DESIGN RESEARCH ON MATHEMATICS EDUCATION: SUPPORTING SECOND GRADERS' ON LEARNING MULTIPLICATION USING STRUCTURED OBJECTS IN INDONESIAN PRIMARY SCHOOL

A THESIS

Submitted in partial fulfilment of the requirements for the degree of
Master of Science (M.Sc)
In
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By :

Fridgo Tasman
NIM 20092812012

GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

APPROVAL PAGE

Research Title	: Supporting Second Graders' on Learning Multiplication Using Structured Objects in Indonesian Primary School
Student Name	: Fridgo Tasman
Student Number	: 20092812012
Study Program	: Mathematics Education

Approved by,

Prof. Dr Zulkardi, M.I.Komp, M.Sc Supervisor I	Dr. Yusuf Hartono Supervisor II

Head of Mathematics Education Department	Director of Graduate School Sriwijaya University
Prof. Dr Zulkardi, M.I.Komp, M.Sc NIP 19610420 198603 1 002	Prof. Dr. dr. H..M.T Kamaluddin, M.Sc., SpFK. NIP 19520930 198201 1 001

DATE OF APPROVAL : MAY 2011

DESIGN RESEARCH ON MATHEMATICS EDUCATION: SUPPORTING SECOND GRADERS' ON LEARNING MULTIPLICATION USING STRUCTURED OBJECTS IN INDONESIAN PRIMARY SCHOOL

A THESIS

Submitted in partial fulfilment of the requirements for the degree of Master of Science (M.Sc)
In
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

By :
Fridgo Tasman
NIM 20092812012

Approved by Examination Committee	**Signature**
Prof. Dr. Zulkardi, M.I.Komp., M.Sc **Sriwijaya University**	-------------------
Dr. Yusuf Hartono **Sriwijaya University**	-------------------
Dr. Jaap den Hertog **Freudenthal Institute, Utrecht University**	-------------------
Prof. Dr. R.K. Sembiring **IP-PMRI**	-------------------
Dr. Frans van Galen **Freudenthal Institute, Utrecht University**	-------------------
Dr. Dolly van Eerde **Freudenthal Institute, Utrecht University**	-------------------
Dr. Darmawijoyo **Sriwijaya University**	-------------------
Dr. Ratu Ilma Indra Putri, M.Si **Sriwijaya University**	-------------------

GRADUATE SCHOOL
SRIWIJAYA UNIVERSITY
MAY 2011

STATEMENT PAGE

I am who sign below:

Name	: Fridgo Tasman
Place, Date of Birth	: Padang, 12 April 1986
Study Program	: Mathematics Education
Student's Number	: 20092812012

Make the real statement with:

1. All data, information, interpretation and statement presented in this thesis, except those being mentioned the source are my own result of observation, research and thoughts with the direction of supervisor.
2. Thesis written was original and never been proposed or used as requirement for academic degree at Sriwijaya University and other colleges.

This thesis statement is made with truth and if the future, there is trouble found in above statement, I agree to receive academic sanctions such as cancellation of academic degree I acquired through the submission of this thesis.

Palembang, Mei 2011

Fridgo Tasman

NIM. 20092812012

Curriculum Vitae

Author is the first son of 4 siblings. His father named Drs. Tasman, M.Si and his mother named Nurlis S.Pd. Author was born in Padang, West Sumatera, Indonesia at April 12, 1986.

In 1998, author completed his study in primary school in SD Negeri 06 Kampung Lapai, Padang. In the same year, the author directly continues his education to junior high school at SMP Negeri 12 Padang and completed in 2001. The author continues his study to senior high school at SMA Negeri 2 Padang completed in 2004. In 2004, the author accepted as a student of Padang State University, Science and Mathematics Faculty, at Mathemematics department and completed in 2008.

In 2009, the author received scholarship 'International Master Program on Mathematics Education (IMPoME)' from the corporation among IP-PMRI (Institut Pengembangan-Pendidikan Matematika Realistik Indonesia), DITJEN DIKTI, and Nuffic NESO. This scholarship is involved two universities in Indonesia, Sriwijaya University and Surabaya University and one university in Netherlands, Utrecht University.

The author accepted as students in Mathematics Education department in International Master Program on Mathematics Education, Post Graduate Program, at Sriwijaya University in International level which had chance to study one year in Utrecht University at Freudenthal Institute.

ABSTRACT

This research aimed to develop classroom activities that support students in learning Multiplication. Design research was chosen as an appropriate means to achieve this goal. Sequences of instructional activities are designed and developed based on the investigation of students' learning processes. Students' actual learning was compared with our conjectured in our Hypothetical Learning Trajectory (HLT). Around fifty-six students and two teachers in elementary school Indonesia (MIN 2 Palembang) involved in this research. The result of the teaching experiment showed that describing structured objects activity could stimulate students to see the configuration of objects, when students saw the configuration of objects. Through emergent modelling, students had idea to count in groups and did repeated addition as a strategy to determine the total number of objects. From the repeated addition, the idea 'add so many times' and the word 'times of', leads students to represent the repeated addition into multiplication sentence. Based on the result, it is recommended to provide structured objects for students, let them to see its configuration and let them to mathematize it when students learn multiplication.

Keyword: *Design research, Repeated addition, Multiplication sentence, Structured objects*.

ABSTRAK

Penelitian ini bertujuan untuk mengembangkan aktivitas kelas yang mendukung siswa dalam pembelajaran perkalian. *Design Research* dipilih sebagai cara yang tepat untuk mencapai tujuan. Aktivitas-aktivitas instruksional didisain dan dikembangkan berdasarkan pada investigasi proses belajar siswa. Proses belajar siswa yang terjadi dikelas dibandingkan dengan prediksi-prediksi yang telah dirancang pada hipotesis lintasan belajar. Sekitar lima puluh enam siswa dan dua guru sekolah dasar di Indonesia (MIN 2 Palembang) terlibat dalam penelitian ini. Hasil dari percobaan pembelajaran menunjukkan bahwa aktivitas mendeskripsikan benda-benda yang telah tersruktur secara berkelompok dapat mendorong siswa untuk melihat konfigurasi dari objek-objek, dan ketika mereka melihat konfigurasi dari objek-objek, melalui pemodelan yang siswa buat, siswa memiliki ide untuk menghitung objek-objek tersebut secara berkelompok dan mengunakan penjumlahan berulang sebagai salah satu strategi untuk menentukan banyak objek secara keseluruhan. Dari penjumlahan berulang, ide untuk menjumlah yang banyak dan kata 'kali' membawa siswa untuk merepresentasikan penjumlahan berulang kepada kalimat perkalian. Berdasarkan hasil tersebut, direkomendasikan untuk menyediakan objek-objek yang telah tersusun secara berkelompok, membiarkan siswa untuk melihat konfigurasinya dan membiarkan siswa untuk mematimatikakannya ketika siswa sedang belajar perkalian.

Kata Kunci : *Design Research, Penjumlahan berulang, Kalimat perkalian, Benda benda yang telah terstruktur*

Summary

Many students identify multiplication as multiplication table where their focus only on memorizing the table. It was very natural since they learned in mechanistic way. According to Van Hauvel et al (2001), the point for students in mechanistic way is learning multiplication table and this is done by rote memorisation. The problem with this approach becomes clear that Armanto (2002) found that most of students (60% out of 42 students) in Indonesia had a lack of memorizing multiplication table. Furthermore the students memorize the multiplication problem without its meaning.

Consequently, it is important that students develop their understanding of multiplication. The acquisition of multiplication begins with a counting process (Coney et al 1988). It shows that it is important to give students such a situation for them to count such as providing structured objects, groups of objects, let them explore and mathematize it. Therefore this research is aimed to develop classroom activities that support students in learning multiplication.

Design research was chosen for achieving the goal and answering the research question which consists of three phases, namely; a preparation and design phase, a teaching experiment and a retrospective analysis. In the design phase we started with clarification of learning goal and combined with anticipatory thought how to reach the learning goal in the class application. The result is our conjectures of instructional activities which consist of learning goal for students, planned instructional activities and the tools that we used and conjectured of learning process which is called Hypothetical Learning Trajectory (HLT). While in the teaching experiment, the instructional activities are tried and revised. We collect the data to answer our research question, how does the role of structured objects evolve when students learning multiplication, and in the retrospective analysis all collected data would be analyzed and would be compared with our conjectured in HLT. The exploration would be refined to form a new cycle in emergence of a local instructional theory. Around fifty-six students involved in this research which divided into two class, 2B class and 2D class, and two teachers in elementary school (Madrasah Ibtidaiyah Negeri 2 Palembang).

The result of the teaching experiment shows that describing structured objects could stimulate the students to see the configuration of objects. When they tried to describe the configuration of objects, they developed the language that related to multiplication such as 'bags of' and 'boxes of'. Through this activity students had to consider the number of groups and the number of elements in each group simultaneously. Here they constructed the idea of unitizing. Seeing the configuration of objects and knowing that the number of objects in each group is same provoked the students to count in groups. They did skip counting, repeated addition or regrouped the repeated addition to determine the total number of objects in the bags or in the boxes that they saw.

In order to come to multiplication sentence, students had to develop the language that related to multiplication. They started with 'bags of' and 'boxes of', then it develop into 'group of'. When they had to determine the total objects in the 'group of', the 'group of' develop into 'times of' which is connected to the idea 'add so many times' or the repeated addition that they had. After that we introduce multiplication symbol '×' to the students. Therefore they can symbolize a times of b, into $a \times b$.

For some students, transformed the repeated addition that they had into multiplication sentence, a×b, is not easy. They tended put the number of multiplier and multiplicand in the wrong order in multiplication sentence. We found that it is because they influenced with the Indonesian language. For example, when they had the repeated addition, 5+5+5+5+5+5+5, they tended to say it in bahasa ‘ limanya tujuh kali’. Because the five (limanya) comes before seven (tujuh) in their word made them tended to put in multiplication sentence 5 × 7. The findings suggest that students need a bridge to transform repeated addition into multiplication sentence. The word ‘… times of…’ provides a bridge to students to transform the repeated addition into multiplication sentence. However, understanding the meaning of that word became important. Students had to have knowledge about the word ‘times’ that they usually hear in daily life first.

After students are able to represent the repeated addition as multiplication sentence, understanding the property of multiplication became important parts. Structuring the objects and let the students make the connection between one and other multiplication facts lead the students to the property of multiplication such as distributive property and commutative property of multiplication. However in this research, we found that students needed more activity to explore those properties of multiplication.

In short, multiplication is not memorizing table, but multiplication is a counting process for students. Therefore we recommended providing structured objects to the students when they learned multiplication, let them to see the configuration of the objects, and let them to mathematize it. When they had several strategy to count, let them shows their strategy in the class discussion and let them to justify it.

Ringkasan

Banyak siswa mengidentikkan perkalian dengan tabel perkalian dimana fokusnya adalah untuk menghafal tabel perkalian itu. Hal tersebut sangat biasa karena mereka belajar dengan cara mekanistik. Berdasarkan Van Hauvel dkk (2001), inti dari pembelajaran secara mekanistik adalah tabel perkalian dan itu dapat dilakukan dengan cara menghafal. Masalah dengan pendekatan ini menjadi jelas bahwa Armanto (2002) menemukan bahwa kebanyakan siswa (60% dari 42 siswa) di Indonesia tidak hafal dengan tabel perkalian, terlebih lagi siswa menghafal tabel perkalian tanpa mengetahui arti dari perkalian itu sendiri.

Oleh karena itu, penting bagi siswa untuk mengembangkan pemahamannya tentang perkalian. Kemahiran dengan perkalian dimulai dari proses berhitung (Coney dkk, 1988). Itu menunjukkan bahwa, penting untuk memberikan siswa situasi untuk berhitung seperit menyediakan objek-objek yang telah terstruktur, kelopok objek-objek, dan membiarkan siswa untuk berekplorasi dan mematimatikakannya. Oleh karena itu, penelitian itu bertujuan untuk mengembangkan aktivitas kelas yang dapat mendukung siswa dalam belajar perkalian.

Design Research dipilih untuk mencapai tujuan dan menjawab pertanyaan penelitian yang terdiri dari tiga fase yang dimanakan dengan; fase persiapan dan fase desain, fase pembelajaran di kelas, dan fase analisis retrospektif. Pada fase pendisainan, kami mulai dengan menentukan tujuan pembelajaran dan dikombinasikan dengan pemikiran antisipasi tentang bagaimana untuk mencapai tujuan dalam pelaksanaan. Hasilnya adalah dugaan-dugaan dari aktivitas instruksional yang terdiri dari tujuan pembelajaran, rencana aktivitas instruksional dan alat-alat yang akan digunakan dalam pembelajaran yang mana hal-hal tersebut kami sebut dengan hipotesis lintasan belajar (HLT). Sedangkan fase pembelajaran di kelas, aktivitas-aktivitas instruksional dicobakan dan direvisi. Kami mengumpulkan data-data untuk menjawab pertanyaan penelitian, bagaimana peranan objek-objek yang tersusun secara terstruktur berkembang ketika siswa belajar perkalian, dan pada fase analisis retrospektif, data-data yang terkumpul akan dianalis dan dibandingkan dengan dugaan-dugaan pada HLT. Setelah data terkumpul dan dianalisis, maka akan menghasilkan siklus baru dalam teori instruksional local. Sekitar lima-puluh enam siswa terlibat dalam penelitian ini yang terdiri atas 2 kelas, yaitu kelas 2B dan 2D dan dua orang guru pada sekolah dasar (Madrasah Ibtidaiyah Negeri 2 Palembang)

Hasil dari pelaksanaan pembelajaran menunjukkan bahwa aktifitas mendeskripsikan objek-objek terskruktur dapat mendorong siswa untuk melihat konfigurasi dari objek-objek tersebut. Ketika mereka mencoba mendeskripsikan konfigurasi dari objek-objek tersebut, mereka mengembangkan bahasa yang terkait dengan perkalian seperti ‘kantong isi’ dan ‘kotak isi’. Melalui aktivitas ini siswa harus memperhatikan banyaknya kelompok dan banyaknya objek pada masing-masing kelompok secara bersamaan. Disini mereka mengkonstruksi ide dari *unitizing*. Melihat konfigurasi dari objek-objek dan mengetahui jumlah objek pada masing-masing kelompok, dapat medorong siswa untuk berhitung secara berkelompok. Mereka melakukan bilangan loncat, penjumlahan berulang atau mengelompokkan kembali penjumlahan berulang untuk menentukan banyaknya objek secara keseluruhan di dalam bungkus atau kotak yang mereka lihat.

Untuk sampai kepada kalimat perkalian, siswa harus mengembangkan bahasa yang terkait dengan perkalian. Mereka mulai dengan dengan ‘kantong isi’ dan ‘kotak isi’, kemudian bahasa itu berkembang menjadi ‘kelompok dari’. Ketika mereka harus

menentukan banyaknya seluruh objek pada ‘kelompok dari’ tersebut, ‘kelompok dari’ berkembang menjadi ‘kali nya’ yang dihubungkan dengan ide ‘menjumlah yang banyak’ atau penjumlahan berulang yang mereka punya. Setelah itu, symbol perkalian ‘×’ sehingga mereka dapat menyimbolkan ‘*a kali b-nya’* dengan $a \times b$

Bagi beberapa siswa, merubah bentuk penjumlahan berulang kepada kalimat perkalian, $a \times b$ tidaklah mudah. Mereka cenderung untuk terbalik meletakkan angka yang merepresentasikan jumlah kelompok (*multiplier*) dan jumlah objek pada masing-masing kelompok (*multiplicand*) pada kalimat perkalian. Kami menemukan hal ini disebabkan oleh pengaruh Bahasa Indonesia yang mereka gunakan. Sebagai contoh, ketika mereka memiliki penjumlahan berulang, 5+5+5+5+5+5+5, mereka cenderung untuk menyebutnya dalam Bahasa ‘ limanya tujuh kali’. Karena lima disebutkan terlebih dahulu dari pada tujuh didalam kata, maka mereka cenderung menuliskannya dalam kalimat perkalian 5 x 7. Hasil temuan menyarankan bahwa siswa membutuhkan penghubung untuk mengubah bentuk penjumlahan berulang ke kalimat perkalian. Kata ‘ ... kali ...nya’ merupakan salah stu penghubung untuk mengubah penjumlahan berulang ke kalimat perkalian. Akan tetapi, pemahaman akan arti dari kata ‘kali’ tersebut merupakan hal yang penting. Siswa harus memiliki pengetahuan terlebih dahulu tentang kata ‘kali’ yang biasa mereka dengar dalam kehidupan sehari-hari.

Setelah siswa dapat menuliskan penjumlahan berulang sebagai perkalian, pemahaman tentang sifat-sifat perkalian menjadi yang penting. Menstruktur objek-objek dan membiarkan siswa untuk menghubungkan fakta-fakta perkalian satu dengan yang lain dapat membawa siswa kepada sifat-sifat dari perkalian tersebut seperti sifat komutatif dan distributive pada perkalian. Akan tetapi dalam penelitian ini, kami menemukan bahwa siswa membutuhkan beberapa aktivifitas lagi untuk mengekplorasi sifat-sifat dari perkalian tersebut.

Secara singkat, perkalian bukanlah hafalan tabel, tatapi adalah proses perhitung bagi siswa. Untuk itu kami menyarankan untuk menyediakan objek-objek yang telah tersusun kepada siswa ketika mereka belajar perkalian, biarkan mereka melihat konfigurasi dari objek-objek tersebut dan biarkan mereka mematematikakannya. Ketika siswa memiliki beberapa strategi untuk menghitung, biarkan mereka menujukkan strategy mereka di diskusi kelas dan biarkan mereka memilih cara yang terbaik menurut mereka sendiri.

"You have to endure caterpillars if you want to see butterflies"

- Antoine De Saint -

I dedicated this thesis with great love to many people surrounding me.

This thesis is my special dedication to:

My lovely parents and my great family for their never ending support.

Also, this thesis is dedicated to someone special in my heart and her family for their great supports during my study.

PREFACE

First of all, I am very thankful to Allah swt, for all the great things He gave to me. In this opportunity, I would also like to say thanks to all people who supported me in doing my research, gave contribution and inspiration in writing this thesis and helped me in finishing my study.

1. I would like to thank to Ibu Prof. Badia Parizade, the rector of Sriwijaya University.
2. I also would like to thank to Bapak Prof. Dr. dr. H.M.T. Kamaluddin, M.Sc., SpFK, the director of graduate school of Sriwijaya University.
3. I also would like to thank to Prof. Dr. Zulkardi, M.I.Komp. M.Sc as the head of mathematics education department in graduate school of Sriwijaya University for organising IMPoME program.
4. I am so glad to be supervised by Prof. Dr. Zulkardi, M.I.Komp. M.Sc, Dr. Yusuf Hartono and Dr. Jaap den Hertog because they gave supervision and assistance in writing this thesis. Therefore, I would like to express my gratitude to them.
5. The idea of this research was greatly influenced by my discussion with Nisa Figueiredo, Dr. Dolly van Eerde and Prof. Koeno Gravemeijer. Thank you very much to them.
6. I would never join this program if Prof. Robert Sembiring, Prof. Marteen Dolk and the PMRI Team did not launch this program in 2009.
7. It was Dr. Ratu Ilma, Dr. Darmawijoyo, and all lecturers and staffs in Mathematics Education Department of Unsri Postgraduate Program who gave a lot of supports during my study in Palembang.
8. I would never go to Utrecht University without support from Nuffic – NESO and Dikti. Thanks to Pak Dede Herlan, Bu Desi Dimas, Bu Shita Wulandari and all NESO staffs, also to Pak Ridwan, Pak Muchlas, Bu Istri, Bu Martha and all Dikti staffs for arranging scholarship for us.
9. Special thanks also for Prof. Dr. Ahmad Fauzan, Dr. Muliyardi, M.Pd and Lutfian Almash, M.Si. and all of my lecturers in Universitas Negeri Padang who introduced me to PMRI and who gave me recommendations to the PMRI Team as one of the candidates of the IMPoME 2009 scholarship awardees.
10. I also would like to thank to Bapak Budiman, S.Pd.I, MM.Pd as headmaster of MIN 2 Palembang who allow me to do research in that school.
11. It was Bu Nasrel and Bu Nurhayati, the classroom teachers in grade 2 of MIN 2 Palembang, who gave a lot of contributions during the teaching experiment.
12. Last but not least, I would like to thank to all my friends in IMPoME and IMPoME'10 Utrecht University – Universitas Sriwijaya for the days we spent together, I would never forget all your kindness.

I wish this thesis, as the result of my research, can give contribution to improve the mathematics education in Indonesia, especially in the multiplication domain in grade 2 primary school.

Palembang, Mei 2011

Fridgo Tasman

Chapter I
Introduction

A. Research Background

Basic multiplication facts are considered to be foundation for all further calculation in mathematics (Wong et al 2007). They form the basis of learning such as division, multi-digit multiplication, fractions, ratios, and decimals. It shows the importance of basic multiplication facts for the students in their learning processes, because they will have difficulties in the next lesson when they do not understand the basic multiplication fact.

According to Kroesbergen (2002) multiplication instruction can start once students have mastered the basic addition and subtraction skills. Normally, multiplication is initiated in the second grade. In Indonesia, the students also start to learn multiplication in grade 2 after they have learned addition and subtraction up to hundred.

In Indonesia, the learning and teaching process of multiplication is still in mechanistic way, the teacher explains the mathematics operation and procedure, give some examples, and asks the pupils to do other similar problems (Armanto, 2002). The point for students in learning multiplication in mechanistic way is learning multiplication tables (Van den Hauvel-Panhuizen et al, 2001) and this is done by rote memorisation. The problem with this approach becomes clear in the following. Armanto (2002) found out in his research on an Indonesian class that most students (60% out of 42 students) had a lack of memorizing multiplication table. Furthermore the students memorize the multiplication table without any idea of its meaning. Memorizing multiplication facts without any idea behind, is not a productive way to learn multiplication as it does not give them a chance to exploit useful number relationships (Van den Hauvel-Panhuizen et al, 2001).

When children learn arithmetic, it is essential that they not only learn number facts (such as multiplication tables) and algorithms but also develop a conceptual understanding of relevant underlying mathematical principles (Squire et al, 2004). Learning the "tables" by rattling them off repeatedly can obstruct the mastery of multiplication facts (Ter Heege 1985). The students need a greater understanding of the process of multiplication as well as when and how to use the multiplication facts (Caron, 2007).

Consequently, it is important that students develop their understanding of multiplication. Many researchers (Gelman, 1972; Ginsburg, 1977; Hughes, 1981; Carpenter and Moser, 1984) have shown that children posses considerable mathematical understanding prior to any formal instruction and this understanding is derived from everyday situations to which the children have been exposed (Anghileri, 1989). This is also in line with the idea of Freudenthal (1991) that proposed the need to connect mathematics to reality with the students' everyday situation.

The acquisition of multiplication begins with a counting process (Coney et al 1988). It shows that it is important to give students such a situation for them to count such as providing structured objects, objects that already arranged in group, and let them explore and mathematize it. This situation provides a wonderful starting point in the process of understanding multiplication.

B. Research Question

Considering the situation that described before, this research is aimed to develop classroom activities that support students in learning multiplication. In order to support the growing process of second graders' understanding of multiplication, this research tries to answer the following research question;

How can structured objects promote students in learning multiplication?

Chapter II
Research Framework

This chapter gives a theoretical framework that underlies this research. This theoretical framework was elaborated to construct groundwork of this research. Literature about multiplication was studied to identify the basic concepts that are required to help students understand multiplication. Moreover, this literature was useful in designing instructional activities. The theory affects this research by connecting the definitions and research experiences.

A. Multiplication

None of the mathematical operations, not even addition and subtraction, is understood as spontaneously as multiplication (Freudenthal 1983). Multiplicative term such as "times" precede multiplication as arithmetical operation. The term "times" is related to the language that students usually hear in daily life. The term "times" means iterating the unit, for example, 3 km is 3 times as long as 1 km if the unit is 1 km, 6 apples is 3 times as many as 2 apples if the unit is 2 apples. Eventually it serves as a tool for thought as starting point to learn multiplication.

Traditionally, multiplication is introduced to students as a way to represent quantities of things that come in groups (Van Galen and Fosnot (2007). For example, someone has 3 bags of 6 candies. To know the total number of candies that someone has means that the students have to count now by group instead of one by one for an efficient count. This is difficult for the students because they have a different idea with the prior knowledge that they already learned (Dolk and Fosnot 2001). The prior knowledge that the students already learned is that number is used to represent a single unit, for example six represents six candies. But in this situation they have to consider six candies in one bag. They have to understand that six can simultaneously be one – one bag of six candies – furthermore they have in front of them three groups of six candies to count. This means that they have to unitize the unit which is called unitizing. Unitizing is thinking of group of things as a unit (Van Galen and Fosnot, 2007).

The idea of unitizing is a big idea in multiplication, because it underlies the developmental progression for multiplication. Schifter and Fosnot (1993) define big ideas as "the central, organizing ideas of mathematics – principle that define mathematical order" (stated in Dolk and Fosnot, 2007). These ideas are called "big" because they are critical to mathematics and because they are big leaps in the development of children reasoning.

To come to multiplication, the term "times" first connects to the idea "add so many times" (Van Hauvel-Panhuizen et al, 2001). When students add so many times, this situation represents familiar procedure which is students are able to perform multiplication (Coney et al, 1988). When students are counting using repeated addition with long strings of repeated addition, this can be tedious and difficult for students. The students often combine a group to make addition easier (Van Galen and Fosnot, 2007). For example 8 groups of 4, students might make 4 groups of 8, transform these into 2 groups of 16. This idea is called by Van Galen and Fosnot as regrouped repeated addition and they determine this idea as one of the big ideas when students learn multiplication.

Other big ideas when students learn multiplication are, according to Van Galen and Fosnot (2007), the distributive property of multiplication and the commutative property of multiplication. These two ideas are related to make connection between one of the other(s) multiplication facts. For example, in the distributive property of multiplication the students can know the product (8 x 5) by adding the product of (5 x 5) and (3 x 5), for commutative property of multiplication, the students do not need to calculate the product 5 x 8 if they already know 8 x 5 from the multiplication table of 5. These two ideas can be as backup strategies when students can not memorizing the multiplication table.

When students learn multiplication by mathematizing – the human activity for organizing and interpreting reality mathematically – their reality, mathematical models become important. Models are the "things" that mathematicians use for interpreting situations mathematically by mathematizing objects, relations, operations and regularities (Lesh et al, 2004). Sometimes students need to modify or extend them by integrating, differentiating, revising, or reorganizing their initial interpretation. According to Dolk and Fosnot (2001), they interpret models as tools for thought. It often begins simply as representations of situation or problems by the students. For example, students may initially represent the situation of 3 bags of 3 apples with a drawing such as make a circle and write number 3 on it, therefore the circle and the number on it as a model of situation for students. When the students are asked to count how many apples in 3 bags where in each bags consist of 3 apples, they might put it in number line, count with skip counting 0, 3, 6, 9 and realize that 9 is the total number of apples in the 3 bags.

Multiplication can take the following appearances in contextual situation, such as group of varying types such as bags, boxes, and a rectangular pattern (Van den Hauvel-Panhuizen et al, 2001). These appearances are very important because they are underlying the basic structure of multiplication and they offer insight into the properties of multiplication which is important for calculation.

Barmby et al (2009) has shown that the rectangular pattern such as the array representation is a key representation for multiplication in elementary school students. The array representation encourages students to develop their thinking about multiplication as a binary operation with row and column representing two inputs. Initially, students structure array as one dimensional path, where they can see the structure in one dimension (row or column) but not both (Dolk and Fosnot, 2001).

Van Galen and Fosnot (2007) stated that multiplication for students start with repeated addition, but structuring the situation can lead naturally to the strategy such as doubling and partial product. It shows the importance to structure the situation so that can provoke the students to find efficient strategy to count the product of multiplication.

When students learn multiplication, Kroesbergen (2002) suggests that it is important to give students sequential stage in the instructional activities: concrete objects (e.g., beads, block), semi concrete (e.g., pictures, representation), and abstract (e.g., numerals, symbols), to help the students develop their understanding of multiplication. The purpose in giving the concrete objects, semi concrete and abstract, to the students is to make the instructional activities real and meaningful for them which is in line with one of the tenet of Realistic Mathematics Education (RME). In the present study, a sequence of instructional activities is developed to help students develop understanding of multiplication by using realistic mathematics education approach.

B. Realistic Mathematics Education

According to Freudenthal, in his book Revisiting Mathematics Education ; China Lecture (1991).

> Mathematics has arisen and arises through mathematising. Mathematising is mathematising something – something non-mathematical or something not yet mathematical enough, which need more, better, more refined, more perspicuous mathematising. Mathematising is mathematising reality, pieces of reality. Mathematising is didactically translated into reinventing, the reality to be mathematised is that of the learner, the reality into which the learner has been guided, and mathematising is the learner's own activity. (P.66)

To help students mathematize reality, the tenets of Realistic mathematics education (RME) offer clues and design heuristics that were also applied in this research.

Five Tenets of Realistic Mathematics Education

Realistic Mathematics Education (RME) has five tenets or principles (Treffers, (1987) in Gravemijer, K. Van den Hauvel, M & Streefland 1990) that were also applied in this research. The tenets and application in this research are described below;

1. Constructions stimulated by concreteness.

   This research does not starts in the formal level but starts with a situation that is experientially real for students with purpose that it will make meaningful for the students because the students can explore and construct the mathematical idea with it. Therefore, in our first instructional activity, we give students concrete objects. We shows to them pens that already in the group of 3. We ask the students to describe the pens to their friend who can not see the objects. The purpose of this activity is to develop appropriate language that related to multiplication.

2. Developing mathematical tools to move from concreteness to abstraction.

This tenet of RME is bridging from a concrete level to a more formal level by using models and symbols. Students' informal knowledge as the result of their experience needs to be developed into formal knowledge. The teacher helps the students by guiding them while students mathematizing their reality. In one of our instructional activity, counting tiles, we ask students to make their representation of complete tiles that arrange in rectangular pattern. Consequently, the class discussion will be held to encourage the students making their *model-of* situation and move to *model-for* for their mathematical reasoning. Therefore the rectangular model presents as the students model of situation for the students. When students ask to represent the situation in multiplication sentence, repeated addition comes as model for their mathematical reasoning. After that the multiplication sentence introduces to them with connected with the idea "add so many times".

3. Stimulating free production and reflection

The idea of this tenet is to raise the levels must be promoted by reflection, which means thinking about one's own thinking. Students' own construction or production assumed will be meaningful for them. During the activities and class discussion the students' construction are used to guide them to the next level, or more formal level. The students' strategies in each activity were discussed in the following class discussion to supports students' acquisition of multiplication.

4. Stimulating the social activity of learning by interaction

   Because the learning process takes place in the social school environment, this situation makes the students have interaction between each other. This interaction is a kind of social process. The understanding of the lesson can be come from students' interaction with each other, when they communicate their work and thought in the social interaction in the classroom. In this research, the students do the activities in the group of three or four. After they discuss in group, the class discussion are held to make they share their idea with other students.

5. Intertwining learning strands in order to get mathematical material structured

   This principle of instruction concerns intertwining learning strands. Intertwining learning strands means that the topic that the students learn should have relation with other topics. This tenet suggests that to integrate various mathematics topics in activity. In learning multiplication, understanding addition plays an important role.

