SUPPORTING FIFTH GRADERS IN LEARNING MULTIPLICATION OF FRACTION WITH WHOLE NUMBER

MASTER THESIS

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09715002

SURABAYA STATE UNIVERSITY

POST GRADUATE PROGRAM

MATHEMATICS EDUCATION STUDY PROGRAM

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Submitted to Post Graduate Program of Surabaya State University
in partial fulfilment of the requirements for the degree of Master of Science
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This master thesis is dedicated to:

My beloved family

for their love, patience and endless prayers

that have always become the source of my spirit and energy

to cope with the struggle of this post graduate academic life

ABSTRACT

Khairunnisak, Cut. 2011. *Supporting Fifth Graders in Learning Multiplication of Fraction with Whole Number*. Master Thesis, Mathematics Education Study Program, Master Program of Surabaya State University. Supervisor: (I) Prof. Dr. Siti Maghfirotun Amin, M.Pd., and (II) Prof. Dr. Dwi Juniati, M.Si.

Keywords: multiplication of fraction with whole number, RME, daily life situations, extend the understanding, initial knowledge, design research

The meaning of multiplication of fraction with whole number is difficult to understand by students. They tend to think that multiplication makes something bigger. Meanwhile, in multiplication of fraction with whole number the result can be either bigger or smaller. Even though students already studied about multiplication of a fraction by a fraction, it was still uneasy for them to understand the topic. Therefore, this research aimed to develop a local instruction theory to support students to extend their understanding of the meaning of multiplication of fraction with whole number.

This research was a design research, which designed and developed by applying the five tenets of Realistic Mathematics Education (RME) in order to support the learning process of multiplication of fraction with whole number, so that students can achieve a better understanding about the topic. Daily life situations, such as preparing Indonesian menus and fair sharing, were used as contexts in developing a sequence of instructional activities to reach the learning goals of multiplication of fraction with whole number. The teaching experiment was conducted two times, namely the first cycle and the second cycle of teaching experiment. The participants of this research were students and a mathematics teacher of grade 5 of one elementary school in Surabaya. Some students of one class were involved in the first cycle, with the aim to see how the designed Hypothetical Learning Trajectory (HLT) works. After some revisions, the revised HLT then implemented for all students of another class that parallel with the first one.

The students involved in this research have learned about multiplication of a fraction by a fraction. Most of them were accustomed to work in a formal level by using algorithms to multiplication of fraction problems. As the result, this research found that students' initial knowledge influenced their learning process. They tend to use formal algorithms to solve daily life situations given. Students' learning process started by exploring contextual situation about fair sharing, where the students extended their understanding that fractions are related to division and multiplication. One of the indicators showing the students have extended their understanding can be seen from the more varied representation and reasoning they gave about the strategies used to solve the problems.

ABSTRAK

Khairunnisak, Cut. 2011. *Supporting Fifth Graders in Learning Multiplication of Fraction with Whole Number*. Tesis, Program Studi Pendidikan Matematika, Program Pascasarjana Universitas Negeri Surabaya. Pembimbing: (I) Prof. Dr. Siti Maghfirotun Amin, M.Pd., and (II) Prof. Dr. Dwi Juniati, M.Si.

Kata Kunci: perkalian pecahan dengan bilangan bulat, pendidikan matematika realistik, situasi dalam kehidupan sehari-hari, memperluas pemahaman, pengetahuan awal, *design research*

Makna perkalian pecahan dengan bilangan bulat adalah sesuatu yang sulit dimengerti oleh siswa. Mereka cenderung berpikir bahwa perkalian itu menghasilkan bilangan yang lebih besar, sedangkan dalam perkalian pecahan dengan bilangan bulat hasilnya dapat berupa bilangan yang lebih besar atau lebih kecil. Walaupun siswa sudah pernah belajar tentang perkalian pecahan dengan pecahan, mereka masih saja sulit untuk memahami topik tersebut. Oleh karena itu, penelitian ini bertujuan untuk mengembangkan suatu *local instructional theory* untuk mendukung siswa-siswa untuk memperluas pemahaman mereka tentang makna perkalian pecahan dengan bilangan bulat.

Penelitian ini adalah suatu *design research*, yang didesain dan dikembangkan dengan mengaplikasikan kelima karakteristik dari pendidikan matematika realistik (*Realistic Mathematics Education*) untuk mendukung proses pembelajaran perkalian pecahan dengan bilangan bulat, sehingga siswa dapat mencapai tingkat pemahaman yang lebih baik tentang topik tersebut. Situasi dalam kehidupan sehari-hari digunakan sebagai konteks dalam mengembangkan rangkaian aktifitas instruksional untuk mencapai tujuan pembelajaran perkalian pecahan dengan bilangan bulat. Peserta dari penelitian ini adalah siswa kelas 5 dari suatu Sekolah Dasar di Surabaya, beserta seorang guru matematika dari kelas tersebut. Beberapa orang siswa dari suatu kelas ikut serta dalam siklus pertama, dengan tujuan untuk melihat bagaimana rancangan hipotesis dari trayektori pembelajaran (*Hypothetical Learning Trajectory*) berjalan. Setelah melalui beberapa revisi, *HLT* tersebut kemudian diimplementasikan pada semua siswa dari kelas 5 yang lain.

Siswa yang ikut serta dalam penelitian ini sudah belajar tentang perkalian pecahan dengan pecahan. Kebanyakan mereka terbiasa bekerja pada tahap formal, menggunakan algoritma untuk menyelesaikan permasalahan perkalian pecahan. Sebagai hasilnya, penelitian ini menemukan bahwa pengetahuan awal siswa mempengaruhi proses pembelajaran mereka. Mereka cenderung menggunakan algoritma untuk menyelesaikan permasalahan dalam kehidupan sehari-hari yang diberikan. Proses pembelajaran siswa dimulai dengan mengekplorasi situasi kontekstual tentang pembagian adil, dimana siswa memperluas pemahaman mereka bahwa pecahan berkaitan dengan pembagian dan perkalian. Salah satu indikator yang menunjukkan bahwa siswa telah memperluas pemahamannya dapat dilihat dari semakin bervariasinya representasi dan penalaran mereka tentang strategi yang mereka berikan untuk menyelesaikan permasalahan.

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CHAPTER I
INTRODUCTION

1.1 Background

The algorithm for multiplication of two fractions seems easy to teach and to learn, since we only have to multiply numerator with numerator to get the numerator of the product, and multiply denominator with denominator to get the denominator of the product (Reys et al, 2007). Multiplication with fraction itself is a difficult idea for students to understand as they tend to associate multiplication with making something bigger (TAL Team, 2008). Meanwhile, in multiplication involving fraction, the result can be smaller. For instance, when we multiply $\frac{1}{2}$ by 3, the result is $\frac{3}{2}$, which is, smaller than 3. In addition, we tend to differentiate the word of multiplication symbol "$\times$" (Streefland, 1991), we use word *"kali"* (times) for the amount greater than one, and for the amount less than one we tend to use the word *"dari"* (of).

According to Armanto (2002), mathematics in Indonesia is taught in a very formal way and teachers merely transfer their knowledge to students in the learning process, they teach with practising mathematical symbols and emphasizing on giving information and application of mathematical algorithm. Students are taught how to use the algorithm to multiply fraction with whole number without emphasizing on the meaning behind it.

Meanwhile, if students learn to perform these operations using only rules, they probably will understand very little the meaning behind them. Students may know how to multiply fraction with whole number as $3 \times \frac{1}{2}$ or $\frac{1}{2} \times 3$ if they have studied the rules, but still not be able to interpret the idea in the real world as basis for solving problems (Copeland, 1976). However, once they forget the rules, students cannot solve problems about multiplication of fraction with whole number (Kennedy, 1980). Further, according to an informal interview before this present research conducted, the teacher said that even though the students have already studied about multiplication of a fraction by a fraction, it still uneasy for them to understand the topic.

Considering the issues mentioned before, the researcher proposed that it would be better if students learn by understanding about the meaning of multiplication of fraction with whole number, rather than only know how to use the algorithms for it. Consequently, the researcher would like to support students to extend their understanding of the subject.

Realistic Mathematics Education (RME) is a theory of mathematics education emerged in the Netherlands in the 1970s that focuses on the importance of students' understanding. Inspired by the philosophy of RME, one group, called *Pendidikan Matematika Realistik Indonesia (PMRI)* Team, developed an approach to improve mathematics learning in Indonesian schools to achieve a better understanding (Sembiring et.al, 2008). One of the principles of RME is the use of contextual situations. According to Kennedy (1980) and TAL Team (2008), many contexts can be used to develop the meaning of fractions multiplication with whole number, for instance recipes with fractions and fair sharing. However, the contextual problems that would be used had to be adjusted to school context and the initial knowledge of students.

1.2 Research Questions

Considering the background described in the previous subsection, thus, the researcher composed two research questions as in the following.

a) How does students' initial knowledge influence students in learning multiplication of fraction with whole number?
b) How to support students to extend their understanding of multiplication of fraction with whole number?

1.3 Aims of the Research

Regarding to the background and the research questions, thus the aim of this research was to examine the effects of students' initial knowledge in learning multiplication of fraction with whole number. Further, the present research was also aimed to develop instructional activities in order to support students to extend their understanding of the meaning of multiplication of fraction with whole number.

1.4 Definition of Key Terms

In order to help readers to follow the idea presented in this research, then some important terms used in this research will be explained as follows.

1. Initial knowledge

   In this research, initial knowledge refers to students' previous knowledge, which is about students' knowledge on multiplication of a fraction by a fraction.

2. Fraction

   Fraction is any number that can be expressed as such a ratio, written $\frac{m}{n}$, where m and n are integers, m is not a multiple of n, and n is not zero (Collins Web Linked Dictionary of Mathematics)

3. Multiplication of fraction with whole number

   Multiplication of fraction is multiplication involving a fraction, can be classified to *a whole number times a fraction*, *a fraction times a whole number*, and *a fraction times a fraction* (Schwartz and Riedesel, 1994)

   This research focused more to multiplication of fraction with whole number, consisting of *multiplication of whole number by fraction* (for example $3 \times \frac{1}{2}$) and *multiplication of fraction by whole number* (for example $\frac{1}{2} \times 3$).

4. Understanding

   According to Hiebert and Carpenter (1992), we can understand something if we can relate or connect it to other things that we know.

5. Extend the understanding of multiplication of fraction with whole number

   Extend the understanding means broaden the connection between ideas, facts, or procedures to the topic that was not learned yet. Since the students participated in this research already studied about *multiplication of a fraction by a fraction*, then the students should broaden their understanding to the *multiplication of fraction with whole number*. One of the indicators that show students' understanding can be seen from the way they explore variety types of computations, such as computing a fraction of some distance. However, it is more important that students can relate it to new situations or problems. Another indicator is that when students can make representation and give reason about strategies they used to solve problem.

6. Hypothetical Learning Trajectory

   A Hypothetical Learning Trajectory (HLT) consists of learning goal for students, mathematical tasks to promote students' learning, and hypotheses about the process of students' learning (Simon and Tzur, 2004)

7. Local Instructional Theory

   A Local Instructional Theory (LIT) is defined as a theory that provides a description of the imaged learning route for a specific topic, a set of instructional activities, and means to support it (Gravemeijer, 2004 and Cobb et al, 2003 and Gravemeijer, 1994 in Wijaya, 2008).

1.5 Significances of the Research

The significances of the present research concerned to theoretical and practical significance for teachers and researchers. Regarding to the theoretical significance, this research offers a grounded instructional theory for learning multiplication of fraction with whole number. For practical significance, the present research provides an overview to the researcher and other researchers about how to design a sequence of instructional activities for learning multiplication of fraction with whole number. Further, this research offers a framework for teaching and engaging students in a sequence of instructional activities in order to support their understanding.

1.6 Assumptions

The participants of this research are fifth graders and a mathematics teacher of one private school in Surabaya. It was assumed that the students were serious in doing tasks given. Further, the second assumption was that the teacher also serious in conducting the teaching and learning process.

CHAPTER II
THEORETICAL FRAMEWORK

This chapter provides theoretical framework as basis for this research. Since the research was conducted in Indonesia, then the researcher provided short overview about multiplication of fraction in Indonesian curriculum. The researcher studied literature related to multiplication of fraction with whole number in order to gain more information on how is the development of students in learning this subject. Then, the literature was used as basis for designing a sequence of instructional activities about multiplication of fraction with whole number. However, since the researcher did not find any ready-to-used instructional activities about the topic, then the researcher selected some theoretical elements from research on *learning fractions* and *multiplication in general* to be applied and to be adapted as the instructional activities in this research.

Since the research was developed based on realistic mathematics education, then the researcher also studied literature related to realistic mathematics education. Emergent perspective was provided as basis for interpreting classroom discourse and communication.

2.1 Multiplication of Fraction with Whole Number

2.1.1 Fractions in Indonesian Curriculum

Based on Indonesian curriculum (Depdiknas, 2006), fractions has been introduced since grade 3 in elementary school. The competences related to fractions that have to be mastered by students are described in Table 2.1 below.

Table 2.1 Fractions in Indonesian Curriculum

Grade	Standard Competence	Basic Competence
3	Understanding simple fraction and the use in solving problems	- Recognizing simple fractions - Comparing simple fractions - Solving problems related to simple fractions
4	Using fractions in solving problems	- Explaining the definition of fractions and its order - Simplifying different types of fractions - Adding fractions - Subtracting fractions - Solving problems related to fractions
5	Using fractions in solving problems	- Converting fractions into percentages and decimals forms and vice versa - Adding and subtracting various forms of fractions - ***Multiplying and dividing various forms of fractions*** - Using fractions in solving ratio and scale problems

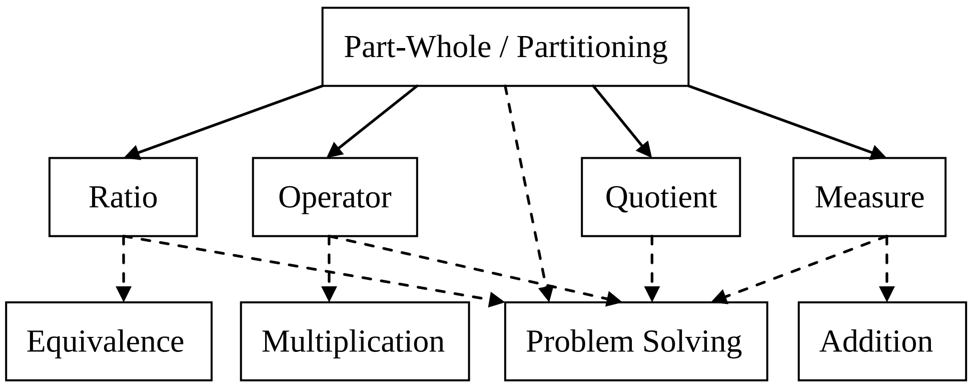
2.1.2 Different Interpretations of Fractions

Based on the curriculum showed in the Table 2.1, before learning about multiplication of fractions, students have to master some pre-knowledge such as the meaning of fractions, addition of fractions, etcetera. According to Freudenthal (1983), fractions can be described as fracture, comparer, and fraction in an operator. He said that fractions appear when the whole is split, cut, sliced, broken, or coloured in some equal parts. He also implied fractions as compared objects, which are separated from each other. Further, Lamon (in Anderson & Wong, 2007) differentiated fractions interpretation as in the following table.

Table 2.2 Different Interpretations of Fractions $\mathbf{\frac{3}{4}}$

Interpretations	**Example**
Part/Whole	3 out of 4 equal parts of a whole or collections of objects
Measure	$\frac{3}{4}$ means a distance of 3 ($\frac{1}{4}$ units) from 0 on the number line
Operator	$\frac{3}{4}$ of something, stretching or shrinking
Quotient	3 divided by 4, $\frac{3}{4}$ is the amount each person receives
Ratio	3 parts cement to 4 parts sand

However, Kieren (in Charalambous and Pitta-Pantazi, 1983) considered the part-whole relationship as the landmark for the other four sub-construct: measure, ratio, quotient, and operator. As development of Kieren’s idea, Behr, et al. (in Charalambous and Pitta-Pantazi, 1983) stated that the part-whole relationship encompasses the distinct sub-construct of fractions. Further, they connected the sub-construct to the process of partitioning as visualised in Figure 2.1.



Notes:
- Solid arrows suggest established relationship among fractional constructs and operations
- Dashed arrows depict hypothesized relationships

Figure 2.1 Link among Different Interpretations of Fractions According to Behr, et al. (in Charalambous and Pitta-Pantazi, 1983)

2.1.3 Understanding Multiplication of Fraction with Whole Number

The need for understanding in learning, teaching and assessing mathematics is very important (NCTM, 1991&1995). Learning with understanding is crucial because something learned by understanding can be used flexibly, be adapted to new situations, and be used to learn new things (Hiebert et.al, 1997). Students need flexible approaches that can be adapted to new situations, and they need to know how to develop new methods for new kind of problems. According to Hiebert and Carpenter (1992), we can understand something if we can relate or connect it to other things that we know. For example, students can understand the multiplication of 6 by $\frac{1}{4}$ if they can relate this to other things that they know about multiplication and the meaning of the fraction $\frac{1}{4}$.

Based on the Table 2.1, multiplication of fraction is introduced in grade 5, where students should be able to multiply various forms of fractions to solve daily life problems. Multiplication with fraction itself could be classified in three cases, namely *a whole number times a fraction*, *a fraction times a whole number*, and *a fraction times a fraction* (Schwarz and Riedesel, 1994). However, this present research focused on the first two types only because the students in this research already learned about how to multiply *a fraction by a fraction*.

Fosnot and Dolk (2002) stated that one of the big ideas of fraction is *fractions are connected to division and multiplication*. For instance, three fourths is *three divided by four*, or *one divided by four three times*. According to the Figure 2.1, the operator sub-construct can be used as a help for developing understanding of multiplication of fractions (Charalambous and Pitta-Pantazi, 1983). Moreover, Freudenthal (1983) said that the operator aspect is more important for fractions than it is for natural numbers because fractions show the operation aspect from the start.

Schwartz and Riedesel (1994) stated that the idea behind *multiplication of a whole number by a fraction* is quite close with *multiplication in whole number*. Fosnot and Dolk (2001) said that the multiplication symbol (×) itself is constructed to represent the actions of iterating equivalent-sized groups, and while this symbol is initially developed to represent mathematical ideas, they become tools and mental images to think with.

2.2 Learning Sequences of Multiplication of Fraction with Whole Number

Initially it will be easier for students to learn about fractions when they first start it with developing their understanding about benchmark fractions that friendly for them, such as halves, thirds, fourths, and then perhaps they can relate it to the more complicated fractions such as sevenths, two-thirds, etc (Reys et.al, 2007 and Barnett et al, 1994). If students are given such simple fraction as finding $3 \times \frac{1}{2}$ or $\frac{1}{2} \times 3$, they might be able to solve it by drawing or using materials.

Schwartz and Riedesel (1994) suggested to give problems about *multiplication of a whole number by a fraction* first because it is conceptually close to *multiplication of two whole numbers*. Since the multiplication of two whole numbers can be interpreted as repeated addition, students also tend to use the repeated addition to solve multiplication of whole number by fraction. Freudenthal (1983) proposed to ask such question as *"How can you say this in other ways?"* after giving some question that can be solved by repeated addition such as $\frac{2}{3} + \frac{2}{3}$ and $\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ so that the word *times* can be elicit.

According to Fosnot and Dolk (2002), when students explore the operation of multiplication with fractions, students have to consider two wholes, that there are relations on relations. Fosnot and Dolk gave example of solution given by one of their subjects of research, called Nora. When Nora was asked to divide five candy bars fairly with six children, she divided the first three bars in half, the fourth bar into quarters, and for the last bar she was faced the sixth of a half. The half is now the whole, which is why it is said $\frac{1}{6}$ of $\frac{1}{2}$.

2.3 Realistic Mathematics Education

Realistic Mathematics Education (RME) is a theory of mathematics education emerged in the Netherlands in the 1970s that focus on the importance of students' understanding. In order to gain students' understanding in multiplication of fraction with whole number, the researcher referred to five tenets of realistic mathematics education (Treffers, 1978; Gravemeijer, 1997) described as follows.

1) Contextual situation

In order to develop intuitive notions that can be basis for concept formation, a rich and meaningful context should be explored. Based on RME, a rich and meaningful context can be used as a starting point in the process of learning multiplication of fractions with whole number. In this research, the context of preparing a number of Indonesian menus was proposed as starting point for learning multiplication of fraction with whole number.

2) *Using models and symbols for progressive mathematizations*

The development from intuitive, informal, context-bound notions towards more formal mathematical concepts is a gradual process of progressive mathematization. Students' informal knowledge as the result of experience-based activities needs to be developed into formal knowledge of multiplication of fraction with whole number. A variety of models, schemes, diagrams, and symbols can support this process. Providing these instruments are meaningful for the students and have the potential for generalization and abstraction. In this research, the use of pictorial model, bar model, and number line model was assumed to support students' learning process. The researcher conjectured that problems such as fair sharing, preparing number of menus, and measuring activities could provoke students to use those kinds of models.

3) *Using students' own constructions and productions*

It was assumed that students' own constructions and productions are meaningful for them. Freudenthal (1991) saw mathematics as a human activity, students should have a right to invent and to develop their own strategies and ideas. Therefore, using students' constructions and productions is an essential part of instruction. In each activity of this research, students were free to use any strategy or model to solve contextual problem. Teachers can underlie their instructional sequences based on students' level of thinking that can be seen from the models and symbols used to solve the multiplication of fractions with whole number problem. Giving open questions can provoke students to use their own strategy to solve the problems based on their level of thinking.

4) *Interactivity*

Through small groups or whole-class discussions, students can learn and share ideas each other. Students can get more insight about multiplication of fraction with whole number through observing and discussing about each other strategies and can use it as scaffolding to develop their own understanding. Therefore, the students were provoked to work in small group when solving the given problems. Mathematical congress also conducted in order to encourage more interactions among every element of teaching and learning process.

In a mathematical congress or a class discussion, Cooke & Bochholz and Doorman & Gravemeijer (in Wijaya, 2008) stated that teacher plays an important role in orchestrating social interaction to reach the objectives both for individual and social learning. Further, Wijaya (2008) elaborated five roles of teacher in the class discussion as 1) providing students an opportunity to present their idea; 2) stimulating social interaction; 3) connecting activities; 4) eliciting mathematical concept; and 5) asking for clarification.

5) *Intertwinement*

It is important to consider an instructional sequence in its relation to other learning strands. We cannot separate multiplication of fraction with whole number with some other learning strands such as multiplication in whole number and addition of fractions. Therefore, the researcher underlay the instructional activities in learning multiplication of fraction by whole number with students' pre-knowledge about multiplying two whole numbers, that multiplication can be represented as repeated addition.

2.4 Emergent Perspective

Emergent perspective is used for interpreting classroom discourse and communication (Gravemeijer and Cobb, 2006).

2.4.1 Social Norms

Social norms refer to the expected ways of acting and explaining that appear through a process of mutual negotiation between teacher and students.

Examples of norms for whole class discussion in the experiment class included the responsibility of the students to explain and give reason to their solutions, to try to understand the other's explanation, and to pose questions if they do not understand it.

2.4.2 Socio-mathematical Norms

Socio-mathematical norms differ with social norms in the way of acting in whole class discussions that are specific to mathematical, that is about multiplication of fraction with whole number. The examples of socio-mathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, and an acceptable mathematical explanation and justification. In solving problems about multiplication of fraction with whole number, there might be some different approach, for example, the use of pictorial representation, formal symbols, etc. Students could decide the easier strategy for them. The students' personal beliefs about what contribution that acceptable, different, sophisticated or efficient encompass the psychological correlate of the socio-mathematical norms.

2.4.3 Classroom Mathematical Practices

Mathematical practice is described as the standard ways of acting, communicating, and symbolizing mathematically at a given moment in time. If in the socio-mathematical norms students could justify which mathematical solution that could be accepted, mathematical practices were focus on particular mathematical ideas. An indication that a certain mathematical practice has been started is that explanations relevant to it have gone beyond justification.

Individual students' mathematical interpretations and actions constitute the psychological correlates of classroom mathematical practices.

2.5 Hypothetical Learning Trajectory

According to Simon and Tzur (2004), Hypothetical Learning Trajectory (HLT) consists of the learning goal for students, the mathematical tasks to promote students' learning, and hypotheses about the process of students' learning. The researcher elaborated the HLT based on the theoretical framework used in this research, henceforth called Initial HLT.

Considering the literature presented previously, the learning sequences were started by activities that have multiplication as repeated addition as the idea behind and then moved to activities in which students could not use repeated addition to solve the problems. Thus, the researcher composed a sequence of instructional activities for multiplication of fraction with whole number as in Table 2.3 and visualisation of the initial HLT can be seen in Appendix A.

Table 2.3 Sequence of Instructional Activities in Initial HLT

Learning Goal	Mathematical Idea	Activity
1. Students are able to represent problems embedded with friendly fractions	- Introduction to fractions - Addition of fractions	Retelling and drawing benchmark fractions - Fair sharing activity - Measuring context
2. Students are able to move from repeated addition to multiplication	- Multiplication of fraction as repeated addition of fractions - Inverse of unit fractions	- Preparing 6 *lontong* of $\frac{1}{2}$ cup rice - Preparing *opor ayam* from 4 chickens, each chicken need $\frac{3}{4}$ litre of coconut milk. - *Mini lesson*: listing the result in a table

Learning Goal	Mathematical Idea	Activity
3. Students are able to change the whole of fractions	- Pieces do not have to be congruent to be equivalent - Relations on relations	- Sharing 5 cakes for 6 people fairly
4. Students are able to relate fractions to multiplication and division	- Fractions are connected to division and multiplication	- Sharing 3 cakes for 4 people - Sharing $\frac{3}{4}$ cake to 3 people
5. Students can develop their sense that the result of multiplying fraction by whole number can be smaller	- Fractions as operator - Fractions are connected to division and multiplication	- Measuring the length of something - Keeping track
6. Students can get more insight about the commutative property of multiplication of fraction	- Commutative property of multiplication of fraction with whole number	- *Mini lesson*: listing the result of multiplication of fraction with whole number such as $3 \times \frac{1}{2}$ and $\frac{1}{2} \times 3$

Further, the sequence of instructional activities for learning multiplication of fraction with whole number presented in the Table 2.3 will be described in detail as in the following.

2.5.1 Representing Fractions

Each student has his own interpretation about some problem. Therefore, before asking students to solve problems, teacher should ask them to interpret the problems by retelling the situation and making drawings of it. It is aimed to ensure that students will not be confused by the given problems.

Goal:

- Students are able to represent various fraction problems

Mathematical Ideas:

- Introduction to fractions
- Addition of fractions

<u>Activity 1: Retelling and Drawing Fractions</u>

In this activity, every student listens to some situation told by teacher. After that, they have to retell and draw it individually on a paper. Here are some problems to be given to them.

1) *"One day before Lebaran, Sinta visited her grandma's house, around eight kilometres from her house. Unfortunately, two kilometres from her grandma's house, her car got flat tire. Rewrite the situation with your own words and draw the position of the car when it got flat tire.*
2) *"Sinta just came back from visiting her grandma's house. She got a cake to be shared with her two siblings. Imagine how Sinta shares it with her siblings and then draw your imaginations on the paper"*
3) *a) One day, mother wants to make chocolate puddings. For one pudding, mother needs $\frac{1}{4}$ kg of sugar. Draw the situation.*

   b) How if mother wants to make two Puddings? Draw it also.

4) *Everyday teacher makes a brownies cake that cut into twenty-four pieces to be sold in a small canteen nearby teacher's house. Yesterday, quarter part of*

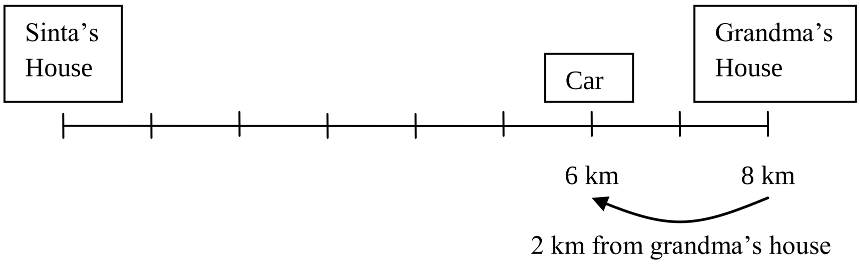
the cake was unsold. Write the situation with your own words and then make representation of the situation.

Conjectures of Students' Strategies in Retelling and Drawing Activity

Since introduction of fractions already taught in grade three, it is expected students to be able to make representation of problems given above. The conjectures about students' answers for those problems are described as follows.

Problem number 1)

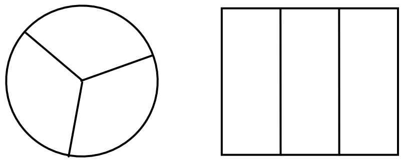
- There is a probability that students will draw a line to represent the road. The road will be divided into eight parts as below.



- However, students might be only draw a line without dividing it into eight parts and directly move two steps back to 6 kilometres.
- There also a probability that students cannot make a representation of the problem because they are not used to do it.

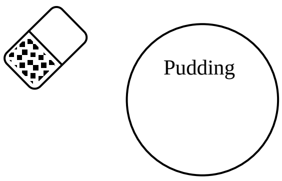
Problem number 2)

Usually a cake is in round or square shape. Therefore, the researcher conjectured that students would represent the cake as follows.



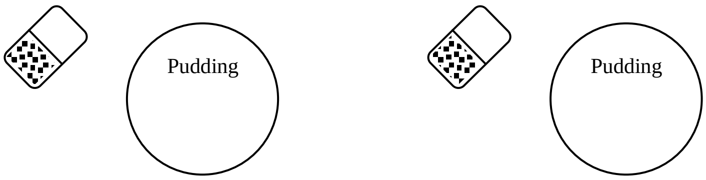
Problem number 3a)

Students might draw a circle to represent the pudding and a cup with a line in the middle as representation of half kg of sugar.



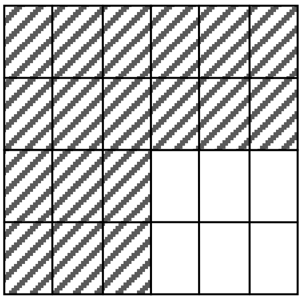
Problem number 3b)

Since in number 3a) students already made the representation, therefore in problem 3b) they probably would make the same picture as what they have drawn in 3a) as many as two times.



Problem number 4)

For problem number 4), it is conjectured that students would leave the quarter part of cake without colour and shade the other part that already sold out.



Discussion of Retelling and Drawing Activity

Retelling and making pictorial representation can show us about the level of students' understanding toward the given problems. Therefore, this activity conducted to train students to be accustomed to represent their thinking in written form.

