SUPPORTING STUDENTS' UNDERSTANDING OF PERCENTAGE

MASTER THESIS

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SURABAYA STATE UNIVERSITY

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MASTER THESIS

A Thesis Submitted to Surabaya State University Postgraduate Program

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in Mathematics Education Program

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DEDICATION

This thesis is dedicated to my parents, my sisters and my brothers who give me never ending love and supports.

It is also dedicated to my fiancé who has been a great source of my motivation and inspiration.

I Love You

ABSTRACT

Rianasari, Veronika Fitri. 2011. *Supporting Students' Understanding of Percentage*. Master Thesis, Mathematics Education Study Program, Master Program of Surabaya State University. Supervisor: (I) Prof. I Ketut Budayasa, Ph.D., and (II) Sitti Maesuri Patahuddin, Ph.D.

Keywords: *design research, percentage, understanding, pre knowledge, contextual situations, bar models*

Many researches revealed that pupils often find difficulties to understand percentage although they are able to recite percent as per hundred and carry out the computations correctly. This might happen due to the way in which the learning percentage has been taught that tends to focus on procedures and recall instead of getting a real understanding of percentage. Considering this facts, this research aimed to develop a local instructional theory to support students to extend their understanding of percentage. This research used design research method as an appropriate means to achieve the research aim.

In Realistic Mathematics Education approach, in which the basic concept is rooted from Freudenthals' idea, mathematics is the activity of organizing matter from reality. Therefore, mathematics should be experientially real for the students. Consequently, in this research the instructional activities were designed through exploring some contextual situations in which percentages play role.

This research involved students and mathematics teachers of grade 5 in SD Laboratorium UNESA and SD BOPKRI III Demangan Baru Yogyakarta. The research conducted in SD Laboratorium UNESA was conducted in two cycles and it aimed at trying out the initial hypothetical learning trajectory. The research in SD Laboratorium UNESA became the preliminary researches of the research in SD BOPKRI III. In this research, the students involved were the students who have learned about percentage. Therefore they had pre knowledge about percentage. Most of the students were get used to work in formal level in solving percentage problems.

The result of the teaching experiment showed that students' pre knowledge influenced their learning process. In some cases most of the students tended to do procedural computations, which probably did not make sense for them, instead of using their common sense. The result of the teaching experiment also showed that investigating contextual situations in which percentage play a role could stimulate students to extend their understanding of percentage. Students' learning process in extending their understanding was started from developing sense of *fullness* of percentage, which afterwards continued to constructing the meaning of percent itself. Based on this basic knowledge, they were given opportunities to construct sense of percentage as a relative value and also to apply this knowledge in a situation in which the percentages are needed. Furthermore, the students expanded their knowledge to percentage greater than a hundred. Through the learning process, the students also got opportunities to explore some strategies such as using benchmark percentages or using bar model. These strategies seemed to make sense for them rather than procedural algorithm strategies.

ABSTRAK

Rianasari, Veronika Fitri. 2011. *Upaya-Upaya yang Mendukung Pemahaman Siswa Mengenai Persentase.* Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. I Ketut Budayasa, Ph.D., dan (II) Sitti Maesuri Patahuddin, Ph.D.

Kata-kata Kunci: *penelitian desain, persentase, pemahaman, pemahaman awal, situaasi kontekstual, model bar*

Banyak penelitian menunjukkan bahwa siswa sering menemukan kesulitan untuk memahami persentase meskipun mereka bisa mengungkapkan bahwa persen adalah per seratus dan dapat melakukan perhitungan dengan benar. Hal ini mungkin terjadi karena pembelajaran persentase cenderung berfokus pada prosedur dan ingatan dan bukan pada pemahaman mengenai persentase. Menyadari fakta-fakta ini, penelitian ini bertujuan untuk mengembangkan teori instruksional lokal untuk mendukung siswa untuk memperluas pemahaman mereka mengenai persentase. Penelitian ini menggunakan metode penelitian desain sebagai metode yang tepat utnuk mencapai tujuan penelitian.

Dalam pendekatan Pendidikan Matematika Realistik, yang konsep dasarnya berakar dari ide Freudenthal, matematika harus berdasarkan pengalaman yang nyata bagi siswa. Sehingga, dalam penelitian ini kegiatan instruksional dirancang melalui eksplorasi situasi-situasi kontekstual yang melibatkan persentase.

Penelitian ini melibatkan siswa-siswa dan guru-guru matematika kelas 5 di SD Laboratorium Unesa dan SD Bopkri III Demangan Baru Yogyakarta. Penelitian di SD Laboratorium Unesa dilakukan dalam dua siklus dan bertujuan untuk mengujicobakan desain pembelajaran awal. Penelitian di SD Laboratorium Unesa menjadi penelitian awal bagi penelitian di SD Bopkri III. Dalam penelitian ini, siswa yang terlibat adalah siswa yang telah belajar persentase. Oleh karena itu, mereka memiliki pengetahuan awal mengenai persentase. Sebagian besar siswa terbiasa memecahkan masalah persentase dengan cara yang formal.

Hasil eksperimen menunjukkan bahwa pengetahuan awal siswa berpengaruh pada proses belajar mereka. Pada beberapa kasus, sebagian besar siswa cenderung melakukan perhitungan prosedural, yang mungkin tidak masuk akal bagi mereka, daripada menggunakan penalaran. Hasil eksperimen juga menunjukkan bahwa investigasi situasi kontekstual yang melibatkan persentase dapat menstimulasi siswa untuk memperluas pemahaman mereka mengenai persentase. Proses belajar siswa dalam memperluas pemahaman mereka dimulai dari mengembangkan *sense* persentase, yang kemudian dilanjutkan dengan mengkonstruksi arti persen. Berdasarkan pemahaman dasar ini, mereka diberi kesempatan untuk mengkonstruksi makna persentase sebagai nilai relatif dan juga menggunakan pengetahuan ini dalam situasi yang membutuhkan persentase. Selanjutnya, siswa-siswa memperluas pengetahuan mereka mengenai persentase yang lebih besar dari seratus. Melalui proses belajar ini, siswa juga mendapatkan kesempatan untuk mengeksplorasi beberapa strategi seperti persentase sederhana atau model bar. Strategi-strategi ini tampak lebih bermakna bagi mereka dibandingkan strategi yang menggunakan perhitungan prosedural.

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Veronika Fitri Rianasari

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CHAPTER I

INTRODUCTION

1.1 Background

Percentage is one of the most widely used mathematical topics in daily life and holds substantial place in the school curriculum for almost any science and social studies (Arthur J. Baroody et al, 1998; James E. Schwartz et al, 1994; Parker and Leinhardt, 1995). The relationship between percentage and other mathematical concepts such as fraction and decimal offers many possibilities to do arithmetic in a flexible way (Galen et al, 2008). Percentages are not only another way of writing down simple fractions, but also derive their right to exist from the limitations of regular fractions; fractions are difficult to compare with each other, and the scale that they provide is rather unrefined (Galen et al, 2008).

Considering the importance of percentage in daily life, percentage has been taught since elementary school. However, many percentage problems indicate that education is primarily focused on procedures and recall instead of getting a real understanding of percentage (Van den Hauvel-Panhuizen, 1994). At the other hand understanding of percentage is necessary to ensure proper interpretation of social studies, science materials, and many situations in daily life (James E. Schwartz et al, 1994). Many students can quickly learn how to calculate percentage accurately because they are familiar with those computations they have already learned in working with fractions and decimals, but they might struggle to explain what percentage is actually stating.

Regarding this fact, there were two important issues that were well-considered as the reasons to design new instructional activities in this research. The first issue is pupils often find it difficult to understand percentage and apply it to solve problems in context (Kouba et al., 1988 in Koay Phong Lee, 1998; Parker and Leinhardt, 1995). The finding in the study of Koay (1998) shows that ability to recite percent as per hundred and carry out the computations correctly does not lead to the ability to interpret and apply the concept in context. Koay found it easier for students to perform computation problems than to explain the meaning of percent and also knowledge of percent was often rigid and rule-bound.

The second issue is about the way of teaching and learning mathematics in Indonesia. Mathematics in Indonesian curriculum tended to be taught in a very formal way; teachers explain the mathematics operation and procedures, give some examples, and ask pupils to do the other similar problems (Armanto, 2002). Formal way of teaching on percentage tends to restrict pupils' creativity and flexibility in their strategies to solve the percent problem. When students solve percent problem (e.g., 75% of 160), most of them know that they have to transform 75% to the fraction 75/100 and then multiply with the 'whole', but if they are asked to explain how they come to the answer, they struggle to explain it. It shows that many of students can remember the 'right rule' for finding percentage but cannot explain what percentage is. Considering this fact, the teaching and learning of percentage need to focus on how understanding of percentage can be taught.

The need to focus on how understanding of percentage can be taught leads to the need of activities that are aimed at supporting students making sense of percentage. This need leads to the third issue namely explorative activities. In Realistic Mathematics Education where teaching is built on the informal knowledge of the students, it is important to give students the opportunity to explore some daily life contexts in which percentages play a role (Van den Hauvel-Panhuizen, 2003). This idea is relevant with Freudenthal's idea that views mathematics as a human activity instead of seeing mathematics as a subject to be transmitted (Freudenthal, 1991). In many books and researches about percentage, there are many problems that can provoke students to think about percentage, such as discount problems, tax problems, increasing or decreasing rate of population and so on. In order to develop instructional activities on percentage for Indonesian students, it is important to adjust those kinds of problems based on Indonesian contexts and situations.

1.2 Research Question

The research question of this study is 'How to support students to extend their understanding of percentage?'.

1.3 Aim of the Research

Considering those three issues in teaching and learning percentage, this research aimed at developing a local instructional theory to support students' to extend their understanding of percentage.

1.4 Definition of Key Terms

In this section, some important terms relating with the research will be explained. This explanation aimed to help the reader to follow the idea of this research.

1. Understanding

   Understanding is the number of linkages that one can make between his/her schema and an abstract or physical object such as a picture, situation, information, or concept. One has a good understanding of an object, if he/she could make many linkages relating to the essential aspects of the object. Skemp (1987) said that to understand something means to assimilate it into an appropriate schema.

2. Support

   Support is any kind of effort of giving encouragement to someone or something to succeed. In this research, support means any kinds of efforts provided by researcher or teacher to stimulate students in learning percentage. The support in this research includes the instructional activities, tools, guidance, and questions posed by teacher.

3. Percentage

   Percentage is relationship based on a one-hundred-part whole and it gives relative measure, not an absolute measure.

4. Extend the understanding of percentage

   Extend the understanding of percentage means constructing the understanding of percentage that is not much explored in the previous learning. In this research, the understanding of percentage that student has to grasp is that a percentage is relationship based on a one-hundred-part whole that gives relative measure. Students do not have to explain this in this manner, but they have to show an awareness of the fact that percentages are always related to something. The awareness of this idea can be shown by some indicators. Students are aware of this idea if and only if the students know that when they divide a whole into ten or a hundred parts then one part represents 1% or 10% of the whole and the students know that they cannot compare percentages absolutely without taking into account to what the percentages refer. With respect to computational goals, a variety of types of computations with percentage are explored, such as compute the part of a whole while the percentage is given. However, it is more important that students are able to use percentage in a situation in which they are needed, when different parts of different wholes have to be compared.

1.5 Significance of the research

The significance of this research concerned with the theoretical significance and practical significance for teachers or researchers. With the respect of theoretical significance, this research contributes to an empirically grounded instructional theory for learning percentage. With the respect of practical significance, this research gives an overview to the researcher and other researchers about how to design an instructional activity for learning percentage and also this research gives an overview to mathematics teachers about how to teach percentage that focus on supporting their understanding.

1.6 Assumptions

This research was built upon some assumptions. The first assumption was that the students were serious in doing the task given during the teaching experiment. The second assumption was that the teacher was serious in conducting the teaching and learning process. The third assumption was that the environment of the school was conducive in supporting the teaching and learning process.

CHAPTER II

THEORETICAL FRAMEWORK

As described in the first chapter, the aim of this research is to develop local instructional theory on supporting students to extend their understanding of percentage for young students, age 10 or 11 years old in 5^{th}grade. For those need, this chapter presents the theoretical framework that aims to provide base of this research. Literatures about percentage, Realistic Mathematics Education (RME), about Indonesia curriculum, and about hypothetical learning trajectory were studied to elaborate this theoretical framework.

2.1 Percentage

Percentage is one of the most widely used mathematical topics in daily life. The many uses of percentage have led to varying interpretations of the meaning of percentage (Parker and Leinhardt, 1995). According to Parker and Leinhardt (1995), percentage can describe part whole relationship and can describe a ratio based upon the meaning of percentage.

2.1.1 Percentage as part whole relationship

Percentage as part whole relationship describes relative value of the part compared to the whole. Percentages are relationships based on a one-hundred-part whole and it gives relative measure, not an absolute measure (Fosnot & Dolk, 2002). Students do not have to explain this in this manner, but they have to show an awareness of the fact that percentages are always related to something and that they therefore cannot be compared without taking into account to what they refer (Van den Heuvel-Panhuizen, 1994).

A common example is a news report that candidate receive 45% of the vote. The percentage describes the subset of people who vote for the candidate compare to the set of all people who vote. Within the part whole relationship meaning, the percentage '100%' is interpreted as the whole. However, percentages greater than 100 are particularly problematic here. Students may struggle to interpret a percentage such as 150% because in this case the part is bigger than the whole. Other problems instead of part-whole problems such as increase-decrease problems are needed to incorporate the broad range of common application.

2.1.2 Percentage as a ratio

Percentage as a ratio describes a comparison between two quantities from different sets or different attributes of the same set. Van den Heuvel-Panhuizen (1994) describes that a key feature of percentage, that one has to understand in order to have insight, is that a percentage is a relation between two numbers or magnitudes that is expressed by means of a ratio. It describes relative amount by which one set is compared to another. For example, suppose the price of an item was originally Rp 12.000,00 and has been increased to Rp 15.000,00. The ratio of the new price to the original price is 150/120; or as stated in percentage, the new price is 125% of the original price.

In this research, the researcher only focused on the meaning of percentage as part whole relationship. Researcher only focused on this meaning because it is the most salient of comparative situations, imaginable for young students in grade 5, and can support students understanding of percentage as relationship based on one-hundred-part whole. Lembke (1991) asserts that an important component of percentage knowledge is the understanding of the concept of the base 100, knowing that percentage always involves comparison of something to 100. That is not coincidental that the scale goes to 100. This is because in this way it is possible to place percentages neatly into our number system so that we can easily convert them into decimal (Galen et al, 2008).

Learning process of percentage should include goals concerning the understanding of percentage and also computational goals. With respect to computational goals, a variety of types of computations with percentage are explored, such as compute the part of a whole while the percentage is given. However, it is more important that students are able to use percentage in a situation in which they are needed, when different parts of different wholes have to be compared (Van den Heuvel-Panhuizen, 1994).

There are many situations in daily life in which percentage play a role. In this research, the contextual situations used as a support in the learning percentage are situations in which percentage plays an important role in comparing proportion. Those contextual situations are discount, concentration of an ingredient, loading in computer process, and other contextual situations in which percentage play a role.

2.2 The Didactical Use of Bar Model

Although the meanings of percentage are diverse, the essence of percentage is proportionality; percentage is used to describe proportional relationship (Parker and Leinhardt, 1995). Percentages indicate the proportion of a specific total that is set to 100 (Galen et al, 2008). This aspect of proportional relationship involving an equivalent relationship between two ratios suggests that there is needed to offer an appropriate model to support student to reason proportionally. The model offered in this research is bar model.

In the learning process of percentage, learning percentage is embedded within the whole of the rational number domain and is strongly entwined with learning fraction, decimals and ratio with the bar model connecting these rational number concepts (Middleton et al in Van den Heuvel-Panhuizen, 2003). The use of model bar in learning percentage is beneficial for students. The first benefit is that bar model has area that makes it easier to talk in terms of “the whole” (Galen et al, 2008). As the second benefit, the bar model gives a good hold for estimating an approximate percentage, especially in cases where the problems concern numbers that cannot be simply converted to an simple fraction or percentage. The third benefit is that the bar model provides the students with more opportunity to progress. This also means that the bar model can function on different levels of understanding (Van den Heuvel-Panhuizen, 2003).

2.3 The strategies to work with percentage

In this research, the students had pre knowledge about strategy to work with percentage. They knew the procedural algorithm to solve percentage problem, especially in computing a part of a whole if the percentage is given. Since the procedural algorithm sometimes did not make sense for them, the students should be given opportunities to explore any number of strategies to work with percentages that are meaningful for them. Based on the research and discussion of Parker and Leinhardt (1999), it has been discussed some strategies. Those strategies are using benchmark percentage, proportional thinking strategies, and additive building-up or splitting strategies. In this research, researcher only focused on using benchmark percentage and additive building-up strategies. As what have been discussed in the previous subchapter, the use of bar model also can be one of the strategies to work with percentage. Therefore, the strategies besides procedural algorithm that the students could use to work with percentage problem are given as follows:

1. Benchmarks percentage strategy

Benchmark of percentages describes a percent that can be replaced by a fraction with numerator of one, known as a unit fraction; for example, 50% of something is ½ of that thing, and 25% of something is ¼ of that thing. By thinking of these benchmark percent values as fractions, students are able to make use of elementary fractional relationships when solving problems (Parker and Leinhardt, 1999).

2. Additive building-up or splitting strategy

Additive building-up or splitting strategy refers to addition of some benchmarks percentage. For example in determining 15% of Rp 100.000,00, first one might find 10% of Rp 100.000,00, and find 5% of Rp 100.000,00, then add those two calculations (10 % of Rp 100.000,00 and 5% of Rp 100.000,00) to get the final result.

3. Using bar model

Bar model can be used as a tool to estimate percentage. The benchmark percentages, fractions, or decimals can be used in this model to solve percentage problems.

2.4 Realistic Mathematics Education

Realistic Mathematics Education (RME) is a domain-specific instruction theory for mathematics education (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). RME is rooted in Freudenthal's interpretation of mathematics as a human activity (Gravemeijer, 1994). Based on Freudenthal's idea, mathematics must be connected to reality, stay close to students and should be relevant to society. The use of realistic contexts became one of the determining characteristics of this approach to mathematics education (Van den Heuvel-Panhuizen, 2003). In this research, daily life percent problems are a set of contextual problem situation for students to learn about percentage.

This section will elaborate the characteristics of RME used as a guideline in designing the instructional activity. The RME approach will be further explained by elaborating five characteristics defined by Treffers (1987) that can be seen as heuristics for instructional design.

1. Phenomenological exploration

In the first of instructional activity, the mathematical activities take place within a concrete context. This mathematical activity should be experientially real for students. In this research, daily life situations in which percentage play an important role such as loading proces, discount, area, and free extra context are employed as the contextual situation for students to learn.

2. Using models and symbols for progressive mathematization

In learning mathematics, varieties of vertical instruments such as models and symbols are offered, explored, and developed to bridge the level difference from a concrete level to a more formal level. In this research, students' informal knowledge as the result of explorative activities has to be developed into formal knowledge of percentage. Students drawing in estimating loading process served as the bases of the emergence of bar model as a tool to work with percentage.

3. Using students' own construction and productions

Giving opportunities for students to explore and contribute various strategies can support students' individual productions. Students' own productions can indicate where they are and how they progress in learning process. In each activity, teacher has a role to provide opportunities for students to explore their own strategy.

4. Interactivity

Interaction among students and between students and teacher can support the development of students' learning process. These interactions are supported by a classroom culture which is conducive. Therefore, one of the tasks of the teacher is to establish the desired classroom culture (Gravemeijer & Cobb, 2006). This social interaction can stimulate students to shorten their learning path and support their own production. In this research, the social interaction is facilitated by group discussion, presentation of students' production, and criticizing strategies among groups.

5. Intertwinement

In designing an instructional activity, it is important to do integration of the various domains. This intertwining of learning strands is exploited in solving real life percent problems, in which problems not only support their understanding of percentage but also support the development of students' number sense; especially the relation between percentage and fraction or decimal. In addition, in this research, the learning of percentage is also intertwined with the area topic.

2.5 Emergent Perspective

Gravemeijer & Cobb (2006) stated that "a key element in the ongoing process of experimentation is the interpretation of both the students' reasoning and learning and the means by which that learning is supported and organized". They contend that it is important to be explicit about how one is going about interpreting what is going on in the classroom. In this research, the framework used for interpreting classroom discourse and communication is the emergent perspective.

