DESIGN RESEARCH ON MATHEMATICS EDUCATION: SPATIAL VISUALIZATION SUPPORTING STUDENTS' SPATIAL STRUCTURING IN LEARNING VOLUME MEASUREMENT

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

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GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

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Date of Approval: May 2011

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State that:

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ABSTRACT

Many prior researches found that most of students in grade five tended to have difficulty in fully grasping the concept of volume measurement. It was revealed the errors that students made on the volume measurement tasks with three dimensional cube arrays are related to some aspects of spatial visualization, such as the skill to "read off" two-dimensional representation of solid objects. For those reasons, a set of activities was designed to help students to relate their spatial visualization ability and how to perceive the three-dimensional structure of the cube arrays. This research was aimed to develop classroom activities with the use of spatial visualization tasks to support students' spatial structuring in learning volume measurement. Consequently, design research was chosen as an appropriate means to achieve this research goal. And realistic mathematics education was chosen as the approach in the teaching-learning process in the classroom. There were Thirty-two students and a teacher of grade five in elementary school in Indonesia, SD Pupuk Sriwijaya Palembang, involved in this research. Our findings suggest that in order to be able to count the volume of an object made of small cubes, the students need to be able to perceive the structures of the units' configuration. Students' spatial structuring abilities provide the necessary input and organization for the numerical procedures that the students use to count an array of cubes. Using spatial structuring strategy allows students to determine the number of cubes in term of layers and then multiple or skip-count to obtain the total number of cubic units. Moreover, we recommend to the teachers in Indonesia to use RME or PMRI approach in their teaching. In our RME classroom, the use of context has stimulated students to think of a way to solve such a problem in which the students could bring their informal knowledge to get ideas in solving such a mathematics problem.

Key concept: volume measurement, spatial structuring, spatial visualization, design research, realistic mathematics education.

ABSTRAK

Banyak peneliti terdahulu menemukan bahwa siswa kelas 5 sekolah dasar memiliki kesulitan dalam memahami konsep pengukuran volume. Hasil penelitian tersebut antara lain menyebutkan bahwa kesalahan yang dilakukan siswa dalam menyelesaikan tugas yang berkaitan dengan susunan kubus satuan berhubungan dengan aspek visualisasi spasial, seperti kemampuan untuk membaca gambar dua dimensi dari benda padat Untuk alasan itu, serangkaian aktivitas di desain untuk membantu siswa menghubungkan kemampuan visualisasi spasial mereka dengan bagaimana mereka memahami struktur dari susunan kubus satuan. Penelitian ini bertujuan untuk mengembangkan kegiatan pembelajaran dengan menggunakan kegiatan yang berhubungan dengan kemapuan visualisasi spasial untuk mendukung kemampuan strukturisasi spasial siswa dalam belajar mengenai pengukuran volume. Oleh karena itu, *design research* dipilih sebagai jenis penelitian yang tepat untuk mencapai tujuan tersebut. Dalam penelitian ini, serangkaian instruksi pembelajaran di desain dan dikembangkan berdasarkan hipotesis proses pembelajaran siswa, dan pendekatan pemelajaran yang diguanakan adalah Pendidikan Matematika Realistik. Tiga puluh dua siswa dan satu orang guru kelas 5 sekolah dasar di Indonesia yaitu SD Pusri-Palembang terlibat dalam penelitian ini. Hasil penelitian ini menyarankan bahwa dalam rangka mengembangkan kemampuan menghitung volume benda yang terbuat atau tersusun dari kubus satuan, siswa sebelumnya harus memahami struktur dari konfigurasi kubus stauan tersebut. Kemampuan visualisasi spasial siswa memungkinkan siswa mengembangkan kemampuan menghitung banyaknya kubus satuan dalam suatu susunan tiga dimensi. Menggunakan strategi strukturisasi spasial memungkinkan siswa menghitung jumlah kubus satuan pada setiap lapisan dalam sebuah sususnan baik dalam baris ataupun dalam kolom kemudian mengalikannya dengan banyaknya lapisan tersebut untuk memperoleh volume total susunan kubus satuan tersebut. Selebihnya, kami merekomendasikan kepada guru di Indonesia untuk menggunakan pendekatan pembelajaran PMRI sebagai pendekatan pembelajaran matematika di kelas. Dalam kelas PMRI kami, penggunaan konteks telat dapat menstimulasi siswa dalam berpikir untuk menemuka cara memecahkan suatu permasalahan matematika. Siswa dapat emmbawa pengetahuan dan pengalaman informalnya untuk mendapatkan ide dalam menyelesaikan masalah.

Kata Kunci: pengukuran volume, strukturisasi spasial, visualisasi spasial, design research, pendidikan matematika realistik.

SUMMARY

Shintia Revina. Design Research on Mathematics Education: Spatial Visualization Supporting Students' Spatial Structuring in Learning Volume Measurement.

In Indonesian primary schools, the volume measurement tasks related to three dimensional cube arrays in grade 5 are usually given directly in the pictorial representation without any concrete activity beforehand. Moreover, it was revealed that the errors that students made on the volume measurement tasks with three dimensional cube arrays are related to some aspects of spatial visualization, such as the skill to "read" two-dimensional representation of solid objects. In that study, the answers students gave to solve the task tended to only count to either the number of faces, or the number of visible small cubes. The students seem not consider about the interior part of the object. There is little research on volume measurement that relates the spatial visualization aspect and how students perceive the three-dimensional structure of the cube arrays. Therefore, in this research we pose a research question, "*how can spatial visualization support students' spatial structuring in learning volume measurement?"*

Sarama & Clements (2009) have emphasized students' spatial structuring ability as an essential factor in learning about volume "packing" measurement. In addition, Ben-Haim et.al (1985) suggests that, in order to be able to count the volume of an object made of small cubes, students need to be able to coordinate and integrate the views of an array either in real blocks arrangement or in drawing representation. On the other hand, the skill to "read" two-dimensional drawing representation of solid objects is a part of the spatial visualization ability (Ben- Haim et.al, *ibid*). Titus & Horsman (2009) define spatial visualization as the ability that involves skill to mentally manipulate and rotate an image into another arrangement and to mentally imagine what is inside of a solid object.

In designing the sequence of instructional activities, we consult to the ideas of realistic mathematics education as follows. The mathematical activity is started from a situation that is experientially real for students. The aim of using contextual situation as preliminary activities is that the students can bring in their (informal) knowledge of the situation. The activities could bridge the students' thinking from a concrete level to a more formal level by using models and symbols. On the other hand, the students are free to discuss what strategies they are going to use in solving the task or problem given. The learning process of students has to be seen as both individual and social process. Therefore, social interaction emerging in the classroom is important part of the whole class performance. In addition, the instructional activities designed not only support learning for volume measurement, but also support the development of spatial ability which is important in geometry domain.

This research was aimed to develop classroom activities with the use of spatial visualization tasks to support students' spatial structuring in learning volume measurement. Consequently, design research was chosen as an appropriate means to achieve this research goal. In this research, there were three phase conducted: preliminary design, teaching experiment and retrospective analysis. We analyzed the data collected during the experimental phase in retrospective analysis: students' works, field notes, and video recording. We compared the students' actual learning and the conjectured HLT.

There were seven activities we designed in our initial hypothetical learning trajectory. They are: "*dodol* packages" activity, "food packages picture" activity, building blocks, count the blocks, predicting the blocks, determining the possible size of the box and "the box capacity" worksheet. In the preliminary experiment, we observed that the students did not have many problems with the tasks. It could be the case that there were problems, but they did not come to the surface yet. We would like to see in the teaching experiment how

students in the real classroom struggled with the activities we designed. Therefore, we improve some activities and add one activity, namely: measuring with different unit sizes.

In teaching experiment, there were seven lessons conducted. In the first lesson, we found that some students thought about linear measurement that is the height, the length or the width of an object or a box, to determine how large an object is. But, some others were aware that in comparing the capacity of two objects, they need a unit to measure the capacity that is the cakes as they said. In the second lesson, we found that some students had difficulty in making a concise drawing of a three-dimensional objects arrangement. Some of them drew separates views of the arrangement and made their interpretation of it, but some others who successfully drew the arrangement as a concise building thought that they had wrong drawing since they found different number of objects between in the real arrangement and in their drawing. These students tended to calculate the number of squares they saw in their drawing. The third lesson was closely related to the second one. The students were asked to make a building based on its drawings from different views. The students who drew separate views of the building could easily recognize what they should do and how different views could refer to a building. Some others found difficulties in interpreting the drawing especially the side view. They built the front and the side view separately and then put it together. It makes them needed more blocks than prepared. Other students tended to built the outer part of the building and then fill in the interior part.

In the “calculate the blocks” activity, some students still perceived the blocks arrangement as unstructured faces. In calculating the number of blocks in the real construction, some calculated the faces of the blocks from the front and on the back and then calculated the faces they could see from the sides, left and right, and also from the top. These students could not perceive the structures of the blocks arrangement as rows or columns. And, in the class discussion, the students who could well perceive the layers always promoted to use it as the abbreviated way in calculating the number of blocks in the arrangement. The activities in lesson 5 seemed as the repetition of the activities in the first lesson. However, in this lesson, the students worked with the cube blocks as we used in lesson 3. We found that some students still have difficulty to imagine what are inside the box. They tried to iterate the blocks along the box but could not find the total. However, most of students tended to cover up the base of the box and then find the number of possible layers. In the sixth lesson, the students learnt that volume of an object can be represented in different numbers depends on the unit size we used in measuring it but its volume stayed constant. If they used bigger units the number will be less and vice versa. The students also realized they could not compare the volumes of two objects unless they measured it with the same units.

Our findings suggest that in order to be able to calculate the volume of an object made of small cubes the students need to be able to perceive the structures of the units’ configuration. Students’ spatial structuring abilities provide the necessary input and organization for the numerical procedures that the students use to calculate an array of cubes. Using spatial structuring strategy allows students to determine the number of cubes in term of layers and then multiple or skip-calculate to obtain the total number of cubic units. Moreover, we recommend to the teachers in Indonesia to use RME or PMRI approach in their teaching. In our RME classroom, the use of context has stimulated students to think of a way to solve such a problem. The students could bring their informal knowledge to get ideas in solving such a mathematics problem. In our class, the situation of packing boxes and the packages arrangement could provoke students to investigate the structures of three-dimension objects in an arrangement.

RINGKASAN

Shintia Revina. Design Research on Mathematics Education: Spatial Visualization Supporting Students' Spatial Structuring in Learning Volume Measurement.

Dalam pembelajaran matematika topik pengukuran volume di kelas 5 sekolah dasar di Indonesia, pengenalan volume biasanya diberikat dalam gambar – gambar susunan kubus satuan tanpa kegiatan konkrit dengan kubus satuan sebelumnya. Selain itu, penelitian terdahulu menemukan bahwa kesalahan yang siswa lakukan dalam menyelesaikan soal tentang mencari volume dari susunan kubus satuan adalah berkaitan dengan beberapa aspek kemampuan visualisasi spasial, seperti keterampilan dalam menginterpretasikan gambar dua dimensi dari benda padat tiga dimensi. Dalam penelitian tersebut disimpulkan bahwa jawaban yang diberikan siswa dalam menyelesaikan soal-soal mengenai volume dari susunan kubus satuan cenderung hanya menghitung jumlah persegi di permukaan yang terlihat, atau jumlah kubus satuan yang terlihat. Para siswa sepertinya tidak mempertimbangkan sisi interior dari susunan kubus satuan tersebut. Hanya sedikit penelitian yang mempelajari tentang pengukuran volume, khususnya mengenai bagaimana kemampuan visualisasi spasial siswa membantu siswa dalam memahami struktur tiga dimensi dari susunan kubus satuan tersebut.Oleh karena itu, penelitian ini mengajukan sebuaah pertanyaan penelitian, "Bagaimana kemampuan visualisasi spasial siswa mendukung kemampuan strukturisasi spasial mereka dalam mempelajari pengukuran volume di kelas 5 sekolah dasar?"

Sarama & Clements (2009) menekankan bahwa kemampuan strukturisasi spasial adalah faktor yang sangat penting dalam mempelajari tentang pengukuran *volume packing*. Di sisi lain, Ben-Haim et.al (1985) menyarankan bahwa agar siswa mampu menghitung volume dari sebuah benda yang tersusun atas kubus satuan, mereka harus dapat mengkoordinasikan dan mengintegrasikan sisi pandang dari susunan tersebut, baik dalam suusnan sebenarnya maupun dalam bentuk gambar. Kemampuan membaca gambar dua dimensi dari benda tiga dimensi adalah bagian dari kemampuan visualisasi spasial (Ben-Haim et.al, *ibid*). Titus & Horsman (2009) mendefinisikan visualisasi spasial sebagai kemampuan yang melibatkan keterampilan unuk memanipulasi dan merotasikan sebuah gambar ke dalam susunan lainnya dan untuk membayangkan keruangan yang ada di dalam sebuah benda padat tiga dimensi.

Dalam merancang rangkaian kegiatan instruksional, kami berpanduan pada ide pendidikan matematika realistik sebagai berikut. Kegiatan matematisasi diawali dengan situasi yang nyata untuk siswa. Tujuan digunakannya situasi kontekstual sebagai kegiatan pendahuluan adalah agar siswa dapat membawa pengetahuan informal mereka dari situasi yang dimaksud. Kegiatan – kegiatan tersebut diharapkan dapat menjembatani pengetahuan siswa dari tahap nyata ke tahap yang lebih formal dengan menggunakan model dan simbol. Di sisi lain, siswa dibebaskan untuk mendiskusikan strategi yang mereka gunakan dalam menyelesaikan masalah yang diberikan. Proses belajar siswa dilihat sebagai proses individu juga sosial. Interaksi sosial yang terjadi adalah bagian yang sangat penting dalam pembelajaran secara keseluruhan. Selain itu, kegiatan yang di desain tidak hanya mendukung siswa dalam belajar pengukuran volume, tetapi juga dalam belajar mengenai geometri.

Penelitian ini bertujuan untuk mengembangkan kegiatan pembelajaran dengan menggunakan kegiatan yang berhubungan dengan kemapuan visualisasi spasial untuk mendukung kemampuan strukturisasi spasial siswa dalam belajar mengenai pengukuran volume. Oleh karena itu, *design research* dipilih sebagai jenis penelitian yang tepat untuk mencapai tujuan tersebut. Terdapat tiga fase yang dilaksanakan, yaitu: *preliminary design, teaching experiment* dan *retrospective analysis*. Kami menganalisis data yang dikumpulkan

selama fase eksperimental dalam *retrospective analysis* yaitu, hasil lembar kerja siswa, catatan lapangan, dan rekaman video. Kami membandingkan pembelajaran siswa yang terjadi di kelas dengan *hypothetical learning trajectory* (HLT) yang kami kembangkan.

Terdapat tujuh aktivitas yang kami desain dalam HLT awal kami, yaitu: kegiatan "kemasan dodol", kegiatan "gambar susunan makanan", kegiatan *building blocks*, menghitung susunan kubus satuan, memperkirakan banyaknya kubus satuan, menentukan ukuran dari sebuah kotak, dan lembar kerja "kapasitas sebuah kotak". Dalam penelitian pendahuluan kami, kami mengamati bahwa siswa tidak memiliki banyak kendala dengan kegiatan yang kami desain. Kemungkinanya adalah sebenarnya ada beberapa masalah, namun belum Nampak ke permukaan. Kami kemudian mencobakannya pada kegiatan pembelajaran sebenarnya di dalam kelas, *teaching experiment*, untuk mengetahui kendala apa yang siswa hadapi sebenarnya. Untuk itu, kami memperbaiki beberapa kegiatan pembelajaran dan menambahkan satu kegiatan, yaitu: mengukur dengan satuan yang berbeda.

Dalam *teaching experiment*,terdapat tujuh pertemuan. Pada pertemuan pertama, kami menemukan bahwa beberapa siswa berpikir mengenai pengukuran linier atau pengukuran panjang dan tinggi dalam menentukan seberapa besar sebuah kotak atau benda. Tetapi, beberapa siswa telah menyadari bahwa dalam membandingkan kapasitas atau sisi dari benda, mereka membutuhkan satuan seperti kue atau makanan yang disusun dalam kotak tersebut. Pada pertemuan kedua, kami menemukan bahwa beberapa siswa menghadapi kesulitan dalam membuat gambar dua dimensi dari susunan benda tiga dimensi. Beberapa siswa menggambar sisi pandang yang berbeda dari susunan itu dan membuat interpretasi dari gambar itu, tetapi beberapa siswa lainnya yang berhasil menggambar susunannya sebagai suatu gambar kesatuan yang utuh memiliki interpretasi yang salah tentang gambar mereka. Menurut mereka jumlah susunan dalam gambar dan sebenarnya berbeda. Mereka cenderung menghitung kotak persegi dalam gambar yang mereka buat. Pertemuan kegita berkaitan erat dengan pertemuan kedua. Kini, siswa diminta untuk membuat bangunan atau susunan dari kubus – kubus satuan yang disipakan berdasarkan gambar yang diberikan. Siswa yang menggambar sisi pandang yang berbeda pada pertemuan sebelumnya, dengan mudah dapat menyadari bagaimana sisi pandang yang berbeda dapat merujuk pada benda yang sama. Beberapa siswa lainya menemukan kesulitan dalam menginterpretasikan gambar, khusunya sisi tampak samping. Mereka membangun susunan dari tampak depan dan tampak samping secara terpisah kemudian menggabungkannya bersama. Hal itu membuat mereka membutuhkan lebih banyak kubus satuan dari yang disediakan. Siswa lainnya cenderung membangun sisi luar dari bangunan kubus stauan dan kemudian mengisi bagian dalamnya.

Dalam kegiatan "menghitung susunan kubus satuan", beberapa siswa masih memahami susunan kubus satuan sebagai permukaan yang tidak terstruktur. Dalam menghitung jumlah kubus satuan, beberapa siswa hanya menghitung bagian muka dan belakang dari kubus satuan kemudian menghitung kubus satuan di setiap sisi, kanan, kiri dan atas. Siswa-siswa tersebut tidak dapat memahami dengan baik bahwa susunan kubus satuan dapat dilihat sebagai susunan baris atau kolom. Dan, dalam diskusi, siswa yang telah dapat memahami susunan kubus satuan sebagai susunan berlapis, baik secara baris maupun kolom, selalu menganjurkan teman – temannya untuk menggunakan strategi tersebut.

Sementara itu, kegiatan dalam pertemuan kelima seperti pengulangan pada pertemuan pertama. Tetapi, dalam pertemuan kali ini, siswa bekerja dengan kubus satuan yang kita gunakan pada pertemuan ketiga. Kami menemukan bahwa beberapa siswa masih menemukan kesulitan dalam membayangkan sisi interior dari sebuah kotak. Mereka mencoba mengiterasi kubus satuan dalam seluruh ruang kotak tetapi tidak dapat menemukan jumlah total dari pengukuran yang mereka lakukan. Tetapi, sebagian besar siswa cenderung menutupi bagian alas kotak kemudian memperkirakan berapa susunan yang mungkin hingga ke bagian atas kotak. Pada pertemuan keenam, para siswa belajar bahwa volume dari sebuah benda dapat direpresentasikan oleh beberapa angka yang berbeda tergantung dari ukuran satuan pengukuran yang digunakan, tetapi volumenya tetap sama. Jika mereka menggunakan satuan yang ukurannya lebih besar, amak jumlahnya akan lebih sedikit, dan sebaliknya. Para siswa juga menyadari bahwa dalam membandingkan volume dari dua benda, mereka harus menggunakan satuan pengukuran yang sama.

Temuan penelitian kami menyarankan bahwa agar siswa mampu menghitung volume dari benda yang tersusun atas kubus satuan, siswa diharapkan mampu memahami struktur dari konfigurasi kubus satuan tersebut. Kemampuan strukturisasi spasial siswa memungkinkan siswa melakukan cara yang terorganisasi dalam menghitung volume dari suatu susunan kubus satuan.

Selebihnya, kami merekomendasikan kepada guru di Indonesia untuk menggunakan pendekatan pembelajaran PMRI sebagai pendekatan pembelajaran matematika di kelas. Dalam kelas PMRI kami, penggunaan konteks telah dapat menstimulasi siswa dalam berpikir untuk menemukan cara memecahkan suatu permasalahan matematika. Siswa dapat emmbawa pengetahuan dan pengalaman informalnya untuk mendapatkan ide dalam menyelesaikan masalah. Dalam kelas kami, situasi susunan kemasan dapat mendorong siswa dalam menginvestigasi struktur dari susunan kemasan tiga dimensi.

"We cannot do great things on this Earth. We can only do small things with great love"
-Mother Theresa-

I dedicated this thesis with great love to many people surrounding me.

This thesis is my special dedication to:

My lovely husband for his never ending support.

Also, this thesis is dedicated to my great family: my mom, my grandpa, my uncle, my aunt and my sister for their great supports during my study.

PREFACE

First of all, I am very thankful to Allah swt. for all the great things He gave to me. In this opportunity, I would also like to say thanks to all people who supported me in doing my research, gave contribution and inspiration in writing this thesis and helped me in finishing my study.

1. I am so glad to be supervised by Dr. Frans van Galen, Prof. Zulkardi and Dr. Darmowijoyo because they gave supervision and assistance in writing this thesis. Also, they supported me by giving suggestions how to design the activities and how to bring the students' struggles to the surface. Therefore, I would like to express my gratitude to them.
2. The idea of this research was greatly influenced by my discussion with Dr. Jaap den Hertog, Dr. Dolly van Eerde and Prof. Koeno Gravemeijer. Thank you very much to them.
3. I would never join this program if Prof. Robert Sembiring, Prof. Marteen Dolk and the PMRI Team did not launch this program in 2009.
4. It was Dr. Ratu Ilma, Dr. Yusuf Hartono, and all lecturers and staffs in Mathematics Education Department of Unsri Postgraduate Program who gave a lot of supports during my study in Palembang.
5. I would never go to Utrecht University without support from Nuffic – NESO and Dikti. Thanks to Pak Dede Herlan, Bu Desi Dimas, Bu Shita Wulandari and all NESO staffs, also to Pak Ridwan, Pak Muchlas, Bu Istri, Bu Martha and all Dikti staffs for arranging scholarship for us.
6. Special thanks also for Drs. Swida Purwanto, M.Pd. and Dra. Pinta Deniyanti Sampoerno, M.Si, my lecturers in Universitas Negeri Jakarta who introduced me to PMRI and who gave me recommendations to the PMRI Team as one of the candidates of the IMPoME 2009 scholarship awardees.
7. It was Bu Rima Yunidar, the classroom teacher in grade 5 of SD Pusri, who gave a lot of contributions during the teaching experiment.
8. Last but not least, I would like to thank to all my friends in IMPoME Utrecht University – Universitas Sriwijaya for the days we spent together. I would never forget all your kindness.

I wish this thesis, as the result of my research, can give contribution to improve the mathematics education in Indonesia, especially in the volume measurement domain in grade 5 elementary school. And I hope other researchers can continue the unfinished part of this study as I discussed and recommended in the last chapter of my work to give even better contributions.

Palembang,   Mei 2011

Shintia Revina

TABLE OF CONTENT

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

CHAPTER I

INTRODUCTION

Measurement has been a central component of primary and secondary school curriculum around the world. During the primary grades, measurement is an important connection from number and operations to algebra and geometry (Charlesworth & Lind, 2010). In grade 5, volume is the focal point of the measurement education. However, Voulgaris and Evangelidou (2003) reported that students in grade 5 and 6 often have difficulty in fully grasping the concept of volume. In those grades is the transition period from primary to secondary education when more abstract methods for measuring volume are introduced.

Sarama & Clements (2009) revealed that what makes a measure of volume difficult is that it requires students to build their competence in spatial structuring, because the cubic unit in volume must be defined, coordinated and integrated in three-dimension. In particular, Ben – Haim et al. (1985) indicated the errors that students in grades 5-8 made on the volume measurement tasks with three dimensional cube arrays are related to some aspects of spatial visualization, such as the skill to "read" two-dimensional representation of solid objects. In that study, the answers students gave to solve the task tended to only count to either the number of faces, or the number of visible small cubes. The students seem not consider about the interior part of the object.

However, there is little research on volume measurement that relates the spatial visualization aspect and how students perceive the three-dimensional structure of the cube arrays. The structure of three-dimensional cubes array can be seen in terms of layers, either in rows or columns. In counting the number of cubes in a three-dimensional array, this structure allows students to determine the number of cubes in one layer and then abbreviate the enumeration by skip counting or multiplying it with the number of layers.

In doing so, the students need to coordinate and integrate the views of an array to form a single coherent mental model in visualizing the array. It means that students need to practice with concrete tasks in which they can well perceive the constructed views of the organization of a three dimensional rectangular array made of unit cubes before engaging with its pictorial representation.

In Indonesian primary schools, the volume measurement tasks related to three dimensional cube arrays are usually given directly in the pictorial representation without any concrete activity beforehand. Moreover, in teaching volume, teachers in Indonesian school directly give volume formula, length times width times height, and then the students can use it to solve any kind of problem related to volume concept. In that way, the students have no opportunity to use their own strategies to solve the problems. Then, their mathematical knowledge is not based on their common sense. Sembiring, Hadi and Dolk (2008) reported that one of the problems of mathematics teaching in primary education in Indonesia is the students' difficulties to comprehend mathematical concepts and to construct and solve mathematical representations from a contextual problem. The traditional teaching approach applied in the classroom makes mathematics even more difficult to learn and to understand. In this approach, students are drilled to solve the problems by only applying formulas.

It is challenge to improve the mathematics education in Indonesian primary schools. Realistic Mathematics Education offers an opportunity to change mathematics education in Indonesia. In Realistic Mathematics Education (RME), the students are allowed and encouraged to invent their own idea and use their own strategies to solve the mathematical problem given, not merely using the rules to solve it.

Considering the aforementioned issues, this study was aimed to develop classroom activities, which RME underlies its design, with the use of spatial visualization tasks to support students' spatial structuring in learning volume measurement in grade 5 elementary school of Indonesia. It was conjectured and expected that students' understanding can be built upon students' experiences in visualizing and structuring the space, and that therefore the students could gain more insight on how to measure the capacity or volume of that space. We pose a research question, "*how can spatial visualization support students' spatial structuring in learning volume measurement?*"

CHAPTER II

THEORETICAL FRAMEWORK

This chapter provides the theoretical framework related to the key concepts of this research. Literature was studied to find out what former studies have shown about the development of students' understanding of volume measurement. Furthermore, this literature is also useful as a basis to design a sequence of instructional activities about volume measurement. Since it is designed under the Realistic Mathematics Education environment, the literature about realistic mathematics education is also needed to explain and to investigate how the contextual situations could be shifted to more formal mathematics.

A. Volume Measurement

Research in the domain of linear, area and volume measurement has highlighted a number of general principles or concepts that underlie the understanding of measurement, namely unit iteration, conservation, attribute identification and the use of informal and formal units (O'Keefe & Bobis, 2008). The measurement of each attribute leads to a definite unit structure (the pattern formed when the units fill the object to be measured). Volume may be measured in two ways. In one method, the space is filled by iterating a fluid unit which takes the shape of the container. In this method, the unit structure is one-dimensional. In the second method, the space is packed with a three-dimensional array unit which is iterated in the third dimension. To differentiate these two methods, we shall call them volume (filling) and volume (packing) respectively (Curry and Outhred, 2005). Related to measurement of (packing) volume, spatial structuring competence is needed to be built because the unit must be integrated and coordinated in three-dimension.

