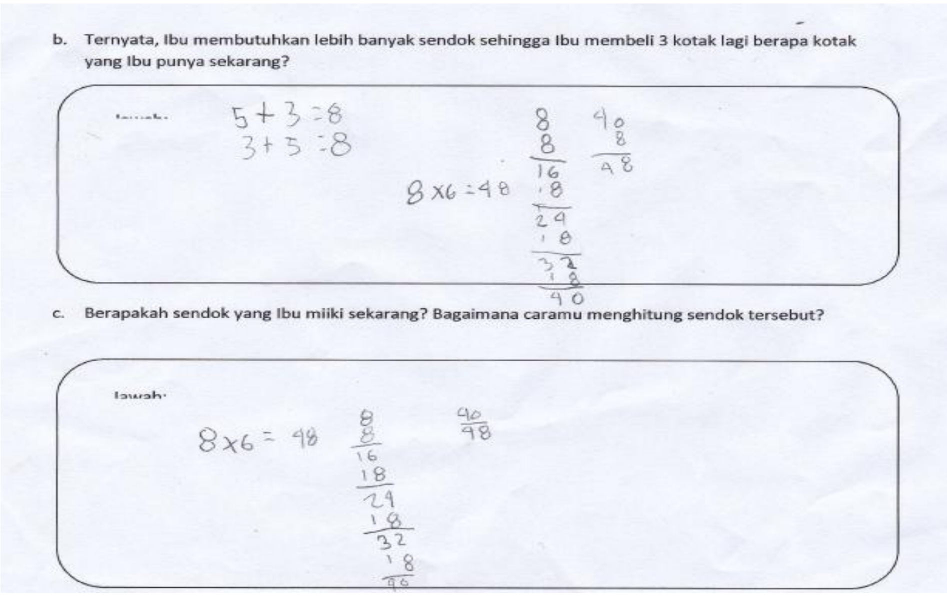


figure 5. 4: Barza's work

Barza seemed to be able to look at the relation of number of spoons in problem a with number of spoons in problem b. He knew that the number of spoons that mother bought in total equals to addition between the numbers of spoons that mother bought in the first time and the number of spoons that mother bought in the second time.

In his worksheet, Barza was able to determine the number of boxes by adding 5 and 3. However, he could not find the numbers of spoons altogether through that situation because he thought that there were 8 boxes altogether with 6 spoons in each box. He used multiplication 8 x 6 and got the result through repeated addition where he added 8 as many 6 times. He seemed to think that the result of 5 + 3 and the result of 3 + 5 are same. He thought that the position of the numbers is not a problem since they have same result. Thus, he knew about commutative - wherever we change position of number, the result is same. That is why he added 8 as many 6 times to find the result of 8 x 6.

During discussion, researcher asked students question: "what can you say about the number of spoons based on situation in the problem?" Among them, Barza was able to use the result of one multiplication and the other multiplication in finding the number of spoons in 8 boxes.

Barza : this is..
These boxes..
Researcher : eh.….listen everybody....
Barza : this is ..(point a pile of spoons in his left side)
There are 5 boxes. One box contains 6. They all are 30.
This one is.. (point a pile of spoons in his right side)
There are 3 boxes. All of them are 18.
So, this is 30 plus 18 that equals to 48

During discussion, Barza realized that he could find the number by adding the number of spoons in 5 boxes and in 3 boxes. He knew that he did not need to get the result by adding 6 from the first box but he could use the number of spoons in 5 boxes which had been found in previous problem with the number of spoons in 3 boxes.

There was a changing of Barza's way that he used repeated addition in his worksheet before he used the result of those multiplications during discussing this spoon problem. It also happened with Agim. At the beginning, Agim was able to calculate the number of spoons by adding the number of spoons in 5 boxes and 3 boxes. It could be seen in his explanation that 5 boxes contain 30 spoons, 3 boxes contain 18 spoons, so there are 48 spoons. However, he could not convince himself that 48 is the result of adding 30 and 18 even he saw it from the result of repeated addition that he made before.

Only one conjecture did not happen that students can translate problem into multiplication which is related to the first problem. Therefore, before students have second activity in the second cycle, they need to maintain the skill that they are able to use multiplication based on the problem. Thus, when they are in the second activity, they can see that the first problem is 5 x 6 and the second is 3 x 6. They are expected being able to relate the result of those multiplications in finding the result of 8 x 6. There is nothing specific changing in the second activity for the second cycle. This activity will be the first activity in the second cycle. In this activity, teacher just needs to pose questions like: "*how do you find 30 and 18? How do you find the total number based on the situation?"* We expect that students can state that they find 30 from 5 x 6 and get 18 from 3 x 6.Meanwhile, they also know that the total numbers altogether equal to the result of 8 x 6. Through those questions, students connect 8 x 6 with 5 x 6 plus 3 x 6.

Students are expected to be able to see number relation where they can use the result of other multiplications in determining the result of a multiplication. In the future, to reach this condition, they need to make arrangement of spoon boxes by themselves and it is not more than two piles. For instance, there are 8 boxes which are provided. Students are asked to arrange them into at least one pile and draw their representation in poster. They work in group and each group has their own arrangement. We expect that the arrangement will vary like in the following table.

Table 2 possibilities of boxes arrangement

The 1st pile	8 boxes	5 boxes	4 boxes	6 boxes	7 boxes
The 2sd pile	0	3 boxes	4 boxes	2 boxes	1 boxes

By arranging the boxes and discussing the possibilities of their arrangement, students are provoked to see number relation among them by making connection among the multiplication in those arrangements. For example, when they know that they get the number of spoons in 8 boxes via 8 x 6, they know that the number of 5 boxes of spoon means 5 x 6, and they also know that the number of 3 boxes of spoons means 3 x 6, they can connect 8 x 6 with 5 x 6 and 3 x 6. Since the results of those multiplications are same, they get that 8 x 6 equals to 5 x 6 plus 3 x 6.

Activity 3

The aim of this activity was that students were able to make structure and calculated through their structure. For this purpose, students were asked to arrange mini chairs in place that was provided and calculate the number of chairs.

This following picture is a place where students arranged mini chairs based on arrangement that they liked.

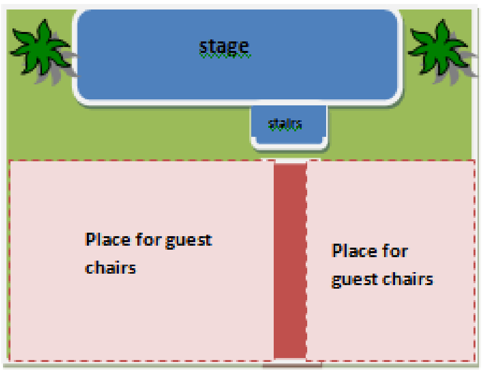


figure 5. 5: places for chairs

When they had this place, they arrange all chairs into two sides. Each side must be fulfilled. When they thought that there was still enough space, they added more chairs. They imagine chair arrangements in Kondangan situation that they have ever seen and matched with the arrangement that they made. Some students put a road to stage in middle so that they have same quantity of chairs in left side and right side. There were also students that arranged the chairs where they had more chairs in the right side than in the left side.

When they were asked about their opinion of chairs, some students said that the chairs in the right side were nice since they arranged the chairs. When they were asked about quantity of chairs they calculated them in each side.

My conjectures were that students are able to do multiplication in finding the number of chairs in the left and the right side and connect the whole multiplication with the part multiplication based on the chair arrangement that they made.

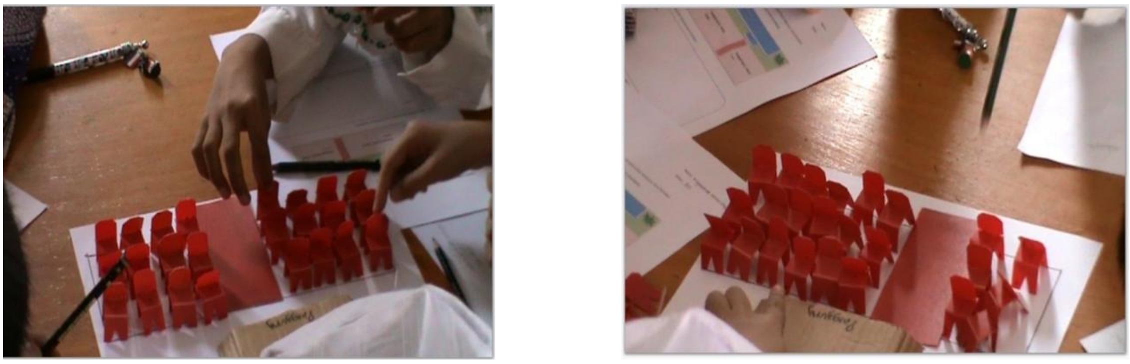


figure 5. 6: students arrange mini chairs

There were 2 arrangements that they made. First, a number of chairs in right side and left side were same that was 12 so the total was 24. Second, chairs in the right side were more than in the left side. There were 18 chairs in right side and 6 in left side. In the first chair arrangement, some students counted the chairs one by one. There were student who used the result of 4 x 3, and double the result so that she get 24. The others tried to find through 8 x 3 and they get 24. In the second chair arrangement, students also used the result of 6 x 3 and 2 x 3 in finding the numbers of chairs in the right and left side. However, there were also students who calculate the chairs through adding 3 as many 6 times.

According to that finding, some students were able to use multiplication to determine the number of chairs although there were also students who used repeated addition. Some students still struggled to translate the problem into multiplication form, whether it was 6 x 3 or 3 x 6. This happened because they did not really understand which one the rows and columns.

The other finding was that there was a student; his name is Arif who was able to use subtraction in determining the numbers of chairs in the right side. He calculated through taking away the number of chairs in the left side from the number of all chairs. He subtracted 24 by 6 so that he got 18.

Arif was believed to be able to use the result of multiplication in the previous problem to determine the number of chairs in the right side in chair arrangement that was different from the previous arrangement. He was able to use subtraction where he subtracted the number of all chairs with the number of chairs in the left side. At the end of discussion, Arif stated that the number of chairs was 18 and he gave multiplication that was suitable with the chair arrangement in the right side.

All conjectures that we expect happened except when they tried to express the chair arrangement into multiplication. They struggled to determine whether it is 6 x 3 or 3 x 6 when they want to calculate the number of chairs in the right side. It also happened when they calculated chairs in the left side in the second arrangement. It was hard for them because they have not really known about rows and columns and how to express it into multiplication.

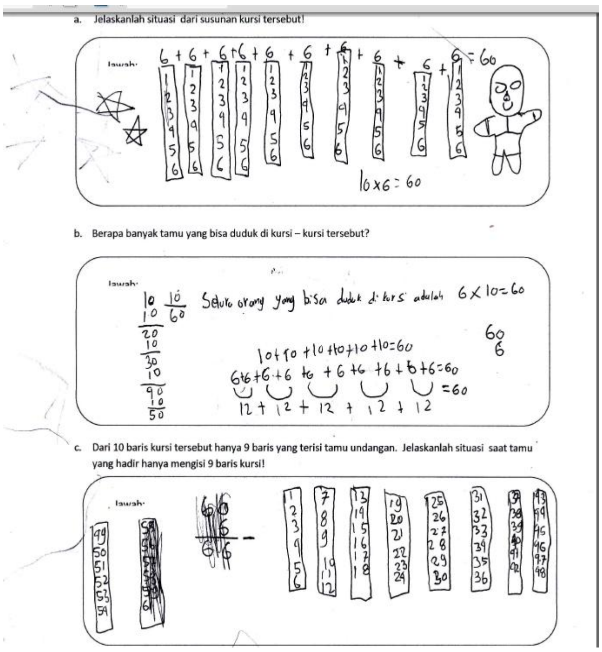
We were planning not to use this activity since they were able to express the relation of part and whole multiplication in activity 2. Moreover, in previous activity, we expect that students decide their own structure. For example, when we have 8 boxes of spoons, students can arrange them into 5 boxes and 3 boxes, 6 boxes and 2 boxes, etc. beside they use number relation, this activity include the way how they make structure by themselves. Therefore, there are two mathematical idea – number relation and structure – that will be reached in one activity – activity 2 that becomes activity 1 in the second cycle.

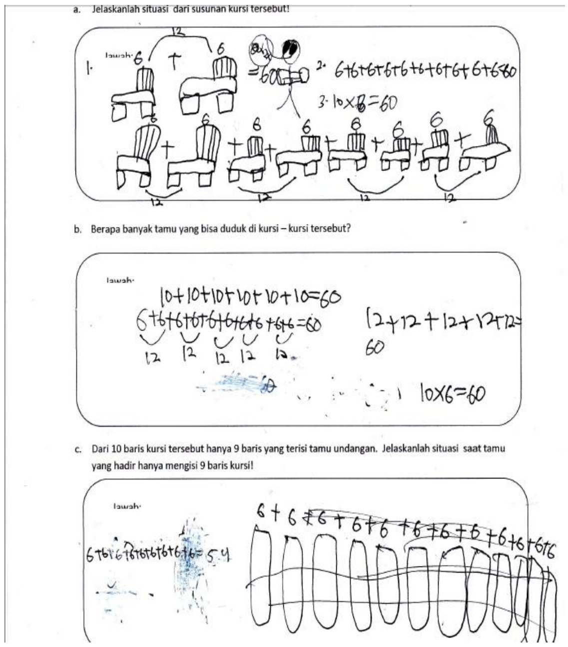
Activity 4

This activity had a purpose that students are able to use subtraction from the result of multiplication by tens to determine the result of multiplication by nine. To reach this purpose, they were given structured problem like following:

a. In a *Kondangan* event, there are 10 rows of chairs which each consist of 6 chairs. How many guests can sit on that chairs?
b. From 10 rows of chairs which consist of 6 coloms of chairs, only 9 rows are fulled. How many attending guests are there?

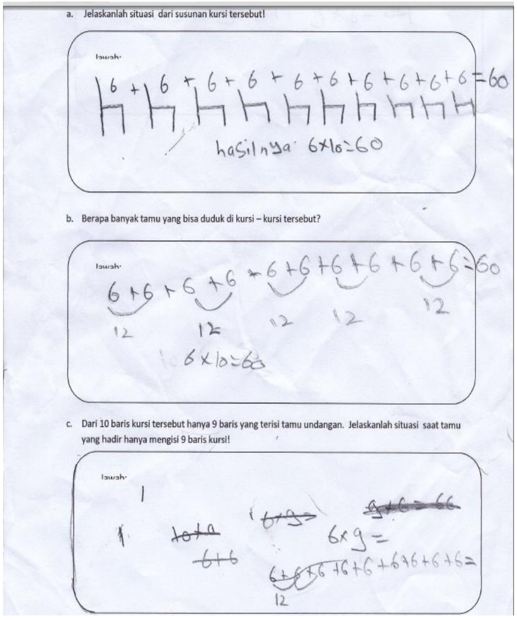
c. In answering problem a, students knew it as 10 x 6 and got the result 60. At the beginning, they were asked to explain the problem through making a draw about chair arrangement. This following are examples of students' works.



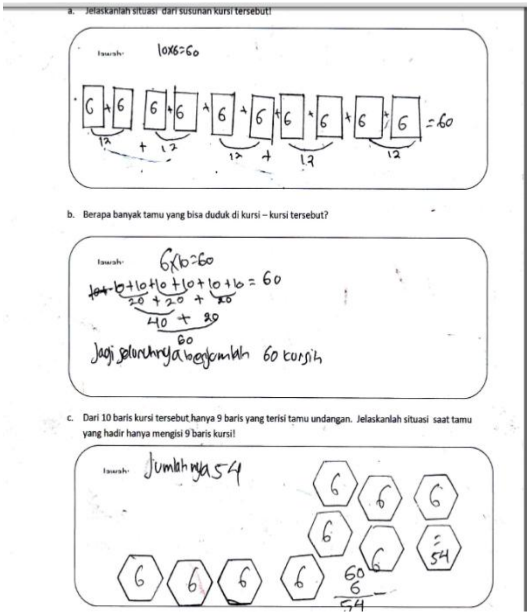


Barza's work

Hasbi's work



Ade's work



Agim's work

figure 5. 7: Barza's, Hasbi's, Ade's, Agim's works

Barza used a rectangle to represent a row and put numbers 1 to 6 on it to represent chairs in one row. He saw the rows in vertical way since he made rectangle and put numbers on it from top to bottom. He numbered the rectangle from 1 to 6 because he knew that there were 6 chairs in one row. Since he knew that there were 10 rows, he made 10 rectangles together with number 1 to 6 on each rectangle. He used multiplication 10 x 6 to find the number of all chairs because there were 10 rows with 6 chairs in one row. As he learned by heart the result of 10 x 6, he knew that the result was 60. However, he also did repeated addition where he added 6 as many 10 times to make sure that the result was 60.

To answer question a), Hasbi categorized 3 steps in his explanation about situation. First, Hasbi made groups of chairs. It can be seen when he drew a chair to represent one row and put 6 as numbers of chairs in one row. It happened because he got information from problem that there were 10 rows with 6 chairs in each row and he tried to describe situation by drawing. Second, he tried to calculate all sixes by adding 6 as many 10 times and he got the result 60. Third, he used multiplication 10 x 6. Perhaps, he knew that there were 6 as many 10 and knew the result of multiplication by heart.

Almost similar with Hasbi, Ade also made group of chairs by drawing a chair as a representation of one row. Because she knew there were 10 rows, she drew 10 chairs together with 6 on each chair. The difference is she thought that there were 6 tens because she saw there were 10 chairs where one chair represented one row. Therefore, she used 6 x 10. However, she also used repeated addition where she added 6 as many 10 times to get the result.

On the other finding, Agim also made group of chairs to represent the situation where he used rectangles together with number 6 on each rectangle. However, he double two consecutive 6 o find the result. At the end, he used multiplication 10 x 6 and wrote down 60 based on the result that he got from addition.

Many ways were done by students in explaining situation. Most of students made drawing and wrote down numbers. They knew that they did not need to draw all chairs because they could represent one chair as one row and the number 6 was used to show the number of chairs in one row. Some students found the result via 6 x 10 by adding 10 as many 6 times. There were also students who tried finding the result of 10 x 6 by adding 6 as many 10 times. They were not sure in translating the problem into multiplication. It could be seen when some students did repeated addition that was not suitable with the multiplication that they used.

How They Calculate Number of Attending Guests

In solving problem b, some students saw the number of guest through the result of 6 x 9. After the teacher retold the problem, there were students that use subtraction in determining the number of attending guest. He subtracted the number of all chair with the number of chairs that were not occupied (see the following figure).