Table 2.1 An Interpretative framework for analyzing individual and collective activity at the classroom level

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about our own role, others' roles, and the general nature of mathematical activity
Socio-mathematical norms	Specifically mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Based on Gravemeijer & Cobb (2006), three aspects of emergent perspective are elaborated. The explanation and discussion of these aspects is relatively brief because the aim is to develop the rationale for the theoretical framework rather than to present detailed analyses of these aspects.

The first aspect of emergent perspective concerns the social norms that are established in the classroom. Social norms refer to expected ways of acting and explaining that is established through negotiation between the teacher and students. The examples of social norms that engaged in classroom included the obligations for students to explain and justify solutions, attempt to make sense of explanation given by other students, and giving opinion indicating agreement and disagreement.

The second aspect is socio-mathematical norms. Socio-mathematical norms refer to the expected ways of explicating and acting in the whole class discussion that are specific to mathematics. The examples of socio-mathematical norms include what count as a different mathematical solution, a sophisticated mathematical solution, and acceptable mathematical explanation and justification.

With the respect of the socio-mathematical norms, students' explanation and reasoning dealing with investigating contextual problems about percentage are explored.

The last social aspect of emergent perspective is classroom mathematical practices. Classroom mathematical practice refers to the normative ways of acting, communicating, symbolizing mathematically that are specific to particular mathematical ideas or concepts. Students interpretation or ideas about percentage, the way they make visual model of percentage, and the way they work with percentage will be analyzed.

2.6 Percentage in the Indonesian Curriculum for 5th grade

In Indonesia National Curriculum, percentage is taught in 5th grade of elementary school. Before students learn about percentage, they have already learned about meaning of fraction and addition and subtraction of fraction in the 3rd and 4th grade. The following table describes learning fraction, decimal, and percentage for 5th graders in the second semester in Indonesian National Curriculum.

Table 2.2 Learning fraction, decimal, and percentage for 5th graders in Indonesian curriculum

Standard Competence	Basic Competence
Number	
5. Using fraction in solving problems	5.1 Converting fraction to **percentage** and decimal form and vice versa 5.2 Adding and Subtracting many forms of fraction 5.3 Multiplying and dividing many forms of fraction 5.4 Using fraction in solving proportion and ratio problems .

Looking at the Indonesia National Curriculum, percentage is taught in close relation to fraction and decimal. Based on observation, the learning of percentage sometimes goes very fast to the formal way and it is taught as another representation of fractional notation. Considering this fact and the need to support students to extend their understanding of percentage, this research focused on the learning of percentage in the 5th grade by emphasizing the understanding of percentage.

2.7 Hypothetical Learning Trajectory

Bakker (2004) said that a design and research instrument that proved usefull during all phases of design research is the so-called 'hypothettical learning trajectory'. A hypothetical learning trajectory (HLT) is the link between an instruction theory and a concrete teaching experiment. Simon (1995, in Bakker 2004) defined HLT as follows:

> The hypothetical learning trajectory is made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process—a prediction of how the students' thinking and understanding will evolve in the context of the learning activities.

Simon (1995, in Simon and Tzur, 2004) described that an HLT consists of the goal for the students' learning, the mathematical task that will be used to promote student learning, and hypotheses about the process of the students' learning. Whereas the teacher's goal for student learning provides a direction for the other components, the selection of learning tasks and the hypotheses about the process of student learning are interdependent. Therefore, an HLT informs researcher and teacher how to carry out a particular teaching experiment (Bakker, 2004). During the preliminary and teaching experiment, HLT was used as a guideline for conducting teaching practice. HLT was also used in the retrospective analysis as guideline and point of reference in answering the research question.

2.8 Hypothetical Learning Trajectory of the Preliminary Researches

2.8.1 Hypothetical Learning Trajectory 1

Implementing many ideas elaborated in the theory above, this research used many daily life contexts such as discount, computer process, and some other contexts in which percentage plays an important role. As the starting point, this research designed an activity to assess informal knowledge of students on percentage. In Realistic Mathematics Education where teaching is built on the informal knowledge of the students, the teaching of percentage could start with assessing what the students already know about percentage (Van den Heuvel-Panhuizen, 1994). After that, the context about computer process, estimating the percentage of loading process, was chosen to construct the meaning of percentage. To give students opportunity to explore the meaning of percentage, some contexts related to part whole relationship were elaborated.

Those kinds of activities were elaborated in a set of instructional activities. This set of instructional activities is divided into six different main activities and it is accomplished in seven days. The instructional activities are presented below:

Table 2.3 The instructional activities in HLT 1

No	Learning Goals	Mathematical ideas	Activities
1	Students are able to recall their informal knowledge of percentage	-	Making poster about the use of percentage in daily life
2	Students are able to construct the meaning of percent as ”so many out of 100”	- Percent means “so many out of 100”. - Percentage describes part whole relationship.	- Estimating percentage of loading process - Shading loading bar if the percentage of loading process is known - Estimating area
3	Students are able to work with benchmark percentages	- Percent means “so many out of 100”. - Percentage describes part whole relationship.	- Estimating area problem - Estimating the number of students
4	Students are able to construct sense of percentage as relative value	- Percentage describes part whole relationship; it describes the relative value of the part compared to the whole. - Percentages are always related to something.	- Investigating two different percentages of discounts.
5	Students are able to use percentage in comparison problem	- Percentage describes relative amount by which one set is compared to another. - Percentages indicate the proportion of a specific total that is set to 100. - Percentage can be used to standardize	Ordering the sweetness of drinks

No	Learning Goals	Mathematical ideas	Activities
		different quantities so they can be compared directly.	
6	Students are able to extend their knowledge to percentage greater than 100	Percentages are not always less than 100, but it can be greater than 100.	- Drawing a chocolate bar with 50% extra free - Investigating the increasing weight of chocolate

The instructional activities for learning percentage that were embedded in the hypothetical learning trajectory will be described in detail as follows:

2.8.1.1 Exploring informal knowledge of percentage

Goal:

- Students are able to show their informal knowledge of percentage.
- Students know some situations in which percentage play role.
- Students realize that percentage is widely used in our daily life.

Activity: Making poster of percentage

Description of the activity:

Before the first lesson, teacher gives homework to students to collect all things around them that show the use of percentage in their daily life. They can collect some advertisements or pictures from magazine or newspaper, or table ingredients that involve percentage, or other information involving percentage. In this first meeting, they have to make a poster about the use of percentage in a group of four or five students. After making the poster, all groups have to show the poster in the classroom and they have to tell what the poster is going about.

Conjecture of students' informal knowledge:

- Students show the use of percentage in discount price. I think most of the students will show the picture about percentage of discount because it is one of the familiar contexts in daily life.
- Students show the percentage of sugar or any substance in ingredient table.
- Students show information involving percent from magazine or newspaper, for example report of vote for candidate in an election.

Class discussion:

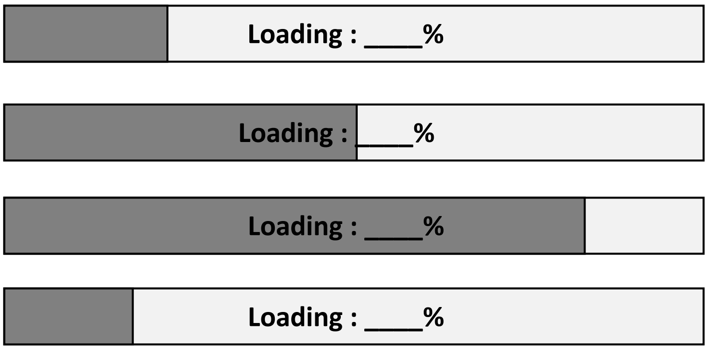
When students tell about the poster, teacher can pose question to determine what they already know about percentage. For example if student make a poster that involve the use of percentage in the discount of clothes in supermarket, teacher can pose such questions "Do you feel happy if there is discount in the price? Why?" or "How if the discount is 90% or only 5%?" to know whether they realizes that percentage close to 100 means almost everything and percentage close to 0 means almost nothing. In the class discussion, teacher also can ask students whether they know the meaning of the symbol "%".Having discussion about the poster of percentage can probe students' informal knowledge of percentage.

2.8.1.2 Constructing the meaning of percent

Goal: Students are able to construct the meaning of percent as "so many out of 100".

Activity: Estimating loading process and shading loading bar

a. Students are asked to estimate how much the percentage of the loading process in each loading bar below:



How much is the percentage of the loading bar in each picture? How do you come to your answer?

b. Students are given three percentages of loading process (10%, 2%, 60%, and 98%), and students are asked to shade part of loading bar that is fully loaded. Here students also have to explain their strategy in drawing the loading process.

Description of the activity:

In this activity teacher asks students about their experience in playing game in computer or play station or in working with computer. Then teacher asks students whether they have seen loading bar in those kind of activities. To make clear about how loading bar works, teacher will show loading bar in the computer and display how it works. There are two different activities in this section.

First Activity

In the first activity, first teacher will display the loading bar and then pauses the process of the loading in a certain moment. While the teacher displays the loading process, he/she hides the percentage. Here, teacher starts to have discussion with the whole class in estimating the percentage of the loading process. In order to give more opportunity for students in estimating the percentage of the loading process, students are given a worksheet contain some pictures of loading process and they have to estimate the percentage.

Second Activity

In the second activity, teacher only gives the percentage of the loading process (10%, 2%, 60%, and 98%), and then he/she asks students to shade the part of the loading bar that is fully loaded. In this activity, students are given a paper containing blank bars so that they can work in the paper. The blank bars given are presented in big size so that students can shade the part of loading bar that is fully loaded as precise as possible.

In these activities, students work in group of four or five students.

Conjecture of students' strategies:

<u>First activity:</u>

- Students might estimate the loading process by using their intuition. For example they might estimate that the percentage of loading process from the first loading process is around 25%. For students who give this answer, teacher can provoke students by giving such question *"How can you make your friends sure that the percentage of the loading process is 20%?"*.
- Students might measure the length of the bar and think of the order of number 1 up to 100 and their position in the loading bar.

- Students might use their known percentage to estimate the percentage of the loading process. In the first loading process, they might recognize that the percentage of the part that is fully loaded is a half of 50%.

<u>Second activity:</u>

- Students make guessing in drawing the loading process.
- Students think the order of the number 1 up to 100 and their position in the loading bar.
- Students measure the picture of the blank bar and then divide it into 100 parts.
- Students might use their known percentage to shade the part of loading bar that is fully loaded.

Class Discussion:

Since in the previous activities students have known many situations in which percentage play role, here teacher introduces another use of percentage in computer process, especially in loading process. In the beginning of activity, teacher can asks students about their interpretation if the percentage in the loading process is very small or otherwise very big close to 100%.

In this class discussion, the first discussion is about the way they estimate the percentage of loading process. In estimating the percentage of loading process, students might have different strategies and answer. They might use their intuition or use their known percentage to estimate the percentage of the loading process.

The next discussion is about how they shade the part of loading bar that is fully loaded if the percentage of loading process is given. This activity is the continuation of the previous activity and it is quite abstract for students because in this activity they have to imagine the situation and then draw the loading process. In shading the loading bar, they are expected to see the relationship between the given percentage and the percentage of the whole loading process (100%) and also are expected to see that the loading bar actually consists of 100 parts, each part represents 1%.

By discussing these activities, students also are expected to develop their sense of the relative value of the part (loading process) compared to the whole. In estimating loading process, relative size of the part is viewed as a sense of fullness of loading process along the linear scale, with 0% means the loading process is started and 100% means the loading process is finished.

Next Activity:

Activity: Estimating the area of fishpond

Teacher says the story about the area of his/her yard in his/her village.

Teacher says that in her/his village, he/she has a house with the total area of the yard is 100 m^2. Teacher says that 10 percent of the yard is used as fishpond. Teacher asks students to estimate the area of the fishpond.

The questions are: How much is the area used as fishpond? How do you come to your answer?

Description of the activity:

Teacher tells the story about the area of the yard in her/his house and the percentage of the area used as fishpond. Then teacher asks students to find the area used as fishpond. In this activity, teacher asks students to make a drawing of the situation. In investigating this problem students work individually.

Conjecture of students' strategy:

- Students might draw a representation of the yard and then divide it into ten parts. After dividing, they conclude that one small part that has area 10 m^2 represent the area of the fishpond.
- Students draw a representation of the yard and then divide it into or a hundred parts. After dividing, they conclude that ten small parts that has area 10 m^2 represent the area of the fishpond.
- Students directly divide the area of the yard (100 m^2) by ten or by one hundred without making drawing.

Class Discussion:

In the mathematical congress of estimating loading process students have already estimated the percentage in the loading process and discuss the meaning of percent. This activity is designed to see whether they really know the meaning of percent. In this activity, the discussion focuses on the way students estimate the area of the fishpond. Students might come up with different strategies to solve it. The meaning of percent can be explored by questions such as *"Why do you divide the area of the yard into ten? Why one small part represents 10%?* or *"Why do you divide the area of the yard into a hundred parts? Why ten small parts represent 10%?*.

2.8.1.3 Working with benchmark percentage

Goal: Students are able to construct the meaning of percent as "so many out of 100" and they were able to use their previous knowledge about bar model as a tool to solve percentage problem.

<u>First Activity:</u> Estimating field area

The problem goes as follows:

Mr. Hadi has a field with the total area is 400 m^2. He plans to plant rice in 75% of the field and the rest is used to plant corn. How much is the area used to plant rice? How much is the area used to plant corn? How do you come to your answer?

Description of the activity:

Teacher tells story about a field of Mr. Hadi that is used to plant rice and corn. After telling the story, then teacher asks students to find the area used to plant rice and corn. In this activity, students will work in a group of four or five students.

Conjecture of students' strategy:

Students' strategies in finding the area used to plant rice:

- Students might draw the field in square or rectangle form and then divide it into some parts. They might divide it into four, or ten, or hundred parts.
- Students will split 75% into 50% and 25%. First they determine 50% of 400 m^2 then they continue to determine 25% of 400 m^2.

Students' strategies in finding the area used to plant corn:

- Students will start with the percentage of area used to plant rice. Since 75% of the field has already used to plant rice, then the area used to plant corn is 25% (100% minus 75%). Then they will determine 25% of 400 m^2 by making drawing of the field.
- After finding the area used to plant rice, students will directly find the area used to plant corn by subtracting the total area of field by the area used to plant rice.

Class Discussion:

Since in the previous activity students have already discussed the meaning of percent, in this activity students are given opportunity to solve a percentage problem to see whether they know the meaning of 75% of a certain amount. In this activity, the given problem is about the area of field that is designed in purpose to give students opportunity to make drawing as a representation of the situation and to make connection with their previous activity.

In this activity, the discussion focuses on the way students estimate the area used to plant rice and corn. Students might come up with different strategies to solve it. In this activity, teacher can provoke students to recall their experience in working with bar or drawing in the previous activity.

<u>Second Activity:</u> Estimating the number of students joining music extracurricular

Here is an announcement about music extracurricular in SD Harapan.

"In SD Harapan, the number of students in grade 5 and 6 is 200 students. From 200 students, 60% of the students join music extracurricular in the school."

Here is conversation between two students that discuss about the announcement:

Rina : Hmm….I think there are many students in grade 5 and 6 interest in music extracurricular. What do you think Budi?

Budi : Yeahh….I think so…

Rina : I think 120 students in grade 5 joining the music extracurricular.

Budi : I don't think so… I think only 100 students joining the music extracurricular.

The question from this story is ‘Do you agree with Rina or Budi? If you agree with Rina, how would you explain your argument to Budi? If you agree with Budi, how would you explain your argument to Rina?’.

Description of the activity:

Teacher shows an announcement about the music extracurricular and then tells the conversation between two students about the announcement. After telling the story, teacher asks students’ opinion about the conversation. In this activity, students discuss the problem in pairs.

Conjecture of students’ thinking:

- Students agree with Rina. They might argue that 60% of 200 students are more than 120 students. It is because 100 students are 50% of 200 students.
- Students agree with Rina. They might argue that 60% of 200 students are same with 50% of 200 students plus 10% of 200 students

  In determining 50% of 200, they might come up with many strategies. They will use halving or partitioning 200 into ten or hundred parts. In determining 10% of 200, they might use the result of 50% of 200 and then divide it by 5 so that they get 10% of 200 or they recall their previous knowledge about finding 10% of 100 m^2. They might divide 200 into ten or one hundred parts. After finding 50% of 200 and 10% of 200, they find that 60% of 200 is equal to 180.
- Students do not have idea about the problem. They confuse because they cannot make a drawing as representation of the situation. Here the problem is about the number of students and not about the area that can visualize in square or rectangle form.

Class Discussion:

Since in the previous activity students have already worked with benchmark percentages such as 50% and 25%, here students are given opportunity to solve a percentage problem with other benchmark percentages. In this activity, students are faced in the situation in which they cannot directly make visual representation of the situation to solve the problem. Here, they have to recall their previous knowledge about the meaning of percent and working with benchmark percentage. The given problem is about the number of students joining music extracurricular that is designed in purpose to give students opportunity to work with percentage as relative value of the part compared to the whole in discrete objects.

In this activity, the discussion focuses on students' reasoning on why they agree with Rina or Budi. Students might come up with different reasoning about it. Teacher can provoke students by posing questions such as *"How do you know that 60% of 200 students are equal to 120 students? Could you make drawing of your strategy?"* . Through discussing the problem, students are expected to see that percentage describes part whole relationship. By investigating the problem students are also expected to consider that in determining the percentage of a certain amount, they can use benchmark percentages to help them to estimate the percentage.

2.8.1.4 Constructing sense of percentage as a relative value

Goal: Students are able to recognize that percentages describe relative value of the part compare to the whole and percentages are always related to something (in this problem percentages relates to prices).

Activity: Investigating discount problem

The discount problem is given as follows:

Two department stores, Ramayana and Matahari, offer discount for their products. Ramayana department store offer 30% discount for toys and Matahari department store offer 40% discount for toys. If you want to buy toys, which store do you prefer?

Description of the activity:

Teacher tells about two advertisements that he/she saw in newspaper. After telling the advertisement, students are given the problem above and they have to discuss the problem in a group of four or five students. When discussing the problem, they have to make a poster describing their opinion and their reasoning.

Conjecture of students' strategy:

- Students will prefer Matahari department store to buy toys because the discount is bigger than in Ramayana so that the price will be cheaper.

  For students who argue like this, teacher can give stimulating question such as "How do you know that the price in Matahari department store is cheaper than in Ramayana while one thing that you know is only the discount?”. If students still argue that the bigger the price the cheaper the price, teacher could ask “How do you convince your friend that the price in Matahari department store is cheaper than in Ramayana department store? Can you give an example of one case?”. From these stimulating questions, it is expected that children will consider that percentage are always related to something.

- Some students maybe will recognize that to determine the price in each department store, one has to know the original price.

  For students who have this opinion, teacher can ask such questions “How if the prices are same?” or “How if the prices are not same?”. From the question, it is expected that students will gain more insight that different reference will lead to different result.

Class Discussion:

From the first activity of making poster, teacher recalls many uses of percentage in daily life and discount context assumed as one of the most familiar contexts for students. In this activity students are given a situation in which there are two different discounts offered in two different stores. This discount context is used to have students discuss what these advertisements really tell them.

In the class discussion, maybe almost all students will argue that *Matahari* gives the best price because the amount of reduction is bigger. Here, teacher does not ask for any argument first, but the teacher gives them opportunity to discuss in groups why they think shopping at *Matahari* is better. After students give argument, teacher can provoke their reasoning through asking questions such

"Are you sure? How do you convince your friend that your answer is true?". In the discussion teacher also give opportunity for students to discuss about the possibilities of the original price of the toys.

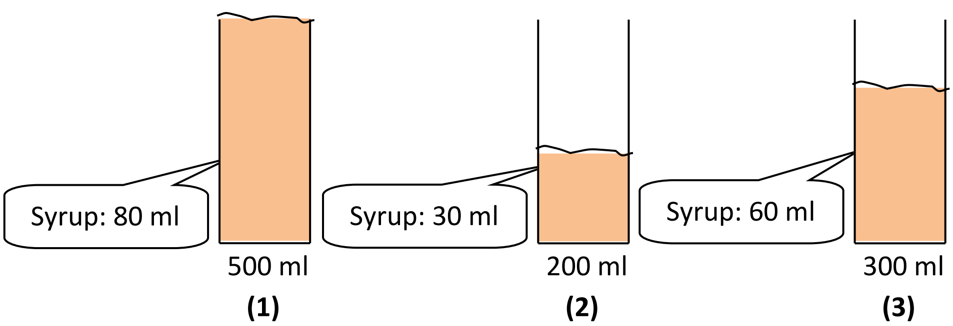
In the case where the original prices are same, it is clear that the price in *Matahari* is cheaper than in *Ramayana*. In the case where the original prices are not same, there are many interesting things to discuss. The main discussion in this case is about the reduced prices (absolute values) and the percentages (relative values) that describe the relationship between the reduced prices with the original prices.

