DESIGN RESEARCH IN GEOMETRY EDUCATION
DEVELOPING SPATIAL ABILITIES IN FIRST GRADE CHILDREN

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Freudenthal Institute
Utrecht University – 2009

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Supervisors

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P R E F A C E

I have been involved in primary mathematics education since I was placed as a junior lecturer in 2005 in the Education of Elementary School Teachers, Surabaya State University, Indonesia. Before, I did a lot work related with mathematics education in higher levels, either as a lecturer in the Mathematics Department or as a private teacher for students in the elementary schools, junior high schools, and senior high schools. During my experience, I had a lot of questions about the use of mathematics in daily life. I found students' difficulties in doing mathematics in higher level. I had been looking for answers of their questions, until one day I realized myself that every difficulty, missing concept and struggling that students faced is started from the basic when they learnt mathematics in the primary school.

I was getting more concerned with mathematics in primary education to think about solution of big problems in mathematics education in Indonesia. I finally found the answer when I discussed with Maarten Dolk in an interview day for my scholarship. The idea of Realistic Mathematics Education (RME), at that time, was totally a new thing for me. However, the basic principles of RME seem to shed a light to get better future for mathematics education in Indonesia and, more importantly, to break all doubts and questions arise. As a mathematics educator, I am fond of to be involved in manner and try to dive to the depth of concepts of RME to be applied in my country. This thesis is one of my starting points to put into real those desires.

Upon the completion of this thesis, my first and greatest thanks to Allah SWT, the Al Mighty of my life that always gives me chances for learning in every single time in my life. I would like to thank to my parents; mom and dad whose prays always follow my step and my success, and for all my brothers and sister: Dedy, Irwan, and Dian that always support my decisions.

The great thankfulness is also addressed to Jaap den Hertog, my supervisor that has given me a lot of help and encouragement to finish my thesis. It is also for Prof. Dr. Ahmad Fauzan, my Indonesian supervisor that helped me a lot during my research and gathering data in Indonesia. Thanks for all your suggestions, your encouragements, and your helps.

For gathering the data, this thesis also involves a great cooperation with SD. At Taqwa in Surabaya. Therefore, I would like to thank for big chances conducting the research to the school principal, Ustadzah Layyin. Every day during the research in school, the teacher and her assistant were always enthusiastic in discussing children's struggles, their achievement, and the following day topics. For them, Rika and Rifa, I really thank for your effort and your time.

I will not forget to say thanks to the PMRI team, especially for Prof. Sembiring and Dr. Siti M. Amin, that gave me the opportunity to study in the Netherlands. Best regards are also for all my six Indonesian friends; Domesia, Meli, Pipit, Lyna, Ariyadi, and Al, for my best colleagues; Suryanti, Dr. Abadi, Susanah, and Dr. Tatag to be the great supporters, and for all staff of Freudenthal Institute. For others that also give me prays, their supports, and their friendly suggestions, although I cannot mention their names one by one, my thankfulness are for them as well.

Last but not least, as a learner, I realize that learning is a never ending process. I do realize that this thesis is not a perfect thing as a scientific literature in mathematics education. However, it is my big hope to give my little experience through this thesis as a reference for the movement of mathematics education, for my beloved country, Indonesia.

Utrecht, 2009

Neni Mariana

A B S T R A C T

This research aims to support the growing process of first graders' spatial abilities especially in orienting and constructing; and explain children's spatial reasoning and levels within orienting and manipulating constructions. During the research period, we found the answers of the research questions; How can first grade children develop their ability in orienting and constructing? What kind of spatial reasoning do first graders use in orienting? Last, How do Van Hiele's levels constitute the development of progression in manipulating constructions? We designed sequential activities of orienting and constructing. The activities of orienting consecutively were: localizing school buildings and classrooms by considering direction between them, relative distances among them, and proportion of buildings' sizes; describing routes both into words and visualizing lines of movements; and examining school buildings' pictures from different point of views to get awareness of the vision lines and different shapes of a building depending on different points of views. The activities of constructing consecutively were: building with wooden cubes to manipulate 4-cube constructions by rotating or mirroring; examining two constructions for reasoning the difference using rotation and mirroring; and drawing basements of constructions and figures to more mentally visualize the shape relations between squares and cubes. The class experiment shows that those sequential activities of orienting could develop children's ability in spatial reasoning. Their reasoning gradually improved from using helps of reality, gestures, representations or drawings, until linguistic representations. The results of constructing show that children in our class experiment gradually shift their levels of thinking in manipulating constructions. We found that there is a shift in Van Hiele's levels. Children are not just at "a" single level if they are doing constructing.

Key words: *Orienting, Constructing, Spatial Reasoning, Van Hiele's Levels of Thinking.*

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1. Introduction

The Indonesian National Curriculum (2006) mostly describes that the basic competencies of geometry in elementary school are about recognizing 2D and 3D shapes and their properties. In the Indonesian classroom situation, children are usually working with shapes, classifying various shapes, and talking about their properties. They are busy drawing geometrical figures, such as triangle, square, cubes, etc, and discussing their properties. Meanwhile, the teacher directly shows them various figures, pictures, or things of geometrical shapes to be explained and discussed in the classroom. This situation makes geometry lacking evidence in their daily life (Fielker, 1979). Children cannot see the relation between the geometry they learn and their environment.

However, the main idea of geometry is about grasping space, the space in which the child lives, breathes, and moves (Freudenthal, in NCTM, 1989). For instance, we can ask children to describe a certain route from their classroom to the library. When they explain it for instance by moving their hands, using direction words, or making a drawing with path line(s), it means that they geometrically conceive ideas of the environment of their school. On the other hand, it also means that they perform their ability in grasping the external world, which is called a spatial ability (Freudenthal, in NCTM, 1989; van Nes & de Lange, 2007).

Many researchers have found correlations between spatial ability and geometry (for example: Casey et al., 2008; Clements & Battista, 1992; Melancon, 1994; Tartre, 1990; Tracy, 1987). They found that children who are able to visualize and have good spatial abilities have more capability in solving geometry. On the other hand, geometry plays an important role for children in developing their logical thinking and their spatial abilities (Fielker, 1979; Tartre, 1990; Tracy, 1987). Those researches show that achievement in geometry is closely related with the level of spatial ability.

In order to contribute to early geometry education in elementary school, this study offers an instructional design of activities for developing spatial abilities. The development is perceived first from the first grade of elementary school children. However, there are different

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[1] A master thesis. Supervised by: Jaap den Hertog (Freudenthal Institute – Utrecht University) and Prof. Dr. Ahmad Fauzan (Medan State University, Indonesia)

[2] A master student of Research and Development in Science Education, Freudenthal Institute, Utrecht University; and a junior lecturer in the Education of Elementary School Teachers, Surabaya State University, Indonesia.

sorts of spatial abilities, and this research focuses on orienting to develop spatial orientation and constructing to develop spatial visualization. Spatial orientation and visualization are essential parts of spatial abilities (Tartre, 1990). We developed those abilities within orienting and constructing, which are important aspects of early geometry (Treffers et al., 1989). For that purpose, we developed a learning trajectory of sequential activities. Therefore, the design activities are divided into two big parts. First, children do sequential activities of orienting to develop their spatial orientation, followed by activities of constructing to develop their spatial visualization.

Freudenthal (1991) suggests that we can use contextual situations to give more meaningful activities in the mathematical classroom. The contextual situation is didactically chosen from phenomena related to the concepts. In the design process of orienting, the context behind concepts leads the development of children's spatial reasoning. Meanwhile, another context is created for a sequence of constructing, and Van Hiele's theory (1986) underpins the progression of abilities in manipulating objects.

The analysis of this study describes changes of children's thinking process within the activities and it builds up a *Local Instructional Theory* (Gravemeijer, 2004) that offers a framework of such activities in developing spatial abilities and spatial reasoning of the first graders.

Hence, to develop the local instructional theory, the aims of this research are:

(1) to support the growing process of first graders' spatial abilities especially in orienting and constructing
(2) to explain children's spatial reasoning and levels within orienting and manipulating constructions

To support the growing process of first graders' spatial abilities, the research tries to answer the following question:

How can first grade children develop their ability in orienting and constructing?

Then, for the second aim, the research tries to answer these questions:

a. What kind of spatial reasoning do first graders use in orienting?
b. How do Van Hiele's levels constitute the development of progression in manipulating constructions?

2. Theoretical Framework

In this chapter, we will give a theoretical framework that underlies this research. It provides the definition of each essential term and defines its applicability. We arrived at each definition based on those used by several researchers and used them in relation to their mathematical content. The theory affects the research by letting us connect the definitions to the research experiences.

2.1. Spatial Ability and Geometry

Spatial is an adjective of the word 'space'. The etymological root of space is *spatium*, which means distance. Didactically, Freudenthal (in NCTM, 1989) defines 'space' as the environment in which the child lives, breathes, and moves. However, our environment has various properties. Which properties are proper for this research? Surely, the answer is all geometrical properties of the environment, such as geometrical shapes of things, directions, or sizes. Freudenthal (2002) takes away all physical properties of space, such as the color, the ornamental details, etc. However, in this research we focus on two physical properties which are *size* (to determine relative proportion on the model) and *shape* (to construct and visualize the reality). In addition, we also focus on two geographical properties of space, which are direction and orientation.

Spatial ability is considered to be an ability to *create* and *manipulate objects* and *orientation in the space* (Gardner, 1983). It may be developed through experiences, sharpening observation skills, solving mazes and other spatial tasks, and exercises in imagery and active imagination.

In this particular research, we expect that children experience those three spatial skills; creating objects, manipulating objects, and orienting in the space. Generally, within the activities children *create* three models of situations, which are a scale model of the school buildings, drawings of routes to a certain destination in the school, and different constructions of 4-cube castles. When they make a scale model and draw routes to a certain direction, it means that they are *orienting in the space*. The space is referred to here is their environment in where they live. They do not need to capture all properties of their environment in the scale model or in the drawing. Our concern is that children are able to capture all geometrical properties of the environment, such as the relative distance between buildings, the relative proportion of buildings' sizes on the model, and the orientation among buildings. Thus, they work physically, mentally, and geographically. Finally, children also learn to *manipulate* *objects*. They manipulate 4 cubes to create as many constructions as possible. Later on, they will realize the role of rotations and mirroring in manipulating the objects.

On the other hand, Diezmann & Watters (2000) explain spatial ability as the ability to invoke and use *particular representations* and *reasoning*. They refer to Einstein, that the particular representations could include diagrams, drawings, maps, and models. In addition, reasoning with spatial representations differs substantively from the sequential reasoning used with linguistic representations, such as text, and involves the use of spatial information to solve problems. Thus, all kind of maps, such as a scale model, are part of representations that can indicate spatial ability. Meanwhile, spatial reasoning can be differentiated into reasoning with spatial representation and linguistic representations.

Making the scale model and the drawing requires children to update relations between three references (Hegarty & Waller, 2004); *intrinsic reference* (among cubes on the scale model or components in the drawings), *egocentric reference* (consciousness of their positions), *environmental frame of reference* (the distances, the proportion of sizes, and the orientation between buildings). Thus, children learn how to relate their experience in the environment with the mathematical concept of space in the drawing and the scale model. Through doing so, the role of spatial reasoning, using either language or representation, sequentially becomes more developed.

Some former researchers distinguished spatial abilities into three essential parts, spatial perception, mental rotation, and spatial visualization (For example: Diezmann & Watters, 2000; Hegarty & Waller, 2004; Lee, 2007; Pentland et al., 2003; Tracy, 1987). However for this study, we refer to Tartre (1990) who only distinguishes two essential parts of spatial abilities, namely orientation and visualization. Spatial orientation is developed within a sequence of activities that requires children to mentally readjust their visual perspective of the school buildings on the scale model. The readjustment is continued until the scale model becomes consistent in its proportions with reality and congruent with orientation in reality. Meanwhile, spatial visualization is developed within a sequence of activities that requires children to mentally move the wooden cubes as they represent their visualization of 4-cube castles.

We also consider the importance of both spatial orientation and visualization. A research by Tartre (1990) has shown that ability of orientation assists children to mentally move or asses the size and shape of part of a figure. Additionally, good orientation gives opportunities for children to have a better knowledge of their environment (Diezmann & Watters, 2000), for example to draw their route from home to school, or to sketch the layout of their home, playground or school building. It provides opportunities for children to geometrically analyze their environments (Jones, 2001).

Meanwhile, the importance of visualization is that it is a crucial component to learn geometrical concepts (Jones, 2001). This study shows that visualization enables children to use wooden cubes (as concrete means) to grapple with abstract images of its basements. In that example, visualization influences children to determine the 2D shape of a 3D object, and vice versa.

So, which role does geometry in this instructional design of developing spatial ability? Before answering that question, we should know what geometry actually is. Etymologically, geometry means measuring the earth (Freudenthal, 2002), but we do not use this etymological definition. We use a didactical definition of geometry, which is understood as *grasping* the physical world (de Lange & van Nes, 2007; Freudenthal, in NCTM, 1989). Heuvel-Panhuizen (2005) argues that this definition is in agreement with the core goal of early geometry in the primary school: developing children's abilities of spatial visualization and reasoning. That goal connects the use of early geometry education in developing children's spatial ability and reasoning.

Furthermore, the learning trajectory described here distinguishes orienting and constructing as the key aspects of geometry (De Moor, 1999; Treffers et al., 1989). Those two key aspects involve activities in space. In this study, orienting seems to play a role as a part of spatial orientation, because when children perform orienting on the buildings, they do not have to mentally move the buildings, only their perceptual perspective of viewing the buildings that is changed or moved (Tartre, 1990). Meanwhile, constructing is considered a part of spatial visualization, because all or part of the cubes as representations of the castle can be moved mentally (Tartre, 1990).

In summary, the theories underlie the design in this study for the first garders. We use two early essential types of orienting recommended by De Moor (1999) and Treffers et al. (1989): localizing and taking point of view. Localizing mainly took place within the context of the school buildings. Through a simple mini lesson, children learn taking point of view, that a certain building can look quite differently when viewed from another side. By making the scale model and describing routes between rooms in the school, children gain experience about rotation and direction, both in the more formal world and with getting more abstract route drawings.

The next learning trajectory is sequential activities of constructing. Constructing is literally conceived as creating 4-cube castles. However, it is expanded towards the abstract by providing pictures of children's 4-cube constructions followed by drawing of their constructions. There is a shift from the real to the mental aspect of constructing. Additionally, we can also see a progression in manipulating constructions. Children are expected to argue with rotating and mirroring, both for manipulating the constructions and for distinguishing them.

2.2. Realistic Mathematics Education

Realistic Mathematics Education (RME) provides five principles (Treffers, 1987) that we use to underpin the instructional design of activities. Their applications in this study are described below:

(1) *Phenomenological exploration*. The mathematical activities take place within a concrete context. The rich phenomena, concrete or abstract, are used to emerge the representations that perform the essential concepts and structures. In this particular research, orienting and constructing take place within a concrete context. The rich phenomena are didactically used to perform the essential concepts. There are two early concepts of activities of orienting; localizing and taking point of view. And for activities of constructing, the essential concepts are about manipulating constructions using transformation. Moreover, we will give a more detailed explanation of the phenomena that underlie those concepts in the section on Didactical Phenomenology.

(2) *Using models and symbols for vertical mathematization*. A variety of 'vertical' instruments such as models, schemas, diagrams, and symbols are offered, explored, and developed to bridge the level difference between the intuitive, informal, context bound towards mathematical concepts. This is a gradual process of vertical mathematization. We offer a scale model of the school buildings and route drawings, explore them, and develop 4-cube constructions as bridge towards some geometrical concepts.

(3) *Using children's own constructions and productions* means that we promote using children's own construction as an essential issue within these 5 principles by the assumption that their own constructions are meaningful for them. During the activities and class discussion of constructing, children create their own constructions of castles and manipulate them. We assume that their own constructions are more meaningful for them for discussing the essential concepts of rotating and mirroring.

(4) *Interactivity.* Children's contributions can be used to unitize different symbols and pictorial representations. By working together, in pairs or in groups, children can learn from each other by the discussion. By working in pairs, children learn from their partner

to unitize personal contribution of the relative distance and relative proportion in the scale model. They also unitize different symbols and pictorial representation on the route drawings.

(5) *Intertwining.* One important thing that has to be considered in designing a sequence of activities is about the relation of the domain with other domains. Mathematics education should lead to useful integrated knowledge. Within the activities, we integrate geometry with other domains, such as measurement, counting, etc. The integrated domains take important roles in the development of children's spatial ability. For example, when children do constructing, the role of counting takes roles in the instructional designs in which children are only allowed constructing a castle with four cubes.

Considering the importance of contextual situations for starting the lesson, we use a fairy tale story about Thumb Princess for the activities of constructing. It is because psychologically, the use of storytelling for a contextual approach can improve geometry skills of children in this age (Casey et al., 2008). For activities of orienting, we use geometrical contexts about inventory school buildings and rooms in it. As mentioned before, the geometrical contexts lead the discussion about direction, orientation, size and shapes.

2.3. Didactical Phenomenology

Freudenthal (1991) suggests that the phenomena by which the concepts appear in reality should be the source of concept formation. There are some phenomena in our daily life that can be didactically extended and brought into the classroom to perform the essential concepts. Therefore, *didactical* phenomenology is defined as the study of mathematical concepts related to phenomena with a didactical interest.

As explained in the previous part, we have two essential concepts of orienting and one essential concept of constructing. Now, we didactically relate each concept with some phenomena. Afterwards, we determine mathematical goals from the didactical phenomenology.

(1) *Localizing*. In localizing the issue is to be able to indicate where something or someone is, or to be able to use localization data in order to find something or someone (Heuvel-Panhuizen, 2005). In order to accomplish those two important issues, we choose an orienting activity, which is making a scale model of school buildings. By making a scale model, children will be able to indicate where something is. Afterwards, they can use the scale model to find routes to some certain destinations. In addition, the scale model also leads children to learn about proportions of buildings' sizes on the scale model. However,

in determining the sizes of the buildings in the scale model, it is not really necessary to give an exact comparison, because the children worked with wooden cubes. As long as they are aware that one cube can be used to represent one room, that is enough. That is why we use the term 'relative proportions of buildings' sizes'.

There are two primordial parameter systems to localise a point consist of distance and direction (Freudenthal, 2002). Those two phenomena appear when we do localizing. However, the distances between buildings as interpreted in the scale model are not necessarily exact. Therefore, we use the term 'relative distance' in the goal(s) of the activities. Meanwhile, the direction is considered through two aspects of orientation, namely static orientation and dynamic orientation. The term direction in *static* orientation is indicated by words such as in front of, next to, in the left/right of, above, below, etc. The term *dynamic* orientation is indicated by destination and words such as to the left/right, straight away, etc. For *dynamic* orientation, another phenomenon emerges, that is a route. When describing a route, children learn both about formal languages of *dynamic* directions and path line(s) to visualize it.

From the above explanation, we formulate some mathematical goals for the instructional design. After a sequence of some orienting activities, we expect children are able to:

- Use direction in determining the relation between buildings' positions.
- Describe directions in explaining a route
- Visualize a route using line(s)
- Determine relative distances among buildings on the scale model
- Determine relative proportions of buildings' sizes on the scale model.

(2) *Taking a point of views*. While they taking a point of views, children must be able to say what can be seen from a certain point, and what cannot. They must be able to indicate whether an object or person is in view of the viewer at a certain point. In addition, they must also be able to imagine and describe how something is seen from a certain position. Therefore, in the instructional design, we choose two activities; a mini lesson about a *hide and seek* game and examining some pictures of the school buildings, and determining shapes of a building construction from different sides. When children learning about taking point of view, they actually learn about a phenomenon; a vision line (Freudenthal, 2002). It is an imaginary line that connects one point (the viewer's position) with another point (the object). Another phenomenon about taking point of view is that the shape of something will seem different when it is seen from a different point.

Therefore, for those purposes, we formulate a goal for this orienting concept: after doing the activities, children are expected to be able to:

- denote a vision line as reasoning for arguing different point of views
- argue different shapes of something if it is seen from different certain points

(3) *Manipulating constructions using transformation.* In manipulating constructions, children learn using three kinds of transformations; rotation, mirroring, and translation. When constructing something, children usually first do that by trial and error, but gradually they become aware with 'pattern' of strategies (Clements et al., 2004). In this study, we expect that they become more aware with the importance of rotating and mirroring to create several different constructions. Furthermore, they also can use those transformations to determine the difference between two constructions. Meanwhile, children also, within their imagination or outside, use translation when they manipulate the constructions to determine their basements. Here, they learn about shape relations, between cubes and squares. Based on the explanation above, we formulate goals for the constructing activities. After doing the constructing activities, we expect that children are able to:

- Use rotation and mirroring to create different constructions
- Use rotation and mirroring to judge the difference between two constructions
- Determine 2D shapes of the basements/sides (squares) from 3D constructions of the wooden cubes model (cubes)

2.4. Van Hiele's Levels in Activities of Constructing

In order to see the growing process of constructing, we refer to the 'visual' levels of Van Hiele (1986). According to the theory of van Hiele, children progress through five levels of thought in geometry. Instead of referring to all those five levels, we only use the first three levels in describing the children's progression of manipulating constructions. Those first three levels are described below:

Level 1: Visual. Initially, children identify and operate on shapes and other geometric configurations according to their appearance. They are not conscious of the properties.

Level 2: Descriptive/Analytic. Upon reaching the second level, children recognize and can characterize shapes by their properties. The product of this reasoning is the establishment of relationships between and the ordering of properties and classes of figures.

Level 3: Abstract/Relational. At this level, children can discover properties of classes of figures by informal deduction. One property can signal other properties, so definitions can be seen not merely as descriptions but as a method of logical organization.

Those three levels are used as the reference of the children's progression in manipulating constructions. Furthermore, we give the explanation about the role of the levels in the methodology of analyzing the data.

The progression moves from making *model of* situations into *model for* situations (Gravemeijer, 1999). One could argue that the scale model of class buildings is not really *a model of* situations, because we provide *too guidance* instructional. By asking the children to represent the building using the cubes and the grid paper as the landscape of the school, this situation does not help children to build up *their own model*. Therefore, we only involve the role of progressive schematization in the activities of constructing, because within the activities of constructing children *create their own models of castles*. Even though the castles should be built with only 4 cubes, the limitation does not prevent the evolvement from *model of* to *model for*. Besides, the limitation helps children to develop their growing process in manipulating the 4 cubes. We expect that we can see the progression of children's strategies in manipulating the 4 cubes. At the end of the activities of constructing, we expect that children are able to use rotation and mirroring as strategies, both for manipulating the cubes and for determining the difference between two constructions.

3. Methodology

3.1. Design Research

Design research is a series of approaches in educational researches, including (but not limited to) design studies, developmental research, formative research, and engineering research (Akker et al, -). Barab and Squire (2004) defined the series with the intent of producing new theories and practices that account for and potentially impact learning and teaching in natural classroom settings. We used this method in this research to design instructional activities to develop conjectured instructional theories about a specific domain, geometry on developing abilities of orienting and constructing in first graders.

We believe that the results of this design will contribute to educational research. We presented three reasons why this research should engage in design research as its research methodology (Edelson, 2002). *First*, it provides a productive perspective for theory development. Within the design process, we made practical considerations such as resources, goals, and constraints to guide us. *Second,* it has typical usefulness of its results. All educational components were involved in the design. The teacher was involved in designing activities for the children, the researcher designed materials for the teacher and the children, and school administrators and the headmaster were also involved in designing policies and systems for teaching and learning; such as determining proper school themes for the school year, or adjusting the school curriculum, etc. Therefore, the usefulness of its results clearly improves the educational system. *Third,* as a consequence of the second reason, we could argue that the design research directly involves the researcher in the improvement of mathematics education.

During the design process, we worked on the basis of the three phases of the design research which are: (1) preparation and design phase, (2) experiment in the classroom, and (3) a retrospective analysis (Gravemeijer & Cobb, 2006). We have done those three phases according to the explanations below:

Preparation and design phase

In this phase, we first tried to formulate research goals for the related domain, which are to support first graders' growing process in orienting and constructing, both for explaining children's spatial reasoning and gradually shifting children's levels of understanding. For doing so, we designed a sequence of activities that included instructional theories about learners and how they learn (Edelson, 2002). We followed some practical activities recommended in the TAL Brochures in designing the earliest sequence of activities. Because of the lack of researcher's experiences working with very young children, we assumed that those activities could also psychologically work with Indonesian first graders. In the sequence of activities, we theoretically conjectured teaching-learning situations that might have happened in the classroom during the implementation of the instructional designs, which is called Hypothetical Learning Trajectory (HLT). Simon (1995) defined an HLT as made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process – a prediction of how the students' thinking and understanding will evolve in the context of the learning activities. We considered those three components as the research instrument that linked between conjectures in the HLT with the real classroom experiments.

In May 2008, we tried out the earliest HLT with 6 to 8 children in the first grade from three parallel classes. The purpose of the try out was to see possible evidence of the first HLT in real situations. As a result of the try out and the classroom observation, we reformulated the HLT and adjusted some considerations, such as the geometrical contexts, working and non-working activities, and social and socio-mathematical norms. The considerations in the second HLT would be applied in the real classroom experiment in July – August 2008.

Experiment in the classroom

During the period of the classroom experiment, we worked with children in the new school year. The revised HLT in the preparation phase proposed a new sequence of activities that we applied in this experiment phase. One could argue that children in the class experiment were psychologically different with children in the preparation phase. They were indeed different. Therefore, during the class experiment we did daily reflections between conjectures in the second HLT with the real classroom experience. The reflections resulted in changes to some activities, and contributed in refining the second HLT and redesigning the instructional activities. Hence, within this design research, we conjectured a local instructional theory in a cumulative cyclic process similar to the figure below.