C. Emergent Modelling

The implementation of the second tenet of RME produced a sequence of models that supported students' acquisition of the basic concept of multiplication. Emergent modelling asks for the best way to represent situation that the students can reinvent or develop their idea about the concept of mathematics (Gravemeijer, 2004[*]). That situation makes emergent modelling is one of the heuristics for realistic mathematics education in which Gravemeijer (1994) describes how *model-of* a situation can become *model-for* for more formal reasoning. There are four levels of emergent modelling. The levels of emergent modelling are shown in the following figure:

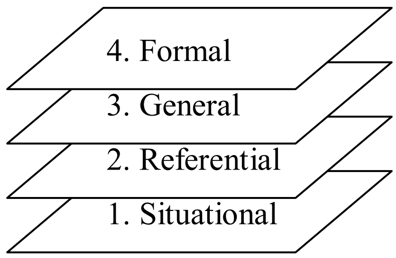


Figure 1 Levels of emergent modelling from situational to formal

The implementation of the four levels of emergent modelling in this research is described as follows;

1. Situational level

   Situational level is the basic level of emergent modelling. In this level domain specific, situational knowledge and strategies are used within the context of situation. In this research, we give the students situation for counting. We expect that the students could find efficient strategy to count such as counting in groups by using the structure of objects.

2. Referential level

   In this level models and strategy refers to situation that sketched on problems. This level also called *model-of*. A class discussion encourages students to shift from situational level to referential when students need to make representation (drawings) as the *model-of* their strategies to count the objects, for example to count the number of breads in 7 bags of 5 breads, the students might draw 7 circles that represent the bags of breads and put number 5 in each circle that represent the quantity of breads in each bag.

3. General level

In this level, a mathematical focus on strategies that dominates the reference of the context, this is also called *models-for*. We expect students could see the structure of objects that supports their strategy to determine the total objects by repeated addition.

4. Formal level

In this level, students work with conventional procedures and notations. In this level the focus of discussion moves to more specifics of models related to the multiplication concept, the students can know what 4x5 is for example.

Chapter III
Research Methodology

A. Design Research

This research was conducted under a design research methodology. The reason is in line with Edelson (2002). First, the design research provides a productive perspective for theory development. Second, the design research has typical usefulness of it results and third, design research directly involves the researcher in the improvement of mathematics education.

Cobb, Confrey, et al. (2003 in Bakker 2004) identify five features that apply to different types of design research. The first is that its purpose is to develop theories about learning and the means that are designed to support learning. We design an instructional theory for students in grade two elementary school and instructional means that support the students understanding in learning multiplication. The second feature of design research is its interventionist nature. The methodology makes the researcher not constrained to improve the design after an experiment cycle has been carried out. The third, Cross-cutting feature is that design research has a prospective and reflective component that need not be separated by an experiment. The researcher confronts conjecture in a prospective with actual learning he/she observe in a reflective part. The Forth feature is the cyclic character of design research. In this cyclic character, the invention and revision occur as iterative process. Conjectures of learning are some time refuted and alternative conjectures can be generated and tested. The fifth crosscutting feature of design research is that the theory is relatively humble in the sense that it is developed for a specific domain; in this research we develop to domain calculation up to hundred especially in multiplications. Therefore it must be general enough to be applicable in different contexts such as different classroom in other counties.

The main objective of design research is to develop theories together with instructional material whereas the main objective of comparative research to evaluate theories or materials (Bakker, 2004). The design research that we use, consist of cycles of three phases. They are;

1. A preparation and design phase

   In our design the first phase we started with clarifications of mathematical learning goals, combined with anticipatory thought how to reach the learning goals in the class applications. The result is our conjectures of instructional activities which consist of three components. They are: (1) learning goal for students, (2) planned instructional activities and the tools that will be used, (3) conjectured of learning processes in which one anticipates how students' thinking and understanding could evolve when instructional activities used in class room (Gravemeijer 2004).

2. A teaching experiment

   In this phase, instructional activities are tried, revised and designed on daily basis during the teaching experiment (Gravemeijer, 2004). In teaching experiment, we try our instructional activities to collect data for answering our research question. After that we revise our instructional activities on daily basis with purpose to develop a well-considered and empirically grounded local instruction theory on how a certain set of instructional activities could work.

3. A retrospective analysis.

In this phase, all collected data are analyzed. Our Hypothetical Learning Trajectory (HLT) is compared with students' actual learning. The exploration is refined to form a new cycle in the emergence of a local instructional theory as shown in figure 2 below.

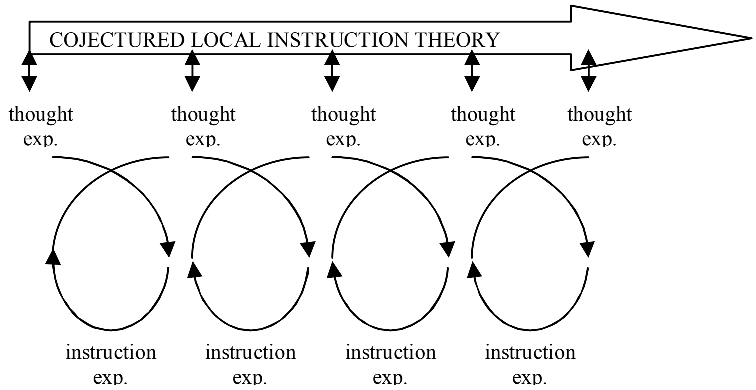


Figure 2. The cyclic process of design research
(Gravemeijer & Cobb, 2006)

B. Research subjects and Timeline of Research

This research had been conducted in MIN 2, Palembang, Indonesia. This school is one of PMRI schools in Palembang. Min 2 Palembang has 4 classes for the second grade namely 2a, 2b, 2c and 2d. When students finished their study in grade one, all of the best students put in 2c and the rest spreads into 2a, 2b, and 2d. We conducted the experiment in two different classes, 2b and 2d, 2b class for preliminary experiment and 2d class for the experimental class. Those two classes consist of 26 to 28 students at the age of 8 to 9 years old. The students in those two classes had already learned about addition and subtraction in the domain up to hundred.

The experiment of this research divided into two parts, preliminary experiment and teaching experiment. In the first part, we tested our HLT in 2b class. We want to investigate the students' thinking of the tasks and problems in the HLT and tested our conjectured about it. In the second part, we improved our initial HLT and then tested it in the 2d class. Based on our explanation above, we summarize the timeline of this research on the table as follows:

DESCRIPTIONS	DATE
Preliminary Design	
Studying literature and designing initial HLT	2 September 2010 – 5 January 2011
Discussion with teacher	26 and 28 January 2011
Preliminary Experiment	
Classroom Observation	29 January 2011
Pre-Test	4 February 2011
Try out "Describing structured objects" Activity	5 February 2011
Try out "Counting Structured objects" Activity	8 February 2011
Try out "Counting dolls" Activity	12 February 2011
Try out "Counting tiles" Activity	15 February 2011
Try out "Counting eggs" Activity	16 February 2011
Try out "Solving multiplication problems" Worksheet	17 February 2011
Analyzing the Preliminary Experiment and Improved the HLT	
Discussion with Teacher	18 February 2011
Preparation for Teaching Experiment	19 – 1 March 2011
Teaching Experiment	
Lesson 1: "Counting structured objects" Activity	2 March 2011
Lesson 2: "Counting Structured objects" Activity	3 March 2011
Lesson 3: "Counting tiles" Activity	7 March 2011
Lesson 4: "Counting eggs" Activity	9 March 2011
Lesson 5: "Counting dolls" Activity	10 March 2011
Lesson 6: "Solving multiplication problems" Worksheet	14 March 2011
Final Assessment	23 March 2011

Table 1. Research Timeline

C. Multiplication in Indonesian curriculum for elementary school grade 2

Students start to learn multiplication in Indonesia since they are in second semester in grade two elementary school. When they start to learn multiplication, they have knowledge about addition and subtraction in the domain up to hundred. Students continue to learn about multiplication in first semester of grade three and continue in first semester in grade 4. Table 2 described multiplication in grade 2 in Indonesian Curriculum.

Standard Curriculum	Basic Competence
The Second Semester of Grade 2	
Numbers 3. Doing two digit multiplication and division	3.1. Doing multiplication which has product in two digits numbers. 3.2. Doing two digits division 3.3. Doing mixed operations

Table 2. Multiplication in grade 2

Multiplication that the students learn in grade 2 is the base for grade 3 and 4. One of the differences between the multiplication in grade 2 and grade 3 or 4 is that the students in grade 3 or 4 work with larger number. It is very important to the students to understand about the concept of multiplication in grade two, so that they can use their knowledge in grade two to starts to explore multiplication with larger number, otherwise the students will have difficulties to continue the lessons.

D. Data Collection

In this study, the data such as video recording, students' works, and field noted were collected during the teaching experiments. We took videotape of the activities and interview some students. We analyzed the data from the video recording and students' works to improve our HLT. More details of data collection of this research described as follows:

1. Video

The students' work and strategies in each activity were observed by video. We also took short discussion with the students and class discussion to investigate students' reasoning for their idea.

2. Written Data

The written data provides information about students' works. These data used for investigating students' achievement. Beside students' works during the teaching experiment, the written data also including field notes, the result of assessments including the final assessment and some notes that collected during teaching experiment.

E. Data Analysis

In retrospective analysis, the data collected were analyzed. In the analysis, the HLT and students' actual learning were compared based on video recording that take in preliminary experiment. The whole video recording was watched and was looked for fragment in which students learned or did not learn what this research conjectured in HLT. The unexpected situations that happen in the class were taken into consideration. After that, the selected fragment was registered for a better organization of the analysis. The part that not relevant with the students' learning was ignored.

The selected fragments were transcribed and the analysis would start by looking at the short conversation and students' gesture in order to make interpretation of students' thinking process. The interpretation was discussed with other researchers.

Other data that also used were teacher' and students' interviews in order to improve the validity of the research (data triangulation). After that for the second opinions of the analysis, the interpretation was asked to the expert in order to analyse intensively and to improve the analysis itself.

The analysis of the lessons was done in two ways; analysis on daily bases and analysis of the whole series of lesson. In daily bases, the analyses focus on how the activities support the intended students' thinking process. While the whole lesson series analyses, focus on the connection between lessons to find out the success for supporting students' learning process.

Finally, the conclusion would be drawn based on the retrospective analysis. These conclusions focus on answering research question in this research and would be given recommendations for the improvement of the HLT, for mathematics educational practice in Indonesia and for further research.

F. Validity and Reliability

The validity concerns the quality of the data collection that collected. The data were collected throughout the learning activities. To guarantee the internal validity of this research, this research used many sources of data, namely video recording of classroom observation, teacher's interview, students' interview and students' work. Having these data, allow this research to conserve the triangulation so that we could control the quality of the conclusions. The research was conducted in a real classroom setting, therefore could guarantee the ecological validity – a form of validity in research study where the methods, materials and setting of the study must approximate the real-life situation that is under investigation.

Improving internal validity, this research transcribed critical episodes of the video recording. Some colleagues involved in analyzing the critical learning episodes. The data were registered to make clear where the data comes from. By documented the analysis, this research takes care of the external validity and the tractability of the research.

The extensive data analysis would make in this research in order to carry out the first cycle of analysis. After that the HLT II compared with the students actual learning. By doing this would show HLT in this research could support students learned multiplication and would give recommendations of how HLT II should be improved for further studies.

Chapter IV
Hypothetical Learning Trajectory

In this research a learning trajectory is defined as a description of the path of learning activities that the students can follow to construct their understanding of multiplication, where in that path considers the learning goal, the learning activities and the conjecture of learning process. The learning trajectory is hypothetical because until we apply our design or until students really work in the problem, we can never be sure what they do or whether and how they construct new interpretations, ideas and strategies.

In this research a set of instructional activities for multiplication were designed and divided into six different activities. In learning multiplication, the second grade students follow these activities: (1) Describing structured objects. (2) Counting structured objects. (3) Counting dolls (4) Counting tiles (5) Counting eggs (6) Solving multiplication problems. Table 3 shows the general overview of hypothetical learning trajectory (HLT) of multiplication in Grade 2 students' elementary school.

Name of Activity	Students activity	Learning Goal	Math Idea	Strategy
1. Describing structured objects	▪ Seeing structure of objects. ▪ saying what they saw and ▪ Writing what they saw on paper.	Students are able to describe the structured objects that they saw	Multiplication language : – … bags of … – … boxes of … – Unitizing	Seeing the structured of objects (How many bags and how many objects in each bag)
2. Counting structured objects	▪ Making representation ▪ Counting structured objects ▪ Writing their strategy to count	Students are able to represent the total number of structured objects into multiplication sentence	Multiplication language : – … groups of … – … times of …	Seeing things not depend on the wrappers of material but seeing things as a group

			Multiplication symbol "x"	Making connection to add so many times
3. Counting dolls	▪ Counting structured objects ▪ Representing the number of objects in multiplication sentence	Students know about distributive property of multiplication.	▪ Distributive property of multiplication ▪ Part whole relationship	▪ Count the number of dolls in racks using repeated addition. ▪ Connecting the number of dolls if the rack were full with the number of dolls in rack and the empty space in rack.
4. Counting the tiles.	▪ Completing the picture ▪ Counting objects in rectangular pattern ▪ Writing their strategy to count ▪ Representing the number of objects in multiplication sentence	▪ Students are able to represent the number of objects in rectangular pattern into multiplication sentence	▪ Structuring (viewing pattern and regularity)	▪ Counting tiles in column ▪ Counting tiles in row
5. Counting eggs	▪ Making representation ▪ Counting objects in rectangular pattern ▪ Writing their strategy to count ▪ Represent the number of objects in multiplication sentence	Students know about commutative properties of multiplication.	Commutative property of multiplication	▪ Counting in row ▪ Counting in column ▪ Connecting the product of two multiplication sentence that they got
6. Solving	▪ Counting	Students are able	Distributive	▪ Repeated

multiplication problems	structured objects ▪ Represent the number of objects into multiplication sentence ▪ Writing their strategy to count.	to solve multiplication problems.	property of multiplication	addition ▪ Doubling ▪ Partial product

Table 3. General overview of HLT

The hypothetical learning trajectory is elaborated in the instructional activities as following:

A. Describing structured objects activity

Learning goal: Students are able to describe the structured objects that they saw.

In this activity we want the students able to describe the configuration of the structured objects that they see. Therefore they are expected able to say and to see, the number of objects in bags/boxes/groups, and the number of objects in each bag/box/group simultaneously. For example: there are 3 groups of 5 breads, there are 3 bags of 5 breads, there are 3 fives breads (they unitize the objects). 3 is the number of bags/groups and 5 is the number of objects in each group.

Description of activity:

The students work with groups of objects. The students have to describe these configurations of objects to another student who can not see the objects. The teacher asks one of the students as a volunteer to draw the objects that he/she does not allow to see. Together the students decide when the drawing is correct. If the drawing do not correct, the students have to find another way to describe in order to make the student that made drawing able to draw the correct drawing.

The teacher shows to the students (except to the student that does not allow seeing the objects) 3 groups of 3 pens, as shown in figure 3 below:

Figure 3. 3 bags of 3 pens

and let one of the students trying to tell to the student who makes drawing and wait him/her finished his/her drawing. Focus of the students who is telling something to the students that made drawing is the quantity of object that he/she saw. At the end of this activity, we expect that the students could say that there are 3 bags of 3 pens, so that the student who made the drawing can make drawing of it, and all of the class can agree of his/her drawing.

After the student are able to make drawing of 3 bags of 3 pens, the teacher can ask another student to make drawing and shows to others students 3 packs of 4 batteries and let the students to describe to the student who can not see the objects.

After doing this activity, the teacher gives worksheet to the students. The worksheet consists of three pictures of structured objects. They are 3 bags of 5 breads, 3 boxes of 6 colas, and 6 boxes of 6 pencils as shown in figure 4 below.

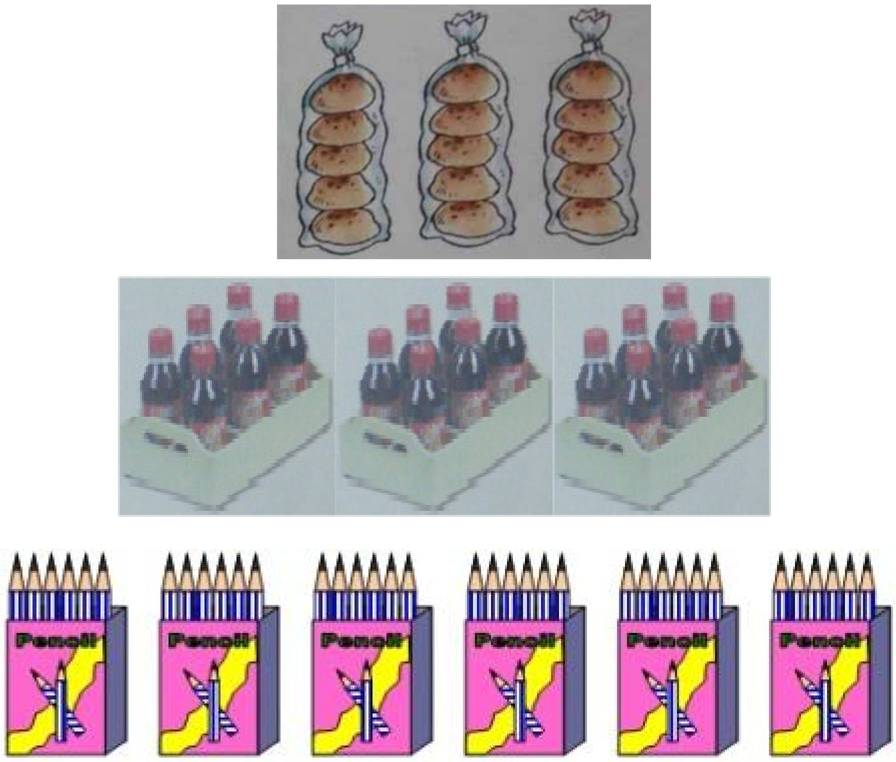


Figure 4. The Structured objects that showed in the students' worksheet

The students are asked to describe those three objects that they saw in those pictures.

To fill the worksheet, the students work in group of 3 or 4, they ask to discuss different way to say it and to practice like the activity that they did before. After the students finish, the class discussion is held. The focus in the class discussion is to find the appropriate language that related to multiplication, such as there 3 groups of 5 breads, there are 3 bags of 5 breads, or there are 3 bags of breads where each bread consist of 5 bread.

Conjecture of Students thinking and Discussion:

When students are saying what they saw to the student who makes representation of it, they might come up with:

- One student might say the name of the objects that they saw, without pay attention to the number of objects and how the objects arranged. Therefore the student who made the drawing might draw a pen. Then the class decides that it was different and they have to think again how to tell it to their friend.
- One student might say the name and the total number of objects, without pay attention to how the objects arranged. Therefore the student who made the drawing might draw the pens as much as his/her friends said to him/her. But the structured of the objects that they see still different and they have to think to tell in different way.
- One student might say the number of bags that they saw without telling how many objects in each bag. It might make the student who made drawing confuses to decide how many objects in each bag and asks how many objects in each bag. Or he/she, just draw the pens in each bag as he/she wants and the others students would say the number of objects in each bag.
- We expect one of the students could say that:
  - There were 3 bags of pens where in each bag consist of 3 pens,
  - There were 3 bags of 3 pens, or
  - There were 3 three pens

  Therefore the student can make representation of the situation in the whiteboard.

After students finished this activity we expect that the students are able to fill the worksheet that we give to them. We expect that they could describe that there were 3 bags/groups of 5 breads, 3 boxes/groups of 6 colas, and 6 boxes/groups of 6 pencils.

B. Counting structured objects.

Learning goal: Students are able to represent the total number of objects in multiplication sentence.

In previous activity students already had language that related to the multiplication such as 3 bags of 5 breads, 3 boxes of 5 colas, or 4 boxes of 6 pencils. In this activity we want to introduce multiplication symbol “x” to the students. We introduce multiplication symbol “x” with the idea “add so many times”. In doing that we give students activity to count groups of objects we expect that they realize that the number of objects in each group is same therefore they can count in groups and did by repeated addition as their strategy to determine the total.

Description of activity:

The activity starts by giving instructional sheet to the students that they have to do in the group of three or four. There are two pictures in the instructional sheet, 7 bags of 5 breads and 9 groups of 5 oranges as shown in figure 5 below.

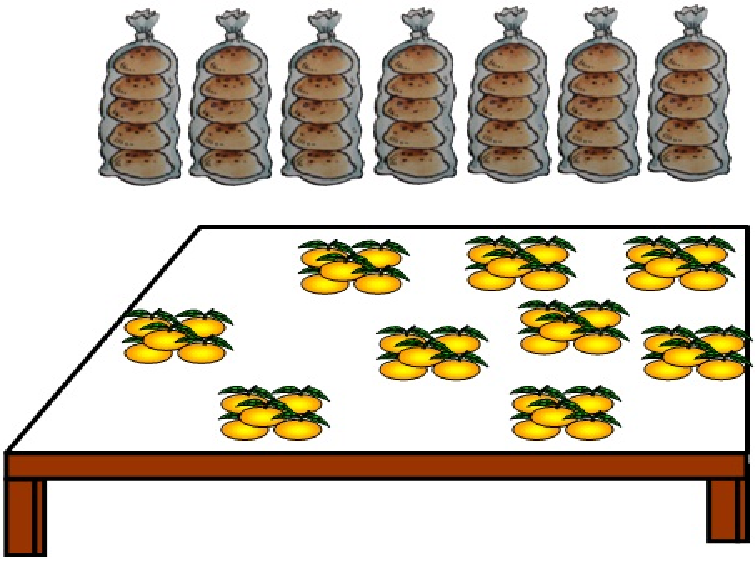


Figure 5. Groups of objects that showed in the instructional sheet for students

The students are asked to make representation of those two pictures on the poster that the teacher gives. How they made their drawing are observed. They also are asked to write on the poster what they saw on the poster like in previous activity (writing the number of objects in groups, for example there are 7 bags of 5 breads) and ask them to count how many objects in those pictures and write their strategy to count the total on the poster. After the students finished, the class discussion is held. The focus in class discussion are introducing multiplication symbol “×”.

Conjecture of Students’ thinking and Discussion:

In this activity the students have to make representation of the picture on their poster. They have to determine the quantity (number) of objects that they saw. When students made their picture, the way they make their picture might be:

- Some groups might make their drawing by one to one correspondence with the picture that they saw. These students do not use the structure of the objects that arranged in the groups.
- Some groups might count the number of bags/groups of objects first and counted the number of objects in each bag. After they know the number of bags/groups they started to make their drawing without doing one to one correspondence with the picture.

When students describe the number of objects that they saw on the picture, the students might come up with:

- Some groups might write the name of the objects that they saw without pay attention to the number of objects and the structured of objects.

- Some groups might write the name and the total number of objects that they saw in the instructional sheet.
- Some groups might write the name of the objects and the number of the bags/groups of objects, without write the number of objects in each bag/group.
- We expected that some groups could write the number of bags/groups and the number of objects in each group.

The students might not have problem to write the number of objects with breads, because they know that those objects covered by bags, we expect they can say that there are 7 bags of 5 breads, but for oranges they might have difficulties in writing it. They might also say that it was in the bags, but in fact there was no bag there. This situation might be become conflict to the students. In this situation the teacher tries to introduce the term "group of" to the students.

After the students finished writing their description, they have to determine the total number of objects. To determine the total number of breads and oranges the students might come up with:

- Counting the number of breads and oranges, one by one. In the picture that they made or in the instructional sheet that they had. These students do not use the structure of objects to do efficient counting.
- Doing repeated addition. Knowing the number of breads and oranges in each bag/group same students might do repeated addition, and to calculate the repeated addition that they have they might be:
  - Counting on or counting back with their fingers.
  - Skip counting by five

- Regrouped the repeated addition 5+5 = 10 and doing skip counting by ten.

We expect that one of the students has idea to short their calculation, for example, if they know the total number of 7 bags of 5 breads, to determine the total number of oranges they can make connection with the number of breads that they had, because the quantity of objects arranged in five, they just add 10 more to know the total number of oranges. If this are happen, it could be discussed in the class discussion.

After students finished with their tasks in the instructional sheet, the class discussion is held. Focus in the class discussion is introducing multiplication symbol to the students. The teacher introduces multiplication symbol to the students in class discussion. When students are describing their strategy to count objects by repeated addition, the teacher tries to give conflict to the students with purpose to introduce multiplication symbol “x” to the students. For example, the teacher can say to the students “oo, it was very long addition. Can you make it simpler?(when the students wrote 5+5+5+5+5+5+5) How about if there are 20 bags of 5 breads, it will be bored to write it, isn’t it?” Now, who have idea to make it simpler?

If there no idea from the students the teacher tries to provoke the students by looking at the long repeated addition that they had, and asking ‘How many times you add the 5?’ (We expect that the students count the 5 and say 7 times) After that the teacher can say to the students ‘can I write the number of breads in 7 times of 5?’ and the teacher introduces the multiplication symbol “x” to the students which replaces the word “times”.

In conclusion the teacher can say to the students that they can represent the number of structured objects by using multiplication sentence, for example, we can say that there are 7 x 5 breads on the picture because there are 7 groups of 5 breads. After that the teacher can give the students worksheet as their practice.

C. Counting dolls

Learning goal: Students know distributive property of multiplication.

We want to make the students know about distributive property of multiplication. In order to do that, we gave the students pictures of dolls that arranged in the group of three in the five rows of rack (shelves). By using the context of doll store want the students realize that 5x3 can be solved by adding (1x3) and (4x3), or any combination of groups of three that add up to 5 groups.

Description of Activity:

In this activity the teacher shows picture of dolls in dolls store as shown in figure 6 below.

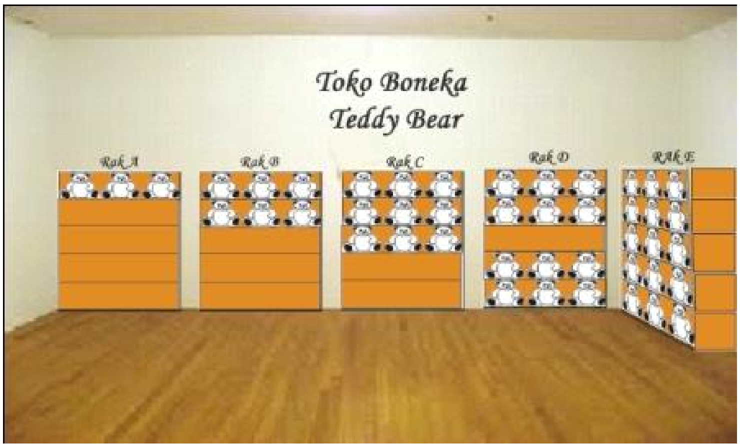


Figure 6 Dolls in Doll Store

The students are asked to make connection among the full rack of doll, the number of dolls in the rack, and the number of dolls in empty space in the rack. The teacher could write (5x3) = (1x3) + (4x3) in the whiteboard and asks the students to give their comment on it. If the do not have idea, the teacher can guide them by asking to the students the number of dolls in the Rak(Rack) E, and how they get it. We expect that some students could answer that there are 15 dolls by adding 3+3+3+3+3. After that the teacher asks the students to represent the number of dolls in full rack in multiplication sentence. When they are able to represent the number of doll in Rak E in multiplication sentence, the teacher asks the students to look at Rak A, and asks how many dolls in Rak A, if that Rak were full of dolls. How many dolls in that Rak and How many dolls they need on order to make Rak A full of doll and tries to put in multiplication symbol as (5x3) = (1x3) + (4x3). After that the teacher gives the students worksheet to discuss in their group.

In the worksheet the students are asked to investigate Rak B, Rak C and Rak D. They have to make connection among the full Rak, the number of doll in the Rak and the number of doll in empty space in the Rak to find others combination of group of three that add up to five groups. After students finished their tasks, the class discussion is held. The students are asked to present their idea in the class and together the class gives comment on it.

Conjecture of students' thinking and discussion:

When the teacher asks to the students how many dolls in the Rak, most of students might count the number of dolls, one by one and get the quantity of dolls in the Rak. Some students might see the structure of the objects and count the total doll by the repeated addition because they see the number of dolls in each row is same. From the repeated addition they expected to be able to transform it into multiplication sentence because they had experienced with transform the repeated addition into multiplication in the activity 2.

When students able to represent the number of dolls in multiplication sentence, they have to make connection among the number of doll in the full Rak, the number of doll in the Rak and the number of doll in the empty space of the Rak and give their conclusion about it.

D. Counting tiles

Learning goal: Students are able to interpret structured objects in rectangular pattern into multiplication sentence.

Description of Activity:

In this activity, the teacher gives instruction sheet to the students. Instructional sheet provides picture of a handyman tiles as shown in figure 7 below. The teacher tells to the students that the handyman tiles was working to install the tiles.

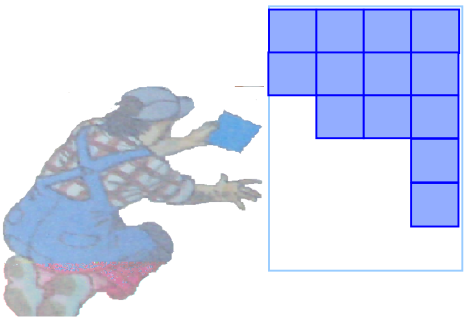


Figure 7. The handyman tiles was working to install the tiles

The students are asked to draw the complete installation of the tiles that he installs, to write their strategy to determine the total number of tiles that they draw and to represent the total number of tiles in multiplication sentence if they are able to do it. The students work in the group of 3 or 4. The teacher gives them a poster to make representation of complete tiles if Pak Toni finished his work. After they finish doing the tasks, the class discussion is held. Focus in the class discussion is representing the number of tiles into multiplication sentence.

Conjecture of students thinking and discussion:

The students are asked to draw the complete installation of the tiles. When students make their drawing they might come up with;

- Some students in their group might imitate the picture in the instruction sheets, and started to complete the tiles in row, or in column.
- Some students in their group might be completed the installation of installation in their instruction sheet as a model for them. After know how the complete installation looks like they might draw in their picture in their poster.
- Some students in their groups might draw directly 4 columns which is 6 tiles in each column because they had mental image of complete installation of the tiles in their head.
- Some students in their groups might draw directly 6 rows which is 4 tiles in each row because they had mental image of complete installation of the tiles in their head.

When students wrote their strategy to count the total number of tiles, the students might come up with:

- Some students might count the tiles one by one. These students do not use the structure of objects to do efficient count. They might also have difficulties to keep track of their counting.

- Some students might count the complete tiles by repeated addition because they know the number of tiles in each row/column is same. They might add
  - 4+4+4+4+4+4, when they counted in row, they might determine the total number of tiles by adding the 4 one by one or by regrouped the repeated addition that they made into 8+8+8.
  - 6+6+6+6, when they counted in column, they might determine the total number of tiles by adding the 6 one by one, or by regrouped the repeated addition that they made into 12+12.

When students represent the total number of tiles into multiplication from repeated addition that they made, they might come up with:

- Some students in their groups might add 4+4+4+4+4+4 and transform it into 6x4. These students know that there were 6 times of the 4 that they add.
- Some students in their groups might add 4+4+4+4+4+4 and transform it into 4x6. These students have difficulties to determine where they have to put the number of multiplier and the number of multiplicand in multiplication sentence.

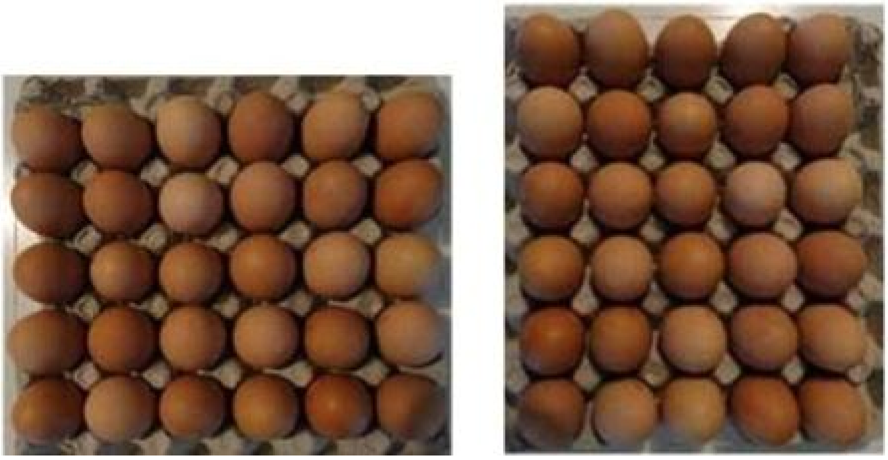
If some students count in column and others count in row, It would be two multiplication sentences that they get, 6 x 4 and 4 x 6. Those two multiplication sentences give them same product, 24. This is might be a conflict for the students, why this can be happen. Therefore the next activity is designed to give students more insight into commutative property of multiplication.

