SUPPORTING STUDENTS' DEVELOPMENT OF THE CONCEPT OF SPEED

MASTER THESIS

Fatimatul Khikmiyah

097785007

SURABAYA STATE UNIVERSITY

MASTER PROGRAM

MATHEMATICS EDUCATION STUDY PROGRAM

2011

SUPPORTING STUDENTS' DEVELOPMENT OF THE CONCEPT OF SPEED

MASTER THESIS

A thesis submitted in partial fulfilment of the requirements for the degree of Master of Science in Mathematics Education at Surabaya State University

Fatimatul Khikmiyah

097785007

SURABAYA STATE UNIVERSITY

MASTER PROGRAM

MATHEMATICS EDUCATION STUDY PROGRAM

2011

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLE

LIST OF APPENDICES

ABSTRACT

Khikmiyah, Fatimatul. 2011. *Supporting Students' Development of the Concept of Speed*. Master Thesis, Mathematics Education Study Program, Master Program of Surabaya State University. Supervisor: (I) Dr. Agung Lukito, M. S and (II) Sitti Maesuri Patahuddin, Ph.D.

Keywords : *The concept of speed, design research, Indonesian fable, toy cars context, ratio table model*

Speed is one of the concepts, which can integrate mathematics and science. Prior researches show that speed is one of the most difficult concepts during the upper grade of elementary school. However, Indonesian elementary students usually learn the concept of speed directly to the formal calculation by introducing them with the formula of speed as distance divided by time and separated from students' experiences.

The main aim of this research is to design a local instruction theory to support students' development in the concept of speed. Hence, design research was chosen as an appropriate means to achieve the goals. A sequence of instructional activities with Indonesian fable and toy cars contexts was designed based on students' development and Realistic Mathematics Education (RME) principles namely 'guided reinvention', didactical phenomenology, emergent modelling and emergent perspective. A teacher and twenty-five students in fifth grade of SD Al-Fatah Surabaya (an elementary school in Indonesia) are involved in this research.

The result of the teaching experiments showed that the movement of toy cars could stimulate the students to learn the proportion of distance and time in speed. Moreover, through ratio table students could the proportion of them in systematic ways and further they could use it as the way to solve the problems as well as for their reasoning in explaining the relation among speed, distance and time

1 INTRODUCTION

Mathematics and science exist in children's surroundings although they do not aware with it. Consider children who lean wide plywood board against many blocks so that it becomes a ramp and start to move with gliding action on it. When their action is too slow, they begin to think how to make it faster. They might explore what happens if they make the steepness vary or use different length of plywood then observe and communicate their exploration to each other. In this example, children engaged in scientific observation, use the mathematical concept of speed, distance, length, height and counting (the number of blocks to support the ramp) as well the concept of ramp in physics. This example also explains how the context of speed gives the opportunity for children to integrate mathematics and science.

In informal situation, children are beginning to understand the meaning of words and phrases like "faster", "slower" and "as fast as" while they are seeing the object moves, a person walks or run, the car travels and so on. They also start to recognize and describe the positions of people or objects, how they move through the space in relation to other people or other objects. Moreover, they will try to describe and to compare the speed using their own words. We can see this when young children for instance told to their friend that "the flash", a superhero in comics, can run as fast as light. Although they use the phrases "as fast as" it does not mean that they know the concept of speed.

In formal situation, the concept of speed introduced to the students since in elementary school may with the consideration that it is a fundamental concept for learning kinematics in junior high school as well calculus in the higher education. However, according to Gravemeijer, et al (2007) the concept of speed is one of the most difficult in the upper grades of primary school. Children meet difficulties in understanding the concept of speed because they must take into consideration of two variables; distance and time (Piaget, 1971., Gravemeijer, et al., 2007). In fact, children have the tendency to look primarily at the distance travelled, and lose sight of the time that is required to travel this distance (Thompson&Thompson, 1992). Moreover, Piaget (1971) explains that children might compare properly the objects but still may not recognize the correct relationship between distance and time. From his four teaching experiments, Thompson&Thompson (1992) see a consistent pattern in children's development of the concept of speed in which they differentiate into four levels. However, only a few studies on the teaching and learning process for the concept of speed in the elementary school. Therefore, more study still needed to observe how students think and reason about the concept of speed.

The important principle in this study is what Freudenthal's idea that views mathematics as a human activity instead of seeing mathematics as a subject to be transmitted (Freudenthal, 1991). His idea then transmitted as Realistic Mathematics Education (RME), which has it origin in the 1970s. The instructional activities that we proposed here based on RME principles and more emphasize on 'guided reinvention', didactical phenomenology, emergent modelling and emergent perspective.

The main aim of the research is to design a local instruction theory to support students' development in the concept of speed, especially for elementary education in Indonesia that more emphasizes on formal mathematics in which the concept of speed given as "distance by time" rigorously. Because speed is distance by time then we can get the distance by multiplying the speed with the time and get the time by divided the distance by speed. Students in upper grade of elementary school in Indonesia mostly know this rule but it is difficult for them to give reasons behind. Hence, the research question of this study is:

How can we support students' development of the concept of speed?

In order to answer that research question, we design a sequence of instructional activities with Indonesian contexts, which facilitate students in developing their own models as one of their reasoning ways to explain the concept of speed and interrelation among speed, distance and time.

Definition of Terms

1. Support
2. Development
3. The concept of speed

   The concept of speed here means speed as a proportion between two different quantities, distance and time.

2 THEORETICAL FRAMEWORK

2.1 THE CONCEPT OF SPEED

In everyday usage, the terms ‘speed’ and ‘velocity’ are interchangeable (Serway, 2009). We may also wonder the use of letter v instead of s for speed. Cassidy et.al, (2002) explain that the concept of speed is closely related to the concepts of velocity, from which the symbol v arises. Speed is perceived as the property of an object that is relative to the ground (Doorman, 2005). Speed indicates how fast something is moving regardless of whether or how it may change the direction. In present study, we use term speed instead of velocity since we only concern with speed as the quantity of motion and this is what the students learn in elementary schools.

Although children have certainly become acquainted with speed, in the context of race, look closely at the speedometer of the car and so forth, the concept of speed is still one of the most difficult quantities in the upper grade of primary school (Gravemeijer, 2007). There are two main issues concern with this fact. First is a conceptual problem. Speed can be called as a compound quantity (Halloun&Hestenes 1985 in Doorman, 2005;Gravemeijer, in press), a rate (Thompson&Thompson, 1992;Abels, M., 2006), or a composite quantity (Gravemeijer, 2007;TAL Team, 2008). These terms refer to a proportion between two different quantities, distance and time. It is also the reason that in physics, speed classified as a derived physical quantity (Ellse et al, 2003). Since speed is a compound quantity then children must pay attention simultaneously to two quantities as they learn this concepts, distance and time. However, children have the tendency to look primarily speed at the distance travelled and lose sight of the time that is required to travel this distance, 40 km/h according to them refers to a distance of 40 km and the unit of time is implicit. This fact might be one of the reasons why students get difficulties in determining the time needed to take 250 meters if the average speed is 40 km/h.

The second issue is a gap between the concepts with daily life situation since the concept of speed refers to a type of imaginary situation. A man driving a car with 40 km/h is assumed would travel 40 kilometres if he drives exactly 1 hour. However, he does not have to drive with this speed for exactly one hour to have a velocity 40 km/h. The speed 40 km/h can represents both instantaneous and average speed. Instantaneous speed means speed at a moment that can be approximated with a time interval that tends to zero meanwhile average speed is related to distance travelled within a time interval (Doorman, 2005). As the children see the tool for measuring speed such as a speedometer in a car, they see instantaneous speed but as soon as we give meaning of speed we use a time interval and lost the instant. This is an aspect that not easy to reach for children.

In four teaching experiments with one fifth grader, four sixth grader and one whole class with 32 seventh grader, Thompson&Thompson (1992) see a consistent pattern in children's development of the concept of speed as follow ;

1. Speed as a distance

   Children begin thinking of speed only of distance-how far one goes(in one unit time). This conception often revealed in the way they express a speed in context-'he is going 2 meters" omitting the time unit (e.g., per second) or expressing an elaborated unit, “he goes 45 miles in an hour”. This image of speed does not involve time as an explicit quantity.

2. Distance increases then time follows

   Children begin to think that speed involves two separate quantities, distance and time. Speed as a distance moved in one time unit. Time is explicit in students’ awareness but the distance is prominent. They come to anticipate that travelling a number of speed-distance (how far one goes in one unit time) produces an identical number of time units but thinking of a number of time units does not automatically evoke an image of an identical number of speed-distance.

3. Distance and time accrue simultaneously

   Distance and time increase simultaneously and continuously. Moving for speed-distance implies moving for one time unit and vice versa. This is the first occasion wherein students reason about speed multiplicatively. The multiplicatively is not numerical but logical-distance and duration co-occur (Piaget, 1970). Here, children can view speed involve a constant proportion. The constancy is in the simultaneous increase of the same amount of distance and same amount of time and vice versa but it does not support an awareness the same proportion of the total distance and time.

4. Speed as a rate

   Distance and time increase simultaneously and continuously and in proportional correspondence. The proportion between the distances in time unit is in the same proportion between the total distance and time.

Researchers have shown that students have the tendency to look primarily at the distance travelled (Thompson&Thompson, 1992; Gravemeijer, 2007) and lose sight the time that is required to travel this distance. Historically, it has also taken a long time before people accepted that the proportion between two different quantities could be used as a unit. Hence, we believe that it is important for children to learn speed first as the type of measurement that involves distance and time. People use a speedometer as an apparatus to measure the speed but we cannot use it to measure the speed of animals or someone who moves. However, we can use the combination of apparatus of measuring distance and time such as measuring tapes and stopwatches to express speed. This situation can be problematical for students in elementary school so that it provokes them to analyse and gives them the opportunity to the process by which mathematics was invented.

Given students' difficulties in understanding the concept of speed and students' development of the concept of speed, in present research we design the instructional activities which in line with Realistic Mathematics Education (RME). By designing and enacting an instructional unit for fifth grade students, we try to create a learning environment in which we can observe how students think and reason about the concept of speed and it relations with distance and time.

2.2 Realistic Mathematics Education

Realistic Mathematics education is rooted in Freudenthal's interpretation of mathematics as an activity (Freudenthal, 1991). Mathematical activity involving solving problems, looking for problems and organizing a subject matter and mathematizing as the main activity (Gravemeijer., 1994). It was Treffers who placed the two ways of mathematizing. He distinguished 'horizontal' and 'vertical' mathematizing. In the case of horizontal mathematizing, mathematical tools are brought forward and used to organize and solve a problem situated in daily life. Vertical mathematizing, on the contrary, stands for all kinds of reorganizations and operations done by students within the mathematical system itself (Van den Heuvel-Panhuizen, 2003). The central principle of RME is that mathematics should always be meaningful to students. The term 'realistic' stresses that problem situation should be 'experientially real' for students. This does not necessarily mean that the problem situations are always encountered in daily life. Students can experience an abstract mathematical problem as real when the mathematics of that problem is meaningful to them (Bakker, 2004).

RME offers heuristic or principles for instructional design in mathematics education ; guided reinvention, didactical phenomenology and emergent modelling (Gravemeijer, 1994). The following session will describe these heuristics.

2.2.1 Guided reinvention

The reinvention principle in RME states that students should be given the opportunity to experience a process similar to the process by which mathematics was invented (Gravemeijer, 1994). It means that it allows the learners to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible. In this present study, we expect that students reinvent the unit of speed that involving the unit of distance and time as the way to quantify something moves. The reinvention principle can be inspired by the history of mathematics and the informal procedure. In order to support the reinvention principle, the appropriate contextual problem that allow for a wide variety of solution procedures need to considered.

Didactical Phenomenology

A didactical phenomenology refers to looking for the situation that creates to be organized by the students. Such an analysis investigates how the concepts we want to teach help organise this situation, and how they can be problematical for the students (Doorman, 2005). The phenomena “beg to be organised” is used as the starting point for education. We use the phenomenon of measuring speed of humans or animals moves in daily life as the starting point since one cannot use a tool like a speedometer to measure but indeed, he can use the combination of apparatus for measuring distance and time to measure the speed.

2.2.2 Emergent Modelling

In the third principle, self-developed model play as the bridge of the gap between the informal knowledge and formal mathematics (Gravemeijer, 1994). In RME, it is expected that the students develop the models by themselves. Materials, visual sketches, paradigmatic situations, schemes, diagrams and even symbols can serve as model (Treffers and Goffree, 1985; Treffers 1987, 1991; Gravemeijer, 1994a in Van Heuvel - Panhuizen, 2003). The models first appear as a *model of* the situation that means very close to the problem situation at hand.

Then gradually it generalized over situations become a model for mathematical reasoning to foster higher-level ideas.

The concept of speed is a type of imaginary situation as we explained before. Here students need materials, visual sketches, schemes, and so on to organize the situation. In present research, we will never know exactly what kinds of models that will appear. However, the sophisticated models for the concepts of speed are models for proportional reasoning, which are the ratio table and the double number line because they both offer a systematic way of writing down the relation between distance and time (Gravemeijer, et al 2005).

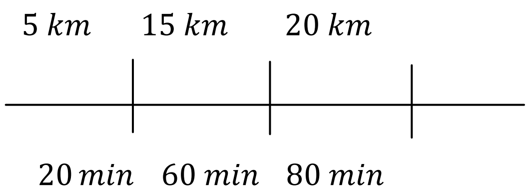
A ratio table is a mental model as well as a work sheet. It plays an important role in the curriculum on proportion (Gravemeijer, 2007). Ratio table is an ideal aid for making handy calculations and gaining insight because the table invites students to write down intermediate steps. Moreover, they explain that the strength of the ratio table is that students can reason with number relationship that they already know. To calculate how long someone will take to cycle 20 km at an average speed of 15 km/h, one can do as :

km	15	5	20
minutes	60	20	80

From the table above we can see clearly every intermediate steps and what they mean. In this case, 15 km in 60 minutes means the same as 5 km in 20 minutes. From the table also students might immediately see that we can go from 15 to 20 with 5 as an intermediate step. Other students might need more intermediate steps to calculate from 15 km to 20 km so that he/she makes a ratio table as shown below:

km	15	30	10	20
minutes	60	120	40	80

Another model that can be used to explain the concept of speed and it relations between distance and time is double number lines. A double number line has the same principle with a ratio table but the position of points in double number line is meaningful. This model allows students to link distance and time together; it is allow students to make accurate calculations and estimates as well. To calculate how long someone will take to cycle as the problem above, a double number line will show as follows:



Above we can see that the intermediate steps that had been used is actually same with the first example in the ratio table; 15 km in 60 minutes means the same as 5 km in 20 minutes hence multiply the proportion of 5 km in 20 minutes by 4 would get 20 km in 80 minutes. All of the models show the same result in which with an average speed of 15 km/h someone need 80 minutes to travel 80 km of distance.

Ratio table and the number line model can work together for the concept of speed and it relations between distance and time. However, these models are not familiar in Indonesia even in mathematics textbook so that we conjecture that it is difficult for students to reinvent by themselves. Nonetheless, the contexts used in the learning process give students opportunity to make the model of the situation. Afterward, teacher can provoke them to make a table that has the same principle with a ratio table but more in vertical forms since students' book provide the lines in horizontal way so that make a vertical table is easier and more natural than a horizontal table.

2.3 EMERGENT PERSPECTIVE

Emergent Perspective is an interpretative framework for analysing or interpreting the classroom discourse and communication. This perspective can be used to understand mathematical learning in the social context of the classroom. In this study, we pay attention mostly in social perspective. In specific, related to social norms and socio mathematical norms. Social norms refer to expected ways of acting and explaining that become established through a process of mutual negotiation between the teacher and students. Meanwhile, the socio-mathematical norms are the ways of explicating and acting in whole class discussion that are specific to mathematics. In other word, it related to the teacher in which what he wants mathematics to be for his or her students. Examples of socio-mathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, and so on (Gravemeijer, 2006)

Before the learning process, we conjectured that both teacher and students have their own beliefs about their roles. During the learning process, it is expected that teacher initiates and develops social and socio-mathematical norms as adjustment with the instructional design. The development of social perspective in the learning process will bring the students to the more democratic in the way of thinking.

2.4 The Concept of Speed in Indonesian Curriculum

Based on Indonesian curriculum, the concept of speed in is taught in the five grade of elementary school. The table describes the concept of speed in fifth grade of elementary schools.

Table 2.1 Learning speed for elementary students' in Indonesian Curriculum

Table 2.1

The First Semester of Grade 5	
Standard Competence	**Basic Competence**
Geometry and Measurement Using measurement of time, angle, distance and speed in solving problems	- Write time notation by using 24 hours notation - Calculation in the unit of time - Measuring the angle - Acquaint with the unit of distance and speed - Solving problems related to time, distance and speed

As explained above that in order to understand the concept of speed as a rate students must first conceive the length of displacement, then perceive a measure of duration and be able to compare the time intervals of different events. Meanwhile, the preliminary concept of its are measurement length and time in which studied since in grade two and the interrelation among unit measurement in metric system is studied in grade 3. Therefore, we assume that students in fifth grades, who are 10 to 11 years old, are able to learn the concept of speed.

3 METHODOLOGY

In this chapter, the researcher will desribe the research methodology, research subject, how data wull be gathered and the methods used to analyze it including the reliability and validity.

3.1 DESIGN RESEARCH METHODOLOGY

As described in the first chapter that the main aim of this research is to develop a local instruction theory for the concept of speed. For this purpose, the present research falls under the general heading of design research. According to Bakker (2004) design research, developmental research, and design experiments all treat design as a strategy for developing and refining theories. Furthermore, he explained that the main objective of design research is to develop theories together with instructional materials. In order to reach the research aim, a sequence activity with Indonesian context was designed to improve educational practises in learning the concept of speed for five graders of elementary school.

A design research which is used consists of cycles of three phases as following:

1. Preparation and design phase
2. Teaching experiment
3. Retrospective analysis

It is evident that the relevant theory about the topic should be studied before designing the research. In this research, the theory came from various sources in mathematics, science, realistic mathematics education, design research and teaching experiment as the initial conjectures for the path of the learning process. Then, the researcher collected Indonesian contexts together with the activities. Indonesian context used as the uniqueness meanwhile the important criterion for selecting the activities were its potential role towards the end of the goal. Both of these resources carried out in the discussion with the supervisors and colleagues in order to make a design of the Hypothetical Learning Trajectory (HLT). This hypothetical learning trajectory assumed could be adjusted with the actual learning process in the teaching experiment.

The teaching experiment will be conducted in the second semester of the fifth grade in the academic year 2010/2011 during three weeks (eight lessons) with 70 minutes of the duration for each lesson. The purpose of this phase is both to test and to improve the conjecture of the learning trajectory that was developed in the previous phase and to collect the data for answering the research questions.

A further phase of the methodology is the retrospective analysis. In this phase, HLT compared with the actual learning process so there might be a revision or reformulation of the HLT as the adjustment with the actual conditions. In order to answer research questions, the whole data then be collected, interpreted, analysed and confirmed.

3.2 RESEARCH SUBJECTS

The study will be conducted with a teacher and twenty-three students in the fifth grade of SD Al-Fatah Surabaya. The students are about 10 to 11 years old. Based on the curriculum, they learn the concepts of time and length as pre-knowledge of the concepts of speed since in the second grade.

3.3 Data Collection

Various data sources are collected in order to follow the learning process and to answer research questions. Hence, the data collection of this research is described as follows:

a. Video

During the teaching experiment, two cameras record videotapes; one camera as a static camera to record the whole class activities and the other camera records the discussion in the group of students. There are also videos of the interviews with the teacher and the students.

b. Written data

The written data from students' work in solving the problems during the class activity and in the final assessment provide more information about students' achievement in learning the concept of speed. Therefore, the written data here include students' work during the teaching experiment, observation sheets, the results of assessments including the final assessment and some notes gathered during the teaching experiment.

3.4 Data Analysis, Reliability and Validity

3.4.1 Data Analysis

The data were analyzed retrospectively use the HLT as the guideline. Meanwhile, the process of data analysis involves making sense out of text and image data. It involves preparing the data for analysis, moving deeper and deeper into understanding the data and making interpretation of the larger meaning of the data (Creswell, 2003). Hence, first the researcher transcribes the episodes that could inform the HLT and answer the research question. Then, the conjectures about the students' learning and views are generated, tested at other episodes and other materials (students work, field notes, and tests). The test for the confirmation and counter the examples. The analyses also discuss with colleagues and the supervisor in order to recheck our interpretation and probably could think alternative interpretations.

3.4.2 RELIABILITY

- Data triangulation

  Data triangulation is built from different data sources, such as the videotaping of the activities, students' work and notes from the observer.

- Intersubjectivity

  Intersubjectivity is applied by involving colleagues in the observations and in analysing the data.

- Trackability

  Trackability means that the reader must be able to track the learning process of the researchers and to reconstruct their study: failures and success, procedures followed, the conceptual framework used, and the reasons to make certain choices must be all reported (Bakker, 2004). In this study, we describe the process of the study systematically to offer other researchers the possibility to retracing our conclusions through the data analyses and teaching experiments.

3.4.3 VALIDITY

- HLT as means to support validity

  In order to support the validity of this study, we use HLT as the formulation and verification of the conjectures about the students' development. The teaching experiments and data analysis are the result of either verification of the conjectures' or in adjustments or new conjectures for subsequent experiments.

- Triangulation

  The validity supported by different methods of data collection as we explained above.

4 HYPOTHETICAL LEARNING TRAJECTORY

The development of an instruction theory according to Bakker (2004) includes both the design of such instructional means and research of how these support in students' reasoning. The first part implies that it is important to design an instructional environment that supports both the learning process and students' reasoning. Dealt with this, Simon in Simon and Tzur, (2004) offered the Hypothetical Learning Trajectory (HLT) as a way to explicate an important aspect of pedagogical thinking involved in teaching mathematics. An HLT consists of the goal for the students' learning, the mathematical tasks that will be used to promote students learning, and hypotheses about the process of the students' learning.

An HLT has different functions depending on the phase of the design research (Bakker, 2004). During the preparation phase, the HLT guides the design of instructional materials that have to be developed or adapted. During the teaching experiment phase, the teacher and the researcher use the HLT as a guideline what to focus on in teaching, interviewing and observing. Meanwhile, the HLT functions as a guideline to determining what the researcher should focus in the retrospective analysis phase.

The end goals of the learning process are students can develop the concept of speed as a compound quantity and use their proportional reasoning in order to explain interrelation among speed, distance and time. Based on these endpoints, we hypothesized the learning trajectory into six phases, reviewing students' informal knowledge of speed, measuring through comparing, Analysing distance and time growth during the movement, developing models and notating speed in a standard unit, converting units of speed and exploring interrelation among speed, distance. As the last part of the sequence, we asses students' development of the concept of speed that they learned before. The following is a description of the Hypothetical Learning Trajectory used in this present research:

4.1 An Indonesian fable for reviewing students' informal knowledge of speed

Goal (s) :

Reviewing students' informal knowledge and a general discussion of motion and speed

An Introduction part :

We can see motion from our daily life, a baby crawls, mother walks, a boy run, animals running fast to catch up its prey, people ride bicycles or motorcycles, cars travel in the street and so on. Human use speed to quantify motion so that it is easy to answer a question such "Which one is faster" and "How fast is something moves?"