In this activity, teacher should read the problem carefully and clearly, so that all students can hear and understand it. In order to make students interested in doing the task, teacher could read the problem as if telling story. While students are working, teacher should pay attention to them and decide what kind of students' answers that will be presented in math congress for the next meeting.

Mathematical Congress for Retelling and Drawing Activity

Math congress is held in the next day so that teacher will have more time to decide what kind of strategies will be presented. It is preferable to present various answers so that students can discuss and determine which strategy that suitable for some problem. After that, teacher can ask students to make representation with more difficult fraction such as $\frac{2}{3}$ or $\frac{3}{5}$ and then the researcher can give them multiplication of fraction with whole number problems.

In this math congress, teacher should focus on students' strategies to represent the problems. It is hoped that students can use those pictorial drawings as model of the situation and later can be model for solving fraction problem.

2.5.2 Moving from Repeated Addition of Fractions to Multiplication

Multiplication of a whole number by a fraction can be represented as repeated addition. Therefore, it is provided one of activities that can guide students to use repeated addition as solution and then it is expected students to come up with multiplication of a whole number by a fraction.

Goal:

- Students can move from repeated addition of fractions to multiplication

Mathematical Ideas:

- Multiplication as repeated addition
- Inverse property of unit fractions

Activity 2: Preparing Number of Menus

"Lontong" is a kind of meal made from rice. Usually, it covered by banana leaves, but in some state it put in a plastic.

In this activity, students work together in small group of 4-5 to solve problem. They have to record their strategy to solve the problem. After they finished their work, they should present it in front of the class and discuss strategies they used.

The problem is as follows.

Mother wants to make some "Lontong" as one of the menus in Lebaran. For making one lontong, mother needs $\frac{1}{2}$ *cup of rice. How many cups of rice needed if mother wants to make 6 lontong?*

Conjectures of Students' Learning Process in Preparing Number of Menus Activity

- In order to solve the problem, it is conjectured that some students will draw picture to represent the *lontong*.

$$\boxed{\tfrac{1}{2}\text{ cup of rice}}\boxed{\tfrac{1}{2}\text{ cup of rice}}\boxed{\tfrac{1}{2}\text{ cup of rice}}\boxed{\tfrac{1}{2}\text{ cup of rice}}\boxed{\tfrac{1}{2}\text{ cup of rice}}\boxed{\tfrac{1}{2}\text{ cup of rice}} = 6\text{ times }\boxed{\tfrac{1}{2}\text{ cup of rice}}$$

Or

$$\underbrace{\boxed{\tfrac{1}{2}\text{ cup of rice}} + \boxed{\tfrac{1}{2}\text{ cup of rice}}} + \underbrace{\boxed{\tfrac{1}{2}\text{ cup of rice}} + \boxed{\tfrac{1}{2}\text{ cup of rice}}} + \underbrace{\boxed{\tfrac{1}{2}\text{ cup of rice}} + \boxed{\tfrac{1}{2}\text{ cup of rice}}}$$

$$= \boxed{1\text{ cup of rice}} + \boxed{1\text{ cup of rice}} + \boxed{1\text{ cup of rice}}$$

$$= 3\text{ times }\boxed{1\text{ cup of rice}}$$

- Students who accustomed to work with formal notation will use repeated addition

$$\frac{1}{2}\text{kg} + \frac{1}{2}\text{kg} + \frac{1}{2}\text{kg} + \frac{1}{2}\text{kg} + \frac{1}{2}\text{kg} + \frac{1}{2}\text{kg} \quad = 1\text{ kg} + 1\text{ kg} + 1\text{ kg}$$

$$= 3\text{ kg of rice}$$

- If students have already known that it is multiplication problem, they will probably use the algorithm for multiplication fraction.

$$6 \times \frac{1}{2}\text{ kg} = \frac{6}{2}\text{ or 3 kg of rice}$$

Discussion of Preparing Number of Menus Activity

When students work with these activities, teacher should pay attention to their strategies. Exploring their reasoning can be one of the ways to lead students into the understanding of the concept.

In this activity, students who solve the problem as the first conjecture already known that repeated addition means multiplication. However, even though they already learned about it, there is still a possibility that they do not know how to multiply six by half cup of rice. Teacher should provoke them to understand that six times half cup of rice means six divide by two as in the next conjecture.

However, although students can solve the problem with formal procedure as in the third conjecture, it does not mean that students already understood about multiplication of a whole number by a fraction. Teacher should explore their reasoning, where the algorithms come from and why it has to be like that.

After that, teacher can give similar problem with different numbers. For example by asking them how much rice needed if teacher wants to make five *lontong*, seven *lontong*, etc. By putting the answers in a list, students can easily see the relation and the pattern and finally they can see that repeated addition means multiplication of a whole number by a fraction, also the algorithm.

After some time discussing about *lontong*, teacher can give other problems that also related to multiplication as repeated addition. For example, teacher can give the following problem.

Beside lontong, teacher also wants to make opor ayam. For one chicken, teacher needs $\frac{3}{4}$ *litre of coconut milk. If teacher has four chickens, how much coconut milk she need?*

In order to solve this problem, students might use the same strategy as when they determine the amount of rice needed to make *lontong*.

<u>Minilesson: Listing the Results in a Table</u>

The goal of this mini lesson is to help students to recognize that repeated addition is a multiplication. After giving some addition problem as $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$, teacher can pose some questions to elicit the word *kali* (times) such as *"How can you say this in other ways?"* or *"How many two thirds?"* Another goal is to develop students understanding about the inverse of fractions. Giving such question as $\frac{1}{2}$ two times, $\frac{1}{4}$ four times, and listing it in a table can help students to recognize the relationship. Therefore, students can get more understanding about the inverse of a unit fraction, so that later they can use this knowledge to solve other problems.

2.5.3 Changing the Whole of Fractions

Goal:

- Students can change the whole of fractions

Mathematical Ideas:

- Pieces do not have to be congruent to be equivalent
- Relations on relations

Activity 3: Fair Sharing

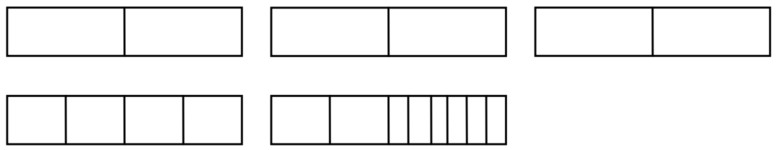
In this activity, students work in pairs to divide some objects given fairly. The example of problem that has to be solved is as follows.

"Yesterday, Aunty gave Saskia 5 Bolu Gulung. Can you help Saskia to divide it fairly for 6 people? How much part of Bolu each person get?"

Conjectures of Students' Learning Process in Fair Sharing Activity

- Students can divide each *Bolu* into six parts, so that each person will get five pieces of $\frac{1}{6}$ *Bolu Gulung*.

- Students might divide *Bolu* as follows.



Discussion of Fair Sharing Activity

When teacher gives such problem to divide some cake or other objects to some people, there is a big possibility that students come up with the first conjecture. They tend to divide each object into the number of people, as conjectured in the first strategy, where they divide each cake into six parts so that they will say that it is $\frac{1}{6}$ of a cake. Thus, since there are five cakes, they can use their previous knowledge about multiplication as repeated addition and get $\frac{5}{6}$ for each person.

Another possibility is students divide the each cake in different parts as the second conjecture. The first three cakes are divided by two so that each piece become half part. Then the fourth cake is divided by four so that each piece become quarter part. Since there are four quarters, they need two more pieces of quarter cake and take it from the fifth cake. The rest of fifth cake is divided in six parts.

2.5.4 Relating Fractions to Multiplication and Division

Goal:

- Students are able to relate fractions to multiplication and division

Mathematical Idea:

- Fractions are connected to division and multiplication

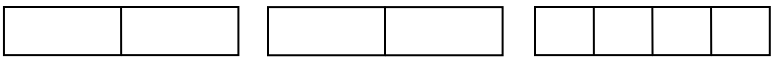
<u>Activity 4: Sharing Three *Bolu* to Four People</u>

The given problem is as follows.

"How to share Bolu Gulung to four people, if you only have three Bolu? How much Bolu each person will get?"

Conjectures of Students' Learning Process in Sharing Three *Bolu* to Four People Activity

- Students can divide each *Bolu* into four parts, so that each person will get three of $\frac{1}{4}$ *Bolu Gulung*.
- Students might divide *Bolu* as follows, so that each person will get $\frac{1}{2}$ and $\frac{1}{4}$ *Bolu*.



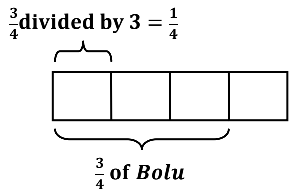
Discussion of Sharing Three *Bolu* to Four People Activity

If students solve the problem as in the first conjecture, in which they divide each *Bolu* into four parts, and by using their previous knowledge about repeated addition, they will say that each person will get $\frac{3}{4}$. Similarly with the second conjecture, students will say that each person will get $\frac{1}{2} + \frac{1}{4}$ that is $\frac{3}{4}$ *Bolu*.

Since in activity 4 students already divided three *Bolu* for four people, in which each person get $\frac{3}{4}$ *Bolu*, then teacher can ask students to share the $\frac{3}{4}$ *Bolu* for three people.

"*If you only have* $\frac{3}{4}$ *Bolu, how you share it to three people? How much Bolu each person will get?*"

Since students already find the three fourth part of the Bolu, it might be easy for them to come up to the answer as



For this activity, teacher can direct students to recognize that fractions are related to multiplication and division, that $\frac{3}{4}$ means $\frac{1}{4}$ three times or 3 divided by 4.

2.5.5 Developing Sense that in Multiplying Fraction by Whole Number, the Result can be Smaller

Goal:

- Students can develop their sense that in multiplying fraction by whole number the result can be smaller

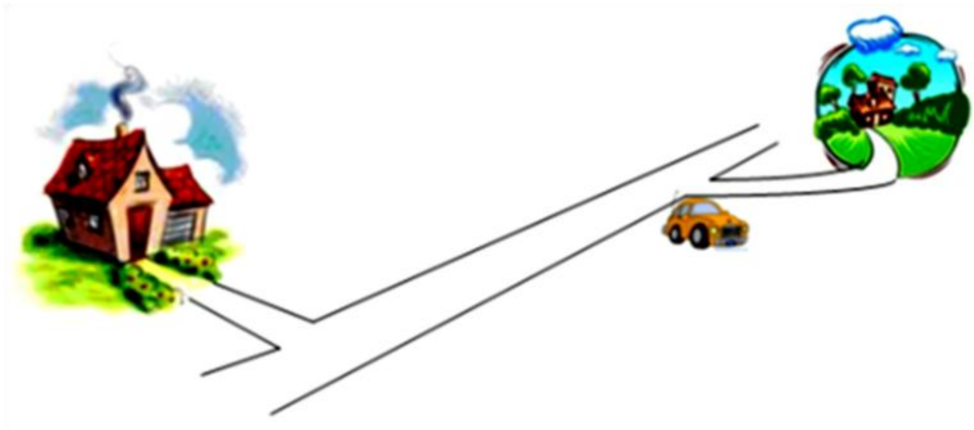
Mathematical Idea:

- Fractions as operator

Activity 5: Measuring Activity

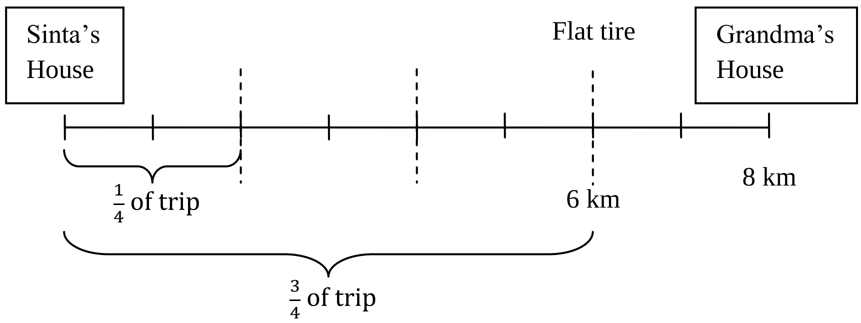
The given problem is as follows.

"One day before Lebaran, Sinta visited her grandma's house, around eight kilometres from her house. Unfortunately, after three-fourth of the trip, the car got flat tire. Can you figure out in what kilometre the car got flat tire?



Conjectures of Students' Learning Process in Measuring Activity

The problem given is quite similar with the one given in the first meeting. Therefore, it is conjectured that students recognize it and use number line to solve this problem.



However, there also a possibility that students work with formal algorithms so that they will come up to the solution as $\frac{3}{4} \times 8$ km $= \frac{3 \times 8}{4}$ km $= \frac{24}{4}$ km $= 6$ km. Some students that used to work in formal notation might be cannot solve the problem if they are not familiar with contextual problem.

Discussion of Measuring Activity

Although students can use multiplication of fraction by whole number correctly, it does not mean that they already understood it. Students who used number line to solve the problem also do not show that they already understood it. There is a possibility that they use the number line model only because they recognize the question with the previous question and directly use the model. Teacher should explore students' reasoning in order to see how deep their insight about the concept is.

In the presentation, teacher can ask students with different strategies to explain their strategies in front of the class. First, teacher can ask some student who used number line to present and to explain it to his friends. Through exploring his strategies, teacher can lead other students to get more insight about the use of number line. After it, teacher can give other problem, for instance giving similar problem with different numbers, and record it in a list so that students can see the pattern of the answer.

Another example of the problem as follows.

Everyday teacher makes a brownies cake that cut into twenty-four pieces to be sold in a small canteen nearby teacher's house. Yesterday, quarter part of the cake was unsold. Can you figure out how many pieces that unsold?

2.5.6 Commutative Property of Multiplication of Fraction

Goal:

- Students can get more insight about the commutative property of multiplication of fraction

Mathematical Idea:

- Commutative property

<u>Mini lesson: Listing the multiplication of fraction</u>

The goal of this mini lesson is to develop students' understanding about commutative properties of multiplication of fraction with whole number. In this activity teacher recalls some problem about multiplication of whole number by fraction and multiplication of fraction by whole number with the same numbers and then record the result in a table so that the properties can be seen easily.

CHAPTER III
RESEARCH METHOD

This chapter describes the methodology and key elements of this research, namely (1) design research methodology, (2) research subjects, (3) data collection, and (4) data analysis, validity and reliability.

3.1 Design Research Phases

As described in Chapter 1, the aim of the present research is to develop a grounded instruction theory for multiplication of fraction with whole number in elementary school. In this research, the researcher was interested in how to help students to extend their knowledge about the meaning of fractions multiplication with whole number. Since the aim of this research was in line with the aim of design research, thus the researcher chose design research as the methodology. Gravemeijer and Cobb (2006) defined design research in three phases, namely preparation for experiment, teaching experiment, and retrospective analysis. These three phases related to the design research will be described as follows.

3.1.1 Preparation for Experiment

The goal of preparation phase (Gravemeijer and Cobb, 2006) is to design a local instructional theory that can be elaborated and refined. Before designing the local instructional theory, the researcher read some literature related to multiplication of fraction with whole number. The researcher then designed a Hypothetical Learning Trajectory (henceforth HLT) consisting of learning goals for students, mathematical tasks to promote students' learning, and hypotheses about the process of students' learning (Simon and Tzur, 2004). After designing mathematical goals of *fractions multiplication with whole number* that are suitable for students in grade 5, the researcher elaborated some activities assumed could support students to get more insight in it. The activities were conducted to be useful to reach the mathematical goals based on the hypotheses of students' thinking and the possible case that might be happen during the learning process. The next step was to test these conjectures in teaching experiment phase.

3.1.2 Teaching Experiment

In this teaching experiment phase, the researcher tested the sequence of instructional activities designed in the preparation phase. In this phase, the designed HLT was used as a guideline for conducting teaching practices. According to Gravemeijer and Cobb (2006), the purpose of this phase is to test and to improve the conjectured local instruction theory developed in the preparation and design phase, and to see how the local instructional theory works. The researcher underlay the teaching experiment based on a cyclic process of (re)designing and testing the instructional activities and the other aspects conducted in the preparation phase.

3.1.3 Retrospective Analysis

Retrospective analysis phase was conducted based on the entire data collected during the experiment. In this phase, the researcher used HLT as a guideline in answering the research questions. After describing general retrospective analyses, the researcher developed a local instruction theory and then addressed it to the more general research topics.

3.2 Research Subjects

The present research was conducted in grade 5 SD Laboratory Universitas Negeri Surabaya. The school is one of the schools that have been involved in *PMRI (Pendidikan Matematika Realistik Indonesia)* project, under the supervision of Surabaya State University.

With the aim to get more space to observe and to explore students learning process, the researcher planned to involve six fifth graders (10-11 years old) from the school and the researcher acted as a teacher. After that, 31 fifth graders (10-11 years old) from another parallel class and the real mathematics teacher of the class would be participants, in order to see how the design works in the real class.

3.3 Data Collection

Data collection was gained through interviewing teacher and students, observing activities in classroom, and collecting students' works that will be described as follows.

3.3.1 Interviews

Before doing experiment for the instructional activities, the researcher interviewed the teacher to gain some information about the socio-mathematical norms in the class and about students' initial knowledge. Furthermore, the researcher asked teacher's comment about the teaching and learning activity. With the intention to know more about students' thinking, the researcher explored students' reasoning through interviews during the lesson and after the class end, if it was needed. The researcher preferred to use face-to-face interviews with teacher and students. In order to reduce any bias responses because of the researcher's presence, the interviews were conducted informally, more like discussions.

3.3.2 Classroom Observations

Before observing the classroom activities, the researcher made some interaction with teacher and students so that they could act and give responses naturally without affected and disturbed by the researcher's presence. During the experiment in the classroom, the researcher observed teacher and students' behaviours. In observing the teaching and learning activity, the researcher used two to three video cameras so that rich data of the activities in the classroom could be collected. One static camera was put in the corner of the class to record whole class teaching and learning processes. Two more video camera acted as dynamic camera, to record some interesting cases in group discussions.

3.3.3 Students' Works

In order to analyze students' thinking process the researcher collected their written work to see their strategies and reasons. The written works consisted of students' worksheets and poster made for class discussions. Collecting students' written works enabled the researcher to save time, thus the researcher could analyze it outside school hours. Based on the written work, the researcher could prepare what kind of questions need to be posed for them, also for preparation of class discussion.

3.4 Data Analysis, Validity, and Reliability

3.4.1 Data Analysis

With the aim of answering the research questions, the researcher analyzed the data collected from the teaching experiment. The progress of students' insight about multiplication of fractions with whole number was observed from the video tapes, interviews, and students' worksheets. Doorman (in Wijaya, 2008) mentioned that the result of design research is not a design that works, but the underlying principles explaining how and why this design works. Therefore, the researcher compared the HLT with students' actual learning to investigate and to explain how students obtained the understanding of multiplication of fraction with whole number.

3.4.2 Reliability

The reliability of this design research was accomplished in a qualitative way. In order to gain the reliability of this research, the researcher used data triangulation and inter-subjectivity. The data triangulation in this research involved videotape, students' works, and field notes. The present research was conducted in two parallel classes, and the researcher reduced the subjectivity by involving the researcher's colleagues to interpret the data collection.

3.4.3 Validity

In order to keep the methodology of this research as valid as possible and to answer the research questions in the right direction, the researcher used the following methods of validity (Wijaya, 2008).

(1) The HLT was used in the retrospective analysis as a guideline and a point of reference in answering research questions, and

(2) The triangulation in collecting data as described before gave sufficient information for the researcher's reasoning in describing the situations and the findings of this research.

CHAPTER IV
RETROSPECTIVE ANALYSIS

In this chapter, the researcher will present HLT 1 as the refinement of the Initial HLT designed in Chapter 2. Based on the data got from the first cycle of teaching experiment, the researcher analysed and improved the HLT 1 to be HLT 2. The HLT 2 then was implemented in the second cycle of teaching experiment. Further, the data got from the second cycle also analysed. The scheme of HLT refinement can be seen in the following picture.

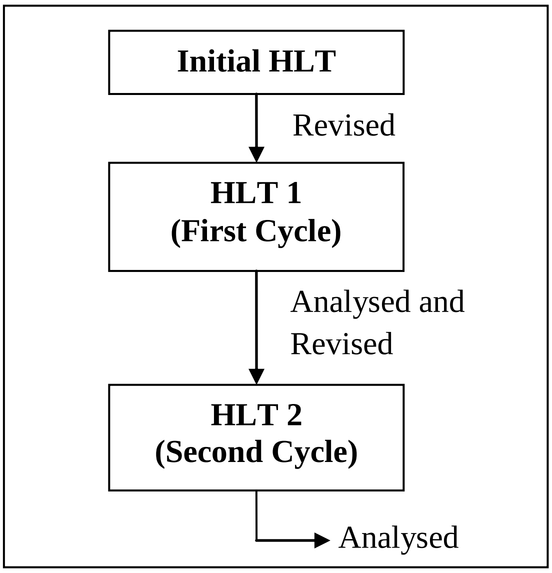


Figure 4.1 Scheme of HLT Changing

In the following section, the researcher will present the observation result and the analyses, both from the first and the second cycle of the teaching experiments.

4.1 HLT 1 as Refinement of Initial HLT

Before conducting the teaching experiment, the researcher made some refinements to the initial HLT. Actually, the order and the path of the initial HLT were not changed, the researcher only made some changing and improvement to the activities and problems that would be given to the students. Therefore, the conjectures for the instructional activities were quite similar to what have been presented in Chapter 2. The rationale behind the refinement of initial HLT to be HLT 1 can be seen in Appendix B. Further, Appendix C shows the visualisation of HLT 1 that was implemented in the first cycle of teaching experiment phase.

4.2 Retrospective Analysis of HLT 1

The teaching experiment for HLT 1 (henceforth first cycle of teaching experiment) was conducted at one of elementary schools in Surabaya on 14 – 24 February 2011. For this first cycle, the researcher offered the sequence of the instructional activities designed, including pre-test and post-test.

The researcher only involved six fifth graders as subject of this first cycle because the researcher need more space to observe and to explore students' learning process and reasoning. The researcher expected to obtain and to explore various reactions toward the instructional activities given. The six fifth graders itself were suggested by the teacher based on their heterogeneous level of competence in learning mathematics.

It was assumed that the students already had sufficient pre-knowledge to learn *multiplication of fraction with whole number*, such as *introduction of fraction* and *addition of fraction*. However, based on the teacher's explanation, the students already learned about the pre-knowledge in the previous grade. They have even learned about multiplication of *a fraction by a fraction* before the first cycle conducted.

Table 4.1 below presented the timeline of the first cycle of this research.

Table 4.1 Timeline of the First Cycle

Date	Activity	Description of the Activity
14-02-2011	- Pre-Test	- Investigating students' initial knowledge
	- Retelling and Drawing Fractions	- Measuring students' understanding about the content of the questions given - Assessing students' representation of introduction of fractions and addition of fractions
16-02-2011	- Preparing Menus	- Focusing on repeated addition of fractions as multiplication of whole number by fraction
	- Colouring	- Focusing on the number of fractions needed to get 1 as the result of multiplication (inverse of unit fraction)
	- Fair sharing	- Focusing on the idea that pieces do not have to be congruent to be equivalent - Relation on relation - Focusing on the relation between fractions to multiplication and division
21-02-2011	- Interviewing students	- Investigating students' reasoning in solving problems given

Date	Activity	Description of the Activity
24-02-2011	- Measuring	- Focusing on the relation between fractions to multiplication and division
	- Post-Test	- Investigating students' knowledge after receiving the instructional activities

In the following, the researcher will elaborate the result of the observations and analyses of the first cycle. The description itself will be divided into pre-test, teaching experiment, and post-test.

4.2.1 Pre-Test of First Cycle

Briefly, the pre-test is aimed to know students' initial knowledge and ability about multiplication of fractions, especially about multiplication of *a whole number by a fraction* and multiplication of *a fraction by a whole number*. In the following, the researcher will describe some remarks got from the pre-test.

4.2.1.1 Students Initial Knowledge of Addition of Fraction and Multiplication of Fraction

Based on the answers in the worksheets, the researcher interpreted that all students could use appropriate algorithms to solve fraction addition problems, whether it had the same denominator or not. Moreover, three of them also converted *improper fractions* to *mixed number*.

Regarding to *multiplication of fraction with whole number* problems, the students showed various strategies. From the six students, only two students gave proper algorithm, meanwhile the other four students mixed up some algorithms. Perhaps, the four students tried to use algorithms of multiplying fraction by fraction because they have already learned it. Nevertheless, they made some mistakes in using the algorithms. While two students made mistakes in converting the whole number, other two students directly multiplied the whole number by the numerator and the denominator. The following figure shows students mistakes in answering question $\frac{1}{4} \times 3$.

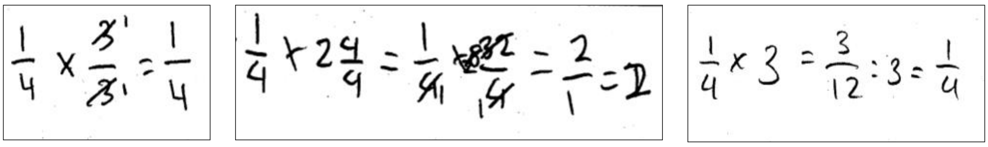


Figure 4.2 Students Mistakes in Solving Multiplication of Fraction with Whole Number

4.2.1.2 Relating Situations to Its Algorithms

From the worksheets, the researcher concluded that all students related one situation to one algorithm only. The difference lay on relating the first and the third situation, as can be seen in the Figure 4.3 below.

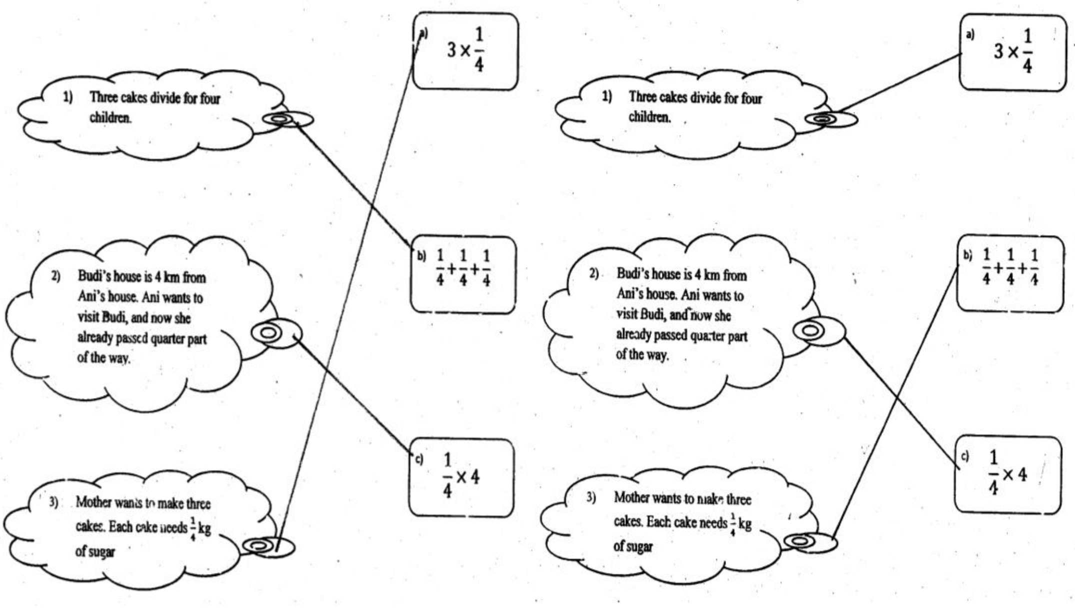


Figure 4.3 Students' Answers of Second Problem of Pre-test

Based on the pictures above, three students related *the first situation to the second algorithm* and *the third situation to the first algorithm*. On the other hand, the other three students related *the first situation to the first algorithm* and *the third situation to the second algorithm*. Perhaps, one of the reasons they only chose one algorithm was the tendency to make one to one correspondence. The students said that they chose the easiest algorithm for the situation.

Another possibility was because the format of questions was not clear enough for them. When the researcher asking them, one of the students realized that the first algorithm is same with the third one and after some time of discussion, all students agreed that both the *first and the third situations* could be related to *the first and the second algorithms*.

4.2.1.3 Relating Algorithms to Situations, Checking the Meaning of Fractions

After relating situations to its algorithms, the students then had to relate algorithms to the suitable situations. From their reactions, the researcher interpreted that the students did not familiar with this kind of problems. Before doing the tasks, they asked us how to answer it. From their answers, the researcher got some remarks about their understanding of fraction, addition of fractions, and multiplication of fraction with whole number.

Based on the students' worksheets, the researcher concluded that only one student who could give suitable situations for all questions. Below are some examples of situations given by Cici (Figure 4.4). However, the researcher interpreted that she might consider $\frac{1}{4} \times 5$ as $\frac{1}{4}$ five times, not $\frac{1}{4}$ of 5.

$\frac{1}{2}+\frac{1}{4}$ →	Chika memakan Setengah dari 1 semangka. Rara memakan $\frac{1}{4}$.	Chika eats half of watermelon. Rara eats $\frac{1}{4}$.
$\frac{1}{4}\times 5$ →	Bu Selfia punya banyak kue. ia punya 5 anak. setiap anak mengambil $\frac{1}{4}$ kue.	Bu Selfia has many cakes. She has 5 children Each child takes $\frac{1}{4}$ cake.

Figure 4.4 Cici Gave Suitable Situations to the Fractions

Another student, Uya, gave situations as if he got task about fraction addition and multiplication of fraction with whole number, as can be seen in Figure 4.5. Perhaps, he only knew fractions as one of subjects in his school.

$\frac{1}{2}+\frac{1}{4}$ →	Pada Saat aku mengerjakan PR aku menemukan Soal $\frac{1}{2}+\frac{1}{4}$ aku menjawab $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}$ $\frac{3}{4}$	When I did my homework, I found question $\frac{1}{2}+\frac{1}{4}$. I answered $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$
$\frac{1}{4}\times 5$ →	Saat ini aku mendapat Soal $\frac{1}{4}\times 5$ Karna aku bisa langsung Jawab $\frac{1}{4}\times\frac{5}{5}$ karna 5 nya Sama aku Jadikan $\frac{1}{1}$. $\frac{1}{1}\times\frac{1}{4}=\frac{1}{4}$	Just now I got question $\frac{1}{4}\times 5$ Because I can, I directly answered it $\frac{1}{4}\times\frac{5}{5}$ Because the 5 is same, I made it $\frac{1}{1}$ $\frac{1}{1}\times\frac{1}{4}=\frac{1}{4}$

Figure 4.5 Uya Related Fractions with School Task

Another one student, Iman, could give suitable situation for fraction addition problem, but when facing by multiplication of fraction with whole number problem, he directly put the question in the situations. Perhaps, he did not have idea about the meaning of multiplication of fraction with whole number. The examples of his answer can be seen in the following figure.