Battista and Clements (1998) in their study found that co-ordination, integration and structuring appear to be required for students in the third, fourth and fifth grades to conceptualize and enumerate the cube units in three dimensional rectangular arrays. A developmental sequence was identified in which at the initial stage students focused on the external aspects of the array and perceived it as an uncoordinated set of faces. At later stage as they reflected on experience of counting or building cube configurations, students gradually become capable of coordinating the separate views of the arrays and they integrated them to construct one coherent and global model of the array.

Voulgaris & Evangelidou (2003) has shown that the development of students' understanding of volume measurement should be seen in a specific step by step sequence. Students need to practice with concrete tasks of increasing structural complexity through which they can acquire personally constructed views of the organization of the three dimensional rectangular arrays made of unit cubes before engaging with two-dimensional representations of divided or undivided rectangular solids. In this way, students move from perceiving the external visible aspects of the object to its internal structural organization in terms of units of volume measurement.

In the present study, a sequence of instructional activities of volume measurement is developed in a specific step by step as suggested by the aforementioned researches. In learning volume measurement, the five grade students experienced the following instructional sequence: (1) comparing the capacity of two objects using unit measurement, (2) relating the external visible part and the interior side of three-dimensional objects, (3) constructing a cube blocks building based on different views pictures, (4) enumerating the number of cube blocks needed to construct such an array, (5) predicting the number of cubes can be packed in an empty rectangular object, and (6) determining the possible sizes of an empty rectangular object which can hold certain number of cube blocks. By experiencing this instructional sequence, it is conjectured that students could gain more insight on how to measure the volume of the rectangular object and could perceive better understanding on the concept of volume measurement.

B. Spatial Structuring and Spatial Visualization in Volume Measurement

Sarama & Clements (2009) have emphasized students' spatial structuring ability as an essential factor in learning about volume "packing" measurement. Students' spatial structuring abilities provide the necessary input and organization for the numerical procedures that the students use to count an array of cubes. Using spatial structuring strategy allows students to determine the number of cubes in term of layers and then multiple or skip-count to obtain the total number of cubic units. In addition, Ben-Haim et.al (1985) suggests that, in order to be able to count the volume of an object made of small cubes, students need to be able to coordinate and integrate the views of an array either in real blocks arrangement or in drawing representation. On the other hand, the skill to "read" two-dimensional drawing representation of solid objects is a part of the spatial visualization ability (Ben- Haim et.al, *ibid*).

In general, spatial visualization can be meant as the ability to mentally manipulate two dimensional and three dimensional figures. Zacks, Mires, Tversky & Hazeltine (2000, in Hegarty and Waller, 2003) conceptualized spatial visualization as the ability to make object-based transformations where only the positions of the objects are moved with respect to the environmental frame of reference whereas the frame of reference of the observer stay constant. In addition, Titus & Horsman (2009) define spatial visualization as the ability that involves skill to mentally manipulate and rotate an image into another arrangement and to mentally imagine what is inside of a solid object.

In particular, Ben-Haim et.al (1985) reported that in a spatial visualization unit of instruction developed for training the students in middle grades about three-dimensional arrays construction, the students are asked to draw flat view of the isometric drawing of a cube building and then count how many cubes in the drawing. The finding of that study suggests that instruction in spatial visualization activities affected students' performance on enumerating the three-dimensional cubes array.

In the present research, spatial visualization was used to support students' spatial structuring in learning about volume measurement. Therefore, the instructional activities designed in the present study involved the spatial visualization tasks to help the students perceive their spatial structuring ability.

C. Realistic Mathematics Education

In the process of doing mathematics, Freudenthal (1991) emphasizes that students should be allowed and encouraged to invent their own idea and use their own strategies. In the other words, they have to learn mathematics in their own way. Freudenthal argued that mathematics is as 'a human activity'. Instead of giving algorithms, mathematics should be taught in the way where students can do and experience to grasp the concepts. Therefore, this study develops an instructional unit on teaching and learning volume measurement in which the students could gain more insight about how to measure the volume of an object through experiencing a sequence of meaningful activities instead of only memorizing the volume formula.

In designing the sequence of instructional activities, we consult to five tenets of realistic mathematics education (Treffers, 1987). On the other hand, the sequence of activities designed in this study is only a part of longer series of learning trajectories in learning volume measurement. And, the descriptions of the five tenets of RME apply to mathematics learning as a process that will take months or years and are not necessarily applicable to a short series of activities. However, in this study, we try to describe the five tenets of RME as following:

1. Contextual situation

In the first instructional activity designed in this study, a concrete contextual situation is used as the preliminary activity. The mathematical activity is started from a situation that is experientially real for students. The aim of using contextual situation as preliminary activities is that the students can bring in their (informal) knowledge of the situation.

2. Bridging by vertical instrument

   The second tenet of RME is bridging from a concrete level to a more formal level by using models and symbols. In the “food packages pictures” activity, the students can develop their own model or symbol to represent the arrangement of the packages in three dimension arrays. In this activity, they also prepared for the next activity in which they work with more formal objects, namely cube unit blocks.

3. Students’ own constructions and productions

   In each activity, the students are free to discuss what strategies they are going to use in solving the task or problem given. In the “building block” activity, they have to make a construction made of cube unit blocks in arrays based on pictures they have. In doing so, they developed the strategies that works for them. In line with experiencing with their own blocks construction, they also have to find their strategies in enumerating the three dimension cube arrays.

4. Interactivity

   The learning process of students has to be seen as both individual and social process. Therefore, social interaction emerging in the classroom is important part of the whole class performance. Working in groups built a natural situation for social interaction such as developing their strategies in drawing the representation of the packages arrangement, in constructing the cube unit blocks and in predicting the number of blocks that could fit in an empty rectangular box.

5. Intertwinement

   Intertwinement suggests integrating various mathematics topics in one activity. The instructional activities designed not only support learning for volume measurement, but also support the development of spatial ability which is important in geometry domain.

The implementation of the second tenet of RME produced a sequence of models that supported students' understanding of the concepts of volume measurement. Gravemeijer (1994) described how models-of a certain situation can become models-for more formal reasoning. We did not go to very formal part of volume measurement education (i.e using standard unit of centimeter cubic or applying formulas of length times width times height), but we try to describe the implementation of the four levels of emergent modeling in the present study as follows:

1. Situational level

   Situational level is the basic level of emergent modeling where domain-specific, situational knowledge and strategies are used within the context of the situation. In this study, comparing the capacity of *dodol* boxes is the contextual situation in which the students solve the problem related to their daily-life situation related to the capacity or volume of an object.

2. Referential level

   The use of models and strategies in this level refers to the situation described in the problem. The referential level is the level of models-of. In the present study, the "food packages pictures" activity encourages students to shift from situational level to referential level when students need to make representations (drawings) as the models-of their strategies.

3. General level

   In general level, models-for emerge in which the students need to develop a model that could be used in different situations. Student-made blocks construction of cube unit blocks could emerge this general level. The students could use the cube unit blocks as model-for to represent any kinds of object that they want to arrange in three dimensional rectangular arrays.

4. Formal level

   As we stated before that we did not really go into very "formal" level. We prefer to use the term "more formal" level instead using "formal" level. In this phase, the students could use their experience with the three previous levels to do reasoning. In "predicting the number blocks" activity and "determining the possible size of the box" activity, the students could focus on the discussion of concepts of units and covering the space.

D. Emergent Perspective

Before starting the process of learning, it is conjectured that the students have their own belief about their own roles, the others' roles, the teacher's roles and the mathematics that students learnt. In this study, during the process of learning, the teacher initiate and develop the social norms that sustain classroom culture characterized by explanation and justification of solution, and argumentation: attempting to make sense of explanation given by others, indicating agreement and disagreement, and questioning alternatives in solutions in which a conflict in interpretation or solution has become apparent (Gravemeijer & Cobb, 2006).

In this research, we focus on the normative aspect of mathematics discussion specific to students' mathematical activity. To clarify this distinction, we use the term socio-mathematical norms rather than social norms. We describe socio-mathematics norms as normative understanding of what counts as mathematically different, mathematically sophisticated, an acceptable mathematical explanation and justification. Students develop their ways of judging, whether a solution is efficient or different, and the teacher is not the only one who decides the acceptable solutions. In this way, socio-mathematical norms are negotiated as the teacher and students participated in the discussions.

E. Volume Measurement in the Indonesian Curriculum

Volume measurement in Indonesia is taught in the fifth grade of elementary school. In grade five, they learned about how to measure volume of a cubical and a rectangular object. The competences have to be mastered by the students are described in the following table:

Table 1. Teaching Volume Measurement in Indonesian Curriculum

Standard Competence	Calculating the volume of cubical and rectangular object and solving daily-life problems related to the concept of volume measurement
Basic Competence	• Calculating the volume of cubical and rectangular object • Solving daily-life problems related to the concept of volume measurement
Indicators	• Identifying three-dimension cubical and rectangular object • Identifying unit of volume measurement • Calculating the volume of object made of cube blocks unit • Calculating the volume of cubical and rectangular object • Solving daily-life problems related to the concept of volume measurement

As shown on the table, the teaching and learning process of volume measurement in the Indonesian curriculum covers some indicators related to what students learn in the sequence of activities designed in this study. In the present study, in order to gain more insight about how to calculate the volume of a cubical or rectangular object, there are some concrete tasks developed before they work with pictorial representation of an object made of cube blocks or solve story problems related to the concept of volume measurement.

CHAPTER III

METHODOLOGY

This chapter describes the methodology of this research. They are research methodology, research subject, data collection, and data analysis, including reliability and validity of the data.

A. Research Methodology

The main aim of this research is to contribute to an empirically grounded instruction theory for volume measurement. In this study we are interested in how spatial visualization can support students' spatial structuring in learning volume measurement. Therefore, a sequence of activities was developed as means to improve educational practices in volume measurement for grade 5 of elementary school in Indonesia. For that purpose, design research was chosen as an approach to answer the research question and achieve the research goals. Gravemeijer & Cobb (2006) define design research by discussing the three phases of conducting a design experiment. The phases in this design research are described as following:

a. Preparing for the Experiment

The goal of preliminary phase of a design research experiment is to formulate a conjectured local instructional theory that can be elaborated and refined while conducting the experiment. In this phase, a sequence of instructional activities containing conjectures of students' strategies and students' thinking was developed. The conjectured hypothetical learning trajectory is dynamic and could be adjusted to students' actual learning during the teaching experiments. Carrying out pre-assessment or pilot experiment before the teaching experiments such as interviews with teacher and students, and whole-class performance assessments are useful to investigate pre-knowledge of the students that would become the research subjects in the upcoming teaching experiment period. Charting this pre-knowledge of the students is important for the starting point of the instructional activities and adjusting the initial Hypothetical Learning Trajectory (HLT).

b. Experimenting in the Classroom

This phase proposes mathematical teaching sequences applied in the class experiments. In this study, the teaching experiments were conducted in six lessons. The teaching experiments emphasize that ideas and conjectures could be adjusted while interpreting students' learning process. Before doing a teaching experiment, teacher and researcher discussed about the upcoming activities. And after each lesson, the researcher and the teacher also made reflection of the whole class performance.

c. Conducting Retrospective Analysis

We analyzed the data that we got from the teaching experiments and used the result of the analysis to develop the next design. The retrospective analysis result became reference to answer the research question.

B. Research Subject and the Timeline of the Research

Thirty two students and a teacher of grade 5 in an Indonesian elementary school in Palembang-Indonesia, that is SD Pusri (Pupuk Sriwijaya) Palembang, were involved in this research. The students were about 10 to 11 years old. SD Pusri Palembang has been involved in the *Pendidikan Matematika Realistik Indonesia* or Indonesian Realistic Mathematics Education project since 2010.

The experiment of this research was divided into two parts, namely preliminary experiment and teaching experiment based on improved HLT. In the first part, there were 5 students involved. We tested some of the activities in our initial HLT to these students. We wanted to investigate the students' thinking of the tasks and problems in the HLT and tested our conjectures about it.

In the second part, we first improved the initial HLT and then test the improved HLT to the whole class. Here, we had the “real” teaching experiment. Those students who involved in the first part also involved in the second phase. We realize that there might be some problems caused by this situation. These students might think that they had already known and experienced the activities and then gave the same answer as in the first phase. However, in the second phase, the students worked and discussed in groups, and then present their ideas in front of their friends. They did not have those activities in the first phase. At that time, we merely asked them about their reactions and their thought about the tasks and the problems without further discussion. We did interview them to ask for more explanation without any justification.

As we mentioned that in the second phase, students worked in groups to discuss their ideas in solving tasks and problems in the activities. From 32 students, there were 7 groups of 4-5 students. And those 5 students worked together in one group. By doing so, there might be 6 new ideas from other groups that firstly experiencing the activities and although those five had already known some activities, they had to discuss the answers among them and then present the result of their discussion, no more as individual. More about the role of these students, we can refer to the theory of constructivism about learning from community.

As Woolfolk (2007) said, social constructivism – such as Vygotsky believe – is a social interaction, cultural tools, and activity that shape individual development and learning. By participating in a broad range of activities with others, learners appropriate the outcomes produced by working together; they acquire new strategies and knowledge in their world. Rather than seeing learning process as individual construction as in constructivism, social constructivism see the learning process as a social constructed knowledge. Hence it is built on what participants contribute and construct it together. There is collaboration between the students during the process of learning and it occurs through socially construction opportunity. It means that the teacher gave equal opportunity to each group in the class to speak out their ideas in the discussion. So, the possibility that they were dominant in the class discussion is very small.

Based on our explanation above about the planning of our research, we can summarize the timeline of this research on the table as follows:

Table 2. Timeline of The Research

DESCRIPTIONS	DATE
Preliminary Design	
Studying literature and designing initial HLT	2 September 2010 – 5 January 2011
Discussion with teacher	25 January 2011
Preliminary Experiment	
Classroom Observation	26 January 2011
Pre-Test	27 January 2011
Try out "Larger Boxes" Activity	28 January 2011
Try out "Picture Packages" Activity	29 January 2011
Try out "Building Blocks" Activity	1 February 2011
Try out "Count the Blocks" Activity	2 February 2011
Try out "Predict the number of blocks" Activity	4 February 2011
Try out "Determine possible sizes of the box" Activity	4 February 2011
Try out "the box capacity" Worksheet	5 February 2011
Analyzing the Preliminary Experiment and Improved the HLT	
Discussion with Teacher	7 February 2011
Preparation for Teaching Experiment	14 – 18 February 2011
Teaching Experiment	
Lesson 1: "Larger Boxes" Activity and class discussion	24 February 2011
Lesson 2: "Picture packages" Activity and class discussion	25 February 2011
Lesson 3: "Building Blocks" and "Count the Blocks" Activity	26 February 2011
Lesson 4: Class Discussion	28 February 2011
Lesson 5: "Predict the number of blocks" Activity and "Determine possible sizes of the box" Activity	1 March 2011
Lesson 6: Measuring volumes with different unit sizes	3 March 2011
Lesson 7: "the box capacity" Worksheet and class discussion	4 March 2011
Final Assessment	4 March 2011

C. Data Collection

In this study, the data such as video recording, students' works and field notes were collected during the teaching experiments. We videotaped the activities and interviewed some students. We analyzed the data from the video recordings and students' works to improve our HLT. More precisely, the data collection of this research is described as follows:

1. Video

   The students' works and strategies in comparing *dodol* boxes, drawing the food packages and building the blocks were observed by video. Short discussion with students and class discussion were recorded to investigate students' reasoning for their idea. The videotaping during the teaching experiments was recorded by a dynamic camera to record the activities in classroom.

2. Written data

   The written data provides information about students' works on the worksheets given in solving the volume measurement problems. These data were used for investigating students' achievement because students' learning processes were observed through videotaping and participating observatory. Besides students' works during the teaching experiment, the written data are also including field notes, the results of assessments including the final assessment and some notes gathered during the teaching experiment.

D. Data Analysis, Reliability and Validity

We analyzed the data collected during the experimental phase in retrospective analysis: students' works, field notes, and video recording. We compared the students' actual learning and the conjectured HLT. Related to video registration, we registered the fragments in the videos in which the learning took place, leave out the irrelevant parts of the videos and transcribe the conversations in the interview sessions with students and teacher. The analysis of the lessons was done by analyzing the daily bases activities and analyzing the whole series of lessons.

The data analysis was accomplished by the researcher with cooperation and review from supervisors to improve the reliability and validity of this research.

1. Reliability

The reliability of this design research is accomplished in qualitative way. The qualitative reliability is conducted in two ways, data triangulation and cross interpretation. The data triangulation in this study involves different sources: the videotaping of the activities, the students' works and field notes. The parts of the data of this research were also cross interpreted with supervisors. This was conducted to reduce the subjectivity of the researcher's point of view.

2. Validity

In this study, the validity refers to internal validity and external validity. The internal validity of this research was kept by testing the conjectures during the retrospective analysis. Meanwhile, the external validity is mostly interpreted as the generalization of the result. The challenge is to preset the results in such a way that others can adjust them in their local contingencies (Bakker, 2004).

CHAPTER IV

HYPOTHETICAL LEARNING TRAJECTORY

Hypothetical learning trajectory is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students work on to achieve the goals), and a hypothetical path that describes the students learning process. Simon (1995, as cited in Gravemeijer et.al, 2003) describes what elements are in the hypothetical learning trajectory:

> "The consideration of the learning goal, the learning activities, and the thinking and learning in which the students might engage make up the hypothetical learning trajectory…"

Therefore, in this chapter we would like to describe the starting point of the students, our learning goals, activities that allow us to reach the goals and the conjectures of students' thinking in the HLT.

Our departure point in this study is students' current knowledge and ability. Most of students in grade 5, of course, have good ability in counting, either with one by one counting or more abbreviated counting such as using structures or skip counting. In calculating the volume of an object made of cube units, the students can count the blocks as unstructured units (one by one) or as structured units in terms of layers either in rows or in columns by skip counting or multiplying.

In grade four, the students already learned about the properties of simple three-dimensional shapes such as cubes and rectangular prism. They have experienced in making representation of those three-dimensional objects into two dimensional drawings. So, they are not new with the cubical or rectangular boxes or its two-dimensional drawing or pictures. We assume that these students also have knowledge or experience with packing situation such as snack or noodles packages situation. We also expect that they are able to estimate the capacity of a box (i.e. how many pieces of snack or noodles, in a certain size, can be put inside a box).

In this HLT, there are several learning goals expected to be reached by the students during the series of lessons in three weeks period. To reach the goals formulated, we designed a sequence of instructional learning for volume measurement which consists of seven activities, which are elaborated as follows:

A. "*Dodol* Packages" Activity

Goal: Students use unit measurement to compare the capacity of two objects.

Description of activity:

The teacher showed two boxes in different shape. One is higher but the other one is wider. The students were asked to find the way to compare those two boxes in order to know which one is larger. There might be some students who had idea about measuring its height or to fill in the boxes with cakes (since the boxes were usually used as cake boxes) Then, the teacher told them that those boxes usually used as packages of *dodol*, one of traditional snacks from Indonesia. Then, she asked the students to help her to figure out which box is the larger. The teacher then asking question, "Now you have some pieces of *dodol* to be packed inside the boxes, then how do you know which boxes is larger?" The teacher provided the sample of pieces of *dodol* as shown in figure 1.

Figure 4. 1. The sample of *dodol*

Conjectures of students' thinking and Discussion:

In the first discussion, they might come up with ideas to put one boxes to the other. However, they can't do that since the size of the boxes is arranged in such manner so that they can't directly compare it. Some students might answer that when you have higher box it means you have larger box. However, it is expected that some students have idea to fill the boxes with something, for example with cakes or snacks because the boxes we showed to the students usually used for food packaging.

In the second tasks, the number of pieces of *dodol* given is restricted so the students can not fully cover up whole space of the box. However, the students might think that the box which can hold more pieces of *dodol* is the larger. The students might come up with these strategies:

- They might try to cover up the boxes and use as many pieces of *dodol* as possible to know how many *dodol* can be put inside. Then, for example, they can say that they could cover half of the space inside the box then they can predict that when they double it they got the number of pieces of *dodol* needed.
- They might cover up the floor of the boxes and find out how many layers needed until the top of the boxes. These students counted the number of pieces of *dodol* needed for one layer and then multiply it.
- Some students may be come up with covering only the border of the floor of the boxes and find out how many they need to cover up the floor (these students apply their knowledge about area in grade 3). And then they do predict or find out how many layers do they need to cover up until the top of the boxes.

However, if the teacher found that it is too difficult for students, teacher can ask the students what the difficulties are and what they need to be able to compare those boxes. The students might come up with ideas to have more pieces of *dodol* so that they can cover up whole space of the boxes. By comparing how many pieces of *dodol* can be put inside the boxes, then they might think that it is the way to be sure which box is larger. The important thing is the discussion. It is expected that in discussion, the students realize that in comparing the capacity of two objects, they need a unit to measure the capacity. By knowing how many units can be put inside in each box, we can compare the capacity of the boxes. Further, it is expected that in determining the capacity of an object, such as a box, we have to emphasize to students that we need to consider not only the height of the box because we have to measure the capacity, the inside/interior part of the object not the length (or height) of it.

B. "Food Packages Picture" Activity

Goal: Students can relate the visible part and interior side of a three-dimensional objects arrangement.

Description of Activity:

There are construction of packages of *teh saring* (tea boxes) or others food packages arrangement on the tables in front of the class.

Figure 4. 2. Sample of the food packages arrangement

The packages are arranged in rectangular arrays with two or three layers high as shown in figure 2. The students worked in groups and each group worked with different constructions of packages. For instance, one group works with construction of *teh saring* boxes, one group works with construction of wafer bars, etc. Actually, the teacher told them that she is assigned to make a report about the new stocks of food for the teachers.

The food packages in front of them are the things she has to report. However, the headmaster wants to have a picture of the packages so he can check the number of packages by seeing the pictures. The task for the students is to help the teacher to draw a representation of the packages arrays from a view in which someone who sees the drawing can know the numbers of packages by only seeing the drawing.

In the last 30 minutes of class period they can put their works on the wall and then they take turn to predict the number of packages in each drawing of the other groups. They have to explain how they get their answer.

Conjectures of students' thinking:

When the students draw a representation of a view in which they can not really see the number of packages in total, they have to consider that every view (front, top and side view) is well represented in their drawing so that the other friends can know the number of packages. We conjectured that some students might try to draw the layers to explain to others about the situation while some students still have difficulties in representing the layers in their drawing.

In the other hand, in making prediction of other friends' drawing, it is expected that they have experience with their own drawing so that they can explain how they enumerate the number of packages in each drawing. To count correctly, they have to relate the part that can be seen (visible) in exterior and the interior side of the packages. By making such a good representation they can give information not only about exterior side of the arrays but also the interior, what are inside, of the arrays arrangement which is not visible.

C. "Building Blocks" Activity

Goal: Students can construct a cube-blocks building based on different views pictures.

Description of activity:

The students got pictures of cubic unit blocks from front, top and side views. They have to make a construction made of cube blocks based on the pictures or drawings given (they work in groups). The students had plenty of cubic unit blocks (the teacher prepared enough materials). Then, the task is to check whether their constructing is suitable with each picture they have.

Conjectures of students' thinking:

- There might be some students who start from front view and then they continued their work by making column to side and fill in the interior until get the right view from side and top.
- There might be also some students who start building from top view, they built layers and just need to see the front view to know how many layers they need.
- There might also some students who really build the block without structuring, they only build the blocks one by one and at the end they match it with the pictures from different views they have.

If the students find difficulties to build the blocks with those different views, teacher can give help by giving a picture in which they can see them (the blocks construction) together. After they built their construction, they were asked to make a drawing of it in which in the drawing they could see all views in a single coherent picture. We predicted that some students might have difficulties in relating those three views together, especially the side and top views.

D. Counting the Cube Blocks

Goal: Students can count the number of cube blocks in a 3D cube blocks construction

Description of activity:

The teacher reminded the students about their previous activity in which they build a construction made of cube unit blocks. Their task in this activity is to count how many cubic blocks unit are in their friends drawing and in the real constructions. They are allowed to take apart their construction and rearrange the construction if needed. The question for them is: *"How do you count the number of cube blocks in your friends' drawing and in their real construction?", "Why do you use that strategy in counting the blocks?", "Do you have another strategy?"*

Discussion:

In the discussion, the teacher reminded about the two previous activities. The students can relate their two previous activities. The one in which they make block constructions based on the views given and the other one in which they enumerate the number of blocks in such a construction. Perhaps, some students are influenced by how they build the blocks in enumerating the cubes array. In this discussion, each group can share their strategy to structure the cube blocks arrays and to count them.

If there are some students who can see the structures in the arrays, they can share with the others who can't see the structures. It is expected that some students can build the structure of arrays in term of layers. Then the teacher can stimulate the other students to see the structures by asking "*How many layers of blocks do you see in their construction*? *How do you know that?"*

By having such a visualization of space, it is expected that the students can relate the visible units and interior side of the arrays. By having those activities, it is also expected that they have enough experience with the structures of cube arrays in the space.

In the next activity which is more formal (more abstract) they have to imagine how to fill an empty box which were packed with cube unit blocks.

E. Predicting the Number of Blocks

Goal: Students are able to estimate the number of cubic units can fit in an empty rectangular object (box).

Description of activity:

Each group of student got an empty box as shown in the picture and some cubic unit blocks. The students have to predict the number of cubic unit blocks can be put into the box. The teacher restricted the number of cube unit blocks used to do the prediction otherwise there might be some students who cover whole space by the cube unit blocks. The task is to solve this problem: "*Estimate how many small cubic unit blocks can be put in the empty box!*"

They are asked to make a written explanation of their prediction (poster, etc). The students can check their answer by covering the space by using the blocks at the end of the activity after all groups already give their answer and explanations.

Figure 4.3 The boxes and the cube blocks

Conjectures of students thinking:

- There might be some students who use some cubic block as the unit to iterate. They need it as the real representation and try to move the block along the space in the box since they are not allowed to cover the space by using all the provided blocks. They can relate to the layers or columns they had experienced with or just count one by one until all the space is filled.

- There might be some students who draw the box in the paper and then also draw the unit cubes in it in such a way that the box looks like a box made of the cubic blocks. They use their mental image of the previous activity to just imagine if there are some cube blocks in the box and then count the arrays as their experience before.

Discussion:

In the discussion, the students can share their strategies in predicting the number of cubic unit blocks can be packed in the given box and whether it is correct prediction or not. If it is correct, they have to explain why they use that strategy, or may be they have another strategy. If it is incorrect, why they think it can be wrong. The explanation and discussion can stimulate the students to really get the meaning of the activity they had. Here, the teacher can introduce the term of volume of an object (box) which represents the number of cubic units can be packed in the interior side of the object. In the discussion, it is also expected that the students can perceive the idea of measuring volume as covering space. It is conjectured that in measuring the volume of an object they arrange a number of similar cubic units to cover the attribute of the measured objects and iterate the cubic unit from one to another end of the measured object.

F. Determining the possible size of the boxes

Description of activity:

The students worked in groups and each group got some cubic unit blocks. The task is to solve this problem: "*If the volume of a box is 24 cubic block units, what is the possible size of the box that can hold all the blocks? Is there only one possibility or more than one? How do you know?"*

Conjectures of Students' thinking:

In solving the first problem, most of students tried to build a rectangular arrays construction of the blocks since they already know that the volume of an object is the number that represents how many cubic unit can fit in inside the object and also since they have some experience with building the blocks. The teacher can also remind them about the term volume itself. The strategy that students use might be as following:

- The students who can perceive the columns or rows (layers) structures might use layers of four (two times two), layers of six (two times three), etc to arrange the 24 blocks (since 4 and 6 are quite familiar to number 24). They can draw representation of their idea to give explanation to the teacher and the other friends.
- Some students just tried and tried until they get a "good" rectangular arrays construction made of the blocks.

