STUDENTS' LEARNING OF COMPARING THE MAGNITUDE OF ONE-DIGIT AND TWO-DIGIT DECIMALS USING NUMBER LINE

A Design Research on Decimals at Grade 5 in Indonesian Primary School

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of
Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:
Puri Pramudiani
NIM 20092812006

GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

APPROVAL PAGE

Research Title	: Students' Learning of Comparing the Magnitude of One-digit and Two-digit Decimals Using Number Line (A Design Research on Decimals at Grade 5 in Indonesian Primary School)
Student Name	: Puri Pramudiani
Student Number	: 20092812006
Study Program	: Mathematics Education

Approved by,

Prof. Dr. Zulkardi, M.I.Komp., M.Sc. Supervisor I	Dr. Yusuf Hartono Supervisor II

Dr. Barbara Ann van Amerom
Supervisor III

Head of Mathematics Education Department,	Director of Graduate School Sriwijaya University,
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. NIP. 19610420 198603 1 002	Prof. Dr. dr. H.M.T. Kamaluddin, M.Sc., SpFK. NIP. 19520930 198201 1 001

Date of Approval: May 2011

STUDENTS' LEARNING OF COMPARING THE MAGNITUDE OF ONE-DIGIT AND TWO-DIGIT DECIMALS USING NUMBER LINE

A Design Research on Decimals at Grade 5 in Indonesian Primary School

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of
Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:

Puri Pramudiani
NIM 20092812006

Approved by Examination Committee	Signature
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. **Sriwijaya University**	----------
Dr. Yusuf Hartono **Sriwijaya University**	----------
Dr. Barbara Ann van Amerom **Freudenthal Institute, Utrecht University**	----------
Prof. Dr. R. K. Sembiring **IP-PMRI**	----------
Dr. Frans van Galen **Freudenthal Institute, Utrecht University**	----------
Dr. Dolly van Eerde **Freudenthal Institute, Utrecht University**	----------
Dr. Darmawijoyo **Sriwijaya University**	----------
Dr. Ratu Ilma Indra Putri, M.Si **Sriwijaya University**	----------

GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

STATEMENT PAGE

I hereby:

Name : Puri Pramudiani

Place of birth : Bandung

Date of birth : October 3, 1985

Academic Major : Mathematics Education

State that:

1. All the data, information, analyses, and the statements in analyses and conclusions that presented in this thesis, except from reference sources are the results of my observations, researches, analyses, and views with the guidance of my supervisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

Palembang, May , 2011

The one with the statement

Puri Pramudiani
NIM. 20092812006

ABSTRACT

This research reports on a new approach of students' learning of comparing the magnitude of decimals. This approach enables students to work with contextual situation within measurement activities (weight and volume measurement) in which number line is used as a model for placing and comparing the magnitude of decimals. Realistic Mathematics Education (RME) underlies this research in the part of designed context and activities. This research has aim to contribute to the classroom activities in learning decimals. Design research was chosen to reach the research goal, and it was conducted in two cycles, namely pilot experiment and teaching experiment. This research involves 7 students in the pilot experiment, 26 students in the teaching experiment, and one teacher of grade 5 in SDN 21 Palembang, Indonesia. The results of this research show that the series of activities could bring the students' learning developed from informal level to pre-formal level in comparing the magnitude of one-digit and two-digit decimals using number line. Based on observation, measurement activities (weight and volume measurement) can promote the students' notion of decimals which, then, provoke the students' thinking into the idea of using number line as a model for placing the magnitude of decimals. It also shows that the number line plays an important role in bridging the experience-based activities into more formal level of mathematics (comparing the magnitude of one-digit and two-digit decimals). In addition, they could realize that the longer digit the decimals, the more precise the measurement. Based on these findings, it is recommended that RME be implemented as an approach of teaching and learning decimals.

Keywords: *decimals, Realistic Mathematics Education (RME), number line, design research.*

ABSTRAK

Penelitian ini melaporkan suatu pendekatan baru pada pembelajaran siswa dalam materi perbandingan besaran bilangan desimal. Pendekatan ini memungkinkan siswa untuk melakukan kegiatan dengan menggunakan situasi yang kontekstual pada kegiatan pengukuran (yaitu pengukuran berat dan pengukuran volume), di mana garis bilangan digunakan sebagai model untuk menempatkan dan membandingkan besaran bilangan desimal. Pendidikan Matematika Realistik Indonesia (PMRI) mendasari penelitian ini pada bagian desain konteks dan aktivitas. Penelitian ini bertujuan untuk memberikan kontribusi terhadap kegiatan pembelajaran di kelas pada materi bilangan desimal. Desain riset dipilih sebagai suatu metode dalam penelitian ini untuk mencapai tujuan penelitian tersebut, dan dilaksanakan dalam 2 siklus yaitu *pilot experiment* dan *teaching experiment*. Yang menjadi target penelitian ini adalah 7 siswa pada *pilot experiment* (siklus pertama), 26 siswa pada *teaching experiment* (siklus kedua) beserta seorang guru kelas 5 di SDN 21 Palembang, Indonesia. Hasil dari penelitian ini menunjukkan bahwa rangkaian kegiatan yang dilakukan dapat membantu perkembangan belajar siswa dari tahap informal ke tahap pra-formal dalam membandingkan besaran bilangan desimal satu angka dan dua angka di belakang koma dengan menggunakan garis bilangan. Hasil observasi menunjukkan bahwa kegiatan pengukuran (berat dan volume) dapat menstimulasi gagasan siswa terhadap bilangan desimal yang kemudian dapat memancing pemikiran siswa terhadap penggunaan garis bilangan sebagai model untuk menempatkan posisi bilangan desimal tersebut. Hasil penelitian ini juga menunjukkan bahwa garis bilangan mempunyai peranan penting dalam menjembatani kegiatan pengalaman belajar siswa ke arah yang lebih formal (membandingkan besaran bilangan desimal satu angka dan dua angka di belakang koma). Selain itu, mereka dapat menyadari bahwa semakin banyak angka di belakang koma, maka semakin teliti hasil dari suatu pengukuran. Berdasarkan hasil penelitian ini, kami merekomendasikan PMRI untuk digunakan sebagai suatu pendekatan dalam pembelajaran desimal.

Kata Kunci: bilangan desimal, Pendidikan Matematika Realistik Indonesia (PMRI), garis bilangan, desain riset.

SUMMARY

There are many researches which have documented students' difficulties in comparing the magnitude of decimals. There are a lot of students' misconceptions that tends to judge the longer the digit decimals, the larger their magnitude. In fact, when students are asked to compare which is the larger number between 12,17 and 12,4, most of them choose 12,17 is larger than 12,4 because 17 is larger than 4.

In the present research, we designed the context and activities which enable students to do the learning process from informal level to pre-formal level in comparing the magnitude of one-digit and two-digit decimals using number line. Realistic Mathematics Education (RME) underlies this research in the part of designed context and activities. In Indonesia, RME has been implemented for over last ten years, namely *Pendidikan Matematika Realistik Indonesia* (PMRI). The context designed in this research is about the precise measurement on the activities (weight and volume measurement). Our hypothesis was that through the series of activities the students can find and explore the notation of decimals (one-digit and two-digit decimals). Using the idea of density of numbers on the number line, we expect that the students would use the idea of partitioning base ten on the number line which enable them to perceive the idea that between two consecutive whole numbers, there are decimals, and between two consecutive one-digit decimals, there are other decimals i.e. two-digit decimals, etc. Finally we expect that the students are able to determine the magnitude of one-digit and two-digit decimals by looking their position on the number line. In addition, we expect that they would realize that the longer digit the decimals, the more precise the measurement.

This research has aim to contribute to the classroom activities in learning decimals. Design research was chosen to reach the research goal, and it was conducted in two cycles, namely pilot experiment and teaching experiment. This research involves 7 students in the pilot experiment, 26 students in the teaching experiment, and one teacher of grade 5 in SDN 21 Palembang, Indonesia. After the first cycle of 6 lessons, activities and some materials were refined in order to improve the designed Hypothetical Learning Trajectory (HLT). The data were collected using video recording, students' work, field notes of classroom observation, and evaluations with the teachers.

The findings in this research are: 1) measurement activities (weight and volume measurement) can promote the students' notion of decimals in which the students could discover decimals and determine their position in between two consecutive whole numbers (on the scale); 2) based on observation, the visualization of the scales containing the sequence of the numbers could provoke the students' thinking into the idea of using number line as a model for placing the magnitude of decimals; 3) Number line plays an important role in bridging the experience-based activities into more formal level of mathematics (comparing the magnitude of one-digit and two-digit decimals). Through observing the position of one-digit and two-digit decimals on the number line, the students used it as their reasoning for showing their magnitude.

Through the contextual situation (weight and volume measurement), the students' learning toward decimals can develop from informal level to pre-formal level. Starting from making representation of the weight scale and measuring cup as *a model of* situation, the students' thinking shifted into the idea of using number line as *a model for* placing the

magnitude of one-digit and two-digit decimals, which finally bring their idea to compare their magnitude by determining the position from left to right comparison on the number line.

The phases of students' learning are 1) Knowing the existing of decimal form through contextual situation (weighing duku and body); 2) Exploring the meaning of one-digit decimals through weighing rice; 3) Using number line as a model for placing the position of one-digit decimals (partition based ten); 4) Exploring the meaning of two-digit decimals through measuring the volume of beverages; 5) Using number line as a model for placing the position of two-digit decimals (partition based tenth of tenth); 6) Getting insight about density of numbers on the number line; 7) Comparing one-digit and two-digit decimals using number line.

Based on the result of post assessment, 69% students shows a good ability in reading the scale, 85% students have a good knowledge of decimals, 77% students could master the idea of density of decimals on the number line, and 77% students could compare and determine the magnitude of one-digit and two-digit decimals. Based on the result of interview, 92% students chose number line as the tool to compare and determine one-digit and two-digit decimals.

Concerning the findings of this research, we give recommendation for further studies about decimals to apply Realistic Mathematics Education as the basic approach for teaching and learning decimals, especially about comparing their magnitude. Even though decimals were known as the abstract number for the students, but in this designed activities we can show that decimals can be taught in a meaningful way within measurement activities (weight and volume measurement). The students could discover decimals by themselves and could develop their ideas to come to the number line as a model for placing their magnitude.

RINGKASAN

Beberapa hasil penelitian telah melaporkan kesulitan-kesulitan siswa dalam membandingkan besaran bilangan desimal. Banyaknya kesalahpahaman siswa yang berpikir bahwa semakin banyak angka di belakang koma, maka semakin besar bilangan desimal tersebut. Oleh karena itu, ketika siswa diminta untuk membandingkan bilangan mana yang lebih besar antara 12,17 dan 12,4, sebagian besar siswa memilih 12,17 yang lebih besar dari pada 12,4 dikarenakan 17 lebih besar daripada 4.

Dalam penelitian ini, kami mendesain konteks dan aktivitas yang memungkinkan siswa untuk melalui pembelajaran dari mulai tahap informal ke tahap yang lebih formal dalam membandingkan besaran bilangan desimal satu angka dan dua angka di belakang koma dengan menggunakan garis bilangan. *Realistic Mathematics Education* (*RME*) mendasari penelitian ini pada bagian desain konteks dan aktivitas. Di Indonesia, *RME* telah diimplementasikan selama kurun waktu 10 tahun terakhir ini, dan dinamakan Pendekatan Matematika Realistik Indonesia (PMRI). Konteks yang didesain dalam penelitian ini adalah tentang ketelitian dalam pengukuran (berat dan volume). Hipotesis kita adalah melalui rangkaian kegiatan yang telah didesain, maka diharapkan siswa akan menggunakan gagasan pembagian garis bilangan per sepuluh yang memungkinkan mereka untuk mendapatkan gagasan tentang kepadatan suatu bilangan dalam garis bilangan (di antara dua bilangan bulat yang berurutan terdapat bilangan desimal satu angka di belakang koma, dan di antara dua bilangan desimal satu angka di belakang koma yang berurutan, terdapat bilangan desimal lainnya, yaitu bilangan desimal dua angka di belakang koma, dan sebagainya). Pada akhirnya kami harapkan siswa dapat menentukan besaran bilangan desimal satu angka dan dua angka di belakang koma. Selain itu, kami harapkan siswa dapat menyadari bahwa semakin banyak angka desimal di belakang koma, maka semakin teliti suatu pengukuran.

Penelitian ini bertujuan untuk memberikan kontribusi terhadap kegiatan pembelajaran di kelas pada materi bilangan desimal. Desain riset dipilih sebagai suatu metode dalam penelitian ini untuk mencapai tujuan penelitian tersebut, dan dilaksanakan dalam 2 siklus yaitu *pilot experiment* dan *teaching experiment*. Yang menjadi target penelitian ini adalah 7 siswa pada *pilot experiment* (siklus pertama), 26 siswa pada *teaching experiment* (siklus kedua) beserta seorang guru kelas 5 di SDN 21 Palembang, Indonesia. Setelah siklus pertama dilaksanakan yang terdiri dari 6 pertemuan, aktivitas dan beberapa bahan ajar diperbaiki salam rangka meningkatkan desain lintasan belajar. Data yang terdiri sari rekaman video, hasil lembar kerja siswa, catatan lapangan observasi kelas, dan evaluasi dengan guru dikumpulkan dan kemudian dianalisis.

Hasil dari penelitian ini menunjukkan bahwa 1) rangkaian kegiatan pengukuran (menimbang berat benda dan mengukur volume minuman ringan) dapat menstimulasi gagasan siswa terhadap bilangan desimal di mana para siswa dapat menemukan bahwa keberadaan bilangan desimal itu berada di antara dua bilangan bulat yang berurutan; 2) berdasarkan hasil observasi, bentuk skala dalam timbangan maupun gelas ukur yang memuat urutan bilangan dapat menstimulasi gagasan siswa terhadap penggunaan garis bilangan sebagai model untuk menempatkan posisi bilangan desimal tersebut; 3) hasil penelitian ini juga menunjukkan bahwa garis bilangan mempunyai peranan penting dalam menjembatani kegiatan pengalaman belajar siswa ke arah yang lebih formal (membandingkan besaran bilangan desimal satu angka dan dua angka di belakang koma). Melalui pengamatan letak (posisi) bilangan desimal satu

angka dan dua angka di belakang koma pada garis bilangan (dari kiri ke kanan), siswa menggunakan hal tersebut sebagai argumen mereka untuk menunjukkan besaran bilangan desimal tersebut.

Melalui situasi yang kontekstual (menimbang berat dan mengukur volume minuman ringan), pembelajaran siswa dalam materi bilangan desimal dapat berkembang dari tahap informal ke tahap pre-formal. Diawali dengan membuat representasi dari timbangan skala dan gelas ukur sebagai model dari situasi, pemikiran siswa berkembang kepada gagasan penggunaan garis bilangan sebagai model untuk menempatkan besaran bilangan desimal satu angka dan dua angka di belakang koma, yang kemudian mampu menggiring pemikiran mereka untuk membandingkan besaran bilangan desimal dengan menentukan posisi bilangan tersebut dari kiri ke kanan pada garis bilangan.

Perkembangan belajar siswa dimulai dari 1) mengenal bilangan desimal (satu angka di belakang koma) melalui situasi yang kontekstual (menimbang duku dan menimbang berat badan); 2) mengeksplorasi bilangan desimal (satu angka di belakang koma) melalui kegiatan menimbang beras; 3) menggunakan garis bilangan sebagai model untuk menentukan posisi besaran bilangan desimal satu angka di belakang koma (partisi berbasis sepuluh); 4) mengeksplorasi bilangan desimal (dua angka di belakang koma) melalui kegiatan pengukuran volume minuman ringan; 5) menggunakan garis bilangan sebagai model untuk menentukan posisi besaran bilangan desimal dua angka di belakang koma (partisi berbasis persepuluh dari persepuluh); 6) mengetahui gagasan tentang kepadatan bilangan pada garis bilangan; dan 7) membandingkan bilangan desimal satu angka dan dua angka di belakang koma dengan menggunakan garis bilangan.

Berdasarkan hasil *post*-penilaian, 69% siswa memiliki kemampuan yang baik dalam membaca skala secara teliti, 85% siswa mempunyai pengetahuan yang baik tentang gagasan bilangan desimal, 77% siswa menguasai gagasan kepadatan bilangan pada garis bilangan, dan 77% siswa dapat membandingkan dan menentukan besaran bilangan desimal satu angka dan dua angka di belakang koma. Berdasarkan hasil wawancara, 92% siswa memilih penggunaan garis bilangan sebagai model atau alat untuk membandingkan bilangan desimal satu angka dan dua angka di belakang koma.

Berdasarkan hasil penelitian ini, kami merekomendasikan PMRI untuk digunakan sebagai suatu pendekatan dalam pembelajaran desimal, khususnya pada materi perbandingan bilangan desimal. Walaupun bilangan desimal dikenal sebagai bilangan yang abstrak bagi siswa, tetapi pada aktivitas yang didesain pada penelitian ini, kita dapat menunjukkan bahwa bilangan desimal bisa diajarkan dengan cara yang bermakna yaitu melalui kegiatan pengukuran (menimbang berat benda dan mengukur volume minuman ringan). Para siswa dapat menemukan sendiri bilangan desimal dan dapat mengembangkan gagasan mereka terhadap garis bilangan sebagai model untuk menempatkan besaran bilangan desimal.

PREFACE

Working in the domain of decimals recorded good experiences for me because I got a lot of insight, not only about how to make a design research but also how to work and learn with young children. These experiences give contribution to my knowledge about how students' thinking and students' learning processes. Working in this domain also gives the contribution for the development of mathematical and didactical activities, because in this research I have found some methods which are enable to be applied either for teachers or for researchers in developing mathematical idea about comparing decimals. From what I have learned, I realize that the necessities in life can't be run well without supporting from the other people. My grateful thanks I give to:

- Prof. Dr. dr. H.M.T. Kamaluddin, M.Sc., SpFK. as the Director of Graduate School in Sriwijaya University.
- Prof. Dr. Zulkardi, M.I.Komp., M.Sc. as the Head of Mathematics Education Department in Graduate School, Sriwijaya University and also as my supervisor I who always gives motivation and becomes the inspiration for many people. No one will doubt toward his contribution for education field, especially for PMRI (*Pendidikan Matematika Realistik Indonesia*) Project which is useful for improving the quality of education in Indonesia.
- Dr. Yusuf Hartono as my supervisor II who is willing to spend many time to guide and support us in order to make a fruitful discussion toward my research. Thank you also for suggesting a lot of "brilliant ideas" when I got stuck doing my research.
- Dr. Barbara Ann van Amerom as my Dutch supervisor who always cares and helps me in every condition. She always gives good contribution toward my research. When I have a problem in doing my work, she does not think that it is only my problem, but also "our

problem". I still keep my promise that I will do my best in conducting my research in order to make our dream come true (finding a better way in teaching decimals in Indonesia).

- Dr. Ratu Ilma Indra Putri, M.Si. as the lecturer in Sriwijaya University who always gives suggestion for adjusting our design research to Indonesian Curriculum.
- Dr. Darmawijoyo as the lecturer in Sriwijaya University who also gives contribution to my knowledge, especially about mathematical idea.
- Dr. Somakim as the lecturer in Sriwijaya University who also support our research with sharing many experiences in teaching mathematics.
- Prof. Dr. R.K. Sembiring who always support International Master Program on Mathematics Education as the part of PMRI Project.
- The lecturers and staffs in Freudenthal Institute, Utrecht University: Jan van Maanen, Jaap den Hertog, Frans van Galen, Dolly van Eerde, Maarten Dolk, Martin Kindt, Jo Nellisen, Mieke Abels, Nisa Figuerido, Dede De Haan, Betty Heijman, Liesbeth, and the other lecturers and staffs who can not be mentioned one by one. Thank you for giving the opportunities to share the knowledge during we are in the Netherlands.
- My lecturers in Indonesia University of Education: Drs. Turmudi., M.Sc., Ph.D; Drs. Suhendra, M.Ed., Drs. Sufyani Prabawanto, M.Ed., Dra. Dian Usdiyana, M.Si., Dr. Elah Nurlaelah, M.Si., Dra. Nurjanah, M.Pd., Aljupri, M.Sc., and the other lecturers who support me in joining this PMRI Project.
- Directorate General of Higher Education (DIKTI) who support the funds for my scholarship during I am in Indonesia and Nuffic Nesso Indonesia who support the funds for my scholarship during I am in the Netherlands.

- Ibu Neti Sudjiati, S.Pd (the teacher in SDN 21 Palembang) and Bapak Yuniman, S.Pd. (the head master in SDN 21 Palembang) who have good cooperation in conducting my research, and also for the students in grade 5A and 5B who involved in my research.
- IMPoME's students 2009 (Unsri-Unesa) who take not only as my colleagues but also as my family who always together finishing our study during this 2 years.
- IMPoME's students 2010 who help giving information and help conducting my research as the observers.
- Dr. Rahmah Johar, M.Pd. who helps sharing the ideas when we are in the Netherlands.
- Pak Abe and Bu Tesi who always assist IMPoME'students in Sriwijaya University.
- My parents (Bapak Dadang Trisula and Ibu Oom Masriah) and also my sister (Firly Fauzia) who always give the best pray for me and become my motivation in order to be a strong person.
- The last, but not the least, my very grateful thanks with million of love I give to my beloved husband (Iwan Sukmana) who makes my life become meaningful. I never thought to sit in this position without supporting from him. That is why I present my thesis and also my efforts especially for him.

I realized that I could not mention the people supporting me one by one, but I hope this thesis can be a representative toward my feelings that I am very happy working in education field. And I hope this study can give a good contribution for improving the education system in Indonesia.

Palembang, May 2011
The writer,

Puri Pramudiani

TABLE OF CONTENTS

LIST OF APPENDICES

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1
INTRODUCTION

The issue about decimals is important because decimals play an important role in mathematics curriculum and in our lives (Hiebert and Wearne, 2002). Decimal number is a central topic that can be applied to other areas of number, e.g. whole numbers, fractions, percentages, etc.(Widjaja: 2008). Moreover, Van Galen et al (2008) stated that decimals are easier to compare than fractions and ratios. When students compare 1,2 and 1,5, those are obvious which is the larger and which is the smaller number rather than they compare $\frac{12}{10}$ and $\frac{6}{4}$.

The extensive studies from around the world on decimals have documented students' difficulties and weak conceptual understanding of decimals from primary to college levels (e.g., Glasgow, Ragan, Fields, Reys, & Wasman, 2000; K. Irwin, 1995; Padberg, 2002; Steinle &Stacey, 1998b, 2001, 2002 in Widjaja, 2008). There are a lot of misconceptions about the magnitude of decimals. Many children often use some implicit and incorrect rules in interpreting decimals (Desmet et al, 2010). For example, students will determine that 12,17 is larger than 12,4 because they reason that 17 is larger than 4. Therefore, the teaching and learning process of decimals need more attention in order to support students' learning, especially in comparing the magnitude of decimals.

However, based on analysis of some Indonesian commercial textbooks, the approach to teaching and learning decimals is very symbolic and no attention is given to creating meaningful referents such as concrete models (Widjaja, 2008). Besides that, usually decimals are only taught as another notation for fractions and percentages and less attention is paid to the magnitude of decimal itself. According to Zulkardi (2002), the

material textbooks in Indonesia contain mainly sets of rules and algorithms and they lack applications that are experientially real to the pupils. In fact, the results of the tests indicated that most pupils lacked understanding of the basic skills that they were supposed to have learned in primary school and in everyday application problems. The meaningful situation is important in order to avoid students' misconception about decimals.

In the present research, we designed the contexts and activities that bring the students into the notion of decimals, and furthermore the students can apply it in comparing the size in which it can be larger or smaller than others (magnitude of decimals). The models used to show the existing of decimals are weight scale and measuring cup which bring them into the idea of number line as a model for placing the magnitude of decimals. More specifically we use density of numbers on number line to show that there are decimals between two consecutive whole numbers and there are other decimals between two different decimals, and so on (Vicki et al, 1999).

Considering to the issues about students' misconception of decimals and also the limitation of the textbook approach in Indonesia, we conduct the research which has aim to investigate the development of students' learning of comparing the magnitude of one-digit and two-digit decimals in grade 5. The research question is:

"How does students' learning develop from informal level to pre-formal level in comparing the magnitude of decimals using the number line?"

In order to answer that research question, we design a sequence of activity using weight and volume measurement activities in which number line is used as a model for placing the magnitude of decimals. Therefore, we specify the research question into following sub research questions:

1. How do measurement activities promote students' notion of one-digit and two-digit decimals?
2. How can scales provoke students to use the number line to position one-digit and two-digit decimals?
3. How can the number line promote students' learning to compare one-digit and two-digit decimals?

CHAPTER 2
THEORETICAL FRAMEWORK

This chapter provides the theoretical framework that was addresses to construct the designing instructional activities. In this research, weight and volume measurement activities were used as experience-based activities and contextual situation in order to support students' learning of comparing the magnitude of one-digit and two-digit decimals. Consequently, literatures about realistic mathematics education were explored in explaining how the experience-based activities as the contextual situations could be shifted into more formal mathematics.

A. Decimal Form (One-digit and Two-digit)

Learning about decimals is an important part in numeracy because it is needed in all walks of life (Vicki et al, 1999). There are some studies stated the meaning of decimals:

1. Widjaja (2008) stated that the word 'decimal' is used to refer to a base ten number that is written with a decimal point.
2. Reys et al (2006) stated that decimals are just another notation for fractions.
3. Vicki et al (1999) stated that decimal is any number written in the notation which uses a point. Here they do not restrict the size of the number to be less than 1. So 5,37 is referred to a "decimal".

In Indonesia, usually we use comma (,) instead of point (.) to represent decimals. Therefore, in this research, we define decimal as the number base ten separated by *comma* (,). For examples 3,5 is read as three *comma* five.

B. Comparing the Magnitude of One-digit and Two-digit Decimals

Van Galen et al (2008) said that the students in primary school may have hardly worked with percentages and decimals. According to Helme&Stacey (1999), there is a high percentage of children that tends to judge longer decimals to be larger. It seems those children read the "decimal" as a part of a whole number, for instance that 4,63 is larger than 4,8 because 63 is larger than 8.

In this research, we investigated students' learning of comparing the magnitude of one-digit and two-digit decimals. According to Freudenthal (1983), magnitude involves the properties of the size such as length, weight, duration, and content. Vicki et al (1999) and Helme&Stacey (1999) stated that comparing the size of decimals is a good task to diagnose difficulties and also it can be addressed common misconceptions about the size of decimals.

To compare decimals, there are many strategies which can be used. Markovits&Even (1999) stated that using number sense and carefully examining the given numbers (e.g., watching the order of magnitudes, using estimation, understanding decimal point, observing that the product of the last digits ends in zero) may successfully lead to a correct answer

Other strategies suggested by Helme&Stacey (1999) are equalizing length with zeros and left to right comparison. In this research, we make boundary into strategy left to right comparison on the number line in which we expect that the students will come up to the idea that the more right position of decimals on the number line, the larger their magnitude. This boundary has aim to make the research more focus and to make a bridge from informal level (experienced-based activities) into more formal level (comparing the magnitude of decimals).

C. Number Line

Building number line is the ideal way to help students understand the ideas related to the numbers (Vicki et al,1999). In this research, developing number line can be one strategy to come up to model for revealing decimals between two consecutive whole numbers and also for showing the order of the numbers. Gravemeijer (1994) stated that the number line subsequently serves as a material basis for the execution of operations. Walle in Suyono (2008) stated that using number line will make students easier to order decimals than converting those into fractions.

There are two ways of drawing number line. It can be drawn vertically or horizontally. In Indonesia, drawing number line usually start from the left to the right horizontally. To develop the skill of determining the position of decimals on the number line, the students can be experienced in the activities of reading the scale in the measurement activities (weight and volume measurement). Vicki et al (1999) stated that scale reading is an important numeracy skill which helps students with the ideas of density of numbers in number line.

D. Density of Numbers on Number Line

According to Vicki et al (1999), density means that between any two consecutive whole numbers there are decimals, and between any two different decimals there are other decimals, and so on. One example of research related to density of numbers on number line is the research of Merenluoto (2003). It shows that the students changed their abstraction of discrete numbers to an operational level of abstraction of density of numbers by adding decimals and moreover. This research suggests that the students need to be aware of how they think about numbers and pay attention to the differences between different kinds of numbers.

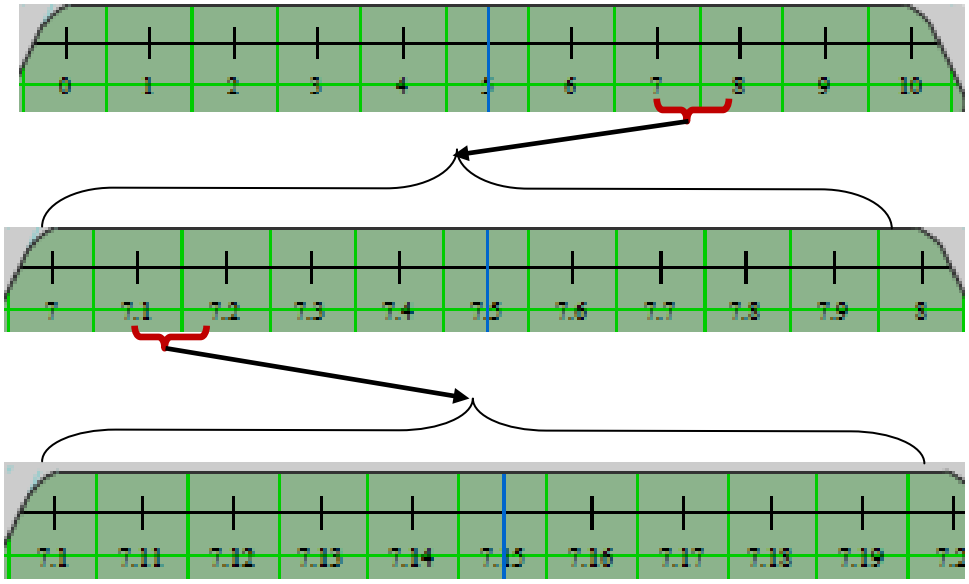


Figure 1. Density of Numbers on Number Line

Here, we expect that the students will use their intuition to come to the idea of density of numbers on number line. Hogarth (1992) specifies the three components that can develop students' intuition i.e. 1) creating awareness, 2) a framework for acquiring specific learning skills, and 3) practice. In the present research, we will assess about students' ability related to skills, knowledge, and the development of students' learning of decimals. According to Nelissen (1999), knowledge is the result of a learner's activity and efforts, rather than of the more or less passive reception of information.

E. Realistic Mathematics Education

To build the conceptual knowledge of decimals, the problems which are given should be meaningful for students. Realistic Mathematics Education (RME) underlies this research in the part of designed context and activities. Zulkardi&Ilma (2006) stated that the context is a main point for students in developing mathematics. Furthermore, they said that the context itself should be meaningful and real for students' mind.

RME is a theory for teaching and learning mathematics that has been developed in the Netherlands since the early 1970's. This approach emphasizes increasing pupils'

understanding and motivation in mathematics (de Lange, 1987; Freudenthal, 1991; Gravemeijer, 1994; Streefland, 1991; in Zulkardi, 2002). In Indonesia, RME is adapted for over last ten years with the support of a group of Dutch math educators to create a new image of mathematics education in primary schools (Sembiring et al, 2010).

As a basis of this research, the RME approach will be defined elaborately through five tenets for Realistic Mathematic Education by Treffers (1987 in Bakker, 2004):

1. Phenomenological exploration.

In order to develop intuitive notions as the basis for concept formation of decimals, in the first activity, we collected the information about students' background knowledge in decimals through playing come closer game.

In the second activity, the students were experienced directly by weighing the fruit (Duku Palembang) and weighing their body in order to find the notation and the sequence of one-digit decimals. In the third activity, the students measured the weight of rice in order to explore the notion of one-digit decimals. In the fourth activity, they measured the volume of beverages in order to explore the notation of two-digit decimals. Through the contexts familiar for students, we expect that those can motivate them to engage in learning process and help them to make mathematics become meaningful.

2. Using models and symbols for progressive mathematization.

The progressive mathematization here means the development from intuitive, informal, context-bound notions toward more formal mathematical concepts (Bakker, 2004). The sequence of designed activities in this research has purpose to bring the students' thinking in order to build self-developed model. Since the form of weight scale and measuring cup contain the scale, we expect those will bring students' thinking into the idea of drawing number line as a model for placing the magnitude of one-digit decimals.

3. Using students' own constructions and productions.

When students can construct number line from their experience in measurement activities, we expect that they can determine the magnitude of decimals by making ten partition of each range. From this strategy, the students could see that there are ten partitions containing one-digit decimals between two consecutive whole numbers, and there are ten partitions containing two-digit decimals between two-consecutive one-digit decimals. From this production, we expect the students can realize that decimal is the number base ten.

4. Interactivity.

From their own production (such as drawing number line or drawing the representation of the scales), we expect that students can make a meaningful discussion and they can share their finding with the others. The role of the teacher here also plays an important part in order to make guided reinvention for students. Freudenthal (in van Nes, 2009) posits the importance of guided reinvention for stimulating mathematization. Finally there will be vertical interaction (between teacher and students) and horizontal interaction (between student with the other students).

5. Intertwinement.

Learning about decimals of course can be a basis which also can be integrated with other domains such as fractions, percentages, proportions, measurement and the development of number sense. When students are able to understand about decimals, it will be easier for them to do calculation such as addition, multiplication, division, etc.

F. Emergent Modelling

Models are primarily used to constitute a concrete point of departure for developing formal mathematics (Gravemeijer, 1994). The development from model of into model for is elaborated in the following four level structures, i.e. situational level, referential level, general level, and formal level.

1. Situational level

The level of the situation; where domain-specific, situational knowledge and strategies are used within the context of the situation (Gravemeijer, 1994). Here, we use the context of weighing scale and measuring the weight of fruit (Duku Palembang), the body, the rice, and the volume of beverages as experience-based activities in order to support students' learning of one-digit and two-digit decimals.

2. Referential level

A referential level; where models and strategies refer to the situation which is sketched in the problem (Gravemeijer, 1994). The accomplishment of the referential level is shown by the use of the stripes on the number line. In this stage, the students are expected to consider a representation of the weight scale and number line as a model of situation of the stripes on the weight scale. Moreover, in this stage number line became the base of the emergence of student-made measuring instruments as the models-of the situation.

3. General Level

A general level; where a mathematical focus on strategies dominates the references to the context (Gravemeijer, 1994). When students have already drawn number line and made the partition base ten or the partition base ten of tenth, then we expect they will come up to model for reasoning in which they use number line to determine the magnitude of decimals

by determining the more right position of decimals on the number line, the larger their magnitude.

4. Formal Level

The level of formal arithmetic; where one works with conventional procedures and notation (Gravemeijer, 1994). In this research, our focus is not really until formal level because we will more focus into the role of number line in comparing one-digit and two-digit decimals (general level). However, this research can be a basis for the next stage (formal level) when the students are able to compare one-digit and two-digit decimals without relying on the number line.

CHAPTER 3
METHODOLOGY

As mentioned in chapter 1, this study has aim to contribute to the classroom activities in learning decimals. Therefore, design research was chosen to acquire the research question and to achieve the research goal. The basic research methodology that will be discussed is: (a) research methodology, (b) research subjects, (c) hypothetical learning trajectory and local instruction theory, (d) data collection, and (e) data analysis including reliability and validity.

In this research, design research was chosen to acquire the research question and to achieve the research goal.

A. Research Methodology

*1. **Preliminary Design***

The first phase starts with formulating mathematical learning goals, combined with anticipatory thought experiments in which one envisions how the teaching-learning process can be conducted in this classroom. This first step results a conjectured local instruction theory that is made up of three components: (a) learning goals for students, (b) planned instructional activities and the tool that was used, and (c) a conjectured learning process in which one anticipates how student's thinking and understanding could evolve when the instructional activities are used in the classroom (Gravemeijer, 2004). In this phase, a sequence of instructional activities (related to decimals) containing conjectures of students' strategies and students' thinking (either in solving the problem or in the discussion) was formulated. The conjectured hypothetical learning trajectory was developed based on

literatures and was adjusted to students' actual learning during the pilot and teaching experiment.

*2. **Pilot Experiment***

The pilot experiment has purpose to test and to see the underlying principles explaining how and why this design works in order to be elaborated and refined for conducting the teaching experiment. This also included the pre-assessment which has aim to assess students' pre-knowledge toward decimals. The try out activities and interview with the teacher and students were held for adjusting the improved Hypothetical Learning Trajectory (HLT).

73 students were involved in the pre-assessment (26 students in class 5A, 23 students in class 5B, and 24 students in class 5C), but only 7 students involved in the series of activities of pilot experiment. The decision of choosing small group because we expect to be more focused to the adjustment of HLT. We expected that choosing 7 students containing various students' abilities (2 high level students, 3 average level students, and 2 low level students) would represent the ability of the other students in whole class.

*3. **Teaching Experiment***

In teaching experiment, instructional activities were tried, revised, and designed on a daily basis during the teaching experiment (Gravemeijer, 2004). The teaching experiment aimed at collecting data for answering the research questions. In this research, the teaching experiments were conducted in six lessons. The teaching experiments emphasize that the mathematical ideas (related to the magnitude of decimals) and conjectures could be adjusted during the teaching and learning process. Before doing the teaching experiment, the teacher and the researcher discussed about the series of activities in order to prepare the

designed activities conducted in the class. After each activity was done, the teacher and the researcher made reflection in order to improve the designed activities and also as a feedback for repairing the weakness or the difficulties in the teaching and learning process. 26 students and a teacher in class 5A, SDN 21 Palembang were involved in this teaching experiment.

4. ***Retrospective Analysis***

In this phase, all data collected during experiment were analyzed. Hypothetical Learning Trajectory (HLT) was compared with students' actual learning.

All activities in the class including the group discussion were recorded on video. This data has aim to compare HLT and students' actual learning. The important fragments were selected and analyzed.

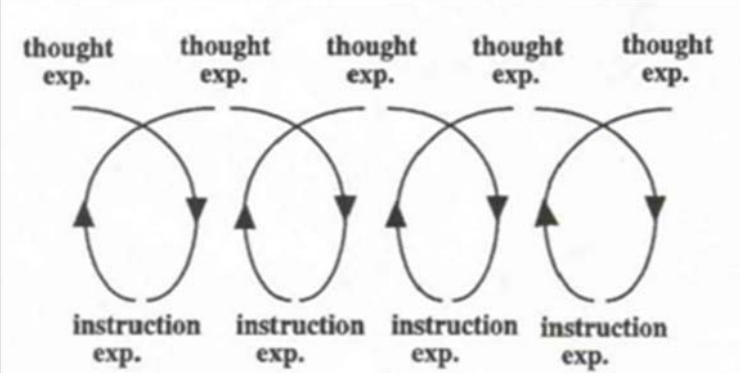


Figure 2: Design research, accumulative cyclic process.

The written data included students' works in each activity, observation sheets, the results of assessments including the final assessment and some notes were also collected and analyzed to see how the instructional activities work for students. Data interview with students and the teacher also were included to know about the feedback of the lesson and the activities in the class.

The result of this research is the underlying principles explaining how and why this design works. The Hypothetical Learning Trajectory served as a guideline in the retrospective analysis to investigate students' learning of comparing the magnitude of one-digit and two-digit decimals.

B. Research subjects

Twenty six students and a teacher for grade 5 SDN 21 Palembang Indonesia became the research subjects in the teaching experiment. This school has been involved in PMRI (*Pendidikan Matematika Realistik Indonesia*) Project since 2010. The students are about 10 to 11 years old, and they had learnt about measurement in grade 1,2,3, and 4, and introduction to fraction in grade 4 semester 2.

C. Decimals in Indonesian Curriculum for 5th Grade

Based on Indonesian curriculum, decimal is taught in grade 5 semester 2 (Kurikulum *Tingkat Satuan Pendidikan Sekolah Dasar*, Depdiknas (2006):

Table 1. Decimals in Indonesian Curriculum Grade 5

Standard Competence	Basic Competence
Numbers 5. Using fraction in problem solving	5.1 Converting fraction into percentages and decimals forms or vice versa 5.2 Adding and Subtracting many forms of fraction 5.3 Multiplying and dividing many forms of fraction 5.4 Using fraction in solving ratio and scale problems

Decimal is taught in relation with fractions and percentages. Sometimes, the teaching and learning of decimal in Indonesia is very formal. In some textbooks, decimal is directly converted from fraction which has denominator ten or one hundred (see figure 3).

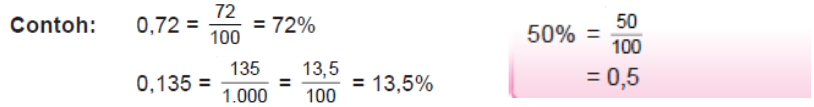


Figure 3: One example about teaching and learning decimals in Indonesia

D. Data Collection

All activities in the class including the group discussion were recorded on the video. This data has aim to compare HLT and students' actual learning. The important fragments were selected and analyzed.

The written data included students' work in each activity. The results of assessments including the final assessment and some notes which were collected during the teaching experiment were also collected and analyzed to see how the instructional activities work for students. Data interview with students and the teacher also were included to know about the feedback of the lesson and the activities in the class.

Table 3.1. The outline of the data

	Activities	**Data Collected**	**Goals**
Preliminary Design (September-December 2010)	Studying literatures and designing initial HLT		
Pilot experiment (February 2011)	Classroom observation in grade 5A, 5B, and 5C	Video recording	- Investigating classroom socio norms
	Pre-assessment in grade 5A, 5B, and 5C	Students' work	Finding students' current knowledge of decimals
	Discussion with the teacher	Field notes	Communicating the designed HLT and teacher guide
	Try out the activities: 1. Activity 1: Playing "*Come Closer*" Game 2. Activity 2: Measuring on Being Precise (precise measurement) 3. Activity 3: Exploring One-Digit Decimals 4. Activity 4: Exploring Two-Digit Decimals 5. Activity 5: Exploring One-digit and Two-digit Decimals on the Number Line 6. Activity 6: Comparing the Magnitude of One-digit and Two-digit decimals 7. Post Assessment (pilot experiment)	- Video recording - Students' work	- Testing the activities - Investigating students' learning of decimals
Revising HLT (February-March 2011)			Elaborating and refining the activities for improved HLT in the teaching experiment
Teaching experiment (March 2011)	Classroom observation 6 lessons 1. Activity 1: Playing "Come Closer" Game 2. Activity 2: Measuring on Being Precise (precise measurement)	- Video recording - Students' work	- Testing the activities - Investigating students' learning of decimals

	Activities	Data Collected	Goals
	3. Activity 3: Exploring One-Digit Decimals 4. Activity 4: Exploring Two-Digit Decimals 5. Activity 5: Exploring One-digit and Two-digit Decimals on the Number Line 6. Activity 6: Comparing the Magnitude of One-digit and Two-digit decimals		
	Final assessment	Students' work	Finding the effect of activities toward students' learning of decimals
	Interview students, teacher, and observers	Video recording and field notes	

E. Data analysis, reliability, and validity

*1. **Data analysis***

The main data that are needed to answer the research question is the videotaping of the activity and interaction between teacher and students and also interaction between one student and the other students during the activity. The students' reasoning also will be investigated during class discussion.