A famous folktale for Indonesian children relates to the concept of speed is a fable 'Kancil dan Siput', so that is why it used in the introduction part of the teaching and learning process. A Kancil known as a mouse deer and siput is a snail. Mouse deer lives in the jungles of Africa, Asia, and many Pacific islands. It is an animal about the size of a cat, has the legs and the tail of a deer, and the face and the body of a mouse-but it is not really a mouse or a deer. The mouse deer eats only plants, but lots of animals eat the mouse deer. Hence, to stay alive, it must be quick and smart. Based on the fable, Kancil challenged snail to compete in running. Snail accepted that challenge on condition that he decided the rules and as the result, the snail won the competition.

During the learning process, first teacher can asks students whether they know the fable, what is ‘Kancil’, what is ‘Siput’ and what is the fable told about the result of the race competition between ‘Kancil and Siput’. It is expected that students will criticize the fable with the idea that it is impossible for ‘Siput’ winning the competition instead of Kancil since normally kancil is faster than siput. Students may also say that there was un-fair competition happened. It is also expected that they come up with the reason behind such as :

- if kancil and siput run at the same starting point and the same time then we’ll can see immediately that kancil will left siput behind.
- siput may start runs first then kancil runs five minutes after, then it is not difficult for kancil to overtake siput.

However, students who did not know what is kancil and siput and how its run may accept that nothing wrong in such situation since it is in a folktale. Teacher can anticipate it by showing the picture of kancil and snail, ask some students to pretend as kancil and siput then ask them walk imitatively. After this introduction part, students are asked to work in a small group (4 or 5 students) for around 10 minutes to solve this following problem.

Problem(s) :

1. Make an order of these animals based on its speed!

2. In your opinion, what is the fastest and animal and discuss the way to compare the speed of the animal in order to know the fastest animal.

Conjectures of students' thinking :

It seems very difficult for the students to order all of the animals. They can determine the slowest animal but to order the entire animal in a correct way then decide the fastest animal is not so easy. However, students might use their informal knowledge such as from daily life, from the article, from the video or television that shows how the animals run etc. In deciding which one is faster between a giraffe and a kangaroo for instance, students might say since a giraffe has long legs than a kangaroo then a giraffe is faster than zebra. Other students might also say that a kangaroo is faster than a giraffe because they saw how these animals ran in the television, etc. There are also possibilities that some students only guessing. In the discussion of the way to compare the speed of the animals, students might come up with the idea of a running competition like in the fable.

Discussion :

It is predicted that there will be various answers in ordering entire animals and deciding the fastest one. Teacher can use this as a starting point for general discussion of speed. Teacher can start with ask the students whether they feel confidence with their answers, if there are various answers in ordering the animals and deciding the fastest one what will they do to get the correct answer? How can they compare the speed of the animals? In addition, why they use that way? To answer the questions students might come up with the idea of searching the speed of the animals in the internet; hold a running competition for the animals, etc.

Human moves and so do the animals. We can see directly who is the fastest in human running competition although the number of participants is so many because they know that they must run in certain distance with certain rule but doing the same for animals is not so easy. Children realize that the fastest is the one who first reach the finish line but it does not mean that they will relate it with the distance and time directly. Hence, teacher should emphasize that the important thing of a running competition is the fastest or the winner is the one who has least time to go to the finish line or with other words at the same distance, the winner needs least time than the others.

Another way to compare the speed of the animals with the same idea with a running competition is measure the time of each animal to reach certain distance one by one. From here, we can see directly how the distance and time involve in comparing the speed. If the second way is not appears during the class discussion then teacher can tell students about the experiment by a British man who measure the speed of cheetah in Kenya. First, he measured off 200-meter dash, a distance that sprinters run throughout the world. The track was on level ground and marked by two posts at either at start or finish. At the start, he tied a piece of white wool yarn between the posts. At the finish, he marked a line on the ground between the two posts. To make sure his stopwatch was correct; he checked it against two stopwatches of the Kenya Athletic Association. Then he picked a quiet, windless day for the experiment. To get ready for each test, the cheetah was held around 17 meters behind the starting line. A truck, with its engine running and ready to go, was 70 meters down the course. The coach stood in the back of the truck. In one hand, he held the stopwatch, in the other a piece of meat that had been shown to the cheetah. He shouted for the cheetah to be released, and started the stopwatch when she broke the yarn at the starting point. A driver revved up the truck to stay ahead of the cheetah until well beyond the finish line. The coach stopped the watch when he saw the chest of the cheetah cross the finish line. Then he threw down the meat to let the cheetah eat it. Two more trials were made, with a thirty-minute rest between them. The times for the three trials were 7.0, 6.9, and 7.2 seconds then he decide that the time needed for the cheetah is more or less 7 seconds.

These two ways are useful in introducing the speed but most of the time it is only focus on the same distance instead of using the same time. However, students can also decide which is the fastest by measuring the animals' distance reached in a given time. The fastest animal is an animal that reach furthest distance in the same time. From the ways to compare the speed of the animals, it is expected that students realize that speed is involving the distance and time, speed in line with distance whereas speed is inversely with time. If something is go faster then the distance travelled is further but it need less time to reach given distance.

4.2 MEASURING THROUGH COMPARING

A Contextual situation:

In this activity, we use 'toy cars' to change the context of animals since students are familiar with toy, it is possible for them to investigate the speed of the toy cars directly and using toy cars we can keep the speed in constant. Next, the term a car used here refers to a toy car.

Goal (s) :

Students can compare the speed of two toy cars

Mathematical ideas :

- Complete motion involves distance travelled and amount of time required to travel that distance.
- Comparing speed means compare the distance and time simultaneously

Problems :

1. Compare the speed of these two toy cars. Which car is faster?
2. Make a drawing of your investigation of those two cars so that other persons know how fast the car compares with the other! How many times is it faster than the other?

Description of the activity(s) :

The students are asked to work in small groups (4 or 5 students). They would given two cars and a stopwatch then discuss and solve the problems for around 30 minutes. Teacher will not tell the way to compare the speed. After that, students will present their work in the class.

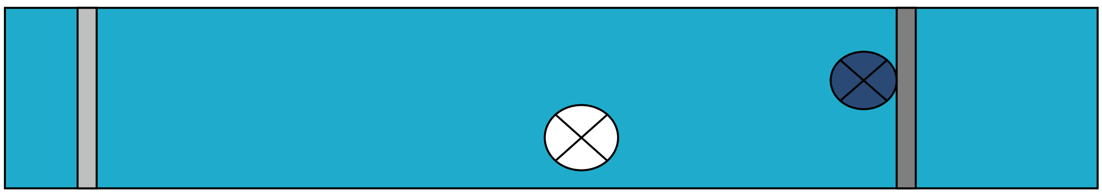
Conjectures of students' thinking and discussion:

In deciding the faster, the students might come up with these strategies:

- Carry out a race competition;
  - Students in the group hold the car, put the car side by side, let the cars go and see the car which overtake the other car is the faster one
  - Students in the group hold the cars, put the car side-by-side, let the cars go and some students in front of them are ready to catch the car. The car that comes first to them is the faster car.
- Compare the speed numerically
  - Students in the group make a path for the cars, let the cars run one-by-one and measure the time needed for each car, the car needs less time is faster than another does.
  - Students in the group hold the cars, put the car side by side then let the cars go. Meanwhile, one student keeps the time and other students stop the car at certain time that they want. The car that goes further is the faster.
  - One Student in the group keep the time, one student hold and let the car goes one by one, and other student stop the car in the time that they choose. After that, they measure the distance travelled by the car. The faster car is the car that goes further in the same time

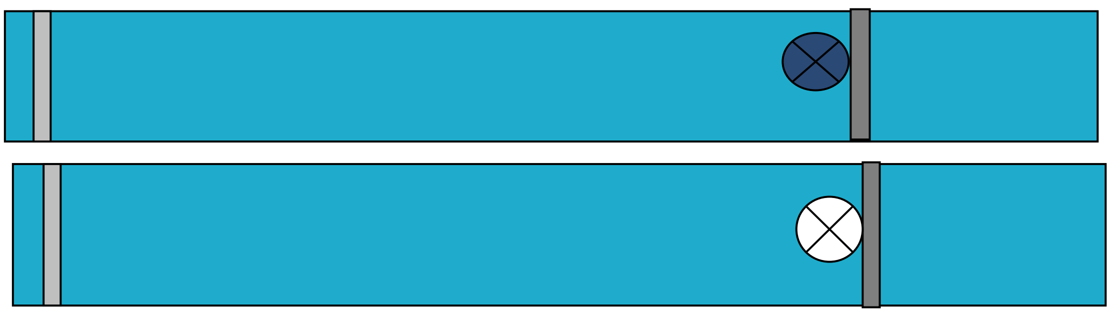
Students are free to choose the way they used to compare the car, so that there might be various ways appeared in the class. Nevertheless, it is important to generalize two possible approaches, namely using the given distance and the given time. By the given distance, the car that need less time is faster. Inversely in the given time, the car travelled further is the faster. Students can see this relation clearly, if they make a drawing of it as mentioned in the second problem. Through the problems, we also expect that children realize that it is important to quantify the speed so that we know exactly how the car goes fast than other. By quantifying the speed students can say for instance, the car A is twice faster than B, three times faster etc. Students might draw as following:

- Drawings of the race path



From the drawing children can explain that car A need less time than B so that it is faster. Teacher can use this drawing to discuss such as if the faster car reach the finish, where is the other car at that time so that children also come up with the idea that the faster car reach further distance in the same time. After that how can children know how much is it faster?, Almost twice, three times and so on. We expect that students will come up with the idea of measuring the distance.

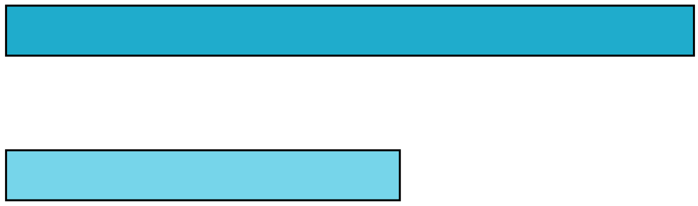
- Drawing of each car travelled for instance 4 meters.



- Make a drawing that after 5 sec for instance:

The distance of car A :

The distance of car B:



The advantage of the drawing above is students can compare in a visual way. We can see that for instance car A is almost twice faster than car B through these drawings. Although not all might appeared in the class, teacher still can continue the discussion like :

- If it is only the first drawing appeared; we know from the drawing that car A reach the finish line earlier so that A is faster than B. Then at the time A reach the finish line, where is the position of the B?
- If it is only the second drawing appeared; we know from the drawing that car A need less time to reach the finish line. Hence A is faster than B. Then, if the cars travelled with the same time, how far is A? Is it have the same distance with B or how?
- If it is only the third drawing appeared; we know from the drawing that car A reach further distance than B. Hence, A is faster than B. Then which car need less time to reach for instance one lap in race competition?

Through the discussion, we expect that students will realize that comparing speed not always by putting side by side. We can compare it by measuring the distance and time simultaneously when something is moving, how much the distance travelled in a given time or vice versa. Then teacher can continue that students can not show precisely how fast something than other from the drawing. It is also difficult for them to explain if they asked to compare many cars for instance if each students in the class or even in the school bring their cars and they want to compare it all. Hence, they need to find the way to state the speed in such a way other people know which car is fastest, the second fastest, the third and so on and how fast it is.

4.3 ANALYSING DISTANCE AND TIME GROWTH DURING THE MOVEMENT

Goal (s) :

- Students can measure and estimate the time needed by toy cars to travel given distance
- Students can draw the situation of the movement of the cars to travel given distance.

Mathematical ideas :

The distance and time increase simultaneously and continuously during the movement.

Problem(s) :

1. Investigate the time needed by the car travelled in 2 meters!
2. Predict how much time it will take if the car continues to travel once around the class! Why is it so?

3. Make a drawing of the car travelled the class and give an explanation in the increasing the distance and time!

Description of the activity :

As the starting point of the activity, teacher will review what the students did in previous lesson in which students compare the speed of two cars and possible approaches to compare the speed. In this activity, students will compare various cars by measuring the time needed for each car to reach certain distance, make a prediction of the time if the speed is constant then analyse the growth of the distance and time during the cars travelled. Teacher asks children to work in small groups then gives them a car (each group has a different car), a stopwatch, a measuring tape and the problem as well. After the group discussion, students will present their works in the class.

Conjectures of students thinking:

- **Problem 1**

- Students will make a path of 2 meters; choose the start and the finish line. One student holds the stopwatch and another student holds the car. After that, they let the car run and stop the stopwatch while the car reaching the finish line. Students may only doing once or more in order to get the valid data.
- Students only make a path of 1 meter; choose the start and the finish line. One student holds the stopwatch and another student holds the car. After that, they let the car run and stop the stopwatch while the car reaching the finish line. The time needed to reach 2 m is twice of the time needed for 1 m path.

- **Problem 2**

- Students measure the length and the width of the class then calculate its perimeter. After that, based on their works in problem 1, students estimate the time needed for the car to go around the class once as follow :

  The car travelled for 2 meters in 5 second

  The length of the class is 6 meters

  The width of the class is 5 meters

  The perimeter of the class is $(2 \times 6) + (2 \times 5) = 22$ meters

  The time needed to around the class once is

  $$(22 : 2) \times 5\ seconds = 11 \times 5 = 55\ seconds$$

- Students measure the perimeter of the class by measuring the whole side of the class. After that, based on their works in problem 1, students make a calculation to estimate the time needed for the car to go around the class once as follow :

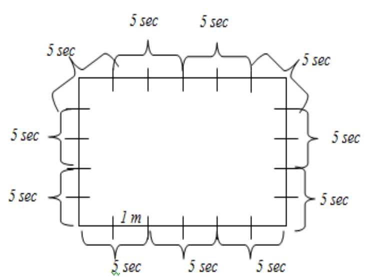
  The car travelled for 2 meters in 5 second

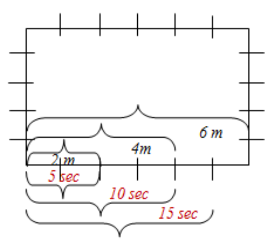
  The perimeter of the class is 22 meters

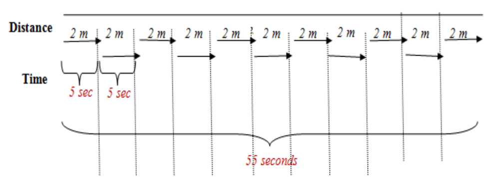
  The time needed to around the class once is

  $$(22 : 2) \times 5\ seconds = 11 \times 5 = 55\ seconds$$

- *Conjectures of students' drawing in Problem 3*







Discussion :

Teacher can start the discussion with students' answers in the first problem. Since there are different cars in each group then the answer also might be vary. From various answers teacher can ask students such as, which car is fastest? If the time needed for a car in-group A is the least one than what will be for the second problem, Will it does so?

After that, teacher can come to the second problem by analysing students' work in determining the perimeter of the class as the distance travelled for the cars. The accuracy is on the discussion instead of children's way of measure the perimeter. However, teacher can also use this to assess students' knowledge in measuring the length as the basic knowledge for distance. The discussion then continues with students' answer in determining the time needed by the cars to round the class. We do not expect that students really let the cars travelling the class to come to the answer. However, we expect that they realize that if they know the time needed by the cars to travel in 2 m and know perimeter as well, then they can predict the time needed to round the class once, even twice, three times and so forth.

Teacher can continue the discussion by analysing students' work in drawing the journey of the car around the class. Through these drawing, we expect that students can see the situation in which during the cars travel, both distance and time growing simultaneously and continuously and the accruals of quantities stand in the same proportional relationship with their total accumulations. If there is more time, students can do an investigation outside by letting the car travelled outside in various distance for instance 6 m, 10 m etc and teacher asked them to measure the time. Through this investigation, we expect that students fell confidence with the prediction of such situation.

4.4 DEVELOPING MODELS AND NOTATING SPEED IN A STANDARD UNIT

Goal(s):

- Students can determine speed in a standard unit
- Students can develop their own models as a tool for solving problems

Mathematical ideas:

- Distance and time have proportional correspondence
- Speed as a compound quantity determine by the distance travelled in a unit time.

Problem(s):

1. Make a long list of the distance travelled by your car after many seconds on the table!
2. How do you determine the speed of the car? Explain your answer!

Description of the activity:

In this activity, children use the result of their investigation in previous lesson. They still works in the same group to solve the problems given around 20 minutes then present their works in the class.

Conjectures of students' thinking:

Problem 1

- Students use the table to structure the list

Distance (meters)	Time (seconds)
2	5
4	10
...	...
24	60
1440	3600

Problem 2

- The speed of the car is 2 meters in 5 minutes
- The speed of the car is 0,4 m/s
- The speed of the car is 10 seconds for 4 meters
- The speed of the car is 24 meters in one minutes or 24 m/minute
- The speed of the car is 1440 meters in one hour or 1440 m/hour

Discussion :

From this activity, students will explore more their prediction in the distance travelled by the car but more focus on the number. Through the presentation of children works, we expect that there are various ways to show how the distance and time increasing during the journey. Whatever the way they used, the numbers shows clearly the increasing multiplicatively of the distance and time. On the other words, it shows proportion between distance and time in which always the same. When the distance increases twice, three times and so forth, then the time increases as well. The proportion of increasing quantities is in the same proportion with the total distance and time. The table also make possible for the teacher to say about the distance travelled by the car in one minute, one hour, two hours or even one day for instance because 60 seconds means 1 minute, 3600 seconds is one hour and so forth.

The discussion on the second problem emphasize on a standard unit of determining the speed. Possible answers such as in the conjectures are correct but some students may know how the speed usually state in daily life as like in metric system. In metric system, speed is usually expressed in kilometers per hour (km/h) or meters per second (m/s). When people use the standard unit of speed then it is easy for them to compare speed around the world. The discussion then focuses on the way to express the speed in m/s.

4.5 CONVERTING UNITS OF SPEED

Goal (s) :

- Students can read and explain what the sign road means and what the speedometer shown
- Students can distinguish between maximum, average and instantaneous speed
- Students can convert the units of speed

Mathematical ideas :

- Instantaneous speed means speed at a moment
- Average speed related to distance travelled within a time interval
- Speed can be expressed in m/s or km/h

Contextual situation :

In the previous activity, students measure the speed of the toy cars. They can express the speed of the toy cars in meter per second (m/s). However, in daily life it is easy to see that speed expressed in kilometres per hour (km/h) for instance in a speedometer and the road sign. Hence, in the first part of the lesson teacher bring the picture of a road sign and a speedometer as the part of the discussion.

Description of the activity(s) :

Teacher starts the activity by giving the poster of a sign road and a speedometer to the students. After that, teacher guides an interactive discussion about it. First he/she can ask the children such as "What is the sign road explain?, What is the speedometer shown? How does the speedometer point to the number, is it fix or changeable? If the speedometer points into 40, could you explain what it means?" After class discussion, teacher then asks students to work in small group to solve the problems below in around 30 minutes. Teacher then discuss the answer in the class by asking the students to explain their answers, posting questions, giving feedbacks and reinforcements on it.

Posters for class discussion

What is the road sign explain?

- What does the speedometer show? If the speedometer points into 40, could you explain what is it mean?
- Can we drive with 40 km/h in 10 minutes?

Conjectures of students' thinking and discussion :

- Conjectures of students' answer in first class discussion

  A sign road poster

  - The sign shows that when we enter the campus area, the speed of the vehicle must be 10 km/h
  - The sign shows that when we enter the campus area, we must walk
  - The sign shows that when we enter the campus area, we can't drive fast
  - The sign shows that when we enter the campus area, the speed of the vehicle must be 10 km/h at maximum

  A speedometer poster

  - First Question
  - Speedometer shows how fast someone rides the motorcycle or the car. If the speedometer points into 40, it means that our speed is 40 km/h at that time and we can reach 40 kilometres of distance in one hour.
  - Speedometer shows the speed of motorcycle or the car. If the speedometer points into 40, it means that it is not very fast.
  - Second Question
  - No, we cannot drive with that speed in 10 minutes since the speed express in one hour
  - No, we cannot drive with that speed because 10 minutes is less than one hour

- Yes, we can because when my father start to move then I see that the pointer in speedometer also start to move
- Yes, we can because the speedometer shows the speed at that time and the pointer sometimes moves up if we go faster and moves down if we go slower

The problem given in the class for reviewing students' knowledge based on the discussion in the previous lesson. Students had stated the speed of toy cars in m/s, for instance if the speed of the car is 3 m/s then the toy cars will travel 3 m in 1 second, 6 m in 2 seconds and so forth. On the discussion, teacher should emphasize that as km/h means that one assumed can reach the distance of 40 km if he travels in one hour, 80 km in two hours and so forth. The difference between the speed of toy cars and the motorcycles, cars etc is in the toy cars, the speed is constant but in motorcycles, cars, etc since sometimes one should aaccelerate or put on the break. However, using mathematics people can predict the time needed or the distance travel if they know the speed.

The discussion is also about maximum, average and instantaneous speed. Teacher can first ask students what they know about maximum speed then emphasize that the sign shows the maximum speed, from the word maximum it means that people are forbidden to enter the area with the speed more than 10 km/h. Meanwhile, the speedometer shows instantaneous speed or the speed only in that time so that it can move up when it goes faster and move down when it goes slower. However, we can estimate the average speed by measuring the distance travelled for a certain time as such in the investigation of the toy cars.

The maximum speed of the motorcycles may more than 140 km/h, instantaneous speed is usually fluctuating and it makes an average speed. Hence, the average speed is always lower than (or equal to) the maximum speed. The average speed can be the maximum speed if for a certain time interval, it moves all the time with the same speed.

Group discussion problem:

1. Arman has jogging as his hobby. He said that last week he ran 60 meters in 15 seconds. Can you predict the Arman's distance in one hour ?
2. Adi likes jogging too. He said that yesterday he ran 50 meters in 12 second. Is Arman's speed higher or lower that Adi's?
3. Find Adi's speed in m/h and km/h?