G. "The Box Capacity" Worksheet

Goal: Students are able to solve problems related to measure the volume of a rectangular object.

Description of activity:

The students worked in pairs and they got a worksheet with some problems related to volume measurement concept. As the result of this activity we hope students were able to apply their knowledge and their experiences in the previous lessons to solve some problems related to measuring volume of rectangular boxes.

For instance, in the worksheet, the students had a question with a picture of soap bars arrangement in the box and are asked to count how many soap bars in the picture, a question related to predict the number of packages can be put inside a box if they know the number of packages in the base of the box and the number of layers needed to cover up the box. There is also a question about the possible height or the possible number of layers of pieces of cakes in a box if they know the volume and a question about calculate the volume of a box if the size of length, width and height of the packages arrangement is given, etc.

Conjectures of students' thinking:

We conjecture that students used their spatial visualization ability in reading the two-dimensional representation or drawing of three dimensional objects arrangement to solve the problem which they have to count the number of soap bars in the drawing. In counting the number of bars, some of them structured it in the term of layers. However, it is also possible that some students still count the bars unstructured, one bar by one bar. They have to apply it also in the next problems. They have to be able to imagine the situation of packing the noodles and packing cakes in the other problems. They can also use their experience in predicting the volume and also have to apply their knowledge about volume as covering space. They can first cover up the base and then iterating the layers until cover up whole space of the box. In the discussion, the students can talk about their strategies in solving the problems. For students who still have questions can ask other friends or the teacher what is still unclear for them.

CHAPTER V

RESTROSPECTIVE ANALYSIS

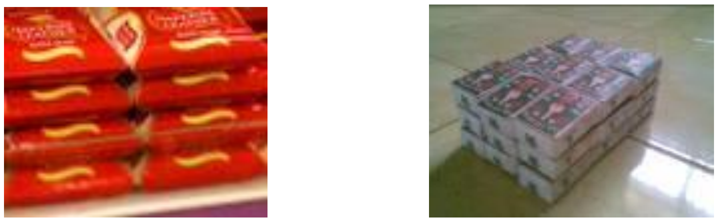
In this chapter, the retrospective analysis of data collected from pre-test, the preliminary experiment, the teaching experiment activities, and final assessment were described. The result of this research is the underlying principles explaining how and why our design works. The hypothetical learning trajectory served as a guideline in the retrospective analysis to investigate and explain students' thinking in learning volume measurement.

A. Pre-Assessment

The pre-test was aimed to know students' current knowledge and ability. In grade four, the students have already learned about the properties of simple three-dimensional shapes such as cubes and rectangular prism. They have experienced in making representation of those three-dimensional objects into two dimensional drawings. So, they were not new with the cubical or rectangular boxes or its two-dimensional drawing or pictures. The teacher's interview signified this fact. She explained that the students had learnt those materials in grade 4.

By giving this pre-test, we wanted to know where our departure point should be. We also wanted to detect what are the difficulties of the students. More specific, the problems in this pre-test wanted to know how the students read off the two-dimensional drawing of three-dimensional object and how they predict the capacity of a box. The students worked in pairs, and the pre-test consists of three problems as follows:

Problem 1: Problem one was about the arrangement of concrete objects. The first was about the arrangement of soap bars and the second was about the arrangement of tea boxes.



(a) (b)

Figure 5.1 Problem 1: The pictures of soap bars and tea boxes arrangement

The students were asked how they calculate the number of soap bars and the number of tea box in the picture 5.1 (a) and (b). Most of the students tended to multiply the rows and the column of the first picture, 4 times 2, but some of them said that they calculate it one by one until get 8 bars. In the second picture, only 6 of 16 students could answer correctly. Some write 9 x 3 or 3 x 9. Some students explained that there are 9 tea boxes on the top, 9 tea boxes on the bottom part and in the middle also 9. Therefore, all together are 27 tea boxes. The other students who gave incorrect answers write that there were 54 tea boxes and some also found that there were 45 tea boxes.

Problem 2: The second problem is about the cube blocks arrangement. It is the pictorial representation of cube blocks arrangement.

(a) (b)

Figure 5.2 Problem 2: Three-dimensional cube arrays

Same as in problem 1, the students were asked how they calculate the number of objects, the cube blocks, in the pictures. Only 2 out of 16 students' answers were correct for the problem in picture 5.2 (a), 9 blocks. Also those students can answer correctly the problem in picture 5.2 (b). One of them explained that he calculate 4 x 3 x 3 and the other one said calculate 12 x 3. The second students saw it in the term of column. One column is 12 and he saw 3 layers so he multiplied it.

The other students had various answer for problem (a) such as: some of them multiply 3 x 3 x 3 makes 27 and some of them answer 15. The students who answered 15 tended to calculate the number of squares they saw in the pictures. For problem (b), the students also gave some different answers, 33 and 54. The students who answer 33 tended to calculate the squares as they did in answering problem (a). Meanwhile the students who answered 54 explained that they calculated the squares on the top, front and right sides as they saw in the picture and then they add the squares in the left and bottom parts. So they have 18 rows – 3 on the top, 3 on the bottom, 4 on the front, 4 on the left and 4 on the right sides – of 3 squares.

From problem 1, we could see that the students seemed have difficulties in determining the number of concrete objects in the pictures of tea boxes. The tea boxes seemed more complicated that the soap bars arrangement On the other hand, they have difficulties in determining the number of blocks in problem 2 which is more abstract. They tended to calculate the number of squares rather than calculate the number of blocks. It indicates that they need more concrete task before they work with pictorial representation of the objects that are arranged in three-dimensional arrays.

Problem 3: The last problem is about how to estimate the capacity of the boxes.

(a) (b)

Figure 5.3 Problem 3: Estimate the number of Blocks

The students were asked to estimate the number of blocks can be put inside those boxes. There are also only 2 pairs of students who are able to answer it. One of them can predict correctly the number of cubes block that can be put inside the box. They explained that they do 9 times 3 in the picture 5.3 (a) and 6 times 3 in the picture 5.3 (b). These students can well perceive the structures of the boxes already. The other one explained that they have 11 cubes and they needed 16 more cubes to fill in the empty space, so all together made 27. However, this student wrote that he needed 18 to cover up the second box but then he added 8 to it. He explained that 8 were the blocks already there. And then they added 18, the blocks needed to cover up whole space of the box, and all together is 26.

The other students answered that they need to cover up the picture (a) with 2 times 8 blocks. They explained that because they saw 8 blocks on the base and then 2 blocks above the base. These students also answered that they need 2 times 6 blocks in picture (b). The answers of the other students indicate that they still had difficulties in predicting or estimating the number of cube blocks that can be put inside a box. Our conjecture was the same as the previous problems that they need more concrete task before working with the pictorial representation of the three-dimensional object arrangement.

B. Preliminary Experiment

We tried out the activities in our initial hypothetical learning trajectory. Based on the pre-test results and our initial hypothetical learning trajectory, we decided to start with some concrete tasks before the students work with pictorial representation of an object made of cube blocks in learning volume measurement. We worked with 5 students and our investigation was focused on finding out how the students compare two boxes with different sizes, draw the three-dimensional arrangement of the objects, read off the two-dimensional pictorial representation of three-dimensional object and how they predict and then determine the volume of a cube or a rectangular object. The students who worked with us were Juan, Aulia, Farish, Fadillah and Syahrul. Based on the teacher, they have different academic ability: Syahrul is representing the high achiever students; Farish, Aulia, Juan and Fadillah are representing the average students. The result of this pilot experiment would give us feedback for the improvement of our hypothetical learning trajectory.

Activity 1: "Larger Boxes" Activity

In the first activity of the preliminary experiment, we tried out the "larger boxes" activity. In this activity, the students were asked to compare the capacity of two objects with different sizes. First, we show them two empty boxes as shown in the figure 5.4 and asked them how to know the larger boxes from those two. We predicted that some students thought that the higher box was the larger and some others might have idea to fill in the boxes with cakes or sweeties because the boxes we shown are usually used as the packages of snacks or cakes.

Then, to follow up the students' idea of putting cakes inside the boxes, we would prepare some pieces of cakes (*dodol.)* We predicted that they compared the boxes by calculating the number of *dodol* can be put inside the boxes and then the box that can contain more is the larger one. We did not give enough number of *dodol* to cover up the boxes so that they had to estimate the number of *dodol* needed.

Box A   Box B

Figure 5.4 Sample of Cartoon and Plastic Boxes

The following is a segment from our video and audio recording.

Researcher : Telling that she has two boxes with different sizes. She showed the boxes. And then she asked the students to figure out how to know which box is the larger one.

All students : (Point out to box A)

Fadillah : This one (point out to box A) because it is wider, and it can contain more.

Researcher : How do you know that?

Fadillah :I saw the size of the width.

Aulia : I saw that the sides are bigger (point to box A), and the other one has shorter.

Researcher : How do you know that?

Syahrul : I measure the sizes, the width and the height.

Farish : I measure the area and then the width and the height.

(while Farish talking, Syahrul put box A on the top of box B)

Researcher : How to know which boxes is larger, Farish?

Farish : Measure the sizes and then put something inside.

Researcher : For example?

Farish : Put cakes.

Researcher : So, to know the larger box you will put the cakes into the boxes?

Juan : This one (point to box A) can contain more pieces of cakes (kue).

Researcher : Are you sure?

Juan : Yes.

Researcher : Ok. Let us prove it.

(Then, the researcher showed them some pieces of dodol, one of traditional snacks from Indonesia. We prepare the "dodol" because we predicted that they will asked for something as unit to put inside the boxes)

Researcher : Do you know what these are?

All Students : *Dodol*.

Researcher : Ok. Now, I have some pieces of *dodol*. Can you estimate how many *dodol* can be put inside each box and then determine which one is larger?

(The researcher gave each student some pieces of dodol)

Researcher :So, how do you know the capacity of this box (box A)?

Farish : I put the *dodol* and then multiply it.

Researcher : So, how many *dodol* there?
Farish : 16
Researcher : 16? So, to fulfill whole space of the box, how many *dodol* do you need?
Farish : 16 times 4 is 64.
Researcher : And how about the other one?
Farish : I put the *dodol* and the same.
Researcher : How many do you need?
Farish : 10 more. (he already had 10 in the box B).
Researcher : How about you Juan how do you know the capacity of the boxes?
Juan : We can fulfill it with the *dodol* so we can know the capacity of the boxes. This one I have 10 and then it can put more above so 20.
Researcher : And the other one?
Juan :The other one I can put 14 so, I need 14 times 4 to the top.
Syahrul : I put *dodol* inside. I multiply 16 times 3.
Researcher :What is 3 means?
Syahrul : The layers.
Researcher : and 16?
Syahrul : On the base.
Researcher : How about the other boxes?
Syahrul : 10 x 2. And 2 is the above part.

Based on our observations, we can make the following conclusions. Throughout this activity, students have shown that they are able to estimate and compare the capacity of the boxes. When we asked which box is larger, all of them directly point out to box A. They had different reasons, such as the sides are bigger, the width and the length are longer, and two of them said that we can put cakes to know the capacity of the boxes.

At first time, they immediately answered that box A is the larger since they can see that the height is higher than that of box B. However, Syahrul, one of the students, try to put box A on the top of the box B. He seemed not sure about his answer that saying box A is the larger because on the other hand he said that he measure the width and the height of the boxes. He put one box on the top of the other and found out that the length of box B is longer while the height and the width are shorter than that of box A. Although the other friends talked about the dimension of length, width and height of the boxes, Farish and Juan had his own way to compare the boxes. They wanted to put cakes inside the boxes and even more, Juan was sure that box A can contain more cakes than box B.

To follow up the students' idea about putting cakes inside the boxes, as we predicted, we gave them some pieces of *dodol*. Because we did not give them enough pieces of *dodol* to fulfill the whole space of the boxes, they have to estimate how many *dodol* they need to cover up each box. All of the students seemed to use layer structure to estimate the capacity of the boxes. Firstly they cover up the base and then predict how many layers that possible. Some of them find out that they can put 14 *dodol* while the other saying 16. It happened because some put the *dodol* very tight one to another while other just put as many as they can. In general, they multiply the number of *dodol* they can put on the base of the boxes with the number of layers that possible until reaching the top of the boxes.

Activity 2: "Picture Packages" Activity

We tried this activity to know how the students visualize the three-dimensional object into the two-dimensional drawing. We arranged 24 tea boxes on the table (see figure 5.5) and then asked them to draw the arrangement on the paper so that the people who see their drawing can understand the situation. The tea boxes were arranged in two layers in which each layer consists of 12 tea boxes as shown in the picture. We conjectured that some students tried to draw the layers to explain to others about the situation while some students still have difficulties in representing the layers in their drawing.

Figure 5.5 Tea Box Arrangement

The tea box arrangement was placed in the center of the students. They saw the arrangement from different angles as shown in the figure. Then, we challenge them to make their own drawing so that they can explain the drawing based on the situation they saw. The result is that the students come up with some different drawings as follows:

Figure 5.6 Syahrul's Drawing of the tea boxes arrangement

Syahrul explained that he saw three parts of the arrangement: the left side, the middle, and the right side (see figure 5.6). In each part he saw 8 tea boxes: 8 on the left, 8 in the middle, and 8 on the right. From his explanation, he tried to visualize the layers of 8 in his drawing. However, in his drawing we can see that he had difficulties in representing the arrangement as a concise building. He separated the boxes into 3 columns – left, middle, and right sides – and 2 rows, front and back sides. In each column, he tried to make 8 by drawing 4 in front and 4 at the back. It indicates that although he is aware of the term of layers, he had difficulties in representing it in his drawing.

Figure 5.7 Farish's Drawing of the tea boxes arrangement

Farish explained that he drawn the twenty-four tea boxes in one layer. We asked for clarification why he drawn all 24 tea boxes on the top rather than 12 tea boxes as he saw from the top and 12 tea boxes as he saw in the bottom. He said that it is easier to calculate if he directly drawn all 24 in one layer (see figure 5.7).

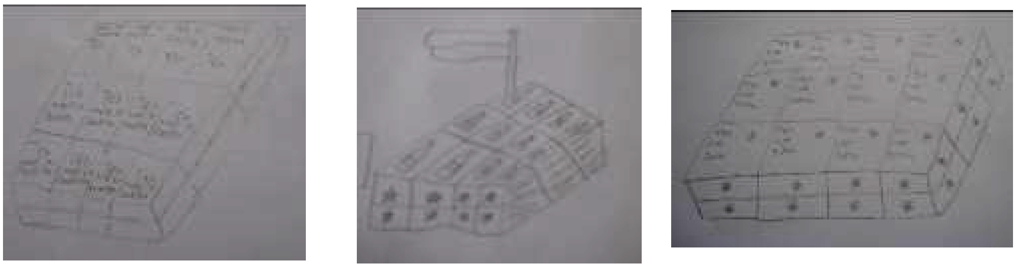


Figure 5.8 Fadillah's, Juan's and Aulia's Drawing of the tea boxes arrangement

Fadillah, Juan and Aulia come up with isometric drawing as shown in the figure 5.8. They are able to figure out that there are two layers in the arrangement. Fadillah said that she can see 12 tea boxes from the top and then in her drawing, she drawn 2 layers so other people can understand that she meant to multiply it by 2. So, the total is 24. In the same way, Juan and Aulia explained that they drawn two layers of 12 on the top and the bottom part so it makes 24 boxes. Aulia emphasized that she multiply 12 by 2 and then makes it 24.

In this activity, we wanted the students to prepare for the next activity namely "Building Blocks" activity. In the next activity, they were asked to construct a building made of cube blocks from the drawings given. The drawings were from different views: front, top and side views. By experiencing this activity, the students represented the three-dimensional objects arrangement into two-dimensional drawing and in the next activity the students have to do vice versa. They were assigned to read off the two-dimensional drawing and build a three-dimensional object arrangement based on that information.

Activity 3: "Building Blocks" Activity

In this activity, the students were asked to build a construction made of cube blocks. The teacher gave the pictures of the construction from side, top and front views to them. Later, after they finish with their construction, they are asked to draw their construction into a single picture in which they can see all three views – side, front and top views – in the drawing. We predicted that some students build the construction first from the top view which makes them possible to build the base of the building and then build the layers until match with front-view picture.

In drawing a single picture of the construction, they might have difficulties in relating those views – side, front and top views together. More specifically, some students might have problem in relating the side and the top views in their drawing.

From our observation, Juan is the one who build the construction from the base. At first, he seemed confused about his construction. He had problem in reading off the pictures. He asked the researcher about how to read off the "side view". He had difficulties to interpret the drawing from that view. He asked, "Where should I start put the blocks in the side views? Do I need to calculate the blocks I saw from the front view in the side views?" He tried to build the outer part of the building by constructing left and front side of it. However, he seemed doubt about his interpretation of the side view. Therefore, he then rebuilt the construction and started from the base then built each layer of the building. He had 24 blocks in which he had to build three layers of 8 blocks. In building his blocks, he made the base, and then the second layer and then the third layer of his building.

Fadillah also build her construction from the top view or build the base first and then side and front views. However, she explained that before she succeeded with her construction, she made a mistake in read off the pictures given. She first build from the front and side views but then the interior part of her construction was empty as shown in figure and she thought that she had not enough blocks to build it. She seemed have difficulty in interpreting the side views picture as Juan had also. But then, she had another problem, interpreting the top view. We can see in figure 5.9 that if we saw it from the top, there is empty space in her building.

Figure 5.9 Fadillah's work of building block activity

Farish did different way from his friends in building the blocks. He had 36 blocks and had to build 4 layers of 9. He first built the front side of his building and then arranged the other blocks into several four-block high buildings (see figure 5.10). Then he built his construction by taking each four-block high building and putting them together until it looked the same from top and side views with the drawing he had.

Figure 5. 10 Farish's work of Building Block activity

Syahrul, and Aulia built their construction first from front and then side views. They then fill in the interior part of their construction after they have the outer part of it. They calculated the number of blocks in each view as they saw in their construction to check whether they had right construction or not.

From this observation, we can make following conclusions. Some students seemed aware of the term of layers in constructing the blocks into a building as asked in the pictures they had. They built the base and then built the layers in rows, as Juan did, or they built the front side and then built the column, as Farish did. However, some also found difficulties in reading off the pictures they had, especially the side view. Most of the students built the outer part of the building first and then fill in the interior part of it. They do this by constructing from the front and then side views. From the example of Fadillah’s work, we can see that because she first built the outer part, she did forget to fill in the interior part of her building and let it empty.

After the students finish with their construction, we asked them to make a drawing that represented their construction. They had this experience in the previous activity in which they made a drawing of the tea boxes arrangement. Fadillah, Juan and Aulia drawn an isometric drawing in the previous activity and also did the same kind in this activity. We found that Aulia turn her construction a little bit so that she could see her construction from all views, not only from the front view as the other friends did. She explained that it makes her easier to draw an isometric drawing. On the other hand, in the previous activity, Farish and Syahrul did not draw an isometric drawing. However, in this activity, they tried to make an isometric drawing, but they told us that they found difficulties to relate the side and the front views in their drawing, especially the top-right side. The students found that they could draw easily the front view but it was not so that easy when they have to connect the side and top views.

Activity 4: Calculate the Blocks

This activity is the continuation of the last activity, building a construction made of cube blocks and making its drawing. First, the students were asked to calculate the number of blocks in their friend’s drawing and then check it by seeing the real construction. In the real construction, the students can touch and tag the blocks while in the drawing they have to imagine the situation of the blocks arrangement. We predicted that the students calculated the blocks both in their drawing and in the real construction by first calculating one layer, either in rows or in columns, and then multiplying it with the number of layers.

Based on our interview, Syahrul explained that he calculated the number of blocks in Juan's drawing by first calculating the number of blocks on the left side and then multiplied it by 2. In Juan construction's there are three layers of 8 blocks. In each layer it was arranged in 2 rows of 4. When he was asked to check his answer, Syahrul separated the blocks in Juan's construction into two columns of 12. He explained that there were 12 blocks on the left and 12 blocks on the right sides, so all together is 24 blocks.

Figure 5. 11. Farish calculated the blocks

Farish, Fadillah, Juan and Aulia had similar way in calculating the blocks. Based on their explanation, they calculated the number of blocks on the top part of the drawing and then calculated the number of layers. Then, they multiplied those two numbers. In the figure 5.11 we can see that Farish took the top part of the building from the bottom part. He explained that he calculated the number of blocks in the top part and multiplied it by two because he had two layers. He separated his building to show us that he had the same number of blocks on the top and on the bottom part.

When they were asked to check their answer, they tagged the blocks on the top of the construction and then tagged one of the blocks in each layer. The students calculated the number of blocks on the top part and then calculate the number of layers on the construction.

Based on this observation, we can conclude that in this stage, the students were aware of the use of layers in calculating the number of blocks both in their drawing and in the real construction. One of them used the columns and the others saw the top part first and then multiplied it with the number of rows or layers they had. In calculating the number of blocks in the drawing, they are no more seeing it as squares which we found in the pre-test. After having some concrete tasks in the previous activities, they are ready to work with the pictorial representation or the drawing of the cube blocks arrangement. However, we would like to improve this activity in the next HLT by giving the students worksheet about calculating the number of blocks in the drawings or pictorial representation of the blocks arrangement.

Activity 5: Predicting the Number of Blocks

There were only 4 students joined this session and also later sessions: Fadillah, farish, Aulia and Syahrul. Juan was ill at that time so that he could not be participate in the activities.

In this activity, we gave each of the students some cube blocks and an empty box. Then, they are asked to estimate the number of blocks can be put inside the box. In the very first activity in this sequence they had already been asked to estimate the number of dodol that they can put inside two boxes to know which box is the larger. And all of them did estimate it by covering the base of the box and then find out the number of layers. Therefore, we predicted that the students would do the same thing, try to cover up the base of the box and then multiply the number of layers.

In this activity, we did not give enough blocks, even to cover up the base. We only gave 11 blocks to each student. We want to know their strategies in estimating the volume of the empty box we gave. The following is the segment in our video recording:

Researcher : So, how do you estimate the number of blocks you can put inside the box?
Syahrul : 8 times 4.

Researcher : How do you get 8 times 4?
Syahrul : 8 on the left, 8 on the right, 8 on the top here and there. (He put 8 blocks on the left side of the base).
Researcher : How about you Farish?
Farish : By multiplied it by two. We need 6 more to fill in the empty space here. (He put 10 blocks on the base and then he explained that he needed 6 more to cover it up). So, if we add all become 32. So, 16 times two. 16 plus 16 or 16 times 2 is 32.
Researcher : And you Aulia?
Aulia : By multiplying 16 times 2.
Researcher : where did you get 16 from?
Aulia : four times four (she pointed out to the blocks that she arranged in the box).
Researcher : So this four and that four, and what is two means?
Aulia : There is two layers.
Researcher : Now, fadillah?
Fadillah : We already have 11, and need five more blocks to cover up this part. (She then put one of the blocks to the top of block on the right corner of the box). And we can have two layers.

Based on our observation, the students tended to cover up the base of the box and then multiply the number of blocks needed to cover it by the number of layers that possible. Although they did not have enough blocks to cover up the base, they first did predict the number of blocks needed on the base and then predict the number of layers. Syahrul perhaps used double structure. He first doubled the eight blocks on the left and then used it to predict the blocks on the right, and then he did the same thing for the top part.

We would like to improve this activity in the next HLT by giving the students worksheet about predicting the number of blocks that can be put inside an empty box in the drawings. We want to know if the students are able to imagine the situation of covering the space of an empty box in the drawing form. With the real blocks and box, we found that they have no difficulties but we did not know how they predict the number of blocks that can be put inside a box in a drawing.

Activity 6 : Determining the possible sizes of the Boxes

In the exercises in the text book about volume measurement tasks, we often found question such as: “If the volume of a box is 24 units, and the area of the base of the box is 6 units. How high is the box?” We want them have preparation to answer such question. And if they have to answer the other way around (i. e knowing the height and asked the possible size of the length and the width of the base) they can also understand the situation.

In this session, we prepared 24 cube blocks for each student. Then, they are asked to make buildings from those 24 blocks. The question for them is: ““*If the volume of a box is 24 cubic block units, what is the possible size of the box that can hold all the blocks? Is there only one possibility or more than one? How do you know?”*

We described the students’ work as follows. Fadillah said that she could build 7 different shapes. She showed some of them such as building of 6 layers of 4 blocks and 4 layers of 6 blocks. Aulia then showed that she could build 2 layers of 12 in which she could arrange the 12 in 4 rows of 3 or 3 rows of 4. Farish then added that when he had 12 on the base, it is also possible to arrange it very long, so that he had two layers of one row of 12. Meanwhile, Syahrul first made two layers of 12 (2 rows of 6) and then he showed that he could rotate it to be a building of 6 layers of 4 (see figure 5.12).

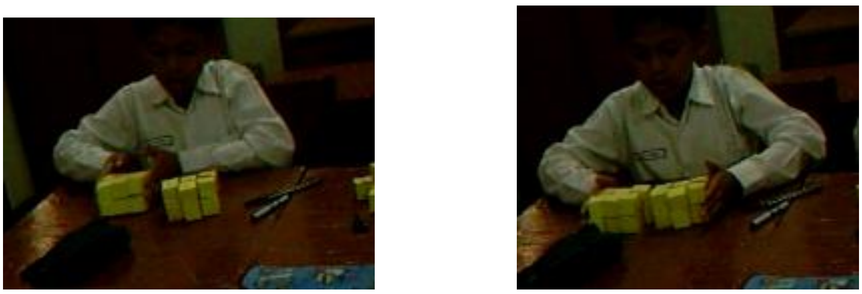


Figure 5. 12. Syahrul moved the blocks

After they finished with their construction, we asked confirmation, what the possible sizes of the boxes that can hold 24 cube blocks are. They then explained that it might be 6 layers high of 4 blocks, 4 layers high of 6 blocks, 3 layers of 8, 2 layers of 12, etc.

From the students' answers, we can see that in determining the possible sizes of a box, they considered about the number of blocks on the base of the box and the number of layers or the height of the box. They realized that a box with volume 24 blocks might have different sizes such as 2 layers high (2 blocks high) of 12 blocks on the base (with 4 blocks long and 3 blocks wide, or 6 blocks long and 2 blocks wide), 4 layers high of 6 blocks on the base, and 6 layers high of 4 blocks on the base.

In the next activity, the students worked in pairs to answer some story-questions related to volume measurement concept in their worksheet. We expected that they were ready to answer the kind of questions which knowing the volume of a box and ask for possible size of the height or the length and the width of the box.

Activity 7: "The Box Capacity" Worksheet

In this activity, we gave a worksheet with two questions. The first question is. "A cake box can hold on 60 pieces of cakes. If on the base of the box there are 12 pieces of cakes, how high the layers of cakes in the box might be?" And the second question is "My mom bought a box of dodol. On the base of the box, there were 5 rows of 4 dodol, and those were arranged 3 layers stacking up. How many dodol are inside the box?" We predicted that they answered those questions by applying their knowledge and experiences in their previous activities. They calculated the number of object on the base and the number of layers in the arrangement. The following are their answer of the first question.