$\frac{1}{2}+\frac{1}{4} \rightarrow$ Ibu menakai gl untuk membuat kue. Ibu ingin membuat dua kue. Setiap kue memerlukan $\frac{1}{4}$ kg.	Mother uses sugar to make cake. Mother wants to make two cakes. Each cake needs $\frac{1}{4}$ kg
$\frac{1}{4}\times 5 \rightarrow$ Ibu tidak suka kepada adik saya karena adik saya meminta $\frac{1}{4}\times5$ sendok gula jika membuat teh	My mother does not like to my sister because she asked $\frac{1}{4}\times 5$ spoons of sugar to make tea.

Figure 4.6 Different Understanding toward Addition of Fraction and Multiplication of Fraction

4.2.1.4 Strategies Used by Students to Solve Contextual Problems

Based on the worksheets, Iman wrote that *half of 48* is *24*. Menawhile, other four students directly divided the number (*48* or *10.000*) by *2*. Just after they had to explain their answer, they wrote it was because half of *48* is *24*. Perhaps, it was because the students did not consider a half as fraction.

The other one student, Uya, considered *half of something* as *something times a half* (for example *half of 48* is $48 \times \frac{1}{2}$). He said that he divided it by 2 because the denominator is 2, and it was valid also for a third. However, when the researcher changed the question to $48 \times \frac{2}{3}$, he answered it 8, because *2 times 3* is 6 and *48 divided by 6* is 8. From the answers, the researcher interpreted that the student did not fully understand about multiplication of fraction with whole number. Perhaps, the numerator 1 in unit fractions means nothing for the students since they directly divided the whole number by the denominator. They did not realize that they have to multiply the whole number by the numerator before dividing it by the denominator or vice versa.

4.2.2 First Cycle of Teaching Experiment

4.2.2.1 Retelling and Drawing Fractions Activity

In this activity, the students had to retell and draw some situations told by the researcher (in this phase, the researcher acted as the teacher). The main goal of this activity is to measure the student's understanding of the contents given, also about their representation of fractions and addition of fractions. It was assumed that when the students understand about the content of questions given, they could retell and make drawing as the representation of it.

Since the introduction of fractions was already learned in grade three, it was expected that the students could represent the problem given, which was about simple fractions. Based on students' answers on the worksheets, the researcher could say that all students could rewrite the story told to them. Although the editorial of the sentences was a bit messy, but they wrote the core of the story. For example, "*Everyday teacher's neighbour makes Brownies 24 pieces. The day after tomorrow* $\frac{1}{4}$ *part of the cake was not sold out.*"

Further, the researcher concluded that the students made proper drawings to represent the situations given. In the following, the researcher will give description of their drawings.

4.2.2.1.1 Determining Car Position when Its Tire Flatted

For this first situation, the researcher made some mistake when telling the situation for the students as "*One day before Lebaran, Sinta visited her grandma's* *house that is 8 km far away from her house. Unfortunately,* ***after 2 km of the journey****, her car got flat tire. Rewrite the situation with your own words and draw the position of the car when it got flat tire."* Therefore, the conjectures made before were adjusted based oon this problem.

Regarding to the drawing they made to represent the problem, all students made a line with Shinta's house in one side and grandma's house in the other side, as what conjectured before. They put a picture of car in some point that indicated the position of the car when the tire flatted.

However, while the other students put the car in the second segment or part from Sinta's house, Uya put the car in the fourth segment (Figure 4.7)

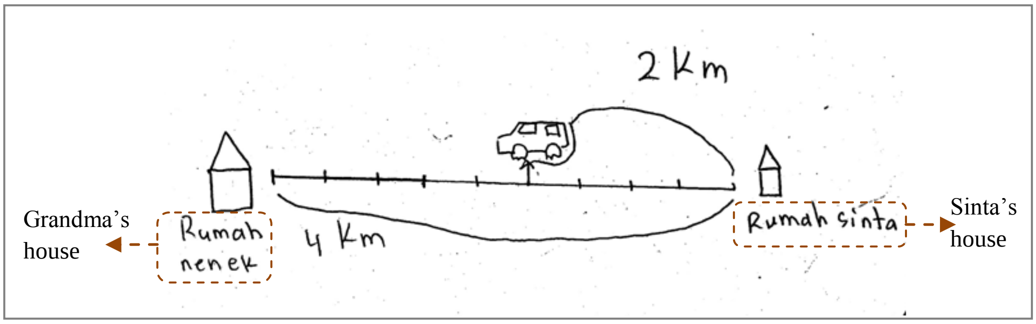


Figure 4.7 Uya's Representation of the Situation, the Car Put in the Fourth Segment

Based on his explanation, he said that he intended to make a line divided to 10 parts and considered it as 4 km. Since 2 km is a half of 4 km, then he drew the car in the middle of the line. However, since he miscounted the parts, the car was not in the middle but was closer to Sinta's house.

4.2.2.1.2 Sharing a Sponge Cake to Three People

As what conjectured, except Uya who made square-shape, the other five students made a circle to represent a cake. They said it was because usually cake was in round-shape. However, they got difficulty to divide the circle to three equal parts. After asking permission to the researcher, they made a square to represent the cake and directly divided it to three parts. Perhaps it was more difficult to divide a circle to three equal parts than to divide a square.

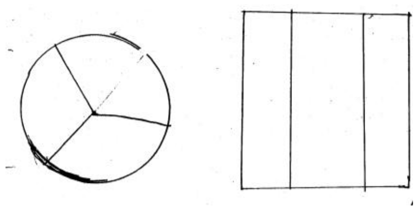


Figure 4.8 Students' Struggle to Divide a Circle to Three Parts Fairly and Draw a Rectangular Shape

4.2.2.1.3 Making Two Chocolate Puddings, Each Pudding Needs Quarter Kilogram of Sugar

Before making representation of two puddings, the students had to make drawing of a pudding that need quarter kilogram of sugar. As what conjectured, when making representation of two puddings that need quarter kilogram of sugar, students made two same representations of what they have drawn when representing one pudding that need quarter kilogram of sugar.

4.2.2.1.4 A Quarter Part of Cake Cut into 24 Pieces

Meanwhile four students shaded 6 parts out of 24 parts to show the number of pieces of brownies that left, one student, Iman, made a rectangle with size 12×12. He said, at that time he was thinking of $12 + 12$. After some time discussed with his friends, he scratched the 10 columns and then shaded 6 parts of the rest to represent $\frac{1}{4}$ of the 24 pieces.

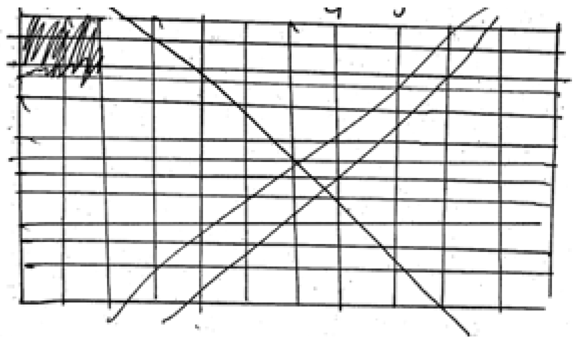


Figure 4.9 Iman's Mistake in Drawing a Cake Divided to 24 Pieces

Almost similar, Cici also made mistake in arranging the pieces. Rather than made a 3×8 rectangular, she made 3×9 ones. Since she shaded 6 parts of it although the number of part was 27, perhaps, she found out the result of $\frac{1}{4}$ of 24 first, and then made drawing of it.

4.2.2.2 Preparing Number of Menus Activity

The mathematical idea in this activity is about *repeated addition of fractions* as *multiplication of whole number by fraction*. In this activity, the students were divided into two groups of three students. The researcher named it as Group A and Group B.

For the first problems, the researcher provided rice and cups as the tool to help students in solving the problem. However, only Group A used the materials given before answering the questions, meanwhile Group B directly answered it. Observation and students' answers in the worksheets indicated that the students directly used algorithms to solve the problems. Just to explain their answer they made drawings, to prove that their answers were true.

Group A solved question 3 – 5 by using repeated addition. Meanwhile, Group B only used it for the first and the fourth problems, and for the rest problems, they wrote multiplication of fraction by whole number. Further, the researcher concluded that the students knew repeated addition means multiplication, but they did not differentiate *multiplication of whole number by fraction* and *multiplication of fraction by whole number*, as showed in the following conversation.

Iman	:	Multiplied by a half
Acha	:	One over two multiplied by twelve
Iman	:	Six
Researcher	:	One over two times twelve, or twelve times one over two?
Acha	:	[Looks at her friends] Twelve.. [Scratches her head] One over two times twelve..
Researcher	:	Why?
Acha	:	One over two. One over two plus one over two plus ... (twelve times) become six. So based on the principle of multiplication, it means multiplication. So, rather than make it difficult, then.. just multiply it..

The interesting strategies appeared when the students filled in the table. They split the numbers to simplify it. Below is the conversation when the researcher explored Group A's strategies to find how many litre of coconut milk needed to make *opor ayam* from 13 chickens.

Uya	:	$3\frac{3}{4}$ plus $6\frac{3}{4}$.
Researcher	:	Why?
Uya	:	Because five plus eight is thirteen. So, $10\frac{2}{4}$.

Cici : Eight plus five is thirteen. So we looked at the result for five, that is $3\frac{3}{4}$ and the result for eight is $6\frac{3}{4}$. So we only need to add it and the result is $10\frac{2}{4}$.

4.2.2.3 Determining Number of Colours Needed

As stated in Appendix B, researcher added one more activity to draw an idea about inverse of unit fractions, that when a unit fraction multiplied by its denominator, the result will be 1. The context used was about knitting yarn, where the students had to determine the number of yarn colours needed to knit one metre knitted if the yarn colour changed every half-metre or quarter-metre.

According to the students' answers on the worksheets, it can be seen that all students gave the same strategy. They made a drawing represented the knitting yarn as can be seen in Figure 4.10.

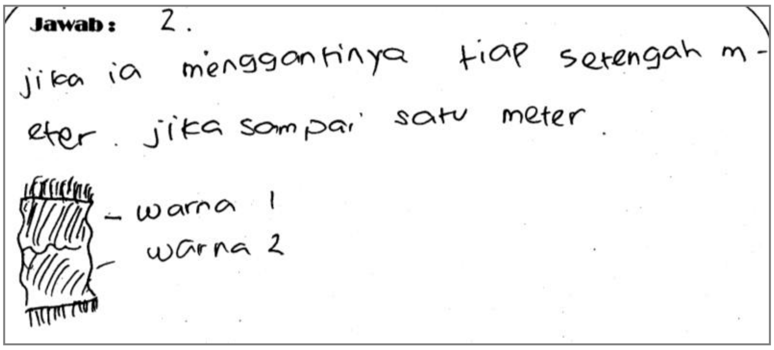


Figure 4.10 Students' Strategy to Find the Number of Colours Needed to Make One Meter of Knitting Yarn

Further, the researcher also offered a context about painting one kilometre of fence. While Group A still used the same strategy as what the have used in the previous problems, Group B directly use the algorithm of multiplication of fraction by whole number, as can be seen in the following figure.

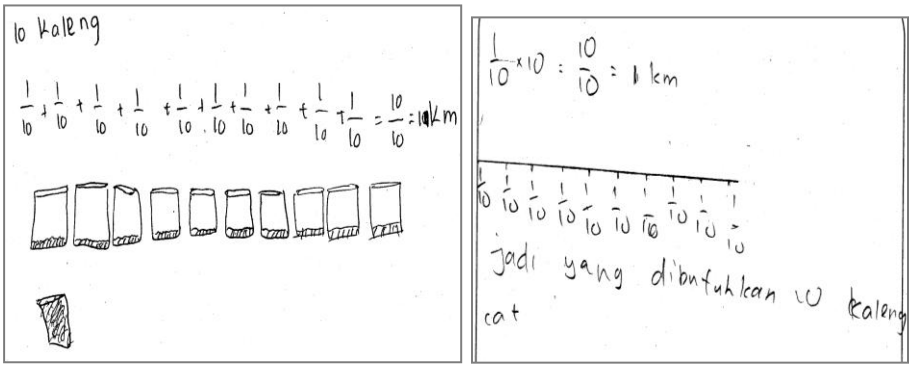


Figure 4.11 Students' Strategies to Find Number of Cans Needed

4.2.2.4 Fair Sharing Activity

In this activity, the students worked in pair because of their request. Another reason was the smaller the groups, the more effective the discussion between the members. Therefore, there were three groups of two students, the researcher named it as Group A, Group B, and Group C.

The first problem given was about dividing five cakes to six people fairly. Actually, for this problem the researcher provided five rectangles as the model of the cakes. After had worked for some time in dividing the rectangles, the students said that drawings were easier for them. However, although the students of Group B made drawing of the five cakes, they still got difficulties to determine how much cakes got by each person.

Figure 4.12 Student Struggled to Cut Five Rectangles to Six Parts Fairly

One of the solutions was out of the conjecture. The student in Group A seemed unfamiliar with partitioning more than one object, therefore they unified the five cakes to be one cake then divided it to six parts, then the result become $\frac{5}{30}$ as in the following figure.

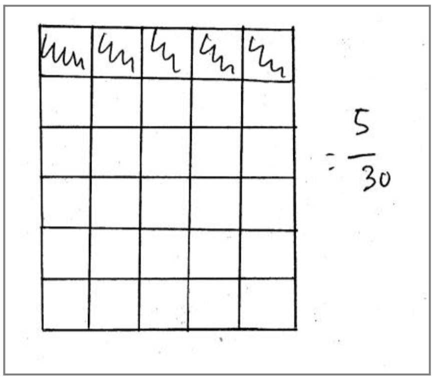


Figure 4.13 Group A Unified the Five Cakes

Another group (Group C) divided each cake to be four parts. However, the students struggled to determine what fraction obtained by each person. At the first, one of the group members said that each person got four parts, they did not consider the distinct whole. After some time discussing, then she differentiated the whole, and got the result, that is $\frac{3}{4}+\frac{1}{12}$, as can be seen un Figure 4.14. Below is the quotation of the discussion.

Researcher	:	So, how much will each person get?
Zia	:	Five
Researcher	:	Comparing to all cakes? How much each person gets?
Zia	:	[Thinking]

Researcher : How much from this cake [points at the first cake]?
Zia : Three fourth
Researcher : From this [points at the last two pieces]?
Zia : Two
Researcher : This [point at one piece in the last cake] was cut to three parts. So how much was one piece? This is [pointed at one piece in the last cake] a quarter and it cut again to be how much parts?
Zia : Three
Researcher : So? How much each person get?
Zia : Four
Researcher : Is it same, the piece here [point at the first cake] and this piece [point at the last small piece]?
Zia : [Counting] Three of one over twelfth
Researcher : How?
Zia : This [points at one piece of the last cake] was divided by three. The other was also divided by three, so there were twelve.
Researcher : So? How much each person get?
Zia : Three fourth plus one over twelve

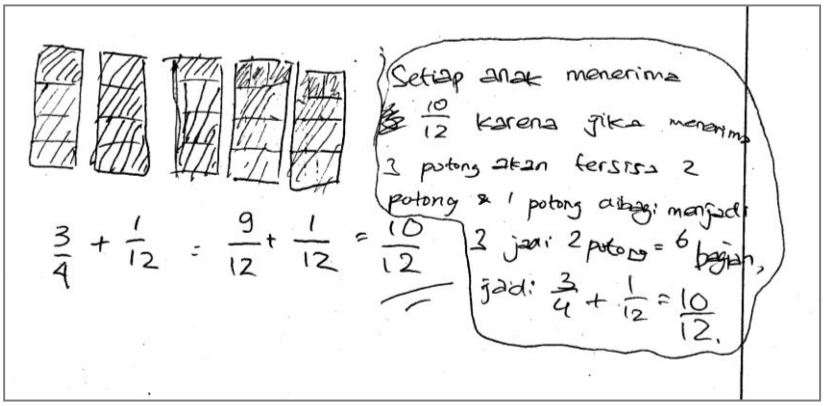


Figure 4.14 Strategy of Group C to Solve 3 Cakes Divided to 6 People

The next problem was about sharing three cakes for four people fairly. From this problem, the researcher intended to draw an idea that fractions are related to multiplication and division. Group A and Group B made the same drawings, but they had different answer. While Group A considered the 12 parts as the whole, Group B saw it per cake. Therefore, Group A answered $\frac{3}{12}$ and Group B answered $\frac{1}{4}$ of each cake so each person got $\frac{3}{4}$. Almost similar, Group C also answered $\frac{3}{4}$ but they took $\frac{3}{4}$ from one cake.

4.2.2.5 Measuring Situation

From this activity, the researcher expected students to draw ideas that fractions is an operator, that fractions are related to multiplication and division. The division of group were still the same with the previous meeting: the students were divided to three groups of two members.

Observation showed that the students calculated $\frac{3}{4}$ of 8 km first before making drawings. When the researcher asked Group C to make drawing, they divided the line to four parts and then counted $\frac{1}{4}$ plus $\frac{1}{4}$ plus $\frac{1}{4}$. Since one part of the line represented 2 km, then they got answer 6 km. Therefore, the distance of the flatted car with aunt's house was 2 km (right side of Figure 4.15).

Similarly, Group A calculated $\frac{1}{4}$ of 8 km and then calculated $\frac{3}{4}$ of it (left side of Figure 4.15). One of member of Group A explained that since a quarter of 8 km is, then if the quarter was added two more, then it became 2 times 3, that is 6.

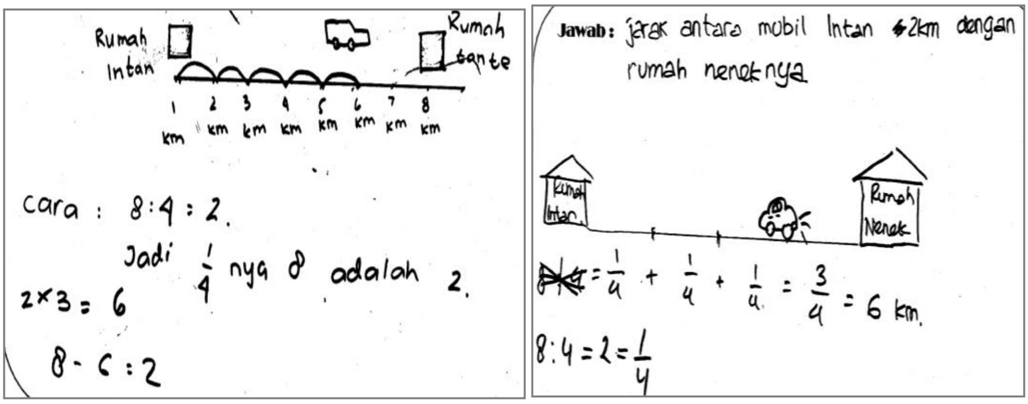


Figure 4.15 Students' Strategies in Solving Measuring Problem

To prove that the students really understood how to find fractions of some length and not only applied algorithms, the researcher gave a line and asked them to find $\frac{5}{6}$ of it. As the answer, the students divided the line to six parts and then multiplied it by 5. From this evidence, the researcher concluded that the students already understood (although the researcher did not make it explicit) that fractions are operator, that fractions are related to division and multiplication.

4.2.3 Post-Test of First Cycle

Questions in the post-test were almost similar with those in the pre-test except for the last question (question 6). Briefly, the researcher could say that all students showed improvement in solving the first five questions that had the same idea with the pre-test. Below are some remarks researcher got from the post-test of the first cycle.

4.2.3.1 Students' Knowledge of Multiplication of Fraction with Whole Number

In solving multiplication of fraction with whole number problem, the students did not made mistake in converting whole number to fraction anymore and could multiply it properly. Iman still used the same method as in the pre-test for solving multiplication of whole number by fraction problem, but in the post-test he made it correctly, he did not make mistake like in the pre-test anymore.

4.2.3.2 Relating Algorithms to Situations, Checking the Meaning of Fractions

Based on the observation, it seems that the students still struggle to find suitable situation for those fractions (especially for multiplication of fraction by whole number problem), since they need long time to finish the problems.

4.2.3.3 Strategies Used by Students to Solve Contextual Problems

All students could solve the problem. When the fraction in the contextual problem was a unit fraction, some of them directly made the denominator as divisor. When the fraction was a proper fraction, then they multiplied it by the number.

4.2.3.4 Relating Situations to Its Algorithms

Based on the answer, it seems that they could relate one situation to more than one algorithm given.

4.2.3.5 Dividing a Number of Cake to Some People

The question was about dividing two cakes (the picture of cake was given) to three people. The picture of two cakes was given in the question. Some students directly partitioned the cakes. The other students made other picture and then divided it. Below are some of their answers.

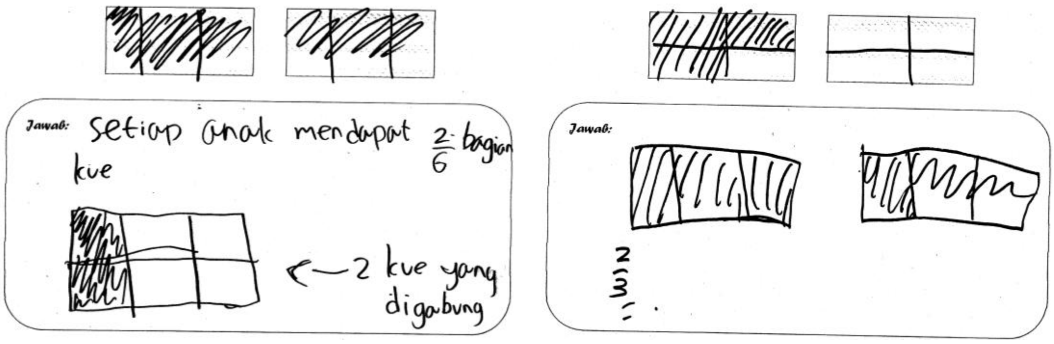


Figure 4.16 Examples of Students' Strategies to Share Two Cakes for Three People

In the left side of Figure 4.16, Iman unified those two cakes and then divided or partitioned it by three. Then he wrote that every person got $\frac{2}{6}$. Meanwhile, in the right picture of Figure 4.16, Cici divided each cake by four parts in at the beginning. Then she said it was wrong so that she made another picture of two cakes and divided it by three. She wrote the result is $\frac{2}{3}$. Based on those two pictures, the researcher deduced some student did not realize that two cakes divided for three people means two divided by three or two third. However, one of the students showed in her answer that she recognized the relation between fraction and division, because she directly used fraction $\frac{2}{3}$ as the result of sharing two cakes for three people.

Acha made other cake pictures (Figure 4.17), each cake had 3 cm of length. Her reason was that if those cakes were unified, then the length would be 9 cm. Therefore, if it was divided by three, then every person would get 3 cm of cake. The researcher interpreted that the student was familiar with measuring activity. This could be proven with her strategy to make cake easier to be divided by giving length on it.

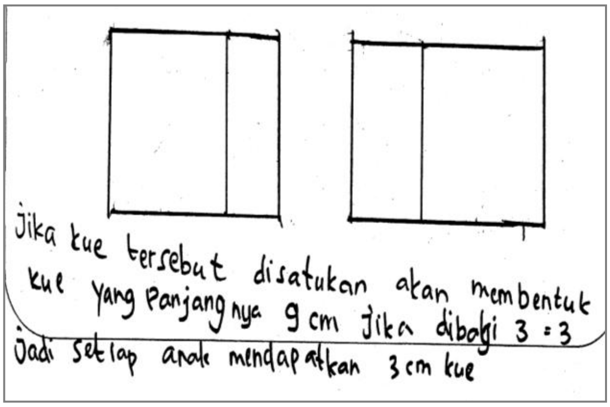


Figure 4.17 Acha's Strategy in Sharing Two Cakes for Three People

4.3 HLT 2 as Refinement of HLT 1

Overall, based on the analysis of the first cycle, the researcher concluded that almost all of the students' answers were in line with the conjectures, with little differences. However, one unpredictable case was that the students could not divide more than one real object (paper as representation of cake) as in Figure 4.12. Moreover, the students were accustomed to use algorithms for solving problems.

Therefore, the researcher decided to revise HLT 1 to be HLT 2. The researcher used fair sharing as the starting point with the expectation that students will not merely apply algorithms. The researcher expected students come to the idea of multiplication of fraction with whole number with this activity.

The conjectures and expectations toward this activity was that students will divided the cakes with various strategies. If they divided each cake with the same number (left side of Figure 4.18), the learning process can continue to the idea that repeated addition of fractions is multiplication of whole number by fraction. Another expectation is that students will divide each cake with different strategies, as the example can be seen in the right side of Figure 4.18.

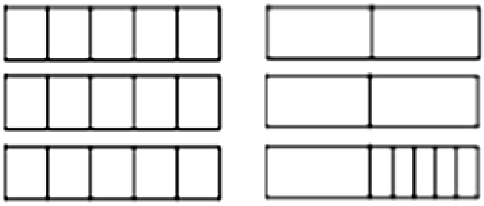


Figure 4.18 Conjectured strategies for sharing three cakes for five people

After solving some contextual problems that were related to multiplication of fraction by whole number and vice versa, then the learning activity can continue to the context about commutative property of multiplication of fraction with whole number. Table 4.2 below presents the HLT 2 for learning multiplication of fraction with whole number and visualised in Appendix D, meanwhile, the detailed problems of each activity can be seen in student's worksheets (Appendix G).

Table 4.2 HLT 2 in Learning Multiplication of Fraction with Whole Number

Learning Goal	Mathematical Idea	Activity
1. Relating fractions to multiplication and division	- Pieces do not have to be congruent to be equivalent - Fractions are related to division and division	- Sharing 1 cake for some people fairly - Sharing 3 cakes for some people fairly
2. Considering some object and its part as fractions 3. Developing sense that the result of multiplication of fraction by whole number can be smaller	- Fractions are connected to division and multiplication - Fractions as operator	- Dividing part of a cake to some people - Finding fractions of some pieces of cake - Measuring activity

Learning Goal	Mathematical Idea	Activity
4. Moving from repeated addition of fractions to multiplication of whole number by fraction	- Multiplication of fraction as repeated addition	- Preparing a number of menus - Listing the result in table
5. Finding inverse of unit fractions	- Inverse property of unit fractions	- Determining number of yarn colours needed to make colourful knitting
6. Recognizing the commutative property of fractions multiplication with whole number	- Commutative property of multiplication of fraction with whole number	- Comparing the length of ribbons

4.4 Retrospective Analysis of HLT 2

The second cycle of teaching experiment was conducted in of elementary schools in Surabaya, by implementing HLT 2 for the thirty-one students of another parallel class. The teacher for this teaching experiment was the real mathematics teacher of the class, meanwhile the researcher and some colleagues acted as observer. However, sometimes the researcher acted as the teacher with the intention to get some experiences about teaching in a quite big class, therefore the design of activities could be more acceptable.

In analysing the data got from the second cycle, the researcher first made groups of students' answers. Then, the researcher analyzed some interesting answers or strategies. The analysis was elaborated in each stage of learning goal, to explain how students obtained the better understanding of multiplication of fraction with whole number so that could be generalized for instructional design.

Table 4.3 Timeline of the Second Cycle

Date	Activity	Description of the Activity
16-02-2011	- Pre-Test	- Investigating students' initial knowledge
02-03-2011	- Fair Sharing	- Focusing on the idea that pieces do not have to be congruent to be equivalent - Focusing on the relation between fractions to multiplication and division
04-03-2011	- Finding fractions of some part of cake	- Focusing on relation on relation - Focusing on the whole of fraction
07-03-2011	- Finding fractions of some pieces of cake	- Focusing on the relation between fractions to multiplication and division
	- Determining the length of journey	- Focusing on fraction as operator - Multiplication of a fraction by a whole number
	- Preparing Number of Menus	- Focusing on the idea that repeated addition of fractions as multiplication of a whole number by a fraction
09-03-2011	- Determining Number of Colour needed	- Focusing on the number of fractions needed to get 1 as the result of multiplication (inverse property of unit fraction)
	- Comparing the length of ribbons	- Focusing on the commutative property of multiplication of fraction with whole number
14-03-2011	- Post-Test	- Investigating students' knowledge after receiving the instructional activities

In the following, the researcher will elaborate the result of the observation and analysis of the second cycle.

4.4.1 Pre-Test of Second Cycle

As written in the beginning of Subsection 4.4, the participants for this teaching experiment were 31 students. However, because of some reason, only 26 students who followed the pre-test, meanwhile the other 5 students did not attend the class at that time.

Briefly, the pre-test was aimed to know students' knowledge and ability about multiplication of fractions, especially about multiplication of *whole number by fraction* and multiplication of *fraction by whole number*. In the following, the researcher will describe some remarks got from the pre-test. In analysing the students' answers, the researcher first grouped the students' strategies and then presented and discussed different strategies.

From the students' answers, the researcher saw that sometime the students used the same strategies to solve similar type of questions. Thus, in this case, the researcher only presented one of the answers as the example. For example, when students used similar strategy to solve two questions about fraction addition with the same denominator, the researcher then only presented one of the answers of those questions as the example.

4.4.1.1 Students' Strategies to Solve Addition of Fraction

Based on the answers in the worksheets, the researcher interpreted that almost all students could use proper algorithms to solve *fractions addition with the same denominator*. However, one student, Ana, seemed tend to equate the denominator, for example $\frac{3}{5}+\frac{3}{5}+\frac{3}{5}=\frac{15}{125}+\frac{15}{125}+\frac{15}{125}=\frac{45}{125}:5=\frac{9}{5}$. Although the denominators already had the same denominator, she still multiplied the denominators and then simplified the result.

There were only four students made some mistakes in solving *fractions addition with the same denominator* problems, with the following description.

Arul perhaps did not understand about it, since he directly rewrote one of the fractions as the answer, for example, $\frac{2}{3}+\frac{2}{3}=\frac{2}{3}$. However, the researcher did not have enough data to make some interpretation why he answered it like that. Meanwhile, another student, Wandi, made mistake when simplifying the fraction $\left(\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=\frac{2}{1}\right)$. The rest two students perhaps only made miscounting since they could solve other question properly.

Regarding to *fractions addition with unlike denominator*, three students perhaps had misunderstanding. Wandi perhaps directly added the numerators without equating the denominator first, and perhaps he chose the largest denominator as the denominator of the result $\left(\frac{4}{5}+\frac{3}{4}=\frac{7}{5}=1\frac{2}{5}\right)$. Another student, Emi, besides adding the numerators, also added the denominators $\left(\frac{3}{5}+\frac{2}{7}=\frac{5}{12}\right)$. Meanwhile, Arul seemed to use cross-addition to solve the problems, for example $\frac{2}{8}+\frac{1}{4}=\frac{6}{9}$.