Conjectures of students' answer and discussion :

Problem 1

The answer will vary, sample answer are :

- Students use a vertical table in two columns to solve the problem

Distance	**Time**
$60\ m$	$15\ sec$
$120\ m$	$30\ sec$
$240\ m$	$60\ \text{sec} = 1\ min$
$14400\ m = 14{,}4\ km$	$60\ \text{min} = 1\ hour$
$28{,}8\ km$	$2\ hours$

Hence Arman's distance in one hour is $14{,}4\ km$

- Students use a vertical table in four columns to solve the problem

Distance (km)	Distance (m)	Time (sec)	Time (hour)
-	60	15	-
-	120	30	-
-	240	60	-
14,4	14400	3600	1
28,8	-	-	2

Hence Arman's distance in one hour is 14,4 km

Problem 2

Students might come up with the methods as they proposed in the previous problems. They already have the table of Arman's distance and time so that they continue with the table for Adi

- Students use a vertical table in two columns

Distance	Time
$50\ m$	$12\ sec$
$250\ m$	$60\ \mathrm{sec} = 1\ min$
$15000\ m = 15\ km$	$60\ \mathrm{min} = 1\ hour$

Hence, Adi's distance in one hour is $15\ km$

- Students use a vertical table in four columns

Distance (km)	Distance (m)	Time (sec)	Time (hour)
-	50	12	-
-	250	60	-

15	15000	3600	1
30	-	-	2

Adi's speed is 15 km/h meanwhile Arman's speed 14.4 km/h hence Adi is faster than Arman's

Problem 3

Based on students' calculation in problem 1, they conclude that Adi's speed is 15000 m/h or 15 km/h

Discussion :

Teacher may discuss briefly all the various ways from the students. Then the focus of the discussion is about the effective ways in converting the speed. The tables above can use as a handy way to solve the problems and explain more about their reasoning. After the discussion of the problems, teacher can propose another question such as "will Arman really be able to run that distance in one hour?" in order to discuss further about human speed in running. Answer from the children might be varying. The sample answers are :

- No, he can cot run exactly in that distance because he will be exhausted
- No, it is not likely that Arman will run distance in one hour because it is hard for him to keep up the speed for a whole hour.
- Yes, because he has experiences in running a long distance

Through this questions teacher can review the discussion in the beginning of the lesson and emphasizes that the calculation does not predict the arrival time in a precise manner, but do realize calculations are useful tool in making estimations.

4.6 Exploring relation among speed, distance and time

Goal(s) :

Further exploration of the relation among speed, distance and time

Mathematical idea(s):

- Speed is directly proportional to distance
- Speed is inversely proportional to time

Problem(s):

1. A rabbit runs 12 km in 15 minutes. Determine the speed of the rabbit in km/h and explain your answer!
2. Mother went to the market by motorcycle with average speed of 30 km/h speed. If she arrived in the market in 10 minutes then how far does the market from her house?
3. During the semester holidays, Chika and her family who lives in Surabaya go to the Jatim Park in Malang that is 90 km far. If Chika's father drove the car with average speed of 40 km/h, how can you predict the time needed for the travel? If they go to travel at 7 o'clock in the morning, what time they will be arrived in Jatim park?

Description of the activity :

At this activity, students again work in group of 4 or 5 for around 30 minutes then write their answer in a poster. After that, students will present their posters in the class.

Conjectures of students' answers :

Problem 1

Since in the previous lesson students know the table then students might use this to solve the problem

- Students use a vertical table in two columns in one steps

Distance	**Time**
$12\ km$	$15\ min$
$48\ km$	$60\ \text{min} = 1\ hour$

Hence, the speed of the rabbit is 48 km/h

- Students use a vertical table in two columns in two steps

Distance	**Time**
$12\ km$	$15\ min$
$24\ km$	$30\ \text{min} = \frac{1}{2} hours$
$48\ km$	$60\ \text{min} = 1\ hour$

Hence, the speed of the rabbit is 48 km/h

- Students use a vertical table in three columns in one step

Distance (km)	**Time (min)**	**Time (hour)**
12	15	-
48	60	1

Hence, the speed of the rabbit is 48 km/h

- Students use a vertical table in three columns in two steps

Distance (km)	Time (min)	Time (hour)
12	15	-
24	30	-
48	60	1

Hence, the speed of the rabbit is 48 km/h

Problem 2

Students use the table to solve the problems given

- Students change the speed of 30 km/h into the table of so many distance in so many time then use a table to solve the problem

Distance	Time
$30\ km$	$1\ h = 60\ minutes$
$5\ km$	$10\ minutes$

Hence, the distance of the market from mother's house is $5\ km$

- Students use a vertical table in three columns in one step

Distance (km)	Time (min)	Time (hour)
30	60	1
5	10	$-$

Hence, the distance of the market from mother's house is $5\ km$

Problem 3

Students use the table to solve the problems given

- Students change the speed of 40 km/h into the table of so many distance in so many time then use a table to solve the problems

Distance	Time
$40\ km$	$1\ h = 60\ minutes$
$10\ km$	$15\ minutes$
$90\ km$	$135\ minutes$

✓ Hence, the time needed for the travel is 135 minutes or 2 hours and 15 minutes

✓ If they go to travel at 7 o'clock in the morning then they will be arrive in Jatim park at 09.15

- Students use a vertical table in three columns

Distance (km)	Time (min)	Time (hour)
40	60	1
10	15	$-$
90	135	$2\ hours\ and\ 15\ minutes$

✓ Hence, the time needed for the travel is 135 minutes or 2 hours and 15 minutes

✓ If they go to travel at 7 o'clock in the morning then they will be arrive in Jatim park at 09.15

Discussion :

Teacher can discuss students' answer during the presentation of each group and after. During the presentation of each group teacher can provoke students to explain more about their answers, ask more to clarify, make a conclusion and so on. After the presentation from the whole group teacher can compare the answers that displayed through the posters then generalize.

5 RETROSPECTIVE ANALYSIS

In this chapter, first the researcher will present the tryout of HLT as described in previous chapter (from now on, we call this HLT as HLT 1). Based on the findings during the try out, the researcher concludes that the sequence of learning in HLT 1 is quite appropriate in supporting students' development of the concept of speed. However, the researcher also finds that some activities and problems need to be improved or revised in order to reach better results. Hence, in the second part of this chapter the researcher proposes second HLT (we called it as HLT 2) as the improvement of HLT 1. In the last part of this section, the researcher will present the teaching experiment phase and its analyses.

5.1 Try Out of Hypothetical Learning Trajectory 1

The tryout of HLT 1 was conducted at SD Muhammadiyah GKB-Gresik from February until March 2011. SD Muhammadiyah GKB Gresik is a private school, located in Gresik, a regency in East Java, Indonesia. It is called as "full day school" because the school activities run from 07.00 o'clock in the morning until 16.00 in the afternoon from Monday to Friday. Meanwhile, on Saturday every student must join extracurricular activities or lesson club.

Based on the Indonesian curriculum, the concept of speed is studied in fifth grade of elementary school. Meanwhile, the design was proposed for students who never learned the concept of speed before. Since curriculum deputy and the mathematics teacher of SD Muhammadiyah GKB - Gresik explained that students in fifth grade already learned the concept, the researcher chose the fourth graders who are 9 to 10 years old as the subjects of the research. Seven fourth graders who join the 'mathematics club', an extra lesson for students who are interested in mathematics, are involved in the tryout of HLT 1. Hence, the try out of HLT 1 run only once a week. These students are Thoriq, Nisa, Indra, Lita, Vinda, Aulia and Nur (These are pseudonyms). The teacher explained that the achievement levels of students varied. Two students, namely Thoriq and Nisa, represent high achievers, three students Indra, Lita, and Vinda are average students; Aulia and Nur are low achievers. We assume that students who are involved have enough pre-knowledge to learn the concept. In order to support the assumption, there was a pre-test and the first activity that aim to review students' pre-knowledge. During the try out, the researcher acts as the teacher. Meanwhile, two teachers as the observers. One of them is a mathematics teacher. During this session, we tried all of the activities in the HLT 1 as well as the test before and after the try out.

5.1.1 Daily Life Situations and An Indonesian Fable As the Context

The main goal of the first activity is to review students' informal knowledge of motion and speed and to conduct a general discussion of them. The researcher chooses those contexts because it is predicted that students are familiar with these so that they would be motivated to follow the lesson. The activities divided into three parts; introduction, discussion in groups and then continue with a whole group discussion. In the beginning of the lesson, teacher raised the questions how students go to school as the topic of the discussion. The term 'speed' appeared from students during the discussion when teacher compared two students' ways to go to school. Students then mentioned any objects have a speed; for instance, cars, motorcycles, trucks, bus, and airplanes. Instead of vehicles, another interesting findings is when one student generalize that something moves because due to some force has a speed. Moreover, he gave examples of it. The following is the segment of a video recording.

Researcher : *I want to ask Nur. Nur, how did you go to the school this morning?*
Nur : *By bicycle*
Researcher : *What about you, Thoriq?*
Thoriq : *My father dropped me*
Nur : (he) *was dropped by a motorcycle*
Thoriq : *Kevin used a bicycle, Mom*
Researcher : *Kevin went by a bicycle and Thoriq went by a motorcycle. What are the differences?*
Thoriq : *The speed*
Vinda : *Yes Mom, the speed*
Researcher : *If the speed is different, what would be also different?*
Vinda : *Tired and not*
Thoriq : *Yes Mom, tired and not. That's right. If it is long.... then we'll feel tired*
....
Researcher : *What do have a speed?*
Aulia : *Cars, motorcycles, trucks, and buses*
Indra : *Airplanes*
Researcher : *What else instead of vehicles?*
Indra : *Forces Mom... Forces. Because forces make something moves*
Researcher : *Wait, would you explain again to your friends?*
Indra : *Forces make something moves*
Researcher : *Oh.... forces makes something moves. So what is the relationship with a speed?*
Indra : *When something falls from the trees.....*
Researcher : *What is happening when something moves?*
Indra : *It has a speed*
Researcher : *Can you mention anything else that has a speed also?*
Indra : *Someone who falls from the helicopter*
....
Researcher : *What about someone who walks, does it have a speed?*
Students : *Yes, it has*
Researcher : *Anything else?*
Thoriq : *Someone who runs*

The topic then turned into a fable related to the speed. Most of the students knew the fable because they gave comments about it. Moreover, Thoriq explained the fable to his friends clearly. Another student, namely Nisa, criticized the fable by explaining that if it is in the real world, then Kancil would win the race competition instead of Siput. This student also gives her reason by using the distance and time. Below is a segment from the discussion

Researcher	*: Based on the fable, Siput won the competition instead of Kancil. What if it is in the real world?*
Nisa	*: Yeah..... Kancil*
Researcher	*: Nisa, why did you say that Kancil would win?*
Nisa	*: Siput, from here to there [one of her fingers pointed to the table in one side and another one pointed to another side] Siput needs more than five minutes*
Researcher	*: What do you mean from here to there?*
Nisa	*: One table*
Researcher	*: What about Kancil?*
Nisa	*: Only by one jump*

After the discussion, the researcher gave the students a set of animal pictures and asked them to work in groups of two or three to answer the questions given in Students' worksheet. There are seven students and the researcher divided them into three groups; Thoriq and Nur, Indra and Aulia, whereas Vinda, Nisa and Lita in the third group. Students would work for 15 minutes to finish their works. For the first questions in which students order ten animals based on their speed, we conjectured that it is quite easy to decide the fastest and the slowest animals but it is difficult for them to order the entire animals correctly. The conjecture is proved by students' answers in which all of the groups wrote that the slowest animal is a garden snail. One of the groups gave the same order for a garden snail and a tortoise as the slowest animals. Two groups decided that the fastest animal is a horse. Another group decided that a horse and an ostrich have the same speed so that both are the fastest animals. Students' answers on paper show that no group ordered the animals in the same order.

As explained above, two groups decided that a horse is the fastest animal and one group decides a horse and an ostrich are the fastest. In the second question, students are asked to rewrite the fastest animal(s) and to give reason behind. It is predicted that the body of animals would influence their answer. In deciding which one is faster between a giraffe and a kangaroo, for instance, students might say since a giraffe has long legs than a kangaroo then a giraffe is faster than a kangaroo. The conjecture happened because a group wrote that a horse is the fastest animal because it is frisky, another group wrote a horse is the fastest animal because it was used for the transportation in the past. Meanwhile, a group who decided a horse and an ostrich are the fastest animals wrote that it is because they have long legs and nails.

For the way to compare the speed of the animals, one group come up with the idea of a running competition as mentioned in the fable. Meanwhile, another group wrote that they would use a stopwatch to compare the speed of the animals. Although these groups did not elaborate their answers in the paper, they explained it further during the presentation as described in the segment below

(In the middle presentation of the group of Thoriq and Nur)

Nur	*: Number 3. Eh.... How can we prove that one animal is faster than other animals? Use a stopwatch*
Researcher	*: Why?Why we can use a stopwatch?*
Thoriq	*: Because.......*
Researcher	*: How, How is the way?*
....	
Thoriq	*: For the horse, the horse first. It starts to run until here then measure the time needed by a stopwatch. After that, the snail also measured. Measured the time, Like that*
Researcher	*: So, for instance we made the horse run. We measured the time by.......*

Thoriq : *A stopwatch*
Researcher : *Second......*
Thoriq : *It is also with the snail. Use a stopwatch too..... until here. It would be a long time. Use a stopwatch to measure the time. For instance if a horse, how many seconds .... five seconds or ten seconds. For the snail, it is usually 1 minute or two minutes. So, the difference is the time*

(In the middle presentation of the group of Indra and Aulia)
Indra : *Number 3. How can we prove that one animal is faster than other animals? Explain your answer. By holding a race competition*
Researcher : *Race competition. How is it? Listen to Indra's answer (talked to other students) He used a race competition. How do animals race? It sounds exciting*
Indra : *Later, a horse, an ostrich, a kangaroo, a zebra, a giraffe......*
Researcher : *He eh, All............*
Indra : *A rabbit, a mouse, a chicken, a tortoise, a snail, a bird all............ are raced. There would be paths and they would run*
....
Researcher : *There is start and finish in race competition. So...... how is the distance that they must cover?*
Indra : *The distance is same but they must have their own path*
Researcher : *So, how can we know what animal is the fastest?*
Indra : *The animal which arrived in the finish first*

5.1.2 'Measuring Through Comparing' Activity

In this activity, students are asked to compare the speed of two toy cars. Through this activity, students have opportunities to apply what they discuss in the previous activity. Students may learn that complete motion involves distance and amount of time required to travel that distance so that when students compare the speed, they actually compare the time and the distance simultaneously. Students learn also how to act and think as scientist. They have problems so that it is important for them to make conjectures and then do an experiment to prove their conjectures. After doing the experiment and find the solutions, students also practise to communicate their findings to other persons.

The researcher made two groups of students, one group consists of four students and the other group consists of three students. Students in the first group are Nisa, Lita, Nur and Indra. Meanwhile, Thoriq, Vinda and Aulia are in the second group. The researcher then gave every group a stopwatch, two toy cars, a measuring tape, a worksheet, a poster paper and markers. After that, the researcher asked the students to work for 10 minutes to solve the problem. The initial problems in the HLT are :

1) Compare the speed of these two toy cars. Which car is faster?
2) Make a drawing of your investigation of those two cars so that other persons know how fast the car compares with the other! How many times it is faster than the other is?

After the discussion with the supervisors, the problems reformulated as follows:

1) Which car is faster between those two cars? Why?
2) How fast is that car comparing with another?
3) Explain your investigation that you did in order to decide the faster car!
4) Make a drawing of your investigation of those two cars so that other persons know which car is faster and how the difference of the speed!

The reformulated problems were given during the tryout of HLT 1.

Based on the classroom observation, both of the groups compared the speed of the car by measuring the time needed for each of the car to travel the same distance. The difference is the path that they used to run the cars. Students in first group used tables as the path. First, Lita take out the tape, put on a table from one side to another. Meanwhile, Nisa hold the cars and Nur hold a stopwatch. After that, Nisa let the first car run. Lita is at one side and Indra at the opposite side. Nur stopped the stopwatch when the car reached the opposite side of the table. Students in this group did the experiment more than two times and in every experiment, the role of the students is interchangeable. After deciding the time needed by the first car to travel the table and make a note of it, students continued with the second car. Students in this group tried also to make a race for the cars once but they again let the car run one by one and measure the time for each car.

FIGURE 4.1 FIRST GROUP COMPARED THE SPEEDS OF THE TOY CARS

Students in the second group used the floor as the path for the cars. First, Vinda took out the tape and measured two meters of the floor. Vinda put the tape laid down on the floor then hold the stopwatch. Thoriq held the car and let the car run. When the car reached the given distance, Vinda stopped the stopwatch and said loudly the time needed by the car to travel the distance to her friends. The same experiment also continued for the second car. They repeated the experiment until two times. The role of the students in the group is interchangeable during the experiment.

FIGURE 1.2 SECOND GROUP COMPARED THE SPEEDS OF THE TOY CARS

Based on students' answers on the poster, the researcher concludes that both of the groups used the same approach to compare the speed of the cars, namely using the given distance. By the given distance, the car that needs less time is the faster. The differences are on the path and the distance that they used. Students knew that with the same distance, the faster car is the one that needs less time. However, it seems that they did not know the inverse in which in the given time, the car travelled farther is the faster. In order to convince it, the researcher involved herself in the group discussion. Below is a segment from the discussion in the second group

Researcher : *For instance, I want to compare the speeds of the cars. I will not measure the distance first. I will let the car run but with the limitation of time*
The cars would only run for two seconds

Thoriq : *If the time is more than two seconds then it means that it is slower. If the time is less than two seconds then it is faster*

Vinda : *No, it is not like that. The time is limited*

Researcher : *Yes, the time is limited*

Vinda : *So...... A car run here [pointed to the table from one point to another point as the start and the finish line of the car]. The teacher doesn't know (the distance) after two seconds, the car must be stopped. After that, where is the other [looking for the other car]"*

Researcher : *For the other...?*

Vinda : The other, Mom, for instance. It doesn't reach here [pointed to the finish point of the first car]. But it is only until here [pointed to the table which is the distance is less than the previous one]. So it is not the same.
Researcher : What is not the same?
Vinda : The distance
Researcher : Oh, the distance is not the same. So, how is the distance of the faster one if the time is limited?
Thoriq : The one, which............ what is that?
Vinda : [smile]
......
Researcher : The car run .... srrttttt [took a car and moved the car by her hand and put in certain point] until here.
[the researcher put another car]
Vinda : That car (the car in researcher's hands) is only until here [Vinda and Thoriq pointed to the table with the distance is less than the stopped point of the car by the researcher]
Researcher : Ok, you move the car
Thoriq : [took the car and let it run]
Vinda : [stopped the car in a certain point on the table that she pointed before- the distance is less than the distance travelled by the previous car]
Researcher : One car is here and another car is here [pointed to both of the cars consecutively]. So, which one is faster?
Vinda : Faster....... ehm...
Thoriq : No, Mom
Researcher : Ok, what is the difference?
Vinda and Aulia : The distance
Thoriq : The speed
....
Researcher : So, if the car is faster then with the same time......
Vinda and Aulia : The distance is longer
Vinda : That is the conclusion
Researcher : What?
Vinda : Yeah.... if the car is faster then the distance is longer. If the car is slower then the distance is shorter

From the various data, the researcher concluded that students indeed could compare the speed of the cars so that they could decide the faster car. However, it does not mean that they pay attention to the distance and time simultaneously. Students tended to use the same distance and the time is quite often implicit. The faster car is the car that needs less time (with the same distance). The idea that in the given time, the car travelled farther is the faster did not appeared in the class until the researcher involved in the students' discussion.

Although there is a group who ever try a race competition for the cars, students' drawings for the fourth problem as shown in the poster are quite similar. Every group drew a path of the car and gave the distance of it, a car in every path and time needed for every car. The first group drew the cars not in line position. Based on the interview with students, it does not mean that the car in front is the faster. A student in the group explained that the pictures were taken in the different times. Different with the first group, students in the second group drew the cars in line position. A student in this group explained that what they drew is the position of the car at a start line.

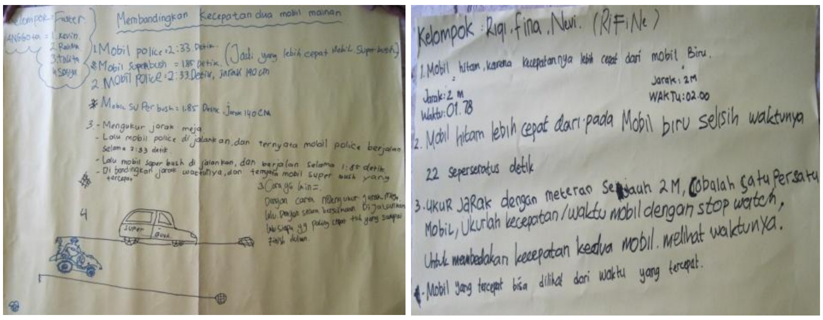


Figure 5.3 Students' posters in comparing the speed of two toy cars

5.1.3 ANALYSING DISTANCE AND TIME GROWTH DURING THE MOVEMENT

Through the activity, we expect that students can measure and estimate the time needed by a toy car to travel certain distance and then make drawings of the situation of the car's movement. In order to reach the goals, students are asked to work in groups to solve the problems.

For the first problem, students are asked to measure the time needed by the car to reach 2 m of distance. Based on the investigation during the group discussion session, only first conjecture appeared in the class. As happened in the previous activity, the difference is the path that students use. For the first time, students in 'RARIALFI' group try to use a table as the path. When they realized that the length of the table is less than 2 meters, they turn to use the floor as the path and used wall as the end for the car. One student, namely Thoriq, let the car run and Vinda caught the car when it reached the wall. Another student, Nur, measured the time. At the poster, they wrote that the time needed for the car to reach 2 m of distance is 3 seconds. They wrote 3 seconds as '00:03 seconds'. Moreover, Vinda said that 3 seconds is an approximate of 02:92 seconds.

Students in the second group used the floor as the path directly. Nisa and Indra measured 2 m of the floor. Nisa hold a measuring tape at one side and Indra at the other side. Nisa hold the measuring tape at right hand and the same time hold the car at left hand. After that, she let the car run to Indra. Another student, Lita hold a stopwatch and measured the time. They wrote their answer at the poster that the time is 3 seconds. When the researcher asked whether the time is exact or not, Nisa said that 3 seconds are the approximate of 02:64 seconds.

In the second problem, students are asked to make an estimate of the time needed for the car to travel once around the class. In this paragraph, the researcher will describe what and how students answer for the second problem. Students in

'RARIALFI' group first measured the width of the classroom. Two students, namely Thoriq and Vinda did it. Meanwhile, Nur wrote the result on a notebook and Aulia prepared the poster. Vinda and Thoriq measured the width of the classroom by measuring the floor at the backside. Since the length of a measuring tape only 5 m at maximum, at the first time they made a sign of 500 cm at the floor by making a small line. After that, they measured the rest. As the result of their measurement, Vinda said to Nur that the width is 730 cm. Nur then wrote the result on her notebook. Vinda and Thoriq continued to measure the length of the classroom by measuring the right side (from the students' point of view) of the wall. As before, they measured 500 m and continued with the rest. Thoriq was confused whether the length of the wall is 680 cm or 608 cm. He said that the length of the class is 500 cm plus 108 cm so that it is 608 cm in total. The researcher then asked him "are you sure with what you said?" Vinda then said to Thoriq that the length of the rest is 180. Hence, it should be 680 cm. Thoriq then thought and said "Oh ya, 500 add with 180 is 680". Nur again wrote the result on her notebook. After that, they wrote on their poster the length and the width of the classroom. Below is their answer to the second problem on the poster.

The length of the class = 1360 cm
The width of the class = 1460 cm
2820 cm = 28 m

Hence, the time needed for the car to travel the class once a is 09 : 40

During the interview with the students in this group, they explained that the length of the class is the total length of the two sides of the class. Hence, it comes from

680 cm + 680 cm. Meanwhile, the width of the class that they wrote on the second line is the total length of the front and the backside. Hence, it is 730 cm + 730 cm, which is 1460 cm. Furthermore, they said that 28 m here is also an approximate of 28,20 m.

Figures 5.3 Students measured the length of the classroom

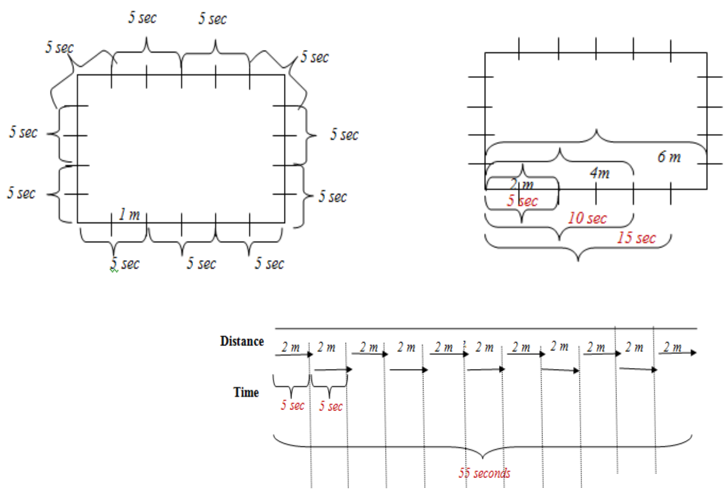
Students in the second group, Nisa, Indra and Lita measured the length and the width of the classroom by measuring the floor. At the first time, Lita measured the length of one tile. After that, she counted the number of tiles in one line. She said to her friends that the number of tiles is 23. Then she tried to multiply 23 by 30 but she did not continue. Nisa asked her to measure the floor by a measuring tape. They measured the line of the floor inside instead of the outer side of the class. Like the first group, they measured 5 m at the first time and then continued with the rest. They used the same way to measure another side. Nisa and Lita then calculated the perimeter of the class on the whiteboard then wrote it on their poster. Here is their answer for to second problem on their poster.

The perimeter of the class is = 2 x (700+736)
= 2 x 1476
= 2872 cm
= 28,72 m

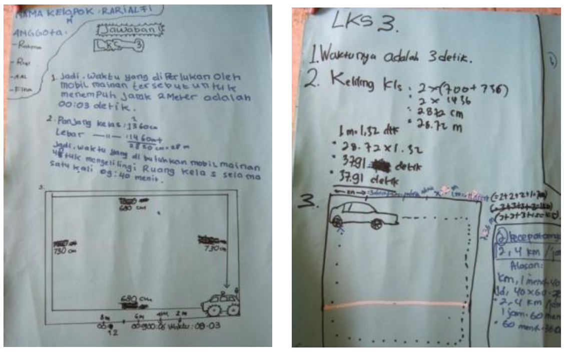
1 m = 1,32 seconds
= 28,72 x 1,32
= 37,91 seconds

During the interview, Lita explained that 28,72 m is the perimeter of the classroom. The perimeter is two times length plus width. Furthermore, since to travel 1 m the car needs 1, 32 seconds it needs 28, 72 x 1, 32 seconds to travel once around the class which is 28, 72 m in long.

For the third problem, the researcher made conjectures as follow:



Meanwhile, we can see students' drawing on the posters as follows:



Figures 5.4 Students’ posters on third activity

Based on the description above, it seems that students did not get difficulties in measuring the distance and the time using the tools given. The distance given in the problem is 2 m but the measuring tape provides only ‘cm’ and ‘inch’. However, students in both groups immediately knew that they have to measure 2 m as 200 cm. For the time, students knew that a stopwatch displayed numbers for minutes, second and the unit below. Although they did not know the name of the unit below seconds, they could make an approximate of the time on seconds as explained before. In making an estimate of the time when the distance is changed to great extent, students tended to make calculation instead of thinking of the increasing of the time and the distance during the car is travelling.

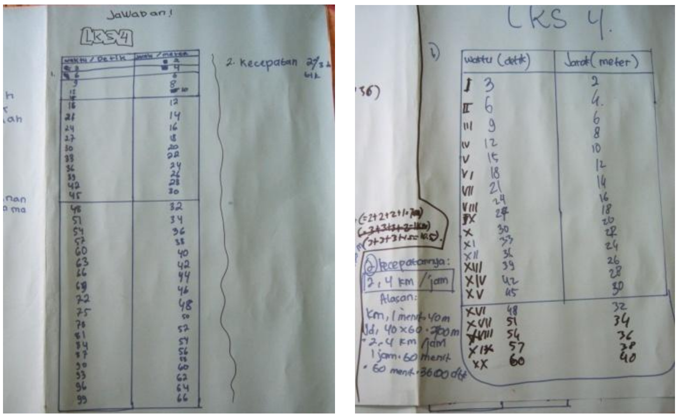
This activity makes students very active because all of the students are involved in it. We can see also how students work together in order to solve the problems. Nevertheless, it seems that it made students busy with practical problems such as the accuracy of the measurement, the way to calculate the perimeter and so on instead of thinking about mathematics. Hence, the researcher concludes that the activity does not give enough support for students to analyse how the distance and time grow during the movements will be.

5.1.4 Developing models and notating speed in a standard unit

This activity is the continuation of the third activity because students used data from their investigation previously. Students are asked to make a long list of the distances travelled by the car after some numbers of seconds. Students are asked also to determine the speed of the car based on the table they made. The groups of the students are unchanged from the previous activity.

In making a long list of the distance and the time, both groups have the same data in which their cars need 3 seconds to travel 2 m of distance. Since the example are given so that the table that students make also very similar except the length of the table that they made. The first group made the table until 33 rows whereas the second group until 20 rows. Hence, the time for these groups are 99 and 60 seconds consecutively.

In notating the speed of the car, the first group seems to have difficulties in it. At the first time, one student in the group, namely Thoriq, said that he did not know how to make it into km/h since he never learns about it. According to him, it is difficult because they should expand the table to include many more rows. It seemed that he tried to make the table until 1 hour. After the discussion with the researcher, students in the first group wrote that the speed of the car is 2 m / 3 seconds. Different from the first group, students in the second group determined the speed of the car in a standard unit, km/h. Knowing the distance at 60 seconds (1 minute) they can calculate the distance in one hour by multiplying it with 60. Below is students' answer on the poster



Figures 5.5 Students' poster in the fourth activity

The observation on the learning process in the class indicates that a ratio table proposed here seems doable for students. Using the table, it is easy for students to see the proportion between distance and time. Students see immediately that when the distance increases two times, three times, four times, etc then the time also increases with the same multiple and vice versa. We can see it also in the second figure above in which the students give number one until 20 in the left side so that in the last row they wrote that for 60 seconds the distance travelled by the car is 40 m.

Students have a tendency to determine the speed in km/h, instead of m/sec. The researcher made the conclusion based on students' answers in the second group and an explanation given by Thoriq as explained above. Probably it is because the unit of speed, km/h, is more familiar to Indonesian students. The unit of km/h can be seen very easy in daily life situation such as in a speedometer either in a car or in a motorcycle, in road signs, in daily life conversation, in textbooks and so on.

5.1.5 Converting Unit of Speed

The researcher started the activity by guiding a discussion about the posters of a road sign and a speedometer. Based on the observation all of the students are familiar with those posters and students are involved in the discussion actively. When the researcher showed the road sign, students explained that the road sign means that anyone cannot have a speed more than 10 km/h and they immediately said that the speed is very slow. All of the students know the name and the function of a speedometer. When the teacher asked about ‘km/h’ which is included in the poster, some of the students said that it shows ‘km per hour’ but some of them said that it is ‘km per jam (hour in Indonesia)’. Students also explained that km/h is a unit of speed, when a speedometer is pointing to 40, for instance, it means that the speed of that vehicle at that time is 40 km/h. Another student explained that 40 km/h means that in one hour he/she can reach 40 km of distance. The researcher reacted to this answer by asking whether someone needs one hour to get the speed of 40 km/h, all of the students said ’no’, but they get difficulties to give reasons. Hence, the researcher explains that 40 km/h means if someone drives the car with that speed that it could be predicted that he/she will reach 40 km for every one hour, 80 km for 2 hours and so on. However, it is quite difficult to reach exactly that distance because if we travel, then the speed is usually not constant. One student, namely Talita, explained that when someone accelerates then the speed will increase and the speed will decrease if someone decelerates, for instance, when traffic light is red and we want to stop. Students also said that it is an instantaneous speed. The discussion then continued by discussing about instantaneous and average speed. The researcher then told a story in which she went to school by a motorcycle. During the trip, the speed of her motorcycle was changed sometimes. The researcher then asked how she can determine the speed of her motorcycle. When the researcher gave the number, students were busy to calculate. The researcher then asked the students to ignore the number, one student, namely Nisa, immediately said that she need to consider the distance and the time. However, the term 'average speed' introduced by the researcher.

After the discussion, students work in groups of three or four to solve the problem as shown in Students' Worksheet 5. Below are the problems and students' answers

Problem 1

Arman has jogging as his hobby. He said that last week he ran 60 meters in 15 seconds. Can you predict the Arman's distance in one hour?

Students' answers in the first group :

$$60\,m \qquad / \qquad 15\,seconds$$
$$\times 4 \qquad\qquad \times 4$$
$$240\ \times 60 = 14400\ \ m\,; \quad 60\text{ sec } = \ 1\text{ minutes } \times 60 = 1\ hour\ or\ 3600\ seconds$$

Students also explained that they multiplied the distance and the time two times, by 4 then by 60. By 4 because they wanted to make the time into one minute and then by 60 because they wanted to convert one minute into one hour. Therefore, the distance was multiplied as well. Students in this group also made a table in order to check their answer. The table will be shown in the figure of students' poster.

Students' answer in the second group :

$60\ m \ : \ 15\ seconds \quad \times \ 3600\ seconds$
$= 4\ m\ \times 3600\ seconds$
$= 14.400\ m\ or\ 14{,}4\ km/h$
Hence, Arman's distance in one hour is 14, 4 km or 14.400 m

The students explained that the distance in every second is $4\ m$; because one hour is $3600\ seconds$ so they multiply $4\ m$ by 3600

Problem 2

Adi likes jogging too. He said that yesterday he ran 50 meters in 12 seconds. Is Arman's speed higher or lower than Adi's?

Students' answer in the first group :

First, students in this group divide 3600 by 12 so that they got 300. After that, they multiply 300 by 50 so that they get 15.000. After that they conclude that Adi is faster because Adi's distance is farther than Arman's.

Students' answer in the second group :

$50\ m \ : \ 12\ seconds \quad \times \ 3600\ seconds$
$= 4{,}2\ m\ \times 3600\ seconds$
$= 15.120\ m\ or\ 15{,}12\ km$

Adi : $15{,}120\ km/h$
Arman : $14{,}4\ km/h$

Problem 3

Find Adi's speed in m/h and km/h?

Students' answer in the first group :

Adi's speed is $15000\ m/3600\ sec\ /1\ h$

Adi's speed is $15\ km/h$

Students' answer in the second group :

$15.120\ m/h$

$15{,}120\ km/h$

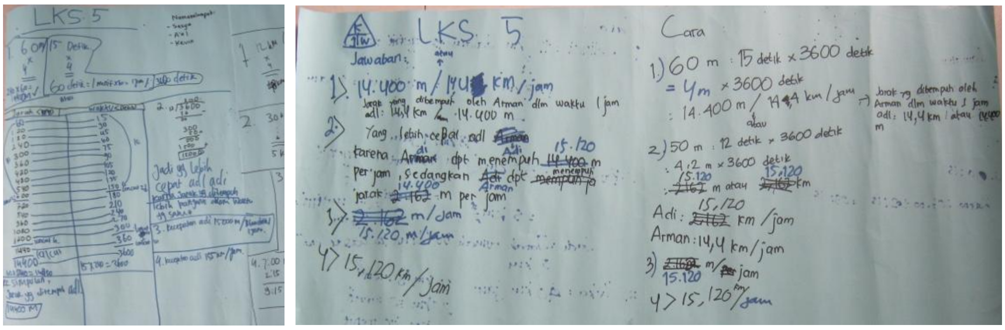


Figure 5.6 Students' posters on fifth activity

Discussion in the beginning of the activity provokes students to be involved in it. The researcher predicts that it is probable because students are quite familiar with a road sign and a speedometer. The unit of km/h is also familiar to them. Hence, students immediately know the meaning of 'km/h' although the unit is written in English version. A speedometer also stimulates students to imagine the change of speed during someone's trip by imagining the moves of a pointer in it. Hence, a speedometer helps students to learn instantaneous speed.

Based on the description above, the activity and the problems from the researcher point of view give enough support to students to understand converting unit of speed. It is also leading students to make advances in using the table of proportion between distance and time in speed.

5.1.6 Exploring Relation among Speed, Distance and Time

The main aim of this activity is to explore relationship among speed, distance and time. Students are asked to solve three different problems. At the first problem, students were given the distance and the time and then asked to determine the speed. At the second problem, the students are given the speed and the time and they are asked to determine the distance. Meanwhile, in the third problem students are asked to determine the time if the distance and the speed are given.

In the HLT, my conjecture was that students would use the table to solve the problems. After that, by multiplying or dividing the time, the distance or the speed they would find the answers. However, students' answers in the posters that they made on this activity show that students already used the table in a sophisticated way. They did not write the 'real' table with rows and columns but their ways to calculate as if they made the table in mind.

Here are the group answers toward the problems given :

Problem 1 :

A rabbit runs 12 km in 15 minutes. Determine the speed of the rabbit in km/h and explain your answer!

Students in the first group wrote :

$$12\ km\ /\ 15\ minutes$$
$$\times 4 \qquad \times 4$$
$$48\ km/1\ \ jam$$

The rabbit can reach 48 km of distance in one hour.
The speed is 48 km/h

Students in the second group wrote :

$$The\ rabbit:\ 12\ km\quad in\quad 15\ minutes$$
$$\times 4 \qquad\qquad \times 4$$
$$48\ km \qquad 60\ minutes = 1\ hour$$

48 km / 1 h

Problem 2 :

Mother went to the market by motorcycle with average speed of 30 km/h. If she arrived in the market in 10 minutes then how far does the market from her house?

Students in the first group wrote :

$$30\ km\ /\ h$$
$$: 6 \qquad : 6$$
$$5\ km \qquad 10\ minutes$$

Hence, Mrs. Mirna in 10 minutes can reach 5 km of distance

Students in the second group wrote :

Mrs. Mirna : $30\ km\ /\ hour\ 60\ minutes$

$$: 6 \qquad : 6$$
$$5\ km \qquad 10\ minutes$$

Hence, the distance of Mrs. Mirna's house to the market is 5 km

Problem 3 :

- ✓ During the semester holidays, Chika and her family who live in Surabaya go to the Jatim Park in Malang, that is 90 km far. If Chika's father drove the car with average speed of 40 km/h, how can you predict the time needed for the travel?

  Students in the first group wrote:

  $40\ km\ /\ 1\ hour$
  $:4 \qquad\qquad :4$
  $10\ km \qquad 15\ minutes$
  $\times 9 \qquad\qquad \times 9$
  $90\ km \qquad 135\ minutes\ or\ 2\ hours\ 15\ minutes$

  Hence, the time needed is 2 hours 15 minutes

Students in the second group wrote :

Chika : $30\ km\ /\ hour \qquad 10km\ /\ 15minutes$
$20km\ /\ 30minutes$
$30km/\ 45\ minutes$
$40km/\ 60\ minutes$
$50km/\ 75\ minutes$
$60km/\ 90\ minutes$
$70km/\ 105\ minutes$
$80km/\ 120\ minutes$
$90km/\ 135\ minutes$

Hence, the time needed is 2 ¼ hours or 2 hours 15 minutes

- ✓ If they go to travel at 7 o'clock in the morning, what time will they arrive in Jatim Park?

  Students in the first group wrote :

$$\begin{array}{r} 7.00 \\ \underline{2.15} \\ 9.15 \end{array} +$$

Hence, they will arrive in the place at 9.15 because 7 o'clock plus 2 hours and 15 minutes is 9 : 15 minutes

Students in the second group wrote :

$$09 : 15$$

Because $07 : 00 + 02 : 15 = 09 : 15$

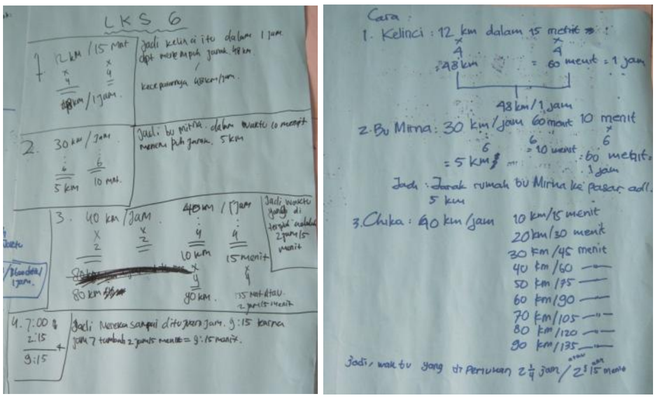


Figure 5.7 Students' posters on 6th activity

Based on students' answers for the problems, it seems that students are able to develop a ratio table. In the previous lesson, the tables that students made are simple. They made the table, then expand the table with consecutive numbers, multiplied by 2, 3, 4, consecutively. Meanwhile, in these problems students did not make the table anymore. They did not multiply or divide the proportion step by step by consecutive numbers. Students directly multiply or divide the proportion by an appropriate number.

Although the problems given in this activity are quite artificial, it is quite appropriate to reach the goals. Hence, we can see how the students develop their skills in explaining relations among speed, distance and time. The researcher will keep the context and the problems. The context are familiar to the students.

Meanwhile, the problems are understandable and its variety can provoke students to explore more about the proportion among the distance and time.

5.2 Conclusion of Try Out of HLT 1

The observation toward the tryout of HLT 1 brings us to the conclusion that in general the HLT is in line with the students' actual learning process although some conjectures did not appeared. Hence, the researcher will keep the sequence of learning goals in HLT 1 because it is predicted that it is quite appropriate in supporting students' development of the speed concept. However, the researcher also found that some activities and mathematical tasks are needed to be improved or be revised in order to reach better results.

The use of daily life situation and an Indonesian fable in the context bring the students to the idea of motion, speed, distance and time. In the previous session, we can see how the students give various examples of the moving objects, vehicles, humans' activities, and even moving objects because of a force. The differences in the way to go to school provoke students to the term 'speed'. In comparing the speed of the animals, students' intuition as well their existing knowledge play a big role. Although there are only three groups in the class, all the conjectures about the way to compare the speed appears; using a race competition and measure the time for each animal to reach a certain distance.

In many activities, students did not have difficulties in solving the problems although we also found that students have struggles because the researcher did not provide the path for the cars. Some mathematical tasks make students busy with the practical problems, instead of thinking about the mathematics itself. One group has a difficulty in notating the speed although they know the distance and the time. Students have a tendency to determine the speed in standard unit. Students are more familiar with the unit of speed in km/h, instead of m/sec.

Interesting findings during the observation in the 'Developing models and notating speed' activity is how the ratio table is doable for Indonesian students. They are not familiar with a ratio table. Therefore, the researcher proposed the table to the students. Nonetheless, through the observation we can see how this model helps students to learn the proportion of distance and time. From the table, students see immediately that when the distance increases two times, three times, four times, etc then the time will also increase with the same multiple and vice versa.

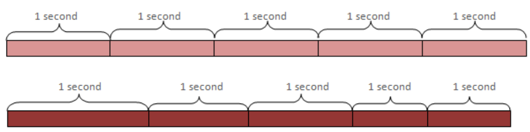
In the last two activities, the tasks given are more in developing students' ability in calculation. Although students can solve the problems, the researcher is not sure whether all of the students are involved in the discussion since each group consist of four or three students. Hence, in teaching experiment phase the researcher recommends making smaller groups on this activity.

5.3 Refinement of Hypothetical Learning Trajectory 1

Based on the findings in the first activity, the researcher will only use the context of an Indonesian fable in the second HLT and we will call this HLT as HLT 2. Using two contexts at the same time, it makes the discussion too abroad. In HLT 2 the students will be asked to make a poster for their answer. Hence, the set of animal pictures would be given with larger size. The improvement will be applied on the problems' design. We consider to change the second problem. Inspired by one student's reasoning when she was criticizing the fable, the researcher will ask the students to decide which one is faster than the two animals (Kancil and Tortoise) and ask them to give reason. We also realize that it is also important to improve the quality of a whole class discussion. In the teaching experiment phase, teacher could add the discussion by making a generalization that speed is related to the motion. Hence, moving object has a speed.

Students' difficulties in comparing the speed of the toy cars without the path make us consider providing the students with the path for the cars. We can make the path, for instance, from the paper. By providing it, we expect that we can minimize practical problems. We also conclude that students indeed could compare the speed of the cars so that they could decide which car is faster. However, it does not mean that they pay attention to the distance and time simultaneously. The role of the teacher is very important to stimulate the discussion. Teacher can stimulate the discussion by providing a conflict situation with a race competition for the car with the different distances and then asking students' comments about the situation. After that, teacher can demonstrate how two cars race against each other and then ask the students to decide which one is faster. From the discussion, it is expected that students can conclude that when we compare the speeds of the cars we can use either the same distance or the same period of time. Hence, if the distances or the time is different then we must consider the proportion of them.

In 'Analysing distance and time growth' activity, students are very active. It can also intertwine the concept of speed and the concept of perimeter. However, the activity cannot reach the goals optimally. The activities in measuring the time and the perimeter of the classroom make students busy with practical problems such as the accuracy of the measurement, the way to calculate the perimeter and so on. In order to improve the activity, the researcher changes it with 'A Paper Tape Activity'. We will provide each group with a path for the car with different lengths. Teacher asks the students to measure the distance and the elapsed time for one car. Each group will have different cars and different lengths of the path so that they have different data. After that, the teacher asks the students to decide the fastest car among them. If no one of the students finds the way to compare the data, teacher can ask the students to measure the distance of the car in one second. By knowing the distance in one second, students can continue by making a sign of the distance every second. Through the sign that they make, it is expected that students realize that the distance and the time is growth simultaneously. In the last part of the 'Paper Tape Activity' teacher will facilitate a discussion about constant and variable speed. Teacher could make a story about two groups of students who did an investigation to their toy cars. They make paper tapes as shown below:



Teacher will stick those paper tapes in the white board and pose questions to the students whether the paper shows the speed of the car and what the students' opinion is about the speed of those toy cars.

Students tend to think the unit of speed in km/h probably because the unit km/h can be seen very easy in daily life situation such as in a speedometer either in a car or in a motorcycle, in sign roads, in daily life conversation, in textbook and so on. Although students are not very familiar with the unit of speed in m/sec, the researcher will continue to introduce it in the teaching experiment because the unit m/sec is closely related to the experiment that students did. It is also useful for students because they will learn this unit in higher education. Hence, it will enhance students' knowledge.