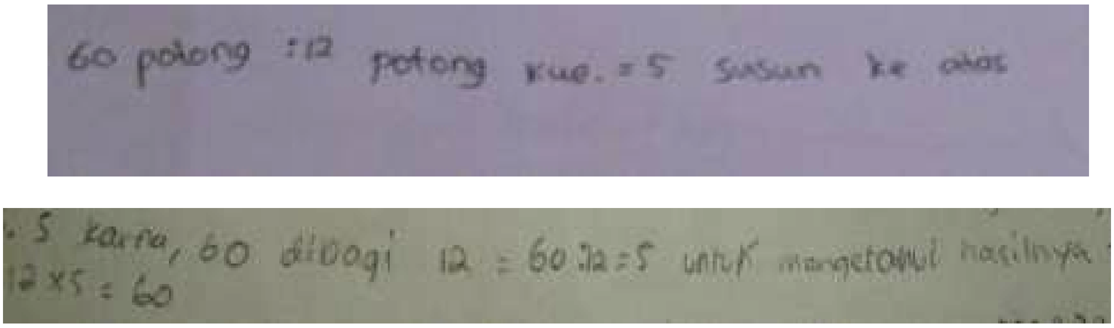


Figure 5. 13 Aulia’s and Fadillah’s answers to question 1

Fadillah and Aulia divided 60 by 12 and got 5. Aulia explained that the result is 5 layers stacking up. They tended to perceive that 12 is the number of cakes in each layer (see figure 5.13). So, to know how many layers in the box, they divide the total number of cakes by the number of cakes in each layer.

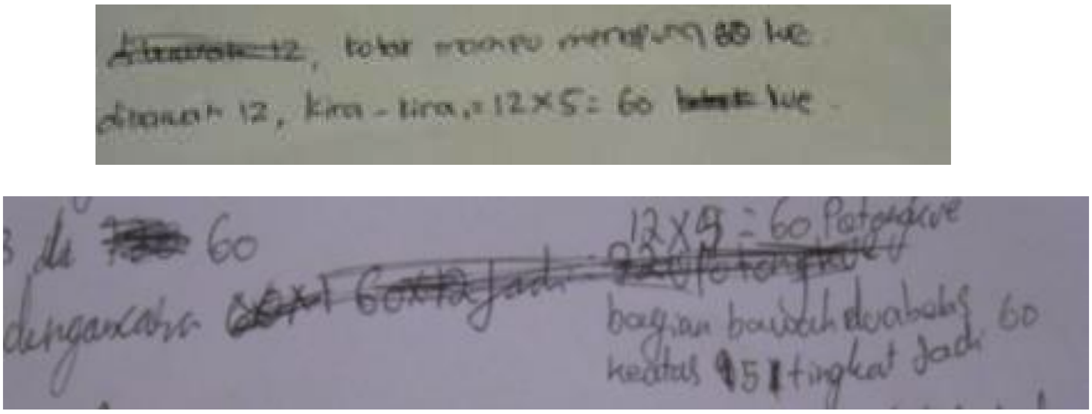


Figure 5. 14 Syahrul’s and Farish’s answers to question 1

Syahrul and Farish solved the problem by multiplying 12 with 5 and the result is 60. Syahrul explained that he “estimated” the number of layers. Meanwhile, on Farish paper, we can see that he first multiply 60 by 12 (see figure 5.14). But then he cross them out and his final answer is that “On the base are twelve, to the top are 5 layers so all is 60”.

And, their answers of the second question are:

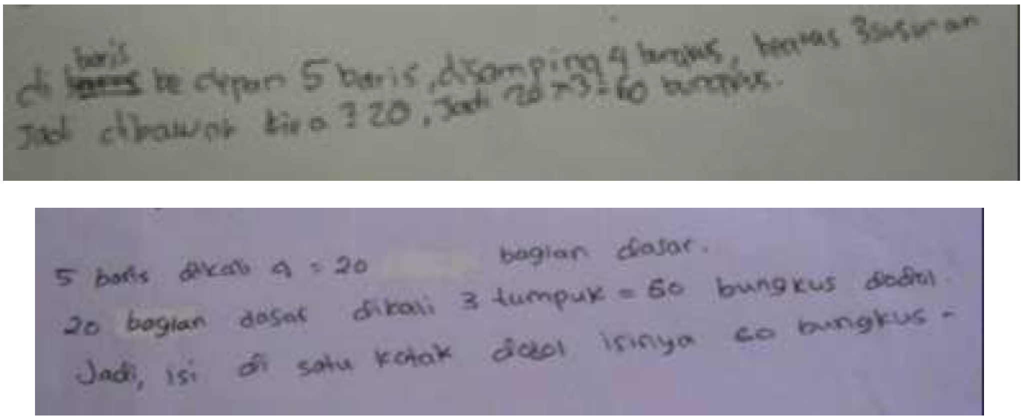


Figure 5. 15 Syahrul's and Aulia's answer to question 2

Syahrul and Aulia explained that they could have 20 dodol on the base. Because there are 3 layers stacking up, they then multiply it by 3. So, the result is 60 dodol. Syahrul again said about "it is estimated that …" (see figure 5.15). He seemed perceive the action of packing up the box as estimating activities.

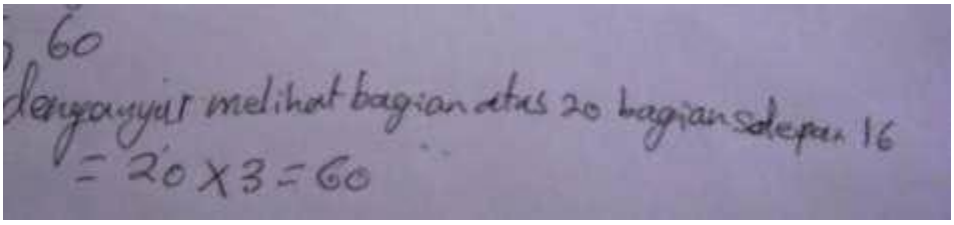


Figure 5. 16 Farish's answer to question 2

In figure 5.16 above, we could see that Farish tended to imagine the top side and he could see 20 dodol then he imagine the front side and then he multiplied 20 times 3 is 60. He tended to apply his experience in building the blocks because he talked about top and front views of the dodol arrangement.

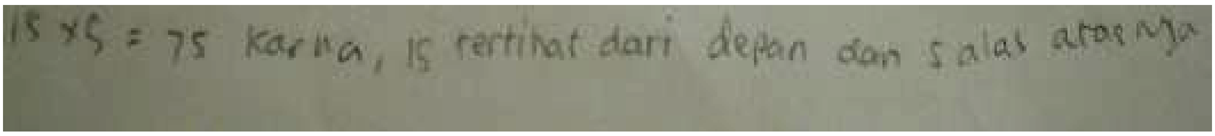


Figure 5. 17. Fadillah's answer to question 2

Fadillah's answer is 75. From her answer in figure 5.17 we observed that she tended to estimate that there are 15 dodol from the front and because the length of the arrangement is 5 so that she multiplied 15 by 5. She seemed did the same thing with Farish, apply her experience about the blocks construction. However, instead of multiplied with the width of the arrangement in the left or right side, which is four dodol, she multiplied the number of dodol in the front with five, the length of the arrangement, again in the front side. On the other hand, in the bottom of her answer sheet, we found that she made two drawings: a drawing of 3 rows of 5 squares and she call it "front", and then a drawing of 4 rows of 5 squares and she called it "base". But, she could not interpret it well. She could not imagine the different views as a concise building.

Based on students' written works, we can draw some conclusions. Students used different strategies in solving the problems. Some of them use layer structures to solve those problems. There are also some students who apply their experience in the previous activities such as in predicting or estimating the number of blocks and in building the cube blocks from different views pictures.

C. Conclusion of the Preliminary Experiment

Based on our observation in the preliminary experiment we can draw following conclusions. In the first activity, comparing the boxes, some students thought that they can compare the boxes by only measure the sizes, length or width or height. They found that the width and the height of box A were longer than those of box B but not the length. The length of box B was longer; it made them doubt of their answer that box A is larger. However, the idea to fill in the boxes with cakes and then calculate how many cakes inside each box can convince themselves and the others in determining the larger box from those two.

In the other activities in the sequence, we could see that most of them are struggling in making a representation or drawing of a three-dimensional object, in interpreting the side view of a construction if we making separates views (i. e. top, side and front views), and in relating the drawing from those different views.

Syahrul and Farish had difficulties in making a drawing of a three-dimensional object. They could not represent the situation in their drawing well although they then could explain to others what they meant in their drawing. Juan and Fadilla had difficulties in interpreting the drawing of the side view when they worked in the "building blocks" activity. Juan seemed doubt with his interpretation of that view, because he thought that the blocks he built in the front are not calculated as part of the side view. But then his construction was not suitable with the drawing he had and he rebuilt it and start from the top view (base). Fadilla did almost the similar thing. She built the front and then the left side of the building and then put it together. However, the result was she had a building with empty space in the center of it. When she realized that she made a mistake, she then rebuilt it also from the base. In the last activity, we could see that Fadilla still had difficulties in relating different views of the building. In solving second problem in the worksheet given, she can translate the situation in the story into drawings of front and top views of the arrangement. However, she did not succeed in relating those views into a concise construction.

In seeing the structures of the cube blocks in a box, the students could think in the term of layers, either in columns or in rows. They tended to multiply the number of blocks needed on the base and then multiply it with the number of layers. We could not really see the students' struggles in calculating or predicting the number of blocks when they had real blocks and real boxes. Therefore, we would like to add worksheets in those activities so that we could see their strategies in doing that without help of the real objects anymore.

In this preliminary experiment, we observed that the students did not have many problems with the tasks. It could be the case that there were problems, but they did not come to the surface yet. We would like to see in the teaching experiment how students in the real classroom struggled with the activities we designed.

D. The Improved Hypothetical Learning Trajectory

Based on the results of our preliminary experiment, we would like to improve and adjust the activities in our initial HLT. In the first activity, we restricted the number of *dodol* given. We did not give enough number of *dodol* even to cover up the base of the boxes. Because all five students in the preliminary design can easily estimate the number of *dodol* needed when we gave them those number of *dodol.* We wanted to know how they estimate the number of *dodol* needed to cover up the boxes when they do not have enough number of *dodol* needed on the base.

We would also like to improve some activities as we mentioned before, "calculate the number of blocks" and "predict the number of blocks" activity, by giving worksheets at the end of the activities. It was aimed to see the reasoning of the students in calculating or predicting the number of blocks needed in the form of drawing. In the preliminary experiment, we found the students are able to calculate and to predict the number of blocks needed in the real box and with using real cube blocks but we could not get information whether they are also able to do it by only imagine the situation, in the drawings.

In "determining possible sizes of the boxes" activity, we would like to make a little adjustment. We would prepare three boxes with different shapes and different sizes but with the same volume (i. e. the volume is 48 cube-block units). We would ask the students to determine which boxes have the same volume and why they think so. They could use the cube blocks unit to help solving the problem given.

In “the box capacity” worksheet, we only tested 2 out of 5 questions we prepared. In the next HLT, we would give all 5 questions in the worksheet, as we plan. The students would work in pairs and then discuss their answers in the class discussion.

Based on our discussion, (researcher and supervisors) during this phase, we decided to add one more activity at the end of our sequence of activities. One additional activity in our next HLT was named “Measuring the box with different units” activity. Analyzing this activity was aimed to know how the students perceive the concept of units in measuring the volume.

In this activity, we prepared two different boxes in which one, yellow box, is larger than another, blue box There were also two different units, yellow units which are bigger and blue units. The students were asked to predict the number of yellow units can be arranged in yellow box and also blue units in blue box. The students would find that blue box can hold more units than yellow box. The blue box can hold 32 blue units and yellow box can hold 12 yellow units.

In the discussion, the teacher could ask question: Is it true if I say blue box is larger since it can hold more units? Let’s see together these boxes, which one is larger? The students might think that it is. But, some of them realized that the yellow box is larger. Then, the teacher can ask: “But why the number of blocks in blue box is more and in yellow box is less?” The students then would realize that because the units used to measure the yellow box is bigger so the number would be less. This activity could provoke the students to realize that if they had different units with different sizes, they could not directly compare the boxes. Because the sizes of the yellow units were bigger so the number would be less and vice versa for the blue units. Then, the teacher asked to do crosscheck. She can ask the students to predict the number of blue units can be arranged in yellow box and vice versa. It was aimed to provoke the students that in comparing the volume of objects they should measure with the same units.

It was also expected that they can see the inverse relation between the unit size and the number of units. The bigger the size of a unit, the less number of the unit was needed to cover up a space. Hence, it was expected that students would rethink about the capacity of larger box after they realized it. If students still do not realize the teacher can give some stimulating question such as: "*Measure both boxes using only the yellow units and then with only the blue units. What do you think about the result of those measurements? Why there is different number of "yellow" and "blue" units there*?"

It was expected that students realize that the capacity of an object can be represented in different number if they use different size of unit measurements. This actually related to the concept of conservation of interior volume in which that the volume of the object stays the same but the size of unit measurement is different.

We discussed with the teacher about all changes we made. The teacher gave some suggestions and we tried to adjust our initial HLT together. We would like to test this version of our HLT. We named it "Improved HLT". We would see how the improved HLT worked in the teaching experiment.

E. Teaching Experiment

In this section, we compared our improved HLT and students' actual learning process during the experimental phase. We investigated how and if the HLT supported students' learning. We looked to the video recordings and selected some critical moments. We also analyzed their written works such as posters, and worksheets as another source. We analyzed every day lesson to investigate what students and teacher do, how the activities work, and how the material contributed to the lesson. We also looked with broader view and searched for connections between the lessons and tried to find out how earlier lessons supports the following ones. The result of the retrospective analysis in this teaching experiment would be used to answer our research questions.

Lesson 1: Comparing the Capacity

Goal: In this lesson, we designed activities in which the students can bring in their (informal) knowledge of the situation of the volume packing.

The Lesson:

We gave them two empty boxes to be compared, namely carton box and plastic box. The width and the height of carton box were longer than those of plastic box but the length was shorter. We predicted that in comparing those boxes, the students compared or measured the sizes. But, since not all of the dimensions of the carton box are longer than that of plastic box so they would be a bit doubt about that way of comparing the boxes. However, since the boxes we prepared are usually used as cakes or food packages, there might be some students who have ideas to fill in the boxes with cakes or foods and then calculate the number of cakes can be put in each box. We expected that the students can bring in their informal knowledge about packing the food or cakes to solve this problem.

We also provided pieces of *dodol* (traditional snacks from Indonesia) as the cakes or food to be put inside the boxes. This is to follow up the ideas of comparing those two boxes by calculating the number of cakes in each box. However, we did not give enough number of *dodol* to cover up whole space of boxes, even not enough to cover up the base of the boxes. They were asked to estimate the number of *dodol* needed to cover up each box. We then can also see the initial phase how the students perceive the structures of the *dodol* arrangement in the boxes. In the preliminary experiment, we found that all five students had no difficulties in perceiving the structures of the *dodol* arrangement in the empty box. However, we predicted that there would be some students who saw the *dodol* arrangement as unstructured object. These students would calculate the *dodol* one by one until the box is fully packed with the *dodol*. But, as we mentioned before, there would also be students who can already imagine the structures of *dodol* in the boxes and calculate it in the terms of layers.

The teacher started the lesson by asking students' experience seeing the arrangement of water cups, noodles or chocolate packages etc. This was aimed to bring their informal knowledge about the packing or arrangement situation. Then, the teacher showed two boxes with different sizes and gave each group those boxes. Then, the students were asked to compare those two. As we predicted, all students directly pointed out to the carton box which is wider and higher. When they were asked to figure out their reason why they thought that it was larger than another, their answers were varied. Some of them said that because the carton box was higher, some of them said that it was higher and wider, but no one said about the length, and the teacher also did not react to this. The other students said about fill in the boxes with food or cakes and then compare the quantity of the food they put in each box, as we predicted.

Then, the teacher told that she had some pieces of cakes, *dodol*, to be put inside the boxes, and then she gave each group 5 *dodol*. To prove their answer that the carton box was larger than plastic box then the students were asked to estimate the number of *dodol* they needed to cover up each box. The group work was continued by group presentation. Each group was asked to tell the other groups how they estimate the number of *dodol* needed.

Dwi and her group found that carton box was larger than plastic box. They found that they needed around 56 dodol to cover up the carton box and needed around 21 to cover up the plastic box. They estimated the number of dodol in each box by iterating it along the space of the boxes. When the teacher asked her to present their idea, Dwi tended to iterate the dodol one by one until the whole space of the boxes were covered as shown in figure 5.18.

Figure. 5. 18. Dwi estimated number of dodol in plastic box

The second group who presented their idea was Gina's group. They also found that the carton box was larger than the plastic box since the carton box could hold more number of dodol than the plastic box, 54 to 18. This group asked for more dodol. They had five and asked for another 10. The teacher then gave them those number and curious what they want to do with it. In estimating the number of dodol in the carton box, they first arranged 9 dodol horizontally and 6 dodol vertically on the base of the box as shown in figure 5.19. Then, they explain that to get the number of dodol needed to cover up the box, they multiplied 9 by 6, makes 54. They did the same way to estimate the number of dodol in the plastic box. They arranged 9 dodol horizontally and 2 dodol vertically on the base and got 18 as the number of dodol needed to cover up whole box. Then, the teacher asked Gina to come to in front of the class to tell the others about her groups' thought and other students were asked to give comments of what Gina explained.

Figure. 5. 19. Gina showed her groups' work

Gina	: Here is one, two, three, four, five, six, seven, eight, nine.(she pointed out to the dodol in the boxes)
Teacher	: Yes.
Gina	: And here are six. Nine times six. So, fifty four.
Teacher	: So, to cover up this box, Gina estimated it by… Nine, how do you get nine?
Gina	: (Pointed out to the dodol in the boxes)
Teacher	: The front. Then six? Oh, from the back.

(She then showed the box as we could see in the figure to other students. The teacher then asked other students about what they think about Gina's strategy)

Teacher	: How do you think about Gina's strategy? Who can help? Raise your hand!

(Some students raised their hand. Syahrul then come to in front of class)

Teacher	: Yes, Syahrul, please.
Syahrul	: Here are nine and there are six. So, I added them become fifteen. To the top, I estimated there are 4 stacks. So, all is fifteen times four equal to forty, uhm,, fifty, uhm, sixty.
Teacher	: Syahrul said he added, 9 plus 6 is..
Students	: Fifteen.
Teacher	: Then Syahrul said that there are 4 layers. So, 15 times 4 is …
Students	: Sixty.
Teacher	: What do you think about Syahrul's strategy?
Students	: Correct.

From that fragment, we could see that the students justify and argue each other answers. The other students help Gina's group by explaining that the numbers should be added not multiplied. Gina then realized herself about her mistakes. After that fragment, the teacher then asked Gina to estimate the number of dodol can be put in the plastic box. And she applied her new knowledge from her friends in estimating the number of dodol in the second box. She added the number of dodol on the base, 9 plus 2 is 11. Then, she put one dodol on the top of the arrangement on the base as in figure 5.20, she multiplied 11 by 2, because she said that there are two layers to the top.

Figure. 5.20. Gina estimated the number of dodol in plastic box

Conclusion:

Throughout this lesson, we could see that to know how large a box is or to compare two boxes with different sizes the students tended to measure the length, the width, and the height of it. They thought about linear measurement. But, some others who had informal knowledge about packing box give ideas about fill in the boxes with some quantities such as food or cakes. In the discussion with the teacher, the students realized that in comparing the capacity of two objects, they need a unit to measure the capacity that is the cakes as they said. By knowing how many units or cakes can be put inside in each box, they could compare the capacity of the boxes.

In estimating the number of dodol can be put in each box, they had different ideas. Some tried to iterate the dodol as the unit along the empty space of the box. Some others had difficulty in imaging the structures of the arrangement and then just multiply the number of dodol arranged on the base. It might be because they could move in the dodol on the base but they have to merely imagine what might be above the base. Also, there were some students who could perceive the structures well. They could think in the term of layers in seeing the structures of the dodol arrangement in the boxes. These students could give ideas to other students to do abbreviation in estimating the number of dodol in each box. They showed that they only need to cover up the base of it and then find the possible height of layers. Other students who were not yet able to see the structures of the layers in the arrangement were helped by the following series of lessons we designed.

Lesson 2: Pictures of the Packages

Goal: In this lesson, we designed activities in which the students were asked to make a drawing from an arrangement of objects in a three-dimensional array. In making the drawing, it is expected that the students can relate the visible part and interior side of a three-dimensional objects arrangement.

The Lesson:

The teacher started the lesson by telling a story that she was assigned to make a report about the new stocks of the food and tissue packs for breakfast of the teachers. The food packages in front of them were the things she has to report. However, the headmaster wanted to have a picture of the packages so she can check the number of packages by seeing the pictures. The task for the students was to help the teacher to draw a representation of the packages arrays from a view in which someone who sees the drawing can know the numbers of packages by only seeing the drawing.

There were four different arrangements on their tables: tissue packs, tea boxes, and two different kinds of wafer bars as in figure 5.21. The students were asked to draw the nearest food arrangement with their chairs. The packages were arranged in three-dimensional arrays and the students had to make its representation in a paper. We predicted that some students made isometric drawings as we saw in the preliminary design and some others had difficulties in representing the objects as a concise building. It was also conjectured that some students drew squares as representation of the blocks that they saw either from the front or from the top of the objects and then they made mistakes when we asked to calculate the number of objects in their drawing.

Figure. 5.21 Food Packages Arrangement

In our observation, we found two groups of students made drawings from separates views. We did not predict this happened because in the preliminary experiment we did not find this tendency. When we asked them how they know the number of objects in their drawings, the first group explained their drawing as shown in the figure below:

Figure. 5.22. Group 4 drawing

They explained that they saw 4 tea boxes in the bottom part and they saw two stacks in front and in the back. So, they multiply those become 8 and it was what they saw from the top because there were three box high, then they multiply it by 3 and all together is 24 (see figure 5.22). The other group who drew the same objects with them also drew the tea boxes arrangement from separates views as shown in figure below:

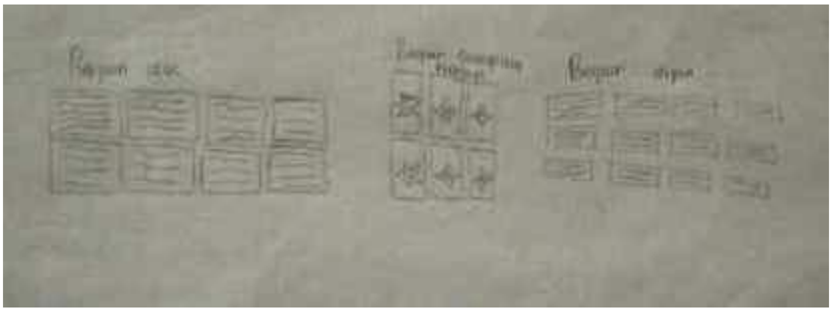


Figure. 5.23. Dwi's group drawing

The group work was continued by group presentation. The following fragments were chosen because the students could give their reasoning why they made such a drawing and how others should interpret their drawings.

Dwi : This is drawing from top, right side and front.
Teacher : Yes. Good. And then?
Dwi : The top is multiplied by the right side. So, eight times three equals to 24 tea boxes.
Teacher : Yes. Good. Give applause for Dwi.

Dwi and her group drew separates views of the tea boxes arrangement as shown in figure 5.23. They drew the tea boxes from three views but from the fragment, in interpreting the drawing she only saw two views: top and right side. From their drawing of right side, there are 2 rows of three tea boxes. Then, to interpret her drawing, she said that we have to look at the top, eight boxes, and then multiply with the tea boxes she saw from the right side, three. So, she interpreted her drawing as 24 tea boxes.

The other group who explained their drawing was group 6. We could not hear very clear their voice in the video, so we could not transcribe it. In the video, we can only see the drawing as in figure 5.24 and one of them, Syahrul was explaining. However, based on our observation, they explained that they made a building of wafer arrangement. There were three rows of 12 wafers. So, all together is 24. They are the group who work with us in the preliminary design. They had experience in drawing the objects arrangement and saw a lot more isometric drawings.

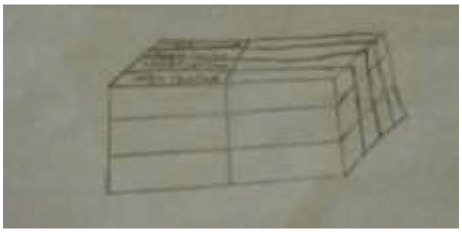


Figure. 5.24 Syahrul's group drawing

The last group who explained their drawing was group 7. They told the other friends that they had wrong drawing because they calculate that there were 18 tissue packs on the table but they had 21 tissue packs in their drawing as in figure 5.25. This was as we conjectured that is some students might make mistakes in interpreting their drawing.

Figure. 5.25. Amel's group drawing

Amel : In our drawing there are 3 times 3 equal to 9, 3 times 2 equal to 6 and 3 times 2 equal to 6. Nine plus twelve is 21. But we calculate there were 18. Who wants to help our group to fix our drawing?

Teacher : Students, Amel wants to ask you to help her find the right thing?

(Some of the students raised their hand)

Teacher : Yes, Fadilla. Fadilla please come to front. Please speak loudly.
Fadilla : *(She thought for a while and tagged the drawing and saw the tissue arrangement in Amel's drawing)*
Teacher : How do you calculate? The first row?
Fadilla : Five
Teacher : Are you sure? How many in the first row?
Fadilla : Nine
Teacher : And how many rows there?
Amel : Two

In the fragment, we observed that Amel and her group tended to calculate the number of squares in their drawing. That's why they thought that they made wrong drawing. They had difficulties in perceiving the structures of the tissue packs in their drawing. However, Fadilla seemed influenced by how Amel calculate the drawing of tissue packs. She said there were five packs of tissue in the drawing in each column. However, then when the teacher asked once more, she changed her answer become nine. And then Amel said that there were two rows of nine. This discussion could help Amel realized that there was nothing wrong with their drawing. The way to calculate is the mistakes they did.

Amel's group did wrong interpretation of their own drawing. However, it would be more difficult when we have to interpret others drawing. Therefore the teacher had ideas to asked students how they interpreted other group drawings. Then, the teacher asked one of the students, Yudha, to interpret Dwi's group drawing and Syahrul's group drawing. He said that he saw 28 objects in Dwi's drawing (see figure 5.23). The teacher asked Dwi if Yudha answered correctly, and the group said it was not correct because there were 24 tea boxes, not 28. He seemed to calculate 8 tea boxes from the front view and added 6 tea boxes from the side view and multiply it by two since there were two rows of it. He tended to be influenced by Dwi who explained that she interpreted the drawing from the top and side views. However, he did not pay much attention when Dwi explaining. However, Yudha then changed his answer become 26. Although the others told him that there were 24 tea boxes, but he did not listen. When he changed his answer become 26, he said 8, 14, 26. He seemed to add all the squares he saw in Dwi's group drawing, 26 tea boxes. Then, Yudha continued to calculate the object in Syahrul's group drawing. He answered 24. He explained that he could see 12 objects from their drawing and all is 24.

Conclusion:

Throughout this lesson students could make representation of the arrangement of three-dimensional objects. It is difficult for most of the students to make a drawing of the objects arrangement as a concise building. They had their own strategies to visualize the situation they saw. Also, they had different interpretations of a drawing. One drawing is easier to interpret than the other one. Even, one group of students thought that they made wrong drawing because they could not well perceive the structures of the objects in their drawing. This activity has provided a bridge for students to develop their thinking process. Later, in the next activity they would have to make arrangement of objects from drawings given.

Lesson 3: Building Blocks and calculate the blocks

Goal: There are two activities in this lesson: Building Blocks and calculate the blocks. The aims of the activities in this lesson are: firstly, to support students in seeing the structures of the blocks construction from different views and to let them find the way to calculate the number of blocks in a three-dimensional arrangement.

The Lesson:

In this lesson, the students experienced two activities, namely building blocks activity and calculate the number of blocks activity. Firstly, the teacher remind the students about yesterday lesson in which each group made a drawing of an arrangement of tea boxes, tissue packs or wafer bars. Then, each group got a box contains some cube blocks and an instruction sheet.