Through this activity, students are expected to realize that percentages describe relative value of the reduced price compare to the original price. Students also expected to explore that percentages are always related to something (prices) and cannot be compared without taking into account to what they refer because reduction in percentage gives relative value, not an absolute value.

2.8.1.5 Using percentage in comparison problem

Goal: Students are able to use percentages in a situation in which percentages are needed; when different parts of different wholes have to be compared.

Activity: Ordering the sweetness of drinks



Can you order those three drinks from the sweetest drink to the least sweet? To make sure that your answer is true, how would you explain your answer to your friends?

Description of the activity:

The activity uses three glasses having different volume of drink. Each drink contains of different amount of syrup. In this activity, teacher asks students to observe those drinks and report their observation data dealing with the volume of drinks and syrup in those glasses. After the children observe those glasses, then teacher asks students to order the sweetness of each drink. In this activity, students work in group of 4 or five students.

Conjecture of students' strategies:

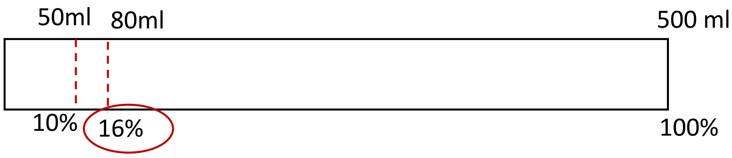
- Some students maybe will order those drinks based on the volume of the syrup without considering the volume of the drink. For students who give this solution, teacher can pose stimulating question *"How about the volume of the drink? Does the volume of the drink not influence the sweetness of those drinks?*. From the question, it is expected that students will consider the volume of the water in determining the sweetness of those drinks.
- Since the question is determining the sweetness of each drink, students may think about the concentration of syrup in each glass. Students might draw the glasses and then estimate the concentration of syrup in each glass by using simple fraction.

  In estimating the concentration of syrup, students will compare the volume of syrup with the volume of drink. Then students might come up with fractions that are difficult to compare. They find that the volume of syrup in the first glass is 4/25 of the volume of the drink, the volume of syrup in the second glass is 3/20 of the volume of the drink, and the volume of syrup in the third glass is 1/5 of the volume of the drink.

For students who come to this answer, teacher can ask question *"Why do you get difficulties in comparing those fractions? Do you have idea how to make those fractions become easy to compare?"*. From the question, it is expected that students will recognize that different denominators make those fractions become difficult to compare so that they have to make it equal. In order to make it become equal, those numbers are easy to compare in base of 100.

- Students might estimate the concentration of syrup in each glass by using percentage. In estimating the percentage, they might use a bar like the bar in the loading process problem. The example of their strategy goes as follows:

The percentage of syrup in the first glass



They might estimate that the volume of syrup in the first glass is 16% of the volume of the drink. By using bar, they also find that that the volume of syrup in the second glass is 15% of the volume of the drink and the volume of syrup in the third glass is 20% of the volume of the drink. For students who give this kind of solution, teacher can pose question "*Why do you prefer percentage in this problem?*". It is expected that students will consider that using percentage make them easy to compare the kind of proportional problem.

Class Discussion:

In this activity, researcher expects the children to invent percentage as the answer to a problem. The researcher place the children in situation in which comparing proportion by using fraction makes things obscure. In this class discussion, the first discussion is about the way students order those three drinks from the sweetest to the least sweet. They might only determine the sweetness of the drinks based on the volume of syrup or they compare the volume of the syrup with the volume of the drink. If students only focus on the volume of syrup, it means that they only look at the absolute amount of syrup without comparing to the volume of water.

Next discussion is about how students order those three drinks from the sweetest to the last one if they compare the volume of syrup with the volume of the drink. Students might come up with different strategies to solve it. By investigating the problem and discussing different strategies used by students, students are expected to see that percentages make them easy to compare proportions. The mathematical idea behind this activity is that percentage describes relative value of the part compared to the whole and it can be used to standardize different quantities so they can be compared directly.

2.8.1.6 Exploring percentage greater than 100

Goal: Students are able to extend their knowledge to percentage greater than 100

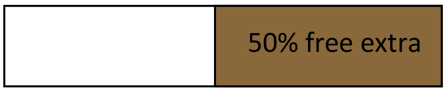
First Activity: Drawing a chocolate bar with 50% extra free

Teacher tells students about the advertisement that says ‘Now available a new product of *Silver Queen* chocolate with 50% extra free from the previous product'. Teacher asks students to draw the new product of *Silver Queen* chocolate.

Description of the activity:

Teacher tells the advertisement and then asks students to draw the chocolate with 50% free extra in their own paper. In this activity they work individually.

Conjecture of students drawing:

- 

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Class Discussion:

In this activity, researcher place students on different problem, not on a 'part-whole' problem that has already done in the previous activities, but in an 'increase-decrease' problem. In drawing the new product of *Silver Queen* chocolate, students might have different drawings. Teacher can use different students' drawings as discussion matter to discuss the '50% extra'. This problem is designed to prepare students to expand their knowledge about percentage greater than 100. This activity is expected can bridge students to expand their knowledge from percentage less than 100 to percentage greater than 100.

Second Activity: Investigating the increasing weight of chocolate

The problem is given as follows:

The old product of *Toblerone* chocolate has weight 200 gram. There is new product of *Toblerone* having weight 150% of the old product. What do you think about the new product of *Toblerone*? Does it increase or decrease? How much the weight of the new product?

Description of the activity:

In discussing the problem, students work in pairs. After they discussed in pair, teacher asks some students to present their answer.

Conjecture of students' thinking:

- Students will think that the weight of the new product of *Toblerone* increases.
- Students will split 150% into 100% and 50%. After they split the percentage, students will recognize that this situation is the same with 50% free extra problem that they have done before. To find the weight of the new product of *Toblerone*, students will determine 50% of 200 gram that is 100 gram and then add this result with the weight of the old product that is 200 gram, so that they get 300 gram as the result.
- Students don't have any idea about it. They might confuse because they cannot make drawing that represent the situation.

Class Discussion:

In the activity, students are given a problem in which percentage greater than 100 plays role. In investigating this problem, students can recall their knowledge about percentage less than 100. Through investigating this activity, students are expected to consider that percentages are not always less than 100 but it can be greater than 100 and they are also expected to consider that they can use their knowledge about percentage less than 100 to investigate percentage problem in which the percentage is greater than 100.

2.8.2 Hypothetical Learning Trajectory 2

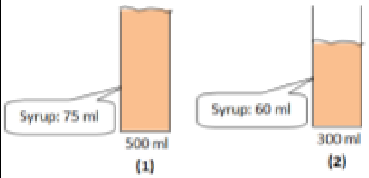
In this section, the researcher presented the hypothetical learning trajectory 2 (HLT2) as the refinement of HLT 1 that is presented in the subchapter 2.6 above. The HLT 1 above was implemented in the first cycle of experiment in SD Laboratorium Unesa. The analysis of the first cycle of experiment was presented in appendix 1. Based on the analysis of the first cycle of experiment, the researcher improved and adjusted some activities in HLT 1. The HLT 2 as the refinement of HLT 1 was also implemented in SD Laboratorium Unesa but in other group. The description of HLT 2 was presented in the table 2.4 below.

Table 2.4 The Outline of HLT 2

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
1	- Students are able to recall their informal knowledge of percentage. - Students know some uses of percentage in daily life	-	Making poster about the uses of percentage in daily life.	-	(The conjectures are the same as the conjectures in HLT1)
2	Students are able to construct *sense* of percentage	Percentage close to 100 means almost 'all' and percentage close to 0 means almost 'nothing'.	- Shading loading bar if the percentage of loading process is known Problem Students were given four loading bars and they had to shade it based on the percentage of loading process (10%, 5%, 60%, and 95%). - Estimating percentage of loading process Problem: Students are asked to estimate how much the	In the second activities, the researcher designed loading bars with various lengths.	- Students just estimate how much the part that is fully loaded - Students might measure the length of the bar by using ruler, and divide the bar into five, ten or twenty parts.

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
			percentage of the loading process in each loading bars.		
3	- Students are able to construct the meaning of percent as ”so many out of 100”. - Students are able to work with benchmark percentages	- Percent means “so many out of 100”. - Percentage describes part whole relationship.	- Investigating area problem Problems: 1. Pak Rahmat has a house with the total area of the yard is $100m^2$. He makes a fishpond in the yard with the area is $15m^2$. Make some drawings of the yard and shade the part of the yard used as the fishpond! How much the percentage of the area of the yard used as the fishpond? 2. Pak Budi has a field with the total area is 200 m^2. Some part of the field used by Pak Budi to plant banana tree. The total area used to plant the banana tress is 150 m^2. Make some drawings of the field and shade the part of the field used to plant banana tree! How much the	In this activities, the researcher provided students with paper grid so that can help them in making drawing, to estimate the percentage, and especially to support them to explore the meaning of percentage as “so many out of 100”.	- Students will draw the yard or the field in rectangle or square form. They might use one square in the grid paper to represent $1m^2$. - In shading the part of the yard or the field, they might just use estimation. - Students might shade 15 squares to represent 15 m^2 or shade 150 squares to represent 150 m^2. - In determining the percentage of the area used as the fishpond or to plant banana tree, students might measure the length of the drawing and divide it into ten or twenty parts, so that they get the area representing 10% or 5%.

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
			percentage of the area part of the field used to plant banana tree?		- Students might compare the number of all squares and the number of the shaded squares. For example: The number of total squares is 100, the number of the shaded squares is 15, then the percentage of area used to as a fishpond is 15/100 and it equals to 15%. - Students might use algorithm to find the percentage.
4	Students are able to construct sense of percentage as relative value	- Percentage gives relative measure, not an absolute measure. - Percentage is an operator so that it always related to something and the operation of multiplication is involved.	- Investigating two different percentages of discounts. Problem: Istana and Sriwijaya shop are two shoes shops that offer different discounts. Istana shop offered 20% discount and Sriwijaya shop offered 25% discount for their product. *'If you want to buy shoes, which shop do you*	In this activity, the researcher adjusted the percentages because the researcher wanted students to use benchmark percentage to solve a problem and not merely use algorithm.	- Students will prefer Sriwijaya shop because the discount is bigger than those in Istana shop so that the price will be cheaper. - Some students might realize that the original prices in both shops might be different. - Students might prefer Istana shop because of

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
			prefer?'.		other considerations such as the quality or the distance of the shop.
5	Students are able to use percentage in comparison problem	- Percentage describes relative amount by which one set is compared to another. - Percentages indicate the proportion of a specific total that is set to 100. - Percentage can be used to standardize different quantities so they can be compared directly.	- Ordering the sweetness of drinks Problem: Students are given two drinks illustrated as follows:  Do you think both drinks have the same sweetness? If not, which drink is the sweeter drink?	Since the findings in the first try out showed that the problem in HLT 1 was complicated for students, then the researcher simplified the problem. Here, the number of glasses is reduced and the number in problem is simplified. These adjustments aims to help student to not constrained with the problem otherwise to help student to cope with mathematical level.	- Some students might determine the sweeter drink based on the volume of the syrup. - Students just guess to determine the sweeter drink. - Students might compare the volume of syrup and the volume of drink by using fraction or percentage
6	Students are able to extend their knowledge to	Percentages are not always less than 100, but it can be greater	- Drawing a chocolate bar with 50% extra free *'The new product Choco*		(The conjectures are the same as the conjectures in HLT1.)

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
	percentage greater than 100.	than 100	*chocolate have 50% extra free from the previous product'* Draw the old and the new product of chocolate Choco! - Investigating the increasing weight of chocolate Problem: The old product of *Fresh* toothpaste has weight 200 gram. The new product of *Fresh* toothpaste is now available with the weight is 150% of the weight of the old product. What do you think about the new product of *Fresh*? Does it increase or decrease? How much the weight of the new product?		

CHAPTER III

RESEARCH METHOD

The issues that will be discussed in this chapter are: (1) design research methodology, (2) research subjects, (3) data collection, and (4) data analysis, reliability, and validity.

3.1 Design Research Methodology

The main goal of this research is to investigate how to support students to extend their understanding of percentage. For this need, this research used a type of research method namely design research for achieving the research goal. Design research is a type of research methods aimed to develop theories about both the process of learning and the means that are designed to support that learning (Gravemeijer & Cobb, 2006).

Gravemeijer & Cobb (2006) define what design research is by discussing the three phases of conducting a design research. Those three phases are preparation and design phase, teaching experiment, and retrospective analysis.

1. Preparation and design phase

This phase proposes to formulate a local instructional theory that can be elaborated and refined while conducting the intended design research. The crucial issue in this phase is that of clarifying its theoretical intent (Gravemeijer & Cobb, 2006). In this phase, the researcher was inspired by studying literature about percentages, realistic mathematics education, and design research. During the preparation and design phase, the researcher established the learning goals for the students, develops a sequence of instructional activities and conjectures of students' strategy and students' thinking. In the end of this phase, the result was a formulation of what is called a hypothetical learning trajectory (HLT).

2. Design experiment

In this phase, the instructional activities are tried, revised, and re (designed) during the experiment (Gravemeijer & Cobb, 2006). In this phase HLT functions as the guidelines for the teacher and researcher. In this research, the experiment were conducted in cyclic processes of trying, revising and re (designing) the instructional activities. The experiment was conducted in three cycles. The first two cycles became the preliminary research of the third cycle of research (that later is called teaching experiment). The first cycle was conducted in three days and the second and third cycles were conducted in three weeks. During the experiment, the researcher adjusted the HLT.

3. Retrospective analysis

In the last phase, the researcher analyzed all data during the experiment. In analyzing the data, HLT is used as guidelines and points of reference in answering research question. In this research, the retrospective analysis was conducted during and after the experiment.

At the heart of the design research there is a cyclic process of (re) designing, testing instructional activities, and analyzing the learning process (Gravemeijer & Cobb, 2006). From this cyclic process emerges a local instructional theory that is still potentially revisable. In this research there were three cycles of experiment, in which the first two cycles became the preliminary experiment of the last cycle (teaching experiment). In this research, the researcher limited the number of the cycles based on whether the data derived from the experiment could answer the research question. Looking back to the research question, the data of experiment could answer the research question if the researcher could gather enough data showing that the students extend their understanding of percentage.

3.2 Research Subjects

The research was done in the fifth grade of SD Laboratorium UNESA and SD BOPKRI III Demangan Baru Yogyakarta, Indonesia. Around twenty five students in each school were involved in the experiment. The students are about 10 to 11 years old and they have learned about percentage in the school. SD Laboratorium Surabaya is one of the schools that have been involved in PMRI *(Pendidikan Matematika Realistik Indonesia)* under the supervision of Surabaya State University and SD BOPKRI III Demangan Baru Yogyakarta is one of the schools that have been involved in PMRI under the supervision of Sanata Dharma University.

3.3 Data Collection

The data collection of this research was described as follows:

1. Video

In this research the video data provides the primary data. The video recorded the activities and discussion in the whole class and in some groups of students, and also recorded the interviews with teacher and some students. Discussion with students during the activities and the class discussion were recorded as data source to observe and investigate students’ reasoning in the learning process. Short discussion with teacher was also recorded to know what the teacher’s opinion and idea about the learning process. The videotaping in this experiment will be recorded by two cameras. One camera was used as a static camera to record the whole activities and the other camera was used as a dynamic camera to record the activities in some groups of students.

2. Written data

The written data was provided as an addition to the video data. In this research, the written data includes student’s work, observation sheet, assessments result, and some notes gathered during the experiment. However, written assessments cannot accurately convey students' immediate responses or physical and verbal expressions. Consequently, observation is important here.

3.4 Data Analysis, Reliability, and Validity

The description of the data analysis and the description about how to ensure the reliability and validity of this research are

3.4.1 Data Analysis

Doorman (2005) states that the main result of a design research is not a design that works, but the reason how, why and to what extent it works. Therefore, the initial instructional design was developed, tested, and analyzed to explain how to support students to extend their understanding of percentage. In this research, the data analysis focused on how students extend their understanding by exploring the activities in the instructional design. The main data that were needed to answer the research question were the videotaping of the students activities and the class discussion following each of the activities. In analyzing the data, especially for the transcript of the video and the students' work, the researcher used codes towards those data. Here, coding was a process for categorizing data and this process was used to help the reader to track the data.

3.4.2 Reliability

Reliability has to do with the quality of measurement. There are two types of reliability – internal and external reliability. Bakker (2004) describes that internal reliability refers to the reasonableness and argumentative power of inferences and assertions. Bakker (2004) also defines that external reliability means that the conclusions of the study should depend on the subjects and conditions, and not on the researcher. In this research, ensuring the reliability was improved by doing two following ways.

a. Data triangulation

Data triangulation relies upon gathering data from multiple sources during the experiment. Considering the importance of data triangulation, the data in this research was gathered by videotaping the learning experiment, collecting students' works, and collecting notes from observation and interview.

b. Cross interpretation

Cross interpretation was conducted by discussing data gathered during the teaching experiment with the supervisors and the colleagues. The cross interpretation was conducted in attempt to minimize the subjectivity of the researcher's point of view.

3.4.3 Validity

Internal validity refers to the quality of the data collection and the soundness of the reasoning that has led to the conclusion. External validity is mostly interpreted as the generalizability of the result (Bakker, 2004). In this research, the validity was kept by doing two following ways:

a. Testing the conjecture

The validity of this research was kept by testing the conjecture during the restrospective analysis.

b. Trackability of the conclusions

The teaching experiment was documented by videotaping the learning experiment, collecting students' works, and collecting notes from observation and interview. By using this data, the researcher was able to describe detailed information of the reasoning that leads to the conclusion. This information enables the reader to track the reasoning that underpins the conclusion.

CHAPTER IV

THE IMPROVED HYPOTHETICAL LEARNING TRAJECTORY

In this chapter, the researcher described the hypothetical learning trajectory for the teaching experiment. In designing this HLT, the researcher implemented the ideas elaborated in the theoretical framework and analysed the preliminary researches. In this research, the preliminary researches included the first and the second experiment in SD Laboratorium Unesa. Since the HLT used in the teaching experiment was derived from the refinement of HLT 1 and HLT 2, then this improved HLT was called HLT 3. The set of instructional activities in HLT 3 is divided into five different main activities and it is accomplished in six meetings. The instructional activities in HLT 3 are presented in the table below:

Table 4.1 The instructional activities in HLT 3

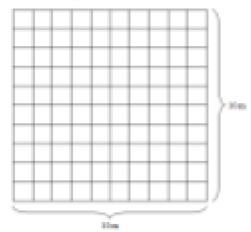
No	Learning Goals	Mathematical ideas	Activities
1	Students have sense about percentage	- Percentage close to 100 means almost ‘all’ while percentage close to 0 means almost ‘nothing’.	- Estimating percentage of loading process - Shading loading bar if the percentage of loading process is known
2	Students are able to construct the meaning of percent as ”so many out of 100”	- Percent means “so many out of 100”. - Percentage describes part whole relationship.	- Estimating area problem - Estimating the number of discrete objects
3	Students are able to construct sense of percentage as relative value	- Percentage describes part whole relationship; it describes the relative value of the part compared to the whole.	- Investigating two different percentages of discount.

No	Learning Goals	Mathematical ideas	Activities
		- Percentages are always related to something.	
4	Students are able to use percentage in comparison problem	- Percentage describes relative amount by which one set is compared to another. - Percentages indicate the proportion of a specific total that is set to 100. - Percentage can be used to standardize different quantities so they can be compared directly.	Comparing two drinks and determining the sweeter drink
5	Students are able to extend their knowledge to percentage greater than 100	Percentages are not always less than 100, but it can be greater than 100.	- Drawing a chocolate bar with 50% extra free - Investigating the increasing weight of chocolate

The instructional activities that were embedded in HLT 3 will be described in the following Table 4.2.