For the data video, the fragments which were relevant with students' learning were selected. And then, the transcriptions were made based on that fragment. In this phase, we looked our conjectures and students' actual learning for the retrospective analysis.

The students' works also were analyzed. This is emphasized into the strategies used by the students. For the post assessment, we gave analysis not only to the strategies used by students but also to the level of accomplishment by giving the percentages of students who could solve the problem and gave their reasoning correctly. The observers and the researcher made some notes during the teaching and learning process toward the designed activities and the students' attitude (including verbal and non-verbal behaviour).

2. ***Reliability***

The qualitative reliability was conducted in two following ways:

a. Data triangulation

The data triangulation includes different data sources, i.e. data interview, observation data, and document analysis (students' works). All activities were video recorded and the students' works were analyzed.

b. Cross interpretation

The parts of the data of this research (especially the video data) were also cross interpreted with the colleagues, the supervisors, and also the other researchers. 14 people (including master students and lecturers) were involved in cross interpretation toward the fragment of the video and students' works in quality boost on 28th of April 2011. This was conducted to reduce the subjectivity of the researcher's point of view.

3. ***Validity***

There are two kinds of data analysis in validity: internal validity and external validity. Internal validity contains a 'normal' conversation, interviews, and triangulation. External validity contains generalization and it depends on similarity context of the study and new situations. To keep the methodology of this research as valid as possible and to answer the research question, the following methods of validity are used in the data analysis (Wijaya, 2008):

a. HLT as means to support validity

The HLT was used in this retrospective analysis as a guideline and a point of reference in answering the research question. This aims to connect and evaluate the conjectures to the collected data and to prevent data bias.

b. Trackability of the conclusions

The teaching and learning process was documented by video recordings, observation sheets, and students' works. With this data, we describe the situation and the findings in order to get the information for our reasoning which will bring to our conclusions.

CHAPTER 4
HYPOTHETICAL LEARNING TRAJECTORY

The instructional activities were designed to investigate students' learning of comparing the magnitude of one-digit and two-digit decimals using number line. The process of designing instructional activities in the classroom contains two important points, namely *hypothet*ical learning trajectory and local instruction theory.

Hypothetical Learning Trajectory (HLT) is a part of planning mathematics lesson which consists of the goal for the students' learning, the mathematical tasks that will be used to promote student learning, and hypotheses about the process of the students' learning. According to Gravemeijer (2004), in designing an instructional activity, a teacher should hypothesize and consider students' reaction to each stage of the learning trajectories toward the learning goals. This hypothesize is elaborated in a day-to-day basis of a planning for instructional activities that is called as hypothetical learning trajectory. A hypothetical learning trajectory consists of learning goals for students, planned instructional activities, and a hypothesized learning process.

The goals for the students are:

1) To make connections between daily life situation (measurement activities) and mathematical idea (the notion of decimals).
2) To explore the notation of one-digit and two-digit decimals.
3) To get an insight about the density of numbers on the number line.
4) To be able to compare the magnitude of one-digit and two-digit decimals from their relative position on the number line.

The mathematical goals in each activity were elaborated in the instructional design.

Local instruction theory is defined as a theory that provides a description of the envisioned learning route for a specific topic, a set of instructional activities and means to support it (Gravemeijer, 2004 and Cobb et al, 2003; Gravemeijer, 1994 and Gravemeijer in Doorman, 2005; in Wijaya, 2008). From the local instruction theory, a teacher could design a hypothetical learning trajectory for a lesson by choosing instructional activities and making conjectured learning process of the students. This local instruction theory encompasses both provisional instructional activities, and a conjectured learning process that anticipates how students' thinking and understanding might evolve when the instructional activities are employed in the classroom (Gravemeijer and Cobb, 2006).

Table 3: Overview of Learning of Comparing One-Digit and Two-Digit Decimals

Content Areas	Goals	Activities	Conjectures	Concept
Students' pre knowledge of numbers	Students are able to determine the numbers between the other numbers	Playing Come Closer Game	- Some students might think that there is no number between two consecutive whole numbers - Some students has already perceived an idea of the notion of decimals based on their daily life experiences	Knowledge of numbers
The existing of decimals in between two consecutive whole numbers	Students are able to find decimals through weighing activity	Measuring the weight of the things (duku and body) using weight scale and digital weight scale	The students will find decimals through weight measurement activities	One-digit decimals
The meaning of one-digit decimals (partition base ten)	Students are able to explore one-digit decimals and put them on the number line	Measuring the weight of rice	- The students are able to find the sequence of one-digit decimals and put them on the number line correctly - The students get difficulties in sequencing one-digit decimals and assumed that 0,10 might come after 0,9 - The students can explore that 0,1 is one over ten and 0,5 is a half or five over ten based on their position on the number line	The relative position of the magnitude of one-digit decimals
The meaning of two-digit decimals (partition base tenth of tenth)	Students are able to explore two-digit decimals and put them on the number line	Measuring the volume of beverages	- The students are able to find the existing of two-digit decimals in between two consecutive one-digit decimals.	The relative position of the magnitude of two-digit decimals
Getting insight about the density of numbers on the number line	Students are able to explore one-digit and two-digit decimals and	Playing Spaceflight game	The students are able to find that between two consecutive whole numbers, there are decimals; and	Density of decimals

Content Areas	Goals	Activities	Conjectures	Concept
	put them on the number line		between two consecutive one-digit decimals there are other decimals (two-digit decimals)	
Comparing the magnitude of one-digit and two-digit decimals	Students are able to compare the magnitude of one-digit and two-digit decimals	Solving contextual problems related to one-digit and two-digit decimals	- The students are able to compare one-digit and two-digit decimals using the strategy of left to right comparison on the number line - The students compare one-digit and two-digit decimals using formal procedures (by making the links between decimals and fractions) - The students might still think that the longer digit decimals, the larger their magnitude.	The magnitude of one-digit and two-digit decimals

Our hypothesis is through these designed activities, the students' learning can develop from informal level (measurement activities) to pre-formal level (comparing the magnitude of one-digit and two-digit decimals using number line). In this chapter, the hypothetical learning trajectory will be elaborated in following manners:

A. Activity 1: Playing "*Come Closer*" Game

Mathematical goal:

Students are able to determine the numbers between the other numbers

1. Activities (Playing Come Closer Game)

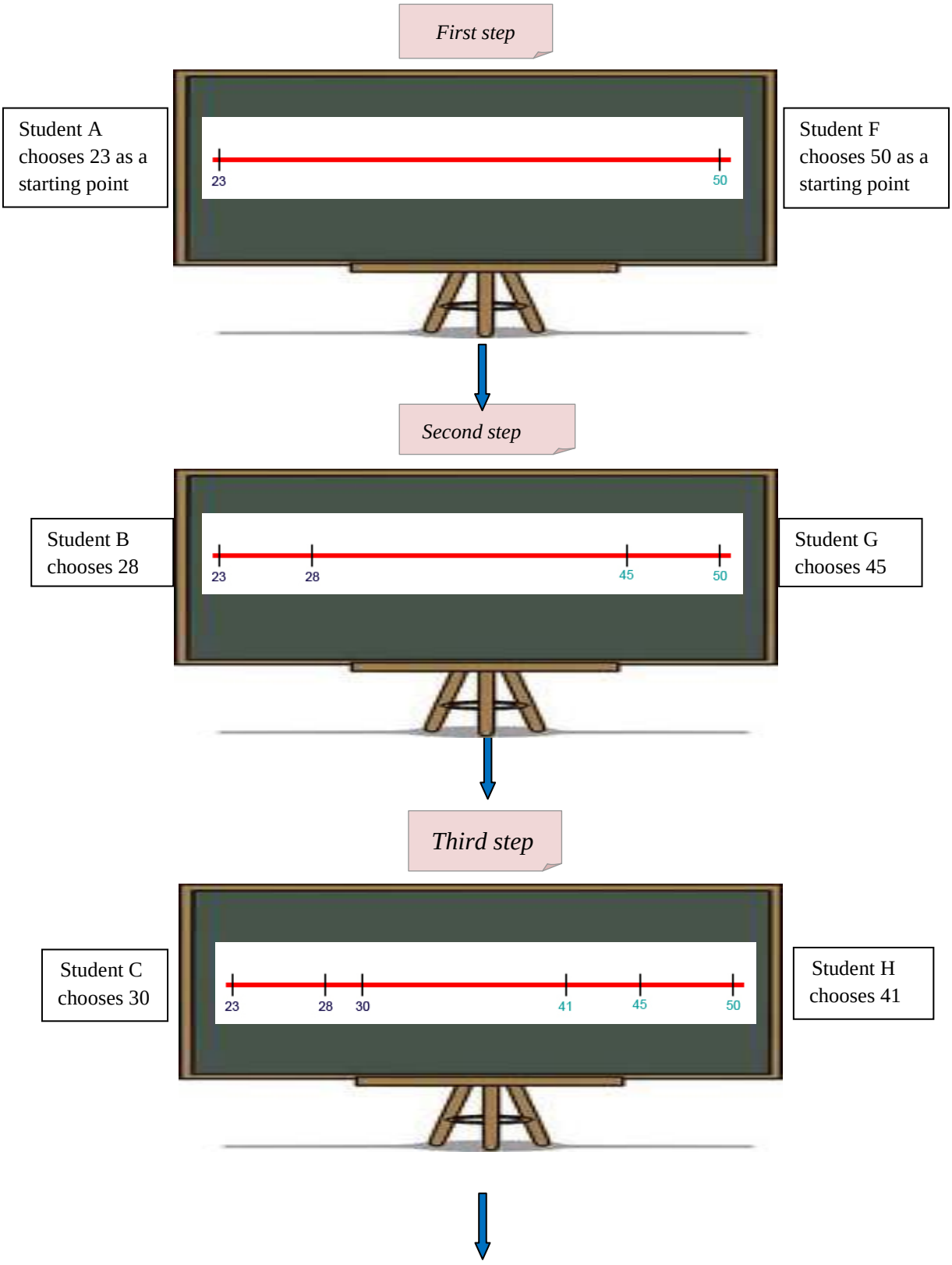
In this activity, we investigate students' pre knowledge toward the numbers through playing the game, namely "*Come Closer*" game. This game enables students to determine the numbers between the other numbers. This game is played by two groups. Each group contains five students.

Group A (Student A, B, C, D, E)	**Group B (Student F, G, H, I, J)**

- The teacher tells the rule of the game is that both groups have to come closer by suggesting the numbers between two whole numbers in the range from 1 through 100. One group goes forward, and the other group goes backward. The students are free to suggest any kind of numbers but it is not allowed to cross each other. All members in each group must have opportunity to suggest the numbers. The group who is not able to nominate the next number, it means that they lose.
- First, each group suggests one number as a starting point, but their opponent should not know about the chosen number. They may write on the empty card. After the teacher

allows to open the number, then all students will know what number chosen by each group. For example, group A chooses 23 and group B chooses 50 as a starting point.

- Then, they make a line on the blackboard.
- Both groups will come closer on the opposite direction by suggesting the number (distance point) and marking that number in the line on the black board.
- They make a toss to decide who will take the first turn, and the opponent takes the next turn after that, and then they take turns again; and so on until they face the condition such in figure 4:


First step
Student A chooses 23 as a starting point
23
50
Student F chooses 50 as a starting point
Second step
Student B chooses 28
23
28
45
50
Student G chooses 45
Third step
Student C chooses 30
23
28
30
41
45
50
Student H chooses 41

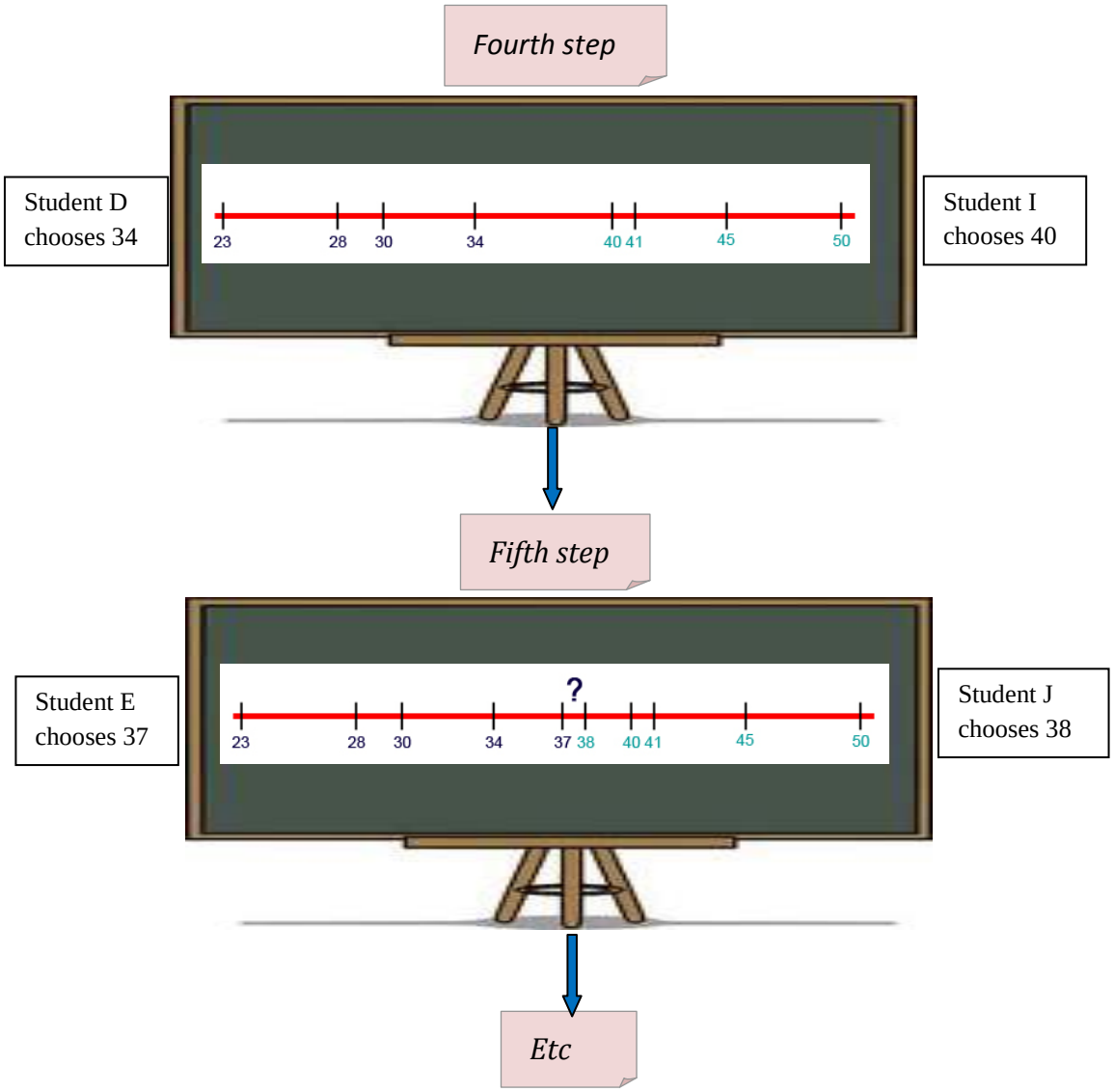


Figure 4: Playing come closer games

2. *Conjectures of students' thinking*

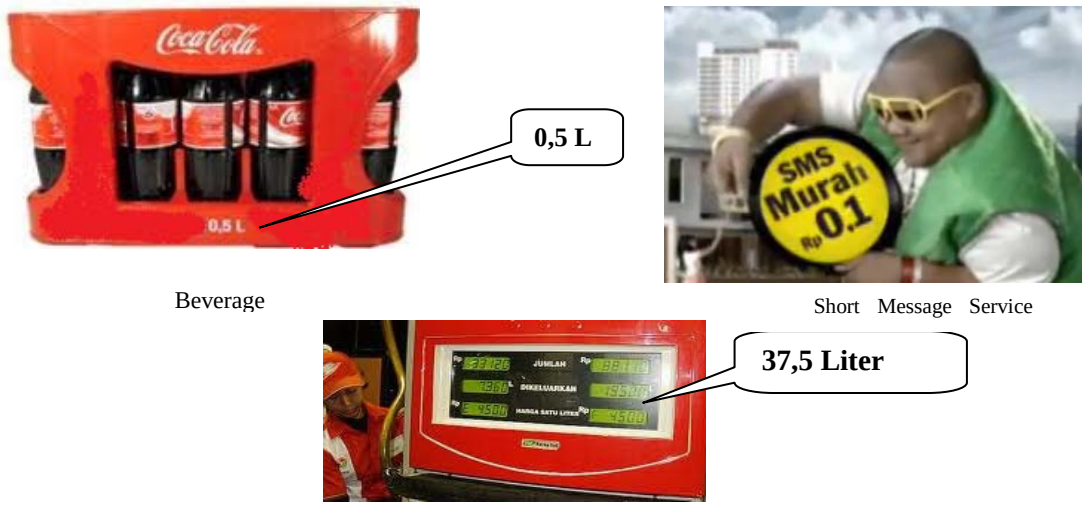
When the closer numbers are in consecutive whole numbers, for example 37 and 38, student E might think that his/her group is lost because (s)he has not chance to suggest the other numbers, and (s)he thinks that the game is ended. Actually, when (s)he proposes fraction such as 37½ or decimals such as 37,5; 37,2; 37,8, etc. (s)he still has chance to continue the game and suggesting the other numbers besides whole numbers. However,

since it is conjectured that the students will only think about whole numbers, then probably the game will be ended until that.

3. ***Discussion***

The teacher lets the students to decide whether they will end the game in that position or try to continue the game with suggesting the other numbers using fractions or decimals. Through this activity, we can see the pre-knowledge of students. When they are not able to suggest the other numbers between 37 and 38, it means that they are not aware that there should be other numbers in between two consecutive whole numbers (either fractions or decimals). Officially, in grade IV the students have already learned about fraction and simple decimal form as division of two whole numbers, but do they realize their magnitude and their relative position on the number line? Therefore, through this activity we expect that we can assess how far the students' knowledge about numbers and their magnitude.

If there is no student who has any idea to give decimals in between two consecutive whole numbers, then the teacher can provoke the question such as: *"We have seen that the game is ended because student E can't nominate the other numbers between 37 and 38. What do you think? Are you sure that there is no number between 37 and 38? Why do you think like that? Explain your reasoning!"* Some students might reason because there is no number between them. Then the teacher can further ask the students: "*How if I nominate the next number in this position?* (The teacher gives the mark in the position between the number 37 and 38) *Can it be?"* Some students might answer *"yes, it can"* but the other students might answer: *"No, it can't be?"* When there are two different answers, let them discuss first and give their own reason. After that, the teacher leads the discussion by showing the posters containing comma numbers in daily life. For example: *"Have you seen these pictures? What do you think about the numbers in these pictures?"*



Beverage

Short Message Service

Gas Station

Figure 5: Posters containing decimals in daily life

For figure 5, some students might have already known that 0,5 L is zero comma five liter; Rp 0,1 is zero comma one rupiah; and 37,5 liter is thirty seven comma five liter. May be they have heard them in their daily life or through the advertisement on the television. But do they understand about that? If no one is able to explain that, then the teacher asks the students (for example)*: "For pictures c, it is stated that 37,5 liter. What do you think about 37,5 liter? If I buy 37 liter gasoline, that is more or less than 37,5 liter? How about if I buy 38 liter gasoline, that is more or less than 37,5 liter?"* After this discussion, we expect the students will realize that 37,5 liter is more than 37 liter but less than 38 liter.

*4. **Reflection of the activities***

Finally, the teacher asks the students: *"So, what is your conclusion about the game that is already played?"* Some students might realize that Student E should still have opportunities to suggest the numbers, e.g. 37,5. Starting from what the students have known and how far their pre-knowledge toward decimals, the teacher can continue the next

activity in order to encourage the students' thinking that there are decimals between two consecutive whole numbers.

B. Activity 2: Precise Measurement Activities

Mathematical goal:

Students are able to read the number between labelled numbers on a scale

1. ***Activities (Measuring the weight of Duku Palembang using weight scale and digital weight scale)***

In order to encourage the students who think that there is no number between two consecutive whole numbers and also to introduce decimal form then the students will be experienced in the activities based on contextual situation. The activity is measuring the weight of Duku Palembang. Duku Palembang is the typical fruit from Palembang, Indonesia. Usually, when people buy it, it is packed in a bunch, so to determine the price, it is measured in kilogram. In this designed activity, we will set the weight of duku not in whole numbers but in *comma numbers*. So, we expect that the students will find that decimal is between two consecutive whole numbers.

The students work in small group containing 4 or 5 students in each group. The teacher asks the students to weigh some bunches of Duku in different weight and report it precisely. Since the requirement data should be as precise as possible, therefore if the scale points to the unlabeled numbers (e.g. between 0 kg and 1 kg), then they have to estimate and draw it precisely. In this case, we expect the students will draw the representation of weight scale as a model of situation.

Figure 6: Weight Scale

After the students measured the weight of Duku, the students are asked to write the result and draw the scale on the poster and report it as a group. After the students measure the weight of Duku using weight scale, then they are asked to measure those duku using digital weight scale.

In the previous activity, the students have measured the weight of the fruits; it is more than a half. Now, in this activity, the students will see that the actual weight for those Duku is 0,5 kg. The teacher asks the students*: "Which of you know about what kind of number is that?"* If no one knows, then the teacher says that it is decimal number (*comma number*).

2. ***Conjectures of students' thinking:***

For example, the weight of the fruit measured by students is in between 0 kg and 1kg:

a. Some students might draw what exactly is shown on the weight scale
b. Some students might draw number line to represent the weight scale

Two possible answers above will be discussed as a starting point to come up to the number line as a model for placing the relative position of decimals which will be elaborated further in the next activity (measuring the weight of rice).

Each group puts their poster containing the result of the weight of duku. After that, each group is given the opportunity to present and to explain about the scale drawn on their

poster. When the needle shows to the position between 0 and 1, it is conjectured that the students will answer that it is more than 0 but less than 1. Then, the teacher can provoke the question: "*What do you think? This group states that "It is more than 0, but less than 1." Which of you can determine precisely how much is its weight?"*

Our conjecture is that the students will think that it is a half kilogram. However, may be there are some students who answer that it is more than a half kilogram. The teacher can ask them why they think like that. Some students might determine a half by determining the representation of the weight scale and knowing that it is a half. To know what precisely the scale is, the measurement activity will be continued using digital weight scale.

When the students compare the result of measurement using weight scale and digital weight scale, they will be aware that in between two consecutive whole numbers there are decimals. They will think further that when the needle is in between 0 and 1, and it shows in the middle , then it is probably 0,5 (shown in digital weight scale).

After that, the teacher asks each group to report all weight of those duku abd draw it on the poster. For example:

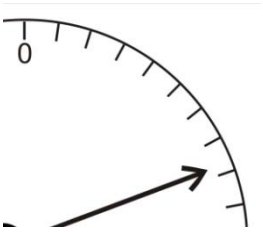


Figure 7: The representation of the weight scale

3. ***Reflection of the activities***

The students are asked about their finding in the weighing activity. If in this activity there are some students who already know about the term of "decimal", then the teacher can discuss it and asks the other students about what the meaning of decimal. It is

conjectured that in this activity, the students will only say that decimal is the number containing comma (point). After that, the teacher can elaborate the discussion by asking: *"What are the advantage using decimals in daily life?"* We expect that the students will give the answer: *"One of the advantages/benefit using decimals is that we can measure the weight precisely/accurately."* Furthermore, the teacher asks the students; *"What do you think? Is it only applicable for measuring the weight?"* We expect that the students will realize that decimal is not only useful for measuring the weight, but also it can be useful for measuring the length, the height, the temperature, and the volume very precisely because sometimes when we measure them, it is not shown in the whole numbers.

C. Activity 3: Exploring One-Digit Decimals

Mathematical goal: Students are able to explore one-digit decimals and put them on the number line

*1. **Review the Previous Activity***

Before the activity is begun, the students are asked to describe about their findings in the previous activity. It is expected that all students know about decimal through measurement activity and they realize that decimal exist in between two consecutive whole numbers. The teacher asks the students: *"What is the main food in Indonesia?"* All students might say: "*Rice*". Then, the teacher asks the students: "*How much the weight of rice that you usually eat in one day?"* The answer may vary. Some students might answer: *"0,5 kg."* Then, the teacher further asks: *"How about the weight of rice that you usually eat once?"* The answer may vary. Some students might say: *"0,1 kg."* Then, the teacher asks: "*What is the meaning of 0,1 kg? And what is the meaning of 0,5 kg?"* It is conjectured that the students still get difficulties to explain it.

*2. **Activities***

The teacher provides some packs of rice in different weights. The students are asked to investigate those weights. One volunteer student is asked to do the experiment in front of the class, and the other students observe it. There will be 10 packs of 0,1 kg rice and two packs of 0,5 kg rice. One volunteer student will be given opportunity to measure using weight scale. Then (s)he will see how much kg there are.

*3. **Discussion and conjectures of students' thinking***

The teacher asks the students: "*What do you think after you weigh that rice using weight scale*?" For 0,5 kg pack of rice, some students realize that when two packs of 0,5 kg were measured, the weight becomes 1 kg. Therefore they conclude that 0,5 kg is a half kilogram. The teacher can continue ask: *"How do you know that it is a half kilogram?"* Some students might answer: *"It is because when we weigh two packs of zero point five, the weight will become 1 kilogram, so 1 pack of 0,5 kg is a half kilogram"*.

After that, the students were asked to weigh several packs of 0,1 kg rice. From this activity the students will also realize about the sequence of one digit decimals. In this activity, they can see on the weight scale that 1 pack is equal to 0,1; 2 packs of 0,1 kg is equal to 0,2 kg, 3 packs of 0,1 kg is equal to 0,3 kg; etc. After they do experiment measuring the nine packs of 0,1 kg (0,9 kg), then the teacher asks the students: "*What number that might come up when we put ten packs of 0,1 kg?"* It is conjectured that some students will answer that it is 0,10 kg because they have already noticed that the sequence of packs of 0,1 kg is 0,1; 0,2; 0,3; ….,0,8; 0,9; then might be 0,10 kg after that. However, some students might already know that ten packs of 0,1 kg is equal to 1 kg. Therefore, it will be proven through measuring it directly either in weight scale or digital weight scale

that ten packs of 0,1 kg is 1 kg. We expect that the students will realize that 0,1 kg is a tenth of 1 kg. During the experiment is done by volunteer student in front of the class, the other students write and draw the weight shown on the weight scale.

When the students have already come up into the idea of *model-of* in the previous activity either by drawing representation of the weight scale or by drawing number line, then the students are asked to do the task. The task is to put the numbers that they find in the weighing rice activity (e.g. 0,5; 0,1; and 1) on an empty number line. From this task, we can see whether they put the position of the numbers precisely or just estimate them. The strategies that might be used by students are:

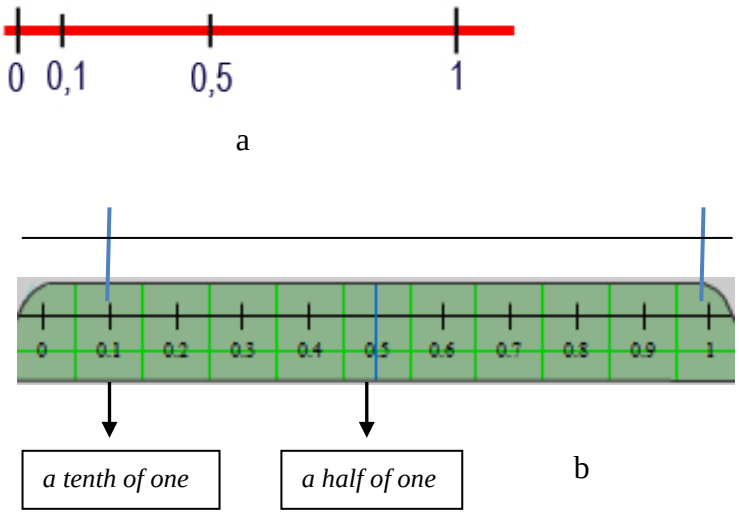


Figure 8: a. Making estimation; b. Putting on number line precisely

4. ***Reflection of the activities***

In this activity, it is expected that the students will be able to explore the notation of one-digit decimals such as 0,5 is a half of 1; and 0,1 is a tenth of 1. Some students might think that those numbers are the numbers less than 1. So, when they are asked "*Is there*

any number between 0 and 1?" Now they might answer "*Yes*", 0,1; 0,2; 0,3; 0,4; 0,5; etc. are the numbers between 0 and 1.

To strengthen that concept and to keep that in students' mind, one action that can be done is by singing "Decimal Song":

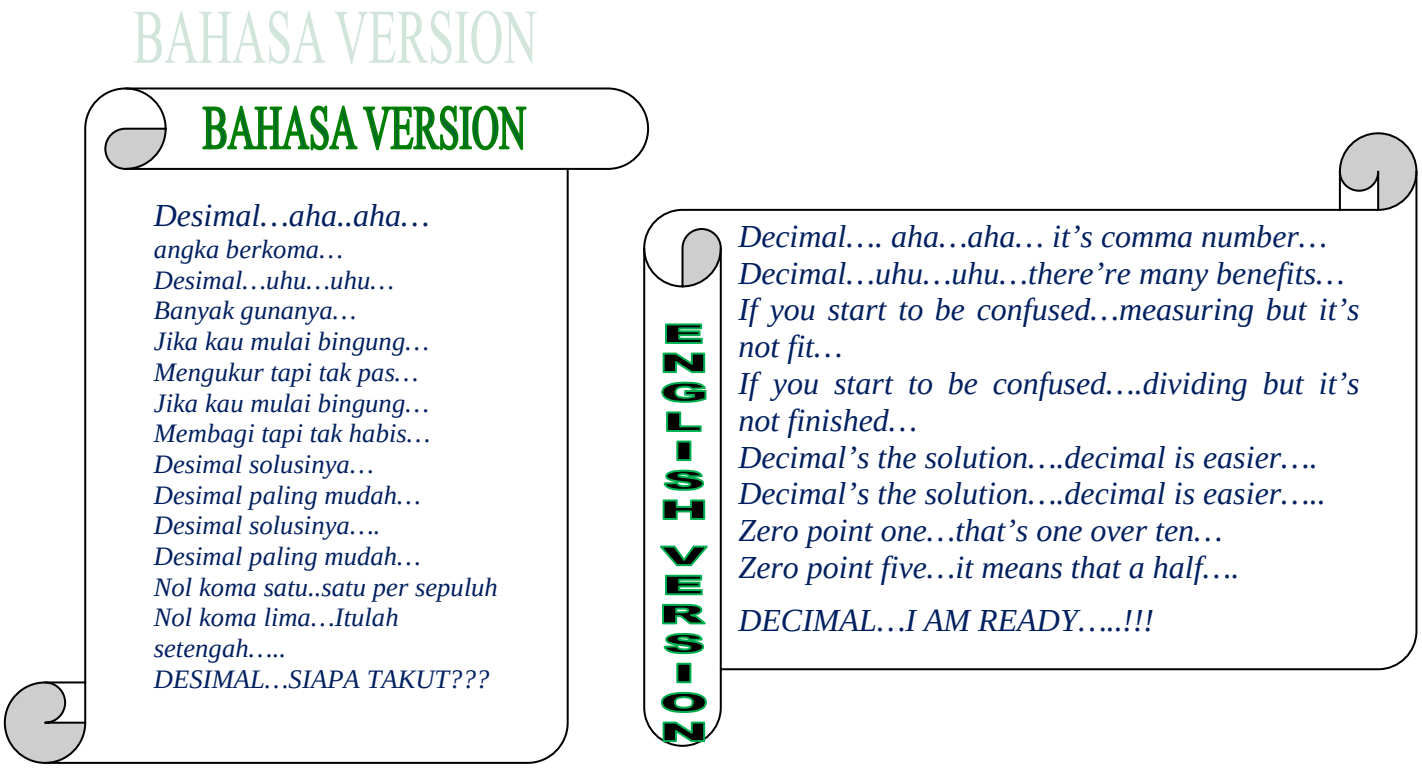


Figure 9: Decimal song

D. Activity 4: Exploring Two-Digit Decimals

Mathematical goal:

Students are able to explore the notion of two digit decimals and put them on the number line

Description of The Activity

1. *Review Previous Activity*

The teacher reviews previous activity by asking the students to sing “Decimal song”. From the previous activity, they have already known about the notion of one digit decimals is in between two consecutive whole numbers and also they know the meaning of simple decimals (0,5 is a half or five over ten; 0,1 is one over ten). In this activity they will be experienced with exploring the notion of two digit decimals. The teacher asks the students: *“Yesterday I bought two beverages that have different volume. Beverage A is 0,25 liter and beverage B is 0,5 liter. What do you think? Which is more, beverage A (0,25 liter) or beverage B (0,5 liter)?”* It is conjectured that some students will answer that 0,25 liter is more than 0,5 liter because they assumed that 25 is larger than 5. The teacher provides two different bottles which have no label, so the students do not know how much it is.

2. *Activities*

The teacher brings the coca cola in 0,25 liter and 0,5 liter. The teacher asks one volunteer student to do the experiment in front of the class. It will be shown that:

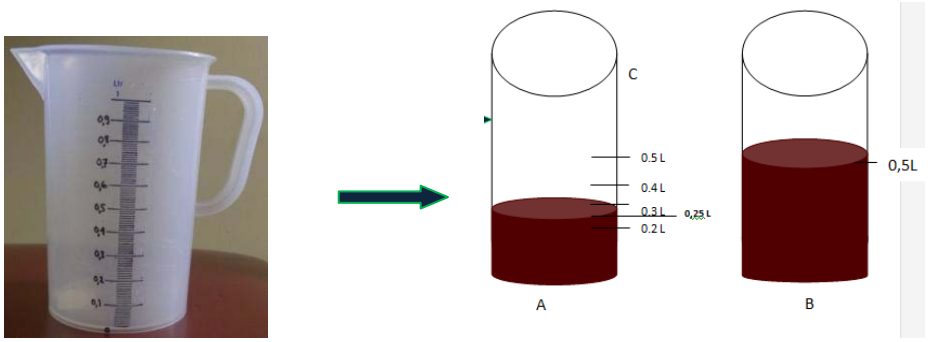


Figure 10: Measuring the volume of beverages

3. ***Discussion and conjectures of students' thinking***

After volunteer student do the experiment, then the teacher asks the students: *"What do you think now? How if I compare 0,25 liter and 0,5 liter beverages? Which one is more?"* After observing the volume of beverages in the cups, then we expect the students will realize that 0,25 is less than 0,5. They can see in measuring cup that the position of 0,25 liter is in between 0,2 liter and 0,3 liter. Then, the teacher asks the students: *"Why it should be written as 0,25 liter, not as 0,2 liter?"* Then probably some students will answer *"It is to be more precise (accurate).*" It is same with the case in weight measurement activity, when the scale is in between 0 and 1 and it is in the mid point, then it might be 0,5. Now, the students can see the fact that when the volume is more than 0,2 ad is still less than 0,3, then it probably in two digit decimals (in this case 0,25).

4. ***Reflection of the activities***

The teacher asks the students: *"What can you conclude from this activity?"* We expect that the students will realize that between two consecutive one digit decimals there are two digit decimals. Besides that we expect the students also realize that the longer digit decimals, the more precise the measurement.

Finally, the teacher asks the students:

"If I have 0,1; 0,5; 0,25 Liter, where will you put those numbers in number line?" We expect that the students will draw and put those numbers in number line:

After that, students also will be experienced by placing the decimals in the number line, such as:

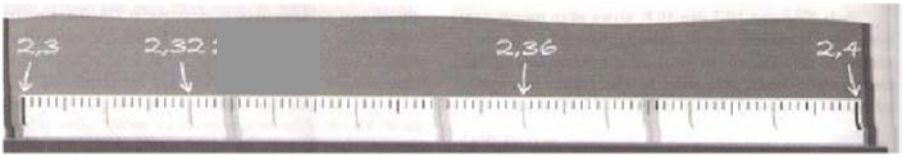


Figure 11: The example of problem

E. Activity 5: Exploring One-digit and Two-digit Decimals on the Number Line

Mathematical goal:

Students are able to explore one-digit and two-digit decimals on the number line

1. Review the Previous Activities

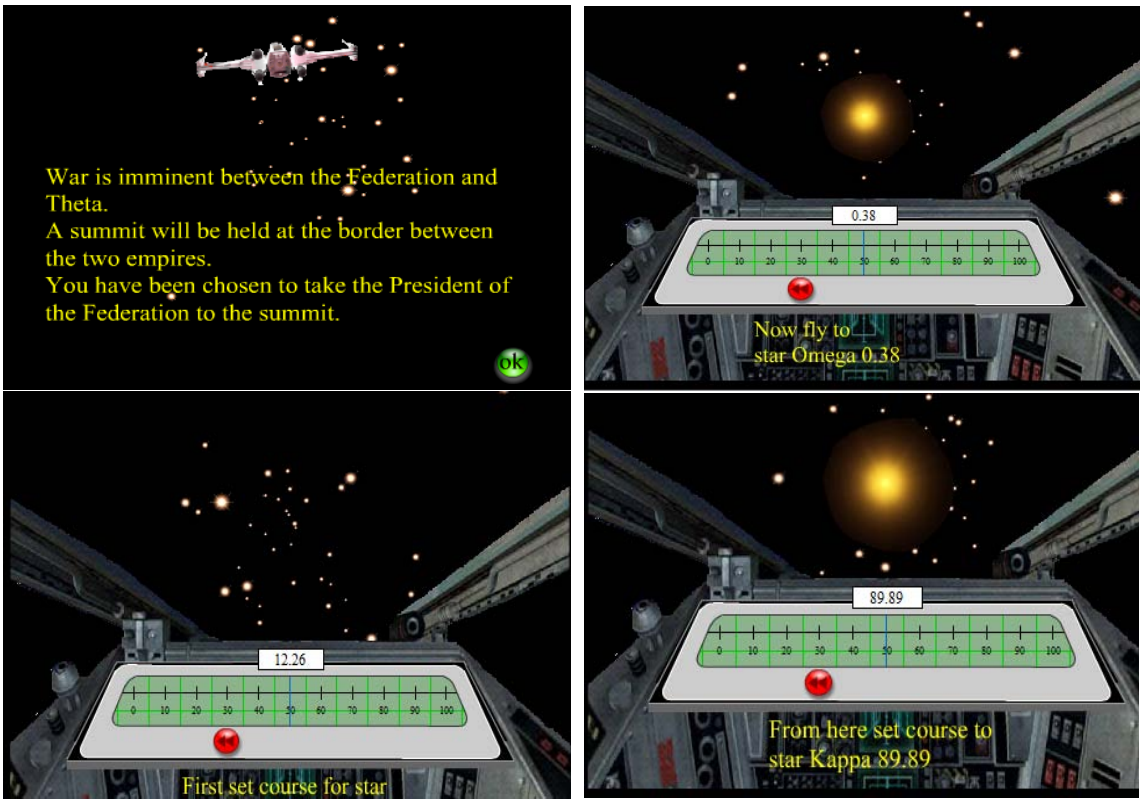
The students are asked to explain about the mathematical ideas that they have already learned in the previous activities. It is emphasized that in weighing duku and body, they have found that in between two consecutive whole numbers there are decimals. In the measuring the volume of beverages, they have found that in between two consecutive one digit decimals there are other decimals, i.e. two-digit decimals.

2. Activities

The activity is playing the games, namely "*Spaceflight*". The students will practice to explore the numbers including whole numbers, one digit, and two digit decimals using number line. In this game, they have to explore about the certain point (decimals) to find the message code. We will assess how far their notion toward one-and two-digit decimals. When they can find the numbers quickly and certainly, it means that their notion is better than the other students who explore the numbers slowly and doubtfully. This game is

online and can be found in: http://www.fi.uu.nl/toepassingen/03127/task3.html designed by Frans van Galen.

Each student plays the game in one computer. They will be brought into computer room. When student has finished playing the game with finding the message code correctly, then (s)he raises his/her hand, and then the teacher checks his/her work. If it is correct, the teacher writes how long the time of him/her to complete the game.



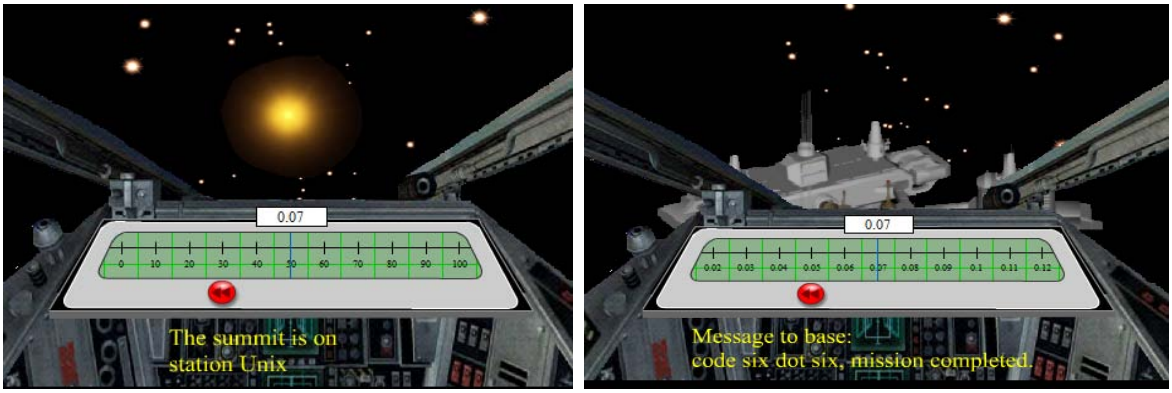


Figure 12: Spaceflight game

3. ***Discussion***

After all students finish playing the game, the teacher leads them into discussion by asking*: "What do you think about the game that you play? How about the numbers in that game?"* We expect that students will realize that when we zoom in the number line, there is density of numbers in that number line.