E. Counting eggs

Learning goal: Students know about commutative properties of multiplication.

Description of Activity:

In this activity, the instructional sheet is given to the students. The students discuss the tasks in the instructional sheet in the group of three or four. Instructional sheet provides two pictures of eggs in eggs carton as shown in figure 8 below.



(a) (b)

Figure 8 Two egg cartons

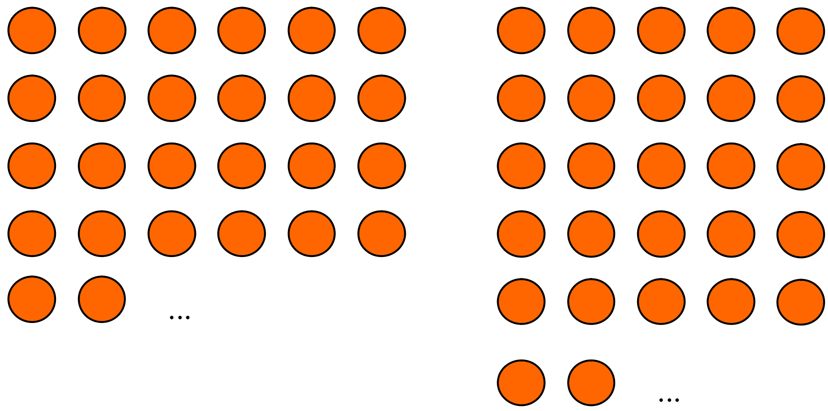
We ask the students, what they can say about those two pictures. After that we give the students tasks. The tasks for the students are:

1. Make representation of those two picture on their poster
2. Writing their strategy to count the total number of egg in the eggs carton (a and b).
3. Represent the number of eggs in multiplication sentence.

Conjecture of Students thinking and Discussion:

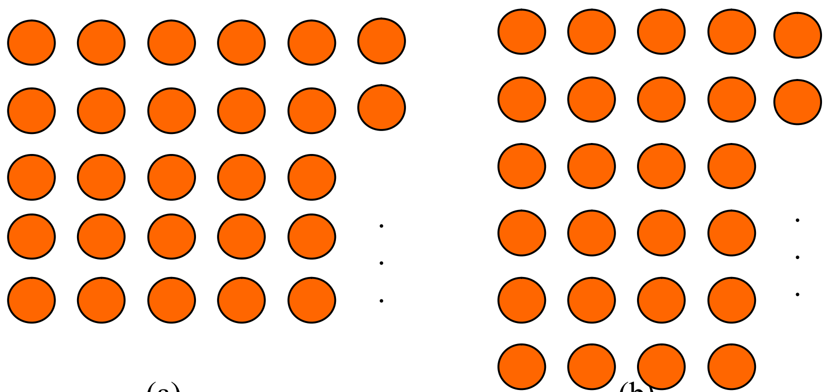
When student are making representation of the picture, the students might draw a circle that represents the eggs. The students might draw the eggs in the row, or in the column as shown in figure 9 below:

Students' representation of the eggs expected could provoke them to count in row.



(a) (b)

Students' representation of the eggs expected could provoke them to count in column.



(a) (b)

Figure 9 Conjectured of students' representation

To count the total number of eggs, the students might come up with:

- Counting the egg one by one. These students do not use arrangement of the egg in the egg carton. These students might have difficulties to keep track of their counting.
- Counting the egg by using the structure of eggs arrangement. The students already made representation of the eggs, when they are making representation we expect that they realize and use the fact that every row or column consist of the same number of egg. Therefore when they count they can count by group.

The way they count might be in row:

- Repeated addition. In picture (a), they know that there are 6 eggs in one row, and there are 5 rows, therefore they add 6 + 6 + 6 + 6 + 6, the way that they add might be one by one, or doing regrouped repeated addition, they add 6 + 6 which is equal 12 and then the add 12 + 12 and add 6 and at the end they get 30.
- Repeated addition. In picture (b), they know that there are 5 eggs in one row, and there are 6 rows, therefore they add 5 + 5 + 5 + 5 + 5 + 5. The way they add might be one by one, or doing regrouped repeated addition. They know that 5 + 5 is 10 and then they add 10 + 10 + 10 and they get 30.

The way they count might be in column:

- Repeated addition. In picture (a), they know that there are 5 eggs in one column, and there are 6 columns, therefore they add 5 + 5 + 5 + 5 + 5 + 5. The way they add might be one by one, or doing regrouped repeated addition. They know that 5 + 5 is 10 and then they add 10 + 10 + 10 and they get 30.

- Repeated addition. In picture (b), they know that there are 6 eggs in one column, and there are 5 columns, therefore they add 6 + 6 + 6 + 6 + 6, the way that they add might be one by one, or doing regrouped repeated addition, they add 6 + 6 which is equal 12 and then the add 12 + 12 and add 6 and at the end they get 30.

After the students finished counting the number of eggs, we expect that they know to represent the number of the eggs in multiplication sentence, because they already have experience to transform the repeated addition that they have into multiplication sentence. Because they get the same product of two multiplication sentence, we expected that they can make relation between those two multiplication product and can conclude that 6x5=5x6

After the students finish doing their tasks, the class discussion is held. The focus in the class discussion is how to interpret the number of eggs into multiplication sentence and their reasoning to conclude 5x6=6x5. We expect that by doing this activity the students realize that the number of eggs do not change when we turn around the carton on the picture.

F. Solving multiplication problems

Learning goal: Students are able to solve multiplication problems.

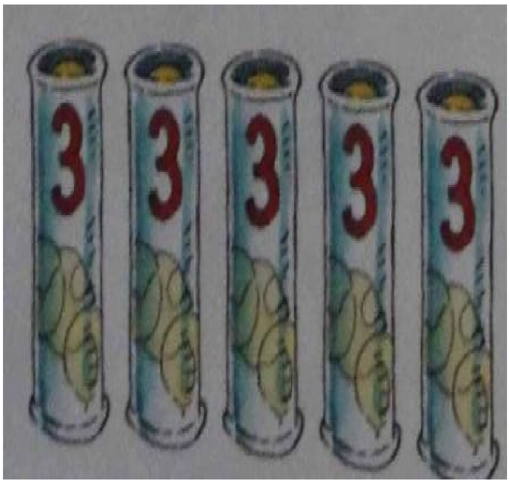
Description of activity;

In this activity we give the students worksheet. In the worksheet they should interpret the pictures that showed in the worksheet into multiplication sentences. They have to write their strategy to determine the total objects in the pictures. We designed the problems in such way that provokes the students to do efficient counting. For example; in the problems we give the students picture of three groups of 5 breads, and in the next question we also give them picture 6 groups of 5 breads. We expect the students not count the object by doing repeated addition (5 + 5 + 5 + 5 + 5 + 5) to know how many breads in the 6 groups of breads, but we expect them to relate the number fact that they already know, for example, they know that 3 groups of 5 breads are 15 and to know 6 groups of 5 breads they can make double of it and symbolized in multiplication (6x5) = (3x5) + (3x5). We also gave the students picture of 4 boxes of 3 balls, and 5 boxes of 3 balls, we expect when students had done 4 boxes of 3 balls the students just add three more to count 5 boxes of 3 balls and able to represent it into (5x3) = (4x3) + (1x3).

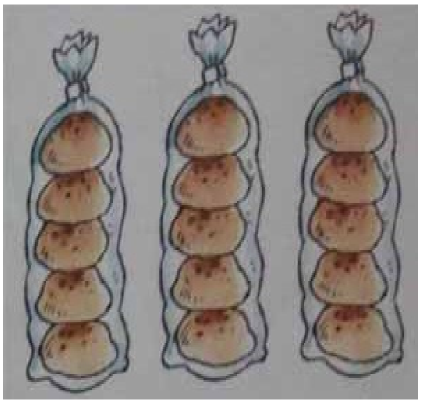
Figure 10 shows the pictures in the students' worksheet (Some Pictures are taken from Book Wis en Reken):

a.

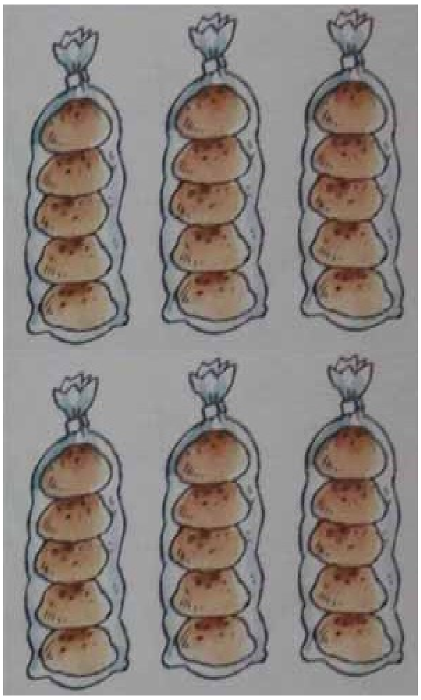
b.



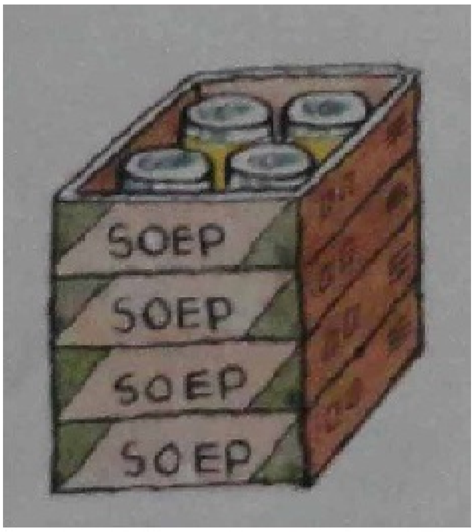
c.



d.



e.



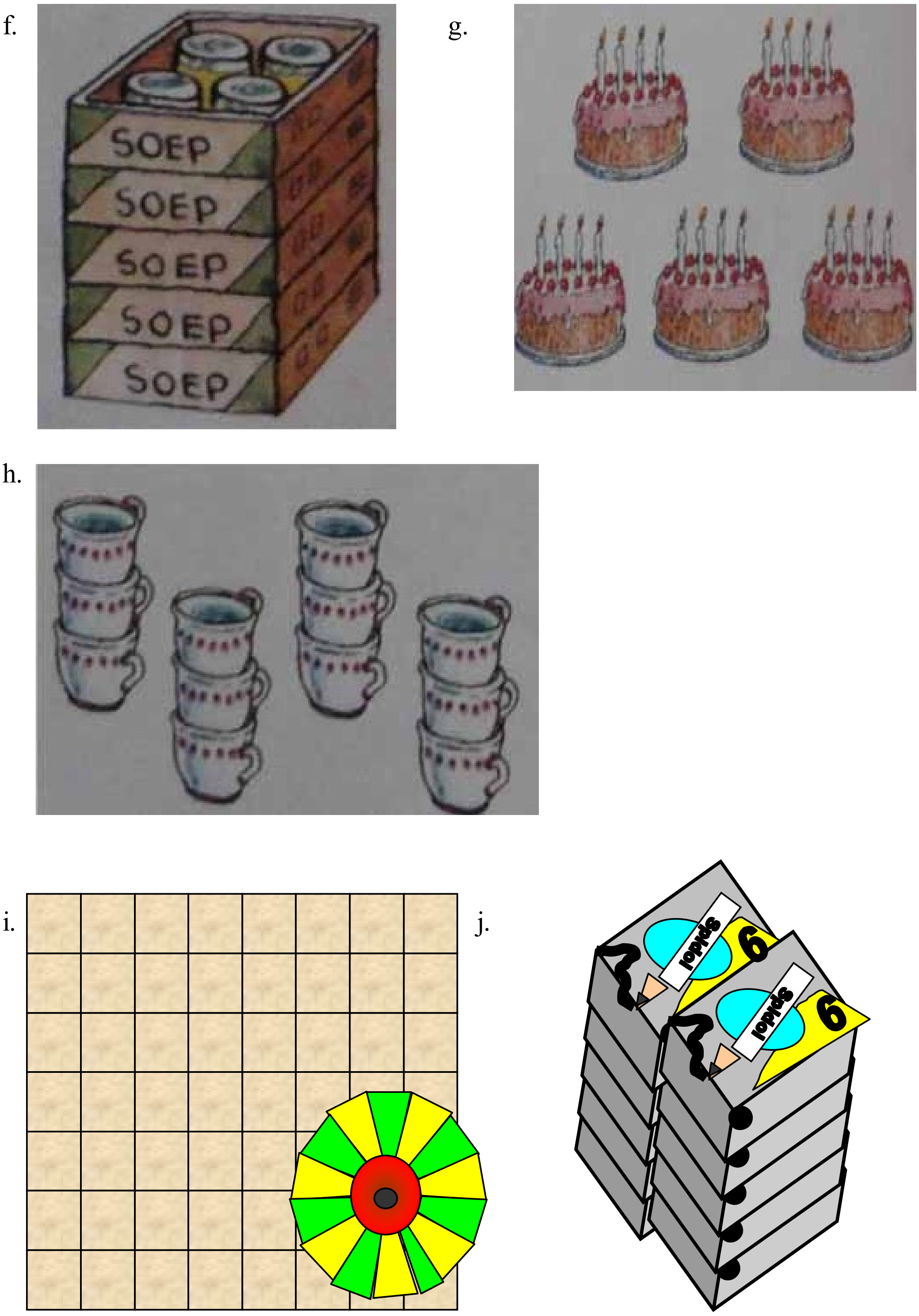


Figure 10. Pictures of structured objects in the students' worksheet

Conjecture of students thinking and discussion;

When students are interpreting the picture into multiplication symbols, the students might come up with;

- Some students might come up with multiplication symbol by seeing the number of bag/box/group and the number of objects in each bag/box/group. They transform it into multiplication sentence by put the number of bag/box/group as multiplier and the number of objects in each bag/box/group as multiplicand.
- Some students might come up with multiplication symbol by seeing the repeated addition that they made, they know it were 4 times of 5 (for example) and transform it into 4x5.

In determine the total objects in the pictures, the students might come up with;

- Some students might come up with repeated addition. (3 + 3 + 3 + 3 by adding the 3 one by one or regrouped the 3+3 into 6 and got result 12) for example.
- Some students might come with partial product. For example; in problem a, students already know that there are 4 boxes of balls, where in each box consist of 3 balls and the total of ball there is 12, or in formal way 4 x 3 is 12 then for b, they realize that there are 5 boxes, and they decide to add 3 more because in one box consist of 3 balls or in formal way they can symbolized it as 5 x 3 = (4 x 3) + (1 x 3).
- Some students might come up with doubling, for example in solving problem d and j. For example in problem d, students already know the number of bread in three bags where in each bag consist of 5 breads. To

know how many breads in sixes bags they made double of it and transform it into multiplication symbol 6x5=(3x5)+(3x5).

After the students finished their tasks, the class discussion is held. The focus in the class discussion are the students reasoning why they can represent the number of objects in multiplication sentence and finds efficient strategy to count the product of multiplication.

Chapter V
RETROSPECTIVE ANALYSIS

Analysis of data collected from pre-test, the preliminary design experiment, the teaching experiment and the final assessment are discussed in this section. The result of this research is underlying principles explaining how and why our design works. Our hypothetical learning trajectory served as a guideline in the retrospective analysis to investigate and to explain students' thinking in learning multiplication in grade 2 elementary school.

A. Pre-Test

The purpose to do this test was to assess students' initial knowledge and ability, more specifically to know students' knowledge and ability about some idea related to the multiplication such as group, their skill and ability to count structured objects, and their knowledge about multiplication language such as times. We also interested to know whether or not the students can interpret the number of structured objects in multiplication sentence.

In Indonesia curriculum, multiplication are given to the students after they learned addition and subtraction up to hundred. They expected mastered on those two topics. The interviewed teacher said that the students already learned about addition and subtraction in grade 1 and first semester of grade 2 but some of the students still have difficulties in those two topics.

To assess students' initial knowledge and ability, we gave to the students 4 problems as follows:

Problem 1

In this problem, students were asked to arrange oranges that spread on the table as shown in figure 11. The students asked to arrange the oranges in order to make easier counting. They asked to make a drawing of their arrangement and gave reason for their arrangement. How they counted after made the drawing was observed.

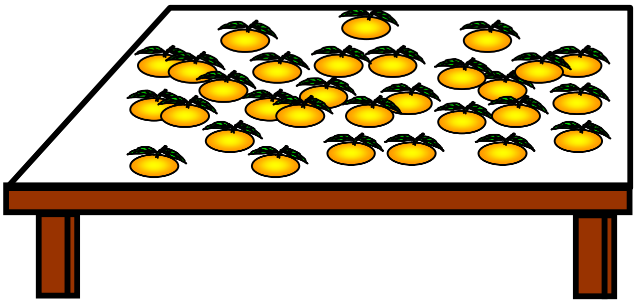


Figure 11. Orange that spread on the table

When we gave the problem to the students, all of students directly counted oranges one by one, some of them, 4 out of 25 students, directly put numbers on oranges in order to keep track of their counting. When they finished counting, they started to make their drawing. Most of the students, 21 out of 25 students, made scratch on the orange as the sign that they already moved the orange on the table to their picture as shown in figure 12.

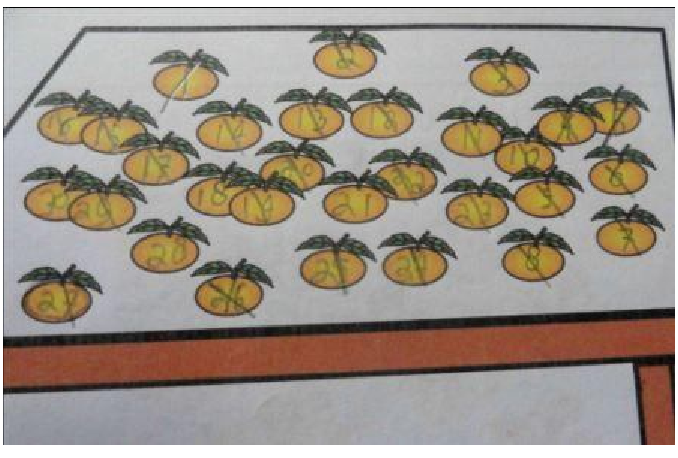


Figure 12. Students' put number on oranges and put scratch on it.

There was one student that missed to draw one orange on his drawing. It happened because he forgot to make scratch on one orange as shown in figure 13. From this student, we concluded that the students made their drawing by making one to one correspondence with the original picture.

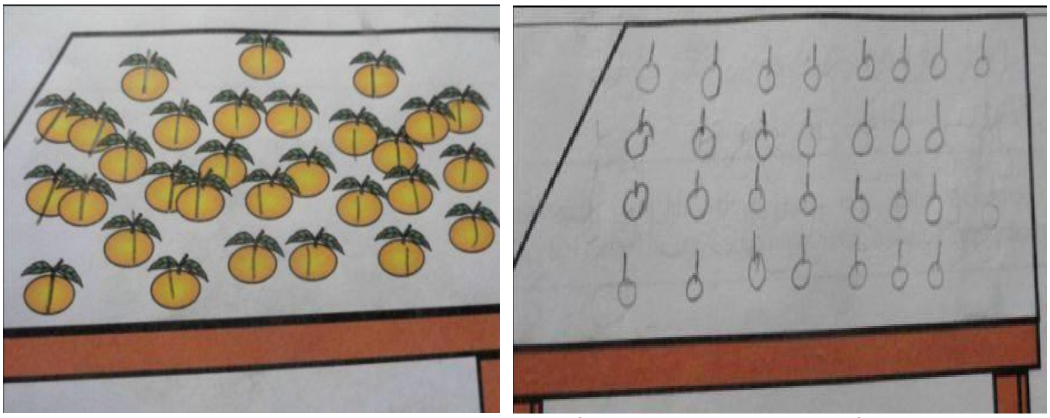
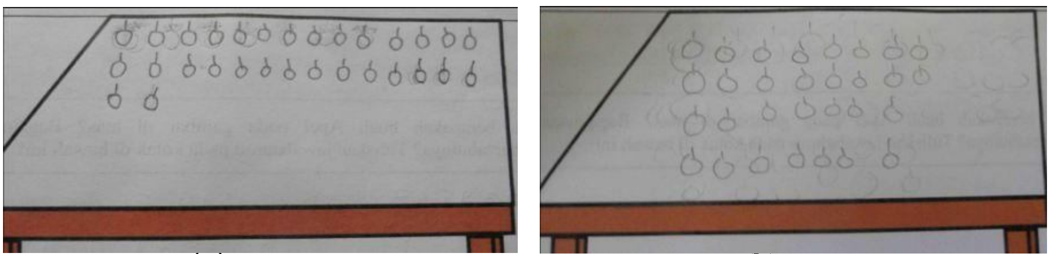


Figure 13. The scratch on oranges influence the drawing of the student

None of the students arranged the oranges in the group model as we expected, but they arranged the oranges in rectangular model. Some students arranged in row, they made drawing of oranges on the table until the edge of the table. After that they made the same arrangement for the second row until they had 30 oranges. Some students arranged the oranges in column or in row. They drew the oranges on the table until the edge of the table and made same arrangement for the second column until they had 30 oranges as shown in figure 14.



(a) (b)

Figure 14. Students' arrangement of oranges in row and in column
(a) student' arrangement in row, (b) student' arrangement in column

Some students arranged the orange in the 5 column of 6, some students arranged the orange in the 6 column of 5 as shown in figure 15 below

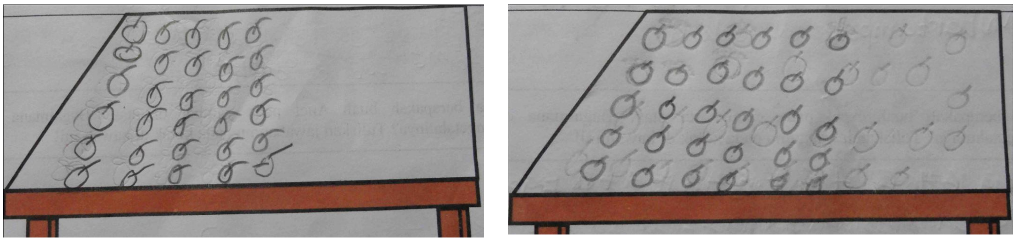


Figure 15. Students’ arrangement of oranges

When we asked students’ reason to make arrangement, most of students write to make easy to count, to make good view, and to be tidy.

Based on our observation, most of students counted the oranges one by one, after they arranged it. This shows that students did not use the arrangement that they made to count, like one of the students works’ that shown in figure 16 below.

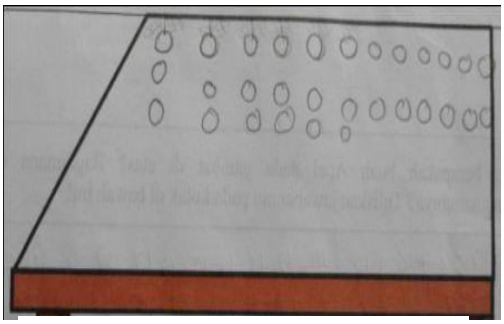


Figure 16.a Student’s drawing

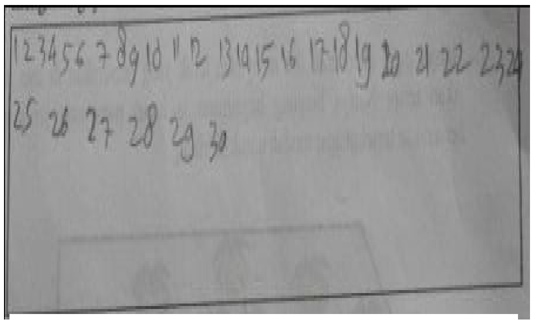


Figure 16.b Student’s way to count

Figure 16. Student’s works

This student drew three rows in her drawing, 2 rows consist of 12 oranges and one row consists of 6 oranges. We expected that the student did doubling and added 12+12+6 to determine the total number of oranges but she tended to count the oranges one by one.

Only some students, 6 out of 25 students, used the arrangement that they made, for example the one who made drawing of oranges in 6 rows of 5 oranges, He counted by five with adding 5+5+5+5+5+5 and they got the number of orange 30 as shown in figure 17 below.

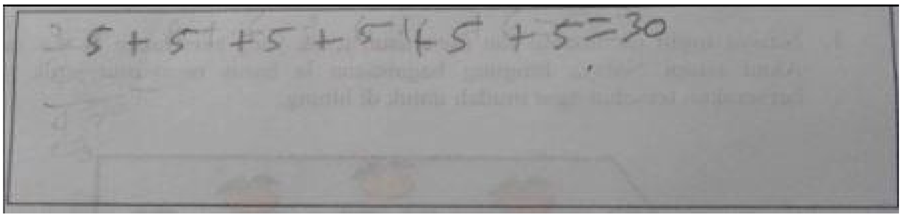


Figure 17. Students' counted the oranges by fives

From this problem, we concluded that students still tended to count the objects one by one. They tended to make their drawing by one to one correspondence with original picture that they saw. However, most of them did not have difficulties with the order of number/ordinal and cardinal aspects – the total amount of objects being counted which is indicated by the last number mentioned - of counting. Only some of students that use the arrangement that they made to make easier counting. They arranged the objects in row or in column. Then they counted the total objects by doing repeated addition by fives. In fact these students are grouping the objects in order to do efficient counting.

Problem 2

In this problem, students were asked to count groups of objects. We gave the students picture of 8 groups of 6 apples and asked the students what they saw on the picture and how many apples that they saw. How they determine the total number of apples was observed. The picture of apples as shown in figure 18 below:

Figure 18. 8 groups of 6 apples

The students gave various answers about the picture that they saw. Most of students described what they saw and their knowledge about apple such as "The taste of apples is delicious, I saw that those apples are still fresh and those apples have its wrapper, I saw apples arranged in good way, I saw apples with the red colour, apples have vitamins". Some students wrote about the arrangement of apples, such as 8 bags of apples where in each bag consist of 6 apples. There was also student wrote that 7 bags of apples where in each bag consist of 6 apples. This student skipped to count one bag of 6 apples.

Based of our observation, most of the students counted the number of apples one by one, they pointed to the apples on the picture and count it. By counting the numbers of apples one by one, students had to take care of the objects that they counted. They had to know which pictures that they had already counted in order to get the right total. Most of students that counted the number of apples one by one got the total number of apples on the pictures are 48 apples, while some of them got the number of apples on the picture are 42 apples. It shown that students have difficulties to keep track of their counting, they skip one bag of the apples as shown in figure 19 below.

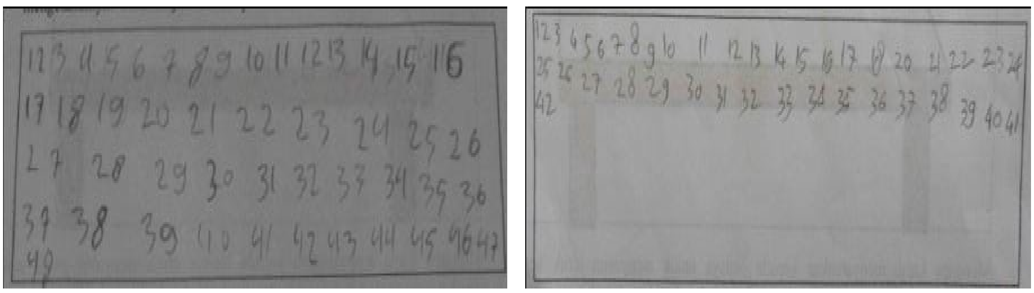


Figure 19. Students' counting the number of apples one by one

There were also students that used the arrangement of apples which is in the group of 6 to determine the total number of apples. Those students know that there were 8 bags of apples where in each bag of apples consist of 6 apples, therefore in order to determine the total number of apples they were doing repeated addition, 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6, like Daffa' work as shown in figure 20 below.

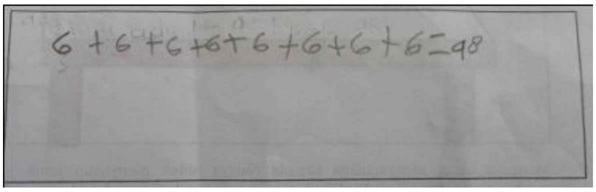


Figure 20. Student' doing repeated addtion

It was interesting to know how Daffa got 48. From his paper that shows in figure 21 below

Figure 21. Daffa' work

We know that Daffa tended to count the apples one by one rather than doing addition. He did not work with mathematics in formal level that suggests him to use symbol and add 6+6+6+6+6+6+6+6. But he tended to work with the contexts that we gave to him, he put number on the apples that represent the quantity of apples. We realized that it was natural since the students can see the objects and can count it one by one.

There was also student counted by group. He counted the Apples by 6. He knew that the number of Apples in each bag consist of 6 Apples and they were 8 bags of Apples then he did repeated addition to know the total number of apples as shown in figure 22 below.

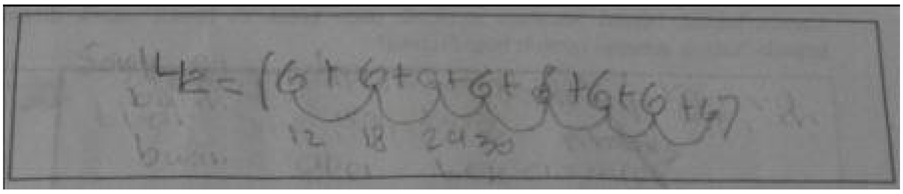


Figure 22. Student' add the 6 one by one

Figure 22 showed the student added the 6 one by one, but he had difficulty to keep track of his counting, the students miss one more 6 to add that make him got the wrong total.

There as also one student that counted the total number of Apples by group of 12. He saw 12 Apples as a group and counted the total by repeated addition of 12 as shown in figure 23.

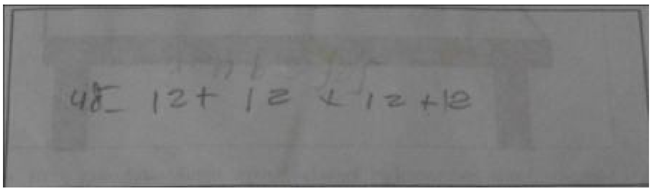


Figure 23. Student' got 48 apples by adding the 12

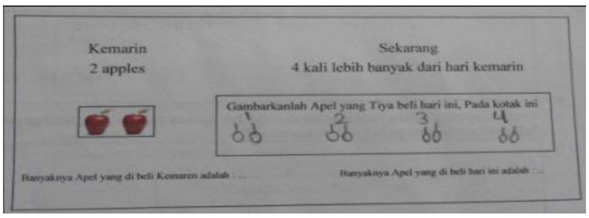
This student was regrouped the 8 bags of 6 Apples into 4 groups of 12 Apples to make easier counting for him.

From this problem, we concluded that most of students tended to count objects one by one, even the objects already arranged in group. They tended to tag the Apples on the picture as a sign for them that they already counted that Apple. Only some students counted by group, they counted the Apples by repeated addition or regrouped repeated addition.

Problem 3

In this problem students were asked to determine the number of objects 4 times as much as of certain objects. We interested to know about students knowledge about the word “times”. We asked to the students if someone bought 2 apples yesterday, and today she buys 4 times as much as yesterday. How many apples that the one buy today? To help students we gave them illustration, we asked the students to make draw 4 times as much as the number of apples that someone bought yesterday.