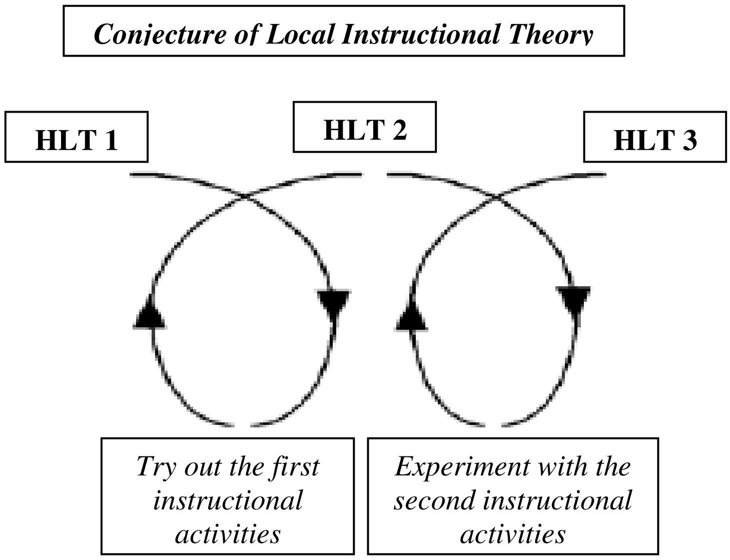


Figure 3.1 A Cumulative cyclic process of this research

A retrospective analysis

The last phase was to analyze the results of classroom experiments. We used the first HLT to analyze the daily try out. As a result of the analysis, we refined HLT and redesigned instructional activities. Afterwards, we did the second cyclic and again analyzed the daily class experiments to refine the HLT and redesign the activities. We focused on the goal(s) achievements of each activity and conjectures in the HLT as the guidance in the analysis.

3.2. Overview of the Data Collection and Experimental Subjects

The first grade classroom that was the subject of this research was one of three first-grade classrooms at a private school, SD At Taqwa in Surabaya, Indonesia. The class consisted of 27 children. The majority of the children was from middle class backgrounds. Most of them were not able to write and read yet. Therefore, we minimized writing assessment and focused more on daily activities. The classroom teaching experiment took place over a period of a month, from July to August 2008.

The school is a private school that develops its own curriculum and has a thematic approach in each school year. They combined their own curriculum with the Indonesian National Curriculum. All lessons and teachers in the school should follow the school curriculum and base the lesson on that year's school theme. This school has never been involved in the PMRI[3] project before.

Within this research, the teacher and her teacher assistant for the class experiment had her first experience using RME as an approach in teaching mathematics. However a week after the headmaster permitted us to conduct the research in her school, a PMRI project team presented this approach in the context of building blocks. The teacher has attended the presentation. Therefore, she already had some understanding of how RME works in the mathematics classroom.

The research team (including me as the main researcher, one video cameraman, one photographer one, and assistant observer who is in charged for the minutes meeting) had developed a good relationship with the teacher and her assistant. The research team occasionally inserted commentary and questions during a whole-class experiment. The children accepted the research team's contributions as if they were visiting the teachers.

In the sample episodes presented in this thesis and throughout the conversation and discussions, we use the word *teacher* collectively to refer to both the teachers and the researcher. However, the classroom teacher took primary responsibility for leading all instructions. She clearly shared the research goals and the importance of basing instructional decisions on children's understandings. Therefore, we consider her an irreplaceable member of the research team.

Each of the data sources elaborated subsequently contributed in its own way to the analysis that will be explained more in the next subchapter. Data were collected from three different sources, the data from: classroom session, formal, and informal meetings with the teachers (Stephan & Cobb, -).

Classroom Sessions

We collected data from classroom session in three modes: video recordings, field notes, and pictures of student works. We recorded all daily lessons using moving video camera by a

[3] PMRI is the stands from *Pendidikan Matematika Realistik Indonesia* [Indonesian Realistic Mathematics Education]. It is a project of Mathematics Education in Indonesia, elaborated with the Netherlands since 2001, to implement an innovation about Realistic Mathematics Education as an approach in teaching-learning mathematics. Source: www.pmri.co.id. Consulted in 31 October 2008.

video cameraman, notes by an observer, and pictures by a photographer. We use the triangulation those data collections to keep the internal validity within this research. We mostly separated children into three large groups, so that every group was assigned one observation team.

Formal and informal Meetings

In May 2008, we observed the school situation and daily lessons in the classroom. We formally met with the headmaster in order to collect some school documents to adjust the contextual situations to the school theme of the year, and the mathematical topics to the school curriculum (because normally geometry is taught in the second semester, while this research was conducted in the first semester). We also recorded the daily math lesson in two classrooms of the first grade to see how the teachers usually teach mathematics. We also took 6 to 8 children from three parallel classrooms to be interviewed and to try out the activities of HLT 1. We used the results of the try out and the exploratory interviews to refine the conjectures in the HLT and redesign the activities and the written tests.

The critical and interesting situations were discussed with the observers and the teachers to improve the internal reliability and were tested with the HLT. After each classroom session, the research team met with the teacher and her assistant for approximately 2 hours to plan instructional activities for the following day. Often during these informal meetings, we observed the video of the day together. Afterwards, the researcher offered observations of children's reasoning and made conjectures about individual and collective mathematical activities for the following day. However, before we offered the hypotheses, in her opinion the teacher could make any decision whether the children were ready yet for the following day's activity by reflecting on today's activity.

Time	Class (# Students)	Type of Observation/ Experiment	Data Collection	# of Lessons	Purpose(s) of Collected data
Before conducting HLT 1 in May 2008	—	School Observation	Pictures of School Buildings	—	for adjusting the context of HLT 1 and some figures in the post test
			School Theme		
	1A (25) grade 1 semester 2	Class Observation	video recording	1	for observing daily math lesson, social norms, and socio-math norms
	—	Interviews with the teacher	audio recording	—	
	1A (25) grade 1 semester 2	Class Observation	pretest	1	for observing children pre-knowledge and redesigning the pretest
2 weeks in May 2008	1A (8) grade 1 semester 2	1st teaching experiment	video recording, route drawings	6	for trying out the activities and redesigning HLT 1 to HLT 2
	1B (6) grade 1 semester 2		video recording, basement drawings, post test	4	for trying out the activities, redesigning HLT 1 to HLT 2, and redesigning the post test
	1C (6) grade 1 semester 2		video recording, route drawings	1	for digging other reasoning of reading routes.
In the end of July 2008	—	School Observation	Pictures of School Buildings	—	for adjusting the context of HLT 2 and some figures in the post test
	1B (27) grade 1 semester 1	Class Observation	video recording	1	for observing daily math lesson, social norms, and socio-math norms
	1B (25) grade 1 semester 1	Class Observation	pretest	1	for observing children pre-knowledge
In August 2008	1B (27) grade 1 semester 1	2nd teaching experiment	video recording, route drawings, notes of the observers, teachers' oral reports, pictures, post test	6	for designing HLT 3

Table 3.1 Overview of Data Collection

3.3. Description of Intended Data Analysis

As was explained before, we used retrospective analysis (in chapter 5) to analyze the data. We chose a small episode of each lesson that individually or classically showed the growing process of children's understandings. We kept the retrospective analyses in three components of the HLT by answering these questions in each lesson:

(1) *The learning goals*

- Did children already achieve the learning goals within the activity?
- How did we refine the learning goals if they were not achieved yet?

(2) *The learning activities*

- Which moments (could be individual moments or classical moments) showed the growing process of children's understandings?
- What mathematical concepts did children learn within the activities?

(3) *The hypothetical learning process*

- Did conjectures from the HLT happen in the class experiments?
- Did there any other spatial reasoning emerge?
- On which van Hiele's levels were the children?

In order to answer the research questions (in Chapter 6), we gathered and concluded information from the retrospective analyses. Therefore, this study will offer three outputs as answers of the research questions:

a. A conjectured instructional theory of sequence(s) of activities that can build first graders' ability in orienting and constructing.
   We would compare the three HLTs and suggest activities from the second HLT and third HLT as lesson sequences for the similar typical classroom, because those last two HLTs were built up from the try out and experiments in real classroom situations.
b. Various spatial reasoning that children use during activities of orienting.
   We listed all spatial reasoning that children used within each orienting lesson. Afterwards, we would classify and draw conclusions about different kinds of spatial reasoning in orienting.
c. Descriptions of Van Hiele's levels that constitute the development of progression in manipulating constructions.
   To see the development of progression within the activities of constructing, we fitted the analyses to the first three levels of Van Hiele, which are: *visual level, descriptive/analytic level,* and *abstract level* (van Hiele, 1986). However, Clements & Battista (1992) believe there is a shift between Van Hiele's levels, and that children are not just "at" a single level. Thus, instead of putting a child on one level of Van Hiele, we preferred to describe children's understanding on their constructions using Van Hiele's levels as the preferences.

To keep the external reliability of this research, the conclusions of this study should depend on the subject and conditions, and not on the researcher. To generalize the results, the problem is how we can generalize the results from these specific contexts as to be useful for other contexts. Gravemeijer & Cobb (2001) suggest an important way to do that is by framing issues as examples of something more general. The extent of this generalization of the results is called external validity (Bakker, 2004).

4. Hypothetical Learning Trajectory

The purposes of this research, as mentioned in the introduction chapter, are to (1) support the growing process of first graders' spatial ability, especially in orienting and constructing, and (2) explain children's spatial reasoning and levels within orienting and manipulating constructions. For those purposes, we designed a sequence of activities that consisted of conjectures of learning processes. As mentioned in the section on didactical phenomenology, we didactically explained the phenomena of three essential concepts of orienting and constructing.

The Hypothetical Learning Trajectory (HLT) in this chapter is the first version of HLTs in the whole design. We followed some activities from the TAL Brochure (Hauven-Panhuizen, 2005) which are suitable in Indonesian contextual situations. In addition, we also considered the learning goals in the part of didactical phenomenology within the decision process. We have already collected some learning goals for a sequence of activities. We use context as the approach to develop mathematical concepts. We use the context of buildings outside of the school within activities of orienting and castles of Thumb Princess within activities of constructing. The contexts effect the progression of mathematical ideas. In designing the HLTs, the sequences of activities consecutively unfolds as follows.

(1) Localizing environment.
    a. Creating model of environment. Children start placing cubes representing buildings by considering *direction between them (both static and dynamic), relative distances among them, and proportion of buildings' sizes.*
    b. Reasoning with the model. Children explain routes to certain destinations by using model as references of reality.

(2) Extending the activity of orienting on localizing environment to taking point of view. Children discuss different places of a viewer that could affect different shapes of a thing. Furthermore, they emerge a vision line between a viewer and an object to be the reason of taking point of views.

(3) Building with wooden cubes.
    a. Manipulating constructions. Instead of using the environment as references of the construction, children create their own constructions using 4 cubes as many as possible. They start learning to manipulate 4 cubes to get various shapes.

   b. Using rotation and mirroring. Children manipulate the cubes more structurally by rotating or mirroring a construction to get another shape.

(4) Reasoning with rotation and mirroring. Children explain the differences between two constructions using rotation or mirroring as the reasons.

(5) Manipulating objects more mentally.

   a. Shapes relations. Children use their mental orientation to guess the sides/basement of a construction. It is getting more mentally when the constructions are in figures.

   b. Visualize their mental figures. Children draw what is drawn in their mind about the basement of a given building figures.

In this first HLT, we designed eleven activities for nine day lessons. The activities were underpinned by the above skeleton. The activities in this HLT 1 consecutively are that: *localizing cubes that represent buildings, describing routes, localizing cubes that represent rooms,* and *drawing basement of the school buildings* determine the first skeleton; *examining pictures of the school buildings,* and *playing hide and seek* determine the second skeleton; *constructing 4-cube constructions* determines the third skeleton; *examining the differences between two constructions* determines the fourth skeleton; finally, *drawing sides of 4-cube constructions, drawing basements of 4-cube constructions and determining the height of the buildings,* and *guess possible basements of a given figure* determine the last skeleton.

Consequently, each activity brings one or several goals that are related with the mathematical ideas. By localizing cubes that represent buildings, children learn about direction between buildings and relative distances among them. Thus, we formulated two goals, which are that after doing the activity, children are able to *use directions in determining relation between buildings,* and *determine relative distance among them on the scale model*.

In describing routes, children learn about how to describe it into words and drawings. Therefore, the goals are that they have to be able to *describe directions in explaining a route among two buildings* and *to visualize a route using line(s)*. When localizing cubes that represent rooms, they learn again about direction between rooms and relative proportions of buildings' sizes on the scale model. Thus, we formulated two goals: children are able to *use directions in determining relation between rooms*, and *determine relative proportions of buildings' sizes on the scale model*. When they draw basements of the school buildings, they learn shape relations between 3D constructions and their basements. Therefore, the goal for them is *to determine 2D shapes of basement (squares) from its 3D constructions on the scale model*.

Within two activities, examining pictures of the school buildings and playing hide and seek, children learn about taking different points of views. Thus, we expect them to be able to *use a vision line as reasoning for arguing different points of views,* and *argue different shapes of the school buildings from different points of views*.

For the last five activities of constructing, we consecutively formulated the goals. Children are able to *manipulate shapes of 4-cube constructions* in the activity constructing 4-cube constructions. We expect them to be able to *use rotation and mirroring as reasoning the differences or the similarity between two constructions* in the activity reasoning the differences or the similarity between two constructions. They learn about shape relations in drawing sides/basements of constructions. Therefore, the goals are *to determine 2D shapes of sides/basements of the constructions (squares) from its 3D constructions, and more mentally visualize the basements of a given figures*.

In summary, we generally can see the relations between the skeleton of sequence, the activity, and the mathematical concepts in this first HLT, from the scheme below.

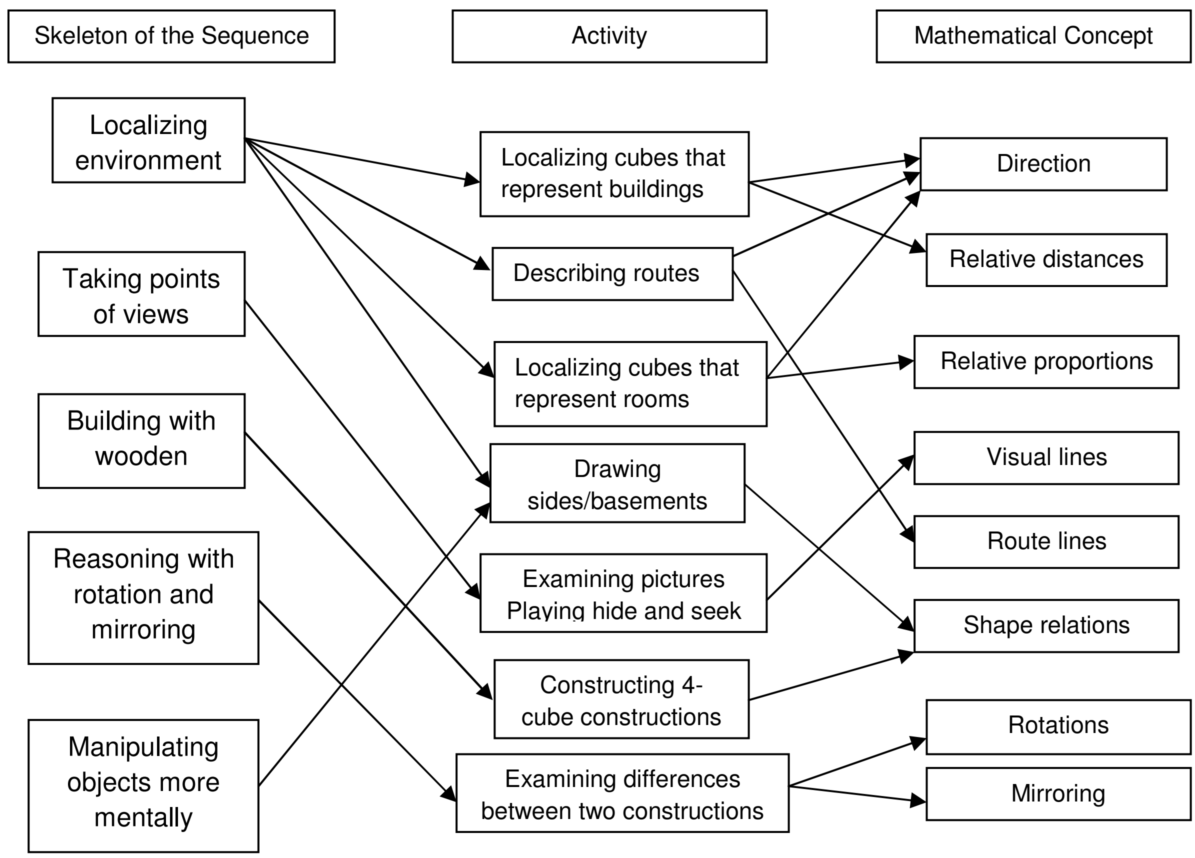


Figure 4.1. Scheme of the Instructional Design HLT 1

The explanation of each day activity in HLT 1 was arranged into four general parts, which are *goal(s) of the activity, starting point(s)*, *intended activity,* and *conjectures of the learning process* (under supervision den Hertog). The *goal(s) of the activity* are the learning goals that contain mathematical concepts that children should achieve after doing the activity. The *starting point(s)* explains children's pre-knowledge or some basic knowledge that is required for doing the activity. In the *intended activity*, we motivated the activity in the relation with their learning goals. We also stated what children can mathematically learn within the activity. Last, we described some conjectures of the learning process in *conjectures of the learning process*.

Day 1: *How do the school's neighborhoods look like?*

The activity: Localizing cubes that represent buildings

Goals of the activity: after doing the activity, we expect that children are able to (1) use direction in determining the relation between buildings' positions surrounding the school buildings, and (2) determine relative distances among buildings on the scale model.

Starting point(s): to reach the goals, we assume that children have informal languages to express directions and informal experience to determine relative distances between two things, for example when they play marbles. Besides, they also have already known condition of the environment and remember it in order to easily determine buildings' positions on the scale model.

Intended activity: children represent buildings with cubes on a grid paper. With the school as a starting point, children place other cubes on the grid paper. One cube represents one building. By doing so, children more focus on learning *direction among buildings* and *relative distances between buildings*. Children could indicate directions by the words. Children initially develop some certain words are firstly built up in this activity, especially the words like: in front of, behind, next to, and in the left/right of. Not only learning about the formal mathematical languages to express the direction, but children also firstly learn about congruently corresponding between position of each building in reality with its position on the scale model in the sense of its related direction with another building.

The second mathematical concept is about *relative distance*. When children put two cubes on the scale model that represent positions of two building in reality, they also have to place those two cubes in the relative distances, because the distances indicate relative positions in reality compared to positions in the scale model. However, the distances between buildings that interpret in the scale model are not necessarily exact.

Conjectures of the learning process: Children may use their hands and gestures to describe the direction by pointing to a certain point. However, if they can point to the right direction as in the reality, it already indicates a good sign of direction awareness. Children are only expected to indicate how the buildings are situated in relation to other buildings from within the school. This inventory is not limited by listing the buildings only. Other objects such as road, bus stop, railway, etc may also be located. Our conjectures are they struggle on the congruency between the direction in the reality and the direction on the scale model. Children may come up with different relative distances on the scale model. We expect that, at least, they are able to proportionally place the cubes as close as in the reality.

Day 2: *How do you get from school to…?*

The activity: Describing Routes

Goals of the activity: after doing the activity, we expect that children are able to (1) describe directions in explaining a route among two buildings. The description could be both, by drawing the route or by orally explaining into the words of *dynamic* directions, such as to the left/right, straight away, etc, and by drawing a route. Thus, we formulate the second goal is they also are able to (2) visualize a path of ways using line(s).

Starting point(s): the direction awareness from the previous activity is useful in this activity. The awareness will help children to move now to the *dynamic* orientation, which is about mapping direction.

Intended activity: after children localized all buildings surrounding the school, they now considered how the way to get there. While children are describing the route, they must know where they are and to what direction they will go. The teacher can introduce the activity by giving question such as: "*how would you walk from the school's gate to the entrance of the hospital?*". First, we expect that children can explain a route to a certain destination into words. Here, they learn to be more aware that the destination words in this activity are more *dynamic* than those in the previous day. Besides, children are allowed to draw the route on the grid paper. Drawing a route on a paper means drawing the paths of ways and/or drawing directions to where we go. Here, children learn how to visualize the path of ways and the direction. They have to make their route clear to the entire class. The teacher can emphasize the importance of clear direction by asking, *'Is the route correct? Is it easy to follow by everybody?'*. If it is still enough time, a game will be performed. The teacher will hand out a card to each pair. The card says which building the children have to think of. Each pair will perform a role play. A child draws a route on a piece of blank paper with the school building as the starting point and a building as the departure point, and another child determine which building will be at the end of the route.

Conjectures of the learning process: when children describe directions in explaining a route, children may only use the gestures of movement. Children move their hands to show the movement of the walker and to emphasize the direction to where the walker goes, whilst in the previous activity the gesture is only by pointing to a certain point. Another possibility is some children may mention street names or other reference names by using directions such as “Second Street to the right”, “Third Street to the left”, and “along the way straight ahead”. In that case, children use orienting concepts to explain a route. Children may still struggle in putting those directions into words. However, the scale model of the previous day may be used to be a model for helping children showing the route. They are allowed to draw the route on the grid paper. The route drawings may be different between each other children.

Day 3: *Where is our classroom in the school?*

The activities: Localizing cubes that represent rooms and Drawing Basement of the School Buildings

Goals of the activity: after doing the activity, children are able to: (1) use direction in determining relation between rooms’ positions of the school buildings, (2) determine relative proportions of buildings’ sizes on the scale model, and (3) determine 2D shapes of basement (squares) from its 3D constructions on the scale model.

Starting point(s): we assume that children already have experience in describing direction, both in formal direction language and in the drawing, from the two previous activities. Besides, they need to be able to memorize all rooms in the school, because that enables them to easily localize those rooms proportionally on the scale model.

Intended activity: within this activity, children initially develop some other words of directions, especially the words like: above and below. We ask them a question: *‘Where is the position of our classroom in the school?’* This question issues to answering the goals of this activity. In order to answer the question, we expect that children firstly aware that one cube does not fit to represent the whole school building. Thus, they would change the representation, at least, one cube for representing one classroom. Here, they learn about how to determine relative proportion of the building’s size. Gradually, they change the scale model within the shapes of other buildings. The discussion is not necessarily to be very detail, but the height and the area that covers the buildings should be proportional with the actual buildings. In this case, sense of scale – the ability to decide relative width, height, and length on the scale model so that it is closely near the reality – was firstly built up.

The next part is that children have to make a drawing of the scale model. For doing so, the teacher starts explaining the condition where children have to move the scale model because the desk will be needed for another lesson, but the scale model must not taken apart. The teacher's explanation will stimulate a discussion concerning how to re-build the scale model once it is taken apart. This task aims to answer the third goal. It encourages children to record the buildings' position on the scale model and what they look like. By doing so, they learn about the relation between cubes and squares for a certain building construction. Besides, children will learn to read a construction drawing of a ground plan by making one. They can derive the shape and size of the buildings and their mutual distances from the drawing.

Conjectures of the learning process: As in the first activity, we expect that they realize the congruency between the direction of rooms in the reality and the direction on the scale model. By starting the lesson with 'Where our classroom in the school is', a conflict will be about the shape of the school. We expect them reasoning the proportion using the sizes in the reality, such as: The supermall is higher, so we put more cubes on it, and since the school building is broader, we put more cubes on its area so that our class will be there.

A possible strategy for the next task of 2D scale model is: Every time someone takes a building of the scale model, they draw squares on the underlying paper and mark how many cubes were on the grid section. And then, put signs for certain buildings, like H on the place of the hospital, S on the place of the school, etc. So, when the scale model is taken apart completely, the drawing is finished at the same time. The struggle may be to determine height of the buildings on the drawings.

<u>Day 4</u>: *Mini Lesson – Where do you have to hide? Where do you have to stand?*

The activities: Examining Pictures of the School Buildings, and Playing Hide and Seek

Goal(s) of the Activity: after doing this mini lesson, children are able to (1) use a vision line as reasoning for arguing different point of views, and (2) argue different shapes of the school buildings from different point of views.

Starting point(s): children already have experience in playing *hide and seek* in their daily life. We assume all children ever have that experience, because in Indonesia *hide and seek* is a favorite game. Another requirement is that they know every detail of the school buildings, so that they can recognize pictures of a school building from different points of views.

Intended activity: the most important idea from the previous activity that has essential contributions for this activity is about drawing basement shapes. By the drawing, beside children learn about relation between squares and cubes, they also learn about different view of the buildings, which is its basement. Children did not need to be aware when they drew it, but it unconsciously gave them experience about taking point of view which is required for today activity. Before doing the mini lesson, children can play hide and seek at the playground. Children play in 4 big groups and play one round. Each group has to make a strategy before playing, by determining in the place where they have to hide if the seeker stands at a certain position in the playground. On a piece of paper, children are asked to draw their strategies and localize position of each player. At the end of this game, they will realize that in determining the position of each player, they learn on making a straight line as reasons whether one could be seen or not by the seeker.

After playing, the children will go back to the classroom and start following the mini lesson. The teacher will show several pictures of the school building taken from many point of views and for each picture the teacher will ask: *'Do you recognize this picture? What is this building?'*. The questions will remind the children that the pictures are pointed to their school building. Another leading question afterwards is: '*All of the pictures are about the same building, which is the school building. Why does it look different?*'The teacher will explain that the buildings look different because the pictures are taken from different positions. One is from the backyard and the other is from the playground; etc. The next question in the mini lesson is: *'Where do you have to stand to get this view of our school buildings?'* This question will stimulate children to be more aware with one-to-one correspondence between a certain point of viewer and a certain orientation of the school building, where the correspondence is denoted by a straight line. However, the awareness can be analyzed from the discussion.

Conjectures of the learning process: My first conjecture is that when children play *hide and seek,* they will realize that in determining the position of each player, they make a straight line as reasons whether one could be seen or not by the seeker. The straight line can be drawn on a drawing strategy, or it can be indicated by their hands when they point to a certain location. The struggle may be about reasoning using that straight line as a vision line.