The teacher can asks the students: "*Please rewrite in your work sheet, where is the position of*: 12,26; 89,89; 0,38, and 0,07. After that the teacher asks some students to write it on the blackboard and the other students correct it.

4. ***Reflection of the activities***

The teacher asks the students: "What *can you conclude about this activity?"* We expect that the students will feel for sure that between two consecutive whole numbers there are one digit decimals, and between two consecutive one digit decimals there are two digit decimals. We expect through this knowledge, it can support the students' learning toward decimals itself, especially when they are asked to compare one-and two-digit decimals. In the next activity, we will provide the contextual problem related to comparing the magnitude of one-and two-digit decimals.

Then, the teacher gives the other problems:

a. *Find the numbers which are smaller than 3 but larger than 1,4 as much as possible and put them on number line!*
b. *Find the numbers which are larger than 45,3 but smaller than 73,91 as much as possible and put them on number line!*

F. Activity 6: Comparing the Magnitude of One-digit and Two-digit decimals

Mathematical goal:

Students are able to compare the magnitude of one-digit and two-digit decimals

*1. **Review the Previous activities***

The students are asked to explain the mathematical ideas that they have already learned from activity 1 to activity 5 in which we expect that they will be able to master the idea of decimal is related to measurement (precise measurement), one-digit and two-digit decimals, and the idea of density of numbers on number line.

*2. **Activities***

In this activity, the students are experienced with solving the contextual problem related to comparing one-digit and two-digit decimals. This activity also has purpose to know the development of students' learning from activity 1 to activity 5.

The teacher gives the problem below:

a. *Andi wants to buy the milk in the market. There are two kinds of milk packaged in the bottle which have the same volume. Milk A contains 9,2 gram sugar, and milk B contains 9,17 gram sugar. What do you think? Explain your reasoning!*

b. *Ayuk goes to the market to buy the meat. There are 3 Kiosk which offer the same price for 1 pack of meat. The weight for each pack in each kiosk can be seen in below figure. Which one that should be bought by Ayuk? Explain your reasoning!*

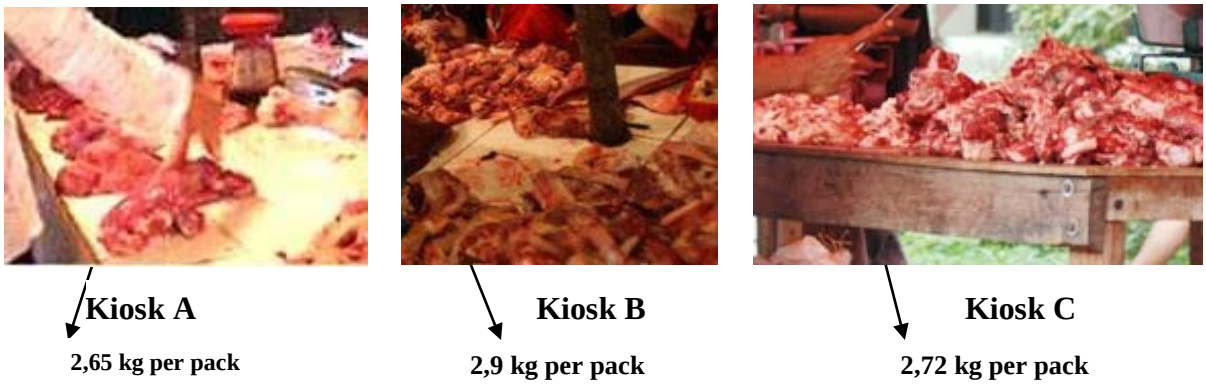


Figure 13: The problem in activity 6

c. *Order the following numbers from the smallest into the largest numbers:*

2,76; 2,5; 73,1; 56,89; 44,44; 89,9; 5,8; 89,89; 23,6; 2,25

3. ***Conjectures of students' thinking***

Problem 1

Some students will answer milk B is sweeter than milk A because they think that 17 is larger than 2 (whole number thinkers). However, they can also reflect to the previous activity that 0,33 in number line is in between 0,3 and 0,4. Therefore, they might think that 9,17 in number line is in between 9,1 and 9,2. Then, they will conclude that milk A is sweeter than milk B because 9,17 < 9,2.

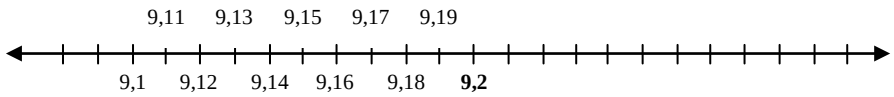


Figure 14: Number line.

Problem 2

- For students who give the correct answer, may be they have known about the magnitude of one-and two-digit decimals. Therefore they can compare that 2,9 > 2,72 > 2,65 because when they put those numbers in number line, then it should be:

Figure 15: Conjecture of making number line

- There is also possibility that the students who give the correct answer maybe they think when they have 2,9; 2,72; and 2,65 kilogram, then they convert those numbers into gram, so they will have 2900 gram, 2720 gram, and 2650 gram. It shows that the meat in Kios B is the heaviest one because 2900 > 2720 > 2650. They may show that using number line:

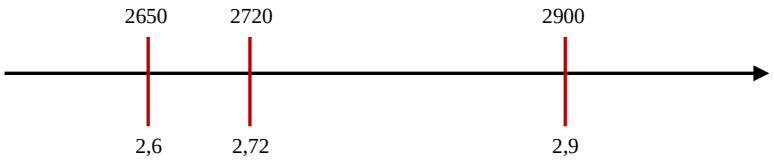


Figure 16: Conjecture of converting the unit of weight

So, they conclude that 2,9 > 2,72 > 2,65

The important thing here is that students can compare and place decimals in number line correctly.

- For students who don't give the correct answer, maybe they still think the comma numbers like in the whole numbers, 72 is larger than 65 and 9. For students who still don't understand about the concept of magnitude of decimals

at the end of activity, they should be given more attention in order to know what are their difficulties and in which activity they couldn't follow the instruction.

Problem 3

Finally we expect that the students can make connection between formal level (comparing magnitude of one-and two-digit decimals) and informal level (measurement activities reasoning).

The expected answer from the students is:

The order of decimals from smallest to largest is:

2,25; 2,5; 2,76; 5,8; 23,6; 44,44; 56,89; 73,1; 89,89; 89,9;

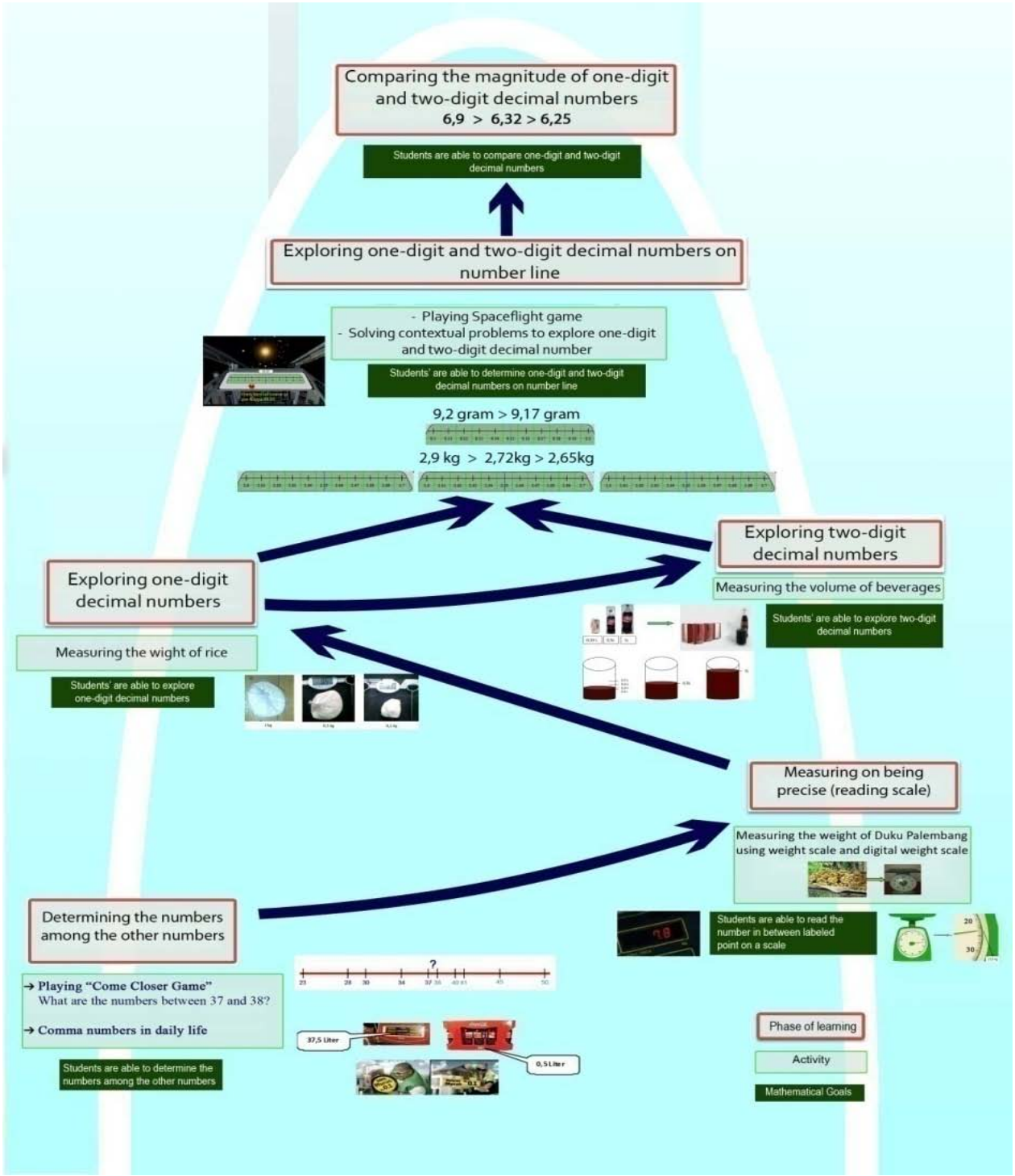


Figure 17: Visualization of initial HLT

CHAPTER 5
RETROSPECTIVE ANALYSIS

In this chapter, the retrospective analysis from pilot experiment and teaching experiment are discussed. The result of this research is the underlying principles explaining how and why this design works. The Hypothetical Learning Trajectory served as a guideline in the retrospective analysis to investigate students' learning of comparing the magnitude of decimals.

A. Pilot experiment

The pilot experiment was conducted in 4 steps: 1) pre assessment, interview, try out the activities, and post assessment. In this pilot experiment, the series of activities were tested in order to make adjustment for improved HLT in the teaching experiment.

*1. **Pre-assessment***

The pre-assessment is aimed to assess students' pre-knowledge toward decimals. The pre-assessment was given to the whole class (26 students in class 5A, 23 students in class 5B, and 24 students in class 5C) but only 7 students involved in the series of activities of pilot experiment. The decision of choosing small group because we expect to be more focused to the adjustment of HLT. We expected that choosing 7 students containing various students' abilities (2 high level students, 3 average level students, and 2 low level students) would represent the ability of the other students in whole class.

Based on the result of pre-assessment and interview, 7 students from class 5B were selected to follow the series of activities in pilot experiment. For the teaching experiment, the researcher chose different class. After discussed with the teacher, we chose class 5A as

the research subject for the teaching experiment because the ability in that class tends to average level.

In problem 1, the students were asked to tell about the pictures containing decimals. In this problem, most of them answered correctly and they recognized the decimal form, for example 19,5 is nineteen point five; 0,5 is zero point five; 0,1 is zero point one. They said that they had ever seen them in their daily life, either in television or in banner on the street. However, there were some students who still did not know about those numbers. They just said 0,5 as zero and five.

In problem 2, the students were asked to read the scale precisely. There were three pictures given to the students (see figure 18).

a b c

Figure 18: The problem in pre-assessment

For figure a, most students just mentioned the shown number (120) without considering the accuracy. For figure b, most students could not mention the number because they did not recognize what the picture was. For figure c, most students assumed that it was a clock. They assumed that it was 7.30. Actually, we expected that the answer is: *"The scale which is more than 7 but less than 8*." This is because its picture was designed by the researcher, not be taken from the actual weight scale. This suggests for the next step (teaching experiment), the pictures used should contain the situation which is familiar for Indonesian students.

In problem 3 the students were asked to compare two beverages in which beverage A contains 12,4 gram sugar and beverage B contains 12,17 gram sugar. Almost all of them

answered that beverage B was sweeter than beverage A because they assumed that 12,17 was larger than 12,4. From 73 students who joined the pre-assessment, there were only two students who gave the answer that beverage A was sweeter than beverage B (Sebastian and Zulfa). At first time, Sebastian gave the answer that 12,4 was more than 12,17 because $12{,}4 = 12\frac{4}{10}$ and $12{,}17 = 12\frac{17}{100}$. It seemed that he already knew about the relation between fractions and decimals. However, when the researcher interviewed him, he changed his mind. He said that beverage B was sweeter than beverage A because 12,17 was larger than 12,4. From that interview the researcher assumed that even though he knew that $12{,}4 = 12\frac{4}{10}$ and $12{,}17 = 12\frac{17}{100}$, but in fact he did not fully understand with that concept, and he could not imagine where the place of 12,4 and 12,17 were on the number line. Therefore, he suddenly changed his mind because he was not quite sure with it.

The interesting case also occurred with Zulfa's answer. He gave the answer that beverage A was sweeter than beverage B because he thought that gram = *garam* (in Bahasa, *garam*= salt).

Researcher : *"For the problem number 4, it was stated that beverage A contains 12,4 gram sugar; beverage B contains 12,17 gram sugar. Why did you say that beverage A was sweeter than beverage B?"*
Zulfa : *"Because beverage A was mixed with the sugar."*
Researcher : *"How about beverage B, it was mixed with the sugar or not?"*
Zulfa :*"Yes…but it was mixed also with the salt."*
Researcher : *"Why do you think that it was mixed with the salt? Which statement mentions it?"*
Zulfa : *"This one…" (*pointing out to the word 12,17 gram sugar*)*

Figure 19: Interview with Zulfa

Researc : *"How about beverage A, it was mixed with the salt also?"*
Zulfa : *"No.."*
Researcher : *"So, in your opinion gram means "garam" (salt)?"*
Zulfa : *"Yes"*

Researcher : *"Ok, how about this, if you have two numbers, 12,4 and 12,17, which is the larger than the others?"*
Zulfa : *"Twelve point seven teen"*
Researcher : *"Why?"*
Zulfa : *"Because twelve point seven teen is larger than twelve point four."*
Researcher : *"How do you know?"*
Zulfa :*"From the digit after point."*
Researcher : *"What happen with the number after point?"*
Zulfa : *"Because this number (*pointing out to the number after point in 12,17*) is larger than this number (*pointing out to the number after point in 12,4*)."*
Researcher : *"Which one do you mean?"*
Zulfa : *"17 is larger than 4"*

4. Andi ingin membeli minuman kaleng di sebuah toko. Di sana terdapat dua macam minuman kaleng yang mempunyai ukuran sama. Minuman kaleng A mengandung 12,4 gram gula dan minuman kaleng B mengandung 12,17 gram gula. Bagaimana menurut pendapatmu? Manakah yang lebih manis? Tuliskan alasanmu!

Figure 20: The question of pre-assessment number 4

The reason of Zulfa was really unpredictable. Apparently, the correct answer for Zulfa is not due to his knowledge of the magnitude of decimals, but about the language. Furthermore, he assumed beverage A did not contain the salt because in the problem (see figure 20), the word gram was in different line with 12,4.

The result of interview with Zulfa also becomes feedback for the teaching experiment. In the next step, we have to consider also about the language and the style of writing. We have to avoid the ambiguous word for the students and we have to convince that the word used is really meaningful for students.

After doing pre-assessment, the students played "Come Closer" game. The result of this game shows how far the students' pre-knowledge toward the numbers. It will determine the next stage of activities. For example, if until the end of the game, there is no student that realizes that there are numbers (decimals) in between two consecutive whole numbers, it gives feedback for us that the students are not quite familiar with decimals and their position on number line. It suggests that in the next activities (weighing activities) the

students should be more encouraged with the existing of decimals through discussing the form and how we call it (i.e 2,5 is two point five). However, if in the teaching experiment, there are some students who already know about decimal form and how they should call it, we don't need to discuss it anymore. We just focus into where the position of decimal forms in the measurement activities is.

2. Try out the activities

Seven students were selected to become the research subjects. They were Sebastian, Bintang, Puteri, Mita, Zulfa, Farhan, and Ari. This selection was based on the observation of the researcher, observers, and also the teacher of that class .We assumed that these students could represent the whole class containing high, average, and low level students.

a. Activity 1: Playing "Come Closer" Game

Mathematical goal:

Students are able to determine the numbers between the other numbers

1) Description of the Activity

As the initial of activity, the students were asked to count from 73 through 88. Then, they were asked to count from 99 through 75. Five of them raised their hand and mentioned the numbers fluently. This starting point has purpose to motivate the students in determining the numbers between the other numbers.

After that, the students started to play "Come Closer" game. This game was played by two groups containing three students for each group. They named their group as Melati and Scorpion. Besides that there was a jury which has the duty to judge the game. The rule of the game was that they have to suggest the numbers between two numbers decided by jury. One group suggested the numbers forward, and the other group suggested the numbers backward, but they were not allowed to cross each other.

In session I, the students suggested the numbers between 0 and 100 without putting the strategies in order to win. However, when the game was played in session II, Melati group put their strategies to beat their opponent. The jury asked them to suggest the numbers between 10 and 500. When the Scorpion group suggested 100, Melati directly suggested 99 (see figure 21).

Figure 21: Come Closer Game Activity

When the last position was in between 99 and 100, *Scorpion* group seemed to be confused suggesting the other numbers, because they just thought about whole numbers. Scorpion tried to suggest the other whole numbers, but the jury said that it was not allowed because those were in the opponent area. No one of them come to fractions or decimals. Finally, they concluded that Scorpion group was lost because they could not suggest the other numbers between 99 and 100.

2) Discussion

After both group played the game, the discussion was held between the researcher and the students.

Reseacher : *"Is there any other numbers between 99 and 100?"*
Puteri : *"No, there is no number between them."*

Researcher : *"Why?"*
Puteri : *"Because there is no other numbers which can be suggested."*

The other students seemed quiet when the researcher asked about it. Then the researcher asked the students one by one. Farhan, Zulfa, and Bintang also agree with Puteri. They said that there was no number between 99 and 100. However, Ari said that there is number between 99 and 100.

Researcher : *"What number is it?"*
Ari : *"98"* (showing that 98 was in the right of 99)
(Then, their friends said that it was not correct.)
Sebastian : *"I know the answer!"*
Researcher : *"What number is it?"*
Sebastian : *"90"* (When he showed the position of 90, he seemed confused. Finally he realized that 90 was not in between 99 and 100)
Researcher : *"Is there any other numbers between 99 and 100?"*
All students : *"No, there is no number between them."*
Researcher : *"Are you sure?"*
All students : *"Yes, for sure."*

Because all students thought that there was no number between them, then the researcher showed the poster of someone who filled the gasoline at the gas station. The volume of gasoline stated was 99,5 L.

Researcher : *"Do you know what kind of picture is this?"*
All students : *"Yes…it is gas station"*
Researcher : *"What is the number in this picture?"*
Zulfa : *"It is ninety nine point five liter."*
Researcher : *"What is ninety nine point five liter?"*
Farhan : *"The gasoline."*
Researcher : *"Ninety nine point five is the number or not?"*
All students : *"Yes, it is the number."*
Researcher : *"If we put ninety nine point five in that number line, where will you put it?"*

First, Ari draw 99,5 liter was in the position 99. However, their friend said that it was not correct. After that, Bintang corrected it. He put 99,5 was in the middle of 99 and 100.

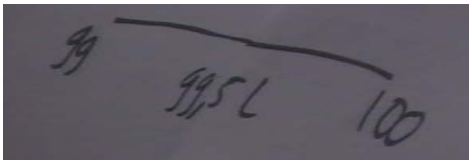


Figure 22: Bintang drew 99,5 in between 99 and 100

Finally all students realized that there was number between 99 and 100.

3) Reflection of Activity 1

In general, the result of activity 1 was suitable with the conjectures in Hypothetical Learning Trajectory. We observed that the students did not realize about the density of numbers on the number line in which decimals are in between two consecutive whole numbers. It seems that the students only knew decimals as other numbers beside whole numbers. At the end of activity, when the researcher showed the poster containing decimals, then finally the students realized that there were decimals in between two consecutive whole numbers. Indeed, this game can assess the pre-knowledge of students toward the numbers.

At the end of activity, the students were given the problems. One of them was in the following problem:

1. *In the sing competition 2010, it has already been announced that the first, second, and third winner are Ipin, Upin, and Ipan respectively. Based on the information, the score for Ipin is 8, and the score for Ipan is 7. What are the possibilities of Upin's score? Explain your reasoning!*

Figure 23: The end assessment of activity 1

Almost all students could determine that between two consecutive whole numbers there were decimals. For the problem 1, there were only Farhan and Ari that could not give the answer correctly. Farhan answered the score of Upin was 6. Ari answered the score of

Upin was 70. The other students answered 7,5. Sebastian answered there were many possibilities, i.e: 7,1; 7,2; 7,3: 7,4; 7,5; 7,6; 7,7; 7,8, and 7,9. The researcher interviewed Farhan and Ari. Farhan thought that Upin was the third winner, therefore he answered that the possible score for Upin was 6. Ari still thought about whole numbers for the possible score of Upin. She did not come into decimals as the numbers between two consecutive whole numbers. Therefore, the researcher concluded that on the next activity, these two students should be given more attention in order to be convinced about the existing of decimals.

b. Activity 2: Precise Measurement Activities

Mathematical goal:

Students are able to read the number between labelled points on a scale

In this activity the students were asked to measure the weight of body (using weight scale and digital weight scale) and measure the weight of the fruit (duku). Before the activity was begun, the students were asked to retell about the previous activity. All students could remember that between two consecutive whole numbers there were comma numbers (decimals).

1) Description of the Activity: Measuring the Weight of Body and Duku

This activity has purpose to know the ability of the students in measuring the things precisely and also to introduce the existing of decimals in the measurement activity. First, the students determined the weight as a whole number, for example, 34 for the weight of Ari. Then, the researcher asked further: *"Are you sure?"* Some students said that it was more than 34, and the other students said that it was less than 34. However, when they measured using digital weight scale, all of them could see that the weight of Ari was indeed 33,4 (see figure 25-28).

Figure 24: Measuring the weight of body

It was suitable with the conjectures in HLT that almost all students made the representation of weight scale as a model of situation (see figure 26-27).

Figure 25: Body weight scale

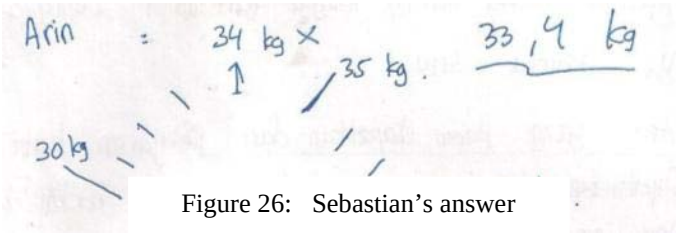


Figure 26: Sebastian's answer

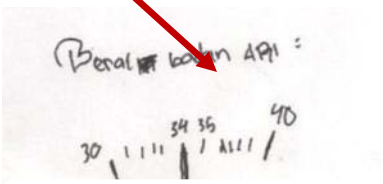


Figure 27: Farhan's answer

Figure 28: Body digital weight scale

After the students finished measuring their weights, then they were asked to measure the weight of some bunches of Duku (the original fruit from Palembang, Indonesia).

Researcher : "Which of you have bought Duku?"
(All students raised their hand)
Researcher : "How much that you usually buy?"
Sebastian : "1 kilogram"
Mita : "2 kilogram"
Researcher : "Only 2 kilogram?"
Zulfa : "2,6 kilogram"
Farhan : "2,5 kilogram"
Puteri : "2,9 kilogram"

At first time, the shown number on the weight scale was covered by white sticker, so the students did not know how much it was. For example, the scale pointed to the position in the middle of 0 and 1 (see figure 29).

Figure 29: Measuring the weight of Duku

Researcher : "How much it is?"
Zulfa : "One, two, three, four, five…" (Counting the lines on the weight scale)
Sebastian : "It is a half kilogram."
Zulfa : "Four kilogram"
Farhan : "Four point nine (4,9) kilogram"
Researcher : "How much this one?" (Pointing to label 0 kg)
Students : "Zero kilogram"
Researcher : "How about this?" (Pointing to label 1 kg)
Students : "One kilogram."
Researcher : "So, how much this one?" (Pointing to pointer point on the scale)
Sebastian : "A half kilogram"
Researcher : "How do you know?"
Sebastian : "Because it is in between 0 and 1."
Researcher : "Is there any other opinion?"
Sebastian : "Zero point five."

Researcher : "How do you know it is zero point five?"
Farhan : "Because there are one, two, three, four, five."
Researcher : "Can you show that to us?"
*Farhan : (*Counting the lines on the weight scale and said: *"one, two, three, four, five")*
Researcher : "Is there any different answer with zero point five?"
Zulfa : "No" (The other students were quiet)
Researcher : "Ok…we will proof it." (The researcher opened the covered sticker)

Finally, all students could see that the actual weight of that Duku was zero point five kilograms. They were happy because their answer was correct. The other students also did the same thing with different weight of Duku, such as: 0,6; 0,3; 0,4; 0,1; and 0,2.

Figure 30: Mita counted the lines on the weight scale to determine the weight of Duku

At the end of activity, the students were asked to explain about the result of the weight both using weight scale and digital weight scale. Most of them only estimated the weight using whole numbers (more or less). However, when they saw using digital weight scale, they realized that what they reported was not as precise as the actual weight. In digital weight scale they could see decimals. Therefore, the students concluded that one of advantages of using decimals was to make the result of measurement more precise.

2) Reflection of Activity 2

The problem in activity 2 was that the researcher could not find the fruit digital weight scale. The researcher just found the fruit weight scale with decimals (see figure 31). The

digital weight scale found only for weighing something heavy such as body weight. So, it was quite difficult if we weigh the fruit less than 1 kg. Sometimes the weight was not appeared. Therefore, the researcher occupied this problem by adding the activity with measurement the weight of the body in order to make the measurement of digital weight scale easier. At least, the students were recognized with the decimal form before they estimate the scale on the weight scale precisely.

Figure 31: The weight scale with decimals

In general, the result of activity 2 was suitable with the conjectures in Hypothetical Learning Trajectory. In measuring body weight, they estimated the weight (more or less) using weight scale, and then they determined precisely using decimals. In measuring the weight of duku, first they determined it was a half through observing the position in the middle between 0 and 1. Surprisingly, their reason of giving the answer 0,5 was because they counted the lines on the weight scale in which the total lines from 0 to 1 was 10. Therefore, they could find that the fifth line was 0,5, the sixth line was 0,6; the first line was 0,1; the third line was 0,3; the fourth line was 0,4; the second line was 0,2; etc. Finally

they could realize that the advantage of using decimals was to give the result of measurement as precise as possible.

c. Activity 3: Exploring One-Digit Decimals

Mathematical goal: Students are able to explore one-digit decimals and put them on the number line

1) Description of the activity: measuring the weight of rice

The students were asked to explain what they have learned in the previous activity. All of them could describe that they have found decimals in the weight measurement activity. In this activity, the students were asked to weigh the rice in order to explore the meaning of one-digit decimals.

Researcher : "When you or your mother bought the rice from the shop, how many that you usually bought?"
Sebastian : "1 big pack"
Farhan : "20 kilogram"
Zulfa : "10 kilogram"
Researcher : "How many kilograms that you usually eat for once?"
Farhan : "0,1 kg"
Sebastian :"0,5 kg"
Researcher : "Ok, now we will measure the rice which usually we eat."

First, Mita measured one pack of 0,5 kg rice.

Researcher : "Zero point five is how much of one kilogram?"
Sebastian : "It is five over ten, one over two."
Researcher : "How do you know?"
Sebastian : "From the equation zero point five is five over ten and then we simplified it become one over two."

It seemed that Sebastian already knew about converting fractions into decimals like he did in the pre-assessment. Therefore, the researcher asked the other students who probably still did not know. The researcher asked Zulfa. And it was true that Zulfa could not describe what zero point five is, even though Sebastian has already mentioned it. Beside Zulfa, the other students also could not describe it.

Therefore, the researcher took another pack of rice which has the same weight. Puteri measured again the weight of that rice. Finally, the students could find that two packs of 0,5 kg rice is 1 kilogram. However, when the researcher asked again: *"How do you know that it is a half kilogram?"*, Puteri was quiet for a while. Then, the researcher asked her to write it on the whiteboard (see figure 32).

Figure 32: Measuring the weight of rice

Researcher : "Why it is a half?"
Puteri : "Because the needle shown was on the number a half."
Researcher : "What do you think?" (The researcher asked the other students)
Farhan : "Because it was 0,5."
Then, the researcher emphasized that:
Researcher : "We have seen that if we measured one pack, the weight is how much?"
Students : "Zero point five kilogram"
Researcher : "We have seen that if we measured two packs, the weight is how much?"
Students : "One kilogram"
Researcher : "So, one point five is how much of one kilogram?"
Sebastian : "A half"
Researcher : "Why?"
Sebastian : "Because a half of one is zero point five." (There was no student who came up to the reason that because two packs are equal to 1 kg, therefore 1 bag is equal to a half).
Researcher : "Ok, if we have number line between 0 and 1, where will you put 0,5?"

Bintang drew the position of 0,5 is in the middle of 0 and 1, and he wrote $\frac{1}{2}$ is equal to $\frac{5}{10}$ (see figure 33). It means that Bintang already knew about the concept of equal fraction.

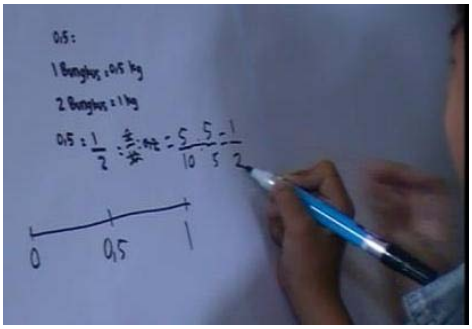


Figure 33: Bintang wrote 0,5= ½ = 5/10

Based on Bintang's answer, the other students followed that 0,5 is a half of 1 kilogram.

At that time, when we asked: *"How many packs of 0,5 kg in order to make 1 kg?"* All of them answered correctly: *"2 packs."* When we asked: *"So, what is your conclusion? 1 pack (0,5 kg) is how many?"* They answered: *"a half"*. However, when we asked: *"Why?"*, all of them were quiet. After discussion with the others, all students knew that 0,5 is a half, but there was no student who was able to give the reason. Based on the observation, before Bintang answered 0,5 is a half, he saw the position of the needle on the weight scale showing that 0,5 was exactly in the middle of 0 and 1. It seemed Bintang knew that when the position was exactly in the middle of something, then it should be a half. Actually, it was not enough as the concrete reason. We expected that the students give the reason from the situation (*1 bag is a half because 2 bags are equal to 1 kg*). We assumed that the students got difficulties in formulating the sentences and how to communicate the reason.

After that, the students were asked to measure ten packs of rice which have the same weight. First the rice measured was 0,1 kg. When two packs of 0,1 kg was measured, the scale became 0,2 kg. Three packs of 0,1 kg was 0,3 kg. The researcher asked Ari to write and predict the sequences of those same packed rice starting from 1 pack to 10 packs. She

wrote 1 pack was 0,1 kg; two packs was 0,2 kg; three packs was 0,3 kg; …, eight packs was 0,8 kg; nine packs was 0,9 kg; then ten packs was 0,10 kg (see figure 34).

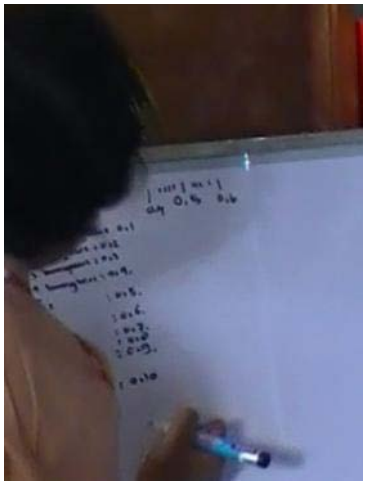


Figure 34: Ari wrote the sequence of one-digit decimals

When the researcher asked the other students: *"Do you agree with Ari?"* Some of them said: *"No, it should be 1 kg."* So, there were two answers: 0,1 and 1 kg. To prove which one was correct, the students measured ten packs of 0,1 kg, and the result indeed was 1 kg. Then, they concluded that 0,1 is one over ten (see figure 35).

Figure 35: Putri wrote 0,1= 1/10

After Ari realized that her answer was not correct, the researcher asked her:

Researcher : "Why do you think that 10 packs of 0,1 is 0,10 kg?"
Ari : "Because that rice was 10 packs, so I assumed that it should be 0,10 (kg)."
Researcher : "What do you think Bintang?"
Bintang : "It should be 1 (kg) because after 0,9 is 1, not 0,10."

At the end of activity, the students could determine the sequence of decimals between two consecutive whole numbers correctly (see figure 36)

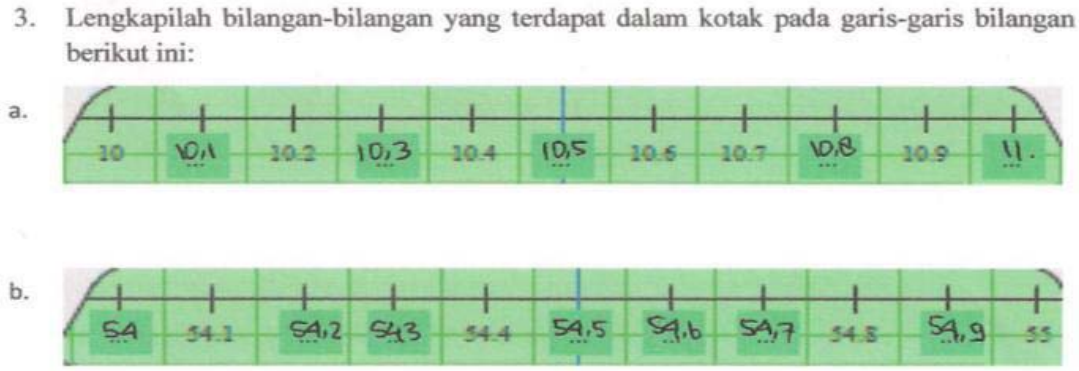


Figure 36: Puteri's answer

To strengthen that concept and to keep that in students' mind, the students sang "Decimal Song":

Figure 37: The students sing "Decimal Song"

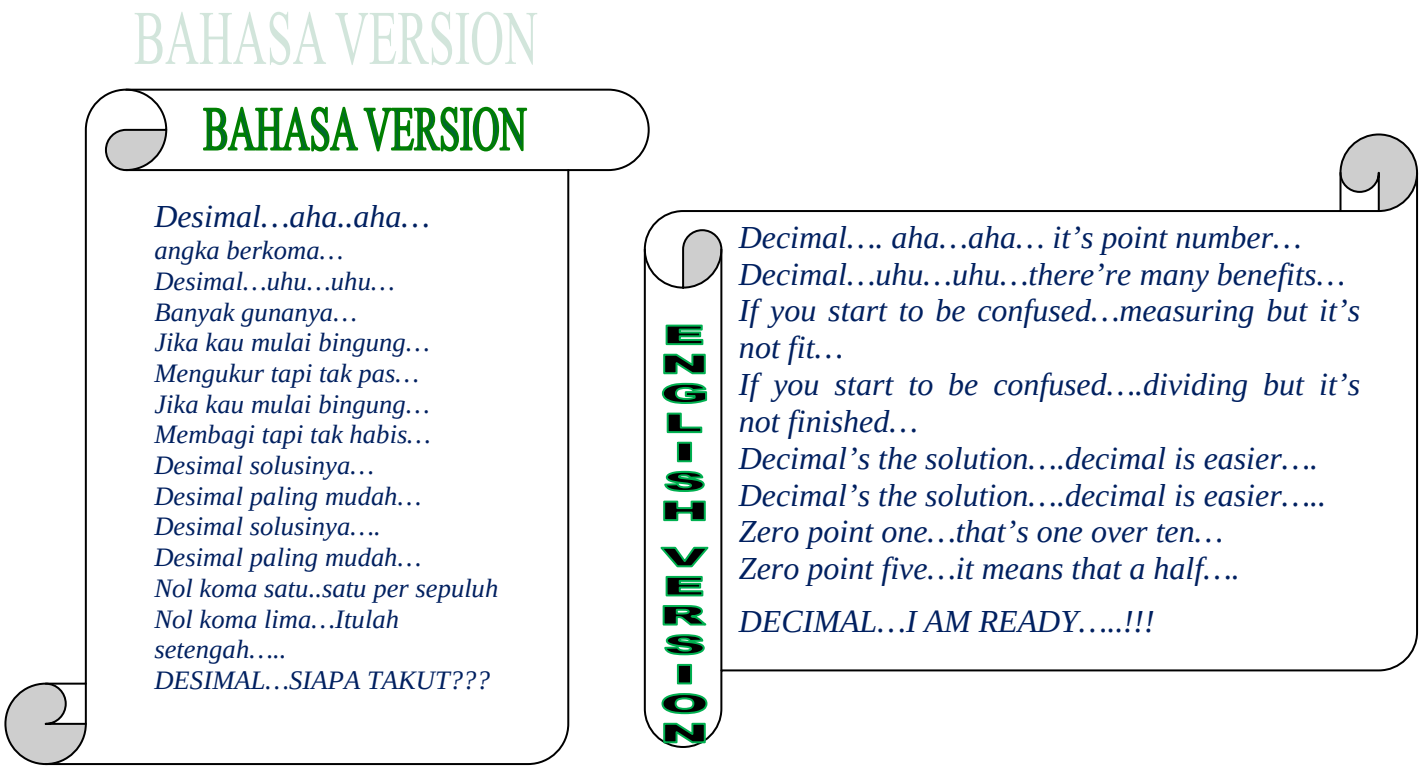


Figure 38: Decimal song text

(This song also can be found in http://www.youtube.com/watch?v=h30HoiU8tGw)

2) Reflection of activity 3

In this activity, the struggles of the students were giving the reason of why 0,5 is a half kilogram and why 0,1 is one over ten. The researcher thinks that it was not enough to convince the students that 0,5 is a half by observing two packs of 0,5 is 1, ten packs of 0,1 is 1. However, the strategy of Farhan and Mita gives the inspiration for the researcher. They determined 0,5 through counting the lines from 0 to the fifth line. Because they observed the needle was in between 0 and 1, consequently when it pointed to the first line, then it should be 0,1; the second line was 0,2; etc.

The students' difficulties in giving the reason of why 0,5 is a half suggest that in the teaching experiment, we have to strengthen the position of decimals on number line (see figure 39).

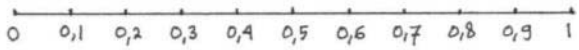


Figure 39: Decimals on Number line

After discussed with the teacher and the supervisors, to strengthen this concept we can make the relation between decimals and fractions, such as:

0,1 is in the first stripe from ten stripes overall (one out of ten) = $\frac{1}{10}$

0,5 is in the fifth stripe from ten stripes overall (five out of ten) = $\frac{5}{10}$

Consequently,

0,2 = $\frac{2}{10}$ because it is in the second stripe from ten stripes overall (two-tenth)

0,3 = $\frac{3}{10}$ because it is in the third stripe from ten stripes overall (three-tenth)

0,4 = $\frac{4}{10}$ because it is in the fourth stripe from ten stripes overall (four-tenth)

Therefore, it can be concluded that decimal refers to a base ten number that is written with "comma" number.

Our focus here is not about converting decimals into fractions and vice versa, but we tried to embed the concept of decimals using the strategy of partition base ten and their position on the number line, which, then, will become the basis for the idea of decimal fractions (for high level students).

Because the idea of partitioning base ten as the basis for decimal fractions is quite new, so as the preparation for teaching experiment, we tried to give the problem about converting decimals into fractions and vice versa. This extra problem has aim to see how far their ability in making relation between decimals and fractions. Some of the students were able to do that, but the others still did the mistakes.

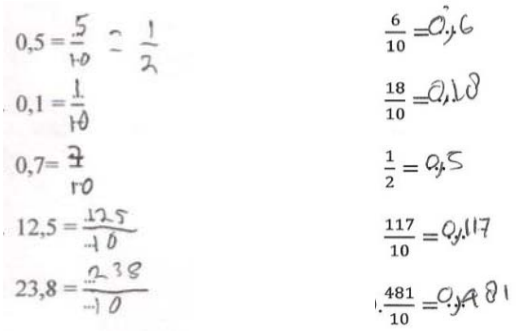


Figure 40: The example of students' work in making relation between decimals and fractions

d. Activity 4: Exploring Two-Digit Decimals

Mathematical goal:

Students are able to explore the notion of two-digit decimals and put them on the number line

1) Description of the Activity

From the previous activity, the students have already known about the notion of one digit decimals were in between two consecutive whole numbers and also they knew the meaning of simple decimals (0,5 is a half; 0,1 is a tenth). In this activity they were asked to explore the notion of two digit decimals.

Figure 41: The researcher showed the poster containing two-digit decimals

First, the students were asked about two different volume of beverages (*"Which is more, beverage A (0,25 liter) or beverage B (0,5 liter)?"* Farhan said 0,25 liter. Bintang, Mita, and Ari said 0,5 liter. Farhan assumed that the larger bottle was 0,25, and the smaller bottle was 0,5 liter. Then, they measured the volume of beverages in larger bottle and the smaller bottle in the measuring cup.

Figure 42: Measuring The Volume of Beverage Activity

However, when Mita measured the larger bottle, it was indeed 0,5 liter, and automatically the second bottle was 0,25 liter. Through observing the position of each height, the students could see that 0,5 is more than 0,25. Besides that, the students could see that 0,25 was in between 0,2 and 0,3 (see figure 43).

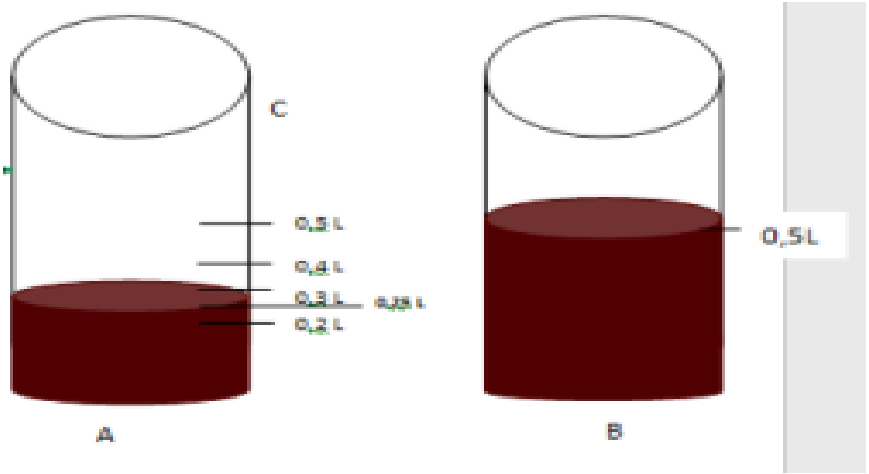


Figure 43: The Volume of 0,25 liter and 0,5 liter

(The detailed activity for the teaching experiment is explained on page 91).

From this activity, the students could see that 0,5 was more than 0,25. Then, interestingly Sebastian could find that 0,25 is a half of 0,5 because when he added 0,25+0,25= 0,50, and he said that zero after point did not contain the meaning. He crossed 0 after point in 1,0 (see figure 44).

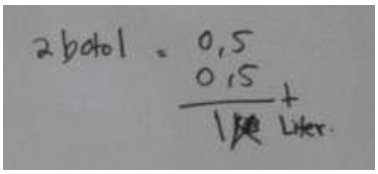


Figure 44: Sebastian' answer

It was also suitable with the conjectures in HLT that the students could find that 0,25 was in between 0,2 and 0,3 (see figure 45). In the previous activity, the students have experienced with measuring the rice and known the sequence of decimals starting from 0 kg until 1 kg, in which there were ten fragments i.e. (0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9, and 1. Therefore, Farhan divided the line became 10 fragments and he continued writing the sequence starting from 0,21 to 0,29. He concluded that between two consecutive whole numbers there were decimals, and between two consecutive one-digit decimals there were other decimals (two-digit decimals), etc. It seemed Farhan used the strategy of determining decimals refer to base ten (see figure 45).

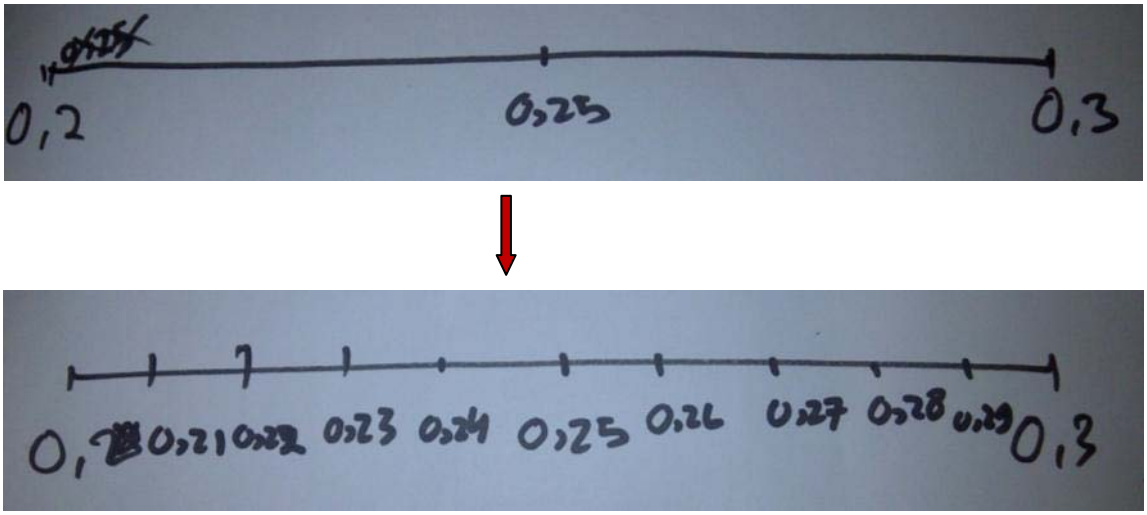


Figure 45: Farhan' answer

Because Sebastian gave the explanation that zero after *comma* (point) in decimals is meaningless, the researcher tried to ask further.

Researcher : "Why after 0,29 is 0,3?"
(The students were quiet for a while)
Researcher : "What number after 0,29 should be?"
Sebastian : "0,30"
Researcher : "Why this is 0,3 , not 0,30?"
Sebastian : "Because zero after point did not contain the meaning."
Researcher : "Can you explain it in front of the class?"
Sebastian : "In the last activity we can see that 1 bottle is 0,5; two bottles is 1,0. In decimals, zero after comma did not contain the meaning, so we can cross it. Same with this (he pointed out to figure 45), *0,2 here is same with 0,20.* (he added zero after 0,2). *So, we can cross the zero become 0,2* (he crossed the zero become 0,2).

Researcher : "You said that 0 after comma did not contain the meaning?"
Sebastian : "Yes…"
Researcher : "So, in your opinion 1,00 did not contain the meaning, isn't it?"
Sebastian : "Yes…"
Researcher : "How about the zero in 0,01, it contains the meaning or not?"
All students : "Yes, there is."
Researcher : "How about the zero in 0,1?"
Sebastian : "Yes, there is."
Researcher : "So, what do you think, which one is larger; 0,1 or 0,01?"
Sebastian : "0,1"
Some students : "0,01"
Researcher : "Why?"

(Sebastian seemed quiet for a while)
Researcher : "If I have a line, where will you put 0,1 and 0,01?"
(Sebastian seemed quiet for a while. Then the researcher provoked him.)
Researcher : "See the number line that Farhan drew! It contains how many fragment (lines)? Let's count together!"
All students : "One, two, three, four, five, six, seven, eight, nine, ten."
Researcher : "Now, how do you draw the number line to put 0,1 and 0,01?"
(First, Sebastian put 0,5 in the middle between 0 and 0,1.See figure 46)

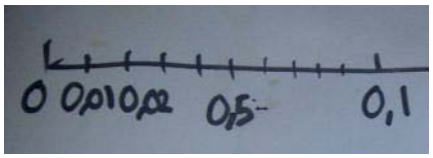


Figure 46: Sebastian's explanation

Researcher : "What do you think? Is it true that 0,5 is in between 0 and 0,1?"
Some students : "No."
Researcher : "If you see what Farhan did, where is the position of 0,5 and where is the position of 0,1?"
(Sebastian pointed that 0,5 was in the right position of 0,1)
Researcher : "So, what do you think, what number should be in the middle of 0 and 0,1?"
(Sebastian wrote 0,05, and after that he made the sequence consisting ten fragments starting from 0,01 to 0,09 in between 0 and 0,1 (see figure 47)

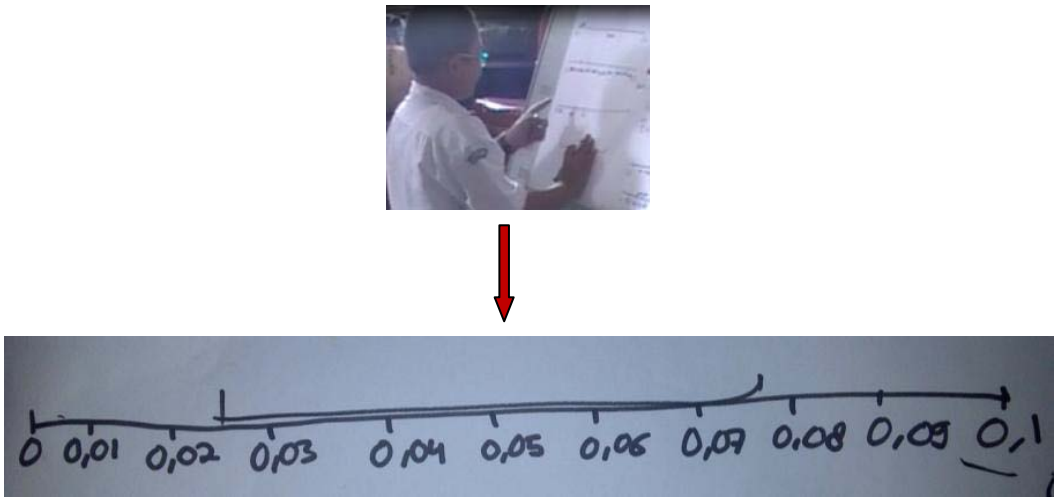


Figure 47: Sebastian's answer

We observed that at first time Sebastian got difficulties in comparing one-digit and two- igit decimals, such as 0,1 and 0,01. However, after he drew number line, he could see that two-digit decimals were in between one-digit decimals.

When the students were given the assessment at the end of activity, five of them (Sebastian, Bintang, Farhan, Zulfa, and Puteri) could put one-digit and two-digit decimals correctly (see figure 48). However, there were two students (Ari and Mita) who still have misconception about that (see figure 49).

The correct answer:

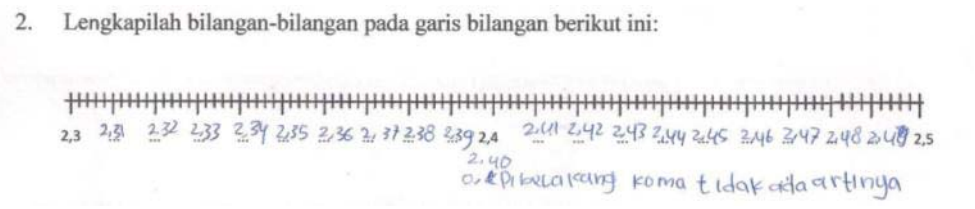


Figure 48: Farhan's answer

The incorrect answer:

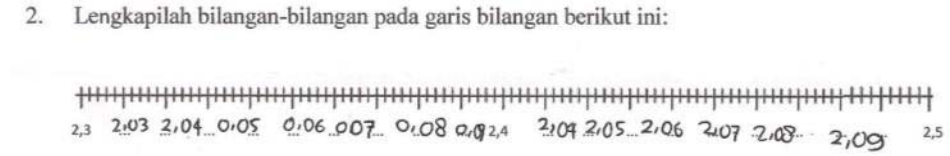


Figure 49: Mita's answer

For the students who gave the incorrect answer, we would give more attention in the next activity in order to encourage them about the idea of density of decimals.

2) Reflection of Activity 4

In order to make the students more understand the position of two digit decimals, in the teaching experiment, we would add the various volume of beverages, not only 0,5 and 0,25, but also the other dosages such as 0,35; 0,6; 0,45; etc. Therefore, they could see the position of two-digit decimals were in between one-digit decimals.

If the students still got difficulties, we would raise the discussion deeper about this. When the students still feel confused, we would give visualization of number line as an imitation of the stripes in the measuring cups (see figure 50). For example, in between 0,3 and 0,4 there are ten fragments (small lines) that make the sequence becomes 0,3; 0,31; 0,32; 0,33; 0,34; 0,35; 0,36; 0,37; 0,38; 0,39, and 0,4. We would let the students reinvent by themselves by provoking the question such as: "*What number that might come up in the stripes (lines) after 0,3?"* Through their experience of measuring the volume of beverages, we expect that the students would come up to the idea of two-digit decimals.

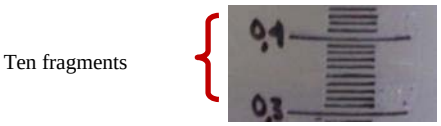


Figure 50: The number line

If it is necessary, we would provide the visualization that one-digit decimal number is "tenth" and two-digit decimal number is "hundredth" because if we zoom the range on the number line, there are ten fragments (lines) between two consecutive whole numbers, and if we zoom further two consecutive one-digit decimals, there are another ten fragments (lines). The model used is ruler (see figure 51).

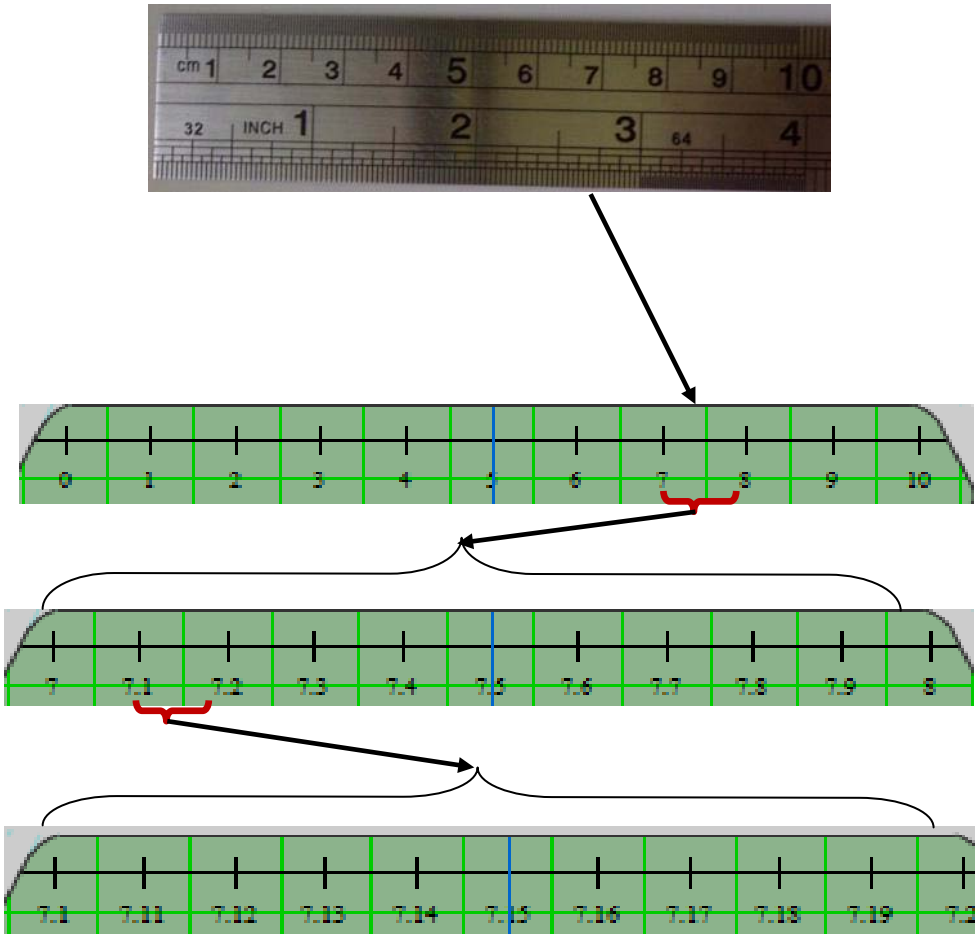


Figure 51: Density of Numbers on the Number Line

The issue of zero after point (*comma*) is meaningless was outside of the researcher's prediction. There was one student (Sebastian) who was able to give that reason. He raised the discussion among the students whereas the other students could see that 0,2 was the same with 0,20. Therefore when they put the numbers between 0,2 and 0,3, it should be started from 0,21; 0,22; 0,23...to 0,29. The number after 0,29 was 0,3 because 0,3 was the same with 0,30.

We asked Sebastian: *"How do you know that zero after comma did not contain the meaning?"* He said: *"I knew from my parents."* We asked the other students whether they agree or not with Sebastian's answer, but all of them were quiet. Finally Ari and Zulfa said that they agreed with Sebastian.

We realized that there were many aspects influenced the development of students' ability in understanding the mathematical concept. Sebastian seemed more advanced than the others because he studied at home with his parents before he studied certain topics in the school. We heard that he also follows the course in one College in Palembang.

Even though the issue of zero after point (*comma*) was not part of the HLT but this case is possible to occur among the discussion with students. Probably there would be a student like Sebastian who assumes that zero after comma is meaningless. However, we discussed with the teacher when this issue occurs in the teaching experiment, we would bring the students to come back to the concept of the position of one-digit and two-digit decimals on the number line.

For example, when students write the sequence after 0,19, some students might write 0,2; but the other students might write 0,20. Then, we will raise the discussion by asking the students: *"0,2 and 0,20 are similar or not?"* Some students might say that those were same, but the other students might say that those were different. Then, we will review activity 4 in which when they count the long lines (10 fragments of one-digit decimals), *"0,2 is in what stripe?"* We expect that the answer is: *"The second stripe from 10 stripes overall"*. It makes 2 out of 10 (0,2=$\frac{2}{10}$). At the same time, when the students count all stripes (including the small stripess), *"0,2 is in what stripe?"*. We expect that the answer is: *"The twentieth stripe from hundred strips overall."* It makes 20 out of 100 (0,20=$\frac{20}{100}$). So, the conclusion is 0,2=$\frac{2}{10} = \frac{20}{100} = 0{,}20$. As mentioned on page 66, converting the decimals into fractions and vice versa is not main focus on this research, but it would probaby occured for the high level students. The focus for average students is about the concept of decimals as the number base ten and their position on the number line.

e. Activity 5: Exploring One-digit and Two-digit Decimals on the Number Line

Mathematical goal:

Students are able to explore one-digit and two-digit decimals on the number line

1) Description of the Activity

In this activity, the students were asked to find the secret code by playing the game of Spaceflight game: (http://www.fi.uu.nl/toepassingen/03127/task3.html designed by Frans van Galen). They enjoyed it because they were quite familiar with playing the game like in the *Play Station*. To find the secret code, they have to pass four steps i.e finding the position of 12,26; 89,89; 0,38; and 0,07. The range given was in between 0 and 100.

The Spaceflight game has purpose to give visualization to the students that when they zoom the range between two consecutive whole numbers, there will be decimals. This software is good enough to provide the concept of density of numbers on the number line.

At first time, we predicted that one student could pass the four steps in five minutes. However, 4 from 7 students needed 10 minutes to play the game. They have difficulties in finding two-digit decimals in between whole numbers on the wide range. From 7 students, there was only Sebastian who was able to find the secret code. It means that he could pass four steps. Farhan could pass three steps. Bintang could pass two steps. Puteri, Mita, Ari, and Zulfa only could pass one step.

2) Reflection of the Activity

Based on the suggestion from the observers (Kurnia, Tari, and Anton), for the next step in the teaching experiment, the range of the first step should not be too wide (not between 0 and 100) but between the consecutive whole numbers (12 and 13). The level of difficulties of the game itself should be developed starting from the easiest problem to the

most difficult problem. The starting point gave influence for the efficiency of playing the game. Therefore, we will start the first problem by determining:

1) 12,26 in between 12 and 13,
2) 89,89 in between 80 and 90;
3) 0,38 in between 0 and 10;
4) 0,07 in between 0 and 100.

Figure 52: The students played Spaceflight game

At the end of activity, the students were asked to give the report of their game through putting the position of each step on the number line. Six of them put the numbers correctly (see figure 53).

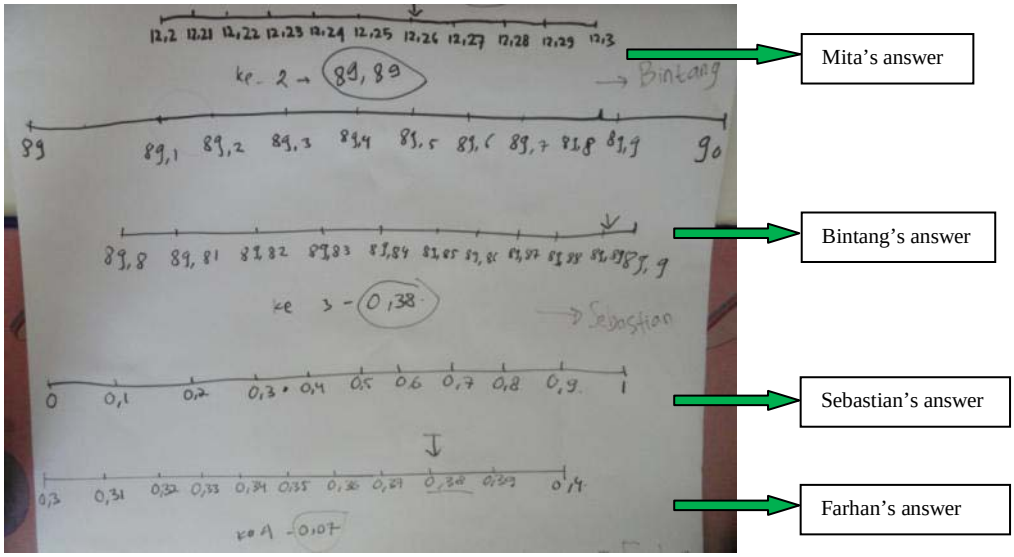


Figure 53: The result of activity 5

There was only Ari who still give the incorrect answer (see figure 54).

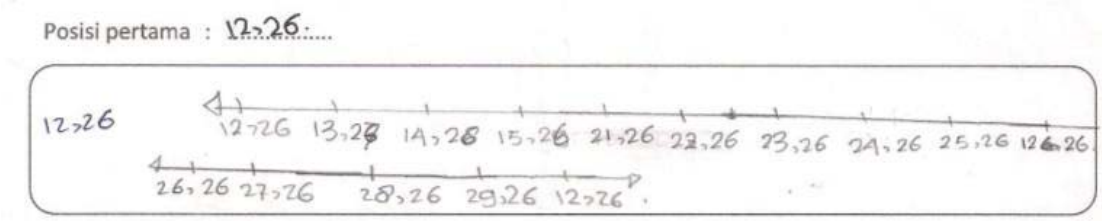


Figure 54: Ari's answer

We observed that in the mathematical congress of activity 5, the students could make the sequence of one-digit and two-digit decimals and put them on the number line (figure 53). They knew that between two consecutive whole numbers there were one-digit decimals, and between two consecutive one-digit decimals there were other decimals (two-digit decimals). It seemed that they now know about the concept of density of numbers on the number line. We expect that the students can use this knowledge to compare one-digit and two-digit decimals, and we can reduce the students' misconception about the longer digit the decimals, the larger their magnitude.

f. Activity 6: Comparing the Magnitude of One-digit and Two-digit Decimals

Mathematical goal:

Students are able to compare the magnitude of one-digit and two-digit decimals

In this activity, the students were asked to solve the following problem:

1. *Andi wants to buy the milk in the market. There are two kinds of milk packaged in the bottle which have the same volume. Milk A contains 9,2 gram sugar, and milk B contains 9,17 gram sugar. What do you think? Which is the sweeter milk? Explain your reasoning!*

2. *Ayuk goes to the market to buy the meat. There are 3 Kiosk which offer the same price for 1 pack of meat. The weight for each pack in each kiosk can be seen in below figure. Which one that should be bought by Ayuk? Explain your reasoning!*

At first time, the students solved the problem individually. For the problem 1, all students (Sebastian, Farhan, Bintang, Zulfa, Mita, Ari, and Puteri) gave the correct answer that beverage A was sweeter than beverage B because 12,4 was larger than 12,17 and they put them on the number line correctly (see figure 56).

For the problem 2, there were four students (Sebastian, Farhan, Bintang, and Puteri) who gave the correct answer and the correct reasoning that the meat in Kiosk B was the heaviest compared to the others. Ari gave the correct answer and gave the interesting reason. She chose Kiosk B because it was the cheapest compared to the others. The reason of Ari probably because she thought that she could get more meat for the same price, so it is cheapest (see figure 55). When in the teaching experiment this kind of reasoning appears, we will bring into class discussion.

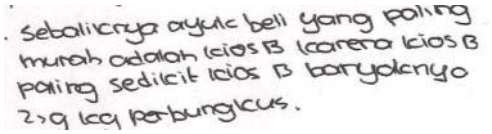


Figure 55: Ari's answer

Mita chose the meat in Kiosk A without giving the reason. Zulfa wrote that he chose Kiosk A because the meat in Kiosk A was the heaviest compared to the others. When the researcher asked him directly, he did not give the reason. However, based on the interview of pre-assessment (see page 49 on fragment line 25), probably he thought that 2,72 is larger than 2,9 because he thought that 72 is larger than 9. However, after the other friends (Ari, Mita, Puteri, and Sebastian) drew the numbers on the number line, Zulfa realized that his answer was not correct.

After the students finished solving the problem individually, the students presented their poster in front of the class. Puteri explained that beverage A was sweeter than beverage B because on the number line they could see that the position of 12,4 was on the right position of 12,17 (see figure 56).

Figure 56: Puteri showed the position of 12,17 and 12,4 on the number line

For the problem 2, Mita and Ari were asked to present their answers in front of the class. First, Mita seemed confused because on the worksheet she chose Kiosk A, but when she drew on the number line she realized that 2,9 > 2,72 > 2,65 because on the number line the position of 2,9 was on the right position of 2,65 and 2,72 (see figure 57).

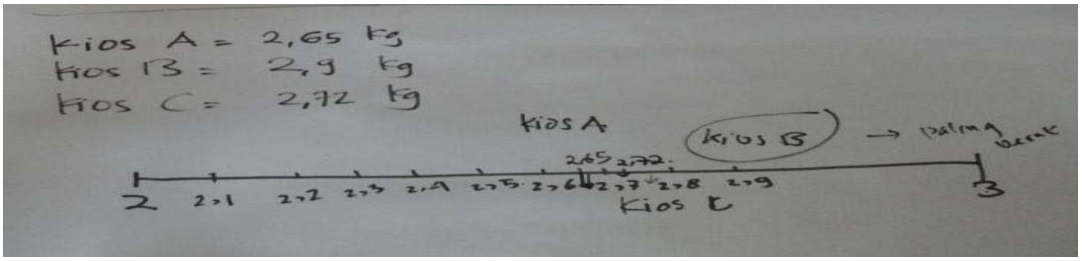


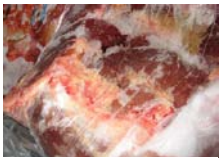
Figure 57: Mita drew the position of 2,65; 2,72. And 2,9 on the number line

3. ***Post Assessment***

The problem of post assessment contained 5 problems (attached in appendix). One of them is described on page 81.

a. The Problem

Atiqah goes to the market to buy the meat. There are 5 Kiosk which offer the same price for 1 pack of meat. The weight for each pack in each kiosk can be seen in below figure. Which one that should be bought by Ayuk? Explain your reasoning!



Kios A
6,5 kg/bungkus

Kios B
6,25 kg/bungkus

Kios C
6,32 kg/bungkus

Kios D
6,9 kg/bungkus

Kios E
6,89 kg/bungkus

Figure 58: The problem of post assessment in pilot experiment

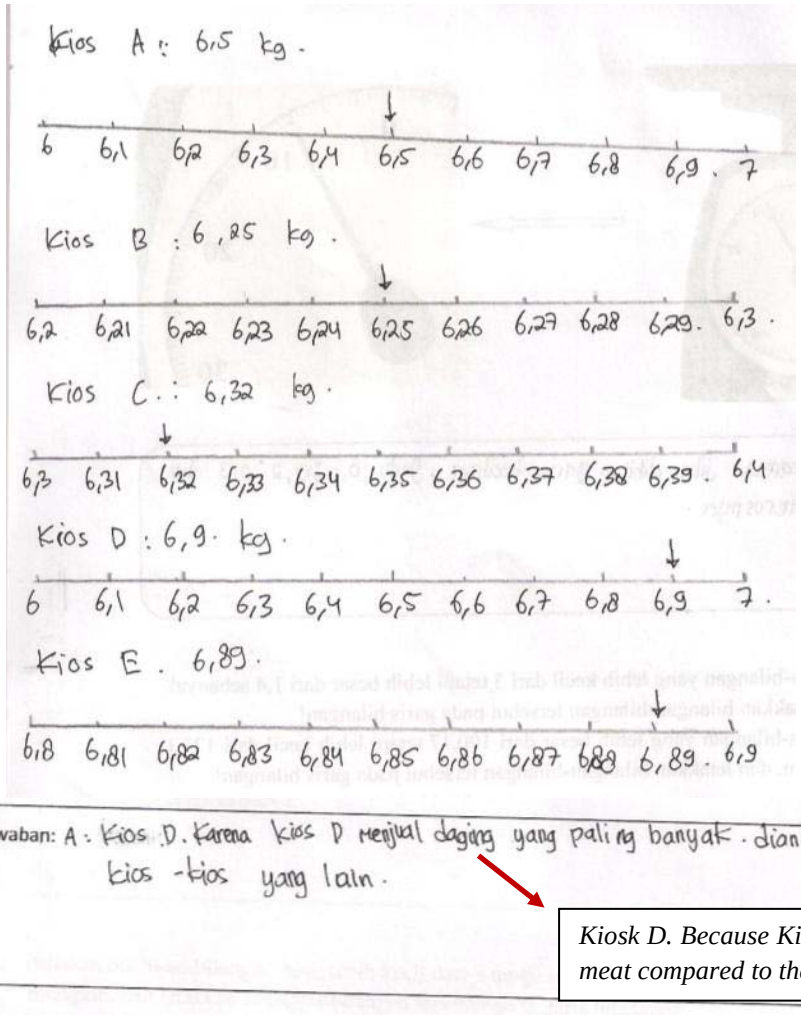


Figure 59: The result of end-assessment of Sebastian

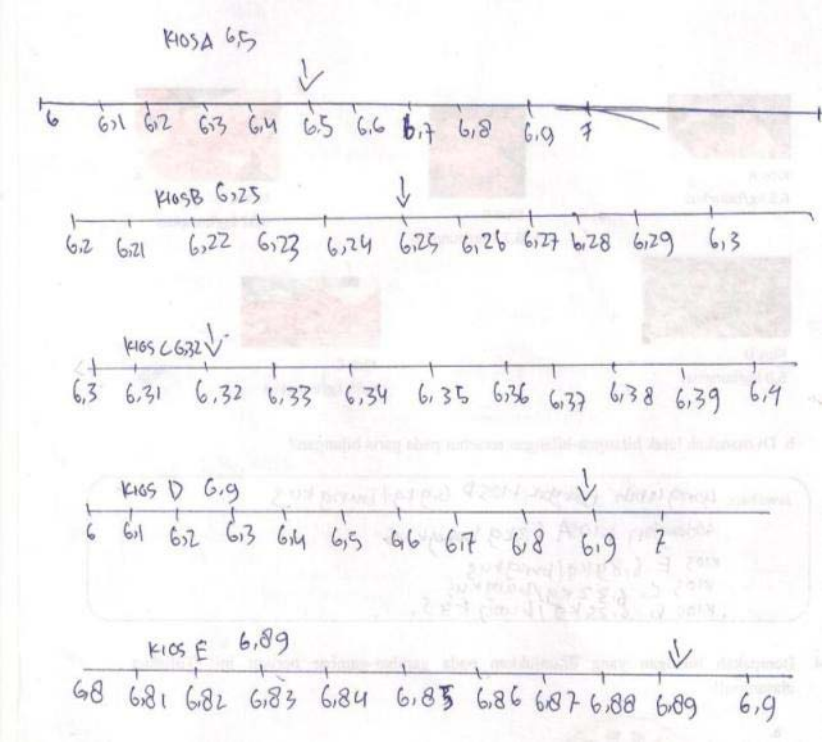


Figure 60: The result of end-assessment of Farhan

The progress from pre -assessment until the end of activity was good enough because all students (high, average, and low level students) could involve actively in the teaching and learning process. 5 out of 7 students (Sebastian, Zulfa, Farhan, Puteri, and Bintang) could determine and compare the magnitude of one-digit and two-digit decimals and they can put them on the number line correctly (see figure 59 and 60). Mita still chose Kiosk A without giving the reason. She just drew number line for Kiosk A. Ari also chose Kiosk A, and she drew the incorrect numbers on the number line (see figure 61).

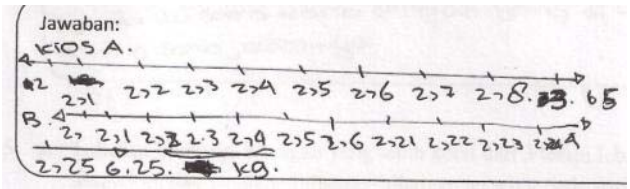


Figure 61: The result of end-assessment of Ari

At the end of post-assessment, the researcher also interviewed the students about the impression of the activities done by them (attached in appendix). Most of them said that

they like the activities and it was meaningful for them. When the researcher asked them to compare the magnitude of one-digit and two-digit decimals without relying on the number line, five of them (Sebastian, Farhan, Bintang, Zulfa, and Puteri) could answer and gave the reason correctly. Two of them (Mita and Ari) still thought that the longer digit the decimals was the larger for their magnitude. It was quite difficult to convince these two students about the magnitude of decimals even though in the series of activities they have experienced with discussion class and given the correct answer (see figure 57). However, in general they have the remarkable progress during the pilot experiment because these two students were brave enough to explain in front of the class even though their answer was still not correct.

b. General Conclusion of the Pilot Experiment Activities

1) Analysis

Before the activities were tried out, we observed that almost all students did not know about the concept of density numbers on the number line. They thought that there was no other number between two consecutive whole numbers. However, after they did the measurement activity, they could realize that there were decimals in between two consecutive whole numbers.

The struggles of the students were that when they weighed the things (body and fruit), some of them could not see the scale precisely. They just thought about the shown number in the scale without considering the accuracy. However, after they compared the result of measurement using digital weight scale, they realized that decimals exist in between two consecutive whole numbers and decimals were needed to measure the things precisely.

One student still thought that the sequence of decimals between 0 and 1 was 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; and 0,10. However, through measurement activity (weighing rice) she could see that ten packs of 0,1 was 1, not 0,10.

Most students still got difficulties giving the reason of why 0,5 is a half and 0,1 is one over ten although they had experienced through weighing activity in which two packs of 0,5 is 1, and ten packs of 0,1 is 1. When this struggle still appear in the teaching experiment, we would occupy them through strengthening the meaning of decimal number refers to base ten (tenth) (see page 88).

Most students still got difficulties in exploring two-digit decimals were in between one-digit decimals. However, through measuring the volume of beverage containing two-digit decimals (such as 0,25 liter), they could see that two-digit decimals were in between one-digit decimals through observing the height of those beverages in the measuring cups. In order to make students more understand, in the teaching experiment they would measure various volume, not only 0,25 liter but also 0,33 liter, 0,45 liter, etc.

Based on the observation in the series of activities and also based on students' works (see figure 33, 36, 45, 47, 48, 53, 56, 57, 59, and 60), almost all students (Sebastian, Farhan, Bintang, Zulfa, Mita, and Puteri) could determine and compare the magnitude of one-digit and two-digit decimals and they can put them on the number line correctly. However, when the researcher interviewed the students at the end of activities, there were only five out of seven students who could compare the magnitude one-digit and two-digit decimals correctly without drawing the number line. They gave the reason through imagining the position of their magnitude. Even though they did not draw the number line, they could determine that 2,9 was larger than 2,72 because on the number line 2,9 was on the right position of 2,72. Whereas 2,72 was in between 2,7 and 2,8 (see the result of

interview on appendix). The students could give the reason by comparing the position from left to right. Based on the literature that mentioned in chapter 2, the strategy of "left to right comparison" is used when comparing from left to right until a digit in one decimal is larger than the corresponding digit in the other. However, two out of seven students (Ari and Mita) still thought that the longer digit the decimals, the larger for their magnitude.

Beside the strategy of "left to right comparison", there were many new and interesting strategies that came up from the students. The issue of zero after comma which was stated by Sebastian also occurred. It means that he used the strategy of "equalizing with zero" (as mentioned also in chapter 2, theoretical framework, page 5). Besides that, Mita determined the sequences of decimals in between two consecutive whole numbers by counting the lines. Farhan determined it by dividing the range became ten fragments (base ten). We will try to find more about literatures that support these strategies.

We observed that through the series of activities 5 out of 7 students could compare the magnitude of one-digit and two-digit decimals because they used their experience of measurement activity (measuring the weight of fruit and rice) to apply the concept of the magnitude of decimals and they formulated their formal strategies using their approach of drawing number line, putting the position of decimals, and finally comparing their magnitude using their strategies (see figure see figure 33, 36, 45, 47, 48, 53, 56, 57, 59, and 60) and the fragment of interview in appendix).

2) About the conjectures

a. For activity 1, it was suitable with the conjectures that all students (Sebastian, Farhan, Bintang, Puteri, Mita, Zulfa, and Ari) said that there were no numbers between 99 and 100 (see the discussion between researcher and the students on page 52 and 53).

Through discussion the poster in daily life containing decimals, finally they realized that in fact there are numbers (decimals) between two consecutive whole numbers. However, showing poster seems that the students were pushed to know the existing of decimals. Therefore, in the teaching experiment, we will let students reinvent decimals by themselves (without showing the poster). We expect that through weighing scale context, they could find that there are decimals in between two consecutive whole numbers. If in the teaching experiment there are some students who already know about the notion of decimals in between two consecutive whole numbers, such as 99,5, 0,5; 0,1, etc, we will more emphasize to the discussion about how and where they place those numbers on number line.

b. For activity 2, it was suitable with conjectures in HLT because all students (Sebastian, Farhan, Bintang, Puteri, Mita, Zulfa, and Ari) made the representation of the weight scale through drawing the sequence of the stripes on the weight scale (see the example of students' drawing on figure 26 and 27, page 56). This conjecture is still kept for the revised HLT (teaching experiment).

c. For activity 3, at first time our conjecture is that the students will understand easily about what the meaning of 0,1; 0,5; are, etc. And we expected that they used the concrete situation (weighing the rice) as their reason to come up to the idea of 0,5 is a half and 0,1 is a-tenth. However, in fact even though they knew that 0,5 is a half and 0,1 is a-tenth but they got difficulties in giving the reason (see the discussion between the researcher and the students on page 61 on fragment line 13).

   Therefore, in the teaching experiment we would keep asking that question and encourage the students that 2 bags are equal to 1 kg, then 1 bag is equal to a half. But, if through that discussion the students still get difficulties to come up to that idea, then we

would support the students by discussing where the position of 0,1 and 0,5 on number line in which *0,1 is in the first stripe from 10 stripes overall 0,1* (one out of ten = $\frac{1}{10}$) and 0,5 is in the fifth stripe from ten stripes overall (five out of ten = $\frac{5}{10}$) (see the improved HLT on page 89). So, our new conjecture is that by encouraging the students that decimal refers to base ten and its position in certain stripe (line) from ten stripes overall, it will make students easier to understand the meaning and the magnitude of decimals.

d. For activity 4, our conjecture was that through measuring the volume of beverage (0,25 liter) in which 0,25 liter is in between 0,2 and 0,3 liter, the students could understand the position of two-digit decimals are in between one-digit decimals. In that activity the students (see Farhan's and Sebastian's answer on figure 45-47, page 70-71) could determine the position of two-digit decimals correctly. However, when they were doing exercise, there was student (see Mita's answer on figure 49 page 72) who still give incorrect answer. So, we will occupy that problem with adding various beverages, not only 0,25 liter but also 0,33 liter, 0,45 liter, etc., and also by supporting students' learning through discussing the position of those numbers on the number line which related to base-ten (a-tenth of a-tenth) (see the improved HLT on page 89).

   Our new conjecture is that by encouraging the students that decimal refers to base ten *(in between one-digit decimals, a-tenth, there were still another ten fragments, two-digit decimals)* and determining its position in certain stripe from hundred stripes overall, it will make students easier to understand the meaning and the magnitude of decimals.

e. For activity 5, our conjecture was that through playing spaceflight game, the students could imagine the idea that when they zoom the range between two consecutive whole numbers there were decimals. And it was suitable with the conjecture because this game is good enough to give the visualization of the idea of zooming-in. However, not all

students could determine the position of decimals correctly It means that we have to reduce the range become not too wide (see description of activity 5 on page 76). Our new conjecture is that by giving the smaller range and developing the level of difficulties of the game starting from the easiest through the difficulties problem (smaller range until larger range) will make the students more effective and challenged in exploring the game.

f. For activity 6, our conjecture is still the same with the preliminary design. We expect that the students will compare one-digit and two-digit decimals through knowing their position on the number line and using their knowledge that two-digit decimals are in between one-digit decimals. Our expectation in the teaching experiment is that the students' misconception (the longer digit digit the decimals, the larger their magnitude) will be reduced.

3) The Improved HLT for the teaching experiment

The pilot experiment showed that the series of activities were doable for students in grade 5. The first activity (playing come closer game) could assess the students' knowledge toward the numbers. The second activity could introduce the existing of decimals in the measurement activity. As mentioned on page 58, because the limitation of the visual aid, the researcher would use the body weight scale and body digital weight scale as the initial activity, and it would be continued by measuring the weight of the fruit using weight scale covered by sticker. The third activity could assist the students to explore one-digit decimals through finding that 0,5 is a half (five over ten) and 0,1 is one over ten through measuring the set packs of rice.

Based on the observation of the researcher and also the discussion with the observers (Kurnia, Tari, and Anton) and the teacher (Ibu Neti), there were some marks that would be modified to improve the HLT in the teaching experiment:

a. In the third activity (exploring one-digit decimals), when the students still got difficulties to give the reason of why 0,5 is a half and 0,1 we can support the students' learning toward decimals refers to the number base ten such in the following manner:

   0,1 is in the first stripe from ten stripes overall (one out of ten) = $\frac{1}{10}$

   0,5 is in the fifth stripe from ten stripes overall (five out of ten) = $\frac{5}{10}$

   Consequently,

   0,2 = $\frac{2}{10}$ because it is in the second stripe from ten stripes overall (two-tenth)

   0,3 = $\frac{3}{10}$ because it is in the third stripe from ten stripes overall (three-tenth)

   0,4 = $\frac{4}{10}$ because it is in the fourth stripe from ten stripes overall (four-tenth), etc.

   To provoke students, we can ask them: *"0,1 is in what line (stripe)?"* We expect that students will answer: *"It is in the first stripe"*. Then, we can ask further: *"From how many stripes overall?"* We expect that students will answer: *"From ten stripes"*.

   Therefore, it can be concluded that the word 'decimal' refers to a base ten number that is written with *comma* numbers (a decimal point) (Widjaja, 2008).

   The emphasizing here is not about converting decimals into fractions or vice versa but to help the students knowing the position of one-digit decimals are in the certain stripe from 10 stripes overall (on the number line).

   We believed that the encouragement of *"0,1 is in the first stripe from ten stripes overall (one out of ten or one over ten)"; "0,5 is in the fifth stripe from ten stripes overall (five out of ten or five over ten or a half)"*, etc. would be easier to be understood by students

which perhaps would lead them into the image of the position of decimals on the number line.

b. In the fourth activity, the students still have difficulties in determining two-digit decimals are in between two consecutive one-digit decimals. To occupy the students' difficulties in exploring two-digit decimals, the concept embedded will be the same with activity 3 (decimal refers to base ten number). The problem in the first cycle was that we could not find the measuring cup which was suitable with my design. Luckily, for the next teaching experiment we find 5 measuring cups in which there are ten fragments of one-digit decimals (between 0 and 1), and between two consecutive one-digit decimals there are another ten fragments (see figure 62). We expect that the students can discover that those are two-digit decimals.

Figure 62: Measuring cup

Our conjecture is that the students will draw number line (either in horizontal or vertical) as a model for reasoning the position of two-digit decimals.

'The same as in activity 3, to make students easier, we will provoke students by asking: *"0,25 is in what stripe?"* We expect that students will answer: *"It is in the 25th stripe"*. Then we can ask further: *"From how many stripes overall?"* We expect that the students will answer *"100 stripes"*.

Therefore, we expect that the students will conclude that 0,25 is in the 25th stripe from 100 stripes overall. 25 out of hundred is $\frac{25}{100}$.

Then, each group will *measure* various volume of beverages, such as 0,35 liter; 0,48 liter, etc. (there will be 5 measuring cup for five groups). We expect that they can see that 0,35 is in the 35th stripe from 100 stripes. When they convert into fraction, 35 out of 100 is $\frac{35}{100}$.

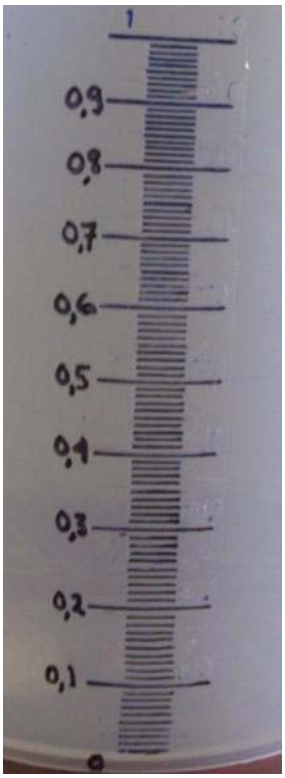


Figure 63: Measuring cup

The emphasizing here is not about converting decimals into fractions or vice versa but to help the students knowing the position of two-digit decimals are in between two consecutive one-digit decimals, and when they look its position that it is in certain stripe from 100 stripes overall.

That is why decimal is *called* "the number refers to base ten" because when there are two consecutive whole numbers and we divide into ten fragments, we can see that there were one-digit decimals. And when we divide the fragment of two consecutive one-digit decimals, there were another ten fragments of two-digit decimals, and so on.

For example:

In between 4 and 5, when those are divided become ten fragments, there are :

4; 4,1; 4,2; 4,3; 4,4; 4,5; 4,6; 4,7; 4,8; 4,9; 5

In between 4,1 and 4,2, when those are divided become another ten fragments there are:

4,1; 4,11; 4,12; 4,13; 4,14; 4,15; 4,16; 4,17; 4,18; 4,19; 4,2

And so on.

We will try to let students reinvent by themselves first. And we expect that the students will come up to this strategy (see Farhan and Sebastian's strategy on page 70 and 71).

c. In the fifth activity, the range of the Spaceflight game would be reduced in order to make the initial game more effective. Therefore, the students can continue to the next step easier (see description of activity 5 on page 77).

d. In the teaching experiment, the ability of students would vary. We would try to set the group discussion containing high, average, and low level students for each group. When there is a class discussion, we will try to give the opportunity to the students who still do not involve actively. Therefore, not only the high level students who dominate the class discussion. We have to make sure that the low level students really understand either in the discussion class or in solving the problem, not because they follow what the other students do. Besides that, to reflect the ability of students, there will be a written task at the end of each activity.

e. In general, there were not so much changed in HLT, but only some activities that will be modified (as mentioned on page 90-93). In order to support students' learning of the magnitude of one-digit and two-digit decimals, we will raise the discussion about decimal refers to base ten through discussing their position on the number line.

f. Based on the findings of this research, we revised the HLT and also formulated the goal in some activities.

The development of students' learning in the revised HLT is:

1) Using the knowledge about numbers

(Goal: students are able to determine the numbers among the other numbers)

2) Knowing the existing of decimal form through contextual situation (weighing duku and body);

   (Goal: Students are able to find one-digit decimals in the measurement activities)

3) Exploring the meaning of one-digit decimals through weighing rice;

   (Goal: Students are able to explore the notation of one-digit decimals and put them on the number line)

4) Using number line as a model for placing the position of one-digit decimals (partition based ten);

5) Exploring the meaning of two-digit decimals through measuring the volume of beverages;

   (Goal: Students are able to explore the notation of two-digit decimals and put them on the number line)

6) Using number line as a model for placing the position of two-digit decimals (partition based tenth of tenth);

7) Getting insight about density of numbers on the number line;

   (Students are able to get insight about the density of numbers on the number line)

8) Comparing one-digit and two-digit decimals using number line.

   (Students are able to compare one-digit and two-digit decimals using the number line).

B. Teaching Experiment

Generally, the retrospective analysis in the teaching experiment was conducted in a similar categorization to the pilot experiment. The analysis was elaborated in each stage of the development of students' learning, instead of in each activity. The analysis is aimed at explaining how students' learning of comparing the magnitude of one-digit and two-digit decimals was supported. Afterwards it could be generalized for the instructional design.

1. The Series of Activities

a. Using the Knowledge about Numbers

Mathematical goal:

Students are able to determine numbers between the other numbers

1) Description of the Activity: Come Closer Game

As mentioned in the pilot experiment, this activity has purpose to know how far the students' pre-knowledge about numbers and decimals. Two groups containing five students and one referee for each group played *Come Closer* game. Each group could play the game by suggesting the numbers between 0 and 100, either forward or backward. In the first five minutes there was no conflict. However, when the range was in between 43 and 44, the teacher asked the students: *"Is there any other number between 43 and 44?"* *Mango* group tried to nominate the other numbers. First Rico nominated 45 after 44, then the teacher provoked the students by asking the referee: *"45 is allowed or not?"* The referee said that it is not allowed because the number given should be in between 43 and 44. Then, Tiara suggested the number: "*40,5*" but she did not put it in between 43 and 44. She put 40,5 after 44 (see figure 64). Then the other students said that it was not correct without giving the reason.

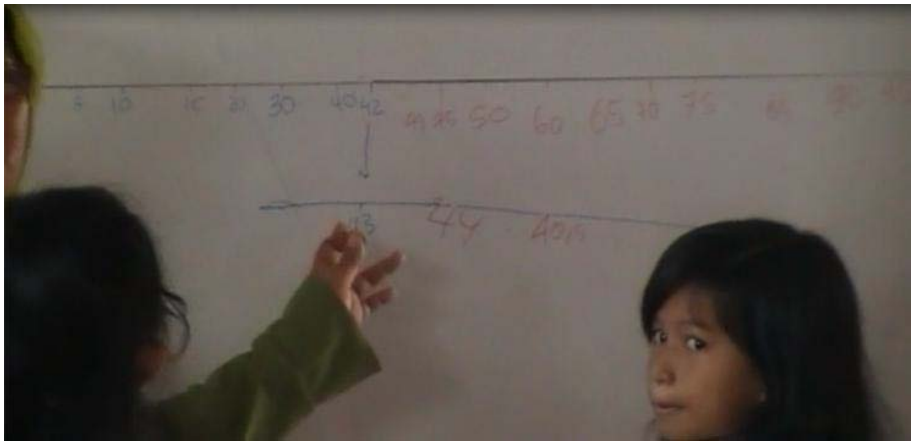


Figure 64: Tiara suggested 40,5 after 44

After that the students were asked to discuss within their groups whether or not there are still other numbers between 43 and 44.

Based on observations, some students assumed that there was no number between 43 and 44. During the students discussed within their group, the researcher interviewed one student, Michell. She also assumed that there is no number between 43 and 44 (see line 17 in the fragment below).

Researcher : "What do you think? Is there any number between 43 and 44?"
Michell : "Yes, there is."
Researcher : "What number is that?"
Michell : "45"
Researcher : "Are you sure that 45 is in between 43 and 44?"
(She was quiet for a while)
Researcher : "Please draw number line and put that numbers (43, 44, and 45) on that number line!"
(Michell drew number line, then she realized that 45 was not in between 43 and 44)
Researcher : "So, what number in between 43 and 44 was?"
Michell : "48"
Researcher : "Can you show me where the position of 48?"
(Michell pointed to the right position of 44)
Researcher : "Is it in between 43 and 44?"
Michell : "No"
Researcher : "So, is there any other number between 43 and 44?"
Michell : "No"
Researcher : "Are you sure?"
Michell : "Yes, for sure."

The interesting case also occurred with Ahmad. He said that there is number between 43 and 44, i.e. "2". After discussing with Cindy, he said that 43,5 is between 43 and 44. However, when he was asked to put it on the number line, he could not do that. He just added (,5) after 43 on the edge of the number line.

When the discussion was raised, most students said that 43,5 is between 43 and 44. The high level students lead the discussion among the group and finally all students agreed that there is 43,5 in between 43 and 44. However, when the teacher asked: *"Where is the position of 43,5?"*, based on the observation, some students were still struggling. When the researcher interviewed Cindy during the discussion with the group, she also could not describe the position of 43,5.

After the discussion was held in each group, Tiara solved the problem in front of the class. She drew and explained that 43,5 in the middle between 43 and 44.

Teacher : *"What kind of number is it?" (*the teacher pointed to 43,5 and asked the other students in the class)
Farhan : *"Decimal"*
Teacher : *"Decimal? What is that?"*
Farhan : *"The number containing comma."*
Teacher : *"Do you agree with Farhan?"*
(Some students said "yes", and the other students were quiet.)

Then, the teacher said that they will explore what kind of number it is in the next activity.

For the reflection of each activity, besides analysis of the learning process and students' thinking, the researcher also collected data through interviewing the teacher and an observer. This interview suggested the improvement for designing next activities which emphasized to the setting of group discussion.

2) Interview with the teacher (Ibu Neti):

Researcher : *"What is your opinion about the teaching and learning process that was held just now?"*
Teacher : *"In my opinion, I am quite satisfied with the result even though there were…still…some students who did not involve actively in the discussion, but we expect for the next activities they will be better than now. The reason of why I feel*

satisfied because mostly the students who at first time did not know about the existing of the other numbers (decimals) between two consecutive whole numbers, now they can learn and know about it."

Researcher : "So, is there any other suggestion in order to improve our design and teaching and learning processes for the next activities?"

Teacher : "I expect for the next activities all students can have the opportunities to try the game and to explain their argument in front of the class."

Researcher : "What do you think about the design of this lesson, is it doable or not?"

Teacher : "Yes, it is doable."

3) Interview with the observer (Pak Yoso):

After interviewed the teacher, the researcher interviewed Pak Yoso (the mathematics teacher in grade 4).

Researcher : "What is your opinion about the teaching and learning process that was held just now?"

Observer : "In general, it was good enough but when the group discussion was held, there were some students who still did not involve actively, give any ideas and share with the others."

Researcher : Is there any other suggestion in order to improve our design and teaching and learning processes for the next activities?"

Observer : "May be for the next activities, the discussion in the group has to be more passionate."

What Ibu Neti and Pak Yoso suggested (see line 4 in the first fragment and line 3 in the second fragment) was true because we also observed that there were only some students who involved actively in the group discussion. There was a problem which was not so important but gave influence to the effectiveness of group discussion. The set of groups was placed by the researcher and the teacher because we want to incorporate various students' ability containing high, middle, and low level students without considering the pleasure of the students to work with whom. As a consequence, the students who felt uncomfortable working with certain persons did not care with the teaching and learning process. In the next activities, we found a better way in order to stimulate the students in the group discussion i.e. by letting them to choose their own

partners in the group discussion, but we noticed first to set the high level student in each group (as the chairperson), and for the rest they could choose by themselves.

4) Analysis

In this activity, some students thought that there is no other numbers between two consecutive whole numbers, but some others did. In both cases, they did not know the position of those numbers on the number line. In accordance with our conjectures that some students would think that there is no other number between two consecutive whole numbers. The other students might already know the existing of decimal number in between them, but it did not guarantee that they understand its position on number line.

We can see the case occurred to Tiara. She started to suggest decimals on the number line, but unfortunately her answer was not correct. She wrote 40,5 after 44. It seems she knew the existing of decimal number but she could not imagine its position and its sequence on the number line. Similarly, Ahmad also said that 43,5 is between 43 and 44, but when he was asked its position on the number line, he could not do that. He just added (,5) at the number 43. The case of Ahmad occurred during the group discussion. It suggested that he has knowledge about decimal number but he did not know the position on the number line.

Even though at the end of activity Tiara could determine the position of 43,5 was in between 43 and 44 (in the middle), but it was through a long discussion with the other students. The finding of this teaching experiment informed that some students have already known about decimals but mostly they did not know where it was because when the teacher brought this problem into the discussion class, the students did not answer directly. They discussed with the others to make sure whether their answer was correct or not, and only some students who involve actively in the discussion class. This suggests that in the

teaching experiment we have to more emphasize to the position of decimal itself on the number line.

b. Contextual Situation (Knowing the Existing of Decimal Form through Weighing The Fruit, Duku)

Mathematical goal:

Students are able to find one-digit decimals on the measurement activities

1) Review the Previous Activity

In this activity the students were asked to measure the weight of Duku (using weight scale in which the pointer points were covered by sticker) and measure the weight of their body (using weight scale and digital weight scale).

Before the activity was begun, the students were asked to retell about the previous activity.

Teacher : "In the previous activity we had played the game, right? What kind of game?"
Students : "Come Closer Game"
Teacher : "Which group was lost?"
Rizkiah : "Mango group"
Teacher : "Why they were lost?"
Sapta : "Because they could not nominate the other numbers."
Teacher : "The number between what numbers?"
(Sapta was quiet for a while)
Rizkiah : "43 and 44."
Teacher : "What number should be in between them?"
Tiara : "43,5"
Dimas : "43,7"
Farhan :" 43,1; 43,8"
Rizkiah : "43,9….43,3; 43,4; 43,5; 43,6; 43,7,43,8, 43,9"
Teacher : "So, what we have learnt in the previous activity?"
Students : "Decimal number."
Teacher : "Decimal number?...What is decimal number?"
Dimas : "The number containing point (comma)."
*Teacher : "Where is decimal number (*based on come closer game*)?"*
Rizkiah : "In the middle."
Teacher : "In the middle of what?"
Tiara : "It is in between two consecutive whole numbers."
Teacher : "Yes, good…so in between two consecutive whole numbers there are….?"
Students : "Decimals."

In the previous activity, the discussion was held until the existing of 43,5 was in between 43 and 44. However, at the initial of this activity Farhan and Dimas mentioned the other numbers beside 43,5 i.e. 43,7; 43,1, and 43,8. And Rizkiah continued it by mentioning the sequence starting from 43,3 till 43,9. We observed that Rizkiah tried to nominate some numbers beside 43,5 and she used the approach of counting. Therefore, she was successful to mention some numbers beside 43,5 even though it has not yet brought into the discussion.

2) Measuring the weight of duku using weight scale and digital weight scale

This activity has purpose to know the ability of the students in measuring the things precisely and also to introduce the existing of decimals in the measurement activity. Each group was asked to weigh two packs of Duku containing different weights. Because the pointer points were still covered by sticker, then each group has to estimate them and gave the reason of why they answered the weight.

As an example, the teacher asked one volunteer student to weigh and to show in front of the class (see figure 65).

Figure 65. Weighing the fruit activity

Teacher : "How much is this?"
*Farhan *) : "A-half kilogram."*
Teacher : "Look that carefully, how much is this?"
Farhan : "A-half kilogram."
Teacher : "Is there any other opinion?"
Sella : "One over two."

Dimas : "Zero point five."
Teacher : "Zero point five? How do you know?"
*Dimas : "Because….because… (*he spoke smoothly*)…if we convert into decimal fraction, a-half is zero point five."*
Teacher : "Dimas said it is zero point five, hmmm…let's see! How do you think?"
(The teacher asked Farhan).
*(*Farhan counted and tagged the long lines in the weight scale and said synchronically with his tagging*: "One, two, three, four, five."*
Teacher : "Look at this…(the teacher asked the other students to pay attention to what Farhan did). *"He (*Farhan*) determined the weight by counting the lines and said one, two, three, four, five. Let us see the actual weight scale by opening the covered sticker!"(*Farhan opened the covered sticker, then he cheered and congratulated Dimas*)*
*): This is not Farhan in the first cycle.

After that, the other students were asked to weigh another pack of Duku. They worked in group containing five or six students. Four groups made the representation of the actual weight scale (see figure 66 as an example). However, there was one group who made number line as a model of situation on the weight scale (see figure 67).

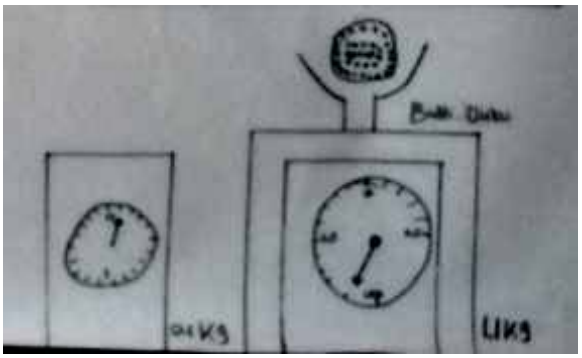


Figure 66: Representation of the weight scale

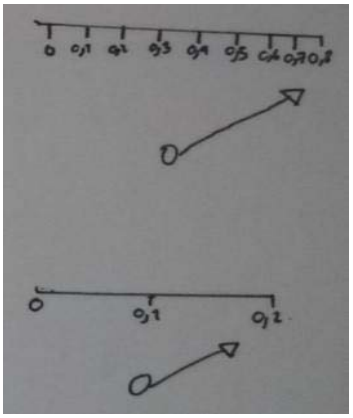


Figure 67: Number line produced by the students

When the students observed the weight scale that they measured, the teacher (Ibu Neti), the researcher, and the observers (Kurnia, Ibu Rahma, Pak Yoso, Ibu Mulyati, and Ibu Suryani) interviewed some students in each group about their findings. Interestingly, there was a conflict when "*Kelengkeng Group*" determined the weight of Duku that they measured (see figure 68).

Figure 68. The scale of 1,1 kg

Rico : *"This is one point five kg, right?" (*he asked to the other student in his group, Febi*)."This is one point five kg, right?....right?" (*he asked again Febi, but Febi still observed the weight scale*).* Rico convinced Febi by saying again: *"1,5 kg, right?"*
Febi : *"One….hmmm….One kg and one."*
Researcher : *"How much?"*
Rico : *"One point five kg"* (he answered for sure)
Febi : *"One kg and one"*
Researcher : *"So, how much?"*
Febi : *"One point one kg."*
Dimas : *"One point five kg."* (he said loudly)
Researcher : *"Which one was correct?"*
Dimas : *"One point five kg."*
Rico : *"One point five kg."*
Febi : *"One point one kg."*
Researcher : *"Which one was correct?"*
(Dimas thought for a while and said): *"One point five kg is here"* (he pointed the pointer 1,5 kg which was still covered by sticker, see figure 69)

Figure 69: Dimas's strategy

Then, Dimas suddenly changed his answer and said: *"This is one point one kg"*
The teacher asked Kelengkeng Group to repeat weighing that duku and asked:
Teacher : *"So, what is your answer?"*
Dimas : *"One point one kg." (*he answered for sure*)*

Teacher : *"How about the second duku?" (*The teacher asked that group to weigh another duku, see figure 70, to see what strategy that they used*)*
Dimas : *"Zero point one."*
Rico : *"Zero point one."*

Figure 70: The scale of 0,1 kg

Another interesting case also occurred with *Orange Group*. When duku measured was in figure 71, Fadhil tried to count the long lines one by one starting from zero, like the other groups did mostly. However, while Fadhil counted and tagged until 0,6 kg, Farhan cut him by saying:

Farhan : *"No Fadhil, we did not necessary to do that. We have known this (*Farhan replaced Fadhil's position and continued counting starting from 1 kg*), this is two (*he tagged to the first line from 1 kg*), three (*he tagged to the second line from 1 kg*), four (*he tagged to the first line from 1 kg*)…So, this is one point four kilogram"*
Researcher : *"So, how much?"*
(Farhan repeated counting and tagging the lines one by one starting from 1 kg and saying):
Farhan : *"One (*he tagged to the first line from 1 kg*), two (*he tagged to the second line from 1 kg*), three (*he tagged to the third line from 1 kg*)…One point three kilogram."*

Figure 71: Farhan's strategy

3) Analysis

Most students determined the weight by counting the long lines. We observed that they determined the first stripe as 0,1 kg, the sixth stripe as 0,6 kg, the eight stripe as 0,8 kg, etc. The conflict occurred to Kelengkeng Group was because Rico and Dimas assumed that each jumping line was 0,5 kg. It was different with Feby's strategy in which he already realized that there was 10 stripes from 0 to 1 and from 1 to 2, so when the needle show to the first stripe from 1 kg, then it should be 1,1 kg.

The misconception of Farhan's answer before he realized that the actual weight scale was one point three kilogram was because he started counting from the edge line and assumed that the line at 1 kg was one, then the next line as two, etc. It seemed that he did measurement toward the lines (not the space between the lines). However, when the researcher asked him to repeat it, then he started counting the space starting from the edge line. Finally he corrected his answer because he realized that when he did measurement, the thing that should be measured was the space between the lines, not only the lines.

After the students wrote their findings in the poster, then they were asked to present their poster in front of the class. While the group discussion goes along, the teacher asked the reason of their answer.

Rizkiah : "The weight of Duku which we measured was 0,3 kg."
Teacher : "Why did you say that it was 0,3 kg?"
Sella : "Because it was in the third line (stripe)."

Finally each group proved their answers by opening the covered sticker in the pointer points.

In this activity, some students made the representation of the weight scale as a model of situation, and some students already made number line as a model for placing the position of one-digit decimals (see figure 66 and 67).

4) Measuring the Weight of Body

Same as the pilot experiment (first cycle), the students were asked to weigh their body. They were divided became 5 groups containing five or six students in each group. One student was chosen as a representative to be weighed, and the other students observed it. We provided 5 weight scales which were set similar with each other and 1 digital weight scale. At first time the students weighed their body using weight scale, and then they reported the data including the situation of the weight scale they measured.

Figure 72: Body measurement activity

Table 4: The result of body measurement activity

Group	**Weight Scale**	**Digital Weight Scale**
Virgo	38 kg	38,4 kg
Orange	30 kg	29,1 kg
Kelengkeng	49 kg	48,1 kg
Mango	37 kg	35,1 kg
Strawberry	31 kg	30,2 kg

When they reported the data using weight scale, all groups reported it using whole number. The result of body measurement using weight scale and digital weight scale for each group were different. When they presented their findings, they said that it was because they did not carefully in observing the weight scale.

From the findings of the groups, there was only Mango group who has the result of measurement which was quite far between weight scale and digital weight scale, i.e. 37 kg and 35,1 kg. After we analyzed from the video, this is because they looked the weight scale from right side position, and it influenced the result because the result might be different when we looked the weight scale from the right/left side and when we looked from the straight side.

Figure 73: Mango group in the body measurement activity

5) Analysis

In body measurement activity using weight scale, the students still reported the data using whole numbers even though the result was not exact in the line at whole number (see figure 72). When the teacher interviewed each group in the group presentation about why the result was different (between weight scale and digital weight scale), most students answered that it was because they did not look that carefully. However, based on our observation, the preciseness was not only the cause which determined the correct or incorrect result. There were many aspects influenced it such as the position of standing when the students observed the result of body measurement (see figure 73), and probably because the distance of the scale was too small (based on the discussion between the

researcher and the teacher). It means that the students still not master about the idea of zooming-in. Actually, when the range between the scale on the weight scale is zoom out, e.g between 38 and 39 in figure 72, it should be there are other small lines (decimals) like in the fruit digital weight scale. As a starting point, it is good that the students still not realized about the idea of zooming-in because in the next activity (fourth and fifth activities) they will be introduced about that concept through measuring the volume of beverages and playing Spaceflight game.

Because the goal of this activity was to introduce the existing of decimals in the measurement activity, this is enough for them to know about decimals toward opening the covered sticker in the fruit digital weight scale and comparing the weight of the body using weight scale and digital weight scale.

c. Exploring the Meaning of One-digit Decimals through Weighing Rice

Mathematical goal: Students are able to explore one-digit decimals and put them on the number line

1) Review the Previous Activity

The students were asked to explain what they have learned in the previous activity. Cindy mentioned that in the previous activity they did weight measurement activity (fruit and body). She continued that Duku measured were 1,3 kg and 0,5 kg. Then Fadhil continued that the result of body measurement activity using weight scale and digital weight scale were different (i.e 30 kg and 29,1 kg) because they did not look it carefully.

Because one of the result of weighing activity that Cindy mentioned was 0,5 kg, then the teacher provoked the students by asking: *"What is the meaning of 0,5 kg?"* Dimas

answered: "*It is a-half*". However, when the teacher asked him about his reason, he was just quiet. We observed that he was able to mention that 0,5 is a half because in the previous activity, he could find the other around when Farhan answered weighed Duku was a-half, he was the only one who answered that the other form of a-half (one over two) was zero point five and he mentioned that zero point five was decimal fraction (see page 102).

This is suitable with our conjectures that the students still did not know about the meaning of zero point five. They just know about the form of decimals (0,5; 0,6; 29,1, etc) toward the weight scale and digital weight scale. Therefore, in this activity they will be asked to explore about the meaning of one-digit decimals toward weighing the rice containing some packs of 0,5 kg and 0,1 kg.

2) Measuring the Weight of the Rice

The teacher started the activity by connecting one-digit decimals with daily life situation.

Teacher : "In the previous activity, we have already discussed that what is the advantage of eating the fruit?"
Students : "In order to be health"
Teacher : "Ok…so what is the main food for us?"
Students : "Rice"
Teacher : "How many kilograms that your mother usually buy?"
Students : "1 kg….2 kg…..1 big pack….3 kg…"
Farhan : "A-half kilogram."
Teacher : "Hmmm…there were many various weights of the rice, now I asked…how many kilograms that your mother usually cook for once?"
Rico : "Three glasses."
Muti : "Five glasses."
Farhan : "Two kilograms."
Teacher : "Yes..it has to be matched with the number of family, isn't it? How if in one family it contains only a few persons. How much that we must cook? Can you estimate them?"
Tiara : "A-half kilogram or 0,5 kg."
Teacher : "Tiara said that a-half kilogram was the same with 0,5 kg. Let's we see whether Tiara's answer was correct or not through measuring these rice directly."

First, Intan measured one pack of 0,5 kg rice but the pointer points were still covered. This has purpose to assess whether all students really understand the previous activity or not. When the weighed rice was in the middle between 0 and 1 (the fifth stripe of 10 stripes between 0 and 1), the teacher asked Intan how much is that. However, Intan was quiet, and the other students said: *"zero point five."* Then, the teacher asked the other students (Rizkiah and Muti) to weigh two packs of rice together (in which each pack was 0,5 kg).

Figure 74: Rizkiah and Muti weighed two packs of 0,5 kg

Teacher : *"How much is this?"* (The teacher pointed to two packs of rice measured)
Students : *"1 kg."*
Teacher : *"So, how much for one pack?"*
Students : *"A half…..zero point five…"*
Teacher : *"How about two packs?"*
Students : *"1 kg"*
Teacher : *"So, what can you conclude? "What is the meaning of 0,5? 0,5 is how much of 1?"*
Farhan : *"Pardon Mom…0,5 is what?"*
Teacher : *"0,5 is how much of 1?"*
Firman : *"A half"*
Teacher : *Which of you can explain your reason?"*

Three students wrote on the blackboard in three different strategies. Dimas drew number line and placed 0,5 in the middle between 0 and 1 (see figure 75). Sella wrote 0,5 was equal to ½ (see figure 76). And Rico used the result of measurement activity as his reason (see figure 77).

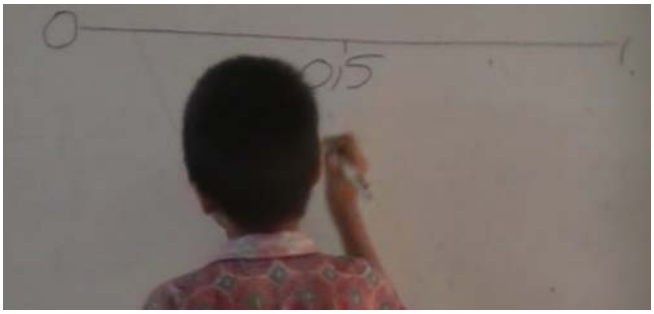


Figure 75: Dimas drew number line

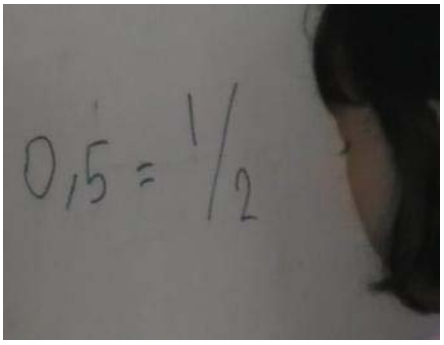
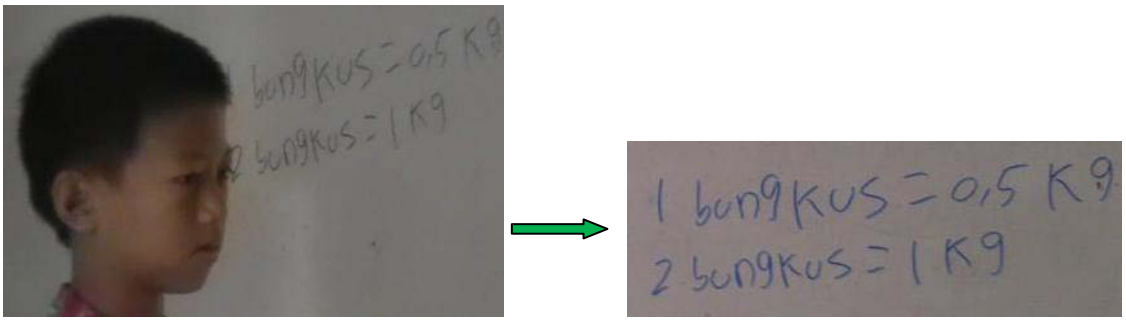


Figure 76: Sella made relation between decimal and fraction



Note: *Bungkus* means pack

Figure 77: Rico used the result of measurement activity as his reason

Rico : *"We have seen that when we weighed one pack, the weight is 0,5 kg, when we added another same pack (0,5 kg), the weight became 1 kg. So, 0,5 kg plus 0,5 kg is 1 kg."*
Teacher : *"What do you think? Is there any different answers with Rico?"*
Students : *"Same.."*

To come up to the relation of the meaning of one-digit decimals, the students were asked to weigh another pack of rice. The teacher provides some packs of rice in which there were 10 packs of 0,1 kg rice.

In order to make connection with daily life situation the teacher asked the students:

Teacher : *"We have weighed the pack of rice which usually we cook in one day (0,5 kg). Now, I ask how much the rice that you usually cook for eat once?"*
Dimas : *"0,3 kg"*
Teacher : *"It could be…is there any other weight?"*
Fifi : *"0,2 kg"*
Teacher : *"It could be…"*
Muti : *"0,1 kg"*

Teacher : "It also could be…"
(There were various weights. All of them were less than 0,5 kg.)

Then, the teacher asked one volunteer student, Kresna, to weigh those packs of rice. Kresna started to weigh one pack. He estimated the weight, then he opened the covered sticker. The first pack weighed was 0,1 kg. And then, when he weighed two packs, the weight was 0,2 kg. Three packs was 0,3 kg. Then, the teacher asked Kresna to write it on the blackboard.

After Kresna reported the three packs of 0,1 kg was 0,3; then the teacher asked him to estimate the weight of four packs until 10 packs of 0,1 kg rice. It was suitable with the conjecture that Kresna made the sequence of its weight using counting approach. After he wrote 9 packs of 0,1 kg was 0,9 kg, then he wrote 10 packs of 0,1 kg was 0,10 kg (see figure 78).

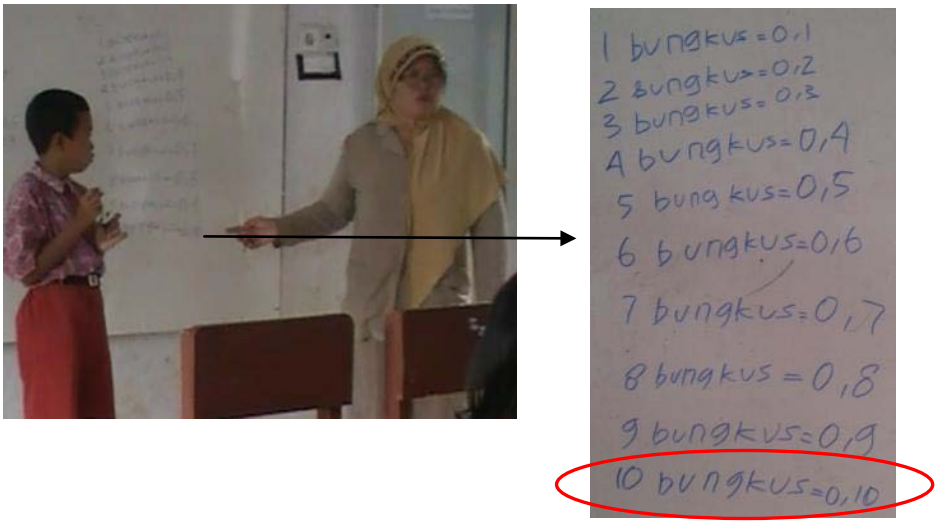


Figure 78: Kresna wrote 10 packs of 0,1 kg rice was 0,10 kg

Teacher : "This is Kresna's answer, how do you think?"
Arissandy : "It was incorrect."
Teacher : "Why?"
Arissandy : "It should be 1 kilogram."
Teacher : "Is there any other opinion?"
(Some students were silent, the others said one kilogram).
Teacher :"We cannot say that it was correct or not before we proof it through measuring the rice directly."

The teacher asked Kresna to weigh the other packs, starting from 4 packs till 10 packs, and the result was 1 kg. The teacher asked the students: *"Which of you can write and draw the situation of what we found (weighing measurement activity)?"* Arrahman drew the sequence of decimals starting from 0 till 1 (see figure 79):

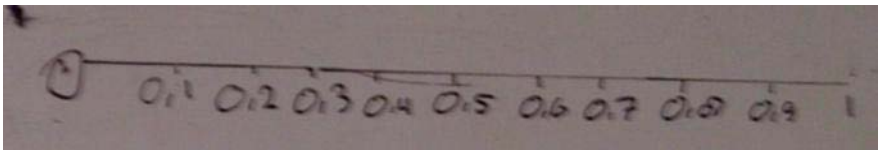


Figure 79: Arrahman drew number line to place the sequence of decimals between 0 and 1

Because Arrahman already came to number line, then the teacher asked the students:

Teacher : "Where is the position of 0,1? At what stripe?"
Some students : "At the second stripe"
Some students : "At the first stripe".
Teacher : "We have already determined those decimals between 0 and 1, so 0,1 is in what stripe?"
Rico :"The first stripe"
Teacher : "From how many stripes overall?

Febi counted and tagged the numbers starting from 0,1 as the first stripe, and he showed in front of the class. Finally he could find that the stripes overall was 10.

Teacher : "1 out of 10 is how much?"
Some students : "one over ten"
Teacher : "Explain! 0,1 is one over ten!"

Farhan bold the number $\frac{1}{10}$ which was written by him. He thought that the word "explain" here meant that his writing should be more clear because in *Bahasa*, explain = *jelaskan*, and the meaning of *jelaskan* can be "*explain the reason"* and also can be "*make it more clear"*. The teacher repeated her question.

Teacher : "Explain the reason of why 0,1 is one over ten!"
Farhan : "Because it is in the first position from ten overall."
Teacher : "Ok…good…how about 0,5?" Where is that position?"

Firman explained in front of the class that 0,5 was in the fifth stripe from ten stripes overall. Therefore, it can be written as $\frac{5}{10}$ which also can be simplified became $\frac{1}{2}$ (see figure 80).

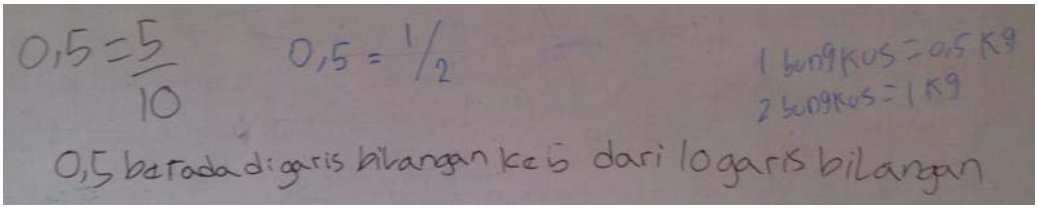


Figure 80: Firman gave the reason of why 0,5=1/2 through determine its position on the number line (base ten).

3) Analysis

Actually, the reason of why we began the activity using 0,5 kg (not 0,1 kg) is to assess whether the students can use their concrete understanding toward the context of weighing the rice or not, because in the pilot experiment, there was no student who could use it as their reason. 0,5 as a half is more familiar in daily life rather than 0,1 as one over ten. Our expectation was that after the students can use their concrete understanding to give the reason of formal notation of 0,5 is a half, then, they will come up into the idea of number line. However, Dimas immediately drawing number line to represent 0,5 is a half. In fact, he could draw correctly but still difficult to give the reason.

Dimas' strategy was same with Bintang's strategy in the pilot experiment (first cycle). They made number line and divided the line became two and determined 0,5 in the middle to represent a-half. As mentioned in the analysis of pilot experiment, the reason of 0,5 was a half through drawing number line was not so clear, but it occurred again in the teaching experiment. The teacher also asked Dimas to explain and made connection between his reason and his strategy (number line), but his reason was not clear (we could not hear his

statements on the video recording). Then, the teacher tried to ask the other students whether there were different strategies with Dimas.

Sella wrote 0,5 was equal to a-half. However, she just wrote on the board without giving the reason. We observed in the second activity when Farhan reported that the pointer point was a-half, then the teacher asked: *"Is there any other opinion?"* We expected that the answer was zero point five, however Sella thought about the other form of a-half, and she answered: "*One over two.*" (see page 101).

Rico wrote that 1 pack was equal to 0,5 kg, and two packs was equal to 1 kg. Then he explained in front of the class that when 0,5 was added with 0,5, the result became 1. That is why 0,5 is a half. From three students' strategies, there was only Rico who used the result of measurement activity as his reason. And he gave an excellent reason by connecting his answer with his concrete understanding.

The idea of the position of 0,1 and 0,5 on number line in which *0,1 is in the first stripe from 10 stripes overall 0,1* (one out of ten = $\frac{1}{10}$) and 0,5 is in the fifth stripe from ten stripes overall (five out of ten = $\frac{5}{10}$) was effective to come up to the magnitude of one-digit decimals. Supporting students' thinking that decimal refers to base ten and its position in the certain line (stripe) from ten lines (stripes) overall make students easier to understand the meaning and the magnitude of decimals.

d. Exploring the Meaning of Two-digit Decimals through Measuring the Volume of Beverages

Mathematical goal:

Students are able to explore the notion of two digit decimals and put them on the number line

From the previous activity, the students have already known about the notion of one digit decimals were in between two consecutive who numbers and also they knew the meaning of simple decimals (0,5 is a half; 0,1 is a tenth). In this activity they were asked to explore the notion of two digit decimals.

Before the activity was begun, the teacher asked the students (without showing the real beverages):

Teacher : "Yesterday I bought two beverages, the volume of first beverage was 0,25 liter, and the volume of the second beverage was 0,5 liter.What do you think? Which one is more? 0,25 liter or 0,5 liter?"
Some students: "0,25 liter."
Some students: "0,5 liter."
(The teacher pointed to one student, Sella to give the reason).
Sella : "I think that 0,25 liter is more than 0,5 liter because 25 is more than 5.
Teacher : "Which of you who agree with Sella's argument?"
(Nine students raised their hand).

Then, the teacher showed two beverages and asked Sella to choose which one was 0,25 liter, and which one was 0,5 liter and draw their position on the number line. Sella chose that the larger bottle was 0,25, and the smaller bottle was 0,5 liter and put 0,25 was on the right position of 0,5 (see figure 81).

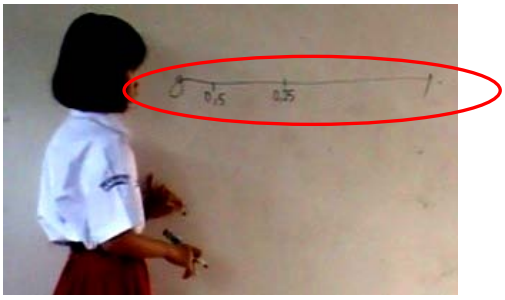



Figure 81: Sella assumed that 0,25 was larger than 0,5

After that, the teacher asked Sella to pour each beverage into the measuring cup. When Sella measured the larger bottle, it was indeed 0,5 liter, and automatically the second bottle was 0,25 liter. Through observing the position of each height, the students could see that 0,5 is more than 0,25. Besides that, the students could see that 0,25 was in between 0,2 and 0,3 (see figure 82).

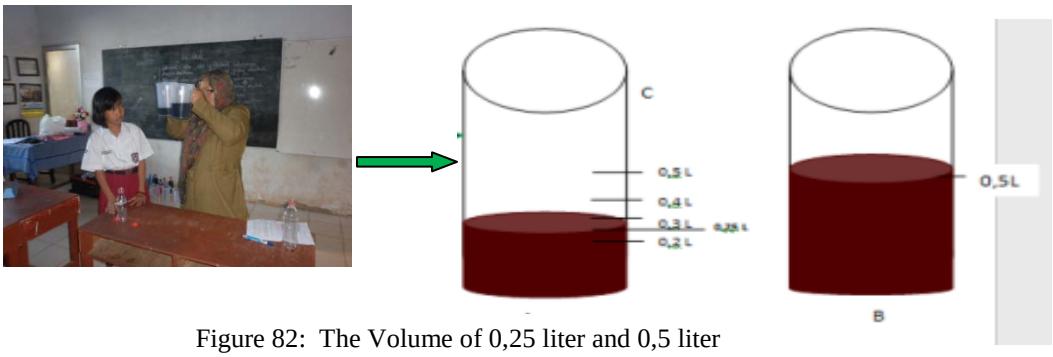
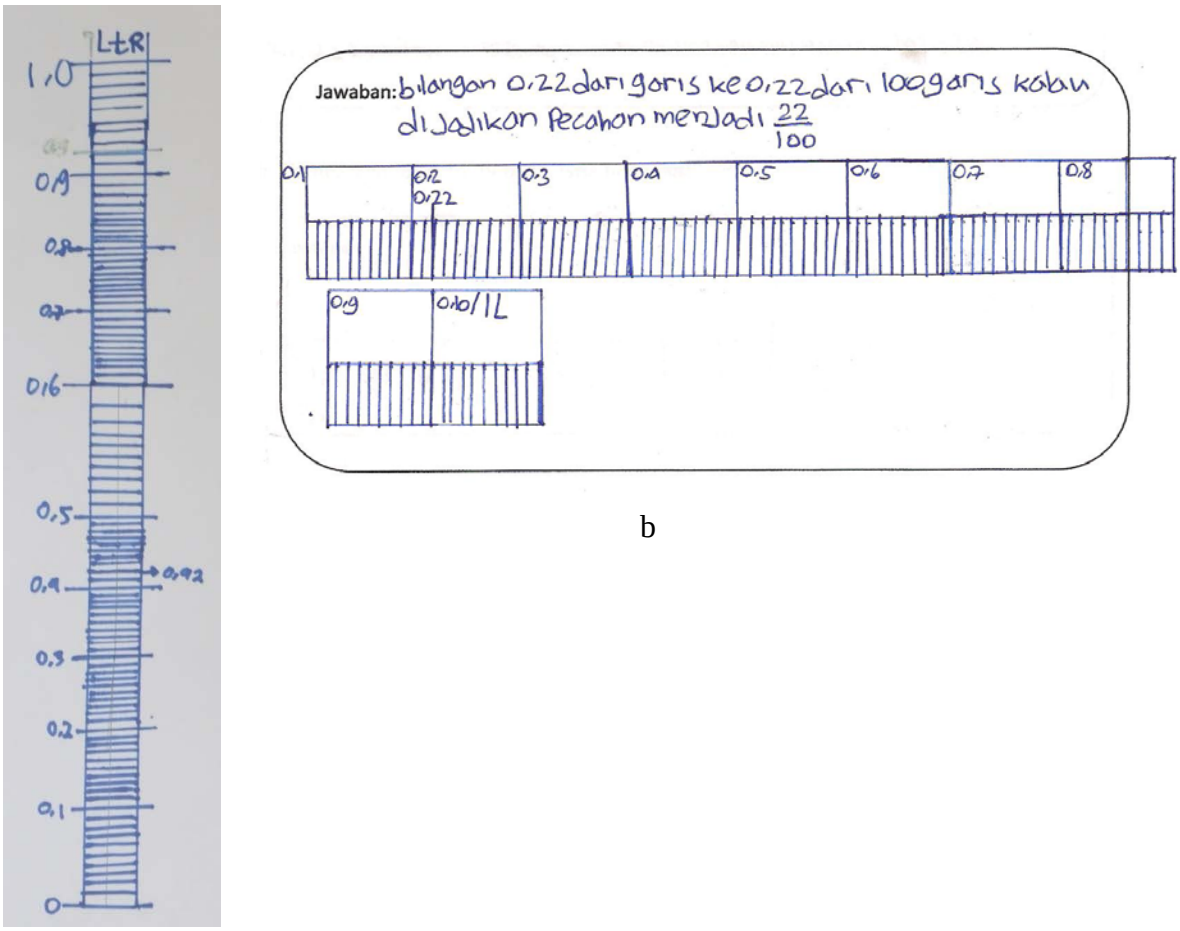


Figure 82: The Volume of 0,25 liter and 0,5 liter

Based on the discussion class, it was suitable with the conjectures in HLT that the students could find two-digit decimals by dividing the range become ten partitions (partition a tenth of a tenth).

In this activity, the students began to consider the representation of the weight scale (containing the arch of the stripes) and the representation of measuring cup (containing the small stripes) as *a model of* situation to represent the sequence of the numbers, including decimals (see figure 83 a&b). The idea of shifting into number line as *a model-for* emerged when the students have done the measurement activities. We observed that they used their experience from measurement activities and considered the visualization of the weight scale and the measuring cup containing the stripes to draw number line as a model for placing the position and determining the magnitude of one-digit decimals (see figure 84).



a b

Figure 83: The representation of measuring cup as a model of situation

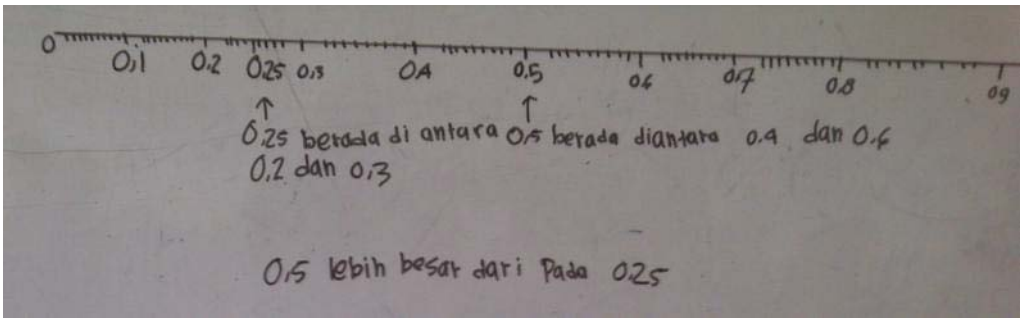


Figure 84: Shifting into number line as a model for placing one-digit and two-digit decimals (partition base tenth of tenth)

In the discussion class, the students concluded that between two consecutive whole numbers there were decimals, and between two consecutive one-digit decimals there were another decimals (two-digit decimals),

After that, each group was asked to measure various dosages of beverages such as 0,35; 0,6; 0,45; etc and they reported their measurement in the poster. It was suitable with the conjectures that all groups made the representation of measuring cup a model of situation and number line as a model for placing the position of the magnitude of decimals.

We provided 1 measuring cup for each group in which there are ten fragments of one-digit decimals (between 0 and 1), and between two consecutive one-digit decimals there are another ten fragments (see figure 85).

Figure 85: Measuring cup

It was suitable with the conjecture that the students drew number line (either in horizontal or vertical) as a model for reasoning the position of two-digit decimals. In the discussion class, the teacher provoked the students by asking:

Teacher :"0,25 is in what stripe?"
Students : "It is in the 25th stripe"
Teacher : "From how many stripes overall?"
Students : "100 lines"

Based on poster that they made, first they made the representation of the measuring cup as a model of situation and placed the position of two-digit decimals in it. Some groups made the representation of measuring cup horizontally and the other group made the representation vertically (see figure 83). Kelengkeng group made extended their ideas

into converting into fraction and reasoned that 0,22 is at the 22^{th} stripe from 100 stripes overall, therefore they conclude that 22 out of 100 is equal to $\frac{22}{100}$ (see figure 83 b).

The teacher encouraged the students through the discussion that is why decimal is *called* "the number refers to base ten" because when there are two consecutive whole numbers and we divide into ten fragments, we can see that there were one-digit decimals. And when we divide the fragment of two consecutive one-digit decimals, there were another ten fragments of two-digit decimals, and so on.

It was suitable with the conjecture that supporting students that decimal refers to base ten *(in between one-digit decimals, a-tenth, there were still another ten fragments, two-digit decimals)* and determining its position in certain stripe (line) from hundred stripes overall make students easier to understand the meaning and the magnitude of decimals.

e. Density of Numbers on the Number Line (Exploring One-digit and Two-digit Decimals on the Number Line)

Mathematical goal:

Students are able to explore one-digit and two-digit decimals on the number line

1) Description of the Activity: Playing Spaceflight Game

In this activity, the students were asked to find the secret code by playing the game of Spaceflight game (http://www.fi.uu.nl/toepassingen/03127/task3.html designed by Frans van Galen). The Spaceflight game has purpose to give visualization to the students that when they zoom the range between two consecutive whole numbers, there will be decimals. This software is good enough to provide the concept of density of numbers on the number line.

Same with the pilot experiment, the students enjoyed playing the game (see figure 86). To find the secret code, they have to pass four steps i.e finding the position of 12,26; 89,89; 0,38; and 0,07. The range given was vary.

1) 12,26 in between 12 and 13,

2) 89,89 in between 80 and 90;

3) 0,38 in between 0 and 10;

4) 0,07 in between 0 and 100.

In average, the students could find the secret code. From 26 students, there were 6 students who cannot find the secret code (Michell, Fifi, Cindy, Amelia, Ahmad, and Rizkiah). This is a remarkable progress from the pilot experiment, because in the first cycle from 7 students, there was only 1 who was able to find the secret code, and now most students (20 out of 26) were able to find it.

The time also was more effective because in the first cycle, it was needed around 10 minutes for one student to find the secret code, but now most students could find the secret code in 5 minutes.

Figure 86: The students played Spaceflight game

2) Discussion Class

At the end of activity, the students were asked to give the report of their game through putting the position of each step on the number line. Six of them put the numbers correctly (see figure 87).

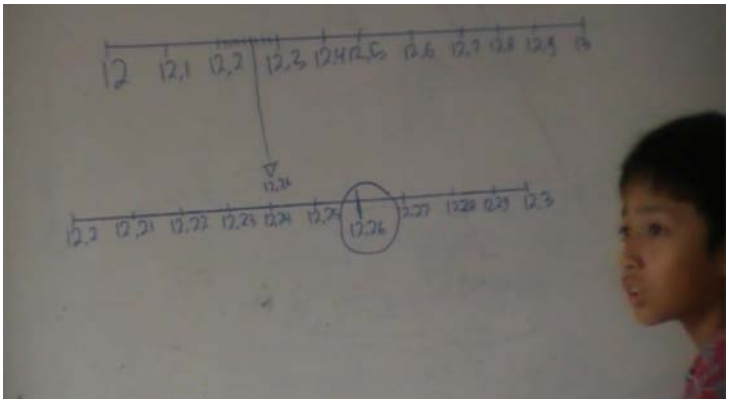


Figure 87: Auliya drew the result of Spaceflight game on number line

Based on students' worksheet, we observed that in the mathematical congress of activity 5, the students could make the sequence of one-digit and two-digit decimals and put them on the number line (figure 87). They knew that between two consecutive whole numbers there were one-digit decimals, and between two consecutive one-digit decimals there were other decimals (two-digit decimals). It seemed that they have mastered the concept of density of numbers on the number line. We expect that the students can use this knowledge to compare one-digit and two-digit decimals, and we can reduce the students' misconception about the longer digit the decimals, the larger their magnitude.

f. Comparing the Magnitude of One-digit and Two-digit Decimals Using Number Line.

Mathematical goal:

Students are able to compare the magnitude of one-digit and two-digit decimals

In this activity, the students were asked to solve the following problem:

Ayuk goes to the market to buy the meat. There are 3 Kiosks which offer the same price for 1 pack of meat. The weight for each pack in each kiosk can be seen in below figure. Which one that should be bought by Ayuk? Explain your reasoning!

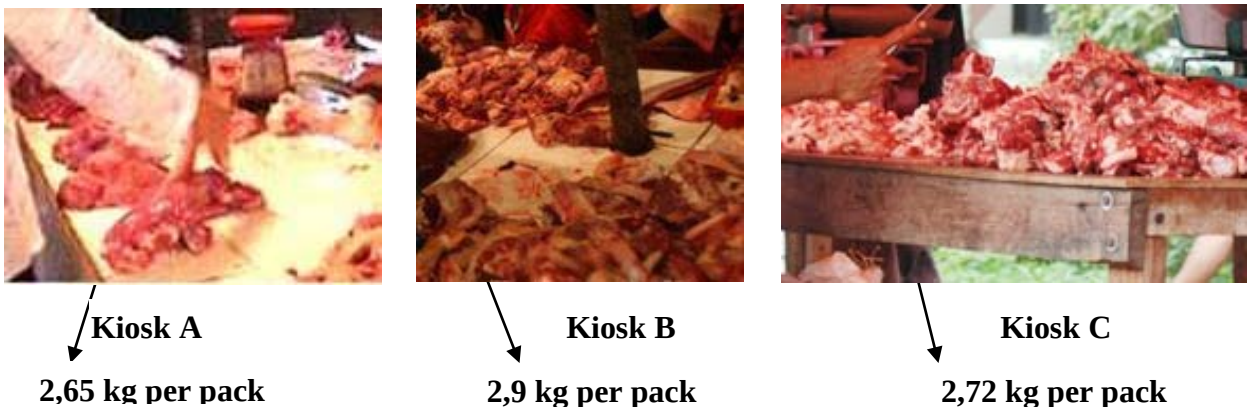
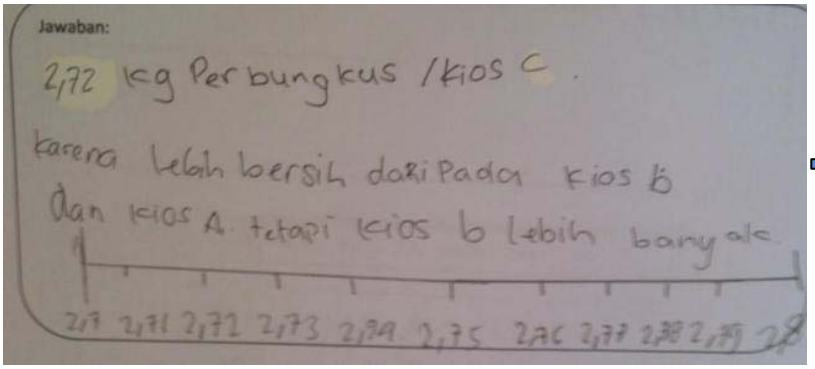


Figure 88: The problem for activity 6

The findings in this activity were more interesting because at first time we thought that the students would choose the Kiosk which sold the heaviest meat. However 2 students (Tiara and Amel) chose Kiosk C because they thought the meat in Kiosk C was the cleanest Kiosk compared to the others. Even though they knew that the heaviest meat was Kiosk B, but because the picture in Kiosk B was not so clear, so they thought that it was dirty (see figure 89).



I choose Kiosk C (2,72 kg per pack) because its meat is the cleanest one compared to the others, even though Kiosk B was the heaviest one.

Figure 89: Tiara's Answer

One student (Intan) chose Kiosk B and gave the reason because it was the cheapest one compared to the others. Three students (Fadhil, Linda, and Arissandy) chose Kiosk C and still gave the reason because it was the heaviest one compared to the others.

Similar with Tiara and Amel, 7 students (Cindy, Rico, Michell, Fifi, Auliya, Nia, and Nova) chose Kiosk A because they thought that the meat in Kiosk A was the cleanest Kiosk compared to the others.

Another issue beside the cleanliness also appeared. This related to the pack of meat. Because the meat in Kiosk A was the only one which was packed, so 3 students (Fifi, Mutia, and Rizkiah) chose the meat in Kiosk A and they gave the reason because the meat in Kiosk A was packed, so probably it was the only one which was suitable with the information on the problem (2,65 kg). Other meats were not provided in the pack.

Teacher : "Which meat that should be bought by Ayuk?"
Students : "Kiosk B.....Kiosk A.....Kiosk C....."
Teacher : "Which of you chose Kiosk B?"
(14 students raised their hand)
Teacher : "Which of you chose Kiosk A?"
(7 students raised their hand)
Teacher : "Which of you chose Kiosk C?"
(5 students raised their hand)
*Teacher : "Please explain your reasoning!" (*in front of the class*)*
Mutia *: "Because the meat in Kiosk B was more."*
Fadhil : "I chose Kiosk C because the meat was very much."
Teacher : "How much is it?"
Fadhil : "2,72."
Fifi : "I chose Kiosk C because the meat was more clean and fresh."

Then, the teacher asked them to draw each weight of the meat on the number line. Finally when the teacher asked three of them drawing each weight of the meat on the number line, then they realized that the meat in Kiosk B was the heaviest one (see figure 90).

Teacher : "Each of you has chosen Kiosk A, B, and C. Which one was the most?"
Students : "Kiosk B...."
Teacher : "Which one?"
Students : "Kiosk B..."
Teacher : "Look at the number line, where is the position of the meat in Kiosk B (2,9)?"
Students : "On the right..."

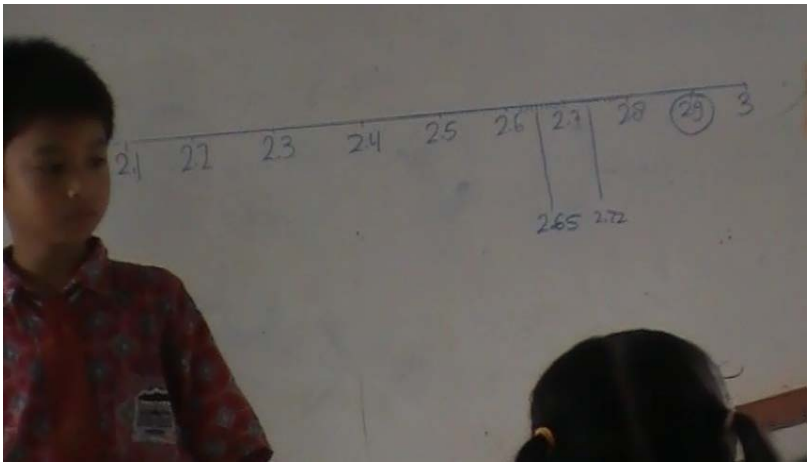


Figure 90: Fadhil, Fifi, and Mutia drew each weight of the meat on the number line

From 26 students, there were 13 students who chose Kiosk B and gave the correct reasoning that the meat in Kiosk B was the heaviest one and they determined their magnitude through drawing number line (see figure 90).

The issue of the cleanliness of the meat was unpredictable. By the chance at that time we have prepared for mathematical congress in which all pictures were clean (see figure 91). The students were asked to discuss in the group containing five or six students to solve the problem similar with the previous problem but containing different weights and different pictures. Then, they were asked to write their solution on the poster. Finally, all group chose Kiosk B because Kiosk B (3,8 kg/pack) was the heaviest one compared to Kiosk A (3,53 kg/pack) and Kiosk C (3,65 kg/pack) (see the result of mathematical congress).

Figure 91: The teacher showed the poster containing 3 Kiosk which sell the meat in different weight

The result of mathematical congress for each group:

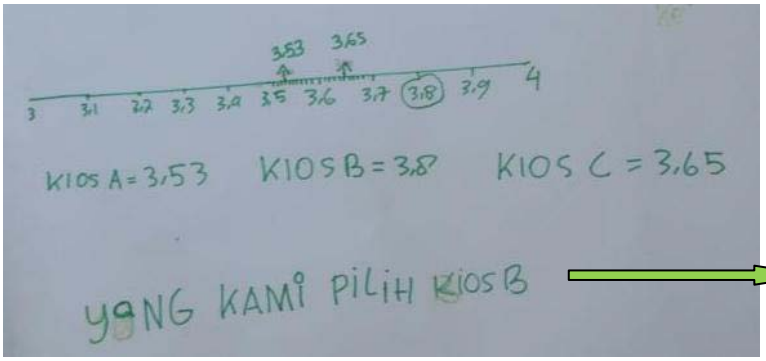


Figure 92: Strawberry group

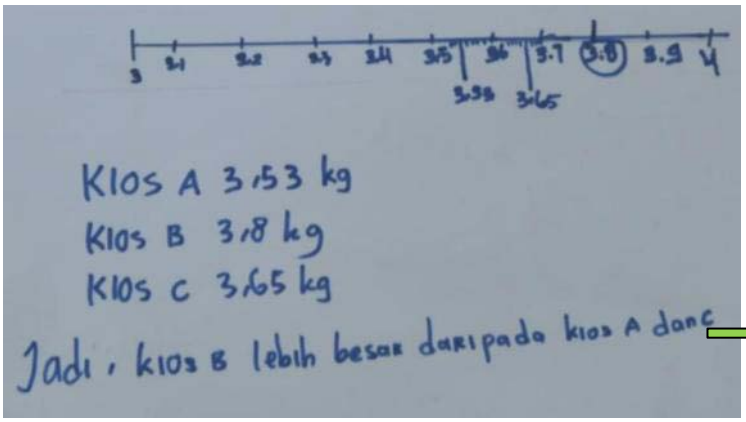


Figure 93: Kelengkeng group

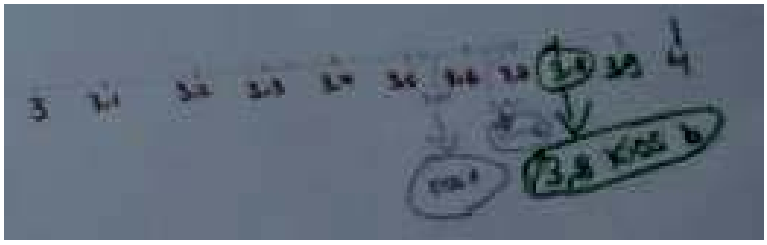


Figure 94: Orange group

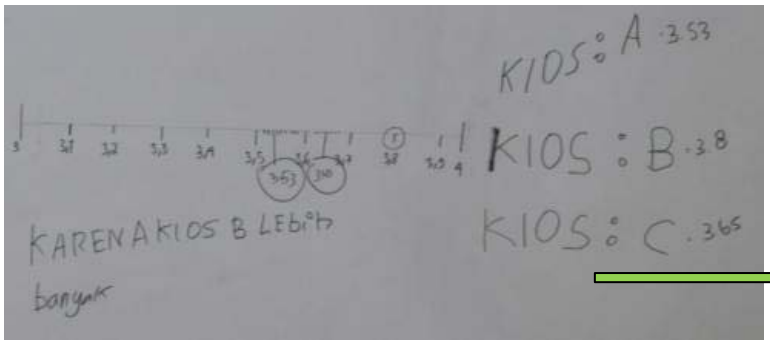


Figure 95: Mango group

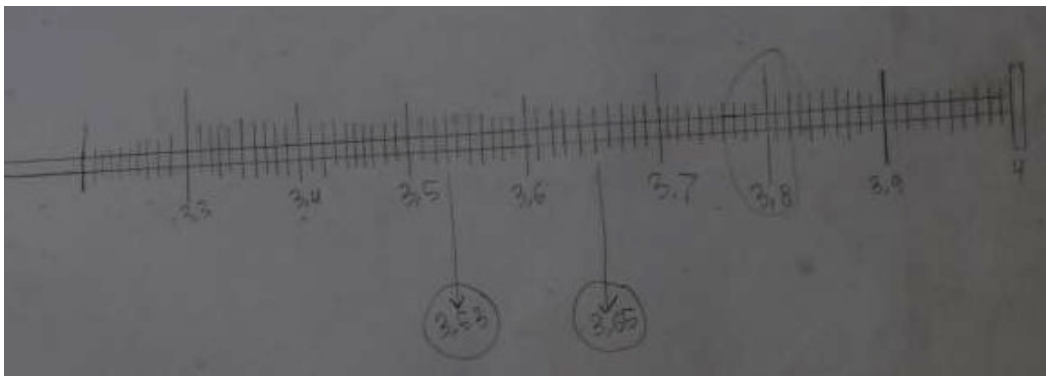


Figure 96: Virgo group

*2. **Post Assessment***

The problem of post assessment contained 5 problems (attached on appendix). They consisted of reading the scale, knowledge of decimals, density of numbers on number line, comparing the magnitude of one-digit and two-digit decimals (contextual problem).

The first problem is aimed assessing students' ability in reading the scale. 69% students show a good ability in reading the scale using their own strategies which were

shown in their reason. The strategies used were 1) looking the position of the scale and determining its position in between two consecutive whole numbers (density of numbers); and 2) counting the lines (stripes). The detailed analysis of this problem is attached on appendix.

The second problem is aimed at assessing students' knowledge of the notion of decimals in between two consecutive whole numbers. 85% students answered the problem correctly. The strategies used were: 1) sequencing the decimals in between two consecutive whole numbers and putting them on the number line; 2) halving strategy (determining a half of two consecutive whole numbers); and 3) sequencing the decimals in between two consecutive whole numbers without drawing number line. The detailed analysis of this problem is attached on appendix.

The third problem is aimed at assessing students' notion of density of decimals on number line. They were asked to determine the numbers in between one-digit and two-digit decimals. 77% students could master the idea of density of decimals on the number line. The strategies used were: 1) dividing the range between two consecutive one-digit decimals become ten partitions and sequencing two-digit decimals; 2) dividing the range between two consecutive one-digit decimals become ten partitions including zero after *comma* in each edge; and 3) counting approach. The detailed analysis of this problem is attached on appendix.

The fourth problem is aimed at assessing students' ability in comparing the magnitude of one-digit and two-digit decimals. 77% students could determine the decimal comparison correctly. The strategies used were 1) left to right comparison on the number line; 2) using the idea of density of decimals; 3) ordering one-digit and two-digit decimals using number line as a tool for placing their magnitude; 2) ordering one-digit and two-digit decimals

without drawing the number line. The detailed analysis of this problem is attached on appendix.

The progress from pre-and post assessment is also quite significant. In the post assessment we included the problem similar with the pre-assessment (but different numbers) i.e. about comparing one-digit and two-digit decimals. The number of percentage of students who answered correctly in the pre-assessment was 4%, whereas in the post-assessment was 96%. It means that the development of students' learning of comparing the magnitude of one-digit and two-digit decimals was quite significant.

C. Discussion

The reflections on basic tenets of RME and the role of the teacher underlie the designed context and the activities. In this section, we highlighted some important issues related to this research.

*1. **Realistic Mathematics Education (RME)***

As mentioned in chapter 2, to support students' learning in decimals, the problems which are given should be meaningful for students. The first tenet of RME is *phenomenological exploration*. In this situation, the students were experienced with measurement activities i.e measuring the weight of the things (fruit and body). This activity could help the students to develop the intuitive notions as the basis for concept of decimals. Besides that, the other measurement activities such as measuring the weight of the rice and the volume of beverages could help the students toward the notion of one-digit and two-digit decimals in meaningful way.

The second tenet of RME is using models and symbols for progressive mathematization. The progressive mathematization here means the development from informal level through more formal mathematical concepts. The sequence of designed activities in this research was successful to bring the students' thinking into their self-developed model. The weight scale and measuring cup can become a model of situation which brought the students into the idea of model for placing the position of decimals (number line). Since the form of weight scale and measuring cup contain the scale, so their thinking shifted from *model of* into making number line as a *model for* placing the relative magnitude of one-digit and two-digit decimals.

The third tenet of RME is using students' own constructions and productions. In the series of activities, most students made the representation of the weight scale when they reported the data of measurement activities. However, there were some students who had already perceived the idea of making number line to place the sequence of decimals. This idea could bring the discussion in which number line plays an important role in determining the relative magnitude of one-digit and two-digit decimals when the students already mastered the concept of density of numbers on the number line. Besides that, number line also could help the students in developing the strategy of left-to right comparison when they compared one-digit and two-digit decimals.

The fourth tenet of RME is interactivity. From their own production (such as drawing number line or drawing the representation of the weight scale), the students could make a meaningful discussion and they could share their finding with the others. The role of the teacher here also plays an important part in making guided reinvention for the students and finally there will be vertical interaction (between teacher and students) and horizontal

interaction (between one student with the other students). The role of the teacher will be explained further in the next section.

The fifth tenet of RME is intertwinement. As mentioned in chapter 2, learning about decimals can be a basis which also can be integrated with other domains such as fraction, percentages, proportion, measurement and the development of number sense. The focus of our study is making connection between decimals and measurement activities. Besides that, we found also some students made the link between decimals and fractions because it is common in Indonesian curriculum that decimals were taught simultaneously with fractions. Through this designed activities we expected that when the students make the link between decimals and fractions, it will not be formal anymore (only converting as a tenth, hundredth, etc) but also create a meaningful situation in which the partition based ten on the number line can become the basis into the idea of denominator ten, hundred, etc.

*2. **Classroom Discussion***

Based on our observation, the socio norms in the class run well. The interaction between teacher and students (vertical interaction) and between one student with the other students (horizontal interaction) occurred during the activities were held. The discussion focused into how they came into the knowledge of decimals. The following excerpt is one of the examples which reflected the classroom discussion:

Teacher : *"How much is this?"* (The teacher pointed to two packs of rice measured)
Students : *"1 kg."*
Teacher : *"So, how much for one pack?"*
Students : *"A half……zero point five…"*
Teacher : *"How about two packs?"*
Students : *"1 kg"*
Teacher : *"So, what can you conclude? "What is the meaning of 0,5? 0,5 is how much of 1?"*
Farhan : *"Pardon Mom…0,5 is what?"*
Teacher : *"0,5 is how much of 1?"*
Firman : *"A half*"
Teacher : *"Which of you can explain your reason?"*

Rico : *"We have seen that when we weighed one pack, the weight is 0,5 kg, when we added another same pack (0,5 kg), the weight became 1 kg. So, 0,5 kg plus 0,5 kg is 1 kg."*
Teacher : *"What do you think? Is there any different answers with Rico?"*
Students : *"Same.."*

Based on the classroom discourse above, it shows the interaction between student, teacher, and the other students. Based on the observation, we noted that most students were involved actively and brave enough to explain their answer in front of the class. Even though in the first activity the group discussion was not run well (based on observer's feedback), but after discussed with the teacher in the following activities we tried to encourage all students to be more active by giving the opportunities to the students who still did not involve actively. The teacher, the researcher, and the observers have some notes to notice which student who already giving contribution. So in the following activities when there were many students raised their hand, the teacher would point to the other students (who were still passive).

3. The Role of the Teacher

The role of teacher in the class discussion will be elaborated in the following manners:

a. Guided reinvention

In the series of activities, the teacher tried to address the guided reinvention for students' learning of comparing the magnitude of one-digit and two-digit decimals through exploring decimal notations using the concrete experiences i.e. weight and volume measurement activities and establishing the position of their magnitude through drawing the number line.

The following fragments are the examples of questions that were used by the teacher to stimulate the students to express their ideas:

- *"Is there any other number between 43 and 44?"*

- *"Zero point five? How do you know?"*
- *"So, what can you conclude? What is the meaning of 0,5? 0,5 is how much of 1?"*
- *Where is the position of 0,1? At what stripe? From how many stripes overall?"*

b. Managing the classroom situation

The weakness of this research is the limitation of the number of media such as fruit weight scale (in activity 2 and 3) and computer (in activity 5). The feedback from the observers also suggested the additional media in order to make the teaching and learning process become more effective. However, it is quite amazing that within the limitation of this media, the teacher could manage the classroom situation in a conducive situation. Based on the observation of the researcher and also from some observers' opinion, most students paid attention when the teacher explained in front of the class. Also when one volunteer student was asked to do the experiment in front of the class, the other students paid attention and gave contribution by giving their ideas when the teacher brought the problem into discussion class.

c. Facilitator

During the activities and the discussion class, the teacher facilitated the students to discuss about different answers and reasons. It was quite different with the traditional teaching that the teacher transmitted the knowledge and gave the instruction directly in one way. The role of the teacher as the facilitator also gave the freedom for the students to share their ideas and could encourage students in giving their reasoning either in group discussion or in front of the class.

CHAPTER 6
CONCLUSION AND RECOMMENDATIONS

A. Conclusion

This chapter presents the conclusion of the research findings in relation to the research question and the recommendations for further studies. Based on the retrospective analysis and the discussion mentioned on chapter 5, we will answer the research question including sub research questions by looking to the series of activities and the development of students' learning in each phase.

Before the activity was conducted, it was suitable with the conjectures in HLT that most students thought that there were no other numbers between two consecutive whole numbers. The struggles of the students were that when they weighed the things, some of them could not see the scale precisely. They just thought about the shown number on the scale without considering the accuracy. However, after they did measurement using weight scale and digital weight scale, they realized that decimals exist in between two consecutive whole numbers, and decimals were needed to measure the things precisely. Through the series of activities, the students could develop their idea into the density of numbers on the number line which bring them into the idea of partitioning base ten and tenth of tenth.

The situational level, where situational knowledge and strategies were used within the context of the situation, emerged in the measurement activities. Through the context of precise measurement, the students could find decimals in weighing body activity by observing the scales that when the needle pointed to the position between two consecutive whole numbers, e.g. between 38 kg and 39 kg, then there should be *comma numbers* (decimals) in it, e.g. 38,4 kg which was appeared on digital weight scale. Similarly, in weighing duku activity, they also found that there were one-digit decimals in between two-consecutive whole numbers (e.g between 0 kg and 1 kg there were 0,1 kg; 0,5 kg; 0,8 kg;

etc.). They found decimals by estimating the scale first and after that they opened the covered sticker to see the actual weight on the scale.

In weighing rice activity, the students started to make the sequence of decimals (one-digit decimals) through counting the long stripes (major scales) using the approach of sequencing whole numbers (the first stripe was 0,1 kg; the second stripe was 0,2 kg; the third stripe was 0,3 kg, ... , the ninth stripe was 0,9 kg). From that invention, the students could find that there were ten partitions containing one-digit decimals in between two consecutive whole numbers which finally could bring the students into the idea of decimal referring to the number base ten, i.e: 0,1 is at the first stripe from ten stripes overall (one over ten); 0,5 is at the fifth stripe from ten stripes overall (five over ten), etc.

The activity of measuring the volume of beverage was aimed to develop students' acquisition for the idea of two-digit decimals. In this activity, the students began to make the representation of measuring cup containing small stripes in between two consecutive one-digit decimals. The visualization of measuring cup could encourage students to perceive the idea that there were two-digit decimals in between two consecutive one-digit decimals. Through this discovery, the students got insight about the idea of density of numbers (between two consecutive whole numbers there are decimals, and between two consecutive one-digit decimals there are other decimals, i.e. two digit decimals, etc.). In addition, they also found the idea that two-digit decimals refer to the partitioning base tenth of tenth (on the number line), e.g. 0,25 is at the twenty fifth stripe from one hundred stripes overall.

The referential level, where models and strategies refer to the situation which is sketched in the problem, was shown when the students made the representation of the weight scale and measuring cup in the measurement activities. In this phase, the students

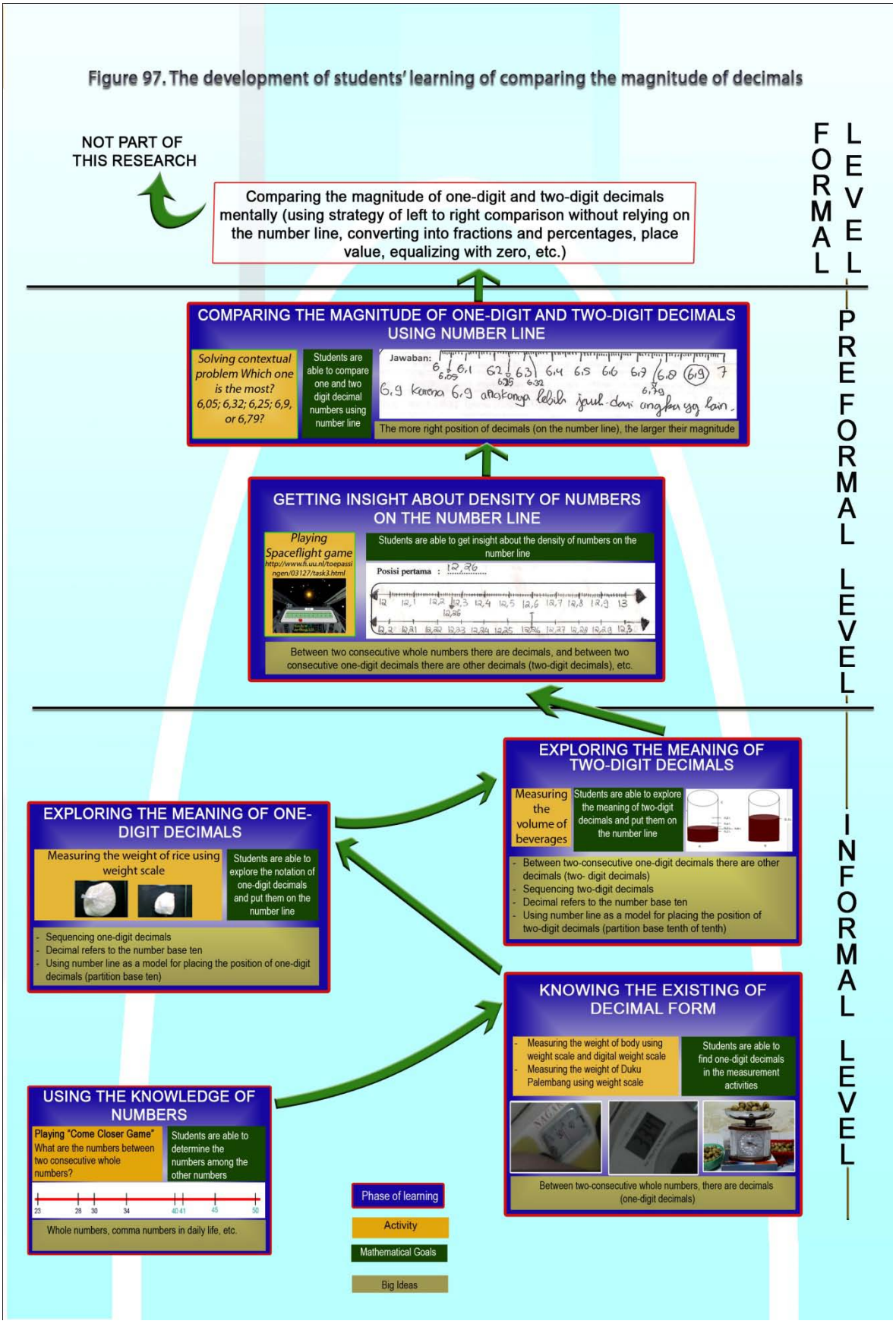
began to consider the representation of the weight scale (containing the arch of the stripes) and the representation of measuring cup (containing the small stripes) as *a model of* situation to represent the sequence of the numbers, including decimals. The idea of shifting into number line as *a model-for* emerged when the students have done the measurement activities. Based on observation and interview with the students, they used their experience from measurement activities and considered the visualization of the weight scale and the measuring cup containing the stripes to draw number line as a model for placing the position and determining the magnitude of one-digit decimals.

Using the number line as *a model for* showed that the students had attained general level of modelling. Through observing the position of one-digit and two-digit decimals on the number line, the students used it as their reasoning for showing the magnitude of one-digit and two-digit decimals. They determined that the more right position of decimals on the number line, the larger their magnitude. In addition, the students could realize that the longer digit the decimals, the more precise the measurement. Therefore, we can show that number line plays an important role in bridging the experience-based activities into more formal level of mathematics (comparing the magnitude of one-digit and two-digit decimals).

We concluded that through the contextual situation (weight and volume measurement), the students' learning toward decimals can develop from informal level to pre-formal level. Starting from making representation of the weight scale and measuring cup as *a model of* situation, the students' thinking shifted into the idea of using number line as *a model for* placing the magnitude of one-digit and two-digit decimals, which finally bring their idea to compare their magnitude by determining the position from left to right comparison on the number line. There were some students' strategies which were outside

from our conjectures, i.e. equalizing with zero and converting the decimals into fractions, but those help the other students into the discussion showing that the strategy of using number line can become the basis for the other formal strategies in comparing the magnitude of decimals. The scheme of the development of students' learning in comparing the magnitude of one-digit and two-digit decimals is summarized on figure 97.

Figure 97. The development of students' learning of comparing the magnitude of decimals



B. Recommendations

As mentioned on chapter 2, Realistic Mathematics Education (RME) underlies this research in the part of designed context and activities. The findings in this research show that measurement activities (weight and volume measurement) can promote the students' notion of decimals in which the students could discover decimals and determine their magnitude by making the partition base ten and observing their position on the number line.

Concerning the findings of this research, we give recommendations for further studies about decimals to apply Realistic Mathematics Education (RME) as the basic approach for teaching and learning decimals, especially about comparing their magnitude. Even though decimals were known as the abstract numbers for the students, but in this designed activities we can show that decimals can be taught in a meaningful way within measurement activities. The students could discover decimals by themselves and could develop their ideas to come to the number line as a model for placing their magnitude. As we discussed on chapter 5, the approach of RME could improve the socio norms, especially in the classroom discussion. We recommend for the teachers in Indonesia to apply RME in teaching mathematics e.g. decimals in order to be more meaningful for students.

We recommend also for Indonesian teachers in grade 5 to include the teaching and learning process about comparing the magnitude of decimals even though it was not included in the curriculum. It can be used to diagnose students' misconceptions about the size of decimals. Besides that, it is helpful for preparing the students when they enter secondary school in which they will encounter many digit decimals (three-digit, four-digit, etc). It is recommended that the teachers not only rely on the formal

approach in textbooks about decimals but also use daily life or meaningful situation for students as a starting point for learning decimals in grade 5.

This research is only an initial bridging from informal level (contextual situation) to pre-formal level mathematics (using number line for comparing the magnitude of one-digit and two-digit decimals), but this can be a basis for further studies to explore the instructional design related to teaching and learning decimals in more formal level e.g. comparing decimals using strategy of left to right comparison without relying on number line. This research can also be connected to other previous researches. Based on the findings of this research, the idea of partitioning base ten on the number line which starting from measurement activities can be a basis for the idea of a-tenth and a-hundredth which will bring them into the idea of common decimal fractions (Van Galen, et al, 2008), place value of decimals (Widjaja,2008), equalizing with zero (Helme&Stacey, 1999), etc.

REFERENCES

Bakker, A. (2004). *Design Research in Statistics Education. On Symbolizing and Computer Tools.* Amersfoort: Wilco Press.

Depdiknas (2006). *Kurikulum Tingkat Satuan Pendidikan Sekolah Dasar*. Jakarta: Depdiknas

Desmet et al. (2010). *Developmental Changes In The Comparison of Decimal Fractions, Centre for Research on The Teaching of Mathematics*, Learning and Instruction, 521-532. Belgium: Centre for Research on The Teaching of Mathematics.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht: Kluwer Academic Publishers.

Gravemeijer. (1994). *Developing Realistic Mathematics Education*. Utrecht: Freudenthal Institute.

Gravemeijer. (2004). *Local Instruction Theories as Means of Support for Teacher in Reform Mathematics Education*. Utrecht: Freudenthal Institute & Department of Educational Research, Utrecht University.

Gravemeijer, K., & Cobb, P. (2006) Design research from the learning design perspective. *Educational design research* (pp. 17-51). London: Routledge

Helme&Stacey (1999). *Can Minimal Support for Teachers Make a Difference to Students' Understanding of Decimals?* Australia: University of Melbourne.

Hiebert and Wearne (2002). *A Model of Students' Decimal Computation Procedures*. University of Delaware: EBSCO Publishing

Hogarth, R.M. (1992). *Educating Intuition*. Chicago and London: University of Chicago Press.

Markovits, Z. & Even, R. (1999). *The Decimal Point Situation: A Close Look at The Use of Mathematics-Classroom-Situations In Teacher Education*. Teaching and Teacher

Education 15, **653-665**. Israel: Centre for Mathematics Education, Oranim School of Education, Tivon, 36006 & Department of Science Teaching, Weizmann Institute of Science, Rehovot, 76100.

Merenlouto, K. (2003). *Abstracting the density of numbers on the number line- a quasi experimental study*. In N. A. Pateman, B. J. Dougherty & J. Zilliox (Eds.),*Proceedings of the 2003 Joint Meeting of PME and PMENA* (Vol. 3, pp. 285 - 292). Honolulu, HI: CRDG, College of Education, the University of Hawai'i.

Nelissen, J.M.C (1999). *Teaching and Learning Thinking Skills.* In Hamers J.H.M, J.E.V. van Luit, B. Csapo (eds). Swets&Zeitlinger Publishers,. Lisse (page 189-215)

Reys, R.E. et al. (2006). *Helping Students Learn Mathematics*. USA: Courier/Kendallville.

Sembiring, R., Hoogland, K., and Dolk, M. *A Decade of PMRI in Indonesia*. Bandung, Utrecht: PMRI Team.

Suyono. (2008). *Matematika Sekolah Dasar dan Menengah Jilid 2, Pengembangan Pengajaran*. Jakarta: Erlangga.

Van Galen, F.V., Figuerido, N, and Keijzer, R. (2008). *Fractions, Percentages, Decimals, and Proportions*, Freudenthal Institute: Sense Publishers.

Van Galen (2010). http://www.fi.uu.nl/toepassingen/03127/task3.html. Utrecht: Freudenthal Institute.

Vicki et al (1999). *Teaching and Learning Decimals*. Australia: Department of Science and Mathematics Education University of Melbourne, University of Melbourne.

Widjaja, W. (2008) *Local Instruction Theory on Decimals: The Case of Indonesian Pre-Service Teachers*. Australia: University of Melbourne.

Wijaya, A. (2008). *Design Research in Mathematics Education Indonesian Traditional Games as Preliminaries in Learning Measurement of Length*. Utrecht University

Zulkardi. (2002). *Developing A Learning Environment on Realistic Mathematics Education For Indonesian Student Teachers*. Enschede: University of Twente.

Zulkardi & Ilma, R. (2006). *Mendesain Sendiri Soal Kontekstual Matematika*, Semarang: Prosiding KNM 13.

TEACHER GUIDE

This teacher guide provides a sequence of activity in exploring the magnitude of one and two digit decimal numbers. There are six activities in which each activity consists of standard competence, basic competence, indicators, goals, materials, instructional activities (starting point, activities, discussion, and reflection of the activities), and assessment.

Students will be experienced with the activities from informal level into more formal level i.e. 1) playing come closer game, 2)measuring the weight of Duku (to explore decimal form), 3) measuring the weight of rice (to explore one digit decimal numbers), 4) measuring the volume of beverages (to explore two digit decimal numbers), 5)playing spaceflight game (to explore one and two digit decimal numbers), and 6)solving contextual problem about comparing the magnitude of one and two digit decimal numbers.

TEACHER GUIDE

Topic	: Decimal Numbers
Grade	: V
Activity	: Playing "*Come Closer*" Game
Time	: 2 X 35 minutes
Meeting	: 1

A. Standard Competence

Using fractions in problem solving

B. Basic Competence

Converting fraction into percentages and decimals forms or vice versa

C. Indicators

1. Students can determine the numbers between the other numbers and putting them on number line
2. Students know about the sequence and the position of whole numbers in number line.

D. The Goal (for students)

1. Students are able to determine the numbers between the other numbers
2. Students are able to put the numbers on number line

E. The Goal (for teacher)

To know how far the background knowledge of students toward whole numbers and decimal numbers

F. Materials

Students' work sheet

G. Instructional Activities

1. Starting Point (5 minutes)

Teacher asks the students: "*Which of you are able to mention the numbers forward from 73 until 88*?"Of course most students will be able to mention it fluently. Then the teacher continues asking: *"Which of you are able to mention the numbers backward from 100 until 90?"* After the students are able to mention them, then the teacher tells the students*: "Today we will play the game, namely "Come Closer" game*."

2. Activities (30 minutes)

a. The teacher explains the rule of the game such in the following manner:

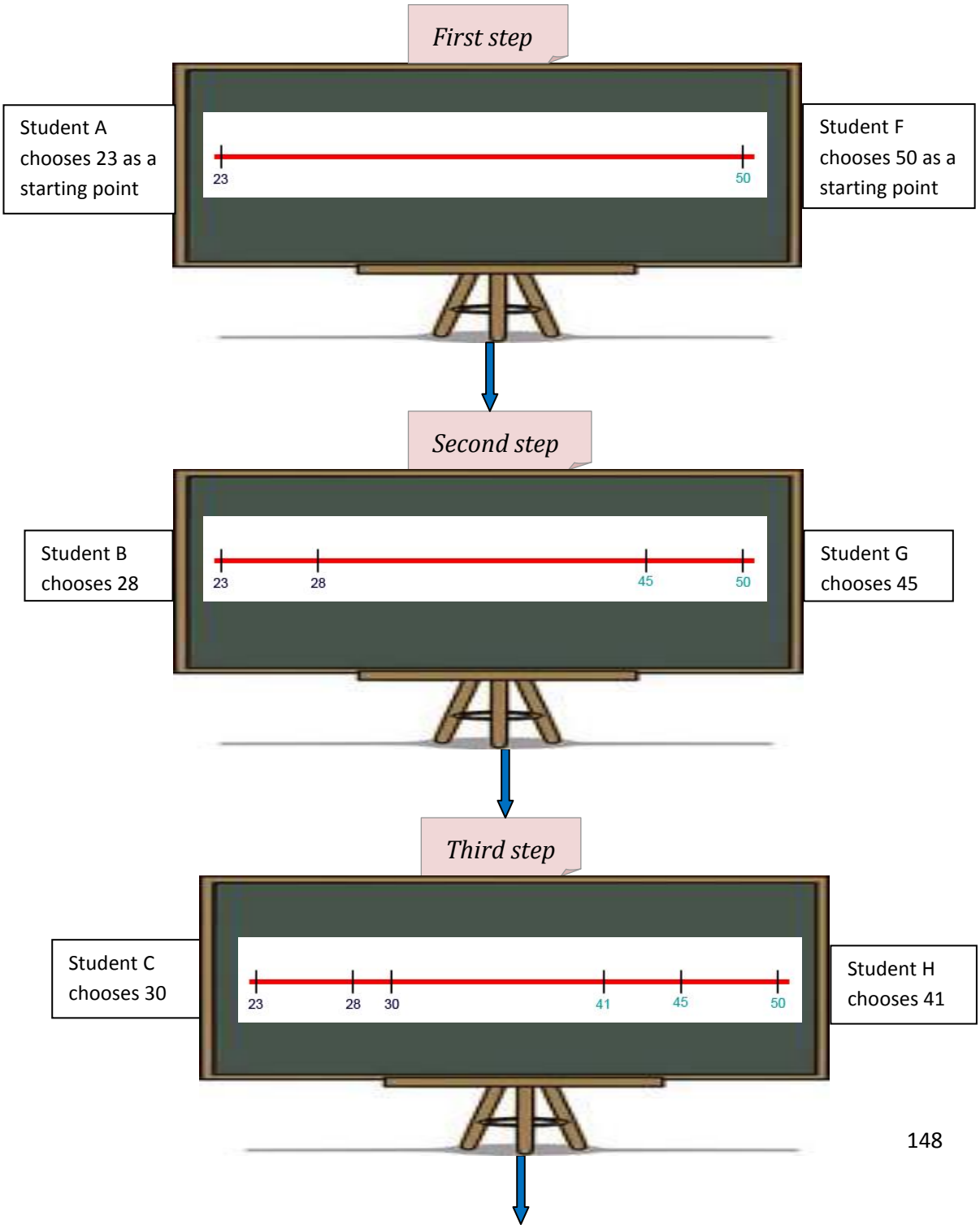
"This game will be played by two groups. Each group contains five students and one referee for each group."

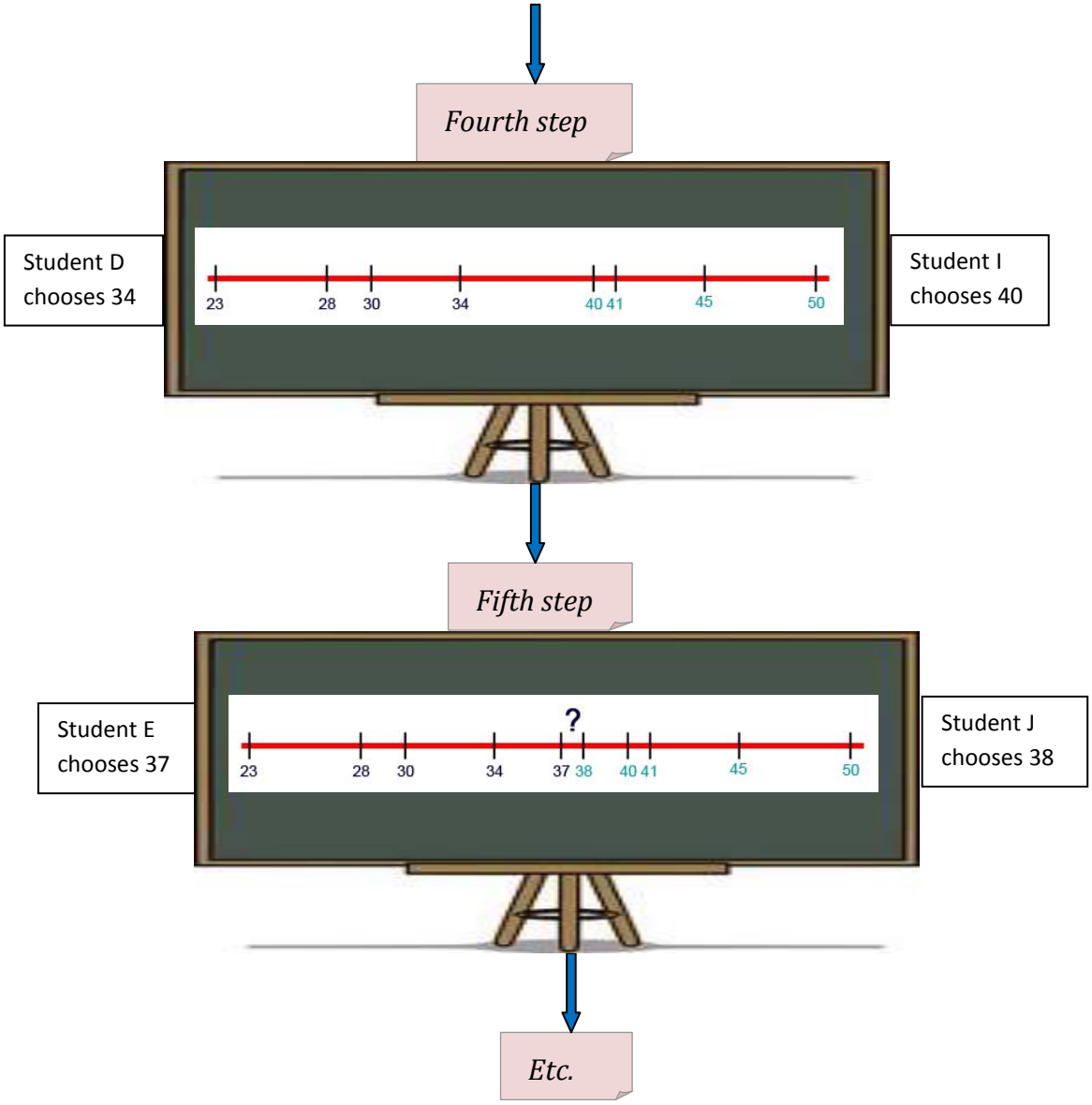
Group A (Student A, B, C, D, E)	VS	Group B (Student F, G, H, I, J)

Both groups have to come closer by nominating the numbers between two whole numbers in the range from 0until 100. One group goes forward, and the other group goes backward. Each group is free to nominate any kind of numbers but it is not allowed to cross each other. All members in each group must have

opportunity to nominate the numbers. The group who is not able to nominate the next number, it means that they lose.

b. First, the referee makes a line on the blackboard.
c. Each group nominates the number as a starting point, One group goes forward and the other group goes backward. For example, group A chooses 23 and group B chooses 50 as a starting point.
d. Then,
e. Both groups will come closer on the opposite direction by nominating the number (distance point) and marking that number in the line on the black board.
f. They make a toss to decide who will take the first turn, and the opponent takes the next turn after that, and then they take turns again; and so on until they face the condition such in the following example:





3. Discussion (30 minutes)

When the closer numbers are in consecutive whole numbers, for example 37 and 38, student E might think that his/her group is lose because (s)he has no chance to nominate the other numbers, and (s)he thinks that the game is ended. In fact, when (s)he proposes fraction such as 37½ or decimal numbers such as 37,5; 37,2; 37,8, etc. (s)he still has chance to continue and nominate the other numbers besides whole numbers. However, since it is conjectured that the students will only think about whole numbers, then probably the game will be ended until that.

The teacher lets the students to decide whether they will end the game in that position or try to continue the game with nominating the other numbers using fractions or decimals. Through this activity, we can see the pre-knowledge of students. When they are not able to nominate the other numbers between 37 and 38, it means that they are not aware that there should be other numbers in between two consecutive whole numbers (either fractions or decimal numbers). Officially, in grade IV students have already

learned about fraction and simple decimal form as division of two whole numbers, but do they realize about the position of fractions and decimals on number line?

The result of "*Come Closer*" game activity shows how far the students' pre-knowledge toward the numbers. It will determine the next stage of activities. For example, if until the end of the game, there is no student realized who realize that there were numbers (decimal numbers) in between two consecutive whole numbers, it give feedback for us that the students are not quite familiar with decimal numbers and their position in number line. It suggests that in the next activities (weighing activities) the students should be more encouraged with the existing of decimal numbers through discussing the form and how we call it (i.e 2,5 is two point five). However, if in the teaching experiment, there are some students who already know about decimal form and how they should call it, we don't need to discuss it anymore. We just focus into where the position of decimal forms in the measurement activities is.

4. Reflection of the activities (5 minutes)

Finally, the teacher asks the students: *"So, what is your conclusion about the game that is already played?"* Some students might answer that student E lose because (s)he cannot nominate the other numbers between 37 and 38. Then, the teacher say: "*In the next activity, we will prove that whether group A indeed lose or not."*

Starting from what they have known and how far their pre-knowledge toward decimal numbers, the teacher can continue to the next activity in order to convince the students that there are decimal numbers between two consecutive whole numbers.

H. Assessment

Kind of assessment : non test (interview and discussion class)

Teacher,	Researcher,
Neti Sutjiati, S.Pd.	Puri Pramudiani
NIP. 196204141983032013	NIM. 20092812006

Head Master

Yuniman, S.Pd.
NIP. 195806121979121007

TEACHER GUIDE

Topic	: Decimal Numbers
Grade	: V
Activity	: Measuring on Being Precise
Time	: 2 X 35 minutes
Meeting	: 2

A. Standard Competence

Using fractions in problem solving

B. Basic Competence

Converting fraction into percentages and decimals forms or vice versa

Measuring on Being Precise

C. Indicators

1. Students are able to measure the things using weight scale
2. Students are able to read the scale between pointer points
3. Students are able to measure the things precisely

D. Goal

1. Students are able to read the unlabeled numbers on the scale.
2. Students are able to measure the things precisely.
3. Students are able to find decimal numbers in the measurement activity.

E. Materials

1. Duku Palembang
2. Fruit weight scale
3. Body weight scale
4. Body digital weight scale
5. Students' work sheet

F. Instructional Activities

1. Starting Point (5 minutes)

First, the teacher asks the students: *"What should you eat in order to be health?"* It is conjectured that the students will answer: *"We have to eat the rice, **fruit**, vegetables, etc*." Then the teacher asks: "What kind of fruit that you usually buy?" The students might say various fruits. Then the teacher asks the students: *"What is the fruit originally from Palembang, Indonesia?"* The students might say "*Duku*".

2. Activities

a. Activity 1: Measuring the weight of Duku Palembang using weight scale (15 minutes)

The teacher tells the students that today we will experience measuring Duku directly, first, using weight scale. Students work in small group containing 4 or 5 students in each group. The teacher asks each group to measure some bunches of Duku in different weight and report it as precise as possible.

It has already been prepared that all weights of Duku are not in whole numbers. Since the requirement data should be as precise as possible, therefore if the scale shows to the unlabeled scale (between pointer points), then the students have to estimate and draw it precisely. In this case, we expect that the students will draw the representation of weight scale in which there is a sequence of numbers as a model of situation. They will measure the weight of Duku using fruit weight scale in which the pointer points are covered by sticker (see the figure below):

First, all pointer points are covered by sticker

b. Discussion of activity 1 (15 minutes)

When the weight of the fruit measured by students is in between 0 and 1 then:

a. Some students might draw what exactly is shown in weight scale:

b. Some students might draw number line to represent the weight scale:

When there is no student who come up into horizontal number line, the teacher can provoke the students by asking: *"If I have this line (empty number line), where will you put that scale?"* From this situation, we expect that the students will make number line as a model for reasoning the position of decimal numbers.

Each group puts their poster containing the result of the weight of Duku. After that, each group is given opportunity to present and explain about the scale drawn on their poster. Some students might determine the numbers by counting the lines. Some students who have heard about comma numbers in their daily life might think that because the numbers are in between 0 and 1, so it probably comma numbers (decimal numbers) such as 0,5; 0,6; 0,7, etc. The other students who have not yet known about decimal numbers might think that it is less than 1 kg.

c. Activity 2: Measuring the weight of body (15 minutes)

After the students measure the weight of Duku using weight scale, then they are asked to measure their body using weight scale and digital weight scale. Then, they are asked to compare it.

d. Discussion of Activity 2 (15 minutes)

The students work in small group containing 5 or 6 students. There will be 5 body weight scale and 1 body digital weight scale. Those weight scales have already been setup similar with each other. In each group, one student become representative to be weighed, and the other students observe and report it. First, each group measures the weight of one representative student using weight scale. It is conjectured that some students will report the data as precise as possible (using decimal numbers). The other students might still report the data using whole numbers because they do not weigh it precisely.

After all students measure their weight, they will compare their result by weighing their body using digital weight scale. Finally, we expect that the students will realize that indeed there are decimal numbers in between two consecutive whole numbers. After the students finished measuring their weight, then they write it on the poster and present it in front of the class.

To reinvent decimal numbers, the students who have already report the data on the poster, then they are asked to measure again their weight using digital weight scale. Now, they could see whether they measure it precisely or not. For example, when they weigh using weight scale, they probably report it as 26 kg, or it is more than 26 kg. However, using digital weight scale they could see that it is indeed 26,4 kg. Same as weighing the fruit, after the students present their report and give their argument of their answer, then they can see that the actual weight scale by opening the covered sticker in the pointer points on the scale.

G. Reflection of the activities (5 minutes)

The teacher asks the students: *"What is the advantage using decimal numbers in daily life?"* We expect that the students will give the answer: "*One of the advantages/benefit using decimal numbers in daily life is that we can know the measurement of the weight precisely/accurately.*" Furthermore, the teacher asks the students; *"What do you think? Is it only applicable for measuring the weight?"* We expect that the students will realize that decimal number is not only useful for measuring the weight, but also it can be useful for measuring the length, the height, the temperature, and the volume very precisely because sometimes when we measure them, it is not shown in the whole numbers.

H. Assessment

Kind of assessment : written test and non test (interview)

Teacher, Researcher,

Neti Sutjiati, S.Pd. Puri Pramudiani
NIP. 196204141983032013 NIM. 20092812006

Head Master,

Yuniman, S.Pd.
NIP. 195806121979121007

EXERCISE
Second Meeting

Name :
Class :
School :

1. a. How much the weight you weighed based on the weight scale? Write and draw your strategy in column below!

Answer:

b. How much the weight you weighed based on digital weight scale?

Answer:

c. Are the result of weight scale and digital weight scale same or not? Explain your reasoning!

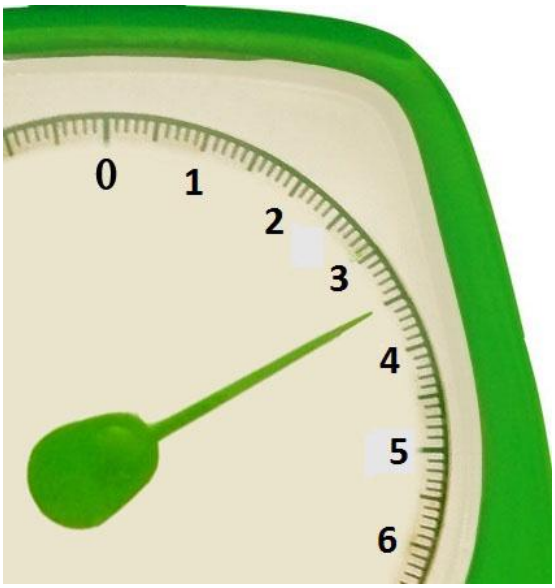
Answer:

2. How much the weight of the fruit (Duku) that you weighed? Write and draw your strategy in column below!

Answer:

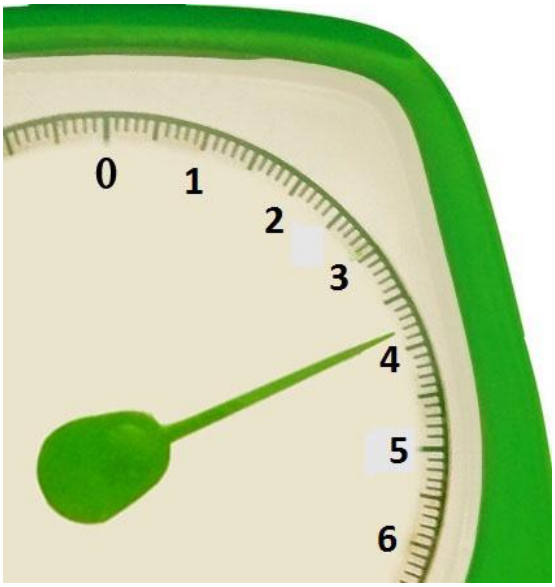
3. W

a.



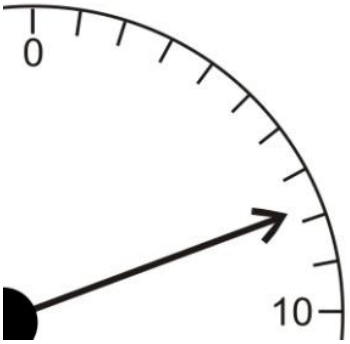
Answer:

b.



Answer:

c.



Answer:

==========**Good Luck**==========

TEACHER GUIDE

Topic	: Decimal Numbers
Grade	: V
Activity	: Measuring The Weight of Rice
Time	: 2 X 35 minutes
Meeting	: 3

A. Standard Competence

Using fractions in problem solving

B. Basic Competence

Converting fraction into percentages and decimals forms or vice versa

C. Indicators

1. Students are able to measure the things precisely
2. Students know about unit fraction (a half, a tenth, etc)
3. Students can explore one-digit decimal numbers

D. Goal

Students are able to explore the notion of one digit decimal numbers and put them on the number line

E. Materials

1. Rice
2. Weight scale
3. Students' work sheet

F. Instructional Activities

1. Starting Point (5 minutes)

The teacher asks the students: *"What is the main food in Indonesia?"* All students might say: "*Rice*". Then, the teacher asks the students: *"How much the weight of rice that you usually eat in one day?"* The answer may vary. Some students might answer: *"0,5 kg."* Then, the teacher further asks: *"How about the weight of rice that you usually eat once?"* The answer may vary. Some students might say: *"0,1 kg."* Then, the teacher asks: *"What is the meaning of 0,1 kg? And what is the meaning of 0,5 kg?"* It is conjectured that the students still get difficulties to explain it.

2. Activities (15 minutes)

The teacher provides some packs of rice in different weights. The students are asked to investigate those weights. One volunteer student is asked to do the experiment in front of the class, and the other students observe it. There will be 10 packs of 0,1 kg rice and two packs of 0,5 kg rice. One volunteer student will be given opportunity to measure using weight scale. Then (s)he will see how much kg there are.

3. Discussion (30 minutes)

The teacher asks the students: *"How much its weight*?" (one pack of 0,1 kg). By observing the position of the line, the students might use their experience in the previous activity that when the line shows to the first line between 0 and 1, then it is probably 0,1 kg. Then, it will be proven by opening the covered sticker. Then, the teacher asks that student to weigh another pack of rice which has the same weight. And it is indeed 0,2 kg. After that, (s)he is asked to weigh the third rice which still have the same weight, and it is indeed 0,3 kg. From this activity, they can see in the weight scale that 1 pack is equal to 0,1; 2 packs of 0,1 kg is equal to 0,2 kg, 3 packs of 0,1 kg is equal to 0,3 kg; etc.

The teacher asks one student to predict that how much the weight that if they weigh 4 packs, 5 packs, … until 10 packs. The teacher asks: "*What number that might come up when I put ten packs of 0,1 kg?"* It is conjectured that some students will answer that it is 0,10 kg because they have already noticed that the sequence of packs of 0,1 kg is 0,1; 0,2; 0,3; ….,0,8; 0,9; then might be 0,10 kg after that. However, some students might already know that ten packs of 0,1 kg is equal to 1 kg because they know that the number after 0,9 is 1. When there is two different answers, the discussion will be raised and it will be proven through measuring it directly using weight scale in which ten packs of 0,1 kg is 1 kg, not 0,10 kg.

In the same manner, the students will weigh the pack of rice that they usually eat for one day (0,5 kg), and they could see that two packs of 0,5 kg is 1 kg. We will encourage the students in order that they think about the position of each weight. For example, the teacher can ask: *"0,1 is in what line?"* We expect that students will answer: *"It is in the first line".* Then, we can ask further: *"From how many lines?"* We expect that students will answer: *"From ten lines"*

0,1 is in the first line from ten lines overall (one out of ten) = $\frac{1}{10}$

0,5 is in the fifth line from ten lines overall (five out of ten) = $\frac{5}{10}$

Consequently,

0,2 = $\frac{2}{10}$ *because it is in the second line from ten lines overall (two-tenth)*

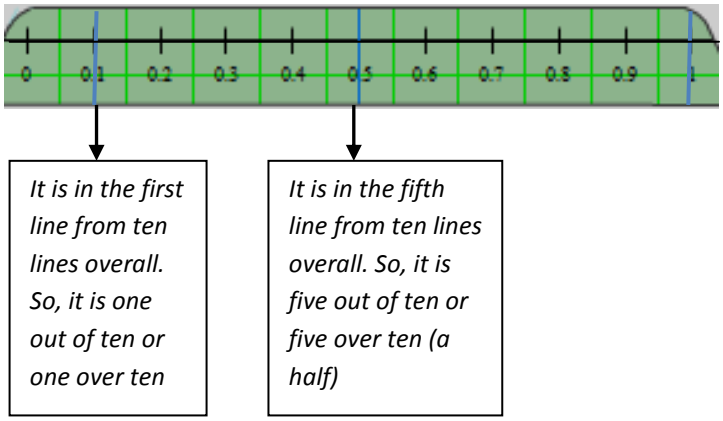
0,3 = $\frac{3}{10}$ *because it is in the third line from ten lines overall (three-tenth)*

0,4 = $\frac{4}{10}$ *because it is in the fourth line from ten lines overall (four-tenth), etc.*

Therefore, it can be concluded that the word 'decimal' refers to a base ten number that is written with a decimal point.

During the experiment is done by volunteer student in front of the class, the other students write and draw the weight shown in weight scale and digital weight scale.

The students are asked to discuss their invention. After that, one student\ is asked to report it in front of the class. When the teacher asks*: "What do you see in the scale when you measure 10 packs of 0,1 kg?" Can you draw it on the board?"* It is conjectured that the student will draw number line such in the following manner:



G. Reflection of the activities (5 minutes)

In this activity, students will recognize about the simple decimal forms in which 0,1 is a tenth of 1 or one over ten and 0,5 is a half of 1 or five over ten;. Some students might think that those numbers are the numbers less than 1. So, when they are asked *"Is there any number between 0 and 1?"* Now they might answer "*Yes*", 0,1; 0,2; 0,3; 0,4; 0,5; etc. are the numbers between 0 and 1.

To strengthen that concept and to keep that in students' mind, one action that can be done is by singing "Decimal Song":

BAHASA VERSION

BAHASA VERSION

Desimal…aha..aha…
angka berkoma…
Desimal…uhu…uhu…
Banyak gunanya…
Jika kau mulai bingung…
Mengukur tapi tak pas…
Jika kau mulai bingung…
Membagi tapi tak habis…
Desimal solusinya…
Desimal paling mudah…
Desimal solusinya….
Desimal paling mudah…
Nol koma satu..satu per sepuluh
Nol koma lima…Itulah setengah…..
DESIMAL…SIAPA TAKUT???

ENGLISH VERSION

Decimal…. aha…aha… it's comma number…
Decimal…uhu…uhu…there're many benefits…
If you start to be confused…measuring but it's not fit…
If you start to be confused….dividing but it's not finished…
Decimal's the solution….decimal is easier….
Decimal's the solution….decimal is easier…..
Zero point one…that's one over ten…
Zero point five…it means that a half….

DECIMAL…WE ARE READY…..!!!

H. Assessment (15 minutes)

Kind of assessment : written test and non test (interview)

Teacher, Researcher,

Neti Sutjiati, S.Pd. Puri Pramudiani

NIP. 196204141983032013 NIM. 20092812006

Head Master

Yuniman, S.Pd.

NIP. 195806121979121007

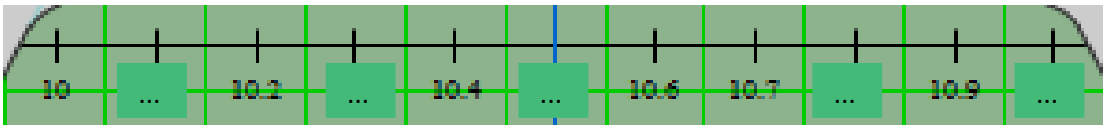
EXERCISE
Third Meeting

Name :
Class :
School :

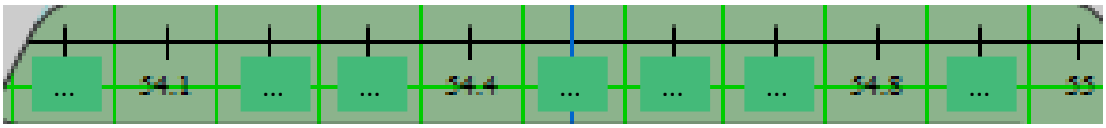
---

1. Convert the decimal numbers below into fraction and vice versa:
   a. 0,5 = $\frac{\cdots}{\cdots}$
   b. 0,1 = $\frac{\cdots}{\cdots}$
   c. 0,7= $\frac{\cdots}{\cdots}$
   d. 12,5 = $\frac{\cdots}{\cdots}$
   e. 23,8 = $\frac{\cdots}{\cdots}$
   f. $\frac{6}{10} = \cdots$
   g. $\frac{18}{10} = \cdots$
   h. $\frac{1}{2} = \cdots$
   i. $\frac{117}{10} = \cdots$
   j. $\frac{481}{10} = \cdots$

2. Find the numbers in the column below!
   a.



   b.



3. Put the numbers below on number line:
   a. 13,7; 12,9; and 12,3
   b. 33,3; 32,6; 33,7; and 32,1

Answer:

==========Good Luck==========

TEACHER GUIDE

Topic	: Decimal Numbers
Grade	: V
Activity	: Measuring The Volume of Beverages
Time	: 2 X 35 minutes
Meeting	: 4

A. Standard Competence

Using fractions in problem solving

B. Basic Competence

Converting fraction into percentages and decimals forms or vice versa

C. Indicators

1. Students are able to measure the volume precisely
2. Students know about the notion of one digit decimal numbers
3. Students can explore the notion of two-digit decimal numbers.

D. Goal

Students are able to explore the notion of two digit decimal numbers and put them on the number line

E. Materials

1. Beverages
2. Measuring cup
3. Students' work sheet

F. Instructional Activities

1. Starting Point (10 minutes)

The teacher reviews previous activity by asking the students to sing "Decimal song". From the previous activity, they have already known about the notion of one digit decimal numbers is in between two consecutive whole numbers and also they know the meaning of simple decimal numbers (0,5 is a half or five over ten; 0,1 is one over ten). In this activity they will be experienced with exploring the notion of two digit decimal numbers. The teacher asks the students: *"Yesterday I bought two beverages that have different volume. Beverage A is 0,25 liter and beverage B is 0,5 liter. What do you think? Which is more, beverage A (0,25 liter) or beverage B (0,5 liter)?"* It is conjectured that some students will answer that 0,25 liter is more than 0,5 liter because they assumed that 25 is larger than 5. The teacher provides two different bottles which have no label, so the students do not know how much it is.

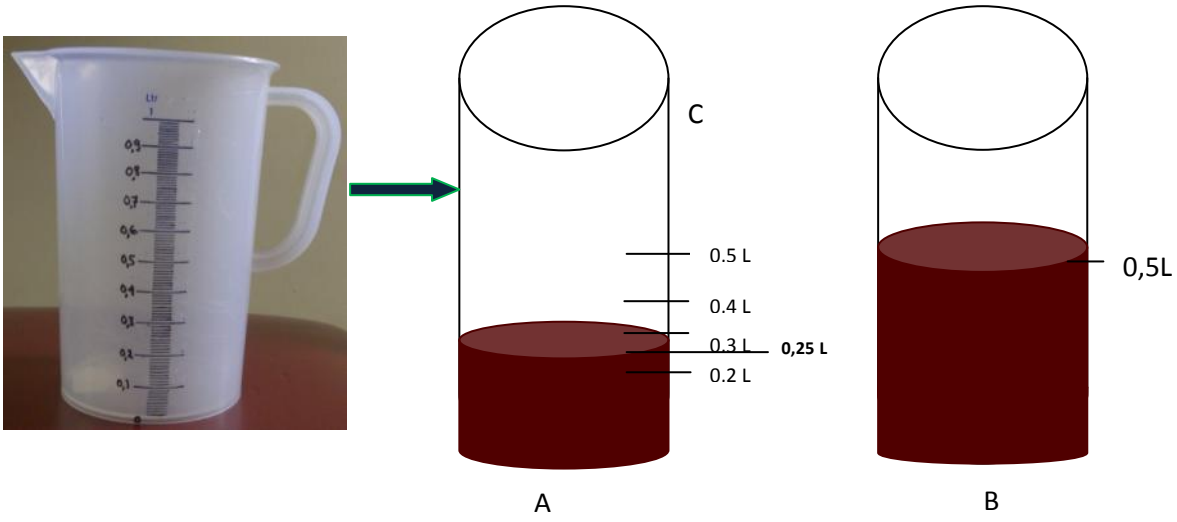
1. Activities (20 minutes)

The students who answer that 0,25 liter is more than 0,5 liter are asked to choose which bottle is 0,25 liter and which bottle is 0,5 liter. It is conjectured that (s)he will answer that the smaller bottle is 0,5 liter and the larger bottle is 0,25 liter. Then, the teacher asks him/her to pour the beverage in the larger bottle (beverage B)

into measuring cup. First, the larger beverage measured is 0,5 liter. Then (s)he will realize that his/her answer is not correct. It means that the smaller bottle should be 0,25 liter. Then, the teacher asks the students: "Where is the position of 0,25 liter?" Some students might answer: *"It is in between 0,2 and 0,3"* Then, the teacher provokes the students: *"In what line is it?"* Some students might count the small lines, and it is indeed the 25th line. The teacher asks them further: *"From how many lines overall?"* Some students might say: *"100 lines"*. The teacher asks: *"25 out of 10 is how much?"* Some students might use their knowledge about fraction that 25 out of 10 is $\frac{25}{100}$. If no student is able to mention it, the teacher can review the previous activity by asking the students: *"5 out of 10 is how much?"* It is expected that the students remember that 5 out of 10 is $\frac{5}{10}$. Therefore, 25 out of 100 is $\frac{25}{100}$.

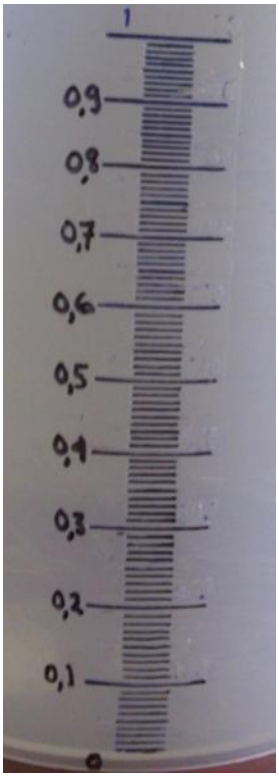
Finally, we expect that the students can realize that one-digit decimal number refers to tenth and two-digit decimal number refers to hundredth. That is why decimal is called "the number refers to base ten" because when there are two consecutive whole numbers and we divide into ten fragments, we can see that there were one-digit decimal numbers. And when we divide the fragment of two consecutive one-digit decimal numbers, there were another ten fragments of two-digit decimal numbers, and so on.

Our focus here is not about converting decimal numbers into fractions and vice versa, however we expect that it can become an initial understanding that the position of one-digit decimal numbers are in between two-consecutive whole numbers, and the position of two-digit decimal numbers are in between one-digit decimal numbers (see figure below).



Our conjecture is that the students will draw number line (either in horizontal or vertical) as a model for reasoning the position of two-digit decimal numbers. After that, each group will measure various volume of beverages, such as 0,35 liter; 0,48 liter, etc. (there will be 5 measuring cup for five groups). We expect that they can see

that 0,35 is in the 35th line from 100 lines. When they convert into fraction, 35 out of 100 is $\frac{35}{100}$.



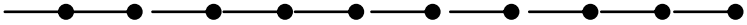
For example:

In between 0 and 1, when those are divided become ten fragments, there are

0; 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; 1

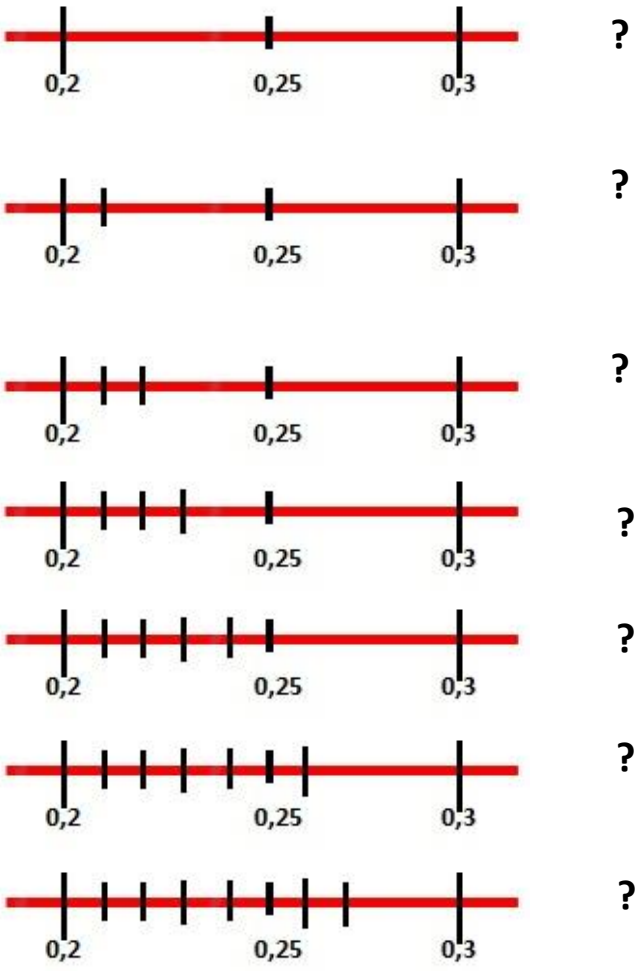
In between 0,1 and 0,2, when those are divided become another ten fragments there are

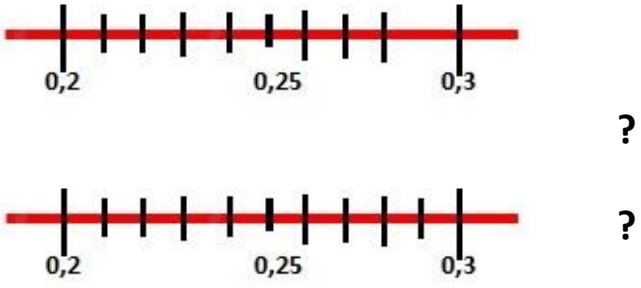
0,1; 0,11; 0,12; 0,13; 0,14; 0,15; 0,16; 0,17; 0,18; 0,19; 0,2



And so on.

Some students might use the strategy of determining a half first, and determine the other numbers beside that (see figure below):





Kita harapkan melalui kegiatan ini siswa dapat menentukan bilangan-bilangan di antara 0,2 dan 0,3 seperti berikut ini:

2. Discussion (20 minutes)

After volunteer student do the experiment, then the teacher asks the students: *"What do you think now? How if I compare 0,25 liter and 0,5 liter beverages? Which one is more?"* After observing the volume of beverages in the cups, then we hope the students will realize that 0,25 is less than 0,5. They can see in measuring cup that the position of 0,25 liter is in between 0,2 and 0,3. Then, the teacher asks the students: *"Why it should be written as 0,25 liter, not as 0,2 liter?"* Then probably some students will answer *"It is to be more precise (accurate).*" It is same with the case in weight measurement activity (activity 2), when the scale is in between 0 and 1 and it is more than a half, then it might be 0,6.

G. Reflection of the activities (5 minutes)

The teacher asks the students: *"What can you conclude from this activity?"* We hope that the students will realize that between two consecutive one digit decimal numbers there are two digit decimal numbers. Besides that we hope the students also realize that the longer digit decimal numbers, the more precise the measurement.

H. Assessment (15 minutes)

Kind of assessment : written test and non test (interview)

Teacher,	Researcher,
Neti Sutjiati, S.Pd.	Puri Pramudiani
NIP. 196204141983032013	NIM. 20092812006

Head Master

Yuniman, S.Pd.
NIP. 195806121979121007

EXERCISE
Fourth Meeting

Name :
Class :
School :

1. Based on the experiment you tried, which is more, 0,25 liter or 0,5 liter? Explain your reasoning!

Answer:

2. a. How much the volume of beverage that your group measured?

Answer:

b. Draw its position on the number line!

Answer:

3. Find the numbers on the number line below!

4. Find the numbers between 15,7 and 15,8 and put them on number line!

Answer:

5. a. Find the position of 7,23 on number line!

Answer:

1. b. Find the position of 14,29 on number line!

Answer:

==========Good Luck==========

TEACHER GUIDE

Topic	: Decimal Numbers
Grade	: V
Activity	: Playing Spaceflight Game
Time	: 2 X 35 minutes
Meeting	: 5

A. Standard Competence

Using fractions in problem solving

B. Basic Competence

Converting fraction into percentages and decimals forms or vice versa

C. Pre Knowledge of Students

1. Students know about the notion and the magnitude of one and two digit decimal numbers
2. Students have experienced in using computer
3. Students can explore one-digit and two-digit decimal numbers

D. Goal

Students are able to explore one and two digit decimal numbers in number line

E. Materials

1. Students' work sheet
2. Computer

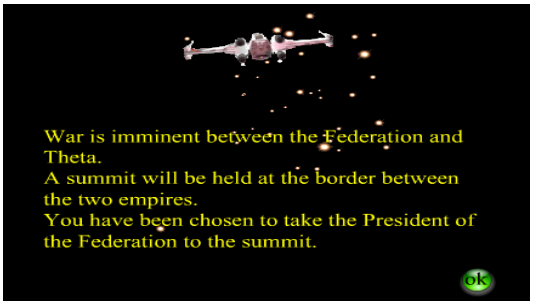
F. Instructional Activities

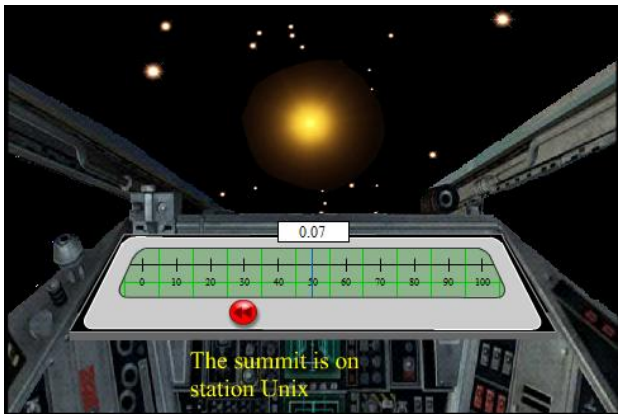
1. Starting Point (5 minutes)

Teacher asks the students to retell about the previous activity that in between two consecutive whole numbers there are decimal numbers and in between two consecutive one-digit decimal numbers there are other decimal numbers (two-digit decimal numbers).

2. Activities (35 minutes)

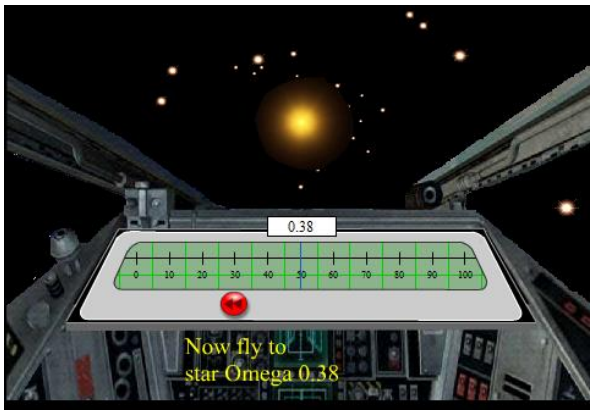
The activity is playing the games, namely "*Spaceflight*". The students will practice to explore the numbers including whole numbers, one digit, and two digit decimal numbers using number line. In this game, they have to explore about the certain point (decimal numbers) to find the message code. We will assess how far their understanding of one and two digit decimal numbers. When they can find the numbers quickly and certainly, it means that their understanding is better than the other students who explore the numbers slowly and doubtfully. This game is online and can be found in http://www.fi.uu.nl/toepassingen/03127/task3.html designed by Frans van Galen.




0.07
The summit is on
station Unix


0.07
Message to base:
code six dot six, mission completed.


89.89
From here set course to
star Kappa 89.89


0.38
Now fly to
star Omega 0.38

Each student plays the game in one computer maximum 5 minutes for each student. To find the secret code, they have to pass four steps i.e finding the position of 12,26; 89,89; 0,38; and 0,07. The range given may vary. The level of difficulties of the game develops starting from the easiest problem to the most difficult problem, i.e:

1) 12,26 in between 12 and 13,
2) 89,89 in between 80 and 90;
3) 0,38 in between 0 and 10;
4) 0,07 in between 0 and 100.

At the end of activity, the students are asked to give the report of their game through putting the position of each step on the number line. We expect that the students could make the sequence of one-digit and two-digit decimal numbers and put them on the number line and also they understand about the concept of density of numbers on the number line in which between two consecutive whole numbers there were one-digit decimal numbers, and between two consecutive one-digit decimal numbers there are other decimal numbers (two-digit decimal numbers). We expect that the students can use this knowledge to compare one-digit and two-digit decimal numbers, and we can reduce the students' misconception that the longer digit decimal number was the larger for its magnitude.

3. Discussion class (15 minutes)

After all students finish playing the game, the teacher leads them into discussion by asking*: "What do you think about the game that you play? How about the numbers in that game?"* We expect that students will realize that when they **zoom** the range between two the numbers in the number line, there is density of numbers in that number line. The teacher can asks the students: *"Please rewrite and draw in your work sheet, where is the position of*: 12,26; 89,89; 0,38, and 0,07. After that the teacher asks some students to write it on the board and the other students correct it.

G. Reflection of the activities (5 minutes)

The teacher asks the students: *"What can you conclude about this activity?"* We hope that the students will feel for sure that between two consecutive whole numbers there are one digit decimal numbers, and between two consecutive one digit decimal numbers there are two digit decimal numbers. We hope through this knowledge, it can bring the students' understanding toward decimal numbers itself, especially when they are asked to compare one and two digit decimal numbers. In the next activity, we will provide the contextual problem related to comparing the magnitude of one and two digit decimal numbers.

H. Assessment (10 minutes)

Kind of assessment : playing spaceflight game on the computer and written test

Teacher,	Researcher,
Neti Sutjiati, S.Pd.	Puri Pramudiani
NIP. 196204141983032013	NIM. 20092812006

Head Master

Yuniman, S.Pd.
NIP. 195806121979121007

EXERCISE
Fifth Meeting

Name :
Class :
School :

Find the secret code in Spaceflight game and draw each position of each stage on number line!

First position : .....

Second position : .....

Third position : .....

Fourth position : .....

What is the secret code (mission completed)?

TEACHER GUIDE

Topic	: Decimal Numbers
Grade	: V
Activity	: Solving Contextual Problem about Comparing The Magnitude of One and Two Digit Decimal Numbers
Time	: 2 X 35 minutes
Meeting	: 6

A. Standard Competence

Using fractions in problem solving

B. Basic Competence

Converting fraction into percentages and decimals forms or vice versa

C. Indicators

1. Students know about the notion and magnitude of one and two digit decimal numbers
2. Students know about density of numbers in number line
3. Students know about the idea of the longer digit decimal numbers, the more precise the measurement
4. Students can compare one-digit and two-digit decimal numbers.

D. Goal

Students are able to compare decimal numbers in one and two digit numbers

E. Materials

Students' work sheet

F. Instructional Activities

1. Starting Point (15 minutes)

The students are asked to retell the activities and what they have learnt from activity 1 through activity 5.

G. Activities (25 minutes)

The teacher gives the problem below:

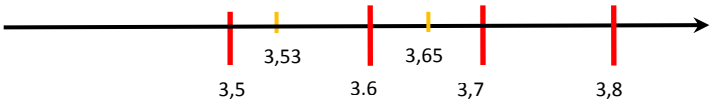
1. *Ayuk goes to the market to buy the meat. There are 3 Kiosk which offer the same price for 1 pack of meat. The weight for each pack in each kiosk can be seen in below figure. Which one that should be bought by Ayuk? Explain your reasoning!*

Kiosk A	Kiosk B	KioskC
3,53 kg/pack	3,8 kg/pack	3,65 kg/pack

H. Discussion and Reflection (30 minutes)

Each group is asked to solve the problem and write on the poster. After that, they are asked to present it in front of the class.

- Some students might give the correct answer and give the correct reasoning that they choose Kiosk B because Kiosk B is the heaviest compared to the others. It is conjectured that they compare it using left to right comparison that determine the right position is the larger. When they give this strategy, it means that they have known about the magnitude of one and two digit decimal numbers. Therefore they can determine that 3,8 > 3,65 > 3,53 because when they put those numbers in number line, then it should be:



- Some students might choose Kiosk B because Kiosk B is the cheapest. With the same price, in Kiosk B they can get more meat.
- There is also possibility that the students who give the correct answer because they think when they have 3,8; 3,65; and 3,53 kilogram, then they convert those numbers into gram, so they will have 3800 gram, 3650 gram, and 3530 gram. It shows that the meat in Kios B is the heaviest one because 2900 > 2720 > 2650.
- For students who don't give the correct answer, maybe they still think the comma numbers like in the whole numbers, 72 is larger than 65 and 9. For students who still don't understand about the concept of magnitude of decimal numbers at the end of activity, they should be given more attention in order to know what are their difficulties and in which activity they couldn't follow the instruction.

Reflection of the activity

Finally we hope that the students can gain their understanding and make the connection between formal level (comparing magnitude of one and two digit decimal numbers) and informal level (measurement activities reasoning).

So, when the students are asked which is the larger between 12,4 and 12,17; now we expect that the students can answer that 12,4 is larger than 12,17 because on the number line 12,4 is in between 12,3 and 12,4 whereas 12,17 is in between 12,1 and 12,2. So, the position of 12,4 is in the right position of 12,17.

I. Assessment

Kind of assessment : written test

Teacher,

Neti Sutjiati, S.Pd.
NIP. 196204141983032013

Researcher,

Puri Pramudiani
NIM. 20092812006

Head Master

Yuniman, S.Pd.
NIP. 195806121979121007

EXERCISE
Sixth Meeting

Name :
Class :
School :

a. Ipin and Upin bought same kind of beverages. The volume of Ipin's beverage is 0,35 liter whereas the volume of Upin's beverage is 0,6 liter. What do you think? Which beverage is more than the other? Explain your reasoning!

Answer:

2. Ayuk goes to the market to buy the meat. There are 3 Kiosk which offer the same price for 1 pack of meat. The weight for each pack in each kiosk can be seen in below figure. Which one that should be bought by Ayuk? Explain your reasoning!

Kios A

3,53 kg per pack

Kios B

3,8 kg per pack

Kios C

3,65 kg per pack

Answer:

3. Find the numbers which is larger than 3,6 but smaller than 4 as much as possible and put them on number line!

Answer:

4. Find the numbers which is larger than 45,3 but smaller than 45,59 as much as possible and put them on number line!

Answer:

a. Arrange these numbers starting from the smallest through the largest numbers:
3,78; 3,5; 83,1; 66,89; 54,44; 79,9; 6,8; 79,89; 33,6; 2,25

Answer:

==========Good Luck==========

APPENDIX

The result of post assessment (second cycle)

For analyzing the result of post assessment, we classified into three categories, i.e. correct answer and correct reasoning, correct answer and incorrect reasoning, and incorrect answer and incorrect reasoning. From 26 students, we classified it based on strategies that they used. See table below:

Mathematical Idea	Correct answer and correct reasoning	Correct answer and incorrect reasoning	Incorrect answer and incorrect reasoning
Reading the scale	- Dimas, Firman, Nova, Sapta answered perfectly. They gave the reason it was 3,7 because it was in between 3 and 4, exactly in the seventh line. - Tiara, Cindy, Fifi, Mutia, Intan used the strategy of counting the lines (stripes). (the needle shown is in the seventh line) - Farhan, Ahmad, Feby, Kresna, Sella determined the scale and gave the reason that it was in between two consecutive whole numbers (3 and 4). - Auliya, Fadhil, and Arissandy gave the reason because it was in the point of 3,7 - Arrahman answered 3,7 and his reason is because it is in 37th line from 10 lines overall. 18 students = 69 %	- Rizky answered that it was 3,7 but her reason because it was more. 1 student = 4%	- Ajeng, Amel, thought that it was 0,7 which is in between 3 and 4 and it was 0,3 between 6 and 7. - Linda, Rico did misscounting. They assumed that the scale was 3,8. Indeed it should be 3,7. - Michell and Rizkiah answered that it was 37. She did not put point (comma) in it. - Nia answered that it was 32. She did not put point (comma) in it. 7 students = 27%
Knowledge of decimal numbers	- Tiara, Ajeng, Arissandy, Farhan, Firman, Rizkiah determined the sequence of	- Rizky answered that Desi's score was 8, Dina's score was 9, and Rika's score was	- Amel made the sequence of decimal numbers starting from 7,11; 7,12; 7,13; 7,14; 7,15 in between 8 and

Mathematical Idea	Correct answer and correct reasoning	Correct answer and incorrect reasoning	Incorrect answer and incorrect reasoning
	decimal numbers between 8 and 9, and putting them on the number line (8,1; 8,2; 8,3…, 8,9). - Same also with Sapta. He drew number line strarting from 8,1 till 8,9. But he restrict his answer for Rika's score was 8,5. - For Kresna, even though he mentioned decimal numbers starting from 8,1 till 8,9 but he gave the reason because Rika's score should be a half from Dina's score and Desi's score. - Cindy, Intan, made the sequence of decimal numbers on number line and determined Rika's score as 8,8 - Arrahman, Feby, Fifi, Michell, Nova answered that Rika's score could be one of 8,1 or 8,2 or 8,3 or 8,4 or 8,5 or 8,6 or 8,7 or 8,8 or 8,9.(they did not draw number line). - Auliya determined Rika's score by drawing number line and he gave the reason because without drawing number line, he could not find Rika's score carefully.	8,5. So, the highest score was 8,5.	9. - Nia put 7 in between 8 and 9. She answered that Rika's score was 7. - Ahmad determined the sequence of decimal numbers between 8 and 9 correctly; however his answer of Rika's score was 7. It means that he still did not understand toward the problem.

Mathematical Idea	Correct answer and correct reasoning	Correct answer and incorrect reasoning	Incorrect answer and incorrect reasoning
	- Dimas made number line containing whole numbers starting from 0 till 17, and he made an arrow in the middle between 8 and 9, and determined 8,5 as Rika's score. - Same with Dimas, but Rico, Fadhil drew number line strating from 0 till 10. - Linda, Mutia, Sella only answered Rika's score should be 8,5 because she was the second winner, whereas the score of the first winner was 9 and the score for the third winner was 8. 22 students = 85%	1 student = 4%	3 students = 11%
Density of numbers on number line	- Tiara, Cindy, Dimas, Farhan, Feby, Nova, Sapta, Sella divided the range between two consecutive one-digit decimal numbers become ten partitions, and determined the sequence of two-digit decimal numbers in them. - Kresna, Firman, also divided each range became ten fragments but he involved zero after comma in each edge. - Ahmad, Ajeng, Arissandy, Intan, made the sequence of two-digit	- Nia and Rizky only drew number line containing one-digit decimal numbers (for problem a). She did not solve the problem b at all containing two-digit decimal numbers.	- Amel and Linda made the sequence starting from 1; 1,4; 1,5; 1,6; 1,7; 1,8; 1,9; 1,10; 1,11; 1,12; 1, 13; 1,14, etc. - Rico did miscounting. After he drew number line starting from 1,1 till 1,9 then he continued with 1,2,1; 1,2,2; 1,2,3; 1,2,4; etc. - Fadhil determined two-digit decimal numbers but he did miscounting. He wrote after 1,49 was 5 not 1,5.

Mathematical Idea	Correct answer and correct reasoning	Correct answer and incorrect reasoning	Incorrect answer and incorrect reasoning
	decimal numbers (involving zero after comma), - Arrahman started from 1,5; 1,6; 1,7; 1,8; 1,9; 1,20, 1, 21; 1,22; 1,23; etc. (correct) - Auliya, Rizkiah first made the sequence of two digit decimal numbers containing zero after comma, and then they erased the zero. - Fifi made the sequence of two-digit decimal numbers but only from 100,17 till 100,36. When the researcher interviewed her why she did not finish her work, she said that because it was too much. - Michell also only drew number line containing one-digit decimal numbers for problem a, and drawing number line starting from 100,11 till 100,19 for problem b. - Mutia also only drew number line containing one-digit decimal numbers for problem a, and drawing number line starting from 100,25 till 100,32 for problem b. 20 students = 77%	2 students = 8%	4 students = 15%

Mathematical Idea	Correct answer and correct reasoning	Correct answer and incorrect reasoning	Incorrect answer and incorrect reasoning
Comparing the magnitude of one-digit and two-digit decimal numbers (contextual problem)	- Tiara determined 12,8 as decimal as tenth, and 12,24 as decimal number as hundredth. - Ahmad, Ajeng, Amel, Arissandy, Dimas, Intan, Kresna, Linda Michell, Sella were only gave the reason that 12,8 was more than 12,24 (without drawing number line). Sella added the reaso of the position of each weights (in between...) - Arrahman answered that 12,8 was more than 12,24 (by drawing number line, and he made the relation between decimal and fraction in which 12,8 contains 80/100, and 12,24 contains 24/100. - Even though the question was what do you think? which is the sweeter milk? But Auliya answered that Jojo should choose beverage B (12,24 gram sugar) because when she chose beverage A (12,8 gram sugar) it will be too sweet. - Cindy, Farhan, Feby, Intan, Rico, Sapta, determined beverage B was sweeter than beverage A by drawing number		Fadhil answered that 12,24 gram sugar was sweeter than 12,8 gram sugar without giving the reason.

Mathematical Idea	Correct answer and correct reasoning	Correct answer and incorrect reasoning	Incorrect answer and incorrect reasoning
	line. - Firman determined beverage B was sweeter than beverage A by drawing number line and he used the strategy of left to right comparison as his reason. - Nova, Fifi, Mutia, Rizkiah determined beverage B was sweeter than beverage A because she said that 12,8=12,80 whereas 12,80 was more than 12,24. - Nia answered that Jojo should buy beverage B containing 12,8 gram sugar because beverage B was more guaranteed. 25 students= 96%	0 student= 0 %	1 student = 4%
Comparing the magnitude of one-digit and two-digit decimal numbers (contextual problem)	- Tiara, Arissandy, Fifi, Nova, Rizkiah, Sapta, determined that 6,9 kg was the heaviest one. They only ordered the numbers without drawing number line. - Amel, Dimas, Farhan, Feby, Intan, Kresna, Linda, Rico, Sella determined that Kiosk D was the heaviest one (6,9 kg) compared to the others, and they drew number line and ordered the numbers completely.	- Ahmad answered that 6,79 was the heaviest one. - Ajeng chose Kiosk A because she thought that 6,05 was the heaviest one compared to the others. - Cindy answered Kiosk D (6,9 kg/pack) but her reason was because it was guaranteed. When Cindy ordered the weights, she made incorrect answer and she put 6,05 after 6,32 and before 6,79. May be she thought that 6,05 like 6,5.	

Mathematical Idea	Correct answer and correct reasoning	Correct answer and incorrect reasoning	Incorrect answer and incorrect reasoning
	- Arrahman's strategy was same with Amel's and he added the strategy of left to right comparison as his reasoning. He also could determine that 6,05 was in between 6 and 6,1, and he made the sequence completely. - Auliya, Firman, Fadhil chose Kiosk D with the reason because it was the heaviest one. And they ordered those weights by drawing number line. - Mutia answered Kiosk D (6,9 kg/pack) but her reason was because it was guaranteed. Mutia ordered the numbers correctly 20 students= 77%	- Michell, Nia, Rizky answered Kiosk D (6,9 kg/pack) was the heaviest oine compared to the others but they ordered the numbers incorrectly: (Michell: 6,79; 6,32; 6,25; 6,9; 6,05) (Nia and Rizky: 6,9; 6,05; 6,25; 6,32; 6,79). 6 students = 23%	0 student = 0%

THE RESULT OF INTERVIEW AT THE END OF ACTIVITY (FIRST CYCLE)

1. Interview with Zulfa

Researcher : *"What did you learn from the series of activities?"*
Zulfa : *"Numbers, decimal numbers…"*
Researcher : *"What is decimal number?"*
Zulfa : *"The numbers containing comma"*
Researcher : *"Can you give the example?"*
Zulfa : *"0,1"*
Researcher : "*What is the meaning of 0,1?"*
Zulfa : "*Mmmm……(*he was quiet for a while and said)*…."One over ten"*
Researcher : "*How about 0,5?"*
Zulfa : *"A half"*
Researcher : *"Why 0,5 is a half"*
Zulfa : (He was quiet around 20 seconds).
Then the researcher asked further
Researcher : "*In the previous time, you know 0,5 is a half from which activity?"*
Zulfa : He was quiet again around 30 seconds. First he said: *"numbers activity"..*but finally he said *"weighing activity."*
Researcher : *"How do you know that 0,5 is a half?"*
Zulfa : (He was quiet again)
Researcher : *"How many packs of rice (0,5 kg) that make 1 kg?"*
Zulfa : *"Two"*
Researcher : *"So, why 0,5 is a half?"*
Zulfa : *"Because 0,5 plus 0,5 is <u>zero</u> over ten, and zero (after comma) did not contain the meaning."*
Researcher : *"How do you know?"*
Zulfa : *"From the activity that we have done"*
(It seemed that he imitated what Sebastian said in activity 4).
Researcher : "*From the weighing activity, you have already found the numbers such as 0,1; 0,2…; 0,8; 0,9…After 0,9 what numbers that might come?"*
Zulfa : *"One"*
Researcher : *"From all series of activities, which activity that you like the most?"*
Zulfa : *"Finding the secret code"*
Researcher : *"Why?"*
Zulfa : *"Because it is interesting."*

2. Interview with Farhan

Researcher : *"What did you learn from the series of activities?"*
Farhan : *"I have learned how to make number line, decimal numbers, and whole numbers…"*
Researcher : *"What is decimal number?"*
Farhan : *"Decimal contains comma, but whole number did not contain comma"*
Researcher : *"Can you give the example?"*
Farhan : *"0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7, etc"*
Researcher : "*And then?"*
Farhan : "*0,8; 0,9; 0,10….ups…..2……"*
Researcher : *"Can you repeat?"*
Farhan : *"Ups….it should be one…0,7; 0,8; 0,9; 1"*
Researcher : *You said 0,10 but now you say 1, which one is correct?"*
Farhan : *"One"*
Then the researcher asked further
Researcher : "*In the previous time, you have found 0,5, what is 0,5?"*

Farhan : *"0,5 is a half from 1?"*
Researcher : *"How about 0,1?"*
Farhan : *"One over ten"*
Researcher : *"How do you know that 0,1 is one over ten?"*
Farhan : *"Because it was divided become ten fragments."*
Researcher : *If there are two Kiosk that sell two meats which have the same price. Kiosk A sells 2,72 kg/ pack and Kiosk B sells 2,9 kg/pack. Which one that you will choose?"*
Farhan : *"2,9"*
Researcher : *"Why?"*
Farhan : "Because 2,9 is larger than 2,72"
Researcher : "How do you know?
Farhan ***: "Because the position of 2,9 is on the right whereas the position of 2,72 is on the left"***
Researcher : *"So, where is the position of 2,9?"*
Farhan : *"2,9 is in between 2,8 and 2,10"*
Researcher : *"From all series of activities, which activity that you like the most?"*
Farhan : *"Making number line"*
Researcher : *"Why?"*
Farhan : *"Because it is interesting."*

3. Interview with Puteri

Researcher : *"What did you learn from the series of activities?"*
Puteri : *"I have experienced with measuring the volume of beverages, weighing the fruit and the rice…"*
Researcher : *"From the result of measurement, what numbers that you got?"*
Puteri : *"0,1; 0,5; 0,3…"*
Researcher : *"What kind of numbers are those?"*
Puteri : *"Decimal numbers"*
Researcher : *"What is decimal number?"*
Puteri : *The numbers containing comma"*
Researcher : *"Are there any other numbers beside that?"*
Puteri : *"Yes…0,11; 0,12; 0,13; 0,14; 0,15; etc.*
Researcher : "*Where is the position of 0,14?"*
Puteri : "*It is in between 0,13 and 0,15."*
Researcher : *How about 0,7? Where is the position of 0,7?"*
Puteri : *"It is in between 0,6 and 0,8"*
Researcher : *"From all series of activities, which activity that you like the most?"*
Puteri : *"Weighing the rice"*
Researcher : *"Why?"*
Puteri : *"Because I knew the weight of the rice."*

4. Interview with Sebastian

Researcher : *"What did you learn from the series of activities?"*
Sebastian : *"I have learned about whole numbers and decimal numbers. Then I learned that in between two consecutive whole numbers there are decimal numbers, and in between decimal numbers there are other decimal numbers."*
Researcher : *"Can you give the example?"*
Sebastian : *"As an example, in between 0 and 1 there are 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8, and 0,9. And then for example in between 0,1 and 0,2 there are 0,11; 0,12; 0,13; 0,14; 0,15; 0,16; 0,17; 0,18; and 0,19"*
Researcher : *"After 0,19 what number that might come?"*

Sebastian : *"0,2"*
Researcher : "*Why not 0,20?"*
Sebastian : "*Because we can ignore the zero after comma, it did not contain the meaning."*
Researcher : *"If I have two numbers, 2,65 and 2,8, which is larger than the other?"*
Sebastian : *"2,8"*
Researcher : "*Why?"*
Sebastian : ***"Because 2,65 is in between 2,6 and 2,7 whereas 2,8 is on its right position of 2,65, therefore 2,8 is larger than 2,65"***
Researcher : *"Can you give another example?"*
Sebastian : *"12,26 and 12,7. 12,7 is larger than 12,26"*
Researcher : *"Why?"*
Sebastian : ***"Because 12,26 is in between 12,2 and 12,3 whereas 12,7 is on its right position of 12,26, so 2,7 is larger than 12,26"***
Researcher : *"From all series of activities, which activity that you like the most?"*
Sebastian : *"Playing Spaceflight game"*
Researcher : *"Why?"*
Sebastian : *"Because we can study and play simultaneously and we can practice to find the numbers quickly, because it was stick on the time."*
Researcher : *"Beside that, what activity that you like?"*
Sebastian : *"Weighing activity."*
Researcher : *"Why?"*
Sebastian : *"Because I never experienced with that when I studied mathematics."*

5. Interview with Ari

Researcher : *"What did you learn from the series of activities?"*
Ari : *"I have learned about decimal number"*
Researcher : *"What is decimal number?"*
Ari : *"The number containing comma"*
Researcher : *"Can you give the example?"*
Ari : *"0,1; 0,01; 0,1; 0,5."*
Researcher : *"What number in between 0 and 1?"*
Ari : "10"
Researcher : *"Hmmm… let's we review the previous activity. If we weigh one pack of 0,1 kg, how much its weight?"*
Ari : "*0,1"*
Researcher : *"How about 2 packs?"*
Ari : "*0,2"*
Researcher : *"How about 3 packs?"*
Ari : "*0,3"*
Researcher : *"How about 9 packs?"*
Ari : *" 0,9"*
Researcher : *"How about 10 packs?"*
Ari : "*0,10"*
Researcher : *"Are you sure?"*
Ari : *"Ups..0,1."*
Researcher : *"What is the meaning of 0,1?"*
Ari : *"One over ten"*
Researcher : *"Why 0,1 is one over ten?"*
Ari : (She was quiet for a while)
Researcher : *"Let's count again the packed of rice measured: 0,1; 0,2…"*
(Ari continued counting)
Ari : *"0,1; 0,2; 0,3; ….; 0,8; 0,9; 1."*

Researcher : *"Before this, you said ten packs are 0,10, now you say 1. Which one is correct?"*
Ari : *"0,10."*

At the end of activity, Ari still thought that the numbers that might come up after 0,9 is 0,10.

6. Interview with Bintang

Researcher : *"What did you learn from the series of activities?"*
Bintang : (He was quiet for a while, then he said) *"I have learned how to make number line, decimal numbers, and whole numbers, and then I knew the weight of the rice, the weight of the fruit"*
Researcher : *"Can you give the example of decimal numbers?"*
Bintang : *"0,1; 0,2; 0,3; 0,4; 0,5; etc"*
Researcher : "*How many digit those are?"*
Bintang : "*One"*
Researcher : *"Beside one-digit, is there any other kind of decimal numbers?"*
Bintang : *"Two-digit"*
Researcher : *"Can you give examples?"*
Bintang : *"0,11; 0,12; 0,13; 0,14; 0,15"*
Then the researcher asked further
Researcher : "*I have question. Andi wants to buy the beverages. There are two beverages. Beverage A contains 12,17 gram sugar and beverage B contains 12,4 gram sugar. Which is the sweeter beverage?"*
Bintang : *"beverage B?"*
Researcher : *"Why?"*
Bintang : *"Because 12,4 is larger than 12,17"*

Bintang is a smart student but he is a quiet person, so sometimes it was difficult to dig his reason.

7. Interview with Mita

Researcher : *"What did you learn from the series of activities?"*
Mita : *"I have learned about decimal numbers…"*
Researcher : *"What is decimal number?"*
Mita : *"The number containing comma"*
Researcher : *"Can you give the example?"*
Mita : *"12,1; 12,2; 12,3; 12,4, etc"*
Researcher : "*How many digit those are?"*
Mita : "*One"*
Researcher : *"Beside one-digit, is there any other kind of decimal numbers?"*
Mita : *"Two-digit"*
Researcher : *"Can you give example?"*
Mita : *"12,21; 12,22; 12,23…"*
Researcher : *If we compare 12,21 and 12,6, which one is larger?"*
Mita : *"12,6"*
Researcher : "*Why?"*
Mita : *"Because it is larger …"*
Researcher : *"If on the number line, which one that on the right position?"*
Mita : *"12,21"*
Researcher : "*You have ever solved this kind of problem. Andi wants to buy the beverages. There are two beverages, beverage A contains 12,17 gram sugar and beverage B contains 12,4 gram sugar. Which is the sweeter beverage?"*

Mita : *"beverage with 12,4 gram sugar?"*
Researcher : *"Why?"*
Mita : *"Because 12,4 is larger than 12,17"*
Researcher *: "So, I ask again, which is larger, 12,6 or 12,21?"*
Mita *: "12,21"*
Researcher *: "Are you sure?"*
Mita *: "Yes."*
Researcher : *"From all series of activities, which activity that you like the most?"*
Farhan : *"Singing the song"*

At the end of activity, Mita still thought that the longer digit decimal number was the larger for its magnitude.

THE RESULT OF INTERVIEW AT THE END OF ACTIVITY (SECOND CYCLE)

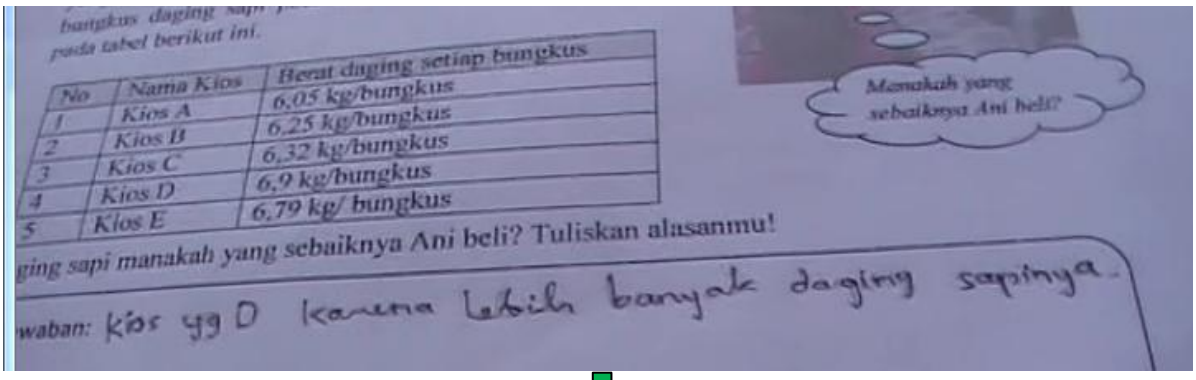
ZERO AFTER COMMA ISSUE

(The interview with Sapta)

Researcher : *"If I have two numbers, 0,2 and 0,20, those are same or not?"*
Sapta : *"Same"*
Researcher : *"Why?"*
Sapta : *"Because sometimes zero after comma contains meaning and sometimes not."*
Researcher : *"Could you give an example of zero after comma which is meaningless?"*
Sapta : *"0,20"*
Researcher : *"Could you give an example of zero after comma which contains meaning?"*
Sapta : *"0,06"*
Researcher : *"How do you know that zero after comma (in 0,20) is meaningless?"*
Sapta : *"I get from the course?"*
Researcher : *"Now let we see each magnitude on the number line!"*
(Sapta drew the sequence of decimals on the number line)
Researcher : *"Now, where is the position of 0,2?"*
Sapta : *"This one..."*(he circled 0,2 on the number line)
Researcher : *"How about 0,20? Where is the position of 0,20?"*
Sapta : *"Mmmm....in between 0,2 and 0,3."*
Researcher : *"Where is the position of 0,25?"*
(Sapta directly made small lines in between 0,2 and 0,3; and then he counted from the edge of 0,2 until the fifth line, and finally he wrote 0,25 in the fifth line between 0,2 and 0,3).
Researcher : *"What is the number that might come up after 0,2?"*
Sapta : *"0,21; 0,22; 0,23; 0,24; 0,25; 0,26; 0,27; 0,28; 0,29, 0,3"*
Researcher : *"So, where is the position of 0,20?"*
Sapta : *"At 0,2."*
Researcher : *"Pardon?"*
Sapta : *"At 0,2; 0,2 is in the second line from ten lines (overall)."*
Researcher : *"How about 0,20? In what line is it?"*
Sapta : *"0,20 is in the 20^{th} line from 100 lines."*
(Sapta wrote 0,20 = 20/100 in the up line; and 0,2=2/10 in the bottom line)
Researcher : *"What makes those different between the numbers in up line and in bottom line?"*
Sapta : *"In up line the zero is different. This one contains zero* (he pointed out to the equation in up line), *and this one did not contain zero* (he pointed out to the equation in bottom line)
Researcher : *"So, what can you conclude, zero after comma contains meaning or not?"*
Sapta : *"Yes, it contains meaning."*
Researcher : *"The meaning of what?"*
Sapta : *"It is a hint to make the equivalence fraction of over hundred, if it contains zero, it means over hundred, if it did not contain zero, it means over ten."*

(The interview with Dimas)

Researcher : *"Why do you chose the meat in Kiosk D?"*
Dimas : *"Because it is the heaviest one."*
Researcher : *"How do you know?"*
Dimas : *"Because 6,05 is in between 6 and 6,1...6,25 is in between 6,2 and 6,3...6,32 is in between 6,3 and 6,4...6,9 is in between 7 and 6,8...6,79 is in between 6,7 and 6,8."*
Researcher : *"How do you know? I did not see you draw number line?"* (see Dimas'answer in figure below)
Dimas : *"I imagined...directly...because 6,9 was the most..."*



Kiosk D because it contains the most meat.

PRE-ASSESSMENT FOR LEARNING DECIMALS GRADE 5

Name :
Class :
School :

1. What do you think about the numbers in the following pictures?

19,5 Liter

Answer:

0,5 L

Answer:

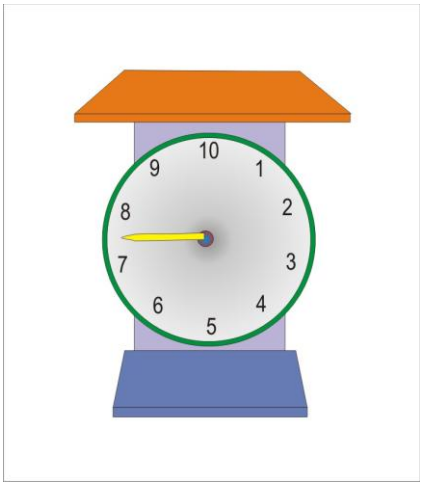
Answer:

Answer:

2. What is the number shown in the following pictures? Explain your reasoning!

Answer:

Answer:



Answer:

3. Andi wants to buy the milk in the market. There are two kinds of milk packaged in the bottle which have the same volume. Milk A contains7,5 gram sugar, and milk B contains 7,21 gram sugar. What do you think? Which is sweeter milk? Explain your reasoning!

Answer:

==========Good Luck==========

Description:

- **For problem number 1:**

The goal of this problem is to know the **students' knowledge** about decimal numbers. I choose those pictures because I am sure that some students have already seen them in their daily life, either in television or in banner on the street. So, I hope the students, at least, are able to mention the comma numbers in those situations, such as in picture 1.a; *the situation is in the gas station in which the filled gasoline is nineteen comma five liter, etc*.

- **For problem number 2:**

The goal of this problem is to know **the skill of students** in reading the scale.

- Are they able to read the unlabeled number? *(e.g "115" in picture 2.a)*
- Are they able to estimate the scale in which the needle shown is in between the labeled numbers (not exactly in the line)? (*e.g 9 plus … in picture 2.b, and 7 plus … in picture 2.c)*. The answers may vary.

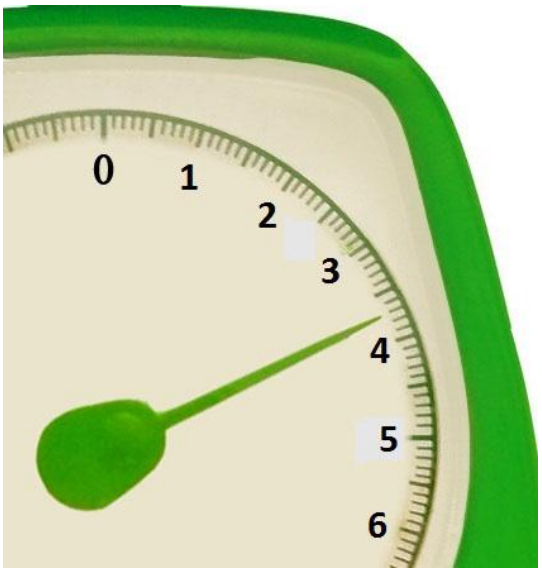
- **For problem number 3:**

The goal of this problem is to know how far students' learning in comparing the magnitude of decimal numbers. What are their strategies and their reasoning?

END-ASSESSMENT FOR LEARNING DECIMALS GRADE 5

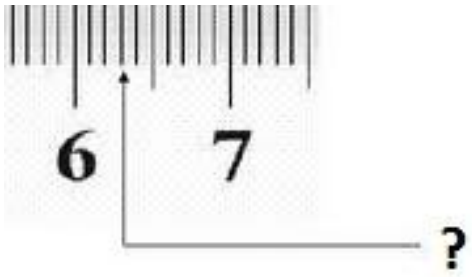
Name :
Class :
School :

1. What numbers shown in the following pictures? Explain your reasoning!



Answer:

It is expected that the students will answer that it is 3,7 because it is in the 7th line between 3 and 4.



Answer:

It is expected that the students will answer that it is 6,3 because it is in the 3th line between 6 and 7.

2. In the Poem Competition , it has been announced:

GRADE	NAME
First Winner	Dina
Second Winner	Rika
Third Winner	Desi

Based on the information, Desi's score was 8 whereas Dina's score was 9. What are the possibilities of Rika's score? Explain your reasoning!

Answer:
It is expected that the students will answer:
Rika is the second winner, so the possibilities of Rika's score are in between 8 and 9 i.e: 8,1; 8,2; 8,3; 8,4; 8,5; 8,6; 8,7; 8,8; 8,9

3. Jojo wants to buy the milk in the market. There are two kinds of milk packaged in the bottle which have the same volume. Milk A contains 12,24 gram sugar, and milk B contains 12,8 gram sugar. What do you think? Which is sweeter milk? Explain your reasoning!

Answer:
It is expected that the students will answer:
Beverage B is sweeter than beverage A because 12,8 is larger than 12,24.
It is also expected that the students will draw 12,8 and 12,24 on the number line correctly.

4. a. Ani goes to the market to buy the meat. There are 5 Kiosk which offer the same price for 1 pack of meat. The weight for each pack in each kiosk can be seen in figure below.

Kios A
6,5 kg/bungkus

Kios B
6,25 kg/bungkus

Kios C
6,32 kg/bungkus

Kios D
6,9 kg/bungkus

Kios E
6,89 kg/bungkus

No	Kiosk	The weight per pack
1	Kiosk A	6,05 kg/pack
2	Kiosk B	6,25 kg/pack
3	Kiosk C	6,32 kg/pack
4	Kiosk D	6,9 kg/pack
5	Kiosk E	6,79 kg/pack

Which one that should be bought by Ani? Explain your reasoning!

Answer:
It is expected that the students will answer that the students will choose Kiosk D because Kiosk D is the heaviest compared to the others.

b. Arrange those Kiosks starting from the Kiosk that sells the lightest meat through the heaviest meat!

Answer:

It is conjectured that the students will use number line as a tool for placing and comparing each weight of the meat.

5. a. Find the numbers which are larger than 1,4 but smaller than 2 as much as possible, then put them on number line!
   b. Find the numbers which are larger than 100,17 but smaller than 100,5 as much as possible, then put them on number line!

Jawaban:

It is conjectured that the students will divide the range into ten fragments to come up to the sequence of one-digit decimal numbers and divide another ten fragments to come up to the sequence of two-digit decimal numbers.

==========Good Luck==========