DESIGN RESEARCH ON DEVELOPING UNIT IN AREA MEASUREMENT FOR GRADE 3 IN INDONESIAN PRIMARY SCHOOL

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:
Kurnia Rahmi Y.
NIM 20092812005

GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

APPROVAL PAGE

Research Title	: Design Research on Developing Unit in Area Measurement for Grade 3 in Indonesian Primary School
Student Name	: Kurnia Rahmi Y.
Student Number	: 20092812005
Study Program	: Mathematics Education

Approved by:

Prof. Dr. Zulkardi, M.I.Komp., M.Sc. Supervisor I	Dr. Yusuf Hartono Supervisor II
Head of Mathematics Education Department,	Director of Graduate School Sriwijaya University,
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. NIP 19610420 198603 1 002	Prof. Dr. dr. H.M.T. Kamaluddin, M.Sc., SpFK. NIP 19520930 198201 1 001

Date of Approval: May 2011

DESIGN RESEARCH ON DEVELOPING UNIT IN AREA MEASUREMENT FOR GRADE 3 IN INDONESIAN PRIMARY SCHOOL

A THESIS

Submitted in Partial Fulfilment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By:

KURNIA RAHMI Y.
NIM 20092812005

Approved by Examination Committee	**Signature**
Prof. Dr. Zulkardi, M.I.Komp., M.Sc. **Sriwijaya University**	----------
Dr. Yusuf Hartono **Sriwijaya University**	----------
Dr. Frans Van Galen **Freudenthal Institute, Utrecht University**	----------
Prof. Dr. R. K. Sembiring **IP-PMRI**	----------
Dr. Darmawijoyo **Sriwijaya University**	----------
Dr. Dolly Van Eerde **Freudenthal Institute, Utrecht University**	----------
Dr. Ratu Ilma Indra Putri, M.Si **Sriwijaya University**	----------

GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

STATEMENT PAGE

I hereby:

Name : Kurnia Rahmi Y.

Place of birth : Bukittinggi

Date of birth : August, 1985

Academic Major : Mathematics Education

State that:

1. All the data, information, analysis, and the statement in analysis and conclusion that presented in this thesis, except from reference sources in the result of my observation, research, analysis, and view with the guidance of my advisors.
2. The thesis that I had made is original of my mind and has never been presented and proposed to get any other degree from Sriwijaya University or other Universities.

This statement was truly made and if in other time that found any fouls in my statement above, I am ready to get any academic sanctions such as, cancelation of my degree that I have got through this thesis.

Palembang, May 2011

The one with the statement

Kurnia Rahmi Y.
NIM. 20092812005

ABSTRACT

In area measurement students have difficulties in learning area measurement. Students mostly focus on applying formula to find the area of certain shapes without knowing what the area is and why the formula works. It is important to construct the unit for area and the measurement procedures since to quantify the area a unit must be used. Therefore, the aim of this research is to develop classroom activities that support students to learn area measurement. The sequences of activities are developed to gain better understanding of the students in learning area measurement. Design research is chosen as the method of the research. A teacher and students in grade 3 in elementary school (SDN 21) in Palembang Indonesia were involved in this research. The results showed that students learn to measure the area start from identifying the attribute being measured while comparing the quantity of area. Afterwards, the need of unit emerged when they have to quantify the quantity of area. The measurement process with the units gained when the students have experiences with covering activity by using units. Covering activity leads students to mentally partition the region into units and allows students to focus on the process of repeatedly using non standard unit as a tool to measure. Through these activities the students can use the unit to measure the area of two dimensional shapes either regular shape or irregular shapes.

Keywords: *Area measurement, unit, design research, RME*

ABSTRAK

Pada materi pengukuran luas, siswa mengalami kesulitan dalam memahami pengukuran luas. Kebanyakan siswa hanya fokus pada penerapan rumus untuk menentukan luas berbagai bangun datar tanpa mengetahui pengertian luas dan mengapa rumus tersebut dapat berfungsi. Sangat penting untuk mengkonstruksi satuan untuk luas dan proses pengukuran karena satuan digunakan untuk mengukur luas. Oleh karena itu, tujuan dari penelitian ini adalah untuk mengembangkan kegiatan kelas yang mendukung siswa untuk belajar pengukuran daerah. Rangkaian aktivitas dikembangkan untuk mencapai pemahaman siswa yang lebih baik dalam belajar pengukuran luas. *Design research* dipilih sebagai metode penelitian. Guru dan siswa kelas 3 SDN 21 palembang Indonesia terlibat dalam penelitian ini. Hasil penelitian menunjukkan bahwa siswa belajar untuk mengukur luas mulai dari mengidentifikasi sifat-sifat bangun yang akan diukur ketika membandingkan luas. Kemudian siswa menyadari bahwa satuan dibutuhkan ketika harus menghitung besarnya suatu daerah. Proses pengukuran dengan menggunakan satuan secara bertahap diperoleh ketika siswa memiliki pengalaman menutupi daerah dengan menggunakan satuan. Akhirnya siswa dapat menggunakan sebagai sarana untuk mengukur luas bidang datar baik bentuk yang beraturan maupun bentuk yang tidak beraturan.

Kata Kunci: pengukuran luas, satuan, design research, RME

SUMMARY

This research focuses on supporting the third grade students in learning area measurement. Area measurement is based on partitioning a region into equally sized units which completely cover it without gaps or overlaps. It is often that teaching tends to focus on numerical results and ignored the idea of the unit. Many findings show that understanding area measurement is difficult. Research in the field of mathematical education often reveals poor understanding of the processes used for area measurement of plan figure (Zacharos, 2006). Battista (Keijzer, 2008) mentioned that there is no obvious instrument for measuring area. It means that it is important to construct the unit for area and the measurement procedures. According to Gravemeijer, et al (2007) students are expected to master an extensive system of units of area, but they appear to have serious difficulty with applying this knowledge. It is preferable to make students familiar with a number of units and applications that are relevant to daily life, and to place the emphasis on developing concept, rules and procedures. Considering the difficulties of the students about area measurement, we develop classroom activities that support students to learn area measurement. So in this research a series of learning activities are designed to bring students in developing a unit as a means of measuring area. Realistic Mathematics Education underlies this research. This approach has been implemented in Indonesia since over the last 10 years that is called Pendidikan Matematika Realistik Indonesia (PMRI). A teacher and students in grade 3 in elementary school (SDN 21) in Palembang Indonesia that has been joined in PMRI project since 2010 were involved in this research.

Design research is used as a method of the research and it follows the phases of the design research that are: preliminary design, teaching experiment, and retrospective analysis. Video recording and students' written works were used to investigate the learning process of the students. Interviews with the students were conducted to get deeper information of their thinking process.

This research was carried out in two cycles namely pilot experiment and teaching experiment. In the pilot experiment the sequences of activities were tried out to small group in order to try out our HLT to see how this design works. We found that some activities did not support to answer our research question. Hence, some activities and some materials such as the question for student worksheets were modified. Pre assessment was conducted to see the pre knowledge of the students and to see our starting point. The results of pre assessment showed that almost students still have no idea about area and they have vague notion about area. It means that it still a lot to be learned.

The teaching experiment was conducted in six activities. The first activity concerns with students' awareness of physically quantity of area. This activity showed that some students were aware to the physical quantity of area. Students compared figures given by putting the one to the top of the other and then looking at which piece sticks out. Some of them also perceive the idea of conservation of area by rearranging the shape to determine which one is bigger.

The awareness of the attribute of area was made for the second activity but they worked with measurement units. It is expected that they can compare the objects by using identical units. Only few students recognized the existence of the unit in the figure. They realized that the unit is not identical and it is difficult to compare by using unidentical units.

In the third activity, the students were expected to use the unit to compare. In this lesson, students are able to use their own unit to cover the shape in comparing the area. However, some students choose unit that physically resemble with the region they were

covering. In this manner, they only focus on the process of repeatedly using a unit in and it seems they did not use the unit to compare. It might be because the question is which baking tray that can be put more cookies. Therefore, they did not pay attention to the size of baking tray. Some students were not aware of gaps and overlap in covering. In this level, these students only focus on counting the unit and did not get what is the area. However, Experience in covering with non standard units helps students to develop the concept of unit iteration and structuring arrays with row and column structuring.

In the fourth activity, students had to find the area of a baking tray with different kinds of unit by covering the baking tray. They cut and rearrange the shape in order to cover the region. In this way they can accept the concept of conservation of area. This is obvious from the fact that the process of measuring improve in this activity. They also consider about the accuracy of counting and find the inverse relationship of the unit.

In the fifth activity, Students were asked to find the area by using given unit. They were expected to be able to find the area either by partition a region and then count the unit or by using multiplication. In here, students can partition the region by fitting the square with the existing square in each row or column. Even some of them did not think to make arrays but directly multiply the number of unit in the side of rectangular shape.

In the last activity, the students were asked to compare the area of irregular shapes. Comparing the area of irregular shapes encouraged them to use unit in helping them finding the area of each shape. They were able to overcome the partial unit in irregular shapes. They combined partial unit together to form whole units and then count the unit used. In this case, students can use square as a unit in estimating the area of irregular shapes.

As a conclusion, students learn to measure the area start from identifying the attribute being measured while comparing the quantity of area. Afterwards, the need of unit emerged when they have to quantify the quantity of area. The measurement process with the units gained when the students have experiences with covering activity by using units. Covering activity leads students to mentally partition the region into units and allows students to focus on the process of repeatedly using non standard unit as a tool to measure. Through these activities the students can use the unit to measure the area of two dimensional shapes either regular shape or irregular shapes.

RINGKASAN

Penelitian ini difokuskan untuk mendukung siswa kelas 3 SD dalam belajar pengukuran luas. Pengukuran luas pada dasarnya mempartisi suatu daerah menjadi satuan-satuan yang berukuran sama tanpa celah ataupun saling tumpang tindih. Namun seringkali pengajaran cenderung fokus pada hasil perhitungan dan mengabaikan konsep dari satuan. Banyak penelitian yang menunjukkan sulitnya memahami konsep luas. Penelitian di bidang pendidikan matematika seringkali menunjukkan kurangnya pemahaman mengenai proses pengukuran luas (Zacharos, 2006). Battista (Keijzer, 2008) menyebutkan bahwa tidak adanya alat pengukuran yang jelas untuk luas. Oleh karena itu satuan untuk mengukur luas dan prosedur pengukuran penting untuk dikembangkan. Menurut Gravemeijer (2007) siswa diharapkan untuk mengusai pengembangan untuk satuan luas namun mereka kesulitan untuk menerapkan pengetahuan ini. Siswa sebaiknya diperkenalkan dengan satuan yang dekat dengan kehidupan mereka sehari-hari serta konsep, aturan dan prosedur dalam pengukuran. Mengingat kesulitan siswa belajar tentang pengukuran area, kami mengembangkan kegiatan kelas yang mendukung siswa untuk belajar pengukuran daerah. Maka dalam penelitian ini serangkaian kegiatan belajar yang dirancang untuk membawa siswa dalam mengembangkan unit sebagai sarana untuk mengukur daerah. *Realistic Mathematics Education* atau yang lebih dikenal Pendidikan Matematika Realistik Indonesia (PMRI) melandasi penelitian ini. Pendekatan ini telah diterapkan di Indonesia sejak 10 tahun terakhir. Guru dan siswa kelas 3 di SDN 21 Palembang Indonesia dilibatkan pada penelitian ini. Sekolah ini telah bergabung dengan proyek PMRI sejak tahun 2001.

Design research dipilih sebagai rancangan penelitian. Rancangan penelitian ini terdiri dari tiga fase yaitu: tahap persiapan, eksperiment kelas dan tahap analisis. Rekaman video dan hasil kerja siswa digunakan untuk menyelidiki proses belajat siswa. Wawancara dengan para siswa dilakukan untuk mendapatkan informasi yang lebih mendalam tentang proses berpikir mereka.

Penelitian ini dilakukan dengan dua siklus, yaitu *pilot experiment* dan *teaching experiment.* Pada pilot experiment serangkaian kegiatan di dalam HLT (*Hypothetical Learning Trajectory*) diujicobakan pada kelompok kecil untuk melihat sejauh mana rancangan tersebut berhasil. Ternyata beberapa aktivitas tidak mendukung untuk menjawab pertanyaan penelitian. Oleh karena itu beberapa aktivitas dan beberapa materi seperti pertanyaan pada lembar kerja siswa diubah untuk meningkatkan kualitas HLT. Penilaian awal dilakukan untuk melihat pengetahuan awal siswa dan melihat dimana langkah awal untuk mengajarkan luas. Hasil dari penilaian ini menunjukkan bahwa sebagian besar siswa masih kebingungan dalam memahami arti luas. Ini berarti masih banyak yang perlu mereka pelajari.

Pembelajaran pada *teaching experiment* dilakukan dalam enam aktivitas. Aktivitas pertama terkait pada pandangan siswa mengenai kuantitas fisik luas. Siswa ditugaskan untuk membandingkan dan mengurutkan bidang datar berdasarkan ukurannya. Aktivitas ini menunjukkan beberapa siswa menyadari kuantitas fisik luas. Mereka membandingkan gambar bidang datar dengan meletakkan gambar yang satu di atas gambar yang lainnya kemudian bagain yang masih bersisa. Beberapa diantara mereka juga menyadari konsep dari konservasi luas dengan menggunting dan mengatur ulang gambar tersebut.

Kesadaran siswa tentang sifat dari luas diadakan pada aktifitas selanjutnya namun siswa juga bekerja dengan satuan pengukuran. Diharapkan mereka dapat membandingkan luas dengan menggunakan satuan yang sama. Hanya sedikit siswa yang menyadari adanya

satuan pada gambar yang diberikan. Mereka menyadari bahwa satuan yang mereka gunakan berbeda dan sulit untuk membandingkan jika menggunakan satuan yang berbeda.

Pada aktivitas ketiga siswa diharapkan dapat menggunakan satuan untuk membandingkan. Pada aktivitas ini siswa dapat menggunakan satuan mereka sendiri. Namun pada kegiatan ini beberapa siswa memilih satuan yang menyerupai daerah yang akan mereka tutupi. Dengan cara ini, mereka hanya focus pada proses perulangan dengan menggunakan satuan dan tidak menggunakan satuan untuk membandingkan. Hal ini mungkin disebabkan karena pertanyaan yang diberikan berupa loyang kue manakah yang dapat memuat lebih banyak kue. Oleh karena itu mereka tidak memperhatikan ukuran loyang kue. Beberapa siswa juga tidak memperhatikan adanya celah dan tumpang tindih ketika menutupi objek. Pada level ini, siswa hanya fokus pada penghitungan satuan dan masih belum paham mengenai luas. Namun, pengalaman menutupi objek dengan satuan yang tidak baku membantu siswa dalam mengembangkan konsep iterasi satuan dan penyusunan bersusun baris dan kolom.

Pada aktivitas keempat siswa ditugaskan untuk menentukan luas benda dengan menggunakan satuan yang berbeda. Mereka menggunting dan menyusun kembali satuan agar satuan tersebut dapat muat di dalam objek yang diukur. Dengan cara ini mereka dapat menerima konsep konservasi luas. Hal ini jelas bahwa kemampuan proses pengukuran siswa meningkat pada kegiatan ini. Mereka juga memperhatikan keakuratan penghitungan dan menemukan hubungan kebalikan dari ukuran satuan.

Pada aktivitas kelima, siswa diminta untuk menemukan luas dengan menggunakan satuan yang telah diberikan. Mereka diharapkan dapat menemukan luas baik dengan mempartisi daerah kemudian menghitung banyaknya satuan yang ada pada daerah tersebut ataupun dengan menggunakan perkalian. Dalam hal ini, siswa dapat mempartisi daerah dengan meyesuaikan persegi-persegi satuan dengan persegi-persegi satuan yang telah ada pada setiap baris atau kolom. Bahkan beberapa siswa tidak membuat gambar susunan persegi tetapi langsung mengalikan banyaknya satuan yang ada pada sisi bangun persegi panjang.

Pada kegiatan terakhir, para siswa diminta untuk membandingkan luas daerah bentuk yang tidak beraturan. Kegitan ini mendorong mereka untuk menggunakan satun dalam membantu mereka menemukan luas masing-masing bentuk. Dalam hal ini. Mereka dapat mengatasi satuan yang tidak utuh pada bentuk yang tidak beraturan. Siswa menggabungkan secara mental satuan yang tidak utuh untuk membentuk satuah yang utuh dan kemudian menghitung banyaknya satuan yang digunakan. Dapat dikatakan siswa menggunakan persegi sebagai satuan dalam memperkirakan luas bentuk yang tidak beraturan.

Sebagai kesimpulan, siswa belajar untuk mengukur luas mulai dari mengidentifikasi sifat-sifat bangun yang akan diukur ketika membandingkan luas. Kemudian siswa menyadari bahwa satuan diperlukan untuk menyatakan besarnya suatu daerah. Proses pengukuran dengan menggunakan satuan diperoleh siswa ketika adanya pengalaman dengan kegiatan menutupi daerah dengan menggunakan satuan. Kegiatan menutupi menuntun siswa untuk mempartisi daerah menjadi satuan-satuan dan memungkinkan siswa fokus pada proses iterasi menggunakan satuan yang tidak baku sebagai sarana untuk mengukur luas. Melalui kegiatan ini siswa dapat menggunakan satuan sebagai sarana untuk mengukur luas bidang datar baik bentuk yang beraturan maupun bentuk yang tidak beraturan.

PREFACE

It was very great experience working with young children to learn mathematics. This thesis is realization of how to have fun with students in learning area measurement. It would not have been possible without assists and supports from many people surrounding me. I would like to express gratitude to them all.

- I am very grateful to the teacher, Ibu Mulyati, who has given her time and effort for this research, students in SDN 21 Palembang who was involved in this research, principal and teachers in SDN 21 Palembang. This research could not be carried out without your help.
- I am so proud supervised by Prof. Zulkardi, Dr. Yusuf Hartono and Frans van Galen who always assist me to improve and sharpen my ideas. Thank you for fruitful discussions in guiding to finish my research.
- To all lecturers and staffs in Freudenthal Institute Utrecht University Prof. Jan van Maanen, Dr. Jaap den Hertog, Dr. Dolly van Eerde, Prof. Koeno Gravemeijer, Martin Kindt, Prof. Jo Nellisen, Dr. Mieke Abels, and the other lecturers thank you for sharing knowledge during my study in the Netherlands.
- I also would like to thank lecturers and staffs in Sriwijaya University, Dr. Ratu Ilma, Dr. Darmawijoyo, Dr. Somakim who gives encouragement and assistance throughout my study in Palembang.
- I thank Prof. Dr. dr. H.M.T. Kamaluddin, M.Sc., SpFK. as director of graduate school Sriwijaya University.
- I also want to thank Directorate General of Higher Education (DIKTI) and Nuffic Neso Indonesia for funding my study.

- I Thank Prof. Dr. R.K. Sembiring, Prof. Marteen Dolk and PMRI team who launch IMPoME program.
- I wish to thank Prof. Dr. Ahmad Fauzan, Dr. Muliyardi, M.Pd., Lutfian Almash M.Si, Hendra Syarifuddin M.Si and all of my lecturers in Padang State University (UNP) who supports me to join this program.
- My special thanks to my dear fellow students for being my family. You all give beautiful moment in my life. The second batch of IMPoMe's students, thank you for helping during the research and also during the conference.

I dedicate this research to my family especially my parents who gave me strength when I am weak and gave me spirit when I am desperate, who always pray for all the best for me.

Palembang, May 2011

Kurnia Rahmi Yuberta

TABLE OF CONTENTS

LIST OF FIGURES

.

LIST OF APPENDICES

LIST OF TABLE

CHAPTER I
INTRODUCTION

Measurement is one of subjects in mathematics which is often experienced in daily life. People deals with measuring situations every days such as measure how much ingredient put when cooking, how far the school from the student's house or how large a garden. Interpreting the number is important to communicate the result of measuring to the other. It means that competence in measuring is needed to teach in primary school. According to Reys et al (2007) measurement should be included in school mathematics because of its power to help students to see the usefulness of mathematics in everyday life. Measurement also can be used to help students learn other topics in mathematics.

One of issues in teaching measurement is that measurement is directly taught at the formal level of young children as an isolated concept (Wijaya, 2008; Castle & Needham, 2007; Kamii & Clark, 1997 and van de Walle & Folk, 2005). In Indonesia, teaching and learning in measurement mostly focuses on the using of formula without take care of development at how the formula is constructed. Fauzan (2002) states that one of fundamental problem in Indonesia is that most of the learning objectives only focus on memorizing facts and concepts, and computational aspects (i.e. applying formula). We can see how difficult it is for the students to differentiate the formula such as area and volume because they learn a 'ready-to-use' system, even though this system is actually the result of a long process of developing suitable tools such as useful units of measurement, a measurement system and suitable formulas (Gravemeijer, et al, 2007).

However, this research is focused on supporting students in learning area measurement for grade 3 in elementary school. Many findings show that to learn area

measurement is difficult. Research in the field of mathematical education often reveals poor understanding of the processes used for area measurement of plan figure (Zacharos, 2006). Battista (Keijzer, 2008) mentioned that there is no obvious instrument for measuring area. It means it is important to construct the unit for area and the measurement procedures. According to Gravemeijer, et al (2007) students are expected to master an extensive system of units of area, but they appear to have serious difficulty with applying this knowledge. It is preferable to make students familiar with a number of units and applications that are relevant to daily life, and to place the emphasis on developing concept, rules and procedures.

Area measurement is based on partitioning a region into equally sized units which completely cover it without gaps or overlaps. Although the idea of unit is fundamental, the experiences with the unit through covering the space are often overlooked and it tends to focus on numerical result (Cavanagh, 2007). As a result students do not have opportunities to make sense the concept of area.

The aim of this research is to develop classroom activities that support students to learn area measurement. So in this research a series of learning activities are designed to bring students in developing a unit as a means of measuring area. Hence, research question in this research is *How can students learn to measure area?* This research is specified into two sub research questions as following:

1. *How can comparing and covering activities bring students to develop the notion of a measurement unit for area?*
2. *How do students use a measurement unit to measure area?*

CHAPTER II
THEORETICAL FRAMEWORK

A. Area Measurement

Area is an amount of two-dimensional surface that is contained within a boundary (Clements and Sarama, 2009). According to Simon and Blume (1994) in Zacharoz (2006), the study of area involves two steps: considering the area as a quantity and evaluating that quantity. To measure the quantity of area a unit must be chosen so that the number of those units that is takes is the size of an object (Cross et al, 2009). Physical quantity can be seen while the experiences offer the students to compare area of objects (Heuvel-Panhuizen, 2005). In this way, intuitive awareness of measuring emerges in many students. For area measurement, comparing can emerge the relative statement such as 'bigger than' or 'larger than'. It can give a range of possibilities but is not accurate enough for describing the objects because relative statement cannot tell how big or how large the object is. However, exact statements can be made by using a unit of measurement used. In here, the students start to develop the need of unit to measure when they have to evaluate the quantity.

Cavanagh (2007) stated that area measurement is based on partitioning a region into equally sized units which completely cover it without gaps or overlaps. However, students probably are not thinking about measuring as covering space (Clements, 2004). Therefore, students should be involved to investigate covering regions with a unit of measure which completely covers it without gaps or overlaps and quantifying that covering. Moreover, finding the area of a region can be thought of as tilling a region with two-dimensional unit of measure (Clements, 2004). Furthermore,

Zacharoz (2006) suggested that area be measured using two-dimensional units such as plane figures (e.g. squares and rectangles) and a square unit is usually selected for overlapping rectangles and other figures with right angles.

Gravemeijer et al (2007) stated that when learning area, it is important for students to construct the units and the measurement procedures by themselves. The measurement process that can be used to plan instruction is as following (Reys et al, 2007):

1. Identify the attribute by comparing objects

To measure with understanding, children should know what attribute they are measuring. Three types of comparisons which can build understanding of attributes are perceptually, directly and indirectly through references. For area, it can be compared by sight (perceptually) if the differences are large enough and the shapes are similar enough. Direct comparison means that children compare two regions where one of the regions can fit within the other. They can cut out the region to easily compare without changing the area. According to Lehrer students have difficulties accepting that when they cut a given region and rearrange its parts to form another shape, the area remain the same (Clements, 2004). When children have some idea of conservation of area that a region can be cut and rearrange without changing the area, they can give many solutions to find the area. If the objects cannot be moved to place one on top of the other, children can trace the objects and use this representation to make an indirect comparison.

2. Choose a unit

After children compare the region they must answer how the accuracy of their answer. In doing so, they need a unit to compare whether by using non standard unit or standard unit. When choosing the unit, there are some concepts develop over time (Reys et al, 2007). The size of the unit chosen depends on the size of the object and on the accuracy needed. There is an inverse relation between the size of a measuring unit and the number of units needed to measure some characteristic (Cross et al, 2009). In other words, the smaller the unit the more accurate and the more units are required. The students also need to know the measurement may be easily if the same unit is used. Besides choosing the unit of measurement, students have to iterate units to cover areas without gaps and overlaps, and to count the units (Kordaki, 2002)

3. Comparing the object to unit

Measuring with units is comparing an object with a unit and find how many units would be equal to that object. To compare the object to unit, identical unit is needed to communicate the result to the other in which every unit used is the same (Michaels et al, 2008). For example, someone says a region is 25 square units mean that every square is exactly the same.