There are two activities in 'Converting the unit of speed' activity, that is discussion relate to the poster and solving problems. We can see how the posters are good resources to discuss the speed; a maximum speed, an instantaneous and an average speed. Hence, in the teaching experiment the researcher will keep the posters as the introduction part of the lesson. In order to improve the learning process, the researcher will show three different posters; a road sign and two posters of a speedometer. The first speedometer will show the initial position of the pointer (the speed of the vehicle is zero). Meanwhile, the second poster will show when the pointer points to certain number, for instance, number 40. The problems given are meant to support students to understand the converting unit of speed. It is also leading students to make advances in using the table of proportion between distance and time in speed. However, since in HLT 2 we will use "paper tapes activity" before "converting units of speed activity", we consider replacing the problem. In "paper tapes activity", each group will have their own data in which their toy car runs in several cm or m in several seconds. Based on the data, we will ask the students to determine the speed in "cm/sec" and then students are asked to continue the table until they find different units of speed, such as m/sec, m/minute, m/h and km/h.

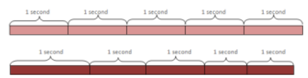
'Exploring relation among distance, speed and time' is the last activity, either in the initial HLT or the improved HLT. The goal of this activity is to explore further the relation among the speed, distance and time. Our findings during the observation show that the problems given in this activity give an opportunity to the students to develop ratio tables. We know that on the first cycle, students during this activity did not make the table anymore. They did not multiply or divide the proportion gradually by consecutive number. They directly multiply or divide the proportion by an appropriate number. The variety of the problems can provoke students to explore more about the proportion between the distance and time; hence, the researcher will keep the activity and the problems.

The table below will show what kinds of changes either in the activity or the problems in HLT 1 as the base for HLT 2.

Table of the changes of HLT 1

NO	TITLE OF THE ACTIVITIES	AIMS	MATHEMATICAL IDEAS	ACTIVITIES / PROBLEMS OF HLT 1	ACTIVITIES / PROBLEMS OF HLT 2
1	**Reviewing Students' Informal Knowledge**	Reviewing Students' Informal Knowledge of Speed	• Every moving object has a speed • Speed involves distance of movement and time to reach the distance	**Problem(s) :** 1.Make an order of given animals based on their speed! 2.In your opinion, what is the fastest animal(s)? Give your reason! 3.Discuss the way to compare the speed of the animals in order to know the fastest animal.	**Problem(s) :** 1.Make an order of given animals based on their speed! 2.Who is faster between a rabbit and a tortoise? Why? 3.How can we determine that one animal is fastest than others?
2	**Measuring through comparing**	Students can compare the speed of two moving objects	• Something moves faster than another if it needs less time to reach the same distance • Something moves faster than another if it is goes farther within the same period of time	**Activity :** Students investigate which car is faster among two toy cars **Materials :** Two toy cars, a measuring tape, a stopwatch **Problem(s) :** 1. Which car is faster between those	**Activity :** Unchanged **Materials :** Two toy cars, a measuring tape, a stopwatch, tracks **Problem(s) :**

				two cars? Why? 2. How fast is that car comparing with another? 3. Explain your investigation that you did in order to decide the faster car! 4. Make a drawing of your investigation of those two cars so that other persons know which car is faster and how the difference of the speed!	Compare the speed of two toy cars. Which one is faster? Explain your answer!
3	**Analysing distance and time growth**	• Students can compare the speed of many toy cars • Students can analyse distance and time growth	• Speed is a proportion between distance and time • Constant speed means same proportion in the whole trips • In variable speed the proportion of the distance and time changes during the	**Activity :** Measure the perimeter of the classroom and make a prediction of time **Problem(s) :** 1. Investigate the time needed by the car travelled in 2 meters! 2. Predict how much time it will take if	**Activity(1) :** **"Paper Tapes" Activity** *Learning Goal(s):* • Students can compare the speed of the car by measuring the ditance in a unit of time • Students can analyse distance and time growth *Problem(s) :* • Measure the time and the distance

			trips.	the car continues to travel once around the class! Why is it so?	travelled by the car • Find the way to compare the speed of the toy cars • Determine the distance travelled by the toy car for every one second **Activity (2):** **Class Discussion** *Problem :* Two groups of students make a paper tape of the distance travelled as follows:  Discuss how is the speed based on those paper tapes!
4	**Paper tapes activity**	Students use models (e.g table) to determine the		**Activity :** Students expand the table	**Activity (1) :** Notating Speed in a standard unit *Learning Goal(s):*

		proportion between distance and time		**Problem(s) :** 3. Make a long list of the distance travelled by your car after several seconds on the table! 4. How do you determine the speed of the car? Explain your answer!	Students can compare the speed by notating the speed of each moving object *Problem(s) :* Notate the speed of the toy car **Activity (2) :** Developing models **Learning Goal(s):** Students use models (e.g table) to determine the proportion between distance and time **Problem(s) :** Make a table of the distance and time of your car
5	**Converting Unit of Speed**	Students can convert the unit of speed (e.g cm/sec, m/sec, km/h)	Speed is determined by the distance travelled in a unit time	**Activity :** All of the students solve the same written problems in group of four or five **Problem(s) :** 1. Arman has jogging as his hobby. He	**Activity :** Students convert the speed of a car in cm/sec into other units that they want (every group uses their previous data) **Problem(s) :** Expand your table and notate the speed

				said that last week he ran 60 meters in 15 seconds. Can you predict the Arman's distance in one hour? 2. Adi likes jogging too. He said that yesterday he ran 50 meters in 12 seconds. Is Arman's speed higher or lower that Adi's? 3. Find Adi's speed in m/h and km/h?	of your toy car in various units of speed (e.g cm/sec, m/sec, km/h)
6	**Exploring relation among speed, distance and time**	Students can predict either the distance or the time if the speed is given	Ratio table determines the proportion between the distance and time when the speed is assumed to be constant	**Activity :** Students solve written problems in groups of four or five **Problem(s) :** *1.* A rabbit runs 12 km in 15 minutes. Determine the speed of the rabbit in km/h and explain your answer! *2.* Mother went to the market by motorcycle with average speed of 30 km/h speed. If she arrived in the market in 10 minutes then how far	**Activity :** Students solve written problems in pairs **Problem(s) :** Unchanged

				does the market from her house? 3. During the semester holidays, Chika and her family who lives in Surabaya go to the Jatim Park in Malang that is 90 km far. If Chika's father drove the car with average speed of 40 km/h, how can you predict the time needed for the travel? If they go to travel at 7 o'clock in the morning, what time they will be arrived in Jatim park?	

5.4 Hypothetical Learning Trajectory 2

In this section, the researcher will present the Hypothetical Learning Trajectory which is implemented in the teaching experiment phase.

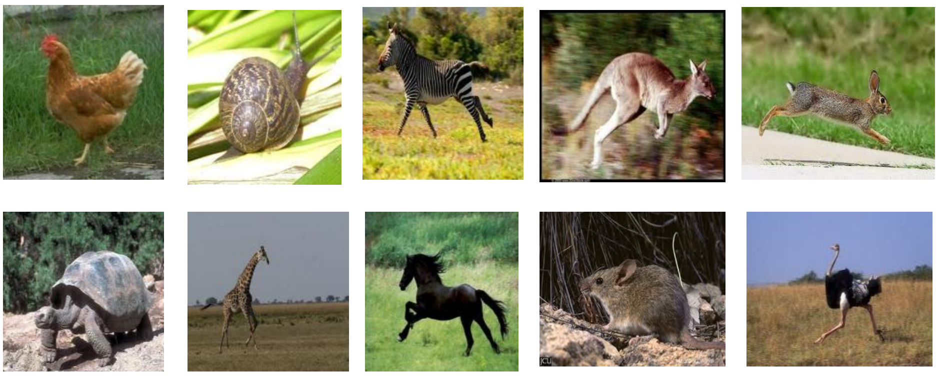
Activity 1 – An Indonesian Fable as the context

Goal (s) :

Reviewing students' informal knowledge and a general discussion of motion and speed

Problem(s) :

1. Make an order of these animals based on their speed!



2. Which one is faster? A rabbit or a tortoise? Why?
3. How can we determine that one animal is faster than the others?

Conjectures of students' thinking :

1. Students choose a garden snail or a tortoise as the slowest animals. For the fastest animals, students might choose a horse, a zebra, a kangaroo, a giraffe or a rabbit as the fastest animals. It seems very difficult for the students to order all of the entire animals in the same way.
2. A rabbit is faster than a tortoise.

The body of the animals or how the pictures show those animals' run might influence students' reasoning.

3. - Students come up with the idea of a race competition
   - Students measure the time needed by each animal to reach certain distance

Activity 2 – Measuring Through Comparing

Goal (s) :

Students can compare the speed of two toy cars

Mathematical idea(s) :

- Something moves faster than another if it needs less time to reach the same distance
- Something moves faster than another if it goes farther within the same time

Problems :

The problem will not be given in written form. Teacher gives it orally to the students before they works in the groups.

Conjectures of students' thinking :

Students will not get difficulties in deciding which car is faster. Nevertheless, their way to compare the speed might vary; the researcher will provide the students with the tracks for the cars. However, the number of tracks given are different. Some group will receive single track and others group receive double tracks. Hence, the students come up with the idea of race competition or measure the time needed for the car to finish the track.

Activity 3 – Analyse the distance and time growth

Goal(s) :

- Students can compare the speed by notating the speed of each moving object
- Students can differentiate constant and variable speed

Mathematical ideas :

- Speed is a proportion between distance and time
- Constant speed means same proportion in the whole trips
- In variable speed the proportion of the distance and time changes during the trips.

Problem(s) :

The problems given orally to the students

1. Measure the time and the distance travelled by the car
2. Find ways to compare the speed of the toy cars

Conjectures of students' thinking :

1. Students will not get difficulties in measuring the time and distance travelled by the car.
2. - Students propose to use the same distance
   - Students propose to use the same time

Activity 4 – "Paper Tapes Activity"

Activity(s):

- Notating Speed in a standard unit

- Developing models

Learning Goal(s):

- Students can compare the speed by notating the speed of each moving object
- Students use table to determine the proportion between distance and time

Problem(s) :

1. Notate the speed of the toy car
2. Make a table of the distance and time of the car travelled in several times

Conjectures of Students' Thinking :

1. Students notate the speed in standard or non standard units
2. Students use the table in vertical or horizontal forms

Activity 5 – Converting Unit of Speed

Goal (s) :

Students convert the speed of car in cm/sec into other units that they want (every group uses their previous data)

Problem(s):

Expand your table and notate the speed of your toy car in various unit of speed (e.g cm/sec, m/sec, km/h)

The researcher also refines the assessment for the last part of the teaching experiment as presented in the following table :

Table 5.1 Refinement of the Assessment

AIMS	MATHEMATICAL IDEAS	ASSESSMENT
Reviewing Students' Informal Knowledge of Speed	• Every moving object has a speed • Speed involves distance of movement and time to reach the distance	-
Comparing the speed of two moving objects	• Something moves faster than another if it needs less time to reach the same distance • Something moves faster than another if it goes farther within the same time	1. Upin and Ipin run from the school to their house. The distance is 900 m. Upin needs 8 minutes to arrive home. Meanwhile, Ipin needs 9 minutes. Who is faster? Why? 2. Dudu and Didi, the ants, run from their place into a cake. During 3 seconds, Dudu walks for 15 cm. Meanwhile, Didi walks for 17 cm as shown in the picture. Who is faster? Why?
Comparing the speed by notating the speed of each moving object	Speed is determined by the distance travelled in a unit of time	3. Rino did an investigation to measure the speed of his toy cars. a. This car runs for 300 cm during 4 seconds. Determine the speed of the car in cm/sec! Explain your answer! b. This car runs for 400 cm during 5 seconds. Determine the speed of the car in cm/sec! Explain your answer! c. From Rino's investigation, which car is faster? Why?
analysing distance and time growth	Speed is a proportion between distance and time • Constant speed • Variable speed	Fajar runs in a field. Every 10 seconds, he stops for a while then continue to run. After 6 stops, he finish his run. a. Draw the path of Fajar if his speed is varying (give a sign for the stop)

<table>
<tr>
<td></td>
<td></td>
<td>b. Draw the path of fajar if the speed is un-constant (give a sign for the stop)
c. If Fajar runs with average speed of 12 km/h, what does it mean? Explain your answer!</td>
</tr>
<tr>
<td>Using models (e.g table) to determine the proportion between distance and time</td>
<td>Ratio table determine the proportion between the distance and time when the speed is assumed to be constant</td>
<td rowspan="2">Nita cycling with average speed of 20 m/sec.
a. Complete the table of distance and time of Nita’s trip.
<table><tr><td>Distance (m)</td><td>...</td><td>...</td><td>....</td><td>....</td><td>....</td><td>....</td></tr><tr><td>Time (detik)</td><td>1</td><td>4</td><td>5</td><td>6</td><td>60</td><td>3600</td></tr></table>
b. Based on the table, determine the speed in km/h!</td>
</tr>
<tr>
<td>Converting the units of speed (e.g cm/sec, m/sec, km/h)</td>
<td>Speed in a standard way can be notated by cm/sec, m/sec, km/h, etc</td>
</tr>
<tr>
<td>Predicting either the distance or the time if the speed is given</td>
<td>• Speed is directly proportional to distance
• Speed is inversely proportional to time</td>
<td>1. Ari goes from his house to the library. The distance from the house to the library is 3 km. If he rides his motorcycle with average speed of 30 km/h, in how many minutes he will arrive at the library? Explain your answer!
2. Father goes to the office by driving his car with average speed of 60 km/h. If he needs ¼ hour to travel, what is the distance from his house to the office? Explain your answer!</td>
</tr>
</table>

5.5 Teaching Experiment

In the following sections, the researcher will compare the HLT 2 with the real learning process in the class. The data collections during the teaching experiment are varied; from the class observation, video recording, students' work, pre-test and final test, and interview with the teacher or the students. The researcher will structure analyses based on the mathematics idea instead of the activity.

5.5.1 Students' Informal Knowledge of Speed

The main goal of the first activity is to review students' informal knowledge of motion and speed and to conduct a general discussion of it. We use a fable as the context in this activity because we believe that it would motivate the students to be involved in the learning process.

As the introduction part of the activity, teacher asked students about a fable, consisting of a race competition in it. This opening question stimulates the students to think and to participate in the discussion. They immediately said about "Kancil and Kura-Kura". Further, one student said that this story is in one episode in "Upin and Ipin" TV series. It is out of our conjectures that the students mentioned "Kancil and Kura-Kura" instead of "Kancil and Siput" in the textbooks. It could be understood because this TV series is presented every day. It is very popular recently and it might influences students' thinking. A story in the television is more interesting for the students than a story in the textbook. Imagine the story in the television might easier for the students because they can visualize the story with colourful and interesting figures.

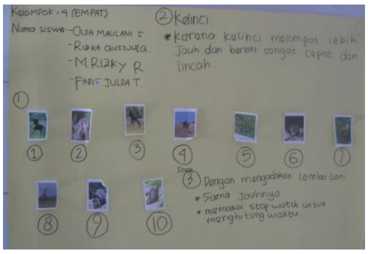
The researcher got an interesting finding when the teacher was asking the students to retell the story. One student who was known as a very low achiever (not only in mathematics but in also other subjects), namely Haris, told the story fluently. He remembered every single detail in the story; he imitated the voice of every character very well. Sometimes, he also acts as the character in the story. Haris is a new student, he moved to this school because in his prior school he has to stay in fourth grade. Hence, his parents moved him to SD Al-Fatah and asked the teacher to put him in fifth grade. All of the students and the teacher were amazed with his way of telling the story. It was concluded because the class are very quite when Haris told the story. Haris got also a very big applause from the teacher and his friends when he finished the story.

Figure 5.8 Haris while telling the story

Asked about the result of a race competition between ‘Kura-kura and Kelinci’ in the story, all of the students answer that Kura-kura is the winner. Naufal then explained that Kelinci failed because Kelinci was sleeping during the race. In the class discussion, the students said that if the rabbit was not sleeping then he would be the winner because he usually runs faster than a tortoise. A term 'fast', 'faster' which is closed to the speed are familiar to the students.

The activity was continued by group discussion in which students make poster of their answers to the students' worksheet. The first problem is comparing 10 animals based on ther speed, the second problem is to determine which one is faster between a rabbit and a tortoise. In the last problem, students are asked to find the way to compare the speed of all the animals so that they know which one is faster than others. Below are some of students' posters :

Translation of figure :
2. A rabbit, because a rabbit walks / runs by jumping, meanwhile a tortoise walks by his feets which are very slow
3. We see the speed of his runs
Holds a race competition



Translation of figure :
2. A rabbit, because a rabbit jumps farther and runs very fast and frisky
3. Holds a race competition
** the same distance*
** using a stopwatch to measure the time*

Translation of figure :
2. A rabbit, because a rabbit are frisky and fast
3. - runs
- the faster need less time
- using a stopwatch to measure the speed of their run

Figure 5.9 Students' answer in comparing the speed of the animals

In the first problem, we expect that students will use their instinct or their knowledge while ordering the animals based on their speed. It does not matter whether or not they order all of the animals correctly as long as they realize that a snail is the slowest animal and the other animals are faster than it. By looking at the pictures of the animals, students might imagine how those animals run so that they can predict which are faster than the others are. Although not all the groups order the animals in the same order, five of six groups decide that the fastest animal is a horse and the slowest animal is a snail. Students' answers show that they have good instinct about speed, in this case, a speed of animals.

It is proved also in the second answer in deciding which one is faster between a rabbit and a tortoise. They totally agree that a rabbit is faster than a tortoise. Although no group gives their reasoning by relating it with the distance or time that might the animals have while they are running, it gives a little bit overview what the informal knowledge that students have about speed. For the students, speed is influenced by the structure of the object. One student explains that a tortoise is slower than a rabbit because rabbit has a small body meanwhile a tortoise has a skull in his back so that it makes him walks slowly. Something which is faster identical with jumping instead of walking, faster also closed to something frisky.

In the third problem, we ask students to find ways to compare the speed of the animals so that someone knows which one is faster than the others are. Three groups mentioned run as the way, meanwhile three other groups mentioned hold a run competition. Moreover, a group explains that the distance for a race competition must be the same and they can measure the time by a stopwatch so that the one who need less time is the winner.

After finishing the posters, students then stick the posters on the wall. Teacher guides the students to discuss the poster by reading students' answers and asking their reasoning. Students were involved in the discussion although not all of them give opinion. They still look shy while answering teacher's questions. When they feel doubt, they do not want to raise their hands as a sign to give their opinion or to answer the questions. In order to improve the discussion, teacher gives motivation to the students many times.

Every moving object has a speed

The researcher uses the context of a fable in the first activity in order to review students' informal knowledge about speed. However, it is not rich enough to make a generalization of every moving object has a speed. Through the discussion with the teacher, we conclude that it is important to add another context and the researcher chooses daily life situation as the context of the discussion. Hence, in the second day, the teacher conducted a discussion with an expectation that through many examples from the daily life situation students will come to it. Below is a segment from the class discussion.

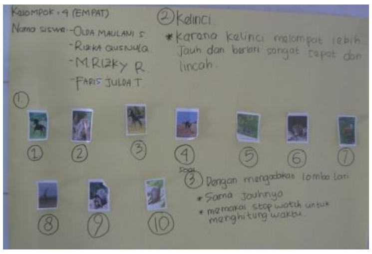
Teacher : In your opinion, what has a speed?
N1 : Animals
Teacher : Animals, ya....
Students : Living things
Teacher : Animal, living things, what else?
Students : Human beings, vehicles
Teacher : Animals, living things, human beings, vehicles, that's good
How do you know that for instance human beings have a speed [raise his right hand]?

N1	*: (when) they run*
Teacher	*: Run...... How do you know that vehicles have a speed?*
Students	*:* (when) *it drove*
Teacher	*: How do you know that animals have a speed?*
Students	*: Run*
Teacher	*: Run... same with human beings run right? It means that they moves right? from one place to another*
	(a few minutes later)
Teacher	*: Anything else that has a speed? for instance a chair, does it has a speed?*
Students	*: No*
Teacher	*: No, what else beside animals, human beings, vehicles?*
H	*: Wind*
Teacher	*: Ok, wind, water, ... water in the river, right? what about toy cars?*
Students	*: Ya...*
Teacher	*: What if the toy cars stand still?*
Students	*: No*
Teacher	*: So, it must move to have a speed right?*
Students	*: Yes*

The segment above shows that it is not difficult for the students to give examples of speed from daily life situations. The examples that they give are varying; not only limited to human beings but also to vehicles and wind. Although the conclusion of the discussion came from the teacher, in the researcher point of view it does not mean that they do not understand it. Students need only to think more about the question from the teacher because they are not accustomed to have a discussion in mathematics class. From the interview with the teacher then cross-checked with students, it is the first time for them to learn mathematics with such organization.

Speed Involves distance of movement and time to reach the distance

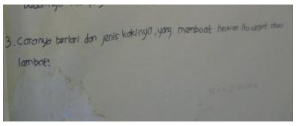
The idea of speed involving distance and time appears first on students' poster. Olda's group wrote that in order to compare the speed of animals they could hold a race competition. They also explained that the distance in the race competition should be the same. After that, one can measure the time by using a stopwatch. Below is Olda's group poster:



Translation of figure 5. :
2. A rabbit because a rabbit jumps farther dan runs very fast and frisky
3. Holds a race competition
** the same distance*
** using a stopwatch to measure the time*

Figure 5.10 Olda's group answer on the second and the third problem

However, there were also students who ignore the distance and time in their reasoning as shown in the figure below



Translation of figure 5.11 :
The way is running and the type of the feet which makes the animals run fast or slow

Figure 5.11 Student's answer without considering the distance and time

Realizing this condition, in the last part of the discussion of the first activity, teacher improvises the discussion by giving an example about students' activity in sport lesson. The following are the improvisation of the teacher in the class.

Students' sport teacher namely bu Menik asks them to sprint. Two students, namely Naufal and Nasha in a race competition of it. At the finish line, the teacher knows that Nasha is the winner. The teacher then asked the reason of it. One student, Hafidz explained that it is because Nasha arrived in the finish first. Again, the distance and the time are not in student's reasoning. Hence, the teacher gives another condition. Below is a segment from the class discussion.

Teacher	*: Or like this. Naufal first run until he arrives at the finish line. After that, Nasha runs until the finish line. And so that all of the students in the class. But, Bu Menik know who is the fastest among them. How does she know?*
Un-known	*: Measure*
Hafidz	*: Using a stopwatch*
Teacher	*: Using a .... ?*
Students	*: Using a stopwatch*
Teacher	*: Using a stopwatch. So, if we want to compare the speed we can also use a stopwatch right? Every student runs one by one. Then bu Menik knows who is the fastest and the slowest by?*
Students	*: A stopwatch*
Teacher	*: So, we can use a stopwatch to compare the..... ?*
Students	*: Speed*

(two minutes later)

Teacher	*: How if Nasha runs for 100 m in one second, meanwhile Naufal runs for 200 m in one second. Who is the winner? [teacher makes a drawing on the blackboard]*
Students	*: Naufal*
Teacher	*: Why Naufal?*
Naufal	*: Because the distance is farther*
Teacher	*: Because the distance is farther. If the distance of Naufal also 100 m, how long will he arrive at the finish line?*
Students	*: 30 seconds*
Teacher	*: 30 seconds or a half minutes right? So, it is not only the time that should be considered, but also........*
Students	*: The distance*

From the students' work and the segment above, we can see that the idea that speed involves distance and time already exist in students' mind. However, when giving reason orally, most of the time only one of them said explicitly. In the next activity, students were comparing the speed of two toy cars. Every group compares the speed of toy cars successfully and in the discussion students come to the conclusion that speed is influenced by the distance and time as shown in the following segment.

Teacher : *Let's make a summary of it. From the investigation, that we did what we already know?*
Students : *The distance*
Ifah : *Speed is influenced by the distance and time.*

It needs more time for the students to realize that speed involves the distance and time because students can compare the speed sometimes only by seeing some moving objects without measuring the time or the distance as in the context of a race competition. They know that the fastest is the one who reach the finish first without considering the time. However, by providing the tools of measuring the distance and time and doing the investigation, students come to that idea.

5.5.2 Comparing Speed: When something is faster than another?

If we ask students to compare the speed of two moving objects, it is not a big problem for them; they can compare properly and it happened during the teaching experiment. When students are asked to compare the speed of two toy cars, in-group either of four or five students or during the class discussion, they did it correctly. Meanwhile, students' reasoning in comparing speed can be classified into two mathematical ideas that we want to develop.

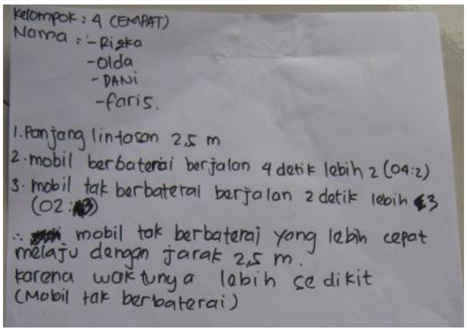
Something moves faster than another if it needs less time to reach the same distance

Different from the try out session, in the teaching experiment the researcher provides the students with the tracks for the toy cars. Two groups have two tracks each; meanwhile the other four have a single track. The different numbers of the track are on purpose; students will come up with various strategies to compare the speed of toy cars. In the conjectures, we predicted that students who have double tracks would use a race competition; meanwhile, the group with a single track will measure the time needed for each car.

Using the track that they have, all of the groups decide the faster car correctly. However, the strategies that they use are similar. They measure the length of the track, run the car one by one and measure the time needed for every car to finish the track even for the group that has double track. From the time they measured, they can decide which car is faster. The following figure shows the investigation by the students.

Figure 5.12 Students while doing the investigation

After the investigation, students wrote the result in the paper as shown in the figure below

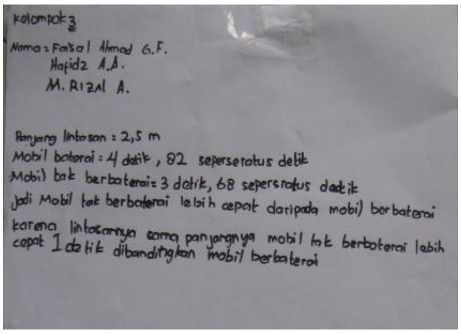


Translation of figure :

1. *The length of the path is 2,5 m*
2. *A battery car runs 4 seconds and 2 (04 : 2)*
3. *A non battery car runs 2 seconds and 3 (02 :3)*

Hence,
a non battery car is faster, runs with 2,5 m of distance.
Because it needs less time (A non battery car)

Figure 5.13 Riska's group work in comparing the speed of two toy cars



Translation of figure
The length of the path is 2,5 m
A battery car runs 4 seconds , 82 perhundreth seconds
A non battery car runs 3 seconds , 68 perhundreth seconds
Hence, a non battery car is faster than a battery car because the length of the track is same, a non battery car 1 second faster compare with a battery car
Because it needs less time (A non battery car)

Figure 5.14 Hafidz's group work in comparing the speed of two toy cars

Given a measuring tape and a stopwatch, it is clear for the students that speed is involving the distance and time. All of the students measure the length of the track and the time needed for every car to travel the distance. They decide the faster car carefully. In giving the reason, some groups only write the time and the distance for every car. Meanwhile, complete reasoning was given by Hafidz's group as shown in the figure above.

Something moves faster than another if it is goes farther within the same time

Even without measuring precisely the length of the path and the time, students still could decide which one is faster. During the second activity, after all of the groups compare the speed of two toy cars and present their works, teacher conducts a class discussion. In the class discussion teacher runs two toy cars (the black and the yellow cars) on the tracks against each other and asks students to determine which of these car is faster.

Teacher laid two tracks on the floor, two students hold a different car in the different side and run those car at the same time. All of the students stood up around the tracks. After both cars were arrived in the finish line, the teacher start the discussion as follow

Teacher : *Which car is faster?*
It against each other here. Which car is faster, where is it Fal?
Nasha : *This ...... this [touch to the black car]*

Figure 5.15 Students run the cars against each other

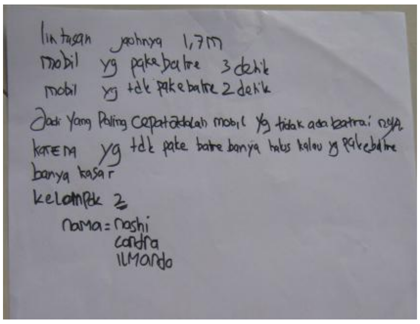
Teacher : *The black or the yellow one?*
Nasha : *The black*
Teacher : *Why the black one?*
Hafidz : *Because it (the black car) is nearer to the finish*
Teacher : *Ok, because it is nearer to the finish. What about this one (the yellow car)*
Students : *It is farther*
Researcher : *Farther to the.... ?*

Students *: To the finish*
Researcher *: It is more ....*
Ifa *: Slower*
Researcher *: How if we measure the position of the car from the initial point of the movement?*
Teacher *: If we look at the position .... How far it moves?*
Researcher *: It moves from what to what?*
Students *: measure, measure hey*
Teacher *: From here to here*
Naufal *: Measure*
Teacher *: Ok, measure*
How long is it?
Students *: one and a half*
Teacher *: One hundred and forty nine. Ok, this one is one hundred and forty nine what about another one .... the yellow car*
[Students measure the distance travelled by the yellow car]
Teacher *: One hundred and ....*
Students *: One hundred and thirteen*
Teacher *: So, if we look at the distance travelled, it means that it goes farther*
Students *: But, the distance is the same. Oh ya, but it has not arrived to the finish yet*
Teacher *: Why?*
Students *: but the length of the distance is the same*
Teacher *: That's Ok. So, which one is faster?*
Students *: the yellow car*
Teacher *: What is the first reason?*
Students *: It is nearer to the finish*
Teacher *: The second reason?*
Students *: It is far from the start than the yellow car*

Hafidz gave his reason, because the black car is nearer to the finish line than the yellow car. Perhaps Hafidz's reasoning is still influenced by the situation of a race competition. Hafidz prefers refering to the finish line as preference of the movement instead of thinking about the distance of the movement itself; that is the distance from the initial position.

Different from others, Ilmando's group reasoning is still influenced by the structure of the car while comparing the speed of two toy cars. They write the distance and the time for each car; they decide correctly, which one is faster.

However, in giving reason they mentioned that the faster car is a battery car because a non-battery car has a smooth tyre whereas the battery car has a coarse tyre. Below is Ilmando's group work.



Translation of the figure :
The distance of the track is 1,7 m
A battery car 3 seconds
A non battery car 2 seconds
Hence the faster car is a non battery car because a non battere car has a smooth tyre meanwhile the battere car has a coarse tyre.

Figure 5. 16 Ilmando's group reason in comparing the speed of two toy cars

The description above shows that students can compare the speed properly either by measuring the distance or not. The reason that students give explicitly mentioned the distance and the time. Students come also to the conclusion that by using the same distance, the faster is the one who need less time. Meanwhile, by using the same time the faster is the one who goes farther than another.

5.5.3 Speed determined by the distance travelled in a unit time

In the third activity, students again worked in a group of four or five. There are six groups in the class. Every group had a toy car, a measuring tape, a stopwatch and a single track. The lengths of every track given to the students are different. We call this as 'paper tapes activity'. In this activity, students are asked to measure the distance and the time of their car. For the time, teacher asked to omit per hundred seconds. Students did the investigation outside the class as shown in the following figure

Figure 5.17 Students measure the time and the distance of the toy cars

All of the students came together to the tracks that have been prepared before. They worked cooperatively. With four or five students in the group, there are job ceiling among them. One student held the car, one student held the stopwatch, two students measured the length of the track, meanwhile another student wrote the result of their investigation. Students' roles are sometimes interchangeable to each other.

None of the students got difficulties in measuring the distance and the time. After doing their investigation, they wrote the results on their paper tapes. Below is the result of their investigation:

NO	GROUP	DISTANCE	TIME
1	Group 1	150 cm	3 seconds
2	Group 2	250 cm	5 seconds
3	Group 3	350 cm	6 seconds
4	Group 4	450 cm	6 seconds
5	Group 5	500 cm	9 seconds
6	Group 6	600 cm	13 seconds

Finishing the measurement, teacher asked the students to find the way to compare the speed of all of the toy cars so that they can determine the fastest car among them. All of the students have a discussion in their group. They wrote the result from the whole group in a paper. For around 8 minutes, no group gives the answer. It seems very difficult for them to compare the speed of the whole cars although the teacher come to the some groups and try to help them by relating with their previous activity in comparing two-toy cars use either the same distance or the same time. Realizing students' difficulties, teacher proposed his way. He explained that he would use the same time for all and the time that he chose is one second. Hence, he asked the students to predict the distance of each toy car in one second.

The researcher is involved also in the group discussion. She came to the some group and tried to lead the students to the conclusion that in order to compare all of the speed, students can determine the distance of each toy cars in one second. In Anang's group the researcher drew the line and wrote that the distance is 100 m for two seconds. In the conversation, one of the students can show the length for one second but get difficulties in giving the reason The following is a conversation between the researcher and Anang after 10 minutes the researcher left his group.

Anang	*: [come to the researcher and show his group's work] Miss Fatim. I found it.*
Researcher	*: What's up, Nang?*
Anang	*: The car in fourth group is the fastest, 75*
Researcher	*: This...* [look at Anang's group work]
Anang	*: The first and the second are same*

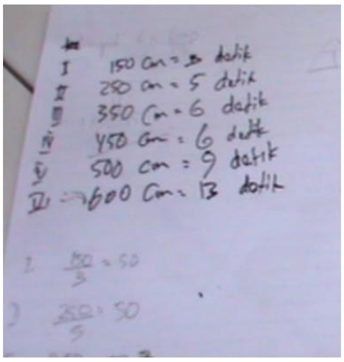


Figure 5.18 Anang’s group work

Researcher	*: so, fifty is* (the distance) *in* every [point to the number 50 as the result of the division of 150 by 3]*?*
Anang	*: Three seconds, wait.....one second*
Researcher	*: One second, so which one is fastest?*
Anang	*:* (the car in) *fourth group*
Researcher	*: Because.... what is 50 means here?*
Anang	*:50 for every one second*
Researcher	*: What is 50 here?*
Anang	*:50 cm in every one second*

All of the students divided the distance by the time in every group. When they finished the calculation, teacher then asked them to put their car in the track as if the cars travel for one second. First, students measured the distance that they want using a measuring tape. After that, they put the car on it. Without request from the teacher Riska’s group made a sign in the paper tape for the distance travelled by the car in one second.

Figure 5.19 Riska and friends measure the distance and put the car as if it travels for one second

After that, teacher asked the students to make signs of distances travelled by the car for every one second. All of the groups measured the distance without doing real investigation. For the first sign (students make lines as the sign), they measured the distance based on their calculation. After that, they continued to measure again using the line that they made before as the starting point.

Figure 5. 20 Ifah and friends make signs for the position of the car in every one second

When the students finished their work, teacher conducted a discussion. Using paper tapes that students made, teacher asked the students about the distance and time growth during the car travel. Teacher took the result from group 1 to start the discussion. Below is a vignette from the discussion

Teacher : *Start from group 1, what is the distance for every one second?*
Students : *50 cm*
Teacher : *For 2 seconds?*
Students : *100* (centimetres)
Teacher : *For 3 seconds?*
Students : *150 cm*

(a few minutes later)

Teacher	*: What about group 2, what is the distance of it* (for every one second)*?*
Students	*: 50*
Teacher	*: Group 3*
Students	*: 58,3*
Teacher	*: So it is faster (than the first and the second group)* *What about this, group 4. Riska. What is the distance for every one second?*
Students	*: 75 cm*
Teacher	*: So, it is faster right. What about this group?*
Students	*: 55,6*
Teacher	*: What about this* [point to the paper tape of group 6]*?*
Students	*: 46,2*
Teacher	*: So, which one is faster?*
Students	*: Group 4*
Teacher	*: Why?*
Students	*: Because it travels for 75 seconds in every one second*

The conflict happened when the students are asked to determine the fastest car. Meanwhile, they have different data in which the distance and the time for every car are different, except, group 3 and group 4 who have the same time for the distance given. We can see it while students cannot find the way for sometime until the teacher and the researcher come to the students' group to have a discussion with them. Students then come to the idea that they have to divide the total distance by the time so that they know the distance in one second. Without giving the formula of speed, students show speed by dividing the total distance by the time.

Understanding the distance and time growth is a kind of thinking experiment. However, the last vignette above shows that, it seems that paper tapes quite effective for students to visualize the distance and the time growth during the car travelled. Knowing the distance in one second, students immediately show the distance in two seconds, three seconds and so on by making signs (lines) in paper tapes. These lines divided the paper tapes into some parts, which is the same as the total time of the movement. Since students made the lines by themselves, they realized that the length of every segment is equal. We can see it from the vignette above when the teacher asked the distance for one second, two seconds, and three seconds above.

Students notated speed in standard unit in the next meeting. Teacher asked the student to write the speed of their car. Teacher split every group into two in this task. Based on students answer we can see that all of the students use a standard way to notate the speed, namely in cm/sec, below are students' work in notating the speed

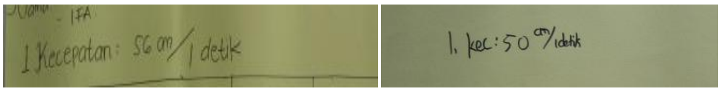


Figure 5.21 Students notate the speed in standard unit

Converting the unit of speed

From the figure above, we can see that students notate the speed of toy car in cm/sec. After the discussion about constant and variable speeds, teacher made a ratio table in horizontal form as a structured way to write the proportion between the distance and the time. After that, teacher asked the students to make their own table based on their investigation of their paper tapes in the previous meeting. Using the table, students had to show the speed in other units, for instance m/sec, km/minutes, and km/h.

Using the table, all of the students can convert the unit of speed. However, the steps that they use are different as well the number of units of speed that they determine. Below is one of student's work in converting the unit of speed by showing the time in a series at the first time.

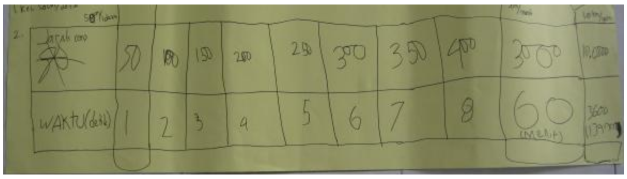


Figure 5.22 Leo and Chandra's work in converting the unit of speed

The figure above shows that at the first time, Leo and Chandra show the proportion between distance and time by adding the distance in every column to the right with 50 and adding the time by 1. Then they jump from 8 seconds into 60 seconds or one minute. It might be the case because they wanted to convert the unit of speed into cm/minute or m/minute. Unfortunately, it is not clear enough about their way to find the distance in one minute.

Different from Leo and Chandra, Nadya and Ifa did not write the time consecutively. Below are their answers

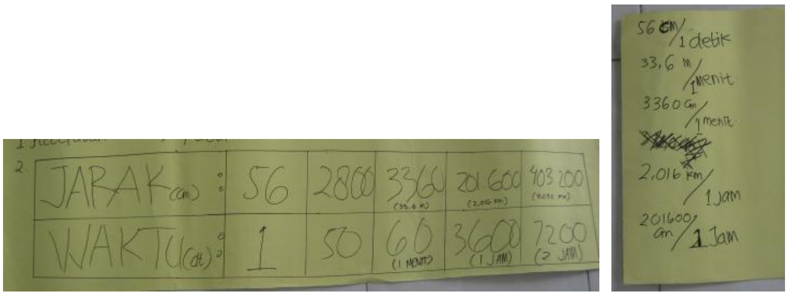


Figure 5.23 Nadya and Ifa's work in converting the unit of speed

From Nadya and Ifa's work, we may see that they are more advanced in using the ratio table than Leo and Chandra because from the first, Nadya and Ifa jump from one second into 50 seconds. After that, they jump to one minute, one hour and two hours consecutively. In their work, they wrote five different units of speed in cm/sec, m/minute, cm/minute, km/h and cm/h.

It seems that all of the students can use the ratio table to convert the unit of speed because all of the students convert the units of speed at least in cm/minutes or m/minutes. Some of the student's work above represent various answers from various students. Leo and Chandra represent the students with average and low achievers, meanwhile Nadya and Ifa are an average and a high achiever consecutively.

5.5.4 Speed is a proportion between a distance and time

At the first stage of understanding speed as a proportion of a distance and time, students first should know that speed is influenced by the distance and time. In the teaching experiment, students come to this idea during the class discussion in which they display the result of the whole group in comparing the speed of toy cars as explained in the previous section. Meanwhile, students understand the speed as the distance travelled of something moves in a unit of time since in the 'paper tapes' activity.

At the next day, teacher displayed two paper tapes on the whiteboard. Those paper tapes are the result of the investigation by group 3 and group 4 in previous meeting. Teacher stands in front of the whiteboard and all of the students sit in their chair. After that, he conducted a discussion about those paper tapes. Below is a segment from the class discussion in which students explain speed as a distance travelled in a unit of time

Teacher : Give attention to this track, they already made lines in it (to differentiate one segment from other segments).

Who is (the member of) group 3?

Students : [some students raise their hands]

Teacher : Group 4?

Students : [some students raise their hands]

Teacher : Can any one of you explain what it means for one segment here [touch to the first segment], group 4 especially........ does any one want to give help to the group 4?

Hafidz : [raises his right hand]

Teacher : Ok..... great help (is given) by group 3. Hafidz, what does it mean here?

Students : The distance, the distance travelled by the car in one second

Teacher : How long is the distance?

Students : 75

Teacher : 75 cm. That's great. He explained that the distance travelled by the (toy) car in one second is 75 cm
What if we (mentioned) reversely?
The distance of the toy car is 75 cm in one second
What is that, the distance travelled in one second?

Students : Speed

Teacher : Right, it is a speed. So the speed of that car is 75 cm in one second
What about this? [touch the second segment in the first paper tape]

Figure 5.24 teacher guide a discussion about paper tapes made by the students

(a few minutes later)

Teacher : what about this [touch the first segment in the second paper tape]
How (the speed) of it?

Students : The speed of the car is 58, 3 cm in one second

From the vignette above, we can see that students see the speed as the distance travelled per unit time. Later, students' understanding toward the proportion of the distance and time develops along the learning process. We can see also students' understanding on the proportion of the distance and time in constant and variable speed in the following section.

Constant and Variable Speed

The discussion about the idea of constant and variable speeds was conducted in the fourth meeting. After two paper tapes from students' work above, teacher remove and change them with another paper tape. The researcher made this paper tape and it illustrates a constant speed. This paper tape consists of seven segments of the same length. Teacher stands in front of the class and the students sit in groups of four or five. Below is the continuation of the discussion

Teacher : *look at these [touching the first four segments]. (the length of them are) equal, right?*
Students :*yes, they are equal*
Teacher :*what does it show [touching one segment in the beginning of the paper tape]?*

Figure 5.25 teacher shows a paper tape

Student : *The distance*
Teacher : *If we give the time, for instance, for every second...... so what does it become?*
Student : *The speed*
Teacher : *So, the distance travelled from here to here [touch the first then the second line of the first segment] during one second, how we call it?*
Student : *The speed*
Teacher : *does (the length of them are) equal?*

Student : *(yes, they are) equal......*
Teacher : *So, how is the speed here?*

Hafidz : The speed is constant
Teacher : 'tetap' (constant in Indonesian) or constant. The speed is 'tetap' or constant. If the length as far this, then it will always fixed. How if the car is faster? What will happen with the drawing? [raises his right hand as a sign to invite the students to be involved in the discussion]

(After two minutes)
Teacher : Ok, Anang
Anang : It goes right [moves his right hand to the right]
Teacher : here, So..... the distance is more ...... what?
Student : Farther
Teacher : Larger ..... farther, right?
So, if it is faster then the distance will be farther.
Teacher : If the car is slower than this [touch the first segment], what will happen? Naufal
Naufal : [moves his right hand close to his face twice indicates shorter distance]
Teacher : The lines change into the left. It is shorter, right?
It could be this far in one second [moves his hand into the paper tape and touch shorter part of one segment in the paper tape] this far, this far .... shorter.

The vignettes above show how the teacher tries to guide the students to understand about constant speed with the aid of paper tapes. The phrase ***"the speed is constant***" from Hafidz shows that the term 'constant speed' already exists in students' mind. By the guidance from the teacher, students realize that in constant speed the distance in every second is the same. The researcher conjectures it is because they made this kind of paper tape in previous activity. In the last vignette above, we can see Anang and Naufal predict successfully the distance if the toy car goes faster or slower. It seems that students realize that when the speed changes, the distance travelled in a unit of time also changes. In the following vignette, the teacher continued the discussion with a variable speed.

Figure 5.26 teacher shows two different paper tapes

[a few minutes later, the teacher sticks second paper tape with seven segments in which two segments are shorter, three of them are of the same and the others are longer]

Teacher	:	*What is the difference from this? [touch the second paper tape]* *I want you to think* *Good......*
Wira	:	*The distance*
Teacher	:	*The distance. Why is the distance?*
Wira	:	*The distance is shorter*
Teacher	:	*This one is shorter [touch at the first segment in the second paper tape]. What about this [touch at the second segment in the second paper tape]?*
Students	:	*Longer*
Teacher	:	*This? [touch the third segment in the second paper tape]*
Students	:	*Shorter*
Teacher	:	*This? [touch the fourth segment in the second paper tape]*
Students	:	*Longer*
Teacher	:	*So ....., what? Riska?*
Riska	:	*The distance is different...... the distance is different (speak louder than before)*
Teacher	:	*The distance is different. If we measure for every second, one second, one second, one second, one second. What does it mean?* *It means that .... how is the speed of the lower tape?*
Students	:	*It is different*
Teacher	:	*Any other term for different..?*
Ifah	:	*The speed is changeable*
Teacher	:	*The speed is changeable or not constant*

We can see that in order to understand a constant and a variable speeds, students are able to use their experience in previous activity to analyse the paper tapes. Every paper tapes illustrate the total distance by the toy cars and every segment illustrates the distance in every second as students made in previous activity. Since students already know that speed determine the distance travelled in a unit time, then students also explain that the paper tapes also illustrate the speed.

Students did not have an experience in making paper tapes for variable speeds. However, they could conclude that the second paper tape shows variable speed under the guidance by the teacher. The teacher guided the students by asking the different lengths of the segments on those paper tapes and asking students' opinion about it. Although it is the first time for the teacher in teaching mathematics using RME approach, in the researcher point of view the teacher is successful in conducting the class discussion.