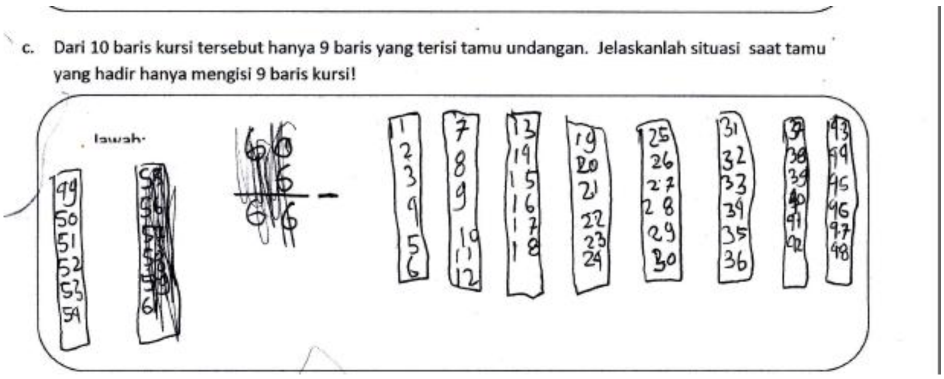


figure 5. 8: Barza determines number of guests

In his worksheet, Barza struggled in determining the result of that subtraction. He crossed the result because he found the number larger than 60 and may be he thought that the number might be smaller than 60. Therefore, to make sure, he drew all chairs in 10 rows and crossed 1 row. Finally, he could find that the result was 54. By making the representation, Barza could find the number of chairs through investigation from situation.

This following is transcript of student's conversation when Barza wanted to make drawing after he was stuck.

Agim : *six times six*
Barza : *wait for moment, if there are 9 rows which are fulfilled. It means… (pointing the figure that he drew) one, two, three, four, five, six, seven, eight, nine, ten. If there are only 9 then just subtract 6.*
Agim : *66*
Barza : *yes, yes.*
A few minute later, Barza tried to do downward subtraction.
Barza : *I am dizzy (crossing out the numbers that have been written)*
Aii.. I answer through using rows..
I use drawing again, may I?
One, two, three, four,..
(Drawing in his worksheet)

According to that transcript, it seems Barza realized that he just needed to subtract 6 from the number of all chairs to determine the number of guest. However, he struggled in finding exact result. He could determine it since he made drawing and used number in his drawing. Some students also did what Barza did but the figures that they drew were different. Some students added 6 as many 9 times. The others could subtract 6 from 60.

One conjecture did not happen that they draw the chairs into rows and columns. Nevertheless, they make drawing and put number on it. They did it because they imagined that there were 10 rows with 6 chairs in each row.

To support students to see the relation between multiplicand by 10 and multiplicand by 9, teacher needs to retell the problem in structured way. First, teacher pose question like:"How many chairs did we get in problem a?" by this question, students remind the number of all chairs in 10 rows that contains 60 chairs. Then, teacher say that guests who attended only fulfilled 9 rows out of 10 rows and pose question:"How many guests did attend in that event? We expect that students think that there is one empty row. It means 6 chairs are not used. Therefore, they think that the number of guests is same with the result of subtraction between the number of all chairs and the number of empty chairs. In this case, they get 54 by subtracting 60 with 6.

To help students to see connection between multiplicand by 10 and multiplicand by 9, teacher can pose a question like: *"How did we get 60?"* we expect that students know it from 10 x 6 since there are 10 rows with 6 chairs in each row. Then, more question like: *"How did we get 6? What multiplication is it?"* Students think that it is 1 x 6 since there is one empty row where it should be fulfilled by 6 people. The other question is: *"How about 54? What multiplication is it if we look at situation in the problem?"* we expect that they state 9 x 6 since guests fulfilled 9 rows out of 10 rows. Thus, they make connection that 9 x 6 equals to 10 x 6 minus 1 x 6.

Activity 5

The purpose of this activity was that students could solve multiplication problem with larger number than 10 by splitting the multiplication. They were given a problem like following:

For the need of *Kondangan* event, mother bought 10 boxes of spoons where there are 6 spoons in each box. Evidently, she thinks that it is not enough so that she buys 3 more boxes.

a) How many boxes of spoons that she buys?
b) How many spoons are there? How do you find the result?

The problem was told one by one. The first problem, they were asked to determine the number of spoons in 10 boxes that mother bought in the first time. Then, they had to determine the number of spoons that mother bought in the second time. They were also asked the number of spoons in all boxes. The following figures are examples of students' work

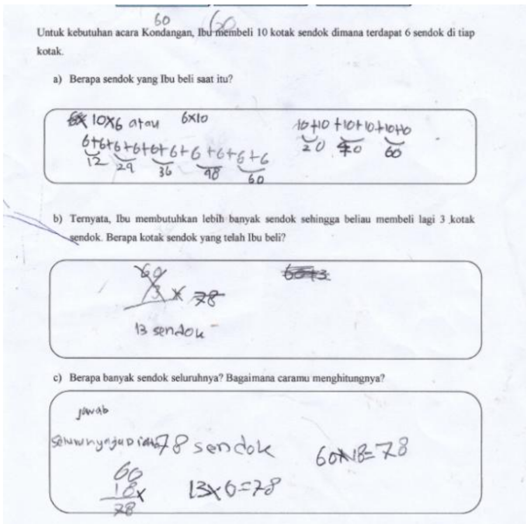


figure 5. 9 (a): Arif determines the number of guests

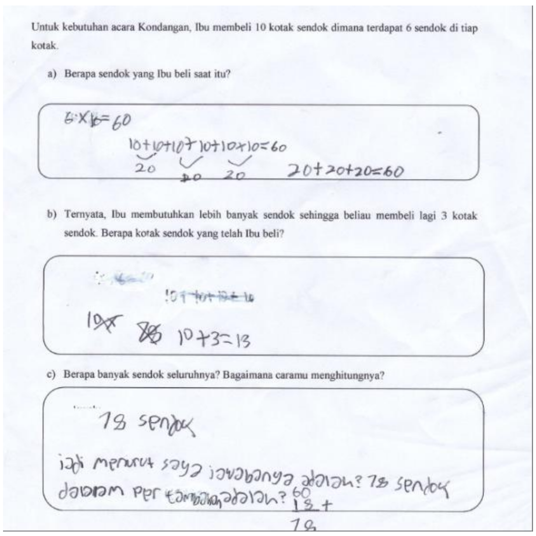


figure 5. 9 (b): Hasbi determines the number of guests

Arif thought that number of spoons equals to result of 10 x 6 since he knew that there were 10 boxes of spoons. However, he also thought that he could find via 6 x 10 since addition by 10 as many 6 times is easy to do. With same reason, Hasbi also looked for the solution from 6 x 10. Although they calculated in two different form of multiplication, they were able to find the same result by using repeated additions that were suitable with the multiplication. Hasbi had already known the result from multiplication but he also included repeated addition. It happened because he wanted to make sure that result was 60.

Hasbi knew that the number of all spoons that mother bought is same with the number of spoons that mother bought in the first time which is added with the number of spoons that mother bought in the second time.

Arif thought to add boxes that mother bought in the first time with 3 more boxes but he wrote 60. However, he knew that 60 is the number of spoons in 10 boxes. He also thought that there are 18 spoons in 3 boxes and got 78. He might add 60 and 18 in getting 78. At the end, he realized that the question is about the number of all boxes not all spoons. Therefore, he wrote 13.

Both Arif and Hasbi looked for the number of all spoons by adding the number of spoons in 10 boxes with the number of spoons in 3 boxes. They knew that there were 60 spoons in 10 boxes and 18 spoons in 3 boxes. Therefore, they knew that there were 78 spoons altogether as the result of addition between 60 and 18.

This following is transcript of conversation when students are able to relate the result of part and whole multiplication in the problem.

Researcher *: let us imagine, how many boxes are altogether?*

Barza *: 13 boxes*
Researcher *: And then, how many spoons in one box?*
Barza *: one box contains 6*
Researcher *: And then?*
Agim *: thirteen times six*
Barza *: Oo, 13 x 6*
Researcher *: So, 13 x 6 equals to..*
Agim & Barza *: seventy eight*
Researcher *: Based on the problem, what is the meaning of 13 x 6?*
Agim *: there are 6 as many 13 times.*
Researcher *: hmmm,*
First, what is mother doing? Second..
Barza *: Me (raising up his finger)*
Researcher *: Barza*
Barza *: Mother wanted to buy spoons. She bought 10 boxes, Nah, 10 boxes contain 60. Then, she bought 13 boxes.. eh, 3 boxes. Now, it becomes 13 boxes.*
Researcher *: Yes, so 13 boxes equals to 10 boxes..*
Barza *: times 6*
Researcher *: times 6, then..*
Barza *: it is added by 3*
Researcher *: then*
Barza *: times 6*
Once more Barza explained to the others
Barza *: these are 10 boxes. One box contains 6 so it becomes 60. Then, mother bought 3 more boxes. It contains 18. So 10 x 6 plus 3 x 6 equals to seventy eight*

At the beginning, students were not able to find the result of 13 x 6 through using the result of two multiplications because they got used to find through repeated addition. When they were asked to imagine the situation that was described in the problem, they could use the result of 2 multiplications to calculate the total number. Although, it needed long enough time discussion, they finally could realize that they could add the result of 10 x 6 and the result of 3 x 6 in finding the result of 13 x 6. The hard part was when they tried to connect parts and the whole multiplication. In this case, they could relate the result of 10 x 6 and 3 x 6 to determine the result of 13 x 6. It was hard part because they did not realize the relation between the results of those multiplications. Through relating to situation, they are able to use the result of two multiplications to find the result of whole multiplication.

Teacher can pose question like: *"How did we get 60?"* This question is purposed to remain students that it is the result of 10 x 6. Furthermore, teacher pose question*:"How about 18?"* we expect that students know that it is the result of 3 x 6. Then, we ask students*:"How many boxes are altogether?*" *If there are 13 boxes with 6 spoons in each box, how many spoons are there?"* Since the result of 13 x 6 equals to the result of addition between the result of 10 x 6 and 3 x 6, we expect that students connect those multiplications and know that 13 x 6 equals to 10 x 6 plus 3 x 6.

What they did was same with all my conjectures except no student used algorithm in finding the result of 13 x 6. Some of them used repeated addition and some of them were able to split the number into 10 x 6 and 3 x 6.

G. Conclusion of Pilot Experiment

Based on observation in pilot experiment, we make conclusion as follow:

In activity 1, students used repeated addition in finding result of multiplication. The difficulty for students is that they were not good enough in translating problem into multiplication. To help them, teacher asked students to make drawing and pose some questions which purpose in order students know the meaning of multiplication related to situation in problem.

In activity 2, students are able to use number relation where they can involve the result of one problem in finding the solution of the other problem. However, they have difficulty in connecting the results that they get with its multiplications. To help them, teacher also pose question which purpose in order they remain and formulate multiplication based on situation in problem.

Students have struggled in determining multiplication based on row and column in chair arrangement that they made. This happened because they did not really understand about array – the role of rows and columns. To help students to use structure, we are planning to use group situation like activity 2 rather than use rectangular situation. Since the purpose of this activity is that students are able to use structure and this purpose can be covered in activity 2, this activity might be not to be used in the second cycle.

There were many ways that students used in explaining situation in activity 4. They are able to find the result of multiplicand by nine by relating to the result of multiplicand by 10. The difficulty is that they were not able to formulate the connection of those multiplications. To help students, teacher can pose some questions in order they see their connection.

To solve multiplication with larger number than 10, students are able to split multiplicand into tens and ones. However, they also have problem in connecting whole multiplication with part multiplication. Similar with previous activity, teacher can help them by posing some questions in order they can see their connection.

Considering students to be able to do flexible calculation in solving multiplication by splitting strategy, there might be needed additional activities where students work in more formal ways. Those activities are purposed to maintain students to involve multiplication that they have already known in solving multiplication problem using splitting strategy.

H. Revised Activities for Second Cycle

At the beginning, there were 5 activities which were designed. After we tried out the designed activities in the first cycle and analyzed them, there are some improvements in some activities. According to our observation and analysis, there is activity that needs to be done outside the second cycle since this activity maintains the basic condition of students before they acquaint with splitting level. There are also some addition activities that we think they need to be included. Therefore, we do revision which is purposed to improve HLT in order we can reach research goal in better way.

Activity 1

Since repeated addition is not our main focus in this research, all activities which purpose to maintain students to be advanced in repeated addition are done outside cycle. Therefore, this activity will not be involved in the second cycle.

Activity 2

Activity 2 becomes activity 1 in the second cycle. This is our starting point to develop students' mental calculation through doing activity which purposes in order students to be able to use number relation. Based on definition of mental calculation that students are able to do flexible and skillful way related to number relation and its characteristics, students acquaint to use number relation in this activity. Beside number relation, students are expected to use structure in this activity where they are asked to arrange spoon boxes which are provided into some parts and they calculate boxes based on structure that they made. For example, when they have 8 boxes of all spoons, they make two piles of these boxes like 5 boxes and 3 boxes. They calculate the number of spoons in 5 boxes and 3 boxes using multiplications. Then, they make connection among those multiplications.

Activity 3

Since this activity purposes in order students can use structure and this purpose is included in activity 2, this activity will not be used.

Activity 4

This activity supported students to see relation between multiplicand by 9 and multiplicand by 10. However, this activity needs to be improved. Especially, when students have difficulty in translating number that they get into multiplications and make connection between multiplication by 9 and multiplication by 10, teacher can support them by posing some question like: *"How do we get the result (for instance, 60 where they get it from 10 x 6)?"*

Activity 5

This activity also supported students to split numbers in multiplication. Similar with activity 4, students need to be supported by posing some questions since they had difficulty in relating the number that they get with the multiplication. However, we need activity in order students are able to complete multiplication table by using number relation.

There are also three additional activities that are more formal than previous activities. Thus, 2 of 5 activities will be used and 3 other activities will be improved become 4 activities. Therefore, there are 6 activities that will be implemented in the second cycle. They are described in the following table:

Table 3: Improved Activities for the Second Cycle

No.	Activity	Goal	Expected result
1.	**Using two multiplications for**	Students are able to use the result of two multiplications in determining the result of a	When student have a multiplication, For instance, 8 x 6, they can split it into 5 x 6 plus

	One Multiplication	multiplication.	3 x 6
2.	**Knowing Relation between multiplication by 10 and by 9**	Students can use the result of multiplication by 10 in determining the result of multiplication by 9	Students can find the result of 9 x 7 by subtracting 7 from the result of 10 x 7.
3.	**Working with model for**	Students are able to draw rectangle model as a model for representing situation	Students can draw rectangle to describe a pile of boxes of chair arrangements.
4.	**Determining One larger number and one smaller number Multiplications**	Students can determine multiplication "before" and multiplication "after" of a multiplication	When they have a multiplication, e.g. 5 x 8, the can determine the result of 4 x 8 and 6 x 8.
5.	**Using known multiplication for another multiplication**	Students are able to use multiplication that they have known in determining the result of a multiplication	In finding the result of 6 x 8, they use multiplication that they know, e.g. 5 x 8 and add more 8 into it.
6.	**Completing Multiplication table**	Students can complete multiplication table, multiplication by 11 and by 12 by using number relation.	In solving 11 x 7, students split it into 10 x 7 plus 1 x 7

I. Teaching Experiment

In this section, we compared our improved HLT and students' actual learning process during the experimental phase. This phase was done in 6 activities and involved as many 28 students of 3A. We investigated how and if the HLT supported students' learning. We looked to the video recordings and selected some critical moments. We also analyzed their written works such as posters, and worksheets as another source. We analyzed every day lesson to investigate what students and teacher do, how the activities work, and how the material contributed to the lesson. The result of the retrospective analysis in this teaching experiment will be used to answer our research questions.

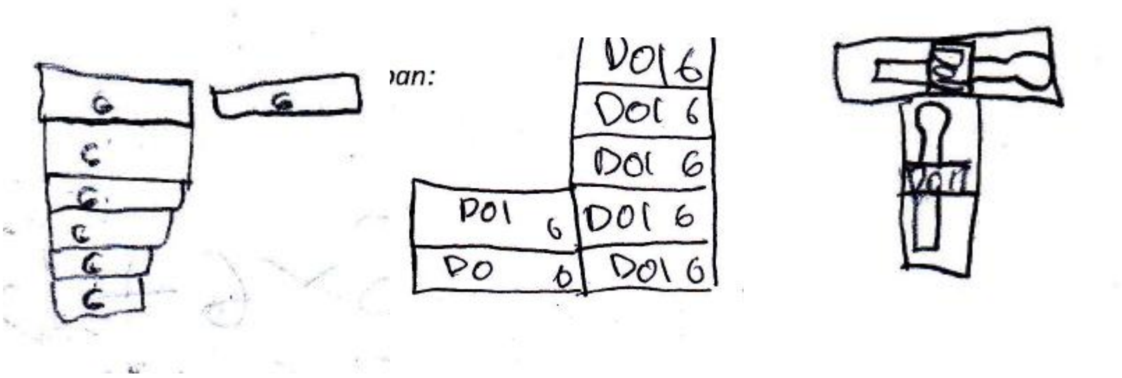
Activity 1: Using Two Multiplications for One Multiplication

This activity purposed in helping students to be able to use result of two multiplications in determining the result of a multiplication. We designed activities in which students can use their informal knowledge in solving problem. Teacher started by asking their experience related to Kondangan situation. This aimed to recall their informal knowledge about what objects that they usually find in that situation like pans, spoons, chairs etc. After they mentioned some equipment including spoons teacher said that all spoons were arranged in their boxes when the party was over. To reach our goal, students were asked to arrange all spoon boxes. For each group, we gave them 7 spoon boxes to be arranged into two piles. We predicted that they had two piles of boxes with different number of boxes in each pile since the number of boxes was odd. It happened. When they saw there were 7 boxes and were asked to arrange them, they tried to make same number in each pile. Because the number of boxes was odd, they arranged them into 4 boxes and 3 boxes at the beginning. After teacher posed a question to see another possible arrangement, they were able to make another combination. Therefore, each group had different arrangement.

After they arranged boxes themselves, they were asked to draw the boxes in their worksheet. This purposed to know the way they represent situation.

figure 5. 7: students count boxes before they make drawing

Some students, at the first time, counted the number of boxes in each pile to know how many boxes that they need to draw. Because there were two piles, they drew boxes in each pile. They drew boxes based on what they saw. The following are examples of their drawings.



(i) (ii) (iii)

figure 5. 8: Ribi's drawing (i), Dwi's drawing (ii), and Abdulrahman's drawing (iii)

Ribi's group arranged 2 boxes in the first pile and 5 boxes in the second pile. He drew rectangle with a written "Dol" and number 6 to represent a box which had brand "Dol" and contained 6 spoons. Because there were 2 boxes in the first pile, he made two rectangles. Then, he drew 5 rectangles to sign that there were 5 boxes in the second pile.

Dwi also did what Ribi did. She signed 6 boxes together with number 6 in each rectangle – a mark signed one box consisted of 6 spoons – in the first pile and one kind of rectangle in the second pile. Looking at her drawing, she seemed starting to draw from top to bottom since position of one box in the second pile was in line with the highest box in the first pile. It happened because she knew the number of boxes in each pile after she counted them. Then, she drew the boxes as many as the number that she knew without consideration about the position of boxes in reality.

Different with Ribi and Dwi, if we look at Abdulrahman's drawing, his group seemed that they only had two boxes in their arrangement. In fact, it was not. They arranged 3 boxes in the first pile and 4 boxes in the second pile. He drew like this since he saw the boxes from top view. Because there were 2 piles, he made two rectangles as representation of those two piles. There were 3 boxes in one rectangle and 4 boxes in the other rectangle.