4. Find the number of units

There are three ways to find the number of units. The first is by counting units. This may be done by merely counting the units or by using addition and multiplication to assist in that counting. If the students have been using non standard units, then using standard units should be easy for them. They will have a good understanding of the

process of measuring, so the purpose should be to give them a feel for standard unit. The second is by using an instrument. Before measuring the area, an instrument is used to measure a certain some dimension. The third is by using formula. The skill of using formula should be developed but it should not take the place of careful development in measuring process. Teaching the topic area in traditional mathematics causes students to think that area of rectangular shapes is always the product of two lengths by applying the formulas (Fauzan, 2002). However, in reality we mostly deal with irregular shapes. It means that the idea of approximation to find the area of irregular shapes needs to be introduced to the students. So that students think that that the area is not only belong to rectangular shape.

5. Report the number units

A measurement must include both the number and the unit chosen. This step requires students to report the result of measurement, both the number and the units used.

According to Clements and Stephan (2004) some of the basic concepts of area measurement are partitioning, unit iteration, conservation, structuring an array and linear measurement. Partitioning is the mental activity of slicing up an object into the same sized units. It involves mentally seeing the object as something that can be partitioned in to smaller area. Unit iteration is the process of finding how many units would match the attribute of the measured object. The concept of conservation of area is also an important idea that a region can be cut and rearrange its parts to another form, the area still the same. When the students can use iteration of equal unit to measure area, they need to structure the unit into an organized array to achieve multiplicative thinking in determining the area. Developing the ability to use two linear dimensions to

build the idea of a two dimensional space is also important in learning area measurement.

B. Realistic Mathematics Education

The designing of local instructional theory in this research is inspired by Realistic Mathematics Education. Realistic Mathematics Education has been developed in the Netherlands since 1970s (de Lange, 1996). This approach has been implemented in Indonesia since over the last 10 years that is called Pendidikan Matematika Realistik Indonesia (PMRI). Many schools have been involved in developing PMRI including the school that we will work with.

According to Freudenthal, mathematics must be connected to reality through problem situations. The term "*reality*" means that the problem situation must be experientially real for students. In realistic mathematics education (RME), students should be given opportunity to reinvent mathematics based on their own strategies.

The local instructional theory in this research in line with five tenets of realistic mathematics education that have been defined by Treffers (1987, in Bakker, 2004) as following:

a. *Phenomenological exploration or the use of contexts*

   In this study, students are involved to explore mathematics from a real situation that has meaning to develop basic concepts of mathematics. In learning area measurement, comparing the size of objects around can be the starting point in learning process.

b. *Using models and symbols for progressive mathematization*

The second tenet of RME gives a bridge from a concrete level to more formal level. Models, symbols, schemas, and diagrams can support the development of students' thinking from concrete level to formal level. Using non standard unit measurement that is chosen by students can be a model to support the students' thinking.

c. *Using students' own constructions and productions*

Students can use their own strategies to solve a problem that have meaning for themselves. Hence, using students' constructions and productions is promoted as an essential part of instruction. Students can use their own production when they choose their own unit in covering shapes.

d. *Interactivity*

The learning process of the students is not only as an individual process but also as social process. In this research, we ask students to work with small group so that they can share their ideas to their friends and can learn from each other in discussion. In class discussion, it could also encourage more interactions among every element in the class.

e. *Intertwinement*

It is important to consider an instructional sequence in its relation to other domains. When students learn about area measurement, it is also support other domain such as multiplication and geometry.

C. Emergent Perspective

Emergent perspective is a framework for interpreting classroom discourse and communication (Gravemeijer and Cobb, 2006). It is used to understand mathematical learning as it occurs in the social context of the classroom. This research will carry out

social perspective that related to social norms, and sociomathematical norms. Social norms refer to expected ways of acting and explaining that become established through a process of mutual negotiation between the teacher and students. Meanwhile, the socio-mathematical norms are the ways of explicating and acting in whole class discussion that are specific to mathematics.

Socio norms in this research include the role of the students to follow the instruction of the teacher, the responsibility of them to explain and justify their solutions, try to hear and understand the explanation of others, and to pose questions if they do not understand it. Whereas, the socio-mathematical norms include how the students find different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution. The students develop personal ways of judging whether their solution is efficient or different. For example, in solving the problem to count how many square unit in certain area, students can decide which solution that easier for them whether they count one by one or use multiplication.

D. Area Measurement in Indonesian Curriculum for Elementary School

Measurement in Indonesia Curriculum has been taught since in the first grade of elementary school. In the first grade students learn about comparison of length as the base of linear measurement. In second grade, students learn how to use measuring instruments both non-standard and standard instruments. In the third grade students begin to learn about area measurement. The table below describes topic measurement for grade 3 in Indonesia curriculum.

Table 1: Indonesian curriculum for the second semester of grade 3

The second Semester of Grade 3	
Standard Competence	**Basic Competence**
Geometry and Measurement 5.Calculating perimeter and area of square and rectangular, and its application in problem solving	5.1 calculating perimeter of square and rectangle 5.2 calculating area of square and rectangle 5.3 Solving problems related to perimeter and area of square and rectangle

Based on the table above, teaching and learning of area measurement focuses on square and rectangle. In this research, the activities are designed to gain insight of what area is, how to measure the area and the unit to measure the area.

CHAPER III
METHODOLOGY

This chapter describes the methodology of the research. The main issues are: research methodology, research subject, data collection and data analysis including the reliability and validity.

A. Research Methodology

The aim of this research is to develop classroom activities that support students to learn area measurement. Therefore, the sequences of activities are design to develop a unit to measure area and to interpret the measurement results. For this purpose, design research is chosen for achieving the research goal and answering the research question. Three phases of design research are discussing in this research. According to Gravemeijer and Cob (2006), the three phases of conducting a design experiment, as following:

a. Preliminary Design

In this phase, the result is a formulation of what is called *a conjectured local instruction theory*, that is made of three components for area measurement: learning goals for students; planned instructional activities and the tools that will be used; and a conjectured learning process to anticipates how students' thinking in the learning process of area measurement.

b. Teaching Experiment

In this phase, instructional activities are tried, revised, and designed on a daily basis during this experiment. The purpose of teaching experiment is to test and

improve the conjectured local instruction theory that was developed in preliminary phase, and to develop an understanding of how it works.

c. Retrospective Analysis

In this phase, all data during teaching experiment are analyzed. Hypothetical Learning Trajectory (HLT) is used in the retrospective analysis as guidelines in answering the research questions. The HLT will be compared with students' actual learning to investigate and to explain how students gain insight in developing unit in area measurement.

B. Research Subject and Timeline of the Research

This research was conducted in elementary school in Palembang, Indonesia, namely SDN 21 Palembang. However, this school is new to mathematics realistic education because it has been involved in PMRI project since 2010. A teacher and students in grade 3 were involved in this research. The students were about 8 or 9 years old and they have learnt about linear measurement and multiplication in grade 2.

This research was carried out in two cycles namely pilot experiment and teaching experiment. In the pilot experiment the sequences of activities were tried out to small group that involved 9 students. The design of HLT was tried out to see how it works and to investigate the students' thinking about the problems so that we can improve the HLT. The improved of initial HLT then was tested to another class in the second cycle. This involved whole class that consisted of 36 students.

The timeline of this research on the tables as follows:

Table 2: The outline of the data

Activity	Date
Preliminary design	
Studying literature and designing initial HLT	September 2010 – January 2011
Discussion with teacher	February 2011
Pilot experiment	
Observation in grade 3A	February 2011
Pre assessment in grade 3D	February 2, 2011
Pre assessment in grade 3A	February 3, 2011
Tryout in grade 3D	
Activity 1: Telling the size of the cakes	February 5, 2011
Activity 2: Choosing the chocolate	February 7, 2011
Activity 3: Cookies in baking trays	February 8, 2011
Activity 4: Unit Investigation	February 9, 2011
Activity 5: Tiles in living room	February 10, 2011
Activity 6: The area of swimming pool	February 11, 2011
Post assessment in grade 3D	February 12, 2011
Teaching Experiment	
Activity 1: Telling the size of the cakes	March 2, 2011
Activity 2: Choosing the chocolate	March 3, 2011
Activity 3: Cookies in baking trays	March 7, 2011
Activity 4: Unit Investigation	March 9, 2011
Activity 5: Tiles in living room	March 10, 2011
Activity 6: The area of islands	March 12, 2011
Post assessment in grade 3A	March 16, 2011

C. Data Collection

In this research, various data source were collected to investigate the learning process of the students and to answer research question. The data collection of this reseach is described as follows :

1. Video Recording

The learning process and students activities in measuring area are recorded by using cameras. Students' strategies and discussion during the lesson are easy to observe from video. Observation and interview with the students were carried out in this research. The students were interviewed to gain more information about their strategies and their reasoning to solve the problem in learning activities.

2. Written Data

The written data from students' work in solving the problems provide more information about students' achievement in learning area measurement. The written data included students' work during teaching experiment, the results of assessments including the final assessment and some notes gathered during the teaching experiment.

D. Data Analysis, Reliability and Validity

1. Data Analysis

The Data were collected and analyzed from the teaching experiment. The process of data analysis involves making sense out of text and image data that is preparing the data for analyses, conducting different analysis, moving deeper and deeper into understanding the data, representing the data, and making an interpretation of the larger meaning of the data (Creswell, 2003). Based on that, researcher transcript the video recording and interview that can give information in order to answer research question. Then, conjectures in learning trajectory were compared with students' actual learning. Analyzing the data involves colleagues and supervisor in order to avoid subjectivity point of view.

2. Reliability

Reliability of the data was gathered in qualitative reliability. The qualitative reliability was conducted in two ways, data triangulation and cross interpretation. In this research, triangulation involved three different sources of gathering data namely video recording, interview and written data. These three sources were used to check interpretation of data gathered. The data gathered also cross interpreted with supervisors in order to reduce the subjectivity of the researcher's point of view.

3. Validity

The validity concerns on the quality of the data collection and the conclusion based on the data. Hypothetical learning trajectory was used as guideline to answer research question. Learning process of the students is recorded and students' answer also collected to gain data. With this extensive data, the situation and the findings were described in detail to give sufficient information for reasoning in conclusions.

CHAPTER IV
HYPOTHETICAL LEARNING TRAJECTORY

A design and research instrument that proved useful during all phases of design research is called hypothetical learning trajectory (HLT) (Barker, 2004). An HLT consists of the goal for students' learning, the mathematical tasks that will be used to promote students learning, and hypothesis about the process of students' learning (Simon and Tzur, 2004).

In the initial hypothetical learning trajectory, students are expected to gradually achieve some mathematical goals as following:

- Students are able to identify the attribute of area
- Students are able to compare and order the area
- Students are able to compare area by using same kind of unit
- Students are able to use unit to compare the area of shape
- Students are able to count the unit used in easy way
- Students are able to determine the area of two dimensional shape by using unit given
- Students are able to find the area of irregular shapes

Based on the goals above, instructional activities are design on learning trajectories. The instructional activities are designed in four phases in which students are involved to do some activities.

The elaborated of the initial hypothetical learning trajectory as follows:

A. Identifying the Attributes

Concept:

a. Conservation of area

Goals:

- Students are able to identify the attribute of area

- Students are able to compare and order the area

Mathematical activity: Telling the size of the cake

In this activity, students will tell about cakes that are brought by teacher. There are two different sizes of cakes with the same thickness. Students are asked what they can tell about those two cakes and what the differences between those cakes. The students will discuss about the differences especially about the size of the cakes. After that, students are given figure of three pieces of cakes with different size. They are asked to order the cake in the figure based on the size. They will be provided with scissor if they feel need to cut the figure (See figure 1)

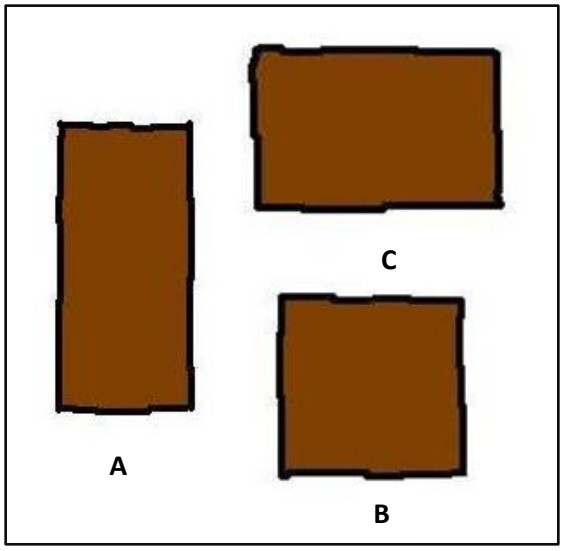


Figure 1: The cakes with different size

The tasks are:

1. The figure above is three pieces of cakes. Sort the cake based on the size.
2. What is your strategy to sort the cakes? Explain your answer.

Conjectures:

In discussion about the size of two cakes, students can compare the cakes by sight. They will react about the size in which one is big and another one is small. Then they have to explain why they think the cake is big or small. The students might think that the side of the big cakes is longer than another one. They might also think that the big cake can be cut into small pieces like the small one. Then, to order the cake in the figure, students might order from the smaller to the biggest or another around. Then, they have to explain why they think so. The problem is how they decide which one is the biggest/the smallest since the figures do not differ much in size so that they cannot compare by sight. Some children might say that the biggest is the longest one, but they have to consider the width of the cake. To decide which one is the biggest/the smallest they might directly compare the cake by cutting the figure and putting one cake on the top of other cakes. Then, they will see part of the cakes that sticks out. Students can reshape the cake in such way one cake is covered by another so that they can see which one is the biggest. They might compare the cake one by one, for example compare cake A and cake B first, after they know which cake is bigger, they will compare it with cake C. after that they can order the cakes from the biggest to the smallest or another around.

Discussion:

In discussion students usually begin by describing the sizes of objects as big and small. They gradually learn to discriminate in what way an object is big or small. They will use specific term such as long, short, large, wide, etc. By describing the size of objects as big and small, students can develop awareness of what area is, and of the range of words that can be used to discuss it. In here, students will use words that represent

quantity or magnitude of attribute by comparing the differences of the cakes based on the size. For the next task, student must order the cakes based on the size that they have discussed before. Conflicts will emerge when students compare the figure by cutting one cake and putting on the top of another but the biggest cake cannot be said certainty. What they have to do is reshape the cake so that one cake can cover another cake, so that the biggest can be said certainty. Through this problem, students become more aware that the larger piece of cakes, deals with the largest area. During this activity, students can acquire experiences with comparing strategies related to physical quantity area. The use of words such as greater, larger and smaller will focus on the attribute of area. It is also expected that they will realize that the area of a plan object does not change if it is reshaped.

b. Identical unit

Goal:

- Students are able to compare area by using same kind of unit

Mathematical activity: Choosing the chocolate

In this activity, students will work in small group (3 or 4 students). They will pretend to buy chocolate. There are two chocolate that have same price but different in size (See figure 2). The figure of the chocolates will be given to them. Students are asked to choose which chocolate that they want to buy. After that they will present their work in front of class and compare their method in choosing the chocolate with other groups. The figure of the chocolates as following:

Figure 2: The chocolates with different sizes

The tasks are:

1. The price is those chocolate above is same. If you want to buy one of those chocolates, which one would you buy? Explain your answer.
2. Describe the method that you use in choosing the chocolate that you want to buy?
3. How many unit chocolate in each chocolate? Explain your answer.

Conjecture:

Some students might think that the biggest is the best choice, but there is possibility that some students choose the small chocolate. It is become a good discussion among them because they have to explain their decision. To decide which one is big/small, they might think that the chocolate that has more units chocolate is the big one. Then, they will count the number of unit chocolate in each chocolate. They will find that the first chocolate has 24 chocolates unit and the second has 16 chocolates unit. However, if they cut the figure of the chocolate and put the first on the top of the second chocolate like what they do in the first activity, they will find that the second chocolate is bigger that the first chocolate. To convince which one is big/small, they could use the small unit of the chocolate and measure another one with them or the other way around by iterating the unit so that the object can be compared.

Discussion:

It is expected that there will be a discussion in group to decide which chocolate is big/small. It will be a conflict for students when they have different answer in discussion. Some group might decide the first chocolate is the biggest since they count the number of unit in each chocolate. Other groups maybe have different answer because they compare directly by cutting and putting on the top of another and decide the second chocolate is the biggest. It is expected that students will discuss about the unit used in each chocolate. They cannot compare the chocolates because the units are different. So, students can understand that the area of objects can be easy to compare if the similar unit is used.

B. Comparing Area

Concept: Partitioning, unit iteration

Goal:

- Students are able to use unit to compare the area of shape

Mathematical activity: Cookies in baking trays

In this activity, students tell about cookies that they usually eat. They will talk about the place to put the cookies while the chef baked those cookies. There are two baking trays that will show to the students. Those baking trays have different size, one is little wider and one is little longer. Students will ask to determine which baking tray that containing more cookies. At the first time, teacher lead the class discussion what should they do for comparing the baking tray. Students cannot compare them just by looking or by comparing directly because baking tray cannot be cut. So, they need another tool to compare baking trays. After the students come up with the unit, they

will work on small group to compare the baking tray. Each group will get two pieces of cardboard as representing of baking trays.

The tasks are:

1. Which baking tray could you put the most cookies?
2. Describe your strategy in comparing the baking tray!

Conjecture:

Some students might just estimate the biggest baking tray since they do not know how to compare. Some students might think that they only need to draw the form of the cardboard on paper and cut to compare, but they have to describe how big their baking tray to answer the question number 3. So, they need number of unit to explain how big it is. They could use any kind of units to measure the baking tray such as small paper/sticky papers and arrange them in baking tray and then counting how many are used. While covering, they may cover the paper with overlapping or with gap. On the other hand, some students might look carefully the paper that should not overlap, or might estimate the gap of the rest part of cardboard. Students who compare the baking tray with small unit will find that they need more units to cover the baking trays and the other hand, students who work with big units will find that they need few units.

Discussion:

It is expected that students can use a third object to compare the baking trays such as their hand or small paper/sticky paper because they cannot compare the baking tray directly. In the previous activity, students already know that they have to compare by using the same unit. While comparing, it is expected that they use same kind of unit so

that they can compare the baking trays. Then they have to count how many unit needed to cover each baking tray. When measuring the object with different unit, they will realize that the larger the unit the fewer are required. Also, they realize that to determine how big the baking tray, the unit that they used has to be not overlap and all parts are covered.

C. Square Unit Measurement

Concept: Structuring space, unit iteration, square unit

Goals:

- Students are able to explain that area is measured by using square units

Mathematical activity: Unit investigation

In the previous activity, students are asked to find how big their baking tray is. In this activity students will be asked to investigate which shape is suitable to measure the baking tray. The students are provided many kind of paper such as rectangles, squares, triangles and circles. Students work in small group (4 or 5 students) and each group is given one cardboard and many kind of papers to measure. The question for them is: which shape that appropriate to measure the area of the cardboard? Explain your answer.

Conjecture:

Students might cover the cardboard by using circles, but there is still space that uncovered. When they covered with triangle, they might have many variations to arrange the triangles. Some students might think that the appropriate shapes are rectangle and square. But, if they ask to cover the cardboard with rectangle in different

direction, they will find that it hard to count. After they compare with square, they will find that no matter how they arrange the square, it always well ordered.

Discussion:

It is expected that students realize that square is appropriate unit to measure the area rather than other unit. It is not wise to cover the baking tray with circle because it cannot cover all surfaces. Also, it is not efficient to work with triangle because the students will have many variations to arrange the triangles. It will be nice to use rectangle because it can cover the surface and easier to count, but the students still have variation in arranging the rectangle if the rectangle arrange in different direction. So, the best choice is square because no matter they arrange the square paper, it is always well ordered so that it will be easy to count. Through this problem, students will understand how useful it is to work with square rather than other units. Because square can cover all surface, easy to arrange and easy to count if the students use multiplication. When the students understand why they have to measure the area by using square unit, it will be easy for them to use standard unit measurement as a square unit such as square centimeters, square meters, etc. But in here, the standard unit has not yet introduced to the students.

D. Find the Number of Unit

a. *Concept:* multiplication

Goal:

- students are able to count the unit used in easy way

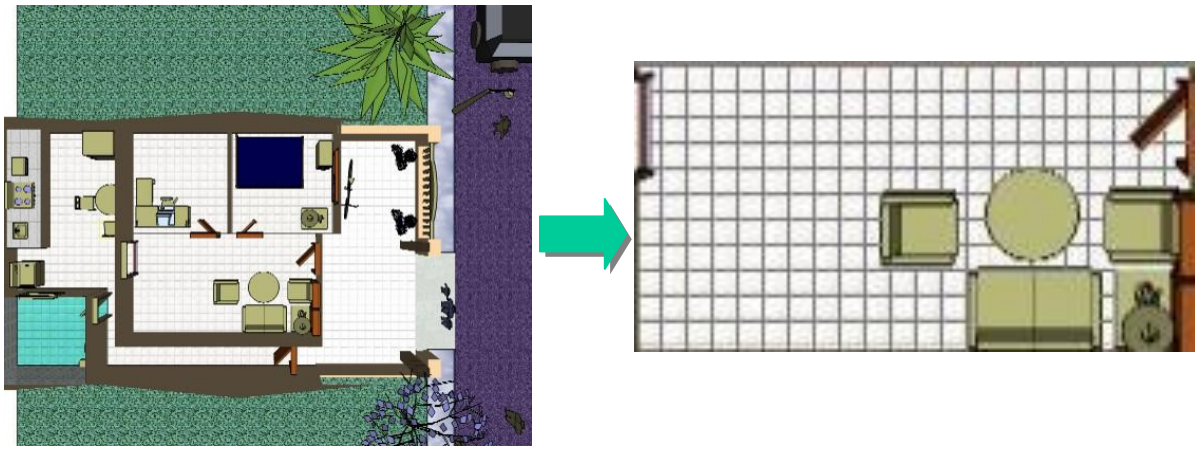
Mathematical activity: Tiles in living room

Students will tell about the sketch of Eko's house. They are told that Eko's parents want to replace the tiles of the living room with the new one. Eko wants to know the area of his living room. The students will be asked to help Eko to find the area of the living room.

The tasks are:

1. The figure below is the sketch of Eko's house. What is the area of Eko's living room?
2. How do you know that? Explain your answer.
3. Find the easiest way to determine the area of the living room. Explain your answer.

The sketch of the living room as following:



The sketch of the house *The sketch of living room*

Figure 3: The sketch of the living room

Conjecture:

Student could count the tiles to determine the area of living room. They might count the tiles one by one. But they will difficult to count the tiles because there are many tiles and there is furniture in the figure. Students could also count the tiles in rows then

count how many rows because they realize that each row has same number of square, they will multiply it by the number of rows. They could also count the tiles in column and then count how many columns in the figure then multiply it.

Discussion:

It is expected that the students can use multiplication to count the tiles in the figure. It will be a conflict for them to count the tiles because not all tiles can be seen. The students can imagine the tiles and realize that the pattern of the tiles is unchanged. It will be easy to count if they think that they have to find the number of tiles on the edge of the room. It is expected that they will use multiplication as an easy way to count the tiles.

b. Concept: conservation

Goal:

- students are able to find the area of irregular shapes

Mathematical activity: The area of swimming pool

In the previous activity students only deal with regular shape problem. In here, students are asked to find the area of irregular shape. Students are given a figure of swimming pool that has irregular shape and then they have to find the area of that swimming pool (See figure 4).

The tasks are:

1. The figure below is a sketch of swimming pool. Find the area of the swimming pool!
2. How do you know that? Explain your answer.