Most of the students, 19 out of 25 students, know that the meaning of times is the iteration of the unit. They know that someone buy 2 apples yesterday, therefore they have to draw 4 times as much as 2 apples. The students know the number of apples that they have to iterate was 2 apples. Therefore 4 times as much as 2 apples let them to draw 8 apples, even students write numbers above their picture or put the apple in the boxes to make they sure that they was really draw 4 times as much as 2 apples as we can see on figure 24 below.



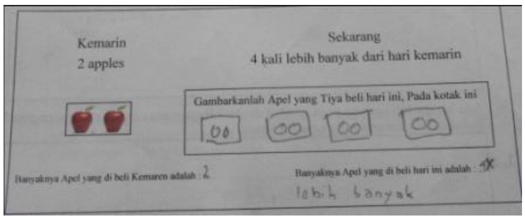


Figure 24. Students’ drawing of 4 times as much as 2 apples

Some of the students, 6 out of 25 students did not know what the meaning of times is. It can be seen from the students drawing. Some students only draw 4 Apples. These students did not know what 4 times as much as yesterday means. These students did not use the information that the one bought

2 apples yesterday. They did not know that the unit that they had to iterate 4 times. Therefore they just drew 4 apples as shown in figure 25.

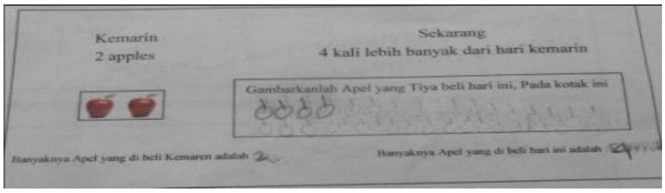


Figure 25. Student' drawing 4 apples as 4 times as much as 2 apples

From this problem, we concluded that most of students already know about the word "times". They knew that "times" means iterating the unit. Here they students had to unitize the objects. Some students still do not know the meaning of times. They did not know the unit that they had to iterate. Some of them just iterate one Apple, in fact the unit should be two Apples.

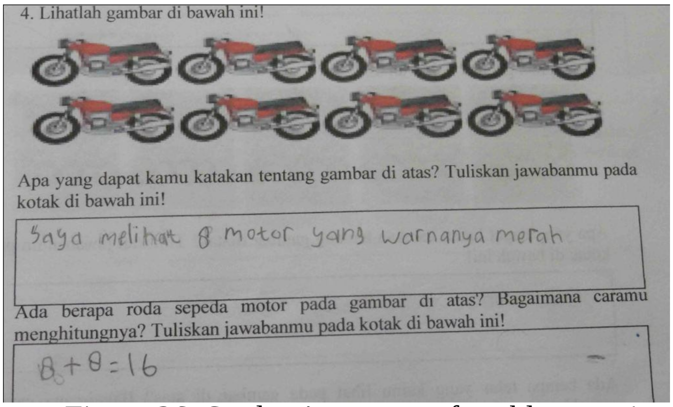
Some students did the problem very well. It shows from their answers that they tended to put two Apples in the box and put number above it. The two Apples became new unit for them and they had to iterate that unit four times because the problem asked them to make four times as much as two Apples. This situation shows that the students constructed one of big ideas in multiplication, unitizing.

Problem 4

This problem was about counting structured objects. In this problem we gave students picture of 8 motor cycles which is arranged in two rows, where in each row consist of 4 motor cycles. We asked them to write what they saw on the picture and we asked them to determine the number of wheels of all motor cycle on that picture.

When we asked the students what they saw on the picture, some students answered that they saw motor cycle, 8 motor cycles which has the red colour. When we asked them how many wheels of all motor cycles, 10 students answered 16, 10 students did not answer this question because they did not have time, and 5 students answered 2. The students, who had answered 2, did not realize that we asked to them the total number of wheels on 8 motor cycles. They thought that we just asked the number of wheels on one motor cycle.

Some students, 4 out of 18 students that got 16, got 16 from adding 8 + 8 as shown in figure 26. These students grouped the number of wheels of motorcycle into 8 and 8. They did it because they saw the number of wheels of motor cycle in a row and by did counting on till they had 16 wheels.


4. Lihatlah gambar di bawah ini!

Apa yang dapat kamu katakan tentang gambar di atas? Tuliskan jawabanmu pada kotak di bawah ini!

Saya melihat 8 motor yang warnanya merah

Ada berapa roda sepeda motor pada gambar di atas? Bagaimana caramu menghitungnya? Tuliskan jawabanmu pada kotak di bawah ini!

8 + 8 = 16

Figure 26. Student's answer of problem no 4

From the pre-test that we gave to the students, we concluded that the students did not have difficulties with counting the objects one by one, most of them knew the sequences of numbers. So they had good starting point to learn multiplication.

B. Preliminary Experiment

At this stage, all designed activities were tried out involving 28 students. The purposes to try out our design are to find out how this design works and to test our conjectures about students thinking and learning processes. The result of this preliminary experiment would give us feedback to improve our hypothetical learning trajectory.

Activity 1: “Describing structured objects” activity

In the first activity of the preliminary experiment, we tried out the “Describing structured objects” activity. The goal of this activity was to make the students able to describe the configuration of the objects that they saw. Through this activity we expect appropriate language that related to multiplication such as *a* ***bags of*** *b*, *a* ***boxes of*** *b* will appear where *a* is the number of groups and *b* is the number of object in each group. We expected after describing what they saw, the students will realize that the number of objects in each bag, box, and pack was same.

In this activity, one student was asked to make picture of what his/her friends said on the whiteboard. The student, who made picture, did not see the structured objects that we saw to others students. The others students who saw the objects will tell to their friend what they saw.

In this activity and the others 5 activities that we had designed, the teacher asked the students to work in group. The teacher divided the class into 7 groups. The names of the groups were Jeruk, Leci, Nanas, Durian, Apel, Mangga, and Anggur. This activity started by showing three bags of three pens to the students.

The following is a segment from our video and audio recording.

Teacher : Pak Farid (the researcher) will show you something. (Farid walked around the class and showed to the students 3 groups of 3 pens as shown in figure 27). Raised your hand if you want to speak!

Figure 27. Farid showed 3 groups of 3 pens to the students

Students : Pen

From the segment above, as our conjectured, students just said the name of the objects that we saw to them. They did not care with the structure of the objects or the configuration of the objects. Here, the role of the teacher was very important to provoke the students to see the structure of the objects. The following is a segment from our video recording.

Teacher	: Only pen? Or is that something than pen?
Student	: Three pens, I saw three pens.
Teacher	: Raised your hand if you want to speak! (Some students raised their hand) Nanas, do you want to speak? Durian, do you want to speak? (None of the students from Nanas and Durian groups raised their hand so the teacher asked those groups whether they want to speak or not, but no response from those two groups) Jeruk, you want to speak? (Aidil Rasyid from Jeruk group raised his hand). Ok, Aidil Rasyid, What did you see?
Aidil Rasyid	: Pen
Teacher	: Only pen? ok, Riska (the one that made picture in front of the class) please draw what your friend said.
Riska	: (Draw a pen) as shown in figure 28.

Figure 28. Riska was drawing one pen as her friend said

(after Riska finished made her drawing)

Teacher	: Is what Riska draw suitable with Aidil Rasyid said?

Students	: No.
Teacher	: I think Riska was correct, Aidil Rasyid said Pen and Riska draw a pen. But with the things that you saw is that correct or not?
Students	: No
Teacher	: Who said no? Ok, group Mangga, C'mon Kurni (a member of mangga group) what did you saw?
Kurni	: Three bags of pen

From the segment above, we can see that the teacher tried to provoke the student to see the structure of the objects by saying 'is that something than pen?' Together the students reacted by saying 'three pens'. Then the teacher tried to build classroom social norm - Socio norm refers to the expected ways of acting and explaining within interaction and negotiation between teacher and students - in the class that the students had to raise their hand if they want to share their opinion.

It was not easy to motivating the students to come to the structure of the objects. It shows from the video segment above that again, the students just said the name of the objects when the teacher asked their opinion about the things that they saw and the teacher let the student who made drawing, Riska, to draw what their friend said. Eventually, Riska just draw a pen, because she did not know how many pens that she had to draw.

The important question comes from the teacher to provoke the students to look at the structure of the objects by saying 'is that what Riska draw suitable with Aidil Rasyid said?' That question asked the students to look for the structure of the objects in order to make Riska could draw the objects that they saw correctly. Suddenly the students reacted with the question that the teacher gave. They tried to say in different way. Now, they said that they see the three bags of pens.

As our conjectured, the students would say that the number of bags or the number of groups that they saw, without telling the number of pens/objects in each group. This situation made their friend, Riska, had difficulties to decide the number of pens in each bags as shown in following segment of our video recording.

(Riska looks confuse to draw and she shows 3 of her fingers to the teacher, after that one of the students said 'draw the pen first')
Teacher : Three bags of pen, how many pen in the bag? (pointing to Kurni)
Kurni : Three (Kurni quiet for a while)
Kurni : Three, threes (then Riska directly draw her picture)
Other Students : Three, three, three
(Riska draw three pens, and the others students said the bag that asked Riska to draw the bag, Riska seem confuse to draw the bag, how the bag is and the teacher said you can draw the box then Riska draw a box with three pens in it and then stop)
Teacher : Are you finished? Three bags. How many bag that you draw?
(Riska quite for a while)
Students : 2 times more!
(Riska continue her picture and finish her picture)
(The teacher asked the students whether Riska picture is correct or not or same with the things that they saw, and all of the students said that it was correct and they agree with Riska's picture. Riska' work are shown in figure 29 below)

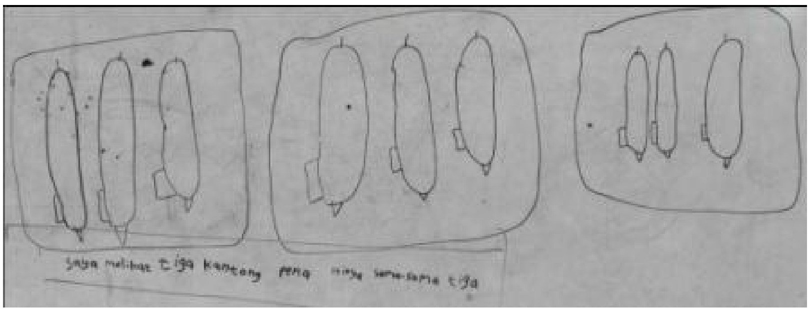


Figure 29. Riska's work

From the segment above, we can see that the teacher had difficulties to give motivation to the students to describe the number of bags and the number of pens in each bag simultaneously. Therefore the teacher tried to help the students by asking 'how many pen in the bag'. Then the students finally can give the structure of the objects to Riska by saying 'three, three, three' that makes Riska able to draw the objects well.

The students are expected to see the structure of the objects that arranged in the groups of three. By doing this activity the students constructed the idea of unitizing, where they had to describe the number of groups and the number of elements in each group to their friend who made drawing. Note Kurni's language. He said "three threes." He unitizes the unit. He saw there were three sets/groups of three pens. He has constructed the big idea of unitizing.

The video segment also showed that students already know the word "times". It was shown when Riska draw one bags of three pens, and look confused, her friends tried to help them by saying say 2 times more. It also shown that the students have feeling that the word "times" means iterating the unit. Here in this case the unit was one bag of 3 pens.

When Riska finished with their drawing, together the class concluded their description of objects that they saw. They concluded that they saw three bags of pens where in each bag consist of three pens. After that, the teacher challenged the students to do like Riska did. Now, Richi were asked to do the same thing like Riska. In that moment, three bags of 4 batteries are showed to the students.

The students did improvement to describe the objects. They did not say only the name of objects, but now, they tended to say the number of groups/bags of objects that they saw without saying the number of objects in each group/bag. The students directly told to Richi that they saw three bags of batteries and Richi directly draw three bags of batteries where in each bag consist of 3 batteries.

Richi thought that the number of batteries would be three, because the number of pens that Riska drew was also three. Therefore he made three bags of three batteries. When Richi finished with his drawing, the teacher asked the students whether Richi's drawing same with what they saw, and directly the students realize that each bag consists of four batteries. It made Richi added one more batteries in each bag and he wrote what his friend said to him as shown in figure 30 below.

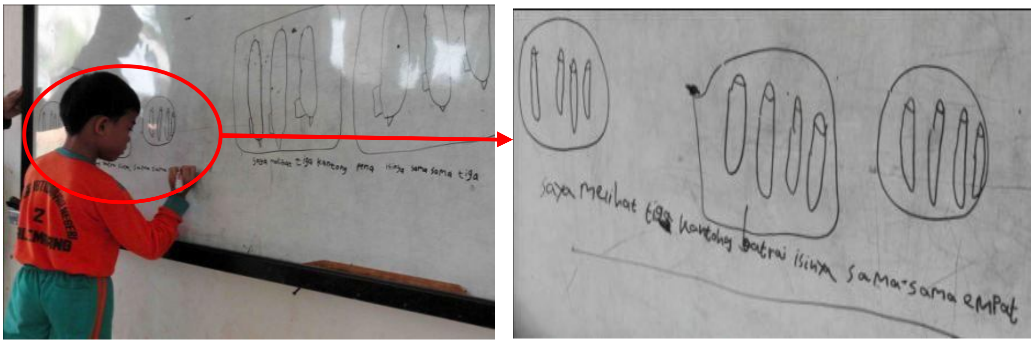


Figure 30. Richi was writing what his friend said to him

We also gave the students worksheet, after they finished 'describing structured objects' activity. From the students' worksheet, most of the students, 21 out of 27 students, can describe the number of objects that they saw as shown in figure 31,

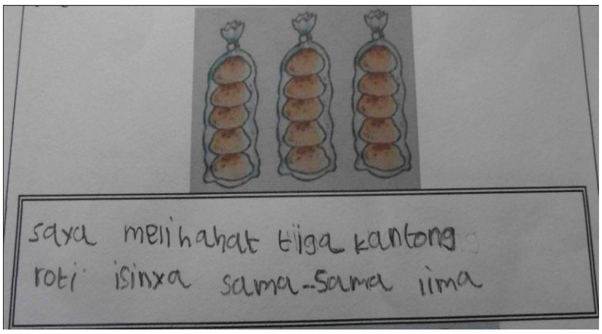


Figure 31. Student' work " I saw three bags of breads
where each bag consist of 5"

but some of the students, 6 out of 27 students, still had difficulties to describe the structured objects that they saw. They just wrote that the number of objects in each bag/box, but forgot to write how many bag or box that they saw as shown in figure 32 below

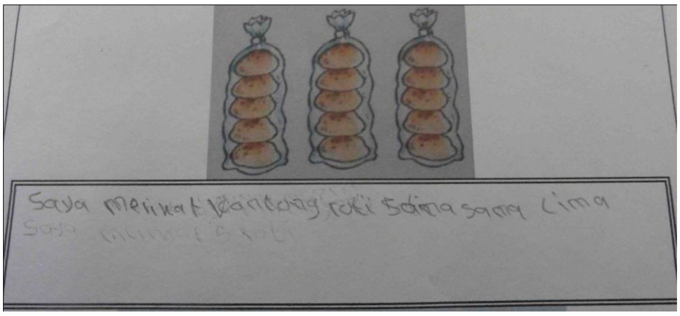


Figure 32. Student' work " I saw bag of breads where each bag consist of 5"

Throughout this activity, students were provoked to see the structure of the objects, the number of groups and the number of elements in each group. From their worksheet, we concluded that students were able to describe the number of structured objects, and they started to develop the language that related to multiplication, where they can see and describe the structured objects. They were able to say what they saw for example three bags of pens, where in each bag consist of 3 pens. The segment of our video and audio recording shown that the students constructed one of big idea in multiplication, unitizing, where in that segment we can see that the students tried to unitize the unit where they can say *three threes* which showed that they can see simultaneously three of pen as a unit and there were 3 group of that unit.

Activity 2: "Counting Structured Objects" Activity

We tried this activity to introduce multiplication symbol "x" to the students. Multiplication symbol "x" is introduced by connecting the idea "add so many times". The goal of this activity is to make the students able to represent the number of structured objects into multiplication sentence. In order to reach our goal, we showed to students two pictures, 7 bags of 5 breads and 9 groups of 5 oranges. We asked students to make representation of those two pictures. Our purpose to ask the students to make representation is to make students sure that the number of objects in each group was same and we expected that it will provoke them to add to determine the total. How the students count structured objects that we gave to them was observed.

As we predicted, some students made their representation not with one to one correspondence with the picture for example the bread, they tended to count the number of bread in one bag only, then they counted the number of bags and they started to make their representation. In the middle of their representation, to make sure, they counted the bags again and counted the bags on their poster and continued if they were not finished. When the students just counted the bag of bread it shown that they unitized the breads, The 5 breads in one bag, became one unit. Therefore in order to complete their representation on the poster, they just needed to count the bag and remembered in their head that each bag consists of equal number of objects.

Experiencing with previous activity, describing structured objects, we conjectured that students were able to describe the objects that they saw. We found all groups of students were able to describe what they saw. They stated the number of bags and the number of objects in each bag. However, for the oranges, they did not say in group, but they tended to say it in bag.

In class discussion the teacher tried to introduce the term 'group' to the students. The following is a segment from our video recording.

Teacher	: Look at the picture that your friends made (Mangga's work)? Is the number of breads and oranges in your friends' picture (students representation) same with the picture that I gave to you? Is that equal or not?
Students	: Equal
Teacher	: Is that the number of this equal or not? What is the name of this (Pointing the picture that the students made).
Students	: group
Teacher	: Do we agree with this, group?

Students : Agree
Teacher : Is the number of objects in each group same?
Students : Same.

The segment showed that the students knew the objects arranged in group, they agreed not to say in bag, but they agreed to say it in group where in each group consist of equal number of objects. Here they developed the language, from *a* bags of *b*, into *a* group of *b*.

After describing the structured objects that they saw, students had to count the total number of breads and oranges. As our conjectured, some students counted the number of breads and the number of oranges one by one in their drawing. These students did not use the fact that the numbers of objects in each group was same. We also conjectured that some students might count the objects by group and did repeated addition, or skip counting by fives or regrouped repeated addition. We found that some students counted the number of objects by group. They did skip counting by five as shown in following segment from our video recording.

Researcher : Ok, this one (pointing the repeated addition, 5 + 5 + 5 + 5 + 5 + 5 + 5, that the students made) how did you count it?
Students : by fives
Researcher : Show me please!
Students : (Pointing to the picture of bread that they made) five, ten, fifteen, twenty, twenty five, thirty, thirty five.

Some students were counting the number of objects by ten as shown in our following video recording.

Researcher : Ok Riska, Tried it (tried to count).
Riska : (Pointing to repeated addition, 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 that she made) ten (pointing to two of fives), twenty (moves her finger to the next two of fives), thirty (moves her finger to the next two of fives), forty (moves her finger to the next two of fives), (quiet for a while and said) forty fives.

From the students poster as shown in figure 33, and our observation none of the students use the fact that they already know as we expected, for example, they already count 5 + 5 + 5 + 5 + 5 + 5 + 5 which is equal 35, but to count 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5, the students started again to count the 5 from the beginning, they did not add two of 5 more to get result by using the number fact that they already count. That happened because the students counted different objects. It shows that the students still work in context. They did not work with mathematics in formal level that suggests them to add two of fives more.

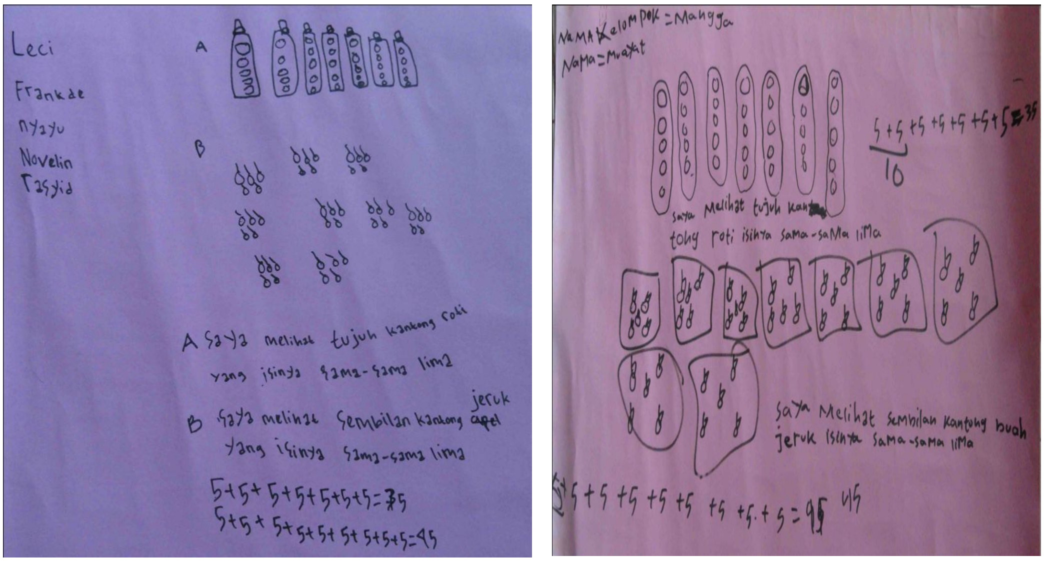


Figure 33. Students' posters

After counting long repeated addition, the teacher tried to introduce multiplication symbol to the students. The following is a segment of our video audio recording.

Teacher	: Look at this, how many the 5 here? (pointing the repeated addition 5 + 5 + 5 + 5 + 5 + 5 + 5 of Mangga's work)
Students	: 7 times
Teacher	: How many times?
Students	: 7 times
Teacher	: ok 7 times of the 5, How about this? (Pointing the repeated Addition, 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 of Mangga's work)

Students : 9 times
Teacher : 9 times? Right? (pointing the repeated addition, 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 and count with the students, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Teacher : How about if we have 50 times of 5?
Students : It will be hard teacher.
Teacher : In mathematics, we can use symbol to make our writing became simple. Ok, how many the five in this picture?(pointing, the picture of 7 groups of 5 breads).
Students : 7 times
Teacher : (Writing , 5 + 5 + 5 + 5 + 5 + 5 + 5) how many times the 5? Lets, count together, 1,2,3,4,5,6,7. in mathematics the symbols for times is “silang” (cross/”×”)what is the symbol for times?
Students : “silang”

From the segment the students already know the word “times”. The students can say that there were 7 times of the 5 and 9 times of 5. The teacher also give the emergent of multiplication symbol to the students by saying ‘how about if we had 50 times of 5’ and suddenly the students know it will be hard, bored and useless to write it down. Then the teacher introduced the multiplication symbol ‘×’ to the students.

At the end of the class, the teacher gave worksheet to the students. From their worksheet we concluded that the students did not have difficulties to represent the structured objects into multiplication sentence. It showed from their worksheet that most of the students, 24 out of 27 students are able to represent the structured objects into multiplication sentence as shown in figure 34 below.

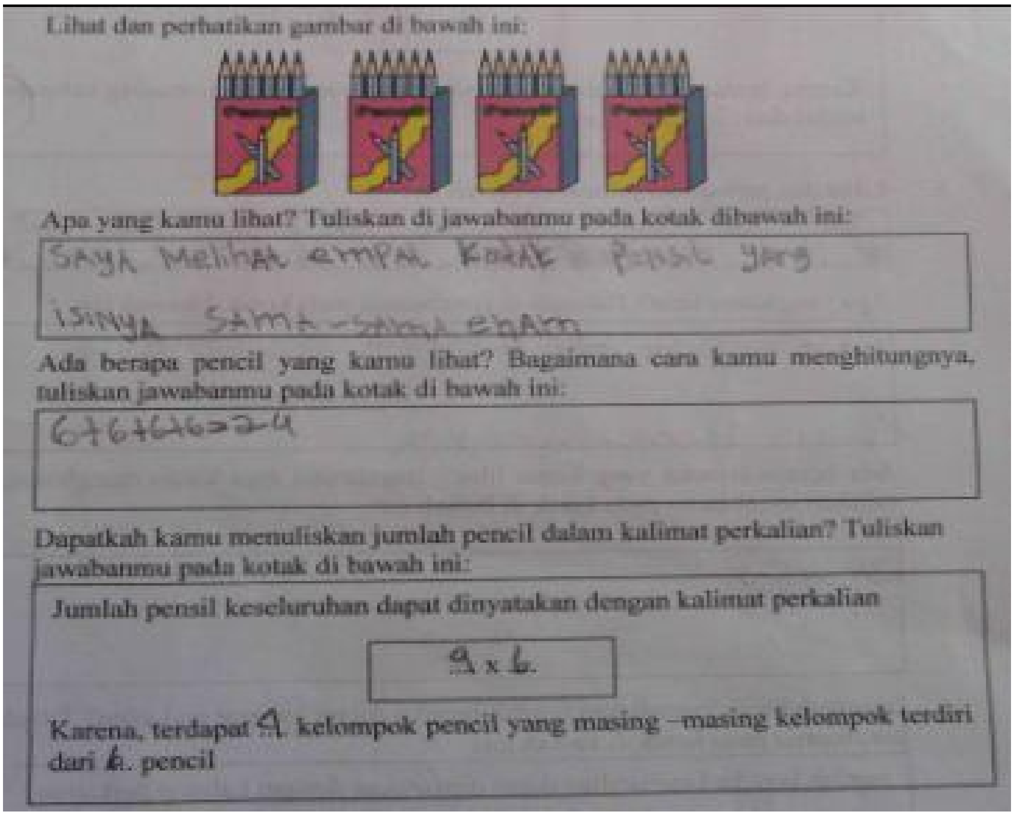
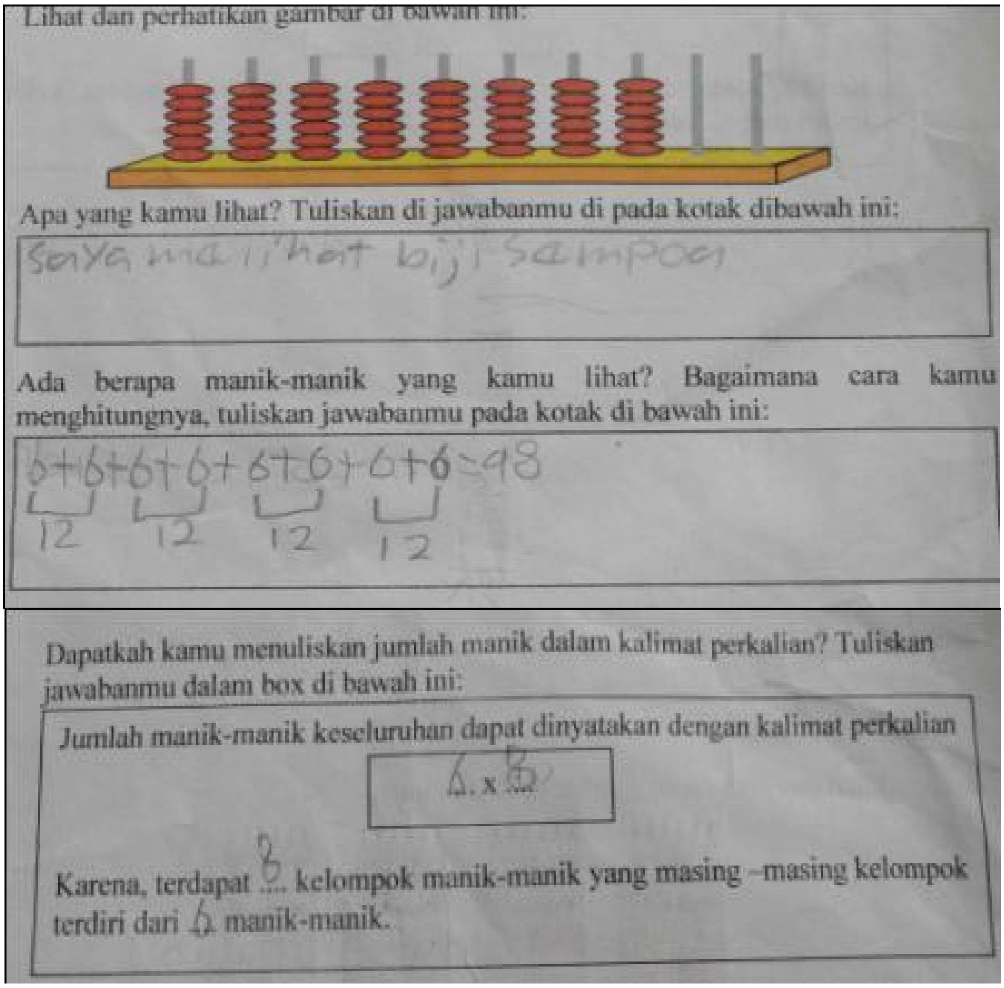


Figure 34. Student' worksheet that showed the student was able to represent the structured objects into multiplication sentence

Some students, 3 out of 27 students, have difficulties to represent the structured objects into multiplication sentence, as shown in figure 35 below. The student struggled to transform the repeated addition into multiplication sentence. They can represent the total number of objects by using repeated addition, 6+6+6+6+6+6+6+6, and regroup 6+6 became 12 and then adding 12+12+12+12 and got result 48, but when they had to put in multiplication sentence they transform the repeated addition into 6 × 8. Based on our observation, this happened because this student tended to put it in word, *enam-nya delapan kali* in Bahasa, the six are eight times, not like the things that we discussed in class, *delapan kali enam-nya,* 8 times of the 6.


Lihat dan perhatikan gambar di bawah ini:

Apa yang kamu lihat? Tuliskan di jawabanmu di pada kotak dibawah ini:

saya melihat biji sampoa

Ada berapa manik-manik yang kamu lihat? Bagaimana cara kamu menghitungnya, tuliskan jawabanmu pada kotak di bawah ini:

6+6+6+6+6+6+6+6=48
12 12 12 12

Dapatkah kamu menuliskan jumlah manik dalam kalimat perkalian? Tuliskan jawabanmu dalam box di bawah ini:

Jumlah manik-manik keseluruhan dapat dinyatakan dengan kalimat perkalian

6 x 8

Karena, terdapat 8 kelompok manik-manik yang masing –masing kelompok terdiri dari 6 manik-manik.

Figure 35. Student' worksheet that showed the student had difficulties to represent the structured objects into multiplication sentence

In this activity, our goal is to introduce multiplication symbol "×" to the students as another way to represent repeated addition. By giving the structured objects, the objects that already arranged in the group, students can see the number of groups and the number of objects in each group. When students had to determine the total, the structure of objects provokes students to count in group.

Repeated addition comes as a strategy for students to count the groups of objects. The class discussion and the language lead them to transform the repeated addition into multiplication. For example if there were 7 bags of 5 breads, to determine the total number of bread students count by group. They did repeated addition 5 + 5 + 5 + 5 + 5 + 5 + 5. Where the 5 is represent the quantity of the breads. The class discussions lead them to find another way to represent the repeated addition into multiplication. The word '*a* times of *b*' became a bridge for them to come to multiplication symbol '×'. Therefore they can represent the total number of objects into multiplication sentence 7×5.

Activity 3: "Counting Doll" activity

The purpose to do this activity is to introduce distributive property of multiplication to the students. We wanted the students had feeling that they can broke apart the multiplication sentence. In this activity we gave the student a picture as shown in figure 6.

The teacher started the lesson by asking the students to look at *Rak (the shelve) E* and asked how many row doll in that *Rak*. All of the students answer 15. As our conjecture, students know the number of dolls in the Rak by counting one by one and counting by group. Some students answer that they counted one by one and some of students knew by seeing the structure of the objects that arranged in the group of three. They knew that it was 15 dolls by adding 3 + 3 + 3 + 3 + 3. Together with they teacher, they represent the repeated addition that they had into multiplication sentence 5 × 3.