Furthermore, students were asked to calculate number of spoons in the first pile and the second pile. In determining the number of spoons, they saw number of boxes in one pile. When they knew them and realized that there were 6 spoons in one box, they used multiplication to find the number of spoons. For example, they arranged two piles of boxes with 2 boxes in one pile and 5 boxes in the other pile. They know that one box contain 6 spoons and they had 2 boxes. Therefore, they know the number of spoons in two boxes was equals to the result of 2 x 6. At the beginning, some student might saw the first pile consisted of 5 boxes therefore they use multiplication 5 x 6 to find the number of spoons. The other students saw that there were 2 boxes. Therefore, the number of spoons was equal to the result of 2 x 6. It happened because they had different decision in determining which one the first pile and the second pile. After discussion, they agreed to decide one pile with two boxes as the first pile and the other as the second pile.

In determining the number of spoons altogether, some students multiplied 7 x 6 since they saw the boxes altogether. They knew that there were 7 boxes with 6 spoons in each box. Therefore they calculated them by multiplying 7 x 6. Some other students saw that the number of all spoons was equal to the number of spoons in the first pile which was added with the number of spoons in the second pile.

In her presentation, Mutia told about number of spoons in each pile of boxes which were arranged by her group. Her group arranged 6 boxes in the first pile and 1 box in the second pile. However, she calculated all spoons by multiplying 7 x 6 because there were 7 boxes altogether with 6 spoons in each box. Ribi did not agree with Mutia's reason since she had two piles of boxes where each pile had different number of boxes but she calculated number of all spoon by multiplying 7 x 6 – the way when she look at the number of boxes altogether.

Mutia said that she get the number of all spoons from the result of 7 x 6. Teacher asked respond of the other students.

Teacher : Anyone?
Ribi : teacher, I don't agree
Teacher : why do you not agree?
Ribi : because it cannot be 7 x 6.
Teacher : why?
Ribi : because the number of boxes in this part (point to the pile which contain 6 boxes) is 6 and in this part (point to pile which contain one box) is 1. It must be 6 x … what is it? 6 x … what is it?Mm..

(laughing)
Teacher : could you write down on blackboard?
Ribi write down his reason why it cannot be 7 x 6
Teacher : what should it be?
Ribi : 6 x 6
Teacher : 6 x 6?
Ribi : this is 6 boxes (point to the first pile) and this is 1 (point to the second pile)
Teacher : but the question is how many spoons altogether..
Ribi : if you ask me how many spoons altogether then ..what is it? (scratching his head)
Teacher : can you write down?

He wrote down 1 box and 6 boxes equals to 42
Teacher : can you write down their multiplication?
What is the multiplication for the first pile?
Ribi : 1 x 1
Teacher : are you sure?
Ribi : eh, 1 x 6
Teacher : then, how about the second pile?
Ribi : 6 x 6
Teacher : how do we know the number of spoons altogether?
Ribi : that is 6 x 1 .. eh.. 6 x 7
Teacher : 6 x 7? It is same with 7 x 6, isn't it?
Ribi : yes because it is 6 (point to the second pile)
Teacher : Hmm.. how is it?
Ribi : (try to skip count) 6, 12..
Teacher : then..
Ribi : 12, 18…18..
Mutia : 21
Ribi : 24.. it is 24, isn't it?
Teacher : then..
Ribi : 30, 36 (finish all boxes in the second pile), 36..
Teacher : do you still have more?
Ribi : plus 6.. eh 36, 36
(use his fingers) 37, 38, 39, 40, 41, 42
Teacher : so,
Ribi : so, it should be 6 x 7
Teacher : 6 x 7? How many boxes do we have?
Ribi : we have 7 boxes
Teacher : they are 7 boxes, aren't they? And each box contain 6 spoons
Ribi : how many spoons are there in each box? (he ask Mutia)

Mutia : *6*
Teacher : *what do you think about her answer? It is right, isn't it?*
Ribi : *(nod)*

Ribi justify and argue Mutia's reason. He seemed to know the way to find the number of all spoons via using multiplication in each pile. It can be seen when he did not agree with Mutia's opinion when she saw the number of boxes altogether. He thought that the number of all spoons can be found by adding the number of spoons in each pile. However, he struggled to formulate multiplication in each pile based on the situation of boxes which were arranged into two piles. Finally, because he did not know how to formulate multiplication from situation of boxes in those two piles, he stated that their multiplication was 6 x 7. When he got the reason why Mutia's opinion worked, he finally accepted her opinion.

In other presentation, Bahrul explained how his group solved problem. Their work was described in poster below.

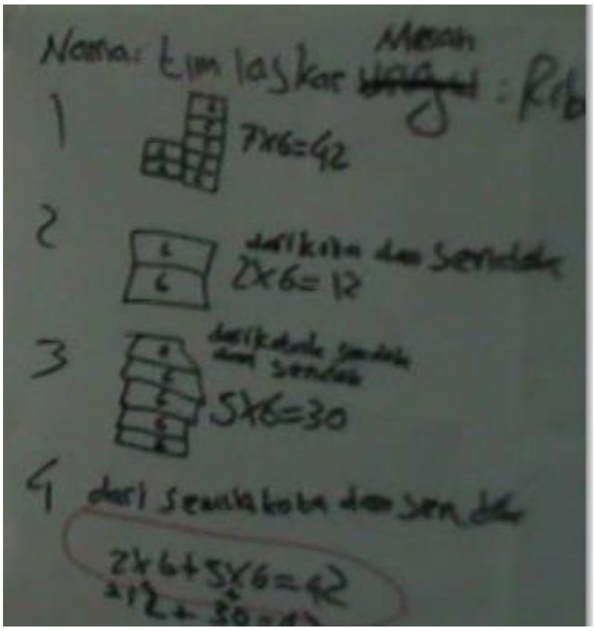


figure 5. 9 Bahrul's poster

Bahrul with his friends in one group arranged 2 boxes in the first pile and 5 boxes in the second pile. Because they know the number of all boxes was 7 boxes with 6 spoons in each box, they automatically wrote 7 x 6 as its multiplication. Based on questions that they had in their worksheet, Bahrul made multiplication based on boxes in each pile and made their connection to get the total number of spoons. To find the number of spoons altogether, he added the number of spoons in the first pile with the number of spoons in the second pile.

Teacher looked at similarity in the result between Laskar Merah's work (Bahrul's group) and Laskar Ungu's work (Mutia's work), both of them got 42 as the number of spoons altogether. Therefore, teacher posed question to stimulate students to see connection between multiplication that Mutia's group had and two multiplications that Bahrul's group made. Bahrul himself tried to make their connection. He knows the meaning of 7 x 6 that there are 7 boxes with 6 spoons in each box. However, he stucked to relate to multiplications that he made. It was hard for him since he did not realize the meaning of their differences.

Abdulrahman tried to make their connection. He wrote down that 2 x 6 plus 5 x 6 is same with 7 x 6 which is equal to 42. He was able to formulate connection among those multiplications because he thought that the result of those multiplications was same. To make sure that other students also know this similarity, teacher asked students to formulate another multiplication that they made in their group. Since there were different arrangements in some groups, they were asked to see whether they also could apply it in multiplications that they made.

Teacher : can you make connection between multiplication 7 x 6 and multiplications in your group?
Some students : yes, we can

Teacher : *please.. Who can make their connection? come on..*
Some students discuss

Dwi : *I can do it, Mam. I understand.*
6 x 6, 1 x 6..
I understand.. (write down on paper)

She wrote 6 x 6 plus 1 x 6 equals to 24 plus 6 on her paper meanwhile Bahrul tried to help Laskar orange to formulate the multiplications like following figure.

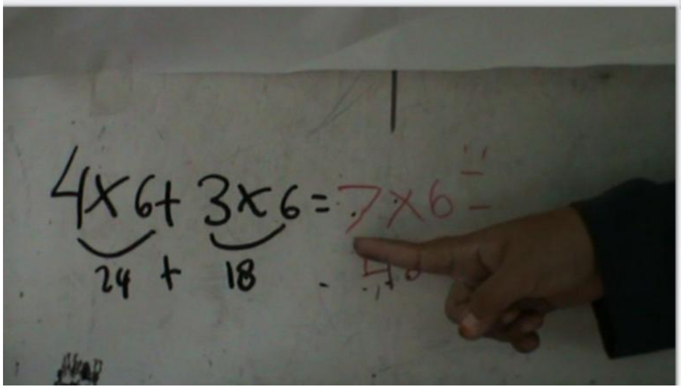


figure 5. 10: Bahrul use number relationship in multiplication

figure 5. 11: Dwi tries to formulate connection of multiplication that she and her group use

While she observed Bahrul working on blackboard, Dwi realized that she also could formulate connection among multiplications which she and her friends made. At the beginning, she and her friend saw boxes altogether and calculated them via multiplication 7 x 6. During discussion, she seemed to know that she also could find the result by using multiplication of boxes in each pile. She was able to formulate the multiplication when she sat in her chair. However, she lost it when she was asked to write down on blackboard. It happened because she just followed what Bahrul did without really understand the meaning of number that she wrote. Intan tried to help her.

Teacher *: who can help Dwi?*
Intan *: (Raise her hand)*
(write down on blackboard)
I think 6 x 6 plus 1 x 6 equals to 7 x 6. 6 plus 1 equals to 7 times 6 and the result is 42.
Teacher *: why do you use 6, 1, and 7? What is their connection?*
Intan *: 6 plus 7 ..eh..6 plus 1 equals to 7. 7 is boxes and 6 is content of a box.*

Intan disagreed and argued Dwi's way. She knew that Dwi and her friends made two piles of boxes. One pile consisted of 6 boxes and the other is one box. Therefore, she stated that the number of spoons altogether was same with the number of spoons in the first pile added by the number of spoons in the second pile. She knew that addition of multiplication in each pile was equal to the result of whole multiplication, 7 x 6. She formulated and told the meaning of number in those multiplications.

Through this lesson, we could see that students use both whole multiplication and part multiplications to know how many all spoons. Some of them saw all boxes and use whole multiplication to find the number of spoons altogether. However, some students also could see the number of spoons in each pile and added them with the other number of spoons in the other pile. By calculating the number of spoons in each pile and adding them, they could find the number of spoons altogether. The only difficulty that they had is when they tried to formulate and connect between whole multiplication and part of multiplications based on box arrangement that they made. For them who were not able to see the use of two multiplications in finding the number of all spoons, they were helped by illustrating with the real boxes that they arranged. By asking them to see boxes in each pile and formulating its multiplication, they see that they could use those multiplications by adding them in determining the number of spoons altogether.

Activity 2: Knowing Relation Between Multiplication by 10 and Multiplication by 9

At the beginning of this meeting, teacher reflected what students have learned from previous lesson. From reflection, some students were able to make connection from whole multiplication - in this case, 7 x 6 - to parts of multiplication like 6 x 6 plus 1 x 6, 5 x 6 plus 1 x 6, or 3 x 6 plus 4 x 6.

This activity purposed to help students to see relation between multiplication by 10 and multiplication by 9. To reach that goal, the teacher started the lesson by telling story about Kondangan situation. Teacher told about guest chairs and attending guests. Task for students was that they were asked to calculate how many chairs that are provided when teacher gave information about the number of rows and the number of chairs in each row. They were also asked to calculate the number of guests who attend in that Kondangan event through calculating the number of chairs which were fulfilled by guests.

There were 10 rows of chairs with 8 chairs in each row (see problem 2 in students' worksheet meeting - 2 in appendix B). We predicted students were able to determine the number of all chairs since they can use multiplication by ten and this multiplication is multiplication that they have already known. In fact, it happened. They were able to find the number of all chairs through multiplication by ten. They automatically know the result of this multiplication.

Most students used multiplication 10 x 8 as an easy way to find the number of chairs altogether although some students, at the first time, drew situation of chairs to help them know the problem. They could find number of chairs easily via 10 x 8 since they knew this kind of multiplication by heart. However, there were also some students who tried to use their previous knowledge about relating two multiplications in finding the result of a multiplication like Meylin did.

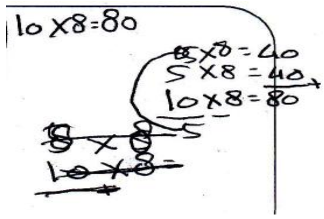


figure 5. 12: Meylin's work

Meylin seemed to know the result of 10 x 8 since she wrote down this multiplication at the beginning (see multiplication on the top of figure). However, she also tried to find the result of 10 x 8 via adding two multiplications 5 x 8. At the first time, she tried to get what number when she subtracted 10 by 5. Then, she got 5. She knew that 5 x 8 is 40 and she added 5 x 8 with other 5 x 8 to get the result of 10 x 8. Melinda used her knowledge which she got from the previous meeting that she could find the result of a multiplication from the result of another multiplication.

After the first problem was solved, teacher continued telling her story. She said 9 of 10 rows were fulfilled by guests. Students were asked to draw the situation and to find out how many guests who attended to that event. We predicted that students drew situation of chair arrangement based on rows and columns like Nikma and Rizky did.

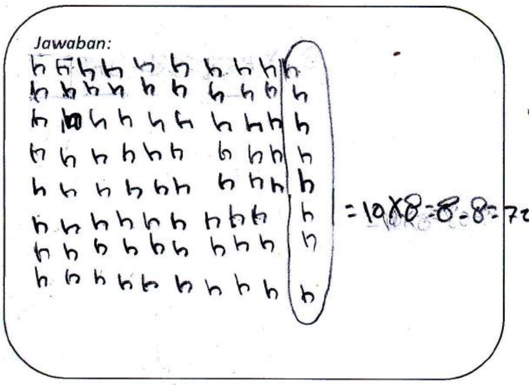


figure 5. 14 (a): Nikma's work

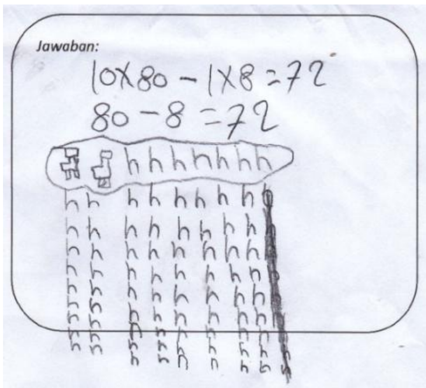


figure 5. 13 (b): Rizky's work

Nikma saw rows of chairs from left to right meanwhile risky did from top to bottom. They had different perception about position of rows. When they knew that only 9 of 10 rows were fulfilled by guests they signed one row of chairs meaning that there was an empty row. Since they could differentiate rows and columns, they knew which part that they needed to sign as an empty row. It is different with what Arina did.

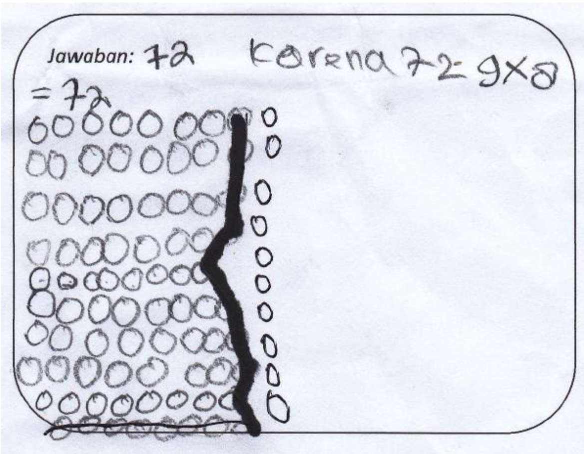


figure 5. 14: Arina's work

Arina made circles to represent all chairs and arranged them into rows and columns. She saw an empty row was one line from top to bottom. It meant she cross one column of 8 columns. In fact, she made rows from top to bottom. Therefore, she had 10 rows of chairs which each consisted of 7 chairs. She did not realize what she did was different with situation in the problem since she could not differentiate between rows and columns. To make sure what she was doing, teacher interviewed her. In interview, she realized that she should not cross circles from top to bottom but it should be from left to right. Therefore, she repaired the arrangement by making new circles from top to bottom and crossed one last row of circles from left to right.

Besides making array as representation of chair arrangement, there were also students who made group situation of the arrangement like Ribi and Bahrul did.

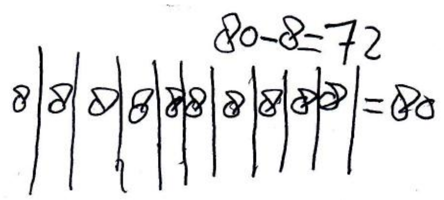


figure 5. 18 (a): Ribi's work

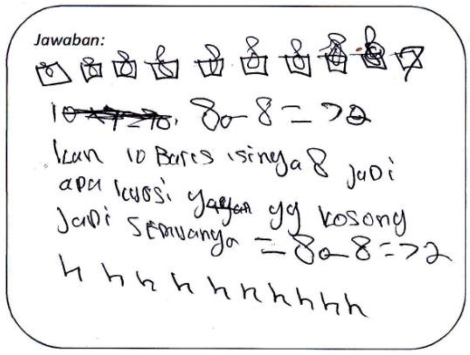


figure 5. 15 (b): Bahrul's work

In interview, Ribi explained his work. Ribi signed rows by making vertical line to differ one row to the other row and wrote 8 to mark that each row contained 8 chairs. When he knew there were 80 chairs altogether and only 9 of 10 rows were fulfilled by guests, he took away 8 in the left side. Therefore, he subtracted 80 with 8 and got 72. Because he knew all guests fulfilled 9 rows of chairs and each row contained 8 chairs, he also used multiplication 9 x 8. However, he found the result of multiplication via subtraction between the number of all chairs and the number of empty chairs.

Different with Ribi, Bahrul made rectangle to represent row. There were 10 rectangles because he knew there were 10 rows. Since there were 8 chairs in each row, he marked 8 in each rectangle as the number of chairs in one row. When he knew there was an empty row since 9 of 10 rows were fulfilled by guests, he crossed one rectangle. He subtracted the number of chairs in 10 rows with the number of empty chairs. Therefore, he knew the number of attending guests were 72 people.

Arina also used multiplication 9 x 8 as an easy way in determining the number of attending guests. She seemed to be able to use two multiplications to determine the result of 9 x 8. She knew the result of 8 x 8 and added 8 to it after she subtracted 9 with 8 to find what number that she needed to add to the result of 8 x 8.

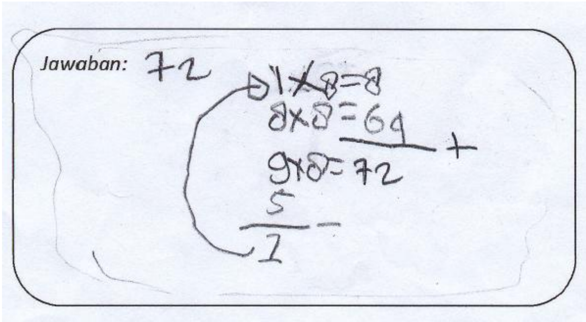


figure 5. 16: Arina tries to find the result of 9 x 8

Rizky saw that he could use multiplication to find the number of attending guests after he drew situation of chairs (see Rizky's work). When he knew the number of attending guests equals to the result of subtraction between the number of all chairs and the number of empty chairs, he formulated multiplication related to that subtraction. He tried to find what multiplication that the result was 80 based on the situation. He overlapped more 0 when he wrote 10 x 8 because he oriented to 80, the number of all chairs. However, he was able to see multiplication which the result was 8 based on the situation that is 1 x 8. What Rizky did have been discussed in class and influenced the other students.