In the instruction, they were asked to construct a building made of the cube blocks as they saw in the drawings. The drawings are from separated views: top view, side view and front view. They have to relate those views to make a concise building of the cube blocks. Every group got different number of blocks and had to build different building. The students had to write down their steps in making the construction and then explained to others. They were also asked to draw their building on the paper. The drawings would be used in the second activity, calculate the blocks.

As we found in the preliminary experiment, we conjectured that some students were able to make good construction as asked in instruction. Perhaps some of them will build from the base and then build layers or some others would make outer parts of the building from each view, front and side and then filled in the inside part to see the top side. However, some of them might make wrong construction based on the views given. It is hard work for them to relate the pictures of an object from different views as one concise object.

Group 1

Group 4

Group 7

Group 5

Group 6

Figure. 5.26 Students' building blocks

Rafli's groups explained that they build first from the front view and then the side view. They filled in the interior part of their building after they had the outer parts of the building. Some others build the layers, after they had the first row of the building (the front side), then they build the second and the other layers as many as they saw in the side view. However, one group of students, group 7, said that they needed more blocks because they only had outer part of their building (see figure 5.26), and the blocks were not enough to make the building as instructed in the drawing. After all students finished working, the teacher asked that group to explain their construction.

Amel	: We started with the front and then building the side.
Teacher	: Then, why you have unfilled part in the building?
Amel	: I forget.
Teacher	: Forget? Or you did not have enough blocks to build it?
Daffa	: Yes, we need more blocks.
(The teacher then looked at the instruction sheet)	
Teacher	: Now, look at your instruction sheet. Does it look similar or something different?
Amel	: Yes, different. From the front is the same. From the left side also the same but from the top is different.
Teacher	: Look carefully from the side view. How many columns are there?
Amel	: two in the drawing and three on the table.
Teacher	: So, how to fix it?
(Fikra and Amel then rebuilt their construction and then it looked exactly the same with the drawings on their sheet. They now constructed two columns in the side view)	
Teacher:	So, now is it the same?
Students	: Yes.

From that fragment, we observed that the group wrongly interpreted the side view of the building. They saw two columns of 4 blocks in the side view in the drawing, but indeed they build 3 columns of 4 blocks. They did not calculate the blocks in front as part of the side view. They perceived it as separate buildings. When the teacher asked them to fix their building into the right one as instructed in the sheet, then they realize that it must be a concise building which could be seen from different views, not separately built.

The lesson then continued to the second activity. The teacher asked the students to come to the nearest neighbor-group and then calculate their blocks in the real construction and in their drawing. We conjectured that some students were able to calculate the number of blocks correctly by using the layers structure either they first calculate the blocks on the base or calculate the number of blocks in column then multiply it with the number of layers. However, there might be students who still have difficulties in perceiving the layers structure. Some students might make mistakes such as calculating the blocks by adding the blocks from every view they could see, it is from the front, at the back, from the top and from the left-right sides of the construction.

The teacher asked some students to calculate it and there was an interesting moment as we conjectured which is transcribed in the following fragment.

Teacher : Group 4, do you get the same number between the real blocks and the drawing of group 5(see figure 5.27)?
Students : No.
Teacher : How many in the construction?
Tasya : 40.
Teacher : How did you get 40?
Tasya : (Pointed out to the blocks in the front)
Teacher : How about you Rafif?
(Rafif calculate one by one and pointed out to every single blocks)
Rafif : 42.
Teacher : How did you get 42?
Rafif : I add all.
(The teacher again asked Tasya to calculate the blocks but Tasya looked frustrated with calculating the blocks. Therefore, the teacher stopped asking her about the blocks but then she asked Tasya to calculate the blocks in the drawing)

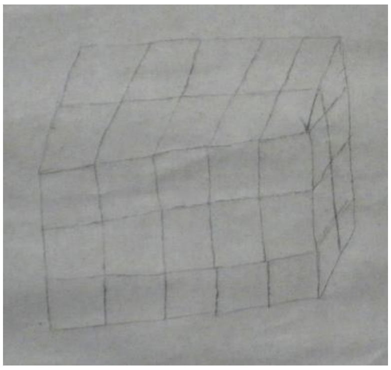



Figure 5.27 Blocks construction and its drawing of group 5

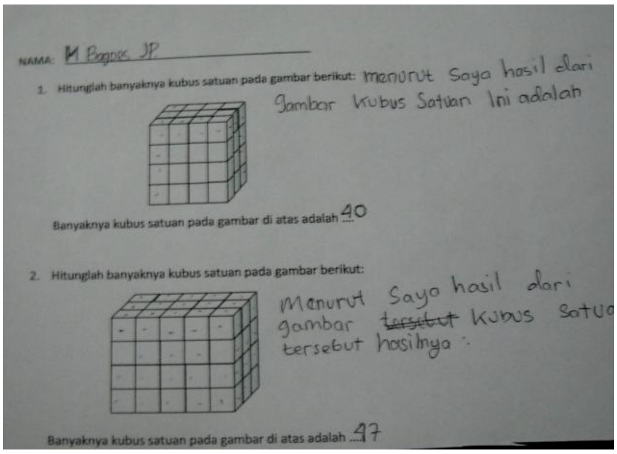
Teacher : Now, in the drawing. How many Tasya you looked in the drawing?
Tasya : 32?
Teacher : How did you get 32?
Tasya : I add all.
Teacher : Now, why you have different number of blocks in the drawing and in the real blocks?

Tasya : Because we have different number of blocks. In the drawing there are 8 and in the construction there are 6 from the side.

In the fragment, we observed that Tasya and Rafif could not well perceive the structures of the blocks. Rafif tended to calculate the blocks from the front, the back, the left and the right side. That’s why he found 42 blocks rather than 30 blocks, the number of blocks in the construction. Tasya also had difficulties in perceiving the structures of the blocks. In calculating the real blocks, she could calculate the number of blocks in the front, 15. But then she did not directly multiply it by two, the number of layers she saw in the side view. She then tried to calculate the blocks one by one and got 40. She tended to calculate the number of blocks in front, in the back, and added the top part she saw, so all together was 40. But she could not explain it. Then, she was influenced by the teacher question and changed every time. She could not decide which her answer was. She looks frustrated in calculating the number of blocks in the construction. In the drawing, she calculated the squares. She told that there were 32 blocks. She said that the drawing was wrong. There should be 6 blocks in the left side not 8 as she saw in the drawing. If we looked carefully to the drawing, the group who drew it did not meant to draw 8 blocks, but they had difficulty to connect the side and the top part of the building in their drawing. Her problem in wrongly interpreted drawing was the same with Amel’s problem in the last meeting. Amel tended to calculate the number of squares in the drawing and not the number of blocks.

The teacher did not ask the right answer from that group. Indeed, she did not ask other students to help the group. She also looked frustrated with that group. Then, the teacher asked other students to calculate the number of blocks in other construction and other drawing. She then pointed to Rizki. Rizki explained that he found there were 36 blocks in the construction of group 2 and also in their drawing. He explained that he calculate it by multiplying 4 by 3 times 3 in the construction. He seemed to calculate the number of blocks from the front and then saw the number of rows. And in the drawing he did the same thing. He found that there was the same number of blocks both in the real construction and in the drawing. Based on his group explanation, they built the blocks constructed from the front wall and then built the blocks to the back. His strategy in calculating the blocks indicates that he was influenced by his strategy in building the blocks. He tended to calculate the number of blocks in front and then calculate the number of layers in the construction.

At the end of lesson, we gave them a worksheet, “calculate the blocks” worksheet. On the worksheet, the students are asked to calculate the number of blocks in two drawings. They are the drawings of objects made of cube blocks. As we found in the pre-test, it was conjectured that most of the students calculated the number of squares rather than the number of blocks in the pictures. Here are some of students’ answers of the worksheet:



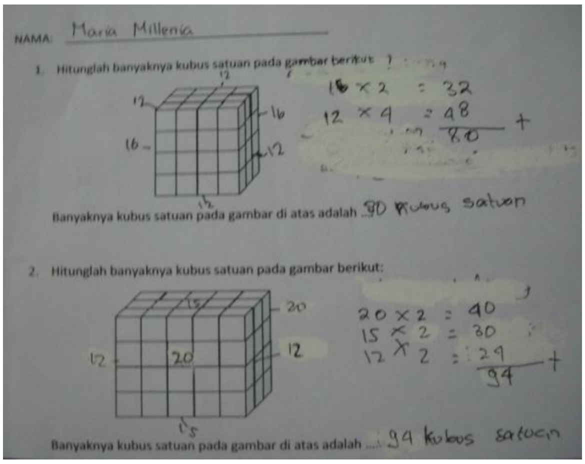


Figure.5.28. Maria and Bagus Answers to the Worksheets “Calculate the Blocks”

From figure 5.28 above, we observed that Bagus tended to calculate the number of squares on the drawing. The dots in his paper strengthen that tendency. And Maria tended to double the number of squares on the drawing. That’s why her answer is two times Bagus answer. However, what interesting in Maria’s answer is that she was able to visualize the part of arrangement at the back, at the bottom and in the left side which we could not see in the drawing. Perhaps, she did not fully understand the task. We asked the students to calculate the number of cube blocks in the arrangement. But, perhaps, she understood it as calculating the number of squares instead of the number of cube blocks. In the next lesson, class discussion, we would ask for clarification about their thought.

Conclusion:

Throughout this lesson, we can see that some of the students had difficulty in relating separate views and build a concise building of it. Some students could not perceive if a building could be seen in different visible views from the front, top, and side. However, the building is stayed the same. They had to construct the visible and also the invisible (interior) part of the building to get a concise building. We could also see that some students struggled in perceiving the structures of the three-dimensional objects arrangement both in the real blocks and in the drawing. They calculated one by one and calculate the blocks in every side. They could not see the structures of the arrangement in every column or in every row. However, some others were already able to see and calculate the structures of the blocks arrangement it in terms of layers. In the next lesson, the students had class discussion in which they shared ideas about their strategy in calculating the number of blocks either in the real construction or in the drawings.

Lesson 4: Class Discussion

Goal: In this lesson, the teacher reminded the students about the last activities: building blocks and calculate the blocks. They had a discussion about some of their friends' strategies in calculating the blocks in the construction they have made and also the blocks in the drawing. There might be some students who able to perceive the structures of the construction well. We conjectured that these students use layers structure that is calculating the number of blocks on the base or on the column and then do skip calculating or multiply it with the number of layers. The other students can learn from their friends how to see the structures of the arrangement so that they could find their own way or strategy in calculating the blocks in an arrangement either in the real construction or in the drawing.

The Lesson:

The teacher wanted to raise discussion about students' strategies in calculating the number of blocks in the arrangement. Therefore, first she asked Rafif's group who made mistakes in doing it and then asked other students what they thought about it and showed her or his strategy that works in determining the number of blocks in any construction.

On the table in front of the class, there was a construction of 30 blocks as shown in the figure 5.27. It was the construction which was built by Rafif's group. The teacher then asked Rafif to calculate the number of blocks in the construction in front of the class. In the following fragment we could see the process of students' thinking:

Teacher : Rafif, how did you calculate the number of blocks?
Rafif : I calculate one by one.
Teacher : Where did you start?
Rafif : From the side part.
Teacher : How many in the side?
Rafif : Nine (He did not explain why he got nine). … (very weak voice)
Teacher : Then, you multiplied it by two, right? So, nine times two?
Students : Eighteen.
Teacher : Then, how many in front?
(Rafif started to recalculate the blocks. His lips and his eyes seemed to point together to the blocks in front of him)
Rafif : Fifteen
Teacher : So, rafif calculate there are 9 plus 9, 18. And 15 plus 15 is 30. So, 30 plus 18, how many?
Tasya : 38 eh, 48.
Teacher : Students, what do you think about Rafif strategy in calculating?
Students : Incorrect.
(Some students raised their hands)
Teacher : Yes, fadilla.
(Fadilla come closer to the blocks construction and then she pointed her fingers to the construction. She seemed calculate the number of blocks on the top part, and then calculate the number of layers)
Fadilla : I calculated the top part, 10. And I multiplied by 3, so 30.
Teacher` : How many in the top part, fadilla?
Fadilla : Ten *(with her finger)*
Teacher : Students, what do you think about fadilla's strategy?
Students : Correct.

From that fragment, we observed that Rafif had difficulty in perceiving the structures of the cube blocks arrangement. He calculated the blocks in front and in the back but then again he calculated the blocks in the left and right sides as we analyzed. In calculating the blocks, he always calculated it one by one. He tended to see it as unstructured arrays. He could not see the layers either in rows or in columns.

However, Fadilla explained that she could see the top part as the first layer and then he multiplied it by three, the number of layers. From our observation in the previous activity and also in the preliminary experiment, Fadilla built the blocks started from the base and continued until all the blocks were used. Same as the evidence we found in Rizki's strategy, she tended to be influenced by her strategy in building the blocks construction to calculate the layers in rows.

From that example, we observed that the students learnt about their friends' mistakes and their friends' strategies in calculating the number of blocks in the real construction. Most of the students could help in determining which strategy is correct and which one is incorrect. The lesson was continued by discussing about how to count the number of blocks in the drawing.

The teacher drawn an object made of a cube blocks in the white board in front of the class. The drawing was the first question they need to answer in the worksheet. The teacher then asked some of the students to present their ideas in front of class.

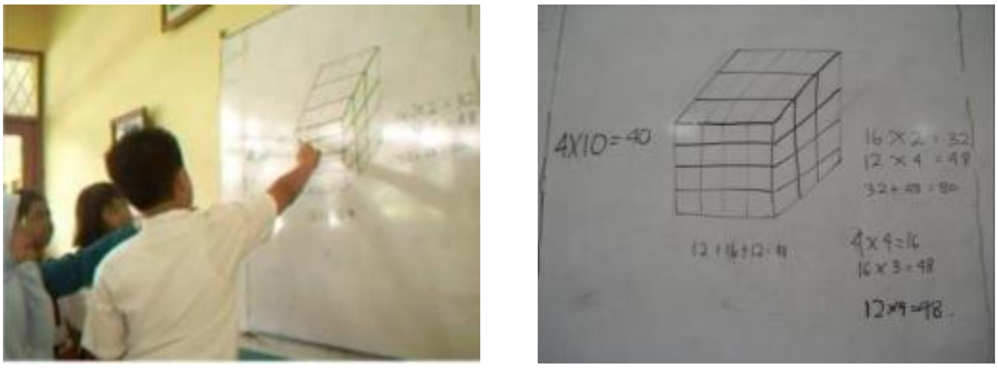


Figure. 5.29. Students explained their answers
of the worksheet" calculate the blocks"

Maria explained that she calculated the number of squares in front, 16, and then the same will be at the back, 16. It makes 32. Then, at the top 12, the same will be in the bottom, 12. And, the same was also in both sides. So, 4 times of 12 makes 48. All together were 80. Actually, Maria was able to visualize the squares at the back, in the bottom and in the other side of the drawing. However, perhaps for her the instruction “Calculate the number of block units in the picture!” was not so clear. She thought about the squares unit rather than the block units as we conjectured.

Nadia calculated differently, she wrote 4 times 10. She could not explained well in front of class but duing our observation, we talked with her and she explained that she calculated the number of squares in the border, 4 squares from left to right and then 3 from front to back and then 3 to the top. All makes 10 and because there are 4 layers high, she multiplied 4 times 10. She did in the same way to answer the second question.

Bagus explained that he also got 40. He explained that he added 12 from the top, 16 squares from the front and 12 squares from the side (as in figure 5.29). Then, the teacher asked the students to determine which answers is/are correct. The following fragment is the discussion:

Teacher : Listen carefully. In front of you, there are three answers of your friends: Nadia, Bagus and Maria. What do you think about their answers?
(Some students said that Nadia and Bagus are correct. But some others said differently. They said that all answers are incorrect).
Teacher : So, what do you think? Dinda what do you think?
(Dinda come to in front of the classroom)
Dinda : I multiply 4 by 4 in front.
Teacher : Yes, write it down.
Dinda : Then I multiplied it by 3, 48.
Teacher : Now, do you think Dinda is correct?
Students ; Yes, she is.
Teacher : Do others have different ideas?
Syahrul : 12 times 4.
Teacher : What does 12 means?
Syahrul : From the top we saw 12 blocks and there are 4 layers. So, all is 48.

From the fragment, we observed that some students still had difficulties in seeing the structures of the blocks arrangement in the drawing. Even, when they are asked to determine which strategy is correct, some of them agreed with Bagus and Nadia. However, in the discussion, they can share ideas about how they calculate and perceive the number of blocks in the drawing. Dinda and Syahrul promoted the using of layers structures in calculating the blocks in the drawing to their friends.

Conclusion:

Throughout the discussion about how they calculate the number of blocks in the real construction and in the drawing, we could see that some of the students had difficulty not only in interpreting a drawing but also they had problem with their spatial ability in perceiving the structures of the blocks even in the real construction. We expected that by having this discussion could help the students to better perceiving the structures of the unit blocks arrangement in the three-dimensional arrays. The students learnt from each other about how to see the structures of the blocks arrangement either in the real arrangement or in the drawing.

Lesson 5: Predicting the number of the blocks and determine the possible sizes of the boxes

Goal: In this lesson, we designed activities in which the students were ready to learn about volume. They could reflect on the previous activities they experienced with. It was expected that after they were familiar with the blocks arrangement and know how to calculate the blocks in a three-dimensional construction, they would be able to make estimation to the number of block units needed to cover up such an empty box. It relates to the meaning of volume “packing” of an object, the number of smaller objects, cube blocks or cube units can be put or can be packed inside the object. In the final section of this lesson, the students were asked to predict the volumes of some boxes with different sizes. Those boxes are different in sizes but could contain the same number of blocks. It was conjectured that the students could realize that the units to measure volume is in three-dimension not in one-dimension or linear such as measure only the length or only the height.

The Lesson:

In the previous lessons, the students worked with boxes, packages arrangement, cube blocks and its drawings. In this lesson, we also prepared boxes and cube blocks. In the first activity of this lesson, we gave them a box and four cube blocks. We asked them to predict the number of cube blocks needed to cover up the box. This activity looks similar to the activity with dodol boxes. The different thing is that now they have cube blocks rather than dodol packages. By observing the students doing this activity, we would like to know if the previous lesson could help the students who had difficulties in perceiving the structures of the objects arrangement in three-dimensional arrays. We predicted that some students still saw the arrangement as unstructured objects but we predicted that the class discussion about dodol packages and the discussion about blocks construction could promote the using of layers either in columns or in rows in calculating or in estimating the number of objects in an arrangement or inside a box.

The teacher started the lesson by telling the students that today she would give a box and a few blocks to each group. Then, the students are asked to estimate the number of cube blocks needed to cover up the box. We observed that during working with the box and the blocks they asked us if it is the same thing with estimating the number of dodol needed to cover up the carton and plastic boxes. They seemed to recall their strategies in that previous activity, but some others just try to solve the problem given.

After five minutes or so, the teacher asked the students to share their strategies in predicting the number of cube blocks can be put inside the box. The teacher then asked Anggi to tell her strategy. The following is the fragment between Bu Rima, the teacher, and Anggi:

Teacher : How do you estimate it? Please, speak loudly Anggi.

(Anggi iterated the cube blocks along the base. She only had four blocks. She first arranged the four blocks into rows and then moved it to get next row until the space was traced).

Teacher : Do you count every row?

(Anggi then nodded her head).

Teacher : So, how many do you have so far?
Anggi : Eight
Teacher : Then, the next? Yes, smart girl now, you are in which row?
Anggi : Three
Teacher : So, now how many
Anggi : Twelve.
Teacher : Now, in which row?
Anggi : The fourth row.
Teacher : So, how many?
Anggi : Sixteen.
Teacher : Yes, now what is the next?

(Anggi then arranged the blocks to the top of the box made column. She arranged three blocks vertically and one block unused. She kept doing to iterate the blocks and made another column of three blocks).

Teacher : How many blocks to the top, Anggi?
Anggi : Nineteen.
Teacher : To the top, Anggi? So, how many?
Anggi : Twenty, twenty two.
Teacher : So, how many the totals?

(Anggi did not give answer. She thought for a while and then continued iterating the blocks for more than 2 minutes but still could not estimate the volume of the box. But then her friend in group, Dinda helped Anggi. Dinda explained that Anggi can just multiply 16 to 3 and makes 48).

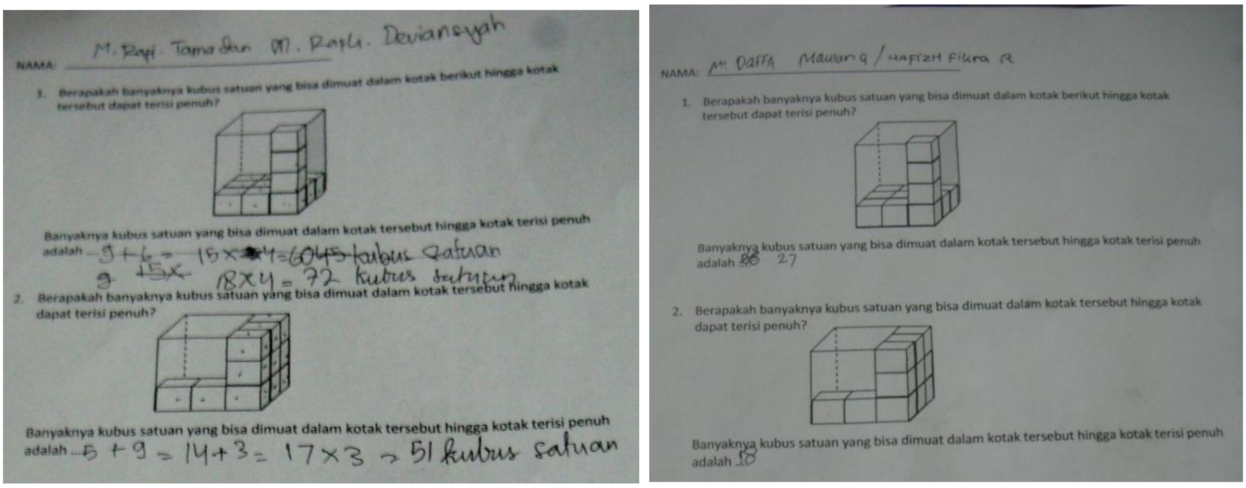
Based on our observation, Anggi tried to iterate the blocks along the base and imagine if she could iterate the blocks to the top until all the space was filled. She did very well when she iterated the blocks along the base. However, she looked doubt when she iterated the blocks to the top of the box. When the teacher asked how many blocks to the top, she did not answer three, as expected by teacher and also by me; indeed she answered nineteen. She tended to add the 16 blocks she had already to the three blocks she arranged later. Then she added three blocks in the next column become twenty two. She kept doing that but she could not decide where she had to stop iterated the blocks. That is why she could not determine what the total number of blocks can be packed in the box. She also had problem with her calculation because she counted twice the blocks on the base. She added all become 16 blocks but then when she arranged it to the top, she calculated also the block on the base. She seemed aware that the structure of the units is three dimensions, that to estimate the volume of the box she needed to measure it by iterating the units, but she could not coordinate and integrate the unit structure. She tended to see it as unstructured arrangement. Therefore, she iterated the blocks one by one instead of used layer structures in measuring or estimating the volume of the box. However, Dinda was one of the students who promote the use of layer strategy to solve the problem. She could imagine what is inside of the box if it was fully packed. She could perceive that the arrangement is in the layers sixteen structures. There were 16 blocks on the base and three layers makes 48.

The teacher then asked other students about their way in predicting the number of blocks needed. Most of the students seemed to cover up the base and then multiplied the number of blocks on the base and the number of layers as Dinda did. One of the students, Rafli, said that he imagined covering up the base first. He added four blocks plus four blocks, repeated it and got 16. Then he estimated the number of blocks can be arranged to the top, that is 3. And all together is 16 times 3, 48, more or less as Dinda's strategy.

Then, the teacher asked if other students had different strategies. But, most of students seemed to do the same thing, multiplied the number of blocks on the base with the number of layers. Then, the teacher told the students that in mathematics, we called the volume of the box is 48. She introduced the term "volume". When the students are asked what volume is, most of them said that volume is the content. Some said about the number of cube blocks can be put inside the box.

Today lesson was divided into two sections because the time was between the break. The first section was closed by giving worksheet about predicting the volume. The students worked in pairs to solve 4 questions on the worksheet. In the worksheet, the students were asked to estimate the number of cube blocks needed to cover up whole space of the box. We predicted that some students still had difficulties in working with drawing or representation. We conjectured that they might do mistakes such as still perceiving the blocks in the drawing as the squares and then multiply with the number of layers since most of them already able to use layers strategy in predicting the real blocks or had wrong interpretation of the number of layers itself.

The following are some of students' answers to solve question number 1 and 2. There was no discussion about this worksheet but by looking to students written work, we would like to know the thinking of other students that did not speak out during today lesson.



(a) (b)

Figure. 5.30. Examples of students' answers to the worksheet "predicting the blocks"

We tried to analyze their works as in figure 5. 30 as follows: Rafi and Rafli tended to calculate the number of squares they saw in the drawing (see the dots they made) and then multiplied it with the number of layers. These students tended to use the layers in calculating but they could not well perceive the number of blocks needed on the base or in the column in the form of drawing. However, we found that they were able to estimate the number of blocks needed in the real box with real blocks. The other students, Daffa and

Fikra did not give any explanation on their sheet. When we asked their explanation, their tendency is to calculate the blocks on the base or in the column and then skipped calculating or multiplied with the number of layers. However, they did mistakes in perceiving the number of layers. They tended to saw 3 layers of 9 blocks in the first drawing, instead of 4 layers as it is, equals to 27, and 2 layers of 9 in the second drawing equals to 18.

From their written works, most of the students already used the layers as the abbreviation to calculate the number of blocks needed to cover up the box. However, some of them had problem in seeing the structures of the blocks arrangement inside the box either still calculate the squares or wrongly interpreted the number of layers.

As we stated in our goal of this lesson, we would like to know how the students understand the situation that when they compared the volume of objects then they found that the objects are different in shapes and sizes but have same volume. We conjectured that some students still had perception that the unit used in measuring volume is one dimensions as they measured length or height. But, we believe that most of them could perceive it as three dimensions.

The teacher then continued the lesson by giving the students two more boxes. So, they now had three boxes with different sizes as shown in figure 5.31. She asked the students to determine from those three which boxes had the same volume. Actually, those three boxes had same volume, 48 cube blocks, but the size of the length, width and the height are different.

Figure. 5.31 Teacher showed the three boxes

Based on our observation, the students tried to fill in the boxes with the cube blocks. They estimated the number of blocks can be arranged in each box. After discussed with their groups, the teacher asked the students to share their ideas with others.

In the discussion, Rafi Tama explained to the teacher that his group found that there are two boxes which had the same volume. He explained that two of the boxes had the same height, 3 blocks high. And on the base they could put the same number of blocks, 16. So, the volume must be the same. But, the other box which height is 2 could not be same with the other two. The teacher then asked other groups to give their ideas. Dwi then raised her hand and explained her thought. Dwi told that her group found that all boxes can hold 48. She showed that she estimated the number of blocks needed on the base in each box and then multiplied with the number of layers. She used different strategy from the first activity in estimating the number of dodol. Last time she calculated it one by one and iterated the dodol along the empty space in the box but now, she used the layers to abbreviate her prediction.