For the second question in the mini lesson, other children may think they are different buildings. To check their arguments, children may go out of the classroom and find it out. At the end, the whole class will conclude that all the buildings in the pictures are the same building, the school building. The following is the discussion that is expected to come up for the third question:

Teacher: Where do you have to stand to see the school in this way? (Show figure by figure)
Children: We have to stand in the middle of playground (for instance)
Teacher: How do you know?
Child 1: because it shows the big door of this building at the center.
Child 2: when you draw a line from the door to the possible place of a viewer, then you can see that the viewer should be in the middle of the playground.

Different possible answers can come from the children. Whatever the answers are, children achieve something although it might be in different levels of thinking. Most children may come up only with the first reason as the child 1 in the discussion example above does. These children are in the level on reasoning with reality and may unconsciously make one-to-one correspondence with a straight line. Meanwhile, children with reasoning similar to child 2 are in the higher level. They realize the connection between two points in a space by using more formal way to describe it.

Day 5: *Different building constructions*

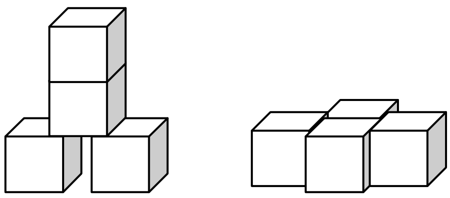
The activity: Constructing 4-Cube Constructions

Goal(s) of the Activity: after doing the activity, children are able to manipulate shapes of 4-cube constructions.

Starting point(s): children have experiences constructing with Lego, as mentioned in the Kindergarten curriculum in Indonesia.

Intended activity: In this activity, children construct different buildings from the same amount of cubes. The teacher can introduce the task by this example of story:

'Act as a city designer. The government asks you to make a special city. There are only 4 cubes to construct each building. How many possible constructions are there? Neither a hole nor a bridge, as the depicted in the following picture, will be allowed…'

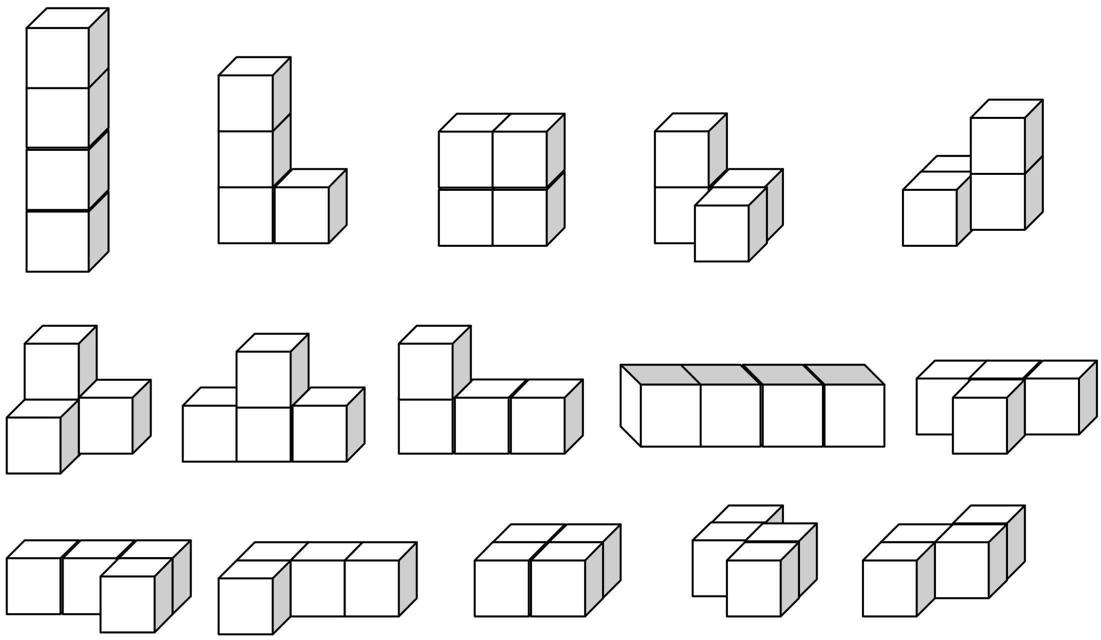


Children start working in pairs, so they can discuss together how to build the buildings. They have to figure out how many possible different constructions appear. Thus, they have to determine together whether the buildings are the same or not. Teacher can help them to understand the task by building two same buildings on the table in which one building is rotated so it looks different. The question to the children is: 'Are they the same or are they different?'. Hence, in this activity children start developing their ability to manipulate constructions. We expect that the children will discover that certain buildings that look different are actually the same once it is rotated.

Conjectures of the learning process: Each pair of children could build only flat buildings. The cubes are laid next to each other and not on top of each other. The teacher anticipates this condition by showing to them other pairs' constructions that are not only flat buildings and asks, 'Look what kind of houses they are building'. Other pairs may spontaneously rotate a building and figure out that they are the same after the rotation. Thus, rotating could be a strategy for some children to notice that some buildings are the same or not.

The requirement that all buildings should be different also stresses some explanation beforehand. It is possible that some children may be able to put the reason into good words, for instance, 'It is only positioned differently, but it is the same'. Thus, the discourse may be about determining whether two constructions are different or not.

These are all possible buildings for 4-cubes:



It would be a good achievement if children can come up to this number of constructions and can also reason that this is the maximum number with four cubes. However, the teacher should not force them to reach this result.

Day 6: *Math Congress – How many possible constructions?*

The activity: Examining the Differences between Two Constructions

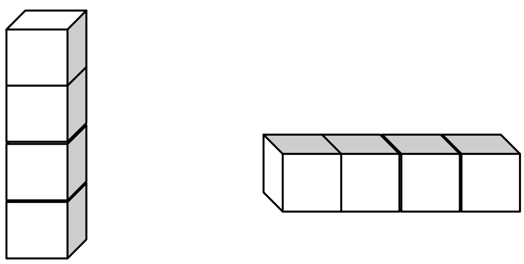
Goal(s) of the activity: after the math congress, children are able to use rotation and mirroring as reasoning the differences or the similarity between two constructions.

Starting point(s): children experience in the previous activity about determining the differences between two constructions. In the previous day, they also may use rotation and mirroring to construct 4-cube constructions. This evidence could be a good starting point for this math congress.

Intended activity: As a continuation of the previous activity, this activity provides class discussion to talk about all possible buildings that appear during the class activity. Children share their experiences and discoveries with the whole class. First, they have to express how they made the buildings. And then, they also have to explain whether the buildings are the same or not, and why is that so. The teacher starts the discussion by assigning, ‘Remember how you built the buildings’.

The discussion about the buildings arise a number of debates:

(1) Rotated buildings are the same with its original?



(2) Mirror buildings are the same each others?

There may be buildings that form a mirror image of each other, as the figure below:

In this activity, children learn that rotated constructions are the same with its original, and mirror constructions are different with its original. The first two constructions were rotated constructions. To get another one, we could rotate the other one 90 degree rotation along y axis. It was not necessary to inform the children about this kind of rotation. The most important thing was whether they aware about the rotation. Different with rotated buildings, the mirror buildings are always different; at least, if they are not symmetrical. If you have two mirror buildings and you rotate one of them, it will never become the same as the other. They are not interchangeable.

Conjectures of the learning process: in this activity, children may struggle on arguing the difference between two constructions. Some children may think that the first rotated constructions are identical, some may not. Teacher can pose question, ‘Let’s see what happens when you rotate them, but you are not allowed to lift them. Don’t lift them up and then put them down horizontally. Are they the same buildings?' Some children may agree, and some may not. The teacher could help them by giving hint, 'If you lived in those buildings, in what ways could you walk around?'. Children could argue using reality, 'The first building is a tower because you can walk upstairs, but the second is not'. For the mirroring buildings, children may rotate one building to see whether they are the same or not.

Day 7: *View it differently?*

The activity: Drawing Sides of 4-Cube Constructions

Goal(s) of the activity: After doing the activity, children are able to determine 2D shapes of sides of the constructions (squares) from its 3D constructions.

Starting point(s): children already have experiences to draw the basement figures in the third activity and they are already able to see different shapes of a building from the fourth activity.

Intended activity: Children, in this activity, experience whether a building looks different when it is seen from different sides. The teacher starts the activity by asking an imagination: *The day is getting dark. It is going to difficult to recognize each building in the dark.* The teacher draws one side of a building and shadows it. *Which building is it? Remember, it is dark and you can only see one side of the building. Who sees the building that is drawn on the blackboard on the table in the same way?* In the dark, one particular silhouette will not do to indentify a building. By finding out which houses can fit to a certain view, children learn how to interpret views. Another task, then, is to do the role play in pairs. One child draws one side of a building and the other guesses which building it is. First, both of them have to pick up some buildings and draw sides of each building as tasks for the other. Then, they play role to guess which building is. In the end of this activity, each pair produces some 2D shapes of sides of each building they made.

Conjectures of the learning process: In this activity, constructing is connected to orienting, especially to take a point of view. In the dark, one particular silhouette will not do to indentify a building. By finding out which houses can fit to a certain view, children learn how to interpret views. Children figure out which building it is by comparing silhouette on the blackboard with the buildings on the table. Come children may walk around the table to see the buildings from different points of views.

Day 8: *How is the basement and how high is a building?*

The activity: Drawing Basements of 4-cube Constructions and Determining the Height of the Buildings

The goal of the activity: children are able to determine 2D shapes of basements of the constructions (squares) from its 3D constructions.

Starting point(s): children already have experience on rotating a construction and see it from different sides. That is a good starting point for children to imagine shape of the basement, the hidden part of a construction.

Intended activity: Now, children learn how to determine the basement so that other can imagine how the building will be. The issue is about the height number of building on the basement. To determine a proper basement of a building, the top view is another central issue. When children have looked at several sides, they have experienced that from every point of view, certain parts are not visible. For example, if children look at the construction from the front, they cannot see the cubes that are behind the visible cubes. From the top, they can see those hidden cubes, but then, there are some others they cannot see. The children gradually will come up with the basement figures of all buildings with the height numbers. For the rest of times, children can play role in pairs. A child makes the basements with the height numbers and the other constructs the building. Moreover, because the height numbers, children know immediately how many cubes they need for the constructions.

Conjectures of the learning process: a common struggle of this activity is that most children may not automatically think of making basement with the height numbers. By pointing in a certain square, children may give correct answer of height numbers based on the construction they see. Children may use a strategy by rebuilding the block construction using those basements, it can be easily seen whether the drawing is correct.

Day 9: *Mini lesson – Guess the possible basements*

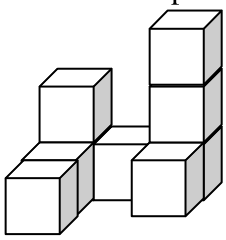
The Activity: Drawing Possible Basements of A Given Figure

The goal of the activity: after doing this mini lesson, children are able to more mentally visualize the basements of a given figures.

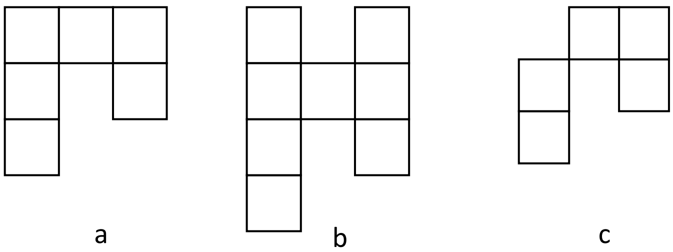
Starting point(s): this mini lesson requires the idea of shapes relations between 3D constructions and 2D shapes that were done in the two former activities.

Intended activity: In this mini lesson, children have to imaginatively visualize the given building figure to guess its possible basement. The teacher starts the mini lesson with some building figures and asks children to guess how the basement will be. The main mathematical idea of this activity is still about shapes relations. The difference is that children have to move to the more abstract level. Since they do not work with concrete materials anymore, they have to convince the others by using spatial reasoning of their answers. Thus, children learn to mentally rotate or translate a given building in the figures to find out its possible basement.

Conjectures of the learning process: The different answers may arise since the building figures are shown only from one view so that there will be possible cubes covered by the others. Children may struggle on imagining how the complete shape of the building so that they can determine its basement. For one example of the building figures is below



From that example building figure, we expect children will come up with various basements that are based on different reasoning.



a b c

Child 1 that comes up with basement (a) could reason that the construction clearly shows all cubes on it, thus this will be a proper basement.

Child 2 that comes up with basement (b) could argue that since some sides are high and we cannot see the whole building figure from other directions, it will possibly be more cubes behind the high building that are not visualized.

Child 3 that comes up with basement (c) could reason that the cubes, that cannot be seen, do not necessarily exist.

In conclusion, the summary of HLT 1 can be seen from the table below:

Day	Name of the Activity	Activity	Conjectures of	
			Learning Process	Struggles
1	How do the school's neighborhoods look like?	Localizing cubes that represent buildings	1. Using hands and gestures to describe directions 2. Listing other objects 3. Having various relative distances	Congruency between direction or distance on the scale model and those in the reality
2	How do you get from school to … ?	Describing Routes	1. Using only the gestures of movement 2. Using words of direction and other references 3. Having various route drawings	Putting directions into words
3	Where is our classroom in the school?	Localizing cubes that represent rooms	1. Using hands and gestures to describe directions 2. Listing other objects 3. reasoning the proportion using the buildings' sizes in the reality	A conflict is about shapes of the school buildings
		Drawing basement of the school buildings	1. Sliding the construction before drawing squares and marking number of cubes on it 2. Giving names for certain buildings	Determining height of the buildings
4	Mini lesson - Where do you have to hide? Where do you have to stand?	Playing *Hide and Seek* Examining pictures of the school buildings	1. Indicating a visual line by pointing to a certain location 2. Reasoning using reality 3. Using a line as a connection between two points	Using a straight line as reasoning of a vision line
5	Different Building Constructions	Constructing 4-cube constructions	1. Constructing only flat buildings 2. Constructing various buildings, not only flat	Determining whether two constructions are diferent or not
6	Math Congress - How Many Possible	Examining the differences between two constructions	1. Using reality for reasoning the differences 2. Rotating a construction to see the differences	Arguing the difference
7	View it differently?	Drawing sides of 4-cube constructions	Walking around the table to see the constructions from different point of views	Comparing silhouette on the board with the buildings on the table
8	How is the basement and how high is a building?	Drawing basements of 4-cube constructions and determining the height of the buildings	1. Pointing to a certain square before marking the height 2. Rebuilding the constructions to check whether the drawing is correct	Not automatically thinking of making basement with height numbers
9	Mini lesson - Guess the possible basements	Drawing possible basements of a given figure	Drawing: (1) only all clearly shown cubes on the construction; (2) possible hidden cubes; (3) all hidden cubes are not drawn	imagining how the complete shape of the building in the figure

Table 4.1. Overview of the HLT 1

5. Retrospective Analysis

5.1. Observation and Refinement of HLT 1

When we tried out the first instructional designs in May 2008, not all activities could be tried out. Based on the observations of school conditions and classroom situations, we made considerations and some changes for adjusting instructional activities of HLT 1:

(1) Contexts of the activities of orienting.

In the first HLT, the contexts in activities of orienting are about buildings surrounding the school. We use the importance of recognizing some important places (such as, the hospital, the police office, etc) surrounding the school to be the reasons for children to do the activities. At the time we designed the first HLT, we did not know yet the real school conditions. After the observations, we saw that there was no building surrounding the school. The school is in the middle of people housings. Thus, we changed the contexts of surrounding buildings of the school. Instead of inventory buildings outside of the school, that do not exist, we utilized school buildings inside to be explored and inventory by the children. We found that this context was more meaningful for children in that school, because it is a new school with still continuously renovated. That condition brings the needs to remember the important rooms in the school, and to explore them within the scale model. Thus, the names of some activities were changed, for instance: activity in the day 1 became *My School Buildings* rather than *How do the school's neighborhoods look like?*, or in the activity of describing routes, we named it *Make a Route to an Important Place in the School* instead of *How do you get from school to…?*.

(2) Context-based rearrangement of the activities of orienting

In the first HLT, we arranged the instructional activities based the recommendation in the TAL Brochures. However, we also considered consistency of the goals among those activities. After some considerations, we rearranged some activities based on the context. We made the new arrangements in the sense of developing context. We saw that this approach is more suitable for our children, because they more easily grabbed the mathematical concepts if the contexts continuously follow the concepts. We postponed the activity of *describing routes* and directly did *localizing cubes that represent rooms* after *localizing cubes that represent buildings*. Then we put *describing routes* in the end of orienting.

(3) Removal activities and separated activities because of the effectiveness

We separated two activities which are in HLT 1 they were in the third day, *Localizing cubes that represent rooms* and *Drawing Basement of the School Buildings*, into two days

When we tried out the activites, we took a whole day to do *Localizing cubes that represent rooms*. Thus, we did *Drawing Basement of the School Buildings* in the following day, in which we gave it a new name, *Removable Buildings*.

The similar case was with *Examining Pictures of the School Buildings* and *Playing Hide and Seek*. It took more than one day to do those activities. In this refined design, for the efficiency of times, we decided to choose *Examining Pictures of the School Buildings* rather than *Playing Hide and Seek*, because this mini lesson was more powerful to reach the goals. However, removing *Playing Hide and Seek* did not influence the trajectory, because both have the same goals.

Drawing Sides of 4-Cube Constructions did not work to be tried out, because the classroom was so bright by the sunlight and we could not make it dark. It was difficult for children to imagine the shadow. Therefore, we decided to remove that activity. However, the essential goal of that activity was not really ignored, because in my opinion it actually has the same goal with *Drawing Basements of 4-cube Constructions*.

All explanation of the changes above can be simplified in the scheme 5.1. The red signs indicated the changes.

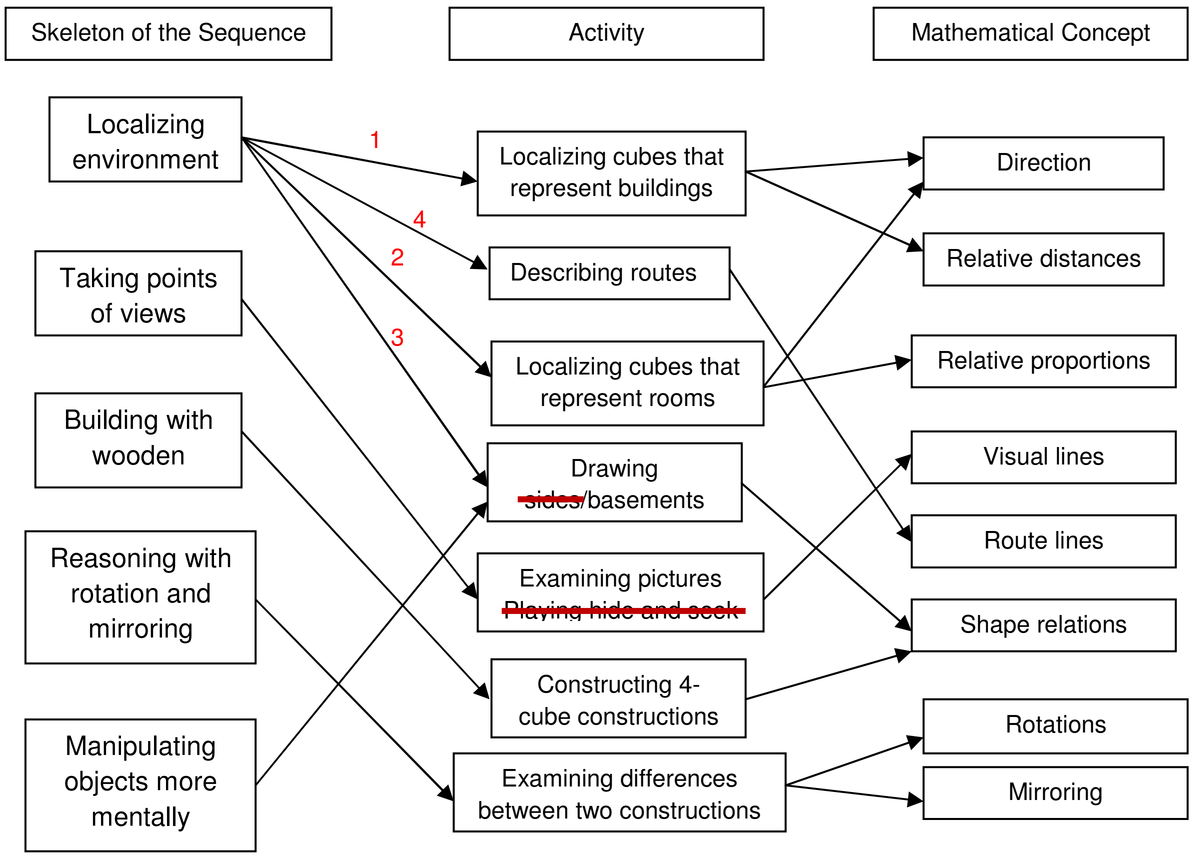


Figure 5.1. Scheme of the Instructional Design of Refined HLT

5.2. Try Out of the Refined HLT and HLT 2

Now, we will explain some interesting findings in the tryout of each activity. The findings are the data that will be retrospectively analyzed. We use the results of the analysis in these tryout activities to enrich the conjectures of refined HLT, which is HLT 2. We arrange the report of this tryout into parts: (1) *description of the classroom session*, (2) *justification of the HLT 1,* chosen data from the classroom session that justify conjectures in HLT 1, (3) *other findings*, new interesting findings that could enrich conjectures in HLT 2 in the (4) *retrospective analysis*. Finally, after explaining the tryout we will give an overview of the refined HLT, the HLT 2. The overview will keep all conjectures in the first HLT, but we add more conjectures based on our findings in this tryout activity.

Day 1: *My School Buildings*

The activity: Localizing cubes that represent buildings

Goals of the activity: after doing the activity, we expect that children are able to (1) use direction in determining the relation between buildings' positions surrounding the school buildings, and (2) determine relative distances among buildings on the scale model.

Description of the classroom session

The teacher started the lesson by informing children a contextual situation: "*In August, there will be new comers in Grade 1. You will become second graders. As the second graders, you know precisely each place in this school. You know where the toilet is, where the library is, and other important places in this school building. But, the new comers, they have not known about them all yet. Would you please help them to inform those important places? Do you have any idea how the way to help them?"*

Children seemed very enthusiastic to help the new comers, but they had no idea how to do that. We gave them hint by firstly inventory the buildings of the school. Children mentioned name of the school buildings one by one, until we dealt with 4 main buildings that needed to be listed. We put a cube on the middle of the paper grid and told them that it represented the school building. With the school as the starting point, children were asked to place other buildings on the grid. When children at work, three of four pairs represented one building by one cube as what we expected. A pair of children, Augi and Kreshna, did differently. They started with building three floor class building using three arranged cubes above. In the class discussion, we arose Augi's ideas and asked him and his partner to present their construction.

Justification of the HLT 1

Some children worked already with more cubes. Augi and Kreshna built the class building by arranging three cubes above. In the class discussion, we asked him and his partner to present their idea.

Augi : we built it in three cube heights
Teacher : tell them why did you do so, Kreshna
Kreshna : because this building consists of three floors.

As our conjectures in HLT 1, some children described the direction by pointing to a certain point. A conversation with Rafee justified that conjecture, when we asked him to explain why he placed a certain cube to represent the principal building.

Teacher : Why should it be that one?
Rafee : see… the principal's building is there (he pointed to a certain direction), in front of this building. So, the cube that represents it should be precisely in front of the cube that represents the current building.

No child listed other objects but the buildings. No evidence that justified the conjectures, that children would capture other objects, such as road, bus stop, etc on the grid paper. Meanwhile, the issue about related distance also arose as what we expected in HLT 1. We can see that in this following conversation.

Teacher : Where shall we place the *masjid*?
Kreshna : (He put a cube on the right orientation. The *masjid* is next to the current building)
Rafee : No, it is too far. It should be here (He changed the cube closer)
Teacher : Why is it not here? (I moved the cube a little bit further)
Rafee : No, it is not there. It should be here (He moved it closer again), because the *masjid* should be closer to this current building.

The struggle in the first HLT was happened. Children mostly found difficulties to congruently correspond between directions on the scale model and direction in the reality.

Teacher : show me, where the principal's building is in reality
Rafee : there, **in front of** this building (his hand pointed to a certain direction)
Bagas : (He glanced at their scale model and pointed to the wrong cube)
This one represents the principal's building.
Rafee : (He looked at that cube and thought in a while)
No, it should be another cube. This cube (He pointed to the right cube)

Other findings

We found that some children had difficulties to understand the word 'building'. They had different meanings of that word. We can see that in this conversation.

I : How many buildings do we have?
Bagas : Two

This building (He pointed to floor) and that building (he pointed to principal's building)

Kreshna: We also have a renovated building (while pointing to the building's position)

Oscar : No, I think we have six buildings. There we have two (He pointed the principal's building)… here we have three (He pointed to the current building), and there (He pointed to the renovated building) … . (Paused) Oh yes, we only have four.

Retrospective Analysis:

From some evidences, we can conclude that children have achieved the learning goals of this first activity. When Rafee localized the principal's building, he pointed to a certain direction in reality before he realized that it should be congruent with its direction on the scale model. The similar case was happened to other children. They used the words like, in front of, next to, etc to indicate directions in determining relation between buildings' positions. The second goal was also achieved. Our conversation with Kreshna and Bagas supported the conclusion that children were aware with the importance of relative distance when they do localizing. The words like, 'too far' and 'should be closer' were examples to indicate the awareness.

In this activity, children learned how to express directions into words. They also learned how to equal between directions in reality and directions on the scale model. In the conversation between the teacher, Rafee, and Bagas showed that. Rafee clearly mentioned the word 'in front of' when he explained relation between the class building and the principal's building. Meanwhile, he also pointed to its direction in the reality to emphasize it. However, Bagas struggled to match those directions. What Rafee did by pointing the direction in reality helped them to finally realize that they should put the cube on the same direction.