In this lesson, most students used multiplication as a handy way. However, some students found the result by connecting two multiplications. They involved previous knowledge where they could use two multiplications in finding the result of a multiplication. Some of them added two multiplications and the others were able to use subtraction. The difficulty that they had is when they formulate form of multiplication for situation that they had. To help them, teacher asked students who were able to formulate the multiplications to explain and to discuss with the other students.

Activity 3: Working with Model For

The goal of this activity was that students were able to represent situation in more general. To reach this goal, teacher started this meeting by remaining students about situation in previous lesson. After students drew situation by themselves, for example, they drew a pile which consisted of 6 spoon boxes by making 6 rectangles as representation of those boxes, teacher drew representation of situation in more general by making a rectangle which has same height with those 6 rectangles. By doing this, we expected that students realized to make model for situation where they describe situation in more general. In this case, they only made one rectangle which has same height with a pile of 6 rectangles.

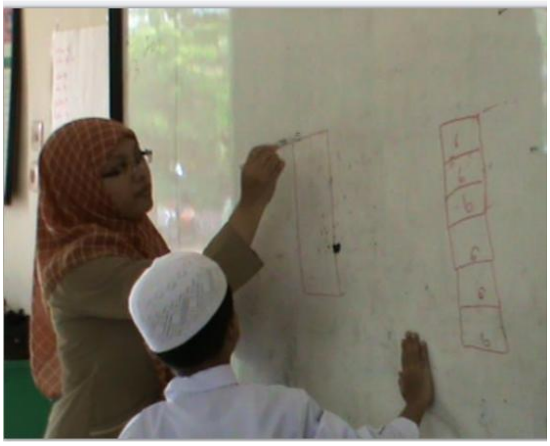


figure 5. 17: teacher introduces model for situation

To see how students worked with this model, teacher gave a problem (see students' worksheet meeting – 3 in appendix B). Students were asked to determine height of a pile of 5 boxes where one box consisted of 8 donat. Then, they were asked to determine the height of 6 boxes in the same figure that they made. Most students made kind of rectangle model to represent situation. When they were asked to describe how high a pile of 5 boxes, some students drew kind of some rectangles to represent the boxes like what Pajri, Aldino, and Meylin did.

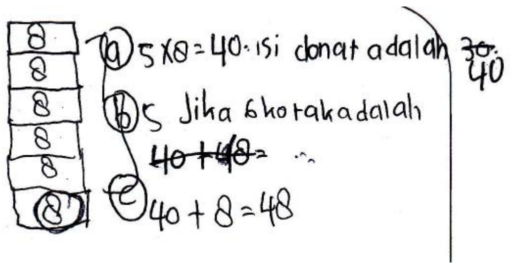


figure 5. 19 (a): Aldino's drawing

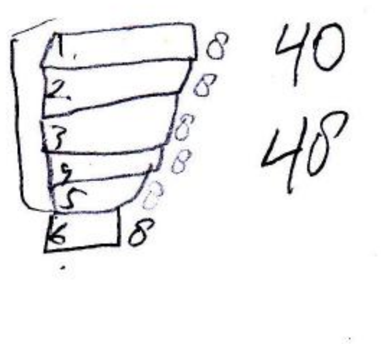


figure 5. 18 (b): Fajri's drawing

Aldino signed the height of 5 boxes by making 5 rectangles in one pile as representation of those boxes. When he was asked to describe the height of 6 boxes, he just added one more rectangle at the bottom of his drawing. It was also what Fajri did. He drew 5 rectangles and marked them from 1 until 5 together with number 8 which described that there were 8 donat in each rectangle. Then, he added one more rectangle together with number 6 and 8 to sign the sixth box with 8 donat inside. However, there were also some of students separating their drawing of 5 boxes and 6 boxes like what Meylin did.

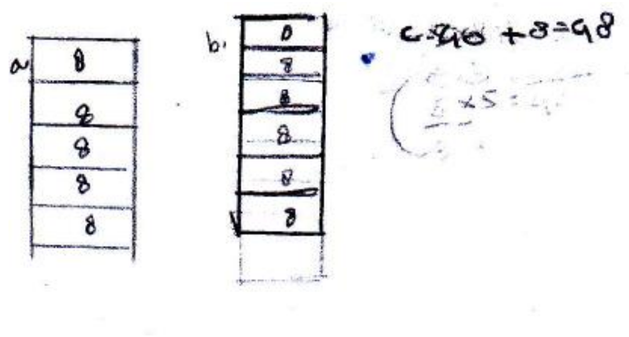


figure 5. 19: Meylin's drawing

It happened since she did not focus with instruction in worksheet. She just thought about drawing 6 boxes without paying attention to instruction. She answered questions on each number of problems. Therefore, she had two drawings with different number of boxes.

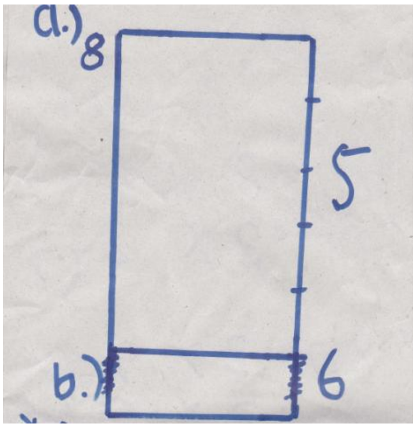


figure 5. 20: Intan's group drawing

It is different with Intan's drawing. At the beginning, Intan drew a rectangle together with number 8 on the top of rectangle and number 5 beside the rectangle. 8 means number of donat in one box and 5 means 5 boxes. To sign there were 5 boxes, she made some lines on the right side of rectangle which is closed to number 5. It seemed that she prefer to make a rectangle with some marks on it in representing a pile of 5 boxes rather than drew 5 rectangles as representation of 5 boxes. To draw a pile of 6 boxes, she added one more rectangle at the bottom of figure that she made before. She seemed to be able to make model for situation since she drew boxes in more general than what Aldino, Fajri, and Intan did. She did not draw one rectangle as one box but she drew a rectangle where she thought it had same height with a pile of 5 boxes. For a pile of 6 boxes, she just added one smaller rectangle as representation of one addition box.

To determine the number of all donat, there were similarity among Aldino, Fajri, Meylin, and the other students. They added 40 – the number of donat in 5 boxes – with 8 – the number of donat in one box. Because they were asked to determine the number of donat in 5 boxes in the first time, they involved it to determine the number of donat in 6 boxes.

To provoke students to draw model for situation, teacher gave one more problem (see students' worksheet meeting – 3 in appendix B). Situation which was used in this problem was different from situation in the previous problem. It was a kind of rectangle situation meanwhile the previous one was group situation. It still related to situation which was used in previous meeting where teacher told about chair arrangement and attending guests in kondangan event (see problem 2 in students' worksheet meeting - 3 in appendix B)

In the next problem, students were asked to draw arrangement of 10 rows of chairs where one row consisted of 7 chairs. After that, they were asked to draw attending guests who fulfilled 9 rows of chairs. In drawing the situation, some students still focused to work in each number of questions since the problem which was given consisted of some questions. It implied they made drawing on each number of questions. Therefore, they had two drawings like what Dwi Mutia did.

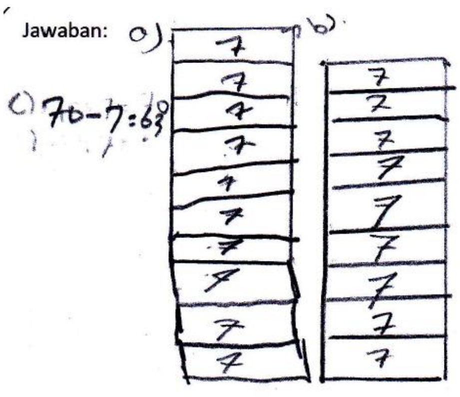


figure 5. 21: Dwi draws situation

Dwi worked based on each number of question. In question a), she drew 10 rectangles which are arranged in one pile together with number 7 in each rectangle. One rectangle means one row and number 7 means number of chairs in one row. Because there were 10 rows, she made 10 rectangles as representation of 10 rows of chairs. For question b), she redrew situation where attending guests sit on 9 rows of chairs. It happened since she did not pay attention with instruction in question b) where she was asked to determine the position of guests who fulfilled 9 rows of chairs in the same figure that she made in question a). She just thought about drawing 9 rows of chairs without considering the instruction in the question b).

Seeing what some students did, teacher remained them about the instruction of questions. Therefore, some students realized that they just needed to make one drawing to describe situation in question a) and b) like what Dwi Amelia and Intan did.

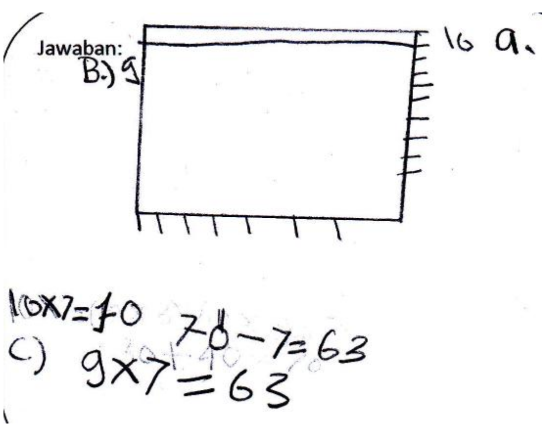


figure 5. 26 (a): Dwi Amelia's work

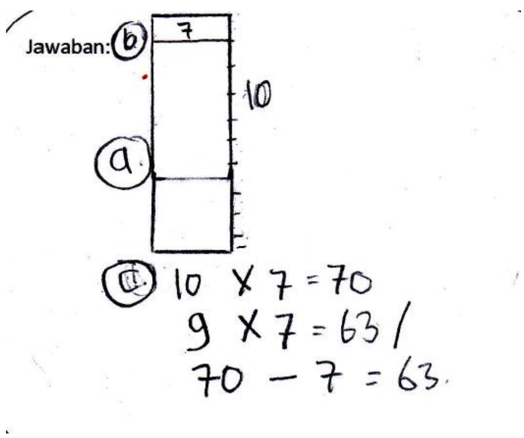


figure 5. 22 (b): Intan's work

Dwi signed a rectangle as representation of chair arrangement of 10 rows. Because there are 10 rows, she made 10 spaces on right side of and she also made 7 spaces on bottom of rectangle to mark that there are 7 chairs in a row. To show that attending guests fulfilled 9 rows, she made a line which passed the ninth line on the right side of rectangle counting from bottom of rectangle. To determine number of all chairs, she used multiplication 10 x 7. The result of 10 x 7 was subtracted with 7 to find the result of 9 x 7.

Almost similar with Dwi, Intan also made a rectangle to represent chair arrangement. She also made some lines on right side of rectangle. She made 10 lines and gave number 10 closed to right side of rectangle since there were 10 rows of chairs. She lined the ninth lines counting from bottom of rectangle to describe that guests fulfilled 9 rows of chairs. She gave number 7 on the top of rectangle to mark that there were 7 chairs in each row. To determine number of attending guest, she used multiplication 9 x 7 after she find the result of 10 x 7 and subtracted it with 7 – a number of empty row in that arrangement.

In this lesson, most students used rectangle model to describe situation. This happened since teacher introduced this model at the first time. Although teacher did it, some students still described situation by their ways using rectangle model. They drew rectangles to represent both group situation like a pile of boxes and rectangle situation like chair arrangement. The only unexpected thing happened was that some students made each drawing in each number of questions. This happened because they want to give answer for each question. Knowing this condition, teacher remained students about instruction in the question that asked them to draw in same figure that they made in previous question.

Activity 4: Determining One Larger Number Multiplication and One Smaller Number Multiplication

At the beginning of lesson, teacher remained students about consecutive numbers. For instance, they told what number before and after 6. They said they were 5 and 7. Then, teacher related to multiplication. Students were asked what multiplication before and after a certain multiplication. For example: when teacher mentioned 5 x 6, they could determine multiplication before and after 5 x 6 in multiplication table, there were 4 x 6 and 6 x 6. It happened after teacher gave assumption about multiplication 5 x 6. She assumed that 5 x 6 means 5 boxes of 6 donat in each box. By relating to context, students knew multiplication before 5 x 6 was 4 x 7 since they assumed that they had 4 boxes – one less box than 5 boxes. They also knew multiplication after 5 x 6 was 6 x 6 since they assumed that they had 6 boxes – one more box than 5 boxes. Then, Students played with some cards where they need to write down “before and after” multiplication of a certain multiplication which was known. Each group had different known multiplications. For example: Laskar Orange – name of a group – had multiplication 10 x 7. Here, they had to determine multiplications before and after 10 x 7 in multiplication table by writing them on the cards. By doing this, students tried to find the result of 9 x 7 and 11 x 7 by relating to result of 10 x 7.

figure 5. 23: Laskar Orange writes multiplications on cards

At the first time, students of Laskar Orange were able to determine multiplication before 10 x 7. That was 9 x 7. However, when they wanted to determine multiplication after 10 x 7 they got stuck until teacher posed them some questions.

Anita : before 10 is 9 and this is after 10 (point to card below multiplication 10 x 7)
Teacher : yes, what is number after 10?
Anita : eleven
Teacher : yes
Anita wrote down multiplication 11 x 7 on a card
Meylin : wow, that is a lot
Teacher :what do you think. If 10 x 7 is 70 then how about 11 x 7?
(Silent)
Teacher : Let us think. For 10 boxes, we have 70. For 11 boxes, how many addition box do we get?
Anita : ten. Seventy seven
Teacher : how do you know?
Anita : ten times 7 plus 7 (write 77 on a card)

Anita seemed to know the number before and after 10. However, she was doubt when she wanted to state 11 as number after 10 until teacher asked her opinion about it. Finally, she formulated multiplication after 10 x 7 was 11 x 7. However, she, Meylin, and Dwi got new problem since they did not know result of this multiplication. Teacher tried to help by relating to a context. She assumed that when they had 10 boxes they have 70. Then, she asked number of addition box if they had 11 boxes. This was purposed in order they thought that they need to add contain of one more box to the number of objects in 10 boxes. In this case, students were able to get the result 77 since they knew the number of objects in 11 boxes was equals to the number of objects in 10 boxes plus one more seven – the number of objects in one box.

Teacher gave new problems (see students' worksheet meeting – 4 in appendix B). They were given opportunity to determine multiplications and the result among those multiplications. When they knew one result of those multiplications, they could determine the result of two other multiplications like what Intan and rizky did.

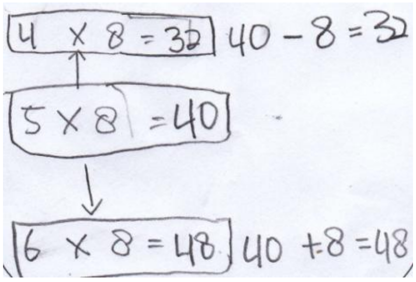


figure 5. 28 (a): Intan solves problem 1

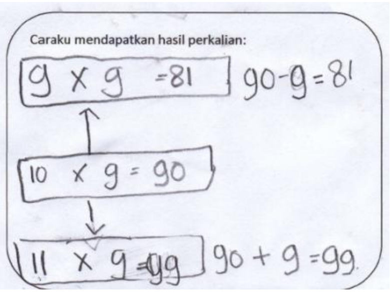


figure 5. 28 (b): Intan solves problem 2

When Intan saw there was only multiplication 5 x 8 which was known, she determined multiplications before and after 5 x 8. There were 4 x 8 and 6 x 8. She knew the result of 5 x 8 is 40 and she used it to find the result of the other multiplications. She added 8 and 40 so that she got 48 as the result of 6 x 8. She found 32 as the result of 4 x 8 by taking away 8 from 40 – the result of 5 x 8. Most students did what Intan did. Since they knew by heart the result of 5 x 8, they used it to find the result of two other multiplications. It was also what she did in answering problem 2. Since she knew the result of 10 x 9 by heart, she used it to find result of 9 x 9 and 11 x 9. She added one more 9 to the result of 10 x 9 and got 99. Meanwhile, she subtracted 90 with 9 since this multiplication was one number less than multiplication 10 x 9.

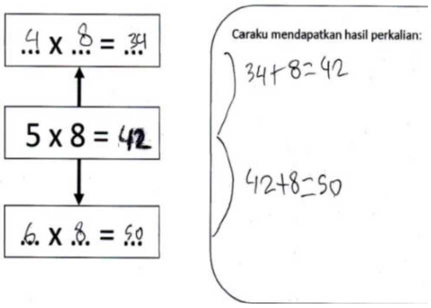


figure 5. 29 (a): Rizky solves problem 1

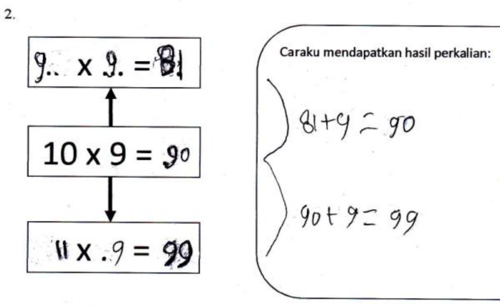


figure 5. 29 (b): Rizky solves problem 2

Different with Intan, Rizky started from the smallest multiplication after he determined the other two multiplications. However, he miscalculated in determining the result of this multiplication. He found the result of 4 x 8 was 34. It happened maybe because of his inadvertence. For the next multiplication, he added 8 and 34 – the result which was gotten in the previous multiplication – and he got 42 as the result. He did the same thing when he determined the result of 6 x 8 – 42 plus 8 makes 50.

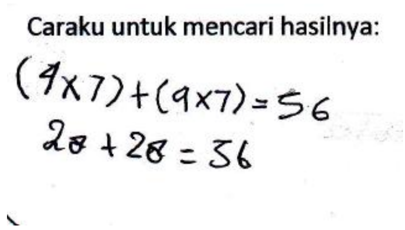
It was also what he did in the next problem (see problem 2 in students' worksheet meeting - 4 in appendix B). He started with the first multiplication and could state result of 9 x 9. Then, consecutively, he added 8 to the result of 9 x 9 to find the result of 10 x 9 and added more 8 to the result of 10 x 9 to find the result of 11 x 9.

In this lesson, students decided which number changed and which number was constant in determining multiplication before and after a multiplication. Most students changed the first number in multiplication and let the second be constant. This happened because teacher illustrated with situation. For example, when they had 5 x 8, teacher assumed there were 5 boxes with 8 donat in each box. Then, teacher asked if they had 4 boxes or 6 boxes. Therefore, most students determined multiplication before 5 x 8 was 4 x 8 and multiplication after 5 x 8 was 6 x 8.

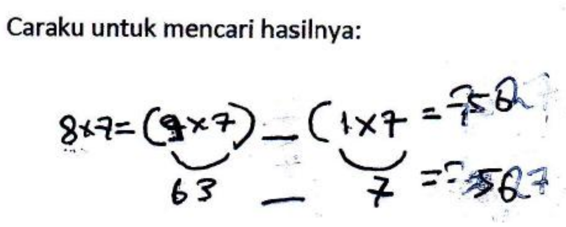
All students started from multiplication that they knew and used it to find the result of other multiplication. Most of time, they started from multiplication that had already stated in the cards and their worksheet. However, there were also some students started from the smallest multiplication. For students who had struggled, teacher helped them by relating their problem to context situation. Therefore, students could think through situation.