The teacher then asked other students about what they thought about Rafi and Dwi thought. Then, Fadilla said that although the boxes had different sizes but the volumes are the same. The teacher then asked other students if they could compare the volume of the boxes by only measuring the height of it as Rafi Tama and his group did. Most of the students said that it could not be happen. Then, Gina explained that we should also look to the length and the width of it, not only the height. To know the volume, she told that we have to fill in the boxes with the cube blocks, first the base and then the height.

The teacher then asked the students to prove if the volume of those boxes were the same. She prepared 48 cube blocks in front of class. Dwi came to in front of the class and showed to the other students that in the first box she could arrange 4 rows of 4 blocks and she could make 3 arrangement of it. Then, in the second box with the same number of blocks, she could arrange 4 rows of 6 blocks in two layers. And to arrange the third one, she first added some blocks to the back of two rows of existing building and built the top layer from the remained blocks from the other two rows as we see in figure 5.32. Then, the teacher asked if other students could make another arrangement of the other possible box sizes. Fadilla raised her hand and then showed that they could also have a 4 layers-high box in which there were 12 blocks in each layer, 4 rows of 3 blocks.

Figure. 5.32. Dwi showed the different arrangement of the blocks

From what Dwi and Fadilla were doing, we observed that they were able to make an object-based transformation, where the objects could be arranged in different shapes but with the same volumes. This is also the idea of conservation of volume. The students were aware that although the blocks were moved in into another box or another shape, but if they did not take away any single number of blocks from the existing number of blocks, the volume would stay constant.

Conclusion:

Throughout this lesson, we could make following conclusions. In this stage, most of students considered about the using of layers to do abbreviated calculation or to estimate the number of blocks in an arrangement or in a box. Most of the students no more think about linear measurement in measuring the volume. They realized that measuring volume is about three-dimension. In this lesson also students learnt about the conservation of volume, that is although we moved in the blocks or we reshaped the blocks, if we still use the same number of blocks, the volume will stay the same. Some of them were aware that the boxes with different sizes might have same volumes since the number of blocks needed to cover it up is the same. In the next lesson, the students had other activities about the conservation in which they had to compare the volume of two boxes with different unit sizes.

Lesson 6: Measuring with different unit sizes

Goal: This lesson was aimed to investigate the students perception about the size of the units used in measuring volume of an object. How they compare two objects if they have different units measurement. It is also expected that they realized about the inverse relation of the units measurement that is the smaller its size the more it numbers and vice versa.

The Lesson:

In the previous lesson, the students did a lot of comparing the volume of objects, but by using the same units. In this lesson the students were asked to compare volume of objects with different unit measurement. We would like to know how they perceive the idea of volume and the idea of units used in measuring the volume.

We prepared two different boxes in which one, yellow box, is larger than another, blue box as in figure 5.33. There were also two different units, yellow units which are bigger and blue units. The students were asked to predict the number of yellow units can be arranged in yellow box and also blue units in blue box. Before letting the students to work with the boxes and the units, the teacher showed the boxes and posed question, “which one looks larger?” and all students immediately answered the yellow box.

In the discussion, the students found that blue box can hold more units than yellow box. The blue box can hold 32 blue units and yellow box can hold 12 yellow units. the following are the fragment in class discussion.

Figure. 5.33. The two boxes with the yellow units and blue units

Teacher : From those two boxes which one contains more blocks?
Students : Blue box
Teacher : How many?
Students : Thirty two.
(the teacher then wrote it down in the board)
Teacher : And how many in yellow box?
Students : twelve
Teacher : Is it true if I say blue box is larger? Let's see together these boxes, which one is larger?
Students :Yellow.
Teacher : But why the number of blocks in blue box is more and in yellow box is less?
Students :Because it is bigger.
(Teacher repeated her question)
Gina : Because the units in yellow box are bigger
Teacher : Gina said that yellow units are bigger and how about the blue units?
Students : smaller.
(The class discussion was interrupted because somebody knocked the door)
Teacher : So, it is because the units are bigger. And the blue box can hold more blocks because the blocks were smaller.

From the fragment, we observed that the students realized that they had different units with different sizes so that they could not directly compare the boxes. Because the sizes of the yellow units are bigger so the number will be less and vice versa for the blue units. Then, the teacher asked to do crosscheck. She asked the students to predict the number of blue units can be arranged in yellow box and vice versa. The teacher wanted to provoke the students that in comparing the volume of objects they should measure with the same units.

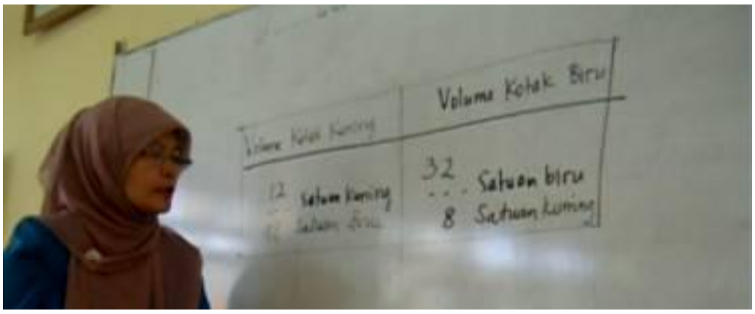


Figure. 5. 34. The table of students' findings
on measuring with different unit sizes

We observed that the students found that the number will change with the unit they used. From the table written on the board as in figure 5.34, the students could see that the capacity of an object can be represented in different number if they use different size of unit measurements. This actually relates to the concept of conservation of interior volume in which that the volume of the object stays the same but the size of unit measurement is different. Moreover, one of the students, Fadilla told that the larger boxes not always contains more units, it depends on the size of the units. She seemed to resume the table on the board that in the first row she saw yellow box hold less and in the second row she saw yellow box hold more. However, she did not compare what if the units were the same.

Conclusion:

Throughout this lesson we can conclude that the students could gain more insight about what a volume of a rectangular box is. The volume of an object represents the number of units can be put inside the object. However, if they used different unit size, they will get different numbers but the volumes stayed the same. If they compared two objects with different units measurement, it was difficult. So, they first had to determine which unit they would use in measuring those two objects.

They also learnt that the volume of an object was determined by the units used to measure it. In measuring a box, when they measured it with bigger size they will get less number of units and if they measured it with smaller units they will get more number of blocks.

Lesson 7: Worksheet and Discussion

Goal: This lesson was aimed to investigate how the students solve such a words problems related to volume conception. We expected that the students could reflect on the lesson series they had followed. It was about making a drawing of volume packing situation, calculating the number of objects in a three-dimensional arrangement in a drawing, estimating the volume of a box, estimating the sizes of the box, etc.

The Lesson:

In this lesson, we expected that the students were ready to solve problems related to volume "packing" concepts. However, we predicted that some students had difficulties in understanding and visualizing the situation given in the story. The other difficulties they might found is to relate the story with the concept of volume they had learnt.

The students had to answer 4 questions on the worksheet prepared. They worked in pairs so that they could discuss their answers. The questions are:

(1) On the base of a noodle box we can put 6 packages of noodles. To the top of the box, there might be 5 layers high of the packages. How do you know how many noodle packages can be put in the box?
(2) Vina wants to buy a box of chocolate wafers. From the top of the box, we can see 8 bars of wafers arranged, and from the front of the box, we can see 6 wafers arranged. How do you know how many wafer bars were arranged in the box?
(3) A cake box can hold 72 pieces of cakes. If on the base of the box, we can put 18 pieces of cakes, how many layers that might be there?
(4) Ms. Ani bought a box of tea. In the box, there are 4 layers high of tea arrangement in which in each layer there are 3 rows of 6 tea. How do you know how many tea are there?

The following are the resume of some of students' answers to the questions on the worksheet:

Table 3. Examples of students' answers to the worksheet "The box capacity"

Question No.	Examples of some answers	
	Correct	Incorrect
1.	Bagus: 6 x 5 = 30 Rafi: 6 on the base times 5 to the top is 30.	Anggi: 6+5 = 11
2.	Dinda: She first made a drawing. On the bottom part of her paper. In her drawing, there were 2 columns of 3 blocks in front and four layers to the back. Then, she wrote 3 x 2 = 6 x 4 = 24.	Nisa: 8 x 6 = 48 Rafli: Front part times top part. So, 6 x 8 = 48 Ridho: 8 x 2 = 16
3.	Farish: 72 : 18 = 4 layers. Daffa:18 x 4 = 72.	Tasya: 72 + 18 = 90
4.	Yudha: 3 x 6 = 18 x 4 = 72 Fikra: 3 x 6 x 4 = 72	Anggi: 3 x 6 = 18 + 4 = 22 Nadia: 3 + 6 + 4 = 13

After break, the students had a discussion about their answers. The teacher asked some students to explain their answers in front of class. They gave argument and justify each other answers.

We observed that from table 5.1. for question number 1, most of students correctly answer the question. Anggi is the only one who added the numbers rather than multiplied it. The students told Anggi that they should not add the 6 and 5. They can add 6 noodles plus 6 noodles plus 6 until 5 times makes 30 noodles, but not added 6 and 5 become 11. In answering question number 2, most of students answered same as Nisa did, 8 x 6 makes 48. So, when the teacher asked their comments about how was her answer, most of the students said that it was correct. However, the teacher then asked other students to give other solution. From the worksheet, we knew that Ridho made a drawing of his interpretation of the problem in the back of his paper but he was shy to explain. In his drawing, he could well interpret the story into a drawing. The only mistake he did was calculating the layers as two layers rather than three layers of 8. It makes him only had 2 layers of 8 and makes 16. When the teacher asked about other solution, Dinda would like to give her reason. She made the same drawing as Ridho and could well interpret it as 4 layers of 6 and makes 24. She said that in her drawing as in figure 5.35 everyone could see representation of 6 wafer bars from front and 8 wafer bars from the top as mentioned in the question.

Figure 5. 35 Dinda explaining her answer of problem 2 in the worksheet “the box capacity”

In discussing question number 3, the teacher asked Daffa to explain. She thought his answer was 72 layers of cakes so it was incorrect. But, then Daffa explained that his answer was 4 layers of cakes. He seemed to do skip calculating of 18 until got 72, because he found it was enough to do until 4 times then he wrote 18 x 4 equal to 72. In different way but give same result, Farish said that he divided 72 by 18 and got 4 layers. But there were still some students who answered incorrectly like Tasya. She tended to do the same strategy as Anggi, added the number of cakes on the base and the total number of cakes makes 90 cakes on total. From question number 4, we could see how students perceive the structures of the tea arrangement. Some of them did the same as Yudha, first calculated the number of arrangement in one layer and then multiplied it with the number of layers.

Conclusion:

Throughout this lesson, we concluded that most of students could perceive the arrangement as structured objects in term of layers. They were able to interpret the questions as action to do skip calculating or multiply the number of arrangement in one layer with the number of layers to get the total number of the arrangement. They used their previous knowledge and experience in working with the blocks, with the views of the building blocks, etc.

F. Analysis throughout All Lessons

In this analysis, we looked at all lessons and searched for connections between them. We focused on the students' learning trajectory throughout those lessons as we wanted to see if the activities have supported students in learning volume measurement.

In the first lesson, we found that some students thought about linear measurement that is the height, the length or the width of an object or a box, to determine how large an object is. But, some others were aware that in comparing the capacity of two objects, they need a unit to measure the capacity that is the cakes as they said. By knowing how many units or cakes can be put inside in each box, they could compare the capacity of the boxes. In estimating the number of dodol can be arranged in a box, some students iterated the units (dodol) along the empty space until they thought that it was fully covered. However, because we did not give enough dodol, some students could only imagine the number of dodol on the base but they could not imagine what on the top of it. But the students, who could well perceive the structures of the arrangement even from this first lesson, help those students by promoting the use of layers in estimating the number of dodol needed.

In the second lesson, we then asked the students to make a drawing of packages arrangement. We found that some students had difficulty in making a drawing of a three-dimensional objects arrangement as a concise building. Some of them drew separates views of the arrangement and made their interpretation of it, but some others who successfully drew the arrangement as a concise building thought that they had wrong drawing since they found different number of objects between in the real arrangement and in their drawing. These students tended to calculate the number of squares they saw in their drawing. They could not well read off their drawing. However, other students who could well interpret the two-dimensional representation of the objects helped those students by showing that it was not squares as they thought but it was tissue packs or tea boxes drawn on a paper.

The third lesson was closely related to the second one. The students were asked to make a building based on its drawings from different views. The students who drew separate views of the building could easily recognize what they should do and how different views could refer to a building. Some others found difficulties in interpreting the drawing especially the side view. They built the front and the side view separately and then put it together. It makes them needed more blocks than prepared. Other students tended to built the outer part of the building and then fill in the interior part. By doing that, we expected that they could relate the visible or the external views of the arrangement with the invisible part that is inside the arrangement so that they could have a coherent image of it.

In the "calculate the blocks" activity, some students still perceived the blocks arrangement as unstructured faces. In calculating the number of blocks in the real construction, some calculated the faces of the blocks from the front and on the back and then calculated the faces they could see from the sides, left and right, and also from the top. These students could not perceive the structures of the blocks arrangement as rows or columns. Other students who could not perceive the layers structures but was able to calculate correctly the number of blocks tended to calculate the blocks one by one. And, in the class discussion, the students who could well perceive the layers always promoted to use it as the abbreviated way in calculating the number of blocks in the arrangement. These students built the blocks configuration by first building the base or the front wall and then continued built the layers. We observed that their strategies in building the blocks influenced them in calculating the blocks.

In interpreting a drawing, most of students still had difficulty in read it off. Although in the second activity they already had discussion about that but most of the students still tended to calculate the number of squares rather than saw it as blocks arrangement. Even more, some of them doubled the number of squares since they thought about the front, the back, the left and the right side, the top and the bottom part. In the discussion, the students discuss about the proper way to read of the drawing of an object made of blocks arrangement. Most of them were likely to calculate the blocks on top and then calculated the number of rows, and some others calculated the blocks from the right side and then calculated the columns.

The activities in lesson 5 seemed as the repetition of the activities in the first lesson. However, in this lesson, the students worked with the cube blocks as we used in lesson 3. We found that some students still have difficulty to imagine what are inside the box. They tried to iterate the blocks along the box but could not find the total. However, most of students tended to cover up the base of the box and then find the number of possible layers. However, in solving the problem in the worksheet about predicting the number of blocks, almost half of the students still could not well perceive either the number of layers or the number of blocks on the base. But, almost all of them used the layers as the way to estimate or to calculate the number of blocks needed, no more one by one calculating.

In comparing the volume of three boxes, there were some students who still thought about linear measurement. They only compared the height of the boxes. However, most of students did estimate the number of blocks needed in each box first and then compare the total number of blocks, the volumes. Some of them concluded that measuring volume is calculating the number of unit needed to cover up the space inside the object and not merely measuring the length or the height of the object. Some others stated that although the boxes had different sizes those had same volume since the number of blocks can be put inside were the same. Based on our observation, some students realized that in the first and second box, they had same number of blocks on the base and also had same number of layers. Some students were able to imagine the objects-based transformation of the blocks configuration, and also capable of reasoning about it by explaining the transformation of the units arrangement.

In the sixth lesson, the students learnt that volume of an object can be represented in different numbers depends on the unit size we used in measuring it but its volume stayed constant. If they used bigger units the number will be less and vice versa. The students also realized they could not compare the volumes of two objects unless they measured it with the same units because they found that the larger box could also hold less number of units since it was filled in with the bigger unit size than the smaller box which was filled in with smaller units.

In the last lesson, they discussed about their interpretation and solutions of the questions related to the volume packing concepts. To solve the problems, they applied their knowledge and experience in previous 6 lessons. They could explain others about their reason and why they thought it was the correct solutions. In our observation, most of students could well imagine the situation in the stories of each problem. They interpreted it into drawings and then saw it as packing situation of structured arrangement. Most of the students tended to calculate the number of objects in one layer and then the number of layers as the way to calculate the number of objects in the arrangement.

G. End Assessment

At the end of the series of lessons in the teaching experiment, we conducted an assessment to see if our activities could support the students in learning the volume measurement. There were 8 problems in the assessment. The problems were about predicting and calculating the volumes of boxes or arrangement of objects. The students worked in pairs. There were 32 students, so we analyzed 16 students' answers.

To analyze this end assessment, we first made an analysis table. We looked at each item (each problem) and see what strategies students used to solve the problem. We grouped the answers into correct and incorrect answers to determine the proportion of the number of students who could correctly answer the questions. Then, we tried to describe their tendency in solving the problems in this section. Before that, we would like to give brief explanation about the assessment as follows.

Problem 1 and 2 are aimed to promote using structure of layers in solving the problems related to determine the “packing” capacity or volume of objects in an object (a box). Problem 3 and 4 are aimed to assess how the students “read off” (visualize) two dimensional representation of the three-dimensional arrangement. Problem number 5 and 7 were aimed to know how the students predict the number of objects could be arranged in an empty space. It could also assess how they perceive the spatial structure of the arrangement. Problem 6 is aimed to assess students if they can apply their knowledge and skill in visualizing the situation of the arrangement of objects in a box and then calculate the number of blocks in their drawing. Problem 8 are aimed to know how they use their mental image to solve the problem related to determine the volume or capacity of an object (box). The students were not given any drawings as in the others problems nor the clue about number of objects in one layer. It is also to asses if they perceive the concept of measuring volume as covering space.

Problem 1

In this problem, we gave a problem about a box of noodles packages. If there were 14 packs could be seen from the top and based on information from the seller there were 3 layers arrangement on the box, then how many noodles pack all together? This problem was aimed to see students’ awareness of using the layers to calculate the total number of noodles packs in the arrangement.

The analysis table showed that 15 out of 16 students’ answers were correct. The students were able to estimate the number of noodles packages arranged in the box. However, we realized that this did not indicate students’ ability to structure the arrangement since we told them the number of the noodles packs in one layer and also the number of layers already. We observed that most of students multiplied the number of packs on the top and the number of layers, 14 times 3 makes 42.

Problem 2

In this problem, there was a cake box in which we could see ten cakes arranged in 2 rows of 5 cakes. We told the students that there were 40 cakes on the box. The question was, “how high was the cakes arranged in the box?”

The analysis table showed that 11 out of 16 students’ answers were correct. The students were able to estimate the number of layers when they knew the total number of cakes and the number of cakes in one layer. Some students divided 40 by 10 makes 4 and some others multiplied 10 by 4 makes 40. The first group students seemed to perceive that the total number of cakes is the multiplication of the number of layers and the number of cakes in one layer. So, to get the inverse, they divided the numbers. The second group seemed to do skip calculating of ten until 40, and then they conclude that they need 4 times of 10 to get 40. As we mentioned there were 11 correct answers or 5 incorrect answers. Two out of 5 explained that they could have 8 layers since the cakes were arranged in 2 of 5 rows. So, they divided 40 by 5 makes 8. Two others divided 40 by 20 makes 2. They seemed to calculate the number of cakes they could see and then multiplied it with the number of rows, 2, so they have 20 as the number of cakes in one layer. One other student even multiplied 40 by 2 makes 80 layers.

Problem 3

There was a picture of an arrangement of tea boxes. There were 27 tea boxes arranged. We would like to know if the students could read off the pictures of the arrangement.

Based on our analysis table, there were 10 out of 16 answers were correct. This problem is likely with the problem in our pre-test. In the pretest less than half of the students could answer it correctly. And, in this test more than 60 percents of the students could answer it correctly. It indicates that most of the students had learnt something. The students answered that there were 9 times 3 equal to 27 tea boxes in the picture. The students use the layers to determine the total number of tea boxes in the picture.

Problem 4

Problem 4 was similar to the third problem. However, they had to calculate the cube blocks rather than tea boxes. In the problem, there were 9 blocks on the base and stacked 4 layers up. So, all together is 36 blocks.

There were also 10 out of 16 students' answers were correct. We can conclude that there were no difference between concrete objects like tea boxes or the cube blocks arrangement for the students after they become familiar with the cube blocks arrangement. In the pre-test, only 2 out of 16 students' answers were correct. It indicates that the students perception had already changed from saw it as unstructured faces (before involving in the series of lessons) to structured layers arrangement (after the lesson series). It also indicates that the class discussion about how to calculate the blocks arrangement had great influence to the students. Most of the students answered 12 times 3 makes 36 blocks. These students tended to first calculate the number of blocks in front side and then multiplied it with the number of layers to the back, 3. The other students calculated the number of blocks they saw from the top that is 9 and then multiplied with the number of layers, 4 and all is 36.

Problem 5

This problem was also likely with a problem in the pre-test. The students were asked to predict the number of blocks in the picture. It was known that there were 5 rows of three blocks on the base and there were 3 layers high. So, the number of blocks needed to cover up the box was 45.

Based on our analysis table, there were 10 of 16 answers were correct. It was also big improvement, from 2 (in the pre-test) to 10 out of 16 correct answers. All of these students calculated the number of blocks on the base and then multiplied with the layers, 15 times 3 makes 45. They seemed influenced by the activity in predicting the number of blocks. In the activity, most of the students also calculated the number of blocks on the base first.

Problem 6

The problem was about the arrangement of cakes in a plastic box. We told the students that the cakes were arranged in such manner so that they could see 12 cakes from front and 9 cakes from the top of the plastic box so in total there were 36 cakes. They were asked to imagine the situation, make its drawing and then calculate the number of cakes in the arrangement (in the box). Actually, the drawing was exactly the same as the drawing in question 4 but we did not think the students realized it. This problem was likely with one of the problems in the seventh activity, items 2.

There were only 7 out of 16 answers were correct. These students could interpret and draw the situation and then do correct calculation. The students tended to calculate the number of blocks on the top, 9, and then multiplied with the number of layers, 4. Or, the number of blocks from front, 12, and the number of layers (in column), 3. However, most of students, 9 other students, tended to calculate those numbers, 12 times 9 makes 108. Most of students still could not well perceive the separates view of the arrangement.

As we found in our activities that most of students had difficulty in making the drawing and interpreting different views.

Problem 7

In problem 7, there was a box of dodol. The picture was really familiar for them since it was really the box and the dodol they used in the first activity. They were asked to determine the number of dodol in the box. There were 14 out of 16 answers were correct. Most of them try to cover up the base that is 2 rows of 7 or 7 rows of 2 dodol, 14, and then multiplied it with the number of layers that is 4 makes 56. In the class discussion last time, the students found 70 dodol and some others found 60. They did not remember the final number but they seemed greatly influenced by the series of activities they involved in.

Problem 8

In the last problem, we only tell a story without any drawing. In the story we told them that there was an arrangement of wafer bars in a box. The wafers were arranged in 3 rows of 5 on the base and 6 layers stacking up until fully covered.

Based on the analysis table, there were 13 out of 16 answers were correct. Most of the students could perceive the situation of packing as covering space. More students could answer this question rather than problem number 5 in which they had drawing of it. The contextual problem was more make sense for them. All of these students calculated the number of wafer bars on the base, since it was mentioned in the story, and then multiplied it with the height of the arrangement, 6 bars, and it makes 90 wafer bars in total

From the end assessment, we could draw following conclusions. Most of the students still had difficulty in perceiving the separates views of the arrangement, drawing and interpreting it. The students needed more discussion about that part. However, the students were greatly influenced by the activities they followed such as class discussion about calculate the blocks, dodol boxes, and predicting the number of blocks. More than

60 percents of students showed that after followed the series of the lessons they could well perceive the three-dimensional objects arrangement as structured layers arrangement.

CHAPTER VI

CONCLUSION AND DISCUSSION

The aim of this research is to develop classroom activities that support students in learning volume measurement. In this research, we posed a research question: "*how can spatial visualization support students' spatial structuring in learning volume measurement?"* In this chapter, first we answer the research question based on our retrospective analysis, and then we reflect on some issues in this study and elaborate recommendation for further studies to improve the mathematics education in Indonesia.

A. Answer to Research Question

"How can spatial visualization support students' spatial structuring in learning volume measurement?"

To answer this question, we looked at the sequence of learning activities and investigate the role of spatial visualization tasks serve in each sequence of students' learning. Then, we can conclude how the role evolves during the activities.

1. The Role of Spatial Visualization Task in Each Sequence of the Students' Learning

This research hypothesized that the students will not employ the layers structures in calculating the three-dimensional unit configuration unless they realize that it is a structured arrangement. Therefore, in lesson 1 we conducted an activity to first evoke students' awareness of the needed of units in measuring capacity of an object and then in estimating the number of units arranged inside the object, they have to be able to imagine what might be inside that object or how many units needed. Through this activity, some students realize that they should iterate the units (dodol) along the empty space until they thought that it was fully covered. Because we restricted the number of units used, some students were aware of using layers as the way to estimate the number of units (dodol) needed. These students were able to see it as structured objects already but most of students needed more times to come up to that idea. They still saw it as unstructured arrangement. They needed to practice with more concrete tasks of increasing structural complexity through which they can acquire personally constructed views of the organization of the three dimensional rectangular arrays. In lesson 2, we found evidence that in visualizing the arrays into drawing, some of them could represent the arrangement from different views. They were aware that the arrangement could not be seen only from one side. However, at this initial stage students focused on the external aspects of the array and perceived it as an uncoordinated set of faces.

Therefore, in lesson 3 and 4 we conducted activities in which students experienced of building and calculating cube configurations. We found evidence that the building blocks activity has helped the students to coordinate the separates views of the arrays. And it influenced the students in calculating the blocks arrangement. Some of students tended to calculate the number of faces on different views but some others who were able to coordinate and integrate the different views could see that it was an arrangement of layers. These students built the construction from the base to the top or from the front to the back. So, these students only calculate the number of blocks on the top or on the front or from the left or right side and then calculate the number of its layers.

In lesson 5, we conducted activity in which students had to predict or estimate the number of blocks needed to cover up a box. It looked as repetition of activity in lesson 1. However, in this stage we conjectured that they would reflect on the previous activities. Our findings showed that the students gradually become capable of coordinating the separate views of the arrays and they integrated them to construct one coherent model of the array. Based on our findings, most of students were aware of using layers in predicting the number of cube blocks needed to cover up the boxes. They tended to first calculate the number of blocks in the row (base) or on the base and then do skip calculating or multiplied it with the number of layers.

In later activities, we conducted the activities in which they have to investigate the boxes with different sizes but with same volumes and investigate the boxes which were measured with different unit sizes. This actually related to the concept of conservation of volume. To be able to perceive the different configurations of the same volumes or the structures of unit sizes, the students needed to be able to make object-based transformations where only the positions of the objects are moved. To reason that three boxes with different sizes could have same volume by only predicting the volumes, the students need to be able to mentally imagine the transformation of the unit configuration. The students could reason that with the same number of blocks they could have different configuration with different shapes. For example, if they have a three layers-high building with 16 blocks on the base, they could arranged it on 2 rows of 8 blocks or 4 rows of 4 blocks and if they wanted to have more blocks on the base they have to make compensation such as had less number of layers. Our finding showed that this activity could provoke students to deepen their understanding of the structures of the unit arrangement.

In investigating the structures of the unit sizes, the activity we conducted asked the students to predict the volume of different boxes with different unit sizes. In our class, the puzzlement happened when the students found that the larger box hold less number of units. However, then they realized that they could not compare the objects if they measured with different units. The students were able to found the inverse relation of the number of units and its size. The bigger a unit is, the less number will be needed and the smaller the unit the more numbers will be needed. This activity has also brought issues that the students could measure the volume of an object with different units and will result different numbers. So, the volume can be represented in different numbers depends on the unit size they used in measuring it but its volume stayed constant.