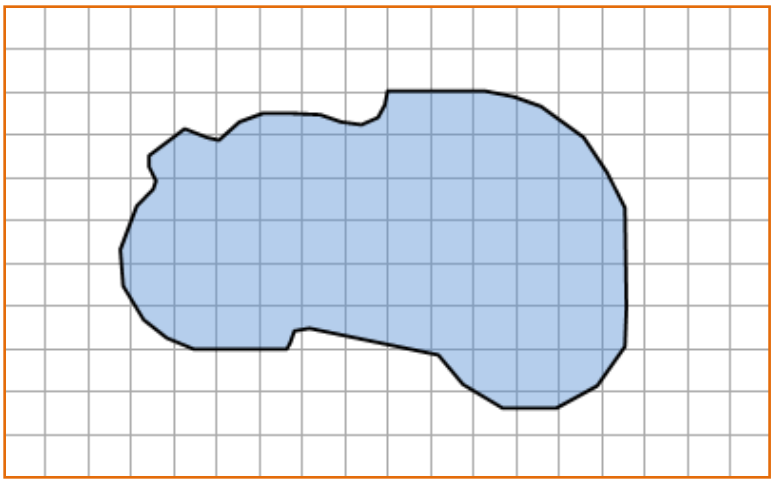


Figure 4: The sketch of swimming pool

Afterward, students will do some exercises to find the area of some irregular shapes such in the following figure:

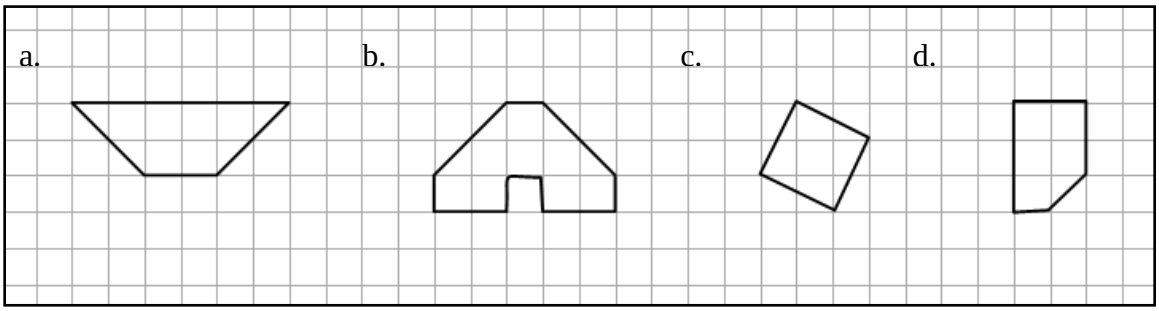


Figure 5: Exercises of irregular shapes

Conjecture:

To know the area of the swimming pool, students could count the square tiles that cover the swimming pool. But it difficult to count how many square in it because the swimming pool has irregular shape and there are many squares unit which not fully cover the swimming pool. Students might only count the full squares that cover the swimming pool without considers the squares that not fully cover the swimming pool. Some of them might estimate by adding mentally the squares which is not full with the

other. They also could count one if the square cover more than a half and not count if the square cover only less than a half.

Discussion:

In this activity, students deal with irregular shape that can make them aware in which the area not only for regular shape like square and rectangular. To find the area of irregular shape students can estimate the area by using unit given. So, the unit can be used to determine the area of irregular shapes. The students have opportunity to see the partial unit can be made to be a unit. While the method does not give an exact area, the result is usually close.

CHAPTER V
RETROSPECTIVE ANALYSIS

Data gathered from pre test, pilot experiment, teaching experiment and final assessment are analyzed in this chapter. The results of this chapter are described as underlying principles explaining how and why this design works. Hypothetical learning trajectory is compared with students' actual learning as guidelines to investigate and to explain how students learn area measurement and develop unit as a means of measuring area.

A. Pre assessment

Pre assessment aimed at seeing the pre knowledge of the students. Students in class 3A and 3D were involved in this pre test. In this test, students had to answer four problems (See appendix A). Problem one consists of two questions. Question a, students were given figure of two field. One is longest and one is little wide. They were asked to tell anything about those two figures. This question aimed at investigating how students describe the size, whether they can use their word to describe the size of objects. Many students described those figures by telling the name of the shape, such as square and rectangle. For example, Indri wrote that Pak Saleh's rice field is a rectangle and Pak Ridwan's rice field is a square. Yessi described the figures by telling that Pak Saleh's rice field is less than Pak Ridwan's rice field and Pak Ridwan's rice field is more than Pak Saleh's rice field. The other two students described by other words. Oka described the figure by telling Pak Saleh's rice field is longer than Pak Ridwan's rice field and Agnes wrote Pak Saleh's rice field is wider than Pak Ridwan's rice field. Meanwhile, Arif and Habib could describe the figure more specific. Arif wrote that Pak Saleh's rice field is longer and smaller, Pak Ridwan's rice field is shorter and bigger and

Habib wrote that Figure A is longer than Figure B but figure B is bigger than figure A. Only few of them described the figure by using words big and small. It makes sense that they still did not have an idea about area yet.

Question b, the students had to determine which field has the largest yield. It aimed at seeing whether they have a sense the attribute of area as two dimensional shapes. Most of students know that figure B has the largest yield. However, many of them cannot give their reason why they think so. They only said that Pak Ridwan's field has the largest yield and Pak Saleh's field has little yield. Only few of them can give reason that Pak Ridwan's field is bigger than Pak Saleh's field.

Problem two aimed at knowing student's strategy in counting the tiles. The sketch of room with furniture in it was given to the students. Although they cannot see all tiles in the figure, it is expected that they count all the tiles in the figure and see mentally that all room is covered by tiles. Most of the students counted the tiles one by one. After interviewing some students, only one student counted the tiles by using multiplication. However, there are some students did not count the hidden tiles. They only counted the visible tiles. Meanwhile, the other students did not count the tiles. They only described the sketch of the room like what they did for problem number one. It seems they did not read the question although the teacher has explained the problem to them. Students who read the question counted the tiles one by one.

Problem three aimed at seeing whether the students can distinguish perimeter and area. For this purpose different shapes of swimming pool were given to the students and they are asked to find which swimming pool is the biggest. In solving this problem, most of the students did not give their reason why they choose the biggest swimming pool. After interviewed some students, three of them stated that A is bigger than B by counting the square

in it. One of students, Vincent, wrote that the figure A is bigger because its square is more than B as can be seen in figure 6. He counted the squares in each swimming pool and compared the number of the squares.

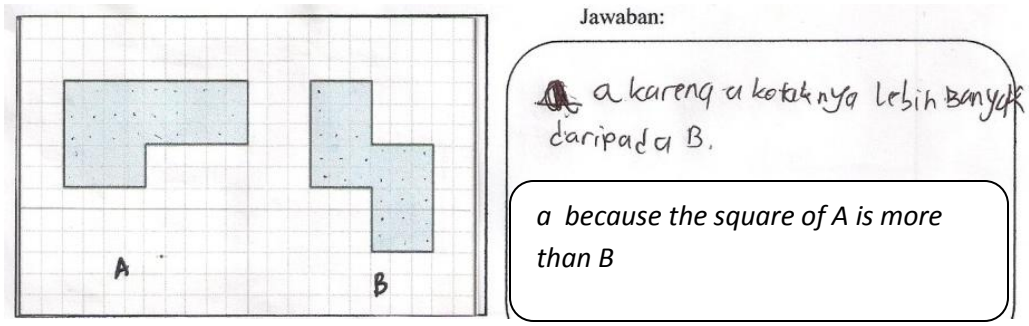


Figure 6: Student's work in counting the units

Some of them chose the bigger based on perimeter of the figure rather than area. Based on their explanation they decided to choose B as bigger because they showed that they measured the perimeter of those two figures. Nadia was asked to explain her answer why she choose figure B. She said that she used ruler to measure the side of each swimming pool (See figure 7). It seems she think that measure the length around the figure is a way to compare which swimming pool is bigger.

Figure 7: Using ruler in measuring

In line with Nadia, Putriza said that B is longer than A while point the perimeter of figure B. it seems that she was thinking that the biggest swimming pool is the longest

perimeter. Arif also explained that B is bigger than A by pointing the perimeter of the figures. It seems that he was thinking that the biggest swimming pool is the longest perimeter.

Problem four aimed at investigating whether the students count the square which is not intact or they ignore that. Four figures of irregular shape were given to them and they have to find how many squares in each shape. Almost all students counted all tiles in each figure although the square is not intact. They did not consider that they should count the square within boundaries of the shape. They did not see that the triangle in the shape can be combined become one square. Only one of them, Mirza, explained that he got his answer by adding a half square became one square (See figure 8).

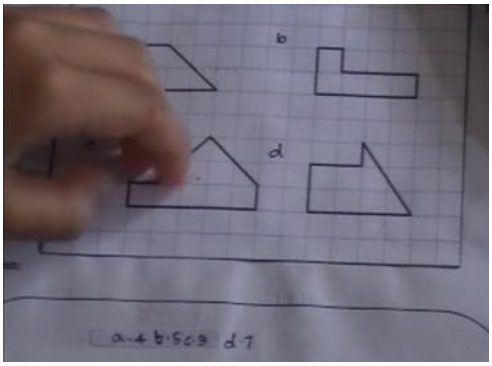


Figure 8: counting the partial units

The pre test showed that students still have a vague notion of area. They still have no word to describe the area and some of them still confuse between area and perimeter by revealing that the area is the length around the figure. It means that it still a lot to be learned related to area. Therefore, for the next cycle, some problems in post test will be modified much more similar to the pretest. In this way we can see how students' understanding has grown in understanding area measurement.

B. Pilot Experiment

The initial HLT was tried out in pilot experiment to find out how the designs work and how the students react about the problem given. Conjectures in our HLT were compared to the students' answers in actual learning process. Data obtained were analyzed to support the adjustment of the initial HLT. Nine students were involved in this pilot experiment. They have different academic ability with high achiever, average achiever and low achiever students.

1. Activity 1: Telling the Size of the Cakes

The aim of this activity was to investigate the ability of students to identify the attribute of area by comparing and ordering the area. For this purpose, students were given a figure of three cakes (see page 17) and they were asked to order the figure based on the size of the cakes. The students worked in pairs so that they can create effective discussion and it allows them to give more contributions to the group.

Before they compare and order the figures, the students were provoked to build vocabulary related to area. Two cakes with different size were shown to them and they are asked to describe the size of the cakes. As we predict, they only compare by sight. But then they were asked why they think so. But they were difficult to describe why the cake is big or small. They answered that one is square and another is rectangle. They also said that the side of one cake is longer than another cake.

To compare and order the figure of the cakes, we predicted that students will cut the figure, put it on the top of the other and see the rest of each figure to compare and order the figures. However, some students just guessed which one is the biggest and which one is the smallest by comparing by sight. One of students' answers in their worksheet: *Figure A is longer, figure C is rather bigger whereas figure B is shorter.* Students also explained their

answer by telling the shape of the cakes. They wrote: *Figure A is bigger than figure B and C because figure A is rectangle, figure B is square and figure C is medium square,* and state the order: *From the smallest B, C, A, from the biggest A, C, B.*

Based on students' answer, they can describe the figure by using the words bigger, smaller or shorter**.** They also described the size of object by telling the shape of the object, such as rectangle is bigger than square. They mention the name of shape seems that they did not have words in explaining why A is bigger than B and C since they also state the order of the cake based on the size. It means that the name of the shape only their way to explain the comparison.

In the figure, the big one is the long one so that they give reason the big one is the longest one. Therefore, the figure of the cake will be modified so that students can give more varied reasons for their answer, not only because the cakes is long but also the cake has two side that has to be considered. Students also did not get used to give reasoning therefore they have difficulties to give a reason for their work. When they were asked which one is bigger, they just replied that this one is big while pointed to the big cake. Before the activity comparing the three figures of cakes students had an activity to describe the real cakes that is brought by teacher but it seems did not help them in comparing the figure. Because the cake has height while the figure is a plan figure. Therefore, for the next cycle students will be given two invitation birthday cards which have same design but different in size. Students are asked to compare which card needs more paper. It is expected that students can have an idea to put the card on the top of another. Moreover, it can provoke them to give reason that one card has more rest if the cards are attached to each other.

During discussion in group, no student wanted to cut the figure or try to put one on the top of another as what in conjectures is, even though they saw scissors. Whereas, it is expected

that they can gain sense that they want to measure a region by putting one on the top of other instead just measure the side of cakes. Students were asked how to convince that their answer is true. Students kept silent and researcher offered to use scissors.

Researcher : Who can convince me that this (pointing figure A) is the biggest? I don't believe that this one is the biggest.
Students : (silent)
Nisa : That is a square (figure B)
Researcher : Can we use scissors?
Nadia : Yes, we can
(Nisa cut figure B and then put it on figure C, see figure 9)
Nisa : Nah, isn't it. Small (pointing piece of figure B)
Nadia : Try this (pointing figure A)
(Nisa put piece of figure B on the figure A)
Nisa : This one is longer (pointing figure A)
Researcher : Now, order those figures based on the size

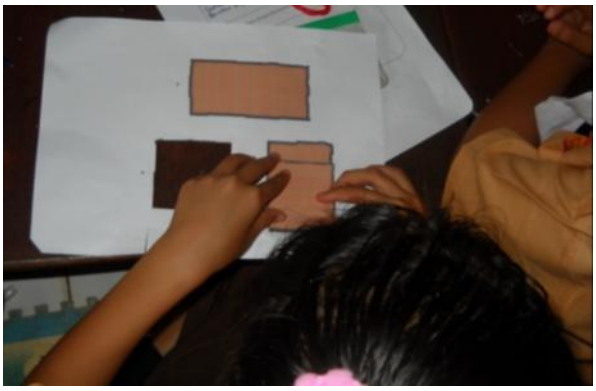


Figure 9: Putting the slice of cakes on the top of another

While cutting the figure, some students still did not know what to do with the figure. They only see how the other friends work with the figures and then did the same thing like their friends did. When comparing figure A and B, Nadia came up with unit which is appear in the figure accidentally because the print out of the figure leave a mark like lines.

Researcher : What do you think which cake that can be shared to the most people?
Students : A
Researcher : Are you sure that figure A is the biggest?
Students : yes
Researcher : why figure A is the biggest?
Nadia : if we cut, it will be bigger

(she tried to cut the figure but her friend forbade to do that)
Nadia : Nah, there is line in this. (put slice of figure B) One, two, three, four, five, six, seven, eight. (put slice of figure A) One, two, three, four, five, six, seven, eight, nine , ten, eleven, twelve.
Researcher : so, which one is the biggest?
Students : A
Researcher : Now, we compare B and C. which one is bigger?
Student : B
Firman : C
Nadia : Let us count. One, two three, four, five, six, seven. B is medium and C is smaller.
(in fact, C is bigger than B)
Researcher : let us prove it. If we cut it which one is bigger?
Students : C
Researcher : if we stick with each other which one is bigger?
Nisa : C
Researcher : do you agree all?
Students : Yes
Researcher : Nadia. How do you think?
Nadia : C
Researcher : Are you sure?
Nadia : (nodding)
Researcher : why?
Nadia : C is rectangle
Researcher : did you see the rest?
Nisa : yes, because C has more rest

From the conversation above, students finally found that the biggest cake is the cake that has more rest although some of them were difficult to explain that. They give reason by telling the shape because they did not know how to explain. In short, in this activity students have a sense of the attribute of area that is the quantity inside two dimensional shapes since they noticed the quantity inside surface of an object.

2. Activity 2: Choosing the Chocolate

In this activity, students were asked to choose a chocolate that they want to buy if two different chocolate but different in size have same price. The goal of this activity is to investigate the ability of students to compare the area of object by using identical unit for both objects. Our conjecture is students use the small unit of the chocolate and measure another one

with them or the other way around by iterating the unit. So, the object can be compared by using the same kind of units. Students were given a figure of two chocolates (see figure 10) and worksheet.

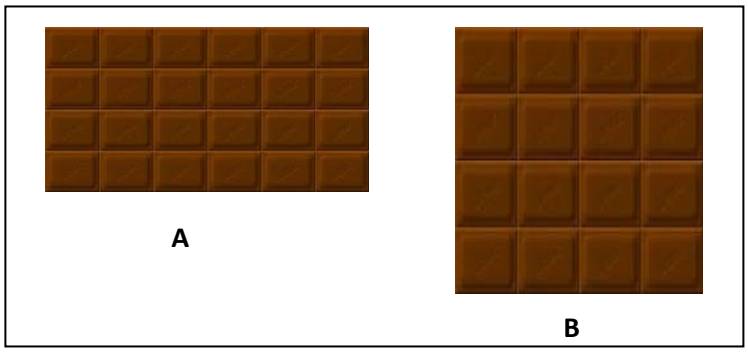


Figure 10: The Chocolates

For the first time, all students give answer directly after seeing the figure of the chocolate.

Researcher : *In the shop, there are two chocolates that have the same price.*
Nadia : *How much the price?*
Researcher : *The price is the same. If you want to buy the chocolate, which chocolate that you want to buy?*
Students : *B*
Nisa : *It is better A because that is a lot.*
Researcher : *look at the picture. Which one would you buy?*
Nisa : *A*
Yasrib : *B*
Researcher : *Yasrib, why do you choose B?*
Yasrib : *because it is a lot*
Researcher : *what is a lot?*
Yasrib : *its slab*
Nisa : *But A is more, A is bigger*
Nadia : *Yes*
Researcher : *How do you know?*
Nadia : *A, the slab is more than B.*
Researcher : *Yasrib, tell to the other. Which chocolate that you want to buy?*
Yasrib : *B*
Researcher : *why do you choose B?*
Yasrib : *(silent)*
Nadia : *Just say that the shape is same with his body.*
Nisa : *Yes, he is big so the chocolate also big.*
Researcher : *so, why do you choose B?*
Yasrib : *Because it is big*

Researcher : which one is big?
Yasrib : the slab
Researcher : how about A?
Yasrib : the slab is small
Nisa : But it has more slabs.
Researcher : How much?
Students : Twenty four
Nadira : fourteen (nadira pointing figure B but then she change her mind become sixteen after count it again, see figure 11)

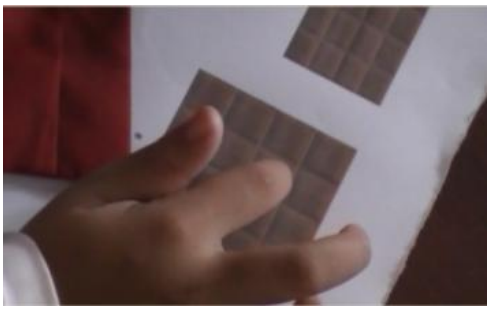


Figure 11: Counting the slab of chocolate B

After the students know how many slabs in each chocolate they were asked which one is bigger. They said that A is bigger. Researcher asked them to relook the figure and asked which one is bigger. Nadira did not agree with her friend. To prove it, they agreed to cut the figure. The students noticed that the chocolates have different size of slab, but they still have difficulties to make a conclusion that to compare the chocolate they need the same size slab of chocolate. Nadhira was interviewed separately to know more about her answer.

Researcher : Which one is bigger A or B?
Nadira : B because A is cut into small part and B is cut into big part.
Researcher : How do you prove that B is bigger than A?
Nadira : Because A is cut into small part, B is cut into big part. If B is cut into small part, B has more slabs.

From the observation, Nadhira could see that the small slab of chocolate can be used as unit to measure chocolate so that the big chocolate can be said certainty. But, not all students realized that they need same unit for both chocolates in comparing.

3. Activity 3: Cookies in Baking Tray

This activity aimed at investigating how the students create their own unit to cover a shape. This activity was begun from comparing two baking trays. The students were asked to find out which baking tray could put the most cookies. Our conjecture is students try to put something as cookies on the baking tray such as paper. For the first time, all students immediately said that the long one can put the most cookies because that is long. And then we tried to provoke them to prove what they said. Nisa said that those two baking tray have to compare by stick them to each other. And then Nadia said that the longest one has the more rest. But she cannot say it surely because the rest is almost close. Students then discuss with their group how to prove it. We conjectured that students will need a tool to compare the two baking tray. Mirza and his friend tried to measure the baking tray by putting them close to each other. Meanwhile, Nisa and her group attached the baking tray together and see the rest. Nadia iterate her hand on the rest of baking tray and say "but it can be more" but she did not count it. Afterward she made a square with her finger on the baking tray (See figure 12).

Figure 12: Making units by hand

The fragment below is the interview with Nadia.

Researcher : What do you make with your finger? What is that?
Nadia : It is the cookies
Researcher : So, do you need cookies?
Nadia : Yes

Researcher gave them a various sticky paper as cookies. We also predicted while covering, students look carefully the paper that should not overlap or may cover the paper with overlapping or with gap. However, students arranged the sticky paper very carefully on the baking tray to avoid overlap even they decorate the cookies with color sticky paper. But then they realized the decoration is not useful.

Nadia and her friend used different sticky paper to cover the baking tray. They also covered the rest that cannot be covered by big sticky papers with the small one. Nadhira counted the sticky paper by numbering the paper. She began confuse when she counted the sticky paper that cover the baking tray because the sticky papers are different. Researcher asked her how about that. She still confused and researcher asked other students how to compare two baking tray if they use different size of sticky paper. They answered baking tray A because it can put more cookies. Researcher said that "but the cookies are different". They still did not realize that they have to use same kind of unit. Then they are reminded about previous lesson that to compare the chocolates they need same slabs of chocolate. However, for the baking tray, they still cannot conclude that they also need the same cookies. After given cross questions, the students finally perceived that they have to use the same kind of cookies to compare the baking trays.

We can conclude that students perceive that they want to compare the quantity of baking trays to decide which baking tray can put more cookies. To compare the baking tray, they partition region by covering the baking trays with small papers as units.

4. Activity 4: Unit Investigation

The goal of this activity is to investigate students reasoning what kind of shape that appropriate to cover the object. To do so, students were given a cardboard as baking tray and

small paper with different shape such as circle, triangle, rectangle and square. They have to find what shape that appropriate to cover the baking tray. We predicted that students find that square is the suitable unit because other shapes cannot cover all surface of baking tray. Students worked in group to solve this problem. At the first time Firman's group of the students chose triangle to cover their baking tray. They seem difficult to arrange the triangles since they quarrel while covering. And then, Firman and Yasrib argue about the paper that stuck to each other. However, they cannot give their reason why they think so. The fragment below is interview with Firman and Yasrib.

Researcher	*: What about this one, you see, this one stuck together. What do you think? Can we do that?*
Yasrib	*: No, we cannot*
Firman	*: Yes, we can*
Researcher	*: What do you think Yasrib?*
Yasrib	*: No, it is not allowed (still quarrel with Firman)*
Researcher	*: Why?*
Yasrib	*: Look this one is come out (point to the triangle that comes out from baking tray)*
Researcher	*: How about that?*
Yasrib	*: Just make it like this (arrange the triangle inside the baking tray)*
Researcher	*: But they stuck together. What do you think Firman?*
Firman	*: It is allowed*
Researcher	*: Why is it allowed?*
Firman	*: In order to become fit*
Researcher	*: What do you think Indri?*
Indri	*: It is allowed*
Researcher	*: So, it means that we can put like this (put the triangle on the top of the other)*
Firman	*: The cookies is going bad (put the triangle outside)*
Yasrib	*: Let them like that (repair the arrangement of the triangle)*
Researcher	*: Why is it not allowed?*
Yasrib	*: No, it is not allowed*
Researcher	*: Why?*
Yasrib	*: I do not know*

Furthermore, Firman decided to repeat covering the baking tray with triangle. Then researcher asked him if he want to cover it by using triangle again. He nodded and researcher offered him to use other shapes to know which shape that suitable to cover the baking tray.

Researcher : How about circle?
Firman : Yes we can
Indri : No
Firman : So, which one?
Yasrib and Indri : the square
Firman : Oh, yes

In discussion students presented they work in front of their friends. There are only two groups and all group said that square is the suitable shape to cover the baking tray. But only one group gave their reason in their worksheet. They said triangle and square cannot cover all baking tray because there is some rest that is not covered. Meanwhile, they said that the rectangle has the most rest. It is because they arrange the rectangle in one way so that it leaves much rest that is not covered.

After seeing the work of students, it is anxious that the students only think that square is the only one unit for area instead suitable for measuring the area. Therefore, for the next cycle we change the discussion so that students realize that there are many kind of units called non standard measurement that can be used in measuring the area. If we use these units of measurement for same object, it does not give exact or uniform number of unit measurement. So, the goal becomes students are able to explain that using non standard unit measurement do not give exact or uniform unit measurement for same object. In this way, the students also learn how to reveal the quantity of area by giving name of the unit and realize the inverse relationship between the number of units and the size of the unit.

5. Activity 5: Tiles in the Living Room

In this activity students were asked to count how many tiles in the sketch of living room. It is purposed to investigate how they count the tiles even though in the sketch there are furniture so that some tiles are hidden. Our conjecture is the students use multiplication as an easy way to count the unit because there are so many tiles and some tiles are hidden. We also

predicted that they will count the tiles one by one. Another conjecture is the students can see the hidden tiles but some of them did not count the hidden tiles.

To count the tiles, some students made dot in each square and some of them labeled by number in each square. Nadhira made mistakes in labeling (See figure 13). She labeled one number for two squares like in the following figure. She made 27 with 2 in one square and 7 in one square. At the end she answered the tiles are 244 tiles.