A struggle about languages also had important role in refining the HLT. Children actually had different meanings about the word 'building'. The meanings may come to different interpretations about building. We, as the researcher, have a meaning of a building as something that has walls and a roof. So, we determined the *masjid* and the renovated building also as buildings, because they have those properties. However when we asked children number of the school buildings, they came up with various answers. For Bagas, there were only two buildings in the school, the class building and the principal's building. It may be because for him, a building should have official purpose and it should be a complete building. Therefore, for him the *masjid* is not a building because it has religion purposes, and the renovated building is not a building either because it has not completed yet. For Kreshna, the renovated building has completed the properties of a building, even though it was still renovated. Meanwhile, Oscar has different interpretation of a building. When he said 'two' for the principal's building, that is because that building has two floors. And 'three' for the current building, that is because it has three floors. He was confused between building and floor.

In the class discussion, when we asked Augi to present his class building, he came up with a reasoning that was related with reality. In the first HLT, we expected that this kind of reasoning emerged in the second activity. Thus, this evidence was good to use as starting the following day activity and to inspire other children about how to reason the proportion.

Day 2: *Where Our Classroom Is*

The activity: Localizing cubes that represent rooms

Goals of the activity: after doing the activity, children are able to: (1) use direction in determining relation between rooms' positions of the school buildings, and (2) determine relative proportions of buildings' sizes on the scale model.

Description of the classroom session

The previous day try out emerged Augi and Kreshna's buildings that was different with others. This difference was brought to the floor to start the lesson in the following day: "*Look… Augi and Kreshna's work is different with the others. When you see your own, can you determine where our classroom is?* (Children shouted: No). *Well, can anybody reconstruct the buildings so everybody can see where our classroom is?*"

We guided them by listing the rooms one by one from the ground floor. Beginning with class 1A, they placed the cube of it. Now on, one cube represented one room in the school. At the rest, children worked in pairs to reconstruct their school building so that each important room was recorded on their scale model. Children seemed more enthusiastic, because they worked with more cubes. They were able to record each room in detail places.

During the discussion, Rafee and Bagas presented their work. Afterwards, other pairs were asked to read Rafee and Bagas' scale model. Using the scale model, they had to determine which cubes represent some other important places in the school.

Justifications of the HLT 1

There was a conflict about the shape of the school, as our conjectures in the first HLT. It was not about the proportional sizes of the buildings, but it was about a detail shape of toilet's locations that is not in a line with classrooms' positions.

Teacher : are you sure with this position?
Augi : yes
Teacher : kreshna, would you please go outside, where is it really?
Kreshna : (He went outside the classroom and checked the real situation)

It is correct in that direction, but it should be a little bit not in a line with the classrooms.

Teacher : where should it be?

Kreshna : (He moved the cubes that represent toilets a little bit not in a line as it in the reality is)

They did not reason the sizes of the buildings using the reality, but they still used the reality as reasoning of shaping the scale model.

Other findings

Some children created the whole school model. They used the cubes to represent other references in the school, such as *pendopo*, the gate, the parking area, etc.

Retrospective Analysis

This activity did not bring many struggles for children. Inventory all the rooms in the school seemed easy for them, because they were already familiar with the detail of the school. They already memorized all rooms in the school and this was good starting point to do this activity. All children had the same idea that a cube represents a room as in the instruction. No child tried to put more cubes for bigger rooms. Therefore, children easily achieved the goals of this activity.

Beside they constructed four main buildings, they started creating the whole school model and adding more surrounding building in school area, such as *pendopo*, the gate, the parking area, etc. This fact brings the idea that a cube does not only use to represent a close room in the school, but it can also be used to represent an open room as the parking area. A pair of children that constructed a gate using cubes shows evidence of children thinking about representing the cube. Constructing cubes to be a gate means that cubes are used as representation of lines, not only as representation of rooms.

Day 3: *Removable Buildings*

The activity: Drawing Basement of the School Buildings

Goals of the activity: after doing the activity, children are able to determine 2D shapes of basement (squares) from its 3D constructions on the scale model.

Description of the classroom session

We introduced the condition in where they have to move the scale model since the desk is needed for another lesson. "*Yesterday, you all did a good job. However, all buildings have been removed from the desk. That is why we need to reconstruct the building again. But, can you imagine? How tired we are if we have to reconstruct all buildings over and over again. Besides, another problem is how to keep all buildings in exact positions as they were before. Would you guys have any idea? How can we easily rebuild the buildings in the same* *positions on the grid papers?*". The idea to draw the basements of each building came from the children.

When children worked in pairs, they produced different drawings (see appendix 1). At the end, we together with the children concluded the easy way to sign the basement. Since children had different ways of drawing the basements, we asked to the whole class to determine which the easiest drawing was to see the condition of our school. Most of them chose the work of Oscar and Quin, because for them it was the easiest to read and the simplest one.

Justifications of the HLT 1

All children did the same strategy to sign the paper. Every time someone took a building of the scale model, they drew a sign on the underlying paper to mark buildings' positions. A nice example from Augi and Kreshna was that firstly, they only drew squares and gave name for each building.

Teacher : I saw you give name for each basement beside draw the square of it. Is that important enough?
Kreshna : yes, it makes us know what building above this basement.

Children, as our conjectures, felt difficult to come up with the idea about the issue of how high the building is. In detail drawing when we posed this issue, they had no idea how to sign the basement so that everybody knows how many floors the building has by only watching the basement drawing.

Other findings

We found various drawings that out of our expectations (see appendix 1). Since the ideas of giving signs came from the children, the discussion did not produce any clear ideas about how it should be when we asked their ideas.

Kreshna : Ahh…I see. We can give signs on the cubes.
Teacher : how?
Kreshna : (Puzzling) I do not know …
Oscar : Hmmm…we have to remember the exact places of the cubes
Teacher : yes, that's right. Is there anybody else with different idea?
Rafee : Make drawings … . We can draw on the grid paper.
Teacher : What a great idea, what kind of drawings can you use?
Rafee : Draw signs on the paper grids right below the cubes placed.

After this following conversation, Augi and Kreshna completed the drawing.

Teacher : but how do you know that a certain building has higher floors?
Augi : the names help us. For this building (He took cubes that represent the current building), it is named 'our classroom' so we know that it should have three floors.
Teacher : how do other people know that?

Augi : (He drew three crosses to represent the number of floors)
Teacher : what are these arrows for?
Kreshna : to emphasize that here and there we have buildings.

At the end, we concluded the easy way to sign the basement.

Teacher : what we have learned today? How are easy ways to recognize the school buildings on this grid paper?
Kreshna : give a sign
Rafee : give more signs for more floors
Augi : give a name

Retrospective Analysis

The contextual situation led them to create a basement for each building. The conversation between the teacher and children showed us that they agreed making drawing on the grid paper to determine positions of removable buildings. They closed discussion by an agreement to draw a sign on each building's location. However, each child had their own meaning about "the sign". Therefore, we found various signs from their drawings (see appendix 1). We can conclude those various signs below:

(1) Any symbols and name of the building, without clearly determining the height (for example: Salsa and Fia's drawing). We can see in their drawing, basements figures are not only areas covered by cubes but they widened each basement figures to give space for title the name of buildings. However, they emphasized the exact covered area using check on each square on where they built cubes.

(2) Crosses on each square as many as the cubes above it (for example: Rafee and Bagas' drawing). We assume that they indirectly realized the relation between cubes and squares. However, the awareness of shapes relations becomes less because they worked on a grid paper that is already squared. We suggested that for this kind of children, you can ask them to reconstruct the scale model on a blank paper so that it more encourages them to draw squares rather than other signs.

(3) Squares and other symbols, such as crosses (for example: Oscar and Quin's). In their drawing, they firstly decided to make a square for representing the basements. During the activity, Oscar saw Rafee and Bagas' work. They insisted him that their way in signing the basement was the correct one. Oscar drew the crosses under their influence, but he did not really understand that the crosses could help them to determine the height of the building.

(4) Squares, clear name of each building, crosses as many as above cubes, and some arrows (for example: Augi and Kreshna's). From the conversation between them and the teacher, firstly they argue that giving name of each building was enough to make people

know the height of the building. With their limit explanation, we assume that they visualized the direction of each building using the arrows. They did not have exact languages to explain the directions, but by using the arrow they made other people know that two buildings (principal's building and class building) are facing each other. In the higher level, this kind of arrow can be found in a map as wind direction which makes everybody knows the exact direction of something. Small arrows that they drew in each building inform entries of each building. This interesting finding supposes those children in higher level of visualizing directions. Besides, they also achieved the goal of this activity by drawing squares in representing the basements of cubes.

In the conclusion, giving arrow does not become an important issue in children's point of view. Because for them, without the arrow, all those three requirements are enough to recognize that the drawing is the scale model of their school buildings. Therefore, most of children thought that Oscar and Quin's drawing was the best one.

Some evidence shows us that generally children achieved the goal of this activity. Although some children used other symbols, instead of drawing squares, we assume that they actually realized the shape relations between cubes and squares, because the symbols were drawn on each square of building's location on the grid paper. The paper was already squared. Therefore, they thought that drawing squares were not really necessary.

Day 4: *Make a Route to an Important Place in the School*

The activity: Describing Routes

Goals of the activity: after doing the activity, we expect that children are able to (1) describe directions in explaining a route among two buildings into words, and (2) visualize a path of ways using line(s).

Description of the classroom session

The teacher started the lesson by performing a contextual situation: "*Today's lesson is to more help your new comers to know the ways to go to some important places in this school. For doing so, I will give each pair a name card. In that card there is a name of a place in our school, for example: the library, the park area, etc. Your duty is drawing a route for them to show the way from your classroom to that certain place.*" Each pair randomly chose a name card. They worked in pair to draw the route, from their classroom to the written place in the card. They produced various route drawings and various ways to express the directions. After they finished the route drawing, they had to read and explain it by describing the directions into words.

Justifications of HLT 1

Mostly, for the first time explaining the route, children faced difficulties to put directions into words. To handle that difficulty, some children used their route drawing to explaining the directions. As our conjectures in HLT 1, some of them used words of directions and other references to make their explanation more clear, like Oscar and Quin when they explained a route from the classroom to the principal's room.

Teacher : can you help Quin? Do we directly go down stairs from class 1A?
Oscar : no, we need to walk a little bit. Then, we turn to this (he made bend of his hand to show the direction of turning), there you can find the stairs to down stairs.
Quin : from 1A, we go down stairs. Here, in the middle is the yard. Go through the yard, you will find the principal's room.

The similar evidence was when Augi explained the route from the classroom to the library.

Augi : from class 1A, we walk like this (He made bend of his hand that shows turning to the right). Then, go down stairs, afterwards you can turn to the left. Go on straight away, and turn to the right. Go up stairs. If you turn there will be other stairs, go up. There will be the library, but you need turn to the right.

Other findings

Rafee and Bagas used their drawing to help them explaining the route directions.

Rafee : this is class 1A so turn to this, and this, turn this, and here. (While showing the route on the paper by their hands)
Bagas : this to this to this … (while drawing a path line on the route drawing)

As we expected in the first HLT, children produced various route drawings. Our conjectures about how various they are were not clear yet. However, by children's drawings (see appendix 2), we can complete those conjectures.

Retrospective Analysis

We found four different levels of explaining a route.

(1) By pointing to the route drawing as a help to give more explanation. Rafee was firstly in this level when he explained about a route from the classroom to the park area.

(2) By drawing a path line on the route drawing. Bagas did it to emphasize and try making his explanation more clear.

(3) By combining gestures of the movement to describe the direction of turning and mentioning other references. Oscar and Quin could not express the words "to the left/right". Therefore, they indicated the directions by bending their hands. Afterwards, they also used other references, such as stairs, the yard, etc to indicate a point of the walker's arrival after the turnings.

Children in those three levels were struggling on putting directions into words, as our conjectures of their struggles in HLT 1 within this activity.

(4) By combining formal languages of directions and other references (as mentioned in the HLT 1). Augi did it very well. Although he firstly still bent his hand to express the turning direction, afterwards he could perfectly explain the direction using formal words and other references.

From those four evidences, we see that nobody use only their gestures of movement when they described a route. The first conjecture of HLT 1 did not emerge in this tryout classroom. Children always combined their gestures with words.

When children drew a route, we found that all of them use different lines to represent roads of the path. By comparing their drawings (see appendix 2), the only one similarity is the path lines. However, there were three specific drawings of a route they made:

First, only using a straight-single line to visualize the path line, as Salsa's

Second, using two curves to visualize the way of path, as Oscar's, Augi's, and Rafee's

Third, using other references beside the line(s). For example, Rafee drew the *masjid* to emphasize that they will through it, even though the *masjid* is not the starting point or the end point of the route.

From aforementioned explanations, we assumed that children could achieve the goals of this activity. Within the activity, they already learnt to express the direction into words and drawing path lines. Those indicate that they grew their spatial reasoning, both by visualizing and by finding proper words to express directions.

Day 5: *Mini Lesson – Where Do You Have to Stand?*

The activities: Examining Pictures of the School Buildings

Goal(s) of the Activity: after doing this mini lesson, children are able to (1) use a vision line as reasoning for arguing different point of views, and (2) argue different shapes of the school buildings from different point of views.

Description of the classroom session

We started the lesson by informing children about this situation: "*Yesterday, I took some pictures of your school when you were in holiday. Now, I'm wondering where I was standing when I took those pictures. Would you help me to guess where we have to stand to see our school from this point of view?*". We consecutively showed some pictures of the school buildings (see appendix 3), while children had to guess where they had to stand so that they could see parts of the school exactly as in the pictures.

Justifications of the HLT 1

No evidence could justify the conjectures in HLT 1. The discussion went in the wrong way.

Other findings

The class discussion was led only by 'where' questions, as below:

Teacher : first picture, where do we have to stand for seeing this part of the school?
Bagas : in front of our classroom
Kreshna : no, we have to stand in front of the school, outside.
Teacher : how about the second picture?
Kreshna : we have to stand on the yard
Teacher : great! And next picture
Augi : we have to stand in front of the class
Teacher : which class?
Kreshna : more precise, it is in the corridor of the classroom
Teacher : well done kreshna. How about this fourth?

And so on….

Retrospective Analysis

After doing this mini lesson, we reconsidered what we have done. We realized that we only asked questions and only hear the right answers. However, we should have asked the reason of children's answers. That makes the goal of this activity is difficult to measure whether it was already achieved or not. Considering what we should have done to dig more understanding of children's thinking, we should do:

First, the 'why' questions that is powerful to dig children's reasoning, for instance, we are standing in front of the class because we see the *masjid* in the left of this picture, etc.

Second, drawing or descriptions for children's answer, so that children do not only mention about other references names surrounding an object in the pictures, but they can also argue that the differences because a vision line from different point of views of the objects. This is the crucial issue in this activity.

Children could not achieve the goals in this activity. We could not know children's thinking either. However, for this last activity of orienting, we decided to use conjectures in HLT 1 in the refined HLT, even though they were not justified in this tryout session. Now, we had to continue to the next part of another trajectory about orienting.

Day 6: *Different 4-Cube Castles for Thumb Princess*

The activity: Constructing 4-Cube Constructions

Goal(s) of the Activity: after doing the activity, children are able to manipulate shapes of 4-cube constructions.

Description of the classroom session

A story about Thumb Princess began the activity: "*Have you ever heard the story of Thumb Princess? Once upon times, there is a Thumb Princess. She is named so because she is as small as a thumb. Thumb Princess is so pretty. She likes traveling very much. One day she meets an Elf Prince and they are married. They are so happy. Thumb Princess wants to build castles, a lot of castles for their children. However, the king gives a rule for the castle buildings. That is they are not more than 4 cubes for each building. Would you please help Thumb Princess and Elf Prince to figure out as many as different constructions of castles with 4 cubes?*"

Children produced different numbers of 4-cube castles.

Justifications of the HLT 1

A small clips on the video recording shows that Alsa firstly constructed only flat castles. Then, her partner's construction, Rara, inspired her to construct high castles. This evidence justified our conjectures in the first HLT. However, in this tryout we did not find any evidence about using rotation as the way of constructing the buildings, and children's arguments that two rotated buildings are the same either.

As the conjecture, some children faced difficulties on determining the differences between two constructions. The difficulty was not about two rotated constructions or two mirroring constructions, but it was about many similar constructions. Most of them only constructed 4 cubes, but they did not realize that they made one construction more than once.

Other findings

We found that children did trial and error in the constructions. No child really realized that they could rotate or mirror a construction to get another construction. We found some children were difficult to understand the instructions. They did not realize that the castle was limited with only 4 cubes, for example Michelle and Sheila. At the beginning, these girls built many castles with more than 4 cubes. They created their own castles. In the end, this pair could build 16 4-cube castles, but some pairs of the constructions were same.

Teacher : what a very high building! How many castles do you have so far?
Michel : these are two castles
Teacher : how many cubes do you use for each castle? Do you remember the rules? We have to be saved and use only 4 cubes for each castle. See… you use more than 4 cubes. Can you reconstruct your castles?
Michel : sure (She started using 4 cubes to construct one castle)

After some times, they enthusiastically built many 4-cube castles. Some of their constructions were same after rotating.

Retrospective Analysis

Some clips show trial and error when children constructed the castles. Later on, we use this evidence to enrich the conjecture for the refined HLT. Some conjectures did not appear in this try out activity, because we did not encourage children to reason the difference between two rotated constructions or two mirroring constructions. We allowed them to manipulate the cubes with their own ways. Therefore, the goal of this activity was not really achieved yet. However, we continued with the following activity and observed whether they could realize the role of rotations and mirroring or not.

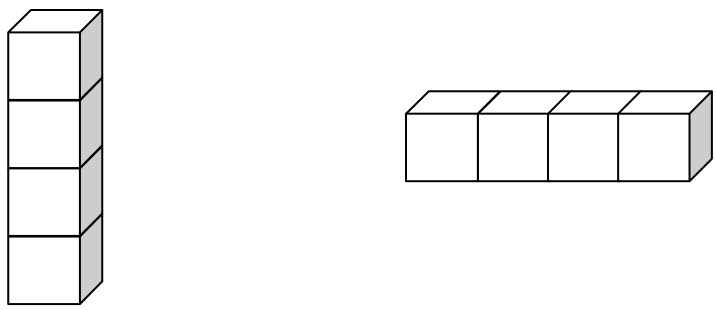
Day 7: *A Math congress - different or not?*

The activity: Examining the Differences between Two Constructions

Goal(s) of the activity: after the math congress, children are able to use rotation and mirroring as reasoning the differences or the similarity between two constructions.

Description of the classroom session

We opened today math congress with this opening: "*Yesterday, all of you did good jobs. You have constructed many castles for Thumb Princess. Do you remember that we have to make different castles? It is to save the area of the kingdom. So, same castles should be removed and leave only one of them.*" Afterwards, we started the discussion by constructing two castles below that will be confronted.



"*Yesterday, I saw these two castles were made by all pairs, the high castle and the long castle. Do you think they are same or different? If you have houses like those castles, do you think the houses are different or not?*". Those questions challenged children in the discussion.

Afterwards, we showed these two constructions, and following by a question: Are they same or not?

Justifications of the HLT 1

A conversation between the teacher and two girls, Alsa and Michel, justified the first conjecture in HLT 1 that children use reality as reasoning the difference between two first constructions. They argued with imaginatively thought that the constructions as real buildings, when we asked them their difference.

I : why?
Alsa : because the first one has an elevator.
I : so, are they same or not?
Alsa and Michel: no…

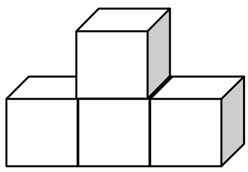
For the second constructions, Michel argued that they are different.

Michel : I think they are different
I : why do you think they are different?
Michel : because the first one has **two floors in the right**, and the second one has **two floors in the left**.

Other findings

In a small clip, we found an interesting discussion between the teacher, Arsyi and Alsa when they argued that the second constructions are the same.

Arsyi : yes, they are same
I : why?
Arsyi : because both of them have the same shapes.
I : how do you think, Alsa and Rara?
Alsa : yes, they are same
I : why?
Alsa : because both consist of **three cubes below and one cube above**.
I : okay, how do you think about this building … (I constructed another building below)



Do you think it is same with the previous two?
Rara : no, they are different.
I : but you said that it will be same if it has three cubes below and one cube above. I think this new building has those requirements.
Rara : it is different because **the above cube is in the middle**.

Retrospective Analysis

We realized that the questions in the first discussion were about the real buildings. It makes a big difference if we talk in the real perspective, because the real buildings cannot be rotated. That is why children thought that those two constructions are different. On the other side, if we talked them as constructions, we could argue using rotation if they are actually the same constructions.

Michel argued that the second constructions are different. She used the words **two floors in the right** and **two floors in the left** to argue the difference. Those words indicate that she used the shapes of the constructions that seem different, as reasoning of the difference. The similar reasoning came from Alsa, her partner. Alsa argued that those constructions have the same shapes. The words **three cubes below and one cube above** indicated she used the shapes of the constructions to determine the difference. Although Alsa and Michel used the same way of reasoning, which was using shape of the constructions, they came up with different arguments. Michel thought that the constructions are same, but Alsa did not. The word "floors" that Michel used indicates that she referred to the reality, and imagined the constructions as real buildings. Thus, she argued those are different.

The two second constructions are also about rotated building. One could argue that those two constructions are mirror constructions in which its mirror plane is parallel. However, we defined those constructions as rotated constructions because they are easily seen can be rotated along x axis. In this second discussion about similarity issue, I guided the discussion focusing on the shapes of constructions and not relating them with real buildings.

Day 8: *How is the basement and how high is the building?*

The activity: Drawing Basements of 4-cube Constructions and Determining the Height of the Buildings

The goal of the activity: children are able to determine 2D shapes of basements of the constructions (squares) from its 3D constructions.

Description of the classroom session

We started the activity by continuing the story as a contextual situation: "*Today, Thumb Princess has a new problem. Her castles should be removed and reconstructed. In order to remember how the castles were taken place, what can we do to help her? Now, try to make their basements. Basement is the basic of a castle, how the shapes under the castle's area. As what Arsyi said that we can slide a castle first, and then draw its basement to remember its previous place. Can you all do that?"*

Arsyi : we can slide the castles to other places and remember their previous places.

After each child finished her/his basement drawings, we asked them to do a role play. Each pair exchanged a role with her/his partner. A child in a pair should build castles on basements that her/his partner made and vice versa. This role play was going well. They worked together cooperatively. A basement maker told his/her partner whether his/her construction above the basement was wrong or right.

Justifications of the HLT 1

Each child was sliding a castle before (s)he made its basement. Each pair produced different model of basements. Most of them did not face difficulties in sliding and drawing the basements' shapes, until we posed another issue about how high the castle is. Every child was struggling about the height issue. Discussions among pairs helped the rest. Akzal helped Arsyi, Rara helped Alsa, and Michel helped Shella.

Other findings

We discussed with children about what kind of signs can be used to determine the height of the building. A child, Akzal, suggested an interesting idea.

Teacher : is there anybody who has an idea how we can know how many floors above the basement?
Akzal : I think we need give a sign in the square.
Teacher : what kind of sign?
Akzal : like a number, that represents how many floors above.
Teacher : could you explain more?
Akzal : like in this castle (pointing to a castle as in appendix 5 number 1), we can write number '4' inside the square. It means there are four cubes above this square.

Retrospective Analysis

In this activity, a conjecture of HLT 1 in the fourth day activity about drawing basement of the school buildings emerged again in this activity. The conjecture mentioned that children may mark the square with numbers that represent height of the building on the basement. The conversation with Akzal shows that the idea of giving number on the basement drawing could come from the children. Akzal clearly explained how to draw a proper basement so that everybody knew and imagined how the building on it. He combined the idea of drawing squares with giving numbers. His explanation showed that he mentally has a good ability to see the relations between the square of basements with the cubes on it.

Day 9: *A Mini Lesson - Draw the Possible Basements*

The Activity: Drawing Possible Basements of A Given Figure

The goal of the activity: after doing this mini lesson, children are able to more mentally visualize the basements of a given figures.

Description of the classroom session

The teacher began the lesson with: "*You constructed many castles with very various constructions. Afterward, you drew basements of the castles. Yesterday, I made some* *drawings of your castles. Now, I want you to guess, how the basement is from each castle drawing that I will show to you. Today, you have to work individually, not in pair anymore. It is okay to have different drawing of one castle. We will discuss it later on.*"

We gave each child a grid paper and castle figures (see appendix 5). Firstly, they worked individually. After they finished, we started the mini lesson by discussing the figures one by one, their results of the basements and their reasons behind the drawings. During children at work, we posed question and led discussion to understand their thinking of the drawings. In general, children did not have any difficulties to do the task.

Justifications of the HLT 1

By looking at children's drawings in appendix 6, we can see that all children drew only all clearly shown cubes on the constructions. They did not think about possible hidden cubes may exist in the figures. Thus, only the first conjecture in HLT 1 was justified with this evidence.

Other findings

There were two children, Alsa and Arsyi, who faced difficulties to interpret the 3D figures. Alsa drew four squares vertically rather than one square for the basement of the first picture. She changed her drawing after she looks at Rara's drawing. She was not confident with her answer, though.

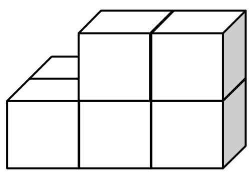
Teacher : Alsa, how many squares as its basement? Do you think it's four or only one?
Alsa : four (but in her drawing she has one square)
Teacher : why?
Alsa : because it has four cubes

Meanwhile, Arsyi drew the basements of castle number 5 as below:

1		
1	**2**	**2**

Teacher : Arsyi, what does those '2' mean?
Arsyi : it has two floors above
Teacher : where is its above?
Arsyi : (He pointed to two cubes that lay down side by side)

We helped him by giving him cubes and asking him to construct the castle above his basement. And his castle became:



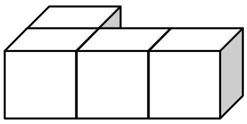
Teacher : is that the castle we want?
Arsyi : No… it's going wrong then (puzzling)
Teacher : (we took away two cubes above) if I take away these two cubes, is it the same castle as in the figure?
Arsyi : Yes, it is the same
Teacher : is everybody agreed?
Akzal : no
Teacher : why do you not agree, Akzal?
Akzal : (he moved the laid down cube above so that it seems as picture 5) because it should be above, here, not laying down there. The picture shows the building has two floors, does not lay down like that.