2. Conclusion

In short, this research has shown that the spatial visualization tasks play an important role in students' learning trajectory. The ability to read off the drawings of three-dimensional objects arrangement, the skill to mentally imagine what is inside a three-dimensional object, the skill to coordinate and integrate different views of an arrangement into a coherent model of an array and the ability to make object-based transformation were needed to be built to support students in perceiving the structures of the unit of volume measurement arrangement. In learning volume measurement we identified that at the initial stage students focused on the external aspects of the array and perceived it as an uncoordinated set of faces. At later stage as they reflected on experience of calculating or building cube configurations, students gradually become capable of integrating the views to construct one coherent model of the array.

Our findings suggest that in order to be able to calculate the volume of an object made of small cubes the students need to be able to perceive the structures of the units' configuration. Students' spatial structuring abilities provide the necessary input and organization for the numerical procedures that the students use to calculate an array of cubes. Using spatial structuring strategy allows students to determine the number of cubes in term of layers and then multiple or skip-calculate to obtain the total number of cubic units.

B. Reflection on the Important Issues

In this research we not only focused on students' thinking process but also we observed how activities helped the students in building their mental model of the situation, how the discussion help the low achieving students to learn, and how the role of the teacher in supporting the students learning.

1. Realistic Mathematics Education

In designing the sequence of instructional activities, we consult to some ideas of realistic mathematics education. On the other hand, the sequence of activities designed in this study is only a part of longer series of learning trajectories in learning volume measurement. And, the descriptions of the principals of RME apply to mathematics learning as a process will take months or years and perhaps could not be applicable to a short series of activities as in our research.

But, we would like to describe our findings so far as follows. In this study, comparing the capacity of *dodol* boxes is the contextual situation in which the students solve the problem related to their daily-life situation related to the capacity or volume of an object. The students could bring their informal knowledge about packing situation to solve the problem in the lesson. On the other hand, the "food packages pictures" activity encourages students to shift from contextual situation to more abstract level. The students were asked to make representations (drawings) as the models-of the situation they perceived.

In the building block activity, the students could use the cube unit blocks as model-for to represent any kinds of object that they want to arrange in three dimensional rectangular arrays. And in the later lessons, the students could use their experience with the three previous levels to do reasoning. In "predicting the number blocks" activity and "determining the possible size of the box" activity, the students could focus on the discussion of concepts of units and covering the space.

2. Classroom Discussion

In this classroom, the teacher and the students were struggling to develop a constructive discussion. There were 32 students in the class. We observed that in the discussions, not all students participated. Only few of them engaged with the discussion while some others busy doing something out of the lesson. The teacher accepted that the culture of discussion was not familiar with the students. They had discussion in other subject as in natural or social science but hardly ever in mathematics. We could say that it was new for the students to have these socio mathematical norms.

During the lessons, the students worked in small groups of four to five students. They first discussed in their group to solve such a problem in each lesson and then in the class discussion they told others about the result of their discussion. The students were free to express their ideas and use their own strategies. The students compared their strategies and then promoted to use abbreviated strategies. The low achievers could have been supported by involving in the discussion. They could learn from others mistakes, their own mistakes and listen to others' more sophisticated strategies.

3. The role of the teacher

In this research, the teacher had been teaching for 27 years, always in grade 4 or 5. She had a lot of experiences in teaching but it was really new for her to conduct a classroom environment as in this research, open and students center. Based on our first day interview with her, she told that it was not usual to have discussion in mathematics lessons as in sciences lessons. However, she is an open-minded person who would like to apply the innovation of the new approach in teaching mathematics in her classroom.

During the experiment, she has shown a good performance in keeping promoting the use of layers in perceiving the structures of the objects arrangement. She has always encouraged students to use abbreviated way to predict or to calculate the blocks arrangement. She asked the students to compare their strategies and find the best one. Look to students' mistakes and try to fix it together. She became the facilitator for students in the learning process. She collected the students' ideas and then asked for judgments from other students about their friends' thoughts. It was not easy for the teacher to manage the students' discussion. However, she has been trying to promote the development of socio-mathematical norms in her classroom.

C. Discussion

In this section, we would like to discuss the most important issue in this research that is volume measurement. As we explained before in chapter 2, in this research, we would like to support students' spatial structuring in learning about volume "packing" measurement.

Then, there are questions from some other researchers, first: We only focused on teaching "packing" volume as separated concept instead of teaching volume measurement as a whole concept that is "packing" and "filling" volume at the same time. How we make sure that the students understand the concept of volume "packing" not as isolated concept from the "filling" volume? And second, what students learn about the concept of volume in this series of lessons? What they perceive about volume of a solid object?

Related to the first question, we realized that it might be dangerous for students when we separated those two terms, volume "packing" and volume "filling",.

Because then, perhaps, they perceive that only cube, cuboids and rectangular prism (as we have in our lessons) have volume while the others shapes such as cylinder, pyramid and sphere do not have any volume. Or, the units must be small cubes and the other units can not be used to measure volume. As we know that in measuring the "filling" volume, the space is filled by iterating a fluid unit which takes the shape of the container in which the unit structure is one-dimensional. On the other hand, in measuring the "packing" volume, the space is packed with a three-dimensional array unit which is iterated in the third dimension.

Based on Indonesia curriculum, in fact, in grade five, the students only learn about the "packing" volume. They had learnt about the "filling" volume in the lower grade, in grade 1. At that stage, they were asked to compare two or three cans in the pictures which have same shapes but different sizes and then determine the order of the cans based on its volume from the smallest to the largest or vice versa. Only later in high school, they will learn more about "volume" as one whole concept. No more separated as "filling" or "packing" volume. They will also learn how to make generalization in calculating the volume with standard units, either with cm^3, m^3, or in liters. So, the series of lesson we designed in this research will help them to have good preparation to come to more formal mathematics in learning about volume measurement.

However, it is possible to add activities in our series of lesson related to teach the filling and volume packing simultaneously. This might be applied in the next cycle of our hypothetical learning trajectory. For example, we propose an activity in which the students are presented with a container full of water and a unit cube said to be made out of iron. Students are asked what would happen to the water when the cube is put into the container. This activity can support students in understanding about displaced volume and also bridge students in relating the filling and the packing volume. It is if they put more and more cube blocks into the container until the container is fully packed with the cube blocks, the water will be come out from the container. It means that the volume of the container can be represented either by the amount of water in liters or by the number of cube blocks inside, and those two are equivalent.

The other question is about the possibility of students' misconception about their understanding of volume measurement in which they might think that only empty space objects which have volume, such as empty boxes or empty bottle, because those objects can be filled or packed by units. Then, the problem will come up when we asked students what they think about volume of a solid object. If we pose question like, "Do you think a tree has volume?' Most of students might answer, "No, trees do not have volume, because we neither can fill in it nor pack it with units". What we can do then? We propose to design one more additional activity in which the students will be given a solid rectangular block measuring, for instance, 20 cm x 30 cm x 40 cm, and a pile of unit cubes. Each student will be asked to find the number of unit cubes needed to construct the block and to explain what was represented by the number they found. By experiencing this activity, we expect that the students could realize that not only empty objects which have volume, but also solid objects. We can measure its volume by knowing the dimensions of the object and compare its volume to the configuration of unit cubes (or other units we can use in measuring the volume) needed to build the solid blocks.

In our series of lessons, we realized that it is only small part of the long learning trajectory of volume measurement. We need to consider more aspects to cover in our lessons when we want to support students in learning volume measurement. Spatial structuring is only one of parts of the aspects students need to learn. We have to design more activities which cover about the concepts of conservation of volume (interior volume and displacement volume) as the basic concepts of volume measurement, the integration between filling and packing volume, and also the conception that volume not only can be measured in empty space objects but also for solid objects such as objects made of woods or irons.

However, at this time, this is what we can contribute to the domain of volume measurement research. We will give recommendation to the next research to give even better contributions.

D. Recommendations

In this section, we would like to give our recommendation related to apply the RME approach in the classroom, teach the volume measurement in grade 5, and suggestion for further studies.

1. Realistic Mathematics Education

In our RME classroom, the use of context has stimulated students to think of a way to solve such a problem. The students could bring their informal knowledge to get ideas in solving such a mathematics problem. In our class, the situation of packing boxes and the packages arrangement could provoke students to investigate the structures of three-dimension objects in an arrangement.

In our classroom, the contributions from the students are highly expected. Students learnt from each other solutions from a class discussion. They compared strategies and discuss the abbreviated strategies could be used to solve such a problem. In our study, we found that the students could appreciate other solutions and then share theirs. Then they judge together if it was right solution or not if it is effective way to solve the problem or not. They were free to use their own way and strategies. In an open environment, the students could construct their own knowledge. However, it takes time and need more efforts to continue the development of the socio-mathematics norms, where students have ability to choose the best strategy. It is important that the teacher and the students keep on the discussion culture and the learning attitudes had been developed.

Therefore, we recommend to the teachers in Indonesia to use RME or PMRI approach in their teaching. This approach allows teachers and students to have discussions and active interactions. And the most beautiful thing in this approach that we see mathematics as human activity which makes the learning more meaningful not just as procedures to follow or rules to apply in solving problems.

2. Volume measurement

In Indonesian primary school, the first time students learn about volume measurement, they are directly asked to calculate the volume of an object made of cube blocks in the form of drawing. As we found most of the students tended to calculate the number of squares rather than number of blocks. Only after doing some exercises about that, the students are usually given the formula of volume: length times width times height. The students have no opportunity to find other way in finding the volume of rectangular prism and cubes. The only way is applying the volume formula to solve the problems in their text book.

Based our findings, the development of students' understanding of volume measurement in grade five, should be seen in a step by step sequence. Students need to practice with concrete tasks before engaging with two-dimensional representations of rectangular solids objects made of cube blocks. In this way, students move from perceiving the external visible aspects of the object to its internal structural organization in terms of units of volume measurement. Later, to solve daily problems related to volume packing, the students could use the layers strategy in calculating the volume of the object with certain sizes. It was not necessary to memorize and then apply the volume formula to solve the problems. The students can construct their own strategy and their own knowledge.

E. Further studies

In our study, we only focus to a specific aspect of volume measurement that is spatial structuring. Later research could also study about other aspects of volume measurement that was little studied in this research such as the concept of conservation of volumes or was not studied like the volumes of other shapes than cube and rectangular prisms.

The findings of our research raised some other questions such as how do the students perceive the concept of conservation of volume? How to support students to relate the concept of volume filling and volume packing measurement? Further research is needed to answer those questions.

REFERENCES

Bakker, A. (2004). *Design Research in Statistics Education. On Symbolizing and ComputerTools*. Amersfoort: Wilco Press

Battista, M. T., & Clements, D. H. (1998). Students' understanding of three-dimensional cube arrays: Findings from a research and curriculum development project. In D. Chazan & R. Lehrer (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 227–248). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Battista, M.T. (2004). Applying cognition-based assessment to elementary school students' development of understanding of area and volume measurement. *Mathematical Thinking and Learning, 6(2),* 185-204.

Ben-Haim, D., Lappan, G., & Houang R.T. (1985). Visualizing rectangular solids made of small cubes: Analyzing and effecting students' performance. *Educational Studies in Mathematics,* 16: 389 – 409.

Charlesworth, R & Lind, K. K. (2010). *Math & Science for Young* Students. Belmont, CA: Wadsworth.

Creswell, J.W. (2003). *Research Design Qualitative, Quantitative, and Mixed Methods Approaches Second Edition.* California: Sage Publications.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultultural perspectives in the context of developmental research. *Educational Psychologist, 31(3/4),* 175-190.

Curry, M., & Outhred, L. (2005). Conceptual understanding of spatial measurement. In P. Clarkson, A. Downton, D. Gronn, A. McDonough, R. Pierce, & A. Roche (Eds.). *Building connections: Theory, research and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 265-272). Sydney: MERGA.

De Lange, J. (1996). Using and Applying Mathematics in Education. In: A.J. Bishop, et al. *International handbook of mathematics education,* Part one. 49-97. Kluwer academic publisher.

Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures.* Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD Beta Press.

Gravemeijer, K., Bowers, J., & Stephan, M. (2003). A hypothetical learning trajectory on measurement and flexible arithmetic. In M. Stephan, J. Bowers, P. Cobb, & K. Gravemeijer (Eds.), *Supporting students' development of measuring conceptions:Analyzing students' learning in social context* (pp. 51-66). Reston: NCTM.

Gravemeijer, K., & Cobb, P. (2006) Design research from the learning design perspective. *Educational design research* (pp. 17-51). London: Routledge

Hegarty, M., Richardson, A. E., Montello, D. R., Lovelace, K., & Subbiah, I. (2002). Development of a self-report measure of environmental spatial ability. *Intelligence*, 30, 425–447.

Hegarty. M. & Waller, D. (2004). A dissociation between mental rotation and perspective-taking spatial abilities. *Intelligence*, 32*:* 175-191

Kozhevnikov, M., & Hegarty, M. (2001). A dissociation between object-manipulation spatial ability and spatial orientation ability. *Memory and Cognition*, 29, 745–756.

Lowrie, T. (2002). The influence of visual and spatial reasoning in interpreting simulated 3D worlds. *International Journal of Computers for Mathematical Learning, 7,* 301-318.

O'Keefe, M., & Bobis, J. (2008). Primary Teachers' perceptions of their knowledge and understanding of measurement. In M. Goos, R. Brown & K. Makar (Ed) *Proceedings of 31st Annual Conference of the Mathematics Education Research Group of Australasia Navigating Currents and Charting Directions,* vol 2: 391-398.

Sarama, J. & Clements, D.H. (2009). *Early Chilhood Mathematics Education Research: Learning Trajectories for Young* Students. New York: Routledge.

Sembiring, R.K., Hadi, S. & Dolk, M.L.A.M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM Mathematics Education*, 40(6), pp. 927-939.

Sumanto, Y., Kurniawati, H., & Aksin, N. (2008). *BSE: Gemar Matematika 5*. Jakarta: Pusat Perbukuan Depdiknas.

Titus, S.J., and Horsman, E. (2009). Characterizing and improving spatial visualization skills: special volume on Research on Thinking and Learning in the Geosciences. *Journal of Geoscience Education*, vol 57: 242-254.

Treffers, A. (1987). *Three Dimensions. A Model of Goal and Theory Description in Mathematics Instruction – The Wiskobas Project*. Dordrecht, The Netherlands: Reidel Publishing Company.

Van Hiele, P. M. (1986). Structure and Insight: *A Theory of Mathematics Education*. Orlando: Academic Press, Inc. (London) Ltd.

Voulgaris S.,& Evangelidou, A. (2003). Volume conception in late primary school students in Cyprus. In Gagatsis, A. & Papastavridis, S.(eds). *Proceedings of the 3rd Mediterranean Conference on Mathematics Education*. (pp.485-493). Greece.

Woolfolk, A. (2007). *Educational Psychology (10th edition)*. Boston: Pearson Education.

Pre-Test

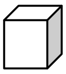
NAME: ____

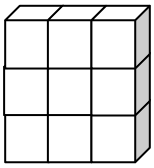
1. a. How do you count the number of soap bars in the picture below:

My answer:

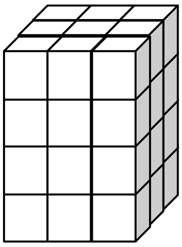
b. How do you count the number of tea boxes in the picture below:

My answer:

2.  is a cube block. How do you count the number of cube blocks in the pictures below:

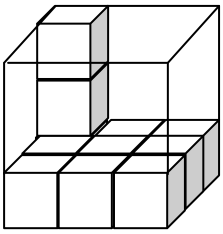
a. 

My answer:

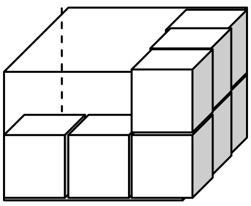
b. 

My answer:

3. How do you predict the number of cube blocks needed to cover up each box in the pictures below:

a. 

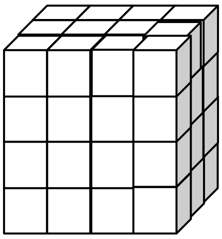
My answer:

b. 

My answer:

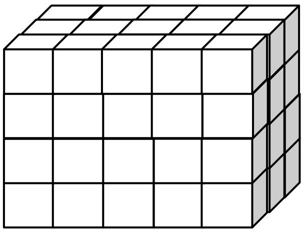
NAME: ____

1. Count how many blocks needed to build this construction:



The number of blocks needed is ….

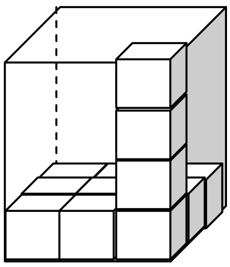
2.Count how many blocks needed to build this construction:



The number of blocks needed is ….

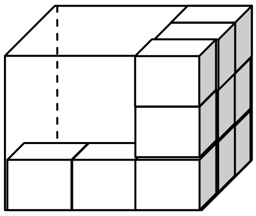
NAME: ____

1. Predict how many blocks needed to cover up the box!



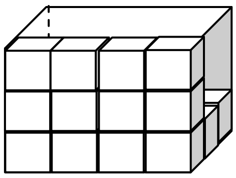
The number of blocks needed to cover up the box is ….

2. Predict how many blocks needed to cover up the box!



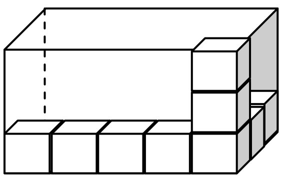
The number of blocks needed to cover up the box is ….

3. Predict how many blocks needed to cover up the box!



The number of blocks needed to cover up the box is ….

4. Predict how many blocks needed to cover up the box!



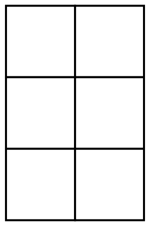
The number of blocks needed to cover up the box is ….

Building Block Activity

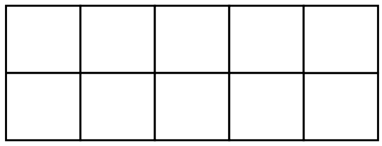
Construct an array made of cube blocks as the picture below:

Front view

Side view



Top view



Tasks:

a. Write down your steps in constructing the cube blocks building on the poster paper you have. For example: I started with building from front view and then … etc.

b. After you have finished building the blocks, please check whether your construction is matching with each picture above.

c. Please make a drawing that represent your cube block building so that its front, top and side view can be seen in a concise drawing.

Worksheet “Measuring Box capacity” Activity

NAME: ________________________________

1. On the base of a noodle box we can put 6 packages of noodles. To the top of the box, there might be 5 layers high of the packages. How do you know how many noodle packages can be put in the box?

2. Vina wants to buy a box of chocolate wafers. From the top of the box, we can see 16 bars of wafers, and from the front of the box, we can see 12 wafers. How do you know how many wafer bars can be put in the box?

3. A cake box can hold 72 pieces of cakes. If on the base of the box, we can put 18 pieces of cakes, how many layers that might be there?

4. Ms. Ani bought a box of tea. In the box, there are 4 layers high of tea arrangement in which in each layer there are 3 rows of 6 tea. How do you know how many tea are there?

END ASSESMENT

Please read carefully the following problems, then solve the problems clearly.

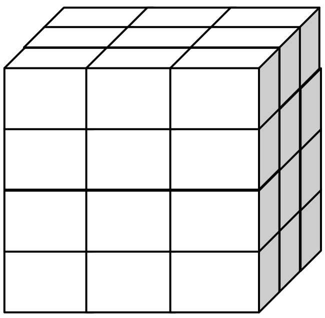
NAME: ____

1. From the top of a noodle box, we can see 14 packs of noodles. Based on the information from the seller, the packs were arranged 3 packs high stacking up. How many packs of noodles are in the box?
   My Answer:

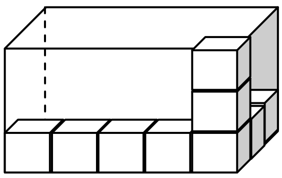
2. Vina wants to buy a box of cakes as shown in the figure. On the box, there is written "40 cakes". How many layers of cakes were arranged in that box?
   My Answer:

3. Mrs. Ani bought a box of tea. In the box, the tea are arranged as in the picture. She is wondering how many tea packs she bought. Can you help her to figure out how many tea packs are there?
   My Answer:

4. How many blocks needed to build this construction?
   My Answer:



5. Predict how many blocks needed to cover up the box!
   My Answer:



6. From the front side of a cake box, we can see 12 pieces of cakes. From the top, we can see 9 pieces of cakes.

   a. How do you draw the situation of the arrangement of the cakes in that box?
   My drawing:

   b. How do you count the number of pieces of cakes in that box?
   My answer:

7. Some pieces of dodol are arranged in a box as shown in the figure: There are two rows of 7 dodol on the base of the box, and it is possible to have 4 layers of it until the top of the box. How do you know how many pieces of dodol are needed to cover up whole space of the box?
   My Answer:

8. Mr. Bagus bought a box of chocolate wafers. In the box, the wafers are arranged in 3 rows of 5 bars in the bottom of the box and 6 bars high stacking up until the top of the box. What is the volume of the box?
   My Answer:

---Good Luck---

Analysis of Students' Answers of the End Assessment

No	Name of students	Number of Problems								Total %	Analysis
		1	2	3	4	5	6	7	8		
Answer Key		42	4	27	36	45	36	56	90		
1.	Gina and Nisa	42	4	27	36	45	36	56	90	100 %	These students have grown up significantly. In the pre-test and in some first activities, their group had difficulties in perceiving the number of dodol arrangement and also in counting the blocks. However, as they involved in the lessons, they now could perceive the structures of the three-dimensional arrangement very well.
2.	Bagus and Rafif	42	4	729	36	32	1296	56	90	62,5 %	They were in the same group with Gina and Nisa. In solving problem 3 and 6, they tended to multiply the numbers. They had difficulty in interpreting the drawings. However, we conjectured that Bagus could correctly answer question 4 because in the class discussion he had experience with that kind of drawing.
3.	Maria and Laudza	42	4	27	58	20	84	56	90	62,5 %	They seemed had difficulty with the non-contextual problem. They could correctly answer all question that in the form of story. They use layers structure to solve the questions. However, they could not perceive the structures of the blocks arrangement but they were able to perceive the structures of cakes, dodol, tea boxes, and wafer bars arrangements.
4.	Zaldi and Afif	42	4	27	27	45	108	-	-	50%	They were in the same group as Maria and Laudza. They tended to have difficulty in interpreting the cube blocks arrangement and in perceiving the different views of the objects. However, they were able to answer question about predicting the blocks. They

											used layers structure in solving problem 5.
5.	Amelia and Dinda	42	5	27	36	24	36	56	90	75%	Amel and Dinda seemed did mistakes in solving problem 5. They did not multiply the blocks on the base, that is 5 and 3 but added it become 8 and then multiplied it with the number of layers. In solving question 2, they saw the number of cakes in one layer as 5 because they were 2 rows of 5 in the arrangement. They did not see it as 10 cakes on the base. However, they also counted the objects in layers. Amel and Dinda had difficulty in reading off pictures of the arrangemenr and also in interpreting their drawing. However, they showed significant improvement in their thought about the three-dimensional objects arrangement.
6.	Daffa and Fikra	42	4	729	46	2340	108	56	90	50%	As Maria and Laudza, these students could not correctly answer the non-contextual problems. They could not perceive the more abstract arrangement as in question 4 to 6. Their difficulties also about interpreting drawing and coordinate the separate views of the arrangement as they didi in answering question 3. They tended to count the number of squares from different views in each drawing and then multiplied the numbers of object they saw from front, top and side. However, they were able to solve the story-like problem with the layers strategy.
7.	Syahrul and Farish	42	4	27	36	45	36	48	80	75%	These students could perceive the structures of the blocks and objects arrangement since first lesson.the only mistakes they did in solving last two questions were wrongly calculating the numbers, that is 15 times 6 as 80. And in solving question 7, they said that

											the drawing was not so clear for them.
8.	Fadilla and Aulia	42	4	27	36	45	36	56	90	100 %	These students could perceive the structures of the blocks and objects arrangement since the very first lesson. Even, fadilla was often giving help to other students in the discussion. From their answers, most of the time they solve the problem by find the number of objects on the base and then multiplied it with the number of layers. However, to count the number of blocks in the drawing as in question 4 and 6, they tended to count the number of blocks in front and then multiplied it with the number of layers to the back. We observed that in the second lesson when she helped Amel's group, the teacher tended to ask her about how many blocks she could see in front and how many layers stacking up were in the drawing. Fadilla seemed influenced by that discussion.
9.	Adelia and Lily	42	4	27	36	45	108	56	90	87,5 %	These students seemed had difficulty in coordinating the separate views of the arrangement. They had difficulty in making the drawing and in question 6, and it was really needed to solve that question. In general Adel and Lily solved the question with finding the number of objects on the base first and multiplied it with the number of layers.
10.	Mitha and Dwi	42	4	27	36	45	36	56	90	100 %	Dwi and Mitha at first time did not use layers strategy in predicting or counting the number of objects in an arrangement. As we knew that they iterated the dodol in the first activity one by one until the space was fully covered. However, they seemed to learn from other friends how to do effective counting. We observed also that they had difficulty in making a drawing of

											the arrangement as concise drawing. However, in solving question 6, Dwi and nisa were able to make a well-interpreted drawing so it makes them easy to interpret it.
11.	Ridho and Landok	42	5	27	3 6	4 5	3 6	5 6	9 0	87,5 %	The same as Amel and Dinda , they saw the cakes arrangement as five on the base. And the same as Daffa and Fikra, they multiplied the number of objects could be seen from front and top.
12.	Rafi and rafli	42	2	71	8 4	4 8	2 1	5 6	9 0	37,5 %	These students still could not perceive the structures of layers arrangement even after involved in the series of lesson. The mistakes that often happened was wrongly interpreted the number of objects in one layer. These students had always counted in term of layers. But, because they could not perceive the structures of the objects on the base or in column, their answers was incorrect. For example, in solving question 3 and 4, they tended to counted the number of squares they saw in the drawing of problem 4, that is 7 squares n each column, and then multiplied it with the number of layers.
13.	Tasya and Nadia	42	2	54	2 8	4 8	2 1	5 6	9 0	37,5 %	They were in the same group with rafi and Rafli. Their problems were quite the same.
14.	Juan	54	4	26	3 6	4 5	3 6	5 6	9 0	75%	The mistakes he did was about wrong calculation. He wrote 14 x 3 but he answered 54 or 9 x 3 as 26. However, overall, Juan did not have many difficulties in perceiving the number of objects in the arrangement.
15.	Anggi	42	8 0	81	3 9	4 5	1 2	5 6	8 1	37,5 %	Anggi was in the same group with Dinda and Amel. However, she seemed to try to use the layers strategy. She needed more supports to learn. From her answers, we could see that she tended to find the number of objects in one arrangement but then added it with the number of layers

16.	Yudha and Rizki	42	4	27	36	45	108	56	90	87,5 %	Our comment to these students was exactly the same as what we give to Adel and Lily.
Number of Correct Answers		**15**	**11**	**10**	**10**	**10**	**7**	**14**	**13**	**The average of the result of the end assessment was 70,31 %.**	

RENCANA PELAKSANAAN PEMBELAJARAN

Topik : Pengukuran Volume
Kelas : 5 (Lima)
Tingkat : Sekolah Dasar

Standar Kompetensi: Menghitung volume kubus dan balok dan menyelesaikan masalah sehari-hari yang berkaitan dengan konsep volume.

Kompetensi Dasar:

- Menghitung volume kubus dan balok
- Memecahkan masalah kehidupan sehari-hari yang berkaitan dengan konsep volume.

Indikator: Siswa dapat mengenal unit pengukuran volume

Tujuan:
Siswa dapat membandingkan isi/kapasitas dari dua buah benda berbentuk kubus atau balok.

Alokasi Waktu:
2 x 35 menit (Pertemuan Pertama)

Alat dan Bahan:
Dua jenis kotak/dus makanan yang berbeda ukuran, kemasan dodol/snack lainnya.

Langkah-langkah Pembelajaran:

Kegiatan Awal: (10-15 menit)

- Siswa duduk dalam kelompok 4-5 orang (kelompok ini telah dibentuk pada pertemuan sebelumnya).
- Siswa akan diberikan motivasi dimana guru akan menceritakan bahwa dalam kehidupan sehari-hari kita sering menggunakan kotak kemasan untuk mengemas makanan dalam acara tertentu. Terkadang kita tidak bisa menentukan seberapa besar kemasan yang kita butuhkan. Dalam pembelajaran kali ini, guru akan menunjukkan 2 buah kotak kemasan, dimana siswa diminta untuk coba membandingkan kedua kotak tersebut, bagaimana cara mengetahui kotak mana yang lebih besar menurut mereka. Siswa diminta untuk coba berdiskusi dengan teman sekelompoknya sekitar 5 menit.

- Siswa diminta untuk mengemukakan pendapat tentang apa yang mungkin mereka lakukan untuk membandingkan kedua kotak tersebut, kotak mana yang lebih besar menurut mereka.
- Berbagai kemungkinan jawaban siswa seperti: mungkin kita bisa mengukur panjang atau lebarnya, atau mengukur tingginya, atau mungkin juga mengisinya dengan kue atau

makanan dan menghitung banyaknya potongan kue yang dapat dimasukkan, dan sebagainya.

- Jika ide-ide tersebut tidak muncul, guru bisa menstimulasi dengan pertanyaan: bagaimana menurutmu, apakah kita bisa membandingkan kedua kotak tersebut dengan memasukkan potongan kue kemudian menghitung banyaknya potongan kue kita butuhkan? Mengapa bisa atau mengapa tidak? Biarkan siswa mengemukakan pendapatnya.

Kegiatan Inti (20-25 menit)

- Setelah siswa berdiskusi bagaimana membandingkan dua buah kotak yang kosong, kini guru bercerita bahwa kedua kotak tersebut biasa digunakan untuk mengemas dodol. Guru memberikan contoh kemasan dodol yang biasa dikemas dalam kedua kotak tersebut.

- Kemudian, guru akan memberikan pertanyaan: "Bagaimana cara kalian mengetahui kotak mana yang lebih besar jika kalian mempunyai kemasan dodol yang akan dikemas didalamnya?"
- Siswa akan mendapatkan kemasan dodol dalam jumlah terbatas. Siswa akan diminta untuk mengira-ngira berapa bungkus dodol yang dibutuhkan hingga kedua kotak tersebut terisi penuh.
- Kemungkinan jawaban siswa adalah: (1) siswa akan meminta dodol dalam jumlah yang cukup hingga kedua kotak terisi penuh. Jika hal ini terjadi, guru dapat menanyakan, berapa buah dodol lagi yang mereka butuhkan dan bagaimana cara mereka mengetahuinya, (2) siswa akan memperkirakan banyaknya dodol yang mungkin disimpan di dalam kedua kotak tersebut dengan cara mengalikan jumlah dodol di bagian dasar kotak dengan tinggi kemasan dodol yang mungkin.

Diskusi Kelas (20-25 menit)

Salah satu perwakilan dari kelompok yang ada diminta untuk mempresentasikan hasil jawaban mereka didepan kelas. Apa saja langkah-langkah mereka dalam membandingkan kotak mana yang lebih besar dan bagaimana mereka mengetahuinya. Guru dapat bertanya jika ada hal-hal yang kurang jelas disampaikan oleh siswa. Guru juga dapat meminta siswa lain untuk member tanggapan dan mempersilahkan kelompok lain yang memiliki cara berbeda dengan kelompok sebelumnya dalam menyelesaikan masalah yang diberikan.

Penutup (5-10 menit)

Di akhir diskusi pada pembelajaran ini, guru dapat memberikan kesempatan kepada siswa jika ada hal-hal yang ingin ditanyakan atau tidak jelas untuk mereka. Guru meminta dua atau tiga orang siswa membuat kesimpulan tentang apa yang mereka dapat atau apa yang mereka pelajari dalam pembelajaran kali ini.

Palembang,   Januari 2011

Guru Kelas 5E,

Rima Yunidar,A.Md

Peneliti,

Shintia Revina

Menyetujui,

Kepala SD Pusri Palembang,

Meliana, A.Md

RENCANA PELAKSANAAN PEMBELAJARAN

Topik : Pengukuran Volume
Kelas : 5 (Lima)
Tingkat : Sekolah Dasar

Standar Kompetensi: Menghitung volume kubus dan balok dan menyelesaikan masalah sehari-hari yang berkaitan dengan konsep volume.

Kompetensi Dasar:

- Menghitung volume kubus dan balok
- Memecahkan masalah kehidupan sehari-hari yang berkaitan dengan konsep volume.

Indikator: Siswa dapat menghitung volume sebuah benda berbentuk kubus atau balok yang terbuat dari kubus satuan.

Tujuan:
Siswa dapat menghubungkan bagian yang terlihat dan bagian yang tidak terlihat dari susunan benda tiga dimensi.

Alokasi Waktu:
2x35 menit (Pertemuan Kedua)

Alat dan Bahan:
Kemasan teh saring, tissue, kemasan makanan/ minuman lainnya.

Langkah-langkah Pembelajaran:

Kegiatan Awal: (5-10 menit)

- Di depan kelas, guru menyiapkan beberapa susunan kemasan makanan dan minuman (bungkus kotak the, wafer, tissue pak).

- Siswa duduk dalam kelompok 4-5 orang.
- Siswa akan diberikan motivasi bahwa di kehidupan sehari-hari kita sering melihat susunan kemasan makanan yang terkadang hanya kita bisa lihat dari samping atau dari bagian atas saja, tahukan kita bagaimana menghitung banyaknya kemasan sebenarnya. Guru memberi contoh gambar di depan kelas.

Kegiatan Inti: (20-25 menit)

- Siswa akan mendengarkan cerita guru bahwa beliau mempunyai tugas untuk melaporkan banyaknya persediaan makanan dan minuman di koperasi sekolah. Selain melaporkan jumlahnya, bu guru juga harus melampirkan gambar sehingga bapak/ibu guru lain dapat mengecek banyaknya persediaan makanan yang ada dengan hanya melihat gambar tersebut.
- Guru meminta siswa untuk membantunya membuat gambar tersebut.

- Setiap kelompok siswa akan ditugaskan untuk menggambar satu susunan kemasan makanan sehingga seseorang yang melihat gambar tersebut akan dapat mengetahui banyaknya jumlah kemasan makanan atau minuman tersebut.
- Siswa juga diminta untuk menghitung banyaknya kemasan yang mereka punya namun tidak menuliskan jumlahnya di kertas poster mereka. Siswa akan berdiskusi selama 10-15 menit untuk menggambar dan kemudian menempel hasil kerja mereka di depan kelas.

Diskusi Kelas: (20-25 menit)

- Salah satu kelompok dapat mengemukakan hasil kerja mereka di depan kelas. Guru dapat bertanya: Dapatkah kalian menjelaskan kepada ibu dan teman-teman yang lain, apa yang kalian gambar? Bagaimana kalian bisa mengetahui banyaknya kemasan makanan/minuman dalam gambar kalian? Bagaimana dengan kelompok lain? Apakah kalian setuju dengan yang dikatakan teman kalian?
- Salah satu siswa dapat maju ke depan dan menghitung banyaknya kemasan dalam gambar tersebut. Kemudian guru dapat meminta satu kelompok lain yang memiliki cara yang berbeda dengan kelompok sebelumnya untuk presentasi dan meminta siswa lain untuk menghitung banyaknya susunan kemasan dalam gambar tersebut.
- Jika siswa kesulitan untuk membuat gambar yang dapat mewakili situasi yang mereka hadapi, guru dan siswa dapat berdiskusi bagaimana cara membuat sebuah gambar dari susunan makanan/minuman sehingga kita bisa melihat banyaknya kemasan baik yang terlihat maupun yang tidak terlihat.

Penutup:

Di akhir diskusi pada pembelajaran ini, guru dapat memberikan kesempatan kepada siswa jika ada hal-hal yang ingin ditanyakan atau tidak jelas untuk mereka. Guru meminta dua atau tiga orang siswa membuat kesimpulan tentang apa yang mereka dapat atau apa yang mereka pelajari dalam pembelajaran kali ini.

Palembang, Januari 2011

Guru Kelas 5E, Peneliti,

Rima Yunidar, A.Md Shintia Revina

Menyetujui,
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Meliana, A.Md

RENCANA PELAKSANAAN PEMBELAJARAN

Topik **: Pengukuran Volume**
Kelas **: 5 (Lima)**
Tingkat **: Sekolah Dasar**

Standar Kompetensi: Menghitung volume kubus dan balok dan menyelesaikan masalah sehari-hari yang berkaitan dengan konsep volume.

Kompetensi Dasar:
- Menghitung volume kubus dan balok
- Memecahkan masalah kehidupan sehari-hari yang berkaitan dengan konsep volume.

Indikator: Siswa dapat menghitung volume sebuah benda berbentuk kubus atau balok yang terbuat dari kubus satuan.

Tujuan:
- Siswa dapat membangun sebuah bangunan yang terdiri dari kubus satuan berdasarkan gambar dari berbagai sisi.
- Siswa dapat menghitung jumlah kubus satuan pada bangunan kubus atau balok dan pada representasinya dalam bentuk gambar.

Alokasi Waktu:
4x35 menit (Pertemuan Ketiga dan Keempat)

Alat dan Bahan:
Kubus satuan dan gambar bangunan kubus dari berbagai sisi (tampak samping, atas dan depan).
Langkah-langkah Pembelajaran:

Kegiatan Awal:
- Guru menyiapkan kubus-kubus satuan dan petunjuk kerja untuk siswa (terlampir).


- Siswa duduk dalam kelompok 4-5 orang.
- Siswa diingatkan bahwa di pertemuan sebelumnya mereka telah membuat gambar dari susunan kemasan makanan dan minuman. Guru dapat memotivasi siswa bagaimana jika siswa ditugaskan sebaliknya, memiliki gambar dan harus menyusun sesuai gambar yang diberikan. Bagaimana pendapat siswa apakah mereka bisa melakukannya?

Kegiatan Inti:
- Masing-masing kelompok akan mendapatkan kubus-kubus satuan (secukupnya) beserta petunjuk kerja yang berisi gambar (terlampir).

- Siswa diminta untuk membuat bangunan sesuai gambar yang diberikan (gambar tampak depan, tampak samping dan tampak atas) dan kemudian membuat sebuah gambar dari bangunan mereka tersebut.
- Siswa akan bekerja selama 10-15 menit untuk menyusun kubus-kubus satuan tersebut dan mencocokannya dengan gambar yang mereka punya. Siswa diminta untuk menuliskan langkah-langkah mereka bekerja di kertas poster.

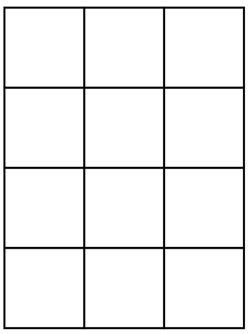
Diskusi Kelas:

- Setelah siswa selesai menyusun kubus-kubus satuan dan mencocokannya dengan gambar yang diberikan serta menuliskan langkah-langkah mereka bekerja pada kertas poster, siswa akan berdiskusi mengenai hasil kerja mereka.
- Salah satu kelompok dipersilahkan mengemukakan hasil kerja mereka dan menjelaskan langkah-langkah mereka membuat bangunan tersebut dan mencocokannya dengan gambar yang mereka punyai.
- Satu atau dua kelompok lain yang memiliki cara berbeda dapat mempresentasikan hasil kerja mereka.
- Guru dapat bertanya kepada kelompok yang presentasi, bagaimana cara mereka menghubungkan gambar tampak depan, atas dan samping yang ada dengan bangunan yang mereka buat?
- Setelah itu, siswa diminta untuk menghitung (1) banyaknya kubus-kubus satuan yang digunakan oleh teman kelompok lain dalam menyusun bangunan kubus milik mereka dan (2) banyaknya kubus satuan pada gambar yang mereka buat.
- Siswa diminta berdiskusi dalam kelompok selama 10 menit, bagaimana cara mereka menghitung banyaknya kubus-kubus satuan tersebut. Apakah mereka punya satu cara dalam menghitungnya atau mereka punya lebih dari satu cara untuk menghitungnya.
- Siswa juga diminta untuk mencocokkan banyaknya kubus satuan yang disusun dengan banyaknya kubus satuan pada gambar. Siswa juga diminta menjelaskan bagimana cara mereka menghitung banyaknya kubus satuan dalam gambar.

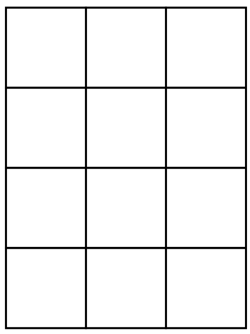
Petunjuk untuk SiswaMenyusun Kubus Satuan

Susunlah kubus-kubus satuan yang tersedia sehingga tampak seperti gambar berikut:

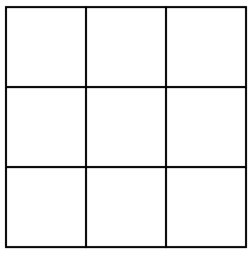
Tampak samping



Tampak Depan



Tampak atas



d. Tuliskan langkah-langkahmu membuat bangunan kubus satuan pada kertas poster yang tersedia! Contoh: saya memulai dari gambar tampak atas kemudian,,, dan seterusnya.
e. Setelah kamu selesai menyusun kubus-kubus satuan tersebut, cocokanlah pekerjaanmu dengan setiap gambar di atas!
f. Buatlah sebuah gambar yang mewakili bangunan kubus yang kamu buat agar bagian depan, atas dan sampingnya bisa dilihat dalam sebuah gambar!

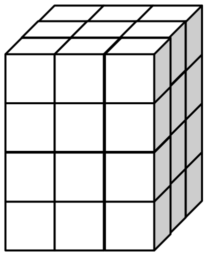
Penutup:

Pada akhir pelajaran, siswa diminta untuk mengerjakan lembar kerja (terlampir).

L*embar kerja siswa*

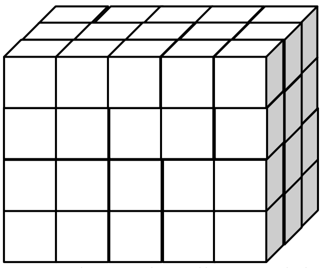
NAMA: ____

1. Hitunglah banyaknya kubus satuan pada gambar berikut:



Banyaknya kubus satuan pada gambar di atas adalah ….

2. Hitunglah banyaknya kubus satuan pada gambar berikut:



Banyaknya kubus satuan pada gambar di atas adalah ….

Palembang, Januari 2011

Guru Kelas 5E, Peneliti,

Rima Yunidar, A.Md Shintia Revina

Menyetujui,

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RENCANA PELAKSANAAN PEMBELAJARAN

Topik : Pengukuran Volume
Kelas : 5 (Lima)
Tingkat : Sekolah Dasar

Standar Kompetensi: Menghitung volume kubus dan balok dan menyelesaikan masalah sehari-hari yang berkaitan dengan konsep volume.

Kompetensi Dasar:

- Menghitung volume kubus dan balok
- Memecahkan masalah kehidupan sehari-hari yang berkaitan dengan konsep volume.

Indikator: Siswa dapat menghitung volume sebuah benda berbentuk kubus atau balok.

Tujuan:

- Siswa dapat memprediksi atau mengestimasi atau memperkirakan jumlah kubus satuan yang dapat disimpan dalam sebuah kotak berbentuk kubus atau balok.
- Siswa dapat menentukan berbagai kemungkinan ukuran kotak berbentuk kubus atau balok dengan volume tertentu.

Alokasi Waktu:
2x35 menit (Pertemuan Kelima)

Alat dan Bahan:
Kotak berbentuk kubus atau balok dan beberapa buah kubus satuan.

Langkah-langkah Pembelajaran:

Kegiatan awal:

- Siswa duduk berkelompok 4-5 orang.
- Guru menyiapkan beberapa kotak kosong dan kubus-kubus satuan secukupnya.

- Siswa akan diingatkan mengenai pembelajaran sebelumnya mengenai cara menghitung kubus-kubus satuan dalam gambar maupun dalam bangunan sebenarnya.
- Guru bertanya kepada siswa bagaimana jika mereka punya sebuah kotak kosong (guru menunjukkannya di depan kelas) dan ingin mengisinya dengan kubus-kubus satuan namun tidak memiliki jumlah kubus-kubus satuan yang cukup. Bagaimana cara mereka mengetahui berapa banyak kubus satuan yang mereka butuhkan?

Kegiatan Inti: *(10-15 menit)*

- Siswa diminta untuk menghitung banyaknya kubus satuan yang mungkin dimasukkan ke dalam sebuah kotak kosong. Guru akan membatasi jumlah kubus-kubus satuan yang dapat digunakan oleh siswa sehingga siswa tidak dapat menutupi seluruh bagian dalam kotak dengan kubus satuan melainkan hanya dengan memprediksi.
- Siswa akan berdiskusi selama 10 menit bagaimana cara mereka memprediksi banyaknya kubus satuan yang bisa disimpan dalam kotak tersebut.

Diskusi Kelas: *(10-15 menit)*

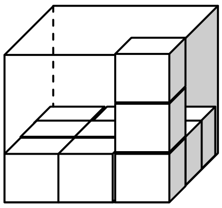
- Dalam diskusi kelas, siswa dapat mendiskusikan mengenai berbagai cara berbeda dalam memprediksi banyaknya kubus-kubus satuan yang harus digunakan agar seluruh bagian dalam kotak terpenuhi oleh kubus-kubus satuan tersebut.
- Berbagai cara yang berbeda memungkinkan siswa untuk menemukan cara yang paling efektif untuk mereka. Guru dapat mengarahkan diskusi sehingga siswa dapat melihat struktur dari kubus-kubus satuan tersebut secara, baris, kolom atau lapisan dari susunan kubus-kubus satuan tersebut dalam memprediksi banyaknya kubus-kubus satuan yang harus digunakan agar seluruh bagian dalam kotak terpenuhi oleh kubus-kubus satuan tersebut.
- Dalam akhir diskusi, guru dapat memperkenalkan istilah volume suatu benda sebagai banyaknya kubus satuan yang menyusunnya. Guru dapat memberikan contoh volume kotak yang baru saja diprediksi oleh siswa, banyaknya kubus satuan yang dapat disimpan didalamnya hingga kotak tersebut terisi penuh merupakan volume kotak tersebut.
- Guru dapat mengarahkan siswa untuk memahami pengukuran volume suatu benda atau suatu kotak sebagai menutup seluruh bagian dalam benda atau kotak tersebut dengan kubus-kubus satuan yang identik dan tanpa celah.
- Kemudian guru menyiapkan tiga buah kotak yang berbeda ukuran dan bentuknya namun memiliki volume yang sama dan kubus-kubus satuan secukupnya.
- Guru mengingatkan siswa mengenai pembelajaran sebelumnya mengenai cara memprediksi banyaknya kubus-kubus satuan yang harus digunakan agar seluruh bagian dalam kotak terpenuhi oleh kubus-kubus satuan tersebut.
- Siswa diminta untuk berdiskusi secara kelompok untuk menentukan kotak mana saja dari ketiga kotak tersebut yang memiliki volume yang sama dan kemudian menjelaskan bagaimana cara mereka mengethauinya.
- Siswa diminta untuk menuliskan hasil diskusi mereka pada kertas poster yang tersedia.
- Siswa diharapkan menemukani bahwa volume suatu benda tidak hanya ditentukan oleh ukuran sisi-sisinya namun oleh banyaknya unit pengukuran yang dapat disusun di dalam benda tersebut.
- Siswa dapat menyadari berbagai benda berbentuk balok dengan ukuran berbeda-beda mungkin akan memiliki volume yang sama.

Penutup:

Siswa diminta menyimpulkan apa yang mereka pahami tentang pengukuran volume suatu benda dan mempersilahkan siswa yang belum memahaminya untuk bertanya dan sebagainya. Pada akhir pelajaran, siswa diminta untuk mengerjakan lembar kerja (terlampir) selama 10-15 menit.

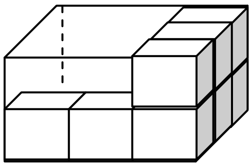
NAMA: ____

1. Berapakah banyaknya kubus satuan yang bisa dimuat dalam kotak berikut hingga kotak tersebut dapat terisi penuh?



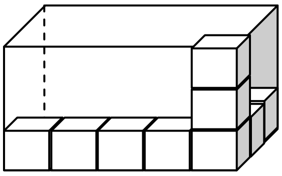
Banyaknya kubus satuan yang bisa dimuat dalam kotak tersebut hingga kotak terisi penuh adalah ….

2. Berapakah banyaknya kubus satuan yang bisa dimuat dalam kotak tersebut hingga kotak dapat terisi penuh?



Banyaknya kubus satuan yang bisa dimuat dalam kotak tersebut hingga kotak terisi penuh adalah ….

3. Berapakah banyaknya kubus satuan yang bisa dimuat dalam kotak tersebut hingga kotak dapat terisi penuh?



Banyaknya kubus satuan yang bisa dimuat dalam kotak tersebut hingga kotak terisi penuh adalah ….

Palembang, Januari 2011

Guru Kelas 5E,

Rima Yunidar, A.Md

Peneliti,

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Menyetujui,
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RENCANA PELAKSANAAN PEMBELAJARAN

Topik : Pengukuran Volume
Kelas : 5 (Lima)
Tingkat : Sekolah Dasar

Standar Kompetensi: Menghitung volume kubus dan balok dan menyelesaikan masalah sehari-hari yang berkaitan dengan konsep volume.

Kompetensi Dasar:

- Menghitung volume kubus dan balok
- Memecahkan masalah kehidupan sehari-hari yang berkaitan dengan konsep volume.

Indikator: Siswa dapat menghitung volume suatu benda dengan berbagai unit pengukuran.

Tujuan:
Siswa dapat mengukur volume suatu benda dengan unit pengukuran yang berbeda.

Alokasi Waktu:
2x35 menit (Pertemuan Keenam)

Alat dan Bahan: unit pengukuran berbeda, kotak dari kertas karton

Langkah-langkah Pembelajaran:

Dalam aktivitas ini, setiap kelompok siswa akan diberikan dua buah kotak, kotak A dan kotak B" yang memiliki ukuran dan bentuk yang berbeda. Guru akan menyediakan dua buah unit pengukuran yang berbeda pula. Kita dapat menyebutnya masing-masing unit kuning dan unit biru. Unit satuan (unit kuning) yang satu lebih besar daripada yang lainnya (unit biru). Tugas untuk siswa adalah:

a. Memperkirakan banyaknya unit kuning yang bisa dimuat dalam kotak A .
b. Memperkirakan banyaknya unit biru yang bisa disimpan dalam kotak B.
c. Membandingkan kotak A dan B. Kotak mana yang lebih besar? Dan Bagaimana cara mereka mengetahuinya.

Berikut jawaban yang mungkin muncul:

- Siswa akan menemukan bahwa jumlah unit kuning yang dibutuhkan untuk mengisi kotak A adalah 12 buah dan jumlah unit biru adalah 32 buah.
- Beberapa siswa akan menyimpulkan bahwa kotak B lebih besar karena dapat menyimpan lebih banyak unit, yaitu unit biru.
- Kemudian mereka juga menemukan bahwa besarnya unit yang digunakan adalah berbeda.
- Guru dapat mengajukan pertanyaan: "Ukurlah volume kedua kotak tersebut hanya dengan unit kuning, kemudian lakukan hal yang sama dengan hanya menggunakanunit biru. Bagaimana hasilnya? Apa pendapatmu, mengapa untuk ukuran kotak yang sama kita mendapatkan hasil pengukuran yang berbeda jika kita menggunakan unit dengan ukuran yang berbeda?

Diskusi:

Diharapkan siswa dapat menemukan hubungan antara besarnya unit dan jumlah unit yang dibutuhkan. Semakin besar unitnya, semkain sedikit jumlah yang dibutuhkan. Kita juga dapat mendiskusikan tentang bagaimana kita mengukur volume suatu benda. Bahwa volume suatu benda dapat diukur dengan menggunakan berbagai macam unit pengukuran. Hal ini akan menghasilkan hasil pengukuran yang berbeda namun mengarah pada volume yang tetap sama.

Palembang,   Januari 2011

Guru Kelas 5E,

Peneliti,

Rima Yunidar, A.Md

Shintia Revina

Menyetujui,

Kepala SD Pusri Palembang,

Meliana, A.Md

RENCANA PELAKSANAAN PEMBELAJARAN

Topik : Pengukuran Volume
Kelas : 5 (Lima)
Tingkat : Sekolah Dasar

Standar Kompetensi: Menghitung volume kubus dan balok dan menyelesaikan masalah sehari-hari yang berkaitan dengan konsep volume.

Kompetensi Dasar:

- Menghitung volume kubus dan balok
- Memecahkan masalah kehidupan sehari-hari yang berkaitan dengan konsep volume.

Alokasi Waktu:
2x35 menit (Pertemuan Ketujuh)

Alat dan Bahan:
Lembar kerja siswa.

Langkah-langkah Pembelajaran:
Siswa diminta mengerjakan lembar kerja berikut secara berpasangan (selama 20-25 menit):
(lembar kerja siswa di pertemuan ketujuh)

NAMA: ____

5. Sebuah kardus mie dapat memuat 6 bungkus mie di bagian dasarnya dan bungkus-bungkus mie tersebut dapat disusun hingga 5 susun ke atas hingga kardus mie terisi penuh. Menurutmu, berapakah banyaknya bungkus mie yang dapat disimpan ke dalam kardus tersebut?
6. Vina ingin membeli satu kotak wafer cokelat. Dari bagian atas kemasan, terlihat 8 bungkus wafer cokelat, dan dari bagian depan kotak terlihat 6 bungkus wafer cokelat. Menurutmu, berapakah banyak bungkus wafer cokelat dalam kotak tersebut?
7. Sebuah kotak kue mampu menampung 72 potong kue. Jika banyaknya kue yang dapat disusun pada bagian permukaan kotak adalah 18 potong kue, berapakah tinggi susunan kue sehingga kotak tersebut terisi penuh?
8. Ibu membeli satu kotak teh. Di bagian permukaan kotak, kemasan teh disusun dalam 3 baris dimana setiap barisnya terdiri dari 6 bungkus teh, dan tinggi susunan teh sebanyak 4 susunan ke atas. Berapa banyak bungkus teh dalam kotak tersebut?

Diskusi

- Guru meminta beberapa perwakilan siswa untuk masing-masing mengemukakan jawaban mereka secara bergantian. Akan ada lima kelompok yang mempresentasikan hasil pekerjaan mereka, namun jika ada kelompok lain yang memang memiliki jawaban yang sangat berbeda dan ingin mendiskusikannya tidak menutup kemungkinan untuk mempersilahkan lebih dari satu kelompok dalam menjawab sebuah soal.

- Siswa dapat mendiskusikan hasil jawaban mereka. Bagaimana mereka menyelesaikan masalah yang ada dalam soal-soal tersebut.
- Siswa dapat membuat gambar atas situasi yang dimaksud kemudian menyelesaikannya berdasarkan pengetahuan dan pengalaman yang mereka lalui pada kegiatan-kegiatan sebelumnya.

	Palembang, Januari 2011
Guru Kelas 5E,	Peneliti,
Rima Yunidar, A.Md	Shintia Revina

Menyetujui,
Kepala SD Pusri Palembang,

Meliana, A.Md