Table 4.2 The outline of HLT 3

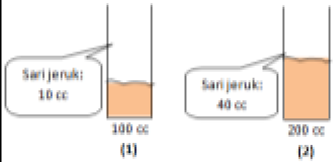
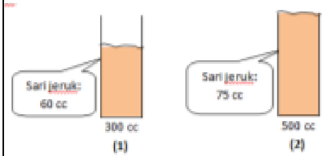
No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
1	Students are able to construct the *sense* of percentage	Percentage close to 100 means almost 'all' and percentage close to 0 means almost 'nothing'.	- Estimating percentage of loading process Students are asked to estimate how much the percentage of the loading process in each loading bars. - Shading loading bar if the percentage of loading process is known Students were given four loading bars and they had to shade it based on the percentage of loading process (95%, 20%, and 11%).	In the first activities, the researcher designed loading bars with different lengths.	- Students just estimate how much the part that is fully loaded - Students might measure the length of the bar by using ruler, and divide the bar into five, ten or twenty parts.
2	- Students are able to construct the meaning of percent as "so many out of 100". - Students are able to work	- Percent means "so many out of 100". - Percentage describes part whole relationship.	- Estimating area problem (by using grid paper) 1. Pak Rahmat wants to build a house in the area of 100 m^2. 10% of the area will be used as fishpond, 75% will be used as the building of the house, and the rest	In the first session of investigating area problem, the students will be given with paper grid that have size 1 cm x 1 cm to support them to explore the meaning of	Estimating area problem (by using grid paper) First problem: - Students might assume that one square represent 1%. - Students might assume that one column or one row represent 10%.

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
	with benchmark percentages		15% will be used as the yard. Below the sketch of the area  a. Coloured the part of the area used as fishpond, building, and yard! b. How much the area used as fishpond, building, and yard? 2. In Kaliurang, Pak Rahmat has a field with the area is 200 m^2. Some parts of the field used to plant chilli. The area of the field used to plant chilli is 150 m^2. Make the drawing of the field and shade the part used to plant chilli. How	percentage as “so many out of 100”. In the next session, the students will not be given grid paper anymore. This aims to give students opportunity to progress. In investigating the percentage of discrete objects, the students will be given a number of pieces of paper to represent candies or chocolates. These pieces of paper will be given to the students who need support to figure out the situation. These pieces of paper also aim to bridge this activity (in which they work with	Second problem: - Students will draw the field in rectangle form. - In shading the field used to plant chilli, they might just use estimation, they might count the squares, or they might measure by using ruler. - In determining the percentage of the area used to plant chilli, students might assume that two small squares represent 1%, so that 150 squares equals to 75%. - Students might measure the length of the drawing and divide it into ten or twenty parts, so that they use benchmark percentages such as 10% or 5%. - Students might compare the number of all squares and the number of the shaded squares. For

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
			much the percentage of the area used to plant chilli? - Estimating area problem (without using grid paper) Problems: 1. Pak Tono has a field with the area is 100 m^2. From the total area, 60 m^2 of the field is used to plant tomato. Draw the sketch of the field and shade the part of the field used to plant tomato! How much the percentage of the area of the field used to plant tomato? Explain your answer! 2. Pak Budi has a field with the area is 200 m^2. From the total area, 180 m^2 is used to plant corn. Draw the sketch of the field and shade the part of the field used to plant corn! How much the	discrete objects) with the previous activity (in which they work with grid paper in investigating continuous objects).	example: The number of total squares is 200, the number of the shaded squares is 150, then the percentage of area used to plant chilli is 150/200 and it equals to 75/100 or 75%. - Students might use algorithm to find the percentage. Estimating area problem (without using grid paper) - Students will draw the field in rectangle form. They might divide the drawing into many small squares (make it similar to grid paper). - They might draw a rectangle and divide it into ten or twenty parts. - In shading the field used to plant tomato or corn, they might just use estimation or they might

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
			percentage of the area of the field used to plant corn? Explain your answer! - Investigating the percentage of discrete objects Problems: 1. Bu Rini sells candies. She sells grape and strawberry candies. Bu Rini has 120 candies that will be sold. 10% of the candies are grape candies. How much grape candies that Bu Rini has? Explain your answer! 2. Bu Sita sells chocolate. From 60 chocolates that will be sold, only 12 chocolates that are unsold. How much the percentage of the chocolate that is unsold? Explain your answer!		measure by using ruler. - In determining the percentage of the area used to plant tomato or corn, students might determine 1% of the drawing or they might measure the length of the drawing and divide it into ten or twenty parts, so that they use benchmark percentage such as 10% or 5%. - Students might compare the number of all squares and the number of the shaded squares. - Students might use algorithm to find the percentage. Investigating the percentage of discrete objects - Students might use the pieces of paper to represent the candies or

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
					chocolates. - Students might group the pieces of paper into ten or twenty groups, so that they get 10% or 5% of the amount. - Students might use algorithm to solve the problem.
3	Students are able to construct sense of percentage as relative value	- Percentage gives relative measure, not an absolute measure. - Percentage is an operator so that it always related to something and the operation of multiplication is involved.	- Investigating two different percentages of discounts. Problem: Istana and Sriwijaya shop are two shoes shops that offer different discounts. Istana shop offered 20% discount and Sriwijaya shop offered 25% discount for their product. *'If you want to buy shoes, which shop do you prefer?'*.	In HLT 2, this activity only focussed on class discussion. The students just discussed their idea among friends and with the teacher, so that not all students have opportunity to share their own idea. In HLT 3, the students will be provided with a worksheet, so that each student will have opportunity to share their own idea.	- Students will prefer Sriwijaya shop because the discount is bigger than those in Istana shop so that the price will be cheaper. - Some students might realize that the original prices in both shops might be different. - Students might prefer Istana shop because of other considerations such as the quality or the distance of the shop.

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
				After they share their idea in a given worksheet, then they will be engaged in a class discussion.	
4	Students are able to use percentage in comparison problem	- Percentage describes relative amount by which one set is compared to another. - Percentages indicate the proportion of a specific total that is set to 100. - Percentage can be used to standardize different quantities so they can be compared directly.	- Ordering the sweetness of drinks Problem: 1. Students are given two drinks illustrated as follows:  Do you think both drinks have the same sweetness? If not, which drink is the sweeter drink? 2.  	In this activity, there were two similar problems that have to be solved. The first problem is simpler than the second problem. The first problem involves simple proportions that aims to help students to not constrained with the problem otherwise to help student to cope with mathematical level.	- Students might use fractions to determine the concentration of extract orange and then compare those fractions by using decimal. - (The other conjectures are the same as the conjectures in HLT1)

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
			Do you think both drinks have the same sweetness? If not, which drink is the sweeter drink?		
5	Students are able to extend their knowledge to percentage greater than 100.	Percentages are not always less than 100, but it can be greater than 100	- Drawing a chocolate bar with 50% extra free Problem: *'The new product Choco chocolate have 50% extra free from the previous product'* Draw the old and the new product of chocolate Choco! If the weight of the old product is 100 gram, how much the weight of the new product? Explain your answer! - Investigating the increasing weight of chocolate Problem: 1. The old product of *Fresh* toothpaste has weight 200 gram. The new product of *Fresh* toothpaste is now	In this activity, we design one extra problem about the expanded field. This problem aims to support student to make progress and to support them to transfer to another context.	- Students might use bar model to calculate 150% of 200 gram or 135% of 300 m^2. - (The other conjectures are the same as the conjectures in HLT1)

No	Learning Goals	Mathematical Ideas	Activities	Explanation	Conjectures
			available with the weight is 150% of the weight of the old product. What do you think about the new product of *Toblerone*? Does it increase or decrease? How much the weight of the new product? 2. Long ago, Pak Budi has a field with having area 300 m^2. Now, Pak Budi has expanded his field, and the area of the recent field is 135% of the area of the previous field. How much the area of the recent field? Explain your answer!		

CHAPTER V

RETROSPECTIVE ANALYSIS

In this chapter, the researcher presented the retrospective analysis from the teaching experiment. In this teaching experiment, HLT 3 as the refinement of HLT 2 was implemented in fifth grade of SD BOPKRI III Demangan Baru Yogyakarta. Twenty five students of fifth grade were involved in this teaching experiment. The characteristic of the students in this teaching experiment was somewhat different from the characteristic of the students in the preliminary experiment. The students were not really active in the class discussion. Most of the students were not brave to share their opinion or idea. In this teaching experiment, the mathematics teacher of the fifth grade became the teacher of the experiment.

Before describing the retrospective analysis of the teaching experiment, the researcher would describe the coding process done during the analysis. At first, the researcher defined the initial name of the data by using a letter. For the figures of the students' work, the researcher used initial 'F' as the code. Besides the codes for figures, the researcher used initial 'T' for teacher, initial 'R' for researcher, initial 'S' for students, and initial 'C1, C2, …, C10' for ten selected students. After that, the researcher used numbers towards the letters to indicate in which phase of the learning process the data is derived and to indicate the order of the data. As the example, the researcher used the code 'S-2-001' to indicate that the data is decribed what students said in the second phase and in the first order.

5.1 Pre test

This pre test was held on March 9th, 2011. The students have learned about percentage in the school. Since the students have learned percentage, then the pre test aimed to see the starting point of the students especially their prior understanding of percentage. It involved investigating students' sense of *fullness* of percentage, the meaning of percent for the students, and the strategy they had to solve percentage problems.

5.1.1 Exploring students' sense of fullness of percentage

The first problem: The first problem aimed to see whether the students have sense about percentage; whether they know that percentage close to 100 means almost 'all' and percentage close to 0 means almost 'nothing'. The problem is about shading the part of a drawing of a field. The Indonesian context about rice field is chosen. The story is about the field of Pak Budi. Pak Budi has a patch of field.

90% of the field is used to plant rice. The figure below is the figure of the field of Pak Budi.

The figure of the field of Pak Budi

The students were asked to shade the part of the field used to plant rice. There were twelve students who were able to shade 90% of the field properly. These students partitioned the field into some parts. They divided the field into ten, twenty or a hundred parts and shaded one or two or ninety parts of it. The other students could not give the correct drawing. Most of the students who could not give the correct drawing used estimation strategy. They shade a third or a quarter or a half or three quarter or the whole field to represent 90% of the field.

Analysis of the first problem: Looking back to the aim of the problem, we could make conclusion that less than a half of the students could shade 90% of the field properly. There were many students who struggled in estimating 90% of the field. It seemed that the students did not perceive that 90% means almost the whole area.

5.1.2 Exploring students' prior knowledge of the meaning of percent

The second problem: The second problem aims to assess whether the students are able to make an assumption in solving a problem, and whether they could determine 10% of a certain amount. The problem is about making a drawing to express the situation *'10 percent of flowers are red'*.

In making the drawing of the situation above, the students drew a number of flowers and shaded some flowers representing red flowers. Only ten students could make the drawing of the situation correctly. These students tended to choose ten, twenty, or fifty as the total number of flowers so that they could determine 10% of that amount. For students who were not able to come to the correct answer, they shaded 10 flowers from the total amount of flowers that they made.

Analysis of the second problem: Back to the aims of this problem, we can conclude that the students were able to make an assumption about the reference of the percentage in solving an open problem. However, only ten out of twenty five students were able to solve the problem correctly. There were many students who were struggling to figure out the situation. For students who shaded 10 flowers from whatever the total amount, it seemed that they treated 10% as the whole number. It indicated that they did not understand the meaning of 10%. This finding showed that the students need a support especially in reconstructing the meaning of percent since they have learned about it before.

The third problem: This problem aims to assess whether the students are able to determine the amount of the part if the percentage and the amount of the whole are given. The familiar context of a sale was chosen. In the end of the year, *Gemilang* book store have a sale. The store offers 25% discount for all books. Santi wants to buy book in the store with the original price is Rp 30.000,00. The students are asked to determine how much money Santi should pay.

In solving the problem, most of the students used algorithm. They transformed 25% into fractional form 25/100 and then multiplied 25/100 with 30.000. Only one student did not use the algorithm; he directly answered the question. It could be either he used estimation or he just guessed the answer. From twenty five students, only seven students could give the correct answer. There were seven students who only answered the reduction price but did not come with the amount of money that Santi should pay. It could be either the students forgot to continue their work or they did not understand the task. The other students could not give the correct answer although they know that they have to multiply 25/100 with 30.000.

Analysis of the third problem: From this result, we could conclude that most of the students could interpret the problem into mathematical language and they could transform percent into fractional notation. It could be either the students were familiar with this kind of problem or they just multiplied both numbers presented in the problem. The result showed that almost three fourth of the students were struggling in solving this problem. Although they were able to interpret the problem into mathematical language, they could not arrive to the correct answer. It might happen because the students tended to do the algorithmic computation and got difficulties in multiplying fraction with a big whole number (multiplying 25/100 with 30.000). Some of the students also did not arrive to the expected answer because they only determined the reduction price and not the final price. The correct algorithm did not success in leading the student to come to the expected conclusion because they might only see the numbers given and put it in the algorithm and they might not pay attention to the context of this problem. This finding indicated that the students tended to use procedural algorithm that probably did not make sense for them instead of using their common sense in solving the problem.

The fourth problem: This problem aims to assess whether students are able to find the percentage of a proportion. The problem is about the proportion of students joining music extracurricular in SD Harapan. The students were given information that the number of students in SD Harapan is 200 students and 40 students join music extracurricular in the school. The students are asked to find the percentage of the students in SD Harapan joining music extracurricular.

Most of the students were not able to solve the problem. Only four students could answer the problem correctly. The students who could correctly answer the problem used division strategy or used benchmark percentage to solve the problem. The students who were not able to solve the problem had different strategies. Six students subtracted 200 by 40, six students divided 200 by 40, two students multiplied 40/100 by 200, four students directly stated the percentage, one student divided 40 by 100, and one student did not give answer.

Some of the students' work will be presented in the following part.

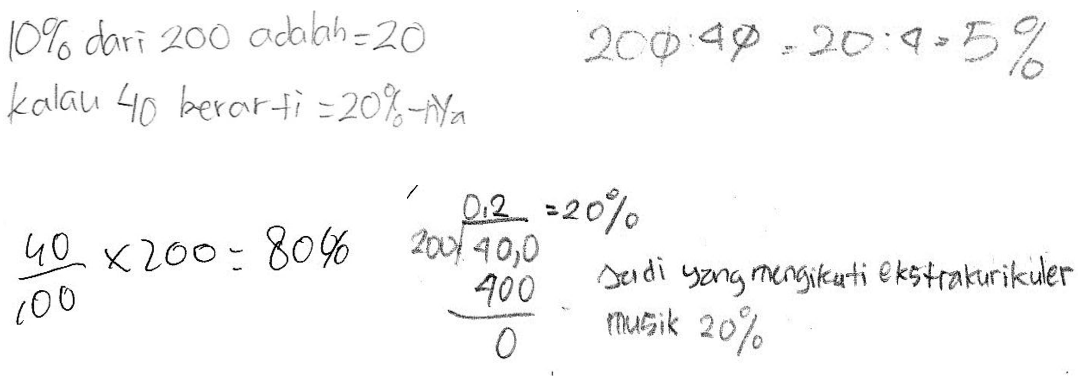
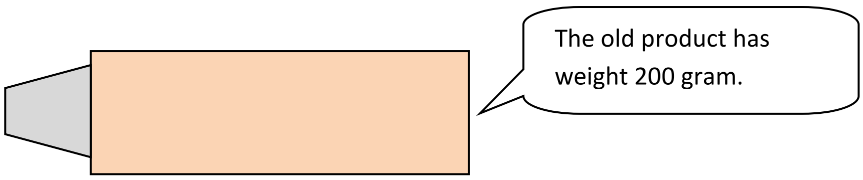


Figure 5.1 Some of students' works in determining percentage

Analysis of the fourth problem: Looking back to the aim of this problem, we could make some conclusions that only few students who were able to find the percentage from a given proportion. Most of the students were not able to find the percentage from a proportion. For students who were able to find the percentage, one student seemed to have an idea about benchmark percentage, and three students seemed to understand the relation between decimal and percentage. The other students got confused in solving this problem. From the example of students' work, there is one case that showed how the students seemed to use the algorithm in computing the part of a whole while the percentage is given in this situation. The student seemed to only focus on both numbers and do some computations with those numbers. Here, we saw that most of the students tended to use procedural computation that did not make sense for them instead of using their common sense.

5.1.3 Exploring students' acquisition in working with percentage greater than a hundred

The fifth problem: This problem aims to assess whether students are able to solve problem involving percentage greater than 100. The problem is going about a toothpaste *Prodent* that has weight 125% of the weight of the old product. The old product of toothpaste *Prodent* has weight 200 gram. The students are asked to draw the new product of *Prodent* and to determine the weight of the new product. Here, the students were given the figure of the old product of toothpaste *Prodent*.



The result showed that most of the student got confused to solve this problem. Only five students were able to give the correct answer by using algorithm. However, only four of them were able to give the correct drawing. The other students failed to arrive to the correct answer. In this problem, most of the students could not draw the new product of toothpaste *Fresh* properly. In drawing the new product of toothpaste *Fresh*, thirteen students drew it in bigger size (but not in a proper way), three students drew it in smaller size, seven students drew in the same size, and the other students did not make the drawing.

Analysis of the fifth problem: Back to the aim of this problem, we could conclude that most of the students got confused in solving percentage problem involving percentage greater than 100. Almost a half of the students did not have sense that percentage greater than 100 indicates that something is increase. It might be that either they have not learned about percentage greater than 100 in the school or they have not enough knowledge about percentage greater than 100.

5.1.4 Exploring students' acquisition in using their prior knowledge to solve a problem

The sixth problem: Since, the students had learned percentage, this problem aims to assess whether students are able to use percentage in solving proportional comparison problem. The problem is about ordering the sweetness of drinks. *Sweet Orange* and *Fruitty* are two kinds of orange drink. Those drinks have the total volumes of drink and volumes of orange juice that are different. The students were asked to determine which drink is the sweeter drink.

Drink	**Total Volume of Drink**	**Volume of extract orange**
Sweet Orange	500 ml	150 ml
Fruitty	400 ml	140 ml

In solving the problem, almost all students answered that *Sweet Orange* is the sweeter drink. Those students had various strategies to solve this problem. Four students divided the total volume of drink by the volume of extract orange. Two students added the total volume of drink and the volume of extract orange. Five students only looked at the volume of extract orange. Seven students only looked at the total volume of drinks, and three students did not give reason. Besides those strategies, there was one student who answered by using percentage. However, he could not give the correct reasoning. He directly answered that the concentration of extract orange in *Sweet Orange* is 15% and the concentration of extract orange in *Fruitty* is 14%. Besides those answers, there was only one student answered that *Fruitty* is the sweeter drink. The student who answered that *Frutity* is the sweeter drink used percentage to answer the problem. His answered goes as follows

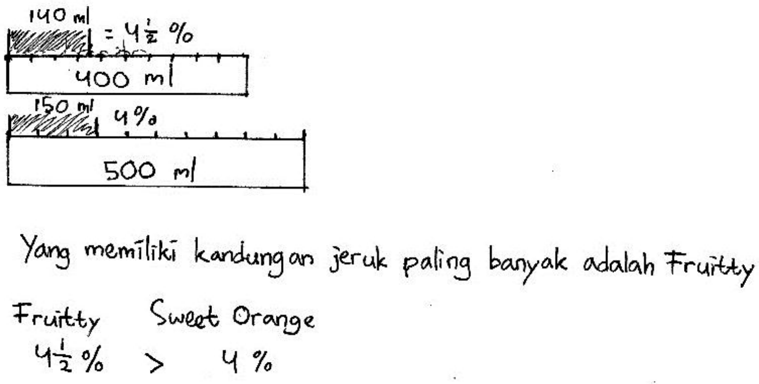


Figure 5.2 Rio’s answer in the pre test in determining the sweeter drink

Analysis of the sixth problem: Looking back to the aim of this problem, we could conclude that most of the students got confused in solving proportional comparison problem. The result showed that most of the students did not realize that the problem is about proportional comparison problem. It could be that either they did not familiar with this kind of problem or they were not able to use their previous knowledge about fraction or percentage to solve this proportional comparison problem. However there were two students that seemed to have idea about determining the concentration of extract orange by comparing the volume of extract orange and the total volume of drink. Those students used percentage to solve the problem, but they could not come to the correct answer. One student who answered that the concentration of extract orange in *Sweet Orange* is 15% seemed to get confused to find percentage from a proportion. It might happen because he did not have fully understanding of percentage. One student who answered that *Fruitty* is the sweeter drink seemed to have an idea to use bar model to solve the problem. However, he failed to come to the correct answer. It could be that either he was not used to using this strategy or he did not do it thoroughly.