4.4.1.2 Students' Strategies to Solve Multiplication of Fraction with Whole Number

In solving multiplication of fraction with whole numbers, the students showed various strategies. The description of their answers as follows.

Six students directly wrote the answer, for example they directly wrote $\frac{3}{4}$ as the answer for question $3\times\frac{1}{4}$. Perhaps, they multiplied the whole number 3 by the numerator 1 and then put the denominator 4 as the denominator of the result.

Other four students converted the whole numbers to be fractions, for example, they changed 3 to be $\frac{3}{1}$ and then used the algorithms to multiply *fraction by fraction* $\frac{3}{1} \times \frac{1}{4} = \frac{3}{4}$. Almost similar, two students also changed the whole number to be fractions and then used the algorithms of solving *multiplication of fraction by fraction*, but they changed the whole number 3 to $\frac{12}{4}$. Perhaps, they wanted to equate the denominator as when solving fractions addition with unlike denominator.

Two other students used *repeated addition of fraction* to solve both *multiplication of whole number by fraction* $\left(3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}\right)$ and *multiplication of fraction by whole number* $\left(\frac{2}{5} \times 3 = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}\right)$.

Four students might mess up multiplication of whole number by fraction with mixed number. For example, in solving $5 \times \frac{2}{3}$, they multiplied the denominator 3 by the whole number 5 and then added the result by the numerator 2, and then put the 3 as the denominator of the result $\left(\frac{13}{4}\right)$. Another student also used the same procedures, but she did not put the denominator and only answered it as $3 \times 4 = 12 + 1 = 13$.

Six students perhaps mixed up the algorithms of multiplication of fraction with whole number to the algorithms to multiply fraction by fraction. They multiplied the whole number to the numerator and then divided it by the multiplication of the whole number by the denominator, for example $5 \times \frac{2}{3} = \frac{10}{15}$.

Meanwhile one more student answered it as $5 \times 3 = 15 \times 2 = 30$. Perhaps he also mixed up some algorithms.

4.4.1.3 Relating Situations to Its Algorithms

The students were asked to relate four given situations to its algorithms. The first situation was about dividing three cakes to four children. As can be seen in Table 4.4, most of the students related the situation to $\mathbf{3:4}$ and/or $\mathbf{\frac{3}{4}}$. Perhaps, the students associated fractions with division.

The third situation was *"Budi's house is 4 km from Ani's house. Ani wants to visit Budi, and now she already passed quarter part of the way".* Most students related it to $\frac{1}{4} \times 4$. Perhaps, the students chose the algorithm because the number (4) in the situations was different with others.

The second situation was about three children, where each of them takes quarter part of a cake. For this situation, most of the students related it to $3 \times \frac{1}{4}$ and/or $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. The researcher interpreted that they partitioned each cake to four parts and took one part for one child. Since the number of children is three, then they added the $\frac{1}{4}$ three times.

Table 4.4 Students' Answers in Relating Situations to Its Algorithms

Sts	1						2						3						4					
	$3\times\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$	$3:4$	$\frac{1}{4}\times 4$	3×4	$3\times\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$	$3:4$	$\frac{1}{4}\times 4$	3×4	$3\times\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$	$3:4$	$\frac{1}{4}\times 4$	3×4	$3\times\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$	$3:4$	$\frac{1}{4}\times 4$	3×4
1		✓					✓									✓		✓			✓		✓	
2		✓	✓	✓			✓	✓	✓								✓				✓			
3			✓				✓							✓									✓	
4				✓					✓								✓		✓					
5				✓			✓										✓		✓					
6																								
7		✓					✓										✓				✓			
8				✓					✓								✓		✓					
9		✓							✓	✓							✓		✓					✓
10		✓					✓										✓				✓			
11				✓			✓										✓			✓				
12				✓					✓								✓		✓					
13		✓							✓								✓		✓					
14		✓					✓										✓				✓			
15		✓					✓										✓				✓			
16		✓					✓										✓				✓			
17		✓					✓										✓				✓			
18	✓	✓	✓	✓			✓	✓	✓	✓							✓				✓			
19				✓						✓							✓				✓			
20				✓			✓	✓				✓					✓				✓			
21				✓			✓										✓				✓			
22						✓			✓					✓					✓					
23		✓					✓										✓				✓			
24				✓		✓			✓								✓		✓	✓				
25			✓				✓									✓				✓				
26			✓					✓		✓		✓					✓		✓					

The fourth situation had similar idea to the second situation: *"Mother wants to make three cakes. Each cake needs $\frac{1}{4}$ kg of sugar."* Similarly with the second situation, most students related it to $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ and/or $3\times\frac{1}{4}$. However, only one student chose the same algorithms both for the second and the fourth situation. Meanwhile, the other students gave different answer with those in the second situation. Perhaps, it was because the students have a tendency to make one to one correspondence. Therefore, after they chose one algorithms, the might be choose different algorithms to other situations.

4.4.1.4 Relating Algorithms to Situations

After relating situations to its algorithms, the students were also asked to write suitable situations for fractions, addition of fraction, and multiplication of fraction with whole number.

1) Fractions

   From the worksheets, the researcher concluded that each student has his own interpretations about fractions. Therefore, the researcher based the interpretation of students' answers on Table 2.2 presented in Chapter 2 of this research.

From the fifteen students who gave clear situations, five students perhaps understood fraction $\frac{2}{3}$ as part-whole relationships, since they gave situations as *"Intan ate 2 of 3 pieces cake"*. Meanwhile, other five students might understand fractions as quotient, since they gave fair sharing situation. For example, they wrote *"two cakes divided to three children"* to represent fraction $\frac{2}{3}$. Three students perhaps considered non-unit fractions as iteration of unit fractions. For example, one of the students wrote *"There are two children. Each of them takes a-third cake"*. Other two students might understand fractions as operator. One of the situations written by the students is *"We have passed $\frac{2}{3}$ of the journey"*.

While the other seven students gave unclear answers and one student did not give answer, the rest three students gave formal algorithms to represent the fractions. For example, they wrote $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$. Perhaps, the students understood fractions in a very formal level.

2) Addition of fractions

Rather than wrote situations as what asked, 11 students directly added the fractions by using algorithms. Four other students also added the fractions first, but after that, they put the result in some situation. For instance, after adding $\frac{1}{4}$ by $\frac{1}{4}$, the students wrote *"I made cake to my Mom. I need $\frac{2}{4}$ flour."*

From the answers, the researcher could not see what is the meaning of addition for them in their daily life.

Nine students gave some situation as *"Father bought* $\frac{1}{4}$ *gram of candies, then he bought another* $\frac{1}{4}$ *gram."* While the students added the same objects (candies), the other two students added two different objects, for example *"*$\frac{1}{4}$ *wheat flour and* $\frac{1}{4}$ *tapioca flour."* Perhaps, the two students did not fully understand about the addition itself, that we cannot add two different entities.

3) Multiplication of fraction with whole number

Regarding to multiplication of fraction with whole number, only eight students gave clear situations. While seven students interpreted it as half of some object as many as three times (i.e. "*There are three children. Each child gets* $\frac{1}{2}$ *part."*), one student interpreted it as half of something (i.e. *"The distance of Rio's house is 3 km and Melati wants to visit him. Melati already passed* $\frac{1}{2}$ *of the way."*). Since the students used the same idea for both $3 \times \frac{1}{2}$ and $\frac{1}{4} \times 5$, the researcher interpreted that perhaps the students did not distinguish those two multiplication of fraction.

4.4.1.5 Strategies Used by Students to Solve Contextual Problems

There are two contextual problems given in the pre-test (question 4 and 5 in Appendix F). In solving those problems, eighteen students directly divided the number of mineral glasses by two, also for the prize of oranges (for instance, student's answer in the upper side of Figure 4.19). Since the fraction $\frac{1}{2}$ is a very common fraction, probably the students already knew that a half means dividing something by two. Other three students used the procedure to multiply a fraction by a whole number, as the example can be seen in the bottom left side of Figure 4.19.

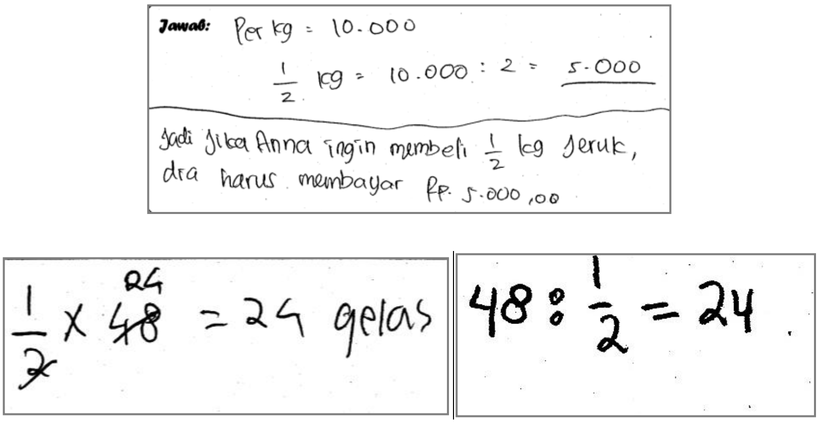


Figure 4.19 Some of Students' Strategies to Solve Contextual Situations

The other five students messed up the algorithms. For example, rather than divided the amount by 2, one student divided it by a half. However, the result became correct (bottom right of Figure 4.19)

4.4.1.6 Finding the Inverse Property of Unit Fractions

The problem offered was about determining number of yarn colour needed to knit one metre of colourful yarn (see Appendix F, question number 6). The strategies used to solve this problem were quite varying. Some students used repeated addition and some other directly used multiplication of fraction with whole number algorithms.

However, some students made some mistakes in solving the problem. It seems that one student directly picked up the numbers in the question and multiplied it, therefore her answer become $1 \times \frac{1}{4}$. As the result, she got $\frac{5}{4}$. Perhaps, she messed up the procedure for multiplying whole number by a fraction with the procedure to convert mixed number to be improper fraction.

4.4.1.7 Commutative Property of Multiplication of Fraction with Whole Number

In order to check students' understanding of the commutative property of multiplication of fraction with whole number, the students were asked whether $6 \times \frac{1}{2}$ is similar to $\frac{1}{2} \times 6$ or not. They then were asked to give reasons by giving daily life situation (see question 7 of Appendix F).

All students said that it was the same. Most of them used formal procedure to prove it. The other students gave situation to prove it, beside the formal procedure. For instance, one student only changed the editorial of the sentence in the situation as in the left side of Figure 4.20. Another student, Aurel, said that it was the same even it was inverted, the multiplication still the same (right side of Figure 4.20)

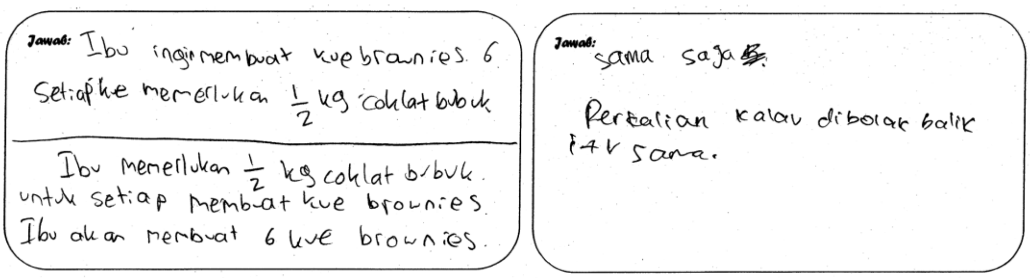


Figure 4.20 Fitri Only Changed the Editorial of Sentences to Prove that $\mathbf{6} \times \frac{\mathbf{1}}{\mathbf{2}}$ is Similar with $\frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{6}$

From the answers, the researcher concluded that the students did not differentiate the meaning of multiplication of a whole number by a fraction and the meaning of multiplication of a fraction by a whole number. They could treat both multiplications as repeated addition.

4.4.2 Second Cycle of Teaching Experiment

4.4.2.1 Fair Sharing Activity to Relate Fractions to Multiplication and Division

In order to achieve the learning goal that fractions are related to multiplication and division, the researcher conducted an activity about fair sharing. The activity was started when the teacher invited some students to show various ways to cut a 'Bolu' cake for four people fairly. Since the researcher provided some real cake to be cut, the students seemed very interested in cutting the cake. Most of the students wanted to go to front of the class to cut the cake.

After some students showed their strategies to divide the cake fairly, the lesson continued with some problems in their worksheet that had to be solved with friends in their groups. In this activity, the students were divided to eleven groups with different number of member: six groups consisting of two members, four groups with three members, and one group of four members.

4.4.2.1.1 Pieces do not have to be Congruent to be Equivalent

The first problem to be solved in the worksheet was sharing a cake to each member of his or her group fairly.

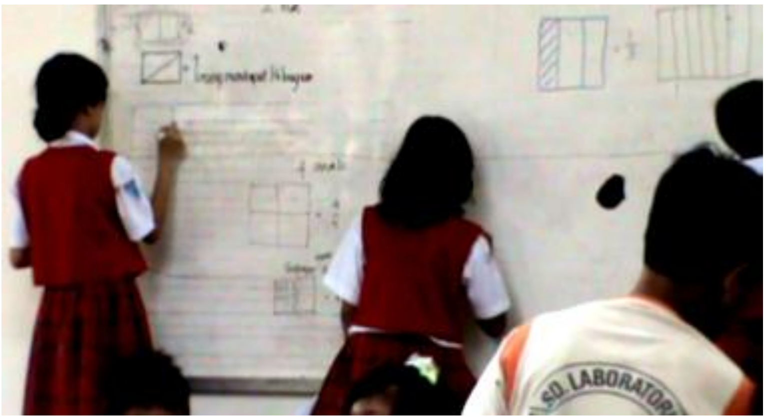


Figure 4.21 Students' Strategies to Share a Cake to Their Group Members

In order to get many fractions, the number of students in each group was different, as described above. Since the students were asked to share the cake in various ways, each group members got different shape of the cake (Figure 4.21). However, all of the students realized that they got fair part with their friends in their group. For them, fair means have the same amount.

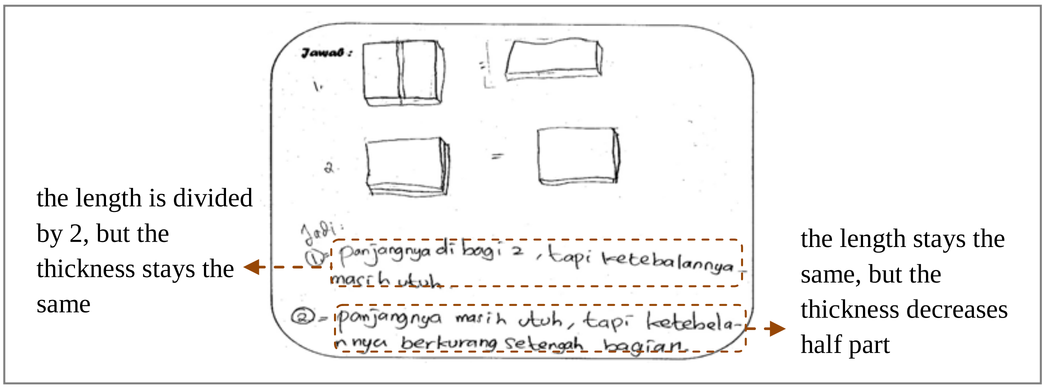


Figure 4.22 Ira's and Emi's Strategies to Share a Cake to Two People

One of the groups seemed attached to the real cake. Although the picture of cake given was in two-dimensional shape, they divided the cake as if it was the real cake as can be seen in Figure 4.22. For the group, even though the pictures have different sizes but the students said that both pieces are fair. Short vignette below shows their understanding of fairness.

Researcher	:	It will be same or not, what you got here [points on the first picture in Figure 4.22] and here [points on the second picture in Figure 4. 22]
Ira	:	[Looks at Emi] It is same, isn't it?
Emi	:	Same
Researcher	:	Why?
Ira	:	Because this [points on the second picture in Figure 4.22] has longer width, but this one [points on the first picture in Figure 4. 22] is shorter but thicker. So, it is same.
Emi	:	Yes, it is same.

4.4.2.1.2 Fractions are Related to Division and Multiplication

In sharing a cake to each member of their group, most of the students could relate fractions to division. As the result of fair sharing, fraction notated by the students was $\frac{1}{\text{the number of group members}}$, some examples of the answers can be seen in Figure 4.23. Most of the students reasoned it was because one cake was divided to the number of person in their groups. In detail, four groups out of six groups that have two members said each person in their groups got the same part, which is $\frac{1}{2}$ of the cake, because the cake was divided to two members. All four groups with three members said that each person got $\frac{1}{3}$ part of the cake because the cake was divided fairly to three people. Another group, that had four members, wrote that each person in their group got $\frac{1}{4}$ part.

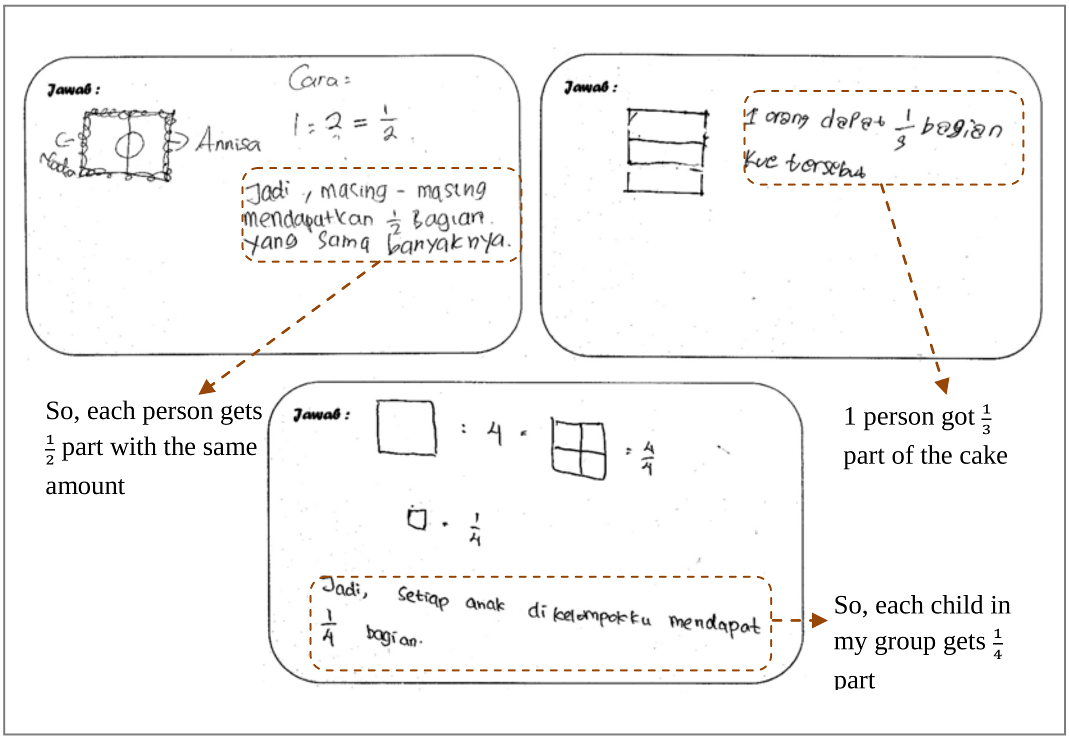


Figure 4.23 Students' Notation of Fair Sharing Result

On the other hand, the rest two groups of two members gave different reason to the answer. One group said that each member got $\frac{1}{2}$ *part of two cake pieces*. Meanwhile, the other one group wrote it as *1 cake divided by* $\frac{1}{2}$, therefore, 1 student got $\frac{1}{2}$ part of cake that had been cut. Perhaps, they got confuse between *the result of the division* and *the divisor*. Perhaps, the students messed up the divisors and the result of division (Figure 4.24).

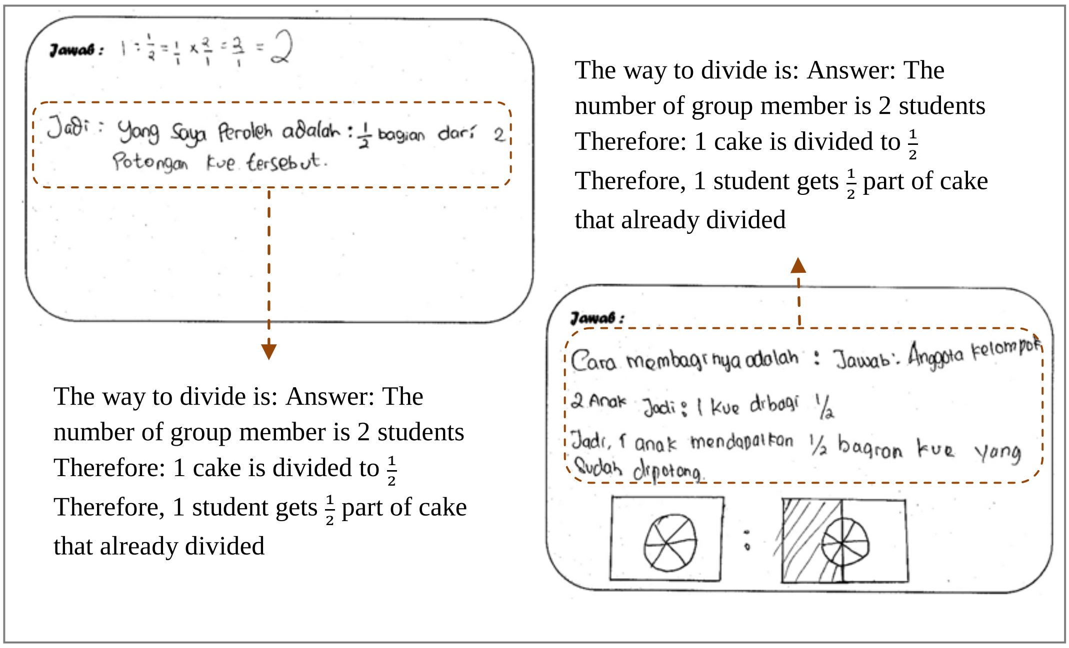


Figure 4.24 Students' Messed Up the Result of Division and the Divisor

Giving different strategies of sharing a cake to three people seemed more difficult to the students. Out of our conjectures, two groups used their knowledge of equivalent fractions to show various ways of dividing a cake to three people. They partitioned the cake as many as the multiplication of three and came with equivalent fractions (Figure 4.25).

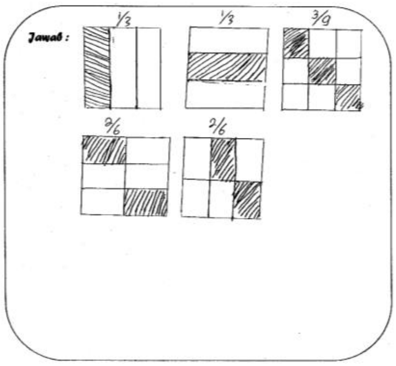


Figure 4.25 Equivalent Fractions as Different Strategies in Sharing a Cake to Three People

The next problem was about sharing three cakes to four people: *"Yesterday, Ani got three brownies from her grandmother. She wants to share it to her three friends. Help Ani to share the three brownies to four people fairly. How much cake got by each person? Explain your answer."*

As conjectured, most of the students partitioned each cake to four parts, but they gave different fractions as the result. The differences lies in the way they notated the amount of cake got by each person, based on the whole of fraction they chose (Figure 4.26). Two groups considered one cake as the whole, therefore each person got $\frac{3}{4}$ of one cake. Another group took one part of each cake and considered it as $\frac{1}{4}$, therefore one person got $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$. The other group considered the three cakes as the whole, therefore they said each person got $\frac{3}{12}$ of three cakes.

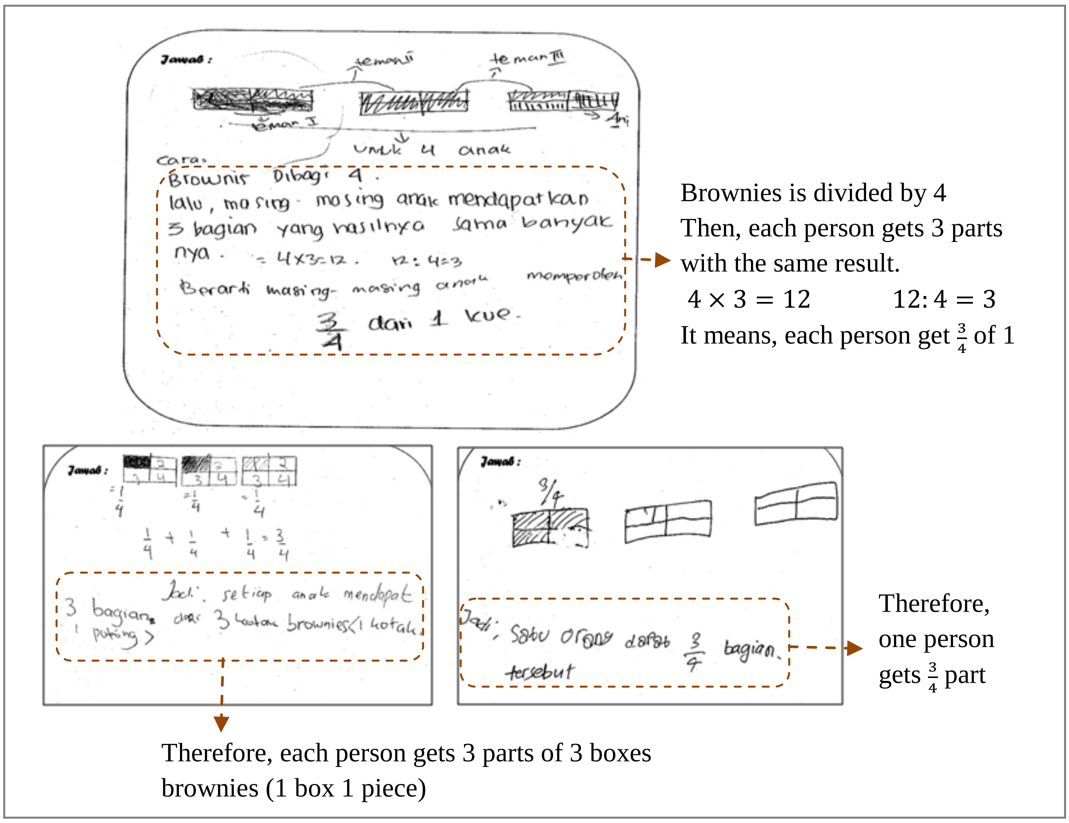


Figure 4.26 Different Wholes of Fractions

Further, when the other groups gave one fraction as the result of fair sharing, one group gave two fractions as the result because they considered two different wholes (Figure 4.26). When the whole is three cakes that partitioned into 12 parts, the fraction was $\frac{3}{12}$. Meanwhile, when the whole is one cake, they got fraction $\frac{3}{4}$.

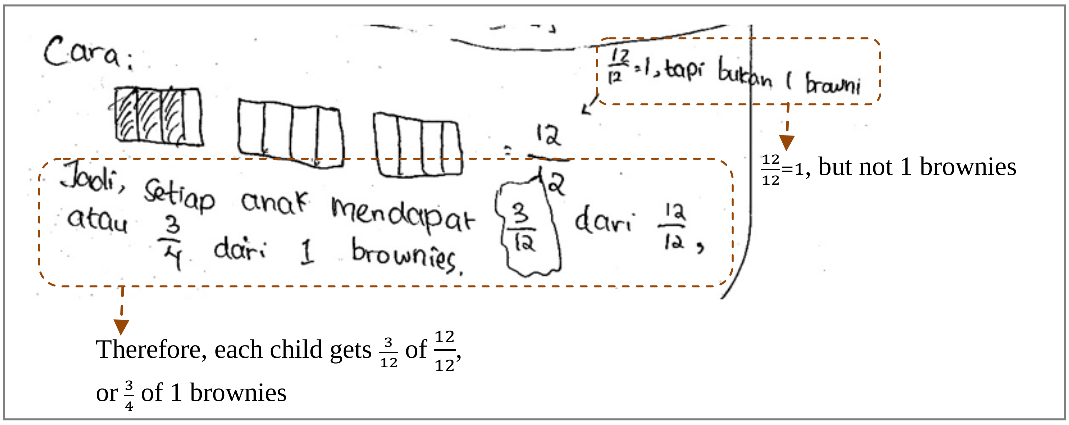


Figure 4.27 One Group Gave Different Answers Based on Different Whole of Fraction

From the descriptions above, it can be seen that all students related fraction to division, meanwhile, the relation between fractions to multiplication did not explicitly appear in this activity. Even though one group (the first picture in Figure 4.26) wrote $\frac{3}{4}$ of 1 cake, which actually means $\frac{3}{4} \times 1$, but the students seems did not realize it. In addition, in the class discussion, the teacher only gave reinforcement that fractions are division by giving some questions such as *"Bu Mar has ten cakes. The number of students in this class is thirty-one. How much part got by each student?"* Therefore, the researcher expected the idea that fractions are also related to multiplication would emerge in the next activity.

4.4.2.2 Considering some Objects and Its Parts as Fractions

The activity offered in the second meeting was finding fractions of some part, which is the continuation of fair sharing activity. In this activity, the students were asked to determine the fractions of some part of cake if the cake were divided for some people. However, before the students working on their worksheets, the teacher conducted class discussion with the aim to recall what the students learned in the previous meeting. After that, the teacher asked one student to draw the strategy he used to divide three cakes to five people (Figure 4.28).

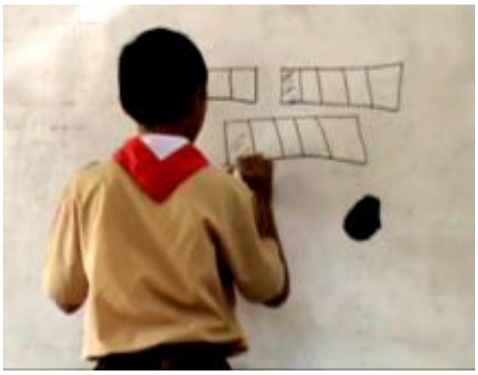


Figure 4.28 One Student Draws His Strategy to Divide Three Cakes to Five People

As can be seen in Figure 4.28, the student partitioned each cake to five parts and shaded one part of each cake. The following vignette shows the discussions between the teacher and the students, about strategy used in Figure 4.28.

Teacher	:	So, each person got one like this [circles the three shaded parts]? So, Ale got this [points at the first shaded part], this [points at the second shaded part], and this [points at the third shaded part]? How much part is this [points at the first shaded part]?
Students	:	One fifth
Teacher	:	How much [points at the second shaded part]?
Students	:	One fifth
Teacher	:	How much [points at the third shaded part]?
Students	:	One fifth
Teacher	:	So how much all?
Students	:	Three fifth

After that, the teacher showed a picture about one different strategy of dividing three cakes to five people (Figure 4.29).