After that the teacher tried to connect the fact that the students knew with others dolls in *Rak* A.

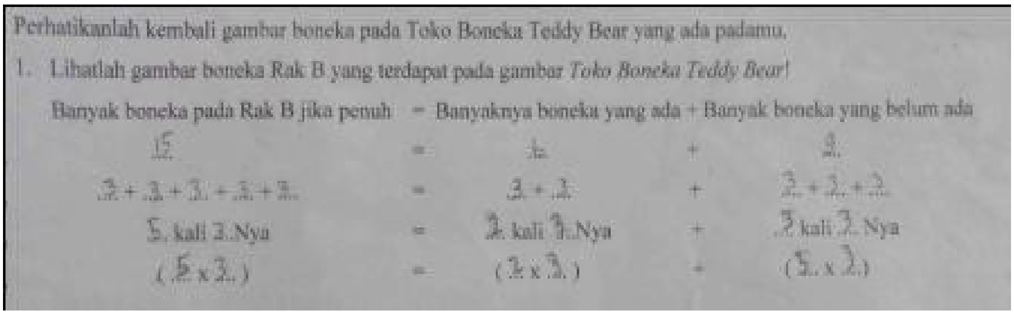
The following is a segment from our video recording.

Teacher	: Now, lets see the dolls in *Rak* A, if the number of dolls in Rak A full, is that same with *Rak* E?
Students	: Same.
Teacher	: Now, how many dolls in *Rak* A
Students	: three

Teacher : How many row that contains dolls in *Rak* A?
Students : one
Teacher : So, in order to make *Rak* A full of dolls how many dolls that we should put in *Rak* A?
Students : Twelve
Teacher : Now, here in *Rak* A, how many times of three in *Rak* A?
Students : One times
Teacher : So, we have (1 × 3), we add with the dolls that we need to complete the *Rak* A. How many rows?
Students : Four
Teacher : How many doll each row consist of?
Students : Three
Teacher : So, 4 times ….
Students : Four times three (the teacher wrote the symbol 4 × 3)

From that segment, the teacher tried to bring students to the idea that they could build the 5 × 3 from (1 × 3) and (4 × 3). They connected the number of dolls in each row, the number of dolls in the Rak and the number of dolls that they needed to make the Rak full of dolls, with the number of dolls if the Rak were full. Here the idea of part whole relationship involved.

For the next construction of 5 × 3 the students had investigated in the worksheet that we gave to them. Figure 36 showed the student' works in counting doll activity.


Perhatikanlah kembali gambar boneka pada Toko Boneka Teddy Bear yang ada padamu.

1. Lihatlah gambar boneka Rak B yang terdapat pada gambar *Toko Boneka Teddy Bear*!

Banyak boneka pada Rak B jika penuh	=	Banyaknya boneka yang ada	+	Banyak boneka yang belum ada
15	=	6	+	9
3 + 3 + 3 + 3 + 3	=	3 + 3	+	3 + 3 + 3
5 kali 3 Nya	=	2 kali 3 Nya	+	3 kali 3 Nya
(5 x 3)	=	(2 x 3)	+	(3 x 3)

Figure 36. Student' works in counting doll activity

The problem for that picture was that the students were asked to see Rak B, and they had to construct the (5 × 3) from the number of dolls in Rak B, and added with the number of dolls that they needed in order to make Rak B full of dolls.

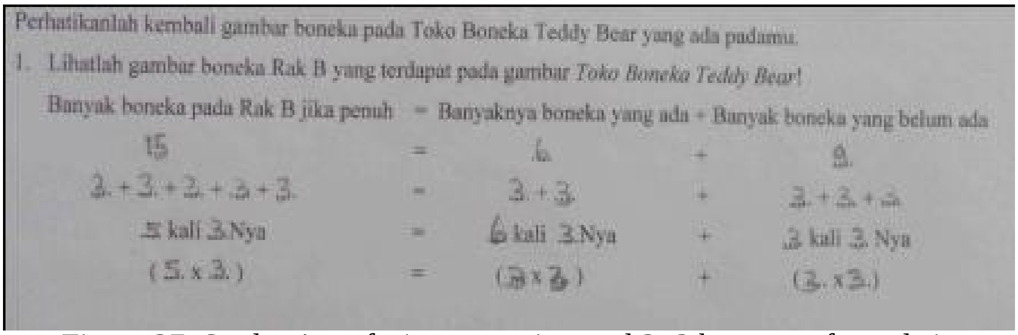
The following is a segment of our video recording that showed students thinking when she made the worksheet:

Researcher	: This rak (poiting to Rak B) if this full how many dolls that we have?
Student	: fifteen.
Researcher	: Why it was fifteen.
Student	: Yes, it was fifteen if it were full.
Researcher	: Why it was fifteen?
Student	: The way I got it!
Researcher	: Yes
Student	: (pointing to the dolls in row) three, three, three, three, three. Fifteen
Researcher	: So it the Rak B full of dolls it will be fifteen dolls, you got if from (pointing to the repeated addition that the students wrote) three,,,
Student	: Three plus three, plus three, plus three, plus three.
Researcher	: Now, how many times the three? (the student counting the three)
Student	: Five times of the three.
Researcher	: Why it were five times of the three?
Student	: (poiting to the repeated addition, 3+3+3+3+3 that she made and counting the three), one, two, three, four, five. (pointing to the word "fives times of the three/*5 kali 3nya*") This is fives and this is the three.

From the segment above, we can see that the student was tried to explain why she can put in word "the repeated addition, 3+3+3+3+3" into word "5 times of the three". She known that the fives is represent the number of the three and the three was the number that she added repeatedly. Therefore she can move to the multiplication symbol 5 × 3. The structure of the objects that arranged in the group of three provoked the students to count by group. She counted the number of dolls by three because in each row consist of three dolls.

She also explained how they got 2 × 3 and 3 × 3 that she made. She got the 2 × 3 because there were 6 dolls on the rak, and she knew by adding 3+3 which is she can put in word 2 times of the 3 and transform it into multiplication sentence 2 × 3 and she got 3 × 3 because she needed 9 dolls in order to make Rak B full of dolls. She got 9 by adding 3+3+3 which is she can put in word 3 times of the 3 and transform it into multiplication sentence 3 × 3.

Some students still had difficulties to interpret the worksheet that we gave to them. This happen because the equal sign (=) in the worksheet made the students confused. When they had to put the 3 + 3 in word, they wrote that 6 times of 3. They tend to add 3+3 that so they put in word 6 times of 3 as shown in figure 37. Therefore the equal sign will be erased in our teaching experiment.


Perhatikanlah kembali gambar boneka pada Toko Boneka Teddy Bear yang ada padamu.

1. Lihatlah gambar boneka Rak B yang terdapat pada gambar *Toko Boneka Teddy Bear*!

Banyak boneka pada Rak B jika penuh	=	Banyaknya boneka yang ada	+	Banyak boneka yang belum ada
15	=	6	+	9
3 + 3 + 3 + 3 + 3	=	3 + 3	+	3 + 3 + 3
5 kali 3 Nya	=	6 kali 3 Nya	+	3 kali 3 Nya
(5 x 3)	=	(3 x 3)	+	(3 x 3)

Figure 37. Student' confusion to put in word 3+3 because of equal sign

From the activity that we had done, we concluded that students can construct the multiplication (5 × 3) from (1 × 3) + (4 × 3) by helping of the structured objects that we showed to them. They were able to represent the number of dolls in the Rak, in multiplication sentence. The relation among full *Rak*, the number of dolls and empty space in the *Rak*, guided them to the distributive property of multiplication.

Activity 4: "Counting Tiles" activity

The goal of this activity is to make the students able to represent the number of structured objects that arranged in rectangular pattern into multiplication sentence. We showed to students the picture of handyman tiles who was working to install the tiles as shown in figure 7.

The students in their group asked to draw the complete installation of tiles, to determine the number of tiles on their picture and to represent the number of tiles in multiplication sentence. Our purpose to ask the students to draw the complete installation of the tiles in order to make the students had felling that the number of tiles in each row or each column was same. When they realized that the number of tiles was same in each column or rows, they expected to count the total number of tiles in groups.

Based of our observation, none of the group of the students directly draws 6 rows and 4 columns. Most of students made their drawing like the picture that we showed to them first and then they started to draw complete installed tiles. As we predicted, the students started to complete the installation of tiles in a column or in a row.

To count the total number of tiles, as we predicted, some students counted the complete installation of the tiles one by one. These students did not use the fact that the number of tiles in each row or each column was same. As our conjectured, some students also counted the total number of tiles by group. They counted in row or in column. They did repeated addition as their strategy to determine the total number of tiles. Therefore they can represent the total number of tiles in multiplication sentence because they had experienced to represent the repeated addition into multiplication sentence in activity two.

The discussion between the students in their group leads them to discuss their strategy to write in their poster. They decided to choose repeated addition as their strategy to determine the total number of tiles in their poster. As a result, 6 out of 7 groups counted the number of tiles by repeated addition, while one of groups count the number of tiles one by one, this group just draw incomplete picture like we shown to them and count the number of tiles that they draw. Some of their posters are showed in figure 38 below.

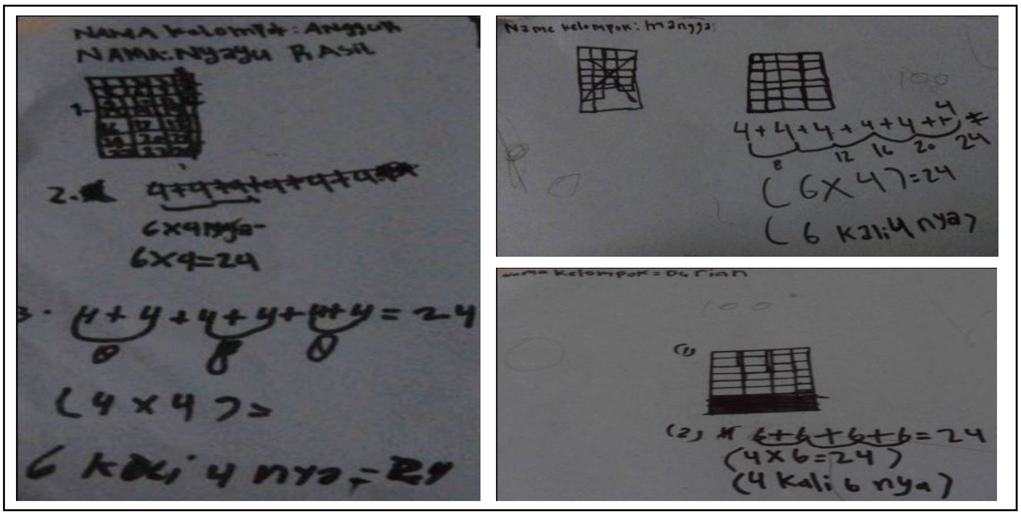


Figure 38. Students' works to represent the number of tiles in multiplication

From figure 38, we can see that the students counted the number of tiles in groups. They counted the number of tiles in a row or in a column and saw the number of tiles in a row or in a column as a group. Therefore the repeated addition arose as their strategy to determine the total. For example, Anggur group(the big one on the figure 38) and Mangga group (the small one in up side) counted the number of tiles by four. These students counted the number of tiles in row, they knew that the number of tiles in one row was four then they had 6 rows therefore they added 4+4+4+4+4+4. To determine the total Anggur group regroup 4+4 and they counted 8+8+8 by adding the 8 one by one, they got result 24 tiles. While Mangga group counted the repeated addition that they made by adding the four one by one. When they had to represent the number of tiles in multiplication sentence, they were able to do it because they already experienced it in the second activity. They wrote in symbol 6 × 4 and put in word 6 times of the 4.

The other group from figure 38, Durian, counted the number of tiles in column. They knew the number of tiles in a column was 6 and they had four columns therefore to determine the total number of tiles they did repeated addition, 6+6+6+6, they added and got the total number of tiles was 24. This group represented the number of tiles in multiplication sentence 4 × 6, and they put in word, 4 times of the 6.

Throughout this activity we can see that students were able to represent the total number of tiles in multiplication sentence. Furthermore they found two ways to represent the number of objects in rectangular pattern in multiplication sentence. Their activity to draw the complete installation of the tiles made them realized that the number of tiles was same in each row or column. When they had the complete installation of tiles, they can determine the total number of tiles. They counted the number of tiles in groups, the number of tiles in a column or in a row became new group. Then repeated addition come as a strategy to determine the total. Students experienced with second activity made them able to transform the repeated addition that they got into multiplication sentence.

Activity 5: "Counting egg" activity

After students know to represent the number of objects in rectangular pattern into multiplication sentence, in this activity they learned about commutative property of multiplication. The goal of this activity is to introduce commutative properties of multiplication that the product of multiplication $a \times b$ is equal with the product of $b \times a$. In this activity we gave the students picture of two eggs cartons, as shown in figure 8. The students asked to make representation of those two pictures on their poster, and determined the total number of eggs in those two pictures.

Based on our observation, before started to make their drawing, some students counted the number of eggs in row or in column. After that they counted how many rows of columns that they had. They directly draw the eggs row by row or column by column on their poster as we predicted. When they made their drawing, they were pointing to the egg in the row on in the column as a mark for them that they were working to draw it.

To count the total number of eggs, as our conjectured, students did repeated addition as their strategy. They counted the number of eggs in row or in column. From their poster, we found that some groups, 2 out of 7 groups, counted the number of eggs in column, like Durian's group as shown in the figure 39 below.

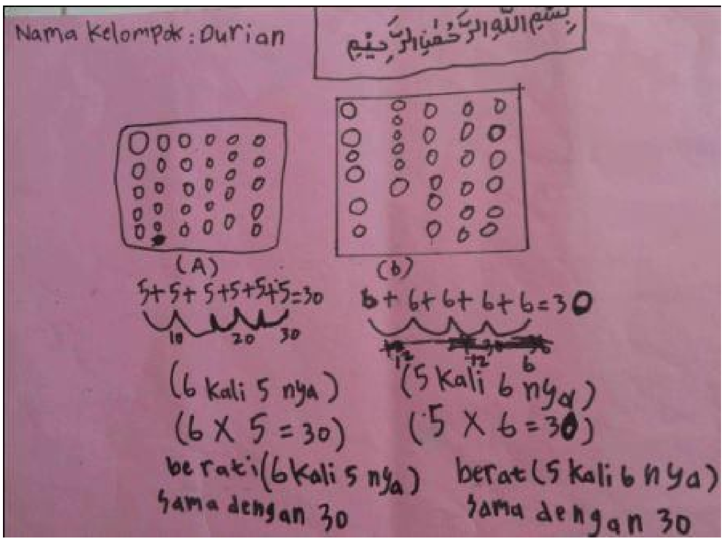


Figure 39. Students' works that they counted the number of eggs in column

From the figure 39, we can see that Durian group counted the number of eggs by group. They grouped the number of eggs in column. They did repeated addition as their strategy to determine the total number of eggs. They counted the number of eggs by regrouping repeated addition. They regrouped 5+5 for picture A and for picture B they regrouped 6+6, to determine the total number of eggs in picture A, they did skip counting by ten and for picture B, they added 12+12+6 and got result 30. They were able to put in word the repeated addition that they made, 6 times of the 5 for picture A, and 5 times of the 6 for picture B and put in multiplication symbol as 6 × 5 and 5 × 6. Durian group did not come to conclusion 5 × 6 = 6 × 5 that we expected. They can not connect the relation between two multiplication sentences that they made.

Some groups, 5 out of 7 groups, counted the number of eggs by group in row. They knew the number of eggs in each row was same, therefore to determine the total number of eggs they did repeated addition as show in figure 40 below.

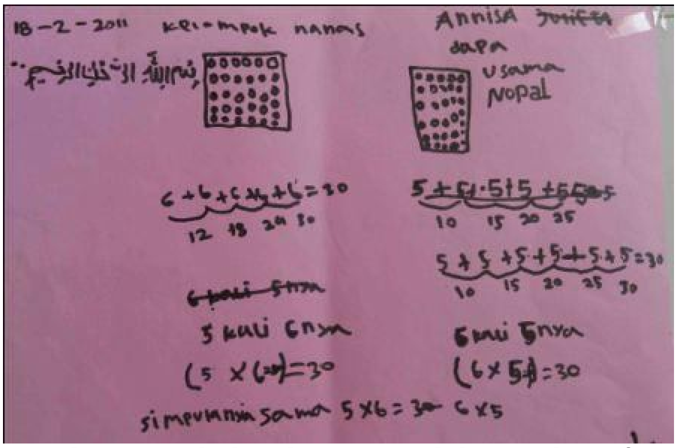


Figure 40. Students' work that they counted the number of eggs in row

From figure 40, Nanas's group did repeated addition as their strategy to determine the total. They did repeated addition 6+6+6+6+6 for picture A, and 5+5+5+5+5+5 for picture B. To determine the total number of eggs they added the 6 one by one for picture A, and they added the 5 one by one for picture B. They were able to put in word 5 times of the 6 and transform it into multiplication symbol 6 × 5 for picture A, and 6 times of the 5 and transform it into multiplication symbol 5 × 6 for picture B. From their investigation, they found that the total number of the eggs in the picture A and picture B was same, thirty. It makes them concluded 6 × 5 = 5 × 6. From data that we had only 4 out of 7 groups are able to conclude that 5 × 6 = 6 × 5 like Nanas did.

As our conjectured, students counted the number of eggs in group and used repeated addition as their strategy to determine the total. They did it because they knew that the number of eggs in each row/column was same. Two pictures that we showed to them gave them two multiplication sentence, 5 × 6 and 6 × 5. Those two multiplication sentences gave them same product. Therefore some group can conclude that 5 × 6 = 6 × 5.

Activity 6: “Solving multiplication problems” activity

In this activity we gave students pictures of structured objects as shown in figure 10. The students had to represent the number of objects in multiplication sentence and determine the product of the multiplication sentence. We gave them certain problems that let them to exploit the number relationship or to make relation one of the other multiplication sentences. For example, we showed to them picture of 4 boxes of 3 balls, and 5 boxes of 3 balls, we expected that the students would add 3 more to determine the total number of balls in 5 boxes of 3 balls because they already counted 4 bottles 3 balls that gave them result 12 balls.

Most of the students, 17 out of 26 students, did not have any difficulties to represent the number of objects to multiplication sentence. While the rest, 6 out of 26 students, still had difficulties to represent the number of structured objects in multiplication sentence. Their difficulties is to decide where they had to put the multiplier (the number of the group), and the multiplicand (the number of objects in each group) in multiplication sentence as shown in the figure 41 below.

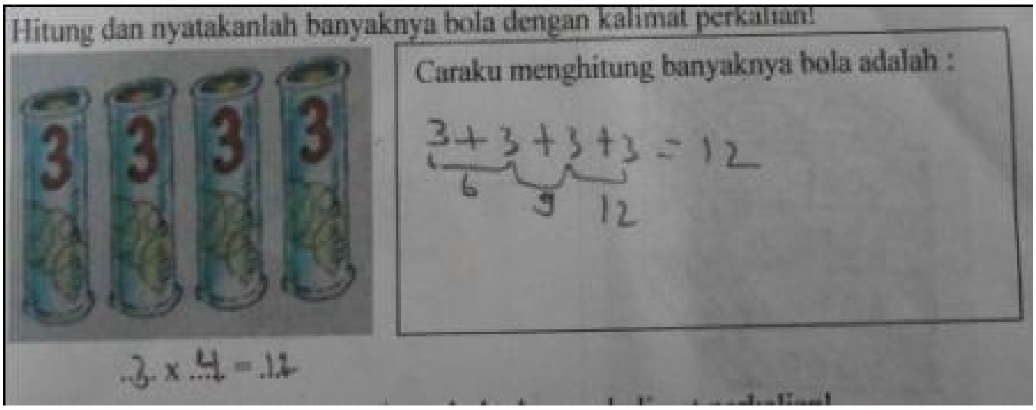


Figure 41. Students difficulties to represent the number of objects into multiplication sentence

As our conjectured, when students asked to count the product of multiplication sentence, students did repeated addition. Most of the students, 20 out of 26 students, calculated the repeated addition by adding the numbers one by one as shown in figure 42 below.

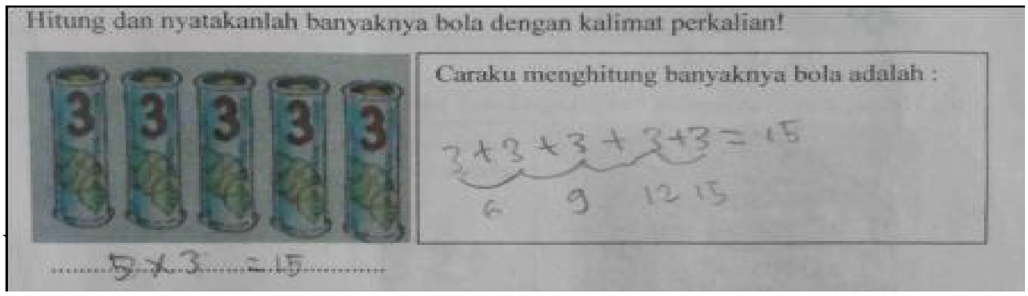


Figure 42. Students' added the 3 one by one

Some of the students, 6 out of 26 students, regrouped repeated addition by adding two numbers in the repeated addition as shown in figure 43 below.

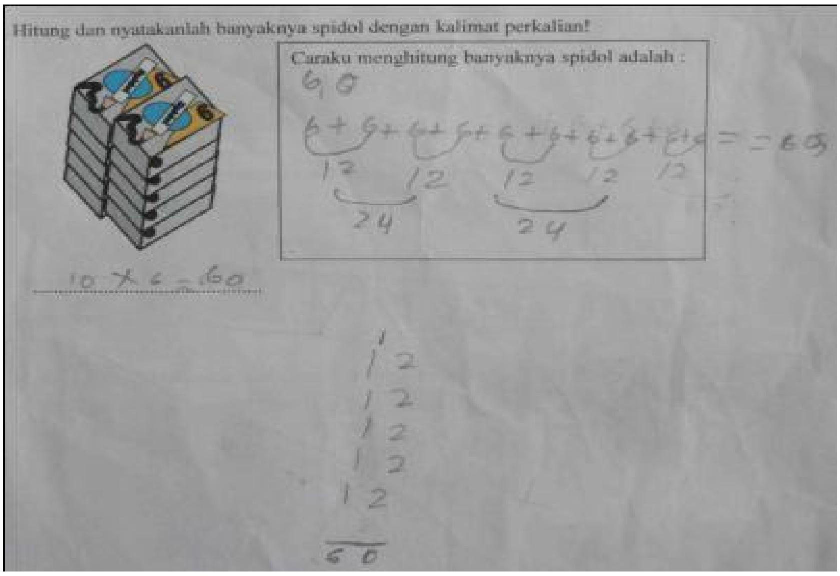


Figure 43. Students regrouped the repeated addition 6+6+6+6+6+6+6+6+6+6

As we showed in figure 43, the student regrouped the repeated addition, 6+6+6+6+6+6+6+6+6+6, into 12+12+12+12+12 and he regrouped again into 24+24+12. To determine the total number of the markers the students calculate 12+12+12+12+12 which gave him result 60.

There was only one student that can make relation of the multiplication sentences by helping of the picture. This student can split the long repeated addition that she made as shown in figure 44 below.

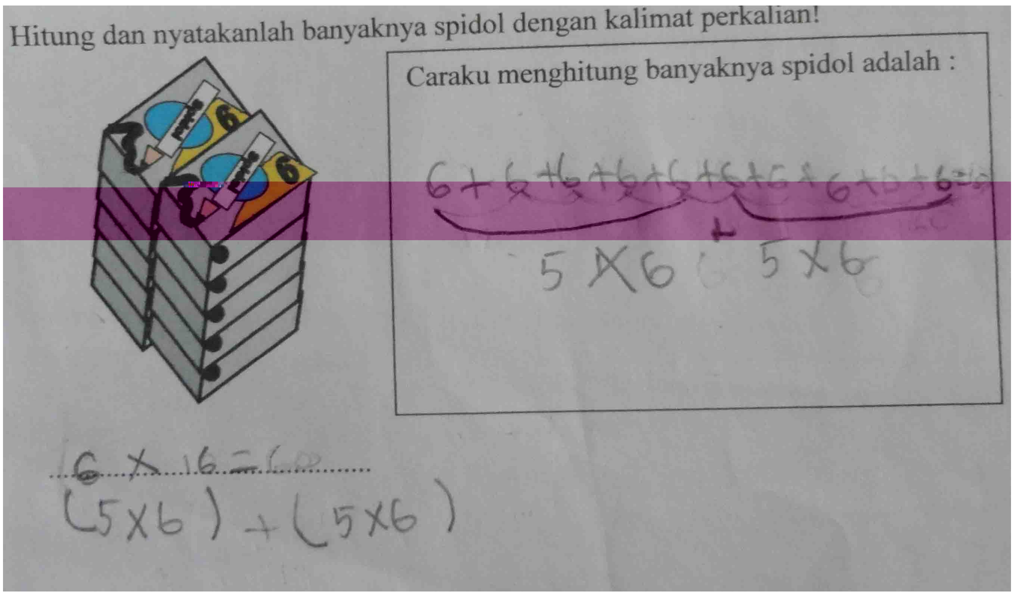


Figure 44. Student split long repeated addition

In figure 44, the student had feeling that she can break apart long repeated addition that she made. But she still confused to represent the total, she represented the total number of markers in that picture as 6 × 6. To know why she made 6x6 we interviewed her. The following is a segment of our video recording:

Researcher : Where did you got 5 × 6?
The student : (pointing to the one side of the box)
The teacher : Count it.
The student : (Pointing to the box in one side and count) one, two, three, four, five.
Researcher : You said, 5 × 6 this one(pointing to one side of the five boxes of markers, and this one (pointing to another side of the five boxes of markers. So, where did you got 6 × 6?
The student : (pointing to the two of number six in the picture)

From the interviewed that we had done with the student we know that the student wrote 6 × 6 because she influenced by the number 6 on the picture that put in one next to each other.

From the activity that we had done, we concluded that most of the students are able to represent the structured objects in multiplication sentence. They counted the structured objects with repeated addition and from the repeated addition they transformed it into multiplication sentence. As we predicted before to determine the total number of objects, students would count the group. They did repeated addition as their strategy to determine the total number of objects, some of them calculated the number that they added one by one and some of them regrouped repeated addition that they made. We expected some students can use the structured of the picture to do efficient counting but there was only one student can use the structured of the picture that we gave but still had misconception to represent it into multiplication sentence.

C. Conclusion of Preliminary Experiment

In the first activity, the observation showed that students were able to describe the number of structured objects that they saw. They developed the word that related to multiplication, such as *a bags of b, a boxes of b*. They saw the objects depends on the wrappers of the objects not depends of how the objects arranged. Therefore for the second activity we introduced to the students the term “group of” that more focus on how the objects arrange.

The task for the students in the first activity is to describe the objects that they saw and we found that the students did not have serious difficulties to describe the objects that they saw, therefore for the teaching experiment, beside describing the number of objects, we also interested to know how they counted the total number of objects that they saw. We interested to know, how students count structured objects that they described.

In the second activity, the observation showed that some students still count the number of structured objects one by one, it happened because they were able to count the objects one by one. Therefore for the teaching experiment we will give the students some pictures of objects in boxes where in the picture they can know and get information the number objects in one box in order to provoke them to count in group. We hope by doing this can provoke the students to do repeated addition.

Before understand why the distributive properties of multiplication works, the students had to know about multiplication sentence. We found that in this activity some students struggled to understand the multiplication sentence. Therefore we wanted to do the activity “counting dolls” after "counting eggs" activity because in those two activities the students were developing their knowledge about multiplication sentence.

In activity counting tiles, the students did not have serious difficulties for representing the number of objects in rectangular pattern into multiplication sentence. Therefore in this activity we did not make any changes.

In counting eggs activity we wanted to improve the picture that we gave to the students, the picture that we gave to the students was not same because of the editing, therefore we wanted to make sure that the picture was the same only one of the eggs carton turn around 90 degrees.

In solving multiplication problem activities, the class discussion did not goes well, therefore we wanted to adjust the time therefore it will be enough time for doing class discussion.

We discussed with the teacher about all changes we made. The teacher gave some suggestion and we tried to adjust initial HLT together. We will test this version of our HLT. We will see how the improved HLT works in the teaching experiment. Figure 45 showed the visualization of the improved HLT.

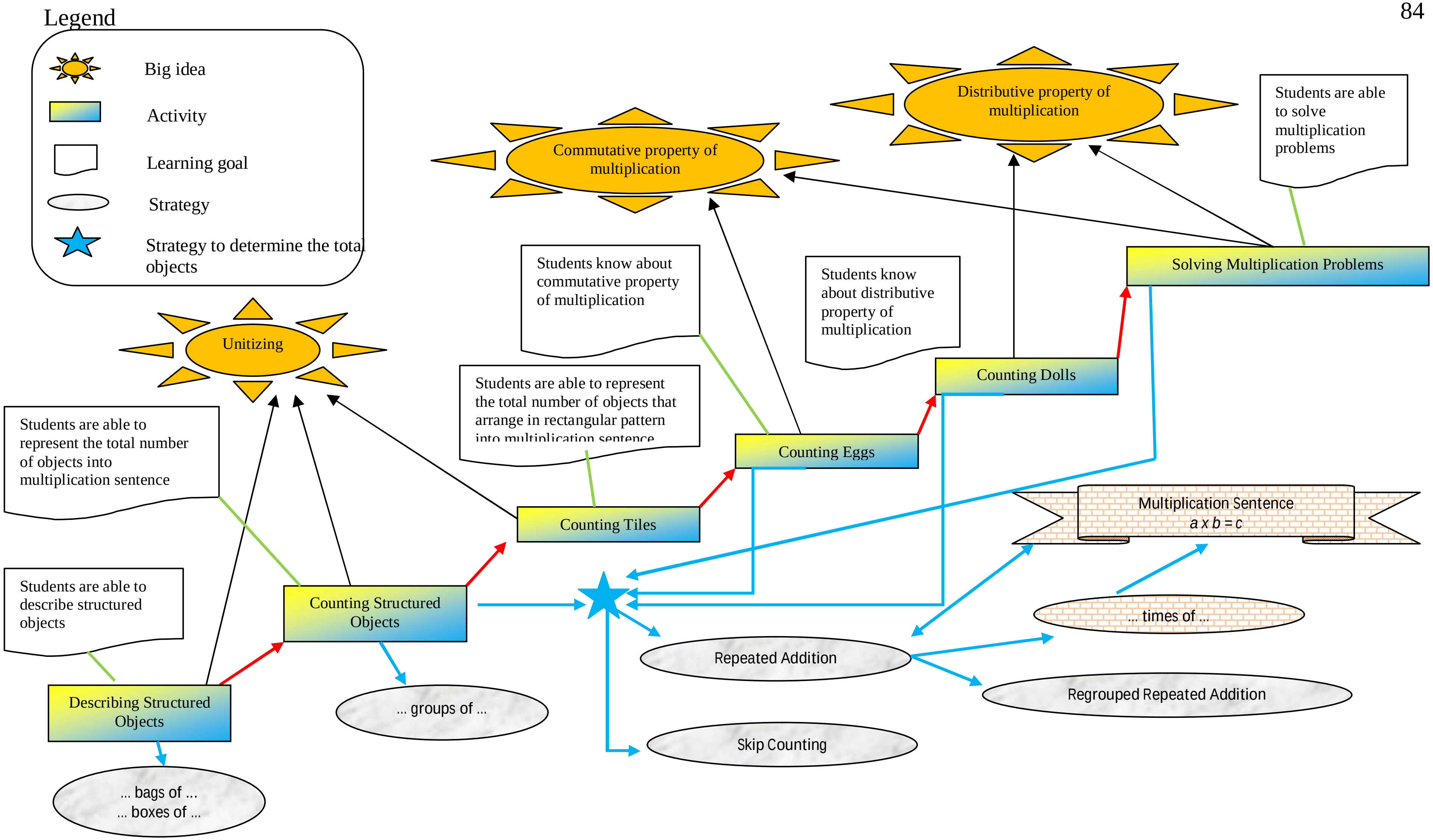


Figure 45. Visualization of the improved HLT

D. Teaching Experiment

This section compared our improved HLT and students' actual learning process during the experimental phase. We investigated how and if the HLT supported students' learning processes. In order to do that, we looked to the video recording and selected some critical moments, analyzed the students written works such as posters and worksheet. We also interviewed the students to know about their mathematical thinking. We analyzed everyday lesson in order to investigate what students and teacher do, how the activities work, and how the material contributed to the lesson. We also look for connections between the lessons and tried to find out how earlier lessons supports the following ones. The result of the retrospective analysis in this teaching experiment will be used to answer our research question.

Lesson 1: Describing Structured Objects

In the first lesson, we designed activities in which the students should describe and find their way to determine the total objects. One student was asked as a volunteer to make drawing what his/her friend said. The one who made drawing did not see the objects, but his/her friends saw the objects. We had shown to students 3 bags of 3 pens and 3 packs of 4 batteries. We expected that the students can describe the configuration of objects, how the objects are arranged. We also expected that the students can realize that the number of objects in each bag, pack, carton, and box is same. Therefore it can provoke them to count by group and used the repeated addition as their strategy to determine the total.

Motivating students to involve in learning process is an important part of learning. By giving a problem or conflict that they had to solve could encourage students to involve, therefore the teacher started the lesson by showing to the students a picture that consist of problem that students had to solve as shown in figure 46 below.

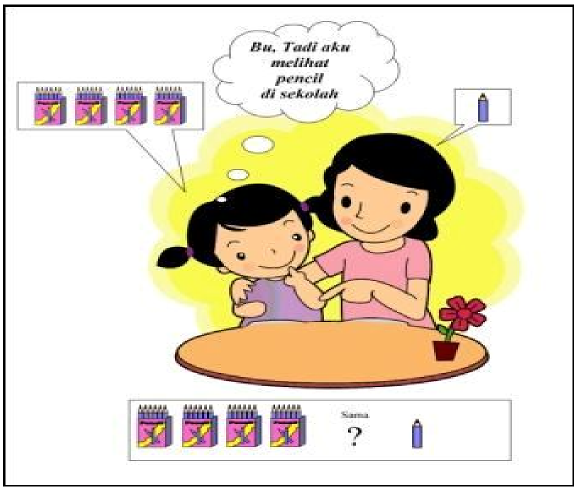


Figure 46. The picture of mother and her daughter.

In the picture 46, the daughter wanted to tell what she saw in the school to her mother. The daughter saw 4 boxes of 6 pencils. She said to her mother that she saw a pencil, then her mother imagine one pencil in their head. The picture tried to give conflict to the students by asking ‘is that what the daughter saw, same with what the mother think?’ which encourages students to describe the objects in details, they have to care with how the objects arranged.

When the students got insight of the problem that they had to describe the configuration of objects, the teacher showed the group of objects and let the students describe to their friend who can not see the objects. At that moment the teacher asked Ade to come to the whiteboard and asked him to make drawing of objects that his friend said. The following is a segment of our video recording:

Teacher	: (The teacher shown, three bags of 3 pens) Now, What I have here?
Students	: Pens, three pens, each three.
Teacher	: How many pens in one box?
Students	: Three

Teacher : (Because Ade looked confuse, the teacher repeats her question) Ok, What is this?
Students : Pens
Teacher : How many boxes of pens?
Students : Three
Teacher : How many pens in each box?
Students : Three.
Teacher : Ok Ade, Do you hear that? Now make the picture of it please!

Before started to make his drawing, Ade made note about what his friends said. He noted that there were three pens 3 boxes as shown in figure 47 below.

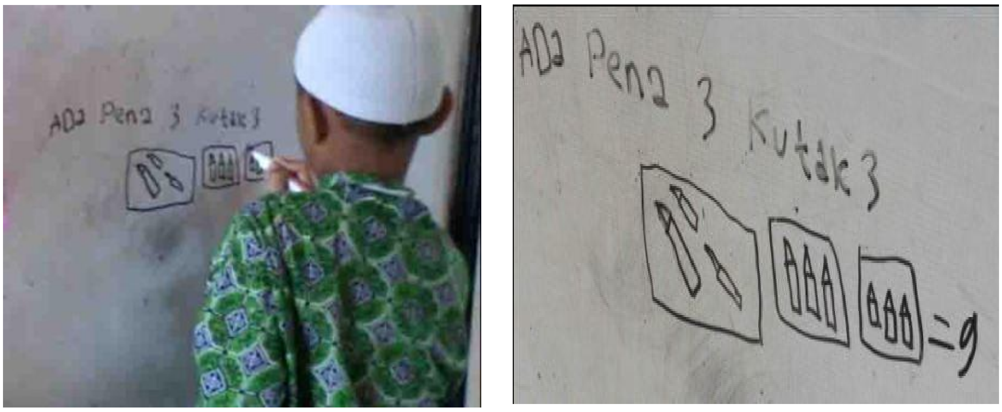


Figure 47. Ade's works

From the segment above, we analyzed that the students realized that the number of objects in each bag is same by saying each three. As our conjectured, First, Students just said the name of the objects that we gave to them. Students' words "pens, three pens, each three" showed that the students struggled to see the configuration of objects. As a result their friend, Ade confused to start his drawing. Realizing that it was difficult for Ade, the teacher tried to give help by asking how many pens in one box and how many boxes there are. After heard the number of groups of objects and the number of objects in each group, Ade made a note for that. He noted, "There were 3 pens, three boxes" and made the model of situation. He can model it well and when the teacher asked him how many pens are there, he directly knew that there were 9 pens, he did mental calculation.

Realizing that the students had difficulties to describe the number of groups and the number of elements in each group simultaneously, the teacher showed 3 bags of 4 batteries and asked the other student to make drawing like Ade did. At that moment, Shella wanted to make drawing of what her friends said. The students directly said to Shella that they saw 4 bags of batteries. After heard that Shella started her drawing, she made three bags of batteries, two bags consist of six batteries, one bag consists of eight batteries as shown in figure 48 below.

Figure 48. Shella was drawing three bags of batteries

When Shella finished her drawing, the teacher asked to the students "did what Shella draw same with the things that they saw?" All of the students answered "It was not same". Adjie, one of the students, raised his hand and said "four, four, four". Adjie come to the whiteboard and told to Shella, "I saw batteries four, four, four". Shella reacted with changes her drawing. She made each bag consist of four batteries.

Ajie, tried to help Shella by saying "four, four, four". Adjie gave Shella the structure of objects, 3 groups of 4. It seems that Adjie was constructed the idea of Unitizing. He tried to tell that there were three units of objects where in that unit consist of four objects. Therefore Shella can change his drawing because she knew how the objects arranged. When Shella finished, the teacher did her role as facilitator, she let the students to decide whether Shella's picture correct or not.

After Shella finished making her drawing and the class agreed with it, the teacher asked the students to determine the total number of batteries. All of the students answered 12, but the teacher wanted to know how the students got twelve, and one of the students, Adjie, showed his strategy he added 4+4+4 and got 12 as shown in figure 49 below.

Figure 49. Adjie got 12 by adding 4+4+4

From the learning process that happened in class that we described above, we concluded that it was not easy for the students to describe the structure of objects. They needed more practiced to see the group and the number of element in each group simultaneously. It shown by their answer when the teacher shown the three bags of three pens. As our conjectured, the students only said the name of the objects that we shown to them. They just said that it was pen. They did not care of the structure of the objects, how the objects arranged, but they are able to see that the number of objects in each bag of pens is same.

At the end of the lesson, the teacher gave the students worksheet. From the students' worksheet, 20 out of 28 students were able to describe the structure of the objects. They were able to describe the number of bag/box with the number of objects in each bag/box simultaneously. They wrote that "I saw six boxes of pencils where in each box consist of 6 pencils as shown in figure 50 below.

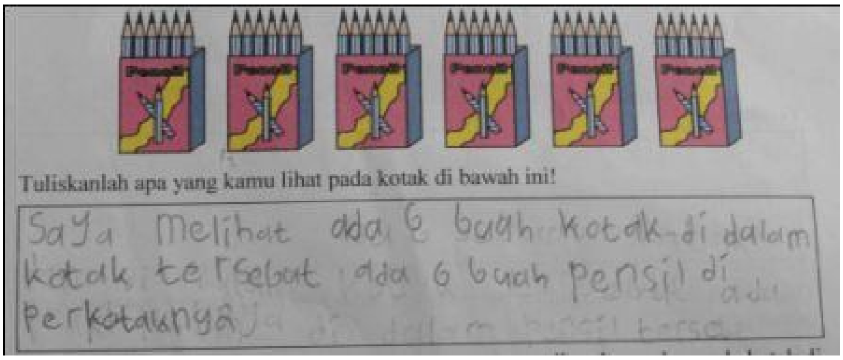


Figure 50. Student's work in describing the structured objects

While 8 out of 28 students still had difficulties to describe the structure of objects, one of the students just wrote that they saw box of pencils as shown in figure 51a, three students just wrote that they saw pencils in the boxes, each box consists of six pencils as shown in figure 51b, and four of students just wrote the number of bag/boxes of the structured objects without mention the number of objects in each bag/box as showed in figure 51c below.

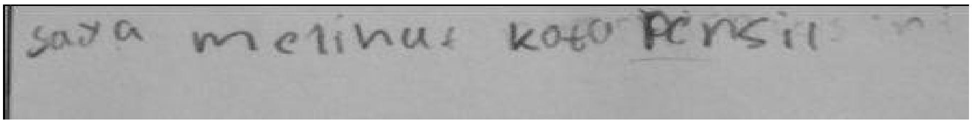


Figure 51a. Student wrote " I saw box of pencils

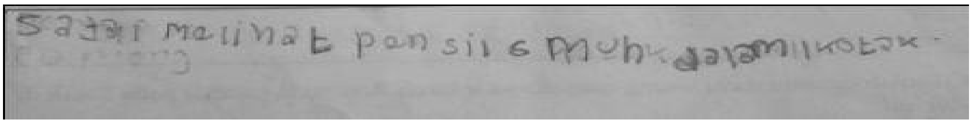


Figure 51b. Student wrote " I saw pencils in the boxes, each box consist of 6 pencils

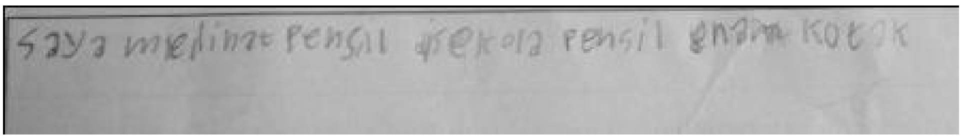


Figure 51c. Student wrote " I saw 6 boxes of pencils in school

Figure 51. Students' difficulties to describe the objects

Based on our observation from our video recording, there were several students' strategies to determine the total objects that showed on the worksheet.

Some students counted the number of total objects by counting on the number of objects one by one on the picture, they counted the objects by pointing the objects on the picture. Most of students counted in groups. They used repeated addition as their strategy to determine the total. They knew that each bag/box of objects consist of equal number of objects, therefore repeated addition arose, for example, Asyfa. She counted the number of breads by repeated addition, she knew that there were six bags of breads therefore she wrote 5+5+5+5+5+5, to determine the total number of breads she used 5 of her fingers and counting back her finger till she had 30. Hafisz also counted the number of structured objects by group and used repeated addition as his strategy, but his ways to determine the total different with Asyfa. Hafisz knew that there were 4 boxes of 7 markers, therefore he wrote 7+7+7+7. To determine the total marker, Hafisz used his fingers, he used 7 of his fingers and counting on her finger till he had 28. Asyfa's and Hafisd's works showed in figure 52 below.

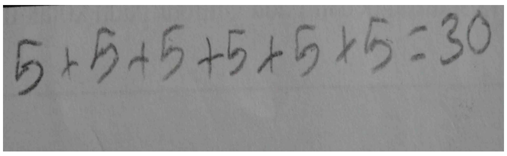


Figure 52a. Asyfa's work

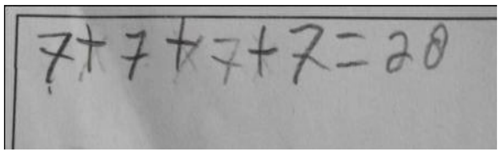


Figure 52b. Hafisz's work

Different with Hafizd and Asyfa, Ade, counted the number of breads by regrouped the repeated addition. When he counted 6 bags of 5 breads, he counted the number of bags in the worksheet, and he matched the number of bags in the worksheet with the repeated addition that he made. He made 5+5+5+5+5+5. After that he determined the total number of breads by regrouped 5+5 which is ten by pointing two of the fives, and counted ten, twenty, thirty.

Throughout this lesson, we concluded that describing the structured objects provoked the students to see the structured of the objects, how the objects arranged.

They had to describe the number of bags/boxes/groups and the number of objects in each bag/box/group simultaneously. They developed the language that related to multiplication such as ‘bags of’, and ‘boxes of’ and the idea of unitizing.

Seeing the structure of objects, how the objects arranged, provoked the students to count in groups and did repeated addition as their strategy because they knew and realized that the number of objects in each bag/box/groups is same. After the students know about repeated addition, the next activity designed in order to introduce multiplication symbol to the students as another way to represent the repeated addition.

Lesson 2. Counting Structured Objects

The students already learned to describe structured objects in activity one, they started to count structured objects that they described. This lesson was started with counting the structured objects. When students counted the number of objects by repeated addition, multiplication symbol would introduce to the students. Multiplication would introduce to the students by connecting with the idea “add so many times”.

The teacher started the lesson by giving tasks for the students. The students worked in the group of four. There were 7 groups in the class. They were Jeruk, Leci, Anggur, Mangga, Apel, Durian and Nanas. The students were asked to make the representation of 7 bags of 5 breads, and 9 groups of 5 oranges that showed in the instructional sheet in their poster. How they made their drawing was observed. They also asked to describe what they saw (7 bags of 5 breads and 9 groups of 5 oranges) and wrote their strategy to determine the total number of objects that they described.

Based on our observation, when the students made their drawing, the students directly drew the objects, they knew the objects arranged in group of five, therefore they made picture of 5 breads in the bags and 5 oranges in the group, in the middle of their drawing, to determine their drawing enough or not, they counting the number of bags on the picture and counting the number of bag/group in their drawing. We concluded that the students tended to count the number of bags/boxes of groups and remember in their head that the number of objects in each bags/boxes/groups is same that showed they constructed the big idea of multiplication, unitizing.

From the students' posters that we observed, all groups were able to describe the structured objects that they saw. They describe the number of groups and the number of objects in each group. It was because they already experienced to describe the structured objects from the first lesson that they got.

To determine the total number of breads, one of the groups, Nanas group, counted the number of breads by group and used repeated addition as their strategy. They wrote in their poster 5+5+5+5+5+5+5. They regroup 5+5 become 10 and counted ten, twenty, thirty, and thirty five. How the students counting shown in following segment of our video recording.

Researcher : How did you get the result? How many bread are there?
Student : Thirty five
Researcher : How did you get it?
Student : (Pointing to their picture of 7 bags of 5 breads and used his two finger pointing in two bags of breads and counting) ten, twenty, thirty, and thirty five(pointing to the last bags with one finger)

From the video segment above, we can see that the students tended to use their picture to count. They also tended to regroup the repeated addition that they had. They did it because they wanted to make efficient counting. It was easy to count by ten for them. It provokes them to regroup 5+5 became ten and did skip counting by ten to determine the total number of objects that they drew.

Interestingly, before the teacher introduce the multiplication symbol to the students, some students in their groups directly represent the repeated addition that they made into multiplication sentence, From 7 groups in the class, 4 groups already represented the repeated addition into multiplication sentence. They were groups, Leci, Apple, Durian and Nanas. Leci group and Apple groups already represent the repeated addition that they made correctly. They knew that the number of breads in the picture that they made are 5+5+5+5+5+5+5, then they can represent it into multiplication sentence because there were 7 times of the 5 and they can made the multiplication symbol 7 × 5. Their works showed in figure 53 below.

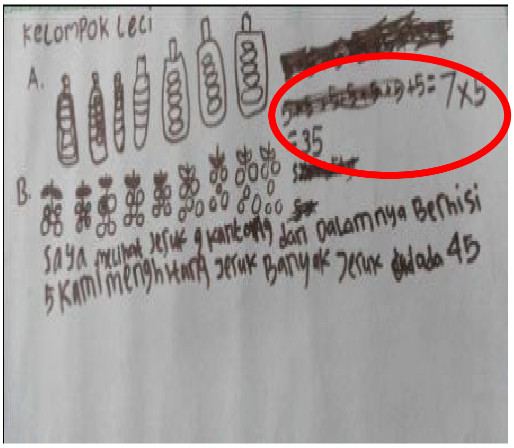


Figure 53a. Leci's works

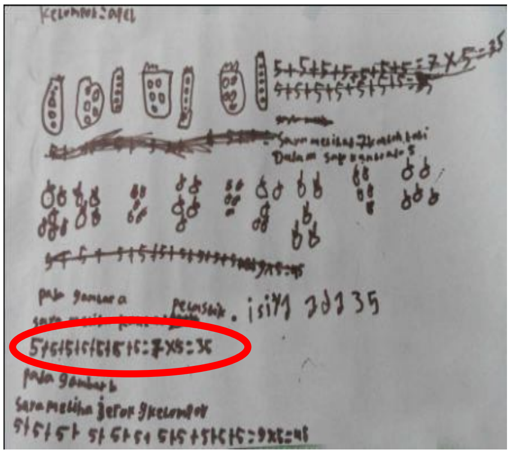


Figure 53b. Apple's works

We can see that Leci and Apel groups counted the number of breads using repeated addition, they made 5 + 5 + 5 + 5 + 5 + 5 + 5 = 7 × 5 = 35 in their poster. The following a segment from our video recording that showed Khadafi's reasoning from Leci group to transform repeated addition into multiplication sentence.

Khadafi : Yeah, teacher, for example

Teacher : What do you mean for example?
Khadafi : This is 7 (pointing to the 7 × 5 that he made), Nah,, there were 7 times of 5. Nah,, (showed five of his fingers) five plus (show another five of his fingers) five is ten, (showed up his two fingers) twenty, (showed up three fingers) thirty,,(showed up four fingers) fourty, (Showed up five fingers) fifty, eh,,,,(quite for a while and counted his 7 finger by ten and said) the result seventy teacher.
Teacher : Seventy, the oranges?

From the segment above, we can see that Khadafi, tried to explain their reason to write 7 × 5. He knew that the 'times' means iterating unit. He know that he iterate the 5 seven times, then he can say there was 7 times of 5 and made him able to transform into multiplication symbol 7 × 5, but he was still struggled to determine the total number of breads. He tried to regroup the repeated addition that he made. He regrouped 5+5 which is ten. Then he counted his 7 fingers by ten, so that they got result 70. It showed that He forgot that one of his fingers represent the 5. After conclude that there were seventy, one of the students from Leci group, Dasti, showed her disagreement. The following is a segment of our video recording.

Dasti : it ware not oranges teacher, it were breads.
Teacher : ok, the breads
Dasti : it were not seventy
Teacher : So, what is your result for that?
Dasti : I did not know.
Teacher : Really, let see,(pointing to one bag that the student draw) how many breads here?
Dasti : It were five there, (she counting the breads that their group made) (pointing two bags of breads in their drawing) Ten, (pointing anothre two bags) twenty, (pointing another two bags) thirty, and (pointing to the last bag of their drawing) thirty five.
Teacher : So, now what are you do not know?
Dasti : (Smile and said) thirty fives.

From the segment above, we can see that Dasti, know that their focus was to count the total number of breads. Therefore she disagreed with the teacher that said it was oranges. She also disagreed with Khadafi that told the number of breads was seventy, but when the teacher asked her opinion, she did not know. The important role for the teacher to provoke the students to count and to prove that Khadafi was not correct showed by asking to the students ‘So, what is your result for that’ and ‘let see, how many the breads here’. The statement ‘let see, how many the breads here’ remained the students that their looking for the total number of breads, therefore the student, Dasti, directly showed that that it was not seventy breads there, but it was thirty five breads that were proven by her counting. The two segments above also showed the usefulness of working in groups. When students working in groups, they can share their ideas, strategies and opinions, the disagreement between them lead them to discussion and together find the solution.

Some groups, 2 groups out of 4 groups that represent the total number of objects by multiplication sentences, Nanas group and Durian group had difficulties to represent repeated addition into multiplication sentence. Their works are shown in figure 54 below.

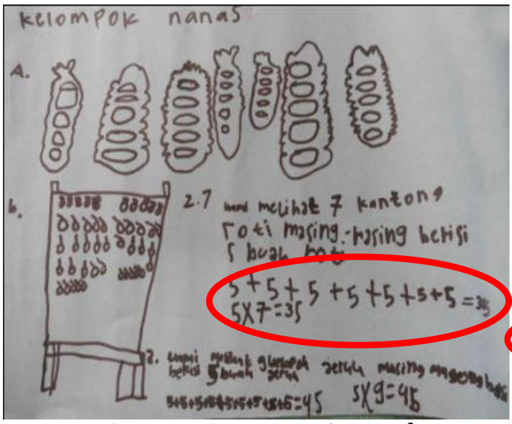


Figure 54a. Nanas’s works

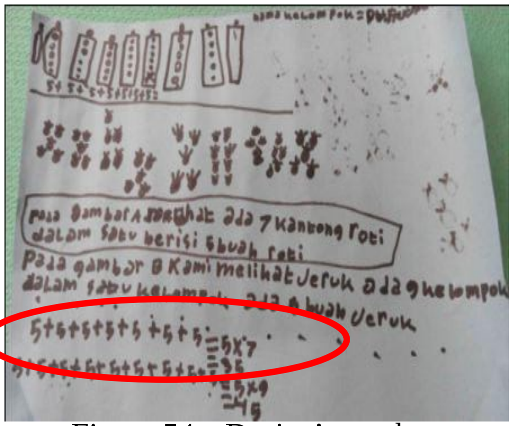


Figure 54a. Durian’s works

From figure 54, we can see that those two groups know that they can determine the number of breads by using repeated addition 5+5+5+5+5+5+5, but when they transform it into multiplication sentence they transform it into 5 × 7. Based on our observation, it was happen because they are influenced by the language. In Bahasa, they said '*limanya tujuh kali'* because the 5(limanya) come before the 7 (tujuh) then they tended to write in multiplication symbol 5 × 7. In order to help the students to transform the repeated addition into multiplication sentence, the class discussion were held.

The class discussion started with showed the poster to the students, The teacher put the poster of 7 bags of 5 breads in the whiteboard and asked to count the total number of breads in that picture. Together with the students the teacher wrote 5+5+5+5+5+5+5. After wrote 5+5+5+5+5+5+5, the teacher asked the students how many five are there, and the students counted one, two, three, four, five, six and seven and concluded that there were 7 times of 5. Then the teacher wrote, '*tujuh kali 5-nya'*. The teacher told to the students that in mathematics the word 'times' / *'kali'* in mathematics is symbolized as '×' so the teacher wrote 7 × 5.

The teacher tried to give the emergent of multiplication symbol to the students. The following is a segment from our video audio recording.

Teacher	: Now, how about if we have 20 times of the 5 (pointing to the repeated addition, 5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5 +5+5+5, that she made) Do we have another way to write it shortly?
Students	: Yes , we have teacher
Teacher	: Ok, who wants to help me?
Shella	: I am teacher.
Teacher	: ok, Shella, please.
Shella	: (Come to the whiteboard and count the 5 in the 5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5+5, then wrote 'ada *20 kali 5nya'* / 'there was 20 times of the 5"

From the segment above, we can see that the teacher tried to tell to the students that multiplication is a simple way to represent the long repeated addition in mathematics. After Shella wrote 'ada 20 kali 5nya' the teacher tried to justified the answer of Shella, to others students. As shown in following segment.

Teacher	: is What Shella did correct or not?

Students : Correct
Teacher : Lets count the five, (pointing to the repeated addition that she made), one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, and twenty. So, there was 20 times of the 5. Now, Who wants to help me to wrote in mathematical symbol? (Arief raised his hand), ok Arif, please!
Arif : (Arif come to the whiteboard and wrote) 20 × 5.

From the segment, we can see that the teacher tried to justify Shella answer to the students by counting the repeated addition. Together they concluded that there were 20 times of the 5. Because the teacher already told to the students that in mathematics the word 'times' is symbolized by '×', it made Arif able to symbolized 20 times of the 5 into 20 × 5.

The teacher challenged the students to represent the total number of oranges into multiplication sentence. As a result, one of the students, Khadafi, came to the whiteboard to do that as showed in figure 55 below.

Figure 55. Khadafi represented the number of oranges in multiplication sentence

When he come to the whiteboard, Khadafi directly made 5+5+5, he stopped for a while and counted the number of groups of oranges in the picture, when he knew that there were 9 groups of oranges, he continued to write the repeated addition 5+5+5+5+5+5+5+5+5. He tried to put in word the repeated addition that he made, '*9 kali 5nya*' / '9 times of 5' and he wrote in multiplication symbol 9 × 5. After Khadafi finished, the teacher asked the other students whether they agree or not, and all of the students agree with Khadafi. The teacher also asked the students if they had questions and none of the students posed question to the teacher or comments to Khadafi.

At the end of the lesson, the teacher gave worksheet to the students. There were three problems in the worksheet. The first problem, we showed to the students 4 boxes of 6 pencils. Most of the students, 26 of 28 students were able to represent it into multiplication sentence 4 × 6. The rest of them, 2 out of 28 students still made it into 6 × 4. We interested to know students reasoning why they represented it into 4 × 6. Therefore we interviewed one the student, the following is the segment of our video recording.

Researcher : Here you wrote 4x6, where do you got it, what do you mean with that?
Student : (She counted the number of pencils in the box) one box consist of six pencils.
Researcher : So, what is the 4 here?
Student : The number of box in the picture.
Researcher : So, what is the 6 here?
Student : The number of pencils in each box.

From the segment above, we can see that the student connected the multiplication symbol that she made, with the objects that we showed to them. It showed that the students tended to connect the multiplication symbol that they had with the situation rather than with the repeated addition that they made.

For the second problem, we showed to the students 8 groups of 6 beads, most of the students, 22 out of 28 students were able to represent it into multiplication sentence, 8 × 6, and 6 out of 28 students represented it into 8 × 6. For the third problem, we showed to the students 3 boxes of 3 dolls. For this problem only 14 out of 28 students are able to represent it into 3 × 3, while some of them answered it 3 × 9, 9 × 3, 3 × 6, 3 × 4. The limitation of time to do the worksheet might make students to do in hurry. Therefore, they not focus to answer the third question.

Throughout this lesson, we can see that the structure of objects provoked the students to count by group. They used repeated addition as their strategy to determine the total objects. The long repeated addition lead the student to make it shorter and symbolized it in multiplication sentence. It was difficult for some of students symbolized the repeated addition into multiplication sentence. The difficulty for students is their confusion in order to determine where they have to put the multiplier (the number of groups) and the multiplicand (the number of elements in each group) into multiplication sentence. We found that it was because of the language, for example, in English they say ‘7 times of 5’, which can easily symbolized into 7×5, but in Bahasa we say ‘5nya 7 kali’ it made the students tended to symbolized it into 5×7.

In general, we concluded that most of the students are able to represent the total number of structured objects that arranged in group model into multiplication sentence. This activity has provided a bridge for students to develop their thinking process. Later, in the next activity they have to represent the structured objects in rectangular pattern into multiplication sentence.

Lesson 3 “Counting Tiles Activity”

The students already experienced two activities, namely describing structured objects activity and counting structured objects activity. In counting structured objects activity the students already had knowledge about multiplication symbol and multiplication sentence in the group model. This lesson started by giving a picture of a handyman tile who was working to install the tiles as showed in figure 7. Students were asked to draw completed installation of the tiles. The completed installation of the tiles would be 4 × 6 tiles or 6 × 4 tiles. How students made complete installation of the tiles, how they determined the total number of tiles, and how they can represent it into multiplication sentence ware observed.

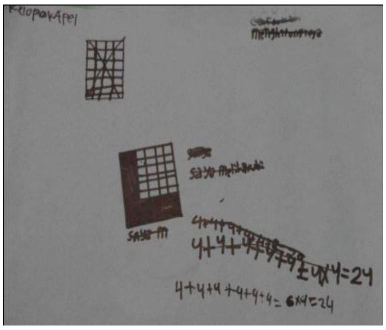
One of the groups, Nanas group, imitated the picture in the instruction sheet as shown in the figure 56 below.

Figure 56. Nanas group imitated the picture in the instruction sheet

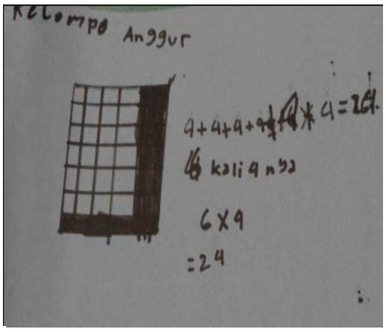
They said to the teacher that they finished making the complete installation of the tiles. The teacher asked them to read the instruction sheet. We observed that they read the instruction and realized that they have to draw the complete installation of the tiles not to draw the same picture as they thought. They continued to draw their picture, as our conjectured they completed their drawing, row by row till they finished. Finally this group succeeded to draw the complete tiles well.

Some groups, like Jeruk group, completed the drawing in the instruction sheet first. They made it as model for them. They realized themselves that it would be easier if they had image of complete installation of the tiles before started to draw in their poster. When they finished with their model, they counted the number of tiles in the first column, and realized that there were 4 columns that consist of 6 tiles in each column. After that, they started to move the picture to their poster.

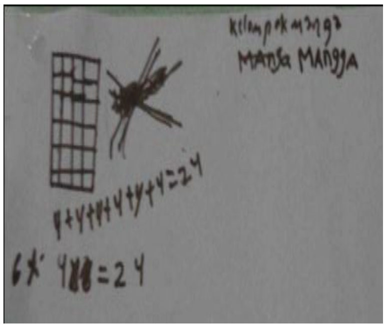
We analyzed that 4 out of 7 groups counted the number of tiles in row, 2 out of 7 groups counted the number of tiles in column, it showed from their strategy to count the total number of tiles by using repeated addition that they made. But one group, Durian group, did not make their strategy to count the total number of tiles, and it made us did not know how this group counted the number of tiles from their poster. Students' posters showed in figure 57 below.



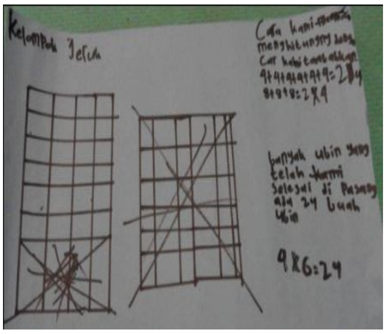
Apel's works



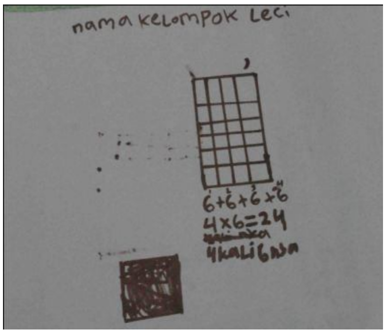
Anggur's works



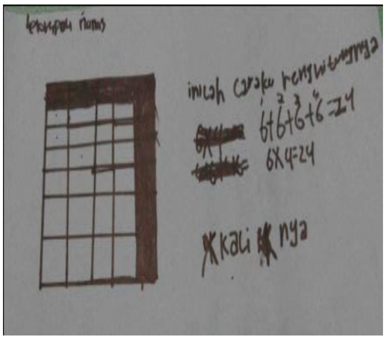
Mangga's works



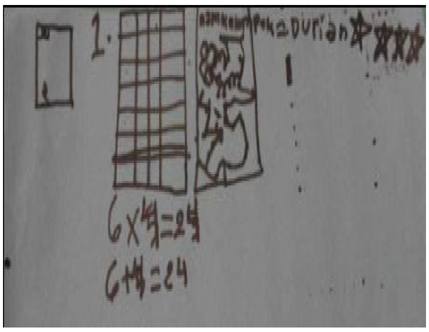
Jeruk's works



Leci's works



Nanas's works



Durian's works

Figure 57. Students' posters of activity counting tiles

Some groups, Jeruk and Nanas, had difficulties to represent the repeated addition into multiplication sentence. It showed from their poster. Jeruk group, counted the number of tiles in row, they know the number of tiles in row consist of 4 tiles, therefore to determine the total number of tiles they did repeated addition, 4+4+4+4+4+4. To count the total they regrouped the repeated addition that they made into 8+8+8. But when they have to represent the number of tiles from repeated addition into multiplication sentence, they made 4×6. They still influenced from the language "*4nya ada 6 kali*" that made them to transform it into 4×6. Nanas group also had difficulties to represent the repeated addition that they had into multiplication sentence like Jeruk group. They counted the number of tiles in column, they knew that there were six column where in each column consist of 6 tiles, therefore to determine the total number of tiles they did repeated addition 6+6+6+6, they counted how many of the six that they had. They tried to put in word in the same way that discussed in lesson two, "*4 kali 6nya*" but they had doubt and decided to erase it and wrote it into 6×4.

To start a fruitful discussion, the teacher asked the students to hang their work in the whiteboard and let them to observe what their friends made and give comment if they have comment on it as shown in figure 58 below.

Figure 58. Students observed their friends' work

As a result from their observation, two of the students from Nanas group, complained with Durian's works. Durian group made their drawing by seven rows, therefore nanas group said to the class it was wrong because the row of the complete tiles must be six rows. Their argumentation was accepted by the whole class, but Nanas group did not said abo

ut the multiplication sentence that Durian group made, and the teacher also let it till another groups complained. But none of the students paid attention on that in that moment.

After had complained from Nanas group, one of the students from Durian group, Shella, complained with Nanas' works as shown in figure 57. The following is a segment from our video recording about student argumentation.

Teacher : Ok class, Shella found mistake from Nanas groups. Ok Shella please!
Shella : What Nanas group did was not correct. It must be 4x6 because there were 4 times of the 6.
Teacher : So it must be?
Shella : four times six (4 × 6)

From the segment above we can see that Shella knew that it was 4 times of the 6. She understood well about the meaning of 'times'. She knew that there were 4 times of 6 and her knowledge about the word 'times' symbolized as '×' in mathematics made her able to symbolized it as 4 × 6.

At the end of lesson, the teacher gave the students a worksheet, in the worksheet students are asked to represent the total number of tiles in multiplication sentence. Figure 59 showed some of students' answer of the worksheet;

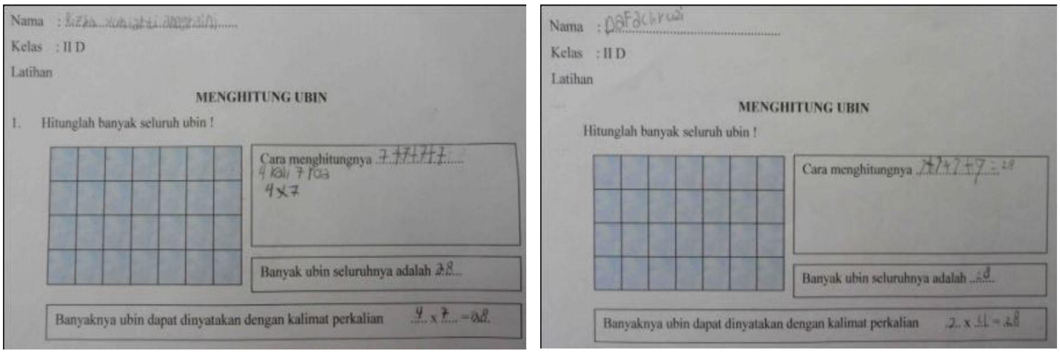


Figure 59. Rizka's and Daffa's Answers in the worksheets "Counting Tiles"

From Figure 59 above, we observed that Riska counted the tiles in the row. She knew that there were four rows and each row consist of 7 tiles. It was proven by her strategy to count the total number of tiles by repeated addition, 7+7+7+7. She was able to represent the repeated addition that she made into multiplication sentence. The word "*4 kali 7nya*" had a function as a bridge for her to come to multiplication sentence 4x7. Daffa also counted the number of tiles in row. He knew that each row consist of 7 tiles and he know that it were 4 rows, therefore to determine the total number of tiles He did repeated addition, 7+7+7+7. But when he had to transform it into multiplication sentence, he transformed the repeated addition into 7×4. He made it because he influenced by the language "*7nya 4 kali*".

Completing the tiles made the students to have information that the number of tiles in each row or column was same. Therefore lead them to count by group as we predicted. They counted the tiles in row or in column and used the repeated addition as their strategy to determine the total. Throughout this lesson, we found that some of students had difficulties to transform the repeated addition into multiplication sentence. The difficulty is much influenced by the language. Most of the students tend to said in word "*7nya 4 kali*" for example, that provokes them to transform in multiplication sentence 7×4. The teacher could help them by writing in word "4 kali 7nya". By using the word "4 kali 7nya" and stress to the students that in mathematics '*kali*' are symbolized by '×', it helps the students to transform the repeated addition into multiplication sentence correctly.

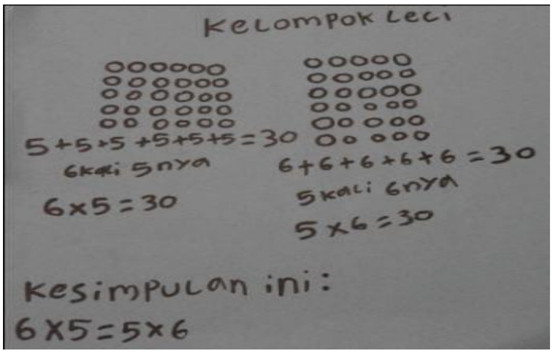
In this lesson, from the students' posters, the students were found two ways of representing the number of tiles in multiplication sentence that depend on their strategy to count the tiles. But it was not discussed in class discussion. The next activity designed to make the students know about the commutative property of multiplication in which they expected able to conclude $a \times b = b \times a$.

Lesson 4. "Counting Egg Activity"

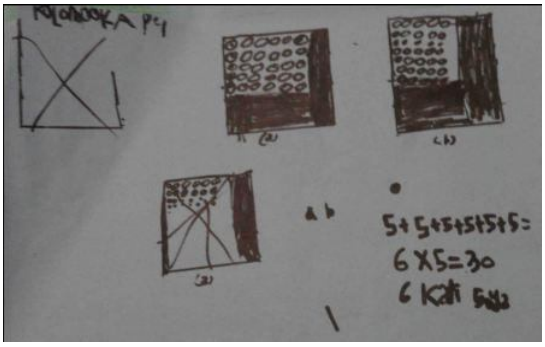
In the previous lesson, students already learned about to represent the total number of objects that arranged in rectangular pattern into multiplication sentence. They found two ways in representing the total number of rectangular pattern that depended on how they counted the number of objects. In this lesson the students learned about the commutative property of multiplication.

The lesson started by giving instructional sheet that the students had to do in their group. Instructional sheet provided two pictures of the eggs cartons. One egg carton was just turn around 90 degrees from another one. The students had to make representation of eggs in the poster that we gave to them. Based on our observation, the students directly draw the egg in the poster that we gave to them. As our conjectured the students draw the eggs in the row, or in the column. Before they started to draw, they counted the number of eggs in the row or column to know how many eggs in one row or column, and started to make drawing of eggs if they already had information about the number of eggs in row or column. In the middle of their drawing they counted the number of row or column that they had in order to decide how many row or column that they needed to draw.

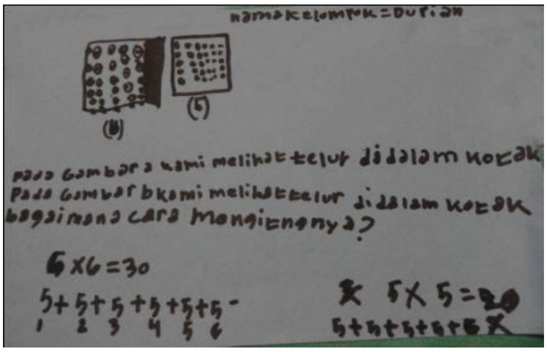
The instructional sheet also asked the students to write their strategy to determine the total number of eggs that they draw. Based on the data that we have, all of the students counted the number of eggs by group in row or in column. They did repeated addition as their strategy to determine the total as shown in figure 58 below.



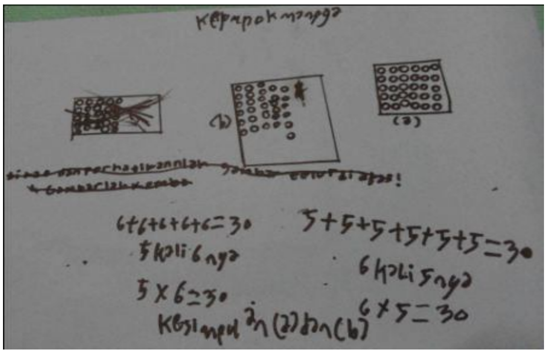
Leci group



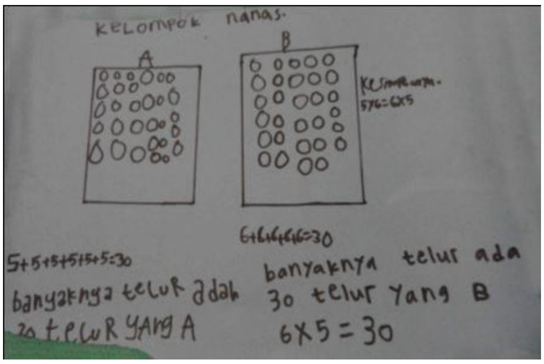
Apel group



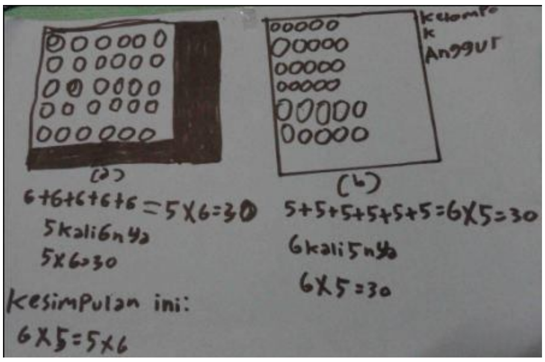
Durian group



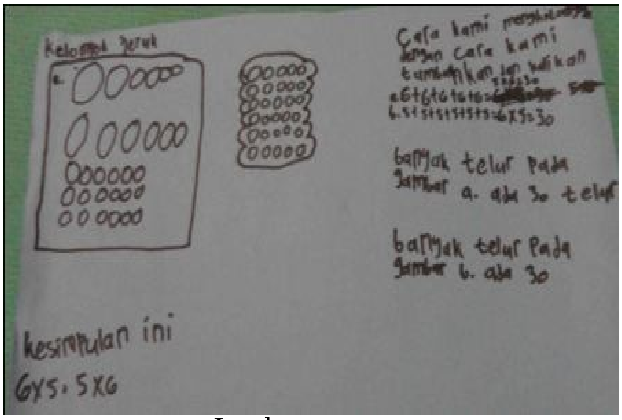
Mangga group



Nanas group



Anggur group



Jeruk group

Figure 60. Students' posters of counting eggs activity

From the students' posters, students represented the total number of eggs by using repeated addition 5+5+5+5+5+5 and 6+6+6+6+6 or 6+6+6+6+6 and 5+5+5+5+5+5. They got those two repeated addition by counting the number of eggs in rows, or in column that depends on their perspective to count the eggs. We analyzed that 5 groups, counted the number of eggs in column. Those groups are Leci, Nanas, Mangga, Apel, Durian. While Anggur and Jeruk counted the number of eggs in row.

Based on our observation, the way of students making their drawing influenced their strategy to count. For example, when students drew the picture in row, they know the total number of eggs in one row. They continued their drawing until they finished. It made them realized that the number of eggs in each row was same, therefore they did repeated addition as their strategy to determine the total. They add the number of egg in row as much as the rows that they had.

There was several students' strategies to count the total number of eggs by repeated addition that they did. As we predicted, some students counted by adding the number that they repeat one by one, some of them did regrouped repeated addition. Based of our observation, students tended to regroup the repeated addition when they counted the fives objects. For example Anggur group, when Ade, one student from Anggur group, finished writing the repeated addition, 5+5+5+5+5+5, He counted the number of the 5 that he had, After knew that he wrote 6 times of 5, he transform the repeated addition that he made into 6 × 5, then let Adjie to count the total number of eggs. Adjie, one of the students from Aggur group, counted the repeated addition 5+5+5+5+5+5, he counted 'five, ten, fifteen, twenty, twenty five, thirty' when his friend Ade was writing that repeated addition. To make sure, Adjie counted it again by pointing to the two rows of 5 eggs, and counted 'ten, twenty, thirty' and concluded 'thirty altogether.

When the students finished with their tasks in instructional sheet, the class discussion held. In the class discussion one of the groups, Jeruk group, presented their works. Jeruk explained their works to the class as shown in figure 61 below, they told that there were 30 eggs in picture A, and 30 in picture b, they explained how they counted the total number of eggs, they counted the eggs in row and did repeated addition as their strategy. From the repeated addition they transform it into multiplication and concluded that 5 × 6 = 6 × 5.

Figure 61. Jeruk group presented their works

After Jeruk presented their works, the teacher asked the students if they had question to Jeruk group. One of the students from Durian group, Shella, asked important question. The following is a segment of our video recording; that showed Shella question.

Shella : (Come to Jeruk group, and observed their works), Where did you got it (Pointing to the conclusion that Jeruk group made ($6 \times 5 = 5 \times 6$)

Researcher : Ok, Kartika (one student from Jeruk group) where did you get it?

The question from Shella, 'where you get it', showed that she wanted to know where the conclusion comes from, Shella asked that because her group did not make the conclusion. Based of our analysis, only 4 groups are able to conclude that $5 \times 6 = 6 \times 5$, they are Anggur, Nanas, Leci and Jeruk. The following segment showed Kartika, a member of Jeruk group, answered Shella question.

Kartika : This is from this (pointing to 6×5, from repeated addition that she made, 5+5+5+5+5+5), and this is from this (pointing to the 5×6, from repeated addition that she made, 6+6+6+6+6).

Researcher : (Shella looked confused), Ok, Now, why do you conclude that it was same, why $6 \times 5 = 5 \times 6$?

Kartika : (pointing to the number 30, from 5×6 and 6×5)

Researcher : Ok, it was same because the number of eggs in those two pictures was 30. Ok, Kartika, why did you say $5 \times 6 = 6 \times 5$?

Kartika : Because it was the 30 eggs in those two pictures.

Researcher : Ok, Shella, do you hear it? They said, because, in these two pictures(pointing to the picture A and Picture B from Jeruk's drawing) there are 30 eggs. Do you agree with that?

Shella : (Nod her head and counted the eggs in those two pictures of Jeruk's drawing one by one, till finished)

Researcher : Ok, Shella, it was same because the number of eggs in these two picture was ….

Shella : Thirty(with smile)

From the segment above we can see that Kartika tried to explain why their groups can conclude $5 \times 6 = 6 \times 5$. She explained to Shella, where their group got 5x6 and 6x5, but it still made Shella confused, because she did not know why it was equal, therefore the researcher tried to make the question clear to Kartika by saying 'why it was equal, why $5 \times 6 = 6 \times 5$'. Kartika knew it was same but she did not tell it directly, she pointing to the number 30, from 5×6 and 6×5 and the researcher tried to make it clear by conclude that it was same because the number of eggs in those two pictures is same. It was 30. But Shella do not agree directly, she counted the number of eggs of Jeruk's drawing one by one, until she was sure that those pictures really consists 30 eggs. After she finished counting and sure, the researcher concluded that it was same because the number of eggs is same.

At the end of the lesson, to make the students sure that the number of egg was same, the teacher cut the poster and turn it around and showed it to the students that it was same and gave conclusion that the number of eggs in 5×6 is same with the number of eggs 6×5, because it was same, there were 30 eggs. The teacher also asked the students to count 7×5 and 5×7 and 9×5 and 5×9 to investigate whether the result of those multiplication sentences gave the same result (product) or not.

Through this lesson, we concluded that the class discussion helped the students to share their idea, so that they can help each other. In the class discussion, students got insight about commutative property of multiplication. Therefore they can conclude that two multiplication sentence can give the same product regardless the order of multiplier and multiplicand in multiplication sentence. It showed from their conclusion that they were able to conclude $5 \times 6 = 6 \times 5$ in this activity.

Lesson 5. "Counting dolls Activity"

In the previous lesson, students already learn about one of the properties of multiplication. In this lesson students will learn about another property of multiplication, distributive property of multiplication. In order to do that, we gave the students pictures of dolls that arranged in the group of three in the five rows of rack (shelves). By using the context of doll store, we want the students realize that

5 × 3 can be solved by adding (1 × 3) and (4 × 3), or any combination of groups of three that add up to 5 groups.

The teacher started the lesson by showed the poster of dolls as showed in figure 4 to the students. The teacher asked to the students how many dolls in Rak E and together they answered 15 dolls. As our conjectured, some students got 15 from adding 3+3+3+3+3, together with the students the teacher transform it into multiplication sentence. The following segment is a segment from our video recording.

Teacher : Now look at Rak A, how many dolls in Rak A?
Students : three
Teacher : In order to make this Rak same with Rak E, or to make this Rak, full of dolls what should we do with the dolls?
Students : We add teacher!
Teacher : Ok, the number of dolls in Rak A adds with the number of dolls in empty space in Rak A. (the teacher wrote it in the whiteboard). Now, in order to make Rak A, full of doll how many dolls that we need to add?
Students : Twelve.
Teacher : How we add it?
Students : three plus three plus tree plus three.

From the segment above, we analyzed that the teacher tried to guide the students to make connection among full Rak, the number of dolls in Rak and the number of dolls in empty space that to make the Rak full of dolls. The students knew that they needed to add the dolls in the Rak to make the Rak full of dolls, they added the number of dolls by adding 3+3+3+3 then they transformed it as 4 × 3. After that the teacher tried to make connection among the number of doll if the Rak were full of dolls, with the number of doll in the Rak and the number of that they needed to add. The teacher together with the students tried to conclude that they can construct (5 × 3) by adding (1 × 3) the number of dolls in the Rak, plus (4 × 3) the number of dolls that they needed to add as shown in figure 62 below.

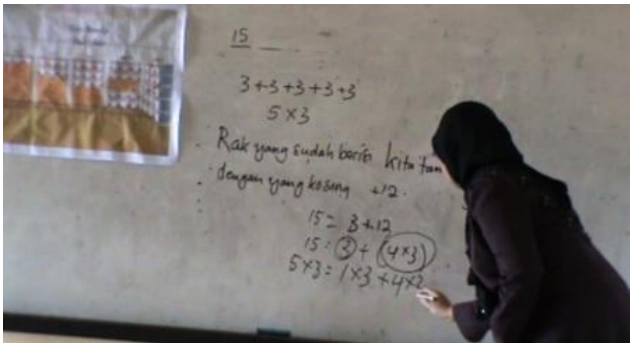
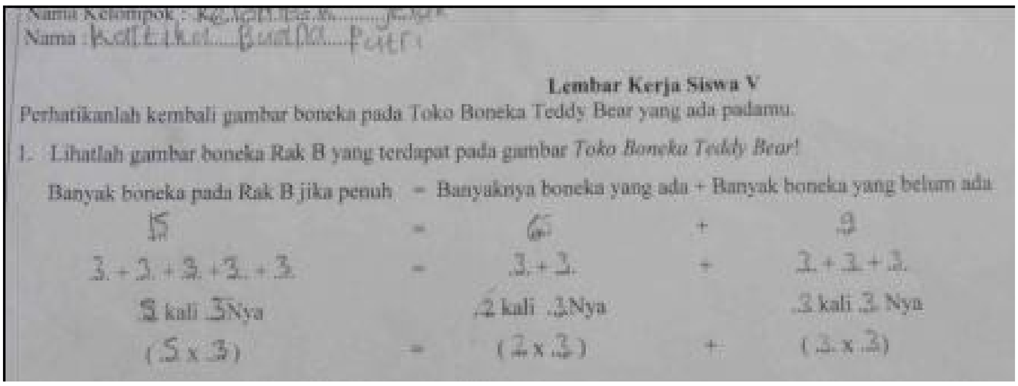


Figure 62 Teacher wrote their conclusion that (5 × 3) = (1 × 3) + (4 × 3)

After the teacher gave conclusion that they can make (5 × 3) from adding (1 × 3) and (4 × 3) by showed from the poster of dolls in Rak a, the teacher asked the students to work in group to do the worksheet to investigate that number of dolls in another Rak. When they finished, some of the students are asked to present their idea in front of the class.

Kartika, one of the students from Jeruk group, wanted to present their idea in front of the class. She wrote what she did in the worksheet to the whiteboard as shown in figure 63 below.


Nama Kelompok : [illegible]
Nama : Kartika Budha Putri

Lembar Kerja Siswa V

Perhatikanlah kembali gambar boneka pada Toko Boneka Teddy Bear yang ada padamu.

1. Lihatlah gambar boneka Rak B yang terdapat pada gambar *Toko Boneka Teddy Bear*!

Banyak boneka pada Rak B jika penuh	=	Banyaknya boneka yang ada	+	Banyak boneka yang belum ada
15	=	6	+	9
3 + 3 + 3 + 3 + 3	=	3 + 3	+	3 + 3 + 3
5 kali 3Nya		2 kali 3Nya		3 kali 3 Nya
(5 x 3)	=	(2 x 3)	+	(3 x 3)

Figure 63 Kartika’s works

After she finished the teacher asked to the class if they have comment or questions to Kartika as describe in following segment:

Teacher : Ok, who wanted to give comment on the things that Kartika have been written in the whiteboard?
(Siska raised her hand, and come to the whiteboard and asked)

Siska : Where did you get the 6 here?(pointing to the number 6 that Kartika wrote.

Teacher : Ok class, Siska asked Kartika, where she got 6. Ok Kartika please explain to your friends.

Kartika : The six from the dolls in Rak B (pointing the dolls in Rak B in the poster)

From the segment and from Kartika's work, we analyzed that Kartika can explain that she got the number six from the picture of dolls in Rak B, then from the picture she got six from add 3+3 then she transformed it into multiplication sentence 2 × 3. She knew that she needed add 9 more dolls to make the Rak B full of dolls, she got 9 by adding 3+3+3 and she put in word 3 times of the 3 and put it in multiplication sentence 3 × 3. She can conclude that (5 × 3) = (2 × 3) + (3 × 3).

Adjie, one student from Anggur group, also wanted to present her work in the class discussion. In the class, he went to the whiteboard and wrote his finding. Adjie's work showed in following figure 64 below.

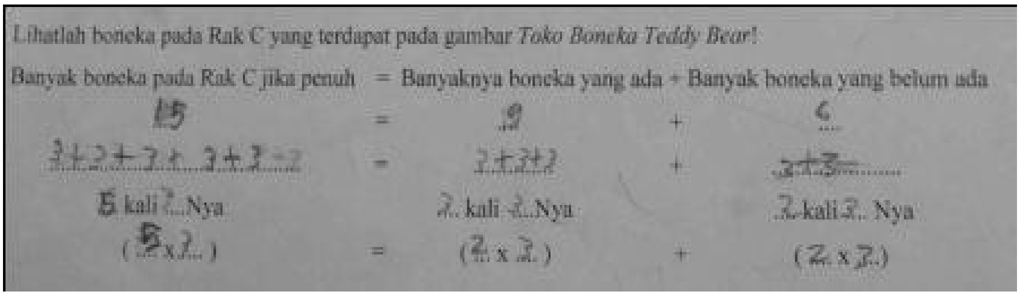


Figure 64 Adjie's works

Adjie wrote what he had in the worksheet in the whiteboard, in order to save times the teacher helped Adjie to write Adjie's works. After finished, the teacher asked the students if they had question to Adjie. The teacher tried to provoke the students to pay attention to the multiplication symbol 2 × 3 that Adjie got from 3+3+3. The students complained that it was 3 × 3 not 2 × 3. The teacher asked another student from Anggur group to help Adjie, Ade come to help Adjie, Ade explained that they got nine from the number of dolls in Rak C, he knew it was nine from 3+3+3 which is he can change into 3 × 3 not 2 × 3, he gave reason that Adjie miss typed to write the multiplication symbol.

From Adjie's work, we can see that class discussion gave students an opportunity to justify their answers. It let the students to discuss and to find the solution. The teacher let the students to decide whether their friend work correct or not by posing question whether they agreed or not and asked their reason why they did not agree and gave their solution.

In order to know about students thinking, we gave the students worksheet and we interviewed them. Figure 65 showed Kartika's works.

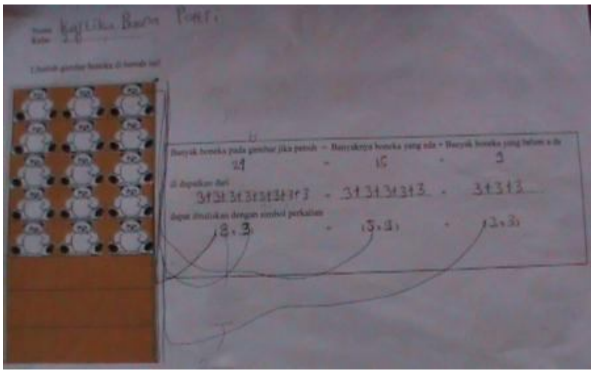


Figure 65 Kartika's works of counting dolls worksheet

From figure 65 we can see that Kartika able to represent the repeated addition that she made into multiplication sentence. The relation among full Rak and the number of dolls in the Rak and the number of dolls in empty space in the Rak, leads her to the conclusion 8 × 3 = (5 × 3) + (3 × 3). In order to know how she got it we interviewed her. The following segment is a segment from our video recording:

Researcher : Why do you made, eight times three (8 × 3) is equal with fives times there (5 × 3) plus three times three (3 × 3)?
Kartika : Can I make a stretch here? (She wanted to make illustration)
Researcher : Yes, of course
Kartika : Because the 8 is from these Rak (she gave mark for 8 rows of the Rak) and the 3 is from the dolls
Researcher : Ok, from the number of dolls in each row and then why it can be these (pointing to 5 × 3 that she made) plus these (pointing

to 3 × 3 that she made).

Kartika : The fives (take a line from the fives in 5 × 3 that she made), from the number of dolls in this Rak (make a line in five row in the picture). While this three (take a line from the 3 in 3 × 3 that she made) from this (make a line in three row of empty space in picture)

From the segment above, we analyzed that Kartika involved part whole relationship to understand why she can construct (8 × 3) = (5 × 3) + (3 × 3). She explained that 8 × 3 is a whole by giving the line in 8 rows of the Rak. She tried to explain that she can make the whole by adding two parts of that (5 × 3) and (3 × 3).

Throughout this lesson, we concluded that by structuring the problem to the students and let them to make the relation between one to the others multiplication facts that they had can help the students to know about distributive property of multiplication. Through this lesson students can realize that they can construct 5×3 from (1 × 3) and (4 × 3), or from (2 × 3) and (3 × 3) from the context that we gave to them.

Lesson 6 “Solving Multiplication Problems Worksheet”

In this lesson, the students worked with groups of objects, we showed to them 10 pictures of structured objects as shown in figure 10 and asked them to work in group of 3 or 4. The students are asked to represent the structured objects in the picture in multiplication sentence. They also asked to write their strategy to determine the total number of objects in picture.

Based on our observation, most of students did repeated addition to determine the total number of objects in the worksheet. They are several ways of students to determine the total number of objects from repeated addition that they made. Some students knew that to determine the total number of objects they had to do repeated addition, but some of them still counted the total objects one by one, For example, Nur, she knew that to determine the total objects from 5 boxes of 3 balls she had to do repeated addition, to determine the total number she used the contexts that we gave to her, she counted the objects one by one by pointing to the picture and counting on the objects as shown in figure 66 below.

Figure 66 Nur was counting the balls in the picture

Some students counted the repeated addition that they made by skip counting, For example, Hafisz, he wrote in his answer sheet repeated addition 4+4+4+4=16 to count 4 boxes of 4 soap. When we asked to him how he got 16 he did skip counting, ‘eight, twelve, sixteen’. He started from eight because he knew that 4+4 is eight then he continue by doing skip counting by four.

Most of students tended to regroup the repeated addition that they had. For example Ade and Nutrisa, The following is a segment from our video recording that showed how Ade and Nutrisa counted the number of balls in 4 boxes of 3 balls.

Researcher : Can I see your work? Where did you get twelve?
Ade : From here (pointing to 4 boxes of 3 balls)
Researcher : Ok, show me, how you get 12.
Ade : there plus three plus three plus three
Researcher : you directly know it 12?
Ade : four times three, twelve.
Researcher : Where did you get your 12?
(Nutrisa, wanted to help)
Nutrisa and Ade : From all of balls
Researcher : Ok, show me please.
Ade : six plus six
Researcher : Ok, which one the six that you mean? This is three not six

(pointing to one box of 3 balls)

Nutrisa : This is six (pointing to two boxes) and this is six (pointing to another two boxes, so twelve altogether.

From the segment we analyzed that students knew that the twelve is the total number of balls in 4 boxes of 3 balls. It showed from their statement when we asked to them where they got 12. Students answered from all of balls. They knew that twelve is the quantity of balls in 4 boxes of 3 balls. They also tended to regroup the repeated addition that they made. They wrote 3+3+3+3, but to determine the total they calculate 6+6 because they knew that 2 boxes of 3 balls consists of 6 balls, therefore they could regroup their repeated addition 3+3+3+3 into 6+6.

Most of the students were able to represent the total number of objects in multiplication sentence. They tended to put the multiplication in word first before they symbolized that as shown in figure 67 below.

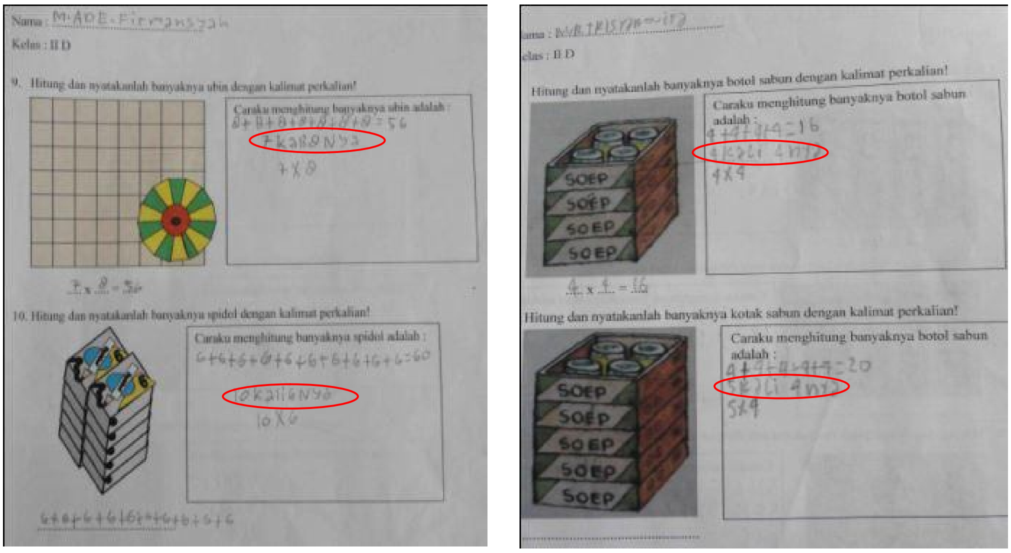


Figure 67 Students tended to put in word the repeated addition before symbolized it in multiplication sentence.

One student, low achieving student, still had difficulties to represent the total number of objects into multiplication sentence. Student’s works showed in figure 68 below.

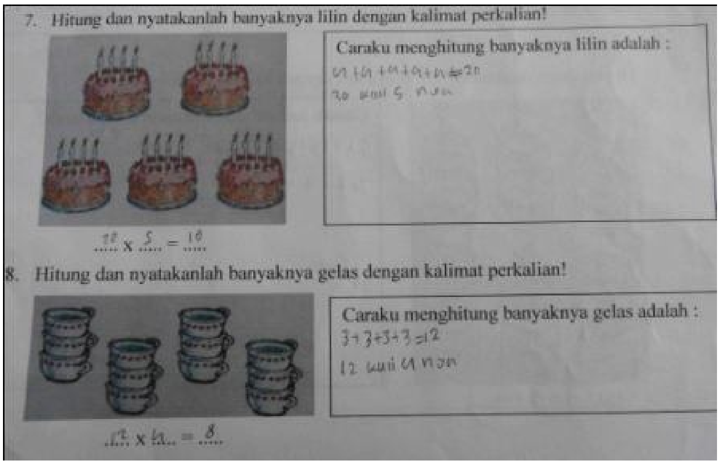


Figure 68. Student's works who did not know the meaning of times

From figure 68 we can see that the student did repeated addition to determine the total number of candles and cups in the picture. She had difficulties to represent the repeated addition that she made into multiplication sentence. It seemed that this student did not know the meaning of 'times', she tried to put in word the repeated addition, 4+4+4+4+4, into 20 times of the 5 and transformed it into multiplication symbol 20 × 5. She got 20 from her calculation of the repeated addition that she made, she got 5 from the number of cakes (the number of groups) and she got the product of the multiplication symbol that she made by subtracting 20 and 5 that she put 15 as the product of multiplication that she made. This student needed more discussion in the idea of the word 'times'.

Based of our observation when students did the worksheet, none of the students used the fact that that they knew to find efficient strategy to count the total number of objects, for example. The students already got 12 from 4 boxes of 3 balls, but none of them directly know that 5 boxes of 3 balls consist of fifteen balls by adding 3 more balls. However, they tended to counted again by adding 3+3+3+3+3. Therefore the teacher asked the students to present their idea in the class. To provoke the students to used the facts that they already knew to find another product of multiplication sentence.

Siska, one of the students from Leci group, presented their group works. Siska wrote what they made in the worksheet in the whiteboard as shown in figure 69 below.

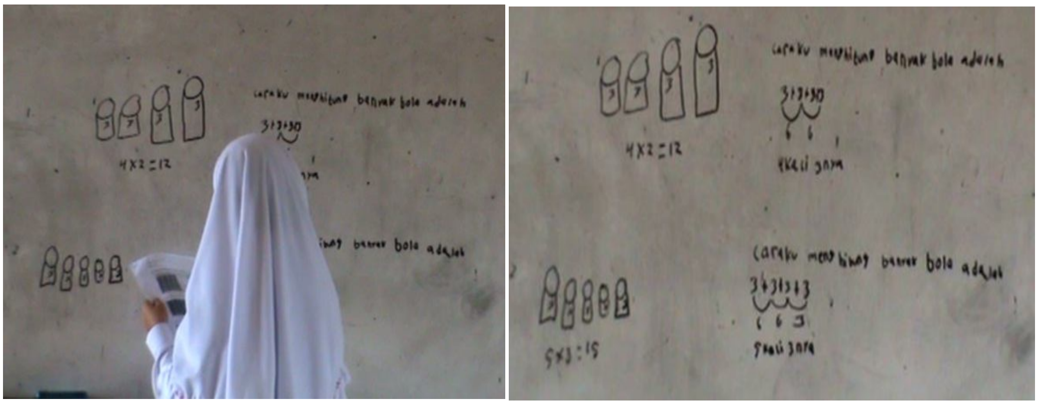


Figure 69. Siska's works

When Siska finished writing her work in the whiteboard, the teacher asked Siska to explain it. Siska explained to her friends that she got the 3 from the picture that she made. She wrote that there were 4 times of the 3. To determine the total, she regrouped the repeated addition that she had, 3+3+3+3, into 6+6 and got 12 as a total. So that she wrote in multiplication symbol 4 × 3 = 12. After Siska explained her works, the teacher provoke the students to make the connection between two problems that Siska did. The following segment is a segment from our video recording.

Teacher : Ok, looked at picture number one. How many boxes in the picture?
Students : four
Teacher : Ok it, means how many times the three?
Students : four
Teacher : Ok, we have four times of the three (Wrote in the whiteboard 3+3+3+3). Number two, how many boxes that we have?
Students : fives
Teacher : Ok, it means we have five times of the three (Wrote in the

whiteboard 3+3+3+3+3). Now, number one! Same or not with number two?

Students : No
Teacher : What is the things that make it not same?
Students : The box
Teacher : Ok the box, Number one we have four boxes and number two we have 5 boxes. In order to make it same what should we do with the box in number one?
Students : we add one more box.

From the segment above we can see that the teacher tried to provoke the students to make connection between problem number one and number two. By asking the number of the boxes in those two numbers of problems and asking what should they do in order to make it same and the students know that they had to add one more box.

When students said add one more box, the teacher asked the students to represent into multiplication sentence and the students was able to represent it into multiplication sentence 1 × 3. Furthermore the teacher together with the students found 5 × 3 by add 4 × 3 and 1 × 3 as shown in figure 70 below.

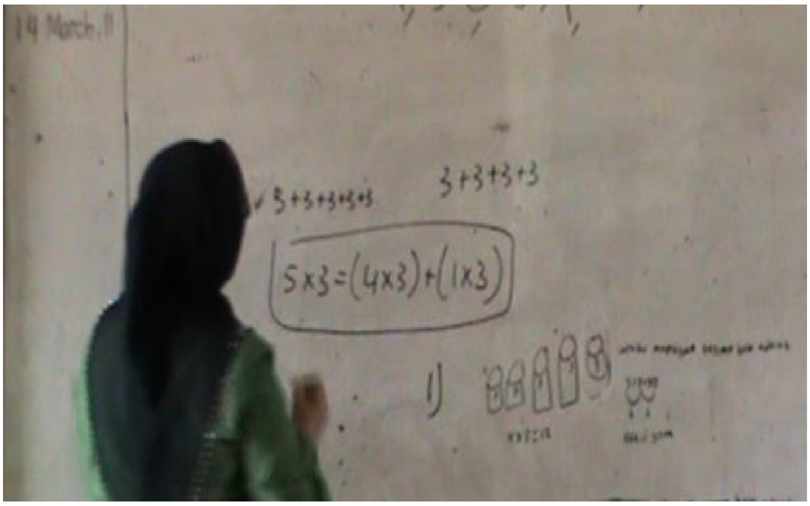


Figure 70. Teacher wrote 5 × 3 = (4 × 3) + (1 × 3) that she found with the students

The teacher also asked the students to make relation of 4 boxes of 4 soaps and 5 boxes of 4 soaps. Two students from Nanas group, Arif and Daffa, come to the whiteboard and present their idea. Arif drew 4 boxes of 4 soaps, and Daffa drew 5 boxes of 4 soaps. Daffa knew that to construct 5 × 3, he could add one more box. Therefore he could conclude that 5 × 4 = (4 × 4) + (1 × 4) as shown in figure 71 below.

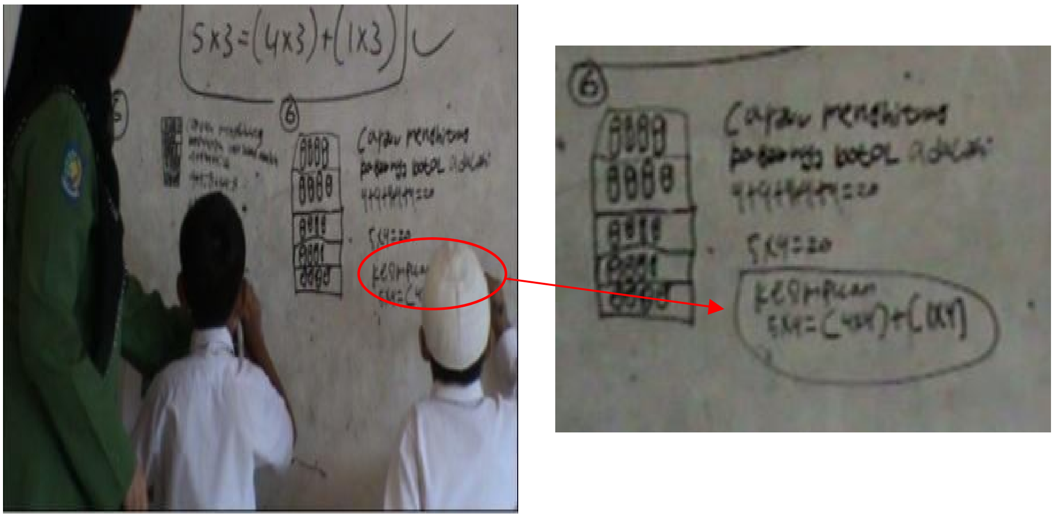


Figure 71. Daffa concluded that he could make 5 × 4 from (4 × 4) and (1 × 4)

At the end of the lesson, Kartika, from Jeruk group, presented their idea as shown in figure 72 below.

Figure 72 Kartika presented her works

She explained that there were 10 boxes of 6 markers. To determine the total number of markers she did repeated addition. She transformed the repeated addition that she made into 10 × 6. She gave reason that 10 was the number of box and 6 was the number of markers in each box. She also found interesting idea that she can conclude that 10×6 can be constructed by adding $(5 \times 6) + (5 \times 6)$. She explained that (5×6) is one side and (5×6) is another side and together there were 10×6.

From Kartika explanation, we analyzed that she constructed the big idea about distributive property of multiplication. She involved part whole relationship. She knew that there are 10 boxes altogether where each box consisted of 6 markers. In fact, Kartika also found the idea of doubling. She knew that there were 10 boxes is a double of 5 boxes.

Throughout this lesson, we concluded that giving the students structured objects provoked the students to count in groups. They did repeated addition as their strategy to determine the total and transform it into multiplication sentence. In this lesson we found that most of the students are able to represent the total number of structured objects into multiplication sentence. As we predicted, students counted the total number by doing repeated addition. They determined the total from the repeated addition in their own way, some students did counting on, some students added the number that they repeated one by one, and most of students regrouped the repeated addition that they made by adding two numbers and made new repeated addition or continue their calculation. After they got the total number of objects, they tended to put in word the repeated addition that they made and transform it into multiplication sentence. The class discussion provoke them to connect the multiplication facts that they already knew that leads them to come to efficient way to calculate the total number of objects from the repeated addition that they made.

E. Analysis throughout All Lessons

In this analysis, we looked at all lessons and searched for connections between them. We focussed on the students' learning trajectory throughout those lessons as we wanted to see if the activities have supported in learning multiplication.

In the first lesson, we found that the students had difficulties to describe structured objects. They did not describe the number of groups and the number of elements in each group simultaneously. The activity 'describing the structured objects' provoked the students to describe the structure of objects, how many groups and how many elements in each group. This activity made the students have to describe to their friend (who can not see the objects and make drawing of his/her imagination about the objects from the information that he/she got from his/her friends), the structure of the objects. Here they had to unitize the objects. They had to see the number of groups and the number of elements in the group simultaneously. If they did not said the number of groups and the number of elements in each group, their friend would have difficulties to make her drawing or He/she would draw different pictures with they saw. By seeing and describing the structured objects the students realized that the number of objects in each group was same that provoked them to count by group and use repeated addition as their strategy to determine the total. In this lesson, we found that the students developed the language that related to multiplication such as *a* bags of *b* or *a* boxes of *b*.

In second lesson, 'counting structured objects', the students were asked to count structured objects in the picture. Besides counting the structured objects, the students are asked to make drawing of structured objects and described the objects that they drew. Their experience with first activity made them able to describe the objects that they made. In this lesson, they developed the language, move from '*a* bags of *b*', '*a* boxes of *b*' to '*a* groups of *b'*, they tended to say the objects arrange in group. When they described the structured objects they realized that the number of objects in each group was same. Therefore the repeated addition arose as a strategy to determine the total number of objects. In making drawing and determine the total number of objects, we found that the students did not have any serious difficulties. But, when we asked them to represent the total number of objects in multiplication, some of them had difficulties to symbolize it. Their difficulties it where they had to put the number of groups (the multiplier) and the number of elements in each group (the multiplicand). The teacher could give the students help by tried to connect the repeated addition that they had with the idea "add so many times" how many times they add certain numbers and the teacher could provoke them to put in word, "*a* times of *b*", "*a kali b-Nya*". Where *a* is the number of *b* that they repeat and *b* is the certain number that they repeat. After put it in word like that most of students were able to put in multiplication symbol $a \times b$.

In the third lesson, the students had to represent the total number of objects in that arranged in rectangular pattern into multiplication sentence. Students had experienced to represent repeated addition into multiplication in the second lesson. The students can see the objects arranged in "group of" where they can see that the number of objects in each group is same then repeated addition arose. But in this lesson, they did not see the group of objects anymore, they would see the structure of objects that arranged in row and in column.

In order to help them had feeling that the number of objects in each row/column was same, we asked the students to complete the drawing of rectangular tiles. Some students directly draw the tiles as the picture that we gave to them, they started to complete the number of tiles in column or in row. When they completed the tiles in a row or in a column, they realized that the number of tiles in each row or column was same, therefore the repeated addition arose. From the repeated addition that they made, they transform it in multiplication sentence. To count the total number of tiles, some of the students still counted the number of tiles by counting on one by one, some of them did skip counting, some of them add the number that they repeated one by one, and some of them tended to regroup the repeated addition that they had to determine the total in efficient way.

In forth lesson, students learned about commutative property of multiplication. Students already experienced with rectangular model that they did in third lesson. In this lesson, students worked with picture of two eggs cartons where one egg carton turn around 90 degrees. The students are asked to make representation of eggs in the egg cartons and represent it into multiplication sentences. Because they had experienced with rectangular pattern, students interpreted the structure of eggs in row or in column. They drew the eggs row by row, or column by column. They tended to do repeated addition to determine the total number of eggs. They represented the repeated addition that they made into multiplication sentence. At the end they got two multiplication sentences. When they counted the number of eggs using repeated addition, they got the number of eggs in those two eggs cartons is same. They also got two multiplication sentences from those two eggs cartons, 5×6 and 6×5. We asked them to make conclusion and some of them can conclude that $5 \times 6 = 6 \times 5$ because the number of eggs in those two eggs carton was same. The equal sign that they made means the product of those two multiplication sentences was same.

The fifth lesson was about distributive property of multiplication. Students already experienced to represent the total number of objects in multiplication sentence that arranged in row and in column. In this activity we showed to students a picture of dolls that arranged in a row of shelves where in a row of shelves contained three dolls. In the picture, there were fives Rak(shelves) of dolls, where in each Rak consisted different number of dolls. The students are asked to make connection among the number of dolls in full Rak, the number of dolls in the Rak and the number of dolls that they need in order to make the Rak full of dolls. The part whole relationship involved in this lesson, the number of doll in full Rak is a whole and the number of dolls in Rak and in the empty space of Rak is the parts. When students knew that the number of dolls in each row in the Rak was same, therefore they can transform the situation into repeated addition. From the repeated addition, they transformed it into multiplication symbol. Because they had to make the Rak full of dolls, they had to add the number of dolls in the Rak, they added the number of dolls in the *Rak*, and got the *Rak* which is full of dolls. Seeing the relation of full *Rak* and number of dolls in the *Rak*, and the number of dolls that they had to add, made the students realized that they can construct the multiplication (5×3) from the number of dolls in five rows of three dolls from adding (1×3) from the number of dolls in the *Rak* and (4×3) from the number of dolls that they had to add in order to make the *Rak* full of dolls for example.

In the sixth lesson, we gave the students worksheet. They had learned about distributive property of multiplication. In this lesson, they had to transform the structured pictures that we gave to them into multiplication sentence. They had to write how they got the total number of structured objects that we gave to them. We designed the problems in such a way in order to provoke the students use the fact that they knew, for example, we showed to them 3 bags of 5 breads, and after that we showed to them 6 bags of 5 breads, we expected that they used their calculation in 3 bags of 5 breads in order to know the total number of 6 bags of 5 breads so they can concluded $(6 \times 5) = (3 \times 5) + (3 \times 5)$. But none of them did like that. They tended to count again the total number of breads in 6 bags of 5 breads by repeated addition. Through this lesson, we concluded that most of the students did not have difficulties to interpret the total number of structured objects in multiplication sentence.

F. End Assessment

At the end of series of lessons in the teaching experiment, we conducted an assessment to see if our activities could support the students in learning multiplication. There were 5 problems in the assessment. The problems were about representing and the structured objects into multiplication sentence and counting the total number of objects. There were 28 students attended the post-test that we had done.

To analyze this end assessment, we made an analysis table, we looked at each problem and see what strategies students used to solve the problem. We grouped the answers into correct and incorrect answers to determine the proportion of the number of students who could correctly answer the questions. Then we tried to describe their tendency in solving problems in this section.

Problem 1

In this problem, we showed to the students picture 7 bags of 5 breads, we asked them to describe what they saw on the picture, writing down their strategy to determine the total number of breads and representing the total number of breads in multiplication sentence. This problem was aimed to know students attitude to describe structured objects, to know students strategy to determine the total of objects and to know their ability to put the structured objects into multiplication sentence.

Based on the data that we had, Most of students, 19 out of 27 students, were able to describe the objects by saying the number of the bags (groups) and the number of breads (objects) in each bag (group), 4 students just described the number of objects in the groups, 2 students just wrote the name of the objects and 2 students wrote the total number of breads that they saw.

All of students did repeated addition to determine the total number of objects. Some students, 13 out of 27, calculated the repeated addition that they made by adding the number that they repeat one by one and 14 out of 27 students regrouped the repeated addition that they made by adding 5+5, and counted by ten. From 27 students, only one students not correct with his calculation, he miss to count one bag of breads.

Most of students, 20 out of 27 students, were correct to represent the total number of breads in multiplication sentence, while 7 of them still do not correct. They tended to put the number of groups and the number of elements in each group in wrong order in multiplication sentence.

Problem 2

In this problem we showed to the students, 5 boxes of 6 markers. We asked them to represent the total number of markers in the picture in multiplication sentence. This problem was aimed to know students ability to interpret the structured objects into multiplication sentence.

Based of on the data that we had, only 9 out of 27 students were correct to represent the total number of markers in the boxes into multiplication sentence. Most of students, 18 out of 27 were not correct to represent the total number of markers in multiplication sentence. They tended to put the number of group and the number of elements in each group in wrong order in multiplication sentence. The students were influenced by number six on the picture. They see number 6 on the picture and there were 5 boxes of markers. Number 6 that they saw made the students tended to put in multiplication sentence 6×5.

We also asked to write their reason, why they can put the number of markers in multiplication sentence. Most of the students who were correctly represent it into multiplication sentence answered because there was 6+6+6+6+6, some students answered because there were 5 times of the 6. From their answer we concluded that they tended to transform the multiplication sentence from the repeated addition that that they got.

Problem 3

In this problem, we showed to the students a picture of tiles. We showed to them 6 rows of 7 tiles, or we can say 7 columns of 6 tiles. We asked the students to write their strategy to determine the total number of tiles and put it in multiplication sentence. We expected that they can represent the total number of tiles in two multiplication sentences that depend on their strategy to count the tiles. This problem was aimed to assess student ability to represent the total number of objects that arranged in rectangular pattern into multiplication sentence.

Based of the data that we had, most of students, 18 out of 27 students were able to represent the total number of tiles in multiplication sentence, further more they also found two ways to represent the total number of tiles in the picture and they did the correct calculation. While one student correct in representing the total number of tiles in multiplication sentence but wrong in calculation the total number of tiles. The rest, 8 out of 27 students, were incorrect to interpret the total number of tiles in multiplication sentence.

Problem 4

In this problem we gave the students, two multiplication sentences, we asked the students to looking for the pair of those two multiplication sentences that have equal product. This problem was aimed to know student knowledge about commutative property of multiplication.

Based on the data that we had, most of students, 24 out of 27 students, were correct to find the pair of two multiplication sentences that we gave to them. While 3 out of 27 did not correct to find the pair of two multiplication sentence that we gave them.

Problem 5

In this problem, we showed to the students a picture of dolls in the Rak(shelves). In the Rak, there were 7 rows and each row contains 3 dolls. Fives rows were full of dolls and 3 Rows were empty. This problem was aimed to construct distributive property of multiplication, we wanted the students can found that they can construct 7×3 by adding (5×3) from the number of dolls in rak, and (2×3) from the number that needed in order to make the Rak full of dolls.

Based on the data that we had, Some students, 13 out 27 students were correct, they can found that 7 × 3 can be constructed from (5 × 3) and (2 × 3). While 14 out of 27 students were not correct to do this problem, most of them were wrong in symbolizing the multiplication that they made, for example, they knew that the total number of dolls can be represented in multiplication sentence 7 × 3, but they were wrong in symbolizing the number of dolls in the Rak, or in the empty space of the Rak. Most of them wrote 3 × 5 for the number of dolls in the Rak and 3 × 2 for the number of dolls in the empty space of the Rak.

From the end assessment, we could draw following conclusions. Some of the students still had difficulties in learning multiplication. Most of students were wrong in transforming the repeated addition into multiplication sentence. It shows that the students needed more discussion in that part. However, most of the students were greatly influenced by the activities that they follow. Most of them were able to describe the objects now, they were able to see the number of groups and the number of elements in each group simultaneously. They constructed the idea of unitizing. From describing the objects, they did repeated addition to determine the total number of objects and transformed the repeated addition that they had into multiplication sentences. They also knew about commutative property of multiplication and distributive property of multiplication. However they still need more discussion in that part too.

G. Discussion

Classroom discussion is one of the important parts in our HLT. The idea of multiplication is discussed in that event, therefore the contribution from the students are highly expected in our class. However, the class that we took for teaching experiment did not used to do classroom discussion. It makes the teacher and students struggled to develop a constructive discussion. Based on our observation, not all students participated in the discussion. Only some students engaged in the classroom discussion while some others busy doing something out of lesson.

In this research, the teacher had a lot of experiences in teaching, but the classroom environment that we did was really new for her. She usually explained the material to the students and gave the students practices. But in this research, we gave the students problems or tasks, let them discuss in their group and gave them chance to present their idea in the class discussion. Therefore the role of teacher as orchestrator in the classroom discussion becomes one of important parts.

During the teaching experiment the teacher has shown a good performance in stimulating social interaction. She become an orchestrator in for students in learning processes, she collected the students' ideas and let the students to present their idea. However it was not easy for the teacher to manage the students' discussion, but she has been tried to do it.

Chapter VI

Conclusion and Recommendation

A. Conclusion

This chapter present the conclusion of the research findings in relation to the research question and recommendation for further studies. To answer our research question, '*How can structured objects promote students in learning multiplication?'* we looked at the sequences of learning activities and investigate what role of the structured objects serve in each sequence of students' learning. After that we can conclude how the role evolves during the activities.

This research hypothesized that students will not employ the structure unless they realize the benefit of structuring for counting and arithmetic. Therefore in lesson 1, 'Describing structured objects' was designed to evoke students' awareness of structure in which students learn to recognize the structure. Through this activity students start to use the structure for their mathematical reasoning.

The awareness of structure has given a basis for students to employ structure for further counting. Students' ability to see the number of groups and the number of objects simultaneously, provide a necessary input and organization for numerical procedures that students use to determine the total. We found that students started to use the structure by grouping. They started to count objects in groups and used several strategies to count the total. Some students did skip counting, by pointing to their representation, some students did repeated addition and some students did regrouped repeated addition that they had to make addition easier.

In line with Gravemijer (1994), this research shows that the structure have triggered students to employ the model of a real situation for solving problem. The schematized picture such as three bags of pens and three bags of four batteries allowed students to relate to real pens and batteries. Gradually, students were able to see the schematized picture separately from real objects and used it as a model for counting by group. This shown students' ability to use model as a bridge to move from concreteness to abstraction.

In order to come to multiplication sentence, students had to develop the language that related to multiplication. They started with *'bags of'* and '*boxes of*', then it develop into '*group of'*. When they had to determine the total object, it develop into '*times of*' which is connected to the idea '*add so many times'* or the repeated addition. After that multiplication symbol '×' was introduced to them that make them able to symbolize '*a times of b*' into $a \times b$.

For some students, transformed the repeated addition that they had into multiplication sentence, $a \times b$, is not easy. They tended put the number of multiplier and multiplicand in the wrong order in multiplication sentence. We found that it is because they influenced with the Indonesian language. For example, when they had the repeated addition, 5+5+5+5+5+5+5, they tended to say it in *bahasa ' limanya tujuh kali'*. Because the five (*limanya*) comes before seven (*tujuh*) in their word made them tended to put in multiplication sentence 5×7. The findings suggest that students need a bridge to transform repeated addition into multiplication symbol. The word '… times of…' provides a bridge to students to transform the repeated addition into multiplication sentence. However, understanding the meaning of that word became important. Students had to have knowledge about the word 'times' that they usually hear in daily life first.

After students are able to represent the repeated addition as multiplication sentence, understanding the property of multiplication became important parts. This research shows that structuring the objects and let the students to make the connection between one and the others multiplication facts lead the students to the property of multiplication such as distributive property and commutative property of multiplication. However they needed more activity to explore those properties of multiplication.

B. Recommendations

This section would like to give recommendation about RME approach in the classroom, about teaching multiplication in grade 2, and suggestion for further studies.

1. Realistic mathematics education.

In our RME classroom, the contexts plays important role to stimulate the thinking process of students. The students could bring their informal knowledge to get ideas in solving mathematics problems. One of the contexts that can be used when students learning multiplication is describing structured objects to who can not see the objects. This context can provoke the students to investigate the configuration of objects, which provokes the students to count in groups. This situation leads them to the idea of multiplication. But the most important is providing the situation for students to count since multiplication is a counting processes.

The contributions from the students are highly expected in the class. Stimulating the social interaction among the students in order to make them learned from each others solutions in the class discussion became one of important parts.

Giving the freedom to the students to present their idea, and let them together to decide whether the problem that they are solved correct or not could stimulate the free production and reflection. Therefore we recommended the teacher to develop socio-mathematics norms like that.

2. Multiplication

In Indonesia primary school, in the first time students learn about multiplication, they are directly got repeated addition which is transformed into multiplication with an equal sign. Students had to memorize the multiplication table without any idea what is that. This research shows that multiplication is a counting process for students. This finding support the previous work from Coney et al (1988) that the acquisition of multiplication for students starts with counting process, not just a memorizing table. Students need sequences of activity to get insight into multiplication.

The development of students' understanding of multiplication should be seen in step by step sequence. Students need to practice with concrete tasks, they need to see the configuration of groups of objects before they can represent in into multiplication sentence. When they see the configuration of objects, they know the number of groups and the number of elements in each group, it provokes them to count in group and the repeated addition arises as a strategy to determine the total of objects.

3. Further Studies

In our study, we only focus on a specific aspect of understanding multiplication especially to help them to come to multiplication symbols. Later research could also study other aspects of multiplication that was little studied in this research such as understanding the property of multiplication either commutative property or distributive property of multiplications.

The findings of our research raised some other questions such as how do the students use and get insight into the commutative property and distributive property in multiplication in order to shortened their calculation? Further research is needed to answer that question.

References

Anghileri, Julie. (1989). *An Investigation of Young Children's Understanding of Multiplication*. Educational Studies in Mathematics 20: 367 – 385. Netherland: Kluwer Academic Publishers.

Armanto, Dian. (2002). *Teaching Multiplication and Division Realistically in Indonesian Primary Schools:A Prototype of Local Instructional Theory*. Univesrity of Twente, Enschede: Doctoral Disertation.

Bakker, A. (2004). *Design Research in Statistics Education on Symbolizing and Computer tools*. Freudenthal Institute, Utrecht-the Netherlands.

Barmby, P, Harries, T and Higgins Steve. (2008). *The array representation and primary children's understanding and reasoning in multiplication*. Springer Science.

Buys Kees et al. (2000). *Wis en Reken*. Gronigen: Bekadidact Baarn.

Caron, Thomas A. (2007). *Learning Multiplication The Easy Way*. North Carolina; Heldref Publications.

Cooney, J.B and Swanson, H Lee. (1988). *Acquisition of Mental Multiplication Skill: Evidence for the Transition Between Counting and Retrieval Strategies.* Journal on Cognition and Instruction, 5 (4) : 323 – 345. Lawrence Erlbaum Associates, Inc.

Dolk, M and Fosnot. (2001). Young Mathematicians at Work; Constructing Multiplication and Division. USA: Heinemann.

Edelson, D. C. (2002). *Design Research: What we learn when we engage in design.* Journal of the Learning Sciences, 11, 105 – 121.

Freudenthal, H. (1991). Revisiting Mathematics Education. China Lecture. Dordrecht: Kluwer Academic Publisher.

Freudenthal, H. (2002). Didactical Phenomenology of Mathematical Structures. New York, Boston, Dordrecht, London, Moscow: Kluwer Academic Publisher.

Fosnot, Catherine Twoney. (2007). *The Big Dinner, Multiplication with the Ratio Table*. USA:First Hand Heinamann.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD Beta Press.

Gravemeijer, K. (2004*). *Creating Opportunities to Reinvent Mathematics*. ICME 10 2004. Regular Lecture.

Gravemeijer, K. (2004). *Local Instruction Theories as Means of Support for Teachers in Reform Mathematics Education.* Utrecht : Freudenthatl Institute and Department of Educational Research Utrecht University.

Gravemeijer, and Cobb. P. (2006). Design Research From a Learning Design Perspective. In J, Van Den Akker, K Gravemeijer, S, McKenney, & N. Nieveen(Eds.), Educational Design Research (pp. 17 – 51). New York: Routledge.

Gravemeijer, K, Van den Hauve, M & Streefland, L. (1990). *Contexts free productions tests and Geometry in Realistic Mathematics Education.* Reseachgroup for Mathematical Education and Education computer Center State University of Utrecht, The Netherlands.

Heege, H. Ter. (1985). *The Acquisition of Basic Multiplication Skills.* Educational Studies in Mathematics 16 (1985) 375 – 388. D. Raidel Publishing Company.

Kroesbergen, E. Helena. (2002). Mathematics ecuation for low-achiving students effects of different instructional principles on multiplication learning. Universiteit Utrecht: Doctoral Disertation.

Kroesbergen, E. H and Van Luit, J.E.H (2002). *Teaching multiplication to low math performer: Guided versus structured instruction.* Instructional Science.

Squire, S, Davies, C and Bryant P. (2004). *Does the cue help? Children's understanding of multiplicative concepts in different problems contexts.* British Journal of Educational Psychology (2004). 74, 515 – 532.

Van den Hauvel-Panhuizen and Tal Team. (2001). *Children Learn Mathematics.* Freudenthal Institute, Utrecht University, the Netherlands.

Van Galen, F and Fosnot. (2007). Groceries, Stamps, and Measuring Strips Early Multiplication. USA:First Hand Heinamann.

Wong, M and Evans D. (2007). *Improving Basic Multiplication Fact Recall for Primary School Students.* Mathematics Education Research Journal 2007 (Vo. 19, No 1, 89 – 106).

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

.

LIST OF APPENDICES