5.1.5 Conclusion of the pre test

The pre test of the teaching experiment showed that the students in grade 5 in SD BOPKRI III Demangan Baru Yogyakarta already have had prior knowledge about percentage. They were able to convert percentage into fraction and they knew the algorithm to solve percentage problem although not all students did the algorithm in proper way. However, it does not mean that they understand the concept of percentage. Some students did not have sense of the *fullness* of percentage. Many students struggled to find an amount if the percentage is given or to find the percentage from a proportion. Most of the students also got confused in solving a problem involving percentage greater than 100. Therefore, the researcher could conclude that the students still needed support to help them to acquire the concept of percentage as well as the strategies used to solve percentage problem.

5.2 Retrospective Analysis of the HLT 3

5.2.1 Developing sense of fullness of percentage

At the beginning of the lesson, the application program of loading bar was displayed in the front of the class so that the students could saw and observed how loading process works in a computer program. The application program was designed in interactive way so that the students could be engaged in class discussion. When the application was displayed and when the percentage of loading process was hidden, the students were invited to estimate the percentage of loading process. The students contributed by giving opinion about their guess of the percentage. The activity then continued with estimating the percentage of some drawings of loading process and shading the part of loading bar that is fully loaded. The students worked in group of four to estimate the percentage and to shade the bar of loading process. The classroom mathematical practice that emerged in these activities was recognizing the *fullness* of percentage.

In this activity, the students were engaged in a class discussion to discuss about the characteristic of loading process and about the meaning of the percentage showed in loading process. The researcher showed the application of loading process in a computer, displayed the application of loading process, and invited the students to be aware that percentages close to 0 means almost nothing and percentages close to 100 almost everything.

R-1-001	: Where do you usually look loading (process)?
S-1-001	: in games
R-1-002	: in games...and then?
S-1-002	: in internet
R-1-003	: what else?
S-1-003	: in hand phone
R-1-004	: Are there percentages (in the loading)?

S-1-004 : Yes, there are
R-1-005 : The percentage is started from what number?
S-1-005 : zero
R-1-006 : zero until?
S-1-006 : zero until a hundred
R-1-007 : not until a thousand?
S-1-007 : no
R-1-008 : [Displaying the application program of loading process] There is a loading (process) started from zero. We click 'play' and then it (the percentage) runs until?
S-1-008 : a hundred
R-1-009 : We will check whether it runs until a hundred.
[After a few minutes]Yaa...then a game appears. Since we will not play this game, then we will close this game. We just saw that the percentage runs from zero until? [The application of loading process shows how the percentage of loading process runs from zero until a hundred and after that a game appears]
S-1-009 : a hundred
R-1-010 : We will do it again. We click play and we click pause. What is the number? [The researcher displays again the application of loading process. She pause the loading in 58%.]
S-1-010 : Fifty eight
R-1-011 : Fifty eight. How much approximately is the shaded part?
S-1-011 : a half [some students keep quite and wait their friends to answer the question]
R-1-012 : a half.... Then we click 'play' again. If ninety? What does it mean if we are playing a game?
S-1-012 : The game will start soon [some students keep quite and wait their friends to answer the question]
R-1-013 : Does the shaded part is almost full?
S-1-013 : Yeah
R-1-014 : If the loading is 100%, how about the shaded part?
S-1-014 : full
R-1-015 : What does it mean if we are playing a game?
S-1-015 : The game starts.

The socio and socio mathematical norms held a big influence in this activity. In the discussion, most of the students kept quiet and waited for the other fellow to answer the question (S-1-011; S-1-012). From the conversation above, the researcher saw that the researcher took a big role during the discussion by always guiding the students to answer each question. There was no place for the students to think about different answers and to do negotiation among them to discuss acceptable answer and justification. Based on these findings, the researcher could conclude that the students did not have enough experience in exploring the *fullness* of percentage in loading process. Therefore the researcher did not enough arguments to conclude whether the students have already perceived the sense of *fullness* of percentage.

The activity then continued with estimating the percentages of some drawings of loading process. In estimating the percentage, each student worked in a given worksheet and they could discuss with their friends in group. There were many strategies used by the students to estimate the percentage of loading process such as measuring the length of loading bar, guessing, combining guessing and measuring, and dividing the bar into ten parts.

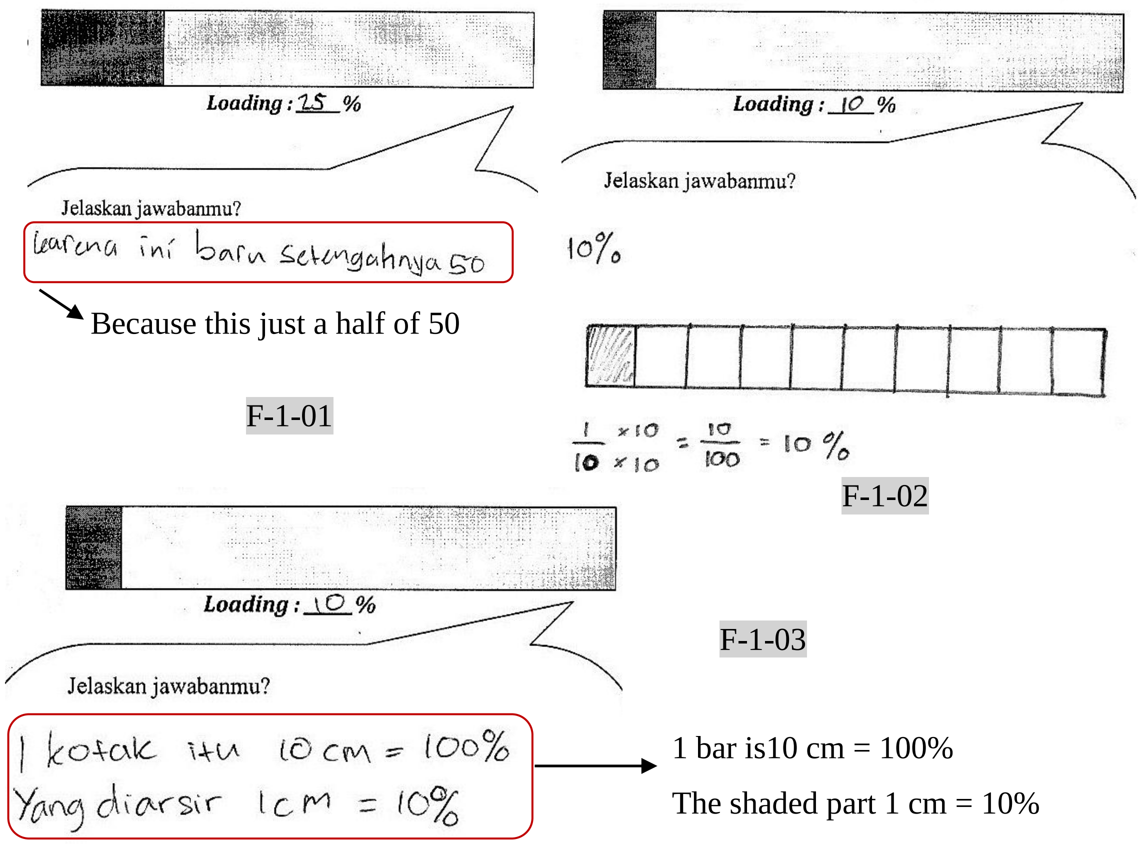


Figure 5.3 Various students' strategies to estimate the percentage of loading process

The Figure 5.3 above showed that the students were able to estimate the percentage of loading process. For the students who only guessed the percentage (F-1-01), there were two conjectures drawn out from this strategy. The first possibility was that the students had strong imagination about how loading process works and the second possibility was that the students did not consider the need of precision in measurement. For the students who divided the bar into ten parts (F-1-02), there was a conjecture drawn out from this strategy. The conjecture was that the students could relate their previous experience in working with shaded area to learn about fraction as part whole relationship. There was also a conjecture drawn out from measuring strategy. For the students who measured the length of the loading bar (F-1-03), the conjecture was that the students consider the need of precision in measurement. Although the previous activity could not show that the students have already perceived the sense of *fullness* of percentage, in this activity the students seemed to have already perceived the sense of *fullness* of percentage.

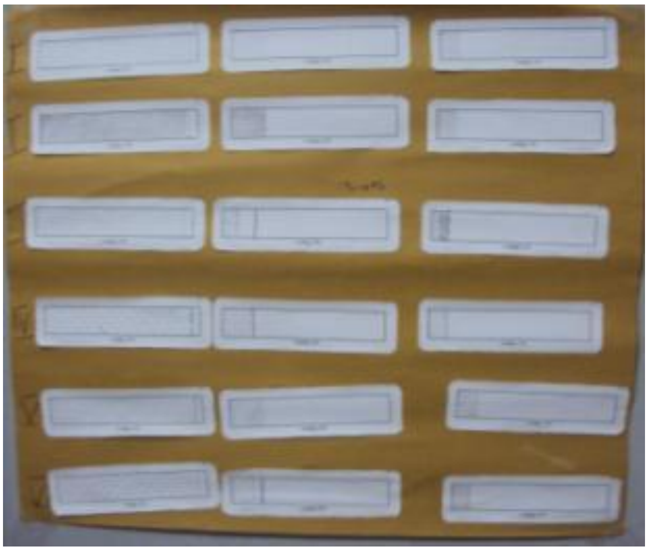
In the end of this activity, a conflict emerged when the students were asked to estimate the percentage of loading bar that has different length; the length of the loading bar is 20 cm and the shaded part has length 5 cm. Around a half of the students were able to solved this problem. Other students could not give the correct answer. Some students answered that since 1 cm represent 10% then the percentage of the loading process is 50%. There were two conjectures drawn out from this answer. The first possibility was that the students did not consider that the length of the bar in this problem was different from the previous problems.

The second possibility was that the students did not completely consider that the percentage of loading process can be represented by different drawings. The following vignette showed how the teacher helped a student who wrote 50% as the answer.

T-1-001	: How much approximately is the length of the shaded part?
C1-1-001	: 5 (cm)
T-1-002	: Ehmm... If the shaded area is as big as this? [Pointing to the middle of the bar]
C1-1-002	: Fifty percent
T-1-003	: [Pointing to his answer] So...Is the shaded part as big as this (the middle of the bar)?
C1-1-003	: **No**
T-1-004	: How much approximately is the shaded area?
C1-1-004	: Ehmm...this fifty [pointing to the middle of the bar]
T-1-005	: How much is the percentage of loading process from this (the beginning of the bar) until this (the end of the bar)?
C1-1-005	: A hundred
T-1-006	: If the loading only end until this [pointing to the middle of the bar]?
C1-1-006	: Fifty percent
T-1-007	: Hmmm...so how much is it (the shaded part)?
C1-1-007	: **Twenty five** (percent)
T-1-008	: Give explanation for your answer on your worksheet
C1-1-008	: (He writes 'loading = 25% because it close to 25%')

From that answer it showed that the student (C1) corrected his previous answer. It seemed that the student could estimate the percentage of loading process although the length is different. At first he got confused because this problem looks similar with the previous problems. It indicated that the students need more experience in estimating percentage of loading bars with various lengths.

The activity is continued with shading the part of loading bar that is fully loaded. The teacher gave the empty bars to the students and the percentages of loading process. In this activity, the students worked in the same group. After finishing their work, all groups stuck their drawing in a poster paper.



F-1-04

Figure 5.4 Students' work in shading the loading bar

From the figure above, it showed that almost all groups could shade the loading bar properly. Most of them used measuring strategy; they divided the length of the bar into ten or twenty parts so that they knew the length of the bar representing 5% or 10%. Here, they started to consider the use of benchmark percentage (10% and 5%).

The following vignette shows how the teacher engaged the students to conclude what they have learned through investigating loading process.

T-1-009	: If you remember on what you have done, if the shaded area goes to the right side, then it close to?
S-1-016	: close to 100
T-1-010	: What does it mean?
C2-1-001	: It will start soon
T-1-011	: How about the shaded part if it will close to 100?
C2-1-003	: Almost full
T-1-012	: Otherwise, if the shaded close to the left side?
S-1-017	: Close to 0
T-1-013	: What does it mean?
S-1-018	: It will be empty
T-1-014	: How about the shaded part?
C3-1-001	: Short
C2-1-004	: Small

T-1-015 : Can you conclude by yourself?
C4-1-001 : The shaded area close to the left side it will close to zero. The shaded part close to the right side it will close to 100.

In the short vignette above, it appeared that the students preferred to say ‘zero’ instead of ‘zero percent’ or ‘a hundred’ instead of ‘a hundred percent’ (S-1-016; S-1-017). It seemed that the symbol percent was not so meaningful for them. They just paid attention to the number of the percentage. However, the visualization of how loading process works was very important in stimulating students to see the relation between the percentage and the shaded area representing part that is fully loaded (C4-1-001).

From investigating loading process and the class discussion, the students can imagine how the percentage runs along the scale as well as the shaded part representing the fully loaded area. Students also commenced to explore the basic concepts of percentage that is the meaning of percentage as part whole relationship and also commenced to acquire the idea of benchmark percentage (F-1-02; F-1-03). From these activities, most of the students seemed to consider the need of precision in measuring (F-1-03; F-1-04). This is a good finding that can be used to construct the meaning of percent in the next discussion. Overall, in this activity most of the students had already perceived the sense of *fullness* of percentage.

5.2.2 Constructing the meaning of percent

From the pre test result, it showed that the students tended to use algorithm to solve percentage problem. It happened because they have learned about percentage before. However, many students often got confused in using the procedure to solve the problem. It indicated that the students did not completely grasp the concept of percentage. The concept of percentage, that one has to grasp in order to have understanding of percentage, is that that percentage indicates part whole relationship in which there were one hundred parts within a whole. The students did not necessarily have to explain in this manner. However, they have to show an awareness of the fact that if one divides a whole into one hundred parts then one small part represents 1% of the whole, or if one divides a whole into ten parts then one small part represents 10% of the whole.

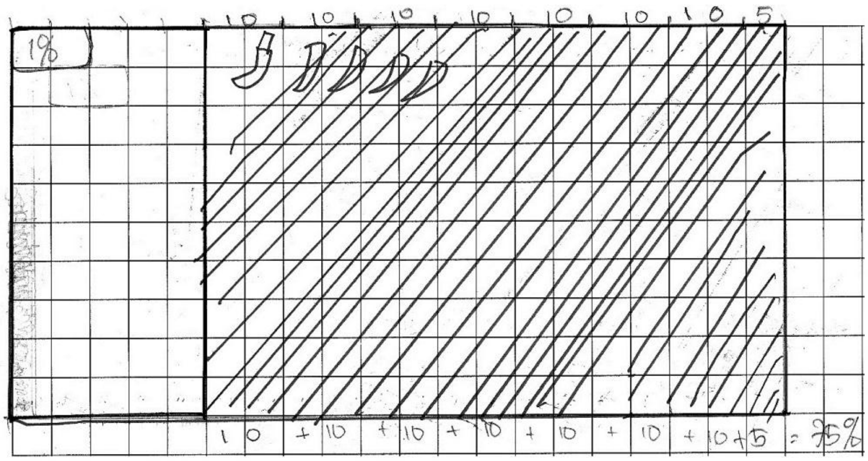
There were three tasks in this phase. First, the students were asked to investigate two area problems by drawing a sketch in a grid paper. In the second activity, they also were asked to investigate two area problems but in this activity they have to make a sketch in a blank paper. Third, the students were given two percentage problems dealing with discrete objects. On the first task, most of the students could perform well. They could explore the idea of percent as 'so many out of 100' and they started to develop bar model. In this activity, they used the grid paper to estimate the percentage. However, the researcher made a highlight of this accomplishment. Since in the first activity the students seemed to rely on the grid paper, we were afraid that the students got difficulties when they have to transfer to other contexts. In the second activity, the students used their previous experience in working with grid paper to estimate the percentage. Here, some students started to use bar model as a tool to solve the problem. In the third activity, most of the students made an arrangement of the pieces of paper to estimate the percentage. Here, most of them made groups of ten.

The first activity: investigating area problems using grid paper

There were two problems posed in this activity. The problems were given as follows:

1. Pak Rahmat wants to build a house in the area of 100 m^2. 10% of the area will be used as fishpond, 75% will be used as the building of the house, and the rest 15% will be used as the yard. Make the sketches of the house and determine the area used as fishpond, the building of the house, and the yard!
2. Pak Rahmat has a field with the area is 200 m^2. Some parts of the field used to plant chilli. The area of the field used to plant chilli is 150 m^2. How much the percentage of the field used to plant chilli!

In solving the first problem, most of the students argued that since there were one hundred squares that represent 100%, then one square is 1%. However in answering the area used as fishpond, the building of the house, or the yard the students tended to use 'square' as the unit area instead of using 'm^2'. It might happen maybe because the squares on the grid paper were more real in their mind rather than using standard unit area. In the second problem, most of the students argued that two squares represent 1%. They came to this idea maybe because they realize that the second sketch is twice bigger than the first sketch. There was an interesting finding in this second problem. There were five students that seemed to develop their own model bar to solve the problem. The figure below is an example of students' work that started to develop bar model.



F-2-01

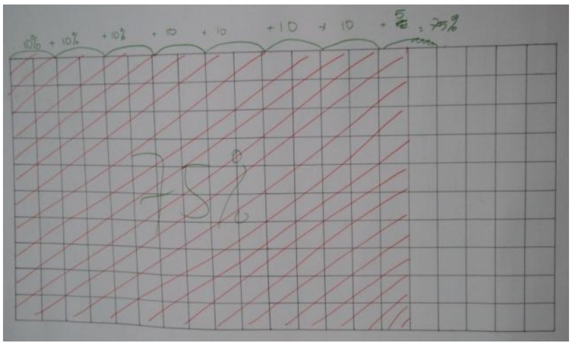
Figure 5.5 Bar model used to estimate percentage of the area used to plant chilli

From the Figure 5.5 above, it showed that the student used their drawing to solve the problem. Since the student reasoned that two squares represent 1% then they argued that two columns (consist of twenty squares) represent 10% although they tended to write '10' instead of '10%'. Since this drawing is the representation of the field and they used the representation to solve the problem, then the researcher might conclude that those students started to develop their own bar model to solve the problem. These findings showed that the students seemed to perceive the meaning of percent as *'so many out of a hundred parts'*.

Since in this activity the students made two sketches; the sketch of house and the sketch of a field, then the teacher prepared two sketches in big size for class discussion so that all students could see it clearly. The sketches were made based on the students' drawing.

F-2-02

The sketch of the first problem



F-2-03

The sketch of the second problem

Based on the drawings above, the teacher engaged the students to observe and compare both sketches. The teacher stimulated the students to perceive the meaning of percent.

T-2-001 : Now you see that this [pointing to the shaded part in F-2-03] is 75% and this [pointing to the red shaded part in F-2-02] is 75%. This [pointing to the red shaded part in F-2-02] is the sketch of the house of Pak Rahmat and this [pointing to the shaded part in F-2-03] is the sketch of his chilli field. What can you conclude from this?

S-2-001 : [the students keep quite]

T-2-002 : This [pointing to the shaded part in F-2-03] is 75% and this [pointing to the red shaded part in F-2-02] is also 75%....but..... Come on...

C5-1-001 : The chilli field is larger because the number of squares is 200. The number of squares in the first problem is 100.

T-2-003 : Ooo..yaa...

C4-1-001 : (Raising his hand)

T-2-004 : Come on Bayu

C4-1-002 : The larger area is the chilli field because the total area is 200 m^2 and that [pointing to F-2-02] the total area is 100 m^2. In the book (worksheet) for the area 100 m^2, one square is one percent or 1 m^2. For the area 200 m^2, two squares are **1m^2** or one percent.