Figure 4.29 Teacher Shows One Strategy of Dividing Three Cakes to Five People

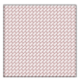
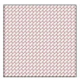
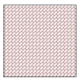
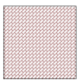
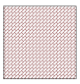
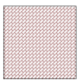
The teacher then asked how much part gotten by each person based on the picture. After some time discussion (as can be seen in the following conversation), then the students answered each person got $\frac{1}{2}$ from the first cake and $\frac{1}{10}$ from the last cake. $\frac{1}{10}$ part itself came because the students divided the half part in the left to be 5 pieces also, therefore there were 10 pieces.

Teacher	:	So, each person got how much cake? This, each person got this [points at the shaded part of first cake in Figure 4.29] and this [points at one of the shaded parts of third cake in Figure 4.29]
Students	:	.....
Teacher	:	How much part is this [points at the shaded part of the first cake]?
Students	:	A half
Ema	:	How about six tenth?
Teacher	:	How come?
Ema	:	That are five. Then, that are ten. So, it is six tenth.
Teacher	:	Ema said, six tenth [writes $\frac{6}{10}$]. How much is this [points at the shaded part of the first cake]?
Ema	:	Five
Teacher	:	How much part is this?
Ema	:	Oh, a half.
Teacher	:	A half [writes $\frac{1}{2}$ below the $\frac{6}{10}$]. Got this [points at one of the shaded parts of the third cake] means addition, right? How much part is this?

Students : One tenth
....
Teacher : So, a half plus a tenth. Is it the same with three fifth? Is it the same with Ale’s answer?
Students : Same.

The lesson continued by worksheet to be solved in groups. The worksheet itself was divided to three types, LKS A, LKS B, and LKS C. The differences lay on the numbers of the first three problems, as can be seen in Table 4.5, so that the students could solve many problems in the limited time.

Table 4.5 Differences of the First Three Numbers of LKS A, LKS B, and LKS C

LKS A	**LKS B**	**LKS C**
1. Yesterday, mother made one cake as in the following.  Today, the cake only **quarter** part of the cake left. Draw the cake yesterday and tomorrow	1. Yesterday, mother made one cake as in the following.  Today, the cake only **half** part of the cake left. Draw the cake yesterday and tomorrow	1. Yesterday, mother made one cake as in the following.  Today, the cake only a **third** part of the cake left. Draw the cake yesterday and tomorrow
2. Today mother wants to share the left cake to her **three** children. How much part got by each child, comparing to one cake? Explain your answer	2. Today mother wants to share the left cake to her **four** children. How much part got by each child, comparing to one cake? Explain your answer	2. Today mother wants to share the left cake to her **five** children. How much part got by each child, comparing to one cake? Explain your answer
3. Tomorrow mother have a plan to invite **12** orphans and mother wants to serve Black Forest like made yesterday. 	3. Tomorrow mother have a plan to invite **8** orphans and mother wants to serve Black Forest like made yesterday. 	3. Tomorrow mother have a plan to invite **15** orphans and mother wants to serve Black Forest like made yesterday. 

LKS A	LKS B	LKS C
Make drawing of how you divide the cake for **12** orphans. How much part got by each orphans?	Make drawing of how you divide the cake for **8** orphans. How much part got by each orphans?	Make drawing of how you divide the cake for **15** orphans. How much part got by each orphans?

4.4.2.2.1 The Whole of Fractions if some Part of Cake Divided to some People

Students' worksheets showed that all students partitioned the cake into two, three, or four parts, according to the problems they got. However, in solving the second problem, they gave different strategies. The researcher will describe their answers as follows.

Out of the conjectures, 6 out of 12 groups directly used the algorithms of fraction division by whole number, for instance, $\frac{1}{4}: 3 = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$. Perhaps, the students used fraction division algorithms because they associated *sharing* with *division*. Further, based on the teacher explanation, one of the reasons might be because the students already learned about fraction division.

The other five groups did not realize that the whole of fraction was not one cake anymore, but part of it. For example, when they were asked to find fraction if half cake divided to four people, they gave $\frac{1}{4}$ as the answer. Just after got some support from either the teacher or the researcher, two of the groups changed their answer in the worksheets (Figure 4.30).

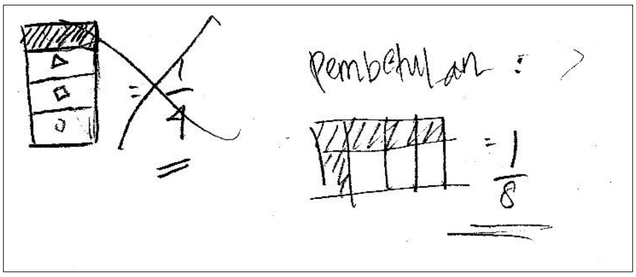


Figure 4.30 Revision of Student's Answer in the Worksheet

Meanwhile, the rest one group gave very different answer as in Figure 4.31. They partitioned the one cake to 15 parts, and also for the 'one third' cake. They said each person got $\frac{3}{15}$ if the whole is one cake, and if the whole is a-third cake, each person also got $\frac{3}{15}$ but the pieces is smaller than what got from one cake. Perhaps, the students realized that the pieces size is different, but they might not be able to relate it to the whole.

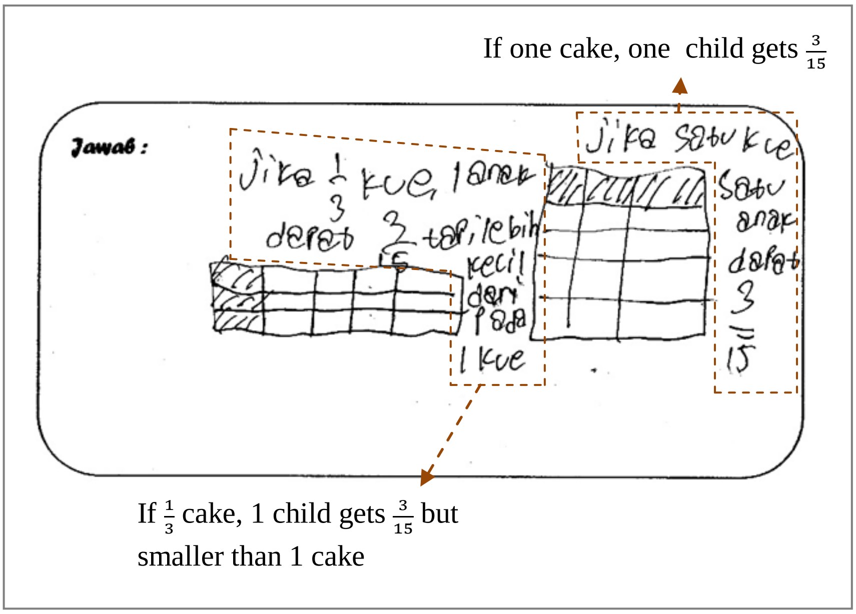


Figure 4.31 One Group Differentiates the Size of the Answer

The class discussion was held then to discuss the various students' answers, the focus lay on the whole of fractions. The following vignette shows the discussion between the teacher and the students in class discussion, when discussing the fraction got if half part of a cake divided to four people fairly.

Teacher	:	This is [points on the half part in Figure 4.32] half part left. The half part is divided to 4. Therefore, each student got $\frac{1}{4}$. Nah, this $\frac{1}{4}$ came from 1 cake or half cake?
Students	:	Half cake
Teacher	:	A quarter of half cake [writes $\frac{1}{4}$ of $\frac{1}{2}$ cake].

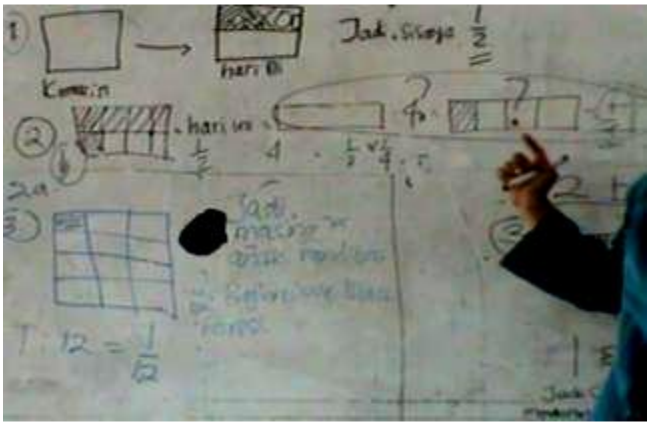


Figure 4.32 Class Discussion to Find the Whole of Fraction

4.4.2.2.2 Finding Fractions of some Pieces of Cake Activity

The lesson then continued with the next problems as follows.

Bu Anto made a cake divided to 10 pieces.

a. If $\frac{1}{5}$ part of the cake has been eaten, how many pieces is that? Explain your answer.

b. How if $\frac{2}{5}$ part that has been eaten? How many pieces is that?

Figure 4.33 Problem to Find Fractions of Some Pieces of Cake

Looking to the students' worksheets, the researcher found that six groups gave the same strategy in solving *question a)* and *question b)* in Figure 4.33. Three out of the six groups used algorithms of multiplication of fraction with whole number, for example they wrote $10 \times \frac{1}{5} = 2$ for question a and $10 \times \frac{2}{5} = 4$ for question b. One of the students said that it was because the word 'of' means multiplication. The rest three groups used the idea of equivalent fractions as $\frac{1^{\times 2}}{5^{\times 2}} = \frac{2}{10}$. Since they had to multiply the denominator 5 by 2 to get 10, then they also had to multiply the numerator 1 by 2 and got 4. It also similar for question b, $\frac{2^{\times 2}}{5^{\times 2}} = \frac{4}{10}$.

On the other hand, the other five groups gave different strategies to solve those two questions. The researcher then looked to videotapes and found that one of the groups used different strategies because they got some guidance from the researcher for question a. Because of they did not have enough time, then the answer of question b still the same. The answer of one group before and after the guidance can be seen in Figure 4.34.

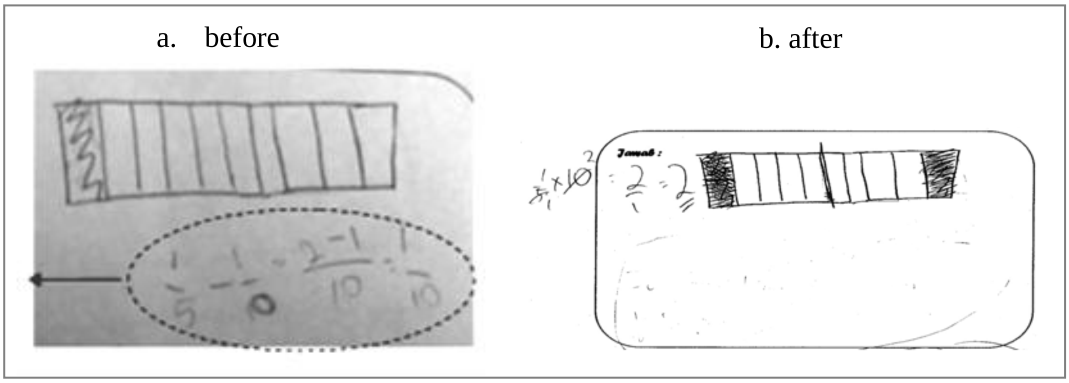


Figure 4.34 Students' Answer before and after the Guidance

The following conversation happened when the researcher asked the students to explain their reasoning through their answer in Figure 4.34a

Putri	:	This is one cake divided to ten. Because of one-fifth, then one is taken.
Researcher	:	One taken from?
Putri	:	From one over.. eh, from.. from five.. Because divided to five [points on the first five pieces] and five [points on the rest five pieces]
Researcher	:	So?
Putri	:	....

(Few minutes later)

Ara	:	Actually, all is.. one-tenth. But, this, one-fifth of the cake have been eaten. Because the numbers are not similar ($\frac{1}{5}$ and $\frac{1}{10}$), then we subtract it become one-tenth.
Researcher	:	Why you subtract one-fifth by one tenth?
Ara	:	Because.. because..

From the vignette above, the researcher interpreted that the students did not have a clue how to find one-fifth of ten pieces. Their confusion might be happen because they did not familiar with this kind of problem, where they had to find fraction of a collection of something. Further, the researcher interpreted that the sign 'minus' come from the word 'taken'. Perhaps, the students associated the word 'taken' to subtraction because the object was gone.

In order to help the students to answer the question, the researcher then gave some guidance by asking some question as can be seen in the following conversation.

Researcher	:	What is a half of ten pieces?
Ara	:	It means.. Four-tenth
Researcher	:	Why is it four-tenth?
Ara	:	A half means ... (no answer)

By asking about another question using a half as benchmark fraction, the researcher expected the students could answer it. However, the students still could not give the answer. Unfortunately, because of lacking of data, the researcher could not give interpretation why the students answer four-tenth. Further, the conversation continued as follows.

Ara	:	It is one-fifth [points on the fraction $\frac{1}{5}$ in the question], then divided by five [points on the first five pieces], by five [points on the first five pieces]. We only take this [points on the shaded piece]

(Few minutes later)

Researcher	:	You said it was divided by two, wasn't it?
Ara	:	Ya
Researcher	:	Then, these are five [points on the first five pieces] and five [points on the last five pieces]. Which one do you take?
Wandi	:	This one [points on the shaded piece]
Researcher	:	How about this? This is five [points on the first five pieces]. How about these [points on the last five pieces]? Do you remove it?
Putri	:	Not remove, only leave it

It seems that the students still did not have a clue how to find one-fifth of the ten pieces. Therefore, the researcher gave another question. They were asked to find one-fifth of fifteen circles. After sometime thinking, one of the student, Ara, used algorithms of multiplication of fraction to find $\frac{1}{5} \times 15$.

(Few minutes later)

Ara	:	[Computes $\frac{1}{5} \times 15$]
Researcher	:	Why you multiplied it?
Ara	:	Because, my teacher said if there is a word 'of' then it means multiplication.
Researcher	:	Who said that?
Ara	:	My private teacher

(Few minutes later)

Ara	:	This sentence (question on the worksheet) has word 'of'. Then it means we have to multiply it, not subtract it.
Researcher	:	Ya, multiplied by?
Ara	:	[computes $\frac{1}{10} \times 10$]

From the conversation above, it seemed that once the student realized that it was about multiplication of fraction with whole number, the student directly used formal algorithms to solve it. In order to make sure the students really understand what they have done, the researcher asked them to prove their answer by using the picture, as in the following conversation.

Researcher	:	You already had one (piece). What should we do then?
Ara	:	.... I don't know. If using picture, I don't know.
Researcher	:	This already took five [points on the first five pieces], this five points [points on the last five pieces], do we remove it?
Ara	:	No
Putri	:	..... Then, we take one more?
Researcher	:	Let's think. If we take one more piece, it becomes?
Putri	:	Two
Researcher	:	Two over?
Putri	:	Two fifth
Researcher	:	Oh ya? This is one, this is one, and then it becomes two of how many pieces?
Putri	:	Two-tenth
Researcher	:	Two-tenth is similar to?
Ara	:	One-fifth.

Researcher : Is it true?
Ara : Yes
Researcher : Then, what else?
Putri : So, we shade this? One more? [Shades one more piece as what can be seen in Figure 4.34b]

As the researcher said before, the group above did not have time to change their answer of question b, since they were asked to present their answer in front of class. In the class discussion, they explained what they already discussed with the researcher. They also said that the word ‘of’ means multiplication.

4.4.2.2.3 Multiplication of Fraction in Measuring Context

The next problem offered was about determining the length of journey of Amin and his uncle (see question 7 in Appendix G). Based on the result of the problem, the researcher saw that the students gave two different results. Seven groups considered that the journey of Amin and his uncle is $\frac{2}{3}$ of 150 m, meanwhile, five groups considered is as $\frac{1}{3}$ of 150 m. However, since it was because the editorial of the question that a little bit unclear, then the researcher treated both results are correct. Therefore, in the following the researcher will give our interpretation about students’ strategies only.

In solving the problem, ten out of twelve groups used the algorithm of multiplication of fraction with whole number as in left side of Figure 4.35. The following interview were between researcher and one of the groups. The researcher concluded that once the students realized about the meaning of word ‘óf’, which was stated in the class discussion before, the students directly used algorithms of multiplication of fraction with whole number.

Researcher : The distance is 150 m from his house. After two third journey, what is the meaning of it?
Ira : Two thirds of 150.
Researcher : Then?
Ira : So, it means $\frac{2}{3}$ multiplied by 150.

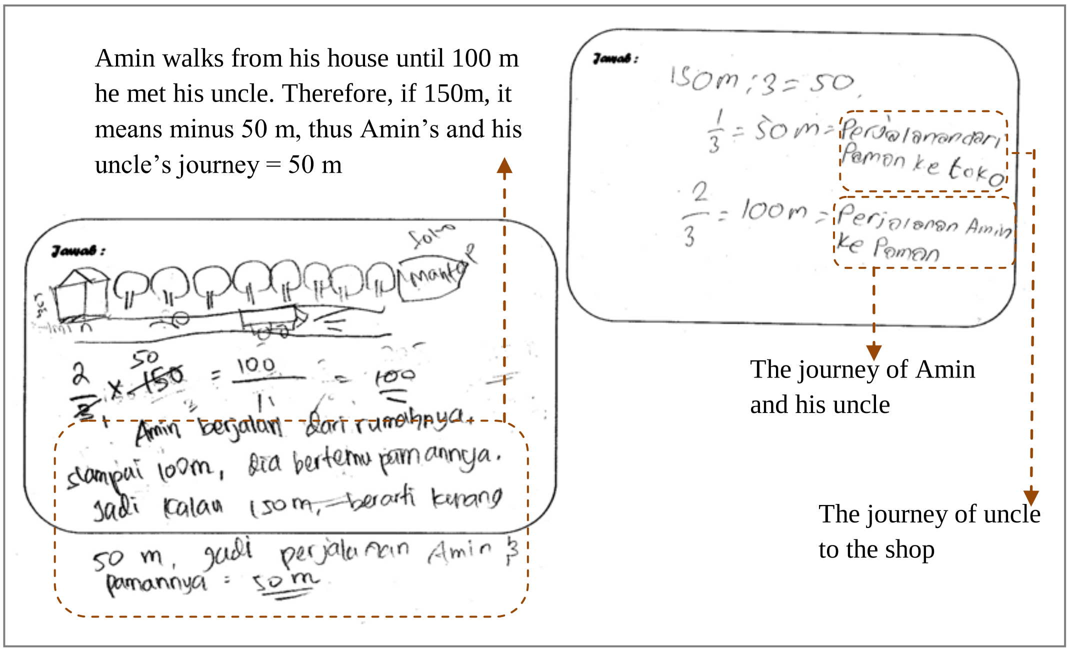


Figure 4.35 Students' strategies to find the length of journey Amin and his uncle

Further, the researcher found one interesting solution from one of the ten groups that used algorithms of multiplication of fraction with whole number. While the other students drew a line or bar to represent the road, this group represented the situation by array with size 25×6 squares (see Figure 4.36). Based on Figure 4.36, the researcher interpreted the students made the drawing after calculated the answer. Therefore, the drawing was not related to the situation.

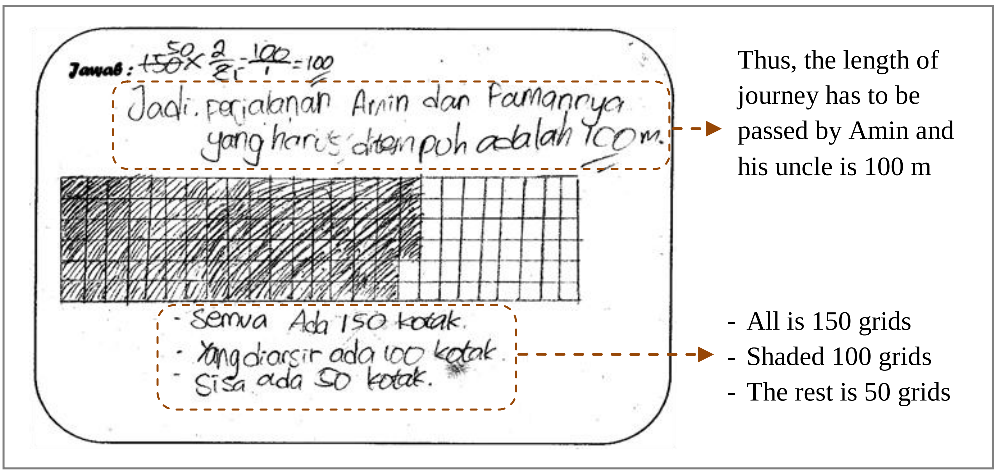


Figure 4.36 One Group Represented Road as Array

In the class discussion, the group presented their answer. The vignette below shows their reasoning of the answer.

Iis	:	[Points on the answer on whiteboard, see Figure 4.37] 150 divided by 3 is 50. 50 multiplied by 2 is 100.
Teacher	:	Why did you multiply it?
Iis	:	[Takes the worksheet from her friend and read the question silently] Because the question is, how meter the journey.

From the vignette above, Iis seems only knew how to use algorithms to multiply a whole number by a fraction, but she did not know what the reason is. The discussion then continued when the teacher provoke her to give more clear answer, by asking her to read the question again as in the following vignette.

Teacher	:	Is it just because the question about how meter the journey? Please give clear answer. Why it was multiplied? Try to read the question.
Iis	:	[Reads the question loudly] Amin was asked to buy ingredients in ‘Mantap’ shop that 150 meter of his house (and so forth)
Ria	:	Of [while hearing Balqis read the word ‘of’]
Iis	:	[Continues read the questions loudly]
Teacher	:	Before Ria have said it. What is it?
Ria	:	Of

Figure 4.37 One Group Presented Their Answer in front of Class

4.4.2.3 Repeated Addition as Multiplication of Whole Number by Fraction

The researcher offered the activity of preparing a number of menus to draw the mathematical idea that repeated addition of fractions as multiplication of whole number by fraction. Since the real teacher had to do something dealing with her health, then for this meeting, the researcher became the teacher.

In the beginning of the meeting, the teacher showed picture of a *lontong*. The teacher said that to make a *lontong*, the ingredient needed was a half cup of rice. Then, some students were invited to make a number of *lontong* by using the tool and ingredient prepared by the teacher.

The lesson then continued with group discussion, where the students had to solve some problems about determining ingredient needed to make a number of *lontong* and *opor ayam*. In solving the problems in the worksheets, all of the answers were in line with the conjectures made. Some students used repeated addition and the other students directly used the algorithm of multiplication of fraction with whole number. The strategies used by the students can be seen in Figure 4.38.

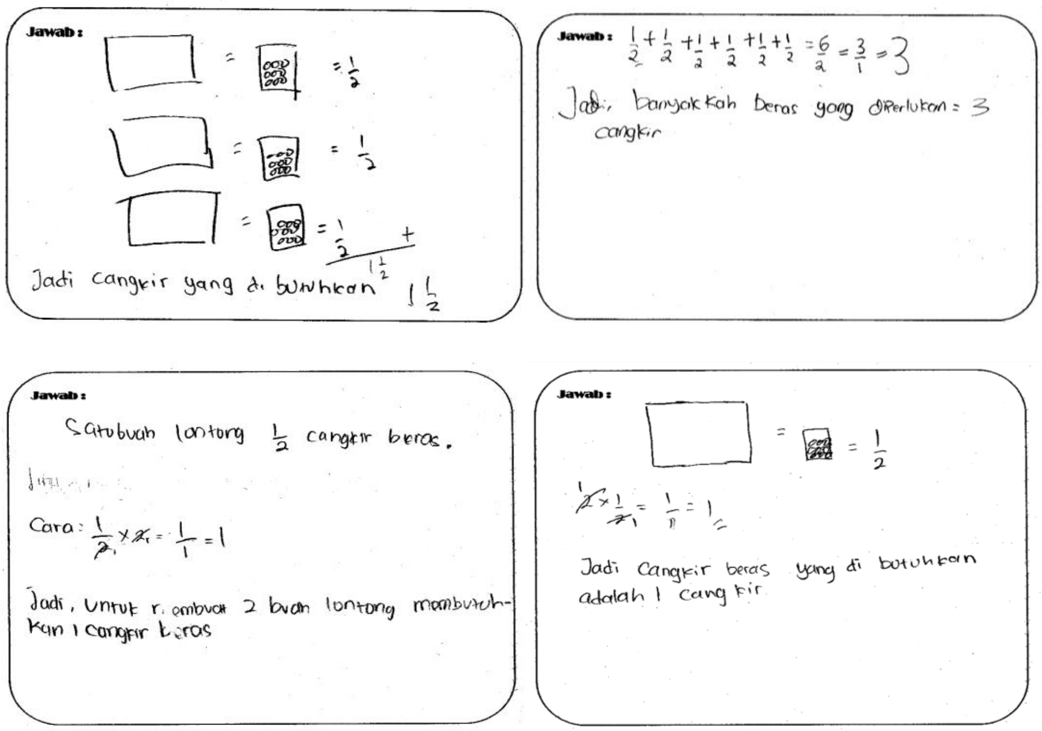


Figure 4.38 Students' Strategies to Solve Preparing Number of Menus Problems

Based on the observation, it seemed that the students already proficient enough to solve the problems without the help of real object as rice. It could be because they were familiar with this kind of problems and because it was close to the multiplication of whole numbers.

4.4.2.4 Colouring Activity to Find the Inverse of Unit Fractions

In order to gain the idea of inverse of unit fractions, the researcher offered a context about knitting yarn. The students had to determine how many colours of yarn needed to make 1 meter of colourful knitting. Based on the students' worksheet, all students gave similar answer to the first and the second question (the questions can be seen in Appendix G4). In the following, the researcher will present discussion among members in one group to answer the first question.

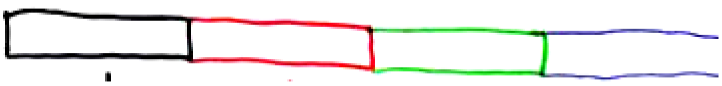
Fitri : [Shows her drawing to her friends]

[Reads question] Adek makes colourful knitting.
Every half meter, half means divided by two [makes a line in the middle of drawing]
[Continues reading question] If she already used two colours, this [points on the left part of the drawing] is been used, then this [points on the right part of the drawing].
Therefore...
[Continues reading question] How long is Adek's knitting now?
One meter.
Is it right? Please count it.

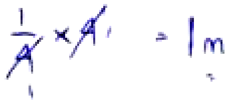
Arul : How many times is it? Is it two (times)?
[Arul writes $\frac{1}{2} \times 2 = 1$]

Further, the researcher presents conversation between the researcher and one group when they were solving the second problems in the worksheet (the final answer of the group can be seen in Figure 4.39).

Ida : [Draws rectangles by using black, red, green, and blue marker]



Researcher : How long is this [points on the black rectangle] ?
Ida : A quarter
Researcher : Write it
Ida : [Writes $\frac{1}{4}$ behind each rectangle]
Fitri : One over four times four.. [Ida writes $\frac{1}{4} \times 4$] One.. one [Ida scratches the fours and replaced it by 1] equal to 1



Researcher : What quarter is this [points at the black $\frac{1}{4}$ in Figure 4.38] ?

Ida : Meter [writes m on the right side of each $\frac{1}{4}$]

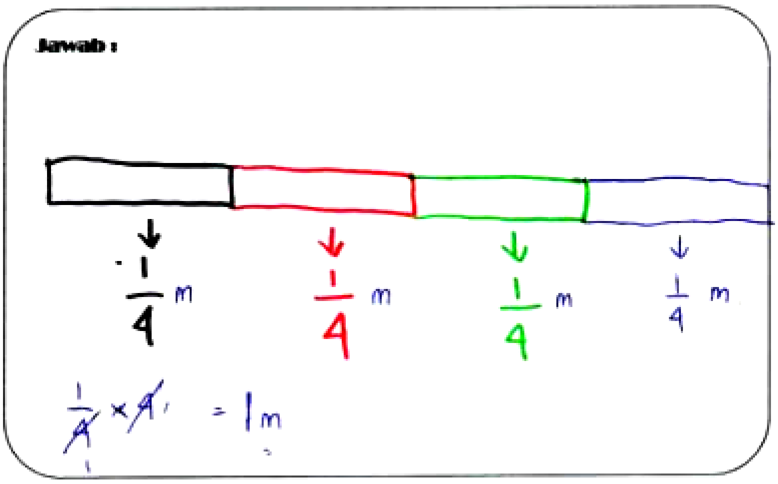


Figure 4.39 Students' Answer of Colouring Activity

The researcher then asked the group to make conclusions about what they have done as what questioned in number 3.

Researcher : Number 3
Ida : [Looks at the first answer]
Fitri : If the denominator ... [silents and smiles]
Researcher : Continue.. If the denominator..
Fitri : If.. [smiles].. What is the conclusion, Ida?
The answers of number 1 and 2 are same, even though the multiplied fraction is different.
Fitri : [Dictates the conclusion while Ida writing]
Fitri : If the denominator is different, but the numerator.. numerator.. numerator of the multiplied fraction is same, then the result is one

One of the students in the vignette above, Fitri, is one of low achievers in the class. However, as can be seen in the vignette, she could draw a 'good' conclusion of the numbers.

4.4.2.5 Comparing the Length of Ribbons to Recognize the Commutative Property of Multiplication of Fraction with Whole Number

In this last activity, the students were expected to gain the idea of commutative property of fractions multiplication with whole number through comparing the length of ribbons activity. The problems offered can be seen in Appendix G4, number 5 and 6.

As the tool, the students were given *one blue ribbon that has 30 cm of length* and *three pink ribbons that has 10 cm of length*. Before giving the ribbon, the teacher explained that 10 cm of the ribbons means 1 meter.

Unfortunately, rather than came to the idea of commutative property, students in one group needed much more support. The following vignette shows how students struggled to solve the second problem by the guidance of observer.

Observer	:	So, how is the story? Could you retell the story to me?
Lia	:	After ... [reads the task]
Observer	:	So, this ribbon [hangs a pink ribbon] has one meter long. Then, each child got how much?
Dika	:	[Points on the task] One over four
Yura	:	One fourth meter
Observer	:	So, how much is one fourth of this [shows the pink ribbon]? The length of this is 1 meter. Then, each child gets one fourth meter. How do you find a quarter from this ribbon?
Dika	:	Divide it
Observer	:	How many?
Dika	:	Divide it by three
Observer	:	If.. Quarter meter.. This is one meter.. How much the one over four?
Lia	:	Folded it

Observer	:	How many?
Lia	:	Three
Observer	:	How? [Gives the ribbon to Lia]
Lia	:	[Folds the ribbon]

From the vignette above, the researcher interpreted that the students even could not show a quarter of a ribbon. Rather than folded the ribbon to four, they said they had to fold it to three parts. Perhaps, it was because they saw the number "3" in the question. The conversation then continued as follows. The observer tried to make the students realised about "a quarter"

Observer	:	Become three or.. but each of them got quarter meter..
Dika	:	[Folds the ribbon]
Observer	:	If quarter meter, dividing by?
Dika	:	Dividing by four
Observer	:	Divided by..
Lia	:	Four
Observer	:	So, this [points at one part of the folded ribbon] small piece, how many meter?
Dika	:	A half.. Eh, a quarter
Lia	:	A quarter
Observer	:	A quarter.. Then we collect it. Cut it. This one [gives a ribbon to Dika]
Dika	:	[Cuts the ribbon]

After the students find the quarters, the discussion continued to determine the total ribbon got by the three daughters.