Ratio table to determine the proportion between distance and time on speed

After conducting a discussion about constant and variable speeds using paper tapes, teacher tried to make a transition from the paper tapes to a ratio table model. Teacher asked the students about the length of one segment as if the total length of 'constant speed paper tape' is 280 cm and the time is 7 seconds. Most all of the students shout that the way to calculate the length is by dividing the total distance with the time so that they got 40 cm. After that, teacher continued with two, three, four seconds and so on. The following is a vignette of the discussion.

[teacher stands in front of the whiteboard with two paper tapes on the board]

Milda	*: forty (forty is the length of one segment in a constant paper tape if the distance is 280 cm and the time is 7 seconds)*
Teacher	*: forty. So, in one second it is forty (cm). What if it is in two seconds, how long is it? ..... I want the others. ....*
Olda	*: eighty*
Teacher	*: How long in three seconds? Oh ..... it is forbidden to make noisy. This one [point to a student in front]*
Student	*: one hundred and twenty*
Teacher	*: What if it is in five seconds? Ok, Izur.*
Students	*: two hundreds.*
Teacher	*:The last. What if it is in ten seconds. So it is more than this paper tapes right?*
Students	*: Nasha, sir.*
Teacher	*: Nasha*
Nasha	*: Four hundreds*
Teacher	*: It is four hundreds for one second, right?, because how much for one second, forty right? so that for ten seconds is four hundreds What if it is one minute? How many seconds in one minute?*
Student	*: sixty*
Teacher	*: sixty seconds. How much is it in one minute? You can calculate in your paper*

After that, teacher made a table on the white board. The table, in horizontal form, consists of two rows and many columns. Teacher wrote the distance (cm) in the first column and the first row meanwhile the time (seconds) in the first column of the second row. Teacher then filled the second and the third column by the distances in one second, two seconds, five seconds and ten seconds. After that teacher asked the students to continue the table.

Figure 5.27 Teacher made a ratio table on the whiteboard

Teacher	*: My question before, if the time is one minute, how long is the distance?*
Students	*: (raise their hands)*
Teacher	*: Hafidz, write it besides (the next column). If it is one minute or sixty seconds how long is the distance?*
Hafidz	*: (come to the white board and write 2400 in the next column)*
Teacher	*: Ok. Two minutes, how long is it? Wira*
Wira	*:[wrote 120 in the time's row and 4800 in the distance's row]*

The vignette above shows the moment of the transition from the paper tapes model to the ratio table model. With paper tapes and guidance from the teacher, it looks easy for the students to show the distance in many different times. The paper tapes could help the students to visualize the distance growth for every second. Meanwhile, in ratio table the distance and the time given not in order. Someone could jump from one second directly to the ten seconds, for instance, to the one minute, one hour and so on. Although the students never know the ratio table before, it seems that students did not get difficulties in using the ratio table. We can see how the students use ratio table in the section of converting the unit of speed and exploring interrelation among speed, distance and time.

5.5.5 Interrelation among speed, distance and time

The goal of the last meeting is that students would understand interrelation among speed, distance and time. In order to reach the goal, we provide the students with the task as the following:

1. A rabbit runs 12 km in 15 minutes. Determine the speed of the rabbit in km/h and explain your answer!
2. Mother went to the market by motorcycle with average speed of 30 km/h. If she arrived in the market in 10 minutes then how far is the market from her house?
3. During the semester holidays, Chika and her family who lives in Surabaya go to the Jatim Park in Malang, that is, 90 km far. If Chika's father drove the car with average speed of 45 km/h, how can you predict the time needed for the travel? If they leave to travel at 7 o'clock in the morning, what time will they arrive at the Jatim park?

The researcher predicted that students would use tables to solve the problems. The tables made by the students would vary and be developed from the initial forms in previous meeting since there would be 13 groups in the class. Moreover, students are free to choose the way to solve the problems. The groups of students in the class consist of 12 groups in pairs and one student works individually. Students wrote their answer in the poster and in the last part there was a discussion on students' answers.

The following is a vignette of a student who gives his reason for the first problem on the presentation in front of the class.

Teacher	*: Hafidz, explain your answer to your friends*
Hafidz	*: 12 km in fifteen minutes*
Teacher	*: 12 km in fifteen minutes. How does it become a quarter hour?*
Hafidz	*: because 15 minutes is a quarter hour*
Teacher	*: after that?*
Hafidz	*: we make it into a half hour so that we add it [point to the 12 on the blackboard] with 12 and become 24*
Teacher	*:Ok, then....*
Hafidz	*: We add (the time) with 15 minutes to become three fourth hours and we add (the distance) 12 more become 36*
Teacher	*: Ok, then.....*
Hafidz	*: then, three fourth hours is 45 minutes so that 45 minutes plus a quarter (hour) is to be one hour and we add again by 12 so that it becomes 48*
Teacher	*:if we skip these two [make a cross sign for 24/30 and 36/45] how can we find (the answer) ?*
(a few seconds)	
Hafidz	*: we multiply by 4*

Hafidz's resoning is based on his answer on the poster. He rewrite his answer on the white board except the clock as the model. Different with Hafidz's group, Naufal's group use a ratio table as their reasoning. Meanwhile, Anang explain by write the proportion among distance and time. Below are those students' works

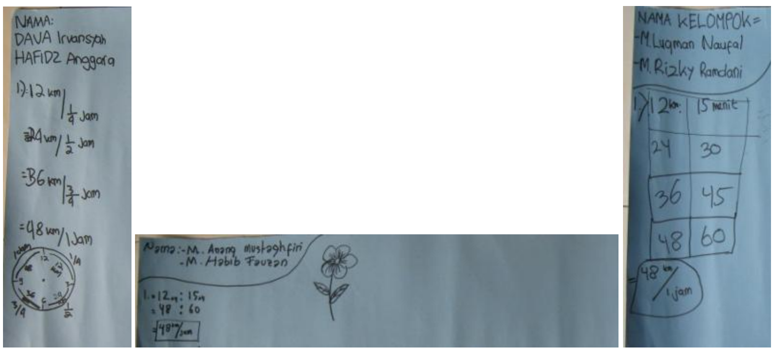


Figure 5.28 Students explain the proportion of the distance and time

Considering Hafidz's reasoning and students' works above, we can conclude that students understand how the proportion of the distance and time in speed. Every time they multiply the time then the distance also increases based on those multiplier and vice versa. Hafidz's and Naufal's works show that they have a similar step to solve the problem. They have 3 steps; by adding the distance and time by 12 and 15 in every step respectively. Meanwhile, Anang's group use proportion and the similarity of the proportion so that they come to the answer directly.

For the second problem, most of the students use a ratio table. There are two types of table that they use; tables in horizontal and vertical forms. Meanwhile, there were two groups using the proportion for the first problem, namely Anang's and Faisal's groups. Below are students' posters on the second problem.

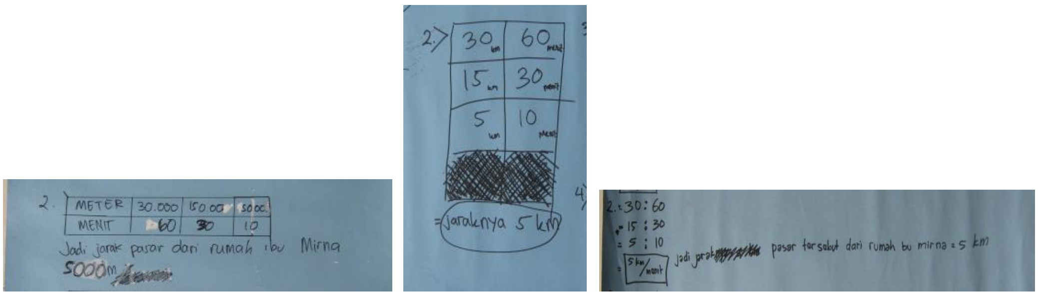


Figure 5. 29 Students answer in predicting the distance

For the third problem, most of the students use a ratio table as well. Like the second problem, there are two types of ratio table that they use. Meanwhile a proportion appears again in Anang's and Faisal's works. The following vignette is an example of a student's explanation in solving the third problem. This student used a ratio table and he explained her reason during the presentation in front of the class.

Naufal	*: [read the third problem in front of the white board]*
Teacher	*: Ok, explain your answer Give attention please! (to all students in the class) What does it mean?*
Naufal	*: The average speed is........ (repeats the problem)*
Teacher	*: Average speed, how fast is it?*
Naufal	*: 45 km per hour. If one hour is 45 km, then 2 hours is 45 multiplied by 2, that is 90 so that the time needed is 2 hours*
Teacher	*:Ok, good. Come back to your seat*

Naufal multiplied the number of the distance and time in the ratio table instead of adding it. When he multiplied the time by two then he also multiplied the distance by two so that he came to the answer. Students who could answered the third problem also answered the fourth problem correctly.

There is a consistency of students' way in answering the problems. Students who use a ratio table for the first problem use it also on the second and the third ones. There was also a consistency in the types of ratio table that used by the students. The students who use a proportion are consistencet also in it. From thirteen groups of the students in the class, only four groups who could not finish their work. Olda is a student who got difficulties in solving the problems given. The researcher came to her group and tried to investigate her understanding.

Olda	*: [make a ratio table with horizontal type].*
Researcher	*: Ok, put what is known on the table. The distance and the time*
Olda	*: [write the distance on the frst row and the time on the second row. After that 12 km and 15 minutes in the second column respectively].*
Researcher	*: (the distance) 12 km and the time?*
Olda	*: 15 minutes*
Researcher	*: if (the time) is 30 minutes, what is the distance, Lafif? (Lafif did not give the answer)*
Researcher	*: Olda (Olda did not give the answer)*

(a few minutes later)	
Researcher	*: Ok, imagine that an ant walks. If it walks 3 cm in 2 minutes, for instance, and it walks again in 2 minutes what is the distance?*
Olda	*:6 cm*
Researcher	*: 6 is only for this or all of it?*
Olda	*:all of it*
Researcher	*:ok, all of it. So the time for here to here is.....*
Olda	*: four*
Researcher	*:now, it walks again and the time is*
Olda	*: 6 minutes*
Researcher	*: so, the distance is*
Olda and Lafif	*: 9 cm*

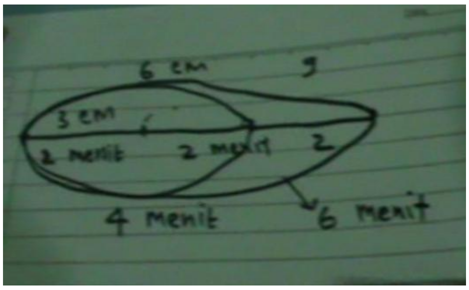


Figure 5.30 The researcher's draw to help Olda and Lafif

The descriptions above show that most of the students understand the interrelation among speed, distance and time. They did not get difficulties in determining the speed when the distance and the time are given, or determining the distance if the speed and the time are given, determining the time when the speed and the distance are given as well. In determining the relation, most of the students use a ratio table. The researcher predicted that it was heppened because a ratio table is a handy way for them. Using a ratio table, the proportion between the distance and the time is clear and every step that they made in the ratio table has meaning for them.

5.5.6 Assessment

Pre-test of the students

In order to discover pre-knowledge of the students, pre-test was conducted before students are involved in the series of activities. Based on the result of the pre-test, the researcher tries to ensure herself whether or not the design she prepared is appropriate for the students. The pre-test was conducted on March 25th 2011. 22 of 25 students are involved in the pre-test because these two students were absent at that time. The researcher will present the result of the pre - test per question.

The problems given in the pre test consist of six problems as the following:

1. Last semester holiday, Rahmad visited her grandmother who lives in a village in Mojokerto. First, he went with a public transportation and travelled for 12,5 km. After that, he walked along 500 m to reach his grandmother's house. What is the distance that he had covered in total? Give your reason!
2. Dina starts to study at 18.00 and finishes at 19.00.
    a. How long does Dina study? Explain your answer!
    b. How many minutes is it? Explain your answer!
    c. How many seconds is it? Explain your answer!
3. Have you ever seen the tools below? What is it for? Give an explanation of the word inside!

4. How can we determine the speed of a moving toy car? Explain your answer!
5. If someone said "I drive my car with an average speed of 40 km/h" What does it mean? Explain your answer in the box given!
6. Mrs. Aisyah goes to school by a motorcycle. The distance from her house to the school is about 3 km. If she wants to drive her motorcycle with average speed of 45 km/h, how long does she take to arrive at the school? Give your reason!

For the first problem, students' answer show that 16 out of 22 (73 %) students know that in order to calculate the total distance, they must add 500 m to 12,5 km. It indicates that most of the students know what the distance is. The rest of students were multiplying or dividing the numbers. From 16 students above, 7 students try to make the unit equal whether in km or in m. However, only three of them answer correctly. It is out of the researcher conjecture that students get difficulties in converting the unit of distance.

Although most of the students cannot answer the problem correctly, it can conclude that most of them know the term 'distance'. Probably, the term 'distance' is used also in their daily life. Distance, according to them indicates how far something goes. Students learned the unit of length and its equivalence since they are in grade three. However, the unit km and m in the problem seem not to give meaning for some students so that they only add those numbers without converting the unit. It is probably because of their way of learning before. Students must recognize stairs of length unit, if they want to convert a unit into the lower unit, they must multiply the number by ten or put a zero behind the number. On the contrary, if they want to convert a unit into the upper unit, they must divide by ten or remove the last zero of the digit number. Students, most of the time, forgot to multiply or divide by ten, what they recognize is putting or removing the zero number. Hence, they write 12,5 km as 12,5000 m when converting the distance.

The aim of second problem is to assess students' knowledge about time, duration and their ability in converting the unit of time. 18 of 22 students (82 %) answer correctly that Dina studies for 1 hour. Some students get the answer by subtracting 18.00 from 19.00 so that they get 1. Some of them write that 18.00 o'clock is equivalent to 6 o'clock in the evening and 19.00 o'clock is equivalent to 7 o'clock. One student also illustrates it by making a drawing of a clock. Hence, they conclude that Dina studies for 1 hour. Below is one of student's answers.

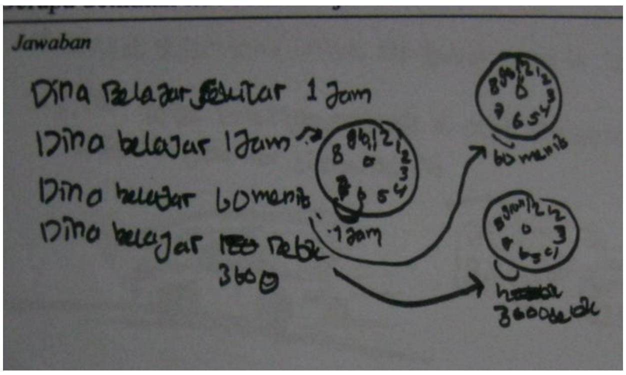


Figure 5.31 Dava's answer on the first problem

It seems that most of the students know that 1 hour is equivalent to 60 minutes. Furthermore, one student gives a reason it is because a minute hand rotates for 60 times in 1 hour. Nevertheless, many students still get difficulties in converting 1 hour into seconds. From 18 students who answer that Dina studied for 1 hour, only 7 of them who convert into seconds correctly.

Only 7 students who write that they have ever seen a speedometer. It does not mean that they did not know a speedometer because 17 students write correctly the use of a speedometer. It shows two possibilities. Students might ignore the first question or they forgot to answer the first question on the number three. Two students write explicitly km/h as the unit of speed even one student give an example as follows:

The hand points to the number 20, it means that the motorcycle is ride along 20 km during one hour

Fourth problem was to check students' way to determine the speed of something moves. For this problem, three students explain by measuring the time and the distance. One of them also wrote the formal way as:

$$v = \frac{s}{t}$$

Some students wrote that they can measure the speed but they gave wrong way to measure; for instance, one student explained by putting the speedometer inside the toy car. Meanwhile, the rest said that they could not measure the speed with various reasons.

From the interview with the student who uses the formula to show the speed, the researcher knows that this student get the formula from her mother, her mother is an elementary teacher. Meanwhile, for other students it could be understood that they get difficulties in answering the question because officially they did not learn the concept of speed yet.

In answering the fifth problem, three students explain 40 km /h meaning that someone will reach 40 km during one hour, 2 students explain 40 km/h as high speed, 8 students only rewrite the problems, 4 students give wrong answers and 5 students did not give their answer for the fifth problem. Students' answer show that although the unit of time 'km/h' exist in daily life situation, it has no meaning for the students. However, we expect that after the learning process, students will develop their understanding.

The last problem was to check students' skill in explaining relationship among speed, distance and time. The researcher assumed that the problem is difficult for students who have never learned the concept of speed before as students in SD Al-Fatah. The conjecture was proved because it seems that students only operate on the numbers in the problems. There are students who multiply, divide, or subtract the numbers. Even student who know the formal procedure for the speed write correct procedure but has a mistake when concluding.

From the pre-test we conclude that these students, in general, are able to start to learn the concept of speed because they already have pre-knowledge of speed. Nonetheless, the discussion with the teacher makes us conclude that it is important for students to refresh their skill in converting the unit of distance and time. Hence, before the teaching experiment the teacher gives two meetings for it.

During one meeting students learn the unit of length and its conversion and another meeting for time and its conversion.

Final test of the students

During seventh lesson, students made a final test with seven tasks, which are described in this section. The result of the final test is only part of the assessment to get the general overview of students' development after learning the concept of speed. Twentythree of twentyfive students were involved in the final test.

First task : Comparing with the same distance

Upin and Ipin run from the school to their house. The distance is 900 m. Upin arrives at home in 8 minutes. Meanwhile, Ipin arrives at home in 9 minutes. Who is faster? Why?

Among 23 students, only one student, namely, Haris who gives wrong answer. Other students wrote that Upin is faster than Ipin and the reason that they gave are:

- ✓ Upin runs faster, the time needed for Upin is less than Upin in the same distance

  Students who give this reason consider the distance and the time being influenced by the speed. They understand that within the same distance, the one is faster if it needs less time than others do.

- ✓ 900 is divided by 8 and 9

  This student compares the speed of two moving objects successfully. The researcher predicted that the student uses the calculation to find the speed although he did not write the result. It is not clear enough whether this student understand that within the same distance, the one is faster if it needs less time than others do or not.

- ✓ The time for Upin is less than Ipin

  Since the distances are equal, these students only consider the time needed to reach the distance.

- ✓ Upin arrives at home first

  The students are still influenced by the situation so that they ignore the distance and the time. The reason that they give indicates that although the student can compare the speed, they still reasoning like the very beginning

Most of the students can compare the speed with the same distance. Although the students did not mention the distance and time explicitly in their reasoning, it does not indicate that they do not understand that speed involves distance and time because they might only consider the time if the distance given are equal.

Second task : Comparing with the same distance

Dudu and Didi, the ants, run from their place into a cake. During 3 seconds, Dudu walks for 15 cm. Meanwhile, Didi walks for 17 cm as shown in the picture. Who is faster? Why?

The aim of this task is to assess whether students understand that within the same time, the faster is the one who goes farther within the same time. Most of the students compare the speed correctly and only five students who give wrong answers to this problem. Although the problem sets the same distance, there are three students calculate the distance as if the periode of time is equal, two students use one second, meanwhile another student uses two seconds. By comparing the distance, they come to the conclusion.

One student, who is known as the high achiever; namely, Anang made a mistake in answering the problem. Moreover, he explained that Dudu is faster because Dudu is nearer to the finish. Anang considered the distance to end point instead of the distance from the initial point.

Third task : Comparing by determining the speed

Rino does an investigation to measure the speed of his toy cars.

a. This car runs for 300 cm during 4 seconds. Determine the speed of the car in cm/sec! Explain your answer!
b. This car runs for 400 cm during 5 seconds. Determine the speed of the car in cm/sec! Explain your answer!
c. From Rino’s investigation, which car is faster? Why?

Regarding this problem, almost more than a half of students in the class can determine the speed in cm/sec so that it were 75 cm/sec for the first car and 80 km/h for the second car. Some of these students have wrong conclusion in determining the faster car.

Fourth task : Constant and variable Speeds

Fajar runs in a field. Every 10 seconds, he stops for a while then continue to run. After 6 stops, he finishes his run.

a. Draw the path of Fajar if the speed is constant (give a sign for the stops)
b. Draw the path of Fajar if the speed is not constant (give a sign for the stops)
c. If Fajar runs with average speed of 12 km/h, what does it mean? Explain your answer!

Most of the students can visualize the constant and variable speed. For the constant speed, they made a segment with the same distance by measuring with a ruler to make sure that the distances are equal.

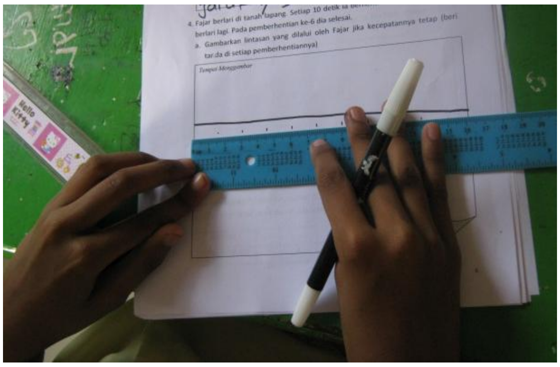


Figure 5.32 Riska measures the distance to draw the path for constant speed

Fifth task : ratio table and notating speed in a standard unit

Nita is cycling with average speed of 20 m/sec.

a. Complete the table of distance and time of Nita's trip.

Distance (cm)						
Time (sec)						

b. Based on the table, determine the speed in km/h

Almost half of the students complete the table correctly. Students who give wrong answer make mistake in calculation because their mistake occurs mostly in the last two columns, which show the distance in 60 seconds and 3600 seconds or one hour. These conjectures also proved by students' calculation as shown in the students' work on the paper.

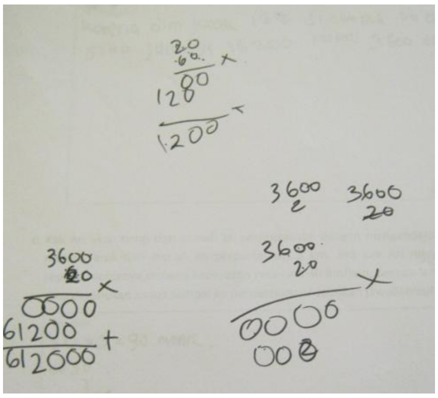


Figure 5.33 Student's calculation on a paper

Students who give correct answer to (a) give correct answer to (b). Meanwhile, five students did not give any answer.

Sixth task: Predict the time if speed and distance are given

Ari goes from his house to the library. The distance from the house to the library is 3 km. If he rides his motorcycle with average speed of 30 km/h, in how many minutes will he arrive at the library? Explain your answer!

Only three students give correct answer; that is 6 minutes. A half of the students give wrong answers and the rest give no answer. Students who give correct answer use a ratio table to find the solution. Below is some of students' work.

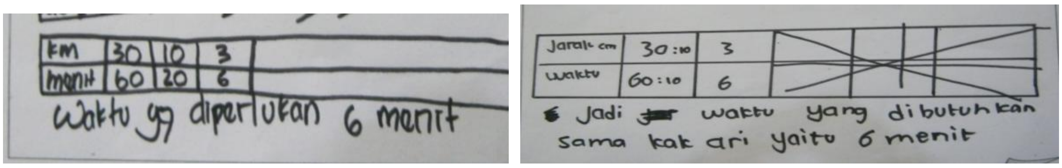


Figure 5.34 Students' work on sixth problem

Seven task : *Predict the distance if speed and time are given*

Father goes to the office by driving his car with average speed of 60 km/h. If he needs ¼ hour to travel, what is the distance from his house to the office? Explain your answer!

The number of students who give correct answers is the same as the number of the students who give wrong answer. Meanwhile, nine of the students give no answer. Like in the sixth problems, students use the ratio table to solve the problem.

From students' answer, we can conclude that the majority of the students were able to show that speed is a proportion among distance and time. The number of students who give correct answer in each task shows the level of the difficulties of the task. Hence, we can also see that many students still get difficulties in converting the unit of speed and explaining interrelation among speed, distance and time. It happened because these mathematical ideas are on the higher level.

6 CONCLUSSION AND DISCUSSION

The main components in this chapter are conclusions as the answers of the research questions and a discussion on some issues in this research and recommendations for the teaching and learning as well for the future research. As mentioned before that the purpose of present research is to contribute to an empirically grounded instruction theory for mathematics education. An instruction theory, in short, is a theory of how students can be supported in learning a specific topic (Bakker, 2004); in our case, the concept of speed for students in elementary school. Hence, the contribution to such instruction theory will be presented in the last part of the conclusions.

6.1 CONCLUSSION

The main research questions mentioned in the introduction part of this research is *"how can we support students' development of the concept of speed"*. In order to answer it, there are two sub-research questions reformulated as follow :

1. What kind of contexts, means and instruction support students' development of the concept of speed?
2. How the models are used to elicit students' development of the concept of speed?

In order to answer the first research question, the summary of analysis of the whole learning sequence as described in subchapter 5.5 would be used. Meanwhile, the second research question will be answered by focusing on the use of models in the learning process. In the last part of the conclusion, the local instruction theory for the teaching and learning the concept of speed in grade fifth of elementary school will be presented.

Answer to the first sub research question

What kind of context, means and instruction support students' development of the concept of speed?

There were three contextual situation; an Indonesian fable, toy cars, and daily life activities have been used in this design research. The summary of the use of these context in the sequence of the learning activities as elaborated in subchapter 5.5.1 to 5.5.6 will be used to answer the first research question.

As explained above that at the initial the context is about an Indonesian fable "Kancil and Siput" in which during the learning process the context is shifted into the part of "Upin and Ipin" story in which the animals involved in the race competition are a rabbit and a tortoise. Using the story, the context seemed real for the students. However, the latter does not mean that the connection to real life is not important. It only implies that the contexts are not necessarily restricted to real-world situations. The fantasy world of fairy tales and even the formal world of mathematics can be very suitable contexts for problems, as long as they are 'real' in the students minds (Van den H. Panhuizen, 2003).

The description in sub-chapter 5.5.1 shows that the context could motivate the students to be involved in the learning process. They give comments towards the story and work together in solving the task given. It does not mean the context is the central point of the learning process. However, together with the designing task and the assistance by the teacher we can review students' informal knowledge about speed. Most of the students choose a race competition as a common way to compare speed. The strength of the context in one hand shows the weakness in other hand. Although students can compare the speed, they seem still not aware of the distance and time. Meanwhile, the "story" context is not rich enough to be generalized.

The context of toy cars most of the time is used in the learning sequences, that is, starting from comparing the speed of two toy cars, analysing distance and time growth, developing models and converting the unit of speed activities. The context of toy cars is connected very near to the children lives since most of the children have experiences in playing toy cars.

When children are asked to figure out the faster car between two toy cars, they actually could use diverse strategies. They could hold a race competition, measure the time of the cars within the same distance or measure the distance within the same time although only the second strategy appeared during the teaching experiment. It is probably because that strategy most of the time is used in the daily life situation, for instance a sprinter who wants to compare his speed with his previous exercise. The use of toy cars in comparing speed activity can build two mathematical ideas at the same time; something moves faster than another if it needs less time to reach the same distance and something moves faster than another if it goes farther within the same time as described in subsection 5.5.2

The toy cars context is also used in analysing distance and time growth activity. The concept of speed as the researcher means in the design is the speed which is assumed to be constant so that the proportion in every second always remains the same. Hence, the researcher chooses toy cars as the context because the speed of toy cars is constant. In subchapter 5.5.3 we can see how the students find the way to determine the distance for every second when the total distance and time are given. We can also see how students made segments on paper tapes so that they could show the distance travelled by the car for every second. Through the paper tapes, students can also visualize that the distance and time growth in such a manner.

The main aim of paper tapes activity is students can use the models to determine the proportion between the distance and time. The models proposed in the design are bar model and ratio table model. The bar model refers to the paper tapes signed by the students in this activity. Meanwhile, ratio table model is a model which is explained in chapter 2. How the model used in supporting students' development of the concept of speed will be elaborated in the next section.

Students use the data from their investigation on the toy cars to convert the unit of speed. Ratio table is another way to write the proportion of the distance and time. However, it needs higher level of thinking to make it than in making bar model because ratio table is usually not presented in the order. We can see in the previous chapter that students did not get difficulties in converting the unit of speed. It is probably because students are involved in the development of the models, from bar to ratio table model.

In exploring relation among speed, distance and time activity, daily life situations were used in the context. In the first problem, the researcher used the animals. Meanwhile, in the second problem, the situation of mother's activity and the family trip are used in the last problem. Presenting the daily life situation in the last sequence, we expect that students could use what they learn to model the reality. We know that the concept of speed is a kind of thinking experiment because constant speed is almost impossible in daily life situation. We may see it in subchapter 5.5.5 in which students determine the speed if the distance and the time are given, predict the distance if the speed and the time are given, predict the time when the speed and the distance are given as well.

Answer to the second sub research question

How the models are used to elicit students' development of the concept of speed?

In this sub-section, the use of models to elicit students' development of the concept of speed would be described as answers to the second research question. Within RME, models are seen as representations of problem situations, which necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation (Van den Heuvel-Panhuizen, 2003). It means that the term model should not be taken too literally. It can also concern a model situation, a scheme, a description, or a way of noting (Gravemeijer et al, 1994). The researcher classified that there were mainly two models which appeared during the lesson series; namely, the bar model and ratio table model. The bar model refers to the paper tapes signed by the students which shows the distance travelled by each toy car for every second. Meanwhile, ratio table model is a model that was described in previous section (2.2.3).

In the lesson series, paper tapes at the first time were used at the second activity when the students are asked to compare the speed of two toy cars. We use this activity to build informal, preschool and out school knowledge of the students of the concept of speed. At this activity students got to work on the task easily. Naturally, students use this paper tapes as the track for the toy cars. Students run the car on it and by measuring the time (students use the same distance in their group) of each car so that they can determine which car is faster. Although the paper tapes appear only as the track for the car at the first time, later for the students the length of the paper tapes indicate the total distance travelled by the car as they wrote in their answer sheet.

Observation during the next activity in which students are asked to compare all of the speed of toy cars in the class (there were six toy cars with five different length of the tracks) shows that the use of paper tapes are more complete than of those in previous. While being asked to determine the distance travelled by the car for every second, students made segments separated by lines on the paper tapes as many as the number of seconds which is needed by the car. If the car needs two seconds to travel the total distance then the number of segments are two. If the car needs five seconds to travel the total distance then the number of the segments are five and so on. As an example, Riska's group measured that the total distance is 250 cm and the total time is 5 seconds. Hence, they made five segments on the paper tapes. They measured 50 cm from the initial side then made the line on it. In making the second line, they start to measure from the first line instead of the initial side. For the students, every segment that they made became a way to show the distance travelled for every second besides the calculation that they made before. Hence, the paper tapes are used not only as the total distances but also the distance travelled by the car for every second.

We may say that at this stage this model starts to meet the bridging function between the informal and the formal level of thinking. The bar model here constituted a context-specific model of the situation; ‘model of’. Later it could be generalized over the situations and becomes then a model that can be used to organize related and new problem situations and to reason mathematically, ‘model for’(Van den Heuvel-Panhuizen, 2003).

The following step is shown in the class discussion in which teacher shows two different paper tapes (one paper tape illustrated constant speed and another paper tape illustrated variable speed). Having experience with the previous activity, students explain that one segment in the bar shows the speed if it is measured for every second. Since, all segments in the first bar are of the same length, students concludes that the first bar shows constant speed meanwhile the second bar shows variable speed. Moreover, when the teacher gives an example as if the length of the first bar is 280 meters within 7 seconds students can calculate that the distance travelled for every second is 40 meter so that 80 meters in 2 seconds, 120 meters in 3 seconds and so on; thus students not only show the distance based on the consecutive seconds. We cannot say exactly whether this is a moment of a shift from a ‘model of’ to a ‘model for’. However, it seems that students start to use the model for their reasoning because based on van den Heuvel-Panhuizen (2003) the real shift of course is made in students’ thinking.

The ratio table models at the first time emerge in the class during ‘converting the unit of speed activity’. Although teacher proposed the ratio table models, most of the students participate actively in the process of model building as shown in the subsection (5.5.4). Both bar model and ratio table model here show the proportion between distance and time in speed. However, ratio table forces the students to developp the higher order thinking because it shows the growth or the proportion of the distance and the time in a more abstract way. Using the bar model, the times are present in a consecutive number and the growth of the distance as well; from one second into two seconds and so forth. Meanwhile, in ratio table students can show the distance in one second then jump to the distance in one hour for instance. As examples, we take Candra’s and Nadya’s group work in converting the unit of the speed of their car as shown in the following figure.

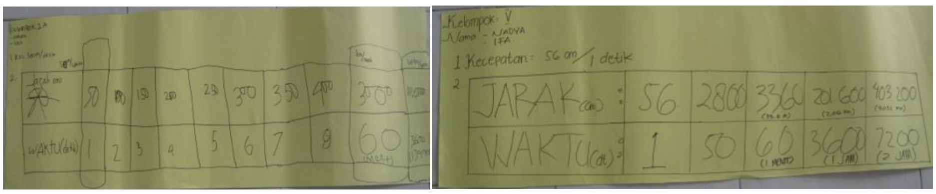


Figure 5.35 Different ways of using ratio table to convert the unit of speed

Through the ratio table that students made, we can see how the table as a tool for thinking and reasoning for them. Students’ poster above show what the students think and what the intermediate steps that the students used to convert the unit of speed. Candra and Leo at the second column wrote the distance is 50 while the time is 1 second. After that, they continue the table by showing the distance in consecutive times until 8 seconds then jump to the 60 seconds or one minute, In other words, the distance is 50 cm for 1 second, 100 cm for 2 seconds, 150 cm for 3 seconds, and so on. It seems that Chandra and Leo are adding the distance by 50 every time they move to the next column except in the last two columns. Different from Candra and Leo, Nadya and Ifah jump immediately from one second to the 50 seconds, then 60 seconds (1 minute), 3600 seconds (1 hour), 7200 seconds (2 hours). Nadya and Ifa probably use multiplication instead of addition.

In the last activity, students are given problems dealing with the relationship among speed, distance and time. Although in the pre-test one student wrote the formula of speed, none of the students who solve the problems use the formula as many Indonesian students did while they are facing those kinds of problems. Many approaches appear in students' work. One of them is a use of a ratio table either in an initial or in a sophisticated way. An initial way here is a ratio table as the teacher shows in the previous activity. Meanwhile, a sophisticated way here is ratio table which was developed by the students from the initial form. In the following figure we can see two different forms of ratio tables. Ifah made the first ratio table in the same form made by the teacher. Ifah made the table in horizontal way, the distance in the first row and the time in the second row. Meanwhile the ratio table made by Naufal and Dani is in vertical form. They did not write the distance and the time anymore. Only the number showed the distance and the time.

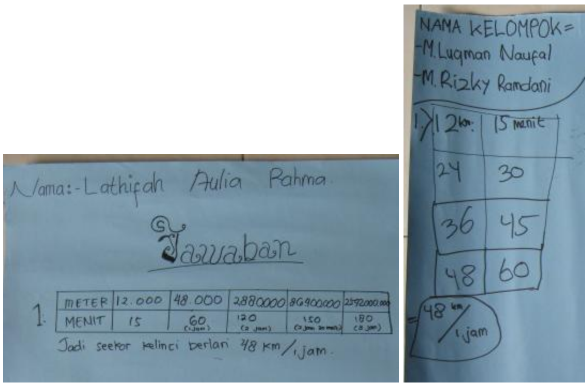


Figure 5. 36 Ratio table made by the students

The figure also shows that there is no fixed strategy to solve the problems; the ratio table allows the flexible approach. From the data given in the problem in which the rabbit runs 12 km in 15 minutes, Ifah changes the distance into 12.000 m. On the next step, she tries to determine the distance during 60 minutes (1 hour) by multiplying 12.000 by 60 so that she got 48.000 m of distance and so forth. Different from Ifah, Naufal and Dani did not change neither the distance nor the time from the initial form. It seems that they multiply both the distance and the time consecutively by 2, 3 and 4.

The learning process shows clearly that the models develop more and more throughout the trajectory, not only in the form of the models but also its function for the students. From the bar model which is close to the situation in the speed to the ratio table which shows number relationship.

As the conclusion, the local instruction theory with respect to the sequence of learning in order to support students' development of the concept of speed in grade fifth of elementary school is summarized in the following table.

Table 6.1 Local instruction theory for the concept of speed

Contextual Situations / Tools	**Activity**	**Possible Models**	**Potential Mathematical topics**
Daily life situations		-	• Every moving object has a speed • Speed involves distance of movement and time to reach the distance
Toy Cars (toy cars, measuring tapes, stopwatches, track for the toy cars)	Comparing speed of two moving objects	-	• Something moves faster than another if it needs less time to reach the same distance • Something moves faster than another if it goes farther within the same time
	Comparing the speed by notating the speed of each moving object	-	Speed is determined by the distance travelled in a unit of time
	Analysing distance and time growth	Bar model	Speed is a proportion between distance and time • Constant speed • Variable speed
	Determine the proportion between distance and time	Bar model	Proportion among distance and time when the speed is assumed to be constant
	Converting the unit of speed	Ratio table,	Speed in a standard way can be notated in cm/sec, m/sec, km/h, etc
Daily life	Predicting either	Ratio table,	• Speed is directly proportional

situations	the distance or the time if the speed is given		to distance • Speed is inversely proportional to time

6.2 Discussion

6.2.1 The use of toy cars in learning the concept of speed

A didactical phenomenology refers to looking for the situation that creates to be organized by the students (Doorman, 2005). We use the phenomenon of measuring speed of toy cars since students cannot use a tool like a speedometer to measure the speed. For the students, it might be a problem for them so that they try to organize the situation. However, in the learning process, students finally could measure the speed of the toy cars by using the combination of the tools for measuring distance and time.

The use of toy cars in learning the concept of speed is not common in Indonesian class. Moreover, students did a kind of experiment outside the classroom. However, we can see the strengths of the context of toy cars in the previous section such as students are being motivated, they are involved actively in the lesson and so on.

The researcher also notes that there is a weakness in the use of toy cars. Using the toy cars, it might be difficult for students to prove that the distance travelled by the car in every second is equal. It happened because it is difficult for students to stop the car exactly in one second so that there are differences in the distance if we exactly did the experiment in every second. Hence, in the teaching experiment, the teacher only asked the students to imagine as if the car stopped for every second instead of do the real experiment. In order to minimize the weakness, it might be useful if we use toy cars which is designed in such a manner that students can set the time of the cars so that it will stop in the time we want.

6.2.2 Teacher's and Students' Interaction

Learning is not merely a solo activity but something that occurs in a society and is directed and stimulated by that socio cultural context (Streefland, 1991). Since the implementation of RME is the first time in SD Al-Fatah Surabaya, it seems quite interesting to observe classroom norms during the learning process. Changing in the learning design might also influence teacher and students' individual beliefs about their own and other's roles.

In order to know how the teacher's experience toward the learning process, the researcher interviewed the teacher in some occasions. Below is a segment of it.

Researcher	:	*Would you explain to me your experiences in teaching mathematics before involving in the research?*
Teacher	:	*My teaching experience started here, from 2002. At that time, I am still in the university. I usually use direct teaching methods in which I tells the students the formula, give the examples then give task to the students and evaluate the results. If there were students who failed I would give extra lesson for them. Like that. It is seldom or I can say never we ask students to work in groups to do some activities*
Researcher	:	*What about the concept of speed?*
Teacher	:	*For speed, usually I tell to the students directly that speed is distance divided by time, then I give difficult problems to the students. If they can do it then it is finished*
Researcher	:	*We use two contexts in the lesson. For the first context, our plan is to use the context of Indonesian fable 'Kancil and Siput' but then it turn into a story of 'Kelinci and Kura-kura' in Upin and Ipin. What is the strength of that context?*

Teacher	:	*Students are more interesting with the story in Upin and Ipin because they ever watched in the television compare with the fable which is presented in the textbook.*
Researcher	:	*Is there any influence to the students' motivation in learning mathematics?*
Teacher	:	*Of course, because it is interesting for the students. First, because most of the students like cartoon films*
Researcher	:	*What is the advantage of the use of toy cars as the context in the learning process of the concept of speed?*
Teacher	:	*Of course it is real for the students. The toy cars as the media in the concept of speed is good.*
Researcher	:	*Is there any difference in students' reasoning before and after the lesson?*
Teacher	:	*First, students' logical thinking develop. Second, they are more confident to share their ideas. However, it needs practices. It is too short to change in six meetings. I see although it is only in a few weeks, they are more active than before*
.....		
Teacher	:	*In general, students are more confident. Take a case of Wira as an example. Before I said that Wira find it is very difficult to learn mathematics because he is usually not active in the learning process. But, yesterday we can see how he was involved in the learning process. He was very active, gave his ideas and answered the questions and so on.*

Besides exploring the teacher's opinion, the researcher also try to conduct an interview with the students in order to know their impressions toward the learning process.

Researcher	:	*Can you explain how do you usually learn mathematics?*
Hafidz	:	*The teacher usually directly gives the formulas, she writes them, gives examples, writes many problems and asks us to solve them*
Wira	:	*The teacher will be very angry if we did not understand about what she explained*
Researcher	:	*Have you learned mathematics as we did together before?*
Students	:	*No*
Researcher	:	*How do you feel when you learn mathematics with a story and toy cars?*
Students	:	*We are very happy*
Ifah	:	*We are very happy but sometimes it is difficult to come to the conclusion*

The first segment gives us a short description how the teacher usually conducts mathematics lesson especially in the concept of speed. It is probably not only in SD Al-Fatah but also in many schools in Indonesia as explained in the introduction part of this study. The students strengthen the teacher's explanation. We can see also how the students are very accustomed with the teacher's way so that they know well what the teacher will do when he/she conducts a mathematics lesson.

Teacher's way of teaching indicates that the teacher is the central point of the learning process. He is the one who establishes whether the learning process is success or not. Here, the role of the teacher in line with an explanation by van de Akker, et all (2006), who said that in traditional mathematics classroom, the role of the teacher is to explain and evaluate, while the social norms include the obligation of the students to try to figure out what the teacher has in mind, and act accordingly.

In the teaching experiment, the researcher offers a class discussion in every day activity because many researches show how the discussion in mathematics class is very important and effective (McGraw, 2002 ;Fricke and NE, 2007). Through the class discussion, we expect that students will not only follow the teacher's way of doing mathematics but they also get the opportunities to think and to present their ideas in solving mathematics problems, as well as to take away skills in social interaction.

The observation showed that the class discussion had a big influence on the learning process. The researcher notes that during the class, the roles of the teacher are gradually changing. The first role of the teacher is providing students an opportunity to present their ideas. Although the teacher tended to take a big role during the discussion, many times he attempted to challenge the students to be involved in the discussion. However, many students still tended to be passive and felt ashamed to present their ideas in the class. Hence, the teacher asked some questions to the students that aim to provide an opportunity for students to present their ideas. The examples of questions that he used are *'How does she know the fastest students?'* or *'What will happen with the drawing (if the car is faster)?'*

The teacher also stimulated social interaction in the class by either making groups of students or asking some questions. Generating micro discussion in a macro discussion in the class can be the first step to stimulate the students to share and discuss their strategies (Wijaya, 2008). The teacher sometimes ask the students to work in pairs or in groups of four or five in order to generate a micro discussion. It was observed that the teacher posed some questions to stimulate social interaction. When the teacher ask *'Explain your answer to your friends'* or *'Write down in the table'* it seems that he tried to encourage students to communicate their ideas either orally or in written form. The teacher sometimes asks also students attention when students are not focusing in the discussion.

Since students usually follow the teacher's way in solving mathematics problems, they get used to give very short answer even only writing the procedures. Hence, during the teaching experiment the teacher many times provokes students to not only give the answers but also provide with their reasoning. The questions such as *'Why did you put a horse as the fastest animal?'* or *'Why the black car is faster (than the yellow car)?'* or *'What is the reason?'* are used by the teacher to provoke students' reasoning.

It is not enough for the teacher to listen the students in the discussion. Hence, the teacher also gives feedback in the discussion so that the students can learn how the discussion is useful for them. The researcher observed that in order to give feedbacks, the teacher appreciates students who present their ideas, gives opinion and solution in the class. He also states his agreements or disagreements towards students' opinion.

The researcher notes that it is not easy for the teacher to conduct a discussion in mathematics class. There are two struggles for him in facilitating the discussion; managing internal dialogue and managing time.

6.3 Recommendations

Reflect on the whole learning process during the teaching experiment, in this section the researcher provides recommendation which is adressed to the practice of teaching and learning and to further research in mathematics education.

The finding that the context of Indonesian fable used in this design does not give enough support to the students underlies the need of giving various situations as the starting point. As the recommendation, we can use daily life situation as the context. Daily life situations might be a fruitful context because it can involve various situations. Meanwhile, students probably have many experiences in it. The ways of students go to school, students' activities in sport lesson, something falls because of force, water flows, and wind and so on might be used as the examples of daily life situations.

From the previous chapter we can also see that only a few students were involved in the discussion. It is not merely because students did not have any idea but sometimes they felt afraid of making mistakes or feel ashamed to say something. Average or low achiever students most of the time are in the categories. Hence, it seems that it is important to strengthen students' self-confidence by giving motivation to the students, letting them to get used to explain their ideas not only in mathematics lesson but also in other subjects; giving appreciation toward students' answers and ensure them that no one will give punishment if they make mistakes.

In order to improve students' reasoning, teacher could minimize "yes" or "no" questions as well as asking the students to continue or repeat what the teacher said. The teacher could train the students with the "how" and "why" questions so that students are not only accustomed to give the answers but also responsible towards the answers that they give.

The teacher who was involved in this research, namely Amir is an experienced mathematics teacher. He was teaching for almost 9 years. However, it is the first time for him to teach using RME principles. Hence, it might inspire other mathematics teachers that it is not difficult for them to implement RME in their class.

Consider Hafidz's and Naufal reasoning in explaining the relationship among speed, distance and time in the sub section 5.5.5, it seems that the concept of speed is a fruitful concept to explore students' additive and multiplicative reasoning. Hence, further research is needed to explore and to improve students' reasoning in mathematics class.