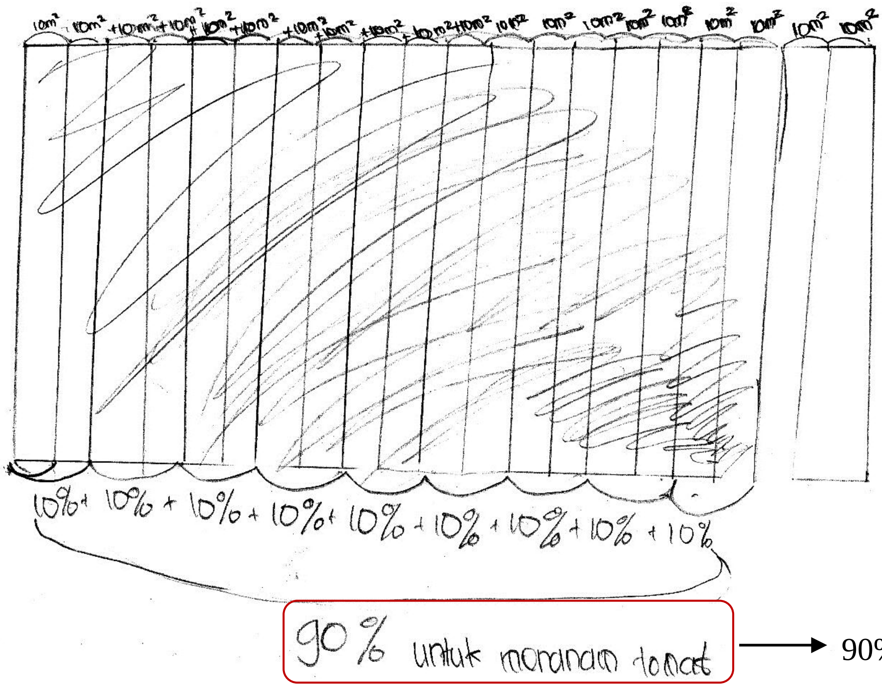
From the reasoning of two students C4 and C5, it seemed that they started to be aware that the same percentage could represent different area because it depends on the total area. The argument of the student in the vignette above showed that the squares in the grid paper served as important tool to help students to estimate percentage (C4-1-002). However, this vignette showed that the students preferred to count the squares rather than the unit measurement of area (C5-1-001; C4-1-002). It might happen because the students were struggling with the concept of area.

The second activity: investigating area problems without grid paper

In the second activity the students were asked to solve the similar task about making the sketch of field used to plant vegetables. Since the students were not provided with grid paper, they had to determine the size of the drawing by themselves in a blank paper. On making drawing of the field, first the students had to know how to make a drawing with a certain area.

From the observation, it showed that the students had different drawings. In drawing the field having area 100 m^2, most of the students drew with size 10 cm x 10 cm and one student drew with size 10 cm x 5 cm. In drawing the field having area 200 m^2, most of the students drew with size 20 cm x 10 cm, one student drew with size 10 cm x 10 cm, and one student drew with size 10 cm x 5 cm. Although they had different sizes of drawing, they were able to make the sketch of the field properly.

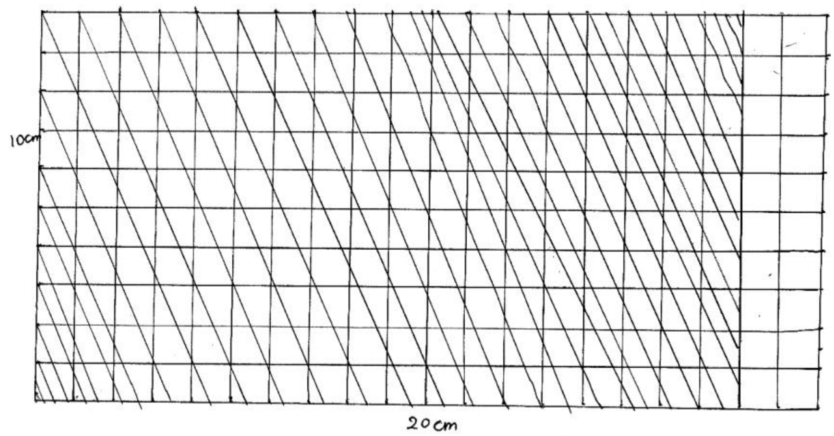
In estimating the percentage of the field used to plant vegetables, the students had various strategies to solve the problems. Those strategies are dividing the drawing in grid pattern, dividing the drawing into ten or twenty parts horizontally, using benchmark percentages such as 1%, 5%, or 10%, and using algorithm. Reflecting this result to the result of the first activity, it showed that the students were able to develop strategies to solve this kind of problem. It seemed that the grid paper in the previous activity was caught well by the students. Below the examples of students' strategies:

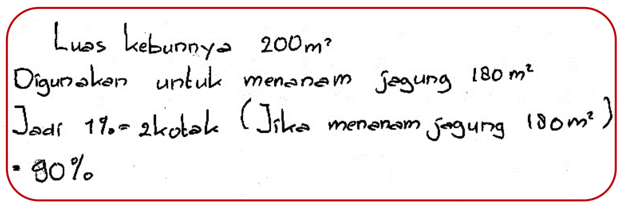


F-2-04

90% to plant tomato

Figure 5.6 One of students' strategies using 10% as the benchmark percentage





The area of the field is 200 m^2

The area used to plant corn is 180 m^2

So 1% = 2 squares (If the area used to plant corn is 180 m^2) = 90%

F-2-05

Figure 5.7 One of students' strategies using grid pattern

From the figures above, it seemed that the students could explore the meaning of percent as *'so many out of a hundred'* (F-2-04; F-2-05). Although they did not say in this manner, they showed an awareness that when they divided a whole into ten parts then one small part represent 10% or when they divided a whole into a hundred parts then one small part represent 1%. The Figure 5.6 showed that the students used bar model as a tool to solve the problem.

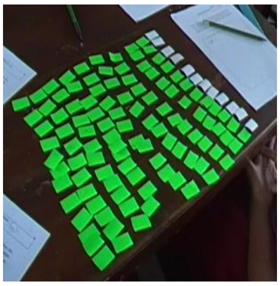
From these findings, it can be concluded that the students were proficient in making a drawing with a certain area (F-2-01; F-2-04; F-2-05). The tool (grid paper) used in the first activity served as good hold for students to explore the meaning of percent and to develop bar model (F-2-01; F-2-04). Therefore, the area context and grid paper could help the students to explore the meaning of percent.

The third activity: investigating percentage problems dealing with discrete object

In this activity, the students were given two options either they want to use some pieces of paper (in the rectangle form) as the representation of the candies or chocolate as the discrete objects in the problems or not. These pieces of paper were given as a tool to support the students on representing the situation and to develop their own bar model. The students were asked to solve two problems. Since both problems were similar, then in this analysis we only focus on the first problem. The students were told that 10% of 120 candies were grape candies, and they were asked to determine the amount of the grape candies.

In solving the problem, most of the students used the pieces of paper as a tool to solve the problem. They arranged the pieces of paper into ten groups.

However, there was one group that did not use the pieces of paper. They directly work with numbers. Those students divided one hundred and twenty with ten so that they got twelve as the answer. The figure below is an example of the students' works that used the pieces of paper to solve the problem.



F-2-06

Figure 5.8 The work of Rani's group

The following vignette showed how the teacher attempted to explore students' reasoning in arranging the pieces of paper.

T-2-005 : How much percent is all of them [pointing to the pieces of paper]?
S-2-002 : one hundred percent
T-2-006 : There are ten equal parts...this is twelve.....and twelve...and twelve...and each part consist of twelve. Which part represents 10%?
S-2-003 : this one [pointing to the left side that were twelve white pieces of paper]
T-2-007 : How do you know?
C6-2-001 : Because the total is ten
C2-2-001 : 120 divided by 10
T-2-008 : There are ten parts. All parts is 100%. So, how much percent is one part?
S-2-004 : Ten percent
T-2-009 : Now if I take two parts, how much percent is it?
C2-2-002 : Twenty
T-2-010 : How much percent is it [pointing to two parts of the pieces of paper] Agung?
C6-2-002 : one hundred
T-2-011 : How much percent is all of them?
C6-2-003 : one hundred
T-2-012 : Now, if I take the white part, how much percent is it?
C6-2-004 : ninety percent
C2-2-003 : The white part
C6-2-005 : oooo the white part is 10%
T-2-013 : If I take two parts, how much percent is it?

C6-2-006 : Twenty percent
T-2-014 : If I take three, how much percent is it?
C6-2-007 : Thirty percent
T-2-015 : If I take four, how much percent is it?
C6-2-008 : Fourty percent
T-2-016 : If I take nine parts
C6-2-009 : That's ninety percent

To solve the problem the students arrange the pieces of paper in such a way that it forms bar model (F-2-06). The vignette above showed that although the students were able to use the pieces of paper to figure out the situation and solve the problem, they were struggling to describe their idea (C6-2-002; C6-2-004). However, the arrangement of the pieces of paper seemed to help them to understand the problem, and it could bridge students' thinking to their previous experience in working with the area problem.

As the conclusion of these activities, the students could explore the meaning of percent as so many out of a hundred through exploring the area problem with the support of grid paper and discrete object problem with the support of pieces of paper. However, the researcher made a highlight of this accomplishment. As the reflection was that this instruction was not really challenge the students to really think about the meaning of percent as so many out of a hundred. Although the students could explore the meaning of percent, they were not challenged to explore this idea. The researcher conjectured that it was caused by two things. The first was that the students were too much assisted by patterns and structures that were provided in the problems. The second was that the numbers involved in the problems were beautiful numbers (multiple of a hundred) so that it was easy to talk about percent.

In the observation, it seemed that in these activities the teacher tried to gave students more opportunities to describe their ideas. By engaging the students to explain their thinking in a group and in the class discussion showed that the teacher established the norm on giving explanation of the answer. However, other social norms also held a big influence in these activities. For example, when the teacher asked them to give conclusion, instead of contributing to give ideas, many students only wait for the other students to respond or when they worked in the group, instead of sharing ideas together, some students only follows the ideas from the clever students.

5.2.3 Constructing sense of percentage as a relative value

After constructing the meaning of percent, constructing the sense of percentage as a relative value was the main issue in this activity. Discount context was chosen as a rich context to bring the students to explore the idea of percentage as a relative value. The teacher told to the students about an advertisement in the newspaper that shows two shoe shops offer different discounts for their product. Here, the students were asked to choose which shop they prefer if they want to buy shoes. The picture of the advertisement is given as follows.

The ways in which the two shops advertise their discount convey the suggestion that the two shops might have different original prices. This clue is given on purpose to alert the students to consider to what the percentages might refer. Below the discussion in the beginning of the lesson:

T-3-001	: There were two shops, this is Istana shop and this is Sriwijaya shop. I am interested because my shoes have broken. Toko Istana offer 20% discount and toko Sriwijaya offer 25% discount. Can you guess which shop I prefer to buy shoes?
S-3-001	: Sriwijaya… Sriwijaya… Sriwijaya
T-3-002	: What is the reason Rita?
C7-3-001	: Because the discount is bigger
T-3-003	: ooo…because the discount is bigger. What else?
S-3-002	: [keep quite]
T-3-004	: Do all of you prefer Sriwijaya?
(A few minutes later)	
T-3-005	: Who prefer Sriwijaya shop?
S-3-003	: [most of the students raise their hand]
T-3-006	: I prefer Istana shop
S-3-004	: haahh??
C5-3-001	: The product is better
T-3-007	: Why?
C5-3-002	: because the product (the quality) is better
T-3-008	: What else?
C4-3-001	: In Sriwijaya the price is more expensive and the price in Istana is cheaper. For example the price in Istana is Rp 100.000,00 and the price in Sriwijaya is Rp 200.000,00. So 25% from Rp 200.000,00 is as much as this, and 20% of Rp 100.000,00 is as much as this.
T-3-009	: Oooo yaaa…

From the vignette above it showed that at first all students preferred to buy shoes in *Sriwijaya* shop (S-3-001). They choose *Sriwijaya* shop because the discount offered is bigger than those in *Istana* shop. It seemed that the students only compared both percentages absolutely. To provoke the students, the teacher made a supposition and she said that she might prefer to buy shoes in *Istana* shop (T-3-006). This supposition helped the students to think other aspects besides percentage of discount that made the teacher prefers *Istana* shop. Dias' opinion about the quality of the product indicated that he use his common sense in preferring something to buy (C5-3-001). Bayu's opinion about the different original prices indicated that he started aware that those percentages relate to the original prices (C4-3-001). After this discussion, the teacher asked the students to discuss the problem with their friend in pair and write their answer in the worksheet containing the pictures of advertisement in *Sriwijaya* and *Istana* shop and the problem.

From the students' answer of the worksheet, it showed that the students had different ideas. Some students prefer to buy shoes in *Sriwijaya* shop, and the other students prefer to buy shoes in *Istana* shop. The amount of the discount, the quality of the product and the availability of the product were the reasons behind their answer. This finding was different from the first finding where all students preferred to buy shoes in *Sriwijaya* because the discount is bigger. It might happen because the teacher asked them to look the pictures of advertisement that is showed in different manner. Almost a half of the students seemed to start aware that they cannot compare those percentages absolutely but they have to relate it to the original price. Therefore, to investigate this finding, the analysis was focused on students' reasoning when they answer the problem.

Students' answer and reasoning in determining which shop they prefer are described as follows:

a. Preferring to buy shoes in *Sriwijaya* shop.

   Students who preferred *Sriwijaya* shop because the discount is bigger seemed to treat the percentage of discount as a quantity rather than as a relation. Kiki was one of the students who argued that he preferred *Sriwijaya* shop because

Sriwijaya shop offered 25% discount whereas *Istana* shop only offered 20% discount. The fact that Kiki still only compared the percentage of discount indicated that he did not consider that percentage in discount gives a relative measure and not an absolute measure. However, there were five students who preferred *Sriwijaya* shop gave assumption that the original prices in both shops are the same. It is clear that these students knew that a percentage is related to something. The students who gave an assumption of the original prices tended to use beautiful numbers such as Rp 100.000,00 or Rp 200.000,00. The students who chose those beautiful numbers were able to take 20% or 25% of those amounts. However, there was a student (Deri) who chose Rp 150.000,00 as the original prices in both shops. Deri made mistake in taking 20% or 25% of Rp 150.000,00. He answered that the new price in *Sriwijaya* shop is Rp 125.000,00 and in *Istana* shop is Rp 130.000,00. He came to this answer because he argued that *'25% of Rp 150.000,00 is Rp 25.000,00 and 20% of Rp 150.000,00 is Rp 20.000'*. This fact showed that the student was struggling over how to take 25% or 20% of an amount. Even though there were some students who able to take 25% or 20% of an amount, it could not give enough argument to say that the students were able to take a certain percent of an amount, since they tended to use such beautiful numbers.

b. Preferring to buy shoes in *Istana* shop.

There were several reasons behind the choice of preferring *Istana* shop. The original prices, the quality, and the availability of the product were the reasons behind their choice. 7 out of 16 students who preferred *Istana* shop argued that they have to consider the original price. Those students seemed to consider that the smaller percentage of discount could give the cheaper price because it depended on the original price. It indicated that they started aware that they cannot compare percentages absolutely but they have to look at the reference of the percentage.

At the end of the lesson, the teacher engaged the students to explore the advantage of using percentage in discount context.

T-3-010	: Now I want to ask you...why don't the shops offer reduction price by using for example discount is ½ of the price or discount ¼ of the price? Why do the shops not use discounts like this? What are these? [pointing to fractions written on the blackboard]
S-3-005	: fractions
T-3-011	: common fractions or for example 0,5 discount. Why do we not use this [pointing to the decimal number] or use this [pointing to the fraction number]?
T-3-012	: Why do we usually use for example discount is 5%? Why?
S-3-006	: [keep quite]
T-3-013	: You can discuss with your friends
T-3-014	: Come on Teguh
C8-3-001	: so people who will buy the product will not be confused
T-3-015	: ooohhh.... so people who will buy the product will not be confused. Why do they confuse?
C5-3-003	: because the fractions are difficult
T-3-016	: ooohh because the fractions
C4-3-002	: [raising his hand]
T-3-017	: How about you Bayu?
C4-3-003	: If the shops use fraction for example a half or a quarter or decimal, people will be fooled so that they will go out from the store
T-3-018	: Why do the shops not use fractions? The shops always use percent... Why is it like that? How about you Sunu, Heri?
C5-3-004	: [raising his hand]
T-3-019	: How about you Dias
C5-3-005	: Because fraction can be changed into percent
T-3-020	: Why do they use percent?
S-3-007	: [keep quite]
T-3-021	: because....
C4-3-004	: percent is per hundredth
T-3-022	: per hundredth....how about fraction?
S-3-008	: [keep quite]
T-3-023	: Students.....Do you think people will get difficulties or not to compare the original price and the reduction price if the shop use fraction as the

discount. For example discount if 1/5, do you think people will get difficulties?

S-3-009 : Yes....they will get difficulties

T-3-024 : Yeah....they will get difficulties to compare the original price and the reduction price.

In the vignette above, the teacher attempted to provoke the students to think about the uses of percentage in discount context (T-3-010; T-3-011; T-3-012). This vignette showed how the students were struggling to communicate their idea. In the observation, we showed that the teacher attempted to give more chance to the students to share their idea but the students tended to be passive. The socio norm also held big influence in this activity. Most of the students tended to wait their fellow to share their idea rather than joined actively in the discussion and only certain students were involved in the class discussion (S-3-006; S-3-007; S-3-008).

In computing the reduction price in both shops, most of the students directly answer without doing any procedural computation and only few students uses procedural algorithm. It might happen because both percentages (25% and 10%) were easily converted into simple fraction and most of the students also choose beautiful number such as Rp 100.000 as the original price so that they could directly determine 20% or 25% of Rp 100.000. From this finding, it showed that the students did not use their previous experience in working with benchmark percentages or bar model to solve percentage problem. However, from the activity, it showed that the students learned something new about the relativity of percentage.

As the conclusion, based on students' answer in the worksheet and their reasoning during the activity, it is showed that almost a half of the students seemed to start aware that in comparing discount they cannot only compare the percentage of discount absolutely but they have to know the reference of the percentage. This finding indicated that those students started to be aware that percentages of discount give relative value and not absolute value so that one could not compare it directly without taking into account to what they refer. However, there were some students who were still struggling with this idea. The students who compared the two percentages absolutely need more help. They must learn that one cannot treat a percentage as an absolute number. At the end of the class discussion the students started to be aware of the advantage of using percentage in discount context. They started to realize that percentage in discount context help people to compare the proportion of the reduction price and the original price. Furthermore, the advantage of using percentage in comparing proportion will be explored further in the next phase of learning.

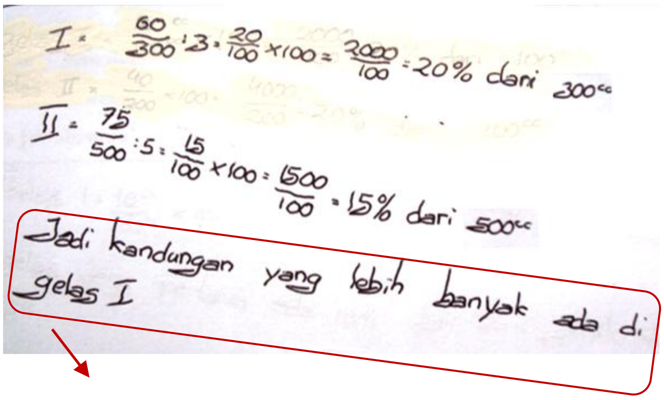
As the reflection, there were some remarks on the problem. The first one concerns the clarity of the problem. Asking for choosing which shop they prefer can make students could be confused. The question is not clear whether it refer to the cheapest price or the biggest amount of the reduction price and whether the students could compare two different products. Another remark concerns on the improvement of the problem. One cannot judge that the students who compared the percentages absolutely really lack of understanding of the relativity of percentage. Students who compared the percentages absolutely might understand the relativity of percentages but they still need extra help in expressing this. To be certain about this, the problem could be improved by extending it with an additional question.

5.2.4 Using percentage in proportional comparison problem

From the result of the pre test, it showed that most of the students got confused in solving proportional comparison problem. It was conjectured that they were not used to solve this kind of proportional comparison problem or they did not realize about the advantage of using percentage in solving this kind of problem. In solving proportional comparison problem in the pre test, most of the students seemed to only compare absolutely the part without comparing to the whole. Consequently, the idea of percentage as a relative value of the part compared to the whole became the main issue in this activity. Here, the students were expected to be able to use their previous knowledge especially about percentage in a situation in which they are needed; when different parts of different wholes have to be compared.

The teacher started the activity by telling a story about making orange juice. Then the teacher showed two orange drinks that consist of different volumes of extract orange and different total volumes of drink. In this activity the students were asked to determine whether both drinks have the same sweetness and if both drinks did not have the same sweetness, they have to determine which drink is the sweeter drink. This activity was continued by working with worksheets contained two similar problems. The difference only on the numbers involved. From the students' answer, it showed that the students made an improvement in solving the first and the second problem. On the first problem, most of the students got confused in solving the problem. Almost a half of the students did not realize that the problem is about proportional comparison problem. They only look at the volume of extract syrup or the total volume of drink and compare it absolutely. However, there were seven students who could answer the problem correctly. Determining the concentration of extract orange by using fraction or percentage or using doubling strategy became the strategies used by seven students to solve the problem. On the second problem, only four students compared absolutely the volume of extract orange. Most of them realized that the problem is about proportional problem so that they could not compare the volume of extract orange absolutely to determine the sweeter drink. However, only nine students were able to solve the problem correctly. These findings indicated that there is a little improvement achieved by the students. These findings also indicated that the students were still struggling to apply their previous knowledge about percentage or fraction or decimal to solve this proportional comparison problem. However, from the correct answer it was difficult to conclude whether those correct solutions reflected that they knew why they have to compare the volume of extract orange and the total volume by using percentage, fraction or decimal and whether the students knew the advantage of using percentage in solving this kind of problem.

One of the strategies used by the students is using percentage. The following excerpt is an example from Bayu who used this strategy.



F-4-01

So, the more content (of extract orange) was in the glass I

Figure 5.9 An example of student' answers using percentage to solve proportional comparison problem

The following vignette showed Bayu's explanation towards his answer.

T-4-001 : Come on Bayu...please explain your answer.

C4-4-001 : In the first glass the extract orange is 60 cc and the total volume is 300cc. So, 60/300 is divided by 3 equals to 20/100 times 100 equals to 20%. Therefore the extract orange in the first glass is 20%.
In the second glass the extract orange is 75 cc and the total volume is 500cc. So, 75/500 is divided by 5 equals to 15/100 times 100 equals to 1500/100 equals to 15%.
Therefore the extract orange in the first glass is 15%.
So, the more content was in the glass I

T-4-002 : So, which glass contains the sweeter drink?

S-4-001 : The first glass.... The second glass...

T-4-002 : Based on Bayu's answer...which glass contains the sweeter drink?

C4-4-002 : The first glass

T-4-003 : Why?

C4-4-003 : Because the percentage in the first glass is bigger

T-4-004 : In the first glass, how much the percentage of the concentration of extracts orange?

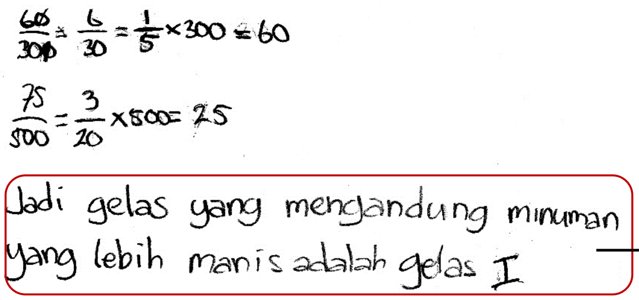
C4-4-004 : 20%

T-4-005 : and in the second glass?

C4-4-005 : 15%

To determine which drink is the sweeter drink, first C4 determined the concentration of the extract orange in each drink by comparing the volume of the extract orange and the total volume of drink (C4-4-001). Then he determined the percentage of the extract orange in both drinks. The bigger the percentage the sweeter the drink is. From his work and from the vignette above, it seemed that the student could use his previous knowledge about percentages in a situation in which percentages are needed. In determining the percentage of the concentration of extract orange, he did the formal calculation (F-4-01). He used this formal computation might be because he was proficient in working with the algorithm.

From the observation, there was an interesting finding. The students compared absolutely the volume of extract orange to determine which drink is the sweeter drink. Gandhang (C9) is one of the students who used this strategy. However, at first he already tried to compare the volume of extract orange and the total volume by using fraction.



F-4-02

So, the glass that contains the sweeter drink is the glass I

Figure 5.10 Gandhang's answer that compared absolutely the volume of extract orange

T-4-006	: Please explain you answer Gandhang
C9-4-001	: The extract orange in the first glass is 60/300. It was simplified into 6/30, and it was simplified again into 1/5. Then 1/5 is multiplied by 300 and equals to 60. The extract orange in the second glass is 75/500. It was simplified into 3/20. Then 3/20 is multiplied by 500 and equals to 25. So, the glass that contains the sweeter drink is the first glass.
T-4-007	: Why?
C9-4-002	: because the extract orange in the first glass is bigger than in the second glass [based on his answer; 60 and 25].
T-4-008	: If we multiply 3/20 by 500, how much the result?
S-4-002	: 75

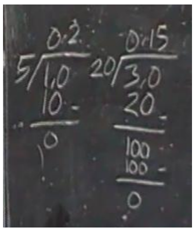
T-4-009 : Now we look on Gandhang's answer. He divided 60 by 300. 60 is the volume of extract orange and 300 is the total volume of drink. Then he simplified 60/300 into 1/5.
In the second glass, he divided 75 by 500. 75 is the volume of extract orange and 500 is the total volume of drink. Then he simplified 75/500 into 3/20.
Here, Gandhang again multiplied1/5 by 300, and multiplied 3/20 by 500, and the result is 60 cc and 75 cc. So?
S-4-003 : It comes back to the problem
T-4-010 : Yeah...it comes back to the problem
T-4-011 : Could we only compare 60cc and 75cc?
C7-4-001 : It can be.
T-4-012 : Why?
C7-4-002 : because 75cc is greater than 60cc

From Gandhang's work, it showed that he used his pre knowledge about equivalent fraction (F4-02; C9-4-001). However, although he was able to carry out the computation correctly, it seemed that the computation did not make sense for him. From the vignette above, the reasoning of Gandhang and Rita showed that they did not perceive the idea of comparing proportionally (C9-4-002; C7-4-002). They tended to compare absolutely the volume of extract orange without considering the total volume of drink.

Considering this fact, the teacher engaged the students in the class discussion to explore the idea of comparing proportionally.

T-4-013 : So we only compare the extract orange? Does the total volume influence the sweetness?
S-4-004 : Yes, it influence the sweetness
T-4-014 : For example, there is one drink with the extract orange is 100 cc, but the total volume is greater than 500 cc. Does the total volume influence the sweetness?
S-4-005 : Yeah
T-4-015 : So, could we only look at the volume of extract orange?
S-4-006 : No
T-4-016 : So, what must be considered to determine the sweetness of those drinks?
S-4-007 : The volume of extract orange and the total volume of drinks.
T-4-017 : From this [referring to Gandhang's answer], what should we compare?
S-4-008 : 1/5 and 3/20
T-4-018 : Yeah...1/5 and 3/20. Now, 1/5 and 3/20...which one is bigger?
S-4-009 : 3/20....... 1/5.....

T-4-019 : Who thinks that 1/5 is bigger than 3/20?
S-4-010 : [almost three fourth of the students raise their hand]
T-4-020 : Who knows the reason?
C5-4-001 : 1/5 is one is divided by five, 3/20 is three is divided by twenty
T-4-021 : Why 1/5 is bigger than 3/20?
C2-4-001 : because one cake divided by five person is bigger than if it is divided by twenty person
T-4-022 : but there is three [pointing to the numerator of 3/20]
C4-4-006 : I know the reason
T-4-023 : come on Bayu (C4)
C4-4-007 : [he writes on the blackboard]



F-4-03

T-4-024 : Here Bayu (C4) uses decimal. Based on his answer, 1/5 equals to 0,2 and 3/20 equals to 0,15. Now, which one is bigger? 0,2 or 0,15?
C4-4-008 : 0,2
S-4-011 : 0,2...... 0,15...
C3-4-001 : I know mom
T-4-025 : come on Rio (C3)
C3-4-002 : [he writes on the blackboard]

F-4-04

T-4-026 : Here, Rio uses cross multiplication. Explain you strategy Rio!
C3-4-003 : 1 is multiplied by 20 and 3 is multiplied by 5
T-4-027 : So?
C3-4-004 : 1/5 is bigger than 3/20
T-4-028 : back to decimal....which one is bigger? 0,2 or 0,15?
S-4-012 : 0,2 [almost all students]....0,15 [only few students]
T-4-029 : how if we use percentage
S-4-013 : 0,2 is 20% and 0,15 is 15%
T-4-030 : 20% and 15%, which one is bigger?
S-4-014 : 20%
T-4-031 : So, which glass contains the sweeter drink?
S-4-015 : the first glass

In the beginning of the short vignette above, the students finally realized the idea of comparing relatively (S-4-006; S-4-007). There were some interesting findings from this discussion. The first finding was that the students were struggling to compare common fractions or decimals (F-4-03). The second finding was that the students were proficient in converting fraction into decimal and also in comparing fractions by using cross multiplication (F-4-04; C3-4-003). Although they could compare fractions by using cross multiplication, they could not give the logic reasoning why 1/5 is bigger than 3/20. This cross multiplication strategy did not seem to make sense for them in comparing fractions. However, the students could easily compare percentages (S-4-014). It might happen because percentages were notated in the similar manner with the notation of whole number.

At the end of the lesson, the teacher stimulated the students to conclude what they have learned through investigating this kind of problem.

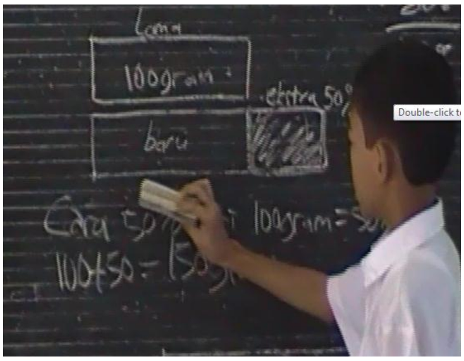
T-4-032	: What do you have learned today?
S-4-016	: Comparison
T-4-033	: If we faced comparison problem, what kind of strategy can be used to solve the problem?
S-4-017	: Using percent.....decimals....common fractions...
T-4-034	: If we can use decimals or common fraction to solve the problem, why most of you used percent?
S-4-018	: [keep quite]
T-4-035	: If we want to compare fractions, decimals, and percents. Which one is easier to compare?
S-4-019	: percent
T-4-036	: Why?
C10-4-001	: because they were easier to be compared
C4-4-009	: more definite

The statement C10-4-001 and C4-4-009 showed that both students seemed to consider the advantage of using percentage in comparing proportions.

As the conclusion, based on the students' answer in the worksheet and students' reasoning during the class discussion, it seemed that around three fourth from the total number of students still had difficulty in solving proportional comparison problem. By the support given by the teacher, the students finally realized that they could not compare directly the volumes of extract orange in determining which drink is the sweeter drink. They finally realized that they should compare the volume of extract orange with the total volume of drink (S-4-006; S-4-007). However, although the students realized the idea of comparing relatively, they still struggled to use their previous knowledge, especially about percentages, in solving this problem. They got difficulties in using their knowledge about percentage in a situation in which they are needed when different parts of different wholes have to be compared. It might happen because the students tended to use formal algorithm instead of using their common sense or using their previous experience in working with benchmark percentage or bar model (F-4-02; F-4-03; F-4-04).

5.2.5 Expanding knowledge to percentage greater than 100

In this phase, the attention of the students moved from working with percentage less than 100 to working with percentage greater than 100. This phase was considered important because percentage greater than a hundred was widely used in daily life. Looking back to the result of the pre test, the result showed that most of the students got difficulties in solving percentage problem involving percentage greater than 100 and almost a half of the students did not have sense that percentages greater than 100 indicate that something is increase. Considering this facts, this phase focus to help the students to perceive the idea that percentages greater than a hundred indicate that something is increase and to help the students to do the flexible computation. The teacher started the activity by telling a story about 50% extra free on a product of tea and asking them about other examples involving extra free. Then the teacher told that there is a new product of *Choco* chocolate having 50% extra free. The teacher asked the students to make the drawing of the old and the new product of *Choco* chocolate and to determine the weight of the new product of *Choco* chocolate if the weight of the old product is given. Only four students had difficulties to solve the problem. Most of the students could give the correct drawing and answer. Below the example of student' reasoning in solving free extra problem:



F-5-01

Figure 5.11 Rio's answer in drawing chocolate having 50% extra free

T-5-001 : Rio, could you explain the answer?

C3-5-001 : This is the old product [pointing to the first drawing]. The old product is 100%...100 gram. If the new product [pointing to the second drawing] has free extra 50% so.... 100% is 100 gram plus 50% is 50gram. Therefore the new product has weight 150 gram.

This finding showed that the students knew the extra free context and realized the reference of the percentage on the extra free (F-5-01; C3-5-001). This activity was expected could bridge the students to explore the idea of percentage greater than a hundred.

The activity then continued by making drawings of the old and the new product of toothpaste *Fresh* in which the new product having weight 150% of the weight of the old product. Here the students also determined the weight of the new product of toothpaste if the weight of the old product is given. Most of the students could give the correct drawings. This finding indicated that that the students started to realize that percentage greater than a hundred indicated that something is increase. However, only ten students could give the correct answer. Most of the students who gave the correct answer use splitting strategy; they split 150% into 100% plus 50%. It indicated that free extra context help the students to see that percentage greater than a hundred (for example 150%) indicated that something have 50% addition, like in the free extra context, so that it caused something is increase. On the other hand, the other students still got confused in solving this problem. The following picture is an example from Gandhang's work that failed to arrive to the correct answer.

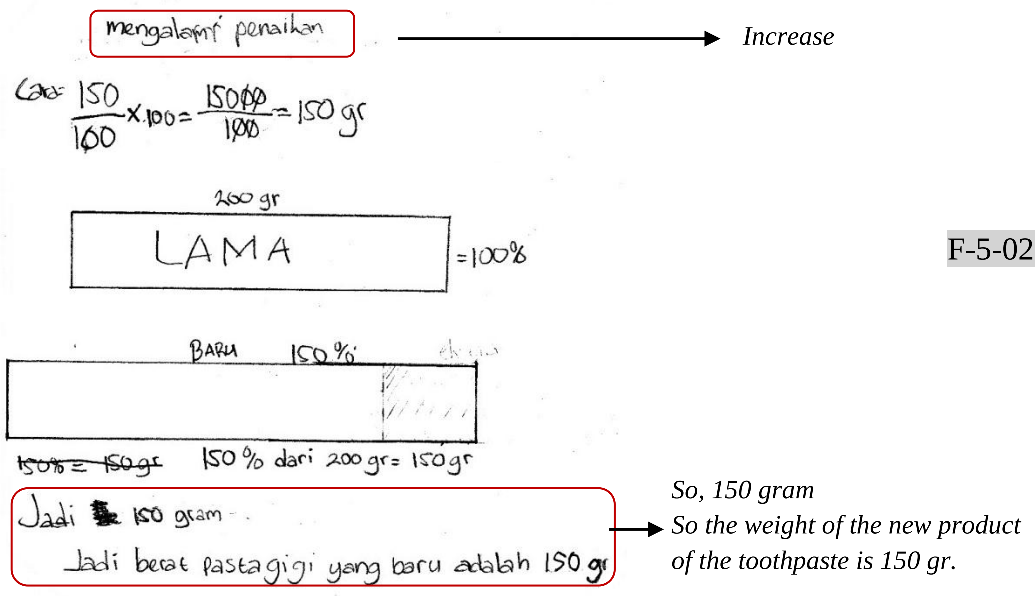
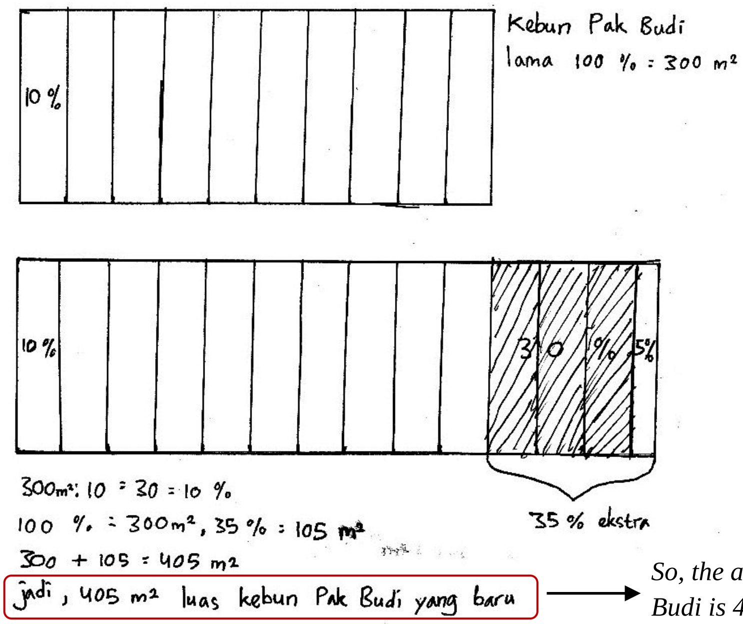


Figure 5.12 An example of incorrect answer in working with percentage greater than a hundred

From the Figure 5.12 above, it showed that the students used formal algorithm to solve the problem. However, it seemed that his computation did not match with his drawing. From Gandhang's drawing we could see that he realized that 150% indicates that something is increase (although he did not draw it properly) but in contrast his computation arrived to the different result. It might happen because Gandhang put the wrong number on the computation. In this case Gandhang could not synchronise his drawing with his result of computation (F-5-02). It seemed that his procedural algorithm did not make sense for him.

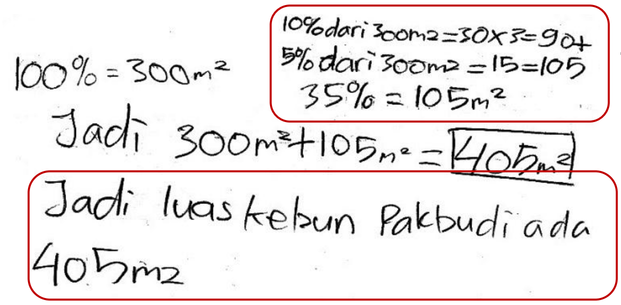
Next, in attempt to strengthen the idea of percentage greater than a hundred, the students were given opportunity to solve a problem involving another benchmark percentage that is 135%. From the students' worksheet, it showed that only a third of the students could solve the problem. There was an interesting finding in this activity. Only three out of eight of the students who could give the correct answer used splitting strategy, they split 135% into 100% plus 35% to help them to ease the calculation. Following are the examples of the students' work using splitting strategy.



F-5-03

So, the area of the new field of Pak Budi is 405 m^2.

Figure 5.13 Rio's answer using bar model and benchmark percentages



10% of 300 m^2 = 30 x 3 = 90 +
5% dari 300 m^2 = 15 = 105
35% = 105 m^2

F-5-04

So, the area of the field of Pak Budi is 405 m^2

Figure 5.14 Rani’s answer using benchmark percentages

To solve the problem, Rio and Rani used 10% and 5% as the benchmark percentage to solve the problem. They split 135% into 100% plus 35%. Both students’ works showed that the students realized that percentage greater than a hundred (for example 135%) indicate that something have 35% addition (F-5-03; F-5-04). In Rio’s answer, he made a drawing of bar that is applied to solve the problem. Here he seemed to use his knowledge about bar model in the previous activities to solve this problem (F-05-03).

Besides using splitting strategy, some students used algorithm to solve the problem. They tended to do formal algorithm because it was their prior knowledge and they were proficient to use this algorithm. Therefore, it was difficult to conclude whether the contexts in this phase help those students to work with percentage greater than a hundred.

As the conclusion of this phase, most of the students started to realize that percentages greater than a hundred indicate that something is increase. From the result of pre test, most of the students had difficulties to do the computation involving percentage greater than a hundred. It might happen because the computation involved big numbers. By using the context of free extra, the researcher expected that the students will be flexible to handle the calculation involving percentage greater than a hundred. However, only few of the students could do the flexible calculation by using splitting strategy. Many students still had difficulties to do the computation. Therefore, it is conjectured that the students still need more practices in investigating this kind of problem.

5.3 Post test

At the end of the lesson, the students completed a post test with seven problems. Those seven problems represented all mathematical ideas that they have learned during the learning process. This post test aimed to find the evidence of their learning process, particularly in investigating to what extent students learned through the learning process. The the critical issues and several remarks will be described in this section.

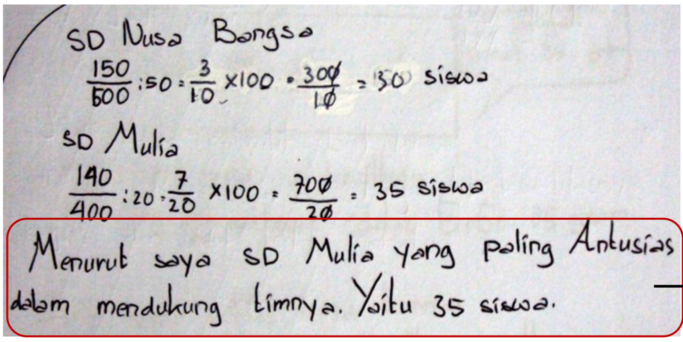
The first critical issue is about the improvement of students' achievement in estimating percentage. On the first problem, the students were asked to shade 80% of the field drawn in rectangular form. Most of the students could shade it properly. There were two different strategies used to shade 80% of the field. Some students divided the drawing into ten parts and shaded 8 parts of it. These students seemed to use 10% as the benchmark percentage to estimate how much 80% is. The other students directly shade the part without partitioning the drawing. This result indicated that the students have sense of the *fullness* of percentage.

The second critical issue is about the meaning of percent as so many out of a hundred. In the post test, there were three different tasks that assess whether the students already perceived the meaning of percent. The result showed that almost a half of the students were able to make visual representation of the situation

'20% of the apples was green' and were able to compute a part of a whole item when the percentage of that part is given. Another result of post test also showed that only 4 out of 24 students were able to determine the percentage of the part if the proportion is given. Of those whose answers were incorrect and whose strategies were obvious in computing a part of a whole item when the percentage of that part is given, almost 70% of the students used formal computation which probably did not make sense to them. This kind of problem, computing a part of a whole item when the percentage of that part is given, could not give the evidence of students' learning process since the students tended to solve it by using formal algorithm. In contrast, of those whose answers were correct and whose strategies were obvious in determining the percentage of the part if the proportion is given, 3 out of 4 students used a rather informal strategy. In the informal strategy, they tried to find 10% or 1% of the whole. For these students, it seemed that they perceive the meaning of the percentage.

The third critical issue is about student's awareness of percentages as relative value. Based on students' answer, only a third of the students understood that one cannot compare percentages without taking into account to what they refer. Five of the students answered that it depends on the quality of the product, three students did not gave the answer, and a third of the students compared absolutely. Of those who made an assumption of the original prices, most of those students used formal computation to find the reduction price; they transformed the percentage into fraction and then multiplied the fraction with the original price.

The fourth critical issue is about students' acquisition in applying the percentages in a situation in which they are needed, when they have to compare different parts of different wholes. Only six students arrived at the correct answer. Of those whose answers were correct, two students just estimated, three students used percentages and one student used decimal. For students who used percentage to solve the problem, one student used formal algorithm and two students used rather informal strategy. In total almost a three fourth of the students failed entirely; they compared the parts absolutely without comparing the part with the whole. Of those whose answers were incorrect, seven students tended to do formal computations with the numbers involved in the problem. One of those computations was given as follows:



I think SD Mulia is more enthusiastic in supporting their team. There were 35 students.

Figure 5.15 One of student's works using formal computation

From the Figure 5.15 above, it showed that the student confused in using the procedural computation to solve the problem. It seemed that the computation did not make sense for him. As the conclusion, it seemed that most of the students did not realize that the problem is about comparing proportional problem and for the students who compare the part with the whole, most of those students tended to use formal algorithm that probably did not make sense for them. Although in the learning process the students had experienced in solving this kind of problem and they realized that they cannot compare the parts absolutely, it seemed that the students cannot relate their previous experience in solving this problem. It indicated that the students could not transfer to different situations.

The fifth critical issue is about students' acquisition in working with percentage greater than a hundred. In the post test, they worked with '175%'.From the students' answer, it showed that only five students could arrive to the correct answer, seven students only could make the visual representation that showed how something increase into 175% and the other students failed entirely. Of those whose answers were correct, two students used benchmark percentages and three students used formal algorithm. The low improvement of students' achievement in working with percentage greater than a hundred could indicate that most of the students were not ready yet to expand their knowledge into percentage greater than a hundred or they need more practices.

5.4 Discussion

In this design research, the characteristics of Realistic Mathematics Education underlay the activities in this research. The implementation of the characteristics of RME will be elaborated on the contextual situation, the classroom culture and the role of the teacher, and the intertwinement of mathematical topics.

5.4.1 Contextual Situation

The first characteristic of RME that is *the phenomenological exploration* served as the base of the sequence of instructional activities. A rich and meaningful context or phenomenon, concrete or abstract, should be explored to develop intuitive notions that can be the basis for concept formation (Bakker, 2004). In this research, the students were confronted with some daily life situations in which percentages play role. Since the students have learned about percentage in the school, the students knew the procedural algorithm to solve percentage problem, especially when they have to compute part of a whole while the percentage is given. Therefore, the researcher should adjust some activities so that they could not directly use their procedural algorithm.

5.4.2 Intertwinement of mathematical topics

According to the fifth characteristic of RME that is *intertwinement*, it is important to consider an instructional sequence in its relation to other domains. In this research, percentage is intertwined with area, estimation, fraction, and decimal.

- Intertwine percentage with area

  In this research, bar model was expected to emerge through investigating area problem. In investigating the percentage of an area, the students could draw a square or rectangle with a certain area and used the drawing to estimate the percentage. Therefore, drawing a certain area can be used to intertwine percentage with area.

- Intertwine percentage with estimation

  In this research, the learning of percentage was started from estimating the percentage of loading process. Therefore, through the learning process, the students were not merely learned about percentage but also they learned about estimation.

- Intertwine percentage with fraction and decimal

  The relationship of percentage with fraction and decimal provide many possibilities to do arithmetic in a flexible way. In estimating with percentage, simple fractions play an important role as reference point. For example when estimating the percentage of loading process, students said that the loading is 50% because the bar that is fully loaded is *a half* of the whole bar. Another example is derived from estimating the concentration of syrup activity. Some students used fractions or decimal to find the percentage of the concentration of syrup.

CHAPTER VI

CONCLUSIONS

This chapter consists of four sections. The first section is about the answer of the research question. The second section is about the local instructional theory of learning percentage. The third section is about the weakness of the research. The last section is about the recommendation for further research. The research question will be answered by summarizing the analysis of the whole learning process and also by analyzing the result of the pre test and post test.

6.1 Answer to the research question

In order to investigate how the students extend their understanding of percentage, there were some aspects that have to be considered. The first aspect is about the actual understanding of the students. Since the students have already learned about percentage, therefore they have actual understanding that later was extended. The second aspect is about the support given during the learning process, and the third aspect is to what extent the understanding of the students. The summary of the result and analysis of students' learning process as elaborated in the previous chapter will be used to answer this research question. The students' learning process of percentage was elaborated in five phases as follows.

1. Developing sense of *fullness* of percentage

The pre test result showed that around a half of the students did not have sense about percentage. They did not perceive that percentage close to 100 means almost 'all' [*see Subchapter 5.1.1.1*]. To support students' sense of fullness of percentage, the familiar context about loading process was chosen. In the learning process, by observing how loading process works, the students could see how percentage runs along the scale from 0 to 100 as well as the shaded part representing the part that is fully loaded (S-1-011; S-1-013; S-1-014). A conflict emerged when the students were asked to estimate the percentage of loading bar that has different length. To support the students, the teacher posed some questions to provoke them to see that the percentage of loading process can be represented by different drawings (T-1-001; T-1-002; T-1-004). As the conclusion, the loading context helped the students to see how the percentage runs along the scale as well as the shaded part representing the fully loaded area (F-1-01; F-1-02; F-1-03; F-1-04). Students also commenced to acquire the idea of benchmark percentages (F-1-04). In general, in this activity most of the students perceived the sense of *fullness* of percentage; they knew that percentage close to zero means almost 'nothing' and percentage close to a hundred means almost 'all'.

2. Constructing the meaning of percent

Although the students were able to recite that percent means per hundredth, the pre test result showed that the students need a support in reconstructing the meaning of percent. The findings also showed that most of the students tended to use procedural computation that probably did not make sense for them instead of using their common sense [s*ee Subchapter 5.1.1.2*].

After perceiving the sense of *fullness* of percentage, the students explored the meaning of percentage as part whole relationship in which there were one hundred parts within a whole. The problems about area and also about discrete objects were chosen to support the students to explore this idea. The grid paper provided in area problems was a tool to support the students to explore the idea of percentage as so many out of a hundred parts (F-2-01; F-2-04; F-2-04). The small pieces of paper in solving discrete objects problems were also a tool to support the students to explore this idea (F-2-06). The grid paper used in the activity not only served as a good hold for students to explore the meaning of percent but also served as a good hold to develop bar model (F-2-06).

3. Constructing sense of percentage as a relative value

In attempt to support the students to explore the idea of percentage as part whole relationship that describes relative value, the familiar situation about discount was chosen to provoke students to explore this idea. According to Van den Heuvel-Panhuizen (1994), students do not have to explain this idea in this manner but they have to show an awareness of the fact that percentages are always related to something. Students' tendency to compare percentages absolutely without considering the reference of the percentages led to the emergence of a conflict (S-3-001). During the class discussion, the teacher gave some guidance, such as giving a supposition, to stimulate the students to come up with the idea of relativity of the percentage (T-3-006; T-3-008). As the conclusion, almost a half of the students seemed to be aware that percentages of discount give relative value and not absolute value so that one could not compare it directly without taking into account to what they refer.

4. Using percentage in solving proportional comparison problem

Although most of the students knew the procedural algorithm to solve percentage problems, it does not mean that they are able to use their knowledge about percentage in a situation in which percentages are needed. The pre test result showed that almost all students got difficulties in solving a problem in which percentages are needed [*see Subchapter 5.1.1.4*]. In the learning process, the students were given opportunities to solve problems in which they have to compare different parts of different wholes. The idea of percentage as a relative value of the part compared to the whole became the main issue in this phase. Here, the students were challenged to use their knowledge about percentage in ordering the sweetness of orange drinks. A conflict emerged when there were some students who compared the volumes of extract orange absolutely (F-4-02; C9-4-002; C7-4-001; C7-4-002). By posing questions by the teacher, the teacher helped the students to come up with the idea of comparing relatively (T-4-009; T-4-013; T-4-015; T-4-016). However, based on the students' answer in the worksheet and students' reasoning during the class discussion and also the result of the pre test, around three fourth of the students still had difficulty in solving proportional comparison problem. They were still struggling to use their previous knowledge, especially about percentages, in a situation in which percentages are needed when different parts of different wholes have to be compared.

5. Expanding knowledge to percentage greater than a hundred

Based on pre test result, almost a half of the students did not have sense that percentage greater than 100 indicates that something is increase and most of the students got confused in solving percentage problems involving percentage greater than 100 [*see Subchapter 5.1.1.3*]. Using the fact that an extra context indicates that there is addition; the students were stimulated to start perceiving the idea of percentage greater than a hundred. By using the context of free extra, the students were expected to be flexible to handle the calculation involving percentage greater than a hundred and not merely used difficult procedural computation that involves big numbers. In fact, only few students could do the flexible calculation by using splitting strategy (F-5-03; F-5-04). Many students tended to use algorithm to solve the problem and there were many students who still had difficulties to do the computation (F-5-02). However, in general the students started to realize that percentages greater than a hundred indicate that something is increase.

Summary

Students' learning process in extending their understanding was started from developing sense of *fullness* of percentage, which afterwards continued to constructing the meaning of percent itself. Based on this basic knowledge, they were given opportunities to construct sense of percentage as a relative value and also to apply this knowledge in a situation in which the percentages are needed. Furthermore, the students expanded their knowledge to percentage greater than a hundred. As the conclusion, the instructional activities could support students to extend their understanding. However, not all students could achieve the complete understanding of percentage. There were many students who were not able to apply their their knowledge in a new situation or to extend their understanding to percentage greater than a hundred.

6.2 Local Instructional Theory

As described in chapter 3, design research is a type of research methods aimed to develop theories about both the process of learning and the means that are designed to support that learning (Gravemeijer & Cobb, 2006). The local instructional theory with respect to the intended activities and conceptual development of the students was summarized in the Table 6.1. Doorman used such a table for describing local instructional theory on modeling motion (Doorman, 2005). The classroom culture and the role of the teacher were essential in this learning process. Therefore those aspects will be briefly discussed in this section.

Table 6.1 Local instructional theory for learning percentage in grade 5

Tool / Contextual situation	Activity	Imagery	Potential mathematical discourse
Loading process (the application program of loading process)	Estimating the percentage of loading process	Signifies that percentage close to 100 means 'almost all' and percentage close to 0 means 'almost nothing'	The *fullness* of percentage

Tool / Contextual situation	Activity	Imagery	Potential mathematical discourse
Area (Grid paper)	Drawing the sketches of house or field	Signifies that if one divides a whole into one hundred parts then one small part represents 1% of the whole, or if one divides a whole into ten parts then one small part represents 10% of the whole	Percent means 'so many out of 100'
Pieces of square paper	Determining the amount of the part of discrete objects from a whole	Signifies the need of benchmark percentages such as 5% or 10% derived from dividing an object into some equal parts	Percent means 'so many out of 100'
Discount (the picture of advertisement)	Comparing different discounts	Signifies that one could not compare the percentages of discount absolutely in determining the cheaper price	Percentage as relative value
Concentration of extract orange (some glasses of orange juice)	Ordering the sweetness of drinks	Signifies that using percentage made one could compare proportions easily	Percentage as relative value
Extra weight	Drawing the product having extra and determining the weight of the product	Signifies that percentage greater than 100 indicate that something is increase	Percentage greater than 100
Weight and area	Solving problem involving percentage greater than a hundred	Signifies the flexible way in computing percentage greater than a hundred	Percentage greater than 100

The classroom culture and the role of the teacher

The means of support encompass potentially productive instructional activities and tools as well as an envisioned classroom culture and the proactive role of the teacher (Gravemeijer & Cobb, 2006). Since classroom norms can differ from one classroom to another, and that they can make a big difference in the nature and the quality of the students' mathematical learning, then one has to consider the characteristic of the envisioned classroom culture (Gravemeijer & Cobb, 2006). Classroom culture would be established through the negotiation between the teacher and the students. Therefore one of the tasks of the teacher is to establish the desired classroom culture.

In the class experiment, the observation showed that the classroom culture had a big influence on students' learning process. In the discussion, sometimes most of the students kept quiet and waited for the other fellow to share their idea. It might happen because the teacher tended to take big role during the discussion. However, sometimes the teacher attempted to give more chance to the students to share their idea but the students still tended to be passive and only certain students were actively involved in the class discussion.

In the class discussion, the teacher did some tasks in order to establish the classroom culture. The first is in providing the students an opportunity to present their idea. According to the third characteristic of RME that is *students' own construction and production*, it is important to give opportunity for the students to explore their own construction and strategy. The questions such as *'How do you know that is is 10%?'* or *'What kind of strategy can be used to solve this* *problem?'* or *'What does it mean?'* are some examples of questions that were used by the teacher to stimulate the students to express their idea. The second task is in stimulating social interaction. According to the fourth characteristic of RME that is *interactivity*, it is important to stimulate students' interaction to shorten their learning path. The question or statement such as *'What else?'* or *'You can discuss with your friends'* are some attempts done by the teacher to stimulate students' social interaction.

6.3 The Weaknesses of the Research

In the learning process, sometimes the students could not achieve the intended learning goal. This might happen because they already knew the procedural algorithm to solve percentage problem. Sometimes the students seemed to think that it was enough that they could solve percentage problem by using the procedural algorithm without trying to make sense of the algorithm. Therefore, the condition of the students in which they have learned about percentage could become the weakness of this research.

The researcher also thought that the instructional activities were too dense. There were many mathematical goals that the students should achieve only in six meetings. This condition was also considered as the weakness of this research.

Besides those conditions, the researcher considered that in designing the instructional activities, the researcher did not consider the learning of the students. In fact, the learning style of the students influenced their learning process. Therefore, this condition might be one of the weaknesses of this research.

6.4 Recommendation

The recommendation in this section will focus on the reflection of the researcher, the revision of some activities, and the recommendation for further research.

6.4.1 Reflection

In this research, the students were students who have learned percentage in the school. They had pre knowledge about percentage that should be considered. Based on observation, the students accustomed to work in formal way; they tended to use procedural algorithm in solving percentage problem. Since they tended to use the algorithm in solving percentage problem, it seemed that some of the the contextual situations did not function as the situations that needed to be mathematized by some students. Therefore, the researcher needs to really know about how far the students' pre knowledge is and how to relate their pre knowledge with the learning process.

After having conducted this experiment, it was considered that the sequence of activities was too dense. There were too many mathematical ideas that have to be achieved only in six meetings. It caused the students did not have enough chance to construct their understanding by having discussion among students so that it influenced the socio-mathematical norm in the classroom.

Besides the weaknesses described above, it was also considered that this learning design could support students to extend their understanding of percentage. In their previous learning, the students seemed to only focus on the computational ability. Through this learning process, the students explored the understanding of percentage that is not much explored in the previous learning. Through this learning process, they explored the meaning of percentage as relative value that describes relationships based on a one-hundred-part whole, they learned the use of percentage in a situation in which the percentages are needed, and also learned about percentage greater than a hundred.

6.4.2 Revision

a. Without grid paper (number complexity)

As the reflection of the making sketches activity witout grid paper, the problems does not really challenge the students to really think about the meaning of percent. It might happen because the numbers involved in the problems were beautiful numbers (multiple of a hundred) so that it was easy to talk about percent. To improve this problem, the researcher thinks that the problem can involve more complex numbers such as ‘320’ or ‘240’.

b. Comparing different discounts activity

In the experiment, the students were given information about two shops that offer different discounts and they were asked to determine which shop they will prefer. Here, the researcher made some remarks. The first is that asking for determining which shop they prefer can make students be confused. The question is not clear whether it refer to the cheaper price or the bigger amount of the reduction price and whether the students could compare two different products. It seems unfair if one compares the cheaper price from different products. To refine this problem, the researcher proposes that both shops only have one same product be on sale and then students are asked to determine which shop gives the cheaper price. The second remark is that the uncertainty in the openness of the problem. We cannot judge that the students who compared the percentages absolutely really lack of understanding of the relativity of percentage. To refine this problem, the researcher thinks that teacher should give additional question in order to give extra help for students in expressing their idea of relativity of percentage. The additional question could be '*Is there any possibility that you will get the cheaper price in Istana shop? If yes, could you give an example?'*.

6.4.3 Recommendation for further research

This section will describe general recommendation that is addressed to both the practice of teaching and learning of percentage and to further research in mathematics education. The recommendation will focus on two issues, namely students' pre knowledge and the learning style of students.

a. Students' pre knowledge

Students' pre knowledge should be well-considered when designing instructional activities. When students come into a classroom, they come with a wide range of experiences. They have their own ideas, knowledge and concepts that are already formed. Some of this pre knowledge may be correct or may be incorrect but needs to be expanded.

In this experiment, the students had already had pre knowledge about percentage. Based on the observation, it showed that they were get used to work in formal level in solving percentage problem by doing such procedural computation that probably did not make sense for them. The findings of this research showed that only few students were flexible to handle computation that seems meaningful for them and the other students still tended to work in formal level. By using this finding, it was difficult to find the evidence of their learning process. It might happen because students' pre knowledge was not well considered. Therefore the learning design should be built upon students' pre knowledge. Building upon pre knowledge is an important part of teaching and learning. This pre knowledge is very important to be acknowledged in order to motivate students' learning process.

b. The learning style of students

Every student learns using a different styles or a combination of learning styles. McNicholas (2008) describes that there are four major categories of learning style. The four major categories are visual/verbal, visual/nonverbal, tactile/kinaesthetic, and auditory/verbal learning style. The visual/verbal learner learns best when information is presented visually and in a written language format. The visual/nonverbal learner learns best when information is presented visually and in a picture or design format. The tactile/kinaesthetic learner learns best when physically engaged in an activity. The auditory/verbal learner learns best when information is presented auditory in an oral language format.

In this research the researcher did not consider this aspect. The findings of this result showed that in certain activity, in comparing two different discounts for example, in which the information is presented auditory in an oral language format, some students did not engage in the discussion. It could happen because those students were not the auditory/verbal learning style learners, so that they might not understand the problem or they were not motivated to discuss the problem. Therefore, besides the pre knowledge, the researcher recommends that for further research the learning style of students have to be considered as well.

As the last part of the recommendation section, the researcher emphasized that the idea of Realistic Mathematics Education was the answer to the need of reforming the teaching and learning of mathematics. One significant characteristic of RME is the focus on the growth of the students’ knowledge and understanding of mathematics.

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