Observer	:	This is a quarter. Ibu bought it to her three children. Show me. Which ribbon is for the first child?
Lia	:	[Puts the small pieces of the ribbon on the paper]
Observer	:	The second child?
Lia	:	[Puts the small pieces of the ribbon on the paper]
Observer	:	The third child?
Lia	:	[Puts the small pieces of the ribbon on the paper]

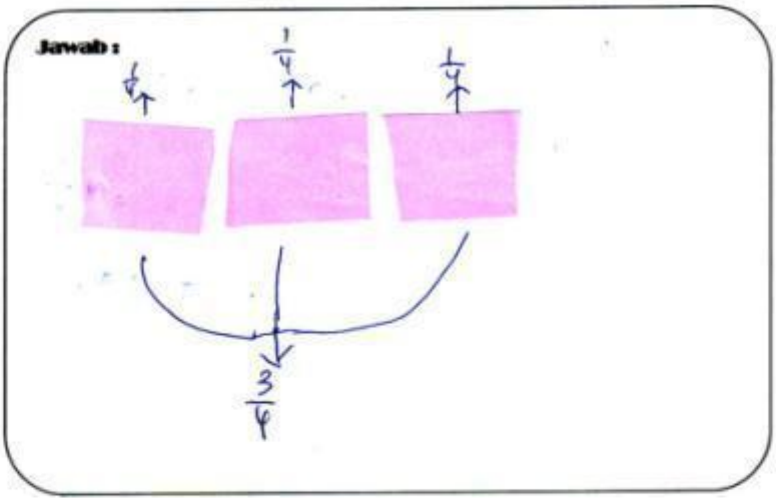


Figure 4.40 Students' Strategy to Find the Total Length of Ribbons

Observer	:	Now, how long is the ribbon bought by Ibu?
Lia	:	Three.. One meter
Observer	:	This is one meter [shows the original pink ribbon] The children are three [points on the three pieces of small ribbon]. What is the total length of the ribbon bought by Ibu?
Lia	:	A quarter
Observer	:	One fourth is this one [points on one pieces of the small ribbon] How if two?
Lia	:	Two fourth
Observer	:	How about three?
Lia	:	Three fourth
Observer	:	So, the total bought by Ibu? Three fourth.. meter.

4.4.3 Post-Test of Second Cycle

The learning series were closed by giving post-test for the students. The eight numbers of question in the post (see Appendix H) intended in assessing to what extend the students' understanding through the learning process. In the following the researcher will describe some remarks got from the post-test. Because of some reason, only 29 students who followed the pre-test, meanwhile the other 2 students did not attend the class at that time.

From the students' answers, the researcher saw that sometime the students used the same strategies to solve similar type of questions. Thus, in this case, the researcher only presented one of the answers as the example. For example, when students used similar strategy to solve two questions about multiplication of fraction with whole number, the researcher then only presented one of the answers of those questions as the example.

4.4.3.1 Students' Strategies to Solve Multiplication of Fraction with Whole Number

In solving multiplication of a whole number by a fraction, the students showed various strategies (see Figure 4.41). There are two items about multiplication of a whole number by a fraction. Since the students gave similar strategies to solve both items, then the researcher will presented their answer for the first item only. Their strategies presented in Figure 4.41 will be described respectively as follows.

14 out of 29 students used the algorithms of multiplication of a whole number by a fraction, they multiplied the whole number 3 by the numerator 2 and then put the denominator 5 as the denominator of the result. They then converted the result as mixed number. One student, Ira, after multiplied the fraction, also represent it as repeated addition. Four students converted the whole number in the form of fraction first and then used procedure for multiplying two fractions. Beside using repeated addition, Ida and Ana also used the same strategy as the four students mentioned before.

However, some students still messed up some procedures. Putri converted the whole number in the form of fraction first and then multiplied both numerator and denominator by 5, after that she simplified it. Perhaps, she tried to equate the denominator as when adding two fractions. Almost similar, perhaps Emi also want to do the same, but she made mistake when multiplying it, she did not multiplied the numerators.

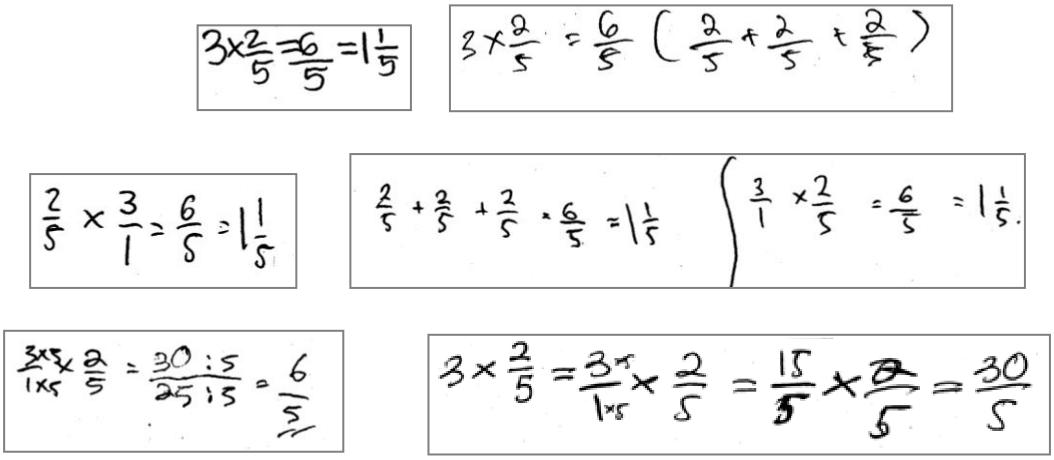


Figure 4.41 Students' Strategies to Solve $\mathbf{3}\times\frac{\mathbf{2}}{\mathbf{5}}$

Regarding to multiplication of a fraction by a whole number, the students also use the same strategy as what they have done when solving multiplication of a whole number by a fraction. Therefore, the researcher will not described it anymore.

4.4.3.2 Relating Algorithms to Situations

Regarding to finding suitable situation for multiplication of fraction with whole number, some students did not differentiate situation for multiplication of a while number by a fraction and a fraction by a whole number (some examples can be seen in Figure 4.42). They gave the similar type of situation for both multiplication problems. 8 of the 15 students gave situations consisting of repeated addition in it. Meanwhile, five students represent the problem by using drawing. They made rectangular shape that divided to some parts as many as, for example 3 times. The rest two students gave situations as a fraction of something. Based on the answers have described, the researcher concluded that for the students, the meaning of multiplication of a whole number by a fraction was the same with that of multiplication of a fraction by a whole number.

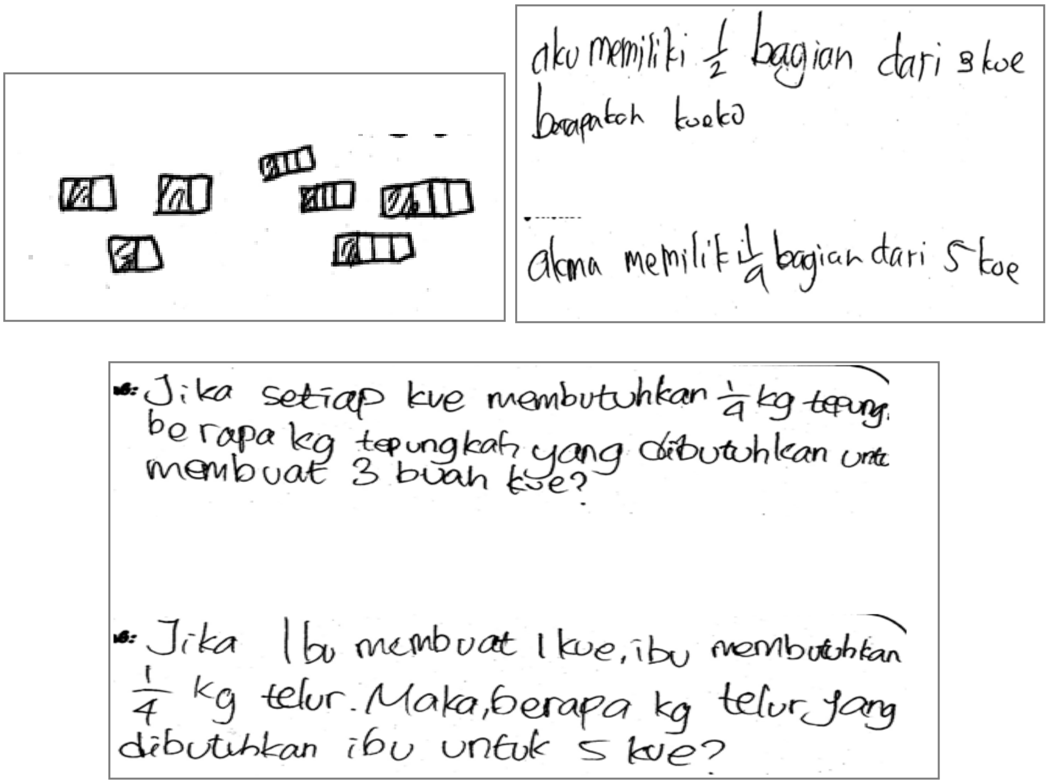


Figure 4.42 Some Students did not Differentiate Situations

4.4.3.3 Commutative Property of Multiplication of Fraction with Whole Number

In order to assess students' understanding of commutative property of multiplication of fraction with whole number, the students were given a situation about determining the length of ribbons (question 6 of Appendix H). Based on the answers, nine students used formal procedure to solve the problem. They multiplied the whole number 3 by the fraction $\frac{1}{4}$ and also multiplied the fraction $\frac{1}{4}$ by the whole number 3.

Eighteen students made drawing represented the problem in order to explain the answers (Figure 4.43). However, they did not use scale in drawing it. Perhaps they made drawing because it was asked in the questions. Therefore, actually from the pictures we could not prove that the lengths were the same. They prove it by using algorithms also.

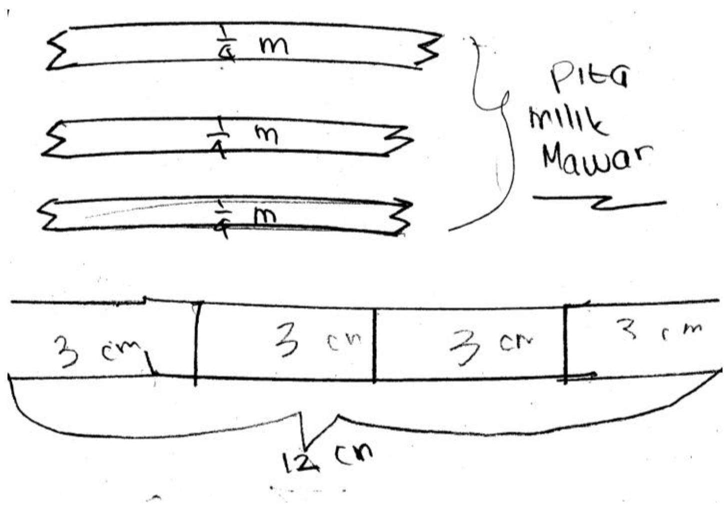


Figure 4.43 Students' Strategy to Show that Three times Quarter Meter is Similar to a Quarter of Three meter

From the answers, the researcher concluded that the students did not differentiate the meaning of multiplication of a whole number by a fraction and the meaning of multiplication of a fraction by a whole number. They could treat both multiplications as repeated addition.

4.5 Discussion: Contextual Situation as Starting Point

Supporting students who were already familiar with formal algorithms needed some adjustment in designing activities and problems so that they can extend their knowledge. The problems offered should be arranged that the students did not realize that it is multiplication of fraction problems. Therefore, they will not merely apply their formal knowledge.

Further, the researcher will discuss about implementation of RME in this design research. As stated in Subsection 2.3, rich and meaningful context as the first tenet of RME can be used as starting point in learning multiplication of fraction with whole number. However, working with students who accustomed to use formal procedure need some adjustment in choosing the situations to be used. The problems given should be organized such that the students did not realize it as multiplication of fraction with whole number. For example, since the students did not recognize that finding fraction of some object as multiplication of fraction by whole number, then the students did not use the formal procedure to solve it. Here, the role of teacher was very important. Another support that can be given to extend their understanding of multiplication of fraction with whole number is by asking them to make representation and to give reasons to prove their answer. The role of teacher to support students will be more elaborate in the following chapter.

CHAPTER V
CONCLUSIONS

In this chapter, the researcher will conclude the whole process of this research to answer the research questions and then pose a local instructional theory on learning multiplication of fraction with whole number. Further, this chapter will presented reflections and recommendations for further research in the field of multiplication of fraction with whole number.

5.1 Answer to the Research Questions

As in the first chapter of this research, there were two research questions. The first research question will be answered by relating the analysis of the pre-test of second cycle as described in Subsection 4.4.1 to students' strategies in solving problems in the teaching experiment phase as described in Subsection 5.4.2. Meanwhile, in order to answer the second research question, the researcher also referred to Subsection 4.4.2 until 4.4.3, but the researcher more focused on what kind of support given to extend students' understanding of each learning phase.

5.1.1 Answer to the First Research Question

The initial knowledge of students more or less affected their learning process in the teaching experiment. Students who were familiar with formal algorithms seemed to have a tendency to use the algorithms in solving problems. Once they know that the question is about multiplication of fraction, they will use the algorithms of multiplication of fraction to solve it. One of the cases can be seen in Subsection 4.4.2.2.2.

Further, since the fifth graders involved in this research already learned about multiplication of a fraction by a fraction, then they changed the whole number to be fraction so that they could use the rules of multiplication of a fraction by a fraction. The problems could appear then when the students made mistakes in converting the whole number or when they did not convert the whole number but directly multiplied the whole number with numerator and denominator (see Subsection 4.4.2.2.2)

Moreover, the knowledge of fraction division also gave influence in their strategy. The students, who associate ‘sharing’ with ‘division’, directly used the algorithms of fraction division when they were faced to activity of sharing some pieces of cake to some people (Subsection 4.4.2.2.1).

5.1.2 Answer to the Second Research Question

As stated in the beginning of Subsection 5.1, in the following the researcher will answer the second research question by focusing on support given in each phase of learning process.

1) Fractions are related to division and multiplication

Since the students are familiar with fair sharing context, then the researcher could use the context as starting point to learn about multiplication of fraction with whole number. The idea that fractions are related to division came when the students were faced to the context, for instance when they were asked to share one cake to a number of people. As the result, they will use unit fraction as

$\frac{1}{\text{the number of group members}}$. Regarding to sharing more than one cakes, the students also tend to relate it to division as $\frac{\text{the number of cakes}}{\text{the number of people}}$. However, as the student already proficient enough regarding to the idea that fraction are related to division, then they do not need much support.

Since the students already learned about *a fraction times a fraction*, then the researcher could use the knowledge to convey the idea of the relation between fraction and multiplication. By asking the students to determine fraction got if some part of cake divided to some people, the student will come *fraction of fraction*, which is actually multiplication of *a fraction by a fraction*.

One of the issues found in fair sharing activity was about the whole of fraction. Even though the students already knew about formal algorithm of multiplication of a fraction by a fraction, they need more support to notate fraction of fraction (see Subsection 4.4.2.2)

Further, asking students to find fractions of something can be used to elicit the idea that fractions are related to multiplication. As in the case of Ara and friends (Subsection 4.4.2.2), one of the supports could be given was by posing such question that provoke students to give reason about their answer. Since they already learned about equivalent fraction, then relating the problem to the pre-knowledge could help them to find the answer without directly used algorithm of multiplication of fraction by whole number. Then, once they recognize the word 'of' means multiplication, they will use the algorithms.

2) Repeated addition of fractions as multiplication of whole number by fraction

Since the idea of multiplication as repeated addition already emerged when learning about multiplication of whole numbers, then the students did not need much support to relate it to multiplication of whole number by fraction, as stated by Schwartz and Riedesel (1994) in Subsection 2.2. However, the use of rice to make *lontong* in the beginning of the meeting seems could draw students' enthusiasm to learn.

3) Inverse property of unit fractions

The researcher hope to elicit the idea of unit fractions by using colouring activity, where the students have to determine the number of yarn colour needed to make one meter of colourful knitting. Since the idea of this activity more or less similar to preparing number of menus activity, then the students also did not need much support in solving the problems. One small support might be given for students in this activity was by giving guidance (see vignette in Subsection 4.4.2.4) to make mathematical conclusion, such as 'when a unit fraction multiplied by its denominator, then the result will be one'.

4) Commutative property of multiplication of fraction with whole number

The students seems already knew that *multiplication of whole number by fraction* ($3 \times \frac{1}{4}$) had the same result as multiplication of *fraction by whole number* ($\frac{1}{4} \times 3$). The support needed was to guide students to understand how to find $3 \times \frac{1}{4}$ and $\frac{1}{4} \times 3$ in some ribbons (Subsection 4.4.2.5).

5.2 Local Instruction Theory for Extending the Meaning of Multiplication of fraction with Whole Number in Grade 5

Local instructional theory is defined as a theory that provides a description of the imaged learning route for a specific topic, a set of instructional activities, and means to support it (Gravemeijer, 2004 and Cobb et al, 2003 and Gravemeijer, 1994 in Wijaya, 2008). In educational practices, a local instruction theory offers teachers a framework of reference for designing and engaging students in a sequence of instructional activities for a specific topic and in this research is a topic about multiplication of fraction with whole number. Since the local instructional theory provides a complete plan for a specific topic, then a teacher could design an HLT by choosing instructional activities and adjusting them to the conjectured learning process of the students. Further, the local instructional theory for teaching and learning multiplication of fraction with whole number in grade 5 of elementary school was summarised in Table 5.1. Since the role of teacher is essential in this interactive process, then it will be discussed in subsection 5.2.1. Furthermore, the weaknesses of this research also will be described in 5.2.2.

Table 5.1 Local Instructional Theory for Multiplication of Fraction with Whole Number in Grade 5

Activity	Tool	Imagery	Potential Mathematics Discourse Topic
Sharing number of cakes to some people fairly	- 'Bolu' Cake, - Figure of Cake	- Signifies the partition of cake	- Fair sharing - Part of a whole - Fractions as quotient - Fraction related to division - Fraction related to multiplication
Dividing part of a cake to some people	Figure of a cake	Signifies the partition of part of cake	- Fair sharing - Part of a whole - Fraction of fraction - Multiplication of fraction by fraction
Finding fractions of some pieces of cake	Figure of a cake partitioned into ten pieces	Signifies the need of the way to elicit number of pieces	- Part of a whole - Equivalent fractions - Fractions as quotient - Fractions as operator - Multiplication of fraction by whole number
Measuring problems	_	Signifies the need of strategy to find a fraction of some length	- Fraction as measure - Multiplication of fraction by whole number
Preparing '*lontong*' and '*opor ayam*'	- Figure of a '*lontong*' - Rice, cup of rice, plastic	Signifies the iteration of fractions	- Repeated addition - Multiplication of whole number by fraction
Determining number of yarn colours needed to make colourful knitting	Colourful marker	Signifies the iteration of fractions	- Repeated addition - Inverse of unit fractions
Comparing the length of ribbons	- Blue ribbon with 30 cm of length - Three pieces of pink ribbon with 10 cm of length	Signifies the need of way to compare the length of ribbons	- Part of a whole - Repeated addition of fraction - Commutative property o multiplication of fraction with whole number

5.2.1 Class Discussion: Teacher's Role and Students' Social Interaction

Students' social interaction in the classroom was stressed in the fourth tenet of RME, *interactivity*. Working in small group enable students to discuss strategies used might scaffold students understanding. In the present research, the researcher observe that sometimes it was more effective when the students work in pair, so that the researcher can reduce the number of students that did not work.

As described in Subsection 2.3, teacher plays an important role in orchestrating social interaction to reach the objectives both for individual and social learning Cooke & Bochholz and Doorman & Gravemeijer (in Wijaya, 2008). Further, the roles of teacher in supporting social interaction found in this research will be described as follows.

In order to stimulate social interaction among the group member, one of the teacher's roles could be by dividing task for each member, when in the group only some students that worked. Further, the teacher could provoke students to discuss their idea so that all member of the group understand it.

Further, the learning process of students can be shortened by, for example, giving different question for each pair and then sharing the strategies in classroom discussion, where the range of social interaction can be broader. The roles of teacher in class discussion of this research could be by offering a chance for students to present their strategies to their friends. In order to stimulating social interaction among all component of classroom, the teacher could pose questions as *'Is it the same with Ale's answer?'* or *'Anyone can help them?'* or *'Before Dinda has said it. What is it?'* etcetera.

The most important goal of a class discussion is transforming students' concrete experiences into mathematical concept as mentioned by Cooke & Buchhlolz in Wijaya (2008). Further, the teacher should also asking for students' clarification in order to investigate students' reasoning about their idea or strategies that could reveal both students' difficulty and achievement in their learning process. For example by asking *"Is it just because the question about how meter the journey? Please give clear answer. Why it was multiplied?"*

5.2.2 The Weaknesses of the Research

During the research, the researcher found some weaknesses of this research. One of the weaknesses related to the preparation of the teaching experiment. Before conducting the teaching experiment, the researcher could not assess students' initial knowledge optimally. The initial HLT was designed to the students who have not learned about multiplication of a fraction by a fraction. Meanwhile, the students have learned about it already. Therefore, some of activities in the HLT have to be changed to be more suitable for them, by considering their actual knowledge.

Another weakness was related to learning styles of the students. In conducting this research, the researcher did not consider about the students' learning styles. Meanwhile, the learning styles might influence the learning trajectory of students. Students who have visual learning style might have different path to achieve the goal of learning process with them who have auditory or kinesthetic of learning style.

5.3 Reflection

The subjects of this research have already learned about multiplication of a fraction by a fraction in the school. They had knowledge about it that should be considered. The researcher really needed to study about students' initial knowledge by conducting proper pre-test. Further, the researcher also had to know how to use the students' initial knowledge to extend their understanding. It is very important to realize about the importance of contextual situation, models, and the influence of socio-mathematical norms to contribute to the students' learning process. Otherwise, the researcher will go back to the old paradigm – where the teacher transfers the knowledge to the students.

Based on the observation, the students accustomed to work in formal way; they tended to use procedural algorithm in solving multiplication of fraction with whole number problem. Since they tended to use the algorithm in solving problems, it seemed that some of the contextual situations did not function as the situations needed to be mathematized by some students. It seemed that the contextual situation only acted as the motivation of the learning process.

After having conducted this experiment, it was considered that the sequence of activities was too compact. Since the students have lack of understanding in the meaning of fractions itself, then some of mathematical ideas were not achieve as expected. Further, the time management should be one of the main considerations. As in the teaching experiment of this research, for some activities, the time was not enough, for example about commutative property that need longer time. It caused the students did not have enough chance to construct their understanding by having discussion among students, especially in the group discussions. Therefore, because of the limited time, the researcher tend to a little bit impatient when seeing students could not give appropriate answer, so that giving them guidance.

5.4 Recommendations

Based on the whole process of teaching multiplication of fraction with whole number, the researcher have some considerations to be recommended for further research in this topic.

One of the recommendations is about discussions in the learning process. The discussion itself can be separated into two, namely group discussion and class discussion. The number of students in one group should be considered carefully. The finding of this research, when the numbers of group member is quite big, then only few students were active in the group discussion. Therefore, one possible solution to this problem could be by making small group, for example two students in one group.

The teacher who was involved in this research is an experienced teacher who has been involved in *Pendidikan Matematika Realistik Indonesia* for long time. Therefore, she was good in conducting class discussion. One of the strategies she used was by asking students with different strategies to present their work in front of class. Since she did not blame students who gave incorrect approach, then the students feels free to share their ideas. Therefore, the researcher could adjust the learning process based on the students' understanding.

The last tenet of RME is about intertwinement. It will be better if the learning process of multiplication of fraction with whole number is intertwined with other topic, for example with the learning of percentages. Therefore, the time allocation could be more efficient and effective.

REFERENCES

Anderson, J., & Wong, M. (2007). Teaching common fractions in primary school: Teachers' Reactions to a New Curriculum. In P. L. Jeffery (Ed) *Proceedings of Australian Association for Research in Education 2006. Engaging Pedagogies* (Vol 1 pp. 1-13). Adelaide, (27-30 Nov 2006)

Armanto, Dian. (2002). Teaching Multiplication and Division in Realistically in Indonesian Primary Schools: A Prototype of Local Instructional Theory. University of Twente.

Bakker, Arthur. (2004). *Design Research in Statistic Education on Symbolizing and Computing Tools*. Utrecht: Freudenthal Institute.

Barnett, Carne; et al. (1994). *Fractions, Decimals, Ratios, and Percents: Hard to Teach and Hard to Learn?* Portsmouth, NH: Heinemann.

Charalombous, Charalombous Y. and Pitta-Pantazi, Demetra. (2005). Revisiting a Theoretical Model on Fractions: Implications for Teaching and Research. In Chick, H. L and Vincent, J.L. (Eds). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp.233-240). Melbourne, PME.

Copeland, Richard W. (1976). How Children Learn Mathematics: Teaching Implications of Piaget's Research. 2nd Edition. New York: MacMillan.

Depdiknas. (2006). *Kurikulum Tingkat Satuan Pendidikan Sekolah Dasar*. Jakarta: Depdiknas.

Fosnot, Catherine T. and Dolk, Maarten L. (2001). *Young Mathematicians at Work Constructing Multiplication and Division*. Portsmouth, NH: Heinemann.

Fosnot, Catherine T. and Dolk, Maarten L. (2002). *Young Mathematicians at Work Constructing Fractions, Decimals, and Percents*. Portsmouth, NH: Heinemann.

Freudenthal, Hans. (1983). *Didactical Phenomenology of Mathematical Structures.* Dordrecht: Reidel.

Gravemeijer, Koeno and Cobb, Paul. (2006). Design Research from a Learning Design Perspective. In Jan van den Akker, et.al. *Educational Design Research*. London: Routledge.

Hiebert, James and Carpenter, Thomas P. (1992). Learning and Teaching with Understanding. In Douglas A. Grouws (Ed). *Handbook of Research on MathematicsTeaching and Learning*, New York: MacMillan.

Hiebert, James et.al. (1997). *Making Sense Teaching and Learning Mathematics with Understanding*. Portsmouth, NH: Heinemann.

Keijzer, Ronald. (2003). *Teaching Formal Mathematics in Primary Education: Fraction Learning as Mathematising Process*. Utrecht: CD-β Press.

Kennedy, Leonard M. (1980). *Guiding Children to Mathematical Discovery*. Second Edition. Belmont, California: Wadsworth Publishing Company, Inc.

Ma, Liping. (1999). *Knowing and Teaching Elementary Mathematics: Teacher's Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

National Council of Teachers of Mathematics. 1991. *Professional Standards for Teaching Mathematics*. Reston, VA: National Council of Teacher of Mathematics.

National Council of Teachers of Mathematics. 1995. *Assessment Standards for School Mathematics*. Reston, VA: National Council of Teacher of Mathematics.

Reys, et.al. (2007). *Helping Children Learn Mathematics*. 8th Edition. New Jersey: John Wiley & Sons, Inc.

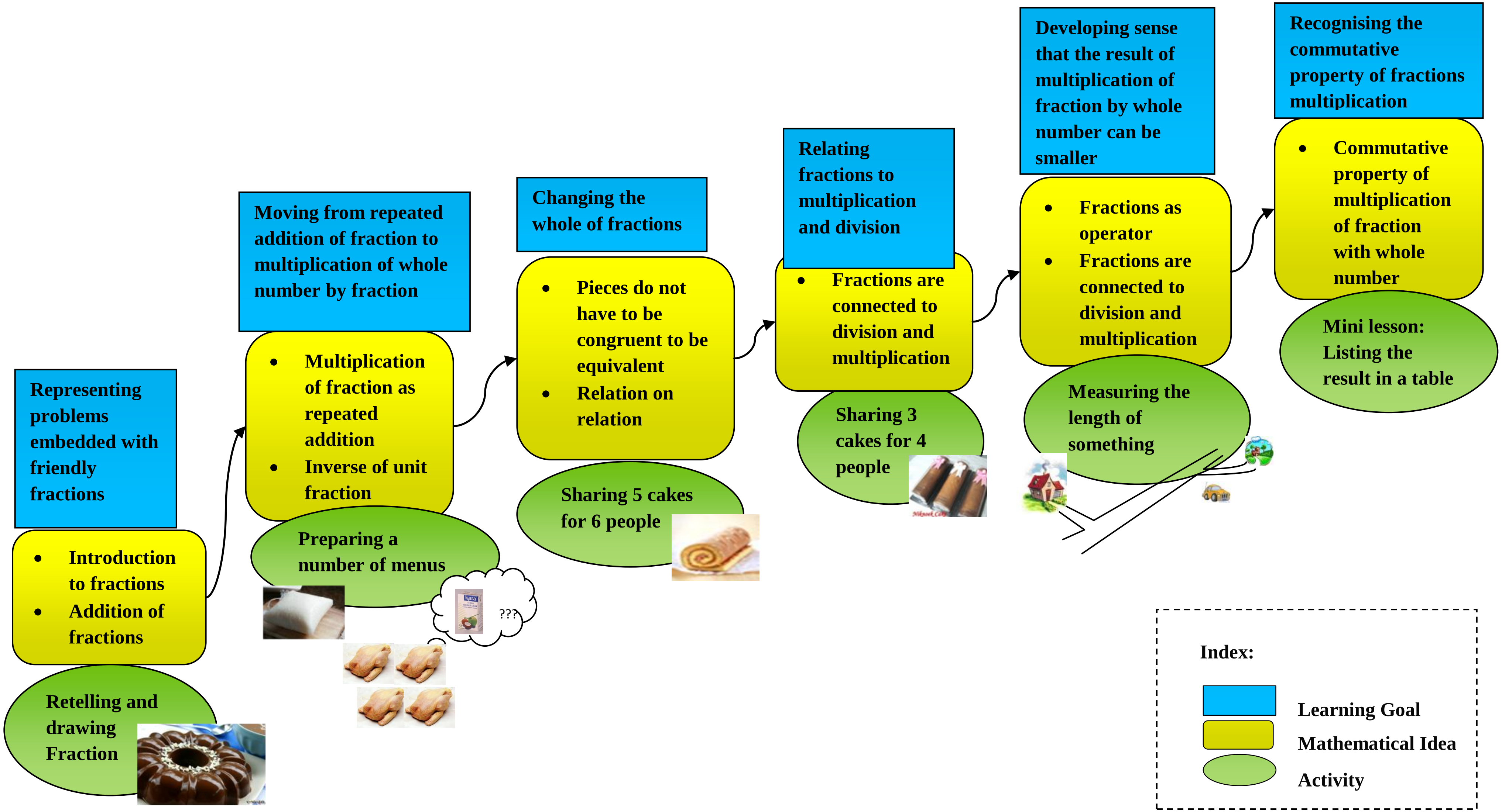
Schwartz, James E. and Riedesel, C. Alan. (1994). *Essentials of Classroom Teaching Elementary Mathematics*. Boston: Allyn and Bacon

TAL Team. (2007). *Fraction, Percentages, Decimal and Proportions*. Utrecht – The Netherlands.