Activity 5 Using Known Multiplication for another Multiplication

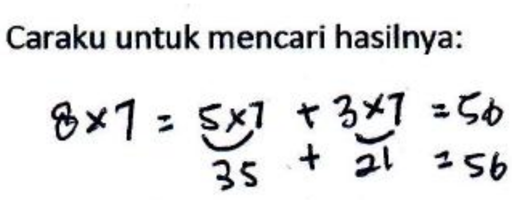
This activity had a purpose that students can use their known multiplication in determining result of a multiplication. To achieve this purpose, teacher gave some multiplications (see students' worksheet meeting 5 in appendix B). Students were asked to solve some multiplications using result of multiplication that they have already known. We expected that they make addition of results of two multiplications. This is what students did. They involved result of multiplication that they know to find the result of a multiplication. These following are examples of how they find result of 8 x 7.



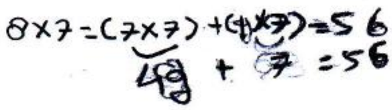
Dwi Amelia's work



Cindy's work



Abdulrahman's work



Meylin's work

figure 5. 30: students find result of 8 x 7

Dwi tried to find the result of 8 x 7 by separating the multiplication into two similar multiplications. In this case, she thought about a half of 8 that is 4. Therefore, she used multiplication 4 x 7. Since she knew the result of 4 x 7, she doubled it to get the result of 8 x 7.

Meylin started from one number multiplication before 8 x 7 that is 7 x 7. Since she knew the result of 7 x 7, she just added 7 more to the result of 7 x 7. It is application of her knowledge that she got from previous meeting purposing to support students to be able to determine multiplication before and after a multiplication. Therefore, when she had a multiplication, she tried to look at multiplication before. If she knew the result, she started from the result of that multiplication.

Different with Meylin, Abdurrahman prefer to use multiplication by 5 since this kind of multiplication is multiplication that have already known by heart. Because 5 plus 3 equals to 8, he added multiplication 5 x 7 with multiplication 3 x 7 to find the result of 8 x 7. It happened since he knew that when he added two results of those multiplications, he got the result of 8 x 7.

If the other students used addition of two multiplications, Cindy used subtraction of two multiplications. She seemed to know result of 9 x 7. Therefore, she started from this multiplication to find result of 8 x 7. Since she learned about determining multiplications before and after a certain multiplication, she applied this knowledge to find the result of 8 x 7. When she knew that multiplication after 8 x 7 is 9 x 7 and recognized the result of 9 x 7, she tried to find out what different they were. Since she knew the different between 9 and 8 was 1 meaning that 1 x 7. She might think that if she had 9 boxes of 7 cakes in each box she got 63. 8 boxes are less 1 box than 9 boxes. She thought to subtract the result of 9 x 7. Therefore, she tried to find the result of 8 x 7 by taking 7 from the result of 9 x 7. This happened because she knew that the result of 8 x 7 should smaller than the result of 9 x 7 and can determine the different between 9 and 8 was 1. Therefore, she subtracted 9 x 7 by 1 x 7 to find the result of 8 x 7.

During students worked on their worksheet, we observed them and found out Meylin who tried to find result of 16 x 6. The following is how she got the result. Making sure what she did, we interviewed her.

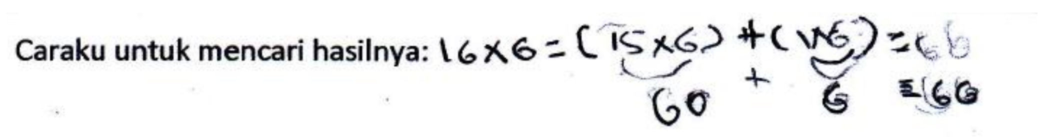


figure 5. 31: Meylin tries to find result of 16 x 6

Teacher : How do you know 15 x 6 is equals to 90? How do you get that?
She erased 90 and replace with 6o.
Teacher : Now it becomes 60, why?
Meylin : ten times six (point to 15 x 6). Ten times six. Ten times six is equal to 60. The rest is five. Five times six makes 30
Teacher : yes, then..
She wrote 66
Teacher : so the result is 66?
You said that 10 x 6 is 60 and 5 times 6 is 30. So, the result is still 60?
Meylin nod
Teacher : iii… let us think again..
Meylin erase 66, become silent for a moment, and use her fingers
Meylin : ten times 6
Teacher : then
Meylin : five times 6
Teacher : how many?
Meylin : 60 eh 160..

Meylin seemed to start with multiplication 15 x 6 and added with multiplication 1 x 6. It happened since she used her previous knowledge about determining multiplication before and after. She looked at multiplication before 16 x 6 that is 15 x 6. Being confidence, she tried to determine its result. It was hard for her since the number of this multiplication was larger than ten. At the beginning, she usually started from multiplication by 5 or 10. Therefore, she tried to find the result of 15 x 6 through splitting them into 10 x 6 and 5 x 6. However, she was not sure when teacher asked about her way in finding 90 as the result of 15 x 6 and changed it to be 60. This happened because she got lost when she explained. She was able to determine result of 10 x 6 and 5 x 6 but she did not add them. She just reminded 60 and stated it as the result of 15 x 6. Believing 60 as the result of 15 x 6, she added it with result of 1 x 6 and got 66 as result of 16 x 6.

Ribi showed how he determined result of 16 x 6. Before that, Intan had presented her work

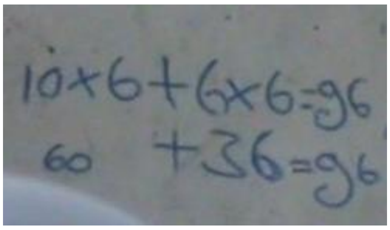


figure 5. 32 (a): Ribi's solves 16 x 6

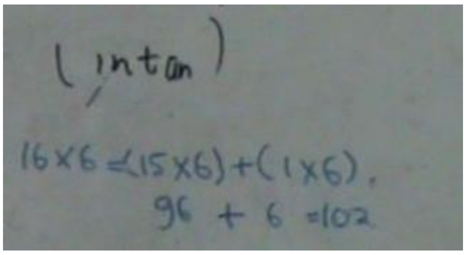


figure 5. 32 (b): Intan solves 16 x 6

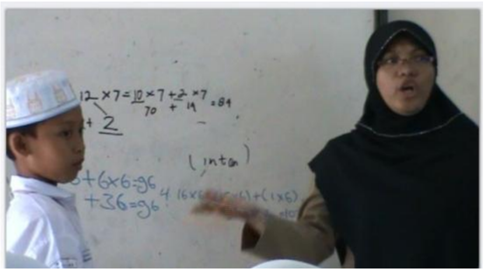


figure 5. 33 (a): teacher manages class discussion

figure 5. 33 (b): teacher asks opinion of the other students

Knowing this difference, teacher asked the other students' opinion.

Teacher : Ribi's result is 96 and Intan's is 102. Why are they different?
Some students raise their hand
Teacher : Dwi Amelia please..
Dwi : The different is six.
Teacher : what do you mean with six? Could you explain it? Mutia, which one do you prefer? Intan's or Ribi's?
Mutia : Intan.
Teacher : do you mean Intan's is the correct one? (Mutia nod) Owh.. could you explain why?
Silent
Teacher : both of them use two multiplications. What multiplication do you know in their works? 10 x 6 or 15 x 6?
Fajri : 10 x 6
Teacher : let us see the question. (write down 16 x 6). Now, does anyone want to ask?
Some students raise their hand
Teacher : Fitri please..
Fitri : where do you get 10 x 6, Ribi?
Ribi : because it is 6 and we take 10
Teacher : owh. So, you split them Ribi?
Ribi : yes

Teacher : Fitri, Ribi split them into 10 and 6. Therefore, he writes 10 x 6. Anyone else?

Adha please..

Adha : where do you get 6 x 6?

Ribi : because the second part is 6 (point to 6)

Teacher : owh.. it is the ones, isn't? so, he got it based on the ones, Adha. Therefore, he writes 6 x 6. Anyone else?

Alfini : how do you get 96?

Ribi : because 60 plus 36 makes 96.

Teacher : yes.

In his presentation, Ribi argued that 16 x 6 can be split into 10 x 6 and 6 x 6. It was done after he recognized that he could separate 16 based on tens and ones. Therefore, he split 16 x 6 into 10 x 6 and 6 x 6. The other students also considered that they could split it into 10 x 6 and 6 x 6 because they know the result of multiplication by 10 rather than result of 15 x 6.

We got that most students used multiplication by 5 and multiplication by 10 for multiplications more than ten since they have already known these multiplications. However, they also recognized multiplication that they know after they look at multiplication before or multiplication after. They could start from that multiplication if they know its result.

Activity 6: Completing Multiplication Table

The purpose of this activity was that students were able to complete multiplication table. In this activity, we expected students were able to complete multiplication by 8 since this multiplication was categorized not easy multiplication for students to be learned.

Teacher gave those multiplications implicitly through playing number cards. 3 number cards, which were given, contained known multiplications. They were multiplication by 1, multiplication by 5, and multiplication by 10. As many 10 empty cards were given as cards which were used to write multiplications that they have to find the result. In their worksheet (see students' worksheet meeting – 6 in Appendix C), after they had a certain number which they got randomly, they had to write down a multiplication in that number and determine its result – every number had a different multiplication. We expected that students started from a multiplication that they have already known to find the result of a multiplication which they have not known. Since teacher gave 3 known multiplications – multiplication by 1, multiplication by 5, and multiplication by 10, they could used them to determine the result of multiplication that they had. This is what happened in class. Most students started from multiplication before or after a multiplication that they had to answer. When they found that multiplication before had already known, they added some numbers to the result of that multiplication or they subtracted some numbers from the result of that multiplication when they found that multiplication after had already known.

figure 5. 34: Widia's group arrange number cards and write down multiplication on it

Since students worked in group, they discussed the way how they could find the result by using multiplication that they know. This was what happened in Widia's group when they wanted to complete all multiplication by 8.

Students wrote down multiplication 8 x 8

Teacher : what is the result of 8 x 8?
Silent
Cindy : seventy one
Teacher : seventy one?
Cindy : eh
Dwi : seventy two
Cindy : o ya, seventy two
Widia : wait a moment, how come 8 x 8 is 72?
Teacher : what do you think Widia? What should it be?
Widia : 72 is result of 9 x 8
Teacher : oo, 9 x 8 is equals to 72. So, what is the result of 8 x 8?
Widia : it is sixty..mm..
Teacher : you said that 9 x 8 is 72. How about 8 x 8?
Silent
Teacher : what do you think? 9 is before or after 8?
Widia : after 8
Teacher : then
Dwi : it should be subtracted
Dwi used her fingers. A minute later, she used paper and pencil.
Cindy : it is 60 eh, 68
Teacher : are you sure?
Cindy : Mam, 5 x 8 is 40, isn't it?
Teacher : yes, then..
Cindy : then, 48, 48, 48,…

Cindy used her fingers
Cindy : 48, 49,50, 51, 52, 53, 54, 55, 56..
57, 58, 59, 60, 61, 62, 63, 64.. sixty..mm. It is sixty four, isn't it mam?
Teacher : yes
Cindy : hey it is sixty four, friends.. (talk to the others)

Students in Widia's group argued that result of 8 x 8 was 72 at the beginning. Widia believed that it was not 72 since she knew72 is the result of 9 x 8 not 8 x 8. However, she was not sure if she knew the result of 8 x 8. Teacher tried to help them by comparing 9 and 8. By posing them a question, Dwi seemed to know that the result of 9 x 8 needed to be subtracted because this multiplication is after 8 x 8. However, Cindy initiated to start from multiplication 5 x 8 since she had already known this kind of multiplication. When she knew 5 x 8 is equals to 40, she continue calculation by skipping number until 48. Because she was difficult to determine the next number, she used her fingers and counted the numbers up until she got the result was 64.

In different group, Intan also tried to find the result of 8 x 8.

Teacher : *how do you find result of 8 x8?*
Intan : *by thinking*
Teacher : *what is the result?*
Silent
Intan : *seventy two*
Teacher : *are you sure?*
Silent

Intan : *sixty four*
Teacher : *how do you get 64?*
Intan : *from 8 x 8*
Teacher : *I saw you said 40 then etc. what does it mean?*
Intan : *no, nothing is important. It is because of 7 x 6 but it is not right.*
Teacher : *so, how do you get that Intan?*
Intan : *I get it from separating number.*
8 x 8 is equals to 5 x 8 plus 3 x 8
Teacher : *how many is 5 x 8?*
Intan : *5 x 8 is equals to 40 and 3 x 8 is equals to 18. So, 40 plus 18 is 54 eh.. 58*
Teacher : *hah? what is the result of 3 x 8?*
Intan : *3 x 8 is 18 eh.. 24*
Teacher : *24?*
Intan : *Yes (nod)*
Teacher : *so..*
Intan : *it is 64.*
Teacher : *ok*

At the beginning, Intan was not sure with her answer since she said that result of 8 x 8 is equals 72. After a moment later, she could find the result of 8 x 8. She argued that she found the result by separating multiplication into two multiplications. She started from multiplication by 5 since she had already known this multiplication. She might think that 8 are equals to 5 plus 3. Therefore, besides involving 5 x 8, she needed to add it with another multiplication. In this case, she tried to find result of 3 x 8. At the beginning, she miscalculated it. Therefore, she got 58 as the result of 8 x 8. Realizing her mistake, she rechecked the result of 3 x 8. When she knew that the result of 3 x 8 is 24 and she also had already known the result of 5 x 8, she added the result of those two multiplications to find the result of 8 x 8. Intan involved her previous knowledge since she used two multiplications to find the result of 8 x 8 - in this case, she used multiplication 5 x 8 and 3 x 8.

Below are two examples of students' work when they tried to find result of 7 x 8.

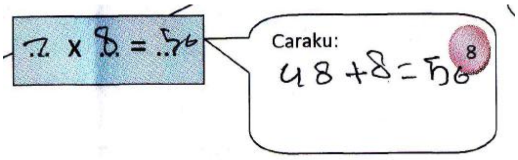


figure 5. 35 (a): Bahrul determines result of 7 x 8

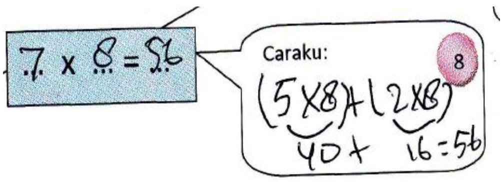


figure 5. 35 (b): Dwi Mutia determines result of 7 x 8

Bahrul started from result of multiplication 6 x 8 to find result of 7 x 8. He justified that he just added one more 8 into the result of 6 x 8 in getting result of 7 x 8 since he had already found the result of 6 x 8 and he knew that multiplication 7 x 8 is multiplication after 6 x 8,

Different with Bahrul, Dwi started from multiplication by 5 since she knew this multiplication and added with another multiplication. In this case, she used 2 x 8. Because she knew that 7 is equals to 5 plus 2, she formulated addition of 5 x 8 and 3 x 8 to find result of 7 x 8. What Bahrul and Dwi did is both of them connected result of two multiplications to find result of a multiplication whether they involved multiplication one number before or multiplication by 5 which they have known. For multiplication more than

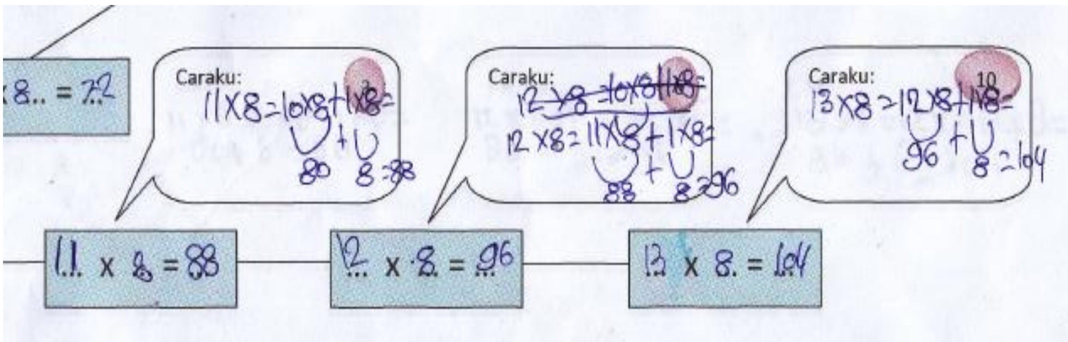


figure 5. 36: rizky use multiplication before to determine a multiplication

multiplication by 10, students started from multiplication before like what Rizky did or used multiplication by 10 like what Dwi did.

To solve each multiplication above multiplication by 10, Rizky started from multiplication before. Since he knew result of multiplication by 10, he used it to find multiplication 11 x 8. Because 11 is one number after 10, he realized that he needed to add one more eight so that he could get the result of 11 x 8. It is also what he did in finding result of 12 x 8 and 13 x 8. However, if we looked at the way he solve 12 x 8 in his worksheet, it seemed that he tried to start from multiplication by 10 at the beginning. It happened because he might think that 12 were not so far with 10 and he also had known the result of 10 x 8. Therefore, he tried to connect multiplication by 10 and another multiplication to get the result. Since he got stuck and it seemed difficult enough for him, he considered another possible way. He back with the way when he found the result of 11 x 8. Since he knew multiplication before 12 x 8 was 11 x 8, he used it and added one more 8 to its result in finding the result of 12 x 8. He also did the same thing in finding result of 13 x 8.

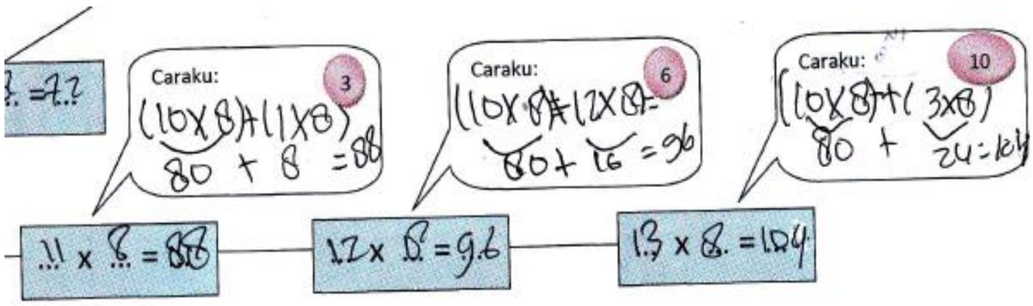


figure 5. 37: Dwi involves multiplication by 10 to determines multiplication more than multiplication by 10

Dwi had different opinion with Bahrul. Because 11 x 8, 12 x 8, and 13 x 8 are outside of multiplication table, she argued that she could use multiplication by 10 to find their results. Since 11, 12, and 13 could be separated into tens and ones and she also had known result of multiplication by 10, she connected multiplication by tens with multiplication by ones to get the result of those multiplications. For instance, when she wanted to find result of 13 x 8, she separated it into 10 x 8 and 3 x 8. Since she knew result of 10 x 8 is 80 and result of 3 x 8 is 24, she added them and got 104 as the result of 13 x 8.

We got that most students used multiplication that they have already known in determining result of a multiplication. They could start from multiplication before, multiplication by 5, or multiplication by 10. It happened since they could see number relation of multiplicand which can be used. By separating a whole multiplication into some parts of multiplications that they know, it could help them to find result of the whole multiplication. The only difficulty that they had is when they determine multiplication by ones after they spilt a whole multiplication into multiplication by tens and multiplication by ones. It happened since students sometimes forgot the result.

J. End Assessment

End assessment was conducted at the end of lesson series. It purposes to see if our activities could support students in developing their mental calculation on multiplication achieving splitting level. There were 5 problems in the assessment (see appendix C). All problems are similar with problems that students got in lesson series. The problems were about determining the result of multiplication using number relation in multiplication.

1. Problem 1

We gave a picture of spoon boxes in this problem (see appendix C). If there were two piles of spoon boxes like in the picture, then how many all spoons are in those boxes? This problem aimed to see awareness of students in using addition of two multiplications in determining the result of a multiplication.

There were 17 out of 28 students who were able to give correct answer. They could determine the number of spoons in each pile by using multiplication. Multiplications that they used in the first and the second pile were added to find the number of spoons in all boxes. They could use result of two multiplications showing the number of spoons in the first and the second pile in determining result of the whole multiplication showing the number of spoons in all boxes since we gave them a figure of spoon boxes which were arranged in two piles. Multiplication which was used depended on the way they saw the boxes. If they show them in total, they used a whole multiplication. However, most students calculated spoons based on each pile. We observed that they multiplied the number of spoons in one box and the number of 5 boxes in the first pile, 5 times 6 makes 30. It was also what they did in the second pile. They multiplied the number of spoons in one box with the number of 3 boxes in the second pile, 3 x 6 makes 18. They added the number of spoons in the first pile and the second piles, 30 plus 18 makes 48.

2. Problem 2

In this problem, teacher told about chair arrangement in a kondangan. There were 9 out of 10 rows fulfilled by guests. The question was: how many guests did attend in that Kondangan event?"

There were 10 out of 28 students whose the answer was correct. They could determine the number of guest who fulfilled 9 rows of chairs by connecting the number of all chairs which were arranged in 10 rows. Some students could state that number of guests were equals to the result of 9 x 8. In determining the result of 9 x 8, some students started from result of 10 x 8. They subtracted it by 8 to get the result of 9 x 8. However, there were also students who used addition of two multiplications to get result of 9 x 8. Some students looked at multiplication before 9 x 8, it is 8 x 8. They split 9 x 8 into 8 x 8 and 1 x 8. They determine result of 8 x 8 and added it with 8. Some students change 9 x 8 into 8 x 9 since they thought that their results are same. Using multiplication by 5, they split 8 x 9 into 5 x 9 and 3 x 9. Then they added the result of multiplication to get result of 8 x 9.

3. Problem 3

We would like to know if students could determine multiplications before and after a certain multiplication. Therefore, teacher gave them a multiplication together with 2 empty spaces as places to write multiplication before and after.

Among 28 students, there were 22 students who could determine result of multiplication "before", 28 students who could determine result of middle multiplication, 23 students who could determine result of multiplication "after", Some students wrote down multiplications before and after 10 x 9 and started from 9 x 9 as the smallest multiplication. At the beginning, they determined result of 9 x 9. In determining result of 9 x 9, some students looked at multiplication before 9 x 9, it is 8 x 9. When they knew its result, they added it with 9. Some students split 9 x 9 into 5 x 9 and 4 x 9. After they get result of each multiplication, they added results of those two multiplications to get result of 9 x 9. There were also some students who started from result of 10 x 9. They knew it by heart. In determining result of multiplication before 10 x 9, they subtracted result of 10 x 9 by 9. They added the result of 10 x 9 with 9 to get result of 11 x 9.

4. Problem 4

Almost similar with problem 3, teacher also gave multiplications which had to determine its result. However, students could start from multiplications that have already known by heart. Students were asked to determine result of 8 x 9, 7 x 8, and 13 x 6.

Among 28 students, there were 16 students who could answer correct question a, 19 students who could answer question b, and 20 students who could answer correct question c. In determining result of each multiplication, some students looked at multiplication before. When they knew the result, they added it with one number. There were also some students using multiplication by 5. For 8 x 9, they spilt it into 5 x 9 and 4 x 9. They also split 7 x 8 into 5 x 8 and 3 x 8. For multiplication 13 x 6, some students used multiplication by 10. They split 13 x 6 into 10 x 6 and 3 x 6. After they get result of each multiplication, they added result of those two multiplications to get result of 13 x 6.

5. Problem 5

We would like to know if students could complete multiplication table. Therefore, teacher gave some parts of multiplication by 7 and students had to determine the result of each multiplication.

Among 28 students, there were 21 students who could answer correct question a, 20 students who could answer question b, 19 students who could answer correct question c, 20 who could answer correct question d, 17 who could answer correct question e, 26 who could answer correct question f, 25 who could answer correct question g, 20 who could answer correct question h, and 19 students who could answer correct question i. Since 3 multiplications were already known. They used it to find result of multiplications which were not known. When they started from this known multiplication, they added one more 7 to get result of the next multiplication. This was also what some students did in determining result of each multiplication. They looked at multiplication before. When they knew it, they added one more 7 in each multiplication. There were also students who used multiplication by 5 since they had known it by heart. For example, they split 8 x 7 into 5 x 7 and 3 x 7. After they get the result for each multiplication, they added the result of those multiplications to get result of 8 x 7. For multiplication more than multiplication by 10, some students still employed multiplication 10. For example, they spilt 12 x 7 into 10 x 7 and 2 x 7. After they found their result, they added them together to get result of 12 x 7. However, there were students who added more 7 to result of 10 x 7 to get result of 11 x 7. They also applied it to get result of 12 x 7 and 13 x 7 – they added one more 7 to result of 11 x 7 to get result of 12 x 7 and added one more 7 to result of 12 x 7 to get result of 13 x 7.

According to end assessment, we conclude that the students were influenced by activities that they followed. As many 60 percents of students could achieve splitting level after they followed the series of activities which were designed. Most of them split multiplications using multiplications that they have already known such as multiplication by 5 and multiplication by 10. Known multiplications were also found after they looked at multiplication before of the certain multiplication.

Discussion Based on Analysis

Some ideas of Realistic Mathematic Education have been considered in designing the sequence of instructional activities such as context situation, model, students' own work, interactivity, and intertwining. However, the sequence of activities designed in this study is only a part of longer series of learning trajectory in developing mental calculation on multiplication. The learning process might take months or a year. It might not be applicable in our research that only took 3 weeks doing a short series of activities. Nevertheless, we would like to describe some finding in our study as follow.

Spoon boxes arrangement and attending guests in *Kondangan* are the contextual situation in this research. In activity 1, students solved problem related the contextual situation.

They could use their informal knowledge about arranging spoon boxes to solve problem in activity 1. The problem given was structured problem where they answer each number of problems. By giving this structured problem, students could see the boxes arrangement as structured objects. Because they saw that there were two piles of boxes, they realized that they could added the number of spoons in each pile to get the number of spoons in all boxes. In activity 1 and 2, there was a transition from contextual situation to more abstract level. Based on contextual situation which was used, students were asked to make drawing of situation. In activity 1, after they worked with real spoon boxes which were arranged, they drew representation of the arrangement. In activity 2, after teacher told story of attending guests who just fulfilled 9 out of 10 rows chairs, they were asked to draw the situation. The drawings that they made were the model of the situation that they imagined.

In activity 3, teacher illustrated to draw rectangle model as model for situation after students were asked to redraw situation in the previous activity. This purposed to help students to draw more general situation in their calculation. In activity 4, students could use their experience that they got in the previous activities in determining multiplication "before" and multiplication "after" of a certain multiplication. For activity 5 and 6, students worked in more abstract way where they had to determine result of multiplication using multiplication that they have already known and completed multiplication table.

Some basic concepts were involved as intertwining in sequence of learning trajectory such as addition, subtraction, and consecutive numbers. Those concepts cannot be separated in developing mental calculation because students use their flexible calculation based on number relation and those concepts relate to number relation. For example, in determining result of 7 x 6, since students know result of 6 x 6 they just need to add one more six to it.

They know that six plus one makes seven. Therefore, they involved those concepts in their mental calculation on multiplication.

Using context situation stimulated students to use their informal knowledge to solve multiplication problems. In this research, the situation of spoon boxes which were arranged into two piles could provoke students to use two multiplications which have same result with a multiplication that they have not known.

Besides talking about context situation, model, students' own work, and intertwining, there was also interactivity in every activity. It could be seen not only when they worked in their group but also when they had classroom discussion. They discussed and decided the way they solve problem because some students might have different opinion.

Classroom Discussion

From information of the teacher and our observation, classroom discussion is usually conducted in mathematics class. In implementation of this study, most students participated in the discussion although we could find a few students who did not contribute. Teacher, most of the time, gave chances to students to pose questions and to tell their idea. By giving opportunity for students to pose question even the simple one, teacher trained students to get used to discuss and express their opinion. This was a form of implementation of socio mathematical norm in their classroom although we found that class became noisy.

In each activity, students worked in their group which consisted of 4 students. They discussed in their group to solve problem before they had classroom discussion. In classroom discussion, some groups which were selected told the way they got result and the other students were given chances to pose questions and tell their opinion. They could express their idea and the way that they used. Teacher managed the discussion and let students to compare some strategies. By discussion, low achieving students could be involved. They could learn from the other students. They argued the other students' opinion and shared their opinion.

The Role of Teacher

Besides giving series of lesson, teacher had important role in supporting the development of students' mental calculation achieving splitting level. She provoked students to see spoon boxes arrangement as structured objects. She helped them to understand problem through relating to situation. She illustrated to make model for situation in order they can represent situation in more general. She managed discussion so that low achieving students could involve in learning. She acted as facilitator when students need. She helped students when they had struggle in solving problem. She encouraged students to use multiplication that they have known. Therefore, students always consider looking for multiplication that they know by splitting an unknown multiplication into some multiplication that they recognize.

Achievement of Splitting Level in Basic Multiplication

At the first time, students learn multiplication as repeated addition. In the process, they have started learning multiplication table. As we found that students most of the time solve multiplication table by using repeated addition. For example, when they had 3 x 8, they add 8 as many 3 times. Furthermore, when we ask them to find result of 4 x 8, they add 8 as many 4 times. They could not see that they can use result of the first multiplication to get result of the second multiplication.

Based on our finding, the development of students' mental calculation achieving to splitting level on basic multiplication should help students to see that they can break down an unknown multiplication into some multiplications that they have known to get its result. To reach this condition, students need to see arranged objects into some parts of arrangement. Later, they realize that they can use result of two known multiplications in determining result of a multiplication. They use multiplications that they have already known such as multiplication by 5 and multiplication by 10 and add the rest multiplication. They split unknown multiplication into multiplication by 5 or multiplication by 10 and the rest multiplication. However, known multiplication can be found by looking at multiplication of one number smaller than unknown multiplication or multiplication of one number larger that unknown multiplication. By involving known multiplications, students can construct their own strategy in determining result in multiplication table without memorizing it.

CHAPTER 6 CONCLUSION

This chapter describes the answer of research question, reflection for some issues in this study, and recommendation for further research.

A. Answer to Research Question

The main research question is: *"How can structures support the development of splitting strategy in doing multiplication by numbers up to 20?"*

There are two sub research questions which are posed, namely:

1. *"How can group situation and rectangular situation lead students to apply splitting strategy?"*
2. *"How can splitting strategy be applied in multiplication by number up to 20?"*

The designed sequence of learning activities was underpinned by tenets of Realistic Mathematics Education (RME) and mental calculation on multiplication. Following the sequence of learning activities starting from structured context could give opportunity for students to achieve splitting level on basic multiplication. Students could see that unknown multiplication can split into some multiplications to find the result of the unknown multiplication.

Answer Sub Research Question 1

In this study, group situation which was used is spoon boxes arrangement and the rectangular situation which was used is guests chair arrangement. Using structured objects like spoon boxes arrangement could lead students to see some parts of arrangement. Thus, they calculate objects by splitting them into the parts. Through discussion, it could promote students to see that a multiplication can be separated into some multiplications because the result of the whole multiplication is same with addition of those multiplications.

In rectangular situation such guest chair arrangement, objects are arranged based on rows and columns. Through arrangement of guest chairs, students could see relation between used objects and unused objects. Because of this condition, they determined the number of used objects by subtracting the number of all objects with the number of unused objects. Therefore, we conclude that giving structured objects, recognizing number relation in multiplication, and maintaining flexible calculation have important roles in supporting the development of mental calculation achieving splitting level.

Answer Sub Research Question 2

Students can apply splitting strategy in determining result of multiplication up to 20 since they perceive that they can involve multiplication that they have already known to determine result of forgotten multiplication in multiplication table. In solving multiplication up to 20, most of them agree that multiplication by 10 was an easy multiplication that they can involve to find result of the multiplication. By splitting the multiplication into multiplication by 10 and multiplication by the rest of numbers, they could determine result of multiplication by number up to 20.

B. Reflection of Important Issues

It is important to know that students have been advanced in basic knowledge such as counting and repeated addition before they acquaint with splitting level. However, it was not easy for them since they have different abilities in learning process. Achieving splitting level in multiplication needs process and it might not cover in 3 weeks of learning activities. According to retrospective analysis and post assessment, there were also any students who were difficult to achieve this level since they were still struggled in understanding basic concept such multiplication as repeated addition.

Based on our observation, teacher had important roles in supporting students to achieve splitting level. She provoked them to see objects as structured objects. She helped them to understand problem through relating to situation. She managed discussion so that low achieving students could involve in learning. She helped students when they had struggle in solving problem. She encouraged students to use multiplication that they have known. Therefore, students always consider looking for multiplication that they know by splitting an unknown multiplication into some multiplication that they recognize. For low achieving students, teacher should help them to get meaning of multiplication by relating to context situation and treat them by giving opportunity to work based on the way that they understand although they still use counting on, skip counting, or repeated addition as long as it make sense for them. At least, when these students can get insight of multiplication as repeated addition themselves, it is a proud moment because they can show that they are able to reach the first level of multiplication. We cannot force them to achieve splitting level meanwhile they do not require the basic condition before they acquaint with this level.

For students who are able to attain this level, splitting strategy promote them to get more insight to multiplication as the way that they can use to determine the amount of objects. Especially, when they want to solve multiplication involving larger numbers, they could see that they can find the result by separating the multiplication into some multiplications that they have already known.

C. Recommendation for Further Studies

This study is only a small part of the development of mental calculation on multiplication. However, because of this small part, it can affect their mental calculation on multiplication by number up to 20. Perhaps, it also applies for multi-digit multiplications.

Therefore, it raises new questions such as: how can splitting strategy be applied on multiplication with larger numbers? To answer the question, further research is needed.

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Appendix A

Name :

Class :

<u>Pre Test</u>

1. Look at this picture below!

Picture (a)

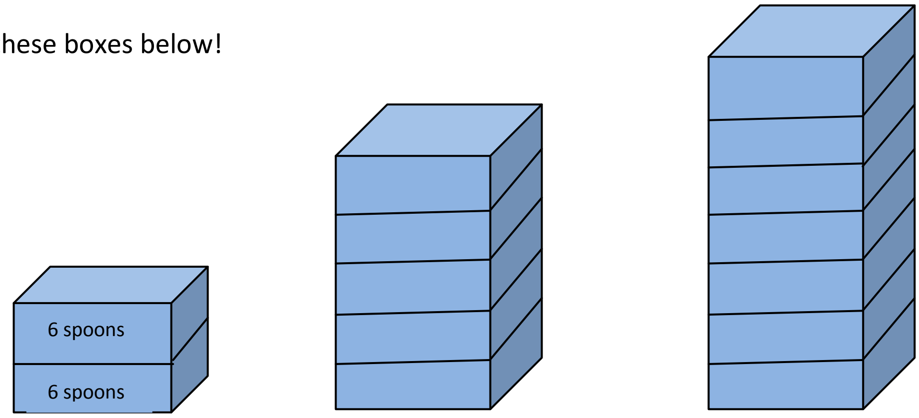
a. Have you ever seen situation like in picture (a)? Where do you usually see that situation?

Answer:

b. What can you explain about objects on the table?

c. How many plates are there?

2. Look at these boxes below!



(a) (b) (c)

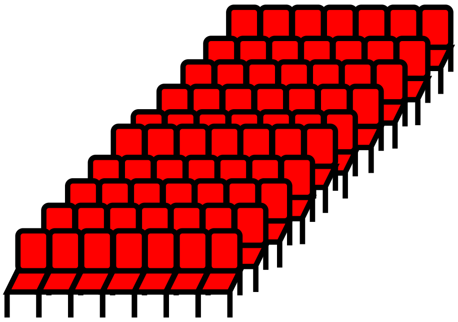
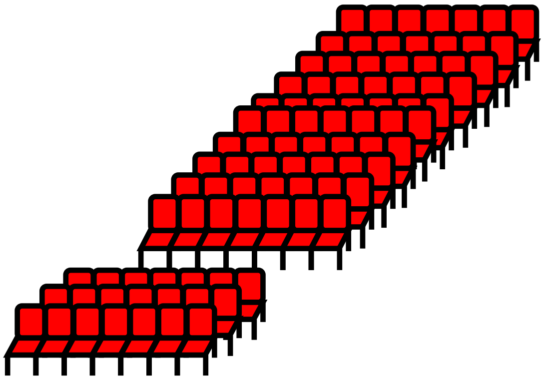
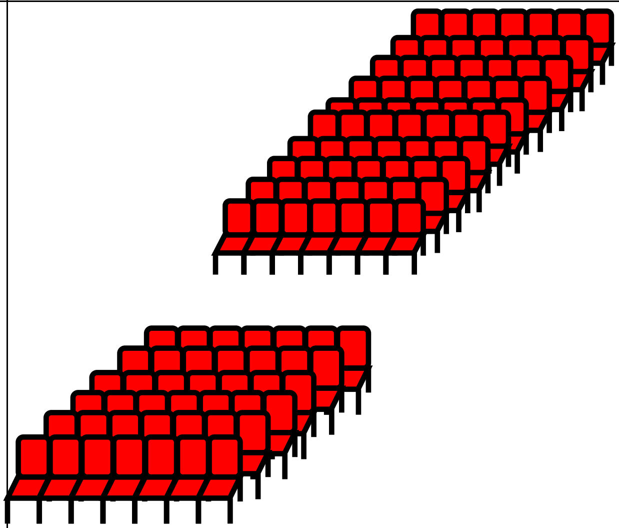
a. what do you think about the boxes?

Answer

b. How many spoons are there in pile a), b), and c)? How do you get the results?

Answer

3. How do you find the number of chairs below? Write down your way in the right side of table!

a.		
b.		
		

Appendix B

STUDENTS' WORKSHEET
Meeting - 1

Name :
Class :

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1. After Kondangan was held, mother kept all spoons which were used in boxes.
- If there were 7 boxes where one bo consisted of 6 spoons, arrange those boxes into two pile of boxes!
- Draw the arrangement of spoon boxes that you made!

Answer:

2. According to spoon boxes arrangement that you made, how many spoons were in the first pile? How do you get the result?

Answer:

3. Ow many spoons were in the second pile? How do you get the reseult?

Answer:

4. How many spoons were altogether? How do get the result?

answer:

STUDENTS' WORKSHEET
Meeting -2

Name :
Class :

---

1. In a Kondangan event, there were 10 rows of guest chairs where one row can contain 8 chairs. How many chairs were provided?

Answer:

2. However, when the event ere held, only one row wasnot fulfilled by guests. How many guests did attend at the moment? Draw situation of chairs which were provided together with attending guests!

Answer:

3. How many guest did attend at the moment? How do you know?

answer:

STUDENTS' WORKSHEET
Meeting-3

Nama :
Kelas :

---

1. a) Draw how high one pile of donat boxes consisting of 5 boxes where each box contain 8 donat!

   b) In the same figure, draw how high a pile of 6 donat boxes!

   c) How many donat are in 5 boxes? How many donat are there in 6 boxes? Explain your answer!

Answer:

2. a) Draw how large an arrangement of 10 rows of chairs with 7 chairs in each row!

   b) In the same figure, draw how many attending guests who sit on 9 rws of chairs!

   c) How many chairs are there in 10 rows? How many guests who sit on 9 rowsof chairs? Explain your answer!

Jawaban:

STUDENTS' WORKSHEET
Meeting-4

Name :
Class :

---

1. Determine multiplications before and after multiplication below!

... x ... =

↑

5 x 8 = ...

↓

... x ... =

My way to get the result:

2.

... x ... =

↑

10 x 9 =

↓

... x ... =

My way to get the result:

STUDENTS' WORKSHEET
Meeting-5

Name :
Class :

Determine the result of multiplication below through multiplication that you have already known!

1. 8 x 7

My way in getting the result:

2. 11 x 8

My way in getting the result:

3. 13 x 7

My way in getting the result:

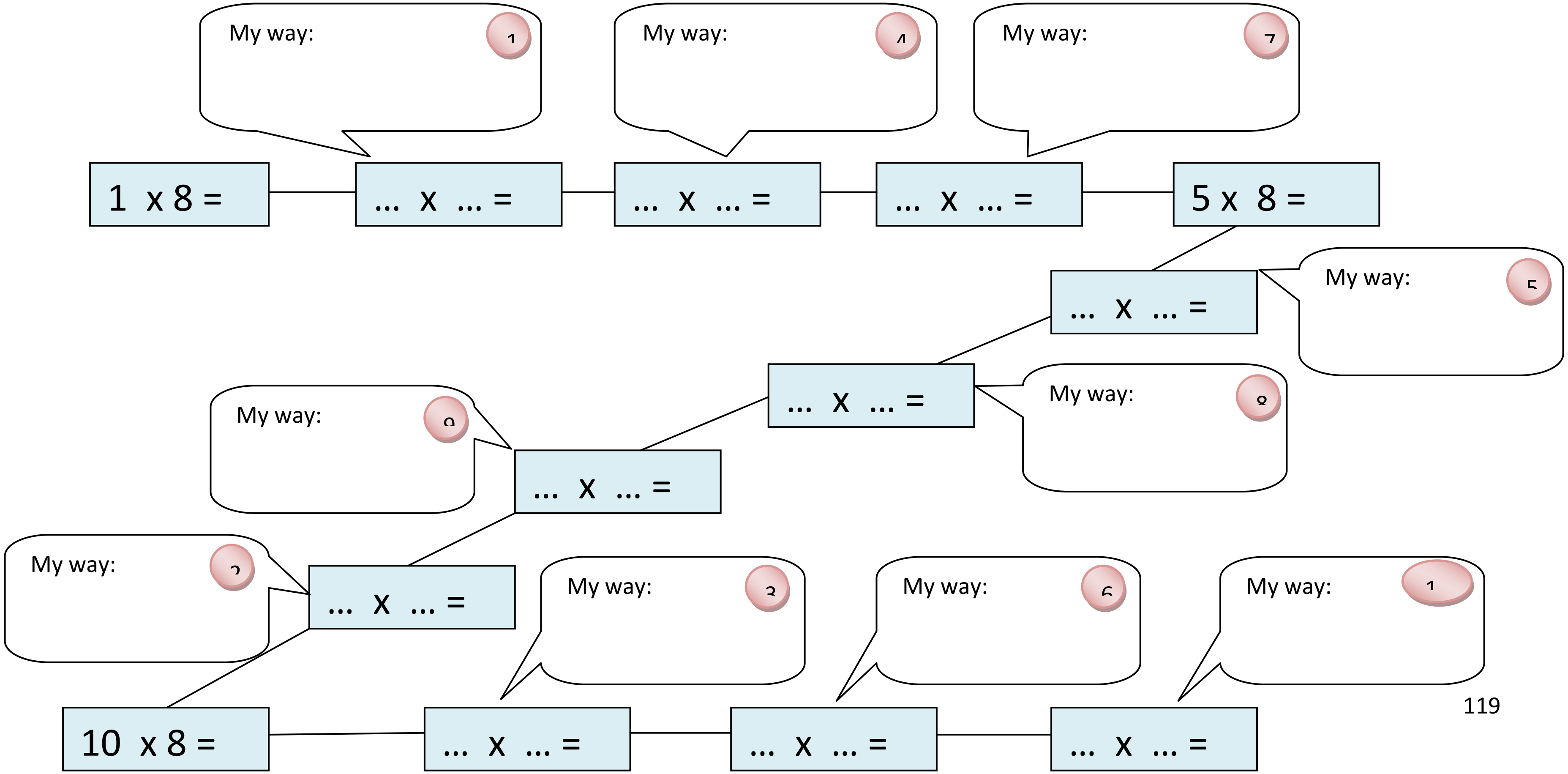
4. 16 x 6

My way in getting the result:

STUDENTS' WORKSHEET
Meeting-6

Name :
Class :

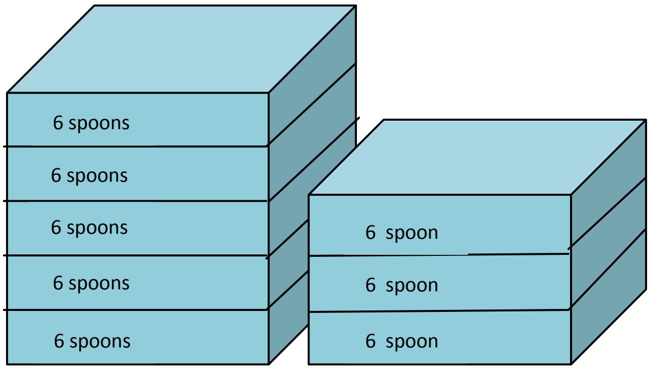
Complete this multiplication table below!



Appendix C

Post Assessment

1. How many spoons are in boxes below?

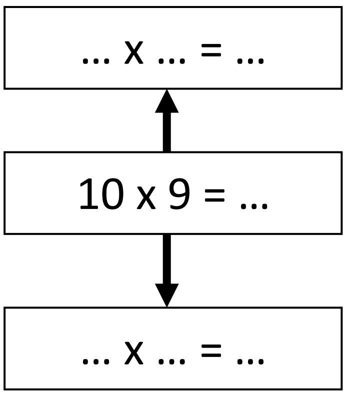


Answer:

2. In a Kondangan event, 9 out of 10 rows of chairs are fulfilled by guests. There are 8 chairs in each row. How many attending guests are there?

Answer:

3. Determine one smaller multiplication and one larger multiplication of multiplication below!



Answer:

4. Determine the result of multiplication in the following using multiplication that you have already known!

a. 8 x 9 =

Answer:

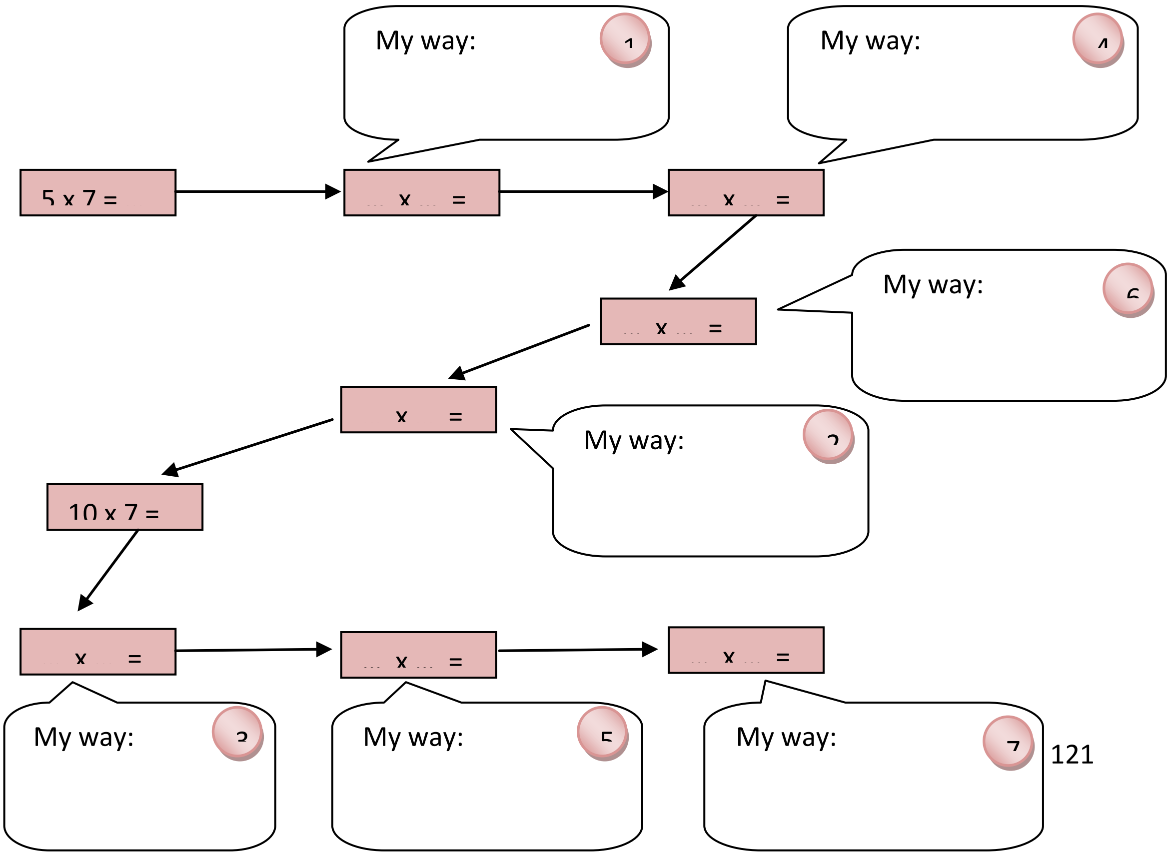
b. 7 x 8 =

Answer:

c. 13 x 6 =

Answer:

5. Complete the multiplication table below!



Appendix D
TEACHER GUIDE

This teacher guide provides guideline of learning activities in the second cycle. It has been revised based on results of pilot experiment. In the first cycle, there are 5 activities that have been tried out. Furthermore, after they were analyzed and improved, they will be implemented in the second cycle. They are described more detail in the following together with what teacher should do in each activity.

Meeting	**: 1**
Topic	**: Multiplication**
Grade	**: III**
Activity	**: Previous multiplication and current multiplication**
Time	**: 2 x 35 minutes**

A. Goal:

Students are able to use the result of multiplication in the previous problem in determining the result of a multiplication.

B. Material

- Spoon boxes
- Students' worksheet

C. Instructional Activity

1. Starting Point (10 minutes)

Teacher states that she has attended a *Kondangan* event. Students are given questions like: "Have you ever gone to *Kondangan*? What does it look like? What can you explain about *Kondangan* situation? What can you find in *Kondangan* event?" Through these questions, students get into learning process indirectly because they tell what they experience related to that *Kondangan* situation. Students will say about food and equipments that exist in that situation. When students state about equipments like spoons, plates, and so on, teacher tell a problem about the need of spoons in a Kondangan event.

2. Main Activity

a. Students are given a problem like:

1) "Mother bought 7 boxes of spoons for the need of Kondangan, where each box consists of 6 spoons. How many spoons did mother buy?
2) "How many spoons did mother have if she bought 8 boxes of spoons where each box consists of 6 spoons?"

b. Problems are given gradually. Students are given time to work on each problem. They are also given a poster and it will be used in presenting their work. In solving problem 1), some students maybe use repeated addition where they add 6 as many 7 times.
c. After they can determine the result of problem 1), they are given problem 2).
d. Students present their works starting from student who still use repeated addition and multiplication 8 x 6 to them who are able to add 6 into the result of 7 x 6. For students who use 8 x 6 = 48, they are asked to explain the way that they use in determining the result of multiplication through posing question like: "What is the meaning of 8 and 6 there? How do you get 48?" Students, who are able to add 6 into the result of 7 x 6, are given a chance to explain and together with teacher formulate that 8 x 6 equals to 7 x 6 plus 1 x 6. These students are expected to be able to influence the other students through discussion. Thus, students can think that they do not need to do repeated addition every time when they want to find the result of a multiplication but they can find via previous multiplication that have already known.

e. Students are given a worksheet which contain a new problem like:

"Mother bought some boxes of spoons where one box contain 6 spoons

1) If mother bought5 boxes, how many spoons did mother buy?
2) However, mother needs more spoons so that she buys 3 more boxes of spoons. How many spoons does mother have?

b. Teacher gives the problem gradually starting from problem 1). In solving problem 1), there might students who are able to use 5 x 6 as handy way to find the result because 5 x 6 is categorized as multiplication by 5 and some students have known this multiplication.
c. After they can solve problem 1), they are given problem 2) where they are asked to find the number of spoons altogether. To find the number of all spoons, they state this problem into multiplication, 8 x 6. Some students can involve the result of 5 x 6 - the result that they got in problem 1) - to get the result of 8 x 6. The difficulty which students have is when they try to connect the whole multiplication with the

parts multiplication. In this case, they are expected to be able to connect 8 x 6 with 5 x 6 and 3 x 6.

d. Teacher can guide students to involve situation in the problem which is given. It can help students in finding connection among 8 x 6, 5 x 6, and 3 x 6 so that they realize that they do not need to do repeated addition to find the result of 8 x 6. They can use the result of multiplication which have already known and add it with the result of another multiplication. In this case, they can use the result of 5 x 6 and 3 x 6 to get the result of 8 x 6.
e. Students present their work starting from students who still use repeated addition until students who are able to use the result of multiplication in the previous problem. When a student presents his work, another student gives respons. Students, who can use the result of multiplication in the previous problem, are expected to be able to influence the other students so that they realize that they can find the result of a multiplication through the result of another multiplication.

f. Reflection

Students are asked to give conclusion when teacher poses question: “What can you get in this meeting?”

D. Assessment: students’ worksheet and interview

Meeting : **2**
Topic : **Multiplication**
Grade : **III**
Activity : **Relation between multiplicand by 10 and by 9**
Time : **2 x 35 minutes**

A. Goal:

Students can use the result of multiplicand by 10 in determining the result of multiplicand by 9

B. Material

Students' worksheet

C. Instructional Activity

1. Starting Point (10 minutes)

Students can reflect what they have learn from the previous meeting that they can determine the result of a multiplication through multiplications which they have already found.

2. Main Activity

a. Students are given a problem like following:

1) Mother bought 10 boxes of spoons where one box consisted of 6 spoons. How many spoons were there?

2) If mother just used 9 boxes, how many spoons did mother use?

b. Students, who work in group, are provided a poster as their worksheet which is also used in their presentation. Almost similar with the problem in the previous meeting, this problem is also given gradually. Students are given time to solve problem a) before they are asked to solve problem b). Most students use multiplication 10 x 6 as a handy way to find the number of spoons. They will not have difficulty in finding the result of this multiplication since they know multiplication by 10.

c. After they are able to solve problem 1), they are given problem 2). In solving problem 2), there might students who just subtract 6 from the result of 10 x 6. They can see that the number of spoons which are used equals to the number of all spoons minus the number of spoons which are not used. Here, they know that they can use the result of multiplication that have been known in the previous problem (10 x 6) in finding the result of 9 x 6

d. Students present their work and the other students give their responds. Teacher arrange the order of presentation starting from students who still use repeated addition until them who can use subtraction from the result of 10 x 6. The difficulty is when they try to connect the result that they get into multiplication. For example, some students know that the result is 54. They get it by doing 60 - 6. Some students struggle to translate 60 – 6 into (10 x 6) – (1 x 6). To help students, teacher can pose question like:" how do we get 60?" this question is purposed to refresh students to give multiplication 10 x 6 based on situation in the problem which is given. Then, teacher poses follow up question like: "How about 6? How is the form of the multiplication?" while students express the intended multiplications, teacher write down them on the blackboard. This is done when there is nothing students who are not able to formulate this connection.

Reflection

Students are asked to give conclusion when teacher poses question: "What can you get in this meeting?"

D. Assessment: students' worksheet and interview

Meeting : **3**
Topic : **Multiplication**
Grade : **III**
Activity : **Working with model for**
Time : **2 x 35 minutes**

A. Goal:

Students are able to draw rectangle model as a model for representing situation

B. Material

Students' worksheet

C. Instructional Activity

1. Starting Point (10 minutes)

Teacher remains situation of spoon boxes arrangement and guest chairs arrangement in the previous meeting.

2. Main Activity

i. One student is asked to make representation of the arrangement. When he or she is able to make the representation using some rectangles, teacher illustrates drawing one rectangle as a model for their calculation.

ii. Teacher gives the other two problems maintaining students to be able to apply this model.

iii. After students work on their worksheet, some students present their works and the other students give their respond.

Reflection

Students are asked to give conclusion when teacher poses question: "What can you get in this meeting?"

D. Assessment: students' worksheet and interview

Meeting	**: 4**
Topic	**: Multiplication**
Grade	**: III**
Activity	**: One larger number and one smaller number multiplication**
Time	**: 2 x 35 minutes**

A. Goal:

Students can determine the result of one larger number multiplication and one smaller number multiplication

B. Material

Students' worksheet

C. Instructional Activity

3. Starting Point (10 minutes)

Teacher reflects what students have learned in the previous meeting. When they are able to state that they can find the result of multiplication through the result of another multiplication, teacher emphasizes by posing a question like: "When I want to answer multiplication; for example, 9 x 7. What can I do?" through this question, we expect that students are able to answer that they can subtract 7 from the result of 10 x 7. In other words, they can use the result of 10 x 7 which is subtracted by 7 to get the result of 9 x 7.

Moreover, teacher needs to remain students about consecutive numbers where they are asked about a number before and a number after. For example, a number before 6 is 5 and a number after 6 is 7. This understanding can be used to determine multiplication of a number above and a number below certain multiplication.

4. Main Activity

i. Teacher provides a card which contains a multiplication. For example, 5 x 8 and some blank cards. They have to determine multiplication of one number below 5 x 8 and multiplication of one number above 5 x 8.

ii. Students are asked to present their work by sticking cards - which contain multiplication that they write – on Styrofoam which is provided. After they stick the cards, they are asked to explain the way that they use to find the result. When one student presents his work, the other students give their responds.

iii. Some multiplications are given as a form of exercises. Then, they present their work like what they do in point b). A challenge which students face is that students will be struggling to decide which numbers to be the benchmark. In multiplication which is given, for instance, 5 x 8, there might students who determine 5 as the benchmark. Some other students choose 8 as the benchmark.

Reflection

Students are asked to give conclusion when teacher poses question: “What can you get in this meeting?”

D. Assessment: students’ worksheet and interview

Meeting : **5**
Topic : **Multiplication**
Grade : **III**
Activity : **From known multiplication to another multiplication**
Time : **2 x 35 minutes**

A. Goal:

Students are able to use multiplication that they have known in determining the result of a multiplication

B. Material

Students' worksheet

C. Instructional Activity

1. Starting Point (10 minutes)

Students are asked to give reflection about what they have learned in the previous meeting. They know that they get used to find the result of multiplication throught the result of another multiplication.

2. Main Activity

i. Some cards which contain certain multiplication are given to students. They are asked to find the result of certain multiplication through the multiplications which they have already known.
For example:
Given a card that contains a multiplication, for example 7 x 6, students are asked to write the result of multiplication on blank cards which are given. There are 2 blank cards which are provided in order they can use two multiplications in determining the result of 7 x 6. For example, they might know the result of 6 x 6. Then, they add 6 to the result of 6 x 6.

ii. One student is asked to stick his work on Styrofoam which is provided and another student gives respond. They are expected to be able to start from multiplication which they have already known. For instance, they know that 6 x 6 equals to 36. They just add 6 to the result of 6 x 6.

iii. Students are given another multiplication as their exercises. Similar with point 2, they present their work and the other students give responds.

Reflection

Students are asked to give conclusion when teacher poses question: “What can you get in this meeting?”

D. Assessment: students’ worksheet and interview

Meeting : 6
Topic : Multiplication
Grade : III
Activity : Completing Multiplication Table
Time : 2 x 35 minutes

A. Goal:

Students can complete multiplication table, multiplication by 11 and by 12.

B. Material

Students' worksheet

C. Instructional Activity

1. Starting Point (10 minutes)

Teacher reflects what students have learned from previous meeting. Students state that they can determine result of multiplication, for example: 7 x 6, by using multiplication that they have known, based on the example, they use result of 6 x 6 since they might know it and just add one more 6.

2. Main Activity

a. Students accept some empty cards and some rolled papers containing a multiplication. Three cards provide a multiplication that they know such multiplication by 1, multiplication by 5, and multiplication by 10.

b. They have to determine result of multiplication in each rolled paper taken randomly. They can involve multiplications which have already been known.

c. Students write down their work on each their worksheet.

d. Some students present their work and the other students give their responds. Teacher helps students if they have difficulty to formulate result that they get.

Reflection

Students are asked to give conclusion.

D. Assessment: students' worksheet and interview

Appendix E

Analysis of Students' Answers of the End Assessment

No	Name of Students	Number of Problems																	Total %
		1	2	3			4			5									
				a	b	c	a	b	c	a	b	c	d	e	f	g	h	i	
	Answer Key	48	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	
1	Dwi Mutia	48	54	81	90	99	72	56	78	43	50	57	64	71	70	77	84	91	64.7
2	M. Rizki Adha	48	80	81	90	99	74	56	18	35	42	46	56	63	70	77	84	101	70.59
3	Dilla Nafasari	48	81	81	90	91	81	62	76	35	50	53	60	67	70	77	84	91	52.94
4	Intan Qurrata Aini	48	80	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	94.12
5	M. Aldino Oktama			81	90	99	142	56	80	35	42	49	56	63	70	77	84	91	76.47
6	Dwi Amelia Anggraini	48	98	81	90	99	72	56	78	35	42	49	56	63	70	77	86	91	88.24
7	Sekar Indah Cahyani		90	82	90	92	54	64	78	35	42	63			70	77	84	91	47.06
8	Meylin Meilinda	48	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	100

9	Nikma Tarisdayani	11	80	79	90	89	99	1102	78	35	42	49	56	72	70	77	84	91	58.82
10	Rayhan A. Bahtiar	48	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	100
11	Rafifa	36	80	81	90	99	72	48	98	35	40				70	77	84	91	52.94
12	Abdul Fathoni Almaghibi	48	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	100
13	Cindy Farizi	65	90	81	90	99	71	80	48	35	42	49	56	63	70	77	84	91	70.59
14	M. Beni Hidayat	48	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	100
15	Ardila A.	36	170	81	90	99	72	48	78	45	51	56	66	86	70	77	74	73	41.18
16	Abdulrahman Ariq Agil	48	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	100
17	Sosriyo	36	19	63	90	72	31	45	63	35	42	49	56	62	70	77	84	111	47.06
18	Alfini Damayanti	48	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	100
19	Azzahra Mawar Daniyah	39	80		90	89	181	56	78		54	94	93	91		82	79	95	17.65
20	Pajri	48	72	81	90	99	72	56	78	35	42	35	66	63	70	77	94	101	88.24
21	Dede Zarkasih	48	90	72	90	92	88	56	65	40	40	50	6	12	70	80	90	100	23.53
22	Arina Alfarini	48	72	81	90	99	90	56	78	35	42	49	56	63	70	77	84	91	94.12
23	Agung Sedayu		80	81	90	99				70	42	46	63	63	70	77			41.18

24	Widya Elsa Fitri	48	98	81	90	99	72	54	78	35	42	49	56						58.82
25	Fadhilah Izzah	50	55	72	90	90	72	56	78	37	48	49	56	63	70	77	84	91	70.59
26	Fitri Rosmala Dewi	48	43	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	94.11
27	M. Azhary	30	72	81	90	99	72	56	78	35	42	49	56	63	70	77	84	91	94.11
28	Rizky Hidayatullah	48	72	81	90	99	73	56	78	30	39	49	56	64	70	77	147	217	64.71
	Number of correct answer	17	10	22	28	23	16	19	20	21	20	19	20	17	26	25	20	19	

The average of the result of the post assessment was 71.85%

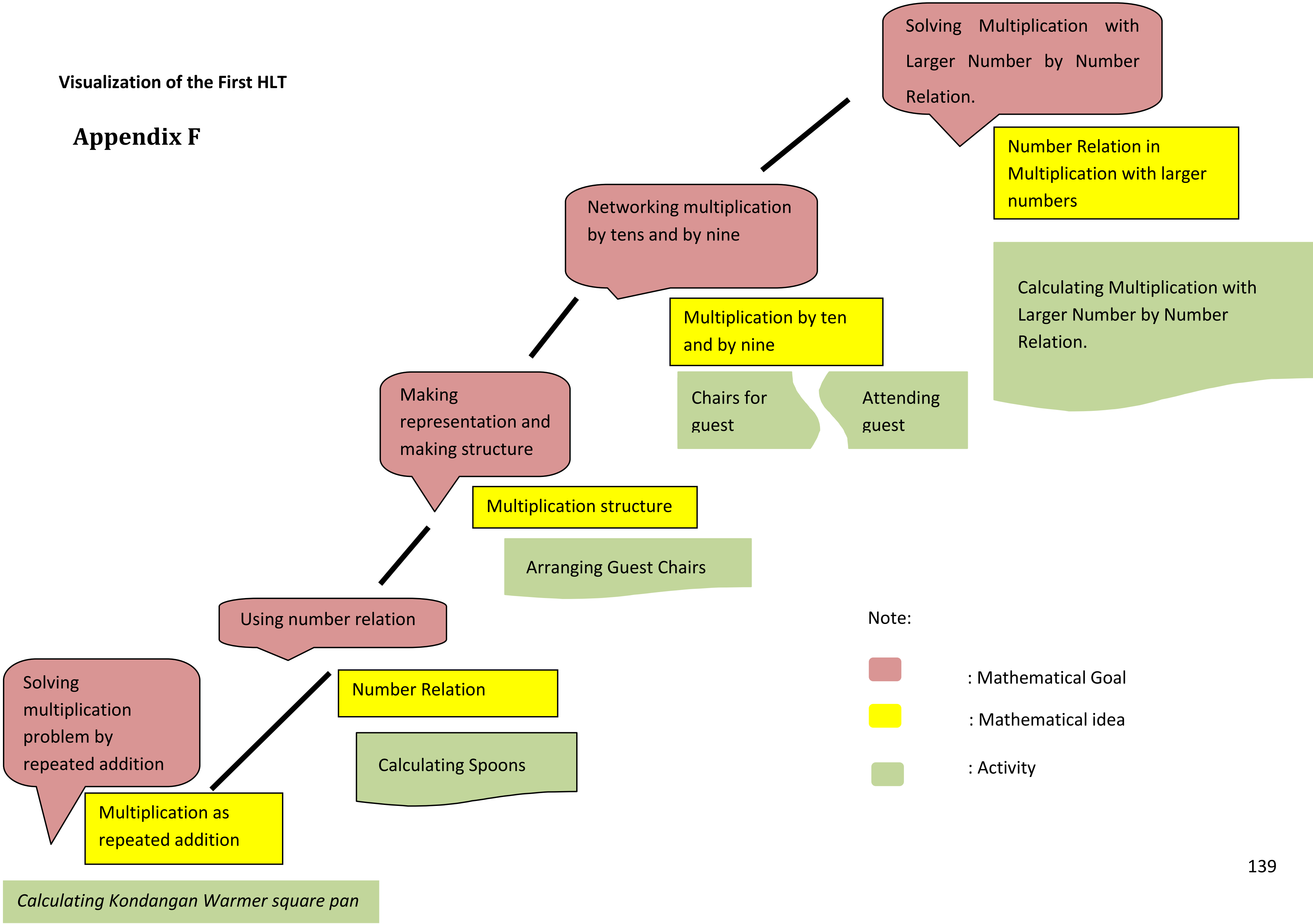
No.	Name	Analisis
1	Dwi Mutia	Mutia could perceive the structure of objects arrangement. She usually involve multiplication by 5 and multiplication by 10 when she solve unknown multiplication by splitting strategy
2	M. Rizki Adha	Rizki sometimes did not get exact result since he miscalculated it. However, he was able to apply splitting strategy in determining result of basic multiplication and multiplication by number up to 20.
3	Dilla Nafasari	This student is not good enough in applying splitting strategy in basic multiplication but she is able to use splitting strategy in determining result of multiplication by number up to 20.
4	Intan Qurrata Aini	From the beginning, Intan could see structure of objects arrangement and she was able to involve multiplication that she known in determining result of unknown or forgotten multiplications. She also usually helped the other students to perceive objects as structured objects.
5	M. Aldino Oktama	This student was still struggled to see objects as structured objects and to use multiplication in determining the number of

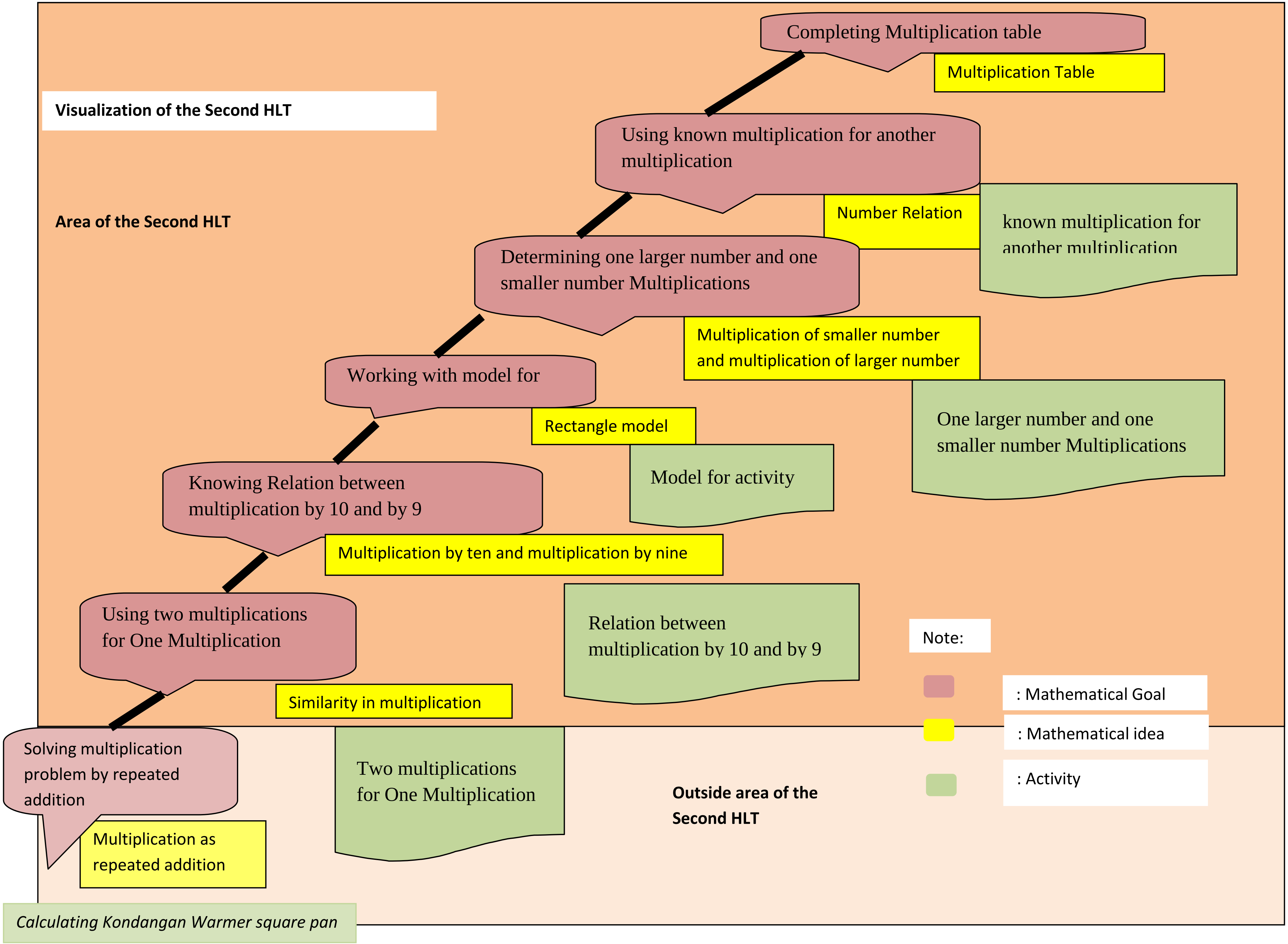
		objects which were arranged in 2 parts.
6	Dwi Amelia Anggraini	The only thing she did was wrong when she misunderstand question 2. She might focus with numbers without really know the meaning of the numbers. She wrote 9 x 10 in determining the number of attending guests. In fact, it should be 9 x 8.
7	Sekar Indah Cahyani	This student saw the number of objects in one group and added them as many the number of group. Although she could not achieve splitting level, she could use repeated addition as the way she determine result of multiplication.
8	Meylin Meilinda	Meylin could perceive the structure of objects arrangement and was able to calculate them in each part of arrangement. She prefer to split multiplication into multiplication by 5 or multiplication by 1 since she has already known these multiplications
9	Nikma Tarisdayani	She might try to find multiplications that she know by looking at multiplication before the unknown multiplication. since it was hard for her to relate the numbers, she could not determine the result of the multiplication. in completing multiplication table, she just add one more number to get result of each multiplication because she saw that the multiplication related with the previous multiplication.
10	Rayhan A. Bahtiar	Similar with Meylin, Rayhan could perceive the structure of objects arrangement. However, he sometimes use algorithm in determining result of multiplication by number up to 20. He did it to make sure that he get the right result.
11	Rafifa	Different with Rayhan, Rafifa could not perceive the arrangement of objects as structured objects that she can split into some parts and calculate them in each part.
12	Abdul Fathoni Almaghibi	At the beginning, e was not sure that he could perceive the structure of objects arrangement. During learning activities, he saw that he could use multiplication by 5 or multiplication by 10 as multiplication that he could involve in determining result of unknown or forgotten multiplication.
13	Cindy Farizi	At the beginning, she was not able to use splitting strategy in determining result of multiplication. However, t the end, she was able to use splitting strategy in completing multiplication table and multiplication by number up to 20.
14	M. Beni Hidayat	Although he could perceive the structure of objects arrangement, he sometimes use algorithm to find result of a multiplication not involving multiplication that he has already known.
15	Ardila A.	She could not perceive the arrangement of objects into some parts and was not able to use multiplication in each part.
16	Abdulrahman Ariq Agil	This students could perceive the structure of objects arrangement since the first activity. He also usulayy helped the other students to se the structure.
17	Sosriyo	At the beginning, Riyo was struggled in understanding multiplication as repeated addition. Although he uld not achieve splitting level until the end of activity, he could apply repeated addition as his way in determining result of multiplications.
18	Alfini Damayanti	Since she could perceive the structure of objects arrangement from the first activity, she could apply splitting strategy in solving

		basic multiplication and multiplication by number up to 20
19	Azzahra Mawar Daniyah	Azzahra also had similar problem with Ardila
20	Pajri	The only mistake he did was about miscalculation. However, Pajri did not have difficulty to see the structure of objects arrangement
21	Dede Zarkasih	Almost similar with Sosriyo, Dede only could improve the way he determine result of multiplication by repeated addition
22	Arina Alfarini	Arina also had similar problem with Pajri
23	Agung Sedayu	Agung had difficulty in following the sequence of learning activities since at the beginning she could not perceive objects as structured objects. He event had not understand the concept of multiplication as repeated addition.
24	Widya Elsa Fitri	Widya take enough long time to perceive the structure of objects arrangement. However, she could do this although she sometimes did miscalculation.
25	Fadhilah Izzah	She was not able to calculate the number of objects in each part of arrangement. In completing multiplication table, she saw that she can use the result of previous multiplication in determining the result of each multiplication.
26	Fitri Rosmala Dewi	Fitri had difficulty in interpreting question 2. Therefore, she got wrong result of calculation. However, overall, she could perceive the structure of objects arrangement.
27	M. Azhary	The only difficulty that he had when he tried to solve 13 x 6 by starting from multiplication before 13 x 6, that is 12 x 6. However, it was also not easy to determine the result of multiplication, he did miscalculation on it.
28	Rizky Hidayatullah	He could perceive the structure of objects arrangement but sometimes he had difficulty in doing calculation on each part of arrangement.

Visualization of the First HLT

Appendix F




Visualization of the Second HLT
Area of the Second HLT
Completing Multiplication table
Multiplication Table
Using known multiplication for another multiplication
Number Relation
known multiplication for another multiplication
Determining one larger number and one smaller number Multiplications
Multiplication of smaller number and multiplication of larger number
One larger number and one smaller number Multiplications
Working with model for
Rectangle model
Model for activity
Knowing Relation between multiplication by 10 and by 9
Multiplication by ten and multiplication by nine
Relation between multiplication by 10 and by 9
Using two multiplications for One Multiplication
Similarity in multiplication
Two multiplications for One Multiplication
Solving multiplication problem by repeated addition
Multiplication as repeated addition
Calculating Kondangan Warmer square pan
Outside area of the Second HLT
Note:
: Mathematical Goal
: Mathematical idea
: Activity