Retrospective Analysis

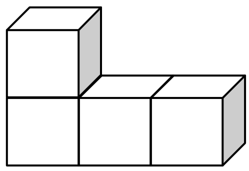
From the new findings, we could conclude two other struggles that might appear in this particular activity.

Struggle 1: Interpreting figures of non-flat constructions.

We found that Alsa had 'top view' interpretations when she drew figures numbers 1, 2, 5, 6, and 7. To understand what in her mind, we gave her cubes to build the castle number 5 and she constructed the cubes as the figures below:



rather than



Here, we can understand that her faults were because she had the wrong interpretation of the 3D figures. She thought the figure number 5 seen from the top. That is why she firstly drew the basement of castle number 5 as below:

1		
1	**1**	**1**

rather than

2	**1**	**1**

The same case with Arsyi, when we took away two cubes above his reconstruction, he thought that the flat construction was right for number 5. However, they achieved the understanding of shape relations between cubes and squares. Beside her 'lay out' interpretation, they understood that the basement should be in square(s) because they looked them as sides of the cubes.

Struggle 2: determining buildings' heights on the basement drawings

Arsyi also had another crucial issue about understanding of number of height. Although he could explain that '2' means that the building on that number has 2 cubes, he did not understand the 'meaning' of two cubes above.

In conclusion, all evidence in this tryout could enrich the previous HLT. We refined the HLT 1, and used other new findings to complete our conjectures in each activity. We used the results to redesign the new HLT, HLT 2. The table 5.2 is the refinement of HLT 1, which is HLT 2.

Day	Name of the Activity	Activity	Conjectures of Learning Process	Conjectures of Struggles
1	My School Buildings	Localizing cubes that represent buildings	1. Using hands and gestures to describe directions 2. Listing other objects 3. Having various relative distances	1. Congruency between direction or distance on the scale model and those in the reality 2. The meaning of a word, *building*
2	Where Our Classrom Is	Localizing cubes that represent rooms	1. Using hands and gestures to describe directions 2. Listing other objects 3. reasoning the proportion using the buildings' sizes in the reality	A conflict is about shapes of the school buildings
3	Removable Buildings	Drawing basement of the school buildings	Sliding the construction before drawing: (1) squares and marking number of cubes on it; (2) Giving names for certain buildings; (3) any symbols without the height; (4) crosses on each square as many as the cubes; (5) complete signs: square, signs of the height, name of building, etc.	Determining height of the buildings
4	Make a route to an important place in the school	Describing Routes	1. Using only the gestures of movement 2. Using words of direction and other references 3. Using the route drawings to describe directions; pointing to it or drawing lines on it. 4. Combining gestures of the movement, words of direction, and other references 5. Having various route drawings; (a) only straight line(s), (b) two curves or bent lines, (c) lines/curves with other references	Putting directions into words
5	Mini lesson - Where do you have to stand?	Examining pictures of the school buildings	1. Indicating a visual line by pointing to a certain location 2. Reasoning using reality 3. Using a line as a connection between two points	1. Using a straight line as reasoning of a vision line 2. Missing the "why" questions in the discussion
6	Different 4-Cube Castles for Thumb Princess	Constructing 4-cube constructions	1. Constructing only flat buildings 2. Constructing various buildings, not only flat 3. Constructing by trial and error	1. Determining whether two constructions are diferent or not 2. Undestanding of 4 cubes as limitations
7	A Math congress - different or not?	Examining the differences between two constructions	1. Using reality for reasoning the differences 2. Rotating a construction to see the differences 3. Using relations among cubes to argue the different shapes between two constructions	Arguing the difference
8	How is the basement and how high is the building?	Drawing basements of 4-cube constructions and determining the height of the buildings	1. Pointing to a certain square before marking the height 2. Rebuilding the constructions to check whether the drawing is correct 3. Sliding the construction before drawing the basements 4. Making squares and marking number of cubes on the basement figures.	Not automatically thinking of making basement with height numbers
9	A Mini Lesson - Draw the Possible Basements	Drawing possible basements of a given figure	1. Drawing: (a) only all clearly shown cubes on the construction; (b) possible hidden cubes; (c) all hidden cubes are not drawn 2. Rebuilding the constructions to check whether the drawing is correct	1. imagining how the complete shape of the building in the figure 2. interpreting figures of non-flat constructions 3. determining building's height on the basement drawings

Table 5.1. Overview of the HLT 2

5.3. General Setting and Pretest before Class Experiment

5.3.1. Pretest

Before the first activity, children in the class experiment took a 30-minute test (see appendix 7). This test was used to get an impression of their prior knowledge and their prior ability in orienting and constructing. We discuss the purposes of making the pretest and the results per question.

The first and second question was about solving maze. They are related with ability of orienting. We designed it based on Indonesian kindergarten curriculum (2006) that mentions that after graduate from Kindergarten, children have to be able to solve simple maze. All children were able to solve the first one, but two children could not solve second maze. They directly drew a line to the finish and could not see the obstacle around the doctor. At that time, we assumed that by being able to solve mazes, children already had a good starting point to do the activities about directions and routes.

The third and fourth questions were based on one familiar game in Indonesia, playing hide and seek. The questions were designed to see their ability on taking points of views. Besides, we tried to dig reasoning within these questions. Most children argued the first house picture was a proper place to hide. Some children chose the second one, but they could argue that the place behind the wall, outside the house, could be a proper place to hide. The evidence indicated a good start for activities taking point of views. However, most children did not write down their reasoning, because they were not able to write yet. They told it to us. Therefore, it made us reconsider the written assessment as an additional assessment, not as the main assessment in this research.

The second part of the pretest was designed to measure children's ability in constructing. In general, we could conclude that children mistakes in those questions were about: (1) interpreting rotated figures, and (2) interpreting 3D figures. Therefore, within the activities of constructing, which were started with concrete materials, we expected that they could improve their ability in manipulating objects by rotating or other transformations, and better interpreting 3D figures.

5.3.2. General Setting of Refinement the HLT 2

Before we conducted the research into class experiment, we made some observation to see the classroom norms, both its social norms and socio-mathematical norms. We considered some changes to adjust our HLT 2 with the class conditions. Some contexts, again, were adjusted with the real situations of the school. For example, because the school is a new school that has been renovating all the times, a building was changed and some rooms' positions were changed as well. A renovated building was being the kindergarten building. However, there was one day school orientation in which children were walking around the school to be introduced all rooms in the school. This is a habit in some Indonesian school every new school year.

We made the changes were because:

(1) Social norms and socio-mathematical norms of the class experiment.

Children in this class were not used to engage in the class discussion. Our observation showed that during the mathematical classroom, they were enthusiastically engaging the competition in the math lesson. When class discussion was started, they were getting noisy and did not pay more attention to whom presenting their ideas. Here, we saw that the social problems were about respectful on sharing ideas and listening to others.

We wanted to change this condition. In the second day activity, we tried to build up a class discussion, because it did not work well. We reduced our standard. Instead of asking them to present the idea in front of the class, we asked them to discuss in pairs most of the times. This affected some designed activities. We removed the discussion in the end of the lesson. As consequence, we more focused on observing the discussion in each pair.

Other norms that affected changes of the design were about the habit of competition and giving award. The teachers were used to give a 'gift' for a child who can behave and answer the questions. In this classroom, children could not stay calm more than 15 minutes. The award controlled them to follow the lesson and the competition kept them paying attention. Therefore, we utilized those habits for refining the activities. We used a competition as an approach for some activities, such as for *localizing cubes that represent rooms, examining pictures of the school buildings, describing routes,* and *constructing 4-cube constructions*. We made competitions for doing those activities and we gave them gifts for whom did them well. However, at the beginning of each competition we always started with a contextual situation to give them meaning for doing the activities.

(2) Unachieved goals

Some goals in some activities could not be achieved well. The goal to determine relative proportions of buildings' sizes on the scale model on the second activity could not be achieved classically. Therefore, we redesigned the following day activity. We made a *competition of localizing place* that included two activities: *localizing cubes that represent rooms* and *describing routes*. We redid the *localizing cubes that represent rooms* and focused on achieving the goal that was not achieved yet in the previous day.

Another goal that was not achieved well was a goal to describe directions in explaining a route among two buildings into words in the third day activity of *describing routes*. Therefore, in the following day we repeated that activity and combined it with *examining pictures of the school building*. After children examining pictures of a school building taken from different points of views, we asked them to describe a route into words from the kindergarten building to that building.

(3) The more limited times in August 2008

Based on Indonesian National Curriculum, geometry should be taught in the second semester. It was really a problem for us, because we worked with children in the first semester. However, the headmaster gave us permission, but she strictly only gave us nine days for doing the activities. We, together with the teachers, arranged the experimental days. Unfortunately, we only could do 6 days out of 9 days. In August, there were holidays for celebrating Indonesian Independence Day. Two days of meeting were used for celebration. And one day meeting was used for monthly assessment, a school program. Thus, we redesigned the activities within 6 days and tried to reach as many goals as possible.

We combined some activities and some goals on several days. On the third day, we did *localizing cubes that represent rooms* and *describing routes* to reach three goals of those activities, which were to (1) determine relative proportions of buildings' sizes on the scale model, (2) visualize a path of ways using line(s), and (3) describe directions in explaining a route among two buildings into words. On the following day, we combined *describing routes* with *examining pictures of the school buildings* to reach the goals, which were to (1) describe directions in explaining a route among two buildings into words, (2) use a vision line as reasoning for arguing different point of views, and (3) argue different shapes of the school buildings from different point of views.

In summary, the changes of the instructional design to refine the HLT in the class experiment were described in the following scheme 5.2. The green signs indicate the changes.

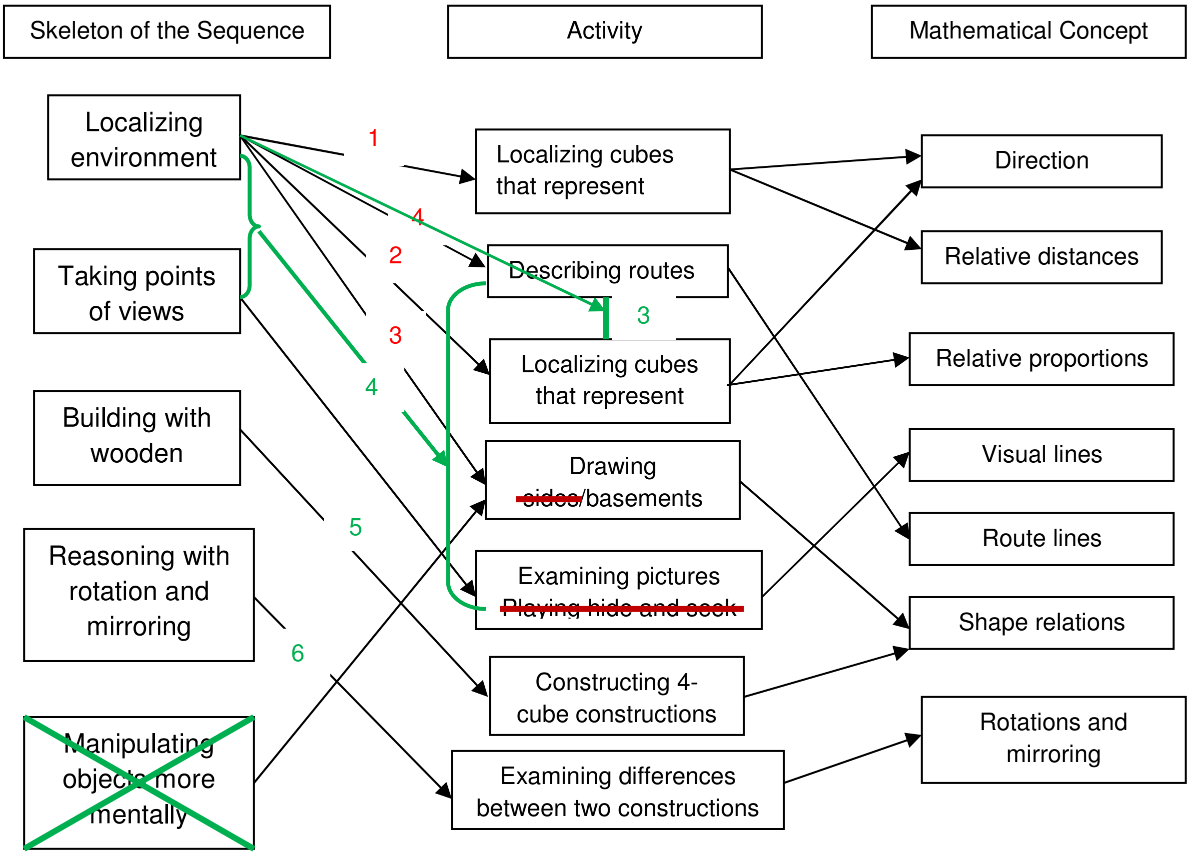


Figure 5.2. Scheme of the Instructional Design of Refined HLT 2

5.4. Comparison Refined HLT 2 with Actual Learning

Day 1: *My School Buildings*

The activity: Localizing cubes that represent buildings

Goals of the activity: after doing the activity, we expect that children are able to (1) use direction in determining the relation between buildings' positions surrounding the school buildings, and (2) determine relative distances among buildings on the scale model.

Description of the classroom session

The teacher brought and showed a red box in front of the class that made all students paying attention and they were curious to know what are inside the box. *This box will be opened if you can behave. Do you want to know what is inside? Who can guess it? Before we open the box, I want to say something. Who did school orientation last week?* Some children raised their hands. *Is anybody that can tell rooms in the school?* Children enthusiastically mention one by one each room in the school. After inventory four main buildings of the school, she started posing a meaningful problem: *Tomorrow, I need your help to guide kindergarten students walking around this school. How to guide them so they will not be lost?* No children listened to her question. The teacher took the red box. *Now, help me to open this box and count it*. Children together counted: one, two, three. The teacher showed a small box of cubes. *I will hand over this small box to whom that can still behave. I need your help to put those four main buildings on this grid paper. We have this building, that (pointing to the kindergarten building), that (pointing to the principal building), and masjid. The buildings are from these wooden cubes.* We handed over the cubes and the grid paper. One pair got a small box of 12 cubes and a piece of grid paper. Children directly started working in pair.

Justifications of HLT 2

Because the teacher only informed that '*The buildings are from these wooden cubes'* without gave any instruction about how to represent one cube, there were many interpretations in children's mind about how one cube being represented. Some children represented one cube as one building. Some of them expected one cube as one floor, so they constructed a building with three floors using three cubes arranged above. Some children already did detail construction in which one cube represented as one room.

Linguistic problems also appeared during the discussion, which was also about the word 'building'. When the teacher concluded that the school has four main buildings, Raihan complained. He asked that the principal building has two buildings; what is in fact it has two floors. This is as the first presumption of possible struggles in HLT2. However, for Raihan was not too difficult to deal with the word 'building', because in the meantime of discussion he changed his mind and agreed that the school only has four main buildings.

Most children described a certain direction by pointing to a certain point, like our conjectures. They also faced the same difficulties on congruently corresponding between directions in the reality and directions in the scale model. As an example, we took a conversation between the teacher and a pair of children, Salma and Fathimah.

Teacher : see…our class building is here (She put a cube on the grid paper to point A, see the picture below this conversation). Where is the position of the *Masjid*?

Salma : here (She pointed to point B, while Fathimah placed the cube)

The position is not correct. It should be on the left side of the class building's position.

Teacher : Where is the kindergarten building?

Salma : Here (She pointed to point C)

They did it in the right of class building's position.

Teacher : where is the principal's building then?

Fathimah : the library?

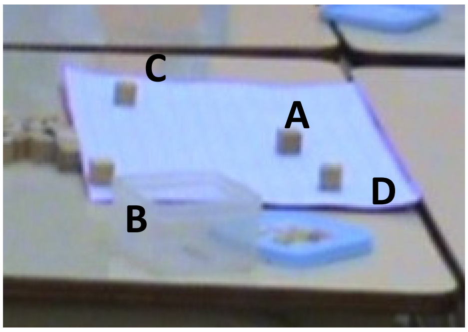
Teacher : yes

Salma : here (She placed the cube to point D)

Teacher : where is the principal's building actually?

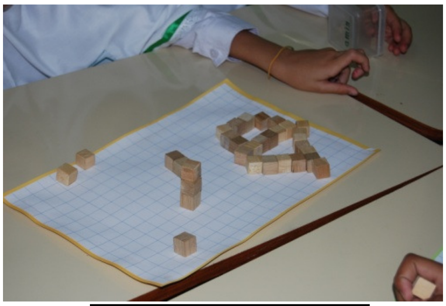
Salma : there (She pointed in the direction of principal's building, in front of the class building)

Teacher : and the *Masjid*?
Salma & Fathimah: (confused)
Teacher : go outside and see how they should be.
Salma went outside and observed the position of the Masjid.
Salma : yeah…it is there
Teacher : where? Okay place the cube.
They realized the mistakes and placed the cubes on the right way.

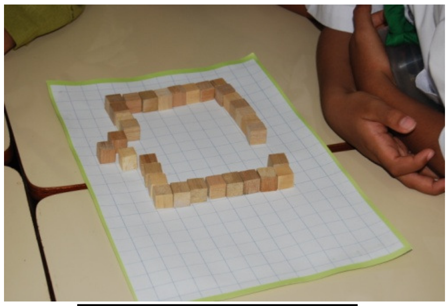


Other findings

Abel and her partner made construction of the *masjid* with 'front view' as in the figure below. The similar case was with Reza and his partner. In the beginning, they used cubes to make the boarder of the school and arranged them with 'top view'.



Abel's construction



Reza's construction

Retrospective Analysis

Now, we discuss more detail the case of Fathimah and Salma. At first, the teacher let children in wrong perception of directions, in her own perception, between class building and the *Masjid*. She kept asking in order to dig children's thinking. She seemed satisfied when they correctly put cube C in the right orientation on the kindergarten position. But then, disorientation of principal's building position appeared. This difficulty was in determining 'facing each other' positions between the *Masjid* and the principal's building. Consequently, this disorientation was appeared from indifference of false. We assume that if the teacher had posted another reasoning question (about why they thought point C was a location for kindergarten building, what relation its direction with the aforementioned direction between A and B) rather than kept asking other building's positions, children might have been more aware with their disorientation and might have not made any other mistakes. However, the following guidance by the teacher helped them to solve their problems, when she asked one of the children to check real condition outside.

For the two pairs of children in the pictures, the cubes for them represent a line rather than a 3D room. In HLT 2, this interpretation was presumed will be appeared in the second day activities in which children will ask to work with more cubes, but in this class this presumption was happened earlier in the first day activity.

We considered the crucial issue in this class was about social norms, in where most children in this class are hyperactive and difficult paying longer attention. This conclusion was based on the observation of video recording of today lesson; in where 20 minute after almost all children finished the tasks, the class situation was out of control again. We did see any opportunities to conduct the class discussion with the children in this class.

However, we tried to conduct class discussion in the following day and saw how it worked. To emerge a conflict to be discussed in the following day, we chose Aisha and Orell to present their construction first and then Anggi and Vira to present their construction. Aisha and Orell were chosen as the first because their construction represented our expectation in the first activity, in where a cube represents a building. Anggi and Vira were chosen, afterwards, because their construction brought an important issue for the second activity, in where a cube represents a room so that everybody more aware about the different height, width, and size of those buildings.

Day 2: *Where Our Classroom Is*

The activity: Localizing cubes that represent rooms

Goals of the activity: after doing the activity, children are able to: (1) use direction in determining relation between rooms' positions of the school buildings, and (2) determine relative proportions of buildings' sizes on the scale model.

Description of the classroom session

The teacher asked Aisha and Orell to present their yesterday construction in front of the classroom. After they finished, the teacher asked to the whole classroom: "*Well, we can see our four main buildings in their construction. But, I'm still wondering, where is our classroom in this kind of construction. Is there anybody else that did different construction yesterday?*". As our expectation, Anggi and Vira arose their hands. The teacher asked them to present their yesterday construction. After finishing their construction, the teacher asked to them: *why do you build like this?*

Anggi	: because we want to know where our classroom is.
Teacher	: then, can you show us where it is?
Vira	: (She pointed to a cube that represents position of the class 1B)
Teacher	: Okay, now all of you have important task to rebuild the school buildings once again. One thing that you have to remember is that it should be clearer in your constructions where some important rooms take place.

Afterwards, children again worked in pairs and started rebuilding the scale model. Some children did the task seriously, but some did not. Instead of doing the task of constructing the school building, they preferred to use the cubes for making any other shapes they want. According to the observer, some children did something else with the cubes. For instance, Raihan and his partner, Jero, used the cubes to play chest. However, this pair finished their task before. Another pair is Salma and Fira, they constructed a building for a party of their favorite cartoon. Some children went outside the classroom and plaid in the playground. The teachers could not handle the situation.

Justification of the HLT 2

An individual achievement, Bryan, showed the evidence of one conjecture about reasoning using the sizes of buildings in reality. Bryan built the principal's building with 4 cubes above, but in fact there were only two floors in the principal's building.

Teacher : It is very high building. Let's see… one, two, three, four. Why is it so high?
Bryan : The rooms are **not short**.
Teacher : Ooo…the rooms are not short. So they are big.
Bryan : (He was nodding)
Teacher : And then, what's next? Where is our classroom?
Bryan : (He pointed a place on the grid paper in front of the principal's building) here…(He put one cube) 1B…(He put one more cube next to the previous one) 1A…(He put another cube on another side of 1B) 1C…

Another conjecture about the struggle on the relative proportion also emerged in this classroom experiment, as an example the conversation below.

Teacher : where is your classroom?
Aisha : (She was silent)
Orell : Here (He pointed to the cube that represents the class building)
Teacher : Aisha, where is your classroom?
Aisha : (She pointed to the same cube)
Teacher : where is 1A class?
Aisha : (She pointed to the same cube again)
Teacher : well, how come is it the same place?
Orell : (He was smiling) see…what I've told you. We have to put more cubes. (He added one more cube next to the first cube to represent the 1A class)
Teacher : Aisha, then where is 1C class?
Aisha : (She added one more cube next to the second cube)

Other findings

We could not find other interesting findings that could enrich our conjectures for refining the HLT.

Retrospective Analysis

In Bryan case, the word "not short" indicates that Bryan has built up his sense of scale. He impressed the bigger rooms by putting more cubes. The impression could not be judged as whether he was able to determine the relative proportion as long as the scale model of the principal's building has not compared yet with other buildings. When Bryan built the class building, he defined one cube as one classroom and put the cubes one by one whilst he did inventory the classrooms one by one. Comparing with the aforementioned scale model of the principal's building he has made, now we can assume that Bryan can determine "relative size", because we now can see that two buildings on the scale model (class building and principal's building) were built with different scales; He built with three cubes for larger rooms and used one cube for classrooms that are smaller.

Although we found few interesting findings, classically the goal of this activity was not achieved. Most children do other activities with the cubes. Therefore, we decided to redefine the HLT for the following day and refined the second activity using another strategy to attract children attention to do the task so that our goal could be achieved. We skipped the third activity of the previous HLT and we formulated a new activity for the following day with the same goal as this second activity. We decided to use a competition and gave children award if they won the competition. To deal with the time, we added one more goal that was describing a route (this goal was in the fourth activity in the previous HLT). The purpose of adding this goal was because we want to know how far children were ready for mentally describing a route, as one of goals of the activity in day 4. Besides, we tried to achieve as many as activities in very limited times we had by combining several activities in one activity that had several goals.

Day 3: *Competition of Localizing an Important Place*

The activities: Localizing cubes that represent rooms; Describing Routes

Goals of the activities: after the competition, children are able to: (1) determine relative proportions of buildings' sizes on the scale model, (2) visualize a path of ways using line(s), and (3) describe directions in explaining a route among two buildings into words.

Description of the classroom session

The class was set into two large groups. Children kept working with their previous partner. We announced the rules of the competition. They were that:

(1) The winner is who the fastest pair that can place a black cube on the proper location correctly.

(2) The winner must explain and convince the others about why (s)he thinks it is placed correctly.

(3) The winner should explain the route from kindergarten building to the place on the name card.

Justifications of HLT 2

For the proportion concepts, children fostered to build a building using more than one cube. In order to determine a right place for the black cube, they built a building more proportionally than the previous day activity. No child used one cube to represent on building during the competition. They did not conflict about shape of the school.

Evidence emerged in which children using the scale model to help them describing a route. We justified this moment is similar with conjectures in HLT 2, in which children using the route drawing to help them describing a route.

Teacher : if you will go from the kindergarten to the library, which route do you go through?
Anggi : (She pointed to the scale model to show the route from Kindergarten Building to the Library)

The second task to draw a route from kindergarten building to a certain room of the school worked as our expectation in HLT 2 (see appendix 8). Some children drew the road of path with a single straight line (such as Reza's). Some of them drew with two lines and other references (such as Daffa's). Some of them used curves or two bent lines to express a turning (as Anggita and Vira's).

Other findings

Most of the time, the teacher asked children to mention the rooms as references when she asked children to explain a route. One small moment below was an example based on her own notes when she did conversation about a route with children.

Teacher : if you will go from the kindergarten to the library, which rooms do you go through?
Ifa : (without pointing and without looking at their scale model)
Play group, stairs, office and class.
Teacher : class? Which class?
Ifa : 4B

A pair of children, Aisha and Orel, drew a single line using arrows on its two sides (see appendix 8). This finding contributed a new conjecture of visualizing path for refined HLT.

Retrospective Analysis

The first goal about the proportion was achieved well. Because they had to localize black cube on a certain room's position in the construction, they fostered to represent a cube for a room and put cubes of a building as many as rooms of the building. However, we lost children's reasoning about the proportions. They did not reason using the sizes of buildings in reality. One logical reason may be because we posed 'why' questions for determining the positions, not for determining the size of the building.

The second goal to visualize the path line(s) could be achieved. Our conjectures of three various ways of visualizing a route were happened in the class experiment. As our explanation above (see the justification part), children expressed the path lines using (1) only a straight line, (2) two curves or bend lines, and (3) lines and adding other references. This evidence strengthened conjectures of the HLT for this particular activity.

The goal to enhance children ability of describing the route using direction words was not achieved. In analyzing the problem, we argue that it was happened because the word that the teacher used in questioning was 'which rooms' instead of 'to which direction'. The questions affected children's answers in where they only did inventory rooms that they passed when they walked through the route. Consequently, we could not clearly see some conjectures in HLT 2, such as their struggles on putting directions into words and how they describing a route. Therefore, we decided to once again bring this goal in the following day activity.

The competition was a good strategy to motivate children in class 1B to achieve the goals. Most children paid more attention to the instructions. They worked and thought about the task in order to be the fastest and the winner in the competition. The black cube was more powerful material that could easily be seen to determine the location of a certain room on the scale model.

Day 4: *A Mini Lesson – Where Do You Have to Stand and How Do You Go There?*

The activities: Describing Routes; Examining Pictures of the School Buildings

Goal(s) of the Activity: after doing this mini lesson, children are able to (1) describe directions in explaining a route among two buildings into words, (2) use a vision line as reasoning for arguing different point of views, and (3) argue different shapes of the school buildings from different point of views.

The following day was a mini lesson about determining point of view of a viewer. The weakness of this activity in the previous HLT was we missed children's reasoning because we only ask about the 'where do you have to stand?' without asking about 'the reason' why they think so. Therefore, for the following day, we notified the teacher to 'why' question for digging what in the children's mind.

Description of the classroom session

The teacher opened the mini lesson by informing the following information: "*Yesterday, Miss Neni took some pictures of our school. Now, I'm wondering where she was standing when she took those pictures. Would you help me to guess where we have to stand to see our school from this point of view?*"

We showed seven pictures of some buildings in the school in this mini lesson (see appendix 9). The mini lesson was started with showing pictures number 1 – 3. These pictures showed one same building, namely the *masjid*, but they were taken from different point of views. The teachers posted questions about:

1. What is the name of the building that you can see from the pictures?
2. In which position did Miss Neni take those pictures?

The issue of this first part was about one same building that can be seen from different positions, which were:

Picture 1: was taken in front of class 4B
Picture 2: was taken from the yard or in front of the principal's building
Picture 3: was taken from kindergarten building, more precisely is in front of Melody Room.

After the teacher showed those three pictures, she asked: "*Hmmm…it's strange, isn't it? The same building, the masjid, but your answers of those three pictures are different, how come? Is there anybody can explain it?*" Afterwards, the teacher questioned to orally describe a route from kindergarten building towards the *masjid*.

In the second part of this mini lesson, the teacher showed picture 4 – 6. One interesting thing from those pictures was they were taken from the same position.Those pictures showed different buildings, which were:

Picture 4: Classroom 4B
Picture 5: Class Building
Picture 6: Kindergarten Building

The issue of this second part of the mini lesson was that from the same position we could view many different buildings, and the differences were depended on to which direction we were standing. After showing the pictures, the teacher was posing a question: "*Hmmmm….this is also strange. We know that Miss Neni was standing on the same place when she took these last pictures, but how can we see different buildings and pictures? Do you have any suggestions for that?*" Lastly, the teacher showed Picture 7, The Library, and she asked them to orally describe a route from kindergarten building to the Library.

Justification of HLT 2

When describing examining the pictures, some children argued using reality. They mentioned some references that were shown in the pictures. We can see that when they discuss about from where picture 2 was taken.

Aksa : from in front of the swaying
Aksa : from in front of … bellow the basket ring
Teacher : how do you know that?
Aksa : because we can see that (he pointed to outside) outside
Reza : because there are basket pole (he pointed to the pictures)

When children asked to argue from where a picture 6 was taken. Some of them pointed to a certain point.

Reza : the picture was taken from there, above (he pointed to a certain point to the second floor of principal's building.
Aksa : it was taken from in front of class 4B, above there (pointed to the same point)
Some children reacted and pointed to the same point.

For describing routes, some evidence justified conjectures in the HLT 2. Some children describe a route from kindergarten to the *masjid* and to the library in several ways.

Anggi : (moved her finger on the desk to describe the route) you can move to here and then straight away, and to here, to here
Iffa : we can go through the door, the stairs, and we pass through class 4B. There is the library (without moving her hand)
Reza : after we go out of the door, we can pass through a playgroup, just walk straight away. Then, you will see the stair, turn to the right and go up stair. After passing through the stair, above you can turn to the right again, walk straight away, pass through the class 4B. Finally, there you can see the library. (He explained by moving his hand)

Other findings

A child, Aksa, suddenly changed the words more formal, when he described a route from kindergarten to the *masjid*.

Aksa : you can go through the play group, then you can turn there, and turn there (he moved his hand by showing a route outside the classroom)
Teacher : I don't understand you, turn there and there. What is there?
Aksa : see (he pointed to the pictures on the board), you can go through the playgroup and then turn to the left and go straight away. Afterwards, you can turn to the left again, there is the *masjid*.

Another finding was about children's argument of the different pictures of the *masjid*.

Reza : because it was pictured from close and from far
Daffa : because the place were taking those pictures are different

Retrospective Analysis

Children came up with various ways of reasoning for those two activities. When examining the pictures of the *masjid*, some children used other reference in reality that was seen in the pictures. Aksa mentioned swaying and basket ring, because one could see those reference in the pictures (see picture number 2 in appendix 9). He further argued that everybody could see those outside. This proves that children may use reality for reasoning. Meanwhile, Reza pointed to the basket pole in the picture to reason the argument. Reza used the picture as model of the real situation as helps for reasoning.

Reasoning on picture 6 emerged proof of the vision line. It was indicated when Reza pointed straight away to a certain direction followed by Aksa and other children. They argued a line connecting a point to another point. They unconsciously indicated a vision line as the reason a point of view.

When describing routes to the *masjid* and the library, some children used their gestures to show the route like Anggi. Some of them mentioned other references that they pass through along the route. One by one the children explained the route. One child more clearly explained a route after another. At the end, Reza showed a perfect explanation that included formal words of directions, other references along the route, and emphasized the explanation with his gestures. Those are indicated the proofs of conjectures in the previous HLT.

Another interesting finding was about the change of Akza. When the teacher asked to describe a route from the kindergarten building to the *masjid*, his words of directions were continuously changed getting more formal during the discussion. Firstly, he only described the route by gestures of movement and the words “turn there”. Finally, he described it more precisely, by mentioning other references along the route, using formal words of directions, and using the picture as helps for emphasize his explanation. The changes were because a stimulating questions from the teacher by asking “What is there?”. This question really affected him to look for a proper word so that everybody knows what he meant by “there”.

From the aforementioned explanation, we can say that children achieved the first two goals of the activity, which are describing routes into words and use a vision line as reasoning of taking a point of views. For the last goal, the achievement could be indicated from the last small moment when children argued different pictures of the *masjid*. Reza and Aksa achieved the goal. Reza was arguing different point of views as reasoning by the words “from close and from far”. Meanwhile, Daffa perfected the explanation. His argument clearly indicates that he realized the different shapes in the pictures were because different points of views in where they were taken.

Day 5: *Different 4-Cube Castles for Thumb Princess*

The activity: Constructing 4-Cube Constructions

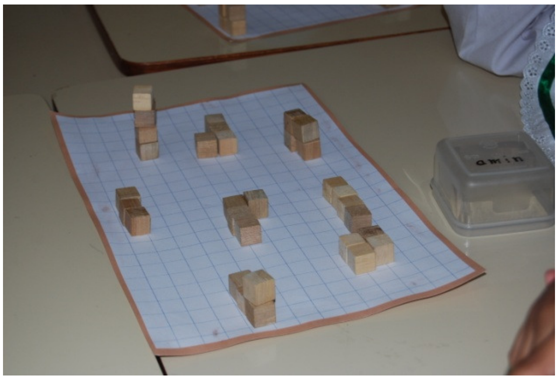
Goal(s) of the Activity: after doing the activity, children are able to manipulate shapes of 4-cube constructions.

Description of the classroom session

The teacher began the lesson by telling a story about Thumb Princess: "*Have you ever heard the story of Thumb Princess? Long time ago, there is a Thumb Princess. She is named so because she is as small as a thumb. Thumb Princess is so pretty. She likes traveling very much. Once upon time, a giant comes to eat the Thumb Prince. She runs to a castle and scream "Help me…. Help me….". An Elf Prince comes helping her and killing the giant. The thumb princess is so thankful to the Elf Prince. She follows the Prince going to his kingdom. In the kingdom, there is an announcement. The king is so happy that his son brings the Thumb Princess. The king announces that there will be a competition to build 4-cube castles for the Thumb Princess. However, the king gives a rule for the castle buildings. That is they are not more than 4 cubes for each building and they must be different each other. Let's follow the competition and get as many as candies from the king. He promises to give you one candy for one creative 4-cube castle.*" Children created various numbers of constructions.

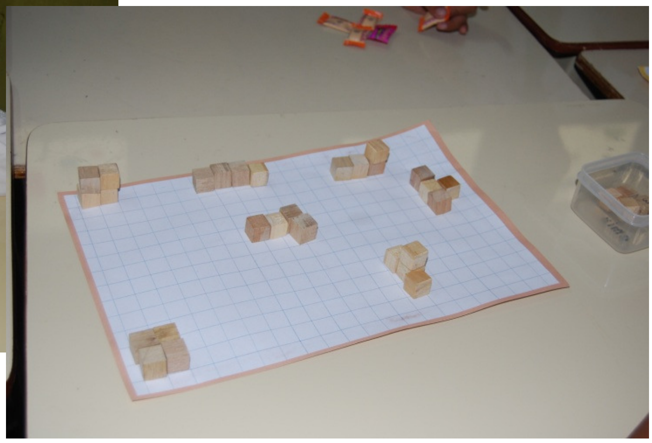
Justifications of the HLT 2

Most children constructed by trial and error. It was indicated, whenever some of them finished one construction, they called the teacher to show it and to judge whether they constructed in a right way.



Children also struggled to determine whether two constructions were different or not. Most children thought that rotated castles are different. It could be seen from the pictures below, as an instance.

There was an interesting finding in this class experiment. A child, Anggi, used rotations to manipulate her constructions. To get another construction, she firstly created a flat castle. Afterwards, for the next castles, she again made the similar flat castle like the first one, and then she rotated it 90 degree anticlockwise to get a high castle. She did it every time she built other castles. We could see her construction in this following picture.

Other findings

There was a pair of children, Raihan and Jero, had interesting ways in manipulating the cubes. They firstly built a vertical castle with four floors. Then, for the next castle, they built the same castle first before he moved one cube in its side to make a totally different castle from the previous one. They continued creating and modified like that to get other different castles.

Retrospective Analysis

We assumed that Raihan and Jero had a very good visual ability. They were able to see shapes relation.

Because the following day was the last day of experiment and it was also the day with only 35 minute mathematics lesson, we decided to use it for conducting math congress about the differences or similarity of the castles. Based on the previous experience, asking children in class 1B for discussing and paying attention to people in front of the class were so difficult. Therefore, we would not use castles construction in front of the class. Instead, we would use pictures of their constructions to attract their attention, because they liked to see their pictures. Now, we moved children from concrete materials towards more abstract materials (from cubes to pictures). The limited time made us limiting the problems to be discussed. We chose two problems that encouraged children spatial reasoning in constructing.

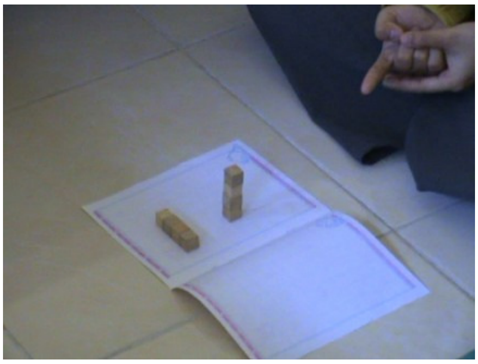
Day 6: *A Math congress - different or not?*

The activity: Examining the Differences between Two Constructions

Goal(s) of the activity: after the math congress, children are able to use rotation and mirroring as reasoning the differences or the similarity between two constructions.

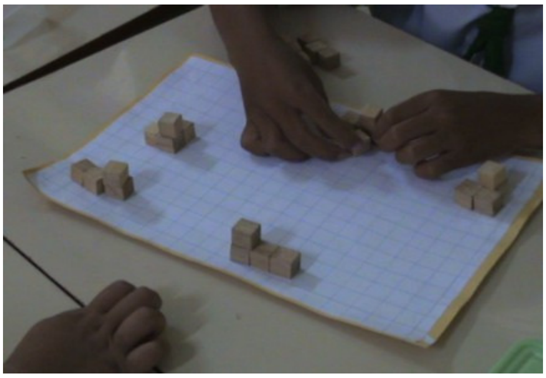
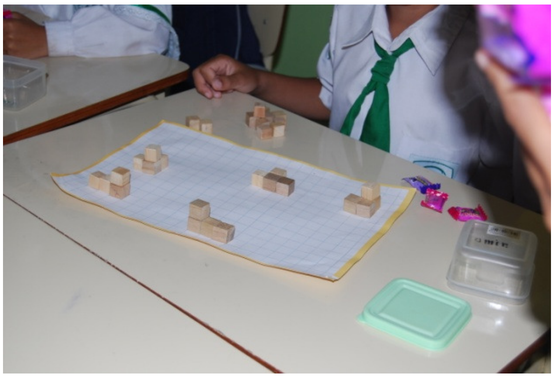
Description of the classroom session

We started the discussion by showing two pictures below.

Yesterday, I saw these two castles were made by some pairs. Who made these two castles yesterday? Do you think they are same or different? If you have houses like those castles, do you think the houses are different or not?

After some discussions, I changed the pictures and continued the math congress with discussing other constructions below: *I also saw some of you have made these two constructions. Whose constructions are they? Are they same or not?*

For the second constructions, there are two boys that give a good answer and reasoning. They are Reza and Orell.

Reza and Orell shared their ideas about those constructions in front of the class.

Teacher : are these two castles same or not?
Reza : yes, they are same.
Teacher : why do think so?
Reza : because **if you move the above cube to the right** then you will get the same castles.
Teacher : hmmmm….but in reality we cannot do that.

Orell walked around the construction and saw it from many different points of view before he shouted.

Orell : I know how to do that… to see its similarity
Teacher : how do you do that?
Orell : just **turn around** one of them, then you will get the same castles.

Both of them are in front of the class to present their strategy.

Retrospective Analysis

From above conversation we can see that Reza and Orell manipulated the constructions to argue the similarity. They argued with different kinds of transformations. Reza manipulated the above cube of a construction by translating it to another side, so everybody could see that it was same with another construction. When he reasoned, the words "**move the above cube to the right**" indicated the translation. This new evidence could be used to enrich our conjectures in the HLT of this activity.

Meanwhile, Orell found another way to reason the similarity. When he walked around and saw the constructions from different points of views, it indicated that he tried to use reality approach in the argumentation. We guess that is because we fore mentioned that in the reality the construction is a building than cannot be moved easily. The walking was a good strategy to reason about the similarity of rotated buildings. However, his reason into words showed that he manipulated the constructions rather than imagined it as real buildings. He used rotation as reasoning of the similarity. It was indicated by the words "**turn around**".

From the evidence, we can conclude that the problems in this math congress were so powerful to reach the goals. Children could use rotations or another kind of transformation, such as translation, to both manipulate the constructions and to reason their similarity.

All the findings and nice evidence within the class experiment were used to refine the HLT 2 and redesign a new HLT, which is HLT 3. The HLT 3 was enriched by some conjectures from two previous HLT. We can see the HLT 3 in the following table 5.2.

Day	Name of the Activity	Activity	Conjectures of	
			Learning Process	Struggles
1	My School Buildings	Localizing cubes that represent buildings	1. Using hands and gestures to describe directions 2. Listing other objects 3. Having various relative distances	1. Congruency between direction or distance on the scale model and those in the reality 2. The meaning of a word, *building* 3. Interpreting cubes, as lines or as rooms
2	Where Our Classrom Is	Localizing cubes that represent rooms	1. Using hands and gestures to describe directions 2. Listing other objects 3. reasoning the proportion using the buildings' sizes in the reality	A conflict is about shapes of the school buildings
3	Competition of Localizing an Important Place	Describing Routes	1. Using only the gestures of movement 2. Using words of direction and other references 3. Using only other references to describe which rooms they pass through the route 4. Using the route drawings to describe directions; pointing to it or drawing lines on it. 5. Combining gestures of the movement, words of direction, and other references 6. Having various route drawings; (a) only straight line(s), (b) two curves or bent lines, (c) lines/curves with other references	Putting directions into words
4	A Mini Lesson - Where do you have to stand and how do you go there?	Examining pictures of the school buildings	1. Indicating a visual line by pointing to a certain location 2. Reasoning using reality 3. Using some references in the pictures as reasoning 4. Using a line as a connection between two points	1. Using a straight line as reasoning of a vision line 2. Missing the "why" questions in the discussion
5	Different 4-Cube Castles for Thumb Princess	Constructing 4-cube constructions	1. Constructing only flat buildings 2. Constructing various buildings, not only flat 3. Constructing by trial and error 4. Manipulating constructions by rotating 5. Manipulating shapes by moving one or several cubes	1. Determining whether two constructions are different or not 2. Undestanding of 4 cubes as limitations
6	A Math congress - different or not?	Examining the differences between two constructions	1. Using reality for reasoning the differences 2. Rotating a construction to see the differences 3. Using relations among cubes to argue the different shapes between two constructions 4. Walking around the rotated constructions to observed the similarity 5. Using translation to argue the similarity	Arguing the difference

Table 5.2. Overview of the HLT 3

5.5. Post Tests

The first version of the post test (Appendix 10) was made based on the earlier version of the pre test. There were two parts of problems. The first part was to check whether activities of orienting enhance children's spatial orientation. The second part was to check whether the previous activities about constructing influence the development of their spatial visualization.

After class experiment, we considered changes of the post test. The changes were considered as the effects of children's progressions during the activities. We formulated totally new assignments for children in order to see whether they accomplished the goals of the activities. Besides, the new post test (see appendix 11) measured whether children can mentally achieve the goals of the activities in more abstract ways. We discuss the purposes of making the post test and the results per question.

The first question represented the first activity about *placing school buildings*. This question provides 2D scale model of the school buildings. The boxes initially represent the buildings in their positions and proportions. Thus, we measured children's ability of orienting by more abstractly determining four main buildings' positions in the boxes. About third quarter of 27 children have a good orientation to solve the problems. The rest still have disorientation problems in determining position between two buildings. However, the failure was because misperceptions of the proportions. They saw the thinnest box was as the kindergarten building, so they determined the left building of it was the class building, which is in fact the thinnest as the principal building.

The second question represented the second activity about *placing rooms in the school*. It provides more detail model of the school buildings with squares as rooms. We stated 1B as the position of their classroom and asked them to determine the library's position related to the direction and the position of 1B. The results showed that most children could determine an exact position of the library. We interviewed some of them who colored a box in another side of an expected box. They reasoned their answer by arguing their perception about where is above and where is below in the 2D figures. We accepted their argumentations because it indicated that they were aware with interpreting directions in 2D figures of the school buildings.

The third question represented the third activity about *describing routes*. More than a half of 27 children drew lines and curves as representation of the road. An interesting findings because we found that 8 of them drew lines and arrow, this indicated they more aware with directions for the drawing. The drawing did not provide chance to describe into words or gestures it. Therefore, to make the drawing more clearly, they added arrow to show the movement and direction to where we go.

The fourth question represented the fourth activity about *taking different point of views*. Nobody faced difficulties to guess the name of the building. For the question "I view it from", about 20 children could describe into formal words, such as in front of the stairs, in front of class 1B, etc. However, the ability of reasoning was still lack. Only about 8 children that could give good reasoning about the 'why' question, both by using references that can be seen from the pictures or by using a word that indicated a vision line, such as "if you look "straight", you will see …"

Finally, the last question represented the sixth activity about examining the differences between two constructions. About 20 children answered that those two pairs of constructions were the same. Most of them argue that the first pair could be rotated or turned around. Most of them argued that the second pair had similarity in their shapes.

5.6. Conclusions

The goal of this research is to develop classroom activities that support the growing process of first graders' spatial abilities especially in orienting and constructing. The contribution of this research will be adding a local instructional theory of teaching early geometry of first graders in Indonesia. The local instructional theory provides knowledge about classroom activities that support students in orienting and constructing.

We have posed three research questions. This subchapter discusses the answers of the research questions which based on our retrospective analysis. Afterwards, in the next chapter we elaborate reflections of this study and suggest recommendation for further studies and improvement of mathematics education in Indonesia.

5.6.1. Answers to the first research question: *How can first grade children develop their ability in orienting and constructing?*

The HLTs provides an answer to this question. The answer provides reconstructions of the earlier HLT. The first reconstruction was tested in the tryout activity with first graders in the end of school year. The second reconstruction was tested in the last teaching experiment in the beginning of school year. We suggest two kinds of HLT. The first one is better tested at the second semester of the first graders. The second is better tested at the first semester of first graders. For a schematic overview of the HLT reconstructions see Figure 5.3.

<u>First Refine HLT: HLT 2</u>

1. *Localizing cubes that represent buildings.*
2. *Localizing cubes that represent rooms.*
3. *Drawing basement of the school buildings.*
4. *Describing a route.*
5. *Examining pictures of the school buildings.*
6. *Constructing 4-cube constructions.*
7. *Examining the differences between two constructions.*

8. *Drawing basements of 4-cube constructions and determining the height of the buildings*.
9. *Drawing possible basements of a given figure*.

Second Refined HLT: HLT 3

1. *Localizing cubes that represent buildings*.
2. *Localizing cubes that represent rooms*.
3. *Describing a route*.
4. *Examining pictures of the school buildings*.
5. *Constructing 4-cube constructions*.
6. *Examining the differences between two constructions*.

Now, in this section we discuss patterns in children's learning and how the instructional activities supported, or failed to support, the learning process we aimed at.

Localizing cubes that represent buildings

The main aims of this activity are to use direction in determining the relation between the school buildings, and determine relative distances among buildings on the scale model. We used a scale model to raise the conflict about directions and relative distances. Children started to express directions. Most of them used their gestures and words. They are mostly struggling in seeing the congruency between directions or distances on the scale model and in the reality. For determining relative distances, each pair had their own perception of how far a distance between two buildings is. Most of them reasoned with listing other objects between two buildings to argue their relative distance.

Localizing cubes that represent rooms

This activity has aims to use direction in determining relation between rooms' positions of the school buildings, and determine relative proportions of buildings' sizes on the scale model. Within the activity, children develop their ability to express the directions. Children also had the same ways of expressing direction as mentioned in the previous activity. However, by listing all rooms in the school this activity consequently brought an issue about relative proportions of the school shapes on the scale model. It brought the conflict on determining the shapes; how big, how high, etc. They reasoned the proportions by mentioning some objects in the school and other references.

Describing a route

This activity was meant to describe directions in explaining a route among two buildings, and visualize a path of ways using line(s). This activity brought two large findings. First, it was about the way in describing direction of movement, such as by only using gestures, words of directions and other references, only other references, route drawing as reasoning, and combining all of those to make the description more clearly. Second, it was about the various route drawings. They could be only a straight line, curves or bent lines, and lines/curves with other references.

Examining pictures of the school buildings

By this activity, children are able to use a vision line as reasoning for arguing different point of views, and argue different shapes of the school buildings from different point of views. Most children usually indicate a visual line by pointing to a certain location. By arguing the different shapes from different point of view, they mentioned other references both in the reality or those were seen in the pictures. This activity failed to help children more aware with a vision line. They could not use a straight line as reasoning by only examining the pictures.

Constructing 4-cube constructions

The aim of this activity is to manipulate shapes of 4-cube constructions. Some evidence showed that children firstly manipulated the 4 cubes by trial and error to get as many as constructions. Some of them that realized a pattern on constructing could draw only flat buildings, rotated buildings, or mirroring buildings. To determine the different constructions were a big issue that could not be main attention in this activity. We assume that was because the children focused on constructing again and again, as many as possible.

Examining the differences between two constructions

After doing this activity, children are able to use rotation and mirroring as reasoning the differences or the similarity between two constructions. Rotation dominantly took role in arguing the similarity. We only showed children rotated constructions, and in the class experiment they also only rotated a construction to get another one. Nobody realized the role of mirroring for getting different constructions. They preferred to manipulate the shapes, some cubes to totally get different constructions.

Drawing basement of the school buildings; Drawing basements of 4-cube constructions and determining the height of the buildings

Those two activities bring the same goal to determine 2D shapes of basements of the constructions (squares) from its 3D constructions. All children did the same strategy to making the basement, which was sliding the construction before they signed the place of its basement. Here, they learnt to come up with various ideas of drawing the basements. Because they worked on a grid paper, this gave more opportunities to express the drawings by other symbols or other signs, such as crosses, check, etc. The difficulty was about determining the height of the buildings on the basement.

Drawing possible basements of a given figure

The activity is aimed to more mentally visualize the basements of a given figures. The ideas of basement drawing and the strategies are similar with the previous activity. Moreover, this activity gave chance to interpret the 3D figures. Some of them interpreted figures of non-flat constructions as the flat constructions. Beside, the issue about the height of the building emerged as a consequence interpreting hidden cubes in the figures.

In short, we claim that children can learn spatial reasoning and using rotation in manipulating the constructions if HLTs similar to the refined above are used. The HLTs offer empirically grounded theory of how children do spatial reasoning and manipulating by rotation. Moreover, the HLTs always need to be adjusted with the local circumstances in where they will be implemented (Barab & Kirshner, 2001). It seems that more times are needed to do our activities, to compensate the limited activities as in HLT 3. Although drawing basements of the school buildings has roles in continuous the activity, one could remove it or combine it with the drawing basement, because they have the same goals. Furthermore, it is worthwhile to try out other contexts as well for achieving the goals in the activity of examining pictures of the school buildings, which are to make children more experience with the importance of a straight line as a visual line.

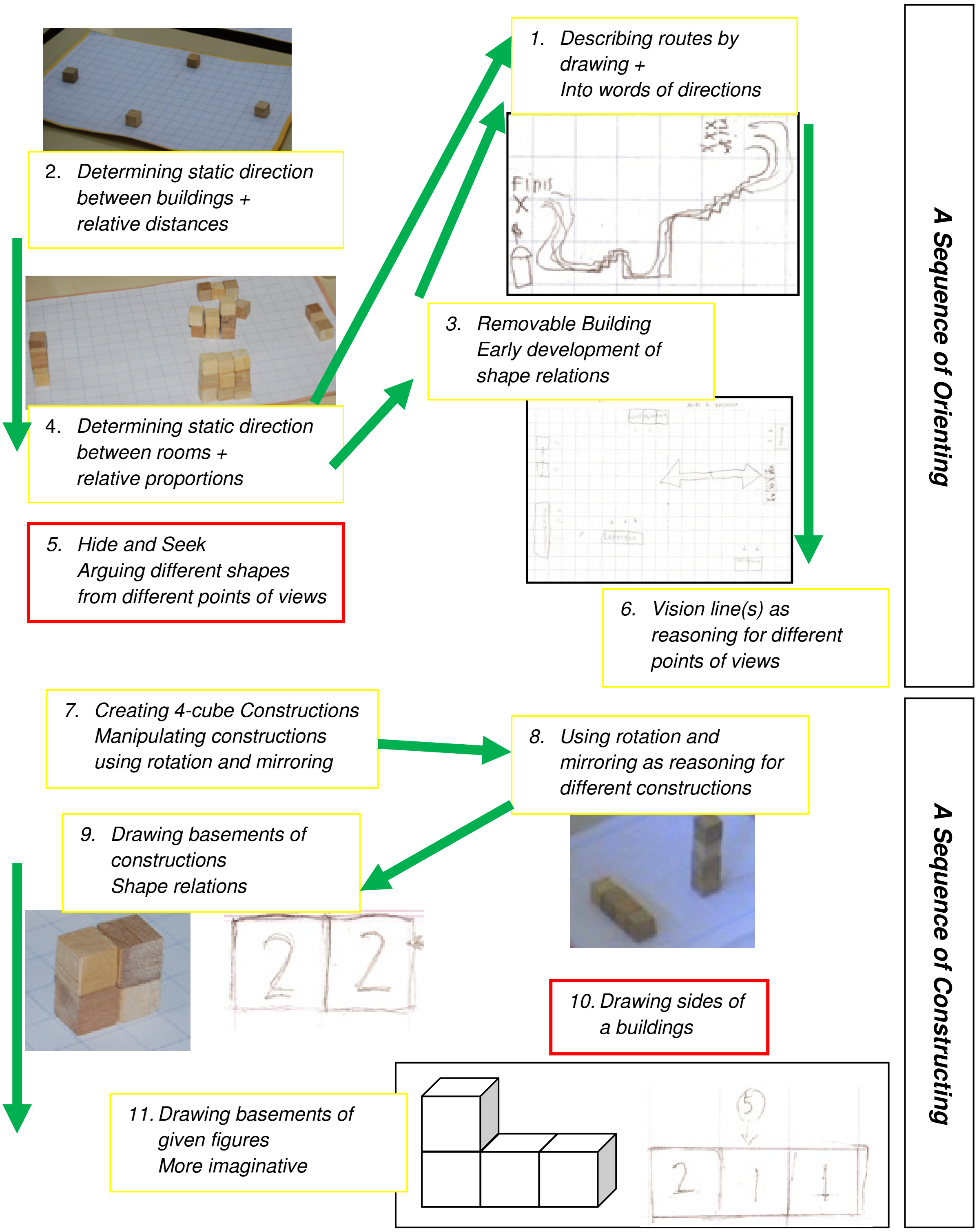


Figure 5.3 Schematic Overview of the reconstructed HLT in Grade 1

5.6.2. Answers to the second research question: *What kind of spatial reasoning do first graders use in orienting?*

To answer this question, we selected and listed all reasoning that children use to do orienting. Afterwards, we make a conclusion to classify all spatial reasoning that emerged during the class experiments. The conclusion will be a contribution to local instructional theory of spatial reasoning in the first grade.

During the research and class experiments, we found many reasoning that children use to explain their thinking process. Based on some evidence in the tryout class and in the class experiment, we list them in each particular goal of orienting.

(1) *Determine directions between buildings*.

a. Children point to a certain direction in the reality
b. Children point to a certain direction in the scale model
c. Children draw an arrow to specify direction of building's position
d. Children explain the reasoning into words of directions.

(2) *Determine relative distances between buildings*.

a. Children use the words like, too far, must be closer, etc, to indicate the relative distance
b. Children mention other references between two buildings. For example, they move the cube of the principal's building further from the class building, because there is a yard between them.

(3) *Determine relative proportion of sizes*.

a. Children use the sizes of buildings in reality
b. Children express reasoning of the relative proportion by words like, not short, so high, big, etc.
c. Children mention other references, beside rooms, to argue the number of cubes to be presented.

(4) *Describing a route into directions*.

a. Children move their hands to show the directions of movement.
b. Children mention other references whilst describe with direction words.
c. Children use the model, such as a route drawing or a scale model, as reasoning of their description.

(5) *Drawing a route*.

a. Children argue with the path of ways (visualized by only a straight line)
b. Children argue with the path of movement (visualized by bend lines or curves)

c. Children argue with the path of movement and directions (visualized by arrow lines)

(6) *Taking different points of views*.

a. Children point to reality as reasons, such as *because it shows the big door of this building at the center*.

b. Children emerge a vision line as reason between two positions, the viewer's and his views.

As a summary from the lists above, we can conclude and classify spatial reasoning of first graders in orienting are:

(1) *Using reality*. We can see that from (1)a., (2)b., 3(a)., 3(c)., (4)b., and (6)a.

(2) *Using gestures*. Children that cannot find proper words for reasoning, they use their hands or other gestures to reason their way of thinking. The lists that show it are (1)a., (1)b., (4)a., and (6)a.

(3) *Using representations, probably drawing or other models and symbols*. The representations help them to explain their reason, since they may not be able to express it into words. List number (1)b., (1)c., (4)c., (5), and (6)b.

(4) *Using proper words of orientation*. Children are able to clearly explain their reasons only into words, both the orientation and adding other references to make the explanation clearer. They do not need other helps such as drawing, gestures, etc for reasoning. We can conclude that from lists number (1)d., (2)a., (3)b., and (4)b.

In summary, the findings can enrich the previous research by Diezmann & Watters (2000). As mentioned in the theoretical framework, they differ spatial reasoning into two big ways of reasoning, with spatial representation and linguistic representation. One could argue that our third conclusion is about spatial representation and our last conclusion is about linguistic representation.

5.6.3. Answers to the third research question: *How do Van Hiele's levels constitute the development of progression in manipulating constructions?*

We answer this question by listing all evidence in constructing. We observe both, their strategy in constructing and their constructions. Afterwards, we classify the evidence in the Van Hiele's levels and make the conclusion of it.

From the classroom practice, we found some evidence of the way children constructed and manipulated the constructions.

(1) Children construct by trial and error

(2) Children construct only flat constructions

(3) Children construct buildings by rotating or translating to get another one.
(4) Children manipulated the shapes to get different construction
(5) Children slide constructions to see the 2D shapes of its basement.
(6) Children mentally manipulate given figures to see their basements.

Now, we discuss one by one the lists above based on the Van Hiele's levels of thinking in the theoretical framework. When children construct only by trial and error, it means that they are not conscious of the properties in configuration of the constructions. Thus, according to Van Hiele, those children are at the first level of thinking. The second evidence also can be categorized in this fisrt level of Van Hiele, because children may continue the following constructions after they identify the appearance of shapes in the first construction, which is flat. The first construction may affect children to make flat for the following constructions. Therefore, they also could be at the first level of Van Hiele.

However, there are children that do change their constructions even though they see others' constructions are not only flat. These children descriptively recognize shapes by flat as their properties. Therefore, in that case, children that construct only flat construction can also be categorized at the second level of Van Hiele. Moreover, children in the third and fourth list can also be at the second level. They establish relationships between constructions and between cubes. Children in the third list establish relationships between rotated constructions because they are rotating one construction to get another one.

Finally, we could categorize the last two lists into the two or the third level of Van Hiele. In these ways of constructing, children establish the relationship between 2D shapes of the basements with 3D shape of the constructions or figures. Besides, they have to discover properties of figures by informal deduction in order to determine the basements. They manipulate the figures more abstract ways to see the relation between squares and cubes.

In summary, as mentioned in the theoretical framework, Clements and Battista (1992) believe there is a shift between Van Hiele's levels. Children are not just at "a" single level if they are doing constructing. Thus, this research gives more proof for the beliefs. The aforementioned explanation shows that the beliefs emerge in our analysis of the evidence in the sequence of constructing.

6. Discussion

In this last chapter, we discuss the reflections of this research. The discussion includes the weaknesses of this research that influence the results of the study. Afterwards, we give recommendation to minimalist the weaknesses for further researches which typically have similar goals and similar classroom conditions.

6.1. Reflections

This research has contribution for local instructional theory of early geometry education in which provides information about classroom activities of supporting the growing process of first graders' spatial ability especially in orienting and constructing. Throughout this research we have tried out activities to foster children learning about orienting and constructing to develop their spatial reasoning and their levels of thinking. In this section, we highlight some important issues based on the findings of this research. First, we highlight a common issue in this particular research that influenced the results. Second, we discuss about new social norms that influences the mathematical classroom in this particular classroom experiment.

6.1.1. Language: As the Common Issue

The common issue is about languages. Most children struggled to understand some instruction because they have misconception with the languages that we used. Although it is not directly related with mathematical content, language that teacher uses to describe the task and to explain the concept enormously influence children thinking and their reactions. If we use languages that are not familiar enough for them such as *route*, or the word 'building' that was misconception for children, it surely gives different reaction that out of our expectation as a teacher.

6.1.2. New Social Norms

In our class experiment, the class discussion could not be run effectively. Before we came to that class, the children had had no habit to listen each other. They were difficult to stay calm and pay attention more than 15 minutes. The teacher always tried playing strategies for each mathematics lessons. However, we want them to learn listening other ideas and respect to differences. In the first day, we tried to conduct a class discussion in the end of lesson, but we ruined by the times. In the beginning of following day, the class discussion was tried to establish. When a pair of children presented their constructions, some children paid attention in the beginning. After few more minutes, the class was getting noisier. Two or three children that really wanted to listen to explanations of the second pair were standing in front of them, while the others did other things. The teacher could not handle this situation.

We could not have much hope based on the second day experience, but we still wanted to teach them respectful to the others. We reduced our standard to educate them the new norms, which are sharing ideas and listening to others. Instead of forcing to conduct a class discussion, we were more focus on the discussion in pairs. We observed how far a child can discuss and share his ideas with his partner. Some examples of small conversation in pairs were already illustrated in the descriptions of class experiments of each day activity.

6.2. Recommendations

In this segment, we give recommendations concerning suggestions based on the reflections. The findings underpin these recommendations.

6.2.1. The Use of Language in Mathematical Practice

For the common issue about the use of languages, we suggest on making the meaning of a word clearer for children, we have to choose proper words for children and relate the words with reality so they can get the meanings. In Indonesian the word building refers to *bangunan* or *gedung*, therefore children had different interpretation when we asked them to mention the main buildings of the school. Children that interpret a building as *gedung* would think the renovated building is not a building, because it has even not finished yet as a complete building. On the contrary, children that interpret a building as *bangunan*, they would think it as parts of the building like, how many floors in it, or as something that cannot be seen as a closed and complete building yet. Therefore, these children categorized the renovated building, the *pendopo*, the parking area also as buildings. What we meant here was a building as both, *bangunan* and/or *gedung*, but we do not use particular parts of building in the definition. So, we do not accept floors as a building, etc.

The explanation above shows that we need to deal with children about meanings of some essential words that we will use in the research. We recommend that to make a pretest or small interview with children in the classroom experiment before conducting the research. The pretest should also include some words that we will use in the research for some essential parts within the activities, so that we can see how far they understand some terms we use or whether they have different meanings of it or they have other informal knowledge about the terms.

6.2.2. New Social Norms for Indonesian Children

We suggest for the similar class condition to focus on discussion in pairs as new strategy to teach children about the norms. We realize that the teacher will be not easy to handle and supervise the discussion within those pairs. However, we have to be in an agreement that by more respectful to listen to others' arguments and share their idea, children will learn more. In addition, they also can master the mathematical concepts by their own way, not from the teacher.

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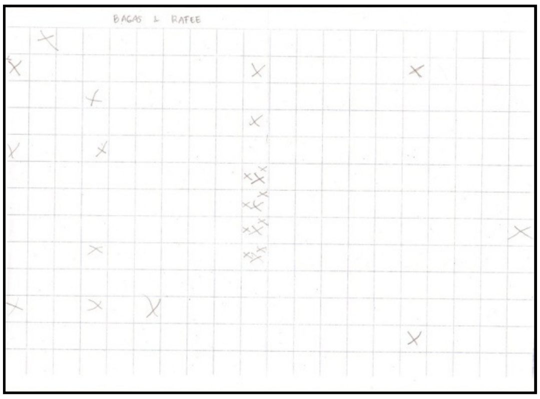
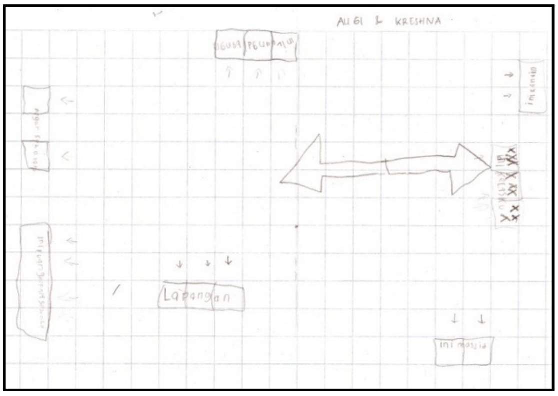
Appendix 1

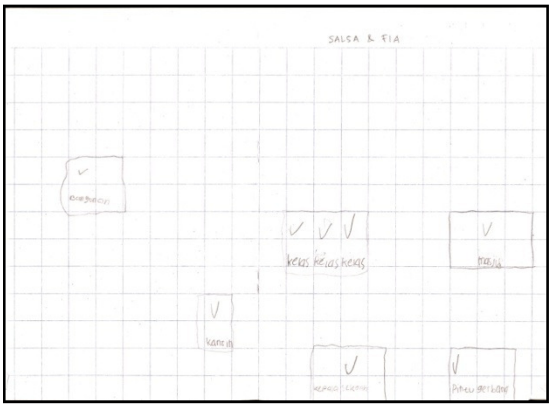
Children's Drawings of 2D Scale Model in the Tryout Class



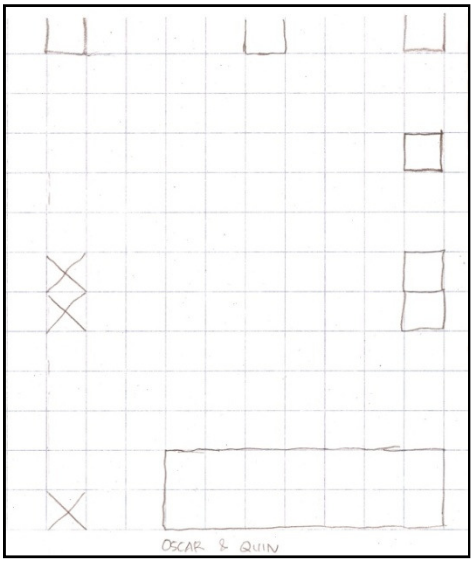
Rafee and Bagas' Drawing



Augi and Kreshna's Drawing



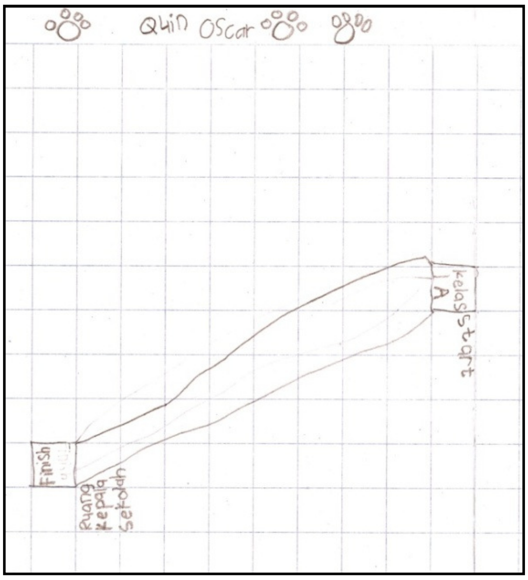
Salsa and Fia's Drawing



Quin and Oscar's Drawing

Appendix 2

Children's Route Drawings in the Tryout Class



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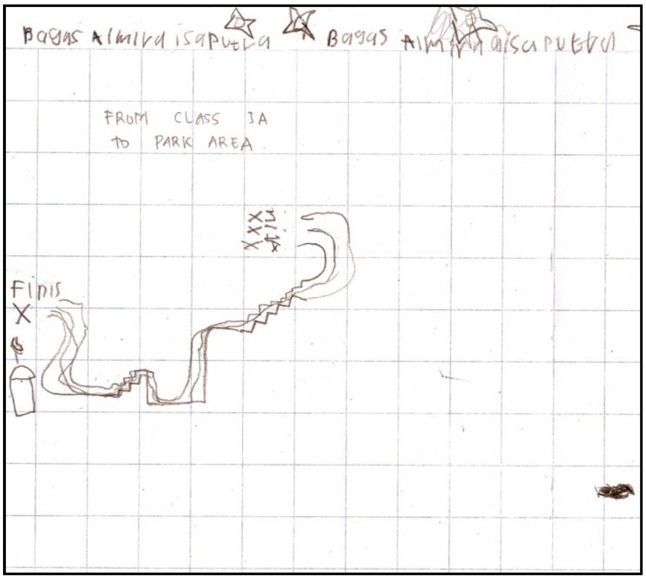

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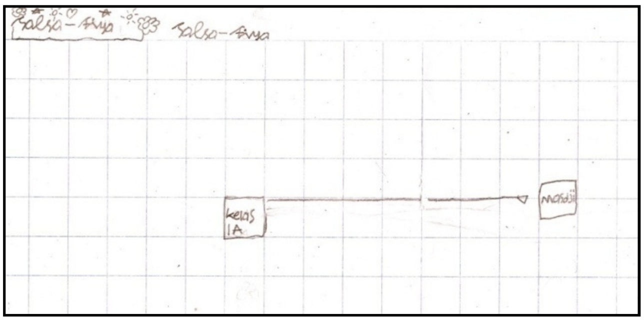

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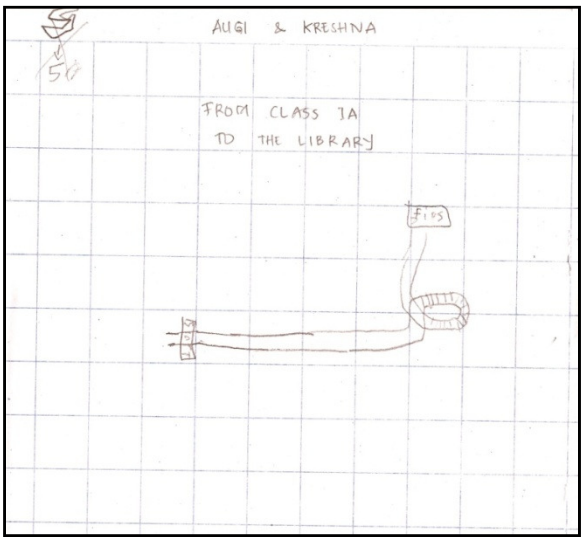

Augi and Kreshna's Drawing

Appendix 3

Pictures of School Buildings in the Mini Lesson of Tryout Class

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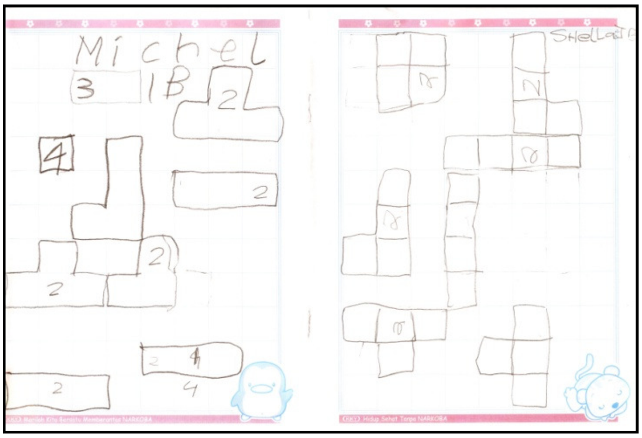
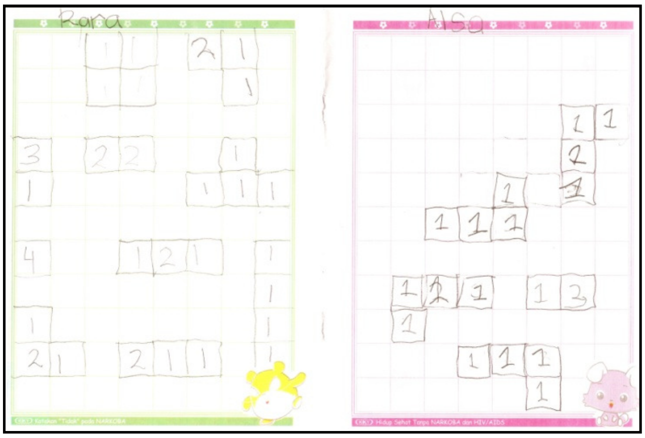
Appendix 4

Figures of Basements by Children in Tryout Class



Michel and Shella's Drawing

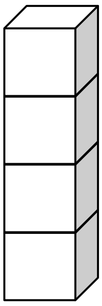
Akzal and Arsyi's Drawing



Rara and Alsa's Drawing

Appendix 5

The Figures of Building in Mini Lesson of Tryout Class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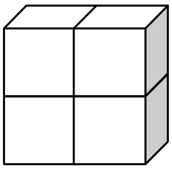
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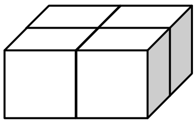
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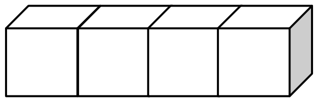
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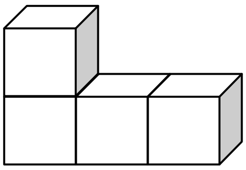
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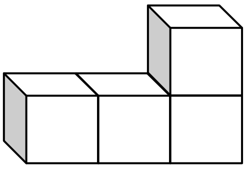
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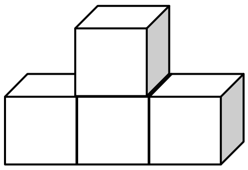
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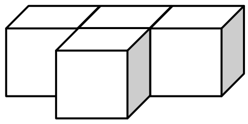
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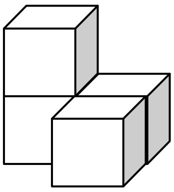
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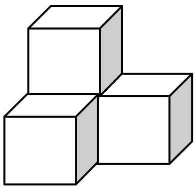
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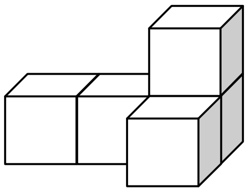
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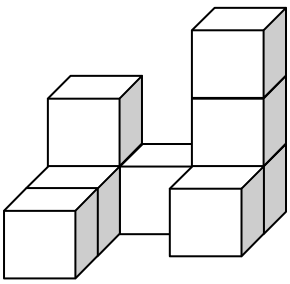
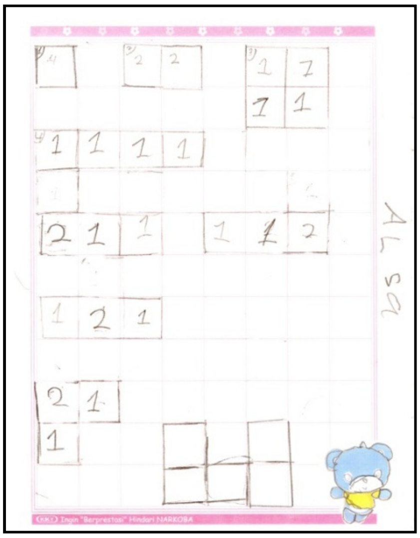

12

Appendix 6

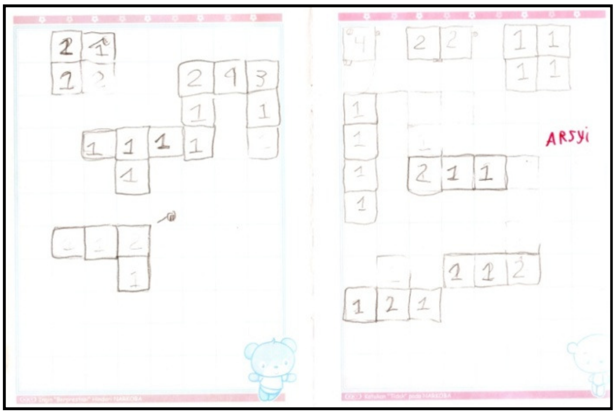
The Results of Children's Drawing in Mini Lesson of Tryout Class

Rara

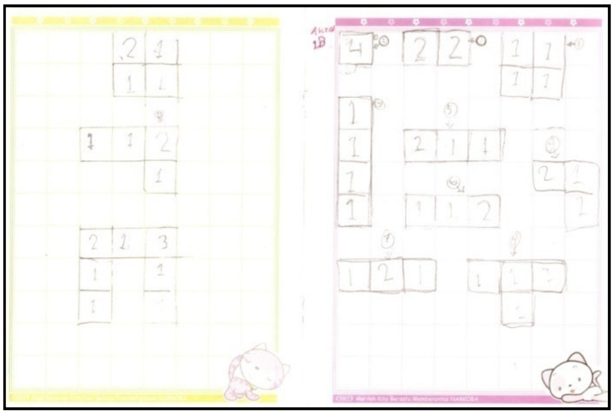
Alsa



Arsyi



Akzal

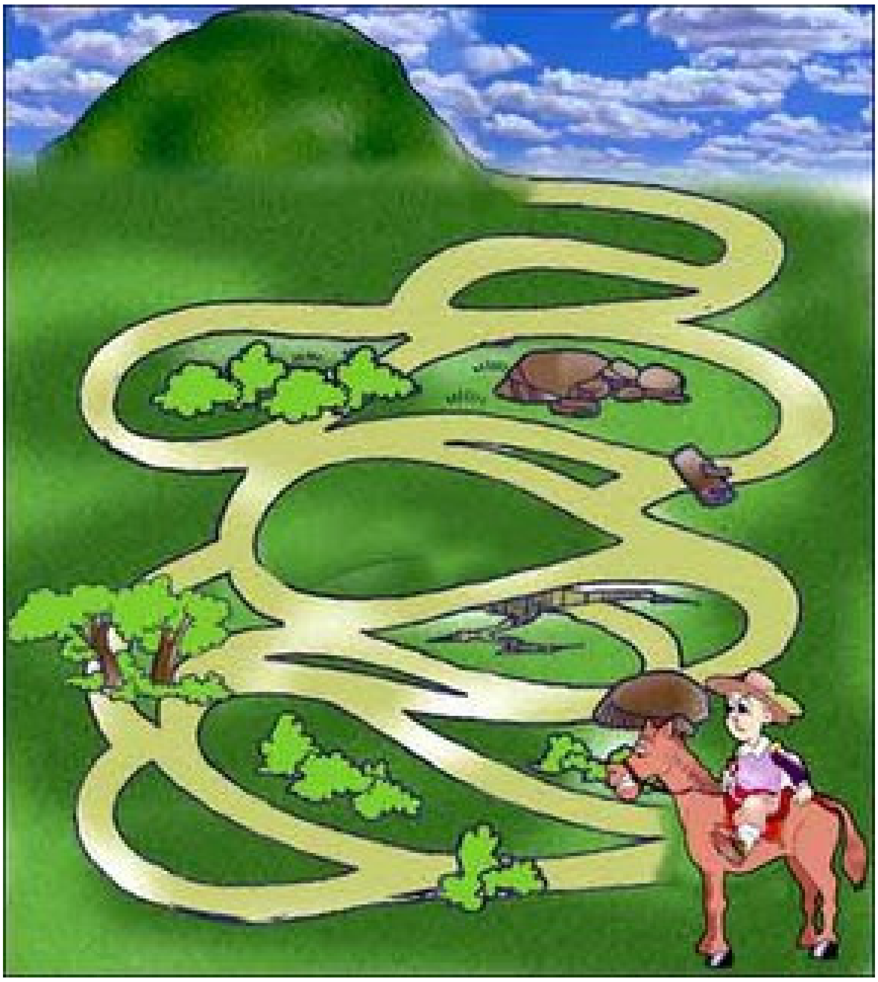


Appendix 7

Pretest Instrument (30 minutes)

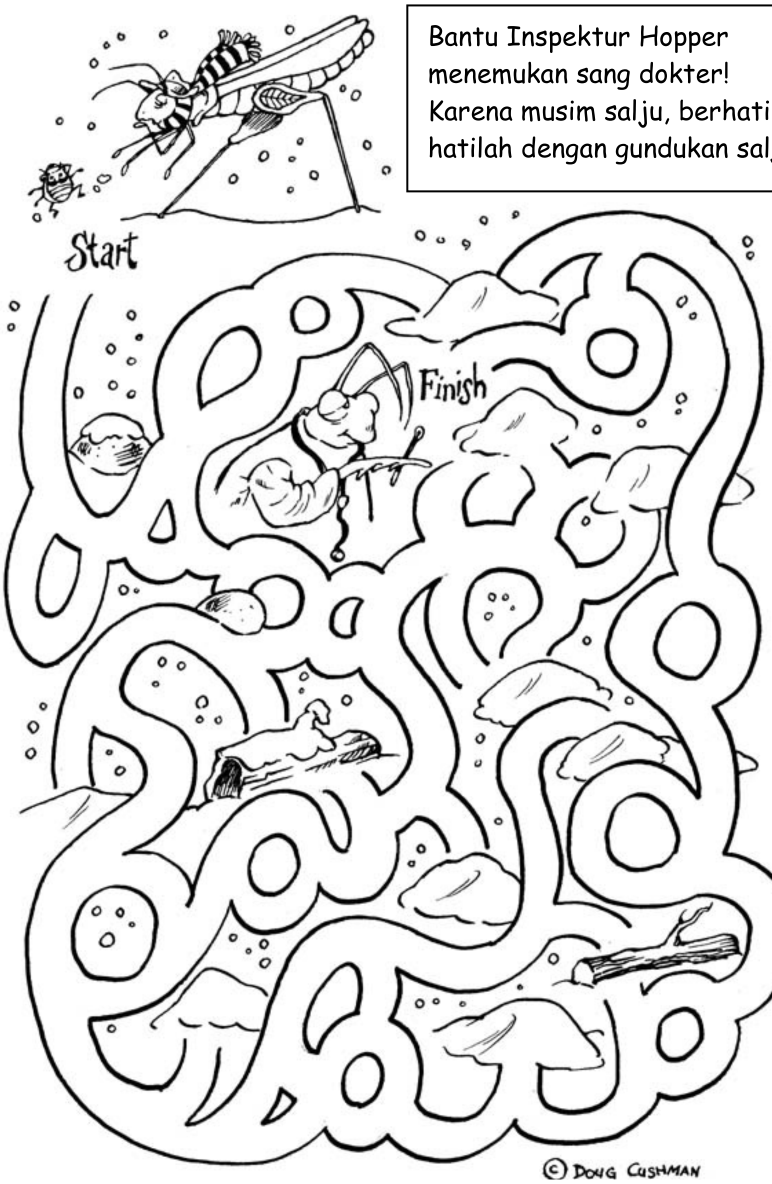
PART 1

1. Tunjukkan jalan terpendek bagi pengendara kuda untuk sampai ke kaki bukit!
   Help the cowboy to reach the hill by showing the shortest way to go there!



2. Jelaskan strategimu untuk membantu Inspektur Hopper. Dari mana kamu memulai langkahmu untuk menemukan jalan keluarnya?
Explain your strategy to help Inspector Hopper. From where do you start your step to find the way out?

Bantu Inspektur Hopper menemukan sang dokter! Karena musim salju, berhati-hatilah dengan gundukan salju



Adapted from: http://www.doug-cushman.com/maze.html

3. Pilih rumah mana yang cocok untuk bermain petak umpet. Kenapa demikian?
   Choose which house is suitable to play hide and seek. Why do you think so?

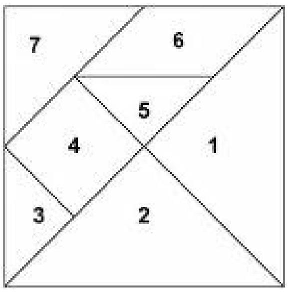
a

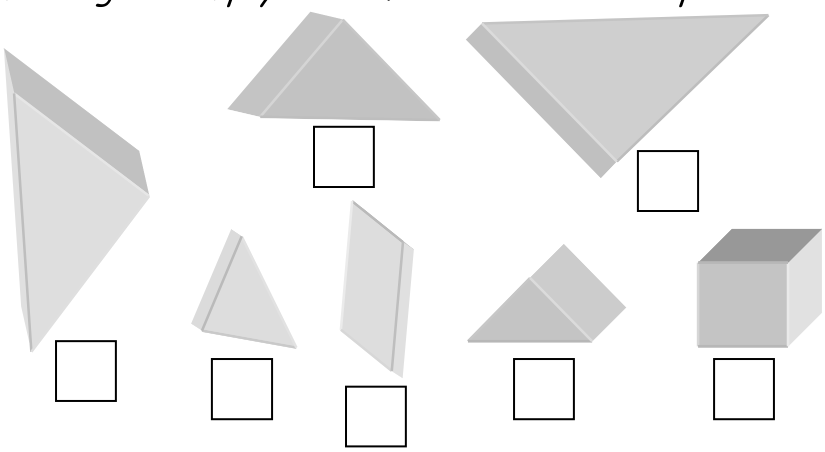
b

4. Di dalam gambar rumah pilihanmu, sebutkan tempat-tempat yang aman untuk bersembunyi! Kenapa kamu beranggapan tempat-tempat itu tepat untuk dipakai bersembunyi?
   In the chosen house, mention all proper places to hide! Why do you think those places are the most appropriate places to hide?

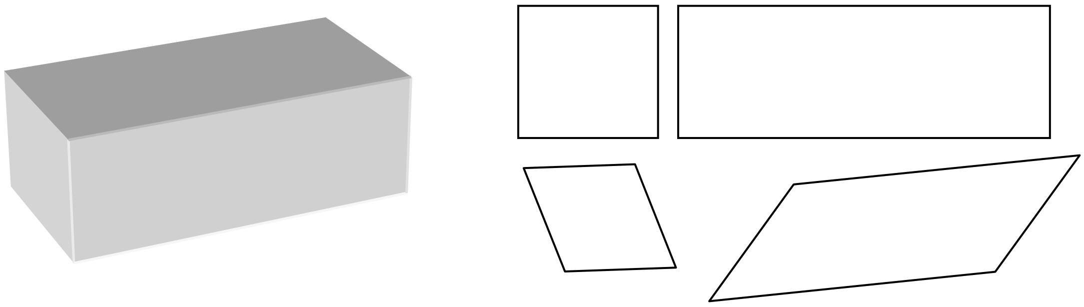
PART 2

5. Beri nomer yang tepat pada kotak-kotak kosong sesuai dengan bentuk bangun-bangun di dalam persegi!
   Match the shapes by numbering the empty boxes for the suitable shape!

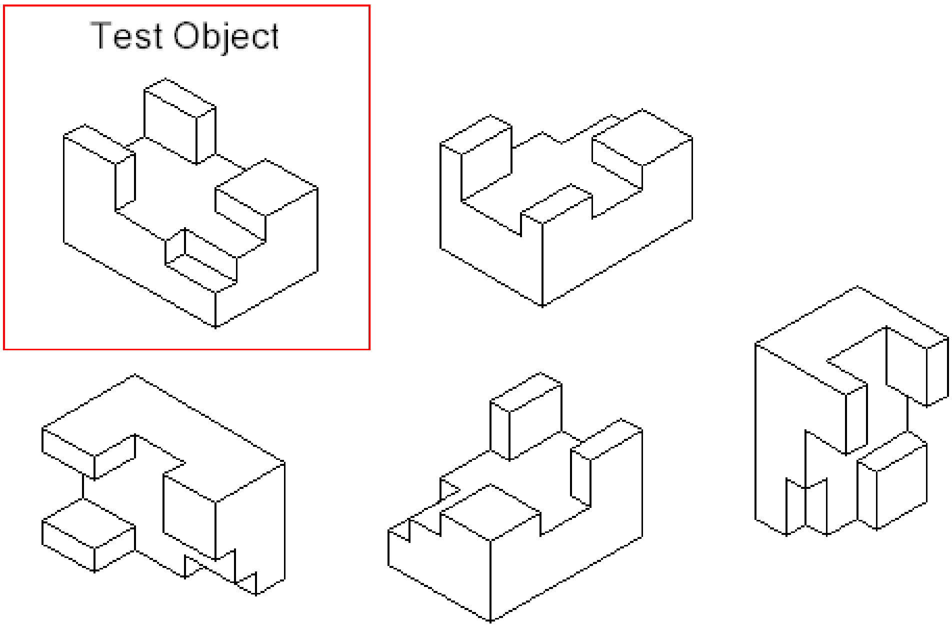




6. Coretlah gambar yang bukan merupakan sisi dari balok di bawah ini! Jelaskan pilihanmu!
   Cross the figures that are not sides of the block below! Explain your choices!



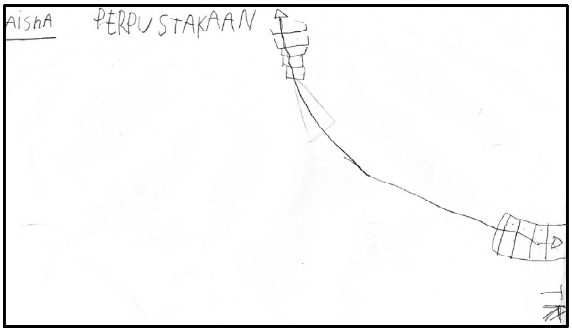
7. Pilih gambar yang tidak sesuai dengan gambar dalam kotak!
   Choose the less suitable figure with the figure inside the box!



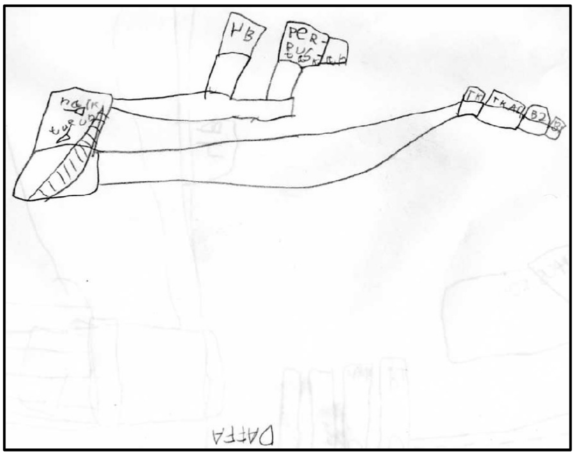
Adapted from: http://www.ul.ie/~mearsa/9519211/newpage2.htm

Appendix 8

Children's Drawings of Routes in the Class Experiment

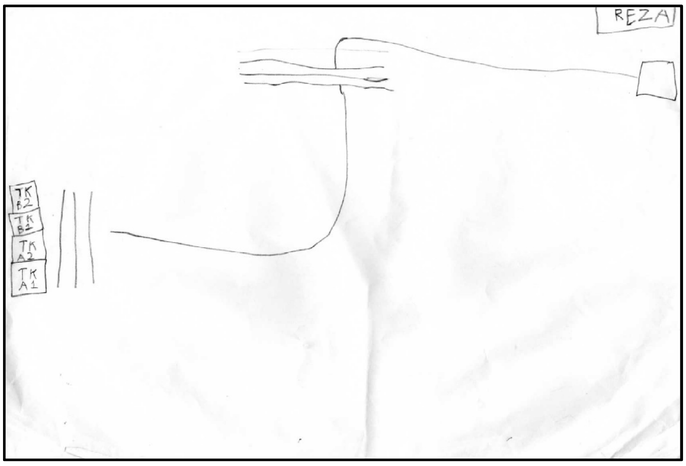


Aisha's Drawing



Daffa's Drawing

Anggita and Vira's Drawing



Reza's Drawing

Appendix 9

Pictures of School Buildings in the Mini Lesson of Class Experiment

1

2

3

4

5

6

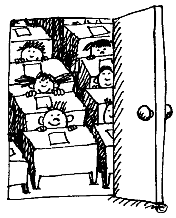
7

Appendix 10

Post Test (Old Version)

Part 1

1. Draw the way to get in your classroom from the gate of the school!



2. Which one is the nearest room from our classroom, the canteen or the library? Why?
3. If you play hide and seek on the playground. Which one is the best place to hide? Why?

The answers of those questions are depended on the school situations, in where I will do my research. Children's answers will be analyzed and more intentioned in the "why" answers. The clearer they describe the real situations, the higher the spatial ability they achieve. The achievement also determines the development of children's spatial ability. That is because the difficulty levels of these problems higher than the problems in the pretest, but they measure the same things and they are made in such away so that children more use their imagination of orienting in space.

4. Where do you have to stand so that you can see these parts of the school building?

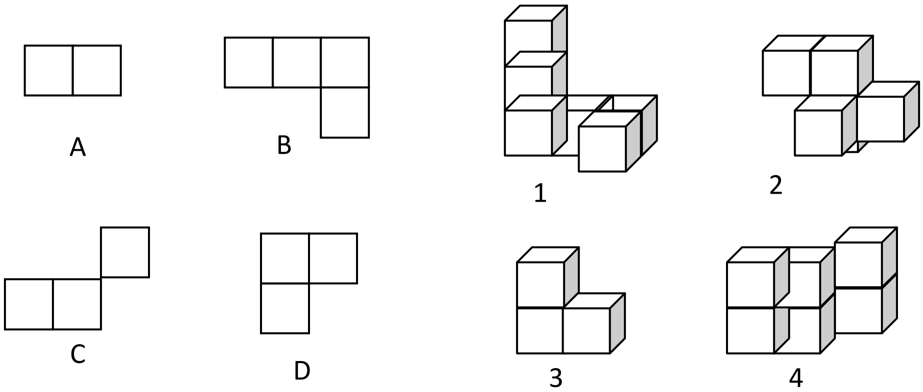
a)

b)

c)

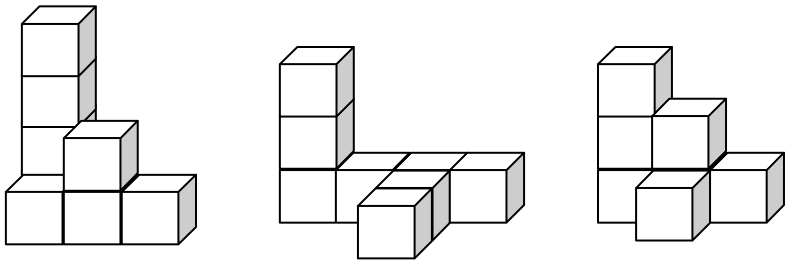
Part 2

1. Match the basement figures below with the suitable buildings.

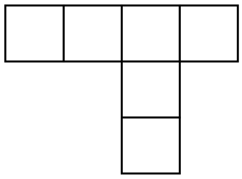


This problem is a close problem that has one correct answer: A – 3, B – 1, C – 4, D – 2. My hypothesis is that children may make mistake by matching D with 3 and C with 2. That could be happened because in this problem they have to mentally move and rotate the building figures so that they can imagine how the basement should be. If there is no mistake, then the spatial ability is successfully developed.

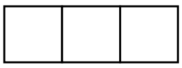
2. Draw the possible basement of these following buildings.



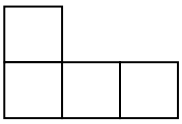
This problem encourages children to relate the given 3D figures with the 2D shapes. For doing so, again, they have to mentally manipulate the objects so that they can reach the good answers. While the second building figure only has one correct answer:



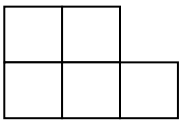
The first and the third figure give more opportunity for variant answers, since there are some cubes that cover the behind cubes. The possible answers for each of those two figures are:

1. a) 

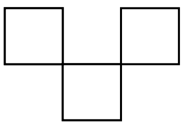
Children who come up with this shape cannot imagine all possible ground floors that are covered behind.

b) 

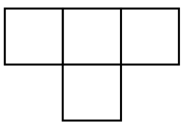
Children who come up with this shape can imagine "existence" ground floor that, with an ideal reason, should exist below the highest floors.

c) 

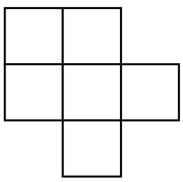
Children who come up with this shape may imagine all possible ground floors that are covered behind.

3. a) 

Children who come up with this shape cannot imagine all possible ground floors that are covered behind.

b) 

Children who come up with this shape can imagine "existence" ground floor that, with an ideal reason, should exist below the highest floors.

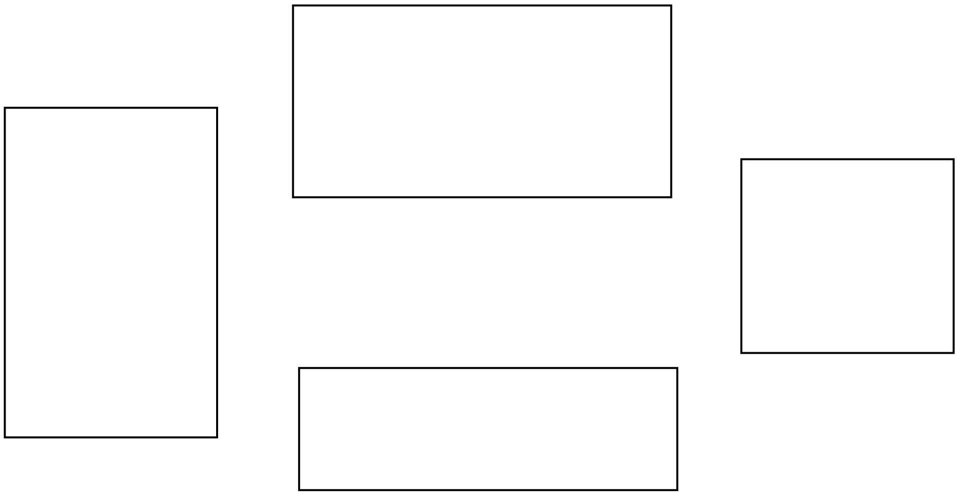
c) 

Children who come up with this shape may imagine all possible ground floors that are covered behind.

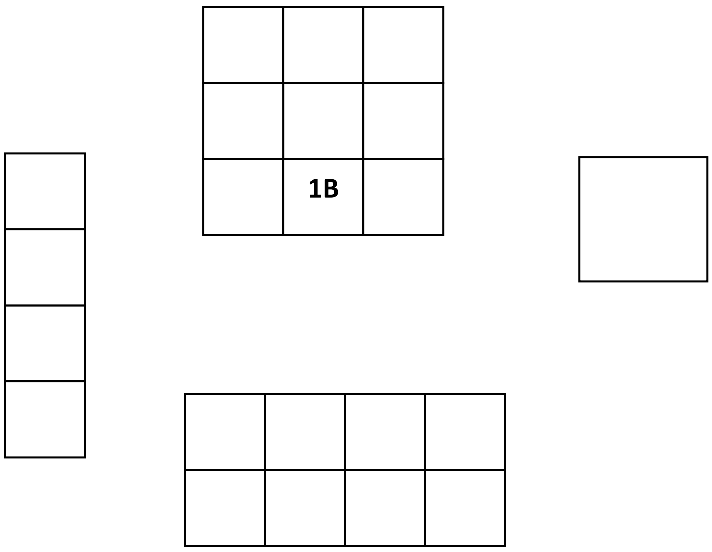
Appendix 11

Post Test (1 jam/1 *hour*)
(New Version)

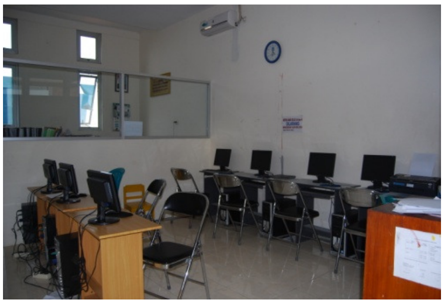
1. Tuliskan nama-nama 4 gedung utama (Gedung TK, Gedung Kelas, Gedung Kepala Sekolah, dan Masjid) di At Taqwa pada kotak-kotak yang tersedia!
   Give names of 4 main buildings (Kindergarten Building, Class Building, Principal's Building, and the Masjid) in At Taqwa, on the proper boxes!



2. Warnailah kotak di bawah ini yang menunjukkan letak perpustakaan!
   Color the box that represents a position of the library!



3. Gambar jalan yang kamu lalui dari gedung TK menuju Ruang Komputer!
   Draw a way that you go through from the Kindergarten Building to the Computer Room!



4. Di mana kamu harus berdiri untuk melihat bangunan sekolah ini? Bagaimana kamu tahu dan yakin?
   Where do you have to stand to view this building? How do you know and sure?

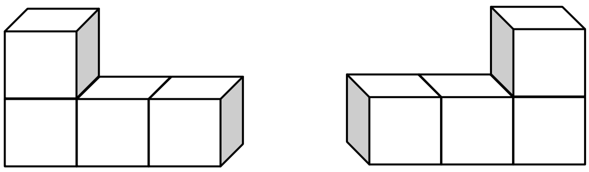
c)

b)

<u>Nama bangunan</u> :
The name of building

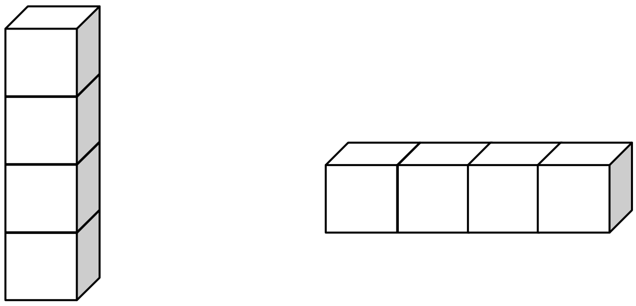
Saya melihat dari :
I view it from
Karena :
Because

5. Perhatikan dua pasang gambar kastil di bawah ini!
Look at these two pairs of castles below!

a) 

Sama atau beda? ……………………………………………………………………..
Same or not?

Kenapa? Karena
Why? Because

b) 

Sama atau beda? ……………………………………………………………………..
Same or not?

Kenapa? Karena
Why? Because