Wijaya, A. (2008). *Design Research in Mathematics Education Indonesian Traditional Games as Preliminaries in Learning Measurement of Length*. Utrecht University

APPENDICES

Appendix A : Visualisation of Initial HLT



Appendix B : Refinements of initial HLT to HLT 1

No	Activity / Problem of Initial HLT	Refinement of Activity / Problem (HLT 1)	Rationale behind the Refinement
1.	**Representing problems embedded with friendly fractions** **Activity 1: Retelling and Drawing Benchmark Fractions** 1) *One day before Lebaran, Sinta visited her grandma's house, around eight kilometres from her house. Unfortunately, two kilometres from her grandma's house, the car got flat tire. Rewrite the situation with your own words and draw the position of the car when it got flat tire.* 2) *Sinta just came back from visiting her grandma's house. She got a cake to be share with her two siblings. Imagine how Sinta shares it with her siblings and then draw your imaginations on the paper.* 3) *a) One day, mother wants to make chocolate puddings. For one pudding, mother needs $\frac{1}{4}$ kg of sugar. Draw the situation.* 	**Representing problems embedded with friendly fractions** **Activity 1: Retelling and Drawing Benchmark Fractions** 1) *One day before Lebaran, Sinta visited her grandma's house, around eight kilometres from her house. Unfortunately, two kilometres from her grandma's house, the car got flat tire. Rewrite the situation with your own words and draw the position of the car when it got flat tire.* 2) *Sinta just came back from visiting her grandma's house. She got a cake to be share with her two siblings. Imagine how Sinta shares it with her siblings and then draw your imaginations on the paper.* 3) *a) One day, mother wants to make chocolate puddings. For one pudding, mother needs $\frac{1}{4}$ kg of sugar. Draw the situation.*	There were no changes to the problems.

No	Activity / Problem of Initial HLT	Refinement of Activity / Problem (HLT 1)	Rationale behind the Refinement
	b) *How if mother wants to make two Puddings?* *Draw it also.* 4) *Everyday teacher makes a brownies cake that cut into twenty-four pieces to be sold in a small canteen nearby teacher's house. Yesterday, quarter part of the cake was unsold. Write the situation in your own words and then make representation of the situation.* 	b) *How if mother wants to make two Puddings?* *Draw it also.* 4) *Everyday teacher makes a brownies cake that cut into twenty-four pieces to be sold in a small canteen nearby teacher's house. Yesterday, quarter part of the cake was unsold. Write the situation in your own words and then make representation of the situation.*	
2.	**Moving from repeated addition to multiplication** **Activity 2: Preparing a Number of Menus** - *Mother wants to make some "Lontong" as one of the menus in lebaran. For making one lontong, mother needs $\frac{1}{2}$ cup of rice. How many cups of rice needed if mother wants to make 6 lontong?* - *Beside lontong, teacher also wants to make opor ayam. For one chicken, teacher needs $\frac{3}{4}$ litre of coconut milk. If teacher has four*	**Moving from repeated addition to multiplication** **Activity 2: Preparing a Number of Menus** 1) A lontong needs $\frac{1}{2}$ *cup of rice. Prepare two" lontong" from the materials provided. How many cup of rice needed? Explain your answer.* 2) *If mother wants to make three "Lontong", how many cups of rice needed by mother? Explain your answer.*	The researcher added some questions to make it smoother. Rather than conducting a mini lesson to listing the result, students were asked to fill in the table on their worksheet.

<table>
<tr><th>No</th><th>Activity / Problem of Initial HLT</th><th>Refinement of Activity / Problem
(HLT 1)</th><th>Rationale behind the Refinement</th></tr>
<tr><td></td><td>chickens, how much coconut milk that she need?

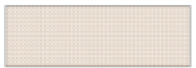
- Mini lesson: listing the result in a table</td><td>3) How many cups of rice needed if mother wants to make 6 lontong? Explain your answer.

4) Beside lontong, mother also wants to make “opor ayam”. For one chicken, mother needs $\frac{3}{4}$ litre of coconut milk. How much coconut milk needed if mother wants to make “opor” from 4 chicken? Explain your answer.


5) How if mother wants to cook “opor” from 5 and 8 chicken? How many litres of coconut milk needed?

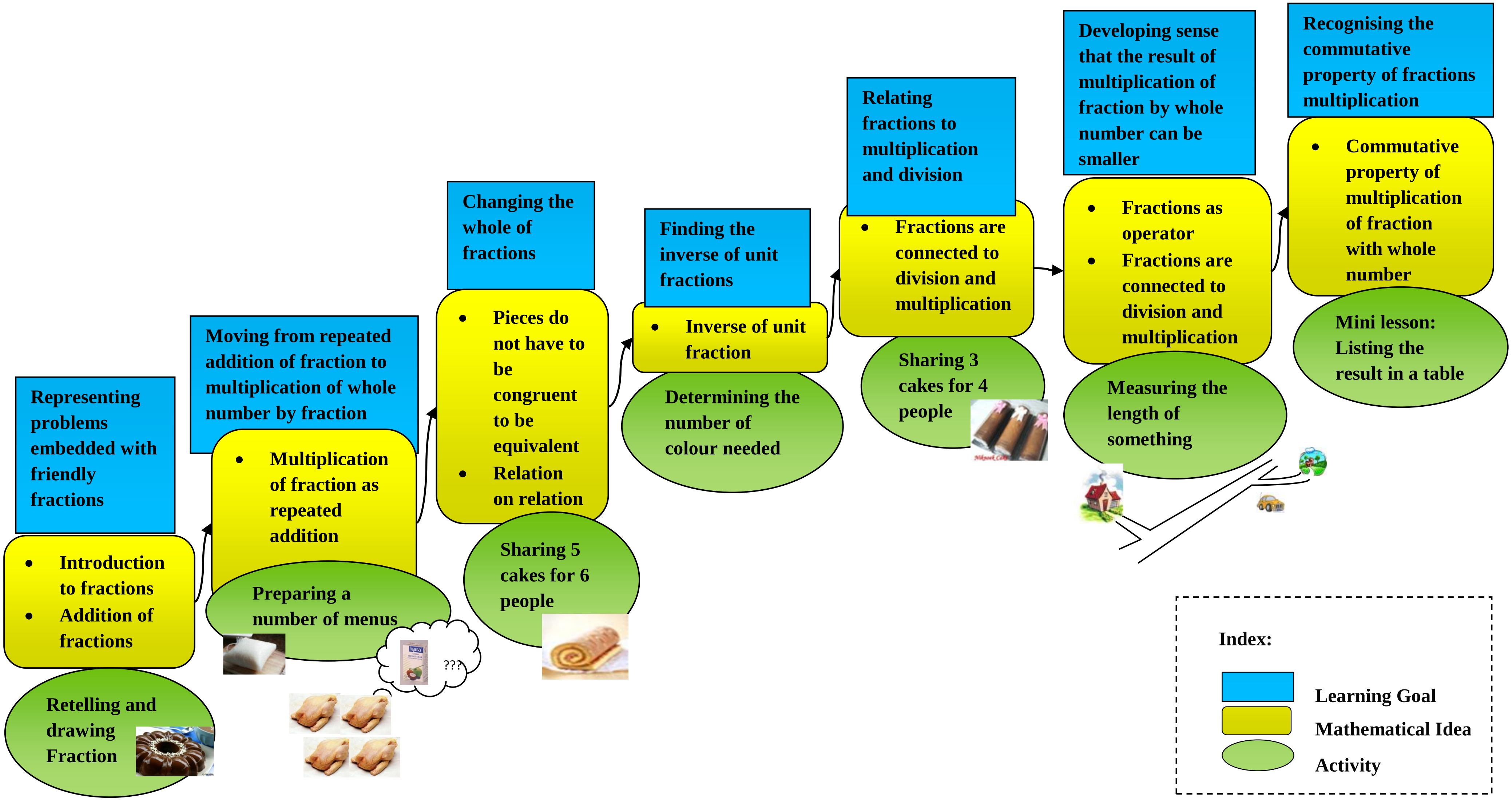
6) Fill in the following table
<table>
<tr><th>Number of 'Lontong'</th><th>Rice Needed</th></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>10</td><td></td></tr>
<tr><td>15</td><td></td></tr>
</table>
<table>
<tr><th>Number of Chicken</th><th>Coconut Milk Needed</th></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>10</td><td></td></tr>
</table></td><td></td></tr>
</table>

No	Activity / Problem of Initial HLT	Refinement of Activity / Problem (HLT 1)	Rationale behind the Refinement
		Finding the inverse of unit fractions **Activity 3: Determining the number of colour needed** 1) a) *As a task, Indah wants to make colourful knitting. She changed the colour of the yarn every half metre. If she already knitted as long as one metre, how many colours have she used? Explain your answer by using picture.* b) *Dian also made colourful knitting but she changed the colour every quarter metre. How many colours have she used if the knitting already one metre long?* *2) Pak Sabar has a coconut garden. He wants to paint one side of the fence. If one can of painting only cover* $\frac{1}{10}$ *km of the fence, how many cans that has to be bought by Pak Sabar? Explain your answer by using picture.*	In the designing phase, the researcher intended to draw the idea of inverse of unit fractions from *preparing menus* activity. However, since in the previous meeting the students tend to directly use algorithm to solve the activity, then the researcher added one more activity about determining the number of colour needed to make colourful things. From this activity, the researcher expected students come to idea that when unit fractions multiplied by its denominator, the result will be 1

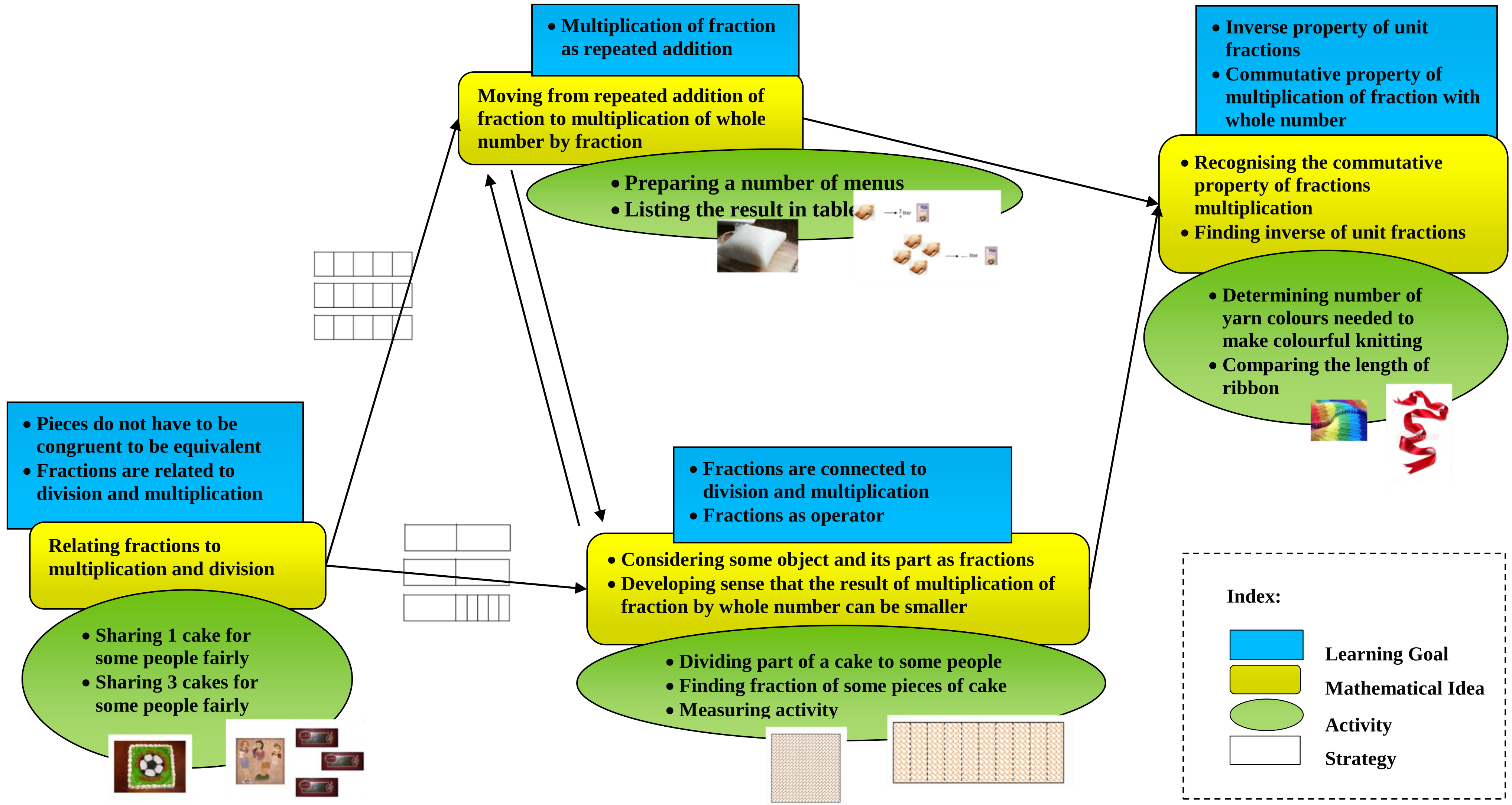
No	Activity / Problem of Initial HLT	Refinement of Activity / Problem (HLT 1)	Rationale behind the Refinement
3.	**Changing the whole of fractions** **Activity 3: Sharing Five Cakes to Six People** 1) *"Yesterday, Aunty gave 5 "bolu gulung" to Saskia. Can you help Saskia to divide it fairly for 6 people? How much parts of Bolu each person get?"*	**Changing the whole of fractions** **Activity 4: Sharing Five Cakes to Six People** 1) *In the holiday, Mamat went to his aunt's house in Jakarta. From there, he got 5 steamed brownies. How you divide the five brownies to 6 people? Give the way you divide it.*	The researcher changed the editorial of the question. Meanwhile, the number stayed the same. Further, the researcher added an instruction to explain the answer because, based on the previous activity, the students would not give explanation if they were not asked to do it.
5.	**Relating fractions to multiplication and division** **Activity 4: Sharing Three "Bolu" for Four People** 2) *"How to share Bolu Gulung to four people, if you only have three Bolu? How much Bolu each person will get?"* 3) "*If you only have $\frac{3}{4}$ Bolu, how you share it with three people? How much Bolu each person will get?*"	**Relating fractions to multiplication and division** **Activity 5: Sharing Three Cakes to Four People** 2) *How you divide three cakes to four people? Make drawing of your way as much as possible. How much cakes got by each person?* 3) *Show $\frac{3}{4}$ parts of cake below.*  *How you divide the $\frac{3}{4}$ parts for 3 people? How much parts got by each person?*	The researcher made instruction to explain the answer more explicit.

No	Activity / Problem of Initial HLT	Refinement of Activity / Problem (HLT 1)	Rationale behind the Refinement
6.	**Developing sense that in multiplying fraction by whole number, the result can be smaller** **Activity 5: Measuring Activity** *"One day before Lebaran, Sinta visited her grandma's house, around eight kilometres from her house. Unfortunately, after three-fourth of the trip, the car got flat tire. Can you figure out in what kilometre the car got flat tire?* 	**Developing sense that in multiplying fraction by whole number, the result can be smaller** **Activity 6: Measuring Activity** *Every Lebaran, Intan always visits her grandma's house that 8 km far away from her house. In this Lebaran, she also wants to visit her grandma. Unfortunately, after* $\frac{3}{4}$ *of the journey, the car got flat tire. Draw the position of Intan's car when the tire flatted. What is the distance between the flatted car and her grandma's house? Explain your answer.*	The researcher changed the editorial of the question to be more explicit and clear.

Appendix C : Visualisation of HLT 1



Appendix D : Visualisation of HLT 2



Appendix E : Rencana Pelaksanaan Pembelajaran (Lesson Plan) of Second Cycle

1. Lesson Plan for the First Meeting

Rencana Pelaksanaan Pembelajaran
(RPP – 1)

Sekolah	:	SD Laboratorium Unesa
Mata Pelajaran	:	Matematika
Materi Ajar / Aspek	:	Perkalian Pecahan dengan Bilangan Bulat
Kelas / Semester	:	V / II
Alokasi Waktu	:	4 Jam Pelajaran (4 × 35 menit)

A. Standar Kompetensi

5. Menggunakan pecahan dalam pemecahan masalah

B. Kompetensi Dasar

5.1 Mengalikan dan membagi berbagai bentuk pecahan

C. Materi Pembelajaran

Pengenalan pecahan

Sebelum mempelajari tentang perkalian pecahan, siswa diajak untuk mengingat kembali tentang pengenalan pecahan. Beberapa hal yang akan dipelajari pada pertemuan kali ini adalah:

- **Potongan tidak harus sama bentuk untuk menjadi adil**
  Dalam pembagian secara adil, potongan tidak harus mempunyai bentuk yang sama untuk dikatakan adil

- **Hubungan antara pecahan dengan perkalian dan pembagian**
  Pecahan itu dapat diperoleh dari proses pembagian dan perkalian. Contohnya tiga perempat itu adalah *tiga dibagi empat* atau *tiga kali dari hasil satu dibagi empat*

D. Indikator

- Membagi satu buah kue menjadi beberapa bagian secara adil
- Membagi tiga buah kue menjadi beberapa bagian secara adil
- Menghubungkan pecahan dengan perkalian dan pembagian

E. Tujuan Pembelajaran

- Siswa dapat membagi sejumlah kue menjadi beberapa bagian secara adil
- Siswa dapat menghubungkan pecahan dengan perkalian dan pembagian

F. Metode Pembelajaran

Tanya jawab, diskusi, tugas kelompok

G. Alat / Bahan / Sumber Belajar

- Kurikulum Tingkat Satuan Pendidikan (KTSP)
- LKS – 1
- Kue bolu
- Pisau
- Spidol

H. Langkah-langkah Kegiatan Pembelajaran

1. Kegiatan Pendahuluan (± 10 menit)

- Guru mengkondisikan kelas dan membagi siswa ke dalam kelompok-kelompok beranggotakan 2, 3 dan 4 orang.
- Guru meminta beberapa orang siswa untuk menceritakan tentang pengalaman mereka ketika berbagi kue.

2. Kegiatan Inti (± 120 menit)

Membagi satu buah kue menjadi beberapa bagian secara adil (±20 menit)

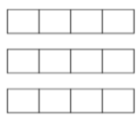
- Guru menunjukkan satu kue bolu di depan kelas.
- Guru meminta beberapa orang siswa ke depan kelas untuk memotong kue tersebut menjadi empat bagian yang sama besar dengan cara yang berbeda-beda.
- Siswa diminta untuk menyelesaikan pertanyaan no.1 pada LKS–1, yaitu untuk menggambarkan berbagai macam cara untuk membagi satu kue kepada setiap anggota kelompoknya. Siswa juga diminta untuk menyatakan berapa bagian yang diterima oleh masing-masing anggota kelompoknya.

Diskusi Kelas (±10 menit)

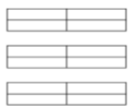
- Dengan bimbingan guru, siswa membandingkan dan mendiskusikan hasil pekerjaan kelompok mereka di depan kelas. Siswa diarahkan untuk melihat dan menyadari bahwa potongan kue tidak harus mempunyai bentuk yang sama untuk dikatakan adil.

Membagi tiga buah kue menjadi beberapa bagian secara adil (±60 menit)

- Siswa diminta menyelesaikan permasalahan selanjutnya yang terdapat pada LKS–1: *"Kemarin, Ani mendapat oleh-oleh tiga buah brownies dari neneknya. Ia ingin membaginya bersama tiga orang temannya. Bantulah Ani untuk membagi ketiga brownies tersebut untuk empat orang secara adil. Jelaskan bagaimana caramu membagi kue-kue tersebut. Berapa bagiankah yang diterima oleh masing-masing anak? Jelaskan jawabanmu."*
- Harapannya, siswa dapat memberikan berbagai macam cara untuk membagi ketiga buah brownies tersebut untuk empat orang. Kemungkinannya, siswa akan membagi kue tersebut seperti pada gambar di bawah ini.



atau



- Siswa kemudian diminta untuk berdiskusi dengan anggota kelompoknya untuk menyelesaikan permasalahan-permasalahan lain yang ada di LKS-1, yaitu untuk membagi tiga buah brownies kepada lima orang anak.
- Jika strategi yang dipergunakan oleh semua siswa sama, maka guru dapat memberi pancingan agar siswa memberikan berbagai strategi yang berbeda (sebagai contoh adalah gambar di atas) kepada siswa

Diskusi Kelas (±30 menit)

- Dengan bimbingan guru, siswa diminta ke depan kelas untuk mempresentasikan dan mendiskusikan bagaimana cara mereka membagi ketiga kue tersebut kepada empat orang anak dan bagaimana cara mereka menyatakan berapa bagian kue yang didapatkan oleh masing-masing anak.
- Begitu pula dengan cara membagi ketiga kue tersebut dibagi kepada lima orang anak.

3. **Kegiatan Penutup (± 10 menit)**
   - Siswa membuat kesimpulan tentang apa saja yang telah mereka pelajari pada pertemuan kali ini. Salah satunya adalah bahwa pecahan merupakan hasil pembagian dan juga berhubungan dengan perkalian

I. Penilaian

- Pengamatan terhadap keaktifan siswa di kelas (diskusi kelompok dan diskusi kelas)
- Pengamatan terhadap pemahaman materi yang dicapai siswa
- Hasil kerja siswa

	Surabaya, 2 Maret 2011
Mengetahui,	Guru Mata Pelajaran
Kepala Sekolah	
………………………..................	………………………...................
NIP / NPP :	NIP :

2. Lesson Plan for the Second Meeting

Rencana Pelaksanaan Pembelajaran
(RPP – 2)

Sekolah	:	SD Laboratorium Unesa
Mata Pelajaran	:	Matematika
Materi Ajar / Aspek	:	- Perkalian pecahan dengan pecahan - Perkalian pecahan dengan bilangan bulat
Kelas / Semester	:	V / II
Alokasi Waktu	:	3 Jam Pelajaran (3 × 35 menit)

A. Standar Kompetensi

5. Menggunakan pecahan dalam pemecahan masalah

B. Kompetensi Dasar

5.1 Mengalikan dan membagi berbagai bentuk pecahan

C. Materi Pembelajaran

- Perkalian pecahan dengan pecahan
- Perkalian pecahan dengan bilangan bulat

Hal yang akan dipelajari pada pertemuan kali ini antara lain adalah:

- **Pecahan sebagai operator**
  Pada pecahan, aspek operator itu sangat penting, karena pecahan sudah menunjukkan aspek ini sejak dari pertama. Aspek operator berarti bahwa pecahan itu sendiri merupakan hasil dari penggabungan perkalian dan pembagian dalam satu operasi (misalnya $\frac{2}{3}$ dari 12 berarti 2 dikali dengan 12 dan kemudian dibagi dengan 3)

D. Indikator

- Menghubungkan pecahan dengan perkalian dan pembagian
- Menentukan suatu bagian jika pecahannya diketahui

E. Tujuan Pembelajaran

- Siswa dapat memahami bahwa pecahan merupakan bagian dari suatu kesatuan, bahwa pecahan itu sangat tergantung pada satuannya
- Siswa dapat memahami pecahan dengan satuan yang berbeda
- Siswa dapat memahami makna dari perkalian pecahan dengan bilangan bulat
- Siswa dapat memahami bahwa hasil dari perkalian pecahan dengan bilangan bulat dapat berupa bilangan yang lebih kecil

F. Metode Pembelajaran

Tanya jawab, diskusi, tugas kelompok

G. Alat / Bahan / Sumber Belajar

- Kurikulum Tingkat Satuan Pendidikan (KTSP)
- LKS – 2 yang terbagi menjadi LKS – 2a, LKS – 2b, dan LKS – 2c
- Gambar strategi pembagian kue
- Spidol

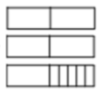
H. Langkah-langkah Kegiatan Pembelajaran

1. Kegiatan Pendahuluan (± 5 menit)

- Guru mengkondisikan kelas dan membagi siswa menjadi beberapa kelompok beranggotakan 2-3 siswa
- Siswa diminta untuk mengingat kembali tentang materi yang sudah dipelajarinya pada hari sebelumnya

2. Kegiatan Inti (± 90 menit)

- Siswa diminta untuk menjelaskan kembali tentang bagian yang diperoleh oleh masing-masing anak, jika ada tiga buah kue untuk lima orang anak. *"Apakah kalian masih ingat dengan permasalahan kemarin? Jika ada tiga kue yang dibagi kepada lima orang anak, berapa bagiankah yang diterima oleh masing-masing anak? Bagaimanakah cara kalian membaginya?"*
- Kemudian, guru menunjukkan gambar berikut dan membimbing siswa untuk berdiskusi tentang bagian yang diperoleh oleh masing-masing anak. *"Jika Ibu membagi ketiga kue tersebut seperti pada gambar ini, berapa bagiankah yang diterima oleh masing-masing anak?"*



- Siswa mungkin memberi jawaban bahwa masing-masing anak akan menerima *2 potong kue* atau $\frac{1}{2}$ *bagian kue ditambah* $\frac{1}{5}$.
- Guru kemudian mengarahkan diskusi ke potongan kue yang kecil-kecil, bahwa satu bagiannya merupakan *seperlima bagian dari setengah kue* atau *sepersepuluh bagian dari satu kue*. Sehingga masing-masing anak tersebut mendapatkan $\frac{6}{10}$ atau $\frac{3}{5}$ bagian kue.
- Siswa kemudian diminta untuk bekerja dengan anggota kelompoknya untuk menyelesaikan soal nomor 1-3 pada LKS-2, yaitu menentukan berapa bagian yang diperoleh oleh beberapa orang anak jika kuenya hanya ada sebagian saja.

  Setiap kelompok mendapatkan angka dan pecahan yang berbeda.
  Contohnya, kelompok yang mendapatkan LKS–2a diminta menentukan berapa bagian yang diterima oleh masing-masing anak, jika seperempat bagian kue dibagi untuk tiga anak.

  Siswa mungkin menyatakan bahwa setiap anak mendapatkan $\frac{1}{3}$ bagian kue. Guru dapat menstimulasi siswa untuk menyadari bahwa sebenarnya setiap anak mendapatkan $\frac{1}{3}$ *bagian dari* $\frac{1}{4}$ *kue*, atau sama saja dengan $\frac{1}{12}$.

  Siswa kemudian diminta untuk menyelesaikan soal nomor 3, dimana mereka harus menentukan berapa bagian yang didapatkan oleh masing-masing anak jika satu kue dibagi untuk 12 orang anak. Kemungkinan besar siswa akan menyatakan bahwa setiap anak mendapatkan $\frac{1}{12}$ bagian kue.

  Begitu pula untuk kelompok-kelompok lain yang mendapatkan LKS–2b dan LKS–2c.

- Siswa kemudian dibimbing untuk membuat kesimpulan secara klasikal dan kemudian diminta untuk menuliskan kesimpulan yang diperoleh tersebut pada tempat yang telah disediakan (LKS–2, soal no.4)
  Kesimpulan yang diharapkan adalah siswa menyatakan bahwa pecahan dari suatu pecahan itu merupakan perkalian pecahan dengan pecahan, sebagai contoh, $\frac{1}{3}$ *dari* $\frac{1}{4}$ *kue* sama saja dengan $\frac{1}{3} \times \frac{1}{4}$, yaitu $\frac{1}{12}$.

- Bersama anggota kelompoknya, siswa diminta untuk menyelesaikan permasalahan-permasalahan lain (soal nomor 5 – 7) yang ada di LKS-2
  Diharapkan siswa menyadari bahwa kata *'dari'* itu berarti perkalian.
- Beberapa siswa diminta untuk mempresentasikan dan mendiskusikan jawabannya di depan kelas.

3. Kegiatan Penutup (± 10 menit)

- Siswa membuat kesimpulan tentang apa saja yang telah mereka pelajari pada pertemuan kali ini. Salah satunya adalah suatu bagian dari suatu kesatuan merupakan perkalian pecahan dengan bilangan bulat, dan bahwa pecahan itu sangat bergantung pada satuannya.