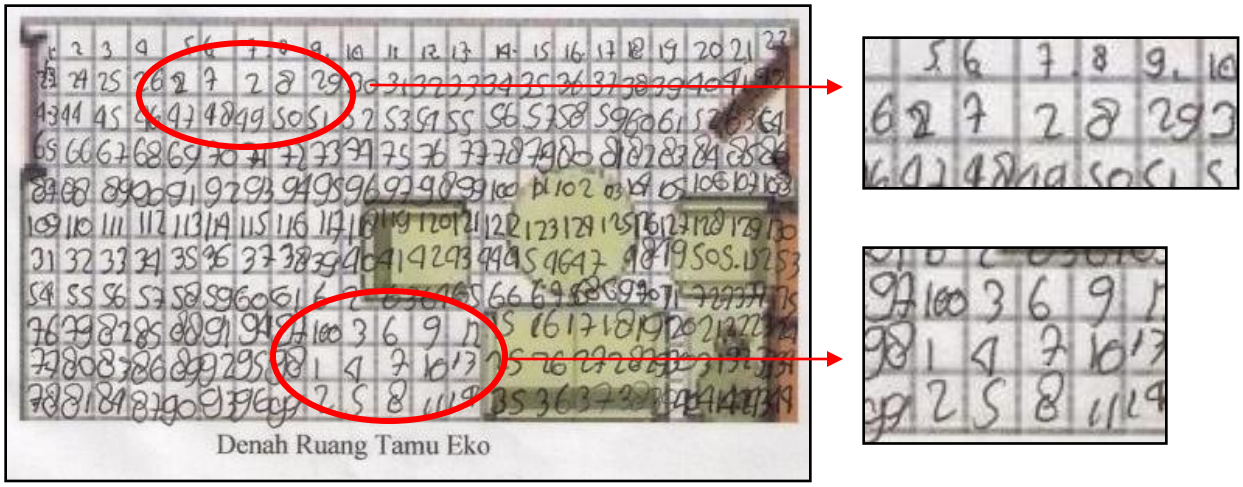


Figure 13: Mistakes in labeling the squares

Firman and Yasrib had problems with counting. Firman counted the tiles one by one and he still confused in counting. He took long time to continue what number after the number he just said. He said "…, seventy eight, seventy nine,…., ninety, ninety one,..." . He realized that his counting was wrong so that he counted it again. We can see many dots that he made in each square like in the figure 14.

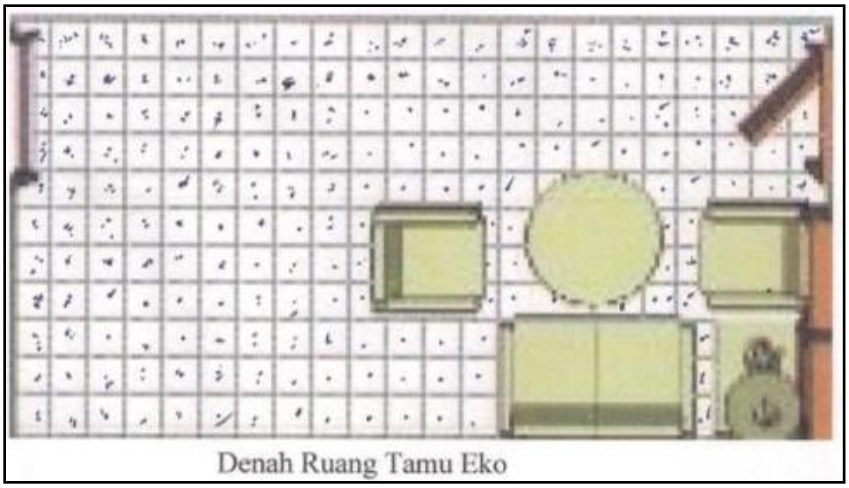


Figure 14: Making dots in counting

In class discussion students were asked whether the tiles under the furniture are counted or not. They said that the tiles are counted. Firman then realized that under the furniture there are tiles that cannot be seen. Afterwards, they were asked how many tiles in one row. They are asked to convince themselves after answered twenty two. Then they counted the first row and got twenty two. They were also asked how many tiles in the second row. Nadhira said "it is also twenty two". Why is it twenty two? Researcher asked furthermore. "Because it is same" Lisa said. It seems she realize that each row consists of the same number of tiles.

All of them agreed that there are eleven rows of tiles in the figure. And each row has twenty two tiles. When they were asked how many tiles all together, Nadhira write 22 eleven times by numbering the number and then she added them. But she made a mistake while counting because she write 244 and erase it became 242 (see figure 15)

At the first time Indri also count the tiles one by one. After she knew that each row has twenty two tiles, she added them by heart and started from 110 plus 22 and added the result with 22. She got 232 as the result but she was not sure about her answer. Then she has an idea to multiply it and she got 242. We can see how she found in the figure 16.

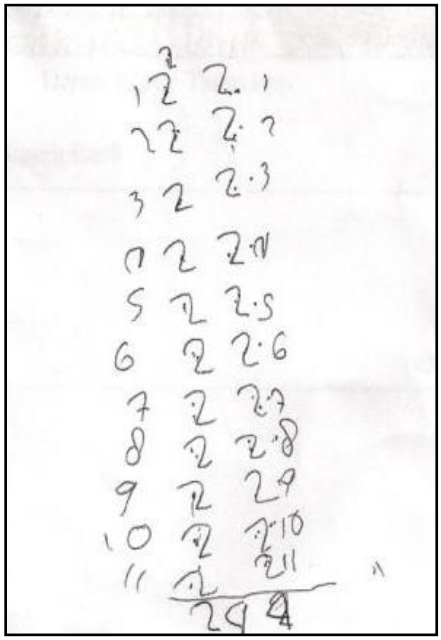


Figure 5.11: Nadhira's answer

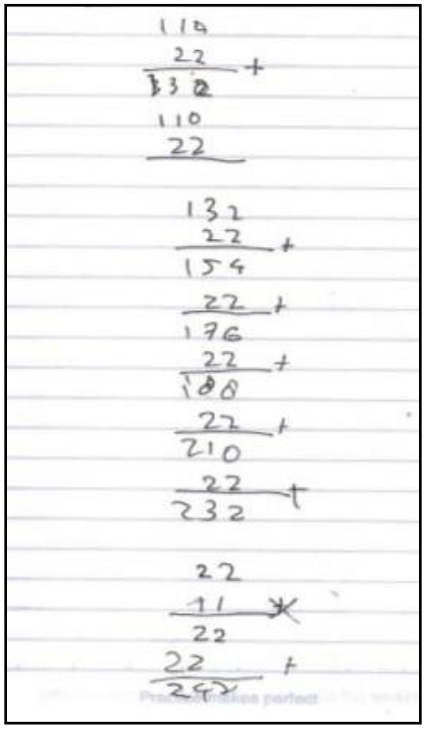


Figure 5.12: Indri's answer

From students' answer, the students have difficulty to count the tiles. It seems using multiplication did not help them in determining the area easily. Multiplication or counting one by one seems hard work to do because they work with large number. Therefore, the problems will be modified for the next cycle. The discussion will be lead into what unit that they are going to use in determining area. In doing so, students will be asked to find the area of the room that is not fully covered by tiles and use the tiles as a unit. It is expected that they not only count the unit in easy way but also learn that the units must be aligned in an array with the same number of units in each row by fitting the unit into the figure. They can perceive the idea of unit iteration and the idea of structuring an array, thus they can understand that area as truly two dimensional. Some might do not think that they need to make the arrays if they can understand in each row or column consist of the same number and they can multiply the number of unit in each side.

6. Activity 6: The Area of Swimming Pool

In this activity students were given the sketch of swimming pool which has irregular shape. They have to determine what the area of that swimming pool is. This activity aimed at investigating the ability of students to find the area of irregular shape, whether they count the square that is not intact or they ignore that. Before this activity students were given the figure of four shapes and they were asked to find the area of each shape. Our conjecture is the students might only count the full squares without consider the square that is not intact. As our prediction, all students ignore the square that is not intact. They just counted the full square.

The students were provoked to count all units by making a triangle which has two squares in it. The students said that the triangle has three squares. Later on, they were told that two triangles can be added become one square and they were asked to look at back their work.

In doing their work, students make a sign like in the figure below. Lisa merged the triangle become a square then count how many square in each figure (See figure 17). But for number c, she only tried to find the pairs of triangle without consider the shape of triangle whether it can be made into square or not.

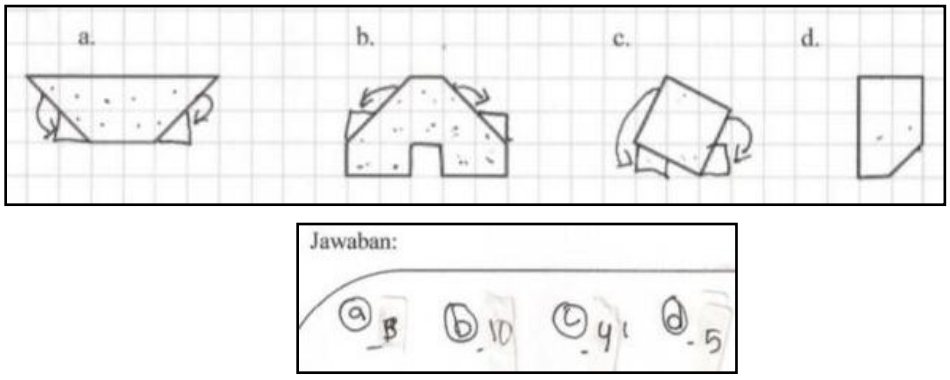


Figure 17: Lisa's answer

When they were asked to find the area of swimming pool, some of them still did not count the square that is not intact. But the other can find the pairs of the squares that were not intact and estimate the area but they did only for some pairs (See figure 18).

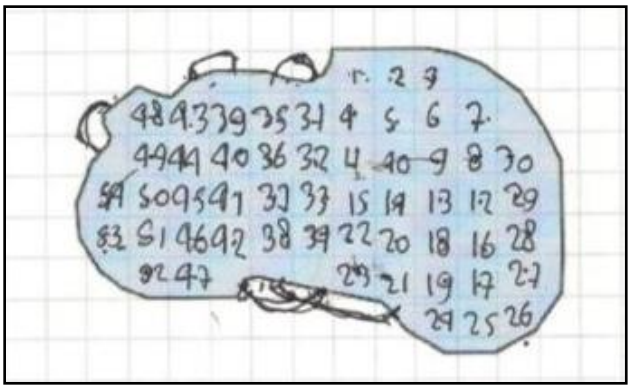


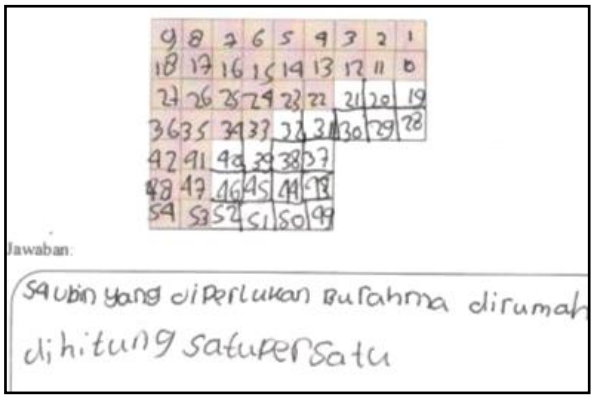
Figure 18: Counting only some pairs of partial units

From the students answers we can see they still have difficulty to determine the area of irregular shape. Conjectures did not emerge may be because students did not realize that the area is the surface contain within a boundary. They only focus on counting the square in the figure without consider that the boundaries of the figures. They should know first that whole surface must be considered if they want to find the area of the shape. Therefore, the exercise will be modified in order the students consider about the boundary of shape by simplify the figure and coloring the figures. The worksheet also will modify by letting students to compare the figure. They will be asked which island is larger so that they can really use squares as a tool in comparing. It is expected that students realize the extra space that have to be considered to determine the area.

7. End Assessment of Pilot Experiment

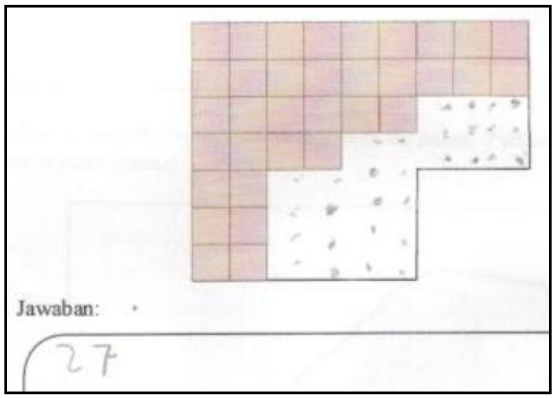
At the end of the series of activities, students were given the task with four questions. This task is aimed to see whether the students can solve the problem related to area of shape

after the series of lesson. Problem one aimed at assessing whether students know to measure the area is by counting the unit and whether they count the area in handy way or just count it one by one. This problem also aimed at seeing whether they can slice up an object into the same size units mentally since the unit is not complete covered the area. If the students understand how to measure the room, they will use the unit given. They also can make partition based on the unit given. The figures 19 are some of their answers for the first question.



54 tiles is needed by Bu Rahma
I counted it one by one

Nadhira's strategy



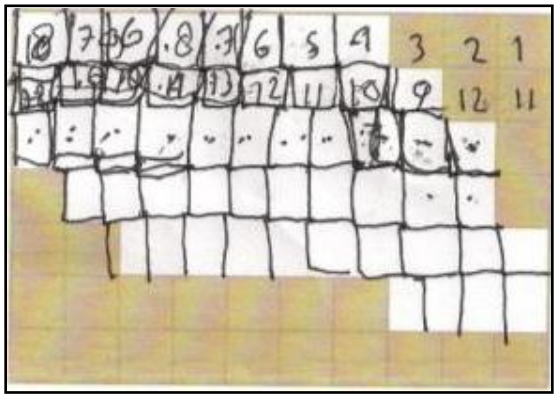
Firman's strategy

Figure 19: Different strategies of students in counting the units

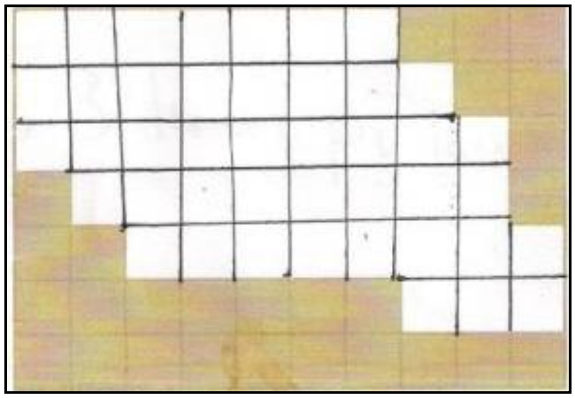
Based on the answer, Nadhira has been able to make her own unit to count the tiles by imitating the unit given. She also labeled the tiles with numbers and found the tiles needed. Whereas, Firman made dot as his unit but his dot is not in line with the unit given. Based on the dot that he made, he just counted how many more tiles to cover the room. It seems he had wrong interpretation of the question given because the question is to determine all tiles in the room. Not only Firman, some students had wrong interpretation of the question. They just

counted how many more tiles to cover the room. However, from the figure above we can see that students can partition the area and make their own unit.

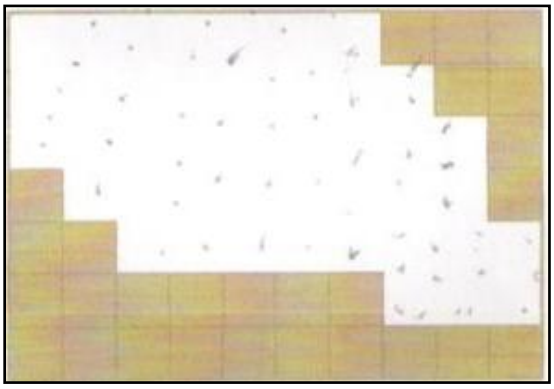
Same with problem one, the aim of problem two is to assess whether the students can slice up an object into the same size units and count the unit. The figure of cakes with some of them have been taken by the guess are given to the students. The problem is to find how many cakes are missing. Students can use many strategies to find how many units are missing. We can see in the figure 20 some of students' answer for this problem.



Nadhira'strategy



Desy's strategy



Firman's strategy

Figure 20: Different strategies of students in partitioning

It is expected that they use multiplication and then subtract by the rest of the cakes. But almost all students solve the problem by first creating the unit and then count the unit one by one. No one of them use multiplication.

Students seem have difficulties to make the unit. They did not use their experience in solving problem number one. We see Nadhira's answer, for the first question she was able to continue the unit given but in the second question she just make square without consider the unit given. Meanwhile, Desy used ruler to make lines continuing the unit given like what she did to answer problem one.

Problem three, students are asked to find the area of an island on grid paper. This problem aimed to assess whether they students can estimate the area of irregular shape into the close result. Most students solved the problem by counting squares in the island. Some of them only counted the intact square, some count all square include the square that is not intact.

But Lisa and Nadia estimated by combining the squares which is not intact with the other such as in the figures 21. We can see that they can estimate the area of the island. Based on the figure above, Lisa and Nadia tried to find pairs of the half square by making arrows for the pairs.

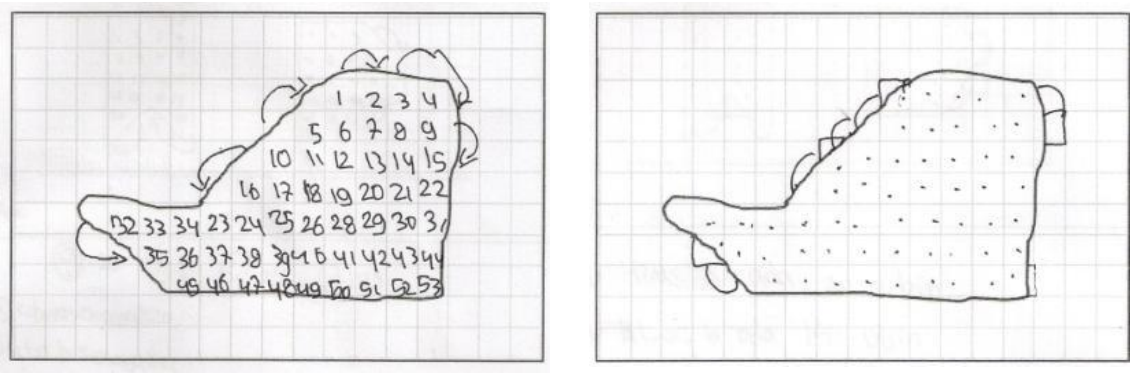


Figure 21: Students' strategies in finding the area of irregular shape

The problem number four also aimed at assessing whether the students can find the area of other shapes beside rectangular shape. If they understand about conservation of area, they will know that they can mentally arrange the unit that is not intact become square unit so

that they can count the number of unit used. Four students counted the square by adding the squares which is not intact with the other. The figures 22 are the answers of students.

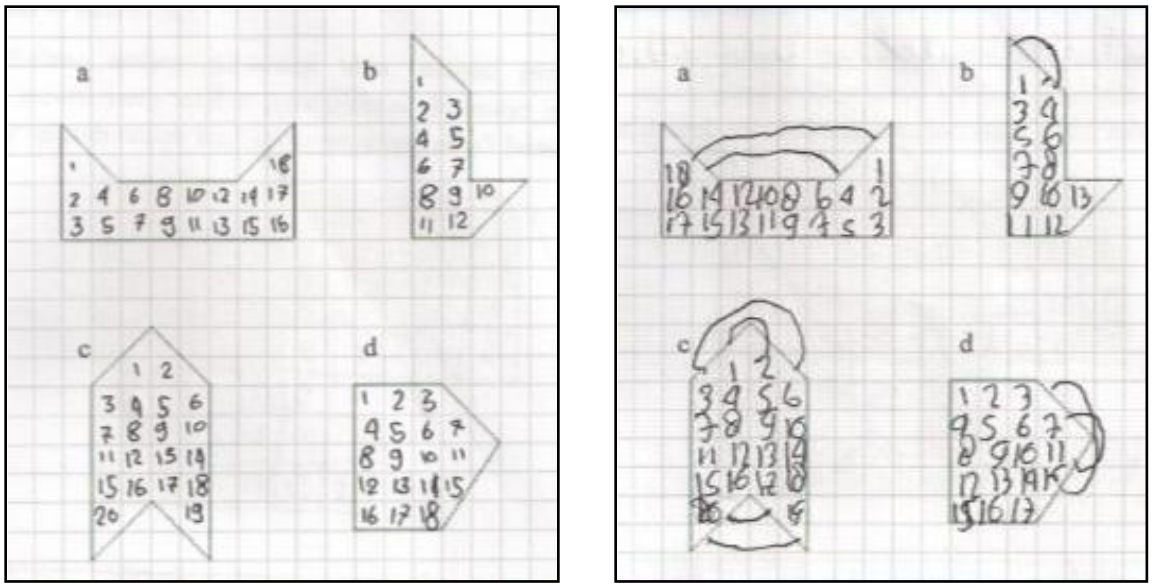


Figure 22: Students' strategies in finding the area of irregular shapes

Same with problem number three, some of them only counted the intact square. The other counted all square included the square that is not intact. It seems they still have difficulty to understand the concept of area that is the quantity within boundaries and they only focus on the number of the square in the shapes.

8. General Conclusion of Pilot Experiment

The observation during pilot experiment showed that students gradually perceive the attribute of area since they see area as two dimensional shapes. They see the quantity inside surface of an object while comparing the cakes and also the chocolate. They also have perceived the need of unit to measure the area. Some of our conjectures match with the actual learning process. Students can iterate the unit and overlapping also observed while students measure the area. However, the students seem still do not understand that the area is a region

within a boundary. In doing the task of irregular shape they only count the intact square and ignore the not intact one within boundaries without trying to rearrange the shapes. Moreover, while doing their work they are still difficult to give reasons for using identical unit in comparing. Therefore, for the next cycle some tasks in the design activities will be modified to improve the design of HLT. Some of mathematical goals also revised adjusted with HLT.

For the first activity, we will modify the figure of three cakes so that students can give varied reason to explain why one cake is bigger than the other. Also, for introduction we change the real cake with invitation cards in order to provoke students to compare the size not only compare by sight but also compare directly by put the one to the top of another.

We also modify the figure of chocolates in second activity so that students can see that two big slabs of chocolate can be made into three small slabs of chocolate. It is expected that students can see the relation between unit given and they can perceive the idea of identical unit to compare the area. In the unit investigation, we change how the students work in group. For the first all kinds of units were given to the students and then they were asked to find out what kind of unit can cover entire surface of baking tray. But the students did not focus to find out. So, for the next cycle we only prepare them one kind of unit for each group and then compare what they found in class discussion.

In determining the area of irregular shape, students still have difficulties to decide whether they have to count the not intact square or not. It seems they have not perceived the idea of conservation area because they are not able to rearrange mentally the shape. Therefore we will improve the discussion with the students by modifying the figure in the worksheet so it can provoke them to differentiate intact square and not intact square. They also have to compare two figures of irregular shape. It is expected that they realize the quantity within boundary when they compare the figures. We adjust the initial HLT (see appendix E) after we

see how students react with series lesson we made to improve our HLT in teaching experiment in order to reach our goals.

C. Teaching Experiment

In teaching experiment, improved HLT was compared with students' actual learning. In this section we investigate how the HLT supports students' learning in area measurement. Video recordings during class discussion and interview, together with students' worksheets are analyzed to find out how students solve problems. The result is then be used to answer research question.

1. Activity 1: Telling the size of the cakes

Measuring with understanding requires that students know what the attribute they are measuring. In the first activity students were engaged with experience to know what they want to measure. First they were shown two cards that were different in size, one is big and another is small. They were asked to tell which one needs more paper. It is expected that they gain a sense what attribute they want to compare and build the vocabulary to describe the size while comparing the cards. All students answered that the bigger card needs more paper. Various reasons were told by students in explaining why they think the big one is big. For example, Faiza said that bigger one is nice to see, Vincent described by telling the letter is bigger, Feni described that the side of the card is bigger than another, Novan said that the angle is greater than another one. Then one of the students, Safira, said that the big one is large. She pasted the paper to each other by fitting the side of the paper and pointed the leftover parts. Then she explained in front of class that the big one has leftover while pointed the leftover parts. It indicates that she aware the sense of area.

The next task is to compare and order three cakes that have different size. The figures have been modified based on our finding in pilot experiment as shown in the figure 23. While comparing three cakes on figure, students still have difficulties to explain their answer in choosing the cakes even thought they had chose big cake and small cake. Actually they have no difficulty to determine that cake B is the smallest because it can be compared by sight. But the conflict emerged when they have to compare cake A and cake C because it cannot be said certainty.

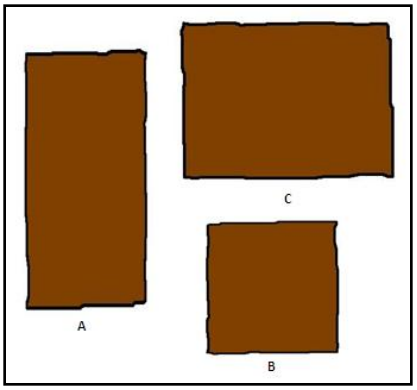


Figure 23: Revision of the figure of cakes

Most students cut the figure and put one to the top of another and see the rest. When they were asked which one is bigger they were difficult to describe bigger or smaller cake. A student, Silpi, argued that cake C is bigger than cake A. It seems she only compare by sight. But when she was asked between B and C she was difficult to explain which cake is bigger. She explained that cake A is bigger because it has more angles. She then realized the angles were not influenced how big the cakes because those cakes have same number of angles. When she was asked further, she put cake A on the top of cake C and arranged them to see which one is bigger. However, she still cannot decide which one is bigger until she realized that the big one is the one which has more rest paper.

Vincent and Sopia used ruler to measure. Vincent who holds the ruler measured the length and the width of each cake even though he used ruler incorrectly by putting one instead of zero as starting point to measure. (See figure 24).

Figure 24: Using ruler to measure

He measured the length and the width, and added them up to get his measurement. From this observation, Vincent already known that he wanted to compare a region by examining the length of two sides of the cakes. In this level, he added the length and the width to reveal the quantity of area. But then after seeing other friends cutting the figure to compare the cakes, Vincent and Sophia decided to do that also. They changed their mind to decide which one is bigger by putting the one to the top of the other and see the rest of paper. They wrote: *"C is bigger than A , A is bigger than B. C is bigger than A because if C is traced, there is remainder parts and A is smaller when it is traced"* . In this manner, they found the way to compare and order the figure and gradually they realize the attribute they want to measure.

Feni and Idris are in the same group but they have different opinion about the cake. Idris said that cake B is bigger than cake A after they compare cake A and cake B by putting cake B on the top of cake A. They were quarrel to decide which one is bigger as in the following fragment.

Idris : We have to order. (After reading the problem in worksheet he put cake A on cake B)
Feni : Look at the rest. Let me do that.
Idris : No, let me. (Grab the piece of cakes)
Feni : Look at the rest.
(Idris put cake A on cake B)
Idris : B is bigger than A.
Feni : No, A.
(Idris insist on his opinion by writing the answer on the worksheet)
Idris : Look at this. (Pointing the rest of cake B)
Feni : Teacher, this one is bigger, isn't it? (Pointing cake A)
Researcher: What do you think?
Idris : B, it has rest
Feni : No, this one is longer (Pointing the rest of cake A)
Idris : Oh yes

Idris known that bigger cake is the one which has rest so he decided cake B is bigger than cake A. However, he only considered about the rest of cake B without pay attention to the rest of cake A. He realized his mistakes after Feni reminded him that cake A has more rest. They pasted up the piece of cake and saw the rest of cake A. Idris then changed his mind after know that bigger cakes is not only has rest but it has more rest than another.

In line with Idris and Feni, Habib found the biggest cakes by cutting the rest of cake C and put the rest on cake A (See figure 25). Then they saw that C still had rest after all cake A was covered.

Figure 25: Cutting the figure of cake C

Researcher : How do you compare cake A and cake C?

Habib : A is longer
Researcher : Do you see the rest?
Habib : No, oh yes (put cake C on cake A)
This one (Pointing the rest of cake C)
Researcher : This one also has rest (Pointing the rest of cake A)
Habib : C is bigger
Researcher : Why is C bigger? If we see A also has rest.
Habib : If we move this (Pointing the rest of cake C) to here (Pointing the rest of cake A) then C is bigger.
Researcher : how do you move it?
Habib : by cutting

Habib cut cake C and then covering all cake A by using the rest of cake C. What Habib did is same with our conjecture. He can imagine that the rest of cake can be move and put on another one. This fact shows he can perceive the concept of conservation area.

In this first activity, most students already get sense about the attribute of area since they already consider that they have to pay attention to the quantity of object while comparing. They gradually learn to discriminate in what way an object is big or small for two dimensional shapes. Some of them also perceive the idea of conservation of area. However, the interesting part of students answer is some of them argued that the figures have different number of angle which influences the area of shapes. It seems the concept of angle is not fully understood by students.

2. Activity 2: Choosing the Chocolate

In general the expectation of this activity was students use identical unit in comparing area. For this purpose, students are given two figures of chocolate that have different size and different number of slabs. Chocolate A is smaller than chocolate B but its slabs are more than chocolate B. They have to choose which chocolate they want to buy. It is expected that they use identical unit for both chocolates to know which chocolate is big or small. A conflict will emerge when they have to determine which chocolate is bigger since they will have different

opinion about the bigger chocolate when students cutting and pasting the chocolates and when they count the slab of the chocolates.

When they were asked which chocolate they want to buy, most students answered that they wanted the big chocolate but there were also students chose the small chocolate. While discussed in group, some students were interviewed to know their reason.

Tara and Albar were interviewed because they have different opinion about the chocolate they want to buy. Tara chose chocolate A because it has many slabs. Meanwhile, Albar chose chocolate B because he said the big slab of chocolate B is fewer. Tara was asked further to know her opinion about the size of chocolate. So, she was asked which chocolate is bigger. She told that chocolate B is bigger than chocolate A but the number of chocolate is less than chocolate A. However, she change her mind after she count the number of both chocolates and said A is bigger than B. This fact shows that Tara still not sure what she mean by big. She seems confuse big mean the size or big mean the number.

Alya noticed that the unit can be transform to another unit. She said that chocolate B is bigger than chocolate A because the slab of chocolate A can be merged become the slab of chocolate B. She was asked to explain her answer as in the following fragment.

Researcher	*: Why B is bigger than A*
Alya	*: because its slab is bigger, if we merge them (the slab of chocolate A)*
Researcher	*: how do you merge them?*
Alya	*: if we break it (slab A)…hm…this is a line (make a line to divide slab B into two). If we divide it will be same (Pointing chocolate A)* (See figure 26)

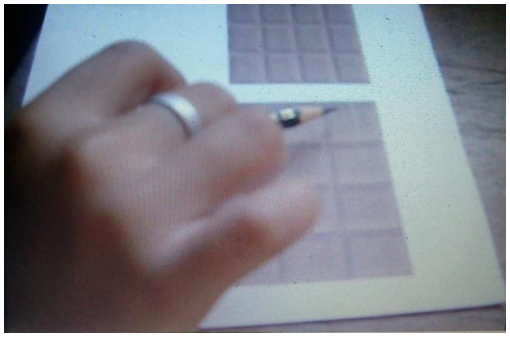


Figure 26: Making a line on slab of chocolate B

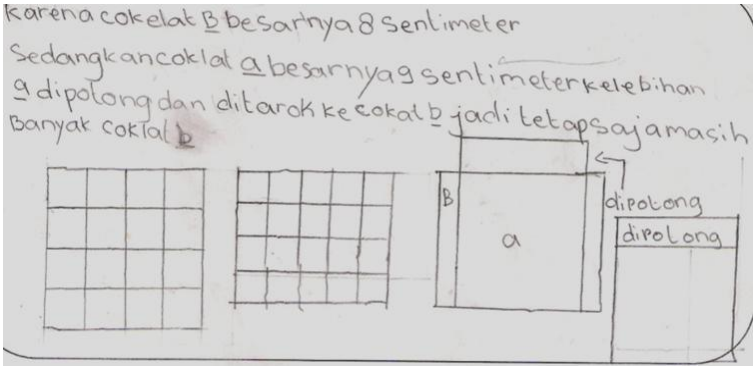
And then Alya make a line to connect the slab of chocolate A with chocolate B. But then she realized that two chocolate B is not same with one chocolate A. This fact shows that Alya noticed chocolate B is bigger because if she used the same chocolate she can get more chocolate than chocolate A by showing that one chocolate B is two chocolate A. Although in fact one slab of chocolate B is not two slabs of chocolate A.

From students answer in their worksheet, most students gave their reason that they chose the bigger chocolate. They compared each slab to find which chocolate is bigger. In line with Alya, Vincent and Sophia stated that chocolate B is bigger than chocolate A because they divided slab of chocolate A into two so that they get larger number even thought the transformation of the number of unit is incorrect. They wrote: *"Chocolate B is bigger. A is smaller and its slab is 20 and B is 16. If one slab B divided into two it become 32. So, chocolate B is bigger".* Safira and Hendra were even more precise in revealing how big the unit compared to another. They stated: *"I choose this chocolate (B) by counting the slabs and also laying one on the top of another. Two slabs of B are three slabs of A".* It means they realize the units are different so it cannot be compared only by directly comparing the number of unit in each chocolate.

Not all students see the slab of chocolates to compare. Some of them tend to cut and paste the figure and see the rest like what they did in the previous activity (See figure 27). Nanda and Fadilah wrote in their worksheet that *"B is bigger than A and A is smaller than B because when it is measured the sides are different"*, they also stated area A is smaller than area B. It is clear that they did not pay attention to the unit of the chocolates.

Figure 27: Cutting the figure of the chocolate

Salwa and Rama also stated chocolate B is bigger than chocolate A, they explained by giving pictures in their worksheet (See figure 28).



Chocolate B is 8 centimeter and A 9 centimeter. The rest of A is cut and put on chocolate B, B is still a lot (big).

Figure 28: Drawing the chocolates to compare

From this figure, Salwa and Rama used ruler to measure sides of chocolates but they seems did not know how the ruler can help them. Since they choose B as the bigger chocolate but they got B smaller in number when they measured with ruler. Then they explained by

cutting and pasting the chocolates to make sure that chocolate B is bigger than chocolate A. These students still think to cut and paste the figure to compare. Although in their figure they made units but they did not consider about that even they used ruler in comparing.

From students' answers, not all students can perceive that using same unit for both objects will be easy in comparing. Some of them can found that beside cutting and pasting they can also count the unit in comparing by noting that the unit should be identical for both chocolates. Meanwhile, some of them tend to cut and paste the figure instead pay attention to the unit. They seem only focus on comparing the quantity by cutting and pasting. Nevertheless, comparing chocolate activity reinforces students to know the attribute of area since they cut and paste the figure to compare the region.

3. Activity 3: Cookies in Baking Tray

Using non standard unit measurement allows students to focus on the process of repeatedly using a unit as a measuring device. Therefore, the aim of this activity is that students are able to use non standard units to compare the area of shapes. In this activity students are given opportunities to cover entire surface by using their own unit. In this activity, students have to compare two cardboards as baking trays that can put more cookies (See figure 29). Students work in group of four so that they can share their ideas and they can have many strategies to solve the problem.

In doing this task, students cannot judge which one is the right one since the size is not much different, baking trays A is longer and baking tray B is rather wide. In this way, students can realize that they need tool to compare so that their measurement can be said certainty. Comparing activity can lead students to develop the need of unit to measure when they have to evaluate the quantity.

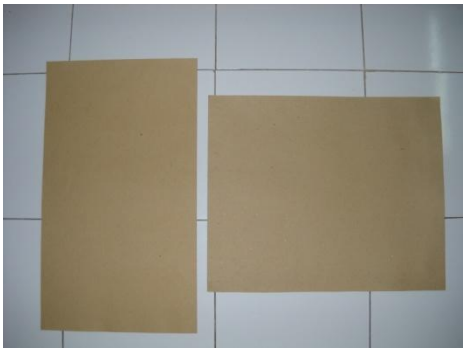


Figure 29: The cardboard as baking trays

At the beginning of this activity, students were asked which baking tray that could put more cookies after they get their cardboard as baking tray. Students had different opinion about that. Some of them raised baking tray A and some raised baking tray B to declare their choice. The teacher asked them how they compare the baking trays. However, it was so difficult to know the reason of the students. They seem shy to give their reason in class discussion. One of students, Vincent, said he can arrange the cakes on the baking tray. Other students said that they could trace the baking tray. Then they were given two size sticky papers as cookies and arrange those on baking trays.

There are two notations when the students covering the baking trays. First, students used different unit in covering. Some groups chose unit that physically resemble with the region they were covering. Feni's group used rectangle paper to cover the longer baking tray and they used square paper to cover another one. They explained that they use that unit because the side of the unit is same. It seems they did not use the unit as a tool for comparing the baking trays but just cover the area by using unit which resemble with it.

Some groups in another around. They used square to cover the longer baking tray and they use rectangle paper to cover another one. Vincent and his group explained in their worksheet: *"Baking trays B has more cookies whereas A has fewer cookies. But cookie A is bigger than cookie B"*. It seems they realized that they use different unit in comparing.

There are four groups out of nine used same unit in covering the baking trays. One of the groups was observed during working in group. For the first time they use different unit but then, Romiza, one of the member asked her friend to cover by using the unit that he used. Faiza and her group used square as their unit, then they were interviewed to know their reason.

Researcher : Why do you use square?
Faiza : It will be counted
Researcher : This one (baking tray A) use square, do you think another also use square?
Faiza : yes
Researcher : why?
Faizq : in order to…
Fadilah : in order to make the same sequence
Researcher : what do you mean by sequence?
Fadilah : the plot of the square is same.

This group seems know that they have to use same unit but they cannot explain that they use the same unit in order compare the baking tray easily.

The second notice is the students confuse whether they can pile up the paper because it cannot fit to cover all parts of baking tray. While covering, Diana, asked whether she can pile up the paper because her paper did not fit to cover the baking tray.

Diana : It is not fit.
Researcher : How do you think if it is not fit?
Diana : I do like this (pile up the paper)
Researcher : Why do you do like that?
Diana : In order become fit
Researcher : Can we do like that?
Diana : No, it is not allowed
Researcher : Why is it not allowed?
Diana : (Silent)

And then Diana's group decided to cut the paper and they are asked to explain what they were doing. They said that they cut the paper in order to make it tidy. And they were asked to count how many units covered the baking tray. Then, they count all units and got twenty three as the

result, include the unit they cut and the paper that stick outward (See figure 30). They were asked what they think about the paper which is stick outward. Diana said it can be cut.

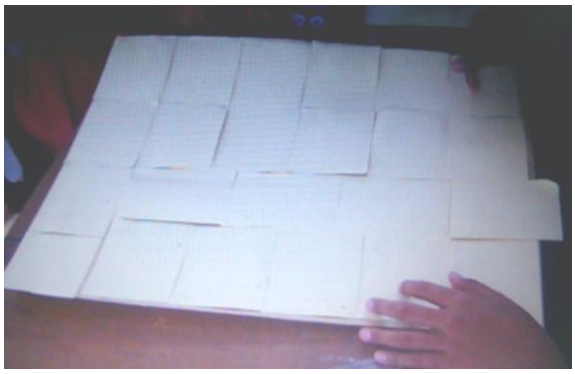


Figure 30: Covering the cardboard

From this observation we can see that Diana's group tried to cover all baking tray with the unit without gap. They realized that the unit must be not overlap so that they cut the rest of paper. In this manner they seem perceive that entire surface of region must be covered with the units. However, they seem did not extend units over the boundaries since they ignore the unit that is overlap in counting units used.

From our observation, we can see that all students aware that a region can be partitioned become small unit and then count those units to reveal the quantity. They also use their own unit and are able to iterate the unit in covering although some of them used non identical unit in covering. They also did not pay attention to the unit that out from boundaries.

4. Activity 4: Unit Investigation

There is an inverse relationship between the number of units and the size of the unit. It is expected that students are able to explain that the larger the unit the fewer are required and vice versa. They should know that using non standard unit measurement do not give exact or uniform unit measurement for same object. By using many kind of units students can realize the unit can be any shapes as long as it can cover all surface.

This activity is similar with the previous activity in which students cover the baking tray to know the area of the baking tray. They still work in group of four. Students were asked to find the area of the baking tray with different kind of units. Conflict emerges when they have to arrange the unit to cover all surfaces of baking tray. They already know from the previous activity that all surfaces must be covered. But not all units fit with the baking tray. So, they have to find the way to make it covered.

Alya and her groups used rectangle to cover their baking tray. She tried to arrange the unit in order it can be fit in that. For the first, she measured by iterating the unit, and she realized that the paper will pile up each other. Then, she changed direction of the cookies. They did not get difficulty covering the baking tray until there are gaps which is not covered because the rectangle paper was not fit on baking tray. Then they covered by folding the paper.

They are asked how many units needed to cover the baking tray. Alya counted the unit one by one started with the intact unit and then counted not intact unit. She counted one number while pointing two folding papers.

Interviewer : How many?
Alya : Twenty two and a half
Interviewer : How do you get a half?
Alya : This one is a half (Pointing a half paper)

However, another group who used the same unit with Alya' group, have difficulty to count how many unit needed. They counted all unit include the stick outward paper. They got twenty five after they count the units.

Researcher : You count this one also? This one is not include covering the baking tray, isn't?
Idris : Yes, it can be folded
Researcher : How many these? (Pointing two half papers)
Idris : Half and half

Researcher : So, how many together?
Idris : One
Researcher : These two we count…
Safira : One
Researcher : Count it again
Safira : Twenty five
Researcher : So, do you count these also? (Pointing the stick out paper)
Safira : Twenty six, twenty seven…
(Safira count the stick out papers by continuing her counting,)
Rizki : Twenty seven and a half

From the fragment above we can see they are difficult to determine how many units needed to cover the baking tray. For the first they ignore the stick out paper but then they seem confuse how to do with the rest of paper.

The same case was also faced by group which used triangle as their unit because there are gaps that was not covered by triangular paper. Revi did not dare to cut the triangle. Therefore, he asked permission to cut the triangle because there are still gaps when he arranged the triangles. Afterwards, he decided to cut the triangle become a half and put it to cover the gaps. However, they were struggling to arrange the half triangle. Other groups who work with triangle also did the same thing by cutting the triangle become a half (See figure 31).

Figure 31: Cutting the triangle

This activity influenced the way they count the unit since they see that one unit can be cut into a half and arrange it to cover the baking tray. So, in counting they did not get

difficulty. By doing this activity students realize they can cut and arrange the unit to cover all surface. In counting the students also pay attention to the unit. They added two half units become one.

When report they result in front of class, Fira and Alya who work with same unit got different result. Therefore, the teacher asked other students about that. They agree that it should be twenty two and a half by counting the work of Alya's group.

To get more information or reasoning from students, nine students were interviewed about the gaps and overlaps after class discussion. We provoked students by giving them a baking tray that not fully covered yet and asked their opinion about that.

Researcher : Can we do like this? (Showed the baking tray that is not fully covered)
Safira : It is not allowed
Researcher : Why?
Feni : Because all baking tray has not fulfilled yet
Safira : Because the baking tray still has rest
Romiza : Because it has not fulfilled yet
Safira : Because the baking tray has not fulfilled yet
Researcher : what should we do?
Hendra : It should be fulfilled
Researcher : What for?
Feni : To know the result
Researcher : Now, if we put like this (Pile up the paper) can we do like this?
Safira : No, we cannot
Researcher : why?
Hendra : because this one still has the rest (point the stacking paper)
Researcher : What do you think?
Safira : The result will be different
Researcher : What is the result?
Safira : twenty
Researcher : what do you think romiza?
Romiza : The result will be the same because there is also a half
Researcher : which one?
Romiza : the pile up paper
Researcher : do you think this one is not counted?
Romiza : yes it is counted. This one is counted as a half (pointing the half part of the unit that is piled up)
Researcher : so, this one we count a half?
Romiza : yes. It will be the same because there are also half parts

From the fragment above, students are able to deal with gaps and overlaps. We can see that students think that all surfaces of baking tray must be covered by sticky paper. In this fragment Romiza also shows that he ignored the pile up paper and only counted the visible units. It indicates he understand that he only need to count the number of unit used in covering area.

After all groups wrote their result on a white board, students make conclusion that the larger the unit the fewer are required and vice versa.

Teacher	*: We can see that the result is different. In your opinion, why the result are different?*
Feni	*: Because the size is not same.*
Teacher	*: yes, because we use rectangle small and big, triangle small and big and square small and big. So, what happen if we use big unit?*
Vincent	*: we need few units*
Teacher	*: what happen if we use small unit?*
Students	*: we need a lot of units.*

From observation and interviewed with students, students realize by using different unit it will get different result. They can see an inverse relationship between the number of units and the size of the unit. They also gradually learn how to deal with gaps or overlapping. In this manner they are able to structure two dimensional shapes into an organized array of unit. This concept will be maintenance in the next activity.

5. Activity 5: Tiles in the Room

In this activity students have to find the area since in the previous activity students are able to find the area of baking tray with non standard unit measurement. In this activity students work individually to raise their confidence to solve the problem by themselves but they can still discuss with the other students. It is expected that the students are able to determine the area of two dimensional shapes by using unit given. To do so, they were given the sketch of room that is not fully covered with tiles. They are challenged to find ways in

determining area such as drawings so that we can see how students are actually reveal in structuring the array.

For the first teacher asked how the students can find the area of the room in the figure. Students said that they can use the square in it. While doing the task, all students are able partitioned blank space of the figures. Most of them partitioned the figured into array and structures. They draw the squares by fitting the square with the existing square in each row. Some of them did not make a draw but just imagine the tiles in their mind while counting. And some made dot in counting. (See figure 32).

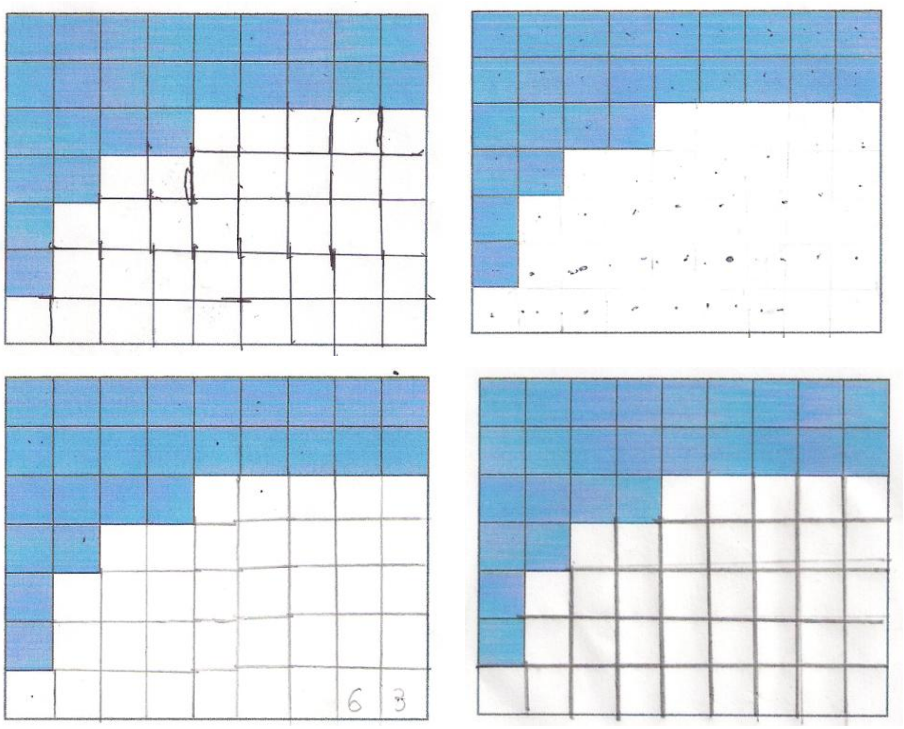


Figure 32: Different strategies of students in partitioning

There are some different results in this activity if we compare the result with the first cycle. In the first cycle most students cannot structure arrays since they cannot fit the square with the existing tiles. In the second cycle most students can structure array by partitioning the blank space and iterate the unit to find the area.

The strategies of students were observed during they were working. Fajri did not draw tiles in his worksheet. He just imagined the tiles by looking the pattern of the tiles. He tagged the blank space while counting (See figure 33). It seems he only counted the tiles to answer the question number two. Because in his worksheet, he stated that the area of the room is seventy two and half cm. It indicates that he still did not understand what area is. However by observing what he was doing in counting the numbers of tiles are needed more he can partition the blank space although he still made mistakes in counting.

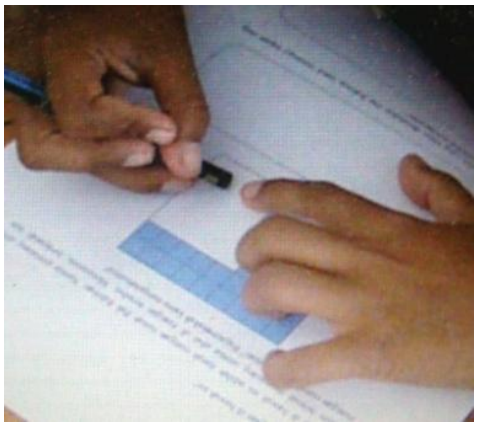


Figure 33: Tagging the blank space while counting

There are three students measured the area by using ruler. In line with fajri, Albar wrote in his worksheet: *"The square on the left side is 6 cm and the right also. The square on the bottom is 7 a half cm and also on the top side"*. In this manner, he reveals the area by measuring the perimeter of the figures. It seems that he has little sense of the attribute of area and it is unclear for him what the area is.

Rizki also counted the tiles by looking the pattern of the tiles. Different with Fajri, she did not tag the blank space or draw squares to fit the square with the existing square in each row. She tagged the existing tiles and counted them repeatedly to find how many more tiles are needed. At the beginning, she made a mistake in interpreting the problem since she only counted how many more tiles are needed. And then she was asked to reread the problem.

Researcher : So what is the question?
Rizki : Area
Researcher : Which one is the area?
Rizki : The area that has not covered yet
Researcher : Are you sure? Which one is pak Rahman' room?
Rizki : This one has not covered yet. All tiles have not put in the room yet.
Researcher : So what is the area of Pak Rahman's room?
Rizki : sixty three

In line with Rizki, Safira did not create arrays in counting the tiles. She just counted the tiles in each side and multiplied.

Researcher : how do you find the answer?
Safira : I multiply
Researcher : what do you multiply?
Safira : this one and this one (Pointing length and width)
Researcher : why do you multiply?
Safira : to make it easy in counting

In this manner, Safira already know that making drawing is not necessary to find the area. We can see that she recognize that the arrays of rows are equivalent so that rows can be repeated instead of drawing the units. She just needs to count how many tiles in each side of the room and then multiply.

Zatun made units continuing the existing units. In her worksheet she first counted the existing unit that is 26 tiles and then counted how many more units are needed that is 37 tiles. At the end she stated that the area is 63 and she got it by using multiplication. At the first we predicted that she will add the units but in fact she used multiplication to determine the area. In this manner, she seems influenced by question number b in which they have to determine how many more units are needed. After doing the task, students are given exercises to know how far they understand the problem. Most students can determine the area of each shape by counting the unit given.

From students' worksheet and interview with students, almost all students can find the area of figure given. They can perceive the idea of unit iteration and the idea of structuring an array.

6. Activity 6: The Area of Islands

In the previous activity, students can find the area of rectangular shapes by using unit as a tool to help them in measuring. In this activity, it is expected that students are able to find the area of irregular shapes. In this task students were given two figures of island that have irregular shapes and they have to find which island is bigger. The figures of the island are modified in such way students deal with conflict in determining which island is bigger. One island has more intact squares than another but it has smaller area. It is expected that students realize square as a unit can be a tool for comparing and they realize all part of units within boundaries need to be considered.

For the first students are given exercises to find the area of irregular shapes. There are 8 different shapes which have determined by students. It is expected that students deal with conflict in which they have to count not intact square and intact square. One of students, Idris, made a mistake in determining the area of the shape. But then he realized by himself after relook the figure. The fragment below is the interview with him.

Interviewer	*: Idris, what is the area of figure number c?*
Idris	*: three and a half*
Interviwer	*: how do you know?*
Idris	*: this, this one is a half (pointing the half part) and suppose it is three….oh… it is incorrect*
Interviewer	*: why?*
Idris	*: it is two. I was wrong*
Interviewer	*: you said before the area is three and a half. So which one is correct?*
Idris	*: two*
Interviewer	*: why are two?*
Idris	*: two because this is three and a half and this one is a half. If we combine the half parts it become two*

Interviewer *: repeat it please! Which one is three and half?*
Idris *: This one. This one, one, two, three, three and half (pointing the colored part). Nah, this one (a half parts) if they are combined it become two*

From this segment we can see that Idris realize his mistake by looking back the colored figure and he considered that he made a mistake in counting the half parts. He was success explaining his reason why the answer is two.

Most of students can solve the problem. They seem did not get any difficulty to determine the area of those figures. It seems the previous activity influence their thinking in determining area in which they cut a unit to cover the gap. Therefore, they combined parts of unit together to form whole units. The fragment below shows how students find the area of irregular shapes.

Teacher *: What is the area for number e?*
Aliya *: Two*
Teacher *: Why two?*
Fathur *: Because there are two colored squares*
Teacher *: Do you all agree with Fathur?*
Faiza *: No, there are half parts*
Teacher *: Which one?*
Faiza *: This one, this one, this one and this one (Pointing a half part)*
Teacher *: So, how many together?*
Faiza *: Two*
Teacher *: Do you agree all?*
Students *: Yes*

We can see that Faiza did not agree with Fathur. He said there are two colored square but Faiza knew that there are four half parts that can be combined become two square units not two colored squares.

The next task is to compare two islands and determine which island is bigger. Most students already know what they have to do to solve the problem but some seem still having a

problem in counting the unit. It seems they did not careful counting the unit so that they skip some units that is not intact.

Romiza : Teacher, can we say tiles instead square in this figure?
Interviewer : Yes, in your opinion which island is wider?
Romiza : I will count it
Interviewer : how do you count?
Romiza : I will count color parts
(romiza count the intact unit first and then count not intact unit by combining the partial units)
Interviewer : How many?
Romiza : Twenty two and a half (In fact the area of the island is twenty four squares)

By observing what Romiza did, it is clear that he can use the square to find the area of island and combine the partial units but he was just not careful in counting the unit. Sophia also did the same thing. She was able to find the area of one island by combining the unit while counting. In her worksheet she wrote: *"The area of Ipin Island is* $21\frac{1}{2}$ *and the area of Upin Island is 22"*. Therefore she made a mistake in determining which island is bigger. From her worksheet we can see that she already considered with not intact unit since she mention a half in her worksheet, however she only was not careful in counting the not intact units.

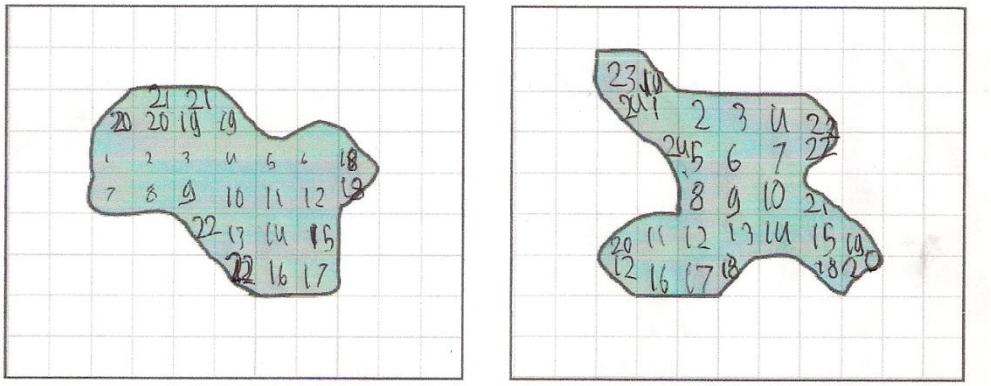


Figure 34: Labeling the figure to find the area of each island

Revi have no difficulty in counting the units. He had an idea to label the squares to help him in counting (See figure 34). Meanwhile the other students counted the squares by

only pointing the square which led them wrong in counting. Revi's answer is suitable with our conjecture that he counted the partial parts become whole unit. He succeeded to find the bigger island since he considered about the extra space of the island that have to be counted. This indicates that he can consider the quantity within boundaries of the area and use the unit to help him in comparing.

Only one student, Albar, ignored whether the unit is intact or not. He got twenty six for area of the first island and eighteen for area of the second Island. He wrote: *"The area of Ipin Island is 26 and Upin Island is 18 so Ipin Island is bigger than Upin Island"*. In class discussion, students argued Albar opinion.

Teacher : Albar what is the area did you get? (Upin Island)
Albar : Eighteen
Teacher : Just now, Revi got twenty four. But Albar got eighteen. So which one is right?
Students : Revi
Teacher : Do you agree all?
Students : Yes
Teacher : Albar, how do you get eighteen?
(Albar shows how he counted the unit of Upin Island. He only counted intact unit)
Albar : I count the squares
Teacher : Do you agree all?
Students : No
Teacher : Why do not you agree?
Eva : He do not count a half parts
Teacher : What are we suppose to do?
Faiza : We count it

From this fragment, we can see that Albar ignored the remaining parts that is not square, while others counted all units include partial parts that can be combined together to form whole units. This indicates Albar had difficulties in considering a unit as the sum of its parts. However, most students seem know what to do to solve the dilemma of counting the extra space of the unit that is not intact.

7. End Assessment of Teaching Experiment

End assessment aimed at seeing whether HLT can support students in understanding area measurement. Five problems were given (See appendix B) to the students with two problems which is similar to pre assessment. It is purposed to see how students' understanding has grown after following the series of lesson. Analysis table was made to analyzed students' answer in describing their tendency solving the problem (See appendix C). The explanation about the end assessment as follows.

Problem 1

This problem aims at investigating how far students can describe area. It is expected that they can distinguish between length and area. Whether they see area as two dimensional shapes or just see the length of the shape. This problem is similar to problem in pre test in which they have to determine which field has larger yield. All students choose the right answer. Most students chose the field by describing their choice. Most of them describe larger field by using words that refers to area such as big and large. These students seem can recognize the attribute what they want to compare. Some students explained based on the shape of the field. e. g., B is bigger than A because B is rectangle and A is square. These students seem difficult to describe the figure in explaining why figure B is bigger or larger.

There are two students explained by making lines in the figure. They made unit to compare those two fields (See figure 35). These students already perceived the attribute of area and can apply what they learn by giving unit in the figure to compare. They made unit then counted the unit used although the units were not identical enough. Compared to pre-assessment, students' answer at this stage has improved significantly. They made units in

comparing. Students also can explain by using word that refers to area despite not all students did that.

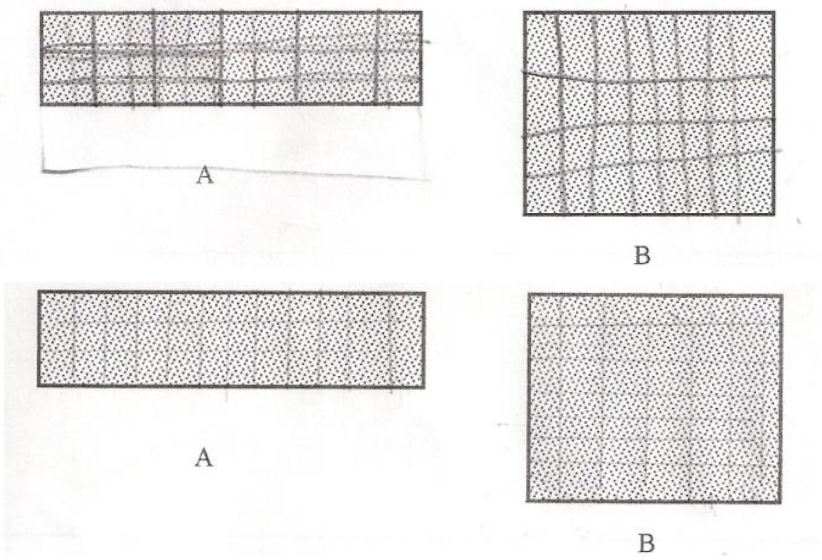


Figure 35: Students made units for each figure

Problem 2

Problem two aims to see whether the students can distinguish perimeter and area. If the students only pay attention to the outside lines of the figure it means they only see the perimeter of the figure. But if they can count the unit inside figure, it means they understand that they measure the quantity of the objects. To see how students' understanding has grown, this problem was made similar with problem on pre assessment. Two figures were given to the students in which those two figures have different area. Figure A has bigger area with shorter perimeter and figure B has smaller area with longer perimeter.

In answering this problem, most of them decided swimming pool A is bigger by counting squares inside swimming pool. They counted the unit in each figure and then compared the number of units. In this manner, we can see that these students distinguished perimeter and area since they revealed the quantity of the surface of two dimensional shapes.

Two students gave their answers without reason. It seems they only compare by sight the figures. One student explained his choice by telling that each sides of the figures is different. Although he choose the right answer, this student seems did not recognize the unit given to compare. He just saw the side of figure in comparing. Another student used ruler to measure but she seems only consider one side of figures to compare. It means that she still did not perceive the attribute what she want to measure. Only one student chose swimming pool B. She explained that figure B is nice to see. We cannot judge this student whether she distinguish area and perimeter. However, if we compare students' answer to the pre assessment, there is much improvement answer given by students. Since in the pre assessment only two students counted the unit to compare the figure given.

Problem 3

Problem three aims at investigating how the students can reveal structuring arrays. Through this problem, we can see whether students consider that the shape provide the number of units in rows and columns or not. To do so, they can make unit that must be aligned in an array with the same number of units in each row by fitting the unit into the figure. So, they can determine the area of the shape for this dimension meaningfully. Most students can make arrays continuing unit given then count the units. In this manner, these students already perceive the idea of unit iteration and learn how to structure arrays. Although they can structure arrays, some students still made mistakes in counting the unit. They seem did not careful in counting the units but the result still close to what we expect. Meanwhile, three students did not make arrays. It seems they imagined the unit and then iterate the units. But they made mistakes in counting to determine the area.

There are two students multiplying the number of unit in each side. One student counted the sides of the figure by labeling the squares in the length and the width (See figure 36). Then she multiplied to get the result but she did not consider that the figure is not rectangle. It might be she is influenced by the fifth activity in which some students use multiplication to determine the area. But in this case the figure is different with the activity. This student seems do not aware why she uses multiplication.

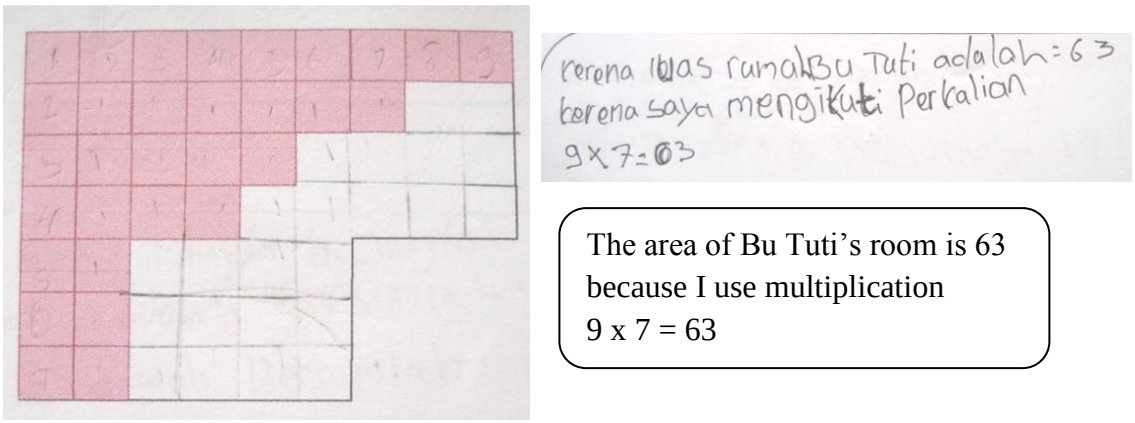


Figure 36: Using multiplication to determine the area

From the students answer we can conclude that students already get the idea of structuring arrays but some of them still difficult in counting the unit. No one of them split the figure in order to help them in counting. However, students can iterate unit in rows and coordinate with columns.

Problem 4

Problem four aims at assessing whether the students can find the area of irregular shape. If they understand how the unit can help them in determining the area they will know that they can mentally arrange the unit that is not intact become square unit so that they can count the number of unit used. From students answer, most students are able to find the area of irregular shape by combine the partial units. But some of them made mistakes in counting the

unit. It seems they did not careful in counting (See figure 37). We can see that this student can combine the partial unit by giving same number for two half units. But for number c, he forgot to label the partial unit.

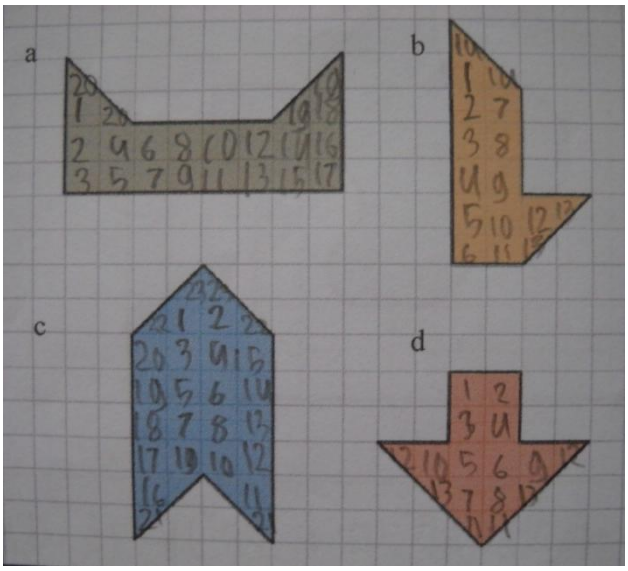


Figure 37: Labeling the irregular shape to find the area

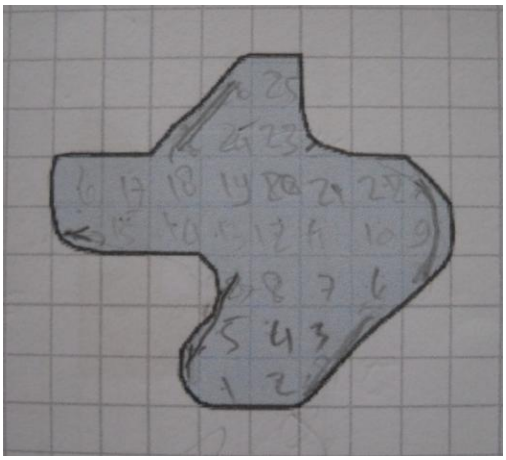
Only one student did count all units although the units did not include within boundaries. This student did not recognize the boundaries of the area since he counted all units although there are some parts of the units that are not included in the area of the problem. There are students that measure the area by using ruler. But he only measured one side of the shapes.

From students answer, we can conclude that most students already measure by using square units and combine parts of the unit together to form whole units. They can solve the dilemma of counting the extra space of the irregular shapes.

Problem 5

Problem five aims at assessing whether students can estimate the area of irregular shape into the close result. In doing so, they have to estimate the area of an island. Most

students were able to estimate the area to the close result by counting the square in the figure and combine the partial unit. One of students, Safira, labeled the intact square with numbers and made signs for pairs of partial units and counted it as one unit. (See figure 38).



luas danau iniada 30 yg utuhnya ada 25 dan yg setengahnya ada 5 saya mengtahuinya dengan cara menghitung

The area of the lake is 30 the there are 25 intact squares and there are 5 half parts.
I know by counting

Figure 38: Combining the partial units

However, same with problem number four, many students still have difficulties in counting the square. They seem did not careful in counting hence they got wrong result. Only one student measured with ruler. This student seems still did not get sense what attribute he wanted to measure. From students' answer for number five, most students can perceive they can use square as unit to estimate the area by counting the unit used in the figure of the lake.

Based on the end assessment, the progress from pre assessment to the end assessment is quite significant. In pre assessment students have vague notion about area since they still confuse between area and perimeter and they also lack the words in describing the area. In the post assessment most students are able to describe the figure by using words that related to area. They also perform their notion about the magnitude of the region by counting the units in figures. They use the unit to find the area of irregular shape. In doing so, they are able to deal with the partial units by combining them to make intact unit.

8. General Conclusion of Teaching Experiment

In this section, the series of activities are analyzed to see how it supports students' learning in area measurement. The teaching experiment was conducted in six activities. The conclusion is drawn as follows.

The first activity concerns with students' awareness of physically quantity of area. The starting point was designed by a situation that involves the area in which students have to compare and order the size of cakes. This activity showed that some students were aware to the physical quantity of area. Students compared figures given by putting the one to the top of the other and then looking at which piece sticks out. Some of them also perceive the idea of conservation of area in which they cut and rearrange the shape to determine which one is bigger by looking at the out-sticking parts. This strategy reinforces the sense of area for students.

The awareness of the attribute of area was made for the next lesson but they also worked with measurement units. It is expected that they can compare using identical units. In this lesson students had to choose which chocolate to buy by comparing their sizes. Students compared the chocolate by putting the one to the top of the other and also count the unit in each chocolate. It is expected that can compare their strategy to compare. Almost all students immediately guess which chocolate is bigger. When asked to prove their guess, most students cut and paste the figure like what they did in the first activity. Only few students recognize the existence of the unit in the figure hence they compare the unit and realize that the unit is not identical and it is difficult to compare by using unidentical units. This is because the figure of the chocolate is too easy compared by sight so that students did not pay attention to the units in the chocolates.

In the third activity, the students had to compare baking trays that can be put more cookies. It is expected that they can use the unit to compare. In this lesson, students are able to use their own unit to cover the shape in comparing the area. However, some students choose unit that physically resemble with the region they were covering. They used different units for different baking tray. In this manner, they only focus on the process of repeatedly using a unit in and it seems they did not use the unit to compare. It might be because the question is which baking tray that can be put more cookies. Therefore, they did not pay attention to the size of baking tray. The question should be which baking tray is bigger so that they can think how to compare by using same kind of units. Some students were not aware of gaps and overlap in covering. They ignore the leftover paper and count all units include the leftover parts as area. In this level, these students did not get what is the area since they only focus on counting the unit. Experience in covering with non standard units helps students to develop the concept of unit iteration and structuring arrays with row and column structuring. They become familiar with the process of measuring that would be used in the next activity.

In the fourth activity, students have to find the area of a baking tray with different kinds of unit by covering the baking tray. Afterwards, they have to compare the result. The students can use their experience in the previous activity to cover the shape with no gaps or overlap. This experience helps them in counting the units used because they know that the cutting parts originate from one unit. In this way they can accept the concept of conservation of area. This is obvious from the fact that the process of measuring improve in this activity. In the previous activity students ignore the half part by counting it as one. But in this activity they consider about the accuracy of counting. They also find that different unit will affect the result of measuring and they realize the inverse relationship of the unit.

In fifth activity, Students were asked to find the area by using given unit. They were expected to be able to find the area either by partition a region and then count the unit or by using multiplication. In here, students can partition the region by fitting the square with the existing square in each row or column. Even some of them did not think to make arrays. They only counted the unit in each side of the rectangle and multiply to determine the area. This indicates students can structure arrays. They are able to understand that the rectangle's dimension has the number of unit in rows and columns so that they can meaningfully determining the area for two dimensional shapes.

In the last activity, the students were asked to compare the area of irregular shapes. Comparing the area of irregular shapes encourage them to use unit in helping them finding the area of each island. They are able to overcome the partial unit in irregular shapes. They combined partial unit together to form whole units and then count the unit used. In this case, students can use square as a unit in estimating the area of irregular shapes.

D. Discussion

In this section, some important issues are discusses to provide the information about classroom activities of supporting students to develop the concepts of unit for area measurement. We highlight some important issues based on the findings of this research.

1. Realistic Mathematics Education

The tenets of RME (Treffers) and the levels of emergent modelling (Gravemeijer) describe mathematics teaching and learning as a long term process. Meanwhile, the design that we made is a part of longer series of the learning trajectories for area measurement. Hence, the description some of these aspects could not be presented in this research.

The design of lesson series in this research was underpinned by some ideas of RME. At the first activity, the students had experiences with the real situation related to their real life to compare and order the size of cake. The figures of cakes are given so that allows students to cut and paste the figure to gain the sense of what area is. The situation to compare the baking tray as a vessel for cookies encourages students to use cookies as a unit for comparing. Using non standard unit measurement that is chosen by students can be a model to support the students' thinking. And In the two last activities the students used their experience in the previous activity in which they have to use the unit to solve the problem.

Considering the last tenet of RME, *intertwinement, s*ome activities used in this research could be developed to reach other mathematical concepts by intertwining with other mathematics topics. The activities that we design with the students not merely support learning for area measurement. These activities also support some basic concepts such as multiplication and geometry. Students can enhance their learning in multiplication through counting the unit in easy way. They also enhance the characteristics of the shapes like rectangle, square and triangle through the unit that they used in covering.

2. Classroom Discussion

The learning process of students is influenced by social process in the classroom discussion besides individual process of the students. In this research the classroom discussion did not run as we expect because the teacher and the students were difficult to build discussion environment. The large number of students where 36 students in this class organized by one teacher causes the discussion is not easy to conduct. Not all students follow the discussion when the other students present their work in front of class. Although they are free to express their idea, some students still not dare to propose their opinion. In other hand, they brave if all

of them together answer teacher's questions so that it is difficult to raise a good discussion. In other words, the students and the teacher not get used to the culture of discussion.

However, all students follow the instruction to do the tasks and they can work together as a group in doing the tasks. For the first and the second activity, students work in pairs to create effective discussion among the students and it allows students to give more contributions to the group. In the third and the forth activity they work in groups of four students so that they can share their ideas and they can have many strategies to solve the problem. Doing tasks in group could have supported the low achiever students by observing the others work. In the fifth and the last activity they work individually to raise their confidence to solve the problem by themselves but they can still discuss with the other students.

3. The Role of Teacher

During the experiment the teacher role was observed in students' learning process in which the students can be engaged and supported their thinking process. The role of the teacher is very essential to bridge the context and mathematical goals and to organize a good discussion. The discussion can encourage students to interact with the other and share their ideas to construct their knowledge in the learning process. Therefore, teacher should be a good leader in provoking students' interaction.

The roles of the teacher that was observed during experiment include helping students in their development by leading the discussion. She presented the problem and posed some questions to the students. The teacher encouraged students to propose their ideas and posed question to justify their answer. The teacher also helped students to enjoy in class by singing together so that the students can relax seeing cameras and stranger people observe them.

However the teacher's belief are still influenced by traditional mathematics teaching, where sometimes she gives direct explanation about the lesson so that students' strategy influenced by the teacher explanation.

CHAPTER VI
CONCLUSION

A. Answering Research Question

Based on the retrospective analysis, we have investigated the activities that lead the students to learn area measurement. The activities bring students to develop the notion of a measurement unit for area. Experiences with physical quantity of area spontaneously gained by comparing activities in which students can aware the attribute of area and develop the range of words that can be used to discuss it. Based on observation, comparing activities lead students to be acquainted with physical quantity of area. The words such as big, large, small when they explain figures indicate that they focus on the attribute of area. Some students showed their ability of the concept of conservation area by reshaping the figure to compare the area. The students who did not reach this level tend to compare by sight after they cut and paste the figure. This strategy indicates students still estimate to compare the area.

The experience with non standard unit is provided to the students so that they start to develop the need of unit to measure when they have to evaluate the quantity. We found that covering activity leads students to mentally partition a region into small units and allows students to focus on the process of repeatedly using non standard unit as a tool to measure. It also provokes the students to construct the idea of the inverse relationship between the size of unit and the number of unit used in covering the region. The students perform their ability in structuring array of the unit while covering. Most students are able to arrange the unit given in such a way fit to other unit without gap or overlaps. However, some students find difficulties in dealing with overlapping space. These students only focus on counting the unit without pay attention to the region they want to cover. After experiencing gaps and overlaps in the iteration

process, gradually students can perceive that entire region must be covered by the units and the area is the number of units within boundaries. Through covering activity, the students are able to develop the unit meaningfully to measure area.

The experiences covering with non standard units help students to use the units to find the area. They use units as references to quantify the magnitude of a region. In doing so, they partition the region by drawing units to structure the units into an organized array and then count the number of units. Meanwhile, some students feel did not need to make a draw because they know that the number of units in each row or column must be unchanged. So that rows or columns can be repeated instead of drawing the units.

The activity with irregular shapes was given to students to investigate how they use unit measurement to determine the area of irregular shape. Most students were able to overcome the partial unit in irregular shapes to evaluate the area of each shape. They combined the partial units together to form whole unit and then counted all unit used within boundaries.

As a conclusion, students learn to measure the area start from identifying the attribute being measured while comparing the quantity of area. Afterwards, the need of unit emerged when they have to quantify the quantity of area. The measurement process with the units gained when the students have experiences with covering activity by using units. Covering activity leads students to mentally partition the region into small units and allows students to focus on the process of repeatedly using non standard unit as a tool to measure. Through these activities the students can use the unit to measure the area of two dimensional shapes either regular shape or irregular shapes.

B. Recommendation

The design of lesson series in this research was underpinned by some ideas of Realistic Mathematics Education (RME). Based on our findings, RME can be used as an approach for teaching and learning mathematics including area measurement. Considering the last tenet of RME, *intertwinement, s*ome activities used in this research could be developed to reach other mathematical concepts by intertwining with other mathematics topics such as multiplication and geometry.

In RME classroom, the social interaction and discussion highly emphasized. The role of the teacher is very essential to bridge the context and mathematical goals and to organize a good discussion. Therefore, teacher should be a good leader in provoking students' interaction. However, the teacher and students we worked with still conform to the RME approach because it is really new for them. More efforts are still needed to continue the development of discussion culture.

In this research, we only focus on non standard unit for area measurement. Although non standard units reinforce basic measurement principles students need to realize that they are limited as a means of communication effectively. The difficulty in communicating the sizes when there is no standard unit measurement can be highlight to help students see the necessity for standard unit.

The findings of our research have raised some new questions such as how to support students to use standard unit measurement to measure the area? What kind of unit that can be count flexibly? How do the students achieve the formula for area? Further research is needed to answer those questions.

REFERENCES

Bakker, A. (2004). *Design Research in Statistics Education. On Symbolizing and Computer Tools*. Amersfoort: Wilco Press.

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.),*Mathematics: Essential for learning, essential for life* (Proceedings of the 21st biennial conference of the Australian Association of Mathematics Teachers, Hobart, pp. 136-142). Adelaide: AAMT

Clements, D. H., and Stephan, M. (2004). *Measurement in Pre-K to grade 2 mathematics.* Mahwah, NJ: Larence Erlbaum Assiciates, Inc.

Clements, D. H, and Sarama, J. (2009). *Learning and Teaching Early Math, the learning Trajectories Approach.* New York, NY: Routledge.

Creswell, J. W. (2003). Research design. Qualitative, Quantitative, and Mixed Approaches. 2nd Edition. London : Sage Publications

Cross, C. T., Woods, T. A., & Scweingruber, H. (2009). *Mathematics Learning in Early childhood*. Washington, DC: The National Academic Press.

de Lange, Jan. (1996). Using and applying mathematics in education. In A.J. Bishop et al. (Eds.), *International Handbook of Mathematics Education*, 49 – 97. The Netherlands: Kluwer Academic Publishers.

Depdiknas. (2006). *Kurikulum Tingkat Satuan Pendidikan Sekolah Dasar*. Jakarta: Depdiknas.

Fauzan, A. (2002). *Applying Realistic Mathematics Education (RME) in Teaching Geometry in Indonesian Primary Schools.* Enschede: PrintPartners Ipskamp.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. Utrecht: Freudenthal Institute.

Gravemeijer, K and Cobb, P. (2006). Design Research from a Learning Design Perspective. In Jan van den Akker, et.al. *Educational Design Research*. London: Routledge.

Gravemeijer, K, Figueiredo, N. Feijs, E., Galen, F van, Keijzer, R. & Munk, F. (2007). *Meten en meetkunde in de bovenbouw. Tussendoelen Annex Leerlijnen Bovenbouw Basisschool. Meten en meetkunde in de bovenbouw. Tussendoelen Annex Leerlijnen Bovenbouw Basisschool*, Utrecht/Groningen: Freudenthal instituut; Wolters-Noordhoff.

Keijzer, R. (2008). *Deca is ten*. Paper presented at the 11th International Conference on Mathematics Education (ICME-11) for *Topic Study Group 2: New developments and trends in mathematics education at primary level*, Monterrey, Mexico.

Kordaki, M & Potary D. (2002). *The Effect of Area Measurement Tools on Pupils' Strategies: The Role of a Computer Microworld*. International Journal of Computers for Mathematical Learning, v7 n1 p65-100.

Michaels, S., Shouse, A. W. and Schweingruben, H. A. (2008). *Ready, set, science!*. Washington, DC: The National Academy of Science.

Reys, R. Lindquist, M. M., Lambdin, D. V., & Smith, N. L. (2007). *Helping Children Learn Mathematics 8th Edition*. New Jersey, NJ: John Wiley & Sons, Inc.

Simon, M. A. & Tzur, Ron. (2004). Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. *Mathematical Thinking & Learning Vol. 6 Issue 2*: *91-104.*

Van de Heuvel-Panhuizen, M. & Buys, K. (2005). *Young Children Learn Measurement and Geometry*. Amersfoort, The Netherland: Drukkerij Wilco.

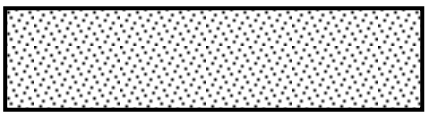
Wijaya, A. (2008). *Design Research in Mathematics Education Indonesian Traditional Games as Preliminaries in Learning Measurement of Length*. Utrecht University.

Zacharos, K. (2006). *Prevailing educational practices for area measurement and students ‟ failure in measuring areas. Journal of Mathematical Behavior 25*, 224-239.

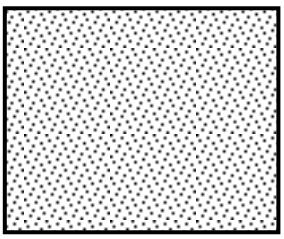
APPENDICES

A. Pre Assessment

1. Two figures below are the rice field that will be harvested. The field of figure A is belong to Pak Saleh and figure B belong to Pak Ridwan . What can you tell about those figures? Which field has the largest yield? How do you know? Explain your answer.



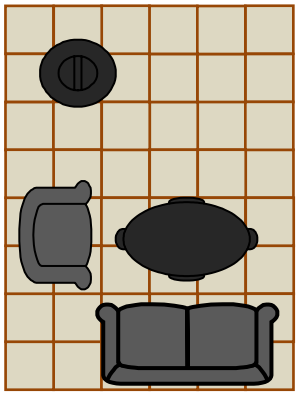
A



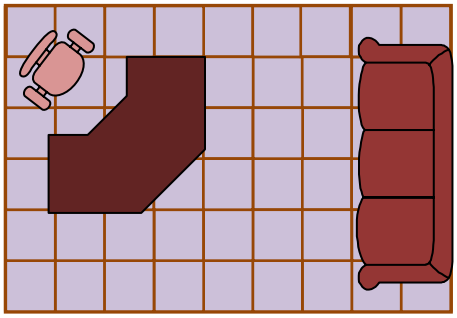
B

2. The figure below is the sketch of rooms that are covered by tiles. How many tiles in each room? How do you know? Explain your answer!

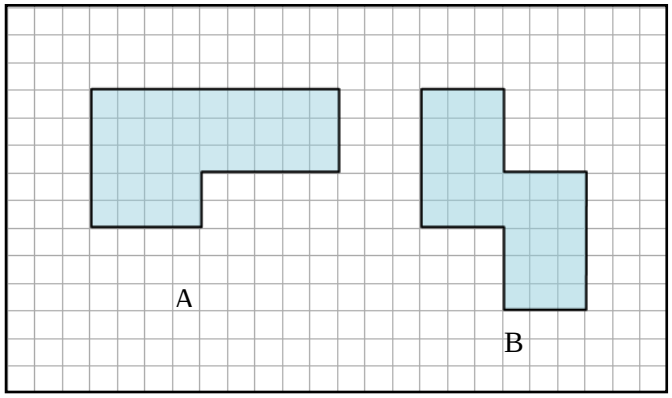
a.



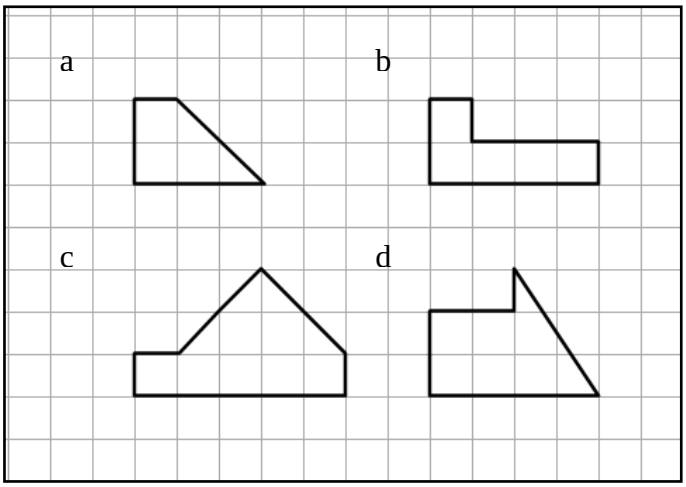
b.



3. The figure below is the sketch of swimming pool. Which one is the biggest? Explain your answer!

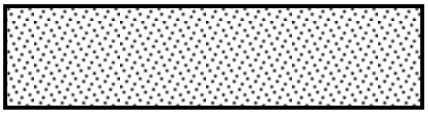


4. How many tiles are covered by each shape below?

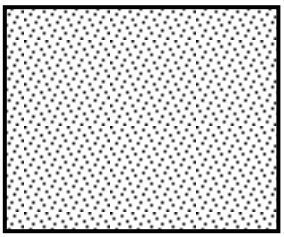


B. End Assessment

1. Two figures below are the rice field that will be harvested. The field of figure A is belongs to Pak Mahmud and figure B belongs to Pak Tarjo. Which field has larger yield? How do you know? Explain your answer.

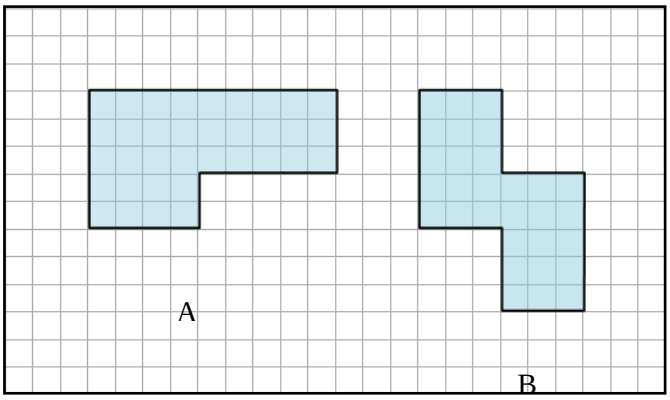


A

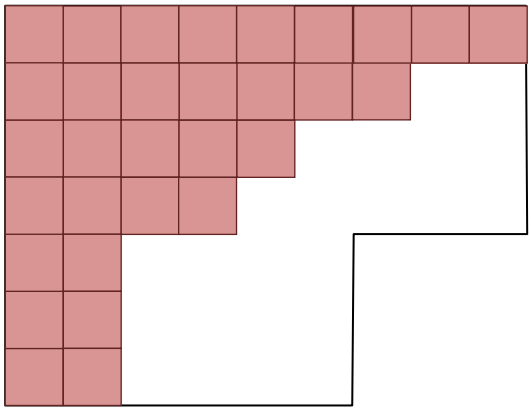


B

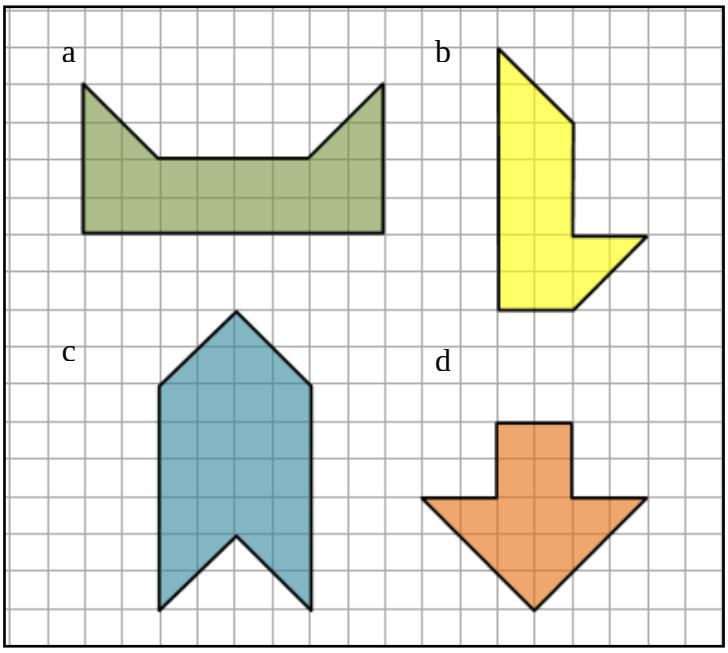
2. The figure below is the sketch of two swimming pools. Which one is bigger? Explain your answer!



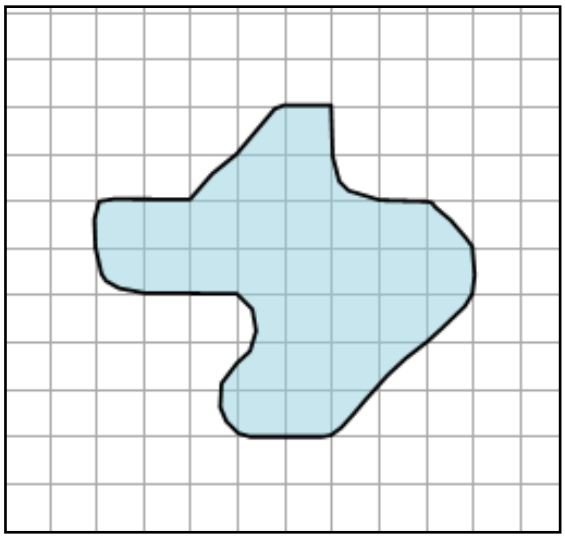
3. Ibu Tuti's house will be fitted with tiles, but the constructor has not finished tiling the room. The figure below is the sketch of Ibu Rahma's room. What is the area of Ibu Tuti's house? Explain your answer!



4. Find the area of each figure below. Explain your answer!



5. The figures below are the sketch of a lake. Estimate the area of this lake. Describe your method to find the answer!



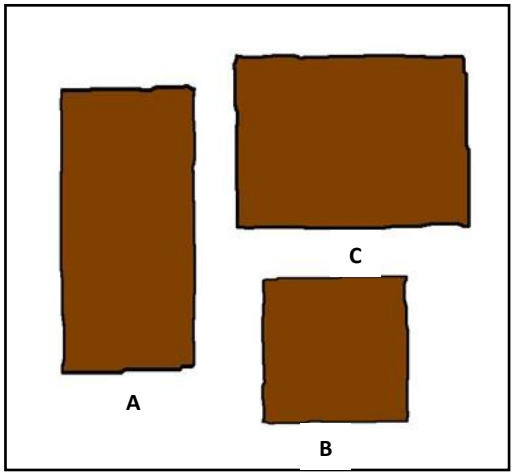
C. Student worksheets

Activity I

Name: _____
Class: _____

Discuss the questions below with your pairs!

1. The figures below are three pieces of cakes. Sort the cakes based on the size!



Answer:

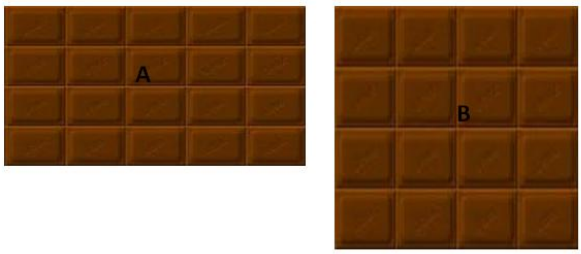
2. What is your strategy to sort the cakes? Use the paper given to help you in explaining your strategy!
   Answer:

Activity II

Name: _____
Class: _____

Discuss the question below with your pair!

1. The figures below are two chocolates. If the price of those chocolate are same, which chocolate do you want to buy? Why do you choose the chocolate?



Answer:

2. What is your strategy in choosing the chocolate you want to buy? Explain your answer! Use the paper given to help you!

Jawaban:

Activity III

Group:
Members:

Discuss the questions below with your group!

1. Look at baking trays in your group. Which baking tray could you put more cookies? Explain your answer!

   Answer:

2. Describe your strategy in comparing the baking tray!

   Answer:

3. How many cookies could you put on the baking trays? Explain your answer!
   Answer:

Activity IV

Group:

Members: 1.
2.
3.
4.

1. What is your unit to cover the baking tray?

   Answer:

2. What is the area of your baking tray? Explain your answer!

   Answer:

3. After comparing the result in class discussion, what do you think about the unit that you used in measuring the baking tray?

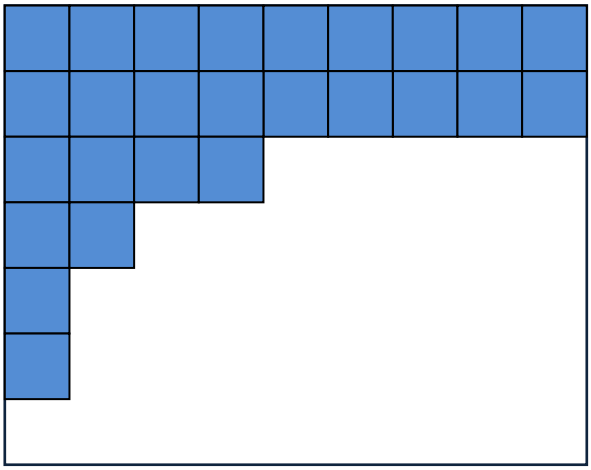
   Answer:

Activity V

Name:
Class:

Answer the questions below!

1. The figure below is the sketch of Pak Rahman's room. However, but the constructor has not finished tiling the room. What is the area of Pak Rahman's room? How do you know?



Answer:

2. How many tiles that must be provided by Pak Rahman to cover the parts that have not been covered with the tiles? Explain your answer!

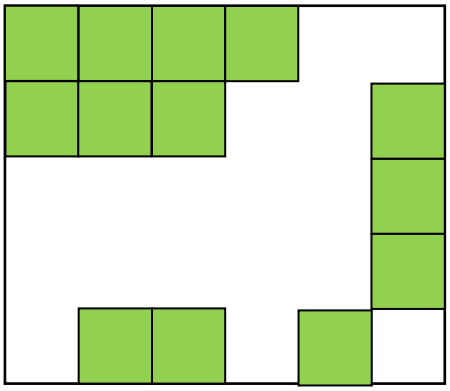
Answer:

Students' Exercise

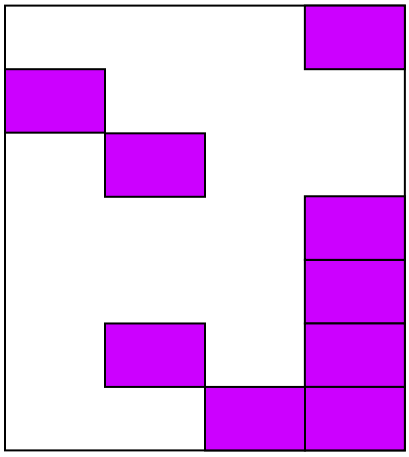
Name:

Class:

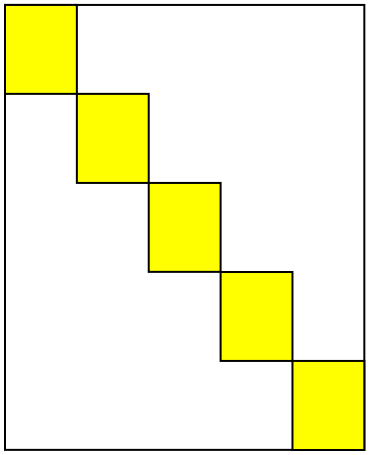
Determine the area of the shapes below!

1. 

Answer:

2. 

Answer:

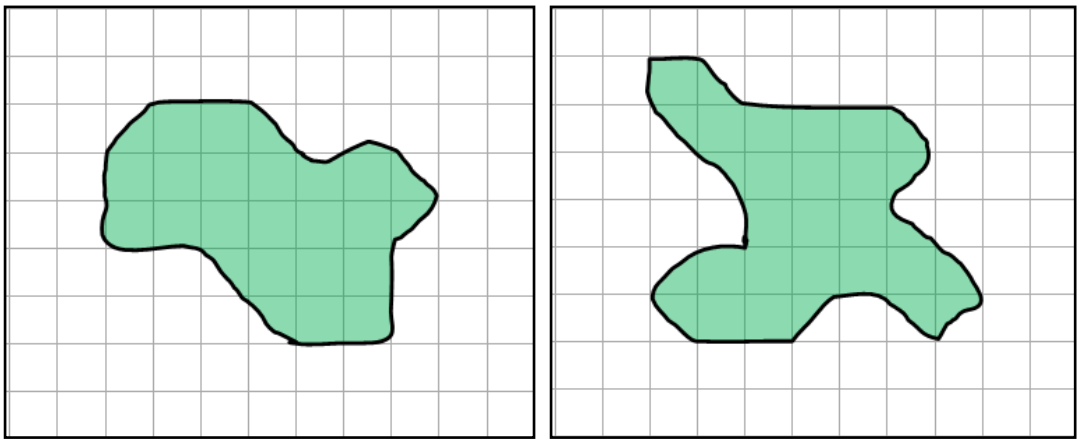
3. 

Answer:

Activity VI

Name:
Class:
Answer the questions below!

1. The figures below are two islands. Which island is bigger? Explain your answer!



Ipin Island Upin Island

Answer:

2. Estimate the area of each island!

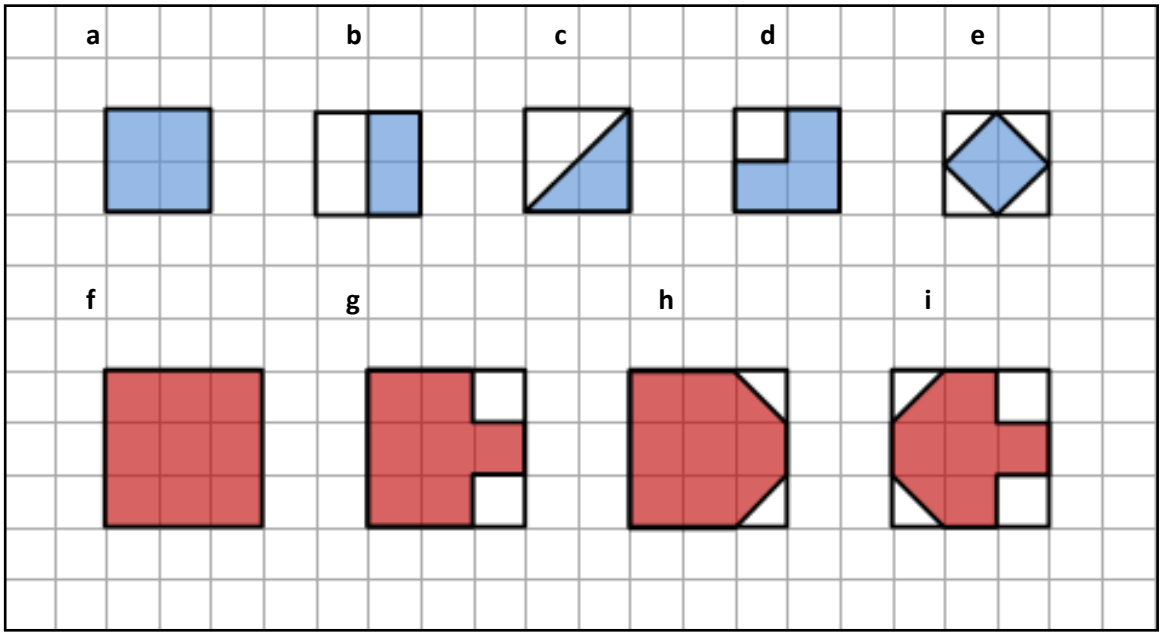
Answer:

Students' Exercise

Name:

Class:

What is the area of the coloring shape? Explain your answer!



Answer:

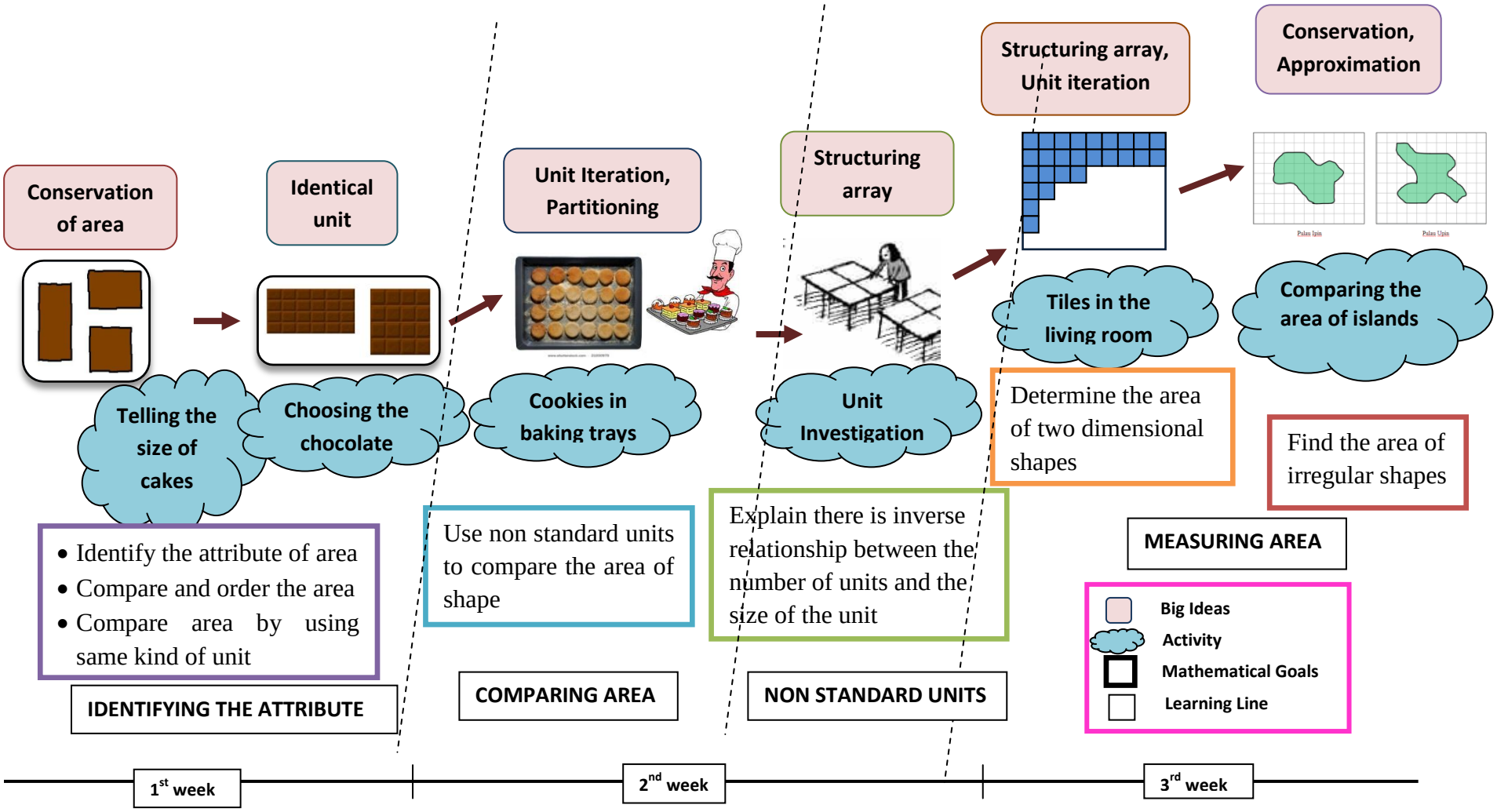
D. Analysis of Students' Answers of the End Assessment

Question	Answer key	Students' answer	Students' strategy	Number of students (out of 34)	Analysis
1	Figure B	Figure B	Explaining by using word large	11 (32,35%)	These students seems get sense the attribute of the object they want to compare since large refers to area
			Explaining by using word big	9 (26,47%)	These students did not specifically describe the area but they seem already know that they want to compare a region.
			Making unit	2 (5,88 %)	These students already perceived the attribute of area and can apply what they learn by giving unit in the figure to compare. They made unit then counted the unit used although the units were not identical enough.
			Explaining by giving the name of shape	3 (8,82%)	These students seems difficult to describe the figure in explaining why figure B is bigger or larger
			Using ruler	2 (5,88 %)	These students seems did not perceive what attribute they want to measure
			Giving answer without reason	7 (20,59%)	These students seems compare by sight the figure without give explanation why they chose their choice
		Figure A	-	-	
2	Figure A	Figure A	Counting the squares	29 (85,29%)	These students seem can distinguish perimeter and area since they count the squares inside figures
			Giving answer without reason	2 (5,88 %)	These students seem only compare by sight to decided which one is bigger
			Using ruler	1 (2,94%)	She seems only consider one side of figures to compare. It means that she still did not perceive the attribute what she want to measure
			Explain the side of figure	1 (2,94%)	He seems still did not know how to compare although he got right answer
		Figure B	Explain without mathematical reasoning	1 (2,94%)	This student seems only focus on the shape of figure

					without consider about the size of the figure
3	54 tiles	54 tiles	Counting the tiles one by one after made arrays or dots	19 (55,88%)	These students can determine the area by making arrays continuing unit given and then count the unit
		other	Counting the tiles but got wrong result	9 (26,47%)	These students also can make arrays in continuing unit given but they seems not careful in counting the unit
			Counting the existing tiles and needed more tiles but no conclusion	3 (8,82%)	These students seem only focused on the square. Without pay attention to the question. However they can make arrays in determining the area
			Using multiplication	1 (2,94%)	This student seems did not recognize that the figure is not rectangle hence she got wrong result
			Only count the tiles needed	1 (2,94%)	This student seems had wrong interpretation about the question. He just pay attention to how many more units needed
			No answer	1 (2,94%)	This student seems did not understand how to do with the question. She just explained the figure without giving any number
4	a. 20 units	20 units	Counting the squares and combine partial units	27 (79,41%)	These students can determine the area of irregular shape. Some of them combined the partial unit and the other counted the unit more than a half as one and ignored the units that were less than a half.
	b. 14 units	14 units		26 (76.47%)	
	c. 24 units	24 units		25 (73,53%)	
	d. 13 units	13 units		28 (82,35%)	
		other	Counting the squares and combine partial unit but got wrong result	3 (8,82%)	These students also can combine partial unit but they seems did not careful in counting
				4 (11,76%)	
				5 (14,71%)	
				2 (5,88 %)	
			Counting all unit	1 (2,94%)	This student did not recognize the boundaries of the area since he counted all unit although there are some parts of the units that are not included in the area of the problem
			Using ruler	1 (2,94%)	This student cannot see the attribute he want to

					measure
			No answer	2 (5,88 %)	These students seem did not read the question since they only describe the figure like what they did for question number 1
5	30 units	30 units	Counting the squares and combine partial unit	19 (55,88%)	These students can estimate to the close result by counting the square in the figure
		other	Counting the squares and combine partial unit but got wrong result	12 (35,29%)	These students also can combine partial unit but they seems did not careful in counting
			Using ruler	1 (2,94%)	This student cannot see the attribute he want to measure
			No answer	2 (5,88 %)	These students seem did not read the question since they only describe the figure like what they did for question number 1

DESIGN RESEARCH ON DEVELOPING UNIT IN AREA MEASUREMENT FOR GRADE 3 IN INDONESIAN PRIMARY SCHOOL



TEACHER GUIDE

Topic	: Area of two dimensional shapes
Class	: 3
Activity	: Telling the size of cakes
Time	: 2 X 35 minutes
Meeting	: 1

B. Basic Competition

Determine the area of square and rectangle

C. Indicator

- Students are able to tell the attribute of objet based on its size.
- Students are able to compare and order plan object based on the area.

D. Goals

- Students are able to identify the attribute of area
- Students are able to compare and order the area

E. Materials

- Scissors
- Student worksheet
- Figure of three cakes

F. Overview

Students will tell about the invitation birthday cards. The cards have the same design but different in size. The students are asked to compare and then discuss about the size of the cards. After they describe the card by using their own words and they know about big and small, students are given a figure of three cakes with different size. They are asked to order the cakes.

G. About Mathematics

Comparing two cards in this activity encourage students to use their own word in describing the objects. In discussion students usually begin by describing the sizes of objects as big and small. They gradually learn to discriminate in what way an object is big or small. They will use specific term such as long, short, large, wide, etc. By describing the size of objects as big and small, students can develop awareness of what area is, and of the range of words that can be used to discuss it. In here, students will use words that represent quantity or magnitude of attribute by comparing the differences of the cakes based on the size. For the next task, student must order the cakes based on the size that they have discussed before. Conflicts will emerge when students compare the figure by cutting one cake and putting on the top of another but the biggest cake cannot be said certainty. What they have to do is reshape the cake so that one cake can cover another cake, so that the biggest can be said certainty. Through this problem, students become more aware that the

larger piece of cakes, deals with the largest area. During this activity, students can acquire experiences with comparing strategies related to physical quantity area. The use of words such as greater, larger and smaller will focus on the attribute of area. It is also expected that they will realize that the area of a plan object does not change if it is reshaped.

H. Planning

1. Apperception (15 minutes)

Students are asked whether they have celebrated their birthday party. The students will talk about their experience in preparing the birthday. They can tell about cakes, dress, balloon, or invitation. Then teacher tell that she bring two invitation birthday cards. These cards have same design but different in size, one is big and one is small. However, teacher does not need to tell them about the size. The teacher just asks which card that needs more paper. The students can tell about the size of the cards. Teacher asks students in giving their argument and how can they convince the other that their argument is true. The questions are:

"Which card that needs more paper?"

"How do you know that card is big?"

2. Main activity (45 minutes)

- Students are given the figure of three cakes and then teacher tell that in a birthday party there are many cakes given to the guess. There are chocolate cakes which also given to guess. Then teacher asks students what they can tell about the three cakes. It is expected that students give answer that the size of the cake is different. And then teacher asks students to order the cakes based on the size.
- Teacher asks: *"look at the figure of the cakes, what you can tell about these three cakes? What about the size? Discuss with your friend how to order the cakes based on the size!"*
- Students are given worksheet. In this activity students work in pairs (20 minutes).
  - Students are asked to compare and order the cakes based on the size of the cakes. To compare the cakes, students can cut the figures because the biggest cannot be said certainly by just look the figures of cakes. Students could also have another strategy to compare the cakes.
  - If the students just guess which cakes is the biggest, middle or the smallest, teacher can ask:
    "How do you know the cake is big?"
- Some pairs present their work in front of class and the other listen and give comments about their work. (25 minutes)

- Students discuss their strategy to compare and order the cakes.

3. **Closing (10 minutes)**

   Students and teacher conclude that to compare two or more objects we can put one to the top of another and see which one that has more rest. The one which has more rest is bigger than another one.

TEACHER GUIDE

Topic	: Area of two dimensional shapes
Class	: 3
Activity	: Choosing the chocolate
Time	: 2 X 35 minutes
Meeting	: 2

A. Basic Competition

Determine the area of square and rectangle

B. Indicator

- Students use same kind of unit to compare the area

C. Goals

- Students are able to compare area by using same kind of unit

D. Materials

- Scissors
- Students worksheet
- Figure of chocolates

E. Overview

Students are given two figure of chocolate with different size. They are asked to choose which chocolate that they want to buy if the price of those chocolate is same. In the previous activity, students are able to compare the area by cutting and pasting the one to the top of another. The same process may be done by students to decide which chocolate is the big or small. The other may just count the slab of chocolate, but the size of slab in each chocolate is different. Can the students find another way to decide which chocolate is big or small other than cutting and pasting? They will discuss about that.

F. About Mathematics

This activity is focused on comparing the chocolate by providing different unit in each chocolate. The different unit in each chocolate will be a conflict for students since they will find different result if they compare with cutting and pasting like what they did in the previous activity. It is expected they will discuss about the unit they used. They cannot decide which chocolate is bigger because the units are different. So, students can see that the area of objects can be easy to compare if the similar unit is used.

G. Planning

1. Apperception (15 minutes)

Teacher reminds students about the previous activity in which they are asked to compare and order three cakes and what strategy they used. Then the teacher asks about

what are their favorite snacks. Students can answer that they like candies, cookies, chocolate etc. Teacher then shows the figure of chocolates. Students can tell what they think about the chocolates. Afterward, some students are asked to draw their chocolate in front of class.

2. Main activity (45 minutes)

- Students are given a worksheet and teacher explains the problem in the worksheet to the students. Teacher asks students' opinion how they decide which chocolate that they want to buy and their reason to choose that chocolate. (5 minutes)
- Students work in pairs to solve the problem in the worksheet. (20 minutes)

- Students might choose bigger chocolate that they want to buy. They could count the slab in each chocolate.
- Teacher can asks students do they sure with their answer and she can remind their strategy in the previous activity to cut and paste the figure to find bigger chocolate.

- Students and teacher discuss the strategy of students to compare the chocolates.(20 minutes)

- Some pairs explain their strategy in choosing the chocolate in front of class.
- Students who cut and paste the figure will find that chocolate B is bigger that chocolate A. but students who count the slab will find chocolate A has more slab than chocolate B. Teacher can ask the other how they think about that problem.
- Students discuss about the unit that they used in comparing the chocolates.
- If the students explain that the slab in each chocolate is different, teacher can asked them to pay attention to each slab of chocolate. And then compare those slabs by cutting and pasting slab of chocolate A and B. teacher asks:
  "What are the differences between these slabs?"
  (It is expected that they say one of side of these chocolates is same but another side is different. Side chocolate A is shorter that side chocolate B. If slab of chocolate B is changed into slab chocolate of A then the number slab of chocolate B is more than the number slab of chocolate A).
- If no student say that to compare the chocolate is needed the same size slab of chocolate then teacher can asks:
  "Can we compare which chocolate is bigger by just count the slab of chocolate?"
  (yes we can, but the size slab of chocolate should be same)
  "What should we noticed in order we can easily compare the size of the chocolates?"
  (The slab of chocolate should be same)

3. Closing (10 minutes)

Students guided by teacher conclude that to compare two chocolate we need the same size of slab to be easy in comparing as well to compare other plan shapes.

TEACHER GUIDE

Topic	: Area of two dimensional shape
Class	: 3
Aktivity	: Cookies on baking trays
Time	: 2 X 35 minutes
Meeting	: 3

A. Basic Competition

Determine the area of square and rectangle

B. Indicator

- Students use their own unit to compare the area of plan abject
- Students count the unit used to cover the area of plan abject

C. Goals

- Students are able to use non standard units to compare the area of shapes

D. Materials

- Cardboards as baking trays
- The figure of baking trays
- Sticky paper

E. Overview

In this activity, students are asked to compare two baking trays, which baking tray can put more cookies. Then students with teacher discuss how they compare those baking trays. Students will be given two cardboards as baking trays which have different size. Students can use small paper to cover the baking tray and count how many used in each baking tray. They can also make grid on the cardboard. Students will work in group 4-5 students in this activity.

F. About Mathematics

In comparing two baking trays, students cannot compare directly because baking tray cannot be cut. Therefore they need tools to compare in order they can sure which baking tray is bigger. They can use small paper as unit and arrange it on the cardboard then compare how many units in each baking tray. When they arrange the small paper, it is expected that all baking tray covered by the unit and no gap or overlap.

G. Planning

1. Apperception (15 minutes)

- Students sit in their group that consist of 4-5 students

- Teacher reminds students about their previous activity in which they compare two chocolate by using same size slab of chocolate so that they can easily compare which one is bigger or smaller.
- Teacher tells about cookies served when *Lebaran Day* and students tell what kind of cookies that they like. Then teacher asks do they make those cookies and what kind of tool to make the cookies. Students can answer many tools such as stove, baking tray etc. Teacher shows figures of two baking trays that have different size and also shows cardboard that represent the baking trays. Students are asked which baking trays can put more cookies. Teacher asks:
  "Which baking tray can put more cookies if we put cookies in it?"
- Teacher asks students' opinion how to compare those two baking trays. Students can give their opinion and the other give comments.

2. Main activity (50 minutes)

- Students work in their group to compare the baking trays. Each group will get two cardboards as baking trays. (25 minutes)
- Students can use their own strategy to compare.
- If there is students ask scissors to cut the cardboard teacher can explain that the baking trays cannot be cut. They can find other strategies to know the answer.
- Students with the teacher discuss the result of some group in front of class. They will discuss about the unit that they used. (25 minutes)
- Some groups present their work in front of class.
- Students with the teacher discuss if the students use different unit to compare the baking trays. Teacher can asks:
  "How if we use different unit to compare these baking trays? Can we?"
  (if we use different unit we are difficult to determine which one is bigger because the unit is different)
- Students with teacher discuss if there is gap or overlap when students cover the baking trays. Teacher can asks:
  "What do you think if these papers overlap? Can we see which one that can put more cookies?"
  (We cannot see which one can be put more cookies because these two trays can be put many cookies no matter how many cookies if we put the cookies overlap)
- *Students discuss how many sticky papers on each baking trays.*

3. Closing (5 minutes)

Students with teacher conclude that the area of baking trays can be revealed by the number of cookies that can be put on it. If the plan object which will be measure cannot be compared directly we can use small objects to measure the area of the plan object.

TEACHER GUIDE

Topic	: Area of two dimensional shapes
Class	: 3
Aktivity	: Choosing the chocolate
Time	: 2 X 35 minutes
Meeting	: 4

A. Basic Competition

Determine the area of square and rectangle

B. Indicator

- Students measure area by using non standard units

C. Goals

- Students are able to explain there is inverse relationship between the number of units and the size of the unit

D. Materials

- Cardboards
- Sticky paper with various shapes (triangle, square, and rectangle)

E. Overview

In the previous activity, students are able to find how big their baking tray is by putting small paper or sticky papers on the baking tray. For this activity students are asked to investigate how big their baking tray is by using non standard unit. To do so, they are provided with triangular paper squared paper and rectangular paper with different size. Then students are asked to find the area of their baking tray by using that shapes.

F. About Mathematics

Many kind of unit can be used to determine the area of two dimensional shape. This activity gives student strong basic in understanding standard unit. They also will realize that a square is not the only unit used to determine the area of two dimensional shapes because other shape also can be used except circle. When measuring the same object with different units, the larger the unit the fewer are required and vice versa. So, by using non standard unit it will have different number of unit if the units are different.

G. Planning

1. Apperception (10 minutes)

- Teacher reminds the previous activity that students cover baking trays to know which baking tray can put more cookies.
- Teacher tells to students that the number of small paper covering the baking tray can be also said the area of baking tray. Teacher says:
  "After we cover the baking tray, we get there are 21 big cookies for baking tray A. we can say that the area of baking tray A is 21 big cookies"
- Students and teacher discuss how if the baking tray also measure by using other shape with different size. Teacher asks:
  "If this baking tray is measured by using triangular shapes, rectangular shape with different size, what is the area of this baking tray?
  "Do you think that the result still same? Let we prove it!"
- Teacher challenge students to find out whether by using different unit will get same result or not.
- Students discuss with their group (4-5 students).

2. Main activity (55 minutes)

- Teacher gives a cardboard as representation of baking tray and also one kind of unit to each group.
- Students work in their group to find out how many sticky papers that can cover the baking tray (20 minutes).
- Some groups present their work in front of class (15 minutes)
  - Every group gives comment to the result of the other groups.
  - In giving the report, teacher reminds students to give the name of unit that they use after the number of the unit. Teacher asks:
    "What is the unit that you used?"
- After they know how many units that can cover the baking tray, students together with teacher compare the result of each group (20 minutes).
  Teacher asks:
  "Why is the result different?
  (Because the unit we used is different so the result is different)
  "What happen if we use small size of unit?
  (The number of unit will be more than the bigger one)
  "What happen if we use big size of unit?
  (The number of unit will be less than the smaller one)

3. Closing (5 minutes)

Together with teacher, students conclude the result of their work. There are many units that can be used to find the area of plan object. When measuring the same object with different units, the larger the unit the fewer are required and vice versa. So, by using non standard unit it will have different number of unit if the units are different.

TEACHER GUIDE

Topic	: Area of two dimensional shapes
Class	: 3
Aktivity	: Choosing the chocolate
Time	: 2 X 35 minutes
Meeting	: 5

A. Basic Competition

Determine the area of square and rectangle

B. Indicator

- Students arrange the unit in determining the area.
- Students count the unit that covering the object.

C. Goals

- Students are able to determine the area of two dimensional shapes by using unit given.

D. Materials

Student worksheet

E. Overview

The teacher shows a sketch of a house that is being fitted with tiles. However, the installation of the tiles has not fully completed yet. Students are asked to tell the picture shown by the teacher. Then students are asked to find out how many tiles are needed to cover the entire room. Students work individually in finding the answer.

F. About Mathematics

This activity focuses on students' strategies in calculating the number of units on two dimensional shapes. Counting the existing unit on the figure in problem may not be easy for students because not all tiles are shown in the figure. It is expected that students can continue to drawing the unit on the figure. Students may also think it is no need to draw the unit because they can just count the unit on the long side and the width of the figure.

G. Planning

1. **Apperception (15 minutes)**

- Teacher reminds the previous activity that there are many units that can be used to find the area of plan object. When measuring the same object with different units, the larger the unit the fewer are required and vice versa. So, by using non standard unit it will have different number of unit if the units are different.

- Teacher asks students to pay attention to their class room. Then she asks students' opinion about the area of their class room. Teacher asks:
  "What do you think about the area of our class room?"
  "What is the unit that you use to know the area?"
  (Students can use the tiles and ceiling in the class room)
- Teacher explains that in this meeting they will find out the area of room on the figure. The tiles in the figure have not fully completed yet. Therefore, they are asked to find out how many tiles are needed to cover the room.

2. Main Activity (45 minutes)

- Students are given the worksheet and they are asked to work individually in solving the problem (20 minutes)
- Students together with teacher discuss about the strategy used by students in solving the problem (25 minutes)
- Some students write their answer on the black board and the other students give their comments
- Teacher tells to students that the long side is called length and the short side is called width

3. Closing (10 minutes)

Students together with teacher conclude that how to determine the area of two dimensional objects that is by using units.

TEACHER GUIDE

Topic	: Area of two dimensional shapes
Class	: 3
Aktivity	: Choosing the chocolate
Time	: 2 X 35 minutes
Meeting	: 6

A. Basic Competition

Determine the area of square and rectangle

B. Indicator

- Students can differentiate an intact unit and not intact unit

C. Goals

- Students are able to measure the area of irregular shapes

D. Materials

Student work sheet

E. Overview

In the previous activity students have already determined the area of regular shape such as square and rectangle. In this activity students will find the area of irregular shape. As an introduction, students will give the exercise to find the area of various shapes. This activity aimed in guiding students to distinguish intact square and not intact square so that later they can estimate the area of shape. Afterwards student are asked to solve a problem in worksheet. The result of students' worksheet will discuss in front of class.

F. About Mathematics

In this activity students will find the area of irregular shapes. It will make students realize that area is not only belonging to rectangular shapes but all two dimensional shapes. Students can estimate to find the area of irregular shapes. Through this activity students also can see that area of two dimensional shape can be cut and arrange mentally without change the area.

G. Planning

1. Apperception (5 minutes)

- Teacher reminds the previous activity about the area of room that not fully covered and how they can find the area of the room. then teacher explain that in this meeting they will find the area of irregular shapes.
- Teacher gives example of the shape beside rectangular shapes.
  "Can you mention another shape beside square and rectangle!"
- Teacher shows some irregular shapes and asks how to find its area.

"To find how to find the area of irregular shapes, please do this exercise first!"

2. Main activity (60 minutes)

- Students are given worksheet and they are asked to work individually (10 minutes)
- Students write their answer on white board and discuss the answer (20 minutes)
  - The other students give their opinion about their friends' answer.
  - Students discuss how many units in each shape and explain whether they count not intact square or not.
- After they know how to do with the not intact square, they are given the second worksheet. They work individually (20 minutes).
- Students together with teacher discuss students' answer (10 minutes)
  - Teacher asks how the students compare which island is bigger.
  - Students discuss if there are students count not intact square as one unit.

3. Closing (5 minutes)

Students together with teacher conclude that not intact square can be combined with the other to be one intact square. So, the area of irregular shape can be determined.