I. Penilaian

- Pengamatan terhadap keaktifan siswa di kelas (diskusi kelompok dan diskusi kelas)
- Pengamatan terhaadp pemahaman materi yang dicapai siswa
- Hasil kerja siswa

Surabaya, 4 Maret 2011

Mengetahui, Kepala Sekolah	Guru Mata Pelajaran
………………………..................	………………………..................
NIP / NPP :	NIP :

3. Lesson Plan for the Third Meeting

Rencana Pelaksanaan Pembelajaran
(RPP 3)

Sekolah	:	SD Laboratorium Unesa
Mata Pelajaran	:	Matematika
Materi Ajar / Aspek	:	Perkalian Pecahan dengan Bilangan Bulat
Kelas / Semester	:	V / II
Alokasi Waktu	:	2 Jam Pelajaran (2 × 35 menit)

A. Standar Kompetensi

5. Menggunakan pecahan dalam pemecahan masalah

B. Kompetensi Dasar

5.1 Mengalikan dan membagi berbagai bentuk pecahan

C. Materi Pembelajaran

Perkalian bilangan bulat dengan pecahan

Catatan :

Penjumlahan pecahan yang berulang dapat juga dinyatakan sebagai perkalian bilangan bulat dengan pecahan

D. Indikator

- Menentukan banyaknya beras yang dibutuhkan untuk membuat beberapa buah lontong
- Menentukan banyaknya santan kelapa yang dibutuhkan untuk membuat opor ayam dari beberapa ekor ayam
- Menggunakan perkalian bilangan bulat dengan pecahan sebagai pengganti penjumlahan pecahan berulang

E. Tujuan Pembelajaran

- Siswa dapat memahami bahwa penjumlahan pecahan yang berulang merupakan perkalian antara bilangan bulat dengan pecahan

F. Metode Pembelajaran

Tanya jawab, diskusi, tugas kelompok

G. Alat / Bahan / Sumber Belajar

- Kurikulum Tingkat Satuan Pendidikan (KTSP)
- LKS – 3
- Gambar lontong
- Beras
- Cangkir beras
- Plastik
- Staples
- Spidol

H. Langkah-langkah Kegiatan Pembelajaran

1. **Kegiatan Pendahuluan (± 5 menit)**
   - Guru mengkondisikan kelas dan membagi siswa menjadi beberapa kelompok beranggotakan 2-3 siswa

- Guru menceritakan pengalamannya ketika membuat lontong untuk disajikan pada arisan keluarga dan menunjukkan gambar lontong yang sudah jadi
- Guru kemudian menunjukkan plastik berisi beras yang kalau dimasak akan menjadi lontong dan mengatakan bahwa setiap plastik membutuhkan $\frac{1}{2}$ cangkir beras

2. **Kegiatan Inti (± 60 menit)**
   - Beberapa orang siswa diminta maju ke depan untuk mempersiapkan empat buah lontong dengan cara mengisi setiap plastik dengan $\frac{1}{2}$ cangkir beras
   - Siswa kemudian diminta untuk menghitung berapa cangkir beras yang dibutuhkannya untuk membuat empat buah lontong tersebut dan menuliskannya pada tabel yang telah disediakan di papan tulis

Jumlah lontong	Beras yang dibutuhkan	Cara yang digunakan
1	$\frac{1}{2}$ cangkir	
2		
3		
4		
5		
6		
10		
15		

Jumlah ayam	Santan kelapa yang dibutuhkan	Cara yang digunakan
1	$\frac{3}{4}$ liter	
2		
5		
8		
13		

   - Siswa kemudian diminta untuk menyelesaikan permasalahan-permasalahan lain yang ada di LKS–3
   - Beberapa siswa diminta untuk melengkapi tabel yang ada di papan tulis berdasarkan jawaban mereka
   - Guru membimbing diskusi sehingga siswa mengerti bahwa penjumlahan pecahan yang berulang merupakan perkalian antara bilangan bulat dengan pecahan.
     Misalnya membahas banyaknya beras yang dibutuhkan untuk membuat 7 buah lontong adalah $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{7}{2}$
     Untuk memunculkan $7\times\frac{1}{2}$, guru dapat menanyakan *"Ada berapa kalikah $\frac{1}{2}$ nya?"*

3. **Kegiatan Penutup (± 5 menit)**
   - Siswa membuat kesimpulan tentang apa saja yang telah mereka pelajari pada pertemuan kali ini. Salah satunya adalah bahwa penjumlahan berulang dari pecahan itu dapat juga dinyatakan sebagai perkalian bilangan bulat dengan pecahan.

I. Penilaian

- Pengamatan terhadap keaktifan siswa di kelas (diskusi kelompok dan diskusi kelas)
- Pengamatan terhadap pemahaman materi yang dicapai siswa
- Hasil kerja siswa

Surabaya, 7 Maret 2011

Mengetahui,
Kepala Sekolah

Guru Mata Pelajaran

…………………………................

NIP / NPP :

…………………………................

NIP :

4. Lesson Plan for the Fourth Meeting

Rencana Pelaksanaan Pembelajaran
(RPP–4)

Sekolah	:	SD Laboratorium Unesa
Mata Pelajaran	:	Matematika
Materi Ajar / Aspek	:	Perkalian Pecahan dengan Bilangan Bulat
Kelas / Semester	:	V / II
Alokasi Waktu	:	3 Jam Pelajaran (3 × 35 menit)

A. Standar Kompetensi

5. Menggunakan pecahan dalam pemecahan masalah

B. Kompetensi Dasar

5.1 Mengalikan dan membagi berbagai bentuk pecahan

C. Materi Pembelajaran

- Invers dari pecahan yang mempunyai angka 1 sebagai pembilangnya
- Sifat komutatif dari perkalian pecahan dengan bilangan bulat

Catatan:

- Invers perkalian dari pecahan satuan adalah penyebutnya sendiri. Invers dari $\frac{1}{a}$ adalah a karena $\left(\frac{1}{a} \times a = 1\right)$
- Perkalian pecahan dengan bilangan bulat mempunyai sifat komutatif $\left(a \times \frac{b}{c} = \frac{b}{c} \times a\right)$

D. Indikator

a) Menentukan panjang rajutan warna-warni, jika setiap $\frac{1}{2}$ meter warna benangnya diganti dengan warna berbeda dan belum pernah dipakai sebelumnya, dan warna yang sudah dipakai adalah 2 jenis.
b) Menentukan panjang rajutan warna-warni, jika setiap $\frac{1}{4}$ meter warna benangnya diganti dengan warna berbeda dan belum pernah dipakai sebelumnya, dan warna yang sudah dipakai adalah 4 jenis.
c) Menentukan banyaknya warna benang yang dibutuhkan untuk membuat hiasan sepanjang satu meter, jika setiap $\frac{1}{5}$ meter, pitanya diganti dengan warna yang berbeda
d) Menggunakan pita yang disediakan untuk membuktikan bahwa $3 \times \frac{1}{4} = \frac{1}{4} \times 3$

E. Tujuan Pembelajaran

- Siswa dapat memahami sifat komutatif pada perkalian pecahan dengan bilangan bulat
- Siswa dapat menentukan invers dari suatu pecahan satuan

F. Metode Pembelajaran

Tanya jawab, diskusi, tugas kelompok

G. Alat / Bahan / Sumber Belajar

- Kurikulum Tingkat Satuan Pendidikan (KTSP)
- LKS–4
- Spidol warna-warni
- Pita merah sepanjang 1 m

- Pita biru sepanjang 3 m
- Lem (double tape)

H. Langkah-langkah Kegiatan Pembelajaran

Kegiatan Pendahuluan (± 5 menit)

- Guru mengkondisikan kelas dan membagi siswa menjadi beberapa kelompok beranggotakan 2-3 siswa
- Siswa diminta untuk mengingat kembali tentang apa yang sudah dipelajarinya pada pertemuan sebelumnya

Kegiatan Inti (± 95 menit)

- Guru memberikan soal-soal yang mirip dengan soal-soal sebelumnya

  Untuk membuat satu kue brownies, Ibu membutuhkan $\frac{1}{5}$ *kg gula pasir. Jika Ibu ingin membuat 3 kue brownies, berapa banyakkah gula yang dibutuhkan oleh Ibu?*
- Guru meminta siswa untuk menyelesaikannya di papan tulis.
- Siswa diminta untuk berdiskusi dengan anggota kelompoknya untuk menyelesaikan soal nomor 1-3 pada LKS–4
- Beberapa kelompok mempresentasikan jawabannya ke depan kelas
- Guru membimbing diskusi sehingga siswa menyimpulkan dan memahami tentang invers dari suatu pecahan yang mempunyai angka 1 sebagai pembilang, misalnya invers dari $\frac{1}{4}$ adalah 4, dan lain-lain
- Harapannya, siswa dapat menyimpulkan bahwa jika suatu pecahan yang mempunyai pembilang satu dikalikan dengan penyebutnya, maka hasilnya adalah 1.
- Siswa diminta untuk menuliskan kesimpulan tersebut pada soal nomor 4 di LKS–4
- Siswa kemudian mengerjakan aktifitas selanjutnya yang ada di LKS–4, yaitu menentukan panjang pita yang dibutuhkan oleh Ibu untuk membuat hiasan pada bajunya
- Selanjutnya, siswa diminta untuk menentukan berapa panjang pita yang dibeli oleh Bu Anto untuk ketiga anaknya, jika setiap anak mendapatkan pita sepanjang seperempat meter.
- Siswa kemudian diminta untuk membandingkan hasil dari kedua aktifitas tersebut untuk menjawab soal nomor 7 pada LKS–4
- Siswa membuat kesimpulan yang diperolehnya dari soal nomor 5, 6, dan 7 pada LKS–4
- Beberapa kelompok mempresentasikan jawabannya di depan kelas
- Guru membimbing diskusi sehingga siswa dapat melihat hubungan antara $\frac{1}{4}$ *meter sebanyak 3 kali* $\left(3 \times \frac{1}{4}\right)$ dengan $\frac{1}{4}$ *dari 3 meter pita* $\left(\frac{1}{4} \times 3\right)$, yaitu $\left(3 \times \frac{1}{4}\right) = \left(\frac{1}{4} \times 3\right)$

Kegiatan Penutup (± 5 menit)

- Siswa membuat kesipmulan tentang apa saja yang sudah dipelajarinya pada pertemuan kali ini, yaitu tentang invers dari pecahan satuan dan juga tentang sifat komutatif dari perkalian pecahan dengan bilangan bulat

I. Penilaian

- Pengamatan terhadap keaktifan siswa di kelas (diskusi kelompok dan diskusi kelas)
- Pengamatan terhadap pemahaman materi yang dicapai siswa
- Hasil kerja siswa

	Surabaya, 9 Maret 2011
Mengetahui,	Guru Mata Pelajaran
Kepala Sekolah	
………………………..................	………………………..................
NIP / NPP :	NIP :

Appendix F : Questions in Pre Test

PRE-TEST

Nama : ....

Kelas : ....

1. Jawablah pertanyaan-pertanyaan berikut

   a. $\frac{2}{3} + \frac{2}{3}$

   Jawab:

   b. $\frac{3}{5} + \frac{3}{5} + \frac{3}{5}$

   Jawab:

   c. $\frac{4}{5} + \frac{3}{4}$

   Jawab:

   d. $\frac{3}{5} + \frac{2}{7}$

   Jawab:

e. $\frac{2}{8} + \frac{1}{4}$

Jawab:

f. $3 \times \frac{1}{4}$

Jawab:

g. $5 \times \frac{2}{3}$

Jawab:

h. $\frac{1}{4} \times 3$

Jawab:

i. $\frac{2}{5} \times 3$

Jawab:

2. Hubungkanlah situasi yang diberikan di sebelah kiri dengan cara membuat garis ke kartu yang cocok di sebelah kanan. Kamu boleh membuat lebih dari satu garis.

1) Tiga buah kue bolu dibagi untuk empat orang anak

2) Ada tiga orang anak. Masing-masing anak menngambil seperempat bagian kue.

3) Rumah Budi berjarak 4 km dari rumah Ani. Ani ingin mengunjungi rumah Budi dan sekarang dia sudah melewati seperempat jalan.

4) Ibu ingin membuat tiga buah kue bolu. Setiap kue membutuhkan $\frac{1}{4}$ kg

a) $3 \times \frac{1}{4}$

b) $\frac{3}{4}$

c) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

d) $3 : 4$

e) $\frac{1}{4} \times 4$

d) 3×4

3. Tulislah beberapa situasi dimana kamu dapat menemukan:

a. $\frac{2}{3}$

Jawab:

b. $\frac{1}{5}$

Jawab:

c. $\frac{1}{4} + \frac{1}{4}$

Jawab:

d. $\frac{1}{2} + \frac{1}{4}$

Jawab:

e. $3 \times \frac{1}{2}$

Jawab:

f. $\frac{1}{4} \times 5$

Jawab:

4. Satu kardus air mineral 'Aira' berisi 48 gelas 'Aira'. Berapa gelaskah yang dimiliki oleh Mita jika ia membeli $\frac{1}{2}$ kardus 'Aira'? Jelaskan jawabanmu.

Jawab:

5. Harga jeruk di pasar adalah Rp10.000,00 per kilogram. Jika Anna ingin membeli $\frac{1}{2}$ kilogram jeruk, berapa rupiahkah yang harus dia bayarkan? Jelaskan jawabanmu.

Jawab:

6. a) Sebagai tugas prakarya, Indah ingin membuat rajutan warna-warni. Dia mengganti warna benangnya tiap setengah meter. Jika ia sudah merajut sepanjang satu meter, berapa macam warna yang telah dia gunakan? Jelaskan jawabanmu dengan menggunakan gambar.

Jawab:

b) Dian juga membuat rajutan. Tapi, ia mengganti warnanya tiap seperempat meter. Berapa warna yang telah dia gunakan jika rajutannya sepanjang satu meter?

Jawab:

7. Apakah $6 \times \frac{1}{2}$ sama dengan $\frac{1}{2} \times 6$? Jelaskan jawabanmu dengan mengunakan contoh dalam kehidupan sehari-hari.

Jawab:

Appendix G : Lembar Kerja Siswa (Worksheet)

1. Worksheet for Fair Sharing Activity

LEMBAR KERJA SISWA
(LKS – 1)

Kelompok :
Kelas :
Nama : 1.
2.
3.
4.
5.
6.

1. a) Bagaimanakah caramu membagi kue berikut kepada setiap anggota kelompokmu? Gambarkanlah caramu tersebut. Berapa bagiankah yang diperoleh oleh masing-masing anggota kelompokmu? Jelaskan jawabanmu.

Jawab :

b) Apakah ada cara lain untuk membagi kue tersebut kepada masing-masing anggota kelompokmu? Gambarkan berbagai macam cara membagi kue tersebut kepada setiap anggota kelompokmu

Jawab :

c) Perhatikan jawabanmu pada soal 1b. Berapa bagiankah yang diterima oleh masing-masing anggota kelompokmu? Apakah sama dengan yang diperoleh pada soal nomor 1a? Jelaskan jawabanmu.

Jawab :

2. Kemarin, Ani mendapat oleh-oleh tiga buah brownies dari neneknya. Ia ingin membaginya bersama tiga orang temannya.

Bantulah Ani untuk membagi ketiga brownies tersebut untuk empat orang secara adil. Jelaskan bagaimana caramu membagi kue-kue tersebut. Berapa bagiankah yang diterima oleh masing-masing anak? Gambarkan dan jelaskan jawabanmu.

Jawab :

3. Bagaimanakah jika ketiga kue tersebut akan dibagi untuk lima orang? Jelaskan caramu membagi kue-kue tersebut. Berapa bagiankah yang akan diterima oleh masing-masing anak? Jelaskan jawabanmu.

Jawab :

4. Apakah yang dapat kamu simpulkan dari permasalahan-permasahan di atas?

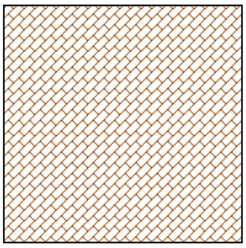
Jawab :

2. Worksheet for Finding Fractions of Something

LEMBAR KERJA SISWA
(LKS – 2a)

Kelompok :
Kelas :
Nama : 1.
2.
3.
4.

1. Kemarin, Ibu membuat satu buah kue Black Forest seperti pada gambar di bawah ini.



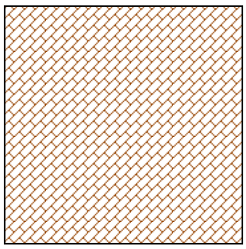
Hari ini, kue tersebut hanya tinggal seperempat bagian saja.
Gambarkanlah keadaan kue kemarin dan hari ini.

Jawab :

2. Hari ini Ibu ingin membagi kue yang masih tersisa tersebut untuk tiga anaknya, berapa bagiankah yang diterima oleh masing-masing anak jika dibandingkan dengan satu kue? Jelaskan jawabanmu.

Jawab :

3. Besok Ibu berencana untuk mengundang 12 anak panti asuhan dan Ibu ingin menyajikan kue Black Forest seperti yang Ibu buat kemarin.



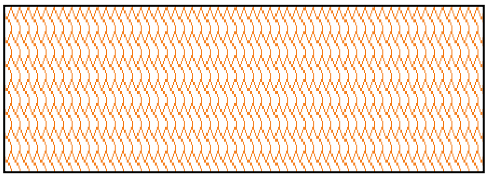
Gambarkanlah caramu membagi kue tersebut kepada 12 orang anak. Berapa bagiankah yang diterima oleh setiap anak?

Jawab :

4. Tuliskan kesimpulan yang kamu peroleh dari diskusi kelas.

Jawab :

5. Bu Anto membuat satu buah kue yang diiris menjadi 10 potong.



a) Jika $\frac{1}{5}$ bagian dari kue tersebut telah habis dimakan, berapa potong kue kah yang telah dimakan tersebut? Jelaskan jawabanmu.

Jawab :

6. Apakah yang dapat kamu simpulkan dari jawaban pada soal nomor 4 dan 5? Apakah hubungan antara $\frac{1}{5}$ dari sepuluh potong kue dengan $\frac{2}{5}$ dari sepuluh potong kue?

Jawab :

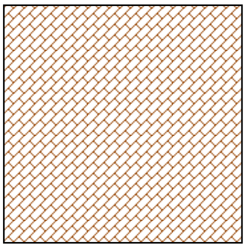
7. Amin diminta untuk membeli bahan kue di toko 'Mantap' yang jaraknya 150 m dari rumahnya. Setelah dua per tiga perjalanan dilaluinya, Amin bertemu dengan pamannya yang bersedia mengantarkannya ke toko 'Mantap'. Berapa meterkah perjalanan Amin dan pamannya? Jelaskan dan gambarkan caramu.

Jawab :

LEMBAR KERJA SISWA
(LKS – 2b)

Kelompok :
Kelas :
Nama : 1.
2.
3.
4.

1. Kemarin, Ibu membuat satu buah kue Black Forest seperti pada gambar di bawah ini.



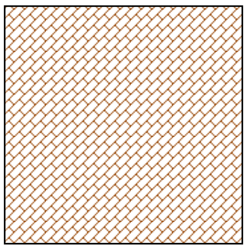
Hari ini, kue tersebut hanya tinggal setengah bagian saja.
Gambarkanlah keadaan kue kemarin dan hari ini.

Jawab :

2. Hari ini Ibu ingin membagi kue yang masih tersisa tersebut untuk empat anaknya, berapa bagiankah yang diterima oleh masing-masing anak jika dibandingkan dengan satu kue? Jelaskan jawabanmu.

Jawab :

3. Besok Ibu berencana untuk mengundang 8 anak panti asuhan dan Ibu ingin menyajikan kue Black Forest seperti yang Ibu buat kemarin.



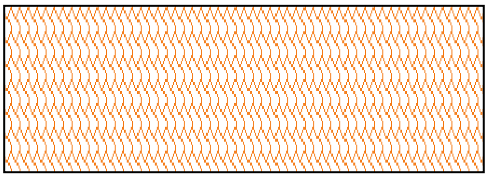
Gambarkanlah caramu membagi kue tersebut kepada 8 orang anak. Berapa bagiankah yang diterima oleh setiap anak?

Jawab :

4. Tuliskan kesimpulan yang kamu peroleh dari diskusi kelas.

Jawab :

5. Bu Anto membuat satu buah kue yang diiris menjadi 10 potong.



a) Jika $\frac{1}{5}$ bagian dari kue tersebut telah habis dimakan, berapa potong kue kah yang telah dimakan tersebut? Jelaskan jawabanmu.

Jawab :

6. Apakah yang dapat kamu simpulkan dari jawaban pada soal nomor 4 dan 5? Apakah hubungan antara $\frac{1}{5}$ dari sepuluh potong kue dengan $\frac{2}{5}$ dari sepuluh potong kue?

Jawab :

7. Amin diminta untuk membeli bahan kue di toko 'Mantap' yang jaraknya 150 m dari rumahnya. Setelah dua per tiga perjalanan dilaluinya, Amin bertemu dengan pamannya yang bersedia mengantarkannya ke toko 'Mantap'. Berapa meterkah perjalanan Amin dan pamannya? Jelaskan dan gambarkan caramu.

Jawab :

LEMBAR KERJA SISWA
(LKS – 2c)

Kelompok :

Kelas :

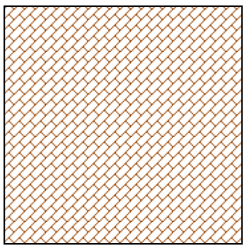
Nama : 1.

2.

3.

4.

1. Kemarin, Ibu membuat satu buah kue Black Forest seperti pada gambar di bawah ini.



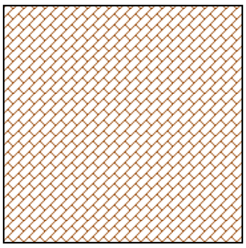
Hari ini, kue tersebut hanya tinggal sepertiga bagian saja.
Gambarkanlah keadaan kue kemarin dan hari ini.

Jawab :

2. Hari ini Ibu ingin membagi kue yang masih tersisa tersebut untuk lima anaknya, berapa bagiankah yang diterima oleh masing-masing anak jika dibandingkan dengan satu kue? Jelaskan jawabanmu.

Jawab :

3. Besok Ibu berencana untuk mengundang 15 anak panti asuhan dan Ibu ingin menyajikan kue Black Forest seperti yang Ibu buat kemarin.



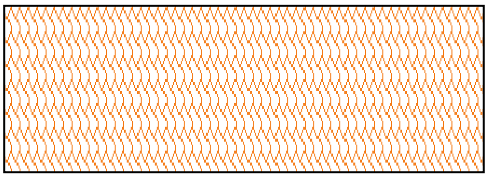
Gambarkanlah caramu membagi kue tersebut kepada 15 orang anak. Berapa bagiankah yang diterima oleh setiap anak?

Jawab :

4. Tuliskan kesimpulan yang kamu peroleh dari diskusi kelas.

Jawab :

5. Bu Anto membuat satu buah kue yang diiris menjadi 10 potong.



a) Jika $\frac{1}{5}$ bagian dari kue tersebut telah habis dimakan, berapa potong kue kah yang telah dimakan tersebut? Jelaskan jawabanmu.

Jawab :

6. Apakah yang dapat kamu simpulkan dari jawaban pada soal nomor 4 dan 5? Apakah hubungan antara $\frac{1}{5}$ dari sepuluh potong kue dengan $\frac{2}{5}$ dari sepuluh potong kue?

Jawab :

7. Amin diminta untuk membeli bahan kue di toko ‘Mantap’ yang jaraknya 150 m dari rumahnya. Setelah dua per tiga perjalanan dilaluinya, Amin bertemu dengan pamannya yang bersedia mengantarkannya ke toko ‘Mantap’. Berapa meterkah perjalanan Amin dan pamannya? Jelaskan dan gambarkan caramu.

Jawab :

3. **Worksheet for Preparing Number of Menus Activity**

LEMBAR KERJA SISWA
(LKS –3)

Hari / Tgl :
Kelompok :
Kelas :
Nama : 1.
2.
3.
4.

1. Satu buah lontong membutuhkan $\frac{1}{2}$ cangkir beras.

   Berapa cangkir beraskah yang dibutuhkan untuk membuat 2 buah lontong? Bagaimanakah caramu menghitungnya? Jelaskanlah jawabanmu.

Jawab :

2. Jika Ibu ingin membuat tiga buah lontong, berapa cangkir beraskah yang dibutuhkan oleh Ibu? Bagaimanakah caramu menghitungnya? Jelaskan jawabanmu.

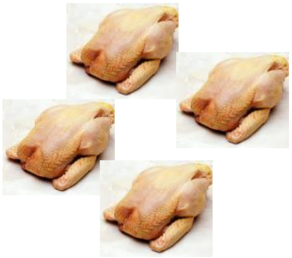
Jawab :

3. Bagaimana jika Ibu ingin membuat 6 buah lontong? Berapa banyakkah beras yang diperlukan? Jelaskan bagaimana caramu menentukan banyaknya beras yang dibutuhkan.

Jawab :

4. Selain membuat lontong, Ibu juga ingin membuat Opor Ayam. Untuk satu ekor ayam, Ibu membutuhkan $\frac{3}{4}$ liter santan kelapa. Berapa banyak santan yang dibutuhkan oleh Ibu jika ia ingin membuat Opor dari 4 ekor ayam? Jelaskan jawabanmu.

$\longrightarrow \frac{3}{4}$ liter



$\longrightarrow$ ….. liter

Jawab :

5. Bagaimanakah jika Ibu ingin memasak opor dari 5 ekor ayam? Berapa liter santankah yang dibutuhkan? Jelaskan. Bagaimana pula jika beliau ingin memasak 8 ekor ayam?

Jawab :

6. Lengkapilah tabel berikut

Jumlah lontong	Beras yang dibutuhkan	Cara yang kamu gunakan
1	$\frac{1}{2}$ cangkir	
2		
3		
4		
5		
6		
10		
15		

Jumlah ayam	Santan kelapa yang dibutuhkan	Cara yang kamu gunakan
1	$\frac{3}{4}$ liter	
2		
5		
8		
13		

7. Apakah yang dapat kamu simpulkan dari tabel pada soal no. 6 di atas?

Jawab :

4. Worksheet for Determining Number of Colour Needed and Comparing the Length of Ribbons Activity

LEMBAR KERJA SISWA
(LKS –4)

Hari / Tgl :

Kelompok :

Kelas :

Nama : 1.

2.

3.

4.

1. Untuk tugas keterampilan, Adek membuat rajutan warna-warni. Setiap $\frac{1}{2}$ meter, ia mengganti warna benangnya dengan warna yang belum pernah dia gunakan sebelumnya. Jika ia sudah menggunakan 2 warna yang berbeda, berapakah panjang rajutan Adek sekarang? Gambarkan dan jelaskan jawabanmu.

Jawab :

2. Ima juga membuat rajutan warna-warni. Setiap $\frac{1}{4}$ meter, ia mengganti warna benangnya dengan warna yang belum pernah digunakannya. Jika ia sudah menggunakan 4 warna yang berbeda, berapa meterkah panjang rajutan Ima sekarang? Gambarkan dan jelaskan jawabanmu.

Jawab :

3. Perhatikan jawabanmu pada soal nomor 1 dan 2 di atas. Apa yang dapat kamu simpulkan?

Jawab :

4. Sebagai tugas prakarya, Indah ingin membuat rajutan warna-warni. Setiap $\frac{1}{5}$ meter, ia mengganti warna benangnya dengan warna yang belum pernah dipakai sebelumnya. Jika ia sudah merajut sepanjang satu meter, berapa jenis warna yang telah dia gunakan? Jelaskan jawabanmu dengan menggunakan gambar.

Jawab :

5. Bu Anto mempunyai pita sepanjang 3 meter. Ia menggunakan $\frac{1}{4}$ dari pita tersebut untuk hiasan pada bajunya.
Dengan mengunakan pita biru yang diberikan oleh gurumu, tunjukkanlah berapa panjang pita yang dibutuhkan oleh Bu Anto.
Berapakah panjang pita tersebut? Jelaskan jawabanmu.

Jawab :

6. Sepulang dari pasar kemarin, Bu Anto membelikan pita untuk tiga orang anaknya. Masing-masing anaknya mendapatkan pita sepanjang $\frac{1}{4}$ meter.
   Dengan menggunakan pita merah yang diberikan oleh gurumu, tunjukkanlah seberapa panjang pita yang dibeli oleh Bu Anto untuk ketiga anaknya tersebut.

Jawab :

7. Apakah panjang pita yang dibutuhkan oleh Bu Anto pada soal nomor 5 sama dengan panjang pita yang dibutuhkannya pada soal nomor 6? Jelaskan bagaimana cara kamu membuktikannya.

Jawab :

Appendix H : Questions in Post Test

POST-TEST

(Waktu : 70 menit)

Nama : ....

Kelas : ....

Hari / Tgl. : ....

1. Jawablah pertanyaan-pertanyaan berikut

   a. $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$

   Jawab:

   b. $\frac{4}{5}+\frac{3}{4}$

   Jawab:

   c. $\frac{3}{5}+\frac{2}{7}$

   Jawab:

d. $3 \times \frac{2}{5}$

Jawab:

e. $5 \times \frac{2}{3}$

Jawab:

f. $\frac{2}{5} \times 3$

Jawab:

g. $\frac{1}{4} \times 5$

Jawab:

2. Hubungkanlah situasi yang diberikan di sebelah kiri dengan cara membuat garis ke kartu yang cocok di sebelah kanan. Kamu boleh membuat lebih dari satu garis.

1) Tiga buah kue bolu dibagi untuk empat orang anak. Berapa bagiankah yang akan diperoleh oleh masing-masing anak?

2) Ada tiga orang anak. Masing-masing anak mengambil seperempat bagian kue. Berapa banyakkah kue yang dimakan oleh ketiga anak tersebut?

3) Ani ingin mengunjungi rumah Budi yang berjarak 3 km. Sekarang dia sudah melewati seperempat jalan. Berapa kilometerkah jarak yang sudah dilalui oleh Ani?

4) Jika setiap bolu membutuhkan seperempat kilogram gula, berapa kilogram gulakah yang dibutuhkan Ibu untuk membuat tiga buah bolu?

a) $3 \times \frac{1}{4}$

b) $\frac{3}{4}$

c) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

d) $3 : 4$

e) $\frac{1}{4} \times 3$

f) $3 - \frac{1}{4}$

3. Tulislah beberapa permasalahan dalam kehidupan sehari-hari di mana kamu dapat menemukan pecahan berikut.

a. $\frac{3}{5}$

Jawab:

b. $\frac{2}{3}$

Jawab:

c. $\frac{1}{4} + \frac{1}{4}$

Jawab:

d. $\frac{1}{2} + \frac{1}{4}$

Jawab:

e. $3 \times \frac{1}{2}$

Jawab:

f. $\frac{1}{4} \times 5$

Jawab:

4. Ibu membutuhkan waktu $\frac{3}{4}$ jam untuk mengukus satu kue bolu. Berapa jamkah yang dibutuhkan oleh Ibu untuk mengukus tiga kue bolu? Jelaskan jawabanmu.

Jawab:

5. Di pasar, keripik ketela dijual seharga Rp24.000,00 per kilogram. Jika Anna ingin membeli $\frac{1}{4}$ kilogram keripik, berapa rupiahkah yang harus dia bayar? Jelaskan jawabanmu.

Jawab:

6. a) Mawar mempunyai 3 helai pita, masing-masing sepanjang $\frac{1}{4}$ meter

b) Melati mendapatkan $\frac{1}{4}$ bagian dari pita yang panjangnya 3 meter

Apakah pita yang dimiliki oleh Mawar sama panjangnya dengan pita yang dimiliki oleh Melati? Jelaskan jawabanmu dengan menggunakan gambar.

Jawab:

7. Sebagai tugas prakarya, Indah ingin membuat rajutan warna-warni. Setiap $\frac{1}{5}$ meter, ia mengganti warna benangnya dengan warna yang belum pernah dipakai sebelumnya. Jika ia sudah merajut sepanjang satu meter, berapa jenis warna yang telah dia gunakan? Jelaskan jawabanmu dengan menggunakan gambar di bawah.

Jawab:

8. Bagilah kedua kue di bawah ini untuk 3 orang anak sehingga setiap anak mendapatkan bagian yang sama. Tunjukkan caramu membaginya dengan membuat garis-garis potongan pada gambar kue yang telah disediakan di bawah ini.
Berapa bagiankah yang didapatkan oleh setiap anak? Jelaskan jawabanmu.

Jawab: